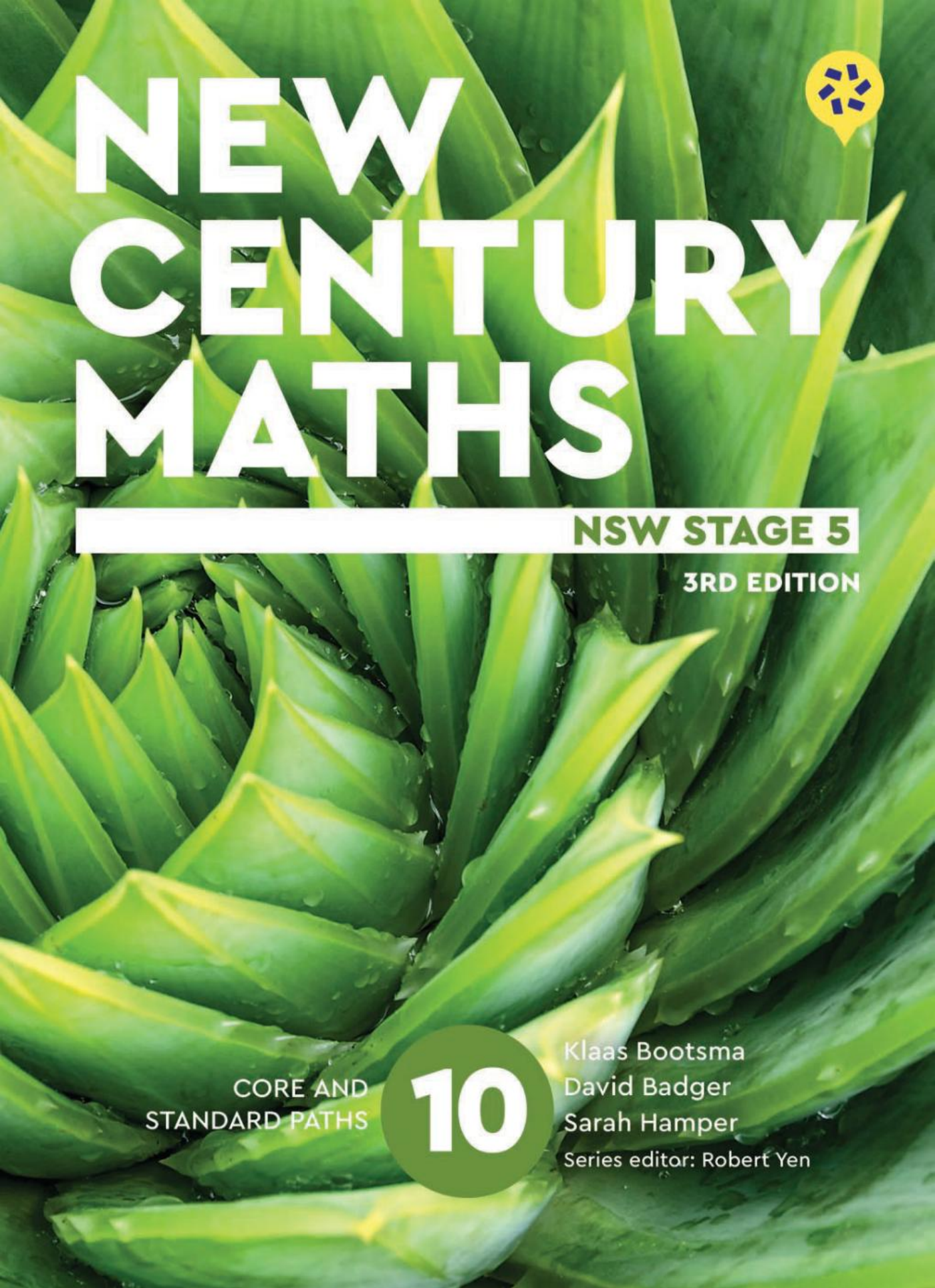
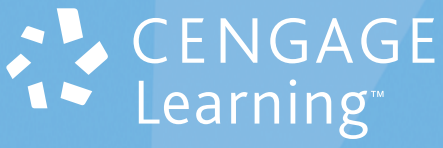

NEW CENTURY MATHS
NSW STAGE 5
3RD EDITION
CORE AND STANDARD PATHS
10
Klaas Bootsma
David Badger
Sarah Hamper
Series editor: Robert Yen



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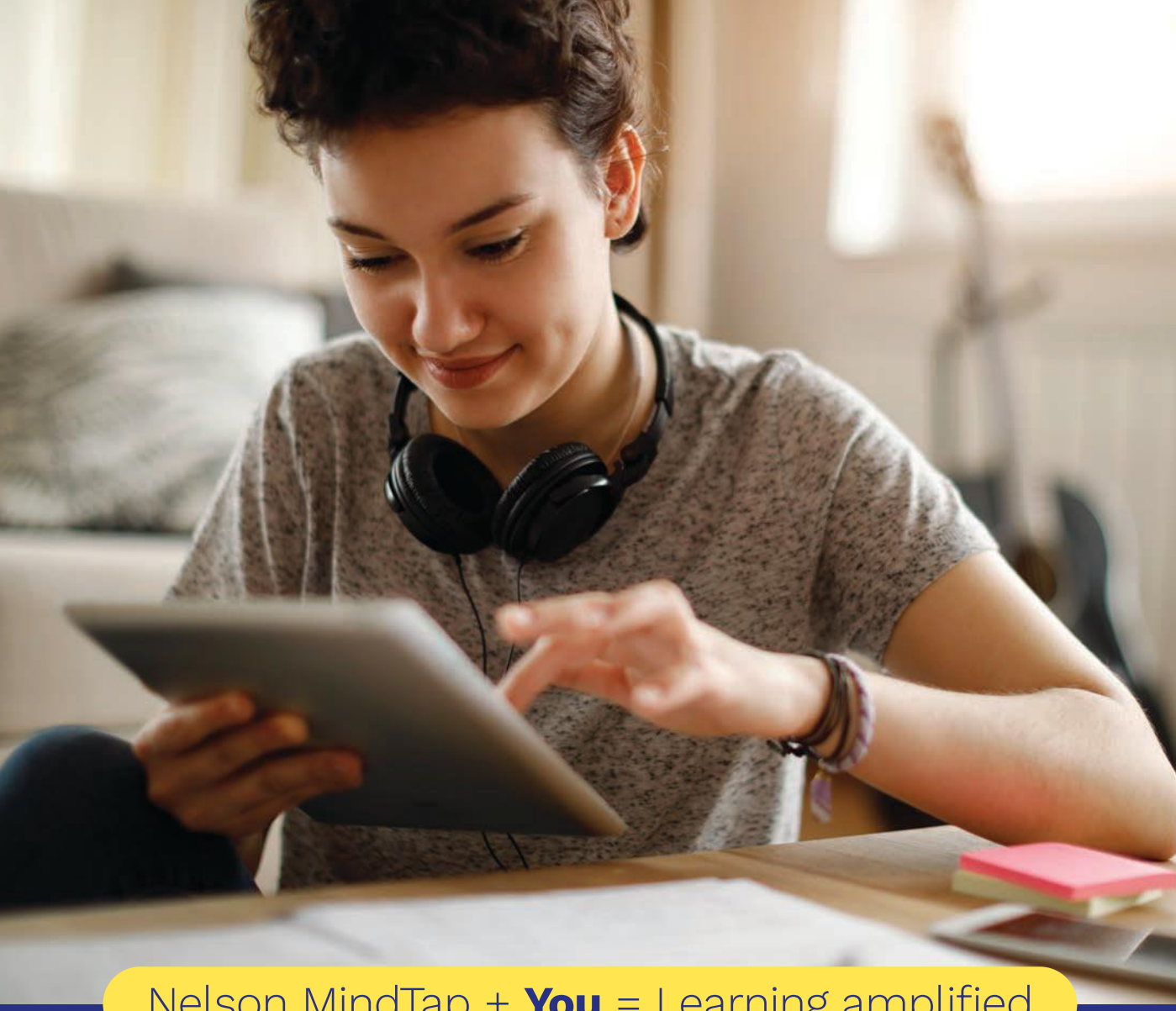
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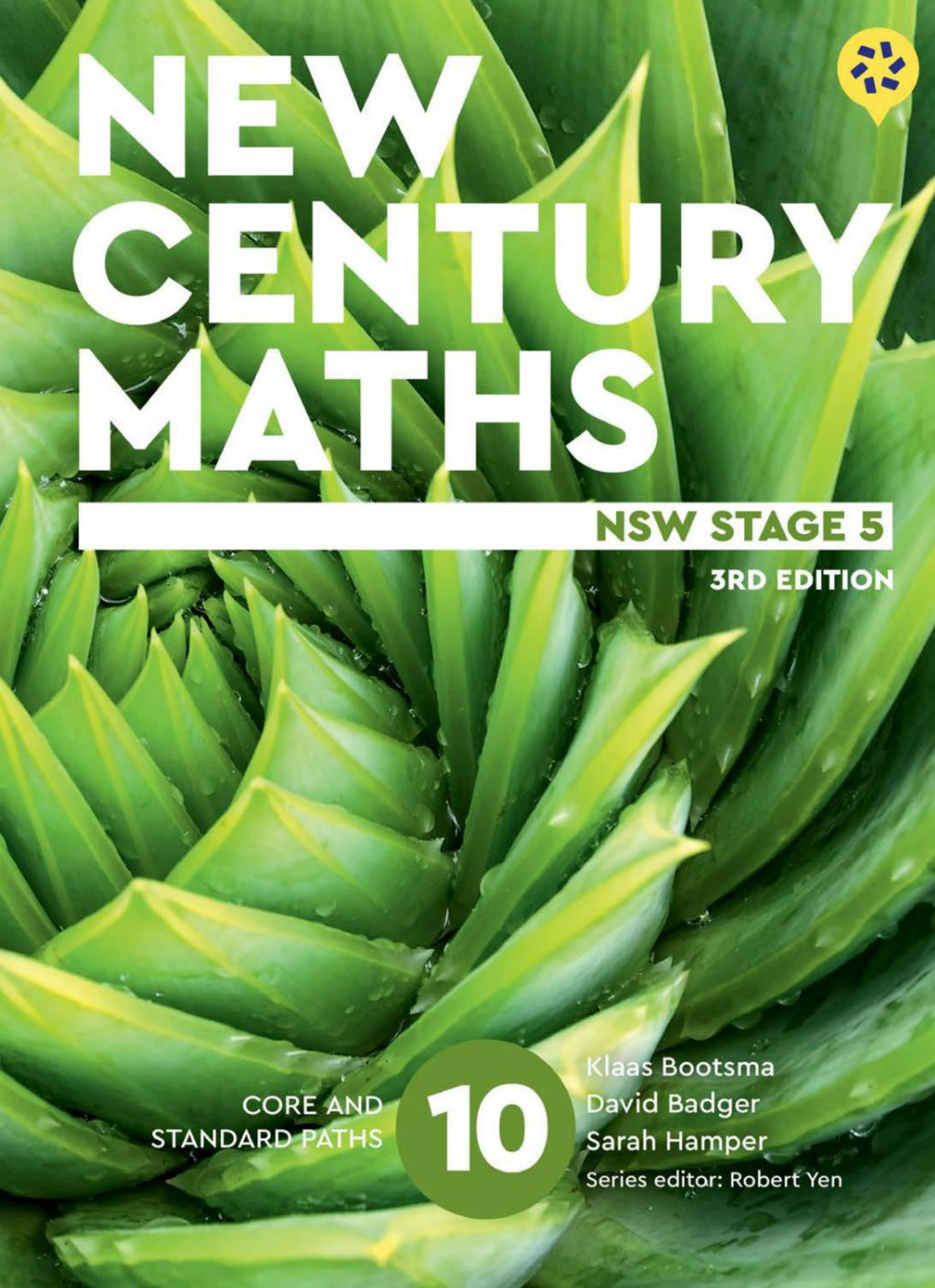


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— **Sarah,** Secondary School Teacher

Nelson MindTap



NEW CENTURY MATHS

NSW STAGE 5

3RD EDITION

CORE AND STANDARD PATHS

10

Klaas Bootsma
David Badger
Sarah Hamper
Series editor: Robert Yen

Nelson

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PREFACE

Teachers, it's time to declutter and start teaching maths again. You asked and we listened.

New Century Maths 7–10 is an original series written by NSW teachers for NSW teachers, focusing on core skills to discourage 'teaching by syllabus dot points', featuring explicit grading of exercise questions, more 'flipped classroom' video tutorials, more applications and problem-solving questions, and worked solutions to every question.

In schools for 30 years, *New Century Maths* is carefully mapped to the NSW syllabus and built on solid pedagogical foundations that integrate into every chapter practical classroom activities, engaging investigations, problem-solving, reasoning, communicating, reflecting, summarising, extension, revision, mental calculation, technology, numeracy and literacy.

We publish 2 levels of mathematics books for Years 9 and 10 (Stage 5), covering similar topics in the same order but at different depths:

- *New Century Maths* covers the Core and most Paths for students heading towards HSC Mathematics Standard courses with the flexibility to change to Advanced during Year 10
- *New Century Maths Advanced* covers the Core and all Paths for students heading towards HSC Mathematics Advanced and Extension courses

This book, *New Century Maths 10*, focuses on the Maths Standard pathway but includes optional Path content for students aspiring towards Maths Advanced (see page vii).

The *Nelson MindTap* online learning platform contains an eBook, quizzes, interactives, 200+ worksheets, 100+ topic tests, worked solutions and much more. We have provided an abundance of resources for teachers to plan and teach for a variety of pathways. *New Century Maths* is clear, concise, fresh and smart. We have designed this series to be user-friendly and uncomplicated so that teachers and students everywhere can pick it up and use straight away. When it comes to teaching in the spirit, breadth and depth of the new syllabus, we've got your back.

About the authors

Klaas Bootsma was head teacher of mathematics at Ambarvale High School and has taught at Lurnea and Grantham high schools. He was a senior HSC marker and has worked on the HSC Advice Line. Klaas also co-wrote *New Century Maths 11–12 Mathematics Standard 2*.

David Badger was principal of Toongabbie Christian School, deputy principal at Mt Annan Christian College and head teacher of mathematics at Eagle Vale High School. His passion is to make mathematics interesting, practical and accessible to all students.

Sarah Hamper is head teacher of mathematics at Cheltenham Girls High School, and has taught at Abbotsleigh, Meriden and Tara Anglican schools. She has an interest in gifted and talented (GAT) students and girls education. Sarah also co-wrote *New Century Maths 11–12 Mathematics Standard 2*.

Series editor **Robert Yen** taught at Hurlstone Agricultural High School. He co-wrote *New Century Maths 11–12 Mathematics Standard 2,* co-edited *Reflections,* the MANSW journal, and now works for Cengage as the mathematics publisher.

Contributing authors

Megan Boltze and **Robert Yen** wrote many of the *Nelson MindTap* worksheets and topic tests.

John Drake, Katie Jackson, Joanne Magner, Scott Smith, Rashmi Bhagwati, Deborah Da Cruz, Monique Ellement, Jade Lori and **Robert Yen** created the video tutorials.

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NUMBER AND ALGEBRA				
Financial mathematics	3	Numeracy and calculation	1	Interest and depreciation
	8	Earning money		
Algebraic techniques	1	Algebra	4	Algebra
			12	Products and factors
Indices	5	Indices	4	Algebra
			13	Surds
Equations	7	Equations	6	Equations
			11	Simultaneous equations
			14	Quadratic equations and the parabola
Linear relationships	11	Coordinate geometry and graphs	2	Graphing lines
Non-linear relationships	11	Coordinate geometry and graphs	7	Graphing curves
			14	Quadratic equations and the parabola
Numbers of any magnitude	5	Indices	3	Surface area and volume
	10	Surface area and volume		
Variation and rates of change	11	Coordinate geometry and graphs	7	Graphing curves
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Trigonometry	2	Pythagoras' theorem	8	Further trigonometry
	4	Trigonometry		
Area and surface area	10	Surface area and volume	3	Surface area and volume
Volume	10	Surface area and volume	3	Surface area and volume
Properties of geometrical figures	13	Similar figures	9	Geometry and networks
Introduction to networks	6	Geometry and networks	9	Geometry and networks
STATISTICS AND PROBABILITY				
Data analysis	9	Analysing data	5	Comparing data
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ABOUT THIS BOOK

Coverage of the syllabus

- *New Century Maths 10* covers the NSW syllabus based on the Australian curriculum, as shown by the table of contents and syllabus grids on the previous pages.
- This book contains Stage 5 Core and Standard Paths content. Path content is marked in red and by *.
- Each chapter begins with a **chapter outline** that includes the Working Mathematically processes covered in each section.

Chapter outline

		Working mathematically				
6.01	Equations	U	F			
6.02	Equations with algebraic fractions*	U	F			
6.03	Quadratic equations $x^2 + bx + c = 0$*	U	F		R	C
6.04	Equation problems	U	F	PS	R	C
6.05	Equations and formulas	U	F		R	C
6.06	Changing the subject of a formula*	U	F		R	C

*PATH

U = UNDERSTANDING

Understanding is 'knowing and relating' maths. It is more than just learning facts. It is deep understanding, seeing how mathematical content is interconnected, knowing 'why' as well as 'how'.

F = FLUENCY

Fluency is 'applying' maths. It is being able to use mathematics competently and effectively. When you are fluent in a language, you have mastered it so that you can improvise and confidently use the correct word or phrase. Fluency in maths is choosing an appropriate skill, method or formula to use at the right place and time.

PS = PROBLEM SOLVING

Problem solving is 'modelling and investigating' with maths. It involves interpreting a rich, elaborate problem, selecting an appropriate strategy or model, solving the problem, then evaluating, communicating and justifying the solution.

R = REASONING

Reasoning is 'generalising and proving' with maths, using higher-order thinking to connect specific facts to general principles, using algebra, logic, proof and justification.

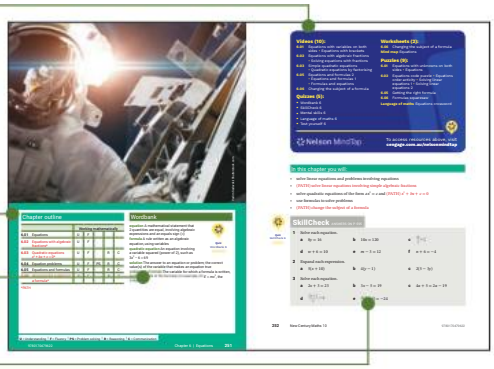
C = COMMUNICATING

Communicating is 'describing and explaining' maths, representing mathematical theory and solutions in words, algebraic symbols, special notations, diagrams, graphs and tables.

- **Understanding** and **Fluency** can be found in every exercise and activity, while **Problem solving**, **Reasoning** and **Communicating** are found in the **Investigations**, **Technology**, **Mental skills**, **Language of Maths** and **Topic overview** activities, and explicitly labelled in every exercise. (see 'In each chapter' next page)

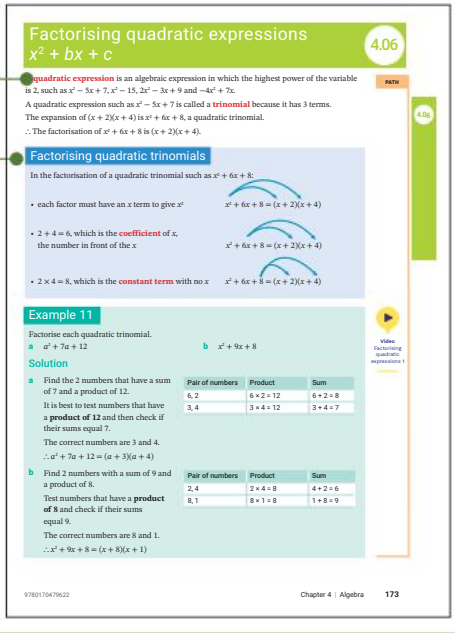
At the beginning of each chapter

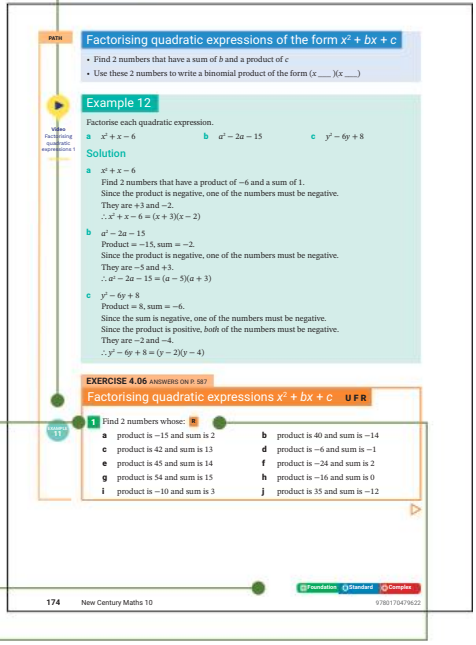
- A listing of *Nelson MindTap* **chapter resources**.
- A **Chapter outline** and **Wordbank** chapter glossary (a full glossary at the back of the book).
- **SkillCheck** reviews prerequisite skills and knowledge for the chapter.



In each chapter

- Important facts and formulas are highlighted in a shaded box.
- Glossary terms are printed in **red**.
- Graded exercises are linked to worked examples and include multiple-choice questions, exam-style problems and realistic applications.

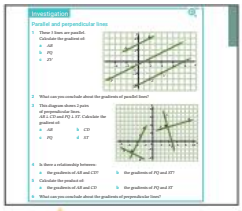




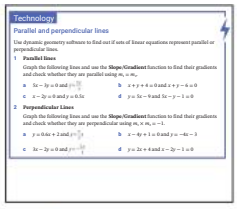
- The exercise questions are colour-coded for level of difficulty:

Foundation | Standard | Complex

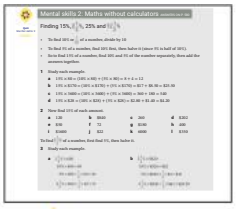
- Specific questions are also labelled **PS** for **Problem solving**, **R** for **Reasoning** and **C** for **Communicating**.



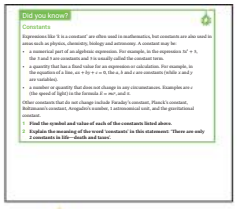
Investigations explore the syllabus in more detail, through group work, discovery and modelling activities.



Technology includes spreadsheets, online calculators, dynamic geometry software and the Internet.



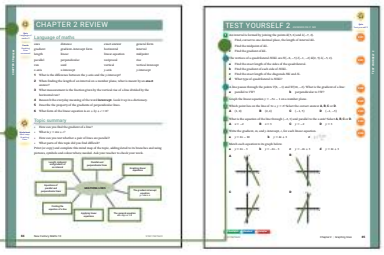
Mental skills reinforce mental calculation strategies ('calculator-free maths').



Did you know? contains interesting facts and applications of the mathematics learned in the chapter.

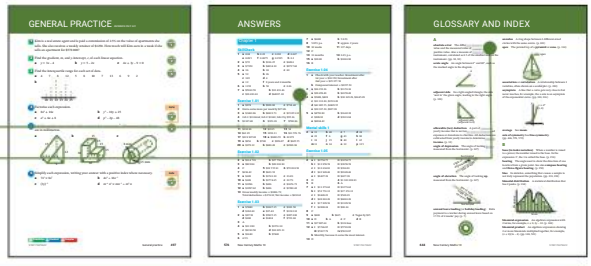
At the end of each chapter

- **Power plus** is an extension/challenge exercise.
- **Language of maths** has a chapter word list and literacy questions.
- **Topic summary** has a mind map activity with downloadable solutions.
- **Test Yourself** contains chapter revision linked to the relevant exercise set.
- **Practice sets** after every 3-4 chapters.



At the end of the book

- **General practice** exercise.
- **Answers** (worked solutions are on the teacher website).
- **Glossary and index.**



Nelson MindTap

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Equation problems

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New Century Maths / Maths in Focus 7–12 series



MATHEMATICAL VERBS

A glossary of 'doing words' commonly found in mathematics problems

analyse: study in detail the parts of a situation
bisect: cut in half
calculate: *see* **evaluate**
classify, identify: state the type, category or feature of an item or situation
comment: express an observation or opinion about a result
complete: fill in detail to make a statement, diagram or table
compare: show how two or more things are similar or different
construct: draw an accurate diagram
convert: change from one form to another, for example, from a fraction to a decimal, or from kilograms to grams
decrease: make smaller
describe: state the features of a situation
estimate: make an educated guess for a number, measurement or solution, to find roughly or approximately
evaluate, calculate: find the value of a numerical expression, for example, 3×8^2 or $4x + 1$ when $x = 5$
expand: remove brackets in an algebraic expression by multiplying, for example, expanding $3(2y + 1)$ gives $6y + 3$
explain: describe why or how
factorise: take out the highest common factor (HCF) of an expression and insert brackets, for example, factorising $5x - 20$ gives $5(x - 4)$. The opposite of **expand.**
give reasons: show the rules or thinking used when solving a problem. See also **justify.**
graph: display on a number line, number plane or statistical graph
hence find/prove: find an answer or prove a result using previous answers or information supplied
identify: *see* **classify.**
increase: make larger
interpret: find meaning in an answer or result
justify: give reasons or evidence to support your argument or conclusion. See also **give reasons.**
measure: determine the size of something, for example, use a ruler to determine the length of a pen
prove that: *see* **show that**
recall: remember and state
reduce (a fraction) to its lowest terms: see **simplify (a fraction)**.
round (a number): find the nearest approximation of a number. For example, 4.3 rounded to the nearest whole number is 4, $12.9598 rounded to the nearest cent is $12.96, 0.166 66 rounded to 3 decimal places is 0.167.
show that, prove: (in questions where the answer is given) use calculation, procedure or reasoning to prove that an answer or result is true
show working: show the steps you used to find the answer
simplify: give a result in its most basic, shortest, neatest form, for example, simplifying a ratio or algebraic expression
simplify (a fraction): reduce the numerator and denominator of a fraction by dividing by their highest common factor (HCF), for example, $\frac{16}{20}$ simplified is $\frac{4}{5}$
simplify (a ratio or rate): reduce the terms or units of a ratio or rate by dividing by their highest common factor (HCF), for example, 10 : 4 simplified is 5 : 2.
sketch: draw a rough diagram that shows the general shape or ideas, less accurate than **construct**
solve: find the value(s) of an unknown variable in an equation or inequality
state: *see* **write.**
substitute: replace a variable by a number and evaluate
verify: check that a solution or result is correct, usually by substituting back into the equation or referring back to the problem
write correct to: See **round (a number).**
write, state: give the answer, formula or result without showing any working or explanation. (This usually means that the answer can be found mentally, or in one step.)

SYMBOLS AND ABBREVIATIONS

$=$	is equal to
$\neq$	is not equal to
$\approx$	is approximately equal to
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
( )	parentheses, round brackets
[ ]	(square) brackets
{ }	braces
$\pm$	plus or minus
-3	negative 3
π	pi = 3.14159...
$0.\dot{1}5\dot{2}$	the recurring decimal 0.152152...
$^\circ$	degree
$42°17'54''$	42 degrees, 17 minutes, 54 seconds
$\angle A$	angle A
$\triangle ABC$	triangle ABC
S 37° W	a compass bearing
217°T	a true bearing
$\parallel$	is parallel to
$\perp$	is perpendicular to
$\sim$	is similar to
$\therefore$	therefore
x^2	x squared, $x \times x$
x^3	x cubed, $x \times x \times x$
$\sqrt{\ }$	square root, radical sign
$\sqrt[3]{\ }$	cube root

$\propto$	is proportional to, varies as
$\{a, b, c\}$	a set with elements a, b, c
{ }, $\varnothing$	the empty set
$\overline{A}$, A^c, A'	the complementary set to set A
$\cup$	union
$\cap$	intersection
$P(E)$	the probability of event E occurring
$P(\overline{E})$	the probability of event E not occurring
LHS	left-hand side
RHS	right-hand side
%	percentage
p.a.	per annum (per year)
sin	sine ratio
cos	cosine ratio
tan	tangent ratio
$\overline{x}$	the mean
σ	the standard deviation
Σ	the sum of
Q_1	first quartile or lower quartile
Q_2	median (second quartile)
Q_3	third quartile or upper quartile
IQR	interquartile range
α	alpha
θ	theta
μ	micro-, mu
m	gradient



1

NUMBER AND ALGEBRA

INTEREST AND DEPRECIATION

The value of an investment increases over time as a result of interest being added to it, whether it be simple or compound interest. On the other hand, the value of assets and items such as cars and office equipment decreases over time due to age and wear-and-tear. Compound interest and depreciation use formulas that involve repeated percentage increase and decrease respectively.



TZIDO SUN/Shutterstock.com

Chapter outline

		Working mathematically				
1.01	Earning an income	U	F	PS	R	
1.02	Income tax	U	F	PS		C
1.03	Simple interest	U	F	PS	R	
1.04	Compound interest	U	F			
1.05	The compound interest formula	U	F	PS	R	
1.06	Term payments	U	F	PS	R	C
1.07	Depreciation	U	F	PS	R	C

Wordbank

allowable deduction A part of a person's yearly income that is not taxed, such as work-related expenses and donations to charities

compound interest Interest calculated on the principal invested as well as on any accumulated interest

depreciation The decrease in the value of items over time due to ageing

future value The final amount of an investment after all interest has been added

net pay Pay received after deductions from gross pay; 'take-home' pay

per annum (p.a.) Per year

principal or **present value** The original amount of money invested or borrowed, for the purpose of calculating interest

repayment or **instalment** The amount of money paid at regular time periods (weekly, fortnightly, monthly) to pay off a loan

simple interest Interest calculated on the original principal invested only

Quiz
Wordbank 1

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Videos (10):

1.01 Annual salaries • Overtime • Commission • Annual leave loading
1.02 Income tax
1.03 Simple interest • Simple interest rate
1.05 The compound interest formula • Compound interest
1.07 Depreciation

Twig video (1):

1.05 Could you owe more than America?

Quizzes (5):

- Wordbank 1
- SkillCheck 1
- Mental skills 1
- Language of maths 1
- Test yourself 1

Skillsheets (3):

SkillCheck Mental percentages • Percentage calculations
1.05 Spreadsheets

Worksheets (9):

SkillCheck Percentage shortcuts
1.01 Earning money
1.02 Income tax tables • Income tax and Medicare levy
1.03 Simple interest • Simple interest table
1.05 Compound interest
1.07 Depreciation
Mind map Interest and depreciation

Puzzles (4):

1.03 Simple interest
1.05 Compound interest with annual rests • Compound interest with non-annual rests
Language of maths Interest and depreciation crossword

Technology (3):

1.05 Comparing interest rates • Simple and compound interest calculator
1.06 Term payments

Spreadsheets (2):

1.05 Comparing interest rates • Simple and compound interest

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

In this chapter you will:

- calculate weekly, fortnightly, monthly and yearly incomes
- calculate wages, salaries, overtime, commission, piecework, royalties and annual leave loading
- use tables to calculate income tax and PAYG tax
- solve problems involving simple interest
- solve problems involving compound interest by repeated percentage increase
- solve problems involving the compound interest formula $FV = PV(1 + r)^n$
- solve problems involving term payments and short-term loans
- solve problems involving depreciation

SkillCheck ANSWERS ON P. 576

1 Convert each percentage to a decimal.

a 4% **b** 22% **c** 18.3% **d** 4.7%

e $9\frac{1}{2}\%$ **f** 6.75% **g** $15\frac{1}{4}\%$ **h** 20%

2 Find:

a 6% of \$1200 **b** 2.5% of \$4650 **c** 12% of \$37 450

3 Increase:

a \$7000 by 5% **b** \$3955 by 2% **c** \$8600 by 1.6%

4 How many months are there in:

a 3 years? **b** 2 years? **c** 5 years?

5 Copy and complete:

a One year = _____ weeks

b One year = _____ fortnights

c One year = _____ days

d 48 months = _____ years

e 84 days = _____ weeks

f 100 months = _____ years _____ months

6 If $P = mvt$, find:

a P when $m = 1600$, $v = 0.072$, $t = 10$

b m when $P = 120$, $v = 0.3$, $t = 8$

c v when $P = 18$, $m = 60$, $t = 5$

7 Evaluate, correct to the nearest cent:

a $\$5000 \times (1.045)^4$

b $\$28\,000 \times (1.03)^6$

c $\$15\,300 \times (1.065)^3$

d $\$32\,400 \times (1.072)^{10}$

Quiz SkillCheck 1

Skillsheets Mental percentages

Percentage calculations

Worksheet Percentage shortcuts

1

1.01 Earning an income

Wages, salaries and overtime

Videos
Annual salaries
Overtime
Commission

Worksheet
Earning money

A **wage** is calculated from the number of hours worked and is usually paid weekly. Wage earners can make more income by working extra hours (overtime).

A **salary** is a fixed annual amount, paid weekly, fortnightly or monthly. Salary earners do not earn overtime pay but can receive benefits such as a computer, company car, expense account, shares in the company or paid medical expenses.



Lyubov Levitskaya/Shutterstock.com

Units of time for wages and salaries

1 year = 12 months
1 fortnight = 2 weeks
1 year = 52 weeks for wage earners
1 year = 52.18 weeks for salary earners

The 2 most common rates of **overtime** pay are:

- **time-and-a-half** = 1.5 × normal hourly rate
- **double time** = 2 × normal hourly rate

Example 1

Tyrone earns a salary of $70 400 p.a. p.a. = per annum = per year
How much does he earn:

a each week? **b** each fortnight? **c** each month?

Solution

a Weekly income = $70 400 ÷ 52.18
= $1349.1759...
≈ $1349.18 Rounded to the nearest cent

b Fortnightly income = 2 × $1349.18 1 fortnight = 2 weeks
= $2698.36

c Monthly income = $70 400 ÷ 12 1 year = 12 months
= $5866.6666...
= $5866.67 Rounded to the nearest cent

Example 2

Noor earns $22.65 per hour at normal rates. Last week, she worked 38 hours at normal rates, 6 hours at time-and-a-half and 3 hours at double time. Calculate Noor's total earnings for the week.

Solution

Normal pay $= \$22.65 \times 38$

$= \$860.70$

Time-and-a-half pay $= 6 \times \$22.65 \times 1.5$ 6 hours

$= \$203.85$

Double time pay $= 3 \times \$22.65 \times 2$ 3 hours

$= \$135.90$

Total earnings $= \$860.70 + \$203.85 + \$135.90$

$= \$1200.45$

Commission, piecework and annual leave loading

Commission is earned by salespeople and agents, and is a percentage of the value of items sold or income made.

Piecework is earned according to the number of items made or tasks completed.

Royalties are paid to artists, authors and musicians for the sale or use of their work. When an artwork appears in a publication, a book is sold or a song or video is played to an audience, the artist or author earns a royalty, a percentage of the fee or price.



iStock.com/Allard1

Annual leave loading or **holiday loading** is extra pay given during annual leave (holidays) and is 17.5% of 4 weeks' normal pay.

Example 3

Georgia is a real estate agent and is paid a commission of 2.5% on the value of apartments she sells. She also receives a weekly retainer of $950. How much will Georgia earn if she sells an apartment for $590 000?

A **retainer** is a fixed amount paid regardless of how many items are sold.

Solution

Commission $=$ 2.5% of $590 000

$= \$14\,750$

Total earnings $=$ commission $+$ retainer

$= \$14\,750 + \$950$

$= \$15\,700$

$\therefore$ Georgia earns $15 700.

Example 4

Tobias is a songwriter who earns a royalty of 9.1 cents per stream for each song of length 5 minutes or less and an extra 1.75 cents per minute (or part thereof) for each song over 5 minutes. Last week, his songs 5 minutes or less were streamed 24 817 times and one song that is 5 minutes 35 seconds was streamed 4516 times. What were Tobias' royalties for the week, correct to the nearest cent?

'Part thereof' means that part of a minute is counted as a whole minute.

Solution

Royalty rate = 9.1 c

Royalty rate for 5 min 35 s song = 9.1 + 1.75 c

= 10.85 c

Tobias' royalties = $24\,817 \times \$0.091 + 4516 \times \$0.1085$ Express cents as dollars

= 2748.333

≈ \$2748.33



Stokkete/Dreamstime.com

Video
Annual leave loading

Example 5

Kham's annual salary is \$70 590. For his Christmas holidays, he received 4 weeks' normal pay plus 17.5% annual leave loading for the 4 weeks. Calculate Kham's:

a normal weekly pay

b annual leave loading

c total pay for the Christmas holiday.

Solution

a Weekly pay = \$70 590 ÷ 52.18

= \$1352.8171...

≈ \$1352.82

b Annual leave loading = $17.5\% \times \$1352.82 \times 4$ 17.5% of 4 weeks' pay

= \$946.974

≈ \$946.97

c Total holiday pay = $(4 \times \$1352.82) + \$946.97$ 4 weeks' pay + leave loading

= \$6358.25

EXERCISE 1.01 ANSWERS ON P. 576

Earning an income U F PS R

Express all answers correct to the nearest cent where necessary.

1 Find the weekly wage for each person.

a Sebeen earns $21.85 per hour and works for 40 hours.

b Connor works 8 hours a day, Monday to Friday, and is paid $23.47 per hour.

c Yoshe works on Monday and Tuesday from 8:30 a.m. until 4:00 p.m. and Thursday from midday until 9:00 p.m., and earns $30.60 per hour.

2 Greta earns $19.56 an hour and works for 31 hours each week. Chandler earns $21.44 per hour for his 27 hours of work. Who earns more per week and by how much?

3 Maggie earns a salary of $180 640 p.a. How much does she earn:

a each week? **b** each fortnight? **c** each month?

4 Rakitu considers 2 jobs, one locally with an annual salary of $57 640 p.a. and the other one in the city with a fortnightly pay of $2320. Calculate the weekly income for each job, determine which one pays more per week, and by how much. R

5 Anan works 38 hours at normal rates, 7 hours at time-and-a-half and 4 hours at double time. Calculate Anan's total earnings if he earns $19.60 per hour at normal rates.

6 Jacqui works 8.5 hours per day from Tuesday to Friday. She is paid $21.78 per hour. She also works on Saturday for 4.5 hours at a special rate of $24.59 per hour. How much did Jacqui earn for the week?

7 Idra works the following hours in a week at the clothing chain *Shop til U Drop*.

Day	Hours worked
Monday	9 a.m. – 5 p.m.
Tuesday	9 a.m. – 4 p.m.
Thursday	10 a.m. – 7:30 p.m.
Friday	10 a.m. – 5 p.m.
Saturday	10:30 a.m. – 5 p.m.

She is paid at the following rates.

Day	Rate of pay
Monday to Friday	$19.62 per hour
Saturday	$23.15 per hour
Thursday after 4:00 p.m.	

What is Idra's total income for the week? PS R

8 Fatimah is paid a commission of 2.5% on the value of goods she sells. She also receives a weekly retainer of $875. How much will Fatimah earn if she sells goods to the value of $41 600 in one week? Select the correct answer **A**, **B**, **C** or **D**.

A $1915 **B** $1061.88 **C** $2187.50 **D** $1018.13

EXAMPLE 1

EXAMPLE 2

EXAMPLE 3

Foundation Standard Complex

9 Nathan is a real estate agent whose commission is calculated on the value of the properties he sells:

- 3% paid on first $300 000
- 1.5% paid on next $250 000
- 0.75% paid on any value thereafter

How much commission did Nathan earn for selling a house for $625 000?

10 Briana designed an app, *KeyFinder*, that sells for $2.49. If she makes 70% profit on the sale price of each app sold, how much would she make from selling 800 units of this app?

11 Spriha is a university student and part-time rideshare driver. Last week, she drove for 16.5 hours and earned an average income of $33.15 per hour from passengers. If her job expenses are $16 per hour, how much did Spriha actually earn from her work last week?

12 Matt charges $60 for each lawn he mows and $45 for trimming hedges in each yard. In a week, he mows 24 lawns and trims 15 hedges. How much does he earn for the week?



Virrage Images/Shutterstock.com

13 *Clean 2 Swim* charges $86 to clean backyard pools. If this business earned $4644 in the first week of summer, how many pools were cleaned? R

14 Jade makes homemade eco-friendly soaps, shampoos and cleaning products. A customer purchases 3 homemade soaps, 2 bottles of shampoo and 3 of the cleaning sprays. How much does Jade receive for these purchases?



Homemade soaps $5.60



Eco-friendly shampoo $12.70



Natural cleaning spray $7.25

Volosina/Shutterstock.com, AlenKadr/Shutterstock.com, Lotus_studio/Shutterstock.com

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9780170479622

15 Maya is a songwriter who earns a royalty of 9.1 cents per stream for each song of length 5 minutes or less and an extra 1.75 cents per minute (or part thereof) for each song over 5 minutes. For one week, the number of times her songs were streamed are shown in this table.

EXAMPLE 4

Song length	Number of times streamed
5 min or less	15 406
Over 5 min up to 6 min	3216
Over 6 min up to 7 min	805

How much will Maya earn in royalties for the week, correct to the nearest cent?

16 A video content creator earns $9.20 per 1000 views on the online video platform *MeScreen*. If Dawson makes a video that is viewed 1 258 344 times in the first week, how much will he earn that week?

17 Tiago earns 19% in royalties on the sale of his novels. If 37 815 paperback copies of his novel are sold for $22.95 each and 16 432 e-books at $17.65 each, calculate his royalty payment.

18 Hasti developed a gaming app for preschoolers and earns 16% in royalties from the sales of the app, which is priced at $7.45 per download.

a Hasti's app was downloaded 7956 times last month. How much did she earn from those sales?

b How many apps must Hasti sell to earn $25 000? R

19 Calculate the annual leave loading for each person if it is 17.5% of 4 weeks' pay.

EXAMPLE 5

a Richard earns $1220 per week

b Ivy earns $2000 per fortnight

c Samir earns $5944 per month

d Ellie earns $46 630 p.a.

20 For his annual holidays, Jake received 4 weeks' normal pay plus 17.5% annual leave loading for the 4 weeks. If Jake's annual salary is $50 725, find his:

a normal weekly pay

b annual leave loading

c total pay for the 4-week holiday.

Did you know?

Residual royalties from old TV shows

Successful TV shows make money for production companies, and usually the writers and actors of these shows receive a share of the profits. They also receive residual royalties if these shows are given a second life through reruns and streaming on a different platform. In 2023, writers and actors in the US went on strike for five months to improve working conditions and achieve fairer distribution of profits received from streaming services.

Old TV series like *Friends* (1994–2004) and *The Office* (2005–2013) are still popular today, with a new generation of younger viewers often binge-watching them. The main cast of *Friends* still each earn 2% of the $1 billion (US) in revenue generated from this series each year, meaning they each receive royalties of $20 million per year.

Compare two different ways actors and writers on TV shows are paid for their work.

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1.02 Income tax

Worksheets
Income tax tables
Income tax and Medicare levy

Not all of a person's income is taxed. If we use some of our income for work-related expenses or to donate money to charities, these amounts are called **allowable deductions** (or tax deductions) and are not taxed. Examples of allowable deductions are tools of trade, uniforms, car/travel expenses, subscriptions to professional organisations and journals.

Income tax

Income tax is calculated on a person's **taxable income**, which is the gross income (total earnings) less all allowable deductions, rounded down to the nearest dollar.

Taxable income = gross income − allowable deductions

Video
Income tax

The more a person earns, the higher the rate of tax to be paid.

Tax rates for Australian residents	
Taxable income	**Tax on this income**
0 – $18 200	Nil
$18 201 – $45 000	19c for each $1 over $18 200
$45 001 – $120 000	$5092 plus 32.5c for each $1 over $45 000
$120 001 – $180 000	$29 467 plus 37c for each $1 over $120 000
$180 001 and over	$51 667 plus 45c for each $1 over $180 000

Source: © Australian Taxation Office for the Commonwealth of Australia

Example 6

Sophia earned $62 348 last financial year and collected bank interest of $440.81. She had allowable deductions of $427.52 in work expenses and $110 in donations to charities.

a Calculate her taxable income.

b Use the tax table to calculate the income tax that Sophie must pay.

Solution

a Taxable income = $62 348 + $440.81 − $427.52 − $110

= $62 251.29

≈ $62 251 Rounded down to the nearest dollar

b According to the table, a taxable income of $62 251 is in the $45 001 − $120 000 tax bracket.

Income tax = $5092 + 0.325 × ($62 251 − $45 000) '32.5c for each $1' means 32.5% or 0.325

= $10 698.575

≈ $10 698.58

PAYG tax and net pay

Income tax deducted from your pay by your employer every payday is called **PAYG (Pay As You Go) tax**. The total amount of PAYG tax paid over the year is usually more than the actual income tax payable, so at the end of the financial year you will receive the difference as a tax refund.

Gross pay is the total amount a person earns or receives, but most workers have a variety of deductions taken from their pay before they receive it, including PAYG tax, superannuation contributions, union fees and health fund payments. The amount of income left after the deductions is called **net pay**.

Net pay

Net pay = gross pay − tax − other deductions

Example 7

Jayden earns a gross pay of $2290.33 per fortnight. His deductions are for PAYG tax, $44.10 for private health insurance and $55.82 for superannuation.

Fortnightly earnings ($)	PAYG tax withheld ($)
2270–2275	418
2276–2279	420
2280–2285	422
2286–2291	424
2292–2297	426
2298–2303	428

a Use the PAYG tax table to find Jayden's PAYG tax per fortnight.

b Calculate Jayden's net pay.

c Calculate Jayden's total deductions as a percentage of his gross income (correct to one decimal place).

Solution

a In the table, $2290.33 falls in the $2286 – $2291 range.

Fortnightly PAYG tax = $424

b Net pay = $2290.33 – ($424 + $44.10 + $55.82)

= $2290.33 – $523.92

= $1766.41

Net pay = gross pay − total deductions

c Total deductions = $523.92

$$\text{Deductions percentage} = \frac{\$523.92}{\$2290.33} \times 100\%$$

$$= 22.8753...\%$$

$$\approx 22.9\%$$

$$\frac{\text{Total deductions}}{\text{Gross pay}} \times 100\%$$

EXERCISE 1.02 ANSWERS ON P. 576

Income tax U F PS C

EXAMPLE 6

1 Suemin earns $49 628 in a year and has allowable deductions of $1930.46.

a Calculate her taxable income, rounded down to the nearest dollar.

b Use the tax table on page 11 to calculate the income tax that Suemin must pay.

2 Aiden is an environmental engineer who had a gross income of $118 742 this year and work-related expenses totalling $4022.80, which are tax-deductible. Calculate Aiden's:

a taxable income, rounded down to the nearest dollar

b income tax.

3 Ellie is a graphic designer who earns an annual salary of $90 541 and has collected $1029.45 in bank interest. She has allowable deductions of $379 for tools related to her work and $287 in donations to charity. Calculate: PS

a Ellie's taxable income **b** the amount of tax payable.

4 Riley the builder had a gross income of $56 922 this year. He is entitled to these tax deductions: tools $1538, training courses $445 and outdoor protective clothing $506. How much should Riley pay in tax? Select the correct answer **A**, **B**, **C** or **D**. PS

A $9775.58 **B** $8966.65 **C** $8157.73 **D** $6884.27

5 Nicola is a nurse earning $87 996 per year. Her allowable deductions are the cost of non-slip footwear $225, the cost of laundering uniforms $1046, and union fees $297.60. How much should Nicola pay in tax? PS

6 Will owns a photography business and earned $228 000 last year. His allowable deductions were website costs $968, photographic equipment $23 672, and travel to photo shoots $15 930. Calculate the amount that Will should pay in tax. PS



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7 Jackson earns a gross weekly income of $1075.26. His weekly deductions are $309.11 PAYG tax, $44.55 for private health insurance and $25.18 for superannuation. Calculate Jackson's net weekly pay. C

8 Elsa earns a gross income of $798.20 per week. Her deductions are $132.44 tax and $32.24 for private health insurance. Calculate Elsa's net income. C

EXAMPLE 7

Use the PAYG table from Example **7** on page 13 to answer questions **9** to **12**.

9 Every fortnight, Mr Singh earns $2278 and pays $22.80 in union fees and $94.10 in superannuation.

- **a** Find how much PAYG tax he pays per fortnight.
- **b** Calculate Mr Singh's fortnightly net pay.
- **c** What percentage (correct to one decimal place) of his gross pay do the deductions make up?

10 Holly earns a gross pay of $2295 per fortnight. Her deductions are PAYG tax, $64.35 for superannuation and $30 for life insurance. Find Holly's:

- **a** PAYG tax
- **b** net pay
- **c** total deductions as a percentage of her gross income (correct to one decimal place).

11 Stefan earns $1148 per week.

- **a** If he is paid fortnightly, what is his fortnightly gross pay?
- **b** Find the PAYG tax that is taken out of his gross pay.
- **c** Stefan's deductions are $141.94 for his health fund and $51.33 for superannuation. Calculate Stefan's net fortnightly pay.

12 Asmita earns a salary of $59 945 p.a. Each fortnight she has deductions of $256.20 for family health insurance and $35 for superannuation taken from her gross income.

- **a** Calculate Asmita's fortnightly gross income.
- **b** How much PAYG tax does she pay per fortnight?
- **c** Calculate Agata's fortnightly net income.

13 Copy and complete this pay slip. PS C

Employee: Ziad Chaker		Hourly pay rate: $21.65	
Hours worked		Deductions	
Normal	38	Tax: $205.72	Other: $168.38
Time-and-a-half	3	Gross weekly income	
Double time	2	Total deductions	
		Net weekly income	

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Technology

Online income tax calculators

The Australian Taxation Office (ATO) website **www.ato.gov.au** has online calculators for income tax and PAYG tax. Visit the website and search 'Simple Tax Calculator' to find the income tax calculator for individuals.

1. Enter the taxable income $63 000 as '63000' (no spaces).
2. Select the current financial year.
3. Select 'Resident for full year' and click 'Next'.
4. The estimated tax payable will be shown on a new screen.
5. Repeat for at least 2 more taxable incomes.
6. Find the PAYG tax calculator and use it to find the PAYG tax payable and net pay for a gross pay of:
   - a $1408 weekly
   - b $2870 fortnightly
   - c $5610 monthly

1.03 Simple interest

Worksheets
Simple interest
Simple interest table

Puzzle
Simple interest

Video
Simple interest

- When you invest money, you receive interest from your investment.
- When you borrow money, you pay interest on your loan.
- The original amount of money invested or borrowed is called the **principal**.
- This interest rate is a percentage of the principal, usually written as a rate **per annum** ('per year'), abbreviated 'p.a.'
- **Simple interest** (or flat rate interest) is interest calculated simply on the original principal.

The simple interest formula

$I = Prn$, where:

I is the simple interest

P is the principal

r is the interest rate per time period, expressed as a decimal

n is the number of time periods

Example 8

Find the simple interest on:

a \$4000 at 3.5% p.a. for 6 years

b \$13 500 at 5.5% p.a. for 7 months

c \$75 640 at 0.42% per month for 2 years

Solution

a $P = \$4000$, $r = 3.5\% = 0.035$, $n = 6$ years

$I = Prn$
$= \$4000 \times 0.035 \times 6$
$= \$840$

$r = 0.035$ per year, $n = 6$ years, so the time period is **years**

b $P = \$13\,500$, $r = 5.5\% = 0.055$, $r = \frac{7}{12}$ years

$I = Prn$
$= \$13\,500 \times 0.055 \times \frac{7}{12}$
$= \$433.125$
$\approx \$433.13$

$r = 0.055$ per year, $n = 7$ *months*, so we must change 7 months to years so that the time period is **years**

rounded to the nearest cent

c $P = \$75\,640$, $r = 0.42\% = 0.0042$,
$n = 2 \times 12 = 24$ months
$I = Prn$
$= \$75\,640 \times 0.0042 \times 24$
$= \$7624.512$
$\approx \$7624.51$

$r = 0.0042$ per month, $n = 2$ *years*, so we must change 2 years to months so that the time period is **months**

rounded to the nearest cent

Example 9

Petra invests \$17 400 for 2 years at 3.75% p.a. flat rate interest. To what final value will her investment grow?

Solution

$P = \$17\,400$, $r = 3.75\% = 0.0375$, $n = 2$ years

Both n and r are in **years**

$I = Prn$
$= \$17\,400 \times 0.0375 \times 2$
$= \$1305$

Value of investment $= \$17\,400 + \$1305$
$= \$18\,705$

Principal + interest

Video
Simple interest rate

Example 10

After 4 years, an investment of \$13 000 has earned \$1092 in simple interest.

What is the annual interest rate?

Solution

$I = \$1092$, $P = \$13\,000$, $n = 4$ years

$I = Prn$

$\$1092 = \$13\,000 \times r \times 4$

$\$1092 = \$52\,000r$

$$n = \frac{\$1092}{\$52000}$$

$= 0.021$

$= 2.1\%$

$\therefore$ Annual interest rate $= 2.1\%$

Video
Simple interest rate

Example 11

For how many months will \$10 000 need to be invested to earn \$250 in simple interest at 3.25% p.a.?

Solution

$I = \$250$, $P = \$10\,000$, $r = 3.25\% = 0.0325$

$I = Prn$

$\$250 = \$10\,000 \times 0.0325 \times n$

$\$250 = \$325n$

$$n = \frac{\$250}{\$325}$$

$= 0.7692...$ years — n is in years, so convert to months

$= 0.7692... \times 12$ months

$= 9.230...$ months

≈ 10 months — rounded *up* to the nearest month

EXERCISE 1.03 ANSWERS ON P. 576

Simple interest U F PS R

EXAMPLE 8

In this exercise, round all money answers to the nearest cent.

1 Calculate the simple interest earned on each investment.

- **a** \$35 000 for 4 years at 3.6% p.a.
- **b** \$26 850 at 1.95% p.a. for 2 years
- **c** \$8200 invested for 5 months at 3% p.a.
- **d** \$6590 invested for 16 months at 0.25% per month
- **e** \$5250 invested for 250 days at 1.04% p.a.
- **f** \$18 400 invested for 3 years at 0.18% per month

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2 Calculate the flat rate interest charged on each loan.

a $1250 for 2 years at 3.5% p.a.

b $18 900 for $5\frac{1}{2}$ years at 5.7% p.a.

c $1.15 million at 4.5% p.a. for 48 months

d $12 000 for 10 months at 0.575% per month

e $9750 for 2.5 years at 0.48% per month

f $24 720 for 136 days at 7.85% p.a.

3 Harry owed $783.26 on his credit card. The credit card company charged him one month's simple interest at 21% p.a. How much interest was he charged? Select the correct answer **A**, **B**, **C** or **D**.

A $13.71 **B** $16.45 **C** $25.38 **D** $37.30

EXAMPLE 9

4 Find the final value of each investment using simple interest.

a $10 000 invested for 3 years at 4% p.a.

b $1500 invested for 18 months at 0.19% per month

c $8500 invested for 3.5 years at 0.25% per month

d $9250 invested for 50 months at 3.75% p.a.

5 Liong borrowed $6000 to go on an overseas holiday, at 12% p.a. flat rate interest for 2 years. Calculate:

a the total interest

b the total amount Liong must repay.



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EXAMPLE 10

6 The interest on a loan of $2500 over 4 years is $450. Calculate the flat rate of interest p.a. R

7 Katy took out a loan for $22 000 over 5 years. If her total loan repayments amounted to $28 400, calculate: PS R

a the interest charged

b the flat rate of interest p.a., correct to 2 decimal places.

8 After 5 years, the interest on a loan of $8000 amounts to $2340. Calculate the annual simple interest rate.

EXAMPLE 11

9 For how many years will $4200 need to be invested to earn $200 interest, if the interest rate is 2.5% p.a.? R

10 How many weeks will it take for $50 000 to earn $750 in interest if the rate is 2.6% p.a.? R

11 How many days will it take for $20 000 to earn $300 in interest if the rate is 4% p.a.? R

12 An online bank offered the following investment to its customers.

- A rate of 2.35% p.a. simple interest for the first 4 months only
- Then the principal and interest reinvested at 0.45% p.a. simple interest

What will Dariya's investment of $3480 be worth after 7 months?
Select **A**, **B**, **C** or **D**. PS R

A $31.21 **B** $374.10 **C** $3511.21 **D** $3854.10

13 For how many months will $20 000 need to be invested to amount to $22 000, if interest is paid at the rate of 0.33% per month? PS R

14 What is the flat rate of interest (as a percentage p.a., correct to one decimal place) when $1650 earns $85 in interest over 2 years? R

15 Toula used a credit card to buy a netbook computer for $799 and some extra accessories for $246. She pays off this debt in 30 days. The credit card charges 22% p.a. simple interest. PS R

a Calculate the simple interest charged.

b How much will Toula pay after 30 days?

16 Bhashine earned $185 interest each year for 4 years on an investment account. At the end of the 4 years, she closed her account and withdrew $6000 in total.
What was the annual flat rate of interest paid into Bhashine's account?
Select **A**, **B**, **C** or **D**. PS R

A 3.1% **B** 3.5% **C** 12.3% **D** 14.1%

Compound interest

1.04

Most investments earn **compound interest** rather than simple interest. With compound interest, the interest earned is **added** to the principal so that next time, the interest is calculated on a larger principal. This means that more interest is earned, because we are also earning interest on the interest we have already earned. The word **compound** means 'combined'.

Example 12

A principal of $23 000 is invested at 4% p.a. interest, compounded yearly for 2 years.

a What is the final value of the investment after 2 years?

b What is the amount of compound interest earned?

Solution

a The interest for each year is calculated separately.

After the first year:

$I = \$23\,000 \times 0.04$

$= \$920$

$\text{Investment} = \$23\,000 + \$920$ Principal + interest

$= \$23\,920$

After the second year:

$I = \$23\,920 \times 0.04$

$= \$956.80$

$\text{Investment} = \$23\,920 + \$956.80$ New principal + interest

$= \$24\,876.80$

b $\text{Compound interest earned} = \text{final value} - \text{principal}$

$= \$24\,876.80 - \$23\,000$

$= \$1876.80$

Notice that compound interest involves repeated percentage increase. In the above example, to calculate compound interest on a principal of $23 000 over 2 years at 4% p.a., we are actually increasing $23 000 by 4% twice.

Adding 4% to the principal is the same as increasing the principal by 4%, which is the same as multiplying the principal by 104% or 1.04.

Investment after 1st year $= \$23\,000 \times 1.04 = \$23\,920$

Investment after 2nd year $= \$23\,920 \times 1.04 = \$24\,876.80$

We can even combine these 2 steps into one step by repeated percentage increases:

Investment after 2nd year $= \$23\,000 \times 1.04 \times 1.04 = \$24\,876.80$

Using repeated percentage increases can simplify our compound interest calculations.

Example 13

A principal of $9000 is invested at 3.7% p.a. compounded yearly over 3 years. What is:

a the final value of the investment after 3 years?

b the compound interest earned?

Solution

a Adding 3.7% interest to the principal is the same as multiplying the principal by 1.037.

$\therefore$ final value after 3 years $= \$9000 \times 1.037 \times 1.037 \times 1.037$

$= \$9000 \times (1.037)^3$

$= \$10\,036.4188...$

$\approx \$10\,036.42$ rounded to the nearest cent

b Compound interest earned = final value – original principal

$= \$10\,036.42 - \$9000$

$= \$1036.42$

EXERCISE 1.04 ANSWERS ON P. 576

Compound interest U F

In this exercise, round all money answers to the nearest cent.

EXAMPLE 12

1 A principal of $23 000 is invested at 5% p.a. interest, compounded yearly over 2 years.

a Copy and complete the following working to calculate the value of the investment after 2 years.

After the first year:

$I = \$23\,000 \times 0.05$ Investment $= \$23\,000 + \$_____$

$= \$_____$ $= \$_____$

After the second year:

$I = \$_____ \times 0.05$ Investment $= \$_____ + _____$

$= \$_____$ $= \$_____$

b Copy and complete the following working to calculate the amount of compound interest earned.

Compound interest earned = final value – principal

$= \$_____ - \$23\,000$

$= \$_____$

2 Finn invests $15 000 at 2.5% p.a. compounded yearly over 3 years. Show all working (as in question **1**) to find:

a the final value of the investment after 3 years

b the total amount of compound interest earned.

3 Selina invests $34 100 at 6.2% p.a. interest compounded yearly over 2 years. Calculate:

a the final value of the investment **b** the compound interest earned.

EXAMPLE 13

4 Use repeated percentage increases to calculate the final value of each investment compounded annually, then calculate the compound interest earned.

a $5000 for 2 years at 4% p.a. **b** $27 800 for 3 years at 2.85% p.a.

c $9600 for 3 years at 5% p.a. **d** $39 500 for 2 years at 3% p.a.

e $18 400 for 4 years at 1.25% p.a.

5 For each investment, calculate the compound interest earned.

a $30 400 at 5% p.a. interest for 3 years.

b $19 150 at 4.2% p.a. interest for 2 years.

c $8750 at 1.75% p.a. interest for 2 years.

d $36 000 at 3.5% p.a. interest for 3 years.

e $18 960 at 6.35% p.a. interest for 5 years.

1.04

Mental skills 1: Maths without calculators ANSWERS ON P. 576

Quiz
Mental skills 1

Percentage of a quantity

Learn these commonly-used percentages and their fraction equivalents.

Percentage	50%	25%	12.5%	75%	20%	10%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$
Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{3}$	$\frac{2}{3}$

Now we will use them to find a percentage of a quantity.

1 Study each example.

a $20\% \times 25 = \frac{1}{5} \times 25$
$= 5$

b $50\% \times 120 = \frac{1}{2} \times 120$
$= 60$

c $12.5\% \times 32 = \frac{1}{8} \times 32$
$= 4$

d $75\% \times 56 = \frac{3}{4} \times 60$
$= \left(\frac{1}{4} \times 60\right) \times 3$
$= 15 \times 3$
$= 45$

e $33\frac{1}{3}\% \times 27 = \frac{1}{3} \times 27$
$= 9$

f $66\frac{2}{3}\% \times 60 = \frac{2}{3} \times 60$
$= \left(\frac{1}{3} \times 60\right) \times 2$
$= 20 \times 2$
$= 40$

2 Now simplify each expression.

a $25\% \times 44$ **b** $33\frac{1}{3}\% \times 120$ **c** $20\% \times 35$ **d** $66\frac{2}{3}\% \times 36$

e $10\% \times 230$ **f** $12\frac{1}{2}\% \times 48$ **g** $50\% \times 86$ **h** $20\% \times 400$

i $75\% \times 24$ **j** $33\frac{1}{3}\% \times 45$ **k** $25\% \times 160$ **l** $10\% \times 650$

m $12.5\% \times 88$ **n** $66\frac{2}{3}\% \times 21$ **o** $20\% \times 60$ **p** $75\% \times 180$

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1.05 The compound interest formula

Worksheet
Compound interest

Puzzles
Compound interest with annual rests

Compound interest with non-annual rests

Technologies
Comparing interest rates

Simple and compound interest calculator

Spreadsheets
Comparing interest rates

Simple and compound interest

Videos
The compound interest formula

Could you owe more than America?

There is a formula for calculating the final amount of an investment earning compound interest. Note the following pattern.

- Final value of \$23 000 at 4% p.a. interest for 2 years = $\$23\,000 \times (1.04)^2$
- Final value of \$9000 at 3.7% p.a. interest for 3 years = $\$9000 \times (1.037)^3$
- Final value of \$18 960 at 6.35% p.a. interest for 5 years = $\$18\,960 \times (1.0635)^5$

Compound interest formula

$FV = PV(1 + r)^n$, where:

FV is the final value **or** **future value** of the investment

PV is the principal, also called the **present value**

r is the interest rate per compounding period, expressed as a decimal

n is the number of compounding periods

The compound interest is then calculated using this formula:

Compound interest = future value − present value

$$I = FV - PV$$

Example 14

For each investment, calculate:

i the future value of the investment

ii the compound interest earned if interest is compounded annually

a \$26 750 is invested at 4% p.a. for 3 years

b \$52 000 is invested at 3.8% p.a. for 5 years

Solution

a i $PV = \$26\,750$, $r = 4\% = 0.04$, $n = 3$

$FV = PV(1 + r)^n$

$= \$26\,750(1 + 0.04)^3$

$= \$26\,750(1.04)^3$

$= \$30\,090.112...$

$\approx \$30\,090.11$

The final value of the investment is \$30 090.11.

ii Compound interest = \$30 090.11 − \$26 750 $I = FV - PV$

= \$3340.11

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b **i** $PV = \$52\,000, r = 3.8\% = 0.038, n = 5$

$FV = PV(1 + r)^n$

$= \$52\,000(1 + 0.038)^5$

$= \$52\,000(1.038)^5$

$= \$62\,659.9597...$

$\approx \$62\,659.96$

ii Compound interest $= \$62\,659.96 - \$52\,000$

$= \$10\,659.96$

Example 15

Calculate the compound interest when $24 500 is invested at 6.3% p.a. for 5 years:

a compounded annually

b compounded monthly.

Solution

a $PV = \$24\,500, r = 0.063, n = 5$

$FV = \$24\,500\,(1 + 0.063)^5$

$= \$24\,500\,(1.063)^5$

$= \$33\,253.1205...$

$\approx \$33\,253.12$

$I = \$33\,253.12 - \$24\,500$

$= \$8753.12$

b Because interest is compounded monthly, r and n must be expressed in months, not years.

$PV = \$24\,500, r = \frac{0.063}{12} = 0.00525$ per month, $n = 5 \times 12 = 60$ months

$FV = \$24\,500(1 + 0.005\,25)^{60}$

$= \$24\,500(1.005\,25)^{60}$

$= \$33\,543.701\,98$

$\approx \$33\,543.70$

$I = \$33\,543.70 - \$24\,500$

$= \$9043.70$

Note: More interest is earned when it is compounded monthly rather than yearly.

Why do you think this is so?

1.05

Video
Compound interest

EXERCISE 1.05 ANSWERS ON P. 576

The compound interest formula U F PS R

In this exercise, round all money answers to the nearest cent.

1 An amount of $13 000 is invested at 5% p.a. interest, compounded annually over 2 years. Which expression represents the future value of the investment?
Select the correct answer **A**, **B**, **C** or **D**.

A $13\,000 \times 0.05 \times 2$ **B** $13\,000\,(1 + 0.05)^2$

C $13\,000 \times (0.05)^2$ **D** $13\,000\,(1 - 0.05)^2$

EXAMPLE 14

2 For each investment, where interest is compounded yearly, calculate:

i the final amount of the investment, A **ii** the compound interest, I, earned.

a $6500 invested at 7% p.a. for 6 years **b** $10 000 invested at 8.5% p.a. for 4 years

c $12 240 invested at 1.6% p.a. for 2 years **d** $34 600 invested at 4.9% p.a. for 5 years

e $8000 invested at 1.75% p.a. for 3 years

3 Calculate the amount of interest earned on an investment of $6500 if it is invested at 2.5% p.a. compounded annually for 8 years. Select **A**, **B**, **C** or **D**.

A $131.14 **B** $832.81 **C** $1300 **D** $1419.62

4 Find the amount of interest earned on one million dollars invested at 14.9% p.a. compounded annually for 6 years.

5 Find the amount of interest charged on a loan of $25 000 if it is borrowed over 10 years at 8% p.a. compounded annually. Select **A**, **B**, **C** or **D**.

A $31 250 **B** $28 973.12 **C** $28 589.72 **D** $20 000

6 Yasmin was given $2000 when she turned 3 years old. Her parents invested it at a 2% p.a. compounded annually. No deposits or withdrawals were made. Which expression can be used to determine how much money Yasmin had in the account when she turned 16? Select **A**, **B**, **C** or **D**. R

A $2000\,(1 + 0.02)^{13}$ **B** $2000\,(1 + 0.2)^{13}$

C $2000\,(1 + 0.02)^{16}$ **D** $2000\,(1 + 0.2)^{16}$

EXAMPLE 15

7 For each investment, calculate:

i the future value **ii** the interest earned.

a $10 000 for 5 years at 2.4% p.a., compounded monthly

b $35 500 for 10 years at 2% per half-year, compounded half-yearly

Half-yearly means 'twice a year' or 'every 6 months'

c $8900 for 2 years at 3% p.a., compounded quarterly

Quarterly means '4 times per year' or 'every 3 months'

d $42 000 for 5 years at 0.225% per month, compounded monthly

e $16 500 for 3 years at 2.6% p.a., compounded half-yearly

f $4900 for 1 year at 0.005% per day, compounded daily



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8 Find the total value of an investment of $4300 over 5 years at 4.6% p.a. interest, compounded every 6 months. Select the correct answer **A**, **B**, **C** or **D**.

A $4817.78 **B** $5384.27 **C** $5397.90 **D** $8506.24

9 **a** Reese invested $6000 for 2 years at a flat rate of 5% p.a. Calculate the amount of interest earned.

b Tegan invested $6000 for 2 years at an interest rate of 5% p.a. compounded annually. Calculate the amount of interest earned.

c Whose investment earned more interest? How much more?

10 A principal of $5000 is invested at 3.5% p.a. for 3 years. Match each compounding period for this investment to its correct expression for the future value of the expression.

a compounded yearly

b compounded half-yearly

c compounded quarterly

d compounded monthly

A $5000\left(1+\frac{0.035}{2}\right)^{3\times2}$

B $5000\left(1+\frac{0.035}{12}\right)^{3\times12}$

C $5000\left(1+\frac{0.035}{4}\right)^{3\times4}$

D $5000\,(1+0.035)^3$

11 Lisa is setting up a trust account for her new grandson Stefan. In 18 years time, she wants the investment to be worth $30 000, to help with the cost of university fees or the purchase of a car. Suppose the interest rate for the account is 2.94% p.a. compounding yearly. PS R

a How much should Lisa invest now to achieve the $30 000 target?

b If Lisa opened a trust account that earns 2.94% p.a. compounding monthly instead, how much less would she need to invest?

12 Zoe is 5 years old and about to start school. Her parents want to invest $25 000, for her high school education expenses, in an account that earns 3% p.a. over 7 years. R

a Calculate the total interest earned if interest is compounded:

i yearly **ii** half-yearly **iii** quarterly **iv** monthly

b Which compounding period should Zoe's parents choose? Why?

13 A principal of $10 000 is invested for 4 years, earning interest at the rate of 3% p.a., compounded monthly. Which expression represents the future value of the investment? Select **A**, **B**, **C** or **D**.

A $10\,000\left(1+\frac{3}{100}\right)^4$

B $10\,000\left(1+\frac{3}{100}\right)^{48}$

C $10\,000\times\left(1+\frac{3}{1200}\right)^4$

D $10\,000\left(1+\frac{3}{1200}\right)^{48}$

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Skillsheet
Spreadsheets

Technology

Comparing simple with compound interest

In this activity, you will compare the interest earned on an investment of $1000 for 10 years at 8% p.a. simple interest and 8% p.a. compound interest, compounded annually.

1 Create this spreadsheet. The present value (*PV*) is entered in cell A1 and the annual interest rates (in decimal form) in cells B1 and C1.

	A	B	C
1	$ 1,000.00	0.08	0.08
2			
3	Years	Simple Interest	Compound Interest
4	1		
5	2		
6	3		
7	4		
8	5		
9	6		
10	7		
11	8		
12	9		
13	10		

2 To calculate the simple interest in column B, in cell B4 enter the formula **=$A$1*$B$1*A4**. Now **Fill Down** from cell B4 to B13.

3 To calculate the compound interest in column C, in cell C4 enter the formula **=$A$1*(1+$C$1)^A4-$A$1**. Now **Fill Down** from cell C4 to C13.

4 Highlight cells A3 to C13. **Insert 'Scatter with Smooth lines and markers'**.

5 When the interest rate is the same, which account pays better interest: simple or compound interest? (Type your answer in cell A15)

6 Now compare the interest earned on an investment of $1000 for 10 years at 9% p.a. simple interest and 7% p.a. compound interest, compounded annually. Change the interest rates in cells B1 (0.09) and C1 (0.07) respectively.

Answer the following questions in the spreadsheet cells indicated in brackets.

7 After how many years did the compound interest rate pay more than the simple interest rate? (A16)

8 How much extra interest did the compound interest rate pay at the end of the 10 years? (A17)

9 Change the interest rate in B1 to 10% (0.1) and C1 to 9% (0.09). How does the change in interest rate affect the amount of interest paid? Include calculations to justify your answer. (A18)

10 Change the interest rate in B1 to 12% (0.12) and C1 to 8.5% (0.085). After how many years did the amount of compound interest earned overtake the amount of simple interest earned? (A19)

11 What is the difference in the amount of compound interest earned for the 10-year period compared to the simple interest investment? Is it a significant amount? Justify your answer. (A20)

Term payments

1.06

Many customers buy expensive household items **on terms**, which means 'paying off' the item by regular instalments over time, after paying a deposit. A **term payments** plan is also called **hire-purchase** because the purchaser actually hires the item until it is completely paid off. Special offers can include interest-free periods, but there may be other conditions such as establishment fees and extra charges if the regular **repayments** are not paid on time. Also, if the purchaser fails to keep up with the payments, higher interest may be charged or the item may be **repossessed** (taken back).



Technology
Term payments

Example 16

Sonia purchases a new fridge and dishwasher package valued at $4925. She pays a 10% deposit and repays the balance in monthly repayments over 3 years. Interest on the balance is charged at a flat rate of 12% p.a.

Find:

a the deposit paid

b the balance owing

c the interest charged on the balance

d the total to be repaid

e the amount of each monthly repayment

f the total price paid for the package.

Solution

a Deposit $= 10\% \times \$4925$

$= \$492.50$

b Balance owing $= \$4925 - \$492.50$ or $90\% \times \$4925$

$= \$4432.50$

c Interest charged on the balance is flat or simple interest.

$P = \$4432.50$, $r = 0.12$, $n = 3$

$I = Prn$

$= \$4432.50 \times 0.12 \times 3$

$= \$1595.70$

d Total to be repaid = balance + interest

$= \$4432.50 + \$1595.70$

$= \$6028.20$

e Number of repayments $= 3 \times 12$ months

$= 36$ months

Monthly repayment $= \$6028.20 \div 36$

$= \$167.45$

f Total price paid = deposit + total repayments

$= \$492.50 + \$6028.20$

$= \$6520.70$

or cash price + interest

$= \$4925 + \$1595.70$

Deferred payment plan

With a **deferred payment plan**, the customer does not make any repayments until a later date, such as after 6 months.

Deferred means 'delayed'.

Example 17



Viktorus/Shutterstock.com

Ilhea and Robert purchase a $3200 home theatre package on a deferred payment plan over 2 years. They make no repayments for the first 3 months, then pay $210 per month.

a Calculate the total repayments.

b How much interest did they pay?

c Ilhea and Robert were also charged the following fees for the plan.

- One-off establishment fee of $25
- Account service fee of $9.95 per month

i How much was paid in fees over the 2-year period?

ii What percentage (correct to one decimal place) of the purchase price was paid in fees?

Solution

a Total cost $= \$210 \times 21$ 2 years − 3 months = 21 months

$= \$4410$

b Interest = total cost − cash price

$= \$4410 - \$3200$

$= \$1210$

c **i** Total fees paid $= \$25 + \$9.95 \times 24$

$= \$263.80$

ii Percentage $= \frac{\$263.80}{\$3200} \times 100\%$

$= 8.24375\%$

$\approx 8.2\%$

EXERCISE 1.06 ANSWERS ON P. 577

Term payments U F PS R C

In this exercise, round all money answers to the nearest cent.

EXAMPLE 16

1.06

1 Gaspard purchases a laundry package of washing machine and dryer valued at $1755. He pays a 10% deposit and repays the balance in 18 months. Interest on the balance is charged at a flat rate of 13.85% p.a. Find: PS R C

- **a** the deposit Gaspard paid
- **b** the balance owing
- **c** the interest charged on the balance
- **d** the total to be repaid
- **e** the amount of each monthly instalment
- **f** the total price paid for the package

2 Freyja bought a backyard plunge pool for $25 500. She paid a 5% deposit and will repay the balance over 5 years, with flat-rate interest charged 8.6% p.a. Calculate: PS R C

- **a** the deposit
- **b** the balance owing
- **c** the interest charged
- **d** the total to be repaid
- **e** the monthly instalment
- **f** the total price paid.

3 Tracy wants to buy a new 3-piece lounge suite that she saw advertised at the lounge sale pictured, costing $6895. R



- **a** How much deposit will she have to pay?
- **b** What will be the flat-rate interest on the balance?
- **c** Find the size of each monthly repayment.

4 Mack pays a $1200 deposit on his holiday and then $185 per month for a year. The cash price of the holiday is $2700.

- **a** How much did Mack pay for his holiday?
- **b** Calculate the interest paid.
- **c** Calculate the balance owing after the deposit was paid.
- **d** Calculate, as a percentage correct to one decimal place, the flat interest rate charged p.a.

5 Mrs Mitchell buys a used car for her daughter by paying a 20% deposit and $275 per month for 4 years. If the car has a cash price of $10 400, find: R C

- **a** the deposit Mrs Mitchell paid
- **b** the balance owing after the deposit is paid
- **c** the total amount Mrs Mitchell repaid in instalments
- **d** the interest charged
- **e** the flat interest rate (p.a.), correct to one decimal place.

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6 Xuan wanted to buy a new smartphone costing $1044. R C

a How much deposit did she pay?

b What is the total amount repaid in repayments for the smartphone?

c Calculate the total interest paid for the year.

d What was the annual flat interest rate for this purchase, correct to 1 decimal place?



Cincila/Shutterstock.com

EXAMPLE 17

7 Diego bought a laptop priced at $1500 on a deferred payment plan: nothing to pay for 3 months and then 9 monthly payments of $210. However, a monthly account fee of $12.95 was added to the plan. PS R

a Find the total cost of the laptop to Diego.

b How much in excess of the cash price was paid?

c What was the flat rate of interest p.a. (correct to one decimal place) charged?

'in excess' means 'above' or 'more than'

8 Tahlia bought a new outdoor setting and BBQ for her backyard that retails for $3899. She paid $500 deposit, no payments for 6 months and then fortnightly payments of $78 for 2 years. PS R

a Find the balance owing after Tahlia had paid the deposit.

b Calculate the total cost of the outdoor setting and BBQ.

c How much in excess of the cash price did Tahlia pay under this plan?

d What is the annual flat rate of interest charged, correct to one decimal place?

9 Sofia bought a home cinema system priced at $2100 on interest-free terms for one year with no repayments for the first 4 months. R

a If Sofia makes 8 equal monthly repayments, what is the amount of each payment?

b There is a service charge of $12.95 every month for this deferred payment plan. What percentage (correct to one decimal place) of the purchase price was paid in service charges?

Investigation

Short-term loans: fast money

Perhaps you have seen ads from finance companies (not banks) offering to lend you cash for a short amount of time (from 16 days to 12 months) if you can't afford something, with quick (or even instant) approval regardless of your financial situation. These small loans are meant to 'tie you over' if you are short of money, but there is always a catch.

In this activity, you will compare the borrowing costs associated with small amounts borrowed as short-term loans.

1 Alice takes out a short-term loan of $700 with an interest rate of 18% per week.

- **a** If she repays the loan in 2 weeks, calculate the total amount Alice will need to repay.
- **b** What do you notice about the amount of interest charged?
- **c** What do you think will happen if Alice cannot repay the loan in 2 weeks?

2 Disha borrows $1500 for one month and pays a flat fee of $30 per $100 borrowed.

- **a** Calculate the total cost of the loan to be repaid.
- **b** What do you notice about the amount of interest charged?
- **c** Suggest a reason a person may need to borrow $1500 for a short time but cannot use a credit card or bank loan.

3

- **a** Do an Internet search of 'short-term loans' and find 2 Australian finance companies that provide short-term loans.
- **b** In addition to charging interest, short-term loans have a range of different fees and costs. List the fees and costs charged by the 2 finance companies that you found.
- **c** List the interest rates of some short-term loans provided by the finance companies.
- **d** List 2 advantages and 2 disadvantages of taking a short-term loan.

4 For a short-term loan of $1500, consider weekly, fortnightly and monthly payments for each of the 2 companies. Copy and complete this table.

Repayments and fees	Company 1	Company 2
Weekly		
Fortnightly		
Monthly		

5 Repeat Question 4 for a short-term loan of $2000.

6 What would be your advice to someone considering a short-term loan? Analyse and compare the loan calculations from the 2 companies you have chosen to support your observations.

1.07 Depreciation

Worksheet
Depreciation

Depreciation is the decrease in value of an item over time, usually something expensive and valuable, called an **asset**. When items lose value because of age or frequency of use, they are said to **depreciate**.

The compound interest formula can be adapted to find the depreciated value of an asset. While compound interest involves repeated percentage increases, depreciation involves repeated percentage *decreases*, so its formula has a minus sign.

Depreciation formula

$S = V_0(1 - r)^n$, where:

S is the final value of the asset, called the **salvage value**

V_0 is the **initial (original) value** of the asset (the value V at time 0)

r is the rate of depreciation per period, expressed as a decimal

n is the number of periods of depreciation

The amount of depreciation is then calculated using this formula:

$$\text{Depreciation} = \text{original value} - \text{final value}$$

Video
Depreciation

Example 18

An accountant's computer system depreciates by 15% each year.

a If the computer system is currently valued at \$2600, what will its value be in 5 years?

b What is the depreciation over this time?

Solution

a $V_0 = \$2600$, $r = 15\% = 0.15$, $n = 5$

$S = V_0(1 - r)^n$

$= \$2600\ (1 - 0.15)^5$

$= \$2600\ (0.85)^5$

$= \$1153.6338...$

$\approx \$1153.63$

The value of the computer system in 5 years will be \$1153.63.

b Depreciation $= \$2600 - \$1153.63$ original value − final value

$= \$1446.37$

9780170479622

Example 19

An industrial oven in a restaurant originally costs $19 800, then depreciates at a rate of 12% p.a.

a Find the salvage value of the oven after 6 years, correct to the nearest dollar.

b Express the salvage value as a percentage of the cost price, correct to one decimal place.

Solution

a $V_0 = \$19\,800$, $r = 0.12$, $n = 6$

$S = V_0(1 - r)^n$

$= \$19\,800\,(1 - 0.12)^6$

$= \$19\,800\,(0.88)^6$

$= \$9195.2009...$

$\approx \$9195$

b Percentage of cost price $= \frac{\$9195}{\$19\,800} \times 100\%$

$= 46.4393...\ \%$

$\approx 46.4\%$

This means that after 6 years, the oven is worth approximately 46% of its original price (or has lost 54% of its initial value).



otnaydur/Shutterstock.com

EXERCISE 1.07 ANSWERS ON P. 577

Depreciation U F PS R C

In this exercise, round all money answers to the nearest cent.

EXAMPLE 18

1 Find the salvage value of a photocopier after 5 years if its purchase price was $2850 and the annual depreciation rate is 20%.

2 **a** Find the value of a car after 7 years if it is purchased new for $49 990 and it depreciates at 12% p.a.

b Find the amount of depreciation over this time.

3 For each item shown in the table, calculate:

i its value after 4 years of depreciation

ii its value after 4 years as a percentage of its original value, correct to one decimal place.

	Item	Original value	Depreciation rate (p.a.)
a	Oven	$1100	12%
b	Fishing boat	$38 500	18%
c	Library	$8460	12%
d	Computer	$2500	20%
e	Furniture	$27 500	15.5%
f	Bike	$2900	22%
g	Electrical tools	$870	17.5%
h	Air conditioner	$2600	9%

EXAMPLE 19

4 A smartphone originally valued at $1729 depreciates at 37% p.a. R

a What percentage (to 2 decimal places where necessary) of the original value remains after:

i 1 year? **ii** 3 years? **iii** 6 years?

b Approximately how long would it take the smartphone to halve its original value?

5 A security system costs a company $12 500 to buy new. It depreciates at a yearly rate of 20%.

a Find the salvage value of the system after:

i 1 year **ii** 2 years **iii** 5 years

b Find the value of the system after 5 years as a percentage of its initial value. Answer correct to one decimal place.



6 Parveen pays $25 490 for a new car. The car will depreciate in value by an average of 11% p.a.

- **a** Find correct to the nearest dollar the market value of the car in 3 years.
- **b** Calculate the amount of depreciation in the car after 3 years.

7 Adam has spent $175 000 on equipment to set up his hairdressing salon. The equipment depreciates at 20% per year. R

- **a** Find the value of the equipment after 4 years.
- **b** Find the amount of depreciation in the equipment after 4 years.
- **c** Find, by trial and error, how long it will take for the value to be under $50 000. Answer in years and months.
- **d** Find the value of Adam's equipment after 9 years as a percentage of its original value, correct to one decimal place.

8 Kamal says that, at 10% p.a. depreciation, a car will lose half its value after 7 years.

Is he correct? Show all working to justify your answer. PS R C

9 Office equipment that is worth $12 000 when new, depreciates at 15% p.a. as shown in the table. PS R C

- **a** How much did the office equipment lose in value in the first year?
- **b** After how many years did the office equipment fall below half its original value?
- **c** By how much did the office equipment depreciate between the 5th and 6th years?
- **d** Will the value of the office equipment ever fall below $100?
- **e** Will the value of the office equipment ever be zero?

Year	Depreciated value
0	$12 000
1	$10 200
2	$8670
3	$7369.50
4	$6264.08
5	$5324.46
6	$4525.79
7	$3846.93
8	$3269.89
9	$2779.40
10	$2362.49
11	$2008.12
12	$1706.90
13	$1450.87
14	$1233.24

Power plus ANSWERS ON P. 577

1 How long, in years and days, will it take an investment of $3000 to earn $500 in simple interest at 4% p.a.?

2 What amount should Owen invest to earn $100 in simple interest if the investment will last for 9 months and the interest rate is 3% p.a.?

3 A principal of $10 000 is invested for 5 years at an interest rate of 5% p.a., with interest compounded weekly. Calculate the future value of the investment.

4 Meghan needs $80 000 in 4 years' time. What present value should she invest now at an interest rate of 6% p.a., with interest compounded annually, to reach her target?

5 A painting appreciates in value at a rate of 3% p.a. while a computer depreciates in value at a rate of 10% p.a. If I bought the painting for $1200 and the computer for $1500 new, what would be their combined value in 5 years time?

6 A bacteria colony is growing at a rate of 20% per hour. If there are 10 000 bacteria now, use the compound interest formula to calculate how many there will be after 1 day. (Give your answer correct to the nearest 10 000.)

7 a You invest $2000 in a bank account at an interest rate of 4% p.a. with interest compounded annually. How long will it take for your investment to double in value?

b If you invested $4000 instead of $2000 at the same interest rate, how long will it take to double in value?

c Does the size of the principal make any difference to the time taken for it to double?

CHAPTER 1 REVIEW

Language of maths

allowable deductions	annual leave loading	compound interest	deposit
depreciation	double time	flat rate	fortnightly
future value	income tax	initial value	overtime
PAYG tax	per annum (p.a.)	present value	principal
quarterly	repayment	salvage value	short-term loan
simple interest	taxable income	term payments	time-and-a-half

1 When investing, why is compound interest better than simple interest?

2 What do the r and n stand for in the formulas $I = Prn$ and $FV = PV(1 + r)^n$?

3 What is another name for flat-rate interest?

4 Which word means a decrease in the value of an item over time?

5 Explain the meanings of the terms **present value** and **future value**.

6 Use a dictionary to find at least 2 different meanings of principal.

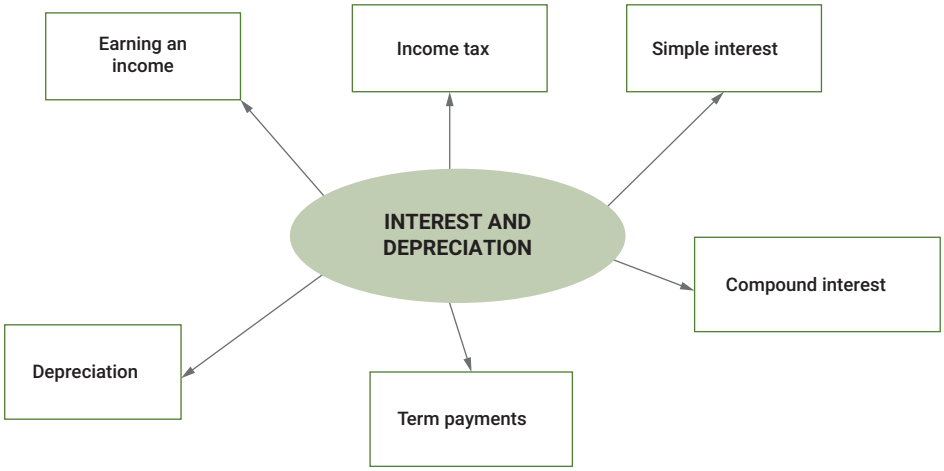
Quiz
Language of maths 1

Puzzle
Interest and depreciation crossword

Topic summary

- Which parts of this chapter were revision of Year 9 knowledge and skills?
- Which parts of this chapter were new to you?
- Do you know how to use the simple interest and compound interest formulas?
- How is income tax calculated?
- How is the depreciation formula similar to the compound interest formula?

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Worksheet
Mind map: Interest and depreciation

Quiz
Test yourself 1

TEST YOURSELF 1

ANSWERS ON P. 577

In this exercise, round all money answers to the nearest cent.

1.01 **1** Alexis is paid a commission of 2.5% on the value of the properties she sells. She also receives a weekly retainer of $1150. How much will Alexis earn if she sells a house for $475 830?

1.01 **2** Caleb earns a salary of $70 400 p.a. How much is he paid each week?

1.01 **3** A supermarket cashier is employed under the following award.

Normal rate: $21.45 per hour	
Normal rate	For 0 to 38 hours worked
Time-and-a-half	For the next 4 hours worked
Double time	For each hour worked after that

Calculate the wage for working:

a 40 hours

b 46 hours.

1.01 **4** Silvia earns royalties from writing cookbooks, at 15.5% of the price of the book.

Christmas sales figures of her latest book are shown in this table.

Version	Retail price	Sales
ebook	$25.95	8415
Hardcover	$45.95	26 500
Softcover	$35.95	15 260

How much did Silvia earn in royalties for these sales, rounded to the nearest cent?

1.01 **5** For his Christmas holidays, Jacan received 4 weeks normal pay plus 17.5% annual leave loading for the 4 weeks. If Jacan's annual salary is $54 920, find:

a his normal weekly pay

b his leave loading

c his total pay for the 4-week holiday.

1.02 **6** Alia earns a salary of $68 650 p.a. Her allowable deductions are donations to charities of $540 and work-related expenses of $385.

a Calculate Alia's taxable income.

b Use the tax table on page 12 to calculate the income tax Alia should pay.

1.03 **7** Calculate the simple interest earned on each investment.

a $20 000 invested for 3 years at 4% p.a.

b $7850 invested at 2.5% p.a. for 15 months

c $4500 invested for 6 months at 0.17% per month

d $25 200 invested for 100 days at 3.45% p.a.

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8 An amount of $5000 is invested at 2.35% p.a. interest, compounded annually over 3 years. 1.04

a What is the total value of the investment after 3 years?

b What is the amount of compound interest earned?

9 Calculate the value of the investment when $34 200 is invested at 3% p.a. for 2 years, with interest compounded annually. 1.05

10 Find the final value if $11 000 is invested for 4 years at 2.4% p.a., with interest compounded monthly. 1.05

11 Find the interest earned when $4895 is invested at 1.95% p.a. for 3 years, with interest compounded annually. 1.05

12 Calculate the interest earned when $46 230 is invested for 9 years at 2.8% p.a., with interest compounded half-yearly. 1.05

13 Yang purchases a furniture package valued at $4875. She pays a 10% deposit and repays the balance in 36 monthly instalments. Interest on the balance is charged at a flat rate of 14.5% p.a. Find: 1.06

a the deposit Yang paid

b the balance owing

c the interest charged

d the total to be repaid

e the amount of each instalment

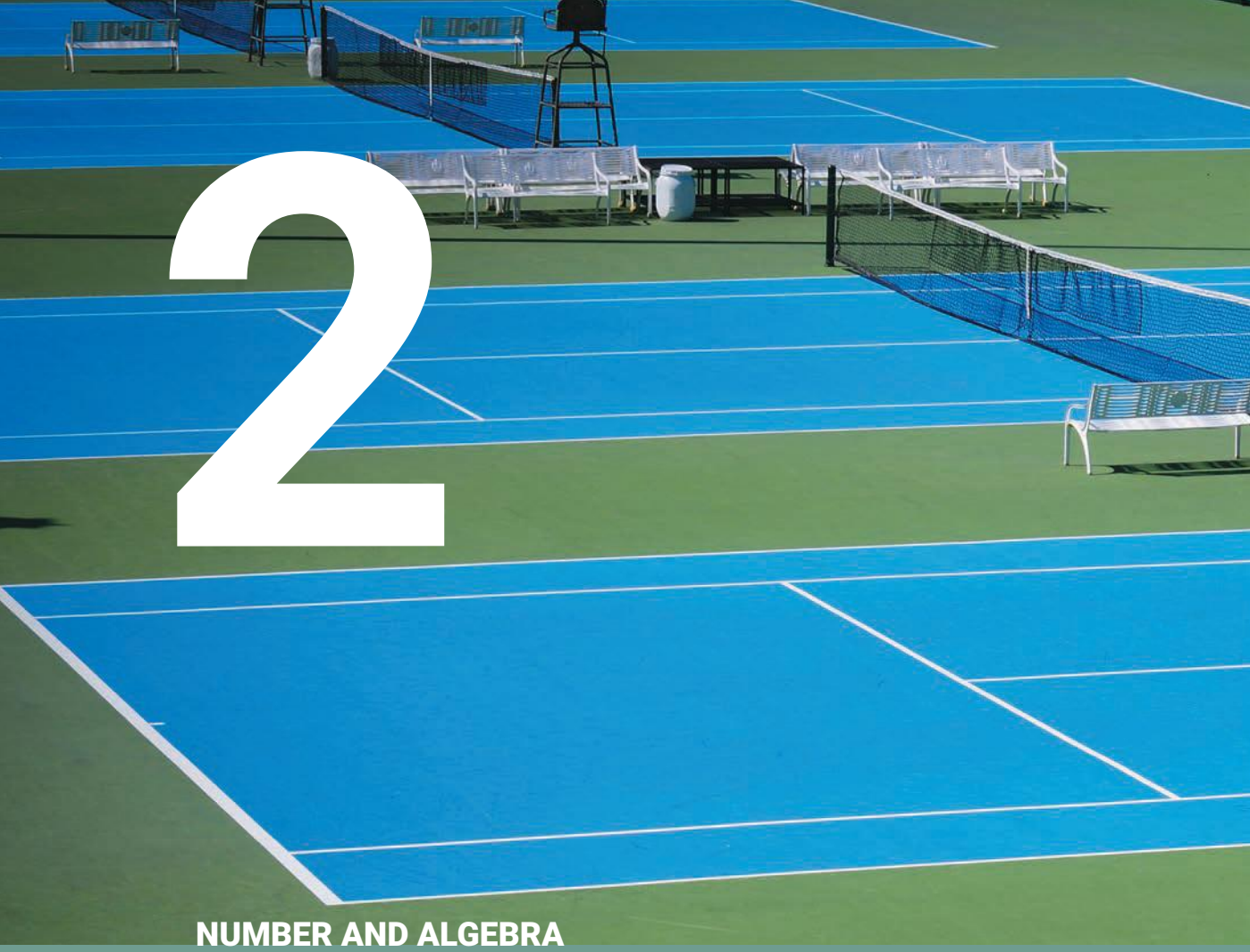
f the total price Yang paid for the package.

14 Bridget bought a new car for $24 990, which depreciates by 10% p.a. 1.07

a Find correct to the nearest dollar the depreciated value of the car after 5 years.

b What is the depreciation over this time?

c Express the depreciated value as a percentage of the original price (correct to one decimal place).



2

NUMBER AND ALGEBRA

GRAPHING LINES

Straight lines are an important part of our environment. We play sport on courts using parallel and perpendicular lines, and skyscrapers would not be standing without straight lines. We can also use straight lines to model different types of data and predict future outcomes.



Chapter outline

	Working mathematically				
2.01 Length, midpoint and gradient of an interval	U	F	PS	R	C
2.02 Parallel and perpendicular lines	U	F			
2.03 Graphing linear equations	U	F		R	C
2.04 The gradient–intercept equation $y = mx + c$	U	F		R	
2.05 The general equation $ax + by + c = 0$*	U	F		R	C
2.06 Finding the equation of a line	U	F		R	C
2.07 Applying linear equations	U	F		R	C
2.08 Equations of parallel and perpendicular lines	U	F		R	C

*PATH

Wordbank

general form Any linear equation expressed as $ax + by + c = 0$, where a, b and c are integers and a is positive

gradient The steepness of a line or interval, measured by the fraction $\frac{\text{rise}}{\text{run}}$

gradient–intercept form Any linear equation expressed as $y = mx + c$, where m is the gradient and c is the y-intercept

linear equation An equation whose graph is a straight line

parallel lines Lines that point in the same direction and have the same gradient

perpendicular lines Lines that cross at right angles (90°) and have gradients whose product is −1

x-intercept The x-value at which a graph cuts the x-axis

y-intercept (or **vertical intercept**) The y-value at which a graph cuts the y-axis

Quiz
Wordbank 2

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Videos (15):

2.01 The gradient of a line • Coordinate geometry • Distance, midpoint and gradient formulas

2.02 Gradients of perpendicular lines

2.03 Graphing linear equations • Testing if a point lies on a line • Horizontal and vertical lines

2.04 Gradient and y-intercept of a line • The gradient-intercept formula • Graphing $y = mx + c$

2.05 The general equation $ax + by + c = 0$

2.06 Finding the equation of a line

2.07 Applying linear functions

2.08 Equations of parallel lines • Equations of perpendicular lines

PhET interactives (4):

2.03 Graphing lines • Function builder: Basics • Function builder

2.04 Graphing slope-intercept

Quizzes (5):

- Wordbank 2
- SkillCheck 2
- Mental skills 2
- Language of maths 2
- Test yourself 2

Skillsheets (2):

SkillCheck Pythagoras' theorem

2.03 Graphing linear equations

Worksheets (11):

2.01 Gradient, midpoint and distance • Intervals match-up • Gradient between 2 points

2.03 A page of lines • A page of number planes • Graphing linear equations • Straight-line equations

2.05 Parallel and perpendicular lines

2.06 Finding the equation of a line • A page of lines

2.08 Writing equations of lines

Mind map Graphing lines

Puzzles (9):

2.01 Finding coordinates for given segment lengths

2.02 Gradients of parallel and perpendicular lines

2.04 Equation of a line • Equations in gradient form

2.05 Linear equations code puzzle

2.06 Straight-line equations

2.08 Linear equations match-up • Equations of parallel lines

Language of maths Graphing lines crossword

Technology (1):

2.01 Midpoint and distance between 2 points

Spreadsheets (2):

2.01 Midpoint and distance between 2 points

2.04 Drawing straight lines: $y = mx + c$

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

In this chapter you will:

- find the distance between 2 points on a number plane
- find the midpoint and gradient of an interval on a number plane
- find the properties of the gradients of parallel and perpendicular lines
- (PATH) find the x- and y-intercepts of a line from its equation
- graph a linear equation on a number plane
- test whether a point lies on a line
- use the gradient–intercept equation of a straight line $y = mx + c$
- (PATH) convert between the general form of a linear equation $ax + by + c = 0$ and the gradient–intercept equation
- find the equation of a line from its graph
- solve problems that can be modelled by linear equations
- find the equation of a line that is parallel or perpendicular to a given line

SkillCheck ANSWERS ON P. 577

Quiz
SkillCheck 2

Skillsheet
Pythagoras' theorem

1 For this number plane, find:

a the midpoint of interval BC

b the midpoint of interval HE

c the length of interval GC

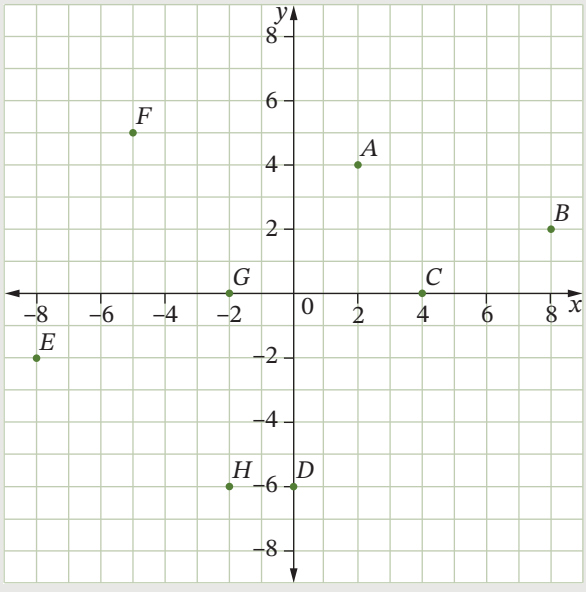
d the length of interval GH

e the lengths of AC and BC, correct to one decimal place

f the type of triangle $\triangle ABC$ is

g the gradient of GE

h the gradient of EH



2 For each linear equation, copy and complete the table of values and graph the equation.

a $y = x - 3$

x	0	1	2	3
y				

b $y = 3x + 2$

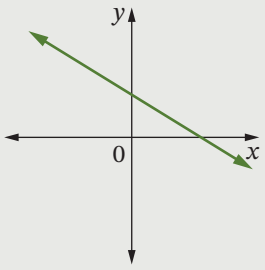
x	−2	−1	0	1
y				

c $y = 1 - 2x$

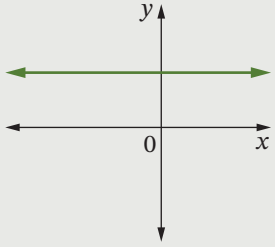
x	−1	0	1	2
y				

3 State whether each line's gradient is positive, negative or neither.

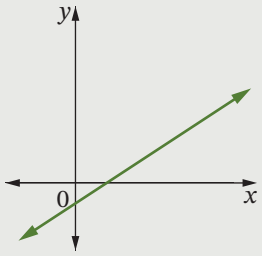
a



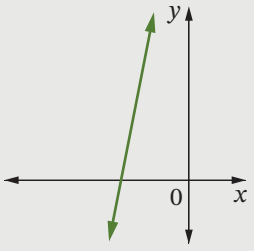
b



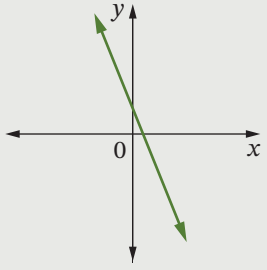
c



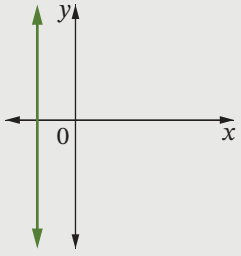
d



e



f



2.01 Length, midpoint and gradient of an interval

Worksheets
Gradient, midpoint and distance

Intervals match-up

Gradient between 2 points

Technology
Midpoint and distance between 2 points

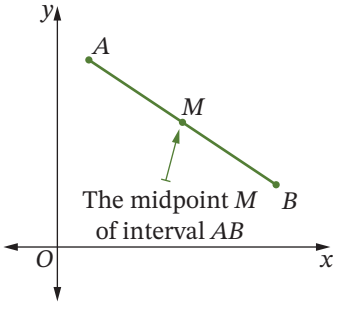
Spreadsheet
Midpoint and distance between 2 points

Video
The gradient of a line

The **length** of an interval AB (or the **distance** between A and B) can be calculated using Pythagoras' theorem if we know the coordinates of A and B.

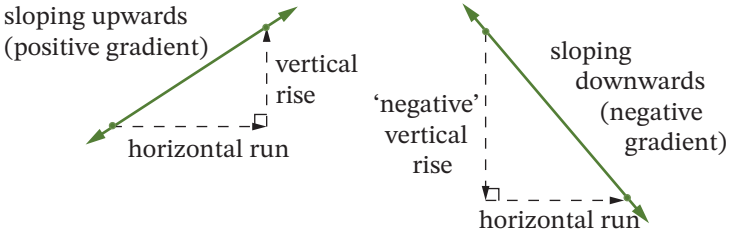
The **midpoint** of an interval AB is the point in the middle of AB or halfway between A and B.

- Its x-coordinate is the average of the x-coordinates of A and B.
- Its y-coordinate is the average of the y-coordinates of A and B.



The **gradient** of an interval measures its steepness. It is given by the formula:

$$m = \frac{\text{vertical rise}}{\text{horizontal run}} = \frac{\text{rise}}{\text{run}}$$



- A line **sloping upwards** has a **positive rise** and **a positive gradient**.
- A line **sloping downwards** has a **negative rise** and a **negative gradient**.
- The **run** is always **positive**.

Example 1

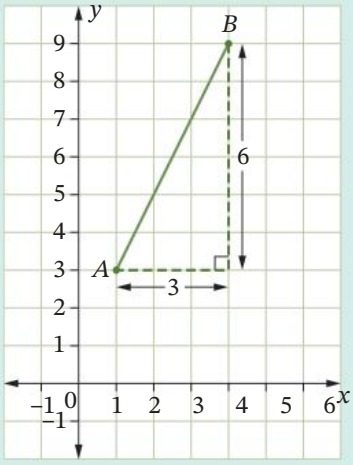
For the interval joining each pair of points given, find:

- **i** the length of the interval, correct to one decimal place
- **ii** the midpoint of the interval
- **iii** the gradient of the interval.

a $A(1, 3)$ and $B(4, 9)$ **b** $P(-5, 8)$ and $Q(3, 6)$

Solution

a **i** Draw a right-angled triangle on the number plane, with AB as the hypotenuse.



Height $= 9 - 3 = 6$ Difference between y-coordinates

Base $= 4 - 1 = 3$ Difference between x-coordinates

$AB^2 = 6^2 + 3^2$ by Pythagoras' theorem

$= 45$

$AB = \sqrt{45}$

$= 6.7082...$

≈ 6.7 units

ii For $A(1, 3)$ and $B(4, 9)$, the average of the x-coordinates is $\frac{1+4}{2} = 2\frac{1}{2}$.

The average of the y-coordinates is $\frac{3+9}{2} = 6$.

$\therefore$ The midpoint of AB is $\left(2\frac{1}{2}, 6\right)$.

From the diagram above, a midpoint at $\left(2\frac{1}{2}, 6\right)$ looks reasonable.

iii The rise is 6 units.

The run is 3 units.

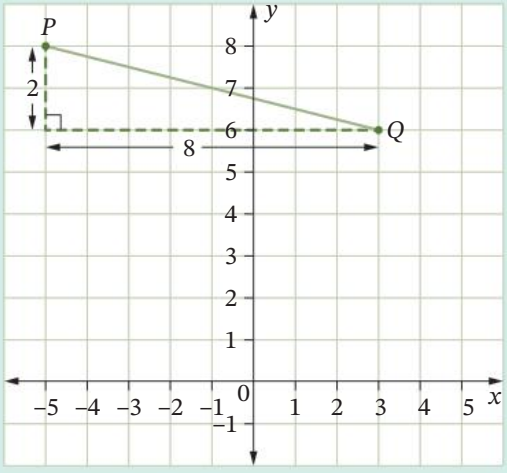
$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{6}{3}$$
$$= 2$$

The gradient is positive, the line is sloping upwards.

b i Draw a right-angled triangle on the number plane with PQ as the hypotenuse.

The height of the triangle is 2 units.

The base of the triangle is 8 units.



$PQ^2 = 2^2 + 8^2$ by Pythagoras' theorem

$= 68$

$PQ = \sqrt{68}$

$= 8.2462...$

≈ 8.2 units

ii For $P(-5, 8)$ and $Q(3, 6)$, the average of the x-coordinates is $\frac{-5+3}{2} = -1$.

From the diagram above, a midpoint at $(-1, 7)$ looks reasonable.

The average of the y-coordinates is $\frac{8+6}{2} = 7$.

$\therefore$ The midpoint of PQ is $(-1, 7)$.

iii The rise is -2 units. Line slopes downwards

The run is 8 units.

$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-2}{8}$$
$$= -\frac{1}{4}$$

The gradient is negative, the line slopes downward.

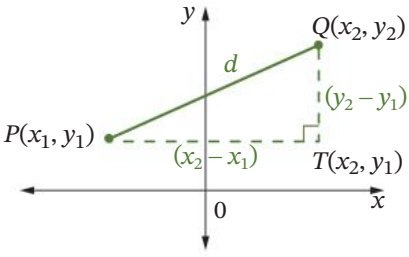
Optional: The distance, midpoint and gradient formulas

The methods for finding the length, midpoint and gradient of an interval can each be summarised by a formula.

The **distance formula** is used to calculate the distance (d) between 2 points $P(x_1, y_1)$ and $Q(x_2, y_2)$, that is, the length of the interval PQ.
By Pythagoras' theorem:

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

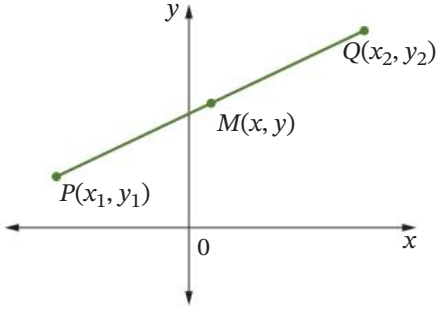
$\therefore d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



The **midpoint formula** gives the coordinates of the point M, the midpoint of the interval joining $P(x_1, y_1)$ and $Q(x_2, y_2)$:

$$M(x, y) \equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

We've used '≡' ('is identical to') rather than '=' because we are referring to the point (x, y), not a number.



The **gradient formula** gives the gradient of the interval or line joining $P(x_1, y_1)$ and $Q(x_2, y_2)$.

Vertical rise = difference in y-coordinates $= y_2 - y_1$

Horizontal run = difference in x-coordinates $= x_2 - x_1$

Gradient, $m = \dfrac{\text{difference in } y}{\text{difference in } x} = \dfrac{y_2 - y_1}{x_2 - x_1}$

Example 2

For the interval joining $P(-5, 8)$ and $Q(3, 6)$ from Example **1b**, use a formula to find:

a the length of the interval, correct to one decimal place

b the midpoint of the interval

c the gradient of the interval.

Solution

For $P(-5, 8)$ and $Q(3, 6)$: $x_1 = -5, y_1 = 8, x_2 = 3, y_2 = 6$.

(x_1, y_1) (x_2, y_2)

a $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ Apply the distance formula

$= \sqrt{(3 - (-5))^2 + (6 - 8)^2}$

$= \sqrt{68}$

$= 8.2462\ldots$

≈ 8.2 units

PATH

Video
Coordinate geometry

2.01

Video
Distance, midpoint and gradient formulas

Puzzle
Finding coordinates for given segment lengths

PATH

b $M(x, y) \equiv \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ — Apply the midpoint formula

$\equiv \left(\frac{-5+3}{2}, \frac{8+6}{2}\right)$

$\equiv (-1, 7)$

c $m = \frac{\text{difference in } y}{\text{difference in } x}$ — Apply the gradient formula

$= \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{6-8}{3-(-5)}$

$= \frac{-2}{8}$

$= -\frac{1}{4}$

Example 3

a Plot the points $A(0, 6)$, $B(5, 6)$, $C(5, 2)$ and $D(-4, 2)$ on a number plane and join them to make the quadrilateral $ABCD$.

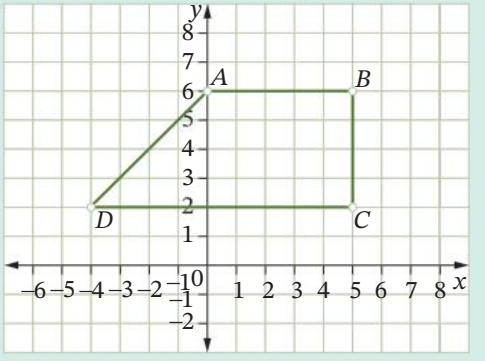
b What type of quadrilateral is $ABCD$?

c Find the exact length of AD.

d Hence find the perimeter of $ABCD$, correct to 2 decimal places.

Solution

a Join the points in the correct order



b Since $AB \parallel CD$, the quadrilateral is a trapezium.

c $AD^2 = 4^2 + 4^2$

$= 32$

$AD = \sqrt{32}$ units — In exact surd form

d By counting grid squares, $AB = 5$, $BC = 4$, $CD = 9$.

Perimeter of $ABCD = 5 + 4 + 9 + \sqrt{32}$

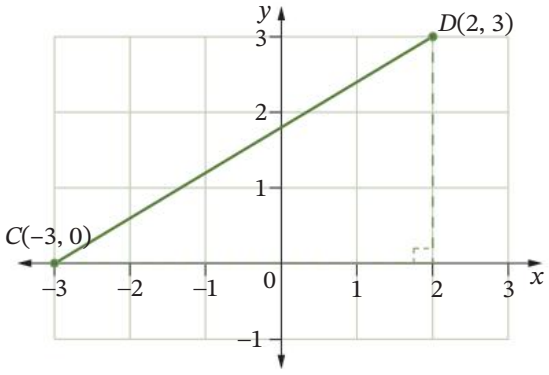
$= 23.656...$

≈ 23.66 units

EXERCISE 2.01 ANSWERS ON P. 578

Length, midpoint and gradient of an interval U F PS R C

Questions **1**, **2** and **3** refer to this diagram of interval *CD*.



1 What is the length of interval *CD*? Select the correct answer **A**, **B**, **C** or **D**.

A 2 units **B** 5.8 units **C** 3.2 units **D** 8 units

2 What is the midpoint of *CD*? Select **A**, **B**, **C** or **D**.

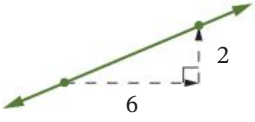
A $(-1, 3)$ **B** $(-5, 3)$ **C** $(-0.5, 1.5)$ **D** $(-2.5, 1.5)$

3 What is the gradient of *CD*? Select **A**, **B**, **C** or **D**.

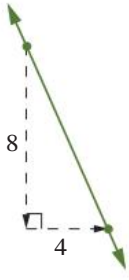
A $\frac{3}{5}$ **B** $-\frac{3}{5}$ **C** $-\frac{5}{3}$ **D** $\frac{5}{3}$

4 Calculate the gradient of each line.

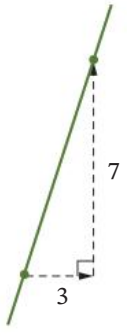
a



b



c



5 For the interval joining each pair of points given, find:

i the length of the interval, correct to one decimal place

ii the midpoint of the interval

iii the gradient of the interval.

a $A(5, 3)$ and $B(7, 2)$ **b** $J(-1, 0)$ and $K(8, 6)$ **c** $M(0, -3)$ and $N(-5, 2)$

d $R(-3, -6)$ and $S(4, -9)$ **e** $A(-7, 2)$ and $B(-5, -8)$ **f** $U(3, -2)$ and $V(7, 2)$

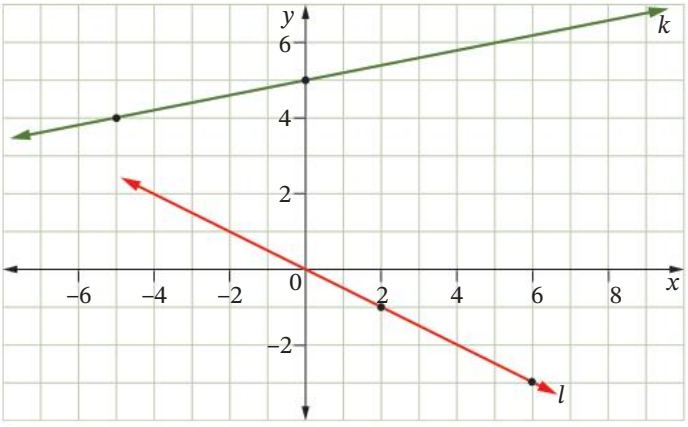
EXAMPLES 1, 2

Foundation Standard Complex

6 Calculate, in exact (surd) form, the distance between each pair of points.

a $(-8, -1)$ and $(0, 4)$ **b** $(12, -6)$ and $(-1, -1)$ **c** $(7, -2)$ and $(-2, -3)$

7 Find the gradient of the lines labelled k and l.



8 Which expression gives the y-coordinate of the midpoint of the interval joining points $(3, 8)$ and $(-1, 5)$? Select **A**, **B**, **C** or **D**.

A $\frac{-1+5}{2}$ **B** $\frac{8+5}{2}$ **C** $\frac{8-5}{2}$ **D** $\frac{5-8}{2}$

EXAMPLE 3

9 The vertices of triangle ABC are $A(-1, -1)$, $B(1, 3)$ and $C(3, 1)$. PS R C

a Draw $\triangle ABC$ on a number plane.

b Find the exact length of each side of the triangle.

c Are any sides of the triangle equal in length?

d What type of triangle is ABC?

e Find the perimeter of $\triangle ABC$, correct to one decimal place.

10 The vertices of quadrilateral $KLMP$ are $K(1, 6)$, $L(7, 2)$, $M(3, -4)$ and $P(-3, 0)$. PS R C

a Draw the quadrilateral on a number plane.

b What type of quadrilateral is $KLMP$?

c Find the gradients of sides KL and PM.

d Find the gradients of sides KP and LM.

e What do you notice about the gradients of the opposite sides of this quadrilateral? What does that mean about those sides?

f Find the exact length of each side of $KLMP$.

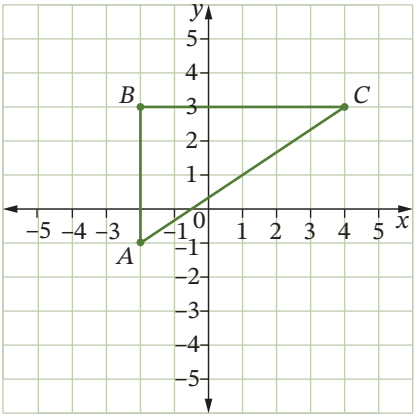
g Find the perimeter of $KLMP$, correct to one decimal place.

h Find the area of $KLMP$.

Foundation Standard Complex

9780170479622

11 This diagram shows a right-angled triangle with vertices $A(-2, -1)$, $B(-2, 3)$ and $C(4, 3)$. R C



a Copy the diagram and find the coordinates of P and Q, the midpoints of BA and BC respectively. Mark P and Q on your diagram.

b Calculate, correct to one decimal place, the lengths of PQ and AC. What do you notice about your answers?

c Find the gradients of PQ and AC. What do you notice about your answers?

Investigation

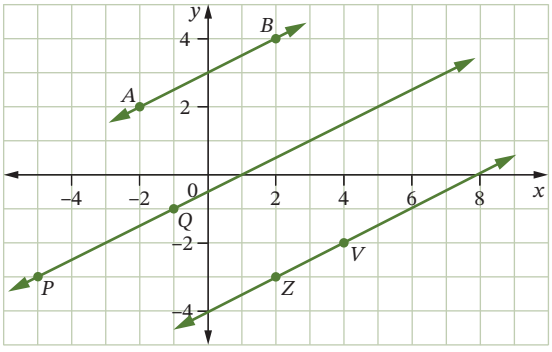
Parallel and perpendicular lines

1 These 3 lines are parallel. Calculate the gradient of:

a AB

b PQ

c ZV

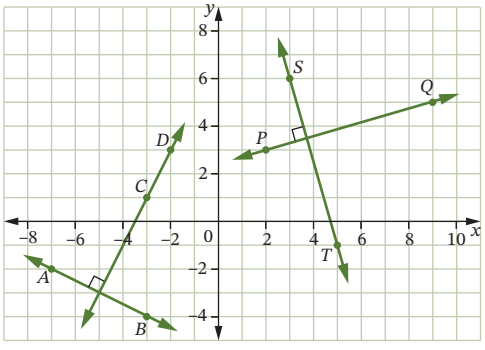


2 What can you conclude about the gradients of parallel lines?

3 This diagram shows 2 pairs of perpendicular lines. $AB \perp CD$ and $PQ \perp ST$. Calculate the gradient of:

a AB **b** CD

c PQ **d** ST



4 Is there a relationship between:

a the gradients of AB and CD? **b** the gradients of PQ and ST?

5 Calculate the product of:

a the gradients of AB and CD **b** the gradients of PQ and ST

6 What can you conclude about the gradients of perpendicular lines?

Foundation Standard Complex

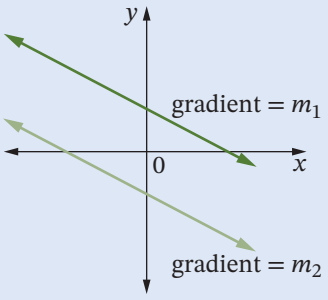
2.02 Parallel and perpendicular lines

Puzzle
Gradients of parallel and perpendicular lines

Parallel lines

Parallel lines have the same gradient.

If 2 lines with gradients m_1 and m_2 are parallel, then $m_1 = m_2$.





Rost9/Shutterstock.com

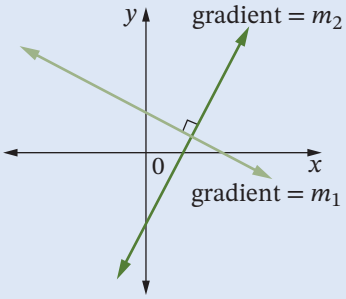
Perpendicular lines

Perpendicular lines have gradients with a product of -1.

If 2 lines with gradients m_1 and m_2 are perpendicular, then

$$m_1 \times m_2 = -1 \text{ or } m_2 = -\frac{1}{m_1}.$$

Note that m_2 is the **negative reciprocal** of m_1.



9780170479622

Example 4

State whether each pair of gradients represent parallel lines, perpendicular lines or neither.

a $m_1 = \frac{1}{2}, m_2 = 2$ **b** $m_1 = 0.4, m_2 = \frac{2}{5}$ **c** $m_1 = 1\frac{3}{5}, m_2 = -\frac{5}{8}$

Solution

a $m_1 \neq m_2$ so the lines are not parallel.

$$m_1 \times m_2 = \frac{1}{2} \times 2$$
$$= 1$$
$$\neq -1$$

so the lines are not perpendicular.

$\therefore$ The lines are neither parallel nor perpendicular.

b $m_2 = \frac{2}{5} = 0.4$

$m_1 = m_2$

$\therefore$ The lines are parallel.

c $m_1 = 1\frac{3}{5} = \frac{8}{5}$

$$m_1 \times m_2 = \frac{8}{5} \times \left(-\frac{5}{8}\right)$$
$$= -1$$

$\therefore$ The lines are perpendicular.

Example 5

Find the gradient of a line that is perpendicular to a line with gradient:

a 2 **b** -3 **c** $\frac{3}{4}$ **d** -0.6

Solution

a $m_1 = 2$

$m_2 = \frac{-1}{m_1}$ for perpendicular lines

$$= \frac{-1}{2}$$
$$= -\frac{1}{2}$$

The negative reciprocal of m_1.

The gradient is $-\frac{1}{2}$.

b $m_1 = -3$

$$m_2 = \frac{-1}{m_1}$$
$$= \frac{-1}{-3}$$
$$= \frac{1}{3}$$

The gradient is $\frac{1}{3}$.

c $m_1 = \frac{3}{4}$

$$m_2 = \frac{-1}{m_1}$$
$$= \frac{-1}{\left(\frac{3}{4}\right)}$$
$$= -\frac{4}{3}$$

The gradient is $-\frac{4}{3}$.

d $m_1 = -0.6 = -\frac{3}{5}$

$$m_2 = \frac{-1}{\left(-\frac{3}{5}\right)} = \frac{5}{3}$$

The gradient is $\frac{5}{3}$.

2.02

Video
Gradients of perpendicular lines

Example 6

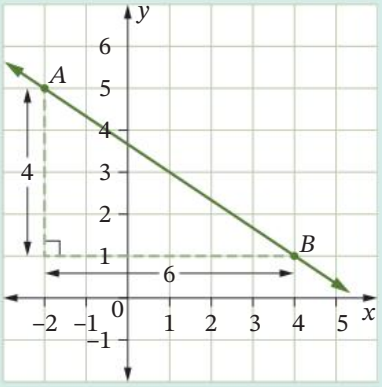
A line passes through the points $A(-2, 5)$ and $B(4, 1)$. What is the gradient of a line:

a parallel to AB? **b** perpendicular to AB?

Solution

Find the gradient of AB by calculating the rise and run.

Sometimes drawing a diagram helps you to understand the problem.



Rise $= 1 - 5 = -4$ Difference between y-coordinates

Run $= 4 - (-2) = 6$ Difference between x-coordinates

Gradient $AB = \frac{-4}{6} = -\frac{2}{3}$ $\frac{\text{rise}}{\text{run}}$

a Any line parallel to AB will have the same gradient as AB.

$\therefore m = -\frac{2}{3}$

b The gradient of a line perpendicular to AB will be given by:

$m = \frac{-1}{\left(-\frac{2}{3}\right)} = \frac{3}{2}$

EXERCISE 2.02 ANSWERS ON P. 578

Parallel and perpendicular lines U F

EXAMPLE 4

1 State whether each pair of gradients represent parallel lines, perpendicular lines or neither.

a $m_1 = \frac{1}{4}, m_2 = 4$ **b** $m_1 = 3, m_2 = -\frac{1}{3}$ **c** $m_1 = 0.5, m_2 = \frac{1}{2}$

d $m_1 = \frac{2}{7}, m_2 = \frac{7}{2}$ **e** $m_1 = \frac{3}{10}, m_2 = 0.3$ **f** $m_1 = 1\frac{1}{5}, m_2 = -\frac{6}{5}$

2 Find the gradient of a line that is parallel to a line with gradient:

a 4 **b** -2 **c** $\frac{1}{3}$ **d** -0.2

Foundation Standard Complex

3 Find the gradient of a line that is perpendicular to a line with gradient:

a 1 **b** -6 **c** -1.5 **d** $\frac{5}{2}$

EXAMPLE 5

4 What is the gradient of a line that is perpendicular to a line with a gradient of 0.8? Select the correct answer **A**, **B**, **C** or **D**.

A 0.2 **B** -0.2 **C** 1.25 **D** -1.25

5 What is the gradient of a line that is parallel to a line that goes through $P(0, 3)$ and $Q(5, -2)$? Select **A**, **B**, **C** or **D**.

A 1 **B** -1 **C** $\frac{1}{5}$ **D** $-\frac{1}{5}$

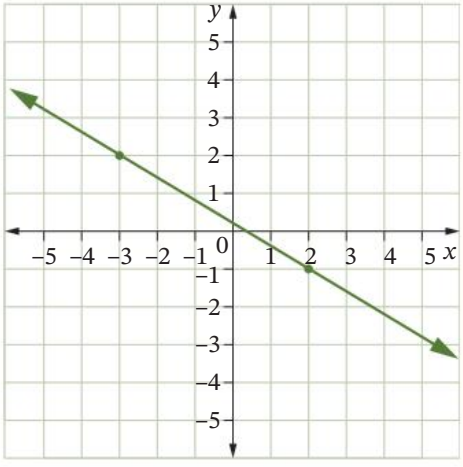
2.02

6 What is the gradient of a line perpendicular to the line shown? Select **A**, **B**, **C** or **D**.

A $\frac{5}{3}$ **B** -5

C $\frac{3}{5}$ **D** $\frac{1}{5}$

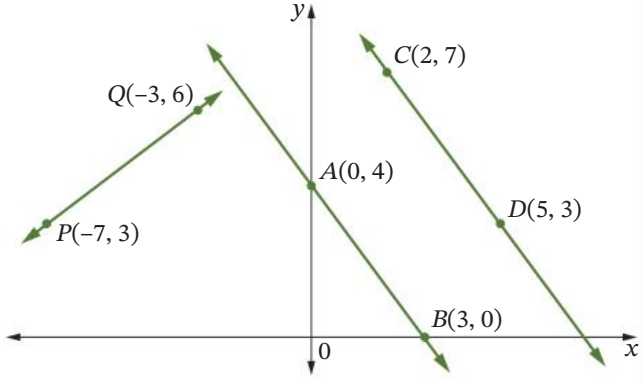
EXAMPLE 6



7 Calculate the gradient of each line shown and test whether:

a $AB \parallel CD$

b $PQ \perp CD$.



8 A line passes through the points $R(-5, 2)$ and $S(1, 4)$. What is the gradient of a line:

a parallel to RS?

b perpendicular to RS?

Foundation Standard Complex

Technology

Parallel and perpendicular lines

Use dynamic geometry software to find out if sets of linear equations represent parallel or perpendicular lines.

1 Parallel lines

Graph the following lines and use the **Slope/Gradient** function to find their gradients and check whether they are parallel using $m_1 = m_2$.

a $5x - 3y = 0$ and $y = \frac{5x}{3}$

b $x + y + 4 = 0$ and $x + y - 6 = 0$

c $x - 2y = 0$ and $y = 0.5x$

d $y = 5x - 9$ and $5x - y - 1 = 0$

2 Perpendicular Lines

Graph the following lines and use the **Slope/Gradient** function to find their gradients and check whether they are perpendicular using $m_1 \times m_2 = -1$.

a $y = 0.6x + 2$ and $y = \frac{5}{3}x$

b $x - 4y + 1 = 0$ and $y = -4x - 3$

c $3x - 2y = 0$ and $y = -\frac{2x}{3}$

d $y = 2x + 4$ and $x - 2y - 1 = 0$

2.03 Graphing linear equations

Video
Graphing linear equations

Interactives
Graphing lines
Function builder: Basics
Function builder

Worksheets
A page of lines
A page of number planes
Graphing linear equations

Skillsheet
Graphing linear equations

A relationship between 2 variables, x and y, whose graph is a straight line is called a **linear relationship**. The expression of that relationship as an algebraic formula, such as $y = 3x + 2$, is called a **linear equation**.

Example 7

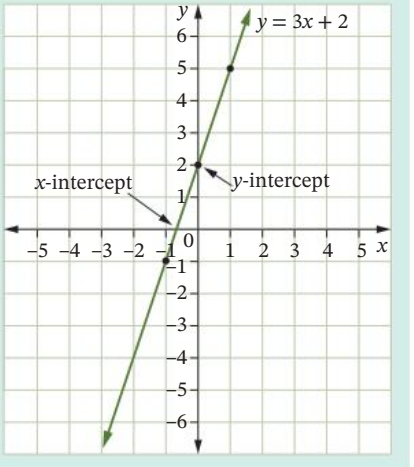
Graph $y = 3x + 2$ on a number plane and find its x- and y-intercepts.

Solution

Complete a table of values. Choose x-values close to 0 for easy calculation and graphing.

x	−1	0	1
y	−1	2	5

Graph (−1, −1), (0, 2) and (1, 5) on a number plane. Rule a straight line through the points, place arrows at each end, and label the line with its equation.



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The x-intercept is the x-value where the line cuts the x-axis and can be found by substituting $y = 0$ into the equation $y = 3x + 2$:

$0 = 3x + 2$

$3x = -2$

$x = -\frac{2}{3}$ which agrees with what's shown on the graph

The x-intercept is $-\frac{2}{3}$.

The y-intercept is the y-value where the line cuts the y-axis and can be found by looking at the graph, or looking at the constant term in the equation $y = 3x + 2$, or substituting $x = 0$ into the equation $y = 3x + 2$ (which has already been done for the table of values).

The y-intercept is 2.

In the above example, every point on the line follows the linear equation $y = 3x + 2$.
For example, $(-1, 1)$, $(0, 2)$ and $(1, 5)$ lie on the line and follow the rule $y = 3x + 2$.

There are an infinite number of points that follow the rule. Arrows on both ends of the line indicate that it has infinite length.

Testing if a point lies on a line

A point lies on a line if its (x, y) coordinates satisfy the equation of the line.



Video
Testing if a point lies on a line



Worksheet
Straight-line equations

Example 8

Test whether each point lies on the line $x - 2y = 5$.

a $(17, 6)$ **b** $(8, -4)$

Solution

- Separate the equation into its left-hand side (LHS) and right-hand side (RHS)
- Substitute the coordinates of the point into both sides
- If LHS = RHS, the point satisfies the equation and so lies on the line
- If LHS $\neq$ RHS, the point does not lie on the line.

a Substitute $x = 17, y = 6$ into $x - 2y = 5$.

LHS $= x - 2y$

$= 17 - 2 \times 6$

$= 5$

RHS $= 5$

LHS = RHS, so $(17, 6)$ lies on the line.

b Substitute $x = 8, y = -4$ into $x - 2y = 5$.

LHS $= x - 2y$

$= 8 - 2 \times (-4)$

$= 16$

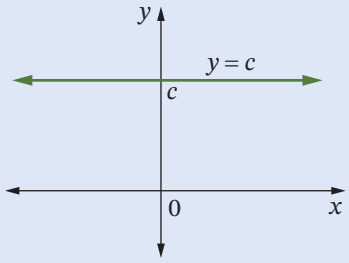
RHS $= 5$

LHS $\neq$ RHS, so $(8, -4)$ does not lie on the line.

Video
Horizontal and vertical lines

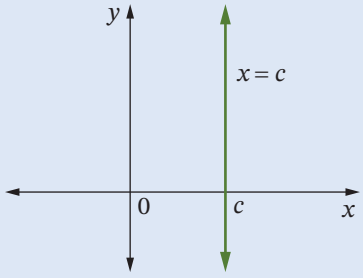
Horizontal and vertical lines

The **equation of a horizontal line** is of the form $y = c$ (where c is a constant number).



Note: The x-axis has equation $y = 0$.

The **equation of a vertical line** is of the form $x = c$ (where c is a constant number).



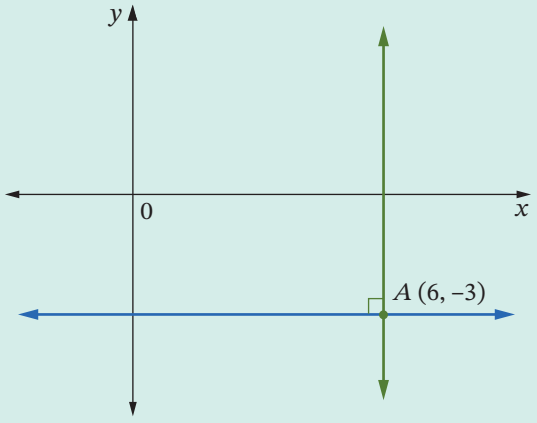
Note: The y-axis has equation $x = 0$.

Example 9

For the graph shown, find the equation of:

a the vertical line

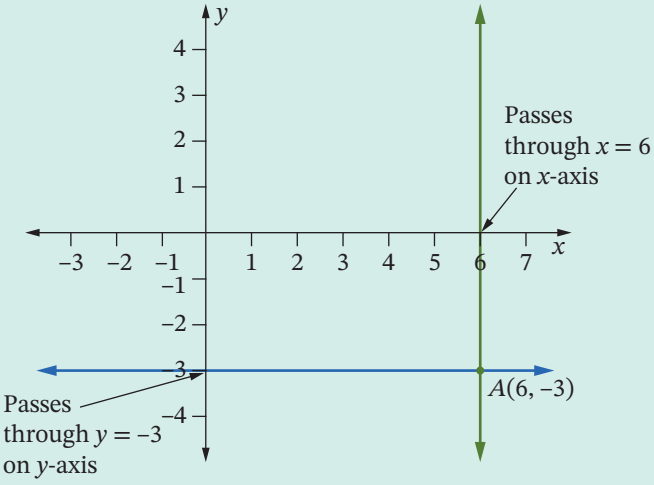
b the horizontal line



Solution

a The vertical line has an x-intercept of 6 and passes though $A(6, -3)$, so its equation is $x = 6$.

b The horizontal line has a y-intercept of -3 and passes though $A(6, -3)$, so its equation is $y = -3$.



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Example 10

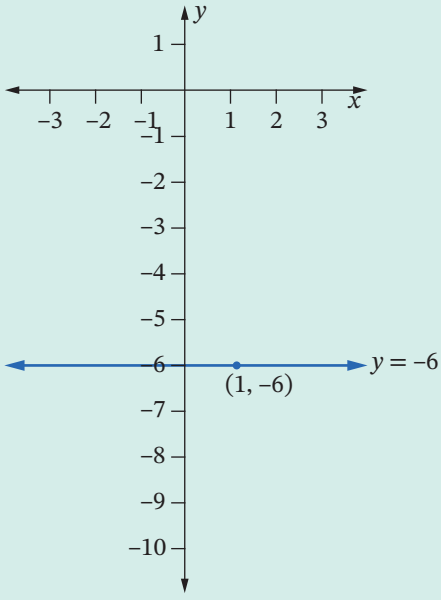
Find the equation of the line that is:

a parallel to the x-axis and passes through the point $(1, -6)$

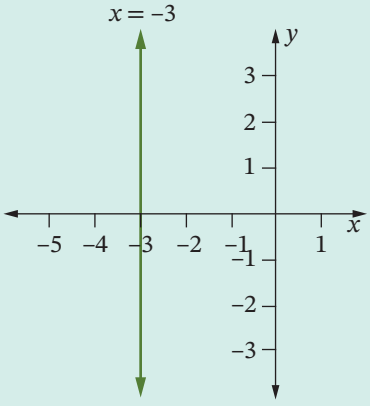
b always 3 units to the left of the y-axis.

Solution

a The equation of the line is $y = -6$.



b The equation of the line is $x = -3$.



EXERCISE 2.03 ANSWERS ON P. 578

Graphing linear equations UFRC

1 Graph each linear equation on a number plane, and find:

i its x-intercept **ii** its y-intercept.

a $y = 3x - 1$ **b** $y = 2x + 5$ **c** $y = -x + 4$

d $y = -2x - 2$ **e** $y = 4x$ **f** $y = \frac{x}{2} + 3$

EXAMPLE 7

2 Test whether the point $(3, -1)$ lies on each line.

a $y = 2x - 5$ **b** $x - y = 4$ **c** $y + 2x = 5$

d $y = x - 4$ **e** $x + y = 5$ **f** $3x + y + 8 = 0$

EXAMPLE 8

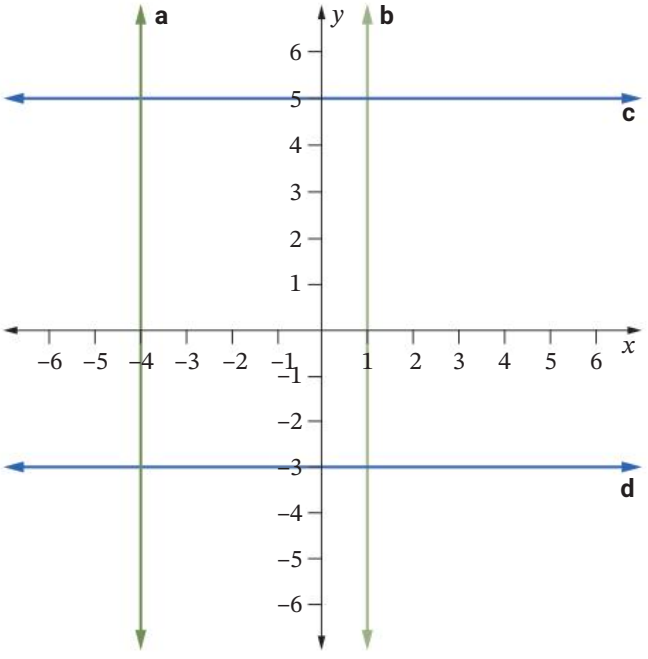
3 Which point lies on the line $y = 6x - 5$? Select the correct answer **A**, **B**, **C** or **D**.

A $(-1, 11)$ **B** $(3, -13)$ **C** $(-2, -17)$ **D** $(-5, 25)$

Foundation Standard Complex

EXAMPLE 9

4 Find the equation of each line shown.



5 Graph each set of lines on a number plane.

a $x = 2\frac{1}{2},\ y = -3,\ y = 1$ **b** $x = 6,\ y = -2,\ x = -\frac{1}{2}$

EXAMPLE 10

6 Find the equation of the line that is: R C

a horizontal and passes through the y-axis at 2

b vertical with an x-intercept of 4

c parallel to the y-axis and passes through the point $(-1, 4)$

d parallel to the x-axis and passes through the point $(0, -2)$

e 3 units above the x-axis

f 1 unit to the left of the y-axis

g drawn through the points $(-1, 6)$ and $(2, 6)$

h drawn through the points $(-1, 8)$ and $(-1, 2)$.

7 Which point lies on the line $4x + y = 1$? Select **A**, **B**, **C** or **D**.

A $(-1, 5)$ **B** $(-2, 7)$ **C** $(6, 9)$ **D** $\left(-\frac{1}{2}, 1\right)$

8 Which line is horizontal and passes through the point $(8, -2)$? Select **A**, **B**, **C** or **D**.

A $y = 8$ **B** $x = 8$ **C** $y = -2$ **D** $x = -2$

9 **a** What is another name for the line $y = 0$? C

b What is another name for the line $x = 0$? C

Foundation Standard Complex

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Technology

Graphing $y = mx + c$

1 Use dynamic geometry software to graph the 4 lines $y = 3x + 2$, $y = 5x + 2$, $y = -2x + 2$ and $y = -0.1x + 2$. Make each line a different colour.

2 Find the gradient (slope) and y-intercept of each line and record your results in a table like this.

Equation	Gradient	y-intercept

3 What do you notice about your results?

4 Repeat the steps above for each set of equations.

a $y = -4x$ $y = -4x + 1$ $y = -4x - 10$

b $y = 2x + 3$ $y = -7x + 3$ $y = 0.2x + 3$

c $y = -x - 1$ $y = -x + 2$ $y = -x - 4$

5 For each set of lines drawn in question **4**, complete a table as shown in Step **2**.

6 What do you notice about each set of lines? Identify any key features of each set of graphs.

2.04 The gradient–intercept equation $y = mx + c$

Videos
Graphing $y = mx + c$
Gradient and y-intercept of a line
The gradient-intercept formula

The gradient–intercept form of a linear equation

The equation of a straight line is $y = mx + c$, where m is the **gradient** and c is the **y-intercept**.

For this reason, $y = mx + c$ is also called the **gradient–intercept form** of a linear equation.

Interactive
Graphing slope-intercept

Puzzles
Equation of a line
Equations in gradient form

Spreadsheet
Drawing straight lines: $y = mx + c$

Example 11

Find the gradient and y-intercept of the line with equation:

a $y = -4x + 9$ **b** $y = 10 - 6x$ **c** $y = \frac{5x+4}{2}$ **d** $3x + 2y - 6 = 0$

Solution

a $y = -4x + 9$ is in the form $y = mx + c$.

$\therefore$ Gradient $m = -4$ and y-intercept $c = 9$.

b $y = 10 - 6x$ can be rewritten as $y = -6x + 10$.

$\therefore$ Gradient $m = -6$ and y-intercept $c = 10$.

c $y = \frac{5x+4}{2}$ can be rewritten as $y = \frac{5x}{2} + \frac{4}{2} = \frac{5x}{2} + 2$.

$\therefore$ Gradient $m = \frac{5}{2}$ and y-intercept $c = 2$.

d $3x + 2y - 6 = 0$ can be rearranged in the form $y = mx + c$.

Make y the **subject** of the equation: 'solve' the equation for y.

$$3x + 2y - 6 = 0$$
$$3x + 2y - 6 - 3x = 0 - 3x$$
$$2y - 6 = -3x$$
$$2y - 6 + 6 = -3x + 6$$
$$2y = -3x + 6$$
$$\frac{2y}{2} = \frac{-3x+6}{2}$$
$$y = \frac{-3x}{2} + 3$$

$\therefore$ Gradient $m = -\frac{3}{2}$ and y-intercept $c = 3$.

PATH

Finding the x- and y-intercepts of a line

We can find the x- and y-intercepts of a line from its equation.

Since any point on the x-axis has a y-coordinate of 0, we can substitute $y = 0$ into the equation to find the x-intercept.

Similarly, any point on the y-axis has an x-coordinate of 0, so we can substitute $x = 0$ into the equation to find the y-intercept.

Finding the x- and y-intercepts of a line

- To find the x-intercept, substitute $y = 0$ and solve the equation
- To find the y-intercept, substitute $x = 0$ and solve the equation

Example 12

Find the x- and y-intercepts of each line.

a $y = 3x - 5$ **b** $2x + y - 8 = 0$

Solution

a For x-intercept, substitute $y = 0$:

$0 = 3x - 5$

$5 = 3x$

$x = \frac{5}{3}$

For y-intercept, substitute $x = 0$:

$y = 3(0) - 5$

$y = -5$

Also, y-intercept for $y = 3x - 5$ must be $c = -5$.

b For x-intercept, substitute $y = 0$:

$2x + 0 - 8 = 0$

$2x = 8$

$x = 4$

For y-intercept, substitute $x = 0$:

$2(0) + y - 8 = 0$

$y = 8$

Example 13

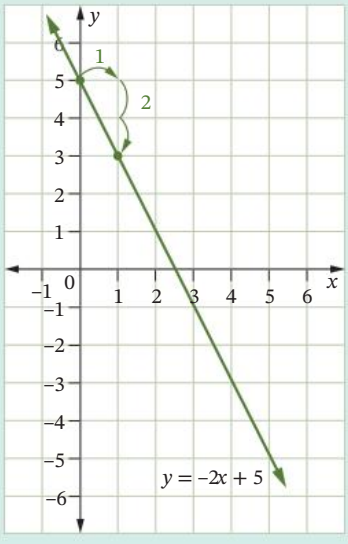
Graph each linear equation by finding the gradient and y-intercept first.

a $y = -2x + 5$ **b** $y = \frac{3}{4}x - 2$

Solution

a $y = -2x + 5$ has a gradient of -2 and a y-intercept of 5.

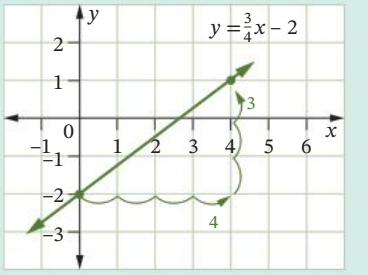
- Plot the y-intercept 5 on the y-axis.
- Make a gradient of $-2 = \frac{-2}{1}$ by moving **across** 1 unit (**run**) and **down** 2 units (**'negative' rise**) and marking the point at (1, 3).
- Rule a line through this point and the y-intercept.



Don't forget to label the line with its equation '$y = -2x + 5$'

b $y=\frac{3}{4}x-2$ has a gradient of $\frac{3}{4}$ and a y-intercept of -2.

- Plot the y-intercept -2 on the y-axis.
- Make a gradient of $\frac{3}{4}$ by moving **across** 4 units (**run**) and **up** 3 units (**rise**) and marking the point at (4, 1).
- Rule a line through this point and the y-intercept.



Example 14

Test whether each line is parallel to $y = -2x + 3$.

a $y = 2x + 3$ **b** $y = -2x + 1$ **c** $y = -2x$ **d** $y = 5x + 3$

Solution

Parallel lines have the **same gradient**. The line $y = -2x + 3$ has the gradient $m = -2$.

a $y = 2x + 3$ has a different gradient, 2, so it is not parallel to $y = -2x + 3$.

b $y = -2x + 1$ has the same gradient, -2, so it is parallel to $y = -2x + 3$.

c $y = -2x$ has the same gradient, -2, so it is parallel to $y = -2x + 3$.

d $y = 5x + 3$ has a different gradient, 5, so it is not parallel to $y = -2x + 3$.

$\therefore$ The lines **b** $y = -2x + 1$ and **c** $y = -2x$ are parallel to $y = -2x + 3$.

EXERCISE 2.04 ANSWERS ON P. 579

The gradient–intercept equation $y = mx + c$ U F R

EXAMPLE 11

1 Find the gradient and y-intercept of each line.

a $y = 3x - 2$ **b** $y = -2x + 7$ **c** $y = x + 4$ **d** $y = 9 - x$

e $y=\frac{3x}{4}+6$ **f** $y = x$ **g** $y=\frac{x}{2}-11$ **h** $y=\frac{2x+18}{3}$

i $y=\frac{-24-x}{3}$ **j** $y = 2(x - 3)$ **k** $11 - 3x = y$ **l** $\frac{2x-7}{2}=y$

PATH

EXAMPLE 12

2 Find the x- and y-intercepts of each line in Question **1**.

3 Find the equation of a line with:

a a gradient of 2 and a y-intercept of 1 **b** a gradient of $\frac{3}{4}$ and a y-intercept of 2

c a gradient of -7 and a y-intercept of 5 **d** a gradient of $-\frac{2}{5}$ and a y-intercept of 3

e $m = -2, c = -3$ **f** $m = -3, c = \frac{1}{2}$

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4 Graph each linear equation by finding the gradient and y-intercept first. EXAMPLE 13

a $y = 2x + 1$ **b** $y = 3x - 2$ **c** $y = 2x$ **d** $y = \frac{x}{2} - 1$

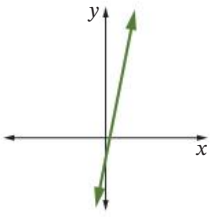
e $y = -2x + 3$ **f** $y = -\frac{3x}{4}$ **g** $y = \frac{-5x+2}{2}$ **h** $y = \frac{3x-20}{5}$

5 Write the equation of a line with a gradient of 2 that passes through the origin. R

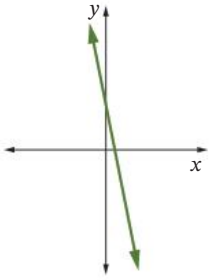
6 Match each equation to its graph below. R

a $y = 4x + 1$ **b** $y = -4x + 1$ **c** $y = -4x - 1$ **d** $y = 4x - 1$

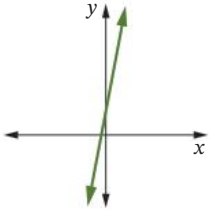
A



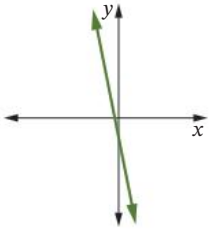
B



C



D



7 For each given line, select the lines that are parallel. There may be more than one answer. R EXAMPLE 14

a $y = x + 6$

A $y = 6x$ **B** $y = 6 - x$ **C** $y = x + 1$ **D** $y = 2x$

b $y = 3x + 10$

A $y = 10x + 3$ **B** $y = 3x - 1$ **C** $y = 1 - 3x$ **D** $y = 4 + 3x$

c $y = \frac{x}{2} + 5$

A $y = 2x - 1$ **B** $y = \frac{x+6}{2}$ **C** $y = 1 - \frac{x}{2}$ **D** $y = x + 2$

d $y = 6$

A $y = 2x + 6$ **B** $y = 6x$ **C** $y = -1$ **D** $y = 10$

e $y = 4x$

A $y = 4x - 2$ **B** $y = 4x + 3$ **C** $y = 4$ **D** $y = 1 - 4x$

f $x = 10$

A $y = 10$ **B** $y = 10x$ **C** $x = 2y$ **D** $x = -6$

8 For each set of linear equations, find a pair of equations whose graphs are parallel lines. R

a $y = 4x + 3$ $y = x + 2$ $y = 4x - 6$ $y = 2x$

b $y = 5x + 1$ $3x - y + 7 = 0$ $y = 3x - 2$ $y = -5x + 2$

Foundation Standard Complex

Quiz
Mental skills 2

Mental skills 2: Maths without calculators ANSWERS ON P. 580

Finding 15%, $2\frac{1}{2}$%, 25% and $12\frac{1}{2}$%

- To find 10% or $\frac{1}{10}$ of a number, divide by 10
- To find 5% of a number, find 10% first, then halve it (since 5% is half of 10%).
- So to find 15% of a number, find 10% and 5% of the number separately, then add the answers together.

1 Study each example.

a $15\% \times 80 = (10\% \times 80) + (5\% \times 80) = 8 + 4 = 12$

b $15\% \times \$170 = (10\% \times \$170) + (5\% \times \$170) = \$17 + \$8.50 = \$25.50$

c $15\% \times 3600 = (10\% \times 3600) + (5\% \times 3600) = 360 + 180 = 540$

d $15\% \times \$28 = (10\% \times \$28) + (5\% \times \$28) = \$2.80 + \$1.40 = \$4.20$

2 Now find 15% of each amount.

a	120	**b**	\$840	**c**	260	**d**	\$202
e	\$50	**f**	72	**g**	\$180	**h**	400
i	\$1600	**j**	\$22	**k**	6000	**l**	\$350

To find $2\frac{1}{2}$% of a number, first find 5%, then halve it.

3 Study each example.

a $2\frac{1}{2}\% \times 600$

$10\% \times 600 = 60$

$5\% \times 600 = \frac{1}{2} \times 60 = 30$

$2\frac{1}{2}\% \times 600 = \frac{1}{2} \times 30 = 15$

b $2\frac{1}{2}\% \times \$820$

$10\% \times \$820 = \$82$

$5\% \times \$820 = \frac{1}{2} \times 82 = \$41$

$2\frac{1}{2}\% \times \$820 = \frac{1}{2} \times \$41 = \$20.50$

4 Now find $2\frac{1}{2}$% of each amount

a	400	**b**	6640	**c**	\$2000	**d**	\$880
e	1500	**f**	\$232	**g**	5400	**h**	\$904

To find 25% of a number, halve the number twice as $25\% = \frac{1}{4}$.

5 Study each example.

a $25\% \times 700$

$50\% \times 700 = \frac{1}{2} \times 700 = 350$

$25\% \times 700 = \frac{1}{2} \times 350 = 175$

b $25\% \times \$86$

$50\% \times \$86 = \frac{1}{2} \times \$86 = \$43$

$\therefore 25\% \times \$86 = \frac{1}{2} \times \$43 = \$21.50$

6 Now find 25% of each amount.

a	2000	**b**	\$80	**c**	18	**d**	\$25
e	\$324	**f**	\$140	**g**	66	**h**	298
i	\$780	**j**	\$1700	**k**	\$126	**l**	1160

To find $12\frac{1}{2}\%$ of a number, find 25% first, then halve it. In other words, halve 3 times because $12\frac{1}{2}\% = \frac{1}{8}$.

7 Study each example.

a $12\frac{1}{2}\% \times 400$

$50\% \times 400 = \frac{1}{2} \times 400 = 200$

$25\% \times 400 = \frac{1}{2} \times 200 = 100$

$12\frac{1}{2}\% \times 400 = \frac{1}{2} \times 100 = 50$

b $12\frac{1}{2}\% \times \$144$

$50\% \times \$144 = \frac{1}{2} \times \$144 = \$72$

$25\% \times \$144 = \frac{1}{2} \times \$72 = \$36$

$12\frac{1}{2}\% \times \$144 = \frac{1}{2} \times \$36 = \$18$

8 Now find $12\frac{1}{2}\%$ of each amount.

a	1280	**b**	\$12	**c**	60	**d**	\$260
e	\$540	**f**	\$250	**g**	304	**h**	1360

The general equation $ax + by + c = 0$

2.05

A linear equation written in **gradient–intercept form**, such as $y = -\frac{3}{4}x + 2$, can also be written in **general form** ($3x + 4y - 8 = 0$). Note that, for the general form, all of the terms on the left-hand side of the equation are written with no fractions, and only 0 is on the right-hand side. Sometimes the general form is neater and more convenient.

PATH



Worksheet
Parallel and perpendicular lines

The general form of a linear equation

The **general form of a linear equation** is written as $ax + by + c = 0$, where a, b and c are integers and a is positive.

PATH

Video
The general equation $ax + by + c = 0$

Puzzle
Linear equations code puzzle

Example 15

Write each linear equation in general form.

a $y = 6x + 2$ **b** $y = -\frac{2}{3}x + 2$ **c** $y = 2x - \frac{3}{5}$

Solution

a $y = 6x + 2$

$0 = 6x - y + 2$ Subtracting y from both sides

$6x - y + 2 = 0$ Swapping sides so that 0 appears on the RHS

b $y = -\frac{2}{3}x + 2$

$3y = 3\left(-\frac{2}{3}x + 2\right)$ Multiplying both sides by 3 to remove the fraction

$3y = -2x + 6$

$2x + 3y = 6$ Adding $2x$ to both sides

$2x + 3y - 6 = 0$ Subtracting 6 from both sides

c $y = 2x - \frac{3}{5}$

$5y = 5\left(2x - \frac{3}{5}\right)$ Multiplying both sides by 5

$5y = 10x - 3$

$0 = 10x - 5y - 3$ Subtracting $5y$ from both sides

$10x - 5y - 3 = 0$ Swapping sides so that 0 appears on the RHS

Example 16

Find the gradient and y-intercept of the line whose equation is $5x + 2y - 10 = 0$.

Solution

Rewrite $5x + 2y - 10$ in the form $y = mx + c$.

Make y the subject, solve for y.

$5x + 2y - 10 = 0$

$2y - 10 = -5x$ Subtracting $5x$ from both sides

$2y = -5x + 10$ Adding 10 to both sides

$y = \frac{-5x + 10}{2}$ Dividing both sides by 2

$y = \frac{-5x}{2} + 5$

Aim to have y on its own on the LHS of the equation.

$\therefore$ Gradient: $m = -\frac{5}{2}$

y-intercept: $c = 5$

Example 17

Find the x- and y-intercepts of each line.

a $3x - 2y + 12 = 0$

b $x + 3y - 7 = 0$

Solution

a For x-intercept, substitute $y = 0$:

$3x - 2(0) + 12 = 0$

$3x = -12$

$x = -4$

For y-intercept, substitute $x = 0$:

$3(0) - 2y + 12 = 0$

$-2y = -12$

$y = 6$

b For x-intercept, substitute $y = 0$:

$x + 3(0) - 7 = 0$

$x = 7$

For y-intercept, substitute $x = 0$:

$0 + 3y - 7 = 0$

$3y = 7$

$y = \frac{7}{3}$

EXERCISE 2.05 ANSWERS ON P. 580

The general equation $ax + by + c = 0$ UFRC

1 Write each linear equation in general form. R C

EXAMPLE 15

a $y = x + 2$ **b** $y = 3x - 1$ **c** $y = 8 + 5x$

d $x + 2y = 3$ **e** $x - 2y = 6$ **f** $y = 8x + 2$

g $y + 3 = 6x$ **h** $2y = x - 6$ **i** $y = \frac{3}{5}x + 2$

2 Find the gradient and y-intercept of the line with equation: R C

EXAMPLE 16

a $2x + y = 6$ **b** $8x - 2y = 10$ **c** $3x - 2y + 4 = 0$

d $y + 2x - 1 = 0$ **e** $2x + y + 5 = 0$ **f** $4x + 3y - 12 = 0$

3 Find the gradient, m, and the y-intercept, c, of the line with equation $x - 3y + 5 = 0$. Select the correct answer **A**, **B**, **C** or **D**.

A $m = -1, c = 5$ **B** $m = \frac{1}{3}, c = \frac{5}{3}$ **C** $m = 1, c = -5$ **D** $m = \frac{1}{3}, c = -\frac{5}{3}$

4 Find the x- and y-intercepts of each line in Question **2**.

EXAMPLE 17

5 Which statement is FALSE about the line whose equation is $3x + y - 6 = 0$? Select **A**, **B**, **C** or **D**. R C

A Its gradient is -3.

B Its y-intercept is -6.

C Its x-intercept is 2.

D It is parallel to the line $y = -3x$.

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2.06 Finding the equation of a line

Video
Finding the equation of a line

Worksheets
Finding the equation of a line

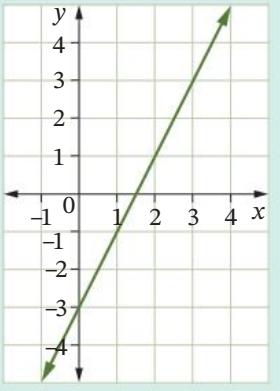
A page of lines

Puzzle
Straight-line equations

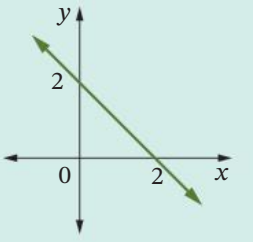
Example 18

Find the equation of each line.

a



b



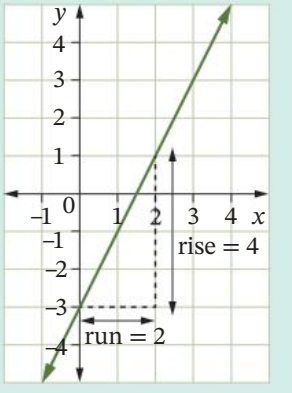
Solution

a Select 2 points on the line to find the gradient, say (0, −3) and (2, 1).

Gradient: $m = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$ from the graph

y-intercept: $c = -3$

∴ The equation of the line is $y = 2x - 3$. $y = mx + c$



We can check that this equation is correct for any point on the line, say (3, 3).

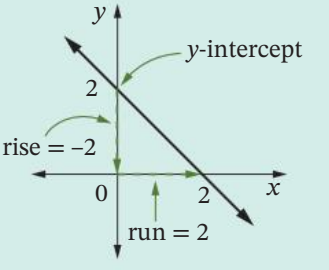
When $x = 3$, $y = 2 \times 3 - 3 = 3$.

b Find the gradient of the line passing through (0, 2) and (2, 0).

Gradient: $m = \frac{\text{rise}}{\text{run}} = \frac{-2}{2} = -1$ from the graph

y-intercept: $c = 2$

∴ The equation of line is $y = -x + 2$. $y = mx + c$

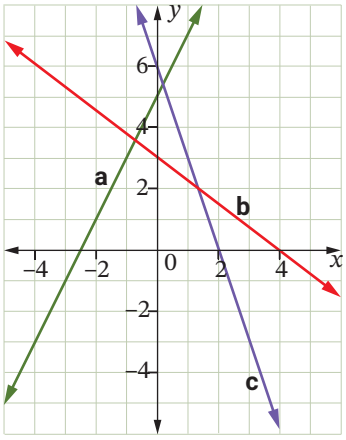


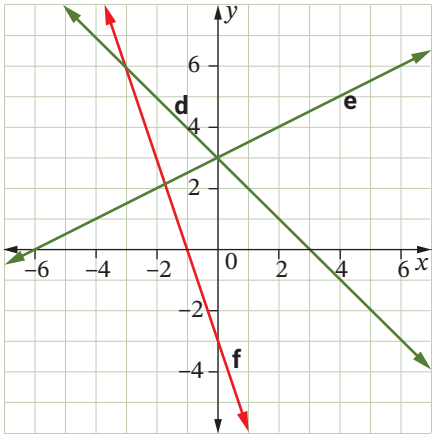
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EXERCISE 2.06 ANSWERS ON P. 580

Finding the equation of a line U F R C

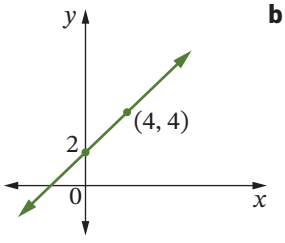
1 Find the equation of each line. R C



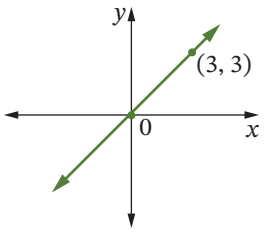


2 Find the equation of each line. R C

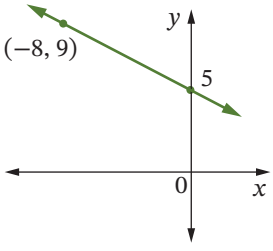
a



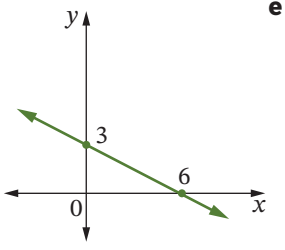
b



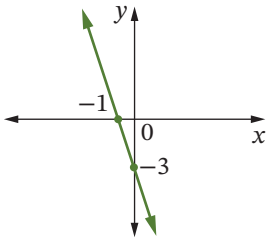
c



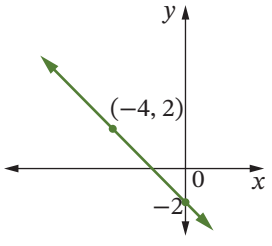
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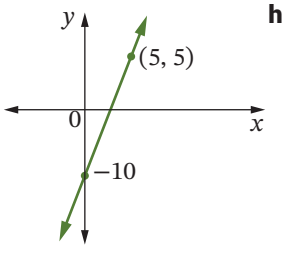
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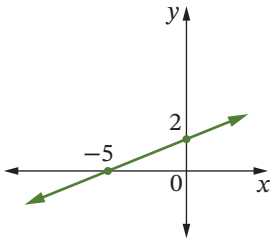
f



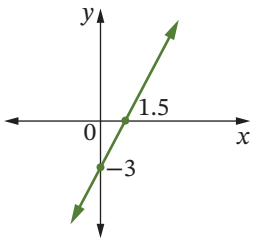
g



h



i

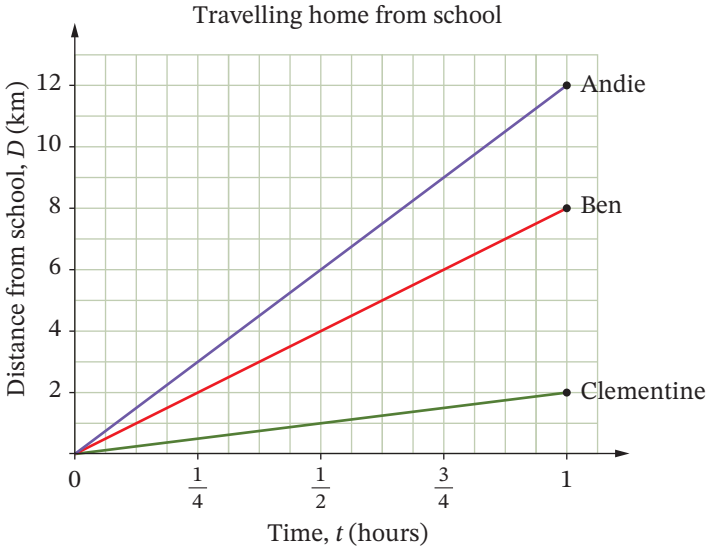


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Investigation

Travelling home

Andie, Ben and Clementine live in a rural area and travel to their nearest school in different ways. This distance-time graph shows their trips home after school.



1. Calculate the gradient of each line.
2. What does the gradient of each line represent?
3. How long did the 3 people take to travel home?
4. What was the speed of each person?
5. They travelled home by walking, jogging or cycling. Which person did what?

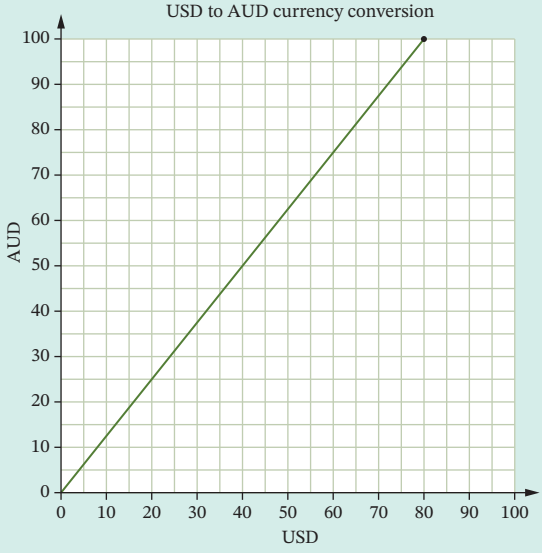
2.07 Applying linear equations

Video
Applying linear functions

Linear equations can be used to model real-life situations such as the cost of a taxi trip or the exchange rate of currencies between 2 countries or the amount of water in a swimming pool when it is being drained. Instead of using x and y as our variables in the linear equation $y = mx + c$, we can use other letters of the alphabet to represent quantities.

Example 19

This conversion graph shows the exchange rate between USD (US dollars) and AUD (Australian dollars)



a Find the gradient and vertical intercept of the line.

b What does the gradient represent?

c Write the equation of the line in the form $A = mU + c$ using U for USD and A for AUD.

d Use the graph to convert:

i 60 USD to AUD

ii 25 AUD to USD

Solution

a Line goes from (0, 0) to (80, 100).

Gradient $= \frac{100-0}{80-0} = 1.25$

Vertical intercept $= 0$ (from graph)

Vertical intercept is the general name for the y-intercept when a variable other than y is used.

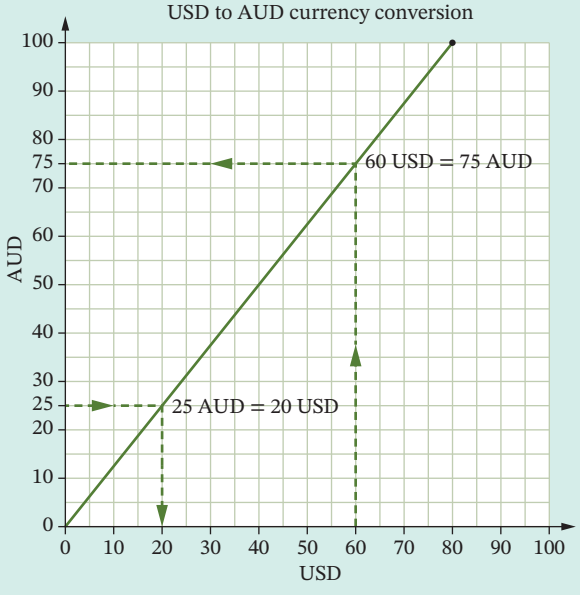
b The gradient, 1.25, represents the exchange rate between USD and AUD (1 USD = 1.25 AUD)

c $A = mU + c$ and we know that $m = 1.25$ and $c = 0$.

We're replacing x with U because U is on the horizontal (x) axis and y with A because A is on the vertical (y) axis.

The equation of the line is $A = 1.25U + 0$, which simplifies to $A = 1.25U$.

d Using the graph:



i 60 USD = 75 AUD

ii 25 AUD = 20 USD

Applying linear equations

- The **gradient** represents the **rate of increase or decrease** in the situation.
- For an increase, the gradient is positive and for a decrease, the gradient is negative.
- The *y*-**intercept** or **vertical intercept** represents the **starting value** in the situation.

EXERCISE 2.07 ANSWERS ON P. 580

Applying linear equations U F R C

The questions in this exercise may also be completed with the aid of a spreadsheet and graphing software.

1 Write an equation in the form $y = mx + c$ using the given change of variables and values. C

a $y \to D, x \to h, m = 30, c = 5$

b $y \to R, x \to t, m = 0.5, c = -8$

c $y \to P, x \to n, m = -50, c = 0$

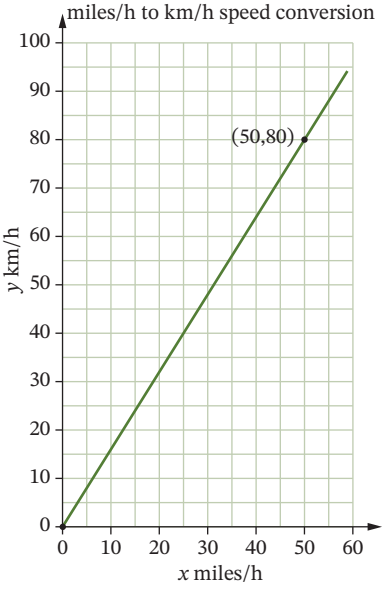
d $y \to F, x \to C, m = 1.8, c = 32$

e $y \to H, x \to w, m = 0.8, c = 0$

f $y \to V, x \to T, m = -40, c = 200$

EXAMPLE 19

2 This conversion graph converts between speeds in miles per hour to km/h. R C



a Use the graph to find what speed in km/h is equivalent to 25 miles/h.

b Find the gradient and vertical intercept of the line.

c What does the gradient represent?

d Find the equation of the line.

3 The distance d km travelled by a car moving at a constant speed of 80 km/h after t hours is given by the equation $d = 80t$. R C

a Copy and complete this table of values using the equation, then graph the equation on a number plane.

Time, *t* hours	0	1	2	3	4
Distance, *d* km					

b Find the gradient of the line.

c What does the gradient represent?

d Use the graph to find:

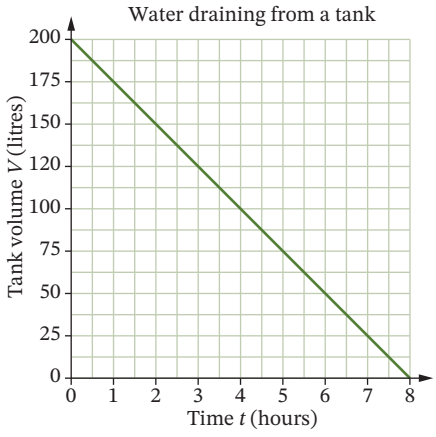
i the distance travelled by the car after $5\frac{1}{2}$ hours

ii the time taken for the car to travel 40 km

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4 Water is draining out of a tank. The volume of water in the tank is shown by this graph. R C



a What is the gradient and vertical intercept of the line?

b What does the vertical intercept represent?

c What does the gradient represent?

d Why is the gradient negative?

e Find the equation of the line using the variables V and t.

5 Vinay is an emergency electrician charges a $100 callout fee plus $75 per hour thereafter. This can be represented by the equation $C = 75t + 100$ where C represents the cost in dollars and t represents the number of hours. R C

a Draw a graph to represent this equation.

b Find the gradient and vertical intercept of the line from the graph and the equation.

c Use the graph or equation to find:

i the charge for a 4-hour job

ii the time taken if the charge was $700

6 Greta bought a new car for $28 000. Its value depreciates at a constant rate of $4000 per year. R C

a Copy and complete this table, then graph the values.

Number of years, n	0	1	2	3
Value, $V	28 000	24 000		

b Extend your graph to find after how many years the car will be worth nothing.

c Find the gradient and vertical intercept of the line.

d What do the gradient and vertical intercept of the line represent?

e Find the equation of the line using V and n.

7 Greta's car has a fuel consumption of 8 L/100 km. This means the car uses 8 litres of fuel for every 100 km travelled.

a Copy and complete this table, then graph the values. R C

Distance travelled, d km	100	200	300	400	500
Petrol used, P L					

b Find the gradient and vertical intercept of the line.

c What does the gradient represent?

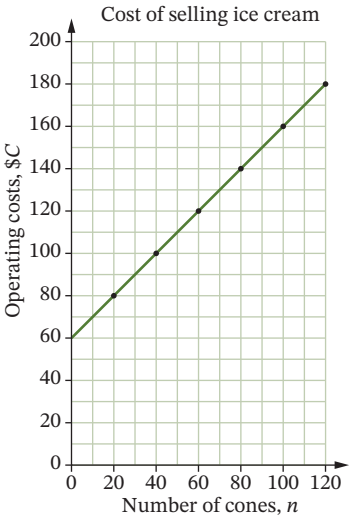
d Find the equation of the line.

e Use the graph to find the amount of petrol used to travel 350 km.

f Verify your answer in e using the equation of the line.

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8 Year 10 sells ice cream at lunchtime as a fund raiser. This graph shows how much it *costs* to set up a stall and produce the ice cream cones. R C



a Before any ice cream is sold, how much does it cost to set up the stall? What costs would this represent?

b By calculating the gradient of the line, determine how much it costs to make each ice cream cone.

c Find the equation of the line.

d Calculate the total cost of making 100 ice cream cones, including the set-up costs.

e If each cone is sold for $5, how much profit would be made from selling 100 ice cream cones?

9 Pavla is saving for her end-of-school holiday. She has her birthday money of $500 and plans to add $50 to that amount each week from her part-time job for the next 2 years. R C

a Copy and complete this table of her savings, then graph these values.

Number of weeks (w)	0	10	20	30	40	50
Savings ($\$S$)						

b What is the vertical intercept and what does it represent?

c What is the gradient of the line and what does it represent?

d Write the equation of the line.

e Use the equation to calculate how much Pavla will save after:

i 27 weeks? **ii** 60 weeks? **iii** 2 years?

d If her goal is $4500, how whole weeks will it take to achieve this amount?

10 The formula $F = 1.8C + 32$ is used to convert temperatures in degrees Celsius (°C) to degrees Fahrenheit (°F). R C

a Copy and complete this table and draw a conversion graph.

Celsius, C	−15	−10	0	10	15
Fahrenheit, F					

b Find the gradient and vertical intercept of the line using the graph and formula.

c Use the formula to convert:

i 6°C to °F **ii** 86°F to °C

Verify your answers using the graph.

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11 Kembla borrowed $1200 from his uncle to buy a kayak. He agrees to pay back $25 a week. R C

a Write an equation to show how much money ($A) Kembla still *owes* after n weeks.

b Use the equation to find the amount owing after:

i 35 weeks **ii** 45 weeks

c Graph the equation.

d Why is the gradient of the line negative?

e Use the graph to verify the vertical intercept and the gradient.

f After how many weeks will the loan be repaid?

Investigation

Sausage sizzle

A local football club is organising a sausage sizzle on Saturday to raise money to buy new equipment. It costs $50 to hire a gas bottle to run the barbecue and each sandwich costs $1 to make. They hope to sell 100 sandwiches.



Alamy Stock Photo/Boaz Rottem

1 Copy and complete this table to show the cost of making sausage sandwiches. Include the cost of hiring the gas bottle.

No. of sandwiches (x)	0	10	20	30	40	50	60	70	80	90	100
Cost ($y)	50	60									

2 Find the linear equation (formula) for y that represents the cost of making x sausage sandwiches.

3 Use an appropriate scale to construct a graph that shows the cost of making from $x = 0$ to 100 sandwiches. Label your axes and give your graph an appropriate title.

4 How much will it cost to make 35 sausage sandwiches?

5 How many sandwiches can be made for $132?

6 How much would it cost to make 120 sausage sandwiches?

7 **a** If the club sold all 100 sausage sandwiches for $5 each, how much money would they take?

b How much profit would the club make?

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2.08 Equations of parallel and perpendicular lines

Worksheet
Writing equations of lines

Puzzles
Linear equations match-up

Equations of parallel lines

Video
Equations of parallel lines

Gradients of parallel and perpendicular lines

If 2 lines with gradients m_1 and m_2 are parallel, then $m_1 = m_2$.

If 2 lines with gradients m_1 and m_2 are perpendicular, then $m_1 \times m_2 = -1$ or $m_2 = -\frac{1}{m_1}$.

Example 20

Find the equation of the line parallel to $y = 8 - 3x$ that passes through the point $(-1, 6)$.

Solution

For $y = 8 - 3x$ (or $y = -3x + 8$), the gradient is $m = -3$.

A line parallel to $y = 8 - 3x$, will also have $m = -3$.

The equation of this line is $y = mx + c = -3x + c$, where c is a constant.

To find the value of c, substitute the point $(-1, 6)$ into the equation:

$y = -3x + c$

$6 = -3 \times (-1) + c$ $\qquad x = -1, y = 6$

$6 = 3 + c$

$c = 3$

$\therefore$ The equation is $y = -3x + 3$.



ngoc tran/Shutterstock.com

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Example 21

Find the equation of the line perpendicular to $3x - 4y + 6 = 0$ that passes through the point $(5, 4)$.

Solution

To find the gradient of $3x - 4y + 6 = 0$, first convert it to the form $y = mx + c$:

$3x - 4y + 6 = 0$

$3x + 6 = 4y$

$4y = 3x + 6$

$y = \frac{3x+6}{4}$

$y = \frac{3}{4}x + \frac{3}{2}$ $\quad y = mx + c$

$\therefore$ Gradient $= \frac{3}{4}$

$\therefore$ Gradient of perpendicular line $= \frac{-1}{\left(\frac{3}{4}\right)}$ $\quad$ The negative reciprocal of $\frac{3}{4}$

$= -\frac{4}{3}$

$\therefore$ The equation of this line is $y = -\frac{4x}{3} + c$

To find the value of c, substitute the point $(5, 4)$ into the equation.

$4 = \left(-\frac{4}{3}\right) \times 5 + c$ $\quad x = 5, y = 4$

$= -\frac{20}{3} + c$

$4 + \frac{20}{3} = c$

$c = \frac{32}{3}$

$\therefore$ The equation is $y = -\frac{4x}{3} + \frac{32}{3}$ or $y = \frac{-4x+32}{3}$ or,

converting to the neater general form:

$3y = -4x + 32$

$4x + 3y - 32 = 0$

Video
Equations of perpendicular lines

2.08

EXERCISE 2.08 ANSWERS ON P. 582

Equations of parallel and perpendicular lines U F R C

EXAMPLE 20

1 Find the equation of the line that is parallel to: R C

a $y = 2x + 9$ and has a y-intercept of 4

b $y = 3x$ and has an x-intercept of -2

c $y = 5 - \frac{x}{2}$ and passes through $(-1, 6)$.

d $2x - y = 6$ and passes through $(5, -2)$

e $y = -5x - 8$ and passes through the midpoint of $(3, -10)$ and $(-5, -6)$

f $2y = x - 3$ and passes through $(6, -7)$

EXAMPLE 21

2 Find the equation of a line that is perpendicular to: R C

a $y = \frac{x}{2}$ and has a y-intercept of -2

b $y = -5x$ and has an x-intercept of 1

c $y = 3x - 1$ and passes through the x-axis at 4

d $y = \frac{x-6}{3}$ and passes through $(1, -6)$

e $x + y - 6 = 0$ and passes through $(-4, 2)$

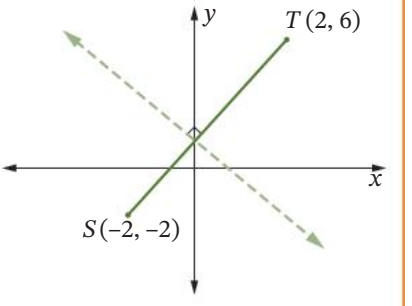
f $3x - y - 9 = 0$ and passes through $(-10, -7)$

3 **a** Find the gradient of interval ST in the diagram.

b Find the midpoint of ST.

c The dotted line is perpendicular to ST and passes through its midpoint. What is its gradient?

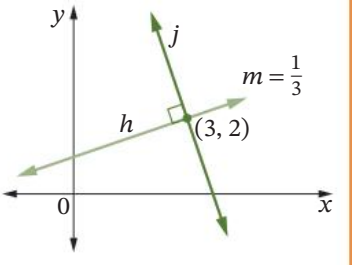
d Find the equation of the dotted line, in the form $y = mx + c$.



4 **a** Find the equation of line h in the diagram.

b Find the gradient of line j (which is perpendicular to line h).

c Find the equation of line j. R C



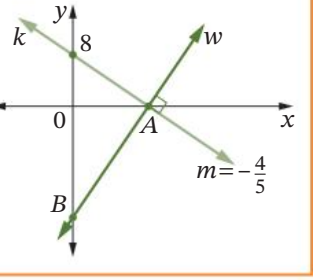
5 **a** Find the equation of line k.

b Find the coordinates of point A.

c Find the gradient of line w.

d Find the equation of line w.

e Find the coordinates of point B. R C



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Did you know?

Constants

Expressions like 'k is a constant' are often used in mathematics, but constants are also used in areas such as physics, chemistry, biology and astronomy. A constant may be:

- a numerical part of an algebraic expression. For example, in the expression $3x^2 + 5$, the 3 and 5 are constants and 5 is usually called the constant term.
- a quantity that has a fixed value for an expression or calculation. For example, in the equation of a line, $ax + by + c = 0$, the a, b and c are constants (while x and y are variables).
- a number or quantity that does not change in any circumstances. Examples are c (the speed of light) in the formula $E = mc^2$, and π.

Other constants that do not change include Faraday's constant, Planck's constant, Boltzmann's constant, Avogadro's number, 1 astronomical unit, and the gravitational constant.

1 **Find the symbol and value of each of the constants listed above.**

2 **Explain the meaning of the word 'constants' in this statement: 'There are only 2 constants in life—death and taxes'.**

Power plus ANSWERS ON P. 582

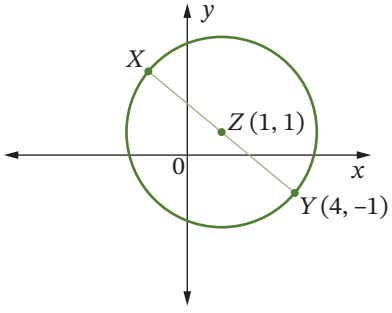
1 A line is drawn through the points $A(0, -2)$ and $B(3, 0)$. The x-coordinate of a point C on AB is 9. Find:

a the gradient of AB **b** the equation of AB **c** the y-coordinate of C.

2 The point $(-1, 6)$ lies on the line $kx + 3y - 13 = 0$, where k is a constant number. Find k.

3 $Z(-1, 3)$ is the midpoint of the interval joining $A(-4, 7)$ and B. Find the coordinates of B.

4 The circle has XY as a diameter and centre Z. What are the coordinates of X?



5 Show that the points (4, 2), (10, –4) and (1, 5) are collinear.

CHAPTER 2 REVIEW

Quiz
Language of maths 2

Puzzle
Graphing lines crossword

Language of maths

axes	distance	exact answer	general form
gradient	gradient–intercept form	horizontal	interval
length	linear	linear equation	midpoint
parallel	perpendicular	reciprocal	rise
run	surd	vertical	vertical intercept
x-axis	x-intercept	y-axis	y-intercept

1 What is the difference between the y-axis and the y-intercept?

2 When finding the length of an interval on a number plane, what is meant by an **exact** answer?

3 What measurement is the fraction given by the vertical rise of a line divided by the horizontal run?

4 Research the everyday meaning of the word **intercept**. Look it up in a dictionary.

5 Describe the property of the gradients of perpendicular lines.

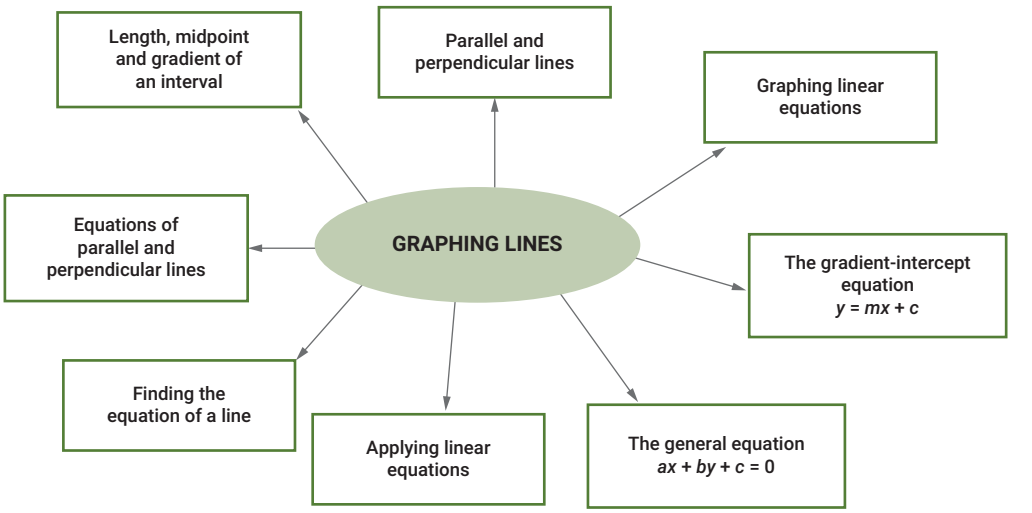
6 What form of the linear equation is $ax + by + c = 0$?

Worksheet
Mind map: Graphing lines

Topic summary

- How can you find the gradient of a line?
- What is $y = mx + c$?
- How can you test whether a pair of lines are parallel?
- What parts of this topic did you find difficult?

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



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TEST YOURSELF 2

ANSWERS ON P. 582

Quiz
Test yourself 2

1 An interval is formed by joining the points $K(5, 6)$ and $L(-7, 2)$. 2.01

a Find, correct to one decimal place, the length of interval KL.

b Find the midpoint of KL.

c Find the gradient of KL.

2 The vertices of a quadrilateral $HJKL$ are $H(-8, -5)$ $J(-1, -2)$ $K(2, 5)$ $L(-5, 2)$. 2.01

a Find the exact length of the sides of the quadrilateral.

b Find the gradient of each side of $HJKL$.

c Find the exact length of the diagonals HK and JL.

d What type of quadrilateral is $HJKL$?

3 A line passes through the points $V(8, -1)$ and $W(10, -2)$. What is the gradient of a line: 2.02

a parallel to VW? **b** perpendicular to VW?

4 Graph the linear equation $y = -5x - 1$ on a number plane. 2.03

5 Which point lies on the line of $3x + y = 2$? Select the correct answer **A**, **B**, **C** or **D**. 2.03

A $(1, 0)$ **B** $(2, 4)$ **C** $(-1, 5)$ **D** $(-1, -5)$

6 What is the equation of the line through $(-2, 3)$ and parallel to the x-axis? Select **A**, **B**, **C** or **D**. 2.03

A $x = -2$ **B** $x = 3$ **C** $y = -2$ **D** $y = 3$

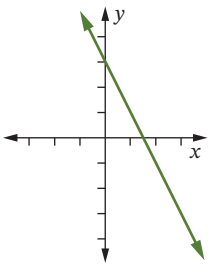
7 Write the gradient, m, and y-intercept, c, for each linear equation. 2.04

a $y = 2x - 10$ **b** $y = 4x + 3$ **c** $y = \frac{4 - 3x}{8}$

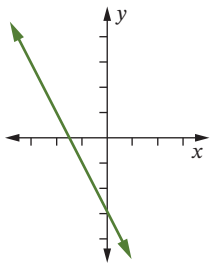
8 Match each equation to its graph below.

a $y = 2x - 3$ **b** $y = -2x - 3$ **c** $y = -2x + 3$ **d** $y = 2x + 3$

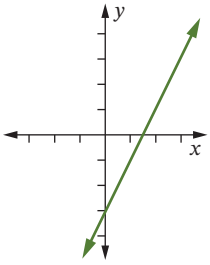
A



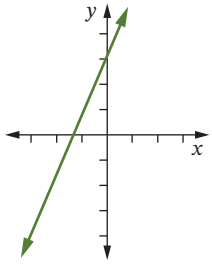
B



C



D



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PATH

9 Find the x- and y-intercepts of each line in Question **7**. 2.04

10 Convert each equation to general form $ax + by + c = 0$. 2.05

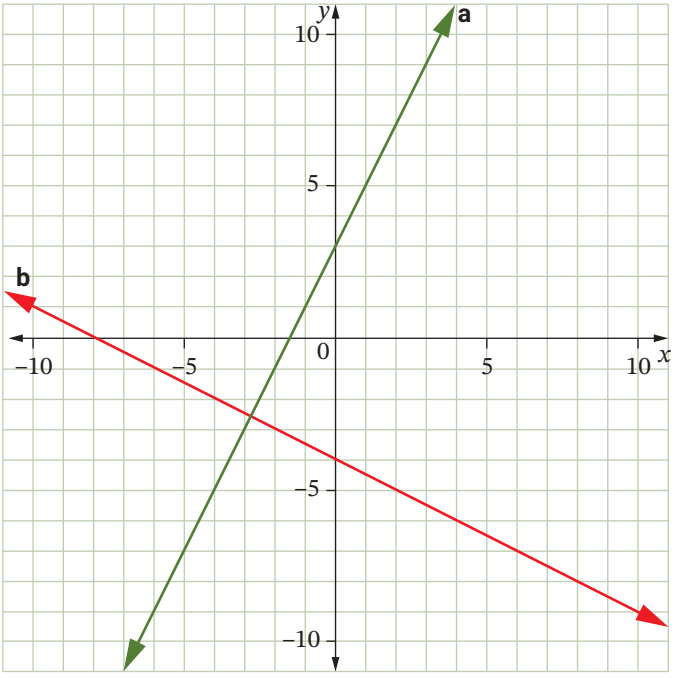
a $y = 3x + 5$ **b** $y = \frac{2x}{5} - 10$ **c** $x = 3y + 6$

11 Rewrite each equation in the form $y = mx + c$, then state the gradient, m, and the y-intercept, c. 2.05

a $x - y + 2 = 0$ **b** $2x - 8y + 8 = 0$ **c** $3x + y - 9 = 0$

12 Find the x- and y-intercepts of each line in Question **11**. 2.05

13 Find the equation of each line and show that they are perpendicular. 2.06



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14 Seth's Self Storage charges an upfront fee of $50 plus $75 a month thereafter to store items. 2.07

a Copy and complete this table of values, then graph the values.

Number of months, n	0	6	12	18	24
Cost, $C					

b Calculate the gradient of the line.

c What is the vertical intercept of the line?

d Find the equation of the line.

e Use the equation or the graph to find:

i the cost of storing items for 5 months

ii the number of months that items can be stored if the cost is $650

15 Find the equation of a line that is: 2.08

a parallel to $y = 3x + 1$ and passes through the x-axis at 2

b perpendicular to $y = \frac{x}{2}$ and passes through the origin.



3

MEASUREMENT AND SPACE

SURFACE AREA AND VOLUME

Some theme parks have wave pools, which are big swimming pools that simulate the movement of the water at a beach. A large volume of water is quickly released into one end of the pool, which produces a large wave that moves from one end of the pool to the other. The excess water in the pool is recycled so that it can be used to produce more waves.



Chapter outline

		Working mathematically				
3.01	Absolute error and percentage error	U	F		R	C
3.02	Rounding error	U	F		R	C
3.03	Areas of composite shapes	U	F	PS	R	
3.04	Surface area of a prism	U	F	PS	R	C
3.05	Surface area of a cylinder	U	F	PS	R	C
3.06	Surface area of a pyramid*	U	F	PS	R	
3.07	Surface areas of cones and spheres*	U	F	PS	R	C
3.08	Surface areas of composite solids*	U	F	PS	R	
3.09	Volumes of prisms and cylinders	U	F	PS	R	
3.10	Volumes of pyramids, cones and spheres*	U	F	PS	R	C
3.11	Volumes of composite solids*	U	F	PS	R	

*PATH

Wordbank

absolute error The greatest possible error of a measurement, equal to half of the precision (smallest unit) on the measuring instrument's scale

cross-section A 'slice' of a solid, taken across the solid rather than along it

curved surface area The area of the curved surface of a solid such as a cylinder or sphere. For example, the curved surface of a cylinder is a rectangle when flattened.

pyramid A solid with a polygon for a base and triangular faces that meet at a point called the apex

rounding error The difference between a calculated answer and an exact answer due to rounding in measurement and calculations

sector A region of a circle cut off by 2 radii, shaped like a piece of pizza

slant height The height of a pyramid or a cone from its apex to its base along a side face rather than its perpendicular height

surface area The total area of all faces of a solid shape

Quiz
Wordbank 3

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Videos (10):

3.04 Surface area of a prism
• Surface areas of prisms

3.05 Surface area of a cylinder

3.06 Surface area of a pyramid

3.07 Surface area of a cone and sphere
• Surface area and volume of a cone

3.09 Capacity of a cylinder
• Volumes of prisms and cylinders

3.10 Volume of a pyramid
• Surface area and volume of a cone
• Volume of a sphere

Twig videos (6):

3.01 Decimal places: Photo finish

3.08 The power of the Sun

3.09 Bees and their hives • The incredible strength of ants • The Menger sponge

3.10 The Pacific Flyer

Quizzes (5):

- Wordbank 3
- SkillCheck 3
- Mental skills 3
- Language of maths 3
- Test yourself 3

Skillsheets (2):

SkillCheck Solid shapes • What is volume?

Worksheets (10):

3.01 Accuracy in measurement 2

3.03 A page of circular shapes

3.04 Surface area 2 • Nets of solids

3.08 A page of prisms and cylinders

3.09 A page of prisms and cylinders • Volumes of solids • Back-to-front problems • Volume and capacity • Biggest volume

3.10 Back-to-front problems (Advanced)

Mind map Surface area and volume

Puzzles (4):

3.03 Area

3.05 Surface area

3.09 Formula matching game

Language of maths Surface area and volume crossword

Technology (2):

3.09 Volume of a box
• Approximating the volume of a cone

Spreadsheet (1):

3.09 Volume of a box

Presentation (1):

3.11 Volumes of shapes

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

In this chapter you will:

- calculate the absolute error and percentage error of a measurement
- identify and investigate rounding error
- calculate the areas of triangles, quadrilaterals, circles, sectors and composite shapes
- calculate the surface areas, volumes and capacities of right prisms and cylinders
- (PATH) calculate the surface areas, volumes and capacities of pyramids, cones, spheres and composite solids

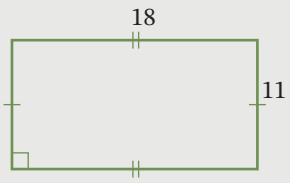
SkillCheck ANSWERS ON P. 583

Quiz
SkillCheck 3

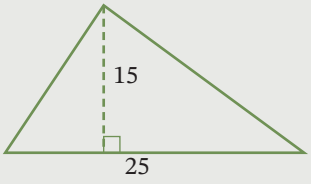
3

1 Calculate the area of each shape. All measurements are in centimetres.

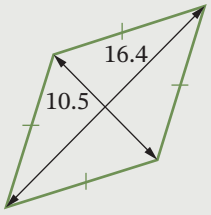
a



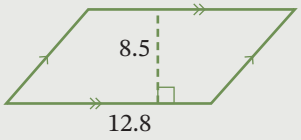
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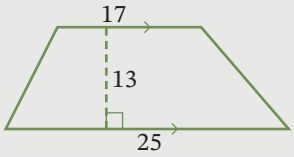
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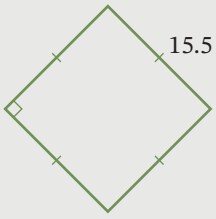
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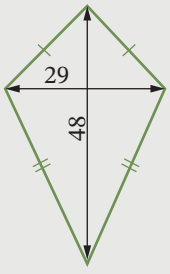
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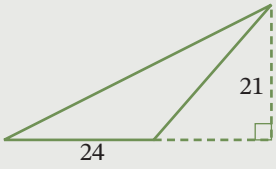
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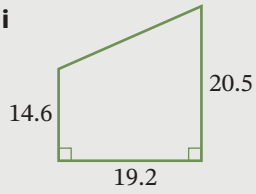
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h

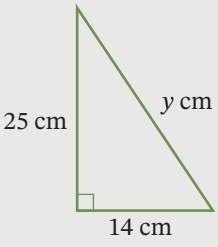


i

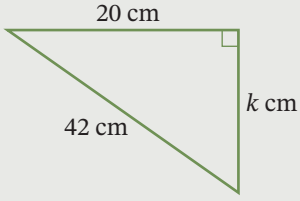


2 Use Pythagoras' theorem to find, correct to one decimal place, the value of each variable.

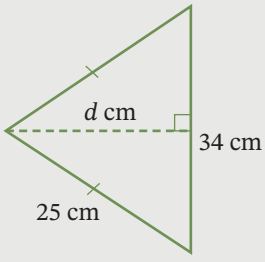
a



b



c



3 For each circle, find correct to one decimal place:

i its circumference **ii** its area.

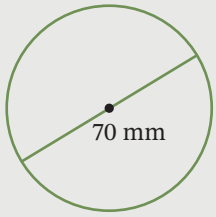
a



b

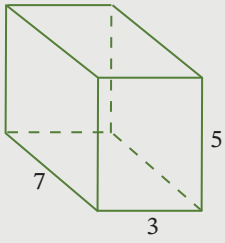


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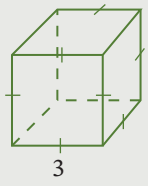


4 Calculate the volume of each solid. All measurements are in metres.

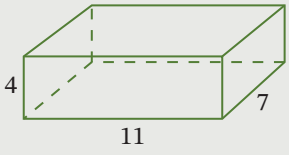
a



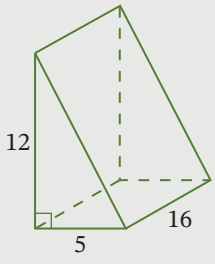
b



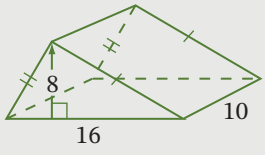
c



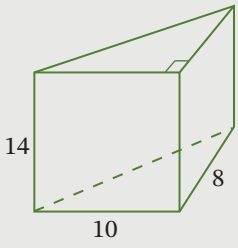
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e



f



Skillsheets
Solid shapes
What is volume?

3.01 Absolute error and percentage error

Video
Decimal places: Photo finish

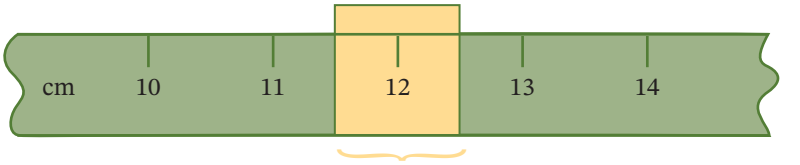
All measurements are approximate. Whether we are using a ruler to measure length, weighing scales for recipe ingredients or a thermometer to measure body temperature, there is some error in every measurement we make.

Absolute error of a measurement

The **absolute error** of a measurement is 0.5 **(half) of the precision** of the measuring instrument's scale.

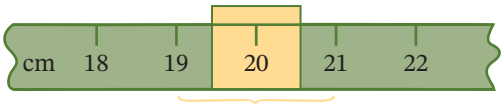
Worksheet
Accuracy in measurement 2

For example, this ruler has a precision of 1 cm, that is, measurements using this ruler are only accurate to the nearest centimetre.



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A measurement of 20 cm on this ruler could actually be any size between 19.5 cm and 20.5 cm.



Strictly speaking, it could not be 20.5 cm because that would round up to 21 cm. The range is actually 19.5 cm up to (but not including) 20.5 cm.

Any measurement in the shaded region should be recorded as 20 cm. The measured length is 20 cm, but the actual length is 20 ± 0.5 cm, meaning '20 cm, give or take 0.5 cm' or '19.5 to 20.5 cm'.

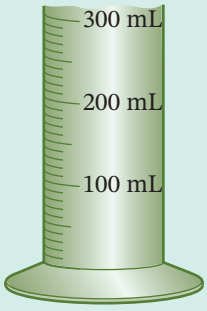
The **absolute error** of this ruler is 0.5 cm, which represents the greatest possible error the actual measurement can differ from the recorded measurement (20 cm), the furthest your measurement could be wrong.

Example 1

Calculate the absolute error of:

a this measuring cylinder

b a measurement of 12.4 cm



Solution

a The cylinder has a precision of 10 mL. — The size of one unit on the scale

Absolute error = 0.5 × 10 mL = 5 mL. — Half of the precision

b A measurement of 12.4 cm has a precision of 0.1 cm. — 12.4 cm is written to the nearest 0.1 cm

Absolute error = 0.5 × 0.1 cm = 0.05 cm.

An absolute error of 1 metre may not be much when measuring the length of a bridge, but it is huge when measuring the length of a bathroom. To understand and compare the sizes of errors when measuring things, we can calculate **percentage error**, the absolute error as a percentage of the measurement.

Percentage error of a measurement

$$\text{Percentage error} = \frac{\text{absolute error}}{\text{measurement}} \times 100\%$$

The smaller the percentage error, the more accurate the measurement, the closer it is to its actual value.

Example 2

The Channel Tunnel is an underwater railway tunnel connecting England and France. It is 50.45 kilometres long.



Gary Perkin/Shutterstock.com

Find the percentage error of this measurement, correct to 2 decimal places.

Solution

Precision $= 0.01$ km

Absolute error $= 0.5 \times 0.01$ km

$= 0.005$ km

Percentage error $= \frac{0.005}{50.45} \times 100\%$ $\quad \frac{\text{absolute error}}{\text{measurement}} \times 100\%$

$= 0.009\,91...$

$\approx 0.01\ \%$

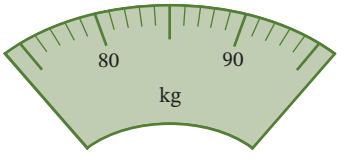
EXERCISE 3.01 ANSWERS ON P. 583

Absolute error and percentage error U F R C

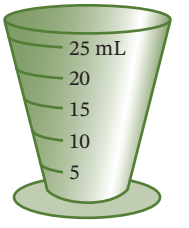
EXAMPLE 1

1 For each measuring instrument, find its precision and absolute error. R C

a



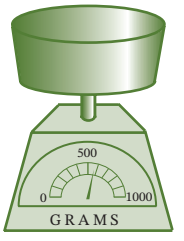
b



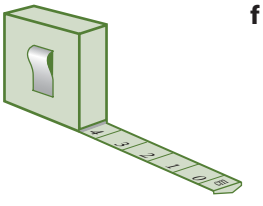
c



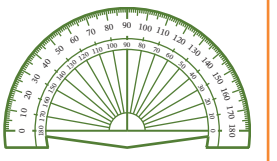
d



e



f



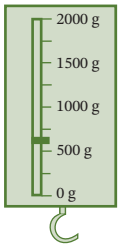
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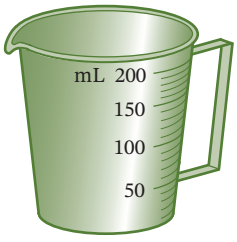
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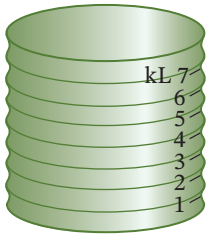
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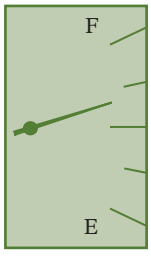
i



j



k



2 Copy and complete this table.

Measurement	Precision	Absolute error
a 58 cm		
b 22.3 mL		
c 31 seconds		
d 175 cm (to the nearest cm)		
e 175 cm (to the nearest 5 cm)		

3 Swara is using a measuring tape marked in millimetres. What is the absolute error of this measurement? Select the correct answer **A**, **B**, **C** or **D**.

A 1 mm

B 0.5 mm

C 1 cm

D 0.5 cm

4 Find the absolute error of each measurement.

a 86 mm

b 2850 L (to the nearest 10 L)

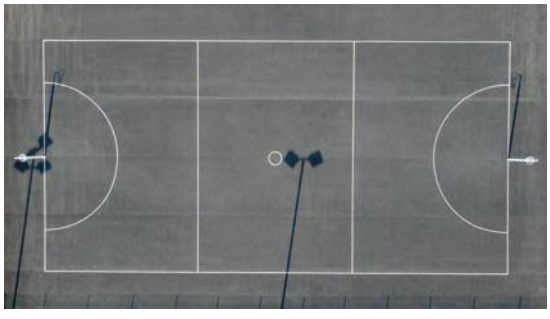
c 9.4 m^2

d 700 kg (to the nearest g)

e 700 kg (to the nearest 10 kg)

f 700 kg (to the nearest 100 kg)

5 A netball court is 15.25 m wide. Calculate the absolute error and hence find the percentage error of this measurement, correct to 3 decimal places.



Richard Newstead/Moment/Getty Images

3.01

EXAMPLE 2

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6 The world's longest river, the Nile river in Egypt, is 6650 km long.

a If this measurement is made to the nearest 10 km, find its absolute error.

b Hence calculate the percentage error, correct to 4 decimal places.



©Frontpage/Shutterstock.com

7 By comparing percentage errors, determine which measurement is Questions 5 and 6 is more accurate, the width of the netball court or the length of the Nile river. R C

8 A timekeeper measures the time it takes an athlete to complete a race to be 32.08 seconds. Calculate the percentage error of this measurement, correct to 4 decimal places.

9 An Olympic marathon is 42.195 km long. Calculate the percentage error of this measurement, correct to 4 decimal places.

10 Which measurement is Questions 8 and 9 is more accurate, the race time or the marathon length? R C

11 *Sweet Juices* is a company using a new machine to bottle its juice. Bottles of juice have a measured capacity with a percentage error of 0.5% of the 500 mL labelled on each bottle. R C

a What is the absolute error of the machine's scale?

b What is the precision of the machine's scale?

c What is the maximum possible capacity?

d What is the minimum possible capacity?

e Which of the following capacities (in mL) could not have been bottled by the machine?

500.05, 498.55, 499.10, 503.10, 502.30, 497.25, 498.50, 502.10

12 The length of a piece of wood is measured to be 8 m. The length of a metal rod is measured to be 8.5 m. By using absolute error in your calculations, show that the piece of wood *could* be longer than the metal rod. R C

13 Why is a measurement of 185.0 cm more accurate than a measurement of 185 cm? R C

14 A cube has a measured length of 4 m. Calculate its maximum possible volume and its minimum possible volume. R C

Rounding error

3.02

Another type of error can occur when performing calculations with measurements, where the measured value or partial answer is rounded too early or severely, resulting in a final answer that is inaccurate (not close to the exact answer). This is called **rounding error**.

Consider the area calculations for purchasing carpet for 3 bedrooms.

Room 1 has dimensions 4.2 m by 3.7 m, Room 2 is 3.9 m by 3.4 m, Room 3 is 3.3 m by 3.3 m.



Pixel-Shot/Adobe Stock

Room 1: $4.2 \times 3.7 = 15.54$ m^2

Room 2: $3.9 \times 3.4 = 13.26$ m^2

Room 3: $3.3 \times 3.3 = 10.89$ m^2

Total area $= 39.69$ m^2

If we were to estimate by rounding measurements to the nearest metre, then:

Room 1: $4 \times 4 = 16$ m^2

Room 2: $4 \times 3 = 12$ m^2

Room 3: $3 \times 3 = 9$ m^2

Total area $= 37$ m^2

Notice that the estimate is too small, and if we had rounded the measurements we would not have bought enough carpet.

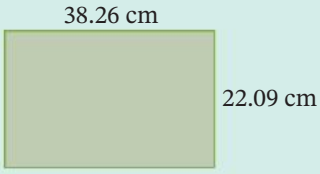
Rounding during calculations can lead to errors in solutions. If approximate values are used in further calculations, the error becomes worse.

The table below shows a number being **rounded** and **truncated** to the nearest whole number, to one decimal place, to 3 decimal places and to 5 decimal places. **Truncate** means to 'cut off' the number at the decimal place regardless of the next digit (or 'rounding down').

10.638 971 564 538 ...	Rounded	Truncated
to the nearest whole number	11	10
to one decimal place	10.6	10.6
to 3 decimal places	10.639	10.638
to 5 decimal places	10.638 97	10.638 97

Example 3

A rectangle has length 38.26 cm and width 22.09 cm.



a Calculate its **perimeter** and area.

b Round the rectangle's length and width to one decimal place and then calculate its perimeter and area with these values and then find correct to one decimal place the percentage error of the perimeter and area.

c Truncate the rectangle's length and width to the nearest whole number, then calculate its perimeter and area with these values, then find correct to one decimal place the percentage error of the perimeter and area.

Solution

a Perimeter $= 2 \times 38.26 + 2 \times 22.09$
$= 120.7$ cm

Area $= 38.26 \times 22.09$
$= 845.1634$ cm^2

b Length $= 38.3$ cm, width $= 22.1$ cm
Perimeter $= 2 \times 38.3 + 2 \times 22.1$
$= 120.8$ cm
Error $= 120.8 - 120.7$
$= 0.1$ cm
Percentage error $= \frac{0.1}{120.7} \times 100\%$
$= 0.0828 \ldots$
$\approx 0.1\%$

Area $= 38.3 \times 22.1$
$= 846.43$ cm^2
Error $= 846.43 - 845.1634$
$= 1.2666$
Percentage error $= \frac{1.2666}{845.1634} \times 100\%$
$= 0.1498 \ldots$
$\approx 0.1\%$

c Length $= 38$ cm, width $= 22$ cm
Perimeter $= 2 \times 38 + 2 \times 22$
$= 120$ cm
Error $= 120.7 - 120$
$= 0.7$ cm
Percentage error $= \frac{0.7}{120.7} \times 100\%$
$= 0.5799 \ldots$
$\approx 0.6\%$

Area $= 38 \times 22$
$= 836$ cm^2
Error $= 845.1634 - 836$
$= 9.1634$
Percentage error $= \frac{9.1634}{845.1634} \times 100\%$
$= 1.0842$
$\approx 1.1\%$

- Notice that the percentage error is higher for area than it is for perimeter: this is due to addition being used to calculate perimeter, whereas by using multiplication to calculate areas, the error is being multiplied.
- Also notice that the percentage error is higher the more we round or truncate the measurements or values. When calculating, it is best to use the exact or most accurate measurements to minimise any rounding errors.

EXERCISE 3.02 ANSWERS ON P. 583

Rounding error U F R C

1 Copy and complete this table to round and truncate the value of π: 3.141 592 6 ...

3.141 592 6 ...	Rounded	Truncated
a to the nearest whole number		
b to 2 decimal places		
c to 4 decimal places		
d to 6 decimal places		

2 A rectangle has length 6.49 m and width 3.82 m.

EXAMPLE 3

- **a** Calculate its area.
- **b** Truncate each measurement to one decimal place, then calculate its approximate area.
- **c** Round each measurement to the nearest whole number, then calculate its approximate area.

3 A circle has radius 7.63 cm.

- **a** Calculate its area correct to 2 decimal places.
- **b** Calculate its area correct to 2 decimal places, using $\pi = 3.14$, then find correct to 2 decimal places the percentage error of the calculated area.
- **c** Round the circle's radius to one decimal place, then calculate its area correct to 2 decimal places, then find correct to 2 decimal places the percentage error of the calculated area.

4 A square has length 11.27 m. R C

- **a** Calculate its perimeter and area.
- **b** Truncate the square's length to one decimal place and use this value to calculate its perimeter and area, then find to one decimal place, the percentage error of the calculated perimeter and area.
- **c** Round the square's length to the nearest whole number and use this value to calculate its perimeter and area, then find to one decimal place the percentage error of the perimeter and area.

5 A rectangle has dimensions 8.25 cm by 6.09 cm. R C

- **a** Calculate its perimeter and area.
- **b** Truncate the rectangle's length and width to one decimal place, then calculate its perimeter and area and find to one decimal place the percentage error of the calculated perimeter and area.
- **c** Round the rectangle's length and width to the nearest whole number, then calculate its perimeter and area and find to one decimal place the percentage error of the perimeter and area.

6 A circle has diameter 8 m, measured correct to the nearest metre. R C

- **a** Calculate the circumference of the circle, correct to 4 decimal places.
- **b** Find the absolute error of the diameter.
- **c** Hence, calculate:
  - **i** correct to 4 decimal places, the smallest possible circumference of the circle
  - **ii** correct to 2 decimal places, the percentage error of the smallest possible circumference
  - **iii** correct to 4 decimal places, the largest possible circumference
  - **iv** correct to 2 decimal places, the percentage error of the largest possible circumference.

7 A right-angled triangle has a base length of 60 mm and a perpendicular height of 90 mm, measured to the nearest mm. R C

- **a** Calculate the length of the hypotenuse, correct to 4 decimal places.
- **b** Find the absolute error of the base length and perpendicular height.
- **c** Hence calculate correct to 4 decimal places the minimum possible length of its hypotenuse and correct to one decimal place its percentage error.
- **d** Calculate the area of this triangle.
- **e** Hence calculate the maximum possible area of the triangle and correct to one decimal place its percentage error.

8 A rectangular swimming pool has side lengths of 3 m and 8 m, correct to the nearest metre. A fence needs to be placed 1 m out from the edge of the pool enclosing the entire pool. Calculate the maximum and minimum perimeter of this fence. R

3.03 Areas of composite shapes

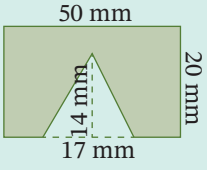
Puzzle
Area

Worksheet
A page of circular shapes

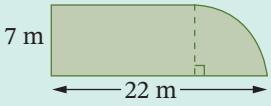
Example 4

Find the area of each composite shape, correct to one decimal place where appropriate.

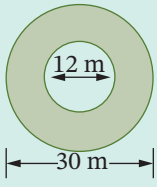
a



b



c



Solution

a $\text{Area} = 50 \times 20 - \frac{1}{2} \times 17 \times 14$
$= 881 \text{ mm}^2$

Area of rectangle − area of triangle

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b The shape is made up of a rectangle and a quadrant.

Radius of quadrant $= 7$ m

Length of rectangle $= 22 - 7$

$= 15$ m

Area of shape = area of rectangle + quadrant

$= 15 \times 7 + \frac{1}{4} \times \pi \times 7^2$

$= 143.4845...$

≈ 143.5 m^2

c This ring shape is an **annulus**, its area is enclosed by 2 circles with the same centre.

Radius of large circle $= \frac{1}{2} \times 30$ m

$= 15$ m

Radius of small circle $= \frac{1}{2} \times 12$ m

$= 6$ m

Area of annulus $= \pi \times 15^2 - \pi \times 6^2$ Large circle − small circle

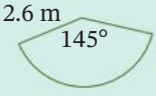
$= 593.7610....$

$= 593.8$ m^2

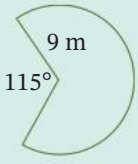
Example 5

Calculate, correct to 2 decimal places, the area of each sector.

a



b



A **sector** is a fraction of a circle 'cut' along 2 radii, like a pizza slice.

Solution

a Area $= \frac{145}{360} \times \pi \times 2.6^2$

$= 8.55385...$

≈ 8.55 m^2

$\frac{145}{360} \times$ area of circle

There are 360° in a circle, but a sector is a fraction of a circle

b Sector angle $= 360° - 115°$

$= 245°$

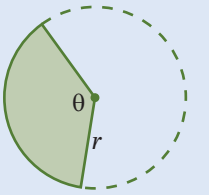
Area of sector $= \frac{245}{360} \times \pi \times 9^2$

$= 173.18029...$

≈ 173.18 m^2

Area of a sector

$$\text{Area of a sector} = \frac{\theta}{360} \times \pi r^2$$



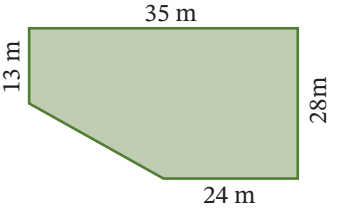
EXERCISE 3.03 ANSWERS ON P. 583

Area of composite shapes U F PS R

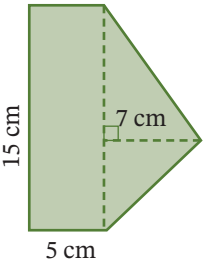
EXAMPLE 4

1 Find the area of each composite shape. PS R

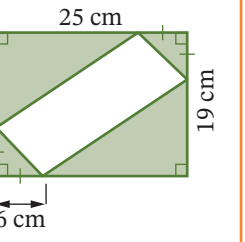
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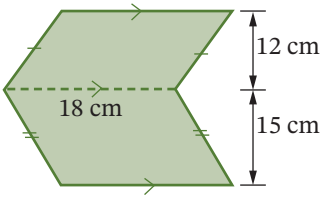
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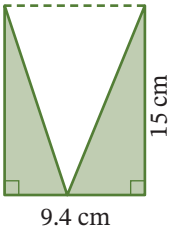
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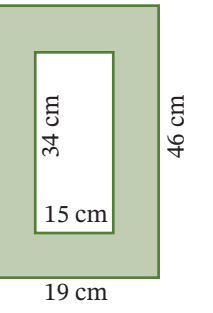
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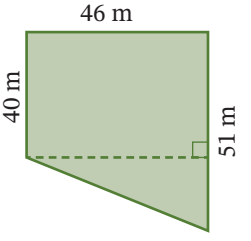
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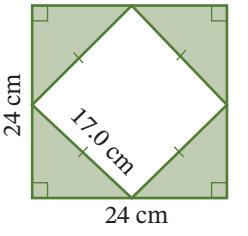
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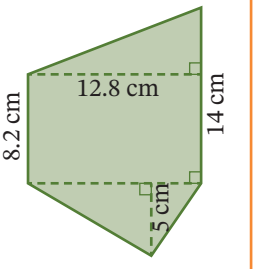
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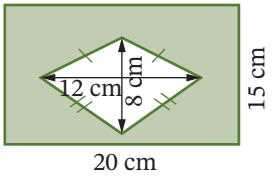
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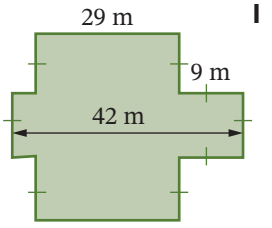
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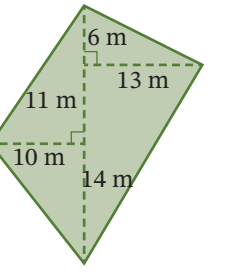
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k



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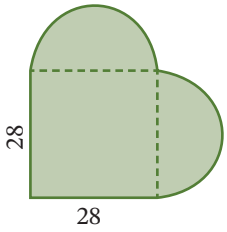


Foundation Standard Complex

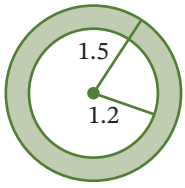
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2 Calculate, correct to one decimal place, the area of each shape. All measurements are in metres. PS R

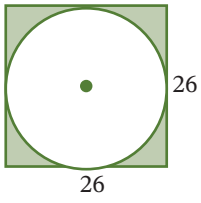
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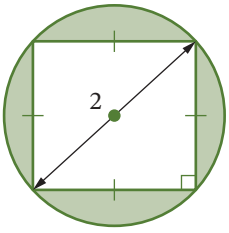
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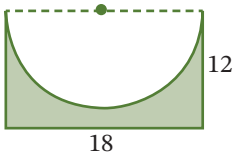
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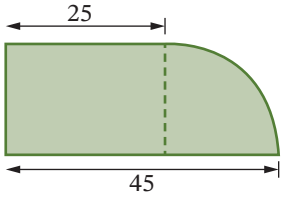
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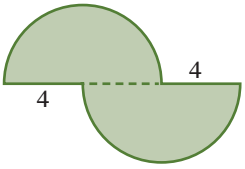
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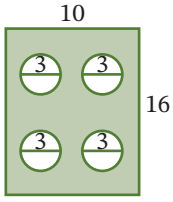
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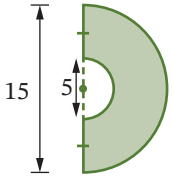
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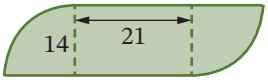
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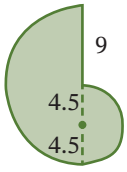
i



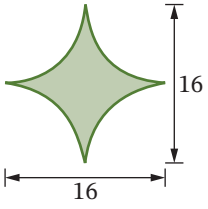
j



k



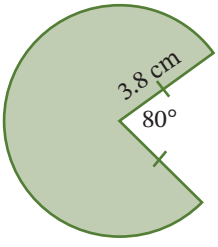
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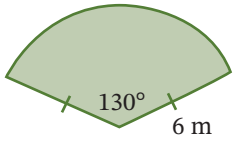
3 Find, correct to one decimal place, the area of each sector.

EXAMPLE 5

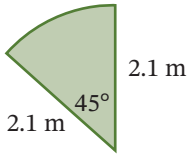
a



b



c



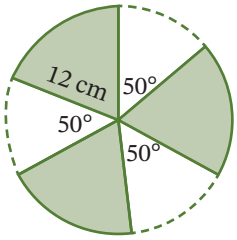
4 A bike tyre has a diameter of 715 mm.

a How far will the bike travel in one revolution of the tyre? Give your answer in metres, correct to 2 decimal places.

b How many revolutions of the tyre are required to travel a distance of 5 km?

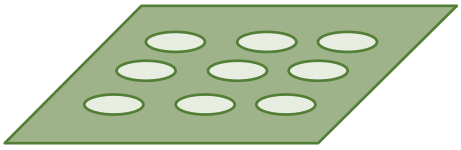
Foundation Standard Complex

5 Calculate the area of the shaded region. Select the correct answer **A**, **B**, **C** or **D**. R



A 362.7 cm²
B 452.4 cm²
C 188.5 cm²
D 263.9 cm²

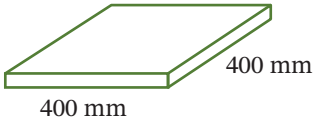
6 A rectangular metal plate with dimensions 2.5 m × 2.2 m has 9 holes of diameter 46 cm drilled in it.



a Find the total area of the holes that have been drilled. Give your answer in m², correct to 2 decimal places.

b What percentage of the metal plate remains?

7 A rectangular courtyard 15 m long and 8 m wide is to be covered with square pavers of side length 400 mm, costing $79.29/m². PS R



a What is the area of one paver, in m²?

b How many pavers will be required to pave the courtyard?

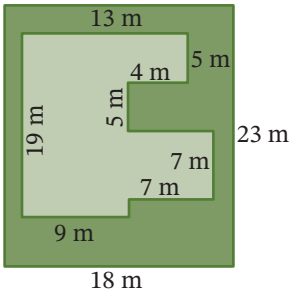
c Calculate the cost of paving the courtyard.

8 A circular sports ground of diameter 140 m has a rectangular soccer pitch measuring 110 m by 70 m inside it. The area outside the soccer pitch is to be painted in the team colour of red.

a Calculate the area that is to be painted red, correct to the nearest m².

b If the cost of paint is $29.50 per 50 m², calculate the cost of painting this area.

9 The diagram shows the floor plan of a house on a block of land. PS R



a Calculate the area of the block.

b Calculate the area taken up by the house.

c What percentage of the area of the block is taken up by the house?

d The area not covered by the house and is to be turfed. Find the cost of turfing the yard at a cost of $11.75/m².

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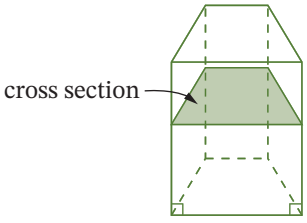
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Surface area of a prism

3.04

A **cross-section** of a solid is a 'slice' of the solid cut *across* it, parallel to its end faces, rather than along it. A **prism** has the same (uniform) cross-section along its length, and each cross-section is a **polygon** (with straight sides).

The trapezoidal prism shown here has cross-sections that are trapeziums.



A right prism

Worksheets
Surface area 2
Nets of solids

3.04

Videos
Surface area of a prism
Surface areas of prisms

Surface area

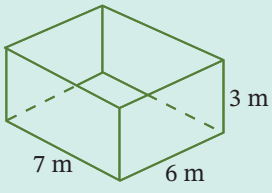
The **surface area** of a solid is the total area of all the faces of the solid. To calculate the surface area of a solid, find the area of each face and add the areas together.

It is often useful to draw the net of a solid when finding its surface area. A net may be used to form an open solid or a closed solid. A sealed cardboard box is an example of a **closed solid**. A cardboard box with the lid removed is an example of an **open solid**. For the surface area of an open solid, we only count the *external* surfaces, not the internal ones (so that we don't count each surface twice).

Example 6

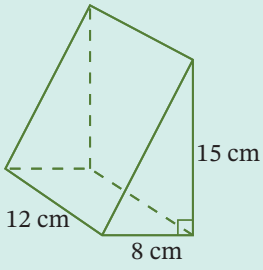
Find the surface area of each prism.

a



Open rectangular prism

b



Closed triangular prism

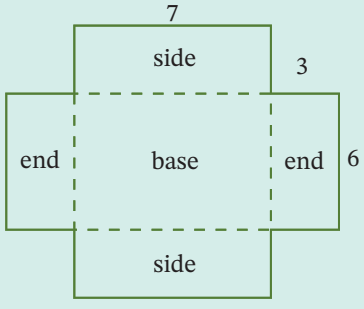
Solution

a This open prism has 5 faces.

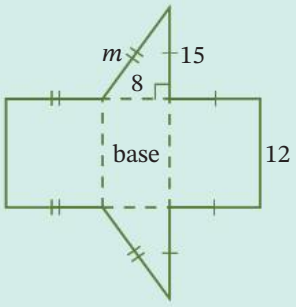
Surface area = 2 ends + 2 sides + base

$= 2 \times (3 \times 6) + 2 \times (3 \times 7) + (6 \times 7)$

$= 120 \text{ m}^2$



b This closed prism has 5 faces: 2 identical triangles (front and back) and 3 different rectangles.



Using Pythagoras' theorem to find m, the hypotenuse of the triangle:

$m^2 = 8^2 + 15^2$

$= 289$

$m = \sqrt{289}$

$= 17$

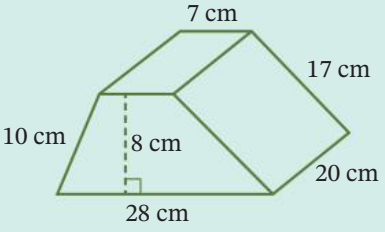
Surface area = 2 triangles + 3 rectangles

$$= 2 \times \left(\frac{1}{2} \times 8 \times 15 \right) + (17 \times 12) + (8 \times 12) + (15 \times 12)$$

$= 600 \text{ cm}^2$

Example 7

Calculate the surface area of this trapezoidal prism.



Solution

This trapezoidal prism has 6 faces:

2 identical trapeziums (front and back) and 4 different rectangles.

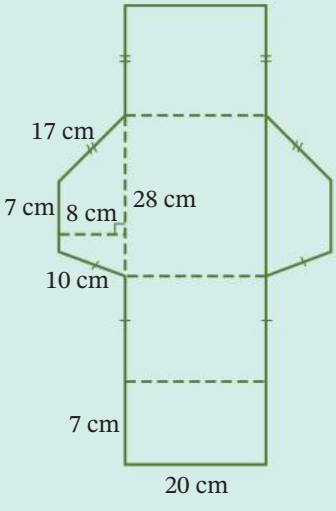
Area of each trapezium $= \frac{1}{2} \times (7 + 28) \times 8$

$= 140 \text{ cm}^2$

Surface area $= (2 \times 140) + (20 \times 7) + (20 \times 10)$

$+ (20 \times 28) + (20 \times 17)$

$= 1520 \text{ cm}^2$



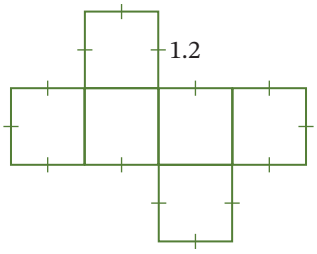
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EXERCISE 3.04 ANSWERS ON P. 583

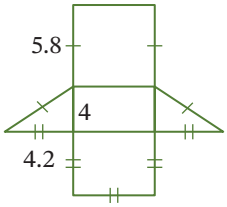
Surface area of a prism U F PS R C

1 Identify the prism that each net represents, then calculate the surface area of the prism. All lengths are in metres. R C

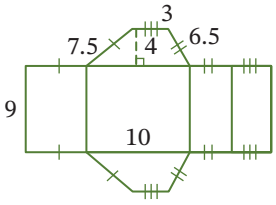
a



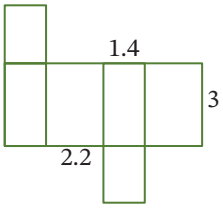
b



c



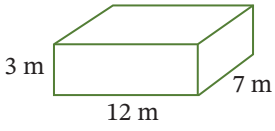
d



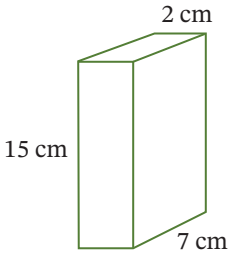
2 Find the surface area of each prism.

EXAMPLE 6

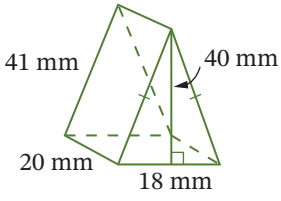
a



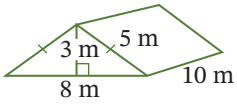
b



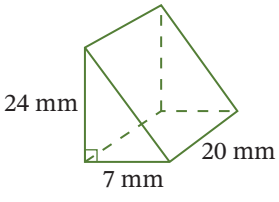
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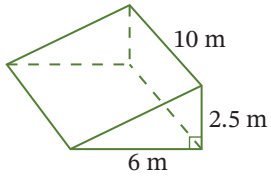
d



e

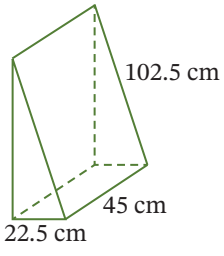


f



3 Calculate the surface area of this triangular prism.
Select the correct answer **A**, **B**, **C** or **D**.

A 12 375 cm²
B 11 250 cm²
C 10 125 cm²
D 12 431.25 cm²

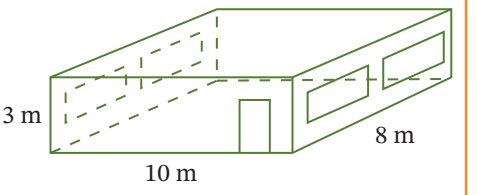


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4 This classroom is being renovated. Find: PS

a the area of the floor to be carpeted and the total cost, at $55 per square metre.

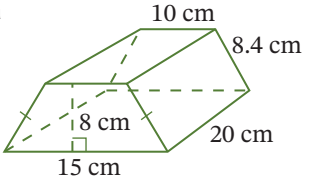
b the ceiling and wall area to be painted, if the room contains 4 windows, each 2.5 m by 1.5 m, and a doorway 2 m by 0.8 m.



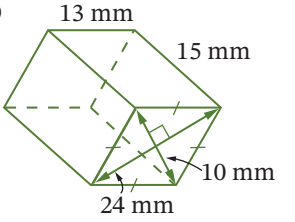
EXAMPLE 7

5 Calculate the surface area of each prism.

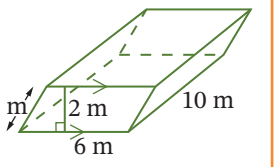
a



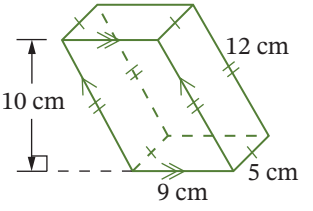
b



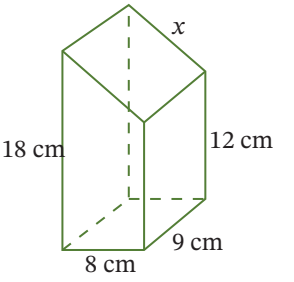
c



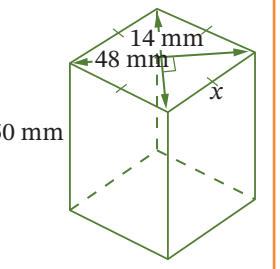
d



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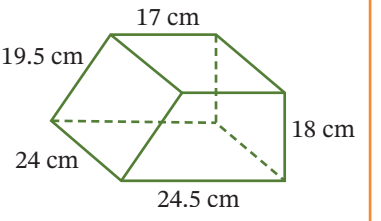
f



6 Calculate the surface area the trapezoidal prism. Select **A**, **B**, **C** or **D**.

A 10 584 cm^2 **B** 2643 cm^2

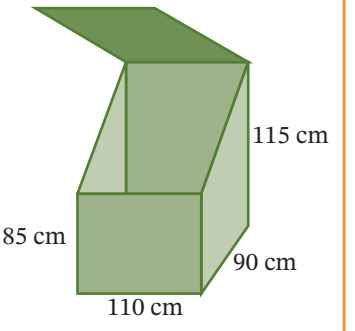
C 2082.75 cm^2 **D** 8964 cm^2



7 The wooden toy box is in the shape of a trapezoidal prism.

a Calculate how much timber is required to make the toy box, correct to the nearest cm^2.

b If the price of the timber is $25/m^2, calculate the cost of making the box.



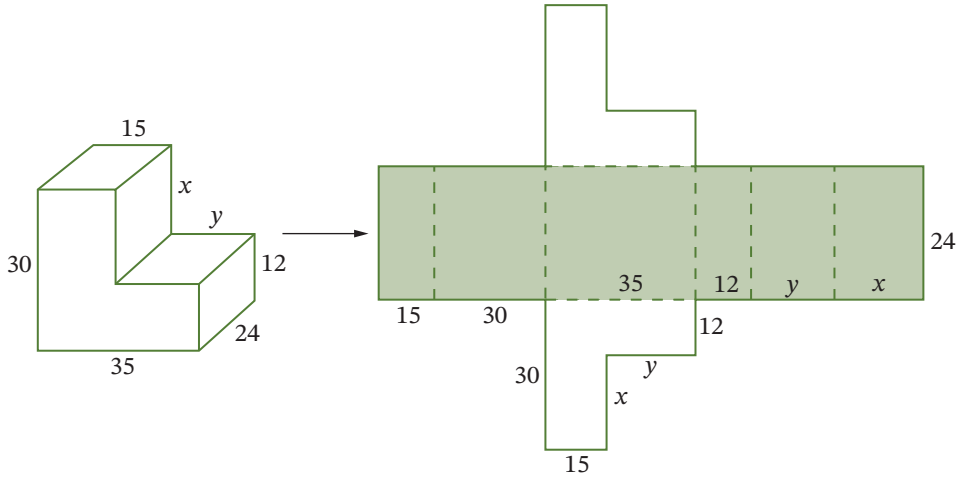
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Investigation

A surface area shortcut

1 Consider this L-shaped prism and its net. We will find its surface area.



a Find x and y.

b This prism has 8 faces: 2 'L-shaped' ends and 6 rectangles. Instead of calculating the areas of the 6 rectangles separately, we can combine them into one long rectangle, as shaded in the net above. The length of the rectangle is the same as the perimeter of the L-shape. What is the length of this long rectangle?

c What is the area of this long rectangle?

d Copy and complete:

Length of shaded rectangle = p _____ of the L-shape.

e Find the surface area of the prism by copying and completing the following:

Surface area = 2 'L-shaped' ends + shaded rectangle

$= 2 \times (15 \times 30 + 20 \times 12) +$ _____

= _____

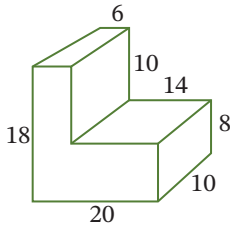
2 From question **1**, it can be seen that the surface area of any prism with end faces of area A and perpendicular height (distance between end faces) h can be calculated using the formula:

$$SA = 2A + Ph$$

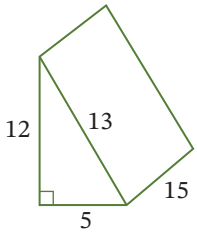
where P = perimeter of the end face.

Use this method to calculate the surface area of each prism. All measurements are in centimetres.

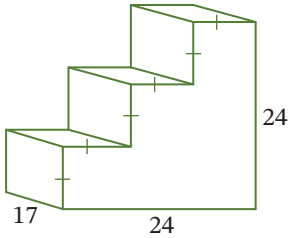
a



b



c

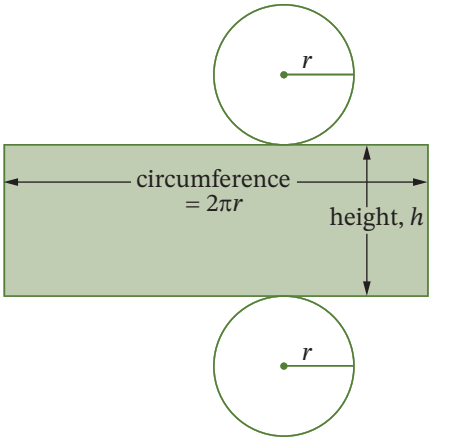


3.05 Surface area of a cylinder

Puzzle
Surface area

Video
Surface area of a cylinder

A closed cylinder has 3 faces made up of 2 circles (the circular ends) and a rectangle (the curved surface). The length of the rectangle is the **circumference** of the circular end, while the width of the rectangle is the height of the cylinder.



Surface area of a cylinder = area of 2 circles
+ area of rectangle

$$SA = 2 \times \pi r^2 + 2\pi r \times h$$
$$= 2\pi r^2 + 2\pi rh$$

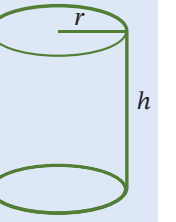
Surface area of a closed cylinder

$$SA = 2\pi r^2 + 2\pi rh$$

where r = radius of circular base

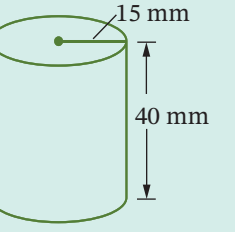
h = perpendicular height

The area of the 2 circular ends = $2\pi r^2$
and the area of the curved surface = $2\pi rh$.



Example 8

Find, correct to the nearest mm², the surface area of this cylinder.



Solution

Surface area = area of 2 ends + area of the curved surface $\quad r = 15, h = 40$

$= 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 15^2 + 2 \times \pi \times 15 \times 40$

$= 5183.627...$

≈ 5184 mm²

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Example 9

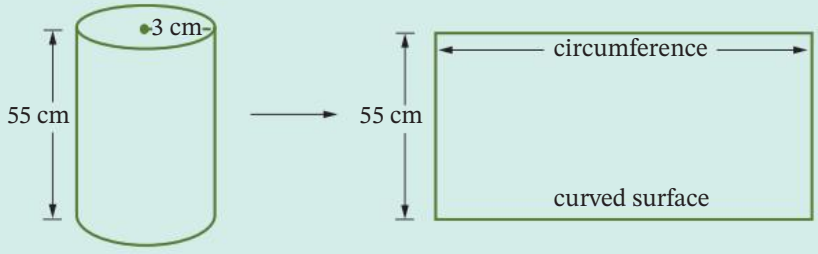
Find, correct to 2 **significant figures**, the surface area of:

a a cylindrical tube, open at both ends, with radius 3 cm and height 55 cm

b an open half-cylinder with radius 0.65 m and length 2.4 m.

Solution

a



Surface area = curved surface

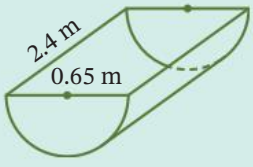
$= 2\pi rh$ $\quad r = 3$ and $h = 55$

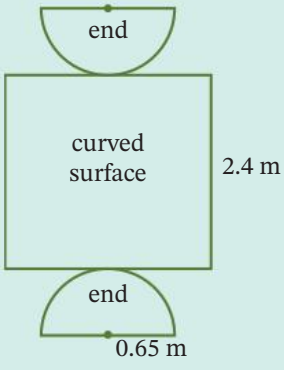
$= 2 \times \pi \times 3 \times 55$

$= 1036.725...$

$= 1000 \text{ cm}^2$

b





Surface area = 2 semicircle ends $+ \frac{1}{2} \times$ curved surface

$= 2 \times \left(\frac{1}{2} \times \pi \times 0.65^2 \right) + \frac{1}{2} \times (2 \times \pi \times 0.65 \times 2.4)$

$= 6.2282...$

$\approx 6.2 \text{ m}^2$

3.05

EXERCISE 3.05 ANSWERS ON P. 583

Surface area of a cylinder U F PS R

EXAMPLE 8

1 Calculate, correct to one decimal place, the surface area of a cylinder with:

a radius 1.4 m, height 2.2 m

b diameter 45 cm, height 65 cm

c diameter 9 cm, height 24 cm

d radius 1.3 m, height 3.8 m

2 Find, correct to the nearest whole number, the curved surface area of a cylinder with:

a radius 1.5 m, height 3.75 m

b diameter 27 cm, height 41 cm

3 A container of potato chips is a cylinder with diameter 7 cm and height 23.2 cm. Calculate its surface area, correct to one decimal place.

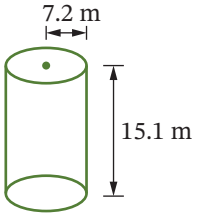
EXAMPLE 9

4 Find the surface area of a cylinder, open at one end with diameter 12 mm and length 15 cm. Select the closest answer **A**, **B**, **C** or **D**. R

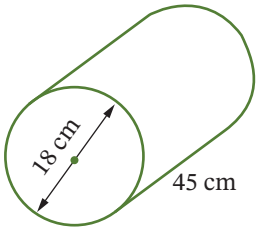
A 678.6 cm^2 **B** 1017 cm^2 **C** 6107.3 mm^2 **D** 5768.0 mm^2

5 Calculate, correct to the nearest whole number, the surface area of each solid. R

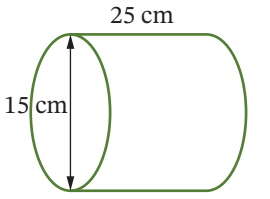
a closed cylinder



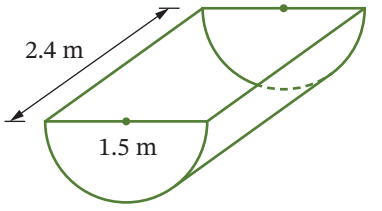
b closed cylinder



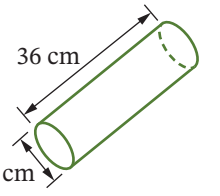
c cylinder with one open end



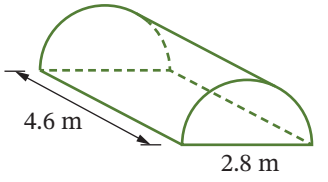
d half cylinder with open top



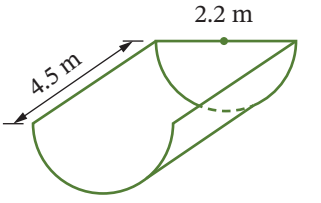
e cylinder open both ends



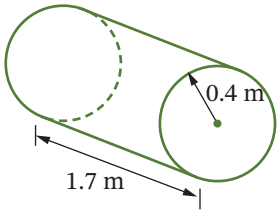
f closed half cylinder



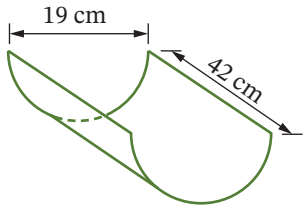
g half cylinder with open top, one end open



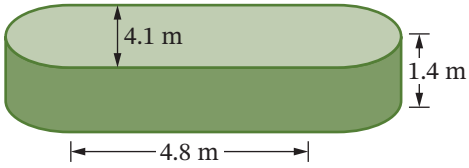
h closed cylinder



i half cylinder, open both ends



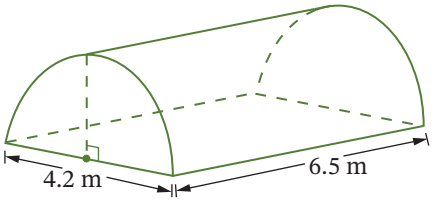
6 The inside of the swimming pool, including the floor, is to be repainted. Find: PS R



a the area to be repainted, correct to one decimal place

b the number of whole litres of paint needed if coverage is 9 m^2 per litre.

7 The diagram shows a tent to be made in the shape of half a cylinder. Find: PS R



a the area of the floor of the tent

b the surface area of the tent, excluding the floor.

c the total cost of materials for the tent if the material for the flooring costs \$18.50/m^2 and the canvas for the tent costs \$21.75/m^2.



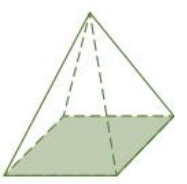
Alamy Stock Photo/Robert Wyatt

Foundation Standard Complex

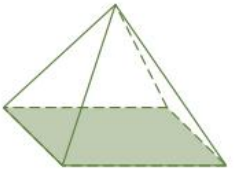
3.06 Surface area of a pyramid

PATH

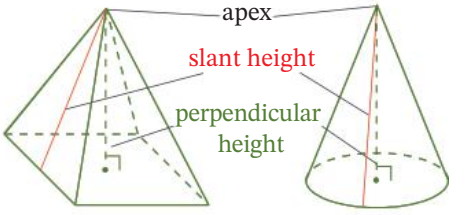
A **pyramid** is a solid shape with a polygon for its base and triangular faces that meet at a point or vertex called its **apex**. Like a prism, a pyramid is named by the shape of its base.


Square pyramid


Triangular pyramid


Rectangular pyramid

A **cone** is a solid shape with a circular base and a curved surface that also has an **apex**. However, a cone is not a pyramid because its base is not a polygon (a circle does not have straight sides).



The **slant height** of a pyramid or cone is the height from the apex to the base, along a side face. It is different from the **perpendicular height** of a pyramid or cone, which is the perpendicular distance from the apex to the base.

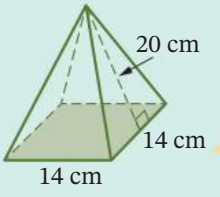
The surface area of a pyramid is calculated by adding the areas of its base and its triangular faces.

Video
Surface area of a pyramid

Example 10

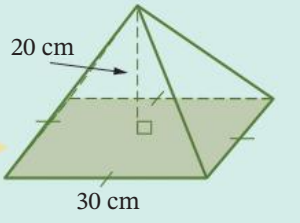
Find the surface area of each square pyramid.

a



This pyramid has a slant height of 20 cm

b



This pyramid has a perpendicular height of 20 cm

Solution

a Surface area = area of square base + area of 4 triangular faces

$$= 14 \times 14 + 4 \times \frac{1}{2} \times 14 \times 20$$
$$= 756 \text{ cm}^2$$

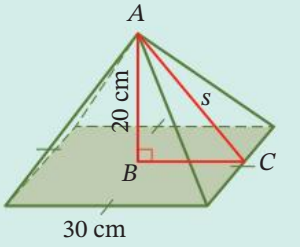
b First find the slant height, s, using Pythagoras' theorem.

In $\triangle ABC$, $s^2 = 20^2 + 15^2$ $\quad BC = \frac{1}{2} \times 30$

$$= 625$$
$$s = \sqrt{625}$$
$$= 25 \text{ cm}$$

Surface area $= 30 \times 30 + 4 \times \frac{1}{2} \times 30 \times 25$

$$= 2400 \text{ cm}^2$$



9780170479622

PATH

3.06

Example 11

A rectangular pyramid with base 10 cm by 8 cm has a perpendicular height of 15 cm. Find its surface area, correct to one decimal place.

Solution

First find the slant heights AP and AQ.

$AP^2 = AX^2 + XP^2$

$= 15^2 + 4^2$ $\quad XP = \frac{1}{2} \times 8$

$= 241$

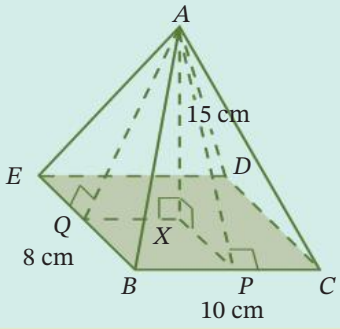
$AP = \sqrt{241}$ cm

$AQ^2 = AX^2 + XQ^2$

$= 15^2 + 5^2$ $\quad XQ = \frac{1}{2} \times 10$

$= 250$

$AQ = \sqrt{250}$ cm



It is better to leave the lengths of AP and AQ in surd form rather than round them to decimals so that the final answer is accurate.

Surface area = area of rectangle base + 2 × Area $\triangle ABC$ + 2 × Area $\triangle ADC$

$= 10 \times 8 + 2 \times \frac{1}{2} \times 10 \times \sqrt{241} + 2 \times \frac{1}{2} \times 8 \times \sqrt{250}$

$= 361.7328...$

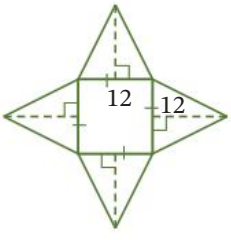
$\approx 361.7 \text{ cm}^2$

EXERCISE 3.06 ANSWERS ON P. 584

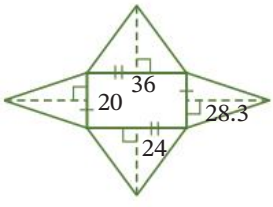
Surface area of a pyramid U F PS R

1 Find the area of each net and hence the surface area of the corresponding pyramid. All measurements are in centimetres.

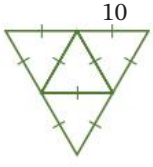
a



b



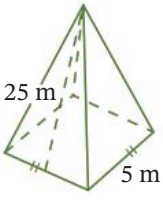
c



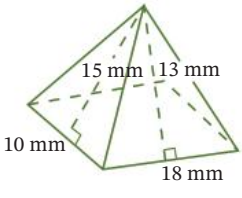
2 Find the surface area of each pyramid.

EXAMPLE 10

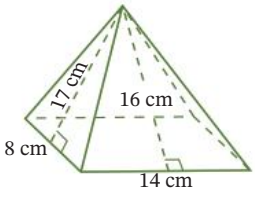
a



b



c

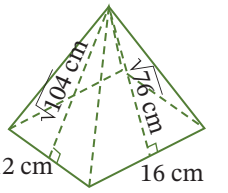


Foundation | Standard | Complex

PATH

3 Find the surface area of this pyramid.
Select the closest answer **A**, **B**, **C** or **D**.

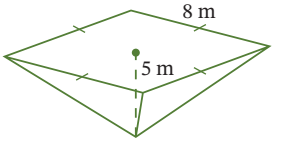
A 288.0 cm²
B 453.9 cm²
C 459.8 cm²
D 471.0 cm²



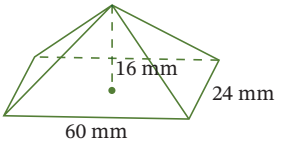
EXAMPLE 11

4 Calculate, correct to one decimal place, the surface area of each pyramid. R

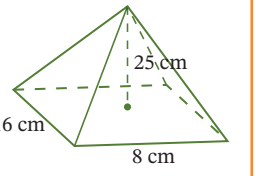
a



b

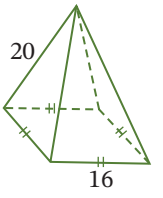


c

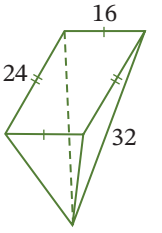


5 Calculate, correct to the nearest square centimetre, the surface area of each pyramid.
All measurements are in centimetres. PS R

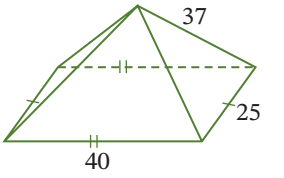
a



b



c



6 The great pyramid of Khufu (or Cheops) in Egypt has a height of 147 m, and each side of its square base measures 230 m. Find its surface area (excluding the base), correct to the nearest square metre. PS R

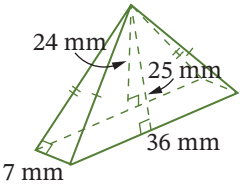
7 The Louvre Art Museum in Paris, France, has a large pyramid at its entrance (built in 1989). It reaches a height of 21.6, has a square base of length 34 m and its sides are covered in rhombus and triangular glass pieces. Calculate the surface area of the glass pyramid (to the nearest m³). PS R



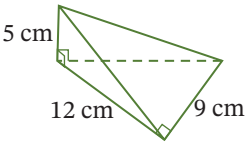
vichie81/Shutterstock.com

8 Calculate, correct to one decimal place, the surface area of each pyramid. PS R

a

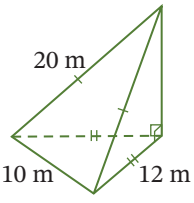


b



All faces are right-angled triangles

c



9 A square pyramid has a surface area of 4704 m² and a base area of 1764 m². Find: R

a the length of its base
b the area of each triangular face
c the slant height of each triangular face
d the perpendicular height of the pyramid.

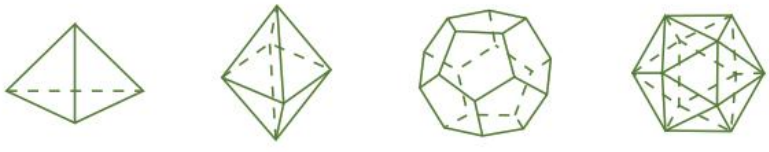
Foundation Standard Complex

9780170479622

Did you know?

The Platonic solids

The cube is called a **regular polyhedron** or **Platonic solid** because it has 6 identical faces. The more formal name for a cube is **regular hexahedron**, 'hex' meaning 6. There are only 6 possible regular polyhedrons: the other 5 are shown below. Each face on a regular polyhedron is a **regular polygon.** The regular tetrahedron, for example, is a pyramid of equilateral triangles.



Regular tetrahedron
4 faces

Regular octahedron
8 faces

Regular dodecahedron
12 faces

Regular icosahedron
20 faces

The tetrahedron, cube and octahedron occur in nature in the form of certain crystals.

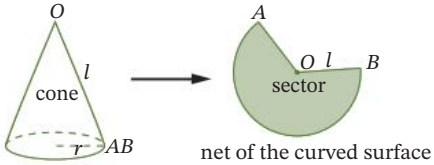
What are the shapes of the faces of each Platonic solid shown?

Investigation

PATH

The surface area of a cone

The net of a cone is made up of a circle (for the base) and a sector of a circle (for the curved surface). The second diagram below shows the curved surface of a cone.



cone

net of the curved surface of the cone

We can use this fact to find a formula for the curved surface area of a cone. Suppose the cone has base radius r and slant height l. Then the sector must have radius l.

Looking at the sector, the major arc length AB is a fraction of the circumference of the circle and the area of the sector is a fraction of the area of the circle. They should be the same fraction, so:

$$\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Arc length}}{\text{Circumference}}$$

1 The major arc length AB is the same as the circumference AB of the base of the cone in the first diagram. Write an algebraic expression for the circumference of the circle in the first diagram.

2 Write an algebraic expression for the circumference of the complete circle in the second diagram.

3 Write an algebraic expression for the *area* of the complete circle in the second diagram.

4 $\frac{\text{Area of sector}}{\text{Area of circle}} = \frac{\text{Arc length}}{\text{Circumference}}$ becomes $\frac{\text{Area of sector}}{\pi l^2} = \frac{2\pi r}{2\pi l}$

Copy and complete: $\therefore$ Area of sector $= \frac{2\pi r}{2\pi l} \times \pi l^2 =$ _____

PATH

5 But the area of the sector is equal to the curved surface area of the cone.

Copy and complete:

Surface area of cone = area of circular base + area of curved surface

= _____ + _____

3.07 Surface areas of cones and spheres

PATH

Surface area of a cone

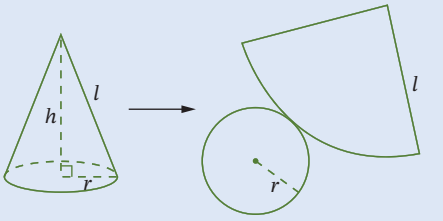
$$SA = \pi r^2 + \pi rl$$

where r = radius of circular base

l = slant height

The area of the circular base = πr^2

and the area of the curved surface = πrl

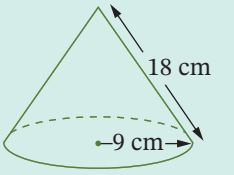


Example 12

For this cone, find correct to one decimal place:

a the curved surface area

b the total surface area.



Solution

$r = 9$ cm and $l = 18$ cm

a Curved surface area = πrl

$= \pi \times 9 \times 18$

$= 508.9380...$

≈ 508.9 cm^2

b Total surface area = $\pi r^2 + \pi rl$

$= \pi \times 9^2 + \pi \times 9 \times 18$

$= 763.4070...$

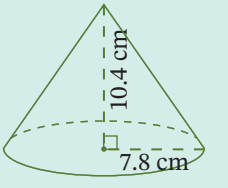
≈ 763.4 cm^2

Videos
Surface area of a cone and sphere

Surface area and volume of a cone

Example 13

Find, correct to 2 decimal places, the surface area of this cone.



Solution

First calculate the slant height:

$l^2 = 7.8^2 + 10.4^2$

$= 169$

$l = \sqrt{169}$

$= 13$

Surface area = $\pi r^2 + \pi rl$

$= \pi \times 7.8^2 + \pi \times 7.8 \times 13$

$= 509.6919...$

≈ 509.69 cm^2

A **sphere** is a ball shape, completely round.
A **hemisphere** is half a sphere.

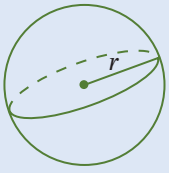
PATH

3.07

Surface area of a sphere

$SA = 4\pi r^2$

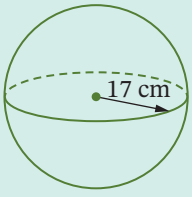
where r = radius of the sphere



Note that the surface area of a sphere is 4 times the area of a circle that shares its centre.

Example 14

Find, correct to 2 decimal places, the surface area of this sphere.



Video
Surface area of a cone and sphere

Solution

Surface area $= 4\pi r^2$

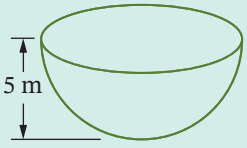
$= 4 \times \pi \times 17^2$

$= 3631.6811...$

$= 3631.68 \text{ cm}^2$

Example 15

Find the surface area of this closed hemisphere in exact form, in terms of π.



Solution

Surface area = Area of circular base + Curved surface area

$= \pi r^2 + \frac{1}{2} \times 4\pi r^2$

$= \pi r^2 + 2\pi r^2$

$= 3\pi r^2$

$= 3 \times \pi \times 5^2$

$= 75\pi \text{ m}^2$

Answers written as surds or in terms of π are exact because they are not decimal approximations.

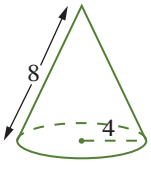
EXERCISE 3.07 ANSWERS ON P. 584

Surface areas of cones and spheres U F PS R C

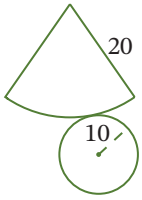
EXAMPLE 12

1 Calculate, correct to the nearest cm^2, the curved surface area of each cone.
All measurements are in centimetres.

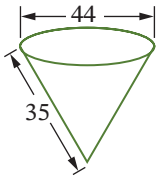
a



b



c

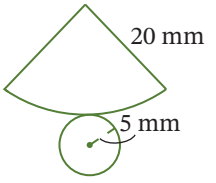


Foundation | Standard | Complex

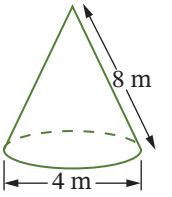
PATH

2 Find, correct to one decimal place, the total surface area of each cone.

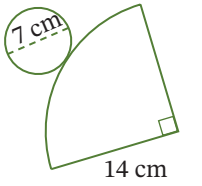
a



b



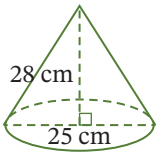
c



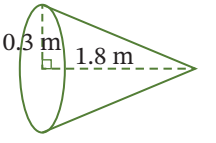
EXAMPLE 13

3 Calculate, correct to one decimal place, the surface area of each cone. R

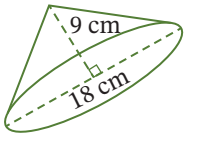
a



b

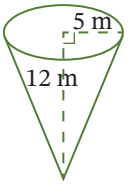


c

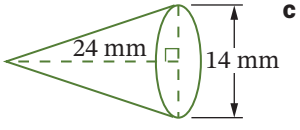


4 Find in exact form (in terms of π) the total surface area of each cone. R C

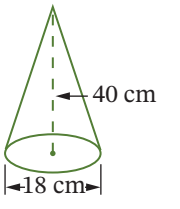
a



b



c



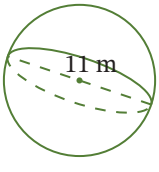
EXAMPLE 14

5 Calculate, correct to one decimal place, the surface area of each sphere.

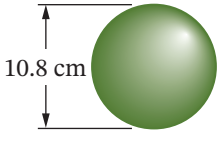
a



b



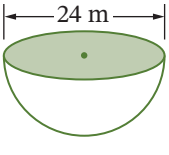
c



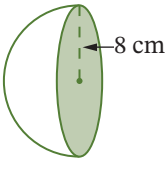
EXAMPLE 15

6 Find in exact form the surface area of each closed hemisphere. R C

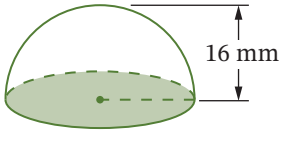
a



b



c



7 Calculate the surface area of a closed hemisphere with radius 1.4 m.
Select the closest answer **A**, **B**, **C** or **D**. R

A 12.32 m² **B** 18.47 m² **C** 24.63 m² **D** 49.26 m²

8 Find, correct to the nearest square metre, the surface area of: PS R

a a sphere with diameter 10 m

b an open cone with base radius 10 m, slant height 20 m

c an open hemisphere with radius 10 m

d a cone with base diameter 10 m, perpendicular height 20 m.

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PATH

3.07

9 The Earth has a radius of approximately 6378 km. PS R C

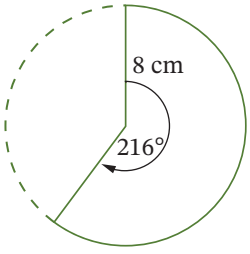
a Calculate the surface area of the Earth in scientific notation, correct to 3 significant figures.

b If 70.9% of the Earth's surface is covered by water, calculate the land area of the Earth's surface, correct to 3 significant figures.

c The land area of Australia is 7 692 000 km^2. Calculate Australia's land area as a percentage of the Earth's land area (correct to 2 decimal places).

10 Find the amount of sheet metal needed to form a conical funnel of base radius 30 cm and vertical height 50 cm, allowing for a 0.5 cm overlap at the join. PS R C

11 A cone is made from a sector of a circle with radius 8 cm and central angle 216°. Find: PS R C



a the length of the arc of the circle which forms the circumference of the cone's base

b the radius of the cone's base

c the slant height of the cone

d the total surface area of the cone, including the base.

12 Find the radius of each solid if it has a surface area of 6000 mm^2. Give your answer correct to 3 significant figures. R

a a sphere **b** a closed hemisphere **c** an open hemisphere

13 A cone has a surface area of 2000 cm^2. If the area of its base is 150 cm^2, find: R

a the radius of its base **b** its slant height **c** its perpendicular height

Mental skills 3: Maths without calculators ANSWERS ON P. 584

Quiz
Mental skills 3

Time differences

1 Study each example.

a What is the time difference between 11:40 a.m. and 6:15 p.m.?

From 11:40 a.m. to 5:40 p.m. = 6 hours

Count: '11:40, 12:40, 1:40, 2:40, 3:40, 4:40, 5:40'

From 5:40 a.m. to 6:00 p.m. = 20 min

From 6:00 p.m. to 6:15 p.m. = 15 min

6 hours + 20 min + 15 min = 6 hours 35 min

OR:



Foundation Standard Complex

b What is the time difference between 20:30 and 01:20?

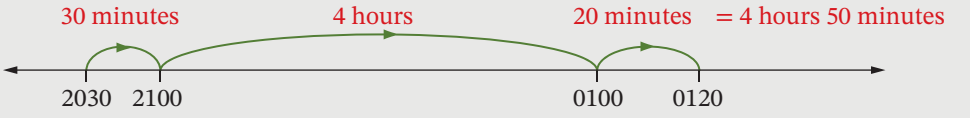
From 20:30 to 00:30 = 4 hours (24 − 20 = 4)

From 00:30 to 01:00 = 30 min

From 01:00 to 01:20 = 20 min

4 hours + 30 minutes + 20 minutes = 4 hours 50 minutes

OR:



2 Now find the time difference between:

a 11:10 a.m. and 7:40 p.m.

b 6:20 p.m. and 12:00 midnight

c 4:45 p.m. and 8:10 p.m.

d 2:35 a.m. and 10:50 a.m.

e 1:05 p.m. and 12:30 a.m.

f 9:35 a.m. and 11:15 a.m.

g 04:25 and 09:35

h 14:40 and 20:25

i 7:55 a.m. and 3:50 p.m.

j 2:40 p.m. and 10:20 p.m.

3.08 Surface areas of composite solids

Video
The power of the Sun

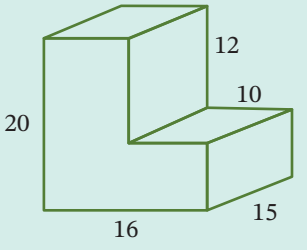
Worksheet
A page of prisms and cylinders

When calculating surface areas of composite solids, remember **not** to include the areas common to both solids.

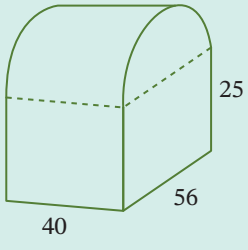
Example 16

Find, correct to one decimal place, the surface area of each solid. All measurements are in centimetres.

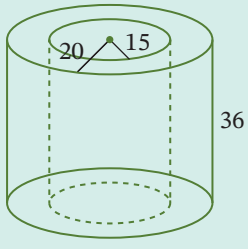
a



b

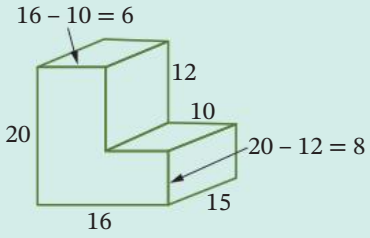


c



Solution

a This prism has 8 faces: 2 identical L-shapes (front and back) and 6 different rectangles.



Area of L-shape $= 16 \times 20 - 10 \times 12$

$= 200 \text{ cm}^2$

Surface area $= (2 \times 200) + (6 \times 15) + (12 \times 15) + (10 \times 15) + (8 \times 15) + (16 \times 15) + (20 \times 15)$

$= 1480 \text{ cm}^2$

Note that the 6 rectangles can also be thought of as one long rectangle of width 15 cm.

Length of long rectangle = perimeter of L = 72

Surface area $= (2 \times 200) + (72 \times 15)$

$= 1480 \text{ cm}^2$

b The solid is made up of a half-cylinder (3 faces) and a rectangular prism (5 faces).

Surface area of half-cylinder
= 2 semi-circular ends + curved surface area

$= 2 \times \frac{1}{2} \times \pi \times 28^2 + \frac{1}{2} \times 2 \times \pi \times 28 \times 40$

$= 5981.5924... \text{ cm}^2$

Radius of semi-circle $= \frac{1}{2} \times 56$

$= 28$

Do not round this partial answer, or the final answer will be inaccurate.

Surface area of rectangular prism $= (2 \times 40 \times 25) + (2 \times 56 \times 25) + (40 \times 56)$

$= 7040 \text{ cm}^2$

Total surface area $= 5981.5924... + 7040$

$= 13\,021.5924...$

$\approx 13\,021.6 \text{ cm}^2$

c The hollow cylinder is made up of 2 annulus (ring) faces, an outside curved surface area and an inside curved surface area.

Surface area of annulus faces $= 2 \times (\pi \times 20^2 - \pi \times 15^2)$ $\quad 2 \times$ area between 2 circles

$= 1099.5574... \text{ cm}^2$

Outside curved surface area $= 2 \times \pi \times 20 \times 36$

$= 4523.8334... \text{ cm}^2$

Inside curved surface area $= 2 \times \pi \times 15 \times 36$

$= 3392.9200... \text{ cm}^2$

Total surface area $= 1099.5574... + 4523.8934... + 3392.9200...$

$= 9016.3709...$

$= 9016.4 \text{ cm}^2$

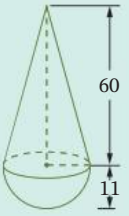
3.08

PATH

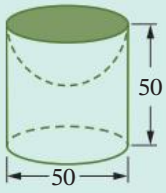
Example 17

Find, correct to the nearest square centimetre, the surface area of each solid.
All measurements are in centimetres.

a



b



Solution

a Surface area = curved surface area of cone + curved surface area of hemisphere

$$= \pi rl + \frac{1}{2} \times 4\pi r^2$$

$r = 11$, $h = 60$ and $l = ?$

$l^2 = 11^2 + 60^2$

$= 3721$

$l = \sqrt{3721}$

$= 61$ cm

Surface area $= \pi \times 11 \times 61 + \frac{1}{2} \times 4 \times \pi \times 11^2$

$= 2868.2740 \ldots$

≈ 2868 cm^2

b Surface area = curved surface of cylinder + circular base + curved surface of hemisphere

$= 2\pi rh + \pi r^2 + \frac{1}{2} \times 4\pi r^2$

$= 2\pi rh + 3\pi r^2$ $\qquad \pi r^2 + 2\pi r^2 = 3\pi r^2$

$= 2 \times \pi \times 25 \times 50 + 3 \times \pi \times 25^2$ $\qquad r = \frac{1}{2} \times 50 = 25$

$= 13\,744.4678 \ldots$

$\approx 13\,744$ cm^2

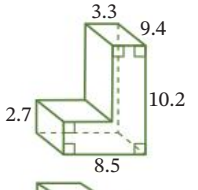
EXERCISE 3.08 ANSWERS ON P. 584

Surface areas of composite solids U F PS R

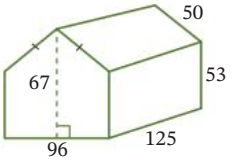
EXAMPLE 16

1 Find the surface area of each solid. All measurements are in centimetres. PS R

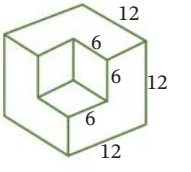
a



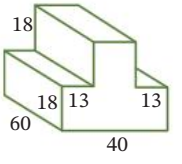
b



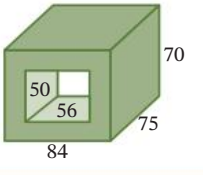
c



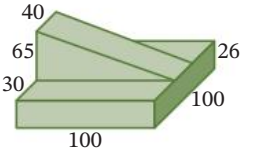
d



e



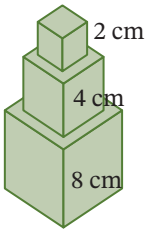
f



Foundation Standard Complex

9780170479622

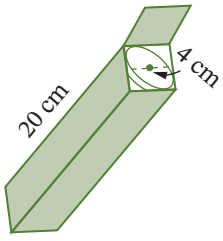
2 3 cubes of length 2 cm, 4 cm and 8 cm are glued on top of each other. Calculate the surface area of the new solid. PS R



3 Circular cracker biscuits of diameter 4 cm are packed in a cardboard box of length 20 cm.

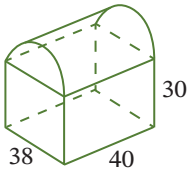
a Calculate the surface area of the box.

b How much cardboard would be saved if the biscuits were packed into a cylindrical box?

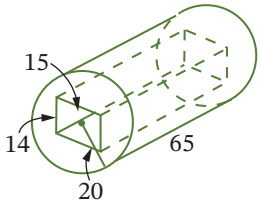


4 Find, correct to one decimal place, the surface area of each solid. All measurements are in centimetres. PS R

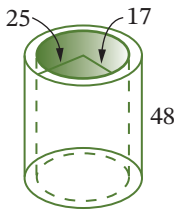
a



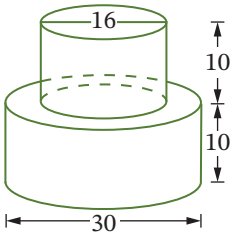
b



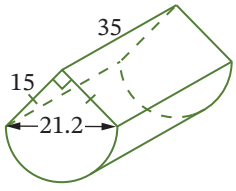
c



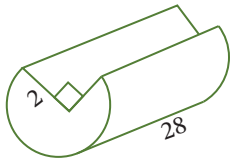
d



e



f

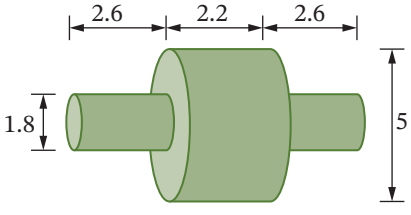


5 Calculate the surface area of the solid, correct to one decimal place. All measurements are in centimetres. R

Select the correct answer **A**, **B**, **C** or **D**.

A 86.0 cm^2 **B** 103.2 cm^2

C 108.3 cm^2 **D** 113.4 cm^2



Foundation Standard Complex

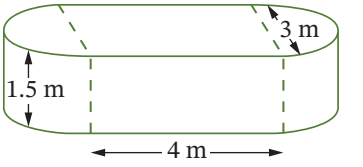
6 A wedding cake with 3 tiers rests on a table. Each tier is 6 cm high. The layers have radii of 20 cm, 15 cm and 10 cm respectively. Find the total visible surface area, correct to the nearest cm². PS R



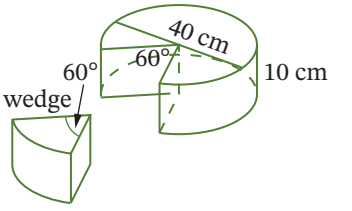
Shutterstock.com/Brian Barney

7 **a** Find, correct to 2 decimal places, the total external area of the wall of this above-ground swimming pool.

b Calculate the area of the water surface, correct to the nearest m². PS R



8 A wedge of cheese is cut from a cylindrical block of height 10 cm and diameter 40 cm. Find the total surface area of the wedge, correct to 2 decimal places. PS R



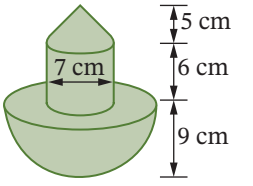
PATH

EXAMPLE 17

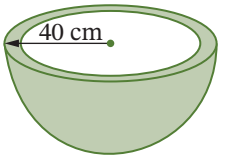
9 Calculate the surface area of the solid. Select **A**, **B**, **C** or **D**. PS R

A 923.98 cm² **B** 962.46 cm²

C 736.62 cm² **D** 669.51 cm²



10 Calculate the total surface of a hemispherical bird bath that is 1 cm thick and has an outer radius of 40 cm. PS R

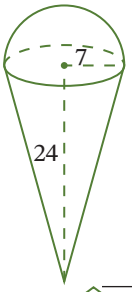


Foundation Standard Complex

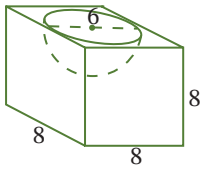
9780170479622

11 Find, correct to one decimal place, the surface area of each solid. All measurements are in centimetres. PS R

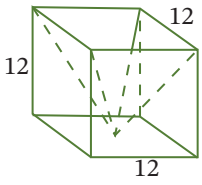
a



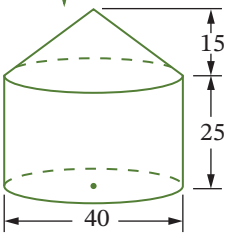
b



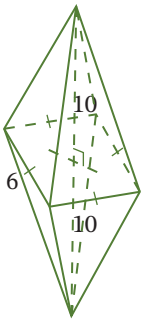
c



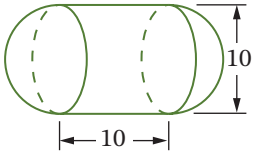
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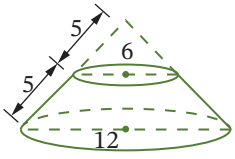
e



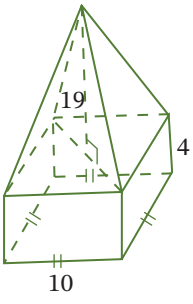
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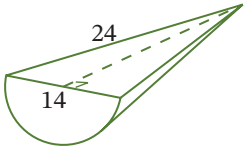
g



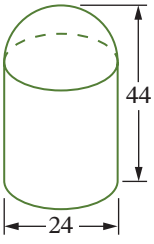
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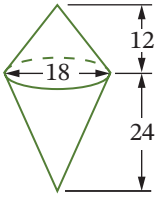
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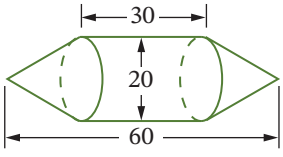
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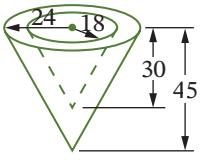
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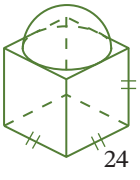
l



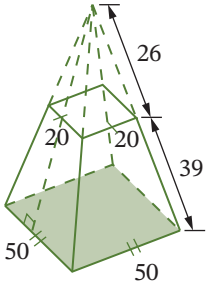
m



n



o



Foundation | Standard | Complex

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3.09 Volumes of prisms and cylinders

Worksheets
A page of prisms and cylinders
Volumes of solids
Back-to-front problems
Volume and capacity
Biggest volume

Technology
Volume of a box

Spreadsheet
Volume of a box

Puzzle
Formula matching game

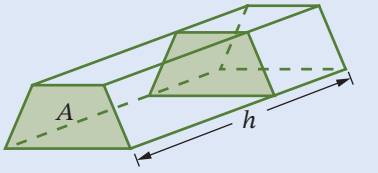
Videos
Capacity of a cylinder
Volumes of prisms and cylinders
Bees and their hives
The incredible strength of ants
The Menger sponge

Volume of a prism

$V = Ah$

where A = area of base

h = perpendicular height

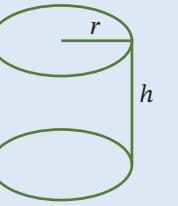


Volume of a cylinder

$V = \pi r^2 h$

where r = radius of circular base

h = perpendicular height



Volume and capacity

1 cm^3 contains 1 mL

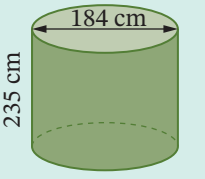
1 m^3 contains 1000 L or 1 kL

Example 18

For this cylinder, calculate:

a its volume correct to the nearest cm^3

b its capacity in kL, correct to 1 decimal place.



Solution

a Radius $= \frac{1}{2} \times 184$

$= 92$ cm

$V = \pi \times 92^2 \times 235$

$= 6\,248\,753.451...$

$\approx 6\,248\,753$ cm^3

b Capacity $= 6\,248\,753$ mL

$= 6\,248\,753 \div 1000 \div 1000$ kL

$= 6.248\,753$ kL

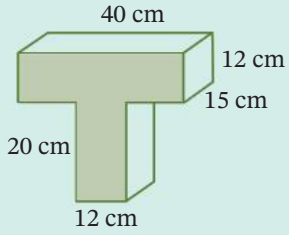
≈ 6.2 kL

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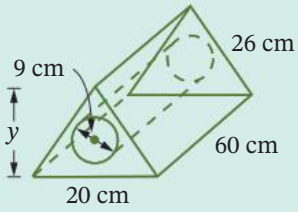
Example 19

Find, correct to the nearest whole number, the volume of each solid.

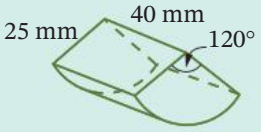
a



b



c



Solution

a

$A = 40 \times 12 + 20 \times 12$

$= 720 \text{ cm}^2$

$V = Ah$

$= 720 \times 15$

$= 10\,800 \text{ cm}^3$

b Cross-section is a triangle minus a circle.

$26^2 = y^2 + 10^2$

$y^2 = 26^2 - 10^2$

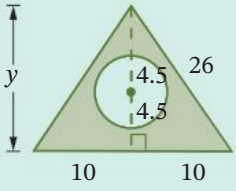
$= 576$

$y = 24 \text{ cm}$

Radius of circle $= \frac{1}{2} \times 9 = 4.5$

$A = \frac{1}{2} \times 20 \times 24 - \pi \times 4.5^2$

$= 176.3827... \text{ cm}^2$



$V = Ah$

$= 176.3827... \times 60$

$= 10\,582.9649...$

$\approx 10\,583 \text{ cm}^3$

c

$A = \frac{120}{360} \times \pi \times 25^2$

$= 654.498... \text{ mm}^2$

$V = Ah$

$= 654.498... \times 40$

$= 26\,179.938...$

$\approx 26\,180 \text{ mm}^2$

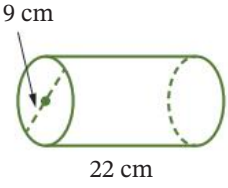
EXERCISE 3.09 ANSWERS ON P. 584

Volumes of prisms and cylinders UFR

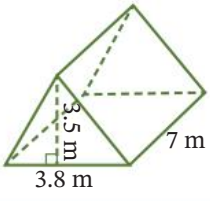
1 Calculate, correct to one decimal place, the volume of each solid.

EXAMPLE 18

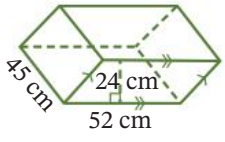
a



b

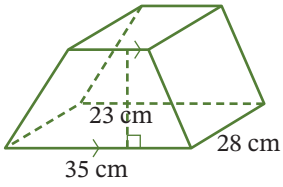


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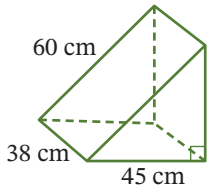


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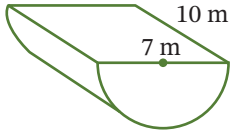
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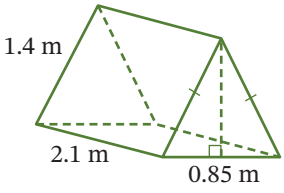
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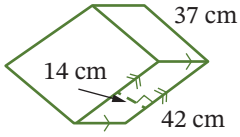
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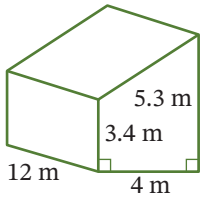
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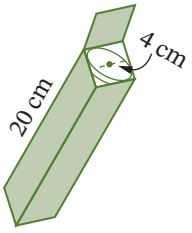
h



i

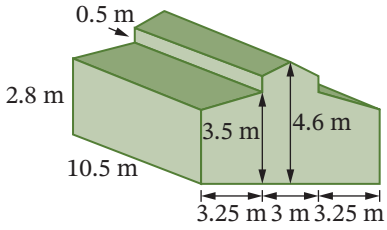


2 Rice crackers of diameter 4 cm are packed in a cardboard box of height 20 cm. Calculate, correct to one decimal place:

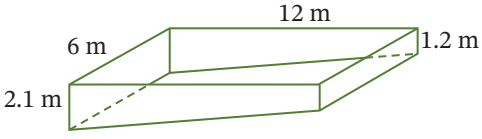


- **a** the volume of the crackers in the box
- **b** the volume of the box
- **c** the percentage of the box that is empty space.

3 Calculate, correct to one decimal place, the volume of the shed. R



4 This swimming pool is 12 m long and 6 m wide. The depth of the pool ranges from 1.2 m to 2.1 m.

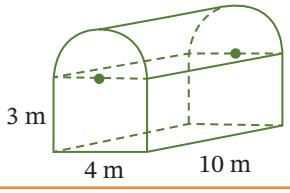


- **a** Calculate the capacity of this pool in litres.
- **b** If the pool is filled so that the water is 8 cm from the top of the pool, calculate the amount of water in the pool (to the nearest litre).

5 An Olympic sized swimming pool is 50 m long, 25 m wide and 2 m deep. What is the capacity of an Olympic pool in litres?

6 **a** Find, correct to 2 decimal places, the volume of this greenhouse.

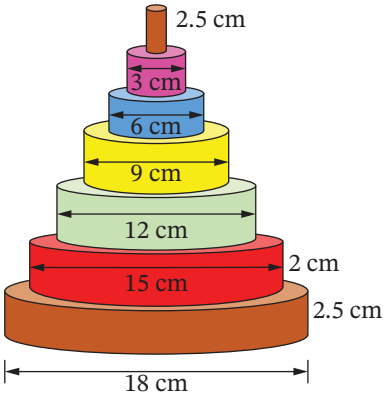
b If this greenhouse costs 0.5c per m^3 per hour to heat, how much is this per day (correct to the nearest cent)?



Foundation Standard Complex

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7 A baby's toy involves placing 5 coloured cylinders on a wooden stick (diameter 1 cm) on a base (diameter 18 cm). The cylinders are of varying diameters with a height of 2 cm. Find the volume of the toy, including the base and peg, correct to one decimal place. R

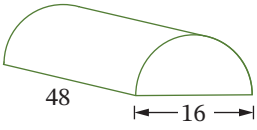


3.09

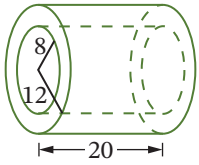
EXAMPLE 19

8 Find, correct to 2 decimal places where appropriate, the volume of each solid. All lengths shown are in centimetres. R

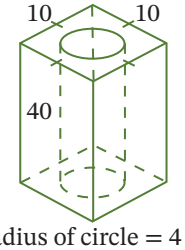
a



b

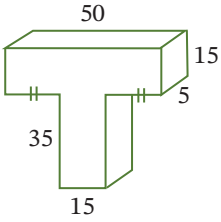


c

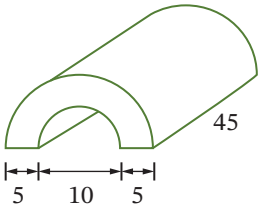


radius of circle = 4 cm

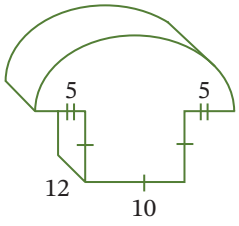
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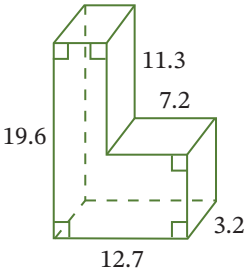
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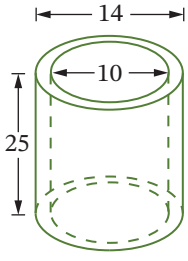
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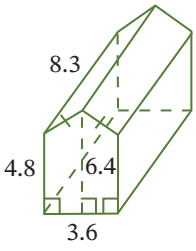
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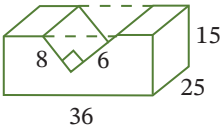
h



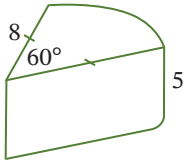
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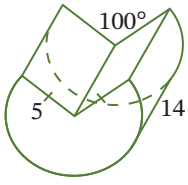
j



k



l



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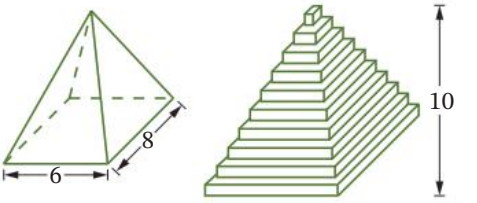
PATH

Technology
Approximating the volume of a cone

Technology

Approximating the volume of a pyramid

In this activity, we will use a spreadsheet to approximate the volume of a rectangular pyramid by slicing it into tiny layers of rectangular prisms of equal thickness.



Let $L = 6$ be the length of the prism, $W = 8$ be the width and $H = 10$ be the height.

The thickness, T, of each layer is given by the formula $T = \frac{H}{\text{number of layers}}$.

Starting at the bottom, the length and width of each layer are decreased by the amounts $\frac{L}{\text{number of layers}}$ and $\frac{W}{\text{number of layers}}$ with each step.

1 Set up your spreadsheet as shown.

	A	B	C	D	E	F
1						
2			Number of layers =			
3	H	L	W	Thickness of layers	Volume of layer	Sum of volumes
4	10	6	8	=$A$4/$D$2	=B4*C4*D4	=E4
5		=B4-$B$4/$D$2	=$C4-$C$4/$D$2			=E5+F4
6						
7						

2 Let the number of layers be 10. Enter 10 in cell D2.

3 Copy each formula down to row 13.

4 Explain the results in cells E13 and F13.

5 How accurate was your result in F13? Explain.

6 Print out your spreadsheet and paste it into your book.

7 Enter 40 (layers) in cell D2 and copy each formula down to row 43.

8 In 1–2 sentences compare your result in F43 with the previous result in F13 from question 4.

9 Enter each value in cell D2 and copy down the formulas as requested.

a 100 (copy down to row 103)

b 200 (copy down to row 203)

c 400 (copy down to row 403)

10 Use the formula $V = \frac{1}{3}Ah$ to calculate the exact volume of the pyramid.

11 Write a brief report about your results in questions **9** and **10**.

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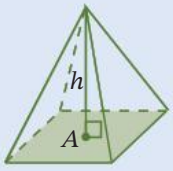
Volumes of pyramids, cones and spheres 3.10

Volume of a pyramid

$$V = \frac{1}{3}Ah$$

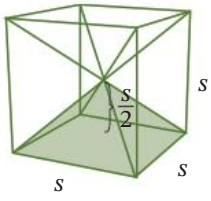
where A = area of the base

h = perpendicular height.



The volume of a pyramid is $\frac{1}{3}$ of the volume of a prism with the same base and height.

This fact is difficult to prove mathematically, but here's a diagram that might help. This is a square pyramid with dimensions s, s and $\frac{s}{2}$, with an identical pyramid balancing on top of it upside-down. Now look at the big cube that surrounds these 2 square pyramids.

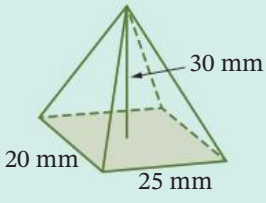


- Can you see how many such pyramids fit into this big cube exactly?
- What is the volume of the big cube?
- So what is the volume of one pyramid?
- Is this the same as $V = \frac{1}{3}Ah$?

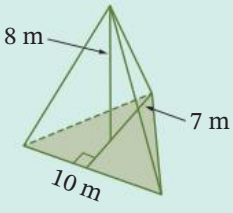
Example 20

Find the volume of each pyramid.

a



b



Solution

a

$A = 20 \times 25$

$= 500$

$V = \frac{1}{3}Ah$

$= \frac{1}{3} \times 500 \times 30$

$= 5000 \text{ mm}^3$

b

$A = \frac{1}{2} \times 10 \times 7$

$= 35$

$V = \frac{1}{3}Ah$

$= \frac{1}{3} \times 35 \times 8$

$= 99\frac{1}{3} \text{ m}^3$

PATH

3.10

Video
The Pacific Flyer

Worksheet
Back-to-front problems (Advanced)

Video
Volume of a pyramid

PATH

Example 21

Find the volume of a square pyramid with base length 22 mm and slant height 61 mm.

Solution

First find h, the perpendicular height of the pyramid.

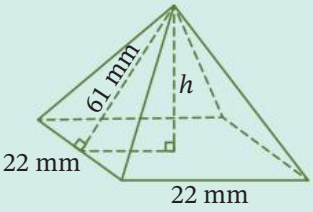
$h^2 = 61^2 - 11^2 = 3600$

$h = 60$ mm

$A = 22^2 = 484$

$V = \frac{1}{3} \times 484 \times 60$

$= 9680$ mm^3



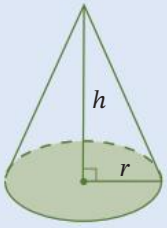
Volume of a cone

Video
Surface area and volume of a cone

$$V = \frac{1}{3}\pi r^2 h$$

where r = radius of the base

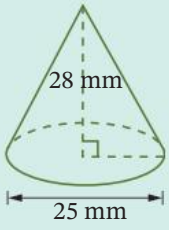
h = perpendicular height.



A cone is like a 'circular pyramid' so $V = \frac{1}{3}Ah = \frac{1}{3} \times \pi r^2 \times h = \frac{1}{3}\pi r^2 h$

Example 22

Find, correct to the nearest cubic centimetre, the volume of this cone.



Solution

$V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 12.5^2 \times 28$ $\quad r = \frac{1}{2} \times 25 = 12.5$

$= 4581.4892...$

≈ 4581 cm^3

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Example 23

A cone has a slant edge of 50 mm and a base radius of 15 mm.
Find its volume correct to 2 significant figures.

Solution

$h^2 = 50^2 - 15^2$

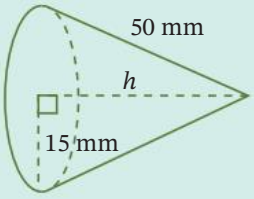
$= 2275$

$h = \sqrt{2275}$

$V = \frac{1}{3} \times \pi \times 15^2 \times \sqrt{2275}$

$= 11\,238.3314...$

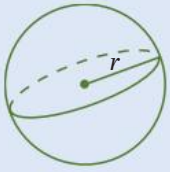
$\approx 11\,000 \text{ mm}^3$



Volume of a sphere

$$V = \frac{4}{3}\pi r^3$$

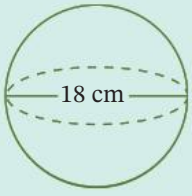
where r = radius of the sphere.



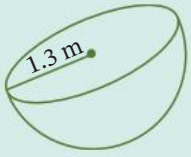
Example 24

Find, correct to 2 significant figures, the volume of each solid.

a



b



Solution

a $V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 9^3$ $\quad r = \frac{1}{2} \times 18 = 9$

$= 3053.6280...$

$\approx 3100 \text{ cm}^3$

b $V = \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \pi \times 1.3^3$

$= 4.6013...$

$\approx 4.6 \text{ m}^3$

Video
Volume of a sphere

PATH

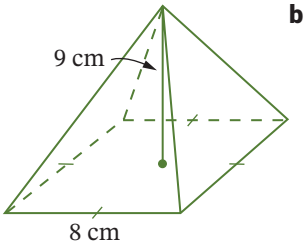
EXERCISE 3.10 ANSWERS ON P. 584

Volumes of pyramids, cones and spheres U F PS R C

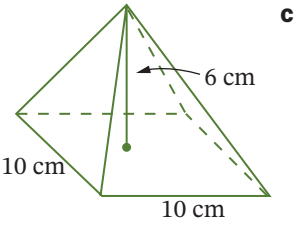
EXAMPLE 20

1 Find, correct to one decimal place, the volume of each pyramid.

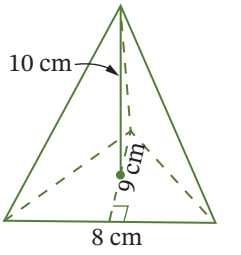
a



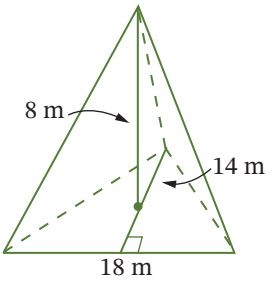
b



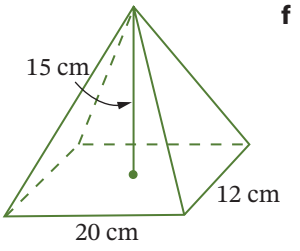
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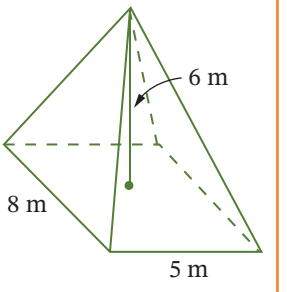
d



e



f

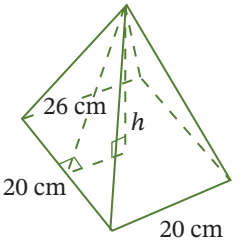


EXAMPLE 21

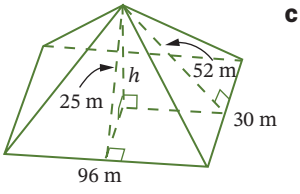
2 For each pyramid, find correct to one decimal place: R

i its perpendicular height **ii** its volume

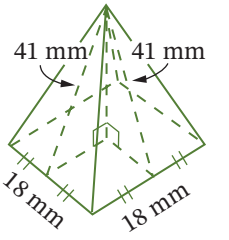
a



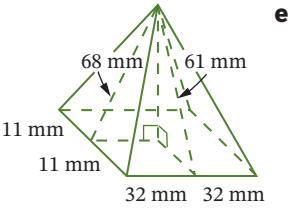
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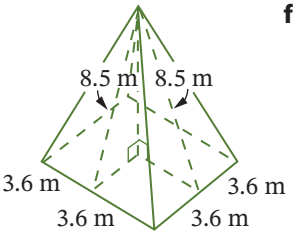
c



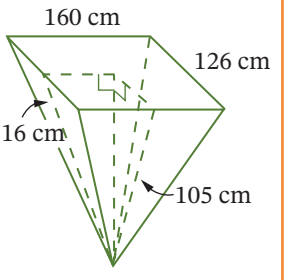
d



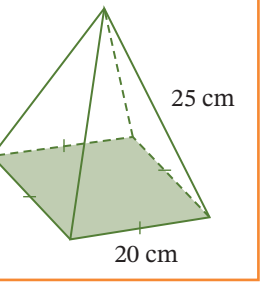
e



f



3 Calculate the volume of this square pyramid.
Select the closest answer **A**, **B**, **C** or **D**. R



A 3055.1 cm^3 **B** 2000 cm^3

C 3333.3 cm^3 **D** 2748.7 cm^3

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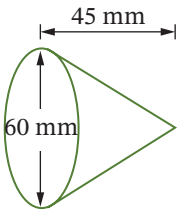
PATH

EXAMPLE 22

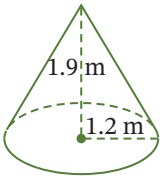
3.10

4 Find, correct to the nearest whole number, the volume of each cone.

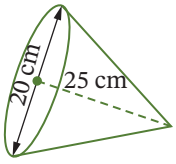
a



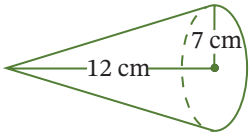
b



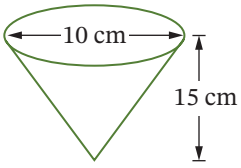
c



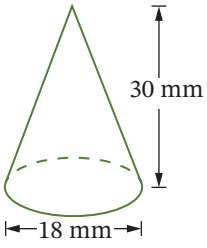
d



e



f

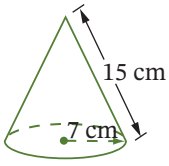


EXAMPLE 23

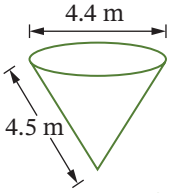
5 For each cone, find correct to one decimal place: R

i its perpendicular height **ii** its volume

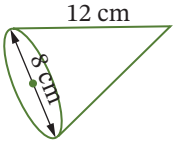
a



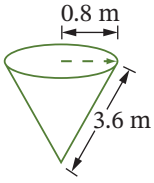
b



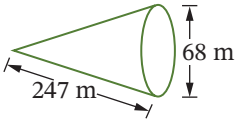
c



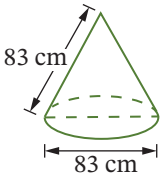
d



e



f



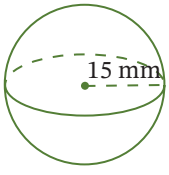
6 Find the volume of a cone with a base diameter of 5.4 cm and a slant height of 9.8 cm. Select **A**, **B**, **C** or **D**. R

A 75 cm^3 **B** 72 cm^3 **C** 288 cm^3 **D** 111.1 cm^3

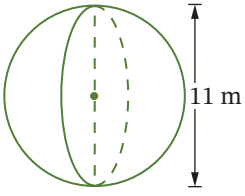
EXAMPLE 24

7 Find the volume of each sphere or hemisphere, correct to one decimal place.

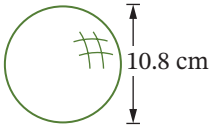
a



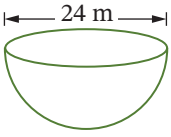
b



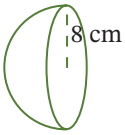
c



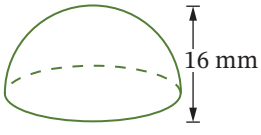
d



e



f

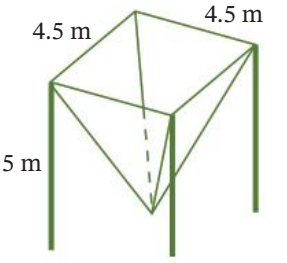


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8 The Earth has a radius of approximately 6378 km. Calculate its volume in scientific notation, correct to 3 significant figures. C

9 A grain hopper is in the shape of a square pyramid. PS R C



a Find the volume of grain it holds when full.

b If there are 750 kg of wheat per m^3, find the mass of grain in the hopper when it is filled to three-quarters of capacity. Give your answer correct to the nearest tonne.

10 A pyramid has a volume of 360 m^3 and a base area of 48 m^2. Calculate its perpendicular height. R

11 A square pyramid has a volume of 800 cm^3 and a perpendicular height of 12 cm. Calculate, correct to one decimal place, the length of its base. R

12 A cone has a volume of 600 m^3 and a base radius of 10 m. Calculate, correct to 1 decimal place, its perpendicular height. R

13 A cone has a volume of 160 cm^3 and a perpendicular height of 20 cm. Calculate, correct to one decimal place, its radius. R

14 The volume of a spherical light is 2664 cm^3. Calculate its diameter. Select **A**, **B**, **C** or **D**. R

A 17.2 cm **B** 8.6 cm **C** 17.0 cm **D** 8.5 cm

3.11 Volumes of composite solids

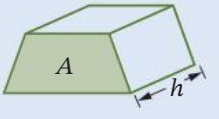
PATH

Presentation
Volumes of shapes

Volume formulas

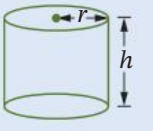
Prism

$V = Ah$



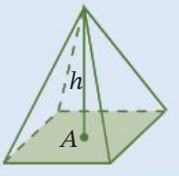
Cylinder

$V = \pi r^2 h$



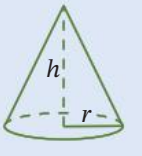
Pyramid

$V = \frac{1}{3}Ah$



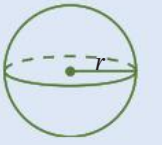
Cone

$V = \frac{1}{3}\pi r^2 h$



Sphere

$V = \frac{4}{3}\pi r^3$



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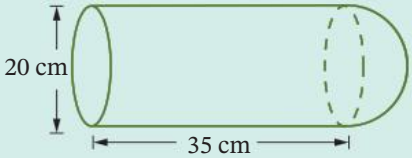
Note that the formulas for **surface area** involve **2 dimensions**, for example, r^2 or rh, while the formulas for **volume** involve **3 dimensions**, for example, r^2h or r^3 or Ah.

Example 25

For this packing capsule, find correct to the nearest whole number:

a its volume

b its capacity in litres



Solution

a Volume $= \pi r^2 h + \frac{1}{2} \times \frac{4}{3}\pi r^3$

$= \pi \times 10^2 \times 35 + \frac{2}{3} \times \pi \times 10^3$

$= 13\,089.9693...$

$\approx 13\,090 \text{ cm}^3$

b Capacity $\approx 13\,090$ mL

$= 13.09$ L

≈ 13 L

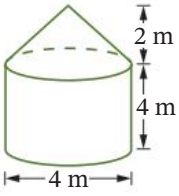
EXERCISE 3.11 ANSWERS ON P. 584

Volumes of composite solids U F PS R

1 This storage tank shown is completely filled with water.

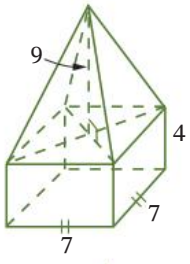
a Calculate to the nearest cubic metre the volume of the tank.

b Find the capacity of the tank, correct to the nearest kilolitre.

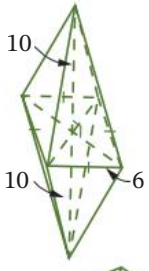


2 Find the volume of each solid. All measurements are centimetres. R

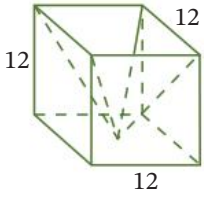
a



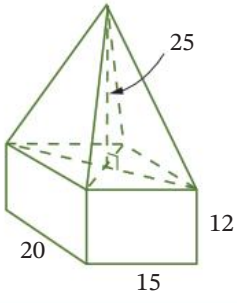
b



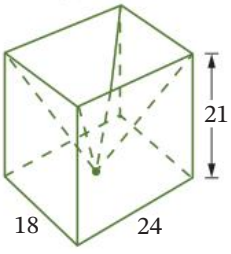
c



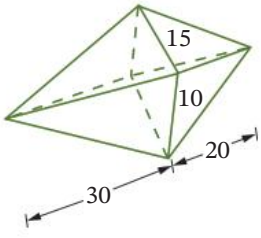
d



e



f



Foundation | Standard | Complex

PATH

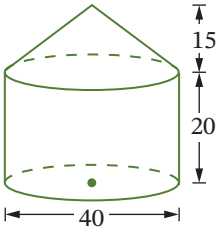
3 For each solid, find:

i the volume (to the nearest cm^3)

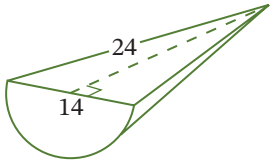
ii the capacity (in litres, correct to 3 decimal places).

All measurements are in centimetres.

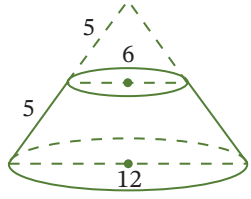
a



b

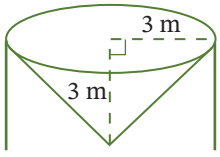


c

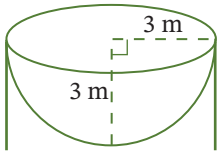


4 A conical tank (A) and a hemispherical tank (B) have measurements as shown. How much more does tank B hold compared to tank A? Answer correct to 2 decimal places.

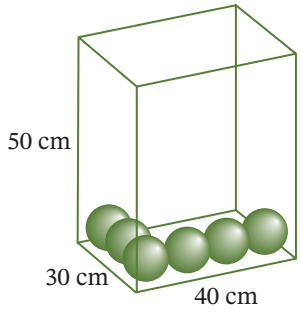
A



B



5 Spherical balls of diameter 10 cm are stacked inside a box as shown. PS R



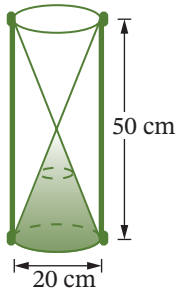
a How many balls will fit in the bottom layer?

b If the balls are stacked in the same manner as the bottom layer until the box is full, how many balls will fit in the box?

c Calculate the volume of the space occupied by the balls when the box is full.

d What percentage of the box is empty space? Give your answer to the nearest whole percentage.

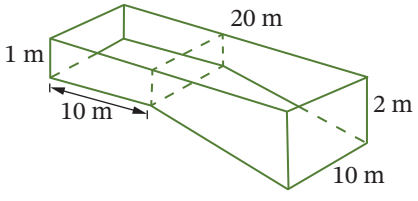
6 The sand in this hourglass takes up $\frac{3}{4}$ of the volume of the bottom cone. PS R



a Calculate, correct to the nearest cubic centimetre, the volume of sand in the hourglass.

b If the sand takes one hour to fall from the top cone to the bottom cone, at what rate is it falling? Give your answer in cm^3/s, correct to 2 significant figures.

7 **a** Calculate the volume of this swimming pool. PS R



b Calculate the capacity of the pool if it is filled to a depth 20 cm from the top.

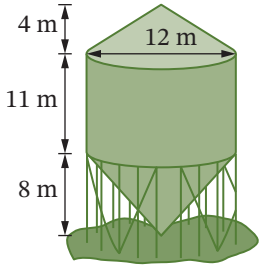
c If water costs \$1.98/kL, find the cost of filling the pool.

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8 **a** Find the capacity of the silo shown, correct to the nearest m^3. PS R

b How many tonnes of barley grain will the silo hold if the density of barley grain is 600 kg/m^3?



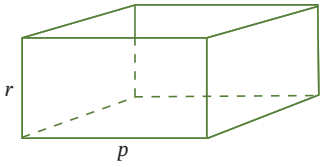
Power plus ANSWERS ON P. 584

1 The total surface area of a cube is 864 cm^2. Find its volume.

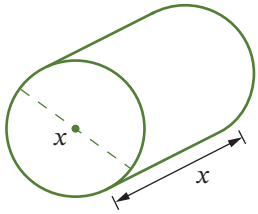
2 A cylinder has a volume of 3619.11 cm^3. Its height is 18 cm. Calculate to the nearest centimetre the radius of its base.

3 Find a formula for the surface area, SA, of:

a this open square prism of base length p and height r

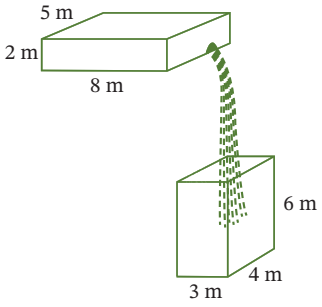


b this cylinder of diameter and height x

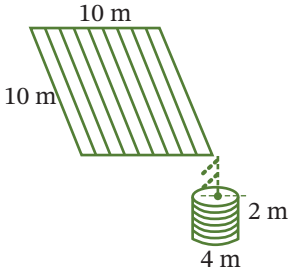


4 The surface area of the curved surface of a can is 27 143.4 mm^2. If its height is 120 mm, find the radius of the can.

5 Water flows from the top tank to the bottom tank at a constant rate. The level of water in the top tank falls at a rate of 15 cm/h. At what rate is the level of water rising in the bottom tank?



6 A 10 m flat square roof drains into a cylindrical rainwater tank with a diameter of 4 m. If 5 mm of rain falls on the roof, by how much (to the nearest millimetre) does the level of the water in the tank rise?



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CHAPTER 3 REVIEW

Quiz
Language of maths 3

Puzzle
Surface area and volume crossword

Language of maths

absolute error	apex	base	capacity
cone	cross-section	curved surface	cylinder
diameter	hemisphere	kilolitre	percentage error
perpendicular height	precision	prism	pyramid
radius	rounding error	sector	slant height
sphere	surface area	truncate	volume

1 What is the difference between rounding and truncating a number?

2 Which word means a 'slice' of a prism or cylinder?

3 Name 3 solids that have a **curved surface area**.

4 What is the formula for the curved surface area of a cone?

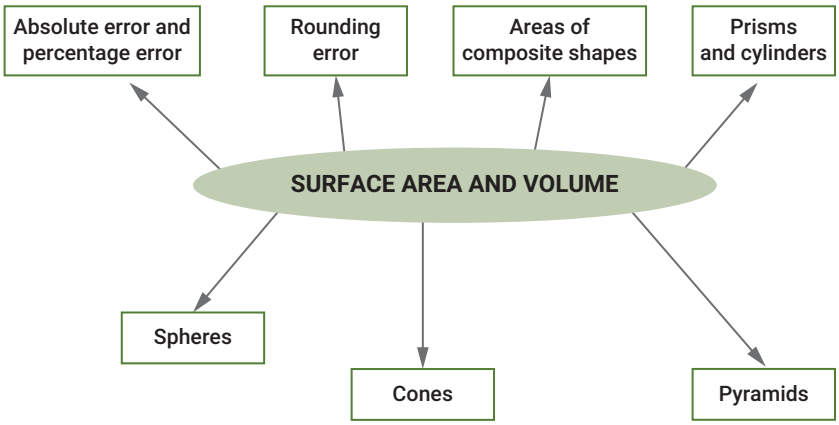
5 Explain the difference between the **perpendicular height** and the **slant height** of a pyramid.

6 What is the formula $V = \frac{1}{3}\pi r^2 h$ used for?

Topic summary

Worksheet
Mind map: Surface area and volume

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

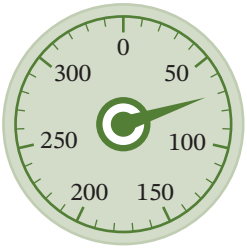


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TEST YOURSELF 3

ANSWERS ON P. 585

1 This weight scale shows a measurement of 70 g. Find the measurement's: 3.01



a absolute error

b percentage error (correct to one decimal place)

2 A plant stem is measured to be 16.0 cm correct to 1 decimal place. What is the percentage error in this measurement? 3.01

A 0.3125 % **B** 0.625 % **C** 3.125 % **D** 6.25 %

3 A triangle has a base length of 5.65 cm and a perpendicular height of 11.31 cm. 3.02

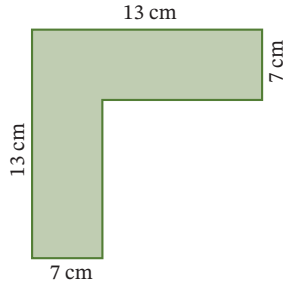
a Calculate its area.

b Round the triangle's base and height to one decimal place, use these values to calculate its area, then find correct to one decimal place the percentage error of the calculated area.

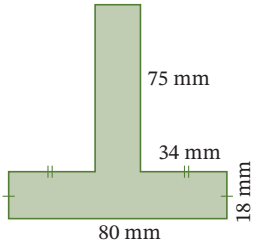
c Truncate the triangle's base and height to the nearest whole number, use these values to calculate its area, then find correct to one decimal place the percentage error of the area.

4 Find the area of each shape. Give your answers correct to one decimal place where necessary. 3.03

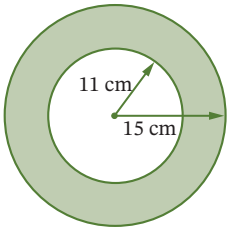
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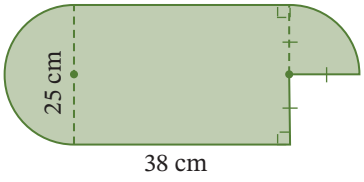
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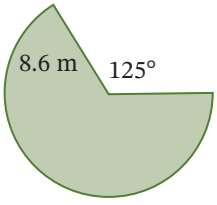
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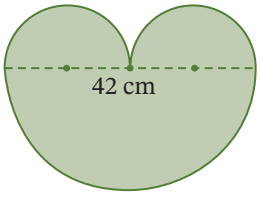
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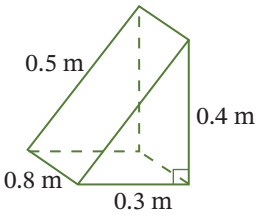


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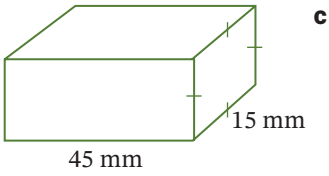
3.04

5 Find the surface area of each prism.

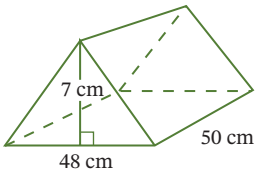
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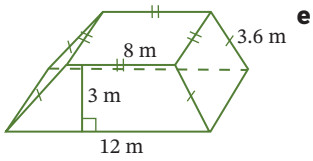
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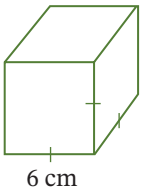
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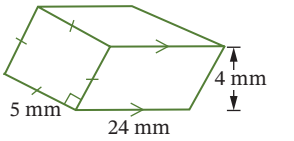
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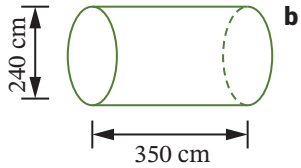
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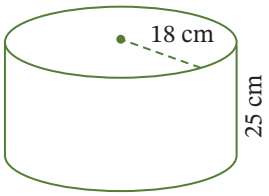
3.05

6 Calculate, correct to one decimal place, the surface area of each solid.

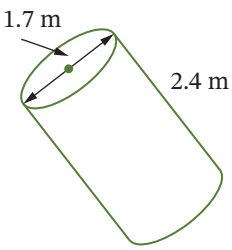
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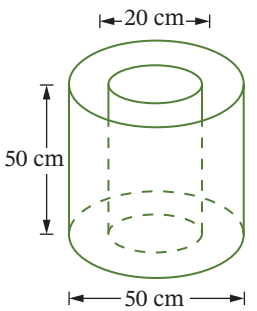


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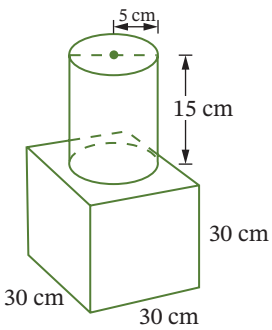


Cylinder, open at one end

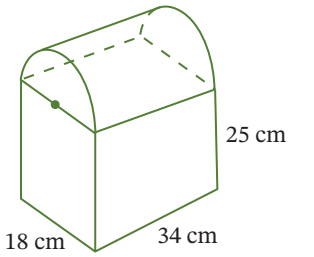
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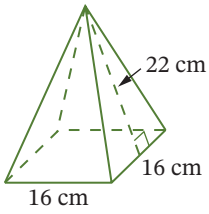
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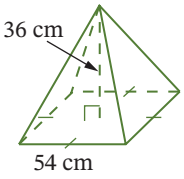
3.06

7 Find the surface area of each pyramid.

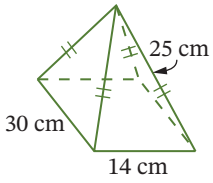
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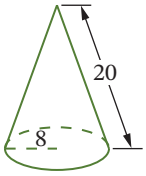
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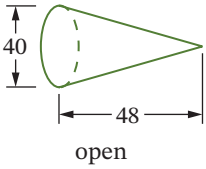
3.07

8 Find the surface area of each solid. All measurements are in metres.

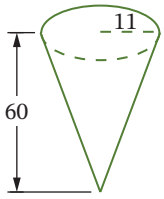
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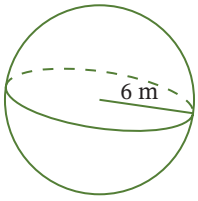
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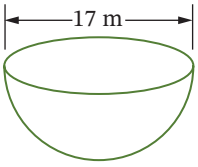
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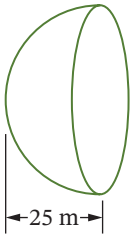
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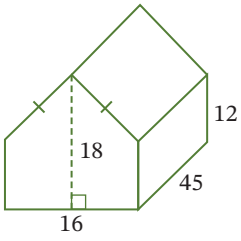
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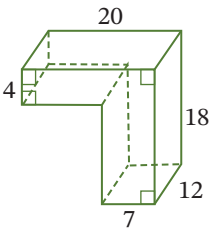
3.08

9 Find, correct to the nearest square centimetre, the surface area of each solid.
All measurements are in centimetres.

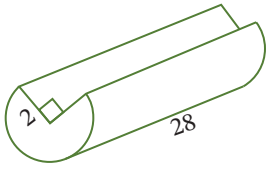
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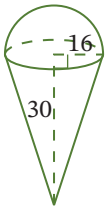
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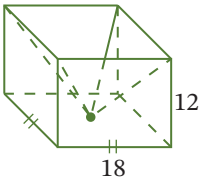
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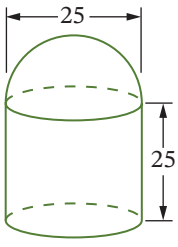
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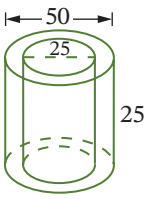


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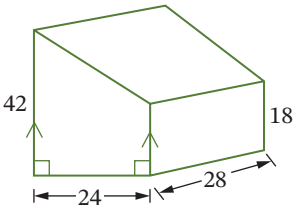
3.09

10 Calculate, correct to nearest cubic metre, the volume of each solid. All measurements are in metres.

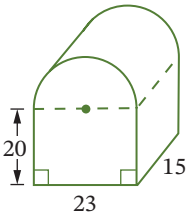
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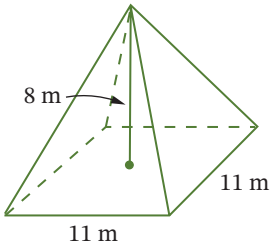


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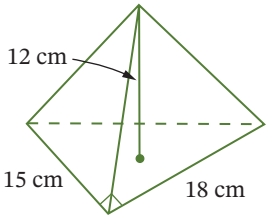
3.10

11 Find, correct to 2 decimal places (where necessary), the volume of each solid.

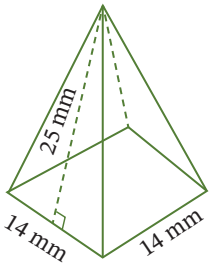
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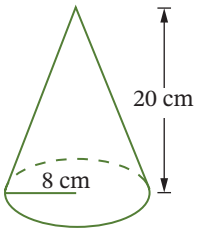
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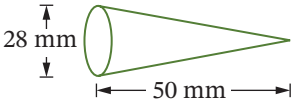
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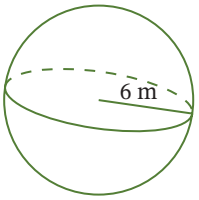
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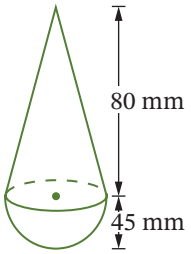
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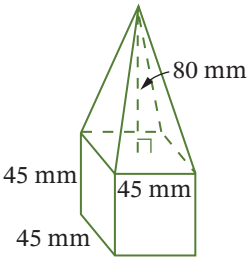
3.11

12 Find, correct to the nearest whole number, the volume of each solid.

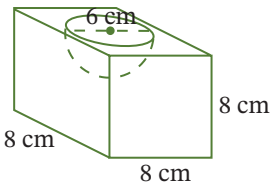
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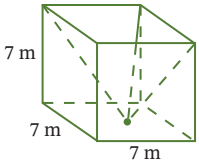
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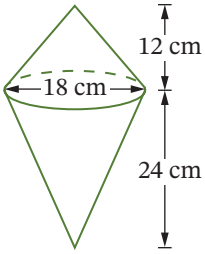
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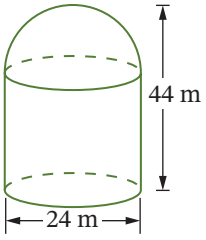
d



e



f



Foundation Standard Complex

9780170479622

PRACTICE SET 1

ANSWERS ON P. 585

1 Cassie earns a salary of $114 750. How much is she paid each week? 1.01

2 An interval is formed by joining the points $M(2, -3)$ and $N(-5, 9)$. 2.01

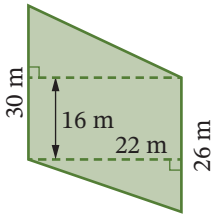
a Find the length of interval MN, correct to one decimal place.

b Find the midpoint of MN.

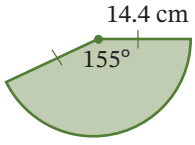
c Find the gradient of MN.

3 Find, correct to one decimal place, the area of each shape. 3.03

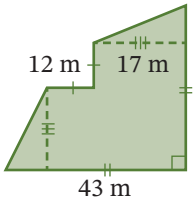
a



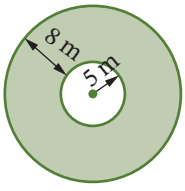
b



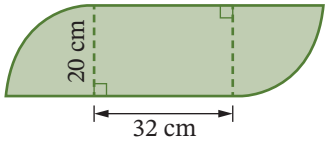
c



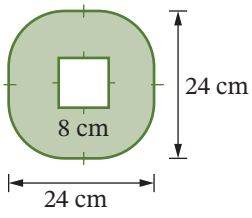
d



e



f



4 Naved is paid 4 weeks' normal pay plus 17.5% annual leave loading for his 4-week holiday. If Naved's salary is $78 580, find his: 1.01

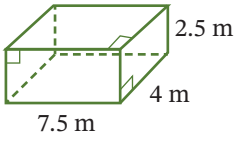
a normal weekly pay

b leave loading

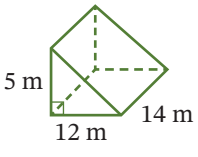
c total pay for the 4-week holiday.

5 Find the surface area of each prism. 3.04

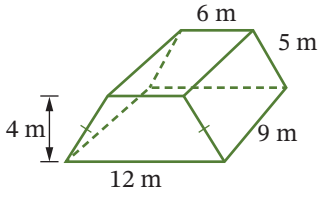
a



b



c



6 There are 1470 students at Callum's school, rounded to the nearest 10. Calculate correct to 2 decimal places the percentage error of this figure. 3.01

Foundation Standard Complex

1.01

7 A call centre operator is employed under the following award.

Normal rate: $28.75 per hour	
Normal rate	For 0 to 36 hours worked
Time-and-a-half	For the next 4 hours worked
Double time	For each hour worked after that

Calculate the wage for working:

a 20 hours **b** 39 hours **c** 45 hours

2.02

8 A line passes through the points $H(5, -3)$ and $K(8, -7)$. Calculate the gradient of the line:

a parallel to HK **b** perpendicular to HK

1.02

9 Weng earns a weekly wage of $1485. She has annual deductions of $1756 for a health fund and $3560 for work expenses.

a Calculate Weng's taxable income.

b Use the tax table on page 11 to calculate the income tax that Weng should pay.

2.03

10 Graph the linear equations $y = 3x - 2$ and $y = -2x + 3$ on a number plane. Where do the lines intersect?

1.03

11 Calculate the simple interest earned on each investment.

a $25 000 invested for 4 years at 3% p.a.

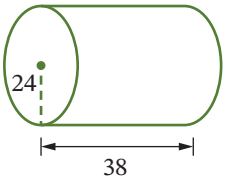
b $500 invested at 1.5% p.a. for 9 months

c $8500 for 2 years at 0.15% per month

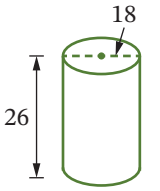
3.05

12 Calculate, correct to one decimal place, the external surface area of each cylinder. All lengths shown are in centimetres.

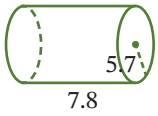
a



b



c



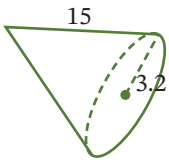
Open at both ends

PATH

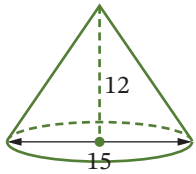
3.07

13 Find, correct to the nearest square metre, the surface area of each solid. All measurements are in metres.

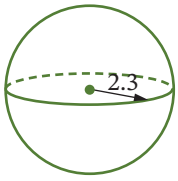
a



b



c



1.04

14 Thanasi invests $12 000 at 4% p.a. interest, compounded annually for 2 years.

a What is the future value of his investment after 2 years?

b What is the amount of interest earned after 2 years?

Foundation Standard Complex

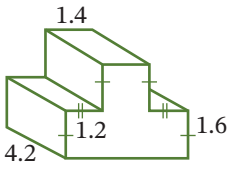
9780170479622

15 Which of the following points lie on the line of $2x + y = 3$?
Select the correct answer **A**, **B**, **C** or **D**. 2.03

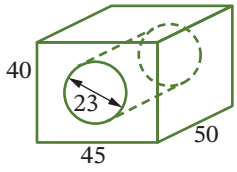
A $(1, 0)$ **B** $(2, -1)$ **C** $(-1, -1)$ **D** $(-1, -5)$

16 Calculate, to nearest cubic metre, the volume of each solid. All measurements are in metres. 3.09

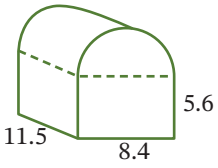
a



b



c



17 Find the equation of the line passing through $(-3, 2)$ that is parallel to the y-axis.
Select **A**, **B**, **C** or **D**. 2.03

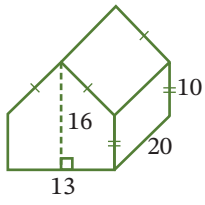
A $x = 2$ **B** $x = -3$ **C** $y = 2$ **D** $y = -3$

18 Jovan purchases a home theatre system valued at $7680. He pays a 10% deposit and repays the balance in 48 monthly instalments. Interest on the balance is charged at a flat rate of 14.5% p.a. Find: 1.06

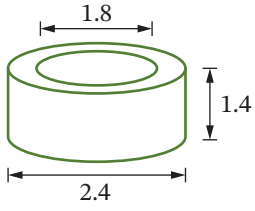
a the deposit paid **b** the balance owing
c the interest charged **d** the total amount to be repaid
e the amount of each instalment **f** the total price Jovan paid for the system.

19 Calculate, correct to nearest square metre, the surface area of each solid.
All lengths shown are in metres. 3.04, 3.05

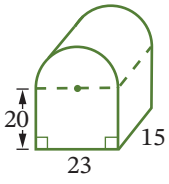
a



b



c



20 Calculate the value of an investment if $46 000 is invested at 4.2% p.a. for 3 years with interest compounded: 1.05

a annually **b** quarterly **c** monthly

21 Find the compound interest earned when $50 000 is invested for 10 years at 6.5% p.a., with interest compounded half-yearly. 1.05

22 Find the gradient, m, and y-intercept, c, for each linear equation. 2.04

a $y = -3x + 8$ **b** $y = 7 + x$ **c** $y = \frac{3-2x}{3}$

PATH

23 Convert each equation to general form $ax + by + c = 0$. 2.05

a $y = 2x - 3$ **b** $2x = y + 5$ **c** $y = \frac{3x}{4} + 6$

24 Rewrite each equation in the form $y = mx + c$. 2.05

a $5x - y + 3 = 0$ **b** $4x + y - 8 = 0$ **c** $2x - 6y + 8 = 0$

Foundation Standard Complex

3.01

25 Measuring with a ruler, Timothee gives a measurement as rounded to 26 cm.

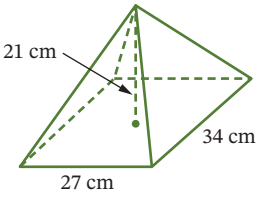
- **a** What is the precision of this ruler?
- **b** What is the absolute error of Timothee's measurement?
- **c** Using a ruler marked in millimetres, Lionel gives the same measurement as 26.8 cm. What is the absolute error of this measurement?
- **d** Calculate correct to 2 decimal places the percentage error of Timothee's and Lionel's measurements.
- **e** Whose measurement is more accurate, Timothee's or Lionel's?

PATH

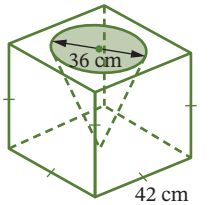
3.11

26 Find, correct to one decimal place, the volume of each solid.

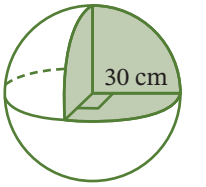
a



b

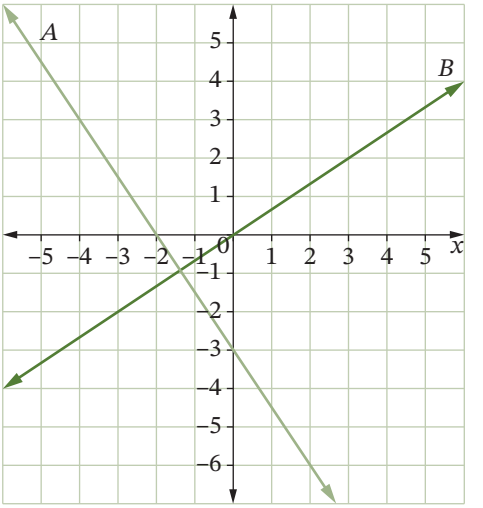


c



2.06

27 Find the equation of each line.



2.07

28 Find the equation of a line that is:

- **a** parallel to $y = 4x - 3$ and has an x-intercept of -8
- **b** perpendicular to $y = -\frac{x}{5} + 3$ and passes through $(0, 0)$.

1.07

29 Annieke purchases a new car for \$39 990, which depreciates by 10% p.a.

- **a** Find the salvage value of the car after 4 years.
- **b** What is the depreciation over this time?
- **c** Express the salvage value as a percentage of the cost price (correct to one decimal place).

Foundation Standard Complex

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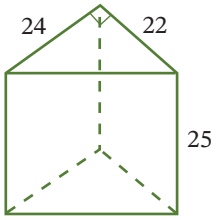
30 Find the gradient, m, and the y-intercept, c, of the line with equation $2x + 5y - 3 = 0$. Select **A**, **B**, **C** or **D**. 2.05

A $m = -2, c = 3$ **B** $m = \frac{3}{5}, c = \frac{2}{5}$

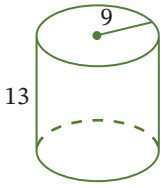
C $m = -\frac{2}{5}, c = \frac{3}{5}$ **D** $m = 2, c = -3$

31 Calculate, correct to the nearest whole number, the volume of each solid. All lengths shown are in centimetres. 3.05

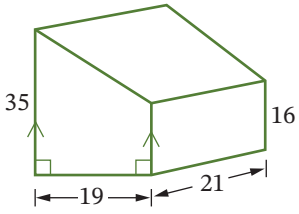
a



b



c



32 Find the equation of the line with: 2.04

a $m = -3, c = 7$ **b** $m = \frac{2}{3}, c = -5$ **c** $m = 0, c = 4$

33 Max's weekly wage is \$1249.20. 1.01

a If he works 36 hours per week, find his hourly rate of pay.

b How much does Max earn per month?

34 A circular swimming pool has a radius of 3.5 m and a depth of 2 m. The pool is to be filled to within 30 cm from the top. 3.09

a Calculate the volume of water in the pool to the nearest m^3.

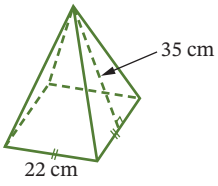
b If water costs \$2.73 per kilolitre, find the cost of filling the pool.

35 What is the flat rate of interest per annum when \$5600 earns \$470 in interest over 3 years? Select **A**, **B**, **C** or **D**. 1.03

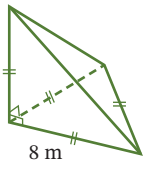
A 2.8% **B** 2.7% **C** 3.0% **D** 27.9%

36 Find the surface area of each pyramid. PATH 3.06

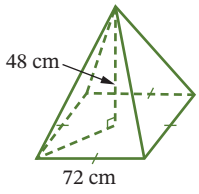
a



b

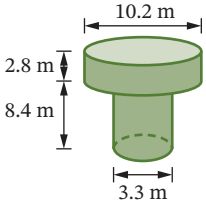


c

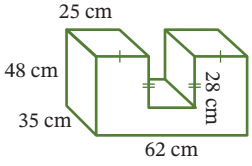


37 Find the surface area of each solid, correct to 1 decimal place. 3.08

a



b



Foundation Standard Complex

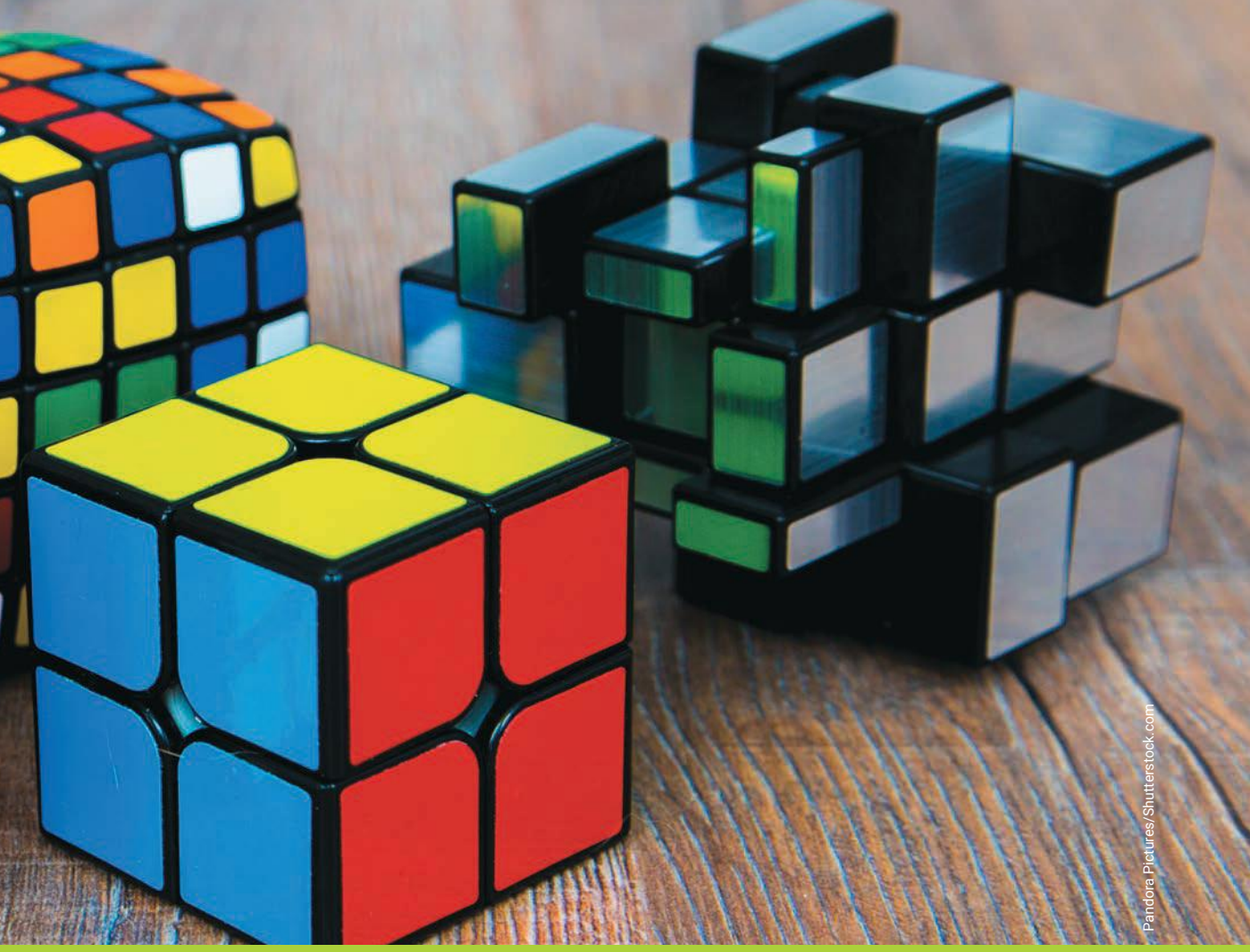


4

NUMBER AND ALGEBRA

ALGEBRA

Rubik's Cube is a puzzle that was invented by Hungarian Professor of Architecture Erno Rubik in 1974. In Hungary, it was originally called the Magic Cube.
The cube has $2^{10} \times 3^7 \times 8! \times 12! \approx 43\,252\,003\,274\,489\,856\,000$ different possible arrangements (8! or '8 factorial' means $8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$). In 2023, US 'speedcuber' Max Park solved a Rubik's cube puzzle in 33 moves in 3.134 seconds!



Chapter outline

		Working mathematically				
4.01	The index laws	U	F		R	C
4.02	Adding and subtracting algebraic fractions	U	F			
4.03	Multiplying and dividing algebraic fractions	U	F			
4.04	Expanding and factorising expressions	U	F			
4.05	Expanding binomial products	U	F		R	C
4.06	Factorising quadratic expressions $x^2 + bx + c$*	U	F		R	

*PATH

Wordbank

binomial An algebraic expression that consists of 2 terms, for example, $4a + 9$, $3 - y$, $x^2 - 4x$

binomial product Binomials multiplied together, for example, $(x + 9)(3x - 4)$

expand To rewrite an expression such as $5(2k - 6)$ without grouping symbols; for example, $5(2k - 6)$ expands to $10k - 30$

factorise To rewrite an expression with grouping symbols, by taking out the highest common factor. Factorising is the opposite of expanding; for example, $9r^2 + 36r$ factorised is $9r(r + 4)$

index laws Rules for simplifying algebraic expressions involving powers of the same base; for example, $a^m \div a^n = a^{m-n}$

quadratic expression An algebraic expression in which the highest power of the variable is 2; for example, $2x^2 + 5x - 3$ or $x^2 + 2$

quadratic trinomial A quadratic expression that has 3 terms, for example, $2x^2 + 5x - 3$

Quiz
Wordbank 4

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Videos (11):

4.01 Numbers and powers • Simplifying with the index laws • Negative indices • Index laws
4.02 Adding and subtracting algebraic fractions • Algebraic fractions
4.03 Algebraic fractions
4.04 Factorising expressions 2
4.05 Expanding binomial products 1 • Expanding binomial products 2
4.06 Factorising quadratic expressions 1

Quizzes (5):

- Wordbank 4
- SkillCheck 4
- Mental skills 4
- Language of maths 4
- Test yourself 4

Skillsheets (3):

4.04 Algebra using diagrams • HCF by factor trees • Factorising using diagrams

Worksheets (9):

4.01 Index laws review
4.03 Algebraic fractions
4.04 Expanding and factorising • Algebra 4
4.05 Mixed expansions • Algebra 2 • Binomial products • Area diagrams

Mind map Algebra

Puzzles (10):

4.01 Indices puzzle • Indices squaresaw
4.02 Algebraic fractions
4.03 Algebraic fractions puzzle • Upside-down fractions
4.04 Factorising puzzle • The distributive law
4.05 Expanding brackets • Trinominoes

Language of maths Algebra crossword

Technology (1):

4.05 Expanding binomials

Spreadsheet (1):

4.05 Expanding binomials

Presentation (1):

4.01 Index laws

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

In this chapter you will:

- apply index laws to numerical expressions with integer indices
- (PATH) apply index laws to algebraic expressions with integer indices
- interpret and use zero and negative indices
- add, subtract, multiply and divide simple algebraic fractions
- (PATH) add, subtract, multiply and divide algebraic fractions with pronumerals in the denominator
- expand and factorise algebraic expressions
- (PATH) factorise algebraic expressions involving terms with indices
- expand binomial products
- (PATH) expand special binomial products
- (PATH) factorise quadratic expressions of the form $x^2 + bx + c$

SkillCheck ANSWERS ON P. 585



Quiz
SkillCheck 4

1 Simplify each expression.

a $g^4 \times g^5$ **b** $r^8 \div r^2$ **c** $(d^5)^3$

d $(-k)^2$ **e** $h \times h^9$ **f** $m^5 \div m$

g a^1 **h** a^0 **i** $3e^2 \times 2e^4$

j $18n^6 \div 6n^2$ **k** $(10w^3)^3$ **l** $25q^0$

m $(vw)^5$ **n** $\left(\frac{c}{p}\right)^3$ **o** y^{-1}

p k^{-2} **q** $\frac{35ad}{20a^2}$ **r** $4y^2 \times \frac{3}{8}y^3$

s $3g^{-2} \times 8g^4$ **t** $\left(\frac{2b}{3h}\right)^2$

2 Evaluate each expression.

a $\frac{a}{5} + \frac{3a}{4}$ **b** $\frac{7p}{2} - \frac{10p}{3}$ **c** $\frac{8}{w} \times \frac{5w}{24}$ **d** $\frac{x}{14} \div \frac{y}{2}$

3 Expand each expression.

a $6m(3m + 11)$ **b** $-5(3g - 8)$ **c** $3w(8y - 4w)$

4 Factorise each expression.

a $4x + 24$ **b** $20 - 15a$ **c** $q^2 + q$

d $18a^2 - 12a$ **e** $-2y - 30$ **f** $-18w + 24$

5 Find 2 numbers whose:

a product is 18 and sum is 9

b product is 8 and sum is -6

c product is -20 and sum is -1

d product is -16 and sum is 6.

4.01 The index laws

Index is another name for **power**. The plural of index is **indices** (pronounced 'in-de-sees'). The following rules are called **index laws**.

Videos
Numbers and powers
Simplifying with the index laws
Negative indices
Index laws

Worksheet
Index laws review

Presentation
Index laws

PATH

The index laws

When **multiplying terms with the same base**, add the powers

$$a^m \times a^n = a^{m+n}$$

When **dividing terms with the same base**, subtract the powers

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$$

When **raising a term with a power to another power**, multiply the powers

$$(a^m)^n = a^{m \times n}$$

When **raising a product of terms to a power**, raise each term to that power

$$(ab)^n = a^n b^n$$

When **raising a quotient of terms to a power**, raise each term to that power

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

Any number **raised to the power of zero** is equal to 1

$$a^0 = 1$$

A number **raised to a negative power** gives a fraction (with a numerator of 1)

$$a^{-n} = \frac{1}{a^n}$$

A number **raised to a power of −1** gives its reciprocal

$$a^{-1} = \frac{1}{a}$$

$$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$$

Puzzles
Indices puzzle
Indices squaresaw

Example 1

Simplify each expression.

a $5p^2 q^3 \times 4p^3 q^4$ **b** $\frac{24e^6 n^{12}}{8en^4}$ **c** $(3t^5)^2$

d $(4d^4 q^2 r)^3$ **e** $\left(\frac{2c^2}{d}\right)^4$ **f** $31x^0 - (31x)^0$

Solution

a $5p^2 q^3 \times 4p^3 q^4 = 5 \times 4 \times p^2 \times p^3 \times q^3 \times q^4$
$= 20p^{2+3} q^{3+4}$
$= 20p^5 q^7$

b $\frac{24e^6 n^{12}}{8en^4} = \frac{{}^{3}\cancel{24}e^6 n^{12}}{{}_{1}\cancel{8}en^4}$
$= 3e^{6-1} n^{12-4}$
$= 3e^5 n^8$

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c $(3t^5)^2 = 3t^5 \times 3t^5$ or $(3t^5)^2 = 3^2 \times t^{5\times 2}$
$= 9t^{5+5}$ $= 9t^{10}$
$= 9t^{10}$

d $(4d^4 q^2 r)^3 = 4^3 d^{4\times 3} q^{2\times 3} r^3$
$= 64d^{12} q^6 r^3$

e $\left(\frac{2c^2}{d}\right)^4 = \frac{(2c^2)^4}{d^4}$
$= \frac{2^4 c^{2\times 4}}{d^4}$
$= \frac{16c^8}{d^4}$

f $31x^0 - (31x)^0 = 31 \times 1 - 1$
$= 30$

4.01

PATH

Example 2

Simplify each expression using a positive index.

a m^{-3} **b** $9x^{-2}$ **c** $(4q)^{-3}$ **d** $\left(\frac{2}{3}\right)^{-1}$

Solution

a $m^{-3} = \frac{1}{m^3}$

b $9x^{-2} = 9 \times x^{-2}$
$= 9 \times \frac{1}{x^2}$
$= \frac{9}{x^2}$

c $(4q)^{-3} = \frac{1}{(4q)^3}$
$= \frac{1}{64q^3}$

d $\left(\frac{2}{3}\right)^{-1} = \left(\frac{3}{2}\right)^1$
$= \frac{3}{2}$

Negative powers of quotients

Consider $\left(\frac{4}{5}\right)^{-2} = \frac{1}{\left(\frac{4}{5}\right)^2} = \frac{1}{\frac{16}{25}}$
$= 1 \div \frac{16}{25}$
$= 1 \times \frac{25}{16}$
$= \frac{25}{16}$
$= \frac{5^2}{4^2}$
$= \left(\frac{5}{4}\right)^2$

Negative powers of quotients

A number **raised to a power of $-n$** gives its reciprocal raised to the power of n.

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n = \frac{b^n}{a^n}$$

PATH

Example 3

Simplify each expression.

a $\left(\frac{4}{3}\right)^{-3}$ **b** $\left(2\frac{1}{2}\right)^{-2}$ **c** $\left(\frac{3a}{b^3}\right)^{-4}$

Solution

a $\left(\frac{4}{3}\right)^{-3} = \left(\frac{3}{4}\right)^{3}$

$= \frac{27}{64}$

b $\left(2\frac{1}{2}\right)^{-2} = \left(\frac{5}{2}\right)^{-2}$

$= \left(\frac{2}{5}\right)^{2}$

$= \frac{4}{25}$

c $\left(\frac{3a}{b^4}\right)^{-2} = \left(\frac{b^4}{3a}\right)^{2}$

$= \frac{b^8}{9a^2}$

EXERCISE 4.01 ANSWERS ON P. 586

The index laws U F R C

EXAMPLE 1

1 Simplify each expression. R C

a $5k^4 \times 3k^7$ **b** $30y^{10} \div 3y^7$ **c** $12p^6 \times \frac{3}{4}p^5$

d $(w^3)^5$ **e** $(2n^3)^5$ **f** $\frac{24h^5}{8h}$

g $3a^2d \times 4a^5d^4$ **h** $(3q^3)^2$ **i** $32w^5y^8 \div 8w^3y^4$

j $(-2c^2)^7$ **k** $\frac{28a^2c^5d^7}{21c^4d^4}$ **l** $(5y^6)^3$

m $4h^3k^2 \times 6hk^3w^5$ **n** $(2d^3g^2)^5$ **o** $\frac{8m^5p^{11}q}{12m^3p^7q}$

2 Simplify each expression. R C

a $(l^3m^5)^6$ **b** $\left(\frac{n}{2}\right)^3$ **c** $7x^0$ **d** $\left(\frac{w^2}{k^3}\right)^5$

e $\left(\frac{2}{3}\right)^0$ **f** $(-8ky^5)^2$ **g** $(16a)^0 - 16a^0$ **h** $\left(\frac{2b}{3d}\right)^4$

i $(5xy^2)^0$ **j** $(-5dy^2)^4$ **k** $\left(-\frac{3k^4}{10}\right)^3$ **l** $-9(a^2b^3)^0$

m $(2p^2q^3r^4)^4$ **n** $(4w^0)^3$ **o** $(-3g^6k)^2$ **p** $3a^0 + (3ab)^0$

3 Evaluate each expression. R C

a 4^0 **b** $(-8)^0$ **c** 7×2^0 **d** $(5 \times 3)^0$

e $(-2)^3$ **f** $(-3)^2$ **g** $(5^2)^2$ **h** $4^0 + 7^0$

i $2^4 \times 2^3$ **j** $(10^2)^0$ **k** $(2^3)^0 - (2^0)^3$ **l** $4^5 \div 4^2$

m $6^2 \div 6^5$ **n** $9^5 \div 9^5$ **o** $(8 \times 3)^0 - 8 \times 3^0$ **p** $5^2 \div 5^0$

q $\left(\frac{2}{5}\right)^0 + \left(\frac{5}{2}\right)^0$ **r** $3^{-2} \div 3^2$ **s** $\left(\frac{3}{4}\right)^0$ **t** $12^{-2} \times 12^2$

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4 Find the value of $(3x)^0 + 3 \times 2^0$. Select the correct answer **A**, **B**, **C** or **D**.

A 6 **B** 1 **C** 7 **D** 4

5 Express each term as a fraction. R C

a 5^{-2} **b** 2^{-5} **c** 20^{-1} **d** 10^{-3} **e** 3^{-4}

6 Simplify $2c^{-3}$. Select **A**, **B**, **C** or **D**.

A $-6c$ **B** $\frac{1}{2c^3}$ **C** $\frac{2}{c^3}$ **D** $\frac{1}{8c^3}$

7 Simplify each expression using positive indices. R C

PATH

EXAMPLE 2

a 8^{-7} **b** 3^{-5} **c** y^{-1} **d** x^{-3}

e $(5b)^{-2}$ **f** $8h^{-3}$ **g** $(ab)^{-1}$ **h** $-pq^{-1}$

i $11w^{-3}$ **j** $(6x)^{-3}$ **k** $a^3 b^{-5}$ **l** mw^{-3}

m $8u^{-3} v^{-4}$ **n** $-2r^6 y^{-5}$ **o** $10e^{-1} f^3$ **p** $\frac{1}{2} g^4 h^{-3}$

q $\frac{3}{4} d^7 n^{-2}$ **r** $(-4c)^{-2}$ **s** $5x^2 y^{-1} w^{-2}$ **t** $2(mp)^{-1}$

8 Simplify each expression. R C

a $10r^{-6}$ **b** $\left(\frac{5}{2}\right)^{-1}$ **c** $\left(\frac{1}{3}\right)^{-1}$ **d** $\left(\frac{1}{x}\right)^{-1}$

e $\left(\frac{k}{3}\right)^{-1}$ **f** $5a^{-2}$ **g** $(6w)^{-2}$ **h** $\left(-\frac{m}{2}\right)^{-1}$

i $\left(\frac{5}{3g^3}\right)^{-1}$ **j** $\left(\frac{2r}{3h}\right)^{-1}$ **k** $m^3 n^2 p^{-2}$ **l** $\left(\frac{5b}{4a^2}\right)^{-1}$

9 Simplify each expression, writing your answer with positive indices. R C

a $(2x^5 y^2)^2 \times 5x^{-4} y^5$ **b** $(3m^8 n^{-2} \times m^{-2} n^3)^4$

c $\left(\frac{6k^3 w^5}{9k^4 w^3}\right)^2$ **d** $(5g^6 y^4)^2 \div 10g^8 y^{-4}$

e $\left(\frac{2a^4 x^4}{3a^5 x^2}\right)^{-3}$ **f** $-6a^2 d^3 \times (-2a^3 d^4)^3$

g $3q^{-5} r^3 \div (6qr^2)^2$ **h** $(4h^3 k^2)^3 \times (hk^5)^2$

i $[24b^5 q^6 \div (-2bq^2)^3]^2$

10 Simplify each expression. R C

EXAMPLE 3

a $\left(\frac{3}{4}\right)^{-2}$ **b** $\left(\frac{2}{3}\right)^{-3}$ **c** $\left(-\frac{1}{10}\right)^{-6}$ **d** $\left(\frac{5}{2}\right)^{-3}$

e $\left(-\frac{4}{3}\right)^{-5}$ **f** $\left(\frac{5}{4}\right)^{-4}$ **g** $\left(2\frac{1}{4}\right)^{-2}$ **h** $\left(1\frac{2}{5}\right)^{-3}$

i $\left(\frac{k}{3}\right)^{-2}$ **j** $\left(-\frac{3}{x}\right)^{-3}$ **k** $\left(\frac{a^2}{5}\right)^{-4}$ **l** $\left(\frac{4}{3g^3}\right)^{-2}$

4.02 Adding and subtracting algebraic fractions

Videos
Adding and subtracting algebraic fractions
Algebraic fractions

Puzzle
Algebraic fractions

Adding and subtracting algebraic fractions

To add or subtract fractions, convert them (if needed) so that they will have the same denominator, then simply add or subtract the numerators.

Example 4

Simplify each expression.

a $\frac{a}{2}+\frac{a}{3}$ **b** $\frac{2x}{5}-\frac{x}{3}$ **c** $\frac{5}{6}+\frac{7b}{15}$

Solution

a $\frac{a}{2}+\frac{a}{3}=\frac{3\times a}{3\times 2}+\frac{2\times a}{2\times 3}$
$=\frac{3a}{6}+\frac{2a}{6}$
$=\frac{5a}{6}$

Common denominator $=2\times 3=6$

b $\frac{2x}{5}-\frac{x}{3}=\frac{3\times 2x}{3\times 5}-\frac{5\times x}{5\times 3}$
$=\frac{6x}{15}-\frac{5x}{15}$
$=\frac{x}{15}$

c $\frac{5}{6}+\frac{7b}{15}=\frac{25}{30}+\frac{14b}{30}$
$=\frac{25+14b}{30}$

The lowest common denominator is 30

EXERCISE 4.02 ANSWERS ON P. 586

Adding and subtracting algebraic fractions UFRC

EXAMPLE 4

1 Simplify each expression.

a $\frac{2k}{7}+\frac{3k}{7}$ **b** $\frac{8x}{11}-\frac{5x}{11}$ **c** $\frac{13m}{15}-\frac{7m}{15}$ **d** $\frac{9g}{5}+\frac{4g}{5}$

e $\frac{3w}{4}-\frac{2w}{5}$ **f** $\frac{a}{3}+\frac{3a}{7}$ **g** $\frac{4c}{5}+\frac{7c}{12}$ **h** $\frac{5u}{2}-\frac{u}{6}$

i $\frac{3}{4k}-\frac{1}{4k}$ **j** $\frac{9}{7x}+\frac{5}{7x}$ **k** $\frac{13}{5h}-\frac{3}{5h}$ **l** $\frac{5}{m}+\frac{7}{2m}$

m $\frac{5}{3a}-\frac{1}{a}$ **n** $\frac{7}{2h}+\frac{4}{3h}$ **o** $\frac{11}{8k}+\frac{4}{5k}$ **p** $\frac{16}{7p}-\frac{5}{3p}$

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2 Simplify each expression.

a $\frac{m}{3}+\frac{n}{5}$ **b** $\frac{k}{7}-\frac{w}{6}$ **c** $\frac{x}{4}-\frac{p}{9}$ **d** $\frac{5}{8}+\frac{h}{3}$

e $\frac{n}{10}-\frac{4}{7}$ **f** $\frac{5y}{2}-\frac{2c}{5}$ **g** $\frac{3q}{11}+\frac{2d}{3}$ **h** $\frac{3a}{8}+\frac{5e}{6}$

i $\frac{2b}{9}+\frac{b}{5}$ **j** $\frac{9y}{7}-\frac{3y}{4}$ **k** $\frac{5c}{3}-\frac{2c}{11}$ **l** $\frac{8g}{5}+\frac{2g}{15}$

m $\frac{4h}{3}+\frac{h}{8}$ **n** $\frac{10x}{7}+\frac{4x}{5}$ **o** $\frac{12}{5}-\frac{8k}{3}$ **p** $\frac{7m}{12}-\frac{2m}{15}$

Multiplying and dividing algebraic fractions

4.03

Multiplying and dividing algebraic fractions

- To multiply fractions, cancel any common **factors**, then multiply the numerators and denominators separately.
- To divide by a fraction $\frac{a}{b}$, multiply by its **reciprocal** $\frac{b}{a}$.



Worksheet
Algebraic fractions

Puzzles
Algebraic fractions puzzle

Upside-down fractions



Video
Algebraic fractions

Example 5

Simplify each expression.

a $\frac{3x}{4}\times\frac{2x}{9}$ **b** $\frac{4}{k}\times\frac{3k}{16}$ **c** $\frac{2}{v}\div\frac{3}{w}$ **d** $\frac{xy}{5}\div\frac{3x}{25}$

Solution

a $\frac{3x}{4}\times\frac{2x}{9}=\frac{3x\times 2x}{4\times 9}$ or $\frac{3x}{4}\times\frac{2x}{9}=\frac{{}^{1}\cancel{3}x}{{}_{2}\cancel{4}}\times\frac{{}^{1}\cancel{2}x}{{}_{3}\cancel{9}}$

$=\frac{6x^2}{36}$ $=\frac{x^2}{6}$

$=\frac{x^2}{6}$

b $\frac{4}{k}\times\frac{3k}{16}=\frac{{}^{1}\cancel{4}}{\cancel{k}}\times\frac{3\cancel{k}}{{}_{4}\cancel{16}}$

$=\frac{3}{4}$

c $\frac{2}{v}\div\frac{3}{w}=\frac{2}{v}\times\frac{w}{3}$

$=\frac{2w}{3v}$

d $\frac{xy}{5}\div\frac{3x}{25}=\frac{\cancel{x}y}{{}_{1}\cancel{5}}\times\frac{{}^{5}\cancel{25}}{3\cancel{x}}$

$=\frac{5y}{3}$

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EXERCISE 4.03 ANSWERS ON P. 586

Multiplying and dividing algebraic fractions U F

EXAMPLE 5

1 Simplify each product.

a $\frac{m}{5}\times\frac{2}{7}$ **b** $\frac{d}{3}\times\frac{h}{4}$ **c** $\frac{2y}{8}\times\frac{2q}{3}$

d $\frac{x}{11}\times\frac{6y}{5}$ **e** $\frac{4k}{7}\times\frac{14v}{4}$ **f** $\frac{5h}{3}\times\frac{9d}{15}$

PATH

g $\frac{12}{v}\times\frac{2}{9}$ **h** $\frac{6}{5b}\times\frac{10b}{18}$ **i** $\frac{4}{5y}\times\frac{7y}{12}$

j $\frac{14}{15p}\times\frac{3hp}{7}$ **k** $\frac{20a}{3k}\times\frac{9a}{5k}$ **l** $\frac{5}{12b}\times\frac{8ab}{15}$

2 Simplify the product $\frac{6ad}{7m}\times\frac{3y}{4d}\times\frac{14gm}{27ay}$. Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{2ak}{7y}$ **B** $\frac{g}{3}$ **C** $\frac{14gm}{9}$ **D** $\frac{7m}{4}$

3 Simplify the quotient $\frac{5xy}{3k}\div\frac{20}{9ky}$. Select **A**, **B**, **C** or **D**.

A $\frac{3y}{4x}$ **B** $\frac{4x}{15y^2}$ **C** $\frac{45k}{60y}$ **D** $\frac{3xy^2}{4}$

4 Simplify each quotient.

a $\frac{h}{5}\div\frac{4}{9}$ **b** $\frac{x}{4}\div\frac{3p}{7}$ **c** $\frac{k}{6}\div\frac{a}{11}$

d $\frac{8my}{27}\div\frac{4m}{9}$ **e** $\frac{3c}{4}\div\frac{5c}{8}$ **f** $\frac{2m}{9}\div\frac{4am}{3}$

PATH

g $\frac{10e}{3}\div\frac{4}{e}$ **h** $\frac{8}{3xy}\div\frac{6x}{5y}$ **i** $\frac{w}{y}\div\frac{4}{y}$

j $\frac{8k}{15gh}\div\frac{12hk}{10g}$ **k** $\frac{6d^2}{25}\div\frac{3d}{10n}$ **l** $\frac{8y}{5}\div\frac{32y^2}{3g}$

5 Simplify each expression.

a $\frac{wy}{5}\div\frac{3y}{20}$ **b** $\frac{d}{3h}\div\frac{k}{6}\div\frac{2d}{hk}$ **c** $\frac{24pq}{7}\div 8p$

d $\frac{x}{4y}\times\frac{12m}{5g}\times\frac{10y}{9x}$ **e** $\frac{2w}{p}\times\frac{3h}{7}\div\frac{4h}{5p}$ **f** $10pg\div\frac{6p}{5}$

g $\frac{4k}{3m}\div\frac{3d}{m}\div\frac{5k}{2m}$ **h** $\frac{3d}{4}\div\frac{d}{a}\times\frac{4d}{15}$ **i** $\frac{8}{3p}\div\frac{4}{a}\times\frac{p}{6a}$

j $15g\div\frac{3g}{4x}$ **k** $\frac{5}{2y}\times\frac{4}{3q}\div\frac{10}{3y}$ **l** $\frac{12}{b}\div\frac{4}{ab}$

6 Simplify the expression $\frac{4km}{5}\div\frac{18d}{7}\div\frac{m}{30}$. Select **A**, **B**, **C** or **D**.

A $\frac{175}{12km}$ **B** $\frac{7km^2}{1350d}$ **C** $\frac{28k}{3d}$ **D** $\frac{4km}{5d}$

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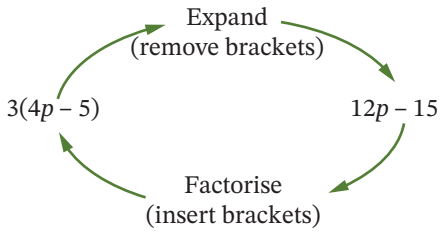
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Expanding and factorising expressions 4.04

Expanding and **factorising** are inverse operations.

When $3(4p - 5)$ is **expanded**, the answer is $12p - 15$.

When $12p - 15$ is **factorised**, the answer is $3(4p - 5)$



Expanding an expression

Multiply each term inside the brackets by the term outside.

$$a(b + c) = ab + ac$$

$$a(b - c) = ab - ac$$

Example 6

Expand each expression.

a $4d(5d + 3)$ **b** $-5(3p - 8)$

Solution

a $4d(5d + 3) = 4d \times 5d + 4d \times 3$ Multiplying out the brackets

$= 20d^2 + 12d$

b $-5(3p - 8) = -5 \times 3p - (-5) \times 8$ Multiplying out the brackets

$= -15p - (-40)$

$= -15p + 40$

Skillsheets
Algebra using diagrams
HCF by factor trees
Factorising using diagrams

Worksheets
Expanding and factorising
Algebra 4

Puzzles
Factorising puzzle
The distributive law

4.04

Example 7

Expand and simplify by collecting **like terms**.

a $-3r^2(4r + 2) - 5r^3$ **b** $9(m - 3) + m(m - 10)$

Solution

a $-3r^2(4r + 2) - 5r^3 = -3r^2 \times 4r + (-3r^2) \times 2 - 5r^3$ Expanding

$= -12r^3 + (-6r^2) - 5r^3$

$= -17r^3 - 6r^2$ Collecting like terms to simplify

b $9(m - 3) + m(m - 10) = 9 \times m - 9 \times 3 + m \times m - m \times 10$

$= 9m - 27 + m^2 - 10m$

$= m^2 + 9m - 10m - 27$ It's neater to place m^2 at the front

$= m^2 - m - 27$ Collecting like terms to simplify



Video
Factorising expressions 2

Factorising an expression

- Find the **highest common factor (HCF)** of the terms and write it outside the brackets
- Divide each term by the HCF and write the answers inside the brackets
- $ab + ac = a(b + c)$ $ab - ac = a(b - c)$
- To check that the factorised answer is correct, expand it.

Example 8

Factorise each expression.

a $18xy^2 - 24xy$ **b** $3m(5 + 2d) + 7(5 + 2d)$ **c** $-5k^2 + 15k$

Solution

a The HCF of $18xy^2$ and $24xy$ is $6xy$.

$\therefore 18xy^2 - 24xy = 6xy \times 3y - 6xy \times 4$ Rewrite the expression using the HCF $6xy$

$= 6xy(3y - 4)$ Write the HCF at the front of the brackets

b The HCF of $3m(5 + 2d) + 7(5 + 2d)$ is $(5 + 2d)$.

$\therefore 3m(5 + 2d) + 7(5 + 2d) = (5 + 2d) \times 3m + (5 + 2d) \times 7$

$= (5 + 2d)(3m + 7)$

c When factorising expressions that begin with a negative term, we use the 'negative' HCF.

The highest 'negative' common factor of $-5k^2$ and $15k$ is $-5k$.

$\therefore -5k^2 + 15k = (-5k) \times k + (-5k) \times (-3)$ $(-5k) \times (-3) = +15k$

$= (-5k)[k + (-3)]$

$= -5k(k - 3)$

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Example 9

Factorise each expression.

a $8a^3 + 4a^2$ **b** $20h^3k + 25h^4k - 10h^2k$

Solution

a The HCF of $8a^3$ and $4a^2$ is $4a^2$.

$\therefore 8a^3 + 4a^2 = 4a^2 \times 2a + 4a^2 \times 1$ Rewrite the expression using the HCF $4a^2$

$= 4a^2(2a + 1)$

b The HCF is $5h^2k$.

$\therefore 20h^3k + 25h^4k - 10h^2k = 5h^2k \times 4h + 5h^2k \times 5h^2 - 5h^2k \times 2.$

$= 5h^2k(4h + 5h^2 - 2)$

EXERCISE 4.04 ANSWERS ON P. 586

Expanding and factorising expressions U F

EXAMPLE 6

1 Expand each expression.

a $5(d + 11)$ **b** $-3(r + 10)$ **c** $7(x - 9y)$ **d** $-4(a - 5w)$

e $-(2 - p^2)$ **f** $-10e(2e^2 + 3)$ **g** $6y(1 + 7y)$ **h** $4xy(3xy - 1)$

i $8rq(2q - r)$ **j** $3ab(4b - 7a)$ **k** $-6h^2(1 - 3h)$ **l** $-5x(5x^2 + 4y)$

m $-(3 + 8a)$ **n** $-2m^2(3m - 4n)$ **o** $5g(3 + 7g^2)$ **p** $-1(5e - 12)$

2 Expand $-2y(5 + 7y)$. Select the correct answer **A**, **B**, **C** or **D**.

A $3y - 5y^2$ **B** $-10y + 14y^2$ **C** $-10y - 14y^2$ **D** $-10y - 5y^2$

3 Use the substitution $x = 2$ to test whether each equation is correct or incorrect.

a $4(x + 10) = 4x + 40$ **b** $5(x - 1) = 5x - 6$ **c** $x(3 - x) = 3x - x^2$

EXAMPLE 7

4 Expand and simplify by collecting like terms.

a $4k(3k - 5) - 9k^2$ **b** $5h - 7h(4 - h)$

c $9w^3 - 3w(5 + 2w^2)$ **d** $24x^3 - 5x^2(2x^2 - 5x)$

e $8 - 3(2 - 7d)$ **f** $4n(3 - 5n) - 6n^2$

g $4y(y - 4) + 5(2y + 1)$ **h** $5(1 - 2a) - a(3 + 4a)$

i $7(3 + 6w) - (5 - 8w)$ **j** $4y^2(5y + 5) + 4(2 - 7y^2)$

k $-v(2v + 7) + 6(v - 1)$ **l** $2(4 - 3a) - a(3 - a)$

m $2c(5c - 1) - 4(7c - 5)$ **n** $3m(m + 5m^2) - m^2(1 - 3m)$

o $4x(2y + 5) - 6y(10 - 3x)$

EXAMPLE 8

5 Factorise each expression.

a $15y - 20$ **b** $21 + 35w$

c $2p + p^2$ **d** $30y - 20y^2$

e $36d^2 + 24d$ **f** $28k^2 - 21k$

g $8(c - 5) - c(c - 5)$ **h** $m(3 + 2m) + 7(3 + 2m)$

i $-q^2 - 36q$ **j** $-8x + 12x^2$

k $b(3b + 5) - 2(3b + 5)$ **l** $-12cd^2 + 8cd$

m $-hn^2 + h^2n$ **n** $-15g^2 - 18g$

o $48q^2 - 54q$

6 Factorise $-16pw + 10xw$. Select **A**, **B**, **C** or **D**.

A $-2w(8p - 10x)$ **B** $-2w(8p - 5x)$

C $-8w(2p - x)$ **D** $-4w(5p + 2x)$

PATH

EXAMPLE 9

7 Factorise each expression.

a $8m^2y^2 - 12my$ **b** $36ab^2c + 27bc$

c $24m^2n - 108mn^2$ **d** $20dg^2 - 35ag$

e $40wy^3 + 24w^2y^2$ **f** $75g^3h^2 - 125gh$

g $-4p^3 - 8p^2 + p$ **h** $6mn^2 + 3mn + 48m^2n$

i $32p^3g + 8pg^2 - 8pg$ **j** $18a^5 - 12a^2 + 15a^4$

k $28m^3h^2 - 21mh^2$ **l** $15kwp - 24wp^2 - 9kw$



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Did you know?

CAPTCHA

For security purposes on a website, have you ever been asked to enter letters, words or numbers that have been displayed in a wavy, difficult-to-read format like this?



This process is called **CAPTCHA**, which stands for **Completely Automated Public Turing Test to Tell Computers and Humans Apart**, invented in 1999. It is designed to ensure that the person accessing the website is a human and not another computer that could be hacking into the site, sending spam or viruses. CAPTCHA uses optical character recognition (OCR), which is a technology that can convert images of text into editable text.

How does CAPTCHA prove that you are a human and not a computer?

What is a Turing test, named after English computer scientist and mathematician Alan Turing?



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Quiz
Mental skills 4

Mental skills 4: Maths without calculators ANSWERS ON P. 587

Estimating answers

A quick way of estimating an answer is to round each number in the calculation.

1 Study each example.

a $55 + 132 - 34 + 17 - 78 \approx 60 + 130 - 30 + 20 - 80$
$= (60 + 20 - 80) + (130 - 30)$
$= 0 + 100$
$= 100$ (Actual answer = 92)

b $78 \times 7 \approx 80 \times 7$
$= 560$ (Actual answer = 546)

c $510 \div 24 \approx 500 \div 20$
$= 50 \div 2$
$= 25$ (Actual answer = 21.25)

2 Now estimate each answer.

a $27 + 11 + 87 + 142 + 64$
b $55 + 34 - 22 - 46 + 136$
c $684 + 903$
d $35 + 81 + 110 + 22 + 7$
e $517 - 96$
f $210 - 38 - 71 + 151 - 49$
g $766 - 353$
h 367×2
i 83×81
j 984×16
k $828 \div 3$
l $507 \div 7$

3 Study each example involving decimals.

a $20.91 - 11.3 + 2.5 \approx 21 - 11 + 3$
$= 13$ (Exact answer = 12.11)

b $4.78 \times 19.2 \approx 5 \times 20$
$= 100$ (Exact answer = 91.776)

c $\frac{37.6+9.3}{41.2-12.7} \approx \frac{38+9}{40-13}$
$= \frac{47}{27}$
$= \frac{50}{30}$
$= 1.6$ (Exact answer = 1.6456...)

4 Now estimate each answer.

a $3.75 + 9.381 + 4.6 + 10.5$
b $14.807 + 6.6 - 7.22$
c 18.47×9.61
d 4.27×97.6
e $\frac{11.07+18.4}{12.2}$
f $\frac{38.18}{17.2-9.6}$
g $\frac{18.46 \times 4.9}{39.72-15.2}$
h $62.13 \div 10.7$
i $(4.89)^2$

Expanding binomial products

4.05

Binomial means '2 terms'. $(m + 8)$ and $(m - 3)$ are **binomial expressions** because they each have exactly 2 terms. $(m + 8)(m - 3)$ is called a **binomial product** because it is a product of 2 binomial expressions.

Example 10

Expand each binomial product.

a $(m + 8)(m - 3)$

b $(4y - 5)(3y - 2)$

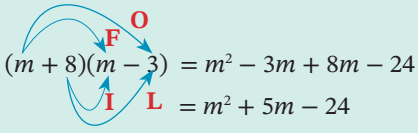
Solution

a $(m + 8)(m - 3) = m(m - 3) + 8(m - 3)$ Each term in $(m + 8)$ is multiplied by $(m - 3)$

$= m^2 - 3m + 8m - 24$ Expanding

$= m^2 + 5m - 24$ Simplifying

One way of remembering which pairs of terms to multiply together in a binomial product is called the **FOIL method**, as shown.



- **F** means multiply the **f**irst terms: $m \times m = m^2$
- **O** means multiply the **o**utside terms: $m \times (-3) = -3m$
- **I** means multiply the **i**nside terms: $8 \times m = 8m$
- **L** means multiply the **l**ast terms: $8 \times (-3) = -24$

b $(4y - 5)(3y - 2) = 12y^2 - 8y - 15y + 10$ Using FOIL

$= 12y^2 - 23y + 10$ Simplifying

Videos
Expanding binomial products 1
Expanding binomial products 2

Worksheets
Mixed expansions
Algebra 2
Binomial products
Area diagrams

Puzzles
Expanding brackets
Trinominoes

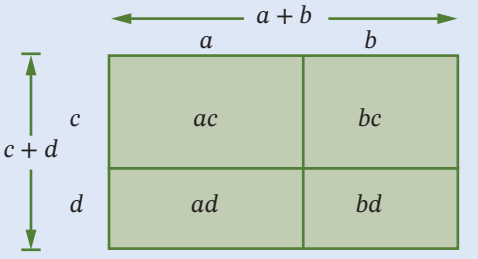
Technology
Expanding binomials

Spreadsheet
Expanding binomials

Expanding a binomial product

Multiply each term in the first binomial by each term in the second binomial.

$$(a + b)(c + d) = ac + ad + bc + bd$$



EXERCISE 4.05 ANSWERS ON P. 587

Expanding binomial products U F R C

EXAMPLE 10

1 Expand each binomial product.

a	$(m + 4)(m + 3)$	**b**	$(w + 5)(w + 5)$	**c**	$(y + 12)(y - 12)$
d	$(h + 7)(h - 9)$	**e**	$(a - 5)(a + 3)$	**f**	$(x - 11)(x - 4)$
g	$(p + 3)(p + 8)$	**h**	$(c - 7)(c - 12)$	**i**	$(g - 1)(g - 2)$
j	$(u - 8)(u + 7)$	**k**	$(m + 4)(m - 10)$	**l**	$(q - 11)(q + 6)$
m	$(d - 5)(8 + d)$	**n**	$(10 - e)(e - 7)$	**o**	$(9 - h)(5 - h)$

2 Expand $(w - 8)^2$. Select the correct answer **A**, **B**, **C** or **D**.

A $w^2 - 64$ **B** $w^2 + 64w$ **C** $w^2 + 16w - 64$ **D** $w^2 - 16w + 64$

3 Expand $(12 - a)^2$. Select **A**, **B**, **C** or **D**.

A $144 - a^2$ **B** $144 - 24a + a^2$

C $a^2 + 24a + 144$ **D** $a^2 + 144$

4 Expand each binomial product.

a	$(3y + 1)(y + 7)$	**b**	$(4k + 9)(3k + 2)$	**c**	$(3m - 5)(m + 3)$
d	$(5p + 3)(2p - 5)$	**e**	$(2w - 3)(w - 8)$	**f**	$(7x + 4)(2x + 3)$
g	$(3b - 4)(3b - 4)$	**h**	$(5a + 6)(5a + 6)$	**i**	$(2q - 7)(5q + 7)$
j	$(6 + 5p)(p - 3)$	**k**	$(4d - 11)(1 + 3d)$	**l**	$(2r - 5)(9 - 4r)$
m	$(7y + 3)(3 - 2y)$	**n**	$(8h - 3)(8h + 3)$	**o**	$(9 - 7w)(7w - 9)$
p	$(4d - 1)(4d + 1)$	**q**	$(f - 1)(1 + 3f)$	**r**	$(6u - 5)(5 - 6u)$

5 Expand $(5 - 3k)(5 + 3k)$. Select **A**, **B**, **C** or **D**.

A $25 - 30k - 9k^2$ **B** $25 - 9k^2$

C $9k^2 - 30k - 25$ **D** $9k^2 - 25$

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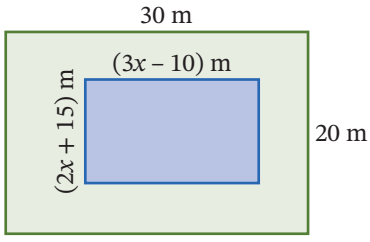
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6 Expand each binomial product and notice any patterns. R C

a	$(x+7)^2$	**b**	$(x-7)^2$	**c**	$(x+7)(x-7)$
d	$(y-6)(y+6)$	**e**	$(y-6)^2$	**f**	$(y+6)^2$
g	$(11+y)(11-y)$	**h**	$(11+y)^2$	**i**	$(11-y)^2$
j	$(3b+7)^2$	**k**	$(3b+7)(3b-7)$	**l**	$(3b-7)^2$

Chapter 12, *Products and factors*, explores these special products more.

7 This diagram shows a house $(3x-10)$ m long and $(2x+15)$ wide on a block of land with dimensions 30 m × 20 m. R C

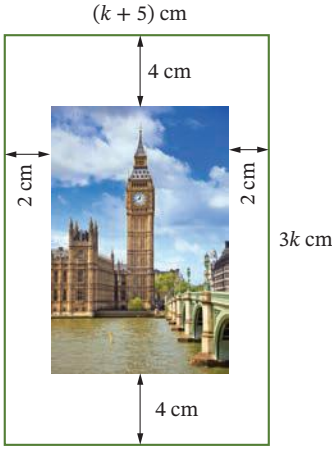


a Write down a binomial expression for the area of the house in square metres.

b Expand and simplify your expression for the area.

c The green area of the block of land not covered by the house is to be turfed. Write a simplified expression for this area in square metres.

8 A photograph frame is $(k+5)$ cm wide and $3k$ cm long. The gap between the photograph and frame is 4 cm at the top and bottom and 2 cm on each side. R C



a What is the area of the photograph frame?

b Write down expressions for the length and width of the photograph.

c Write down a binomial expression for the area of the photograph.

d Expand and simplify your expression for the area of the photograph.

e Find an expression for the area of the frame not taken up by the photograph.

9 A family room in a house is to be extended. The room is a metres long and b metres wide. The length is to be increased by 3 metres and the width by 1 metre. R C

a Write down expressions for the new length and width in metres.

b Write down a binomial expression for the new area of the room in square metres.

c Expand and simplify your expression for the area.

d By how much has the area of the room increased in square metres?

10 Prove that: R C

a $(a-b)^2 = (b-a)^2$

b $(a+b)(a-b) = a^2 - b^2$

c $(a-b)^2 = a^2 - 2ab + b^2$

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PATH

Investigation

Factorising quadratic expressions

1 **a** Show that $(x + 3)(x + 5) = x^2 + 8x + 15$.

b The quadratic expression $x^2 + 8x + 15$ has 3 terms. The coefficient of x (the number in front of the x) is 8. How are the 3 and 5 in $(x + 3)(x + 5)$ related to the 8?

c The constant term in $x^2 + 8x + 15$ is 15, the number with no x at the end. How are the 3 and 5 related to the 15?

2 **a** Expand $(x + 9)(x + 2)$.

b What is the coefficient of x? How are 9 and 2 related to it?

c What is the constant term? How are 9 and 2 related to it?

3 **a** Expand $(x + 8)(x - 3)$.

b What is the coefficient of x? How are 8 and -3 related to it?

c What is the negative constant term? How are 8 and -3 related to it?

4 **a** Expand $(x - 4)(x - 1)$.

b What is the negative coefficient of x? How are -4 and -1 related to it?

c What is the positive constant term? How are -4 and -1 related to it?

5 In the expansion of any binomial product, how are the coefficient of x and the constant term related to the numbers in the binomials?

6 Copy and complete:

a $(x + ___)(x + ___) = x^2 + 5x + 4$

b $(x + ___)(x + ___) = x^2 + 8x + 15$

c $(x + ___)(x + ___) = x^2 + 7x + 12$

d $(x + ___)(x - ___) = x^2 - 4x - 32$

e $(x + ___)(x - ___) = x^2 + 2x - 3$

f $(x - ___)(x - ___) = x^2 - 9x + 20$

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Factorising quadratic expressions $x^2 + bx + c$

4.06

A **quadratic expression** is an algebraic expression in which the highest power of the variable is 2, such as $x^2 - 5x + 7$, $x^2 - 15$, $2x^2 - 3x + 9$ and $-4x^2 + 7x$.

A quadratic expression such as $x^2 - 5x + 7$ is called a **trinomial** because it has 3 terms.

The expansion of $(x + 2)(x + 4)$ is $x^2 + 6x + 8$, a quadratic trinomial.

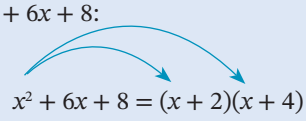
$\therefore$ The factorisation of $x^2 + 6x + 8$ is $(x + 2)(x + 4)$.

PATH

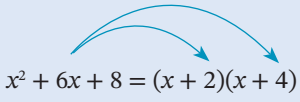
Factorising quadratic trinomials

In the factorisation of a quadratic trinomial such as $x^2 + 6x + 8$:

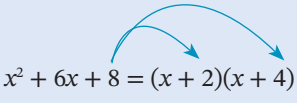
- each factor must have an x term to give x^2



- $2 + 4 = 6$, which is the **coefficient** of x, the number in front of the x



- $2 \times 4 = 8$, which is the **constant term** with no x



Example 11

Factorise each quadratic trinomial.

a $a^2 + 7a + 12$ **b** $x^2 + 9x + 8$

Solution

a Find the 2 numbers that have a sum of 7 and a product of 12.

It is best to test numbers that have a **product of 12** and then check if their sums equal 7.

Pair of numbers	Product	Sum
6, 2	$6 \times 2 = 12$	$6 + 2 = 8$
3, 4	$3 \times 4 = 12$	$3 + 4 = 7$

The correct numbers are 3 and 4.

$\therefore a^2 + 7a + 12 = (a + 3)(a + 4)$

b Find 2 numbers with a sum of 9 and a product of 8.

Test numbers that have a **product of 8** and check if their sums equal 9.

Pair of numbers	Product	Sum
2, 4	$2 \times 4 = 8$	$4 + 2 = 6$
8, 1	$8 \times 1 = 8$	$1 + 8 = 9$

The correct numbers are 8 and 1.

$\therefore x^2 + 9x + 8 = (x + 8)(x + 1)$

Video
Factorising quadratic expressions 1

PATH

Factorising quadratic expressions of the form $x^2 + bx + c$

- Find 2 numbers that have a sum of b and a product of c
- Use these 2 numbers to write a binomial product of the form $(x ___)(x ___)$

Video
Factorising quadratic expressions 1

Example 12

Factorise each quadratic expression.

a $x^2 + x - 6$ **b** $a^2 - 2a - 15$ **c** $y^2 - 6y + 8$

Solution

a $x^2 + x - 6$
Find 2 numbers that have a product of -6 and a sum of 1.
Since the product is negative, one of the numbers must be negative.
They are $+3$ and -2.
$\therefore x^2 + x - 6 = (x + 3)(x - 2)$

b $a^2 - 2a - 15$
Product $= -15$, sum $= -2$.
Since the product is negative, one of the numbers must be negative.
They are -5 and $+3$.
$\therefore a^2 - 2a - 15 = (a - 5)(a + 3)$

c $y^2 - 6y + 8$
Product $= 8$, sum $= -6$.
Since the sum is negative, one of the numbers must be negative.
Since the product is positive, *both* of the numbers must be negative.
They are -2 and -4.
$\therefore y^2 - 6y + 8 = (y - 2)(y - 4)$

EXERCISE 4.06 ANSWERS ON P. 587

Factorising quadratic expressions $x^2 + bx + c$ U F R

EXAMPLE 11

1 Find 2 numbers whose: R

a product is -15 and sum is 2
b product is 40 and sum is -14
c product is 42 and sum is 13
d product is -6 and sum is -1
e product is 45 and sum is 14
f product is -24 and sum is 2
g product is 54 and sum is 15
h product is -16 and sum is 0
i product is -10 and sum is 3
j product is 35 and sum is -12

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PATH

EXAMPLE 12

4.06

2 Factorise each quadratic expression. R

a	$y^2 + 8y + 12$	**b**	$m^2 + 15m + 56$	**c**	$g^2 + 9g + 14$
d	$w^2 + 13x + 36$	**e**	$p^2 + 11p + 24$	**f**	$a^2 + 13a + 42$
g	$e^2 + 12e + 27$	**h**	$n^2 + 6n + 9$	**i**	$c^2 + 10c + 21$

3 Factorise each quadratic trinomial. R

a	$x^2 - 9x + 20$	**b**	$h^2 - 13h + 30$	**c**	$p^2 - 11p + 24$
d	$e^2 - 11e + 30$	**e**	$w^2 - 17w + 72$	**f**	$k^2 - 10k + 9$
g	$m^2 - 16m + 64$	**h**	$u^2 - 5u + 6$	**i**	$d^2 - 12d + 35$

4 Factorise each quadratic expression. R

a	$q^2 - 8q - 20$	**b**	$h^2 - 5h - 36$	**c**	$y^2 + 7y - 44$
d	$x^2 - 2x - 63$	**e**	$u^2 + 9u - 10$	**f**	$e^2 + 7e - 30$
g	$a^2 - a - 110$	**h**	$y^2 + 6y - 27$	**i**	$m^2 - 6m - 7$
j	$c^2 + 7c - 18$	**k**	$k^2 + 3k - 54$	**l**	$r^2 - 9r - 22$
m	$p^2 - 4p - 32$	**n**	$u^2 + 12u - 45$	**o**	$b^2 - 6b - 16$

5 Factorise $a^2 - 11a - 42$. Select the correct answer **A**, **B**, **C** or **D**.

A $(a - 7)(a + 6)$ **B** $(a + 7)(a - 8)$ **C** $(a - 21)(a + 2)$ **D** $(a + 3)(a - 14)$

6 Factorise each quadratic trinomial. R

a	$h^2 - 2h + 1$	**b**	$x^2 + 15x + 50$	**c**	$r^2 + 20r + 96$
d	$a^2 - 3a - 28$	**e**	$u^2 - 7u - 60$	**f**	$y^2 - 18y + 81$
g	$v^2 - v - 56$	**h**	$w^2 - 11w - 60$	**i**	$g^2 + 3g - 18$
j	$p^2 + 14p + 48$	**k**	$e^2 + 7e - 8$	**l**	$x^2 - 19x + 84$

Power plus ANSWERS ON P. 587

1 Simplify each expression using index notation.

a	$13^7 \times (13^5 \div 13^{11})$	**b**	$(8^{-4})^5 \div (8^5)^{-4}$	**c**	$\left(\frac{(7^2)^3}{7^5 \times 7^2}\right)^{-1}$
d	$\left(\frac{1}{10}\right)^{-10} \times \left(\frac{1}{10}\right)^{10}$	**e**	$\left(-2\frac{1}{2}\right)^3$	**f**	$\left(\frac{1}{2}\right)^{-10} - \left(\frac{1}{4}\right)^{-5}$

2 Simplify each expression.

a	$\frac{1}{x} + \frac{1}{y} - \frac{1}{w}$	**b**	$\frac{1}{x} \times \frac{1}{y} \div \frac{1}{w}$	**c**	$\frac{1}{x} \times \left(\frac{1}{y} + \frac{1}{w}\right)$

3 Expand each expression.

a	$(a - b)(a + b - c)$	**b**	$(x - y + 1)^2$	**c**	$t^{-1}(3t^2 + 4t - 1)$

4 Factorise each expression.

a	$x^2 - 133x + 1000$	**b**	$y^2 + 14y - 1800$
c	$b^2 + 82b + 1681$	**d**	$n^2 - 2500$

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CHAPTER 4 REVIEW

Quiz
Language of maths 4

Puzzle
Algebra crossword

Language of maths

algebraic fraction	base	binomial	brackets
coefficient	constant term	denominator	expand
expression	factorise	highest common factor	index law
indices	numerator	product	power
quadratic expression	reciprocal	term	variable

1 Dividing by a fraction is the same as multiplying by its what?

2 Any number raised to the power of 0 is equal to what?

3 What is the difference between **expand** and **factorise?**

4 What power is associated with the reciprocal of a term or number?

5 In the quadratic trinomial $2x^2 - 3x + 6$, what is:

a the constant term? **b** the coefficient of x?

6 Copy and complete:

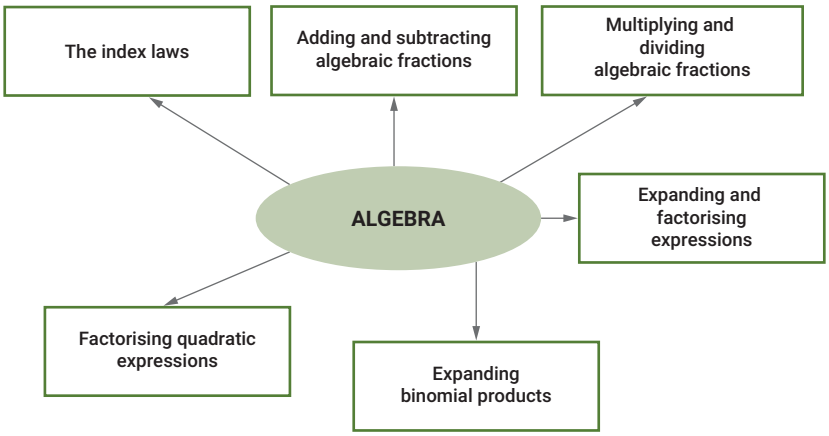
To factorise quadratic expressions of the form $x^2 + bx + c$, first find 2 numbers that have a _____ of b and a _____ of c.

Worksheet
Mind map: Algebra

Topic summary

- What was this topic about? What was the main theme?
- What content was new and what was revision for you?
- Write 3 index laws in both words and symbols.
- Write 10 questions (with solutions) that could be used in a test for this chapter.
  Include some questions that you have found difficult to answer.
- List the sections of work in this chapter that you did not understand. Follow up this work.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



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TEST YOURSELF 4

ANSWERS ON P. 588

Quiz
Test yourself 4

4.01

PATH

TEST YOURSELF 4

1 Simplify each expression.

a $3v^4 w^2 \times 2vw^5$

b $\frac{24t^8h^8}{3th^2}$

c $(5xy^2)^2$

d $\left(\frac{2p}{3}\right)^0$

e $(4k)^{-1}$

f $\left(\frac{5y}{2}\right)^3$

g $8^0 - 2^0$

h $3^{-1} + 4^0$

i $16a^3g^5 \div 18ag^4$

4.01

2 Simplify each expression.

a $(4m)^{-2}$

b $4m^{-2}$

c $\left(\frac{5b^8y^6}{b^2y^3}\right)^4$

d $(4t^4 u^5)^3 \times 8t^2 u$

e $45c^6 d^8 \div (-3cd^2)^2$

f $\left(\frac{45ab^4}{54a^2b^3}\right)^{-1}$

4.02

3 Simplify each expression.

a $\frac{t}{4} - \frac{2t}{5}$

b $\frac{5g}{3} + \frac{3g}{2}$

c $\frac{5x}{16} - \frac{x}{4}$

d $\frac{3b}{7} + \frac{2}{7}$

e $\frac{5w}{8} - \frac{3}{2}$

f $\frac{7}{12} - \frac{y}{5}$

g $\frac{5m}{7} + \frac{3n}{4}$

h $\frac{5p}{6} - \frac{p}{8}$

4.03

4 Simplify each expression.

a $\frac{3}{m} \times \frac{p}{4}$

b $\frac{1}{q} \times \frac{3}{q}$

c $\frac{2d}{3} \times \frac{15y}{14d}$

d $\frac{m}{3} \div \frac{2m}{9}$

e $\frac{5}{2a} \div \frac{4a}{7}$

f $\frac{12md}{5} \div \frac{3m}{10d}$

g $\frac{4xy}{5q} \div \frac{3x}{2y} \times \frac{9q}{4y}$

h $\frac{8ab}{3c} \div \frac{5}{2c} \div \frac{4b}{15c}$

4.04

5 Expand each expression.

a $9(m - 8)$

b $b(10a + b)$

c $-3y(4x^2 - 5y)$

d $8tp(7p - 5t)$

e $-(3n - 10)$

f $-5h^2(3h + 7)$

g $4y(5y - 7h)$

h $-wx(3x - 7w)$

Foundation | Standard | Complex

9780170479622

4.04

6 Expand and simplify each expression.

a $5g - 3g(6 - 7g)$

b $4fg(g - 6f) - 6f^2 g$

c $12(9 - n) - 5(2n + 3)$

d $x^2(6x + x^2) + 2x(3x^3 + x^2)$

e $3(7 - 2y) - 5y(7 - 2y)$

4.04

7 Factorise each expression.

a $8t - 72$

b $b^2 + 36b$

c $-3m - 33$

d $36wr^2 + 28w^2r$

e $-24p + 18q$

f $2(5x - 1) - 3x(5x - 1)$

PATH

4.04

8 Factorise each expression.

a $15xy^2 - 30x^3 y^3$

b $6pt^2 + 12p^2 t - 48p^3$

c $32r^2 s^4 + 12r^4 s^3$

d $50x^4 y^3 - 75x^3 y^4$

e $-8p^3 q^3 + 48p^3 q^6$

f $n(n^2 + 6) - (n^2 + 6)$

4.05

9 Expand each binomial product.

a $(b + 3)(b + 10)$

b $(d + 8)(d - 7)$

c $(t - 6)(9 - t)$

d $(5x + 7)(4x - 3)$

e $(7y - 5)(7y + 5)$

f $(3p - 8)(7p - 2)$

g $(3m + 7)(3m + 7)$

h $(4 - x)(3x + 2)$

i $(6 - 5d)(3 - 2d)$

PATH

4.06

10 Factorise each quadratic expression.

a $y^2 + 10y + 25$

b $x^2 - 21x + 20$

c $n^2 + 8n - 33$

d $a^2 - 11a + 28$

e $m^2 - 5m - 84$

f $p^2 + 3p - 54$

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5

STATISTICS AND PROBABILITY

COMPARING DATA

Is climate change affecting the amount of rainfall in different areas? What are the tourist numbers in different parts of Australia, and how much do they spend? How do we ensure accurate medical testing data to monitor the potential for pandemics such as COVID-19? To answer these questions, sets of data need to be collected and then compared by looking at the shape of their displays or by analysing their measures of centre and spread.



Chapter outline

		Working mathematically				
5.01	The shape of a frequency distribution	U	F		R	C
5.02	Quartiles and interquartile range	U	F			
5.03	Standard deviation	U	F			
5.04	Comparing means and standard deviations	U	F			
5.05	Box plots	U	F		R	C
5.06	Parallel box plots	U	F		R	C
5.07	Comparing data sets	U	F		R	C
5.08	Scatterplots	U	F		R	C
5.09	Line of best fit	U	F		R	C
5.10	Sampling and surveys*	U	F		R	C
5.11	Statistical investigations	U	F	PS	R	C

*PATH

Wordbank

bivariate data Data that measures 2 variables, represented by an ordered pair of values that can be graphed on a scatterplot

box plot (or **box-and-whisker plot**) A graph that shows the quartiles of a set of data and the highest and lowest values; the box contains the middle 50% of values while the lines or 'whiskers' extend to the 2 extremes

five-number summary For a set of numerical data, the lowest value, lower quartile, median, upper quartile, highest value

interquartile range (IQR) The difference between the upper quartile and lower quartiles, IQR = $Q_3 - Q_1$, representing the middle 50% of values

quartiles The values Q_1, Q_2, Q_3 that divide a set of data into quarters (4 equal parts)

scatterplot A graph consisting of dots on a number plane that represent bivariate data

stratified sample A random sample where the proportions reflect the proportions in the population. For example, if 10% of the population is left-handed, then 10% of the stratified sample is left-handed.

Quiz
Wordbank 5

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Videos (13):

5.01 The shape of a frequency distribution • Analysing dot plots
5.02 Interquartile range 1 • Interquartile range 2
5.03 Standard deviation on a Casio calculator
5.05 Box-and-whisker plots • Statistics
5.06 Double box plots
5.07 Comparing data sets • Back-to-back stem-and-leaf plots • Histogram vs box plot
5.08 Scatterplots
5.09 Lines of best fit

Twig videos (2):

5.03 Freak waves
5.08, 5.09 Can eating fish prevent murder?

PhET interactive (1):

5.09 Least-squares regression

Quizzes (5):

- Wordbank 5
- SkillCheck 5
- Mental skills 5
- Language of maths 5
- Test yourself 5

Skillsheet (1):

SkillCheck Statistical measures

Worksheets (15):

5.01 Stem-and-leaf plots • Analysing data
5.02 Interquartile range
5.05 Five-number summaries
5.06 Box-and-whisker plots • Data 1
5.07 Comparing city temperatures • Comparing word lengths • Investigating young drivers
5.08 Scatterplots
5.09 Line of best fit • Trendlines • Data 2 • 2 mm grid paper
Mind map Comparing data

Puzzles (4):

5.01 Statistical match-up
5.05 Mode, median and mean
5.08 Scatterplots matching game
Language of maths Data crossword

Technology (4):

5.01 Skewness
5.05 Five-number summary
5.06 Parallel box plots
5.09 Line of best fit

Spreadsheets (4):

5.01 Skewness
5.05 Five-number summary
5.06 Parallel box plots
5.09 Line of best fit

Presentation (1):

5.06 Analysing data

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

In this chapter you will:

- describe the shape of a statistical distribution in terms of its symmetry and skewness, using terms such as 'skewed', 'symmetric' and 'bimodal'
- calculate quartiles and the interquartile range of a set of data
- calculate the mean and standard deviation of a set of data and compare data sets
- calculate the five-number summary of a set of data and use it to construct a box plot
- construct histograms, dot plots, back-to-back stem-and-leaf plots and parallel box plots to compare 2 or more sets of data
- compare shapes of box plots, histograms and dot plots
- graph bivariate data on scatterplots to investigate patterns and relationships between independent and dependent variables, including strength and direction, association vs causation
- use technology to construct lines of best fit and use them to make predictions
- (PATH) evaluate statistics presented in the media and identify types of samples, sample size and biases in sampling, sources, graphs, interpretation, claims and conclusions
- (PATH) analyse and evaluate media reports of statistical studies

SkillCheck ANSWERS ON P. 588

Quiz SkillCheck 5

Skillsheet Statistical measures

1 For each set of data, find:

i the range

ii the mean (correct to one decimal place)

iii the median

iv the mode

a 15 13 18 14 15 18 23 14 20 16 15

b 8°C 3°C −5°C 2°C −4°C 7°C 3°C 0°C

c

Stem	Leaf
1	0 3 6
2	1 4 4 7 8
3	2 3 4 5 5 7 9
4	0 5 7 8
5	2 6 8

d

Score	Frequency
0	2
1	5
2	8
3	4
4	3
5	1

2 A cricketer scored these numbers of runs in 10 innings.

34 21 78 30 26 19 41 36 16 32

a Find:

i the median **ii** the mean **iii** the range.

b Which score is the outlier?

c **i** Calculate the median, mean and range if the outlier is not included in the scores.

ii What effect does the outlier have on the mean, median and range?

5.01 The shape of a frequency distribution

Worksheets
Stem-and-leaf plots

Analysing data

Puzzle
Statistical match-up

Technology
Skewness

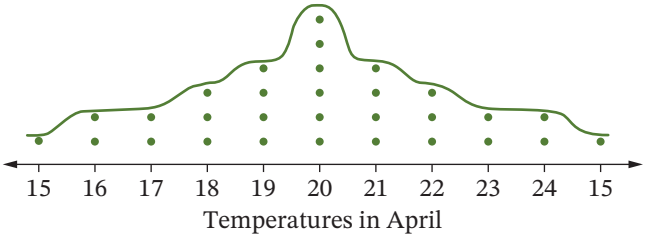
Spreadsheet
Skewness

A **statistical distribution** shows how a **data** set is arranged, especially when graphed. When looking at **histograms, dot plots** and **stem-and-leaf plots**, an overall pattern can be seen from the shape of the display.

The shape of a statistical distribution shows how the data is spread. This can be seen by drawing a curve around the graph or display.

A distribution is **symmetrical** if the data is evenly spread or balanced about the centre.

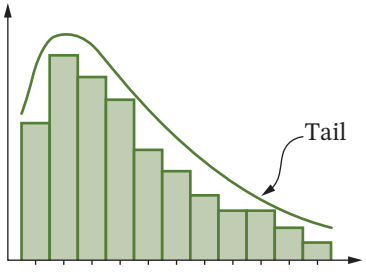
Stem	Leaf
3	0 2 4
4	1 8 9 9
5	2 4 5 6 6 7 8 8
6	0 3 4 5 5 6 7 8 9 9
7	2 4 4 4 5 5 5 5
8	2 8 8 8
9	3 5 7



Videos
The shape of a frequency distribution

Analysing dot plots

A distribution is **skewed** if most of the data is bunched or clustered at one 'end' of the distribution and the other 'end' has a 'tail'.



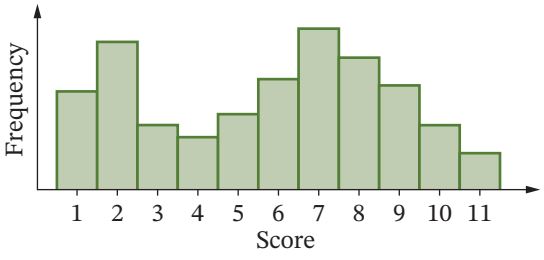
A distribution is **positively skewed** if its tail points to the right (higher values).

Stem	Leaf
0	3 5
1	0 6
2	5 7 8
3	0 3 8 9
4	1 1 2 3 4 8
5	0 0 1 1 2 2 5 5
6	3 5 7 5 6 6 7 7 9
7	0 2 2 4 5

A distribution is **negatively skewed** if its tail points to the left (lower values).

A distribution is **bimodal** if it has 2 peaks. The higher peak is the mode, while the other peak indicates another value that has a high frequency.

For example, this frequency histogram has 2 peaks at 2 and 7 so it is bimodal. The mode, however, is 7.



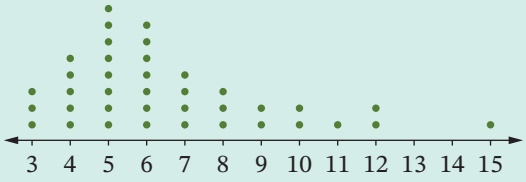
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Example 1

For each statistical distribution:

i describe the shape ii identify any outliers and clusters

a



b

Stem	Leaf
10	4 5
11	3 4 4 9
12	1 2 2 6 8
13	0 1 5 5 7 9 9 9
14	4 5 6 8 8
15	0 0 1 1
16	0 2

Solution

a **i** The shape is positively skewed (tail points towards the higher values).

ii 15 is an outlier and clustering occurs at 5 and 6.

b **i** The shape is symmetrical (the data is balanced about the stem of 13).

ii There are no outliers, but clustering occurs in the 130s.

EXERCISE 5.01 ANSWERS ON P. 588

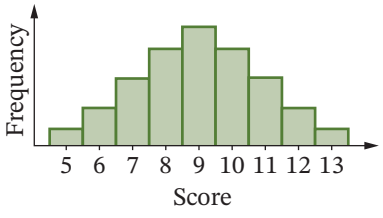
The shape of a frequency distribution U F R C

1 For each statistical distribution: R C

EXAMPLE 1

i describe the shape ii identify any outliers and clusters.

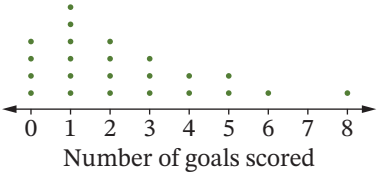
a



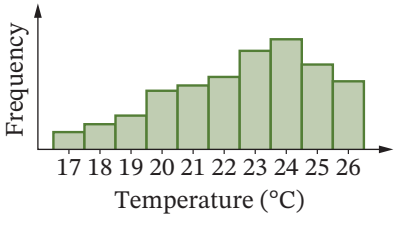
b

Stem	Leaf
2	4 5 6 9
3	1 2 3 3 4 5 7 8
4	0 4 4 6 8 9
5	4 5 5 8
6	0 0 2 3 5 6 7 8 9 9
7	3 5 7 8 8 9 9
8	1 1 3 5 6
9	0 3 5 6

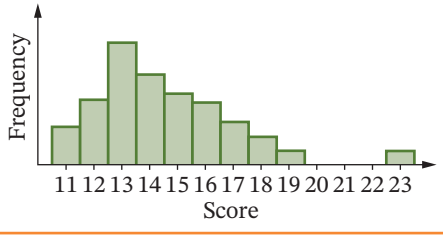
c



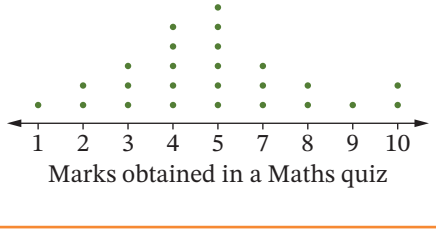
d



e



f



Foundation Standard Complex

g

Stem	Leaf
12	0 2 4 9
13	2 4 6 7 8 8 8
14	3 3 4 4 5 5 8 9 9 9
15	0 1 1 5 7 8 9 9
16	1 1 5 6 7
17	2 4 5 8
18	0 3 9
19	5 8
20	6 8

h

Stem	Leaf
5	3 4 4 6 7 8 9
6	0 0 5 9 9
7	2 4 5 6
8	5 7 8
9	3 3 6 7 8
10	2 4 6 8 8 8 8
11	
12	
13	6

2 These are the final round scores for players in a golf tournament. R C

66	70	67	72	75	72	70	74	75	72
74	72	73	71	71	69	70	71	71	74
72	69	75	73	69	75	73	69	69	67
74	72	72	73	71	73	77	68	72	72

a Arrange the data into a frequency table and construct a frequency histogram.

b Are there any outliers?

c Describe the shape of the distribution.

d Give a possible reason for the shape of the distribution.

e Where does clustering occur?

f Find the mode, the mean and the median and show their position in the histogram.

3 The stem-and-leaf plot shows the number of hours that students spend on their computers during the week. R C

Stem	Leaf
0	1 1 1 1 1 2 2 2 2 3 3 3 5 6 6 7 7 7 7 9 9
1	0 1 1 1 2 4 4 5 6 8 8 9
2	0 5 5 5 8 8
3	0 0 0 1 5
4	0 0

a How many students were surveyed?

b Where does the clustering occur?

c Are there any outliers?

d Describe the shape of the distribution.

e Give a possible reason for the shape of the distribution.

f Find the mean (to one decimal place), median and mode.

4 The following values are the heights (in cm) of 30 Year 8 students. R C

162	155	153	162	182	173	165	165	142	167
164	168	150	155	143	153	123	163	170	169
153	162	161	170	160	162	172	151	160	171

a Arrange the data into an ordered stem-and-leaf plot.

b Describe the shape of the distribution.

c Are there any outliers?

d Where does clustering occur?

e Find the mode, median and mean.

5 The daily maximum temperatures (correct to one decimal place) in Brisbane in November are shown in the stem-and-leaf plot. R C

Stem	Leaf
24	1
25	8
26	0 1 2 7 7
27	0 9 9
28	1 1 2 4 4 5 6 7 8 8
29	0 1 3 3 3
30	6 8 9
31	5
32	8

a Describe the shape of the distribution.

b Are there any outliers?

c What is the mode?

d Find the mean, correct to one decimal place.

e What is the median?

f Find the range.

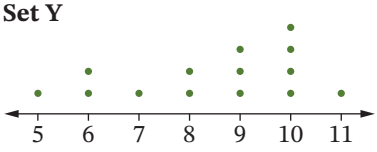
g Is the range a good indicator of the spread of the temperatures? Give reasons.

6 Which statement is true about the sets of data below? Select the correct answer **A**, **B**, **C** or **D**.

Set X



Set Y



A X is symmetrical and Y is positively skewed.

B X is bimodal and Y is negatively skewed.

C X has 2 peaks and the median of Y is 8.

D The median of X and Y is 8.

5.02 Quartiles and interquartile range

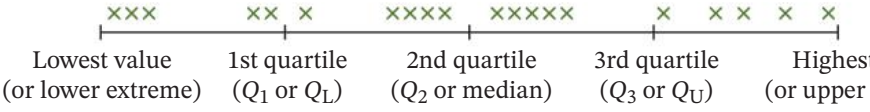
Quartiles

Videos
Interquartile range 1
Interquartile range 2

The **median**, being the middle value, divides a set of data into 2 equal parts (halves).

Quartiles are the values Q_1, Q_2 and Q_3 that divide the set of data into 4 equal parts (quarters).

Data values (in order)



Worksheet
Interquartile range

The **1st quartile** Q_1, also called the **lower quartile** Q_L, is the value that divides the lower 25% of values. $\frac{1}{4}$ of the values lie below Q_1.

The **2nd quartile** Q_2 is the value that divides the lower 50% of values, so it is also the **median**. $\frac{1}{2}$ of the values lie below Q_2.

The **3rd quartile** Q_3, also called the **upper quartile** Q_U, is the value that divides the lower 75% of values from the upper 25% of values. $\frac{3}{4}$ of the values lie below Q_3, $\frac{1}{4}$ of the values lie above it.

Finding the quartiles of a data set

- Sort the values in order, find the median and call it Q_2
- Find the median of the bottom half of the values and call it Q_1 (or Q_L)
- Find the median of the top half of values and call it Q_3 (or Q_U).

Example 2

Find the quartiles for each set of data.

a	9	3	8	7	6	8	4	6	2	10	9		
b	15	18	7	16	23	9	15	20	16	14	13	11	19
c	65	84	75	82	97	70	68	76	93	48			
	79	54	80	79	82	96	63	85	72	70			

Solution

a Arranging the 11 values in ascending order, we have:

2 3 (4) 6 6 (7) 8 8 (9) 9 10

Lower quartile $Q_1 = 4$ | Median $Q_2 = 7$ | Upper quartile $Q_3 = 9$

Q_2 (median) = 7

Q_1 (lower quartile) = 4 — The middle of the 5 values below 7

Q_3 (upper quartile) = 9 — The middle of the 5 values above 7

b Arranging the 13 values in ascending order, we have:

7 9 11 13 14 15 **15** 16 16 18 19 20 23

$Q_1 = \frac{11+13}{2} = 12$ $\quad Q_2 = 15$ $\quad Q_3 = \frac{18+19}{2} = 18.5$

c Arranging the 20 values in ascending order, we have:

48 54 63 65 68 70 70 72 75 76 79 79 80 82 82 84 85 93 96 97

$Q_1 = \frac{68+70}{2} = 69$ $\quad Q_2\,(\text{median}) = \frac{76+79}{2} = 77.5$ $\quad Q_3 = \frac{82+84}{2} = 83$

5.02

The interquartile range

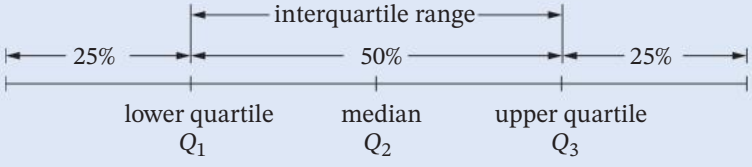
The **range** is a **measure of spread** because it gives an indication of how widely the values are spread in a set of data. Another measure of spread is the **interquartile range**.

The **interquartile range** is the difference between the upper and lower quartiles and so it is the range of the middle 50% of the data.

Interquartile range

Interquartile range (IQR) = upper quartile – lower quartile

$= Q_3 - Q_1$



The interquartile range takes into account the middle 50% of values and ignores very low or very high values (outliers), so sometimes it is better to use than the range as a measure of spread.

Example 3

The number of runs scored by the Sydney Sixers Twenty20 cricket team per match during one season were:

76 143 127 176 142 116 137 104 161 174 149 154 180 137 175

a Find the range.

b Find the interquartile range.

c Which is the better measure of spread of the points scored by the Sixers – the range or interquartile range?

Solution

First arrange the scores in order:

76 104 116 **127** 137 137 142 **143** 149 154 161 **174** 175 176 180

Lower quartile: $Q_1 = 127$ Median: $Q_2 = 143$ Upper quartile: $Q_3 = 174$

a Range $= 180 - 76$
$= 104$

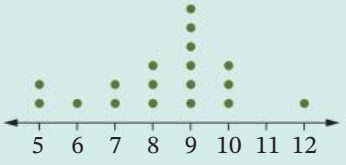
b Interquartile range $= Q_3 - Q_1$
$= 174 - 127$
$= 47$

c The interquartile range is the better measure of spread as the outlier of 76 is excluded. As an outlier, the value of 76 has affected the range, making it very big.

Example 4

Find the interquartile range of each data set.

a



b

Stem	Leaf
1	2 7
2	0 3 4 4 5
3	1 2 2 4 6 8 8 9
4	0 1 3 7
5	1 2

Solution

a There are 18 values, so the median is 'between' the 9th and 10th values, which are both 9s (see diagram).

Median, $Q_2 = \frac{9+9}{2} = 9$

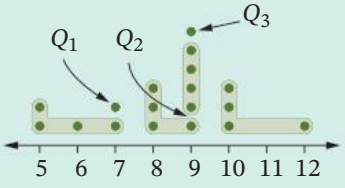
Q_1 is the median of the bottom half of 9 values (the 5th value).

$Q_1 = 7$.

Q_3 is the median of the top half of values (the 14th value).

$Q_3 = 9$.

$\therefore$ IQR $= Q_3 - Q_1$
$= 9 - 7$
$= 2$



b There are 21 values, so the median is the 11th value (see diagram).

$Q_2 = 34$

Lower quartile, $Q_1 = \frac{24+24}{2} = 24$

Upper quartile, $Q_3 = \frac{40+41}{2} = 40.5$

$\therefore$ IQR $= 40.5 - 24$
$= 16.5$

Stem	Leaf
1	2 7
2	0 3 4 4 5
3	1 2 2 4 6 8 8 9
4	0 1 3 7
5	1 2

Q_1 between 5th–6th values
Q_2 11th value
Q_3 between 16th–17th values

EXERCISE 5.02 ANSWERS ON P. 589

Quartiles and interquartile range U F

EXAMPLE 2

1 Find the quartiles for each set of data.

a 3 7 9 5 5 6 2 8 9 7

b 15 19 18 12 20 34 28 18 28 20 23 25

c 34 45 32 38 29 40 37 33 35
30 34 35 38 37 38 31 30 34

EXAMPLE 3

2 Calculate the range and the interquartile range of each data set in question **1**.

3 Calculate the interquartile range for each set of data below.

a 5 6 6 7 8 9 9 10 14 14 15 16

b 2 0 3 5 2 1 0 6 4 3 8 4 2

4 The monthly rainfall figures in mm for Ulladulla one year were:

31 174 288 89 15 123 26 5 8 275 38 58

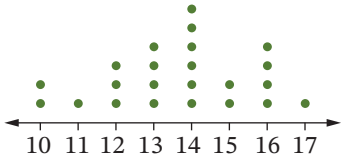
For this data, find:

a the range **b** the interquartile range.

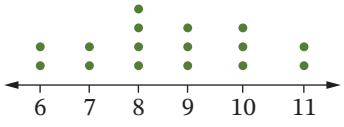
EXAMPLE 4

5 Find the interquartile range for each set of data.

a



b



c

Stem	Leaf
3	2 7
4	0 3 3 5
5	2 4 5 6 7 8 8
6	3 4 7
7	2

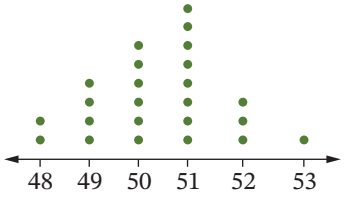
d

Stem	Leaf
1	3 5 8 9
2	0 1 3 3 4 5 6
3	5 8 9 9
4	1 3
5	4

e

Stem	Leaf
10	3 5 5 6 6
11	0 1 2
12	3 4 6 7 8
13	4 7
14	1

f



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6 The pulse rates for a group of students are as follows.

82 81 72 58 79 77 62 66 92 78 80 67 91 75 72 68

a Find the range.

b Find the interquartile range.

c **i** List the values that lie between the lower and upper quartiles.

ii What percentage of values lie between Q_1 and Q_3?

d What percentage of values lie above the lower quartile?

7 The number of points per game scored by a basketball team during one season were:

55 35 49 53 51 55 42 48 63 43 48 48 62

a Find:

i the range **ii** the interquartile range.

b Which is the better measure of spread?

c List the values that lie between Q_1 and Q_3. What percentage of the values is this?

Did you know?

How did statistics begin?



Adam Ján Figeľ/Alamy Stock Photo

In prehistoric times, when the number of people and animals was recorded in pictures and symbols on the walls of caves, a simple form of statistics was being used.

Before 3000 BCE, ancient Babylonians used clay tablets to record crop yields and trade data, and around 2650 BCE the Egyptians 'surveyed' the population and wealth of their country before building the pyramids. Forms of statistics were also used in the Bible in the 'Book of Numbers' and the 'First Book of Chronicles'. Numerical records existed in China before 2000 BCE, and the Greeks (to help collect taxes) held a census in 594 BCE. The Roman Empire was the first government to collect information about the population. In 1086 a census was conducted in England. The information obtained in this census was recorded in the Domesday Book.

Research more about the Domesday Book.
Write a one-page report suitable for a classroom presentation.

Standard deviation

5.03

The **standard deviation** is another measure of spread.
Like the mean, it is calculated using all the values in a data set.

Standard deviation

- The **standard deviation** of a set of data is an average of how different each value is from the mean.
- The symbol for standard deviation is σ or σ_n. σ is the lower-case Greek letter 'sigma'
- Standard deviation has a complex formula so it is best calculated using the calculator's statistics mode.
- It is a better measure of spread than the range and interquartile range because its value depends on every value in the data set.

Example 5

Video
Standard deviation on a Casio calculator

Calculate, correct to 2 decimal places, the standard deviation of each set of data.

a The daily maximum temperature (in °C) in Campbelltown for 2 weeks in January.

45.0	24.5	24.8	29.1	35.0	26.9	31.8
33.8	32.9	23.6	22.1	29.2	27.1	32.7

b The scores of Year 10 students in a Science quiz.

Score	2	3	4	5	6	7	8	9	10
Frequency	2	1	3	3	2	5	6	4	2

Solution

Follow the instructions for the statistics mode (SD or STAT) of your calculator as shown in the tables below.

a

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT 1-VAR	MODE STAT =
Clear the statistical memory	SHIFT 1 Edit, Del-A	2nd F DEL
Enter data	SHIFT 1 Data to get table 45.0 = 24.5 =, etc. to enter in column AC to leave table	45.0 M+ 24.5 M-, etc.
Calculate the standard deviation (σ_x = 5.75)	SHIFT 1 Var σ_x =	RCL σ_x
Return to normal (COMP) mode.	MODE COMP	MODE 0

$\sigma = 5.75$

b

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT 1-VAR SHIFT MODE scroll down to STAT Frequency? ON	MODE STAT =
Clear the statistical memory	SHIFT 1 Edit, Del-A	2nd F DEL
Enter data	SHIFT 1 Data to get table 2 = 3 =, etc. to enter in x column 2 = 1 =, etc. in FREQ column AC to leave table	2 2nd F STO 2 M+ 3 2nd F STO 1 M-
Calculate the standard deviation ($\sigma_x = 2.26$)	SHIFT 1 Var σ_x =	RCL σ_x
Return to normal (COMP) mode.	MODE COMP	MODE 0

$\sigma = 2.26$

EXERCISE 5.03 ANSWERS ON P. 589

Standard deviation U F R C

Note: In this exercise, express all means and standard deviations to 2 decimal places.

EXAMPLE 5

1 Calculate the standard deviation of each set of data.

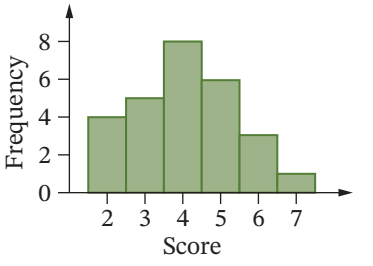
a 5 4 7 8 2 9 10

b 20 23 28 24 19 25 26 24 23

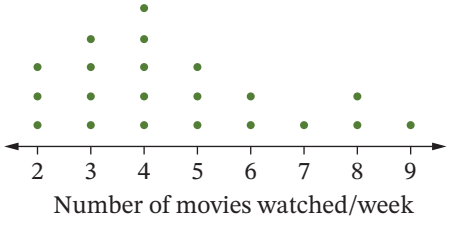
c

x	f
10	2
11	5
12	9
13	8
14	3
15	1

d



e



2 An English class of Year 10 students scored the following marks for their speeches: R C

12 15 14 16 16 12 11 18 7 10
15 14 13 13 18 10 12 12 14 13

a Which mark is the outlier?

b Find the standard deviation of the marks:

i with the outlier **ii** without the outlier.

c What effect does removing the outlier have on the standard deviation?

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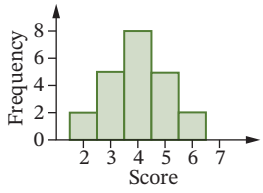
9780170479622

3 For the 3 statistical distributions **A**, **B** and **C** shown, which one has: R

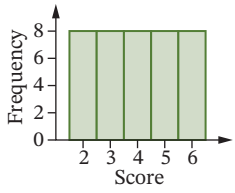
a the highest standard deviation?

b the lowest standard deviation?

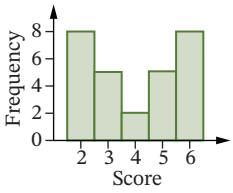
A



B

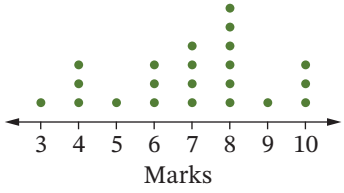


C



4 Find the standard deviation of each data set.

a



b

Stem	Leaf
2	0 2 7
3	5 5 6 8 9
4	1 2 4 5 6 6 7
5	0 3 4 5 9 9
6	1 5 5
7	6

5 The heights of girls in a Year 9 basketball team are: R

151 161 171 175 176 157 175 163 164

a Calculate the mean and the standard deviation of heights of the basketball team.

b Divya joins the basketball team. What is her possible height if the standard deviation:

i increases **ii** decreases?

6 The training times (in seconds) of Harrison sprinting over 100 m were: R

11.2 11.0 10.9 12.3 11.8 11.1 11.4 11.6 11.0

a Find the mean and standard deviation of his training times.

b What training time would Harrison have to do to:

i increase the standard deviation?

ii decrease the standard deviation?

7 Brooke's times (in seconds) for swimming 100 m were: R C

55.7 59.8 58.4 56.7 60.0 55.8 57.4 58.0

An error was made in recording these times and 2 seconds needs to be added to each time.

Which of the following statements is true? Select the correct answer **A**, **B**, **C** or **D**.

A The standard deviation will increase and the mean will stay the same.

B The standard deviation will decrease and the mean will increase.

C The standard deviation will stay the same and the mean will increase.

D The standard deviation and the mean will both stay the same.

Foundation Standard Complex

Investigation

The formula for standard deviation

The formula for the standard deviation of a set of data is $\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$, where x is each data value, $\bar{x}$ is the mean and n is the number of data values.

The steps for calculating standard deviation are:

- Calculate the mean $\bar{x}$
- For every value in the data set, find the difference between the value and the mean, then square this difference: $(x-\bar{x})^2$
- Calculate the average of these 'squared deviations' by adding them and dividing their sum by the number of values: $\frac{\Sigma(x-\bar{x})^2}{n}$
- Calculate the square root of this average: $\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

We will now use this method to calculate the standard deviation of this set of data:

4 5 6 7 2 8 6 5 2

1 Calculate the mean of this data.

2 Copy and complete the table below by finding for each data value its difference from the mean and the square of this difference.

Score, x	4	5	6	7	2	8	6	5	2
$x-\bar{x}$	−1	0							
$(x-\bar{x})^2$	1	0							

3 Find the mean of the 'squared deviations' calculated in the bottom row of the table.

4 The standard deviation is the square root of this mean. Calculate the standard deviation correct to 2 decimal places.

5 Check your answer by calculating the standard deviation using your calculator's statistics mode and comparing both answers.

6 Use the standard deviation formula to calculate the standard deviation of each set of data.

a 5 4 7 8 2 9 10

b 20 23 28 24 19 25 26 24 23

Check your results by using your calculator.

7 The standard deviation is never negative. Explain why.

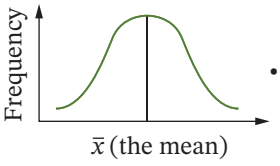
8 If the values of a set of data are all the same, what is the standard deviation? Explain.

Did you know?

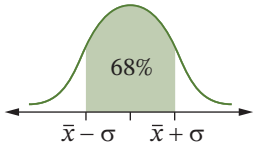
The normal curve

If the heights of all the people in Australia were graphed on a frequency polygon, the graph would be a normal curve, a symmetrical bell-shaped curve that peaks in the middle.

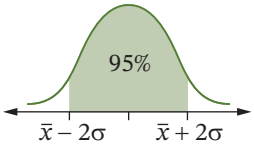
The normal curve has the following features:



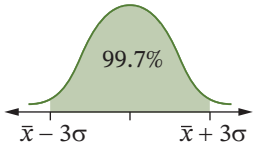
- The mean, median and mode are the same.



- About 68% of scores lie within one standard deviation of the mean.



- About 95% lie within two standard deviations of the mean.



- About 99.7% lie within three standard deviations of the mean.

Measure and analyse the heights of the students at your school. Do the data follow a normal curve?

Video
Freak waves

5.04

Comparing means and standard deviations

5.04

The **mean** and **standard deviation** can be used to compare different sets of data.

Example 6

The heights (in cm) of the girls and boys in a Year 10 PE class at Baramvale High were measured:

Girls:	163	155	171	162	165	158	172	
	166	163	150	160	181	160	156	
Boys:	174	167	164	175	189	145	165	166
	165	168	167	171	169	172	168	

a Calculate, correct to 2 decimal places, the mean and standard deviation for:
i the girls **ii** the boys **iii** the class.

b Which group has the greater spread of heights?

c Is there a significant difference between the heights of girls and boys?

Solution

a Using the calculator's statistics mode:

i Girls: $\bar{x} = 163$ cm, $\sigma_n \approx 7.60$

ii Boys: $\bar{x} \approx 168.33$ cm, $\sigma_n \approx 8.64$

iii Class: $\bar{x} \approx 165.76$ cm, $\sigma_n \approx 8.58$

b The data for the boys in the class has the greater spread of heights as its standard deviation is higher.

c The mean height of boys is greater than that of the girls but the girls have a smaller spread of heights.

Comparing measures of spread

The **standard deviation** is usually the most appropriate measure of spread as it uses all of the values in the data set.

The **range** is the easiest to calculate but its value only depends upon 2 values: the highest and the lowest.

If there are **outliers** in the data set, then the standard deviation and range will be affected by these extreme scores. In this case, the **interquartile range** is the better measure, because it is the range of the middle 50% of scores and so is not affected by outliers.

Example 7

a The ages of the children using a jumping castle and visiting a petting zoo are shown:

Jumping castle:	3	3	4	5	5	6	8	10	18
Petting zoo:	3	4	5	6	6	7	8	8	10

For each set of data, calculate:

i the range **ii** the interquartile range

iii the standard deviation (to 2 decimal places).

b Which is the best measure of spread for each set of data?

Solution

a For the jumping castle:

i Range $= 18 - 3$
$= 15$

ii IQR $= 9 - 3.5$
$= 5.5$

iii $\sigma_n \approx 4.48$

For the petting zoo:

i Range $= 10 - 3$
$= 7$

ii IQR $= 8 - 4.5$
$= 3.5$

iii $\sigma_n \approx 2.05$

b The jumping castle data has an outlier, 18, which affects the range and standard deviation. The interquartile range is the best measure for this data set.

The petting zoo data does not have an outlier, so the standard deviation is the best measure for this data set.

EXERCISE 5.04 ANSWERS ON P. 589

Comparing means and standard deviations U F R C

Note: In this exercise, express all means and standard deviations correct to 2 decimal places.

EXAMPLE 6

5.04

1 The pulse rates (in beats/minute) of a sample of men and women taken at a shopping centre are shown. R C

Men:	68	72	75	73	81	77	69	68	79	83	65	59	60	72	70
Women:	82	61	79	77	75	68	86	81	72	77	78	81	90	83	73

a Find the mean and standard deviation of each group.

b Is there a significant difference between the mean and standard deviation for men and women? Give reasons.

2 The reaction times (in seconds) for the dominant and non-dominant hands of a group of athletes were measured. R C

Dominant hand:	0.41	0.29	0.35	0.42	0.42	0.43	0.39	0.61	0.38
	0.34	0.75	0.34	0.38	0.47	0.34	0.32	0.29	0.30
Non-dominant hand:	0.46	0.34	0.38	0.39	0.39	0.39	0.51	0.50	0.47
	0.40	2.60	0.34	0.39	0.51	0.35	0.37	0.31	0.32

a Find the mean and standard deviation for each data set.

b Is there a significant difference between the results? Explain your answer.

c **i** What are the outliers for the reaction time of the dominant hand?

ii Find the mean and standard deviation without the outliers.

iii What effect does removing the outliers have on the mean and standard deviation?

d Find the mean and standard deviation of the reaction time for the non-dominant hand without the outlier.

e On which group has the removal of outliers had the greater effect on the mean and standard deviation? Justify your answer.

3 The scores of 2 cricket teams were recorded on a back-to-back stem-and-leaf plot. R

Western Tigers		Barrington City
5 2	7	
	8	3
7	9	0 8
8	10	7
	11	4 6
6	12	1 5
9 9 8 5	13	7
7 4	14	6
5	15	6 8

a Find the mean and standard deviation for each team.

b Which team was more consistent with its scores?

4 Vatha and Ana's times for running 100 m trials are shown. R C

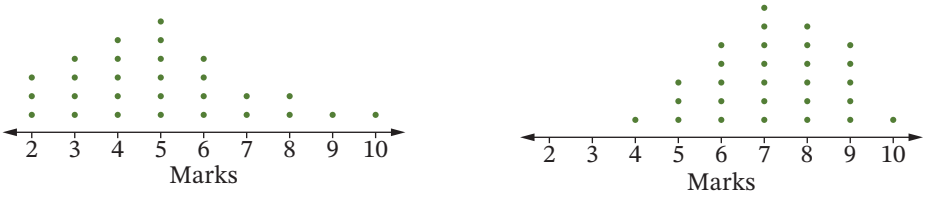
Vatha:	13.0	13.5	14.2	13.7	13.2	14.7	13.5	14.3
Ana:	14.2	13.2	15.1	13.8	14.2	15.2	13.9	13.5

a Find the mean and standard deviation for each runner.

b Which runner is more consistent? Give reasons.

Foundation Standard Complex

5 The dot plots show the test results of a class before and after using a tutorial website.



Which of the following is true?

A Both the mean and standard deviation increased.

B The mean increased and the standard deviation decreased.

C The mean decreased and the standard deviation increased.

D Both the mean and standard deviation decreased.

EXAMPLE 7

6 The marks obtained by students in Maths and Science exams are given below. R C

Maths:	40	72	76	74	60	64	64	59	74	84	62	84	66	64
	71	68	78	63	57	55	73	80	67	86	57	87	62	52
Science:	42	54	61	72	76	54	65	80	39	74	82	54	57	63
	64	75	68	76	81	40	37	43	58	68	67	49	54	62

a For each subject, find:

i the range **ii** the interquartile range **iii** the standard deviation.

b Find the mean for each subject.

c Determine in which subject the students performed better, giving reasons.

7 The points scored per match by the Roosters and the Dragons during a football season were: R C

Roosters:	10	16	8	50	22	38	34	30	16	12	18	38	12	20	18
	36	40	28	42	28	56	22	22	24						
Dragons:	10	6	17	25	19	13	10	18	14	32	0	14	14	16	10
	0	22	18	20	26	18	18	22	19						

a For each team, find:

i the range **ii** the interquartile range

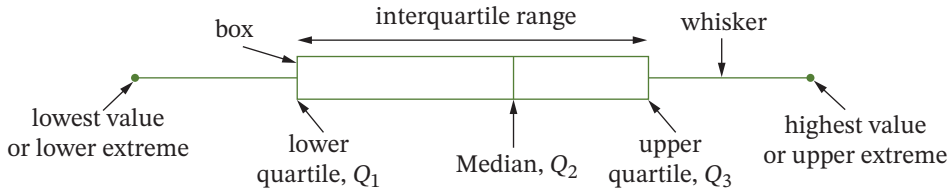
iii the mean **iv** the standard deviation.

b By comparing the means and the measures of spread, decide which was the better team.

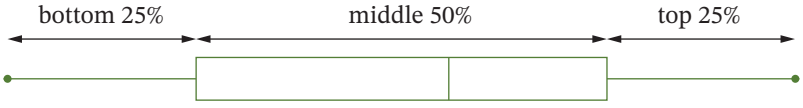
Box plots

5.05

A **box plot** (or **box-and-whisker plot**) displays the quartiles of a set of data and the lowest and highest values (lower and upper extremes).



The 'box' represents the middle 50% of values and the interquartile range, while the 'whiskers' represent the lowest and highest 25% of values.



Five-number summary

A box plot gives a **five-number summary** of a data set:

- the lower extreme (or lowest value)
- the lower quartile, Q_1
- the median, Q_2
- the upper quartile, Q_3
- the upper extreme (or highest value).

Example 8

The number of hours per week that Nick worked at the Big Chicken over summer were:

5 5 4 8 10 3 12

7 7 3 8 8 15

a Find a five-number summary for this data.

b Represent this data on a box plot.



Jeffrey Issac Greenberg 2+/Alamy Stock Photo

Videos
Box-and-whisker plots
Statistics

Worksheet
Five-number summaries

Puzzle
Mode, median and mean

Technology
Five-number summary

Spreadsheet
Five-number summary

Solution

a First arrange the values in order.

3 3 4 5 5 7 7 8 8 8 10 12 15

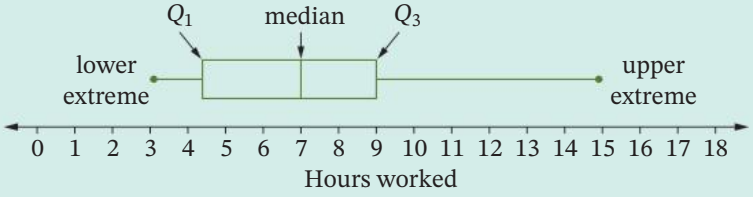
Q_1 Median Q_2 Q_3

Lowest value = 3

$Q_1 = \frac{4+5}{2} = 4.5$ $\quad$ $Q_3 = \frac{8+10}{2} = 9$

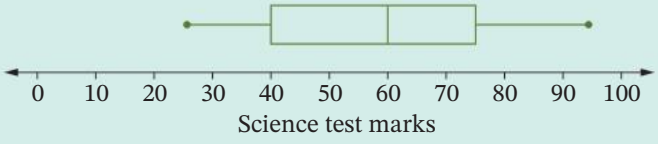
Median = 7 $\quad$ Highest value = 15

b



Example 9

The box plot represents the results of 80 students in a Science test.



a Find the range of the test results.

b Find the median test value.

c What is the interquartile range?

d How many students had a test mark between:

i 25 and 75? $\quad$ **ii** 40 and 60?

e What percentage of students scored more than 75?

Solution

a Range = highest value – lowest value
= 95 – 25
= 70

b Median = 60

c Interquartile range = $Q_3 - Q_1$
= 75 – 40
= 35

d **i** 25 is the lowest value and 75 is Q_3, so 75% × 80 = 60 students had a mark between 25 and 75.

ii 40 is Q_1 and 60 is the median, so 25% × 80 = 20 students had a mark between 40 and 60.

e 75 is Q_3, so 25% of the students scored more than 75.

EXERCISE 5.05 ANSWERS ON P. 589

Box plots UFRC

EXAMPLE 8

1 The number of orders taken per hour at Bernoulli's Pizza on a weekend were:

3	5	1	2	4	6	8	10	7	6
12	15	10	3	5	18	5	8	9	10

a Find the five-number summary for this data.

b Represent this data on a box plot.

2 The daily amount of snow (in cm) that fell at Thredbo during one ski season was:

2	5	5	2	5	7	1	2	2	2	2	2	12
20	12	5	40	50	10	40	13	30	5	35	2	6

a On how many days did it snow?

b Find a five-number summary for this data.

c Represent this data on a box plot.

3 The monthly rainfall figures for Penrith one year were:

98	266	149	94	15	65	19	5	24	34	67	28

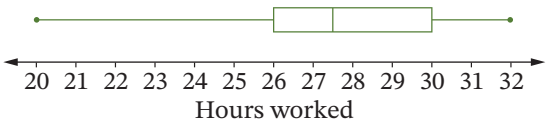
a Find the range.

b Find the five-number summary.

c Represent the data on a box plot.

EXAMPLE 9

4 This box plot represents the number of hours worked in one week by the staff at a supermarket.



a What is the median number of hours worked?

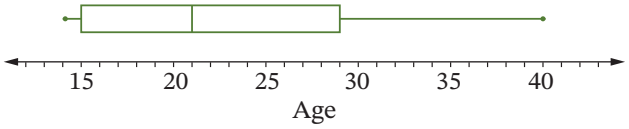
b What is the lower quartile?

c What is the upper quartile?

d Find the interquartile range.

e Estimate the percentage of staff who worked between 26 and 30 hours.

5 The ages of 16 people waiting at a bus stop are displayed in the box plot below.



a What is the range?

b What is the median age?

c Find the interquartile range.

d What percentage of people were aged from:

i 21 to 29?

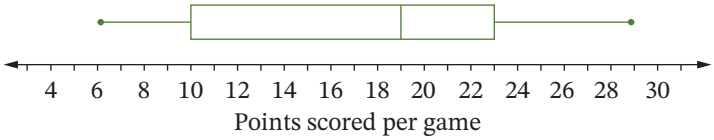
ii 15 to 40?



Getty Images/Mark Kolbe

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6 The box plot shows the number of points per game scored by Ben in 28 basketball games during the season.



a What is the five-number summary for the box plot?

b Find the interquartile range.

c In how many games did Ben score:

i more than 19 points? **ii** between 19 and 23 points?

iii less than 10 points? **iv** at least 10 points?

7 For each set of data, find the five-number summary and draw a box plot.

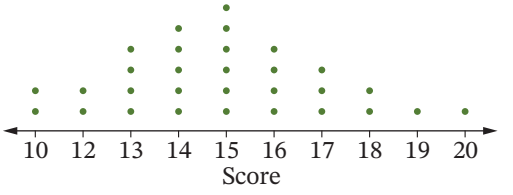
a

Stem	Leaf
2	0 2 3 5
3	3 7
4	4 6 7 8 8 9 9
5	0 1 1 5 6
6	0 3 3 8 8
7	2 5 6
8	5 5 7 8

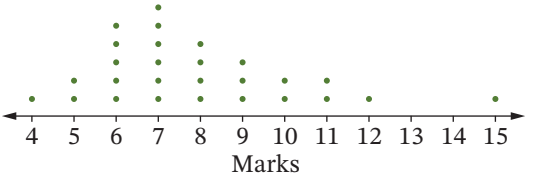
b

Stem	Leaf
3	0 7
4	2 6 6
5	1 2 5 9
6	0 4 7 7 9
7	2 3 5 6 8
8	3 4
9	5

c



8 The results of a general knowledge quiz (out of 15) taken by Year 10 students are displayed in the dot plot. R C



a Find the five-number summary for the dot plot and then draw a box plot.

b Describe the shape of the dot plot and compare it to the shape of the box plot.

c What is the outlier?

d Find the five-number summary for the data in the dot plot without the outlier and draw a box plot.

e Compare the 2 box plots. How are they:

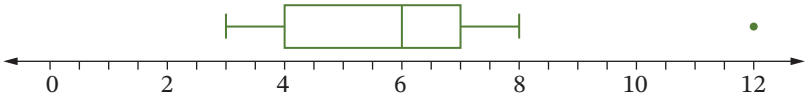
i similar? **ii** different?

Technology

Box plots

Use graphing software or a spreadsheet to draw box plots.

1. Enter this set of data: 3, 3, 4, 4, 5, 6, 7, 7, 7, 8, 12
2. Choose appropriate settings for the scale on your box plot.
3. Your box plot should look similar to the one below.



4. Write down the five-number summary for this data set.
5. Now enter the marks of an English exam completed by 2 classes 10A and 10B and create a box plot for each class:

   10A: 21, 81, 33, 58, 67, 76, 64, 74, 56, 60, 54, 74, 49, 83, 66

   10B: 77, 63, 63, 35, 51, 42, 54, 55, 71, 43, 41, 41, 40, 76, 72
6. Complete a five-number summary for each class.
7. What is the IQR for each class?
8. Which class had the highest mark?
9. Which class had the lowest mark?
10. Which class performed better? Give reasons for your answer, including explanations using the five-number summaries you found in step 6.

Parallel box plots 5.06

Parallel box-and-whisker plots can be used to compare 2 or more sets of data. They are drawn on the same scale, but above each other.

Example 10

Two sprinters run the following times (in seconds) over 100 metres.

Sam:	10.9	10.5	11.0	9.9	10.7	10.5	10.0	11.2	11.5	10.3
Jesse:	11.0	11.4	10.1	9.8	10.8	11.4	10.7	10.3	11.1	11.6

a Find the five-number summary for each sprinter.

b Draw parallel box plots to display the data for both sprinters.

c Find the interquartile range for each sprinter.

d Find the range for each sprinter.

e Which sprinter is more consistent? Justify your answer.

Video
Double boxplots

Worksheets
Box-and-whisker plots
Data 1

Presentation
Analysing data

Technology
Parallel box plots

Spreadsheet
Parallel box plots

Solution

a

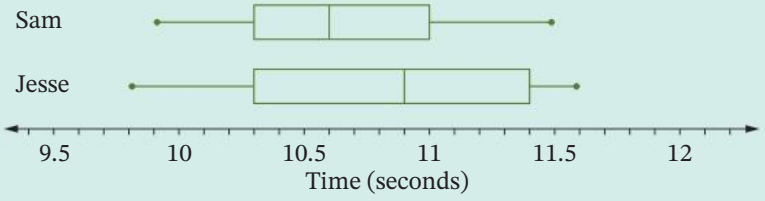
Sam: 9.9 10.0 10.3 10.5 10.5 10.7 10.9 11.0 11.2 11.5

Lowest value; Q_1; $Q_2 = \frac{10.5+10.7}{2} = 10.6$; Q_3; Highest value

Jesse: 9.8 10.1 10.3 10.7 10.8 11.0 11.1 11.4 11.4 11.6

Lowest value; Q_1; $Q_3 = \frac{10.8+11.0}{2} = 10.9$; Q_3; Highest value

b



c Interquartile range for Sam = 11.0 − 10.3 = 0.7

Interquartile range for Jesse = 11.4 − 10.3 = 1.1

d Range for Sam = 11.5 − 9.9 = 1.6

Range for Jesse = 11.6 − 9.8 = 1.8

e Sam is the more consistent sprinter, since both the range and interquartile range of his times are lower than those of Jesse.

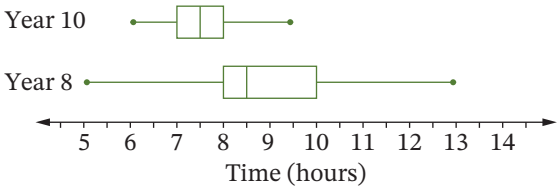


iStock.com/IPGGutenbergUKLtd

EXERCISE 5.06 ANSWERS ON P. 590

Parallel box plots U F R C

1 The parallel box plots show the number of hours of sleep that Year 8 and Year 10 students usually have on a school night.



a For each Year group, find:

i the range **ii** the median **iii** the interquartile range.

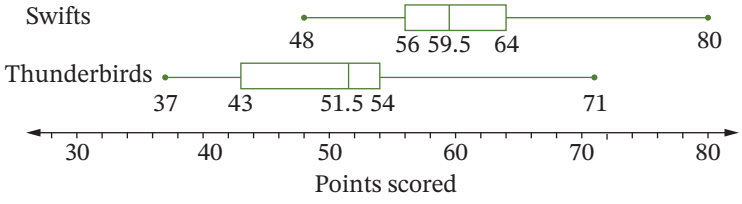
b What percentage of students usually had at most 8 hours of sleep on a school night in:

i Year 8? **ii** Year 10?

c 40 students in both Year 8 and Year 10 were surveyed. How many students usually had at least 10 hours of sleep in:

i Year 8? **ii** Year 10?

2 The number of points scored per match by the Adelaide Thunderbirds and the NSW Swifts netball teams during a season are shown in the parallel box plot. R C



a Find the range of points scored by:

i the NSW Swifts

ii the Adelaide Thunderbirds.

b What is the median number of points scored for each team?

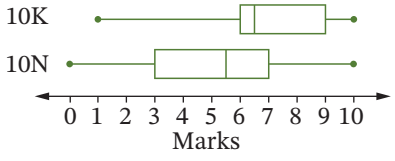
c Find the interquartile range for each team.

d Which team is more consistent?

e Which team performed better? Give reasons.


Matt King/Getty Images

3 The box plots show the test results of students from 2 different classes. R C



a Find the range of marks for each class.

b Find the median mark for each class.

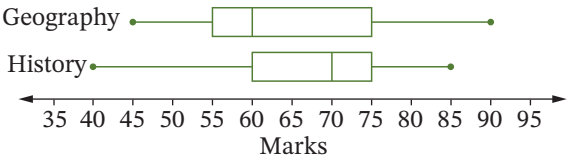
c Find the interquartile range for each class.

d Which class is more consistent?

e Find the percentage of students who scored 6 or more in 10K.

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4 In a Year 10 class of 28 students, the marks for History and Geography tests were displayed using a double box plot. R



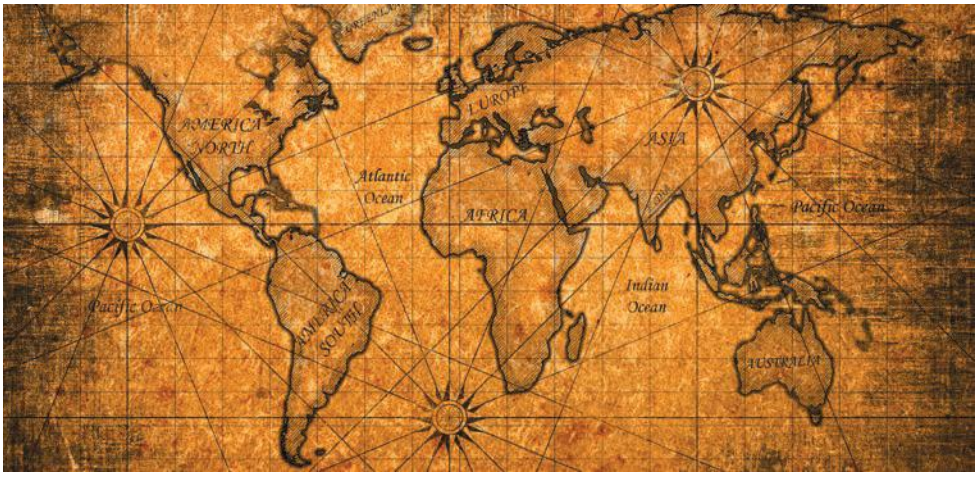
Which statement below could be true? Select the correct answer **A**, **B**, **C** or **D**.

A In Geography, more students scored between 60 and 75 than between 55 and 60.

B 14 students scored the same or more in History than the median mark in Geography.

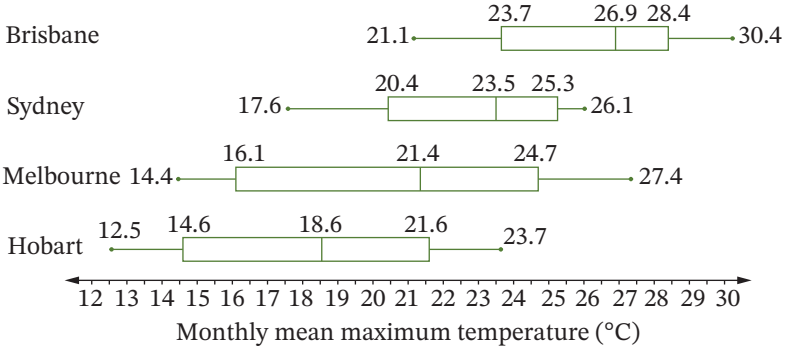
C More students scored 60 or more in History than they did in Geography.

D The interquartile range for Geography is 5 less than the interquartile range for History.



Dmitry Rukhlenko/Shutterstock.com

5 The monthly mean maximum temperatures for 4 state capitals are shown in the box plots. R C



a Find the median, range and interquartile range for each city.

b Which capital city had the most spread in temperature?

c Which capital city had the highest mean monthly temperatures? Justify your answer.

d Which city is warmer – Sydney or Melbourne? Give reasons.

e Which city was more consistent – Sydney or Melbourne? Give reasons.

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EXAMPLE 10

5.06

6 The number of text messages received by a group of students in one hour are as follows. R C

Male:	2	0	3	0	1	2	5	6	2	1	3	2	3	7	4
Female:	4	5	6	3	7	5	8	7	4	2	4	5	10	4	3

a Find the five-number summary for each sex.

b Draw parallel box-and-whisker plots to display the data.

c Find the interquartile range for each sex.

d Find the range for each sex.

e Compare the number of text messages that males and females receive. Are there any significant differences between the spread of the 2 sets of data?

7 Students in a PE class had their heights measured (in cm). R C

Male:	174	167	164	175	189	145	165	166	165	167	171	169
Female:	163	155	171	162	165	183	172	175	166	163	150	186

a Find the five-number summary for each group and draw a parallel box plot to display the data.

b Find the range and interquartile range for each group.

c How does the spread of heights of male students compare with the spread of heights of female students?

8 Students at a university were asked whether their frequency of exercise was high or low and then had their pulse taken. R C

Low:	90	78	80	84	70	66	92	80	80	77	64	88
High:	96	71	68	56	64	60	50	76	78	49	68	74

a Find a five-number summary for each group and then draw parallel box plots to show the information.

b Find the range and interquartile range for each group.

c Compare the spread between the 2 groups. Are there significant differences between them?

d Which group had the lower pulse rates?

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9 The average maximum monthly temperatures (in °C) for Hobart and Darwin are shown. R C

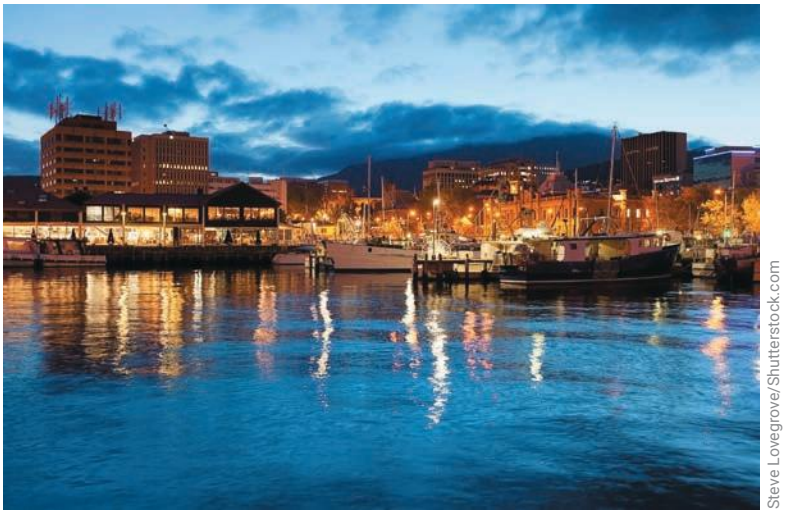
Hobart:	22.7	22.3	20.9	18.2	15.3	13.0	12.5	13.5	15.4	17.4	19.1	20.8
Darwin:	31.8	31.5	32.0	32.7	32.1	30.7	30.6	31.4	32.6	33.3	33.3	32.7

Source: © Bureau of Meteorology

a Find the five-number summary for each city and draw a parallel box plot.

b Find the range and interquartile range for each city.

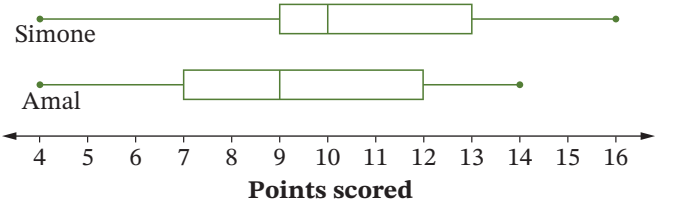
c Which city had more consistent average maximum monthly temperatures? Give reasons.



Steve Lovegrove/Shutterstock.com

Hobart harbour

10 These box plots show the numbers of points scored by 2 basketball players during the season. R C



a Which player has the most points scored for a single game?

b What is the range of the points scored by each player?

c By just looking at the range, which player would seem to be more consistent? Justify your answer.

d Find the median score of each player.

e Find the interquartile range for each player.

f Which player is more consistent?

g Estimate the percentage of games in which Simone scored 9 or 10 points.

h Estimate the percentage of games in which Amal scored more than 12 points.

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Mental skills 5: Maths without calculators ANSWERS ON P. 591

Multiplying by 9, 11, 99 and 101

We can use expansion when we multiply by a number near 10 or near 100.

1 Study each example.

a $25 \times 11 = 25 \times (10 + 1)$
$= 25 \times 10 + 25 \times 1$
$= 250 + 25$
$= 275$

b $14 \times 9 = 14 \times (10 - 1)$
$= 14 \times 10 - 14 \times 1$
$= 140 - 14$
$= 126$

c $32 \times 12 = 32 \times (10 + 2)$
$= 32 \times 10 + 32 \times 2$
$= 320 + 64$
$= 384$

d $7 \times 99 = 7 \times (100 - 1)$
$= 7 \times 100 - 7 \times 1$
$= 700 - 7$
$= 693$

e $27 \times 101 = 27 \times (100 + 1)$
$= 27 \times 100 + 27 \times 1$
$= 2700 + 27$
$= 2727$

f $18 \times 8 = 18 \times (10 - 2)$
$= 18 \times 10 - 18 \times 2$
$= 180 - 36$
$= 144$

2 Now evaluate each product.

a	16×11	**b**	33×11	**c**	29×9	**d**	45×9
e	62×11	**f**	7×101	**g**	18×101	**h**	36×99
i	19×8	**j**	45×12	**k**	21×102	**l**	6×98
m	32×9	**n**	7×99	**o**	39×101	**p**	71×12

Quiz
Mental skills 5

5.07

Comparing data sets 5.07

Example 11

The back-to-back stem-and-leaf plot shows the results in Year 10 Maths and Science tests.

Maths		Science
5 2	3	6 8
8 6 3 0	4	4 6
8 7 7 4 1	5	1 5 9
8 8 7 6 6 3 2 0	6	0 2 8 9
6 5 4 2 1 1	7	2 3 4 4 5 8 8
6 4 3	8	0 0 2 4 5 6 7 8 9
6 0	9	0 4 4

a Find the mean mark (correct to one decimal place) for each subject.

b Find the median for each subject.

c Find the range and interquartile range for each subject.

d For each subject:

i describe the shape

ii identify any outliers and clusters.

e In which subject have the students performed better? Justify your answer.

Videos
Comparing data sets

Back-to-back stem-and-leaf plots

Worksheets
Comparing city temperatures

Comparing word lengths

Investigating young drivers

Solution

a Mean for Maths $= \frac{1919}{30}$
$= 63.9666...$
≈ 64.0

Mean for Science $= \frac{2151}{30} = 71.7$

b Median for Maths $= \frac{66+66}{2} = 66$

Median for Science $= \frac{74+75}{2} = 74.5$

c Range for Maths $= 96 - 32 = 64$
Interquartile range $= 74 - 54 = 20$

Range for Science $= 94 - 36 = 58$
Interquartile range $= 85 - 60 = 25$

d **i** The results for Maths are symmetrical, while the results for Science are negatively skewed.

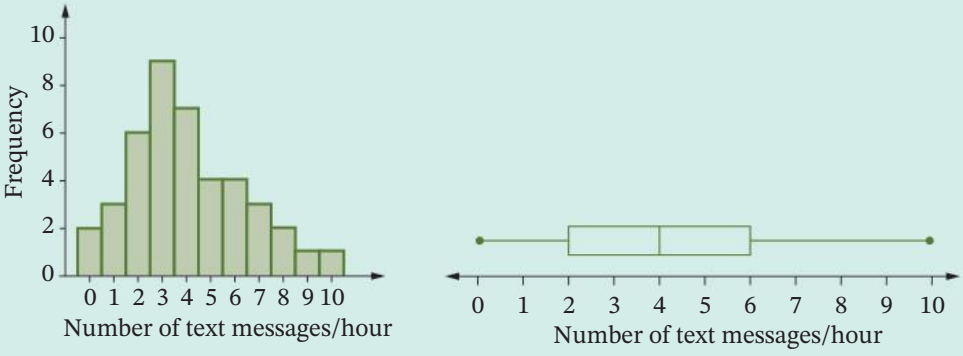
ii No outliers. There is some clustering for the Maths results in the 60s and 70s, and in Science the clustering occurs in the 70s and 80s.

e The students have performed better in Science as the mean and median for Science are greater than the mean and median for Maths. This can also be seen by the shape of the data on the stem-and-leaf plot.

Video
Histogram vs box plot

Example 12

The number of text messages received per hour by a group of teenagers are displayed in the frequency histogram and the box plot below.



a How many teenagers received more than 6 text messages per hour?

b Find:

i the mode

ii the median

iii the range

iv the interquartile range.

c The shape of the distribution is positively skewed. How is this shown by:

i the frequency histogram?

ii the box plot?

d According to the box plot, what percentage of teenagers received 2 or more text messages?

e What information is better seen on:

i the frequency histogram?

ii the box plot?

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Solution

a Number of teenagers receiving more than 6 text messages
$= 3 + 2 + 1 + 1$ Using the frequency histogram
$= 7$

b i Mode $= 3$ Using the frequency histogram

ii Median $= 4$ Using the box plot

iii Range $= 10 - 0$ Using the frequency histogram or box plot
$= 10$

iv Interquartile range $= 6 - 2$ Using the box plot
$= 4$

c i The tail of the frequency histogram leans towards the higher values.

ii The length of the box plot to the right of the median (Q_2) is greater than its length to the left of the median.

d $Q_1 = 2$, so 75% of teenagers received 2 or more text messages per hour.

e i The mode and information regarding the number of text messages received by teenagers can be determined from the frequency histogram.

ii The median, quartiles and interquartile range are easily determined from the box plot.

EXERCISE 5.07 ANSWERS ON P. 591

Comparing data sets U F R C

EXAMPLE 11

1 The back-to-back stem-and-leaf plot shows the amount of cash (in dollars) carried by a sample of Year 11 students at Nelson Senior High. R C

Boys		Girls
5 5 3	0	5 5 6 8 9
8 5 5 2 0	1	0 2 2 5 5 8 8 9
9 6 5 5 5 0 0	2	0 5 6 8 8 8
8 5 5 4 3 2 0 0	3	0 1 4 5 6
5 4 4 2 2 0	4	0 0 5 6
6 6 5 4 3	5	0 3 5
4 2 2	6	5 5 8
5	7	0 4

a Find the mean amount of cash (correct to the nearest cent) carried by each group.

b Find the median amount of cash carried by each group.

c Find the range and interquartile range for each group.

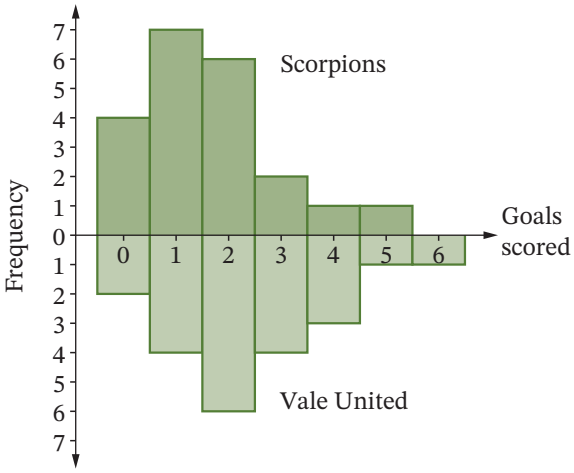
d For each group:

i describe the shape **ii** identify any outliers and clusters.

e Who generally carries more cash – boys or girls? Justify your answer.

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2 The back-to-back histogram shows the number of goals scored by 2 football teams during a season. R C



a How many games were played by each team?

b How many goals were scored by

i the Scorpions

ii Vale United?

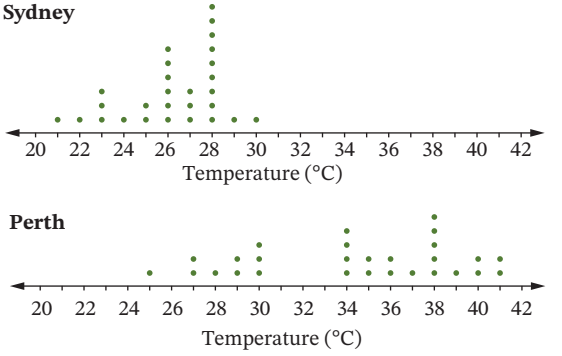
c Find the mean number of goals scored by each team.

d What is the range for each team?

e Describe the shape of each team's results.

f Which team performed better? Give reasons.

3 The daily maximum temperatures for Sydney and Perth in February are shown. R C



a Find the mean (to 1 decimal place), median and modal temperatures for each city.

b Find the range and interquartile range of temperatures for each city.

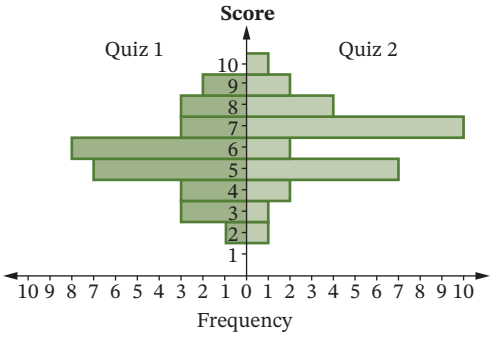
c Describe the distribution shape of the temperatures for each city and identify any outliers or clusters.

d Compare the temperatures in Sydney and Perth. Comment on **measures of centre** (the mean, median and mode), and measures of spread (range and interquartile range).

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4 The results for 2 quizzes taken by a Year 10 History class are shown. R C



a How many students are in the Year 10 History class?

b Find the mean and mode for each quiz.

c Find the median for each quiz.

d For each quiz, find the:

i range **ii** interquartile range.

e Describe the distribution for each quiz, identifying any clusters and outliers.

f Are there significant differences between the results of the 2 quizzes? Justify your answer.

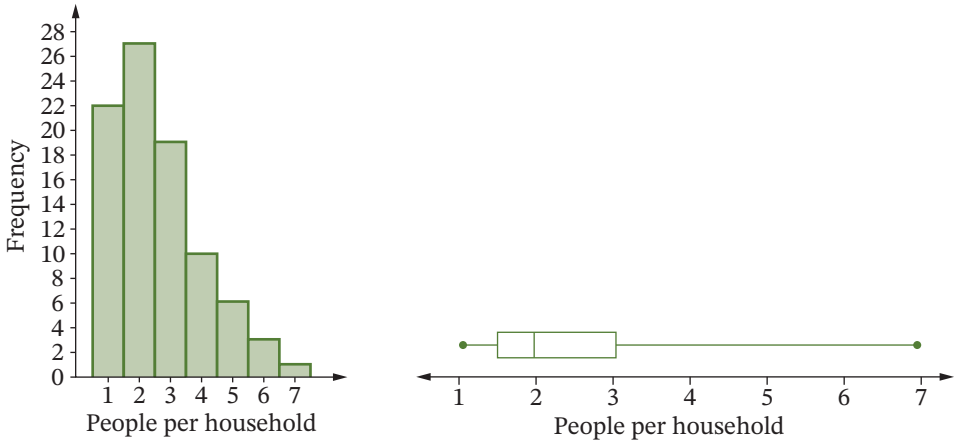


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EXAMPLE 12

5 A survey to determine the number of people per household was conducted in several shopping centres. The results are shown in the frequency histogram and box plot. R C



a How many households had 3 or more people?

b Find the:

i mode **ii** median

iii range **iv** interquartile range.

c Describe the shape of the distribution.

d According to the box plot, what percentage of households had 2 or more people?

e Clustering occurs at 1 to 3 people per household. How is this shown on the:

i frequency histogram? **ii** box plot?

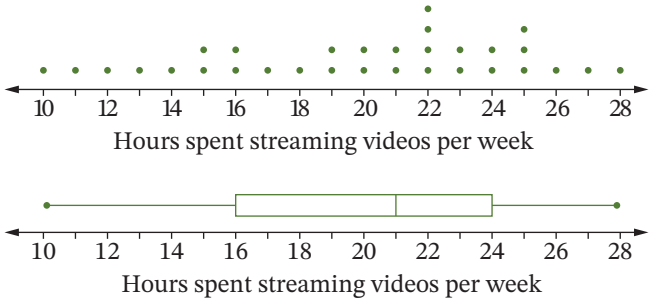
f What information is better seen on the:

i frequency histogram? **ii** box plot?

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6 The dot plot and box plot show the number of hours that Year 10 students spent streaming videos during one week. R C



a How many students streamed videos for:
- **i** fewer than 15 hours per week?
- **ii** more than 20 hours per week?

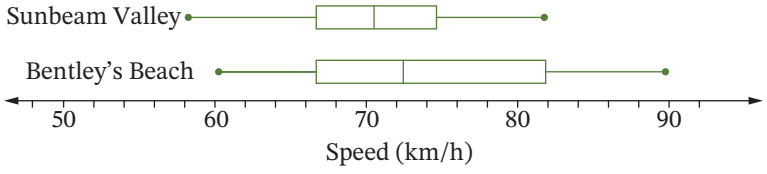
b Find the:
- **i** mode
- **ii** range
- **iii** interquartile range.

c What is the shape of the distribution? How is this shown by the:
- **i** dot plot?
- **ii** box plot?

d Which display of data, the dot plot or box plot, is better for finding the:
- **i** mode?
- **ii** median?
- **iii** number of students who streamed videos for 25 hours?
- **iv** interquartile range?

7 The speeds of cars were monitored along a main road in 2 different places. The results are shown in the back-to-back stem-and-leaf and parallel box plots. R C

Sunbeam Valley		Bentley's Beach
8	5	
9 8 8 7 4 3 3 3 2 0	6	0 0 1 2 3 5 5 7 8 9
9 9 6 5 5 4 4 3 3 2 2 1 1 0 0 0	7	0 0 2 2 3 3 5 5 5 6 6
2 0 0	8	0 2 3 4 5 5 5 8
	9	0



a Find the range, median and interquartile range for each place.

b What is the shape of the distribution for each place?

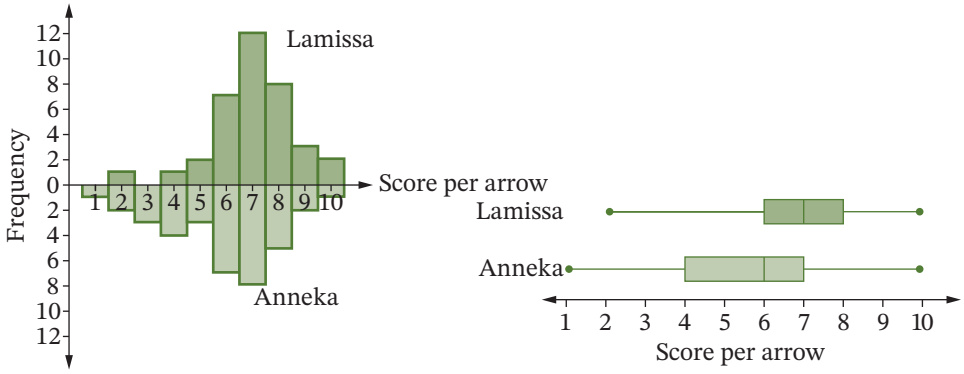
c Are there any clusters or outliers in either place?

d According to the box plot, what percentage of drivers in Bentley's Beach drive faster than all drivers in Sunbeam Valley?

e In which place do drivers generally drive faster? Give a possible reason for your answer.

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8 Lamissa and Anneka shoot arrows at a target 50 m away during an archery contest. They scored 10 for a bullseye down to 1 for the outer ring. Their results are displayed in the back-to-back histogram and the parallel box plots. R C



a How many arrows did Lamissa and Anneka shoot?

b Find the mode and median score per arrow for each contestant.

c Find the range and interquartile range for each contestant.

d Describe the shape of the distribution for each contestant.

e According to the box plots, on what percentage of arrows shot was a score of 6 or less achieved by:

i Lamissa? **ii** Anneka?

f Who was the better archer during this contest? Justify your answer by referring to the measures of central tendency and spread.



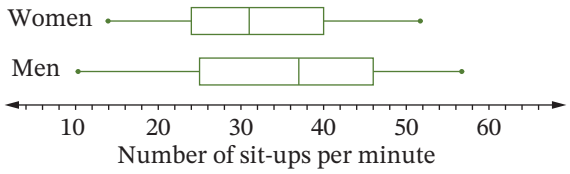
Ivan Smuk/Shutterstock.com

Foundation Standard Complex

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9 The number of sit-ups per minute completed by men and women at the Full On Fitness Centre are displayed in the back-to-back histogram and parallel box plots. R C

Women		Men
8 7 5 4	1	0 6 7 9 9
9 9 9 8 8 7 4 4 3 3 1 0	2	0 2 3 4 4 5 5 7 7 8
7 6 5 5 5 4 3 2 1 0 0	3	0 2 4 5 6 6 7 7 8 8 8 8 9
7 5 4 3 2 0 0	4	1 3 4 6 6 6 6 6 7 7 9
2 1 0	5	0 1 3 4 7 7



a Why would a dot plot be an inappropriate way to display the data shown above?

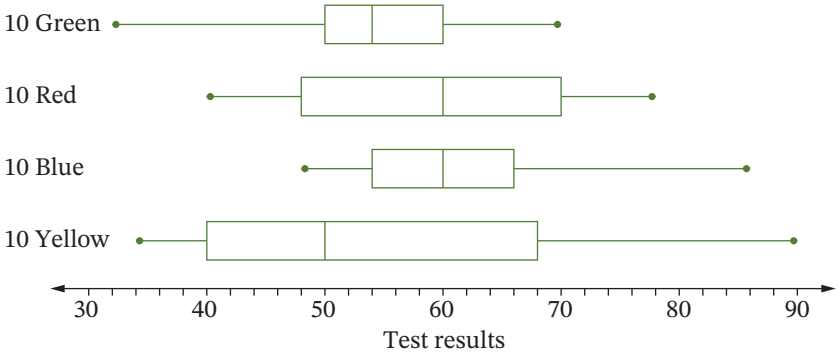
b What is the median number of sit-ups per minute completed by each group?

c Find the range and interquartile range for each group.

d Describe the shape of the distributions for each group.

e Which group has more spread in the number of sit-ups completed per minute? Give reasons for your answer.

10 The results of a Maths test given to 4 Year 10 classes are shown. R C



a What is the range of test results for:

i 10 Yellow? **ii** 10 Blue?

b For which class are the test results:

i positively skewed? **ii** negatively skewed? **iii** symmetrical?

c Which class had:

i the lowest interquartile range? **ii** the highest test score?

iii the highest median?

d Which class had the best test results overall? Give reasons.

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5.08 Scatterplots

Worksheet
Scatterplots

Puzzle
Scatterplots matching game

Bivariate data is data that measures 2 variables, such as a person's height and arm span (distance between outstretched arms). Bivariate data is represented by an ordered pair of values that can be graphed on a **scatterplot** for analysis.

A **scatterplot** is a graph of points on a number plane. Each point represents the values of the 2 different variables and the resulting graph may show a pattern that may be linear or non-linear. If there is a pattern, then a relationship may exist between the 2 variables. This relationship is called an **association** or **correlation**.

Example 13

The heights and arm spans of a group of students are shown in the table.

Height, *H* cm	162	182	153	145	172	163	150	142	183	145	192	171
Arm span, *S* cm	158	185	145	143	174	165	151	141	181	158	191	178

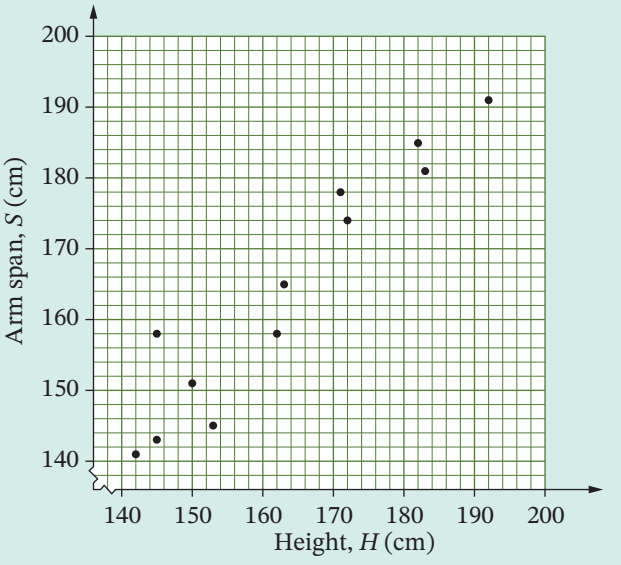
a Plot the data on a scatterplot.

b Describe the pattern of the plotted points.

c Describe the association between the students' heights and arm spans.

Solution

a



b The points form a linear pattern.

c As the heights of students increase, their arm spans tend to increase.

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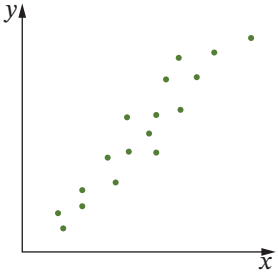
Strength and direction of linear associations

The linear pattern will indicate the **strength** and **direction** of the association between the 2 variables. The strength of an association between 2 variables can be described as:

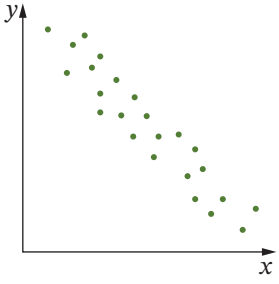
- **strong** if the points are close together
- **weak** if the points are more spread out
- **perfect** if all points lie on a straight line.

The direction of an association can be described as **positive** or **negative**:

x and y have a **positive association** or **correlation** if y increases as x increases.



x and y have a **negative association** or **correlation** if y decreases as x increases.



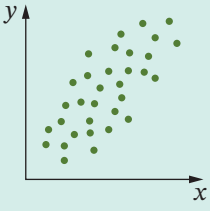
5.08

Video
Scatterplots

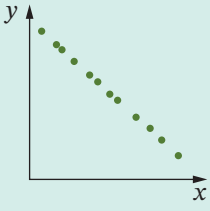
Example 14

Describe the strength and direction of the association shown in each scatterplot.

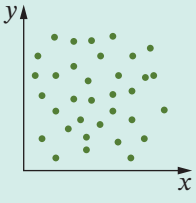
a



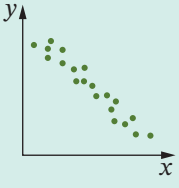
b



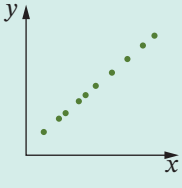
c



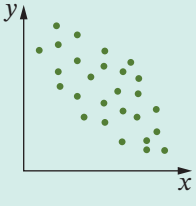
d



e



f



Solution

a	Weak positive association	The points can be seen to form a line, but they are spread out
b	Perfect negative association	The points lie on a decreasing straight line
c	No association	The points are very spread out with no pattern
d	Strong negative association	The points can be seen to form a decreasing line and they are close together
e	Perfect positive association	The points lie on an increasing straight line
f	Weak negative association	The points can be seen to form a decreasing line, but they are spread out

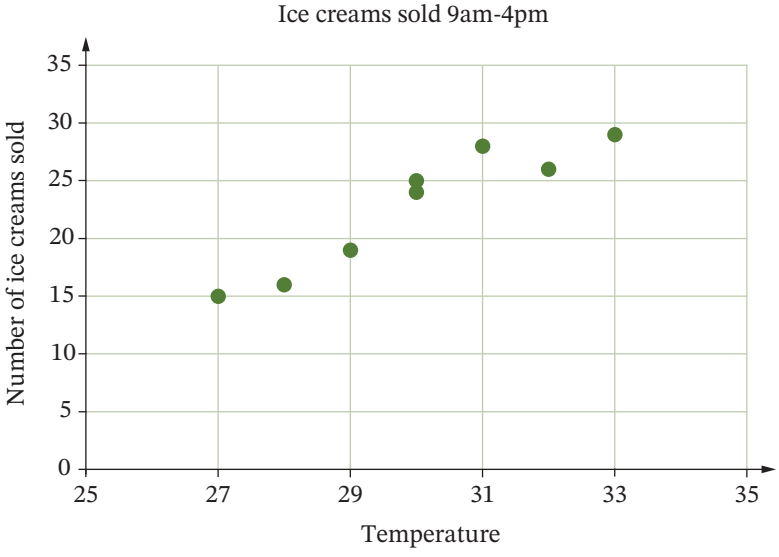
Video
Can eating fish prevent murder?

Association and causation

An association between 2 variables means that the variables have some relationship, but not necessarily a direct one. It may not mean that one variable *causes* the other variable (this is called **causation**), or that a change in one variable directly affects the other. For example, at a beach in summer, there may be a strong positive association between the number of ice creams sold and the number of people becoming sunburnt (as the number of ice creams sold increases, the number of sunburnt people increases), but that doesn't mean that ice cream sales directly cause sunburn. Indirect and external factors such as the beach temperature probably have a direct effect on both ice cream sales and sunburn. A common saying among statisticians and scientists is 'Association (or correlation) does not imply causation'.

The data in this table and scatterplot compares the hourly temperature at a beach and the number of ice creams sold at the beach café during that hour.

Temperature (°C)	27	28	30	31	33	32	30	29
Ice creams sold	15	16	25	28	29	26	24	19



In this case, a **causal relationship** does exist, showing causation between temperature and ice cream sales, as the increase in temperature directly causes more ice creams to be sold.



FatCamera/E+/Getty Images

Example 15

For each pair of variables, determine whether the association is a causal relationship.

a As the amount of road traffic increases, the longer the car trip takes

b The taller a person, the heavier they are

c As a student spends more hours per day playing computer games, they achieve lower marks in class tests

d The more rooms in a family home, the more cars owned by that family

Solution

a Causal: the increase in traffic directly affects how long the car trip takes

b Not causal: a person could maintain the same weight as they grew taller because of fat and muscle distribution, they have a slender (skinny) figure, good diet

c Causal: the student spends more time on computer games and has less time to study, causing them to score lower in tests

d Not causal: while more rooms imply more people in the family, they could be children, aged people or others who don't drive

Dependent and independent variables

If a variable y depends on the value of the variable x, then y is called the **dependent variable** and x is called the **independent variable**. For example, stride length (the length of a person's walking step or pace) depends on the person's height, so stride length is the dependent variable and height is the independent variable. When graphing, the dependent variable is shown on the vertical (y-) axis while the independent variable is shown on the horizontal (x-) axis.

EXERCISE 5.08 ANSWERS ON P. 592

Scatterplots U F R C

1 The heights and handspans of a group of students are shown in the table. C

EXAMPLE 13

Height, H cm	168	175	175	156	160	173	171	180	185	175	182	180
Handspan, S cm	20.0	21.1	17.6	16.5	17.5	19.0	20.8	22.5	25.0	23.0	20.2	21.1

a Plot the data on a scatterplot.

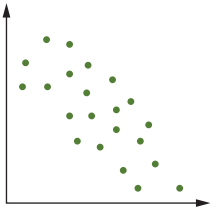
b Describe the pattern of the plotted points.

c Describe the association between the students' heights and their hand spans.

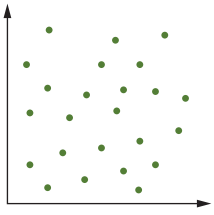
2 Describe the strength and direction of the relationship shown in each scatterplot. C

EXAMPLE 14

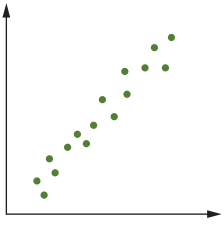
a



b



c



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3 Describe the strength and direction between height, H and handspan, S in question **1**. C

4 The height and stride length measurements of some students are shown in the table. R C

Height, H cm	174	160	158	180	169	172	171	171	148	190	166	173
Stride length, L cm	72.2	64.0	66.4	74.7	70	71.5	70.9	71.2	61.4	78.9	68.0	71.9

a Explain why stride length is the dependent variable.

b Graph this data on a scatterplot.

c Describe the pattern of the plotted points.

d Describe the association between the students' heights and stride lengths.

e Describe the strength and direction of the relationship.

f Predict the stride length of a student who is 175 cm tall.

5 The table lists the points scored for and against each NRL team one season. R C

a Graph this data on a scatterplot.

b Is the pattern of the points linear?

c Describe the strength and direction of the association between points scored for and points scored against.

Points scored for, F	Points scored against, A
568	369
579	361
559	438
497	403
597	445
545	536
445	441
481	447
405	438
506	551
449	477
448	488
462	626
497	609
409	575
431	674

6 Year 10 students were surveyed on the number of hours in a week they spent doing homework and the number of hours they spent on the computer. R C

Homework, H	2	15	12	5	4	2	4	15	14	5	2	5	20	4	2	11
Computer, C	25	30	18	35	6	30	20	22	6	40	8	3	20	30	5	8

a Plot the points on a scatterplot.

b Describe the strength and direction of the association between hours spent doing homework and hours spent on the computer.

EXAMPLE 15

5.08

7 For each pair of variables, determine whether the association is a causal relationship. R

- **a** Increase in scarf-wearing and the number of hot chocolates sold at a ski resort
- **b** The size of a bushfire and the number of firefighters managing it
- **c** Hours of usage of a mobile phone and the amount of charge remaining on its battery
- **d** Decrease in daylight and number of road accidents
- **e** The price of fuel and the amount of fuel petrol bought by a person

8 Which one of these examples best shows causation?
Select the correct answer **A**, **B**, **C** or **D**. R

- **A** Time spent watching films at the cinema and time spent at a café
- **B** Increase in eating healthy food and increase in level of happiness
- **C** Number of rainy days in Spring and number of picnics held during Spring
- **D** Students' test results in History and their test results in Science

9 A survey was conducted to see whether there was a relationship between height and the age of students in a high school. R C

Age, *A* (years)	14	16	15	13	11	14	17	15	12	11	14	16	13	18
Height, *H* (cm)	162	174	182	162	132	173	187	160	154	145	165	171	151	181

- **a** Graph the points on a scatterplot.
- **b** Which variable could be considered as the dependent variable? Give reasons.
- **c** Describe the strength and direction of the association between the age and height of students.



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Technology

Scatterplot patterns

Investigate one of the following pairs of bivariate data for a group of students or people. You will need instruments (measuring tapes and/or trundle wheels) and stopwatches to help you collect your data.

- Height vs arm span
- Reaction time vs hours of sleep
- Stride length vs 50 m sprint time

1. Enter your data into a spreadsheet. Graph it using Scatter with Smooth Lines and Markers.
2. Analyse your graph. What type of linear relationship does it show? Positive or negative? Strong or weak?
3. Write a brief summary describing the relationship between the 2 variables.



Kim Karpeles/Alamy Stock Photo

5.09 Line of best fit

Worksheets
Line of best fit
Trendlines
Data 2

Interactive
Least-squares regression

Technology
Line of best fit

Spreadsheet
Line of best fit

If 2 variables x and y show a strong linear relationship when graphed on a scatterplot, the linear relationship can be approximated by drawing a **line of best fit** through the points and finding its equation $y = mx + c$. This line can be done on paper but it is easier to graph it using technology such as a spreadsheet, dynamic geometry or graphing software.

Line of best fit

- It represents most or all of the points as closely as possible.
- It goes through as many points as possible.
- It has roughly the same number of points above and below it.
- It is drawn so that the distances of points from the line are as small as possible.

A line of best fit can be used to predict what might happen:

- between the points on the scatterplot, within the range of data (this is called **interpolation**, pronounced 'in-terp-o-lay-shun'), or
- beyond the points on the scatterplot, outside the range of data (this is called **extrapolation**, pronounced 'ex-trap-o-lay-shun').

Example 16

The arm span and right foot size of 12 Year 10 students were measured:

Arm span, S (cm)	177	179	162	182	181	171	161	176	175	190	168	165
Right foot size, F (cm)	25	26	24	28	27	25	23	25	24	30	24	24

a Graph the points on a scatterplot and construct a line of best fit.

b Find the equation of the line of best fit.

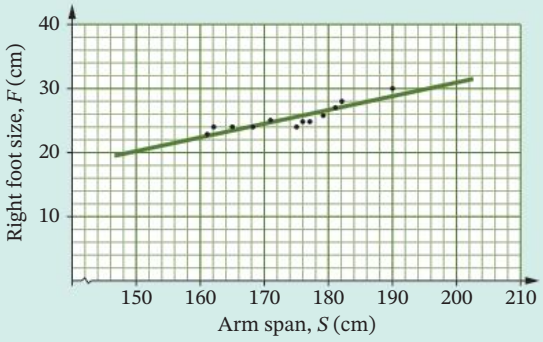
c Use the equation to estimate the foot size of a student with an arm span 173 cm.

d Use the graph to interpolate the foot size of a Year 10 student with an arm span of 185 cm.

e Use the graph to extrapolate the arm span of a Year 10 student who has a foot size of 22 cm.

Solution

a



b Use the formula $y = mx + c$ to find the equation of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{27 - 20}{181 - 150}$$ using 2 points on the line (150, 20) and (181, 27)

$$= \frac{7}{31}$$

$$\approx 0.226$$

$$y = 0.226x + c$$

$20 = 0.226(150) + c$ using the point (150, 20) to find c

$20 = 33.9 + c$

$c = -13.9$

$y = 0.226x - 13.9$

$F = 0.226S - 13.9$ x and y replaced by S and F respectively

c When $S = 173$ cm,

$F = 0.226 \times 173 - 13.9$

$= 25.198$ cm

A Year 10 student with an arm span of 173 cm would have a foot size of 25.198 cm.

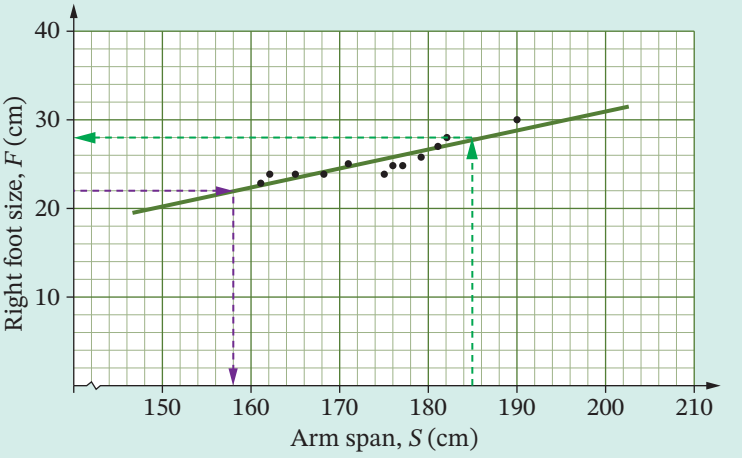
Videos
Lines of best fit

Can eating fish prevent murder?

5.09

d From the graph, a Year 10 student with an arm span of 185 cm would have a foot size of 28 cm.

This is **interpolating** because we are reading from the graph **between** the given points.



e From the graph, a Year 10 student with a foot size of 22 cm would have an arm span of 158 cm.

This is **extrapolating** because we are reading from the graph **outside** the given points.

Worksheet
2 mm grid paper

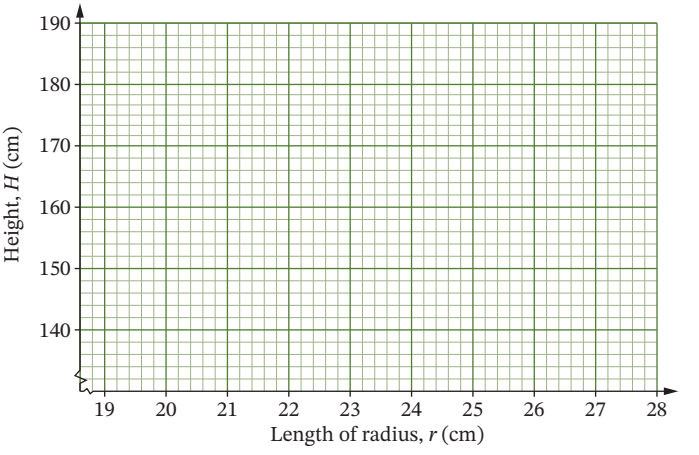
EXAMPLE 16

EXERCISE 5.09 ANSWERS ON P. 593

Line of best fit U F R C

1 Forensic scientists can estimate people's heights from the lengths of their bones such as the tibia, femur, humerus and radius. This table gives the heights of females and the lengths of their radius bones. R

Length of radius, r (cm)	25.2	22	23	22.5	21.8	26.2	20.4	23.5	24.3	21.4
Height, H (cm)	173	158	165	161	158	179	152	167	169	156



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a Use graph paper with axes as shown to plot the points on a scatterplot and construct a line of best fit.

b Find the equation of the line of best fit.

c Use your equation to find the height of a female whose radius is 25 cm long.

d If the radius is of length 27 cm, use the line of best fit to predict the height of the female.

2 The heights and shoe size of a group of Year 11 students were measured and recorded in the table.

Height, H (cm)	175	174	177	180	179	176	170	175	179	180	178	183	178	173	179	174
Shoe size, S	10.5	10	10	12	11	9.5	7.5	9	11.5	12.5	11	12.5	12	9.5	10.5	9

a Graph the points on a scatterplot and construct a line of best fit.

b Find the equation of the line of best fit.

c Use the equation to estimate the shoe size of a student whose height is 172 cm.

d Use the graph to interpolate the shoe size of a student who is 181 cm tall.

e Use the graph to extrapolate the shoe size of a student with height 185 cm.

3 The air temperature, T (°C) was measured at various heights, h (m) above sea level. R

Height, h (m)	0	500	1000	2000	2500	4000	5900	7500	10000
Temperature, T (°C)	20	14	8	3	−5	−13	−20	−35	−50

a Graph the points on a scatterplot and construct a line of best fit.

b Find the equation of the line of best fit.

c Use the equation to estimate the temperature at a height of 1500 m.

d Use the graph to find the height above sea level for a temperature of −10°C.

4 The results obtained by 18 Year 10 students in Maths and Science exams are shown. R C

Maths	59	52	72	85	75	45	65	64	62	58	78	90	40	70	50	45	82	50
Science	65	54	67	83	75	39	59	64	60	56	80	95	38	65	48	48	85	51

a Graph the points on a scatterplot and construct a line of best fit.

b Simone missed the Science test but obtained a 80% in her Maths exam. Use the line of best fit to predict Simone's Science result.

c If Mario obtained 96% in the Science exam, predict what result he might have achieved in the Maths exam.

5 Angelicki is measuring the amount by which a spring is stretched when different masses are hung from the spring for a Science experiment. Her results are shown. R C

Mass, M (g)	10	20	25	30	35	40	50
Spring stretch, S (cm)	5.9	11.2	12.3	14.8	17	22.4	25.2

a Graph the points on a scatterplot and construct a line of best fit.

b Use the line of best fit to predict the length the spring stretches for a mass of 45 g.

c What mass would have to be attached to stretch the spring 28 cm?

d Are there limitations to using the line of best fit to predict the length the spring is stretched by different masses?

6 The men's 100 m world record times for running are shown in the table. R C

Year	1964	1968	1983	1988	1991	1994	1996	1999	2005	2006	2007	2008	2009
Time (s)	10.06	9.95	9.93	9.92	9.86	9.85	9.84	9.79	9.77	9.76	9.74	9.69	9.58

a Graph the points and construct a line of best fit.

b Use the line of best fit to predict the time to run the 100 m in 2030.

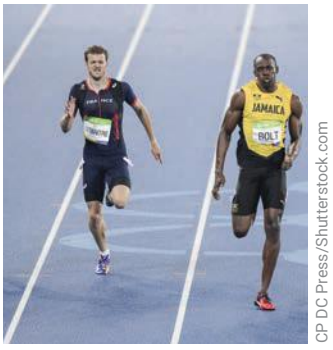
c What are the limitations of using the line of best fit to predict times to run 100 m?

Did you know?

Lightning Bolt

The current record of 9.58 seconds for running 100 metres is still held by Jamaica's Usain Bolt, who achieved this in 2009 at the World Championships in Berlin, Germany. In 2016, American sprinter Justin Gatlin achieved the time of 9.45 seconds on the Japanese television show *Kasupe!*, but this record was disallowed. In the same year, at the Rio Olympics, Usain Bolt won the gold medal with a winning time of 9.81 seconds, Justin Gatlin won silver with 9.89 seconds, and Canadian Andre de Grasse won bronze with 9.91 seconds.

Find out why Justin Gatlin's record time was disallowed in 2016.



CP DC Press/Shutterstock.com

Technology

Lines of best fit

In this activity, we will use a spreadsheet to create a scatterplot and graph a line of best fit. The heights of men and the lengths of their femur bone are recorded in the table below.

Length of femur, *f* (cm)	40	42.9	44.2	46.1	46.8	47	48.4	50.3	51.2	57.2
Height, *H* (cm)	162	165	164	173	174	178	179	182	186	200

1 Enter the data from the table into a spreadsheet. Type **Length of femur** in cell A1 and **Height** in B1.

2 To graph a scatter plot, select all the values in cells B1 to K2, and under the **Insert** menu, select **Scatter** and **Scatter with Straight Lines and Markers**.

3 To draw the line of best fit, select one of the points on the scatterplot and right-click. Select **Add Trendline**, **Linear and Display Equation on chart**, then **Close**.

4 Check your answers to questions **1** to **3** from Exercise 6.09 using a spreadsheet.

Investigation

Australian Bureau of Statistics

The Australian Bureau of Statistics (ABS) is the official organisation in charge of collecting data for government departments. The data collected covers many areas – from population, employment, weekly earnings, weight and obesity in adults, to health of children in Australia.

Visit the ABS website www.abs.gov.au to answer the following questions.

1 a What is the current population of Australia?

b What is the predicted population for:

i 2030? ii 2040? iii 2050?

c What is Australia's rate of population increase?

2 Go to 2021 Census Data by Location, and then to Data and analysis.

a What was the population in NSW and its increase from 2016?

b Which state had the:

i largest increase in population?

ii smallest increase in population?

Sampling and surveys 5.10

PATH

We live in a world of 24-hour news, whether it is from newspapers, TV or the Internet, that often quote results from surveys. When survey data is used in the media we need to consider:

- where the news comes from and what **samples** the statistics are based on
- who supplied the information
- the number of samples and the sample size used
- the way in which the collected data has been presented.



Maxx-Studio/Shutterstock.com

Example 17

What concerns could be raised about the following claim?

'The Daily Sun newspaper reports that it has an average issue readership of 1.39 million and that its Travel section has a readership of 1.46 million.'

Solution

The newspaper is reporting about its own readership, so it may be biased. It also states that its Travel section has a higher readership than its issue readership.

PATH

Example 18

The weights (in kg) of a large group of 18–20-year-olds attending university are:

57	58	62	84	64	74	57	55	56	90
68	63	49	66	63	65	60	60	46	70
85	60	70	41	73	75	67	63	70	85
51	49	75	77	87	54	60	75	58	68
55	65	66	57	85	75	56	60	62	75
74	58	51	62	50	55	71	57	58	100
72	58	103	64	52	55	80	96	45	87
81	80	48	54	65	54	59	50	78	60
74	70	64	59	72	78	104	63	102	95

a How many students were in the group?

b Randomly select 4 groups of 10 and for each sample calculate:

i the mean **ii** the median **iii** the interquartile range.

c Use your results to estimate the mean, median and interquartile range of the population from your 4 samples.

d Compare your estimates to the mean, median and interquartile range of the population.

Solution

a There were 90 students in the group.

b Randomly select 4 samples of 10 from the 'population'.

Sample 1:	90	63	75	48	74	85	51	96	60	78
Sample 2:	62	75	103	64	65	54	55	54	60	75
Sample 3:	68	70	57	52	78	74	60	63	58	87
Sample 4:	72	54	52	80	45	87	49	77	54	58

The statistics for each group are:

Sample 1: $\bar{x} = 72$ median = 74.5 interquartile range = 25

Sample 2: $\bar{x} = 66.7$ median = 63 interquartile range = 20

Sample 3: $\bar{x} = 66.7$ median = 65.5 interquartile range = 16

Sample 4: $\bar{x} = 62.8$ median = 56 interquartile range = 25

c Taking averages, population statistics estimates are:

$$\text{Mean} = \bar{x} = \frac{72 + 66.7 + 66.7 + 62.8}{4} = 67.1 \quad \text{(correct to 1 decimal place)}$$

$$\text{Median} = \frac{74.5 + 63 + 65.5 + 56}{4} = 64.8 \quad \text{(correct to 1 hbox decimal place)}$$

$$\text{Interquartile range} = \frac{25 + 20 + 16 + 25}{4} = 21.5$$

d The statistics for the population are:
Mean, $\bar{x} = 66.9$ (correct to 1 decimal place)
Median = 64
Interquartile range = 18
The estimates for the mean, median and interquartile range compare very favourably with the population statistics.

Sampling methods

PATH

5.10

In statistics, the **population** refers to all the items under investigation or being studied, and a census is a survey of the entire population. If the population is too large or difficult to survey, then a **sample** of the population is selected instead. The sample must be representative of the population, by being **random, unbiased** and **large**.

Sampling is done to estimate as accurately as possible the characteristics of the population. The size of the sample chosen is important, because if it is too large or takes too much time it can become very expensive to collect data. If the sample size is too small, it may not be representative of the population at all. The larger the sample, the more accurate the results of the statistical investigation.

There are 4 types of samples, described below.

In a **random sample**, each item is chosen completely at random from the population, for example, selecting numbers out of a box or using a computer to generate random numbers to choose people or items for the sample.

In a **systematic sample**, the first item is chosen at random and all other items are then chosen at regular intervals, for example, every 20th mobile phone is taken from a factory to be tested or the driver of every 10th car passing on a road is chosen for a random breath test. 'Systematic' means a 'system' is used to select items.

In a **stratified sample**, the sample is formed by selecting accurate proportions of subgroups (or strata) within a population. For example, if a survey is being conducted about changes to the school uniform, it is important to represent all year levels of the school population in the correct proportions. If 19% of the school population are in Year 7, then 19% of the sample should be Year 7 students. 'Stratified' means 'in layers', like stratified rock. Stratified sampling can be more expensive and time-consuming to implement, but it can provide the most accurate representation of a population.

In a **self-selected sample**, people volunteer themselves to participate in a survey, so the sample is not randomly chosen nor necessarily representative of the population, for example, voting on an online poll on the best restaurant in town. Self-selected samples contain the most bias, so online surveys and reviews open to anyone to comment are unreliable and cannot be trusted 100%.

Example 19

What type of sample would you use to investigate each issue?

a The opinion of Year 10 students on new times for the school timetable.

b The charge time of a power bank.

c The amount of time spent on social media by Australian high school students.

d The popularity of a new tax system.

Solution

a A random sample provides a fair representation of Year 10 students.

b A systematic sample will ensure power banks produced at different times will be tested.

c A stratified sample will ensure that students of all ages, areas and cultures will be included.

d A stratified sample will ensure that people from all areas are included.

PATH

Example 20

A factory produces 240 car tyres in a day. At what interval should one tyre be selected to obtain a systematic sample of 15 tyres?

Solution

Divide the size of the population by the size of the sample required: $\frac{240}{15} = 16$

Every 16th item should be selected.

Example 21

James wants to use a stratified sample of 110 people to survey the school population about the canteen menu. If there are 60 staff and 900 students at his school, how many staff and students should be chosen for the sample?

Solution

School population $= 60 + 900 = 960$

Teachers in the sample: $\frac{60}{960} \times 110 = 6.875$

≈ 7

Students in the sample: $\frac{900}{960} \times 110 = 103.125$

≈ 103

7 teachers and 103 students should be chosen for the sample. Note that $7 + 103 = 110$.

EXERCISE 5.10 ANSWERS ON P. 594

Sampling and surveys U F R C

EXAMPLE 17

1 A TV network surveys 300 people in shopping centres between 9 a.m. and 11 a.m. to get feedback on its new game show.

a How is this survey biased?

b Suggest a better method for getting feedback about its game show.

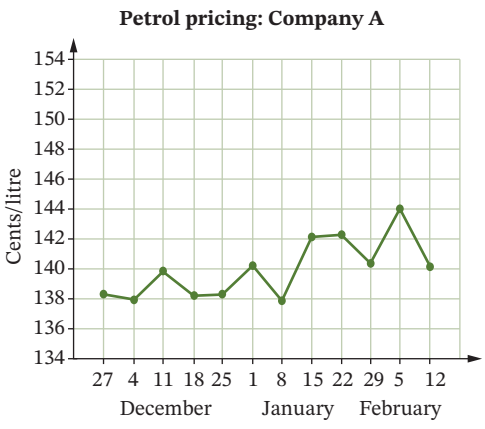
2 A report about hot water systems recommended a heat pump system. The report stated that people in Queensland who had the heat pump hot water system saved 30% on their electricity bills. The company is using this information to advertise their product in NSW and Victoria. How might this information be unsuitable for people in NSW and Victoria? Give reasons.

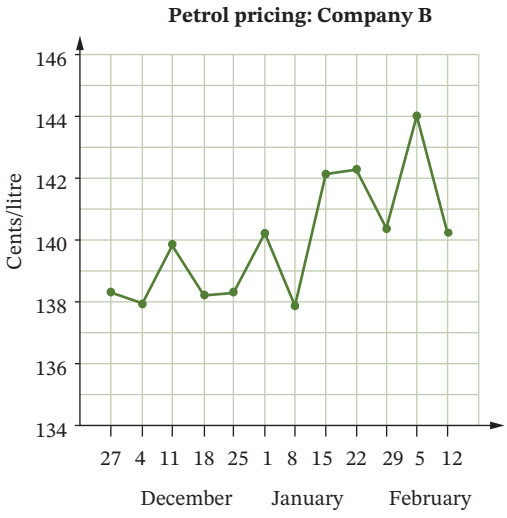


Foundation Standard Complex

9780170479622

3 Two reports on petrol prices over a 12-week period were written by 2 different companies. Each company used line graphs to show the price of petrol for the same period. R C





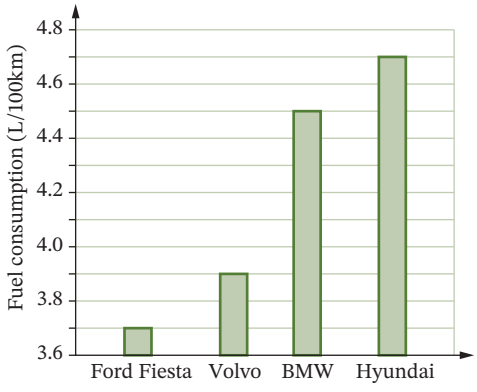
a What message is being suggested about petrol prices in the line graph presented by:

i Company A?

ii Company B?

b How could both graphs be improved to give a true picture of changing petrol prices?

4 A report on the diesel fuel consumption of different cars was published in a motoring magazine, featuring this misleading graph. R C



a What is the graph suggesting about the fuel consumption of the different cars?

b What is the difference in fuel consumption between the:

i Ford Fiesta and the Volvo?

ii Ford Fiesta and the Hyundai?

iii BMW and the Hyundai?

c How should the graph be redrawn so that it is not biased towards the Ford Fiesta and the Volvo?

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PATH

5 A company sells a new app. After 3 months, they conduct a survey and customers are asked to rate the product as Excellent, Good or Satisfactory. Is the survey biased? Justify your answer. R C

6 A market research company working for a car manufacturer needs to determine the most popular car colours. R C

a Give an example of a biased question for this survey.

b What other information should the market research company use, apart from the survey, to determine the most popular colour car?

EXAMPLE 18

7 a Randomly select 4 samples of 10 weights from the population shown in Example **18** on page 232, and for each sample calculate the: R C

i mean **ii** median **iii** interquartile range.

b Use your results to estimate the mean, median and interquartile range of the population from your 4 samples.

c How do the statistics of your samples compare to the mean, median and interquartile range of the population?

d How do the estimated statistics compare to the population statistics?

8 a Repeat the process of question **7** by taking 2 samples of size:

i 5 **ii** 15 **iii** 20

b Do the sample statistics become more accurate and move closer to the population statistics as sample size increases? R C

EXAMPLE 19

9 What type of sample would you use to investigate each issue? R C

a Testing a new vaccine on patients

b Finding the most popular streamed TV program this week

c Investigating how many Year 10 students have a part-time job

d Surveying customers on their online experience with *BuzzShop*

e Determining the most popular phone brand

f Investigating the most popular ice cream flavour of 8-year-olds

10 A sports club has 608 members. There are 365 females and 243 males. Determine whether each sample of sports club members is random, systematic or stratified. C

a Choose every 6th person on the membership list

b Choose 51 of the females and 34 of the males

c 85 names chosen by computer from the membership list

EXAMPLE 20

11 A factory makes 300 light globes in a day. How often should a globe be selected for a quality control check if a systematic sample of 15 globes is required? R C

12 Explain how a systematic sample of 20 students could be chosen from a school of 340 students, to participate in a survey.

13 Which one of the following is an example of a systematic sample? Select **A, B, C** or **D**.

- **A** 30 customers who bought concert tickets are chosen by a random number generator
- **B** The first 30 people who bought concert tickets are chosen
- **C** 15 males and 15 females who bought concert tickets are chosen
- **D** Every 8th customer who bought concert tickets was prompted to complete a survey

14 A regional public transport provider surveys 80 commuters about the efficiency of the bus network. This table shows the number of commuters in each sector of the region.

Lake	Beach	Central	Hills
2560	3400	5125	4487

If the sample is to be stratified, how many 'Beach' commuters should be included?

15 The student population of Nelson MindTap College is shown in this table.

Year	7	8	9	10	11	12
Number of Students	120	132	146	155	162	178

If a survey is to be given to 50 students at the school, how many students from each year should be chosen if a stratified sample is used?

PATH

EXAMPLE 21

5.11

Statistical investigations 5.11

PATH

Governments, businesses and other organisations collect and use statistics to plan and make intelligent decisions for the future. For example:

- the government wants to know how many people use public transport
- a babygoods manufacturer wants to test the safety of a new high chair
- a coffee shop chain wants to decide where to open its next store
- an international relief agency needs to find out which countries require aid most urgently
- an electronics company wants to know when to introduce a new digital device.

You should have enough knowledge and experience now to evaluate statistical studies and carry out an investigation of your own.

There are 6 stages in a statistical investigation.

1. Posing questions

Before an investigation can begin, the researcher needs to determine clearly the key questions of the study: the main aims, ideas and topics of interest. What is the theory to be researched and tested? This is called the **hypothesis**.

2. Collecting data

According to the Australian Bureau of Statistics (ABS), **data** are 'observations or facts which when collected, organised and evaluated become **information**'. Collecting data usually requires the most work and time, when the researcher goes out 'into the field' to find the necessary

PATH

facts, through surveys, interviews, questionnaires, observations, tests, experiments and existing records. They must be efficient, objective, ethical (honest) and unbiased. Samples must be random, representative and large.

3. Organising data

Once the raw data have been collected, they are converted into meaningful information for further study. This involves grouping, ordering or classifying the data in different ways, such as into categories, lists, tables or spreadsheets.

4. Summarising and displaying data

Any patterns or trends are clearer when displayed graphically, such as on a line graph or stem-and-leaf plot, or summarised by measures of centre and spread. These calculated values are called **summary statistics**. Use appropriate data visualisations (graphs, tables, infographics) to illustrate the data.

5. Analysing data and drawing conclusions

Analyse means to examine in closer detail, and analysing data is the most important stage. This involves comparing statistics to notice any significant results, answering the questions posed in the first stage and making important decisions and recommendations.

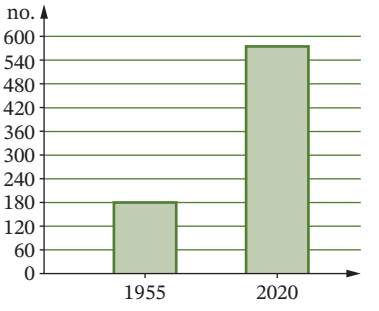
6. Writing a report

Finally, the researcher writes a report on the investigation, summarising clearly the project's aims, methods, findings, insights and conclusions.

EXERCISE 5.11 ANSWERS ON P. 594

Statistical investigations U F PS R C

1 This graph compares the number of passenger vehicles per 1000 people in Australia in 1955 and 2020.



Source: www.abs.gov.au

a How many passenger vehicles per 1000 people were there in 1955?

b What was the percentage increase in the rate between 1955 and 2020?

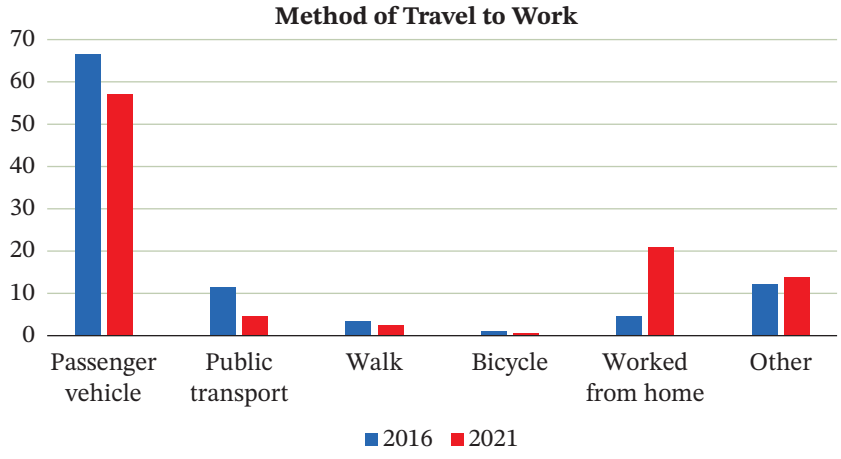
2 Visit the Australian Bureau of Statistics (ABS) website www.abs.gov.au and search for Motor vehicle census.

a What was the total number of vehicles registered last year?

b How many passenger vehicles were registered last year?

c What was the average annual growth rate over the last 5 years?

3 This graph compares the types of commuter transport used by Australians in 2016 and 2021. C



Source: www.abs.gov.au

a What percentage of people used a passenger vehicle to get to work in 2016 and what change has occurred from 2016 to 2021?

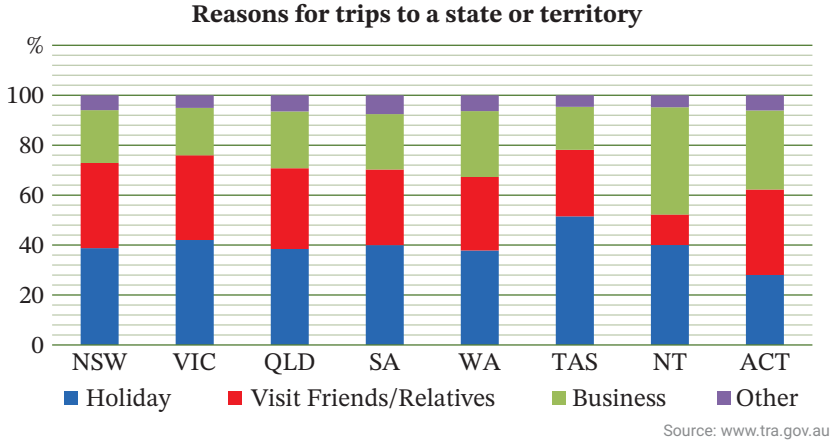
b What percentage of Australians used public transport in 2021? Perform an Internet search to see whether this has changed in recent years.

c Which mode of transport showed the largest percentage decrease from 2016 to 2021?

d List 3 advantages for using public transport.

e Which category showed the biggest increase? Why do you think this occurred?

4 This graph shows the reasons Australians went on interstate trips one year. R C



Source: www.tra.gov.au

a Which state/territory had the highest proportion of trips for holidays?

b What percentage of people visited South Australia to see friends/relatives?

c Which state/territory showed the highest percentage of visiting friends and relatives and which showed the lowest? Give possible reasons for your answer.

PATH

d The percentage of people taking business trips to Sydney and Melbourne is higher than the percentage of people taking business trips to Tasmania. Give a possible reason for this.

e Which state/territory had the lowest percentage of people visiting for a holiday? Give a possible reason for this.

5 Summarise your answers to questions **1** to **3** in a brief report about passenger vehicle use in Australia. Using your results, indicate what action governments (Federal, State and local) should do in terms of building roads, accident research and consideration of the environment. R C

6 'Life expectancy at birth is one of the most widely-used and internationally-recognised indicators of population health. High life expectancy at birth indicates low levels of infant mortality, a safe environment in which to live, a good health care system, sufficient food, and the adoption of preventative health measures.' PS R C

Source: Australian Bureau of Statistics, Life Expectancy Trends Australia.

Investigate life expectancy trends in Australia, using ABS and life insurance websites. Investigate data such as:

- life expectancy graphs or tables at birth and for different ages
- comparison of life expectancy between Australia and other countries
- comparison of life expectancy between Australian states and territories
- infant mortality rates
- causes of death (such as cancer)
- life expectancy of Aboriginal and Torres Strait Islander Australians
- implications of higher life expectancy on health costs, disabilities associated with aging population, aged-care facilities, pensions.

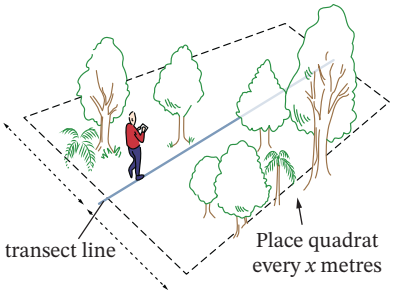
7 Is Australia becoming a warmer continent? Investigate this by looking at data from the Australian Bureau of Statistics and the Australian Bureau of Meteorology www.bom.gov.au. PS R C

8 Investigate vaping and alcohol use by teenagers in Australia. Include tables and graphs in your report. Refer to the National Drug Strategy Household Survey (www.nationaldrugstrategy.gov.au) and search 'alcohol and teenage statistics in Australia' on the Internet. PS R C

Investigation

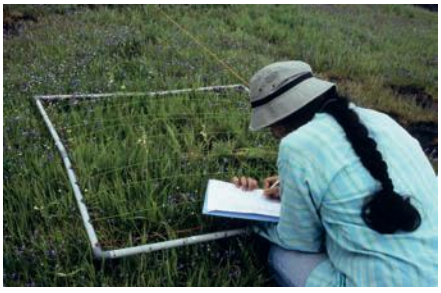
Sampling and biodiversity

Biodiversity is a measure of the number and variety of plant and animal species living in an ecosystem. In this investigation, we will look at how sampling methods can be used to track the biodiversity of native species in an area. A method used by environmental scientists involves taking samples regularly at the same locations to detect changes.



A **transect** is a straight line through a natural landscape, along which measurements are taken.

A **quadrat** is a rectangular frame that serves as a sample, within which species are counted and the biodiversity is surveyed. It can be placed at intervals along the transect line in order to collect data.


ephotocorp/Alamy Stock Photo

Work in groups of 2–3 students. You will need your notebook, a pen, measuring tape or trundle wheel, and a quadrat. A quadrat can be made using 4 plastic straws and some wool to make a frame or you can use a hula hoop.

1 As a class, create a transect, at most 10 metres long (in a straight line) in your school, such as on a section of lawn, bush or within a garden area.

2 The first group lays their quadrat on the ground at the start of the transect line. Other groups will survey at 1 m intervals along the line. It is important to record detailed observations of the surroundings and characteristics as accurately as possible. Each group draws a simple drawing of key features along or near the transect. Include the direction of north, nearby structures (buildings or sports goals), large plants (trees or shrubs), rocks, gravel or pathways across the transect.


Martyn F. Chillmaid/Science Source

3 Students observe how many different plants they can see. Count the number of common elements within the frame of the quadrat and record in your notebook. If practical, collect small samples of the items you find within the quadrat.

Quadrat location: _____



Number of plants in each portion of quadrat

Quadrat Section	Species 1 _____	Species 2 _____	Species 3 _____	Species 4 _____
1				
2				
3				
4				
Total				

4 After completing the field work, return to the classroom and collate the group data.

Population density of plants in a community

Class Group	Species 1 (Total)	Species 2 (Total)	Species 3 (Total)	Species 4 (Total)
1				
2				
3				
4				
5				
6				
:				
Total				
Population Density	/m^2	/m^2	/m^2	/m^2

5 What common plants did you identify?

6 Which plant did you see the most? The least?

7 What other common elements did you find?

8 Were there any unusual or unique plants or items in this area?

9 Draw appropriate graphs of the data collected.

10 Calculate the percentage of quadrats each species identified appears in.

11 Identify any changes which could be made to increase the biodiversity in this part of the school.

Power plus ANSWERS ON P. 594

1 The strength and direction of the association between 2 variables can be measured by the correlation coefficient (r).

a Between which 2 values does the correlation coefficient lie? Research this.

b What is the strength and direction of the association if the correlation coefficient is 0?

c Write a possible value for the correlation coefficient for each association described.

i perfect positive **ii** weak negative **iii** strong negative

2 The standard deviation of a data set 1 2 2 2 and 8 is $\sqrt{d}$. Find the value of d.

3 The following values are the test results on a History exam for a class of 20 students.

13 14 16 12 14 16 18 13 15 10

9 15 13 14 13 10 8 14 16 14

a Find the mean, median and mode.

b Find the range and interquartile range.

c An error was made in recording the values and 4 marks need to be added to each value. What effect will this have on the statistics calculated in parts **a** and **b**?

CHAPTER 5 REVIEW

Language of maths

association	bivariate data	box plot	causation
dependent variable	extrapolation	five-number summary	independent variable
interpolation	interquartile range	line of best fit	measure of spread
negatively skewed	positively skewed	random	sample
scatterplot	self-selected	standard deviation	stratified
strong	symmetrical	systematic	weak

1 What is represented by the 'whiskers' on a **box plot**?

2 Random breath testing is an example of what type of sampling?

3 What are the 5 things found in a **five-number summary**?

4 Describe a statistical distribution that is **positively-skewed**.

5 What type of graph is used to represent **bivariate data**?

6 Give 2 examples of how statistics can be misleading.

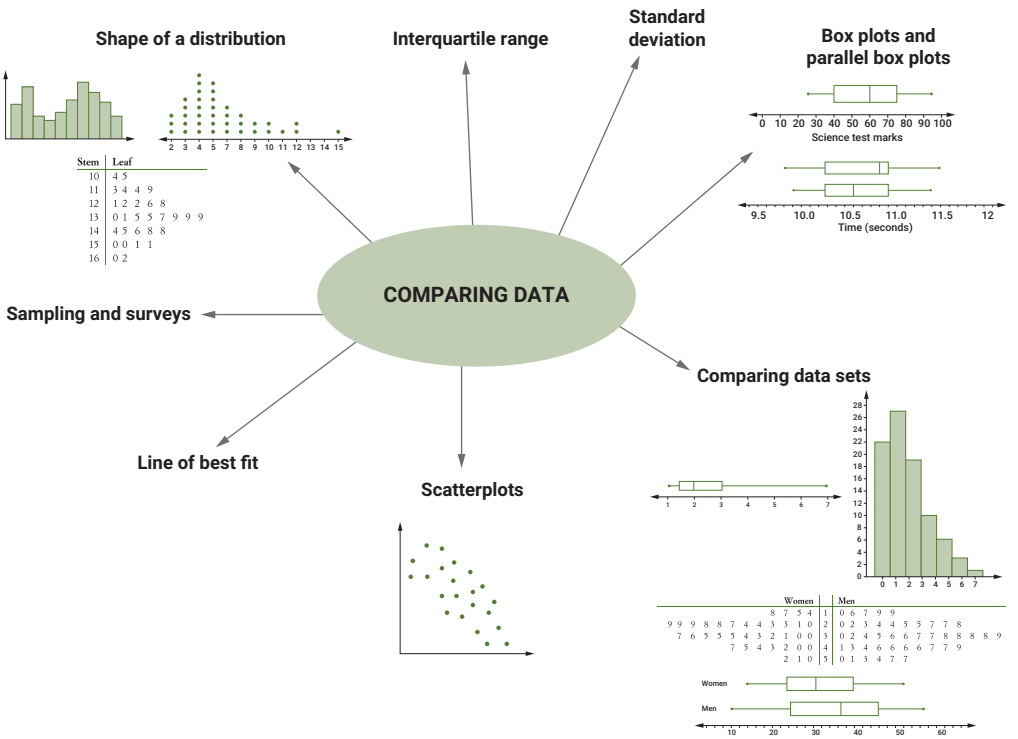
Quiz
Language of maths 5

Puzzle
Data crossword

Topic summary

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

Worksheet
Mind map: Comparing data



Quiz
Test yourself 5

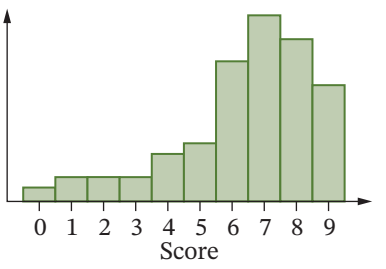
TEST YOURSELF 5 ANSWERS ON P. 594

5.01

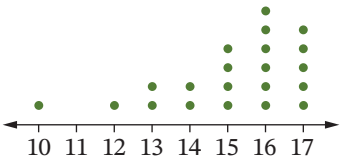
1 For each statistical distribution:

i describe the shape **ii** identify any outliers and clusters.

a



b



c

Stem	Leaf
3	0 1 2
4	1 3 4 4 5 6
5	0 4 5 7 8
6	3 7 8
7	0 1
8	4
9	8

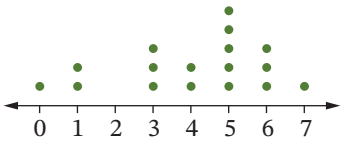
5.02

2 Find the interquartile range of each set of data.

a 5 8 8 10 12 13 14 15 18

b 24 15 23 28 20 20 18 30 21 18

c



d

Stem	Leaf
3	0 1 2
4	3 5 8 8 9 9 9
5	4 5 6 6 8
6	0 1 3 7
7	2

e

Score	Frequency
10	3
11	8
12	15
13	18
14	10
15	5

5.03

3 The reaction times (in seconds) of a sample of truck drivers were measured:

0.34 0.35 0.34 0.37 0.42 0.45 0.43 0.29 0.38 0.40
0.37 0.62

a Find, correct to 2 decimal places, the mean and standard deviation.

b Find the range and interquartile range.

c Which is the best measure of spread for this set of data? Justify your answer.

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4 The Health exam results for a class of PE students are shown here. 5.04

Girls:	83	78	63	84	65	51	76	69	42	84	60	64	92	73	32
Boys:	65	34	75	68	56	63	79	55	68	52	49	85	64	58	54

a Find the mean and standard deviation of both groups.

b Which group performed better in the exam? Give reasons.

5 The number of goals scored by the Under-18s Vale soccer team are: 5.05

2 0 0 4 2 1 1 2 3 2 3 7 4 3 1 0 4 2

a Find the range and interquartile range of the scores.

b Find the five-number summary for the data.

c Draw a box plot for the data.

6 The pulse rates of students were taken before and after exercising. The results were: 5.06

Before exercise:	78	80	66	70	56	64	68
	65	50	76	80	70	70	59
After exercise:	141	140	89	95	110	126	84
	82	90	88	146	98	96	92

a Find the five-number summary for the pulse rates before and after exercise.

b Construct parallel box plots for the 2 sets of data.

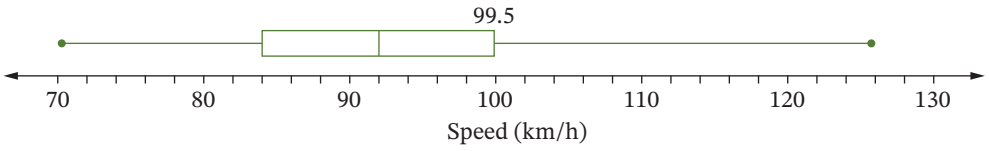
c Find the range and interquartile range of the pulse rates:

i before exercising **ii** after exercising.

d Compare the 2 sets of pulse rates. Are there significant differences between them? Justify your answer.

7 The speeds of cars (in km/h) were monitored between 1:00 and 1:30 p.m. on a main road. The results are shown in the stem-and-leaf plot and box plot. 5.07

Stem	Leaf
7	0 3 7 9
8	0 2 2 3 5 6 8 8 9
9	0 0 1 3 5 5 7 8 9 9 9
10	0 0 4 4 6
11	0 1
12	6



a Which display best indicates:

i skewness? **ii** clustering and outliers?

b Why would a dot plot be unsuitable for displaying the data?

c Find the:

i median **ii** interquartile range.

d What percentage of cars were travelling at a speed of at least 92 km/h?

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5.08

8 11 boxes containing 60 oranges each were placed in cold storage for different periods. After storage, the number of good oranges in each box was counted.

Weeks in storage (*W*)	2	5	12	8	14	6	5	9	10	3	11
Number of good oranges, (*N*)	58	50	33	40	28	50	52	38	35	55	33

a Which is the independent variable? Give reasons.

b Graph this data on a scatterplot.

c Describe the pattern of the plotted points.

d Describe the association between the number of weeks in storage and the number of good oranges.

e Describe the strength and direction of the association between the variables *W* and *N*.

5.08

9 For each pair of variables, determine whether the association is a causal relationship. R

a The number of sunny days this week and a person's level of happiness

b A person's height and their arm span

c The time it takes to walk to school and the distance from home to school

d The age of people and their reaction time

e The distance travelled by a car and the amount of petrol used

f The interest rate of home loans and the number of new home loans

5.09

10 The heights and weights of 15 people were measured.

Height, *H* (cm)	152	160	179	180	185	174	165	150	145	142	155	153	175	155
Weight, *W* (kg)	50	65	72	77	81	77	65	57	48	53	61	67	72	56

a Graph the points on a scatter plot and construct a line of best fit.

b Find the equation of the line of best fit.

c Use the equation to estimate the weight of a student who is 170 cm tall.

d Use the graph to interpolate the weight of a student with height 163 cm.

e Use the graph to extrapolate the height of a student who weighs

i 85 kg **ii** 45 kg

5.10

11 An advertisement in a magazine states that a product is 75% fat-free.

a What impression is the advertisement trying to make about the product?

b What doesn't the advertisement say about the product?

12 Nikhita investigates how often Year 10 students at her school play sport. She selects every 6th student on the Year 10 list to survey. There are 186 Year 10 students at her school. 5.10

a What is the name of the sampling method Nikhita has chosen to use?

b Write down one advantage and one disadvantage of this sampling method.

13 There are 550 students at Cengage Languages High School. They all study either French, German or Spanish. 5.10

Language	French	Spanish	German
Number of students	181	146	223

Data is to be collected from a sample of 40 students stratified by the language they study. Calculate the number of French students in the sample.



6

NUMBER AND ALGEBRA

EQUATIONS

In 1962, when astronaut John Glenn was preparing to be the first American to orbit the Earth, he called upon mathematician Katherine Johnson (1918–2020) to check by hand the complex calculations being performed on NASA's new electronic computers. Johnson worked for NASA for 35 years and her story was told in the 2016 book and film *Hidden Figures*. During her career, she solved equations that guided the paths of spacecrafts and helped the US send the first astronauts to the moon in 1969. Building on the ancient achievements of the Egyptians and Babylonians, Johnson opened the door to space travel and future space tourism.



Chapter outline

		Working mathematically				
6.01	Equations	U	F			
6.02	Equations with algebraic fractions*	U	F			
6.03	Quadratic equations $x^2 + bx + c = 0$*	U	F		R	C
6.04	Equation problems	U	F	PS	R	C
6.05	Equations and formulas	U	F		R	C
6.06	Changing the subject of a formula*	U	F		R	C

*PATH

Wordbank

equation A mathematical statement that 2 quantities are equal, involving algebraic expressions and an equals sign (=)

formula A rule written as an algebraic equation, using variables

quadratic equation An equation involving a variable squared (power of 2), such as $3x^2 - 6 = 69$

solution The answer to an equation or problem; the correct value(s) of the variable that makes an equation true

subject of a formula The variable for which a formula is written, on the left side of the formula, for example, for $E = mc^2$, the subject is E

Quiz
Wordbank 6

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Videos (10):

6.01 Equations with variables on both sides • Equations with brackets

6.02 Equations with algebraic fractions • Solving equations with fractions

6.03 Simple quadratic equations • Quadratic equations by factorising

6.05 Equations and formulas 2 • Equations and formulas 1 • Formulas and equations

6.06 Changing the subject of a formula

Quizzes (5):

- Wordbank 6
- SkillCheck 6
- Mental skills 6
- Language of maths 6
- Test yourself 6

Worksheets (2):

6.06 Changing the subject of a formula

Mind map Equations

Puzzles (9):

6.01 Equations with unknowns on both sides • Equations

6.02 Equations code puzzle • Equations order activity • Solving linear equations 1 • Solving linear equations 2

6.05 Getting the right formula

6.06 Formulas squaresaw

Language of maths Equations crossword

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

In this chapter you will:

- solve linear equations and problems involving equations
- (PATH) solve linear equations involving simple algebraic fractions
- solve quadratic equations of the form $ax^2 = c$ and (PATH) $x^2 + bx + c = 0$
- use formulas to solve problems
- (PATH) change the subject of a formula

Quiz
SkillCheck 6

SkillCheck ANSWERS ON P. 595

1 Solve each equation.

a $8y = 16$ **b** $10x = 120$ **c** $\frac{m}{5} = 2$

d $w + 6 = 10$ **e** $m - 3 = 12$ **f** $n + 6 = -4$

2 Expand each expression.

a $5(x + 10)$ **b** $4(y - 1)$ **c** $2(5 - 3y)$

3 Solve each equation.

a $2x + 3 = 23$ **b** $3x - 5 = 19$ **c** $4a + 5 = 2a - 19$

d $\frac{3x+2}{5} = 4$ **e** $4(2 - x) = -24$

Equations

6.01

Example 1

Solve each equation.

a $3m - 6 = 12$

b $5 - 2a = 3a$

c $9x + 10 = 7x - 6$

d $5(p + 6) = 3p + 5$

Solution

a $3m - 6 = 12$

$3m - 6 + 6 = 12 + 6$ Adding 6 to both sides

$3m = 18$

$\frac{3m}{3} = \frac{18}{3}$ Dividing both sides by 3

$m = 6$

Check:

LHS $= 3 \times 6 - 6 = 12$

RHS $= 12$

LHS = RHS

b $5 - 2a = 3a$

$5 - 2a + 2a = 3a + 2a$ Adding $2a$ to both sides

$5 = 5a$

$5a = 5$

$\frac{5a}{5} = \frac{5}{5}$ Dividing both sides by 5

$a = 1$

Check:

LHS $= 5 - 2 \times 1 = 3$

RHS $= 3 \times 1 = 3$

LHS = RHS

c $9x + 10 = 7x - 6$

$9x + 10 - 7x = 7x - 6 - 7x$

$2x + 10 = -6$

$2x + 10 - 10 = -6 - 10$

$2x = -16$

$\frac{2x}{2} = \frac{-16}{2}$

$x = -8$

d Expand the LHS:

$5(p + 6) = 3p + 5$

$5p + 30 = 3p + 5$

$5p + 30 - 3p = 3p + 5 - 3p$

$2p + 30 = 5$

$2p + 30 - 30 = 5 - 30$

$2p = -25$

$\frac{2p}{2} = \frac{-25}{2}$

$p = -12\frac{1}{2}$

Puzzles
Equations with unknowns on both sides

Equations

Videos
Equations with variables on both sides

Equations with brackets

6.01

Example 2

Solve each equation.

a $4(y+1)+3(y-5)=8$

b $\frac{5d}{3}+4=15$

c $\frac{6-m}{5}=4$

d $\frac{4w+3}{7}=5$

Solution

a Expand and collect like terms.

$4(y+1)+3(y-5)=8$

$4y+4+3y-15=8$

$7y-11=8$

$7y-11+11=8+11$

$7y=19$

$\frac{7y}{7}=\frac{19}{7}$

$y=2\frac{5}{7}$

Check:

$\text{LHS}=4\left(2\frac{5}{7}+1\right)+3\left(2\frac{5}{7}-5\right)=8$

RHS = 8

LHS = RHS

b $\frac{5d}{3}+4=15$

$\frac{5d}{3}+4-4=15-4$

$\frac{5d}{3}=11$

$\frac{5d}{3}\times 3=11\times 3$

$5d=33$

$d=\frac{33}{5}$

$d=6\frac{3}{5}$

c $\frac{6-m}{5}=4$

$\frac{6-m}{5}\times 5=4\times 5$

$6-m=20$

$6-m-6=20-6$

$-m=14$

$m=-14$

d $\frac{4w+3}{7}=5$

$\frac{4w+3}{7}\times 7=5\times 7$

$4w+3=35$

$4w+3-3=35-3$

$4w=32$

$w=\frac{32}{4}$

$w=8$

EXERCISE 6.01 ANSWERS ON P. 595

Equations U F

EXAMPLE 1

1 Solve each equation.

a $\frac{k}{6}=10$

b $w+3=-6$

c $5y-1=9$

d $3a+10=25$

e $2x+6=22$

f $15a-2=13$

g $12-r=18$

h $7w-10=32$

i $9y-6=-24$

j $11-6a=-10$

k $7u=u+32$

l $5a=a-7$

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2 Solve $6x - 3 = 27$. Select the correct answer **A**, **B**, **C** or **D**.

A $x = 4$ **B** $x = 5$ **C** $x = 10$ **D** $x = 18$

3 Solve $10 - 2a = 20$. Select **A**, **B**, **C** or **D**.

A $a = -15$ **B** $a = 8$ **C** $a = 32$ **D** $a = -5$

4 Solve each equation.

a $5y + 10 = 3y + 30$ **b** $8a + 20 = 4a + 10$

c $6y - 1 = 3y + 14$ **d** $12a + 30 = 5a + 9$

e $5y + 3 = 8y - 21$ **f** $14x - 20 = 8x - 14$

g $9y + 1 = 3y - 5$ **h** $15x - 15 = 8x - 85$

i $8m - 10 = 5 - 2m$ **j** $18 - 3y = 6 - 2y$

k $1 - 7a = 10 + 2a$ **l** $11 - 5x = 3x + 43$

5 Solve $4y = y - 15$. Select **A**, **B**, **C** or **D**.

A $y = -3$ **B** $y = \frac{7}{4}$ **C** $y = -5$ **D** $y = 11$

6 Solve each equation.

a $3(x - 6) = 30$ **b** $5(m + 10) = 80$

c $2(5y + 3) = 46$ **d** $3(y + 2) = 5y - 10$

e $5(y + 4) = 3y + 6$ **f** $10(x - 3) = 5(x + 5)$

g $2(3m + 6) = 4(m - 1)$ **h** $5(2a + 7) = 5(4 - a)$

i $3(1 - 2y) = 18 - 3y$

7 Solve $2(y - 3) = 5 + 4y$. Select **A**, **B**, **C** or **D**.

A $y = -9$ **B** $y = -5$ **C** $y = -\frac{11}{2}$ **D** $y = -\frac{1}{2}$

8 Solve each equation.

a $3(d + 3) + 4(d + 1) = 15$ **b** $3(y - 1) + 5(y + 4) = 10$

c $7(k + 1) + 2(k - 6) = 3$ **d** $5(g - 3) + 2(g - 2) = 4$

e $6(2h + 3) + 5(h - 3) = 9$ **f** $2(1 + p) + 3(4 + p) = 5$

9 Solve each equation.

a $\frac{4k}{5} = 8$ **b** $\frac{2d}{6} = 7$ **c** $\frac{w+7}{3} = 8$

d $\frac{3m}{4} + 6 = 10$ **e** $\frac{7+a}{8} = 3$ **f** $3 + \frac{2d}{7} = 9$

g $\frac{9-y}{5} = 3$ **h** $8 - \frac{4x}{3} = 5$ **i** $10 = \frac{9b}{7}$

j $\frac{x+1}{4} + 2 = 10$ **k** $\frac{y-2}{5} - 6 = 3$ **l** $\frac{m+2}{5} - 1 = 3$

m $\frac{4h-3}{2} = 7$ **n** $\frac{2e+11}{4} = 6$ **o** $\frac{12-5y}{3} = 9$

p $\frac{8h+3}{5} = 7$ **q** $\frac{9a}{4} - 5 = 7$ **r** $\frac{9-3c}{7} = 8$

6.01

EXAMPLE 2

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Investigation

Make your own equation

Here are 2 equations that have the same solution, $x = 6$:

$5x - 1 = 23 + x$ and $\frac{3x+12}{10} = 3$

1 For each solution below, make up 2 equations that have that solution.

a $x = 4$ **b** $x = \frac{1}{2}$ **c** $x = 10$

d $x = 1.5$ **e** $x = 0$ **f** $x = -2$

2 Compare your answers with those of other students. Check that each equation is correct.

6.02 Equations with algebraic fractions

PATH

Puzzles
Equations code puzzle

Equations order activity

Solving linear equations 1

Solving linear equations 2

Example 3

Solve each equation.

a $\frac{5k+4}{3} - 7 = 12$

b $\frac{3(p-3)}{2} + 6 = 1$

Solution

a

$\frac{5k+4}{3} - 7 = 12$

$\frac{5k+4}{3} - 7 + 7 = 12 + 7$

$\frac{5k+4}{3} = 19$

$\frac{5k+4}{3} \times 3 = 19 \times 3$ Multiply both sides by 3

$5k + 4 = 57$

$5k + 4 - 4 = 57 - 4$

$5k = 53$

$\frac{5k}{5} = \frac{53}{5}$

$k = 10\frac{3}{5}$

b

$\frac{3(p-3)}{2} + 6 - 6 = 1 - 6$

$\frac{3(p-3)}{2} = -5$

$\frac{3(p-3)}{2} \times 2 = -5 \times 2$ Multiply both sides by 2

$3(p - 3) = -10$

$3p - 9 = -10$ Expand the LHS

$3p - 9 + 9 = -10 + 9$

$3p = -1$

$\frac{3p}{3} = \frac{-1}{3}$

$p = -\frac{1}{3}$

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For equations with more than one fraction, multiply both sides by a common multiple of the denominators to remove the fractions.

PATH



Videos
Equations with algebraic fractions

Solving equations with fractions

6.02

Example 4

Solve each equation.

a $\frac{2m}{3}-\frac{m}{2}=2$ **b** $\frac{2a+4}{5}=\frac{2}{3}$ **c** $\frac{3x-5}{4}=\frac{x+1}{7}$

Solution

a

$$\frac{2m}{3}-\frac{m}{2}=2$$

$$6\left(\frac{2m}{3}-\frac{m}{2}\right)=6\times 2$$

Multiply both sides by 6, the lowest common multiple (LCM) of 3 and 2

$$6^{2}\left(\frac{2m}{3_{1}}\right)-6^{3}\left(\frac{m}{2_{1}}\right)=12$$

$$4m-3m=12$$

$$m=12$$

Check by substituting that this solution is correct.

Note: We can also solve this equation by simplifying the LHS first:

$$\frac{2m}{3}-\frac{m}{2}=2$$

$$\frac{4m}{6}-\frac{3m}{6}=2$$

$$\frac{m}{6}=2$$

$$m=12$$

However, the first method may be faster and easier because operations with fractions are not required.

b

$$\frac{2a+4}{5}=\frac{2}{3}$$

$$15^{3}\left(\frac{2a+4}{5_{1}}\right)=15^{5}\left(\frac{2}{3_{1}}\right)$$ LCM = 15

$$3(2a+4)=10$$

$$6a+12=10$$

$$6a=-2$$

$$a=\frac{-2}{6}$$

$$a=-\frac{1}{3}$$

c

$$\frac{3x-5}{4}=\frac{x+1}{7}$$

$$28^{7}\left(\frac{3x-5}{4^{1}}\right)=28^{4}\left(\frac{x+1}{7^{1}}\right)$$ LCM = 28

$$7(3x-5)=4(x+1)$$

$$21x-35=4x+4$$

$$17x=39$$

$$x=\frac{39}{17}$$

$$x=2\frac{5}{17}$$

PATH

EXERCISE 6.02 ANSWERS ON P. 596

Equations with algebraic fractions U F

1 Solve each equation. Select the correct answer **A**, **B**, **C** or **D**.

a $\frac{3y}{4}=6$

A $y=9$ **B** $y=8$ **C** $y=7$ **D** $y=6$

b $\frac{a+1}{2}=3$

A $a=4$ **B** $a=5$ **C** $a=6$ **D** $a=7$

c $\frac{x-1}{4}+2=10$

A $x=49$ **B** $x=37$ **C** $x=33$ **D** $x=3$

EXAMPLE 3

2 Solve each equation.

a $\frac{2y-3}{5}+2=7$ **b** $\frac{7m+5}{2}-3=8$ **c** $\frac{4+3x}{4}-5=2$ **d** $\frac{7+2k}{6}+3=12$

e $6+\frac{3w+1}{4}=10$ **f** $4+\frac{6a-5}{3}=9$ **g** $\frac{1-4e}{2}-8=3$ **h** $6+\frac{3-2x}{3}=0$

i $\frac{5-4p}{6}+2=5$ **j** $\frac{7-3y}{4}+1=5$ **k** $\frac{2(x+1)}{5}=10$ **l** $\frac{3(m-2)}{4}=6$

m $\frac{8(n+1)}{3}+2=4$ **n** $\frac{5(1-n)}{2}-1=3$ **o** $\frac{4(1+d)}{3}+1=7\frac{1}{3}$ **p** $8+\frac{3(w-4)}{5}=14$

q $\frac{4(d-5)}{7}-3=4$ **r** $4=\frac{6(3-m)}{5}-4$ **s** $1+\frac{4(7-h)}{3}=13$ **t** $5=2+\frac{3(1-2g)}{7}$

EXAMPLE 4

3 Solve each equation.

a $\frac{2k}{3}=\frac{5}{4}$ **b** $\frac{3w}{10}=\frac{2}{5}$ **c** $\frac{5x}{2}=-\frac{10}{3}$ **d** $\frac{5k-1}{4}=\frac{3}{5}$

e $\frac{5}{4}=\frac{4m+1}{7}$ **f** $\frac{1-2y}{5}=\frac{8}{3}$ **g** $\frac{n-7}{6}=\frac{n+3}{4}$ **h** $\frac{y+2}{5}=\frac{y-1}{2}$

i $\frac{a+5}{3}=\frac{a-1}{8}$ **j** $\frac{p+2}{5}=\frac{p-5}{2}$ **k** $\frac{4k-3}{5}=\frac{2k+7}{4}$ **l** $\frac{3y+2}{3}=\frac{2y+1}{4}$

m $\frac{w}{5}+\frac{w}{2}=7$ **n** $\frac{5a}{7}-\frac{2a}{3}=2$ **o** $\frac{2e}{3}+\frac{e}{2}=8$ **p** $\frac{3a}{2}+\frac{4a}{5}=1$

q $\frac{2y}{5}-\frac{y}{3}=4$ **r** $\frac{a}{3}+\frac{3a}{4}=2$ **s** $\frac{5m}{4}-\frac{3m}{10}=1$ **t** $\frac{4h}{3}+\frac{h}{5}=3$

u $\frac{5y-2}{7}=\frac{3y+5}{3}$

4 Solve each equation. Select **A**, **B**, **C** or **D**.

a $\frac{4m}{5}-\frac{m}{3}=2$

A $m=10$ **B** $m=12$ **C** $m=\frac{30}{7}$ **D** $m=\frac{4}{3}$

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b $\frac{m+1}{2}=\frac{3+2m}{5}$

A $m=1$ **B** $m=5$ **C** $m=\frac{5}{3}$ **D** $m=\frac{2}{3}$

5 Use substitution to match each solution with its equation.

a $x=18$ **b** $x=-1$ **c** $x=3$ **d** $x=4$

A $\frac{3x+1}{2}=5$ **B** $\frac{2x}{3}-\frac{x}{2}=3$ **C** $\frac{x-1}{3}=\frac{x+1}{5}$ **D** $2x-3=x-4$

PATH

Quadratic equations $x^2 + bx + c = 0$ 6.03

An equation in which the highest power of the variable is 2 is called a **quadratic equation**, for example, $x^2 = 5$, $3m^2 + 7 = 10$, $d^2 - d - 6 = 0$ and $4t^2 - 3t = 8$.

The quadratic equation $x^2 = c$

The quadratic equation $x^2 = c$ (where c is a positive number) has 2 solutions:

$x = \pm\sqrt{c}$ (which means $x = \sqrt{c}$ and $x = -\sqrt{c}$).

Example 5

Solve each quadratic equation.

a $m^2 = 16$ **b** $3x^2 = 75$ **c** $4m^2 - 12 = 0$

Video
Simple quadratic equations

Solution

a $m^2 = 16$

$m = \pm\sqrt{16}$ Finding the square root of both sides

$= \pm 4$

b $3x^2 = 75$

$x^2 = \frac{75}{3}$ Dividing both sides by 3

$x^2 = 25$

$x = \pm\sqrt{25}$ Finding the square root of both sides

$= \pm 5$

c $4m^2 - 12 = 0$

$4m^2 - 12 + 12 = 0 + 12$ Adding 12 to both sides

$4m^2 = 12$

$m^2 = \frac{12}{4}$ Dividing both sides by 4

$m^2 = 3$ Finding the square root of both sides

$m = \pm\sqrt{3}$ Leave the answer as a surd (in exact form)

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Example 6

Solve $7x^2 - 88 = 0$, writing the solution correct to one decimal place.

Solution

$7x^2 - 88 = 0$

$7x^2 = 88$

$x^2 = \frac{88}{7}$

$x = \pm\sqrt{\frac{88}{7}}$

$x = \pm 3.54562...$

$\approx \pm 3.5$

PATH

The quadratic equation $x^2 + bx + c = 0$

To solve quadratic equations of the form $x^2 + bx + c = 0$, factorise $x^2 + bx + c$ into binomial products.



Video
Quadratic equations by factorising

Example 7

Solve $x^2 + 5x + 6 = 0$.

Solution

To factorise $x^2 + 5x + 6$, find 2 numbers that have a sum of 5 and a product of 6.

The correct numbers are 2 and 3.

$x^2 + 5x + 6 = 0$

$(x + 2)(x + 3) = 0$

The LHS has been factorised into 2 binomials, $(x + 2)$ and $(x + 3)$, whose product is 0.

If 2 numbers have a product of 0, then one of the numbers *must* be 0.

$\therefore x + 2 = 0$ or $x + 3 = 0$

$\therefore x = -2$ or $x = -3$

$\therefore$ The solution to $x^2 + 5x + 6 = 0$ is

$x = -2$ or $x = -3$.

Check:

When $x = -2$,

LHS $= (-2)^2 + 5 \times (-2) + 6 = 0$

RHS $= 0$

LHS = RHS.

When $x = -3$,

LHS $= (-3)^2 + 5 \times (-3) + 6 = 0$

RHS $= 0$

LHS = RHS.

Example 8

Solve each quadratic equation.

a $x^2 - x - 2 = 0$ **b** $u^2 + 3u - 28 = 0$

c $3m^2 = 6m$ **d** $w^2 - 10w + 25 = 0$

Solution

a $x^2 - x - 2 = 0$

Find 2 numbers that have a sum of -1 and a product of -2.

They are -2 and 1.

$(x - 2)(x + 1) = 0$

$\therefore x - 2 = 0$ or $x + 1 = 0$

$\therefore x = 2$ or $x = -1$

$\therefore$ The solution to $x^2 - x - 2 = 0$ is

$x = 2$ or $x = -1$.

Check:

When $x = 2$,

LHS $= 2^2 - 2 - 2 = 0$

RHS $= 0$

LHS = RHS.

When $x = -1$,

LHS $= (-1)^2 - (-1) - 2 = 0$

RHS $= 0$

LHS = RHS.

b $u^2 + 3u - 28 = 0$

Find 2 numbers that have a sum of 3 and a product of -28.

They are 7 and -4.

$(u + 7)(u - 4) = 0$

$\therefore u + 7 = 0$ or $u - 4 = 0$

$\therefore u = -7$ or $u = 4$

$\therefore$ The solution to $u^2 + 3u - 28 = 0$ is $u = -7$ or $u = 4$.

c $3m^2 = 6m$

$3m^2 - 6m = 0$

This requires a simpler factorisation as there are only 2 terms, both involving m.

You cannot divide both sides by m because if $m = 0$, then you are dividing by 0.

$3m(m - 2) = 0$

$\therefore 3m = 0$ or $m - 2 = 0$

$\therefore m = 0$ or $m = 2$

$\therefore$ The solution to $3m^2 - 6m = 0$ is $m = 0$ or $m = 2$.

d $w^2 - 10w + 25 = 0$

Find 2 numbers that have a sum of -10 and a product of 25.

They are -5 and -5.

$(w - 5)(w - 5) = 0$

$\therefore w - 5 = 0$ or $w - 5 = 0$

$\therefore w = 5$ or $w = 5$

$\therefore$ The solution to $w^2 - 10w + 25 = 0$ is $w = 5$ (only one solution).

PATH



Video Quadratic equations by factorising

6.03

PATH

Note: Quadratic equations of the form $ax^2 + bx + c = 0$ can be found in Chapter 14, *Quadratic equations and the parabola*, an optional Advanced chapter.

EXERCISE 6.03 ANSWERS ON P. 596

Quadratic equations $x^2 + bx + c = 0$ UFRC

EXAMPLE 5

1 Solve each quadratic equation, writing the solutions as surds if necessary.

a $m^2 = 144$ **b** $x^2 = 400$ **c** $y^2 = 35$

d $k^2 - 169 = 0$ **e** $y^2 - 1 = 0$ **f** $w^2 - 24 = 0$

g $x^2 + 10 = 14$ **h** $t^2 - 9 = 7$ **i** $\frac{a^2}{2} = 8$

j $4k^2 = 180$ **k** $3w^2 = 300$ **l** $d^2 + 60 = 204$

m $\frac{k^2}{2} = 7$ **n** $\frac{w^2}{10} = 2.5$ **o** $4x^2 = 1$

p $\frac{m^2}{4} = 10$ **q** $5y^2 = 5$ **r** $2p^2 + 3 = 21$

s $\frac{3k^2}{2} + 1 = 13$ **t** $\frac{y^2}{5} - 2 = 18$

EXAMPLE 6

2 Solve each equation, correct to one decimal place where necessary.

a $5m^2 - 20 = 0$ **b** $\frac{4a^2}{9} = 36$ **c** $m^2 = 28$

d $9m^2 - 2 = 32$ **e** $9k^2 + 10 = 13$ **f** $\frac{2x^2}{5} = 23$

g $\frac{k^2}{16} = 6$ **h** $\frac{3k^2}{10} = 27$ **i** $6y^2 = 0.726$

j $3a^2 + 11 = 267$ **k** $2y^2 - 14 = 63$ **l** $\frac{2w^2}{5} - 1 = 19$

3 Solve each quadratic equation. Select the correct answer **A**, **B**, **C** or **D**.

a $x^2 = 121$

A $x = 12, -12$ **B** $x = 11, -11$ **C** $x = 10, 11$ **D** $x = 12, -11$

b $9m^2 - 1 = 35$

A $m = 3, -3$ **B** $m = 2, -2$ **C** $m = 8, -8$ **D** $m = 9, -9$

Foundation Standard Complex

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PATH

EXAMPLE 7

EXAMPLE 8

6.04

4 Solve each equation.

a $x^2 + 3x + 2 = 0$ **b** $y^2 + 5y + 4 = 0$ **c** $y^2 + 16y + 48 = 0$

d $x^2 + x - 12 = 0$ **e** $x^2 + 2x - 3 = 0$ **f** $x^2 + 3x - 40 = 0$

5 Solve each equation.

a $x^2 - x - 30 = 0$ **b** $x^2 - 8x + 16 = 0$ **c** $x^2 - 5x - 66 = 0$

d $d^2 - 2d = 0$ **e** $x^2 - 3x - 10 = 0$ **f** $n^2 + 4n = 0$

g $k^2 - 7k = 0$ **h** $y^2 = 5y$ **i** $v^2 = 12v$

6 Explain why the quadratic equation $x^2 = -25$ has no solutions. R C

7 State which of these quadratic equations have no solutions. Give reasons. R C

a $x^2 = -9$ **b** $2k^2 + 5 = 9$ **c** $3m^2 + 8 = 4$

d $\frac{9w^2}{2} - 1 = 1$ **e** $4 + \frac{d^2}{3} = 8$ **f** $\frac{5a^2}{2} + 3 = 2$

8 Solve each quadratic equation. Select **A**, **B**, **C** or **D**.

a $x^2 + 4x - 60 = 0$

A $x = -10, 6$ **B** $x = 12, -5$ **C** $x = 10, -6$ **D** $x = -12, 5$

b $q^2 + 3q = 0$

A $q = 3, -3$ **B** $q = 6, -3$ **C** $q = 0, -3$ **D** $q = 0, 3$

Equation problems 6.04

Example 9

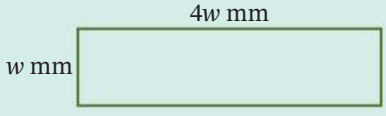
A rectangle is 4 times as long as it is wide. The perimeter of the rectangle is 180 mm. Find the dimensions of the rectangle.

Solution

Let the width of the rectangle be w mm. Then the length is $4w$ mm.

$\therefore$ Perimeter: $w + 4w + w + 4w = 180$

$$10w = 180$$
$$w = \frac{180}{10}$$
$$= 18$$



$\therefore$ The width of the rectangle is 18 mm.

Its length is $4w = 4 \times 18 = 72$ mm.

Check: Perimeter $= 18 + 72 + 18 + 72 = 180$ mm.

Foundation Standard Complex

Example 10

Paris is 7 years older than Amy. 10 years from now, the sum of their ages will be 43.

How old are they now?

Solution

Let x = Amy's age now.

Then Paris' age now = $x + 7$.

Break the information into 'Now' and 'In 10 years time'

	Now	In 10 years time
Amy	x	$x + 10$
Paris	$x + 7$	$x + 7 + 10 = x + 17$

In 10 years time:

Sum of ages: $(x + 10) + (x + 17) = 43$

$$2x + 27 = 43$$
$$2x = 16$$
$$x = 8$$

Amy is 8 years old now.

Paris is $8 + 7 = 15$ years old now.

Check: In 10 years time, the sum of their ages will be $18 + 25 = 43$.

EXERCISE 6.04 ANSWERS ON P. 596

Equation problems U F PS R C

For each question, write an equation and solve it to answer the problem. PS R C

EXAMPLE 9

1. The longer sides of an isosceles triangle are twice as long as the shorter side. The perimeter of the triangle is 90 mm. Find the lengths of the sides of the triangle.

2. The length of a rectangle is 3 times as long as its width. The perimeter of the rectangle is 152 mm. Find its dimensions.

3. The length of a rectangle is 3 more than twice its width. Find the dimensions of the rectangle if its perimeter is 84 cm.

4. The sum of 3 **consecutive** integers is 186. Find the integers.

EXAMPLE 10

5. Tyson is 9 times the age of his daughter, Charlotte. In 5 years, he will be 4 times the age of Charlotte. How old are they now?

6. When 15 is subtracted from 3 times a certain number, the answer is 63. What is the number?

7. The product of 2 and a number is the same as 12 minus the number. Find the number.



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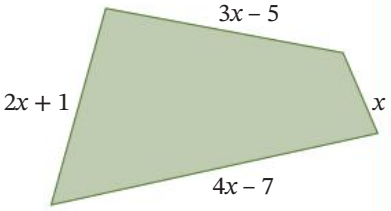
Foundation Standard Complex

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8 The sum of the present ages of Vatha and Chris is 36. In 4 years time, the sum of their ages will equal twice Vatha's present age. How old are they now?

9 4 consecutive integers have a sum of 858. Find the 4 integers.

10 The perimeter of the block of land shown is 344 m.
Measurements shown are in metres.
Find the value of x.



11 Manori's bag contained 10-cent and 20-cent coins. She had 202 coins, with a total value of $31.90. How many 20-cent coins did Manori have?

12 If 17 more than a number is 5 more than 3 times the number, what is the number?

13 The sum of Scott's age and his mother Kait's age is 45. In 5 years time, 3 times Scott's age less 9 will be the same as Kait's age. Find the present ages of Scott and Kait.

14 One angle in a triangle is double the smallest angle, and the third angle in the triangle is 5 more than 4 times the smallest angle. Find the size of each angle in the triangle.

6.04

Mental skills 6: Maths without calculators ANSWERS ON P. 596

Quiz
Mental skills 6

Multiplying and dividing by 5, 15, 25 and 50

It is easier to multiply or divide a number by 10 than by 5. So whenever we multiply or divide a number by 5, we can double the 5 (to make 10) and then adjust the first number.

1 Study each example.

a **To multiply by 5**, halve the number, then multiply by 10.

$$\begin{aligned} 18 \times 5 &= 18 \times \frac{1}{2} \times 10 \text{ (or } 9 \times 2 \times 10) \\ &= 9 \times 10 \\ &= 90 \end{aligned}$$

b **To multiply by 50**, halve the number, then multiply by 100.

$$\begin{aligned} 26 \times 50 &= 26 \times \frac{1}{2} \times 100 \text{ (or } 13 \times 2 \times 100) \\ &= 13 \times 100 \\ &= 1300 \end{aligned}$$

c **To multiply by 25**, quarter the number, then multiply by 100.

$$\begin{aligned} 44 \times 25 &= 44 \times \frac{1}{4} \times 100 \text{ (or } 11 \times 4 \times 25) \\ &= 11 \times 100 \\ &= 1100 \end{aligned}$$

d **To multiply by 15**, halve the number, then multiply by 30.

$$\begin{aligned} 8 \times 15 &= 8 \times \frac{1}{2} \times 30 \text{ (or } 4 \times 2 \times 15) \\ &= 4 \times 30 \\ &= 120 \end{aligned}$$

Foundation | Standard | Complex

e **To divide by 5**, divide by 10 and double the answer. We do this because there are 2 5s in every 10.

$140 \div 5 = 140 \div 10 \times 2$

$= 14 \times 2$

$= 28$

f **To divide by 50**, divide by 100 and double the answer. This is because there are 2 50s in every 100.

$400 \div 50 = 400 \div 100 \times 2$

$= 4 \times 2$

$= 8$

g **To divide by 25**, divide by 100 and multiply the answer by 4. This is because there are 4 25s in every 100.

$600 \div 25 = 600 \div 100 \times 4$

$= 6 \times 4$

$= 24$

h **To divide by 15**, divide by 30 and double the answer. This is because there are 2 15s in every 30.

$240 \div 15 = 240 \div 30 \times 2$

$= 8 \times 2$

$= 16$

2 Now evaluate each expression.

a	32×5	**b**	14×5	**c**	48×5	**d**	18×50
e	52×50	**f**	36×25	**g**	28×5	**h**	12×25
i	12×15	**j**	22×35	**k**	$90 \div 5$	**l**	$170 \div 5$
m	$230 \div 5$	**n**	$1300 \div 50$	**o**	$900 \div 50$	**p**	$300 \div 25$
q	$1000 \div 25$	**r**	$360 \div 45$	**s**	$210 \div 15$	**t**	$360 \div 15$

6.05 Equations and formulas

PATH

Puzzle
Getting the right formula

A **formula** is an equation that describes a relationship between variables. For example, the formula for the perimeter of a rectangle is $P = 2(l + w)$, where P is the perimeter, l is the rectangle's length and w is the width. Because the formula is for the perimeter, P is called the **subject** of the formula and it is the variable on its own on the left side of the '=' sign.

Example 11

The cost of a trip, C, in dollars, for a ride share company is $C = 5 + 2.4d$, where d is the distance travelled, in kilometres.

a Find the cost of a trip if the distance travelled is 15 km.

b Find the distance travelled if the cost of the trip was $78.20.



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Solution

a When $d = 15$:

$C = 5 + 2.4 \times 15$

$= 41$

The cost was $41.

b When $C = 78.20$:

$78.20 = 5 + 2.4d$

$73.20 = 2.4d$

$d = \frac{73.20}{2.4}$

$= 30.5$

The distance travelled was 30.5 km.

Example 12

The surface area of a sphere is given by the formula $A = 4\pi r^2$, where r is the radius.

Find (correct to one decimal place):

a the surface area of a sphere with radius 2.8 cm

b the radius of a sphere with surface area 40 m^2.

Solution

a When $r = 2.8$:

$A = 4 \times \pi \times 2.8^2$

$= 98.520...$

≈ 98.5

The surface area of the sphere is 98.5 cm^2.

b When $A = 40$:

$40 = 4\pi r^2$

$4\pi r^2 = 40$

$r^2 = \frac{40}{4\pi}$

$= 3.183...$

$r = \sqrt{3.183}$ $r > 0$ because the radius is positive

$= 1.784...$

≈ 1.8

The radius of the sphere is 1.8 m.

PATH

Videos
Equations and formulas 2
Equations and formulas 1
Formulas and equations

6.05

Video
Equations and formulas 2

PATH

EXERCISE 6.05 ANSWERS ON P. 596

Equations and formulas U F R C

EXAMPLE 11

1 The formula for the perimeter of a rectangle is $P = 2(l + w)$. R

a Find the perimeter of a rectangle with length 10 cm and width 16 cm.

b Find the width of a rectangle whose perimeter is 58 m and length is 12 m.

2 A formula for converting speed expressed in m/s (metres/second) to a speed expressed in km/h is $k = 3.6M$, where M is the speed in m/s. Convert each speed to km/h. R C

a 10 m/s **b** 24 m/s **c** 50 m/s

3 A car is travelling at a speed of 110 km/h on a freeway. Use the formula from question **2** to calculate how fast this is in m/s.

4 The formula for converting temperatures in °F to °C is $C = \frac{5}{9}(F - 32)$.
Express each temperature in °C, correct to the nearest degree.

a 80°F **b** 32°F **c** 212°F **d** 102°F

5 Use the formula from Question 4 to express each temperature in °F.

a 30°C **b** 40°C **c** 15°C **d** 45°C

6 The average of 2 numbers, m and n, is $A = \frac{m+n}{2}$. If 2 numbers have an average of 28 and one of the numbers is 13, use the formula to find the other number. R

EXAMPLE 12

7 Pythagoras' theorem is $c^2 = a^2 + b^2$, where c is the length of the hypotenuse in a right-angled triangle and a and b are the lengths of the other 2 sides. Find, correct to one decimal place where necessary:

a c, if $a = 5$ and $b = 10$ **b** a, if $c = 41$ and $b = 40$ **c** b, if $c = 20$ and $a = 10$.

8 The formula for the circumference of a circle is $C = 2\pi r$, where r is the radius.
Find, correct to one decimal place: R C

a the circumference of a circle with radius 2.4 m

b the radius of a circle whose circumference is 200 cm.

9 The body mass index (BMI) of an adult is $B = \frac{M}{h^2}$, where M is the mass in kilograms and h is the height in metres. Find, correct to one decimal place: R

a the BMI of Leonie who is 1.85 m tall and has a mass of 72 kg

b the mass of Dane with a BMI of 24, who is 2.1 m tall.

10 The volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius. Find, correct to one decimal place: R

a the volume of a sphere with radius 3.2 cm

b the radius of a sphere with volume 500 m^3.

Foundation Standard Complex

11 The average speed in km/h of a car is given by the formula $S = \frac{D}{T}$, where D is the distance covered in kilometres and T is the time taken in hours. Find, correct to the nearest whole number: R

a the average speed of a car that takes 4.5 hours to travel a distance of 420 km

b the distance travelled, if a car maintains a speed of 87.2 km/h for 5 hours

c the time taken, if a distance of 650 km is covered at a speed of 91 km/h.

12 The cost, C (in dollars), of a hire car is $C = 75 + 2.5d$, where d is the number of kilometres travelled. Calculate: R

a the cost of hiring a car to travel 350 km

b the distance travelled, if the cost is $135.

13 The surface area of a closed cylinder is given by the formula $SA = 2\pi r^2 + 2\pi rh$. Calculate, correct to one decimal place: R

a the surface area of a cylinder with radius 2.1 m and height 3.5 m

b the height of a cylinder with surface area 1255.38 cm^2 and radius 9 cm.

PATH

Changing the subject of a formula 6.06

In the formula $v = u + at$, v is the **subject** of the formula. When the formula is rearranged so that one of the other variables becomes the subject, the process is called **changing the subject of the formula**. To change the subject of a formula, use the same rules as for solving an equation. The answer is not a number but an algebraic expression (another formula).

PATH



Video
Changing the subject of a formula



Puzzle
Formulas squaresaw

Worksheet
Changing the subject of a formula

Example 13

a For the formula $v = u + at$, change the subject to a.

b Make x the subject of the formula $d = 3xy - k$.

c If $m = \frac{3d - w}{p}$, change the subject of the formula to w.

Solution

a To make a the subject of the formula, solve it like an equation for a.

$v = u + at$

$u + at = v$ Swapping sides so the new subject a appears on the LHS

$at = v - u$ Subtracting u from both sides

$a = \frac{v - u}{t}$ Dividing both sides by t

Foundation Standard Complex

PATH

b

$d = 3xy - k$

$3xy - k = d$ Swapping sides

$3xy = d + k$ Adding k to both sides

$x = \frac{d+k}{3y}$ Dividing both sides by $3y$

c

$m = \frac{3d-w}{p}$

$mp = 3d - w$ Multiplying both sides by p

$mp + w = 3d$ Adding w to both sides

$w = 3d - mp$ Subtracting mp from both sides

EXERCISE 6.06 ANSWERS ON P. 596

Changing the subject of the formula U F R C

EXAMPLE 13

1 Make y the subject of each formula. R C

a $3x + y = 5$ **b** $x + 4y = 7$ **c** $d - 6y = 10$

d $g + by = x$ **e** $A = M - xy$ **f** $m = \frac{3y}{d}$

g $\frac{2+wy}{5} = x$ **h** $T = 10gy$ **i** $\frac{y-p}{2k} = d$

j $m = 4h - ay$ **k** $\frac{B-ay}{g} = 6$ **l** $V = \frac{3e+4y}{2}$

m $R = \frac{1}{3}Ty$ **n** $B = \frac{4}{wy}$ **o** $h = \frac{6}{y-1}$

2 For the formula $S = ut + \frac{1}{2}at^2$, change the subject to:

a u **b** a

3 The formula for the area of a trapezium is $A = \frac{1}{2}h(a+b)$. Which of the following is the correct formula for h? Select the correct answer **A**, **B**, **C** or **D**. R C

A $h = \frac{A}{2(a+b)}$ **B** $h = \frac{2A}{a} + \frac{2B}{b}$ **C** $h = \frac{2A}{a+b}$ **D** $h = \frac{2A}{2a+b}$

4 What is the correct formula for $P = 2(L + W)$ if L is the subject? Select **A**, **B**, **C** or **D**. R C

A $L = P - 2W$ **B** $L = \frac{P-2W}{2}$ **C** $L = \frac{P}{2} - 2W$ **D** $L = \frac{P}{2W}$

Foundation Standard Complex

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PATH

5 The cost ($\$C$) of a taxi trip is $C = 3.6 + 2.2d$, where d is the distance travelled in kilometres. R C

a Make d the subject of the formula.

b If the cost of a taxi trip was \$31.10, find the number of kilometres travelled.

6 Change the subject of each formula to the variable shown in brackets. R C

a $y = mx + c \ [m]$ **b** $A = lbh \ [b]$ **c** $V = \frac{1}{3}Ah \ [h]$

d $V = \frac{\pi r^2 h}{3} \ [h]$ **e** $3x + k = 4 - 2x \ [x]$ **f** $P = \frac{d+n}{h} \ [n]$

6.06

Power plus ANSWERS ON P. 597

1 Solve each equation.

a $\frac{3(1-y)}{5} = 4 - 2y$ **b** $\frac{50}{2y} = 10$ **c** $\frac{2m+5}{3} - 1 = m + 4$

2 If $y = \frac{ab+cd}{e}$, find d if $y = -12$, $a = -1$, $b = -8$, $c = 7$ and $e = 4$.

3 Randall is 10 years older than Tarni. In 3 years' time, Randall will be twice as old as Tarni. How old are Randall and Tarni now?

4 One-third of a number added to one-sixth of a number is 18. What is the number?

CHAPTER 6 REVIEW

Quiz
Language of maths 6

Puzzle
Equations crossword

Language of maths

brackets	check	equation
expand	factorise	formula
fraction	inverse operation	LHS
lowest common multiple (LCM)	quadratic equation	RHS
solution	solve	square root
subject	substitute	variable

1 What type of equation has 2 as the highest power of x?
Write an example of this type of equation.

2 Write the subject of the formula $C = \frac{5}{9}(F - 32)$.

3 Find the lowest common multiple of 6 and 9.

4 Why is it possible for a **quadratic equation** to have more than one solution?

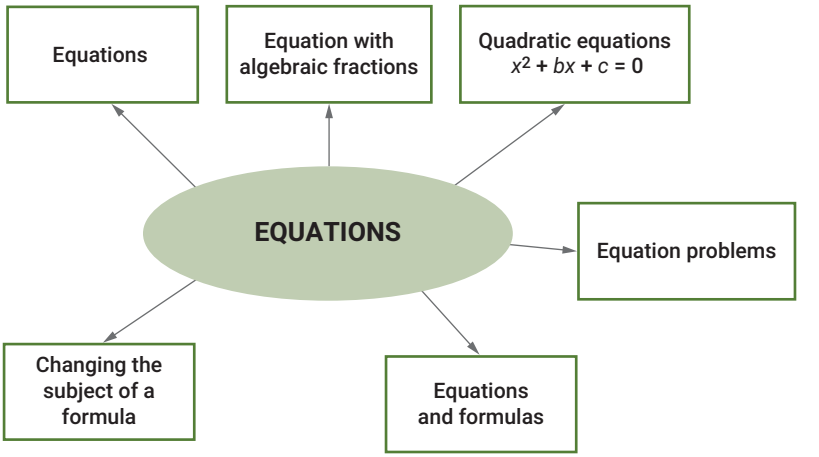
5 When checking the solution to an equation, you need to show that 'LHS = RHS'.
What does that mean?

6 What is the inverse operation to squaring a number?

Worksheet
Mind map: Equations

Topic summary

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



9780170479622

TEST YOURSELF 6

ANSWERS ON P. 597

Quiz
Test yourself 6

6.01

1 Solve each equation.

a $3a + 10 = 43$ **b** $8y + 5 = 2y + 21$

c $2a - 12 = 6a$ **d** $9 - 2y = 5 + 2y$

e $3(m - 2) = 27$ **f** $2(2a + 1) = 3(a + 10)$

g $5(h + 1) + 3(h - 2) = 12$ **h** $4(2y + 1) + 3(1 + 4y) = 20$

PATH

6.02

2 Solve $\frac{2(p-1)}{3} = 4$. Select the correct answer **A**, **B**, **C** or **D**.

A $p = 7$ **B** $p = 11$ **C** $p = 5$ **D** $p = 4$

6.02

3 Solve each equation.

a $\frac{3w+2}{5} = 4$ **b** $\frac{y}{5} = \frac{7}{4}$ **c** $\frac{2a+1}{2} = \frac{3a-1}{4}$

d $\frac{3m+5}{6} = \frac{10-m}{3}$ **e** $\frac{4x-7}{3} = \frac{2x+1}{8}$ **f** $\frac{5-7p}{5} = \frac{4p+3}{2}$

g $\frac{2s}{3} - \frac{s}{6} = 2$ **h** $\frac{x}{10} + \frac{x}{2} = 1$ **i** $\frac{5k}{4} - \frac{2k}{5} = 7$

6.02

4 Solve $\frac{4d}{3} + \frac{2d}{5} = 7$. Select the correct answer **A**, **B**, **C** or **D**.

A $d = 9\frac{1}{3}$ **B** $d = 4$ **C** $d = 4\frac{1}{26}$ **D** $d = 8$

6.03

5 Solve each quadratic equation.

a $y^2 = 4$ **b** $p^2 - 100 = 0$ **c** $4x^2 = 40$

d $3m^2 - 3 = 0$ **e** $\frac{2w^2}{5} = 20$ **f** $x^2 + 8x + 7 = 0$

g $h^2 - 8h - 9 = 0$ **h** $u^2 + 4u - 77 = 0$ **i** $k^2 + 5k = 0$

j $m^2 - 2m = 0$ **k** $b^2 + 20b + 100 = 0$ **l** $w^2 = 9w$

6.04

6 The sum of 4 consecutive numbers is 374. Find the 4 numbers.

6.04

7 Corinna is 3 times older than her daughter, Isla. Five years ago, she was 4 times Isla's age. What are their ages?

TEST YOURSELF 6

Foundation | Standard | Complex

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6.05

8 The braking distance (in metres) of a bicycle travelling at a speed of v metres/second is $d = \frac{v(v+1)}{2}$. Calculate the braking distance when the speed of the bicycle is 15 m/s.

9 The volume of a pyramid is given by the formula $V = \frac{1}{3}Ah$, where A is the area of the base and h is the perpendicular height of the pyramid. Find:

a the volume of a pyramid with a base area of 48 mm^2 and a perpendicular height of 10 mm

b the base area of a pyramid with a volume of 500 m^3 and a perpendicular height of 5 m.

PATH

6.06

10 Make a the subject of each formula.

a $y = ax + b$ **b** $P = \sqrt{\frac{a}{m}}$ **c** $M(1 + a) = 1 - a$

Foundation Standard Complex

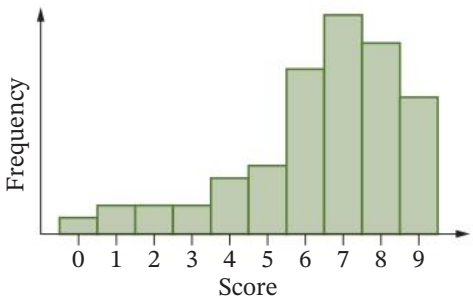
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PRACTICE SET 2 ANSWERS ON P. 597

1 For each statistical distribution: 5.01

i describe the shape **ii** identify any outliers and clusters.

a



b

Stem	Leaf
3	0 1 2
4	1 3 4 4 5 6
5	0 4 5 7 8
6	3 7 8
7	0 1
8	4
9	8

2 Simplify each expression. 4.01

a $(3nm^2)^4$ **b** $5p^4q^2 \times 4qp^5$ **c** $\frac{32a^5b^6}{8ab^3}$

d $(2h)^0$ **e** 3^{-3} **f** $\left(\frac{5}{2}\right)^3$

3 Solve each equation. 6.01

a $5b + 12 = 37$ **b** $25y - 6 = 7y + 21$

c $3(m - 2) = 27$ **d** $2(2a + 1) = 3(10 - 4a)$

4 Find the interquartile range of this set of data. 5.02

24 15 23 28 20 20 18 30 21 18

5 The maximum daily temperatures (°C) in Perth for the first 14 days in April were: 5.05

27.1 24.1 23.7 24.6 26.5 31.8 33.6 32.6 34.0 35.6 39.5 26.2 26.7 24.9

a Find the range and interquartile range for the temperatures.

b Find the five-number summary for the data.

c Draw a boxplot for the data.

d Identify any outliers.

6 Simplify each expression. PATH 4.01

a $(3y)^{-2}$ **b** $3y^{-2}$ **c** $\left(\frac{4x^6y^6}{x^3y^2}\right)^3$

d $(2h^4k^5)^3 \times 3h^2k$ **e** $48v^5w^8 \div (-4vw^2)^2$ **f** $\left(\frac{36a^4b^2}{48ab^4}\right)^{-1}$

Foundation Standard Complex

4.02

7 Simplify each expression.

a $\frac{3t}{8}+\frac{t}{8}$ **b** $\frac{4x}{15}-\frac{x}{3}$ **c** $\frac{3g}{4}-\frac{2g}{5}$ **d** $\frac{6k}{5}-\frac{k}{2}$

PATH

4.03

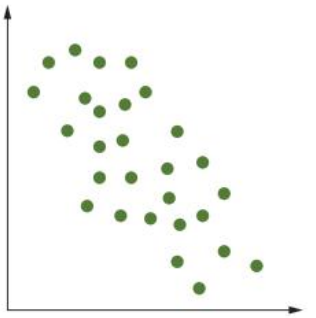
8 Simplify each expression.

a $\frac{x}{7}\times\frac{6}{y}$ **b** $\frac{m^3}{8}\div\frac{m^2}{16}$ **c** $\frac{21e}{10d^2}\times\frac{15d}{7e}$ **d** $\frac{12pv}{15}\div\frac{4p}{10v}$

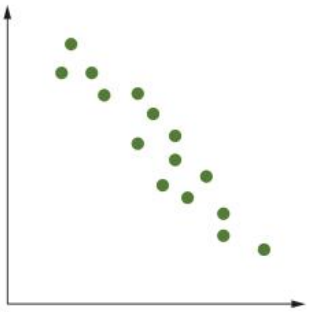
5.08

9 Describe the strength and direction of the association shown in each scatterplot.

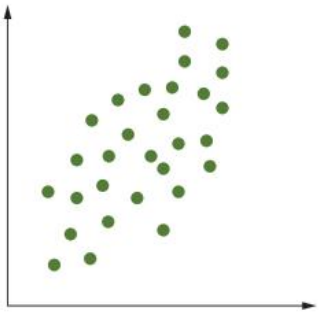
a



b



c



6.02

10 Solve each equation.

a $\frac{2a+3}{4}=5$ **b** $\frac{k}{7}=\frac{5}{4}$

PATH

c $\frac{2w}{5}-\frac{w}{10}=3$ **d** $\frac{1}{4}(3g+2)=\frac{1}{2}(2g-1)$

4.04

11 Expand each expression.

a $7(y-9)$ **b** $-n(8n+m)$

c $-4(5w-6)$ **d** $4a^2b(6a+7b)$

5.06

12 The masses of students (in kg) was recorded before and after an exercise program.

Before program	142	141	90	96	111	127	85	80	91
After program	99	101	77	81	85	96	79	77	70

a Find the five-number summary before and after the exercise program.

b Construct parallel boxplots for the 2 sets of data.

c Find the range and interquartile range for student weights:

i before the program **ii** after the program.

d Are there significant differences between the 2 sets of data? Justify your answer.

Foundation Standard Complex

9780170479622

13 Expand and simplify each expression. 4.04

a $3ab(b - 6a) - 6a^2b$ **b** $4(9 - 3r) - 6r(8 - 3r)$

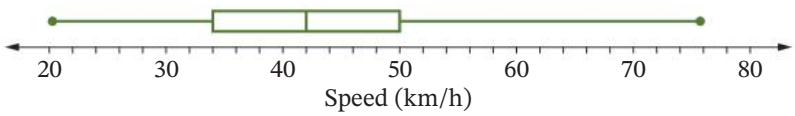
14 Factorise each expression. 4.04

a $7w + 42$ **b** $p^2 - 25p$

c $-4n - 44$ **d** $3(7a - 2) - 5a(7a - 2)$

15 The speeds of cars (in km/h) were monitored during 2:30 to 4:00 p.m. in a school zone. The results are shown in the stem-and-leaf plot and boxplot below. 5.07

Stem	Leaf
2	0 3 7 9
3	0 2 2 3 5 6 8 8 9
4	0 0 1 3 5 5 7 8 9 9
5	1 1 4 4 6
6	0 1
7	6



a Which plot gives the better indication of:

i skewness? **ii** clustering and outliers?

b Find the median.

c Find the interquartile range.

16 Solve each quadratic equation, correct to one decimal place where necessary. 6.03

a $p^2 - 144 = 0$ **b** $9y^2 = 36$ **c** $5x^2 - 5 = 0$

d $\frac{t^2}{4} = 25$ **e** $3x^2 - 70 = 0$ **f** $\frac{3w^2}{5} = 23$

17 Expand and simplify each expression. 4.05

a $(q + 3)(2q + 9)$ **b** $(3f + 8)(f - 7)$

c $(4g - 7)(5g - 3)$ **d** $(3x - 7)(3x + 7)$

18 Factorise each quadratic expression. PATH 4.06

a $r^2 + 11r + 24$ **b** $y^2 - 31y + 30$

c $x^2 + 9x - 36$ **d** $t^2 - t - 72$

5.08

19 10 boxes containing 60 apples each were placed in cold storage for different periods. After each period of storage, a box was withdrawn and the number of good apples was counted.

Weeks in storage (W)	2	3	5	6	7	8	9	10	12	14
Number of good apples (N)	56	53	48	48	50	39	36	30	34	26

a Which is the independent variable? Give reasons.

b Graph this data on a scatterplot.

c Describe the association between the weeks in storage and the number of good apples.

d Describe the strength and direction of the association between the variables W and N.

PATH

6.03

20 Solve each quadratic equation.

a $x^2 + 8x + 7 = 0$

b $h^2 - 8h - 9 = 0$

c $u^2 + 4u - 77 = 0$

d $w^2 = 9w$

6.04

21 Solve each problem using an equation.

a The sum of 3 consecutive numbers is 63. Find the 3 numbers.

b One angle in a triangle is double the size of the smallest angle, and the third angle in the triangle is 60° more than 3 times the size of the smallest angle.
Find the size of each angle.

6.05

22 The volume of a pyramid is given by the formula $V = \frac{1}{3}Ah$, where A is the area of the base and h is the perpendicular height of the pyramid. Find:

a the volume of a pyramid if the base area is 96 mm^2 and the perpendicular height is 15 mm

b the base area of a pyramid if its volume is 600 cm^3 and its perpendicular height is 12 cm

c the perpendicular height of a pyramid if its volume is 48 m^3 and its base area is 40 m^2.

5.10

23 A researcher surveys every 10th person walking past the Sydney Opera House on Monday morning to investigate public opinion about climate change.

a What type of sampling has been conducted?

b Why is this sample biased?

c What could be done to remove bias from the sample?

5.04

24 The test results for 2 groups of students are shown.

Girls: 78 73 58 79 60 46 71 64 37 79 55 59 87 68 27

Boys: 70 39 80 73 61 68 84 60 73 60 54 90 69 63 59

a Find correct to one decimal place the mean and standard deviation of each group.

b Which group performed better in the exam? Give reasons.

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PATH
6.05
5.09

25 Change the subject of each formula as indicated

a $D = \frac{M}{V}$ to M **b** $y = mx + c$ to m **c** $t = \sqrt{\frac{n}{3}}$ to n

26 The heights and shoe sizes of 10 people were measured.

Weight, W (kg)	60	78	82	88	90	94	75	66	58	65
Height, H (cm)	142	168	184	170	180	178	170	158	150	146

a Graph the points on a scatterplot and construct a line of best fit.

b Find the equation of the line of best fit.

c Use the graph to extrapolate the weight of a person with a height 130 cm.

d Use the graph to interpolate the height of a person who weighs 80 kg.



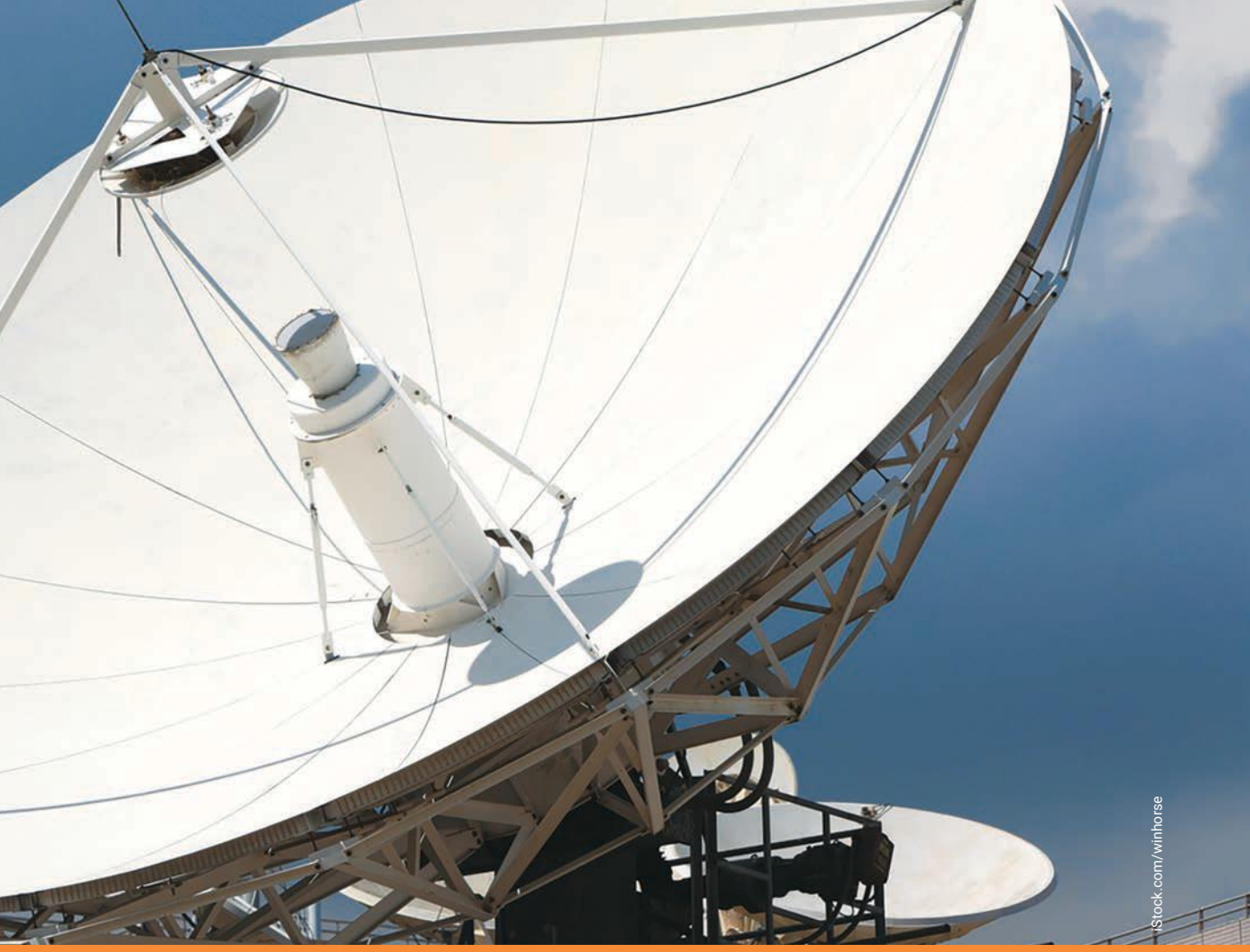
7

NUMBER AND ALGEBRA

GRAPHING CURVES

When an object is thrown upwards, its path is a curve called a parabola. The shape and length of the path will depend on the initial speed of the object. Furthermore, car headlights and satellite dishes all use mirrors or reflectors that have the shape of a parabola.

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Chapter outline

		Working mathematically				
7.01	Direct variation*	U	F	PS	R	C
7.02	Inverse variation*	U	F	PS	R	C
7.03	Conversion graphs*	U	F		R	C
7.04	Graphing quadratic equations $y = ax^2 + c$	U	F		R	C
7.05	Applying quadratic equations	U	F		R	C
7.06	Graphing hyperbolas $y = \frac{k}{x} + c$*	U	F		R	C
7.07	Graphing exponential equations $y = a^x + c$	U	F		R	C
7.08	Exponential growth and decay	U	F		R	C
7.09	Identifying graphs	U	F		R	C

*PATH

Note: See 14.05 for Graphing quadratic equations $y = ax^2 + bx + c$

Wordbank

asymptote A line that a curve gets very close to but never touches, for example, the x-axis is an asymptote of the exponential curve

direct variation or **direct proportion** A relationship between 2 variables of the form $y = kx$, where k is a constant; for example, if $y = 8.5x$, then y is directly proportional to x

exponential equation An equation involving a variable as a power, such as $y = 3^x$, whose graph is an exponential curve

inverse variation or **inverse proportion** A relationship between 2 variables of the form $y = \frac{k}{x}$, where k is a constant; for example, if $y = \frac{50}{x}$, then y is inversely proportional to x

parabola A U-shaped curve that is the graph of a quadratic equation

quadratic equation An equation involving a variable squared (power of 2), such as $y = 3x^2 - 6$, whose graph is a curve called a parabola

Quiz
Wordbank 7

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Videos (8):
- **7.01** Direct proportion • Direct linear variation
- **7.02** Inverse variation • Inverse proportion
- **7.04** The parabola $y = ax^2 + c$
- **7.05** Applying quadratic functions
- **7.06** Graphing hyperbolas
- **7.07** The exponential curve

***Twig* videos (2):**
- **7.02** The heartbeat formula
- **7.07** The emperor's chessboard

***PhET* interactive (1):**
- **7.04** Graphing quadratics

Quizzes (5):
- Wordbank 7
- SkillCheck 7
- Mental skills 7
- Language of maths 7
- Test yourself 7

Worksheets (7):
- **7.02** Direct and inverse proportion • Variation problems
- **7.03** Currency conversion graph
- **7.04** Graphing parabolas
- **7.06** Graphing hyperbolas
- **7.07** Graphing exponentials
- **Mind map** Graphing curves

Puzzles (2):
- **7.09** Matching graphs
- **Language of maths** Graphing curves crossword

Technology (2):
- **7.04** Investigating parabolas 1
- **7.04, 7.06** Curve sketcher

Spreadsheets (2):
- **7.04** Investigating parabolas 1
- **7.04, 7.06** Curve sketcher

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

In this chapter you will:

- (PATH) solve problems involving direct and inverse variation
- (PATH) read and interpret conversion graphs
- graph parabolas of the form $y = ax^2$ and $y = ax^2 + c$
- graph exponential curves of the form $y = a^x$
- solve problems that can be modelled by quadratic and exponential equations
- (PATH) graph hyperbolas of the form $y = \frac{k}{x}$
- match graphs to their equations
- use technology to find points of intersection between a line and a curve

Quiz
SkillCheck 7

SkillCheck ANSWERS ON P. 598

1 If $A = 2x^2 - 3$, find A if:

a $x = 1$ **b** $x = 4$ **c** $x = 0$ **d** $x = -6$

2 If $y = 5^x$, find y if:

a $x = 4$ **b** $x = 5$ **c** $x = 0$ **d** $x = -2$

3 If $S = \frac{120}{t}$, find S if:

a $t = 10$ **b** $t = 35$ **c** $t = 2.5$ **d** $t = 1.8$

Direct variation 7.01

Two variables are **directly proportional** to each other if one variable is a constant multiple of the other; when one variable changes, the other one changes by the same factor.
This relationship is called **direct variation** or **direct proportion**.

PATH

7.01

Videos
Direct proportion
Direct linear variation

Direct variation

If y is **directly proportional** to x, written as '$y \propto x$', then $y = kx$, where k is a constant (number) called the **constant of variation** or **constant of proportionality**.

- A direct linear relationship exists between x and y
- If x increases (or decreases), y increases (or decreases)
- If x is doubled (or halved), y is doubled (or halved)
- Another way of saying 'y is directly proportional to x' is 'y **varies directly** as x'
- The graph of direct variation is a straight line going through (0, 0) with gradient k

Example 1

The distance (d) in metres travelled by a cyclist on a road bike is directly proportional to the number of rotations (r) of its wheels. After 750 rotations, a distance of 1600 m is travelled.

a What distance will be travelled after 1200 rotations?

b How many full rotations will be needed to cover 30 km?


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Solution

a d is directly proportional to r, or $d \propto r$.

$\therefore d = kr$ Forming a variation equation

To find k, substitute the information given for r and d.

When $r = 750$, $d = 1600$:

$1600 = k \times 750$ Finding k

$k = \frac{1600}{750}$

$k = 2.133...$

$\therefore d = 2.133... r$ Rewriting the equation $d = kr$

PATH

When $r = 1200$, Solving the problem

$$d = 2.133... \times 1200$$
$$= 2560 \text{ m}$$

After 1200 rotations, the distance travelled will be 2560 m.

b When $d = 30 \text{ km} = 30\,000 \text{ m}$ Distance must be in metres

$$30\,000 = 2.133... \times r$$
$$r = \frac{30\,000}{2.133...}$$
$$= 14\,062.5$$
$$\approx 14\,063 \text{ rotations}$$ Rounding up for full rotations

For a distance of 30 km, there will be 14 063 rotations.

Solving direct variation problems

1 Identify the 2 variables (say x and y) and form a variation equation, $y = kx$
2 Substitute values for x and y to find k, the constant of variation
3 Rewrite $y = kx$ using the value of k
4 Substitute a value for x or y into $y = kx$ to solve the problem

Example 2

M varies directly as n. If when $n = 6$, $M = 103.8$, find M when $n = 14.2$.

Solution

$M = kn$ Forming a proportion equation

To find k, substitute $n = 6$ and $M = 103.8$.

$103.8 = k \times 6$ Finding k

$$6k = 103.8$$
$$k = \frac{103.8}{6}$$
$$= 17.3$$

$\therefore M = 17.3n$ Rewriting the equation $M = kn$

Substitute $n = 14.2$ to find M. Solving the problem

$$M = 17.3 \times 14.2$$
$$= 245.66$$

EXERCISE 7.01 ANSWERS ON P. 598

PATH

Direct variation U F PS R C

EXAMPLE 1

7.01

1 Match each statement with its variation equation (k is the constant of variation). C

a The distance, D, travelled is directly proportional to the time, T.

b The wage, W, earned is directly proportional to the hours, h, worked.

c The wedding cost, C, varies directly as the number, n, of guests.

d The interest, I, earned varies directly as the size of the deposit, D

A $C = kn$ **B** $I = kD$ **C** $D = kT$ **D** $W = kh$

2 The distance, D, travelled by Connor, a marathon runner, varies directly as time, T. PS R C

Time, T (min)	1	2	3
Distance, D (m)	190	380	570

a Find the constant of variation and write the equation for D.

b How far, in kilometres, will Connor run in:

i 20 minutes? **ii** 45 minutes?

c How long would it take Connor to run 12.35 kilometres? Answer in hours and minutes.

3 Mehta's earnings for working a shift at the local nursery are directly proportional to the number of hours she works. Yesterday, she earned $304.30 for working an 8.5-hour shift. PS R C

a If Mehta's earnings are represented by E, and the number of hours worked is represented by h, find the constant of variation and write an equation for E.

b How much will she earn for working a 7-hour shift?

c How many hours did she work today if she earned $196.90 for the shift?



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Foundation Standard Complex

PATH

4 The amount of interest, I, earned for one year on an investment account varies directly as the size of the deposit, D. PS R C

a If Caterina earns \$826.50 interest on an investment of \$14 250, find the variation equation for I, including the constant of variation.

b Hence, how much will she earn on an investment of \$9000?

c If she doubles the size of her investment in **b**, how much will she earn in interest?

EXAMPLE 2

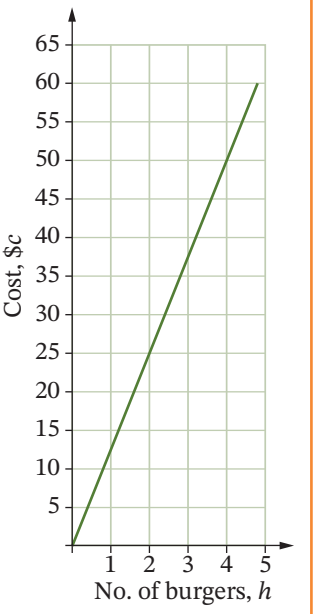
5 S varies directly as t. If when $t = 14$, $S = 106.4$, what is the value of S when $t = 0.3$? Select the correct answer **A**, **B**, **C** or **D**. R C

A 2.28 **B** 27.72 **C** 36.12 **D** 446.88

6 Find the linear formula for b in terms of a for this table of values.

a	4	8	12	16	20
b	10	20	30	40	50

7 The line graph shows that the cost of hamburgers purchased from the local takeaway store depends directly on the number of burgers purchased. PS R C



a Copy this table and use the graph to complete it.

No. of burgers, h	1	2	3
Cost, c (\$)			

b Find the variation equation to represent the relationship between the cost (\$$c$) and the number of burgers (h).

c If Kendall buys 6 hamburgers, what is the total cost of the hamburgers?

d The total cost of one order of hamburgers is \$137.50. How many hamburgers were ordered?

e Find the gradient of the line. How is it related to the constant of variation?

8 K varies directly as L. If $L = 9.5$ when $K = 1045$, what is the value of K when $L = 1.65$? Select **A**, **B**, **C** or **D**. R C

A 0.015 **B** 93.7 **C** 181.5 **D** 1708.575

9 A linear relationship exists between the mass of a car (m kg) and its rate of fuel consumption (F L/100 km). PS R C

a Find the variation equation for F if a 1500 kg car uses fuel at a rate of 9 L/100 km.

b Find the fuel consumption of a 2500 kg car.

10 For an object that is cooling, the drop in temperature varies directly as time. If the temperature drops 8°C in 5 minutes, how long would it take to drop 10°C? Select **A**, **B**, **C** or **D**. PS R C

A 6.25 min **B** 7 min **C** 12.8 min **D** 16 min

11 The weight of an astronaut on Mars is proportional to his weight on Earth. A 72 kg astronaut weighs 27.4 kg on Mars. PS R C

a Calculate how much a 60 kg astronaut weighs on Mars, correct to 1 decimal place.

b If an astronaut weighs 32 kg on Mars, calculate his weight on Earth, correct to 1 decimal place.



Esteban De Armas/Shutterstock.com

PATH

7.02

Inverse variation 7.02

Two variables are **inversely proportional** to each other if when one variable increases, the other one decreases by the same factor. This relationship is called **inverse variation** or **inverse proportion**.

The table below shows the different speeds of a car (s km/h), and the time it takes to travel 100 km (t min). As the speed increases, the time taken decreases.

Speed (s km/h)	50	60	80	100
Time (t min)	120	100	75	60

Inverse variation

If y is **inversely proportional** to x, written as '$y \propto \frac{1}{x}$', then $y = \frac{k}{x}$, where k is a constant (number) called the **constant of variation** or **constant of proportionality**.

- If x increases, y decreases ('inverse' means 'opposite')
- If x decreases, y increases
- If x is doubled, y is halved
- If x is halved, y is doubled
- Another way of saying 'y is inversely proportional to x' is 'y **varies inversely** as x'

PATH

Worksheets
Direct and inverse proportion

Variation problems

Videos
Inverse variation

The heartbeat formula

Foundation Standard Complex

PATH

Example 3

The time (t) in minutes taken by a car to travel 100 km is inversely proportional to the speed (s km/h) of the car, as shown in the table above. At 50 km/h, the time taken is 120 minutes.

a Find the constant of variation and the inverse variation equation for t.

b How long did the car take to travel 100 km at:

i 40 km/h? **ii** 110 km/h?

c Find the car's speed if it took 45 minutes to travel 100 km.

Solution

a t is inversely proportional to s.

$\therefore t = \frac{k}{s}$ Forming an inverse variation equation

To find k, substitute the information given for s and t.

When $s = 50$, $t = 120$: Finding k, the constant of variation

$120 = \frac{k}{50}$

$k = 120 \times 50$

$= 6000$

The constant of variation is 6000.

$\therefore t = \frac{6000}{s}$ Rewriting the equation $t = \frac{k}{s}$

b **i** When $s = 40$, $t = \frac{6000}{40} = 150$ min

At 40 km/h, the trip takes 150 min (or 2 h 30 min).

ii When $s = 110$,

$t = \frac{6000}{110}$

$= 54.5454...$

≈ 55 min

At 110 km/h, the trip takes 55 min.

c When $t = 45$,

$45 = \frac{6000}{s}$

$45s = 6000$

$s = \frac{6000}{45}$

$= 133\frac{1}{3}$ km/h

For a travel time of 45 min, the speed must be $133\frac{1}{3}$ km /h.

Solving inverse variation problems

1 Identify the 2 variables (say x and y) and form a variation equation, $y = \frac{k}{x}$

2 Substitute values for x and y to find k, the constant of variation

3 Rewrite $y = \frac{k}{x}$ using the value of k

4 Substitute a value for x or y into $y = \frac{k}{x}$ to solve the problem

Example 4

The temperature, T (in degrees Celsius), of the air is inversely proportional to the height, h (in metres), above sea level. At 600 m above sea level, the temperature is 8°C.

a What is the temperature at 1000 m above sea level?

b Graph the relationship between temperature and height above sea level, for heights between 0 and 5000.



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PATH

Video
Inverse proportion

7.02

Solution

a T is inversely proportional to h.

$T = \frac{k}{h}$ Forming a variation equation

Substitute $h = 600$ and $T = 8$ to find k.

$8 = \frac{k}{600}$ Finding k

$k = 8 \times 600$

$= 4800$

$\therefore T = \frac{4800}{h}$ Rewriting the equation $T = \frac{k}{h}$

When $h = 1000$, Solving the problem

$T = \frac{4800}{1000} = 4.8°C$

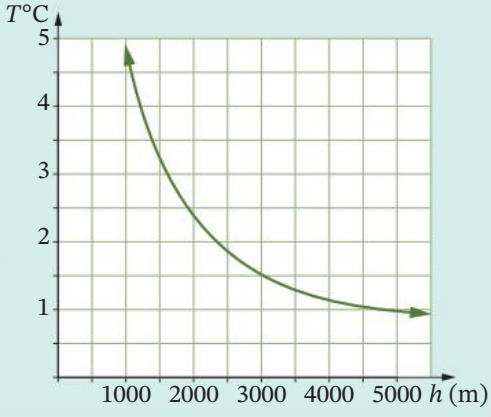
The temperature at a height of 1000 metres above sea level is 4.8°C.

b Draw a table of values for $T = \frac{4800}{h}$

h	1000	2000	3000	4000	5000
T	4.8	2.4	1.6	1.2	0.96

Note that as h increases, T decreases.

This graph is called a **hyperbola**.



PATH

Example 5

P varies inversely as q. If when $q = 4$, $P = 38$, find P when $q = 5$.

Solution

$P = \frac{k}{q}$ — Forming a variation equation

Substitute $q = 4$, $P = 38$

$38 = \frac{k}{4}$ — Finding k

$k = 38 \times 4 = 152$

$\therefore P = \frac{152}{q}$ — Rewriting the equation $P = \frac{k}{q}$

Substitute $q = 5$ to find P. — Solving the problem

$P = \frac{152}{q} = 30.4$

EXERCISE 7.02 ANSWERS ON P. 598

Inverse variation U F PS R C

1 Match each statement with its inverse variation equation (k is the constant of variation). C

a The travel time, T, is inversely proportional to the speed, S.

b The temperature, T, above sea level varies inversely as the altitude, h.

c The cost, C, per person of hiring a bus varies inversely as the number, n, of people.

d The time, T, taken to move house is inversely proportional to the number, n, of helpers.

A $T = \frac{k}{n}$ **B** $T = \frac{k}{S}$ **C** $T = \frac{k}{h}$ **D** $C = \frac{k}{n}$

EXAMPLE 3

2 The time taken, T hours, to travel from Sydney to Melbourne varies inversely as its average speed, s km/h. PS R C

a If it takes 11.5 hours at an average speed of 80 km/h, find the constant of variation and the variation equation for T.

b If the average speed is increased to 90 km/h, how long will the journey take? Answer in hours and minutes.

c Find the average speed needed to complete the trip in 10 hours.

EXAMPLE 4

3 Air temperature, T (in degrees Celsius), varies inversely as its height, h (in metres), above sea level. At 150 m above sea level, the temperature is 30°C. PS R C

a Find the constant of variation and write a variation equation for T.

b What is the temperature at:

i 300 m above sea level? **ii** 2500 m above sea level?

c What is the height above sea level when the temperature is:

i 8°C? **ii** 22.5°C?

d Graph the relationship between temperature and height above sea level. Use T on the vertical axis and h on the horizontal axis with $h = 0, 500, 1000, 1500, ..., 3000$.

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PATH

7.02

4 The maximum number of diners, N, in Yen's restaurant is inversely proportional to the amount of floor space, S m², allocated to each diner. If 1.4 m² is allowed per diner, the restaurant can have 80 diners. PS R C

a Write a variation equation for N.

b How many diners could the restaurant contain if only 1.1 m² was allocated per person?

c How much space (correct to one decimal place) is allocated to each diner if the restaurant has a maximum capacity of 55 diners?

d During the COVID-19 pandemic of 2020, social distancing measures introduced to minimise the spread of the virus allocated 4 m² to each diner. How many diners could Yen's restaurant serve under this restriction?

5 The rate of vibration of a string varies inversely as its length. A string that is 330 mm long vibrates at 196 Hz (hertz). What length of string will vibrate at 500 Hz? Select the correct answer **A**, **B**, **C** or **D**. PS R C

A 129 mm **B** 297 mm **C** 323 mm **D** 842 mm

6 Which equation represents this table of values? Select **A**, **B**, **C** or **D**. R

x	2	5	8	10
y	2.5	1	0.625	0.5

A $y = \frac{10}{x}$ **B** $y = \frac{5}{x}$ **C** $y = \frac{2.5}{x}$ **D** $y = \frac{1}{x}$

7 K is inversely proportional to L. If $L = 2$ when $K = 7$, find K when $L = 15$. R

EXAMPLE 5

8 Ankit believes that at a train station, the number of people waiting on the platform is inversely proportional to the time until the next train arrives. According to his model, when there are 15 people waiting, the train will arrive in 4 minutes. PS R C

a When will the train arrive if there are 25 people waiting?

b How many people are waiting at the station 12 minutes before the train arrives?



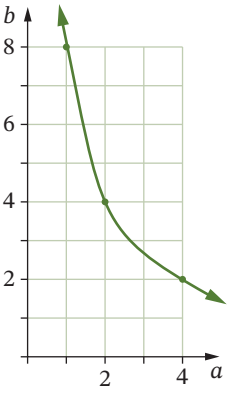
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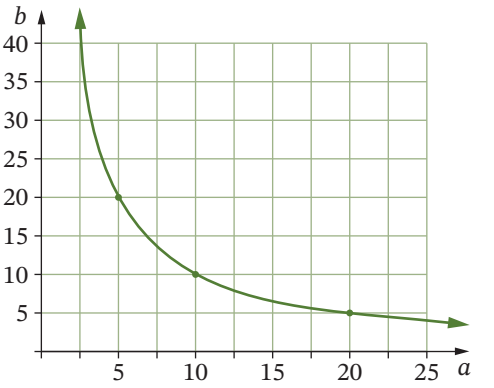
PATH

9 Each graph shows an inverse relationship between a and b. Find each variation equation. R C

a



b



10 The time taken to travel from Sydney to Brisbane varies inversely as the average speed. A train travelling at an average speed of 70 km/h takes 14 h 7 min. How long would the same trip take if the average speed of the train was: PS R C

a 50 km/h? **b** 75 km/h?

11 The amount of time it takes Sonia to paint her house is inversely proportional to the number of friends she has to help her. When she has 5 friends helping, the job takes 4 days. PS R C

a How long will it take if she has 3 friends helping?

b How many friends must she have to help her to paint the house in 2 days?

12 The microwave oven cooking time (t seconds) for a meal varies inversely as the oven's power rating (W watts). For an 800 W microwave oven, the cooking time is 300 seconds. PS R C

a Find the variation equation for t.

b What is the cooking time for the meal in a 1200 W microwave oven?

c Find the power rating (to the nearest 100 W) of the microwave oven if the cooking time is:

i 6 min 20 s **ii** 2 min 40 s

13 The pressure (P atmospheres) of an amount of gas varies inversely as its volume (V litres). For a particular gas, its pressure is 5.2 atm if its volume is 4.5 L. PS R C

a Find the variation equation for P.

b Calculate the volume of the gas when its pressure is 2.8 atm.

c If its volume increases to 8 L, what is its pressure?

Conversion graphs

7.03

A **conversion graph** is used to convert from one unit to another; for example, miles to kilometres, or Australian dollars to US dollars. It usually contains one straight line that begins at the origin (0, 0), so it has an equation in the form $y = kx$ and is an example of **direct variation**.

PATH

Worksheet
Currency conversion graph

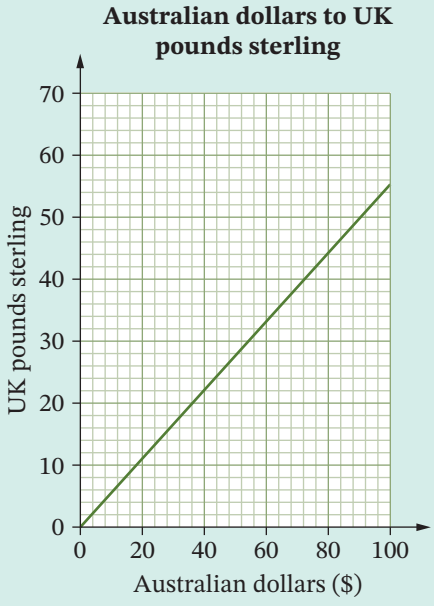
Example 6

Exchange rates change daily, but suppose the exchange rate between the Australian dollar and the UK pound sterling is \$A1 = £0.553, then \$A100 = £55.30 sterling. These values are used to draw this conversion graph.

Use the graph to convert:

a \$A50 to pounds

b £10 to Australian dollars.



Solution

Reading from the graph:

a \$A50 ≈ £28

b £10 ≈ \$A18

PATH

EXERCISE 7.03 ANSWERS ON P. 599

Conversion graphs U F R C

EXAMPLE 6

1 Use the graph on the previous page for this question.

a Convert to pounds:

i $A40

ii $A88

b Convert to Australian dollars:

i £18

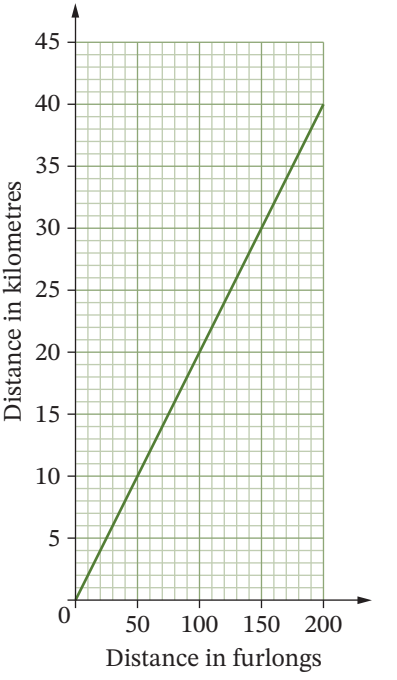
ii £60

c In 2015, $A1 = £0.50. R C

i How much less was $A40 worth in UK pounds sterling in 2015 than it is using this more recent conversion graph?

ii How much more was £60 worth in Australian dollars in 2015 for UK tourists visiting Australia, than it is using this more recent conversion graph?

2 The furlong is an imperial measure once used to measure length. This conversion graph shows distances in furlongs converted to kilometres.



a Convert to kilometres:

i 10 furlongs

ii 100 furlongs

iii 170 furlongs

b Convert to furlongs:

i 10 km

ii 25 km

iii 36 km

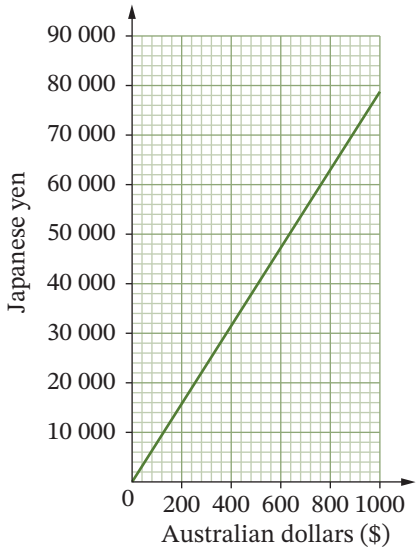
c Use an answer from part **a** to convert 300 furlongs to kilometres. R

d Use an answer from part **b** to convert 100 kilometres to furlongs. R

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3 This graph converts between Australian dollars and Japanese yen (¥).



a Convert to Japanese yen:

i $200

ii $800

iii $1000

b Convert to Australian dollars.

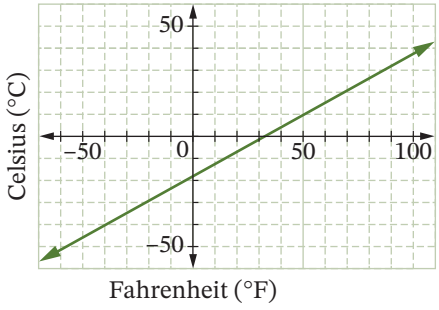
i ¥20 000

ii ¥60 000

iii ¥72 000

4 This graph converts temperatures between degrees Fahrenheit and degrees Celsius. Convert:

Degrees Fahrenheit to degrees Celsius



a 0°F to °C

b 50°F to °C

c 80°F to °C

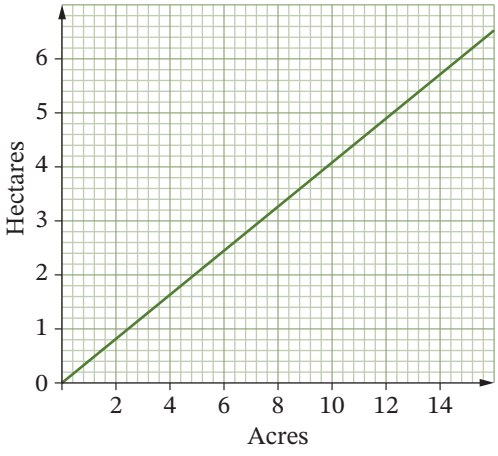
d 0°C to °F

e −10°C to °F

f 30°C to °F

5 This conversion graph converts between acres and hectares. The acre is an imperial measure of land area.

Converting acres to hectares



a Use the graph to convert 12 acres to hectares.

b A garden has an area of 5 acres. What is this area in hectares?

c Use the graph to convert 4.4 hectares to acres.

d Mr Ferguson has a property with an area of 5 hectares. How big is this in acres?

e A rectangular playing field measures 250 m by 128 m.

i What is the area of the field in square metres?

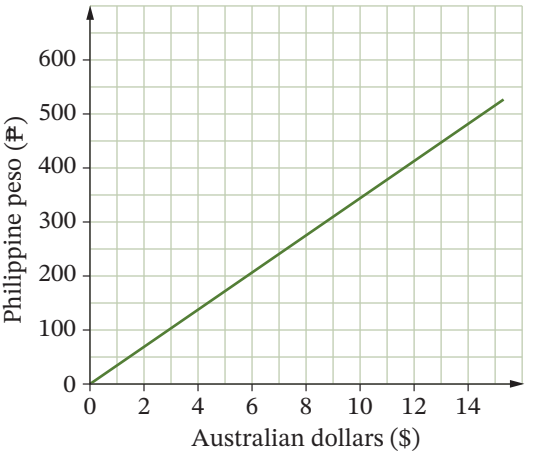
ii What is the area of the field in hectares?

iii What is the area of the field in acres?

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6 This graph is used to convert Australian dollars to Philippine pesos (₱).



a Change each Australian dollar amount into Philippine pesos.

i $15

ii $6

iii $9

b Change ₱350 to Australian dollars.

c How many Australian dollars would you receive for ₱200?

d Estimate the number of pesos you should get for $120. R

7.04 Graphing quadratic equations $y = ax^2 + c$

Worksheet Graphing parabolas

Technology Investigating parabolas 1

Curve sketcher

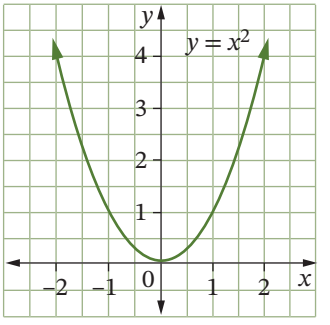
Spreadsheets Investigating parabolas 1

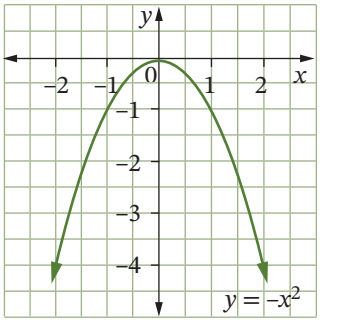
Curve sketcher

Interactive Graphing quadratics

An equation in which the highest power of the variable is 2 is called a **quadratic equation**, for example, $y = 2x^2 - 5$, $y = x^2 + 7x + 12$ and $y = -5x^2$. The graph of a quadratic equation is a smooth U-shaped curve called a **parabola**.

The graphs of $y = x^2$ and $y = -x^2$ are shown below.





Concave up (looks like a smile ☺)

Minimum value of the parabola is $y = 0$.

Concave down (looks like a frown ☹)

Maximum value of the parabola is $y = 0$.

Both parabolas have one axis of symmetry, the y-axis, and a turning point, called the **vertex**, at the origin (0, 0).

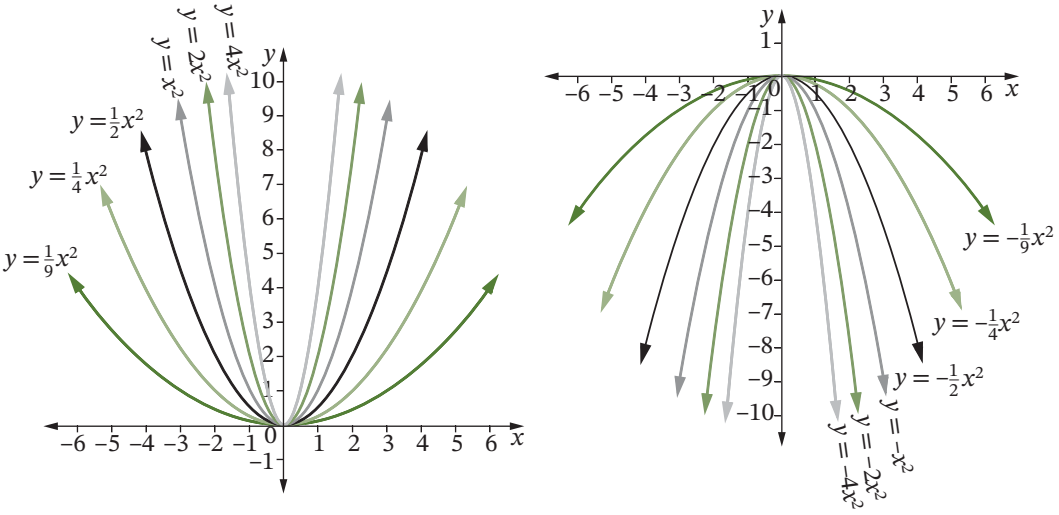
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The graph of $y = ax^2$

For the graph of $y = ax^2$, where a is a constant (number), the size of a (the **coefficient** of x^2) affects whether the parabola is 'wide' or 'narrow'.

As the size of a increases, the parabola becomes 'narrower' and as the size of a decreases, the parabola 'widens'. If a is negative, then the parabola is concave down.



The graph of $y = ax^2$ is a parabola with the y-axis as its **axis** (of symmetry) and $(0, 0)$ as its **vertex**.



Neale Cousland/Shutterstock.com

The graph of $y = ax^2 + c$

For the graph of $y = ax^2 + c$, where a and c are constants, the effect of c is to move the parabola up or down from the origin. Also, c is the y-intercept of the parabola.

Video
The parabola
$y = ax^2 + c$

Example 7

Graph each set of quadratic equations on the same number plane, showing the vertex of each parabola.

a $y = x^2, y = x^2 - 4, y = x^2 + 2$

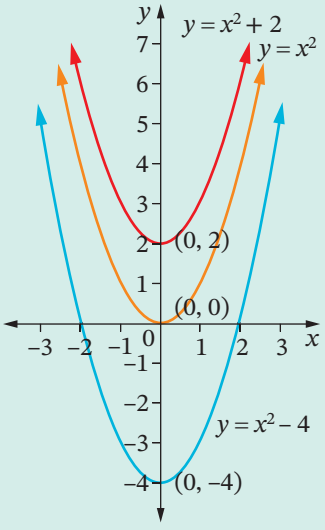
b $y = -x^2, y = -x^2 - 4, y = -x^2 + 5$

Solution

a First draw the graph of $y = x^2$. Its vertex is at (0, 0).

The graph of $y = x^2 - 4$ is like that of $y = x^2$, but moved 4 units down. Its vertex is at (0, −4).

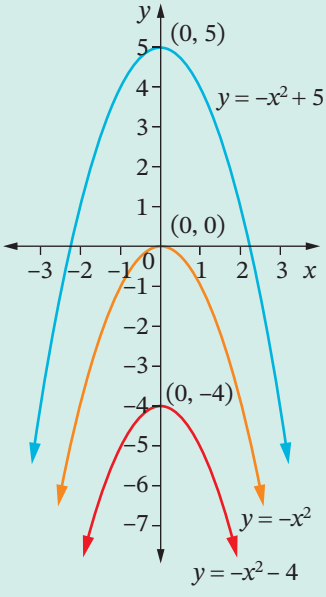
The graph of $y = x^2 + 2$ is like that of $y = x^2$, but moved 2 units up. Its vertex is at (0, 2).



b The graph of $y = -x^2$ is the graph of $y = x^2$ reflected across the x-axis. Its vertex is at (0, 0) as well.

The graph of $y = -x^2 - 4$ is like that of $y = -x^2$, but it is moved 4 units down. Its vertex is at (0, −4).

The graph of $y = -x^2 + 5$ is like that of $y = -x^2$, but it is moved 5 units up. Its vertex is at (0, 5).



Note:

- In part **a**, all parabolas are concave up, because a is positive (1)
- In part **b**, all parabolas are concave down, because a is negative (–1)
- For $y = ax^2 + c$, the y-intercept of the parabola is c

Example 8

For the graph of each quadratic equation, state:

i whether the parabola is wider or narrower than the graph of $y = x^2$

ii whether the parabola has moved up or down when compared to the graph of $y = x^2$

iii the y-intercept.

a $y = 3x^2 - 1$ **b** $y = \frac{1}{3}x^2 + 2$

Solution

a i The coefficient of x^2 is 3, while the coefficient of x^2 in $y = x^2$ is 1.

$\therefore$ The parabola will be narrower than $y = x^2$.

ii The constant term is -1.

$\therefore$ The parabola has moved down.

iii The y-intercept is -1.

Vertex at $(0, -1)$

b i The coefficient of x^2 is $\frac{1}{3}$.

$\therefore$ The parabola will be wider than $y = x^2$.

ii The constant term is 2.

$\therefore$ The parabola has moved up.

iii The y-intercept is 2.

Vertex at $(0, 2)$

Example 9

A parabola has the equation $y = 2x^2 + 3$.

Find the x-coordinate of the point on the parabola that has a y-coordinate of:

a 11 **b** 165.

Solution

a Substitute $y = 11$ into $y = 2x^2 + 3$.

$11 = 2x^2 + 3$

$8 = 2x^2$

$2x^2 = 8$

$x^2 = \frac{8}{2}$

$= 4$

$x = \pm\sqrt{4}$

$= \pm 2$

This means that there are 2 points on the parabola with a y-coordinate of 11.

They are $(2, 11)$ and $(-2, 11)$.

b Substitute $y = 165$ into $y = 2x^2 + 3$

$165 = 2x^2 + 3$

$162 = 2x^2$

$2x^2 = 162$

$x^2 = \frac{162}{2}$

$= 81$

$x = \pm\sqrt{81}$

$= \pm 9$

This makes sense because a parabola is symmetrical: (2, 11) and (–2, 11) are on either side of the y-axis.

EXERCISE 7.04 ANSWERS ON P. 599

Graphing quadratic equations $y = ax^2 + c$ UFRC

This exercise can be completed using graphing technology.

EXAMPLE 7

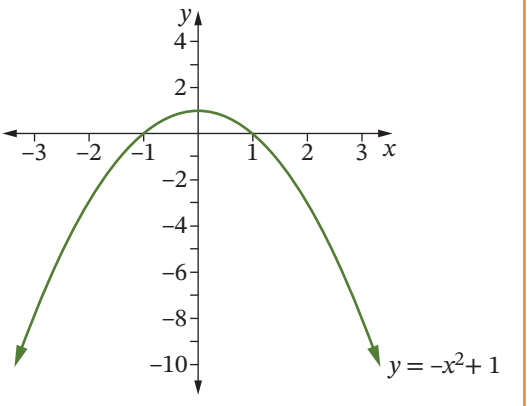
1 **a** Graph each quadratic equation, showing the vertex of each parabola.

$y = x^2$ $y = -x^2$ $y = x^2 + 2$ $y = -2x^2$ $y = x^2 - 1$

b State which graphs you have drawn in part **a**: C

i are concave up **ii** are concave down **iii** have a turning point at (0, 0).

2 Which statement is false about this parabola? Select **A**, **B**, **C** or **D**. C



A Its axis of symmetry is the x-axis.

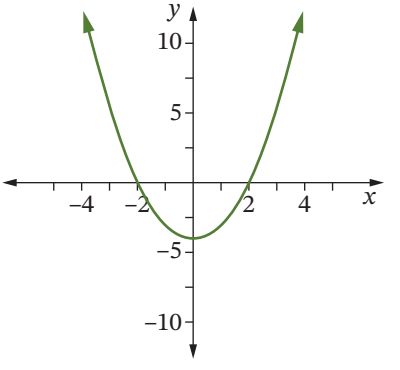
B It is concave down.

C Its vertex is (0, 1).

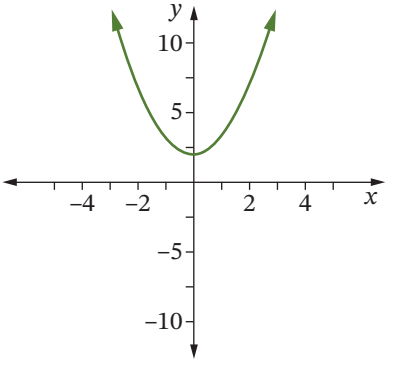
D It has a maximum value.

3 Which diagram shows the graph of $y = x^2 - 2$? Select **A**, **B**, **C** or **D**. R

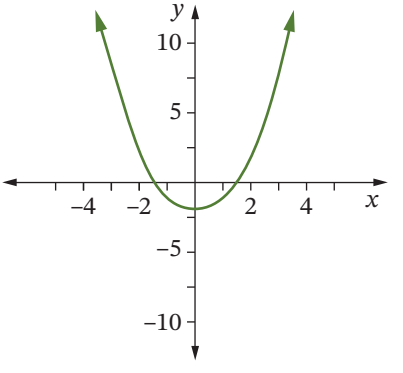
A



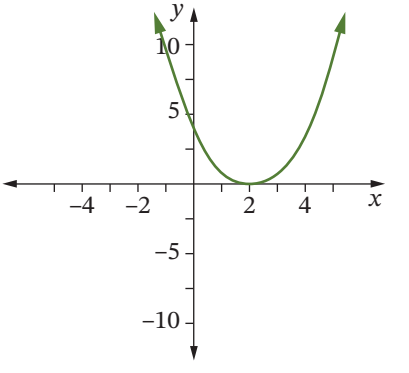
B



C



D

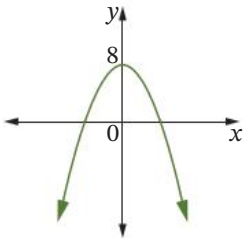


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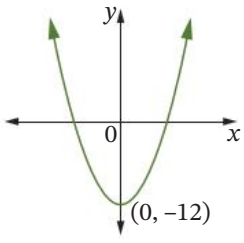
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4 Match each graph with its correct quadratic equation. R C

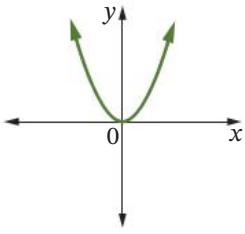
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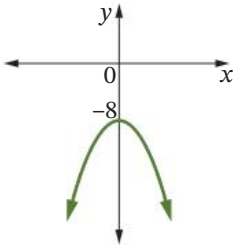
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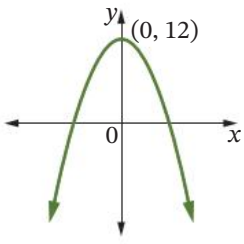
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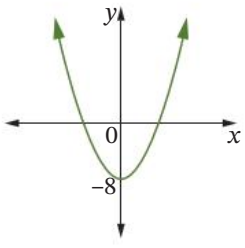
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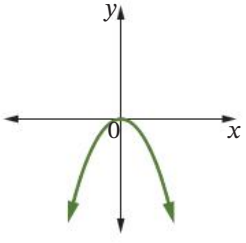
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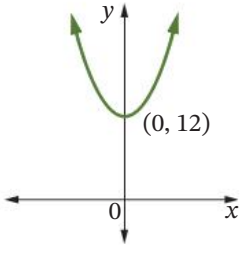
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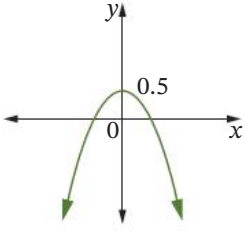
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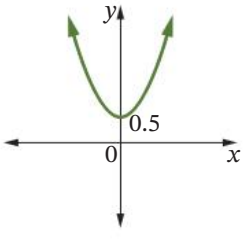
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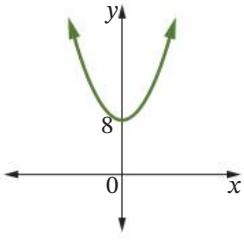
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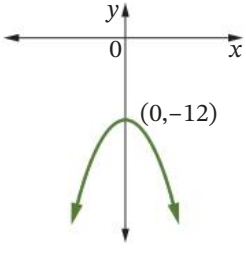
j



k



l



A $y = x^2$ **B** $y = -x^2$ **C** $y = x^2 - 8$ **D** $y = -12 - x^2$

E $y = \frac{1}{2} + x^2$ **F** $y = 8 - x^2$ **G** $y = 8 + x^2$ **H** $y = -x^2 + \frac{1}{2}$

I $y = x^2 - 12$ **J** $y = 12 - x^2$ **K** $y = -x^2 - 8$ **L** $y = x^2 + 12$

7.04

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5 Find the equation of each parabola described, in the form $y = x^2 + c$ or $y = -x^2 + c$ (where c is a constant). R C

a vertex $(0, 0)$, concave down

b concave up, turning point $(0, 0)$

c axis of symmetry $x = 0$, maximum $y = \frac{1}{4}$

d concave down, maximum $y = -9$

e turning point $\left(0, \frac{1}{2}\right)$, concave down

f axis of symmetry y-axis, minimum $y = 9$

6 a Graph $y = 2x^2 + 1$ after copying and completing this table. R C

x	−2	−1	0	1	2
y					

b State the coordinates of the vertex.

c Is the parabola concave up or concave down?

d What is its minimum value?

7 a Graph $y = -3x^2 + 2$ after copying and completing this table. R C

x	−2	−1	0	1	2
y					

b Find the vertex.

c Write the equation of its axis of symmetry.

d Find its maximum value.

8 Which statement is false about the graph of $y = 4x^2 - 1$? Select **A**, **B**, **C** or **D**. R C

A Its axis of symmetry is $y = 0$.

B It is concave up.

C The vertex is $(0, -1)$.

D It has a minimum value of $y = -1$.

EXAMPLE 8

9 For the graph of each given quadratic equation, state: R C

i whether the parabola is wider or narrower than the graph of $y = x^2$

ii whether the parabola has moved up or down when compared to the graph of $y = x^2$

iii the y-intercept.

a $y = 2x^2 + 3$

b $y = \frac{1}{2}x^2 + 1$

c $y = 6x^2 - 5$

d $y = 0.2x^2 - 12$

EXAMPLE 9

10 A parabola has the equation $y = x^2 - 5$. Find the x-coordinates for the points on the parabola that have a y-coordinate of:

a 11

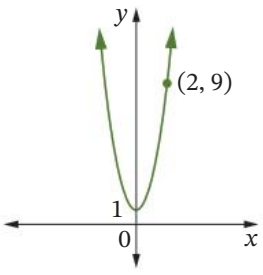
b 116.

Foundation Standard Complex

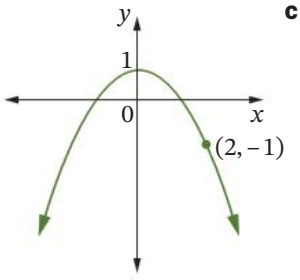
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11 Match each graph with its correct quadratic equation. R C

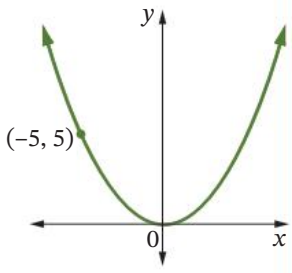
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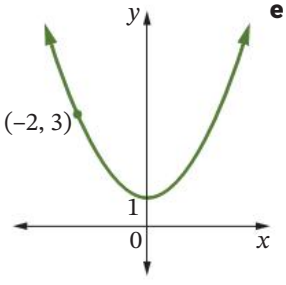
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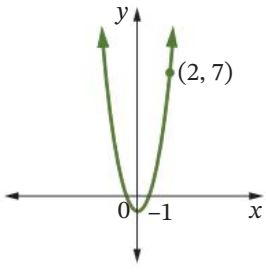
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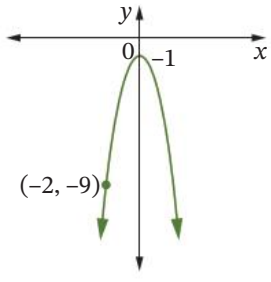
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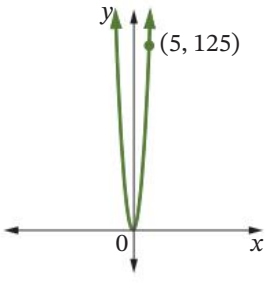
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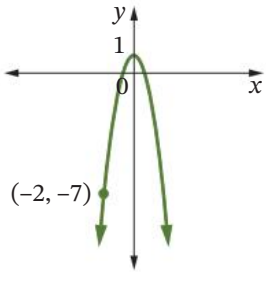
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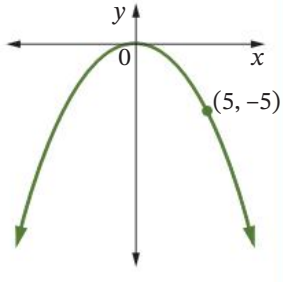
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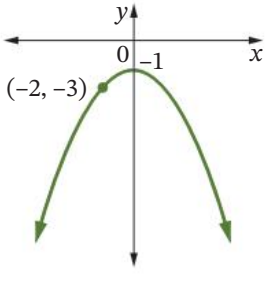
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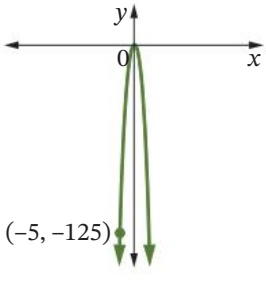
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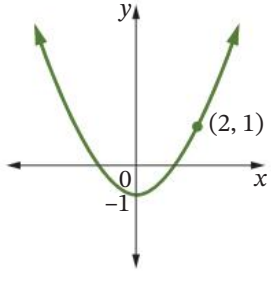
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k



l



A $y = 5x^2$ **B** $y = 2x^2 + 1$ **C** $y = \frac{1}{2}x^2 - 1$ **D** $y = \frac{1}{5}x^2$

E $y = 2x^2 - 1$ **F** $y = -5x^2$ **G** $y = -\frac{1}{2}x^2 + 1$ **H** $y = -\frac{1}{5}x^2$

I $y = -2x^2 - 1$ **J** $y = \frac{1}{2}x^2 + 1$ **K** $y = -2x^2 + 1$ **L** $y = -\frac{1}{2}x^2 - 1$

12 A parabola has the equation $y = 2x^2 + 3$. Find the x-coordinates of the points on the parabola that have a y-coordinate of:

a 165 **b** 395.

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Did you know?

Parabolas in architecture

There are many examples of parabolas in architecture and engineering.

The Notre Dame Cathedral in Paris, France is almost 900 years old and has flying buttresses on the outside that have the shape of parabolas.



Bill Perry/Shutterstock.com

Bridges also often use parabolic curves in their construction. One modern application is the cables used in the suspension of the Golden Gate Bridge in San Francisco.



iStock.com/MasterLu

Find 2 different uses of parabolas in real-life constructions and create a presentation involving them.

Mental skills 7: Maths without calculators ANSWERS ON P. 599

Quiz
Mental skills 7

Multiplying decimals

1 Study each example.

a $3 \times 8 = 24$, so $3 \times 0.8 = 2.4$

0 dp + 1 dp = 1 dp (dp = decimal places)

The number of decimal places in the answer is equal to the total number of decimal places in the question. Also, the answer sounds reasonable because, by estimation:

$3 \times 0.8 \approx 3 \times 1 = 3$ $(2.4 \approx 3)$

b $6 \times 5 = 30$, so $0.6 \times 0.5 = 0.30 = 0.30$

1 dp + 1 dp = 2 dp

By estimation, $0.6 \times 0.5 \approx 0.5 \times 0.5 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = 0.25$ $(0.3 \approx 0.25)$

c By estimation, $0.07 \times 0.3 \approx 0.07 \times \frac{1}{3} \approx 0.02$ $(0.021 \approx 0.02)$

2 Now evaluate each product.

a	0.7×5	**b**	12×0.2	**c**	0.4×0.3	**d**	$(0.6)^2$
e	8×0.1	**f**	0.03×0.9	**g**	4×0.05	**h**	1.1×8
i	0.3×0.8	**j**	0.2×0.06	**k**	9×0.2	**l**	0.07×0.4

3 Study each example.

Given that $15 \times 23 = 345$, evaluate each product.

a $1.5 \times 2.3 = 3.45$

1 dp + 1 dp = 2 dp (*Estimate* $1.5 \times 2.3 \approx 2 \times 2 = 4$)

b $150 \times 0.23 = 15 \times 10 \times 0.23 = 15 \times 0.23 \times 10 = 3.45 \times 10 = 34.5$

0 dp + 2 dp = 2 dp

(*Estimate* $150 \times 0.23 \approx 150 \times 0.2 = 150 \times \frac{1}{5} = 30$)

c $0.15 \times 2300 = 0.15 \times 23 \times 100 = 3.45 \times 100 = 345$

2 dp + 0 dp = 2 dp

(*Estimate* $0.15 \times 2300 \approx 0.2 \times 2300 = \frac{1}{5} \times 2300 = 460$)

4 Now given that $39 \times 17 = 663$, evaluate each product.

a	3.9×17	**b**	39×170	**c**	39×0.17	**d**	0.39×1.7
e	3.9×1.7	**f**	390×1.7	**g**	3.9×0.17	**h**	3.9×170
i	3900×1.7	**j**	39×1.7	**k**	39×0.017	**l**	0.39×0.17

7.05 Applying quadratic equations

Video
Applying quadratic functions

Parabolas can be seen and are used in our everyday lives. When water is sprayed upwards from a fountain or hose, or when a ball is thrown or kicked, the path of the object is a parabola.

Example 10

A ball is thrown and its height (h metres) at any time (t seconds) is given by the quadratic equation $h = 2t - 0.5t^2$.

Quadratic equations can have different variables other than x and y.

a Complete this table of values to graph the equation.

Time, t (seconds)	0	1	2	3	4	5
Height, h (metres)						

b What is the maximum height of the ball?

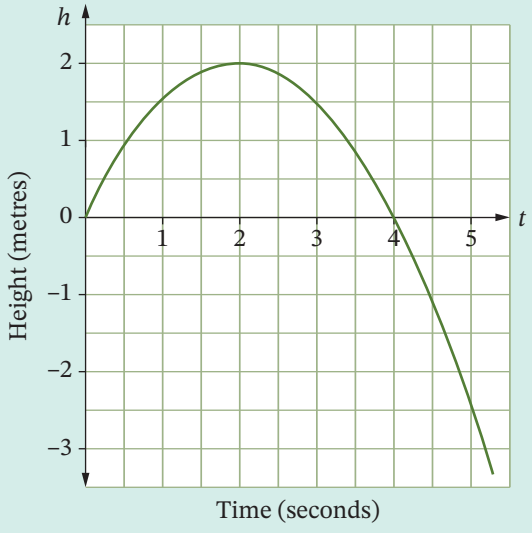
c Find the total time the ball is in the air.

d Use the graph to find when the height of the ball is 1.5 m.

Solution

a

Time, t (seconds)	0	1	2	3	4	5
Height, h (metres)	0	1.5	2	1.5	0	−2.5



Note: This is not the graph of the path of the ball. It is the graph of the height of the ball at different times, t seconds. We can ignore that part of the graph after $t = 4$ below the t-axis because the height cannot be negative.

b Maximum height is 2 m. From table or graph, $h = 2$

c 4 s. When $h = 0$ again, at $t = 4$

d The ball is at a height of 1.5 m at 1 and 3 s.

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EXERCISE 7.05 ANSWERS ON P. 600

Applying quadratic equations U F R C

EXAMPLE 10

7.05

1 A ball is thrown upwards and its height, h metres, after t seconds is given by the quadratic equation $h = 3.5t - 0.49t^2$. R C

a Copy and complete this table of values to graph the equation.

Time, t (seconds)	0	1	2	3	4	5	6	7	8
Height, h (metres)									

b What is the maximum height of the ball and when does it reach this height?

c At what times does the ball reach a height of 3 m? Explain why there are 2 answers.

2 The shape of cables hanging between 2 towers 100 m apart on a suspension bridge is modelled by the equation $h = 10 + 0.004x^2$ where h is the height, in metres above ground and x is the horizontal distance in metres from the centre of the bridge. R C

a Copy and complete this table, then graph the equation.

x	−50	−40	−30	−20	−10	0	10	20	30	40	50
h											

b How tall are the towers at either end of the cables?

c What is the height of the cables at their lowest point?

d How far from the centre is the height of the cables 15 m?

3 A stone is dropped from a cliff and its height (h metres) at any time (t seconds) is given by $h = 80 - 4.9t^2$. R C

a Graph the quadratic equation for values of t from 0 to 5.

b What is the height of the cliff?

c Use your graph to find the height of the stone after 3 seconds.

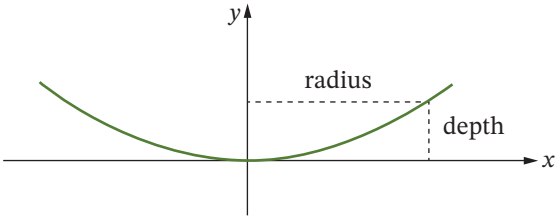
d When will the stone hit the ground?

e How long after it is dropped is the stone 5 metres above the ground? Answer to the nearest 0.1 s.



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4 The inside of a large radio telescope is the shape of a parabola and can be modelled by the equation $y = 0.0091x^2$, where y metres is the depth and x metres is the radius. R



a Copy and complete this table of values.

Radius, x (metres)	−40	−30	−20	−10	0	10	20	30	40
Depth, y (metres)									

b Graph the equation, using a scale of 1 cm = 10 m on the x-axis and 1 cm = 5 m on the y-axis.

c If the radius of the radio telescope is 32 m, use your graph to find its depth.

d Find the radius of the radio telescope that has a depth of:

i 1 m **ii** 2.5 m

5 A ball is dropped from the top of a building 200 m high and the distance, d metres, it falls after t seconds is given by the equation $d = 5t^2$. R

a Copy and complete this table of values and graph the equation.

Time, t (seconds)	0	1	2	3	4	5	6	7
Distance, d (metres)								

b How far has the ball dropped after:

i 2 seconds? **ii** 5 seconds?

c How far did the ball drop in:

i the 3rd second? **ii** the 4th second?

d After how many seconds did the ball hit the ground (correct to 1 decimal place)?

e After how many seconds was the ball 80 m *above the ground*?

6 A parabolic reflector has the equation $y = 0.2x^2$, where x (cm) is the radius of the reflector and y (cm) is the depth.

a Copy and complete this table and graph the equation.

Radius, x (cm)	−5	−4	−3	−2	−1	0	1	2	3	4	5
Depth, y (cm)											

b If the radius of a reflector is 2.5 cm, what is its depth?

c If the depth of a reflector is 4.5 cm, what is its radius?

7 At a water fountain, the height of the water being sprayed up follows the equation $h = 1.25 + 4t - 0.85t^2$, where h (metres) is the height of the water and t (seconds) is the time. R C



iStock.com/Petra Richli

a Copy and complete this table, then graph this equation.

Time, t (seconds)	0	1	2	3	4	5	6
Height, h (metres)							

b What is the vertical intercept of the graph and what does this value represent?

c How long does it take for the water to leave the fountain to when it hits the ground?

d What is the maximum height the water reaches?

e How long does it take for the water to reach its maximum height?

8 The height h metres above ground of a skydiver jumping from a plane is given by $h = 3000 - 5t^2$, where t seconds is the time after jumping.

a Copy and complete this table to graph the equation. Use a scale of 1 cm = 5 s on the horizontal axis and 1 cm = 500 m on the vertical axis.

Time (t seconds)	0	5	10	15	20	25
Height (y metres)						

b What was the height of the plane when the skydiver jumped?

c What was the height of the skydiver after 15 seconds?

d If the parachute is opened at a height of 1200 m, how long will it take the skydiver to drop to this height?

7.05

Investigation

PATH

Graphing $y = \frac{1}{x}$

1 Copy and complete this table for $y = \frac{1}{x}$. Explain why no y value exists for $x = 0$.

x	−5	−4	−3	−2	−1	−0.5	−0.2	−0.1	0	0.1	0.2	0.5	1	2	3	4	5
y									–								

2 Hence graph $y = \frac{1}{x}$ on a number plane.

3 There are 2 parts or 'branches' to your graph. In which quadrants of the number plane are the branches?

4 Use your graph to explain what happens to the y value as x becomes very large.

5 Explain what happens to the y value as x approaches 0.

6 The graph of $y = \frac{1}{x}$ has 2 axes of symmetry. Draw them on your graph.

7 Copy and complete the table from question **1** for $y = -\frac{1}{x}$.

8 Hence graph $y = -\frac{1}{x}$ on a number plane.

9 How does the graph of $y = -\frac{1}{x}$ compare with that of $y = -\frac{1}{x}$?

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Technology

Graphing $y = \frac{k}{x}$

1 Use graphing technology to graph each equation.

a $y = \frac{1}{x}$ **b** $y = \frac{2}{x}$ **c** $y = \frac{5}{x}$ **d** $y = \frac{10}{x}$

2 Compare the graphs from question **1**. What happens to the graph of $y = \frac{k}{x}$ as k increases?

3 Graph $y = \frac{2}{x}$ and $y = -\frac{2}{x}$ and compare them.

4 Graph $y = \frac{4}{x}$ and use **Trace** to complete this table of values.

x	1	2	5	10	100	200	1000
y							

5 What happens to the y values when the x values become very large?

6 For $y = \frac{4}{x}$ use the **Trace** function to complete this table of values.

x	0.0001	0.01	0.1	0.5	1	5
y						

7 What happens to the y values when the x values become very small and close to 0?

7.06 Graphing hyperbolas $y = \frac{k}{x} + c$

PATH

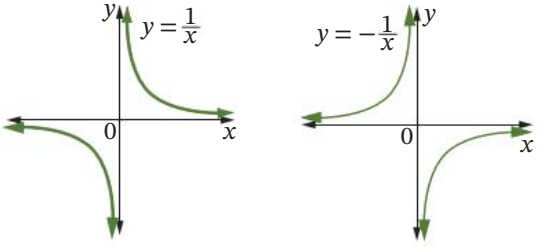
Worksheet
Graphing hyperbolas

Technology
Curve sketcher

Spreadsheet
Curve sketcher

The graph of $y = \frac{k}{x}$, where k is a constant, is a curve with 2 branches called a **hyperbola** (pronounced 'hy-perb-o-la').

The graphs of $y = \frac{1}{x}$ and $y = -\frac{1}{x}$ are shown.



The hyperbola $y = \frac{k}{x}$

- The hyperbola has 2 separate branches in different quadrants.
- If k is positive, the graph is in the 1st and 3rd quadrants.
- If k is negative, the graph is in the 2nd and 4th quadrants.
- The graph has 2 axes of symmetry: their equations are the straight lines $y = x$ and $y = -x$.
- The graph has rotational symmetry of 180° about (0, 0).
- The higher the value of k, the further the hyperbola is from the x- and y-axes.
- As x becomes very large, y gets closer to 0.
- As x becomes closer to 0, y gets very large.
- The graph gets very close to the x- and y-axes but never crosses them. The x- and y-axes are called **asymptotes** because the graph approaches them but never touches them.

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Example 11

Graph each hyperbola and mark the coordinates of one point on the curve.

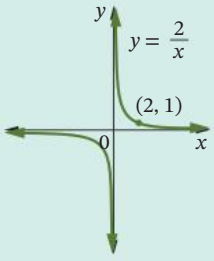
a $y = \frac{2}{x}$ **b** $y = -\frac{3}{x}$

Solution

a Let $x = 2$ Choosing any value of x

$$y = \frac{2}{2}$$
$$= 1$$

A point on the curve is (2, 1).

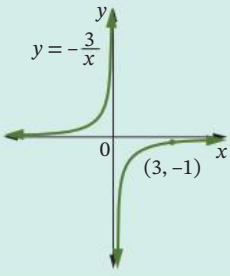


b Let $x = 3$

$$y = \frac{3}{3}$$
$$= -1$$

A point on the curve is (3, –1).

As $k = -3$ is negative, the hyperbola is in the 2nd and 4th quadrants.



The hyperbola $y = \frac{k}{x} + c$

The graph of $y = \frac{k}{x} + c$ is the hyperbola $y = \frac{k}{x}$ translated c units up (or down if c is negative).

The hyperbola $y = \frac{k}{x - b}$

The graph of $y = \frac{k}{x-b}$ is the hyperbola $y = \frac{k}{x}$ translated b units to the right (or left if b is negative).

Example 12

Graph each hyperbola, find any intercepts and mark the coordinates of one point on the curve.

a $y = \frac{2}{x} + 1$ **b** $y = -\frac{3}{x-2}$

Solution

a Let $x = 2$ Choosing any value of x

$$y = \frac{2}{2} + 1$$
$$= 2$$

A point on the curve is (2, 2).

PATH

Video
Graphing hyperbolas

7.06

PATH

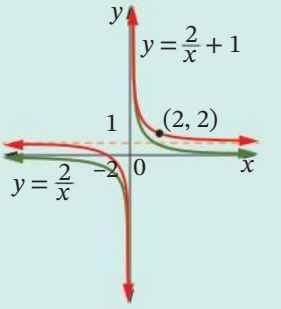
The graph of $y = \frac{2}{x} + 1$ is the graph of $y = \frac{2}{x}$ translated up 1 unit. This means that the horizontal asymptote is now at $y = 1$.
An x-intercept now occurs when $y = 0$.

$0 = \frac{2}{x} + 1$ Multiplying both sides by x

$0 = 2 + x$

$x = -2$

The x-intercept is -2.



b Let $x = 3$

$y = -\frac{3}{3-2}$

$= -3$

A point on the curve is $(3, -3)$.

The graph of $y = -\frac{3}{x-2}$ is the graph of $y = -\frac{3}{x}$ translated right 2 units. This means that the vertical asymptote is now at $x = 2$.

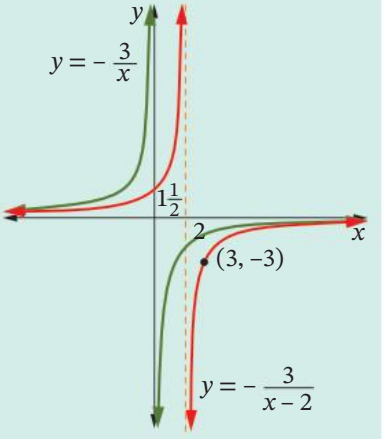
A y-intercept now occurs when $x = 0$.

$y = -\frac{3}{0-2}$

$= \frac{-3}{-2}$

$= 1\frac{1}{2}$

The y-intercept is $1\frac{1}{2}$.



EXERCISE 7.06 ANSWERS ON P. 601

Graphing hyperbolas $y = \frac{k}{x} + c$ UFRC

1 **a** Copy and complete this table for $y = \frac{2}{x}$.

x	−3	−2	−1	0	1	2	3
y				–			

b Graph $y = \frac{2}{x}$, showing the coordinates of one point on the hyperbola.

c On your diagram, draw in the axes of symmetry for the hyperbola.

d What are the equations of these axes?

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PATH

EXAMPLE 11

7.06

2 Graph each hyperbola and mark the coordinates of one point on the curve.

a $y = \frac{4}{x}$ **b** $y = -\frac{2}{x}$ **c** $y = \frac{3}{x}$

3 **a** The distance from Sydney to Melbourne is close to 1000 km. Using the equation $s = \frac{1000}{t}$, copy and complete the following table that relates time (t hours) and speed (s km/h) for the trip. Round your answers to the nearest km/h.

t	1	2	3	...	10
s					

b Hence graph the equation $s = \frac{1000}{t}$.

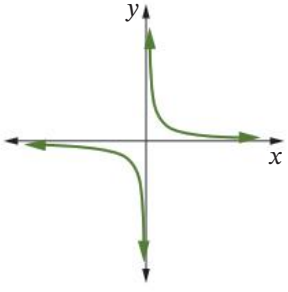
c Why are the values for t only positive numbers? Explain why t cannot be equal to 0. R C

d If the time is doubled, is the speed halved? Use the information from your graph to support your answer. R C

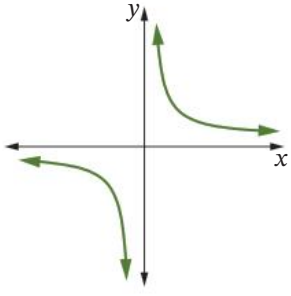
4 Match each equation with its graph. R

a $y = \frac{5}{x}$ **b** $y = -\frac{2}{x}$ **c** $y = \frac{1}{x}$ **d** $y = -\frac{5}{x}$

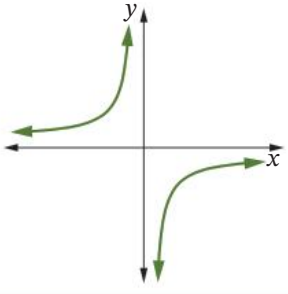
A



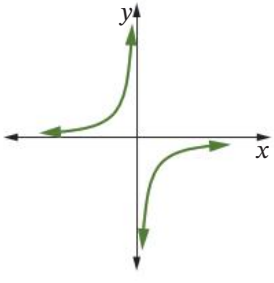
B



C



D



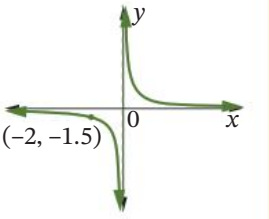
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5 This curve is a hyperbola of the form $y = \frac{k}{x}$.

a Find the value of k.

b Hence state the equation of this hyperbola.



EXAMPLE 12

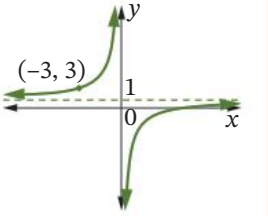
6 Graph each hyperbola and mark the coordinates of one point on the curve.

a $y = \frac{1}{x} + 2$ **b** $y = -\frac{2}{x} - 3$ **c** $y = \frac{2}{x-1}$ **d** $y = \frac{-3}{x+2}$

7 This curve is a hyperbola with equation $y = \frac{k}{x} + c$.

a Find the values of c and k.

b Hence state the equation of this hyperbola.



8 Sarah and David want to buy a rectangular block of land that has an area of 800 m^2. There are several blocks available with this area. R C

a Copy and complete this table that relates the length (L metres) and width (W metres) of the block of land.

L	10	20	30	...	100
W					

b What is the formula for W?

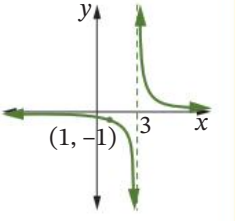
c Explain why the length or width cannot be equal to 0 metres.

d Graph the formula for W.

e What happens to the width as the length continues to increase? How is this shown on the graph?

f What happens to the width as the length approaches 0? How is this shown on the graph?

9 Which equation best represents this hyperbola?
Select the correct answer **A**, **B**, **C** or **D**.



A $y = \frac{2}{x-3}$ **B** $y = \frac{-1}{x+3}$

C $y = \frac{2}{x} + 3$ **D** $y = -\frac{1}{x} - 3$

10 Find the asymptotes of the hyperbola with equation: R C

a $y = \frac{k}{x}$ **b** $y = \frac{k}{x} + c$ **c** $y = \frac{k}{x-b}$ **d** $y = \frac{k}{x-b} + c$

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Investigation

Graphing $y = 2^x$

This activity can be completed using graphing technology.

1 Copy and complete this table of values for $y = 2^x$.

x	−3	−2	−1	0	1	2	3	4
y								

2 Graph the points from the table and join them with a smooth curve. The equation $y = 2^x$ is called an exponential equation and its graph is called an exponential curve (exponent means 'power').

3 Graph $y = 2^{-x}$ in a similar way.

4 Compare the graphs of $y = 2^x$ and $y = 2^{-x}$. Describe any similarities and differences.

5 The y-intercept of any graph with equation $y = a^x$ (where a is a positive constant) is always 1. Explain why.

6 The graph of $y = 2^x$ is increasing. Is the graph of $y = 2^{-x}$ increasing or decreasing? Give reasons.

7 Describe what happens to the graph of $y = 2^x$ when:

a x approaches a large positive number

b x approaches a large negative number.

Technology

Exponential curves

Use graphing technology to complete this activity.

1 Graph $y = 2^x$ using the software. Adjust the colour or line thickness if you need to.

2 Now graph these exponential curves, using different colours.

$y = 2^{-x}$, $y = -2^x$, $y = -2^{-x}$, $y = 2^x + 1$, $y = 2^x - 1$

3 **a** Which graphs are similar?

b Identify any features such as y-intercepts.

c Which graphs are similar as:

i x becomes larger? **ii** x becomes smaller?

4 Repeat steps **1** to **3** for these exponential curves:

$y = 3^x$, $y = 3^{-x}$, $y = -3^x$, $y = -3^{-x}$, $y = 3^x + 1$, $y = 3^x - 1$

7.07 Graphing exponential equations $y = a^x + c$

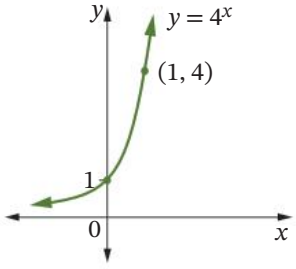
Videos
The exponential curve
The emperor's chessboard

Worksheet
Graphing exponentials

An equation of the form $y = a^x$, where a is a positive constant and the variable x is a power, is called an **exponential equation**, for example, $y = 5^x$, $y = 2^x$ and $y = 3^x$. The graph of an exponential equation is called an **exponential curve**.

The table of values and graph of $y = 4^x$ is shown.

x	−2	−1	0	1	2	3	4
y	$\frac{1}{16}$	$\frac{1}{4}$	1	4	16	64	256



- The y-intercept of $y = a^x$ is always 1 since $a^0 = 1$.
- As x increases (to the right, in the positive direction), a^x becomes very large. Graphically, this means that the graph of $y = a^x$ increases sharply with a steep positive gradient.
- As x decreases (to the left, in the negative direction), a^x approaches 0. This means that the graph of $y = a^x$ flattens out and approaches the x-axis as x becomes a large negative number. The x-axis is an **asymptote** because the curve approaches it but never touches it.
- The exponential curve is always above the x-axis because the value of a^x is always positive.

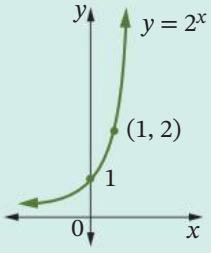
Example 13

Sketch each exponential equation and mark the y-intercept on each curve.

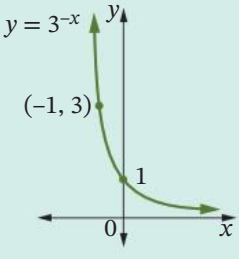
a $y = 2^x$ **b** $y = 3^{-x}$

Solution

a
- The y-intercept of $y = 2^x$ is 1
- When $x = 1$, $y = 2$, so (1, 2) is a point on the curve
- As x increases (to the right in the positive direction), 2^x becomes very large (steep positive gradient)
- As x decreases (to the left in the negative direction), 2^x approaches 0
- The x-axis is an asymptote.



b
- The y-intercept of $y = 3^{-x}$ is 1
- When $x = -1$, $y = 3$, so (−1, 3) is a point on the curve
- As x decreases (to the left in the negative direction), 3^{-x} becomes very large (steep negative gradient)
- As x increases (to the right in the positive direction), 3^{-x} approaches 0
- The x-axis is an asymptote.



Note that the graph of $y = 3^{-x}$ (and of $y = a^{-x}$ in general) is decreasing, and is actually a reflection of $y = 3^x$ in the y-axis.

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EXERCISE 7.07 ANSWERS ON P. 603

Graphing exponential equations $y = a^x + c$ U F R C

Some of this exercise may be completed using graphing technology.

EXAMPLE 13

7.07

1 **a** Graph each exponential equation on the same axes. R C

i $y = 2^x$ **ii** $y = 3^x$ **iii** $y = 5^x$

b What is the y-intercept of each curve?

c Describe what happens to the graph of $y = a^x$ as a increases.

2 **a** Graph $y = 4^x$ and $y = 4^{-x}$ on the same axes.

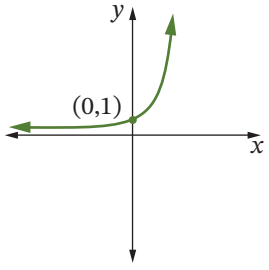
b Copy and complete:

i The reflection of $y = 4^x$ in the y-axis is ...

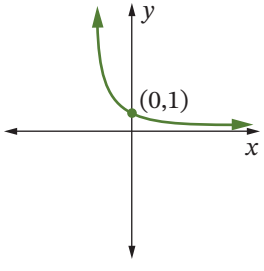
ii The reflection of $y = a^x$ in the y-axis is ...

3 Which graph represents $y = 2^{-x}$? Select the correct answer **A**, **B**, **C** or **D**. R C

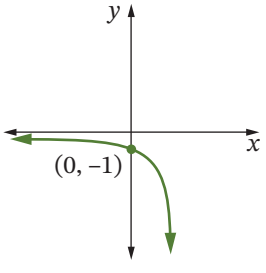
A



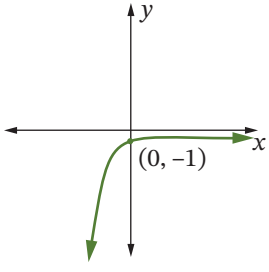
B



C



D



4 **a** Graph $y = 2^x$ and $y = -2^x$ on the same axes.

b How are the 2 graphs related? R C

c Copy and complete: The reflection of $y = a^x$ in the x-axis is ...

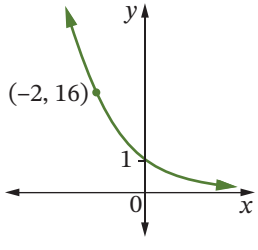
5 Graph $y = 3^x + 1$ and $y = 3^x - 1$ on the same axes and describe how they are related. R C

6 Sketch each exponential curve, showing the y-intercept.

a $y = 2^x$ **b** $y = 3^{-x}$ **c** $y = -4^x$

d $y = -2^{-x}$ **e** $y = 4^x + 1$ **f** $y = 4^x - 1$

7 Find an exponential equation for this graph. R C



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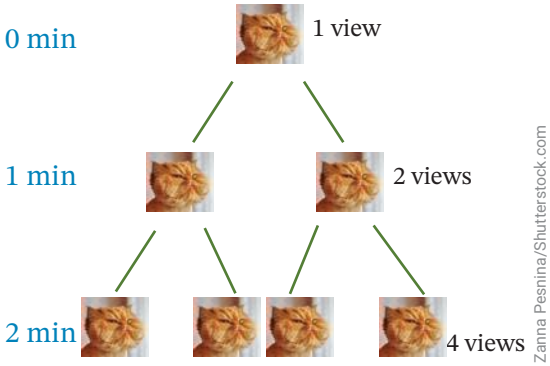
Investigation

Videos going viral

Use a spreadsheet to help you with this investigation.

With the invention of social media and social networks, we can now communicate with each other instantly. However, what often starts off as a simple message between friends or an online video post can quickly multiply at an alarming rate, be seen by the general public worldwide and 'go viral' (spread like a virus).

Suppose a video of an angry cat is shared with a friend who one minute later shares it with 2 other friends. This occurs every minute so that every minute the number of *new* views doubles. This is an example of something growing exponentially, according to the formula $y = 2^x$, where x represents number of minutes.



Zanna Pesnina/Shutterstock.com

1 Use a spreadsheet to calculate the number of new views after:

a 15 min **b** 30 min **c** 1 hour **d** 2 hours

e 4 hours **f** 10 hours **g** 20 hours **h** 30 hours

2 Find how long it will take until the total number of new views reaches:

a 64 **b** 256 **c** over 500 **d** over 1000

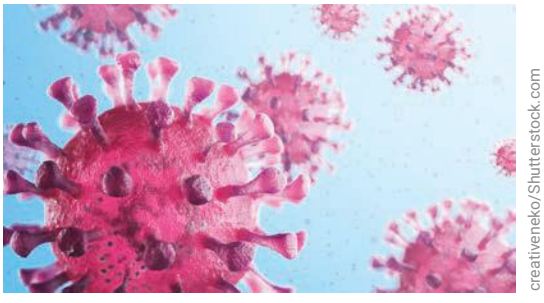
e over 3000 **f** over 10 000 **g** over 1 million **h** over 4 million

3 Use the spreadsheet's **Graph Option** to graph the viral pattern. Describe the shape of the graph.

Did you know?

Exponential growth

When an increase can be described using an exponential equation, it is called exponential growth. Examples include the growth of a virus, population (people or animals) and compound interest investments.


creativeneko/Shutterstock.com

Population growth is monitored in different countries through the fertility (birth) and mortality (death) rates as well as migration. The data collected for these figures can often be modelled as an exponential function. By modelling the changes in population, predictions of future changes in population can be simulated and towns and cities can prepare for possible expansion in the numbers of schools, hospitals, housing and other necessary infrastructure.

At what rate is the population of Australia growing?
What about the world's population?

Investigation

Graphing $y = 3(2^x)$

This activity can be completed using graphing technology.

1 Copy and complete this table of values for $y = 3(2^x)$.

$3(2^x)$ means $3 \times (2^x)$

x	−3	−2	−1	0	1	2	3	4
y								

2 Graph the points from the table and join them with a smooth curve.

3 Graph each exponential equation in a similar way.

a $y = 1.5(2^x)$ **b** $y = 4(3^x)$ **c** $y = 0.5(4^x)$

4 Compare the graphs $y = 3(2^x)$, $y = 1.5(2^x)$, $y = 4(3^x)$ and $y = 0.5(4^x)$. Describe any similarities and differences.

5 Graph each exponential equation and describe any similarities and differences.

a $y = 5(2^{-x})$ **b** $y = 4(3^{-x})$ **c** $y = 0.5(4^{-x})$ **d** $y = 2(5^{-x})$

6 **a** What are the y-intercepts of the graphs in Question 1?

b What are the y-intercepts of the graphs in Question 2?

c What is the y-intercept of any graph with equation $y = k(a^x)$ or $y = k(a^{-x})$?

Exponential growth and decay 7.08

Any quantity that increases according to the exponential equation $y = k(a^x)$, where $a > 1$, is showing **exponential growth**, increasing slowly at first and then more quickly.

Examples of exponential growth are population growth, an investment increasing with compound interest, and the number of people or computers affected by a virus.

Any quantity that decreases according to the exponential equation $y = k(a^{-x})$, where $a > 1$, is showing **exponential decay**, decreasing quickly at first and then more slowly.

Examples of exponential decay are the cooling of a hot liquid, the depreciating value of an asset and radioactive material decay.

We have studied compound interest and depreciation in Chapter 1, *Interest and depreciation*.

Example 14

The future value $\$FV$ of an amount of \$80, invested at 5% p.a. interest compounded annually, after n years is given by the formula $FV = 80(1.05)^n$.

a Complete this table of values, correct to the nearest dollar.

n	0	10	20	30	40	50
FV						

b Draw the graph of the equation. Use a scale of 1 cm = 10 years on the x-axis and 1 cm = \$100 on the y-axis.

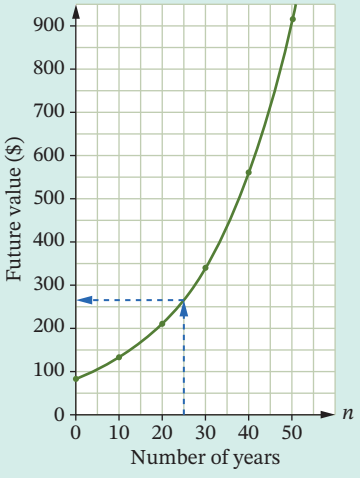
c Use your graph to find the future value of the investment after 25 years.

Solution

a

n	0	10	20	30	40	50
FV	80	130	212	346	563	917

b, c



Using the graph, the value of the investment after 25 years is about \$270.

(Using the formula, the value of the investment is \$270.91)

Example 15

The value $\$V$ of a car which depreciates annually is given by the equation $V = 50\,000\,(1.25)^{-n}$, where n is the number of years.

a Graph the equation for values of n from 0 to 5.

b What is the vertical intercept and what does it represent?

c Use the graph to estimate the value of the car after 3.5 years.

d Use the equation to determine when the car's value will drop to below \$10 000.

Solution

a Completing a table of values with V rounded to the nearest \$100, using the formula $V = 50\,000\,(1.25)^{-n}$.

n	0	1	2	3	4	5
V	50 000	40 000	32 000	25 600	20 500	16 400

b The vertical intercept is 50 000, which is the original value of the car.

c From the graph, the value of the car after 3.5 years is \$23 000.

(Using the formula, the value of V is \$22 897)

d Use guess-and-check:

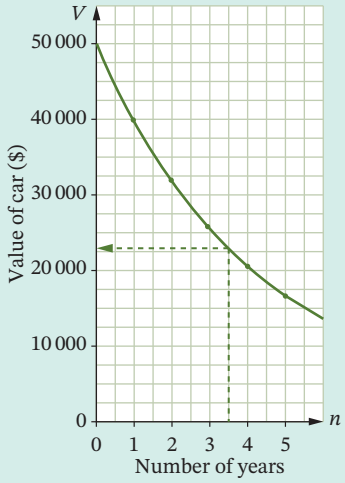
$n = 5, V = 16\,400$

$n = 6, V = 13\,107$

$n = 7, V = 10\,485$

$n = 8, V = 8388$

The value of the car will be less than \$10 000 after 8 years.



EXERCISE 7.08 ANSWERS ON P. 605

Exponential growth and decay U F R C

EXAMPLE 14

1 The population of a small town grows according to the equation $P = 1200\,(1.095)^n$, where P is the population and n is the number of years after the end of 2025. R C

a Copy and complete the table of values correct to the nearest 10.

n	0	1	2	3	4	5	6	7	8	9	10
P											

b Graph the exponential equation for n from 0 to 10. Use a scale of 1 cm = 2 years on the horizontal axis and a scale of 1 cm = 400 people on the vertical axis.

c What is the vertical intercept and what does it represent?

d Use a guess-and-check method to find in what year the population will be triple the original population.

2 Bacteria cells grow by continually dividing into 2 cells. So 1 cell will divide into 2 cells, the 2 cells divide into 4 cells and so on. The number of bacteria cells (B) present can be found using the exponential equation $B = 10(2)^n$, where n is the number of times the cells divide. The cells divide every 20 minutes. R C

a Copy and complete this table of values and then graph the equation.

Use a scale of 1 cm = 1 division on the horizontal axis and 1 cm = 100 bacteria on the vertical axis.

n	0	1	2	3	4	5	6
B							

b How many bacteria were there initially?

c How many bacteria were present after:

i 2 hours? **ii** 3 hours? **iii** 5 hours?

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3 The future value of $1500 invested at 6.5% p.a. interest compounded annually for n years is given by the equation $FV = 1500(1.065)^n$. R C

a Copy and complete this table correct to the nearest dollar.

n	0	2	4	6	8	10
FV						

b Graph the equation using a scale of 1 cm = 2 years on the horizontal axis and 1 cm = $200 on the vertical axis

c What is the vertical intercept and what does it represent?

d Use your graph to find the what the future value is after:

i 3 years **ii** 7 years

EXAMPLE 15

4 A ball is dropped from a window and the height (H cm) it reaches each bounce (x) is given by the equation $H = 400(1.55)^{-x}$. R C

a Copy and complete this table of values (to the nearest whole number).

x	0	2	4	6	8	10
H						

b Graph the equation using a scale of 1 cm = 2 bounces on the horizontal axis and 1 cm = 50 cm on the vertical axis.

c What is the vertical intercept and what does it represent?

d Use the graph to find the height of the ball on the 5th bounce.

e After which bounce will the height be less than 20 cm?

f Use the equation to find the height of the ball on the 14th bounce.

5 The value of a car that depreciates annually is given by the equation $V = 35\,000(1.111)^{-n}$, where n is the number of years. R C

a Graph the equation for values of n from 0 to 8.

b What is the vertical intercept and what does it represent?

c Use the graph to estimate the value of the car after 3.5 years.

d Use the equation to determine when the car's value will drop to below $10 000.

6 Atmospheric pressure (A), measured in millibars (mbar) decreases with altitude (h, metres) and can be calculated using the exponential equation $A = 1013\left(\frac{25}{22}\right)^{-0.001h}$. R C

a Copy and complete this table of values and draw the graph. Use a scale 1 cm = 1000 m on the horizontal axis and 1 cm = 100 mbar on the vertical axis.

h	0	1000	2000	3000	4000	5000	6000	8000	10 000
A									

b What is the vertical intercept and what does it represent?

c Use your graph to find the atmospheric pressure at:

i 2500 m **ii** 5100 m **iii** 9000 m

d Use your graph to find the altitude if the atmospheric pressure is:

i 700 mbar **ii** 500 mbar **iii** 350 mbar

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e Use the graph to find the atmospheric pressure at the summit of:

i Mt Kosciuszko (2228 m) **ii** Mt Everest (8848 m)

f Use the equation to find the atmospheric pressure at a height of 15 000 m.

7 The half -life of caffeine in the human body is about 5 hours. This means that if you consume 20 mg of caffeine, then after 5 hours you will still have 10 mg in your body.

An equation for the amount of caffeine (C mg) left in the human body t hours after drinking a cup of coffee (about 230 mL) is $C = 95(2)^{-0.2t}$. R C

a Copy and complete this table of values and graph the equation.

t	0	5	10	15	20	25
C						

b How much caffeine was originally in the cup of coffee?

c Use the graph to find how much caffeine will be in the human body after:

i 7 hours **ii** 16 hours **iii** 24 hours

d After how many hours will the amount of caffeine be less than 20 mg?

8 House prices ($P \times \$100\,000$) in a city are increasing according to the equation $P = P_0(1.15)^n$ where $P_0 \times \$100\,000$ is the original price of a house and n is the number of years after the end of 2025. Note that P and P_0 are values multiplied by \$100 000. R C

a The original price of a house is \$500 000.
Copy and complete this table, correct to one decimal place.

n	0	1	2	3	4	5
P (in \$100 000s)	5					

b Draw the graph of the equation.

c What is the vertical intercept and what does it represent?

d Use the graph to find the value of the house at the end of 2030.

9 A certain radioactive substance decays such that its mass (M kg) after t years is given by the equation $M = 100(2.718)^{-0.016t}$. R C

a Copy and complete this table of values correct to one decimal place.

t	0	10	20	30	40	60
M (kg)						

b What was the original mass of the radioactive substance?

c Draw the graph of the exponential equation. Use a scale of 1 cm = 10 years on the horizontal axis and 1 cm = 10 kg on the vertical axis.

d Use your graph to find the mass remaining after 25 years.

e The half-life of the substance is the time it takes to decay to half of the original mass. Use the graph to determine the half-life of the substance.

f Use the equation to find correct to 2 decimal places what amount of the substance remains after:

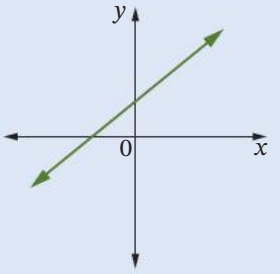
i 100 years **ii** 200 years **iii** 300 years

7.09 Identifying graphs

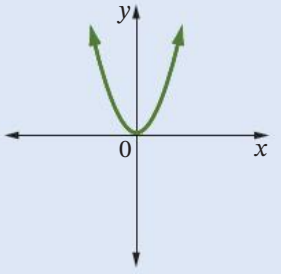
Graphs

Puzzle
Matching graphs

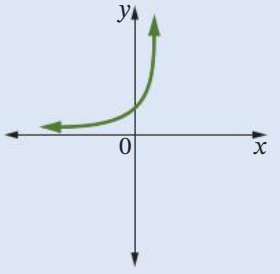
Straight line $y = mx + c$ or $ax + by + c = 0$



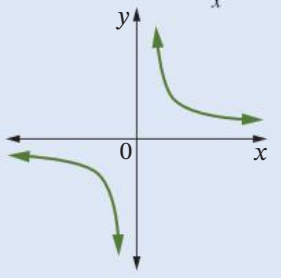
Parabola $y = ax^2 + c$



Exponential curve $y = a^x$



Hyperbola $y = \frac{k}{x}$



Note: The hyperbola $y = \frac{k}{x}$ is PATH content.

Example 16

State whether each equation represents a straight line, a parabola, a hyperbola or an exponential curve.

a $y = 3x^2 - 1$ **b** $y = 5x + 7$ **c** $y = -\frac{1}{2}x^2 - 10$

d $y = \frac{2}{x}$ **e** $y = 3^x$

Solution

a $y = 3x^2 - 1$ is a parabola because it is of the form $y = ax^2 + c$.

b $y = 5x + 7$ is a straight line because it is of the form $y = mx + c$.

c $y = -\frac{1}{2}x^2 - 10$ is a parabola because it is of the form $y = ax^2 + c$.

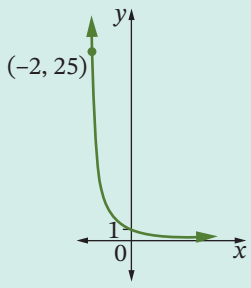
d $y = \frac{2}{x}$ is a hyperbola because it is of the form $y = \frac{k}{x}$.

e $y = 3^x$ is an exponential curve because it is of the form $y = a^x$.

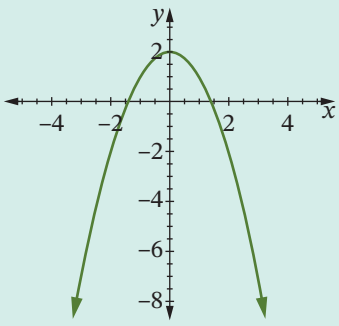
Example 17

Match each graph with its equation.

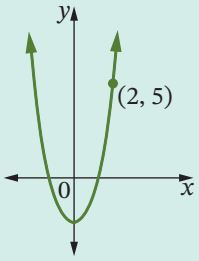
a



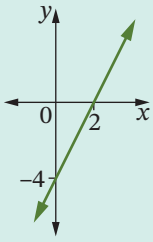
b



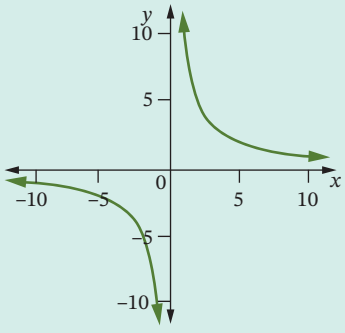
c



d



e



A $y = 2x - 4$ **B** $y = \frac{9}{x}$ **C** $y = 2x^2 - 3$

D $y = 5^{-x}$ **E** $y = -x^2 + 2$

Solution

When matching graphs to equations, the coordinates of a point on the graph may need to be substituted into the equation to verify that the equation represents the graph.

a This is an exponential curve. The only possible match is **D**, $y = 5^{-x}$

Test point: $(-2, 25)$

LHS $= 25$

RHS $= 5^{-(-2)} = 5^2 = 25 =$ LHS

b This is a concave down parabola with a y-intercept of 2. The only possible match is **E**, $y = -x^2 + 2$

c This is a concave up parabola that matches with **C**, $y = 2x^2 - 3$

Test point: $(2, 5)$

LHS $= 5$

RHS $= 2 \times 2^2 - 3 = 5 =$ LHS

d This is a straight line with a y-intercept of -4 that matches with **A**, $y = 2x - 4$

e This is a hyperbola that has points $(3, 3)$ and $(-3, -3)$.

The only possible match is **B**, $y = \frac{9}{x}$

EXERCISE 7.09 ANSWERS ON P. 607

Identifying graphs U F R C

Note: The hyperbola is PATH content.

EXAMPLE 16

1 For each equation state whether its graph is a straight line (L), a parabola (P), an exponential curve (E) or a hyperbola (H). R C

a $y = 9x^2 - 4$	**b** $y = 9x$	**c** $y = 9^x$	**d** $y = 9$
e $y = \frac{12}{x}$	**f** $y = 3x - 8$	**g** $y = 3x^2 - 8$	**h** $y = 2x + 5$
i $y = -x^2 + 6$	**j** $y = 10^{-x}$	**k** $y = 7x^2 + 2$	**l** $y = -\frac{2}{x}$

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EXAMPLE 17

7.09

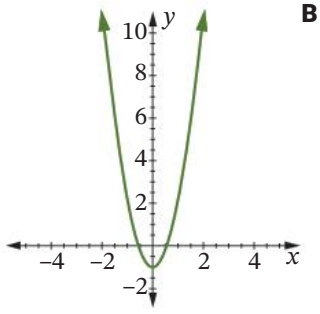
2 Match each equation with its graph. R C

a $x = 4$ **b** $y = -\frac{1}{2}x + 1$ **c** $y = 1 - x^2$ **d** $y = 5$

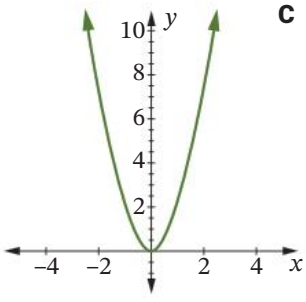
e $y = 3x^2 - 1$ **f** $y = 3^x$ **g** $y = -\frac{4}{x}$ **h** $y = 3^{-x}$

i $y = 2x^2$ **j** $y = 9x^2 - 4$ **k** $y = 3x - 2$ **l** $y = \frac{6}{x}$

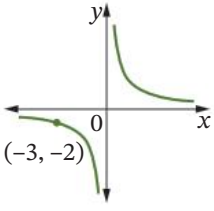
A



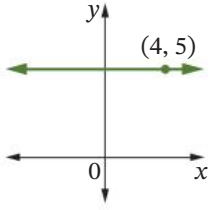
B



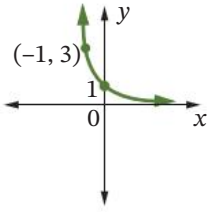
C



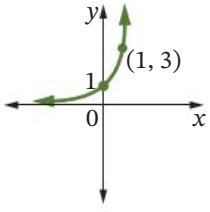
D



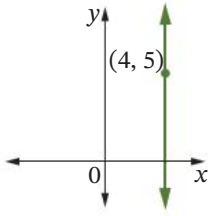
E



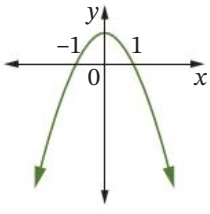
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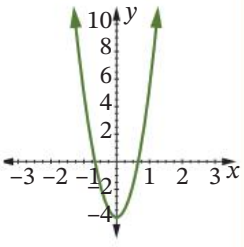
G



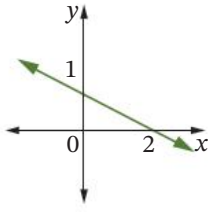
H



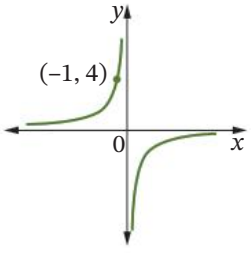
I



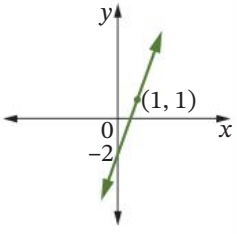
J



K



L



3 Sketch each equation, showing a point on the curve.

a $y = x^2 - 3$ **b** $y = 5^x$ **c** $y = -x^2 + 4$ **d** $y = 4 - x$

e $y = \frac{1}{2}x^2$ **f** $y = -2x + 4$ **g** $y = -\frac{5}{x}$ **h** $y = 10^{-x}$

4 Find the y-intercept of the graph of each equation.

a $y = 3^x$ **b** $y = 2x^2 + 3$ **c** $y = -7x^2 - 6$ **d** $y = 5^{-x}$

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Technology

Points of intersection

In this activity we will use graphing technology to find the point of intersection of a line and a curve.

1 Graph these equations of a line and parabola on the same screen.

$y = 3x - 4$ and $y = 2x^2 + x - 16$

2 How many points of intersection are there between the line and the parabola?

3 Click on each point of intersection to display its coordinates.

4 Graph each pair of equations to find the points of intersection of the line:

a $y = -x + 8$ and parabola $y = x^2 - 8x + 8$

b $x + y = 4$ and the exponential curve $y = 2^x$

c $y = 6x - 10$ and the parabola $y = 9x - x^2$

d $y = x - 3$ and hyperbola $y = -\frac{2}{x}$

e $y = x + 6$ and parabola $y = 5x^2$

f $15x + 8y = 17$ and exponential curve $y = 4^{-x}$

g $x - 2y = -6$ and hyperbola $y = \frac{8}{x}$

h $y = 2x - 1$ and hyperbola $y = \frac{15}{x}$

i $2x + y = -1$ and parabola $y = x^2 - 2x - 5$

j $x + 6y = 19$ and exponential curve $y = 3^x$

Technology

Identifying graphs

1 Use graphing technology to graph each equation and classify it as either a straight line (L), a parabola (P), a hyperbola (H) or an exponential curve (E).

a $y = 2x$ **b** $y = \frac{7}{x}$ **c** $y = x^2 + 1$

d $y = 2^x$ **e** $y = 2x^2 + 3$ **f** $y = 4 - 2x$

g $y = -\frac{1}{x}$ **h** $y = 2 - x$ **i** $y = 4 - x^2$

j $y = 5^{-x}$ **k** $y = -3x^2$ **l** $y = 3^x$

2 Without using graphing technology, classify each equation.

a $y = 3x - 2$ **b** $y = x^2 + 3$ **c** $y = 2^x + 1$

d $y = -\frac{4}{x}$ **e** $y = 4x^2 - 1$ **f** $y = 3^x - 2$

g $y = 4^x - 1$ **h** $y = 3x^2 - 4$ **i** $y = 10 - 2x^2$

j $y = -2x^2$ **k** $y = \frac{5}{x}$ **l** $y = -4 - 2x^2$

3 Check your answers to question **2** by drawing each equation using graphing technology.

4 State briefly in words how you distinguish between each type of equation in question **2**.

5 Use graphing technology to find the x-intercepts and y-intercepts (if they exist) of the graphs in questions **1** and **3**. Provide approximate answers where necessary.

Power plus ANSWERS ON P. 608

1 On the same set of axes, draw the graph of each equation.

a $y = x^2$ **b** $y = (x + 1)^2$ **c** $y = (x - 2)^2$

2 On the same set of axes, draw the graph of each equation.

a $y = -x^2$ **b** $y = -(x - 3)^2$ **c** $y = -(x + 2)^2$

3 For the graph of the parabola $y = (x + a)^2$, describe the effect on the graph of different values of a.

4 Draw the graph of each equation.

a $xy = 1$ **b** $y(x + 2) = 1$ **c** $y(x - 3) = 1$

CHAPTER 7 REVIEW

Quiz
Language of maths 7

Puzzle
Graphing curves crossword

Language of maths

asymptote	axis	coefficient	concave down
concave up	constant	conversion graph	curve
decay	direct variation	exponential	growth
hyperbola	inverse variation	parabola	proportional
quadratic	table of values	turning point	variable
vertex	vertical intercept	x-intercept	y-intercept

1 What is the coefficient of x^2 in the quadratic equation $y = 3x^2 + 10$?

2 What is the graph of $y = \frac{k}{x}$ called?

3 True or false: The exponential curve $y = 2^x$ passes through the point (0, 0).

4 In the variation equation $y = \frac{k}{x}$, which is the **constant of variation**?

5 Write down the equation of a parabola that is concave down and has a y-intercept of 3.

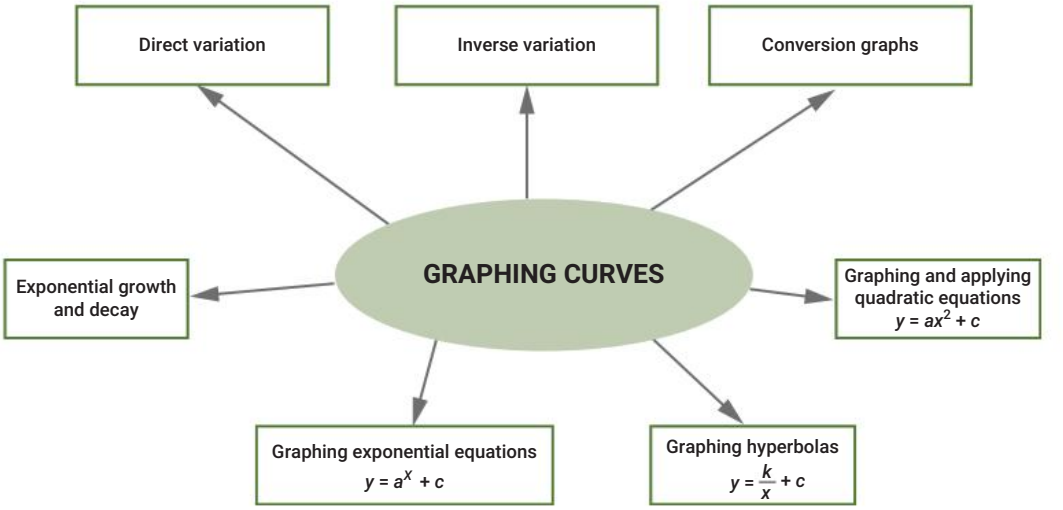
6 What is the asymptote of the exponential curve $y = a^x$?

Worksheet
Mind map: Graphing curves

Topic summary

- Which parts of this chapter were new to you?
- What is the difference between direct and inverse variation?
- Do you know the equations of a parabola, hyperbola and exponential curve, and how to graph them?
- Explain how the graph of $y = 2x^2 + 3$ is different from the graph of $y = -2x^2 + 3$. How are they similar?

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



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TEST YOURSELF 7 ANSWERS ON P. 609

1 H is directly proportional to t. If when $t = 12$, $H = 138$, find H when $t = 27$.

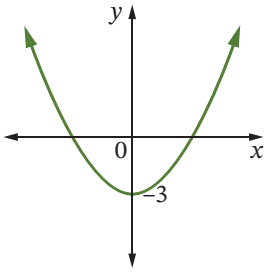
2 Air temperature, T (in degrees Celsius), varies inversely as its height, h (in metres), above sea level. At 400 m above sea level, the temperature is 15°C.
What is the temperature at 600 m above sea level?

3 The graph in Example **6** on page 293 converts Australian dollars to UK pounds sterling. Use the graph to convert:

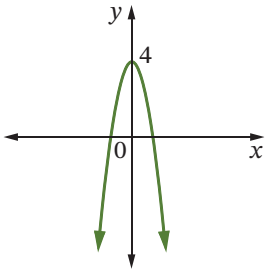
a \$A70 to £ **b** £56 to Australian dollars

4 Match each graph to its correct equation.

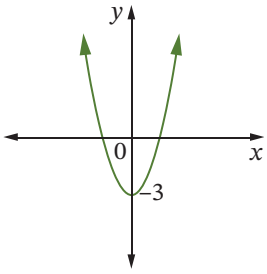
a



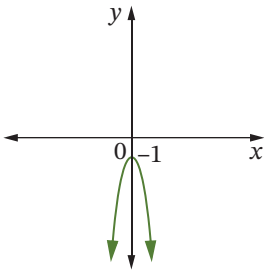
b



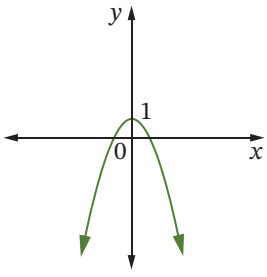
c



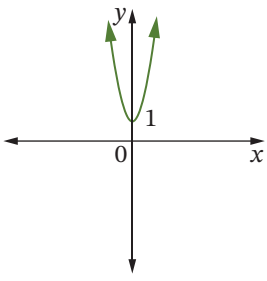
d



e



f



A $y = x^2 - 3$ **B** $y = 3x^2 + 1$ **C** $y = \frac{1}{2}x^2 - 3$

D $y = -x^2 + 1$ **E** $y = -4x^2 - 1$ **F** $y = 4 - 3x^2$

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7.05

5 A stone is dropped from the top of a building and its height (h) in metres after t seconds is given by the quadratic equation $h = 70 - 4.9t^2$.

a Copy and complete this table of values and graph the equation.

Use a scale of 1 cm = 1 second on the horizontal axis and 1 cm = 10 m on the vertical axis.

Time (t seconds)	0	1	2	3	4
Height (h metres)					

b What is the vertical intercept and what does it represent?

c What is the height of the stone after 1.5 seconds?

d How far does the stone fall in the 3rd second?

e Use the graph to determine the time it takes for the stone hit the ground.

PATH

7.06

6 Graph each hyperbola, showing the coordinates of one point on the curve.

a $y = -\frac{3}{x}$ **b** $y = \frac{8}{x}$ **c** $y = \frac{2}{x+1}$ **d** $y = -\frac{4}{x}$

7.07

7 Graph each exponential equation.

a $y = 4^x$ **b** $y = 4^{-x}$ **c** $y = -4^x$ **d** $y = -4^{-x}$

7.08

8 Simone is given some money on her 5th birthday, and this amount is doubled each birthday until she turns 21. The amount of money ($\$A$) she receives each birthday is modelled by the exponential equation $A = 5(2^n)$, where n is the number of years after her 5th birthday.

a Copy and complete this table of values to graph the equation.

n (years)	0	1	2	3	4	6	8	10
A (cents)								

b How much money did Simone receive on her 5th birthday? How is this shown on your graph?

c How much did Simone receive on her 9th birthday (in dollars)?

d At what age will Simone receive more than $1500?

e How much will Simone receive on her 21st birthday?

7.08

9 The salvage value of a car ($\$V$) after n years can be calculated using the equation $V = 12\,000\,(1.24)^{-n}$.

a Copy and complete this table of values, correct to the nearest $10.

n	0	1	2	3	4	5
V						

b Use a scale of 1 cm = 1 year on the horizontal axis and 1 cm = $1000 on the vertical axis to graph the equation.

c What is the vertical intercept and what does it represent?

d How much does the value of the car drop in the:

i 1st year? **ii** the 4th year?

e Use guess-and-check to find when the value of the car falls below $1000.

10 Match each equation with its correct graph.

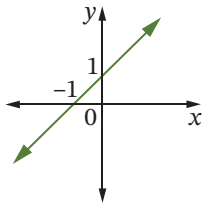
a $y = \frac{1}{4}x^2$ **b** $y = 3^x$ **c** $y = -2x^2 - 1$

d $x = -5$ **e** $y = -3x^2$ **f** $y = 2x^2 - 1$

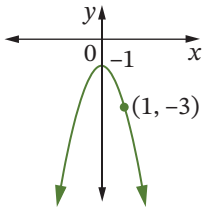
g $y = 3^{-x}$ **h** $y = -\frac{8}{x}$ **i** $y = x^2$

j $y = x + 1$ **k** $y = -5$ **l** $y = -2 - 2x$

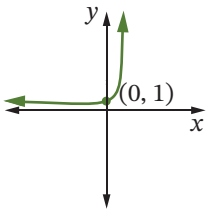
A



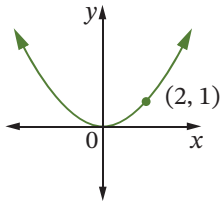
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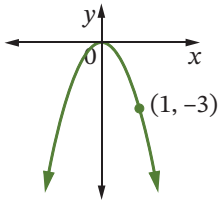
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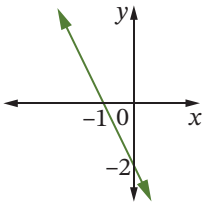
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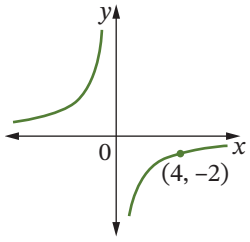
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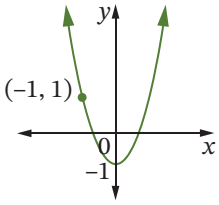
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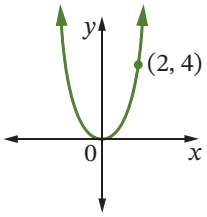
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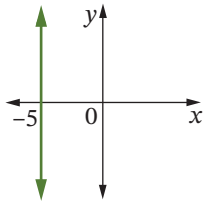
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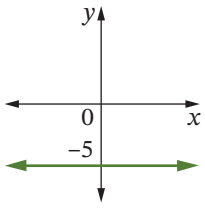
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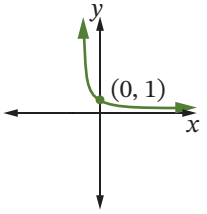
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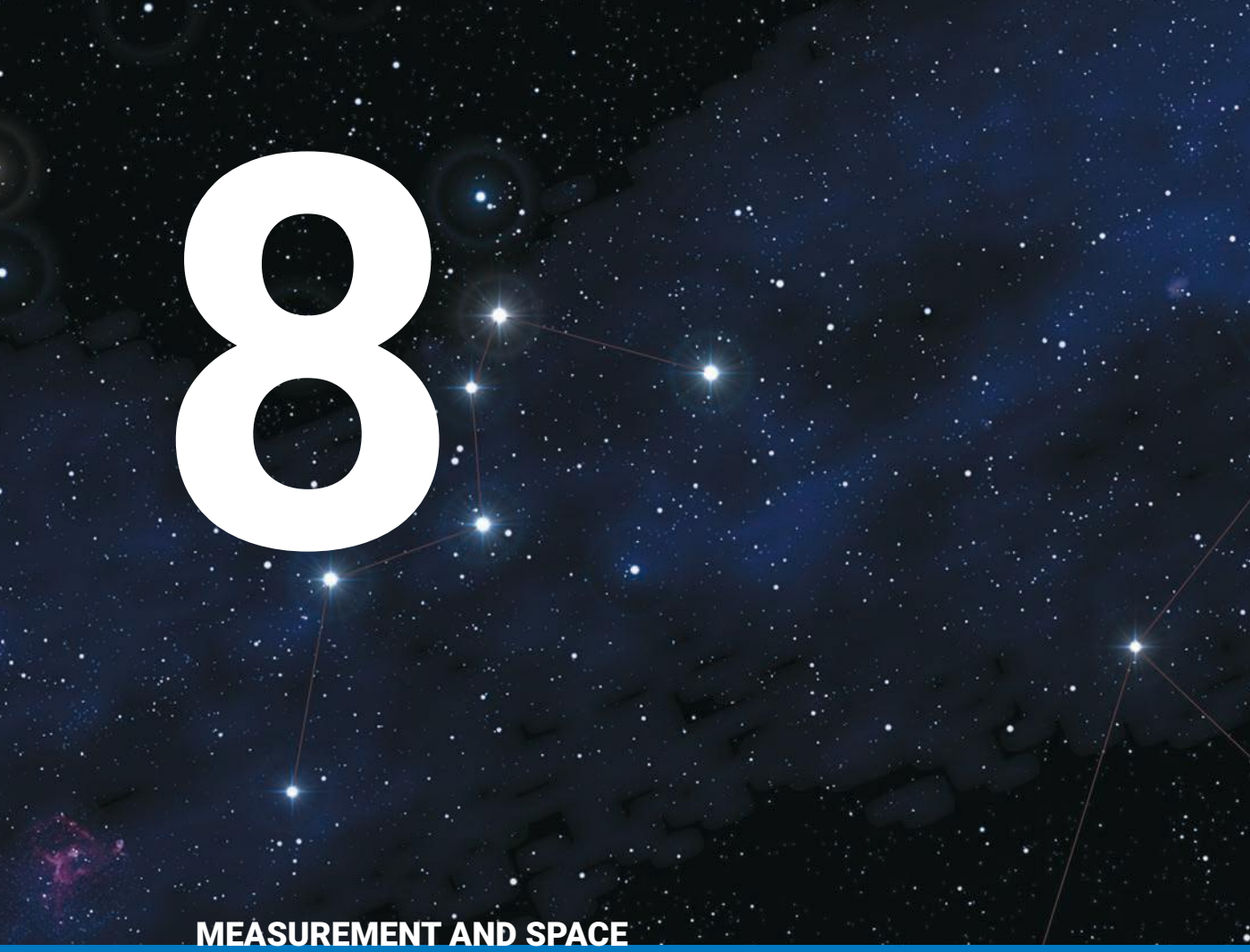


L



Foundation | Standard | Complex

9780170479622



8

MEASUREMENT AND SPACE

FURTHER TRIGONOMETRY

Trigonometry is a branch of mathematics that uses the relationship between the angles and sides of triangles. Trigonometry is used in navigation when locating the position or bearing of a destination from a known location, and finding the distances between places that cannot be physically measured.

Trigonometry helps us understand physical phenomena that are periodic or cyclic, such as tidal movements, phases of the moon, average monthly temperatures and sound waves. It is also used in construction, engineering, physics and gaming.



Chapter outline

		Working mathematically				
8.01	Right-angled trigonometry	U	F	PS		C
8.02	Angles of elevation and depression	U	F	PS		
8.03	Bearings	U	F	PS	R	C
8.04	Problems involving bearings	U	F	PS		C
8.05	Pythagoras' theorem and trigonometry in 3D*	U	F	PS	R	C
8.06	The sine rule*	U	F	PS	R	
8.07	The sine rule for angles*	U	F	PS	R	C
8.08	The cosine rule*	U	F	PS	R	C
8.09	The cosine rule for angles*	U	F	PS	R	
8.10	The area of a triangle*	U	F	PS	R	
8.11	Problems involving the sine and cosine rules*	U	F	PS	R	C

*PATH

Wordbank

Quiz
Wordbank 8

angle of depression The angle of looking down, measured from the horizontal

angle of elevation The angle of looking up, measured from the horizontal

bearing The angle used to show the direction of one location from a given point

cosine rule A rule that relates the 3 sides and one of the angles of any triangle:
$c^2 = a^2 + b^2 - 2ab \cos C$

included angle The angle between 2 known sides

minute (') A unit for measuring angle size, $\frac{1}{60}$ of a degree

sine rule A rule that relates the sides of any triangle to the sine of their opposite angles: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

theta (θ) A letter of the Greek alphabet used as a variable for angles

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Videos (24):

SkillCheck Trigonometry • Trigonometry on a calculator • Rounding angles

8.01 The trigonometric ratios • Finding an unknown side • Finding the hypotenuse • Finding an unknown angle • Trigonometry

8.02 Trigonometry 2 • Angles of elevation and depression • Angle of depression 1 • Angle of depression 2

8.03 Three-figure bearings

8.04 Bearings • Trigonometry 2 • True bearings • Problems involving bearings

8.06 The sine rule

8.07 Obtuse angles using the sine rule • The sine rule

8.08 The cosine rule

8.09 The cosine rule for angles 2 • The cosine rule • The cosine rule for angles 1

8.10 The sine area formula

8.11 The sine and cosine rules

Quizzes (5):

- Wordbank 8
- SkillCheck 8
- Mental skills 8
- Language of maths 8
- Test yourself 8

Worksheets (15):

SkillCheck Trigonometric calculations • Investigating trigonometric ratios

8.01 Trigonometry review • Trigonometry problems

8.03 A page of bearings • NSW map bearings • 16 points of the compass

8.04 Elevations and bearings

8.06 Discovering the sine rule

8.07 Sine rule problems

8.09 Cosine rule problems

8.11 Finding an unknown side • Finding an unknown angle • The sine and cosine rules

Mind map Further trigonometry

Puzzles (10):

8.01 Trigonometry match-up • Trigonometric equations 1 • Trigonometry squaresaw • Trigonometry: Finding angles • Solving triangles • Trigonometry equations 2

8.03 Bearings match-up

8.05 Pythagorean two-step problems

8.11 The sine and cosine rules

Language of maths Further trigonometry crossword

Presentations (2):

8.02 Applications of triangles

8.05 2D and 3D applications of trigonometry

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

In this chapter you will:

- use trigonometry to solve problems involving right-angled triangles, to find unknown sides and angles in right-angled triangles where the angle is measured in degrees and minutes
- solve trigonometry problems involving angles of elevation and depression, and bearings
- (PATH) use Pythagoras' theorem and trigonometry to solve three-dimensional problems
- (PATH) use the sine and cosine rules to find an unknown side or angle in any triangle
- (PATH) use the formula $A = \frac{1}{2}ab\sin C$ to find the area of a triangle with side lengths a, b and included angle C

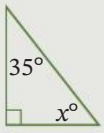
SkillCheck ANSWERS ON P. 611

1 Solve each equation.

a $\frac{x}{5} = 7$ **b** $\frac{h}{4} = 8.3$ **c** $\frac{45}{y} = 9$

2 Find the value of each variable.

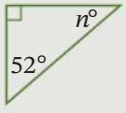
a



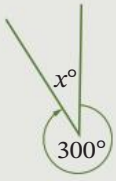
b



c



d



e

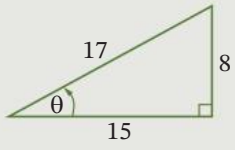


f

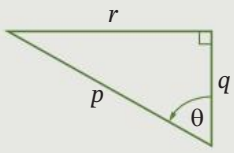


3 For each triangle, name the hypotenuse, opposite and adjacent sides for angle θ.

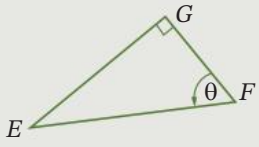
a



b



c



4 Evaluate each expression correct to 4 decimal places.

a cos 32° **b** sin 50.9° **c** 200 tan 18°

d tan 8° 45′ **e** 14 sin 87° 40′ **f** $\frac{13}{\cos 18° 27'}$

5 Round each angle to the nearest degree.

a 64° 27′ **b** 25° 43′ **c** 12° 8′ 50″

6 Round each angle to the nearest minute.

a 50° 19′ 26″ **b** 31° 55′ 55″ **c** 64° 18′ 30″

7 Convert each angle to degrees and minutes, correct to the nearest minute.

a 45.8° **b** 33.175° **c** 5.346°

Quiz
SkillCheck 8

Worksheets
Trigonometric calculations
Investigating trigonometric ratios

Videos
Trigonometry
Trigonometry on a calculator
Rounding angles

8

8.01 Right-angled trigonometry

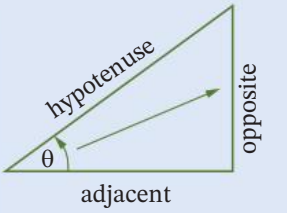
There are 3 special fractions called **trigonometric ratios** that relate the lengths of 2 sides of a right-angled triangle: **sine**, **cosine** and **tangent**.

Video The trigonometric ratios

Puzzle Trigonometry match-up

The trigonometric ratios

Ratio	Abbreviation	Meaning
sine	sin	$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$
cosine	cos	$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$
tangent	tan	$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$



You can remember the 3 ratios using **SOH-CAH-TOA** (pronounced 'so-car-toe-ah'):

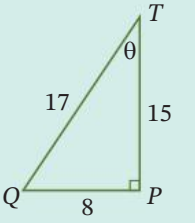
$$\sin = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{O}{H} \quad \text{SOH}$$

$$\cos = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{A}{H} \quad \text{CAH}$$

$$\tan = \frac{\text{opposite}}{\text{adjacent}} = \frac{O}{A} \quad \text{TOA}$$

Example 1

In $\triangle PTQ$, find $\sin\theta$, $\cos\theta$ and $\tan\theta$.



Solution

For angle θ, opposite = 8, adjacent = 15, hypotenuse = 17.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{17} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{15}{17} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{8}{15}$$

Videos Finding an unknown side; Finding the hypotenuse; Trigonometry

Finding an unknown side

We can use a trigonometric ratio to calculate the length of an unknown side in a right-angled triangle if one other side and angle are known. We need to select the correct ratio that links the given angle to the unknown side and known side.

Finding an unknown side in a right-angled triangle

1. Identify the 2 labelled sides and decide whether to use sin, cos or tan
2. Write an equation using the ratio, the given angle and the variable
3. Solve the equation to find the value of the variable



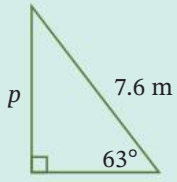
Puzzle
Trigonometry equations 1

8.01

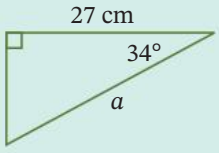
Example 2

Find the value of each variable, correct to one decimal place.

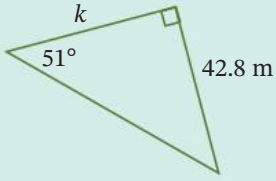
a



b



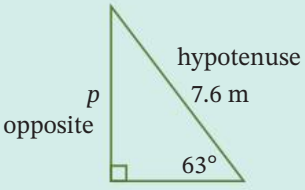
c



Solution

a SOH, CAH or TOA?

The marked sides are the opposite (O) side and the hypotenuse (H), so use sin.



$$\sin 63° = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$= \frac{p}{7.6}$$

$$\sin 63° \times 7.6 = \frac{p}{7.6} \times 7.6$$ Multiplying both sides by 7.6
$$7.6 \sin 63° = p$$

$$p = 7.6 \sin 63°$$ Swapping sides
$$= 6.7716...$$
$$\approx 6.8$$

From the diagram, a length of 6.8 m looks reasonable.

b 27 cm is adjacent, a is hypotenuse, so use cos.

$$\cos 34° = \frac{27}{a}$$ Note that the variable a appears in the denominator of the equation

$$\cos 34° \times a = \frac{27}{a} \times a$$ Multiplying both sides by a
$$a \cos 34° = 27$$

$$\frac{a \cos 34°}{\cos 34°} = \frac{27}{\cos 34°}$$ Dividing both sides by cos 34°
$$a = \frac{27}{\cos 34°}$$
$$= 32.5678...$$
$$\approx 32.6 \text{ cm}$$

Note that when the unknown appears in the denominator of an equation, it can swap positions with the trigonometric ratio, so that $\cos 34° = \frac{a}{27}$ becomes $a = \frac{27}{\cos 34°}$.

c 42.8 m is opposite, k is adjacent, so use tan.

$\tan 51° = \frac{42.8}{k}$ k appears in the denominator

$k = \frac{42.8}{\tan 51°}$ Swapping the position of k with tan 51°

$= 34.6587...$

≈ 34.7 m

Alternative method

To avoid having k in the denominator, we could use tan with the third angle of the triangle.

3rd angle $= 180° - 90° - 51°$

$= 39°$

$\tan 39° = \frac{k}{42.8}$

$k = 42.8 \tan 39°$

$= 34.6587...$

≈ 34.7 m

Videos
Finding an unknown angle
Trigonometry

Finding an unknown angle

A scientific calculator can also be used to find an unknown angle, θ, if the trigonometric ratio of the angle is known, for example, if $\sin \theta = 0.9063$.

An unknown angle can be found using the $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ keys on the calculator. These are the **inverse sin**, **inverse cos** and **inverse tan** functions, found by pressing the SHIFT or 2nd F key before the sin, cos or tan key.

Worksheets
Trigonometry review
Trigonometry problems

Puzzles
Trigonometry squaresaw
Trigonometry: Finding angles
Solving triangles
Trigonometry equations 2

Example 3

a If $\sin \alpha = \frac{5}{8}$, find the value of angle α, correct to the nearest degree.

b If $\tan X = 2.1572$, find the value of angle X, correct to the nearest minute.

Solution

a $\sin \alpha = \frac{5}{8}$

$\alpha = 38.6821...°$ On a calculator: SHIFT sin 5 ▭ 8 =

$\approx 39°$ Note: ▭ or $a^{b/c}$ is the fraction key

b $\tan X = 2.1572$ SHIFT tan 2.1572 =

$X = 65.1292...$

$= 65° 7' 45.35''$ °'" or DMS

$\approx 65° 8'$

Finding an unknown angle in a right-angled triangle

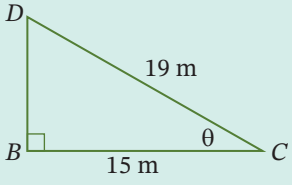
1. Identify 2 known sides and decide whether to use the sin, cos or tan ratio.
2. Write an equation using the ratio, the angle and the 2 sides as a fraction.
3. Use the calculator's inverse trigonometric function to find the size of the angle.



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Example 4

Find the size of angle θ, correct to the nearest degree.

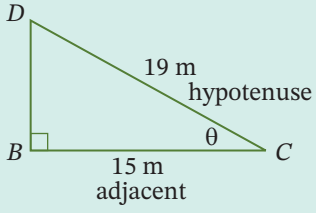


Solution

SOH, CAH or TOA?

The known sides are the adjacent (A) side and the hypotenuse (H), so use cos.

$\cos\theta = \frac{15}{19}$



$\theta = 37.8636...$

$\approx 38°$

On a calculator: SHIFT cos 15 ▭ =

From the diagram, an angle size of 38° looks reasonable.

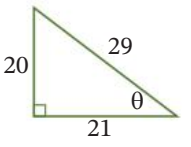
EXERCISE 8.01 ANSWERS ON P. 611

Right-angled trigonometry U F PS C

EXAMPLE 1

1 For each marked angle, find the sine, cosine and tangent ratios. C

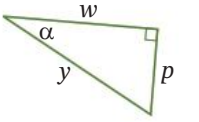
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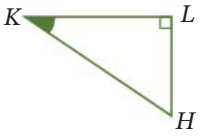
b



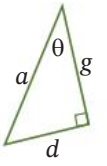
c



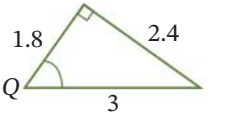
d



e



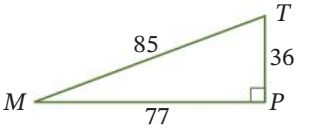
f



2 For the triangle shown, find:

a cos T **b** tan M

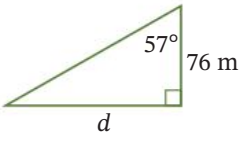
c sin T **d** sin M



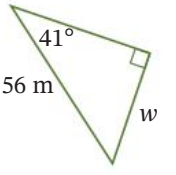
EXAMPLE 2

3 Find the value of the variable in each triangle, correct to one decimal place.

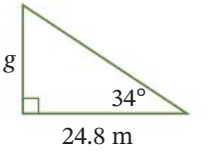
a



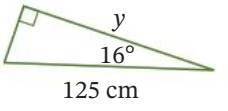
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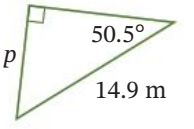
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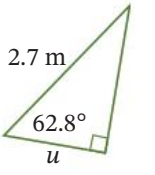
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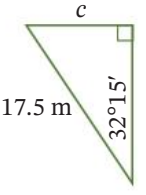
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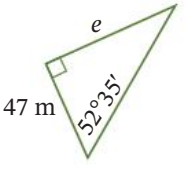
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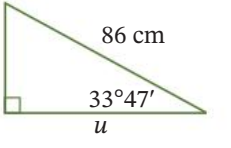
g



h



i



EXAMPLE 3

4 Find the size of angle θ, correct to the nearest degree.

a $\tan\theta = 1.85$ **b** $\sin\theta = 0.6432$ **c** $\cos\theta = \frac{\sqrt{3}}{2}$ **d** $\tan\theta = 7.1$

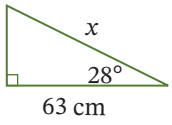
e $\sin\theta = \frac{3}{4}$ **f** $\cos\theta = \frac{2}{17}$ **g** $\tan\theta = \frac{1}{\sqrt{3}}$ **h** $\sin\theta = \frac{1}{\sqrt{2}}$

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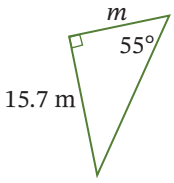
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5 Find the value of each variable, correct to one decimal place.

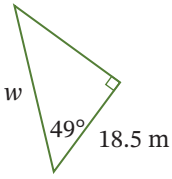
a



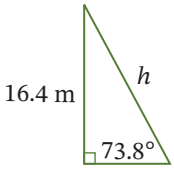
b



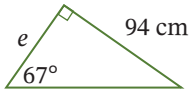
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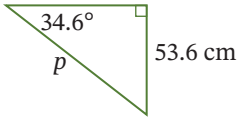
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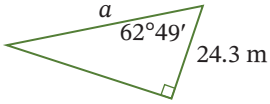
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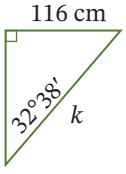
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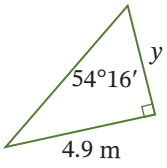
g



h

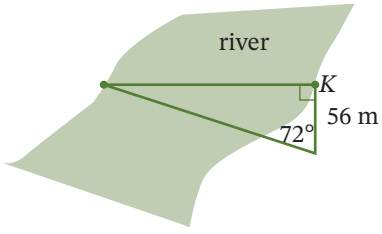


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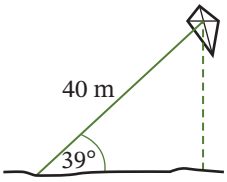


6 Find each length or distance, correct to one decimal place. PS

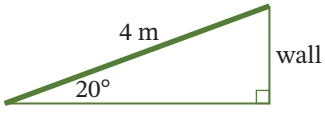
a The width of the river at the point K



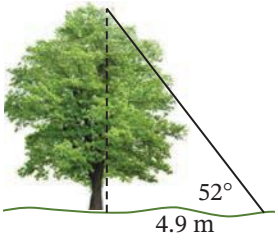
b The height of the kite



c The distance the base of the ramp is from the wall.



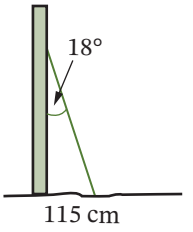
d The height of the tree



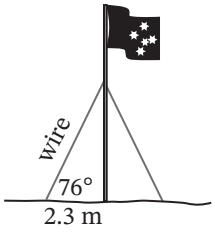
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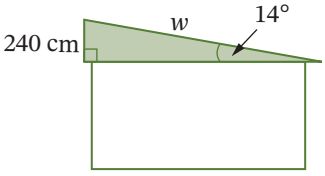
e The distance the ladder reaches up the wall.



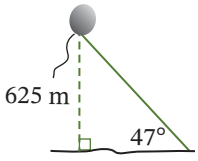
f The length of the support wire.



g The length of the sloping roof.



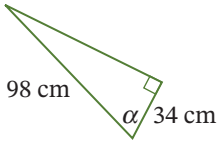
h The distance a balloon has drifted from its starting point.



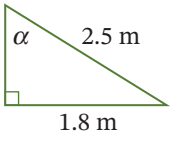
EXAMPLE 4

7 Find the size of angle α, correct to the nearest minute.

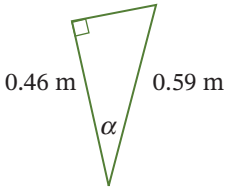
a



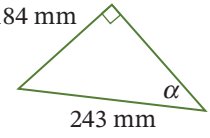
b



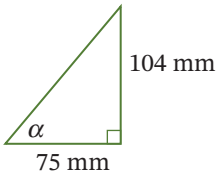
c



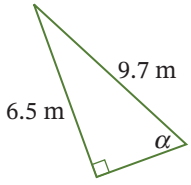
d



e

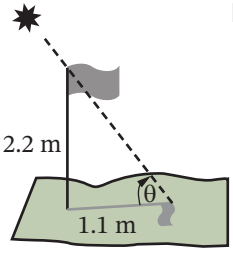


f

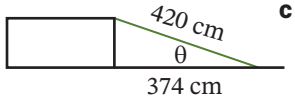


8 Find the size of angle θ, correct to the nearest degree.

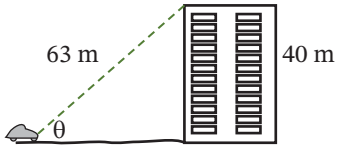
a



b



c



9 A ladder 4.3 m long is leaning against a wall. If the angle between the ladder and the wall is 28°, how far up the wall does the ladder reach? PS

Foundation Standard Complex

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10 The entrance to a bank has a ramp for wheelchair access that is 3.6 m long. If the ramp is inclined at an angle of 10° to the ground, what is the height of the entrance (to the nearest cm)? PS

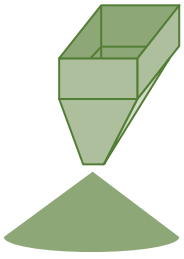
For questions **11** to **15**, give your answers correct to the nearest degree.

11 A ladder 6.8 m long is placed against a wall. If the foot of the ladder is 2.1 m from the base of the wall, find the angle at which the ladder is inclined to the ground. PS

12 A walking trail falls 7 m for every 130 m travelled along the trail. At what angle is the trail inclined to the horizontal? PS

13 A tree that is 15 m tall casts a shadow that is 19.6 m long. Find the angle of the sun from the ground. PS

14 When sand is poured from a hopper at a steady rate it forms a conical pile. The height of the pile is 3 m and its diameter is 8.6 m. Calculate the angle of repose of the sand (the angle the sloping side makes with the horizontal base) PS



15 Calculate the take-off angle of a passenger plane if its altitude after 30 seconds is 600 m and it has flown a distance of 2500 m. PS



Shutterstock.com/ Ryan Fletcher

16 A rectangular gate has a diagonal brace that makes an angle of 35° with the top of the gate. The height of the gate is 1200 mm. Calculate the length of the diagonal brace. Select the correct answer **A**, **B**, **C** or **D**. PS

A 1465 mm **B** 1714 mm **C** 2092 mm **D** 2134 mm

17 A stretch of road rises 220 m for every 3 km travelled along the road. Find the angle at which the road is inclined to the horizontal. PS

18 A roof has a pitch of 18° 26′. The height of the roof is 4.1 m. Calculate the overall width of the roof (correct to one decimal place). PS



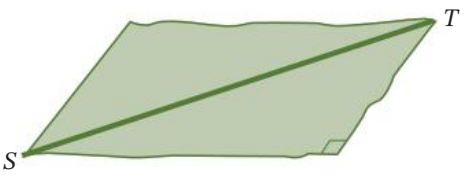
19 A glider is descending at an angle of 25° to the horizontal. The length of its flight path until it lands is 3.5 km. What was the altitude (to the nearest 0.1 km) of the glider? PS



COLOMBO NICOLA/ Shutterstock.com

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20 The diagonal path of a large rectangular reserve makes an angle of 32° with the longer side of the reserve. The width of the reserve is 230 m. PS



a Calculate the length of the path (correct to the nearest metre).

b How much shorter is walking along the path from S to T than walking along the 2 edges of the reserve?

21 $\triangle PQR$ is right-angled at Q, $PR = 15.3$ m and $\angle P = 23°$. Find the length of QR, correct to 2 decimal places.

22 $\triangle TPM$ is right-angled at M, $MT = 39.3$ cm and $\angle T = 44.7°$. Find the length of PT, correct to one decimal place.

23 In $\triangle WYX$, $\angle Y = 90°$, $\angle W = 74°\ 25'$ and $XY = 245$ mm. Find the length of WY, correct to the nearest millimetre.

24 $\triangle KLM$ is right-angled at L, $KL = 19$ cm and $LM = 21$ cm. Find $\angle K$, correct to the nearest degree.

25 In $\triangle PTW$, $\angle T = 90°$, $PW = 22.3$ m and $TW = 7.6$ m. Find $\angle W$, correct to the nearest degree.

26 In $\triangle ABC$, $\angle B = 90°$, $BC = 0.8$ m and $AC = 1.3$ m. Find $\angle C$, correct to the nearest minute.

Did you know?

The Greek alpha-bet

Here are 8 letters (in lower-case and capitals) from the Greek alphabet.

α, A alpha	β, B beta	γ, Γ gamma	δ, Δ delta
θ, Θ theta	φ, Φ phi	σ, Σ sigma	ω, Ω omega

The ancient Greeks had a great influence on the development of mathematics. It is traditional to use Greek letters as variables, particularly in geometry and trigonometry.

1 **Find out how many letters there are in the Greek alphabet, and name each one.**

2 **Compare the Greek alphabet with our Roman alphabet.**

3 **Can you see where the word *alphabet* comes from? Explain how it originated.**

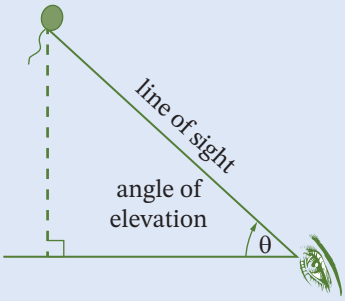
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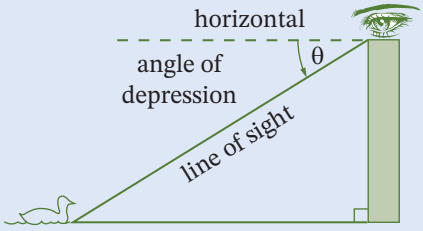
Angles of elevation and depression 8.02

Angles of elevation and depression

The **angle of elevation** is the angle of looking **up**, measured from the horizontal.

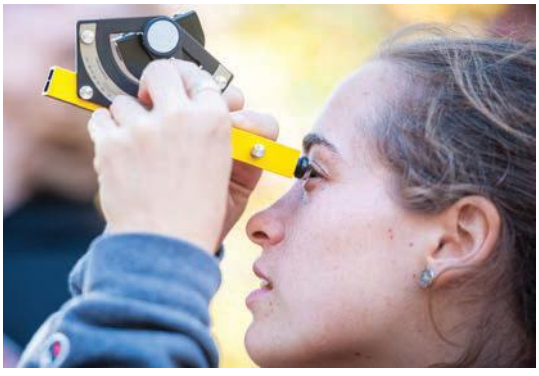


The **angle of depression** is the angle of looking **down**, measured from the horizontal.



Problems involving angles of elevation and depression usually require the tan ratio in their solutions.

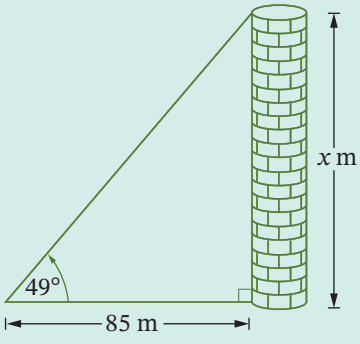
An instrument for measuring an angle of elevation or depression is a **clinometer**. It is like a protractor with a sighting tube attached.



Getty Images/VW Pics

Example 5

The angle of elevation of a chimney is 49° at a distance of 85 m from its base. Find the height of the chimney, to the nearest metre.



Solution

Let the height be x metres.

$$\tan 49° = \frac{x}{85}$$

$$x = 85 \tan 49°$$

$$= 97.7813...$$

$$\approx 98$$

The height of the chimney is 98 m.

Videos
Trigonometry 2
Angles of elevation and depression

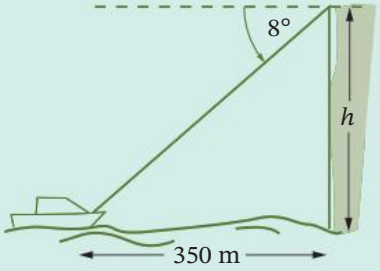
Presentation
Applications of triangles

8.02

Videos
Angles of elevation and depression
Angle of depression 1
Angle of depression 2

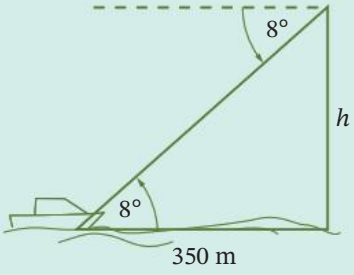
Example 6

The angle of depression of a boat from the top of a cliff is 8°. If the boat is 350 m from the base of the cliff, calculate the height of the cliff, correct to the nearest metre.



Solution

By alternate angles on parallel lines, the angle of elevation of the top of the cliff from the boat is also 8°.



$$\tan 8° = \frac{h}{350}$$

$$h = 350 \tan 8°$$

$$= 49.1892...$$

$$\approx 49$$

The height of the cliff is 49 m.

Alternative method:

The third angle in the triangle (adjacent to the angle of depression) = 90° – 8° = 82°.

$$\tan 82° = \frac{350}{h}$$

$$h = \frac{350}{\tan 82°}$$

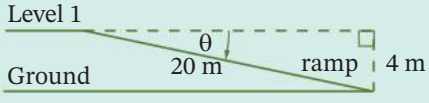
$$= 49.1892...$$

$$\approx 49$$

The height of the cliff is 49 m.

Example 7

The ramp from one level to the next in a car park is 20 m long and drops 4 m. Find the angle of depression of the ramp, to the nearest degree.



Solution

$$\sin\theta = \frac{4}{20}$$

$$\theta = 11.5369...°$$

$$\approx 12°$$

The angle of depression of the ramp is 12°.

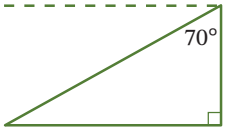
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EXERCISE 8.02 ANSWERS ON P. 612

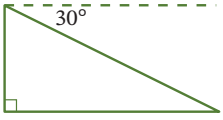
Angles of elevation and depression U F PS

1 Copy each diagram, mark the angle of elevation θ and find its size.

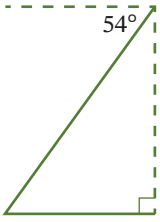
a



b

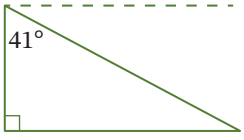


c

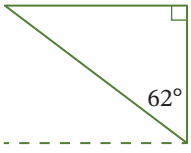


2 Copy each diagram, mark the angle of depression θ and find its size.

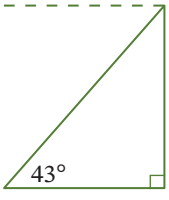
a



b

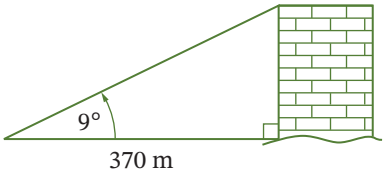


c

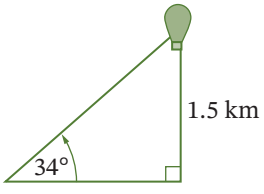


3 Millie stands 370 m from the base of a building. Using a clinometer, she finds that the angle of elevation of the top is 9°. Find the height of the building, to the nearest metre.

EXAMPLE 5

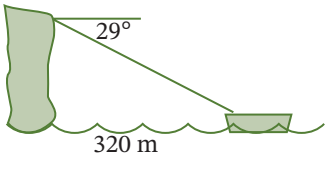


4 The angle of elevation of a weather balloon at a height of 1.5 km is 34°. How far (to the nearest metre) is the observer from being directly under the balloon?

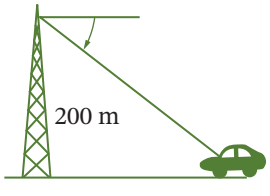


5 A raft is 320 m from the base of a cliff. The angle of depression of the raft from the top of the cliff is 29°. Find the height of the cliff, to the nearest metre. PS

EXAMPLE 6



6 From the top of a 200 m tower, the angle of depression of a car is 48°. How far is the car from the foot of the tower? PS



7 A 275 m radio mast is 1.7 km from a school. Find, correct to the nearest degree, the angle of elevation of the top of the mast from the school. PS

EXAMPLE 7

8 A monument 24 m high casts a shadow 20 m long. Calculate, correct to the nearest degree, the angle of elevation of the sun at this time of day. PS

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9 In a concert hall, Ethan is sitting 20 m from the stage by line of sight. He is also 5 m above the level of the stage. At what angle of depression is the stage? Answer to the nearest minute. PS

10 A plane is 340 m directly above one end of a 1000 m runway. Find, correct to the nearest minute, the angle of depression to the far end of the runway. PS



verzellenberg/ Shutterstock.com

11 An observer 174 cm tall is standing 11.6 m from the base of a flagpole. The angle of elevation to the top of the flagpole is 43°. How high is the flagpole, to the nearest cm? PS

12 A flagpole is mounted on top of a tall building. At a distance of 250 m from the base of the building, the angles of elevation of the bottom and top of the flagpole are 38° and 40° respectively. Calculate the height of the flagpole, correct to one decimal place. PS

13 A news helicopter hovers at a height of 500 m. The angles of depression of a fire moving in the direction of the helicopter are first 10° and then 15°. How far (to the nearest metre) has the fire moved between the 2 observations? PS

14 The angle of elevation to the bottom of a transmission tower on a hill from an observer 1.8 km away from the base of the hill is 5°. The angle of elevation to the top of the tower from the observer is 6.8°. Find the height of the tower to the nearest metre. PS

8.03 Bearings

Worksheets
A page of bearings

NSW map bearings

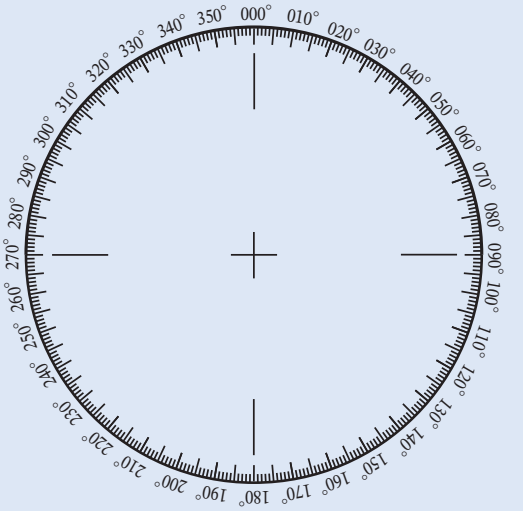
16 points of the compass

Puzzle
Bearings match-up

Bearings are used in navigation. A **bearing** is an angle measurement used to describe precisely the direction of one location from a given reference point.

True bearings

True bearings, also called **three-figure bearings**, use angles from 000° to 360° to show the amount of turning measured **clockwise from north 000°**. Note that the angles are always written with 3 digits.



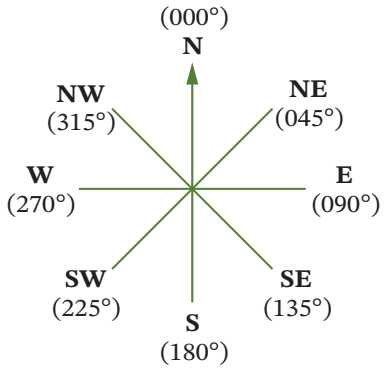
True bearings

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This **compass rose** shows the true bearings of 8 points on the compass. A bearing of due east is 090°, while a compass direction of southwest (SW) is 225°.

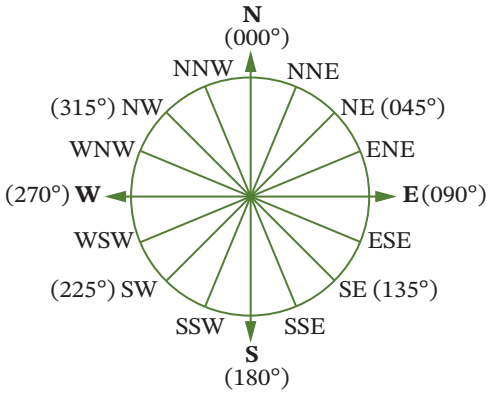
- Bearings from 000º to 090º are in the NE quadrant
- Bearings from 090º to 180º are in the SE quadrant
- Bearings from 180º to 270º are in the SW quadrant
- Bearings from 270º to 360º are in the NW quadrant



8.03

This diagram shows the 16 points of the compass.

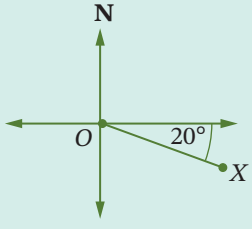
N = north	NE = northeast
E = east	SE = southeast
S = south	SW = southwest
W = west	NW = northwest
NNE = north-northeast	ENE = east-northeast
ESE = east-southeast	SSE = south-southeast
SSW = south-southwest	WSW = west-southwest
WNW = west-northwest	NNW = north-northwest



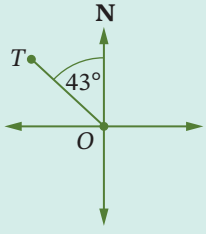
Example 8

Find the true bearing of each point from O.

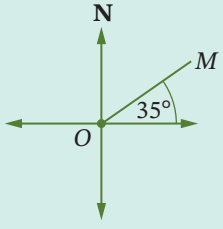
a



b



c



Solution

a Bearing of X from O is $90° + 20° = 110°$.

b Bearing of T from O is $360° - 43° = 317°$.

c Bearing of M from O is $90° - 35° = 055°$

This is sometimes written as 110°T to indicate that it is a True bearing.

Must be written as a 3-digit angle

Video
Three-figure bearings

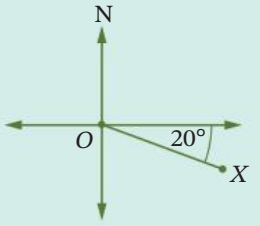
Compass bearings

Compass bearings use acute angles measured from the North-South axis towards East or West, such as N20°E or S57°W. They are named according to the quadrant in which they are located, and are used in land surveying.

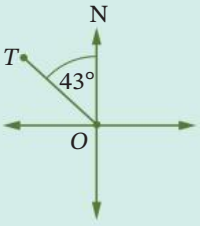
Example 9

Write the compass bearing of each point from O.

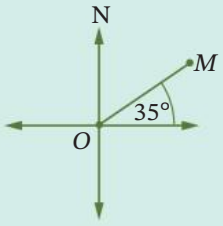
a



b



c



Solution

a X is in the southeast (SE) quadrant. From south, it is $90° - 20° = 70°$ towards east.
The compass bearing of X from O is S70°E.

b T is in the northwest (NW) quadrant. From north, it is 43° towards west.
The compass bearing of T from O is N43°W.

c M is in the northeast (NE) quadrant. From north, it is $90° - 35° = 55°$ towards east.
The compass bearing of M from O is N55°E.

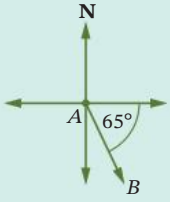
Video
Three-figure bearings

Example 10

Sketch point B on a compass rose if B has a bearing of 155° from A.

Solution

Draw the compass rose on the point where the bearing is measured from.
155° is between 90° and 180°, so B is in the southeast (SE) quadrant.
$155° - 90° = 65°$, so B is 65° from east (E).



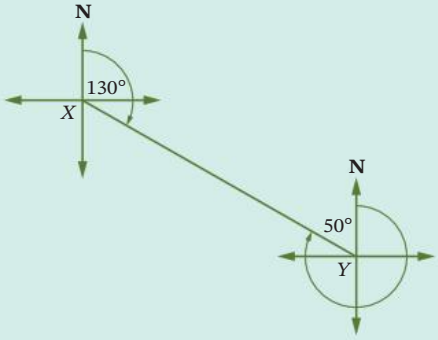
Video
Three-figure bearings

Example 11

The bearing of Y from X is 130°. What is the bearing of X from Y?

Solution

Sketch the bearing of Y from X.
On the same diagram, draw a compass rose at Y and find $\angle NYX$.
$\angle NYX = 50°$ (co-interior angles, $NX \parallel NY$)
$\therefore$ Bearing of X from $Y = 360° - 50°$
$= 310°$



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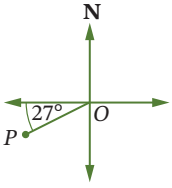
EXERCISE 8.03 ANSWERS ON P. 612

Bearings UFRC

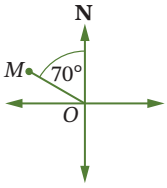
1 Find the true bearing of each point from O. C

EXAMPLE 8

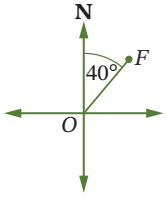
a



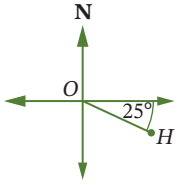
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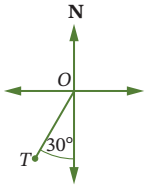
c



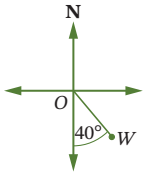
d



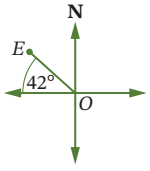
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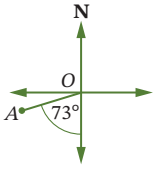
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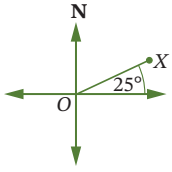
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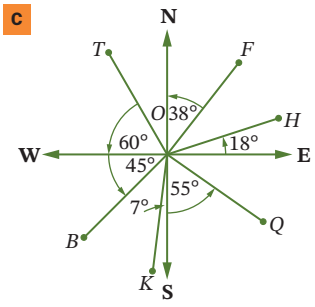
h



i



2 What is the true bearing of each direction or point from O? C



a	N	**b**	E	**c**	S
d	W	**e**	F	**f**	Q
g	T	**h**	B		
i	H	**j**	K		

3 Find the compass bearing of each point from O in Question **1**. C

EXAMPLE 9

4 Find the true bearing of each compass direction. C

a	SW	**b**	E	**c**	NE	**d**	ESE
e	NNW	**f**	W	**g**	SE	**h**	WSW

5 Sketch each bearing on a compass rose.

a	220°	**b**	060°	**c**	260°	**d**	125°
e	N10°W	**f**	S87°W	**g**	S9°E	**h**	N32°E

6 **a** What is the compass direction halfway between northwest and west?

b What is the compass bearing of this compass direction? C

7 Sketch P on a compass rose if P has a bearing of:

EXAMPLE 10

a	132° from T	**b**	260° from M	**c**	335° from X	**d**	010° from K

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EXAMPLE 11

8 If the bearing of P from A is 060°, what is the bearing of A from P? R

9 The bearing of T from Y is 100°. What is the bearing of Y from T? R

10 What is the (smallest) angle between:

a S and SW? **b** NE and SE? **c** E and NW?

d NE and SSW? **e** E and SSW? **f** SW and WNW?

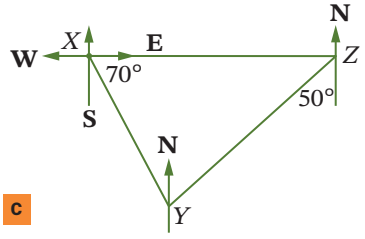
11 The compass bearing of H from M is N65°W. Find the compass bearing of M from H. R C

12 Draw a diagram for each situation described. C

a A plane flies on a bearing of 280° for 150 km and then another 250 km on a bearing of 080°.

b A cyclist travels 15 km due east and then 20 km on a S45°W bearing.

13 For this diagram, find the true bearing of: C



a Y from Z

b X from Z

c Y from X

14 For the above diagram, find the compass bearing of: C

a X from Y

b Z from Y

c Z from X

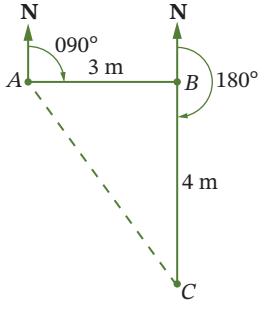
Investigation

Compass walks

You need: a directional compass and a tape measure or trundle wheel.

This activity can also be done in the classroom using scale drawings, graph paper, a ruler and a protractor.

A triangular walk



1 Starting at A, walk due east for 3 m to B.

2 From B, walk due south for 4 m to C.

3 How far is C from A?

4 What is the bearing of:

a A from C?

b C from A?

A square walk

1 Starting at P, walk at a bearing of 045° for 8 m to Q.

2 From Q, walk at a bearing of 315° for 8 m to R.

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3 From R, walk at a bearing of 225° for 8 m to S.

4 How far is S from P?

5 What is the bearing of:

a P from S? b S from P?

A pentagonal walk

1 Starting at U, walk at a bearing of S50°E for 4 m to V.

2 From V, walk at a bearing of N40°E for 7 m to W.

3 From W, walk at a bearing of N40°W for 4.8 m to X.

4 From X, walk at a bearing of west for 4.5 m to Y.

5 How far is Y from U?

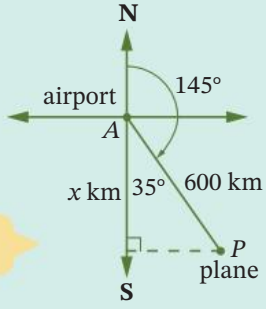
6 What is the bearing of:

a U from Y? b Y from U?

Problems involving bearings 8.04

Example 12

A plane leaves an airport and flies 600 km on a bearing of 145°. How far south of the airport is the plane (to the nearest km)?



Solution

To find x, create a right-angled triangle.

Let x km = distance south

$\therefore \angle SAP = 180° - 145°$ (angles on a straight line)

$= 35°$

$\cos 35° = \frac{x}{600}$

$x = 600 \cos 35°$

$= 491.4912...$

≈ 491

The plane is 491 km south of the airport.

Worksheet
Elevations and bearings

Videos
Bearings
Trigonometry 2
True bearings

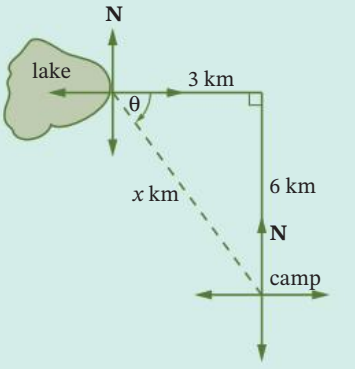
Video
Problems involving bearings

Example 13

From camp, Sumaiyaa walks due north for 6 km, then 3 km due west to a lake.

a How far is Sumaiyaa from the camp, correct to the nearest 0.1 km?

b What is the bearing of the camp from the lake (to the nearest minute)?



Solution

a Let x = distance from camp.

$x^2 = 6^2 + 3^2$ by Pythagoras' theorem

$= 45$

$x = \sqrt{45}$

$= 6.708...$

≈ 6.7 km

Sumaiyaa is 6.7 km from the camp.

b Note angle θ in the diagram.

$\tan\theta = \frac{6}{3}$

$\theta = 63.434...°$

$= 63° 26' 5.82"$

$\approx 63° 26'$

Bearing of camp from lake

$= 90° + 63° 26'$

$= 153° 26'$

EXERCISE 8.04 ANSWERS ON P. 613

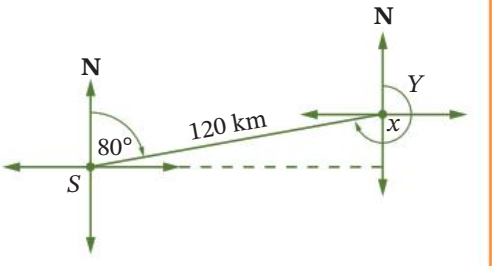
Problems involving bearings U F PS C

EXAMPLE 12

1 A yacht leaves Sydney and sails 120 km on a bearing of 080°.

a How far north (to the nearest km) of Sydney is the yacht?

b What is the bearing of Sydney from the yacht?

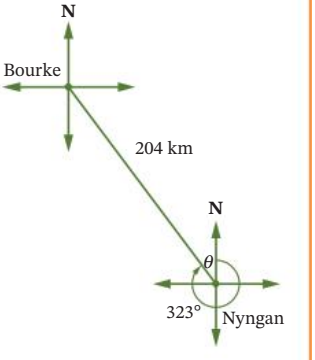


2 Declan leaves Nyngan and drives 204 km to Bourke. The bearing of Bourke from Nyngan is 323°.

a Find the value of θ.

b How far north (to the nearest km) of Nyngan is Bourke?

c What is the bearing of Nyngan from Bourke?



Foundation Standard Complex

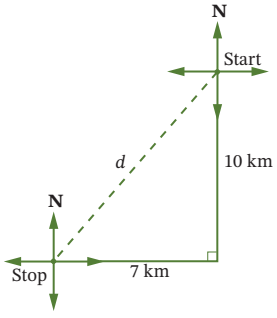
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3 The distance 'as the crow flies' from Sydney to Wollongong is 69 km. If the bearing of Wollongong from Sydney is 205°, calculate:

- **a** how far south Wollongong is from Sydney (correct to one decimal place)
- **b** how far east Sydney is from Wollongong (correct to one decimal place)
- **c** the compass bearing of Sydney from Wollongong.

4 Jana cycles 10 km due south, then 7 km due west.

- **a** How far (correct to one decimal place) is Jana from her starting point?
- **b** What is her true bearing from the starting point, correct to the nearest degree?



5 A triathlete cycles 20 km on a SSE bearing to the finish line.

- **a** How far (to the nearest km) has the triathlete travelled in a southerly direction?
- **b** What is the compass bearing of the starting point from the finish line? PS C

6 A hiking group walks from Sandy Flats to Black Ridge (a distance of 20.9 km) in the direction 078°. They then turn and hike due south to Rivers End, then due west back to Sandy Flats. How far have they hiked altogether (to the nearest 0.1 km)? PS C

7 A triangular orienteering run starts at Alpha and passes through the checkpoints of Bravo and Charlie before finishing at Alpha. Bravo is 8.5 km due east of Alpha, and Charlie is 10.5 km due south of Bravo. PS C

- **a** Calculate, correct to 3 decimal places, the distance from Charlie to Alpha.
- **b** Find the compass bearing of Alpha from Charlie to the nearest degree.

8 A plane takes off from Darwin at 10:15 a.m. and flies on a bearing of 150° at 700 km/h. PS C

- **a** How far (to the nearest km) due south of Darwin is the plane at 1:45 p.m.?
- **b** What is the bearing (correct to the nearest degree) of Darwin from the plane?

9 A fishing trawler sails 30 km from port on a bearing of S60°E until it reaches a submerged reef. How far (to the nearest km) is the port: PS C

- **a** north of the reef?
- **b** west of the reef?

10 2 racing pigeons are set free at the same time. The first bird flies on a course of N40°E while the second bird flies on a course of S50°E. PS C

- **a** The first bird flies 200 km until it is due north of the second bird. Find their distance apart, correct to 2 decimal places.
- **b** How far has the second bird flown?

11 2 horse riders start from the same stable. The rider of the black horse goes due west for 5.5 km and stops. The rider of the chestnut horse travels in a direction of N57°W until he is due north of the black horse. How far did the rider of the chestnut horse travel? Answer correct to 2 significant figures. PS C

12 2 ships leave from the same port. One ship travels on a bearing of 157° at 20 knots. The second ship travels on a bearing of 247° at 35 knots. (1 knot is a speed of 1 nautical mile per hour.) PS C

a How far apart are the ships after 8 hours, to the nearest nautical mile?

b Calculate the bearing of the second ship from the first, to the nearest minute.

13 A plane flies 1200 km on a NE compass bearing and then changes direction to a bearing of NW and flies for a distance of 450 km. PS C

a How far, correct to the nearest km, is the plane from its starting point?

b What is the compass bearing of the plane from its starting point, to the nearest minute?

14 A fishing boat leaves Sydney Harbour on a bearing of ESE and after 15 nautical miles, changes direction to a bearing of SSW and travels a further 10 nautical miles.

Find the true bearing of Sydney from the boat's final position, correct to the nearest minute. PS C

8.05 Pythagoras' theorem and trigonometry in 3D

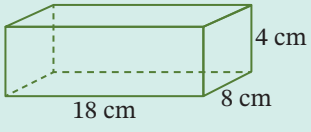
PATH

Puzzle
Pythagorean two-step problems

Presentation
2D and 3D applications of trigonometry

Example 14

A wooden rectangular box has dimensions 18 cm × 8 cm × 4 cm.



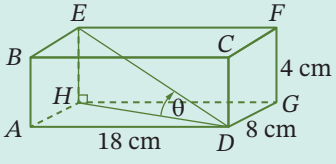
a Find, correct to one decimal place, the length of the longest pencil that can lie flat in the base of the box.

b Find, correct to one decimal place, the length of the longest diagonal of the box.

c Find, correct to the nearest degree, the angle that the longest diagonal makes with the base of the box.

Solution

Label the box as shown. *HD* is the length of the longest pencil that can lie flat in the base of the box, while *ED* is the longest diagonal of the box.



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a Using the right-angled triangle DAH:

$HD^2 = DA^2 + AH^2$

$= 18^2 + 8^2$

$= 388$

$HD = \sqrt{388}$

$= 19.6977...$

≈ 19.7 cm

The longest pencil that can lie flat in the base of the box is 19.7 cm.

b $ED^2 = HD^2 + HE^2$

$ED^2 = \left(\sqrt{388}\right)^2 + 4^2$

$= 404$

$ED = \sqrt{404}$

$= 20.0997...$

≈ 20.1 cm

The longest diagonal of the box is 20.1 cm.

c In the right-angled triangle EHD, θ is the angle that the longest diagonal makes with the base of the box.

$\tan\theta = \frac{EH}{HD}$

$= \frac{4}{\sqrt{388}}$

$\theta = 11.4789...°$

$\approx 11°$

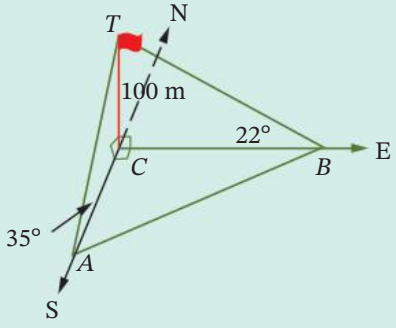
The longest diagonal makes an angle of 11° with the base of the box.

or use sin or cos since ED is also known

4 $\frac{\square}{\square}$ $\sqrt{\square}$ 388 SHIFT tan =

Example 15

A 100 m flagpole is observed from 2 different locations. From point A, due south of the flagpole the angle of elevation of the top of the flagpole is 35°; from point B, due east of the flagpole the angle of elevation is 22°. Find the distance between A and B, correct to the nearest metre.



Solution

There are 3 right-angled triangles in this diagram. To find AB, we must first find AC and CB using trigonometry.

In $\triangle ACT$,

$\tan 35° = \frac{100}{AC}$

$AC = \frac{100}{\tan 35°}$

$= 142.8148...$

Do not round yet.

In $\triangle BCT$,

$\tan 22° = \frac{100}{CB}$

$CB = \frac{100}{\tan 22°}$

$\approx 247.5086...$

In $\triangle ABC$,

$AB^2 = AC^2 + CB^2$

$= (142.8148...)^2 + (247.5086...)^2$

$= 81656.6166...$

$AB = \sqrt{81656.6166...}$

$= 285.7562...$

≈ 286 m

From the diagram, 286 m seems to be a reasonable answer.

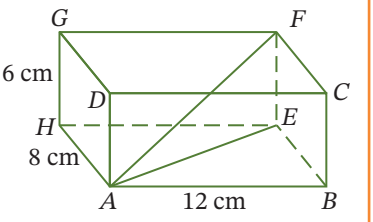
PATH

EXERCISE 8.05 ANSWERS ON P. 613

Pythagoras' theorem and trigonometry in 3D U F PS R C

EXAMPLE 14

1 For this rectangular prism, calculate:

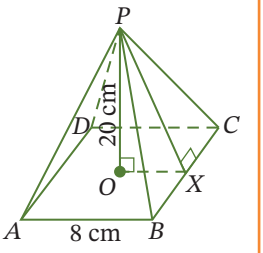


- **a** the length of AE in surd form
- **b** the length of AF, correct to one decimal place
- **c** the size of $\angle FAE$, correct to the nearest degree.

2 Sketch a diagram of a cube of length 20 cm, then find:

- **a** the exact length of the longest diagonal on any face
- **b** the length of the longest diagonal of the cube, correct to 2 decimal places
- **c** the angle that the longest diagonal makes with the base, correct to the nearest minute.

3 The diagram shows a square pyramid with base length 8 cm and perpendicular height 20 cm. PX is the slant height of the pyramid. Calculate: R



- **a** the length of PX, correct to 2 decimal places
- **b** the angle of inclination of PX, correct to the nearest degree.

4 A cone has a base diameter of 2.8 m and a slant height of 2.5 m. Find the angle that the cone makes with the vertical at the top of the cone. R

5 A fruit juice container is a square prism with dimensions 8 cm by 3 cm by 3 cm. R

- **a** Find, correct to one decimal place, the length of the longest drinking straw that fits inside the container.
- **b** To the nearest degree, what angle does the longest straw make with the vertical?

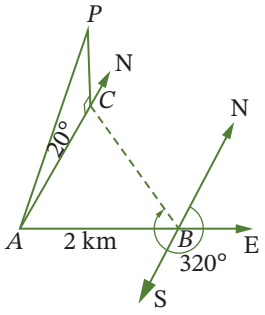
6 From a point X, 37 m from the base of a tree, the angle of elevation is 55° while the angle of elevation of the tree, from a point Y due east of the tree, is 25°. Find, correct to the nearest metre: PS R

- **a** the height of the tree
- **b** the distance of the tree from point Y

EXAMPLE 15

7 A 15 m flagpole stands on level ground. From point P, due west of the flagpole the angle of elevation of the top of the pole is 38°. From point Q, due north of the flagpole, the flagpole has an angle of elevation of 25°. Find PQ, correct to one decimal place. PS R

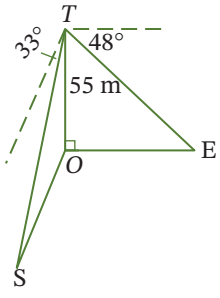
8 From a point, A, at the base of a mountain, the mountain peak, P, is due north and has an angle of elevation of 20°. From a point, B, 2 km due east of A on the same level, the mountain peak has a bearing of 320°. PS R



a What is the size of $\angle CBA$?

b Calculate the height, PC, of the mountain correct to the nearest metre.

9 From the top of her 55-metre office building, Madison observes 2 cars parked at ground level. The angle of depression of the car due east of the building is 48° and the angle of depression of the car parked due south of the building is 33°. PS R



Calculate, correct to the nearest metre, how far:

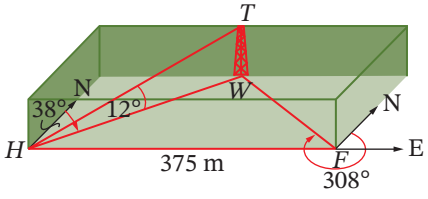
a each car is from the building

b the cars are apart.

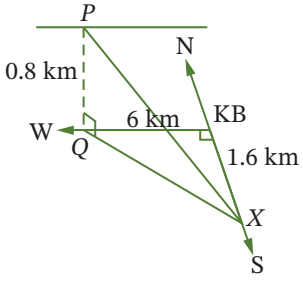
10 Hassan observes a transmission tower at an elevation angle of 12° and bearing 038°. Fatima stands 375 m due east of Hassan and observes the tower at a bearing of 308°. PS R

a Find the sizes of the angles of $\triangle FHW$.

b Find the height of the tower TW, correct to one decimal place.



11 A plane flies 6 km due west of Keira Bay at a constant height of 800 m. Xander sees the plane from his house 1.6 km south of Keira Bay.



Find, correct to the nearest degree: PS R C

a the bearing of the plane from Xander

b the angle of elevation of the plane from Xander.

Presentation
2D and 3D applications of trigonometry

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PATH

12 Sophia observes a plane due north at an angle of elevation of 40°. The plane is flying due east at an altitude of 4000 m and at a speed of 200 km/h. PS R C

a Find the angle of elevation and the bearing of the plane from Sophia after 2 minutes.

b What is the distance (to the nearest m) from Sophia to the plane after 2 minutes?

13 From a lighthouse 95 m above sea level, a boat is observed on a compass bearing of SSE and at an angle of depression of 2°. Another boat is seen on a bearing of ENE and at angle of depression of 1.5°. PS R C

a Calculate the distance (correct to one decimal place) between the boats.

b What is the bearing of the first boat from the second boat?

Quiz
Mental skills 8

Mental skills 8: Maths without calculators ANSWERS ON P. 613

Divisibility tests

How can you tell if a number is divisible by 2? Look at its last digit. If that digit is 2, 4, 6, 8 or 0, then the number is divisible by 2 (that is, it is even).

How can you tell if a number is divisible by 5? If its last digit is 0 or 5, then the number is divisible by 5.

These are examples of **divisibility tests**—rules for checking whether or not a number is divisible by a certain number. The table below shows some common divisibility tests.

A number is divisible by:	if:
2	its last digit is 2, 4, 6, 8 or 0
3	the sum of its digits is divisible by 3
4	its last two digits form a number divisible by 4
5	its last digit is 0 or 5
6	it is even and the sum of its digits is divisible by 3
9	the sum of its digits is divisible by 9
10	its last digit is 0

1 Study each example.

a Test whether 748 is divisible by 2, 3 or 4.

- Last digit is 8 (even), ∴ 748 is divisible by 2
- Sum of digits = 7 + 4 + 8 = 19, which is not divisible by 3, ∴ 748 is not divisible by 3
- 48 is divisible by 4, ∴ 748 is divisible by 4 (748 ÷ 4 = 187)

b Test whether 261 is divisible by 5 or 9.

- Last digit is 1, not 0 or 5, ∴ 261 is not divisible by 5
- 2 + 6 + 1 = 9, which is divisible by 9, ∴ 261 is divisible by 9. (261 ÷ 9 = 29).

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c Test whether 570 is divisible by 4, 6 or 10.

- 70 is not divisible by 4, ∴ 570 is not divisible by 4
- 570 is even and 5 + 7 + 0 = 12, which is divisible by 3, ∴ 570 is divisible by 6 (570 ÷ 6 = 95)
- Last digit is 0, ∴ 570 is divisible by 10 (570 ÷ 10 = 57)

2 Test whether each number is divisible by 2, 3, 5 or 6.

a	250	**b**	189	**c**	78	**d**	465
e	1024	**f**	840	**g**	715	**h**	627

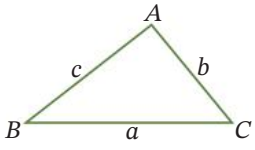
3 Test whether each number is divisible by 4, 9 or 10.

a	144	**b**	280	**c**	522	**d**	4170
e	936	**f**	726	**g**	342	**h**	5580

The sine rule

8.06

The angles of a triangle are labelled with capital letters while the sides are labelled with lowercase letters. By convention, we use a to label the side opposite $\angle A$, b to label the side opposite $\angle B$, and so on.



There is a relationship between each angle in a triangle and its opposite side. The longest side is always opposite the largest angle, the next smallest side is opposite the next smallest angle and so on. This relationship is called the **sine rule**.

PATH

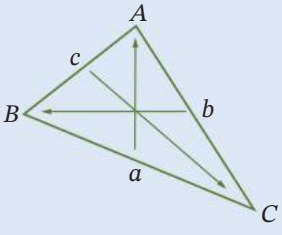
Worksheet
Discovering the sine rule

The sine rule

For any triangle ABC:

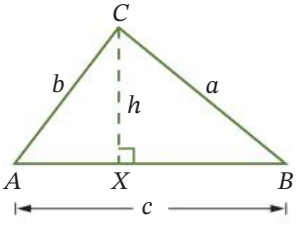
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The ratios of the sides in a triangle to the sine of their opposite angles are equal.



Proof:

In $\triangle ABC$, draw CX for the perpendicular height, h, of the triangle. CX divides $\triangle ABC$ into 2 right-angled triangles.



In $\triangle AXC$, $\sin A = \frac{h}{b}$

$\therefore h = b \sin A$

In $\triangle BXC$, $\sin B = \frac{h}{a}$

$\therefore h = a \sin B$

$\therefore a \sin B = b \sin A$

PATH

$$\frac{a\sin B}{\sin B}=\frac{b\sin A}{\sin B}$$ Dividing both sides by $\sin B$

$$a=\frac{b\sin A}{\sin B}$$

$$\frac{a}{\sin A}=\frac{b\sin A}{\sin B\sin A}$$ Dividing both sides by $\sin A$

$$\frac{a}{\sin A}=\frac{b}{\sin B}$$

By drawing the perpendicular from A to BC, it can be shown that $\frac{b}{\sin B}=\frac{c}{\sin C}$

$$\therefore\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

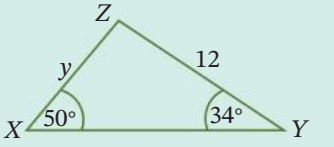
The sine rule allows us to apply trigonometry to any triangle, not just right-angled triangles.

The sine rule is used in problems involving 2 sides of a triangle and the 2 angles opposite them.

Video
The sine rule

Example 16

Find y, correct to one decimal place.



Solution

$$\frac{a}{\sin A}=\frac{b}{\sin B}$$ Sides and opposite angles

$$\frac{y}{\sin 34°}=\frac{12}{\sin 50°}$$

$$y=\frac{12\sin 34°}{\sin 50°}$$

$$=8.7596\ldots$$

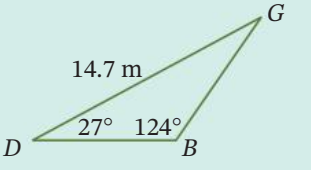
$$\approx 8.8\text{ cm}$$ From the diagram, an answer of 8.8 cm looks reasonable

Example 17

In $\triangle BDG$, $\angle B=124°$, $\angle D=27°$ and $DG=14.7$ m. Find BD, correct to one decimal place.

Solution

Draw a diagram.



$$\frac{BD}{\sin G}=\frac{14.7}{\sin 124°}$$

$\angle G=180°-27°-124°$ (angle sum of a triangle)

$=29°$

$$\frac{BD}{\sin 29°}=\frac{14.7}{\sin 124°}$$

$$BD=\frac{14.7\sin 29°}{\sin 124°}$$

$=8.5963\ldots$

≈ 8.6 m

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EXERCISE 8.06 ANSWERS ON P. 613

PATH

The sine rule U F PS R

1 Evaluate each expression, correct to one decimal place.

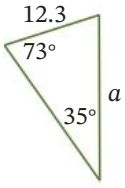
a $\dfrac{14.7\sin 64°}{\sin 46°}$ **b** $\dfrac{34.5\sin 33.4°}{\sin 115.7°}$ **c** $\dfrac{69\sin 107°\,33'}{\sin 38°\,47'}$

EXAMPLE 16

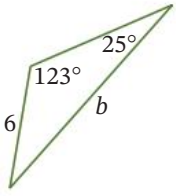
8.06

2 Find the value of each variable, correct to 2 decimal places.

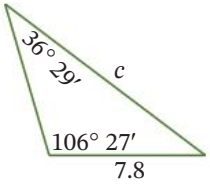
a



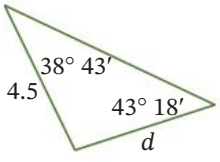
b



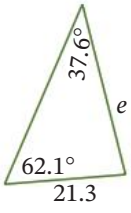
c



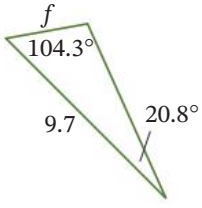
d



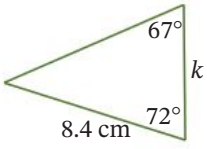
e



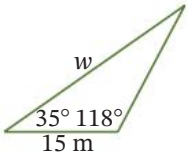
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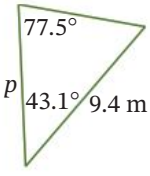
g



h



i

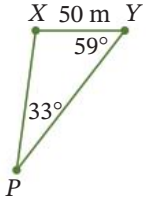


3 In $\triangle PTM$, $\angle P = 67°$, $\angle M = 52°$ and $PM = 12.5$ m. Find the length of PT. Select the closest answer **A**, **B**, **C** or **D**.

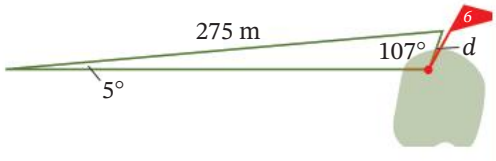
A 10.7 m **B** 14.6 m **C** 11.3 m **D** 13.9 m

EXAMPLE 17

4 X and Y are 2 light towers 50 m apart on one side of a park. P is a light tower on the other side of the park. If $\angle Y = 59°$ and $\angle P = 33°$, find PX correct to the nearest metre. PS R



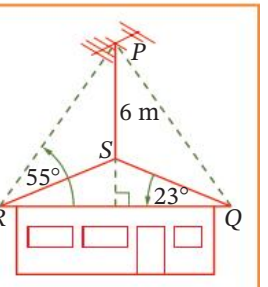
5 A golfer drives a ball 275 m at an angle of 5° off centre. The ball lands at an angle of 107° from the hole. Calculate the distance of the ball from the hole, correct to the nearest metre. PS R



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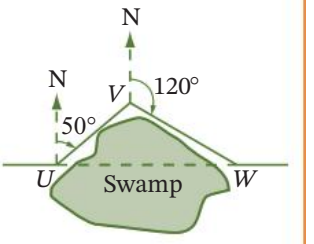
6 A 6 m television antenna is mounted on a roof pitched at an angle of 23°. It is supported by 2 wires, PQ and PR, inclined at 55° to the horizontal. PS R



a Show that $\angle PSR = 113°$.

b Calculate the length of the wire PR, correct to the nearest centimetre.

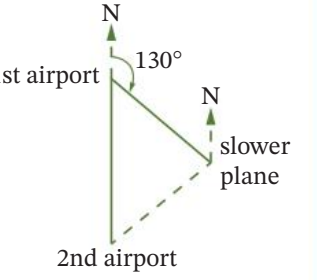
7 To avoid a swamp, Jesinta runs 70 m on a bearing of 050° to V. She then turns and runs to W on a bearing of 120°. If W is directly east of U: PS R



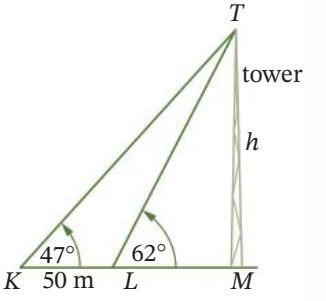
a find $\angle UVW$

b calculate UW, correct to one decimal place.

8 2 planes leave the airport at the same time. One flies due south at 400 km/h and lands at a second airport after $1\frac{1}{2}$ hours. The other flies on a bearing of S50°E and after $1\frac{1}{2}$ hours is at a bearing of N75°E from the second airport. How far (to the nearest km) is the slower plane from the second airport? PS R



9 The angle of elevation of a tower from a point L is 62°. From a point K, 50 m further from the tower, the angle of elevation is 47°. PS R



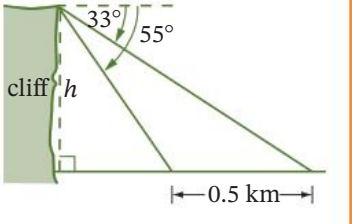
a Use the sine rule in $\triangle KTL$ to show that $TL = \frac{50\sin 47°}{\sin 15°}$.

b Let the height of the tower be h. In the right-angled $\triangle LMT$, show that $TL = \frac{h}{\sin 62°}$.

c Hence show that $h = \frac{50\sin 47°\sin 62°}{\sin 15°}$

d Hence calculate the height of the tower, correct to one decimal place.

10 From the top of a cliff, the angles of depression of 2 boats at sea that are 0.5 km apart are 55° and 33°. PS R



a Let the height of the cliff be h. Show that $h = \frac{0.5\sin 33°\sin 55°}{\sin 22°}$.

b Hence calculate the height, to the nearest metre.

11 A yacht leaves Fremantle and sails on a bearing of 312° for 650 km. It then changes direction and sails on a bearing of 235° until it is due west of Fremantle. How far is the yacht from Fremantle (to the nearest km)? PS R

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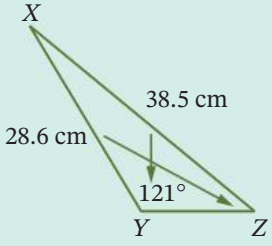


Hilke Maunder/Alamy Stock Photo

The sine rule for angles 8.07

Example 18

Find angle Z, correct to the nearest degree.



Solution

$$\frac{28.6}{\sin Z} = \frac{38.5}{\sin 121°}$$

Sides and opposite angles

$$\frac{\sin Z}{28.6} = \frac{\sin 121°}{38.5}$$

Inverting both sides so that Z is in the numerator

$$\sin Z = \frac{28.6 \sin 121°}{38.5}$$
$$= 0.636...$$
$$Z = 39.55...$$
$$\approx 40°$$

On a calculator: SHIFT sin Ans =

From the diagram, an answer of 40° looks reasonable

PATH

Worksheet
Sine rule problems

Video
Obtuse angles using the sine rule

PATH

Obtuse-angled answers

If sin θ is positive, angle θ could be 2 different values, one acute and one obtuse.

However, these 2 values are related: they are supplementary angles (adding to 180°).

For example, if sin θ = 0.5, then θ = 30° or 180° – 30° = 150°.

You can check on your calculator that sin 30° = sin 150° = 0.5.

When using SHIFT sin on your calculator, it will give the acute angle θ only.

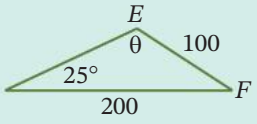
For the obtuse angle, you must calculate 180° – θ.

The sine ratio of supplementary angles

$\sin(180° - \theta) = \sin\theta$

Example 19

Find θ correct to the nearest minute if it is an obtuse angle.



Solution

$$\frac{200}{\sin\theta} = \frac{100}{\sin 25°}$$

$$\frac{\sin\theta}{200} = \frac{\sin 25°}{100}$$

$$\sin\theta = \frac{200\ \sin 25°}{100}$$

$$= 0.845\ldots$$

$\theta = 57.697\ldots$

But θ is obtuse, so:

$\theta = 180 - 57.697\ldots$

$= 122.3027\ldots$

$= 122°\ 18'\ 9.77''$

$\approx 122°\ 18'$

The ambiguous case (when there are 2 possible answers)

When we use the sine rule to find an angle, it is possible to find both an acute angle and an obtuse angle as solutions. Likewise, there could be 2 possible triangles: one acute-angled, the other obtuse-angled. However, the obtuse-angled triangle may not be possible. We need to check that the sum of the angles in the triangle is not greater than 180°.

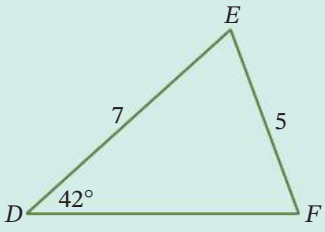
Example 20

a In $\triangle DEF$, $\angle D = 42°$, $d = 5$ cm and $f = 7$ cm. Find $\angle F$ correct to the nearest degree.

b In $\triangle LMN$, $\angle M = 130°$, $LN = 15$ cm and $LM = 7$ cm. Find $\angle N$, correct to the nearest degree.

Solution

a Draw a rough diagram.



$$\frac{7}{\sin F} = \frac{5}{\sin 42°}$$

$$\frac{\sin F}{7} = \frac{\sin 42°}{5}$$

$$\sin F = \frac{7 \sin 42°}{5}$$

$$= 0.93678\ldots$$

$$F = 69.5181\ldots$$

$$\approx 70°$$

But F could be obtuse.

$$F = 180° - 70°$$

$$= 110°$$

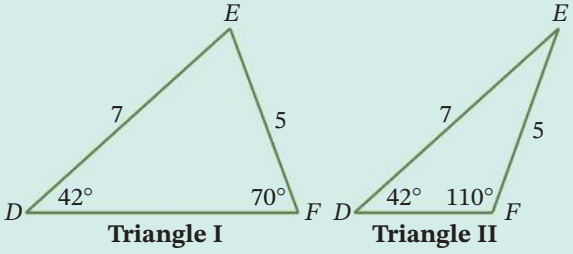
Checking the third angle of the obtuse-angled triangle:

$$\angle E = 180° - 42° - 110°$$

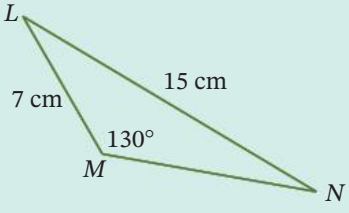
$$= 28°$$

$\therefore$ The obtuse-angled solution is possible.

$\therefore \angle F = 70°$ or $110°$



b Draw a rough diagram.



$$\frac{\sin N}{7} = \frac{\sin 130°}{15}$$

$$\sin N = \frac{7 \sin 130°}{15}$$

$$= 0.3574\ldots$$

$$N = 20.9459\ldots$$

$$\approx 21°$$

PATH

Video
The sine rule

8.07

PATH

But N could be obtuse.

$N = 180° - 21°$

$= 159°$

Checking the third angle of the obtuse-angled triangle:

$\angle L = 180° - 130° - 159°$ Impossible

$= -109°$

$\therefore$ The obtuse-angled solution is not possible.

$\therefore \angle N = 21°$

EXERCISE 8.07 ANSWERS ON P. 613

The sine rule for angles U F PS R C

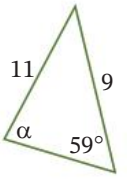
1 Find the acute angle X in each equation, correct to the nearest degree.

a $\sin X = \frac{5.3 \sin 123°}{9.7}$ **b** $\sin X = \frac{39 \sin 85°29'}{64}$ **c** $\sin X = \frac{467 \sin 63.8°}{518}$

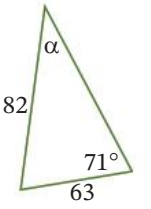
EXAMPLE 18

2 Find α in each triangle if α is acute, correct to the nearest 0.1 degree.

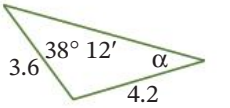
a



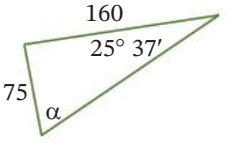
b



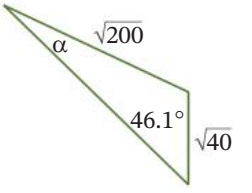
c



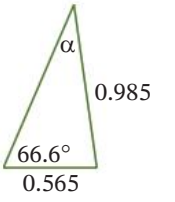
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e



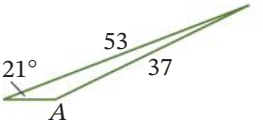
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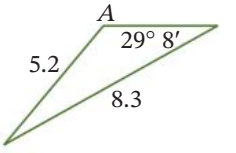
EXAMPLE 19

3 Find the size of $\angle A$ to the nearest minute if $\angle A$ is obtuse.

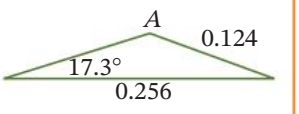
a



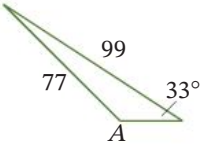
b



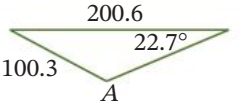
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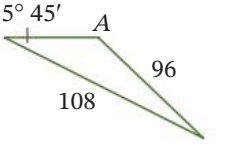
d



e



f

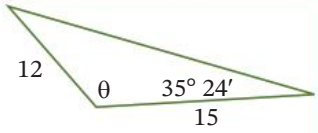


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4 Find the size of θ. Select the correct answer **A**, **B**, **C** or **D**.



A 46° 24′ **B** 98° 12′

C 133° 36′ **D** 81° 48′

5 In $\triangle BGP$, $\angle G = 121°$, $BG = 5.2$ m, and $BP = 12.8$ m. Find $\angle B$ (to the nearest degree).

6 Find all possible angles for each triangle, correct to the nearest degree. R C

EXAMPLE 20

a In $\triangle PQR$, $\angle P = 35°$, $p = 8$ cm, and $q = 10$ cm. Find $\angle Q$.

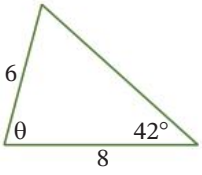
b In $\triangle UVW$, $\angle W = 95°$, $w = 16$ cm, and $v = 10$ cm. Find $\angle V$.

c In $\triangle XYZ$, $\angle Y = 24°$, $y = 3.4$ km, and $z = 5.7$ km. Find $\angle Z$.

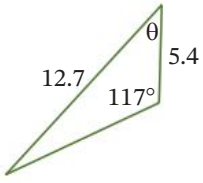
d In $\triangle DEF$, $\angle E = 37°$, $e = 107$ mm, and $d = 121$ mm. Find $\angle D$.

7 Find θ in each triangle correct to the nearest degree, given that θ is acute.

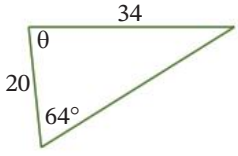
a



b



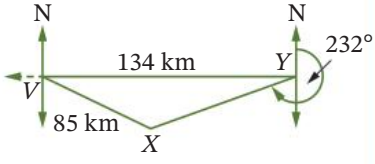
c



8 Max walked 5.6 km due east from his campsite. He then turned and walked on a bearing of S33°W until he was 6 km from the campsite. What is the bearing of the campsite from Max? Select **A**, **B**, **C** or **D**. PS R C

A N52°E **B** N71°W **C** N19°W **D** N52°W

9 Yorker (Y) is 134 km due east of Vendor (V). Xena (X) is on a bearing of 232° from Yorker and is also 85 km from Vendor. $\angle VXY$ is an obtuse angle. Find the bearing of Xena from Vendor. PS R C



8.08

The cosine rule

8.08

PATH

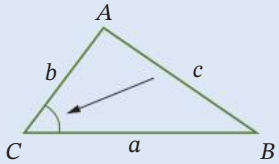
The **cosine rule** is a relationship between the 3 sides of a triangle and one of its angles. It is an extension of Pythagoras' triangle that can be applied to any triangle, not just right-angled ones.

The cosine rule

For any triangle ABC:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

where c is the unknown side, C is the angle opposite c, and a and b are the other 2 sides.

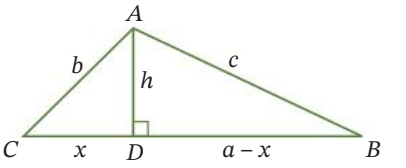


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Proof:

In $\triangle ABC$, draw AD for the perpendicular height, h, of the triangle. AD divides $\triangle ABC$ into 2 right-angled triangles.



Let $CD = x$, $\therefore DB = a - x$

In $\triangle ADC$, $b^2 = h^2 + x^2$

$\therefore h^2 = b^2 - x^2$ [1]

In $\triangle ABD$, $c^2 = h^2 + (a - x)^2$

$\therefore h^2 = c^2 - (a - x)^2$

$\therefore b^2 - x^2 = c^2 - (a - x)^2$

$b^2 - x^2 = c^2 - (a^2 - 2ax + x^2)$

$b^2 - x^2 = c^2 - a^2 + 2ax - x^2$

$b^2 = c^2 - a^2 + 2ax$

$\therefore c^2 = a^2 + b^2 - 2ax$ [2]

In $\triangle ADC$, $\cos C = \frac{x}{b}$

$\therefore x = b \cos C$

Substituting for x in [2]:

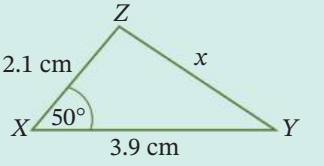
$c^2 = a^2 + b^2 - 2ab \cos C$

The cosine rule can be used in problems involving 3 sides of a triangle and one of the angles.

Video
The cosine rule

Example 21

Find x correct to 2 decimal places.



Solution

$c^2 = a^2 + b^2 - 2ab \cos C$

$x^2 = 2.1^2 + 3.9^2 - 2 \times 2.1 \times 3.9 \cos 50°$

$= 9.091138...$

$x = \sqrt{9.091138...}$

$= 3.01515...$

≈ 3.02 cm

50° is the angle opposite x

From the diagram, an answer of 3.02 cm looks reasonable

EXERCISE 8.08 ANSWERS ON P. 613

The cosine rule U F PS R C

1 Solve each equation for x, correct to one decimal place.

a $x^2 = 8^2 + 9^2 - 2 \times 8 \times 9 \times \cos 38°$

b $x^2 = 11.3^2 + 9.7^2 - 2 \times 11.3 \times 9.7 \times \cos 76.9$

c $x^2 = 17^2 + 20.1^2 - 2 \times 17 \times 20.1 \times \cos 149° 45'$

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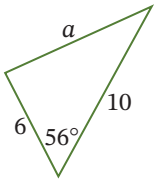
PATH

EXAMPLE 21

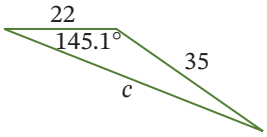
8.08

2 Find, correct to 2 decimal places, the value of each variable.

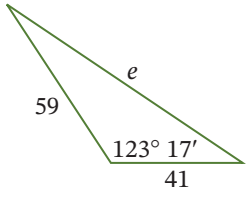
a



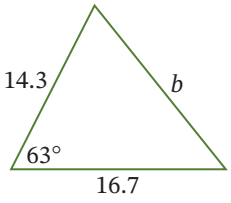
b



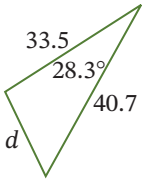
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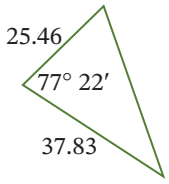
d



e



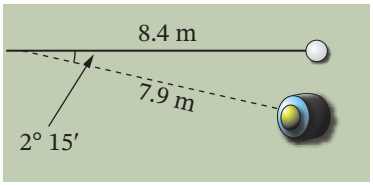
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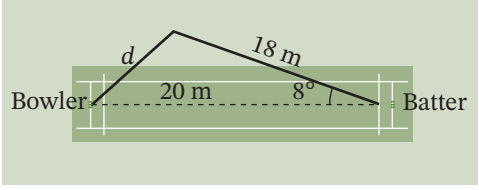
3 In $\triangle DEF$, $\angle E = 38° 46'$, $d = 8.7$ m and $f = 15.6$ m. What is the length of DF?
Select the closest answer **A**, **B**, **C** or **D**.

A 10.4 m **B** 19.5 m **C** 17.9 m **D** 12.8 m

4 In a game of lawn bowls, Jayden is aiming to hit the jack (target ball) 8.4 m away. If he bowls 2° 15′ off centre and his bowl travels 7.9 m, how far is his bowl from the jack? Answer correct to one decimal place. PS R



5 In a cricket match, the distance between the bowler and the batter was 20 m. During one bowl, the batter hit the ball at an angle of 8° to the line of the pitch and the bowler ran and caught the ball after it had travelled 18 m. How far did the bowler run to catch the ball?
Select **A**, **B**, **C** or **D**. PS R



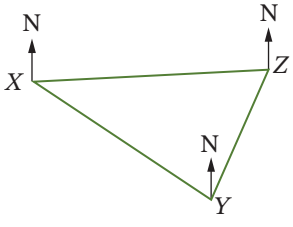
A 1.1 m **B** 2.0 m **C** 3.3 m **D** 4.0 m

6 A yacht sails from X to Y on a bearing of 130° for 4.2 km. It then turns and travels to Z on a bearing of 025° for 2.9 km. PS R C

a Copy the diagram and mark the given information on it.

b Explain why $\angle XYZ = 75°$.

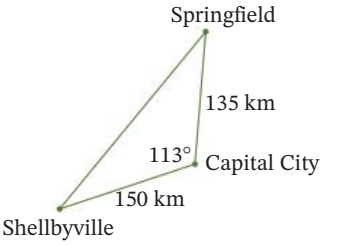
c Calculate the distance XZ, correct to one decimal place.



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7 3 towns are joined by straight roads. What distance (correct to the nearest kilometre) is saved by going directly from Springfield to Shellbyville instead of travelling via Capital City? PS R C



8 **a** What is the value of cos 90°?

b What does $c^2 = a^2 + b^2 - 2ab\cos C$ simplify to if $C = 90°$?

c Hence what happens to the cosine rule when it is applied to a right-angled triangle?

8.09 The cosine rule for angles

PATH

Worksheet
Cosine rule problems

If we rewrite the cosine rule so that cos A is the subject, then we will have a formula for finding an unknown angle when the 3 sides of a triangle are known.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$c^2 + 2ab\cos C = a^2 + b^2$ Adding $2ab\cos C$ to both sides so that cos C appears on the LHS

$$2ab\cos C = a^2 + b^2 - c^2$$

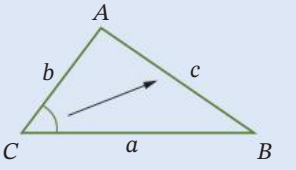
$$\therefore \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

The cosine rule for angles

For any triangle ABC:

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

where C is the unknown angle, c is the side opposite C, and a and b are the other 2 sides.

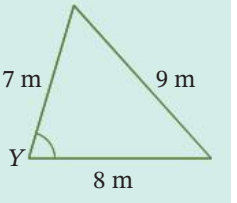


The cosine rule can be used to find an unknown angle if the lengths of the 3 sides are known.

Example 22

Video
The cosine rule for angles 2

Find the size of the marked angle Y, correct to the nearest degree.



Solution

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos Y = \frac{7^2 + 8^2 - 9^2}{2 \times 7 \times 8}$$

$$= \frac{32}{112}$$

9 mm is opposite angle Y

$Y = 73.398...$

$\approx 73°$

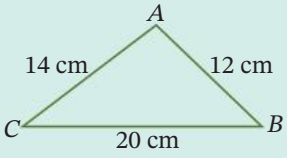
From the diagram, an answer of 73° looks reasonable

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Example 23

Calculate, correct to the nearest minute, the size of the largest angle in this triangle.



Solution

The largest angle is opposite the longest side, so it is $\angle A$.

$$\cos A = \frac{14^2 + 12^2 - 20^2}{2 \times 14 \times 12}$$

$$= \frac{-60}{336}$$

Cos is negative so the angle will be obtuse

$$A = 100.28656...$$

$$= 100° 17' 11.6''$$

$$\approx 100° 17'$$

From the diagram, an answer of 100° 17′ looks reasonable

PATH

Videos
The cosine rule
The cosine rule for angles 1

8.09

EXERCISE 8.09 ANSWERS ON P. 613

The cosine rule for angles U F PS R

1 Solve each equation for X, correct to the nearest degree.

a $\cos X = \frac{12^2 + 14^2 - 15^2}{2 \times 12 \times 14}$

b $\cos X = \frac{5.7^2 + 6.8^2 - 3.7^2}{2 \times 5.7 \times 6.8}$

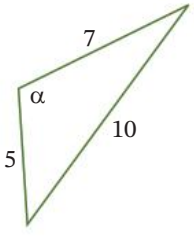
c $\cos X = \frac{5^2 + 6^2 - 9^2}{2 \times 5 \times 6}$

d $\cos X = \frac{9.2^2 + 4.7^2 - 12.8^2}{2 \times 9.2 \times 4.7}$

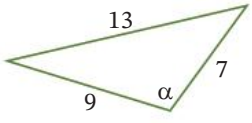
EXAMPLE 22

2 Find the size of α in each triangle, correct to the nearest degree.

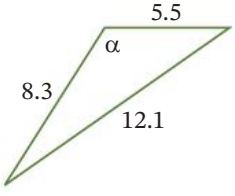
a



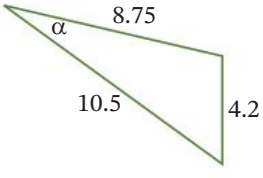
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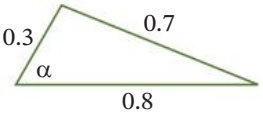
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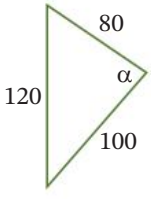
d



e



f



3 In $\triangle ABC$, $a = 8.5$ cm, $b = 12.1$ cm and $c = 17.6$ cm. What is the size of $\angle B$?
Select the correct answer **A**, **B**, **C** or **D**.

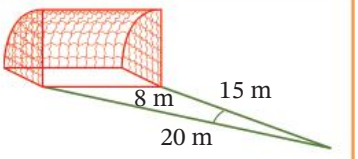
A 25° 40′ **B** 141° 57′ **C** 116° 17′ **D** 38° 3′

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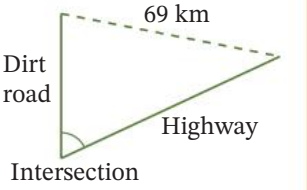
PATH

EXAMPLE 23

4 A soccer goal is 8 m wide. A player shoots for goal (along the ground) when 20 m from one post and 15 m from the other post. Within what angle (correct to the nearest 0.1 degree) must the shot be made for the player to have a chance of scoring a goal? PS R



5 2 cars leave an intersection at the same time. Car *A* drives down the dirt road at 60 km/h and car *B* drives down the highway at 100 km/h. After 45 minutes they are 69 km apart. Find the angle between the 2 roads, correct to the nearest minute. PS R



6 A triangle has sides of 21 m, 17 m and 10 m.
Find the size of the largest angle, correct to the nearest degree. R

7 A plane flies from Alice Springs on a bearing of S65°W. After 450 km, the plane changes to a northwesterly direction for 1100 km. When the plane stops it is 1380 km from Alice Springs. What is the bearing (to the nearest degree) of the plane from Alice Springs? PS R

8.10 The area of a triangle

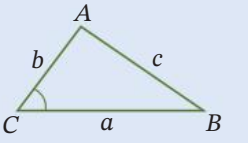
PATH

We already know that the formula for the area of a triangle is $A = \frac{1}{2}bh$, but there is also a trigonometric formula if we know the lengths of 2 sides of the triangle and the size of the **included angle** between them.

The area of a triangle

$A = \frac{1}{2}ab\sin C$

where C is the included angle between sides a and b.



Proof:

In $\triangle ABC$, draw AX for the perpendicular height, h, of the triangle.

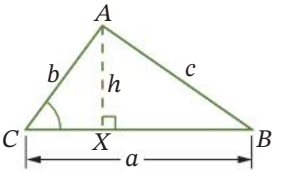
Area of $\triangle ABC = \frac{1}{2}ah$ [1]

But in $\triangle AXC$, $\sin C = \frac{h}{b}$

$\therefore h = b \sin C$

Substituting this into [1]:

$A = \frac{1}{2}ab\sin C$

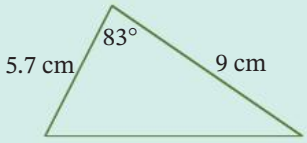


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Example 24

Find, correct to one decimal place, the area of this triangle.



Solution

$A = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 5.7 \times 9 \times \sin 83°$

$= 25.458...$

$\approx 25.5 \text{ cm}^2$

83° is the included angle between 5.7 cm and 9 cm

PATH

Video
The sine area formula

8.10

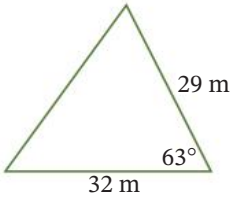
EXAMPLE 24

EXERCISE 8.10 ANSWERS ON P. 613

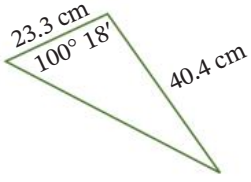
The area of a triangle U F PS R

1 Find, correct to one decimal place, the area of each triangle.

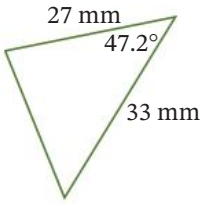
a



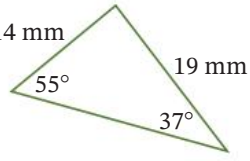
b



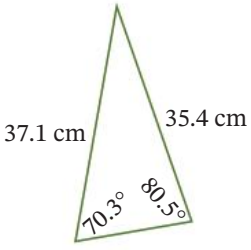
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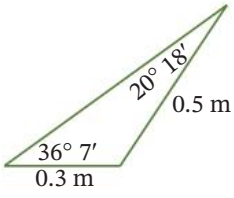
d



e

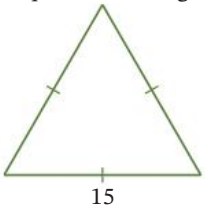


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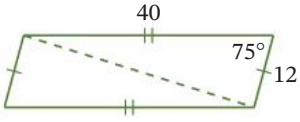


2 Calculate, correct to one decimal place, the area of each shape. All measurements are in metres. R

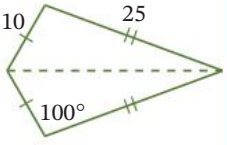
a Equilateral triangle



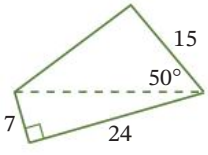
b Parallelogram



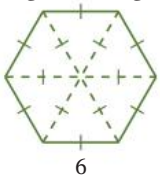
c Kite



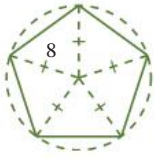
d Quadrilateral



e Regular hexagon



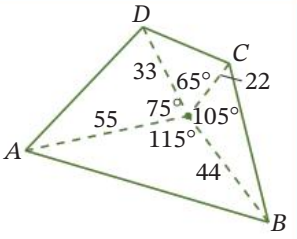
f Regular pentagon inscribed in a circle of radius 8



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PATH

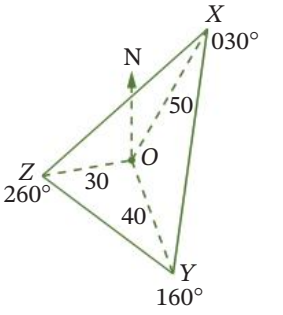
3 The diagram shows the results of a radial survey of a block of land. All distances are in metres. PS R



a Use the cosine rule to find the lengths of AB, BC, CD and AD and, hence, find the perimeter of the block of land (to the nearest metre).

b Use the area formula to find the area of each triangle and, hence, calculate the area of the block of land (to the nearest m^2).

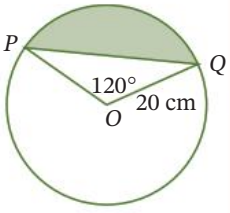
4 The results of a radial survey are shown in the diagram. All measurements are in metres. PS R



a Find the size of $\angle XOY$.

b Calculate, correct to 2 decimal places, the area of $\triangle XOY$.

5 O is the centre of a circle of radius 20 cm. Calculate, correct to one decimal place, the area of: PS R

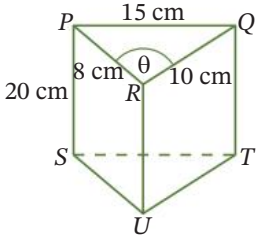


a sector OPQ

b triangle OPQ

c the shaded segment.

6 A triangular prism has base edges of 8 cm, 10 cm and 15 cm, and a height of 20 cm. PS R



a Calculate the size $\angle PRQ$, correct to nearest degree.

b Find the area of $\triangle PQR$, correct to the nearest cm^2.

c Find the volume of the prism.

7 The area of an equilateral triangle is 600 cm^2. Calculate the perimeter of the triangle, correct to one decimal place.

Did you know?

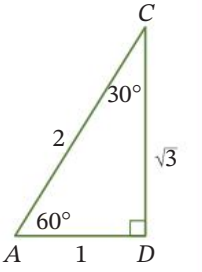
Triangle area formulas

There are many formulas for calculating the area, A, of a triangle:

Heron's formula: given 3 sides a, b, c:

$A = \sqrt{s(s-a)(s-b)(s-c)}$, where s is the semi-perimeter $\frac{a+b+c}{2}$

- Given 2 sides, a, b and the included angle, C: $A = \frac{1}{2}ab \sin C$
- Given one side, a and 3 angles, A, B, C: $A = \frac{1}{2}a^2 \frac{\sin B \sin C}{\sin A}$



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1 **Find the area of this triangle using the 3 formulas, and check that your answers agree.**

2 **Show how the formula $A = \frac{1}{2}a^2\frac{\sin B \sin C}{\sin A}$ can be proved from $A = \frac{1}{2}ab\sin C$ using the sine rule.**

3 **Research other formulas for the area of a triangle.**

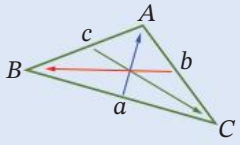
Problems involving the sine and cosine rules

8.11

The sine and cosine rules

The sine rule is used for triangle problems involving 2 sides and the 2 angles opposite them.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

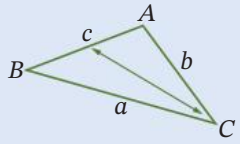


The cosine rule is used for triangle problems involving 3 sides and one angle.

$$c^2 = a^2 + b^2 - 2ab\cos C$$

and

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



PATH

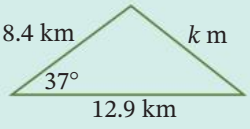
Worksheets
Finding an unknown side
Finding an unknown angle
The sine and cosine rules

Puzzle
The sine and cosine rules

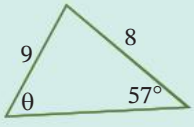
Video
The sine and cosine rules

Example 25

a Find the value of k, correct to one decimal place.



b Find the value of θ, correct to the nearest minute.



Solution

a The problem involves 3 sides and one angle so use the cosine rule.

$k^2 = 8.4^2 + 12.9^2 - 2 \times 8.4 \times 12.9 \times \cos 37°$

$= 63.889...$

$k = \sqrt{63.889...}$

$= 7.993...$

≈ 8.0 m

b The problem involves 2 sides and the 2 angles opposite them, so use the sine rule.

$\frac{\sin\theta}{8} = \frac{\sin 57°}{9}$

$\sin\theta = \frac{8\sin 57°}{9}$

$= 0.7454...$

$\theta = 48.2007...$

$= 48° 12' 2.77''$

$\approx 48° 12'$

PATH

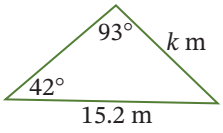
EXERCISE 8.11 ANSWERS ON P. 613

Problems involving the sine and cosine rules U F PS R C

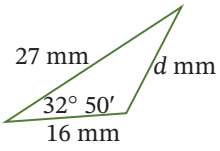
EXAMPLE 25

1 Find, correct to one decimal place, the value of each variable.

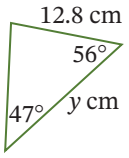
a



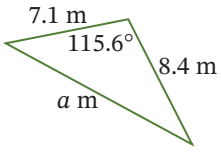
b



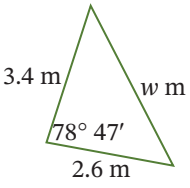
c



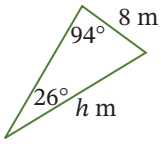
d



e

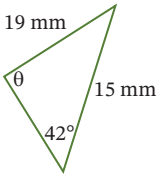


f

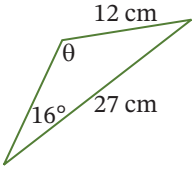


2 Find the value of θ to the nearest degree. Use diagrams to note whether θ is acute or obtuse.

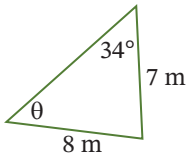
a



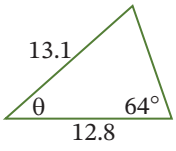
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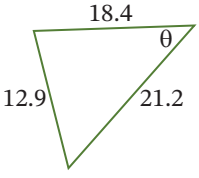
c



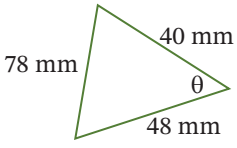
d



e



f

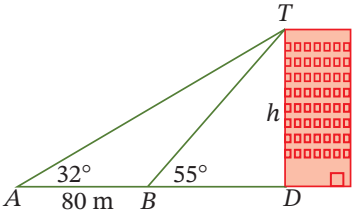


3 The angles of elevation of a building measured from 2 positions 80 m apart are 32° and 55°. PS R C

a Explain why $\angle ATB = 23°$.

b Find, correct to 2 decimal places, the length of BT.

c Hence find the height, h, of the building, correct to the nearest metre.



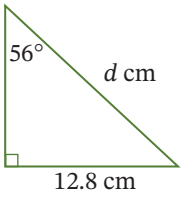
4 **a** What is the value of sin 90°? PS R C

b Find, correct to one decimal place, the value of d using:

i the sine rule

ii the sine ratio for right-angled triangles.

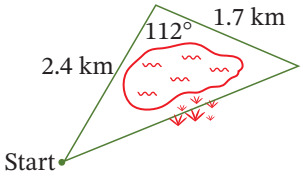
c What do you notice about your results? Give reasons.



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5 Mikayla needs to run around a cross-country course as shown. What is the length of the course, correct to one decimal place? PS R



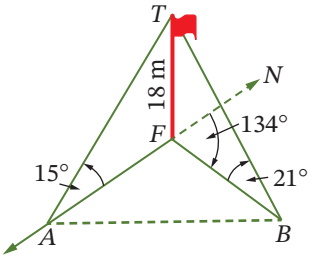
6 A plane flew on a bearing of 150° for 370 km. It then changed direction and flew another 285 km on a bearing of 235°. How far, correct to the nearest km, is the plane from its starting point? PS R

7 From a point A, south of a flagpole, the angle of elevation to the top of the pole is 15°. From B, the angle of elevation to the top of the flagpole is 21°.

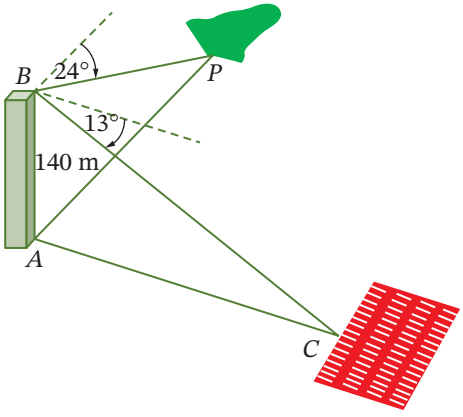
The bearing of B from the flagpole is S46°E. Find correct to one decimal place: PS R

a the length of AF and BF

b the distance between A and B



8 From the top of a building, the angles of depression to a park (P) and a car park (C) are 24° and 13° respectively. The height of the building is 140 m, the park is due north of the building and the bearing of the car park is ESE from the building. Calculate the distance between the park and the car park, correct to the nearest metre. PS R



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Power plus ANSWERS ON P. 614

1 **a** Copy and complete each pair of trigonometric ratios, correct to 3 decimal places.

i $\sin 20° =$ _____, $\cos 70° =$ _____

ii $\sin 47° =$ _____, $\cos 43° =$ _____

iii $\sin 55° =$ _____, $\cos 35° =$ _____

iv $\sin 85° =$ _____, $\cos 5° =$ _____

b What do you notice about each pair of answers in part **a**?

c What do you notice about each pair of angles in part **a**?

d If $\cos 30° \approx 0.8660$ and $\sin \theta \approx 0.8660$, what is the value of θ?

e Copy and complete each equation.

i $\sin 75° = \cos$ ____ **ii** ____ $80° = \cos 10°$

iii $\cos$ ____ $= \sin 72°$ **iv** $\sin 30° =$ ____ $60°$

v $\cos 65° = \sin$ ____ **vi** $\sin$ ____ $= \cos 58°$

f Copy and complete this general rule: $\sin x = \cos$ (_____).

g Use a right-angled triangle with one angle x and sides a, b and c to prove that the above rule is true.

2 A plane is flying at an angle of 15° inclined to the horizontal.

a How far will the plane have to travel along its line of flight to increase its altitude (height) by 500 m?

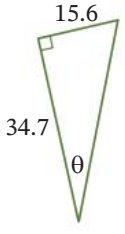
b At what angle must the plane climb to achieve an increase in altitude of 500 m in half the distance needed at an angle of 15°?

3 If $\sin 30° = \frac{1}{2}$, express the following as surds.

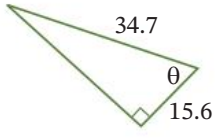
a $\cos 30°$ **b** $\tan 30°$

4 Find the value of angle θ, correct to the nearest second.

a



b



5 By drawing an appropriate triangle, prove that:

a $\tan 45° = 1$ **b** $\sin 45° = \frac{1}{\sqrt{2}}$ **c** $\cos 45° = \frac{1}{\sqrt{2}}$

6 Sketch a right-angled triangle with side lengths a, b and c, where c is the hypotenuse.

Let θ be one of the acute angles. Prove that $(\sin \theta)^2 + (\cos \theta)^2 = 1$.

CHAPTER 8 REVIEW

Quiz
Language of maths 8

Puzzle
Further trigonometry crossword

Language of maths

adjacent	angle of depression	angle of elevation	bearing
clinometer	compass bearing	cosine rule	degree (°)
denominator	horizontal	hypotenuse	included angle
inverse ($^{-1}$)	minute (′)	obtuse	opposite
right-angled	sine rule	supplementary	tangent (tan)
theta (θ)	trigonometric ratio	true bearing	vertical

1 Copy and complete: A **bearing** is an _____ used to describe precisely the _____ of one location from a given reference point.

2 The word **minute** has an alternative pronunciation and meaning. What is its alternative meaning?

3 If 2 angles have a sum of 180°, what special name do we give them?

4 Explain what is meant by an **angle of depression**.

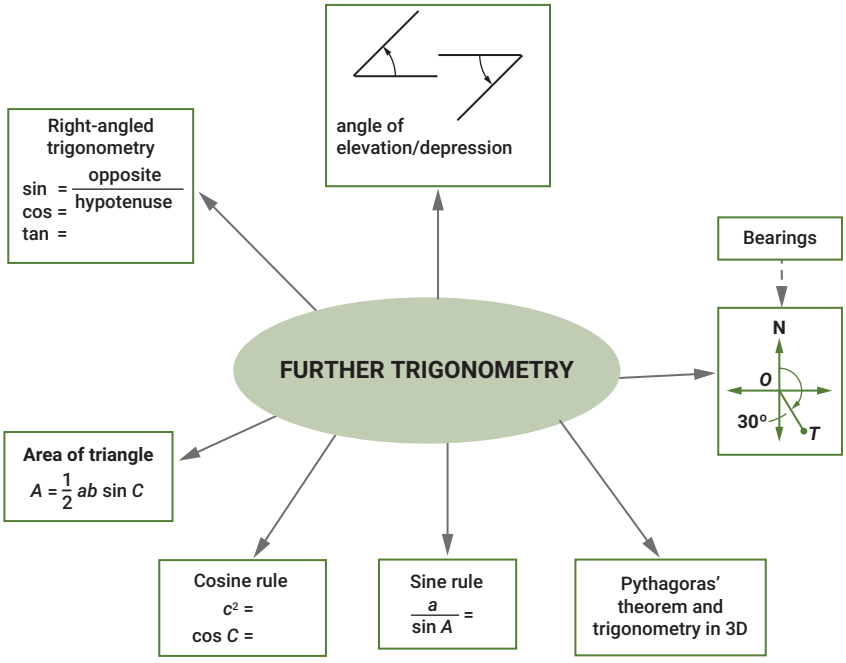
5 What type of bearing is also called a **three-figure bearing**?

6 What is the name given to the angle that is between 2 known sides?

Topic summary

Worksheet
Mind map: Further trigonometry

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

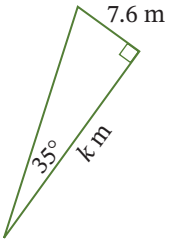


TEST YOURSELF 8 ANSWERS ON P. 614

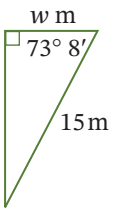
Quiz
Test yourself 8

8.01 **1** Find, correct to one decimal place, the value of each variable.

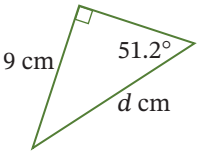
a



b

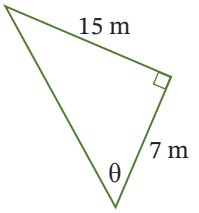


c

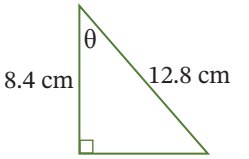


8.01 **2** Find the value of θ, correct to the nearest minute.

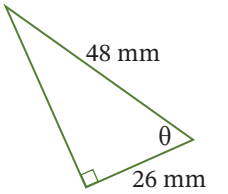
a



b



c



8.01 **3** In $\triangle XWY$, $\angle W = 90°$, $WY = 17.2$ m and $XW = 3.5$ m. Find $\angle Y$, correct to the nearest minute. Select the correct answer **A**, **B**, **C** or **D**.

A 11° 30′ **B** 11° 50′ **C** 11.52′ **D** 11° 31′

8.02 **4** From the top of a 300 m cliff Renee observes a boat at a distance of 1.3 km from the base of the cliff. Find, correct to the nearest degree, the angle of depression of the boat.

8.02 **5** The angle of elevation of a tower roof is 26° at a point 400 m from its base. Find the height of the tower, correct to the nearest metre.

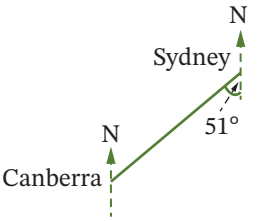
8.03 **6** Sketch point B on a compass rose so that B has a true bearing from A of:

a 220° **b** 020° **c** 120° **d** 310°

8.03 **7** What is the true bearing of:

a Canberra from Sydney?

b Sydney from Canberra?



8.04 **8** A hiking group walked on a bearing of N65°W for 13 km to reach a lookout.

a Find correct to one decimal place how far west they have travelled.

b Find the true bearing the group must walk to return to their starting point.

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9 2 planes leave an airport at the same time. The first travels on a bearing of 206° at 450 km/h. The second travels on a bearing of 296° at 600 km/h.

a How far does each plane travel after 3 hours?

b How far apart are the planes after 3 hours, to the nearest km?

c Calculate, correct to the nearest degree, the bearing of the first plane from the second plane.

8.04

10 A box in the shape of a square prism has a base of 10 cm by 10 cm and is 30 cm tall. Find to the nearest whole number:

a the length of the longest diagonal of the box

b the angle that the longest diagonal makes with the base.

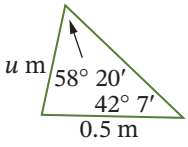
PATH

8.05

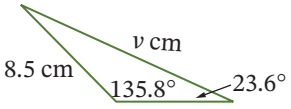
11 Find the value of each variable, correct to one decimal place.

8.06

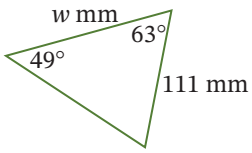
a



b



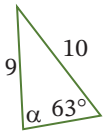
c



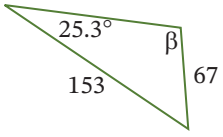
12 Find the value of each variable, correct to the nearest minute.

8.07

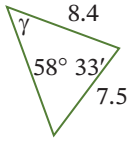
a



b



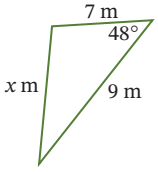
c



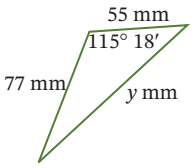
13 Find the value of each variable, correct to one decimal place.

8.08

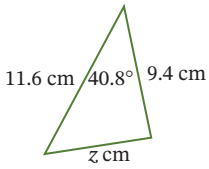
a



b



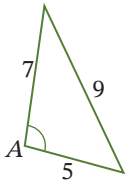
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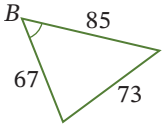
14 Find the size of each marked angle, correct to the nearest degree.

8.09

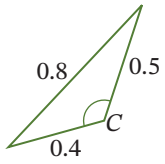
a



b



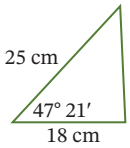
c



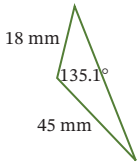
15 Find, correct to the nearest whole number, the area of each triangle.

8.10

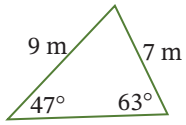
a



b



c



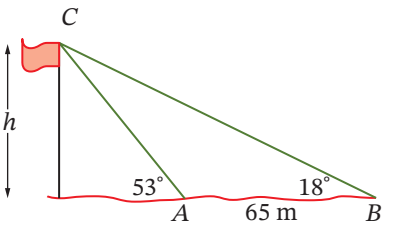
TEST YOURSELF 8

Foundation | Standard | Complex

PATH

8.11 **16** A parallelogram has sides of 12 cm and 6 cm and one interior angle of 65° 35′. Find the length of the longer diagonal, correct to one decimal place.

8.11 **17** The distance from A to B is 65 m.



a Show that $\angle ACB = 35°$

b Hence find the length of AC correct to 2 decimal places.

c Hence find the height of the flagpole correct to one decimal place.

8.11 **18** A boat sails from Perth on a bearing of SSW for 12 nautical miles and then changes to a bearing of WNW and sails a further 42 nautical miles. Find correct to one decimal place:

a how far the boat is from Perth

b the true bearing of the boat from Perth

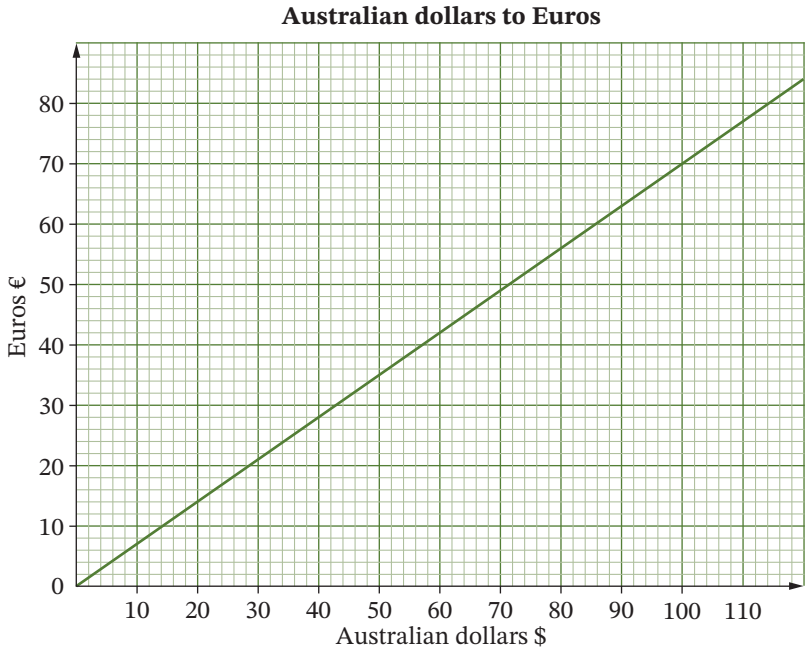
c on what true bearing it will have to sail on its way directly back to Perth

PRACTICE SET 3

ANSWERS ON P. 614

PATH

1 This conversion graph shows the exchange rate between the Australian dollar and the Euro. 7.03



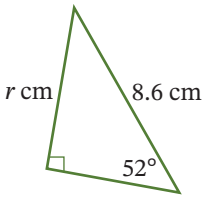
Use the graph to convert:

a A$25 to € **b** €60 to A$ **c** A$140 to €

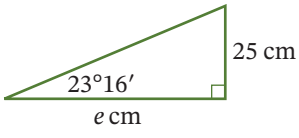
2 T varies directly as h. If $T = 48$ when $h = 5$, find T when $h = 16.5$. 7.01

3 Find the value of each variable, correct to one decimal place. 8.01

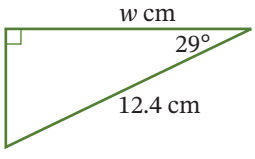
a



b



c



4 **a** Graph each quadratic equation, showing the vertex of each parabola. 7.04

i $y = x^2$ **ii** $y = 4 - x^2$ **iii** $y = 3x^2 + 1$ **iv** $y = 3 - 2x^2$

b State which graphs you have drawn in part **a**:

i are concave up **ii** are concave down **iii** have a turning point at (0, 1).

Foundation Standard Complex

PATH

7.02

5 The time taken, t seconds, to complete a race is inversely proportional to the speed, S m/s, of the sprinter.

a If it takes Leilani 9.8 seconds to complete a race at a speed of 10.2 m/s, find the variation equation for t.

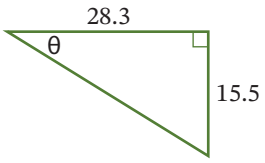
b If it took Leilani 10.5 seconds to complete a similar race, what was her speed?

c If Leilani runs at a speed of 10.3 m/s, how long will it take her to complete the race?

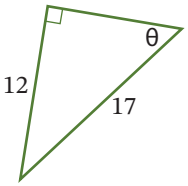
8.01

6 Find θ, correct to the nearest degree.

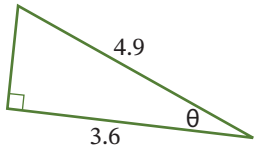
a



b



c



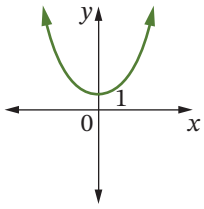
8.01

7 A kite is flying at the end of a string that is 85 m long. The string makes an angle of 57° with the ground. At what height is the kite flying? Answer correct to the nearest metre.

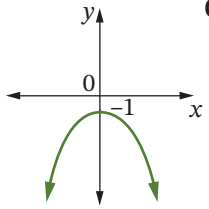
7.04

8 Which of these could be a graph of $y = 1 - 2x^2$? Select **A**, **B**, **C** or **D**.

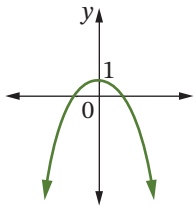
A



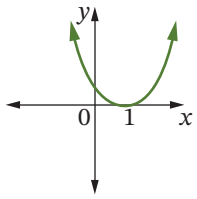
B



C

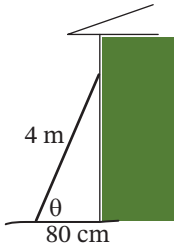


D



8.01

9 A 4 m ladder is placed against the side of a house. The foot of the ladder is 80 cm from the base of the house. Find the angle between the ladder and the ground, correct to the nearest degree.

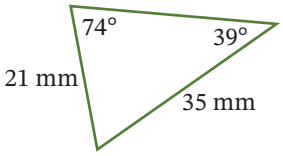


PATH

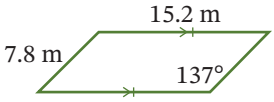
8.10

10 Find the area of each shape, correct to 2 significant figures.

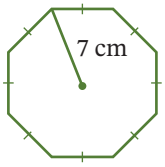
a



b



c



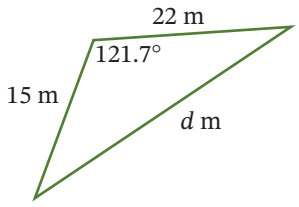
Foundation Standard Complex

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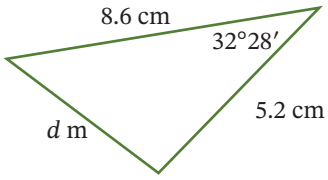
PATH

11 Find the value of d, correct to one decimal place. 8.08

a

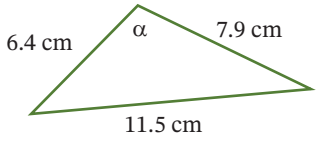


b

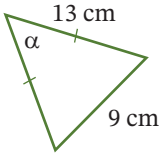


12 Find α, correct to the nearest minute. 8.11

a

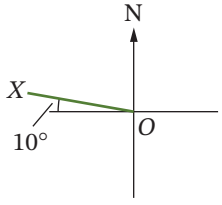


b

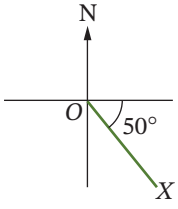


13 Write the true bearing of point X from O in each diagram. 8.03

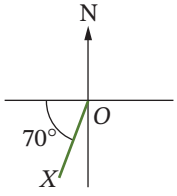
a



b



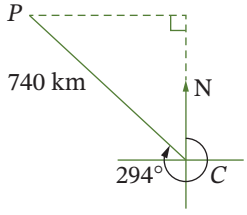
c



14 A plane flies on a bearing of 294° from Canberra for a distance of 740 km. 8.04

a How far north has the plane travelled (correct to the nearest kilometre)?

b What is the bearing of Canberra from the plane's position?



15 A parabola has the equation $y = 4x^2 - 3$. Find the x-coordinates of the points on the parabola that have a y-coordinate of 13. 7.04

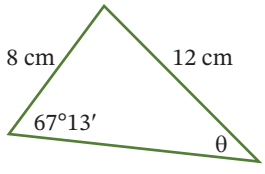
16 Sketch each exponential curve, showing the y-intercept. 7.07

a $y = 10^x$ **b** $y = 2^x - 3$ **c** $y = 4^{-x}$ **d** $y = 5^x + 2$

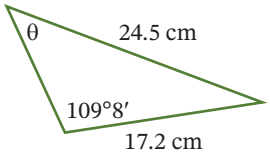
PATH

17 Find θ, correct to the nearest minute. 8.07

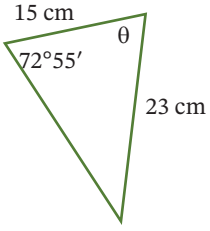
a



b



c



Foundation Standard Complex

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7.09

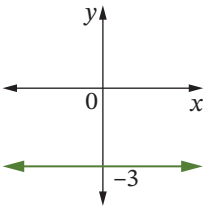
18 Match each equation to its graph.

a $y = 2x^2 - 2$ **b** $x = -3$ **c** $y = 2^{-x}$ **d** $x + y = 1$

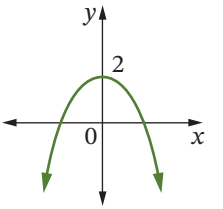
e $y = 2 - x^2$ **f** $y = \frac{8}{x}$ **g** $y = 2x^2$ **h** $y = 3^x$

i $y = -\frac{8}{x}$ **j** $y = x$ **k** $y = -3$ **l** $y = 4^x - 1$

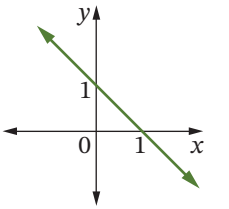
A



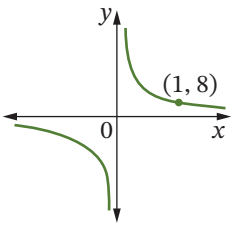
B



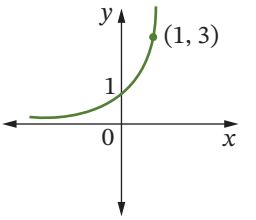
C



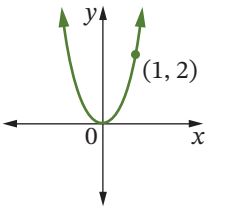
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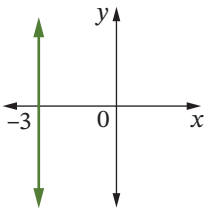
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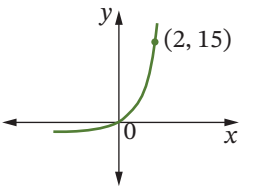
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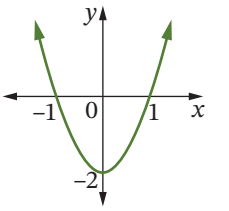
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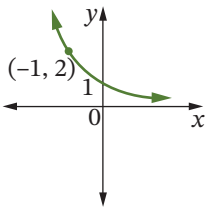
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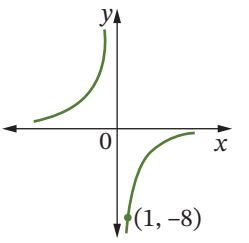
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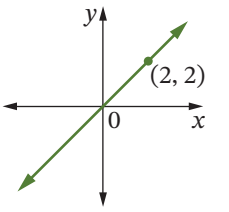
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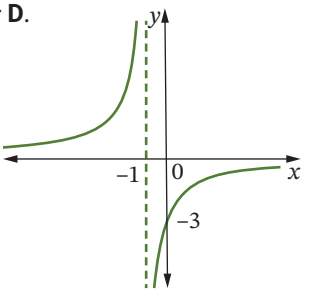
PATH

7.06

19 Which equation best represents this hyperbola? Select **A**, **B**, **C** or **D**.

A $y = \frac{3}{1-x}$ **B** $y = -\frac{1}{x+3}$

C $y = -\frac{3}{1-x}$ **D** $y = -\frac{3}{x+1}$



Foundation Standard Complex

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20 A fishing trawler leaves Ulladulla harbour on a bearing of N80°E for 30 km and then changes direction to a bearing of S53°E and stops when it is due east of the harbour.

How far from Ulladulla, correct to the nearest km, is the trawler?

PATH 8.06

21 Exponential growth or decay?

a The value of a house doubles every 10 years

b A football team loses $\frac{1}{4}$ of its supporters every year because of its poor performance

c The concentration of a drug in a person's body decreases by 10% every hour

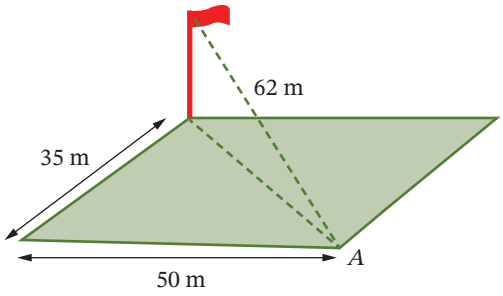
d The number of likes received for a cat video is going viral

7.08

22 A flagpole is at the corner of a rectangular courtyard in a school. The distance from A to the top of the flagpole is 62 metres.

a Calculate, correct to one decimal place, the height of the flagpole.

b Calculate, correct to the nearest minute, the angle of elevation from A to the top of the flagpole.

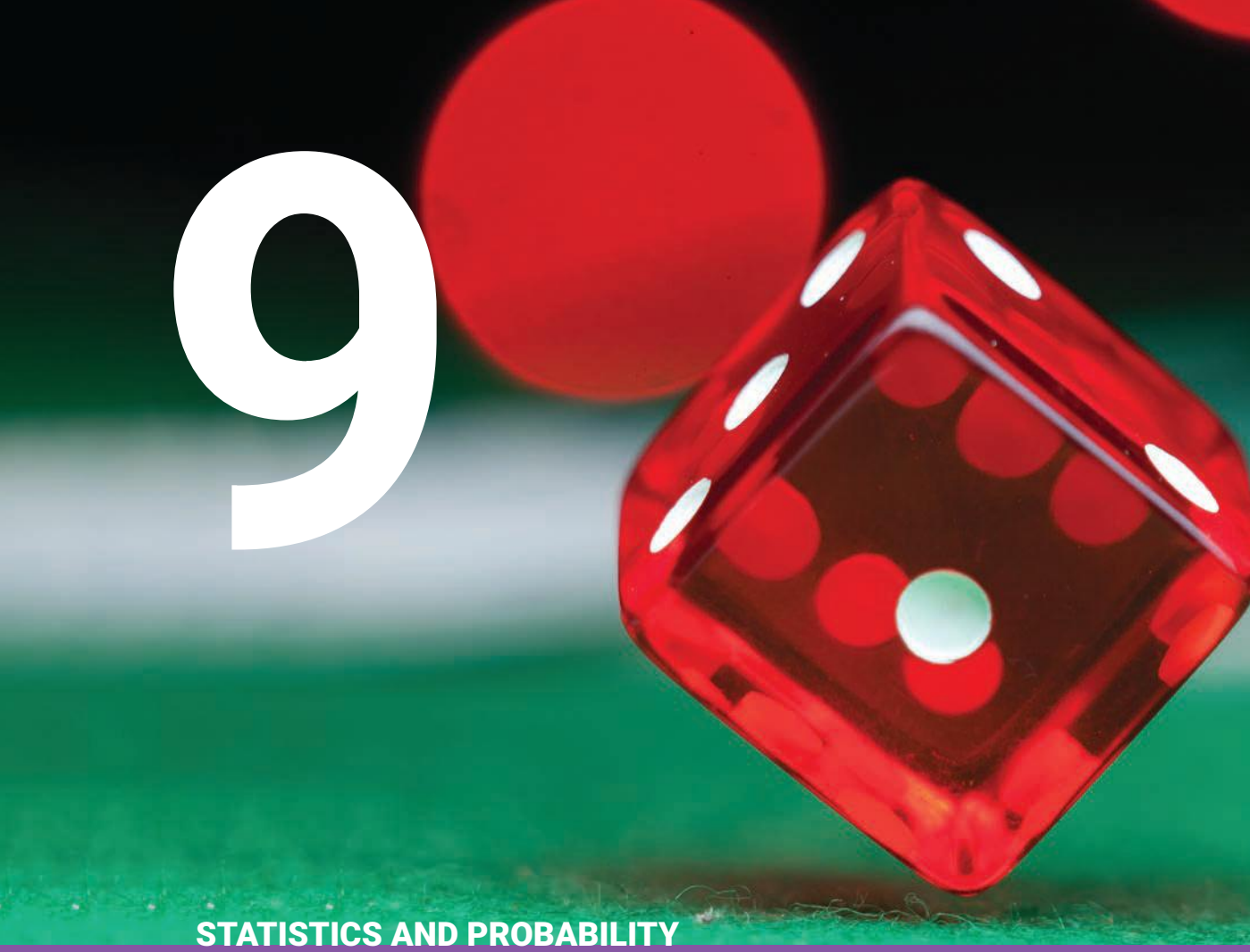


PATH 8.05

23 In $\triangle ABC$, $\angle A = 38°$, $a = 7$ cm and $c = 9$ cm. Find the possible values for the size of $\angle C$ (correct to the nearest degree).

8.07

Foundation Standard Complex



9

STATISTICS AND PROBABILITY

PROBABILITY

Probability theory, the study of chance, began in the 17th century when 2 great mathematicians, Blaise Pascal and Pierre de Fermat, wrote to each other to discuss mathematical problems arising from games of chance. Since then, probability has become an essential branch of mathematics that is used widely in fields such as weather forecasting, finance, insurance, politics and medical testing.



Chapter outline

		Working mathematically				
9.01	Observed probability	U	F	PS	R	C
9.02	Sets and Venn diagrams*	U	F		R	C
9.03	Two-way tables*	U	F		R	C
9.04	Tree diagrams	U	F	PS	R	C
9.05	Selecting with and without replacement	U	F	PS	R	C
9.06	Dependent and independent events	U	F	PS	R	C

*PATH

Wordbank

relative frequency The frequency of an event over repeated trials as a fraction of the total number of trials

set A collection of distinct objects, for example, set $X = \{1, 2, 3, 4, 5, 6\}$

simulation An artificial imitation of a chance experiment, usually using technology, to approximate probabilities of events

tree diagram A diagram of branches for listing all the possible outcomes of a multi-step chance experiment

trial One go or run of a repeated probability experiment, for example, one roll of a die

two-step experiment A chance experiment with 2 steps or stages, such as rolling a pair of dice

two-way table A way of grouping items into 2 overlapping categories, such as gender and the ability to drive a car

Venn diagram A diagram of circles (usually overlapping) for grouping items into categories

Quiz
Wordbank 9

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Videos (10):

9.01 Experimental probability
9.02 Venn diagrams 1 • Venn diagrams 2
9.03 Two-way tables 1 • Two-way tables 2
9.04 Tables and tree diagrams • Tree diagrams 3 • Tree diagrams 1 • Tree diagrams 2
9.05 Selecting with and without replacement

Twig videos (6):

9.01 Freak waves
9.06 The Monty Hall problem • The birthday paradox • The card counter • The prisoner's dilemma • Bayesian robots

PhET interactive (1):

9.01 Plinko probability

Quizzes (5):

- Wordbank 9
- SkillCheck 9
- Mental skills 9
- Language of maths 9
- Test yourself 9

Worksheets (9):

SkillCheck Probability review
9.01 Relative frequencies • Coins probability • Dice probability
9.03 Two-way tables • Two-way probability tables
9.04 Tree diagrams
9.05 Multi-step experiments
Mind map Probability

Puzzles (7):

9.01 Dice probability
9.02 Venn diagrams matching activity • And/or problems • Venn diagrams
9.03 Combined events: Two-way tables
9.04 Combined events: Tree diagrams
Language of maths Probability crossword

Technology (1):

9.01 Long-run proportion

Spreadsheet (1):

9.01 Long-run proportion

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

In this chapter you will:

- calculate expected and observed probabilities, including from simulated chance experiments
- (PATH) describe compound events using terminology such as 'and', exclusive 'or', inclusive 'or' and 'at least'
- (PATH) use sets, set notation, Venn diagrams and two-way tables to represent sample spaces and compound events to solve probability problems
- use tree diagrams and tables (arrays) to represent the sample space of two- and three-step chance experiments, with and without replacement, to solve probability problems
- identify dependent and independent events, and use the product rule for independent events

SkillCheck ANSWERS ON P. 615

1 If a die is rolled, which of the following is more likely? Select the correct answer **A**, **B**, **C** or **D**.

A a number less than 3 **B** an even number

C a number that is 3 or more **D** a number that is a prime number

2 A bag contains 5 10-cent coins, 4 20-cent coins and 3 50-cent coins. A coin is drawn at random from the bag.

a How many outcomes are in the sample space?

b Are the outcomes equally likely?

3 For this spinner, the red sector is twice as large as each of the other sectors. Find the probability that when the spinner is spun, the arrow lands on:



a red

b purple or blue

c not green.

4 A normal die is rolled. What is the probability of rolling:

a a 7? **b** a number less than 7?

5 The probability of Danica hitting a bullseye when playing darts is 0.6. What is the probability of Danica not hitting the bullseye?

6 Aditi bought 10 tickets in a raffle in which 400 tickets were sold and there is only one prize. What is the probability that Aditi will win the prize? Select **A**, **B**, **C** or **D**.

A $\frac{1}{400}$ **B** 2.5% **C** 0.0025 **D** 0.975

7 The probability of cloudy skies tomorrow is 0.85. What is the probability of clear skies?

Quiz
SkillCheck 9

Worksheet
Probability review

9.01

Observed probability 9.01

Probability calculated using the formula:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

is more specifically called **theoretical probability** or **expected probability**.

We can also determine probability based on the results of a trial that has been repeated many times, such as testing the effectiveness of 100 light globes, or rely on past statistics, such as weather patterns or the ages of drivers having car accidents. This type of probability is called **observed probability** or **experimental probability**, which is based on **relative frequency**, the number of times an **event** occurred as a fraction of the total frequency of outcomes.

Videos
Experimental probability

Freak waves

Worksheet
Relative frequencies

Puzzle
Dice probability

Interactive
Plinko probability

Technology
Long-run proportion

Spreadsheet
Long-run proportion

Observed probability

$$P(E) = \frac{\text{number of times the event happened}}{\text{total number of trials}}$$

$$\text{or } P(E) = \frac{\text{frequency of } E}{\text{total frequency}}$$

Expected frequency is the expected number of times an event will occur over repeated trials.

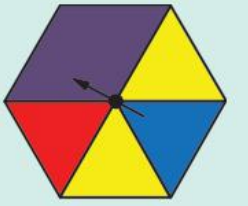
Expected frequency

Expected frequency = theoretical probability × number of trials.

Example 1

This spinner was spun 160 times and the results are shown.

Outcome	Frequency
Red	30
Blue	32
Yellow	50
Purple	48



a Calculate, as a decimal:

i the observed probability that the arrow stops on blue

ii the theoretical probability that the arrow stops on blue.

b Are the observed and theoretical probabilities similar?

c If the spinner is spun 500 times, calculate the expected frequency of spinning purple based on the theoretical probability.

Solution

a i Observed probability:

$$P(\text{Blue}) = \frac{32}{160}$$
$$= 0.2$$

ii Theoretical probability:

$$P(\text{Blue}) = \frac{1}{6}$$
$$= 0.1666...$$
$$\approx 0.17$$

b By comparing the decimals for the 2 answers, we see that the observed and theoretical probabilities are similar.

c Expected frequency of purple $= \frac{1}{3} \times 500$ probability × number of trials

$$= 166.6666...$$
$$\approx 167$$

9780170479622

Example 2

James rolled a die 100 times and recorded the results in a table.

Outcome	Frequency
1	23
2	19
3	11
4	12
5	18
6	17

a Find the observed probability of rolling:

i an even number

ii an even number or a number greater than 4

'Even number or a number greater than 4' is an example of a **compound event**.

iii an even number less than or equal to 4.

b Calculate the probability of rolling a 2 or 3

i as an observed probability **ii** as an expected probability.

c If the die is rolled 100 times, what is the expected frequency of rolling a 2 or a 3?

How does this compare with James' observed frequency?

Solution

a i Rolls of even numbers $= 19 + 12 + 17$ Frequencies of 2, 4, 6

$= 48$

Observed $P(\text{even}) = \frac{48}{100} = \frac{12}{25}$

ii Rolls of even numbers or numbers greater than 4 Frequencies of 2, 4, 6, 5

$= 19 + 12 + 17 + 18$

$= 66$

Observed $P(\text{even or} > 4) = \frac{66}{100} = \frac{33}{50}$

iii Rolls of even numbers less than or equal to 4 Frequencies of 2 and 4

$= 19 + 12$

$= 31$

Observed $P(\text{even and} \leq 4) = \frac{31}{100}$

b i Rolls of 2 or 3 $= 19 + 11$ Frequencies of 2 and 3

$= 30$

Observed $P(2 \text{ or } 3) = \frac{30}{100} = \frac{3}{10}$

ii Expected $P(2 \text{ or } 3) = \frac{2}{6} = \frac{1}{3}$

c Expected frequency of 2 or 3 $= \frac{1}{3} \times 100$ Probability × number of trials

$= 33.333...$

≈ 33

From the table, the observed frequency $= 19 + 11 = 30$, which is close to 33.

Probability simulations

Sometimes it is impractical or impossible to run a probability experiment because it is expensive or time-consuming. In these cases, we can imitate or recreate a chance situation using a computer model or an object like a die or spinner to generate random events. This is called a **probability simulation**, which can be used to calculate observed probabilities and predict outcomes.

Spreadsheets, apps, cards, dice, spinners and coins can be used to simulate random events. Simulations allow many repetitions, often thousands of times, of chance experiments to be conducted. Because it can easily be repeated, an advantage of a simulation is that it can also be used to verify the theoretical probability of an event.

Example 3

How likely is it for a family of 2 children to have both children of the same gender, that is, 2 boys or 2 girls? Design and conduct a probability simulation to determine a solution to this problem. Predict what the probability will be.

Solution

As there are 2 possible outcomes for the gender of a child, it can be modelled by flipping a coin, for example, let heads represent girls (G) and tails represent boys (B).

To simulate a family with 2 children, flip 2 coins together (or flip one coin twice).

The possible outcomes for a family with 2 children are GG, GB, BG and BB.

1 Copy the table below. Flip both coins after deciding which coin represents the first child. Alternatively search for an online coin flipping website or app or use the spreadsheet formula =INT(RAND()*2)+1) as shown in **Technology: Throwing a coin** after Exercise 9.01.

Outcome	Tally	Frequency	Observed probability
GG	|||		
GB	|		
BG	||		
BB	𝍸 ||		
		Total: 30	

2 Flip the 2 coins 30 times and complete the Tally column in the table.

3 Complete the Frequency and Observed probability columns.

4 Copy and complete the following to solve the problem.

$P(\text{same gender}) = P(\text{GG or BB})$

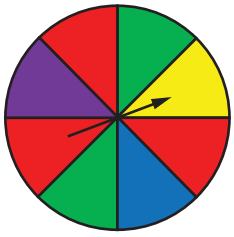
$= _____$

5 Compare your results with other students in your class. Do you notice any similarities or differences? Why?

EXERCISE 9.01 ANSWERS ON P. 615

Observed probability U F P S R C

1 Aashima spun this spinner 200 times and recorded the results. R C



Event	Frequency
Red	85
Green	42
Blue	28
Yellow	15
Purple	30

a Calculate, as a decimal, the observed probability that the arrow points to:

i red **ii** blue **iii** green.

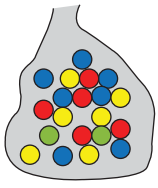
b Calculate, as a decimal, the theoretical probability that the arrow points to:

i red **ii** blue **iii** green.

c Are the observed and theoretical probabilities similar?

d For 200 spins, what is the expected frequency of red or purple based on the theoretical probability? How does this compare with the observed frequency?

2 A bag contains 7 blue, 6 yellow, 5 red and 2 green marbles. Lamisa selects a marble at random, records its colour and then returns it to the bag. Lamisa repeats this process 100 times and the results are shown. R C



Outcome	Frequency
Red	20
Blue	38
Yellow	33
Green	9

a Find the observed probability of selecting a marble that is:

i red **ii** blue

iii yellow **iv** green.

b What is the expected probability of selecting a marble that is:

i red? **ii** blue?

iii yellow? **iv** green?

c Are the observed and expected probabilities similar?

d If the process is repeated 100 times, what is the expected frequency of a selecting a yellow or green marble? How does this compare with the observed frequency?

3 A coin is tossed. R C

a What is the expected frequency of heads if the coin is tossed 100 times?

b Toss a coin 100 times, either by using a real coin or by computer or calculator simulation. Copy this table and record your results in it.

Outcome	Frequency
Head	
Tail	

c Calculate, as a decimal:

i the observed probability of tossing a head

ii the theoretical probability of tossing a tail.

d Are the observed and theoretical probabilities similar?

EXAMPLE 2

4 A die was repeatedly rolled and the results are shown in the table. R C

Outcome	Frequency
1	95
2	119
3	108
4	87
5	78
6	113

a How many times was the die rolled?

b Find the observed probability (as a decimal) of rolling:

i an odd number

ii a number less than 4

iii a 2 or a 3

iv a number less than 4 or an even number.

c Find the expected probability (as a decimal) of rolling:

i an odd number

ii a number less than 4

iii a 2 or a 3

iv a number less than 4 or an even number.

d Compare the observed probabilities to the expected probabilities.

5 Place 5 blue counters, 2 red counters and 3 yellow counters in a bag. Select a counter at random from the bag, note its colour and return the counter to the bag. Repeat this 60 times. Copy this table and record your results in it. R C

Event	Tally	Frequency
Blue		
Red		
Yellow		

a What is the observed probability of selecting:

i a blue counter? **ii** a red counter?

iii a yellow counter? **iv** a red or blue counter?

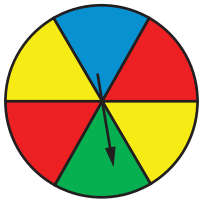
b What is the theoretical probability of selecting:

i a blue counter? **ii** a red counter?

iii a yellow counter? **iv** a red or blue counter?

c Are the observed probabilities similar to the theoretical probabilities?

6 Tara spun this spinner 50 times and the results are shown. R C



Event	Frequency
Red	15
Blue	6
Yellow	24
Green	5

a What is the observed probability (as a decimal) of the arrow stopping on

i red? **ii** blue? **iii** yellow? **iv** green?

b What is the expected probability (as a decimal) of the arrow stopping on

i red? **ii** blue? **iii** yellow? **iv** green?

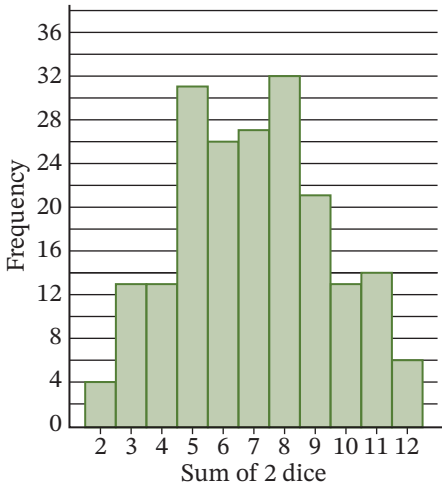
c Are the observed and expected probabilities similar?

d What is the expected frequency of the arrow stopping on a colour that is not yellow? How does this compare with Tara's observed frequency?

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7 2 dice are rolled and the sum of the numbers rolled was recorded in the frequency histogram.



a How many times were the dice rolled?

b Based on these results, what is the observed probability of rolling a sum:

i of 2? **ii** of 7?

iii of 10? **iv** greater than 7?

v less than 7?

vi of 7 or 8?

vii that is even and greater than 6?

8 Children at a shopping mall were asked how they travelled to school. R C

Mode of transport	Frequency
Walk	27
Bus	80
Car	62
Train	21
Bicycle	5
Skateboard	1
Other	4

a How many students were surveyed?

b Based on these results, find the probability that a student chosen at random will:

i walk to school

ii be driven to school

iii catch a bus to school

iv catch a train to school

v ride a skateboard to school.

c What mode of transport could 'Other' include?

d Survey 100 students at your school and make up a table showing the results. How do the results from your school compare with the results from the survey?

9 A die is rolled 100 times. R C

Outcome	Tally	Frequency
1		
2		
3		
4		
5		
6		

a What is the expected probability of rolling a 6? (Express your answer as a fraction and as a decimal.)

b How many times would you expect a 6 to appear if the die was rolled 100 times?

c Simulate rolling a die 100 times and record your results in a table similar to the one shown.

d What is the relative frequency of rolling a 6? (Express your answer as a fraction and as a decimal.)

e How does the expected probability of rolling a 6 compare with the observed probability?

EXAMPLE 3

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10 A school trivia quiz awards winners by using a computer to select a random prize from the following: Rubik's cube, art supplies, lollies, chocolates, squishy toy and ice cream. Greta won 2 prizes. Design and conduct a probability experiment to determine the probability that both prizes were the same. PS R C

11 A spinner for a game has 5 equal sectors with 2 of the sectors labelled WIN. Jayden spins it 3 times and wins the game if he spins WIN twice or more. Design a probability experiment to determine the probability of winning the game. An Internet search of 'adjustable probability spinners' could help you simulate this experiment. PS R C

12 Two out of every 5 people who have a runny nose and a fever have tested positive for a virus. Design a probability simulation to predict the number of people infected by this virus from 1000 tests. PS R C

13 Five out of every 7 people own a pet. Of these pet owners, 80% have a dog. Design a probability simulation to model this situation and calculate the percentage probability that a person selected at random owns a dog. PS R C

14 'If there are 10 horses in a race, the probability of each horse winning is 1 in 10'. Explain why this statement is not true. Give at least 2 reasons. R C



Mikhail Pogosov/Shutterstock.com

Technology

Throwing a coin

Worksheet
Coins probability

9.01

In this activity, a spreadsheet is used to simulate the throwing of a coin. The computer can quickly generate either a 1 (representing heads) or a 2 (tails).



Yeti studio/Shutterstock.com

1. In cell A1, enter the label 'Tossing a coin'.
2. In cell A2, enter the formula **=INT(RAND()*2+1)**, press Enter and either 1 or 2 will appear in the cell.
3. Select cell A2 and **Fill Down** to cell A11 to simulate the tossing of a coin 10 times.
4. Copy the table and in the first blank row record your results for Trials 1–10 (the numbers of heads and tails in your first 10 'tosses').
5. Simulate another 10 tosses of the coin by making the spreadsheet generate another set of random numbers. Do this by placing the cursor between the tops of columns A and B (so that it turns into a 'double arrow') and clicking.
6. Enter the results for Trials 11–20 in your table.
7. Repeat 8 more times so that you have 100 tosses of the coin recorded in the table.
8. **a** For 100 tosses of a coin, what is the expected frequency of heads based on the theoretical probability?
   b Compare this with the actual frequency.
9. Calculate, as a decimal, the theoretical and observed probabilities of tossing heads.
10. Compare your results with those of other students in your class. Briefly explain any differences and why they may have occurred.
11. Combine the results of students in your class to calculate the observed probability of throwing heads, as a decimal. How does this compare with the theoretical probability?

Trial	Number of heads	Number of tails
1–10		
11–20		
21–30		
31–40		
41–50		
51–60		
61–70		
71–80		
81–90		
91–100		
Total		

Worksheet
Dice probability

Technology

Rolling dice

In this activity, we will use a scientific calculator to simulate the rolling of a die.

1 Copy the table shown.

Outcome	Tally	Frequency	Observed probability
1			
2			
3			
4			
5			
6			

2

Casio scientific calculator	Sharp scientific calculator
Enter the formula RanInt#(1,6) as shown: [ALPHA] [.] 1 [SHIFT] [)] 6 [)] [=]	[2nd F] 7 (RANDOM), select 1 R-DICE, [=]

This generates a random number from 1 to 6. Press [=] more times to generate more random numbers. Alternatively, on a spreadsheet, type **=INT(RAND()*6+1)** into a cell and press the Enter key, then the [F9] key repeatedly for more random numbers.

3 Run the simulation 50 times and record the results in the table.

4 Calculate the theoretical probability for each number being tossed on the die.

5 Compare the theoretical probability and observed probability for each number. Are there similarities or differences? How could the observed probabilities be more accurate? Explain.

6 Copy this table for rolling 2 dice and finding their sum.

Sum	Tally	Frequency	Observed probability
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

7 On a scientific calculator, rolling 2 dice is achieved by adding the result of rolling one die twice.

Casio scientific calculator	Sharp scientific calculator
Enter the formula RanInt#(1,6) + RanInt#(1,6) as shown below: ALPHA · 1 SHIFT) 6) + ALPHA · 1 SHIFT) 6) =	Enter the formula R.DICE + R.DICE as shown below: 2nd F 7 (RANDOM), select 1 R-DICE, + 2nd F 7 (RANDOM), select 1 R-DICE, =

This generates the sum of 2 random numbers from 1 to 6.

Press = more times to generate more sums.

8 Run the simulation 50 times and record the results in the table.

9 Which sum is the most likely to occur?

10 Which sum is the least likely to occur? Why?

11 Which sum is theoretically the:

a most likely? **b** least likely?

Why?

12 Compare your answers to questions **9**, **10** and **11** and then compare them with the answers of other students in your class. Write a few sentences explaining your observations.

13 If you ran 50 more simulations, would you expect your results to be similar or different to your first 50 results? Explain.



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9.02 Sets and Venn diagrams

PATH

Videos
Venn diagrams 1
Venn diagrams 2

Puzzles
Venn diagrams matching activity
And/or problems
Venn diagrams

Sets

A **set** is a collection of distinct items, for example, a set of cricket equipment consists of a cricket bat, stumps, a cricket ball, helmet, pads, and so on. Distinct means 'different', so the elements in a set are all different.



atish naik/Shutterstock.com

The members of a set are called its **elements** and they can be listed between **braces** (curly brackets), separated by commas. For example,

- the set of even numbers {2, 4, 6, 8, 10, ...}
- the set of odd numbers {1, 3, 5, 7, 9, ...}
- the set of prime numbers {2, 3, 5, 7, 11, 13, 17, 19, ...}
- the set of multiples of 7 {7, 14, 21, 28, 35, ...}

Consider the set of vowels in the alphabet, set $V = \{a, e, i, o, u\}$.

Then the **complementary set** to set V is the set of letters in the alphabet that are *not* vowels.

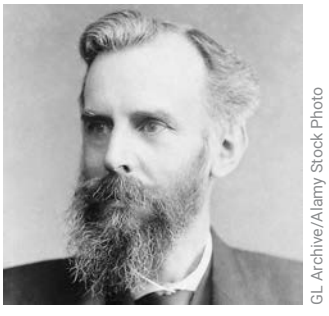
The complementary set to V is written as $\overline{V}$ (or V^C or V').

$\overline{V} = \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$ (the consonants)

$\overline{V}$ means 'not V' or the complement of V.

Venn diagrams

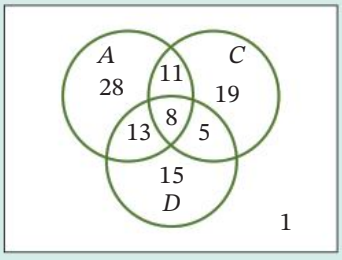
A **Venn diagram** is a diagram of circles (usually overlapping) that is used to represent sets. A rectangle represents the whole group, called the **universal set**, while the circles represent individual sets. Items common to 2 or more sets are placed in the intersection (overlapping region) of the circles. The Venn diagram was invented in 1880 by English mathematician and priest, John Venn (1834–1923).



GL Archive/Alamy Stock Photo

Example 4

This Venn diagram shows the results of a survey on what type of movies – action (A), comedy (C) or drama (D) – students prefer to watch.



a How many students were surveyed?

b How many students preferred to watch 2 types of movies only?

c Calculate, as a decimal, the probability of selecting a student who prefers to watch:

i action movies only

ii action or comedy movies, but not dramas

iii action and drama movies

iv all types.

d A student is chosen from those who like action and comedy movies. What is the probability that they also like to watch drama movies?

e What is the probability of selecting a student who does not like watching any of the 3 movie types?

Solution

a Number of students $= 28 + 11 + 8 + 13 + 5 + 19 + 15 + 1$

$= 100$

b 29 students preferred 2 types of movies only. $11 + 13 + 5 = 29$

c **i** Students preferring action movies only $= 28$ — The region of A that doesn't overlap C or D

$P(\text{action only}) = \frac{28}{100} = 0.28$

ii Students preferring action or comedy only — The regions of A and C that don't overlap with D

$= 28 + 19 + 11 = 58$

$P(\text{action or comedy only}) = \frac{58}{100} = 0.58$

'Action or comedy only' is an example of a **compound event**.

iii Students preferring action and drama $= 13 + 8$ — The regions where A and D intersect

$= 21$

$P(\text{action and drama}) = \frac{21}{100} = 0.21$

iv Students preferring all types $= 8$ — The region where the 3 circles intersect

$P(\text{all types}) = \frac{8}{100} = 0.08$

PATH

d Students preferring action and comedy $= 11 + 8$

$= 19$

Students preferring action and comedy and drama $= 8$

$P(\text{drama if preferring action and comedy}) = \frac{8}{19}$

≈ 0.42

e There is one student who doesn't prefer action, comedy or drama.

$P(\text{not action, comedy or drama}) = \frac{1}{100}$

$= 0.01$

When we combine 2 or more simple events, we get a **compound event**. In the above example, 'action and drama' and 'all types' are compound events.

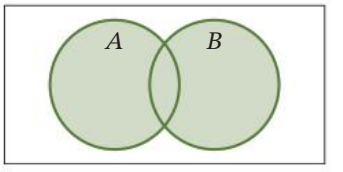
Union and intersection

For 2 sets or events A and B, the compound event '***A* or *B***' means to have A or B or both occurring. For example, 'to drive a car' **or** 'to ride a bus' means to drive a car only, or to ride a bus only, or to do both.

On a Venn diagram, this is represented by the region inside both sets A and B.

In set notation, this is written $\boldsymbol{A \cup B}$ and is called the **union** of sets A and B.

Union means to join, combine or unite.

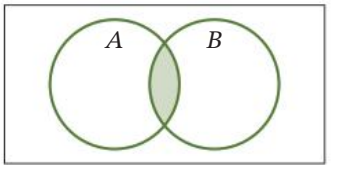


$A \cup B$ ('A union B')

The compound event '***A* and *B***' means to have both A and B occurring together. For example, 'to drive a car' **and** 'to ride a bus' means to do both things.

On a Venn diagram, this is represented by the region where sets A and B overlap.

In set notation, this is written $\boldsymbol{A \cap B}$ and is called the **intersection** of sets A and B.



$A \cap B$ ('A intersection B')

Notice that 'A or B' actually **includes** 'A and B' so this is an example of an **inclusive 'or'**

If 'A or B' didn't include 'A and B', then that would be an **exclusive 'or'**.

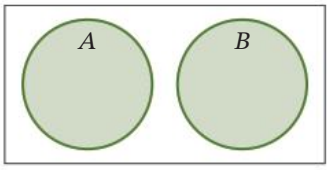
Union and intersection

$A \cup B$ means 'A **union** B' (A **or** B) and is the set of **all** elements in set A and set B.

$A \cap B$ means 'A **intersection** B' (A **and** B) and is the set of elements in **both** sets A and B.

Mutually exclusive events

If A and B are not overlapping, then they are **mutually exclusive** and on a Venn diagram they appear as 2 separate circles. For mutually exclusive sets or events, '***A* or *B***' means A only or B only but not both. For example, 'male' **or** 'female' means to be male, or female, but not both. In this case, 'A or B' **excludes** 'A and B' so this is an example of an **exclusive 'or'**.



Mutually exclusive events:
'A or B' means A or B but not both

Also, $A \cap B = \{\ \}$ has no elements because mutually exclusive sets have nothing in common. $\{\ \}$ is called an **empty set**, also written $\varnothing$.

Example 5

Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$ be the set of integers from 1 to 15.

a From this set, list the elements of each subset.

A **subset** is a set that is part of a larger set.

i $A = \{\text{odd numbers}\}$ **ii** $B = \{\text{even numbers}\}$ **iii** $C = \{\text{multiples of 5}\}$

iv $D = \{\text{prime numbers}\}$ **v** $E = \{\text{factors of 14}\}$

b List the elements of each set described.

i $A \cup C$ **ii** $C \cup D$ **iii** $B \cap C$ **iv** $A \cap D$

v $\bar{D}$ **vi** $C \cup \bar{E}$ **vii** $C \cap \bar{E}$ **viii** $\bar{C} \cap \bar{E}$

Solution

a **i** $A = \{1, 3, 5, 7, 9, 11, 13, 15\}$ odd numbers from 1 to 15

ii $B = \{2, 4, 6, 8, 10, 12, 14\}$ even numbers from 1 to 15

iii $C = \{5, 10, 15\}$ multiples of 5 numbers from 1 to 15

iv $D = \{2, 3, 5, 7, 11, 13\}$ prime numbers from 1 to 15

v $E = \{1, 2, 7, 14\}$ factors of 14 from 1 to 15

b Using the answers from part **a**:

i $A \cup C = \{1, 3, 5, 7, 9, 10, 11, 13, 15\}$ A union C

ii $C \cup D = \{2, 3, 5, 7, 10, 11, 13, 15\}$ C union D

iii $B \cap C = \{10\}$ B intersection C

iv $A \cap D = \{3, 5, 7, 11, 13\}$ A intersection D

v $\bar{D} = \{1, 4, 6, 8, 9, 10, 12, 14, 15\}$ The complement of D (not D)

vi $\bar{E} = \{3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15\}$ Not E

$C \cup \bar{E} = \{3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 15\}$ C union $\bar{E}$

vii $C \cap \bar{E} = \{5, 10, 15\}$ C intersection $\bar{E}$

viii $\bar{C} = \{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, 13, 14\}$ Not C

$\bar{C} \cap \bar{E} = \{3, 4, 6, 8, 9, 11, 12, 13\}$. Intersection of not C and not E

PATH

Example 6

A survey of 110 students at Hamper Valley College showed that 34 students study Art, 65 students study Chemistry, and 23 students study both Chemistry and Art.

a Represent this information on a Venn diagram.

b How many students study Art and Chemistry, but not both?

c What is the probability of randomly selecting a student from this group who takes:

- **i** Chemistry?
- **ii** Art and Chemistry?
- **iii** Art or Chemistry?
- **iv** neither Art nor Chemistry?



Alamy Stock Photo/Ian Shaw

Solution

a S = students surveyed

A = students doing Art

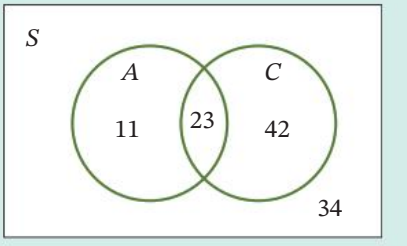
C = students doing Chemistry

There are 23 students who take both Art and Chemistry.

∴ Students doing Art only = 34 − 23 = 11

∴ Students doing Chemistry only = 65 − 23 = 42

∴ Students who take neither Art nor Chemistry = 110 − 11 − 42 − 23 = 34



b Number of students studying Art only or Chemistry only = 11 + 42 = 53

'Art or Chemistry only' is a compound event and an example of 'exclusive or'.

c **i** 65 students take Chemistry.

$P(\text{Chemistry}) = \frac{65}{110} = \frac{13}{22}$

ii $P(\text{Art and Chemistry}) = \frac{23}{110}$ $P(A \cap C)$

iii Number of students who take Art or Chemistry = 11 + 23 + 42 = 76

$P(\text{Art or Chemistry}) = \frac{76}{110} = \frac{38}{55}$ $P(A \cup C)$

iv $P(\text{neither Art nor Chemistry}) = \frac{34}{110} = \frac{17}{55}$ $P(\overline{A \cup C})$

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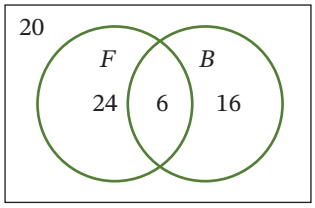
EXAMPLE 4

9.02

EXERCISE 9.02 ANSWERS ON P. 616

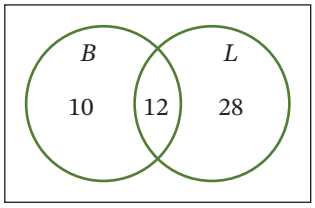
Sets and Venn diagrams U F R C

1 This Venn diagram shows the number of competitors who swam in freestyle (F) and butterfly (B) events at the school swimming carnival. How many students swam in freestyle or butterfly, but not both?
Select the correct answer **A**, **B**, **C** or **D**.



A 28 **B** 34 **C** 40 **D** 60

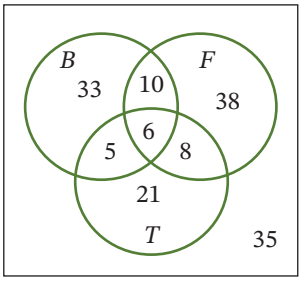
2 50 people were asked whether they had breakfast (B) or lunch (L) today. The results are shown in the Venn diagram. C



a What is the probability of selecting a person from this group who had:
- **i** breakfast?
- **ii** lunch?
- **iii** breakfast but not lunch?
- **iv** breakfast and lunch?
- **v** breakfast or lunch only?

b Of the people who had lunch, find the probability that a person also had breakfast.

3 The Venn diagram shows the number of Year 10 students who play basketball (B), touch football (F) or tennis (T). C

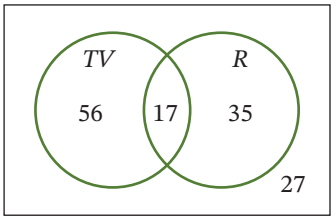


a How many students are in Year 10?

b Find the probability of selecting a student who plays:
- **i** basketball only
- **ii** tennis only
- **iii** touch football and tennis
- **iv** touch football or tennis
- **v** basketball but not touch football
- **vi** all 3 sports.

c Of the students that play touch football, find the probability of selecting a student who also plays tennis.

4 This Venn diagram shows the results of a survey asking people whether they watch TV or read (R) at home. R



a How many people were surveyed?

b Find the probability of selecting a person who watches TV only.

c What is the probability of selecting a person who doesn't watch TV or read?

d Of the people who read, find the probability that they also watch TV.

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EXAMPLE 5

5 Let $K = \{5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22\}$ be the set of integers from 5 to 22. R C

a From this set, list the elements of each subset.

i $M = \{\text{odd numbers}\}$ **ii** $N = \{\text{even numbers}\}$ **iii** $P = \{\text{multiples of 4}\}$

iv $Q = \{\text{square numbers}\}$ **v** $R = \{\text{prime numbers}\}$

b List the elements of each set described.

i $M \cup N$ **ii** $M \cup P$ **iii** $N \cap P$

iv $M \cap Q$ **v** $Q \cap R$

6 Let $X = \{\text{vowels}\}$ and $Y = \{\text{letters from the word MUSICIAN}\}$. R C

a Represent sets X and Y on a Venn diagram.

b List the elements of the set $X \cap Y$.

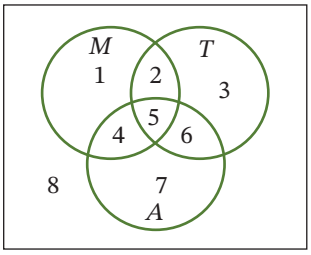
c List the elements of the set $X' \cap Y$.

7 Consider this Venn diagram that has its regions numbered 1 to 8. R C

What regions are included in each set shown below?

a $M \cap A$ **b** $A \cup T$ **c** $\overline{M}$

d $(T \cap M) \cup A$ **e** $A \cap (T \cup M)$ **f** $\overline{T} \cap A$



8 Let $A = \{31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46\}$ with subsets $B = \{\text{multiples of 4}\}$ and $C = \{\text{multiples of 8}\}$. List the elements of each set. R C

a $B \cup C$ **b** $B \cap C$ **c** $B \cup \overline{C}$ **d** $\overline{B} \cap C$

e $\overline{B} \cup C$ **f** $B \cap \overline{C}$ **g** $\overline{B} \cup \overline{C}$ **h** $\overline{B} \cap \overline{C}$

EXAMPLE 6

9 Of the 54 Year 10 Music students, 23 students sing (S), 43 students play a musical instrument (P) and 12 students sing and play a musical instrument. R C

a Show this information on a Venn diagram.

b Find the probability of selecting a Music student who:

i sings or plays an instrument

ii sings only

iii plays a musical instrument only

iv sings or plays an instrument, but not both.

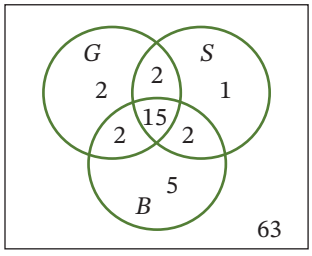
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PATH

9.02

10 This Venn diagram shows the number of countries that won medals at the Winter Olympic Games in Pyeongchang, South Korea in 2018. R



a How many countries competed at these games?

b What is the probability of randomly selecting a country that:

i won only gold medals?

ii won gold, silver and bronze medals?

iii won gold or silver medals, but not bronze?

iv did not win a gold or silver medal?

c Of the countries that won medals, what is the probability of selecting a country that

i won gold medals?

ii won bronze, but no gold or silver?

11 At Riverside College, Year 10 students are asked what language they are studying. 64 students take French (F), 47 students take Japanese (J), 15 students take both French and Japanese, and 27 do not study a language. R C

a How many students are in Year 10?

b Show the information on a Venn diagram.

c How many students studied only one language?

d Find the probability of selecting a Year 10 student at random who studies:

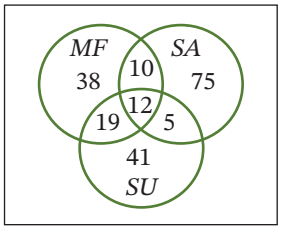
i French but not Japanese

ii Japanese but not French

iii no languages

iv only one language.

12 People were surveyed on the day they preferred to shop: Monday to Friday (MF), Saturday (SA) or Sunday (SU). The results are shown in the Venn diagram. R C



a How many people were surveyed?

b What is the probability of selecting a person who prefers to shop on:

i Monday to Friday?

ii Saturday?

iii Sunday?

iv on the weekend only?

v on Saturday or Sunday?

vi any day (has no preference)?

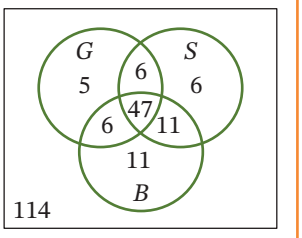
c Find the probability of selecting a person who only prefers Saturday or Sunday, but not both.

d Is it necessary to include the rectangle in this Venn diagram? Give reasons.

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13 This Venn diagram shows the number of countries that won gold, silver, bronze or no medals at the 2021 Olympic Games in Tokyo, Japan.



a Find the total number of countries that competed at these games.

b What is the probability of randomly selecting a country that:

i won a silver medal only?

ii won one medal only?

iii won at least 2 medals?

iv won at most one medal only?

c Out of the countries that won gold medals, find the probability of selecting a country that:

i won gold and silver, but not bronze

ii won gold, silver and bronze.

14 A travel agent surveyed 100 people to find out how many of them had visited Melbourne, M, and Brisbane, B. Of this sample, 31 people had visited Melbourne, 26 people had visited Brisbane and 12 people had visited both Melbourne and Brisbane. Draw a Venn diagram to represent these values and count the number of people who have visited: R C

a Melbourne or Brisbane, $M \cup B$

b Brisbane but not Melbourne, $B \cup M'$

c Melbourne or Brisbane, but not both, $(M \cup B) \cap (\overline{M \cap B})$

d neither Melbourne nor Brisbane, $(\overline{M \cup B})$.

15 24 people go on holidays. 15 go swimming, S, 12 go fishing, F, and 6 do neither activity. Draw a Venn diagram, find the number of elements in $S \cap F$ and describe what the people in this region do. R C

16 At Nelson Cove Senior College, students can study any of the 3 languages, French, F, Japanese, J, and Spanish, S. Of the 35 Year 12 Languages students, 21 study French, 18 study Japanese and 13 study Spanish. Of these, 11 students study both French and Japanese, 7 study both French and Spanish, 6 study both Japanese and Spanish, while one student studies all 3 languages. R C

a Represent the given information on a Venn diagram.

b How many students do not study any of the languages, $\overline{F} \cap \overline{J} \cap \overline{S}$?

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Two-way tables

9.03

A **two-way table** is another way of grouping items into overlapping categories, especially when there are many overlaps that cannot easily be represented by a Venn diagram.

Example 7

Year 11 students at Southbank College were surveyed on whether they had part-time jobs.

	Part-time work	No part-time work
Male	43	27
Female	35	31

a How many students are in Year 11 at Southbank College?

b How many students had part-time work?

c How many male students were in Year 11?

d What is the probability of selecting a student at random that:

i works part-time? **ii** is female and works part-time?

iii is male and doesn't work? **iv** doesn't work?

e What is the probability of selecting a student working part-time given that:

i the student is male? **ii** the student is female?

Solution

a Number of Year 11 students $= 43 + 27 + 35 + 31 = 136$

b Students with part-time work $= 43 + 35 = 78$

c Male students in Year 11 $= 43 + 27 = 70$

d **i** $P(\text{student works part-time}) = \frac{78}{136} = \frac{39}{68}$

ii There are 35 female students who work part-time.

$P(\text{female and part-time}) = \frac{35}{136}$

'Female and works part-time' is a compound event.

iii There are 27 males who don't work.

$P(\text{male and not working}) = \frac{27}{136}$

iv Number of students not working $= 27 + 31 = 58$

$P(\text{not working}) = \frac{58}{136} = \frac{29}{68}$

e **i** There are 70 male students and 43 of them work part-time.

$P(\text{working part-time given that student is male}) = \frac{43}{70}$

ii There are 66 female students and 35 of them work part-time.

$P(\text{working part-time given that student is female}) = \frac{35}{66}$

PATH

9.03

Videos
Two-way tables 1
Two-way tables 2

Worksheets
Two-way tables
Two-way probability tables

Puzzle
Combined events: Two-way tables

PATH

EXERCISE 9.03 ANSWERS ON P. 617

Two-way tables U F R C

EXAMPLE 7

1 People attending the Staying Alive Fitness Centre early on a Saturday morning either went swimming or did a workout in the gym. The numbers are shown in the table. R C

	Swimming	Gym
Male	32	53
Female	24	41

a How many people went to the fitness centre?

b Find the probability that a person selected at random:

i was female and went swimming

ii was male and did a workout in the gym

iii went swimming.

c Find the percentage (to the nearest whole number) of females who did a workout in the gym.

2 Year 10 students at Nusentry High School were given a choice of 2 activities on a wet sports afternoon: bowling or indoor soccer. Their selections are shown in the table. R C

	Bowling	Indoor soccer
Boys	25	48
Girls	43	12

a How many students are in Year 10?

b How many students:

i went bowling? **ii** played indoor soccer?

c What is the probability of randomly selecting a student who went bowling?

d What is the probability of randomly selecting a girl who played indoor soccer?

3 The composition of the Legislative Assembly (the lower house) in the State Parliament is shown in the table. R C

	Liberal/Nationals	Labor	Greens/Other
Male	41	20	5
Female	11	14	6

The Liberal/Nationals are in government and the others are in opposition.

a How many members of parliament (MPs) are there in the Legislative Assembly?

b Find the percentage probability of randomly selecting an MP who is:

i female **ii** male and in the Opposition **iii** Greens/Other.

c What percentage probability of:

i Government MPs are female? **ii** Opposition MPs are female?

d Compare your answers to part **c** and comment on the differences between the 2 results.

4 People were asked to name their favourite takeaway food. The results are shown in the table. R C

	Pizza	Hamburger	Fish and Chips
Men	22	35	18
Women	43	26	6

a How many people were surveyed?

b Find the probability (as a decimal) that a person selected at random:

i is male

ii is female and likes fish and chips

iii likes pizza

iv is male and likes hamburgers.

c If a male is selected at random, what is the probability that his favourite takeaway food is pizza?

5 Year 7 students were asked about their favourite drink. The results are in the table. R C

	Boys	Girls
Water	21	35
Milk	11	12
Juice	15	17
Soft drink	31	18

a How many students were in Year 7?

b What is the probability of randomly selecting a student that:

i prefers water?

ii is a boy and likes milk?

iii is a girl and likes soft drinks?

c What is the probability that if a girl is randomly selected, she prefers water?

6 A survey looked at whether people ate breakfast and whether they exercised regularly. R C

	Exercise	No exercise
Ate breakfast	72	27
Did not eat breakfast	38	63



Shutterstock.com/Ddaxiao Productions

a How many people were surveyed?

b What percentage of people exercised?

c Find the percentage probability of picking a person at random who:

i eats breakfast

ii does not exercise regularly

iii eats breakfast and exercises regularly

iv does not eat breakfast and does not exercise.

d Of the people who exercise regularly, what is the probability of picking someone who eats breakfast?

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PATH

7 Students at Mt Badger College were asked to indicate their preference for dark or milk chocolate in a survey. R C

		Milk chocolate	
		Like	Dislike
Dark chocolate	Like	545	134
	Dislike	157	42

a How many students attended the college?

b What is the probability (correct to 3 decimal places) of selecting a student at random who:

i likes dark chocolate?

ii likes both milk chocolate and dark chocolate?

iii likes dark chocolate, but dislikes milk chocolate?

iv dislikes both dark and milk chocolate?

9.04 Tree diagrams

Video
Tables and tree diagrams

Worksheet
Tree diagrams

Puzzle
Combined events: Tree diagrams

A **two-** or **three-stage experiment** is a **chance experiment** that has 2 or 3 parts or stages, for example:

- rolling 2 or 3 dice
- drawing 2 or 3 prizes in a raffle
- observing the weather each day over a weekend or a long weekend
- throwing 2 or 3 coins together.

A **tree diagram** lists all the possible outcomes of each stage. Branches stretch out to show the possible pathways of outcomes at each stage. An outcomes column at the end of the diagram lists the sample space.

The sample space for two-stage experiments can be displayed using lists, tables or tree diagrams, but the sample space for three-stage experiments is best displayed using a tree diagram.

Example 8

A coin is tossed and a die is rolled.

a Use a table to display the sample space.

b Find the probability of rolling:

i a tail and a 3

ii a head and an even number.

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Solution

a The sample space of a coin is a head (H) and a tail (T).

The sample space for a die is 1, 2, 3, 4, 5 and 6.

The sample space of tossing a coin and rolling a die is shown in the table below.

		Die					
		1	2	3	4	5	6
Coin	H	H1	H2	H3	H4	H5	H6
	T	T1	T2	T3	T4	T5	T6

Using a table ensures that all outcomes are counted

b **i** There are 12 outcomes in the sample space.

$\therefore P(\text{a tail and a 3}) = P(\text{T3}) = \frac{1}{12}$

'Tail and 3' is a compound event.

ii There are 3 outcomes that make up the event a head and an even number: H2, H4, H6.

$\therefore P(\text{a head and an even number}) = \frac{3}{12}$

$= \frac{1}{4}$

Example 9

2 coins are tossed.

a Use a tree diagram to list the sample space.

b Find the probability of tossing:

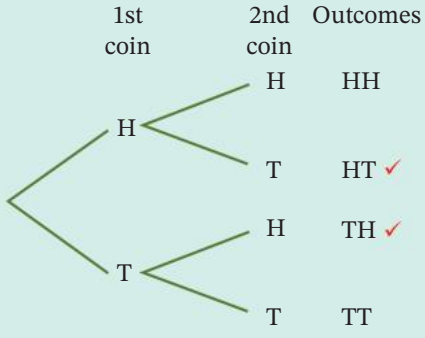
i 2 heads

ii a head and a tail (in any order).

Solution

a There are 2 outcomes for the first coin, followed by 2 outcomes for the second coin. There are $2 \times 2 = 4$ possible outcomes.

Using a tree diagram ensures that all outcomes are counted.



b **i** There is one outcome out of a possible 4 for 2 heads.

$\therefore P(\text{2heads}) = \frac{1}{4}$

ii There are 2 outcomes for a head and a tail (✓ on the tree diagram).

$\therefore P(\text{a head and a tail}) = \frac{2}{4}$

$= \frac{1}{2}$

9.04

Videos
Tree diagrams 3
Tree diagrams 1
Tree diagrams 2

EXERCISE 9.04 ANSWERS ON P. 617

Tree diagrams U F PS R C

EXAMPLE 8

1 A boy and a girl are to be chosen from this shortlist to represent the school at a conference:

Boys: Ben, Christian, Ewan, William.

Girls: Becky, Cassandra, Emily, Marianne, Roz, Siana. R C

a List all the possible pairs of a boy and a girl.

b Find the probability of selecting:

i Christian and Roz

ii a boy and a girl whose names begin with a B or a C

iii a pair that includes Emily.

2 2 coins are tossed together. R C

a Copy and complete the table to find all the outcomes in the sample space.

		1st coin	
		H	T
2nd coin	H		
	T		

b What is the probability of tossing:

i 2 tails? **ii** a head and a tail?

iii at least one head?

3 2 dice are rolled.

a Copy and complete this table to list the sample space. PS R C

		1st die					
		1	2	3	4	5	6
2nd die	1	1, 1	2, 1				
	2						
	3						
	4			3, 4			
	5						
	6						

b How many possible outcomes are there?

c Find the probability of rolling:

i two 6s **ii** at least one 6

iii doubles **iv** 2 even numbers

v at least one 2 **vi** 2 numbers greater than 3

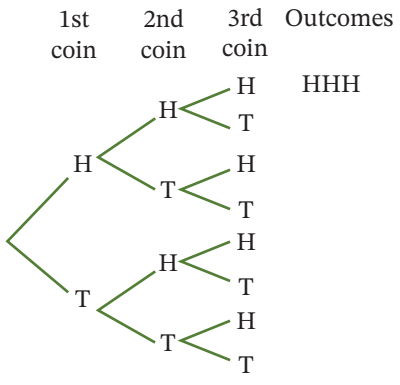
vii one odd and one even number

viii 2 numbers where the first number is greater than the second number.

EXAMPLE 9

9.04

4 3 coins are tossed. PS R C



a Copy and complete the tree diagram.

b How many outcomes are there in the sample space?

c Use the tree diagram to find:

i P(3 heads) **ii** P(2 heads)

iii P(3 tails)

iv P(head, then tail and then head)

v P(2 heads or 3 heads).

d Find the probability of tossing:

i at least 1 tail **ii** at most 2 tails.

e If 3 coins are tossed 200 times, find the expected frequency of:

i tossing 2 heads **ii** tossing no tails.

5 Use a tree diagram to display all possible outcomes when a coin and die are tossed together. R C

6 A 6-sided die and a 4-sided die (numbered 1, 2, 3 and 4) are rolled together. PS R C

a Construct a table to list the outcomes in the sample space.

b How many outcomes are in the sample space?

c Find the probability of rolling:

i doubles

ii 2 even numbers

iii one even and one odd number

iv a pair of numbers that are both less than 4

v a pair of numbers that are both greater than 4.

7 2 dice are rolled and the sum of the 2 numbers is calculated. PS R C

a Copy and complete this table to show all possible sums.

		1st die					
		1	2	3	4	5	6
2nd die	1	2					
	2						
	3				7		
	4						
	5						
	6					11	

b Find the of probability of rolling a sum:

i of 5 **ii** of 12 **iii** of 7

iv that is even **v** less than 2 **vi** more than 7

vii at least 7 **viii** between 4 and 8.

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8 4 coins are tossed. PS R C

a Use a tree diagram to list the sample space.

b Find the probability of tossing:

i 4 heads **ii** 1 head **iii** 2 tails

iv at least 1 tail **v** 2 heads and then 2 tails **vi** not more than 1 tail.

c If 4 coins are tossed 1000 times, find the expected frequency of having:

i 4 heads **ii** 2 heads and 2 tails **iii** at least one tail.

9 The weather on a long weekend will either be fine or rain each day, with each outcome being equally likely. PS R C

a Draw a tree diagram to show the possible outcomes for Saturday, Sunday and Monday.

b What is the probability that:

i it rains on all 3 days?

ii it is fine on 2 of the 3 days?

iii it is fine on Saturday and Sunday, but rains on Monday?

iv it rains on at least one day of the long weekend?

Quiz
Mental skills 9

Mental skills 9: Maths without calculators ANSWERS ON P. 618

Percentage increase and decrease

The fraction equivalents of commonly-used percentages can help us when we need to increase or decrease a number by a percentage.

Percentage	1%	5%	10%	$12\frac{1}{2}\%$	20%	25%	$33\frac{1}{3}\%$	50%
Fraction	$\frac{1}{100}$	$\frac{1}{20}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$

1 Study each example.

a Increase 360 by 25%.

$25\% \text{ of } 360 = \frac{1}{4} \times 360$

$= 360 \div 4$

$= 90$

$360 + 90 = 450$

b Increase \$80 by 5%.

$5\% \text{ of } \$80 = \frac{1}{20} \times \$80$

$= \$80 \div 20$

$= \$4$

$\$80 + \$4 = \$84$

or $10\% \text{ of } \$80 = \$8$

$\therefore 5\% \text{ of } \$80 = \$8 \div 2$

$= \$4$

2 Now increase:

a \$340 by 20% **b** 66 by 50% **c** 150 by $33\frac{1}{3}\%$ **d** \$400 by 1%

e 640 by 5% **f** \$72 by $12\frac{1}{2}\%$ **g** \$470 by 10% **h** 180 by 25%

i 420 by $33\frac{1}{3}\%$ **j** \$80 by 5% **k** \$280 by 25% **l** 70 by 20%

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3 Study each example.

a Decrease \$225 by $33\frac{1}{3}\%$

$33\frac{1}{3}\%$ of \$225 $= \frac{1}{3} \times$ \$225

$=$ \$225 $\div 3$

$=$ \$75

\$225 $-$ \$75 $=$ \$150

b Decrease \$70 by 15%

10% of \$70 $= \frac{1}{10} \times$ \$70 $=$ \$7

$\therefore$ 5% of \$70 $= \frac{1}{2} \times$ \$7 $=$ \$3.50

$\therefore$ 15% of \$70 $=$ (10% $\times$ \$70) + (5% $\times$ \$70)

$=$ \$7 + \$3.50

$=$ \$10.50

4 Now decrease:

a 440 by 25%
b \$300 by 20%
c 2400 by $33\frac{1}{3}\%$
d \$500 by 15%
e \$250 by 10%
f \$120 by 50%
g \$72 by $12\frac{1}{2}\%$
h 80 by 5%
i \$85 by 20%
j \$3800 by 1%
k \$440 by 15%
l \$150 by $33\frac{1}{3}\%$

Investigation

The birth month paradox

A **paradox** is a statement or proposition that seems impossible but is actually true.

1 Copy this table.

Group	Outcome (Y or N)
1	
2	
3	
4	
5	

2 Randomly select a group of 5 people and ask them what month they were born in. If 2 or more people have the same birth month, record a Y in the table for Group 1, otherwise write N.

3 Repeat this process 4 more times, recording your results in the table.

4 Combine your results with those of 6 other students so that you have the outcomes for 30 groups.

5 What fraction of the groups had repeated birth months?

6 Collect the results of another group of 6 students. What fraction of the groups had a repeated birth month?

7 The birth month paradox is that in any randomly selected group of 5 people, the probability that at least 2 people have the same birth month is greater than 0.5. Have your results shown this to be true?

8 Can you show the following?

a For every 23 people selected at random, the probability that at least 2 people will share the same birthday is 50%.

b If 30 people are selected at random, this probability is 70%.

c If 50 people are selected at random, this probability is 97%.

9.05 Selecting with and without replacement

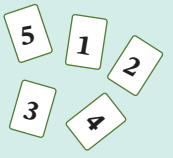
Worksheet
Multi-step experiments

Video
Selecting with and without replacement

In two- and three-stage experiments where an item is selected repeatedly, the outcome of the second or third stage may be affected by the outcome of the previous stage. This depends on whether each selected item is **returned** to the set of items before the next item is selected. If it is, then this is called **selecting 'with replacement'**. If it isn't, then it is called **selecting 'without replacement'**.

Example 10

2 cards are selected at random from a set of cards numbered 1 to 5, to form a 2-digit number.



a Make a list of all possible outcomes if the cards are drawn:
i with replacement **ii** without replacement.

b If the first card is replaced before the second card is drawn, find the probability that the number formed is:
i even **ii** greater than 30 **iii** divisible by 5.

c If the first card is not replaced, find the probability that the number formed is:
i even **ii** odd **iii** less than 20.

Solution

a i The possible outcomes, **with replacement**, are:

		1st digit				
		1	2	3	4	5
2nd digit	1	11	21	31	41	51
	2	12	22	32	42	52
	3	13	23	33	43	53
	4	14	24	34	44	54
	5	15	25	35	45	55

There are $5 \times 5 = 25$ different outcomes possible.

ii The possible outcomes, **without replacement**, are:

		1st digit				
		1	2	3	4	5
2nd digit	1	–	21	31	41	51
	2	12	–	32	42	52
	3	13	23	–	43	53
	4	14	24	34	–	54
	5	15	25	35	45	–

There are fewer outcomes for 'without replacement' because numbers with repeated digits such as 11 and 44 are not allowed.

There are $5 \times 4 = 20$ different outcomes possible.

b i With replacement, there are 10 even numbers.

$P(\text{even number}) = \frac{10}{25} = \frac{2}{5}$

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ii There are 15 numbers greater than 30.

$P(\text{number} > 30) = \frac{15}{25} = \frac{3}{5}$

iii There are 5 numbers divisible by 5. 15, 25, 35, 45 and 55

$P(\text{number divisible by } 5) = \frac{5}{25} = \frac{1}{5}$

c i Without replacement, there are 8 even numbers.

$P(\text{even}) = \frac{8}{20} = \frac{2}{5}$

ii There are 12 odd numbers.

$P(\text{odd}) = \frac{12}{20} = \frac{3}{5}$

iii There are 4 numbers less than 20.

$P(\text{number} < 20) = \frac{4}{20} = \frac{1}{5}$

Probability tree diagrams

For more complex probability problems, the number of possible outcomes can be quite large and the tree diagram very long and detailed. In these cases, it is often possible to construct a simplified tree diagram that has probability values listed on its branches. This smaller 'trimmed' tree diagram is sometimes called a **probability tree diagram**.

Example 11

A bag contains 3 red lollies, 5 blue lollies and 4 green lollies. Two lollies are selected at random from the bag without replacement. Calculate the probability of selecting:

a red lolly and then a blue lolly **b** 2 lollies the same colour.

Solution

First, write down the numbers of lollies and count the total.

3 R

5 B

4 G 12 lollies

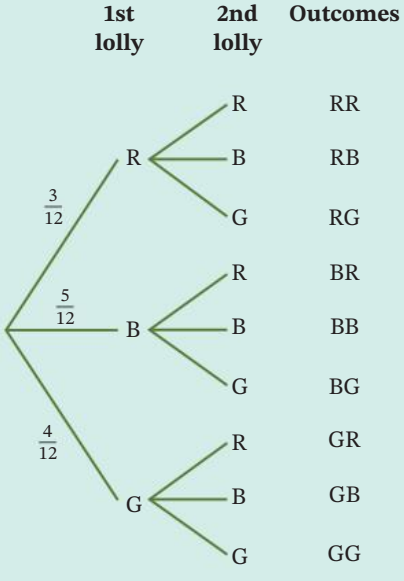
The tree diagram will be 2 stages deep as there are 2 selections.

For the 1st lolly, draw 3 branches for the 3 colours: red (R), blue (B), green (G).

For the 2nd lolly, also draw 3 branches for the 3 colours.

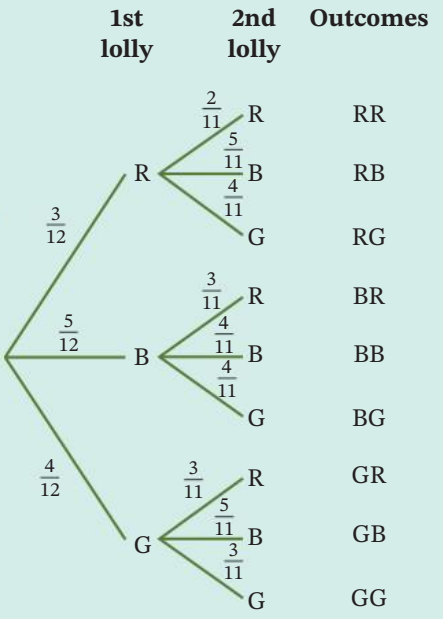
List the $3 \times 3 = 9$ outcomes from RR down to GG.

For the 1st lolly, the possible outcomes are red, blue or green. Write the appropriate probability values out of 12 for each colour: $\frac{3}{12}, \frac{5}{12}, \frac{4}{12}$.



For the 2nd lolly, there are now 11 lollies left. Write the appropriate probability values out of 11 for each colour. Make sure that the probabilities consider the first counter taken out.

If the first counter was red, then there are only 2 red lollies left in the bag, so the probability of another red is $\frac{2}{11}$.



Hint: The fractions in each group of joined branches should add up to 1, for example, $\frac{2}{11}+\frac{5}{11}+\frac{4}{11}=\frac{11}{11}=1$.

a To find the probability of red and then blue, follow the branches that give RB and multiply their probabilities.

$P(\text{RB}) = \frac{3}{12}\times\frac{5}{11}$ 'AND' means we multiply the probabilities

$= \frac{15}{132}$

$= \frac{5}{44}$

b To find the probability of 2 same colours, look for RR or BB or GG.

$P(\text{same colour}) = \text{P(RR or BB or GG)}$ 'OR' means we add the probabilities

$= \text{P(RR)} + \text{P(BB)} + \text{P(GG)}$

$= \frac{3}{12}\times\frac{2}{11} + \frac{5}{12}\times\frac{4}{11} + \frac{4}{12}\times\frac{3}{11}$

$= \frac{6}{132} + \frac{20}{132} + \frac{12}{132}$

$= \frac{38}{132}$

$= \frac{19}{66}$

Probabilities on a probability tree diagram

- For 'AND' events, multiply the probabilities along the branches of each step.
- For 'OR' events, add the probabilities of separate branches together.

EXERCISE 9.05 ANSWERS ON P. 619

Selecting with and without replacement U F PS R C

EXAMPLE 10

1 The positions of captain and vice-captain of a netball team are to be selected from Cate, Amal, Gemma, Josie, Evangeline and Rukshana. R C

a List the possible pairings of captain and vice-captain.

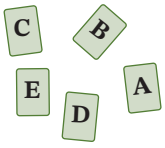
b What is the probability of Amal being captain or vice-captain?

c What is the probability of Evangeline becoming vice-captain?



Alamy Stock Photo/Tom Lindsey

2 2 cards are drawn from a set of cards labelled A, B, C, D and E. PS R C



a Make a list of all possible outcomes if the cards are drawn:

i with replacement **ii** without replacement.

b If the first card is replaced before the second card is drawn, find the probability that:

i both letters are the same

ii both letters are vowels

iii one letter is a vowel and the other is a consonant.

c If the first card is not replaced, find the probability that:

i both letters are vowels

ii one letter is a vowel and the other is a consonant

iii the first letter is a B or a D

iv the last letter is not A.

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3 When staying at a hotel, David and Sarah can select one item from each course of a breakfast menu. R C

1st course	2nd course
Cereal (C)	Bacon and eggs (B)
Fruit (F)	Ham and cheese croissants (H)
Yoghurt (Y)	Pancakes (P)
	Sausages and tomatoes (S)
	Toast and jam (T)

a Copy and complete the table to list all the different 2-course breakfasts available.

		2nd course				
		B	H	P	S	T
1st course	C					
	F					
	Y					

b If one of the combinations of breakfasts is chosen at random, what is the probability that it includes:

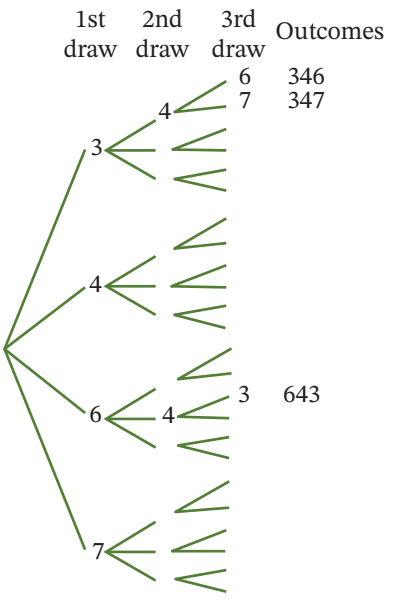
i fruit?

ii cereal but not bacon and eggs?

iii fruit and croissants?

4 The numbers 3, 4, 6 and 7 are written on separate cards and placed in a bag. 3 cards are drawn at random without replacement to form a 3-digit number. PS R C

a Copy and complete the tree diagram to show all 24 possible outcomes.



b Find the probability of forming a number:

i that is even

ii greater than 400

iii between 400 and 700

iv that is even and greater than 400.

5 The numbers 4, 5 and 9 are written on separate cards and placed in a bag. 3 cards are drawn at random with replacement to form a 3-digit number. PS R C

a Use a tree diagram to show all 27 possible outcomes.

b Find the probability of forming a number:

i with all digits the same

ii that is odd

iii less than 500

iv with all digits different.

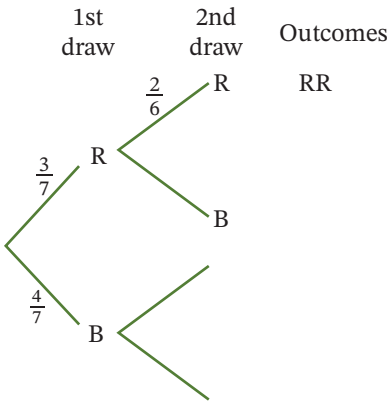
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EXAMPLE 11

9.05

6 A bag contains 3 red and 4 blue marbles. 2 marbles are taken out of the bag without replacement. R C



a Copy and complete the probability tree diagram to list all possible outcomes.

b Find the probability of drawing:

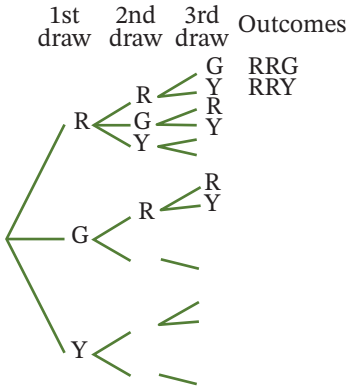
i 2 red marbles

ii 2 blue marbles

iii a blue and a red marble

iv at least one blue marble.

7 A bag contains 2 red marbles, 1 green marble and 1 yellow marble. 3 marbles are drawn from the bag at random without replacement. PS R C



a Copy and complete the probability tree diagram to list the sample space.

b Find the probability of drawing:

i 2 red marbles

ii a red, green, and red marble in that order

iii at least one red marble.

8 A family has 3 children. PS R C

a Use a tree diagram to list all possible outcomes in the sample space.

b What is the probability that the family consists of:

i 3 boys?

ii 3 girls?

iii 2 girls and a boy?

iv a girl and then 2 boys?

9 2 ice cream flavours are selected from vanilla, strawberry and chocolate. PS R C

a Draw a tree diagram to show the sample space if the flavours are selected:

i with replacement (repetition allowed)

ii without replacement.

b If the flavours are selected with replacement, find the probability of selecting:

i 2 identical flavours

ii 2 different flavours

iii no vanilla

iv at least one strawberry flavour.

c If the flavours are selected without replacement, find the probability of selecting:

i 2 identical flavours

ii 2 different flavours

iii no vanilla

iv at least one strawberry flavour.

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10 For a long weekend (Saturday to Monday), the probability of rain on each day is $\frac{2}{5}$. R C

a Draw a probability tree diagram to show the possible outcomes for the 3 days. Label each branch of the tree diagram with its probability.

b For the long weekend, calculate the probability of:

i exactly 2 rainy days

ii no rainy days

iii exactly one rainy day

iv at least one rainy day.

11 James, Max and Phoebe all play tennis. James serves an ace 4 out of 5 times on average, Max can serve an ace 3 out of 4 times and Phoebe 2 out of 3. They each complete a serve at the same time. R C

a Draw a probability tree diagram representing the possible outcomes for James, Max and Phoebe.

b What is the probability that:

i they all serve an ace?

ii none of them serve an ace?

iii James serves an ace but the other 2 players do not?

iv only one of the players serves an ace?

12 Yen has 4 tickets in a raffle where 100 tickets have been sold. 3 tickets will be drawn without replacement for 1st, 2nd and 3rd prize. R C

a Draw a probability tree diagram for winning 1st, 2nd or 3rd prize.

b What is the probability of winning:

i 1st prize?

ii 2nd prize but no other prizes?

iii exactly one of the prizes?

iv none of the prizes?

Investigation

Dependent or independent?

Work in pairs.

You will need: a coin, 3 blue pens and 2 red pens.

1 a i Toss a coin and record the outcome (head or tail).

ii What is the probability of obtaining your outcome?

b i Toss the coin a second time and record the outcome.

ii What is the probability of obtaining the second outcome?

c Is the outcome of the second toss affected by the outcome of the first toss? Is the probability of the second outcome independent or dependent on the first outcome? Justify your answer.

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2 **a** Copy this table.

With replacement	1st draw	2nd draw
Blue		
Red		
	40	40

b Put 3 blue pens and 2 red pens in a bag. Randomly draw a pen from the bag and record its colour.

c Put back the pen you drew and shake the bag. Draw a pen again and record its colour.

d Repeat the procedure from parts **a** and **b** 40 times and record the totals of each outcome in the table.

e Use your results to find:

i P(blue pen drawn first) **ii** P(blue pen drawn second).

f **i** Are your 2 results for part **e** the same?

ii Would you expect the results to be the same? Give reasons.

g Is the outcome of the second draw dependent on the outcome of the first draw?

3 **a** Copy this table.

Without replacement	1st draw	2nd draw
Blue		
Red		
	40	40

b Again, place 3 blue pens and 2 red pens in a bag. Randomly draw a pen from the bag and record its colour.

c Do not return the pen, shake the bag and draw a second pen, recording its colour.

d Repeat the procedure from parts **a** and **b** 40 times and record the totals of each outcome in the table.

e Use your results to find:

i P(blue pen drawn first) **ii** P(blue pen drawn second).

f **i** Are your 2 results for part **e** the same?

ii Would you expect your results to be the same? Give reasons.

g Is the outcome of the second draw dependent on the outcome of the first draw? Compare your results with those of other students in your class.

Dependent and independent events 9.06

Two events are **independent** if the outcome of one event **does not affect** the outcome of the other event. So, one event occurring does not change the probability of the other event. For example, if a coin and a die are tossed together, the 2 events are independent as the outcome on the coin does not affect the outcome on the die. Wearing blue socks and passing a driving test are also independent events.

Two events are **dependent** if the outcome of one event **does affect** the outcome of the other event. So, one event occurring changes the probability of the other event occurring. For example, when selecting 2 coloured pencils from a pencil case without replacement, the 2 events are dependent because the outcome of the second draw is affected by the outcome of the first draw. Raining on a school sports day and sport being cancelled are also dependent events.

Videos
The Monty Hall problem
The birthday paradox
The card counter
The prisoner's dilemma
Bayesian robots

Example 12

A coin and a die are tossed together.

a List the outcomes in the sample space.

b Find:

i $P(\text{head on the coin})$ **ii** $P(\text{even number on the die})$

iii $P(\text{head and even number})$.

c Is $P(\text{head and even number}) = P(\text{head}) \times P(\text{even number})$?

d Are the 2 events dependent or independent?

Solution

a The outcomes are H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5 and T6.

b **i** $P(\text{head}) = \frac{1}{2}$ **ii** $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$

iii $P(\text{head and even}) = \frac{3}{12} = \frac{1}{4}$ H2, H4 and H6

c Yes, since $P(\text{head and even}) = \frac{1}{4}$ and $P(\text{head}) \times P(\text{even}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

d The 2 events are independent since the outcome when tossing a coin does not affect the outcome when rolling a die.

The product rule for independent events

Two events are **independent** if the outcome of one event does not affect the outcome of the other event.

If A and B are 2 independent events, then $P(A \text{ and } B) = P(A) \times P(B)$.

Example 13

A bag contains 3 brown chocolates and 1 white chocolate. 2 chocolates are drawn from the bag, without replacement.

a Find the probability of:

i selecting a brown chocolate with the first draw

ii selecting a brown chocolate with the second draw if the first chocolate was brown.

b Are selecting a brown chocolate with the first and second draws dependent or independent?

Solution

a **i** $P(\text{brown on the first draw}) = \frac{3}{4}$

ii After drawing a brown chocolate, there are 3 chocolates left, of which 2 are brown.

$\therefore P(\text{brown on the second draw}) = \frac{2}{3}$

b The bag contains 2 brown chocolates and 1 white chocolate for the second draw, so $P(\text{brown})$ decreases from $\frac{3}{4}$ to $\frac{2}{3}$. The second event is dependent on the first event.

EXERCISE 9.06 ANSWERS ON P. 621

Dependent and independent events U F PS R C

1 State whether each pair of events are dependent or independent. R C

- **a** Rolling 4 on a die and rolling an even number on another die
- **b** Rolling 6 on a die and rolling 6 again on the same die
- **c** Training hard at soccer and winning a soccer match
- **d** Drawing a red ball from a bag containing red and blue balls, replacing it and then drawing a blue ball from the bag
- **e** Electing a team captain from a group of players and then electing a vice-captain from the same group
- **f** Tossing 2 coins and obtaining a head on the first coin and a head on the second coin
- **g** Finding $20 on the street and getting a phone call from your parent

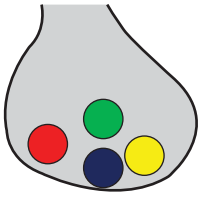
2 In Lotto, 6 balls are drawn without replacement. Are the events of drawing each of the balls dependent or independent events? Give reasons. R C

3 A coin is tossed 3 times and the result is heads each time. R C

- **a** Are each of the 3 coin tosses dependent or independent events?
- **b** The coin is tossed a 4th time. What is the probability of obtaining a head on the 4th toss?

EXAMPLE 12

4 A normal die is rolled and a marble is drawn from a bag containing a yellow marble, green marble, blue marble and red marble. R C





Shutterstock.com/Andrejs83

- **a** Find the probability of:
  - **i** rolling a number less than 3 with the die
  - **ii** drawing a green marble from the bag.
- **b** List the outcomes for rolling the die and drawing a marble from the bag.
- **c** What is the probability of rolling a number less than 3 and drawing a green marble?
- **d** Is $P(\text{rolling a number less than 3}) \times P(\text{drawing a green marble}) = P(\text{a number less than 3 and a green marble})$?
- **e** Are the events of rolling a number less than 3 and drawing a green marble dependent or independent?

5 A red die and a blue die are rolled. R

- **a** Find:
  - **i** $P(5 \text{ on the red die})$
  - **ii** $P(\text{an even number on the blue die})$.
- **b** Hence find $P(5 \text{ on the red die AND an even number on the blue die})$.

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EXAMPLE 13

6 A bag contains 5 red balls and 4 yellow balls. 2 balls are drawn at random without replacement. R C

a What is the probability of drawing a red ball first?

b What is the probability of drawing a red ball on the second draw if the first ball was red?

c Are the 2 events dependent or independent? Give reasons.

7 A bag contains 5 yellow counters and 3 red counters. 2 draws are made with no replacement. Find the probability of drawing: R C

a **i** a yellow counter on the first draw

ii a yellow counter on the second draw after a yellow counter was drawn with the first draw

b **i** a red counter on the first draw

ii a yellow counter on the second draw after a red counter was drawn on the first draw

c **i** a yellow counter on the first draw

ii a red counter on the second draw after a yellow counter was drawn on the first draw

d **i** a red counter on the first draw

ii a red counter on the second draw after a red counter was drawn on the first draw.

8 3 children in a family are all girls. What is the probability that the next child in this family will be a girl? R

9 A pair of dice are rolled. Use the product rule to find the probability of rolling: PS R

a a 6 on the first die and an odd number on the second die

b a 1 on both dice

c a prime number on the first die and a factor of 6 on the second die

d an even number on the first die and a number between 2 and 5 on the second die.

10 The probability that Natalie wins her tennis match is $\frac{13}{20}$. The chance that she will pass her Science exam is $\frac{4}{5}$. What is the probability that Natalie: PS R

a wins her tennis match and passes her Science exam?

b loses her tennis match and passes her Science exam?

11 The probability that it will rain on Saturday is 0.6. The probability that it will rain on Sunday is 0.25. What is the probability that it will rain: PS R

a on both Saturday and Sunday?

b on Sunday only?

c on neither Saturday nor Sunday?

12 A main road has 3 sets of traffic lights, each with a probability of 0.7 of being green. Use the product rule to find the probability of: PS R

a green on the first 2 lights, red (or amber) on the 3rd light

b green on all 3 lights

c red (or amber) on all 3 lights

d only one of the 3 lights being green.

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Did you know?

Lotteries and Lotto

A lottery is a game of chance in which numbered tickets are drawn from tickets that have been sold. Lotteries were introduced by the State Government to raise money for hospitals. The first lottery was drawn on 20 August, 1931 with a first prize of £5000.

Lotteries have been used to celebrate special events and to help finance special projects. The Opera House Lottery, which commenced selling on 25 November 1957, was used to finance the construction of the Sydney Opera House.



Shutterstock.com/Korawee

Other games of chance have been introduced, including Lotto (1979), Instant Scratchies (1982), OzLotto (1994) and Powerball (1996).

Research the probability of winning Lotto, OZ lotto and Powerball.

Power plus ANSWERS ON P. 621

1 Students at Nelson Secondary College were surveyed about which sport they like to watch and what type of movies they like to see.

	Horror/Drama	Fantasy	Comedy	Action
Football	23	34	30	48
Cricket	25	12	45	34
Tennis	8	12	32	17

a How many students were surveyed?

b If a student is selected at random, what is the probability that the student likes to watch:

i horror/drama movies?

ii comedy and football?

iii tennis, but not fantasy?

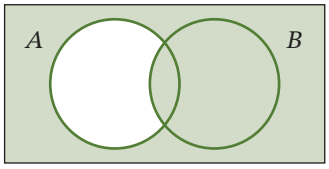
iv action, but not cricket or tennis?

c Given that a student likes to watch football, find the probability that the student also likes to watch action movies.

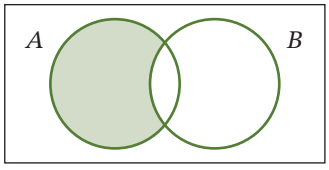
d Of the students who like comedy, what is the probability that they also like to watch cricket?

2 Use set notation to describe the shaded region in each Venn diagram.

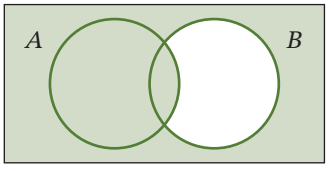
a



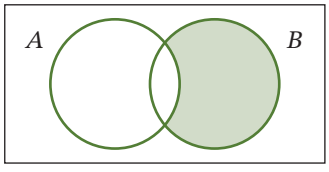
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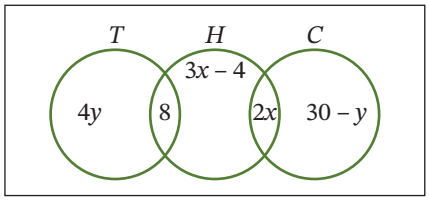
c



d



3 79 students play at least one of the following sports: tennis (T), hockey (H) and cricket (C). The Venn diagram shows the number of students involved in each sport in algebraic form.



a Write a formula for y in terms of x.

b Given that the number of students who played tennis is 8 times as many as those who played both hockey and cricket, find x and y.

c Find the number of students who only played hockey.

4 2 hair ties are drawn without replacement from a bag containing 3 purple and 2 red hair ties.

a Copy and complete the tree diagram to show the sample space.

b Given that the first hair tie drawn is purple, what is the probability that the second hair tie will be:

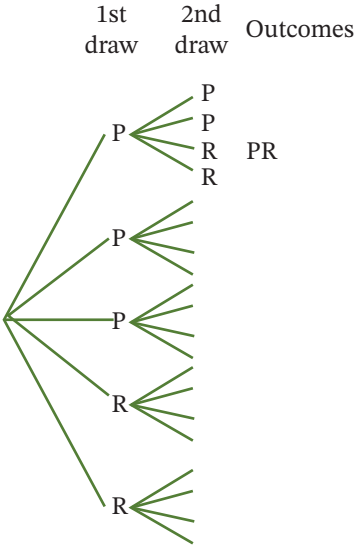
i red? **ii** purple?

c Given that the first hair tie drawn is red, find the probability that the second hair tie is:

i red **ii** purple.

d Given that the drawn hair ties are the same colour, what is the probability that they are:

i both red? **ii** both purple?



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CHAPTER 9 REVIEW

Language of maths

complementary	compound event	dependent event
die/dice	event	expected frequency
expected probability	independent event	intersection
mutually exclusive	observed probability	probability tree diagram
random	relative frequency	sample space
set	simulation	subset
tree diagram	two-stage experiment	two-way table
union	Venn diagram	with/without replacement

1 What is the meaning of **expected frequency**?

2 Which word means to imitate or recreate a chance experiment, usually by a computer model?

3 On a **Venn diagram**, what does the rectangle represent?

4 Give an example of **dependent events**.

5 When are **tree diagrams** used in probability?

6 For 2 events A and B, what is the difference between '**A or B**' and '**A and B**'?

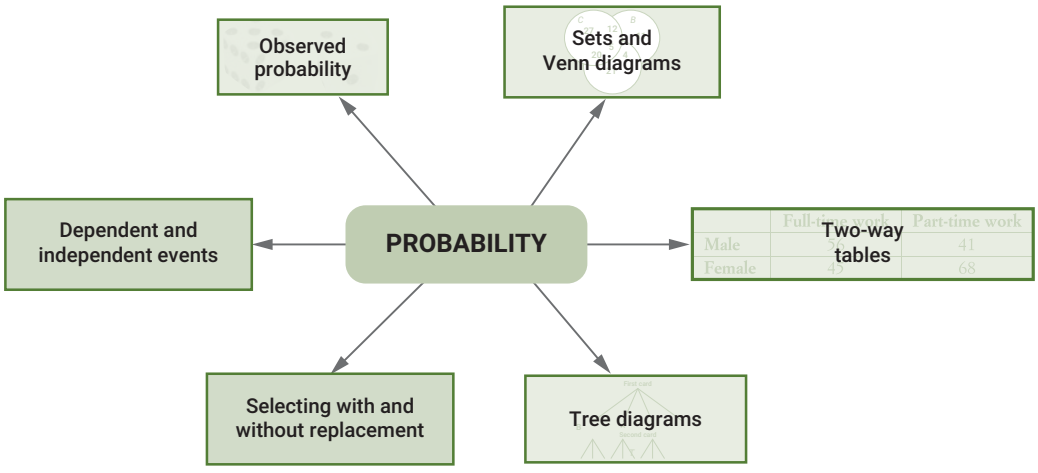
Quiz
Language of maths 9

Puzzle
Probability crossword

Topic summary

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.

Worksheet
Mind map: Probability



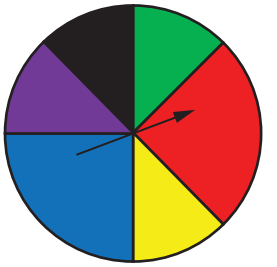
Quiz
Test yourself 9

TEST YOURSELF 9

ANSWERS ON P. 622

9.01

1 For the spinner shown, the red and blue sectors are twice as large as the other sectors. Rafiya spun the spinner 100 times and her results are shown in the table.



Outcome	Frequency
Red	22
Blue	29
Yellow	10
Purple	12
Black	13
Green	14

a What is the observed probability (as a decimal) of the arrow stopping on:

i red? **ii** blue? **iii** yellow? **iv** green?

b What is the expected probability (as a decimal) of the arrow stopping on:

i red? **ii** blue? **iii** yellow? **iv** green?

c Are the observed and expected probabilities similar?

d What is the observed probability of the arrow stopping at purple or black?

e What is the expected frequency of a colour that is not purple or black? How does this compare with Rafiya's observed frequency?

9.01

2 3 coins are tossed 150 times and the number of heads at each trial is recorded in the table.

Number of heads	Frequency
0	20
1	53
2	64
3	13

a Find correct to 3 decimal places the relative frequency of tossing:

i one head **ii** 2 heads

iii 3 heads **iv** at least 2 heads.

b Find correct to 3 decimal places the observed probability of:

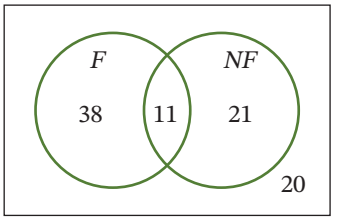
i at least one head **ii** 3 tails.

c Are the answers in part **b** the same or different? Explain why.

PATH

9.02

3 People at a train station were asked whether they prefer to read fiction (F) or non-fiction (NF) books. The results are shown in the Venn diagram.



a How many people were surveyed?

b How many people are in the set:

i $\overline{NF}$? **ii** $F \cup NF$? **iii** $\overline{F} \cap NF$?

c Find the probability of selecting a person from this group who only reads fiction books.

d What is the probability of selecting a person who doesn't read fiction or non-fiction books?

e Of the people who read fiction books, find the probability that they read non-fiction books.

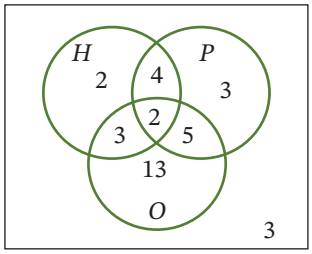
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PATH

9.02

4 This Venn diagram shows the results of a survey on the types of music that school students listen to – Hip Hop/Rap (H), Pop (P) and Other (O), which includes R&B, Dance, Metal, Rock and Independent.



a How many students were surveyed?

b How many students are in the set:

i $H \cap O$? **ii** $H \cup P$? **iii** $(O \cap P) \cup H$?

c Find the probability of selecting a student who likes to listen to all types of music.

d What is the probability of selecting a student who listens to

i Hip Hop/Rap and Pop?

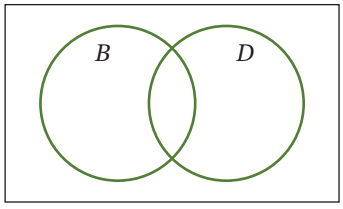
ii Hip Hop/Rap or Pop?

iii Pop music only?

e Why are 3 students in the rectangle, but not in the circles?

9.02

5 Of 20 people in an office, 6 have blue eyes (B), 8 have dark hair (D) and 3 have blue eyes and dark hair.



a Copy and complete the Venn diagram to show the given information.

b What is the probability of selecting a person at random from the office who has:

i blue eyes only? **ii** dark hair?

iii blue eyes and dark hair? **iv** hair that is not dark?

c What is the probability of selecting a person at random who has neither blue eyes nor dark hair?

9.03

6 Students were asked what type of activities they would like to do on a camp.

	Hiking	Rock climbing	Kayaking
Boys	25	38	47
Girls	45	23	22

a How many students were surveyed?

b Find the probability (as a decimal) that a student selected at random:

i likes rock climbing

ii likes kayaking and is a girl

iii is a girl who likes hiking

iv is a boy who likes rock climbing or kayaking.

c If a boy is selected at random, what is the probability that his favourite activity is hiking?

d If a student who likes kayaking is chosen, what is the probability that the student is:

i a boy? **ii** a girl?

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9.04

7 A pair of dice are rolled and the sum of the 2 numbers is calculated.
Find the probability of rolling a sum of 6. Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{1}{5}$ **B** $\frac{1}{9}$ **C** $\frac{5}{6}$ **D** $\frac{5}{36}$

8 A pair of 4-sided dice (numbered 1, 2, 3 and 4) are rolled.

		1st die			
		1	2	3	4
2nd die	1				
	2		2, 2		
	3				
	4			3, 4	

a Copy and complete this table.

b How many possible outcomes are there?

c Find the probability of rolling:

i one odd and one even number

ii 2 even numbers

iii at least one 3

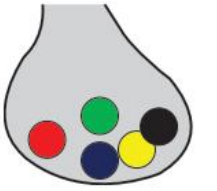
iv 2 numbers less than 3

v a double

vi 2 numbers so that the first number is odd.

9.05

9 2 marbles are drawn from a bag containing a red, a blue, a green, a yellow and a black marble.



a Make up a list to show all the possible outcomes if the marbles are taken:

i with replacement

ii without replacement.

b If the first marble is replaced before the second marble is drawn, find the probability of drawing:

i 2 red marbles

ii 2 marbles of the same colour

iii a yellow and a black marble

iv at least one green marble.

c If the first marble is not replaced, find the probability of drawing:

i a green marble and a yellow marble

ii no red marbles.

9.05

10 The numbers 2, 4, and 7 are written on separate cards and placed in a bag.
3 cards are drawn at random to form a 3-digit number.



a Make up a tree diagram to list the sample space if the cards are drawn:

i with replacement

ii without replacement.

b If the cards are drawn with replacement, find the probability of forming:

i an even number

ii a number less than 400

iii the numbers 222, 444, or 777

iv an odd number greater than 400.

c If the cards are drawn without replacement, find the probability of forming:

i an odd number

ii a number greater than 400

iii a number beginning with 7

iv a number divisible by 4.

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11 A fruit bowl contains 2 apples and 3 oranges. Two pieces of fruit are selected at random, without replacement. 9.05

a Draw a probability tree diagram showing all probabilities on the branches.

b What is the probability of selecting:

i 2 apples? **ii** 2 different fruits? **iii** at least 1 orange?

12 State whether each pair of events are dependent or independent. 9.06

a Tossing a tail from a coin and then a head from the same coin.

b Drawing a ticket in a raffle and winning a first prize and then drawing a second ticket and winning a second prize.

c Electing a president for a cricket club and then electing the vice-president of the cricket club.

d A family's first 3 children are girls and then the 4th child is also a girl.

e Rain in your town or city today and a car accident.

f The Roosters winning a football match and you winning Lotto.



10

MEASUREMENT AND SPACE

GEOMETRY AND NETWORKS

The word ‘geometry’ comes from the Greek word *geometria*, which means ‘earth measuring’. The principles and ideas of geometry and networks are evident in many aspects of our lives. Geometry can be seen in the design of buildings, bridges, roads and transport networks, such as this footbridge over the Yarra River in central Melbourne.



Chapter outline

	Working mathematically				
10.01 Line and rotational symmetry*	U	F		R	C
10.02 Transformations on the number plane*	U	F		R	C
10.03 Similar figures	U	F		R	C
10.04 Finding sides in similar figures	U	F	PS	R	
10.05 Networks*	U	F		R	C
10.06 Planar graphs and Euler's formula*	U	F		R	C
10.07 Eulerian trails and circuits*	U	F		R	C

*PATH

Wordbank

edge (of a network) A line that connects vertices (points) together

Eulerian trail A walk in a network in which every edge is used only once

planar graph A connected network (graph) where the edges do not cross (overlap)

scale factor The amount by which a shape has been enlarged or reduced, equal to $\frac{\text{image length}}{\text{original length}}$

similar To have the same shape but not necessarily the same size, an enlargement or reduction (symbol: ~)

transformation Translation (slide), reflection (flip) or rotation (turn)

vertex (of a network) A point representing an item that is connected by lines called edges.

Quiz
Wordbank 10

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Videos (4):

10.04 Finding an unknown side in similar figures
10.05 Networks
10.07 Eulerian trails • Euler's formula for planar graphs

Twig videos (3):

10.03 The history of the golden ratio
10.05 The seven bridges of Konigsberg • Networks: Labyrinths and mazes
10.07 The seven bridges of Konigsberg

Quizzes (5):

- Wordbank 10
- SkillCheck 10
- Mental skills 10
- Language of maths 10
- Test yourself 10

Skillsheets (2):

10.01 Line and rotational symmetry
10.04 Finding sides in similar triangles

Worksheets (10):

10.01 Symmetry • Symmetry of flat shapes
10.02 Transformations • 5 mm grid paper
10.03 A page of similar figures • Enlargements and reductions • Enlarging a logo
10.04 Finding sides in similar figures
10.07 Eulerian trails and circuits
Mind map Geometry and networks
Test yourself 5 mm grid paper

Puzzles (4):

SkillCheck Finding angles
10.03 Cartoon enlargement
10.04 Similar triangles
Language of maths Geometry and networks crossword

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

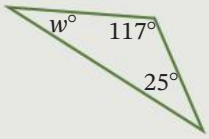
In this chapter you will:

- (PATH) identify line and rotational symmetries in plane shapes
- (PATH) translate, reflect and rotate points and shapes on a number plane using coordinates, where the reflections are across the x- or y- axis and the rotations through multiples of 90° about the origin
- understand the properties of similar figures
- use scale factors to find unknown sides in similar figures
- (PATH) describe networks using appropriate terminology, including vertex, edge, face and degree
- (PATH) describe a planar graph and apply Euler's formula
- (PATH) describe connected graphs, walks, trails, circuits, paths, cycles
- (PATH) identify and investigate Eulerian trails and circuits

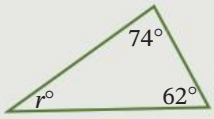
SkillCheck ANSWERS ON P. 623

1 Find the value of each variable.

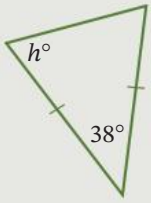
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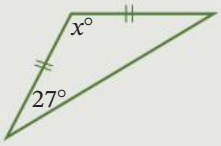
b



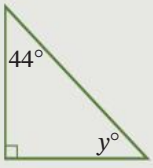
c



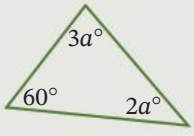
d



e



f



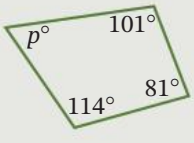
2 Solve each equation.

a $\frac{x}{24} = \frac{5}{8}$ **b** $\frac{m}{7} = \frac{10}{14}$ **c** $\frac{a}{2} = \frac{4}{8}$ **d** $\frac{k}{15} = \frac{2}{3}$

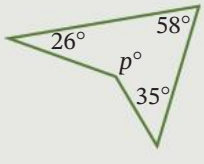
e $\frac{7}{12} = \frac{x}{3}$ **f** $\frac{5}{6} = \frac{n}{4}$ **g** $\frac{t}{2} = \frac{7}{3}$ **h** $\frac{d}{5} = \frac{17}{10}$

3 Find the value of p in each diagram.

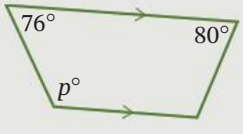
a



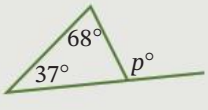
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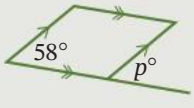
c



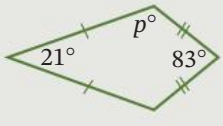
d



e



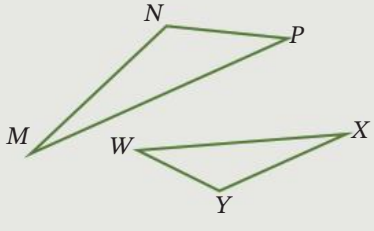
f



4 Triangles MNP and WXY are similar.

a List all pairs of matching angles.

b List all pairs of matching sides.



Quiz
SkillCheck 10

Puzzle
Finding angles

10

10.01 Line and rotational symmetry

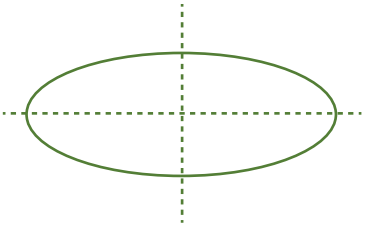
PATH

Skillsheet
Line and rotational symmetry

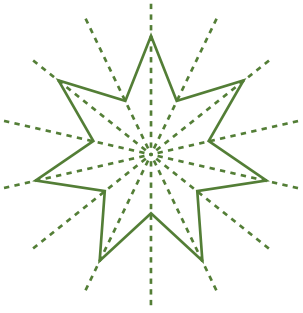
Worksheets
Symmetry

Symmetry of flat shapes

When a shape with **line symmetry** is folded along a line, one half of the shape will fit exactly on top of the other half. The fold line is called the **axis of symmetry**.

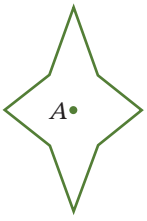


This ellipse has 2 axes of symmetry.

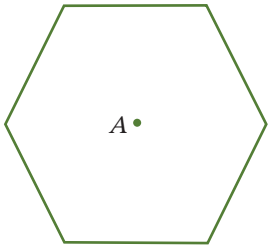


This star shape has 7 axes of symmetry.

When a shape with **rotational symmetry** is rotated about the point, called the **centre of symmetry**, it fits on itself at least once before one complete revolution.



This star shape fits on itself twice in one revolution, so it has rotational symmetry of order 2.



The regular hexagon fits on itself 6 times in one revolution, so it has rotational symmetry of order 6.

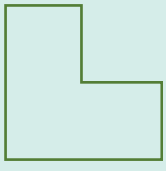
Example 1

For each shape:

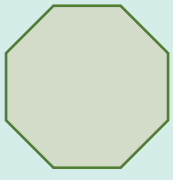
i count how many axes of symmetry it has

ii state whether it has rotational symmetry and the order if it does.

a



b

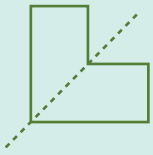


9780170479622

Solution

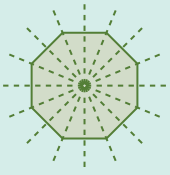
a **i** 1 axis of symmetry

ii no rotational symmetry



b **i** 8 axes of symmetry

ii rotational symmetry of order 8



PATH

10.01

EXERCISE 10.01 ANSWERS ON P. 623

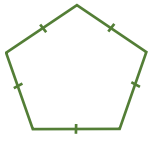
Line and rotational symmetry U F R C

1 Copy each shape and mark its axes of symmetry. R C

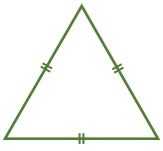
a



b



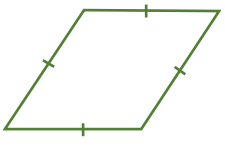
c



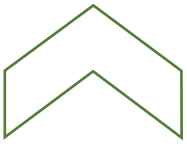
d



e



f



2 State whether each shape has rotational symmetry and the order if it does. R C

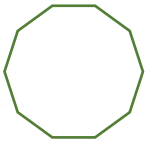
a



b



c



3 Which of these capital letters have: R C

B C H M O P S W X Y Z

a line symmetry? **b** rotational symmetry?

Foundation Standard Complex

PATH

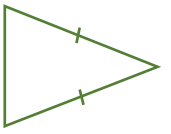
EXAMPLE 1

4 For each shape: R C

i find how many axes of symmetry it has

ii state whether it has rotational symmetry and the order if it does.

a



b



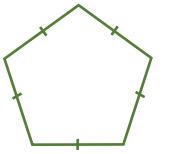
c



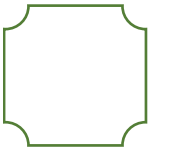
d



e



f



5 For each shape, find: R C

i the number of axes of symmetry

ii the order of rotational symmetry.

a square **b** rhombus **c** regular decagon (10 sides)

d circle **e** kite

6 **a** Find the number of axes of symmetry and the order of rotational symmetry for:

i a scalene triangle **ii** an isosceles triangle **iii** an equilateral triangle.

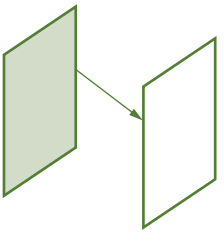
b What do you notice about your answers to part **a**? R C

10.02 Transformations on the number plane

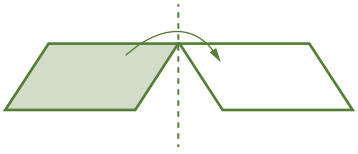
PATH

Worksheet
Transformations

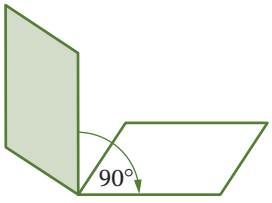
There are 3 types of **transformations**.



translation ('slide')



reflection ('flip')



rotation ('spin')

The shapes have changed position (transformed) but still have the same shape and size.

The original shape is called the **original**.

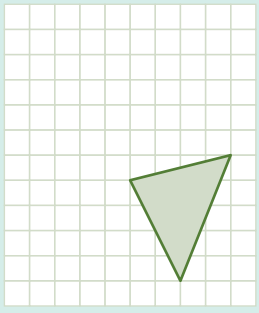
The transformed shape is called the **image**.

Foundation Standard Complex

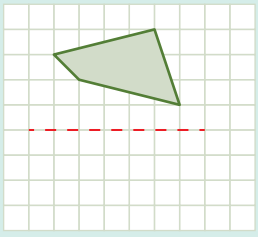
9780170479622

Example 2

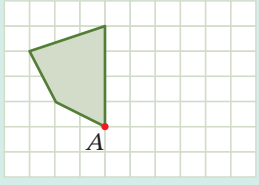
a Translate the triangle 4 units left and 5 units up.



b Reflect this quadrilateral across the dotted line, the **line of reflection**.

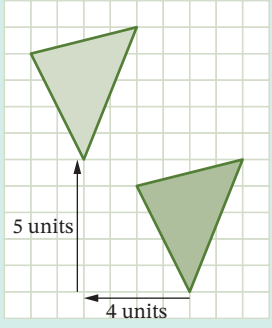


c Rotate the shape 90° in a clockwise direction about the point A, the centre of rotation.



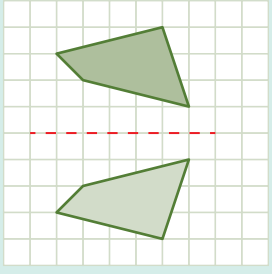
Solution

a



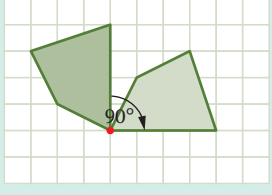
Every point on the original is shifted 4 units left and 5 units up.

b



The original and its mirror image are the same distance from the line of reflection.

c



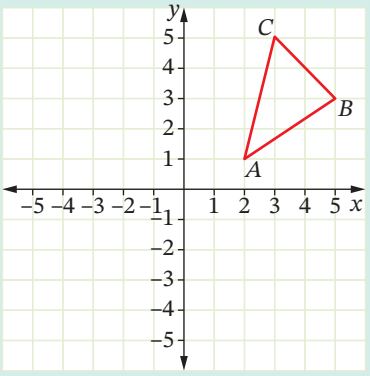
Every point on the original and image are the same distance from the centre of rotation.

PATH

When a point or vertex P of an original shape is transformed, the corresponding point or vertex on the image is labelled P', pronounced 'P-dash' or 'P-prime'.

Example 3

a Reflect $\triangle ABC$ across the y-axis to obtain the image $\triangle A'B'C'$.

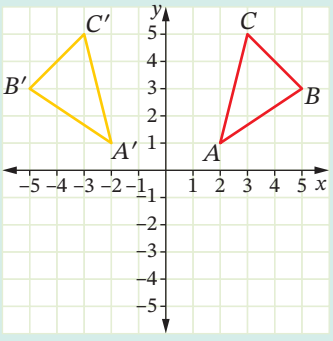


b Compare the coordinates of each vertex of $\triangle ABC$ to its matching vertex in $\triangle A'B'C'$.

c Now reflect $\triangle ABC$ across the x-axis to obtain the image $\triangle A'B'C'$. Compare the coordinates of each vertex of the original to its matching vertex in its image.

Solution

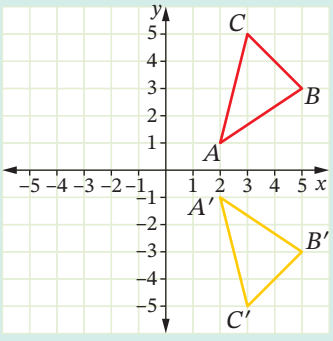
a



b $A(2, 1) \rightarrow A'(-2, 1)$, $B(5, 3) \rightarrow B'(-5, 3)$, $C(3, 5) \rightarrow C'(-3, 5)$

When reflected in the y-axis, the x-coordinate of each vertex changes sign while the y-coordinate stays the same.

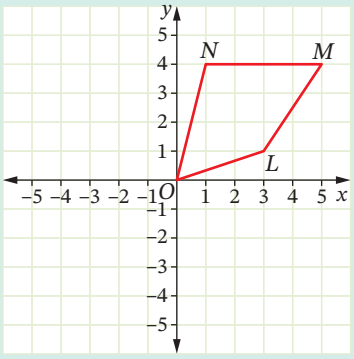
c



$A(2, 1) \rightarrow A'(2, -1)$, $B(5, 3) \rightarrow B'(5, -3)$, $C(3, 5) \rightarrow C'(3, -5)$

When reflected in the x-axis, the y-coordinate of each vertex changes sign while the x-coordinate stays the same.

Example 4



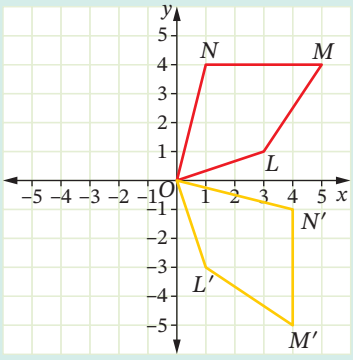
a Rotate $OLMN$ clockwise about O through each angle to obtain $OL'M'N'$:

i 90° **ii** 180° **iii** 270°

b Compare the coordinates of each vertex of $OLMN$ to each vertex of the image $OL'M'N'$ for each of the rotations in part **a**.

Solution

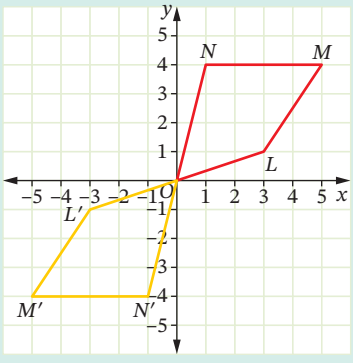
a i



b i $O(0, 0) \rightarrow O(0, 0)$. O is the centre of rotation.

$L(3, 1) \rightarrow L'(1, -3)$,
$M(5, 4) \rightarrow M'(4, -5)$,
$N(1, 4) \rightarrow N'(4, -1)$.

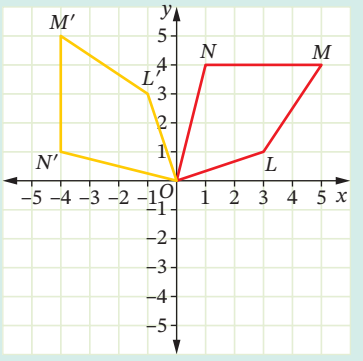
a ii



b ii $O(0, 0) \rightarrow O(0, 0)$,
$L(3, 1) \rightarrow L'(-3, -1)$,
$M(5, 4) \rightarrow M'(-5, -4)$,
$N(1, 4) \rightarrow N'(-1, -4)$.

PATH

a **iii**



b **iii** $O(0, 0) \rightarrow O(0, 0)$
$L(3, 1) \rightarrow L'(-1, 3)$,
$M(5, 4) \rightarrow M'(-4, 5)$,
$N(1, 4) \rightarrow N'(-4, 1)$.

Example 5

Find the new position of the point $K(-2, 5)$ if it is:

a rotated about the origin clockwise through 180°

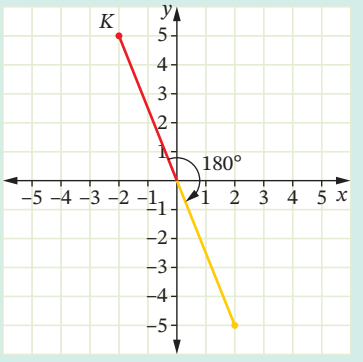
b translated 4 units to the right and 7 units down

c reflected in the y-axis.

Solution

a K is rotated anticlockwise about the origin.

$\therefore$ the new position of K is $(2, -5)$



b The new position of K is $(-2 + 4, 5 - 7) = (2, -2)$.

c K is reflected in the y-axis so the y-coordinate stays the same.

$\therefore$ the new position of K is $(2, 5)$

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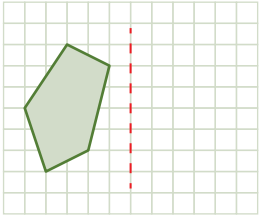
EXERCISE 10.02 ANSWERS ON P. 623

Transformations on the number plane UFRC

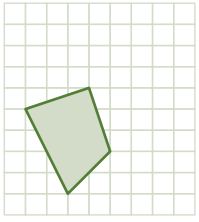
EXAMPLE 2

1 Copy each shape onto grid paper and draw the image after performing the stated transformation.

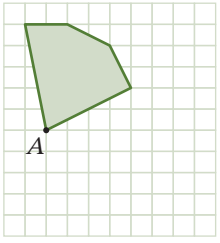
a Reflect across the dotted line



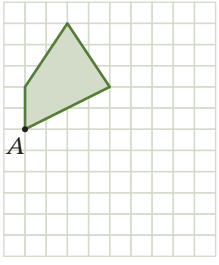
b Translate 3 units right and 3 units up



c Rotate 90° clockwise about A



2 Copy this shape onto grid paper and draw the image of the shape after it has been translated 6 units to the right, 5 units down, followed by an anticlockwise rotation of 90°.



10.02

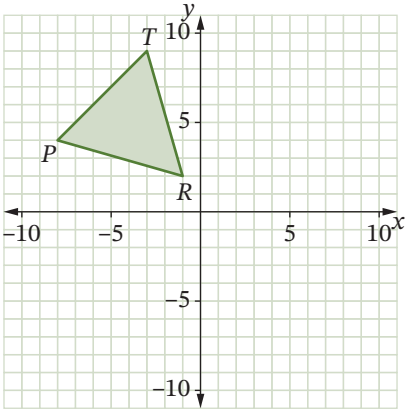
Worksheet
5 mm grid paper

EXAMPLE 3

3 **a** Copy and reflect the shape $\triangle PRT$ across the y-axis to obtain the image $\triangle P'R'T'$.

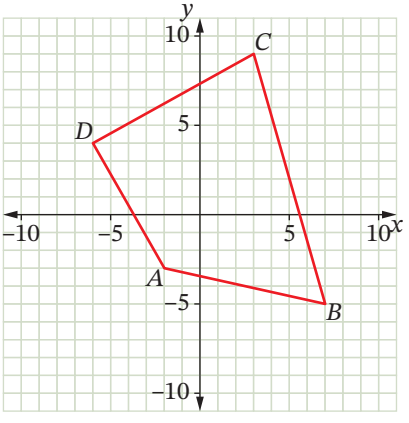
b Compare the coordinates of each vertex of $\triangle PRT$ to its matching vertex in $\triangle P'R'T'$.

c Now copy and reflect $\triangle PRT$ across the x-axis to obtain the image $\triangle P'R'T'$. Compare the coordinates of each vertex of $\triangle P'R'T'$ to its matching vertex in $\triangle PRT$. R C



4 **a** Copy and reflect the quadrilateral $ABCD$ across the x-axis to obtain the image $A'B'C'D'$.

b Write the coordinates of each vertex of $A'B'C'D'$ and compare them to the coordinates of the vertices of $ABCD$. R C

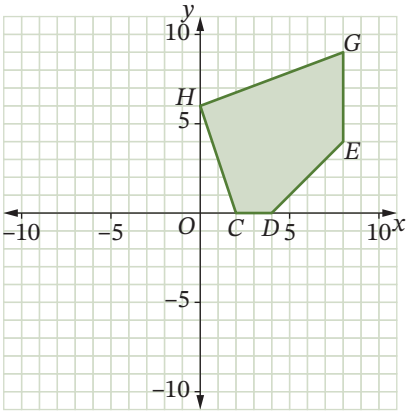


Foundation Standard Complex

PATH

EXAMPLE 4

5 **a** Copy and rotate the shape $CDEGH$ clockwise about O through 90° to create the image $C'D'E'G'H'$.



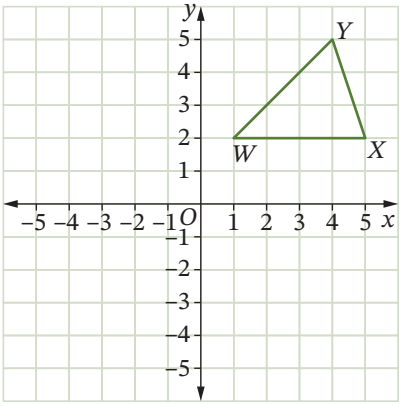
b Write down the coordinates of D' and E' and compare them to the coordinates of D and E.

c Write down the new positions of C, G and H if $CDEGH$ is rotated about O through: R C

i 180° **ii** 270°

6 **a** Copy and rotate $\triangle WXY$ clockwise about O through the following angles to obtain $\triangle W'X'Y'$:

i 90° **ii** 180° **iii** 270°



b Compare the coordinates of the vertices of $\triangle WXY$ to those of $\triangle W'X'Y'$ for each of the rotations. R C

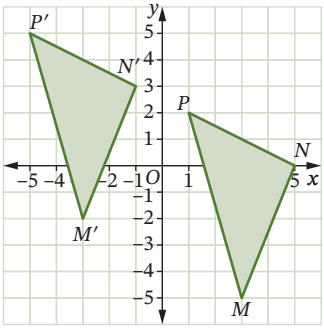
c Copy and rotate $\triangle WXY$ anticlockwise about O through the following angles

i 90° **ii** 180° **iii** 270°

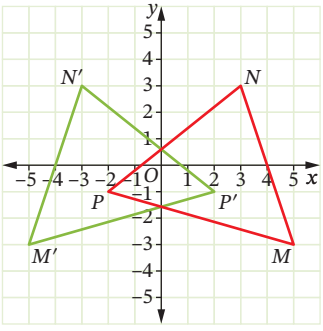
d Which of the images from part **c** are the same as the images from part **a**?

7 Describe the transformations that has been performed on $\triangle MNP$ in each diagram. R C

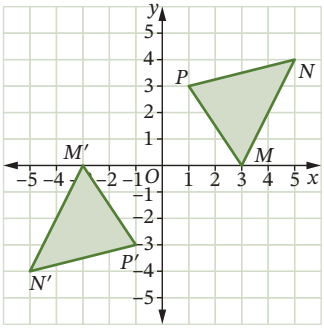
a



b



c

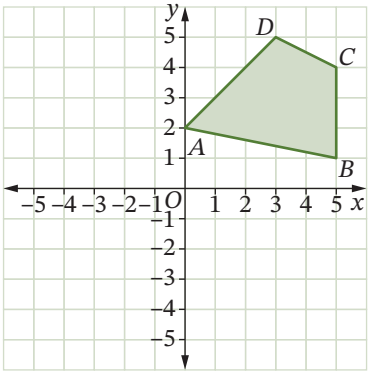


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8 **a** Rotate the quadrilateral $ABCD$ about O through 270° and then reflect the image in the x-axis to obtain $A'B'C'D'$.

b Compare the coordinates of the vertices of $ABCD$ to those of $A'B'C'D'$. R C



EXAMPLE 5

9 The point $T(4, -3)$ is reflected in the x-axis. What are the new coordinates of T? Select the correct answer **A**, **B**, **C** or **D**.

A (−4, −3) **B** (−4, 3) **C** (4, 3) **D** (4, −3)

10 The point $W(-2, -5)$ is rotated through 180° about the origin to obtain the point W'. What are the coordinates of W'? Select **A**, **B**, **C** or **D**.

A (5, 2) **B** (2, 5) **C** (−5, 2) **D** (−2, 5)

11 Find the coordinates of the image point A' if $A(-4, -6)$ is:

a reflected in the x-axis

b reflected in the y-axis

c translated 4 units right and 7 units up

d rotated clockwise about the origin through

i 90° **ii** 180° **iii** 270°

12 What is the new position of $D(3, -8)$ if it is reflected in the x-axis and then rotated clockwise about the origin through 90°? Select **A**, **B**, **C** or **D**.

A (8, −3) **B** (−3, 8) **C** (−8, 3) **D** (−3, −8)

13 The point $M(5, 2)$ is rotated anticlockwise about the origin through 90° and then reflected in the y-axis. What are the new coordinates of M? Select **A**, **B**, **C** or **D**.

A (−2, −5) **B** (5, −2) **C** (−5, −2) **D** (2, 5)

14 Find the new position of $X(-3, -7)$ if it is: R C

a rotated through 180° about the origin

b translated 5 units to the right and 9 units up

c reflected in the x-axis

d rotated clockwise through 90° and then reflected in the y-axis

e translated 6 units to the right, 4 units up and then rotated anticlockwise about the origin through 90°.

15 Find the coordinates of the image point X' if $X(-4, -8)$ has been rotated about the origin through 180° and then reflected in the x-axis. Select **A**, **B**, **C** or **D**.

A (−4, 8) **B** (4, 8) **C** (8, −4) **D** (4, −8)

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PATH

16 The point $T(3, -4)$ is the image of a point after it has rotated through 180° and then has been translated 3 units down and 5 units to the right. Find the original point. R C

17 Find the original point if (1, 5) is the image of a point that has been rotated 90° clockwise about the origin and then reflected in the x-axis.

Technology

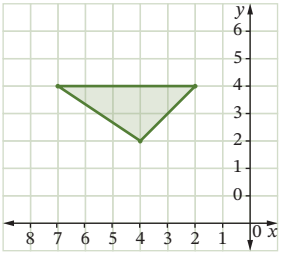
Transformations on the number plane

In this activity you will create reflections, rotations and translations of various shapes using **dynamic geometry software.** Create a number plane that shows all 4 quadrants and values of x and y from –10 to 10.

Activity 1: Reflection

1 Draw a **triangle** with vertices at (–7, 5), (–2, 5) and (–4, 3).

2 Draw a **reflection line on the *y*-axis,** through the points (0, 6) and (0, 0).



3 Reflect the triangle across the line.

4 Click a vertex and drag it. What happens?

5 Draw a new **reflection line** through the points (2, 7) and (–5, –2).

6 Reflect the triangle across this line.

Activity 2: Rotation

1 Draw a **polygon** with vertices at (0, 3), (–2, 3), (–3, 2), (–2, 0) and (0, 3).

2 Select the polygon and rotate it 45° clockwise about the point (0, 3).

3 Repeat step **2**, then rotate the polygon 180° clockwise about (0, 3). What happens?

4 Try other points as the centre point and enter different angles. What happens?

Activity 3: Translation

1 Draw a **polygon** with vertices at (–3, 4), (–3, –1), (1, –1), (1, 1), (–2, 1), (–2, 4) and (–3, 4).

2 Translate the polygon so that the image of (1, 1) is (4, 3).

3 Now translate the shape in 2 other ways.

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Activity 4

1. Construct a **Polygon**.
2. Reflect the polygon about a point.
3. Reflect the polygon about a line.
4. Rotate the polygon by an angle.
5. Translate the polygon by a vector.



Quiz
Mental skills 10

Mental skills 10: Maths without calculators ANSWERS ON P. 625

Dividing a quantity in a given ratio

1 Study this example.

Divide \$5600 between Alice and Peter in the ratio 5 : 3.

Total number of parts = 5 + 3 = 8.

1 part = \$5600 ÷ 8 = \$700

Alice's share = 5 × \$700 = \$3500

Peter's share = 3 × \$700 = \$2100

Check: \$3500 + \$2100 = \$5600 (original amount)

2 Now divide each of these quantities in the given ratio.

a Divide \$150 between Mark and Jenni in the ratio 2 : 1.

b Divide \$2100 between Simon and Sunil in the ratio 4 : 3.

c Divide \$720 between Lisa and Bree in the ratio 2 : 7.

d Divide \$2000 between William and Adriana in the ratio 1 : 3.

e Divide \$4500 between Anne and Pete in the ratio 3 : 2.

f Divide \$3000 between Sharanya and Asam in the ratio 3 : 7.

g Divide \$3600 between Cindy and Carmen in the ratio 5 : 1.

h Divide \$1600 between Nancy and John in the ratio 3 : 5.

i Divide \$990 between Carol and Louis in the ratio 5 : 4.

j Divide \$4000 between Yvette and Andre in the ratio 1 : 4.

k Divide \$4900 between Arden and Ivan in the ratio 3 : 4.

l Divide \$3200 between Tan and Mai in the ratio 5 : 3.

10.03 Similar figures

Video
The history of the golden ratio

Worksheets
A page of similar figures

Enlargements and reductions

Enlarging a logo

Puzzle
Cartoon enlargement

Similar figures have the same shape, but are not necessarily the same size.

When a figure is enlarged or reduced, a **similar figure** is created. The original figure is called the **original**, while the enlarged or reduced figure is called the **image**.



Original Image

The **scale factor** describes by how much a figure has been enlarged or reduced.

Scale factor

$$\text{Scale factor} = \frac{\text{image length}}{\text{original length}}$$

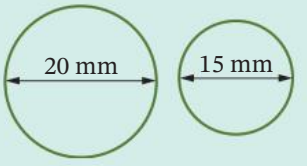
- If the scale factor is greater than 1, then the image is an enlargement.
- If the scale factor is between 0 and 1, then the image is a reduction.

Example 6

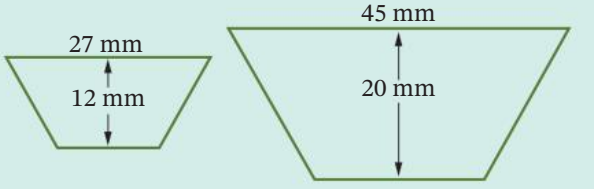
Find the scale factor for each pair of similar figures.

In all questions, assume the left figure is the original and the right figure is the image.

a



b



Solution

a $\text{Scale factor} = \frac{15}{20}$ $\frac{\text{image length}}{\text{original length}}$

$= \frac{3}{4}$

b $\text{Scale factor} = \frac{45}{27}$ $\left(\text{or } \frac{20}{12}\right)$ $\frac{\text{image length}}{\text{original length}}$

$= \frac{5}{3}$

The symbol for 'is similar to' is '~'. When using this notation, we must make sure that the vertices (angles) of similar figures are written in matching order.

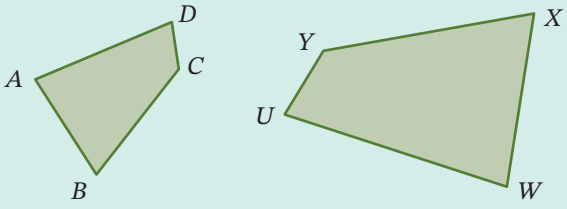
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Properties of similar figures

- Matching angles are equal
- Matching sides are in the same ratio

Example 7

These 2 quadrilaterals $ABCD$ and $UWXY$ are similar.



a List all pairs of matching sides and matching angles.

b Use the correct notation to write a similarity statement relating these 2 quadrilaterals.

Solution

a By rotating the figure $ABCD$, its shape can be matched with $UWXY$.

The pairs of matching sides are:	The pairs of matching angles are:
AB and WX	$\angle A$ and $\angle W$
BC and XY	$\angle B$ and $\angle X$
CD and YU	$\angle C$ and $\angle Y$
AD and WU.	$\angle D$ and $\angle U$.

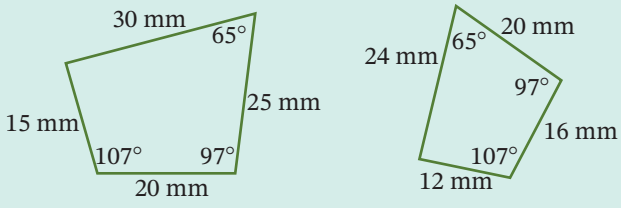
b $\angle A$ matches with $\angle W$, $\angle B$ matches with $\angle X$, $\angle C$ matches with $\angle Y$, $\angle D$ matches with $\angle U$.

$\therefore ABCD \sim WXYU$ Matching order of vertices

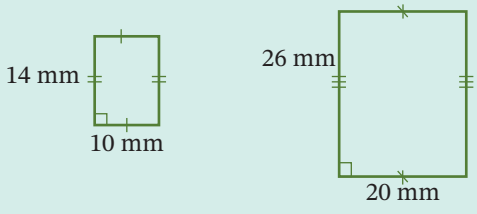
Example 8

Test whether each pair of figures are similar.

a



b



Solution

a For the 2 quadrilaterals, matching angles are equal and the ratios of matching sides are equal.

$\frac{20}{16}=\frac{5}{4}, \frac{25}{20}=\frac{5}{4}, \frac{30}{24}=\frac{5}{4}, \frac{15}{12}=\frac{5}{4}$

$\therefore$ The quadrilaterals are similar.

b For the 2 rectangles, matching angles are equal (90°), but the ratios of matching sides are not equal.

$\frac{10}{20}=\frac{1}{2}$ but $\frac{14}{26}=\frac{7}{13}$

$\therefore$ The rectangles are not similar.

EXERCISE 10.03 ANSWERS ON P. 625

Similar figures U F R C

EXAMPLE 6

1 By measurement, find the scale factor for each pair of similar figures.

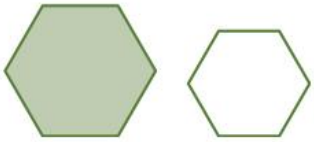
a



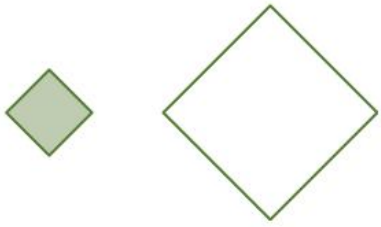
b



c

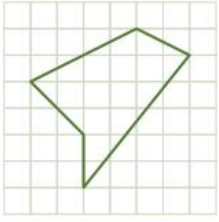


d

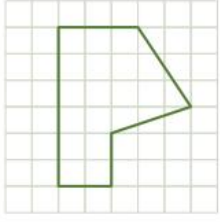


2 Copy each figure onto graph paper and draw its image using the given scale factor.

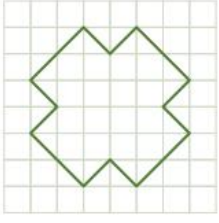
a Scale factor $= \frac{1}{2}$



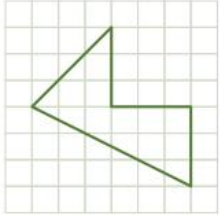
b Scale factor $= 3$



c Scale factor $= 2.5$



d Scale factor $= \frac{2}{3}$



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EXAMPLE 7

10.03

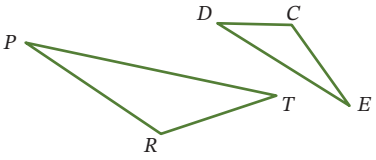
3 For each pair of similar figures: R C

i list all pairs of matching angles

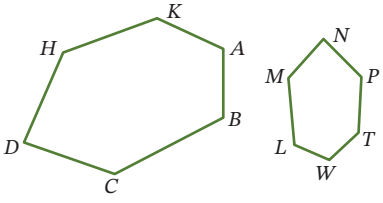
ii list all pairs of matching sides

iii use the correct notation ('~') to write a similarity statement relating them.

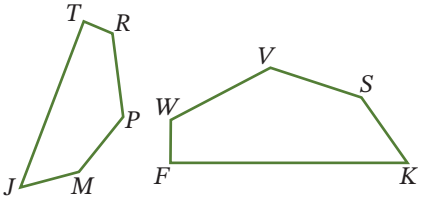
a



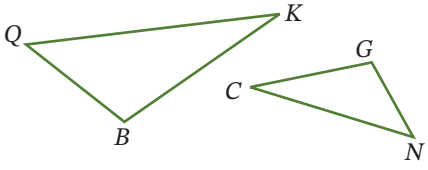
b



c



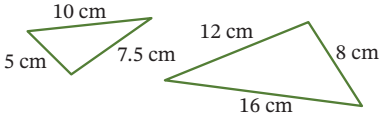
d



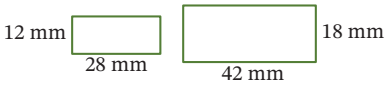
EXAMPLE 8

4 Test whether each pair of figures are similar. R C

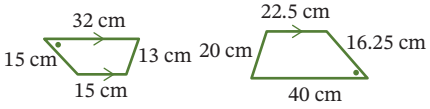
a



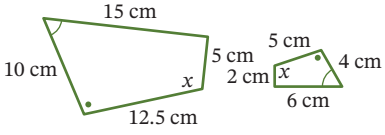
b



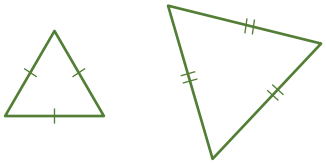
c



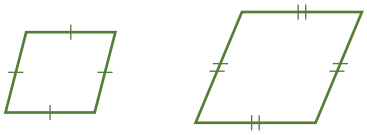
d



e



f



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10.04 Finding sides in similar figures

Video
Finding an unknown side in similar figures

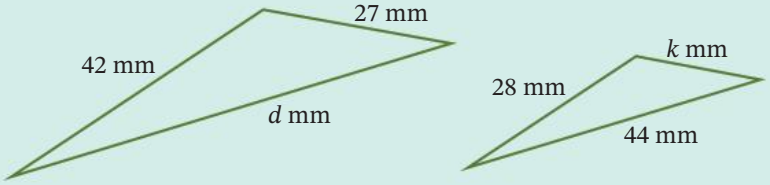
Skillsheet
Finding sides in similar triangles

Worksheet
Finding sides in similar figures

Puzzle
Similar triangles

Example 9

The 2 triangles are similar. Find the values of d and k.



Solution

Since the triangles are similar, the ratios of matching sides are equal.

$\frac{d}{44} = \frac{42}{28}$

$d = \frac{42}{28} \times 44$

$= 66$

$\frac{k}{27} = \frac{28}{42}$

$k = \frac{28}{42} \times 27$

$= 18$

Alternative method:

Scale factor $= \frac{28}{42} = \frac{2}{3}$

$d = 44 \div \frac{2}{3}$

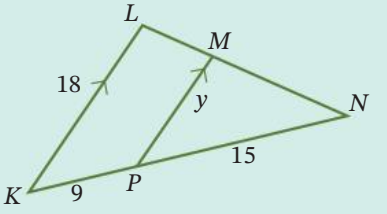
$= 66$

$k = 27 \times \frac{2}{3}$

$= 18$

Example 10

$\triangle KLN \sim \triangle PMN$. Find the value of y.



Solution

$\frac{MP}{LK} = \frac{PN}{KN}$ Ratios of matching sides are equal.

$\frac{y}{18} = \frac{15}{24}$ $KN = 9 + 15 = 24$

$y = \frac{15}{24} \times 18$

$= 11\frac{1}{4}$

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EXERCISE 10.04 ANSWERS ON P. 626

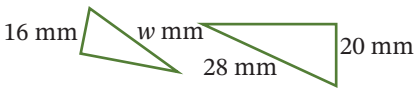
Finding sides in similar figures U F PS R

1 Find the value of each variable in each pair of similar figures. R

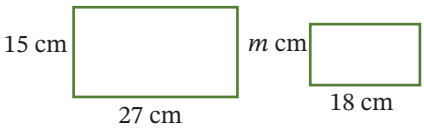
EXAMPLE 9

10.04

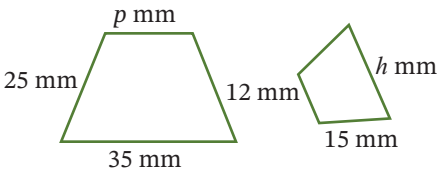
a



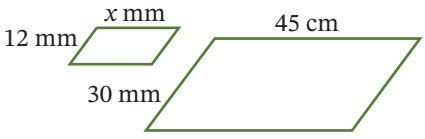
b



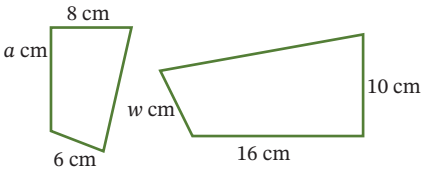
c



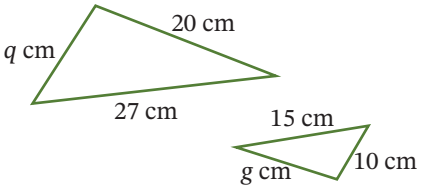
d



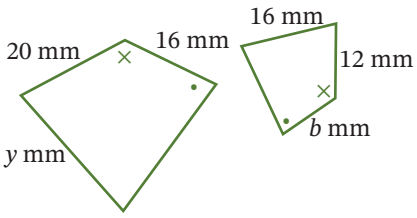
e



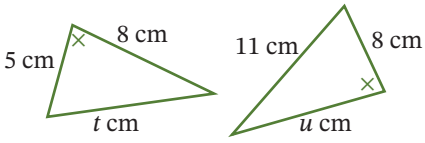
f



g

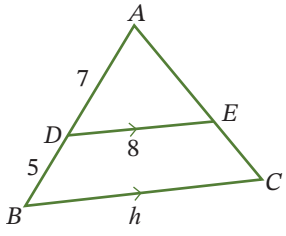


h

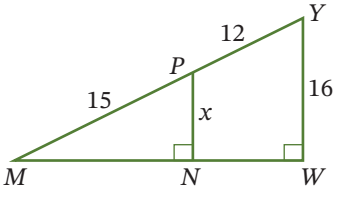


2 $\triangle ABC \sim \triangle ADE$. Find the value of h. R

EXAMPLE 10

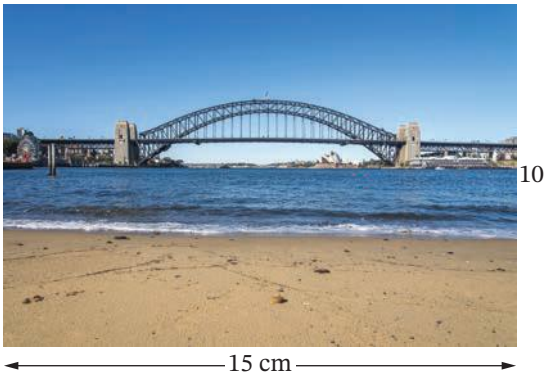


3 $\triangle MNP \sim \triangle MWY$. Find the value of x. R



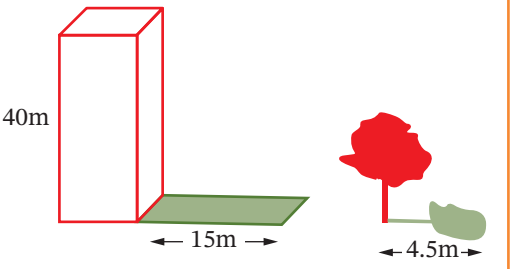
Foundation Standard Complex

4 This photograph of the Sydney Harbour Bridge has been enlarged so that its length is 24 cm. If the dimensions of the original photo were 15 cm × 10 cm, what is the width of the enlargement? R



Taki O/Shutterstock.com

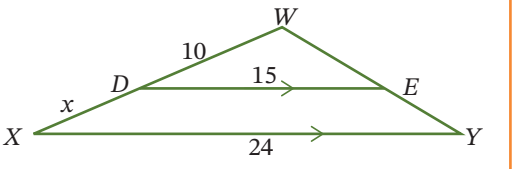
5 A building that is 40 m high casts a shadow 15 m long. At the same time, the shadow of a tree is 4.5 m long. What is the height of the tree? PS R



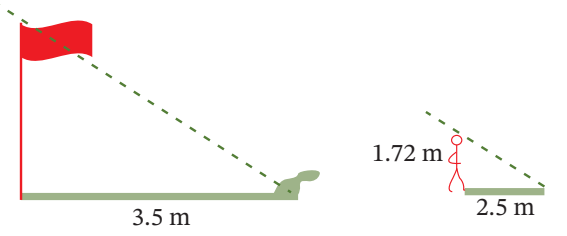
6 $\triangle WXY \sim \triangle WDE$. What is the value of x? Select the correct answer **A**, **B**, **C** or **D**. R

A 4 **B** 6

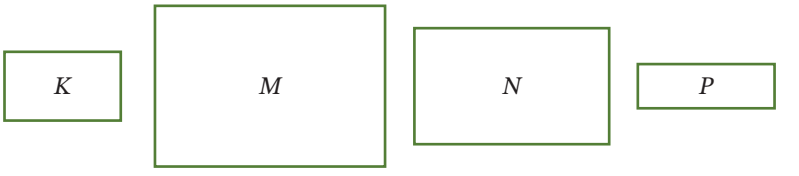
C 8 **D** 10



7 Katrina is 1.72 m tall and casts a shadow that is 2.5 m long. At the same time, a flagpole casts a shadow that is 3.5 m long. How long is the flagpole? PS R



8 Which 2 rectangles are similar? Select **A**, **B**, **C** or **D**.



A M and N **B** K and P **C** M and P **D** K and N

9 A 2 m high fence casts a shadow of 1.4 m. How long is the shadow cast by a pole that is 3.2 m high at the same time? PS R

Foundation Standard Complex

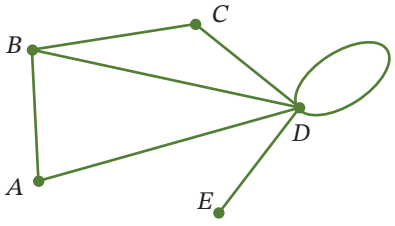
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Networks

10.05

A **network** is a collection of objects connected to each other in some way or system. Examples of networks include bus, rail and air routes, computer systems and family or friend relationships.

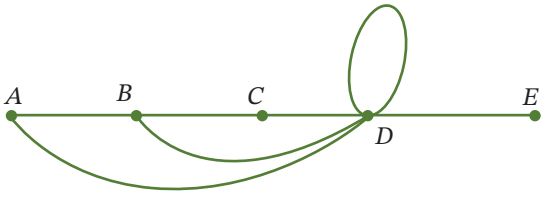
A **graph** is a diagram of a network where the objects are represented by points called **vertices** (or **nodes**), which are joined by lines called **edges**. The study of networks is called **graph theory**.



This graph or network diagram has 5 vertices, *A, B, C, D, E,* and 7 edges, *BC, BD, BA, AD, ED, CD, DD.*

A **loop** is an edge that connects a vertex to itself, such as *DD* in the diagram.

A network can be drawn in different ways as long as it shows the same connections. This diagram represents the same network as above.



PATH

Videos
Networks
The seven bridges of Konigsberg
Networks: Labyrinths and mazes

10.05

Degree of a vertex

The **degree** (or **order**) of a vertex is the number of edges connected to it, with loops counted as 2 edges. An **even vertex** has an even number of edges connected to it. An **odd vertex** has an odd number of edges.

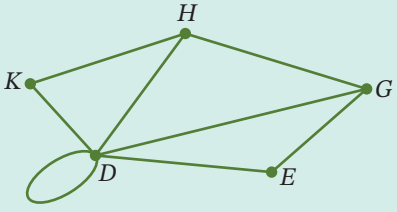
The **sum of the degrees** of all vertices in a network is equal to **twice the number of edges**, because each edge belongs to 2 vertices. This means that the sum of the degrees in a network is always even.

Example 11

a Find the degree of each vertex in this network.

b How many vertices are odd?

c Show that the sum of the degrees is equal to twice the number of edges.



Solution

a

Vertex	*D*	*E*	*G*	*H*	*K*
Degree	6	2	3	3	2

b There are 2 odd vertices.

c Number of edges = 8.

Sum of degrees of vertices = 6 + 2 + 3 + 3 + 2 = 16,

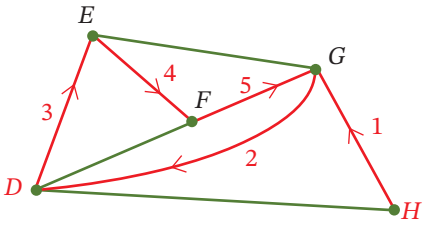
which is twice the number of edges (2 × 8).

PATH

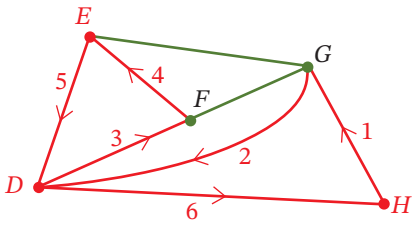
Walks, trails, paths and cycles

A **walk** is *any* route along the edges of a network, and may include repeated vertices or edges. Any walk that starts and ends at the same vertex is called a **closed walk**.

A **trail** is a walk with no repeated edges, such as shown in the network below. However, a vertex can be repeated, such as *G* in *HGDEFG*. A **circuit** is a **closed trail**, such as *HGDFEDH* shown below (starts and ends at *H*).



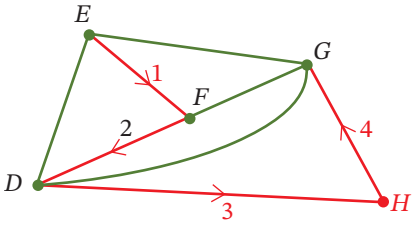
trail



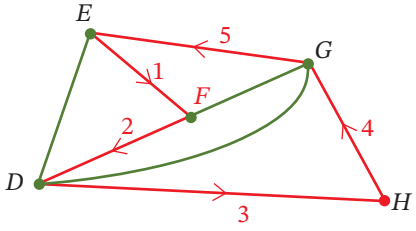
circuit

Think of a bushwalker walking on a trail or circuit and not repeating an edge.

A **path** is a walk with no repeated vertices (or edges), such as *EFDHG* shown in the network below. A **cycle** is a **closed path**, such as *EFDHGE* shown below (starts and ends at *E*, no repeated vertices).



path



cycle

Think of a cyclist riding along a bicycle path or cycle and not revisiting any points.

Example 12

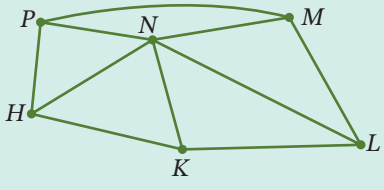
a State whether each walk is a trail, path or neither?

i *KLMPNHK* **ii** *PNMLNHK*

iii *HNMLNKHP* **iv** *LMNPHKNM*

b Write down a circuit that uses the edge *LN*.

c Write down a cycle the starts with *K* and uses the edge *HN*.



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Solution

a **i** *KLMPNHK* is a path since no vertices (and edges) are repeated.

ii *PNMLNHK* is a trail as no edges are repeated but vertex *N* is repeated.

iii *HNMLNKHP* is a trail as no edges are repeated but the vertices *N* and *H* are repeated.

iv *LMNPHKNM* is neither as the edge *MN* is repeated.

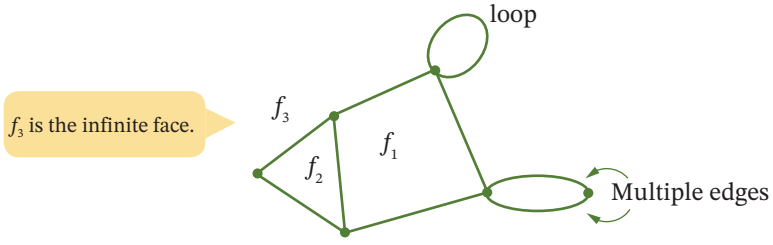
b *KLNMPHK* is a circuit because it starts and ends at *K* with no repeated edges.

c *KHNPMLK* is a cycle because it has no repeated vertices.

Faces of a network

A **face** of a network is a region bounded by edges. Regions formed by loops or multiple edges between the same vertices are *not* faces, but the external (outside) region of a network is a face. The external face is called the **infinite face**.

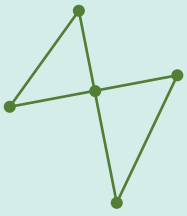
This network has 3 faces, f_1, f_2 and f_3.



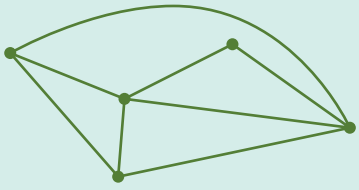
Example 13

Count the number of faces in each network.

a

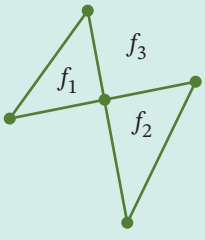


b



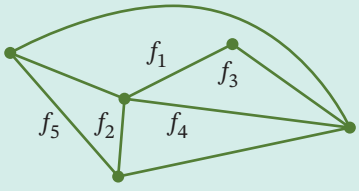
Solution

a



3 faces

b



5 faces

PATH

EXERCISE 10.05 ANSWERS ON P. 626

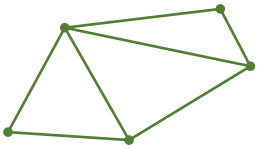
Networks U F R C

EXAMPLE 11

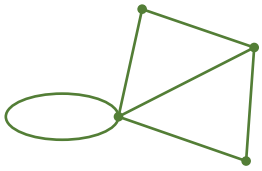
1 For each network, count the number of:

i vertices **ii** edges.

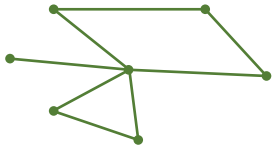
a



b



c

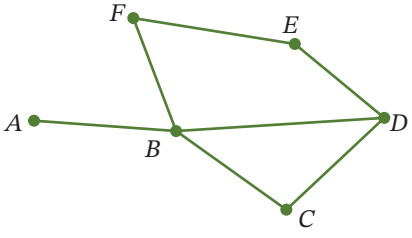


2 For each network: R C

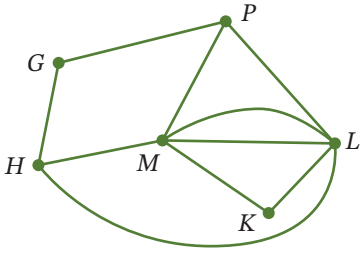
i find the degree of each vertex

ii show that the sum of the degrees is equal to twice the number of edges.

a

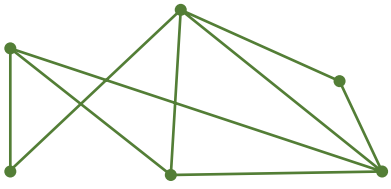


b

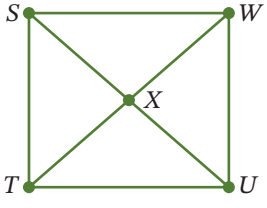


3 What is the sum of the degrees of all vertices in this graph? Select the correct answer **A**, **B**, **C** or **D**.

A 9 **B** 14 **C** 18 **D** 22

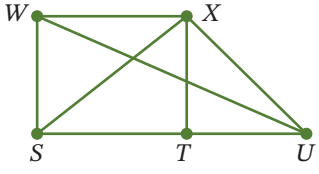


4 Which of these graphs shows the same information as the given network diagram?



Select **A**, **B**, **C** or **D**. R C

A



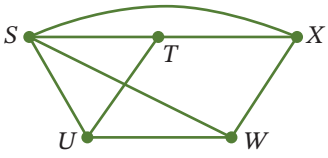
B



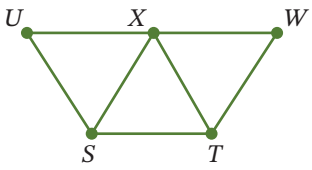
Foundation Standard Complex

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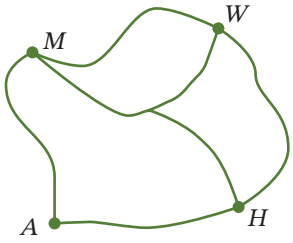
C



D

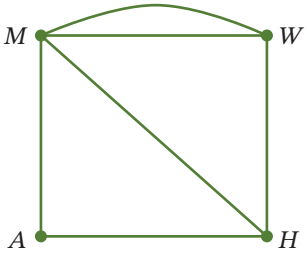


5 Which network diagram represents the map showing the roads that connect 4 towns?

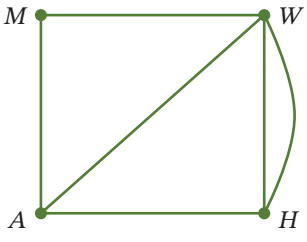


Select **A**, **B**, **C** or **D**. R C

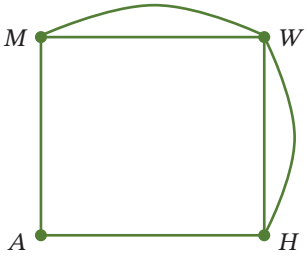
A



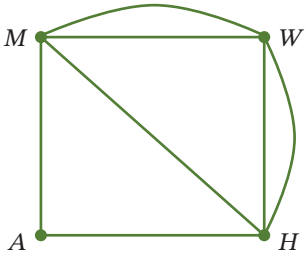
B



C



D



6 Anniek, Belinda, Emily, Mehek and Rima work at the same *J-Mart* store. R C

- Anniek is friends with Belinda, Emily and Rima
- Belinda is friends with Anniek and Emily
- Mehek is friends with Emily and Rima
- Rima is friends with Anniek, Mehek and Emily
- Emily is friends with Anniek, Belinda, Mehek and Rima

a Draw a graph to represent this network of friends.

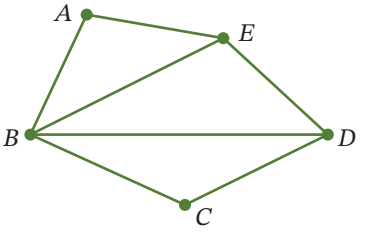
b Who is not a friend with Anniek?

Foundation Standard Complex

PATH

EXAMPLE 12

7 **a** State whether each walk is a trail or path. R C



i *AEBDCB* **ii** *EBDCBAE* **iii** *EDCBAE*

iv *BDCBAED* **v** *DBEABCD* **vi** *DCBEA*

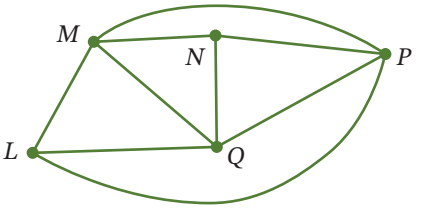
b Which of the walks in part **a** is:

i a circuit (a closed trail)? **ii** a cycle (closed path)?

c Write down a circuit that contain the edges *BC* and *DE*.

d Write down a cycle that contains the edge *CD*.

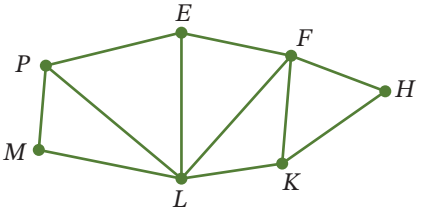
8 Which of the following is a circuit?



Select **A**, **B**, **C** or **D**. R C

A *MPQNMLPM* **B** *PMNPQLP* **C** *QNPMLPN* **D** *LQPLMQP*

9 **a** Which of the following walks is a cycle?



Select **A**, **B**, **C** or **D**. R C

A *LKHFLEPM* **B** *FHEKHFEM* **C** *MLKHFEKM* **D** *EFHKLPE*

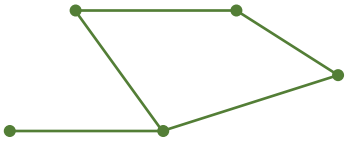
b Write down a cycle that begins at *K* and includes edge *LP*.

Foundation Standard Complex

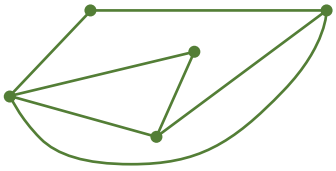
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10 Count the number of faces in each network.

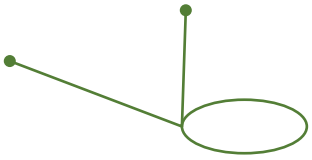
a



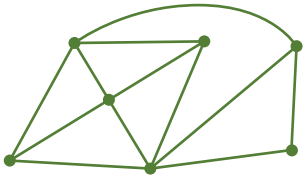
b



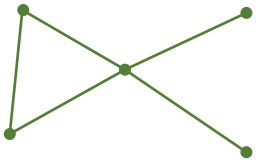
c



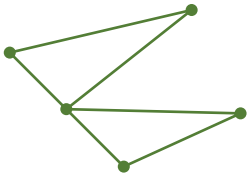
d



e



f



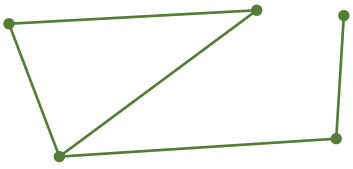
PATH

EXAMPLE 13

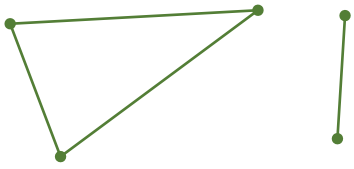
10.06

Planar graphs and Euler's formula 10.06

A network or graph is **connected** if there is a path between any 2 vertices on the network.

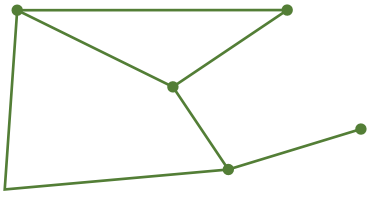


Connected

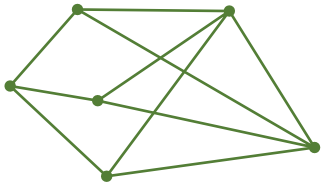


Not connected

A **planar graph** is a connected graph that can be drawn so that no edges cross over. 'Planar' means 2D, on a flat plane, and a planar graph can be drawn on a flat plane so that its edges don't overlap.



Planar graph



A non-planar graph

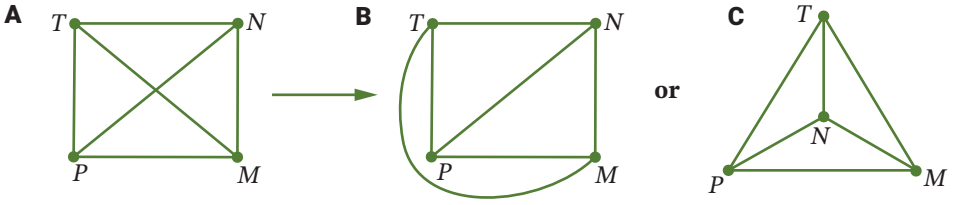
PATH

Video Euler's formula for planar graphs

Foundation | Standard | Complex

PATH

Some graphs may have edges crossing but they are still planar graphs because they can be *redrawn* without any of the edges crossing. For example, Graph **A** below is planar because it can be redrawn as **B** or **C**.

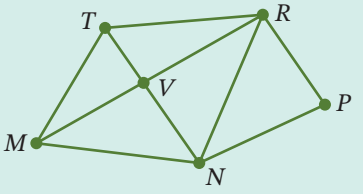


A **non-planar graph** has edges crossing and cannot be drawn so that the edges do not cross.

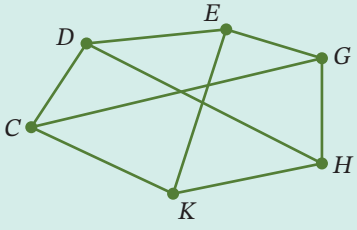
Example 14

State whether each network is a planar or non-planar graph.

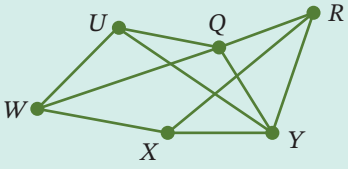
a



b



c

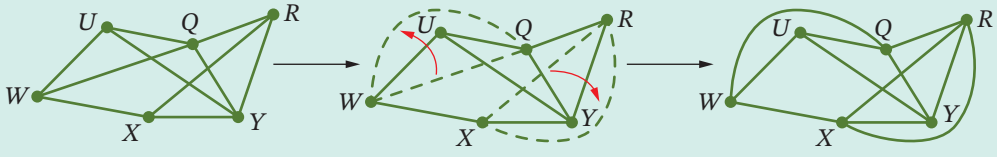


Solution

a A planar graph because no edges cross.

b It is impossible to move the edges so that no edges crossover so this is a non-planar graph.

c This graph can be redrawn by moving XR and WQ as shown.

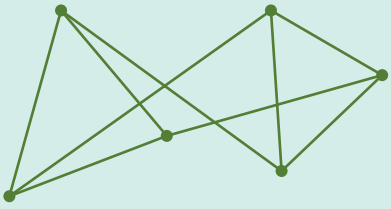


There are now no edges crossing so it's a planar graph.

9780170479622

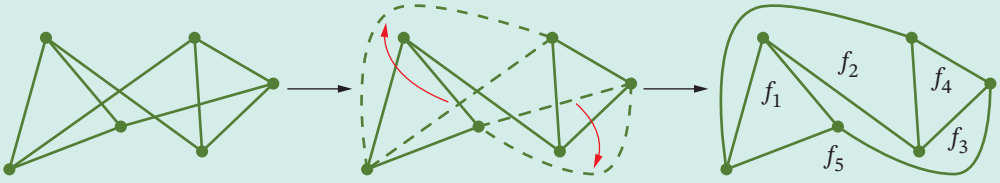
Example 15

Count the number of faces in this planar graph.



Solution

Redraw the graph so that no edges cross.



There are 5 faces.

Euler's formula

The Swiss mathematician Leonhard Euler (pronounced 'oiler') discovered a relationship between the vertices, edges and faces of a planar graph.

Euler's formula for planar graphs

For a graph to be planar,

$$v - e + f = 2$$

where

v = number of vertices

e = number of edges

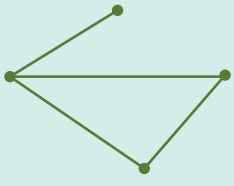
f = number of faces

Example 16

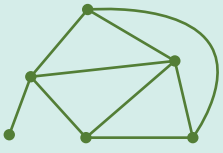
'Verify' means to show that something is true or correct.

Verify Euler's formula for each planar graph.

a



b



PATH

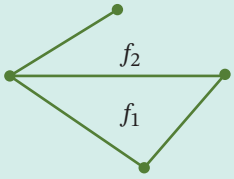
Solution

a $v = 4, e = 4$ and $f = 2$

$\therefore v - e + f = 4 - 4 + 2$

$= 2$

$\therefore$ Euler's formula is true.

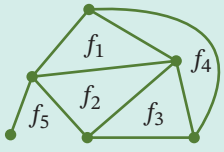


b $v = 6, e = 9$ and $f = 5$

$\therefore v - e + f = 6 - 9 + 5$

$= 2$

$\therefore$ Euler's formula is true.

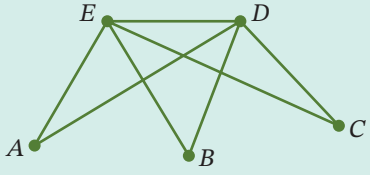


Example 17

Show that the graph is planar by:

a redrawing the graph so no edges cross

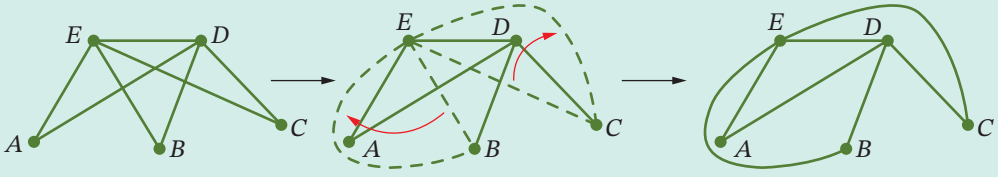
b using Euler's formula.



Solution

a Edge EC crosses edges AD and BD, so move edge EC to the outside of vertex D.
Also, edge EB crosses edge AD so move edge EB to the outside of vertex A.

Since the graph can be redrawn without any edges crossing, it is a planar graph

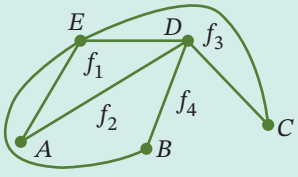


b $v = 5, e = 7$ and $f = 4$

$\therefore v - e + f = 5 - 7 + 4$

$= 2$

$\therefore$ the graph is planar

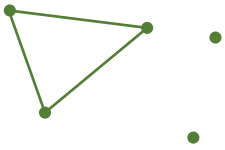


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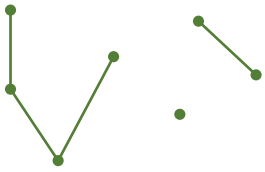
Planar graphs and Euler's formula U F R C

1 What is the smallest number of edges required to make each graph a connected network. R C

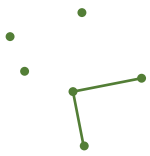
a



b



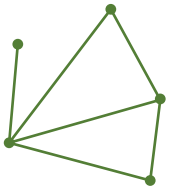
c



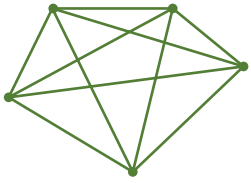
EXAMPLE 14

2 State whether each network is a planar or not planar graph. Give a reason for your answer. C

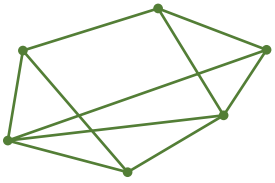
a



b

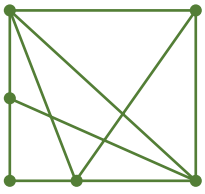


c

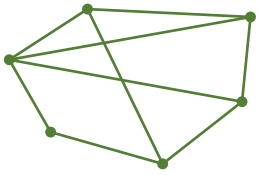


3 Redraw each graph to show that it is planar. C

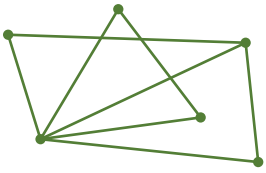
a



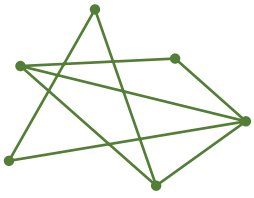
b



c



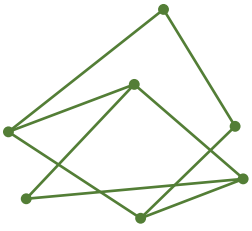
d



e



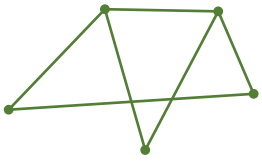
f



EXAMPLE 15

4 Redraw each planar graph to count its number of faces. R C

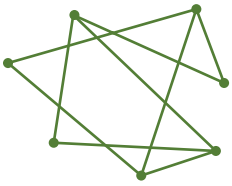
a



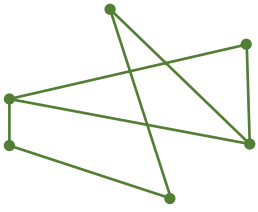
b



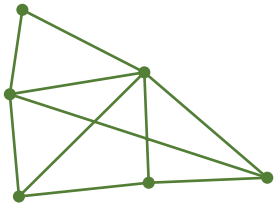
c



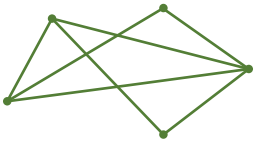
d



e



f



Foundation | Standard | Complex

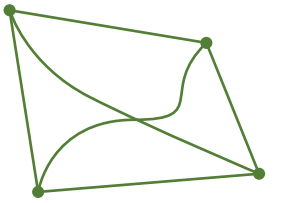
PATH

5 How many faces does this graph have?

Select the correct answer **A**, **B**, **C** or **D**.

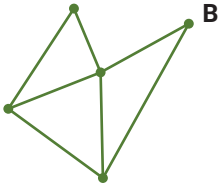
A 2 **B** 3

C 4 **D** 5

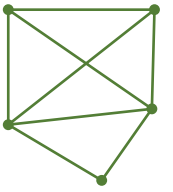


6 Which planar graph has 5 vertices and 5 faces? Select **A**, **B**, **C** or **D**.

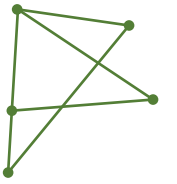
A



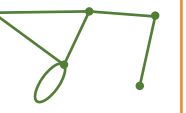
B



C



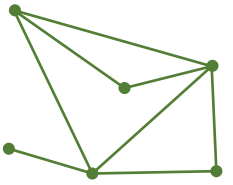
D



EXAMPLE 16

7 Verify Euler's formula for each planar graph. R C

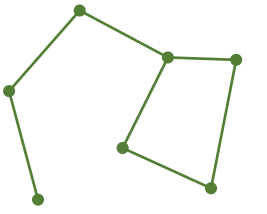
a



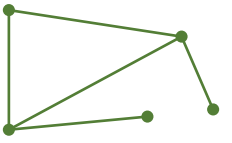
b



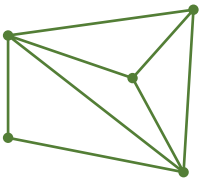
c



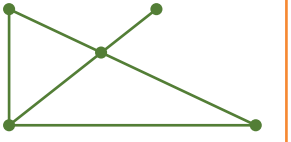
d



e



f

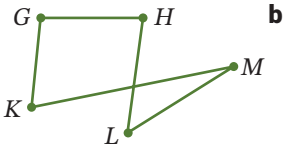


EXAMPLE 17

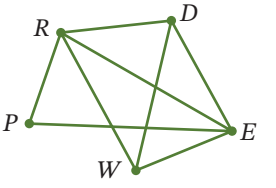
8 Show that each graph is planar by: R C

i redrawing the graph so no edges cross **ii** using Euler's formula.

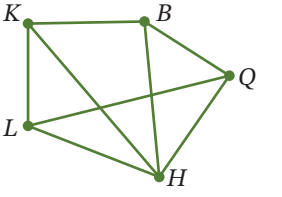
a



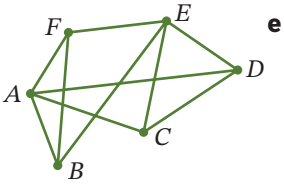
b



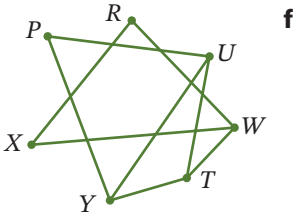
c



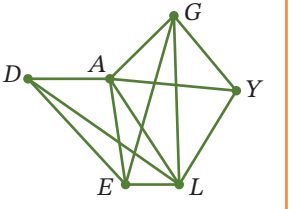
d



e



f



Foundation Standard Complex

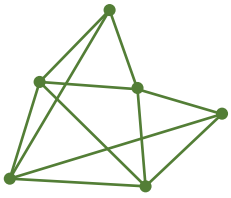
9780170479622

9 For which graph does Euler's formula NOT apply? Select **A**, **B**, **C** or **D**.

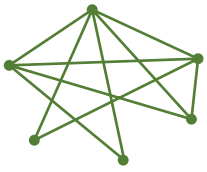
A



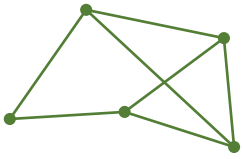
B



C



D



10 A planar graph has 7 vertices and 7 faces. What is its number of edges? Select **A**, **B**, **C** or **D**.

A 9 **B** 12 **C** 14 **D** 15

11 A planar graph has 10 faces and 13 edges. How many vertices has it? Select **A**, **B**, **C** or **D**.

A 3 **B** 5 **C** 8 **D** 11

12 Draw a planar network with 5 vertices and 7 faces.

13 A planar graph has 10 edges. Which of the following could be the number of vertices and faces? Select **A**, **B**, **C** or **D**.

A 9 vertices and 5 faces **B** 5 vertices and 5 faces

C 6 vertices and 7 faces **D** 8 vertices and 4 faces

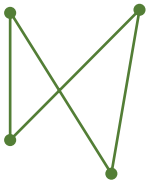
Investigation

Eulerian trails and circuits

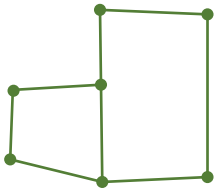
1 Copy each network, then trace over it with your pen or finger using each edge only once without lifting your pen or finger.

Check your work with that of other students.

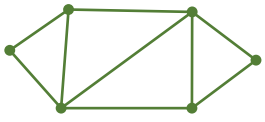
a



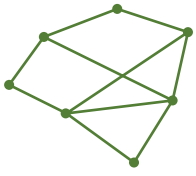
b



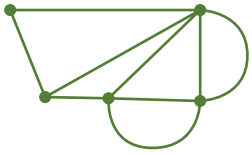
c



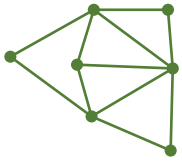
d



e



f

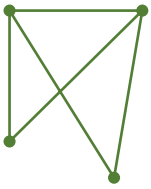


Foundation Standard Complex

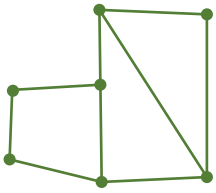
PATH

2 An extra edge has been added to each network in Question 1. Copy each network, then trace over it with your pen or finger using each edge only once. Which networks could you trace?

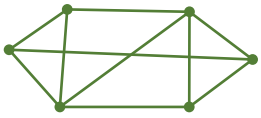
a



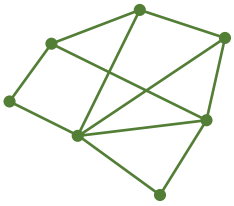
b



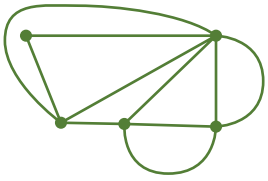
c



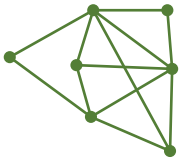
d



e



f



3 Copy and complete the table for each network in questions **1** and **2**.

Network	Number of odd vertices	Are all vertices even? (Y or N)	Is the network traceable? (Y or N)	Does trace begin and end at the same vertex? (Y or N)
1 a	0	Y	Y	Y
b				
c				
d				
e				
f				
2 a				
b				
c				
d				
e				
f				

4 Use the results in the table to copy and complete:

a A network can be traced if it is exactly _____ or _____ odd vertices.

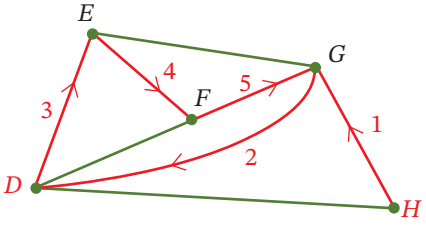
b A network's trace starts and finishes at the same vertex if all vertices are _____.

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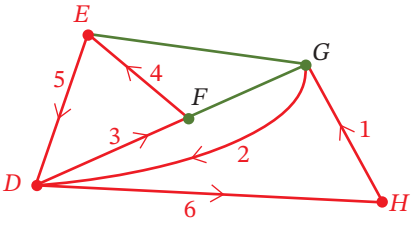
Eulerian trails and circuits

10.07

Remember that a **trail** is a walk in a network with no repeated edges, while a **circuit** is a **closed trail**, starting and ending at the same vertex.

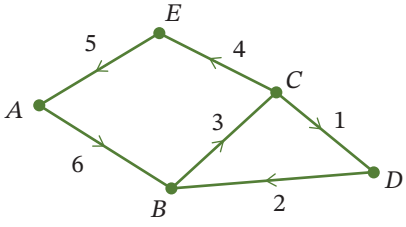


trail

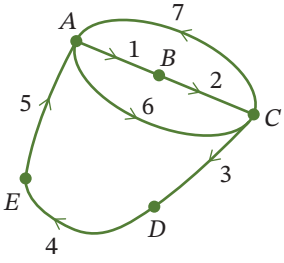


circuit

An **Eulerian trail** is a trail that uses *every* edge once, while an **Eulerian circuit** is a circuit that uses *every* edge once.



Eulerian trail



Eulerian circuit

Eulerian trails

An **Eulerian trail** uses every edge in the network only once.

For an Eulerian trail to exist, the network must have exactly 2 odd vertices.

Eulerian trails start at one odd vertex and finish at the other odd vertex.

PATH

10.07

Video
Eulerian trails

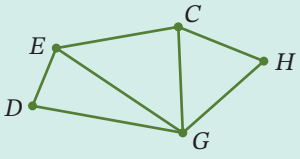
Worksheet
Eulerian trails and circuits

PATH

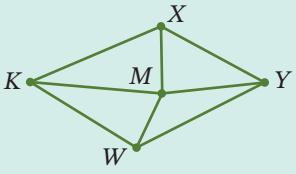
Example 18

Which network has an Eulerian trail?

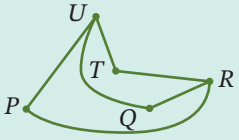
a



b



c



Solution

a This network has 2 odd vertices at C and E so it has an Eulerian trail, such as $EDGHCGEC$.

b This network has 4 odd vertices (degree 3) so it does not have an Eulerian trail.

c This network has 2 odd vertices at U and R (degree 3) so it has an Eulerian trail, such as $UPRQUTR$.

Eulerian circuits

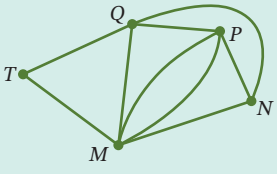
An **Eulerian circuit** uses every edge in the network only once and starts and ends at the same vertex.

For an Eulerian circuit to exist, all the vertices in the network must be even.

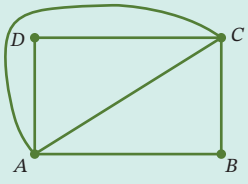
Example 19

Which network has an Eulerian circuit?

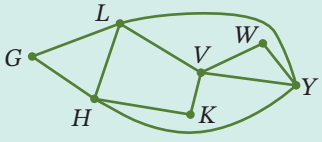
a



b



c



Solution

a Vertex M has degree 5 and vertex N has degree 3 so the network does not contain an Eulerian circuit.

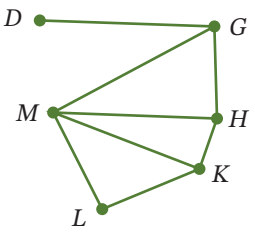
b All vertices are even (B and D have degree 2, A and C have degree 4) so the network has an Eulerian circuit. Two possible Eulerian circuits are $BCADCAB$ and $ABCACDA$.

c All vertices are even (vertices G, K and W have degree 2, vertices H, L, V and Y have degree 4) so the network has an Eulerian circuit.

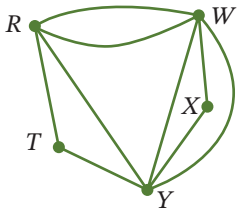
Eulerian trails and circuits U F R C

1 Which network has an Eulerian trail? Give reasons.

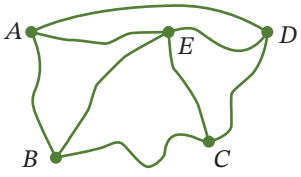
a



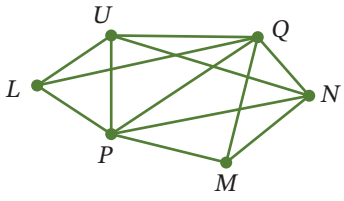
b



c



d



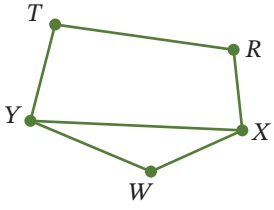
2 For each network: R C

i find the degree of each vertex

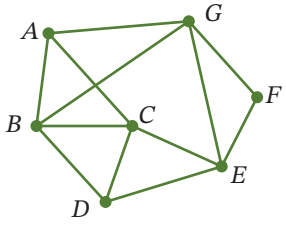
ii explain why the graph has an Eulerian trail

iii copy the graph and show a Eulerian trail on it.

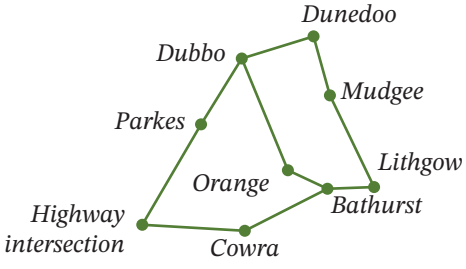
a



b



3 This graph shows the major roads linking cities in Central West NSW.



a Does this road network contain an Eulerian trail? Give reasons.

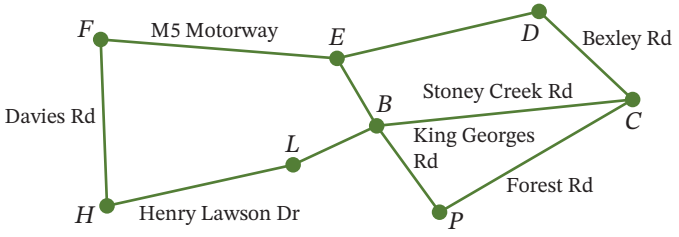
b Where will the trail start and finish?

c What is a possible Eulerian trail?

Foundation Standard Complex

PATH

4 This graph shows a network of major roads in the St George area, south of Sydney. R C

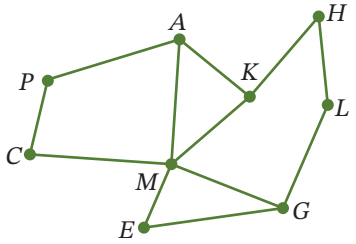


a Explain why the graph has an Eulerian trail.

b If the trail starts at E, where will it finish?

c Copy the graph and show the trail.

5 Which vertices can be joined by an edge so that the network contains an Eulerian trail?



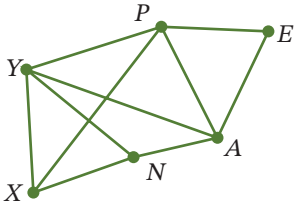
Select the correct answer **A**, **B**, **C** or **D**.

A G and K **B** M and P **C** G and H **D** K and L

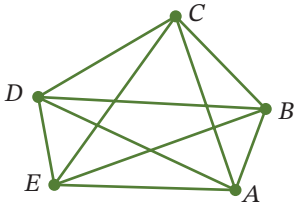
EXAMPLE 19

6 Test whether each network has an Eulerian circuit.

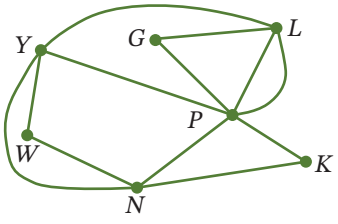
a



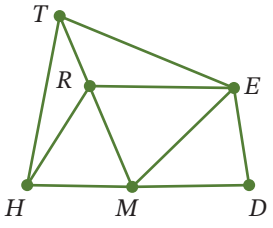
b



c



d



Foundation Standard Complex

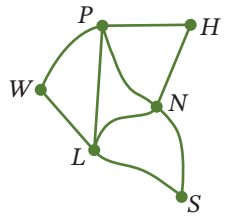
9780170479622

7 **a** Explain why this graph has an Eulerian circuit. R C

b Write down 2 Eulerian circuits.

c Copy the graph to draw the 2 circuits.

d What do you notice about your circuits?

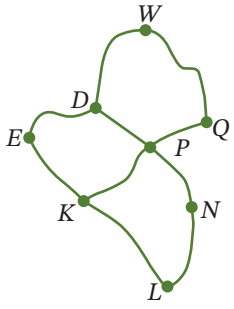


8 A road map for the delivery of milk to cafés and shops is shown by the network. The delivery starts at D and does not use the same road twice. R C

a Explain why the delivery cannot start and finish at D.

b A road is added to the network so that the delivery can start and finish at D. Which 2 vertices would the road join?

c Find 2 suitable routes for the milk delivery.



9 **a** This graph shows the roads connecting 4 towns.

Is it possible to start and finish at the same town after travelling along each road only once?

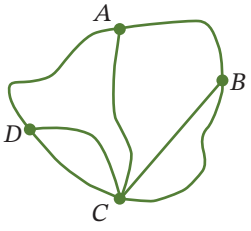
b A second road joining A to D is added.

Is it possible to travel along each road only once and

i start and finish at the same town?

ii start and finish at different towns?

Give reasons. R C



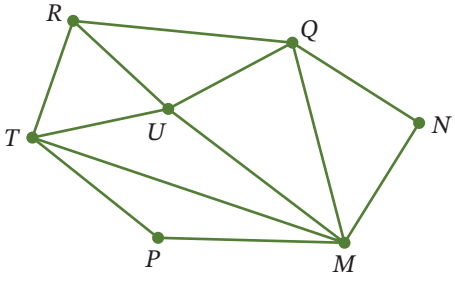
10 This tourist map shows 7 tourist destinations. Marinella and her family want to visit each destination, but use each road only once. R C

a From which destination should they start?

b At which destination will they finish their trip?

c How many destinations will Marinella and her family pass:

i only once? **ii** exactly twice?



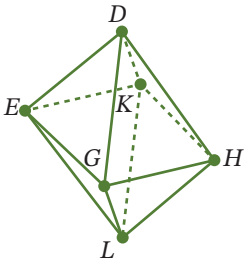
Foundation Standard Complex

PATH

11 An octahedron can be thought of as a 3D network.

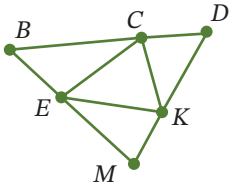
a Show that this network has an Eulerian circuit.

b Starting at D, describe a path that is an Eulerian circuit.

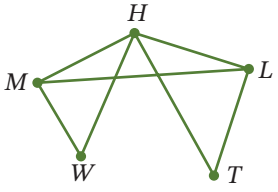


12 Determine which network is an Eulerian trail, an Eulerian circuit or neither. Give reasons for your answers. R C

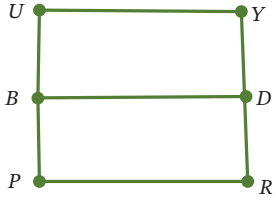
a



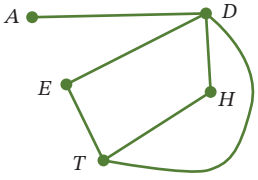
b



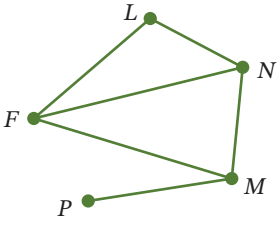
c



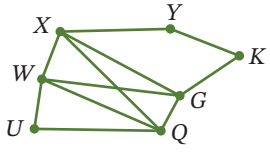
d



e

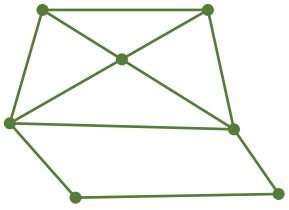


f

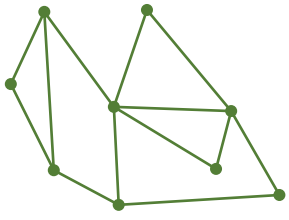


13 Which graph contains an Eulerian circuit? Select **A**, **B**, **C** or **D**.

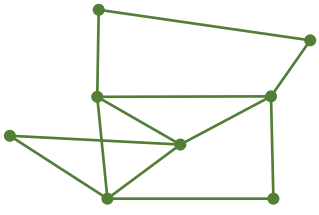
A



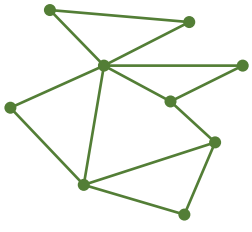
B



C



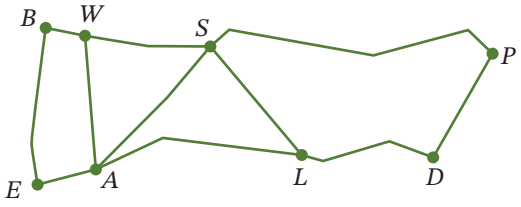
D



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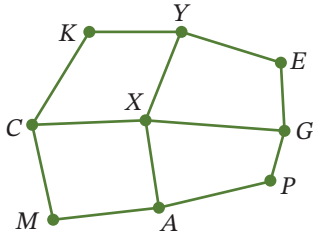
14 Mehek is competing in a 20 km road race and is given a map of the course as shown. To complete the race she must run along each road only once. R C



a At which point would the race start?

b Where would the race finish? Give reasons.

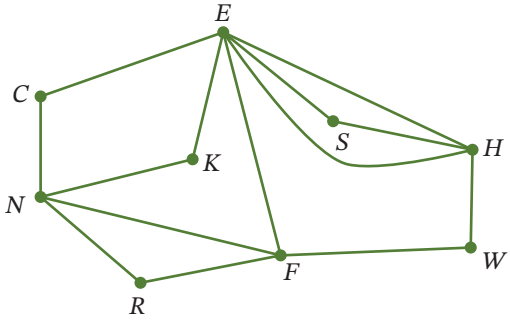
15 a Explain why this graph does not contain an Eulerian trail or circuit. R C



b How many vertices need to be joined for an Eulerian trail to exist in the graph?

c Which vertices should be joined for an Eulerian circuit to exist in the graph?

16 A marketing salesperson travels to stores in 8 different locations. In this network diagram, the vertices represent the locations and the edges represent the roads. R C



a Explain why the salesperson can travel from C, visit every location using each road only once and then return to location C.

b Describe the route the salesperson could take.

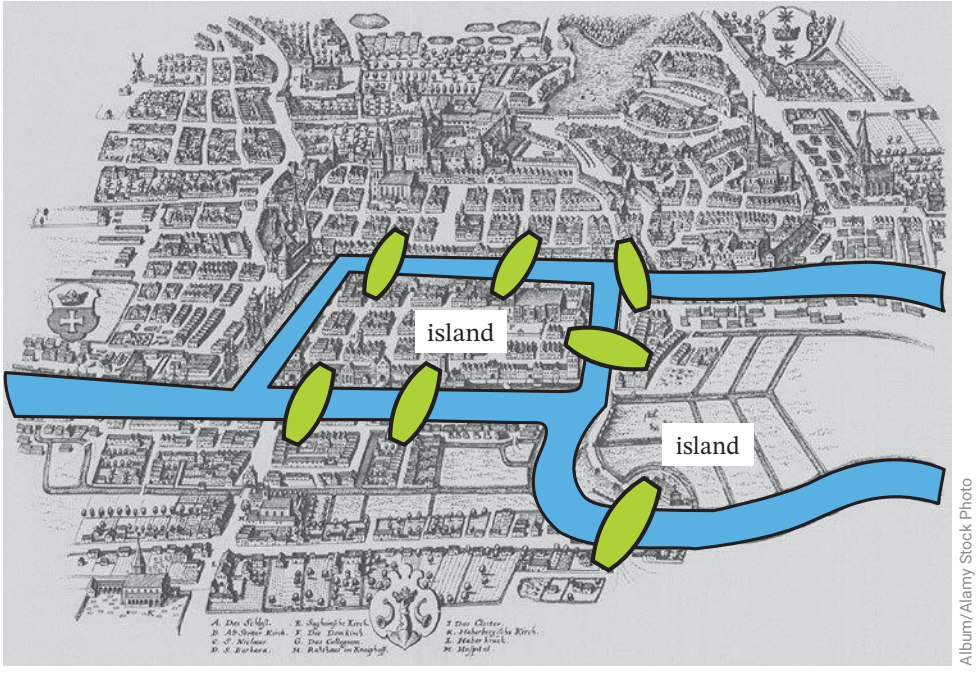
Foundation Standard Complex

Video
The seven bridges of Konigsberg

Did you know?

The 7 bridges of Königsberg

In the Prussian town of Königsberg (which is now Kaliningrad, Russia), there were 7 bridges connecting 2 islands and the banks of the Pregel River. There was a tradition where people would walk through the town on a Sunday and it became a challenge to complete a walk crossing each bridge only once. No person was able to do it but no one could prove that it was impossible.



Album/Alamy Stock Photo

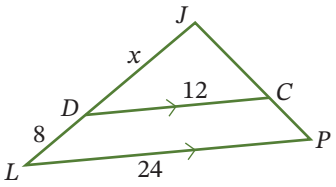
In 1736, the Swiss mathematician Leonard Euler solved the problem by reducing the map to a network. He found that it was impossible to do the walk. Eventually another bridge was built. That is why a walk that crosses every bridge exactly once is called an **Eulerian trail**. Euler developed a new branch of mathematics called **graph theory**, the geometry of networks.

1 **Draw a network to represent the map and explain why the original walk was impossible.**

2 **An eighth bridge was built. Where would the bridge have been placed so that the network could be traversed?**

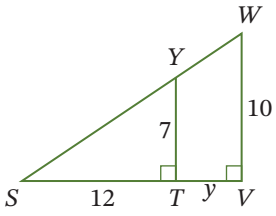
Power plus ANSWERS ON P. 628

1 $\triangle JDC \sim \triangle JLP$. Find the value of x.



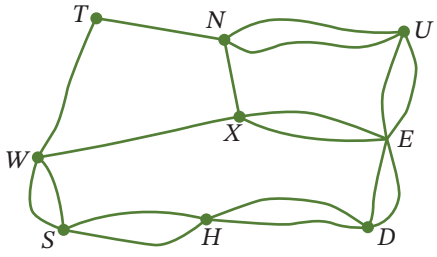
2 $\triangle STY \sim \triangle SVW$.

Find the value of y.



3 Leilani is surveying homes on family size.

Some streets have homes on both sides (shown by double edges on the network diagram) and other streets only have homes on one side only (shown by single edges).



a Explain why an Eulerian circuit can be found on this network.

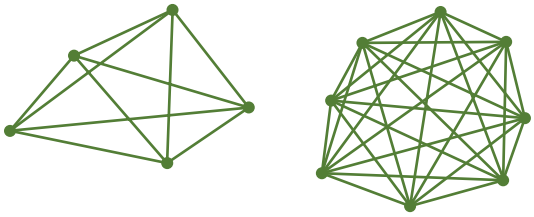
b Write down a circuit that begins at:

i N **ii** H **iii** X

c For each of your circuits from part **b**, write down which vertices occur:

i once in the circuit **ii** 3 times in the circuit.

4 In a polygon, each vertex is joined to all other vertices as shown (all diagonals are shown).



What type of polygon drawn this way will always have an Eulerian circuit?

CHAPTER 10 REVIEW

Quiz
Language of maths 10

Puzzle
Geometry and networks crossword

Language of maths

circuit	cycle	degree	edge
enlargement	Eulerian	face	image
line symmetry	matching	network	original
path	planar graph	reduction	reflection
rotation	rotational symmetry	scale factor	similar (~)
trail	transformation	translation	vertex (node)

1 What is **rotational symmetry**?

2 Describe what happens to a figure that is changed by a scale factor of $\frac{1}{2}$?

3 What is the symbol and meaning of 'is similar to'?

4 What type of graph can be drawn so that its edges don't cross?

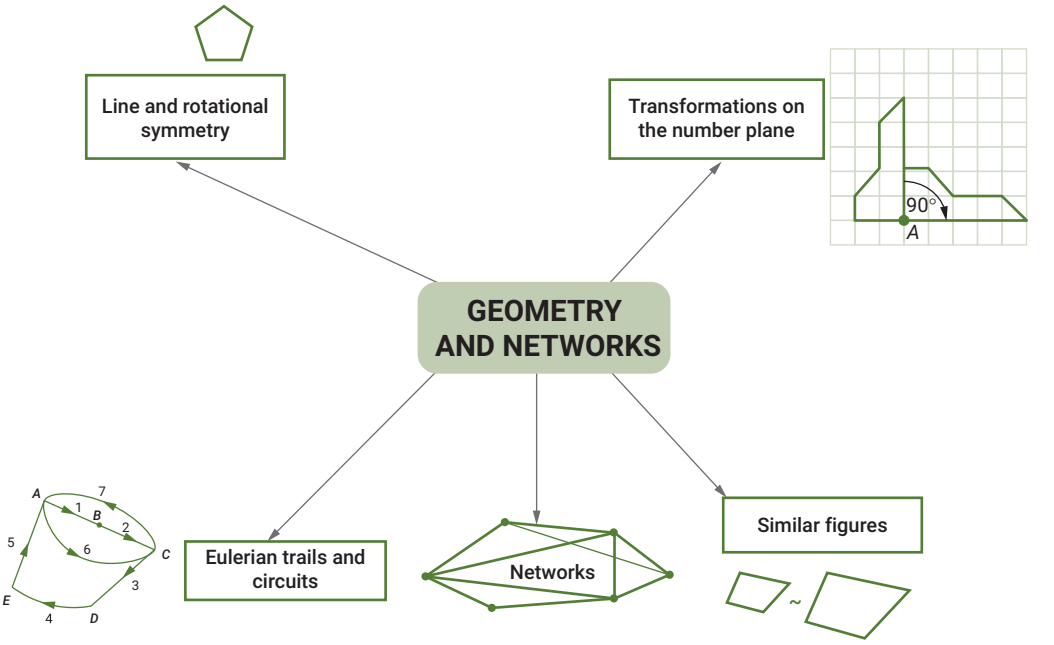
5 Explain what the formula $v - e + f = 2$ means.

6 Explain the difference between an **Eulerian trail** and an **Eulerian circuit**.

Worksheet
Mind map: Geometry and networks

Topic summary

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



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TEST YOURSELF 10

ANSWERS ON P. 628

1 For each shape, find:

i the number of axes of symmetry

ii the order of rotational symmetry

a isosceles triangle **b** regular octagon **c** parallelogram.

2 **a** Copy the quadrilateral $PLMN$ onto grid paper and then draw its image after:

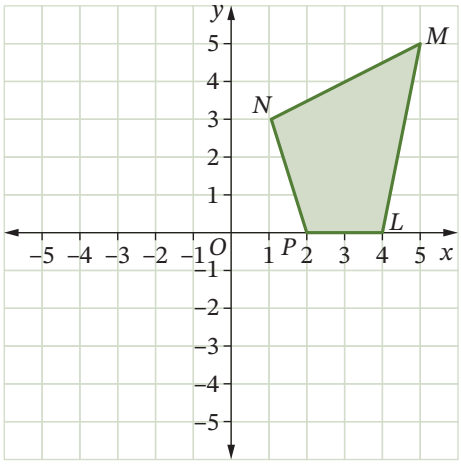
i reflecting it across the y-axis

ii reflecting it across the x-axis

iii rotating it anticlockwise about O through 90°

iv translating it 5 units to the left and 4 units down.

b For each of the above transformations, compare the coordinates of each vertex of the original to its matching vertex in its image.



3 Find the new position of the point $K(-5, 3)$ if it is:

a reflected in the y-axis

b reflected in the x-axis

c rotated clockwise about O through

i 90° **ii** 180° **iii** 270°

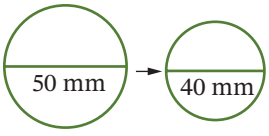
d translated 8 units to the right and 5 units down

e translated 3 units to the right, 2 units up and then rotated 180° about O.

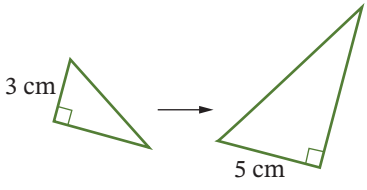
4 The point $W(-4, -2)$ is the image of a point after it has been reflected in the x-axis and then rotated clockwise about O through 90°. Find the original point.

5 Calculate the scale factor between each pair of similar figures.

a



b



PATH

Quiz
Test yourself 10

10.01

Worksheet
5 mm grid paper

10.02

10.02

10.02

10.03

TEST YOURSELF 10

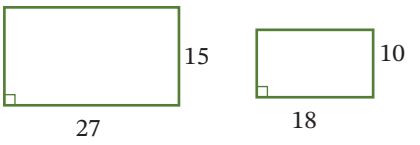
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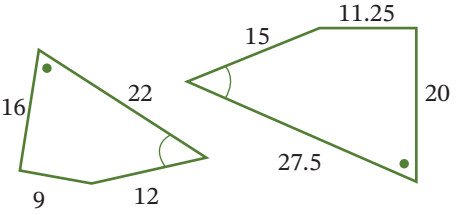
10.03

6 Test whether each pair of figures are similar.

a



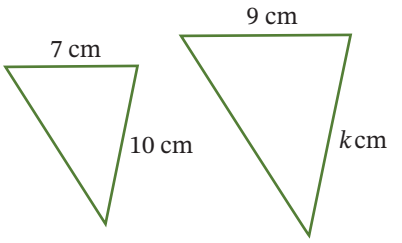
b



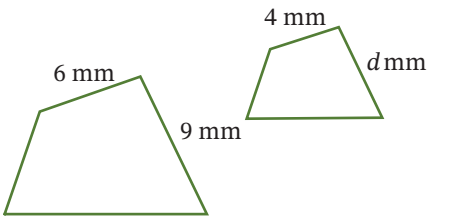
10.04

7 Find the value of the variable in each pair of similar figures.

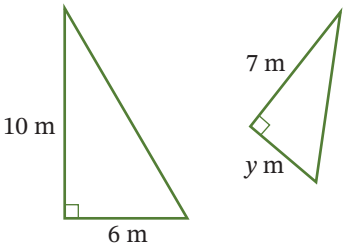
a



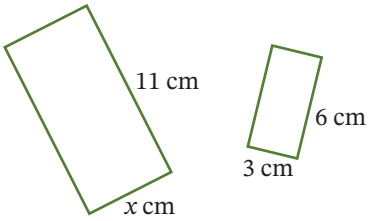
b



c

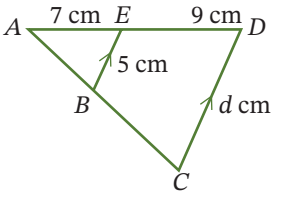


d



10.04

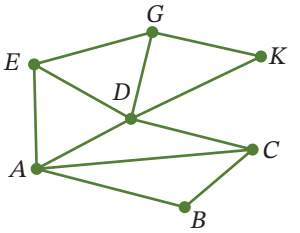
8 If $\triangle ABE \sim \triangle ACD$, find the value of d.



PATH

10.05

9



a State whether each walk is a trail or a path.

i *EABCDKG*

ii *CADKGEABC*

iii *EDKGDAB*

iv *DCBAEGKD*

b Which of the walks in part **a** is a **circuit**?

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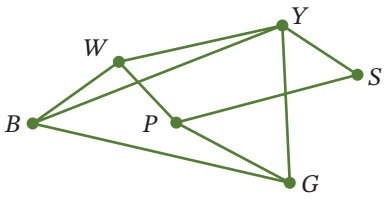
PATH

10 How many faces does this graph have?

Select **A**, **B**, **C** or **D**.

A 5 **B** 6

C 7 **D** 8

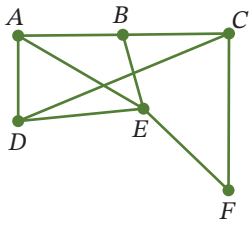


10.05

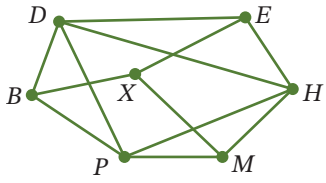
11 Show that the graph is planar by:

i redrawing it so that no edges cross **ii** using Euler's formula.

a



b

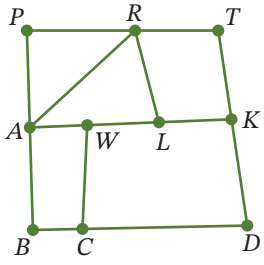


10.06

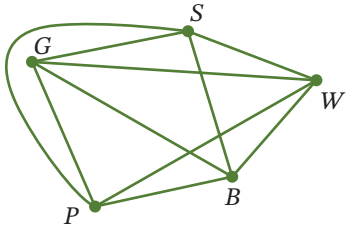
12 Determine whether each network has an Eulerian trail, an Eulerian circuit or neither.

Give a reason for your answer.

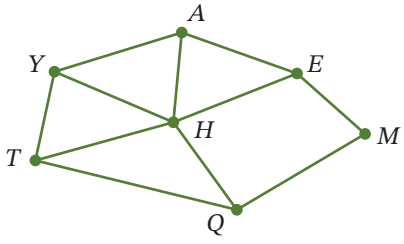
a



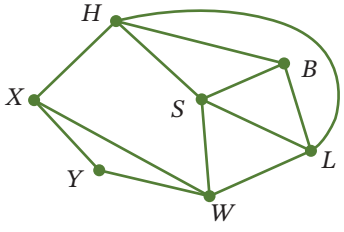
b



c



d

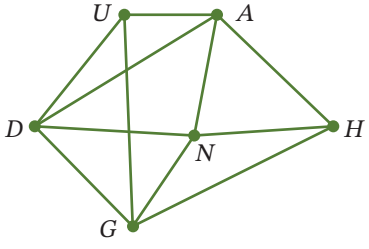


10.07

13 The graph shows how 6 towns are connected by a road network.

Is it possible to start at one town and go along each road once and finish at the same town?

Give a reason for your answer.



10.07

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PRACTICE SET 4 ANSWERS ON P. 630

9.05

1 2 dice are rolled.

a How many outcomes are possible?

b What is the probability of rolling:

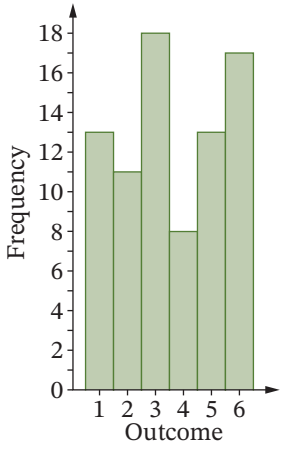
i double 1s?

ii any doubles?

iii 2 numbers both less than 4?

9.01

2 A die was rolled and the outcomes were recorded in this frequency histogram.



a How many times was the die rolled?

b Find the observed probability of rolling:

i a 1

ii an even number

iii a number less than 4

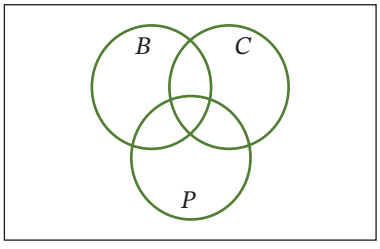
iv at least a 3.

c What is the theoretical probability of rolling a 6? How does this compare with the observed probability of rolling a 6?

PATH

9.02

3 Of 160 Year 11 students at Westvale High, 54 do Biology (B), 75 take Chemistry (C) and 68 study Physics (P). 55 students take both Chemistry and Physics, 20 do Biology and Chemistry and 10 students take all three.



a Copy and complete the Venn diagram to show this information.

b Find the probability of selecting a student who:

i only takes Physics

ii does not do Biology, Chemistry or Physics

iii does Chemistry and Physics but not Biology

iv studies Chemistry or Biology

v only does one Science subject.

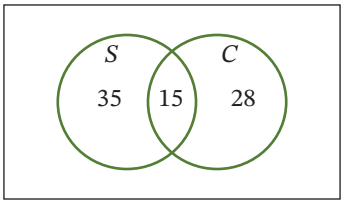
c From the students who do Biology, what is the probability that the student also takes Physics?

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PATH

4 This Venn diagram shows the results of a survey on whether people prefer to watch movies by streaming (S) or at the cinema (C). 9.02



a How many people were surveyed?

b Find the probability of selecting a person from this survey who prefers to watch movies:

i by streaming **ii** only at the cinema.

c What is the probability of selecting a person who likes to stream movies only?

5 For each shape, find: 10.01

i the number of axes of symmetry

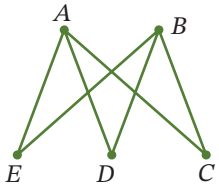
ii the order of rotational symmetry.

a rhombus **b** regular pentagon **c** trapezium

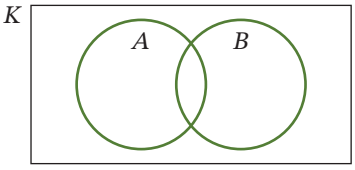
6 Show that this graph is planar by: 10.05

a redrawing it so that no edges cross

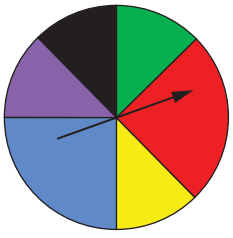
b using Euler's formula.



7 Copy the Venn diagram. If $K = \{\text{factors of } 120\}$, $A = \{\text{prime numbers}\}$ and $B = \{\text{multiples of } 5\}$, place the numbers in the Venn diagram. 9.02

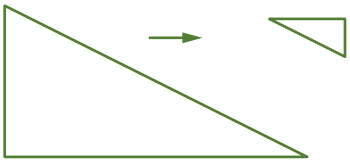


8 Olivia spun this spinner twice, landing on red both times. Is the second outcome dependent or independent of the outcome of the first spin? 9.06

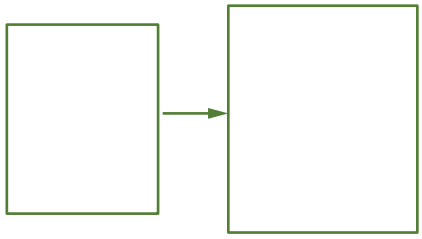


9 By measurement, calculate the scale factor for each pair of similar figures. 10.03

a



b



Foundation Standard Complex

9.01

10 A bag contains 4 red, 5 black and 3 green marbles. Sadiya selects a marble at random, records its colour and then returns the marble to the bag. Sadiya does this process 150 times. The results are shown in the table.

Outcome	Frequency
red	58
black	65
green	27

a Find, correct to 2 decimal places, the observed probability of selecting a marble that is:
i red **ii** black **iii** green.

b Find, correct to 2 decimal places, the expected probability of selecting a marble that is
i red **ii** black **iii** green.

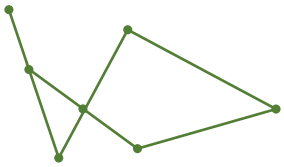
c For which outcome are the observed and expected probabilities most similar?

d What is the expected frequency of selecting a red or green marble? How does this compare with the observed frequency?

PATH

10.05

11 For this network, which one of these statements is true? Select the correct answer **A**, **B**, **C** or **D**.



A The number edges is less than the number of vertices

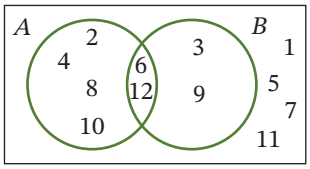
B The sum of the degrees of the vertices is double the total number of edges

C The sum of the number of edges and the number of vertices is even

D The number of edges is the same as the sum of the degrees of the vertices

9.02

12 For this Venn diagram, list the elements of:



a $\overline{B}$ **b** $A \cup \overline{B}$ **c** $\overline{A} \cap \overline{B}$.

10.02

13 Find the new position of the point $T(-3, -2)$ if it is:

a reflected in the x-axis.

b reflected in the y-axis.

c rotated clockwise about O through 90°.

d translated 7 units right and 4 units up.

e rotated 180° about O and then reflected in the y-axis.

9.04

14 A 6-sided die and a 4-sided die are rolled together and the product of the 2 numbers is calculated.

	1	2	3	4	5	6
1						
2						
3						
4						

a Copy and complete this table to show all possible products.

b Find the probability of rolling a product:
i of 20 **ii** of 6 **iii** that is odd
iv of at least 12 **v** less than 10 **vi** from 11 to 23.

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15 Shoppers at a mall were asked whether they had a pet dog or cat. The results are shown in the 2-way table.

	Cat	No cat
Dog	28	35
No dog	32	40

PATH 9.03

a How many shoppers were surveyed?

b Find the probability of randomly selecting a shopper from the survey who does not have a cat or dog.

c What is the probability of randomly selecting a shopper who:

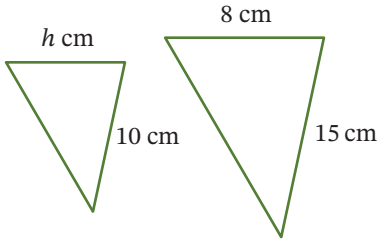
i has only a dog or cat (not both)?

ii has a cat?

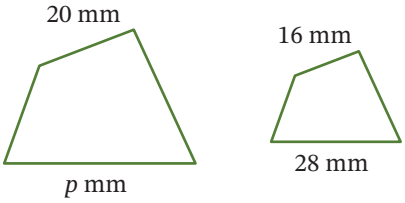
iii does not have a dog?

16 Find the value of the variable in each pair of similar figures. 10.03

a

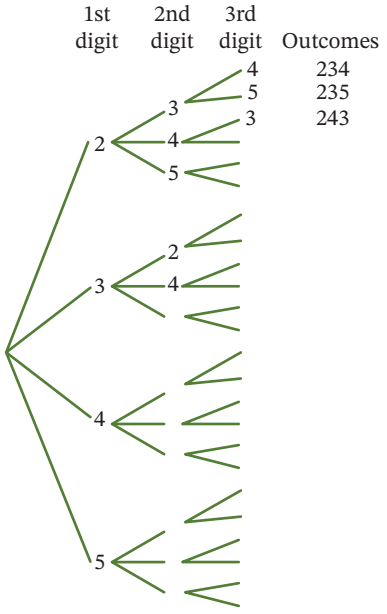


b



17 3 cards are drawn from a set of cards numbered 2, 3, 4 and 5, without replacement, to form a 3-digit number. 9.05

a Copy and complete the tree diagram to list all possible outcomes.



b Find the probability of forming:

i an even number

ii a number ending in 3

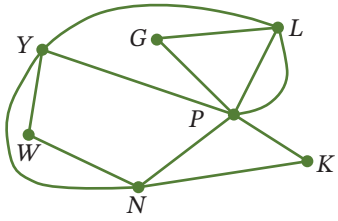
iii a number greater than 400

iv a number between 200 and 500

v a number divisible by 5.

18 **a** Explain why this graph has an Eulerian circuit.

b Write down 2 Eulerian circuits.



PATH 10.07

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9.05, 9.06

19 A bag contains 3 yellow and 2 red marbles. 2 marbles are drawn from the bag, without replacing the marble from the first draw.

a Draw a probability tree diagram representing the sample space for the 2 marbles drawn. Label each branch with its probability.

b Find the probability of:

i selecting 2 red marbles

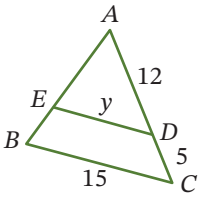
ii selecting at most one yellow marble.

c Are the 2 draws dependent or independent? Justify your answer.

d If the 2 draws are made with replacement of the marble after the first draw, are the draws dependent or independent? Justify your answer.

10.04

20 If $\triangle ABC \sim \triangle AED$, find the value of y (correct to one decimal place).



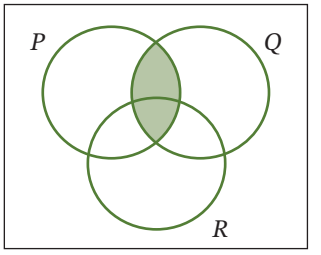
PATH

9.02

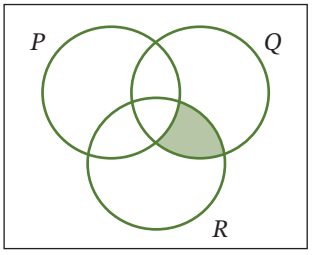
21 Match each set description with its correct shaded region in the Venn diagram.

a $Q \cup R$ **b** $P \cap Q \cap R$ **c** $Q \cap R \cap P'$

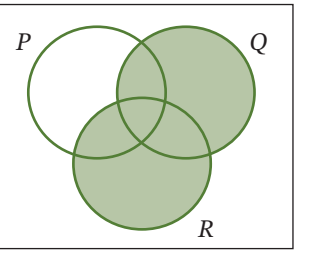
d $P \cap Q$ **e** $Q \cap (P \cup R)'$ **f** $(P \cup Q) \cap R'$



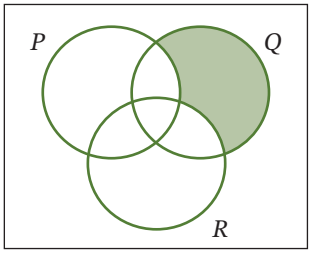
A



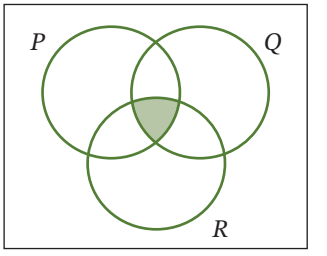
B



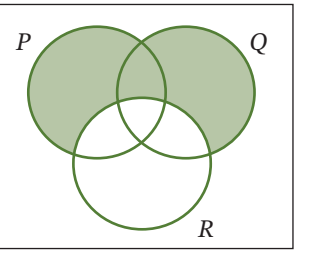
C



D



E



F

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GENERAL PRACTICE

ANSWERS ON P. 631

1 Kim is a real estate agent and is paid a commission of 2.5% on the value of apartments she sells. She also receives a weekly retainer of $1250. How much will Kim earn in a week if she sells an apartment for $578 000?

CHAPTER 1

2 Find the gradient, m, and y-intercept, c, of each linear equation.

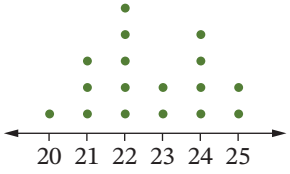
a $y = 3x - 4$ **b** $y = 5 - 2x$ **c** $4x + 3y - 9 = 0$

CHAPTER 2

3 Find the interquartile range for each set of data.

a 4 7 8 12 5 8 10 7 13 6 9 2

b



CHAPTER 5

4 Factorise each expression.

a $8x^2 + 16x$ **b** $y^2 - 10y + 25$

c $a^2 + 6a + 8$ **d** $p^2 - 2p - 48$

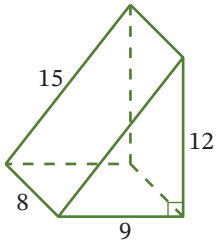
PATH

CHAPTER 4

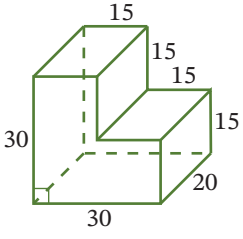
5 Find the surface area of each solid, correct to the nearest mm^2. All measurements shown are in millimetres.

CHAPTER 3

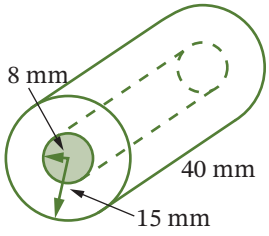
a



b



c



6 Simplify each expression, writing your answer with a positive index where necessary.

a $7x^5 \times 8x^7$ **b** $4x^2 \div 16x^{-3}$

c $(3y)^{-2}$ **d** $m^{-6}\, n^3 \times mn^{-1} \div m^2\, n$

PATH

CHAPTER 4

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PATH

CHAPTER 9

7 Students in Year 10 at Nelson Secondary College were asked if they had studied Japanese.

	Male	Female
Japanese	35	87
No Japanese	67	21

a How many students are in Year 10 at the school?

b What is the probability of selecting a Year 10 student at random who is:

i male and has studied Japanese?

ii female or has studied Japanese?

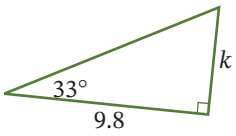
c Find the probability, expressed as a percentage to the nearest whole number, of randomly selecting a male in Year 10 who has not studied Japanese.

d Given that a student is a Year 10 female, what is the probability that she has studied Japanese?

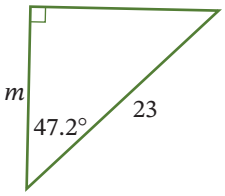
CHAPTER 8

8 Find the value of the variable in each diagram, correct to one decimal place.

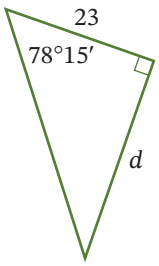
a



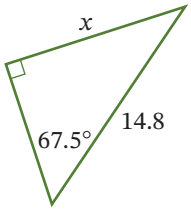
b



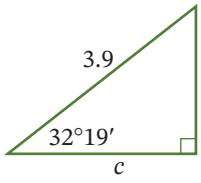
c



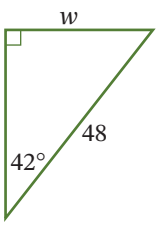
d



e



f



CHAPTER 6

9 Solve each equation.

a $2k - 5 = 8$

b $3(m + 7) = 12$

c $4(x - 3) - 2(x - 1) = 5$

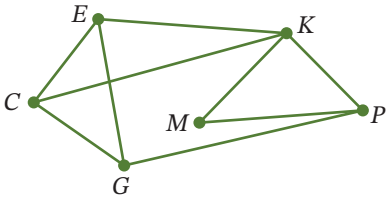
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10 **a** Count the number of edges, vertices and faces in this network.

b Verify Euler's formula for this network.

c Which one of these walks is a cycle? Select **A**, **B** or **C**.



A *PMKEG** **B** *CEGPMKE** **C** *MPGECKM*

PATH
CHAPTER 10

11 The angle of elevation of the top of a tree is 65° at a distance of 12 m from its base. Find the height of the tree, to the nearest metre.



CHAPTER 8

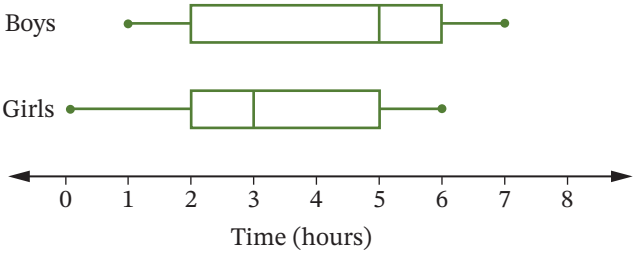
12 Solve each equation.

a $\frac{a-2}{3}=7$ **b** $\frac{4y}{5}-2=3$

c $\frac{3m+8}{2}=-1$ **d** $\frac{5k}{3}-\frac{k}{2}=4$

PATH
CHAPTER 6

13 Students were surveyed about the amount of time they typically spend on the Internet over a weekend. The results for boys and girls are displayed in these parallel box plots.



CHAPTER 5

a Calculate the interquartile range for the boys.

b What is the median amount of time spent on the Internet by the girls surveyed?

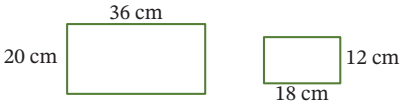
c What percentage of girls usually spend fewer than 5 hours on the Internet over the weekend?

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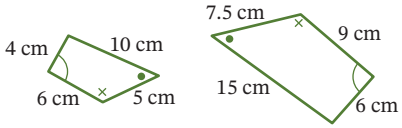
CHAPTER 10

14 Which pair of figures are similar? Select **A**, **B**, **C** or **D**.

A



B



C



D



PATH

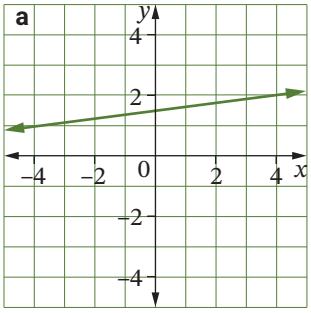
CHAPTER 6

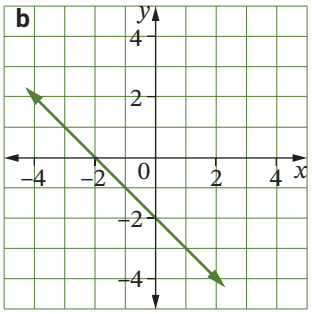
15 Change the subject of the formula $V = \frac{K - 6u}{d}$ to u.

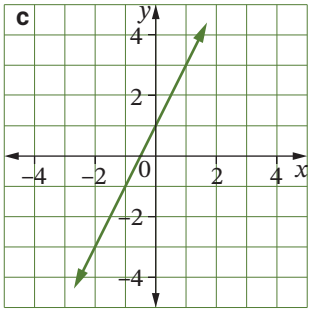
CHAPTER 2

16 For each line, find:

i the gradient **ii** the y-intercept **iii** the equation of the line.







PATH

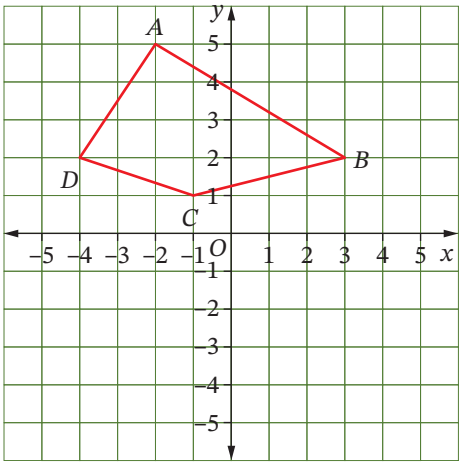
CHAPTER 10

17 **a** Copy and reflect the quadrilateral $ABCD$ across the y-axis to obtain the image $A'B'C'D'$.

b Compare the coordinates of each vertex of $ABCD$ to its matching vertex in $A'B'C'D'$.

c Find the new position of vertex A if it is rotated 90° clockwise about O.

d The vertex D is reflected in the x-axis and then rotated 270° clockwise about O. What is the new position of D?



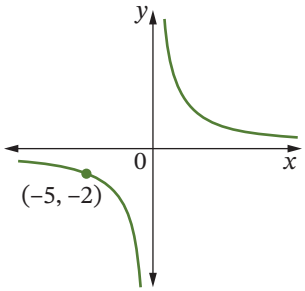
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PATH

CHAPTER 7

18 Find the equation of this hyperbola.



Select the correct answer **A**, **B**, **C** or **D**.

A $y = -\frac{2}{x}$ **B** $y = \frac{10}{x}$ **C** $y = -\frac{10}{x}$ **D** $y = \frac{5}{x}$

CHAPTER 1

19 Calculate the simple interest on each investment.

a \$500 invested at 4% p.a. for 2 years

b \$280 invested at 2.5% p.a. for 7 months

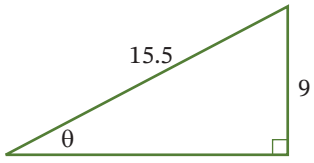
CHAPTER 8

20 Find θ:

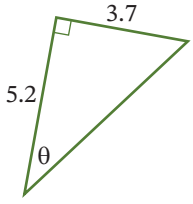
i correct to the nearest degree

ii correct to the nearest minute.

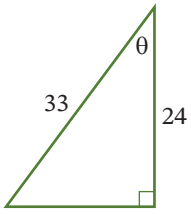
a



b



c



CHAPTER 1

21 Calculate the future value for each investment.

a \$800 invested at 4% p.a. for 4 years, compounded annually

b \$1260 invested at 8% p.a. for 3 years, compounded quarterly

CHAPTER 2

22 For the interval joining each pair of points given, find:

i the length of the interval, correct to one decimal place

ii the midpoint of the interval

iii the gradient of the interval.

a $C(2,1)$ and $D(6, 9)$ **b** $X(-7, -2)$ and $Y(5, 4)$

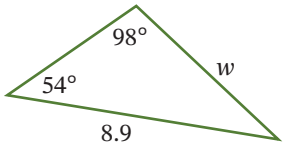
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PATH

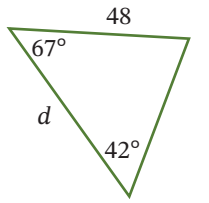
CHAPTER 8

23 Find the value of each variable, correct to 2 decimal places.

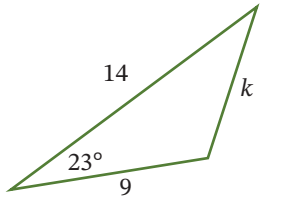
a



b



c



CHAPTER 7

24 Graph each equation on a number plane.

a $y = x^2$

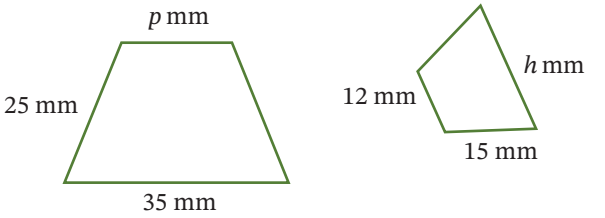
b $y = -x^2 + 3$

c $y = 3^x$

d PATH $y = \frac{12}{x}$

CHAPTER 10

25 Find the value of each variable in this pair of similar figures.



CHAPTER 8

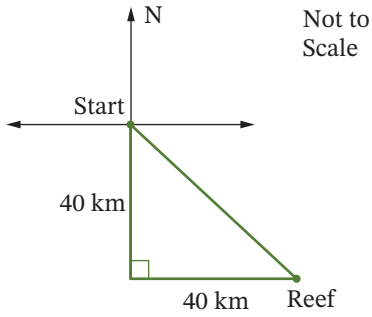
26 Imogen sailed due south for 40 km. Then she sailed due east for 40 km to a reef. What is the bearing of the reef from Imogen's starting point?
Select **A**, **B**, **C** or **D**.

A SW

B NE

C E

D SE



Not to Scale

CHAPTER 9

27 The weather on a long weekend will either be fine or rainy each day, with each outcome being equally likely.

a Draw a tree diagram to show the possible outcomes for Saturday, Sunday and Monday.

b What is the probability that it is fine:

i on all 3 days?

ii on exactly 2 of the days?

iii on at least one of the days?

PATH

CHAPTER 7

28 For an object that is cooling, the drop in temperature is directly proportional to the time. The temperature drops 5°C in 12 minutes. How long will it take to drop 8°C?

Foundation Standard Complex

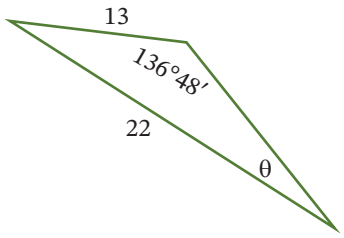
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PATH

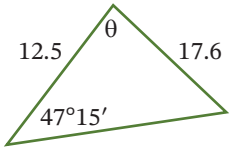
CHAPTER 8

29 Find the size of θ in each triangle, correct to the nearest minute.

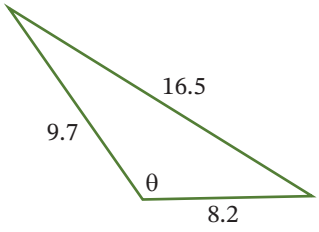
a



b



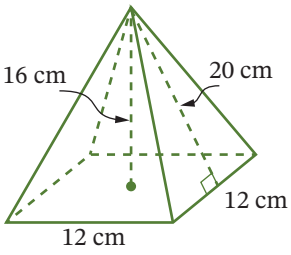
c



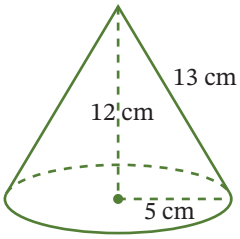
CHAPTER 3

30 Find the surface area and volume of each solid.

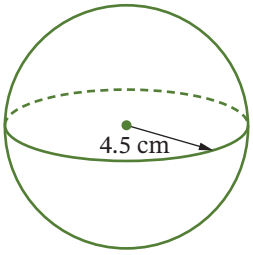
a



b



c



CHAPTER 1

31 Find the purchase price V_0 of a car which has depreciated at a rate of 18% p.a. over 5 years and now has a salvage value of $18 500.

CHAPTER 4

32 Expand each binomial product:

a $(m + 7)(m + 5)$ **b** $(2w - 3)(2w + 5)$ **c** $(3p + 1)(p - 5)$.

CHAPTER 7

33 The value (V) of a computer, t years after being purchased, can be modelled by the exponential equation $V = 3000(1.47)^{-t}$.

a Copy and complete this table of values to graph the equation. Use a scale of 1 cm = 1 year on the horizontal axis and 1 cm = $500 on the vertical axis.

t	0	1	2	3	5	7
V						

b What was the purchase price of the computer?

c What is the value of the computer after 4 years?

d Use guess-and-check to find when the value of the computer will be less than $100.

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11

NUMBER AND ALGEBRA

SIMULTANEOUS EQUATIONS

OPTIONAL PATH TOPIC
RECOMMENDED FOR STAGE 6 MATHEMATICS ADVANCED

Many scientific, natural, economic and social phenomena can be modelled by equations. Often these models consist of more than one equation. For example, when manufacturing milk, equations can be written that describe relationships between quantity, cost and income. These equations can then be solved simultaneously to obtain information about pricing and the quantities that need to be produced and sold to make the most profit.

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Chapter outline

	Working mathematically				
11.01 Solving simultaneous equations graphically*	U	F		R	C
11.02 The elimination method*	U	F			
11.03 The substitution method*	U	F			
11.04 Problems involving simultaneous equations*	U	F	PS	R	C

*PATH

Wordbank

coefficient The numerical part of an algebraic term. For example, in $3x^2 + 7x - 1$ the coefficient of x is 7

elimination method A method of solving simultaneous equations that involves combining them to eliminate one of the variables

Quiz
Wordbank 11

graphical method A method of solving simultaneous equations that involves graphing them on a number plane and identifying the point(s) of intersection

simultaneous equations 2 (or more) equations that must be solved together so that the solution satisfies both equations. For example, $y = 2x + 1$ and $y = 3x$ are simultaneous equations that have a solution of $x = 1$, $y = 3$

substitution method A method of solving simultaneous equations that involves substituting one equation into another equation

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Videos (3):

11.02, 11.03 Simultaneous equations 1 • Simultaneous equations 2

11.04 Simultaneous equations problems

***Twig* video (1):**

11.04 The chase

***PhET* interactive (1):**

11.02 Equality explorer: Two variables

Quizzes (2):

- Wordbank 11
- SkillCheck 11

Worksheets (3):

11.01 Testing simultaneous equations • Intersection of lines

11.04 Simultaneous equations problems

Puzzles (2):

11.03 Simultaneous equations order activity • Simultaneous equations by substitution

Technology (1):

11.01 Simultaneous equations solver

Spreadsheet (1):

11.01 Simultaneous equations solver

Presentation (1):

11.04 Simultaneous equations

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

PATH

In this chapter you will:

- (PATH) solve linear simultaneous equations graphically, including using graphing technology
- (PATH) solve linear simultaneous equations algebraically using the elimination and substitution methods
- (PATH) solve problems using linear simultaneous equations

Quiz
SkillCheck 11

SkillCheck ANSWERS ON P. 632

1 Given the equation $y = 2x + 5$, find y when:

a $x = 0$ **b** $x = 4$ **c** $x = \frac{1}{2}$ **d** $x = -3$

2 Given the equation $y = 4 - 3x$, find y when:

a $x = 5$ **b** $x = 1$ **c** $x = -1$ **d** $x = -\frac{1}{2}$

3 By completing a table of values, graph each equation.

a $y = x + 1$ **b** $y = 3x$ **c** $y = \frac{x}{2} - 1$

d $y = 3 - x$ **e** $x + y = 4$ **f** $2x - y = 5$

4 Test whether the point $(-2, 3)$ lies on the line represented by each equation.

a $y = 1 - x$ **b** $x + y = 3$ **c** $2x - y = 7$

d $\frac{1}{2}x + y = 2$ **e** $y = 3x + 7$ **f** $2y = 3x$

5 **a** Show that the point (2, 5) lies on both the lines $y = 2x + 1$ and $x + y = 7$.

b At what point do these 2 lines intersect?

6 Use the y-intercept and the gradient to graph each equation.

a $y = -2x + 3$ **b** $y = \frac{5}{2}x - 2$ **c** $y = -\frac{4}{3}x + 5$

7 Expand each expression.

a $3(2x + 4)$ **b** $-(x - 5)$ **c** $5(x + 2y - 3)$

8 Solve each equation.

a $6x = 2x + 8$ **b** $5x + 4 = 2x - 7$ **c** $2x + 3(x - 4) = 3$

PATH



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Solving simultaneous equations graphically

11.01

A linear equation such as $3x + 5 = 11$ is in one variable (x) and has only one solution ($x = 2$). However, a linear equation in **2** variables, such as $x + 3y = 5$, has more than one solution (for example, $x = 2$, $y = 1$, or $x = 5$, $y = 0$, and so on). The equation actually has an **infinite number** of solutions.

We will now look at solving 2 equations simultaneously to see if there is a solution that satisfies **both** equations. These are called **simultaneous equations**, which can be solved **graphically** or **algebraically**.

PATH

Worksheets
Testing simultaneous equations

Intersection of lines

Technology
Solving simultaneous equations

Spreadsheet
Simultaneous equations solver

Solving simultaneous equations graphically

- Linear simultaneous equations can be graphed as lines on the same number plane.
- If 2 lines are drawn, the lines will intersect (unless they are parallel).
- At the point of intersection, the x-coordinate and y-coordinate represent the solution to the simultaneous equations.

PATH

Example 1

On the same set of axes, graph $3x + y = 4$ and $x + y = -2$, then solve the equations simultaneously.

Solution

Step 1

Construct tables of values.

$3x + y = 4$

x	0	1	2
y	4	1	−2

$x + y = -2$

x	0	1	2
y	−2	−3	−4

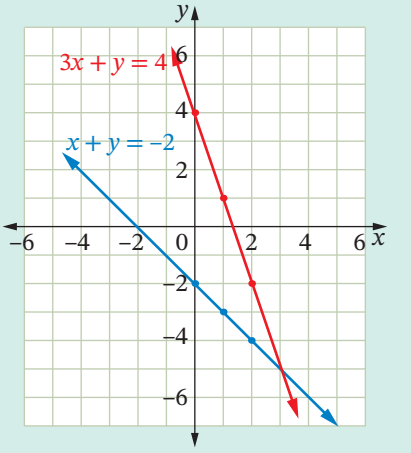
Step 2

Graph the equations.

The lines intersect at $(3, -5)$.

$\therefore$ The solution of the simultaneous equations $3x + y = 4$ and $x + y = -2$ is $x = 3, y = -5$.

Check that $x = 3, y = -5$ satisfies both equations.



EXERCISE 11.01 ANSWERS ON P. 634

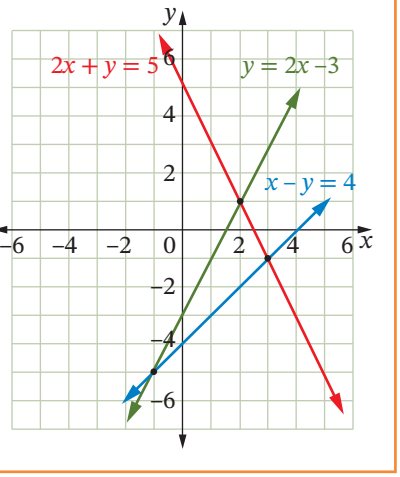
Solving simultaneous equations graphically U F R C

1 Use the graph to write the solution to each pair of simultaneous equations. C

a $x - y = 4$ and $2x + y = 5$

b $2x + y = 5$ and $y = 2x - 3$

c $x - y = 4$ and $y = 2x - 3$



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2 Graph each pair of equations on the same set of axes. Then find the solution to the pair of simultaneous equations.

a $y = 2x$ and $y = 3 - x$
b $y = 2x + 1$ and $y = x - 4$
c $x + y = 3$ and $4x + y = 6$
d $y = -x + 2$ and $y = 3x + 4$
e $y = 2x - 5$ and $y = 5x + 1$
f $2x + y = 6$ and $y = 1 - x$
g $y = 7 - x$ and $y = 3x + 5$
h $x + 2y = 7$ and $2x - y = 4$
i $3x - 2y = 12$ and $x + 2y = 8$
j $y = x + 3$ and $2x - y = 2$
k $5x - y = 5$ and $x + y = 4$
l $5x + 3y = 20$ and $y = x - 4$

3 **a** On the same set of axes, draw the graphs of $y = 1 - 2x$ and $2x + y = 4$.
b Why isn't there a solution to the simultaneous equations $y = 1 - 2x$ and $2x + y = 4$? R C

4 **a** On the same set of axes, draw the graphs of $x + y = 3$ and $2x + 2y = 6$.
b Why are there an infinite number of solutions to the simultaneous equations $x + y = 3$ and $2x + 2y = 6$? R C

PATH
EXAMPLE 1

The elimination method 11.02

Using graphs to solve simultaneous equations can be time-consuming and inaccurate.

Algebraic methods provide a better way of solving them.

There are 2 algebraic methods: the **elimination method** and the **substitution method**.

In the elimination method, equations are added or subtracted to eliminate one of the variables.

PATH

Interactive
Equality explorer: Two variables

Video
Simultaneous equations 1

Example 2

Solve the simultaneous equations $x + 3y = 7$ and $4x - 3y = 13$ using the elimination method.

Solution

Label each equation.

$x + 3y = 7$ [1]

$4x - 3y = 13$ [2]

Since there is the same number of ys in each equation, and since they are **opposite in sign** ($3y$ and $-3y$), add equations [1] and [2] to eliminate the variable y.

$5x = 20$ [1] + [2]

$\frac{5x}{5} = \frac{20}{5}$

$\therefore x = 4$

Substitute $x = 4$ into equation [1] to find the y-value.

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PATH

$x + 3y = 7$

$4 + 3y = 7$

$4 + 3y - 4 = 7 - 4$

$3y = 3$

$\therefore y = 1$

$\therefore$ The solution is $x = 4, y = 1$.

Check that this solution works for both equations [1] and [2].

Example 3

Solve the simultaneous equations $2k + 3m = 9$ and $2k - 5m = 1$.

Solution

Label each equation.

$2k + 3m = 9$ [1]

$2k - 5m = 1$ [2]

Since there is the same number of ks in each equation, and because **they have the same sign** ($2k$ and $2k$), subtract equation [2] from [1] to eliminate k.

$8m = 8$ [1] – [2]

$\frac{8m}{8} = \frac{8}{8}$

$\therefore m = 1$

Substitute $m = 1$ into equation [1] to find the value of k.

$2k + 3m = 9$

$2k + 3 \times 1 = 9$

$2k + 3 = 9$

$2k + 3 - 3 = 9 - 3$

$2k = 6$

$\frac{2k}{2} = \frac{6}{2}$

$\therefore k = 3$

$\therefore$ The solution is $m = 1, k = 3$.

Check that this solution works for both equations [1] and [2].

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Example 4

Solve $3a + 4c = 8$ and $2a - 3c = 11$.

Solution

Label each equation.

$3a + 4c = 8$ [1]

$2a - 3c = 11$ [2]

In this case, neither adding nor subtracting equations [1] and [2] will eliminate a variable. Let's choose to eliminate c. We need to make the **coefficient** of c the same in both equations ($12c$).

The **coefficient** of c is the number in front of the c in the equation

$9a + 12c = 24$ [3] Multiplying both sides of [1] by 3

$8a - 12c = 44$ [4] Multiplying both sides of [2] by 4

$17a = 68$ [3] + [4]

$\therefore a = 4$

Substitute $a = 4$ in [1] to find c.

$3a + 4c = 8$

$3 \times 4 + 4c = 8$

$12 + 4c = 8$

$4c = -4$

$c = -1$

$\therefore$ The solution is $a = 4$, $c = -1$.

PATH

Video
Simultaneous equations 2

11.02

EXAMPLE 2

EXERCISE 11.02 ANSWERS ON P. 634

The elimination method U F

1 For each pair of simultaneous equations, eliminate one variable by adding the equations, then solve the equations.

a $4k + d = 5$
$2k - d = 7$

b $2x - w = 6$
$x + w = 9$

c $3g + 5h = 4$
$2g - 5h = 6$

d $7p - 4n = -20$
$3p + 4n = 10$

e $4q + 3r = 8$
$-q - 3r = 7$

f $-5k - 3x = 8$
$5k + 4x = -3$

g $-4c - 6e = -12$
$4c - 10e = -4$

h $-3y + 5k = 21$
$3y + k = -3$

i $a + 3f = 8$
$-a + 4f = 6$

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PATH

EXAMPLE 3

2 For each pair of simultaneous equations, eliminate one variable by subtracting the equations, then solve the equations.

a	$5k + d = 16$ $3k + d = 4$	**b**	$4a + 3c = 7$ $a + 3c = 4$	**c**	$4h + 3y = 24$ $4h - y = 8$
d	$3x + 5e = 16$ $3x - 2e = -5$	**e**	$4q - 2w = -1$ $7q - 2w = 8$	**f**	$6p + 5c = 39$ $4p + 5c = 31$
g	$5y + 3m = 18$ $2y + 3m = 6$	**h**	$3a + 2r = 8$ $a + 2r = 10$	**i**	$-x + 5w = 8$ $-x + 3w = 4$
j	$4y + 7g = 2$ $4y - 3g = 22$	**k**	$2e - 3n = 14$ $5e - 3n = -1$	**l**	$7k - 5h = 31$ $7k + 3h = 43$

EXAMPLE 4

3 Solve each pair of simultaneous equations.

a	$3w + q = 6$ $2w - 3q = 15$	**b**	$2x + m = 5$ $3x + 2m = 3$	**c**	$2d + 3h = 25$ $d + 4h = -5$
d	$-3g + 2n = 9$ $g + 5n = 14$	**e**	$5m - h = 10$ $m - 3h = 2$	**f**	$2y + 3e = -6$ $5y - 2e = 23$
g	$3q - 2w = 11$ $2q - 5w = 22$	**h**	$5a + 3d = 4$ $4a + 2d = 3$	**i**	$-2p + 3k = 19$ $7p + 4k = 6$
j	$5a + 2f = -14$ $2a - 3f = 2$	**k**	$5r - 3c = 2$ $-3r + 2c = -14$	**l**	$5y - 4x = 1$ $2y - 3x = 6$
m	$3x + 4y = 20$ $2x - 5y = 21$	**n**	$7g + 3h = 39$ $3g + 5h = 26$	**o**	$5w - 3k = 25$ $3w - 7k = 28$

11.03 The substitution method

PATH

With the **substitution method**, **substitute** the x or y variables from one equation into the other equation.

Example 5

Solve the simultaneous equations $y = x + 4$ and $y = 3x - 2$.

Solution

Label each equation.

$y = x + 4$ [1]

$y = 3x - 2$ [2]

Video
Simultaneous equations 1

Puzzles
Simultaneous equations order activity

Simultaneous equations by substitution

Foundation Standard Complex

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Use equation [1] to **substitute for *y*** in equation [2] and solve for x.

$$x + 4 = 3x - 2$$
$$x + 4 - 3x = 3x - 2 - 3x$$
$$-2x + 4 = -2$$
$$-2x + 4 - 4 = -2 - 4$$
$$-2x = -6$$
$$\frac{-2x}{-2} = \frac{-6}{-2}$$
$$x = 3$$

Now substitute $x = 3$ into equation [1] to find y.

$$y = x + 4$$
$$y = 3 + 4$$
$$= 7$$

$\therefore$ The solution is $x = 3$ and $y = 7$.

Example 6

Solve the simultaneous equations $5x + 3y = 9$ and $y = 7 - 3x$.

Solution

Label each equation.

$5x + 3y = 9$ [1]

$y = 7 - 3x$ [2]

Since y is the subject in [2], **substitute equation [2]** into equation [1] to give an equation using x only.

$$5x + 3(7 - 3x) = 9$$
$$5x + 21 - 9x = 9$$
$$-4x = -12$$
$$\frac{-4x}{-4} = \frac{-12}{-4}$$
$$x = 3$$

Now substitute $x = 3$ into equation [2] to find y.

$$y = 7 - 3x$$
$$y = 7 - 3 \times 3$$
$$= -2$$

$\therefore$ The solution is $x = 3$ and $y = -2$.

PATH

11.03

Video
Simultaneous equations 2

PATH

EXERCISE 11.03 ANSWERS ON P. 635

The substitution method U F

EXAMPLE 6

1 Use the substitution method to solve each pair of simultaneous equations.

a $y = 2x + 1$ and $y = x + 3$

b $y = 5 - 2x$ and $y = 3x + 2$

c $x = 3 + 2y$ and $x = 9 - y$

d $y = -x$ and $y = 3x - 8$

e $x = 1 - 4y$ and $x = 2y + 7$

f $x = 2y$ and $x = 6 - y$

EXAMPLE 6

2 Solve each pair of simultaneous equations.

a $y = 2x + 3$ and $3x - y = 6$

b $y = x - 2$ and $3x + y = 18$

c $y = 1 - 4x$ and $4x + 2y = 3$

d $x = 2y - 5$ and $4x - y = -13$

e $x = 3y - 4$ and $5x - 4y = 2$

f $x = 5 - 3y$ and $4y - x = 23$

g $2x - 5y = -1$ and $y = 10 - x$

h $6y - 2x = 9$ and $y = \frac{x+2}{2}$

i $x = \frac{9-y}{3}$ and $3x + 2y = 10$

j $y = 3x + 5$ and $4x - 3y = 1$

11.04 Problems involving simultaneous equations

PATH

Video
The chase

Worksheet
Simultaneous equations problems

Presentation
Simultaneous equations

Sometimes, worded problems can be solved using simultaneous equations. In these situations, following these steps:

- Read the problem carefully
- Identify the variables to be used
- Use the variables to write simultaneous equations from the information given in the problem
- Solve the equations
- Solve the problem by answering in words

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Example 7

At an art show there were 520 guests. If there were 46 more women than men, how many women attended the show?

Solution

Let the number of women attending be w.

Let the number of men attending be m.

$w + m = 520$ [1] 520 people altogether.

$w = m + 46$ [2] 46 more women than men.

Use equation [2] to substitute for w in equation [1].

$m + 46 + m = 520$

$2m + 46 = 520$

$2m + 46 - 46 = 520 - 46$

$2m = 474$

$m = 237$

Substitute $m = 237$ into equation [2] to find w.

$w = 237 + 46$

$= 283$

$\therefore$ There were 283 women who attended the art show.



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PATH

Video
Simultaneous equations problems

Example 8

Masi and Amir spent $1582 on shrubs and trees to plant on their large block of land. Altogether they bought 73 plants. The shrubs cost $19 each while the trees cost $32 each. How many of each plant did they buy?

Solution

Let x be the number of shrubs.

Let y be the number of trees.

$\therefore x + y = 73$ [1]

and $19x + 32y = 1582$ [2]

Neither adding nor subtracting equations [1] and [2] will eliminate a variable.
Let's choose to eliminate x.

We will need to make the coefficient of x the same in both equations ($19x$).

$19x + 32y = 1582$ [2]

$19x + 19y = 1387$ [3] Multiplying both sides of equation [1] by 19.

$13y = 195$ [2] – [3]

$y = 15$

Substitute $y = 15$ in [1] to find the value of x.

$x + y = 73$

$x + 15 = 73$

$x = 58$

So Masi and Amir bought 58 shrubs and 15 trees.

EXERCISE 11.04 ANSWERS ON P. 635

Problems involving simultaneous equations U F PS R C

EXAMPLE 7

1 At a school concert there were 640 guests. There were 70 more women than men. How many of the audience were men? R C

2 At a circus, there were twice as many children as there were adults in attendance. Altogether, 1020 attended the circus. How many were children? R C

EXAMPLE 8

3 Tickets to a concert cost $15 for children and $28 for adults. Altogether, 650 people attended the concert and ticket sales totalled $13 052. Let a stand for the number of adults and c stand for the number of children in the audience. R C

a Explain why the equations $a + c = 650$ and $28a + 15c = 13\,052$ correctly match the information.

b Solve the equations simultaneously to find the number of children that attended the concert.

Foundation Standard Complex

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4 Ashleigh bought a total of 130 meat pies and sausage rolls for the canteen of the local football club. Each meat pie cost her $3 and the sausage rolls were $2 each. Altogether, Ashleigh spent $335. How many sausage rolls did she buy? PS R C

5 Tayyab is 3 times as old as Sejuti. The sum of their ages is 48. How old are Tayyab and Sejuti? PS R C

6 The sum of the ages of Hayley and her mother is 70. The difference between their ages is 38 years. How old is Hayley? PS R C

7 A business bought a total of 60 ink cartridges. Some of them were black, costing $35 each. The others were colour, each costing $49. How many of each type did the business buy if the total cost of the ink cartridges was $2422? PS R C

8 The cost of going to the movies for 3 adults and 5 children is $142, while the cost for 5 adults and 8 children is $231.50. Find the cost for an adult and the cost for a child. PS R C

9 Pete's Pizzas sells Supreme pizzas for $15.90 each and vegetarian pizzas for $13.50 each. If 45 pizzas were sold at lunchtime, totalling $684.30, how many of each pizza were sold? PS R C

10 Ronel bought 4 punnets of strawberries and 7 punnets of blueberries for $40.30 and Nikita bought 5 punnets of strawberries and 2 punnets of blueberries for $26.75. What was the cost of each punnet of strawberries and blueberries? PS R C

11 A money box contains only 20-cent coins and 50-cent coins. Altogether, there are 853 coins in the money box and they amount to $281. Let x be the number of 20c coins and y be the number of $2 coins. R C

a Explain why the equations $x + y = 853$ and $20x + 50y = 28\,100$ correctly match the information.

b Solve the equations to determine the number of 20-cent and 50-cent coins in the money box.

12 The initial cost for producing bottles of fresh orange juice is $135 plus $1.20 for each bottle. The bottles of juice are sold for $3 each. C is the cost in dollars, R is the total sales in dollars and n is the number of bottles produced and/or sold. PS R C

a Explain why the equations $C = 135 + 1.2n$ and $R = 3n$ correctly match the information.

b Copy and complete the tables of values below for both equations.

$C = 135 + 1.2n$

n	0	50	100
C			

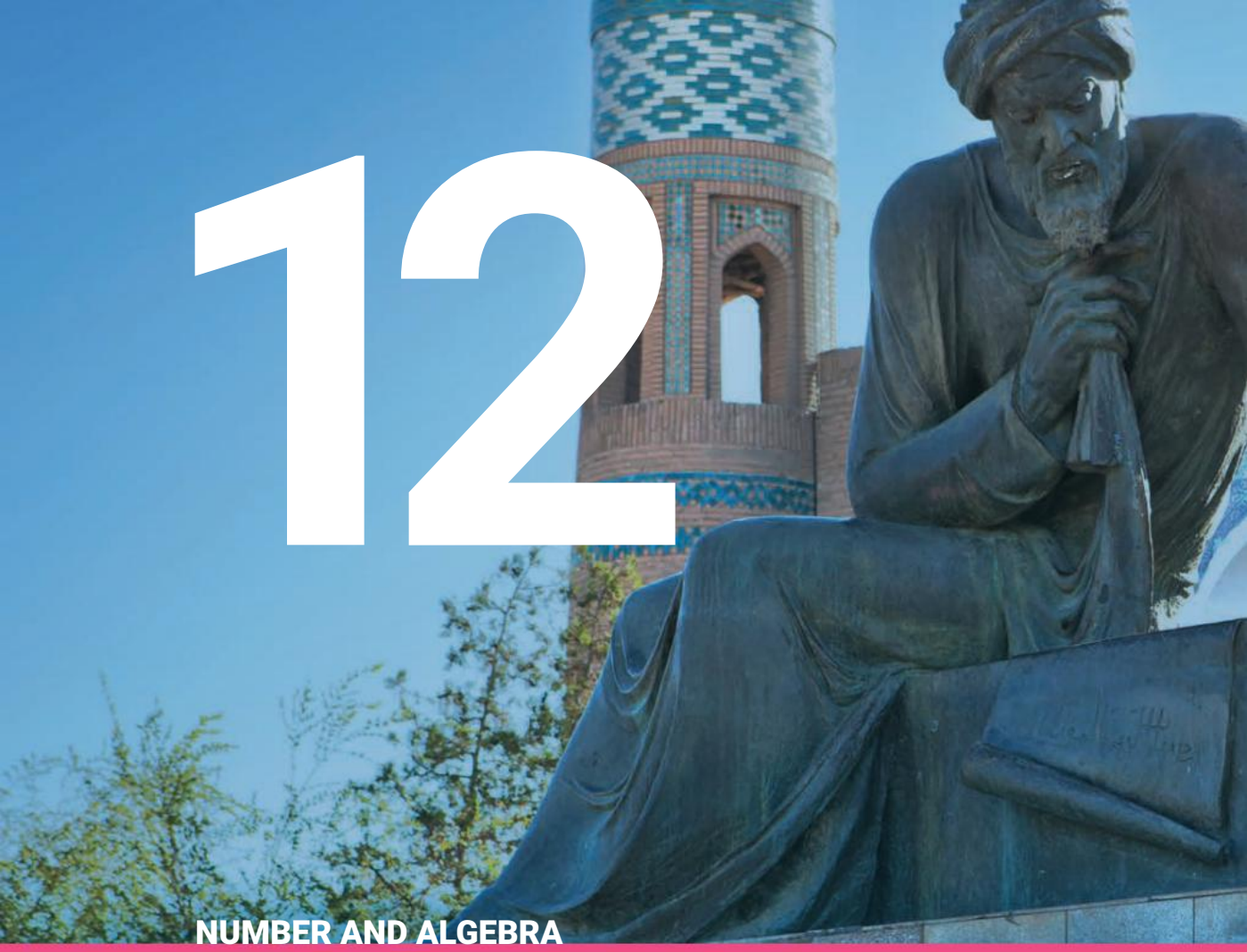
$R = 3n$

n	0	50	100
R			

c Draw the graphs of both equations on the same axes for values of 0 to 100 for n on the horizontal axis and values of $0 to $300 on the vertical axis.

d For what value of n is total sales equal to total cost (the break-even point)?

Foundation Standard Complex



12

NUMBER AND ALGEBRA

PRODUCTS AND FACTORS

OPTIONAL PATH TOPIC
RECOMMENDED FOR STAGE 6 MATHEMATICS ADVANCED

In 825 CE, the Persian mathematician al-Khwarizmi used the Arabic word ‘al-jabr’ to describe the process of adding equal quantities to both sides of an equation. When al-Khwarizmi’s book was translated into Latin and introduced to Europe, ‘al-jabr’ became ‘algebra’ and the word was adopted as the name for the branch of mathematics that uses formulas to describe patterns and relationships in our world. The photo above shows a monument of al-Khwarizmi located in Khiva, Uzbekistan.



Chapter outline

	Working mathematically				
12.01 Perfect squares*	U	F			
12.02 Difference of 2 squares*	U	F		R	C
12.03 Mixed expansions*	U	F		R	C
12.04 Factorising special binomial products*	U	F			
12.05 Factorising quadratic expressions $ax^2 + bx + c$*	U	F		R	
12.06 Mixed factorisations*	U	F		R	
12.07 Factorising algebraic fractions*	U	F		R	

*PATH

Wordbank

binomial An algebraic expression that consists of 2 terms, for example, $4a + 9$, $3 - y$, $x^2 - 4x$

binomial product An algebraic expression showing 2 or more binomials multiplied together, for example, $(x + 9)(3x - 4)$

factorise To rewrite an expression with grouping symbols, by taking out the highest common factor; factorising is the opposite of expanding, for example, $9r^2 + 36r$ factorised is $9r(r + 4)$

highest common factor (HCF) The largest term that is a factor of 2 or more terms, for example, the HCF of $9r^2 + 36r$ is $9r$

perfect square A square number or an algebraic expression that represents one, for example, 64, $(x + 9)^2$

quadratic expression An algebraic expression in which the highest power of the variable is 2, for example, $2x^2 + 5x - 3$ or $x^2 + 2$

quadratic trinomial An algebraic expression that consists of 3 terms, for example, $x^2 + 2x + 6$

Quiz
Wordbank 12

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Videos (6):

12.01 Special binomial products
12.02 Difference of 2 squares • Special binomial products
12.04 Difference of 2 squares
12.05 Factorising quadratic expressions (Advanced) • Factorising quadratic expressions 2 • Advanced algebra

Quizzes (2):

- Wordbank 12
- SkillCheck 12

Worksheets (4):

12.02 Special products
12.03 Mixed expansions • Algebra 6
12.07 Simplifying algebraic fractions

Puzzles (10):

12.01 Factorominoes
12.02 Enough time
12.03 Products and factors squaresaw • Expanding brackets
12.04 Grouping • Difference of 2 perfect squares
12.05 Products and factors squaresaw • Perfect squares • Factorominoes • Factorising puzzle (Advanced) • Trinominoes
12.06 Mixed factorisations • Factorominoes • Products and factors squaresaw

Technology (1):

12.05 Factorising trinomials

Spreadsheet (1):

12.05 Factorising trinomials

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

In this chapter you will:

- (PATH) recognise and expand special binomial products that are perfect squares or the difference of 2 squares
- (PATH) factorise an expression with 4 terms by grouping in pairs
- (PATH) factorise non-monic quadratic expressions, including those that are perfect squares or the difference of 2 squares
- (PATH) factorise and simplify algebraic fractions

PATH

Quiz
SkillCheck 12

SkillCheck ANSWERS ON P. 635

1 Expand each binomial product.

a $(x + 1)(x + 7)$
b $(d + 3)(d - 4)$
c $(t - 5)(t + 5)$
d $(y + 2)(5y - 6)$
e $(7y - 3)(7y - 3)$
f $(5p + 4)(2p - 7)$

2 Factorise each quadratic expression.

a $y^2 + 10y + 25$
b $x^2 - 21x + 20$
c $n^2 + 8n - 33$
d $a^2 - 11a + 28$
e $m^2 - 5m - 84$
f $p^2 + 3p - 54$

Perfect squares

12.01

We know how to expand **binomial products** such as $(k + 3)(k - 7)$. Now we will examine the special binomial product where a binomial is multiplied by itself, for example,

$(x + 4)(x + 4) = (x + 4)^2$ or $(2a - 9)(2a - 9) = (2a - 9)^2$.

16, 49, v^2 and $(y + 5)^2$ are called **perfect squares** because they are square numbers.

PATH



Puzzle
Factorominoes

Perfect squares

The formulas for expanding the perfect square of a binomial are:

$$(a + b)^2 = a^2 + 2ab + b^2$$
$$(a - b)^2 = a^2 - 2ab + b^2$$

Proof

$(a + b)^2 = (a + b)(a + b)$
$= a(a + b) + b(a + b)$
$= a^2 + ab + ba + b^2$
$= a^2 + 2ab + b^2$

$(a - b)^2 = (a - b)(a - b)$
$= a(a - b) - b(a - b)$
$= a^2 - ab - ba + b^2$
$= a^2 - 2ab + b^2$

Example 1

Copy and complete the expansion of each perfect square.

a $(x + 7)^2 = x^2 + ____ + 49$

b $(y - 6)^2 = y^2 - 12y + ____$

c $(5g + 9)^2 = ____ + 90g + 81$

d $(3d - 5)^2 = 9d^2 - ____ + 25$

Solution

a In the expansion,

$_____ = 2 \times x \times 7$ Doubling the product of the 2 terms
$= 14x$
$\therefore (x + 7)^2 = x^2 + 14x + 49$

b In the expansion,

$_____ = 6^2$ The second term squared
$= 36$
$\therefore (y - 6)^2 = y^2 - 12y + 36$

c $_____ = (5g)^2$ The first term squared
$= 25g^2$
$\therefore (5g + 9)^2 = 25g^2 + 90g + 81$

d $_____ = 2 \times 3d \times 5$ Doubling the product of the 2 terms
$= 30d$
$\therefore (3d - 5)^2 = 9d^2 - 30d + 25$

PATH

Video
Special binomial products

Example 2

Expand each perfect square.

a $(n-5)^2$ **b** $(k+4)^2$ **c** $(3y-8)^2$

Solution

a $(n-5)^2 = n^2 - 2 \times n \times 5 + 5^2$
$= n^2 - 10n + 25$

1st term squared – double product + 2nd term squared

b $(k+4)^2 = k^2 + 2 \times k \times 4 + 4^2$
$= k^2 + 8k + 16$

c $(3y-8)^2 = (3y)^2 - 2 \times 3y \times 8 + 8^2$
$= 9y^2 - 48y + 64$

PATH

EXERCISE 12.01 ANSWERS ON P. 635

Perfect squares U F

EXAMPLE 1

1 Copy and complete the expansion of each perfect square.

a $(x+10)^2 = x^2 + _____ + 100$
b $(m-8)^2 = _____ - 16m + 64$
c $(p-t)^2 = p^2 - 2pt + _____$
d $(h+4)^2 = h^2 _____ + 16$
e $(k-9)^2 = k^2 _____ + 81$
f $(8+5f)^2 = 64 _____ + 25f^2$
g $(2d+3)^2 = _____ + _____ + 9$
h $(6a+1)^2 = _____ + 12a + _____$

EXAMPLE 2

2 Expand each perfect square.

a $(m+9)^2$ **b** $(u+3)^2$ **c** $(y-6)^2$
d $(8+k)^2$ **e** $(5-h)^2$ **f** $(7+k)^2$
g $(f+20)^2$ **h** $(q-11)^2$ **i** $(10+t)^2$
j $(x-w)^2$ **k** $(a+g)^2$ **l** $(2m-3)^2$
m $(5x-6)^2$ **n** $(9a+2)^2$ **o** $(3e-4)^2$
p $(5+7b)^2$ **q** $(4-5p)^2$ **r** $(11-2c)^2$
s $(10g+3)^2$ **t** $(3k+11)^2$ **u** $(5+2v)^2$

3 Expand each perfect square.

a $(7h+2k)^2$ **b** $(8a-3y)^2$ **c** $(xy+z)^2$
d $\left(1+\frac{1}{y}\right)^2$ **e** $\left(k-\frac{1}{k}\right)^2$ **f** $\left(w+\frac{3}{w}\right)^2$

4 Use expansion to evaluate each square number without using a calculator.

a $21^2 = (20+1)^2$ **b** $45^2 = (40+5)^2$ **c** $29^2 = (30-1)^2$
d $59^2 = (60-1)^2$ **e** $102^2 = (100+2)^2$ **f** $98^2 = (100-2)^2$

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Difference of 2 squares

12.02

The binomial product $(a + b)(a - b)$ has a special pattern when expanded.

Difference of 2 squares

$$(a + b)(a - b) = a^2 - b^2$$

The answer is called the **difference of 2 squares**.

Proof:

$$\begin{aligned}(a - b)(a + b) &= a(a + b) - b(a + b) \\ &= a^2 + ab - ba - b^2 \\ &= a^2 - b^2 \qquad + ab - ba = 0\end{aligned}$$

When the sum of 2 terms is multiplied by their difference, the answer is the square of the first term minus the square of the second term (the difference of 2 squares).

Example 3

Expand each expression.

a $(d + 3)(d - 3)$

b $(2 + r)(2 - r)$

c $(7x + 2)(7x - 2)$

d $(4k - 5p)(4k + 5p)$

Solution

a $(d + 3)(d - 3) = d^2 - 3^2$
$= d^2 - 9$

b $(2 + r)(2 - r) = 2^2 - r^2$
$= 4 - r^2$

c $(7x + 2)(7x - 2) = (7x)^2 - 2^2$
$= 49x^2 - 4$

d $(4k - 5p)(4k + 5p) = (4k)^2 - (5p)^2$
$= 16k^2 - 25p^2$

PATH

12.02

Worksheet
Special products

Puzzle
Enough time

Videos
Difference of 2 squares

Special binomial products

PATH

EXERCISE 12.02 ANSWERS ON P. 635

Difference of 2 squares U F R C

EXAMPLE 3

1 Expand each expression.

a	$(m + 5)(m - 5)$	**b**	$(c - 10)(c + 10)$	**c**	$(a + 12)(a - 12)$
d	$(6 - y)(6 + y)$	**e**	$(8 - m)(8 + m)$	**f**	$(p + 1)(p - 1)$
g	$(5 + e)(5 - e)$	**h**	$(v + 11)(v - 11)$	**i**	$(w - 3)(w + 3)$
j	$(x - 10)(x + 10)$	**k**	$(q + 7)(q - 7)$	**l**	$(9 - g)(9 + g)$
m	$(b - 2)(b + 2)$	**n**	$(15 - r)(15 + r)$	**o**	$(d + 13)(d - 13)$

2 Expand each expression.

a	$(2h - 3)(2h + 3)$	**b**	$(5r + 4)(5r - 4)$	**c**	$(5b + 8)(5b - 8)$
d	$(4p - 7)(4p + 7)$	**e**	$(3 - 8k)(3 + 8k)$	**f**	$(7x - 5)(7x + 5)$
g	$(2 + 9m)(2 - 9m)$	**h**	$(9k - 4l)(9k + 4l)$	**i**	$(7n + 8m)(7n - 8m)$
j	$(4g - 5h)(4g + 5h)$	**k**	$(7u + 3w)(7u - 3w)$	**l**	$(11a + 3b)(11a - 3b)$
m	$\left(a+\frac{1}{a}\right)\left(a-\frac{1}{a}\right)$	**n**	$\left(\frac{w}{3}-2\right)\left(\frac{w}{3}+2\right)$	**o**	$\left(1-\frac{1}{r}\right)\left(1+\frac{1}{r}\right)$

3 Natalie's age is p years. R C

a What was Natalie's age last year?

b What will Natalie's age be next year?

c Write an expression for (Natalie's age last year) × (Natalie's age next year).

d If (Natalie's age last year) × (Natalie's age next year) is equal to 48, what is Natalie's age?

4 By expressing 31×29 as $(30 + 1)(30 - 1)$, use the difference of 2 squares to find the value of 31×29. R C

5 Use the method of question **4** to evaluate each expression. R C

a 21×19 **b** 51×49 **c** 89×91 **d** 78×82

6 Use 'sum by difference' to evaluate each expression.

a	$15^2 - 14^2$	**b**	$24^2 - 23^2$	**c**	$65^2 - 35^2$
d	$101^2 - 99^2$	**e**	$23^2 - 17^2$	**f**	$50^2 - 48^2$

Foundation Standard Complex

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Mixed expansions 12.03

Example 4

Expand and simplify each expression.

a $(4r + 5)(1 - 2r)$ **b** $(7 + 9x)^2$ **c** $(3d - 10)(3d + 10)$

d $(a + 6)(a - 6) + (a + 12)(a + 3)$ **e** $(m - 2)^2 - (m - 2)(m + 2)$

Solution

a $(4r + 5)(1 - 2r) = 4r(1 - 2r) + 5(1 - 2r)$ Expanding

$= 4r - 8r^2 + 5 - 10r$

$= -8r^2 - 6r + 5$ Rearranging and putting the higher order terms first.

b $(7 + 9x)^2 = 7^2 + 2 \times 7 \times 9x + (9x)^2$ Expand the perfect square.

$= 49 + 126x + 81x^2$

$= 81x^2 + 126x + 49$ Rearranging and putting the higher order terms first.

c $(3d - 10)(3d + 10) = (3d)^2 - 10^2$ Expand into the difference of 2 squares

$= 9d^2 - 100$

d $(a + 6)(a - 6) + (a + 12)(a + 3) = a^2 - 36 + a^2 + 3a + 12a + 36$

$= 2a^2 + 15a$

e $(m - 2)^2 - (m - 2)(m + 2) = m^2 - 4m + 4 - (m^2 - 4)$

$= m^2 - 4m + 4 - m^2 + 4$

$= -4m + 8$

PATH

Worksheets
Mixed expansions
Algebra 6

Puzzles
Products and factors squaresaw
Expanding brackets

12.03

EXERCISE 12.03 ANSWERS ON P. 636

Mixed expansions U F R C

1 Expand and simplify each expression.

EXAMPLE 4

a $(2m - 1)(2m + 1)$ **b** $(y + 4)(5y - 3)$ **c** $(2k - 7)^2$

d $(d + 9)^2$ **e** $(2e - 1)(e + 1)$ **f** $(5a + 4)(5a - 4)$

g $(2 - p)(p - 2)$ **h** $(10 - 6y)(10 + 6y)$ **i** $(h - 3m)^2$

j $(2x - 3)(y + 3)$ **k** $(11a - 4b)(11a + 4b)$ **l** $\left(u - \frac{1}{u}\right)^2$

Foundation Standard Complex

PATH

2 Expand and simplify each expression. R C

a $(m-5)(m+5)+25$ **b** $6y+(y-3)^2+9$

c $(3x+1)(2-x)+2x+4$ **d** $(d+4)^2-8d+5$

e $16+(4k-8)(4k+8)$ **f** $(x-y)^2-(x+y)^2$

g $20a-(a-2)(a-5)+a^2$ **h** $2(f-2)(f+2)$

i $(2h+3)^2-(2h-3)(2h+3)$ **j** $7xy-(2x-3)(y+3)$

3 Expand and simplify each expression. R C

a $(8a-1)(8a+1)-4a^2+1$ **b** $(n+1)^2+2n+3$

c $3(4-u)(4+u)+(u-12)(u+4)$ **d** $(2m-n)^2+(2m+n)^2$

e $(x-2)(x+3)-(x-2)(x+2)$ **f** $2(b-1)^2-(2b-1)^2$

g $(y+1)^2+(y+2)^2+(y+3)^2$ **h** $(x-3)(x+3)+(x+3)^2+(x-3)^2$

i $(5n+3)(5n-3)+(3n-5)(3n+5)$ **j** $2(a-b)(a+b)-(a+b)^2-(a-b)^2$

12.04 Factorising special binomial products

PATH

Factorising by grouping in pairs

Puzzle
Grouping

An **algebraic expression with 4 terms** can often be factorised in pairs, that is, 2 terms at a time, to make a binomial product.

Example 5

Factorise each expression.

a $3ac+2bd+2bc+3ad$

b $4km+6mn-6kp-9np$

c $10xw-6yw-10xp+6yp$

Solution

a $3ac+2bd+2bc+3ad=3ac+3ad+2bd+2bc$ Grouping into pairs for factorising

$=3a(c+d)+2b(d+c)$ Factorising each pair

$=(c+d)(3a+2b)$ Factorising again

b $4km+6mn-6kp-9np=2m(2k+3n)-3p(2k+3n)$ Factorising each pair

$=(2k+3n)(2m-3p)$ Factorising again

c $10xw-6yw-10xp+6yp=2(5xw-3yw-5xp+3yp)$ Factorising all terms first

$=2[w(5x-3y)-p(5x-3y)]$ Factorising each pair

$=2(5x-3y)(w-p)$ Factorising again

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Factorising the difference of 2 squares

We have learned the formula $(a + b)(a - b) = a^2 - b^2$.

If we use this rule in reverse, then the factors of $a^2 - b^2$ are $(a + b)$ and $(a - b)$.

The difference of 2 squares

$$a^2 - b^2 = (a + b)(a - b)$$

Example 6

Factorise each expression.

a $x^2 - 4$ **b** $9 - 16b^2$ **c** $20d^2 - 5a^2$ **d** $y^3 - y$

Solution

a $x^2 - 4 = x^2 - 2^2$
$= (x + 2)(x - 2)$

b $9 - 16b^2 = 3^2 - (4b)^2$
$= (3 + 4b)(3 - 4b)$

c $20d^2 - 5a^2 = 5(4d^2 - a^2)$
$= 5[(2d)^2 - a^2]$
$= 5(2d + a)(2d - a)$

d $y^3 - y = y(y^2 - 1)$
$= y(y + 1)(y - 1)$

EXERCISE 12.04 ANSWERS ON P. 636

Factorising special binomial products U F

1 Factorise each expression.

a $4ab + 5bc + 4ad + 5cd$
b $2xy - 5wy + 2xt - 5wt$
c $9ac + 6bc + 12ad + 8bd$
d $10x^2 + 30 + x^3 + 3x$
e $3a^2 + 3ab + 3ac + 3bc$
f $6rt - 18wt + 6rp - 18wp$
g $14e - 21 + 2de - 3d$
h $hk - h^2 - 2k + 2h$
i $3mn - 6m + pn - 2p$
j $9p^2 - 27 + qp^2 - 3q$
k $fg - fh - 10g + 10h$
l $9kl - 12ml + 9kn - 12mn$
m $2p - 2c - p^2 + pc$
n $l^3 + lm^2 - 3l^2 - 3m^2$
o $a(x + 1) + y(x + 1) - ka - ky$
p $p(a - b) - 2q(a - b) + 3qp - 6q^2$

2 Factorise each expression.

a $w^2 - 9$ **b** $y^2 - 36$ **c** $k^2 - 1$ **d** $m^2 - 121$
e $p^2 - 64$ **f** $c^2 - 100$ **g** $4e^2 - f^2$ **h** $a^2 - 9b^2$
i $16y^2 - 1$ **j** $4 - b^2$ **k** $25 - e^2$ **l** $1 - 16x^2$
m $k^2 - u^2$ **n** $49 - 16m^2$ **o** $b^2 - 121d^2$ **p** $36c^2 - 25k^2$
q $16 - 81h^2$ **r** $25a^2 - 64m^2$ **s** $100 - 49n^2$ **t** $121p^2 - 144q^2$
u $\frac{1}{4} - 25c^2$ **v** $4w^2 - \frac{1}{9}$ **w** $64h^2 - 2\frac{1}{4}$ **x** $1 - m^2 n^2$

PATH

Puzzle
Difference of 2 perfect squares

12.04

Video
Difference of 2 squares

EXAMPLE 5

EXAMPLE 6

Foundation Standard Complex

PATH

3 Factorise each expression.

a $2a^2 - 2b^2$	**b** $7k^2 - 28$	**c** $3 - 75u^2$	**d** $x^3 - 49x$
e $k - 16k^3$	**f** $50q^2 - 2$	**g** $3d^2 - 12v^2$	**h** $5t^5 - 125t^3$
i $2a^2b^2 - 2$	**j** $x^2y^2 - x^2w^2$	**k** $192f^2 - 108g^2$	**l** $45d^2 - \frac{5}{4}$
m $2x^2 - 8a^2$	**n** $100 - 25w^2$	**o** $1\frac{1}{4} - 80e^2$	**p** $9c^2 - 6\frac{1}{4}$

4 Factorise each expression.

a $\frac{p^2}{16} - \frac{x^2}{25}$	**b** $x^2 - \frac{1}{9}$	**c** $\frac{v^2}{64} - \frac{u^2}{81}$
d $\frac{2y^2}{9} - \frac{2m^2}{121}$	**e** $\frac{16a^2}{49} - \frac{25b^2}{4}$	**f** $g^4 - 81$
g $10\,000 - n^4$	**h** $(x + y)^2 - x^2$	**i** $(p - 2q)^2 - (2p + q)^2$
j $\frac{x^2}{4} - \frac{y^2}{36}$	**k** $(a + b)^2 - (a - b)^2$	**l** $25 - (4 - x)^2$

12.05 Factorising quadratic expressions $ax^2 + bx + c$

PATH

In Chapter 4, Algebra, we factorised **quadratic trinomials** of the type $x^2 + bx + c$. For example, the factorisation of $x^2 + 6x + 8$ is $(x + 2)(x + 4)$.

We will now factorise quadratic expressions of the type $ax^2 + bx + c$, such as $6x^2 + 19x + 15$, where x^2 has a **coefficient**.

Puzzles
Products and factors squaresaw
Perfect squares
Factorominoes
Factorising puzzle (Advanced)
Trinominoes

Technology
Factorising trinomials

Spreadsheet
Factorising trinomials

Example 7

Factorise each quadratic expression.

a $3g^2 + 12g - 36$ **b** $48 - 8p - p^2$

Solution

a $3g^2 + 12g - 36 = 3(g^2 + 4g - 12)$ Taking out the HCF of 3 first.

$= 3(g - 2)(g + 6)$ Product = –12, sum = 4

b $48 - 8p - p^2 = -p^2 - 8p + 48$ Rearranging the terms to make the p^2 term first.

$= -1(p^2 + 8p - 48)$ Taking out a common factor of –1

$= -(p + 12)(p - 4)$ Product = –48, sum = 8

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Example 8

Factorise $3x^2 + 8x + 4$.

Solution

There is no HCF, so we need to split up the middle term $8x$.

Find 2 numbers that have a product of 12 and a sum of 8.

$3x^2 + 8x + 4 = 3x^2 + 8x + 4$ (sum of 8; product of 12 (3×4))

The 2 numbers are $+6$ and $+2$, so we will split $8x$ into $6x$ and $2x$.

$\therefore 3x^2 + 8x + 4 = 3x^2 + 6x + 2x + 4$

$= 3x(x + 2) + 2(x + 2)$ Factorising by grouping in pairs

$= (x + 2)(3x + 2)$ Factorising again

PATH

Video
Factorising quadratic expressions (Advanced)

12.05

Factorising quadratic trinomials of the form $ax^2 + bx + c$

- Find 2 numbers that have a sum of b and a product of ac
- Use these 2 numbers to split the middle term bx into 2 terms
- Factorise by grouping in pairs

Example 9

Factorise each quadratic expression.

a $5k^2 - 12k + 4$ **b** $9m^2 - 9m - 4$ **c** $6t^2 + t - 12$

Solution

a $5k^2 - 12k + 4$

$5 \times 4 = 20$

Find 2 numbers that have a product of 20 and a sum of -12.

Since the sum is negative, one of the numbers must be negative.

Since the product is positive, both of the numbers must be negative.

Try $-5, -4$: $-5 \times (-4) = 20, -5 + (-4) = -9$ ✗

Try $-10, -2$: $-10 \times (-2) = 20, -10 + (-2) = -12$ ✓

They are -10 and -2. Split $-12k$ into $-10k$ and $-2k$.

$5k^2 - 12k + 4 = 5k^2 - 10k - 2k + 4$

$= 5k(k - 2) - 2(k - 2)$ Factorising by grouping in pairs

$= (k - 2)(5k - 2)$

Videos
Factorising quadratic expressions 2
Advanced algebra

PATH

b $9m^2 - 9m - 4$

$9 \times (-4) = -36$

Find 2 numbers with a product of –36 and a sum of –9.

Since the product is negative, one of the numbers must be negative.

Try 12, –3: $12 \times (-3) = -36$, $12 + (-3) = 9$ ✗

Try –12, 3: $-12 \times 3 = -36$, $-12 + 3 = -9$ ✓

They are –12 and 3.

$$9m^2 - 9m - 4 = 9m^2 - 12m + 3m - 4$$
$$= 3m(3m - 4) + 1(3m - 4)$$
$$= (3m - 4)(3m + 1)$$

c $6t^2 + t - 12$

$6 \times (-12) = -72$

Find 2 numbers with a product of –72 and a sum of 1.

Try 8, –9: $8 \times (-9) = -72$, $8 + (-9) = -1$ ✗

Try –8, 9: $-8 \times (9) = -72$, $-8 + 9 = 1$ ✓

They are –8 and +9.

$$6t^2 + t - 12 = 6t^2 - 8t + 9t - 12$$
$$= 2t(3t - 4) + 3(3t - 4)$$
$$= (3t - 4)(2t + 3)$$

Example 10

Factorise each quadratic expression.

a $18a^2 - 18a - 8$ **b** $10 - 7x - 12x^2$

Solution

a $18a^2 - 18a - 8 = 2(9a^2 - 9a - 4)$ Taking out the HCF of 2 first

$= 2(9a^2 - 12a + 3a - 4)$ Product = –36, sum = –9

$= 2[3a(3a - 4) + 1(3a - 4)]$

$= 2(3a - 4)(3a + 1)$

b $10 - 7x - 12x^2 = -12x^2 - 7x + 10$ Rearranging the terms to make the x^2 term first

$= -(12x^2 + 7x - 10)$ Taking out a common factor of –1

$= -(12x^2 + 15x - 8x - 10)$ Product = –120, sum = –7

$= -[3x(4x + 5) - 2(4x + 5)]$

$= -(4x + 5)(3x - 2)$ This can also be written as $(4x + 5)(2 - 3x)$

EXERCISE 12.05 ANSWERS ON P. 636

PATH

Factorising quadratic expressions $ax^2 + bx + c$ U F R

12.05

1 Factorise each quadratic expression. Look for the highest common factor first. R

EXAMPLE 7

a	$3m^2 + 9m + 6$	**b**	$2y^2 + 2y - 4$	**c**	$5t^2 - 10t - 400$
d	$5e^4 + 25e^3 - 120e^2$	**e**	$x^3 - x^2 - 110x$	**f**	$4b^2 - 4b - 168$
g	$4w^2 + 4w - 48$	**h**	$3a^3 - 9a^2 - 12a$	**i**	$2e^2 + 18e + 40$
j	$24 - 5t - t^2$	**k**	$42 + u - u^2$	**l**	$28 + 3x - x^2$
m	$12 - b - b^2$	**n**	$7k - 12 - k^2$	**o**	$12x - 35 - x^2$

2 Factorise each quadratic expression. R

EXAMPLE 8

a	$2x^2 + 11x + 5$	**b**	$4m^2 + 13m + 3$	**c**	$5y^2 + 17y + 6$
d	$6u^2 + 19u + 10$	**e**	$2w^2 + 31w + 15$	**f**	$4e^2 + 15e + 9$
g	$8f^2 + 14f + 3$	**h**	$3d^2 + 5d + 2$	**i**	$2b^2 + 9b + 7$
j	$5y^2 + 16y + 11$	**k**	$8g^2 + 26g + 15$	**l**	$6a^2 + 23a + 21$

3 Factorise each quadratic expression. R

EXAMPLE 9

a	$2y^2 - 11y + 12$	**b**	$10k^2 - 19k + 6$	**c**	$6e^2 - 13e + 6$
d	$4b^2 - 13b + 3$	**e**	$6w^2 - 23w + 15$	**f**	$12f^2 - 25f + 12$
g	$15m^2 - 26m + 8$	**h**	$9x^2 - 12x + 4$	**i**	$2a^2 - 23a + 45$
j	$12y^2 - 35y + 8$	**k**	$8d^2 - 34d + 21$	**l**	$4h^2 - 36h + 81$

4 Factorise each quadratic expression. R

a	$5y^2 - 6y - 11$	**b**	$4d^2 - d - 5$	**c**	$2m^2 - 3m - 9$
d	$3t^2 - t - 30$	**e**	$6h^2 - h - 7$	**f**	$2y^2 - 5y - 12$
g	$8a^2 - 2a - 3$	**h**	$15u^2 - 7u - 4$	**i**	$9c^2 - 12c - 5$
j	$6c^2 - 7c - 24$	**k**	$20n^2 - 27n - 14$	**l**	$12x^2 - 7x - 10$

5 Factorise each quadratic expression. R

a	$5m^2 + 2m - 7$	**b**	$6g^2 + g - 12$	**c**	$3p^2 + 4p - 4$
d	$7w^2 + 6w - 1$	**e**	$5y^2 + 14y - 3$	**f**	$3n^2 + 10n - 8$
g	$4b^2 + 9b - 9$	**h**	$8m^2 + 10m - 3$	**i**	$3x^2 + 2x - 16$
j	$8u^2 + 26u - 15$	**k**	$24e^2 + 31e - 15$	**l**	$20h^2 + 11h - 42$

6 Factorise each expression given it is a perfect square. R

a	$81w^2 - 180w + 100$	**b**	$4y^2 + 8y + 4$	**c**	$25h^2 - 40h + 16$

7 Factorise each quadratic expression by first taking out a common factor. R

EXAMPLE 10

a	$6y^2 + 10y - 4$	**b**	$6g^2 + 15g - 36$	**c**	$24e^2 - 28e - 12$
d	$8a^2 - 10a - 12$	**e**	$12u^2 + 20u - 8$	**f**	$-25q^2 - 5q + 6$
g	$-12m^2 + 14m - 4$	**h**	$20 - h - 12h^2$	**i**	$24c^2 + 48c + 18$
j	$15 + 9w - 6w^2$	**k**	$12d^2 + 2d - 30$	**l**	$22x - 12 - 6x^2$

Foundation | Standard | Complex

PATH

8 Factorise each quadratic expression. R

a	$2a^2 + 5a + 3$	**b**	$12m^2 - 32m + 5$	**c**	$4x^2 + 11x - 3$
d	$7w^2 - 8w + 1$	**e**	$4h^2 - 7h - 15$	**f**	$8x^2 - 2x - 3$
g	$5r^2 + 26r + 5$	**h**	$2d^2 - 15d + 7$	**i**	$6n^2 - 7n - 3$
j	$8 - 6m - 9m^2$	**k**	$3 - 2c - 5c^2$	**l**	$15g^2 + 19g + 6$
m	$15 + 14q - 8q^2$	**n**	$3x^2 - 13x + 14$	**o**	$16 - 8d - 3d^2$
p	$42 - 10y - 12y^2$	**q**	$28d^2 - 44d - 24$	**r**	$100k^2 + 80k + 16$

12.06 Mixed factorisations

PATH

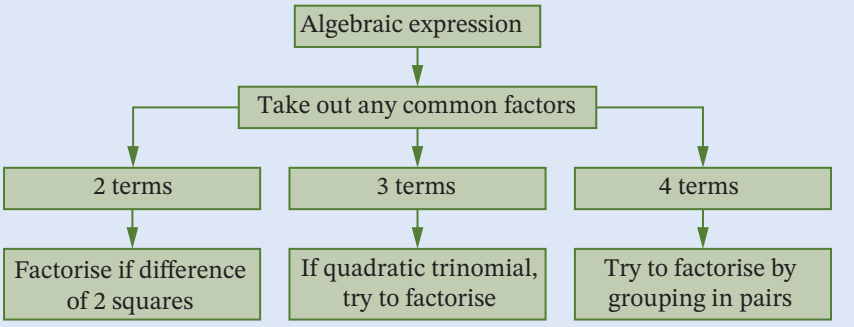
Puzzles
Mixed factorisations

Factorominoes

Products and factors squaresaw

Factorisation strategies

- Look for any common factors and factorise first.
- If there are 2 terms, try factorising using the difference of 2 squares.
- If there are 3 terms, try factorising as a quadratic trinomial.
- If there are 4 terms, try factorising by grouping in pairs.



Example 11

Factorise each quadratic expression.

a $3a^2 - 27$ **b** $5a^2 + 100$ **c** $20b^2 - 52b + 24$ **d** $d^3 - d^2 - d + 1$

Solution

a $3a^2 - 27 = 3(a^2 - 9)$ Taking out the HCF of 3 first

$= 3(a + 3)(a - 3)$ Difference of 2 squares

b $5a^2 + 100 = 5(a^2 + 20)$ 2 terms but not a difference of 2 squares

Foundation Standard Complex

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c $20b^2 - 52b + 24 = 4(5b^2 - 13b + 6)$

$= 4(5b^2 - 10b - 3b + 6)$ Product = 30, sum = –13

$= 4[5b(b - 2) - 3(b - 2)]$

$= 4(b - 2)(5b - 3)$

d $d^3 - d^2 - d + 1 = d^2(d - 1) - 1(d - 1)$ Factorising by grouping in pairs

$= (d - 1)(d^2 - 1)$

$= (d - 1)(d + 1)(d - 1)$ Difference of 2 squares

$= (d - 1)^2(d + 1)$

PATH

12.06

EXAMPLE 11

EXERCISE 12.06 ANSWERS ON P. 637

Mixed factorisations U F R

1 Factorise each expression. R

a $m^2 - 16m + 64$ **b** $3d^2 - 3d$ **c** $3w^2 - 4w - 15$

d $3k - 15 - 5h + hk$ **e** $25y^2 - 64$ **f** $100f^2 - 64$

g $q^2 + 3q - 3pq$ **h** $3 + 2g - g^2$ **i** $24n^2 + 44n - 40$

j $25r^2 - 1$ **k** $b^3 + b^2 + b + 1$ **l** $4x^2 - 20x + 25$

m $4 - d - 5d^2$ **n** $a^3 - a^2 - a + 1$ **o** $8 - 2v^2$

p $mn^2 + mnp + 3mn + 3mp$ **q** $2w^2 - 24w + 72$ **r** $36h^2 + 12h + 1$

2 Factorise each expression. R

a $15r^2 - 31rt - 24t^2$ **b** $4d^2 + 4d + 1$

c $9g^2 - 36k^2$ **d** $e^3 - 3e^2 - 10e$

e $5(p + q)^2 - 125(p - q)^2$ **f** $28x^2 - 7$

g $a^2 - b^2 + 4a - 4b$ **h** $c^3 - 2c^2 - 4c + 8$

i $6a^2 + 13a - 5$ **j** $y^2 - 3y + 5y - 35$

k $18p^2 + 24p + 8$ **l** $1 - 2a - 24a^2$

m $9x^2 - 27x + 18x - 54$ **n** $2a^2b - 6ab - 3a + 9$

o $2a^2 + 12a + 18$ **p** $25u^2 - 10u + 1$

q $4k^2 - 5k - 21$ **r** $48 - 3w^2$

s $3 - 27c^2$ **t** $k^3 + 4k^2 - 16k - 64$

u $5y^3 - 10y^2 + 15y$ **v** $m^3n - 4mn$

w $8 - 2a^2$ **x** $32c^2 - 40c - 12$

3 Factorise each expression. R

a $12e^2 + 13e - 35$ **b** $10y^2 + 19ky - 15k^2$ **c** $16h^4 - 81$

d $w^2 - (6 - w)^2$ **e** $d^3 - 16d + 4 - d$ **f** $u^2 - 8u + 16 - k^2$

12.07 Factorising algebraic fractions

PATH

Worksheet
Simplifying algebraic fractions

Example 12

Simplify each expression.

a $\frac{10a+25b}{5}$ **b** $\frac{9y^2-16}{6y+8}$ **c** $\frac{x^2+x}{-4x-4}$ **d** $\frac{t^2-3t+2}{3t^2-5t-2}$

Solution

a $\frac{10a+25b}{5} = \frac{5^1(2a+5b)}{5^1}$

$= 2a+5b$

b $\frac{9y^2-16}{6y+8} = \frac{(3y+4)(3y-4)}{2(3y+4)}$

$= \frac{3y-4}{2}$

c $\frac{x^2+x}{-4x-4} = \frac{x(x+1)}{-4(x+1)}$

$= -\frac{x}{4}$

d $\frac{t^2-3t+2}{3t^2-5t-2} = \frac{(t-2)(t-1)}{(t-2)(3t+1)}$

$= \frac{t-1}{3t+1}$

Example 13

Simplify each expression.

a $\frac{4}{x^2+x}+\frac{3}{x^2-1}$ **b** $\frac{3m-6}{4}\times\frac{8m}{m^2-2m}$ **c** $\frac{d^2+3d+2}{d^2-9}\div\frac{d^2+d}{3d+9}$

Solution

a $\frac{4}{x^2+x}+\frac{3}{x^2-1} = \frac{4}{x(x+1)}+\frac{3}{(x+1)(x-1)}$ Factorising denominators

$= \frac{4(x-1)}{x(x+1)(x-1)}+\frac{3x}{x(x+1)(x-1)}$ Using common denominators

$= \frac{4x-4+3x}{x(x+1)(x-1)}$

$= \frac{7x-4}{x(x+1)(x-1)}$

b $\frac{3m-6}{4}\times\frac{8m}{m^2-2m} = \frac{3(m-2)}{4_1}\times\frac{^2\cancel{8}\,m}{m(m-2)}$

$= 6$

c $\frac{d^2+3d+2}{d^2-9}\div\frac{d^2+d}{3d+9} = \frac{d^2+3d+2}{d^2-9}\times\frac{3d+9}{d^2+d}$

$= \frac{(d+2)(d+1)}{(d+3)(d-3)}\times\frac{3(d+3)}{d(d+1)}$

$= \frac{3(d+2)}{d(d-3)}$

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Factorising algebraic fractions U F R

EXAMPLE 12

1 Simplify each expression. R

a $\frac{3x+3y}{3}$ **b** $\frac{5}{10t-10r}$ **c** $\frac{ab-ac}{a^2}$

d $\frac{y-1}{1-y}$ **e** $\frac{w^2-16}{w+4}$ **f** $\frac{5d-5w}{d^2-w^2}$

g $\frac{(k+5)^2}{k^2-25}$ **h** $\frac{6c^2-6}{2c+2}$ **i** $\frac{am-an+m-n}{m^2-n^2}$

j $\frac{y^2+9y+20}{2y+10}$ **k** $\frac{k^2-3k-4}{k^2-16}$ **l** $\frac{16a^2-25c^2}{4a^2-9ac+5c^2}$

m $\frac{x^2+4x+4}{x^2-x-6}$ **n** $\frac{1-c-2c^2}{3c^2+2c-1}$ **o** $\frac{ap+4p-2a-8}{2p^2-8}$

p $\frac{c^2-9c+20}{cu-4+c-4u}$ **q** $\frac{h^2+3h+2}{2h^2+4h}$ **r** $\frac{8e^2-14e-15}{4e^3-25e}$

EXAMPLE 13

2 Simplify each expression. R

a $\frac{5}{m(m+1)}+\frac{2}{(m+1)(m+2)}$ **b** $\frac{6}{(w+5)(w+3)}-\frac{4}{w(w+3)}$

c $\frac{3}{(b+2)(b-1)}+\frac{1}{(b-1)(b-3)}$ **d** $\frac{2}{k^2+k}-\frac{3}{k^2-1}$

e $\frac{5}{4h+4}+\frac{3}{h^2+h}$ **f** $\frac{3}{d^2+3d+2}-\frac{4}{d+2}$

g $\frac{3}{r^2-36}-\frac{5}{4r+24}$ **h** $\frac{3}{d^2+2d}+\frac{d}{d^2-4}$

i $\frac{5}{k^2-3k-4}-\frac{k}{k^2-1}$ **j** $\frac{2}{q^2-1}+\frac{3}{q+1}$

k $\frac{4}{a^2-8a+15}+\frac{3}{a^2+4a-21}$ **l** $\frac{2}{y^2+5y+6}-\frac{7}{y^2-y-12}$

m $\frac{5}{e^2+3e+2}-\frac{8}{e^2+9e+14}$ **n** $\frac{3}{6h^2-h-2}+\frac{4}{12h^2-23h+10}$

3 Simplify each expression. R

a $\frac{3m+9}{2}\times\frac{4m}{m+3}$ **b** $\frac{5d-10}{3d-9}\times\frac{5d-15}{8d-16}$ **c** $\frac{4}{e+2}\times\frac{e^2+2e}{8e}$

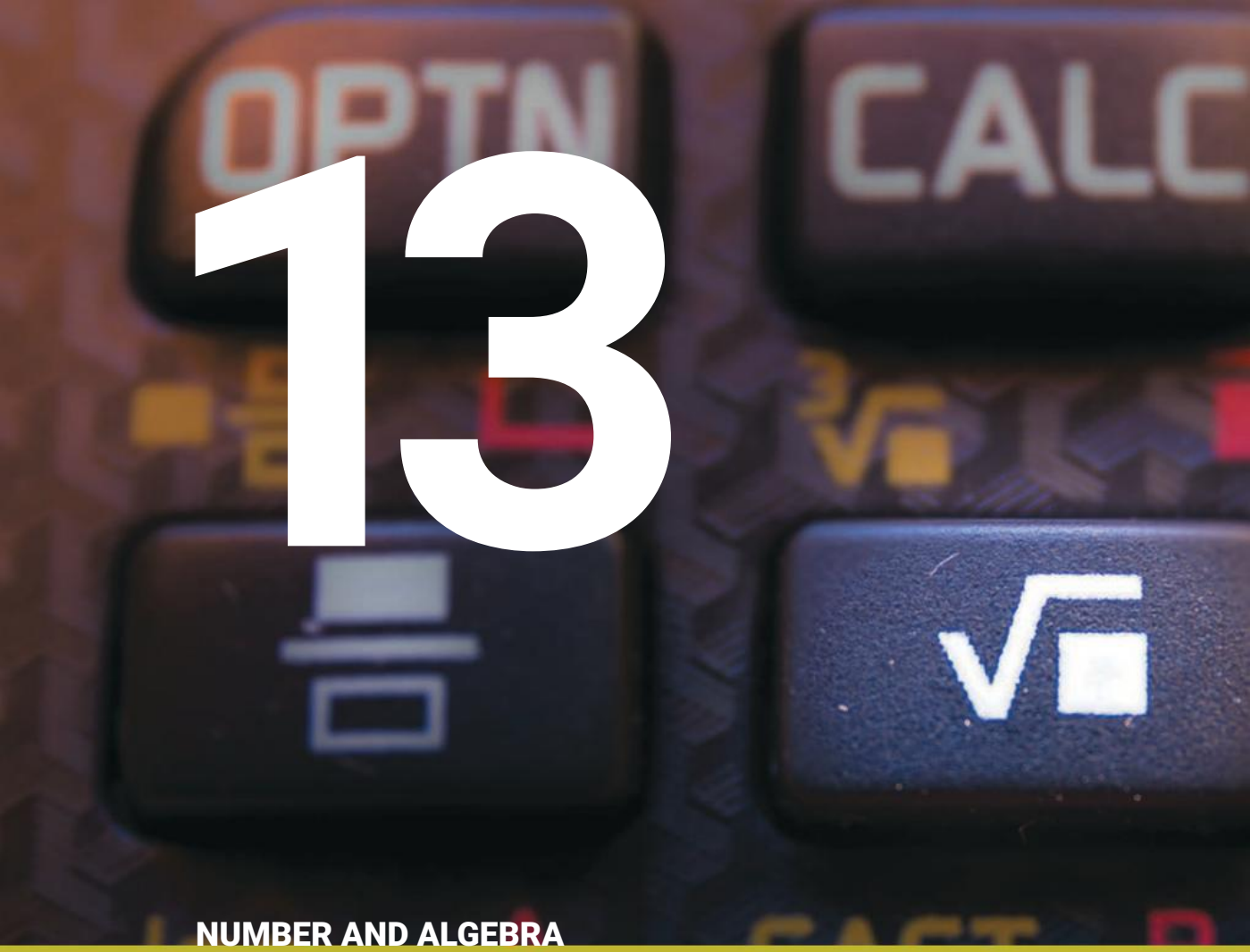
d $\frac{3k+6}{5}\times\frac{10k}{k+2}$ **e** $\frac{5h}{3h+9}\times\frac{6h+18}{h^2+h}$ **f** $\frac{4}{a^2-b^2}\times\frac{3a+3b}{8}$

g $\frac{r+t}{t^2-r^2}\times\frac{r^2-rt}{5r+5t}$ **h** $\frac{20m+16}{7m-7}\times\frac{7m}{5m+4}$ **i** $\frac{p^2+2p+1}{p^2-1}\times\frac{4p-4}{p^2+p}$

j $\frac{y+2}{5y}\div\frac{7y+14}{15y}$ **k** $\frac{5}{x^2-4}\div\frac{15}{2x+4}$ **l** $\frac{4n+8}{n+5}\div\frac{6n+12}{5n+25}$

m $\frac{d^2+d}{d+3}\div\frac{6d}{d^2-9}$ **n** $\frac{1}{f^2-6f+9}\div\frac{4}{f^2-9}$ **o** $\frac{3f+6}{f^2+f-6}\div\frac{f^2-2f-8}{f^2-f-12}$

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13

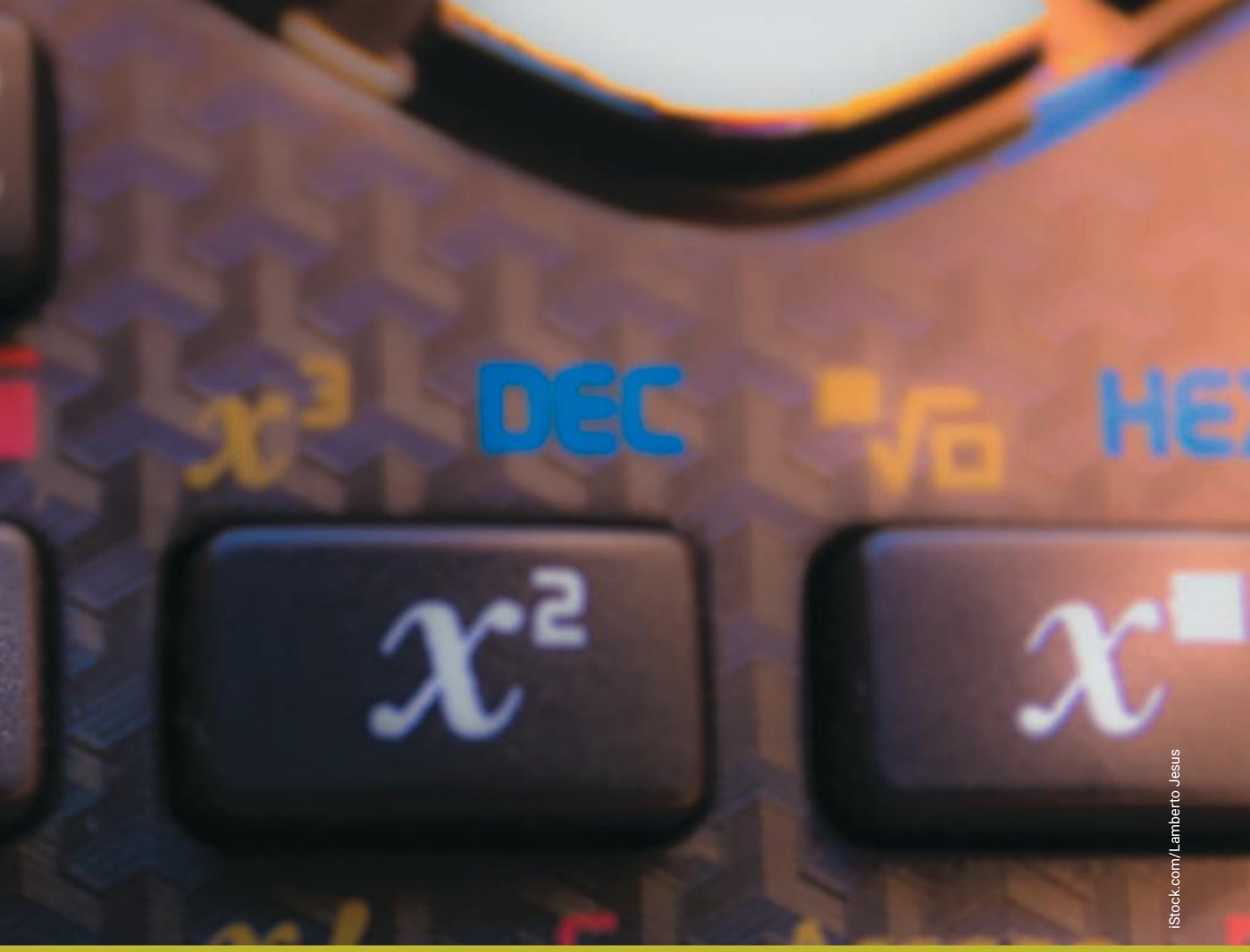
NUMBER AND ALGEBRA

SURDS

OPTIONAL PATH TOPIC
RECOMMENDED FOR STAGE 6 MATHEMATICS ADVANCED

When applying Pythagoras' theorem, we have found lengths that cannot be expressed as an exact rational number. Pythagoras encountered this when calculating the diagonal of a square of side length 1 unit. A **surd** is a square root ($\sqrt{\ }$), cube root ($\sqrt[3]{\ }$), or any type of root whose exact decimal or fraction value cannot be found.

The Pythagoreans explained nature, the universe — in fact everything — in terms of whole numbers. Apparently, they were so upset about the discovery of surds that they tried to keep the discovery a secret. Hippasus, one of the Pythagoreans, was drowned for revealing the secret to outsiders.



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Chapter outline

	Working mathematically				
13.01 Surds*	U	F		R	C
13.02 Simplifying surds*	U	F		R	
13.03 Adding and subtracting surds*	U	F			
13.04 Multiplying and dividing surds*	U	F			
13.05 Binomial products involving surds*	U	F			
13.06 Rationalising the denominator*	U	F			

*PATH

Note: Chapter 12, Products and factors, must be completed before this chapter.

Wordbank

irrational number A number such as π or $\sqrt{2}$ that cannot be expressed as a fraction $\frac{a}{b}$

rational number Any number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$

rationalise the denominator To simplify a fraction involving a surd by making its denominator rational (that is, not a surd)

real number A number that is either rational or irrational and whose value can be graphed on a number line

simplify a surd To write a surd $\sqrt{x}$ in its simplest form so that x has no factors that are perfect squares

surd A square root (or other root) whose exact value cannot be found

Quiz
Wordbank 13

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Videos (4):
13.02 Simplifying surds 2
• Simplifying surds 1
13.05 Binomial products involving surds
13.06 Rationalising the denominator

Quizzes (2):
- Wordbank 13
- SkillCheck 13

Worksheets (2):
13.01 Surds on the number line
13.04 Multiplying and dividing surds

Puzzles (3):
13.02 Simplifying surds
13.03 Surds code puzzle
13.04 Surds

Technology (1):
13.02 Simplifying surds quiz

Spreadsheet (1):
13.02 Simplifying surds quiz

Nelson MindTap

To access resources above, visit
cengage.com.au/nelsonmindtap

In this chapter you will:

- (PATH) describe real, rational and irrational numbers and surds
- (PATH) simplify, add, subtract, multiply and divide surds
- (PATH) expand and simplify binomial products involving surds
- (PATH) rationalise the denominator of expressions of the form $\frac{a\sqrt{b}}{c\sqrt{d}}$

PATH

Quiz
SkillCheck 13

SkillCheck ANSWERS ON P. 637

1 Simplify each expression.

a $(5y)^2$ **b** $(4m)^3$ **c** $(-3x)^2$

2 Expand each expression.

a $5(x + 2)$ **b** $4(y - 3)$ **c** $3(1 + 2w)$

d $2(5 - y)$ **e** $-5(2a + 3)$ **f** $k(1 + 2k)$

3 Select the square numbers from the following list of numbers.

44 81 25 100 75
72 16 50 64 32

4 Expand and simplify each expression.

a $(m + 3)(m + 7)$ **b** $(y + 1)(y - 4)$ **c** $(n - 2)(n - 3)$

d $(2d + 3)(1 + 3d)$ **e** $(1 - 5p)(4 + 3p)$ **f** $(3a + 2f)(a + 5f)$

g $(x + 4)^2$ **h** $(y - 3)^2$ **i** $(2k + 1)^2$

j $(a - 5)(a + 5)$ **k** $(t + 7)(t - 7)$ **l** $(3m + 4)(3m - 4)$

Surds

13.01

PATH

13.01

A **surd** is a square root ($\sqrt{\ }$), cube root ($\sqrt[3]{\ }$), or any type of root whose exact decimal or fraction value cannot be found. As a decimal, its digits run endlessly *without repeating* (like π), so they are neither terminating nor recurring decimals.

$\sqrt{7}$ is read as 'the square root of 7' or simply 'root 7'.

$\sqrt{7}$ is called the **exact value**, $\sqrt{7} \approx 2.64575$ is the **approximate value**.

Rational numbers such as fractions, decimals and percentages, can be expressed in the form $\frac{a}{b}$, where a and b are integers ($b \neq 0$), but surds are **irrational numbers** because they cannot be expressed in this form.

Rational numbers

can be expressed in the form $\frac{a}{b}$

Integers

$\frac{4}{1} = 4, \frac{26}{1} = 26, \frac{-3}{1} = -3$

Terminating decimals

0.5, $7\frac{1}{8} = 7.125$,

16% = 0.16, 1.32

Recurring decimals

$\frac{2}{3} = 0.666\ldots$

$\frac{5}{6} = 0.833\ldots$

$\frac{4}{11} = 0.3636\ldots$

Irrational numbers

cannot be expressed in the form $\frac{a}{b}$

Surds

$\sqrt{5}, -\sqrt{2}, \frac{\sqrt{11}}{3}, 8\sqrt{6}$

Transcendental numbers

Have no pattern and are non-recurring

e.g. $\pi = 3.14159...$,

$\cos 38° = 0.78801...$, $e = 2.71828...$

Example 1

Select the surds from this list of square roots:

$\sqrt{72}$ $\sqrt{121}$ $\sqrt{64}$ $\sqrt{90}$ $\sqrt{28}$

Solution

$\sqrt{72} = 8.4852...$ $\sqrt{121} = 11$ $\sqrt{64} = 8$

$\sqrt{90} = 9.4868...$ $\sqrt{28} = 5.2915...$

So the surds are $\sqrt{72}$, $\sqrt{90}$ and $\sqrt{28}$.

PATH

Example 2

Is each number rational or irrational?

a $4\frac{2}{5}$ **b** $\sqrt[3]{-8}$ **c** $-\sqrt{7}$ **d** $0.\dot{6}$ **e** 5π

Solution

a $4\frac{2}{5} = \frac{22}{5}$ which is in the form of a fraction $\frac{a}{b}$

$\therefore 4\frac{2}{5}$ is a rational number.

b $\sqrt[3]{-8} = -2$ which can be written as $\frac{-2}{1}$

$\therefore \sqrt[3]{-8}$ is a rational number.

c $-\sqrt{7} = -2.645751311\ldots$ The digits run endlessly without repeating.

$\therefore -\sqrt{7}$ is an irrational number.

d $0.\dot{6} = 0.666\ldots$ which is a recurring decimal

$= \frac{2}{3}$ which is a fraction

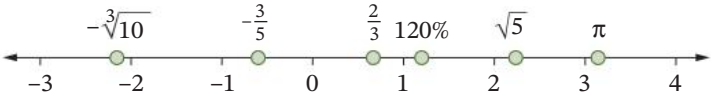
$\therefore 0.\dot{6}$ is a rational number.

e $5\pi = 15.707\ 963\ 27...$ The digits run endlessly without repeating

$\therefore 5\pi$ is an irrational number.

Surds on a number line

The rational and irrational numbers together make up the **real numbers**.
Any real number can be represented by a point on the number line.



$-\sqrt[3]{10} = -2.1544...$ irrational (surd)

$-\frac{3}{5} = -0.6$ rational (fraction)

$\frac{2}{3} = 0.6666...$ rational (fraction)

$120\% = 1.2$ rational (percentage)

$\sqrt{5} = 2.2360...$ irrational (surd)

$\pi = 3.1415...$ irrational (pi)

Example 3

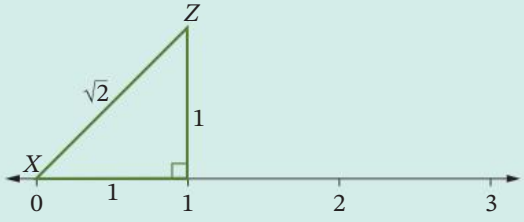
Use a pair of compasses and Pythagoras' theorem to estimate the value of $\sqrt{2}$ on a number line.

Solution

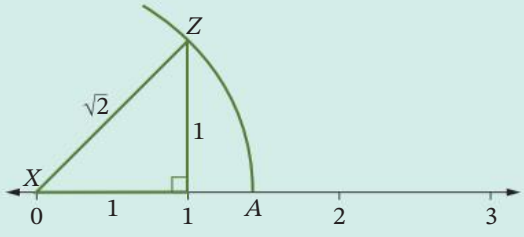
Step 1: Using a scale of 1 unit to 2 cm, draw a number line as shown.



Step 2: Construct a right-angled triangle on the number line with base length and height 1 unit as shown. By Pythagoras' theorem, show that $XZ = \sqrt{2}$ units.



Step 3: With 0 as the centre, use compasses with radius $XZ\left(\sqrt{2}\right)$ to draw an arc to meet the number line at A as shown. The point A represents the value of $\sqrt{2}$ and should be approximately 1.4142...



PATH

Worksheet
Surds on the number line

13.01

EXERCISE 13.01 ANSWERS ON P. 637

Surds U F R C

EXAMPLE 1

1 Which of these numbers is a surd? Select the correct answer **A**, **B**, **C** or **D**. C

A $\sqrt{9}$ **B** $\sqrt{225}$ **C** $\sqrt{160}$ **D** $\sqrt{81}$

2 Which of these numbers is NOT a surd? Select **A**, **B**, **C** or **D**. C

A $\sqrt{77}$ **B** $\sqrt{144}$ **C** $\sqrt{18}$ **D** $\sqrt{200}$

3 Select the surds from this list of numbers. C

$\sqrt{32}$ $\sqrt{33}$ $\sqrt{289}$ $\sqrt{81}$ $\sqrt{4.9}$

$\sqrt{52}$ $\sqrt{121}$ $\sqrt{144}$ $\sqrt{196}$ $\sqrt{200}$

Foundation Standard Complex

PATH

EXAMPLE 2

4 Is each number rational (R) or irrational (I)? R C

a $5.\dot{6}$ **b** $\sqrt{8}$ **c** $\sqrt{4}$ **d** $3\frac{1}{7}$

e $\sqrt[3]{27}$ **f** $1.3\dot{5}$ **g** $\sqrt[3]{-64}$ **h** $27\frac{1}{2}\%$

i $\sqrt{5\times10^3}$ **j** $\frac{3}{11}$ **k** $\frac{\sqrt{50}}{3}$ **l** $\sqrt{\sqrt{4}}$

5 Arrange each set of numbers in descending order.

a $1\frac{4}{7}, \sqrt{2}, \frac{\pi}{2}$ **b** $\sqrt[3]{20}, 2.\dot{6}, 2\frac{7}{9}$

6 Express each real number correct to one decimal place and graph them on a number line.

a $-1\frac{4}{5}$ **b** 74% **c** $\frac{4}{11}$ **d** $-\sqrt{12}$

e $-\sqrt[3]{15}$ **f** $2\frac{5}{9}$ **g** $\frac{\pi}{2}$ **h** 187%

7 Use the method from Example **3** to estimate the value of $\sqrt{2}$ on a number line.

EXAMPLE 3

8 **a** Use the method from Example **3** to estimate the value of $\sqrt{5}$ on a number line by constructing a right-angled triangle with base length 2 units and height 1 unit.

b Use a similar method to estimate the following surds on a number line:

i $\sqrt{10}$ **ii** $\sqrt{17}$

13.02 Simplifying surds

PATH

Puzzle Simplifying surds

Technology Simplifying surds quiz

Spreadsheet Simplifying surds quiz

The square of any real number is always positive (except for $0^2 = 0$), so it is not possible to give the square root of a *negative* number.

The radical symbol $\sqrt{\ }$ stands for the *positive* square root of a number. For example $\sqrt{4} = 2$ (not –2).

The square root of x

For $x < 0$ (negative), $\sqrt{x}$ is undefined.

For $x = 0$, $\sqrt{x} = 0$.

For $x > 0$ (positive), $\sqrt{x}$ is the positive square root of x.

For $x \geq 0$, $\left(\sqrt{x}\right)^2 = \sqrt{x} \times \sqrt{x} = x$ and $\sqrt{x^2} = x$.

Foundation Standard Complex

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PATH

13.02

Example 4

Simplify each expression.

a $\left(\sqrt{7}\right)^2$ **b** $\left(3\sqrt{5}\right)^2$ **c** $\left(-2\sqrt{3}\right)^2$

Solution

a $\left(\sqrt{7}\right)^2 = 7$

b $\left(3\sqrt{5}\right)^2 = 3\sqrt{5} \times 3\sqrt{5}$ $3\sqrt{5}$ means $3 \times \sqrt{5}$

$= 3^2 \times \left(\sqrt{5}\right)^2$

$= 9 \times 5$

$= 45$

c $\left(-2\sqrt{3}\right)^2 = (-2)^2 \times \left(\sqrt{3}\right)^2$

$= 4 \times 3$

$= 12$

The square root of a product

For $x > 0$ and $y > 0$:

$$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$$

A surd $\sqrt{n}$ can be simplified if n can be divided into 2 factors, where one of them is a square number such as 4, 9, 16, 25, 36, 49, ...

Example 5

Simplify each surd.

a $\sqrt{8}$ **b** $\sqrt{108}$ **c** $4\sqrt{45}$ **d** $\dfrac{\sqrt{288}}{3}$

Solution

a $\sqrt{8} = \sqrt{4} \times \sqrt{2}$ 4 is a square number

$= 2 \times \sqrt{2}$

$= 2\sqrt{2}$

b *Method 1*:

$\sqrt{108} = \sqrt{36} \times \sqrt{3}$

$= 6 \times \sqrt{3}$

$= 6\sqrt{3}$

Method 2:

$\sqrt{108} = \sqrt{4} \times \sqrt{27}$

$= 2 \times \sqrt{9} \times \sqrt{3}$

$= 2 \times 3 \times \sqrt{3}$

$= 6\sqrt{3}$

Method 2 involves simplifying surds *twice* ($\sqrt{108}$ and $\sqrt{27}$). Method 1 shows that when simplifying surds, look for the highest square factor possible.

Videos
Simplifying surds 2
Simplifying surds 1

PATH

c $4\sqrt{45} = 4 \times \sqrt{9} \times \sqrt{5}$
$= 4 \times 3 \times \sqrt{5}$
$= 12\sqrt{5}$

d $\frac{\sqrt{288}}{3} = \frac{\sqrt{144} \times \sqrt{2}}{3}$
$= \frac{12\sqrt{2}}{3}$
$= \frac{12^{4}\sqrt{2}}{\cancel{3}^{1}}$
$= 4\sqrt{2}$

EXERCISE 13.02 ANSWERS ON P. 638

Simplifying surds U F R

EXAMPLE 4

1 Simplify each expression.

a $\left(\sqrt{2}\right)^2$ **b** $\left(\sqrt{5}\right)^2$ **c** $\left(3\sqrt{3}\right)^2$ **d** $\left(5\sqrt{10}\right)^2$

e $\left(\sqrt{0.09}\right)^2$ **f** $\left(-2\sqrt{7}\right)^2$ **g** $\left(-3\sqrt{5}\right)^2$ **h** $\left(-5\sqrt{2}\right)^2$

EXAMPLE 5

2 Simplify each surd.

a $\sqrt{50}$ **b** $\sqrt{27}$ **c** $\sqrt{24}$ **d** $\sqrt{54}$

e $\sqrt{243}$ **f** $\sqrt{45}$ **g** $\sqrt{48}$ **h** $\sqrt{200}$

i $\sqrt{96}$ **j** $\sqrt{63}$ **k** $\sqrt{288}$ **l** $\sqrt{108}$

m $\sqrt{75}$ **n** $\sqrt{147}$ **o** $\sqrt{32}$ **p** $\sqrt{242}$

q $\sqrt{162}$ **r** $\sqrt{245}$ **s** $\sqrt{125}$ **t** $\sqrt{512}$

3 Simplify each expression.

a $5\sqrt{50}$ **b** $3\sqrt{8}$ **c** $4\sqrt{27}$ **d** $8\sqrt{98}$

e $\frac{\sqrt{40}}{2}$ **f** $\frac{\sqrt{243}}{9}$ **g** $\frac{\sqrt{28}}{6}$ **h** $3\sqrt{24}$

i $9\sqrt{68}$ **j** $\frac{\sqrt{3125}}{10}$ **k** $\frac{1}{2}\sqrt{72}$ **l** $\frac{3}{4}\sqrt{48}$

m $10\sqrt{160}$ **n** $3\sqrt{75}$ **o** $7\sqrt{68}$ **p** $\frac{\sqrt{52}}{6}$

4 Which surd below is equivalent to $4\sqrt{50}$? Select **A**, **B**, **C** or **D**.

A $8\sqrt{5}$ **B** $20\sqrt{2}$ **C** $8\sqrt{2}$ **D** $20\sqrt{5}$

5 Which surd below is equivalent to $\frac{\sqrt{250}}{10}$? Select **A**, **B**, **C** or **D**.

A $\frac{\sqrt{5}}{10}$ **B** $\frac{\sqrt{10}}{2}$ **C** $2\sqrt{10}$ **D** $5\sqrt{10}$

6 Decide whether each statement is true (T) or false (F). R

a $3\sqrt{7} = \sqrt{21}$ **b** $\sqrt{12} = 6$

c $\left(\sqrt{9.4}\right)^2 = 9.4$ **d** $\sqrt{75} = 5\sqrt{3}$

e $\sqrt{3} \approx 1.7$ **f** The exact value of $\sqrt{10}$ is 3.162 277 8

Foundation Standard Complex

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Adding and subtracting surds

13.03

Just as you can only add or subtract 'like terms' in algebra, you can only add or subtract 'like surds'. You may first need to express all the surds in their simplest forms.

PATH



13.03

Puzzle
Surds code puzzle

Example 6

Simplify each expression.

a $4\sqrt{2}+5\sqrt{2}$ **b** $7\sqrt{3}-2\sqrt{3}$ **c** $5\sqrt{2}-3\sqrt{3}+\sqrt{2}$

d $\sqrt{50}+\sqrt{32}$ **e** $\sqrt{8}-\sqrt{27}+\sqrt{18}$ **f** $5\sqrt{20}-3\sqrt{125}$

Solution

a $4\sqrt{2}+5\sqrt{2}=9\sqrt{2}$

b $7\sqrt{3}-2\sqrt{3}=5\sqrt{3}$

c $5\sqrt{2}-3\sqrt{3}+\sqrt{2}=6\sqrt{2}-3\sqrt{3}$

d $\sqrt{50}+\sqrt{32}=\sqrt{25}\sqrt{2}+\sqrt{16}\sqrt{2}$
$=5\sqrt{2}+4\sqrt{2}$
$=9\sqrt{2}$

Simplifying each surd

e $\sqrt{8}-\sqrt{27}+\sqrt{18}=\sqrt{4}\sqrt{2}-\sqrt{9}\sqrt{3}+\sqrt{9}\sqrt{2}$
$=2\sqrt{2}-3\sqrt{3}+3\sqrt{2}$
$=5\sqrt{2}-3\sqrt{3}$

f $5\sqrt{20}-3\sqrt{125}=5\sqrt{4}\sqrt{5}-3\sqrt{25}\sqrt{5}$
$=5\times2\sqrt{5}-3\times5\sqrt{5}$
$=10\sqrt{5}-15\sqrt{5}$
$=-5\sqrt{5}$

PATH

EXERCISE 13.03 ANSWERS ON P. 638

Adding and subtracting surds U F

EXAMPLE 6

1 Simplify each expression.

a $5\sqrt{7}+2\sqrt{7}$ **b** $3\sqrt{2}-8\sqrt{2}$

c $7\sqrt{5}-\sqrt{5}$ **d** $\sqrt{5}+3\sqrt{5}$

e $5\sqrt{17}-5\sqrt{17}$ **f** $3\sqrt{10}-2\sqrt{10}$

g $4\sqrt{15}-3\sqrt{15}+7\sqrt{15}$ **h** $5\sqrt{6}-2\sqrt{6}-4\sqrt{6}$

i $3\sqrt{3}+4\sqrt{3}-5\sqrt{3}$ **j** $4\sqrt{5}+7\sqrt{5}-\sqrt{5}$

k $8\sqrt{10}-5\sqrt{10}+3\sqrt{10}$ **l** $10\sqrt{3}-3\sqrt{3}-12\sqrt{3}$

2 Simplify each expression.

a $3\sqrt{5}-8+2\sqrt{5}$ **b** $11\sqrt{10}+3\sqrt{2}+2\sqrt{10}$

c $-4\sqrt{3}+5\sqrt{2}-5\sqrt{3}$ **d** $3\sqrt{15}+3\sqrt{2}+4\sqrt{15}+5\sqrt{2}$

e $\sqrt{7}-3\sqrt{5}-4\sqrt{7}+\sqrt{5}$ **f** $4\sqrt{6}-3\sqrt{3}-2\sqrt{6}-5\sqrt{3}$

g $10\sqrt{11}-5\sqrt{3}+3\sqrt{11}+4\sqrt{3}$ **h** $\sqrt{13}+8\sqrt{7}-7\sqrt{13}+3\sqrt{7}$

i $2\sqrt{5}-3\sqrt{7}-2\sqrt{5}-3\sqrt{7}$ **j** $4\sqrt{10}-3\sqrt{5}-4\sqrt{10}$

3 For each expression, select the correct simplified answer **A**, **B**, **C** or **D**.

a $\sqrt{3}+\sqrt{12}$

A $5\sqrt{3}$ **B** $\sqrt{15}$ **C** $2\sqrt{6}$ **D** $3\sqrt{3}$

b $4\sqrt{5}-2\sqrt{125}$

A $-6\sqrt{5}$ **B** $\sqrt{5}$ **C** $-\sqrt{45}$ **D** $-46\sqrt{5}$

4 Simplify each expression.

a $\sqrt{8}+\sqrt{32}$ **b** $\sqrt{108}-\sqrt{27}$ **c** $\sqrt{20}-\sqrt{80}$

d $\sqrt{28}-\sqrt{63}$ **e** $3\sqrt{6}+\sqrt{24}$ **f** $2\sqrt{5}+\sqrt{125}$

g $\sqrt{40}-\sqrt{90}$ **h** $5\sqrt{11}+\sqrt{99}$ **i** $3\sqrt{2}+\sqrt{18}$

j $\sqrt{27}+5\sqrt{3}$ **k** $\sqrt{200}-7\sqrt{2}$ **l** $\sqrt{50}+\sqrt{32}$

m $5\sqrt{3}+2\sqrt{27}$ **n** $3\sqrt{20}-\sqrt{245}$ **o** $7\sqrt{12}-5\sqrt{48}$

p $4\sqrt{27}+2\sqrt{243}$ **q** $3\sqrt{63}-2\sqrt{28}$ **r** $2\sqrt{98}+3\sqrt{162}$

s $-5\sqrt{6}+2\sqrt{150}$ **t** $4\sqrt{50}+3\sqrt{18}$ **u** $5\sqrt{27}-6\sqrt{75}$

v $3\sqrt{112}-2\sqrt{252}$ **w** $\sqrt{32}+\sqrt{8}+\sqrt{12}$ **x** $\sqrt{27}+\sqrt{54}+\sqrt{243}$

y $\sqrt{98}-3\sqrt{20}-2\sqrt{8}$ **z** $3\sqrt{96}-2\sqrt{150}+\sqrt{24}$

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Multiplying and dividing surds

13.04

The square root of products and quotients

For $x > 0$ and $y > 0$:

$\sqrt{xy} = \sqrt{x} \times \sqrt{y}$

$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$

PATH

13.04

Puzzle Surds

Worksheet Multiplying and dividing surds

Example 7

Simplify each expression.

a $\sqrt{3} \times \sqrt{5}$ **b** $\sqrt{10} \times \sqrt{6}$ **c** $3\sqrt{7} \times 5\sqrt{7}$

d $5\sqrt{27} \times 3\sqrt{6}$ **e** $\sqrt{54} \div (-\sqrt{2})$ **f** $\frac{15\sqrt{32}}{5\sqrt{8}}$

Solution

a $\sqrt{3} \times \sqrt{5} = \sqrt{15}$

b $\sqrt{10} \times \sqrt{6} = \sqrt{60}$
$= \sqrt{4} \times \sqrt{15}$
$= 2\sqrt{15}$

c $3\sqrt{7} \times 5\sqrt{7} = 3 \times 5 \times \sqrt{7} \times \sqrt{7}$
$= 15 \times 7$
$= 105$

d $5\sqrt{27} \times 3\sqrt{6} = 5 \times 3 \times \sqrt{27} \times \sqrt{6}$
$= 15 \times 3\sqrt{3} \times \sqrt{6}$
$= 45\sqrt{18}$
$= 45 \times 3\sqrt{2}$
$= 135\sqrt{2}$

e $\sqrt{54} \div (-\sqrt{2}) = -\frac{\sqrt{54}}{\sqrt{2}}$
$= -\sqrt{27}$
$= -\sqrt{9} \times \sqrt{3}$
$= -3\sqrt{3}$

f $\frac{15\sqrt{32}}{5\sqrt{8}} = 3\sqrt{4}$
$= 3 \times 2$
$= 6$

Example 8

Simplify $\frac{5\sqrt{2} \times 4\sqrt{12}}{10\sqrt{8}}$.

Solution

$\frac{5\sqrt{2} \times 4\sqrt{12}}{10\sqrt{8}} = \frac{20\sqrt{24}}{10\sqrt{8}}$
$= 2\sqrt{3}$

PATH

EXERCISE 13.04 ANSWERS ON P. 638

Multiplying and dividing surds U F

EXAMPLE 7

1 Simplify each expression.

a $\sqrt{7}\times\sqrt{2}$ **b** $-\sqrt{5}\times\sqrt{7}$ **c** $\sqrt{6}\times\sqrt{8}$

d $\sqrt{12}\times\sqrt{3}$ **e** $\sqrt{10}\times\left(-\sqrt{5}\right)$ **f** $3\sqrt{3}\times5\sqrt{3}$

g $5\sqrt{10}\times3\sqrt{3}$ **h** $-2\sqrt{7}\times5\sqrt{3}$ **i** $7\sqrt{5}\times4\sqrt{5}$

j $2\sqrt{3}\times\left(-5\sqrt{6}\right)$ **k** $4\sqrt{3}\times\sqrt{27}$ **l** $-3\sqrt{5}\times4\sqrt{10}$

m $-7\sqrt{2}\times4\sqrt{8}$ **n** $\sqrt{18}\times8\sqrt{3}$ **o** $10\sqrt{2}\times2\sqrt{8}$

p $3\sqrt{18}\times5\sqrt{12}$ **q** $3\sqrt{44}\times\left(-2\sqrt{99}\right)$ **r** $5\sqrt{8}\times4\sqrt{40}$

s $8\sqrt{3}\times3\sqrt{54}$ **t** $-8\sqrt{32}\times\sqrt{27}$ **u** $\sqrt{90}\times\sqrt{72}$

v $-5\sqrt{20}\times3\sqrt{8}$ **w** $7\sqrt{18}\times3\sqrt{24}$ **x** $3\sqrt{48}\times2\sqrt{12}$

2 Simplify each expression.

a $\sqrt{15}\div\sqrt{3}$ **b** $\sqrt{18}\div\left(-\sqrt{6}\right)$ **c** $\dfrac{6\sqrt{48}}{2\sqrt{8}}$

d $10\sqrt{54}\div5\sqrt{27}$ **e** $-3\sqrt{98}\div6\sqrt{14}$ **f** $\dfrac{7\sqrt{18}}{\sqrt{2}}$

g $2\sqrt{24}\div4\sqrt{6}$ **h** $\dfrac{\sqrt{128}}{\sqrt{2}}$ **i** $15\sqrt{18}\div3\sqrt{6}$

j $\dfrac{-20\sqrt{10}}{-4\sqrt{5}}$ **k** $36\sqrt{24}\div9\sqrt{8}$ **l** $16\sqrt{30}\div8\sqrt{5}$

m $12\sqrt{14}\div6$ **n** $\dfrac{3\sqrt{2}}{-12}$ **o** $\sqrt{80}\div4\sqrt{5}$

p $5\sqrt{60}\div\sqrt{15}$ **q** $6\sqrt{8}\div3\sqrt{2}$ **r** $\dfrac{-42\sqrt{54}}{6\sqrt{3}}$

s $12\sqrt{63}\div3\sqrt{7}$ **t** $\dfrac{8\sqrt{50}}{2\sqrt{200}}$ **u** $6\sqrt{3}\div\sqrt{243}$

3 Simplify:

a $\sqrt{6}\times\sqrt{6}$ **b** $\sqrt{7}\times\sqrt{7}$ **c** $2\sqrt{3}\times\sqrt{3}$

d $5\sqrt{y}\times3\sqrt{y}$ **e** $\sqrt{x}\times\sqrt{x}$ **f** $\sqrt{a^2}\times\sqrt{a}$

4 Simplify $3\sqrt{2}\times\sqrt{6}$. Select the correct answer **A**, **B**, **C** or **D**.

A 6 **B** $6\sqrt{2}$ **C** $6\sqrt{3}$ **D** $12\sqrt{2}$

5 Simplify $20\sqrt{10}\div5\sqrt{2}$. Select **A**, **B**, **C** or **D**.

A $4\sqrt{5}$ **B** $15\sqrt{5}$ **C** 10 **D** 20

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6 Simplify each expression.

a $\dfrac{3\sqrt{5}\times 4\sqrt{2}}{3\sqrt{40}}$ **b** $\dfrac{3\sqrt{12}\times 8\sqrt{6}}{4\sqrt{27}}$ **c** $\dfrac{5\sqrt{8}\times 2\sqrt{90}}{10\sqrt{24}}$

d $\dfrac{4\sqrt{5}}{2\sqrt{15}\times 5\sqrt{27}}$ **e** $\dfrac{10\sqrt{686}\times 3\sqrt{12}}{5\sqrt{28}\times\sqrt{18}}$ **f** $\dfrac{8\sqrt{80}\times 3\sqrt{2}}{4\sqrt{5}\times 6\sqrt{8}}$

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EXAMPLE 8

13.05

Binomial products involving surds 13.05

Surd expressions involving brackets can be expanded in the same way as algebraic expressions of the form $a(b + c)$ and $(a + b)(c + d)$.

PATH

Example 9

Expand and simplify each expression.

a $\sqrt{3}\left(\sqrt{5}+\sqrt{7}\right)$ **b** $2\sqrt{11}\left(3\sqrt{11}-5\sqrt{2}\right)$

Solution

a $\sqrt{3}\left(\sqrt{5}+\sqrt{7}\right)=\sqrt{3}\times\sqrt{5}+\sqrt{3}\times\sqrt{7}$

$=\sqrt{15}+\sqrt{21}$

b $2\sqrt{11}\left(3\sqrt{11}-5\sqrt{2}\right)=2\sqrt{11}\times 3\sqrt{11}-2\sqrt{11}\times 5\sqrt{2}$

$=6\times 11-10\times\sqrt{22}$

$=66-10\sqrt{22}$

Example 10

Expand and simplify each binomial product.

a $\left(\sqrt{7}+\sqrt{5}\right)\left(3\sqrt{2}-\sqrt{3}\right)$ **b** $\left(3-2\sqrt{10}\right)\left(\sqrt{5}-3\sqrt{2}\right)$

Solution

a $\left(\sqrt{7}+\sqrt{5}\right)\left(3\sqrt{2}-\sqrt{3}\right)=\sqrt{7}\left(3\sqrt{2}-\sqrt{3}\right)+\sqrt{5}\left(3\sqrt{2}-\sqrt{3}\right)$

$=\sqrt{7}\times 3\sqrt{2}-\sqrt{7}\times\sqrt{3}+\sqrt{5}\times 3\sqrt{2}-\sqrt{5}\times\sqrt{3}$

$=3\sqrt{14}-\sqrt{21}+3\sqrt{10}-\sqrt{15}$

b $\left(3-2\sqrt{10}\right)\left(\sqrt{5}-3\sqrt{2}\right)=3\left(\sqrt{5}-3\sqrt{2}\right)-2\sqrt{10}\left(\sqrt{5}-3\sqrt{2}\right)$

$=3\times\sqrt{5}-3\times 3\sqrt{2}-2\sqrt{10}\times\sqrt{5}+2\sqrt{10}\times 3\sqrt{2}$

$=3\sqrt{5}-9\sqrt{2}-2\sqrt{50}+6\sqrt{20}$

$=3\sqrt{5}-9\sqrt{2}-2\left(5\sqrt{2}\right)+6\left(2\sqrt{5}\right)$

$=3\sqrt{5}-9\sqrt{2}-10\sqrt{2}+12\sqrt{5}$

$=15\sqrt{5}-19\sqrt{2}$

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Special binomial products (see Chapter 12)

Perfect squares

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

Difference of 2 squares

$(a + b)(a - b) = a^2 - b^2$



Video
Binomial products involving surds

Example 11

Expand and simplify each expression.

a $\left(\sqrt{7}-\sqrt{5}\right)^2$

b $\left(2\sqrt{3}+3\sqrt{5}\right)^2$

c $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)$

d $\left(3\sqrt{11}+4\right)\left(3\sqrt{11}-4\right)$

Solution

a $\left(\sqrt{7}-\sqrt{5}\right)^2=\left(\sqrt{7}\right)^2-2\times\sqrt{7}\times\sqrt{5}+\left(\sqrt{5}\right)^2$ using $(a - b)^2 = a^2 - 2ab + b^2$

$=7-2\sqrt{35}+5$

$=12-2\sqrt{35}$

b $\left(2\sqrt{3}+3\sqrt{5}\right)^2=\left(2\sqrt{3}\right)^2+2\times2\sqrt{3}\times3\sqrt{5}+\left(3\sqrt{5}\right)^2$ using $(a + b)^2 = a^2 + 2ab + b^2$

$=(4\times3)+12\sqrt{15}+(9\times5)$

$=12+12\sqrt{15}+45$

$=57+12\sqrt{15}$

c $\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)=\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2$ using $(a + b)(a - b) = a^2 - b^2$

$= 5 - 2$

$= 3$

Note that because of the 'difference of 2 squares', the answer is not a surd but a rational number.

d $\left(3\sqrt{11}+4\right)\left(3\sqrt{11}-4\right)=\left(3\sqrt{11}\right)^2-4^2$ using $(a + b)(a - b) = a^2 - b^2$

$= (9 \times 11) - 16$

$= 83$

Binomial products involving surds U F

EXAMPLE 9

1 Expand and simplify each expression.

a $\sqrt{5}(\sqrt{3}+\sqrt{2})$ **b** $\sqrt{6}(\sqrt{2}-1)$ **c** $\sqrt{2}(\sqrt{3}+\sqrt{7})$

d $\sqrt{5}(3\sqrt{2}-\sqrt{5})$ **e** $3\sqrt{2}(\sqrt{2}+2\sqrt{3})$ **f** $-\sqrt{11}(4-\sqrt{5})$

g $2\sqrt{7}(3\sqrt{7}-4)$ **h** $5\sqrt{5}(1+3\sqrt{5})$ **i** $3\sqrt{2}(4\sqrt{2}+\sqrt{3})$

EXAMPLE 10

2 Expand and simplify $(\sqrt{3}+2\sqrt{5})(5\sqrt{2}+\sqrt{3})$. Select the correct answer **A**, **B**, **C** or **D**.

A $20\sqrt{10}$ **B** $2\sqrt{15}+5\sqrt{6}$

C $5\sqrt{6}+3+10\sqrt{10}+2\sqrt{15}$ **D** $5\sqrt{5}+\sqrt{3}+7\sqrt{7}+4\sqrt{2}$

3 Expand and simplify each expression.

a $(\sqrt{5}-3)(2\sqrt{5}+\sqrt{2})$ **b** $(\sqrt{7}-\sqrt{3})(\sqrt{7}+2)$

c $(7\sqrt{3}+2)(4\sqrt{2}+\sqrt{3})$ **d** $(3\sqrt{2}-\sqrt{5})(5\sqrt{2}+2\sqrt{5})$

e $(\sqrt{7}+2\sqrt{11})(3\sqrt{7}+4\sqrt{11})$ **f** $(5\sqrt{3}-2\sqrt{2})(4\sqrt{3}-3\sqrt{2})$

g $(6+2\sqrt{10})(3\sqrt{10}-1)$ **h** $(\sqrt{7}-2\sqrt{5})(3\sqrt{5}+2\sqrt{7})$

EXAMPLE 11

4 Expand $(5+\sqrt{7})^2$. Select **A**, **B**, **C** or **D**.

A 12 **B** 32 **C** $32+10\sqrt{7}$ **D** $32+5\sqrt{7}$

5 Expand and simplify each expression.

a $(\sqrt{5}-\sqrt{3})^2$ **b** $(\sqrt{7}+\sqrt{2})^2$

c $(\sqrt{5}-2)^2$ **d** $(3+\sqrt{10})^2$

e $(5\sqrt{2}+3\sqrt{3})^2$ **f** $(5\sqrt{7}-2)^2$

g $(3\sqrt{2}+2\sqrt{5})^2$ **h** $(2\sqrt{5}+\sqrt{3})^2$

6 Expand and simplify each expression.

a $(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$ **b** $(5+\sqrt{3})(5-\sqrt{3})$ **c** $(6+2\sqrt{7})(6-2\sqrt{7})$

d $(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})$ **e** $(\sqrt{11}-\sqrt{10})(\sqrt{11}+\sqrt{10})$ **f** $(5\sqrt{7}+3)(5\sqrt{7}-3)$

g $(3\sqrt{2}+\sqrt{5})(3\sqrt{2}-\sqrt{5})$ **h** $(4\sqrt{2}-5\sqrt{3})(4\sqrt{2}+5\sqrt{3})$

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PATH

7 Expand and simplify $(5\sqrt{2}-4\sqrt{3})(5\sqrt{2}+4\sqrt{3})$. Select **A**, **B**, **C** or **D**.

A $25\sqrt{2}-16\sqrt{3}$ **B** $10\sqrt{2}+10\sqrt{6}$ **C** 2 **D** 26

8 Expand and simplify each expression.

a $(3\sqrt{7}-5)^2$ **b** $(5\sqrt{2}-4)(\sqrt{2}+5)$ **c** $(2\sqrt{7}+3\sqrt{5})(\sqrt{5}+\sqrt{7})$

d $(4\sqrt{3}+5)^2$ **e** $(4\sqrt{2}+\sqrt{3})(4\sqrt{2}-\sqrt{3})$ **f** $(3\sqrt{10}-\sqrt{2})^2$

13.06 Rationalising the denominator

PATH

If $\sqrt{2}\approx 1.4142$, what is the value of $\frac{3}{\sqrt{2}}$? Fractions containing surds in the denominator are difficult to work with. When approximating the value of $\frac{3}{\sqrt{2}}$, it is difficult to mentally divide by 1.4142. We can overcome this by making the denominator **rational** (that is, **not** a surd).

Numbers that have irrational denominators, such as $\frac{1}{\sqrt{5}}$, $\frac{3}{2\sqrt{7}}$, $\frac{\sqrt{3}}{\sqrt{2}}$, $\frac{5\sqrt{7}}{\sqrt{3}}$ can be rewritten with a rational denominator by multiplying both the numerator and denominator by the surd that appears in the denominator. This method is called **rationalising the denominator**.

Video
Rationalising the denominator

Example 12

Rationalise the denominator of each surd.

a $\frac{3}{\sqrt{2}}$ **b** $\frac{5}{4\sqrt{3}}$ **c** $\frac{8\sqrt{2}}{3\sqrt{5}}$ **d** $\frac{\sqrt{2}+1}{\sqrt{3}}$

Solution

a $\frac{3}{\sqrt{2}}=\frac{3}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$ because $\frac{\sqrt{2}}{\sqrt{2}}=1$

$=\frac{3\sqrt{2}}{2}$

Note: Now it is easier to approximate $\frac{3}{\sqrt{2}}$ by approximating $\frac{3\sqrt{2}}{2}$ and mentally multiplying $\frac{3}{2}$ by 1.4142 than by dividing 3 by 1.4142.

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b $\frac{5}{4\sqrt{3}} = \frac{5}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{5\sqrt{3}}{4 \times 3}$

$= \frac{5\sqrt{3}}{12}$

c $\frac{8\sqrt{2}}{3\sqrt{5}} = \frac{8\sqrt{2}}{3\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

$= \frac{8\sqrt{10}}{3 \times 5}$

$= \frac{8\sqrt{10}}{15}$

d $\frac{\sqrt{2}+1}{\sqrt{3}} = \frac{\sqrt{2}+1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{6}+\sqrt{3}}{3}$

PATH

13.06

EXERCISE 13.06 ANSWERS ON P. 638

Rationalising the denominator U F

EXAMPLE 12

1 By rationalising the denominator, which surd is equivalent to $\frac{2}{\sqrt{6}}$?
Select the correct answer **A**, **B**, **C** or **D**.

A $2\sqrt{6}$ **B** $\frac{\sqrt{6}}{3}$ **C** $\frac{\sqrt{6}}{6}$ **D** $\frac{2\sqrt{6}}{3}$

2 Rationalise the denominator of each surd.

a $\frac{1}{\sqrt{2}}$ **b** $\frac{1}{\sqrt{7}}$ **c** $\frac{1}{\sqrt{3}}$ **d** $\frac{3}{\sqrt{2}}$

e $\frac{2}{\sqrt{7}}$ **f** $\frac{1}{3\sqrt{2}}$ **g** $\frac{1}{2\sqrt{3}}$ **h** $\frac{1}{4\sqrt{7}}$

i $\frac{7}{3\sqrt{5}}$ **j** $\frac{\sqrt{2}}{3\sqrt{5}}$ **k** $\frac{3\sqrt{2}}{2\sqrt{6}}$ **l** $\frac{5\sqrt{3}}{4\sqrt{5}}$

3 Which surd is equivalent to $\frac{\sqrt{3}}{2\sqrt{5}}$? Select **A**, **B**, **C** or **D**.

A $\frac{\sqrt{15}}{10}$ **B** $2\sqrt{15}$ **C** $\frac{\sqrt{15}}{3}$ **D** $\sqrt{5}$

4 Which surd is equivalent to $\frac{\sqrt{27}}{3\sqrt{18}}$? Select **A**, **B**, **C** or **D**.

A $\frac{1}{2}$ **B** $\frac{\sqrt{2}}{2}$ **C** $\frac{\sqrt{5}}{6}$ **D** $\frac{\sqrt{6}}{6}$

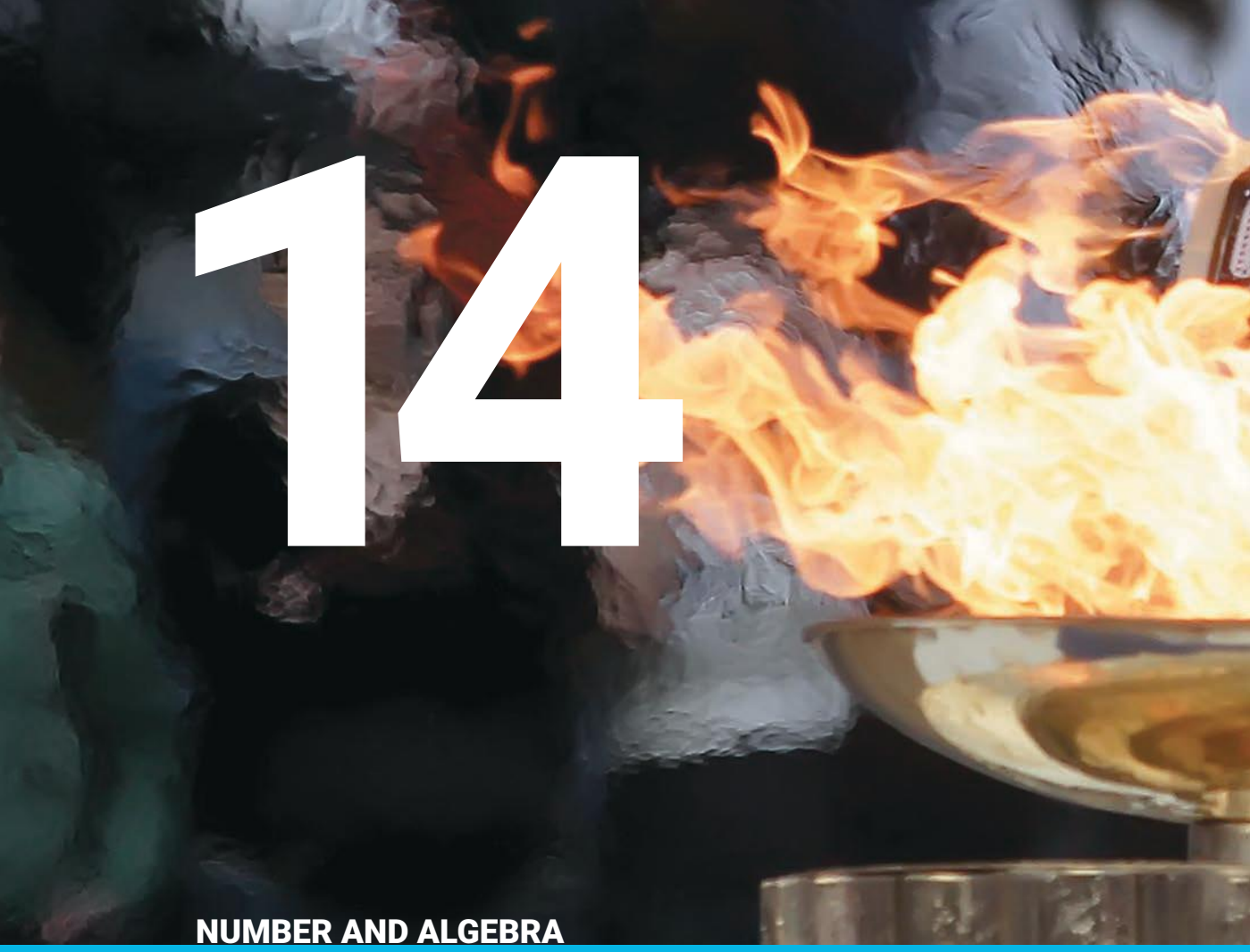
5 Rationalise the denominator of each expression.

a $\frac{\sqrt{2}-1}{\sqrt{2}}$ **b** $\frac{1-\sqrt{5}}{\sqrt{5}}$ **c** $\frac{5+\sqrt{3}}{2\sqrt{2}}$ **d** $\frac{\sqrt{2}-\sqrt{3}}{3\sqrt{6}}$

6 Simplify each expression, giving the answer with a rational denominator.

a $\frac{1}{\sqrt{7}} + \frac{1}{\sqrt{2}}$ **b** $\frac{\sqrt{2}}{\sqrt{5}} + \frac{3}{\sqrt{3}}$ **c** $\frac{3}{2\sqrt{3}} - \frac{1}{\sqrt{2}}$

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14

NUMBER AND ALGEBRA

QUADRATIC EQUATIONS AND THE PARABOLA

OPTIONAL PATH TOPIC
RECOMMENDED FOR STAGE 6 MATHEMATICS ADVANCED

Every 4 years, a few months before the Olympic Games Opening Ceremony, the Olympic flame is lit in a symbolic ceremony near the Temple of Hera, at the sacred site of the Ancient Olympics in Greece.

The flame is lit by traditional means, using the Sun's rays, to demonstrate its purity. To ensure it is lit by natural means, a parabolic mirror is used, which acts as a giant magnifying glass.

The Olympic flame has been used to light the Olympic cauldron for every Olympic Games Opening ceremony, since the Summer Olympic Games held in Berlin, Germany in 1936.


Ververidis Vasilis/Shutterstock.com

Chapter outline

		Working mathematically				
14.01	Quadratic equations $ax^2 + bx + c = 0$*	U	F	PS	R	C
14.02	Completing the square*	U	F		R	C
14.03	The quadratic formula*	U	F			C
14.04	Which method is best?*	U	F	PS	R	C
14.05	Graphing quadratic equations $y = ax^2 + bx + c$*	U	F		R	C
14.06	The axis of symmetry and vertex*	U	F		R	C
14.07	Non-linear simultaneous equations#	U	F		R	C

*PATH

Note: Chapter 12, *Products and factors*, must be completed before this chapter.

Wordbank

axis of symmetry The vertical line that divides a parabola in half and passes through the parabola's vertex

coefficient The number in front of a variable; for example, in $y = 3x^2 + 4x - 6$, the coefficient of x^2 is 3

quadratic equation An equation in which the highest power of the variable is 2; for example, $2x^2 - 12x + 10 = 0$

quadratic expression An algebraic expression in which the highest power of the variable is 2; for example, $3x^2 + 4x - 6$

quadratic formula A formula for solving quadratic equations of the form $ax^2 + bx + c = 0$

solution The answer to an equation, inequality or problem, the correct value(s) of the variable that makes an equation or inequality true

vertex A turning point of a curve; a parabola has one vertex

***x*-intercept** The *x*-value(s) at which a graph intersects the *x*-axis

Quiz
Wordbank 14

U = Understanding | **F** = Fluency | **PS** = Problem solving | **R** = Reasoning | **C** = Communication

Videos (5):
14.01 Quadratic equations by factorising
14.02 Solving quadratic equations
• Simple quadratic equations
14.03 The quadratic formula
• Solving quadratic equations
14.07 Non-linear simultaneous equations

Quizzes (2):
- Wordbank 14
- SkillCheck 14

Worksheets (5):
14.03 Investigating quadratic equations
14.05 Graphing parabolas 2
14.06 Features of a parabola
• A page of parabolas • Parabolas

Puzzles (4):
14.01 Factorising quadratic equations
14.02 Complete the square order activity
14.03 Quadratic equations puzzle
• The quadratic formula

Technology (2):
14.03 The quadratic formula
14.05 Investigating parabolas 1

Spreadsheet (2):
14.03 The quadratic formula
14.05 Investigating parabolas 1

Nelson MindTap

To access resources above, visit
cengage.com.au/nelsonmindtap

In this chapter you will:

- (PATH) solve quadratic equations of the form $ax^2 + bx + c = 0$ by factorising, completing the square and using the quadratic formula
- (PATH) solve problems involving quadratic equations
- (PATH) graph parabolas of the form $y = ax^2 + bx + c$ and identify their features
- (PATH) find the axis of symmetry and the vertex of a parabola
- (PATH) find the point of intersection of a line with a parabola or hyperbola by solving linear and non-linear simultaneous equations

PATH

Quiz
SkillCheck 14

SkillCheck ANSWERS ON P. 638

1 Solve each quadratic equation.

a $4x^2 = 100$ **b** $3m^2 = 12$ **c** $x^2 - 81 = 0$
d $12k^2 = 3$ **e** $4u^2 - 49 = 0$ **f** $7w^2 - 63 = 0$

2 Factorise each expression.

a $16 - m^2$ **b** $d^2 - 121$ **c** $14y - 2y^2$
d $10p^2 + 25p$ **e** $5x^2 - 320$ **f** $18w^2 - 50$
g $k^2 + 5k + 4$ **h** $y^2 - 10y + 16$ **i** $m^2 - m - 56$
j $u^2 + 8u - 65$ **k** $w^2 - 10w + 21$ **l** $x^2 - 2x - 24$

3 If $y = x^2 + 4x - 7$, find the value of y if:

a $x = 1$ **b** $x = -1$ **c** $x = 2$ **d** $x = -\frac{1}{2}$

4 Factorise each expression.

a $3a^2 + 10a + 3$ **b** $5x^2 - 13x - 6$ **c** $6y^2 + y - 40$

d $15t^2 + 7t - 4$ **e** $5v^2 - 32v - 21$ **f** $8y^2 + 34y + 35$

g $15h^2 - 23h + 4$ **h** $12p^2 + 11p - 15$ **i** $16d^2 + 40d + 25$

PATH

14.01

Quadratic equations $ax^2 + bx + c = 0$ 14.01

In Chapter 6, *Equations and inequalities*, we learnt that in a **quadratic equation**, the highest power of the variable is 2; for example, $x^2 = 5$, $3m^2 + 7 = 10$, $d^2 - d - 6 = 0$ and $4y^2 - 3y = 8$.

The general form of a quadratic equation is $\boldsymbol{ax^2 + bx + c = 0}$, where a, b and c are constants (numbers) and $a \neq 0$.

Solving a quadratic equation means finding those values of the variable that satisfy the equation (or make the statement true). When checking solutions, substitute the possible solutions into the equation and show that the left-hand side (LHS) of the equation is equal to the right-hand side (RHS) of the equation.

PATH

Puzzle
Factorising quadratic equations

Solving $ax^2 + bx + c = 0$ by factorising

To solve quadratic equations of the form $ax^2 + bx + c = 0$, we need to factorise the quadratic expression on the LHS (see Chapter 12, *Products and factors*).

Example 1

Solve each quadratic equation.

a $4w(3w + 2) = 0$ **b** $(5m + 2)(2m - 7) = 0$

Solution

a $4w(3w + 2) = 0$

$\therefore 4w = 0$ or $3w + 2 = 0$

$w = 0$ or $3w = -2$

$w = 0$ or $w = -\frac{2}{3}$

b $(5m + 2)(2m - 7) = 0$

$5m + 2 = 0$ or $2m - 7 = 0$

$5m = -2$ or $2m = 7$

$m = -\frac{2}{5}$ or $m = \frac{7}{2}$

$m = -\frac{2}{5}$ or $m = 3\frac{1}{2}$

PATH



Video
Quadratic equations by factorising

Example 2

Solve each quadratic equation. Use substitution to check your solutions.

a $6p^2 - 9p = 0$ **b** $2x^2 - x - 15 = 0$ **c** $6y^2 = -7y + 5$

Solution

a $6p^2 - 9p = 0$ Factorising

$3p(2p - 3) = 0$

$3p = 0$ or $2p - 3 = 0$

$p = 0$ or $2p = 3$

$p = 0$ or $p = \frac{3}{2}$

$p = 0$ or $p = 1\frac{1}{2}$

Check: When $p = 0$, LHS $= 6 \times 0^2 - 9 \times 0 = 0 =$ RHS

When $p = 1\frac{1}{2}$, LHS $= 6 \times \left(1\frac{1}{2}\right)^2 - 9 \times 1\frac{1}{2} = 0 =$ RHS

b $2x^2 - x - 15 = 0$

$2x^2 - 6x + 5x - 15 = 0$ Product $= -30$, sum $= -1$

$2x(x - 3) + 5(x - 3) = 0$

$(x - 3)(2x + 5) = 0$

$x - 3 = 0$ or $2x + 5 = 0$

$x = 3$ or $2x = -5$

$x = 3$ or $x = -\frac{5}{2}$

$x = 3$ or $x = -2\frac{1}{2}$

Check: When $x = 3$, LHS $= 2 \times 3^2 - 3 - 15 = 0 =$ RHS

When $x = -2\frac{1}{2}$, LHS $= 2 \times \left(-2\frac{1}{2}\right)^2 - \left(-2\frac{1}{2}\right) - 15 = 0 =$ RHS

c $6y^2 = -7y + 5$

$6y^2 + 7y - 5 = 0$ Move RHS to LHS to make '$= 0$'

$6y^2 - 3y + 10y - 5 = 0$ Product $= -30$, sum $= 7$

$3y(2y - 1) + 5(2y - 1) = 0$

$(2y - 1)(3y + 5) = 0$

$2y - 1 = 0$ or $3y + 5 = 0$

$2y = 1$ or $3y = -5$

$y = \frac{1}{2}$ or $y = -\frac{5}{3}$

$y = \frac{1}{2}$ or $y = -1\frac{2}{3}$

Check: When $y=\frac{1}{2}$:

$\text{LHS}=6\times\left(\frac{1}{2}\right)^2=1\frac{1}{2}$

$\text{RHS}=-7\times\left(\frac{1}{2}\right)+5=1\frac{1}{2}$

LHS = RHS

When $y=-1\frac{2}{3}$:

$\text{LHS}=6\times\left(-1\frac{2}{3}\right)^2=16\frac{2}{3}$

$\text{RHS}=-7\times\left(-1\frac{2}{3}\right)+5=16\frac{2}{3}$

LHS = RHS

PATH

14.01

EXERCISE 14.01 ANSWERS ON P. 639

Quadratic equations $ax^2 + bx + c = 0$ U F PS R C

EXAMPLE 1

1 Solve each quadratic equation.

a	$(m+7)(m+3)=0$	**b**	$(d-3)(d-7)=0$	**c**	$(y-3)(y+5)=0$
d	$k(k-3)=0$	**e**	$t(t+7)=0$	**f**	$2p(p-3)=0$
g	$w(3w-2)=0$	**h**	$(2n+1)(n-3)=0$	**i**	$(5a-3)(2a-1)=0$
j	$(3x+1)(2x+3)=0$	**k**	$(2c-5)^2=0$	**l**	$(1-2f)^2=0$
m	$(3c+1)(4c+1)=0$	**n**	$(1-2h)(h+1)=0$	**o**	$(5-7e)(1-e)=0$

EXAMPLE 2

2 Solve each quadratic equation.

a	$2y^2+7y+6=0$	**b**	$2g^2+5g+3=0$	**c**	$3d^2+5d+2=0$
d	$5t^2+16t+11=0$	**e**	$2p^2-11p+12=0$	**f**	$10x^2-19x+6=0$
g	$8y^2-2y-3=0$	**h**	$6a^2-5a-4=0$	**i**	$4w^2-7w-15=0$
j	$5c^2+2c-7=0$	**k**	$8e^2+10e-3=0$	**l**	$3q^2+4q-15=0$
m	$4g^2-20g+25=0$	**n**	$18m^2-3m-10=0$	**o**	$16-8w-3w^2=0$
p	$36+3y-3y^2=0$	**q**	$2f^2-24f+72=0$	**r**	$12h^2+3h-9=0$

3 Express each quadratic equation in the form $ax^2 + bx + c = 0$ and solve.

a	$2x^2=x+15$	**b**	$4t(t+2)=5$	**c**	$41u=-8u^2-5$
d	$7m^2=8m-1$	**e**	$p(p-3)=28$	**f**	$(e-2)^2=9$
g	$t(2t-13)=-15$	**h**	$7=6d^2+11d$	**i**	$5h^2=125$
j	$8f^2=4f$	**k**	$6w^2+3=19w$	**l**	$4a(3a+5)-8=0$

4 A certain positive number, plus its square, minus 72, equals 0.
Find the number. PS R C

Foundation | Standard | Complex

14.02 Completing the square

PATH

Video
Solving quadratic equations

Puzzle
Complete the square order activity

Quadratic equations can be solved by using a method called **completing the square**. We try to make the LHS of the equation a perfect square (see Chapter 12, *Products and factors*).

The method of **completing the square** is based on the following results for perfect squares.

$(x + a)^2 = x^2 + 2ax + a^2$

$(x - a)^2 = x^2 - 2ax + a^2$

We note that the last term, a^2, is the square of 'half the coefficient of x'.

Example 3

Find the numbers that complete the square in each equation.

a $x^2 + 10x + ... = (x + ...)^2$

b $x^2 - 14x + ... = (x - ...)^2$

Solution

a The coefficient of x is 10.
Half of 10 is 5, and $5^2 = 25$.
The perfect square is
$x^2 + 10x + 25 = (x + 5)^2$

b The coefficient of x is -14.
Half of -14 is -7, and $(-7)^2 = 49$.
The perfect square is
$x^2 - 14x + 49 = (x - 7)^2$

Video
Simple quadratic equations

Example 4

Solve $(k + 3)^2 = 7$.

Solution

$(k + 3)^2 = 7$

$k + 3 = \pm\sqrt{7}$ Taking the square root of both sides.

$k + 3 - 3 = \sqrt{7} - 3$ **or** $k + 3 - 3 = -\sqrt{7} - 3$

This solution is usually written as:

$k = -3 + \sqrt{7}$ **or** $k = -3 - \sqrt{7}$

Notice that the 2 solutions are surds (not rational), of the form $a + \sqrt{b}$ and $a - \sqrt{b}$.

Example 5

Solve $x^2 + 6x - 2 = 0$ by completing the square.

Solution

Step 1

Move the constant term to the RHS.

$x^2 + 6x = 2$

Step 2

Halve the coefficient of x, square it and then add the square to both sides.

$x^2 + 6x + 3^2 = 2 + 3^2$

$x^2 + 6x + 9 = 11$

Step 3

Express the LHS as a perfect square.

$(x + 3)^2 = 11$

Step 4

Solve the resulting equation.

$x + 3 = \pm\sqrt{11}$ — Taking the square root of both sides.

$x = -3 \pm \sqrt{11}$ — This answer in surd form is called the 'exact answer'.

$x = -3 + \sqrt{11}$ or $x = -3 - \sqrt{11}$

Example 6

Solve $2x^2 - 3x - 4 = 0$, writing the solution correct to 2 decimal places.

Solution

$2x^2 - 3x - 4 = 0$

$2x^2 - 3x = 4$ — Moving the constant term to the RHS.

$x^2 - \frac{3}{2}x = 2$ — Dividing both sides by 2, the coefficient of x^2.

$x^2 - \frac{3}{2}x + \left(-\frac{3}{4}\right)^2 = 2 + \left(-\frac{3}{4}\right)^2$ — Completing the square.

$\left(x - \frac{3}{4}\right)^2 = \frac{41}{16}$ — Expressing the LHS as a perfect square and leaving the RHS as an improper fraction.

$x - \frac{3}{4} = \pm\sqrt{\frac{41}{16}}$ — Taking the square root of both sides.

$x - \frac{3}{4} = \pm\frac{\sqrt{41}}{4}$ — Simplifying the RHS.

$x = \frac{3}{4} \pm \frac{\sqrt{41}}{4}$ — This is the exact answer in surd form.

$x = \frac{3 \pm \sqrt{41}}{4}$

$x = \frac{3+\sqrt{41}}{4}$ or $x = \frac{3-\sqrt{41}}{4}$ — This is an approximate answer in decimal form.

$x = 2.35078...$ or $x = -0.85078...$

$x \approx 2.35$ or $x \approx -0.85$

PATH

EXERCISE 14.02 ANSWERS ON P. 639

Completing the square U F R C

EXAMPLE 3

1 Find the numbers that 'complete the square' in each equation. R C

a $x^2 + 2x + ___ = (x + ___)^2$

b $p^2 - 6p + ___ = (p - ___)^2$

c $m^2 - 8m + ___ = (m - ___)^2$

d $k^2 + 4k + ___ = (k + ___)^2$

e $y^2 - 7y + ___ = (y - ___)^2$

f $w^2 - 3w + ___ = (w - ___)$

g $x^2 + x + ___ = (x + ___)^2$

h $h^2 - 5h + ___ = (h - ___)^2$

i $a^2 + \frac{7}{2}a + ___ = (a + ___)^2$

j $v^2 + \frac{5}{3}v + ___ = (v + ___)^2$

EXAMPLE 4

2 Solve each equation, writing the solution in surd form. R C

a $(d + 3)^2 = 7$

b $(x - 5)^2 = 5$

c $(p + 1)^2 = 10$

d $(y - 1)^2 = 2$

e $\left(m - \frac{1}{2}\right)^2 = 5$

f $\left(t + \frac{2}{3}\right)^2 = 3$

g $(c + 1)^2 = \frac{21}{2}$

h $(w - 3)^2 = \frac{41}{2}$

i $\left(n + \frac{2}{3}\right)^2 = \frac{7}{9}$

j $\left(e - \frac{3}{2}\right)^2 = \frac{71}{4}$

k $(d - 2)^2 = 5$

l $\left(x - \frac{3}{4}\right)^2 = 2$

EXAMPLE 5

3 Solve each equation by completing the square. Leave your answers in exact form. R C

a $h^2 + 2h - 5 = 0$

b $r^2 - 2r - 1 = 0$

c $m^2 + 6m + 2 = 0$

d $w^2 - 4w - 60 = 0$

e $a^2 - 10a - 5 = 0$

f $y^2 + 8y - 3 = 0$

g $p^2 + 12p - 5 = 0$

h $x^2 - 4x + 2 = 0$

i $u^2 + 9u + 14 = 0$

j $d^2 + d - 7 = 0$

k $c^2 - 9c + 2 = 0$

l $e^2 + 5e + 2 = 0$

m $y^2 - 3y - 8 = 0$

n $b^2 - b - 5 = 0$

o $q^2 - 3q + 1 = 0$

p $2g^2 + 7g = 3$

q $2x^2 = 7 - 5x$

r $3f(f + 4) = 6$

EXAMPLE 6

4 Solve each of these by completing the square. Give your answers correct to 2 decimal places if irrational. R C

a $x^2 + 12x + 9 = 0$

b $m^2 - 16m - 7 = 0$

c $g^2 + 4g - 3 = 0$

d $2h^2 + 3h - 7 = 0$

e $5w^2 - 4w - 3 = 0$

f $3y^2 + y - 5 = 0$

g $p(3p + 2) = 8$

h $4e^2 - 4 = e$

i $2n^2 = 5 - 3n$

Foundation Standard Complex

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The quadratic formula

14.03

There is a formula for solving a quadratic equation of the form $ax^2 + bx + c = 0$ that involves the coefficients a, b and c.

The quadratic formula

The solutions to the quadratic equation $ax^2 + bx + c = 0$ are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 7

Solve each quadratic equation using the quadratic formula.

a $6x^2 + x - 2 = 0$

b $3x^2 + 11x + 2 = 0$

Solution

a For $6x^2 + x - 2 = 0$:

$a = 6$, $b = 1$, and $c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-1 \pm \sqrt{1^2 - 4 \times 6 \times (-2)}}{2 \times 6}$$
$$= \frac{-1 \pm \sqrt{49}}{12}$$
$$= \frac{-1 \pm 7}{12}$$

$x = \frac{-1+7}{12}$ or $x = \frac{-1-7}{12}$

$x = \frac{6}{12}$ or $x = \frac{-8}{12}$

$x = \frac{1}{2}$ or $x = -\frac{2}{3}$

b For $3x^2 + 11x + 2 = 0$:

$a = 3$, $b = 11$ and $c = 2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-11 \pm \sqrt{11^2 - 4 \times 3 \times 2}}{2 \times 3}$$
$$= \frac{-11 \pm \sqrt{97}}{6}$$

$x = \frac{-11 \pm \sqrt{97}}{6}$ or $x = \frac{-11 - \sqrt{97}}{6}$ In exact surd form

PATH

14.03

Worksheet
Investigating quadratic equations

Puzzles
Quadratic equations puzzle

The quadratic formula

Technology
The quadratic formula

Spreadsheet
The quadratic formula

Videos
The quadratic formula

Solving quadratic equations

PATH

Example 8

Solve $2x^2 - 6 = 9x$ using the quadratic formula, expressing the answer correct to 2 decimal places.

Solution

$2x^2 - 6 = 9x$

$2x^2 - 9x - 6 = 0$

Writing the quadratic equation in the form $ax^2 + bx + c = 0$

Substitute $a = 2, b = -9, c = -6$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4 \times 2 \times (-6)}}{2 \times 2}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{9 \pm \sqrt{129}}{4}$$

$x = \frac{9 + \sqrt{129}}{4}$ or $x = \frac{9 - \sqrt{129}}{4}$

$x = 5.0894...$ or $x = -0.5894...$

$x \approx 5.09$ or $x \approx -0.59$

EXERCISE 14.03 ANSWERS ON P. 640

The quadratic formula U F C

EXAMPLE 7

1 Solve each quadratic equation using the quadratic formula. Write each solution in exact form. C

a	$x^2 + 6x + 2 = 0$	**b**	$m^2 - 5m - 3 = 0$	**c**	$w^2 - 8w + 3 = 0$
d	$k^2 + 3k - 5 = 0$	**e**	$y^2 - 4y - 1 = 0$	**f**	$p^2 + p - 5 = 0$
g	$u^2 - 7u - 3 = 0$	**h**	$2a^2 + 3a - 7 = 0$	**i**	$5q^2 - 6q + 1 = 0$
j	$3c^2 + 2c - 2 = 0$	**k**	$4e^2 - 5e - 2 = 0$	**l**	$3x^2 + 8x + 2 = 0$
m	$2d^2 - 4d - 5 = 0$	**n**	$3a^2 - 10a - 2 = 0$	**o**	$2t^2 + 3t - 5 = 0$
p	$3y^2 + 8y + 4 = 0$	**q**	$6k^2 - 11k + 5 = 0$	**r**	$2n^2 - 5n - 11 = 0$

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2 Solve each quadratic equation using the quadratic formula, expressing the solution as a surd. C

a	$5y^2 - 9y = 3$	**b**	$3m^2 = 7 - 2m$	**c**	$4x^2 = 3x + 2$
d	$1 - 4k - k^2 = 0$	**e**	$3m^2 - 1 - 3m = 0$	**f**	$1 - 2g - 5g^2 = 0$
g	$8 = 9h - 2h^2$	**h**	$2w + 2 = 3w^2$	**i**	$4p - 3p^2 = -1$
j	$2 - 4u - 5u^2 = 0$	**k**	$6a^2 = 9 - 4a$	**l**	$10 = 3y + 2y^2$

3 Solve each quadratic equation, writing the solutions correct to 2 decimal places. C

a	$k^2 - 9k + 1 = 0$	**b**	$c^2 - 2 = 0$	**c**	$m^2 - 5 = 2$
d	$2n^2 + 2 = 7n$	**e**	$2p^2 + 3p - 4 = 0$	**f**	$6w^2 + 5w - 2 = 0$
g	$3x^2 + 2 - 8x = 0$	**h**	$h^2 = 7 + 2h$	**i**	$1 + x - x^2 = 0$
j	$36 = 17a - a^2$	**k**	$5v^2 - 11 = 0$	**l**	$5c^2 + 8 = 15c$
m	$t^2 = 5(t + 5)$	**n**	$(x - 6)^2 = 3$	**o**	$12 = 2d^2 - 3d$

PATH

14.04

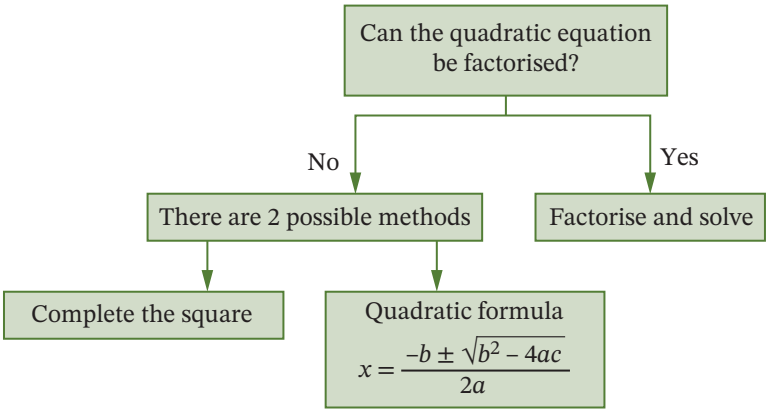
EXAMPLE 8

Which method is best?

14.04

We have learned 3 methods of solving quadratic equations.

Factorising is the simplest, but only some quadratic equations can be factorised, while the **quadratic formula** works for all quadratic equations. **Completing the square** also works for all equations, but the algebra can be messy.



PATH

Foundation Standard Complex

PATH

EXERCISE 14.04 ANSWERS ON P. 640

Which method is best? U F R

1 Solve each quadratic equation using the most suitable method. You may want to try 2 methods to compare your answers and determine how effective the methods were. R

a $x(2x - 3) = 0$ **b** $m^2 = 36$ **c** $3w^2 - 5w - 1 = 0$

d $u^2 - u - 12 = 0$ **e** $(5k - 3)(2k + 7) = 0$ **f** $16 = 25a^2$

g $x(x - 2) = 63$ **h** $n^2 - 6n = 10$ **i** $(a + 3)^2 = 5$

j $(d + 3)(d - 1) = 0$ **k** $t^2 = 4t$ **l** $(2y - 3)^2 = 11$

m $30 - x^2 = 7x$ **n** $6a^2 = 6 - 5a$ **o** $4x = 12x^2 - 3$

2 Solve each quadratic equation using an appropriate method, giving your answers correct to 2 decimal places where necessary. R

a $y^2 = 5y$ **b** $144 = a^2$ **c** $(u - 2)^2 = 11$

d $x^2 - 6x = -9$ **e** $18 + 7x - x^2 = 0$ **f** $x^2 + 10x + 5 = 0$

g $a(2a - 1) = 7$ **h** $2w^2 = 3w + 7$ **i** $(5m - 1)^2 = 18$

j $(3 - 2k)^2 = 27$ **k** $\frac{36}{x} + 1 = 2x$ **l** $\frac{h}{3h + 2} = \frac{h - 3}{4h}$

14.05 Graphing quadratic equations $y = ax^2 + bx + c$

PATH

Worksheet Graphing parabolas 2

Technology Investigating parabolas 1

Spreadsheet Investigating parabolas 1

The graph of a quadratic equation is a smooth U-shaped curve called a **parabola**. We have already graphed simple parabolas of the form $y = ax^2 + c$ in Chapter 7, *Graphing curves*.

Now we will graph parabolas of the form $y = ax^2 + bx + c$.

Example 9

For each quadratic equation:

i complete a table of values and draw the graph

ii find the x-intercepts

iii find the y-intercept

iv solve $y = 0$ and compare the solutions to the x-intercepts.

a $y = x^2 + 6x + 5$ **b** $y = 2x^2 - x - 10$

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Solution

a **i** $y = x^2 + 6x + 5$

x	−6	−5	−4	−3	−2	−1	0	1
y	5	0	−3	−4	−3	0	5	12

ii The x-intercepts are -5 and -1.

iii The y-intercept is 5.

Note that this is the constant term, $c = 5$, in $y = x^2 + 6x + 5$.

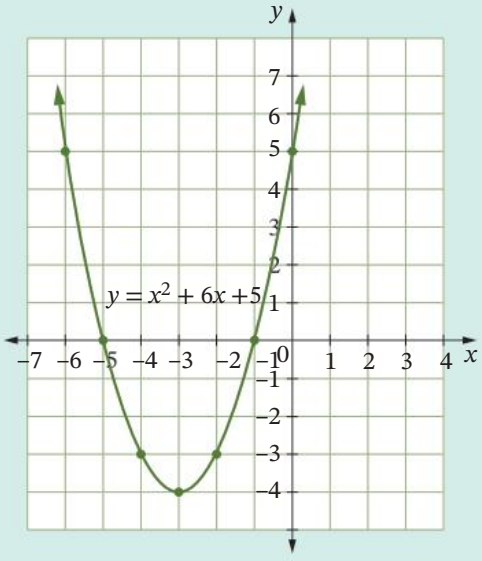
iv $x^2 + 6x + 5 = 0$

$(x + 5)(x + 1) = 0$

$x + 5 = 0$ or $x + 1 = 0$

$x = -5$ or $x = -1$

The solutions to the quadratic equation are the same as the x-intercepts of the graph of $y = x^2 + 6x + 5$.



b **i** $y = 2x^2 - x - 10$

x	−3	−2	−1	0	1	2	3
y	11	0	−7	−10	−9	−4	5

ii The x-intercepts are -2 and $2\frac{1}{2}$.

iii The y-intercept is -10.

Note that this is the constant term, $c = -10$, in $y = 2x^2 - x - 10$.

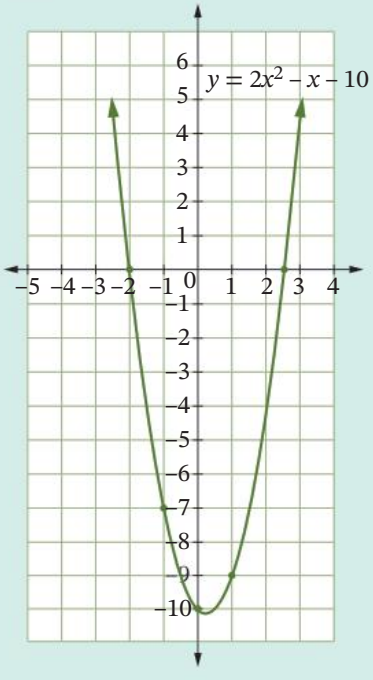
iv $2x^2 - x - 10 = 0$

$(2x - 5)(x + 2) = 0$

$2x - 5 = 0$ or $x + 2 = 0$

$x = 2\frac{1}{2}$ or $x = -2$

The solutions to the quadratic equation are the same as the x-intercepts of the graph of $y = 2x^2 - x - 10$.



PATH

The graph of the parabola $y = ax^2 + bx + c$

- If $a > 0$ (positive), the parabola is **concave up**
- If $a < 0$ (negative), the parabola is **concave down**
- The ***y*-intercept** is where $x = 0$, so it is the value of c
- The ***x*-intercepts** are where $y = 0$, so are the solutions to the quadratic equation $ax^2 + bx + c = 0$
- Because the parabola is symmetrical, the x-coordinate of its vertex is halfway between the x-intercepts.

Example 10

Graph each quadratic equation, showing its x- and y-intercepts.

a $y = x(2x - 5)$ **b** $y = 2x^2 + x - 6$ **c** $y = -2x^2 - 3x + 9$

Solution

a $y = x(2x - 5)$

$= 2x^2 - 5x$

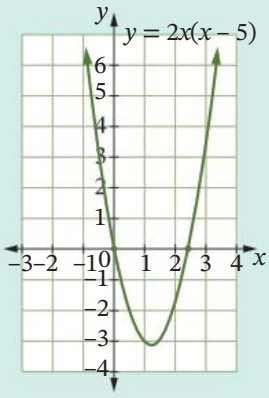
$a = 2 > 0$, so the parabola is concave up

x-intercepts: $x(2x - 5) = 0$

$x = 0$ and $2x - 5 = 0$

$x = 0$ and $x = 2\frac{1}{2}$

y-intercept: $c = 0$



b $y = 2x^2 + x - 6$

$a = 2 > 0$, so the parabola is concave up

x-intercepts:

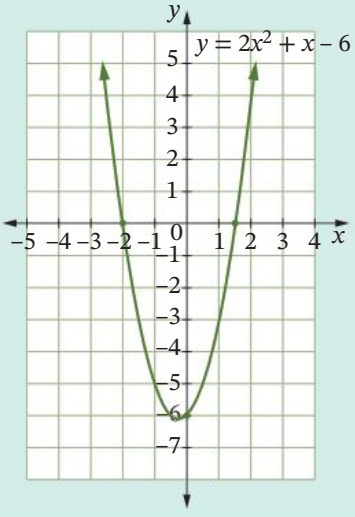
$2x^2 + x - 6 = 0$

$(2x - 3)(x + 2) = 0$

$2x - 3 = 0$ and $x + 2 = 0$

$x = 1\frac{1}{2}$ and $x = -2$

y-intercept: $c = -6$



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c $y = -2x^2 - 3x + 9$

$a = -2 < 0$, so the parabola is concave down

x-intercepts:

$-2x^2 - 3x + 9 = 0$ Dividing by -1

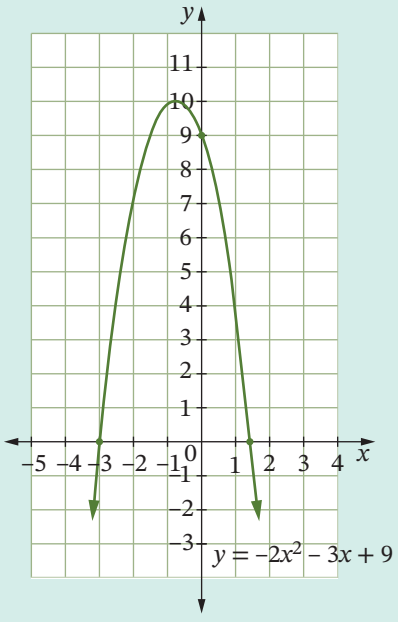
$2x^2 + 3x - 9 = 0$

$(2x - 3)(x + 3) = 0$

$2x - 3 = 0$ and $x + 3 = 0$

$x = 1\frac{1}{2}$ and $x = -3$

y-intercept: $c = 9$



EXERCISE 14.05 ANSWERS ON P. 640

Graphing quadratic equations $y = ax^2 + bx + c$ U F R C

EXAMPLE 9

1 For each quadratic equation: R C

i complete a table of values and draw the graph

ii find the x-intercepts

iii find the y-intercept

iv solve $y = 0$ and compare the solutions to the x-intercepts.

a $y = x^2 + 4x + 3$ **b** $y = 2x^2 + 7x + 3$

c $y = 2x^2 - 3x - 9$ **d** $y = x^2 - 2x$

2 Find the y-intercept of the parabolas with equation:

a $y = 3x^2 - 2x - 5$ **b** $y = 2x^2 + 6x + 3$ **c** $y = 5x^2 - 10x$.

EXAMPLE 10

3 Graph each quadratic equation, showing its x- and y-intercepts.

a $y = x(x - 4)$ **b** $y = 3x(x + 2)$ **c** $y = (x + 3)(x - 5)$

d $y = (x - 2)(2x - 5)$ **e** $y = -(x + 1)(3x + 2)$ **f** $y = x^2 + 6x + 8$

g $y = -x^2 + 4x + 5$ **h** $y = 3x^2 + 11x - 20$ **i** $y = -2x^2 + 5x - 2$

Foundation Standard Complex

14.06 The axis of symmetry and vertex

PATH

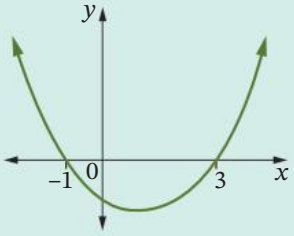
Worksheets
Features of a parabola
A page of parabolas
Parabolas

The axis of symmetry and vertex of $y = ax^2 + bx + c$

- The **axis of symmetry** of the parabola is the vertical line: $x = -\frac{b}{2a}$
- The **axis of symmetry** passes through the point halfway between the 2 x-intercepts of the parabola
- The **vertex** of the parabola lies on the axis of symmetry, so its ***x*-coordinate** is $x = -\frac{b}{2a}$ and its ***y*-coordinate** can be found by substituting the x-coordinate into the equation of the parabola

Example 11

Find the equation of the axis of symmetry of this parabola.



Solution

The x-intercepts are -1 and 3.

The equation of the axis of symmetry is:

$$x = \frac{-1+3}{2}$$
$$= 1$$

Finding the average of the x-intercepts.

Example 12

A parabola has the equation $y = 2x^2 - 4x + 3$.

a Find the equation of its axis of symmetry

b Find the coordinates of the vertex of the parabola.

Solution

a For $y = 2x^2 - 4x + 3$, $a = 2$, $b = -4$ and $c = 3$.

The axis of symmetry is:

$$x = -\frac{b}{2a}$$
$$= -\frac{-4}{2 \times 2}$$
$$= 1$$

$x = 1$ is the equation of the axis of symmetry.

b The vertex lies on the axis of symmetry.

Substitute $x = 1$ in $y = 2x^2 - 4x + 3$

$y = 2 \times 1^2 - 4 \times 1 + 3$

$= 1$

The vertex is (1, 1).

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Example 13

Sketch the parabola $y = 2x^2 + 5x + 1$.

Solution

$a = 2 > 0$, so the parabola is concave up.

y-intercept $= 1$.

$y = 2x^2 + 5x + 1$ cannot be factorised, so we cannot find the x-intercepts precisely.

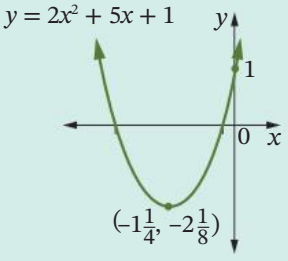
For the vertex:

$$x = -\frac{b}{2a} \quad \text{where } a = 2, \text{ and } b = 5$$
$$= -\frac{5}{2 \times 2}$$
$$= -1\frac{1}{4}$$

Substitute $x = -1\frac{1}{4}$ into $y = 2x^2 + 5x + 1$

$$y = 2 \times \left(-1\frac{1}{4}\right)^2 + 5 \times \left(-1\frac{1}{4}\right) + 1$$
$$= -2\frac{1}{8}$$

$\therefore$ The vertex is $\left(-1\frac{1}{4}, -2\frac{1}{8}\right)$.





JossK/Shutterstock.com

PATH

Graphing the parabola $y = ax^2 + bx + c$

- Use the sign of a (the coefficient of x^2) to determine whether the parabola is **concave up** or **down**
- Find the y-**intercept**, c
- Find the x-**intercepts** where possible by solving a quadratic equation
- Use $x = -\frac{b}{2a}$ to find the **axis of symmetry** and the **vertex**

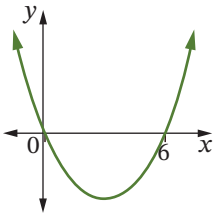
EXERCISE 14.06 ANSWERS ON P. 642

The axis of symmetry and vertex U F R C

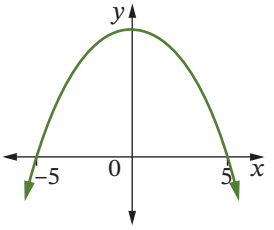
EXAMPLE 11

1 Write the equation for the axis of symmetry in each parabola. R C

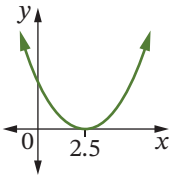
a



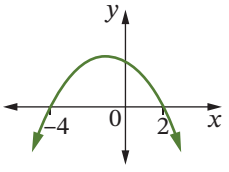
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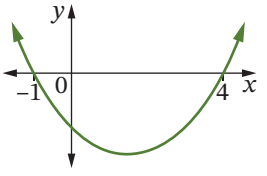
c



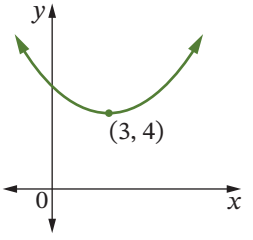
d



e



f



EXAMPLE 12

2 For each parabola with the equation given below:

i find the equation of its axis of symmetry

ii find the coordinates of its vertex.

a	$y = x^2 - 6x + 8$	**b**	$y = -x^2 + 10x - 9$	**c**	$y = x^2 - 2x + 10$
d	$y = -x^2 + 8x + 9$	**e**	$y = -x^2 + x - 25$	**f**	$y = 5x^2 + 40x$
g	$y = 24x - 3x^2$	**h**	$y = 4x^2 + 2x - 1$	**i**	$y = 1 - 3x - 9x^2$

EXAMPLE 13

3 Graph each quadratic equation, showing:

i the x-intercepts (correct to one decimal place where necessary)

ii the y-intercept

iii the equation of the axis of symmetry

iv the coordinates of the vertex

v the concavity.

a	$y = x^2 - 6x - 40$	**b**	$y = x^2 - 3x$	**c**	$y = 2x^2 + 3x + 4$
d	$y = -x^2 + 6x + 5$	**e**	$y = -4x^2 - 12x + 21$	**f**	$y = x^2 - 8x + 3$
g	$y = 5x^2 + 7x + 4$	**h**	$y = 8x - 2x^2$	**i**	$y = -2x^2 + 7x - 6$

Foundation Standard Complex

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Non-linear simultaneous equations

14.07

The points of intersection of a line with a parabola, circle or hyperbola may be found either graphically or algebraically. The algebraic method involves solving simultaneous equations using the substitution method, which is covered in Chapter 11, *Simultaneous equations.*

PATH

14.07

Video
Non-linear simultaneous equations

Example 14

Find the points of intersection of:

a the line $y = 2x - 3$ and the parabola $y = x^2 - 3x + 1$

b the line $y = x + 2$ and the circle $x^2 + y^2 = 4$

c the line $y = x + 5$ and the hyperbola $y = \frac{6}{x}$.

Solution

a $y = 2x - 3$ [1]

$y = x^2 - 3x + 1$ [2]

Use [2] to substitute for y in [1].

$x^2 - 3x + 1 = 2x - 3$

$x^2 - 5x + 4 = 0$

$(x - 4)(x - 1) = 0$

$x - 4 = 0$ or $x - 1 = 0$

$x = 4$ or $x = 1$

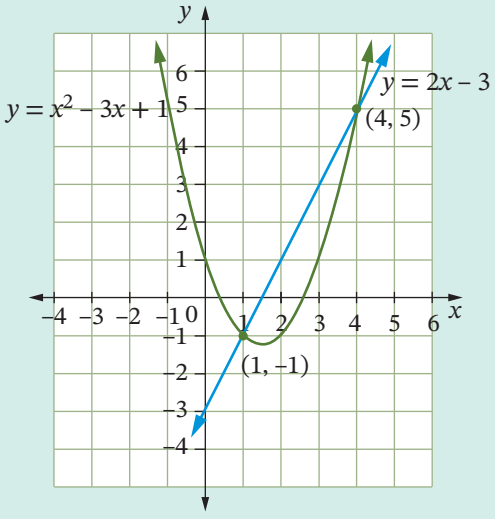
Substitute $x = 4$ and $x = 1$ into [1] to find y.

$y = 2 \times 4 - 3$ and $y = 2 \times 1 - 3$

$= 5$ $= -1$

The solutions are $x = 4, y = 5$ and $x = 1, y = -1$

The points of intersection are (4, 5) and (1, −1).



This solution can also be found by graphing both equations and finding their points of intersection, as shown in the graph.

PATH

b $y = x + 2$ [1]

$x^2 + y^2 = 4$ [2]

Use [1] to substitute for y in [2].

$$x^2 + (x + 2)^2 = 4$$
$$x^2 + x^2 + 4x + 4 = 4$$
$$2x^2 + 4x = 0$$
$$2x(x + 2) = 0$$

$2x = 0$ or $x + 2 = 0$

$x = 0$ or $x = -2$

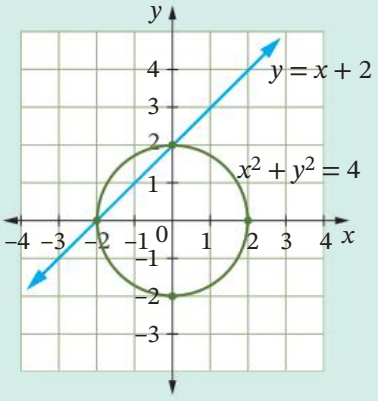
Substitute $x = 0$ and $x = -2$ into [1] to find y.

$y = 0 + 2$ and $y = -2 + 2$

$y = 2$ and $y = 0$

The points of intersection are $(0, 2)$ and $(-2, 0)$.

This solution can also be found by graphing both equations and finding their points of intersection, as shown in the graph.



c $y = x + 5$ [1]

$y = \frac{6}{x}$ [2]

Use [1] to substitute for y in [2].

$$x + 5 = \frac{6}{x}$$
$$x(x + 5) = x \times \frac{6}{x}$$
$$x^2 + 5x = 6$$
$$x^2 + 5x - 6 = 0$$
$$(x + 6)(x - 1) = 0$$

$x + 6 = 0$ or $x - 1 = 0$

$x = -6$ or $x = 1$

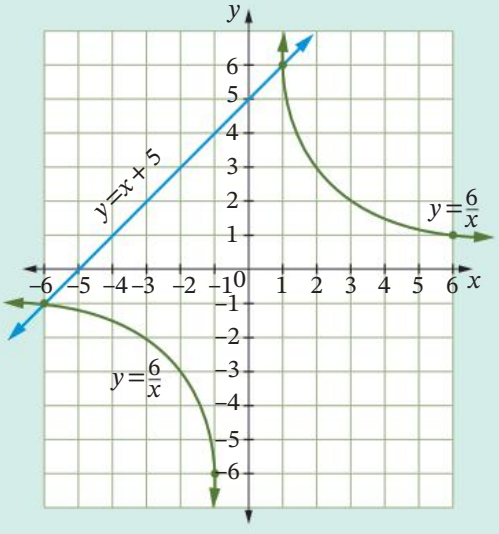
Substitute $x = -6$ and $x = 1$ into [1] to find y.

$y = -6 + 5$ and $y = 1 + 5$

$y = -1$ and $y = 6$

The points of intersection are $(-6, -1)$ and $(1, 6)$.

(Also, see their graphs).



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EXERCISE 14.07 ANSWERS ON P. 643

PATH

Non-linear simultaneous equations U F R C

EXAMPLE 14

14.07

1 Find the points of intersection of the line and the parabola with equations: R

a $y = x^2$ and $y = 5x - 6$

b $y = 2x^2$ and $y = 8x$

c $y = x^2 + 10$ and $y = 2x + 18$

d $y = 5x^2$ and $y = x + 6$

e $y = 3x^2 + 2x + 10$ and $y = 12 - 3x$

f $y = 9x - x^2$ and $y = 6x - 10$

g $y = 6x - 3$ and $y = x^2 - 8x + 46$

h $x + y = 2$ and $y = x^2 - 2x$

2 Find the points of intersection of the line: R

a $y = x - 3$ and the hyperbola $y = -\frac{2}{x}$

b $y = 1 - x$ and the circle $x^2 + y^2 = 1$

c $x + y = 3$ and the circle $x^2 + y^2 = 9$

d $y = x + 5$ and the circle $x^2 + y^2 = 25$

e $y = 5x + 2$ and the hyperbola $y = \frac{7}{x}$

f $2x - y = 1$ and the hyperbola $y = \frac{15}{x}$

g $y = 2x + 5$ and the hyperbola $y = -\frac{3}{x}$

3 Explain why there are no points of intersection between the equations $y = -3x - 1$ and $y = \frac{2}{x}$. Show working to help justify your answer. R C

Foundation | Standard | Complex

ANSWERS

Chapter 1

SkillCheck

1 **a** 0.04 **b** 0.22 **c** 0.183 **d** 0.047
e 0.095 **f** 0.0675 **g** 0.1525 **h** 0.2
2 **a** $72 **b** $116.25 **c** $4494
3 **a** $7350 **b** $4034.10 **c** $8737.60
4 **a** 36 **b** 24 **c** 60
5 **a** 52 **b** 26
c 365 **d** 4
e 12 **f** 8 years and 4 months
6 **a** 1152 **b** 50 **c** 0.06
7 **a** $5962.59 **b** $33 433.46
c $18 481.63 **d** $64937.10

Exercise 1.01

1 **a** $874 **b** $938.80 **c** $734.40
2 Greta earns more per week by $27.48.
3 **a** $3461.86 **b** $6923.73 **c** $15 053.33
4 Job 1: $1104.64; Job 2: $1160; Job 2 by $55.36
5 $1107.40 **6** $851.18 **7** $780.86
8 A **9** $13 312.50 **10** $1394.40
11 $282.98 **12** $2115 **13** 54
14 $63.95 **15** 1852.31 **16** $11 576.76
17 $219 997.02 **18** **a** $9483.55 **b** 20 973
19 **a** $854 **b** $700 **c** $956.87 **d** $625.55
20 **a** $972.12 **b** $680.48 **c** $4568.96

Exercise 1.02

1 **a** $47 697 **b** $5968.53
2 **a** $114 719 **b** $27 750.68
3 **a** $90 904 **b** $20 010.80
4 C **5** $18 555.91 **6** $55 010.50
7 $696.42 **8** $633.52
9 **a** $420 **b** $1741.10 **c** 23.6%
10 **a** $426 **b** $1774.65 **c** 22.7%
11 **a** $2296 **b** $426 **c** $1676.73
12 **a** $2297.62 **b** $426 **c** $1580.42
13 Gross weekly income = $1006.73;
Total deductions = $374.10; Net income = $632.63

Exercise 1.03

1 **a** $5040 **b** $1047.15 **c** $102.50
d $263.60 **e** $37.40 **f** $1192.32
2 **a** $87.50 **b** $5925.15 **c** $207 000
d $690 **e** $1404 **f** $723.04
3 A
4 **a** $11 200 **b** $1551.30
c $9392.50 **d** $10 695.31
5 **a** $1440 **b** $7440
6 4.5%
7 **a** $6400 **b** 5.82%
8 5.85% p.a. **9** approx. 2 years
10 30 weeks **11** 137 days
12 C
13 31 months **14** 2.6% p.a.
15 **a** $18.90 **b** $1063.90
16 B

Exercise 1.04

1 **a** Check with your teacher. Investment after 1st year = $24 150; Investment after 2nd year = $25 357.50
b Compound interest = $2357.50
2 **a** $16 153.36 **b** $1153.36
3 **a** $38 459.48 **b** $4359.48
4 **a** $5408, $408 **b** $30 245.29, $2445.29
c $11 113.20, $1513.20
d $41 905.55, $2405.55
e $19 337.39, $937.39
5 **a** $4791.80 **b** $1642.38
c $308.93 **d** $3913.84
e $6834.42

Mental skills 1

2 **a** 11 **b** 40 **c** 7 **d** 24
e 23 **f** 6 **g** 43 **h** 80
i 18 **j** 15 **k** 40 **l** 65
m 11 **n** 14 **o** 12 **p** 135

Exercise 1.05

1 B
2 **a** **i** $9754.75 **ii** $3254.75
b **i** $13 858.59 **ii** $3858.59
c **i** $12 634.81 **ii** $394.81
d **i** $43 949.46 **ii** $9349.46
e **i** $8427.39 **ii** $427.39
3 D **4** $1 301 018.83
5 B **6** A
7 **a** **i** $11 273.62 **ii** $1273.62
b **i** $52 751.13 **ii** $17 251.13
c **i** $9448.23 **ii** $548.23
d **i** $48 063.26 **ii** $6063.26
e **i** $17 829.56 **ii** $1329.56
f **i** $4990.24 **ii** $90.24
8 C
9 **a** $600 **b** $615 **c** Tegan by $15
10 **a** D **b** A **c** C **d** B
11 **a** $17 807.64 **b** $124 less
12 **a** **i** $5746.85 **ii** $5793.89
iii $5817.79 **iv** $5833.87
b Monthly, because it earns the most interest.
13 D

Exercise 1.06

1 **a** $175.50 **b** $1579.50 **c** $328.14
d $1907.64 **e** $105.98 **f** $2083.14

2 **a** $1275 **b** $24 225 **c** $10 416.75
d $34 641.75 **e** $577.36 **f** $35 916.75

3 **a** $1379 **b** $2316.72 **c** $217.58

4 **a** $3420 **b** $720 **c** $1500 **d** 48%

5 **a** $2080 **b** $8320 **c** $13 200
d $4880 **e** 14.7%

6 **a** $104.40 **b** $1536 **c** $596.40 **d** 63.5%

7 **a** $2045.40 **b** $545.40 **c** 36.4%

8 **a** $3399 **b** $4556 **c** $657 **d** 7.7%

9 **a** $262.50 **b** 7.4%

Exercise 1.07

1 $933.89

2 **a** $20 429.69 **b** $29 560.31

3 **a** **i** $659.66 **ii** 60.0%
b **i** $17 406.69 **ii** 45.2%
c **i** $5073.42 **ii** 60.0%
d **i** $1024 **ii** 41.0%
e **i** $14 020.37 **ii** 51.0%
f **i** $1073.44 **ii** 37.0%
g **i** $403.03 **ii** 46.3%
h **i** $1782.95 **ii** 68.6%

4 **a** **i** 63% **ii** 25.00% **iii** 6.25%
b By trial and error, in approx. 1.5 years.

5 **a** **i** $10 000 **ii** $8000 **iii** $4096
b 32.8%

6 **a** $17 970 **b** $7520

7 **a** $71 680 **b** $103 320
c Approx. 5 years and 8 months.
d 13.4%

8 Yes, it will lose approximately 52% after 7 years.

9 **a** $1800 **b** 5 years **c** $798.67
d Yes, in the 30th year. **e** No

Power plus

1 4 years and 61 days **2** $4444.44

3 $12 838.71 **4** $63 367.50

5 $2276.87 **6** 790 000

7 **a** 18 years **b** 18 years
c No. The size of the interest rate and the number of compounding periods determine how quickly the principal takes to double in value.

Test yourself 1

1 $13 045.75 **2** $1349.18

3 **a** $879.45 **b** $1115.40 **4** $307 619.39

5 **a** $1052.51 **b** $736.76 **c** $4946.80

6 **a** $67 725 **b** $12 477.63

7 **a** $2400 **b** $245.31 **c** $45.90 **d** $238.19

8 **a** $5360.85 **b** $360.85

9 $36 282.78 **10** $12 107.19

11 $291.98 **12** $13 145.47

13 **a** $487.50 **b** $4387.50 **c** $1908.56
d $6296.06 **e** $174.89 **f** $6783.56

14 **a** $14 756 **b** $10 234 **c** 59.0%

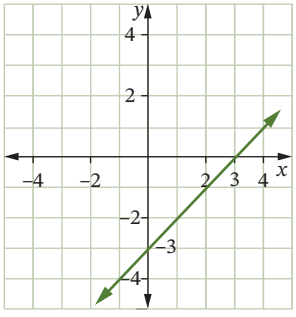
Chapter 2

SkillCheck

1 **a** (6, 1) **b** (−5, −4)
c 6 **d** 6
e $AC = BC = 4.5$ **f** isosceles
g $\frac{1}{3}$ **h** $-\frac{2}{3}$

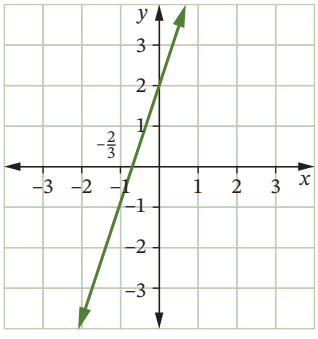
2 **a**

x	0	1	2	3
y	−3	−2	−1	0



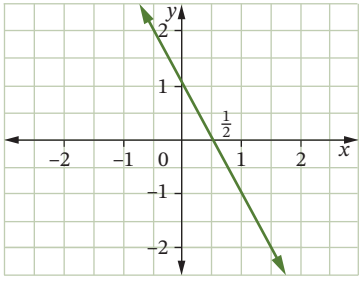
b

x	−2	−1	0	1
y	−4	−1	2	5



c

x	−1	0	1	2
y	3	1	−1	−3



3 **a** negative **b** neither **c** positive
d positive **e** negative **f** neither

Exercise 2.01

1 B **2** C **3** A

4 **a** $\frac{1}{3}$ **b** -2 **c** $\frac{7}{3}$

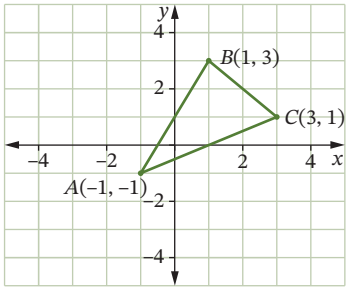
5 **a** **i** 2.2 **ii** (6, 2.5) **iii** $-\frac{1}{2}$
b **i** 10.8 **ii** (3.5, 3) **iii** $\frac{2}{3}$
c **i** 7.1 **ii** (−2.5, −0.5) **iii** -1
d **i** 7.6 **ii** (0.5, −7.5) **iii** $-\frac{3}{7}$
e **i** 10.2 **ii** (−6, −3) **iii** -5
f **i** 5.7 **ii** (5, 0) **iii** 1

6 **a** $\sqrt{89}$ **b** $\sqrt{194}$ **c** $\sqrt{82}$

7 k: $m = \frac{1}{5}$; l: $m = -\frac{1}{2}$

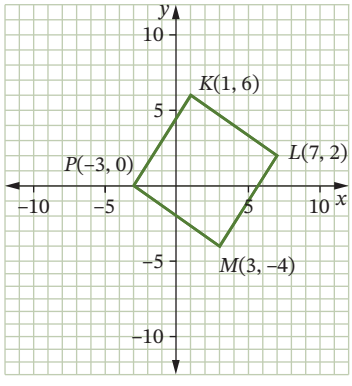
8 B

9 **a**



b $AB = AC = \sqrt{20}$ or $2\sqrt{5}$, $BC = \sqrt{8}$ or $2\sqrt{2}$
c $AB = AC = \sqrt{20}$ or $2\sqrt{5}$
d isosceles
e 11.8

10 **a**



b square
c $m_{KL} = -\frac{2}{3}$, $m_{PM} = -\frac{2}{3}$
d $m_{KP} = \frac{3}{2}$, $m_{LM} = \frac{3}{2}$
e the gradients are equal, they are parallel
f $KL = LM = PM = KP = \sqrt{52}$ or $2\sqrt{13}$
g 28.8 **h** 52 sq. units

11 **a** $P(-2, 1)$, $Q(1, 3)$
b $PQ = 3.6$, $AC = 7.2$, $AC = 2 \times PQ$
c $m_{PQ} = \frac{2}{3}$, $m_{AC} = \frac{2}{3}$; the gradients are equal.

Exercise 2.02

1 **a** neither **b** perpendicular **c** parallel
d neither **e** parallel **f** neither

2 **a** 4 **b** -2 **c** $\frac{1}{3}$ **d** -0.2

3 **a** -1 **b** $\frac{1}{6}$ **c** $\frac{2}{3}$ **d** $-\frac{2}{5}$

4 D **5** B **6** A

7 **a** $m_{AB} = -\frac{4}{3}$, $m_{CD} = -\frac{4}{3}$; $\therefore AB \parallel CD$
b $m_{PQ} = \frac{3}{4}$, $m_{CD} = -\frac{4}{3}$; $\therefore PQ \perp CD$

8 **a** $\frac{1}{3}$ **b** -3

Exercise 2.03

Teacher to check graphs.

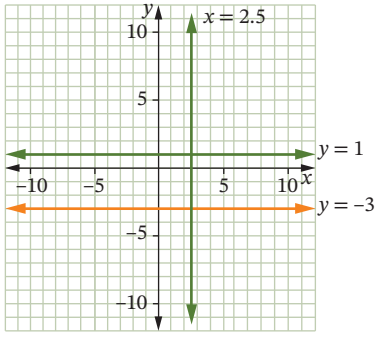
1 **a** **i** $\frac{1}{3}$ **ii** -1
b **i** -2.5 **ii** 5
c **i** 4 **ii** 4
d **i** -1 **ii** -2
e **i** 0 **ii** 0
f **i** -6 **ii** 3

2 **a** no **b** yes **c** yes
d yes **e** no **f** no

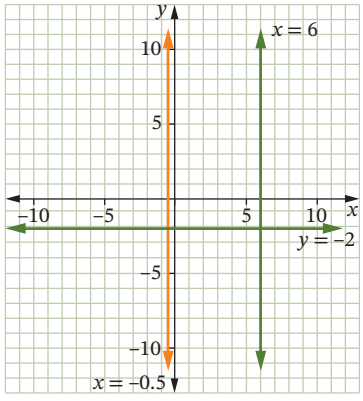
3 C

4 **a** $x = -4$ **b** $x = 1$ **c** $y = 5$ **d** $y = -3$

5 **a**



b



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6 **a** $y = 2$ **b** $x = 4$ **c** $x = -1$ **d** $y = -2$
e $y = 3$ **f** $x = -1$ **g** $y = 6$ **h** $x = -1$

7 A **8** C

9 **a** x-axis **b** y-axis

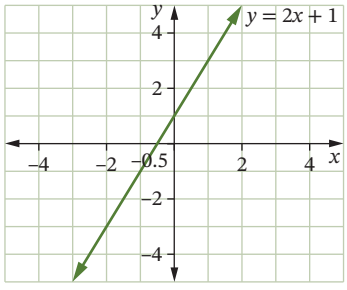
Exercise 2.04

1 **a** $m = 3, c = -2$ **b** $m = -2, c = 7$
c $m = 1, c = 4$ **d** $m = -1, c = 9$
e $m = \frac{3}{4}, c = 6$ **f** $m = 1, c = 0$
g $m = \frac{1}{2}, c = -11$ **h** $m = \frac{2}{3}, c = 6$
i $m = -\frac{1}{3}, c = -8$ **j** $m = 2, c = -6$
k $m = -3, c = 11$ **l** $m = 1, c = -\frac{7}{2}$

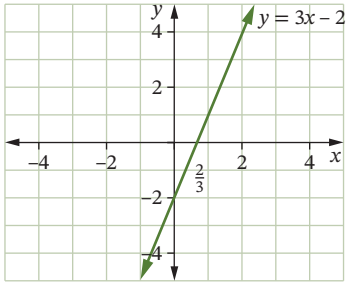
2 **a** $\frac{2}{3}, -2$ **b** $\frac{7}{2}, 7$ **c** $-4, 4$ **d** $9, 9$
e $-8, 6$ **f** $0, 0$ **g** $22, -11$ **h** $-9, 6$
i $-24, -8$ **j** $3, -6$ **k** $\frac{11}{3}, 11$ **l** $\frac{7}{2}, -\frac{7}{2}$

3 **a** $y = 2x + 1$ **b** $y = \frac{3}{4}x + 2$
c $y = -7x + 5$ **d** $y = -\frac{2}{5}x + 3$
e $y = -2x - 3$ **f** $y = -3x + \frac{1}{2}$

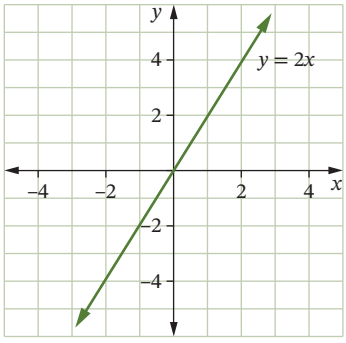
4 **a** $m = 2, c = 1$



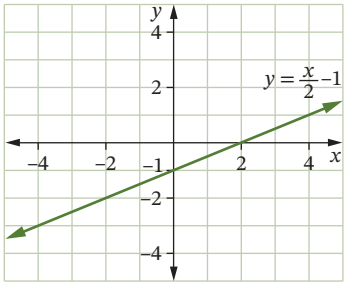
b $m = 3, c = -2$



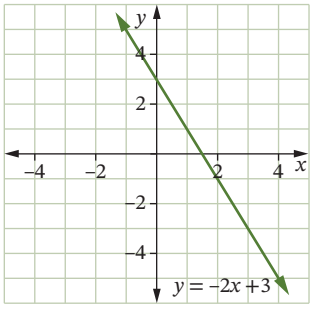
c $m = 2, c = 0$



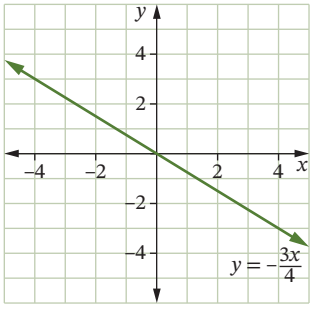
d $m = \frac{1}{2}, c = -1$



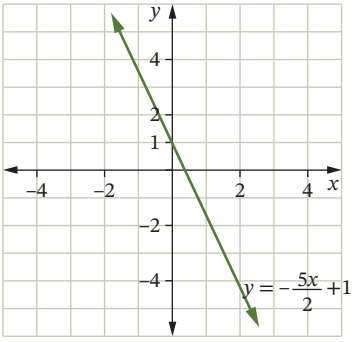
e $m = -2, c = 3$



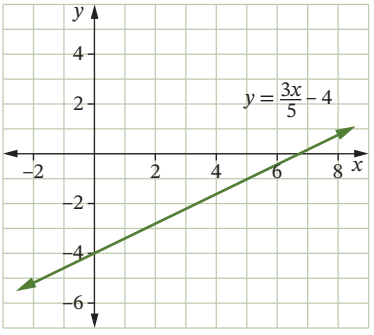
f $m = -\frac{3}{4}, c = 0$



g $m = -\frac{5}{2}, c = 1$



h $m = \frac{3}{5}, c = -4$



5 $y = 2x$

6 **a** C **b** B **c** D **d** A

7 **a** C **b** B, D **c** B
d C, D **e** A, B **f** D

8 **a** $y = 4x + 3, y = 4x - 6$
b $3x - y + 7 = 0, y = 3x - 2$

Mental skills 2

2 **a** 18 **b** \$126 **c** 39 **d** \$30.30
e \$7.50 **f** 10.8 **g** \$27 **h** 60
i \$240 **j** \$3.30 **k** 900 **l** \$52.50

4 **a** 10 **b** 166 **c** \$50 **d** \$22
e 37.5 **f** \$5.80 **g** 135 **h** \$22.60

6 **a** 500 **b** \$20 **c** 4.5 **d** \$6.25
e 81 **f** \$35 **g** 16.5 **h** 74.5
i \$195 **j** \$425 **k** \$31.50 **l** 290

8 **a** 160 **b** \$1.50 **c** 7.5 **d** \$32.50
e \$67.50 **f** \$31.25 **g** 38 **h** 170

Exercise 2.05

1 **a** $x - y + 2 = 0$ **b** $3x - y - 1 = 0$
c $5x - y + 8 = 0$ **d** $x + 2y - 3 = 0$
e $x - 2y - 6 = 0$ **f** $8x - y + 2 = 0$
g $6x - y - 3 = 0$ **h** $x - 2y - 6 = 0$
i $3x - 5y + 10 = 0$

2 **a** $m = -2, c = 6$ **b** $m = 4, c = -5$
c $m = \frac{3}{2}, c = 2$ **d** $m = -2, c = 1$
e $m = -2, c = -5$ **f** $m = -\frac{4}{3}, c = 4$

3 B

4 **a** 3, 6 **b** $\frac{5}{4}$, −5 **c** $-\frac{4}{3}$, 2
d $\frac{1}{2}$, 1 **e** $-\frac{5}{2}$, −5 **f** 3, 4

5 B

Exercise 2.06

1 **a** $y = 2x + 5$ **b** $y = -\frac{3}{4}x + 3$ **c** $y = -3x + 6$
d $y = -x + 3$ **e** $y = \frac{1}{2}x + 3$ **f** $y = -3x - 3$

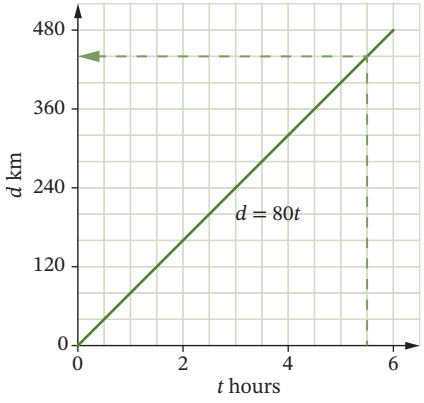
2 **a** $y = \frac{1}{2}x + 2$ **b** $y = x$ **c** $y = -\frac{1}{2}x + 5$
d $y = -\frac{1}{2}x + 3$ **e** $y = -3x - 3$ **f** $y = -x - 2$
g $y = 3x - 10$ **h** $y = \frac{2}{5}x + 2$ **i** $y = 2x - 3$

Exercise 2.07

1 **a** $D = 30h + 5$ **b** $R = 0.5t - 8$
c $P = -50n$ **d** $F = 1.8C + 32$
e $H = 0.8w$ **f** $V = -40T + 200$

2 **a** 40 km/h **b** $\frac{8}{5}$, 0
c The conversion rate between m/h and km/h; 1 m/h = $\frac{8}{5}$ (or 1.6) km/h.
d $y = \frac{8}{5}x$ (or $1.6x$)

3 **a** 0, 80, 160, 240, 320

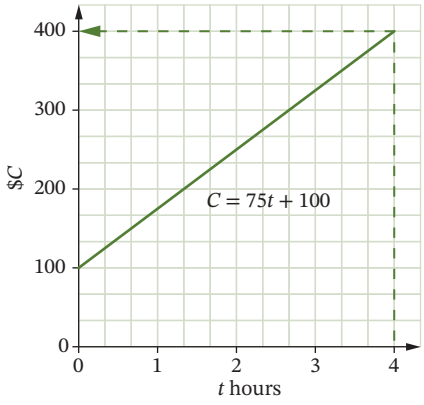


b 80 **c** the speed 80 km/h
d **i** 440 km **ii** 0.5 h (or 30 min)

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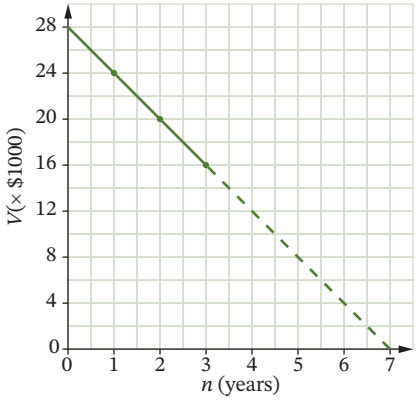
4 **a** −25, 200 **b** initial volume
c the rate at which the volume in the tank is changing, −25 L/h
d the volume is decreasing **e** $V = -25t + 200$

5 **a**



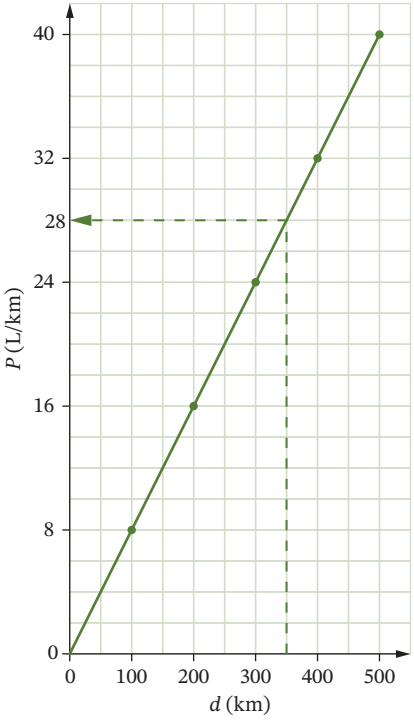
b 75, 100 **c** **i** \$400 **ii** 8 hours

6 **a** 20 000, 16 000
b



c −4000, 28 000
d gradient represents the change in value of the car per year, losing \$4000 per year; vertical intercept represents the original value of the car \$28 000
e $C = -4000n + 28\,000$

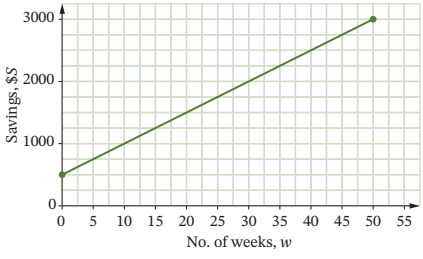
7 **a** 8, 16, 24, 32, 40



b $\frac{2}{25}$ or 0.08, 0 **c** The amount of fuel used for each kilometre, $\frac{2}{25}$ or 0.08 L/km
d $P = \frac{2}{25}d$ (or $0.08d$) **e, f** 28 L

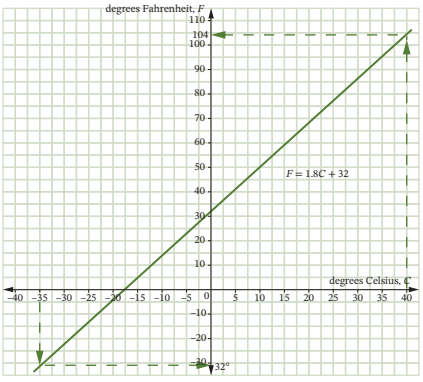
8 **a** \$60, hiring ice cream machine or buying utensils
b \$1 **c** $C = n + 60$
d \$160 **e** \$340

9 **a** 500, 1000, 1500, 2000, 2500, 3000



b 500, birthday money, initial savings
c 50, weekly savings **d** $S = 50w + 500$
e **i** \$1850 **ii** \$3500 **iii** \$5700 **f** 80 weeks

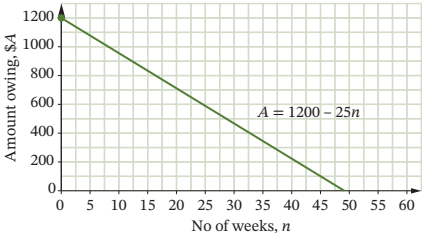
10 **a** 5, 14, 32, 50, 59



b 1.8, 32 **c** **i** 42.8°F **ii** 30°C

11 **a** $A = 1200 - 25n$ **b** **i** \$325 **ii** \$75

c



d Loan amount is decreasing **e** 1200, –25

f 48 weeks

Exercise 2.08

1 **a** $y = 2x + 4$ **b** $y = 3x + 6$ **c** $y = -\frac{1}{2}x + \frac{11}{2}$

d $y = 2x - 12$ **e** $y = -5x - 13$ **f** $y = \frac{1}{2}x - 10$

2 **a** $y = -2x - 2$ **b** $y = \frac{1}{5}x - \frac{1}{5}$ **c** $y = -\frac{1}{3}x + \frac{4}{3}$

d $y = -3x - 3$ **e** $y = x + 6$ **f** $y = -\frac{1}{3}x - \frac{31}{3}$

3 **a** $m = 2$ **b** $M(0, 2)$

c $-\frac{1}{2}$ **d** $y = -\frac{1}{2}x + 2$

4 **a** $y = \frac{1}{3}x + 1$ **b** -3 **c** $y = -3x + 11$

5 **a** $y = -\frac{4}{5}x + 8$ **b** $A(10, 0)$ **c** $\frac{5}{4}$

d $y = \frac{5}{4}x - \frac{25}{2}$ **e** $(0, -12.5)$

Power plus

1 **a** $\frac{2}{3}$ **b** $y = \frac{2}{3}x - 2$ **c** $y = 4$

2 $k = 5$ **3** $B(2, -1)$ **4** $X(-2, 3)$

5 Teacher to check, see worked solutions.

Test yourself 2

1 **a** 12.6 **b** $M(-1, 4)$ **c** $\frac{1}{3}$

2 **a** $HJ = JK = KL = HL = \sqrt{58}$

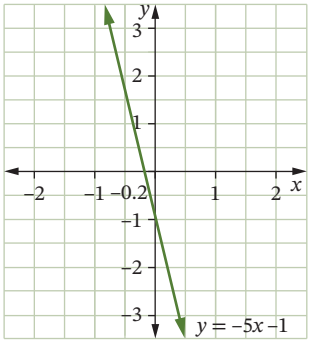
b $m_{HJ} = \frac{3}{7}, m_{JK} = \frac{7}{3}, m_{KL} = \frac{3}{7}, m_{HL} = \frac{7}{3}$

c $HK = \sqrt{200}$ or $10\sqrt{2}$, $JL = \sqrt{32}$ or $4\sqrt{2}$

d rhombus

3 **a** $-\frac{1}{2}$ **b** 2

4



5 C **6** D

7 **a** $m = 2, c = -10$ **b** $m = 4, c = 3$

c $m = -\frac{3}{8}, c = \frac{1}{2}$

8 **a** C **b** B **c** A **d** D

9 **a** 5, –10 **b** $-\frac{3}{4}$, 3 **c** $\frac{4}{3}, \frac{1}{2}$

10 **a** $3x - y + 5 = 0$ **b** $2x - 5y - 50 = 0$

c $x - 3y - 6 = 0$

11 **a** $m = 1, c = 2$ **b** $m = \frac{1}{4}, c = 1$

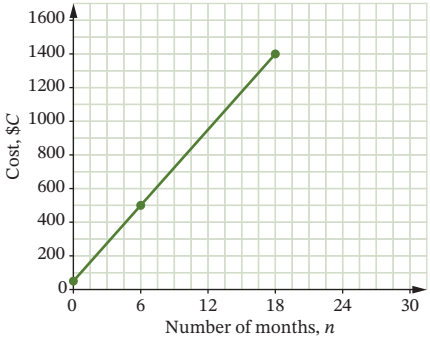
c $m = -3, c = 9$

12 **a** –2, 2 **b** –4, 1 **c** 3, 9

13 **a** $y = 2x + 3$ **b** $y = -\frac{1}{2}x - 4$

The product of their gradients, 2 and $-\frac{1}{2}$, is equal to -1, so they are perpendicular.

14 50, 500, 950, 1400, 1850



b 75 **c** 50 **d** $C = 75n + 50$

e **i** \$425 **ii** 8 months

15 **a** $y = 3x - 6$ **b** $y = -2x$

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Chapter 3

SkillCheck

1 **a** 198 cm^2 **b** 187.5 cm^2 **c** 86.1 cm^2
d 108.8 cm^2 **e** 273 cm^2 **f** 240.25 cm^2
g 696 cm^2 **h** 252 cm^2 **i** 336.96 cm^2

2 **a** 28.7 cm **b** 36.9 cm **c** 18.3 cm

3 **a** **i** 40.2 cm **ii** 128.7 cm^2
b **i** 69.1 cm **ii** 380.1 cm^2
c **i** 219.9 mm **ii** 3848.5 mm^2

4 **a** 105 m^3 **b** 27 m^3 **c** 308 m^3
d 480 m^3 **e** 640 m^3 **f** 560 m^3

Exercise 3.01

1 **a** 1 kg, 0.5 kg **b** 5 mL, 2.5 mL
c 1 min, 0.5 min **d** 100 g, 50 g
e 1 cm, 0.5 cm **f** 1°, 0.5°
g 5 km/h, 2.5 km/h **h** 250 g, 125 g
i 10 mL, 5 mL **j** 1 kL, 0.5 kL
k $\frac{1}{4}$ (or 0.25) tank, $\frac{1}{8}$ (or 0.125) tank

2 **a** 1 cm, 0.5 cm **b** 0.1 mL, 0.05 mL
c 1 s, 0.5 s **d** 1 cm, 0.5 cm **e** 5 cm, 2.5 cm

3 B

4 **a** 0.5 mm **b** 5 L **c** 0.05 m^2
d 0.5 g **e** 5 kg **f** 50 kg

5 0.005, 0.033%

6 **a** 5 km **b** 0.0752%

7 width of the netball court

8 0.0156%

9 0.0012%

10 marathon length

11 **a** 2.5 mL **b** 5 mL **c** 502.5 mL
d 497.5 mL **e** 503.10, 497.25

12 For 8 m wood, absolute error = 0.5 m, actual length between 7.5 and 8.5.
For 8.5 m metal rod, absolute error = 0.05 m, actual length between 8.45 and 8.55.
The metal rod could be at or near 8.45 m while the wood could be longer at or near 8.5 m.

13 A measurement of 185.0 cm has an absolute error of 0.05 cm and means the actual measurement is between 184.95 and 185.05 cm (also percentage error ≈ 0.027%). A measurement of 185 cm has an absolute error of 0.5 cm and means the actual measurement is between 184.5 and 185.5 cm (also percentage error ≈ 0.207%). So the 185.0 cm has less error and is more accurate.

14 91.125 m^3, 42.875 m^3

Exercise 3.02

1 **a** 3, 3 **b** 3.14, 3.14
c 3.1416, 3.1415 **d** 3.141 593, 3.141 592

2 **a** 24.7918 m^2 **b** 24.32 m^2 **c** 24 m^2

3 **a** 182.89 cm^2 **b** 182.80 m^2, 0.05%
c 181.46 m^2, 0.78%

4 **a** 45.08 m, 127.0129 m^2
b 44.8 m, 125.44 m^2, 0.6%, 1.2%
c 44 m, 121 m^2, 2.4%, 4.7%

5 **a** 28.68 cm, 50.2425 cm^2
b 28.4 cm, 49.2 cm^2, 1.0%, 2.1%
c 28 cm, 48 cm^2, 2.4%, 4.5%

6 **a** 25.1327 m **b** 0.5 m
c **i** 23.5619 m **ii** 6.25%
iii 26.7035 m **iv** 6.25%

7 **a** 108.1665 mm **b** 0.5 mm
c 107.4733 mm, 0.6%
d 2700 mm^2 **e** 2737.625 mm^2, 1.4%

8 32 m, 28 m

Exercise 3.03

1 **a** 897.5 m^2 **b** 127.5 cm^2 **c** 283 cm^2
d 486 cm^2 **e** 70.5 cm^2 **f** 364 cm^2
g 2093 m^2 **h** 287 cm^2 **i** 174.08 cm^2
j 252 cm^2 **k** 900 m^2 **l** 356.5 m^2

2 **a** 1399.8 m^2 **b** 2.5 m^2 **c** 145.1 m^2
d 1.1 m^2 **e** 88.8 m^2 **f** 814.2 m^2
g 50.3 m^2 **h** 131.7 m^2 **i** 78.5 m^2
j 601.9 m^2 **k** 159.0 m^2 **l** 54.9 m^2

3 **a** 35.3 cm^2 **b** 40.8 m^2 **c** 1.7 m^2

4 **a** 2.25 m **b** 2226

5 D

6 **a** 1.50 m^2 **b** 72.8%

7 **a** 0.16 m^2 **b** 750 **c** $9514.80

8 **a** 7694 m^2 **b** 154 × $29.50 = $4543

9 **a** 414 m^2 **b** 240 m^2 **c** 58% **d** $2044.50

Exercise 3.04

1 **a** cube, 8.64 m^2 **b** triangular prism, 75.6 m^2
c trapezoidal prism, 295 m^2
d rectangular prism, 27.76 m^2

2 **a** 282 m^2 **b** 298 cm^2 **c** 2720 mm^2
d 204 m^2 **e** 1288 mm^2 **f** 165 m^2

3 A

4 **a** 80 m^2, $4400 **b** 171.4 m^2

5 **a** 1036 cm^2 **b** 1020 mm^2 **c** 204 m^2
d 390 cm^2 **e** 672 cm^2 **f** 5672 mm^2

6 B

7 **a** 60 336 cm^2 **b** Need 7 m^2, cost = $175

Exercise 3.05

1 **a** 31.7 m^2 **b** 12 370.0 cm^2
c 805.8 cm^2 **d** 41.7 m^2

2 **a** 35 m^2 **b** 3478 cm^2

3 587.2 cm^2

4 D

5 **a** 1009 m^2 **b** 3054 cm^2 **c** 1355 cm^2
d 7 m^2 **e** 905 cm^2 **f** 39 m^2
g 17 m^2 **h** 5 m^2 **i** 1253 cm^2

6 **a** 64.4 m^2 **b** 8 L

7 **a** 27.3 m^2 **b** 56.7 m^2
c Cost = 28 × $18.50 + 57 × $21.75 = $1757.75

ANSWERS

Exercise 3.06

1 **a** 432 cm² **b** 2150 cm² **c** 173.2 cm²
2 **a** 275 m² **b** 564 mm² **c** 472 cm²
3 B
4 **a** 166.4 m² **b** 3456 mm² **c** 743.1 cm²
5 **a** 843 cm² **b** 1592 cm² **c** 3116 cm²
6 85 854 m² **7** 1869 m²
8 **a** 1344 mm² **b** 180 cm² **c** 343.4 m²
9 **a** 42 m **b** $A = 735$ m²
c 35 m **d** 28 m

Exercise 3.07

1 **a** 101 cm² **b** 628 cm² **c** 2419 cm²
2 **a** 392.7 mm² **b** 62.8 m² **c** 192.4 cm²
3 **a** 1695.0 cm² **b** 2.0 m² **c** 614.3 cm²
4 **a** 90π m² **b** 224π mm² **c** 450π cm²
5 **a** 2827.4 mm² **b** 380.1 m² **c** 366.4 cm²
6 **a** 432π m² **b** 192π cm² **c** 768π mm²
7 B
8 **a** 314 m² **b** 628 m² **c** 628 m² **d** 402 m²
9 **a** 5.11×10^8 km² **b** 149 000 000 km²
c 5.17% **10** 5525 cm²
11 **a** 30.16 cm **b** 4.80 cm
c 8 cm **d** 193.02 cm²
12 **a** 21.9 mm **b** 25.2 mm **c** 30.9 mm
13 **a** 6.9 cm **b** 85.2 cm **c** 85 cm

Mental skills 3

2 **a** 8 hours 30 mins **b** 5 hours 40 mins
c 3 hours 25 mins **d** 8 hours 15 mins
e 11 hours 25 mins **f** 1 hour 40 mins
g 5 hours 10 mins **h** 5 hours 45 mins
i 7 hours 55 mins **j** 7 hours 40 mins

Exercise 3.08

1 **a** 446.96 cm² **b** 49 270 cm² **c** 864 cm²
d 11 064 cm² **e** 45 160 cm² **f** 40 260 cm²
2 464 cm²
3 **a** 352 cm² **b** 76 cm²
4 **a** 9721.7 cm² **b** 14 031.4 cm² **c** 14 778.1 cm²
d 2858.8 cm² **e** 2793.5 cm² **f** 394.7 cm²
5 B **6** 2953 cm²
7 **a** 26.14 m² **b** 19 m²
8 1028.32 cm² **9** A
10 19 858.0 cm²
11 **a** 857.7 cm² **b** 412.3 cm² **c** 1042.0 cm²
d 5969.0 cm² **e** 250.6 cm² **f** 628.3 cm²
g 282.7 cm² **h** 652.9 cm² **i** 501.6 cm²
j 3769.9 cm² **k** 1148.8 cm² **l** 3017.7 cm²
m 6615.4 cm² **n** 3908.4 cm² **o** 7940 cm²

Exercise 3.09

1 **a** 1399.6 cm³ **b** 46.6 m³ **c** 56 160 cm³
d 17 066 cm³ **e** 33 931.8 cm³ **f** 192.4 m³
g 1.2 m³ **h** 21 756 cm³ **i** 208.8 m³
2 **a** 251.3 cm³ **b** 320 cm³ **c** 21.5%
3 350.4 m³
4 **a** 118 800 L **b** 113 040 L
5 2 500 000 L
6 **a** 182.83 m³ **b** $21.94 per day **7** 1415.7 cm³
8 **a** 4825.49 cm³ **b** 5026.55 cm³ **c** 1989.38 cm³
d 6375 cm³ **e** 5301.44 cm³ **f** 3084.96 cm³
g 536.19 cm³ **h** 1884.96 cm³ **i** 167.33 cm³
j 12 900 cm³ **k** 167.55 cm³ **l** 794.12 cm³

Exercise 3.10

1 **a** 192 cm³ **b** 200 cm³ **c** 120 cm³
d 336 m³ **e** 1200 cm³ **f** 80 m³
2 **a i** 24 cm **ii** 3200 cm³
b i 20 m **ii** 19 200 m³
c i 40 mm **ii** 4320 mm³
d i 60 mm **ii** 28 160 mm³
e i 7.7 m **ii** 133.1 m³
f i 84 cm **ii** 564 480 cm³
3 D
4 **a** 42 412 mm³ **b** 3 m³ **c** 2618 cm³
d 616 cm³ **e** 393 cm³ **f** 2545 mm³
5 **a i** 13.3 cm **ii** 680.7 cm³
b i 3.9 m **ii** 19.9 m³
c i 11.3 cm **ii** 189.6 cm³
d i 3.5 m **ii** 2.4 m³
e i 244.6 m **ii** 296 162.1 m³
f i 71.9 cm **ii** 129 638.4 cm³
6 B
7 **a** 14 137.2 mm³ **b** 696.9 m³
c 659.6 cm³ **d** 3619.1 m³
e 1072.3 cm³ **f** 8578.6 mm³
8 1.09×10^{12} km³
9 **a** 33.75 m³ **b** 19 t
10 22.5 m **11** 14.1 cm
12 5.7 m **13** 2.8 cm
14 A

Exercise 3.11

1 **a** 59 m³ **b** 59 kL
2 **a** 343 cm³ **b** 240 cm³ **c** 1152 cm³
d 6100 cm³ **e** 6048 cm³ **f** 2500 cm³
3 **a i** 31 416 cm³ **ii** 31.416 L
b i 616 cm³ **ii** 0.616 L
c i 264 cm³ **ii** 0.264 L
4 28.27 kL
5 **a** 12 balls **b** 60 balls **c** 31 416 cm³ **d** 48%
6 **a** 1963 cm³ **b** 0.55 cm³/s
7 **a** 250 m³ **b** 210 kL **c** $415.80
8 **a** 1696 m³ **b** 1017 t

Power plus

1 1728 cm³
2 8 cm
3 **a** $2p^2 + 4pr$ **b** $\frac{3\pi x^2}{2}$

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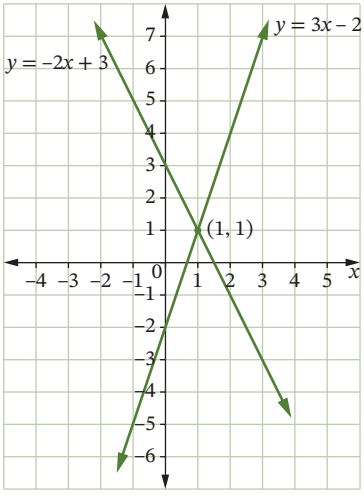
4 36 mm (to the nearest mm)
5 0.5 m/h or 50 cm/h
6 40 mm

Test yourself 3

1 **a** 5 g **b** 7.1%
2 A
3 **a** 31.95075 cm² **b** 32.205 cm², 0.8%
c 27.5 cm², 13.9%
4 **a** 133 cm² **b** 2340 mm² **c** 326.7 cm²
d 1318.2 cm² **e** 151.7 m² **f** 1039.1 cm²
5 **a** 1.08 m² **b** 3150 mm² **c** 5236 cm²
d 277.6 m² **e** 216 cm² **f** 434 mm²
6 **a** 354 371.7 cm² **b** 4863.2 cm² **c** 17.4 m²
d 14 294.2 cm² **e** 5871.2 cm² **f** 4427.8 cm²
7 **a** 960 cm² **b** 7776 cm² **c** 1356 cm²
8 **a** 704 m² **b** 3267 m² **c** 2488 m²
d 452 m² **e** 681 m² **f** 5890 m²
9 **a** 3180 cm² **b** 1268 cm² **c** 395 cm²
d 3318 cm² **e** 1728 cm² **f** 3436 cm²
10 **a** 36 816 m³ **b** 20 160 m³ **c** 10 016 m³
11 **a** 322.67 m³ **b** 540 cm³ **c** 1568 mm³
d 1340.41 cm³ **e** 10 262.54 mm³ **f** 904.78 m³
12 **a** 360 498 mm³ **b** 145 125 mm³ **c** 455 cm³
d 229 m³ **e** 3054 cm³ **f** 18 096 m³

Practice set 1

1 $2199.12
2 **a** 13.9 **b** $\left(-1\frac{1}{2}, 3\right)$ **c** $-\frac{12}{7}$
3 **a** 616 m² **b** 280.5 m² **c** 935 m²
d 452.4 m² **e** 1268.3 cm² **f** 457.1 cm²
4 **a** $1505.94 **b** $1054.16 **c** $7077.92
5 **a** 117.5 m² **b** 480 m² **c** 324 m²
6 0.34%
7 **a** $575 **b** $1164.38 **c** $1495
8 **a** $-\frac{4}{3}$ **b** $\frac{3}{4}$
9 **a** $71 904 **b** $13 835.80
10



11 **a** $3000 **b** $5.63 **c** $306
12 **a** 9349.4 cm² **b** 1979.2 cm² **c** 279.4 cm²
13 **a** 183 m² **b** 510 m² **c** 66 m²
14 **a** $12 979.20 **b** $979.20
15 B
16 **a** 35 m³ **b** 69 266 m³ **c** 860 m³
17 B
18 **a** $768 **b** $6912 **c** $4008.96
d $10 920.96 **e** $227.52 **f** $11 688.96
19 **a** 1352 m² **b** 22 m² **c** 2822 m²
20 **a** $52 042.84 **b** $52 142.72 **c** $52 165.50
21 44 791.90
22 **a** $m = -3, c = 8$ **b** $m = 1, c = 7$
c $m = -\frac{2}{3}, c = 1$
23 **a** $2x - y - 3 = 0$ **b** $2x - y - 5 = 0$
c $3x - 4y + 24 = 0$
24 **a** $y = 5x + 3$ **b** $y = -4x + 8$ **c** $y = \frac{1}{3}x + \frac{4}{3}$
25 **a** 1 cm **b** 0.5 cm
c 0.05 cm (or 0.5 mm)
d Timothee's 2%, Lionel's 0.19% **e** Lionel's
26 **a** 6426 cm³ **b** 59 837.7 cm³ **c** 98 960.2 cm³
27 line A: $y = -\frac{3x}{2} - 3$; line B: $y = \frac{2x}{3}$
28 **a** $y = 4x + 32$ **b** $y = 5x$
29 **a** $26 237.44 **b** $13 752.56 **c** 65.6%
30 C
31 **a** 6600 cm³ **b** 3308 cm³ **c** 10 175 cm³
32 **a** $y = -3x + 7$ **b** $y = \frac{2}{3}x - 5$ **c** $y = 4$
33 **a** $34.70 **b** $5413.20
34 **a** 65 m³ **b** $177.45
35 A
36 **a** 2024 cm² **b** 134.0 m² **c** 13 824 cm²
37 **a** 340.2 m² **b** 14 940 cm²

Chapter 4

SkillCheck

1 **a** g^9 **b** r^6 **c** d^{15} **d** k^2
e h^{10} **f** m^4 **g** a **h** 1
i $6e^6$ **j** $3n^4$ **k** $1000w^9$ **l** 25
m v^5w^5 **n** $\frac{c^3}{p^3}$ **o** $\frac{1}{y}$ **p** $\frac{1}{k^2}$
q $\frac{7d}{4a}$ **r** $\frac{3}{2}y^5$ **s** $24g^2$ **t** $\frac{4b^2}{9h^2}$
2 **a** $\frac{19a}{20}$ **b** $\frac{p}{6}$ **c** $\frac{5}{3}$ **d** $\frac{x}{7y}$
3 **a** $18m^2 + 66m$ **b** $-15g + 40$ **c** $24wy - 12w^2$
4 **a** $4(x + 6)$ **b** $5(4 - 3a)$ **c** $q(q + 1)$
d $6a(3a - 2)$ **e** $-2(y + 15)$ **f** $-6(3w - 4)$
5 **a** 3 and 6 **b** −2 and −4
c 4 and −5 **d** 8 and −2

ANSWERS

Exercise 4.01

1 **a** $15k^{11}$ **b** $10y^3$ **c** $9p^{11}$
d w^{15} **e** $32n^{15}$ **f** $3h^4$
g $12a^7d^5$ **h** $9q^6$ **i** $4w^2y^4$
j $-128c^{14}$ **k** $\frac{4a^2cd^3}{3}$ **l** $125y^{18}$
m $24h^4k^5w^5$ **n** $32d^{15}g^{10}$ **o** $\frac{2m^2p^4}{3}$

2 **a** $l^{18}m^{30}$ **b** $\frac{n^3}{8}$ **c** 7 **d** $\frac{w^{10}}{k^{15}}$
e 1 **f** $64k^2y^{10}$ **g** -15 **h** $\frac{16b^4}{81d^4}$
i 1 **j** $625d^4y^8$ **k** $-\frac{27k^{12}}{1000}$ **l** -9
m $16p^8q^{12}r^{16}$ **n** 64 **o** $9g^{12}k^2$ **p** 4

3 **a** 1 **b** 1 **c** 7 **d** 1
e -8 **f** 9 **g** 625 **h** 2
i 128 **j** 1 **k** 0 **l** 64
m $\frac{1}{216}$ **n** 1 **o** -7 **p** 25
q 2 **r** $\frac{1}{81}$ **s** 1 **t** 1

4 D

5 **a** $\frac{1}{25}$ **b** $\frac{1}{32}$ **c** $\frac{1}{20}$ **d** $\frac{1}{1000}$ **e** $\frac{1}{81}$

6 C

7 **a** $\frac{1}{8^7}$ **b** $\frac{1}{3^5}$ **c** $\frac{1}{y}$ **d** $\frac{1}{x^3}$
e $\frac{1}{25b^2}$ **f** $\frac{8}{h^3}$ **g** $\frac{1}{ab}$ **h** $-\frac{p}{q}$
i $\frac{11}{w^3}$ **j** $\frac{1}{216x^3}$ **k** $\frac{a^3}{b^5}$ **l** $\frac{m}{w^3}$
m $\frac{8}{u^3v^4}$ **n** $\frac{-2r^6}{y^5}$ **o** $\frac{10f^3}{e}$ **p** $\frac{g^4}{2h^3}$
q $\frac{3d^7}{4n^2}$ **r** $\frac{1}{16c^2}$ **s** $\frac{5x^2}{yw^2}$ **t** $\frac{2}{mp}$

8 **a** $\frac{10}{r^6}$ **b** $\frac{2}{5}$ **c** 3 **d** x
e $\frac{3}{k}$ **f** $\frac{5}{a^2}$ **g** $\frac{1}{36w^2}$ **h** $\frac{-2}{m}$
i $\frac{3g^3}{5}$ **j** $\frac{3h}{2r}$ **k** $\frac{m^3n^2}{p^2}$ **l** $\frac{4a^2}{5b}$

9 **a** $20x^6y^9$ **b** $81m^{24}n^4$ **c** $\frac{4w^4}{9k^2}$
d $\frac{5g^4y^{12}}{2}$ **e** $\frac{27a^3}{8x^6}$ **f** $48a^{11}d^{15}$
g $\frac{1}{12q^7r}$ **h** $64h^{11}k^{16}$ **i** $9b^4$

10 **a** $\frac{16}{9}$ **b** $3\frac{3}{8}$ **c** 1 000 000 **d** $\frac{8}{125}$
e $-\frac{243}{1024}$ **f** $\frac{256}{625}$ **g** $\frac{16}{81}$ **h** $\frac{125}{343}$
i $\frac{9}{k^2}$ **j** $-\frac{x^3}{27}$ **k** $\frac{625}{a^8}$ **l** $\frac{9g^6}{16}$

Exercise 4.02

1 **a** $\frac{5k}{7}$ **b** $\frac{3x}{11}$ **c** $\frac{2m}{5}$ **d** $\frac{13g}{5}$
e $\frac{7w}{20}$ **f** $\frac{16a}{21}$ **g** $\frac{83c}{60}$ **h** $\frac{7u}{3}$
i $\frac{1}{2k}$ **j** $\frac{2}{x}$ **k** $\frac{2}{h}$ **l** $\frac{17}{2m}$
m $\frac{2}{3a}$ **n** $\frac{29}{6h}$ **o** $\frac{87}{40k}$ **p** $\frac{13}{21p}$

2 **a** $\frac{5m+3n}{15}$ **b** $\frac{6k-7w}{42}$ **c** $\frac{9x-4p}{36}$ **d** $\frac{15+8h}{24}$
e $\frac{7n-40}{70}$ **f** $\frac{25y-4c}{10}$ **g** $\frac{9q+22d}{33}$ **h** $\frac{9a+20e}{24}$
i $\frac{19b}{45}$ **j** $\frac{15y}{28}$ **k** $\frac{49c}{33}$ **l** $\frac{26g}{15}$
m $\frac{35h}{24}$ **n** $\frac{78x}{35}$ **o** $\frac{36-40k}{15}$ **p** $\frac{9m}{20}$

Exercise 4.03

1 **a** $\frac{2m}{35}$ **b** $\frac{dh}{12}$ **c** $\frac{qy}{6}$ **d** $\frac{6xy}{55}$
e $2kv$ **f** dh **g** $\frac{8}{3v}$ **h** $\frac{2}{3}$
i $\frac{7}{15}$ **j** $\frac{2h}{5}$ **k** $\frac{12a^2}{k^2}$ **l** $\frac{2a}{9}$

2 B **3** D

4 **a** $\frac{9h}{20}$ **b** $\frac{7x}{12p}$ **c** $\frac{11k}{6a}$ **d** $\frac{2y}{3}$
e $\frac{6}{5}$ **f** $\frac{1}{6a}$ **g** $\frac{5e^2}{6}$ **h** $\frac{20}{9x^2}$
i $\frac{w}{4}$ **j** $\frac{4}{9h^2}$ **k** $\frac{4dn}{5}$ **l** $\frac{3g}{20y}$

5 **a** $\frac{4w}{3}$ **b** 1 **c** $\frac{3q}{7}$ **d** $\frac{2m}{3g}$
e $\frac{15w}{14}$ **f** $\frac{25g}{3}$ **g** $\frac{8m}{45d}$ **h** $\frac{ad}{5}$
i $\frac{1}{9}$ **j** $20x$ **k** $\frac{1}{q}$ **l** $3a$

6 C

Exercise 4.04

1 **a** $5d + 55$ **b** $-3r - 30$
c $7x - 63y$ **d** $-4a + 20w$
e $-2 + p^2$ **f** $-20e^3 - 30e$
g $6y + 42y^2$ **h** $12x^2y^2 - 4xy$
i $16rq^2 - 8r^2q$ **j** $12ab^2 - 21a^2b$
k $-6h^2 + 18h^3$ **l** $-25x^3 - 20xy$
m $-3 - 8a$ **n** $-6m^3 + 8m^2n$
o $15g + 35g^3$ **p** $-5e + 12$

2 C

3 **a** Yes **b** No **c** Yes

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4 **a** $3k^2 - 20k$ **b** $-23h + 7h^2$
c $3w^3 - 15w$ **d** $49x^3 - 10x^4$
e $2 + 21d$ **f** $12n - 26n^2$
g $4y^2 - 6y + 5$ **h** $5 - 13a - 4a^2$
i $16 + 50w$ **j** $20y^3 - 8y^2 + 8$
k $-2v^2 - v - 6$ **l** $8 - 9a + a^2$
m $10c^2 - 30c + 20$ **n** $2m^2 + 18m^3$
o $20x + 26xy - 60y$

5 **a** $5(3y - 4)$ **b** $7(3 + 5w)$
c $p(2 + p)$ **d** $10y(3 - 2y)$
e $12d(3d + 2)$ **f** $7k(4k - 3)$
g $(c - 5)(8 - c)$ **h** $(3 + 2m)(m + 7)$
i $-q(q + 36)$ **j** $-4x(2 - 3x)$
k $(3b + 5)(b - 2)$ **l** $-4cd(3d - 2)$
m $-hn(n - h)$ **n** $-3g(5g + 6)$
o $6q(8q - 9)$

6 B

7 **a** $4my(2my - 3)$ **b** $9bc(4ab + 3)$
c $12mn(2m - 9n)$ **d** $5g(4dg - 7a)$
e $8wy^2(5y + 3w)$ **f** $25gh(3g^2h - 5)$
g $p(1 - 8p - 4p^2)$ or $-p(4p^2 + 8p - 1)$
h $3mn(2n + 1 + 16m)$
i $8pg(4p^2 + g - 1)$ **j** $3a^2(6a^3 - 4 + 5a^2)$
k $7mh^2(4m^2 - 3)$ **l** $3w(5kp - 8p^2 - 3k)$

Mental skills 4

Exact answers shown

2 **a** 331 **b** 157 **c** 1587 **d** 255
e 421 **f** 203 **g** 413 **h** 734
i 6723 **j** 15 744 **k** 276 **l** $72\frac{3}{7}$

4 **a** 28.231 **b** 14.187 **c** 177.4967
d 416.752 **e** 2.4156 **f** 5.0237
g 3.6890 **h** 5.8065 **i** 23.9121

Exercise 4.05

1 **a** $m^2 + 7m + 12$ **b** $w^2 + 10w + 25$
c $y^2 - 144$ **d** $h^2 - 2h - 63$
e $a^2 - 2a - 15$ **f** $x^2 - 15x + 44$
g $p^2 + 11p + 24$ **h** $c^2 - 19c + 84$
i $g^2 - 3g + 2$ **j** $u^2 - u - 56$
k $m^2 - 6m - 40$ **l** $q^2 - 5q - 66$
m $d^2 + 3d - 40$ **n** $-e^2 + 17e - 70$
o $h^2 - 14h + 45$

2 D **3** B

4 **a** $3y^2 + 22y + 7$ **b** $12k^2 + 35k + 18$
c $3m^2 + 4m - 15$ **d** $10p^2 - 19p - 15$
e $2w^2 - 19w + 24$ **f** $14x^2 + 29x + 12$
g $9b^2 - 24b + 16$ **h** $25a^2 + 60a + 36$
i $10q^2 - 21q - 49$ **j** $5p^2 - 9p - 18$
k $12d^2 - 29d - 11$ **l** $-8r^2 + 38r - 45$
m $-14y^2 + 15y + 9$ **n** $64h^2 - 9$
o $-49w^2 + 126w - 81$ **p** $16d^2 - 1$
q $3f^2 - 2f - 1$ **r** $-36u^2 + 60u - 25$

5 B

6 **a** $x^2 + 14x + 49$ **b** $x^2 - 14x + 49$ **c** $x^2 - 49$
d $y^2 - 36$ **e** $y^2 - 12y + 36$ **f** $y^2 + 12y + 36$
g $121 - y^2$ **h** $121 + 22y + y^2$ **i** $121 - 22y + y^2$
j $9b^2 + 42b + 49$ **k** $9b^2 - 49$ **l** $9b^2 - 42b + 49$

7 **a** $(3x - 10)(2x + 15)$ **b** $6x^2 + 25x - 150$
c $-6x^2 - 25x + 750$

8 **a** $3k(k + 5) = 3k^2 + 15k$
b length $= 3k - 8$, width $= k + 1$
c $(3k - 8)(k + 1)$
d $3k^2 - 5k - 8$
e $20k + 8$

9 **a** length $= a + 3$, width $= b + 1$
b Area $= (a + 3)(b + 1)$
c Area $= ab + a + 3b + 3$
d Increase in area $= a + 3b + 3$

10 Proof: see worked solutions

Exercise 4.06

1 **a** $-3, 5$ **b** $-10, -4$ **c** $6, 7$ **d** $-3, 2$
e $5, 9$ **f** $6, -4$ **g** $6, 9$ **h** $-4, 4$
i $-2, 5$ **j** $-7, -5$

2 **a** $(y + 6)(y + 2)$ **b** $(m + 8)(m + 7)$
c $(g + 2)(g + 7)$ **d** $(w + 9)(w + 4)$
e $(p + 8)(p + 3)$ **f** $(a + 6)(a + 7)$
g $(e + 3)(e + 9)$ **h** $(n + 3)^2$
i $(c + 3)(c + 7)$

3 **a** $(x - 5)(x - 4)$ **b** $(h - 3)(h - 10)$
c $(p - 3)(p - 8)$ **d** $(e - 6)(e - 5)$
e $(w - 9)(w - 8)$ **f** $(k - 9)(k - 1)$
g $(m - 8)^2$ **h** $(u - 3)(u - 2)$
i $(d - 5)(d - 7)$

4 **a** $(q - 10)(q + 2)$ **b** $(h - 9)(h + 4)$
c $(y + 11)(y - 4)$ **d** $(x - 9)(x + 7)$
e $(u + 10)(u - 1)$ **f** $(e + 10)(e - 3)$
g $(a - 11)(a + 10)$ **h** $(y + 9)(y - 3)$
i $(m - 7)(m + 1)$ **j** $(c + 9)(c - 2)$
k $(k + 9)(k - 6)$ **l** $(r - 11)(r + 2)$
m $(p - 8)(p + 4)$ **n** $(u + 15)(u - 3)$
o $(b - 8)(b + 2)$

5 D

6 **a** $(h - 1)^2$ **b** $(x + 5)(x + 10)$
c $(r + 8)(r + 12)$ **d** $(a + 4)(a - 7)$
e $(u + 5)(u - 12)$ **f** $(y - 9)^2$
g $(v - 8)(v + 7)$ **h** $(w - 15)(w + 4)$
i $(g + 6)(g - 3)$ **j** $(p + 6)(p + 8)$
k $(e + 8)(e - 1)$ **l** $(x - 7)(x - 12)$

Power plus

1 **a** 13 **b** 1 **c** 7 **d** 1 **e** $-\frac{125}{8}$ **f** 0

2 **a** $\frac{wy + wx - xy}{wxy}$ **b** $\frac{w}{xy}$ **c** $\frac{w + y}{wxy}$

3 **a** $a^2 - b^2 - ac + bc$
b $x^2 + y^2 - 2xy + 2x - 2y + 1$
c $3t + 4 - \frac{1}{t}$

ANSWERS

4 **a** $(x-8)(x-125)$ **b** $(y+50)(y-36)$
c $(b+41)^2$ **d** $(n-50)(n+50)$

Test yourself 4

1 **a** $6v^5w^7$ **b** $8t^7h^6$ **c** $25x^2y^4$
d 1 **e** $\frac{1}{4k}$ **f** $\frac{125y^3}{8}$
g 0 **h** $1\frac{1}{3}$ **i** $\frac{8a^2g}{9}$

2 **a** $\frac{1}{16m^2}$ **b** $\frac{4}{m^2}$ **c** $625b^{24}y^{12}$
d $512t^{14}u^{16}$ **e** $5c^4d^4$ **f** $\frac{6a}{5b}$

3 **a** $\frac{-3t}{20}$ **b** $\frac{19g}{6}$ **c** $\frac{x}{16}$ **d** $\frac{3b+2}{7}$
e $\frac{5w-12}{8}$ **f** $\frac{35-12y}{60}$ **g** $\frac{20m+21n}{28}$ **h** $\frac{17p}{24}$

4 **a** $\frac{3p}{4m}$ **b** $\frac{3}{q^2}$ **c** $\frac{5y}{7}$ **d** $\frac{3}{2}$
e $\frac{35}{8a^2}$ **f** $8d^2$ **g** $\frac{6y}{5}$ **h** $4ac$

5 **a** $9m-72$ **b** $10ab+b^2$
c $-12x^2y+15y^2$ **d** $56tp^2-40t^2p$
e $-3n+10$ **f** $-15h^3-35h^2$
g $20y^2-28hy$ **h** $-3wx^2+7w^2x$

6 **a** $-13g+21g^2$ **b** $4fg^2-30f^2g$ **c** $93-22n$
d $8x^3+7x^4$ **e** $10y^2-41y+21$

7 **a** $8(t-9)$ **b** $b(b+36)$ **c** $-3(m+11)$
d $4wr(9r+7w)$ **e** $-6(4p-3q)$
f $(5x-1)(2-3x)$

8 **a** $15xy^2(1-2x^2y)$ **b** $6p(t^2+2pt-8p^2)$
c $4r^2s^3(8s+3r^2)$ **d** $25x^3y^3(2x-3y)$
e $-8p^3q^3(1-6q^3)$ **f** $(n^2+6)(n-1)$

9 **a** $b^2+13b+30$ **b** d^2+d-56
c $15t-54-t^2$ **d** $20x^2+13x-21$
e $49y^2-25$ **f** $21p^2-62p+16$
g $9m^2+42m+49$ **h** $8+10x-3x^2$
i $10d^2-27d+18$

10 **a** $(y+5)^2$ **b** $(x-20)(x-1)$
c $(n+11)(n-3)$ **d** $(a-7)(a-4)$
e $(m-12)(m+7)$ **f** $(p+9)(p-6)$

Chapter 5

SkillCheck

1 **a** **i** 10 **ii** 16.5 **iii** 15 **iv** 15
b **i** 13 **ii** 1.8 **iii** 2.5 **iv** 3
c **i** 48 **ii** 34.3 **iii** 34.5 **iv** 24, 35
d **i** 5 **ii** 2.2 **iii** 2 **iv** 2

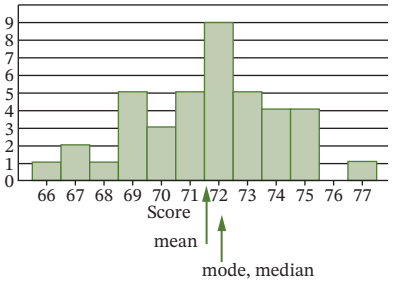
2 **a** **i** 31 **ii** 33.3 **iii** 62
b 78
c **i** Median = 30, mean = 28.3, range = 25.
ii The outlier has increased the median (by 1), the mean (by 5), and the range (by 37).

Exercise 5.01

1 **a** **i** symmetrical
ii clustering at 9, no outliers
b **i** bimodal
ii clustering in the 30s and 60s, no outliers
c **i** positively skewed
ii clustering at 1, 8 is an outlier
d **i** negatively skewed
ii clustering at 23–24, no outliers
e **i** positively skewed
ii clustering at 13, 23 is an outlier
f **i** symmetrical
ii clustering at 4 and 5, no outliers
g **i** positively skewed
ii clustering at 130–150, no outliers
h **i** bimodal
ii clustering at 50s and 100s, 136 is an outlier

2 **a**

Score	Frequency
66	1
67	2
68	1
69	5
70	3
71	5
72	9
73	5
74	4
75	4
76	0
77	1



b no outliers
c negatively skewed
d In golf scoring, the lower the score, the better. In the final round of a tournament you would expect good players, so more low scores.
e clustering at 72
f mode = 72, $\bar{x} = 71.6$, median = 72

3 **a** 46 **b** 1–9 hours (stem of 0)
c no outliers **d** positively skewed
e Most students spend limited time on their computers, and have other commitments and do other activities such as sport. Only a few students spend many hours on the computer during the week.
f Mode = 1, $\bar{x} = 13.9$, median = 11

4 **a**

Stem	Leaf
12	3
13	
14	2 3
15	0 1 3 3 3 5 5
16	0 0 1 2 2 2 2 3 4 5 5 7 8 9
17	0 0 1 2 3
18	2

b Symmetrical

c 123 is an outlier

d Clustering occurs in the 160s

e mode = 162, median = 162, $\bar{x} = 160.2$

5 **a** Bimodal **b** No **c** 29.3

d 28.4 **e** 28.45 **f** 8.7

g Yes, because there aren't any outliers.

6 B

Exercise 5.02

1 **a** 5, 6.5, 8 **b** 18, 20, 26.5 **c** 32, 34.5, 38

2 **a** range = 7, IQR = 3 **b** range = 22, IQR = 8.5

c range = 16, IQR = 6

3 **a** 7.5 **b** 3

4 **a** 283 **b** 128

5 **a** 3 **b** 2.5 **c** 17.5

d 19 **e** 21.5 **f** 1.5

6 **a** 34 **b** 13

c **i** 68, 72, 72, 75, 77, 78, 79, 80 **ii** 50%

d 75%

7 **a** **i** 28 **ii** 9.5

b The interquartile range, as it is not affected by the value of 35.

c 48, 48, 48, 49, 51, 53, 55; 54%

Exercise 5.03

1 **a** 2.66 **b** 2.63 **c** 1.19 **d** 1.33 **e** 2.01

2 **a** 7 **b** **i** 2.64 **ii** 2.28

c Decreases the standard deviation

3 **a** C **b** A

4 **a** 1.99 **b** 13.43

5 **a** $\bar{x} = 165.89$, $\sigma = 8.37$

b **i** less than 157 or greater than 175

ii between 157 and 174

6 **a** $\bar{x} = 11.37$, $\sigma = 0.43$

b **i** less than 10.9, greater than 11.8

ii between 10.9 and 11.8

7 C

Exercise 5.04

1 **a** Men: $\bar{x} = 71.40$, $\sigma = 6.77$

Women: $\bar{x} = 77.53$, $\sigma = 6.96$

b Yes, the mean of women's pulse rates is much higher. The standard deviation for women is slightly higher.

2 **a** Dominant hand: $\bar{x} = 0.40$, $\sigma = 0.11$;

Non-dominant hand: $\bar{x} = 0.52$, $\sigma = 0.51$

b Yes, the mean reaction time and standard deviation of the dominant hand are much lower than the mean and standard deviation of the non-dominant hand.

c **i** 0.61 and 0.75 **ii** $\bar{x} = 0.37$, $\sigma = 0.05$

iii Removing the outliers has reduced the mean from 0.40 to 0.37 and more than halved the standard deviation.

d $\bar{x} = 0.40$, $\sigma = 0.06$

e The removal of the outlier from the dominant hand had the greater effect on the mean and standard deviation as the outlier of 2.60 was a more extreme value than the outliers for the dominant hand.

3 **a** Western Tigers: $\bar{x} = 122.92$, $\sigma = 26.98$;

Barrington City: $\bar{x} = 120.92$, $\sigma = 23.62$

b The Barrington City team is slightly more consistent as the standard deviation is 23.62 compared with 26.98 for Western Tigers.

4 **a** Vatha: $\bar{x} = 13.76$, $\sigma = 0.55$;

Ana: $\bar{x} = 14.14$, $\sigma = 0.66$

b Vatha is more consistent as the standard deviation for her times is significantly lower than the standard deviation for Ana's times.

5 B

6 **a** Maths: **i** range = 47 **ii** IQR = 14 **iii** $\sigma = 10.97$

Science: **i** range = 45 **ii** IQR = 19 **iii** $\sigma = 13.16$

b Maths: $\bar{x} = 67.82$; Science: $\bar{x} = 61.25$

c The students performed better in Maths as the mean was 67.82 compared to 61.25 for Science. The marks for Maths were also more consistent as the IQR and standard deviation were both lower than those of Science.

7 **a** Roosters: **i** range = 48 **ii** IQR = 20

iii $\bar{x} = 26.67$ **iv** $\sigma = 12.55$

Dragons: **i** range = 32 **ii** IQR = 8

iii $\bar{x} = 15.88$ **iv** $\sigma = 7.36$

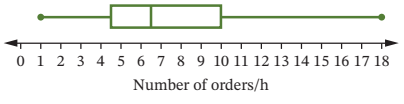
b The range, IQR and the standard deviation for the Dragons are significantly lower than those of the Roosters, which show that the Dragons are more consistent in the number of points they scored per match.

However the mean of the Roosters is significantly greater than the mean of the Dragons, which would indicate they are a better team as they were able to score many more points per match.

Exercise 5.05

1 **a** 1, 4.5, 6.5, 10, 18

b

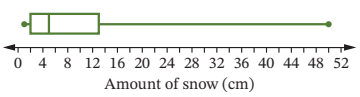


ANSWERS

2 **a** 26

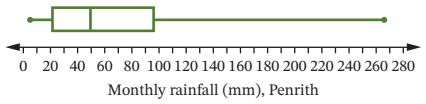
b 1, 2, 5, 13, 50

c



3 **a** 261 **b** 5, 21.5, 49.5, 96, 266

c



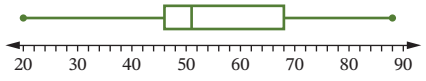
4 **a** 27.5 h **b** 26 h **c** 30 h **d** 4 h **e** 50%

5 **a** 26 **b** 21 **c** 14 **d** **i** 25% **ii** 75%

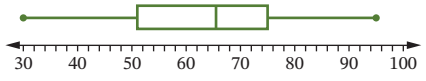
6 **a** 6, 10, 19, 23, 29 **b** 13

c **i** 14 **ii** 7 **iii** 7 **iv** 21

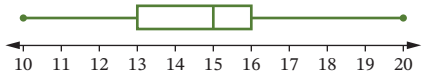
7 **a** 20, 46, 51, 68, 88



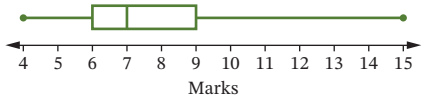
b 30, 51, 65.5, 75, 95



c 10, 13, 15, 16, 20

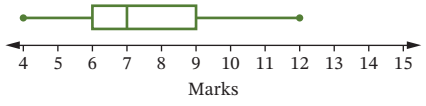


8 **a** 4, 6, 7, 9, 15



b Dot plot is positively skewed. The length of the box plot from the median to the highest value is greater than the length from the median to the lowest value.

c 15 **d** 4, 6, 7, 9, 12



e **i** The box plots are the same up to Q_3.

ii The whisker from Q_3 is reduced without the outlier.

Exercise 5.06

1 **a** **i** Year 10: 3.5; Year 8: 8

ii Year 10: 7.5; Year 8: 8.5

iii Year 10: 1; Year 8: 2

b **i** 25% **ii** 75% **c** **i** 10 **ii** 0

2 **a** **i** 32 **ii** 34

b Swifts: 59.5, Thunderbirds: 51.5

c Swifts: 8, Thunderbirds: 11

d The range for the Swifts is slightly smaller (32) and the IQR = 8 for the Swifts is less than the IQR = 11 for the Thunderbirds indicating that the Swifts are more consistent in their performance.

e The position of the Swifts box plot shows that they scored more points in games, they have a higher median of 59.5 than the Thunderbirds who had a median of 51.5 so the Swifts performed better in the season.

3 **a** 10K: 9; 10N: 10 **b** 10K: 6.5; 10N: 5.5

c 10K: 3; 10N: 4

d 10K – lower range and IQR.

e 75%

4 C

5 **a** Brisbane: 26.9, 9.3, 4.7 Sydney: 23.5, 8.5, 4.9

Melbourne: 21.4, 13, 8.6 Hobart: 18.6, 11.2, 7

b Melbourne – it has the highest range and IQR.

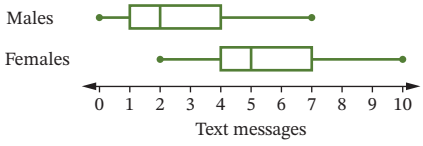
c Brisbane, more than half of the mean monthly temperatures are higher than most of the mean monthly temperatures of the other cities.

d Sydney's median temperature is significantly higher than Melbourne's, so Sydney is the warmer city.

e Sydney has the smaller range and IQR of mean monthly temperatures so it is more consistent.

6 **a** Males: 0, 1, 2, 4, 7; Females: 2, 4, 5, 7, 10

b

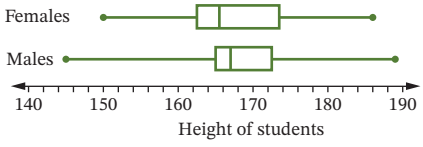


c Males: 3 Females: 3

d Males: 7 Females: 8

e Both are positively skewed, the interquartile range is the same, and the range of females is 1 more that of the males. Females do receive more text messages as the box plot shows that 75% of females receive more messages than 75% of males.

7 **a** Males: 145, 165, 167, 172.5, 189;
Females: 150, 162.5, 165.5, 173.5, 186

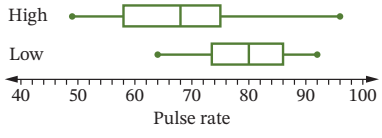


b Male: range = 44; IQR = 7.5

Female: range = 36; IQR = 11

c Male students have a greater range (44 compared to 36), but a smaller interquartile range (7.5 compared to 12).

8 **a** Low: 64, 73.5, 80, 86, 92; High: 49, 58, 68, 75, 96

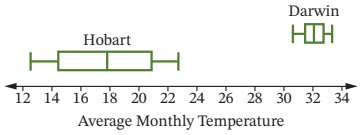


b Low: 28, 12.5; High: 47, 17

c The range and interquartile range of the High Frequency group are both greater than the Low Frequency group.

d The High Frequency group.

9 **a** Hobart: 12.5, 14.4, 17.8, 20.85, 22.7;
Darwin: 30.6, 31.45, 32.05, 32.7, 33.3



b Hobart: 10.2°, 6.45° Darwin: 2.7°, 1.25°

c Darwin average monthly temperatures were more consistent than Hobart, since its range (2.7) was much smaller than Hobart's (10.2), while Hobart's IQR was much larger (6.45) than Darwin's (1.25).

10 **a** Simone **b** Simone: 12; Amal: 10

c Amal, smaller range **d** Simone: 10; Amal: 9

e Simone: 4; Amal: 5

f Not enough information given to make a valid decision. The interquartile range and range only differ by 1.

g 25% **h** 25%

Mental skills 5

2 **a** 176 **b** 363 **c** 261 **d** 405

e 682 **f** 707 **g** 1818 **h** 3564

i 152 **j** 540 **k** 2142 **l** 588

m 288 **n** 693 **o** 3939 **p** 852

Exercise 5.07

1 **a** Boys: \$34.58; Girls: \$31.78

b Boys: \$33.50; Girls: \$28

c Boys: Range = 72, IQR = 25;
Girls: Range = 69, IQR = 30.5

d **i** Boys are positively skewed slightly, girls are positively skewed.

ii There are no outliers, clustering occurs for the boys in the 20–30s and for the girls in the 10–20s.

e Boys generally carry more cash – they have a higher mean than the girls and the shape of the data for girls is more positively skewed.

2 **a** 21 games **b** **i** 34 **ii** 51

c Scorpions: $\overline{x} = 1.6$ goals; Vale United: $\overline{x} = 2.4$

d Scorpions 5, Vale United 6

e The shape of both teams' results is positively skewed. Clustering for Scorpions occurs at 1 and 2 and for Vale United it occurs at 2.

f Vale United performed better as its mean was 2.4 goals/game compared to Scorpions 1.6 goals/game.

3 **a** Sydney: $\overline{x} = 26.2$, median = 26.5, mode = 28
Perth: $\overline{x} = 34.3$, median = 35, mode = 38

b Sydney: Range = 9°, IQR = 3
Perth: Range = 16°, IQR = 8

c The temperatures for Sydney and Perth are both negatively skewed, there are no outliers. Sydney's temperatures are clustered from 26 to 28, while Perth's have no distinct cluster.

d Sydney's temperatures are lower than Perth's, as evidenced by the significantly lower mean, median and mode. The range and interquartile range for Perth are greater than the range and interquartile range for Sydney, indicating greater spread.

4 **a** 30

b Quiz 1: $\overline{x} = 5.6$, mode = 6;
Quiz 2: $\overline{x} = 6.3$, mode = 7

c Quiz 1: 6; Quiz 2: 7

d Quiz 1: **i** Range = 7 **ii** IQR = 2
Quiz 2: **i** Range = 8 **ii** IQR = 2

e Quiz 1: Results are symmetrical with clustering 5–6, no outliers.
Quiz 2: Results are bimodal with clustering at 5 and 7–8, no outliers.

f Scores for Quiz 2 are better than Quiz 1, as the mean of Quiz 2 is higher than the mean of Quiz 1. The spread for both quizzes are similar as there is only a difference of 1 between the ranges and the IQRs are equal.

5 **a** 39

b **i** mode = 2 **ii** median = 2
iii range = 6 **iv** IQR = 1.5

c positively skewed, no outliers

d 50%

e **i** By the highest columns.

ii By the short length of the box when compared to the whole length of the box plot.

f **i** The shape of the distribution, the frequency for each household size and the mode. The mean can also be calculated from the histogram.

ii The shape of the distribution, the median and the quartiles Q_1 and Q_2.

6 **a** **i** 5 **ii** 16

b **i** mode = 22 **ii** range = 18
iii IQR = 24 – 16 = 8

c Negatively skewed.
i The tail of the dot plot goes to the left.
ii The length of the box plot from the lowest score to the median is longer than from the median to the highest score.

d **i** dot plot **ii** box plot
iii dot plot **iv** box plot

7 **a** Sunbeam Valley: range = 24, median = 71, IQR = 8
Bentley's Beach: range = 30, median = 73, IQR = 15

b Sunbeam Valley: negatively skewed (slight)
Bentley's Beach: positively skewed

c Sunbeam Valley's speeds are clustered in the 70s.

d 25%

e Bentley's Beach – higher median, positively skewed. 25% of drivers drive faster than all drivers in Sunbeam Valley. This may be due to more main roads with higher speed limits.

8 **a** 36

b Lamissa: mode = 7, median = 7
Anneka: mode = 7, median = 6

c Lamissa: range = 8, IQR = 8 – 6 = 2
Anneka: range = 9, IQR = 7 – 4 = 3

d Lamissa's distribution of scores is negatively skewed with clustering at 7. Anneke's distribution is negatively skewed with clustering at 6 and 7.

e **i** 25% **ii** 50%

f Lamissa is the better archer. Her median score is higher than Anneke's. According to the box plot, roughly 25% of Lamissa's scores are less than 6 compared to Anneke's 50%, and 50% of Lamissa's scores are equal to or better than 75% of Anneke's. Lamissa's range and IQR are also slightly smaller than Anneke's, which suggests she is the more consistent archer.

9 **a** The range (47) is too large.

b Women: 31 Men: 37

c Women: Range = 38, IQR = 40 – 24 = 16
Men: Range = 47, IQR = 46 – 25 = 21

d Distribution for women is positively skewed. Distribution for men is symmetrical.

e Men have the greater spread in the number of sit-ups completed, as the range and IQR are both greater than those for women.

10 **a** **i** 56 **ii** 38

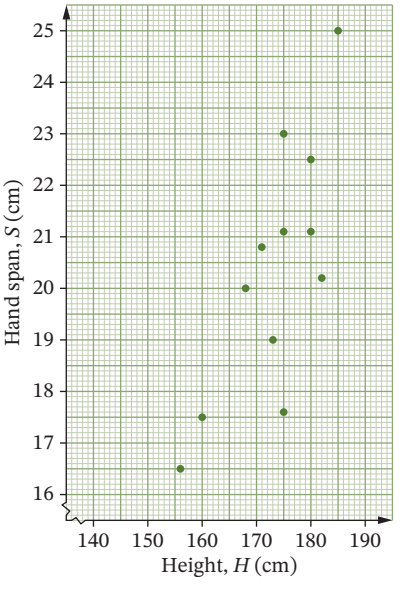
b **i** 10 Blue, 10 Yellow **ii** 10 Green
iii 10 Red

c **i** 10 Green **ii** 10 Yellow
iii 10 Red, 10 Blue

d 10 Blue. It shares the highest median with 10 Red but its lowest score is still higher than 25% of 10 Red's scores.

Exercise 5.08

1 **a**



b linear

c As the heights of students increase, their hand spans tend to increase.

2 **a** weak negative relationship

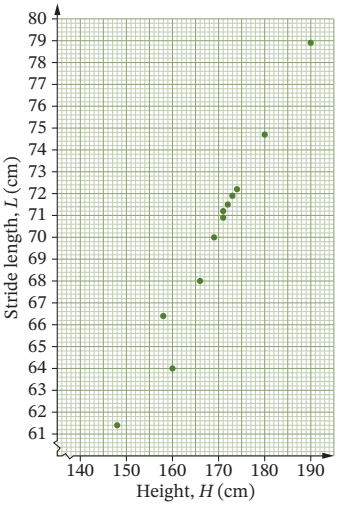
b no relationship

c strong positive relationship

3 Weak positive

4 **a** Stride length depends on a person's height as the taller the person, the longer their legs are.

b



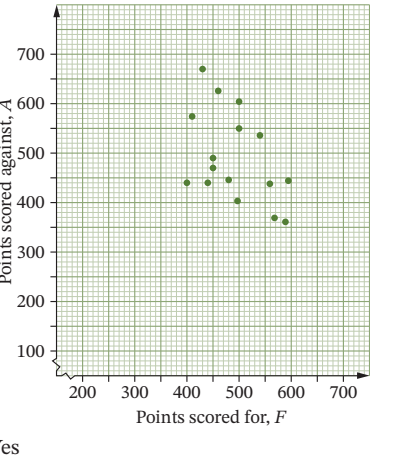
c linear

d Students' stride length increases with height

e strong positive relationship

f 72.5 to 73 cm

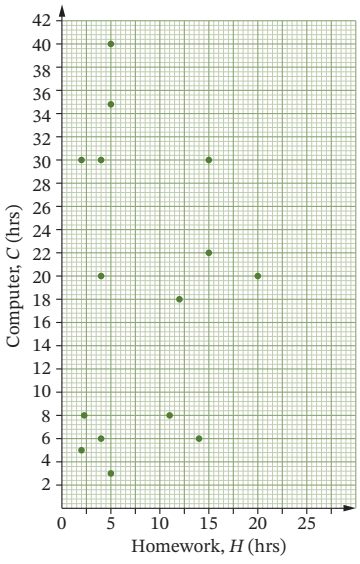
5 **a**



b Yes

c Weak negative relationship

6 a

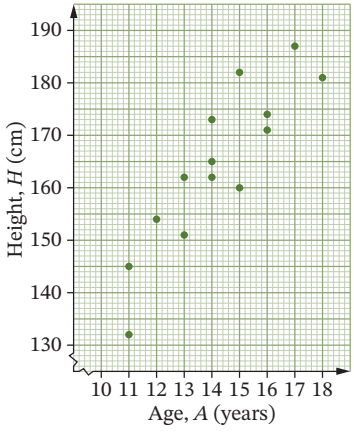


b no relationship

7 a Not causal **b** Causal **c** Causal
d Not causal **e** Not causal

8 C

9 a



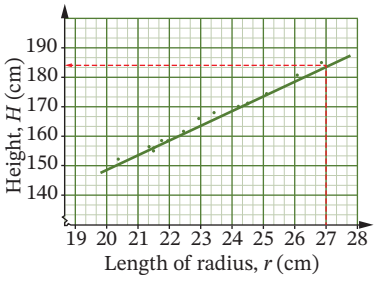
b Height, because its value depends on age (not the other way around).

c weak positive relationship.

Exercise 5.09

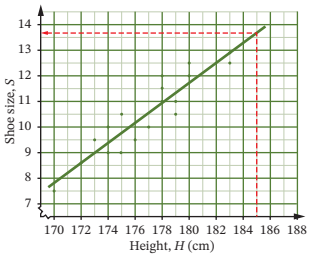
Answers will vary as equations of lines will vary.

1 a



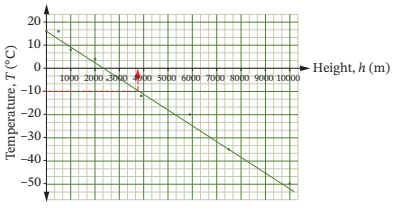
b $H = 5r + 48.5$ **c** 173.5 cm **d** 184 cm

2 a



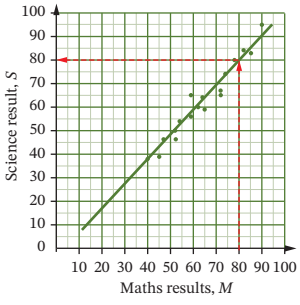
b $S = 0.39H - 59$ **c** 8
d 12 **e** 13.5

3 a



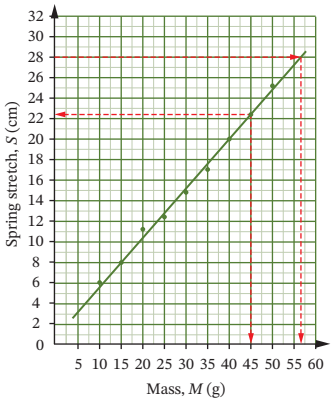
b $T = -0.0068h + 16$ **c** 5.8°C
d 3800 m

4 a



b 80 **c** 95

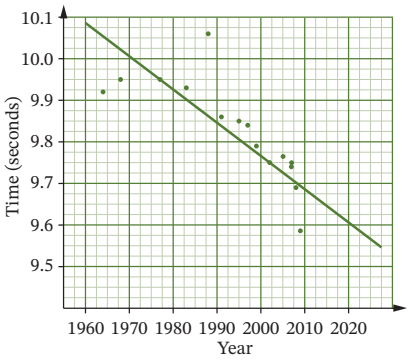
5 a



b 22.4 cm **c** 56 g

d Yes, because a spring has an elastic limit, which is the point at which a spring will not return to its original length as a result of the Mass attached to the spring being too heavy.

6 **a**



b 9.53 s

c There is a limit to how fast a person can run.

Exercise 5.10

1 **a** Just surveying 300 people between 9 a.m. and 11 p.m. in shopping centres only targets a narrow group of people in certain areas.

b The sample needs to be more random and over a large area, not just in shopping centres. A telephone survey should produce more accurate feedback.

2 The report does not say what conditions are needed for the hot water system to work effectively. The temperature in Queensland is much warmer than in NSW and Victoria. Consequently, with the cooler climate in NSW and Victoria, especially in winter, the heat pump system may not provide the savings that people in Queensland obtain.

3 **a** **i** The price of petrol has shown little increase from December to February.

ii The price of petrol has shown marked rises and falls over the period from December to February.

b Both graphs could be improved by starting the vertical scale at 0 cents/litre.

4 **a** That there is a marked difference between the fuel consumption of the different cars.

b **i** 0.2 L/100 km **ii** 1 L/100 km **iii** 0.2 L/100 km

c Begin the scale on the vertical axis with 0 and use a scale of 1 cm = 0.5 L/100 km instead of 1 cm = 0.2 L/100 km.

5 Yes, as there is no option for a customer to rate the app as unsatisfactory or poor.

6 **a** An example of a biased question could be: Which of these colours do you prefer – red, black, silver, blue?

b Apart from surveying people, they need to look at the sales figures of all cars. This will give information about the most popular car colour.

7, 8 Teacher to check.

9 **a** stratified **b** random

c systematic or random

d random or systematic

e random **f** systematic or random

10 **a** systematic **b** stratified **c** random

11 every 20th globe

12 every 17th student from a list

13 D

14 17

15

Year	7	8	9	10	11	12
Number of students	7	7	8	9	9	10

Exercise 5.11

1 **a** approx. 180 **b** approx. 220%

2 Teacher to check

3 **a** 66%, –9 **b** 5%

c public transport **d** Teacher to check

e worked from home, the COVID pandemic in 2020

4 **a** Tasmania **b** 28%

c Highest: Queensland – has a large population and many people move there when they retire

Lowest: Tasmania – has a small population, unlikely to know someone who lives there.

d The ACT contains Canberra, the capital of Australia and a business centre that is easy to reach, whereas it is much further (in distance and time) to travel to Tasmania.

e ACT: for business and government (work-related).

5–8 Teacher to check

Power plus

1 **a** –1 and 1

b There is no relationship between the variables.

c **i** 1 **ii** – 0.2 **iii** –0.8

Other answers possible, teacher to check

2 6.4

3 **a** $\bar{x} = 13.35$, median = 14, mode = 14

b range = 10, IQR = 15 – 12.5 = 2.5

c The mean, median, and mode will increase by 4, the range and the interquartile range remain unchanged.

Test yourself 5

1 **a** **i** negatively skewed

ii clustering at 7 and 8, no outliers

b **i** negatively skewed

ii clustering at 16 and 17, 10 is an outlier

c **i** positively skewed

ii clustering in the 40s and 50s, no outliers

2 **a** 6.5 **b** 6 **c** 2.5 **d** 12.5 **e** 2

3 **a** $\bar{x} = 0.40$, $\sigma = 0.08$

b range = 0.33, IQR = 0.08

c The interquartile range is the better measure as the standard deviation is affected by the outlier 0.62.

4 **a** Girls: $\bar{x} = 67.73$, $\sigma = 16.08$

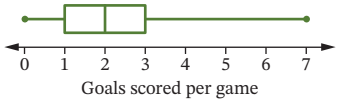
Boys: $\bar{x} = 61.67$, $\sigma = 12.35$

b The girls performed better than the boys as their mean mark was about 6 more than the mean mark of the boys. However the boys marks had less spread than the girls.

5 **a** range = 7, IQR = 3 – 1 = 2

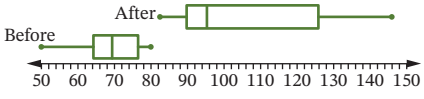
b 0, 1, 2, 3, 7

c



6 **a** Before: 50, 64, 69, 76, 80;
After: 82, 89, 95.5, 126, 146

b



c **i** range = 30, IQR = 12

ii range = 64, IQR = 37

d The pulse rates for after exercise are significantly higher. In fact, all the rates for after exercise are above all the rates for before exercise. The median pulse for after exercise is 95.5 compared to the median pulse of 69 for before exercise. The range and interquartile range are also greater for the after exercise pulse rate.

7 **a** **i** Both **ii** Stem-and-leaf plot

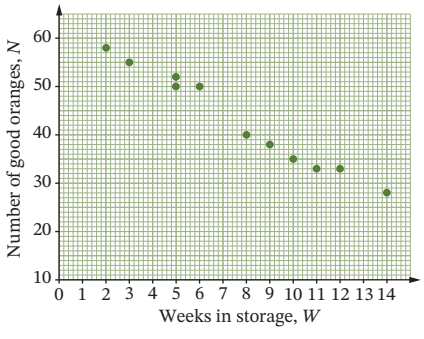
b The range (126 – 70 = 56) is too large

c **i** median = 92 **ii** IQR = 99.5 – 84 = 15.5

d 50%

8 **a** Weeks in storage: this determines how many oranges stay good.

b



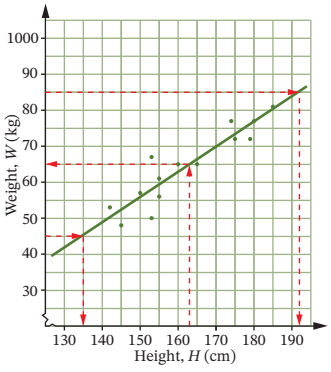
c linear

d The longer the oranges remain in storage, the less good oranges are in the box.

e strong negative correlation

9 **a** Not causal **b** Not causal **c** Causal
d Causal **e** Causal **f** Causal

10 **a**



b $W = 0.7H - 49$ **c** 70 kg **d** 65 kg

e **i** 192 cm **ii** 135 cm

11 **a** That the product is healthy.

b There is no data given on the actual fat content in the product. This should also be stated in terms of a percentage of daily requirement of fat or mg of fat.

12 **a** Systematic

b Answers may vary. Advantage: Selecting every 6th student is objective and not biased, no favouritism. Disadvantage: Could be more complicated to implement, keeping track of order.

13 13

Chapter 6

SkillCheck

1 **a** $y = 2$ **b** $x = 12$ **c** $m = 10$
d $w = 4$ **e** $m = 15$ **f** $n = -10$

2 **a** $5x + 50$ **b** $4y - 4$ **c** $10 - 6y$

3 **a** $x = 10$ **b** $x = 8$ **c** $a = -12$
d $x = 6$ **e** $x = 8$

Exercise 6.01

1 **a** $k = 60$ **b** $w = -9$ **c** $y = 2$
d $a = 5$ **e** $x = 8$ **f** $a = 1$
g $r = -6$ **h** $w = 6$ **i** $y = -2$
j $a = 3\frac{1}{2}$ **k** $u = 5\frac{1}{3}$ **l** $a = -1\frac{3}{4}$

2 B **3** D

4 **a** $y = 10$ **b** $a = -2\frac{1}{2}$ **c** $y = 5$
d $a = -3$ **e** $y = 8$ **f** $x = 1$
g $y = -1$ **h** $x = -10$ **i** $m = 1\frac{1}{2}$
j $y = 12$ **k** $a = -1$ **l** $x = -4$

5 C

6 **a** $x = 16$ **b** $m = 6$ **c** $y = 4$
d $y = 8$ **e** $y = -7$ **f** $x = 11$
g $m = -8$ **h** $a = -1$ **i** $y = -5$

7 C

8 **a** $d = \frac{2}{7}$ **b** $y = -\frac{7}{8}$ **c** $k = \frac{8}{9}$
d $g = 3\frac{2}{7}$ **e** $h = \frac{6}{17}$ **f** $p = -1\frac{4}{5}$

9 **a** $k = 10$ **b** $d = 21$ **c** $w = 17$ **d** $m = 5\frac{1}{3}$
e $a = 17$ **f** $d = 21$ **g** $y = -6$ **h** $x = 2\frac{1}{4}$
i $b = 7\frac{7}{9}$ **j** $x = 31$ **k** $y = 47$ **l** $m = 18$
m $h = 4\frac{1}{4}$ **n** $e = 6\frac{1}{2}$ **o** $y = -3$ **p** $h = 4$
q $a = 5\frac{1}{3}$ **r** $c = -15\frac{2}{3}$

Exercise 6.02

1 **a** B **b** B **c** C

2 **a** $y = 14$ **b** $m = 2\frac{3}{7}$ **c** $x = 8$ **d** $k = 23\frac{1}{2}$
e $w = 5$ **f** $a = 3\frac{1}{3}$ **g** $e = -5\frac{1}{4}$ **h** $x = 10\frac{1}{2}$
i $p = -3\frac{1}{4}$ **j** $y = -3$ **k** $x = 24$ **l** $m = 10$
m $n = -\frac{1}{4}$ **n** $n = -\frac{3}{5}$ **o** $d = 3\frac{3}{4}$ **p** $w = 14$
q $d = 17\frac{1}{4}$ **r** $m = -3\frac{2}{3}$ **s** $h = -2$ **t** $g = -3$

3 **a** $k = 1\frac{7}{8}$ **b** $w = 1\frac{1}{3}$ **c** $x = -1\frac{1}{3}$
d $k = \frac{17}{25}$ **e** $m = 1\frac{15}{16}$ **f** $y = -6\frac{1}{6}$
g $n = -23$ **h** $y = 3$ **i** $a = -8\frac{3}{5}$
j $p = 9\frac{2}{3}$ **k** $k = 7\frac{5}{6}$ **l** $y = -\frac{5}{6}$
m $w = 10$ **n** $a = 42$ **o** $e = 6\frac{6}{7}$
p $a = \frac{10}{23}$ **q** $y = 60$ **r** $a = 1\frac{11}{13}$
s $m = 1\frac{1}{19}$ **t** $h = 1\frac{22}{23}$ **u** $y = -6\frac{5}{6}$

4 **a** C **b** A

5 **a** B **b** D **c** A **d** C

Exercise 6.03

1 **a** $m = \pm 12$ **b** $x = \pm 20$
c $y = \pm\sqrt{35}$ **d** $k = \pm 13$
e $y = \pm 1$ **f** $w = \pm\sqrt{24}$ $\left(\text{or } \pm 2\sqrt{6}\right)$
g $x = \pm 2$ **h** $t = \pm 4$
i $a = \pm 4$ **j** $k = \pm\sqrt{45}$ $\left(\text{or } \pm 3\sqrt{5}\right)$
k $w = \pm 10$ **l** $d = \pm 12$
m $k = \pm\sqrt{14}$ **n** $w = \pm 5$
o $x = \pm\frac{1}{2}$ **p** $m = \pm\sqrt{40}$ $\left(\text{or } \pm 2\sqrt{10}\right)$
q $y = \pm 1$ **r** $p = \pm 3$
s $k = \pm\sqrt{8}$ $\left(\text{or } \pm 2\sqrt{2}\right)$
t $y = \pm 10$

2 **a** $m = \pm 2$ **b** $a = \pm 9$ **c** $m \approx \pm 5.3$
d $m \approx \pm 1.9$ **e** $k \approx \pm 0.6$ **f** $x \approx \pm 7.6$
g $k \approx \pm\ 9.8$ **h** $k \approx \pm 9.5$ **i** $y \approx \pm 0.3$
j $a \approx \pm 9.2$ **k** $y \approx \pm 6.2$ **l** $w \approx \pm 7.1$

3 **a** B **b** B

4 **a** $x = -2, -1$ **b** $y = -4, -1$ **c** $y = -4, -12$
d $x = -4, 3$ **e** $x = -3, 1$ **f** $x = -8, 5$

5 **a** $x = 6, -5$ **b** $x = 4$ **c** $x = 11, -6$
d $d = 0, 2$ **e** $x = 5, -2$ **f** $n = 0, -4$
g $k = 0, 7$ **h** $y = 0, 5$ **i** $v = 0, 12$

6 You cannot take the square root of a negative number.

7 **a, c, f:** cannot find the square root of a negative number.

8 **a** A **b** C

Exercise 6.04

1 18 mm, 36 mm, 36 mm **2** 57 mm, 19 mm
3 13 cm, 29 cm **4** 61, 62, 63
5 Tyson is 27, Charlotte is 3 **6** 26
7 4 **8** Vatha is 22, Chris is 14
9 213, 214, 215, 216 **10** $x = 35.5$
11 117 **12** 6
13 Scott is 11, Kait is 34 **14** 25°, 50°, 105°

Mental skills 6

2 **a** 160 **b** 70 **c** 240 **d** 900
e 2600 **f** 900 **g** 140 **h** 300
i 180 **j** 770 **k** 18 **l** 34
m 46 **n** 26 **o** 18 **p** 12
q 40 **r** 8 **s** 14 **t** 24

Exercise 6.05

1 **a** 52 cm **b** 17 m

2 **a** 36 km/h **b** 86.4 km/h **c** 180 km/h

3 30.6 km/h

4 **a** 27°C **b** 0°C **c** 100°C **d** 39°C

5 **a** 86°F **b** 104°F **c** 59°F **d** 113°F

6 43

7 **a** 11.2 **b** 9 **c** 17.3

8 **a** 15.1 m **b** 31.8 cm

9 **a** 21.0 **b** 105.8 kg

10 **a** 137.3 cm^3 **b** 4.9 m

11 **a** 93 km/h **b** 436 km **c** 7 h

12 **a** \$950 **b** 24 km

13 **a** 73.9 m^2 **b** $h = 13.2$ cm

Exercise 6.06

1 **a** $y = 5 - 3x$ **b** $y = \frac{7-x}{4}$ **c** $y = \frac{d-10}{6}$
d $y = \frac{x-g}{b}$ **e** $y = \frac{M-A}{x}$ **f** $y = \frac{dm}{3}$
g $y = \frac{5x-2}{w}$ **h** $y = \frac{T}{10g}$ **i** $y = 2kd + p$
j $y = \frac{4h-m}{a}$ **k** $y = \frac{B-6g}{a}$ **l** $y = \frac{2V-3e}{4}$

m $y = \frac{3R}{T}$ **n** $y = \frac{4}{Bw}$

o $y = \frac{6}{h} + 1$ or $y = \frac{6+h}{h}$

2 **a** $a = \frac{2S - at^2}{2t}$ **b** $a = \frac{2S - 2ut}{t^2}$

3 C

4 B

5 **a** $d = \frac{C - 3.6}{2.2}$ **b** 12.5 km

6 **a** $m = \frac{y - c}{x}$ **b** $b = \frac{A}{lh}$ **c** $h = \frac{3V}{A}$

d $h = \frac{3V}{\pi r^2}$ **e** $x = \frac{4 - k}{5}$ **f** $n = Ph - d$

Power plus

1 **a** $y = 2\frac{3}{7}$ **b** $y = 2\frac{1}{2}$ **c** $m = -10$

2 $d = -8$

3 Randall is 17, Tarni is 7.

4 The number is 36.

Test yourself 6

1 **a** $a = 11$ **b** $y = 2\frac{2}{3}$ **c** $a = -3$

d $y = 1$ **e** $m = 11$ **f** $a = 28$

g $h = 1\frac{5}{8}$ **h** $y = \frac{13}{20}$

2 A

3 **a** $w = 6$ **b** $y = 8\frac{3}{4}$ **c** $a = -3$

d $m = 3$ **e** $x = 2\frac{7}{26}$ **f** $p = -\frac{5}{34}$

g $s = 4$ **h** $x = 1\frac{2}{3}$ **i** $k = 8\frac{4}{17}$

4 C

5 **a** $y = \pm 2$ **b** $p = \pm 10$

c $x = \pm\sqrt{10}$ **d** $m = \pm 1$

e $w = \pm\sqrt{50}$ $\left(\text{or } \pm 5\sqrt{2}\right)$ **f** $x = -7, -1$

g $h = 9, -1$ **h** $u = 7, -11$ **i** $k = 0, -5$

j $m = 0, 2$ **k** $b = -10$ **l** $w = 9, 0$

6 92, 93, 94, 95 **7** Corinna 45, Isla 15.

8 120 m

9 **a** 160 mm^3 **b** 300 m^2

10 **a** $a = \frac{y - b}{x}$ **b** $a = mP^2$ **c** $a = \frac{1 - M}{M + 1}$

Practice set 2

1 **a** **i** negatively skewed

ii no outliers, clusters at 7 and 8

b **i** positively skewed

ii outlier 98, clustering in 40s and 50s

2 **a** $81n^4 m^8$ **b** $20p^9 q^3$ **c** $4a^4 b^3$

d 1 **e** $\frac{1}{27}$ **f** $\frac{125}{8}$

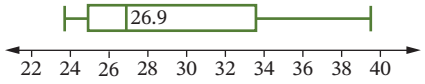
3 **a** $b = 5$ **b** $y = 1\frac{1}{2}$ **c** $m = 11$ **d** $a = 1\frac{3}{4}$

4 6

5 **a** range = 15.8, interquartile range = 8.7

b 23.7, 24.9, 26.9, 33.6, 39.5

c April's maximum temperature first fortnight 2020



d 39.5°C

6 **a** $\frac{1}{9y^2}$ **b** $\frac{3}{y^2}$ **c** $64x^9 y^{12}$

d $24h^{14} k^{16}$ **e** $3v^3 w^4$ **f** $\frac{4b^2}{3a^3}$

7 **a** $\frac{t}{2}$ **b** $\frac{-x}{15}$ **c** $\frac{7g}{20}$ **d** $\frac{7k}{10}$

8 **a** $\frac{6x}{7y}$ **b** $2m$ **c** $\frac{9}{2d}$ **d** $2v^2$

9 **a** weak negative **b** strong negative

c weak positive

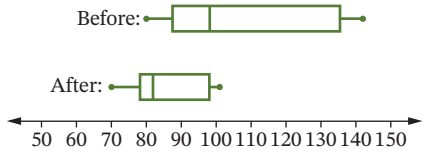
10 **a** $a = 8\frac{1}{2}$ **b** $k = 8\frac{3}{4}$ **c** $w = 10$ **d** $g = 4$

11 **a** $7y - 63$ **b** $-8n^2 - nm$

c $-20w + 24$ **d** $24a^3 b + 28a^2 b^2$

12 **a** Before: 80, 87.5, 96, 134, 142; After: 70, 77, 81, $97\frac{1}{2}$, 101

b



c Before: range = 62, interquartile range = 46.5

After: range = 31, interquartile range = 20.5

d Yes, there is a significant difference with the students' weights decreasing and having less spread after the program. According to the box plot, 75% of the masses after the program are below 50% of the masses before the program. The median has dropped significantly, as well as both the range and the interquartile range, meaning the students have reached more consistent weights.

13 **a** $3ab^2 - 24a^2 b$ **b** $18r^2 - 60r + 36$

14 **a** $7(w + 6)$ **b** $p(p - 25)$

c $-4(n + 11)$ **d** $(7a - 2)(3 - 5a)$

15 **a** **i** stem-and-leaf plot **ii** stem-and-leaf plot

b median = 41 **c** interquartile range = 18

16 **a** $p = \pm 12$ **b** $y = \pm 2$ **c** $x = \pm 1$ **d** $t = \pm 10$

e $x = \pm 4.8$ **f** $w = \pm 6.2$

17 **a** $2q^2 + 15q + 27$ **b** $3f^2 - 13f - 56$

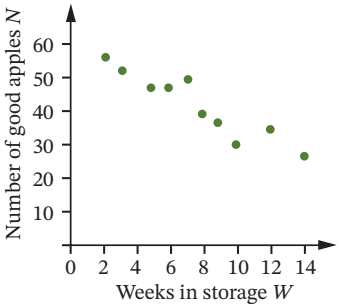
c $20g^2 - 47g + 21$ **d** $9x^2 - 49$

18 **a** $(r + 8)(r + 3)$ **b** $(y - 30)(y - 1)$

c $(x + 12)(x - 3)$ **d** $(t - 9)(t + 8)$

19 a The independent variable is W, the weeks in storage. The number of weeks in storage is set first, after which time the number of good apples is counted.

b



c The number of good apples decreases the longer the apples are in storage.

d There is a strong and negative relationship between the variables W and N.

20 a $x = -7, x = -1$ **b** $h = 9, h = -1$

c $u = -11, u = 7$ **d** $w = 0, w = 9$

21 a 20, 21, 22 **b** 20, 40, 120

22 a 480 mm^3 **b** 150 cm^2 **c** 3.6 m

23 a systematic

b people walking past the Sydney Opera House on Monday morning don't represent the population; more likely to be tourists, retired or unemployed, people living in Sydney/city rather than regional NSW or outside NSW

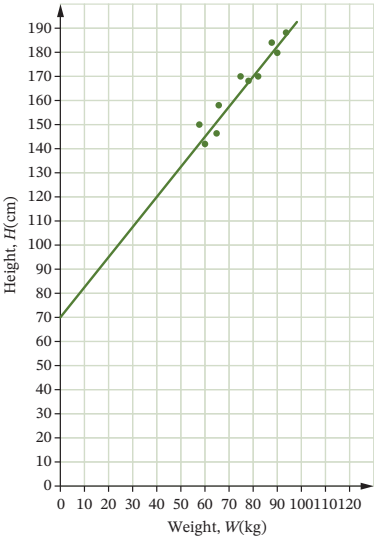
c survey people in all areas at different times of day, including beyond working hours

24 a Girls: 62.7, 16.1; boys: 66.9, 12.2

b The boys performed better as their mean is higher.

25 a $M = DV$ **b** $m = \frac{y - c}{x}$ **c** $n = 3t^2$

26 a



b $H = 1.2W + 70$; similar answers possible

c 48 kg

d 170 cm

Chapter 7

SkillCheck

1 a −1 **b** 29 **c** −3 **d** 69

2 a 625 **b** 3125 **c** 1 **d** $\frac{1}{25}$

3 a 12 **b** $3\frac{3}{7}$ **c** 48 **d** $66\frac{2}{3}$

Exercise 7.01

1 a C **b** D **c** A **d** B

2 a 190, $D = 190T$

b i 3.8 km **ii** 8.55 km

c 1 h 5 min

3 a 35.8, $E = 35.8h$

b $250.60 **c** 5.5 h

4 a $I = 0.058D$ **b** 522 **c** 1044

5 A **6** $b = 2.5a$

7 a

h	c
1	$12.50
2	$25
3	$37.50

b $c = 12.5h$ **c** $75 **d** 11

e 12.5. It is the same.

8 C

9 a $F = 0.006m$ **b** 15 L/100 km

10 A

11 a 22.8 kg **b** 84.1 kg

Exercise 7.02

1 a B **b** C **c** D **d** A

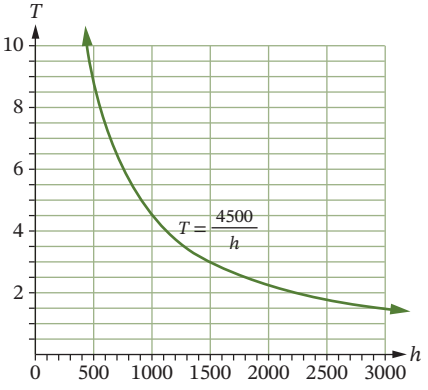
2 a 920, $T = \frac{920}{s}$ **b** 10 h 13 min **c** 92 km/h

3 a 4500, $T = \frac{4500}{h}$

b i 15°C **ii** 1.8°C

c i 562.5 m **ii** 200 m

d



4 a $N = \frac{112}{S}$ **b** 101 **c** 2.0 m^2 **d** 28

5 A **6** B **7** $\frac{14}{15}$

9780170479622

8 **a** 2.4 min **b** 5 people

9 **a** $b = \frac{8}{a}$ **b** $b = \frac{100}{a}$

10 **a** 19 h 46 min **b** 13 h 11 min

11 **a** $6\frac{2}{3}$ days **b** 10 friends

12 **a** $t = \frac{240\,000}{W}$ **b** 200 s

c **i** 600 W **ii** 1500 W

13 **a** $P = \frac{23.4}{V}$ **b** 8.4 L **c** 2.9 atm

Exercise 7.03

1 **a** **i** £22 **ii** £49

b **i** \$33 **ii** \$108

c **i** £22 − £20 = £2 **ii** \$120 − \$108 = \$12

2 **a** **i** 2 km **ii** 20 km **iii** 34 km

b **i** 50 furlongs **ii** 125 furlongs **iii** 180 furlongs

c 60 km **d** 500 furlongs

3 **a** **i** ¥16 000 **ii** ¥63 000 **iii** ¥78 000

b **i** \$250 **ii** \$760 **iii** \$920

4 **a** −18°C **b** 10°C **c** 26°C

d 32°F **e** 14°F **f** 86°F

5 **a** 4.9 ha **b** 2 ha **c** 10.8 acres **d** 12.2 acres

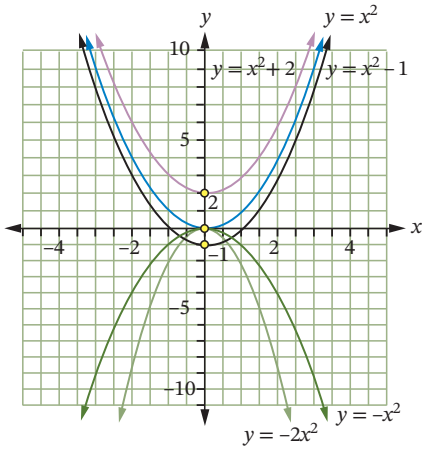
e **i** 32 000 m² **ii** 3.2 ha **iii** 7.8 acres

6 **a** **i** ₱520 **ii** ₱210 **iii** ₱310

b \$10.20 **c** \$5.80 **d** ₱4200

Exercise 7.04

1 **a**



b **i** $y = x^2, y = x^2 + 2, y = x^2 - 1$

ii $y = -x^2, y = -2x^2$

iii $y = x^2, y = -x^2, y = -2x^2$

2 A **3** C

4 **a** F **b** I **c** A **d** K

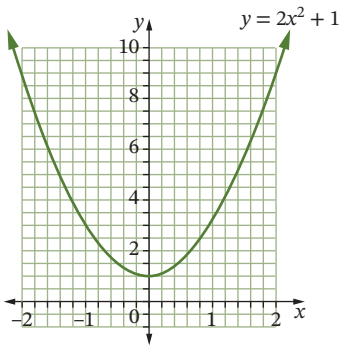
e J **f** C **g** B **h** L

i H **j** E **k** G **l** D

5 **a** $y = -x^2$ **b** $y = x^2$ **c** $y = -x^2 + \frac{1}{4}$

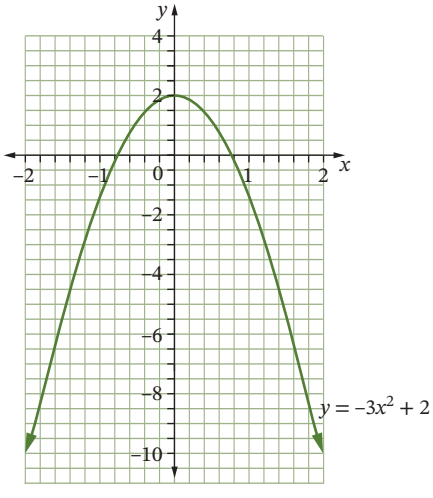
d $y = -x^2 - 9$ **e** $y = \frac{1}{2} - x^2$ **f** $y = x^2 + 9$

6 **a**



b (0, 1) **c** concave up **d** $y = 1$

7 **a**



b (0, 2) **c** $x = 0$ **d** $y = 2$

8 A

9 **a** **i** narrower **ii** up **iii** 3

b **i** wider **ii** up **iii** 1

c **i** narrower **ii** down **iii** −5

d **i** wider **ii** down **iii** −12

10 **a** $x = \pm 4$ **b** $x = \pm 11$

11 **a** B **b** G **c** D **d** J

e E **f** I **g** A **h** K

i H **j** L **k** F **l** C

12 **a** $x = \pm 9$ **b** $x = \pm 14$

Mental skills 7

2 **a** 3.5 **b** 2.4 **c** 0.12 **d** 0.36

e 0.8 **f** 0.027 **g** 0.2 **h** 8.8

i 0.24 **j** 0.012 **k** 1.8 **l** 0.028

4 **a** 66.3 **b** 6630 **c** 6.63 **d** 0.663

e 6.63 **f** 663 **g** 0.663 **h** 663

i 6630 **j** 66.3 **k** 0.663 **l** 0.0663

ANSWERS

Exercise 7.05

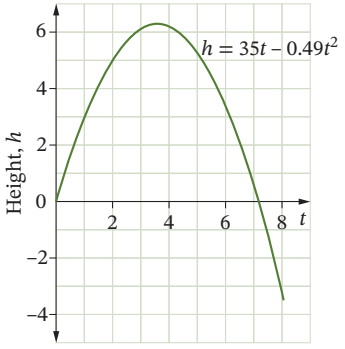
1 **a**

t (seconds)	0	1	2	3	4	5	6	7	8
h (metres)	0	3.01	5.04	6.09	6.16	5.25	3.36	0.49	−3.36

b $h \approx 6.2$ m after 3.6 s

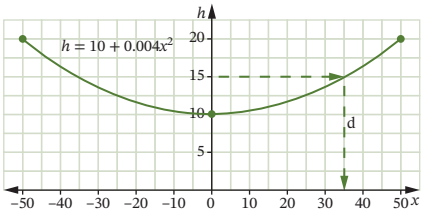
c when $t = 1$, 6.1

The ball is going up at 1 s and is going down at 6.1 s.



2 **a**

x	−50	−40	−30	−20	−10	0	10	20	30	40	50
h	20	16.4	13.6	11.6	10.4	10	10.4	11.6	13.6	16.4	20



b 20 m **c** 10 m **d** 35 m

3 **a**

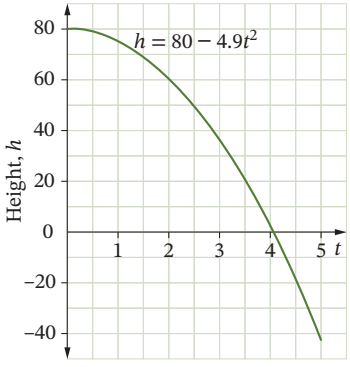
t	0	1	2	3	4	5
h	80	75.1	60.4	35.9	1.6	−42.5

b 80 m

c 35 m

d 4 s

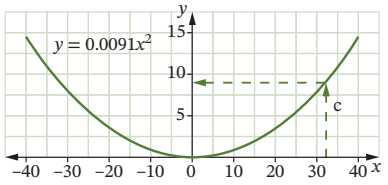
e 3.9 s



4 **a**

x	−40	−30	−20	−10	0	10	20	30	40
y	14.56	8.19	3.64	0.91	0	0.91	3.64	8.19	14.56

b



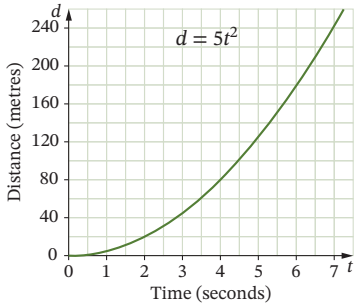
c about 9.2 m

d (using the graph) **i** 10 m **ii** 16 m

(using the equation) **i** 10.5 m **ii** 16.6 m

5 **a**

Time, t (seconds)	0	1	2	3	4	5	6	7
Distance, d (metres)	0	5	20	45	80	125	180	245



b **i** 20 m **ii** 125 m

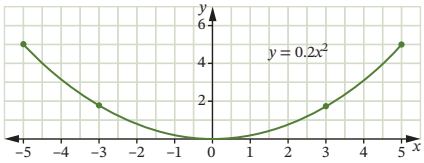
c **i** 25 m **ii** 35 m

d 6.3 s **e** 4.9 s

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6 a

Radius x (cm)	−5	−4	−3	−2	−1	0	1	2	3	4	5
Depth, y (cm)	5	3.2	1.8	0.8	0.2	0	0.2	0.8	1.8	3.2	5

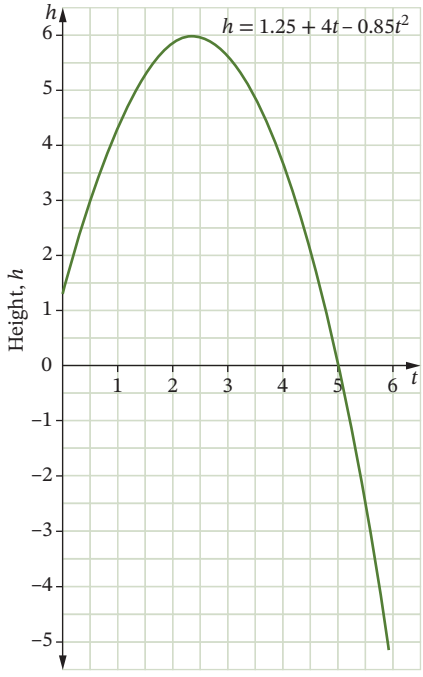


b $d = 1.25$ cm

c $x = 4.74$ cm

7 a

Time, t (seconds)	0	1	2	3	4	5	6
Height, h (metres)	1.25	4.4	5.85	5.6	3.65	0	−5.35



b 1.25, the height from which the water is sprayed

c 5 s

d 6 m

e 2.3 s using the graph, 2.4 s using the equation

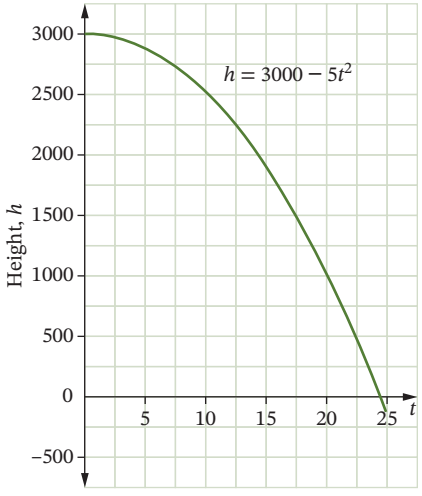
8 a

Time (t seconds)	0	5	10	15	20	25
Height (y metres)	3000	2875	2500	1875	1000	−125

b 3000 m

c 1875 m

d 18.5 s (using graph)
19 s (using equation)

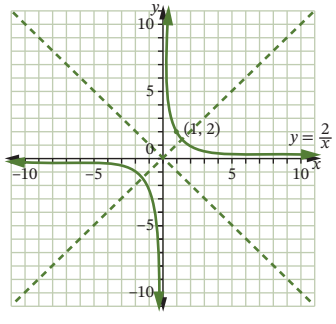


Exercise 7.06

1 a

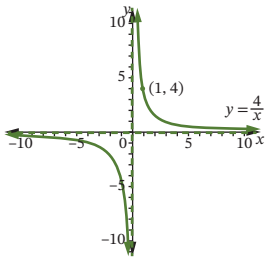
x	−3	−2	−1	0	1	2	3
y	$-\frac{2}{3}$	−1	−2	–	2	1	$\frac{2}{3}$

b, c

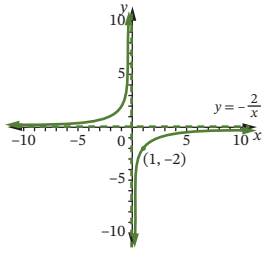


d $y = x, y = -x$

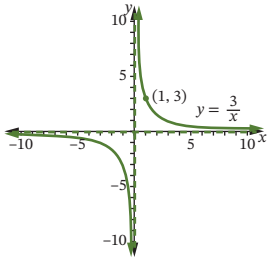
2 **a**



b



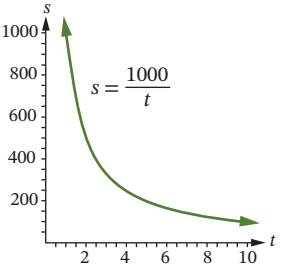
c



3 **a**

t	1	2	3	4	5	6	7	8	9	10
S	1000	500	333	250	200	167	143	125	111	100

b



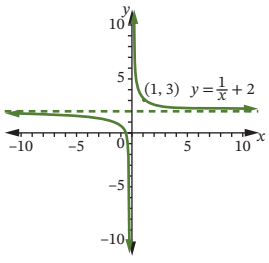
c The time taken is always positive and it is impossible to travel with zero time. Also, you cannot divide by zero.

d Yes, when $t = 2$ h, $s = 500$ km/h and when $t = 4$ h, $s = 250$ km/h.

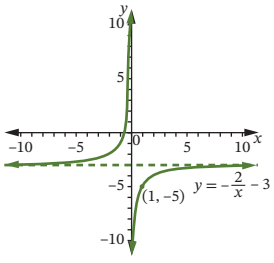
4 **a** B **b** D **c** A **d** C

5 **a** $k = 3$ **b** $y = \frac{3}{x}$

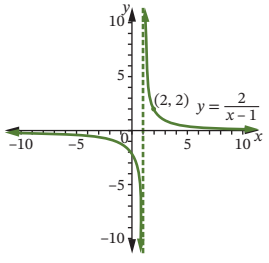
6 **a**



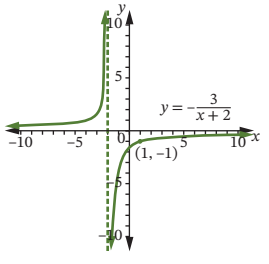
b



c



d



7 **a** $c = 1, k = -6$ **b** $y = -\frac{6}{x} + 1$

8 **a**

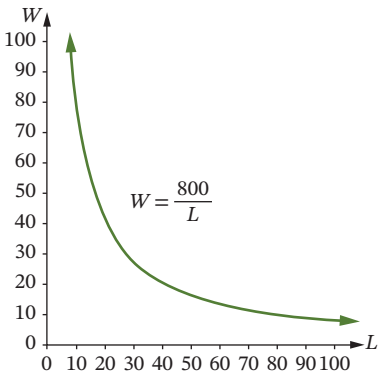
L	10	20	30	40	50	60	70	80	90	100
W	80	40	27	20	16	13	11	10	9	8

b $WL = 800$ or $W = \frac{800}{L}$

c If the length or width equals zero, the block of land doesn't exist.

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d



e As the length increases, the width decreases. The graph flattens out and gets closer to the horizontal axis, but never touches it (an asymptote).

f As the length decreases, the width increases. The graph is steeper and gets closer to the vertical axis, but never touches it (an asymptote).

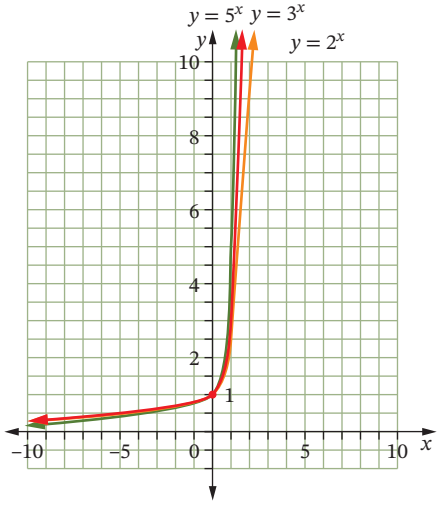
9 A

10 **a** $x = 0, y = 0$ **b** $x = 0, y = c$

c $x = b, y = 0$ **d** $x = b, y = c$

Exercise 7.07

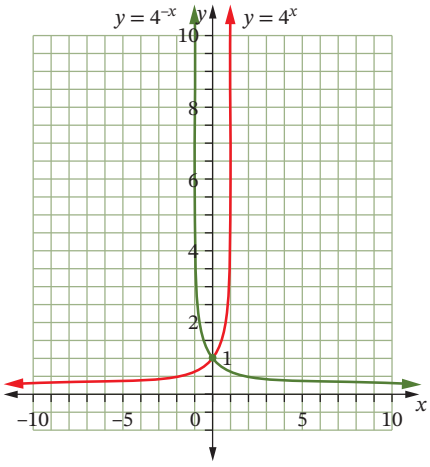
1 **a**



b 1

c becomes steeper, in 1st quadrant

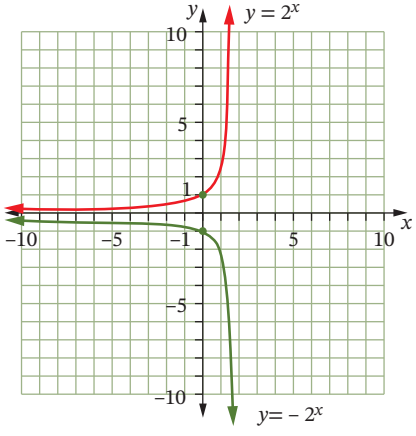
2 **a**



b **i** $y = 4^{-x}$ **ii** $y = a^{-x}$

3 B

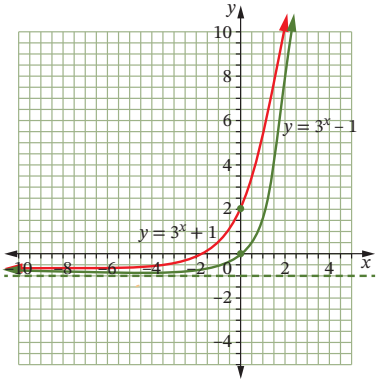
4 **a**



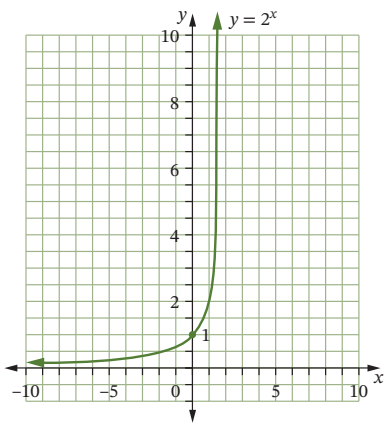
b one is the reflection of the other in the x-axis

c $y = -a^x$

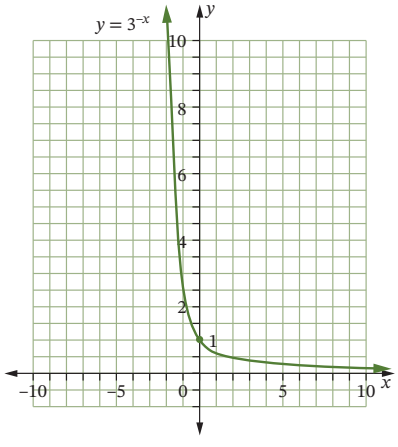
5 Same shape, shifted down 2 units.



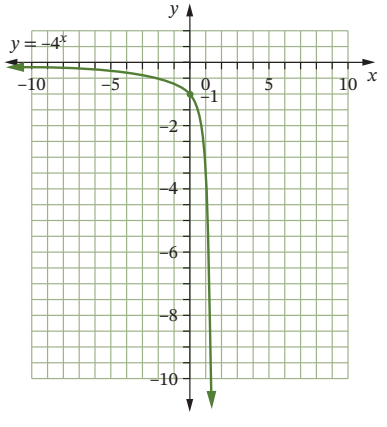
6 **a**



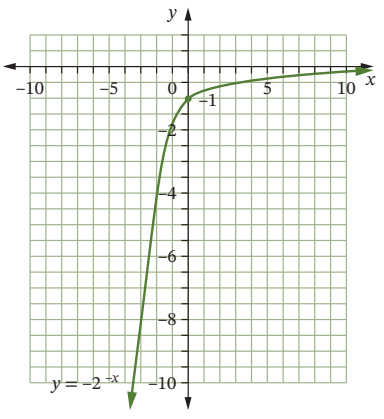
b



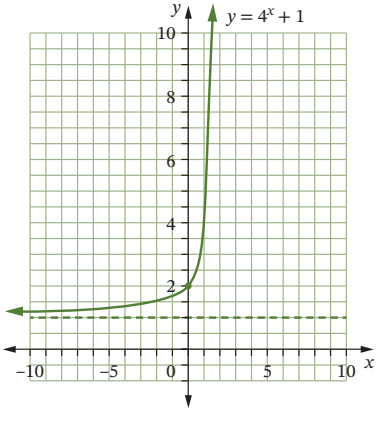
c



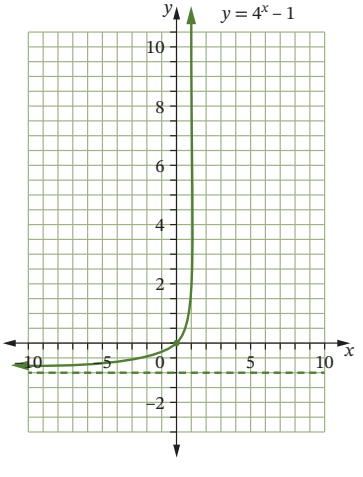
d



e



f



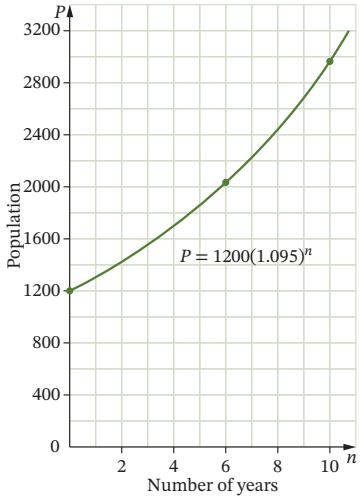
7 $y = 4^{-x}$

Exercise 7.08

1 a

n	0	1	2	3	4	5	6	7	8	9	10
P	1200	1310	1440	1580	1730	1890	2070	2270	2480	2720	2970

b

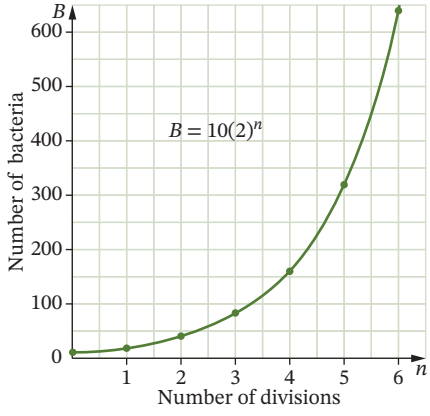


c 1200, the population in 2025.

d $P = 3904$ when $n = 13$ so the population triples during 2038.

2 a

n	0	1	2	3	4	5	6
B	10	20	40	80	160	320	640



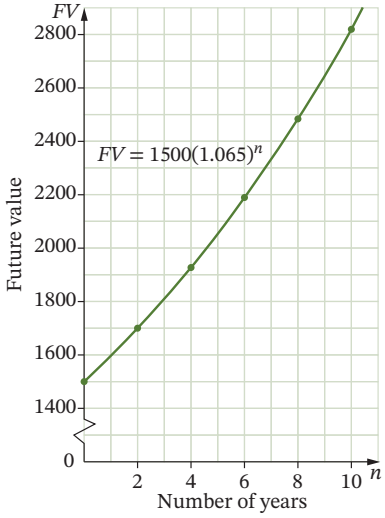
b 10

c i 640 **ii** 5120 **iii** 327 680

3 a

n	0	2	4	6	8	10
FV	1500	1701	1930	2189	2482	2816

b



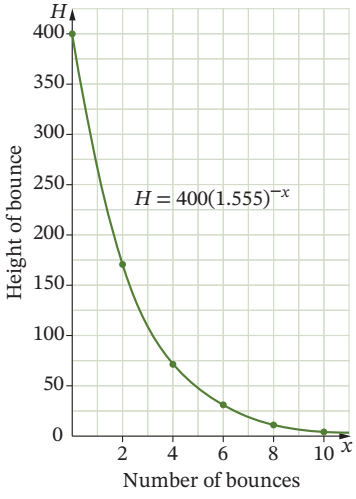
c 1500, the initial investment (principal or present value)

d i \$1800 **ii** \$2330

4 a

x	0	2	4	6	8	10
H	400	166	69	29	12	5

b



c 400, the height in cm from which the ball was dropped.

d 45 cm

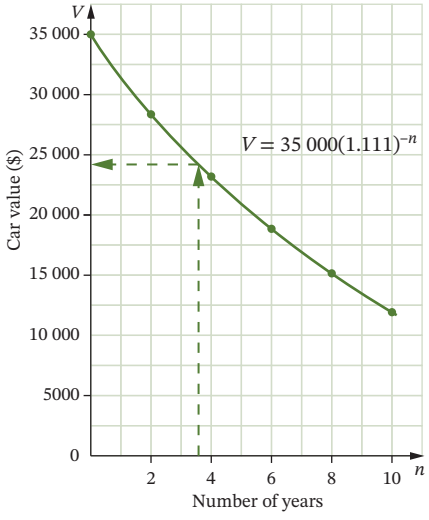
e 7th bounce

f 0.87 cm

ANSWERS

5 **a**

n	0	1	2	3	4	5	6	7	8
V	35 000	31 503	28 356	25 523	22 973	20 677	18 612	16 752	15 078



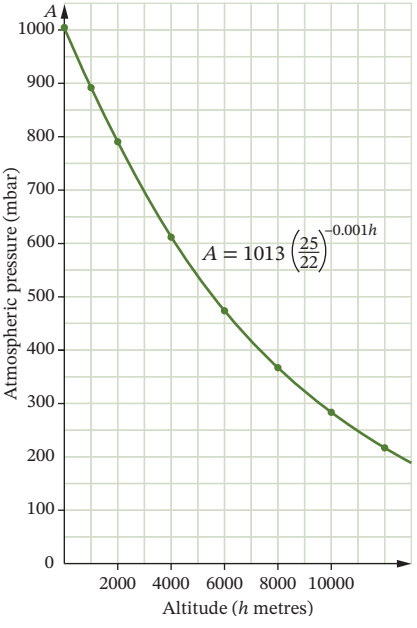
b 35 000, the original value of the car in dollars

c About $24 200

d After 12 years

6 **a**

h	0	1000	2000	3000	4000	5000	6000	8000	10 000
A	1013	891	784	690	607	535	470	364	282



b 1013 mbar, the atmospheric pressure at sea level.

c **i** 750 mbar **ii** 530 mbar **iii** 320 mbar

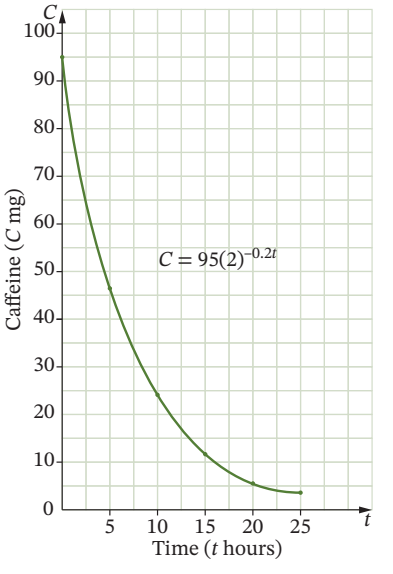
d **i** 2900 m **ii** 5500 m **iii** 8300 m

e **i** 760 mbar **ii** 330 mbar

f 149 mbar

7 **a**

t	0	5	10	15	20	25
C	95	47.5	23.8	11.9	5.9	3



b 95 mg

c **i** 35 mg **ii** 10.2 mg **iii** 3.3 mg

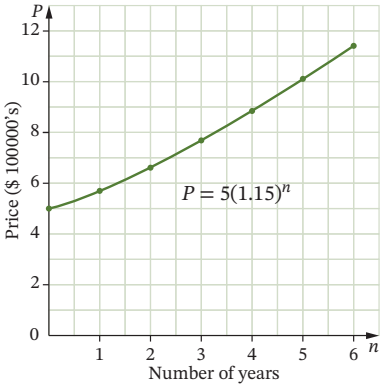
d 12

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8 **a**

n	0	1	2	3	4	6
P (in \$100 000s)	5	5.8	6.6	7.6	8.7	11.6

b



c \$500 000, the original price of the house (at the start of 2025).

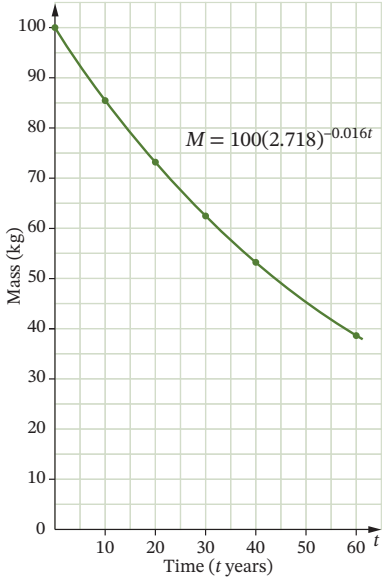
d \$1 010 000

9 **a**

t	0	10	20	30	40	60
M (kg)	100	85.2	72.6	61.9	52.7	38.3

b 100 kg

c



d 68 kg

e 44 years

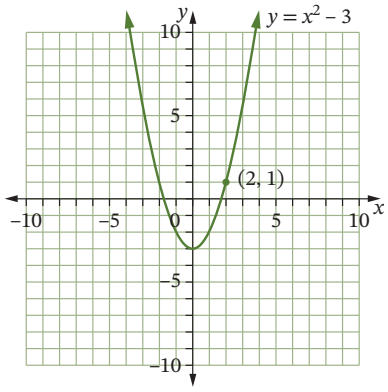
f **i** 20.19kg **ii** 4.08 kg **iii** 0.82 kg

Exercise 7.09

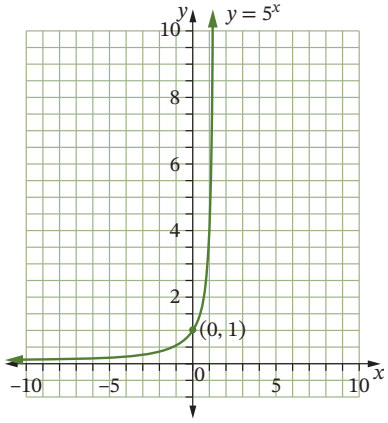
1 **a** P **b** L **c** E **d** L
e H **f** L **g** P **h** L
i P **j** E **k** P **l** H

2 **a** G **b** J **c** H **d** D **e** A
f F **g** K **h** E **i** B **j** I
k L **l** C

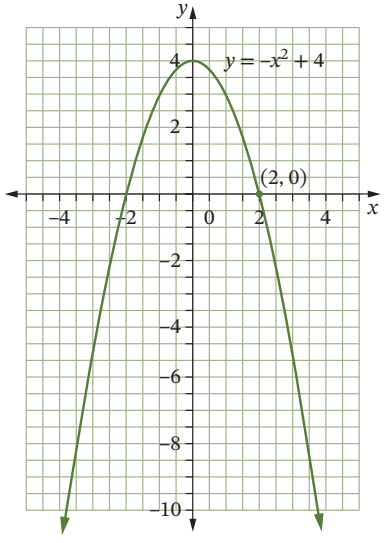
3 **a**



b

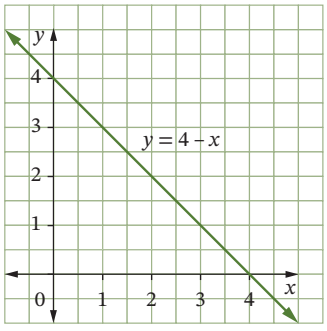


c

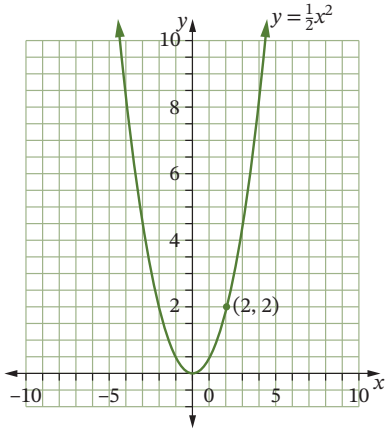


ANSWERS

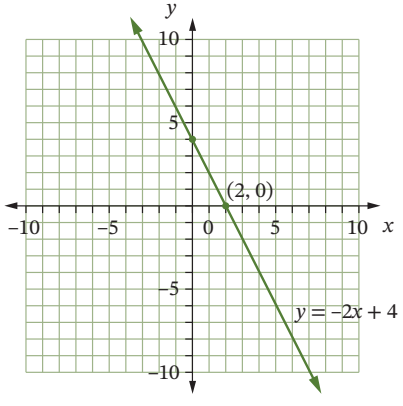
d



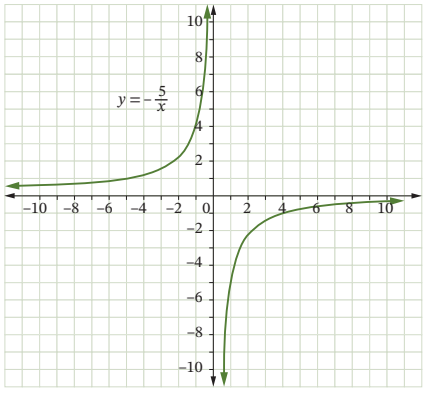
e



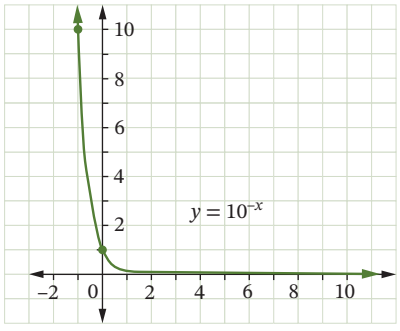
f



g



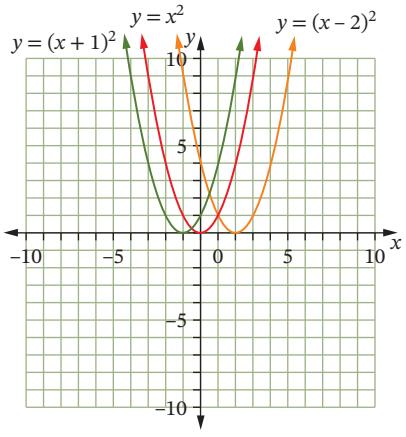
h



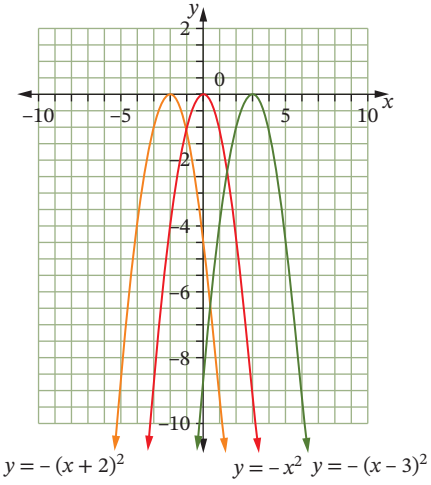
4 **a** 1 **b** 3 **c** −6 **d** 1

Power plus

1



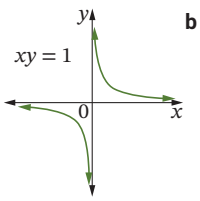
2



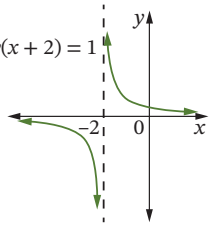
3 If a is positive, the parabola $y = x^2$ is shifted left a units.
If a is negative, the parabola $y = x^2$ is shifted right a units.

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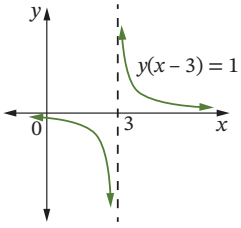
4 **a**



b



c



Test yourself 7

1 $H = 310.5$ **2** 10°C

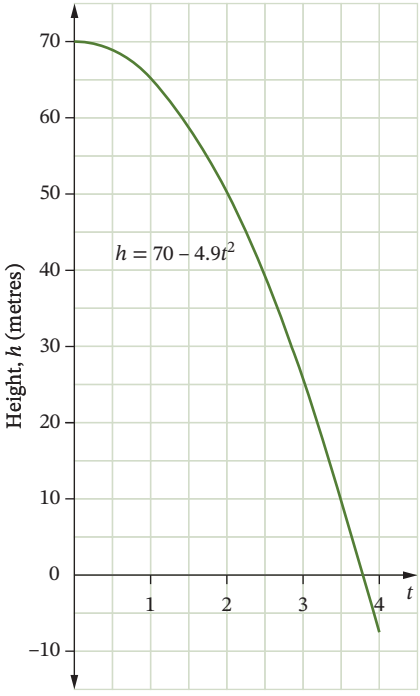
3 **a** £39 **b** $A101

4 **a** C **b** F **c** A

d E **e** D **f** B

5 **a**

Time (t seconds)	0	1	2	3	4
Height (h metres)	70	65.1	50.4	25.9	−8.4



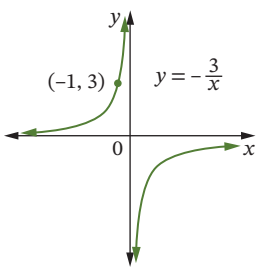
b 70, the height of the building.

c 59 m

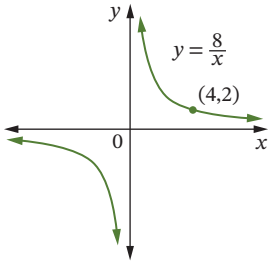
d 24.5 m

e 3.8 s

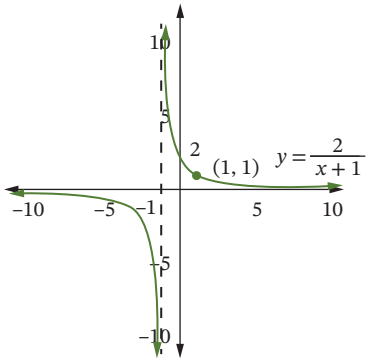
6 **a**



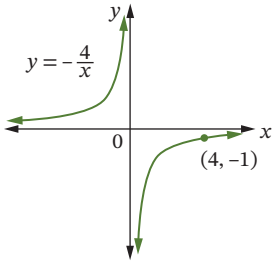
b



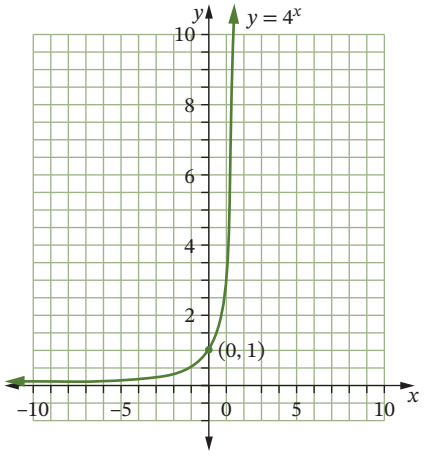
c



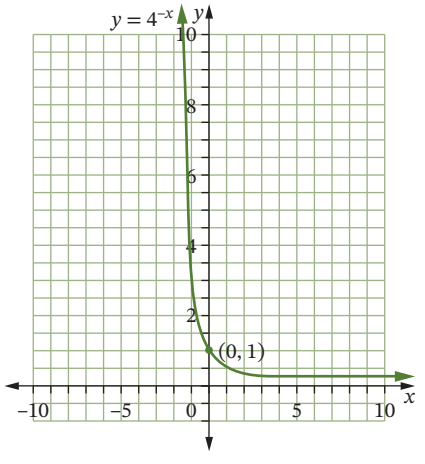
d



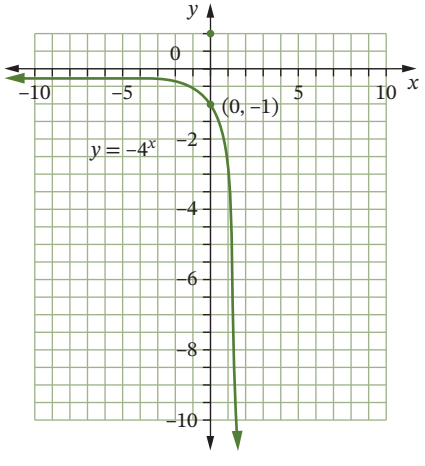
7 **a**



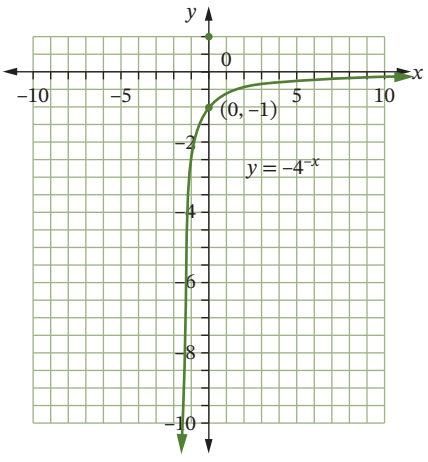
b



c

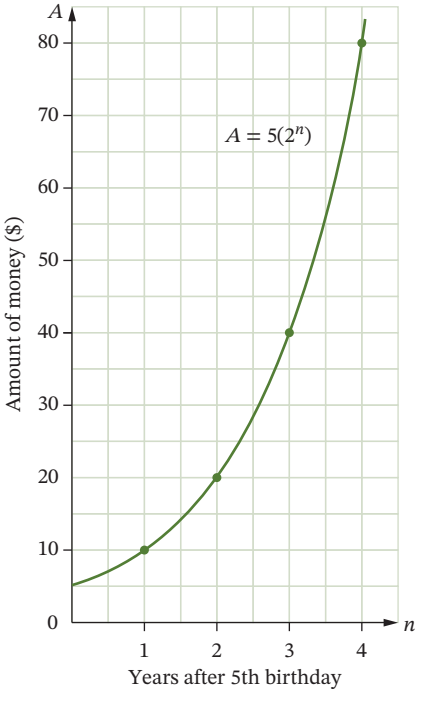


d



8 **a**

n	0	1	2	3	4	6	8	10
A	5	10	20	40	80	320	1280	5120



b $5, the vertical intercept.

c $80 (9th birthday is when $n = 4$)

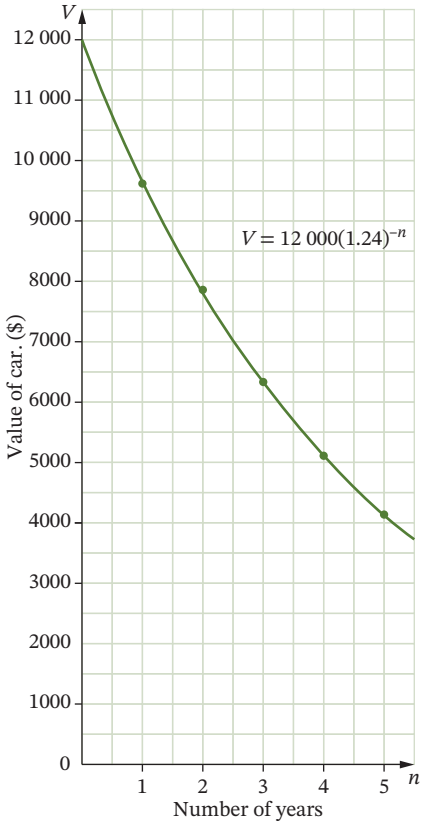
d $2560 (14th birthday is when $n = 9$)

e $327 680 (21th birthday is when $n = 16$)

9 a

n	0	1	2	3	4	5
V	12 000	9680	7800	6290	5080	4090

b



c 12 000, the original value of the car in dollars.

d i $2323 **ii** $1218

e 12 years

10 **a** D **b** C **c** B **d** J
e E **f** H **g** L **h** G
i I **j** A **k** K **l** F

Chapter 8

SkillCheck

1 **a** $x = 35$ **b** $h = 33.2$ **c** $y = 5$

2 **a** $x = 55$ **b** $y = 23$ **c** $n = 38$
d $x = 60$ **e** $x = 50$ **f** $x = 160$

3 **a** hyp: 17, opp: 8, adj: 15
b hyp: p, opp: r, adj: q
c hyp: EF, opp: EG, adj: GF

4 **a** 0.8480 **b** 0.7760 **c** 64.9839
d 0.1539 **e** 13.9884 **f** 13.7044

5 **a** 64° **b** 26° **c** 12°

6 **a** 50°19′ **b** 31°56′ **c** 64°19′

7 **a** 45°48′ **b** 33°11′ **c** 5°21′

Exercise 8.01

1 **a** $\sin\theta = \frac{20}{29}$ $\cos\theta = \frac{21}{29}$ $\tan\theta = \frac{20}{21}$

b $\sin F = \frac{DE}{FE}$ $\cos F = \frac{FD}{FE}$ $\tan F = \frac{DE}{FD}$

c $\sin\alpha = \frac{p}{y}$ $\cos\alpha = \frac{w}{y}$ $\tan\alpha = \frac{p}{w}$

d $\sin K = \frac{LH}{KH}$ $\cos K = \frac{KL}{KH}$ $\tan K = \frac{LH}{KL}$

e $\sin\theta = \frac{d}{a}$ $\cos\theta = \frac{g}{a}$ $\tan\theta = \frac{d}{g}$

f $\sin Q = \frac{2.4}{3} = \frac{4}{5}$ $\cos Q = \frac{1.8}{3} = \frac{3}{5}$ $\tan Q = \frac{4}{3}$

2 **a** $\frac{36}{85}$ **b** $\frac{36}{77}$ **c** $\frac{77}{85}$ **d** $\frac{36}{85}$

3 **a** 117.0 m **b** 36.7 m **c** 16.7 m
d 120.2 cm **e** 11.5 m **f** 1.2 m
g 9.3 m **h** 61.4 m **i** 71.5 cm

4 **a** 62° **b** 40° **c** 30° **d** 82°
e 49° **f** 83° **g** 30° **h** 45°

5 **a** 71.4 cm **b** 11.0 m **c** 28.2 m
d 17.1 m **e** 39.9 cm **f** 94.4 cm
g 53.2 m **h** 215.1 cm **i** 3.5 m

6 **a** 172.4 m **b** 25.2 m **c** 3.8 m **d** 6.3 m
e 353.9 cm **f** 9.5 m **g** 992.1 cm **h** 582.8 m

7 **a** 69°42′ **b** 46°3′ **c** 38°46′
d 49°13′ **e** 54°12′ **f** 42°5′

8 **a** 63° **b** 27° **c** 39°

9 3.8 m

10 0.63 m or 63 cm

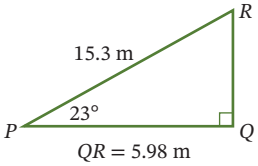
11 72° **12** 3° **13** 37° **14** 35°

15 14° **16** C **17** 4.2° **18** 24.6 m

19 1.5 km

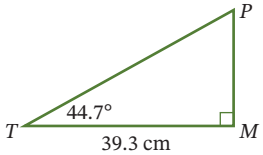
20 **a** 434 m **b** 164 m

21



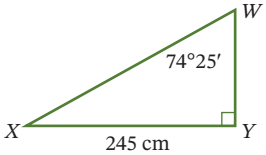
$QR = 5.98$ m

22



$PT = 55.3$ cm

23



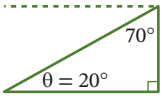
$WY = 68$ mm

24 48° **25** 70° **26** 52° 1′

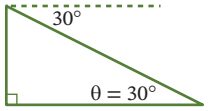
ANSWERS

Exercise 8.02

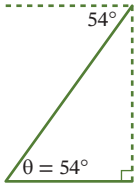
1 **a**



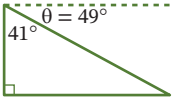
b



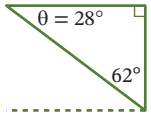
c



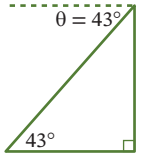
2 **a**



b



c



3 59 m **4** 2224 m or 2.224 km
5 177 m **6** 180 m **7** 9°
8 50° **9** 14° 2′ **10** 18° 47′
11 1256 cm **12** 14.5 m
13 970 m **14** 57 m

Exercise 8.03

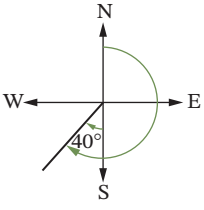
1 **a** 243° **b** 290° **c** 040° **d** 115° **e** 210°
f 140° **g** 312° **h** 253° **i** 065°

2 **a** 000° **b** 090° **c** 180° **d** 270° **e** 038°
f 125° **g** 330° **h** 225° **i** 072° **j** 187°

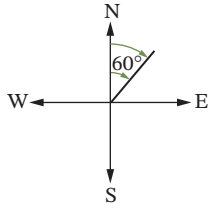
3 **a** S63°W **b** N70°W **c** N40°E
d S65°E **e** S30°W **f** S40°E
g N48°W **h** S73°W **i** N65°E

4 **a** 225° **b** 090° **c** 045° **d** 112.5° **e** 337.5°
f 270° **g** 135° **h** 247.5°

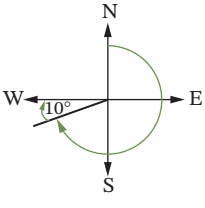
5 **a**



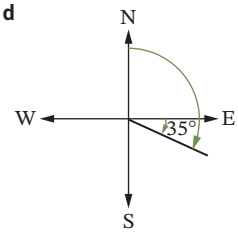
b



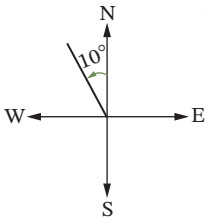
c



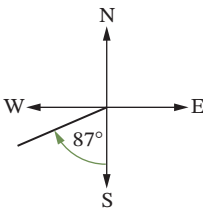
d



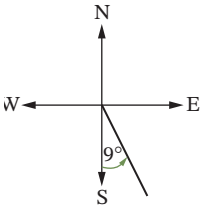
e



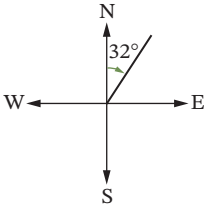
f



g

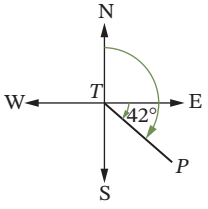


h

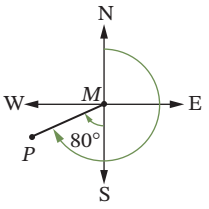


6 **a** WNW **b** N67.5°W

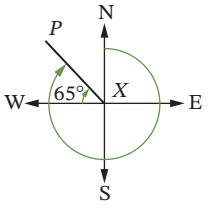
7 **a**



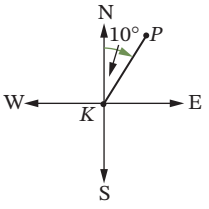
b



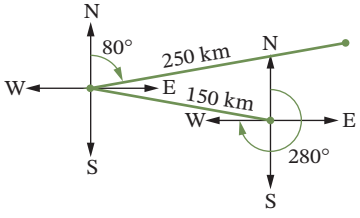
c



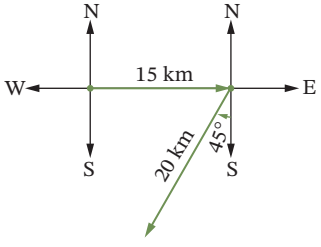
d



8 240° **9** 280°
10 **a** 45° **b** 90° **c** 135°
d 157.5° **e** 112.5° **f** 67.5°
11 S65°E
12 **a**



b



13 **a** 230° **b** 270° **c** 160°
14 **a** N20°W **b** N50°E **c** E

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Exercise 8.04

1 **a** 21 km **b** 260°
2 **a** 37° **b** 163 km **c** 143°
3 **a** 62.5 km **b** 29.2 km **c** N25°E
4 **a** 12.2 km **b** 215°
5 **a** 18 km **b** N22.5°W
6 45.7 km
7 **a** 13.509 km **b** N39°W
8 **a** 2122 km **b** 330°
9 **a** 15 km **b** 26 km
10 **a** 261.08 km **b** 167.82 km
11 6.6 km
12 **a** 322 nautical miles **b** 276° 45′
13 **a** 1282 km **b** N24° 27′E
14 326° 11′

Exercise 8.05

1 **a** $4\sqrt{13}$ cm **b** 15.6 cm **c** 23°
2 **a** $\sqrt{800}$ cm or $20\sqrt{2}$ cm **b** 34.64 cm
c 35°16′
3 **a** 20.40 cm **b** 79° **4** 34°
5 **a** 9.1 cm **b** 28°
6 **a** 53 m **b** 113 m **7** 37.5 m
8 **a** 50° **b** 868 m
9 **a** 85 m, 50 m **b** 99 m apart
10 **a** $\angle WHF = 52°$, $\angle WFH = 38°$ and $\angle HWF = 90°$
b 49.1 m
11 **a** 285° **b** 7°
12 **a** 26°, 054° **b** 9120 m
13 **a** 4.5 km **b** 210° 38′

Mental skills 8

2 **a** 2, 5 **b** 3 **c** 2, 3, 6 **d** 3, 5
e 2 **f** 2, 3, 5, 6 **g** 5 **h** 3
3 **a** 4, 9 **b** 4, 10
c 9 **d** 10
e 4, 9 **f** none of these
g 9 **h** 4, 9, 10

Exercise 8.06

1 **a** 18.4 **b** 21.1 **c** 105.0
2 **a** $a = 20.51$ **b** $b = 11.91$ **c** $c = 12.58$
d $d = 4.10$ **e** $e = 30.85$ **f** $f = 3.55$
g $k = 5.99$ cm **h** $w = 29.17$ m **i** $p = 8.29$ m
3 C **4** 79 m **5** 25 m
6 **b** 1042 cm
7 **a** 110° **b** 131.6 m **8** 561 km
9 **d** 124.7 m **10** **b** 595 m **11** 1104 km

Exercise 8.07

1 **a** 27° **b** 37° **c** 54°
2 **a** 44.5° **b** 46.6° **c** 32.0°
d 67.3° **e** 18.8° **f** 31.8°
3 **a** 149° 7′ **b** 129° 0′ **c** 142° 8′
d 135° 33′ **e** 129° 29′ **f** 162° 13′
4 B **5** 39°
6 **a** 46° or 134° **b** 39°
c 43° or 137° **d** 43° or 137°
7 **a** 21° or 75° **b** 41° **c** 84°
8 C **9** 128°

Exercise 8.08

1 **a** 5.6 **b** 13.1 **c** 35.8
2 **a** $a = 8.30$ **b** $c = 54.52$ **c** $e = 88.41$
d $b = 16.33$ **e** $d = 19.44$ **f** $f = 40.72$
3 A **4** 0.6 m **5** C
6 **a** Teacher to check.
b $\angle XYN = 180° - 130° = 50°$ (co-interior angles on parallel lines)
$\angle XYZ = 50° + 25° = 75°$
c 4.4 km
7 47 km
8 **a** 0 **b** $c^2 = a^2 + b^2$
c With $\cos 90° = 0$, the cosine rule reverts to Pythagoras' theorem.

Exercise 8.09

1 **a** 70° **b** 33° **c** 109° **d** 131°
2 **a** 112° **b** 108° **c** 121°
d 23° **e** 60° **f** 83°
3 D **4** 20.8° **5** 64° 40′
6 99° **7** N71°W

Exercise 8.10

1 **a** 413.4 m^2 **b** 463.1 cm^2 **c** 326.9 mm^2
d 132.9 mm^2 **e** 320.4 cm^2 **f** 0.1 m^2
2 **a** 97.4 m^2 **b** 463.6 m^2 **c** 246.2 m^2
d 227.6 m^2 **e** 93.5 m^2 **f** 152.2 m^2
3 **a** 225 m **b** 2770 m^2
4 **a** 130° **b** 766.04 m^2
5 **a** 418.9 cm^2 **b** 173.2 cm^2 **c** 245.7 cm^2
6 **a** 112° **b** 37 cm^2 **c** 740 cm^3
7 111.7 cm

Exercise 8.11

1 **a** 10.2 **b** 16.1 **c** 17.1
d 13.1 **e** 3.9 **f** 18.2
2 **a** 32° **b** 142° **c** 29°
d 55° **e** 37° **f** 125°
3 **a** $\angle ATB = 55° - 32°$ (exterior angle of a triangle)
b 108.50 m **c** 89 m
4 **a** 1 **b** **i** 15.4 **ii** 15.4
c The results are the same. The sine rule $\frac{d}{\sin 90°} = \frac{12.8}{\sin 56°}$ becomes $d = \frac{12.8}{\sin 56°}$ (since $\sin 90° = 1$), which is the same result when using the sine ratio.
5 7.5 km **6** 486 km
7 **a** $AF = 67.2$ m, $BF = 46.9$ m **b** 48.3 m
8 783 m

ANSWERS

Power plus

1 **a** **i** 0.342, 0.342 **ii** 0.731, 0.731
iii 0.819, 0.819 **iv** 0.996, 0.996
b Each pair of trigonometric ratios has the same value.
c The pairs of angles are complementary (add to 90°).
d 60°
e **i** 15° **ii** sin **iii** 18°
iv cos **v** 25° **vi** 32°
f $\sin x = \cos(90° - x)$
g Teacher to check.

2 **a** 1932 m **b** 31°

3 **a** $\frac{\sqrt{3}}{2}$ **b** $\frac{1}{\sqrt{3}}$

4 **a** 24° 12′ 26″ **b** 63° 17′ 3″

5, 6 Teacher to check.

Test yourself 8

1 **a** 10.9 m **b** 4.4 m **c** 11.5 cm

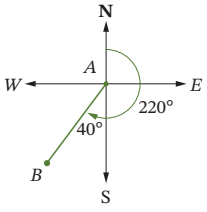
2 **a** 64° 59′ **b** 48° 59′ **c** 57° 12′

3 A

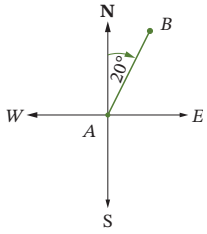
4 13°

5 195 m

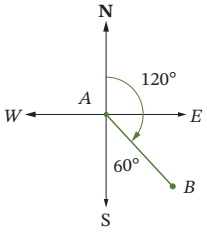
6 **a**



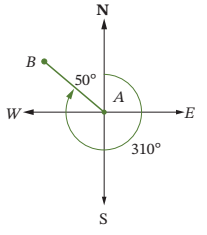
b



c



d



7 **a** 231° **b** 051°

8 **a** 11.8 km **b** S65°E

9 **a** 1st plane 1350 km, 2nd plane 1800 km
b 2250 km **c** 153°

10 **a** 33 cm **b** 65°

11 **a** 0.4 **b** 14.8 **c** 136.4

12 **a** 81° 54′ or 98° 6′ **b** 77° 24′ or 102° 36′
c 49° 37′

13 **a** 6.8 **b** 112.1 **c** 7.6

14 **a** 96° **b** 56° **c** 125°

15 **a** 165 cm² **b** 286 mm² **c** 30 m²

16 15.5 cm

17 **a** Proof: see worked solutions **b** 35.02 m
c 28.0 m

18 **a** 43.7 nautical miles **b** 276.6°
c 096.6°

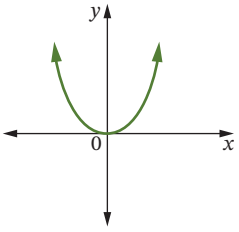
Practice set 3

1 **a** € 17.50 **b** A$86 **c** € 98

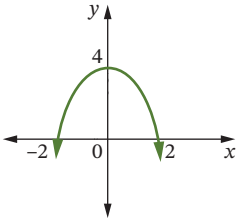
2 158.4

3 **a** 6.8 **b** 58.1 **c** 10.8

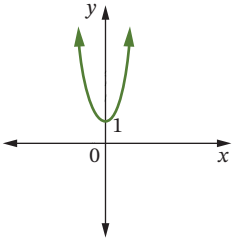
4 **a** **i**



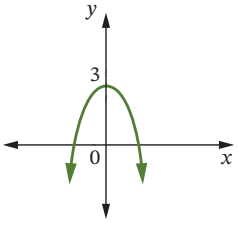
ii



iii



iv



b **i** $y = x^2, y = 3x^2 + 1$
ii $y = 4 - x^2, y = 3 - 2x^2$
iii $y = 3x^2 + 1$

5 **a** $t = \frac{99.96}{S}$ **b** 9.52 m/s **c** 9.70 s

6 **a** 29° **b** 45° **c** 43°

7 71 m

8 C **9** 78°

10 **a** 340 mm² **b** 81 m² **c** 140 cm²

11 **a** 32.5 **b** 5.1

12 **a** 106° 36′ **b** 40° 30′

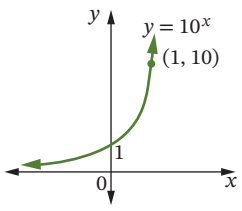
13 **a** 280° **b** 140° **c** 200°

14 **a** 301 km **b** 114°

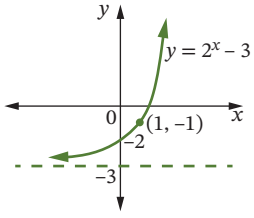
15 $x = \pm 2$

9780170479622

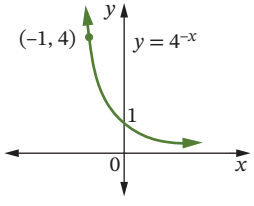
16 a



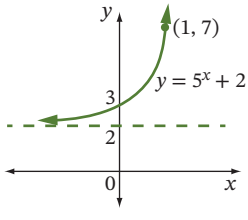
b



c



d



17 a $\theta = 37° 56'$ **b** 41° 33° **c** 68° 31′

18 a I **b** G **c** J **d** C
e B **f** D **g** F **h** E
i K **j** L **k** A **l** H

19 D

20 36 km

21 a growth **b** decay **c** decay **d** growth

22 a 10.9 m **b** 10° 8′

23 52°, 128°

Chapter 9

SkillCheck

1 C

2 a 3

b No, $P(10\text{c coin}) = \frac{5}{12}$,
$P(20\text{c coin}) = \frac{1}{3}$, $P(50\text{c coin}) = \frac{1}{4}$

3 a $\frac{1}{3}$ **b** $\frac{1}{3}$ **c** $\frac{5}{6}$

4 a 0 **b** 1

5 0.4 **6** B **7** 0.15

Exercise 9.01

1 a i 0.425 **ii** 0.14 **iii** 0.21

b i 0.375 **ii** 0.125 **iii** 0.25

c Yes

d Expected frequency = 100. The observed frequency of red or purple is 115, which is more than the expected frequency.

2 a i $\frac{1}{5} = 0.2$ **ii** $\frac{19}{50} = 0.38$

iii $\frac{33}{100} = 0.33$ **iv** $\frac{9}{100} = 0.09$

b i $\frac{1}{4} = 0.25$ **ii** $\frac{7}{20} = 0.35$

iii $\frac{3}{10} = 0.3$ **iv** $\frac{1}{10} = 0.1$

c Yes

d Expected frequency = 40. The expected frequency compares very favourably with the observed frequency of 42.

3 a 50 **b** Teacher to check.

c i Teacher to check. **ii** $\frac{1}{2}$

d Teacher to check.

4 a 600

b i $\frac{281}{600} \approx 0.468$ **ii** $\frac{322}{600} \approx 0.537$

iii $\frac{227}{600} \approx 0.378$ **iv** $\frac{522}{600} = 0.87$

c i 0.5 **ii** 0.5 **iii** 0.33 **iv** 0.83

d The probabilities are similar.

5 a Teacher to check.

b i $\frac{1}{2} = 0.5$ **ii** $\frac{1}{5} = 0.2$

iii $\frac{3}{10} = 0.3$ **iv** $\frac{7}{10} = 0.7$

c Teacher to check.

6 a i $\frac{3}{10} = 0.3$ **ii** $\frac{3}{25} = 0.12$

iii $\frac{12}{25} = 0.48$ **iv** $\frac{1}{10} = 0.1$

b i 0.33 **ii** 0.17 **iii** 0.33 **iv** 0.17

c No, the observed probability of yellow was higher and the observed probability of green was lower.

d Expected frequency of not yellow is 33. This is more than the observed frequency of 26.

7 a 200

b i $\frac{4}{200} = 0.02$ **ii** $\frac{27}{200} = 0.135$

iii $\frac{13}{200} = 0.065$ **iv** $\frac{86}{200} = 0.43$

v $\frac{87}{200} = 0.435$ **vi** $\frac{59}{200} = 0.295$

vii $\frac{51}{200} = 0.255$

8 **a** 200

b **i** $\frac{27}{200} = 0.135$ **ii** $\frac{62}{200} = 0.31$

iii $\frac{80}{200} = 0.4$ **iv** $\frac{21}{200} = 0.105$

v $\frac{1}{200} = 0.005$

c Ferry, light rail (tram)

d Teacher to check.

9 **a** $\frac{1}{6} \approx 0.17$ **b** 16 or 17 times

c, **d**, **e** Teacher to check.

10 Approx 0.17

11 Approx 0.35

12 400

13 Approx 57%

14 Teacher to check. The probability of winning would be different for each horse, due to methods of training, strength of horse and conditions of the racetrack, so it is not a simple probability of 1 in 10.

Exercise 9.02

1 C

2 **a** **i** $\frac{11}{25}$ **ii** $\frac{4}{5}$ **iii** $\frac{1}{5}$ **iv** $\frac{6}{25}$ **v** $\frac{19}{25}$

b $\frac{3}{10}$

3 **a** 156

b **i** $\frac{11}{52}$ **ii** $\frac{7}{52}$ **iii** $\frac{7}{78}$ **iv** $\frac{22}{39}$ **v** $\frac{19}{78}$ **vi** $\frac{1}{26}$

c $\frac{7}{31}$

4 **a** 135 **b** $\frac{56}{135}$ **c** $\frac{1}{5}$ **d** $\frac{17}{52}$

5 **a** **i** $M = \{5, 7, 9, 11, 13, 15, 17, 19, 21\}$

ii $N = \{6, 8, 10, 12, 14, 16, 18, 20, 22\}$

iii $P = \{8, 12, 16, 20\}$

iv $Q = \{9, 16\}$

v $R = \{5, 7, 11, 13, 17, 19\}$

b **i** {5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22}

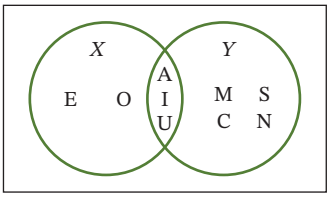
ii {5, 7, 8, 9, 11, 12, 13, 15, 16, 17, 19, 20, 21}

iii {8, 12, 16, 20}

iv {9}

v { }

6 **a**



b {A, I, U} **c** {M, S, C, N}

7 **a** 4, 5 **b** 2, 3, 4, 5, 6, 7 **c** 3, 6, 7, 8

d 2, 4, 5, 6, 7 **e** 4, 5, 6 **f** 4, 7

8 **a** {32, 36, 40, 44} **b** {32, 40}

c {31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46}

d { }

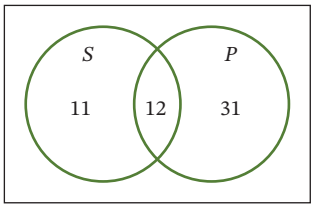
e {31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 45, 46}

f {36, 44}

g {31, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46}

h {31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 45, 46}

9 **a**



b **i** 1 **ii** $\frac{11}{54}$ **iii** $\frac{31}{54}$ **iv** $\frac{7}{9}$

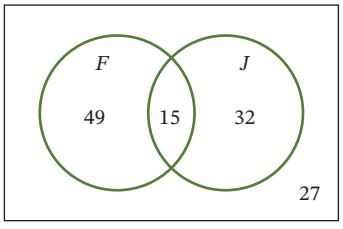
10 **a** 92

b **i** $\frac{1}{46}$ **ii** $\frac{15}{92}$ **iii** $\frac{5}{92}$ **iv** $\frac{17}{23}$

c **i** $\frac{21}{29}$ **ii** $\frac{5}{29}$

11 **a** 123

b



c 81

d **i** $\frac{49}{123}$ **ii** $\frac{32}{123}$ **iii** $\frac{9}{41}$ **iv** $\frac{27}{41}$

12 **a** 200

b **i** $\frac{79}{200}$ **ii** $\frac{51}{100}$ **iii** $\frac{77}{200}$

iv $\frac{121}{200}$ **v** $\frac{81}{100}$ **vi** $\frac{3}{50}$

c $\frac{29}{40}$

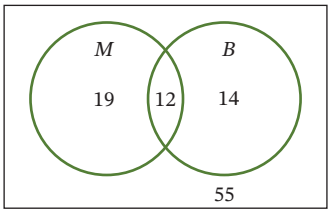
d No, because all the people surveyed indicated a day on which they preferred to shop.

13 **a** 206

b **i** $\frac{3}{103}$ **ii** $\frac{11}{103}$ **iii** $\frac{35}{103}$ **iv** $\frac{68}{103}$

c **ii** $\frac{3}{32}$ **ii** $\frac{47}{64}$

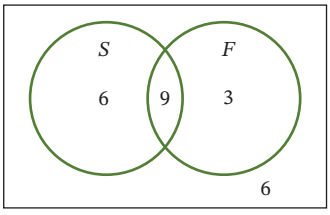
14



a 45 **b** 14 **c** 33 **d** 55

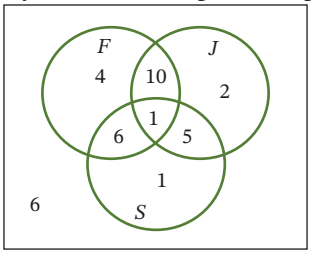
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15



9, they do both swimming and fishing.

16 a



b 6

Exercise 9.03

1 a 150

b i $\frac{4}{25}$ **ii** $\frac{53}{150}$ **iii** $\frac{28}{75}$

c 63%

2 a 128

b i 68 **ii** 60

c $\frac{17}{32}$ **d** $\frac{3}{32}$

3 a 97

b i 32.0% **ii** 25.8% **iii** 11.3%

c i 21.2% **ii** 44.4%

d The percentage of females in the opposition is just more than double that of females in the government.

4 a 150

b i 0.5 **ii** 0.04 **iii** 0.43 **iv** 0.23

c $\frac{22}{75} = 0.293$

5 a 160

b i $\frac{7}{20} = 0.35$ **ii** $\frac{11}{160} \approx 0.069$ **iii** $\frac{9}{80} \approx 0.113$

c $\frac{35}{82} \approx 0.43$

6 a 200 **b** 55%

c i 49.5% **ii** 45%

iii 36% **iv** 31.5%

d $\frac{72}{110} \approx 65.5\%$

7 a 878

b i $\frac{679}{878} = 0.773$ **ii** $\frac{545}{878} = 0.621$

iii $\frac{67}{439} = 0.153$ **iv** $\frac{21}{439} = 0.048$

Exercise 9.04

1 a

		Girls					
		Be	Ca	Em	M	R	S
Boys	Ben	Ben, Be	Ben, Ca	Ben, Em	Ben, M	Ben, R	Ben, S
	C	C, Be	C, Ca	C, Em	C, M	C, R	C, S
	Ew	Ew, Be	Ew, Ca	Ew, Em	Ew, M	Ew, R	Ew, S
	W	W, Be	W, Ca	W, Em	W, M	W, R	W, S

b i $\frac{1}{24}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{6}$

2 a

	H	*T*
H	HH	HT
T	TH	TT

b i $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{3}{4}$

3 a

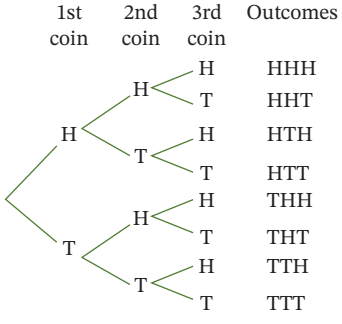
		1st die					
		1	2	3	4	5	6
2nd die	1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
	2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
	3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
	4	1, 4	2, 4	3, 4	4, 4	5, 4	6, 4
	5	1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
	6	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

b 36

c i $\frac{1}{36}$ **ii** $\frac{11}{36}$ **iii** $\frac{1}{6}$ **iv** $\frac{1}{4}$

v $\frac{11}{36}$ **vi** $\frac{1}{4}$ **vii** $\frac{1}{2}$ **viii** $\frac{5}{12}$

4 a



b 8

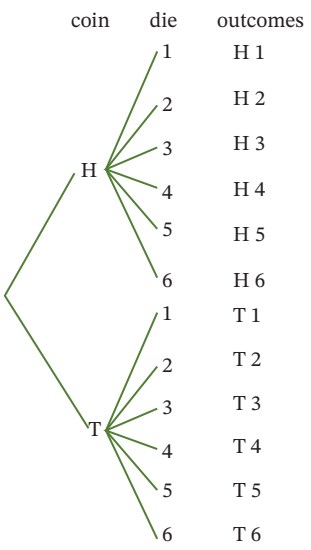
c i $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{8}$ **iv** $\frac{1}{8}$ **v** $\frac{1}{2}$

d i $\frac{7}{8}$ **ii** $\frac{7}{8}$

e i 75 **ii** 25

ANSWERS

5



6 a

		1st die					
		1	2	3	4	5	6
2nd die	1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
	2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
	3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
	4	1, 4	2, 4	3, 4	4, 4	5, 4	6, 4

b 24

c i $\frac{1}{6}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{3}{8}$ **v** 0

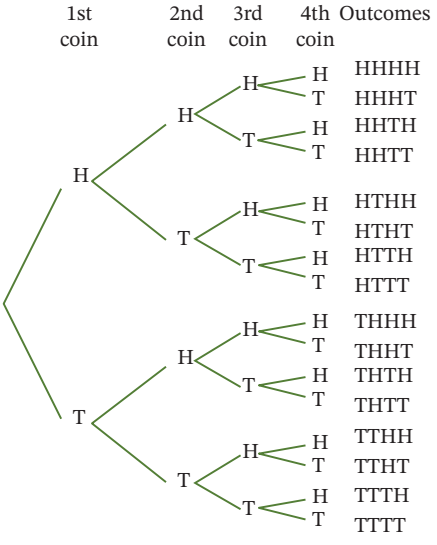
7 a

		1st die					
		1	2	3	4	5	6
2nd die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b i $\frac{1}{9}$ **ii** $\frac{1}{36}$ **iii** $\frac{1}{6}$ **iv** $\frac{1}{2}$

v 0 **vi** $\frac{5}{12}$ **vii** $\frac{7}{12}$ **viii** $\frac{5}{12}$

8 a

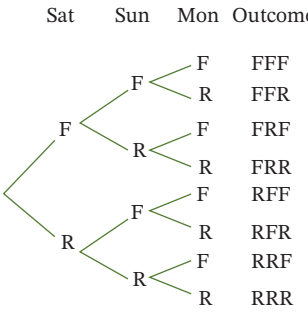


b i $\frac{1}{16}$ **ii** $\frac{1}{4}$ **iii** $\frac{3}{8}$

iv $\frac{15}{16}$ **v** $\frac{1}{16}$ **vi** $\frac{5}{16}$

c i 63 **ii** 375 **iii** 938

9 a



b i $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{8}$ **iv** $\frac{7}{8}$

Mental skills 9

2 a \$408 **b** 99 **c** 200 **d** \$404

e 672 **f** \$81 **g** \$517 **h** 225

i 560 **j** \$84 **k** \$350 **l** 84

4 a 330 **b** \$240 **c** 1600 **d** \$425

e \$225 **f** \$60 **g** \$63 **h** 76

i \$68 **j** \$3762 **k** \$374 **l** \$100

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Exercise 9.05

1 a C = Cate, A = Amal, G = Gemma, J = Josie, E = Evangeline, R = Rukshana

Vice-captain \ Captain	C	A	G	J	E	R
C	–	AC	GC	JC	EC	RC
A	CA	–	GA	JA	EA	RA
G	CG	AG	–	JG	EG	RG
J	CJ	AJ	GJ	–	EJ	RJ
E	CE	AE	GE	JE	–	RE
R	CR	AR	GR	JR	ER	–

30 possible pairings

b $\frac{1}{3}$ **c** $\frac{1}{6}$

2 a i

	A	B	C	D	E
A	AA	AB	AC	AD	AE
B	BA	BB	BC	BD	BE
C	CA	CB	CC	CD	CE
D	DA	DB	DC	DD	DE
E	EA	EB	EC	ED	EE

ii

	A	B	C	D	E
A		AB	AC	AD	AE
B	BA		BC	BD	BE
C	CA	CB		CD	CE
D	DA	DB	DC		DE
E	EA	EB	EC	ED	

b i $\frac{1}{5}$ **ii** $\frac{4}{25}$ **iii** $\frac{12}{25}$

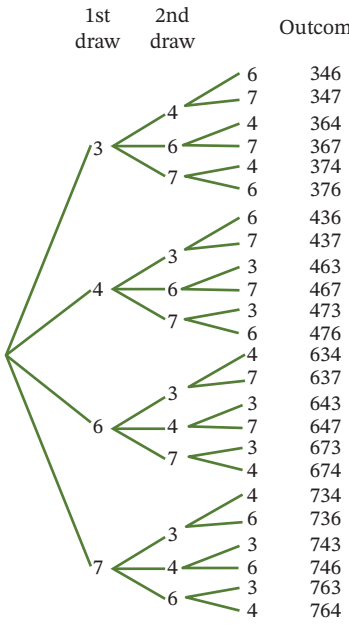
c i $\frac{1}{10}$ **ii** $\frac{3}{5}$ **iii** $\frac{2}{5}$ **iv** $\frac{4}{5}$

3 a

1st course \ 2nd course	B	H	P	S	T
C	CB	CH	CP	CS	CT
F	FB	FH	FP	FS	FT
Y	YB	YH	YP	YS	YT

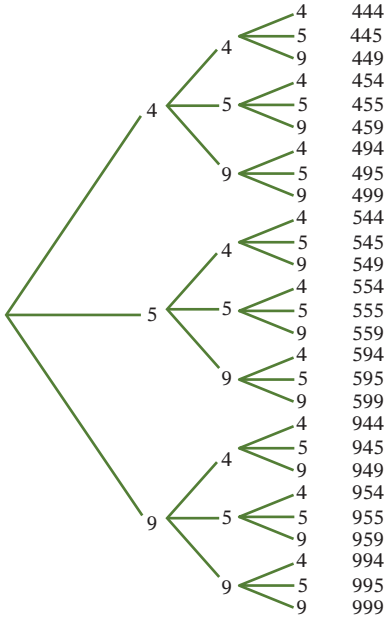
b i $\frac{1}{3}$ **ii** $\frac{4}{15}$ **iii** $\frac{1}{15}$

4 a



b i $\frac{1}{2}$ **ii** $\frac{3}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{3}$

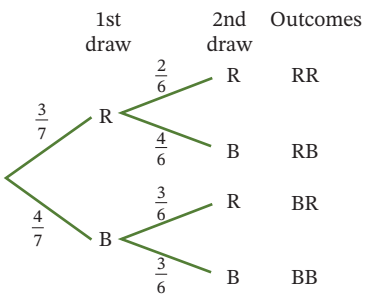
5 a



b i $\frac{1}{9}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$ **iv** $\frac{2}{9}$

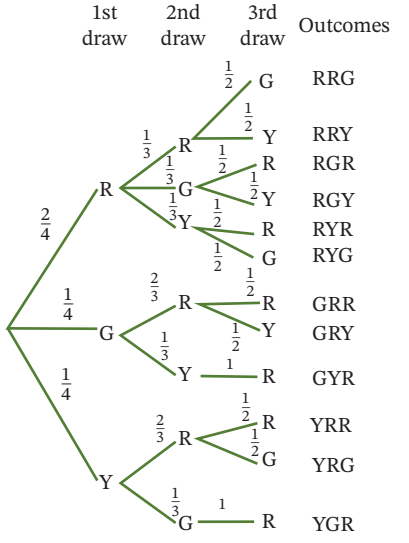
ANSWERS

6 a



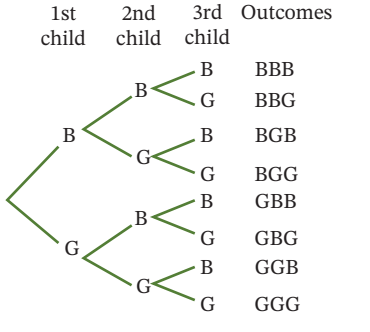
b i $\frac{1}{7}$ **ii** $\frac{2}{7}$ **iii** $\frac{4}{7}$ **iv** $\frac{6}{7}$

7 a



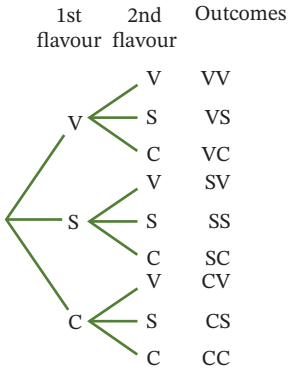
b i $\frac{1}{2}$ **ii** $\frac{1}{12}$ **iii** 1

8 a

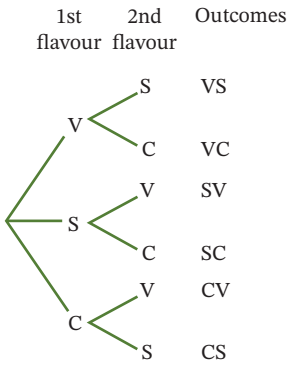


b i $\frac{1}{8}$ **ii** $\frac{1}{8}$ **iii** $\frac{3}{8}$ **iv** $\frac{1}{8}$

9 a i



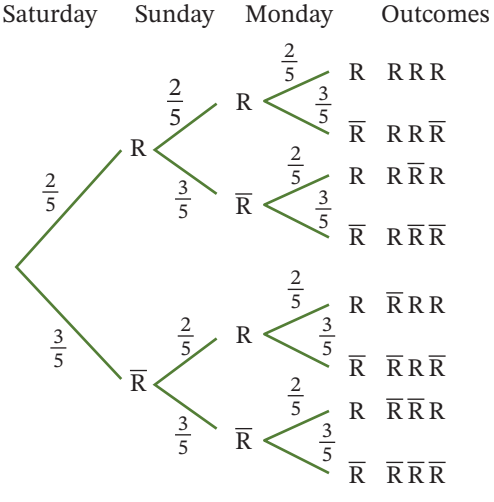
ii



b i $\frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** $\frac{4}{9}$ **iv** $\frac{5}{9}$

c i 0 **ii** 1 **iii** $\frac{1}{3}$ **iv** $\frac{2}{3}$

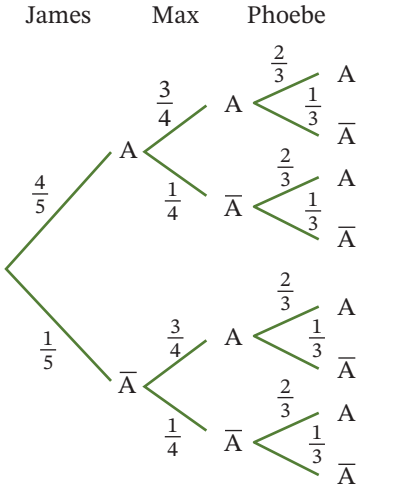
10 a



b i $\frac{36}{125}$ **ii** $\frac{27}{125}$ **iii** $\frac{54}{125}$ **iv** $\frac{98}{125}$

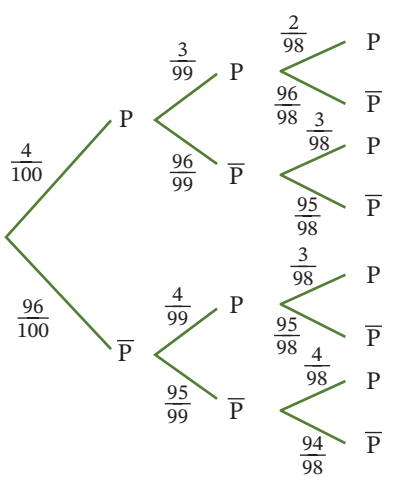
9780170479622

11 a



b i $\frac{2}{5}$ **ii** $\frac{1}{60}$ **iii** $\frac{1}{15}$ **iv** $\frac{3}{20}$

12 a



b i $\frac{1}{25}$ **ii** $\frac{304}{8085}$ **iii** $\frac{304}{2695}$ **iv** $\frac{7144}{8085}$

Exercise 9.06

1 a independent **b** independent **c** dependent
d independent **e** dependent **f** independent
g independent

2 Dependent, as the balls are not replaced when drawn.

3 a independent **b** $\frac{1}{2}$

4 a i $\frac{1}{3}$ **ii** $\frac{1}{4}$

b 1Y, 2Y, 3Y, 4Y, 5Y, 6Y, 1G, 2G, 3G, 4G, 5G, 6G, 1B, 2B, 3B, 4B, 5B, 6B, 1R, 2R, 3R, 4R, 5R, 6R

c $\frac{1}{12}$ **d** Yes, $\frac{1}{3}\times\frac{1}{4}=\frac{1}{12}$

e independent

5 a i $\frac{1}{6}$ **ii** $\frac{1}{2}$

b $\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$

6 a $\frac{5}{9}$ **b** $\frac{4}{8}=\frac{1}{2}$

c Dependent, as the first draw changes the contents of the bag.

7 a i $\frac{5}{8}$ **ii** $\frac{4}{7}$

b i $\frac{3}{8}$ **ii** $\frac{5}{7}$

c i $\frac{5}{8}$ **ii** $\frac{3}{7}$

d i $\frac{3}{8}$ **ii** $\frac{2}{7}$

8 $\frac{1}{2}$

9 a $\frac{1}{12}$ **b** $\frac{1}{36}$ **c** $\frac{1}{3}$ **d** $\frac{1}{6}$

10 a $\frac{13}{25}$ **b** $\frac{7}{25}$

11 a 0.15 **b** 0.1 **c** 0.3

12 a 0.147 **b** 0.343 **c** 0.027 **d** 0.189

Power plus

1 a 320

b i $\frac{7}{40}=0.175$ **ii** $\frac{3}{32}\approx0.094$

iii $\frac{57}{320}\approx0.178$ **iv** $\frac{3}{20}=0.15$

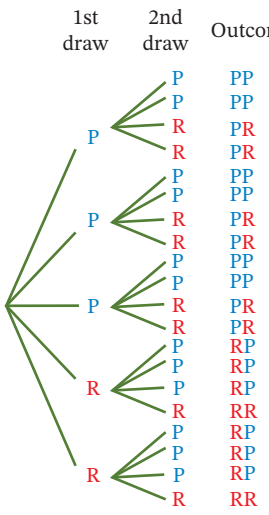
c $\frac{16}{45}$ **d** $\frac{45}{107}$

2 a $\overline{A}\cup B$ **b** $A\cap\overline{B}$

c $A\cup\overline{B}$ **d** $\overline{A}\cap B$

3 a $y=\frac{45-5x}{3}$ **b** $x=3, y=10$ **c** 5

4 a



b i $\frac{1}{2}$ **ii** $\frac{1}{2}$

c i $\frac{1}{4}$ **ii** $\frac{3}{4}$

d i $\frac{1}{4}$ **ii** $\frac{3}{4}$

ANSWERS

Test yourself 9

1 **a** **i** 0.22 **ii** 0.29 **iii** 0.1 **iv** 0.14

b **ii** 0.25 **ii** 0.25 **iii** 0.125 **iv** 0.125

c Yes **d** 0.25

e The expected number of times the arrow stops at a colour not purple or black is 75, which is the same as the observed number of times.

2 **a** **i** 0.353 **ii** 0.427 **iii** 0.087 **iv** 0.513

b **i** 0.867 **ii** 0.133

c Different – at least one head occurring excludes 0 heads occurring, which is the same as 3 tails occurring. The events are complementary.

3 **a** 90 **b** **i** 58 **ii** 70 **iii** 21

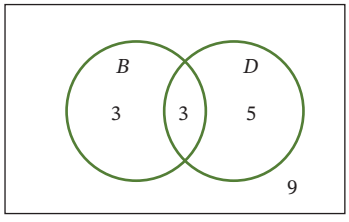
c $\frac{19}{45}$ **d** $\frac{2}{9}$ **e** $\frac{11}{49}$

4 **a** 35 **b** **i** 5 **ii** 19 **iii** 16

c $\frac{2}{35}$ **d** **i** $\frac{6}{35}$ **ii** $\frac{19}{35}$ **iii** $\frac{3}{35}$

e They don't like the types of music mentioned in the survey.

5 **a**



b **i** $\frac{3}{20}$ **ii** $\frac{2}{5}$ **iii** $\frac{3}{20}$ **iv** $\frac{3}{5}$

c $\frac{9}{20}$

6 **a** 200

b **i** 0.305 **ii** 0.11 **iii** 0.225 **iv** 0.425

c $\frac{25}{110} \approx 0.227$

d **i** $\frac{47}{69} \approx 0.681$ **ii** $\frac{22}{69} \approx 0.319$

7 D

8 **a**

		1st die			
		1	2	3	4
2nd die	1	1, 1	2, 1	3, 1	4, 1
	2	1, 2	2, 2	3, 2	4, 2
	3	1, 3	2, 3	3, 3	4, 3
	4	1, 4	2, 4	3, 4	4, 4

b 16

c **i** $\frac{1}{2}$ **ii** $\frac{1}{4}$ **iii** $\frac{7}{16}$

iv $\frac{1}{4}$ **v** $\frac{1}{4}$ **vi** $\frac{1}{2}$

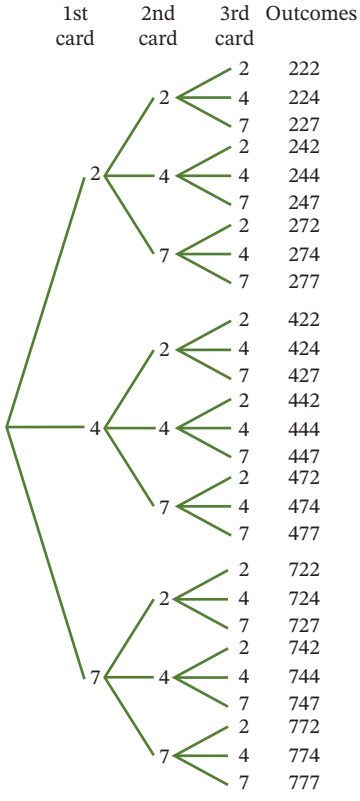
9 **a** **i** RR, RB, RG, RY, RBla, BR, BB, BG, BY, BBla, GR, GB, GG, GY, GBla, YR, YB, YG, YY, YBla, BlaR, BlaB, BlaG, BlaY, BlaBla

ii RB, RG, RY, RBla, BR, BG, BY, BBla, GR, GB, GY, GBla, YR, YB, YG, YBla, BlaR, BlaB, BlaG, BlaY

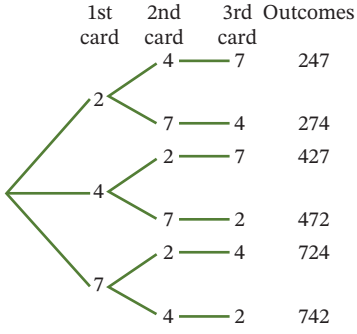
b **i** $\frac{1}{25}$ **ii** $\frac{1}{5}$ **iii** $\frac{2}{25}$ **iv** $\frac{9}{25}$

c **i** $\frac{1}{10}$ **ii** $\frac{3}{5}$

10 **a** **i**



ii

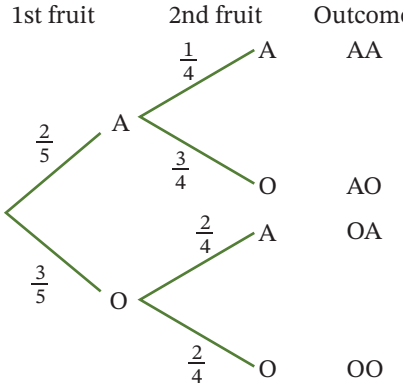


b **i** $\frac{2}{3}$ **ii** $\frac{1}{3}$ **iii** $\frac{1}{9}$ **iv** $\frac{2}{9}$

c **i** $\frac{1}{3}$ **ii** $\frac{2}{3}$ **iii** $\frac{1}{3}$ **iv** $\frac{1}{3}$

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11 a



b i $\frac{1}{10}$ **ii** $\frac{3}{5}$ **iii** $\frac{9}{10}$

12 a independent **b** dependent **c** dependent
d independent **e** dependent **f** independent

Chapter 10

SkillCheck

1 a $w = 38$ **b** $r = 44$ **c** $h = 71$
d $x = 126$ **e** $y = 46$ **f** $a = 24$

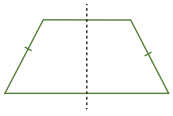
2 a $x = 15$ **b** $m = 5$ **c** $a = 1$ **d** $k = 10$
e $x = 1\frac{3}{4}$ **f** $n = 3\frac{1}{3}$ **g** $t = 4\frac{2}{3}$ **h** $d = 8\frac{1}{2}$

3 a $p = 64$ **b** $p = 241$ **c** $p = 104$
d $p = 105$ **e** $p = 58$ **f** $p = 128$

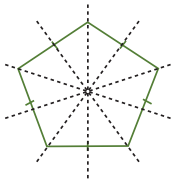
4 a $\angle M$ and $\angle X$, $\angle N$ and $\angle Y$, $\angle P$ and $\angle W$
b MN and XY, NP and YW, MP and XW

Exercise 10.01

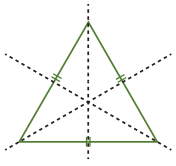
1 a



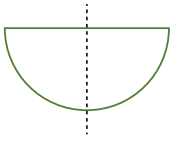
b



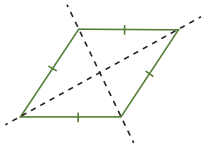
c



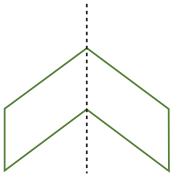
d



e



f



2 a Yes, order 2 **b** Yes, order 4 **c** Yes, order 10
3 a **C, H, M, O, W, X, Y** **b** **H, O, S, X, Z**

4 a i 1 **ii** No
b i 6 **ii** Yes, order 6
c i 1 **ii** No
d i no axis of symmetry **ii** Yes, order 3
e i 5 **ii** Yes, order 5
f i 4 **ii** Yes, order 4

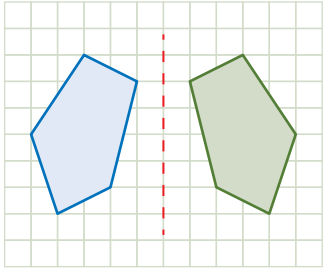
5 a i 4 **ii** order 4
b i 2 **ii** order 2
c i 10 **ii** order 10
d i infinite **ii** order infinite
e i 1 **ii** no rotational symmetry

6 a i no axes of symmetry, no rotational symmetry
ii 1 axis of symmetry, no rotational symmetry
iii 3 axes of symmetry, order of rotational symmetry 3

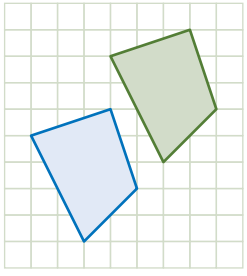
b No rotational symmetry when a shape has 0 or 1 axis of symmetry
For 2 or more axes of symmetry, the order of rotation is the same as number of axes of symmetry.

Exercise 10.02

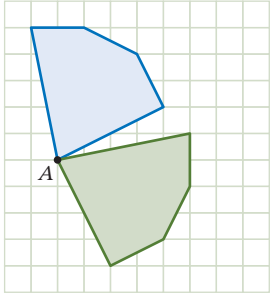
1 a



b

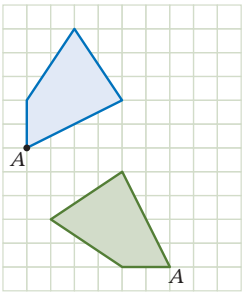


c

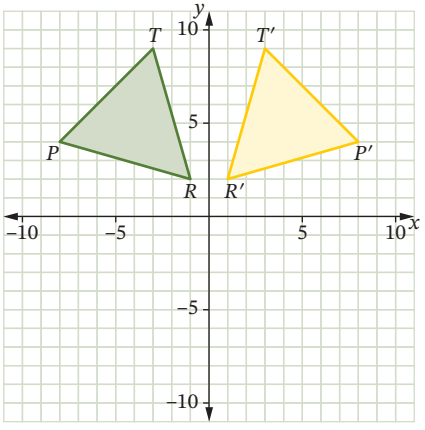


ANSWERS

2



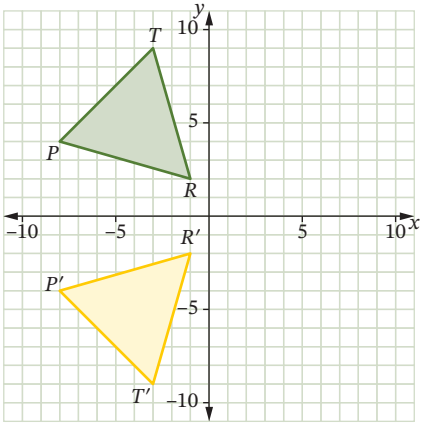
3 **a**



b $P(-8, 4) \rightarrow P'(8, 4)$, $R(-1, 2) \rightarrow R'(1, 2)$, $T(-3, 9) \rightarrow T'(3, 9)$

The x-coordinate of each vertex changes sign, while the y-coordinate stays the same.

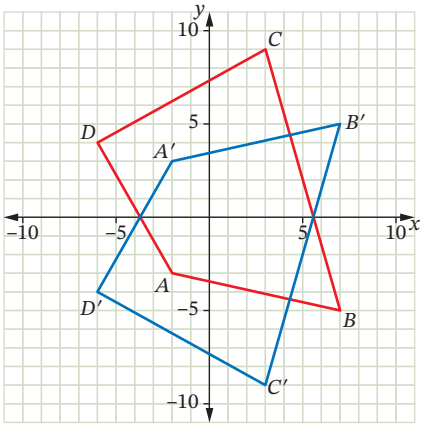
c



$P(-8, 4) \rightarrow P'(-8, -4)$, $R(-1, 2) \rightarrow R'(-1, -2)$, $T(-3, 9) \rightarrow T'(-3, -9)$

The x-coordinate of each vertex stays the same sign, while the y-coordinate changes sign.

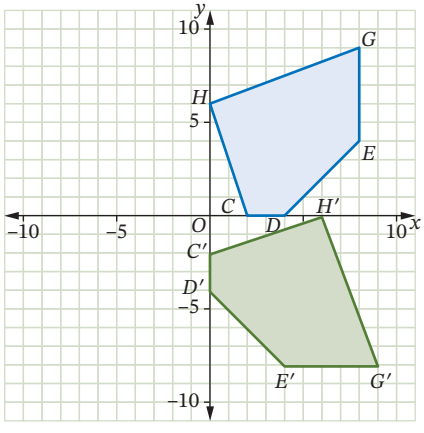
4 **a**



b $A(-2, -3) \rightarrow A'(-2, 3)$, $B(7, -5) \rightarrow B'(7, 5)$, $C(3, 9) \rightarrow T'(3, -9)$, $D(-6, 4) \rightarrow D'(-6, -4)$

The x-coordinate of each vertex stays the same sign, while the y-coordinate changes sign.

5 **a**



b $D(4, 0) \rightarrow D'(0, -4)$, $E(8, 4) \rightarrow E'(4, -8)$.

The x- and y-coordinates swap, with the new y-coordinate changing sign

c **i** $C(2, 0) \rightarrow C'(-2, 0)$

$G(8, 9) \rightarrow G'(-8, -9)$

$H(0, 6) \rightarrow H'(0, -6)$

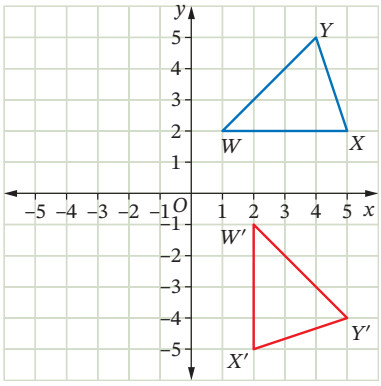
ii $C(2, 0) \rightarrow C'(0, 2)$

$G(8, 9) \rightarrow G'(-9, 8)$

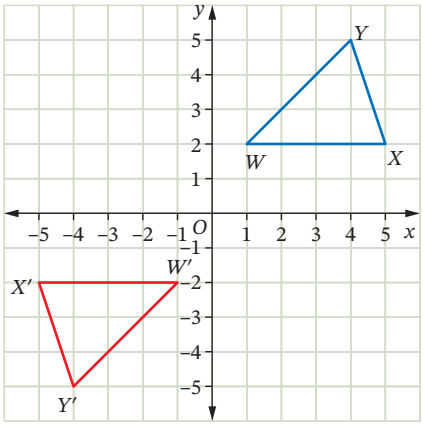
$H(0, 6) \rightarrow H'(-6, 0)$

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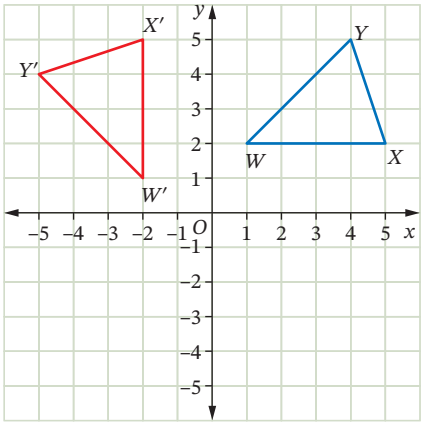
6 a i



ii



iii



b i $W(1, 2) \rightarrow W'(2, -1), X(5, 2) \rightarrow X'(2, -5), Y(4, 5) \rightarrow Y'(5, -4)$

ii $W(1, 2) \rightarrow W'(-1, -2), X(5, 2) \rightarrow X'(-5, -2), Y(4, 5) \rightarrow Y'(-4, -5)$

iii $W(1, 2) \rightarrow W'(-2, 1), X(5, 2) \rightarrow X'(-2, 5), Y(4, 5) \rightarrow Y'(-5, 4)$

c Teacher to check

d Anticlockwise 90° rotation is the same as a clockwise 270° rotation.

Anticlockwise 180° rotation is the same as a clockwise 180° rotation.

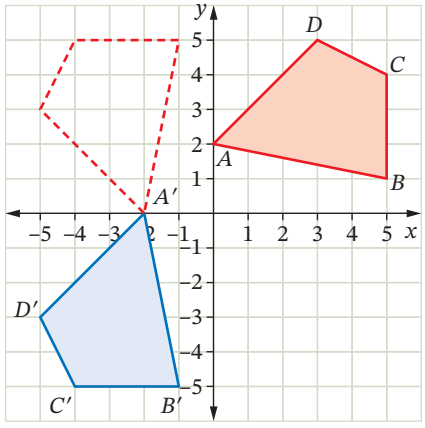
Anticlockwise 270° rotation is the same as a clockwise 90° rotation.

7 a Translation 6 units left and 3 units up.

b Reflection in the x-axis

c Rotation of 180° about O.

8 a



b $A(0, 2) \rightarrow A'(-2, 0), B(5, 1) \rightarrow B'(-1, -5), C(5, 4) \rightarrow C'(-4, -5), D(3, 5) \rightarrow D'(-5, -3)$

9 C

10 B

11 a (−4, 6) **b** (4, −6) **c** (0, 1)
d i (−6, 4) **ii** (4, 6) **iii** (6, −4)

12 A

13 D

14 a (3, 7) **b** (2, 2) **c** (−3, 7) **d** (7, 3) **e** (3, 3)

15 D

16 (2, 1)

17 (5, 1)

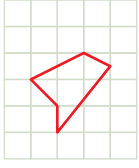
Mental skills 10

2 a \$100, \$50 **b** \$1200, \$900 **c** \$160, \$560
d \$500, \$1500 **e** \$2700, \$1800 **f** \$900, \$2100
g \$3000, \$600 **h** \$600, \$1000 **i** \$550, \$440
j \$800, \$3200 **k** \$2100, \$2800 **l** \$2000, \$1200

Exercise 10.03

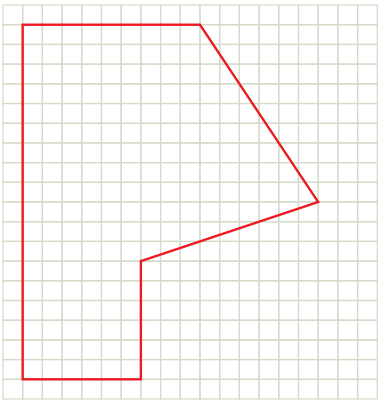
1 a 2 **b** $\frac{2}{3}$ **c** $\frac{4}{5}$ **d** $2\frac{1}{2}$

2 a

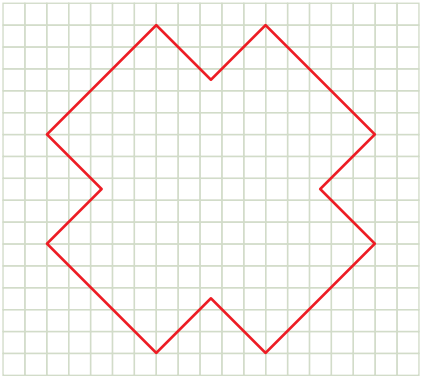


ANSWERS

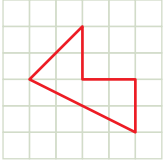
b



c



d



3 **a** **i** $\angle P$ and $\angle E$, $\angle R$ and $\angle C$, $\angle T$ and $\angle D$

ii RP and CE, RT and CD, PT and ED

iii $\triangle PRT \sim \triangle ECD$

b **i** $\angle A$ and $\angle W$, $\angle B$ and $\angle L$, $\angle C$ and $\angle M$, $\angle D$ and $\angle N$, $\angle H$ and $\angle P$, $\angle K$ and $\angle T$

ii WL and AB, LM and BC, MN and CD, NP and DH, PT and HK, TW and KA

iii $ABCDHK \sim WLMNPT$

c **i** $\angle J$ and $\angle F$, $\angle F$ and $\angle T$, $\angle R$ and $\angle W$, $\angle P$ and $\angle V$, $\angle M$ and $\angle S$, $\angle J$ and $\angle K$

ii JT and KF, TR and FW, RP and WV, PM and VS, MJ and SK

iii $JTRPM \sim KFWVS$

d **i** $\angle Q$ and $\angle N$, $\angle B$ and $\angle G$, $\angle K$ and $\angle C$

ii QB and GN, BK and GC, KQ and CN

iii $\triangle QBK \sim \triangle NGC$

4 **a** Yes, $\frac{10}{16}=\frac{7.5}{12}=\frac{5}{8}$ **b** Yes, $\frac{12}{18}=\frac{28}{42}=\frac{2}{3}$

c No, $\frac{32}{40}=\frac{4}{5}, \frac{15}{20}=\frac{3}{4}, \frac{15}{22.5}=\frac{2}{3}, \frac{13}{20}$

d Yes, $\frac{15}{6}=\frac{10}{4}=\frac{12.5}{5}=\frac{5}{2}=2.5$

e Yes, all equilateral triangles are similar.

f No, matching angles are not equal.

Exercise 10.04

1 **a** $w = 22.4$ **b** $m = 10$

c $p = 20, h = 21$ **d** $x = 18$

e $a = 12.8, w = 7.5$ **f** $g=11\frac{1}{9}, q=18$

g $y=26\frac{2}{3}, b=9\frac{3}{5}$ or 9.6

h $u=12\frac{4}{5}$ or 12.8, $t=6\frac{7}{8}$ or 6.875

2 $h=13\frac{5}{7}$ **3** $x=8\frac{8}{9}$

4 $w = 16$ cm **5** 12 m **6** B

7 $h = 2.408$ m **8** D **9** 2.24 m

Exercise 10.05

1 **a** **i** 5 **ii** 7

b **i** 4 **ii** 6

c **i** 7 **ii** 8

2 **a** **i**

Vertex	A	B	C	D	E	F
Degree	1	4	2	3	2	2

ii $14 = 2 \times 7$

b **i**

Vertex	G	H	M	K	L	P
Degree	2	3	5	2	5	3

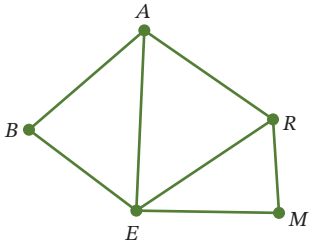
ii $20 = 2 \times 10$

3 C

4 A

5 D

6 **a**



b Mehek

7 **a** **i** trail **ii** trail

iii path **iv** trail

v trail **vi** path

b **i** ii and v are circuits **ii** iii is a cycle

c $DBEABCD$

d $CDEABC$

8 B

9 **a** D **b** $KLPEFHK$

10 **a** 2 **b** 4 **c** 1

d 7 **e** 2 **f** 3

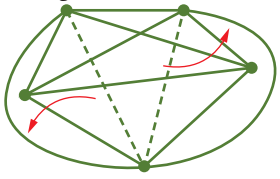
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Exercise 10.06

1 **a** 2 **b** 2 **c** 3

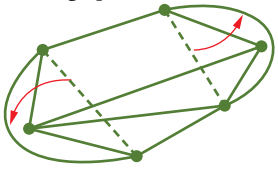
2 **a** Planar graph because no edges cross.

b Non-planar graph as it cannot be redrawn so that edges do not cross.



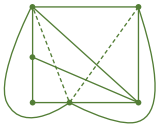
2 edges can be moved as shown but 2 other edges still cross.

c Planar graph as it can be redrawn as shown.

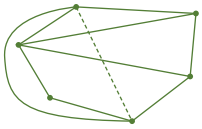


The graph is planar since no edges cross when it is redrawn.

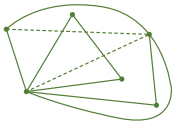
3 **a**



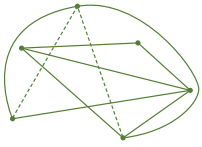
b



c



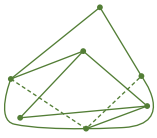
d



e



f



4 **a** 3 **b** 6 **c** 4 **d** 3 **e** 6 **f** 4

5 C

6 B

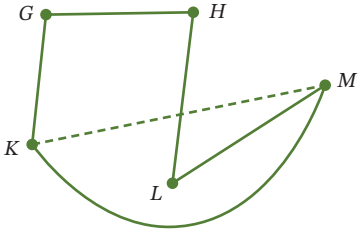
7 $(v - e + f = 2)$

a $6 - 8 + 4 = 2$ **b** $4 - 3 + 1 = 2$

c $7 - 7 + 2 = 2$ **d** $5 - 5 + 2 = 2$

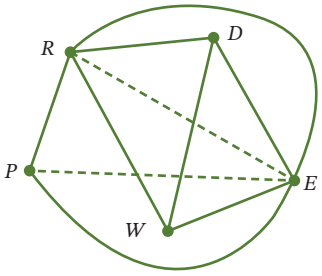
e $5 - 8 + 5 = 2$ **f** $5 - 6 + 3 = 2$

8 **a** **i**



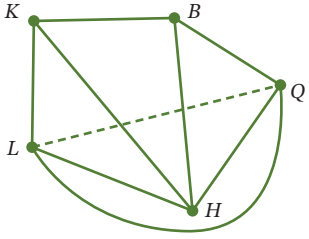
ii $5 - 5 + 2 = 2$

b **i**



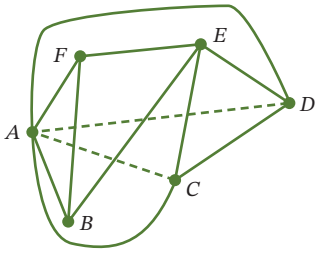
ii $5 - 8 + 5 = 2$

c **i**



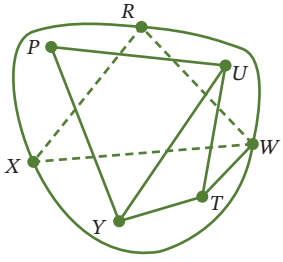
ii $5 - 8 + 5 = 2$

d **i**



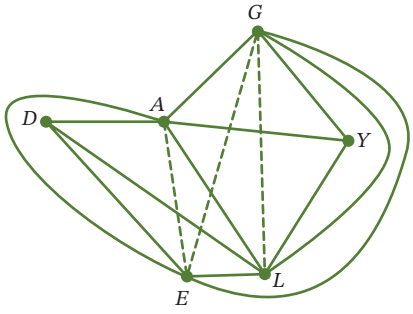
ii $6 - 10 + 6 = 2$

e **i**



ii $7 - 9 + 4 = 2$

f **i**



ii $6 - 12 + 8 = 2$

ANSWERS

9 C
10 B
11 B
12 Teacher to check.
13 D

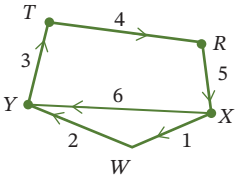
Exercise 10.07

1 **a** No, there are 4 odd vertices.
b Yes, since the network has only 2 odd vertices.
c No, there are 4 odd vertices.
d No, there are 4 odd vertices.

2 **a** **i**

Vertex	*T*	*R*	*X*	*W*	*Y*
Degree	2	2	3	2	3

ii There are exactly 2 odd vertices
iii



The graph shows the Eulerian trail is *XWYTRXY*.

b **i**

Vertex	*A*	*B*	*C*	*D*	*E*	*F*	*G*
Degree	3	4	4	3	4	2	4

ii There are exactly 2 odd vertices
iii Teacher to check the trail on the graph. A Eulerian trail is *ABCAGECDEFGBD*.

3 **a** Yes, since the network only has 2 odd vertices.
b The trail will start at Bathurst and finish at Dubbo or vice versa.
c Bathurst, Lithgow, Mudgee, Dunedoo, Dubbo, Orange, Bathurst, Cowra,
Highway intersection, Parkes, Dubbo.

4 **a** The network only has 2 odd vertices.
b C
c Teacher to check trail on the graph. 2 Eulerian trails are *EFHLBEDCBPC* and *EDCBEFHLBPC* – all trails must start at *E*, finish at *C* and use each road only once.

5 A

6 **a** No, there are 2 odd vertices (*X* and *N*).
b Yes, all vertices are even.
c Yes, all vertices are even.
d No, there are 2 odd vertices (*H* and *T*).

7 **a** All vertices have an even degree.
b Teacher to check graphs. 2 Eulerian circuits are *HPWLSNLPNH* and *NSLWPLNPHN*.
c The circuits are the same length and they start and finish at the same vertex.

8 **a** There are 2 odd vertices – *D* and *K*.
b *D* and *K* (They both become even vertices).
c Two possible routes are *DWQPNLKEDKPD, DEKDPKLNPQWD*.

9 **a** Since all vertices are odd, it is not possible travel along each road once.
b **i** Since there are 2 odd vertices, it is not possible to start and finish at the same town.
ii There are now only 2 odd vertices, so it is possible to start and finish at different vertices.

10 **a** Start at *R* or *M* (the 2 odd vertices).
b Then finish at *M* or *R*.
c Possible routes are *MNQRUQMPTMUTR*, *MPTMNQMUTRUQR* if starting at *M* or *RQNMQURTPMTUM* if starting at *R*.
i 2 destinations - *P* and *N*
ii 4 destinations - *Q*, *R*, *T* and *U*

11 **a** Vertices all have even degree 4.
b One Eulerian circuit is *DHGDKHLKEGLED* – teacher to check other circuits.

12 **a** Eulerian circuit – all vertices are even.
b Eulerian trail – *M* and *L* are the only 2 odd vertices.
c Eulerian trail – *B* and *D* are the only 2 odd vertices.
d Eulerian trail – *T* and *A* are the only 2 odd vertices.
e neither – there are 4 odd vertices and 1 even vertex.
f Eulerian circuit – all vertices are even.

13 C

14 **a** There are only 2 odd vertices which are *W* and *L* so Mehek would start at either *W* or *L*.
b The race would finish at *L* or *W*.

15 **a** There are more than 2 odd vertices.
b 2 odd vertices need to be joined.
c Join *A* to *C* and *G* to *Y* or *A* to *G* and *C* to *Y*.

16 **a** All vertices have an even degree.
b Teacher to check. One possible route is *FRNFEKNCEHSEHWF*.

Power plus

1 $x = 8$

2 $y = 5\frac{1}{7}$

3 **a** Since all vertices are even, an Eulerian circuit exists.
b **i** *NUEDHSWTNXWSHDEXEUN*
ii *HSWTNUEDHSWXNUEXEDH*
iii *XEXNUNTWSHDEUEDHSWX*
(other solutions are possible – teacher to check)
c These answers apply to the above Eulerian circuits.
i *T* occurs once in all three circuits
ii *N* and *E*, *H* and *E*, *X* and *E*

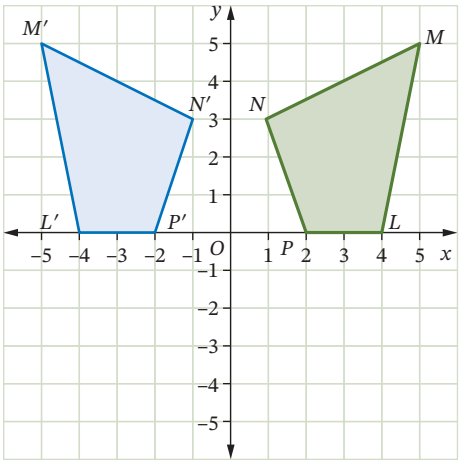
4 A polygon with an odd number of vertices – triangle, pentagon, heptagon, ...

Test yourself 10

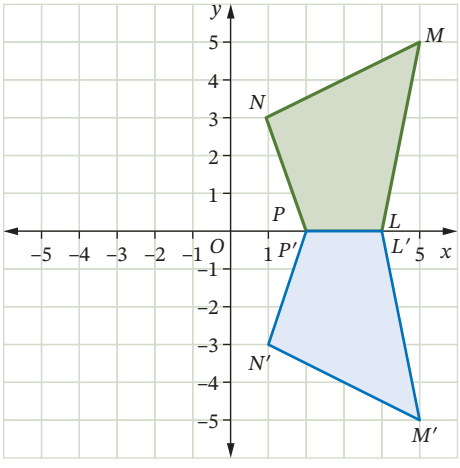
1 **a** **i** 1 **ii** no rotational symmetry
b **i** 8 **ii** 8
c **i** 0 **ii** 2

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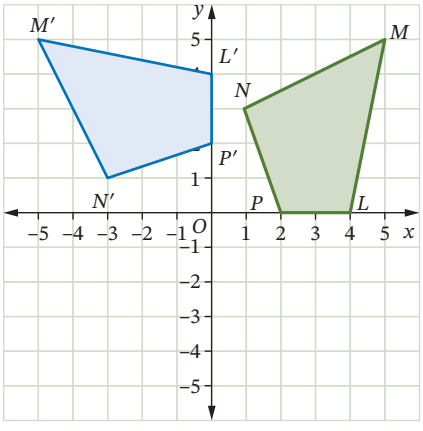
2 a i



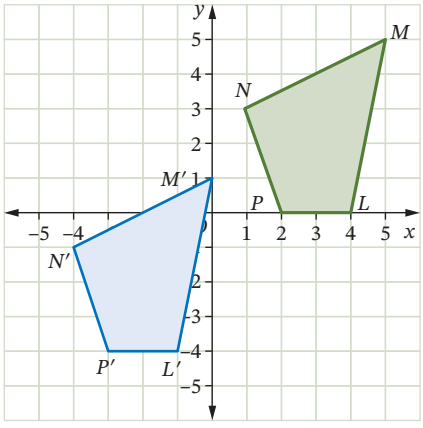
ii



iii



iv



b i $P(2, 0) \to P'(-2, 0)$, $L(4, 0) \to L'(-4, 0)$, $M(5, 5) \to M'(-5, 5)$, $N(1, 3) \to N'(-1, 3)$

ii $P(2, 0) \to P(2, 0)$, $L(4, 0) \to L'(4, 0)$, $M(5, 5) \to M'(5, -5)$, $N(1, 3) \to N'(1, -3)$

iii $P(2, 0) \to P'(0, 2)$, $L(4, 0) \to L'(0, 4)$, $M(5, 5) \to M'(-5, 5)$, $N(1, 3) \to N'(-3, 1)$

iv $P(2, 0) \to P'(-3, -4)$, $L(4, 0) \to L'(-1, -4)$, $M(5, 5) \to M'(0, 1)$, $N(1, 3) \to N'(-4, -1)$

3 a (5, 3) **b** (−5, −3)
c i (3, 5) **ii** (5, −3) **iii** (−3, −5)
d (3, −2) **e** (2, −5)

4 (2, 4)

5 a $\frac{4}{5}$ **b** $\frac{5}{3}$

6 a Yes, $\frac{18}{27} = \frac{10}{15} = \frac{2}{3}$

b Yes $\left(\frac{9}{11.25} = \frac{12}{15} = \frac{22}{27.5} = \frac{16}{20} = \frac{4}{5}\right)$

7 a $k = 12\frac{6}{7}$ **b** $d = 6$

c $y = 4\frac{1}{5}$ or 4.2 **d** $x = 5\frac{1}{2}$ or 5.5

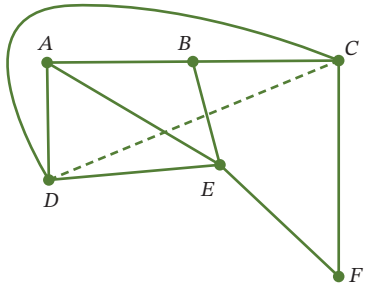
8 $d = 11\frac{3}{7}$

9 a i path **ii** trail **iii** trail **iv** path

b ii is a circuit

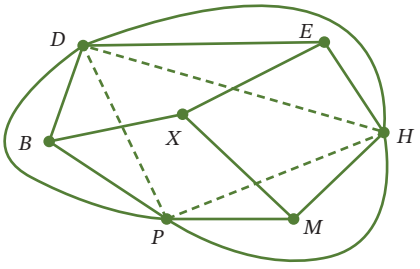
10 A

11 a i



ii $v - e + f = 6 - 9 + 5$
$= 2$

b i



ii $v - e + f = 7 - 12 + 7$
$= 2$

12 a Neither, since there are more than 2 odd vertices the graph is not a Eulerian trail and all vertices are not even, so the graph is not a Eulerian circuit.

b All the vertices are even, so it is a Eulerian circuit.

c Neither, since there are more than 2 odd vertices.

d There are only 2 odd vertices so it is an Eulerian trail.

13 No, since there are 2 odd vertices, it is not possible to start and finish at the same town.
(It is possible to go along each road once if you start at one odd vertex (U or H) and then finish at the other odd vertex.)

Practice set 4

1 a 36

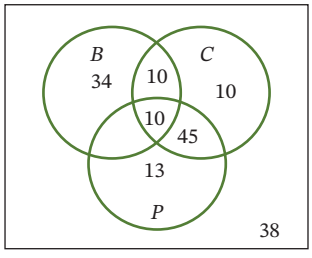
b i $\frac{1}{36}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{4}$

2 a 80

b i $\frac{13}{80}$ **ii** $\frac{36}{80} = \frac{9}{20}$ **iii** $\frac{42}{80} = \frac{21}{40}$ **iv** $\frac{56}{80} = \frac{7}{10}$

c $\frac{1}{6} = 0.1\dot{6}$, which is lower than the observed probability of $\frac{17}{80} = 0.2125$

3 a



b i $\frac{13}{160}$ **ii** $\frac{19}{80}$ **iii** $\frac{9}{32}$

iv $\frac{109}{160}$ **v** $\frac{57}{160}$

c $\frac{5}{27}$

4 a 78

b i $\frac{25}{39}$ **ii** $\frac{14}{39}$

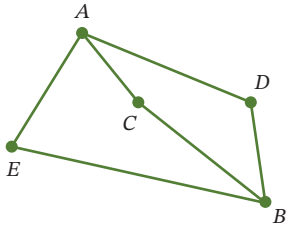
c $\frac{35}{78}$

5 a i 2 **ii** 2

b i 5 **ii** 5

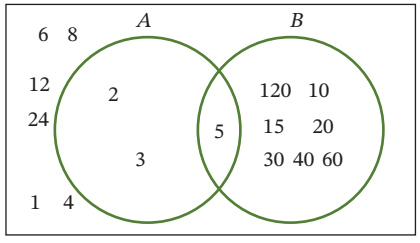
c i 0 **ii** no rotational symmetry

6 a



b $5 - 6 + 3 = 2$

7



8 independent

9 a $\frac{1}{4}$ or 0.25 **b** $\frac{5}{4}$ or 1.25

10 a i 0.39 **ii** 0.43 **iii** 0.18

b i 0.33 **ii** 0.42 **iii** 0.25

c drawing a black marble

d 88, which is close to the observed frequency of 85

11 B

12 $\overline{B} = \{1, 2, 4, 5, 7, 8, 10, 11\}$
$A \cup \overline{B} = \{1, 2, 4, 5, 6, 7, 8, 10, 11, 12\}$
$\overline{A} \cap \overline{B} = \{1, 5, 7, 11\}$

13 a (−3, 2) **b** (3, −2) **c** (−2, 3) **d** (4, 2) **e** (−3, 2)

14 a

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24

b i $\frac{1}{24}$ **ii** $\frac{1}{8}$ **iii** $\frac{1}{4}$

iv $\frac{1}{3}$ **v** $\frac{5}{8}$ **vi** $\frac{7}{24}$

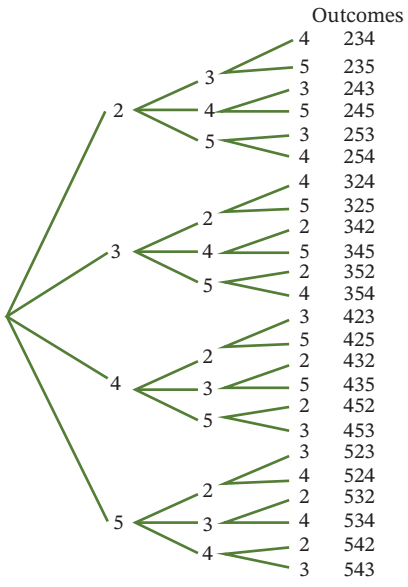
15 a 135 **b** $\frac{40}{135} = \frac{8}{27}$

c i $\frac{67}{135}$ **ii** $\frac{60}{135} = \frac{4}{9}$ **iii** $\frac{72}{135} = \frac{8}{15}$

16 a $h = 5\frac{1}{3}$ **b** $p = 35$

9780170479622

17 a

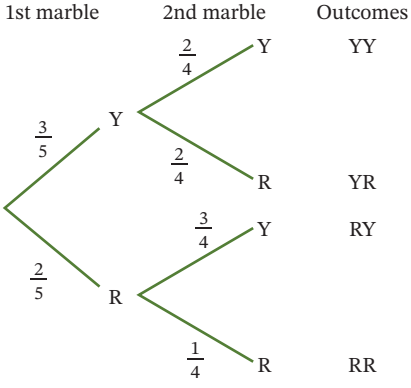


b i $\frac{1}{2}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{3}{4}$ **v** $\frac{1}{4}$

18 a All vertices have an even degree.

b Teacher to check circuits.
2 Eulerian circuits are *GLPKNYLPNWYPG* and *YPNWYLPGLPKNY*

19 a



b i $\frac{1}{10}$ **ii** $\frac{7}{10}$

c Dependent, as the sample space has reduced from 5 to 4.

d Independent, as the number of marbles in the bag remains the same.

20 10.6

21 a C **b** E **c** B **d** A **e** D **f** F

General practice

1 \$15 700

2 a $m = 3, c = -4$

b $m = -2, c = 5$

c $m = -\frac{4}{3}, c = 3$

3 a 4 **b** 2.5

4 a $8x(x + 2)$ **b** $(y - 5)^2$

c $(a + 4)(a + 2)$ **d** $(p + 6)(p - 8)$

5 a 396 mm² **b** 3750 mm² **c** 6792 mm²

6 a $56x^{12}$ **b** $\frac{x^5}{4}$ **c** $\frac{1}{9y^2}$ **d** $\frac{n}{m^7}$

7 a 210

b i $\frac{1}{6}$ **ii** $\frac{143}{210}$

c 32% **d** $\frac{29}{36}$

8 a 6.4 **b** 15.6 **c** 110.6

d 13.7 **e** 3.3 **f** 32.1

9 a $k = 6\frac{1}{2}$ **b** $m = -3$ **c** $x = 7\frac{1}{2}$

10 a 9 edges, 6 vertices, 5 faces

b $6 - 9 + 5 = 2$ **c** C

11 26 m

12 a $a = 23$ **b** $y = 6\frac{1}{4}$ **c** $m = -3\frac{1}{3}$ **d** $k = 3\frac{3}{7}$

13 a 4 **b** 3 h **c** 75%

14 B

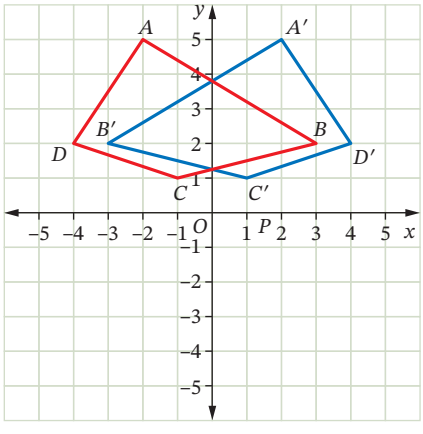
15 $u = \frac{K - Vd}{6}$

16 a i $\frac{1}{8}$ **ii** $\frac{3}{2}$ **iii** $y = \frac{x}{8} + \frac{3}{2}$

b i -1 **ii** -2 **iii** $y = -x - 2$

c i 2 **ii** 1 **iii** $y = 2x + 1$

17 a



b $A(-2, 5) \rightarrow A'(2, 5)$, $B(3, 2) \rightarrow B'(-3, 2)$, $C(-1, 1) \rightarrow C'(1, 1)$, $D(-4, 2) \rightarrow D'(4, 2)$

c (5, 2) **d** (2, −4)

18 B

19 a \$40 **b** \$4.08

20 a i 35° **ii** 35° 30′

b i 35° **ii** 35° 26′

c i 43° **ii** 43° 21′

ANSWERS

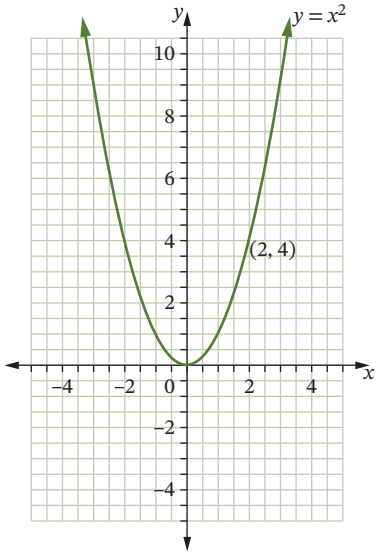
21 **a** \$935.89 **b** \$1597.98

22 **a** **i** 8.9 **ii** (4, 5) **iii** 2

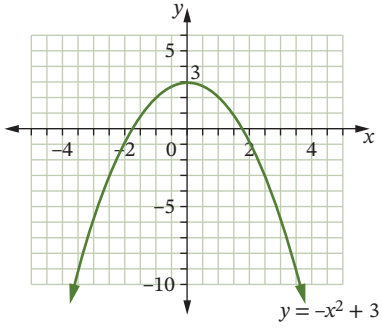
b **i** 13.4 **ii** (−1, 1) **iii** $\frac{1}{2}$

23 **a** $w = 7.27$ **b** $d = 67.83$ **c** $k = 6.71$

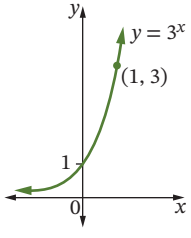
24 **a**



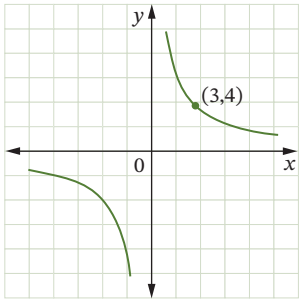
b



c

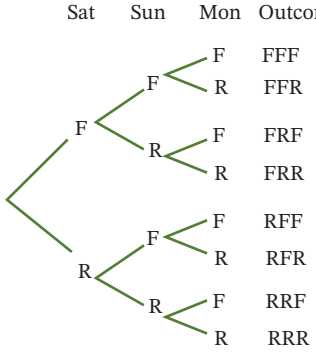


d



25 $h = 21, p = 20$ **26** D

27 **a** F = fine, R = rain



b **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{7}{8}$

28 19.2 min

29 **a** 23° 52′ **b** 101° 19′ **c** 134° 12′

30 **a** $SA = 624$ cm², $V = 768$ cm³

b $SA = 282.7$ cm², $V = 314.2$ cm³

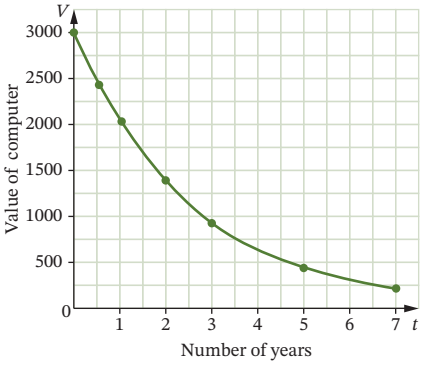
c $SA = 254.5$ cm², $V = 381.7$ cm³

31 \$49 900 (to the nearest dollar)

32 **a** $m^2 + 12m + 35$ **b** $4w^2 + 4w - 15$ **c** $3p^2 - 14p - 5$

33 **a**

t	0	1	2	3	5	7
V	3000	2041	1388	944	437	202



b \$3000

c \$642

d After 9 years the value will be \$94 (to the nearest dollar)

Chapter 11

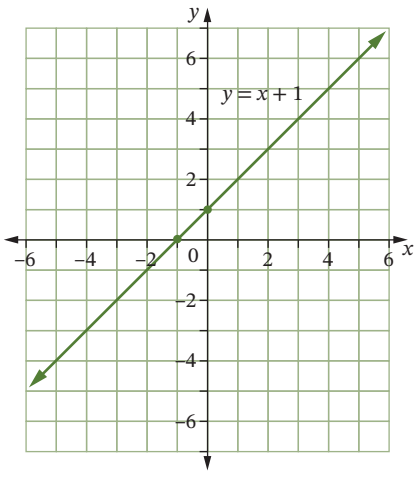
SkillCheck

1 **a** 5 **b** 13 **c** 6 **d** −1

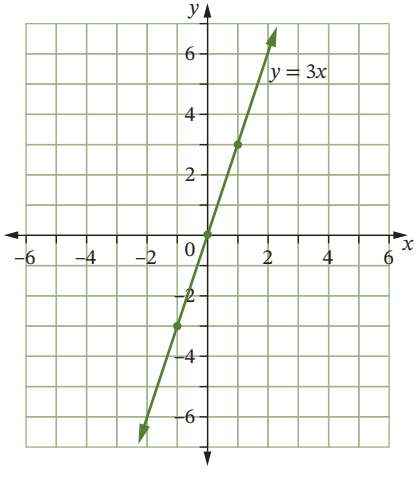
2 **a** −11 **b** 1 **c** 7 **d** $5\frac{1}{2}$

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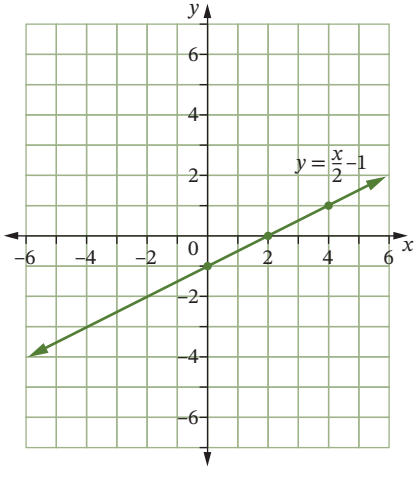
3 **a**



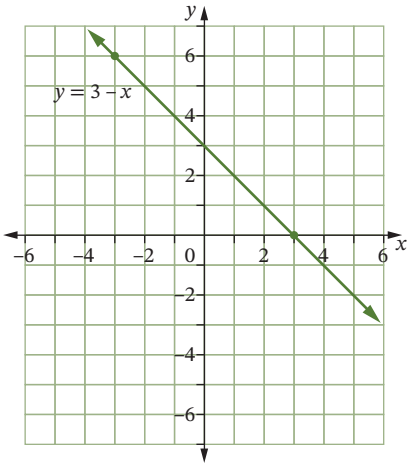
b



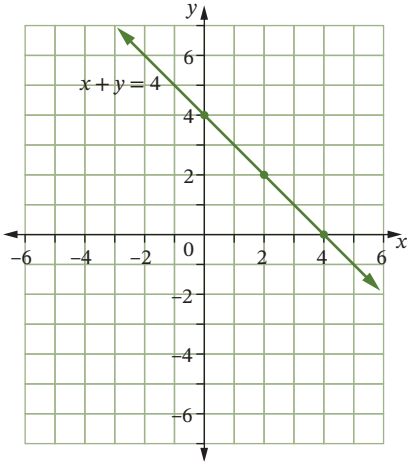
c



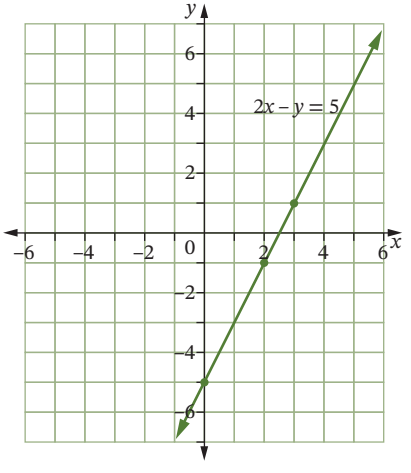
d



e



f

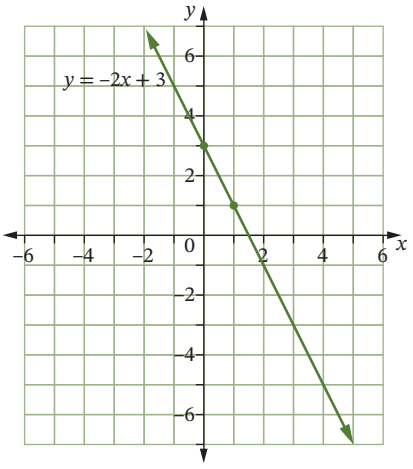


4 (−2, 3) lies on *a*, *d*

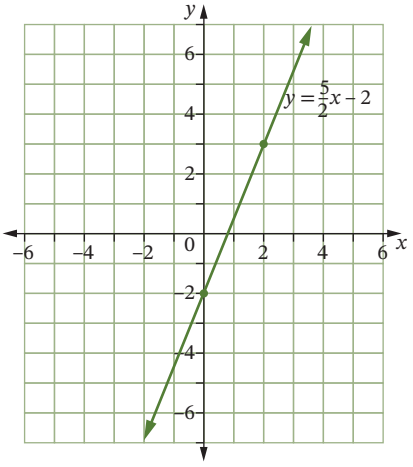
5 **a** For $y = 2x + 1$, when $x = 2$, $y = 2 \times 2 + 1 = 5$
$\therefore$ (2, 5) lies on $y = 2x + 1$
For $x + y = 7$, when $x = 2$, $y = 5$, $2 + 5 = 7$
$\therefore$ (2, 5) lies on $x + y = 7$

b (2, 5)

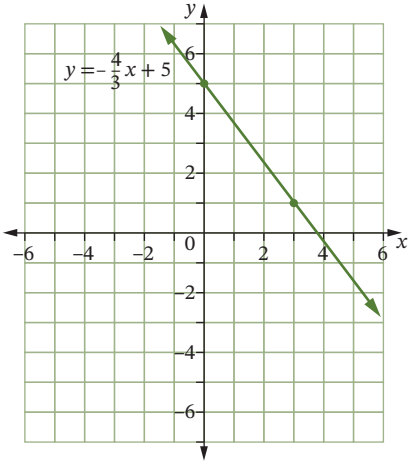
6 **a** $m = -2, c = 3$



b $m = \frac{5}{2}, c = -2$



c $m = -\frac{4}{3}, c = 5$



7 **a** $6x + 12$ **b** $-x + 5$ **c** $5x + 10y - 15$

8 **a** $x = 2$ **b** $x = -3\frac{2}{3}$ **c** $x = 3$

Exercise 11.01

1 **a** $x = 3, y = -1$

b $x = 2, y = 1$

c $x = -1, y = -5$

2 **a** $x = 1, y = 2$ **b** $x = -5, y = -9$

c $x = 1, y = 2$ **d** $x = -\frac{1}{2}, y = 2\frac{1}{2}$

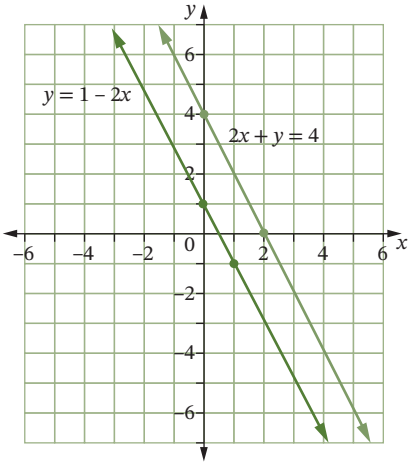
e $x = -2, y = -9$ **f** $x = 5, y = -4$

g $x = \frac{1}{2}, y = 6\frac{1}{2}$ **h** $x = 3, y = 2$

i $x = 5, y = 1\frac{1}{2}$ **j** $x = 5, y = 8$

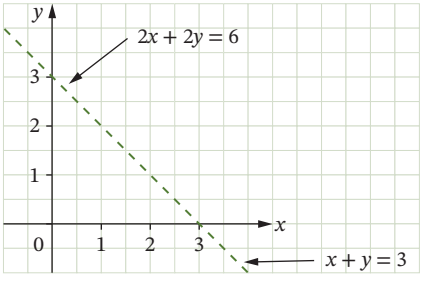
k $x = 1\frac{1}{2}, y = 2\frac{1}{2}$ **l** $x = 4, y = 0$

3 **a**



b The lines are parallel.

4 **a**



b Both equations give the same line, so there are an infinite number of points of intersection.

Exercise 11.02

1 **a** $d = -3, k = 2$ **b** $x = 5, w = 4$

c $g = 2, h = -\frac{2}{5}$ **d** $n = 3\frac{1}{4}, p = -1$

e $q = 5, r = -4$ **f** $k = -4\frac{3}{5}, x = 5.$

g $c = 1\frac{1}{2}, e = 1$ **h** $k = 3, y = -2$

i $a = 2, f = 2$

2 **a** $d = -14, k = 6$ **b** $a = 1, c = 1$
c $h = 3, y = 4$ **d** $e = 3, x = \frac{1}{3}$
e $q = 3, w = 6\frac{1}{2}$ **f** $c = 3, p = 4$
g $m = -\frac{2}{3}, y = 4$ **h** $a = -1, r = 5\frac{1}{2}$
i $x = 2, w = 2$ **j** $g = -2, y = 4$
k $e = -5, n = -8$ **l** $k = 5\frac{1}{2}, h = 1\frac{1}{2}$

3 **a** $q = -3, w = 3$ **b** $m = -9, x = 7$
c $d = 23, h = -7$ **d** $g = -1, n = 3$
e $h = 0, m = 2$ **f** $e = -4, y = 3$
g $q = 1, w = -4$ **h** $a = \frac{1}{2}, d = \frac{1}{2}$
i $k = 5, p = -2$ **j** $a = -2, f = -2$
k $c = -64, r = -38$ **l** $x = -4, y = -3$
m $x = 8, y = -1$ **n** $g = 4\frac{1}{2}, h = 2\frac{1}{2}$
o $k = -2\frac{1}{2}, w = 3\frac{1}{2}$

Exercise 11.03

1 **a** $x = 2, y = 5$ **b** $x = \frac{3}{5}, y = 3\frac{4}{5}$ **c** $x = 7, y = 2$
d $x = 2, y = -2$ **e** $x = 5, y = -1$ **f** $x = 4, y = 2$

2 **a** $x = 9, y = 21$ **b** $x = 5, y = 3$ **c** $x = -\frac{1}{4}, y = 2$
d $x = -3, y = 1$ **e** $x = 2, y = 2$ **f** $x = -7, y = 4$
g $x = 7, y = 3$ **h** $x = 3, y = 2\frac{1}{2}$ **i** $x = 2\frac{2}{3}, y = 1$
j $x = -3\frac{1}{5}, y = -4\frac{3}{5}$

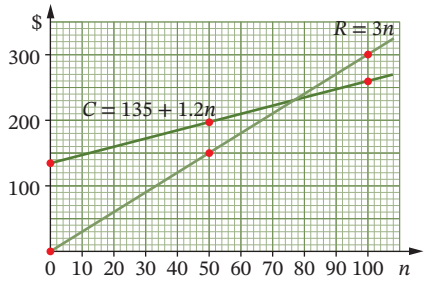
Exercise 11.04

1 285 **2** 680
3 **a** Teacher to check **b** 396
4 55 **5** Tayyab 36, Sejuti 12
6 16 **7** black 37, colour 23
8 Children: \$15.50, Adult: \$21.50
9 Supreme 32, Vegetarian 13
10 Strawberries \$3.95; Blueberries \$3.50
11 **a** Teacher to check.
b 20-cent coins: 485, 50-cent coins: 368
12 **a** Teacher to check.
b $C = 135 + 1.2n$ $R = 3n$

n	0	50	100
C	135	195	255

n	0	50	100
R	0	150	300

c



d $n = 75$

Chapter 12

SkillCheck

1 **a** $x^2 + 8x + 7$ **b** $d^2 - d - 12$
c $t^2 - 25$ **d** $5y^2 + 4y - 12$
e $49y^2 - 42y + 9$ **f** $10p^2 - 27p - 28$
2 **a** $(y + 5)^2$ **b** $(x - 20)(x - 1)$
c $(n + 11)(n - 3)$ **d** $(a - 7)(a - 4)$
e $(m - 12)(m + 7)$ **f** $(p + 9)(p - 6)$

Exercise 12.01

1 **a** $20x$ **b** m^2 **c** t^2 **d** $+ 8h$
e $- 18k$ **f** $+ 80f$ **g** $4d^2, 12d$ **h** $36a^2, 1$

2 **a** $m^2 + 18m + 81$ **b** $u^2 + 6u + 9$
c $y^2 - 12y + 36$ **d** $64 + 16k + k^2$
e $25 - 10h + h^2$ **f** $49 + 14k + k^2$
g $f^2 + 40f + 400$ **h** $q^2 - 22q + 121$
i $100 + 20t + t^2$ **j** $x^2 - 2xw + w^2$
k $a^2 + 2ag + g^2$ **l** $4m^2 - 12m + 9$
m $25x^2 - 60x + 36$ **n** $81a^2 + 36a + 4$
o $9e^2 - 24e + 16$ **p** $25 + 70b + 49b^2$
q $16 - 40p + 25p^2$ **r** $121 - 44c + 4c^2$
s $100g^2 + 60g + 9$ **t** $9k^2 + 66k + 121$
u $25 + 20v + 4v^2$

3 **a** $49h^2 + 28hk + 4k^2$ **b** $64a^2 - 48ay + 9y^2$
c $x^2y^2 + 2xyz + z^2$ **d** $1 + \frac{2}{y} + \frac{1}{y^2}$
e $k^2 - 2 + \frac{1}{k^2}$ **f** $w^2 + 6 + \frac{9}{w^2}$

4 **a** 441 **b** 2025 **c** 841
d 3481 **e** 10 404 **f** 9604

Exercise 12.02

1 **a** $m^2 - 25$ **b** $c^2 - 100$ **c** $a^2 - 144$
d $36 - y^2$ **e** $64 - m^2$ **f** $p^2 - 1$
g $25 - e^2$ **h** $v^2 - 121$ **i** $w^2 - 9$
j $x^2 - 100$ **k** $q^2 - 49$ **l** $81 - g^2$
m $b^2 - 4$ **n** $225 - r^2$ **o** $d^2 - 169$

2 **a** $4h^2 - 9$ **b** $25r^2 - 16$ **c** $25b^2 - 64$
d $16p^2 - 49$ **e** $9 - 64k^2$ **f** $49x^2 - 25$
g $4 - 81m^2$ **h** $81k^2 - 16l^2$ **i** $49n^2 - 64m^2$
j $16g^2 - 25h^2$ **k** $49u^2 - 9w^2$ **l** $121a^2 - 9b^2$
m $a^2 - \frac{1}{a^2}$ **n** $\frac{w^2}{9} - 4$ **o** $1 - \frac{1}{r^2}$

3 **a** $p - 1$ **b** $p + 1$ **c** $p^2 - 1$ **d** 7
4 899
5 **a** 399 **b** 2499 **c** 8099 **d** 6396
6 **a** 29 **b** 47 **c** 3000
d 400 **e** 240 **f** 196

ANSWERS

Exercise 12.03

1 **a** $4m^2 - 1$ **b** $5y^2 + 17y - 12$
c $4k^2 - 28k + 49$ **d** $d^2 + 18d + 81$
e $2e^2 + e - 1$ **f** $25a^2 - 16$
g $-p^2 + 4p - 4$ **h** $100 - 36y^2$
i $h^2 - 6hm + 9m^2$ **j** $2xy + 6x - 3y - 9$
k $121a^2 - 16b^2$ **l** $u^2 - 2 + \frac{1}{u^2}$

2 **a** m^2 **b** $y^2 + 18$ **c** $-3x^2 + 7x + 6$
d $d^2 + 21$ **e** $16k^2 - 48$ **f** $-4xy$
g $27a - 10$ **h** $2f^2 - 8$ **i** $12h + 18$
j $5xy - 6x + 3y + 9$

3 **a** $60a^2$ **b** $n^2 + 4n + 4$ **c** $-2u^2 - 8u$
d $8m^2 + 2n^2$ **e** $x - 2$ **f** $1 - 2b^2$
g $3y^2 + 12y + 14$ **h** $3x^2 + 9$ **i** $34n^2 - 34$
j $-4b^2$

Exercise 12.04

1 **a** $(b + d)(4a + 5c)$ **b** $(y + t)(2x - 5w)$
c $(3c + 4d)(3a + 2b)$ **d** $(10 + x)(x^2 + 3)$
e $3(a + b)(a + c)$ **f** $6(t + p)(r - 3w)$
g $(7 + d)(2e - 3)$ **h** $(h - 2)(k - h)$
i $(3m + p)(n - 2)$ **j** $(9 + q)(p^2 - 3)$
k $(f - 10)(g - h)$ **l** $3(l + n)(3k - 4m)$
m $(2 - p)(p - c)$ **n** $(l - 3)(l^2 + m^2)$
o $(a + y)(x + 1 - k)$ **p** $(a - b + 3q)(p - 2q)$

2 **a** $(w + 3)(w - 3)$ **b** $(y + 6)(y - 6)$
c $(k + 1)(k - 1)$ **d** $(m + 11)(m - 11)$
e $(p + 8)(p - 8)$ **f** $(c + 10)(c - 10)$
g $(2e + f)(2e - f)$ **h** $(a + 3b)(a - 3b)$
i $(4y + 1)(4y - 1)$ **j** $(2 + b)(2 - b)$
k $(5 + e)(5 - e)$ **l** $(1 + 4x)(1 - 4x)$
m $(k + u)(k - u)$ **n** $(7 + 4m)(7 - 4m)$
o $(b + 11d)(b - 11d)$ **p** $(6c + 5k)(6c - 5k)$
q $(4 + 9h)(4 - 9h)$ **r** $(5a + 8m)(5a - 8m)$
s $(10 + 7n)(10 - 7n)$ **t** $(11p + 12q)(11p - 12q)$
u $\left(\frac{1}{2} + 5c\right)\left(\frac{1}{2} - 5c\right)$ **v** $\left(2w + \frac{1}{3}\right)\left(2w - \frac{1}{3}\right)$
w $\left(8h + \frac{3}{2}\right)\left(8h - \frac{3}{2}\right)$ **x** $(1 + mn)(1 - mn)$

3 **a** $2(a + b)(a - b)$ **b** $7(k + 2)(k - 2)$
c $3(1 + 5u)(1 - 5u)$ **d** $x(x + 7)(x - 7)$
e $k(1 + 4k)(1 - 4k)$ **f** $2(5q + 1)(5q - 1)$
g $3(d + 2v)(d - 2v)$ **h** $5t^3(t + 5)(t - 5)$
i $2(ab + 1)(ab - 1)$ **j** $x^2(y + w)(y - w)$
k $12(4f + 3g)(4f - 3g)$
l $5\left(3d + \frac{1}{2}\right)\left(3d - \frac{1}{2}\right)$ or $\frac{5}{4}(6d + 1)(6d - 1)$
m $2(x + 2a)(x - 2a)$ **n** $25(2 + w)(2 - w)$
o $5\left(\frac{1}{2} + 4e\right)\left(\frac{1}{2} - 4e\right)$ or $\frac{5}{4}(1 + 8e)(1 - 8e)$
p $\left(3c + 2\frac{1}{2}\right)\left(3c - 2\frac{1}{2}\right)$ or $\frac{1}{4}(6c + 5)(6c - 5)$

4 **a** $\left(\frac{p}{4} + \frac{x}{5}\right)\left(\frac{p}{4} - \frac{x}{5}\right)$ **b** $\left(x + \frac{1}{3}\right)\left(x - \frac{1}{3}\right)$
c $\left(\frac{v}{8} + \frac{u}{9}\right)\left(\frac{v}{8} - \frac{u}{9}\right)$ **d** $2\left(\frac{y}{3} - \frac{m}{11}\right)\left(\frac{y}{3} + \frac{m}{11}\right)$
e $\left(\frac{4a}{7} + \frac{5b}{2}\right)\left(\frac{4a}{7} - \frac{5b}{2}\right)$ **f** $(g + 3)(g - 3)(g^2 + 9)$
g $(10 + n)(10 - n)(100 + n^2)$
h $y(2x + y)$
i $-(p + 3q)(3p - q)$ **j** $\left(\frac{x}{2} + \frac{y}{6}\right)\left(\frac{x}{2} - \frac{y}{6}\right)$
k $4ab$ **l** $(1 + x)(9 - x)$

Exercise 12.05

1 **a** $3(m + 1)(m + 2)$ **b** $2(y + 2)(y - 1)$
c $5(t - 10)(t + 8)$ **d** $5e^2(e + 8)(e - 3)$
e $x(x - 11)(x + 10)$ **f** $4(b - 7)(b + 6)$
g $4(w + 4)(w - 3)$ **h** $3a(a - 4)(a + 1)$
i $2(e + 5)(e + 4)$ **j** $-(t + 8)(t - 3)$
k $-(u - 7)(u + 6)$ **l** $-(x - 7)(x + 4)$
m $-(b + 4)(b - 3)$ **n** $-(k - 3)(k - 4)$
o $-(x - 5)(x - 7)$

2 **a** $(x + 5)(2x + 1)$ **b** $(m + 3)(4m + 1)$
c $(y + 3)(5y + 2)$ **d** $(2u + 5)(3u + 2)$
e $(w + 15)(2w + 1)$ **f** $(e + 3)(4e + 3)$
g $(2f + 3)(4f + 1)$ **h** $(d + 1)(3d + 2)$
i $(b + 1)(2b + 7)$ **j** $(y + 1)(5y + 11)$
k $(4g + 3)(2g + 5)$ **l** $(3a + 7)(2a + 3)$

3 **a** $(y - 4)(2y - 3)$ **b** $(2k - 3)(5k - 2)$
c $(2e - 3)(3e - 2)$ **d** $(b - 3)(4b - 1)$
e $(w - 3)(6w - 5)$ **f** $(3f - 4)(4f - 3)$
g $(3m - 4)(5m - 2)$ **h** $(3x - 2)^2$
i $(2a - 5)(a - 9)$ **j** $(3y - 8)(4y - 1)$
k $(4d - 3)(2d - 7)$ **l** $(2h - 9)^2$

4 **a** $(y + 1)(5y - 11)$ **b** $(4d - 5)(d + 1)$
c $(2m + 3)(m - 3)$ **d** $(3t - 10)(t + 3)$
e $(6h - 7)(h + 1)$ **f** $(y - 4)(2y + 3)$
g $(2a + 1)(4a - 3)$ **h** $(5u - 4)(3u + 1)$
i $(3c + 1)(3c - 5)$ **j** $(3c - 8)(2c + 3)$
k $(4n - 7)(5n + 2)$ **l** $(4x - 5)(3x + 2)$

5 **a** $(5m + 7)(m - 1)$ **b** $(3g - 4)(2g + 3)$
c $(3p - 2)(p + 2)$ **d** $(7w - 1)(w + 1)$
e $(5y - 1)(y + 3)$ **f** $(3n - 2)(n + 4)$
g $(4b - 3)(b + 3)$ **h** $(4m - 1)(2m + 3)$
i $(3x + 8)(x - 2)$ **j** $(4u + 15)(2u - 1)$
k $(3e + 5)(8e - 3)$ **l** $(4h + 7)(5h - 6)$

6 **a** $(9w - 10)^2$ **b** $4(y + 1)^2$ **c** $(5h - 4)^2$

7 **a** $2(y + 2)(3y - 1)$ **b** $3(g + 4)(2g - 3)$
c $4(2e - 3)(3e + 1)$ **d** $2(a - 2)(4a + 3)$
e $4(u + 2)(3u - 1)$ **f** $-(5q + 3)(5q - 2)$
g $-2(2m - 1)(3m - 2)$ **h** $-(3h + 4)(4h - 5)$
i $6(2c + 3)(2c + 1)$ **j** $-3(w + 1)(2w - 5)$
k $2(2d - 3)(3d + 5)$ **l** $-2(x - 3)(3x - 2)$

8 **a** $(a+1)(2a+3)$ **b** $(2m-5)(6m-1)$
c $(4x-1)(x+3)$ **d** $(w-1)(7w-1)$
e $(h-3)(4h+5)$ **f** $(4x-3)(2x+1)$
g $(r+5)(5r+1)$ **h** $(d-7)(2d-1)$
i $(3n+1)(2n-3)$ **j** $-(3m-2)(3m+4)$
k $-(5c-3)(c+1)$ **l** $(3g+2)(5g+3)$
m $-(4q+3)(2q-5)$ **n** $(x-2)(3x-7)$
o $-(3d-4)(d+4)$ **p** $-2(3y+7)(2y-3)$
q $4(d-2)(7d+3)$ **r** $4(5k+2)^2$

Exercise 12.06

1 **a** $(m-8)^2$ **b** $3d(d-1)$
c $(w-3)(3w+5)$ **d** $(3+h)(k-5)$
e $(5y+8)(5y-8)$ **f** $4(5f+4)(5f-4)$
g $q(q+3-3p)$ **h** $(3-g)(g+1)$
i $4(2n+5)(3n-2)$ **j** $(5r+1)(5r-1)$
k $(b^2+1)(b+1)$ **l** $(2x-5)^2$
m $-(5d-4)(d+1)$ **n** $(a-1)^2(a+1)$
o $2(2+v)(2-v)$ **p** $m(n+3)(n+p)$
q $2(w-6)^2$ **r** $(6h+1)^2$
2 **a** $(3r-8t)(5r+3t)$ **b** $(2d+1)^2$
c $9(g+2k)(g-2k)$ **d** $e(e-5)(e+2)$
e $-20(2p-3q)(3p-2q)$ **f** $7(2x+1)(2x-1)$
g $(a-b)(a+b+4)$ **h** $(c-2)^2(c+2)$
i $(3a-1)(2a+5)$ **j** $(y+7)(y-5)$
k $2(3p+2)^2$ **l** $-(6a-1)(4a+1)$
m $9(x+2)(x-3)$ **n** $(a-3)(2ab-3)$
o $2(a+3)^2$ **p** $(5u-1)^2$
q $(k-3)(4k+7)$ **r** $3(4+w)(4-w)$
s $3(1+3c)(1-3c)$ **t** $(k+4)^2(k-4)$
u $5y(y^2-2y+3)$ **v** $mn(m+2)(m-2)$
w $-2(a+2)(a-2)$ **x** $4(2c-3)(4c+1)$
3 **a** $(3e+7)(4e-5)$ **b** $(2y+5k)(5y-3k)$
c $(2h-3)(2h+3)(4h^2+9)$ **d** $12(w-3)$
e $(d-4)(d^2+4d-1)$ **f** $(u-4-k)(u-4+k)$

Exercise 12.07

1 **a** $x+y$ **b** $\frac{1}{2(t-r)}$ **c** $\frac{b-c}{a}$
d -1 **e** $w-4$ **f** $\frac{5}{d+w}$
g $\frac{k+5}{k-5}$ **h** $3(c-1)$ **i** $\frac{a+1}{m+n}$
j $\frac{y+4}{2}$ **k** $\frac{k+1}{k+4}$ **l** $\frac{4a+5c}{a-c}$
m $\frac{x+2}{x-3}$ **n** $\frac{1-2c}{3c-1}$ **o** $\frac{a+4}{2(p+2)}$
p $\frac{c-5}{u+1}$ **q** $\frac{h+1}{2h}$ **r** $\frac{4e+3}{e(2e+5)}$
2 **a** $\frac{7m+10}{m(m+1)(m+2)}$ **b** $\frac{2w-20}{w(w+3)(w+5)}$
c $\frac{4b-7}{(b-1)(b+2)(b-3)}$ **d** $\frac{-k-2}{k(k+1)(k-1)}$
e $\frac{5h+12}{4h(h+1)}$ **f** $\frac{-4d-1}{(d+2)(d+1)}$
g $\frac{42-5r}{4(r+6)(r-6)}$ **h** $\frac{d^2+3d-6}{d(d+2)(d-2)}$
i $\frac{-k^2+9k-5}{(k+1)(k-1)(k-4)}$ **j** $\frac{3q-1}{(q+1)(q-1)}$
k $\frac{7a+13}{(a-3)(a-5)(a+7)}$ **l** $\frac{-5y-22}{(y+3)(y+2)(y-4)}$
m $\frac{-3e+27}{(e+2)(e+1)(e+7)}$ **n** $\frac{20h-11}{(3h-2)(2h+1)(4h-5)}$
3 **a** $6m$ **b** $1\frac{1}{24}$ **c** $\frac{1}{2}$
d $6k$ **e** $\frac{10}{h+1}$ **f** $\frac{3}{2(a-b)}$
g $\frac{-r}{5(r+t)}$ **h** $\frac{4m}{m-1}$ **i** $\frac{4}{p}$
j $\frac{3}{7}$ **k** $\frac{2}{3(x-2)}$ **l** $3\frac{1}{3}$
m $\frac{(d+1)(d-3)}{6}$ **n** $\frac{f+3}{4(f-3)}$ **o** $\frac{3}{f-2}$

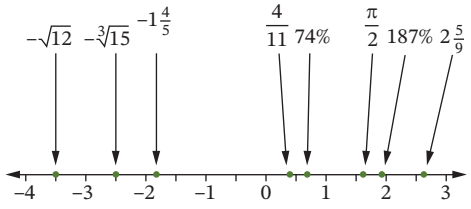
Chapter 13

SkillCheck

1 **a** $25y^2$ **b** $64m^3$ **c** $9x^2$
2 **a** $5x+10$ **b** $4y-12$ **c** $3+6w$
d $10-2y$ **e** $-10a-15$ **f** $k+2k^2$
3 81, 25, 100, 16, 64
4 **a** $m^2+10m+21$ **b** y^2-3y-4
c n^2-5n+6 **d** $6d^2+11d+3$
e $4-17p-15p^2$ **f** $3a^2+17af+10f^2$
g $x^2+8x+16$ **h** y^2-6y+9
i $4k^2+4k+1$ **j** a^2-25
k t^2-49 **l** $9m^2-16$

Exercise 13.01

1 C **2** B
3 $\sqrt{32}, \sqrt{33}, \sqrt{4.9}, \sqrt{52}, \sqrt{200}$
4 **a** R **b** I **c** R **d** R
e R **f** R **g** R **h** R
i I **j** R **k** I **l** I
5 **a** $1\frac{4}{7}, \frac{\pi}{2}, \sqrt{2}$ **b** $2\frac{7}{9}, \sqrt[3]{20}, 2.\dot{6}$
6 **a** -1.8 **b** 0.7 **c** 0.4 **d** -3.5
e -2.5 **f** 2.6 **g** 1.6 **h** 1.9



7 1.4
8 Teacher to check, $\sqrt{5} \approx 2.24, \sqrt{10} \approx 3.16, \sqrt{17} \approx 4.12$

Exercise 13.02

1 **a** 2 **b** 5 **c** 27 **d** 250
e 0.09 **f** 28 **g** 45 **h** 50

2 **a** $5\sqrt{2}$ **b** $3\sqrt{3}$ **c** $2\sqrt{6}$ **d** $3\sqrt{6}$
e $9\sqrt{3}$ **f** $3\sqrt{5}$ **g** $4\sqrt{3}$ **h** $10\sqrt{2}$
i $4\sqrt{6}$ **j** $3\sqrt{7}$ **k** $12\sqrt{2}$ **l** $6\sqrt{3}$
m $5\sqrt{3}$ **n** $7\sqrt{3}$ **o** $4\sqrt{2}$ **p** $11\sqrt{2}$
q $9\sqrt{2}$ **r** $7\sqrt{5}$ **s** $5\sqrt{5}$ **t** $16\sqrt{2}$

3 **a** $25\sqrt{2}$ **b** $6\sqrt{2}$ **c** $12\sqrt{3}$ **d** $56\sqrt{2}$
e $\sqrt{10}$ **f** $\sqrt{3}$ **g** $\frac{\sqrt{7}}{3}$ **h** $6\sqrt{6}$
i $18\sqrt{17}$ **j** $\frac{5\sqrt{5}}{2}$ **k** $3\sqrt{2}$ **l** $3\sqrt{3}$
m $40\sqrt{10}$ **n** $15\sqrt{3}$ **o** $14\sqrt{17}$ **p** $\frac{\sqrt{13}}{3}$

4 B **5** B

6 **a** false **b** false **c** true
d true **e** true **f** false

Exercise 13.03

1 **a** $7\sqrt{7}$ **b** $-5\sqrt{2}$ **c** $6\sqrt{5}$ **d** $4\sqrt{5}$
e 0 **f** $\sqrt{10}$ **g** $8\sqrt{15}$ **h** $-\sqrt{6}$
i $2\sqrt{3}$ **j** $10\sqrt{5}$ **k** $6\sqrt{10}$ **l** $-5\sqrt{3}$

2 **a** $5\sqrt{5}-8$ **b** $13\sqrt{10}+3\sqrt{2}$ **c** $5\sqrt{2}-9\sqrt{3}$
d $7\sqrt{15}+8\sqrt{2}$ **e** $-2\sqrt{5}-3\sqrt{7}$ **f** $2\sqrt{6}-8\sqrt{3}$
g $13\sqrt{11}-\sqrt{3}$ **h** $11\sqrt{7}-6\sqrt{13}$ **i** $-6\sqrt{7}$
j $-3\sqrt{5}$

3 **a** D **b** A

4 **a** $6\sqrt{2}$ **b** $3\sqrt{3}$ **c** $-2\sqrt{5}$
d $-\sqrt{7}$ **e** $5\sqrt{6}$ **f** $7\sqrt{5}$
g $-\sqrt{10}$ **h** $8\sqrt{11}$ **i** $6\sqrt{2}$
j $8\sqrt{3}$ **k** $3\sqrt{2}$ **l** $9\sqrt{2}$
m $11\sqrt{3}$ **n** $-\sqrt{5}$ **o** $-6\sqrt{3}$
p $30\sqrt{3}$ **q** $5\sqrt{7}$ **r** $41\sqrt{2}$
s $5\sqrt{6}$ **t** $29\sqrt{2}$ **u** $-15\sqrt{3}$
v 0 **w** $6\sqrt{2}+2\sqrt{3}$ **x** $12\sqrt{3}+3\sqrt{6}$
y $3\sqrt{2}-6\sqrt{5}$ **z** $4\sqrt{6}$

Exercise 13.04

1 **a** $\sqrt{14}$ **b** $-\sqrt{35}$ **c** $4\sqrt{3}$ **d** 6
e $-5\sqrt{2}$ **f** 45 **g** $15\sqrt{30}$ **h** $-10\sqrt{21}$
i 140 **j** $-30\sqrt{2}$ **k** 36 **l** $-60\sqrt{2}$
m -112 **n** $24\sqrt{6}$ **o** 80 **p** $90\sqrt{6}$
q -396 **r** $160\sqrt{5}$ **s** $216\sqrt{2}$ **t** $-96\sqrt{6}$
u $36\sqrt{5}$ **v** $-60\sqrt{10}$ **w** $252\sqrt{3}$ **x** 144

2 **a** $\sqrt{5}$ **b** $-\sqrt{3}$ **c** $3\sqrt{6}$ **d** $2\sqrt{2}$
e $-\frac{\sqrt{7}}{2}$ **f** 21 **g** 1 **h** 8
i $5\sqrt{3}$ **j** $5\sqrt{2}$ **k** $4\sqrt{3}$ **l** $2\sqrt{6}$
m $2\sqrt{14}$ **n** $-\frac{\sqrt{2}}{4}$ **o** 1 **p** 10
q 4 **r** $-21\sqrt{2}$ **s** 12 **t** 2
u $\frac{2}{3}$

3 **a** 6 **b** 7 **c** 6
d $15y$ **e** x **f** $a\sqrt{a}$

4 C **5** A

6 **a** 2 **b** $4\sqrt{6}$ **c** $\sqrt{30}$
d $\frac{2}{45}$ **e** $14\sqrt{3}$ **f** 2

Exercise 13.05

1 **a** $\sqrt{15}+\sqrt{10}$ **b** $2\sqrt{3}-\sqrt{6}$ **c** $\sqrt{6}+\sqrt{14}$
d $3\sqrt{10}-5$ **e** $6+6\sqrt{6}$ **f** $\sqrt{55}-4\sqrt{11}$
g $42-8\sqrt{7}$ **h** $5\sqrt{5}+75$ **i** $24+3\sqrt{6}$

2 C

3 **a** $10+\sqrt{10}-6\sqrt{5}-3\sqrt{2}$ **b** $7+2\sqrt{7}-\sqrt{21}-2\sqrt{3}$
c $28\sqrt{6}+21+8\sqrt{2}+2\sqrt{3}$ **d** $20+\sqrt{10}$
e $109+10\sqrt{77}$ **f** $72-23\sqrt{6}$
g $16\sqrt{10}+54$ **h** $-16-\sqrt{35}$

4 C

5 **a** $8-2\sqrt{15}$ **b** $9+2\sqrt{14}$ **c** $9-4\sqrt{5}$
d $19+6\sqrt{10}$ **e** $77+30\sqrt{6}$ **f** $179-20\sqrt{7}$
g $38+12\sqrt{10}$ **h** $23+4\sqrt{15}$

6 **a** 1 **b** 22 **c** 8 **d** 2
e 1 **f** 166 **g** 13 **h** -43

7 C

8 **a** $88-30\sqrt{7}$ **b** $21\sqrt{2}-10$ **c** $5\sqrt{35}+29$
d $73+40\sqrt{3}$ **e** 29 **f** $92-12\sqrt{5}$

Exercise 13.06

1 B

2 **a** $\frac{\sqrt{2}}{2}$ **b** $\frac{\sqrt{7}}{7}$ **c** $\frac{\sqrt{3}}{3}$ **d** $\frac{3\sqrt{2}}{2}$
e $\frac{2\sqrt{7}}{7}$ **f** $\frac{\sqrt{2}}{6}$ **g** $\frac{\sqrt{3}}{6}$ **h** $\frac{\sqrt{7}}{28}$
i $\frac{7\sqrt{5}}{15}$ **j** $\frac{\sqrt{10}}{15}$ **k** $\frac{\sqrt{3}}{2}$ **l** $\frac{\sqrt{15}}{4}$

3 A **4** D

5 **a** $\frac{2-\sqrt{2}}{2}$ **b** $\frac{\sqrt{5}-5}{5}$
c $\frac{5\sqrt{2}+\sqrt{6}}{4}$ **d** $\frac{2\sqrt{3}-3\sqrt{2}}{18}$

6 **a** $\frac{2\sqrt{7}+7\sqrt{2}}{14}$ **b** $\frac{\sqrt{10}+5\sqrt{3}}{5}$ **c** $\frac{\sqrt{3}-\sqrt{2}}{2}$

Chapter 14

SkillCheck

1 **a** $x=\pm5$ **b** $m=\pm2$ **c** $x=\pm9$
d $k=\pm\frac{1}{2}$ **e** $u=\pm3\frac{1}{2}$ **f** $w=\pm3$

2 **a** $(4-m)(4+m)$ **b** $(d-11)(d+11)$
c $2y(7-y)$ **d** $5p(2p+5)$
e $5(x-8)(x+8)$ **f** $2(3w-5)(3w+5)$
g $(k+1)(k+4)$ **h** $(y-8)(y-2)$
i $(m-8)(m+7)$ **j** $(u+13)(u-5)$
k $(w-3)(w-7)$ **l** $(x-6)(x+4)$

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3 **a** $y = -2$ **b** $y = -10$
c $y = 5$ **d** $y = -8\frac{3}{4}$

4 **a** $(3a + 1)(a + 3)$ **b** $(5x + 2)(x - 3)$
c $(2y - 5)(3y + 8)$ **d** $(3t - 1)(5t + 4)$
e $(5v + 3)(v - 7)$ **f** $(2y + 5)(4y + 7)$
g $(3h - 4)(5h - 1)$ **h** $(4p - 3)(3p + 5)$
i $(4d + 5)^2$

Exercise 14.01

1 **a** $m = -7$ or -3 **b** $d = 3$ or 7
c $y = -5$ or 3 **d** $k = 0$ or 3
e $t = -7$ or 0 **f** $p = 0$ or 3
g $w = 0$ or $\frac{2}{3}$ **h** $n = -\frac{1}{2}$ or 3
i $a = \frac{1}{2}$ or $\frac{3}{5}$ **j** $x = -\frac{1}{3}$ or $-1\frac{1}{2}$
k $c = 2\frac{1}{2}$ **l** $f = \frac{1}{2}$
m $c = -\frac{1}{3}$ or $-\frac{1}{4}$ **n** $h = -1$ or $\frac{1}{2}$
o $e = \frac{5}{7}$ or 1

2 **a** $y = -2$ or $-1\frac{1}{2}$ **b** $g = -1$ or $-1\frac{1}{2}$
c $d = -1$ or $-\frac{2}{3}$ **d** $t = -2\frac{1}{5}$ or -1
e $p = 1\frac{1}{2}$ or 4 **f** $x = \frac{2}{5}$ or $1\frac{1}{2}$
g $y = \frac{3}{4}$ or $-\frac{1}{2}$ **h** $a = -\frac{1}{2}$ or $1\frac{1}{3}$
i $w = -1\frac{1}{4}$ or 3 **j** $c = 1$ or $-1\frac{2}{5}$
k $e = \frac{1}{4}$ or $-1\frac{1}{2}$ **l** $q = -3$ or $1\frac{2}{3}$
m $g = 2\frac{1}{2}$ **n** $m = -\frac{2}{3}$ or $\frac{5}{6}$
o $w = -4$ or $1\frac{1}{3}$ **p** $y = -3$ or 4
q $f = 6$ **r** $h = -1$ or $\frac{3}{4}$

3 **a** $x = -2\frac{1}{2}$ or 3 **b** $t = -2\frac{1}{2}$ or $\frac{1}{2}$
c $u = -\frac{1}{8}$ or -5 **d** $m = \frac{1}{7}$ or 1
e $p = -4$ or 7 **f** $e = -1$ or 5
g $t = 1\frac{1}{2}$ or 5 **h** $d = -2\frac{1}{3}$ or $\frac{1}{2}$
i $h = \pm 5$ **j** $f = 0$ or $\frac{1}{2}$
k $w = \frac{1}{6}$ or 3 **l** $a = -2$ or $\frac{1}{3}$

4 8

Exercise 14.02

1 **a** $x^2 + 2x + 1 = (x + 1)^2$
b $p^2 - 6p + 9 = (p - 3)^2$
c $m^2 - 8m + 16 = (m - 4)^2$
d $k^2 + 4k + 4 = (k + 2)^2$
e $y^2 - 7y + \frac{49}{4} = \left(y - \frac{7}{2}\right)^2$
f $w^2 - 3w + \frac{9}{4} = \left(w - \frac{3}{2}\right)^2$
g $x^2 + x + \frac{1}{4} = \left(x + \frac{1}{2}\right)^2$
h $h^2 - 5h + \frac{25}{4} = \left(h - \frac{5}{2}\right)^2$
i $a^2 + \frac{7}{2}a + \frac{49}{16} = \left(a + \frac{7}{4}\right)^2$
j $v^2 + \frac{5}{3}v + \frac{25}{36} = \left(v + \frac{5}{6}\right)^2$

2 **a** $d = -3 + \sqrt{7}, -3 - \sqrt{7}$
b $x = 5 + \sqrt{5}, 5 - \sqrt{5}$
c $p = -1 + \sqrt{10}, -1 - \sqrt{10}$
d $y = 1 + \sqrt{2}, 1 - \sqrt{2}$
e $m = \frac{1 + 2\sqrt{5}}{2}, \frac{1 - 2\sqrt{5}}{2}$
f $t = \frac{-2 + 3\sqrt{3}}{3}, \frac{-2 - 3\sqrt{3}}{3}$
g $c = \frac{-2 + \sqrt{42}}{2}, \frac{-2 - \sqrt{42}}{2}$
h $w = \frac{6 + \sqrt{82}}{2}, \frac{6 - \sqrt{82}}{2}$
i $n = \frac{-2 + \sqrt{7}}{3}, \frac{-2 - \sqrt{7}}{3}$
j $e = \frac{3 + \sqrt{71}}{2}, \frac{3 - \sqrt{71}}{2}$
k $d = 2 + \sqrt{5}, 2 - \sqrt{5}$
l $x = \frac{3 + 4\sqrt{2}}{4}, \frac{3 - 4\sqrt{2}}{4}$

3 **a** $h = -1 \pm \sqrt{6}$ **b** $r = 1 \pm \sqrt{2}$
c $m = -3 \pm \sqrt{7}$ **d** $w = -6, 10$
e $a = 5 \pm \sqrt{30}$ **f** $y = -4 \pm \sqrt{19}$
g $p = -6 \pm \sqrt{41}$ **h** $x = 2 \pm \sqrt{2}$
i $u = -7, -2$ **j** $d = \frac{-1 \pm \sqrt{29}}{2}$
k $c = \frac{9 \pm \sqrt{73}}{2}$ **l** $e = \frac{-5 \pm \sqrt{17}}{2}$
m $y = \frac{3 \pm \sqrt{41}}{2}$ **n** $b = \frac{1 \pm \sqrt{21}}{2}$
o $q = \frac{3 \pm \sqrt{5}}{2}$ **p** $g = \frac{-7 \pm \sqrt{73}}{4}$
q $x = -3\frac{1}{2}, 1$ **r** $f = -2 \pm \sqrt{6}$

4 **a** $x = -11.20$ or -0.80
b $m \approx -0.43$ or 16.43
c $g \approx -4.65$ or 0.65
d $h \approx 1.27$ or -2.77
e $w \approx 1.27$ or -0.47
f $y \approx 1.14$ or -1.47
g $p = -2$ or $1\frac{1}{3}$
h $e \approx 1.13$ or -0.88
i $n = 1$ or -2.5

Exercise 14.03

1 **a** $x = -3 \pm \sqrt{7}$ **b** $m = \frac{5 \pm \sqrt{37}}{2}$

c $w = 4 \pm \sqrt{13}$ **d** $k = \frac{-3 \pm \sqrt{29}}{2}$

e $y = 2 \pm \sqrt{5}$ **f** $p = \frac{-1 \pm \sqrt{21}}{2}$

g $u = \frac{7 \pm \sqrt{61}}{2}$ **h** $a = \frac{-3 \pm \sqrt{65}}{4}$

i $q = \frac{1}{5}, 1$ **j** $c = \frac{-1 \pm \sqrt{7}}{3}$

k $e = \frac{5 \pm \sqrt{57}}{8}$ **l** $x = \frac{-4 \pm \sqrt{10}}{3}$

m $d = \frac{2 \pm \sqrt{14}}{2}$ **n** $a = \frac{5 \pm \sqrt{31}}{3}$

o $t = -2\frac{1}{2}, 1$ **p** $y = -\frac{2}{3}, -2$

q $k = \frac{5}{6}, 1$ **r** $n = \frac{5 \pm \sqrt{113}}{4}$

2 **a** $y = \frac{9 \pm \sqrt{141}}{10}$ **b** $m = \frac{-1 \pm \sqrt{22}}{3}$

c $x = \frac{3 \pm \sqrt{41}}{8}$ **d** $k = -2 \pm \sqrt{5}$

e $m = \frac{3 \pm \sqrt{21}}{6}$ **f** $g = \frac{-1 \pm \sqrt{6}}{5}$

g $h = \frac{9 \pm \sqrt{17}}{4}$ **h** $w = \frac{1 \pm \sqrt{7}}{3}$

i $p = \frac{2 \pm \sqrt{7}}{3}$ **j** $u = \frac{-2 \pm \sqrt{14}}{5}$

k $a = \frac{-2 \pm \sqrt{58}}{6}$ **l** $y = \frac{-3 \pm \sqrt{89}}{4}$

3 **a** $k \approx 8.89, 0.11$ **b** $c \approx 1.41, -1.41$

c $m \approx 2.65, -2.65$ **d** $n \approx 3.19, 0.31$

e $p \approx 0.85, -2.35$ **f** $w \approx 0.30, -1.13$

g $x \approx 2.39, 0.28$ **h** $h \approx 3.83, -1.83$

i $x \approx 1.62, -0.62$ **j** $a \approx 14.52, 2.48$

k $v \approx 1.48, -1.48$ **l** $c \approx 2.31, 0.69$

m $t \approx 8.09, -3.09$ **n** $x \approx 4.27, 7.73$

o $d \approx 3.31, -1.81$

Exercise 14.04

1 **a** $x = 0$ or $\frac{3}{2}$ **b** $m = \pm 6$ **c** $w = \frac{5 \pm \sqrt{37}}{6}$

d $u = 4$ or -3 **e** $k = \frac{3}{5}$ or $-\frac{7}{2}$ **f** $a = \pm\frac{4}{5}$

g $x = 9$ or -7 **h** $n = 3 \pm \sqrt{19}$ **i** $a = -3 \pm \sqrt{5}$

j $d = -3$ or 1 **k** $t = 0$ or 4 **l** $y = \frac{3 \pm \sqrt{11}}{2}$

m $x = -10$ or 3 **n** $a = \frac{2}{3}$ or $-\frac{3}{2}$ **o** $x = \frac{1 \pm \sqrt{10}}{6}$

2 **a** $y = 0$ or 5 **b** $a = \pm 12$

c $u \approx 5.32, -1.32$ **d** $x = 3$

e $x = 9, -2$ **f** $x \approx -0.53, -9.47$

g $a \approx 2.14, -1.64$ **h** $w \approx 2.77, -1.27$

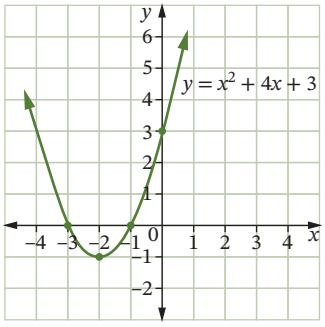
i $m \approx 1.05, -0.65$ **j** $k \approx -1.10, 4.10$

k $x = \frac{9}{2}$ or -4 **l** $h = -6, -1$

Exercise 14.05

1 **a** **i**

X	−3	−2	−1	0	1	2	3
Y	0	−1	0	3	8	15	24

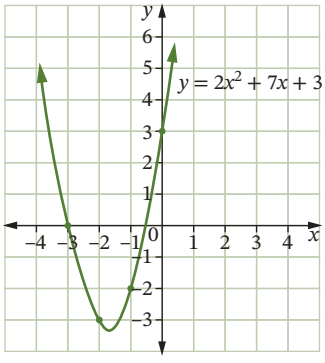


ii -3 and -1 **iii** 3

iv $x = -3$ and -1, same as x-intercepts

b **i**

X	−3	−2	−1	0	1	2	3
Y	0	−3	−2	3	12	25	42



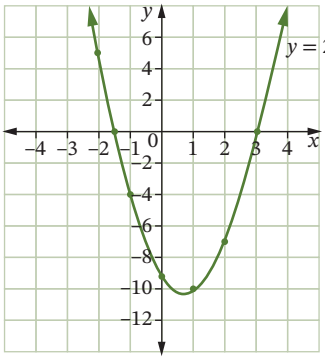
ii -3 and $-\frac{1}{2}$ **iii** 3

iv $x = -3$ and $-\frac{1}{2}$, same as x-intercepts

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c i

X	−3	−2	−1	0	1	2	3
Y	18	5	−4	−9	−10	−7	0

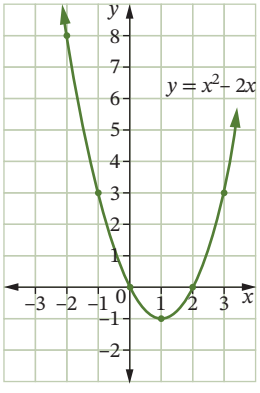


ii $-1\frac{1}{2}$ and 3 **iii** −9

iv $x = -1\frac{1}{2}$ and 3, same as x-intercepts

d i

X	−3	−2	−1	0	1	2	3
Y	15	8	3	0	−1	0	3

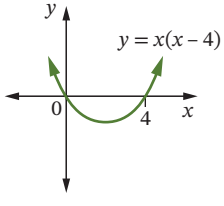


ii 0 and 2 **iii** 0

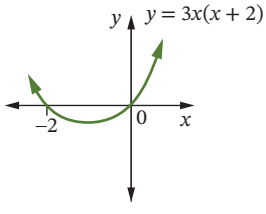
iv $x = 0$ and 2, same as x-intercepts

2 a −5 **b** 3 **c** 0

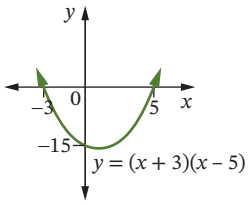
3 a



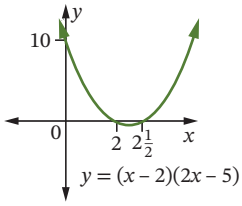
b



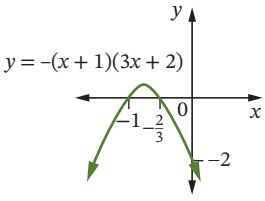
c



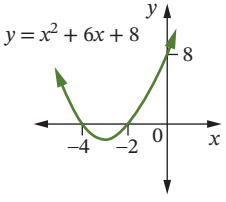
d



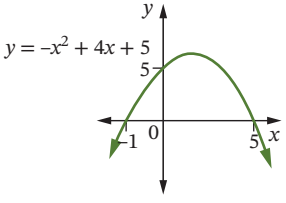
e



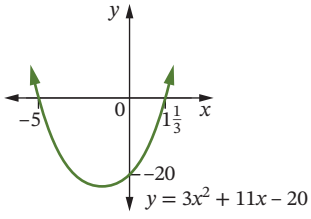
f



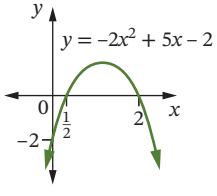
g



h



i



ANSWERS

Exercise 14.06

1 **a** $x = 3$ **b** $x = 0$ **c** $x = 2\frac{1}{2}$

d $x = -1$ **e** $x = 1\frac{1}{2}$ **f** $x = 3$

2 **a** **i** $x = 3$ **ii** $(3, -1)$

b **i** $x = 5$ **ii** $(5, 16)$

c **i** $x = 1$ **ii** $(1, 9)$

d **i** $x = 4$ **ii** $(4, 25)$

e **i** $x = \frac{1}{2}$ **ii** $\left(\frac{1}{2}, -24\frac{3}{4}\right)$

f **i** $x = -4$ **ii** $(-4, -80)$

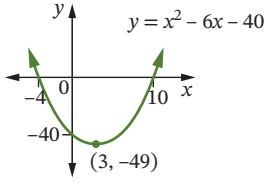
g **i** $x = 4$ **ii** $(4, 48)$

h **i** $x = -\frac{1}{4}$ **ii** $\left(-\frac{1}{4}, -1\frac{1}{4}\right)$

i **i** $x = -\frac{1}{6}$ **ii** $\left(-\frac{1}{6}, 1\frac{1}{4}\right)$

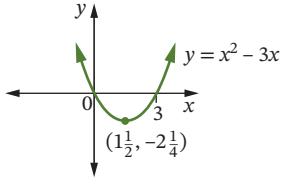
3 **a** **i** $-4, 10$ **ii** -40 **iii** $x = 3$

iv $(3, -49)$ **v** concave up



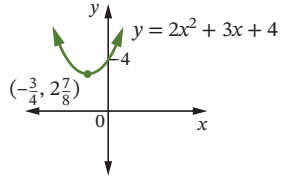
b **i** $0, 3$ **ii** 0 **iii** $x = 1\frac{1}{2}$

iv $\left(1\frac{1}{2}, -2\frac{1}{4}\right)$ **v** concave up



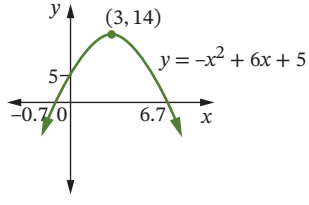
c **i** No x-intercepts. **ii** 4 **iii** $x = -\frac{3}{4}$

iv $\left(-\frac{3}{4}, 2\frac{7}{8}\right)$ **v** concave up



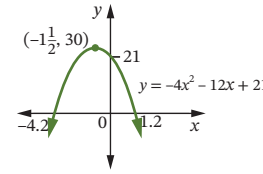
d **i** $-0.7, 6.7$ **ii** 5 **iii** $x = 3$

iv $(3, 14)$ **v** concave down



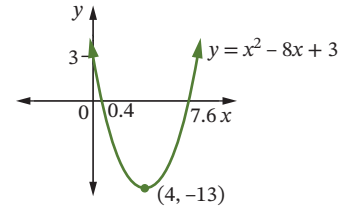
e **i** $-4.2, 1.2$ **ii** 21 **iii** $x = -1\frac{1}{2}$

iv $(-1\frac{1}{2}, 30)$ **v** concave down



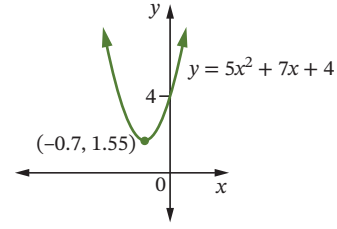
f **i** $0.4, 7.6$ **ii** 3 **iii** $x = 4$

iv $(4, -13)$ **v** concave up



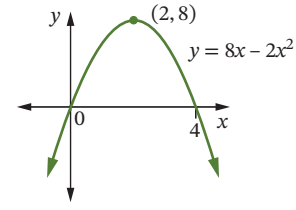
g **i** No x-intercepts. **ii** 4 **iii** $x = -0.7$

iv $(-0.7, 1.55)$ **v** concave up



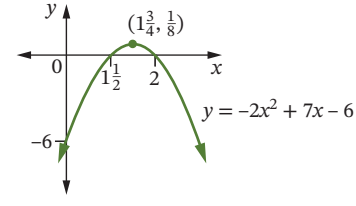
h **i** $0, 4$ **ii** 0 **iii** $x = 2$

iv $(2, 8)$ **v** concave down



i **i** $1\frac{1}{2}, 2$ **ii** -6 **iii** $x = 1\frac{3}{4}$

iv $\left(1\frac{3}{4}, \frac{1}{8}\right)$ **v** concave down



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Exercise 14.07

1 **a** $(3, 9), (2, 4)$ **b** $(0, 0), (4, 32)$

c $(4, 26), (-2, 14)$ **d** $(-1, 5), (1\frac{1}{5}, 7\frac{1}{5})$

e $(-2, 18), (\frac{1}{3}, 11)$ **f** $(-2, -22), (5, 20)$

g $(7, 39)$ **h** $(2, 0), (-1, 3)$

2 **a** $(1, -2), (2, -1)$ **b** $(0, 1), (1, 0)$

c $(0, 3), (3, 0)$ **d** $(0, 5), (-5, 0)$

e $(1, 7), \left(-1\frac{2}{5}, -5\right)$ **f** $(3, 5), \left(-2\frac{1}{2}, -6\right)$

g $(-1, 3), \left(-1\frac{1}{2}, 2\right)$

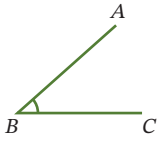
3 There are no solutions for x when solving $-3x - 1 = \frac{2}{x}$, which becomes the quadratic equation $3x^2 + x + 2 = 0$, as $x = \frac{-1 \pm \sqrt{-23}}{6}$. Also, when both equations are graphed, the line $y = -3x - 1$ with gradient -3 and y-intercept -1 does not cross the hyperbola $y = \frac{2}{x}$.

GLOSSARY AND INDEX

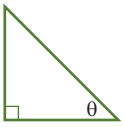
A

absolute error The difference between the actual value and the measured value of a quantity, as a positive value. Also a measure of the precision of an instrument, calculated as 0.5 of the smallest unit on the instrument. (pp. 92, 93)

acute angle An angle between 0° and 90°, such as the marked angle in the diagram.



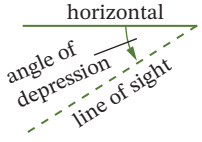
adjacent side In a right-angled triangle, the side 'next to' the given angle, leading to the right angle. (p. 338)



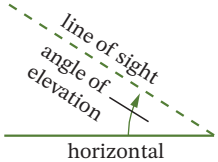
adjacent side
(adjacent to θ)

allowable (tax) deduction A part of a person's yearly income that is not taxed, such as work-related expenses or donations to charities. All deductions are subtracted from yearly income to determine **taxable income**. (p. 12)

angle of depression The angle of looking **down**, measured from the horizontal. (p. 347)



angle of elevation The angle of looking **up**, measured from the horizontal. (p. 347)

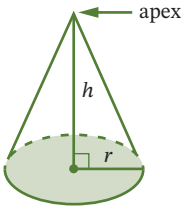


annual leave loading (or **holiday loading**) Extra payment to a worker during annual leave based on 17.5% of 4 weeks' pay. (p. 7)



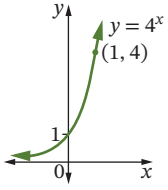
annulus A ring shape between 2 different-sized circles with the same centre. (p. 101)

apex The pointed top of a **pyramid** or **cone**. (p. 114)



association or **correlation** A relationship between 2 variables, often shown on a scatterplot. (p. 220)

asymptote A line that a curve gets very close to but never touches, for example, the x-axis is an asymptote of the exponential curve. (pp. 310, 316)



average See **mean**.

axis of symmetry See **line symmetry**. (pp. 446, 570, 572)

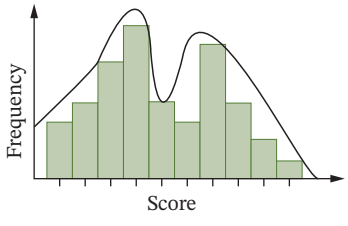
B

base (in index notation) When a number is raised to a power, the number raised is the base. In the expression 3^5, the 3 is called the base. (p. 156)

bearing The angle used to show the direction of one location from a given point. See also **compass bearing** and **three-figure bearing**. (p. 350)

bias In statistics, something that causes a sample to not truly represent the population. (pp. 233, 238)

bimodal distribution A statistical distribution that has 2 peaks. (p. 184)



binomial expression An algebraic expression with 2 terms, for example, $x + 9$, $2y - 12$. (p. 169)

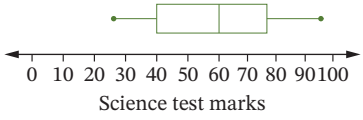
binomial product An algebraic expression showing 2 or more binomials multiplied together, for example, $(x + 9)(3x - 4)$. (pp. 169, 521)

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bivariate data Data that measures 2 variables, such as a person's height and arm span, represented by an ordered pair of values that can be graphed on a **scatterplot** for analysis. (p. 220)

bonus Extra pay for achieving a high quality or volume of work, such as meeting an important quota, goal or deadline.

box plot (or **box-and-whisker plot**) A graph that shows the quartiles of a set of data and the highest and lowest values; the 'box' contains the middle 50% of values while the 'whiskers' extend to the 2 extremes. (p. 201)



C

capacity The amount of material (usually liquid) that a container can hold, measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML). (p. 128) See also **volume**.

categorical data Data that can be classified into categories, such as hair colour, favourite radio station or postcode. Data that is not **numerical**.

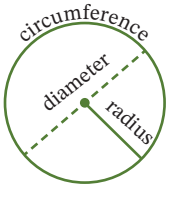
causation When one variable causes the other variable in an association, where a change in one variable directly affects the other. (p. 222)

census A survey of the entire **population** of people or items, not just a survey of a **sample**.

chance experiment An activity or process that involves chance, for example, rolling a die or tossing a coin. (p. 418)

circuit A closed **trail** in a network, a walk that has no repeated edges and starts and ends at the same vertex. (pp. 466, 479)

circumference The perimeter of a circle. $C = \pi d$ or $C = 2\pi r$, where C is the circumference, π is pi, d is the diameter and r is the radius. (p. 110)



cluster A group of data values that are bunched or close together. (p. 184)

coefficient The number in front of a variable in an algebraic term. For example, the coefficient of x in $2x - 5$ is 2. (pp. 173, 297, 511, 528)

commission Pay earned by salespeople and agents, calculated as a percentage of the value of items sold or income made. (p. 7)

compass bearing A bearing that is an acute angle measured from North or South, towards East or West, for example, S25°E. See also **bearing** and **true bearing**. (p. 351)

compass rose A cross-shaped diagram that shows the direction of north. (p. 351) See also **compass bearing**.



complementary event All the outcomes that are *not* the event; the 'opposite' event. For example, the complementary event to rolling 1 on a die is rolling a number that is not 1.

complementary set The set of items that are not in a given set. If A is the set of odd numbers, then the complementary set $\overline{A}$ is the set of numbers that are not odd. (p. 406)

composite shape A shape made up of 2 or more basic shapes. (p. 100)

compound event A chance event that is a combination of 2 or more simple events, for example, 'female or left-handed'. (pp. 397, 407, 408)

compound interest Interest that is calculated as a percentage of the original principal and the accumulated interest. (p. 21) See also **simple interest**.

cone A solid shape with a circular base, a curved surface and an apex. (p. 114)

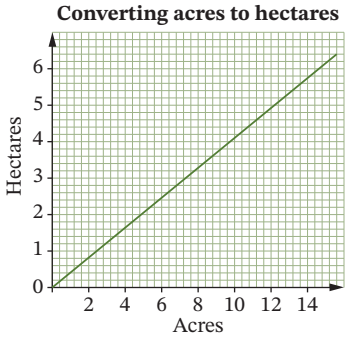
consecutive numbers Any series of integers that follow each other in order; for example, 8, 9 and 10. (p. 264)

constant of variation See **direct proportion**. (pp. 283, 287)

constant term The term in an algebraic expression that is a number only, with no variable. For example, the constant term in $x^2 - 4x + 6$ is 6. (p. 173)

continuous data Numerical data that can be measured on a smooth scale without any gaps, and can take on a full range of values, such as the height of people. Continuous data is measured on a scale without 'gaps', unlike **discrete data**.

conversion graph A line graph for converting between different units or currencies, for example, miles to kilometres, or Australian dollars to US dollars. It usually contains one straight line that begins at the origin (0, 0). (p. 293)



correlation See **association**. (pp. 220, 221)

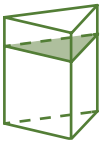
cosine A ratio in a right-angled triangle:

$$\cos\theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}}$$

where θ is an angle. (p. 338) See also **sine** and **tangent**.

cosine rule A rule that relates the 3 sides and one of the angles of any triangle: $c^2 = a^2 + b^2 - 2ab\cos C$. (pp. 338, 371)

cross-section A 'slice' of a solid cut across it rather than along it. (p. 105)



cumulative frequency A progressive or running total of frequencies, the sum of frequencies of a particular data value and all values below it.

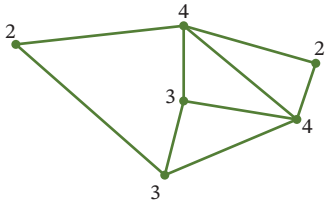
curved surface area The area of the curved surface of a solid such as a cylinder, cone or sphere. (pp. 110, 118)

cycle A closed **path** in a network, a walk that has no repeated vertices (or edges) and starts and ends at the same vertex. (p. 466)

D

data Information, a collection of facts. (pp. 184, 237)

degree (of a vertex) The number of edges coming out of the vertex in a network. Loops are counted as 2 edges. (p. 465)



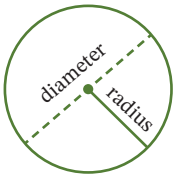
denominator The number below the line in a fraction. The denominator of $\frac{2}{3}$ is 3. (p. 552)

dependent event An event whose outcome (and probability) depends upon the outcome of another event; for example, the colour of the second marble drawn from a bag depends on the colour of the first marble drawn. (p. 431)

dependent variable A variable that depends on another variable for its value. For example, if y depends on x, then the dependent variable is y and the **independent variable** is x. (p. 223)

depreciation The decrease in the value of items over time due to ageing or use. (p. 34)

diameter An interval joining 2 points on the circumference and passing through the centre of a circle, or the length of that interval. The diameter is double the **radius**. (p. 141)

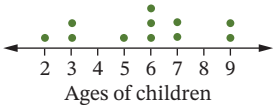


difference of 2 squares An algebraic expression of the form $a^2 - b^2$, that can be factorised into $(a + b)(a - b)$. For example, $x^2 - 25 = (x + 5)(x - 5)$. (pp. 523, 550)

direct variation (or **direct proportion**) A relationship between 2 variables of the form $y = kx$, where k is a constant called the **constant of variation**. For example, if $y = 8.5x$, then y is directly proportional to x. (pp. 283, 293)

discrete data Numerical data that are counted or measured, only taking on distinct, separate values, such as the number of children in a family (0, 1, 2, ...). Discrete data has a scale with 'gaps' or jumps, unlike **continuous data**.

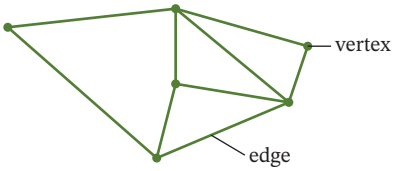
dot plot A graph that uses dots above a number line to show the frequencies of data values. (p. 184)



double time Overtime pay that is calculated at 2 times the normal pay rate. (p. 6)

E

edge (of a network) The connecting lines that join vertices together. (p. 465)



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element The items that are the members of a **set**. (p. 406)

elimination method A method of solving simultaneous equations that involves combining them to eliminate one of the variables. (p. 509)

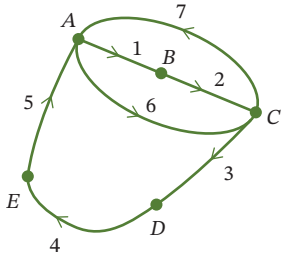
equation A mathematical statement that 2 quantities are equal. For example, $8 + 2 = 10$ or $3b - 7 = 5$. (p. 266)

equilateral triangle A triangle with all 3 sides equal (and all angles 60°).

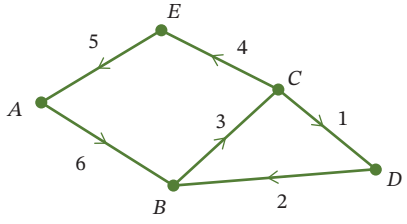


Euler's formula (for planar graphs) $v - e + f = 2$, where v is the number of vertices, e is the number of edges and f is the number of faces. (p. 473)

Eulerian circuit A walk in a network in which every edge is used only once and begins and ends at the same vertex. (pp. 479, 480)



Eulerian trail A walk in a network in which every edge is used only once. (pp. 479, 486)



event In probability, a result involving one or more outcomes. For example, when rolling a die, the event 'rolling an even number' contains the 3 outcomes {2, 4, 6}. (p. 395)

expected frequency The expected number of times an event will occur over repeated trials, calculated by multiplying the probability of the event by the number of trials. (p. 396)

expected probability See **theoretical probability**. (p. 395)

exponential curve The graph of an exponential equation $y = a^x$. (p. 316) See **asymptote** for a diagram.

exponential decay An decrease that can be modelled by an exponential equation $y = b(a^{-x})$, where $a > 1$, decreasing quickly at first and then more slowly. (p. 319)

exponential equation An equation of the form $y = a^x$, where a is a positive constant and the variable x is a power, for example, $y = 4^x$. (p. 316)

exponential growth An increase that can be modelled by an exponential equation $y = b(a^x)$, where $a > 1$, increasing slowly at first and then more quickly. (pp. 318, 319)

F

face (of a network) A region bounded by the edges of the network, and the region outside the network (called the **infinite face**). (p. 467)

factor or **divisor (of a number)** A value that divides evenly into a given number. For example, the factors of 15 are 1, 3, 5 and 15. (p. 161)

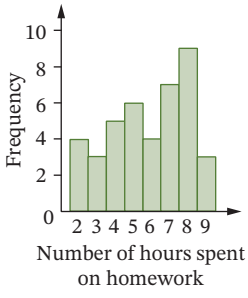
five-number summary For a set of numerical data, the lowest value, lower quartile, median, upper quartile, highest value; used to draw a **boxplot**. (p. 201)

formula (plural: **formulas** or **formulae**) A rule written as an algebraic equation, using variables. The formula for the area of a triangle is $A = \frac{1}{2}bh$. (p. 266)

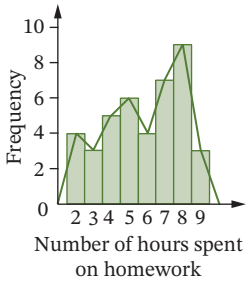
fraction A number written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

frequency The number of times an event occurs in repeated trials of a probability experiment, or the number of times a value appears in a set of data. (pp. 395, 396)

frequency histogram A column graph that shows the frequencies of numerical data. There are no spaces between the columns, and the graph looks like a row of office buildings. (p. 184)



frequency polygon A line graph that shows the frequencies of numerical data. It can be made by joining the midpoints of the column tops of a histogram. The graph looks like a mountain. (p. 197)



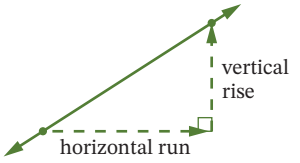
frequency (distribution) table A table listing the frequency of each value in a set of data, with columns for Score (x), Frequency (f) and sometimes Tally and fx.

future value The final amount of an investment after all interest is added. (p. 24)

G

general form of a linear equation The equation of a straight line $ax + by + c = 0$, where a, b and c are integers and a is positive. (p. 69)

gradient The steepness of a line or interval, measured by the fraction $\frac{\text{rise}}{\text{run}}$. (pp. 46, 64, 76)



gradient–intercept form of a linear equation The equation of a straight line $y = mx + c$, where m is the **gradient** and c is the y-intercept. (pp. 64, 69)

graph Another name for a network. The study of networks is called **graph theory**. (p. 465)

gross pay Pay received before tax and other deductions are taken out. (p. 13)

H

hemisphere Half a **sphere**. (p. 119)



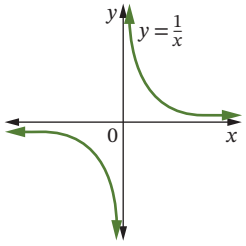
highest common factor (HCF) or **greatest common divisor (GCD)** The largest factor shared by 2 or more numbers or algebraic terms. For example, the HCF of 36 and 8 is 4 and the HCF of $6xy$ and $12y^2$ is $6y$. (p. 164)

hire-purchase See **term payments**. (p. 29)

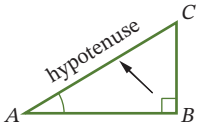
horizontal Going across, sideways, flat. (p. 46)



hyperbola The graph of $y = \frac{k}{x}$, where k is a constant, a curve with 2 branches. (p. 310)



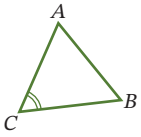
hypotenuse The longest side of a right-angled triangle, opposite the right angle. (p. 338)



I

image A transformed shape after it has been enlarged or reduced. (pp. 448, 458)

included angle The angle between 2 given sides of a shape. For example, the included angle for sides AC and CB in this triangle is $\angle C$. (p. 376)



income tax A tax paid to the government based on the size of a person's income. (p. 12)

independent event An event whose outcome (and probability) does not depend upon the outcome of another event; for example, the number rolled on the second die does not depend on the number rolled on the first die. (pp. 431, 432)

independent variable A variable whose value does not depend on another variable. For example, if y depends on x, then the **dependent variable** is y and the independent variable is x. (p. 223)

index (Plural: **indices**, pronounced 'in-da-sees') See **power**. (p. 156)

index law An algebraic rule for simplifying expressions involving powers of the same base, for example, $a^m \times a^n = a^{m+n}$. (p. 156)

initial At the start or beginning, original. (p. 34)

initial value The original value or purchase price of an asset that is depreciating. (p. 34)

instalment See **repayment**. (p. 29)

interquartile range (IQR) The difference between the upper quartile and lower quartiles,

$$\text{IQR} = Q_3 - Q_1$$

representing the middle 50% of values. (pp. 189, 198)

intersection ($\cap$) The set of elements that are in common from 2 or more sets. $A \cap B$ is the intersection of sets A and B and contains all the elements that both A and B have in common. (pp. 408, 409)

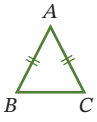
interval A section of a line with a definite length, such as AB shown. (p. 46)



inverse variation (or **inverse proportion**) A relationship between 2 variables of the form $y = \frac{k}{x}$, where k is a constant; for example, if $y = \frac{50}{x}$, then y is inversely proportional to x. (p. 287)

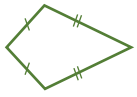
irrational number A number such as π or $\sqrt{2}$ that cannot be expressed as a fraction (rational number). In decimal form, its digits run endlessly without repeating. (p. 539) See also **rational number** and **real number**.

isosceles triangle A triangle with 2 equal sides (and 2 equal angles opposite those sides).



K

kite A quadrilateral with 2 pairs of equal adjacent sides.



L

LHS The left-hand side (of an equation). (p. 69)

like terms Algebraic terms that have exactly the same variables. For example, $5xy$ and $2xy$ are like terms, $3xy$ and $4x^2$ are not like terms. (p. 164)

line of best fit A straight line drawn through the points on a scatterplot that best describes the bivariate data. (p. 226)

line symmetry A plane shape has line symmetry if it can be folded so that one half fits exactly on top of the other half; that is, one half is the mirror-image of the other. The folding line is called the **axis of symmetry** (plural: **axes**). An equilateral triangle has 3 axes of symmetry. See also **rotational symmetry**. (p. 446)

linear equation A formula whose graph is a straight line, or an equation involving a variable that is not raised to a power, such as $2x + 9 = 17$. (pp. 58, 59)

M

mean The average of a set of data, represented by $\bar{x}$, calculated by dividing the sum of the values by the number of values. (p. 197)

measure of centre An average, middle or typical value of a set of data. The 3 measures of centre are the **mean**, **median** and **mode**. (p. 214)

measure of spread A statistical value that describes how the values in a data set are spread, for example, **range** or **interquartile range**. (p. 189)

median The middle value when the values of a data set are arranged in order. If the number of values is even, then the median is the average of the 2 middle values. (p. 188)

midpoint The point in the middle of an interval or halfway between 2 given points. (p. 46)

minute (symbol ′) A measure of angle size. $\frac{1}{60}$ of a degree. $1° = 60′$.

mode The most common or frequent value(s) in a set of data. (p. 184)

mutually exclusive events Events or categories that have no items in common. (p. 409)

N

negatively skewed See **skewed distribution**. (p. 184)

net pay Pay received after deductions from gross pay; 'take-home' pay. (p. 13)

network (or graph) A collection of points called vertices connected together by lines called edges. (p. 465)

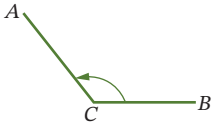
numerator The number above the line in a fraction. The numerator of $\frac{2}{3}$ is 2. (p. 552)

numerical data Data that can be measured or counted, such as a person's height or the number of goals scored. Data that is not **categorical**.

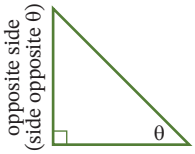
O

observed probability An estimate of expected or theoretical probability; the **relative frequency** of an event in repeated trials of an experiment, found using the formula $P(E) = \frac{\text{frequency of } E}{\text{total frequency}}$ (p. 395)

obtuse angle An angle greater than 90° but less than 180°. (p. 368)



opposite side In a right-angled triangle, the side directly facing the given angle. (p. 338)



outcome In probability, the result of a situation or experiment. For example, when rolling a die, one possible outcome is rolling a 4. (p. 395)

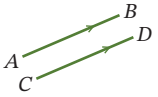
outlier An extreme data value that is much different to the other values in a set. (p. 198)

overtime Time worked beyond normal working hours, such as at night or on weekends, at a higher rate of pay. (p. 6)

P

parabola A U-shaped curve that is the graph of a quadratic equation such as $y = x^2$. (pp. 296, 566)

parallel lines Lines that point in the same direction and do not intersect. $AB \parallel CD$ means 'AB is parallel to CD'. (pp. 54, 58)



parallelogram A quadrilateral in which the opposite sides are parallel.



path A walk in a network that has no repeated vertices (or edges). (p. 466)

PAYG (Pay As You Go) tax Income tax deducted from your pay each payday by your employer. (p. 13)

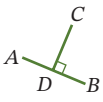
per annum (p.a.) Per year. (p. 16)

percentage error The absolute error of a measurement as a percentage of the actual value, the relative error expressed as a percentage. (p. 93)

perfect square A square number or an algebraic expression that represents one; for example, 64, $(x + 9)^2$, $(a - b)^2$. (p. 521)

perimeter The distance around the outside of a shape. The sum of the lengths of its sides.

perpendicular lines Lines that intersect to form a right angle. $AB \perp CD$ means 'AB is perpendicular to CD'. (pp. 54, 58)



piecework Earnings based on the number of items processed, made or delivered, paid at a rate per item rather than on the number of hours worked. (p. 7)

planar graph A connected network (graph) where the edges do not cross (overlap) (p. 471)

population In statistics, all of the items being studied, the entire group. (p. 233)

positively skewed See **skewed distribution**. (p. 184)

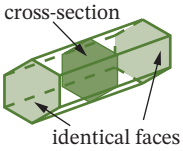
power (or **index**) The number of times a base is multiplied by itself. In 2^5, the power is 5. Also called the *exponent*. (p. 156)

precision 0.5 of the smallest unit on the instrument. (p. 92)

present value or **principal** An amount of money invested or borrowed, on which interest is calculated. (p. 24)

present value or **principal** The starting value of an investment. (p. 24)

prism A solid shape with identical cross-sections and straight sides. (p. 105)



probability The chance of an event occurring, measured as a fraction, decimal or percentage between 0 and 1. (pp. 395, 431)

product The result of a multiplication. The product of 7 and 3 is 21.

pronumeral Another name for **variable**.

pyramid A solid with a polygon for a base and triangular faces that meet at a point called the **apex**. (p. 114)

Q

quadrant (of a circle) A sector that is a quarter of a circle, containing a right angle. (p. 101)



quadratic expression An algebraic expression in which the highest power of the variable is 2; for example, $x^2 - 5x + 7$, $x^2 - 15$, $2x^2 - 3x + 9$ and $-4x^2 + 7x$. (p. 173)

quadratic equation An equation in which the highest power of the variable is 2, that is, a variable squared; for example, $3x^2 - 6 = 69$ or an equation such as $y = 3x^2 - 6$, whose graph is a **parabola**. (pp. 259, 296, 557)

quadratic formula The formula for solving quadratic equations of the form $ax^2 + bx + c = 0$, which is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. (p. 565)

quadratic trinomial (p. 528) See **trinomial**.

quadrilateral Any polygon with 4 sides.



quarterly Occurring regularly 4 times a year, that is, every 3 months. (p. 26)

quartile The values Q_1, Q_2, Q_3 that divide a set of data into 4 equal parts. The 1st quartile Q_1 is the lower quartile, the 2nd quartile Q_2 is the **median**, the 3rd quartile Q_3 is the upper quartile. (p. 188)

R

radius (plural: **radii**) An interval joining the centre of a circle to the circumference, or the length of that interval. The radius is half of the **diameter**. (p. 101)



random In probability, describing a situation where every possible outcome has an equal chance, or is equally likely. (p. 233)

random sampling In statistics, selecting a sample in which every person or item in the population has an equal chance of being selected. A sample should be random to be truly representative of the population. (p. 233)

range In a set of data, the difference between the highest and lowest values. (pp. 189, 198)

rational number A number that can be written as a fraction in the form $\dfrac{a}{b}$, where a and b are integers and $b \neq 0$. (p. 539) See also **irrational number** and **real number**.

rationalise the denominator To simplify a fraction involving a surd by making its denominator rational (that is, not a surd). (p. 552)

real number A **rational** or **irrational number** that can be ordered on a number line. (p. 540)

reciprocal The product of any number and its reciprocal is 1. The reciprocal of any number is found by first writing the number as a fraction and then swapping the numerator with the denominator. The reciprocal of 5 is $\dfrac{1}{5}$ and the reciprocal of $\dfrac{2}{3}$ is $\dfrac{3}{2}$. (p. 161)

rectangle A quadrilateral with 4 right angles. (p. 406)



relative frequency The number of times an event or data value occurred, written as a fraction of the total number of events or data values. (p. 395) See also **observed probability**.

repayment (or **instalment**) The amount of money paid at regular time periods (weekly, fortnightly, monthly) to pay off a loan. (p. 29)

retainer A fixed amount paid to a salesperson before **commission** is added. (p. 7)

rhombus A quadrilateral with 4 equal sides.



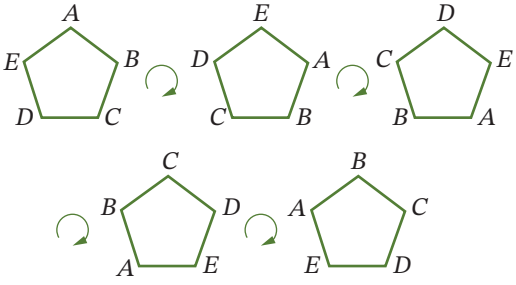
RHS The right-hand side (of an equation). (p. 557)

See also **similarity test**

right-angled triangle A triangle with one 90° angle. (p. 338)

rise Short for 'vertical rise', this is the change in vertical position between 2 points on a line or interval, the number of units 'going up', used with the **run** to calculate the **gradient** of a line or interval. (pp. 46, 66) See **gradient**.

rotational symmetry A plane shape has rotational symmetry if it can be spun around its centre so that it fits onto itself again before making a complete revolution. The centre point is called the **centre of symmetry**. A regular pentagon (pictured) has rotational symmetry, and because it can fit onto itself 5 times during one complete revolution, we say that it has *rotational symmetry of order 5*. See also **line symmetry**. (p. 446)



rounding error Error in measurement and calculation from rounding partial measurements or calculations too early, resulting in inaccurate final answers. (p. 97)

royalty Income earned by recording artists and authors, based on the number of copies of their work that are sold. (p. 7)

run Short for 'horizontal run', this is the change in horizontal position between 2 points on a line or interval, the number of units 'going right', used with the **rise** to calculate the **gradient** of a line or interval. (pp. 46, 66) See **gradient**.

S

salary A fixed yearly amount of money that is paid weekly, fortnightly or monthly, not dependent on the number of hours worked. (p. 6)

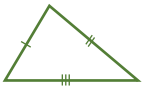
salvage value The value of an asset that is depreciating. (p. 34)

sample In statistics, a group of people or items selected from a population for study. (pp. 231, 233)

sample space In a probability situation, the set of all possible outcomes. (p. 418)

scale factor The amount by which a shape has been enlarged or reduced, equal to $\frac{\text{image length}}{\text{original length}}$. (p. 458)

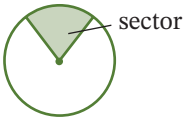
scalene triangle A triangle with no equal sides.



scatterplot A graph of points on a number plane. Each point represents the values of the 2 different variables and the resulting graph may show a pattern. (p. 220)

second (″) A measure of angle size. $\frac{1}{60}$ of a minute. $1' = 60''$.

sector A region of a circle cut off by 2 radii. (p. 101)



self-selected sample A sample in which people choose themselves to participate, such as an online survey. Because the participants are volunteers, a self-selected sample is neither random nor necessarily representative of the population. (p. 233)

set A collection of distinct items called **elements**. (p. 406)

shape of a distribution The way the data in a frequency distribution is spread, can be **symmetrical**, positively **skewed** or negatively **skewed**. (p. 184)

significant figures The meaningful digits in a number that show its level of accuracy, the first non-zero digits; for example, 31 487 000 has 5 significant figures.

similar To have the same shape but not necessarily the same size, an enlargement or reduction. The symbol '~' means 'is similar to'. (p. 458)

simple interest Interest that is calculated as a percentage of the original principal. (p. 16) See also **compound interest**.

simulation An artificial imitation of a chance situation, usually achieved by technology, such as using a computer to predict population growth. (p. 398)

simultaneous equations 2 (or more) equations that must be solved together so that the solution satisfies both equations. For example, $y = 2x + 1$ and $y = 3x$ are simultaneous equations that have a solution of $x = 1$, $y = 3$. (p. 398)

sine A ratio in a right-angled triangle:

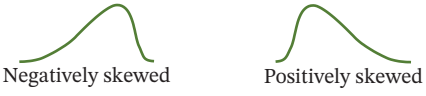
$$\sin\theta = \frac{\text{side opposite to } \theta}{\text{hypotenuse}}$$

where θ is an angle. (p. 338) See also **cosine** and **tangent**.

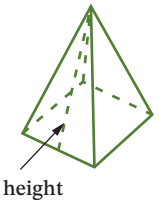
sine rule A rule that relates the sides of any triangle to the sine of their opposite angles:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{(p. 363)}$$

skewed distribution A distribution in which most of the values are clustered at one end, creating a 'tail' at the other end. The tail determines whether the skew is positive or negative. (p. 184) See also **symmetrical distribution**.



slant height The height of a pyramid or cone from its **apex** (top) to its base along a side face rather than its perpendicular height. (p. 114)



solution The answer to an equation, inequality or problem, the correct value(s) of the variable that makes an equation true.

sphere A ball shape, a solid that is completely round. (p. 119)



square A quadrilateral with 4 equal sides and 4 right angles.



standard deviation (symbol σ) A measure of spread that depends on every value in the data set and their mean. (pp. 193, 197, 198)

stem-and-leaf plot A 'number graph' that lists all the data values, in groups. Each value is split into a 'stem' and a 'leaf'. This stem-and-leaf plot shows 12 test scores, from 42 to 82. (p. 184)

Stem	Leaf
4	2 5
5	0 2 8
6	6 7
7	3 5 7 7
8	2

Key, 5|8 stands for 58

stratified sample A sample in which items are chosen by categories that are in proportion to their representation in the population, for example, if 18% of the population speak English as their second language, then in a stratified sample 18% should also speak English as their second language. (p. 233)

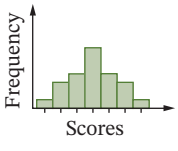
subject of a formula The variable for which a formula is written, the variable on the left-hand side of a formula. The subject of the formula $A = \frac{1}{2}bh$ is A. (p. 269)

substitution method A method of solving simultaneous equations that involves substituting one equation into another equation. (pp. 509, 512)

surd A square root (or other root) whose exact value cannot be found because it is **irrational**, such as $\sqrt{10}$ or $\sqrt[3]{7}$. (p. 539)

surface area The total area of all the faces of a solid shape. (pp. 105, 139)

symmetrical distribution A distribution in which all values are distributed equally on both sides of the centre, its shape having line symmetry. (p. 184) See also **skewed distribution**.



systematic sample A sample in which an item is chosen from the population at regular intervals, for example, every 100th car tested on a factory line. (p. 233)

T

tangent A ratio in a right-angled triangle:

$$\tan\theta = \frac{\text{side opposite to } \theta}{\text{side adjacent to } \theta}$$

tax deduction See **allowable deduction**. (p. 12)

taxable income The part of a person's income that is taxed, equal to yearly income minus allowable deductions. (p. 12)

term (of an expression) A part of an algebraic expression. For example, $b^2 + 6b - 9$ has 3 terms: b^2, $6b$ and -9. (p. 526)

term payments Paying for an expensive item through a loan in which regular instalments are made over time. Also called **hire-purchase**. (p. 29)

theoretical probability or **expected probability** Probability calculated using the formula:

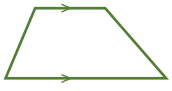
$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$ (p. 395)

three-stage experiment (or **three-step experiment**) A chance experiment with 3 stages, such as tossing 3 coins together. (p. 418)

time-and-a-half Overtime pay that is calculated at 1.5 times the normal pay rate. (p. 6)

trail A walk in a network that has no repeated edges. However, a vertex can be repeated. (pp. 466, 479)

trapezium A quadrilateral with one pair of opposite sides parallel. (p. 105)



tree diagram A diagram of branches for listing all of the possible outcomes of a multi-step chance experiment. (p. 418)

trial One go or run of a repeated probability experiment; for example, one roll of a die. (p. 395)

trigonometric ratio The ratio of 2 sides in a right-angled triangle; for example, sine is the ratio of the opposite side to the hypotenuse. (p. 338)

trinomial An algebraic expression with 3 terms, for example, $3x + 2y - 5$. In a **quadratic trinomial** such as $x^2 + 4x + 6$, the highest power of the variable is 2. (p. 173). See also **binomial** and **quadratic expression**.

true bearing (or **three figure bearing**) A bearing that uses 3-digit angles from 000° to 360° to show the amount of turning measured clockwise from north. (p. 350) See also **bearing** or **compass bearing**.

truncate 'Cutting off' a number at the decimal place regardless of the next digit, or 'rounding down'. (p. 97)

two-stage experiment (or **two-step experiment**) A chance experiment with 2 stages, such as rolling a pair of dice. (p. 418)

two-way table A table that shows the number of items belonging to overlapping categories. (p. 415)

where θ is an angle. See also **sine** and **cosine**.

	Can swim	Cannot swim
Boys	13	2
Girls	9	3

U

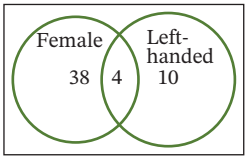
union ($\cup$) The combination of 2 or more sets. $A \cup B$ is the union of sets A and B and contains the elements of A and B combined. (pp. 408, 409)

V

variable A symbol, usually a letter of the alphabet, that stands for a number. Also called a **pronumeral** or **unknown**. (p. 173)

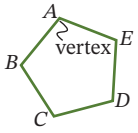
variation See **direct variation**. (pp. 283, 293)

Venn diagram A diagram of circles (usually overlapping) for grouping items into categories. (p. 406)



vertex (or **node, of a network**, plural: **vertices**) A point representing an item that is connected by lines called edges. See **edge** for diagram. (p. 465)

vertex (plural: **vertices)** A corner of a shape, angle or curve. (pp. 296, 297, 570, 572)



vertical Going up and down, at a right angle to the **horizontal**. (p. 46)



vertical intercept See ***y*-intercept** (pp. 75, 76)

volume The amount of space taken up by a solid object, measured in cubic units. (p. 139)

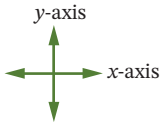
W

walk Any route along the edges of a network. (p. 466)

wage An amount of money paid to people for work, calculated on the number of hours worked. (p. 6)

X

***x*-axis** The horizontal axis of a number plane (running across). (p. 316)



***x*-intercept** The x-value at which a graph cuts the x-axis. (pp. 59, 64, 568, 572)

Y

***y*-axis** The vertical axis of a number plane (running up and down, see diagram above). (p. 64)

***y*-intercept** or **vertical intercept** The y-value at which a line cuts the y-axis. (pp. 59, 64, 75, 76, 568, 572)

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