Mathematics

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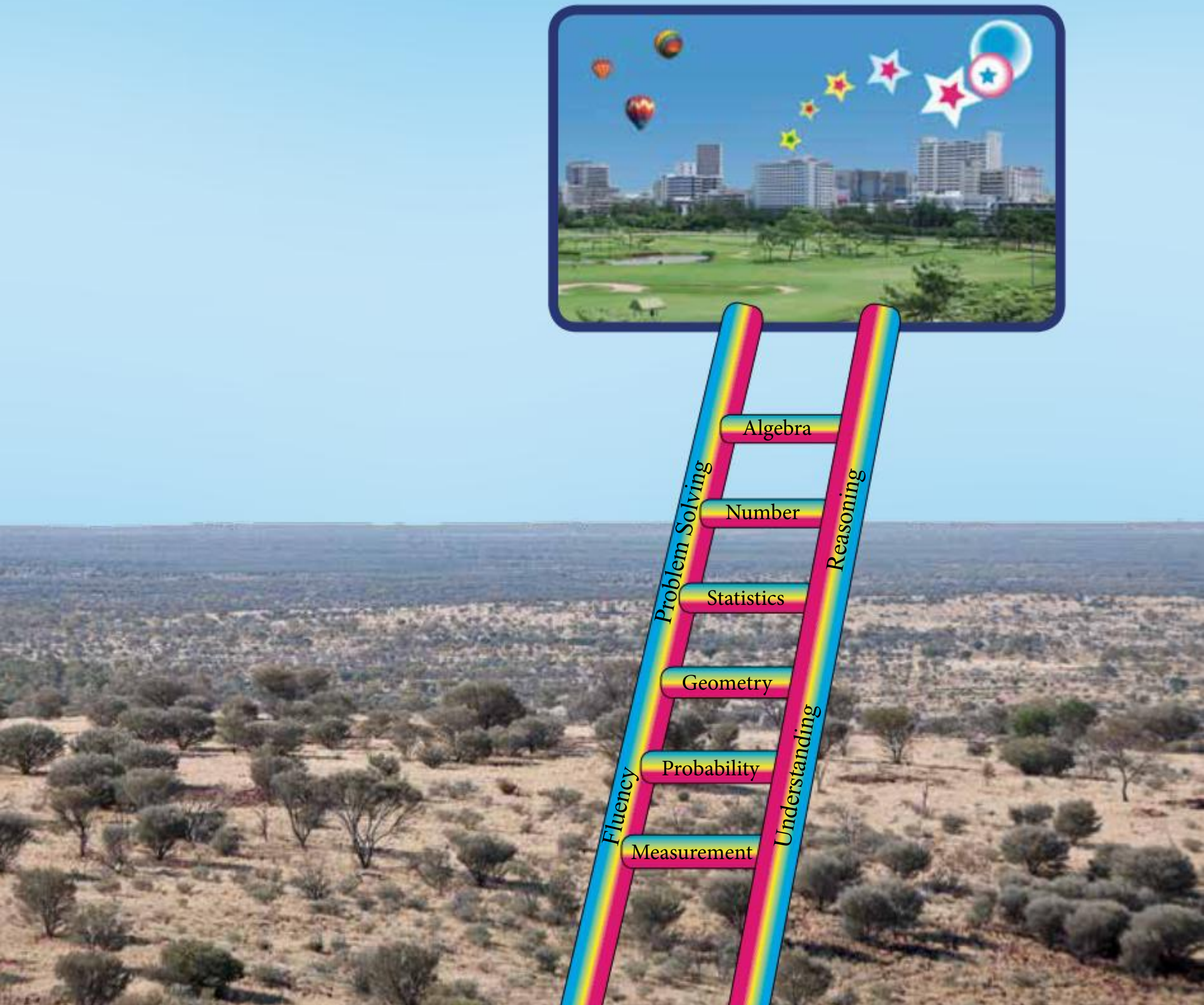

National Curriculum

Dr Terry Dwyer

Mathematics

National Curriculum

9

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The cover: Maths is the pathway to success in our technological economy and society.

"In today's world, economic access and full citizenship depend crucially on math and science literacy." - Robert P. Moses.

"Mathematics makes a significant contribution to our modern society; its basic skills are vital for the life opportunities of the youth. Let us all embrace and encourage the study of this indispensable subject."
- Alfred L. Teye.

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510

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Contents - Overview

Preface

This text has been written for Year 9 students. The aim of the text is to assist students in investigating and understanding the exciting and very important world of Mathematics and to implement the intent of the Australian Mathematics Curriculum.

A literature review of learning from school textbooks was used to enhance the format of this textbook.

Each chapter, apart from Review, contains:

- ★ Numerous worked examples
- ★ Numerous sets of graded exercises
- ★ An open-ended rich task
- ★ Mental computation
- ★ Technology in mathematics
- ★ Investigations
- ★ Puzzles
- ★ NAPLAN questions
- ★ Maths competition preparation
- ★ A mathematics game
- ★ A mathematics trick
- ★ A bit of mathematics history
- ★ Careers using mathematics
- ★ Chapter review

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Resources

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- ★ **Workprogram**
- ★ **Study guides** for each term.
- ★ Detailed **lesson plans** for each term.
- ★ Sample **assessment items.**
- ★ **PDF** of this textbook.

Indices 1

Number and Algebra → Real Numbers

★ Apply index laws to numerical expressions with integer indices.
★ Connect different strategies for simplifying expressions with indices to illustrate the meaning of negative indices.
★ Move fluently between representations of numeric and algebraic terms with negative indices.
★ Apply knowledge of index laws to algebraic terms and simplify algebraic expressions, using both positive and negative integral indices.

Infinite, adj. 1. exceedingly great, unlimited, immeasurably large.

A TASK

- Everyone should know about infinity.
- Every child learns to count to ten, then to twenty, then to one hundred.
- Then what? Infinity of course.

Research infinity and then give a five minute explanation of infinity to the rest of the class (Try to include one of the interesting stories about infinity).

A LITTLE BIT OF HISTORY

- The Romans used "Decies centena milia (ten hundred thousand) as the Roman words for 1 000 000.
- The French, in the 13th century, were the first to use the word 1 million.
- The Indians were the most advanced with large numbers and had by the 7th century defined infinity as having a denominator of zero.

$$\frac{1}{0} = \infty$$

A 1 m cubic block will have 1 million 1 cm cubes.

$\frac{1}{0.1} = 10$ $\frac{1}{0.001} = 1000$ $\frac{1}{0.000001} = 1000000$

WarmUp

Exercise 1.1

Calculate each of the following:

1 1×1	**2** 2×2	**3** 3×3	**4** 4×4
5 10×10	**6** 1×1×1×1	**7** 2×2×2	**8** 3×3×3
9 4×4×4	**10** 5×5×5	**11** 10×10×10	**12** 2×2×2×2
13 3×3×3×3	**14** 10×10×10×10	**15** 5×5×5×5	

16 Copy and complete the following table:

Squares are often used. (Area square = side^2)

Cubes happen now and then. (Volume cube = side^3)

	Square	Cube	Fourth	Fifth
1				
2				
3				
4				
5				

17 10×10

18 10×10×10

19 10×10×10×10

20 10×10×10×10×10

21 10×10×10×10×10×10

22 10×10×10×10×10×10×10

Exercise 1.2

Use a calculator to calculate each of the following:

On some calculators, the power button is y^x.

5×5×5×5×5×5×5x5

On a calculator: 5 ^ 8 = 390625

y^x

1 2×2×2×2×2×2

2 5×5×5×5×5×5×5

3 1×1×1×1×1×1×1×1

4 10×10×10×10×10×10

5 2×2×2×2×2×2×2×2

6 $2^2 \times 2^2 \times 2^2 \times 2^2$

7 $(2^2)^4$

8 2^{2^4}

9 3×3×3×3×3×3×3×3×3

10 $3^3 \times 3^3 \times 3^3$

11 $(3^3)^3$

12 3^{3^3}

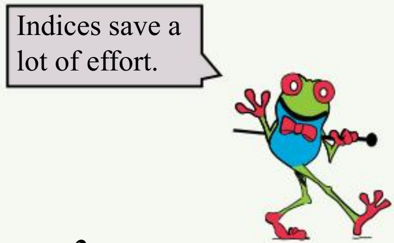
Indices

A convenient way of writing $2\times2\times2$ is 2^3

$$2^3 \quad \begin{array}{l} \leftarrow \text{Index} \\ \leftarrow \text{Base} \end{array}$$

Exercise 1.3
Write the following in index form:

$2\times2\times2\times2\times2$ $= \underline{2^5}$	$a\times a\times a\times a$ $= \underline{a^4}$



1	$4\times4\times4$	**2**	$2\times2\times2\times2$	**3**	$a\times a\times a$
4	$10\times10\times10$	**5**	$b\times b\times b\times b\times b$	**6**	$h\times h\times h$
7	$m\times m\times m\times m\times m$	**8**	$9\times9\times9\times9$	**9**	$3\times3\times3\times3\times3\times3$

Exercise 1.4
Write the following in factor form:

3^4 $= \underline{3\times3\times3\times3}$	b^3 $= \underline{b\times b\times b}$

1	4^3	**2**	b^4	**3**	5^2
4	2^7	**5**	6^2	**6**	m^5
7	x^4	**8**	p^5	**9**	1^4

Exercise 1.5
Write the following in index form:

$2\times2\times2\times4\times4$ $= \underline{2^3\times4^2}$	abbaaab $= \underline{a^4\times b^3}$

1	aabbbaa	**2**	$3\times3\times3\times2\times2$	**3**	abaaababb
4	$2\times3\times2\times3\times2\times3\times2$	**5**	bggggbbbg	**6**	zzzzzzzzzzz
7	ppqrppqqrrp	**8**	$2\times2\times2\times3\times4\times4\times4\times3$	**9**	4gg4g4g4gg

Biochemists study the chemistry of living things in the fields of medicine, agriculture, the environment, and manufacturing.
- Relevant school subjects are English, Mathematics, Chemsitry, Biology.
- Courses normally involve a Degree with a major in chemistry/biochemistry.

Index Law 1

Multiplying Indices:	or	Multiplying Indices:	→	**Index Law 1**
$2^4\times2^2 = 2\times2\times2\times2 \times 2\times2$ $= 2^6$		$2^4\times2^2 = 2^{4+2}$ $= 2^6$		$a^m\times a^n = a^{m+n}$

Exercise 1.6
Simplify and write the following in index form:

$2^3\times2^2 = 2\times2\times2 \times 2\times2 = \underline{2^5}$ **or** $2^3\times2^2 = 2^{3+2} = \underline{2^5}$	$a^2\times a^5 = a\times a \times a\times a\times a\times a\times a = \underline{a^7}$ **or** $a^2\times a^5 = a^{2+5} = \underline{a^7}$

1	$2^2\times2^3$	**2**	$3^3\times3^2$	**3**	$2^4\times2^2$	**4**	$4^4\times4^3$
5	$a^2\times a^2$	**6**	$b^3\times b^2$	**7**	$z^5\times z^3$	**8**	$w^4\times w^3$
9	$2^3\times2^1$	**10**	$t^2\times t^5$	**11**	$5^2\times5^3$	**12**	$10^4\times10^5$
13	$c^4\times c^3$	**14**	$10^2\times10^4$	**15**	$a^2\times a^3$	**16**	$b^5\times b$
17	$2.4^5\times2.4^2$	**18**	$z^3\times z^2$	**19**	$0.5^5\times0.5^4$	**20**	$10^5\times10^2$
21	$2^2\times2^3\times2^2$	**22**	$3^4\times3^3\times3^2$	**23**	$a^2\times a^3\times a^3$	**24**	$u^2\times u^2\times u^3$

Index Law 2

Dividing Indices:	or	Dividing Indices:	→	**Index Law 2**
$a^3\div a^2 = \frac{a\times a\times a}{a\times a} = a$		$a^3\div a^2 = a^{3-2} = a$		$a^m\div a^n = a^{m-n}$

Exercise 1.7
Simplify and write the following in index form:

$2^3\div2^2 = \frac{2\times2\times2}{2\times2} = \underline{2}$ **or** $2^3\div2^2 = 2^{3-2} = \underline{2}$	$a^6\div a^2 = \frac{a\times a\times a\times a\times a\times a}{a\times a} = a\times a\times a\times a = \underline{a^4}$ **or** $a^6\div a^2 = a^{6-2} = \underline{a^4}$

1	$2^5\div2^3$	**2**	$4^5\div4^2$	**3**	$2^4\div2^2$	**4**	$4^4\div4^3$
5	$a^6\div a^2$	**6**	$b^7\div b^2$	**7**	$10^5\div10^3$	**8**	$w^4\div w^3$
9	$2^3\div2^1$	**10**	$t^7\div t^4$	**11**	$5^6\div5^4$	**12**	$10^5\div10^2$
13	$a^4\div a^3$	**14**	$1.2^4\div1.2^2$	**15**	$2^9\times2^9\div2^3$	**16**	$p^3\div p^3$
17	$\frac{m^4}{m^2}$	**18**	$\frac{e^5}{e^2}$				
19	$\frac{c^7}{c^4}$	**20**	$\frac{10^4}{10^2}$				

$m^4\div m^2 = \frac{m^4}{m^2}$

They are the same thing.

Index Law 3

Power Indices:
$(2^3)^2 = (2\times2\times2)^2$
$= (2\times2\times2)\times(2\times2\times2)$
$= 2^6$

or

Power Indices:
$(2^3)^2 = 2^{3\times2}$
$= 2^6$

→

Index Law 3
$(a^m)^n = a^{m\times n}$

Exercise 1.8
Simplify and write the following in index form:

$(b^4)^2 = (b\times b\times b\times b)^2$ $= (b\times b\times b\times b)\times(b\times b\times b\times b)$ $= \underline{b^8}$ **or** $(b^4)^2 = b^{4\times2} = \underline{b^8}$	$3^4\times(3^2)^3 = 3^4\times3^6 = \underline{3^{10}}$ $(b^4)^2b^3 = b^8\times b^3 = \underline{b^{11}}$

1 $(2^2)^3$	**2** $(2^3)^3$	**3** $(3^2)^3$	**4** $(4^2)^3$
5 $(a^2)^2$	**6** $(b^3)^4$	**7** $(t^2)^4$	**8** $(n^2)^5$
9 $(5^2)^2$	**10** $(s^2)^5$	**11** $(m^2)^3$	**12** $(3^4)^2$
13 $(5^3)^3$	**14** $(10^2)^3$	**15** $(10)^3$	**16** $(g^2)^5$
17 $(10^3)^2$	**18** $(h^4)^3$	**19** $(d^2)^4$	**20** $(2^5)^3$
21 $2^2\times(2^2)^3$	**22** $2^2\times(2^3)^3$	**23** $(3^2)^3\times3^3$	**24** $4^4\times(4^2)^3$
25 $d^3(d^2)^2$	**26** $b^4(b^3)^4$	**27** $(t^2)^4t^5$	**28** $n^3(n^2)^5$
29 $(5^2)^2\times5$	**30** $s^2(s^2)^5$	**31** $m(m^2)^3$	**32** $3\times(3^4)^2$

Index Law 4

Zero Index:
$p^3\div p^3 = 1$
or $p^3\div p^3 = p^{3-3}$
$= p^0$
Which must be = 1

or

Zero Index:
$p^0 = 1$

→

Zero Index
$a^0 = 1$

Try 5^0 on your calculator.
Is your answer 1?

Exercise 1.9
Simplify each of the following:

$3^0 = \underline{1}$	$h^0 = \underline{1}$	$3\times5^0 = 3\times1 = \underline{3}$	$5b^0 = 5\times1 = \underline{5}$

1 2^0	**2** 5^0	**3** b^0	**4** k^0
5 a^0	**6** 3^0	**7** 10^0	**8** d^0
9 5×2^0	**10** $3a^0$	**11** 6×4^0	**12** 2×1^0
13 $7w^0$	**14** $3e^0$	**15** 9×3^0	**16** 8×2^0
17 $(5^0)^2\times5$	**18** $s^2(s^0)^5$	**19** $m(m^2)^0$	**20** $3\times(3^0)^2$

Index Law 5

What happens when the index is negative?

$$a^2 \div a^5 = \frac{a \times a}{a \times a \times a \times a \times a} = \frac{1}{a \times a \times a}$$

or

$$a^2 \div a^5 = a^{2-5} = a^{-3}$$

Negative Index

$$a^{-m} = \frac{1}{a^m}$$

Exercise 1.10

Write each of the following using a negative index:

$\frac{1}{10^3} = 10^{-3}$	$\frac{1}{b^5} = b^{-5}$	$\frac{1}{x} = x^{-1}$	$\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$ $\{2^3 = 2\times2\times2 = 8\}$

1 $\frac{1}{10^2}$ **2** $\frac{1}{2^5}$ **3** $\frac{1}{a^4}$ **4** $\frac{1}{10^4}$

5 $\frac{1}{x^7}$ **6** $\frac{1}{x}$ **7** $\frac{1}{10^6}$ **8** $\frac{1}{3^4}$

9 $\frac{1}{4}$ **10** $\frac{1}{9}$ **11** $\frac{1}{27}$ **12** $\frac{1}{16}$

Find the value of each the following:

$2^{-3} = \frac{1}{2\times2\times2} = \frac{1}{8}$ {also = 0.125}	$10^{-4} = \frac{1}{10\times10\times10\times10} = \frac{1}{10000}$ {also = 0.0001}

13 2^{-2} **14** 4^{-1} **15** 2^{-4} **16** 10^{-1}

17 10^{-2} **18** 10^{-3} **19** 10^{-4} **20** 10^{-5}

21 5^{-2} **22** 2^{-5} **23** 0.4^{-2} **24** 1.5^{-2}

Copy each of the following tables and use a calculator to complete them:

25

Power	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
Value			4	2				

26

Power	5^4	5^3	5^2	5^1	5^0	5^{-1}	5^{-2}	5^{-3}
Value			25	5				

27

Power	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
Value			100	10				

Index Law 1

$a^m \times a^n = a^{m+n}$

This only works if the bases are the same.

Exercise 1.11

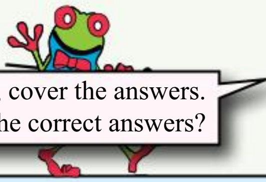
Use the Index Laws to simplify each of the following:

$2^2 \times 2^{-5} = 2^{2+{}^-5}$ $\{a^m \times a^n = a^{m+n}\}$ $= \underline{2^{-3}}$ $\{2 + {}^-5 = {}^-3\}$	$5x^{-4} \times 3x^3 = 15x^{-4+3}$ $\{a^m \times a^n = a^{m+n}\}$ $= \underline{15x^{-1}}$ $\{{}^-4 + 3 = {}^-1\}$

1 $2^3 \times 2^{-6}$	**2** $5^2 \times 5^{-6}$	**3** $3^5 \times 3^{-2}$	**4** $10^4 \times 10^{-5}$
5 $x^7 \times x^{-3}$	**6** $2x^3 \times 3x^{-4}$	**7** $5x^5 \times x^{-6}$	**8** $4x^3 \times 5x^{-5}$
9 $4^8 \times 4^{-6}$	**10** $a^5 \times a^{-6} \times a^4$	**11** $3x^3 \times 2x^{-2} \times 4x^{-4}$	**12** $10^3 \times 10^{-3}$

Index Law 2

$a^m \div a^n = a^{m-n}$



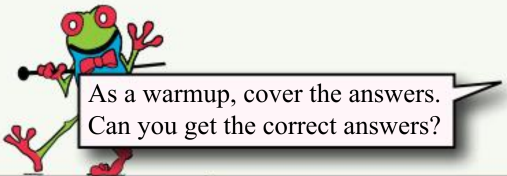
$4 + {}^-5 =$	${}^-1$
${}^-3 + {}^-2 =$	${}^-5$
$5 - {}^-3 =$	8
${}^-2 - {}^-4 =$	2

$5^2 \div 2^{-3} = 2^{2-{}^-3}$ $\{a^m \div a^n = a^{m-n}\}$ $= \underline{2^5}$ $\{2 - {}^-3 = 2+3 = 5\}$	$8x^{-5} \div 2x^3 = 4x^{-5-3}$ $\{a^m \div a^n = a^{m-n}\}$ $= \underline{4x^{-8}}$ $\{{}^-5 - 3 = {}^-8\}$

13 $3^3 \div 3^{-2}$	**14** $2^5 \div 2^{-2}$	**15** $5^{-3} \div 5^2$	**16** $10^{-4} \div 10^{-2}$
17 $x^3 \div x^{-3}$	**18** $x^{-3} \div x^5$	**19** $y^{-3} \div y^{-4}$	**20** $a^3 \div a^{-1}$
21 $4x^5 \div 2x^{-1}$	**22** $12b^4 \div 4b^{-2}$	**23** $2^4 \times 2^3 \div 2^{-2}$	**24** $10^3 \div 10^{-3}$

Index Law 3

$(a^m)^n = a^{m \times n}$



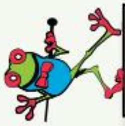
$2 \times 5 =$	10
$4 \times {}^-3 =$	${}^-12$
${}^-3 \times 2 =$	${}^-6$
${}^-5 \times {}^-3 =$	15

$(3^{-2})^4 = 3^{{}^-2 \times 4}$ $\{(a^m)^n = a^{m \times n}\}$ $= \underline{3^{-8}}$ $\{{}^-2 \times 4 = {}^-8\}$	$(x^2)^3 = x^{2 \times 3}$ $\{(a^m)^n = a^{m \times n}\}$ $= \underline{x^6}$ $\{2 \times 3 = 6\}$

25 $(2^{-2})^3$	**26** $(3^{-3})^2$	**27** $(2^2)^{-4}$	**28** $(5^{-2})^{-2}$
29 $(x^2)^3$	**30** $(n^{-3})^2$	**31** $(a^4)^{-3}$	**32** $(y^{-1})^{-5}$
33 $x^{-3} \times (x^{-1})^{-3}$	**34** $(3^{-3})^2 \times (3^{-2})^{-2}$	**35** $a^4 \times a^{-3} \times (a^4)^{-3}$	**36** $(10^{-1})^{-2}$

Summary

Index Law 1 $a^m \times a^n = a^{m+n}$	$2^4 \times 2^2 = 2^{4+2}$ $= \underline{2^6}$	$3x^4 \times 2x^2 = 6x^{4+2}$ $= \underline{x^6}$
Index Law 2 $a^m \div a^n = a^{m-n}$	$5^2 \div 5^5 = 5^{2-5}$ $= \underline{5^{-3}}$	$6a^3 \div 3a^2 = 2a^{3-2}$ $= \underline{2a}$
Index Law 3 $(a^m)^n = a^{m \times n}$	$(7^3)^2 = 7^{3\times 2}$ $= \underline{7^6}$	$(x^3)^2 = x^{3\times 2}$ $= \underline{x^6}$
Index Law 4 $a^0 = 1$	$3^0 = \underline{1}$	$x^0 = \underline{1}$
Index Law 5 $a^{-m} = \frac{1}{a^m}$	$10^{-3} = \underline{\frac{1}{10^3}}$	$x^{-5} = \underline{\frac{1}{x^5}}$



The more problems you work,
the better you become,
the more opportunities.

Exercise 1.12

Use the Index Laws to simplify each of the following:

1	$3^4 \times 3^3$	**2**	$x^2 \times x^3$	**3**	$7^2 \times 7^5$	**4**	$x^4 \times y^3$
5	$a^2 \times a^2$	**6**	$b^3 \times b^5$				
7	$z^5 \times d^2$	**8**	$10^4 \times 10^3$				
9	$2^{-3} \times 2^1$	**10**	$1.5^2 \times 1.5^4$				
11	$5^3 \div 5^2$	**12**	$10^5 \div 10^2$				
13	$d^4 \div d^2$	**14**	$2^2 \div 2^4$	**15**	$a^2 \div a^6$	**16**	$e^5 \div x^6$
17	$1.2^7 \div 1.2^2$	**18**	$z^3 \div z^8$	**19**	$4^{-5} \div 4^4$	**20**	$10^3 \div 10^{-2}$
21	$(3^2)^2$	**22**	$(x^3)^2$	**23**	$(5^2)^3$	**24**	$(x^2)^5$
25	$(a^2)^5$	**26**	$(s^{-2})^5$	**27**	$(m^2)^{-3}$	**28**	$(3^4)^2$
29	$(x^{-2})^{-4}$	**30**	$(10^2)^3$	**31**	$4a^2 \times 3a^3$	**32**	$2b^5 \times 5b^2$
33	$4^5 \div 4^2$	**34**	$10z^3 \div 2z^5$	**35**	$10^5 \times 10^2 \times 10^4$	**36**	$10^5 \times 10^2 \times 10^{-4}$
37	$10^6 \times 10^{-2} \times 10^{-4}$	**38**	$3x^4 \times 2x^3 \div x^2$	**39**	$x^4 \times 6x^{-3} \div 2x^2$	**40**	$(x^{-4})^2 \times 9x^3 \div 6x^2$

The bases aren't the same.
The answer is: $x^4 \times y^3$

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 1.13

1 Spell power.
2 $5 - 7$
3 $3 - {}^{-}4$
4 $10^2 \times 10^3$
5 $z^3 \div z^7$
6 $(2^3)^2$
7 Increase \$6 by 10%
8 I buy a loaf of bread for \$3.45 and a biro for 40c, what is the total?
9 If I paid \$50 deposit and 10 payments of \$20. How much did I pay?
10 I spend \$70 per week on groceries, roughly how much per year?

10% of \$6 = \$0.60

Roughly 50 weeks
$\$70 \times 50 = \$70 \div 2 \times 100$
$= \$35 \times 100$
$= \$3500$

Exercise 1.14

1 Spell simplify.
2 ${}^{-}3 + 5$
3 $1 - {}^{-}2$
4 $10^5 \times 10^2$
5 $x^5 \div x^7$
6 $(4^2)^{-3}$
7 Increase \$8 by 10%
8 I buy a loaf of bread for \$3.35 and 2L milk for \$3.10, what is the total?
9 If I paid \$100 deposit and 10 payments of \$15. How much did I pay?
10 I spend \$120 per week on groceries, roughly how much per year?

1 googol = 10^{100} = 10 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000

Q: Divide 14 sugar cubes into 3 cups of coffee so that each cup has an odd number of sugar cubes.
A: 1,1,12
12 isn't odd!
It's an odd number of cubes to put in a cup of coffee.

Exercise 1.15

1 Spell negative.
2 ${}^{-}4 + {}^{-}5$
3 $5 - {}^{-}1$
4 $10^{-2} \times 10^5$
5 $a^6 \div a^4$
6 $(3^3)^3$
7 Increase \$9 by 10%
8 I buy a loaf of bread for \$3.25 and a book for \$9.50, what is the total?
9 If I paid \$100 deposit and 10 payments of \$35. How much did I pay?
10 I spend \$80 per week on unleaded petrol, roughly how much per year?

NAPLAN Questions

Exercise 1.16

1 Calculate each of the following:

a) 2.35×10^2 **b)** 2.35×10^4 **c)** 2.35×10^7

2 $5.16^3\times2.73^4$ is closest to:

a) 800 **b)** 8 000 **c)** 80 000

3 What is the value of $6a^2$ when $a = {^-2}$?

$({^-2})^2$	$= {^-2}\times{^-2}$	$= 4$
$({^-2})^3$	$= {^-2}\times{^-2}\times{^-2}$	$= {^-8}$
$({^-2})^4$	$= {^-2}\times{^-2}\times{^-2}\times{^-2}$	$= 16$

4 What is the value of $2x^2 + 3x - 2$ when $x = {^-1}$?

5 What is the value of $a^2 + b^2$ when $a = 3$ and $b = {^-1}$?

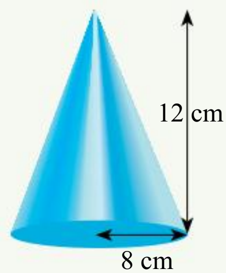
6 Is $y = 3x^3$ a correct rule for y in terms of x?

x	0	1	2	3
y	0	3	24	81

7 Is $y = 2x^2 - x$ a correct rule for y in terms of x?

x	0	1	2	3
y	0	1	6	15

8 The volume of a cone is given by the formula: $V = \frac{\pi r^2 h}{3}$, where r is the radius and h is the height. What is the volume of a cone with r = 8 cm and h = 12 cm?



9 4 200 000 is the same as:

a) 4.2×10^5 **b)** 4.2×10^6 **c)** 4.2×10^7

10 What is the value of: $3\times10^3 + 7\times10^2 + 6\times10^1$?

11 Calculate each of the following:

a) $2^3 - 2^2$ **b)** $3^3 - 3^2$ **c)** $4^3 - 4^2$

12 Which is the same as $3^2\times3^2$

a) $3\times3\times3$ **b)** $3\times3\times3\times3$ **c)** $3\times3\times3\times3\times3$

13 Which is the same as $2^3\times4^2$

a) $2\times3\times4\times2$ **b)** $2\times2\times2\times4\times2$ c) $2\times2\times2\times2\times2\times2\times2$

14 15^2 is between:

a) 150 and 200 **b)** 200 and 250 **c)** 250 and 300

15 Solve for b in:

a) $2^b = 16$ **b)** $2^b=64$ **c)** $3^b = 81$

16 What is the difference between 10^6 and 10^5?

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 1.17

1 Find the value of each of the following:

a) $(0.1)^2$	**b)** $(0.01)^2$	**c)** $(0.001)^2$
d) $(0.1)^3$	**e)** $(0.01)^3$	**f)** $(0.001)^3$
g) $(^{-}1)^2$	**h)** $(^{-}1)^3$	**i)** $(^{-}1)^4$
j) $(^{-}1)^5$	**k)** $(^{-}1)^6$	**l)** $(^{-}1)^{13}$
m) $10 + 10^2 + 10^3$	**n)** $2 + 2^2 + 2^3 + 2^4$	**o)** $10\times10^2\times10^3$
p) $2^2 - 1^2$	**q)** $3^2 - 2^2$	**r)** $4^2 - 3^2$

How many digits in 10.1^6?

$10.1^2 = 102.01$ $3 + 2 = 5$ digits
$10.1^3 = 1030.301$ $4 + 3 = 7$ digits
10.1^4 $5 + 4 = 9$ digits

10.1^6 would have $7+6 = 13$ digits

2 How many digits in?

a) 10.1^7 **b)** 10.1^{37} **c)** 10.3^{11}

3 The following sequence is formed by squaring the previous number and subtracting 5. What is the fifth term?
3, 4, 11, ...

4 Calculate each of the following:

a) $\frac{2^6 \times 2^3}{2^2 \times 2^7}$ **b)** $\frac{3^4 \times 3^3}{3^2 \times 3^4}$ **c)** $\frac{4^5 \times 4^3}{4^3 \times 4^4}$

What is the last digit in 23^{15}?

$3^1 = 3$
$3^2 = 9$ Last digit pattern is: 3, 9, 7, 1, 3, 9, 7, 1, ...
$3^3 = 27$ repeating in blocks of 4
$3^4 = 81$ The index, 15, is one from the fourth repeat
$4^5 = 243$ The last digit in 23^{15} is 7

5 What is the last digit in 3^{32}?
6 What is the last digit in 43^{47}?
7 What is the last digit in 393^{88}?
8 What is the last digit in 94^{71}?

9 Given that a and b can be any positive integer between 2 and 5, what is the largest possible value of $(2a - b)^{(b-a)}$?

Investigations

Investigation 1.1 **Maths Joke?**

An infinite number of Year 9 Maths students go to the Tuckshop. The first goes up and asks, "I'll have a litre of milk, please." Each student, in sequence, says, "and I'll have half of what they had."

The tuckshop person says, "You are all idiots," and gives them two litres of milk.

To understand the joke, calculate how much milk is needed:

$$= 1 + \frac{1}{2} + \frac{1}{2\times2} + \frac{1}{2\times2\times2} + \frac{1}{2\times2\times2\times2} + \ldots\ldots\ldots$$

∞ means infinity, an extremely large number.

or $=1 + 2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} + 2^{-5} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + 2^{-\infty}$

What is the answer?

Investigation 1.2 **Large numbers?**

There are many legends in which the reward for a great deed is something like:
$1 is paid on the first day of the month, and then doubled for each sucessive day of a 31 day month.

$1	$2	$4	$8			

What is the total payment for the month?

If a $1 coin is 9 grams, and the daily reward must be carried without assistance, out of the bank, what is the largest amount that will be paid?

A Couple of Puzzles

Exercise 1.18

1 Give an estimate of: 31.3 x 4.87

2 Make 8.9 appear on the display of a calculator without using the 8 key or the 9 key.

3 A club's middle batsman named Chuck
squared his number of runs just for luck
by subtracting his score and forty-two more
the final result was a duck

How many runs did Chuck score?

4 What is $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ of a pie?

A Game

Knots is played with naughts and crosses on a 4x4 square.
The loser is the person who can't make a move.

1 Take it in turns to place a naught or cross in one of the 16 cells.

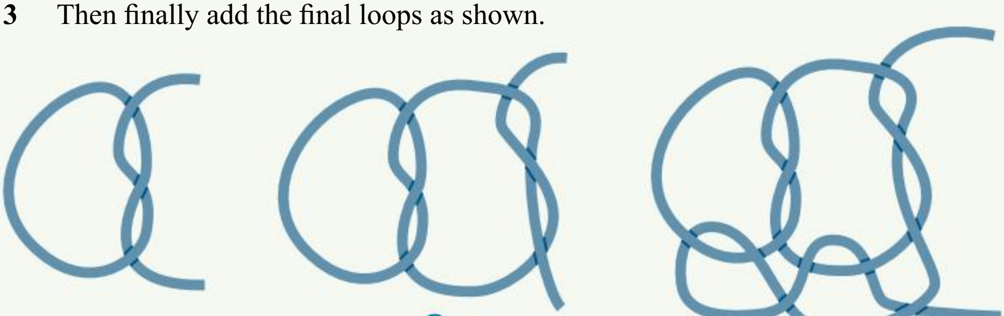
2 A naught cannot be placed above, below, or beside another naught (diagonally is OK).
Similarly for crosses.

O		X	O
	O		
X		X	O
	O		X

3 It is X's turn. Can you find a place to put an X? O has lost the game.

A Sweet Trick

1 With a rope or string make the first knot as shown.
2 Then add the second knot as shown in the middle diagram.
3 Then finally add the final loops as shown.



Get your audience to pull each end of the rope.

Can you guess what happens?

Technology

Technology 1.1

Use a spreadsheet to help with Investigation 1.1.

$$= 1 + \frac{1}{2} + \frac{1}{2\times 2} + \frac{1}{2\times 2\times 2} + \frac{1}{2\times 2\times 2\times 2} + \ldots\ldots\ldots\ldots\ldots$$

or $$=1 + 2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} + 2^{-5} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + 2^{-\infty}$$

	Amount Milk	Running Total
1	1	1
2	0.5	1.5
3	0.25	1.75
4	0.125	1.875
5	0.0625	
6		
7		
8		
9		
10		

= b2/2

= sum(b2:b3)

1.3E−8 means move the decimal point 8 places to the left.
=0.000 000 013
(or make the cell width wider)

Graph the running total.

Technology 1.2

Use a spreadsheet to help with Investigation 1.2.

Day of Month	Reward	Daily Total
1	1	1
2	2	3
3	4	7
4	8	15
5	16	31
6	32	
7		
8		
9		
10		

= 2*b2

= sum(b2:b3)

1.3E+8 means move the decimal point 8 places to the right.
=130 000 000
(or make the cell width wider)

Chapter Review 1

Exercise 1.19

$$d^3$$

Index

Base

1 Write each of the following in index form:

$3\times3\times3\times3 = \underline{3^4}$	$^{-}a\times{}^{-}a\times{}^{-}a = \underline{(^{-}a)^3}$

a) $1\times1\times1\times1$ **b)** $4\times4\times4\times4$ **c)** $10\times10\times10\times10\times10\times10$

d) $g\times g\times g\times g\times g\times g\times g$ **e)** $1.7\times1.7\times1.7\times1.7$ **f)** $^{-}5\times{}^{-}5\times{}^{-}5\times{}^{-}5\times{}^{-}5\times{}^{-}5$

2 Use the Index Laws to simplify each of the following:

$2^2\times2^{-5} = 2^{2+{}^{-}5}$ $\{a^m\times a^n = a^{m+n}\}$ $= \underline{2^{-3}}$ $\{2 + {}^{-}5 = {}^{-}3\}$	$5x^{-4}\times3x^3 = 15x^{-4+3}$ $\{a^m\times a^n = a^{m+n}\}$ $= \underline{15x^{-1}}$ $\{{}^{-}4 + 3 = {}^{-}1\}$

a) $4^3\times4^{-2}$ **b)** $6^2\times6^{-7}$ **c)** $2^4\times2^{-2}$ **d)** $10^6\times10^{-5}$

e) $x^6\times x^{-3}$ **f)** $4a^2\times3a^{-4}$ **g)** $2x^5\times x^{-6}$ **h)** $2x^2\times3x^{-5}$

i) $2^3\times2^{-5}$ **j)** $x^3\times x^{-6}\times x^4$ **k)** $4x^3\times2x^{-2}\times x^{-5}$ **l)** $10^3\times10^{-2}$

$5^2\div2^{-3} = 2^{2-{}^{-}3}$ $\{a^m\div a^n = a^{m-n}\}$ $= \underline{2^5}$ $\{2 - {}^{-}3 = 2+3 = 5\}$	$8x^{-5}\div2x^3 = 4x^{-5-3}$ $\{a^m\div a^n = a^{m-n}\}$ $= \underline{4x^{-8}}$ $\{{}^{-}5 - 3 = {}^{-}8\}$

m) $2^3\div2^{-4}$ **n)** $7^5\div7^{-4}$ **o)** $3^{-3}\div3^2$ **p)** $10^{-5}\div10^{-2}$

q) $n^3\div n^{-3}$ **r)** $x^{-4}\div x^5$ **s)** $p^{-3}\div p^{-4}$ **t)** $a^3\div a^{-2}$

u) $6x^6\div2x^{-1}$ **v)** $8x^4\div4x^{-3}$ **w)** $3^4\times3^4\div3^{-2}$ **x)** $10^4\div10^{-4}$

3 Use the Index Laws to simplify each of the following:

$(3^{-2})^4 = 3^{{}^{-}2\times4}$ $\{(a^m)^n = a^{m\times n}\}$ $= \underline{3^{-8}}$ $\{{}^{-}2\times4 = {}^{-}8\}$	$(x^{-2})^{-3} = x^{{}^{-}2\times{}^{-}3}$ $\{(a^m)^n = a^{m\times n}\}$ $= \underline{2^6}$ $\{{}^{-}2\times{}^{-}3 = 6\}$

a) $(3^{-2})^3$ **b)** $(2^{-2})^{-3}$ **c)** $(y^2)^3$ **d)** $(10^{-1})^{-3}$

e) $6^4\times6^2$ **f)** $x^2\times x^5$ **g)** $10^7\times10^3$ **h)** $2^{-5}\times2^1$

i) $1.2^3\times1.2^5$ **j)** $4^5\div4^2$ **k)** $10^6\div10^2$ **l)** $8.1^3\div8.1^2$

m) $3^{-5}\div3^3$ **n)** $10^9\div10^{-2}$ **o)** $4x^2\times2x^4$ **p)** $2x^5\times5x^3$

q) $10^6\times10^2\times10^{-4}$ **r)** $10^5\times10^{-2}\times10^{-4}$ **s)** $5x^2\times2x^3\div x^2$

t) $2x^5\times6x^{-3}\div2x^2$ **u)** $x^{-3}\times(x^{-1})^{-3}$ **v)** $(x^{-4})^2\times4x^3\div2x^2$

10^6 = 1 000 000 (generally named 1 million).

Chapter Review 2

Exercise 1.20

1 Write each of the following in index form:

d^3 Index Base

$3\times3\times3\times3 = \underline{3^4}$	${}^-a\times{}^-a\times{}^-a = \underline{({}^-a)^3}$

a) $1\times1\times1\times1\times1$ **b)** $2\times2\times2\times2$ **c)** $10\times10\times10\times10\times10$

d) $x\times x\times x\times x\times x\times x\times x$ **e)** $6.9\times6.9\times6.9\times6.9$ **f)** ${}^-3\times{}^-3\times{}^-3\times{}^-3\times{}^-3\times{}^-3$

2 Use the Index Laws to simplify each of the following:

$2^2\times2^{-5} = 2^{2+{}^-5}$	$\{a^m\times a^n = a^{m+n}\}$	$5x^{-4}\times3x^3 = 15x^{-4+3}$	$\{a^m\times a^n = a^{m+n}\}$
$= \underline{2^{-3}}$	$\{2 + {}^-5 = {}^-3\}$	$= \underline{15x^{-1}}$	$\{{}^-4 + 3 = {}^-1\}$

a) $5^3\times5^{-4}$ **b)** $d^4\times d^{-3}$ **c)** $2^5\times2^{-2}$ **d)** $10^7\times10^{-4}$

e) $x^5\times x^{-2}$ **f)** $3c^2\times2c^{-6}$ **g)** $5x^3\times x^{-4}$ **h)** $7x^6\times2x^{-5}$

i) $1^3\times1^{-9}$ **j)** $x^{-7}\times x^3\times x^4$ **k)** $6x^4\times2x^{-3}\times x^{-5}$ **l)** $10^6\times10^{-2}$

$5^2\div2^{-3} = 2^{2-{}^-3}$	$\{a^m\div a^n = a^{m-n}\}$	$8x^{-5}\div2x^3 = 4x^{-5-3}$	$\{a^m\div a^n = a^{m-n}\}$
$= \underline{2^5}$	$\{2 - {}^-3 = 2+3 = 5\}$	$= \underline{4x^{-8}}$	$\{{}^-5 - 3 = {}^-8\}$

m) $3^3\div3^{-6}$ **n)** $w^5\div w^{-8}$ **o)** $2^{-4}\div2^2$ **p)** $10^{-8}\div10^{-3}$

q) $m^3\div m^{-4}$ **r)** $x^5\div x^5$ **s)** $7^{-3}\div7^{-4}$ **t)** $x^3\div x^{-2}$

u) $10x^6\div5x^{-2}$ **v)** $6x^4\div3x^{-5}$ **w)** $2^4\times2^4\div2^{-5}$ **x)** $10^9\div10^{-4}$

3 Use the Index Laws to simplify each of the following:

$(3^{-2})^4 = 3^{{}^-2\times4}$	$\{(a^m)^n = a^{m\times n}\}$	$(x^{-2})^{-3} = x^{{}^-2\times{}^-3}$	$\{(a^m)^n = a^{m\times n}\}$
$= \underline{3^{-8}}$	$\{{}^-2\times4 = {}^-8\}$	$= \underline{2^6}$	$\{{}^-2\times{}^-3 = 6\}$

a) $(2^{-3})^3$ **b)** $(8^{-2})^{-3}$ **c)** $(x^4)^2$ **d)** $(10^{-2})^{-3}$

e) $3^4\times3^2$ **f)** $x^7\times x^5$ **g)** $10^5\times10^6$ **h)** $4^{-5}\times4^4$

i) $6.7^3\times6.7^9$ **j)** $5^5\div5^3$ **k)** $10^6\div10^5$ **l)** $2.1^3\div2.1^5$

m) $3^{-6}\div3^3$ **n)** $10^8\div10^{-2}$ **o)** $2x^2\times6x^5$ **p)** $3x^7\times5x^3$

q) $10^4\times10^2\times10^{-5}$ **r)** $10^7\times10^{-3}\times10^{-5}$ **s)** $7x^3\times2x^3\div x^4$

t) $5x^4\times6x^{-3}\div2x^2$ **u)** $(3^{-3})^2\div(3^{-2})^{-2}$ **v)** $a^4\times a^{-3}\div(a^5)^{-3}$

10^9 = 1 000 000 000 (generally named 1 billion).

Algebra 1 2

Number and Algebra → Patterns and algebra

★ Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.
★ Understand that the distributive law can be applied to algebraic expressions as well as numbers, and understanding the inverse relationship between expansion and factorisation.
★ Extend and apply the index laws to variables, using positive integral indices.

A TASK

Can you solve: $x^2 + 2x = 15$?

Quadratic equations play a very important part in solving thousands of problems in our modern society.

Research quadratic equations.

The quadratic equation : $x^2 - 5x = {}^-6$ has two solutions: $x= 2$ and $x = 3$.

$2^2 - 5\times 2 = {}^-6$

$3^2 - 5\times 3 = {}^-6$

A LITTLE BIT OF HISTORY

Algebra has its origins in the work of mathematicians in ancient Babylonia.

There are many Babylonian maths tablets that have survived 2000 years.

The Babylonian solution to the quadratic equation:

$x^2 + bx = c$ was: $x = \frac{-b}{2} + \sqrt{\left(\frac{b}{2}\right)^2 + c}$

The clay tablets were written on with a blunt reed leaving wedge shaped indents. Cuneiform tablet means the writing is wedge shaped.

Algebra Warmup

Algebra is fundamental to solving millions of real world problems.

No big deal. Looks pretty simple to me.

Exercise 2.1

Simplify the following expressions:

$3x + 2x = \underline{5x}$	$5y - 7y = \underline{{}^{-}2y}$	$9b + 5b - b = \underline{13b}$

1 $2x + 4x$	**2** $5a - 3a$	**3** $3c + 6c$
4 $7x - 2x$	**5** $12c + 7c$	**6** $3z - 5z$
7 $19y + 8y$	**8** $4w - 6w$	**9** $2x + 6x$
10 $10x - 3x$	**11** $5b - 10b$	**12** $d + 3d$
13 $8a + 5a$	**14** $7x - 9x$	**15** $3m + 6m$
16 $5x + x + 3x$	**17** $8h - 3h - 2h$	**18** ${}^{-}6x + 2x + 3x$
19 $9x + 2x - 3x - x$	**20** $8a + 2a - 4a - 5a$	

The key is to only join together the terms that are alike.

$7a + 5 - 4a = \underline{3a + 5}$	$5y^2 + 4y + y - 2y^2 = \underline{3y^2 + 5y}$

21 $3x + 4 + 2x$	**22** $7 + 4b + 2b$
23 $8 + 5x - 2x$	**24** $4b - 4a + 2b$
25 $8x + 5x + 7$	**26** $7a + 4b - 4a + 2b$
27 $6x + 3d - 5x + d$	**28** $9 + x + 3x - 7$
29 $8b^2 - 4 + 5b^2 + 9$	**30** $7x^3 + 5 - 5x^3 + 3$
31 $8xy + 2xy + 6 - 5$	**32** $6as^2 + 5d^5 - 2as^2 + 2d^5$
33 $3x + 4 + 2x - 6x$	**34** ${}^{-}14x + 6y + 9x - 8y + y$

Multiply the numbers.
Multiply the letters.

Exercise 2.2

Simplify the following expressions:

$4 \times 2x = 4 \times 2 \times x$ $= \underline{8x}$	$3d \times 5e = 3 \times d \times 5 \times e$ $= \underline{15de}$	$10b \times \frac{1}{5} = 10 \times \frac{1}{5} \times b$ $= \underline{2b}$

1 $3 \times 5x$	**2** $2 \times 7a$	**3** $3 \times 6m$
4 $4 \times 3x$	**5** $3 \times 5b$	**6** $p \times 4$
7 $3x \times 9$	**8** $2f \times 4n$	**9** $7x \times 2y$
10 $5h \times 2b$	**11** $8t \times 3d$	**12** $g \times 7k$
13 $\frac{m}{2} \times 4n$	**14** $10r \times \frac{p}{2}$	**15** $8m \times \frac{k}{4}$

Multiply the numbers.
Multiply the letters.

Index Law 1

$a^m \times a^n = a^{m+n}$

+ times − = −
− times + = −
− times − = +

$4a \times 3a = 4 \times 3 \times a \times a$ $= \underline{12a^2}$	$5mn \times {}^{-}2m^2n = 5 \times {}^{-}2 \times m \times m^2 \times n \times n$ $= \underline{{}^{-}10m^3n^2}$

m x m^2 = m x m x m = m^3

16 $4x \times 3x$ **17** $3d \times 4d$
18 $5a \times 3a$ **19** $7d \times 3d$
20 $5x \times {}^{-}2x$ **21** $2x \times {}^{-}3x$ **22** ${}^{-}3x \times 4x$
23 ${}^{-}4a^2 \times {}^{-}3a$ **24** $6p \times {}^{-}2p^3$ **25** ${}^{-}9w \times 3w^2$
26 $8s^3 \times 2s^2$ **27** $4x^2 \times 5x \times 2x$ **28** $3e \times e^2 \times 2d$
29 $5mn \times {}^{-}3m^2n$ **30** $7pn \times {}^{-}4p^2n$ **31** $4ab \times {}^{-}6a^2b$
32 ${}^{-}3p^2d \times {}^{-}2pd$ **33** ${}^{-}4h^2 \times {}^{-}4h$ **34** ${}^{-}4a^2b^2c \times {}^{-}5a^2bc$

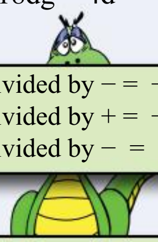
$9a \div 6a$ and $\frac{9a}{6a}$ and $\frac{9}{6}$ and $\frac{3}{2}$ are the same thing.

Exercise 2.3
Simplify the following expressions:

$8x \div 2 = \frac{8x}{2}$ $= \underline{4x}$	$9a \div 6a = \frac{9a}{6a}$ $= \underline{\frac{3}{2}}$	$18ay \div 4a = \frac{18ay}{4a}$ $= \underline{\frac{9y}{2}}$

1 $8a \div 2$ **2** $12x \div 3$ **3** $6c \div 2$
4 $15x \div 5$ **5** $14y \div 7$ **6** $8n \div 4$
7 $24k \div 6$ **8** $30d \div 10$ **9** $9x \div 6$
10 $20x \div 15x$ **11** $14g \div 4g$ **12** $6y \div 4y$
13 $12ay \div 4a$ **14** $14de \div 4e$ **15** $18dg \div 4d$

Divide the numbers.
Divide the letters



+ divided by − = −
− divided by + = −
− divided by − = +

$${}^{-}6x^5 \div 4x^2y = \frac{-6x^5}{4x^2y} = \underline{\frac{-3x^3}{2y}}$$

Index Law 2

$a^m \div a^n = a^{m-n}$

Calculators are very good at handling fractions:
See Technology 2.1

16 ${}^{-}8x \div 4$ **17** $6p \div {}^{-}3$ **18** ${}^{-}12y \div {}^{-}2y$
19 ${}^{-}4g \div 2g$ **20** ${}^{-}12a^5 \div {}^{-}4a^2$ **21** $8v^7 \div {}^{-}4v^4$
22 ${}^{-}12a^7y \div {}^{-}4a^3$ **23** $14de^4 \div {}^{-}7e^2$ **24** ${}^{-}18dg \div 4d$
25 ${}^{-}16b^8c^2 \div 24b^6$ **26** ${}^{-}28a^5d^3 \div {}^{-}12d^2$ **27** ${}^{-}24d^5w^6z \div 36d^3w^3$

Distributive Law

The distributive law:
$a(b + c) = ab + ac$

Each term inside the brackets: **b** and **c** is multiplied by the term outside the brackets: **a** to give: **$ab + ac$**

ie., **a** is distributed through the brackets.

Exercise 2.4

Expand each of the following:

Multiply each inside term by the outside term.

$4(a + 3) = \underline{4a + 12}$	$3(2b - 5) = \underline{6b - 15}$

1 $4(b + 3)$	**2** $5(c + 2)$	**3** $2(a + 7)$
4 $3(g + 1)$	**5** $6(h + 5)$	**6** $7(n + 4)$
7 $2(2z - 4)$	**8** $5(2s - 4)$	**9** $3(4d - 3)$
10 $4(3f - 7)$	**11** $8(2a - 5)$	**12** $6(5h - 6)$

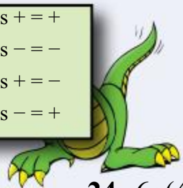
$^{-}4(a + 3) = \underline{^{-}4a - 12}$	$^{-}3(2b - 5) = \underline{^{-}6b + 15}$

13 $^{-}2(a + 3)$	**14** $^{-}5(r + 2)$	**15** $^{-}2(c + 4)$
16 $^{-}5(2m - 4)$	**17** $^{-}3(y + 2)$	**18** $^{-}9(w + 6)$
19 $^{-}3(2c - 4)$	**20** $^{-}4(3e - 4)$	**21** $^{-}5(4v + 3)$

$5w \times 3w = 15ww = 15w^2$

$5w(3w - 2m) = \underline{15w^2 - 10mw}$

+ times + = +
+ times − = −
− times + = −
− times − = +



22 $t(2t + 3)$	**23** $3z(4z + 5)$	**24** $6g(4g + 5)$
25 $2d(3d - 4)$	**26** $3n(2n - 4)$	**27** $m(7m - 2)$
28 $^{-}t(2t + 3)$	**29** $^{-}3z(4z - 2)$	**30** $^{-}6g(4g + 5)$
31 $^{-}p(3 + 2p)$	**32** $^{-}4e(3e - 2c)$	**33** $^{-}4u(2b - 4u)$

Exercise 2.5

Simplify each of the following by expanding and then collecting like terms:

$8(2x + 3) + 5x + 7$	$3(5a - 2) + 3a - 9$
$= 16x + 24 + 5x + 7$	$= 15a - 6 + 3a - 9$
$= \underline{21x + 31}$	$= \underline{18a - 15}$

1 $2(x + 3) + 3x + 5$	**2** $4(x - 3) + 2x - 1$
3 $3(a - 4) + 9a + 13$	**4** $6(2b + 6) - 10b + 3$
5 $5(x - 2) - 3x - 10$	**6** $2(5x - 2) + 5x + 4$
7 $4(y - 8) + 4y - 10$	**8** $2y(3y + 1) + 8y^2 + 3y + 2$
9 $t(2t + 3) + 5t^2 + 6t$	**10** $3z(4z + 5) + 15z^2 + 10z$

Distribute - to spread out, to cover everything.

The Distributive Law: $a(b + c) = ab + ac$

Exercise 2.6

Simplify each of the following by expanding and then collecting like terms:

$3(x + 2) + 2(x + 4)$	$^-5 \times 3 = -5$
$= 3x + 6 + 2x + 8$	$^-5(x + 3) + 3(x - 1)$
$= \underline{5x + 14}$	$= {}^-5x - 15 + 3x - 3$
	$= \underline{{}^-2x - 18}$
	$3 \times -1 = -3$

1 $2(x + 3) + 3(x + 1)$

2 $^-5(x + 2) + 2(x + 4)$

3 $2(c + 4) + 3(c + 3)$

4 $^-4(d + 5) + 3(d + 1)$

5 $5(h + 1) + 2(h + 3)$

6 $^-2(x + 7) + 2(x - 2)$

7 $3(m + 4) + 2(m + 2)$

8 $3(y - 2) + 2(y - 3)$

9 $5(w - 2) + 3(w + 4)$

10 $^-3(c - 4) + 3(c + 2)$

11 $4(e - 4) + 3(e + 5)$

12 $^-5(v + 3) + {}^-2(v - 4)$

13 $^-t(2t + 3) + t(3t - 4)$

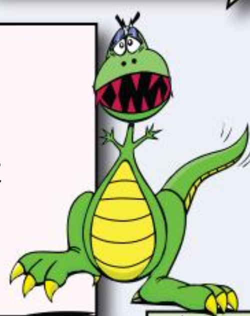
14 $^-3z(4z - 2) + {}^-2(z - 4)$

+ times + = +
+ times − = −
− times + = −
− times − = +

Multiply each inside term by the outside term.

The Distributive Law:

$$a(b + c) = ab + ac$$



+ times + = +
+ times − = −
− times + = −
− times − = +

$4(x + 3) - 3(x + 4)$	$^-4 \times -1 = 4$
$= 4x + 12 - 3x - 12$	$^-4(x - 1) - 2(x - 4)$
$= \underline{x}$	$= {}^-4x + 4 - 2x + 8$
$-3 \times 4 = -12$	$= \underline{{}^-6x + 12}$
	$-2 \times -4 = 8$

15 $3(x + 4) - 2(x + 2)$

16 $^-4(x - 1) - 2(x - 2)$

17 $2(x + 5) - 3(x + 4)$

18 $^-5(y + 2) - 2(y + 6)$

19 $4(a + 1) - 6(a - 2)$

20 $^-2(b + 2) - 2(b - 3)$

21 $^-5(n - 1) - 3(n - 4)$

22 $^-7(y - 1) - 5(y - 2)$

Distributive Law

The Distributive Law:

$$a(b + c) = ab + ac$$

Multiply each inside term by the outside term.

Exercise 2.7

Simplify each of the following by expanding and then collecting like terms:

$(x + 5)(x + 4)$	$(x + 3)^2 = (x + 3)(x + 3)$
$= x(x + 4) + 5(x + 4)$	$= x(x + 3) + 3(x + 3)$
$= x^2 + 4x + 5x + 20$	$= x^2 + 3x + 3x + 9$
$= \underline{x^2 + 9x + 20}$	$= \underline{x^2 + 6x + 9}$

1 $(x + 1)(x + 2)$ **2** $(x + 2)(x + 1)$

3 $(x + 3)(x + 1)$ **4** $(x + 1)(x + 4)$

5 $(x + 2)(x + 4)$ **6** $(x + 1)^2$

7 $(x + 2)^2$ **8** $(x + 3)^2$

9 $(2x + 1)(x + 1)$ **10** $(2x + 1)(x + 2)$

+ times + = +
+ times – = –
– times + = –
– times – = +

Simplify each of the following by expanding and then collecting like terms:

$(x + 5)(x - 3)$	$(x - 4)^2 = (x - 4)(x - 4)$
$= x(x - 3) + 5(x - 3)$	$= x(x - 4) - 4(x - 4)$
$= x^2 - 3x + 5x - 15$	$= x^2 - 4x - 4x + 16$
$= \underline{x^2 + 2x - 15}$	$= \underline{x^2 - 8x + 16}$

11 $(x + 3)(x - 1)$ **12** $(x + 4)(x - 1)$

13 $(x + 2)(x - 2)$ **14** $(x + 4)(x - 4)$

15 $(x + 5)(x - 1)$ **16** $(x + 2)(x - 3)$

17 $(x + 1)(x - 3)$ **18** $(x + 2)(x - 3)$

19 $(x - 1)^2$ **20** $(x - 2)^2$

21 $(x - 3)^2$ **22** $(x - 5)^2$

23 $(2x + 3)(x - 1)$ **24** $(3x + 1)(2x - 2)$

25 $(2x - 2)(x + 1)$ **26** $(3x - 1)(2x - 2)$

Factorisation

The inverse of distribution is called factorisation. In factorisation, the highest common factor is taken from each term.

Factorisation:

$$ab + ac = a(b + c)$$

The common term, a, is taken out and put at the front.

Exercise 2.8
Factorise each of the following:

$2x + 6$ $= 2(x + 3)$	$8a + 4$ $= 4(2a + 1)$	$6x^2 + 9x$ $= 3x(2x + 3)$

1	$2a + 6$	**2**	$2b + 4$	**3**	$2c + 10$
4	$5x + 10$	**5**	$3m + 6$	**6**	$4n + 8$
7	$3p + 9$	**8**	$5d + 20$	**9**	$7h + 35$
10	$6a + 3$	**11**	$10u + 5$	**12**	$9r + 3$
13	$15x + 3$	**14**	$18g + 6$	**15**	$21s + 7$
16	$5p + 40$	**17**	$3n + 27$	**18**	$35x + 5$

Algebra is an essential tool in thousands of careers and is fundamental to solving millions of problems.

A bit of factorisation preparation.

Exercise 2.9
Find the highest common factor of each of the following pairs of terms:

3a and 6a The factors of 3a are: 3, a The factors of 6a are: 2, 3, 6, a The highest common factor = 3a	4ef and 8fg The factors of 4ef are: 2, 4, e, f The factors of 6a are: 2, 4, 8, f, g The highest common factor = 4f

1	3x and 6x	**2**	4a and 8ab	**3**	6ab and 10a
4	4b and 6ab	**5**	3xy and 9y	**6**	5s and 10d
7	3g and 12	**8**	12e and 4	**9**	8ab and 12abc
10	6p and 14pq	**11**	4p and 16	**12**	8a and 2b
13	6 and 12y	**14**	5xy and 15yz	**15**	15ef and 27fg
16	16g and 8	**17**	16h and 64hij	**18**	14rt and 35t

Factorisation

Factorising is the inverse of distributing.

The highest common factor, a, is taken out and put at the front.

Factorisation:

$$ab + ac = a(b + c)$$

Exercise 2.10

Factorise each of the following:

$4x - 12$	$6ab - 8b$	$7t^2 - 5t$
$= \underline{4(x - 3)}$	$= \underline{2b(3a - 4)}$	$= \underline{t(7t - 5)}$

1 $4a - 12$	**2** $2b - 6$	**3** $5x - 15$
4 $3g - 6$	**5** $7w - 28$	**6** $3d - 6$
7 $10a - 4$	**8** $10s - 5$	**9** $15bc - 3c$
10 $6u - 8$	**11** $8pq - 6p$	**12** $12x - 8$
13 $10uv - 15u$	**14** $18b - 6$	**15** $14c^2 - 21c$
16 $16h^2 - 12h$	**17** $21d^3 - 14d$	**18** $24p^3 - 12p^2$

+ times + = +
+ times − = −
− times + = −
− times − = +

Exercise 2.11

Factorise each of the following:

$^-5x - 30$	$^-6x^2 + 15x$	$^-20p^3 - 12p^2$
$= \underline{^-5(x + 6)}$	$= \underline{^-3x(2x - 5)}$	$= \underline{^-4p^2(5p + 3)}$

For these problems, take out the negative common factor.

1 $-5z - 10$	**2** $-2a - 10$	
3 $-3x - 12$	**4** $-6q - 18$	
5 $-5d - 20$	**6** $-4b - 16$	
7 $-9m - 18$	**8** $-2n - 22$	**9** $-5b - 35$
10 $-2d + 6$	**11** $-3w + 27$	**12** $-6p + 36$
13 $-4x^2 + 12x$	**14** $-5g^2 + 25g$	**15** $-2e^2 - 26e$
16 $-8b^2c + 2b$	**17** $-6q^2 + 3q$	**18** $-15ao^2 + 5o$
19 $-6a^3 + 15a$	**20** $-12t^4 - 15t^2$	**21** $-6p^5 - 36p^3$

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 2.12

1 Spell Distributive.
2 Simplify: $2a + 6a$
3 Expand: $3(x - 2)$
4 Factorise: $6x + 4$
5 $5 - 7$
6 $10^2 \times 10^3$
7 $(2^3)^2$
8 Complete: $4\times17 = 4(10 + 7) = 4\times10 + 4\times7 =$
9 134×11
10 I pay \$300 per week on rent, roughly how much per year?

Roughly 50 weeks
$\$300\times50 = \$300\div2\times100$
$= \$150\times100$
$= \$15\ 000$

$134\times11 = 1474$
Write the 1st and last numbers: 1.........4
Sum each consecutive pair: 1+3=4, 3+4=7
Insert these between the 1st and last: 1474

Exercise 2.13

1 Spell Factorise.
2 Simplify: $8x + 3x$
3 Expand: $4(x - 3)$
4 Factorise: $9x + 6$
5 $^-2 - 4$
6 $10^5 \div 10^3$
7 $(x^2)^5$
8 Complete: $5\times28 = 5(20 + 8) = 5\times20 + 5\times8 =$
9 2416×11
10 I pay \$360 per week on rent, roughly how much per year?

Small opportunities are often the beginning of great enterprises - Demosthenes.

Exercise 2.14

1 Spell Algebra.
2 Simplify: $6y + 4y$
3 Expand: $^-4(x - 2)$
4 Factorise: $6x + 8$
5 $5 + {}^-9$
6 $10^4\times10^2$
7 $(x^3)^3$
8 Complete: $3\times42 = 3(40 + 2) = 3\times40 + 3\times2 =$
9 6251×11
10 I pay \$480 per week on rent, roughly how much per year?

In what month do people eat the least? February – it's the shortest month.

NAPLAN Questions

Exercise 2.15

The value of $\frac{5x}{3x-4}$ when $x = 3$? $= \frac{5 \times 3}{3 \times 3 - 4}$ $= \frac{15}{5} = \underline{3}$	The value of $3x^2 - 2x + 5$ when $x = {}^{-}1$? $= 3\times({}^{-}1)^2 - 2\times{}^{-}1 + 5$ $= 3\times1 + 2 + 5$ $= \underline{10}$

1 If $x = 4$, what is the value of $\frac{5x}{2x-3}$?

2 $y = 10 - 3x$, what is the value of y when $x = 2.5$?

3 What is the value of $3x^2 - 5x + 1$ when $x = {}^{-}1$?

4 $A = 3d^2$ What is the value of A when $d = 5$?

5 $M = \frac{5a}{9b}$, what is the value of M when $a = 1.8$ and $b = 0.2$?

6 Given that $P = 50b + 250$, what is b when $P = 600$?

7 $25\phi = 30$ What is the value of ϕ?

One method is to substitute values for x until both expressions are the same.

8 $y = 3x + 2$
$y = 4x - 1$ What value of x satisfies both of these equations?

9 $y = 5x - 2$
$y = 3x + 6$ What value of x satisfies both of these equations?

10 A rule for a pattern is multiply by four and then add 3. The first three numbers of this pattern are: 7, 11, 15, ... What is the fifth number in this pattern?

11 A rule for a pattern is to add six and then divide by five. The first three numbers of this pattern are: 1.4, 1.6, 1.8, ... What is the fifth number in this pattern?

12 If $a = 3$, what is the value of $6a$?

13 Expand: $2(5x + 1)$

14 Expand: ${}^{-}3(4a - 1)$

15 What is the value of $a^2 + b^2$ when $a = {}^{-}2$ and $b = 3$?

16 What is the value of $5x^2$ when $x = {}^{-}2$?

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

Order of Operations:

1 ( ) brackets first.

2 × and ÷ from left to right.

3 + and − from left to right.

Exercise 2.16

1 Evaluate each of the following:

a) $1 + 2 \times 3 - 4$

b) $15 - 12 \div 3$

c) $2 \times 5 - 6 \div 2$

d) $(10 + 2) \times 5 - 5$

e) $((((1 - 2) - 3) - 4) - 5)$

f) $6 - (5 - (4 - (3 - (2 - 1))))$

2 Simplify each of the following:

a) $2^3 \times 2^2$

b) $2^5 \div 2^3$

c) $2^2 \div 2^3 \times 2^4$

d) $3^7 \div 3^9 \times 3^2$

$2^7 \div 2^5$

$= 2^{7-5}$

$= \underline{2^2 \text{ or } 4}$

3 Simplify each of the following:

a) $5x + 3y - 3x + 2y$

b) $3x - 4y - 5x + y$

c) $(2a + b) - (a - 3b)$

d) $4(x - 2) - 3(x + 5)$

e) $(x - 1) - (1 - x)$

f) $2x(3x - 1) + 5x^2$

$2(x - 1) - 3(x - 4)$

$= 2x - 2 - 3x + 12$

$= \underline{{}^{-}x + 10}$

4 What is the next term in the following sequence? $64, 2^6, 4^3$, ...

5 What is the next term in the following sequence? $729, 3^6, 9^3$, ...

6 If $2^{(x+2)} = 16$, what is the value of x?

7 If $4^{(2x-1)} = 8$, what is the value of x?

8 If x and y are positive numbers, which of the following is the largest?

a) $(x + y)^2$

b) $x^2 + y^2$

c) $x^2 + xy + y^2$

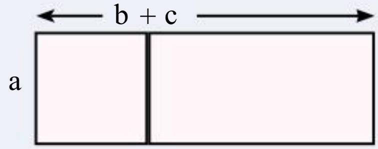
Stockbrokers buy and sell shares and bonds for clients.

- Relevant school subjects are Mathematics and English.
- Courses usually involve a Universtity Bachelor degree with a major in commerce/finance.

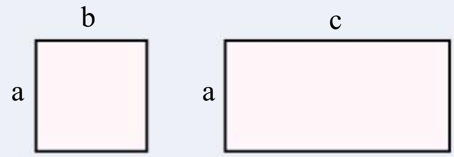
Investigations

Investigation 2.1 **The Distributive law: $a(b + c) = ab + ac$**

1 Write an algebraic expression for the area of the rectangle shown on the right.



2 Write an algebraic expression for the area of each of the two rectangles shown on the right and sum them together.



3 What do you notice?

Investigation 2.2 **The Distributive law: $a(b + c) = ab + ac$**

Use centicubes or counters to prove the distributive law for each of the following instances:

1 $2(3 + 1) = 2x3 + 2x1$

2 lots of (3 + 1) is the same as 2 lots of 3 and 2 lots of 1

2 $3(2 + 1)= 3x2 + 3x1$

3 $2(4 + 2) = 2x4 + 2x2$

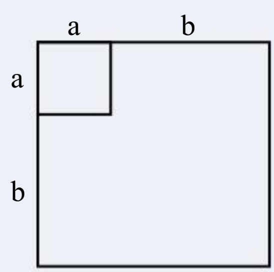
4 $4(3 + 2) = 4x3 + 4x2$

Investigation 2.3 $\mathbf{(a + b)(a + b) = a^2 + 2ab + b^2}$
$\mathbf{(a + b)(a - b) = a^2 - b^2}$

Use one of the above methods to prove either:

$$\mathbf{(a + b)(a + b) = a^2 + 2ab + b^2}$$

or $$\mathbf{(a + b)(a - b) = a^2 - b^2}$$

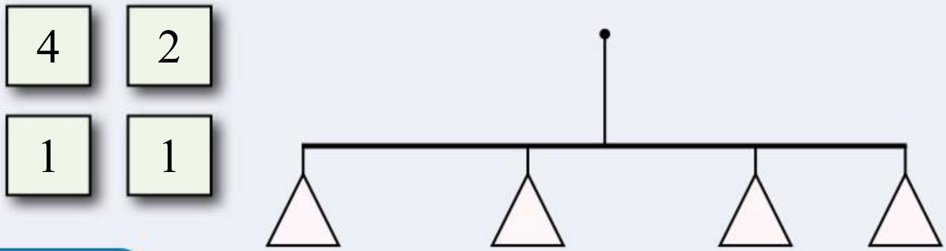


A Couple of Puzzles

Exercise 2.17

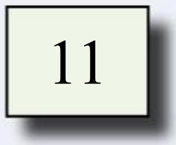
1 Who am I? If you subtract me from 56 and then double the answer, the result is 26.

2 Put the four weights on the pans so that the balance balances.



A Game

Guess



1 One player thinks of a number from 1 to 20 and writes it on a piece of paper.

2 The other player asks a series of questions to find out the number.
The first player can only answer Yes or No.

1	Is the number even?	**NO**
2	Is the number >10?	**YES**
3	Is the number > 14?	**NO**
4	Is the number = 11?	**YES**

3 Switch players.

The winner is the player asking the least number of questions.

A Sweet Trick

1	Ask your audience to write down	6
	two numbers less than 20.	13
2	They add 1st to 2nd to make a 3rd.	19
3	They add 2nd to 3rd to make a 4th.	32
4	Repeat until there are ten numbers.	51
		83
		134
		217
		351
		568

5 Ask them to total all ten numbers.

Before they finish you call out the answer: 1474

How? Multiply the seventh number by 11 (134×11). Why?

Technology

Technology 2.1 Simplifying Fractions
Scientific calculators are excellent in working with fractions:

1 Simplify $\frac{15}{35}$ $\boxed{15}$ $\boxed{a\frac{b}{c}}$ $\boxed{35}$ $\boxed{=}$ $\boxed{3r7}$ meaning $\frac{3}{7}$

2 Simplify $\frac{18}{4}$ $\boxed{18}$ $\boxed{a\frac{b}{c}}$ $\boxed{4}$ $\boxed{=}$ $\boxed{4r1r2}$ meaning $4\frac{1}{2}$

To change to a vulgar fraction: $\boxed{2ndF}$ $\boxed{a\frac{b}{c}}$ to give $\boxed{9r2}$ ie $\frac{9}{2}$

3 Use a scientific calculator to simplify the following ratios:

a) 3 : 9 **b)** 9 : 12 **c)** 16 : 24
d) 2.1 : 3.5 **e)** 14.4 : 12.6 **f)** 256 : 1024

Technology 2.2 Expanding and Factorising
Graphics calculators are capable of expanding and factorising:

1 Choose **expand** from the algebra menu.
2 Enter the algebraic expression: $3(4x - 5)$ to produce $12x - 15$

1 Choose **factor** from the algebra menu.
2 Enter the algebraic expression: $12x - 15$ to produce $3(4x - 5)$

Technology 2.3 The Distributive Law and Factorising
There are a considerable number of resources about the Distributive Law and factorising on the Internet.

Try some of them.

Technology 2.4 Substitution
Use a spreadsheet to check your answers to previous exercises:

Use any substituting value other than 0

	A	B
1	Substituting value	3
2	2x + 6	12
3	2(x + 3)	12

Enter the first expression =2*B1 + 6

Enter the second expression =2*(B1 + 3)

Chapter Review 1

Exercise 2.18

$5y - 7y = {}^{-}2y$

1 Simplify the following expressions:

a) $5x - 2x$ **b)** $7x + 3x$ **c)** $3x - 5x$

d) $6 + 9x - 2x$ **e)** $4b - 4a + 3b$ **f)** $3rs^2 + 5x^5 - 2rs^2 + 3x^5$

g) $5 \times 3x$ **h)** $3x \times 5$ **i)** ${}^{-}3a \times 2a$

j) ${}^{-}4x^2 \times {}^{-}5x$ **k)** $5y \times {}^{-}2y^3$ **l)** $2x^2 \times 3x \times 2x$

m) $20x \div 5$ **n)** $21y \div 7$ **o)** ${}^{-}8g \div 2g$

p) ${}^{-}12x \div 3$ **q)** ${}^{-}14y \div {}^{-}2y$ **r)** $18ab \div 4a$

s) ${}^{-}10a^5 \div {}^{-}4a^2$ **t)** ${}^{-}24d^5e^6f \div 36d^3e^3$

${}^{-}3(2y - 5) = {}^{-}6y + 15$

2 Expand each of the following:

a) $3(x + 2)$ **b)** $5(a + 4)$ **c)** $6(y + 7)$

d) $2(3d - 4)$ **e)** $7(3h - 4)$ **f)** $3(2x - 5)$

g) ${}^{-}5(3t - 4)$ **h)** ${}^{-}2(y + 3)$ **i)** ${}^{-}8(p + 4)$

j) ${}^{-}x(2x + 3)$ **k)** ${}^{-}3x(4x - 2)$ **l)** ${}^{-}6m(4m + 5)$

m) ${}^{-}g(2 + 5g)$ **n)** ${}^{-}4x(3x - 2y)$ **o)** ${}^{-}4x(2y - 4x)$

3 Simplify each of the following by expanding and then collecting like terms:

a) $2(x + 3) + 5(x + 4)$ **b)** ${}^{-}2(x + 5) + 4(x + 1)$

c) $5(y - 2) + 2(y + 4)$ **d)** ${}^{-}4(y - 3) + 3(y + 2)$

e) ${}^{-}p(2p + 1) + p(3p - 4)$ **f)** ${}^{-}5b(4b - 2) + {}^{-}2(b - 1)$

g) $(x + 3)(x + 2)$ **h)** $(x + 1)^2$

i) $(x + 5)(x - 1)$ **j)** $(x + 1)(x - 2)$

k) $(x + 2)(x - 2)$ **l)** $(x + 3)(x - 3)$

m) $(2x - 2)(x + 1)$ **n)** $(3x - 1)(2x - 3)$

$12ab - 8a = 4a(3b - 2)$

4 Factorise each of the following:

a) $5x + 15$ **b)** $3y + 6$ **c)** $4a + 16$

d) $18x + 3$ **e)** $18e + 6$ **f)** $14d + 7$

g) $4n - 8$ **h)** $2m - 6$ **i)** $5x - 15$

j) $16y - 4$ **k)** $10x - 5$ **l)** $15ab - 3a$

m) $6t - 10$ **n)** $12x - 8$ **o)** $8uv - 6u$

p) $14f^2 - 12f$ **q)** $21x^3 - 14x$ **r)** $24p^3 - 18p^2$

s) $-5a - 15$ **t)** $-2b - 8$ **u)** $-2c + 6$

v) $-4x^2 + 12x$ **w)** $-4g^2 + 24g$ **x)** $-2x^2 - 20x$

y) $-6x^3 + 15x$ **z)** $-12h^5 - 15h^2$

$-9x^5 - 12x^2 = {}^{-}3x^2(3x^3 + 4)$

Chapter Review 2

Exercise 2.19

$5y - 7y = {}^{-}2y$

1 Simplify the following expressions:

a) $9x - 2x$ **b)** $5x + 4x$ **c)** $4x - 8x$

d) $3 + 6x - 3x$ **e)** $9b - 3a + 2b$ **f)** $6fg^2 + 3z^4 - 5fg^2 + 3z^4$

g) $6 \times 2x$ **h)** $8x \times 2$ **i)** ${}^{-}5a \times 3a$

j) ${}^{-}2x^2 \times {}^{-}3x$ **k)** $4k \times {}^{-}2k^3$ **l)** $3x^2 \times 4x \times x$

m) $24x \div 6$ **n)** $28o \div 7$ **o)** ${}^{-}6h \div 2h$

p) ${}^{-}16x \div 4$ **q)** ${}^{-}12a \div {}^{-}6a$ **r)** $20mn \div 4m$

s) ${}^{-}14a^6 \div {}^{-}7a^4$ **t)** ${}^{-}24a^5e^6t \div 20a^4e^3$

${}^{-}3(2y - 5) = {}^{-}6y + 15$

2 Expand each of the following:

a) $2(x + 5)$ **b)** $4(z + 3)$ **c)** $5(y + 3)$

d) $6(2w - 4)$ **e)** $9(3c - 5)$ **f)** $4(2x - 1)$

g) ${}^{-}4(3v - 2)$ **h)** ${}^{-}6(r + 1)$ **i)** ${}^{-}7(u + 5)$

j) ${}^{-}a(4a + 3)$ **k)** ${}^{-}5x(2x - 3)$ **l)** ${}^{-}6d(2d + 3)$

m) ${}^{-}p(2 + 3p)$ **n)** ${}^{-}4y(3x - 2y)$ **o)** ${}^{-}4y(2y - 4x)$

3 Simplify each of the following by expanding and then collecting like terms:

a) $3(x + 1) + 2(x + 4)$ **b)** ${}^{-}3(x + 2) + 4(x + 3)$

c) $6(w - 3) + 2(w + 5)$ **d)** ${}^{-}2(f - 3) + 5(f + 2)$

e) ${}^{-}b(2b + 5) + b(3b - 4)$ **f)** ${}^{-}4g(4g - 5) + {}^{-}3(g - 1)$

g) $(x + 2)(x + 3)$ **h)** $(x + 1)^2$

i) $(x + 4)(x - 1)$ **j)** $(x + 2)(x - 1)$

k) $(x + 1)(x - 1)$ **l)** $(x + 2)(x - 2)$

m) $(2x - 1)(x + 1)$ **n)** $(3x - 1)(2x - 1)$

$12ab - 8a = 4a(3b - 2)$

4 Factorise each of the following:

a) $3x + 15$ **b)** $2a + 8$ **c)** $6b + 12$

d) $15c + 3$ **e)** $18d + 9$ **f)** $12e + 6$

g) $3f - 9$ **h)** $3g - 6$ **i)** $5h - 15$

j) $20m - 4$ **k)** $15x - 5$ **l)** $15bc - 5b$

m) $12u - 10$ **n)** $14v - 8$ **o)** $10st - 6s$

p) $12r^2 - 15r$ **q)** $18x^4 - 12x$ **r)** $12y^4 - 18y^2$

s) $-5b - 20$ **t)** $-2d - 10$ **u)** $-3c + 6$

v) $-6x^5 + 12x$ **w)** $-6f^2 + 24f$ **x)** $-5x^2 - 20x$

y) $-12x^3 + 15x$ **z)** $-18h^7 - 15h^2$

$-9x^5 - 12x^2 = {}^{-}3x^2(3x^3 + 4)$

Area 3

Measurement & Geometry ➡ Using units of measurement

★ Calculate the areas of composite shapes.
★ Understand that partitioning composite shapes into rectangles and triangles is a strategy for solving problems involving perimeter and area.
★ Analyse nets of prisms and cylinders to establish formulas for surface area.
★ Calculate the surface area of cylinders and right prisms and solve related problems.
★ Become fluent with calculation of area and identify that area is used in the workplace and everyday life.

A TASK

Allen's Rule suggests that animals from colder climates usually have shorter limbs and smaller ears. The reason being that a smaller surface area to volume ratio reduces heat loss.

Research Allen's Rule

- What is Allen's Rule.
- Find examples that may support Allen's Rule.
- Design an experiment to demonstrate Allen's Rule.
- What advantages would a high surface area to volume ratio be to an animal or plant?

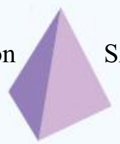
A LITTLE BIT OF HISTORY

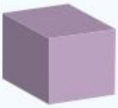
1847 Bergmann's Rule suggests that large animals are found in colder climates and small animals are found in warmer climates.

1877 Allen's Rule suggests that animals with a smaller surface area to volume ratio are better able to survive colder climates.

1937 Hesse's Rule suggests that animals with a larger heart to body weight are found in colder climates compared to animals in warmer climates.

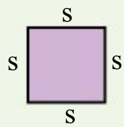
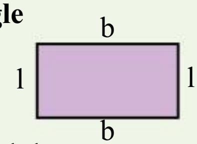
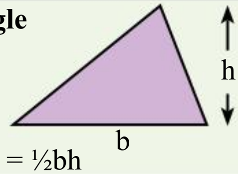
Surface area/volume ratio

Tetrahedron  SAV= 7.2

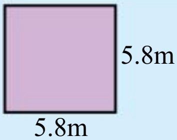
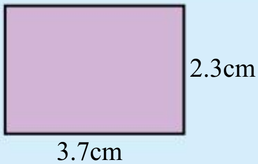
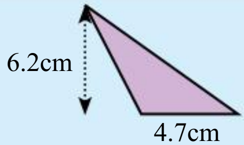
Cube  SAV=6

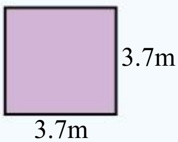
Sphere  SAV=4.8

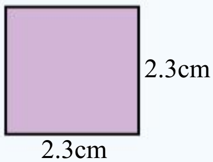
Area Warmup

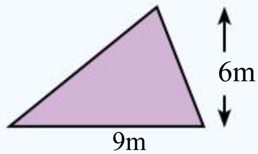
Square	**Rectangle**	**Triangle**
		
Area = s^2	Area = l×b	Area = ½bh

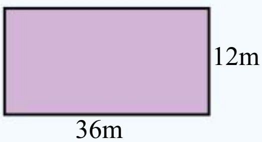
Exercise 3.1
Calculate the area of each of the following shapes:

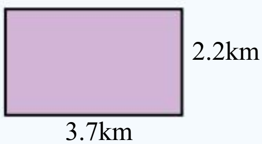
		
Area = s^2 = $(5.8\text{m})^2$ = $33.64\ \text{m}^2$	Area = l×b = 3.7cm × 2.3cm = $8.51\ \text{cm}^2$	Area = ½bh = 0.5×4.7cm×6.2cm = $14.57\ \text{cm}^2$

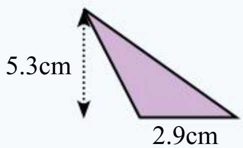
1 

2 

3 

4 

5 

6 

7 A kitchen bench top is 1.4 m by 2.9 m. How many square metres of laminate is needed to cover the top of the bench?

8 A rectangular paddock is 124 m by 111 m. What is the area of the paddock in square metres and hectares (1 hectare = 10 000m^2)?

A hectare is the area of a square 100 m by 100 m.

9 A triangular road sign has a base of 20 cm and a perpendicular height of 32 cm. What is the area of the road sign?

10 A bedroom is 3.3 m by 2.8 m. How many square metres of carpet is needed to cover the floor of the room?

11 A paddock, in the shape of a triangle, has a base of 648 m and a perpendicular height of 457 m. What is the area of the paddock in square metres and hectares?

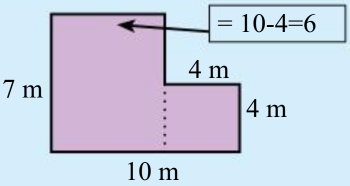
12 The builder wants to put a 2 m wide concrete path around the outside of a 12 m square building. What is the area of the path?

Composite Shapes

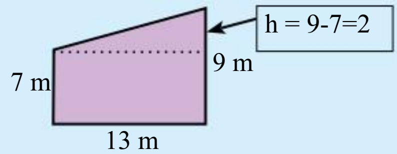
Composite shapes can be squares, rectangles, and triangles composed together.

Exercise 3.2

Calculate the area of each of the following composite shapes:

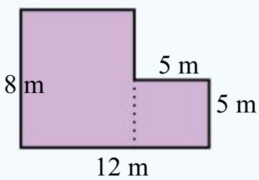


Area $= \text{rectangle} + \text{square}$
$= lb + s^2$
$= 7\times6 + 4\times4 \text{ m}^2$
$= 42 + 16 \text{ m}^2$
$= \underline{58 \text{ m}^2}$



Area $= \text{triangle} + \text{rectangle}$
$= \tfrac{1}{2}bh + lb$
$= 0.5\times13\times2 + 13\times7 \text{ m}^2$
$= 13 + 91 \text{ m}^2$
$= \underline{104 \text{ m}^2}$

1



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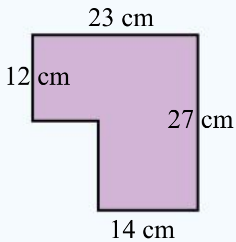

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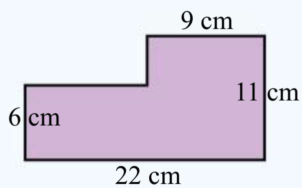

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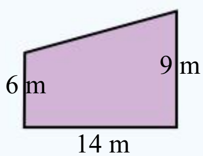

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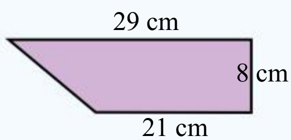

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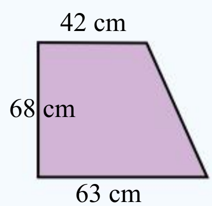

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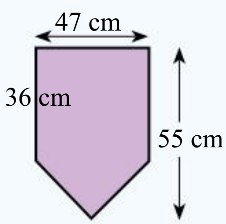

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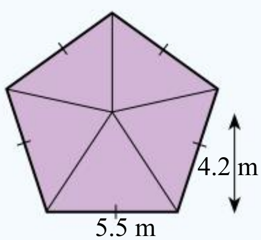

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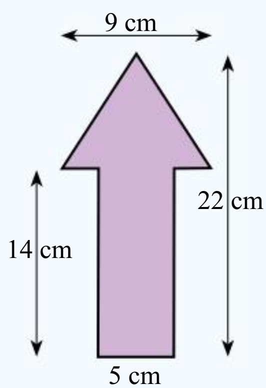
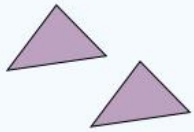
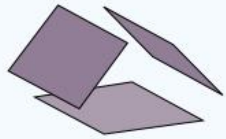
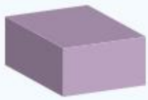
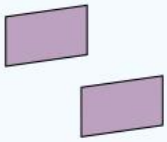
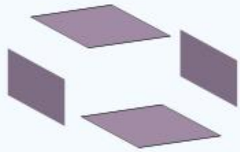


Prisms

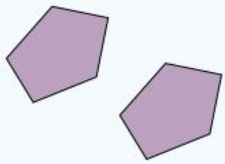
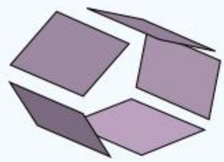
Prisms are solid, or hollow, objects with two identical ends and rectangular sides.

Triangular Prism = 2 triangular ends + 3 rectangular sides

Rectangular Prism = 2 rectangular ends + 4 rectangular sides

Pentagonal Prism = 2 pentagonal ends + 5 rectangular sides

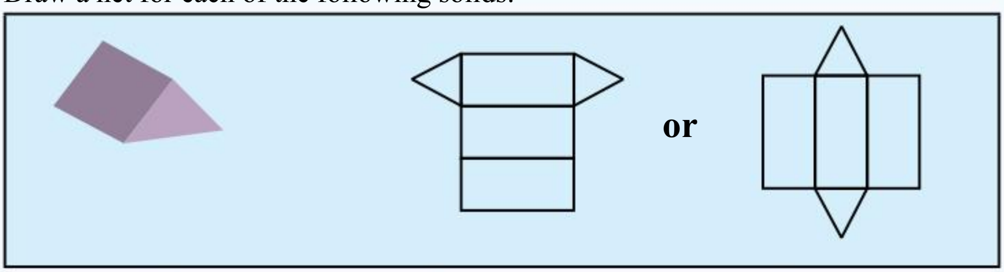
Exercise 3.3

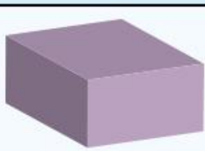
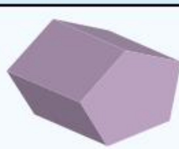
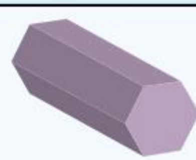
Copy and complete the following table:

Prism	Ends	Sides	Total faces
Triangular prism	2 triangles	3 rectangles	5
Rectangular prism	2 rectangles		
Pentagonal prism			
Hexagonal prism			
Heptagonal prism			
Octagonal prism			

Exercise 3.4

Draw a net for each of the following solids:



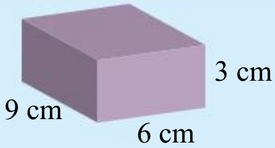
1  **2**  **3** 

Surface Area

The surface area of a solid is the total area of each face of the solid.

Exercise 3.5

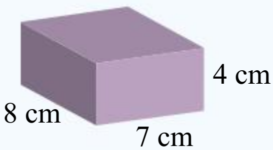
Find the surface area of each of the following prisms:



$$\begin{aligned}\text{Surface area} &= 2 \text{ ends} + 2 \text{ sides} + (\text{top} + \text{bottom})\\ &= 2\times6\times3 + 2\times3\times9 + 2\times(6\times9)\ \text{cm}^2\\ &= 36 + 54 + 108\ \text{cm}^2\end{aligned}$$

$\underline{\text{Surface area} = 198\ \text{cm}^2}$

1



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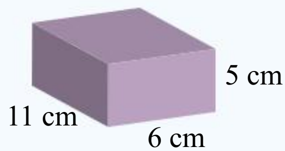

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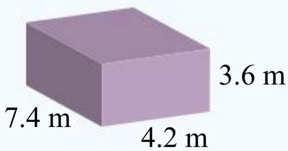

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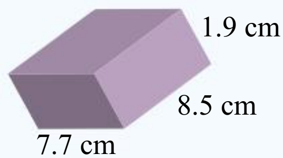

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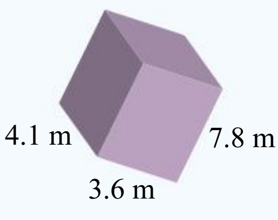

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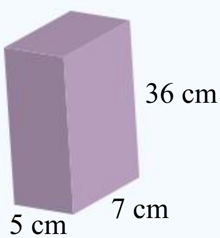
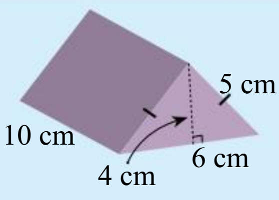
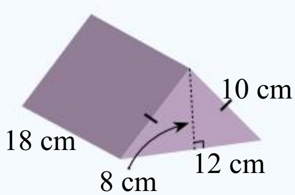




$$\begin{aligned}\text{Surface area} &= 2 \text{ ends} + 3 \text{ sides}\\ &= 2\times\tfrac{1}{2}\times6\times4 + 5\times10 + 5\times10 + 6\times10\ \text{cm}^2\\ &= 24 + 50 + 50 + 60\ \text{cm}^2\end{aligned}$$

$\underline{\text{Surface area} = 184\ \text{cm}^2}$

7



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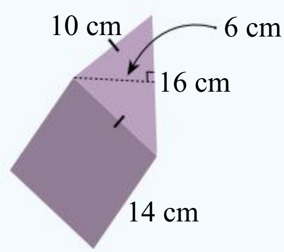
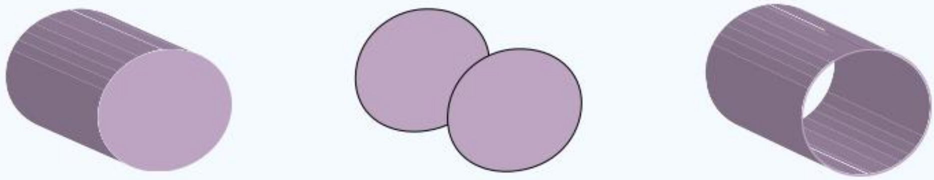


Cylinders

Prisms are solid, or hollow, objects with two identical circular ends and a tube side.



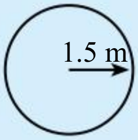
Circular prism = 2 circular ends + 1 tube

Circle Review

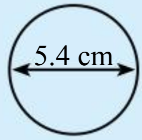
Exercise 3.6

Calculate the circumference of each of the following circles (2 decimal places):

$$C = 2\pi r = 2 \times \pi \times 1.5 = \underline{9.42\text{ m}}$$



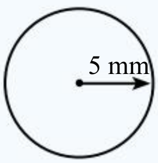
$$C = \pi D = \pi \times 5.4 = \underline{16.96\text{ cm}}$$



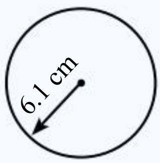
Using the calculator: $C = 2 \times \pi \times 1.5$

[2] [×] [π] [×] [1.5] [=]

1



2



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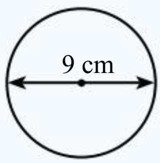

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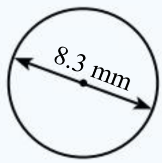
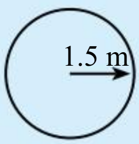


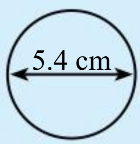
Exercise 3.7

Calculate the area of each of the following circles (Correct to 2 decimal places):

$$A = \pi r^2 = \pi \times 1.5^2 = \underline{7.07\text{ m}^2}$$



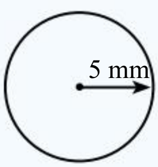
$$A = \pi r^2 = \pi \times (5.4 \div 2)^2 = \underline{22.90\text{ cm}^2}$$



Using the calculator: $A = \pi \times (5.4 \div 2)^2$

[π] [×] [( ] [5.4] [÷] [2] [ )] [x^2] [=]

1



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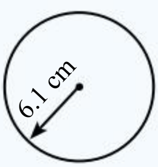

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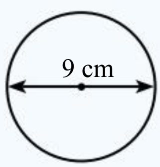

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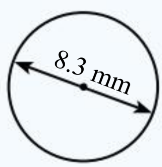
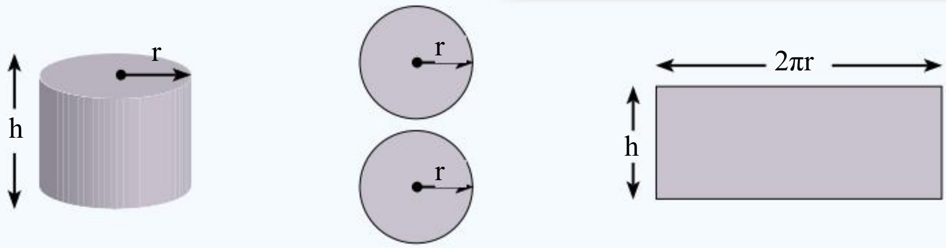


Cylinder Surface Area

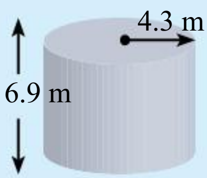
Surface area = 2 circles + rectangle.



Circular prism	=	2 circular ends	+	1 rectangle
Surface area	=	$2\times \pi r^2$	+	$2\pi r\times h$

Exercise 3.8

Find the surface area of each of the following cylinders:



Surface area = area of 2 circles + area of rectangle

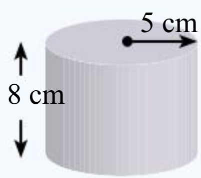
$= 2\times\pi r^2 + 2\pi r\times h$

$= 2\times\pi\times 4.3^2 + 2\pi\times 4.3\times 6.9\ \text{m}^2$

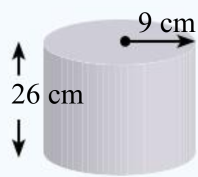
$= 116.18 + 186.42\ \text{m}^2$

Surface area $= 302.60\ \text{m}^2$

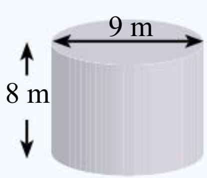
1



2



3



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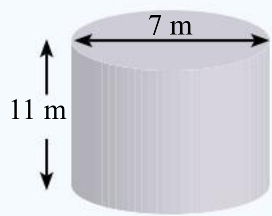

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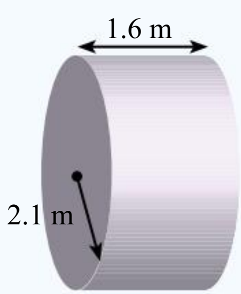
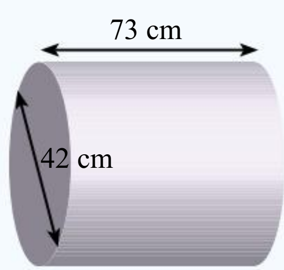


6



All Together

The surface area of a solid is the total area of each face of the solid.

Exercise 3.9

1 The four sides and the top of the 20 foot container are to be given two coats of paint.

20' Container Outside Dimensions	
Length	6.05 m
Width	2.44 m
Height	2.59 m

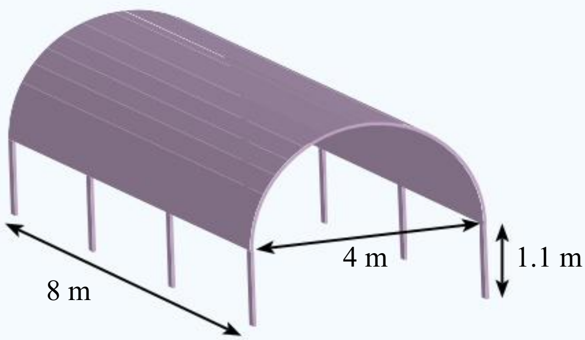
a) What is the surface area of the four sides and the top?

b) How much paint is needed for two coats if 1 litre of paint will, on average, cover 15 m^2.

c) What is the cost of paint @ $63.00 for 4L?

2 Estimate the cost of painting the outside of the water storage tank.
Assume that the painting (including labour, materials, scaffolding, etc) will cost $120/m^2.

3 Calculate how many rolls of 70% shade cloth is needed to completely cover the greenhouse design.



70% shade cloth
1.83 m wide Roll
(50m length)

$180

Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 3.10

1 Spell Rectangular Prism.

2 What is the formula for the area of a triangle?

3 What is the area of a rectangle 4 m by 8 m?

4 What is the formula for the area of a circle?

5 Simplify: $3a - 5a$

6 Expand: $3(x - 2)$

7 Factorise: $6x + 4$

8 $10^2 \times 10^3$

9 $10^7 \div 10^3$

10 I buy a USB stick for \$12.30 with a \$20 note. How much change?

Why is 6 afraid of 7?
Because 789

Exercise 3.11

1 Spell Surface Area.

2 What is the formula for the area of a rectangle?

3 What is the area of a triangle height = 4 m and base = 7 m?

4 What is the formula for the circumference of a circle?

5 Simplify: $4b \times 3b$

6 Expand: $2(3x - 1)$

7 Factorise: $6x + 9$

8 $a^4 \times a^2$

9 $b^2 \div b^6$

10 I buy a DVD for \$14.80 with a \$20 note. How much change?

Success is ninety-nine percent failure - Soichiro Honda.

Exercise 3.12

1 Spell Cylinder.

2 What is the formula for the area of a triangle?

3 What is the area of a rectangle 7 cm by 6 cm?

4 What is the formula for the area of a circle?

5 Simplify: $5a - 8a$

6 Expand: $2(5x - 2)$

7 Factorise: $4x + 10$

8 $10^7 \times 10^3$

9 $(a^{2)3}$

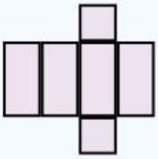
10 I buy \$35.60 of petrol with a \$50 note. How much change?

NAPLAN Questions

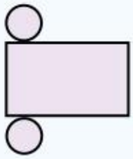
Exercise 3.13

1 Either make or draw a diagram of what each of the following nets will make.

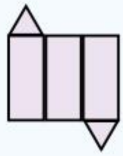
a)



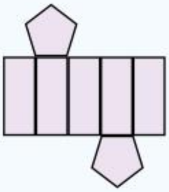
b)



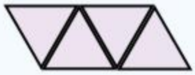
c)



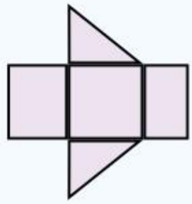
d)



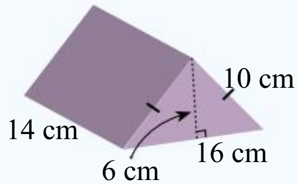
e)



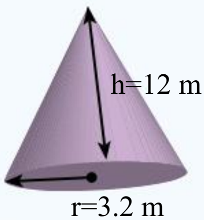
f)



2 What is the total surface area of the triangular prism?



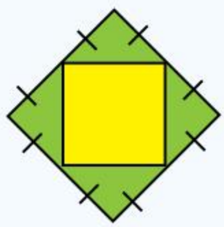
3



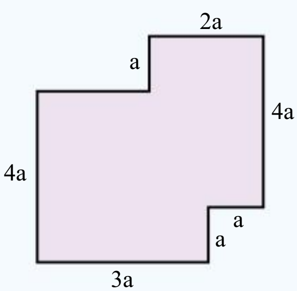
Find the volume of the cone. $V = \frac{\pi r^2 h}{3}$

4 The area of the rectangle is 54 m^2.
What is the length of the rectangle?

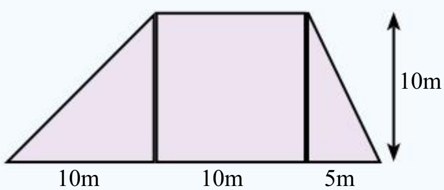
5 What is the ratio of the area of the inner yellow square to the area of the larger green square?



6 What is the area of the floor plan?



7 What is the area of the trapezium?



Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

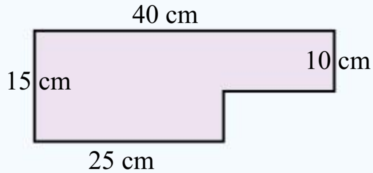
Exercise 3.14

1 The area of a square is 169 cm^2.
What is its perimeter?

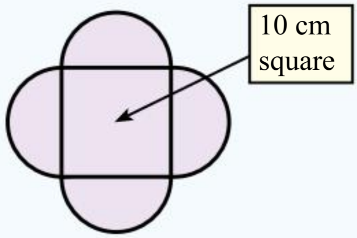
2 Each floor of an office block has rectangular floors 32 m by 15 m.
If the total office space is 3840 m^2, how many floors in the building?

3 Find the area of each of the following shapes:

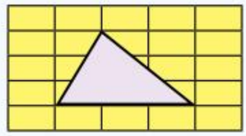
a)



b)



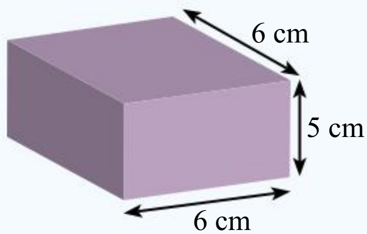
4 The overall yellow rectangle is 10 cm by 10 cm.
What is the area of the triangle?



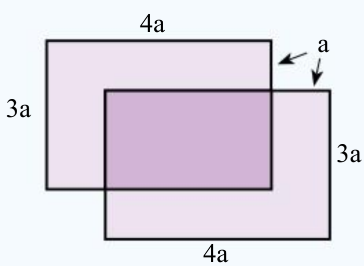
5 If the length of a rectangle has been doubled and the width tripled, what has happened to the area of the rectangle?

6 If the perpendicular height of a triangle has been tripled, what has happened to the area of the triangle?

7 How many 1 cm × 2 cm × 3 cm blocks can be cut from the block shown?

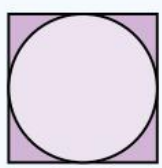


8 The two rectangles overlap.
What is the area of the overlap?



9 A circle is placed within a square as shown.
What is the ratio of the area of the circle to the area of the square?

a) 1:2 **b)** 2:π **c)** π:4

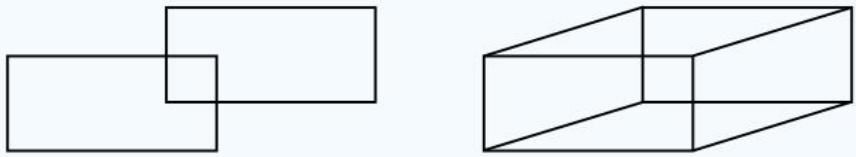


Investigations

Investigation 3.1 Drawing 3D shapes

Rectangular prism

a) Draw two rectangles **b)** Join the vertices



Use a similar technique to draw:

a) a triangular prism.

b) a cylinder.

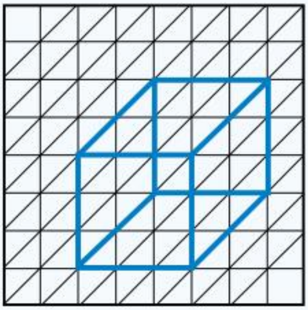
c) a pentagonal prism.

Investigation 3.2 3D shapes and Oblique Grids

Use oblique grid paper, isometric grid paper, and isometric dot paper to draw prisms.

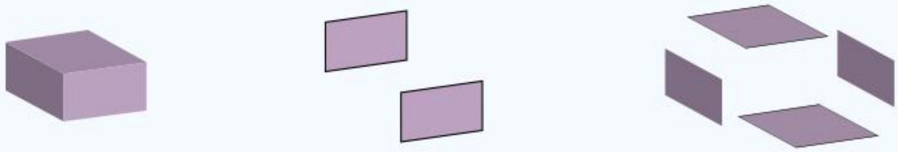
Which do you prefer?

These papers are available on the Internet.



Investigation 3.3 Rectangular prisms

Bring a rectangular box from home.



1 Cut the rectangular faces from the box.

2 Count the number of rectangular faces.

3 Measure the rectangles and calculate their area.

4 Calculate the surface area of the box.

Investigation 3.4 Newspaper revenue

1 Select a local newspaper.

2 Find the newspaper advertisement rates (or use the ad rate card shown).

3 Calculate the total revenue from the ads.

Advertising Rates

Community Newspapers

Full page = 16 modules

$320 per module

Technology

Technology 3.1 Surface Area Spreadsheets

Make a spreadsheet to check your answers to the earlier exercises.

Enter the formula:
=2*b2*c2+2*c2*d2+2*b2*d2

Prism	Length	Width	Height	Surface Area
Rectangular	6.01	2.4	2.7	74.262

Enter the formula:
=2*pi()*c2*b2+2*pi()*c2^2

Prism	Length	Radius	Surface Area
Cylinder	4.92	1.2	46.14371

Technology 3.2 Prism Activities

Search the Internet for some of the many prism activities.

Use search phrases such as:

a) 'Interactive geometry prisms'
b) 'Polyhedra models'
c) '3d objects nets'

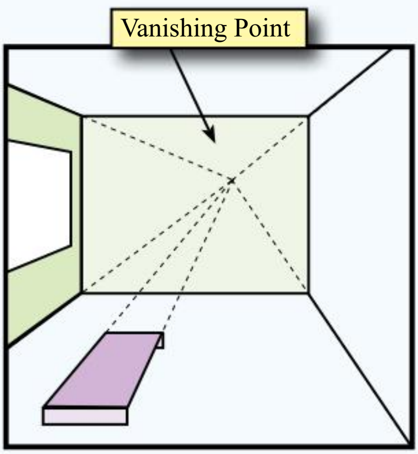
"Mum, will you do my maths homework?"
"No, it wouldn't be right."
"Well, you could try."

Technology 3.3 3D Sketching

a) Find 3D sketching software on the Internet and use the software to sketch/design in three dimensions.
Use a search phrase such as '3d sketching'.

b) Use an Isometric drawing tool to draw 3D shapes.
Use a search phrase such as 'Isometric drawing tool'.

c) The use of a Vanishing Point makes drawings/sketches look more realsitic.
Take part in an online tutorial about Vanishing Point drawings.
Use a search phrase such as 'Vanishing point drawing'.

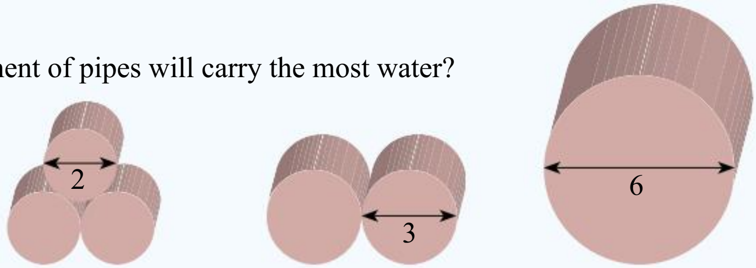


A Couple of Puzzles

Exercise 3.15

1 If $10^{-2} = \frac{1}{10^2} = \frac{1}{100} = 0.01$, what is 10^{-3}

2 Which arrangement of pipes will carry the most water?



A Game

Double then nothing is a dice game in which the first person to 100 is the winner.

1 Roll a pair of dice.
Your score is the sum of the two top faces of each die.

2 While it is your turn you can roll the dice as often as you like and you can keep totalling the score for each throw. However, **if you throw a double** then your turn is scored at **zero** and it is the next person's turn.

A Sweet Trick

1	Without looking, ask your audience to choose three different numbers from 1 to 9.	2 6 3
2	Have them write the three numbers in descending order.	632
3	Have them reverse the digits and find the difference.	632 − 236 = 396
4	Ask them to tell you either the first digit or the last digit.	3

You tell them that the other two digits are 9 and 6.

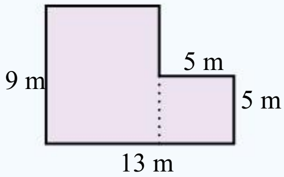
The middle digit is always 9. The other two digits always sum to 9.

Chapter Review 1

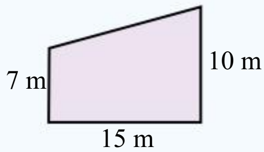
Exercise 3.16

1 Calculate the area of each of the following composite shapes:

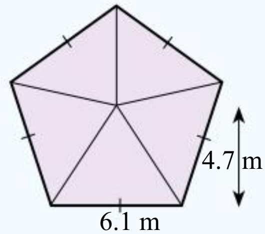
a)



b)

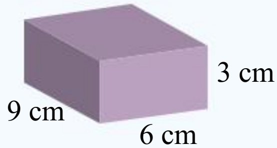


c)

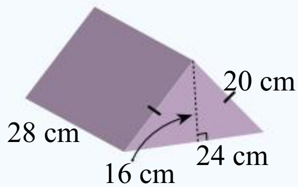


2 Find the surface area of each of the following prisms:

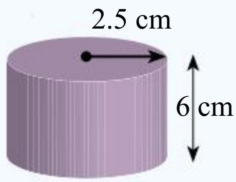
a)



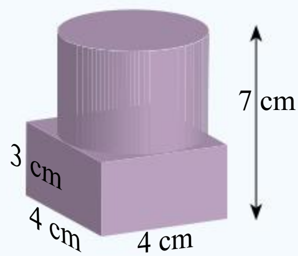
b)



c)



d)



3 The four sides and the top of the 40 foot container are to be given two coats of paint.

40' Container Outside Dimensions	
Length	12.19 m
Width	2.44 m
Height	2.59 m

a) What is the surface area of the four sides and the top?

b) How much paint is needed for two coats if 1 litre of paint will, on average, cover 15 m^2.

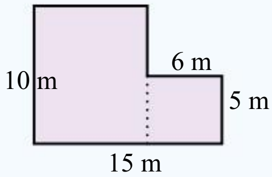
c) What is the cost of paint @ $74.00 for 4L?

Chapter Review 2

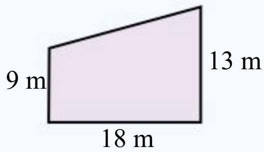
Exercise 3.17

1 Calculate the area of each of the following composite shapes:

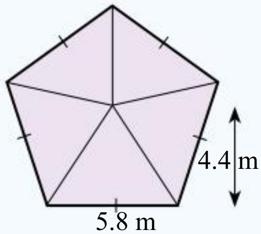
a)



b)

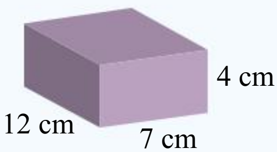


c)

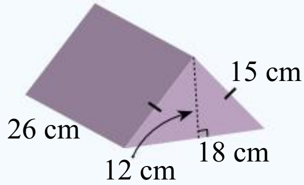


2 Find the surface area of each of the following prisms:

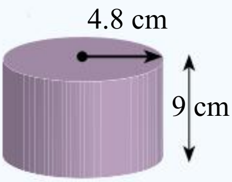
a)



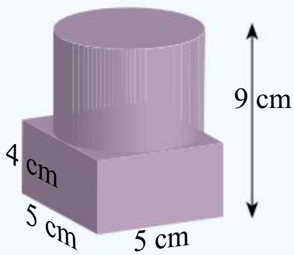
b)



c)



d)



3 Estimate the cost of painting the outside of the water storage tank.
Assume that the painting (including labour, materials, scaffolding, etc) will cost \$110/m^2.

Linear & Non-Linear Graphs

4

Number and Algebra ➞ Linear and non-linear relationships

★ Sketch linear graphs using the coordinates of two points.
★ Determine linear rules from suitable diagrams, tables of values and graphs and describe them both using words and algebra.
★ Sketch parabolas, hyperbolas, circles.

A catenary is the shape of a cat's tail?

A TASK

Catenary means chain, and refers to the shape of a chain hanging between two supports.

A simplified equation of the catenary is: $y = \dfrac{(e^x + e^{-x})}{2}$

- What is e?
- Use the formula to graph a catenary (Technology 4.3).
- Research practical applications of the catenary.
- Take photos of different kinds of catenaries.
- Publish your findings - classroom wall?

A LITTLE BIT OF HISTORY

1638 Galileo describes a hanging chain as similar in shape to a parabola.

1690 Huygens first uses the term catenaria.

1691 Leibniz, Huygens and Bernoulli derive the equation of the catenary after Bernoulli issued a challenge.

A spider's web has many catenaries.

Linear Rules

A linear rule increases by the same amount each step.

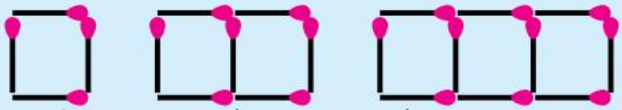
Exercise 4.1

For each of the following patterns:

a) Count the number of matches or dots needed for each step, extend the pattern and summarise the pattern in a **table**.

b) Write a **rule** for the pattern.

c) **Check** that the rule is correct:



Table

Step	1	2	3	4	5
Matches	4	7	10	13	16

Rule: Matches = 3×step + 1 or m = 3s + 1

Check. When step = 5, Matches = 3×5+1 = 16 ✓

Extra 3 matches for each step
3×step
First step is 3+1 (= 4 matches).
matches = 3×step+1

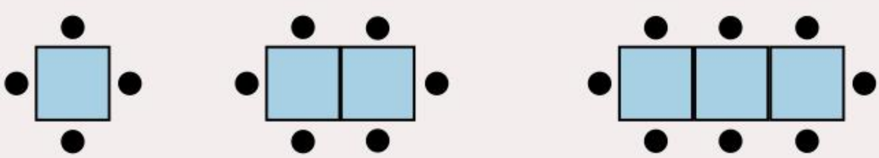
1

2

3

4

5



Linear Rules

A linear rule increases by the same amount each step.

Exercise 4.2

Write a rule for each of the following tables of values:

x	1	2	3	4	5	10	20
y	¯6	¯4	¯2	0	2	12	32

Extra 2 for each x → **2x**

First step (¯6) is 2−8. → **y = 2x − 8**

Check your answer:
y = 2x − 8
y = 2×5 − 8 {Checking for x=5}
y = 2 ✓

1

x	1	2	3	4	5	10	20
y	4	6	8	10	12	22	42

2

x	1	2	3	4	5	10	20
y	4	7	10	13	16	31	61

3

x	1	2	3	4	5	10	20
y	¯5	¯3	¯1	1	3	13	33

4

x	1	2	3	4	5	10	20
y	¯7	¯4	¯1	2	5	20	50

x	1	2	3	4	5	10	20
y	14	11	8	5	2	¯13	¯43

Subtract 3 for each x → **¯3x**

First step (14) is ¯3+17. → **y = ¯3x + 17**

Check your answer:
y = ¯3x + 17
y = ¯3×10 +17 {Check x=10}
y = ¯13 ✓

5

x	1	2	3	4	5	10	20
y	15	13	11	9	7	¯3	¯23

6

x	1	2	3	4	5	10	20
y	5	2	¯1	¯4	¯7	¯22	¯52

7

x	1	2	3	4	5	10	20
y	3	¯1	¯5	¯9	¯13	¯33	¯73

8

x	1	2	3	4	5	10	20
y	¯7	¯5	¯3	¯1	1	11	31

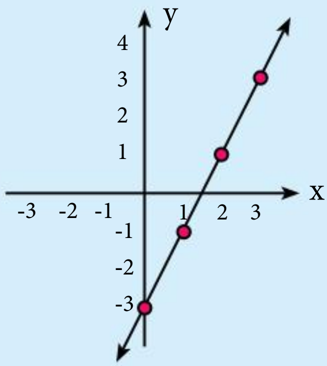
Algebra is an essential tool in thousands of careers and is fundamental to solving millions of problems.

Linear Rules

The graph is linear because the points are in a line.

Exercise 4.3

Write a rule for each of the following graphs:



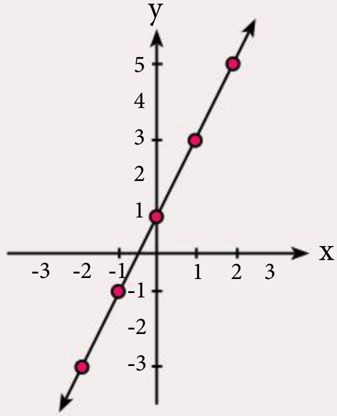
Write the coordinates in a table:

x	0	1	2	3
y	-3	-1	1	3

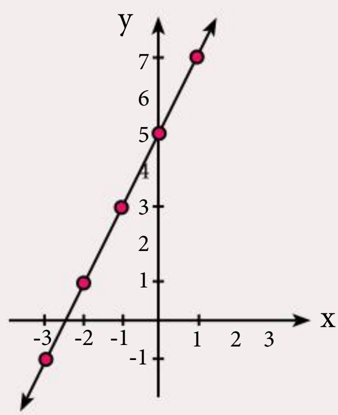
Increase of 2 each x → **2x**

Step 1 $(^{-}1)$ is 2−3. → **y = 2x − 3**

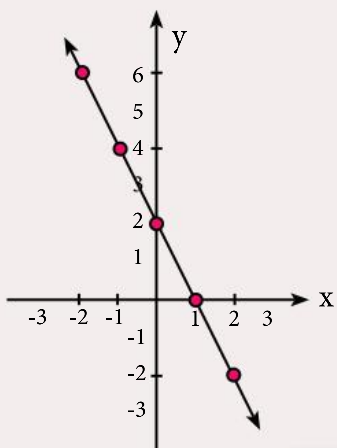
1



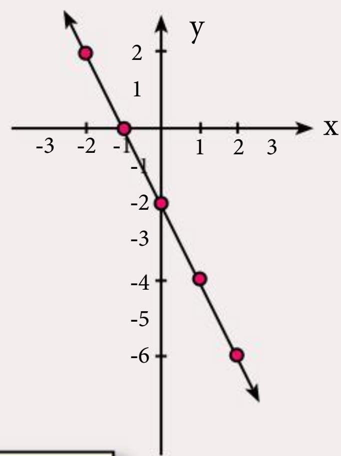
2



3



4



Can you write the rule without having to complete a table?

Sketching Linear Graphs

A linear graph increases by the same amount each step.

Exercise 4.4

For each of the following:

a) Copy and complete the following table.

b) Draw a graph of the linear rule.

x	-2	-1	0	1	2
y = 2x + 1					

When $x = {}^{-}2$, $y = 2\times{}^{-}2 + 1 = {}^{-}3$

When $x = {}^{-}1$, $y = 2\times{}^{-}1 + 1 = {}^{-}1$

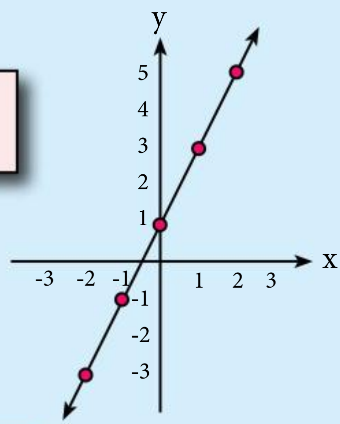
When $x = 0$, $y = 2\times0 + 1 = 1$

When $x = 1$, $y = 2\times1 + 1 = 3$

When $x = 2$, $y = 2\times2 + 1 = 5$

$2\times{}^{-}2$ = two lots of ${}^{-}2$
$= {}^{-}2\ {}^{-}2$
$= {}^{-}4$

x	-2	-1	0	1	2
y = 2x + 1	-3	-1	1	3	5



Plot the points in the table.

The x first, then the y.

1

x	-2	-1	0	1	2
y = x + 2					

2

x	-2	-1	0	1	2
y = x + 3					

3

x	-2	-1	0	1	2
y = 2x + 5					

4

x	-2	-1	0	1	2
y = x − 1					

5

x	-2	-1	0	1	2
y = 2x − 3					

6

x	-2	-1	0	1	2
y = 3x + 1					

7

x	-2	-1	0	1	2
y = −x + 2					

8

x	-2	-1	0	1	2
$y = {}^{-}3x + 4$					

9

x	-2	-1	0	1	2
y = 40x + 60					

10

x	-2	-1	0	1	2
y = 25x − 10					

Linear Graphs

Linear Graphs are of the form:

$$y = mx + c$$

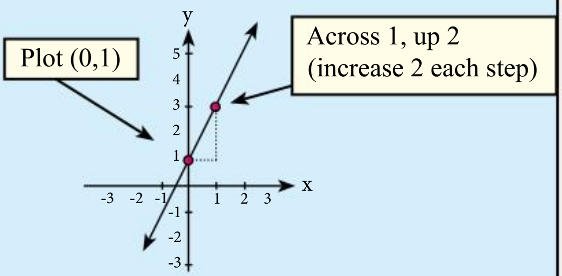
a) m is the increase each step.
b) c is the value of y when x = 0 ie (0,c).
c) The power of x and y is 1: $y^1 = mx^1 + c$

Exercise 4.5

Draw a quick sketch of each of the following:

$y = 2x + 1$
$y = mx + c$

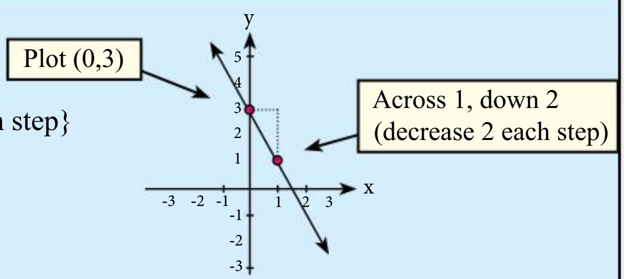
$m = 2$ {increases by 2 each step}
$c = 1$ {a point is (0,1)}



1	$y = 2x + 2$	**2**	$y = 3x + 1$	**3**	$y = 2x + 5$
4	$y = 3x + 2$	**5**	$y = 2x - 1$	**6**	$y = 5x + 2$
7	$y = 2x - 3$	**8**	$y = 4x + 5$	**9**	$y = x - 2$ ← m = 1

$y = {}^{-}2x + 3$
$y = mx + c$

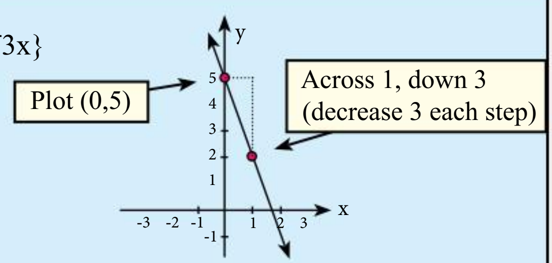
$m = {}^{-}2$ {decreases by 2 each step}
$c = 3$ {a point is (0,3)}



10	$y = {}^{-}2x + 2$	**11**	$y = {}^{-}3x + 3$	**12**	$y = {}^{-}2x + 4$
13	$y = {}^{-}3x + 1$	**14**	$y = {}^{-}2x + 5$	**15**	$y = {}^{-}2x - 1$
16	$y = {}^{-}2x - 2$	**17**	$y = 3x + 2$	**18**	$y = {}^{-}x - 1$ ← $m = {}^{-}1$

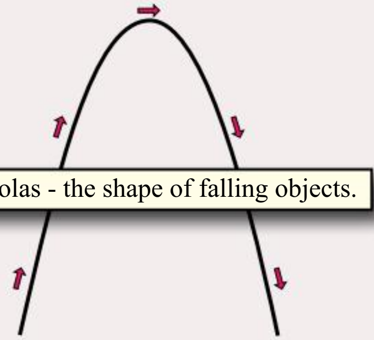
$3x + y = 5$
$y = {}^{-}3x + 5$ {inverse of 3x is ${}^{-}3x$}
$y = mx + c$

$m = {}^{-}3$ {decreases by 3 each step}
$c = 5$ {a point is (0,5)}



19	$2x + y = 1$	**20**	${}^{-}2x + y = 1$	**21**	$x + y = 3$
22	$x + y = 4$	**23**	$x + y = 5$	**24**	$3x + y = 2$
25	${}^{-}2x + y = {}^{-}2$	**26**	${}^{-}3x + y = {}^{-}1$	**27**	$4x + 2y = 6$

Non-Linear Graphs



Exercise 4.6

For each of the following:

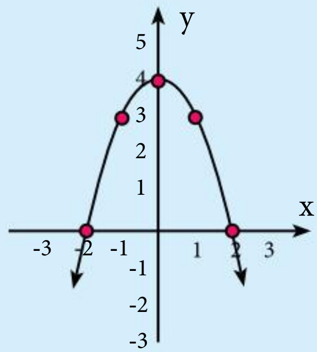
a) Copy and complete the table.
b) Plot the points.

$y = {}^-x^2 + 4$

x	$^-2$	$^-1$	0	1	2
$y = {}^-x^2{+}4$					

When $x = {}^-2$, $y = {}^-({}^-2 \times {}^-2) + 4 = {}^-4 + 4 = 0$
When $x = {}^-1$, $y = {}^-({}^-1 \times {}^-1) + 4 = {}^-1 + 4 = 3$
When $x = 0$, $y = {}^-(0 \times 0) + 4 = 0 + 4 = 4$
When $x = 1$, $y = {}^-(1 \times 1) + 4 = {}^-1 + 4 = 3$
When $x = 2$, $y = {}^-(2 \times 2) + 4 = {}^-4 + 4 = 0$

x	$^-2$	$^-1$	0	1	2
$y = {}^-x^2{+}4$	0	3	4	3	0



b) Plot the points.

1

x	-2	-1	0	1	2
$y = x^2$					

2

x	-2	-1	0	1	2
$y = x^2 + 1$					

3

x	-2	-1	0	1	2
$y = x^2 + 2$					

4

x	-2	-1	0	1	2
$y = x^2 + 3$					

5

x	-2	-1	0	1	2
$y = x^2 - 1$					

6

x	-2	-1	0	1	2
$y = x^2 - 2$					

7

x	-2	-1	0	1	2
$y = x(x + 1)$					

8

x	-2	-1	0	1	2
$y = x(x - 1)$					

Parabola:

$\mathbf{y = ax^2 + bx + c}$

If a is positive:

If a is negative:

Non-Linear Graphs

Circles - Points at a constant distance from the centre.

$\mathbf{x^2 + y^2 = a^2}$

Exercise 4.7

For each of the following:

a) Check that the table is correct.

b) Plot the points.

$x^2 + y^2 = 4$

x	$^-2$	$^-1$	0	1	2
$x^2 + y^2 = 4$	0	$\sqrt{3}$ or $^-\sqrt{3}$	2 or $^-2$	$\sqrt{3}$ or $^-\sqrt{3}$	0

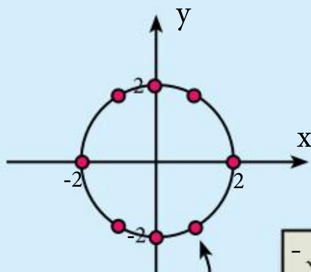
When $x = 1$, $y = {}^-\sqrt{3}$

$x^2 + y^2 = (1)^2 + ({}^-\sqrt{3})^2$

$= 1 + 3$

$= 4$ ✓

Thus $(1, {}^-\sqrt{3})$ is a point on $x^2 + y^2 = 4$



$^-\sqrt{3} = {}^-1.7$

1

x	$^-2$	$^-1$	0	1	2
$x^2 + y^2 = 5$	1 or $^-1$	2 or $^-2$	$\sqrt{5}$ or $^-\sqrt{5}$	2 or $^-2$	1 or $^-1$

2

x	$^-3$	$^-2$	0	2	3
$x^2 + y^2 = 9$	0	$\sqrt{5}$ or $^-\sqrt{5}$	3 or $^-3$	$\sqrt{5}$ or $^-\sqrt{5}$	0

3

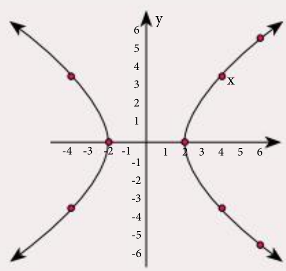
x	$^-4$	$^-2$	0	2	4
$x^2 + y^2 = 16$	0	$\sqrt{12}$ or $^-\sqrt{12}$	4 or $^-4$	$\sqrt{12}$ or $^-\sqrt{12}$	0

4

x	$^-4$	$^-2$	2	4	6
$x^2 - y^2 = 4$	$\sqrt{12}$ or $^-\sqrt{12}$	0	0	$\sqrt{12}$ or $^-\sqrt{12}$	$\sqrt{32}$ or $^-\sqrt{32}$

Hyperbolas:
$\mathbf{x^2 - y^2 = a^2}$

The shadow of the tip of a stick traces out a hyperbola on the ground over the course of a day.



Mental Computation

The majority of everyday problems are solved mentally by adults.

Exercise 4.8

1 Spell Linear.

2 What is the linear rule for:

x	0	1	2	3
y	-3	-1	1	3

3 Roughly sketch the rule: $y = 2x + 3$

4 What is the formula for the area of a triangle?

5 What is the area of a rectangle 4 m by 8 m?

6 Simplify: $3a - 7a$

7 Expand: $2(x - 3)$

8 Factorise: $6x + 4$

9 $10^2 \times 10^3$

10 14×15

$$\begin{aligned} 14\times15 &= 14\times(10+5) \\ &= 14\times10 + 14\times5 \\ &= 14\times10 + 14\div2\times10 \\ &= 140 \quad + 70 \\ &= 210 \end{aligned}$$

Exercise 4.9

1 Spell Parabola.

2 What is the linear rule for:

x	0	1	2	3
y	5	3	1	⁻1

3 Roughly sketch the rule: $y = 3x + 2$

4 What is the formula for the area of a circle?

5 What is the area of a triangle height = 6 m and base = 7 m?

6 Simplify: $7m - 3m$

7 Expand: $3(x - 2)$

8 Factorise: $3x + 9$

9 $10^5 \times 10^2$

10 18×15

Student: Pi r squared.
Baker: No! Pies are round, cakes are square!

Exercise 4.10

1 Spell Hyperbola.

2 What is the linear rule for:

x	0	1	2	3
y	-2	1	4	7

3 Roughly sketch the rule: $y = 2x - 3$

4 What is the formula for the circumference of a circle?

5 What is the area of a rectangle 5 m by 7 m?

6 Simplify: $6b - 9b$

7 Expand: $5(x + 3)$

8 Factorise: $5x + 10$

9 $10^4 \div 10^2$

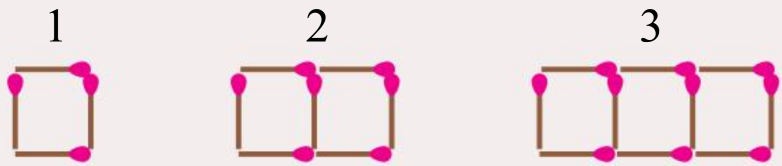
10 22×15

One may walk over the highest mountain one step at a time - John Wanamaker.

NAPLAN Questions

Exercise 4.11

1 What is the rule for the number of matches?



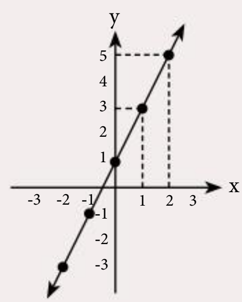
2 A rule is: $y = 5 - 2x$. What is the value of y when $x = 2.25$?

3 A rule is: $y = 5 - 2x - x^2$. What is the value of y when $x = {}^{-}2$?

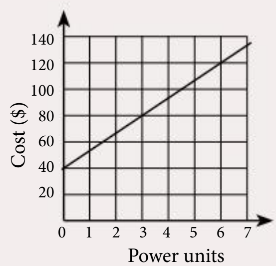
4 Write a linear model of the following plumber's fee:

Hours (h)	1	2	3	4
Fee ($F)	90	140	190	240

5 Write the linear rule represented by this graph.



6 What is the rule for the power cost?



7 Which rule would produce the following table?

a) $y=x+1$ **b)** $y=2x-1$ **c)** $y=x^2-3$

x	0	1	2	3
y	$^{-}1$	1	3	5

Don't do too much in your head.
Pen and paper work will get better results.

8 A rule is: $y = 2a^2 - 4b$. What is the value of y when $a = 2$ and $b = {}^{-}2$?

9 A rule is: $y = 5x^2 - 4$. What is the value of y when $x = {}^{-}3$?

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 4.12

1 An approximate formula for converting kilometres, k, to miles, m, is $m = \frac{5k}{8}$. How many miles in 32 kilometres?

2 Does the point ($^{-}2$,1) lie on the line: $y = 2x + 5$?

3 The point (100, 8) lies on the line $20x + y = 2008$.
Find another point that lies on this line?

4 What is the equation of each of the following lines?

> Each step increases by 2 and it cuts the y-axis at (0,$^{-}3$).
> m=2 c=$^{-}3$
> $y = 2x - 3$

a) Each step increases by 2 and it cuts the y-axis at (0,1).
b) Each step decreases by $^{-}2$ and it cuts the y-axis at (0,$^{-}1$).

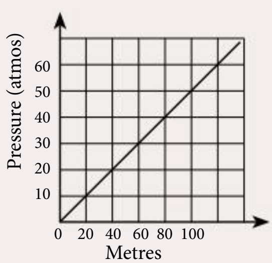
> Write a mathematical model of the following printing costs:
>
> Let b = no. of thousands
> Each step increases by \$1500: Thus C = 1500b
> When b = 0, cost=\$4000
>
> Thus **C = 1500b + 4000**

No. books	Cost
2 000	$7 000
3 000	$8 500
5 000	$11 500
10 000	$19 000

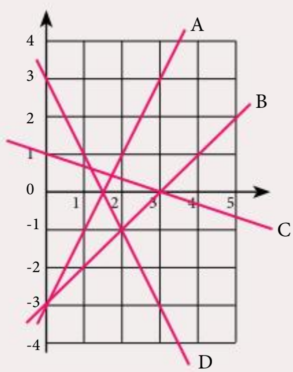
5 Write a mathematical model of the following printing costs:

No. books	Cost
3 000	$12 000
5 000	$15 000
7 000	$18 000
10 000	$22 500

6a) The following graph shows the relationship between water depth, in metres, and pressure, in atmospheres. What is the rule?



6b) Which line represents $3y = {}^{-}6x + 9$?



Investigations

Investigation 4.1 **A falling rock?**

The distance a rock falls when dropped is given by the parabola: $d = 4.9t^2$, where d is the distance in metres dropped and t is the time in seconds.

Example:
A stone is dropped into a well and a splash sound is heard 2.5 seconds later.

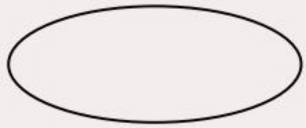
Distance $= 4.9t^2$
$= 4.9 \times 2.5^2$ m
$= 30.6$ m

Investigate

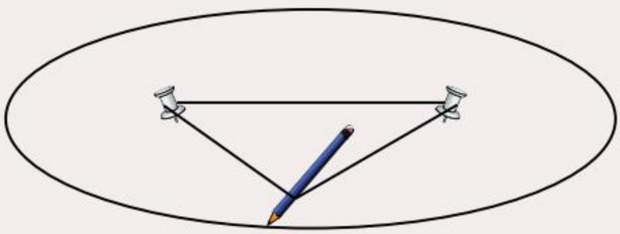
Dropping stones.

Investigation 4.2 **The Ellipse**

The Earth moves around the Sun in an elliptical orbit.



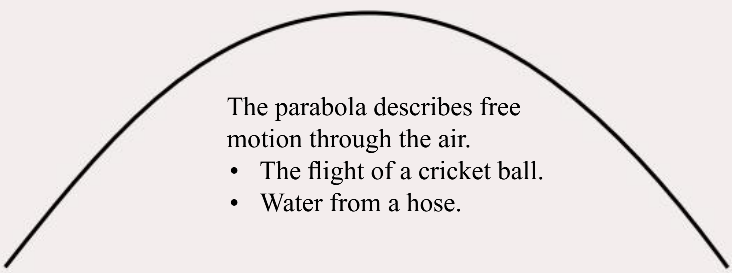
Use the method shown below to draw an ellipse.



Ellipse:
Investigate the formula:

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

Investigation 4.3 **The Parabola**



The parabola describes free motion through the air.

- The flight of a cricket ball.
- Water from a hose.

The parabola has many practical applications. Investigate one of them.

Civil Engineers design, build, and maintain bridges, dams, railways, buildings, airports etc.
- Relevant school subjects are English, Mathematics, and Science.
- Courses usually involve a Universtity engineering degree.

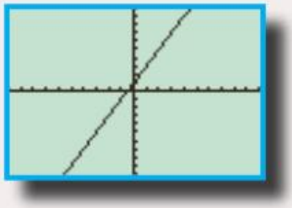
Technology

Technology 4.1
Use a Graphics Calculator to plot the rules in Exercise 4.5.

x	-2	-1	0	1	2
y = 2x + 1					

Press **Y =** and enter the equation eg., 2x + 1

Press **Graph** to see a plot of the equation



Press **Table** to see a table of the values

x	-2	-1	0	1	2
y = 2x + 1	-3	-1	1	3	5

Technology 4.2
Use a Graphics Calculator, as above, to experiment with non-linear tables of data (See Exercises 4.6 and 4.7).

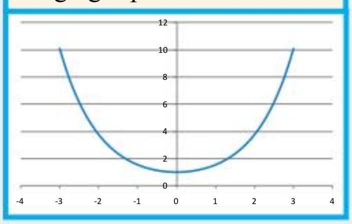
Technology 4.3
Use a Spreadsheet to experiment with a Catenary.

x	-3	-2	1	0	1	2	3
$y = 0.5(e^x + e^{-x})$	10.1	3.8	1.5	1	1.5	3.8	10.1

Enter the formula:
= 0.5*(EXP(B2)+EXP(-B2))

A catenary is the shape of a hanging rope.

Use the Chart (Scatter) to plot the points:



Technology 4.4 Applets
There are a very large number of "graph, function, plotter" applets on the Internet.

There are also applets that let you experiment with parabolas, hyperbolas, ellipses.

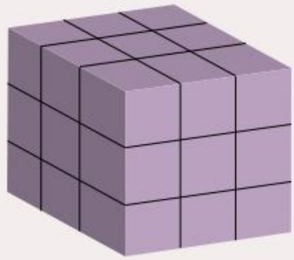
A Couple of Puzzles

Exercise 4.13

1 Who am I? If you double me and then add twenty-five, the result is seventy-three.

2 If aΔb = a$\times$b − 6, what is 4Δ3?

3 A cube has been made by gluing twenty-seven smaller cubes together.
How many faces have glue on them?



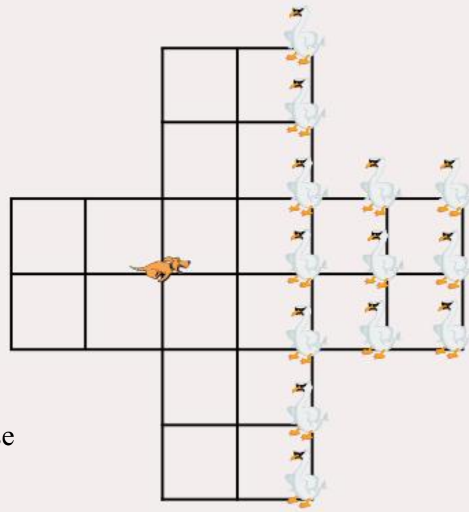
A Game

Fox and Geese is a two player game. One person is the fox, and the other person looks after the 13 geese.

Taking turns, the fox and a goose move along a line to the next intersection.

The fox kills a goose by jumping over the goose onto a vacant intersection, in line, on the other side of the goose. The goose is removed from the game.

The geese try to herd the fox into a corner where the fox cannot make a move. A goose cannot jump over the fox.



A Sweet Trick

1	Ask your audience write down any number and then enter the number into a calculator.		729
2	Add 17	729+17	= 746
3	Multiply by 64	746×64	= 47744
4	Multiply by 25	47744×25	= 1193600
5	Subtract 22176	1193600−22176	= 1171424
6	Divide by 1 600	1171424÷1600	= 732.14
7	Subtract the original number	732.14−729	= 3.14

This will give the value of π every time.
Make up a story - This is another way of working out the value of π?

They need to press '=' after each calculation.

Chapter Review 1

Exercise 4.14

1 Write a rule for the following pattern:

2 Write a rule for each of the following tables:

a)

x	1	2	3	4	5	10	20
y	4	6	8	10	12	22	42

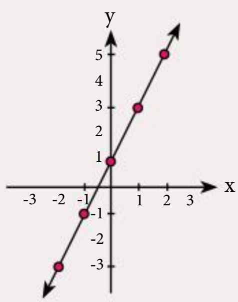
b)

x	1	2	3	4	5	10	20
y	5	2	$^{-}1$	$^{-}4$	$^{-}7$	$^{-}22$	$^{-}52$

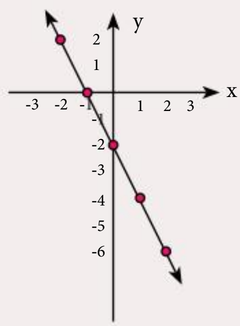
A pessimist says the glass is half empty. An optimist says the glass is half full. An engineer says, "Why all the wasted space?"

3 Write a rule for each of the following graphs:

a)



b)



4 Copy and complete each of the following tables and draw a graph of the rule:

a)

x	-2	-1	0	1	2
$y = 2x - 3$					

b)

x	-2	-1	0	1	2
$y = {}^{-}3x + 4$					

c)

x	-2	-1	0	1	2
$y = x^2 - 1$					

d)

x	-2	-1	0	1	2
$y = x^2 - 2$					

5 Draw a sketch of:

a) $y = 2x + 2$ **b)** $y = 2x - 1$ **c)** $y = {}^{-}2x + 4$

6 Sketch $x^2 + y^2 = 9$.

x	$^{-}3$	$^{-}2$	0	2	3
$x^2 + y^2 = 9$	0	$\sqrt{5}$ or $^{-}\sqrt{5}$	3 or $^{-}3$	$\sqrt{5}$ or $^{-}\sqrt{5}$	0

Chapter Review 2

Exercise 4.15

1 Write a rule for the following pattern:

2 Write a rule for each of the following tables:

a)

x	1	2	3	4	5	10	20
y	5	8	11	14	17	32	62

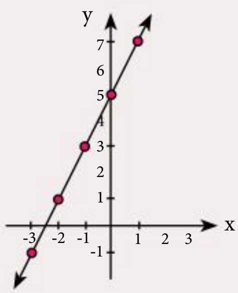
b)

x	1	2	3	4	5	10	20
y	7	3	$^{-}1$	$^{-}5$	$^{-}9$	$^{-}29$	$^{-}69$

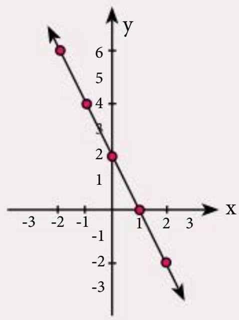
If your attack is going well, you have walked into an ambush - Murphy's Laws of Combat.

3 Write a rule for each of the following graphs:

a)



b)



4 Copy and complete each of the following tables and draw a graph of the rule:

a)

x	-2	-1	0	1	2
$y = 2x - 1$					

b)

x	-2	-1	0	1	2
$y = {}^{-}3x + 2$					

c)

x	-2	-1	0	1	2
$y = x^2 - 1$					

d)

x	-2	-1	0	1	2
$y = 2x^2 - 3$					

5 Draw a sketch of:

a) $y = 2x + 1$ **b)** $y = 2x - 2$ **c)** $y = {}^{-}2x + 2$

6 Sketch $x^2 + y^2 = 16$.

x	$^{-}4$	$^{-}2$	0	2	4
$x^2 + y^2 = 16$	0	$\sqrt{12}$ or $^{-}\sqrt{12}$	4 or $^{-}4$	$\sqrt{12}$ or $^{-}\sqrt{12}$	0

Review 15

Chapter 1 Indices 1

$2\times2\times2 = 2^3$ ← Index, ← Base

$(^-2)^2 = {}^-2\times{}^-2 = 4$
$(^-2)^3 = {}^-2\times{}^-2\times{}^-2 = {}^-8$
$(^-2)^4 = {}^-2\times{}^-2\times{}^-2\times{}^-2 = 16$

Index Law 1

$a^m \times a^n = a^{m+n}$

Index Law 2

$a^m \div a^n = a^{m-n}$

Index Law 3

$(a^m)^n = a^{m\times n}$

Zero Index

$a^0 = 1$

Negative Index

$a^{-m} = \dfrac{1}{a^m}$

Chapter 2 Algebra 1

$7a + 5 - 4a$
$= \underline{3a + 5}$

$3d \times 5e$
$= 3 \times d \times 5 \times e$
$= \underline{15de}$

$5mn \times {}^-2m^2n$
$= 5 \times {}^-2 \times m \times m^2 \times n \times n$
$= \underline{{}^-10m^3n^2}$

$18ay \div 4a$
$= \dfrac{18ay}{4a}$
$= \underline{\dfrac{9y}{2}}$

Distributive Law: a(b + c) = ab + ac

$3(2b - 5)$
$= \underline{6b - 15}$

${}^-4(x - 1) - 2(x - 4)$
$= {}^-4x + 4 - 2x + 8$
$= \underline{{}^-6x + 12}$

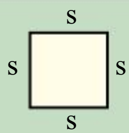
Factorisation: a(b + c) = ab + ac

$4x - 12$
$= \underline{4(x - 3)}$

${}^-6x^2 + 15x$
$= \underline{{}^-3x(2x - 5)}$

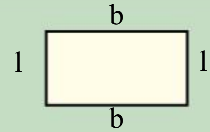
Chapter 3 Area

Square



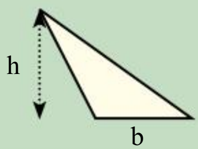
Area = $s \times s = s^2$

Rectangle



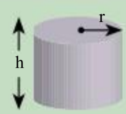
Area = $l \times b$

Triangle



Area = $\frac{1}{2}bh$

Cylinder



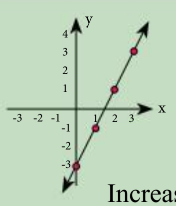
Surface Area = $2\pi r^2 + 2\pi rh$

Chapter 4 Linear & Non-Linear Graphs

Linear Graphs are of the form:

y = mx + c

a) m is the increase each step.
b) c is the value of y when x = 0 ie (0,c).



Increase of 2 each x → **2x**
When x=0, y=$^-3$ → **$\underline{y = 2x - 3}$**

Review 1

Exercise 5.1 Mental computation

1 Spell Cylinder.

2 What is the linear rule for:

x	0	1	2	3
y	-2	0	2	4

3 Roughly sketch the rule: $y = 2x + 1$

4 What is the formula for the area of a triangle?

5 What is the area of a rectangle 3 m by 4 m?

6 Simplify: $5a - 7a$

7 Expand: $3(x - 2)$

8 Factorise: $6x + 4$

9 $10^2\times10^5$

10 14×15

$$\begin{aligned} 14\times15 &= 14\times(10+5) \\ &= 14\times10 + 14\times5 \\ &= 14\times10 + 14\div2\times10 \\ &= 140 \quad + 70 \\ &= 210 \end{aligned}$$

Exercise 5.2

1 Write each of the following in index form:

d^3 Index Base

$3\times3\times3\times3 = \underline{3^4}$	$^{-}a\times{}^{-}a\times{}^{-}a = \underline{(^{-}a)^3}$

a) $1\times1\times1\times1$ **b)** $4\times4\times4\times4\times4\times4$ **c)** $10\times10\times10\times10\times10$

d) $a\times a\times a\times a\times a$ **e)** $1.3\times1.3\times1.3\times1.3$ **f)** $^{-}2\times{}^{-}2\times{}^{-}2\times{}^{-}2\times{}^{-}2$

2 Use the Index Laws to simplify each of the following:

$2^2\times2^{-5} = 2^{2+^{-}5}$ $\{a^m\times a^n = a^{m+n}\}$ $= \underline{2^{-3}}$ $\{2 + {}^{-}5 = {}^{-}3\}$	$5x^{-4}\times3x^3 = 15x^{^{-}4+3}$ $\{a^m\times a^n = a^{m+n}\}$ $= \underline{15x^{-1}}$ $\{^{-}4 + 3 = {}^{-}1\}$

a) $3^4\times3^{-2}$ **b)** $4^2\times4^{-5}$ **c)** $2^4\times2^{-3}$ **d)** $10^7\times10^{-5}$

e) $x^7\times x^{-3}$ **f)** $3x^4\times x^{-6}$ **g)** $3x^4\times3x^{-5}$ **h)** $4x^6\times2x^{-2}\times x^{-5}$

$5^2\div2^{-3} = 2^{2-^{-}3}$ $\{a^m\div a^n = a^{m-n}\}$ $= \underline{2^5}$ $\{2 - {}^{-}3 = 2+3 = 5\}$	$8x^{-4}\div2x^3 = 4x^{^{-}4-3}$ $\{a^m\div a^n = a^{m-n}\}$ $= \underline{4x^{-7}}$ $\{^{-}4 - 3 = {}^{-}7\}$

i) $2^3\div2^{-5}$ **j)** $9^5\div9^{-4}$ **k)** $4^{-2}\div4^2$ **l)** $10^{-5}\div10^{-3}$

m) $x^{-3}\div x^5$ **n)** $a^{-5}\div a^{-4}$ **o)** $b^3\div b^{-2}$ **p)** $12x^5\div4x^{-3}$

Index Law 1	Index Law 2	Index Law 3	Zero Index
$a^m\times a^n = a^{m+n}$	$a^m\div a^n = a^{m-n}$	$(a^m)^n = a^{m\times n}$	$a^0 = 1$

3 Use the Index Laws to simplify each of the following:

$(3^{-2})^4$	$= 3^{-2\times4}$	$\{(a^m)^n = a^{m\times n}\}$	$(x^{-2})^{-3}$	$= x^{-2\times-3}$	$\{(a^m)^n = a^{m\times n}\}$
	$= \underline{3^{-8}}$	$\{{}^{-}2\times4 = {}^{-}8\}$		$= \underline{2^6}$	$\{{}^{-}2\times{}^{-}3 = 6\}$

a) $(2^{-2})^3$ **b)** $(3^{-2})^{-2}$ **c)** $(x^2)^4$ **d)** $(10^{-2})^{-3}$

e) $4^4\times4^2$ **f)** $x^3\times x^5$ **g)** $10^6\times10^3$ **h)** $2^{-5}\times2$

i) $1.3^3\times1.3^5$ **j)** $10^5\div10^2$ **k)** $5x^3\times2x^4$ **l)** $(x^{-4})^2\times9x^3\div2x^2$

4 Simplify the following expressions:

$\boxed{5y - 7y = {}^{-}2y}$ $\boxed{{}^{-}3x\times4x = {}^{-}12x^2}$

a) $7x - 2x$ **b)** $4x + 3x$ **c)** $2x - 5x$

d) $4 + 9x - 3x$ **e)** $3d - 4a + 3d$ **f)** $4xy^2 + 5a^5 - 2xy^2 + 6a^5$

g) $2\times3x$ **h)** $4x\times5$ **i)** ${}^{-}5n\times2n$

j) ${}^{-}3x^2\times{}^{-}4x$ **k)** $3y\times{}^{-}2y^3$ **l)** $2x^2\times5x\times2x$

m) $20x\div4$ **n)** ${}^{-}15x\div3$ **o)** ${}^{-}10b\div{}^{-}2b$

p) $14ab\div4a$ **q)** ${}^{-}10x^5\div{}^{-}4x^2$ **r)** ${}^{-}12d^6e^5f \div 36d^3e^3$

5 Expand each of the following:

$\boxed{{}^{-}3(2y - 5) = {}^{-}6y + 15}$

a) $5(x + 3)$ **b)** $2(h + 3)$ **c)** $5(d + 2)$

d) $2(4g - 3)$ **e)** $8(2x - 3)$ **f)** $7(2x - 3)$

g) ${}^{-}4(2p - 3)$ **h)** ${}^{-}6(c + 1)$ **i)** ${}^{-}x(x + 4)$

j) ${}^{-}3x(4x - 2)$ **k)** ${}^{-}3x(2x - 4y)$ **l)** ${}^{-}4x(3y - 2x)$

$\boxed{3(x - 2) + 2(x + 5) = 3x - 6 + 2x + 10 = 5x + 4}$

6 Simplify each of the following by expanding and then collecting like terms:

a) $3(x + 2) + 5(x + 1)$ **b)** ${}^{-}2(x + 3) + 5(x + 1)$

c) $2(y - 3) + 4(y + 2)$ **d)** ${}^{-}4(y - 1) + 3(y + 5)$

e) ${}^{-}t(3t + 1) + t(2t - 2)$ **f)** ${}^{-}5x(3x - 2) + {}^{-}2(x - 1)$

g) $(x + 2)(x + 3)$ **h)** $(x + 1)^2$

i) $(x + 4)(x - 1)$ **j)** $(x + 1)(x - 1)$

k) $(x + 2)(x - 2)$ **l)** $(2x - 1)(2x - 3)$

7 Factorise each of the following:

$\boxed{-9x^5 - 12x^2 = {}^{-}3x^2(3x^3 + 4)}$

a) $5x + 10$ **b)** $3x + 6$ **c)** $4x + 16$

d) $18a + 3$ **e)** $24r + 6$ **f)** $14y + 7$

g) $4x - 12$ **h)** $2y - 4$ **i)** $5x - 15$

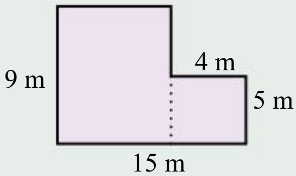
j) $6p - 10$ **k)** $10w - 8$ **l)** $10ab - 5a$

m) ${}^{-}5x - 15$ **n)** ${}^{-}2x - 6$ **o)** ${}^{-}2x + 8$

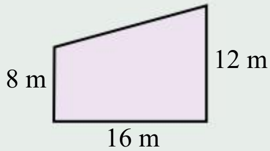
p) $10b^2 - 12b$ **q)** $14x^3 - 21x$ **r)** $24x^3 - 18x^2$

8 Calculate the area of each of the following composite shapes:

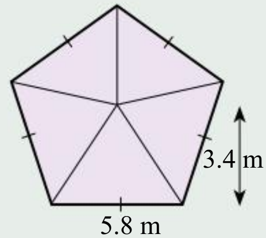
a)



b)

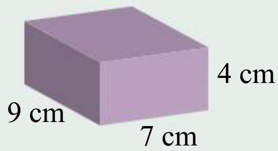


c)

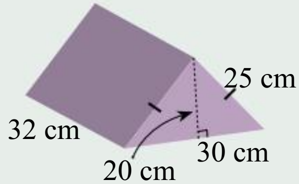


9 Find the surface area of each of the following prisms:

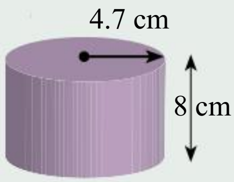
a)



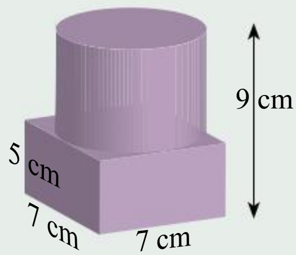
b)



c)



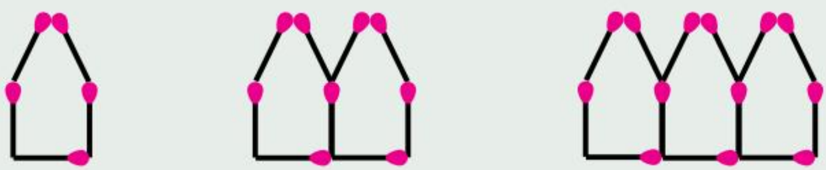
d)



10 The four sides and the top of the shed are to be given two coats of paint. How much paint is needed for two coats if 1 litre of paint will, on average, cover 15 m^2. What is the cost of paint @ $74.00 for 4L?

Shed Outside Dimensions	
Length	12.19 m
Width	2.44 m
Height	2.59 m

11 Write a rule for the following pattern:

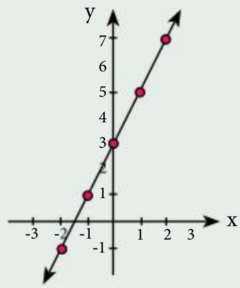


12 Write a rule for the following table:

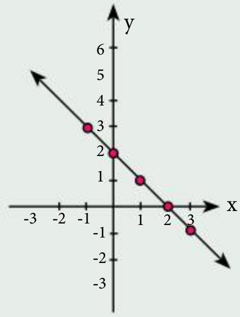
x	1	2	3	4	5	10	20
y	9	4	$^{-}1$	$^{-}6$	$^{-}11$	$^{-}36$	$^{-}86$

13 Write a rule for each of the following graphs:

a)



b)



14 Copy and complete each of the following tables and draw a graph of the rule:

a)

x	-2	-1	0	1	2
$y = 2x - 2$					

b)

x	-2	-1	0	1	2
$y = {}^-2x + 4$					

c)

x	-2	-1	0	1	2
$y = x^2 - 1$					

d)

x	-2	-1	0	1	2
$y = x^2 - 2$					

15 Draw a sketch of:

a) $y = 2x + 3$
b) $y = 2x - 1$
c) $y = {}^-2x + 4$

A tourist sees a black sheep and says "The sheep are black." The mathematician says "There is at least one sheep, and one side of that sheep is black."

16 Sketch $x^2 + y^2 = 5$.

x	$^-2$	$^-1$	0	1	2
$x^2 + y^2 = 5$	1 or $^-1$	2 or $^-2$	$\sqrt{5}$ or $^-\sqrt{5}$	2 or $^-2$	1 or $^-1$

Exercise 5.3 Mental computation

1 Spell Parabola.

2 What is the linear rule for:

x	0	1	2	3
y	-5	-2	1	4

3 Roughly sketch the rule: $y = 2x + 1$
4 What is the formula for the area of a circle?
5 What is the area of a triangle, height = 5 m and base = 8 m?
6 Simplify: $9x - 7x$
7 Expand: $3(x - 2)$
8 Factorise: $6x + 8$
9 $10^5 \times 10^3$
10 24×15

I'm a great believer in luck, and I find the harder I work the more I have of it - Thomas Jefferson.

Review 2

Exercise 5.4

$$d^3$$

Index

Base

1 Write each of the following in index form:

$3\times3\times3\times3 = \underline{3^4}$	$^{-}a\times^{-}a\times^{-}a= \underline{(^{-}a)^3}$

a) $1\times1\times1$ **b)** $6\times6\times6\times6$ **c)** $10\times10\times10\times10\times10$

d) $x\times x\times x\times x$ **e)** $9.2\times9.2\times9.2\times9.2$ **f)** $^{-}4\times^{-}4\times^{-}4\times^{-}4\times^{-}4$

2 Use the Index Laws to simplify each of the following:

$2^2\times2^{-5} = 2^{2+^{-}5}$ $\{a^m\times a^n = a^{m+n}\}$ $= \underline{2^{-3}}$ $\{2 + {}^{-}5 = {}^{-}3\}$	$5x^{-4}\times3x^3 = 15x^{^{-}4+3}$ $\{a^m\times a^n = a^{m+n}\}$ $= \underline{15x^{-1}}$ $\{^{-}4 + 3 = {}^{-}1\}$

a) $2^4\times2^{-2}$ **b)** $5^3\times5^{-7}$ **c)** $3^4\times3^{-2}$ **d)** $10^8\times10^{-5}$

e) $x^7\times x^{-3}$ **f)** $4x^5\times x^{-3}$ **g)** $2x^2\times4x^{-5}$ **h)** $4x^3\times3x^{-2}\times x^{-5}$

$5^2\div2^{-3} = 2^{2-^{-}3}$ $\{a^m\div a^n = a^{m-n}\}$ $= \underline{2^5}$ $\{2 - {}^{-}3 = 2+3 = 5\}$	$8x^{-4}\div2x^3 = 4x^{^{-}4-3}$ $\{a^m\div a^n = a^{m-n}\}$ $= \underline{4x^{-7}}$ $\{^{-}4 - 3 = {}^{-}7\}$

i) $3^3\div3^{-6}$ **j)** $6^5\div6^{-4}$ **k)** $2^{-5}\div2^2$ **l)** $10^{-7}\div10^{-3}$

m) $x^{-4}\div x^8$ **n)** $a^{-3}\div a^{-4}$ **o)** $b^3\div b^{-2}$ **p)** $9x^4\div3x^{-3}$

Index Law 1	**Index Law 2**	**Index Law 3**	**Zero Index**
$a^m\times a^n = a^{m+n}$	$a^m\div a^n = a^{m-n}$	$(a^m)^n = a^{m\times n}$	$a^0 = 1$

3 Use the Index Laws to simplify each of the following:

$(3^{-2})^4 = 3^{^{-}2\times4}$ $\{(a^m)^n = a^{m\times n}\}$ $= \underline{3^{-8}}$ $\{^{-}2\times4 = {}^{-}8\}$	$(x^{-2})^{-3} = x^{^{-}2\times^{-}3}$ $\{(a^m)^n = a^{m\times n}\}$ $= \underline{2^6}$ $\{^{-}2\times^{-}3 = 6\}$

a) $(2^{-2})^3$ **b)** $(3^{-2})^{-3}$ **c)** $(b^2)^3$ **d)** $(10^{-1})^{-4}$

e) $4^4\times4^3$ **f)** $x^3\times x^5$ **g)** $10^2\times10^3$ **h)** $2^{-6}\times2^1$

i) $3.2^3\times3.2^4$ **j)** $10^9\div10^2$ **k)** $5x^2\times3x^2$ **l)** $(x^{-3})^2\times8x^3\div2x^2$

4 Simplify the following expressions:

$\boxed{5y - 7y = {}^{-}2y}$ $\boxed{^{-}3x\times4x = {}^{-}12x^2}$

a) $5x-4x$ **b)** $8x+3x$ **c)** $3x-6x$

d) $2+8x-2x$ **e)** $5b-4b+3b$ **f)** $7x^2+5y^5-2x^2+3y^5$

g) $9\times3x$ **h)** $3x\times7$ **i)** $^{-}3b\times2b$

j) $^{-}3x^2 \times {}^{-}5x$ **k)** $4x \times {}^{-}2x^3$ **l)** $5x^2 \times 3x \times 2x$

m) $12x \div 4$ **n)** $^{-}12x \div 4$ **o)** $^{-}12n \div {}^{-}2n$

p) $14xy \div 8x$ **q)** $^{-}6b^5 \div {}^{-}4b^2$ **r)** $^{-}36x^5y^6z \div 24x^3y^3$

$^{-}3(2y - 5) = {}^{-}6y + 15$

5 Expand each of the following:

a) $4(x + 3)$ **b)** $3(a + 2)$ **c)** $6(b + 1)$

d) $3(2c - 1)$ **e)** $5(3d - 4)$ **f)** $5(3e - 2)$

g) $^{-}2(3x - 4)$ **h)** $^{-}7(x + 4)$ **i)** $^{-}x(4x + 3)$

j) $^{-}2x(3x - 1)$ **k)** $^{-}4x(3x - 2y)$ **l)** $^{-}4x(2y - 4x)$

$3(x - 2) + 2(x + 5) = 3x - 6 + 2x + 10 = 5x + 4$

6 Simplify each of the following by expanding and then collecting like terms:

a) $2(x + 4) + 5(x + 3)$ **b)** $4(x + 5) + {}^{-}2(x + 1)$

c) $6(x - 2) + 2(x + 3)$ **d)** $^{-}4(w - 3) + 3(w + 2)$

e) $^{-}a(2a + 1) + a(3a - 4)$ **f)** $^{-}4b(3b - 2) + {}^{-}2(b - 1)$

g) $(x + 1)(x + 3)$ **h)** $(x + 1)^2$

i) $(x + 4)(x - 1)$ **j)** $(x + 2)(x - 2)$

k) $(x + 3)(x - 3)$ **l)** $(3x - 1)(2x + 3)$

$-9x^5 - 12x^2 = {}^{-}3x^2(3x^3 + 4)$

7 Factorise each of the following:

a) $5x + 10$ **b)** $3x + 9$ **c)** $4x + 8$

d) $18p + 3$ **e)** $12v + 6$ **f)** $15w + 5$

g) $4m - 8$ **h)** $2n - 6$ **i)** $5o - 20$

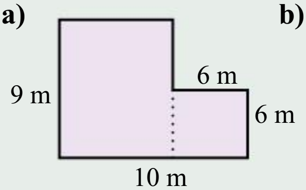
j) $6x - 10$ **k)** $12x - 8$ **l)** $8xy - 6x$

m) $-5x - 20$ **n)** $-2x - 8$ **o)** $-2x + 6$

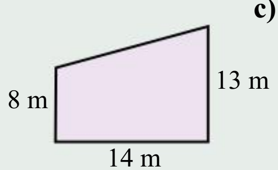
p) $14b^2 - 12b$ **q)** $21x^3 - 14x$ **r)** $24x^3 - 12x^2$

8 Calculate the area of each of the following composite shapes:

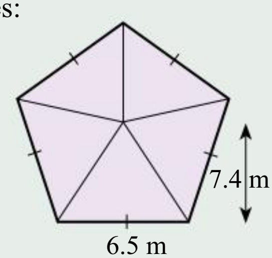
a)



b)

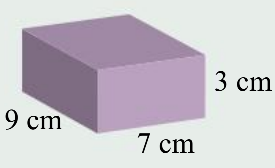


c)

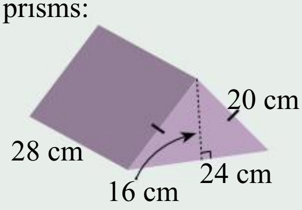


9 Find the surface area of each of the following prisms:

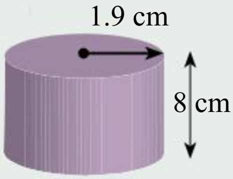
a)



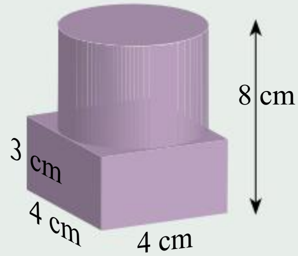
b)



c)



d)



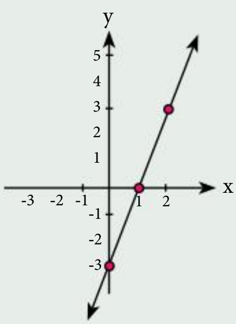
10 Write a rule for the following pattern:

11 Write a rule for the following table:

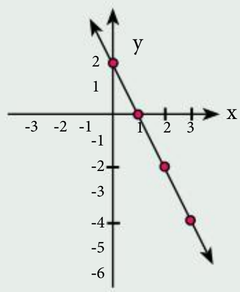
x	1	2	3	4	5	10	20
y	$^{-}7$	$^{-}3$	1	5	9	29	69

12 Write a rule for each of the following graphs:

a)



b)



13 Copy and complete each of the following tables and draw a graph of the rule:

a)

x	-2	-1	0	1	2
$y = 3x - 1$					

b)

x	-2	-1	0	1	2
$y = {}^{-}2x + 3$					

c)

x	-2	-1	0	1	2
$y = x^2 + 3$					

d)

x	-2	-1	0	1	2
$y = x^2 - 1$					

14 Draw a sketch of:

a) $y = 2x + 1$

b) $y = 2x - 2$

c) $y = {}^{-}2x + 1$

> Before you build a better mousetrap, it helps to know if there are any mice out there - Mortimer B Zuckerman.

15 Sketch $x^2 + y^2 = 9$.

x	$^{-}3$	$^{-}2$	0	2	3
$x^2 + y^2 = 9$	0	$\sqrt{5}$ or $^{-}\sqrt{5}$	3 or $^{-}3$	$\sqrt{5}$ or $^{-}\sqrt{5}$	0

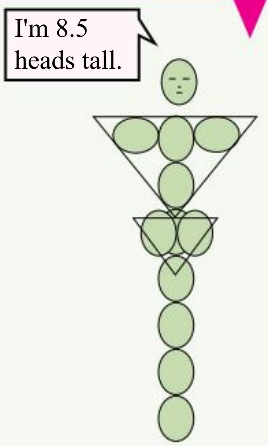
Proportion 6

Number and Algebra → Real Numbers

★ Solve problems involving direct proportion.

★ Explore the relationship between graphs and equations corresponding to simple rate problems.

★ Understand the difference between direct and inverse proportion, identifying these in real-life contexts and using these relationships to solve problems.



A TASK

A common problem for artists is drawing in proportion. The basic unit for drawing the human figure is the 'head' - the distance from the top of the head to the chin.

Investigate the following human drawing proportions:

Average - 7.5 heads tall.
Noble - 8 heads tall.
Heroic - 8.5 heads tall (bigger chest, longer legs).

A LITTLE BIT OF HISTORY

The human figure has been drawn since prehistoric times. The human figure is probably the most difficult subject for an artist.

- Ancient Greeks carved human figures in marble.
- 1487 - Leonardo da Vinci suggests ideal human proportions with the Vitruvian Man.
- 19th century European human figures drawn with clothes.
- Drawing the human figure in the 21st century is considered mandatory for an artist.

Leonardo da Vinci used the Golden Proportion in his painting of the Mona Lisa.

Warmup

A ratio can be written as 3: 10, a fraction $\frac{3}{10}$, a decimal 0.3, a percentage 30%

Exercise 6.1

Write the following comparisons as ratios:

17 people passed the test and 4 failed.	
a) What is the ratio of pass to fail?	17 : 4
b) What is the ratio of fail to pass?	4 : 17
c) What is the ratio of pass to the total?	17 : 21
d) What is the ratio of fail to the total	4 : 21

The mathematical symbol for ratio is

1 The Maths class has 13 girls and 11 boys.

a) What is the ratio of girls to boys?
b) What is the ratio of boys to girls?
c) What is the ratio of girls to the total number in the class?
d) What is the ratio of boys to the total number in the class?

2 Last month there were 24 sunny days and 7 cloudy days.

a) What is the ratio of sunny to cloudy days?
b) What is the ratio of cloudy to sunny days?
c) What is the ratio of sunny to the total number of days in the month?
d) What is the ratio of cloudy to the total number of days in the month?

3 Write each of the following ratios as a fraction, a decimal and a percentage:

	fraction	decimal	percentage
1 : 4	$\frac{1}{4}$	0.25	25%
5 : 2	$\frac{5}{2}$	2.5	250%

Make a percentage by multiplying by 100.

a) 1 : 5	**b)** 1 : 2	**c)** 1 : 4	**d)** 3 : 5
e) 1 : 10	**f)** 2 : 5	**g)** 2 : 10	**h)** 4 : 5
i) 7 : 10	**j)** 3 : 10	**k)** 6 : 10	**l)** 5 : 10
m) 8 : 10	**n)** 5 : 5	**o)** 9 : 10	

4 Write each of the following fractions as a ratio, a decimal and a percentage:

a) $\frac{1}{2}$	**b)** $\frac{3}{10}$	**c)** $\frac{4}{5}$	**d)** $\frac{1}{4}$
e) $\frac{3}{4}$	**f)** $2\frac{1}{2}$	**g)** $3\frac{1}{10}$	**h)** $6\frac{3}{5}$

Warmup

Exercise 6.2

Simplify the following ratios:

15 : 9

$$= \frac{15}{9}$$

$$= \frac{3 \times 5}{3 \times 3}$$

$$= \frac{5}{3} \qquad = \underline{5 : 3}$$

4.5 : 5.5

$$= \frac{4.5 \times 10}{5.5 \times 10}$$

$$= \frac{45}{55}$$

$$= \frac{5 \times 9}{5 \times 11}$$

$$= \frac{9}{11} \qquad = \underline{9 : 11}$$

8b : 4b

$$= \frac{8b}{4b}$$

$$= \frac{8}{4}$$

$$= \frac{2}{1} \qquad = \underline{2 : 1}$$

1 2 : 4	**2** 3 : 6	**3** 5 : 10
4 3 : 12	**5** 2 : 8	**6** 5 : 20
7 2.5 : 1.5	**8** 2.4 : 1.8	**9** 5 : 1.5
10 2.8 : 2.1	**11** 4.0 : 1.2	**12** 8a : 12a
13 2c : 8c	**14** 9x : 15x	**15** 1.44y : 7.2y

Exercise 6.3

Express each of the following as a ratio and simplify

500g to 2 kg

= 500 g to 2000 g

= 500 : 2000

$$= \frac{500}{2000}$$

$$= \frac{1 \times 500}{4 \times 500}$$

$= \underline{1 : 4}$

40 mins to 1 hour 30 mins

= 40 mins to 90 mins

= 40 : 90

$$= \frac{40}{90}$$

$$= \frac{4 \times 10}{9 \times 10}$$

$= \underline{4 : 9}$

First make the units the same.

Calculators are good at simplifying ratios (Technology 6.1).

1 200 g to 1 kg	**2** 600 g to 3 kg	**3** 500 g to 3 kg
4 2 kg to 100 g	**5** 1.5 kg to 500 g	**6** 2.5 kg to 1.5 kg
7 30 mins to 1 hour	**8** 20 mins to 1 hour	**9** 50 mins : 2 hours
10 3 hours to 30 mins	**11** 1.5 hours to 4 hours	**12** 3 hours to 1 hr 40 mins
13 600 mm to 1.2 m	**14** 3.5 m to 700 mm	**15** 500 mm to 2.5 m
16 45 km to 15 km	**17** 32 km to 48 km	**18** 4.5 km to 1.5 km
19 50 L to 75 L	**20** 20 L to 32 L	**21** 2.4 L to 3.2 L
22 600 mL to 1.2 L	**23** 2 L to 400 mL	**24** 1.2 m to 60 cm
25 80 cm to 2.4 m	**26** 40 cents to $2	**27** $3 to 60 cents

Proportion

When two ratios are equal they are said to be in **proportion**.

Exercise 6.4

Which of the following pairs of ratios are in proportion?

$$4:6 \quad \text{and} \quad 10:15$$
$$= \frac{4}{6} \qquad\qquad = \frac{10}{15}$$
$$= \frac{2\times 2}{2\times 3} \qquad\qquad = \frac{5\times 2}{5\times 3}$$
$$= 2:3 \qquad\qquad = 2:3$$

The ratios are equal. They are in proportion.

1	4 : 2 and 6 : 3	**2**	8 : 4 and 10 : 5	**3**	12 : 6 and 14 : 7
4	6 : 2 and 6 : 3	**5**	12 : 4 and 9 : 3	**6**	15 : 5 and 14 : 7
7	5 : 20 and 2 : 8	**8**	10 : 4 and 15 : 6	**9**	12 : 8 and 9 : 6
10	2.0 : 0.5 and 8 : 2	**11**	2.5 : 0.5 and 10 : 2	**12**	6 : 4 and 1.8 : 1.2
13	1 : 0.5 and 2.1 : 0.7	**14**	20 mins to 1 hour and 30 mins to 1 hour 30 mins		

Travel 120 km in 80 minutes and travel 180 km in 2 hours.

120 km	in	80 mins
180 km	in	120 mins
$\frac{120}{180}$	and	$\frac{80}{120}$
2:3	and	2:3

The ratios are equal. They are in proportion.

A **ratio** compares quantities of the **same kind**.

Write the quantities with the **same units under each other**.

15 Travel 200 km in 3 hours and travel 600 km in 9 hours.

16 Travel 90 km in 80 minutes and travel 99 km in 88 minutes.

17 2 tonnes are loaded in 3 hours and 6 tonnes are loaded in 9 hours.

18 Travel 100 km on 8 litres of petrol and travel 200 km on 16 litres of petrol.

19 Travel 80 km on 6 litres of petrol and travel 60 km on 4 litres of petrol.

20 4 kg costs \$2 and 6 kg costs \$3.

21 4k costs \$12 and 3.5 kg costs \$10.50.

22 6 parcels of weight 9 kg and 24 parcels of weight 36 kg.

23 \$AU200 exchanged for \$US204 and \$AU500 exchanged for \$US510

24 120 cm tall at 12 years and 160 cm tall at 20 years.

Proportion

Proportion means that two ratios are equal.

Proportion
2:3 = 4:6

$\frac{2}{3} \; \frac{4}{6}$

or 2×6 = 3×4

Proportion

If $\frac{a}{b} = \frac{c}{d}$ $\qquad \frac{a}{b} \; \frac{c}{d}$

then $ad = bc$
or $\quad bc = ad$

Cross-multiplication is useful in solving proportion problems. $\frac{a}{b} \; \frac{c}{d}$

Exercise 6.5

Assuming proportionality, solve the following problems.

If one Australian dollar, \$AU1, can be exchanged for \$US1.03, how many Australian dollars can be exchanged for \$US236?

\$AU1 for \$US1.03
x for \$US236

$$x \times 1.03 = 1 \times 236 \quad \{\text{cross multiply}\}$$
$$x = \frac{1 \times 236}{1.03} \quad \{\div \text{ by } 1.03\}$$
$$\underline{x = \$229.13} \quad \{\text{calculator}\}$$

If a farmer can grow 132 tonnes of corn on 30 hectares, how many hectares is needed to grow 1500 tonnes of corn?

132 t for 30 ha
1500 t for x ha

$$x \times 132 = 1500 \times 30 \quad \{\text{cross multiply}\}$$
$$x = \frac{1500 \times 30}{132} \quad \{\div \text{ by } 132\}$$
$$\underline{x = 341 \text{ ha}} \quad \{\text{calculator}\}$$

1 If \$AU1 can be exchanged for \$US1.06, how many Australian dollars can be exchanged for \$US1280?

2 The wheel on an electric motor completes 1800 revolutions in 3 minutes. How many revolutions will it complete in 8 minutes?

3 A car uses 7 litres of petrol to travel 100 km. How much petrol is needed to travel 325 km?

4 A car uses 6.8 litres of petrol to travel 100 km. How far will the car travel on 52 litres of petrol?

5 The design requires a small gear to large gear ratio of 1 : 3. If the large gear has 45 teeth, how many teeth on the small gear?

6 100 g contains 2.3 g of fat. How much fat in 250 g?

7 The concrete requires a mix of 6 bags of sand to one bag of cement. How many bags of cement need to be mixed 35 bags of sand?

8 10 mL of custard powder should be added to 500 mL of milk to make custard. How much custard powder should be added to 2 L of milk?

9 If it takes 4 minutes to run 800 m, how long will it take to run 4 km?

10 If 40 ha of land is valued at \$450 000, what will be the valuation of 180 ha?

Direct Proportion

Direct proportion means an increase in one quantity will cause a similar increase in another quantity.

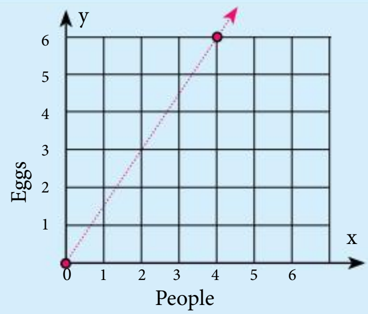
Direct proportion test.
Double one quantity and the other quantity will double.

Exercise 6.6
Draw a graph and write a rule for each of the following:

A pizza scrambled egg recipe uses 6 eggs to serve 4 people.

People (x)	0	4
Eggs (y)	0	6

0 eggs serves 0 people.



y = mx + c

Increase of 6 for 4 steps
so m = 6/4
= 1.5
c = 0 {cuts y axis at 0}

Eggs = 1.5×people

1 For each 40 km the rally car uses 8 litres of fuel.

0 distance, 0 fuel.

Distance (x)	0	40
Fuel (y)	0	8

2 For each 60 km the truck uses 12 litres of fuel.

Increase 12 for 60
m = 12/60 c = 0
m = 0.2 ∴ Fuel = 0.2×Distance.

3 15 mg of children's Dosumedrol for every 10 kg.

4 It was suggested that 75 kg of lime per 5 hectares would reduce the soil acidity.

5 The 250 ha paddock yielded 375 tonnes of cotton.

6 The price for cotton was 450 cents for 1 kg.

7 Sound can travel 3.4 km in 10 seconds.

Direct Proportion because:
3.4×2 km in 10×2 seconds.
6.8 km in 20 seconds works.

Direct Proportion

Exercise 6.7

Solve the following Direct Proportion problems by:

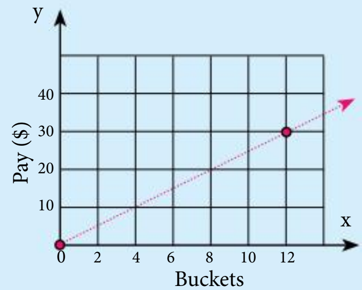
a) Finding the rule.

b) Using the rule to solve the problem.

Mark was paid $30 for picking 12 buckets of tomatoes.
How many buckets would Mark need to pick for $20?

y = mx + c Increase of 30 for 12 steps
so m = 30/12
= 2.5
c = 0 {cuts y axis at 0}
y = 2.5x {or pay = 2.5×buckets}

When pay = $20
20 = 2.5×buckets
20÷2.5 = buckets {inverse of × is ÷}
8 = buckets



1 Jess picked 19 buckets of tomatoes in two hours.
How many buckets of tomatoes would Jess be expected in 7 hours?

2 The car can travel 100 km on 6 litres of petrol.
How far will the car travel on 25 litres of petrol?

3 Aaron can run 4 km in 20 minutes.
How far can Aaron be expected to run in 1.5 hours?

4 Megan can run 3 km in 14 minutes.
How long would it take Megan to run 10 km?

5 The infusion pump was set to give 300 mL of medication over 120 minutes.
How much medication would be given in 15 minutes?

6 The gravy recipe suggests 2 tablespoons of flour for enough gravy for 8 people. How many tablespoons of flour is needed to serve 50 people?

30g butter
2 tbsp flour
3 cups chicken stock
1 tbsp sherry

7 If the electricity tariff is 18 cents per kilowatt-hour,
what is the cost for using 65 kilowatt-hours (kWh) in one day?.

8 If a normal household uses 5200 kWh of electricity in four months,
how much energy would be used, on average, in 1 week?

Inverse Proportion

Inverse proportion means an **increase in one** quantity will cause a similar **decrease in another** quantity.

Inverse proportion example.

10 people can build a house in 24 days.
20 people .... 12 days.
30 people .... 8 days.
40 people .... 6 days.

People	Days
10	24
20	12

$10\times24 = 20\times12$

People	Days
a	c
b	d

$ac = bd$

Exercise 6.8

Assuming inverse proportion, solve the following problems.

If 20 people can build a house in 30 days, how long would it take 25 people to build the same house? 20 people 30 days 25 people x days $20\times30 = 25\times x$ $20\times30\div25 = x$ $24 = x$ 25 people would take 24 days to build the house.	It will cost \$30 per person if there are 45 people on the charter bus. If there are 40 people how much will it cost? \$30 45 people \$x 40 people $30\times45 = x\times40$ $30\times45\div40 = x$ $33.75 = x$ If there are 40 people, it will cost \$33.75 per person.

1 If 5 people can build a house in 20 days,
how long would it take 20 people to build the same house?

2 If 12 people can build a house in 15 days,
how many people will it take to build the same house in 10 days?

3 It will cost \$120 per person if there are 15 people on the charter plane.
If there are 10 people on the plane what would be the cost per person?

4 It will cost \$90 per person if there are 20 people on the charter bus.
How many people would be needed to reduce the cost per person to \$60?

5 Travelling from A to B will take 30 mins at an average speed of 80 km/h.
How long will it take at an average speed of 100 km/h?

6 Travelling from A to B will take 30 mins at an average speed of 80 km/h.
What average speed would be needed to reduce the time to 20 mins?

7 If it takes 24 hours for a hose with a water flow of 15 litres per minute to fill a tank, how long will it take to fill the tank if the water flow is increased to 20 litres per minute?

Money and Proportion

Cross-multiplication is useful in solving proportion problems. $\frac{a}{b} \times \frac{c}{d}$

Proportion
2:3 = 4:6
$\frac{2}{3} = \frac{4}{6}$
or $2\times6 = 3\times4$

Proportion
If $\frac{a}{b} = \frac{c}{d}$ $\quad \frac{a}{b} \times \frac{c}{d}$
then ad = bc
or bc = ad

Exercise 6.9
Assuming proportionality, solve the following problems.

If 2.4 metres of garden fencing costs \$68.40, what is the cost of 10.6 metres? 2.4 m costs \$68.40 10.6 m costs x $2.4\times x = 10.6\times68.40$ {cross multiply} $x = \frac{10.6\times68.40}{2.4}$ {÷ by 2.4} $x = \$302.10$ {calculator}	If \$AU10 969 can be exchanged for MYR35 420 (Malaysian Ringgits), how many Malaysian Ringgits can be exchanged for \$AU45 000? \$AU10 969 for MYR35 420 \$AU45 000 for x $10\,969\times x = 45\,000\times35\,420$ $x = \frac{45\,000\times35\,420}{10\,969}$ {÷ 10969} $x = \text{MYR}145\,310$

1 1.6 kilograms of rump steak cost \$32.60.
a) What is the cost of 2.3 kg?
b) How much steak for \$15?

Direct proportion means an increase in one quantity will cause a similar increase in another quantity.

2 45 kg of garden mulch costs \$68.50.
a) What is the cost of 130 kg?
b) How much garden mulch for \$100?

3 1.55 square metres of cloth cost \$8.35.
a) What is the cost of 4.8 square metres of cloth?
b) How much cloth for \$20?

4 If \$NZ3610 can be exchanged for JPY238 937 (Japanese Yen).
a) How many New Zealand Dollars can be exchanged for JPY1000?
b) How many Japanese Yen can be exchanged for \$NZ1000?

5 1000 size 23/15 staples cost \$12.78.
a) How many staples for \$100?
b) What is the cost of 5000 staples?

Direct proportion test.
Double one quantity and the other quantity will double.

6 If \$SG967 (Singapore Dollar) can be exchanged for \$AU733.22.
a) How many Singapore Dollars can be exchanged for \$AU1000?
b) How many Australian Dollars can be exchanged for \$SG1000?

Mental Computation

The better you become at mental athletics the better you can think.

Exercise 6.10

1. Spell Proportion.
2. Simplify 15 : 10
3. A car uses 7 litres to travel 100 km. How much petrol is needed to travel 300 km?
4. 10 people build a house in 50 days How many can build the house in 20 days?
5. What is the linear rule for:

x	0	1	2	3
y	-4	-1	2	5

6. Roughly sketch the rule: $y = 2x + 3$
7. What is the formula for the circumference of a circle?
8. Simplify: $7a - 5a$
9. Expand: $3(x + 2)$
10. Factorise: $5x + 10$

Patient: I swallowed a clock some time ago.
Doctor: Why didn't you see me earlier?
Patient: I didn't want to alarm you.

Exercise 6.11

1. Spell Inverse.
2. Simplify 8 : 12
3. A car uses 8 litres to travel 100 km. How much petrol is needed to travel 400 km?
4. 20 people build a shed in 5 days How long will it take 4 people to build the shed?
5. What is the linear rule for:

x	0	1	2	3
y	6	4	2	0

6. Roughly sketch the rule: $y = 2x - 1$
7. What is the formula for the area of a circle?
8. Simplify: $4a - 6a$
9. Expand: $5(x - 2)$
10. Factorise: $6x + 8$

Inverse Land?
What is 7 times 6? 42
What is 6 times 7? 24

Exercise 6.12

1. Spell Cross-multiplication.
2. Simplify 15 : 9
3. A car uses 6 litres to travel 100 km. How much petrol is needed to travel 150 km?
4. 20 people build a house in 30 days How many can build the house in 10 days?
5. What is the linear rule for:

x	0	1	2	3
y	-2	2	6	10

6. Roughly sketch the rule: $y = {}^{-}2x + 4$
7. What is the formula for the circumference of a circle?
8. Simplify: $8a - 3a$
9. Expand: $6(x + 3)$
10. Factorise: $6x + 10$

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 6.13

1 A pedestrian is walking at a speed of 6 km/h.
How far does the pedestrian walk in 10 minutes?

Hint:
Make the units the same.

6 km in 60 mins
x km in 10 mins

$6\times10 = x\times60$

$\frac{6\times10}{60} = x$

Walk 1 km in 10 mins

2 A person is running at a speed of 12 km/h.
How far does the person run in 20 minutes?

3 A car is travelling at a speed of 80 km/h.
How far does the car travel in 10 seconds?

4 Mowing is quoted at \$93.50/ha.
What would be the cost of mowing 1 km^2?

5 Harvesting standing coarse grain is quoted at \$186.50/ha.
What would be the cost of harvesting 450 acres (1 acre = 0.405 hectares)?

6 At idle speed, the crankshaft of an engine typically rotates at 800 revolutions per minute?
How many degrees does the crankshaft turn in 1 second?

800×360° in 60 secs
x° in 1 sec

$800\times360\times1 = x\times60$

$\frac{800\times360}{60} = x$

Rotates 4800° in 1 second.

7 A fan spins at 1200 revs per minute.
In one second, how many degrees does the fan turn?

8 Karen can pick 12 buckets of tomatoes in 1 hour.
Megan can pick 10 buckets in 40 minutes and Eun-Young can pick 8 buckets in 30 minutes.
How long would it take the three people to pick 100 buckets of tomatoes?

Karen: 12 buckets in 1 h.
Megan: 15 buckets in 1 h.
Eun-Y: 16 buckets in 1 h.

Together: 43 buckets in 1 h.
100 buckets in x.

9 Matthew can pick 14 buckets of tomatoes in 1 hour. Adam can pick 10 buckets in 50 minutes and Aaron can pick 9 buckets in 30 minutes.
How long would it take the three people to pick 100 buckets of tomatoes?

10 Pipe A can fill a tank in 120 minutes.
Pipe B can fill the tank in 180 minutes.
Pipe C can fill the tank in 90 minutes.
If the three pipes are used together, how long will it take to fill the tank?

Actuaries analyse all kinds of data. For example, actuaries may design life insurance policies and premiums based on calculations involving economic trends, unemployment, illness, accident, and death probabilities.
An actuary is a kind of super mathematician and is paid accordingly.

- Relevant school subjects are English, Mathematics, Science.
- Courses normally involve a University degree and post graduate study.

Investigations

Investigation 6.1 Direct Proportion

1 Draw a circle.

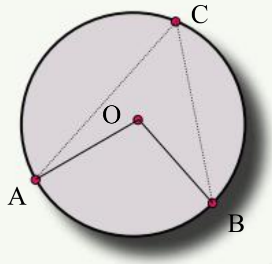
2 Mark two points, A and B, on the circumference.

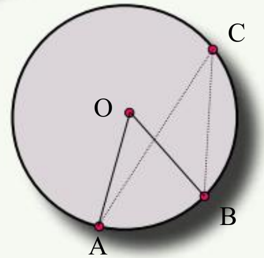
3 Join A and B to the centre O.

4 Mark C on the circumference, and join A and B to C.

5 Measure angles AOB and ACB.

6 What do you notice?
Does the size of the circle make a difference?
Investigate why?





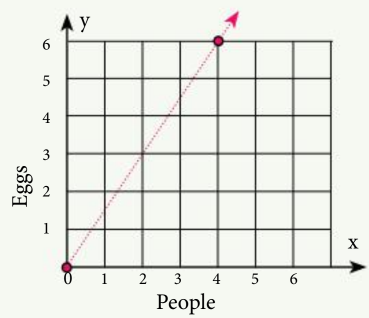
Circle	1	2	3	4	5	6
∠AOB						
∠ACB						
ratio						

Investigation 6.2 Inverse Proportion

The graph of Direct Proportion is a straight line.

Eggs (x)	0	6
People (y)	0	4

0 eggs serves 0 people.



Investigate

The graph of Inverse Proportion?

Inverse proportion example.
10 people can build a house in 24 days.
20 people .... 12 days.
30 people .... 8 days.
40 people .... 6 days.

A Couple of Puzzles

Exercise 6.14

1 Desley has been complaining that she paid \$18 500 in tax last financial year. If her tax rate was 31.5%, what was Desley's nett income last financial year?

2

What is the secret number?

- It is a fraction.
- The decimal equivalent has one decimal place.
- The numerator is a square number.
- The denominator is between 10 and 20.
- The fraction is between 0.4 and 0.8

A Game

Estimate. Players, or teams, take turns in estimating the average of a set of numbers. Sets of numbers are written on cards.

1 A card is selected at random. The player, or team, estimates the average of a set of numbers written on the card.

	28	18	
20	47	13	45
49	38	8	41
33	37	48	46
9	48	6	18
16	7	35	12
		34	24

2 The estimate is scored:

Within 1 of the correct answer:	5 points
Within 2 of the correct answer:	4 points
Within 3 of the correct answer:	3 points
Within 4 of the correct answer:	2 points
Within 5 of the correct answer:	1 point

3 Highest score, after five or ten estimates, wins.

A Sweet Trick

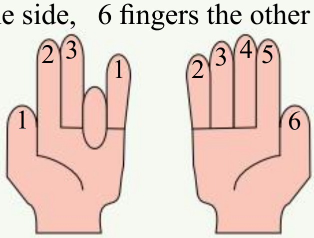
1 Show your audience your 9 times hand calculator

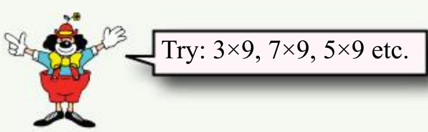
2 Ask your audience to choose a multiplicant from 1 to 9. 4

3 Bend over the fourth finger from the left

4 The fingers left give the answer. 9×4 = 36

3 fingers one side, 6 fingers the other = 36





Technology

Technology 6.1 Simplifying Ratios

Calculators, with $\boxed{a\frac{b}{c}}$, are very good at simplifying ratios:

1 $\frac{6}{9}$ $\boxed{6}$ $\boxed{a\frac{b}{c}}$ $\boxed{9}$ $\boxed{=}$ $\boxed{2r3}$ meaning $\frac{2}{3}$

2 $\frac{36}{60}$ $\boxed{36}$ $\boxed{a\frac{b}{c}}$ $\boxed{60}$ $\boxed{=}$ $\boxed{3r5}$ meaning $\frac{3}{5}$

Technology 6.2 Direct Proportion

Setup a spreadsheet to automatically solve direct proportion problems.

	A	B	C
1	$AU		$US
2	1	for	1.03
3	x	for	236
4			
5	x	=	$229.13

Enter formula:
=a2*c3/c2

If one Australian dollar, $AU1, can be exchanged for $US1.03, how many Australian dollars can be exchanged for $US236?

$AU1 for $US1.03
x for $US236

$x \times 1.03 = 1 \times 236$ {cross multiply}

$x = \dfrac{1 \times 236}{1.03}$ {÷ by 1.03}

$x = \$229.13$ {calculator}

Technology 6.3 Inverse Proportion

Use a spreadsheet to graph Inverse Proportion.

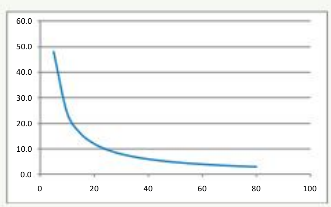
1 Enter points (product = 240).

2 Graph points.

Inverse proportion example.
10 people can build a house in 24 days.
20 people .... 12 days.
30 people .... 8 days.
40 people .... 6 days.

	A	B
1	People	Days
2	5	48.0
3	10	24.0
4	15	16.0
5	20	12.0

Enter formula:
=240/a2



Technology 6.4 Proportion Games

Search the Internet for some of the many Proportion games and applets.
Use search phrases such as: 'proportion game applet', 'inverse proportion applet'.

Chapter Review 1

Exercise 6.15

1 Simplify the following ratios:

a) 3 : 15 **b)** 6 : 8 **c)** 15 : 20

d) 12 : 18 **e)** 2.4 : 1.8 **f)** 5 : 1.5

2 Which of the following pairs of ratios are in proportion?

a) 4 : 2 and 10 : 5 **b)** 12 : 4 and 9 : 3 **c)** 12 : 6 and 14 : 7

d) Travel 200 km in 3 hours and travel 600 km in 9 hours.

e) 2 tonnes are loaded in 5 hours and 6 tonnes are loaded in 15 hours.

3 Draw a graph and write a rule for each of the following:

a) For each 40 km the rally car uses 10 litres of fuel.

b) Sound can travel 3.4 km in 10 seconds.

4 Assuming proportionality, solve the following problems.

a) If \$AU1 can be exchanged for \$US1.08, how many Australian dollars can be exchanged for \$US1280?

b) Aaron can run 4 km in 20 minutes.
How far can Aaron be expected to run in 50 minutes?

c) A car uses 6.8 litres of petrol to travel 100 km.
How far will the car travel on 52 litres of petrol?

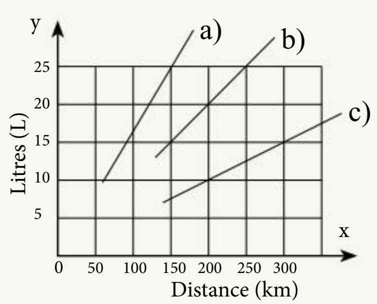
5 Assuming inverse proportion, solve the following problems.

a) If 5 people can build a house in 20 days,
how long would it take 20 people to build the same house?

b) It will cost \$50 per person if there are 20 people on the charter bus.
How many people would be needed to reduce the cost per person to \$40?

c) Travelling from A to B will take 30 mins at an average speed of 80 km/h.
What average speed would be needed to reduce the time to 20 mins?

6 From the graph, find the litres per 100 km fuel consumption for each vehicle.



Chapter Review 2

Exercise 6.16

1 Simplify the following ratios:

a) 10 : 5 **b)** 6 : 4 **c)** 12 : 8

d) 20 : 15 **e)** 1.2 : 1.5 **f)** 3 : 1.5

2 Which of the following pairs of ratios are in proportion?

a) 6 : 2 and 9 : 3 **b)** 5 : 4 and 10 : 2 **c)** 15 : 5 and 12 : 4

d) Travel 300 km in 4 hours and travel 450 km in 6 hours.

e) 6 tonnes are loaded in 4 hours and 10 tonnes are loaded in 6 hours.

3 Draw a graph and write a rule for each of the following:

a) For each 50 km the rally car uses 10 litres of fuel.

b) The 250 ha paddock yielded 325 tonnes of cotton.

4 Assuming proportionality, solve the following problems.

a) If \$AU1 can be exchanged for \$US0.89, how many Australian dollars can be exchanged for \$US750?

b) Aaron can run 4 km in 20 minutes.
How far can Aaron be expected to run in 2 hours?

c) A car uses 7.3 litres of petrol to travel 100 km.
How far will the car travel on 64 litres of petrol?

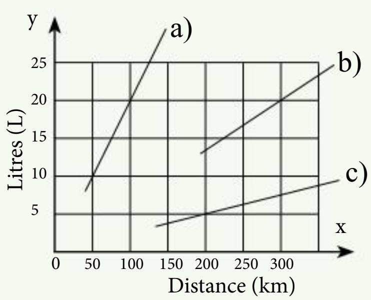
5 Assuming inverse proportion, solve the following problems.

a) If 10 people can build a house in 30 days,
how long would it take 25 people to build the same house?

b) It will cost \$65 per person if there are 30 people on the charter bus.
How many people would be needed to reduce the cost per person to \$57?

c) Travelling from A to B will take 40 mins at an average speed of 90 km/h.
What average speed would be needed to reduce the time to 30 mins?

6 From the graph, find the litres per 100 km fuel consumption for each vehicle.



Pythagoras' Theorem

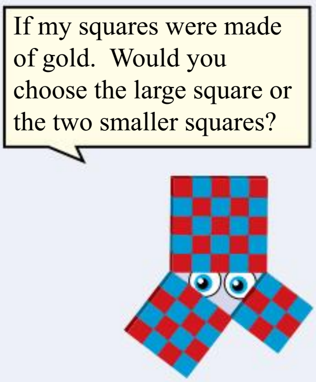
7

Measurement and Geometry ➞ Pythagoras and Trigonometry

★ Investigate Pythagoras' Theorem and its application to solving simple problems involving right-angled triangles.

★ Understand that Pythagoras' Theorem is a useful tool in determining unknown lengths in right-angled triangles and has widespread applications.

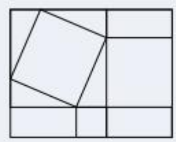
★ Recognise that right-angled triangle calculations may generate results that can be integral, fractional or irrational numbers known as surds.



A TASK

A popular puzzle is the Pythagorean Puzzle.

- Research the puzzle.
- Make your own pieces.
- Demonstrate that you can make and solve your own puzzles.



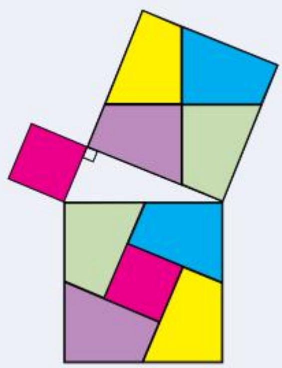
A LITTLE BIT OF HISTORY

1900-1600 BC — A Babylonian Tablet contains the Theorem.

560-480 BC — Pythagoras' Theorem: In a right-angle triangle $c^2=a^2+b^2$.

300 BC — Euclid supplies two different proofs and states the converse that if $a^2+b^2=c^2$ then a right-angled triangle.

Present day — Hundreds of proofs exist. The Theorem has thousands of applications.



Pythagorean Triads

Early civilisations knew that the following ratios produced right-angled triangles:

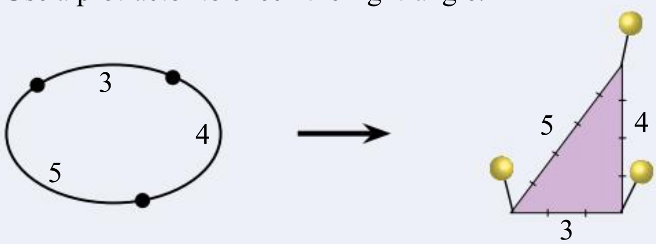
3:4:5

5:12:13

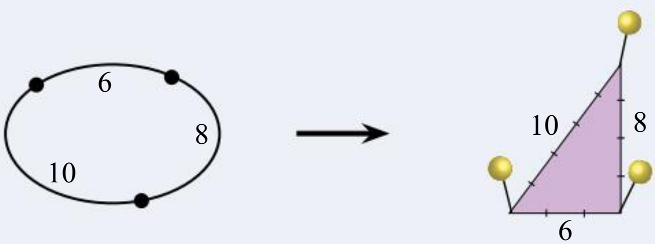
And that is how buildings were built square.

Exercise 7.1

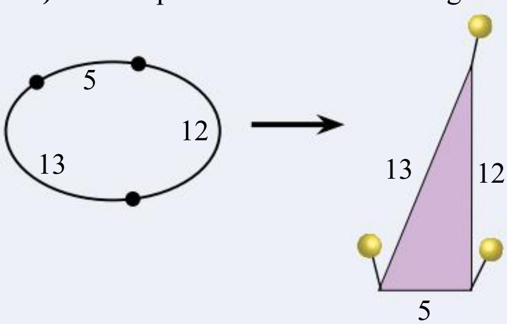
1 Use the 3:4:5 ratio to make a right-angled triangle.
- **a)** Make a loop of string with knots at 3, 4, 5 intervals.
- **b)** Hold tight at the knots to make a triangle with sides 3, 4, 5.
- **c)** Use a protractor to check the right-angle.



2 Use the 6:8:10 ratio (double 3:4:5) to make a right-angled triangle.
- **a)** Make a loop of string with knots at 6, 8, 10 intervals.
- **b)** Hold tight at the knots to make a triangle with sides 6, 8, 10.
- **c)** Use a protractor to check the right-angle.



3 Use the 5:12:13 ratio to make a right-angled triangle.
- **a)** Make a loop of string with knots at 5, 12, 13 intervals.
- **b)** Hold tight at the knots to make a triangle with sides 5, 12, 13.
- **c)** Use a protractor to check the right-angle.

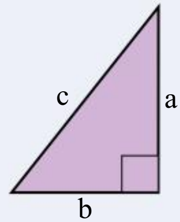


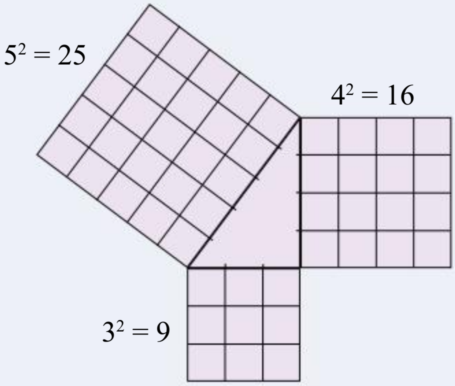
A few Pythagorean Triads
3:4:5
6:8:10
9:12:15
5:12:13
10:24:26
15:36:39
8:15:17
16:30:34

Pythagoras' Theorem

In any right-angled triangle:
The square on the hypotenuse is equal to the sum of the squares on the other two sides.

$c^2 = a^2 + b^2$





$$\begin{aligned} c^2 &= 5^2 \\ &= 25 \\ a^2 + b^2 &= 3^2 + 4^2 \\ &= 9 + 16 \\ &= 25 \end{aligned}$$

Thus $c^2 = a^2 + b^2$
The triangle is right-angled.

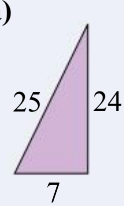
Exercise 7.2

1 Copy and complete the following table:

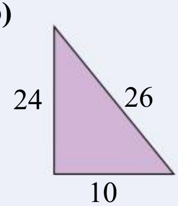
					Right-angled?
a=3	b=4	$a^2+b^2=25$	c=5	$c^2=25$	yes. $c^2 = a^2 + b^2$
a=4	b=5	$a^2+b^2=41$	c=6	$c^2=36$	no. $c^2 \neq a^2 + b^2$
a=6	b=8	$a^2+b^2=$	c=10	$c^2=$	
a=5	b=12	$a^2+b^2=$	c=13	$c^2=$	
a=7	b=15	$a^2+b^2=$	c=16	$c^2=$	

2 Which of the following triangles are right-angled triangles?

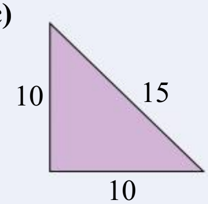
a)



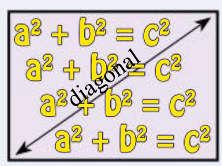
b)



c)



3 A rectangular picture frame measures 58 cm by 67 cm with a diagonal of 88.6 cm. Is the picture frame square?



4 A carpenter needs to know whether a door frame is square.
If the door frame is square then a door can be fitted.
The door frame measures 810 mm by 2000 mm and the diagonal is 2140 mm.

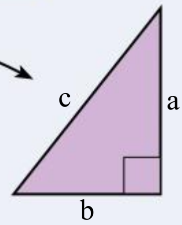
5 A 2.3 m ladder is leaning against a wall. The bottom of the ladder is 0.8 m from the wall and the top of the ladder is 2.16 m up the wall. Is the wall vertical?

6 A rectangular gate measures 1.2 m by 2.3 m with a 2.4 m diagonal. Is the gate square? If not, should the diagonal be longer or shorter?

Hypotenuse

$c^2 = a^2 + b^2$

In any right-angled triangle:
The square on the hypotenuse is equal to the sum of the squares on the other two sides.

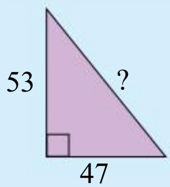


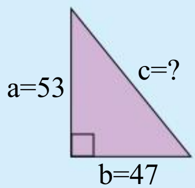
The hypotenuse is the longest side.
It is opposite the right-angle (90°).

Exercise 7.3

1 Find the length of the hypotenuse in each of the following:

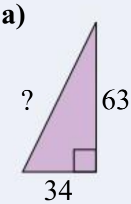
First add a, b, c



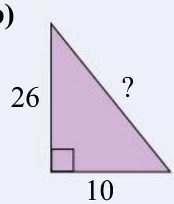


$$
\begin{aligned}
c^2 &= a^2 + b^2\\
c^2 &= 53^2 + 47^2\\
c^2 &= 5018\\
c &= \sqrt{5018}\\
c &= \underline{70.84}
\end{aligned}
$$

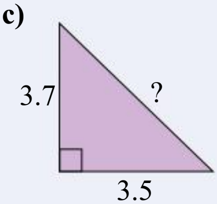
a)



b)



c)

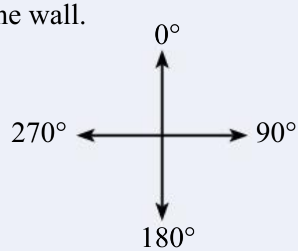


2 A farmer wishes to check that the gate is square. What should be the length of the diagonal if the gate is 3.4 m by 1.2 m?

3 The window frame needs to be square to accept a 844 mm by 1173 mm window. What should the length of the diagonal of the window frame?

4 The diagonal of a TV screen describes the size of the TV.
For example, a 68 cm TV has a screen with a 68 cm diagonal.
Calculate the size of each of the following TV screens:
a) 54 cm by 41 cm
b) 39 cm by 31 cm
c) 13.7 inches by 10 inches (Computer monitor)

5 A ladder lies against a vertical wall. The bottom of the ladder is 2.1 m away from the wall, and the top of the ladder is 5.6 m up the wall.
What is the length of the ladder?

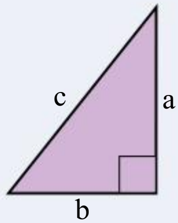


6 A plane travels from A to B on a bearing of 90° at a speed of 150 km/h for 30 mins. The plane then travels from B to C on a bearing of 180° at a speed of 200 km/h for 90 mins. What is the distance between A and C?

The Shorter Sides

In any right-angled triangle:
The square on the hypotenuse is equal to the sum of the squares on the other two sides.

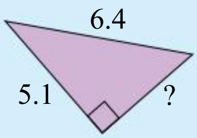
$a^2 + b^2 = c^2$

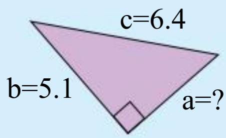


Exercise 7.4

1 Find the length of the unknown in each of the following:

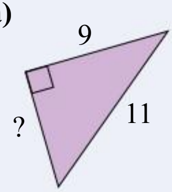
First add a, b, c



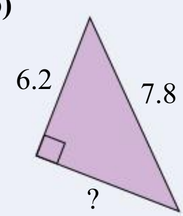


$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
a^2 + 5.1^2 &= 6.4^2 \\
a^2 &= 6.4^2 - 5.1^2 \\
a^2 &= 14.95 \\
a &= \sqrt{14.95} \\
a &= 3.87
\end{aligned}
$$

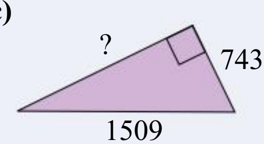
a)



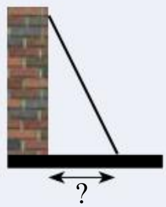
b)



c)

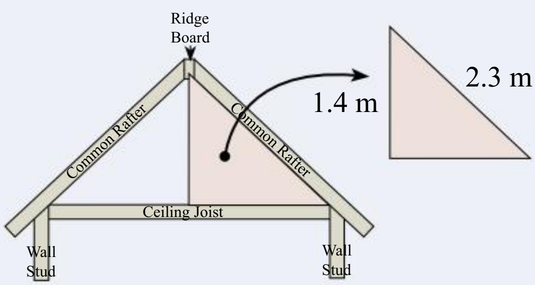


2 A 2.8 m ladder is to be laid against a wall so that the top of the ladder is 2 m up the wall. How far out from the base of the wall should the ladder be placed?



Rounding?
See Technology 7.1

3 Calculate the length of the ceiling joist.



4 How far up a building will a 25 m ladder reach if the base of the ladder must be 4 m out from the base of the wall?

5 A kite on a 75 m length of string is vertically above a point 22 m from the person flying the kite. How high is the kite?

Length of a line

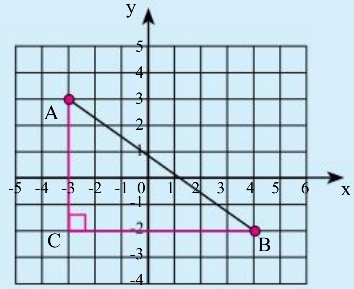
Exercise 7.5

Find the length of AB

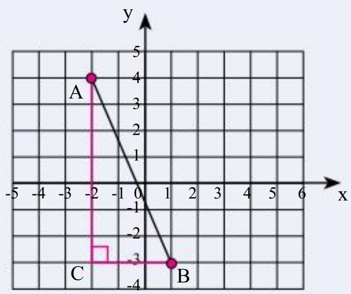
A (-3, 3) and B(4, -2)

AC = 5 and CB = 7

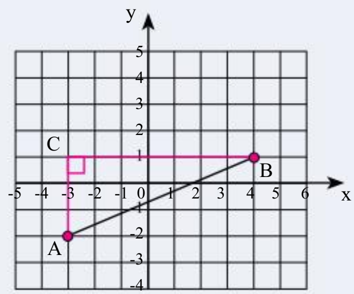
$$\begin{aligned} AB^2 &= AC^2 + CB^2 \\ &= 5^2 + 7^2 \\ &= 74 \\ AB &= \sqrt{74} \text{ or } 8.60 \end{aligned}$$



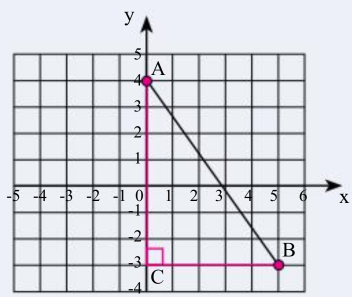
1



2



3



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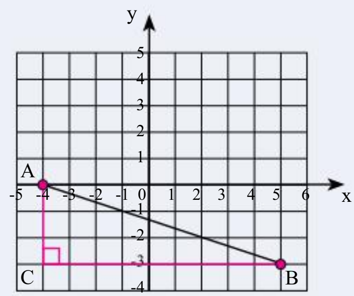


Plot the points first.

Rounding?
See Technology 7.1

5 A(3,2), B(6,5)

6 A(1,1), B(4,5)

7 A(-4,3), B(4,2)

8 A(5,2), B(-5,-3)

9 A(-3,-4), B(5,4)

10 A(-1,5), B(2,-3)

11 A(1,1), B(5,5)

12 A(1,2), B(4,6)

13 A(-4,2), B(4,2)

14 A(5,0), B(0,5)

15 A(-3,3), B(2,3)

16 A(2,0), B(2,-3)

Our Number System

Real Numbers

Complex Numbers

Rational Numbers

Can be expressed as a ratio a/b where a and b are integers.

Irrational Numbers

Cannot be expressed as a ratio a/b (a and b are integers).

Integers.
⁻3, 5, ⁻18, 0, 3, 9

Surds.
√2 = 1.4142135...
√3 = 1.7320508...

a + bi
a and b are real numbers.
i is the imaginary part.

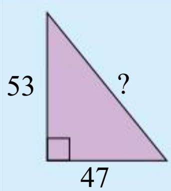
Fractions.
1/2, 3/4, 5/3, 6/7

Special numbers.
π = 3.1415926...
e = 2.7182818...

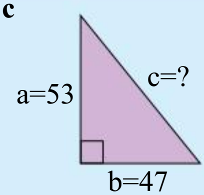
Decimals.
(Either terminate or recurr.)

Exercise 7.6

Calculate the unknown and describe the result as rational or irrational (If rational describe the result as integral, fractional, or decimal):

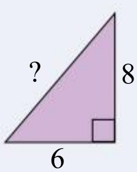


First add a, b, c

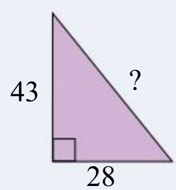


$c^2 = a^2 + b^2$
$c^2 = 53^2 + 47^2$
$c^2 = 5018$
$c = \sqrt{5018}$
The result is irrational.

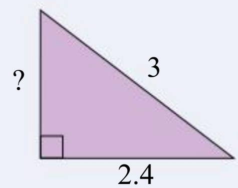
1



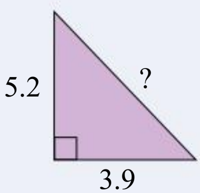
2



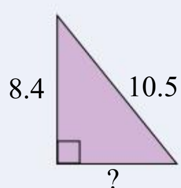
3



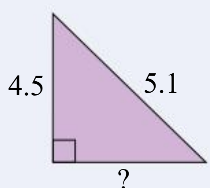
4



5



6

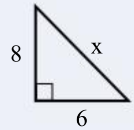


Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 7.7

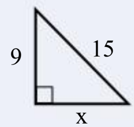
1. Spell Pyhtagoras.
2. What is Pythagoras' Theorem?
3. Is {6,8,10} a Pythagorean triad?
4. Find x
5. Simplify 5 : 10
6. Karen can walk 2 km in 20 mins.
   How long will it take her to walk 5 km?
7. The trip to work takes 30 mins at 80 km/h.
   How long will it take at 60 km/h?
8. Simplify: 7a − 5a
9. Expand: 3(x + 2)
10. 46×5



$$\begin{aligned}46\times5 &= 46\times10\div2\\ &= 460\div2\\ &= 230\end{aligned}$$

Exercise 7.8

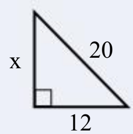
1. Spell Hypotenuse.
2. What is Pythagoras' Theorem?
3. Is {9,12,15} a Pythagorean triad?
4. Find x
5. Simplify 8 : 6
6. Seb can run 3 km in 15 mins.
   How long will it take him to run 5 km?
7. The trip to work takes 40 mins at 80 km/h.
   How long will it take at 100 km/h?
8. Simplify: 9x − 7x
9. Expand: 5(x − 2)
10. 64×5



Why are mathematicians afraid of driving a car?
The width of the road is negligible compared to its length.

Exercise 7.9

1. Spell Theorem.
2. What is Pythagoras' Theorem?
3. Is {12,16,20} a Pythagorean triad?
4. Find x
5. Simplify 12 : 8
6. Jess can walk 5 km in 60 mins.
   How long will it take her to walk 3 km?
7. The trip to work takes 30 mins at 60 km/h.
   How long will it take at 90 km/h?
8. Simplify: 6x − 5x + 3x
9. Expand: 4(x + 5)
10. 36×5



You may be disappointed if you fail, but you are doomed if you don't try - Beverly Sills.

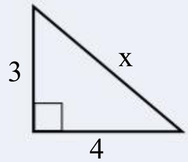
Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

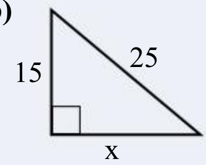
Exercise 7.10

1 Find x.

a)



b)



Show that 3a, 4a, 5a is a Pythagorean triad.

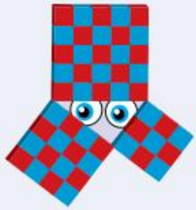
$c^2 = (5a)^2$ $\qquad a^2 + b^2 = (3a)^2 + (4a)^2$

$c^2 = 25a^2$ $\qquad = 9a^2 + 16a^2$

$\qquad = 25a^2$

$\therefore c^2 = a^2 + b^2$

Thus 3a,4a,5a is a Pythagorean triad.



2 Show that 3m, 4m, 5m is a Pythagorean triad.

3 Show that 5a, 12a, 13a is a Pythagorean triad.

4 If p, q, r is a Pythagorean triad, show that ap, aq, ar is also a pythagorean triad.

Find the value of x.

In triangle A

$y^2 + 7^2 = 8^2$

$y^2 = 15$

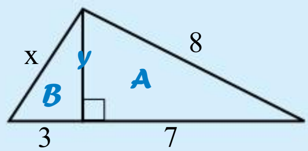
In triangle B

$x^2 = y^2 + 3^2$

$x^2 = 15 + 9$

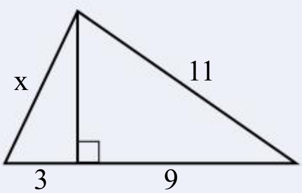
$x^2 = 24$

$x = \sqrt{24}$ or $2\sqrt{6}$

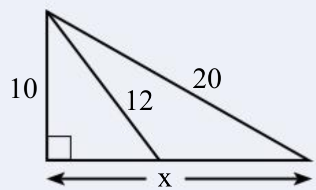


5 Find the value of x.

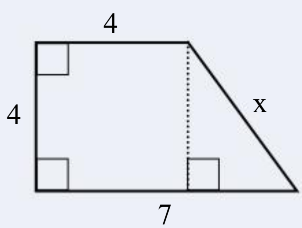
a)



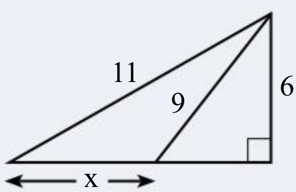
b)



c)



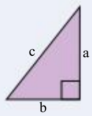
d)



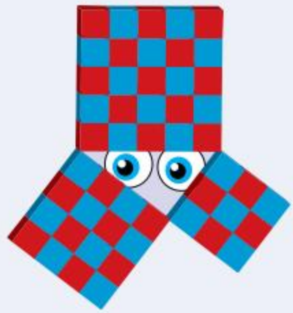
Investigations

Investigation 7.1 Proof of Pythagoras' Theorem

In any right-angled triangle:
The square on the hypotenuse is equal to the sum of the squares on the other two sides.



$c^2 = a^2 + b^2$



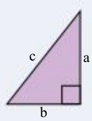
There are many proofs of Pythagoras' Theorem.
Use the Internet to select one of the proofs and demonstrate it to others.

Investigation 7.2 Pythagorean Triples

When all three sides of a right-angled triangle are integers, their lengths form a Pythagorean triple.
3, 4, 5 is a Pythagorean triple.

Integers are whole numbers within the set:
$Z = \{..... {}^-4, {}^-3, {}^-2, {}^-1, 0, 1, 2, 3, 4, ...\}$

$c^2 = a^2 + b^2$



Pythagorean triple formula

Select two integers n and m with n > m.

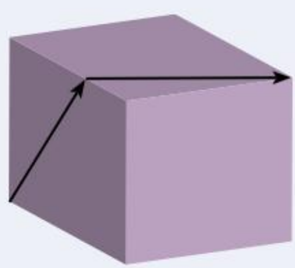
$a = n^2 - m^2$
$b = 2nm$
$c = n^2 + m^2$

Use the Pythagorean triple formula to generate Pythagorean triples.

Investigation 7.3 Shortest Surface Path

Investigate

The shortest surface path on a cube



Technology

Technology 7.1 **Square Root**

There are at least two ways of finding the square root with a calculator.

1 $\sqrt{9.2}$ [√] [9.2] [=] [3.033150] which is 3.03 to 2 decimal places.

2 $9.2^{0.5}$ [9.2] [y^x] [=] [3.033150] which is 3.03 to 2 decimal places.

$$\sqrt{x} = x^{\frac{1}{2}} = x^{0.5}$$

Rounding to two decimal places, first look at the third decimal place:

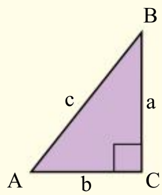
56.231694 ↑ less than 5 thus **56.23**	27.01769 ↑ 5 or more thus **27.02**	1.07276 ↑ less than 5 thus **1.07**	4.79634216 ↑ 5 or more thus **4.80**

Technology 7.2 **Triangle Solvers**

The Internet has a considerable number of 'triangle solvers'.
Use one of them to solve previous problems.

If a= 4, b=3

c = 5
A = 53.13°
B = 36.87°
C = 90.00°
Area = 6
Perimeter = 12



Technology 7.3 **Pythagoras Spreadsheet**

Setup a spreadsheet to solve right-angled triangles.

	a	b	c
1	3	4	5

Enter the formula:
=sqrt(a1^2+b1^2)

Technology 7.3 **Pythagoras' Theorem**

Search the Internet for some of the many Pythagoras' Theorem interactive games and applets. These are very useful in understanding Pythagoras' Theorem.

Petroleum Geologists explore the earth's surface to predict possible locations of oil and natural gas.

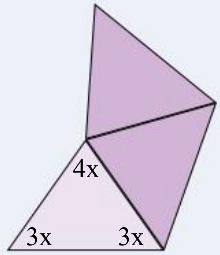
- Relevant school subjects are Mathematics, Chemistry, Physics, English.
- Courses usually involve a University Bachelor degree.

A Couple of Puzzles

Exercise 7.11

1 What is my age? My age is twice the number of faces on a cube plus three times the number of edges on a cube.

2 The pentagon can be made with this isosceles triangle. What is the size of x?



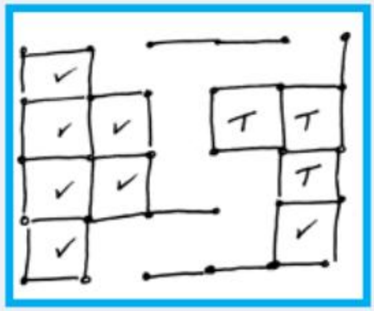
A Game

Box is played on a grid of dots.

1 Players take turns to draw a line to join two adjacent dots.

2 If a turn completes a box then the player is allowed to draw another line to join another two adjacent dots.
Mark on your completed box

3 When all dots have been joined, total the number of boxes.



A Sweet Trick

Number off your audience from 1 to the number in your audience. While you are not looking or out of the room, have your audience put a rubber band on someone's finger.

Ask your audience to:

1	Multiply the number of the person with the string by 2.	Person 5 $5 \times 2 = 10$
2	Add 3.	$10 + 3 = 13$
3	Multiply the result by 5.	$13 \times 5 = 65$
4	If the string is on the right hand add 8. If the string is on the left hand add 9.	$65 + 8 = 73$
5	Multiply by 10.	$73 \times 10 = 730$
6	Add the number of the finger (The thumb = 1).	$730 + 3 = 733$
7	Add 2.	$731 + 2 = 735$
8	Ask them to tell you the answer.	735

Mentally subtract 222. The remainder gives the answer.

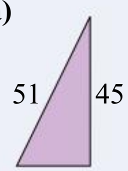
$735 - 222 = 513.$
5 person number 5
1 right hand (2 left hand)
3 on third finger

Chapter Review 1

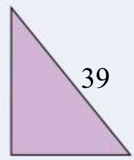
Exercise 7.12

1 Which of the following triangles are right-angled triangles?

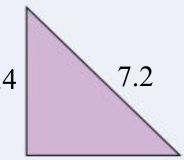
a)



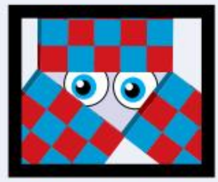
b)



c)



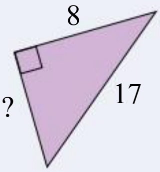
2 A rectangular picture frame measures 28 cm by 23 cm with a diagonal of 36 cm. Is the picture frame square?



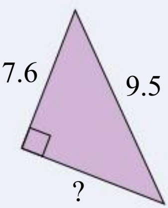
3 A carpenter needs to know whether a door frame is square. If the door frame is square then a door can be fitted. The door frame measures 810 mm by 2000 mm and the diagonal is 2158 mm.

4 Find the length of the unknown in each of the following:

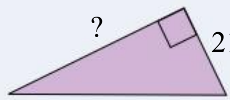
a)



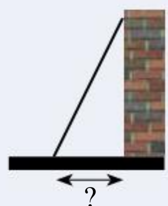
b)



c)

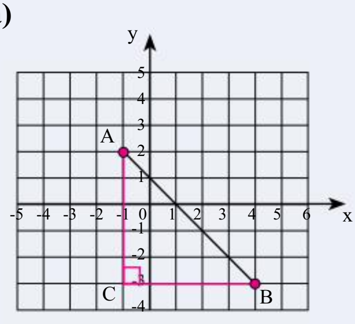


5 A 3.2 m ladder is to be laid against a wall so that the top of the ladder is 2.5m up the wall. How far out from the base of the wall should the ladder be placed?

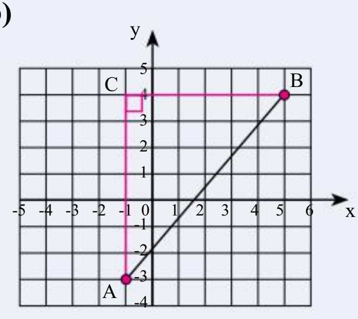


6 Find the length of AB.

a)



b)



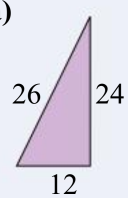
c) A(5,2), B(6,5) **d)** A(1,-1), B(4,4) **e)** A(-4,-3), B(4,3)

Chapter Review 2

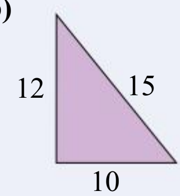
Exercise 7.13

1 Which of the following triangles are right-angled triangles?

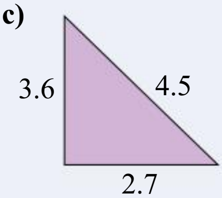
a)



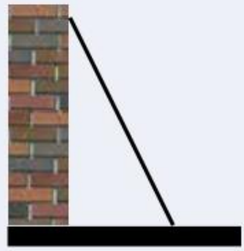
b)



c)



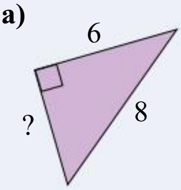
2 A 2.2 m ladder is leaning against a wall. The bottom of the ladder is 0.7 m from the wall and the top of the ladder is 2.06 m up the wall. Is the wall vertical?



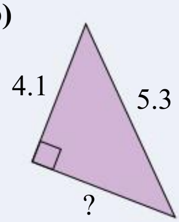
3 A rectangular gate measures 1.4 m by 2.5 m with a 2.79 m diagonal. Is the gate square? If not, should the diagonal be longer or shorter?

4 Find the length of the unknown in each of the following:

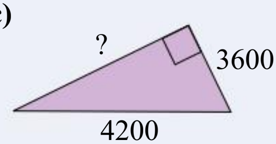
a)



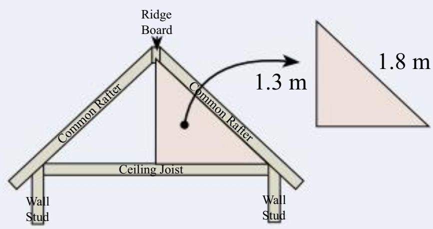
b)



c)

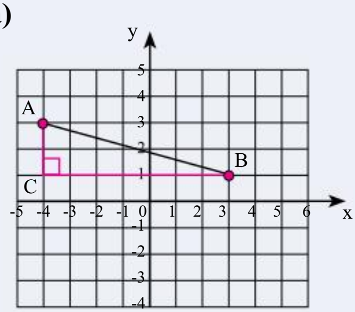


5 Calculate the length of the ceiling joist.

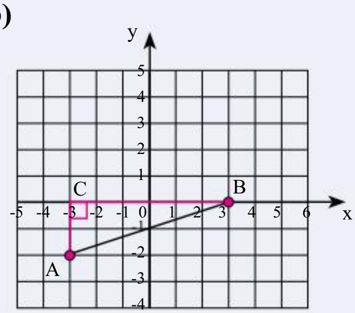


6 Find the length of AB.

a)



b)

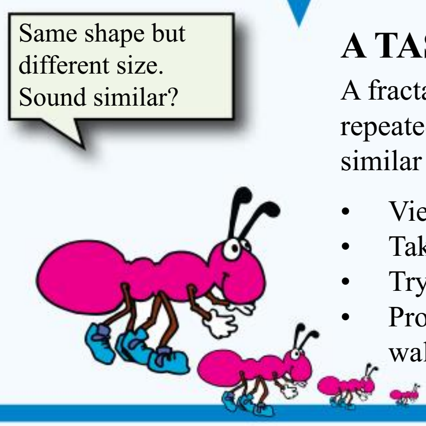


c) A(3,2), B(6,4) **d)** A(-1,1), B(-4,4) **e)** A(-4,3), B(3,0)

Geometry 8

Measurement & Geometry - Geometric reasoning

★ Use the enlargement transformation to explain similarity and develop the conditions for triangles to be similar.
★ Solve problems using ratio and scale factors in similar figures.



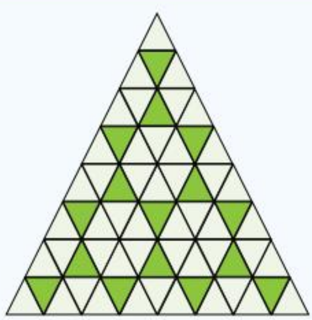
A TASK

A fractal is a geometric shape that can be repeatedly split into parts. Each part being similar to the original shape.

- View some online fractal videos.
- Take part in some online fractal activities.
- Try some of the fractal generators.
- Produce your own fractal for the classroom wall.

A LITTLE BIT OF HISTORY

1670 Liebnitz considered repetition of similar shapes.
1904 Koch mathematically defined the Koch Curve.
1915 Sierpinski constructed the Sierpinski triangle.
1975 Mandelbrot used the term fractal to describe shapes consisting of similar shapes.

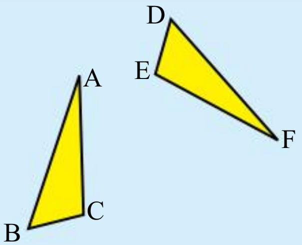


Congruent Triangles

> **Congruent triangles** have exactly the **same shape and size**.
> They fit exactly on top of each other.
> The symbol for congruence is $\equiv$ or $\cong$

Exercise 8.1

Given that the following pairs of triangles are congruent,
Correctly name them (**angles and sides must match**):

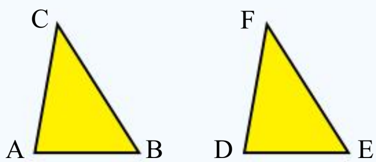


$\angle A = \angle F$
$\angle B = \angle D$
$\angle C = \angle E$

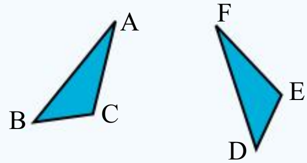
$\therefore \triangle ABC \equiv \triangle FDE$

> If the angles are in matching order then the sides will be in matching order.
> And vice versa.

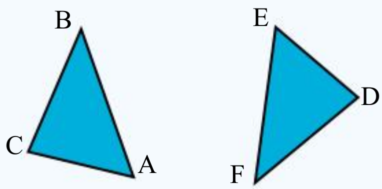
1



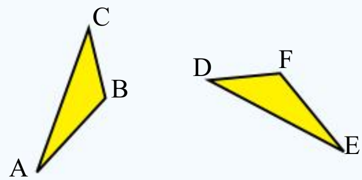
2



3



4

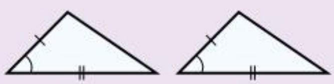


Two triangles are congruent if:

- three sides on one triangle are congruent with matching sides on the other.

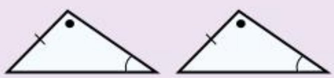
SSS

- two sides and the *included* angle on one triangle are congruent with the two sides and *included* angle on the other.



SAS

- two angles and a side on one triangle are congruent with the two angles and matching side on the other.



AAS

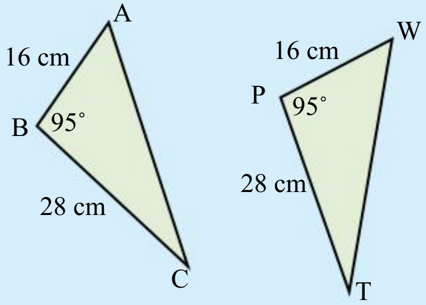
Two right-angled triangles are congruent if:

- hypotenuse and a side on one triangle are congruent with hypotenuse and matching side on the other.

RHS

Tests for Congruent Triangles

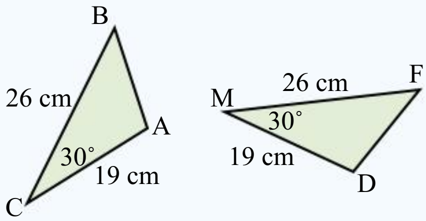
Exercise 8.2 Use the tests for congruence to test whether the following pairs of triangles are congruent:



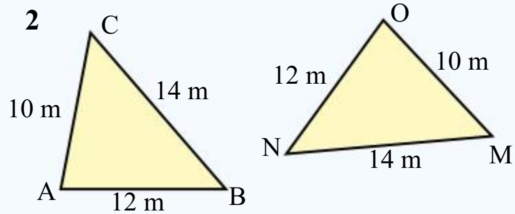
side AB = side WP
$\angle B = \angle P$ {angle inbetween}
side BC = side PT

$\therefore \triangle ABC \equiv \triangle WPT$ {**SAS**}

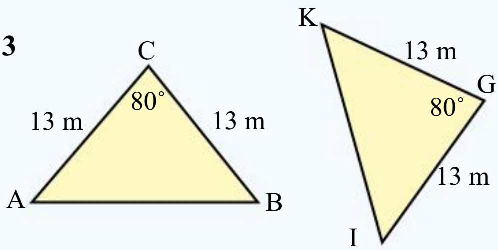
1



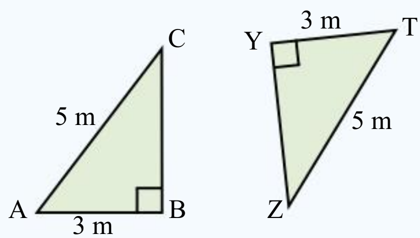
2



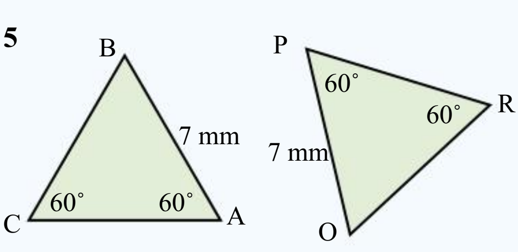
3



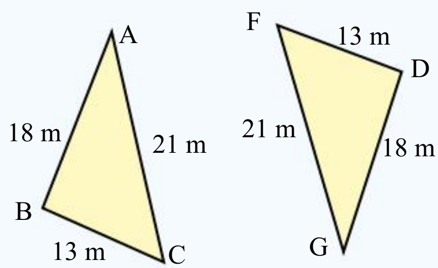
4



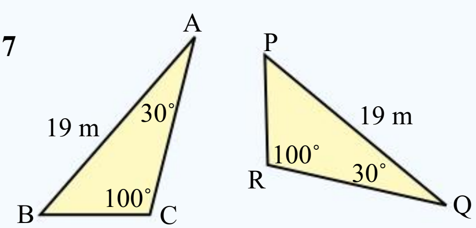
5



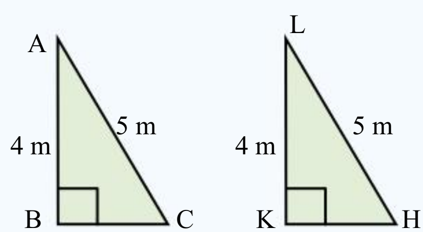
6



7



8



Similarity Transformation

A similarity transformation may change both position and size but **keeps the same shape**.

Measure to check that the original image has been doubled in size.

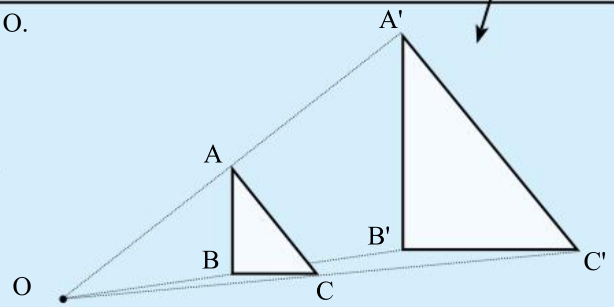
Exercise 8.3

1 Enlarge the original image by a scale factor of two.

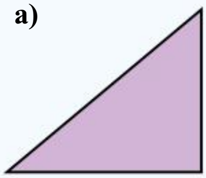
a) Choose an external point O.
b) Draw lines to image and extend same distance

$$\text{Scale} = \frac{OA'}{OA} = \frac{OB'}{OB} = \frac{OC'}{OC}$$

$$\text{Scale} = \frac{2}{1}$$

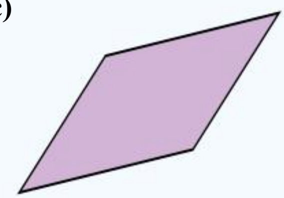


a)

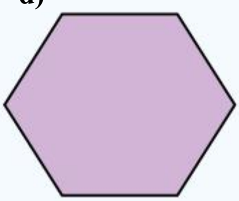


b)

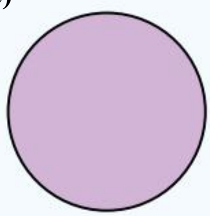
c)



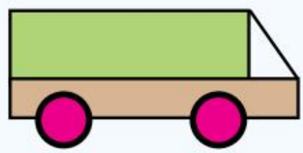
d)



e)



f)

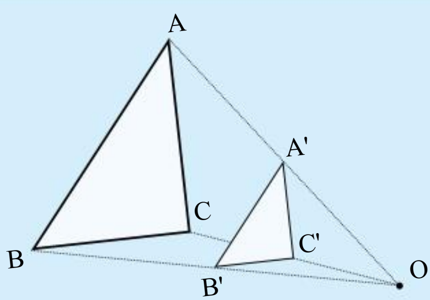


2 Reduce the above original images using a scale factor of ½.

a) Choose an external point O.
b) Draw lines to image and find the midpoints.

$$\text{Scale} = \frac{OA'}{OA} = \frac{OB'}{OB} = \frac{OC'}{OC}$$

$$\text{Scale} = \frac{1}{2}$$



Similarity Transformation

A similarity transformation may change both position and size but **keeps the same shape.**

Measure to check that the original image has been doubled in size.

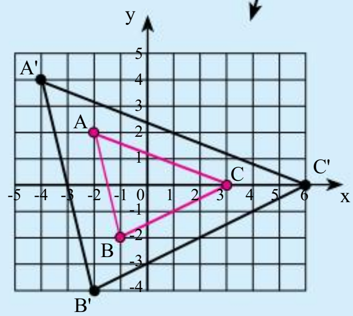
Exercise 8.4

1 Enlarge the original triangle by a scale factor of two.

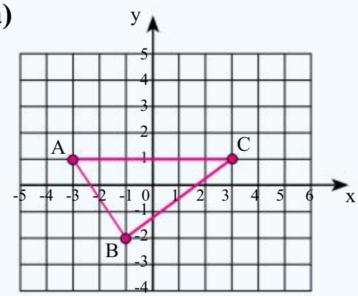
1 A (-2,2) double to A' (-4,4)
2 B (-1,-2) double to B' (-2,-4)
3 C (3,0) double to C' (6,0)

$$\text{Scale} = \frac{OA'}{OA} = \frac{OB'}{OB} = \frac{OC'}{OC}$$

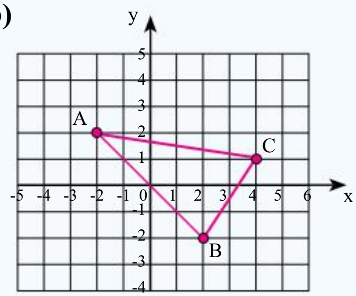
$$\text{Scale} = \frac{2}{1}$$



a)



b)



The enlargement should have a similar shape.

c) A(-3,2), B(6,5), C(2,-2) **d)** A(1,1), B(4,5), C(-4,2)

2 Enlarge each following triangle by a scale factor of three.

a) A(-3,2), B(6,5), C(2,-2) **b)** A(-3,-4), B(2,-4), C(2,2)

3 Reduce each following triangle using a scale factor of ½.

a) A(-3,2), B(6,5), C(2,-2) **b)** A(1,2), B(4,6), C(6,2)

4 Enlarge each following triangle using a scale factor of 1½.

a) A(-3,2), B(6,5), C(2,-2) **b)** A(-4,-2), B(2,3), C(-3, 3)

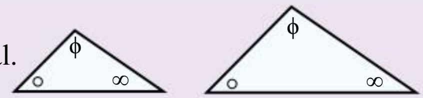
Similar Triangles

Similar triangles have **exactly the same shape** but not necessarily the same size.
The corresponding **angles are equal**.
The corresponding sides have the same scale factor.

The symbol for similarity is $\sim$.

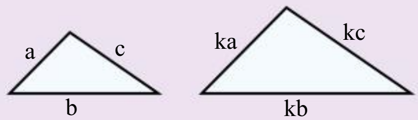
Two triangles are similar if:

► The three matching angles are equal.



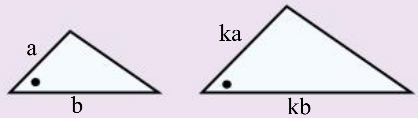
AAA

► The three matching sides are in the same ratio



SSS

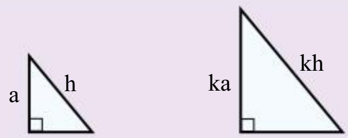
► Two matching sides are in the same ratio and the *included* angles are equal



SAS

Two right-angled triangles are similar if:

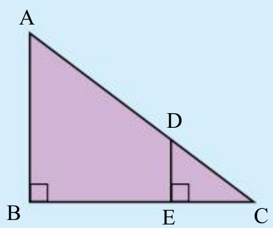
► The hypotenuse and a matching side are in the same ratio.



RHS

Exercise 8.5

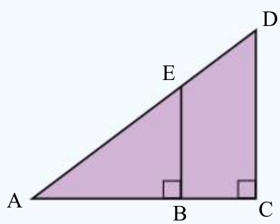
Prove that $\triangle ABC \sim \triangle DEC$



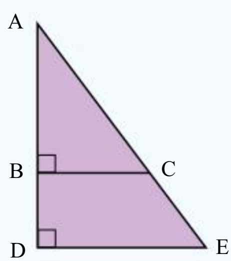
$\angle C = \angle C$ {common angle}
$\angle B = \angle E$ {both 90°}
$\angle A = \angle D$ {3rd angle must be equal}

$\therefore \triangle ABC \sim \triangle DEC$ {**AAA**}

1 Prove that $\triangle ABC \sim \triangle DEC$

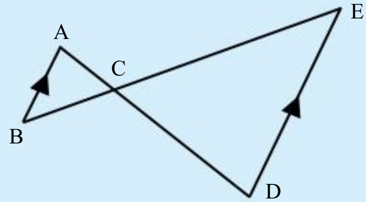


2 Prove that $\triangle ABC \sim \triangle DEC$



Tests for Similar Triangles

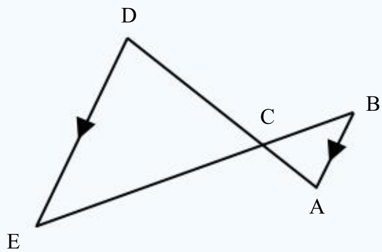
Prove that $\triangle ABC \sim \triangle DEC$



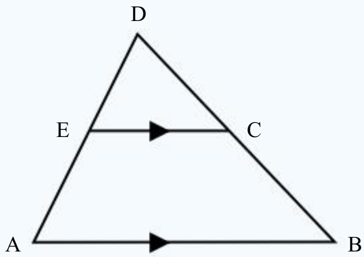
$\angle A = \angle D$ {alternate angle}
$\angle B = \angle E$ {alternate angle}
$\angle BCA = \angle ECD$ {vertically opposite}

$\therefore \triangle ABC \sim \triangle DEC$ {**AAA**}

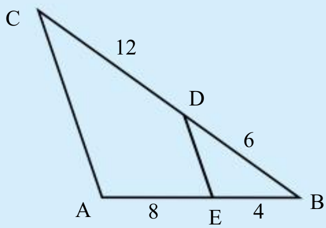
3 Prove that $\triangle ABC \sim \triangle DEC$



4 Prove that $\triangle DAB \sim \triangle DEC$



Prove that $\triangle ABC \sim \triangle EBD$



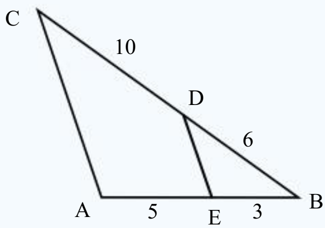
$$\frac{AB}{EB} = \frac{12}{4} = \frac{3}{1}$$

$$\frac{BC}{BD} = \frac{18}{6} = \frac{3}{1}$$

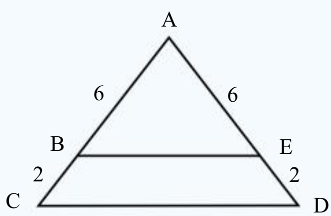
$\angle B = \angle B$ {common included angle}

$\therefore \triangle ABC \sim \triangle EBD$ {**SAS**}

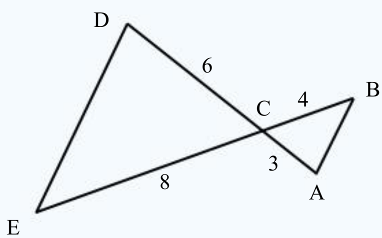
5 Prove that $\triangle CAB \sim \triangle DEC$



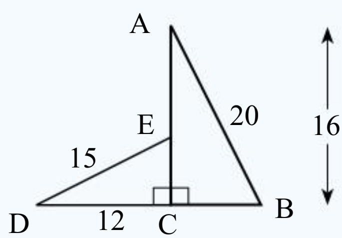
6 Prove that $\triangle ABE \sim \triangle ACD$



7 Prove that $\triangle DEC \sim \triangle ABC$

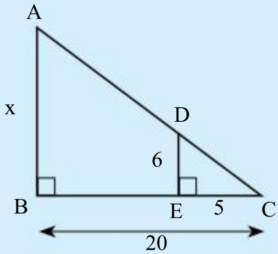


8 Prove that $\triangle ABC \sim \triangle DEC$



Similar Triangles

Exercise 8.6
Find the length of the unknown.



$\triangle ABC \sim \triangle DEC$ {AAA}

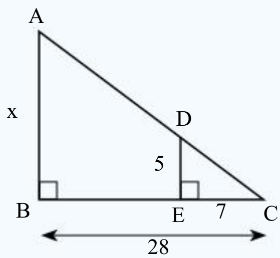
$\frac{AB}{DE} = \frac{BC}{EC}$ {same scale factor}

$\frac{x}{6} = \frac{20}{5}$

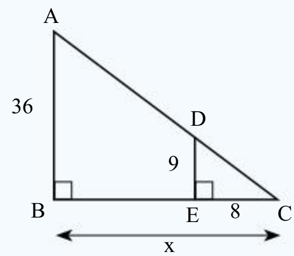
$x = \frac{20}{5} \times 6$ {inverse of ÷ is ×}

$\underline{x = 24}$

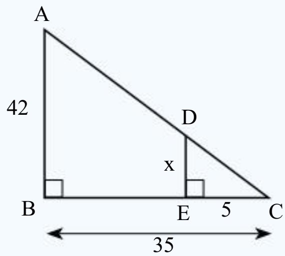
1



2



3



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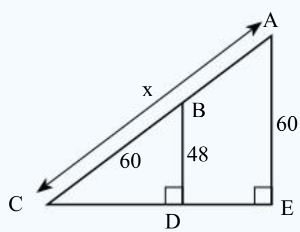
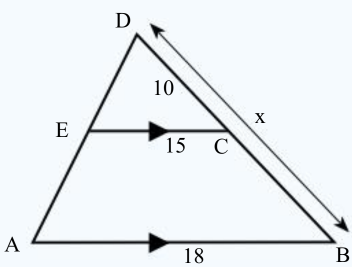
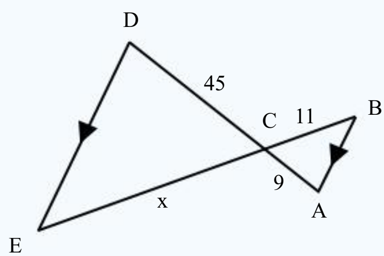


5



6



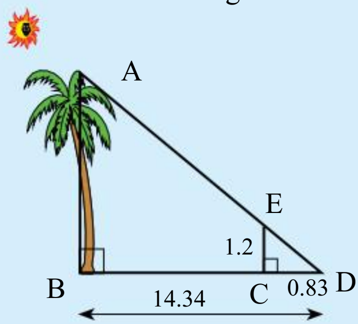
Building Site Managers usually supervise construction sites to ensure the building is proceeding as planned.

- Relevant school subjects are Mathematics and English.
- Courses usually involve a Certificate or Diploma in building.

Similar Triangles

Exercise 8.7

A 1.2 metre stick casts a 0.83 metre shadow at the same time a tree casts a 14.34 m shadow. What is the height of the tree?



$\triangle ABD \sim \triangle ECD$ {AAA}

$\frac{AB}{EC} = \frac{BD}{CD}$ {same scale factor}

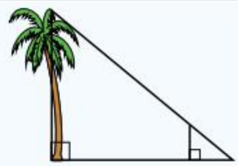
$\frac{AB}{1.2} = \frac{14.34}{0.83}$

$AB = \frac{14.34}{0.83} \times 1.2$ {inverse of ÷ is ×}

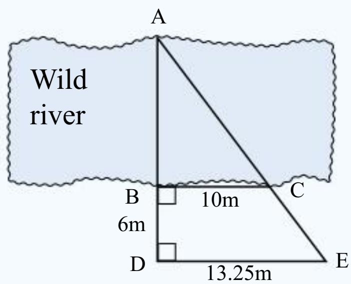
$AB = 20.73$

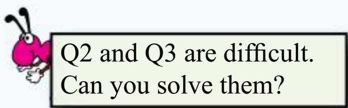
The tree is 21 m high

1 A 1.5 metre stick casts a 0.62 metre shadow at the same time a tree casts a 5.4 m shadow. What is the height of the tree?

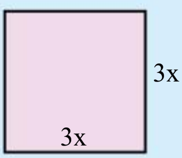


2 In trying to calculate the width of a wild river, the following diagram was produced. What is the width of the river at AB?





3 The following figures are similar and the ratio of corresponding sides is 3:1. What is the ratio of the perimeter? What is the ratio of the area?



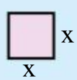
Perimeter $= 3x+3x+3x+3x$

$= 12x$

Area $= s^2$

$= (3x)^2$

$= 9x^2$



Perimeter $= x+x+x+x$

$= 4x$

Area $= s^2$

$= x^2$

Perimeter:

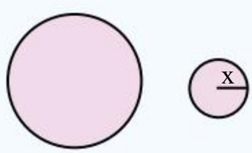
$\text{ratio} = \frac{12x}{4x} = 3:1$

Area

$\text{ratio} = \frac{9x^2}{x^2} = 9:1$

a)

b)



Mental Computation

Mental computation can make problems easier and quicker.

Exercise 8.8

1 Spell Congruent.
2 What does AAA mean?
3 What is the symbol for similar?
4 Scale factor? One side 4, corresponding side 12.
5 Is {3,4,5} a Pythagorean triad?
6 Two sides in a right-angled triangle are 1 and 3. Hypotenuse?

$h^2=3^2+1^2$
$h^2=9+1$
$h=\sqrt{10}$

7 Simplify 15 : 12
8 Tim can run 4 km in 20 mins.
How long will it take him to run 5 km?
9 Simplify: $9x - 7x + 3x$
10 I buy a \$5.90 magazine with a \$10 note, how much change?

A cat has nine tails.
Proof:
No cat has eight tails.
A cat has one tail more than no cat.
Therefore, a cat has nine tails.

Exercise 8.9

1 Spell Similar.
2 What does SSS mean?
3 What is the symbol for similar?
4 Scale factor? One side 3, corresponding side 12.
5 Is {6,8,10} a Pythagorean triad?
6 Two sides in a right-angled triangle are 1 and 2. Hypotenuse?
7 Simplify 20 : 15
8 Jess can run 5 km in 30 mins.
How long will it take her to run 3 km?
9 Simplify: $4x - 8x + 2x$
10 I buy a \$9.10 magazine with a \$20 note, how much change?

The greatest mistake you can make in life is to be continually fearing you will make one -- Elbert Hubbard.

Exercise 8.10

1 Spell Transformation.
2 What does AAS mean?
3 What is the symbol for similar?
4 Scale factor? One side 8, corresponding side 4.
5 Is {9,12,15} a Pythagorean triad?
6 Two sides in a right-angled triangle are 1 and 1. Hypotenuse?
7 Simplify 12 : 18
8 Megan can run 5 km in 35 mins.
How long will it take her to run 4 km?
9 Simplify: $10x - 3x - 8x$
10 I buy a \$15.30 magazine with a \$20 note, how much change?

Competition Questions

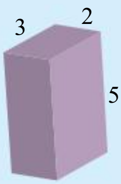
Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 8.11

1 A building casts a 15 m shadow at the same time as Aaron casts a 1.2 m shadow. Aaron is 1.8 m tall. What is the height of the building?

2 If the height of the mother is 145 cm, what is the height of the child?

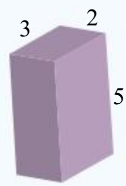
The dimensions of the rectangular prism is doubled.
What has happened to the surface area?



Original Surface Area
$A = 2(2\times3)+2(2\times5)+2(3\times5) = 62$
Doubled Surface Area
$A = 2(4\times6)+2(4\times10)+2(6\times10) = 248$

Ratio of Doubled to Original = 248 : 62 = 4 : 1

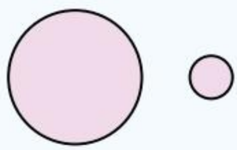
3 The dimensions of the rectangular prism is tripled. What has happened to the surface area?



4 The dimensions of the rectangular prism is doubled. What has happened to the volume?

5 The dimensions of the rectangular prism is tripled. What has happened to the volume?

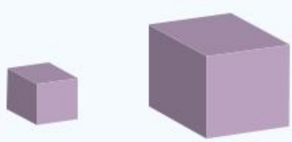
6 The two circles are similar with a scale factor of 3. If the radius of the smaller circle is r cm, what is the area of the larger circle?



7 The two squares are similar with a scale factor of 2. If the perimeter of the smaller square is p, what is the perimeter of the larger square?

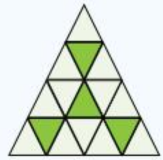
8 The two squares are similar with a scale factor of 2 : 3. If the perimeter of the smaller square is p, what is the area of the larger square?

9 The two cubes are similar with a scale factor of 2 : 5. If the side of the smaller square is s,
a) what is the volume of the larger cube?
b) what is the surface area of the larger cube?



Technology

Similar figures have **exactly the same shape** but not necessarily the same size.
The corresponding **angles are equal**.
The corresponding sides have the same scale factor.

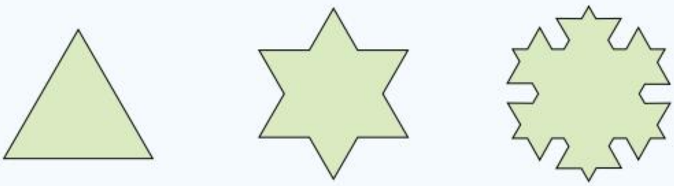


Technology 8.1 Fractals

A fractal is a geometric shape that can be split into parts.
Each part being similar to the original shape.

Fractals are found in nature. They have applications in soil mechanics, seismology, medicine and artwork.

a) Draw the first four iterations of the Koch snowflake'



Each iteration produces smaller similar shapes.

Start with an equilateral triangle
Repeat three times:
Add triangles a third the size to each side.

b) Use Internet software to draw iterations of the Koch Snowflake.
Use search phrases such as 'Koch Snowflake' with 'applet', 'interactive' etc.

c) Use Internet software to produce some of the beautiful Mandelbrot sets and Julia sets.

Technology 8.2 Sierpinski Gasket

a) Draw the first four iterations of the Sierpinski Gasket.

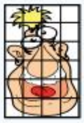
Start with a triangle
Repeat three times:
Find the midpoint of each side.
Connect them to form a similar triangle.

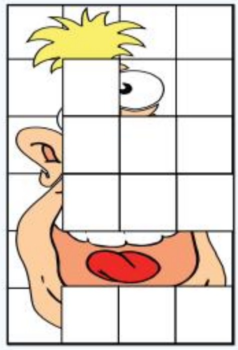
b) Use Internet software to draw iterations of the Sierpinski Gasket.
Use search phrases such as 'Sierpinski Gasket' with 'applet', 'interactive' etc.

Investigations

Investigation 8.1 Enlarge a Cartoon

1 Select a cartoon.
2 Draw grids on the cartoon (6×4?).
3 Draw a larger (6×4?) grid (3 times the size?).
4 Copy each section onto the larger grid.



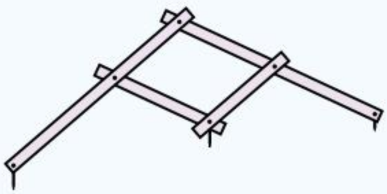


How much larger is the area?

Investigation 8.2 Use a Pantograph

1 Make or purchase a Pantograph.
2 Use the Pantograph to enlarge a drawing.
3 Are the figures similar?
(Check the ratio of corresponding sides.

Construction instructions are on the Internet.

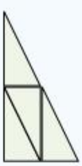


How does it work?

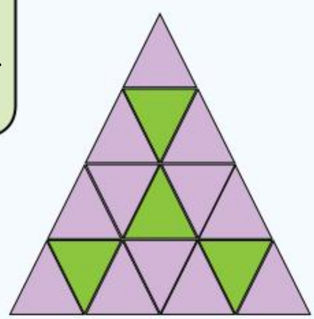
Investigation 8.3 Tessellation

Investigate

Shapes that can make larger versions of themselves



Equaliteral triangles?
Isosceles triangles?
Right-angled triangles?
Squares? Rectangles?
Rhombuses? Kites?



A Couple of Puzzles

Exercise 8.12

1 How many different numbers can you make using the digits 1, 2, 3?

2 Two friends pressed enough olives to fill an 8 litre barrel. If they only have a 5 litre container and 3 litre container, how can they arrange to have 4 litres each?

A Game

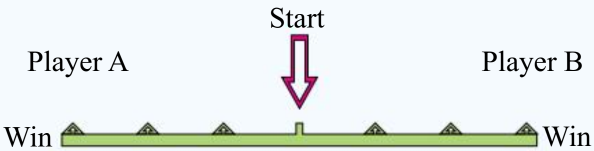
Tug of war is a two player game in which each player starts with a total of 50. The winner is the person who moves the marker from the **start** to a **win**.

1 Each player secretly writes down a number (Each player then subtracts the number from their total).

2 The player with the highest number moves the marker one place towards their win.

3 The above two steps are repeated.

If your total becomes 0 then you have no moves left.



A Sweet Trick

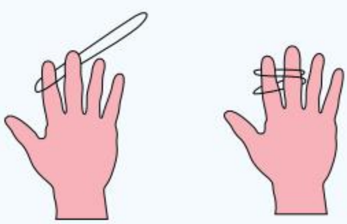
Set up the following trick:

1 Wrap an elastic band around the index finger.

2 Then under and around the middle finger.

3 Loop it onto the index finger again

4 Ask your audience to trap the elastic band by grasping the end of either finger.

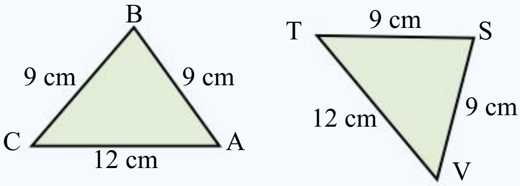


You drop the band off the end of the free finger. The band, amazingly, is on the free finger.

Chapter Review 1

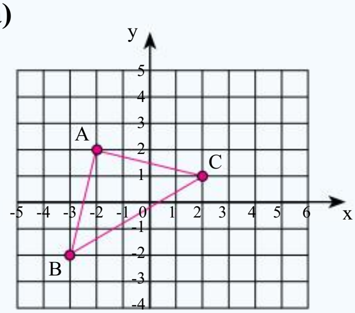
Exercise 8.13

1 Use the tests for congruence to test whether the following pair of triangles are congruent:

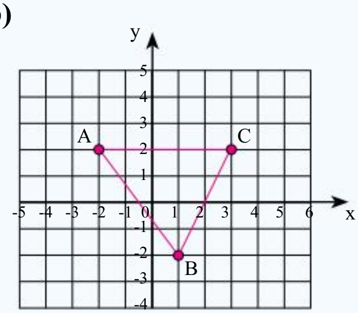


2 Enlarge the original triangle by a scale factor of two.

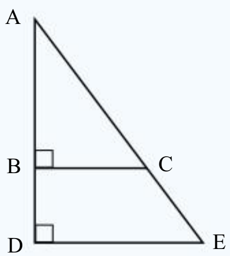
a)



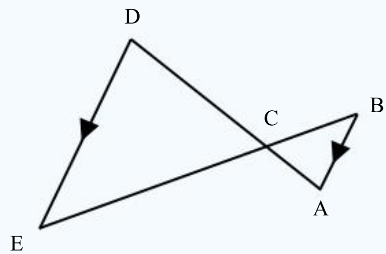
b)



3 Prove that $\triangle ABC \sim \triangle ADE$

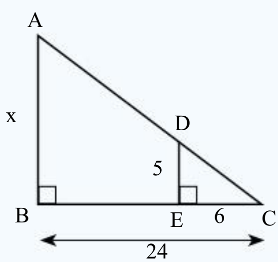


4 Prove that $\triangle ABC \sim \triangle DEC$

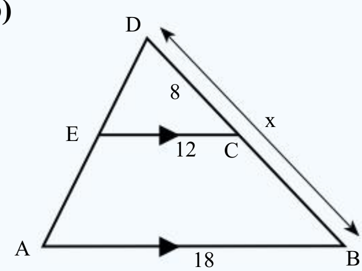


5 Find the length of the unknown.

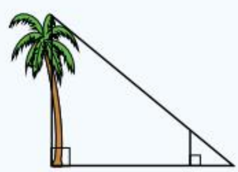
a)



b)



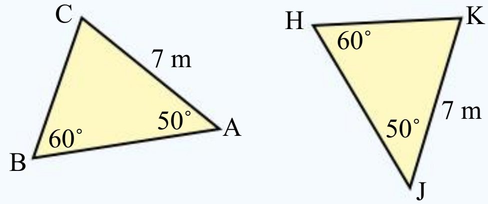
6 A 1.2 metre stick casts a 0.80 metre shadow at the same time a tree casts a 4.5 m shadow. What is the height of the tree?



Chapter Review 2

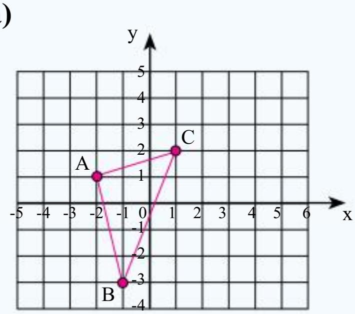
Exercise 8.14

1 Use the tests for congruence to test whether the following pair of triangles are congruent:

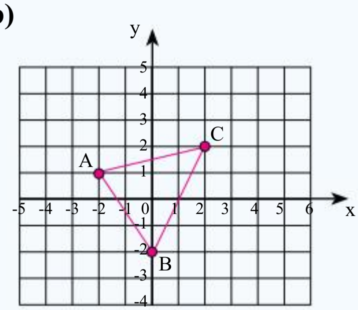


2 Enlarge the original triangle by a scale factor of two.

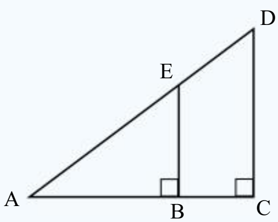
a)



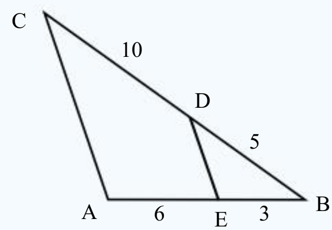
b)



3 Prove that $\triangle ABE \sim \triangle ACD$

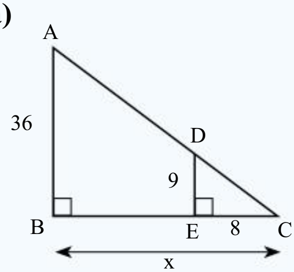


4 Prove that $\triangle ABC \sim \triangle EBD$

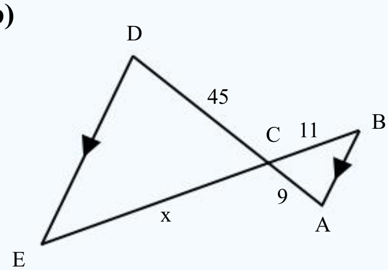


5 Find the length of the unknown.

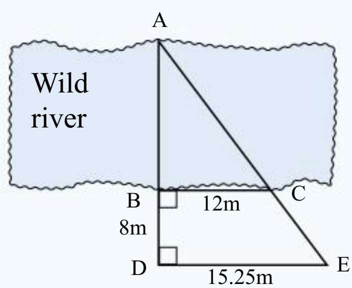
a)



b)



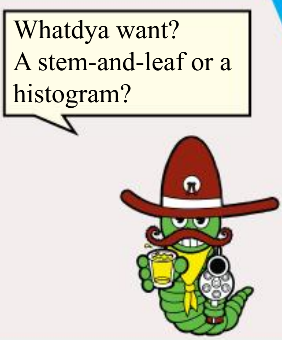
6 In trying to calculate the width of a wild river, the following diagram was produced. What is the width of the river at AB?



Statistics 9

Statistics and Probability ➡ Data representation and interpretation

★ Identify everyday questions and issues involving at least one numerical and at least one categorical variable, and collect data directly from secondary sources.
★ Construct back-to-back stem-and-leaf plots and histograms and describe data, using terms including 'skewed', 'symmetric' and 'bi modal'.
★ Compare data displays using mean, median and range to describe and interpret numerical data sets in terms of location (centre) and spread.



A TASK

Show your class how to use a graphics calculator or Internet applet to help produce a stem-and-leaf plot.

- Research graphics calculator and stem-and-leaf plots.
- Practice with the problems in Exercise 9.5.
- Plan your lesson.
- Show your class.

A LITTLE BIT OF HISTORY

1900s Bowley used stem-and-leaf plots for initial data analysis.

1977 Tukey used computer technology of the time to produce stem-and-leaf plots. Stem-and-leaf plots then became popular ways of showing the shape of data and were easy to produce via computer.

Now Modern computer graphic capability has reduced the use of stem-and-leaf plots.

1	46
2	0233
3	1455567
4	00235677899
5	2234489
6	2688
7	03

3 | 1 means 31

A stem-and-leaf plot showing symmetrically shaped data.

Descriptive Statistics

The first step in analysing data is to describe the data using descriptive statistics.

The **Mean**
describes the **middle** of the data.

The mean is also called the **average**.
The mean is heavily affected by extreme scores.

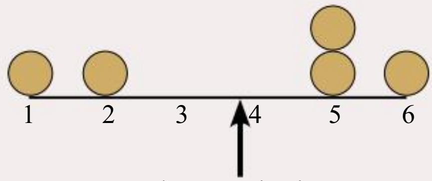
Exercise 9.1
Find the mean of each of the following set of scores:

1, 2, 5, 5, 6

$$\text{mean} = \frac{\text{sum of scores}}{\text{number of scores}}$$

$$\text{mean} = \frac{1+2+5+5+6}{5}$$

mean = 3.8



Balance point is 3.8

1	1, 2, 4, 5, 5, 6	**2**	1, 2, 5, 5, 6, 130
3	1, 1, 2, 2, 2, 3, 4, 4	**4**	1, 5, 5, 2, 6, 4
5	41, 45, 45, 42, 46, 44	**6**	9.1, 9.5, 9.5, 9.2, 9.6, 9.4
7	231, 235, 235, 232, 236, 234	**8**	$^{-}1$, $^{-}5$, $^{-}5$, $^{-}2$, $^{-}6$, $^{-}4$
9	4.2, $^{-}3.6$, 1.4, $^{-}2.8$, 3.7,	**10**	$\frac{3}{4}$, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{2}$

The **Median**
is the **middle** of a set of scores.

The median ignores extreme high scores and extreme low scores.

Exercise 9.2
Find the median of each of the following set of scores:

2, 7, 5, 1, 3, 5, 7 Put the scores in ascending order 1, 2, 3, $\boxed{5}$, 5, 7, 7 Median = 5 {5 is in the middle }	2, 7, 5, 1, 3, 5, 7, 4 Put the scores in ascending order 1, 2, 3, $\boxed{4, 5}$, 5, 7, 7 Median = 4.5 {Average of 4 & 5}

1	1, 3, 5, 2, 7, 7, 5	**2**	1, 3, 5, 2, 7, 7, 500
3	51, 53, 55, 52, 57, 57, 55	**4**	1, 2, 3, 4, 1, 2, 4, 3
5	6.2, 9.8, 3.6, 3.2, 3.1, 3.3	**6**	21, 24, 23, 23, 24, 56
7	$^{-}5$, 2, 3, $^{-}2$, $^{-}4$, 3	**8**	1, $^{-}3$, 4, $^{-}2$, $^{-}1$, 2, 1

The Range
describes the **spread** of the data.

The range is the simplest description of the spread of the data.

Exercise 9.3

Find the range of each of the following set of scores:

1, 2, 3, 4, 4

Range = largest − smallest
Range = 4 – 1
Range = 3

The range is the difference between the smallest and the largest.

1 46, 48, 45, 29, 56, 27
2 4, 5, 2, 3, 6, 5, 1
3 2.3, 4.7, 3.2, 2.6, 3.5, 2.5
4 11, 13, 13, 15, 11, 12, 17, 14
5 632, 635, 636, 632, 631
6 $^-2$, $^-3$, $^-5$, $^-4$, $^-1$, $^-2$, $^-6$
7 5.2, $^-6.1$, 5.8, $^-3.2$, $^-4.6$
8 ¾, ¼, ½, ¼, ¼, ½

Exercise 9.4

Calculate the mean, median, and range for each of the following:
If the mean and median differ by a reasonable amount, try to explain the difference.

Recent house price sales: $450k, $520k, $480k, $1250k, $530k.

$$\text{mean} = \frac{\text{sum of scores}}{\text{number of scores}}$$

$$\text{mean} = \frac{450 + 520 + 480 + 1250 + 530}{5}$$

mean = $646k

In ascending order:
$450k, $480k, $520k, $530k, $1250k
Median = $520

Range = 1250 − 450
Range = $800k

The larger price of $1250k has affected the mean. The median is less affected and is probably the better measure of the middle.

1 Recent house price sales: $450k, $520k, $480k, $480k, $530k.

2 Recent house price sales: $340k, $420k, $430k, $370k, $420k, $1310k.

The mean is the usual measure of the middle unless there are extreme values.

3 This week's 400 m times: 92 s, 95 s, 97 s, 93 s, 94 s, 94 s, 88 s

4 Test scores: 7, 8, 8, 7, 6, 9, 9, 10, 7, 8, 10, 10, 6, 8, 8, 9, 7, 9, 9, 8, 10, 6, 8

5 The ages of the people on the bus: 13, 15, 15, 14, 13, 16, 13, 14, 14, 15, 39

Stem-and-Leaf Plots

A Stem-and-Leaf Plot is **simple**, shows the **shape** of the data and puts the data in **order**.

Exercise 9.5

Use a stem-and-leaf plot to represent the following data.
Also find the mean, median, and range.

The test scores of 13 students: 45, 37, 46, 48, 39, 42, 45, 42, 23, 37, 45, 46, 41

in order: 23, 37, 37, 39, 41, 42, 42, 45, 45, 45, 46, 46, 48

Test scores (4|5 is 45)

Stem	Leaf
2	3
3	7 7 9
4	1 2 2 5 5 5 6 6 8

$\text{mean} = \dfrac{\text{sum of scores}}{\text{number of scores}}$

$\text{mean} = \dfrac{536}{13}$

mean = 41.23

median = 42

range = 48 − 23
range = 25

1 The test scores of 10 students: 76, 83, 85, 78, 82, 66, 83, 79, 84, 78, 94

2 The Melbourne daily maximum temperature for the last week:
17.0, 18.6, 19.8, 20.2, 18.7, 20.3, 20.6,

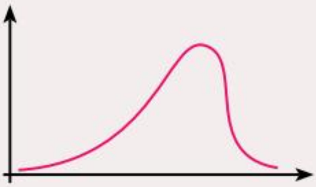
3 Number of container ships per month at the Port of Brisbane:
65, 65, 64, 74, 80, 73, 77, 72, 72, 70, 71, 69, 63

4 Number of twenty foot container units per month exported from Flinders Ports in South Australia:
8200, 9500, 11000, 9400, 10000, 9300, 11000, 9600, 7500

5 Ages of Australian Prime Ministers at first taking office:
52, 47, 37, 59, 46, 53, 53, 40, 53, 52,
59, 44, 47, 57, 55, 60, 57, 68, 56, 63,
56, 45, 53, 48, 57, 50, 49

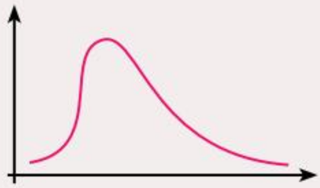
6 The weekly house rents ($): 348, 362, 352, 385, 351,
363, 375, 351, 349, 351,
348, 350, 356, 345, 349
352, 349, 374, 350, 349

Stem-and-Leaf Plots



'Negative skew'
or 'Left skew'.
The left tail is longer.
There are few low values.

The data is
'Symmetrical'.
mean = median



'Positive skew'
or 'right skew'.
The right tail is longer.
There are few high values

Exercise 9.6
Describe the shape of the following stem-and-leaf plots:

Test scores (4|5 is 45)

2	3
3	7 7 9
4	1 2 2 5 5 5 6 6 8

The shape is a negative skew.

There are few low values.

1

Exports (9|3 is 93)

7	5
8	2
9	3 4 5 6
10	0
11	0 0

2

Ships (7|2 is 72)

6	3 4 5 5 9
7	0 1 2 2 3 4 7
8	0

3

Age (5|2 is 52)

3	7
4	0 4 5 6 7 7 8 9
5	0 2 2 3 3 3 3 5 6 6 7 7 7 9 9
6	0 3 8

4

Rent (35|2 is 352)

34	5 8 8 9 9 9 9
35	0 0 1 1 1 2 2 6
36	2 3
37	4 5
38	5

5

Test marks (6|6 is 66)

6	6
7	6 8 8 9
8	2 3 3 4
9	4

6

Temperature (18|6 is 18.6)

17	0
18	6 7
19	8
20	2 3 6

Histograms

Histograms:
Grouped data on the horizontal axis.
Frequencies on the vertical axis.

Because Histograms group data, some data value is lost and thus the measures are not as accurate.

Exercise 9.7
Use a stem and leaf plot to represent the following data.
Also find the mean, median, and range.

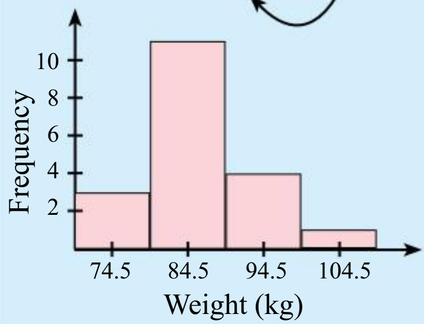
Team individual weights: 99, 85, 80, 83, 87, 88, 96, 92, 75, 88, 76, 103, 87, 83, 83, 86, 84, 79, 95.

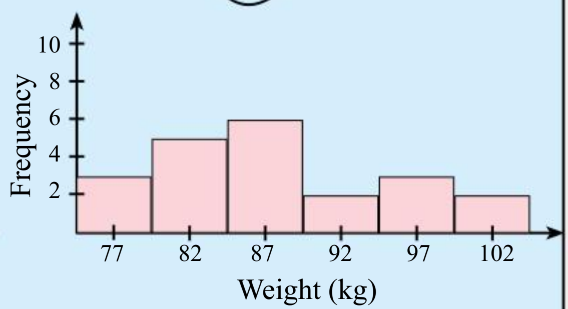
Choose groupings so that there are not too many groups, nor too few groups (5 to 12 groups).

Each of these groupings are ok. Which do you prefer?

Weight	No.
70 - 79	3
80 - 89	11
90 - 99	4
100 - 109	1

Weight	No.
75 - 79	3
80 - 84	5
85 - 89	6
90 - 94	1
95 - 99	3
100 - 104	1





$$\text{mean} = \frac{\text{sum of scores}}{\text{number of scores}}$$

$$\text{mean} = \frac{74.5\text{x}3 + 84.5\text{x}11 + 94.5\text{x}4 + 104.5\text{x}1}{19}$$

$$\text{mean} = \frac{1635.5}{19}$$

Mean = 86.08

Use the midpoint of the group.

Median = 84.5

Range = 104.5 − 74.5
Range = 30

$$\text{mean} = \frac{\text{sum of scores}}{\text{number of scores}}$$

$$\text{mean} = \frac{77\text{x}3 + 82\text{x}5 + 87\text{x}6 + 92 + 97\text{x}3 + 102}{19}$$

$$\text{mean} = \frac{1648}{19}$$

Mean = 86.74

Use the midpoint of the group.

Median = 87

Range = 102 − 77
Range = 25

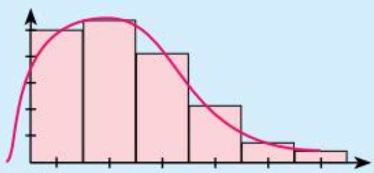
1 Team individual weights: 99, 85, 80, 83, 87, 88, 96, 92, 75, 88, 76, 103, 87, 83, 83, 86, 84, 79, 95.

2 Daily pollution measurements: 636, 644, 647, 648, 721, 450, 476, 645, 622, 580, 660, 539, 487, 549.

Histograms

'Negative skew' The left tail is longer. There are few low values.	**'Symmetrical'**. mean = median	**'Positive skew'** The right tail is longer. There are few high values	**'Bimodal'** Two distinct peaks.

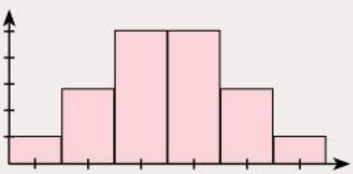
Exercise 9.8
Describe the shape of the following histograms:



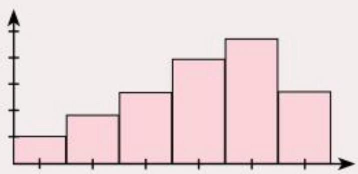
The shape is a positive skew.

There are few high values.

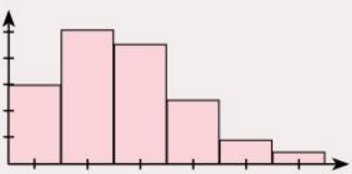
1



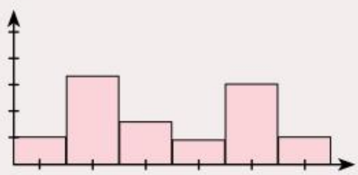
2



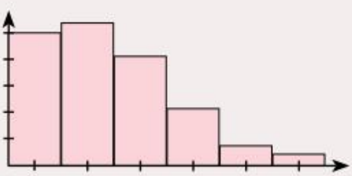
3



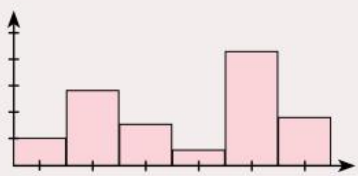
4



5



6



Comparative Analysis

Exercise 9.9

Use a back-to-back stem-and-leaf plot to represent the following data.
Use the mean, median, and range to compare the data.

9A Test marks							9B Test marks						
68	79	97	78	83	65	72	74	72	89	67	78	93	82
74	57	63	96	82	67	84	79	69	67	60	80	73	79

First, put the data in order:

9A 57, 63, 65, 67, 68, 72, 74, 78, 79, 82, 83, 84, 96, 97

9B 60, 67, 67, 69, 72, 73, 74, 78, 79, 79, 80, 82, 89, 93

9A

$$\text{mean} = \frac{\text{sum of scores}}{\text{number of scores}}$$

$$\text{mean} = \frac{1065}{14}$$

Mean = 76.07

median = (74+78)÷2

Median = 76

range = 97 − 57

Range = 40

Test marks (7|8 is 78)

7	5	
8 7 5 3	6	0 7 7 9 9
9 8 4 2	7	2 3 4 8 9 9
4 3 2	8	0 2 9
7 6	9	3

9B

$$\text{mean} = \frac{\text{sum of scores}}{\text{number of scores}}$$

$$\text{mean} = \frac{1062}{14}$$

Mean = 75.86

median = (74+78)÷2

Median = 76

range = 93 − 60

Range = 33

Comment. The stem-and-leaf plot shows that 9A and 9B have a similar slightly positive skew shape. The means of both classes are almost identical and the medians are the same. Thus both classes have no difference on central measures. The range is a little different but not enough to suggest that there is a difference in class performance. Both classes are very similar on this test.

1

9C Test marks							9D Test marks						
78	69	94	73	66	63	71	59	61	84	77	83	82	92
71	74	65	56	87	59	86	86	79	75	73	88	72	74

2 Fitness measurements of a class immediately after Summer and immediately after Winter.

After Summer							After Winter						
69	71	74	53	82	80	66	63	45	77	84	56	57	69
71	67	50	78	73	60	75	70	52	54	67	71	67	71
68	43	81					69	55	47				

3 While researching the strength of concrete beams made from different mixes, the following samples were obtained.

Mix 1 (thousands of psi)							Mix 2 (thousands of psi)						
5.4	5.2	6.2	5.0	4.8	5.2	5.2	5.2	6.4	6.0	5.0	6.4	5.8	5.4
5.0	4.8	5.2	5.4	4.8	5.2	5.6	5.6	6.2	5.8	6.0	5.4	5.2	5.8

Comparative Analysis

Exercise 9.10

Use a compound histogram to represent the following data.
Use the mean, median, and range to compare the data.

9C Test marks							9D Test marks						
78	69	94	73	66	63	71	59	61	84	77	83	82	92
71	74	65	56	87	59	86	86	79	75	73	88	72	74

Group the data.

9C	No.
50 - 59	2
60 - 69	4
70 - 79	5
80 - 89	2
90 - 99	1

9D	No.
50 - 59	1
60 - 69	1
70 - 79	6
80 - 89	5
90 - 99	1

9C

$$\text{mean} = \frac{\text{sum of scores}}{\text{number of scores}}$$

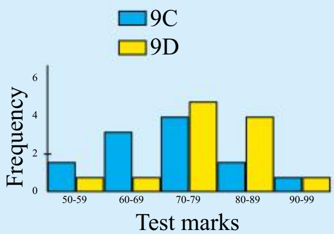
$$\text{mean} = \frac{1012}{14}$$

Mean = 72.29

Median = 74.9

range = 94.9 − 54.9

Range = 40



9D

$$\text{mean} = \frac{\text{sum of scores}}{\text{number of scores}}$$

$$\text{mean} = \frac{1085}{14}$$

Mean = 75.86

Median = 74.9

range = 94.9 − 54.9

Range = 40

Comment. The histograms show that 9C has a slight positive skew while 9D has a slight negative skew. The mean of 9D is higher than the mean of 9D while the medians and ranges are the same. Thus 9D has a slighly better performance on this assessment than 9C.

1

9E Test marks							9F Test marks						
78	69	94	73	66	63	71	59	61	84	77	83	82	92
71	74	65	56	87	59	86	86	79	75	73	88	72	74

2 Weekly earnings of full-time employees:

Female ($)					Male				
1250	960	1100	1020	1330	1430	1310	1400	1730	1170
1170	940	930	1650	1090	1580	2010	1330	1340	1490
820	910	1200	1320	710	1070	1310	1600	1460	1740

3 Mean monthly rainfall, mm, for Sydney and Jakarta (capital of Indonesia).

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sydney	110	122	128	125	123	131	97	86	71	76	85	82
Jakarta	300	300	220	140	130	95	60	45	54	110	140	210

Mental Computation

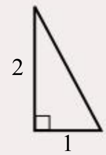
Many everyday problems are solved mentally.

Exercise 9.11

1 Spell Symmetrical.
2 What is the average of 10 and 19?
3 If there are 9 numbers, where is the median?
4 What is the range?
5 What does AAA mean?
6 Scale factor? One side 3, corresponding side 12.
7 Two sides in a right-angled triangle are 1 and 2. Hypotenuse?
8 Simplify 18 : 12
9 The car uses 7 litres of petrol for 100 km.
How far will 21 litres take the car?
10 I bought 3 loaves of bread @ \$2.60 each. Cost?

9 numbers in order.
median=(9+1)÷2=5
The median is the 5th number.

$h^2=2^2+1^2$
$h^2=4+1$
$h = \sqrt{5}$



2 + 2 = 5 ????
2.4 + 2.4 = 4.8
rounding to the nearest integer,
2 + 2 = 5.

Exercise 9.12

1 Spell Histogram.
2 What is the average of 20 and 29?
3 If there are 11 numbers, where is the median?
4 What is the mean?
5 What does SSS mean?
6 Scale factor? One side 5, corresponding side 15.
7 Two sides in a right-angled triangle are 1 and 1. Hypotenuse?
8 Simplify 24 : 18
9 The car uses 8 litres of petrol for 100 km.
How far will 20 litres take the car?
10 I bought two 2 litres of milk @ \$2.80 each. Cost?

Those who can't laugh at themselves leave the job to others - Anon.

Exercise 9.13

1 Spell Median.
2 What is the average of 30 and 39?
3 If there are 10 numbers, where is the median?
4 What does bimodal mean?
5 What does SAS mean?
6 Scale factor? One side 6, corresponding side 18.
7 Two sides in a right-angled triangle are 1 and 3. Hypotenuse?
8 Simplify 30 : 12
9 The car uses 9 litres of petrol for 100 km.
How far will 27 litres take the car?
10 I bought 5 loaves of bread @ \$2.60 each. Cost?

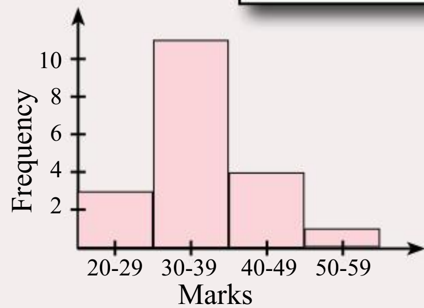
10 numbers in order.
median=10÷2=5th and 6th
The median is the average of the 5th number and the 6th number..

Competition Questions

Prepare for mathematics competitions and build maths muscle at the same time.

Exercise 9.14

1 If the marks are doubled, draw a histogram to show the new results.



2 Aaron played a series of soccer matches. His descriptive statistics for the number of goals scored per match is shown. Give an example of a series of matches that would produce these statistics.

	Mean	Median	Range
Aaron	2	0	6

Find the average of $\frac{1}{3}$ and $\frac{2}{9}$

$$=\frac{1}{2}\left(\frac{1}{3}+\frac{2}{9}\right)=\frac{1}{2}\left(\frac{3}{9}+\frac{2}{9}\right)=\frac{1}{2}\left(\frac{5}{9}\right)=\frac{5}{18}$$

3 Find the average of $\frac{1}{4}$ and $\frac{1}{2}$.

4 Find the average of $\frac{1}{4}$ and $\frac{1}{8}$.

5 The average of four numbers is 36. If 6 is subtracted from each number, what is then the average of the four numbers?

6 The average of eight numbers is 48. If 2 is subtracted from each number, what is then the average of the eight numbers?

A person has an average of 48 after three tests. What mark must the person get on the fourth test so that the average of the four tests is 50?

$48=\frac{\text{total}}{3}$ $\qquad$ $50=\frac{\text{total}}{4}$

Total after 3 tests = 144 $\qquad$ Total after 4 tests = 200

200 − 144 = 56

Thus need 56 on the fourth test.

7 Karen has an average of 83 after ten tests. What mark is needed in the next test to raise her average to 85?

8 Eun-Young has averaged 85% in the six maths test so far this year. Eun-Young would like to finish the year with an overall average of 90% or better. Is this possible with four tests left?

9 The average of five numbers is 4. When the sixth number is added, the average is 5. What is the sixth number?

Investigations

Investigation 9.1 **Undertake real-life research.**

1 Form a team and brainstorm an appropriate problem or issue.

2 Plan the research
- Define the overall research question.
- Define subset research questions.
- Decide how to obtain data to answer the research questions.
- Consider ways of ensuring that the data collection is unbiased.
- Consider the equipment needed for the research.

3 Conduct the research
- Rehearse the data collection method.
- Collect the data.

A backward poet writes inverse.

4 Analyse the data
- Look for errors/outliers and decide what to do with the errors/outliers.
- Calculate the appropriate descriptive statistics.
- Choose an appropriate method of presentation (Histograms etc).

5 Report the conclusions
- Match the analysis with the research questions.
- Do the answers to the research questions indicate further research questions?

Investigation 9.2
Collect Facebook activity data from a Year 12 or Year 10 form class or maths class and the Facebook activity from your own form class or maths class.
Construct compound histograms or stem-and-leaf plots.

a) Are the shapes for the other class and your class similar?
Would you have expected the shapes to be similar or different?

b) Is the data symmetrical?
Does the mean of your class have a value close to the value of the median of your class?
Does the mean of the other year's class have a value close to the value of the median of the other class?

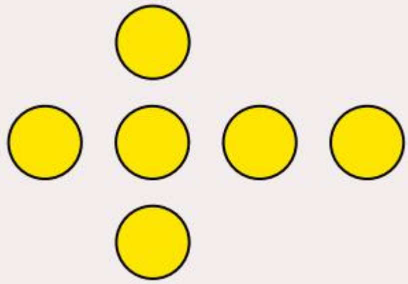
Investigation 9.3 **Online statistical activities**
There are a large number of national and international online statistical activities for Year 9 students. These activities generally involve the collection of data from your class to form a large national data set or international data set. This then forms the basis of infomed research on many relevant topics.
Become involved in one of these activities.

Meteorologists study the atmosphere to forecast the weather.
- Relevant school subjects are Mathematics, Science, and English.
- Courses usually involve a science degree.

A Couple of Puzzles

Exercise 9.15

1 Move just one coin so that there are two rows with four coins in each row.



A Game

Kayles, described in a 14th century manuscript, is played with a row of skittles or bowling pins. The winner is the person who knocks over the last skittle. (Take it in turns to take one counter or take two counters next to each other).

O	O	O	O	O	O	O	O	O	O	O	O	O	O	O

A strategy can be deduced by working with a smaller number of counters.

O	O	O

Play Kayles against the computer. Find a Kayles applet on the Internet.

With three counters, the best first move strategy is to take the middle counter.
The other player is then forced to take one from either end, leaving the last counter for you.

A Sweet Trick

Use this trick to show off your astonishing memory.

1 Make up a set of cards similar to the following:

1 **7 189 763**	2 **4 370 774**	3 **1 561 785**
4 **8 640 448**	5 **5 831 459**	6 **2 022 460**
7 **9 101 123**	8 **6 392 134**	9 **3 583 145**

2 Ask your audience to choose a card and tell you the card number.

3 You write out the seven-digit number written on the card.

The secret

a) Multiply the number of the card by 17 — 4x17 = 68

b) Start the number by reversing the digits. — 86

c) Add the last two terms to get the third. (Use the last unit only) — 8640448

8+16=14

6+4=10

Technology

Technology 9.1 The Calculator

Scientific calculators will calculate descriptive statistics such as mean and sum.

1 Change the calculator mode to Stat or SD

2 Clear the calculator.

2 Enter a number then press M+

3 Repeat entering a number and then pressing M+

4 Find the $\bar{x}$ button, this is the mean.

5 Find the $\sum x$ button, this is the sum of the numbers.

Check your calculator's manual for Statistical calculations

Technology 9.2 The Spreadsheet

Most spreadsheets will calculate a massive number of descriptive statistics **and** draw a frequency polygon:

1 Enter a set of data into the spreadsheet.

2 Enter formulas for the mean, median, and mode.

The Internet has tutorials for all kinds of spreadsheet graphs.

	A	B	C
1	2	mean =	2.6
2	2	median =	2
3	2	mode =	2
4	3		
5	4		

=mean(A1:A5)

=median(A1:A5)

=mode(A1:A5)

Technology 9.3 The Graphics calculator

A graphics calculator will automatically calculate a large number of descriptive statistics **and** draw a frequency column graph:

1 Select the STAT menu, EDIT, and enter data into one of the lists.

2 Return to the main screen.

3 Select the STAT menu, Calc, 1_Stats and enter L1.

L1	L2	L3
2		
2		
2		
3		
4		

→

1_Stats

$\bar{x}$ = 2.6

median = 2

n = 5

$\sum x$ = 13

+ heaps more.

To draw a frequency column graph:

1 Enter the numbers in L1 and the frequency in L2.

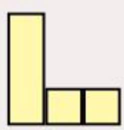
2 Use STATPLOT to set up the graph Xlist= L1 and Frequency = L2.

3 Use Zoom and ZooMSTAT to fit the graph if necessary.

4 TRACE.

L1	L2	L3
2	3	
3	1	
4	1	

→



Chapter Review 1

Exercise 9.16

1 Calculate the mean, median, and range for each of the following:

a) The ages of the people on the bus: 15, 14, 15, 15, 14, 15, 15, 15, 15, 42

b) Annual Wages: $45k, $52k, $48k, $48k, $53k.

2 Describe the following data distributions using terms such as skew, symmetrical, or bimodal.

a)

Test marks (6|4 is 64)

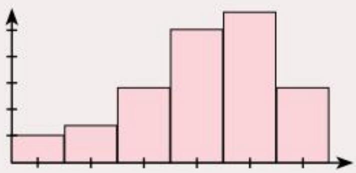
5	2
6	3 4 9
7	2 3 3 4 6
8	4 4 7
9	5

b)

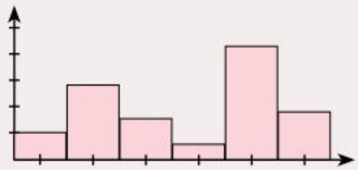
Rent (35|5 is 355)

35	0 5 5
36	0 0 0 5 5 5 5
37	0 5
38	5
39	5

c)



d)



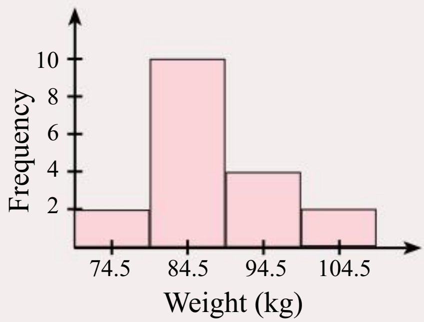
3 Find the mean, median, and range of each of the data displays:

a)

Test scores (4|5 is 45)

2	3
3	7 7 9
4	1 2 2 5 5 5 6 6 8

b)



4 The skin elasticity of 48 people at a workplace is measured. 24 of the people work mainly outdoors, while the other 24 people work mainly indoors. Analyse the data and make a comment.

Skin elasticity (sun-exposed)						Skin elasticity (sun-protected)					
76	56	74	58	82	67	48	88	78	55	44	81
79	55	81	39	65	81	127	67	67	48	31	76
24	45	54	42	38	23	92	105	74	64	80	77
93	42	81	75	86	12	48	59	99	57	70	54

Chapter Review 2

Exercise 9.17

1 Calculate the mean, median, and range for each of the following:

a) Heart rates: 84, 73, 83, 55, 48, 62, 65, 61, 72

b) Cholesterol levels: 4.4, 5.1, 4.2, 3.8, 4.1.

2 Describe the following data distributions using terms such as skew, symmetrical, or bimodal.

a)

Exports (9|3 is 93)

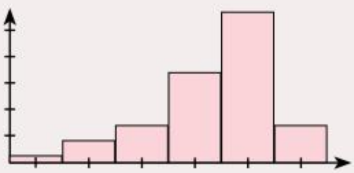
7	5
8	2 4 4 6 7 7
9	3 5
10	0 4 6 6 8 9
11	0 0

b)

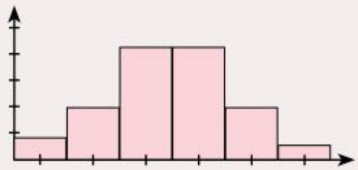
Ships (6|2 is 62)

5	0 2 4 5
6	2 3 5 5 5 7 9
7	4 6
8	2

c)



d)



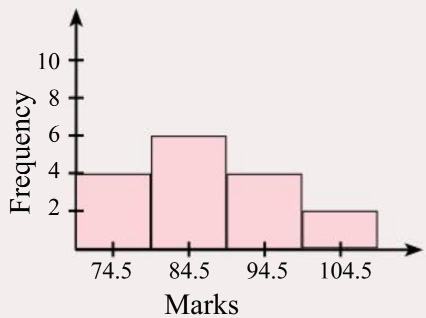
3 Find the mean, median, and range of each of the data displays:

a)

Test scores (4|5 is 45)

7	5 7
8	0 3 5 8 8
9	1 2 6

b)



4 Patients with multiple rib fractures were asked to provide a pain score one hour after receiving one of two analgesic drugs. A high score indicates a high level of pain. Analyse the data and make a comment.

Pain score (Analgesic A)						Pain score (Analgesic B)					
11	17	8	5	14	10	14	9	5	10	10	14
7	7	10	10	8	4	13	5	15	10	5	7
7	5	11	6	12	10	12	9	10	12	15	13
5	9	13	3	10	15	10	8	5	15	10	12
5	7	9				8	5	9			

Review 2 10

Chapter 6 Proportion

When two ratios are equal they are said to be in **proportion**.

Proportion

If $\frac{a}{b} = \frac{c}{d}$ $\quad \frac{a}{b} \times \frac{c}{d}$

then $ad = bc$

or $bc = ad$

Inverse proportion means an **increase in one** quantity will cause a similar **decrease in another** quantity.

Inverse Proportion

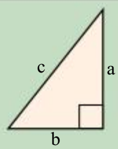
a — c

b — d

$ac = bd$

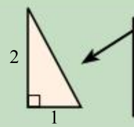
Chapter 7 Pythagoras

In any right-angled triangle:
The square on the hypotenuse is equal to the sum of the squares on the other two sides.



$$\boxed{c^2 = a^2 + b^2}$$

The hypotenuse is the longest side.
It is opposite the right-angle (90°).



$h^2=2^2+1^2$
$h^2=4+1$
$h = \sqrt{5}$

Chapter 8 Geometry

Similar triangles have **exactly the same shape** but not necessarily the same size.

The corresponding **angles are equal**.

The corresponding sides have the same scale factor.

The symbol for similarity is ~.

Two triangles are similar if:

AAA The three matching angles are equal.
SSS The three matching sides are in the same ratio.
SAS Two matching sides are in the same ratio and the *included* angles are equal.
RHS The hypotenuse and a matching side of a right-angled triangle are in the same ratio.

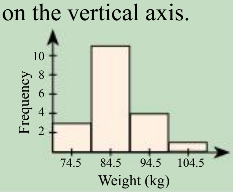
Chapter 9 Statistics

A Stem-and-Leaf Plot is **simple**, shows the **shape** of the data and puts the data in **order**.

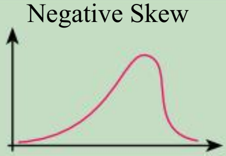
Test scores (4|5 is 45)

2	3
3	7 7 9
4	1 2 2 5 5 5 6 6 8

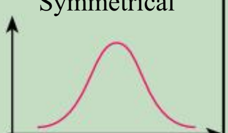
Histograms: Grouped data on the horizontal axis. Frequencies on the vertical axis.



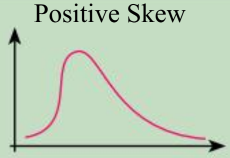
Negative Skew



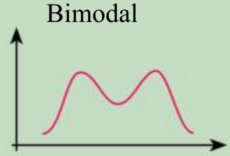
Symmetrical



Positive Skew



Bimodal



Review 1

Exercise 10.1 Mental computation

1 Spell Symmetrical.

2 What is the average of 10 and 19?

3 If there are 9 numbers, where is the median?

4 What is the range?

5 What does SSS mean?

6 Scale factor? One side 4, corresponding side 12.

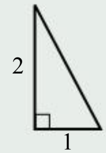
7 Two sides in a right-angled triangle are 1 and 2. Hypotenuse?

8 Simplify 12 : 16

9 The car uses 8 litres of petrol for 100 km.
How far will 12 litres take the car?

10 I bought 5 loaves of bread @ \$2.60 each. Cost?

9 numbers in order.
median=$(9+1)\div 2=5$
The median is the 5th number.



$h^2=2^2+1^2$
$h^2=4+1$
$h=\sqrt{5}$

Exercise 10.2

1 Simplify the following ratios:

a) 5:15 **b)** 6:10 **c)** 12:20

d) 15:18 **e)** 1.6:2.4 **f)** 3:1.5

2 Which of the following pairs of ratios are in proportion?

a) 6:2 and 15:5 **b)** 8:2 and 9:3 **c)** 12:6 and 14:7

d) 2 tonnes are loaded in 5 hours and 6 tonnes are loaded in 15 hours.

3 Draw a graph and write a rule for each of the following:

a) For each 50 km the rally car uses 10 litres of fuel.

b) Sound can travel 3.4 km in 10 seconds.

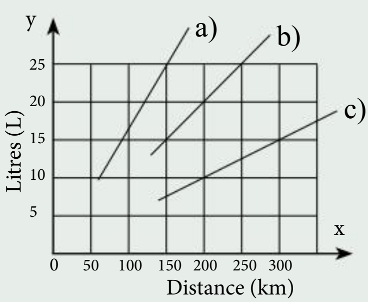
4 Assuming proportionality, solve the following problems.

a) If \$AU1 can be exchanged for \$US1.04, how many Australian dollars can be exchanged for \$US130?

b) Riley can run 4 km in 20 minutes.
How far can Riley be expected to run in 50 minutes?

5 If 5 people can build a house in 30 days, how long would it take 20 people to build the same house (assuming inverse proportion)?

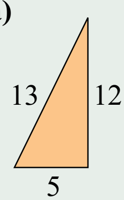
6 From the graph, find the litres per 100 km fuel consumption for each vehicle.



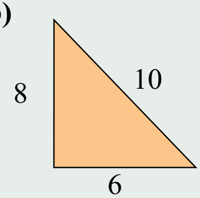
A mistake is simply another way of doing things - Katharine Graham.

7 Which of the following triangles are right-angled triangles?

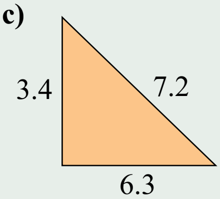
a)



b)



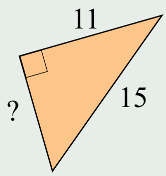
c)



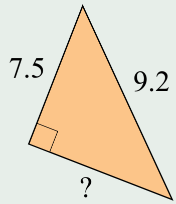
8 A carpenter needs to know whether a door frame is square.
If the door frame is square then a door can be fitted.
The door frame measures 810 mm by 2000 mm and the diagonal is 2151 mm.

9 Find the length of the unknown in each of the following:

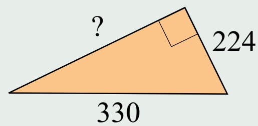
a)



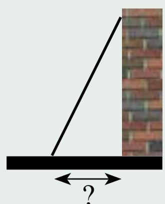
b)



c)

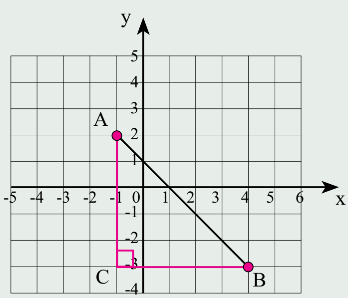


10 A 3.1 m ladder is to be laid against a wall so that the top of the ladder is 2.3m up the wall. How far out from the base of the wall should the ladder be placed?

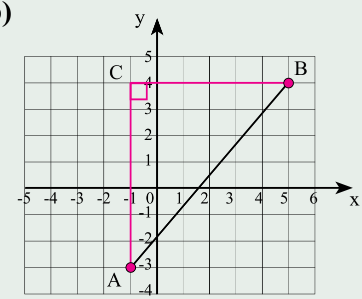


11 Find the length of AB.

a)

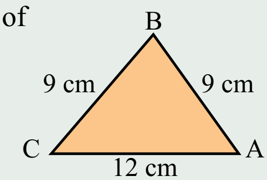


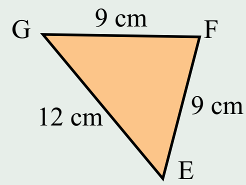
b)



c) A(1,2), B(3,5) **d)** A(4,-1), B(2,5) **e)** A(-3,-2), B(-1,4)

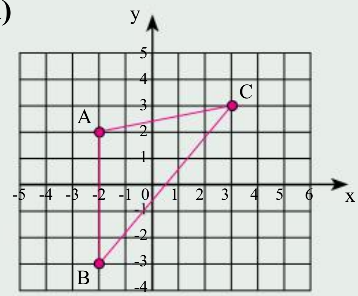
12 Use the tests for congruence to test whether the following pair of triangles are congruent:



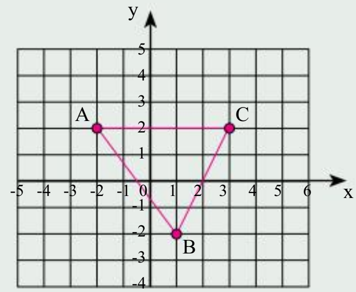


13 Enlarge the original triangle by a scale factor of two.

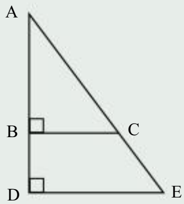
a)



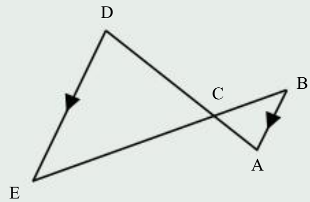
b)



14a Prove that △ABC ~ △ADE

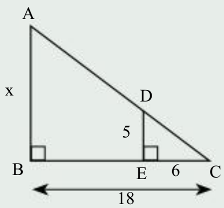


14b Prove that △ABC ~ △DEC

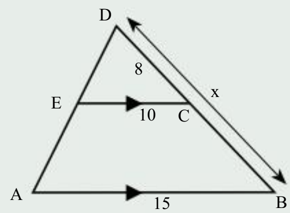


15 Find the length of the unknown.

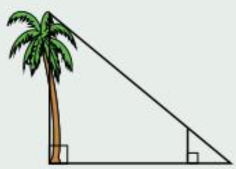
a)



b)



16 A 1.4 metre stick casts a 0.70 metre shadow at the same time a tree casts a 5.6 m shadow. What is the height of the tree?



17 Calculate the mean, median, and range for each of the following:

a) The ages of the people on the bus: 15, 15, 15, 15, 14, 15, 13, 15, 15, 53

b) Annual Wages: $47k, $57k, $54k, $62k, $53k.

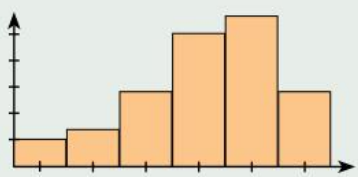
18 Describe the following data distributions using terms such as skew, symmetrical, or bimodal.

a)

Rent (35|5 is 355)

35	0 5 5
36	0 0 0 5 5 5 5
37	0 5
38	5
39	5

b)



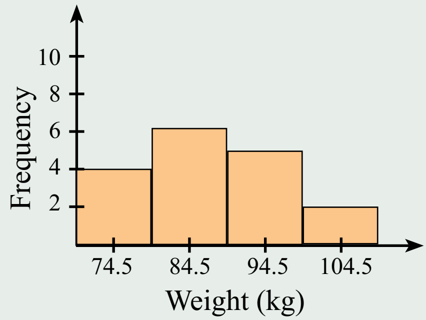
19 Find the mean, median, and range of each of the data displays:

a)

Test scores (8|2 is 82)

6	3
7	0 4 9
8	1 2 2 7 8
9	3

b)



20 Investigating the use of heat treatment on sprains, a research scientist collected the following data on the time, in days, taken for a muscle sprain to heal. Analyse the data and make a comment.

Sprains (with heat treatment)						Sprains (without treatment)					
15	15	14	20	16	18	21	21	20	22	16	25
15	15	18	18	16	12	17	20	19	22	19	20
18	16	17	16	19	14	18	19	22	17	16	20
19	17	15	12	16	14	20	24	21	23	18	22
18	22	17				18	16	19			

Review 2

Exercise 10.3 Mental computation

1 Spell Pythagoras.
2 What is the average of 20 and 29?
3 If there are 10 numbers, where is the median?
4 What is the range?
5 What does AAA mean?
6 Scale factor? One side 4, corresponding side 12.
7 Two sides in a right-angled triangle are 1 and 1. Hypotenuse?
8 Simplify 15 : 12
9 The car uses 6 litres of petrol for 100 km.
How far will 15 litres take the car?
10 I bought 4 loaves of bread @ $2.60 each. Cost?

Exercise 10.4

1 Simplify the following ratios:

a) 15:5 **b)** 12:4 **c)** 16:8

d) 15:10 **e)** 1.8:1.5 **f)** 5:1.5

2 Which of the following pairs of ratios are in proportion?

a) 6:2 and 9:3 **b)** 5:4 and 10:2 **c)** 15:5 and 12:4

d) Travel 300 km in 4 hours and travel 450 km in 6 hours.

3 Draw a graph and write a rule for each of the following:

a) For each 50 km the rally car uses 15 litres of fuel.

b) The 200 acre paddock yielded 400 tonnes of corn.

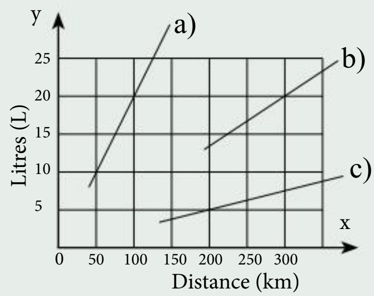
4 Assuming proportionality, solve the following problems.

a) If \$AU1 can be exchanged for \$US0.90, how many Australian dollars can be exchanged for \$US810?

b) A car uses 6.5 litres of petrol to travel 100 km.
How far will the car travel on 55 litres of petrol?

5 It will cost \$55 per person if there are 25 people on the charter bus.
How many people would be needed to reduce the cost per person to \$50?

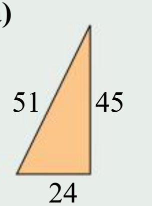
6 From the graph, find the litres per 100 km fuel consumption for each vehicle.



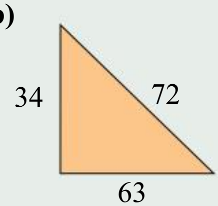
A good head and a good heart are always a formidable combination - Nelson Mandela.

7 Which of the following triangles are right-angled triangles?

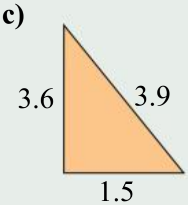
a)



b)



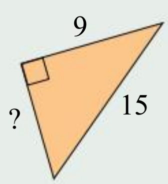
c)



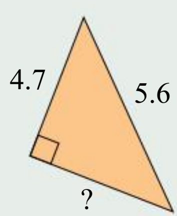
8 A rectangular gate measures 1.4 m by 2.5 m
with a 2.79 m diagonal. Is the gate square?
If not, should the diagonal be longer or shorter?

9 Find the length of the unknown in each of the following:

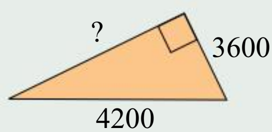
a)



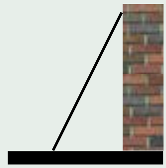
b)



c)

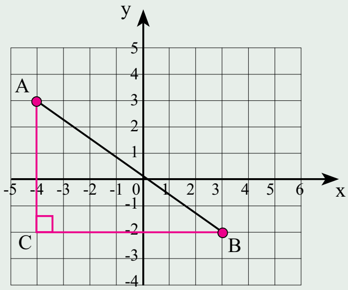


10 A 1.2 m ladder is leaning against a wall. The bottom of the ladder is 0.4 m from the wall and the top of the ladder is 1.13 m up the wall. Is the wall vertical?

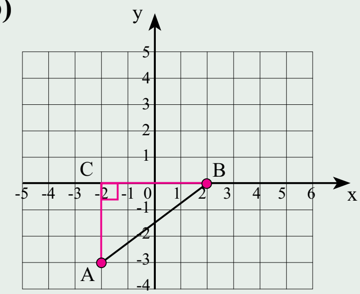


11 Find the length of AB.

a)

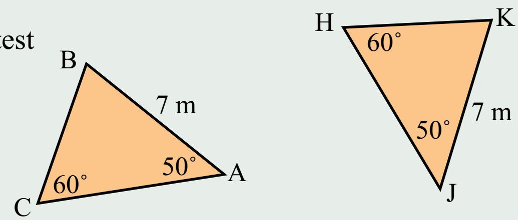


b)



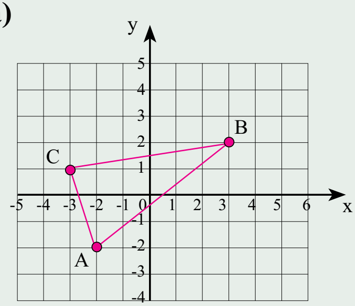
c) A(5,2), B(6,5) **d)** A(1,-1), B(4,4) **e)** A(-4,-3), B(4,3)

12 Use the tests for congruence to test whether the following pair of triangles are congruent:

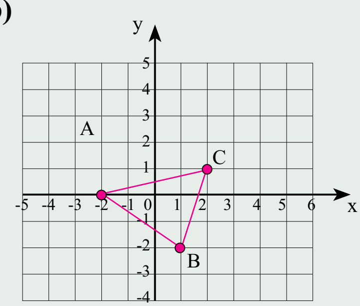


13 Enlarge the original triangle by a scale factor of two.

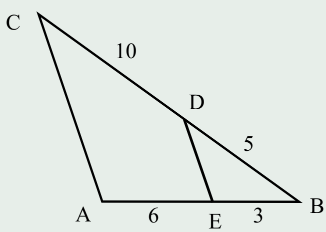
a)



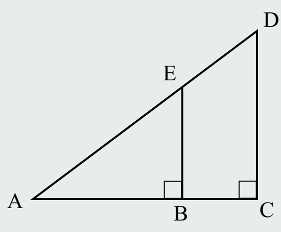
b)



14 Prove that △ABC ~ △EBD

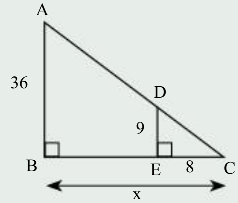


4 Prove that △ABE ~ △ACD

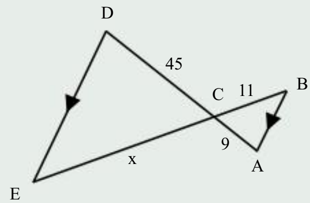


15 Find the length of the unknown.

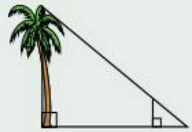
a)



b)



16 A 1.5 metre stick casts a 0.65 metre shadow at the same time a tree casts a 3.6 m shadow. What is the height of the tree?



17 Calculate the mean, median, and range for each of the following:

a) Car speed: 56, 45, 54, 55, 53, 62, 49, 51, 72

b) Arm span: 155, 174, 164, 168, 159, 172.

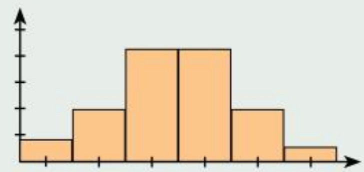
18 Describe the following data distributions using terms such as skew, symmetrical, or bimodal.

a)

Pallets (3|2 is 32)

2	0 2 4 5
3	2 3 5 5 6 9 9
4	4 6
5	2

b)



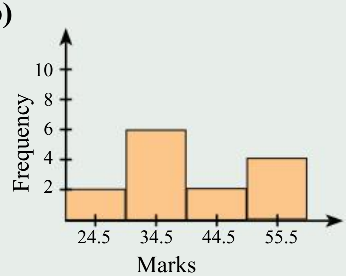
19 Find the mean, median, and range of each of the data displays:

a)

Test scores (1|8 is 18)

0	7
1	8 9
2	4 5 5 6 8 8 8

b)



20 Data was collected on daily student absence data during the flu season before and after a handwashing campaign.
Analyse the data and make a comment.

Absences (before)						Absences (after)					
41	46	55	64	57	66	45	47	52	49	57	52
53	57	44	60	47	49	45	44	43	41	43	31
52	41	32	35	33	24	38	33	36	27	33	23
35	20					30	21				

Indices 2

11

Number and Algebra → Real Numbers

★ Express numbers in scientific notation.
★ Understand that the use of index notation is an efficient way of representing numbers and symbols and has many applications, particularly in science.
★ Represent extremely large and small numbers in scientific notation, and numbers expressed in scientific notation as whole numbers or decimals.
★ Apply index laws to numerical expressions with integer indices.
★ Apply knowledge of index laws to algebraic terms and simplify algebraic expressions, using both positive and negative integral indices.

Measurement and Geometry → Units of Measurement

★ Investigate very small and very large time scales and intervals.
★ Investigate the usefulness of scientific notation in representing very large and very small numbers.

I'm off to compete in the Mental Calculation World Cup.

A TASK

Design, develop, and evaluate a survey to test the support for the following statement:

"Mental computation is so important that it should be taught and tested as a part of every maths lesson".

A LITTLE BIT OF HISTORY

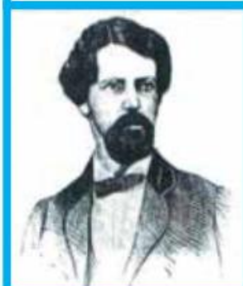
Zerah Colburn (1804-1840) showed awesome mental powers at the age of just eight years old.

Zera could instantly give the product of two numbers each of four digits but hesitated if both numbers exceeded 10 000.

Zera was asked for 8^{16}
in a few seconds, he replied '281 474 976 710 656'.

Asked for the factors of 247 483 he replied 941 and 263.
Asked for the factors of 171 395 he gave 5, 7, 59 and 83.
Asked for the factors of 36 083 he said there were none.

Zera was thought to be mentally retarded until seven.



Warmup

A convenient way of writing $2\times2\times2$ is 2^3

Index (the 3)
Base (the 2)

Exercise 11.1

Write the following in index form:

$2\times2\times2\times2\times2$ $= 2^5$	$a\times a\times a\times a$ $= a^4$

1 $2\times2\times2$ **2** $8\times8\times8\times8$ **3** $a\times a\times a\times a\times a$
4 $10\times10\times10$ **5** $b\times b\times b\times b\times b$ **6** $h\times h\times h$
7 $m\times m\times m\times m\times m$ **8** $10\times10\times10\times10$ **9** $3\times3\times3\times3\times3\times3$

Exercise 11.2

Write the following in factor form:

10^4 $= 10\times10\times10\times10$ $= 10\ 000$	b^3 $= b\times b\times b$

1 4^3 **2** b^4 **3** 5^2
4 2^7 **5** 6^2 **6** m^5
7 x^4 **8** p^5 **9** 1^4

Exercise 11.3

Write the following in index form:

$2\times2\times2\times4\times4$ $= 2^3\times4^2$	abbaaab $= a^4\times b^3$

1 aabbbaa **2** $3\times3\times3\times2\times2$ **3** abaaababb
4 $2\times10\times2\times10\times2\times10\times2$ **5** bggggbbbg **6** zzzzzzzzzzz
7 ppqrppqqrrrp **8** $2\times2\times2\times3\times4\times4\times4\times3$ **9** 10gg10g10g10gg

Exercise 11.4

Simplify the following:

$10^4\times10^2$ $= 10\times10\times10\times10\times10\times10$ $= 10^6$	$(2\times10^2)^3$ $= 2\times10^2\times2\times10^2\times2\times10^2$ $= 2^3\times10^6$

1 $a^2\times a^3$ **2** $3^3\times3^4$ **3** $a^4\times a^2$
4 $10^3\times10^3$ **5** $b^5\times b$ **6** $10^2\times10^3\times10^4$
7 $(3\times10^2)^3$ **8** $(a\times10^2)^3$ **9** $(3^2\times10^3)^2$

Index Law 1

Multiplying Indices:		Multiplying Indices:
$2^4 \times 2^2 = 2\times2\times2\times2 \times 2\times2 = 2^6$	or	$2^4 \times 2^2 = 2^{4+2} = 2^6$

→

Index Law 1

$a^m \times a^n = a^{m+n}$

Exercise 11.5

Simplify and write the following in index form:

$10^3\times10^2 = 10\times10\times10 \times 10\times10 = \underline{10^5}$	$a^2\times a^5 = a\times a \times a\times a\times a\times a\times a = \underline{a^7}$
or $10^3\times10^2 = 10^{3+2} = \underline{10^5}$	**or** $a^2\times a^5 = a^{2+5} = \underline{a^7}$

1	$10^2\times10^3$	**2**	$4^2\times4^2$	**3**	$3^4\times3^3$	**4**	$10^4\times10^3$
5	$x^2\times x^4$	**6**	$x^3\times x^2$	**7**	$x^5\times x^2$	**8**	$x^4\times x^3$
9	$2^3\times2^1$	**10**	$a^2\times a^7$	**11**	$3^2\times3^3$	**12**	$10^3\times10^6$
13	$b^4\times b^3$	**14**	$10^2\times10^7$	**15**	$c^3\times c^3$	**16**	$d\times d^4$
17	$6.3^5\times6.3^2$	**18**	$e^3\times e^6$	**19**	$0.1^5\times0.1^4$	**20**	$10^5\times10^3$
21	$10^4\times10^3\times10^2$	**22**	$z^4\times z^3\times z^2$	**23**	$2^2\times2^3\times2^3\times2^4$	**24**	$y^2\times y^2\times y^3$

$b = b^1$
$10 = 10^1$

Index Law 2

Dividing Indices:		Dividing Indices:
$a^3 \div a^2 = \frac{a\times a\times a}{a\times a} = a$	or	$a^3\div a^2 = a^{3-2} = a$

→

Index Law 2

$a^m \div a^n = a^{m-n}$

Exercise 11.6

Simplify and write the following in index form:

$10^3\div10^2 = \frac{10\times10\times10}{10\times10} = \underline{10}$	$a^6\div a^2 = \frac{a\times a\times a\times a\times a\times a}{a\times a} = a\times a\times a\times a = \underline{a^4}$
or $10^3\div10^2 = 10^{3-2} = \underline{10}$	**or** $a^6\div a^2 = a^{6-2} = \underline{a^4}$

1	$10^5\div10^3$	**2**	$10^4\div10^2$	**3**	$2^6\div2^2$	**4**	$3^5\div3^3$
5	$x^5\div x^2$	**6**	$a^4\div a^2$	**7**	$10^7\div10^3$	**8**	$b^4\div b^3$
9	$10^3\div10$	**10**	$n^8\div n^4$	**11**	$2^6\div2^4$	**12**	$10^6\div10^2$
13	$y^7\div y^3$	**14**	$0.2^4\div0.2^2$	**15**	$10^9\times10^5\div10^6$	**16**	$x^3\div x^3$
17	$\frac{10^5}{10^3}$	**18**	$\frac{x^7}{x^4}$				
19	$\frac{a^7\times a^2}{a^4}$	**20**	$\frac{10^7\times10^3}{10^4\times10^6}$				

$m^4\div m^2 = \frac{m^4}{m^2}$

They are the same thing.

Index Law 3

Power Indices:

$(2^3)^2 = (2\times2\times2)^2$
$= (2\times2\times2)\times(2\times2\times2)$
$= 2^6$

or

Power Indices:

$(2^3)^2 = 2^{3\times2}$
$= 2^6$

Index Law 3

$(a^m)^n = a^{m\times n}$

Exercise 11.7

Simplify and write the following in index form:

$(b^4)^2 = (b\times b\times b\times b)^2$ $= (b\times b\times b\times b)\times(b\times b\times b\times b)$ $= \underline{b^8}$ **or** $(b^4)^2 = b^{4\times2} = \underline{b^8}$	$10^4\times(10^2)^3 = 10^4\times10^6 = \underline{10^{10}}$ $(b^4)^2b^3 = b^8\times b^3 = \underline{b^{11}}$

1	$(b^2)^3$	**2**	$(b^3)^2$	**3**	$(b^3)^3$	**4**	$(10^2)^3$
5	$(10^2)^2$	**6**	$(a^3)^4$	**7**	$(3^2)^4$	**8**	$(2^2)^4$
9	$(10^2)^2$	**10**	$(x^2)^5$	**11**	$(y^2)^3$	**12**	$(5^5)^2$
13	$(4^3)^3$	**14**	$(10^2)^3$	**15**	$(10)^3$	**16**	$(a^2)^5$
17	$(10^3)^3$	**18**	$(c^4)^3$	**19**	$(d^2)^5$	**20**	$(0.3^5)^3$
21	$10^2\times(10^2)^3$	**22**	$2^2\times(2^3)^3$	**23**	$(x^2)^3\times x^3$	**24**	$z^5\times(z^2)^3$
25	$10^3(10^2)^2$	**26**	$b^5(b^3)^4$	**27**	$(2^2)^42^5$	**28**	$m^3(m^3)^5$
29	$(3^2)^2\times3$	**30**	$p^2(p^2)^3$	**31**	$10(10^2)^3$	**32**	$10\times(10^4)^2$

$10 = 10^1$

The more practice you get the easier it becomes.

Index Law 4

Zero Index:

$p^3\div p^3 = 1$

or $p^3\div p^3 = p^{3-3}$
$= p^0$

Which must be = 1

or

Zero Index:

$p^0 = 1$

Zero Index

$a^0 = 1$

Try 5^0 on your calculator.
Is your answer 1?

Exercise 11.8

Simplify each of the following:

$10^0 = \underline{1}$	$h^0 = \underline{1}$	$3\times5^0 = 3\times1 = \underline{3}$	$5b^0 = 5\times1 = \underline{5}$

1	10^0	**2**	2^0	**3**	x^0	**4**	a^0
5	b^0	**6**	3^0	**7**	4^0	**8**	y^0
9	3×10^0	**10**	$4c^0$	**11**	9×2^0	**12**	3×1^0
13	$8m^0$	**14**	$2h^0$	**15**	5×10^0	**16**	6×0.01^0
17	$(2^0)^2\times2$	**18**	$p^2(p^0)^3$	**19**	$10(10^4)^0$	**20**	$10\times(10^0)^3$

Index Law 5

What happens when the index is negative?

$$a^2 \div a^5 = \frac{a \times a}{a \times a \times a \times a \times a} = \frac{1}{a \times a \times a}$$

or

$$a^2 \div a^5 = a^{2-5} = a^{-3}$$

Negative Index

$$a^{-m} = \frac{1}{a^m}$$

Exercise 11.9

Write each of the following using a negative index:

$\frac{1}{10^3} = \underline{10^{-3}}$	$\frac{1}{b^5} = \underline{b^{-5}}$	$\frac{1}{10} = \underline{10^{-1}}$	$\frac{1}{10000} = \frac{1}{10^4} = \underline{10^{-4}}$

1 $\frac{1}{10^2}$ **2** $\frac{1}{10^5}$ **3** $\frac{1}{b^4}$ **4** $\frac{1}{3^5}$

5 $\frac{1}{x^3}$ **6** $\frac{1}{10}$ **7** $\frac{1}{10^4}$ **8** $\frac{1}{2^6}$

9 $\frac{1}{100}$ **10** $\frac{1}{1000}$ **11** $\frac{1}{8}$ **12** $\frac{1}{27}$

Simplify and write the following in index form:

$10^2 \times 10^{-3} = 10^{2-3} = \underline{10^{-1}}$	$10^{-3} \div 10^{-4} = 10^{-3--4} = 10^{-3+4} = \underline{10}$

13 $10^{-2} \times 10^3$ **14** $10^{-2} \times 10^5$ **15** $10^6 \div 10^{-2}$ **16** $10^{-2} \div 10^{-2}$

17 $3^{-4} \times 3^3$ **18** $10^{-4} \times 10^5$ **19** $x^5 \div x^{-4}$ **20** $10^{-2} \div 10^5$

21 $x^6 \times x^{-4}$ **22** $10^3 \times 10^{-4} \times 10^2$ **23** $2^6 \div 2^2$ **24** $3^5 \div 3^3$

25 $x^{-5} \times x^2 \times x^4$ **26** $a^4 \times a^{-9} \times a^3$ **27** $10^{-7} \div 10^3$ **28** $b^{-4} \div b^{-3}$

$(10^{-2})^4 = 10^{-2\times 4} = \underline{10^{-8}}$	$9(10^0)^{-3} = 9 \times 10^{0\times -3} = 9 \times 1 = \underline{9}$

29 $(10^{-2})^3$ **30** $(5^{-3})^3$ **31** $(2^2)^{-4}$ **32** $(10^{-2})^{-2}$

33 $(x^2)^3$ **34** $(10^{-3})^2$ **35** $(b^4)^{-3}$ **36** $(y^{-1})^{-6}$

37 3×10^0 **38** $4(c^{-2})^0$ **39** $9(10^0)^{-3}$ **40** 3×1^0

$4 \times 10^5 \times 2 \times 10^{-3} = 8 \times 10^{5-3} = \underline{8 \times 10^2}$	$6 \times 10^{-2} \div (2 \times 10^{-5}) = 3 \times 10^{-2--5} = 3 \times 10^{-2+5}$ $= \underline{3 \times 10^3}$

41 $3 \times 10^2 \times 2 \times 10^3$ **42** $6 \times 10^6 \times 3 \times 10^{-3}$ **43** $9 \times 10^{-2} \div (3 \times 10^{-6})$ **44** $4 \times 10^3 \div (2 \times 10^{-4})$

45 $4 \times 10^4 \times 3 \times 10^5$ **46** $2 \times 10^{-4} \times 3 \times 10^{-2}$ **47** $x^5 \times 4x^{-3} \div 2x^2$ **48** $(x^{-2})^2 \times 6x^3 \div 2x^2$

Scientific Notation

Very large numbers.

Index notation is easier and quicker.

The Sun has a mass of:
2 000 000 000 000 000 000 000 000 000 000 kg.
2×10^{30} kg

Exercise 11.10
Write in scientific notation:

314 000 $= \underline{3.14\times10^5}$	4 670 000 $= \underline{4.67\times10^6}$

6 5 4 3 2 1
4 670 000

1	375 000	**2**	582 000	**3**	160 000
4	5 100 000	**5**	6 200 000	**6**	7 400 000
7	47 000	**8**	29 000	**9**	81 300
10	83 000 000	**11**	48 100 000	**12**	7 620 000
13	91 000	**14**	76 000 000	**15**	20 000 000

16 The power station produces 250 000 000 000 watts.

17 The speed of light, 299 800 000 m/s.

18 The distance from the Earth to the Sun, 149 000 000 000 m.

19 The diameter of the Earth, 12 700 000 m

20 Estimated number of stars in the Milky Way, 100 000 000 000 stars.

Exercise 11.11
Write as ordinary numbers:

5.7×10^4 $= \underline{57\ 000}$	9.13×10^7 $= \underline{91\ 300\ 000}$

1 2 3 4 5 6 7
91 300 000

1	10^3	**2**	3×10^5	**3**	8×10^4
4	4×10^5	**5**	3.3×10^2	**6**	8.1×10^8
7	6.3×10^7	**8**	7.11×10^7	**9**	10^9
10	3.879×10^5	**11**	9.324×10^6	**12**	7.8214×10^9
13	2.1012×10^{12}	**14**	5.5×10^8	**15**	2.3×10^4

16 The amount of water in the dam, 3.2×10^8 litres.

Scientific Notation

Very small numbers.

Index notation is easier and quicker.

Electric charge of an electron:
0.000 000 000 000 000 000 160 2 coulombs.
1.602×10^{-19} coulombs

Exercise 11.12
Write in scientific notation:

= 0.000 174 $= 1.74\times10^{-4}$	= 0.000 081 $= 8.1\times10^{-5}$

1 2 3 4 5
0.000 081

$0.001 = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$

1	0.000 031	**2**	0.000 006 3	**3**	0.001
4	0.000 8	**5**	0.000 856	**6**	7 400 000
7	0.05	**8**	0.26	**9**	0.491
10	0.001	**11**	0.000 1	**12**	0.000 01
13	0.009 1	**14**	0.000 066	**15**	0.000 000 0939

16 A virus has a length of: 0.000 000 025 m.

17 Light travels one metre in: 0.000 000 003 s.

18 Wavelength of green light: 0.000 000 52 m.

19 Mass of an electron: 0.000 000 000 000 000 000 000 000 000 9 g

20 Mass of a proton: 0.000 000 000 000 000 000 000 001 67 g

Exercise 11.13
Write as ordinary numbers:

$= 2\times10^{-4}$ = 0.000 2	$= 4.5\times10^{-8}$ = 0.000 000 045

1 2 3 4 5 6 7 8
0.000 000 045

1	2×10^{-1}	**2**	7×10^{-2}	**3**	5×10^{-3}
4	1×10^{-4}	**5**	6.7×10^{-5}	**6**	3.2×10^{-6}
7	8.35×10^{-4}	**8**	9.91×10^{-9}	**9**	1.04×10^{-8}
10	6.172×10^{-6}	**11**	4.304×10^{-7}	**12**	6.826×10^{-12}
13	9.013×10^{-9}	**14**	6.5×10^{-12}	**15**	3.3×10^{-15}

16 The muon has a mean lifetime of 2.2×10^{-6} seconds.

Scientific Notation

Exercise 11.14

Light travels 3×10^8 metres in one second.
How far will light travel in one minute?

3×10^8 m in 1s
x m in 60s

$$\begin{aligned} x\times1 &= 3\times10^8\times60 \quad \text{\{cross multiply\}} \\ x &= 180\times10^8 \\ x &= \underline{1.8\times10^{10}\text{ metres}} \end{aligned}$$

See Technology 11 for calculator use.

Using a calculator:
$\boxed{3}\ \boxed{\text{EXP}}\ \boxed{8}\ \boxed{\times}\ \boxed{60}\ \boxed{=}$

1 Light travels 3×10^8 metres in one second.
- **a)** How far will light travel in one minute?
- **b)** How far will light travel in one hour?
- **c)** How far will light travel in one day?

2 A light year is used to describe distances in space.
A light year is the distance light will travel in one year.
Light travels 3×10^8 m/s (ie., 3×10^8 metres in one second).
How far does light travel in a light year?

3 The Moon is approximately 3.8×10^8 m from the Earth.
How long will it take to get from the Earth to the Moon at a speed of 1800 m/s (ie 1800 metres in 1 second)?

4 Alpha Centauri, our nearest star, is approximately 4.3 light years away from the Earth. How long will it take to a spacecraft to get from the Earth to Alpha Centauri at a speed of 100 000 km/h?

Light travels 3×10^8 metres in one second.
How long does it take light to travel 1 metre?

3×10^8 m in 1s
1 m in xs

$$\begin{aligned} x\times3\times10^8 &= 1\times1 \quad \text{\{cross multiply\}} \\ x &= \frac{1}{3\times10^8} \\ x &= 0.000\,000\,003\text{ metres} = \underline{3\times10^{-9}\text{ s}} \end{aligned}$$

Using a calculator:
$\boxed{1}\ \boxed{\div}\ \boxed{3}\ \boxed{\text{EXP}}\ \boxed{8}\ \boxed{=}$

5 Light travels 3×10^8 metres in one second.
How long does it take light to travel 100 metres?

6 Light travels 3×10^8 metres in one second.
How long does it take light to travel 1 km?

7 The circumference of the Earth is approximately 4×10^4 km and the Earth rotates once every 24 hours. What is the speed of the surface of the Earth in metres per second ($\text{speed} = \frac{\text{dis tan ce}}{\text{time}}$)?

Scientific Notation

Exercise 11.15

Find the circumference of the Earth
(Radius = 6.38×10^{6} m).

$C = 2\pi r$

$C = 2\pi\times6.38\times10^{6}$

$C = 40086722.26$ m

$C = 4\times10^{7}$ m

Using a calculator:
$\boxed{2}\ \boxed{\times}\ \boxed{\pi}\ \boxed{\times}\ \boxed{6.38}\ \boxed{\text{EXP}}\ \boxed{6}\ \boxed{=}$

1 Find the circumference of the Moon (Radius = 1.74×10^{6} m).

2 Find the circumference of the Sun (Radius = 6.95×10^{8} m).

3 Find the circumference of a classical electron orbit (Radius = 2.82×10^{-15} m).

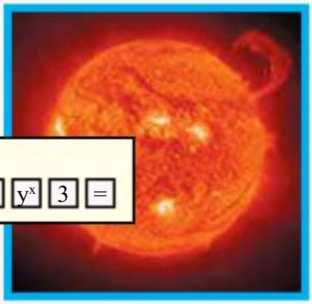
Find the volume of the Earth
(Radius = 6.38×10^{6} m).

$$V = \frac{4\pi r^3}{3}$$

$$V = \frac{4\pi(6.38\times10^{6})^3}{3}$$

$$V = 1.09\times10^{21}\ \text{m}^3$$

Using a calculator:
$\boxed{4}\ \boxed{\times}\ \boxed{\pi}\ \boxed{\times}\ \boxed{(}\ \boxed{6.38}\ \boxed{\text{EXP}}\ \boxed{6}\ \boxed{)}\ \boxed{y^x}\ \boxed{3}\ \boxed{=}$



4 Find the volume of the Moon (Radius = 1.74×10^{6} m).

5 Find the volume of Mars (Radius = 3.38×10^{6} m).

6 Find the volume of the Sun (Radius = 6.95×10^{8} m).

7 Are the Earth, Moon, and Mars composed of similar material?

a) Find the density of Earth (Mass = 5.975×10^{24} kg).

b) Find the density of Moon (Mass = 7.35×10^{22} kg).

c) Find the density of Mars (Mass = 6.42×10^{23} kg).

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

Our Universe

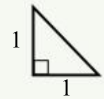
1.4×10^{10} years old?
1.3×10^{11} galaxies?
5×10^{22} stars?
9×10^{26} metres in width?

Mental Computation

Mental computation helps people prepare for problems in everyday life.

Exercise 11.16

1 Spell Scientific.
2 $10^3 \times 10^4$
3 Write in scientific notation: 570 000
4 Write in scientific notation: 0.000 49
5 What is the average of 2, 2, 2, 4, 6?
6 What is the median of 2, 2, 2, 4, 6?
7 What does SSS mean?
8 Two sides in a right-angled triangle are 1 and 1. Hypotenuse?
9 Simplify 15 : 9
10 The car travels 50 km in 40 s. At this speed, how long will it take the car to travel 150 km?



$h^2=1^2+1^2$
$h^2=1+1$
$h = \sqrt{2}$

Even if you fall on your face, you're still moving forward - Victor Kiam.

Exercise 11.17

1 Spell Notation.
2 $10^{-3} \times 10^4$
3 Write in scientific notation: 2 340 000
4 Write in scientific notation: 0.000 023
5 What is the average of 2, 2, 3, 4, 5?
6 What is the median of 2, 2, 3, 4, 5?
7 What does SAS mean?
8 Two sides in a right-angled triangle are 1 and 2. Hypotenuse?
9 Simplify 16 : 12
10 The car travels 50 km in 50 s. At this speed, how long will it take the car to travel 200 km?

Incoming fire has the right of way - Murphy's Laws of Combat.

Exercise 11.18

1 Spell Measurement.
2 $10^{-3} \div 10^4$
3 Write in scientific notation: 5 540 000 000
4 Write in scientific notation: 0.000 000 6
5 What is the average of 2, 2, 3, 3, 6?
6 What is the median of 2, 2, 3, 3, 6?
7 What does AAA mean?
8 Two sides in a right-angled triangle are 3 and 1. Hypotenuse?
9 Simplify 25 : 15
10 The car travels 60 km in 40 s. At this speed, how long will it take the car to travel 180 km?

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 11.19

1 Find the value of each of the following:

a) $(0.1)^2$

b) $(0.01)^2$

c) $(0.001)^3$

d) $(0.0001)^2$

e) $10\times10^2\times10^3$

f) $10\times10^2\times10^3\times10^4\times10^5$

g) $2\times2^2\times2^3\times2^4\times2^5$

h) $1 + 4/100 + 6/10000$

$(0.01)^2 = (10^{-2})^2 = 10^{-4}$
or $= \underline{0.0001}$ {4 decimal places}

2 Which is the largest?

a) $3\times3^2\times3^3$ **b)** $(3\times3^2)^3$ **c)** $3^2\times3\div3^{-2}$

3 Light travels 3×10^8 metres in one second.
How far will light travel in 1 minute?

What is the last digit in: $3^{12}\times4^{15}$?

$3^1 = 3$
$3^2 = 9$
$3^3 = 27$
$3^4 = 81$
$3^5 = 243$

Last digit pattern is: 3, 9, 7, 1, 3, 9, ...

Last digit in 3^{12} is 1

$4^1 = 4$
$4^2 = 16$
$4^3 = 64$
$4^4 = 246$

Last digit pattern is: 4, 6, 4, 6, 4, ...

Last digit in 4^{15} is 4

Last digit in $3^{12}\times4^{15}$ is $1\times4 = 4$

4 What is the last digit in: $3^{13}\times4^{16}$?

5 What is the last digit in: $2^{16}\times4^{43}$?

6 What is the last digit in: $5^{19}\times4^{13}$?

7 What is the last two digits in: 5^{555}?

8 Given that a and b can be any of 2, 3, 4, and 5,
what is the largest possible value of $(b-a)^{(b-a)}$?

Ship's Officers operate as an intermediary between the captain and the crew. They supervise the deck crew as they load and unload cargo.

- Relevant school subjects are English, Mathematics, Physics.
- Courses normally involve a cadetship with a shipping company.

Investigations

Investigation 11.1 1 million

What is the height, in metres, of a million sheets of paper 0.25 mm thick?

What is the height, in metres, of a million dollars worth of $50 notes if each $50 note is 0.25 mm thick?

What is the height, in metres, of a million dollars worth of $10 notes if each $10 note is 0.25 mm thick?

How far will a million steps take you?

Investigation 11.2 Micrometre

A micrometre is one millionth of a metre.

Investigate

Ways in which people can understand the size of a micrometre?

centimetre = 0.01 m
millimetre = 0.001 m
micrometre = 0.000 001 m
nanometre = 0.000 000 001 m
picometre = 0.000 000 000 001 m

Investigation 11.2 Scientific Notation

	Prefix
10^{18}	exa
10^{15}	peta
10^{12}	tera
10^{9}	giga
10^{6}	mega
10^{3}	kilo
10^{2}	hecto
10	deca
10^{-1}	deci
10^{-2}	centi
10^{-3}	milli
10^{-6}	micro
10^{-9}	nano
10^{-12}	pico
10^{-15}	femto
10^{-18}	atto

Investigate

Practical applications of **very large** numbers.

Investigate

Practical applications of **very small** numbers.

A Couple of Puzzles

Exercise 11.20

1 If you lived 1 billion seconds, how old would you be?

1 billion (common)
$= 1$ thousand million
$= 10^9$
$= 1\ 000\ 000\ 000$

1 billion (old meaning)
$= 1$ million million
$= 10^{12}$
$= 1\ 000\ 000\ 000\ 000$

2 Sal, Mal, and Al each belong to a different club. One is in the maths club. One is in the chess club. One is in the science club. The chess club member is the youngest. Mal is older than the maths club member. The maths club member's brother would like to join the maths club. Sal is an only child. Name the person in each club

	Maths	Chess	Science
Sal			
Mal			
Al			

A Game

Alphabet is played by two people with the 26 letters of the alphabet.
The winner is the person who crosses out Z.

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

Starting at A, take it in turns to cross out 1, 2, 3 or 4 letters. The letters must be in order.

Play a couple of games and try to determine a winning strategy.

A Sweet Trick

1 Ask your audience to choose a number from 1 to 10. $\quad 6$
2 Add 10. $\quad 6+10 = 16$
3 Double. $\quad 2\times16 = 32$
4 Subtract 4. $\quad 32-4 = 28$
5 Double. $\quad 2\times28 = 56$
6 Divide by 4. $\quad 56\div4 = 14$

Ask for their answer and subtract 8 to get their number.
$14 - 8 = 6$

An algebraic explanation:

$$= \frac{2(2(x+10)-4)}{4}$$

$$= \frac{2(2x+20-4)}{4}$$

$$= \frac{4x+32}{4}$$

$$= x + 8$$

Technology

Technology 11.1 Calculators and Indices

$2^3 \times 3^2 =$ $\boxed{2}\boxed{y^x}\boxed{3}\boxed{\times}\boxed{3}\boxed{y^x}\boxed{2}\boxed{=}$

$= 72$

The power button can be: $\boxed{y^x}$ or $\boxed{\wedge}$

Technology 11.2 Calculators and Scientific Notation

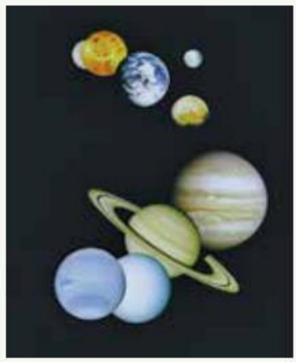
Light travels 3×10^8 metres in one second.
How far will light travel in one day?

3×10^8 m in 1s
x m in $24\times60\times60$ s {1 day = $24\times60\times60$ s }

$x\times1 = 3\times10^8\times24\times60\times60$ {cross multiply}

$\boxed{3}\boxed{\text{EXP}}\boxed{8}\boxed{\times}\boxed{24}\boxed{\times}\boxed{60}\boxed{\times}\boxed{60}\boxed{=}$

$x = 2.59\times10^{13}$
Light travels 2.59×10^{13} metres in one day.



a) How far will light travel in one minute?
b) How far will light travel in one hour?

Technology 11.3 Calculators and Scientific Notation

Find the volume of the Earth
(Radius = 6.38×10^6 m).

$$V = \frac{4\pi r^3}{3}$$

$$V = \frac{4\pi(6.38\times10^6)^3}{3}$$

$\boxed{4}\boxed{\times}\boxed{\pi}\boxed{\times}\boxed{(}\boxed{6.38}\boxed{\text{2nd}}\boxed{\text{EE}}\boxed{6}\boxed{)}\boxed{\wedge}\boxed{3}\boxed{\div}\boxed{3}\boxed{\text{Enter}}$

$V = 1.09\times10^{21}$ m^3
The volume of the Earth is 1.09×10^{21} cubic metres.

a) Find the volume of the Moon (Radius = 1.74x106 m).

b) Find the volume of Mars (Radius = 3.38x106 m).

c) Find the volume of the Sun (Radius = 6.95×10^8 m).

Chapter Review 1

Exercise 11.21

1 Write each of the following using a negative index:

a) $\frac{1}{10^3}$ **b)** $\frac{1}{10^4}$ **c)** $\frac{1}{a^7}$

2 Simplify and write the following in index form:

a) $10^{-3}\times10^5$ **b)** $10^{-2}\times10^4$ **c)** $10^3\div10^{-7}$

d) $10^{-4}\times10^{-3}$ **e)** $10^2\times10^{-4}\times10^6$ **f)** $x^4\times x^{-8}\times x^3$

g) $(10^{-2})^3$ **h)** $(2^{-3})^3$ **i)** $(5^2)^{-3}$

j) $(10^{-2})^{-3}$ **k)** $(a^2)^3$ **l)** 8×1^0

m) $5\times10^2\times3\times10^3$ **n)** $2\times10^6\times3\times10^{-4}$ **o)** $6\times10^{-4}\div(3\times10^{-6})$

p) $3\times10^{-4}\times5\times10^{-3}$ **q)** $x^6\times4x^{-3}\div2x^2$ **r)** $(x^{-2})^2\times8x^4\div2x^2$

3 Write in scientific notation:

a) 95 000 **b)** 520 000 **c)** 16 000 000

d) 0.000 01 **e)** 0.000 005 2 **f)** 0.005

g) 12 grams of carbon has 60 000 000 000 000 000 000 000 molecules.

h) The virus has a length of: 0.000 000 04 m.

4 Write as ordinary numbers:

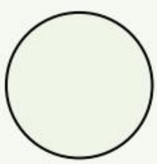
a) 10^2 **b)** 6.2×10^7 **c)** 9.51×10^8

d) 2×10^{-3} **e)** 7.42×10^{-4} **f)** 4.55×10^{-6}

g) The population of Indonesia is approximately 2.3×10^8.

h) Mass of an electron: 9×10^{-28} g

5 Find the circumference of Mars (Radius = 3.4×10^6 m).



Circumference
(Distance around the outside.)
$C=2\pi r$

6 Find the volume of Mars (Radius = 3.4×10^6 m).

Volume
(Space occupied by the sphere.)
$V=\frac{4\pi r^3}{3}$

7 What is the last digit in: $3^{13}\times4^{16}$?

Chapter Review 2

Exercise 11.22

1 Write each of the following using a negative index:

a) $\frac{1}{10^2}$ **b)** $\frac{1}{10^5}$ **c)** $\frac{1}{x^9}$

2 Simplify and write the following in index form:

a) $10^{-2}\times10^{3}$ **b)** $10^{-5}\times10^{3}$ **c)** $10^{6}\div10^{-2}$

d) $10^{-5}\times10^{-3}$ **e)** $10^{3}\times10^{-4}\times10^{5}$ **f)** $x^{5}\times x^{-7}\times x^{2}$

g) $(10^{-3})^{3}$ **h)** $(5^{-3})^{4}$ **i)** $(2^{2})^{-4}$

j) $(10^{-3})^{-3}$ **k)** $(y^{2})^{5}$ **l)** 9×1^{0}

m) $2\times10^{4}\times3\times10^{3}$ **n)** $5\times10^{7}\times3\times10^{-3}$ **o)** $4\times10^{-4}\div(2\times10^{-5})$

p) $2\times10^{-3}\times4\times10^{-3}$ **q)** $a^{6}\times4a^{-4}\div2a^{2}$ **r)** $(x^{-2})^{3}\times8x^{4}\div2x^{5}$

3 Write in scientific notation:

a) 430 000 **b)** 23 000 **c)** 200 000 000

d) 0.001 **e)** 0.000 072 **f)** 0.000 09

g) Jupiter has a diameter of 140 000 000 m..

h) The bacteria has a length of: 0.000 006 m.

4 Write as ordinary numbers:

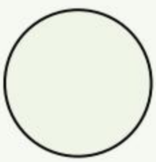
a) 10^{4} **b)** 5.5×10^{6} **c)** 8.01×10^{9}

d) 4×10^{-3} **e)** 1.42×10^{-6} **f)** 3.05×10^{-12}

g) The population of Australia is approximately 2.3×10^{7}.

h) The average size of a nucleus in an animal cell is 9×10^{-7} metres.

5 Find the circumference of Jupiter (Radius = 7.0×10^{7} m).



Circumference
(Distance around the outside.)
$C=2\pi r$

6 Find the volume of Jupiter (Radius = 7.0×10^{7} m).

Volume
(Space occupied by the sphere.)
$V=\frac{4\pi r^3}{3}$

7 What is the last digit in: $3^{16}\times4^{16}$?

Trigonometry 1 — 12

Measurement and Geometry ➡ Pythagoras and Trigonometry

★ Use similarity to investigate the constancy of the sine, cosine and tangent ratios for a given angle in right-angled triangles.
★ Develop an understanding of the relationship between the corresponding sides of similar right-angled triangles.
★ Apply trigonometry to solve right-angled triangle problems.
★ Understand the terms 'adjacent' and 'opposite' sides in a right-angled triangle.

And with a torch it even works at night.

A TASK

A sundial measures the time using the position of the sun. Use the Internet to find a sundial design (there are hundreds of different designs).

- Make your sundial.
- Test your sundial.
- Demonstrate your sundial to your class.

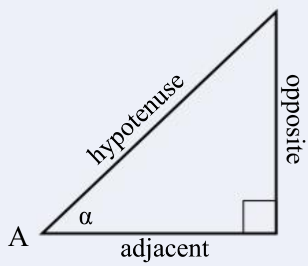
A LITTLE BIT OF HISTORY

The Babylonians, around 3 000 years ago, measured angles in degrees, minutes, and seconds.

Around 2 000 years ago, Indian astronomers developed trigonometry based on a sine function. The Indian sine function was the length of the opposite side for a given hypotenuse.

Muslim scientists had tables for sine and tangent that were extremely accurate (1 part in 700 million).

When calculus was invented, around 300 years ago, trigonometric functions became much more important in many more pure and applied mathematical applications.



$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

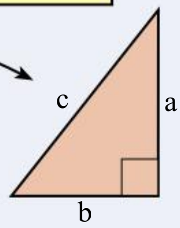
$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

Pythagoras' Theorem

$c^2 = a^2 + b^2$

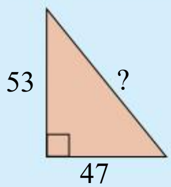
In any right-angled triangle:
The square on the hypotenuse is equal to the sum of the squares on the other two sides.

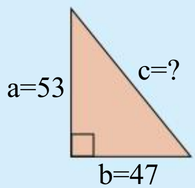


The hypotenuse is the longest side.
It is opposite the right-angle (90°).

Exercise 12.1
Find the length of the hypotenuse in each of the following:

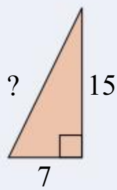
First add a, b, c





$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 53^2 + 47^2 \\ c^2 &= 5018 \\ c &= \sqrt{5018} \\ c &= 70.84 \end{aligned}$$

1



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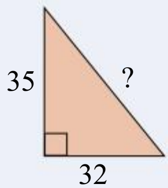

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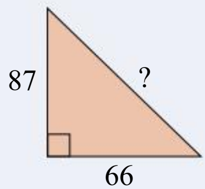

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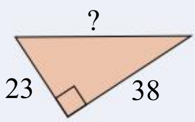

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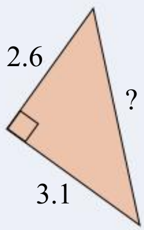

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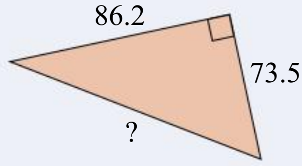
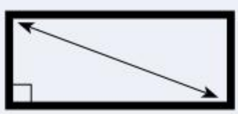
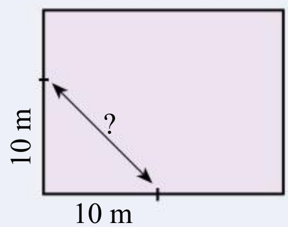


7 A 3m wide by 1.4 m rectangular gate needs a diagonal brace to keep it rigid.
What should be the length of the diagonal?



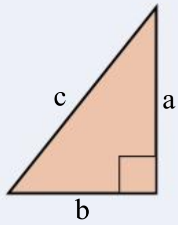
8 A builder checks the right-angle of a slab corner by making marks 10 m out from each corner.
How far apart should the marks be?



Pythagoras' Theorem

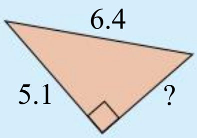
In any right-angled triangle:
The square on the hypotenuse is equal to the sum of the squares on the other two sides.

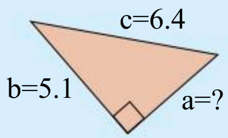
$a^2 + b^2 = c^2$



Exercise 12.2
Find the length of the unknown in each of the following:

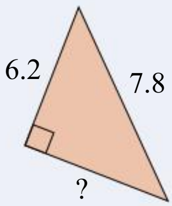
First add a, b, c





$$\begin{aligned} a^2 + b^2 &= c^2 \\ a^2 + 5.1^2 &= 6.4^2 \\ a^2 &= 6.4^2 - 5.1^2 \\ a^2 &= 14.95 \\ a &= \sqrt{14.95} \\ \underline{a} &\underline{= 3.87} \end{aligned}$$

1



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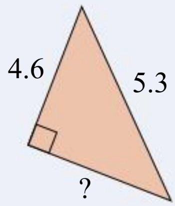
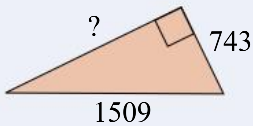


3

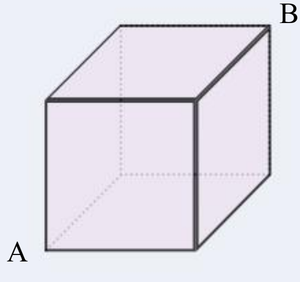


Rounding to two decimal places, first look at the third decimal place:

56.231694 ↑ less than 5 thus **56.23**	27.01769 ↑ 5 or more thus **27.02**	1.07276 ↑ less than 5 thus **1.07**	4.79634216 ↑ 5 or more thus **4.80**

4 A 45 m tower is supported by guy wires.
The guy wires are attached to the top of the tower and anchored to the ground out from the tower.
If the guy wires are 150 m long, how far out from the tower are they anchored?

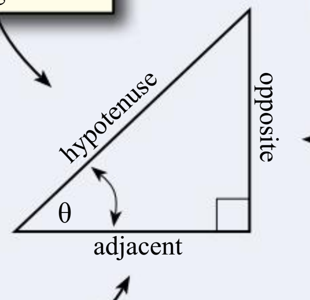
5 What is the distance from A to B on the 10 cm cube?



Naming Sides

The **hypotenuse** is the longest side.
It is opposite the right-angle.

Opposite is 'opposite' the angle.



Adjacent means 'next to' the angle.

Small letters from the Greek alphabet are sometimes used to represent angles.

Alpha	α
Beta	β
Theta	θ
Phi	φ
Lamda	λ

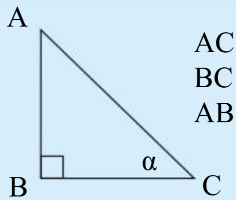
Exercise 12.3

For each of the following triangles, name

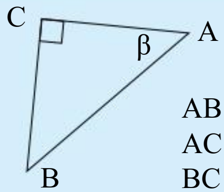
a) the hypotenuse.

b) the side adjacent to the angle.

c) the side opposite the angle.

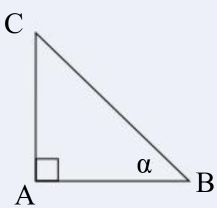


AC is the hypotenuse.
BC is adjacent.
AB is opposite.



AB is the hypotenuse.
AC is adjacent.
BC is opposite.

1



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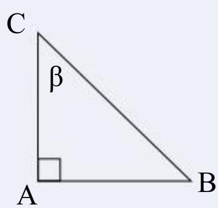

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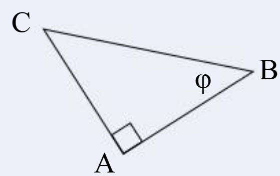

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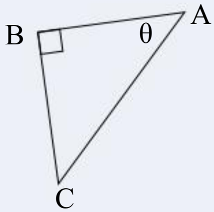

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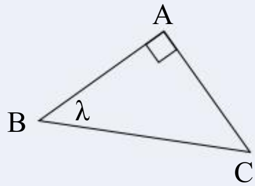

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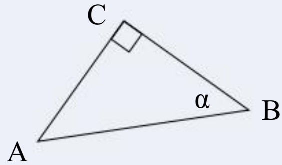
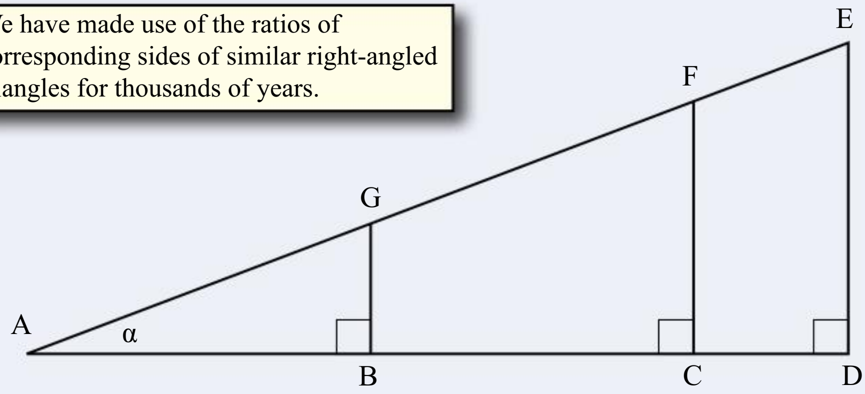


Trigonometry

We have made use of the ratios of corresponding sides of similar right-angled triangles for thousands of years.



You will need a ruler to measure each line.

Exercise 12.4

1 Copy and complete the following table:

Triangle	**Opposite**	**Adjacent**	$\frac{\text{opposite}}{\text{adjacent}}$
△ABG	1.7 cm	4.6 cm	1.7 ÷ 4.6 = **0.37**
△ACF			
△ADE			

$\tan A = \frac{\text{opposite}}{\text{adjacent}}$

α = 20°
Use your calculator:
tan 20 =

2 Complete the following table:

Triangle	**Opposite**	**Hypotenuse**	$\frac{\text{opposite}}{\text{hypotenuse}}$
△ABG	1.7 cm	4.9 cm	1.7÷4.9 = **0.35**
△ACF			
△ADE			

$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$

α = 20°
Use your calculator:
sin 20 =

3 Complete the following table:

Triangle	**Adjacent**	**Hypotenuse**	$\frac{\text{adjacent}}{\text{hypotenuse}}$
△ABG	4.6 cm	4.9 cm	4.6÷4.9 = **0.94**
△ACF			
△ADE			

$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$

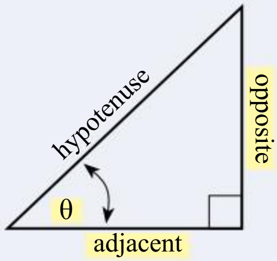
α = 20°
Use your calculator:
cos 20 =

The Tan Ratio

Trigonometry was developed thousands of years ago to solve the many problems in surveying, engineering, architecture, astronomy, etc, etc, etc.

Trigonometry n. branch of mathematics dealing with the relationships between angles and sides of triangles.

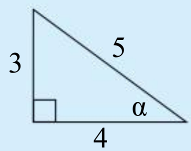
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



Exercise 12.5
Find tan α and the size of the angle α.

3 is opposite α

4 is adjacent to α



$$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\alpha = \frac{3}{4} = 0.75$$

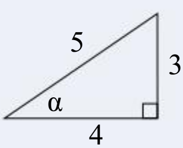
$\alpha = \tan^{-1} 0.75$
$\alpha = 36.9°$

Use your calculator:
2ndF | $\tan^{-1}$ | 0.75 | =

$\tan^{-1}$ means 'an angle whose tan is'.
Thus $\tan^{-1}$ 0.75 means 'an angle whose tan is 0.75' (which is 36.9°).

Make sure your calculator is on **degrees**.

1



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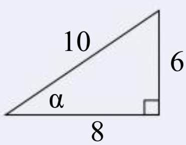

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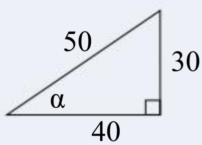

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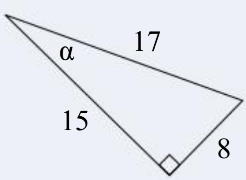

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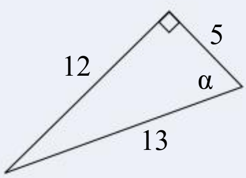

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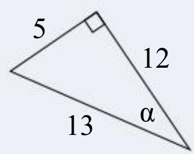


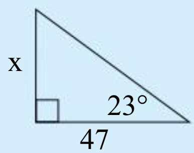
The Tan Ratio

Trigonometry can be used to find a side after knowing a side and an angle in a right-angled triangle.

Trigonometry is used millions and millions of times every day.

Exercise 12.6

Find x in each of the following right-angled triangles:



$$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 23 = \frac{x}{47}$$

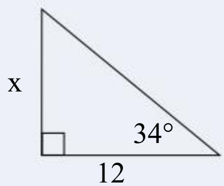
$\tan 23 \times 47 = x$ {inverse of ÷ is ×}

$\underline{19.95 = x}$

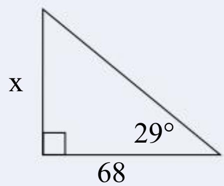
Make sure your calculator is in degrees (deg).

$\boxed{\text{tan}}\ \boxed{23}\ \boxed{\times}\ \boxed{47}\ \boxed{=}$

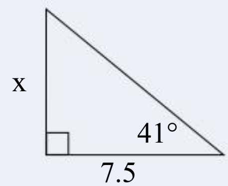
1



2



3



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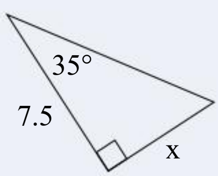

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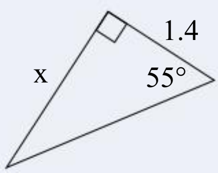

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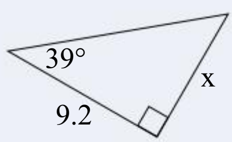
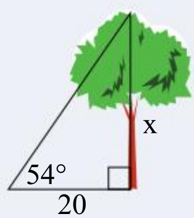
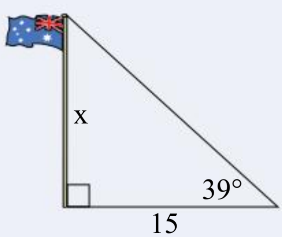


7 20 m out from the base of a tree, a clinometer measures the angle of elevation to the top of the tree as 54°. Find the height of the tree.



8 15 m out from the base of a flagpole, a clinometer measures the angle of elevation to the top of the flagpole as 39°. Find the height of the flagpole.



The Tan Ratio

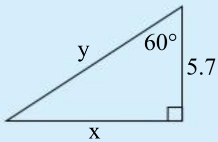
Trigonometry can be used to find a side after knowing a side and an angle in a right-angled triangle.

Pythagoras' Theorem can be used to find the third side after knowing two sides in a right-angled triangle.

In any right-angled triangle: The square on the hypotenuse is equal to the sum of the squares on the other two sides.

Exercise 12.7

Find the unknown sides:

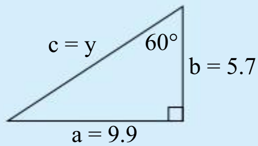


$$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 60 = \frac{x}{5.7}$$

$$\tan 60 \times 5.7 = x$$

$$\underline{9.87 = x}$$



$$c^2 = a^2 + b^2$$

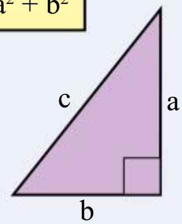
$$y^2 = 9.9^2 + 5.7^2$$

$$y^2 = 130.5$$

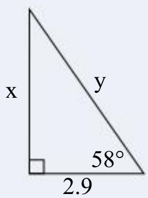
$$y = \sqrt{130.5}$$

$$\underline{y = 11.40}$$

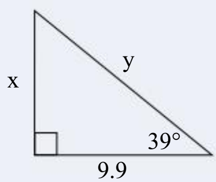
$c^2 = a^2 + b^2$



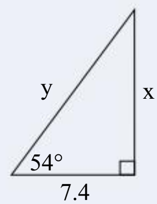
1



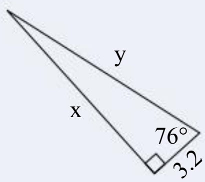
2



3



4



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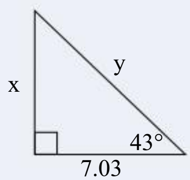

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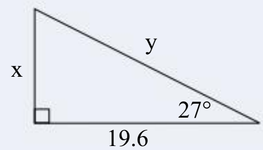


7 A student with a clinometer, is lying on the ground 4.6 m out from the base of a flagpole. If the clinometer reads 45°, what is the height of the flagpole?

8 6.2 m out from the base of a tree, a clinometer measures the angle of elevation to the top of the tree as 34°. Find the height of the tree.

9 4.3 m out from the base of a building, a clinometer measures the angle of elevation to the top of the building as 45°. Find the height of the building.

10 The angle of elevation of the top of a tower from a point 37 m out from the base of the tower is 53°. Find the height of the tower correct to one decimal place.

The Tan Ratio

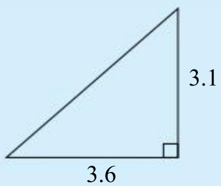
The angles in a triangle sum to 180°

Trigonometry can be used to find an **angle** after knowing two sides in a right-angled triangle.

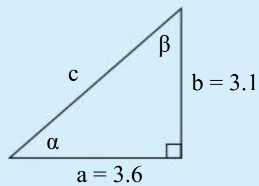
Pythagoras' Theorem can be used to find the third side after knowing two sides in a right-angled triangle.

Solve means 'find **all** unknowns'.
A triangle has **3 sides** and **3 angles**.

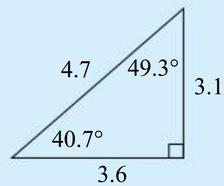
Exercise 12.8
Solve the following triangles:



$$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan\alpha = \frac{3.1}{3.6}$$
$$\alpha = \tan^{-1}(3.1 \div 3.6)$$
$$\underline{\alpha = 40.7°}$$

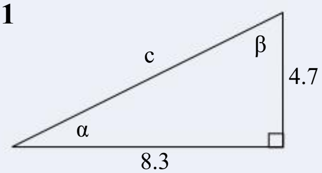


$$c^2 = a^2 + b^2$$
$$c^2 = 3.6^2 + 3.1^2$$
$$c^2 = 22.57$$
$$c = \sqrt{22.57}$$
$$\underline{c = 4.75}$$



sum of angles = 180°
$$\alpha + \beta + 90 = 180$$
$$40.7 + \beta + 90 = 180$$
$$\beta = 180 - 40.7 - 90$$
$$\underline{\beta = 49.3°}$$

1



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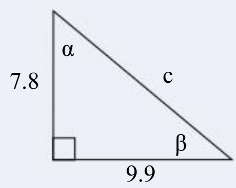

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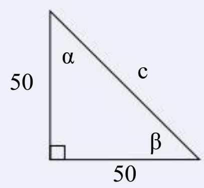

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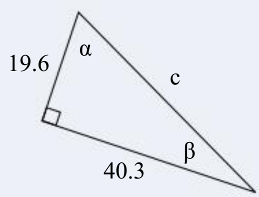

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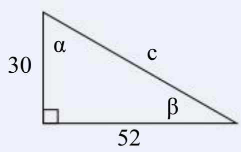

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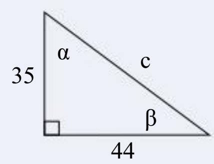


$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

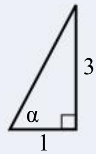
The **tangent ratio** is one of several ratios involving the relationships between the sides and angles of triangles.
Sin and cos are in Chapter 17.

Mental Computation

Mental computation gives you practice in thinking.

Exercise 12.9

1 Spell Trigonometry.
2 What is the tan ratio?
3 In the triangle, what is tanα?



4 If one angle in a right-angled triangle is 30°, what is the third angle?
5 Two sides in a right-angled triangle are 1 and 3. Hypotenuse?

$c^2=1^2+3^2$
$c^2=1+9$
$c=\sqrt{10}$

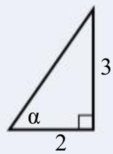
6 Write in scientific notation: 54 000
7 Write in scientific notation: 0.003 2
8 $10^6 \div 10^4$
9 What is the average of 2, 2, 3, 4, 5?
10 16×25

16×25
= 4×4×25
= 4×100
= 400

If you can dream it, you can do it - Walt Disney

Exercise 12.10

1 Spell Tangent.
2 What is the tan ratio?
3 In the triangle, what is tanα?

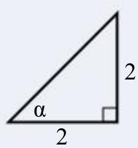


4 If one angle in a right-angled triangle is 60°, what is the third angle?
5 Two sides in a right-angled triangle are 2 and 3. Hypotenuse?
6 Write in scientific notation: 170 000
7 Write in scientific notation: 0.000 14
8 $10^6 \div 10^3$
9 What is the average of 1, 2, 3, 4, 5?
10 20×25

All of us could take a lesson from the weather. It pays no attention to criticism..

Exercise 12.11

1 Spell Pythagoras.
2 What is the tan ratio?
3 In the triangle, what is tanα?



4 If one angle in a right-angled triangle is 40°, what is the third angle?
5 Two sides in a right-angled triangle are 2 and 2. Hypotenuse?
6 Write in scientific notation: 3 000 000
7 Write in scientific notation: 0.000 000 9
8 $10^9 \div 10^6$
9 What is the average of 2, 3, 3, 4?
10 24×25

Conveyancers compile the documentation needed for the sale and purchase of real estate.

- Relevant school subjects are English and Mathematics.
- Courses usually involve a diploma or business degree.

Competition Questions

$\sqrt{0.0009} = 0.03$

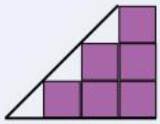
Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 12.12

1. What is the square root of 400?
2. What is the square root of 4?
3. What is the square root of 0.04?
4. What is the square root of 0.0004?
5. What is the gradient of the ramp, the thick line, in each of the following?

gradient=slope=tangent ratio.
Assume each block is square.

a)



b)

c)

6. One angle in a right-angled triangle is 37°, what is the size of the other two angles?
7. Two sides of a triangle are 6 cm and 3 cm. Can the third be 10 cm?
8. Two sides of a triangle are 6 cm and 3 cm. Can the third be 2 cm?

A right-angled isosceles triangle has an area of 18.
What is the length of the hypotenuse?

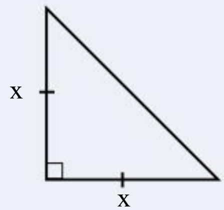
$$\text{area} = \frac{\text{base} \times \text{height}}{2}$$

$$18 = \frac{x^2}{2}$$

$$36 = x^2$$

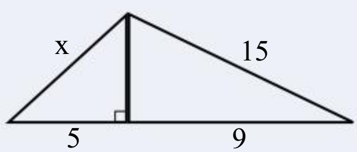
$$6 = x$$

$$c^2 = a^2 + b^2$$
$$c^2 = 6^2 + 6^2$$
$$c^2 = 72$$
$$c = \sqrt{72}$$
$$c = \sqrt{(36 \times 2)}$$
$$c = 6\sqrt{2}$$



An isosceles triangle has two equal sides.

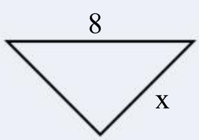
9. A right-angled isosceles triangle has an area of 50. What is the length of the hypotenuse?
10. A right-angled isosceles triangle has an area of 32. What is the length of the hypotenuse?
11. Find the value of x in the following diagram:



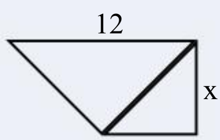
An isosceles triangle has two equal angles of 45° opposite the equal sides.

12. All angles in the following diagrams are either 45° or 90°. Find x.

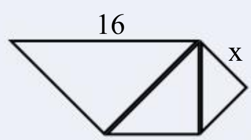
a)



b)



c)



A Couple of Puzzles

Exercise 12.13

1 Complete the following:

$1 + 3 + 5 =$

$1 + 3 + 5 + 7 =$

$1 + 3 + 5 + 7 + 9 =$

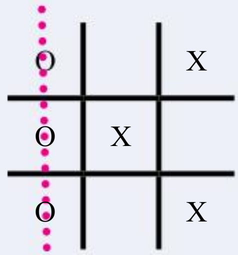
$1 + 3 + 5 + 7 + 9 + 11 =$

$1 + 3 + 5 + 7 + 9 + 11 + 13 =$

$1 + 3 + 5 + 7 + ... + 97 + 99 =$

A Game

Diox is a two player game based on naughts and crosses. The winner is the first person to have three Os or three Xs in a row, column, or diagonal as in the original game of naughts and crosses.



Players take turns throwing a die.

- an even number means the player must place an X
- an odd number means the player must place an O

A Sweet Trick

The Mobius strip

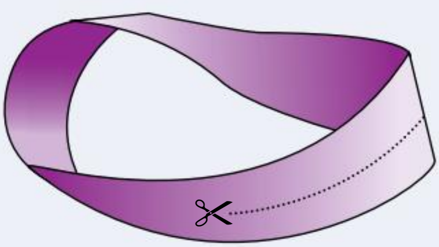
1 Obtain a long strip of paper that is about 5 cm wide.

2 Make a loop with a half twist and tape the two ends together.

3 Ask your audience what they would expect if you cut the strip of paper in half along the middle of the strip of paper.

4 Cut along the middle of the loop and produce a larger loop and not two loops as would be expected.

- What happens if you cut along the middle of the larger loop again?
- What happens if you cut a loop with a full twist?



Why did the chicken cross the Mobius strip? To get to the same side.

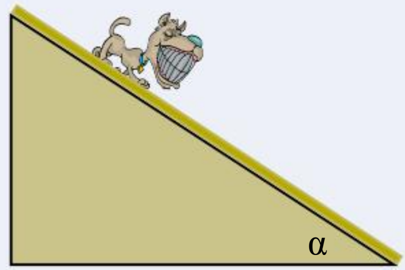
The Mobius strip is the source of a number of puzzles based on the half twist making the inside surface and the outside surface the same.

Try it by drawing a line along the outside that is also the inside.

Investigations

Investigation 12.1 Slope

The tan ratio is used to measure slope or gradient.



1 What is the angle at which an object begins to slip down the slope (Use the tan ratio to calculate the angle)?

2 Compare this angle with other objects.

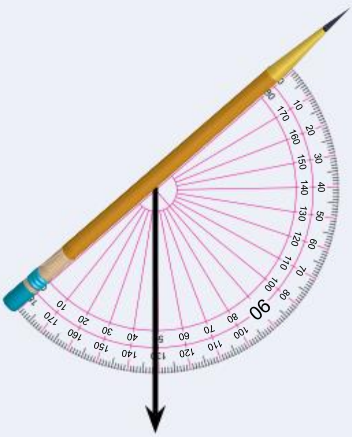
3 Why the difference?

Investigation 12.2 Natural Slope?

Investigate

The angle of natural slopes.

Investigation 12.3 Find Heights

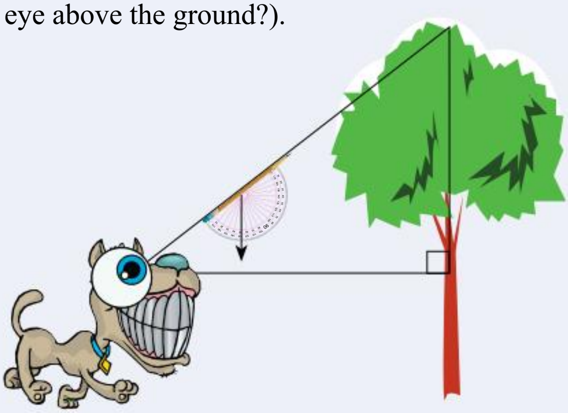


1 Make a clinometer using a straw, a protractor, a small weight on the end of a string, and sticky tape.

2 Measure a distance out from the base of a tree or flagpole.

3 Aim the clinometer at the top of the tree and measure the angle of elevation.

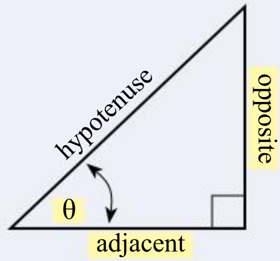
4 Use the tan ratio to calculate the height of the tree (What about the height of your eye above the ground?).



Technology

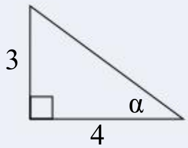
Use a spreadsheet to solve the previous exercises.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



Technology 12.1 The Tan Ratio and the Spreadsheet

a) Given the opposite and adjacent, find the angle.

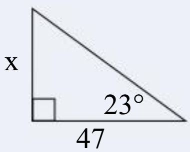


	a	b	c	d
1	Opposite	Adjacent	Tan α	α
2	3	4	0.75	36.87

Enter the formula:
=atan(c2)*180/pi()

The *180/pi() is needed to convert radians to degrees.

b) Given the angle and adjacent, find the opposite.



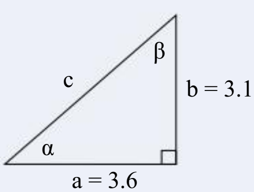
	a	b	c
1	Angle	Adjacent	Opposite
2	23	47	19.95

Enter the formula:
=tan(a2*pi()/180)*b2

The *pi()/180 is needed to convert degrees to radians.

a) Given the opposite and adjacent, solve the triangle.

	a	b	c	d	e
1	Opposite	Adjacent	α	β	Hypotenuse
2	3.1	3.6	40.73	49.27	4.75



Enter the formula:
=atan(a2/b2)*180/pi()

The *180/pi() is needed to convert radians to degrees.

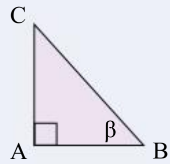
Enter the formula:
=180−90−c2

Enter the formula:
=sqrt(a2*a2+b2*b2)

Chapter Review 1

Exercise 12.14

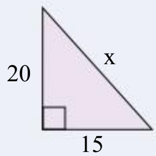
1 For the adjacent triangle, name:



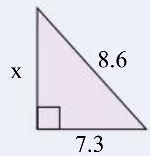
a) the hypotenuse.
b) the side adjacent to the angle.
c) the side opposite the angle.

2 Use Pythagoras' Theorem to find the unknown:

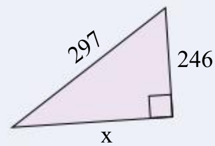
a)



b)

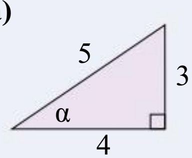


c)

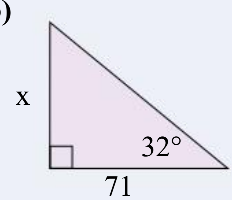


3 Find the unknown in each of the following triangles:

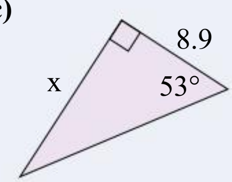
a)



b)



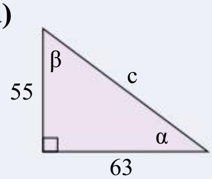
c)



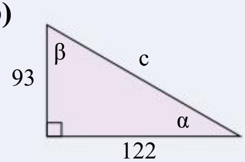
Solve means 'find **all** unknowns'.

4 Solve the following triangles:

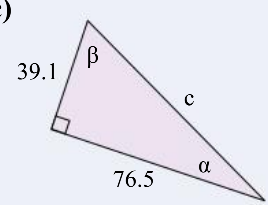
a)



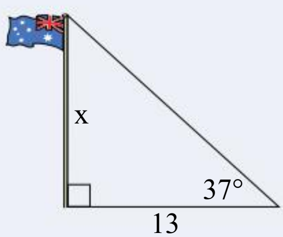
b)



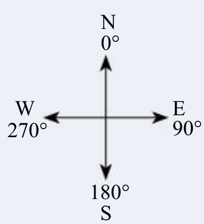
c)



5 13 m out from the base of a flagpole, a clinometer measures the angle of elevation to the top of the flagpole as 37°. Find the height of the flagpole.



6 A ship sails due north for 15 km, then on a bearing of 160° until the ship is due east of its starting point. How far is the ship from its starting point?



Chapter Review 2

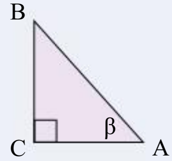
Exercise 12.15

1 For the adjacent triangle, name:

a) the hypotenuse.

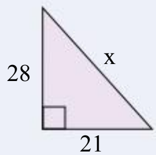
b) the side adjacent to the angle.

c) the side opposite the angle.

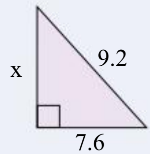


2 Use Pythagoras' Theorem to find the unknown:

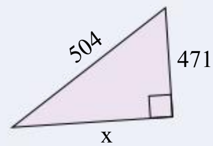
a)



b)

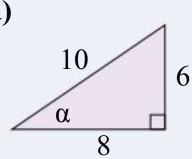


c)

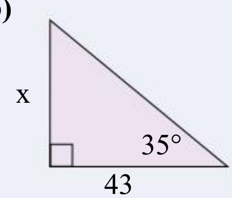


3 Find the unknown in each of the following triangles:

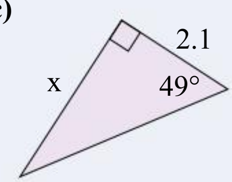
a)



b)



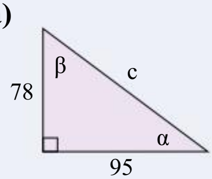
c)



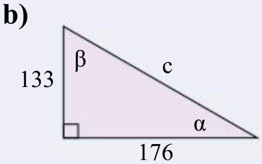
Solve means 'find **all** unknowns'.

4 Solve the following triangles:

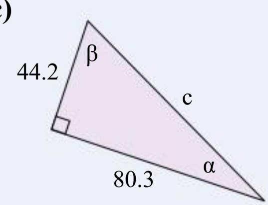
a)



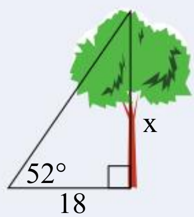
b)



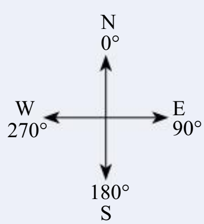
c)



5 18 m out from the base of a tree, a clinometer measures the angle of elevation to the top of the tree as 52°. Find the height of the tree.



6 A ship sails due East for 60 km, then on a bearing of 225° until the ship is due south of its starting point. How far is the ship from its starting point?



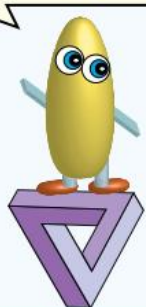
Volume

13

Measurement and Geometry ➡ Using units of measurement

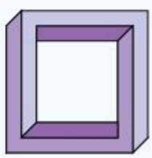
- ★ Calculate the volume of cylinders and solve related problems.
- ★ Solve problems involving the volume of right prisms.
- ★ Build on the understanding of volume to become fluent with calculation, and identify that volume relationships are used in the workplace and everyday life.

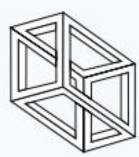
Paradox, n. 1. a statement appearing to be absurd but containing a truth. 2. a thing showing a contradiction.



A TASK

- Research visual/optical paradoxes/illusions.
- Make a poster for the classroom.





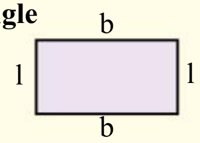
A LITTLE BIT OF HISTORY

Euler (1707-1783), a famous mathematician, has written that artists are quite skilled at using visual illusions in their work. Euler suggested that the whole of the art of painting is based on visual illusions.

Ptolemy (90-168), an influential Greek astronomer famous for developing theory from experimental data - the scientific method, was intrigued by the increasing size of celestial bodies as they approach the horizon - The magnificent size of a full moon as it arises in the east.

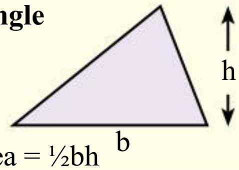
Area Warmup

Rectangle



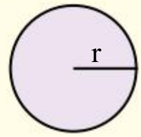
Area = l×b

Triangle



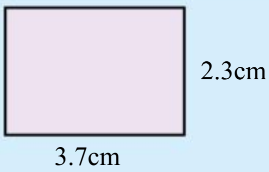
Area = ½bh

Circle

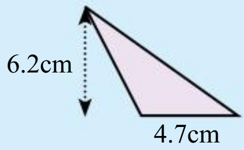


Area = πr^2

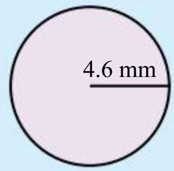
Exercise 13.1
Calculate the area of each of the following shapes:



Area = l×b
= 3.7cm × 2.3cm
= 8.51 cm²

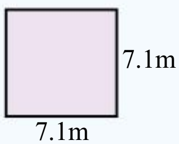


Area = ½bh
= 0.5×4.7cm×6.2cm
= 14.57 cm²



Area = πr^2
= π×4.6mm×4.6mm
= 66.48 mm²

1



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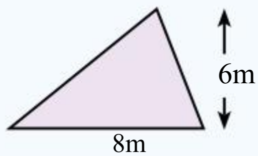

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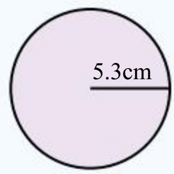

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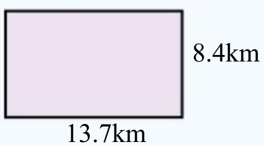
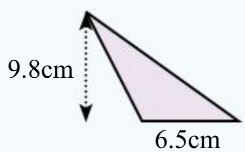
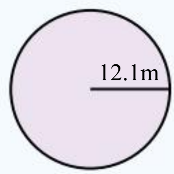


5



6



7 A rectangular paddock is 120 m by 140 m. What is the area of the paddock in square metres and hectares (1 hectare = 10 000m²)?

8 A paddock, in the shape of a triangle, has a base of 230 m and a perpendicular height of 370 m. What is the area of the paddock in square metres and hectares?

9 The sprinkler sprays water in a circle with a radius of 4.5 m. Calculate the area covered by the sprinkler.

> A hectare is the area of a square 100 m by 100 m.

10 The pizza dish has a diameter of 29 cm. Calculate the area of the pizza dish.

> The diameter is twice the radius.

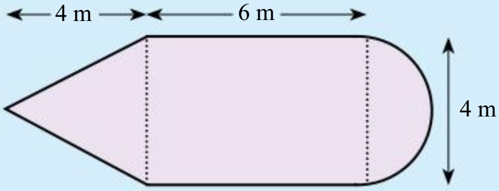
11 A classroom has a length of 6 m and a width of 10 m. The floor of the classroom is to be covered with carpet tiles. If each carpet tile has a length of 30 cm and a width of 45 cm, how many carpet tiles are needed to cover the floor?

Composite Shapes

Composite shapes can be rectangles, triangles, and circles composed together.

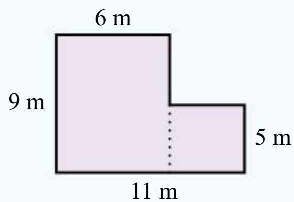
Exercise 13.2

Calculate the shaded area of each of the following composite shapes:



Area	= Area of triangle	+ Area of rectangle	+ Area of circle
	= ½bh	+ l×b	+ πr^2
	= ½×4×4	+ 6×4	+ $\pi \times 2^2$
	= 8	+ 24	+ 12.57
Area	= 44.57 m^2		

1



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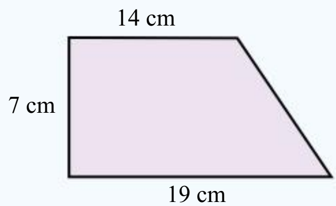
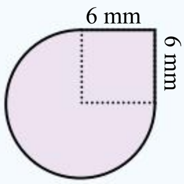


3



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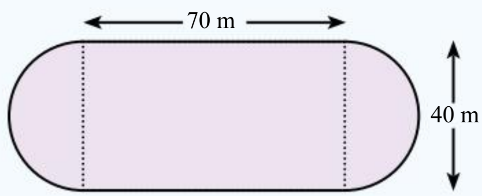
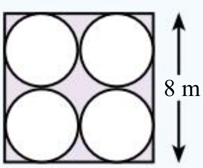
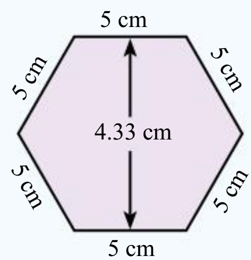


5



6



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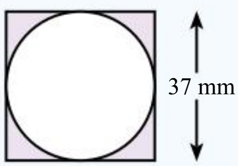

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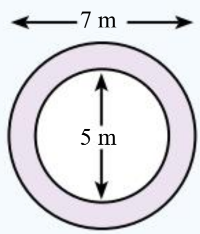


Prisms

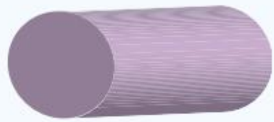
A **prism** has the same cross-section along its length.

The volume is the space occupied by the prism.

If the cross-section is a triangle, it is a triangular-based prism

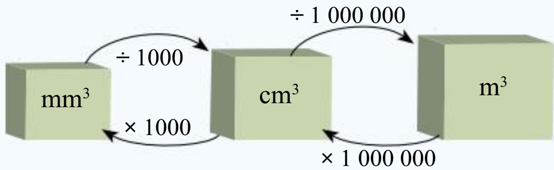
If the cross-section is a rectangle, it is a rectangular-based prism

If the cross-section is a circle, it is a cylinder.



Units of Volume

$1\text{cm}^3 = 1000\text{mm}^3$
$1\text{m}^3 = 1\ 000\ 000\text{cm}^3$

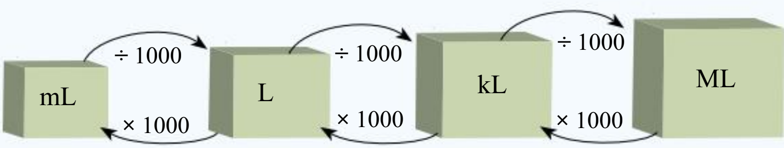


Exercise 13.3

1 Complete the following unit conversions:

8500 mm³ to cm³ $8500\text{ mm}^3 = 8500 \div 1000\text{ cm}^3$ $= \underline{8.5\text{ cm}^3}$	2.3 m³ to cm³ $2.3\text{ m}^3 = 2.3 \times 1\ 000\ 000\text{ cm}^3$ $= \underline{2\ 300\ 000\text{ cm}^3}$

a) 6700 mm³ to cm³ **b)** 900 mm³ to cm³
c) 35 000 000 cm³ to m³ **d)** 2 400 000 cm³ to m³
e) 4.9 m³ to cm³ **f)** 1.9 m³ to cm³
g) 8.3 cm³ to mm³ **h)** 0.6 cm³ to mm³



1L = 1000mL
1kL = 1000L
1ML = 1000kL

MegaLitre

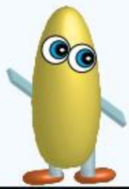
2 Complete the following unit conversions:

6500 mL to L $6500\text{ mL} = 6500 \div 1000\text{ L}$ $= \underline{6.5\text{ L}}$	2.8 ML to L $2.8\text{ ML} = 2.8 \times 1\ 000\ 000\text{ L}$ $= \underline{2\ 800\ 000\text{ L}}$

a) 4200 mL to L **b)** 61.3 ML to L
c) 5800 L to kL **d)** 720 000 L to ML
e) 7.3 kL to L **f)** 0.61 kL to L

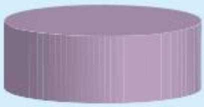
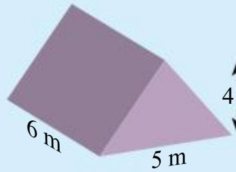
Volume of Prisms

Prisms are three-dimensional shapes that have a constant cross-section.

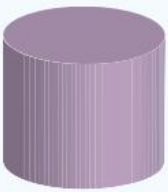


Exercise 13.4

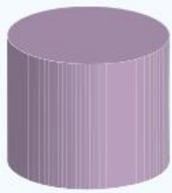
Find the volume of each of the following prisms:

		
V = Area of base × height $= \pi r^2 \times h$ $= \pi \times 3^2 \times 2$ $V = 56.55 \text{ cm}^3$	V = Area of base × height $= l \times b \times h$ $= 4 \times 2 \times 9$ $V = 72 \text{ m}^3$	V = Area of base × height $= ½bh \times h$ $= ½ \times 5 \times 4 \times 6$ $V = 60 \text{ m}^3$

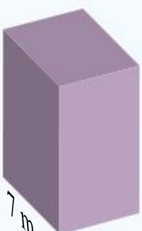
1



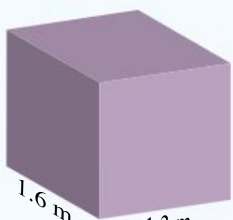
2



3



4



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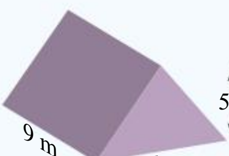

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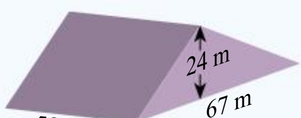
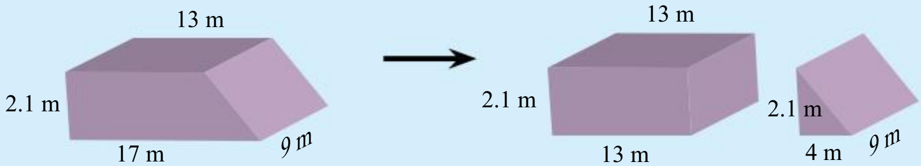


Composite Solids

Exercise 13.5

Calculate the volume of each of the following solids:



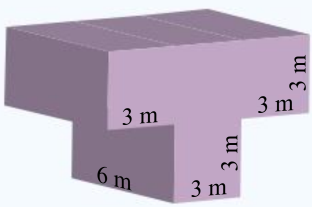
Volume $=$ Volume rectangular prism $+$ Volume triangular based prism

$= \quad l \times b \times h \quad + \quad \frac{1}{2}bh \times h$

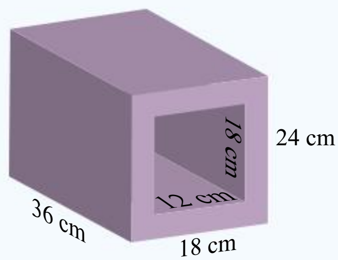
$= \quad 13 \times 9 \times 2.1 \quad + \quad \frac{1}{2} \times 4 \times 2.1 \times 9$

Volume $= 283.5\ m^3$

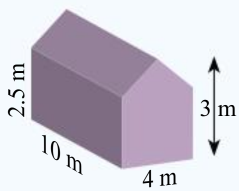
1



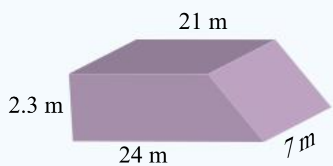
2



3

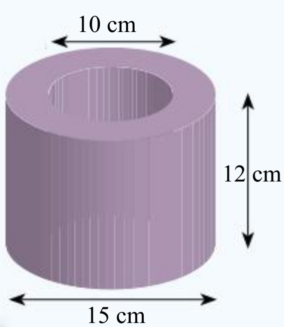


4



5

6



Volume, n. 1. amount of space contained by a three-dimensional object.
SI unit is cubic metre (m^3).

1 L = 1000 mL
1L = 1000cm^3 of water
1L = 1 kg

Practical Applications

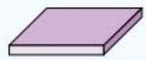
Exercise 13.6

What is the cost of concrete for the shed slab?
The slab is 7.2 m by 2.4 m by 15 cm. Concrete delivered is $180 per cubic metre.

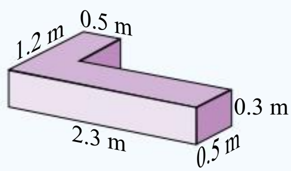
V = Area of base × height
  = l × b × h
  = 7.2 × 2.4 × 0.15   {15 cm = 0.15 m}
V = 2.59 m³ Thus need 3 cubic metres of concrete

Cost = 3×$180 = $540. The concrete for the slab will cost $540

1 What is the cost of concrete for the shed slab?
The slab is 6.8 m by 3.4 m by 18 cm.
Concrete delivered is $205 per cubic metre.

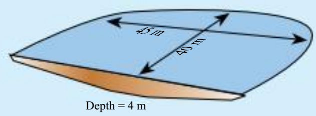


2 What is the cost of concrete for the garden feature?
Concrete delivered is $210 per cubic metre.



Estimate the volume of the dam.

0.4 is a correction for the slope of the sides of the dam.



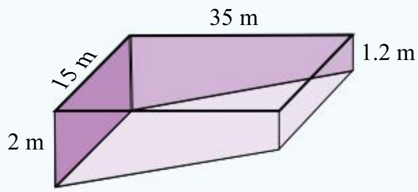
V = 0.4×Area of surface×depth
  = 0.4 × l × b × d
  = 0.4 × 45 × 40 × 4
V = 2880 m³ = 2 880 000 litres
The volume is estimated at 2.9 megalitres.

$1 m^3 = 1000$ L

3 Estimate the volume of a dam whose surface approximates a rectangle 67 m by 38 m and with a depth of 5 m (use a correction factor of 0.4 for the slope of the sides of the dam)?

4 The water tank has a diameter of 3.3 m and a height of 2.2 m
(Some of the tank is in the ground).
How much water will the tank hold?

5 How many litres of water is needed to fill the swimming pool?

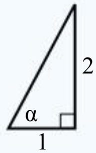


Mental Computation

You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 13.7

1 Spell Cylinder.
2 Area of a circle formula?
3 Volume of a prism?
4 How many litres in a megalitre?
5 In the triangle, what is tanα?



6 Two sides in a right-angled triangle are 1 and 2. Hypotenuse?

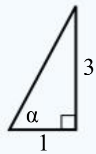
$c^2=1^2+2^2$
$c^2=1+4$
$c = \sqrt{5}$

7 Write in scientific notation: 5 000
8 Write in scientific notation: 0.000 2
9 $10^7 \div 10^4$
10 23×9

23×9
$= 23 \times (10-1)$
$= 230-23$
$= 207$

Exercise 13.8

1 Spell Rectangular based Prism.
2 Area of a circle formula?
3 Volume of a prism?
4 What is the weight of 1 L of water?
5 In the triangle, what is tanα?

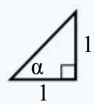


6 Two sides in a right-angled triangle are 1 and 3. Hypotenuse?
7 Write in scientific notation: 80 000
8 Write in scientific notation: 0.000 03
9 $10^7 \times 10^4$
10 34×9

"The volume of a prism is area of base by height," said Jess with some capacity.

Exercise 13.9

1 Spell Triangular based Prism.
2 Area of a circle formula?
3 Volume of a prism?
4 How many litres in a kilolitre?
5 In the triangle, what is tanα?



6 Two sides in a right-angled triangle are 1 and 1. Hypotenuse?
7 Write in scientific notation: 9 000 000
8 Write in scientific notation: 0.000 000 1
9 $10^5 \div 10^7$
10 46×9

Turn your wounds into wisdom - Oprah Winfrey.

Competition Questions

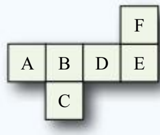
Build maths muscle and prepare for mathematics competitions at the same time.

Exercise 13.10

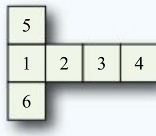
1 60 centicubes are glued together to form a rectangular based prism. If the area of the base is 20 cm^2, what is the height of the prism?

2 60 centicubes are glued together to form a rectangular based prism. If the perimeter of the base is 14 cm, what is the height of the prism?

3 If the net is folded to form a cube, which letter is opposite C?



4 The net is folded to form a cube, If the three numbers at each corner are added, what is the largest sum?



5 A box measures 20 cm by 30 cm by 40 cm. What is the volume, in litres, of the box?

6 A box of baking soda measures 3 cm by 5 cm by 7 cm. (1 teaspoon = 5 cm^3). How many teaspoons of baking soda are expected to be in the box?

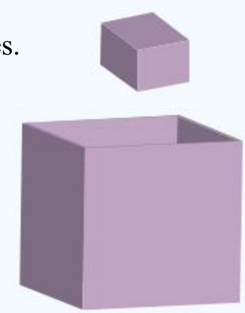
25 mm of rain on a flat roof puts 2 500 L of water into the tank. If all of the rain on the roof goes into the tank, what is the area of the roof?

$$\begin{aligned}
\text{Volume} &= 2\,500 \text{ L} \\
\text{Area of roof} \times \text{depth} &= 2\,500 \text{ x } 1000 \text{ cm}^2 \\
\text{Area of roof} \times 2.5 \text{ cm} &= 2\,500\,000 \text{ cm}^2 \\
\text{Area of roof} &= 2\,500\,000 \div 2.5 \text{ cm}^2 \\
&= 1\,000\,000 \text{ cm}^2 \\
&= \underline{100 \text{ m}^2}
\end{aligned}$$

An olympic pool holds about 1 megalitre of water.

7 20 mm of rain on a flat roof puts 2 000 L of water into the tank. If all of the rain on the roof goes into the tank, what is the area of the roof?

8 A 4 metre square-based tank has water to a depth of 4 metres. If a cube of side 2 metres is placed in the tank, what is then the level of water in the tank?

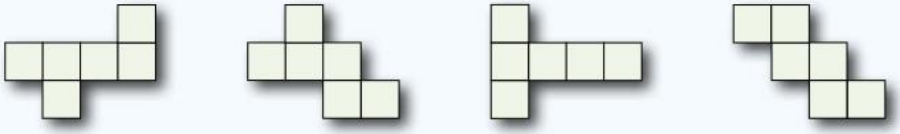


9 A 6 metre square-based tank has water to a depth of 4 metres. If a cube of side 2 metres is placed in the tank, what is then the level of water in the tank?

Investigations

Investigation 13.1 Nets of Cubes

Which of the following nets will fold to form a cube?
Make copies and try them.



Investigate

How many different nets can be folded to make a cube?

Investigation 13.2 How big is 1 cubic centimetre (cm^3)?

To become familiar with the volume of a cubic centimetre, make a 1 cm by 1 cm by 1 cm box or use centicubes.
Use the cubic centimetre to estimate volumes in your classroom:

- The volume of a calculator.
- the volume of a pencil case.
- etc.

How close were your estimates to the actual volumes?

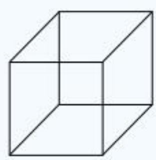
If you have centicubes, use the centicubes to make a 5×5×5 box and show that the volume is 125 cm^3.

Investigation 13.3 How big is 1 cubic metre (m^3)?

To become familiar with the space of a cubic metre, make a 1 m by 1 m by 1 m frame
Use the cubic metre to estimate volumes in your classroom:

- The volume of a desk.
- The volume of the classroom.

How close were your estimates to the actual volumes?



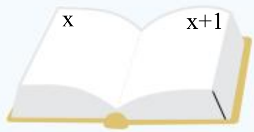
A cubic metre of water weights 1 tonne (1000L).

Investigation 13.4 The Soma cube

Investigate

The Soma cube

A Couple of Puzzles



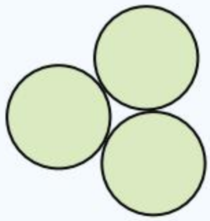
Exercise 13.11

1 When a book is open the sum of the two pages is 15.
What is the number of the next page.

$$x+x+1=15$$

2 A drum is full. When 40 L is taken out of the drum,
the drum is one-third full. How much will the drum hold?

3 How many coins, of the same size,
are needed to 'ring' a coin?



A Game

Dart maths.
The first to get a total of exactly 301 is the winner
(two to five players).

1 When it is your turn, throw a dice three times.

2 Then try to use ( ), +, –, ×, or ÷ with your
three numbers to progress towards the
exact total of 301.

$$5^{(2+1)} = 125$$

A Sweet Trick

1 Ask your audience to draw a 3×3 box
around any 9 numbers on a calendar

2 You tell them the sum is 144

3 They take ages to check the sum
on their calculator.

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

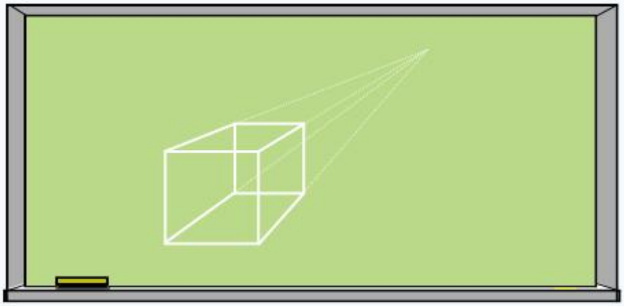
You just multiply the centre
number by 9. 16×9 = 144.

One way to multiply by 9:
Multiply by 10 then subtract
the number.
16×9 = 16×10−16 = 144

Technology

Technology 13.1 Drawing cubes

- ☞ Use a search phrase such as "drawing cubes" or "drawing cubes perspective" to find online tutorials for drawing cubes.
- ☞ When you feel that you are good at drawing cubes, learn to draw cylinders.
- ☞ Now try your hand at drawing triangular based prisms and other prisms..



Technology 13.2 Drawing Prisms

- ☞ Interactive geometry software or dynamic geometry environments are about making and manipulating geometric objects.
- ☞ Use a search phrase such as "interactive geometry software" to find a list of such software.
- ☞ Experiment with one or two of them.

Technology 13.3 Applets

- ☞ Use a search phrase such as 'volume applet' or 'interactive volume' to find one of the many applets on the Internet. Experiment with them.
- ☞ Use a search phrase such as 'volume calculator' to find one of the many volume calculators on the Internet. Experiment with them.
- ☞ Use a search phrase such as "java soma cube" to find an applet that lets you experiment with the Soma cube.

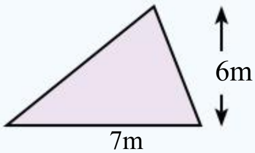
Aerospace Engineers design, develop, manufacture, and maintain aeroplanes, helicopters, spacecraft, missiles, etc.
- Relevant school subjects are English, Mathematics, Physics, and Chemistry.
- Courses generally involves a University engineering degree.

Chapter Review 1

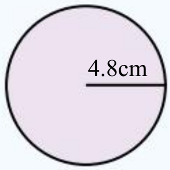
Exercise 13.12

1 Calculate the area of each of the following shapes:

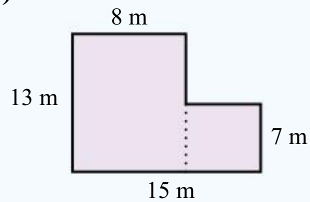
a)



b)



c)



2 Make the following unit conversions:

a) 7200 mm^3 to cm^3 **b)** 58 000 000 cm^3 to m^3

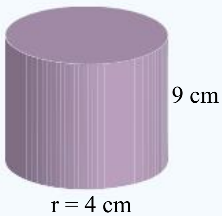
c) 2.7 m^3 to cm^3 **d)** 0.9 cm^3 to mm^3

e) 4600 mL to L **f)** 410 000 L to kL

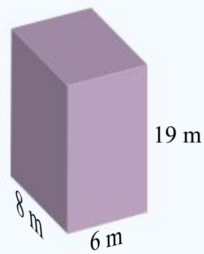
g) 1.1 kL to L **h)** 0.25 L to mL

3 Calculate the volume of each of the following solids:

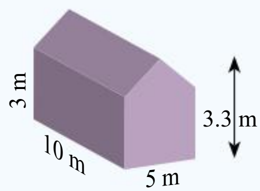
a)



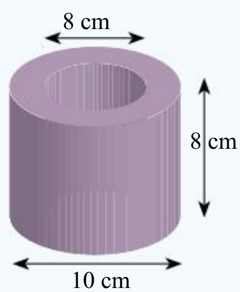
b)



c)

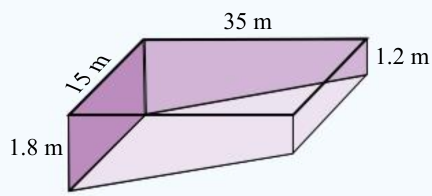


d)



4 What is the cost of concrete for the shed slab?
The slab is 6.4 m by 3.8 m by 15 cm.
Concrete delivered is $190 per cubic metre

5 How many litres of water is needed to fill the swimming pool?

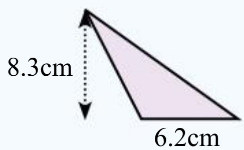


Chapter Review 2

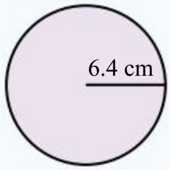
Exercise 13.13

1 Calculate the area of each of the following shapes:

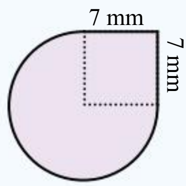
a)



b)



c)



2 Make the following unit conversions:

a) 92 000 mm^3 to cm^3 **b)** 9 800 000 cm^3 to m^3

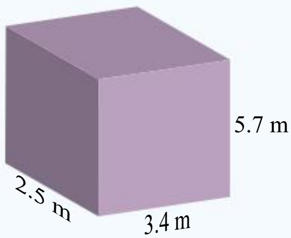
c) 9.8 m^3 to cm^3 **d)** 0.6 cm^3 to mm^3

e) 5600 mL to L **f)** 350 000 L to kL

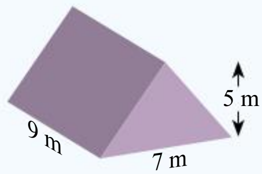
g) 4.1 kL to L **h)** 0.49 L to mL

3 Calculate the volume of each of the following solids:

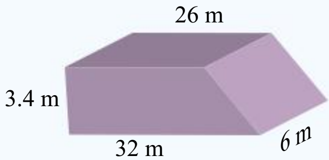
a)



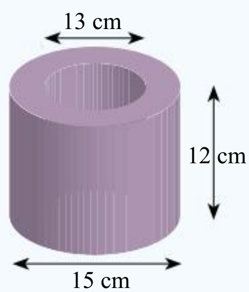
b)



c)

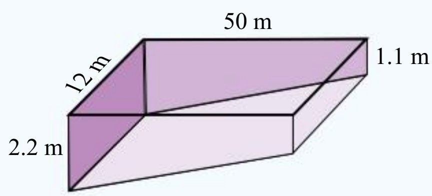


d)



4 What is the cost of concrete for the shed slab?
The slab is 9.2 m by 3.1 m by 21 cm.
Concrete delivered is $180 per cubic metre

5 How many litres of water is needed to fill the swimming pool?



Probability 14

Statistics and Probability ➞ Chance

- ★ List all outcomes for two-step chance experiments, both with and without replacement using tree diagrams or arrays.
- ★ Assign probabilities to outcomes and determine probabilities for events.
- ★ Calculate relative frequencies from given or collected data to estimate probabilities of events involving 'and' or 'or'.
- ★ Posing 'and', 'or', 'not' and 'given' probability questions about objects or people.
- ★ Collect data to answer the questions using Venn diagrams or two-way tables.

A TASK

Is it OK for a family to spend \$50 each week playing Gold Lotto?

Organise a class debate:
Two teams, three speakers per team, debate the topic. A toss of a coin usually decides which team, Affirmative, agrees with the topic, and which team, Negative, disagrees with the topic.
Four minutes is normally given to each speaker with a warning bell at the end of three minutes and two bells at four minutes. The teams speak in the following order:

1 First Affirmative **2** First Negative	Define the topic. Introduce the team's main argument. Rebut the opening main argument 'why it isn't true.'
3 Second Affirmative **4** Second Negative	Further support of team's argument. Some rebuttal. Rebuttal of affirmative's argument.
5 Third Affirmative **6** Third Negative	Rebuttal of opposition. Summary of team's argument. Convincing concluding statement.

A LITTLE BIT OF HISTORY

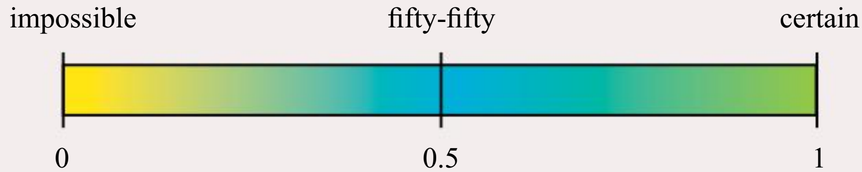
Sir Francis Bacon (1561-1626), the father of deductive reasoning, was the first to use inductive thinking as a basis for scientific procedure.

Thomas Bayes (1702-1761) provided the first mathematical basis for inductive reasoning and developed Bayesian probability theory.


Sir Francis Bacon

Warm Up

Probability is the chance of something happening.



Exercise 14.1

1 Copy the above probability scale and add each of the following to the scale.

a) The day after Friday will be Saturday.
b) We have Buckley's chance of winning the match.
c) Throw a coin and it will show a tail.
d) You will use a computer today.
e) We have a 3 in 4 chance of picking the winner.
f) The probability of winning first prize is $\frac{1}{8000000}$ or 0.000 000 125.

$$\text{Probability} = \frac{\text{Number of possible outcomes}}{\text{Total number of outcomes}}$$

A standard 6-sided die is thrown. What is the probability of each of the following happening?
a) 5 **b)** odd **c)** <3

Sample space = {1,2,3,4,5,6}

a) $P(5) = \underline{\frac{1}{6}}$ {there is one 5}

b) $P(\text{odd}) = \frac{3}{6} = \underline{\frac{1}{2}}$ {1,3,5 are odd}

c) $P(<3) = \frac{2}{6} = \underline{\frac{1}{3}}$ {1,2 are <3}

Eight balls are numbered 1 to 8. What is the probability of selecting a ball that shows:
a) 2? **b)** even? **c)** >6?

Sample space = {1,2,3,4,5,6,7,8}

a) $P(2) = \underline{\frac{1}{8}}$ {there is one 2}

b) $P(\text{even}) = \frac{4}{8} = \underline{\frac{1}{2}}$ {2,4,6,8 are even}

c) $P(>6) = \frac{2}{8} = \underline{\frac{1}{4}}$ {7,8 are >6}

2 A six-sided die is thrown. What is the probability of each of the following?

a) 3 **b)** even **c)** odd
d) not even **e)** <3 **f)** not <3
g) >6 **h)** a number divisible by 3

3 A bag contains five blue marbles and three red marbles.
What is the probability of taking a marble from the bag that is:

a) red? **b)** blue? **c)** not red?
d) not blue? **e)** blue or red? **f)** white?

Theoretical Probability

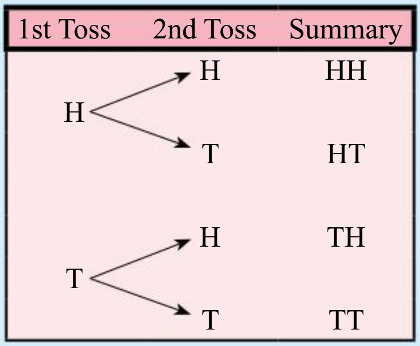
$$\text{Probability} = \frac{\text{Number of possible outcomes}}{\text{Total number of outcomes}}$$

Tree diagrams and two-way tables are useful when calculating the theoretical probability of multi-outcome and compound events.

Exercise 14.2

Assuming that the chance of a head or tail is equal, use a tree diagram and two-way table to determine the following theoretical probabilities for tossing two coins:

a) P(2 heads) **b)** P(1 head & 1 tail) **c)** P(2 tails)



Coin 1

Coin 2	H	T
H	HH	HT
T	TH	TT

P(HH) $= \frac{1}{4} = \underline{0.25}$

P(HandT) $= \frac{2}{4} = \underline{0.5}$

P(TT) $= \frac{1}{4} = \underline{0.25}$

1 Assuming that the chances of a head or tail is equal, use a tree diagram and two-way table to determine the following theoretical probabilities for the tossing of two coins:

a) P(2 heads) **b)** P(1 head & 1 tail) **c)** P(2 tails)

2 Assuming that the chances of a girl or boy being born is equal, use a tree diagram and a two-way table to determine the following theoretical probabilities for a family of two children:

a) P(2 girls) **b)** P(1 girl & 1 boy) **c)** P(2 boys)

3 Assuming that the chances of a girl or boy being born is equal, use a tree diagram to determine the following theoretical probabilities for a family of three children:

a) P(3 girls) **b)** P(2 girls & 1 boy)
c) P(1 girl & 2 boys) **d)** P(3 boys)

4 Assuming that the chances of a head or tail is equal, use a tree diagram to determine the following theoretical probabilities for the tossing of four coins:

a) P(4 heads) **b)** P(3 heads & 1 tail) **c)** P(2heads & 2tails)
d) P(1 head & 3 tails) **e)** P(4 tails)

Experimental Probability

$$\text{Experimental probability} = \frac{\text{frequency of event}}{\text{total frequency}}$$

Experiments are useful in indicating the chance of something happening.

Exercise 14.3

A die was tossed 30 times:

Frequency of 1s = 4
Frequency of 2s = 6
Frequency of 3s = 4
Frequency of 4s = 4
Frequency of 5s = 5
Frequency of 6s = 7
TOTAL FREQUENCY= 30

$P(1) = \dfrac{\text{frequency of 1}}{\text{total frequency}} = \dfrac{4}{30} = 0.13$

$P(2) = \dfrac{\text{frequency of 2}}{\text{total frequency}} = \dfrac{6}{30} = 0.20$

$P(3) = \dfrac{\text{frequency of 3}}{\text{total frequency}} = \dfrac{4}{30} = 0.13$

$P(4) = \dfrac{\text{frequency of 4}}{\text{total frequency}} = \dfrac{4}{30} = 0.13$

$P(5) = \dfrac{\text{frequency of 5}}{\text{total frequency}} = \dfrac{5}{30} = 0.17$

$P(6) = \dfrac{\text{frequency of 6}}{\text{total frequency}} = \dfrac{7}{30} = 0.23$

1 Investigate the experimental probability of tossing a die. Toss a die 60 times.

Experimental probability changes from trial to trial.

	60 throws	600 throws
P(1)	$= \dfrac{\quad}{60} = 0.$	$= \dfrac{\quad}{600} = 0.$
P(2)	$= \dfrac{\quad}{60} = 0.$	$= \dfrac{\quad}{600} = 0.$
P(3)	$= \dfrac{\quad}{60} = 0.$	$= \dfrac{\quad}{600} = 0.$
P(4)	$= \dfrac{\quad}{60} = 0.$	$= \dfrac{\quad}{600} = 0.$
P(5)	$= \dfrac{\quad}{60} = 0.$	$= \dfrac{\quad}{600} = 0.$
P(6)	$= \dfrac{\quad}{60} = 0.$	$= \dfrac{\quad}{600} = 0.$

Theoretical probability of tossing a die.

$S = \{1,2,3,4,5,6\}$

$P(1) = \dfrac{1}{6} = 0.17 \qquad P(2) = \dfrac{1}{6} = 0.17$

$P(3) = \dfrac{1}{6} = 0.17 \qquad P(4) = \dfrac{1}{6} = 0.17$

$P(5) = \dfrac{1}{6} = 0.17 \qquad P(6) = \dfrac{1}{6} = 0.17$

2 Increase the number of trials from 60 to 600, using a table similar to this table:

Technology 14.1 may be useful.

3 Combine your results with other members of your class. As the number of trials increase, does the experimental probability get closer to the theoretical probability?

Experimental Probability

$$\text{Experimental probability} = \frac{\text{frequency of event}}{\text{total frequency}}$$

If the experiment is fair then with a large number of trials, the experimental probability and the theoretical probability should be similar.

Exercise 14.4

Two dice were tossed and the total noted.
This was repeated 100 times.

Total	**Frequency**	**Experimental Probability**
2	3	P(2) = 3/100 = 0.03
3	7	P(3) = 7/100 = 0.07
4	11	P(4) = 11/100 = 0.11
5	10	P(5) = 10/100 = 0.10
6	13	P(6) = 13/100 = 0.13
7	20	P(7) = 20/100 = 0.20
8	6	P(8) = 6/100 = 0.06
9	16	P(9) = 16/100 = 0.16
10	6	P(10) = 6/100 = 0.06
11	7	P(11) = 7/100 = 0.07
12	1	P(12) = 1/100 = 0.01
Total =	100	

Most people don't believe that the chances of a total of 7 are so high.
Actually 6 in 36 or about 17 in 100. Why?

Technology 14.2 may be useful.

1 Investigate the experimental probability of the totals obtained by tossing two six-sided dice 100 times.
Use a procedure similar to the above example.

2 Combine your results with other members of your class.
As the number of trials increase, does the experimental probability get closer to the theoretical probability?

"One die, two dice," explained Jack, taking a chance.

Theoretical probability of totals when tossing two dice.

S = {2,3,4,5,6,7,8,9,10,11,12}

$P(2) = \frac{1}{36} = 0.03$ $\quad P(3) = \frac{2}{36} = 0.06$ $\quad P(4) = \frac{3}{36} = 0.08$ $\quad P(5) = \frac{4}{36} = 0.11$

$P(6) = \frac{5}{36} = 0.14$ $\quad P(7) = \frac{6}{36} = 0.17$ $\quad P(8) = \frac{5}{36} = 0.14$ $\quad P(9) = \frac{4}{36} = 0.11$

$P(10) = \frac{3}{36} = 0.08$ $\quad P(11) = \frac{2}{36} = 0.06$ $\quad P(12) = \frac{1}{36} = 0.03$

Theoretical Probability

$$\text{Probability} = \frac{\text{Number of possible outcomes}}{\text{Total number of outcomes}}$$

Tree diagrams and two-way tables are useful when calculating the theoretical probability of multi-outcome and compound events.

Exercise 14.5

Use a table to determine the following theoretical probabilities of the totals when tossing two dice:

		Dice 1					
		1	2	3	4	5	6
Dice 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

a) $\text{P(total of 7)} = \frac{\text{No. total of 7}}{\text{Total}} = \frac{6}{36} = \frac{1}{6} = 0.17$

b) $\text{P(total of 12)} = \frac{\text{No. total of 12}}{\text{Total}} = \frac{1}{36} = 0.03$

c) $\text{P(total} > 10) = \frac{\text{No. total} > 10}{\text{Total}} = \frac{3}{36} = \frac{1}{12} = 0.08$

1 Use a table to determine the following theoretical probabilities when tossing two dice.

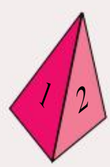
a) P(total of 2)	**b)** P(total of 3)	**c)** P(total of 4)
d) P(total of 5)	**e)** P(total of 6)	**f)** P(total of 7)
g) P(total of 8)	**h)** P(total of 9)	**i)** P(total of 10)
j) P(total of 11)	**k)** P(total of 12)	**l)** P(total of 1)
m) P(total > 10)	**n)** P(total > 9)	**o)** P(total > 7)
p) P(total < 3)	**q)** P(total < 5)	**r)** P(total < 7)
s) P(total is even)	**t)** P(total is odd)	**u)** P(total divisible by 3)

2 Use a table to determine the following theoretical probabilities when tossing a six-sided die and a coin.

a) P(a 6 and a H)	**b)** P(a 6 and a T)
c) P(a 4 and H)	**d)** P(even and T)
e) P(odd and H)	**f)** P(even)

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

3 Use a table to determine the following theoretical probabilities when tossing a six-sided die, with faces numbered 1 to 6, and a four sided-die numbered 1 to 4.



a) P(total of 1)	**b)** P(total of 2)	**c)** P(total of 3)
d) P(total of 4)	**e)** P(total of 5)	**f)** P(total of 6)
g) P(total of 7)	**h)** P(total of 8)	**i)** P(total of 9)
j) P(total of 10)	**k)** P(total of 11)	**l)** P(total of 12)
m) P(total < 5)	**n)** P(total > 7)	**o)** P(total is odd)

Theoretical Probability

$$\text{Probability} = \frac{\text{Number of possible outcomes}}{\text{Total number of outcomes}}$$

Exercise 14.6

Determine the following theoretical probabilities when tossing a six-sided die and a coin (Use a tree diagram or two-way table to list all possible outcomes).

a) P(a 3 and a H) **b)** P(a 3 or a H)

Sample space →

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

1,H	2,H	3,H	4,H	5,H	6,H
1,T	2,T	3,T	4,T	5,T	6,T

1,H	2,H	3,H	4,H	5,H	6,H
1,T	2,T	3,T	4,T	5,T	6,T

P(a 3 and a H) = $\frac{1}{12}$ = 0.08

P(a 3 or a H) = $\frac{7}{12}$ = 0.58

AND means the intersection.
The 3 **and** the Head.

OR means add together.
The 3s **together** with the Heads.

1 Determine the following theoretical probabilities when tossing a six-sided die and a coin (Use a tree diagram or two-way table to list all possible outcomes).

a) P(a 5 and a H) **b)** P(a 5 or a H)

c) P(a 1 and a T) **d)** P(a 1 or a T)

1,H	2,H	3,H	4,H	5,H	6,H
1,T	2,T	3,T	4,T	5,T	6,T

1,H	2,H	3,H	4,H	5,H	6,H
1,T	2,T	3,T	4,T	5,T	6,T

e) P(even and a H)
{The evens **and** the heads}

f) P(even or a H)
{The evens **together** with the heads}

g) P(odd and a T)
{The odds **and** the Tails}

h) P(odd or a T)
{The odds **together** with the tails}

2 Determine the following theoretical probabilities when tossing an eight-sided die and a coin (Use a tree diagram or two-way table to list all possible outcomes).

a) P(a 7 and a T) **b)** P(a 7 or a T)

c) P(a 3 and a H) **d)** P(a 3 or a H)

e) P(even and a T) **f)** P(even or a T)

g) P(odd and a H) **h)** P(odd or a H)

Theoretical Probability

$$\text{Probability} = \frac{\text{Number of possible outcomes}}{\text{Total number of outcomes}}$$

OR means add together.
AND means the intersection.

Showing all outcomes with a two-way table or tree diagram makes problem-solving easier.

Exercise 14.7

Use a table to determine the following theoretical probabilities of the totals when tossing two dice:

a) P(5 or odd)

Mark the 5s **together** with the odds

Dice 2 \ Dice 1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$\text{P(5 or odd)} = \frac{\text{no of 5s or odds}}{\text{total}} = \frac{18}{36} = \underline{0.5}$$

b) P(<7 and even)

Mark the <7 **and** even

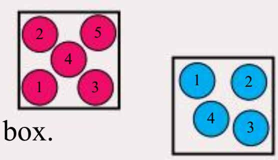
Dice 2 \ Dice 1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$\text{P}(< 7 \text{ and even}) = \frac{\text{no of} < 7 \text{ and even}}{\text{total}} = \frac{9}{36} = \underline{0.25}$$

1 Determine the following theoretical probabilities of the **totals** when tossing two dice.

a) P(7 or odd) **b)** P(7 and odd) **c)** P(8 or even)
d) P(8 and even) **e)** P(<4 or odd) **f)** P(<4 and even)
g) P(>8 or even) **h)** P(>8 and odd) **i)** P(odd or divisible by 3)
j) P(odd or even) **k)** P(odd and even)

2 Box A has 5 marbles numbered from 1 to 5.
Box B has 4 marbles numbered from 1 to 4.
Determine the following theoretical probabilities of the **totals** when selecting one marble from each box.



a) P(a total of 4) **b)** P(a total of 5)
c) P(an odd total) **d)** P(an even total)
e) P(3 and odd) **f)** P(3 or odd)
g) P(<3 and even) **h)** P(<3 or even)
i) P(odd and divisible by 3) **j)** P(odd or divisible by 3)

Web Designers design, create, and maintain web pages for a large range of clients.
- Relevant school subjects are Mathematics and English.
- Courses range from Certificate to Bachelor degrees.

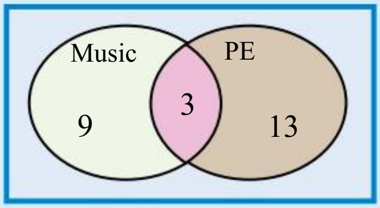
Venn Diagrams

Venn diagrams are a good way of representing probability.

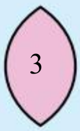
Exercise 14.8

In a class of 25 students, 12 students study music, and 16 students study PE.
Find the probability that a student:

a) studies music and PE.
b) studies music or PE.
c) does not study music
d) studies PE given the student studies music.

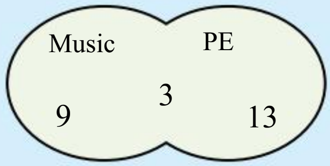


a) music **and** PE



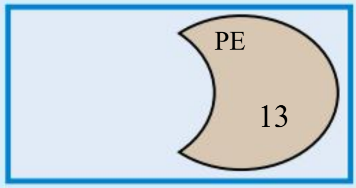
$= \frac{3}{25} = 0.12$

b) music **or** PE



$= \frac{25}{25} = 1$

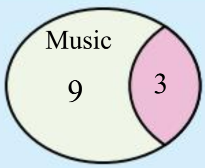
c) **not** music



$= \frac{13}{25} = 0.52$

d) PE given music

$= \frac{3}{12} = 0.25$



given music means look at the **music students**

1 In a class of 25 students, 12 students study music, and 17 students study PE. Draw a Venn Diagram and find the probability that a student:

a) studies music and PE.
b) studies music or PE.
c) does not study music.
d) studies PE given that the student studies music..

PE and music = 12+17−25

2 In a class of 23 students, 12 students play cricket, and 15 students play netball. Draw a Venn Diagram and find the probability that a student:

a) plays cricket and netball.
b) plays cricket or netball.
c) does not play cricket.
d) plays netball given that the student plays cricket.

3 In a class of 29 students, 27 students passed Maths, and 23 students passed English. Draw a Venn Diagram and find the probability that a student:

a) passed maths and English.
b) passed maths or English.
c) did not pass Maths.
d) passed English given that the student also passed maths.

Mental Computation

Mental computation helps you learn how to think.

Exercise 14.9

1 Spell Probability.

2 A six-sided die is tossed, what is P(2)?

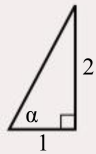
3 A six-sided die is tossed, what is P(>4)?

4 Two six-sided dice are tossed, P(total=2)?

5 Area of a circle formula?

6 Volume of a prism?

7 In the triangle, what is tanα?



8 Two sides in a right-angled triangle are 1 and 2. Hypotenuse?

$c^2=1^2+2^2$
$c^2=1+4$
$c=\sqrt{5}$

9 Write in scientific notation: 5 000

10 14×15

14×15
= 7×2×15
= 7×30
= 210

Exercise 14.10

1 Spell Experimental.

2 A six-sided die is tossed, what is P(3)?

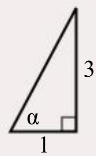
3 A six-sided die is tossed, what is P(<3)?

4 Two six-sided dice are tossed, P(total=10)?

5 Area of a circle formula?

6 Volume of a prism?

7 In the triangle, what is tanα?



8 Two sides in a right-angled triangle are 1 and 3. Hypotenuse?

9 Write in scientific notation: 670 000

10 16×15

"Mathematically, OR means together," said Ella and Noah collectively.

Exercise 14.11

1 Spell Theoretical.

2 A six-sided die is tossed, what is P(5)?

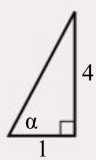
3 A six-sided die is tossed, what is P(<4)?

4 Two six-sided dice are tossed, P(total=7)?

5 Area of a circle formula?

6 Volume of a prism?

7 In the triangle, what is tanα?



8 Two sides in a right-angled triangle are 1 and 4. Hypotenuse?

9 Write in scientific notation: 0.000 091

10 18×15

Don't taunt the alligator until after you've crossed the creek - Dan Rather.

Competition Questions

Prepare for mathematics competitions and build maths muscle at the same time.

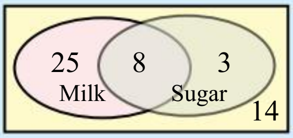
Exercise 14.12

In a survey of 50 coffee drinkers, 33 have milk and 11 have sugar with their coffee. 8 have both milk and sugar. Find the probability that:

a) a coffee drinker has sugar only..

b) a coffee drinker has neither milk nor sugar.

Step 1: Draw a Venn diagram



Step 2: Calculate the probabilities

P(sugar only) = $\frac{3}{50} = 0.06$

P(neither milk nor sugar) = $\frac{14}{50} = 0.28$

1 The hospital data showed that of the 80 patients, 29 patients had the A antigen, 20 had the B antigen. 6 patients had both the A and B antigens.
Find the probability that:

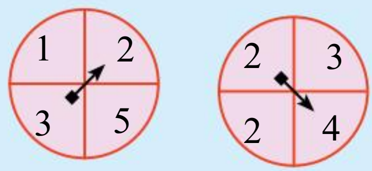
a) a patient had the B antigen only.

b) a patient had no antigen (ie., neither the A nor B antigen).

The two spinners show a total of 7.

a) How many different ways can a total of 5 be obtained?

b) If the spinners are spun again, what is the probability of a total of 4?



First draw up a two-way table

Spinner 1 \ Spinner 2	2	2	3	4
1	3	3	4	5
2	4	4	5	6
3	5	5	6	7
5	7	7	8	9

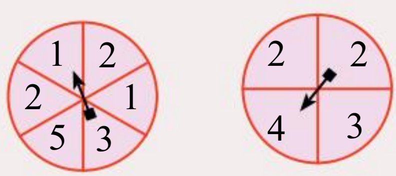
a) A total of 5 in 4 ways.

b) P(total=4) = 3/16 = 0.19

2 The two spinners show a total of 5.

a) How many different ways can a total of 5 be obtained?

b) If the spinners are spun again, what is the probability of a total of 4?



3 Three boys and two girls are randomly arranged in a row.
Find the probability that:

a) the two girls are together.

b) a boy is on each end.

c) a girl is on each end.

Hint: Draw diagrams.

Investigations

Investigation 14.1 A Blue Moon?

Use the Internet to research the following questions:

a) Once in a blue moon. What is a blue moon?

b) Buckley's chance. Who was Buckley?

Investigation 14.2 Research a gambling game.

There have been thousands and thousands of gambling games throughout the centuries.

The one certainty about gambling is that the odds are against the gambler.

a) Select a gambling game.

b) Investigate the theoretical probabilities of a gambler winning.

c) Simulate the game a large number of times.

d) Report your findings.

Investigation 14.3 Tossing a Coin and a Die.

Investigate

Is it true that:

$$P(\text{a 3 and a H}) = \frac{1}{12} = 0.08\ ?$$

$$P(\text{a 3 or a H}) = \frac{7}{12} = 0.58\ ?$$

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T

Investigation 14.4 Probability Simulators.

Investigate

Online probability simulators.

Why were economists created?

So meteorologists would't look so bad.

Technology

Technology 14.1

Use the Rand (random) on a calculator to simulate the throwing of a die.

A throw = 2ndF Rand x 6 + 0.5 = {Round the answer}

The random function gives a number between 0 and 1. This is scaled to give a number from 1 to 6.

Repeatedly pressing = may give a sequence of random numbers.

Technology 14.2

Use a spreadsheet to simulate the tossing of a coin. This is a great way to estimate the experimental probability of a head or tail. Use the Help to find out about the Round function and the Rand function.

Press F9 to get a new set of random numbers.

	1
	1
	2
	2
	1
	1
	1
	2
	2
	1
No of heads	6

Enter the formula =Randbetween(1,2)

If tails=1, heads=2
To count heads enter:
=Countif(b1:b10,2)

Technology 14.3

Use a spreadsheet to simulate the throwing of a die. This is a great way to estimate the experimental probability of the numbers 1, 2, 3, 4, 5, 6.

Enter the formula =Randbetween(1,6)

5		
4		
5	No 1s	86
6	No 2s	89
1	No 3s	73
3	No 4s	77
3	No 5s	91
5	No 6s	84
2		
1		

If using 500 rows enter: =CountIF(a1:a500,1)

If using 500 rows enter: =CountIF(a1:a500,6)

A Couple of Puzzles

Exercise 14.13

1 What is the value of: $2^{2^{2^{2}}}$

2 Put a positive integer in each box so that:

a) The sum of all five digits is 15.

b) The first digit is the difference between the last two digits.

c) The second digit is one less that the third digit.

3 Two cylinders are exactly the same size, have exactly the same mass, look exactly the same. One is hollow and one is solid.
What is the easiest way to tell them apart?

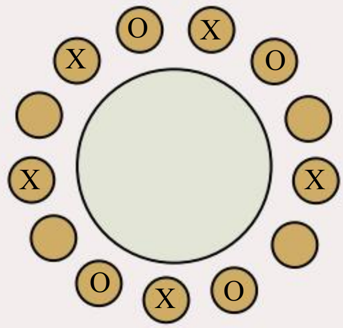
A Game

Crots is played with naughts and crosses on the seats around a table.
The loser is the person who can't make a move.

1 Take it in turns to place a naught or cross on one of the seats around the table.

2 A naught cannot be placed beside another naught.
Similarly for crosses.

3 It is naught's turn.
Can you find a place to put an O?
X has lost the game.



A Sweet Trick

1 Ask your audience to draw a 5×4 box around any 20 numbers on a calendar.

2 You tell them the sum is 270.

3 They take ages to check the sum on their calculator.

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

You quickly give the answer of 270.

Add the smallest and largest numbers, then multiply by 10.
$(1+26)\times 10 = 270$

Chapter Review 1

Exercise 14.14

1 Assuming that the chances of a head or tail is equal, use a tree diagram and two-way table to determine the following theoretical probabilities for the tossing of two coins:

a) P(2 heads) **b)** P(1 head & 1 tail) **c)** P(2 tails)

2 Determine the following theoretical probabilities when tossing a six-sided die and a coin (Use a tree diagram or two-way table to list all possible outcomes).

a) P(a 2 and a T) **b)** P(a 2 or a T)

c) P(a 6 and a H) **d)** P(a 6 or a H)

Hint: Draw up a two-way table first.

3 Determine the following theoretical probabilities of the **totals** when tossing two dice.

a) P(4 or odd) **b)** P(4 and odd) **c)** P(9 or even)

d) P(9 and even) **e)** P(<5 or odd) **f)** P(<5 and odd)

g) P(>9 or even) **h)** P(>9 and odd) **i)** P(odd or divisible by 5)

j) P(odd or even) **k)** P(odd and even)

4 In a class of 26 students, 22 students passed Maths, and 19 students passed English. Draw a Venn Diagram and find the probability that a student:

a) passed Maths and English.
b) passed Maths or English.
c) did not pass Maths.
d) passed English given that the student also passed Maths.

5 Mia's previous weekly maths scores, out of 50, are shown below:

41	43	44	37	48
39	42	39	45	48
46	46	42	40	48

'25 is a pass,' said Mark in passing.

Using this data, what is the probability that Mia's next result will be

a) greater than 45?
b) over 40?

6 **a)** What is the probability that a two-digit number contains a 5?

b) What is the probability that a two-digit number contains a 0?

Chapter Review 2

Exercise 14.15

1 Assuming that the chances of a girl or boy being born is equal, use a tree diagram and a two-way table to determine the following theoretical probabilities for a family of two children:

a) P(2 girls) **b)** P(1 girl & 1 boy) **c)** P(2 boys)

2 Determine the following theoretical probabilities when tossing an eight-sided die and a coin (Use a tree diagram or two-way table to list all possible outcomes).

a) P(a 8 and a H) **b)** P(a 8 or a H)
c) P(odd and a T) **d)** P(odd or a T)

Hint: Draw up a two-way table first.

3 Determine the following theoretical probabilities of the **totals** when tossing two dice.

a) P(3 or odd) **b)** P(3 and odd) **c)** P(11 or even)
d) P(11 and even) **e)** P(<4 or odd) **f)** P(<4 and even)
g) P(>7 or even) **h)** P(>7 and odd) **i)** P(odd or divisible by 3)
j) P(odd or even) **k)** P(odd and even)

4 In a class of 22 students, 11 students play cricket, and 14 students play netball. Draw a Venn Diagram and find the probability that a student:
a) plays cricket and netball.
b) plays cricket or netball.
c) does not play cricket.
d) plays netball given that the student plays cricket.

5 Jack's previous weekly maths scores, out of 50, are shown below:

23	22	25	29	24
18	18	20	28	29
28	25	28	21	22

'24 is almost a pass,' said Tim roughly.

Using this data, what is the probability that Jack's next result will be
a) greater than 20?
b) a fail (<25)?

6 **a)** What is the probability that a two-digit number contains a 8?
b) What is the probability that a two-digit number contains a 1?

Review 3 15

Chapter 11 Indices 2

$2\times2\times2 = 2^3$ ← Index (3), Base (2)

Scientific Notation

$460\ 000 = 4.6\times10^5$

$0.000\ 007\ 1 = 7.1\times10^{-6}$

Negative Index

$a^{-m} = \frac{1}{a^m}$

Index Law 1

$a^m\times a^n = a^{m+n}$

Index Law 2

$a^m\div a^n = a^{m-n}$

Index Law 3

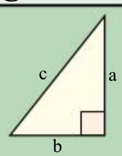
$(a^m)^n = a^{m\times n}$

Zero Index

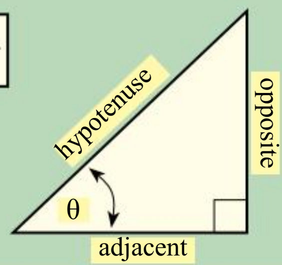
$a^0 = 1$

Chapter 12 Trigonometry 1

$c^2 = a^2 + b^2$



$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

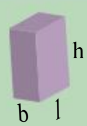


The angles in a triangle sum to 180°

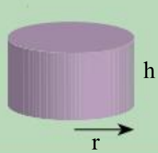
Chapter 13 Volume

A **prism** has the same cross-section along its length.

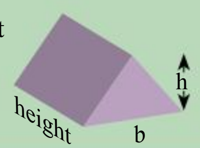
V = Area of base × height
= l × b × h



V = Area of base × height
= $\pi r^2 \times h$



V = Area of base × height
= ½bh × height



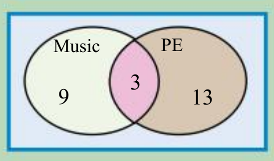
Chapter 14 Probability 1

$\text{Probability} = \frac{\text{Number of possible outcomes}}{\text{Total number of outcomes}}$

$\text{Experimental probability} = \frac{\text{frequency of event}}{\text{total frequency}}$

Dice 2 \ Dice 1	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

	1	2	3	4	5	6
H	1,H	2,H	3,H	4,H	5,H	6,H
T	1,T	2,T	3,T	4,T	5,T	6,T



Review 1

Exercise 15.1 Mental computation

1 Spell Theoretical.

2 A six-sided die is tossed, what is P(3)?

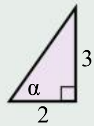
3 A six-sided die is tossed, what is P(>3)?

4 Two six-sided dice are tossed, P(total=4)?

5 Area of a circle formula?

6 Volume of a prism?

7 In the triangle, what is $\tan\alpha$?



8 Two sides in a right-angled triangle are 2 and 3. Hypotenuse?

9 Write in scientific notation: 0.000 45

10 24×15

Exercise 15.2

1 Write each of the following using a negative index:

a) $\frac{1}{10^2}$ **b)** $\frac{1}{10^5}$ **c)** $\frac{1}{x^4}$

2 Simplify and write the following in index form:

a) $10^{-2}\times10^{5}$ **b)** $10^{5}\div10^{-3}$ **c)** $10^{-5}\times10^{-3}$

d) $x^{3}\times x^{-8}\times x^{3}$ **e)** $(10^{-2})^{-4}$ **f)** 3×1^{0}

g) $4\times10^{7}\times3\times10^{-4}$ **h)** $9\times10^{-5}\div(3\times10^{-6})$ **i)** $x^{4}\times4x^{-3}\div2x^{5}$

3 Write in scientific notation:

a) 430 000 **b)** 670 000 **c)** 55 000 000

d) 0.000 1 **e)** 0.000 031 **f)** 0.003

g) 16 grams of oxygen has 60 000 000 000 000 000 000 000 molecules.

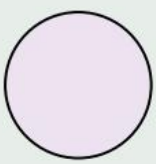
4 Write as ordinary numbers:

a) 10^{4} **b)** 3.2×10^{9} **c)** 4.01×10^{7}

d) 5×10^{-3} **e)** 8.2×10^{-5} **f)** 4.5×10^{-6}

g) Mass of an electron: 9×10^{-28} g

5 Find the circumference of Earth (Radius = 6.4×10^{6} m).



Circumference
(Distance around the outside.)
$C=2\pi r$

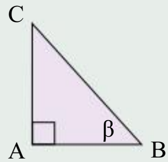
6 Find the volume of Earth (Radius = 6.4×10^6 m).

Volume
(Space occupied by the sphere.)

$$V = \frac{4\pi r^3}{3}$$

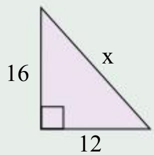
7 What is the last digit in: $3^{11}\times4^{15}$?

8 For the adjacent triangle, name:
a) the hypotenuse.
b) the side adjacent to the angle.
c) the side opposite the angle.

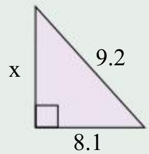


9 Use Pythagoras' Theorem to find the unknown:

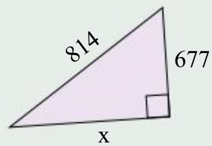
a)



b)

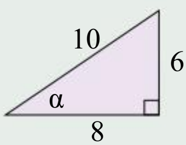


c)

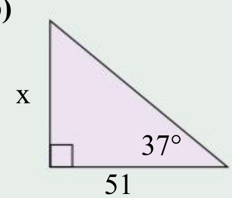


10 Find the unknown in each of the following triangles:

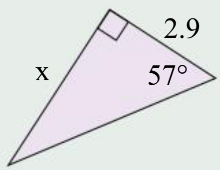
a)



b)



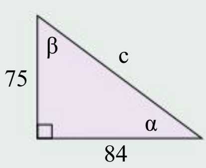
c)



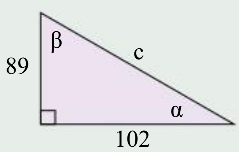
Solve means 'find **all** unknowns'.

11 Solve the following triangles:

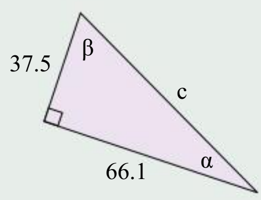
a)



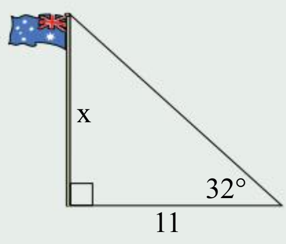
b)



c)

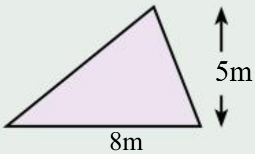


12 11 m out from the base of a flagpole, a clinometer measures the angle of elevation to the top of the flagpole as 32°. Find the height of the flagpole.

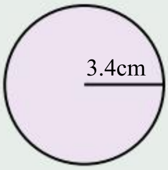


13 Calculate the area of each of the following shapes:

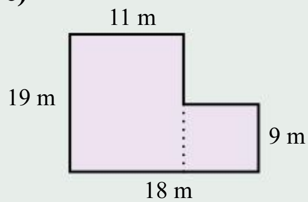
a)



b)



c)



14 Make the following unit conversions:

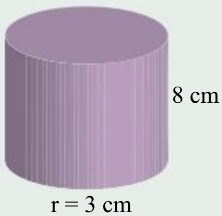
a) 7.3 m^3 to cm^3 **b)** 0.6 cm^3 to mm^3

c) 7600 mL to L **d)** 540 000 L to kL

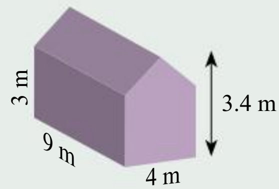
e) 6.1 kL to L **f)** 0.32 L to mL

15 Calculate the volume of each of the following solids:

a)

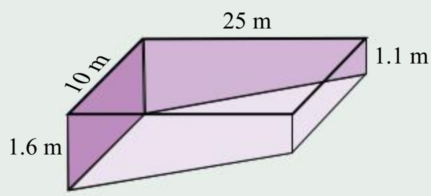


b)



16 What is the cost of concrete for the shed slab? The slab is 5.6 m by 2.8 m by 17 cm. Concrete delivered is \$185 per cubic metre

17 How many litres of water is needed to fill the swimming pool?



18 Assuming that the chances of a head or tail is equal, use a tree diagram and two-way table to determine the following theoretical probabilities for the tossing of two coins:

a) P(2 heads) **b)** P(1 head & 1 tail) **c)** P(2 tails)

19 Determine the following theoretical probabilities when tossing a six-sided die and a coin (Use a tree diagram or two-way table to list all possible outcomes).

a) P(a 4 and a T) **b)** P(a 4 or a T)

c) P(a 5 and a H) **d)** P(a 5 or a H)

20 Determine the following theoretical probabilities of the **totals** when tossing two dice.

a) P(10 or odd) **b)** P(10 and odd) **c)** P(<4 or even)

d) P(<4 and even) **e)** P(odd or even) **f)** P(odd or divisible by 3)

21 In a class of 23 students, 17 students passed Maths, and 15 students passed English. Draw a Venn Diagram and find the probability that a student:

a) passed Maths and English.

b) passed Maths or English.

c) did not pass Maths.

d) passed English given that the student also passed Maths.

Hint: Draw up a two-way table first.

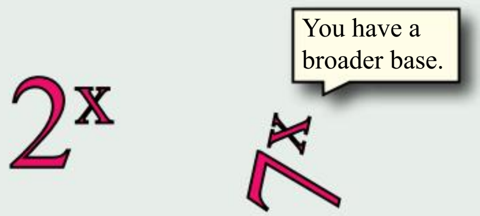
22 Mark's previous weekly pizza sales are shown below:

651	717	749	730	662
716	711	733	732	704
687	677	697	665	673

Using this data, what is the probability that Mark's next week's result will be

a) greater than 700?

b) over 730?



Review 2

Exercise 15.2 Mental computation

1 Spell Trigonometry.

2 A six-sided die is tossed, what is P(6)?

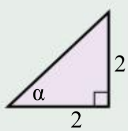
3 A six-sided die is tossed, what is P(>3)?

4 Two six-sided dice are tossed, P(total=10)?

5 Area of a circle formula?

6 Volume of a prism?

7 In the triangle, what is tanα?



8 Two sides in a right-angled triangle are 2 and 2. Hypotenuse?

9 Write in scientific notation: 0.000 74

10 34×15

Exercise 15.3

1 Write each of the following using a negative index:

a) $\frac{1}{10^5}$ **b)** $\frac{1}{10^9}$ **c)** $\frac{1}{x^3}$

2 Simplify and write the following in index form:

a) $10^{-6}\times10^{4}$ **b)** $10^{7}\div10^{-3}$ **c)** $10^{-3}\times10^{-3}$

d) $x^{5}\times x^{-8}\times x^{3}$ **e)** $(10^{-2})^{-3}$ **f)** 9×1^{0}

g) $5\times10^{6}\times3\times10^{-3}$ **h)** $6\times10^{-4}\div(3\times10^{-6})$ **i)** $x^{3}\times6x^{-3}\div2x^{5}$

3 Write in scientific notation:

a) 30 000 **b)** 4 800 000 **c)** 310 000

d) 0.003 **e)** 0.000 62 **f)** 0.000 000 63

g) Wavelength of green light: 0.000 000 52 m.

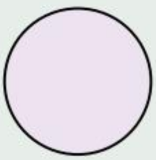
4 Write as ordinary numbers:

a) 10^{3} **b)** 4.2×10^{6} **c)** 5.11×10^{9}

d) 2×10^{-4} **e)** 7.1×10^{-7} **f)** 3.2×10^{-9}

g) The distance from the Earth to the Sun, 1.49×10^{11} m.

5 Find the circumference of Venus (Radius = 6.05×10^{6} m).



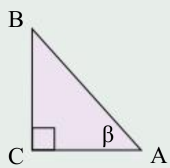
Circumference
(Distance around the outside.)
$C=2\pi r$

6 Find the volume of Venus (Radius = 6.05×10^{6} m).

Volume
(Space occupied by the sphere.)
$$V=\frac{4\pi r^3}{3}$$

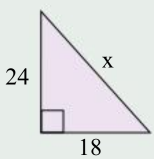
7 What is the last digit in: $3^{16}\times4^{23}$?

8 For the adjacent triangle, name:
a) the hypotenuse.
b) the side adjacent to the angle.
c) the side opposite the angle.

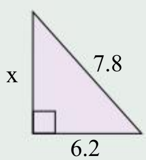


9 Use Pythagoras' Theorem to find the unknown:

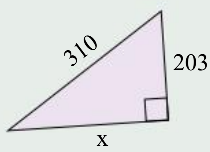
a)



b)

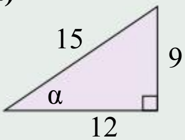


c)

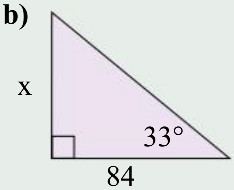


10 Find the unknown in each of the following triangles:

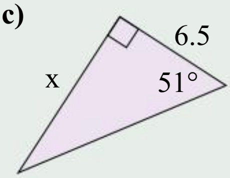
a)



b)



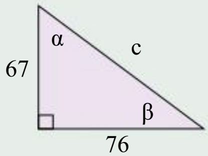
c)



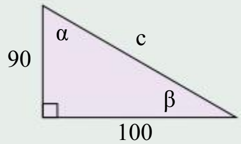
Solve means 'find **all** unknowns'.

11 Solve the following triangles:

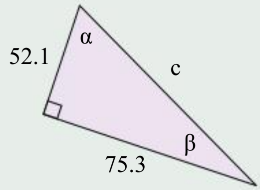
a)



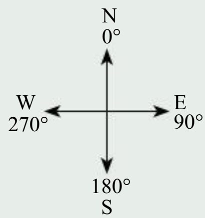
b)



c)

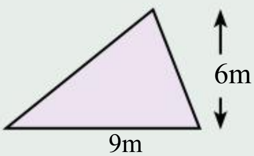


12 A ship sails due south for 65 km, then on a bearing of 45° until the ship is due east of its starting point. How far is the ship from its starting point?

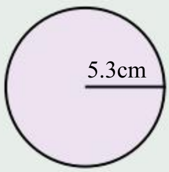


13 Calculate the area of each of the following shapes:

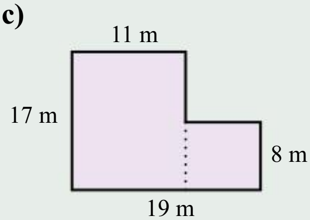
a)



b)



c)



14 Make the following unit conversions:

a) $8.2\ m^3$ to cm^3

b) $0.5\ cm^3$ to mm^3

c) 8200 mL to L

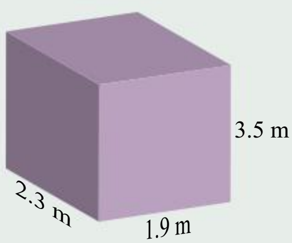
d) 630 000 L to kL

e) 9.2 kL to L

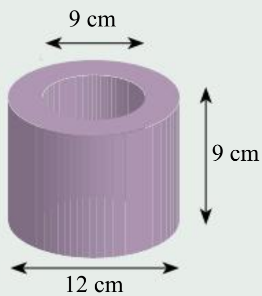
f) 0.42 L to mL

15 Calculate the volume of each of the following solids:

a)

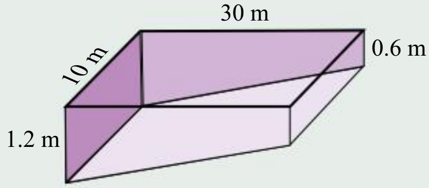


b)



16 What is the cost of concrete for the shed slab?
The slab is 6.4 m by 2.6 m by 18 cm.
Concrete delivered is $185 per cubic metre

17 How many litres of water is needed to fill the swimming pool?



18 Assuming that the chances of a head or tail is equal, use a tree diagram and two-way table to determine the following theoretical probabilities for the tossing of two coins:

a) P(2 heads) **b)** P(1 head & 1 tail) **c)** P(2 tails)

19 Determine the following theoretical probabilities when tossing a six-sided die and a coin (Use a tree diagram or two-way table to list all possible outcomes).

a) P(a 1 and a T) **b)** P(a 1 or a T)
c) P(a 3 and a H) **d)** P(a 3 or a H)

20 Determine the following theoretical probabilities of the **totals** when tossing two dice.

a) P(5 or odd) **b)** P(5 and odd) **c)** P(<10 or even)
d) P(>10 and even) **e)** P(odd or even) **f)** P(odd or divisible by 3)

21 In a class of 21 students, 16 students passed Maths, and 15 students passed English. Draw a Venn Diagram and find the probability that a student:
a) passed Maths and English.
b) passed Maths or English.
c) did not pass Maths.
d) passed English given that the student also passed Maths.

22 Jess's previous weekly customer numbers are shown below:

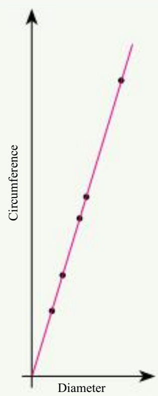
73	61	67	76	58	69	77
59	77	71	65	57	70	63
57	53	67	60	65	63	75

Using this data, what is the probability that Jess's next week's result will be
a) greater than 70?
b) less than 60?

Coordinate Geometry 16

Number & Algebra ➞ Linear & non-linear relationships

★ Find the distance between two points located on a Cartesian plane using a range of strategies, including graphing software.
★ Investigate graphical and algebraic techniques for finding distance.
★ Find the midpoint and gradient of a line segment (interval) on the Cartesian plane using a range of strategies.
★ Investigate graphical and algebraic techniques for finding midpoint and gradient.



A TASK

"A pi equation"

- Have your group collect as many different circular objects as possible (plates, saucers, lids, jars, coins etc.
- Measure the diameter and circumference of each object.
- Plot the points on a graph.
- Work out the gradient of the line.
- Because for circles: $C = \pi d$, the gradient is π and the y-intercept is 0. Is the gradient of your line $= \pi$?
- Make a powerpoint and show it to your class.

A LITTLE BIT OF HISTORY

Carl Frederick Gauss (1777-1855) is often referred to as the 'Prince of Mathematics'.

1801 Gauss used a couple of observations of the planetoid Ceres to predict its exact position after it had been lost to astronomers.

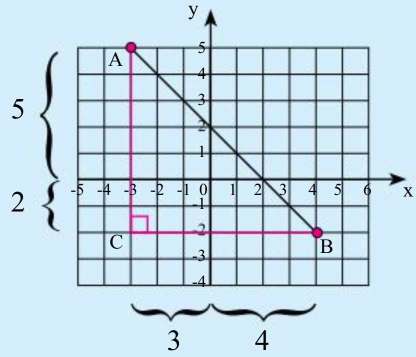
1810 Gauss solved six linear equations to mathematically describe the orbit of Pallas. His method is now known as Gaussian elimination.

Distance Between 2 Points

Exercise 16.1

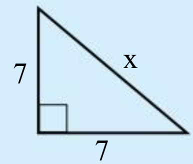
Find the distance between A and B.

A(-3, 5) and B(4, -2)



$$AC = 5+2 = 7$$

$$CB = 3+4 = 7$$

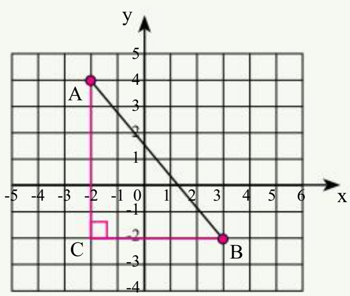


$$AB^2 = AC^2 + CB^2 = 7^2 + 7^2 = 98$$

$$AB = \sqrt{98} = \sqrt{49 \times 2} = 7\sqrt{2} \text{ or } 9.90$$

Rounding?
See Technology 16.1

1



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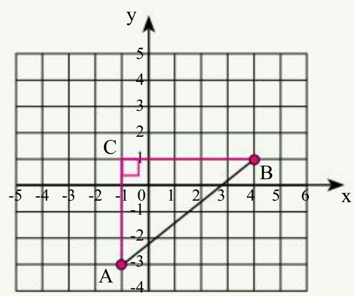

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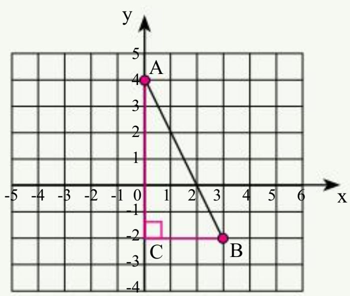

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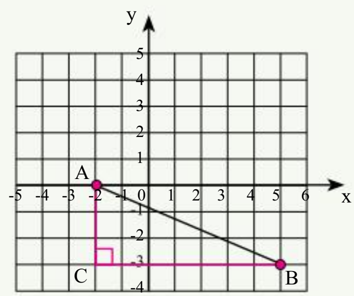


Plot the points first.

Exercise 16.2

Find the distance between the two points given.

1 A(3,2), B(5,3) **2** A(2,2), B(4,3) **3** A(1,2), B(2,4)

4 A(5,3), B(3,6) **5** A(3,4), B(6,1) **6** A(1,1), B(4,4)

7 A(3,3), B(-1,-1) **8** A(1,2), B(-4,3) **9** A(4,3), B(-2,-3)

10 A(2,0), B(-1,-3) **11** A(-2,5), B(-4,6) **12** A(-2,-1), B(2,1)

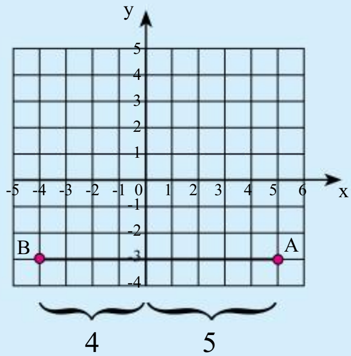
13 A(-4,0), B(-1,2) **14** A(-2,-1), B(-4,-2) **15** A(5,-2), B(2,-3)

A(5, -3), B(-4, -3)

$$AB = 4+5$$
$$= \underline{9}$$

or

$$AB = 5 - {}^{-}4$$
$$= 9$$



Can you find the distance without plotting the points?

16 A(5,2), B(3,2) **17** A(2,1), B(4,1) **18** A(5,3), B(1,3)

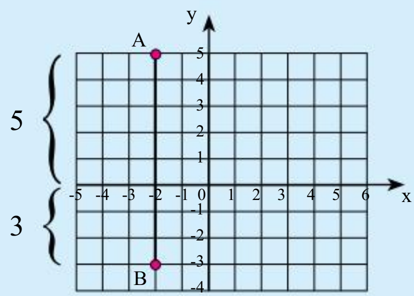
19 A(4,-1), B(3,-1) **20** A(3,-2), B(-1,-2) **21** A(-1,-4), B(-2,-4)

A(-2, 5), B(-2, -3)

$$AB = 5+3$$
$$= \underline{8}$$

or

$$AB = 5 - {}^{-}3$$
$$= 8$$



Can you find the distance without plotting the points?

22 A(3,2), B(3,5) **23** A(1,4), B(1,2) **24** A(4,1), B(4,2)

25 A(-5,2), B(-5,-3) **26** A(-3,-4), B(-3,4) **27** A(-1,-5), B(-1,-3)

Midpoint

The midpoint is halfway.

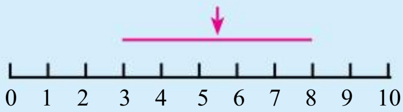
Exercise 16.3

Find halfway between:

3 and 8

Halfway $= \frac{3+8}{2}$

$= \underline{5.5}$



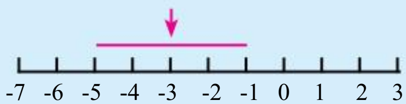
Add
then divide by 2.

1 2 and 6 **2** 5 and 9 **3** 6.2 and 13.7

-5 and -1

Halfway $= \frac{^-5 + ^-1}{2}$

$= \underline{-3}$



Add
then divide by 2.

4 -1 and -7 **5** -3 and 4 **6** 6 and -5

Exercise 16.4

Find the midpoint of AB

A(1,5) and B(4,2)

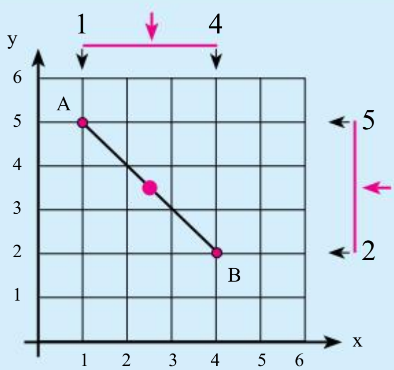
x-coordinates

midpoint $= \frac{1+4}{2} = 2.5$

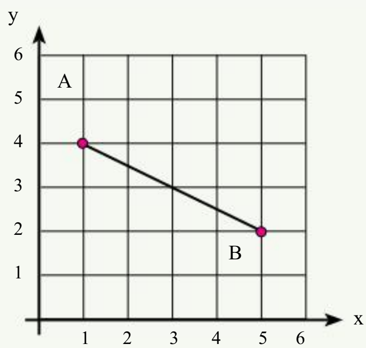
y-coordinates

midpoint $= \frac{5+2}{2} = 3.5$

The midpoint of AB is (2.5, 3.5)



1



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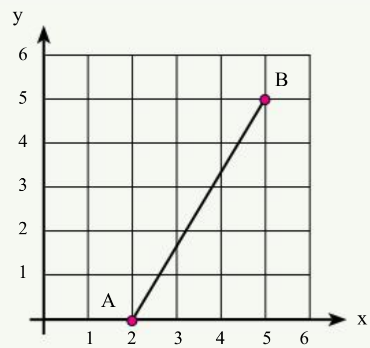


A line segment is a part of a line that is bounded by two end points.

Exercise 16.5

Find the midpoint of the line segment AB.

A(-3, 5) and B(4, -2)

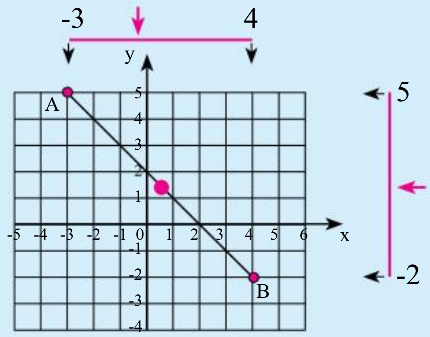
x-coordinates

midpoint = $\dfrac{^{-}3+4}{2}$ = 0.5

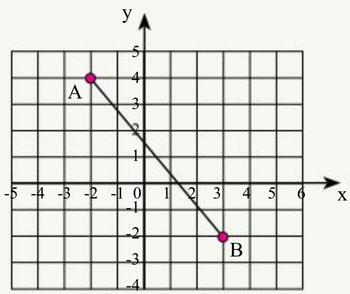
y-coordinates

midpoint = $\dfrac{5+{}^{-}2}{2}$ = 1.5

The midpoint of AB is (0.5, 1.5)



1



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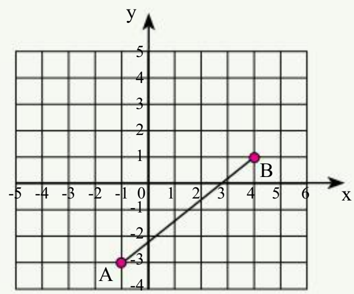

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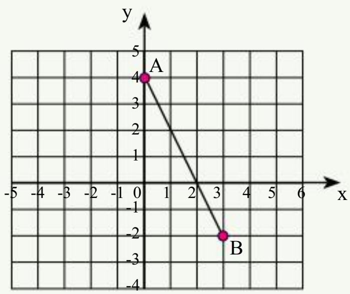

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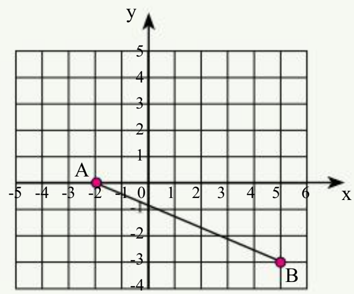


Plot the points first.

5 A(1,4), B(2,3)	**6** A(2,2), B(3,3)	**7** A(5,5), B(2,2)
8 A(2,1), B(4,7)	**9** A(3,6), B(6,1)	**10** A(5,1), B(4,0)
11 A(2,3), B(-2,-2)	**12** A(5,1), B(-2,3)	**13** A(2,3), B(-5,-1)
14 A(3,0), B(-7,-8)	**15** A(-3,1), B(-2,4)	**16** A(-1,-2), B(3,1)
17 A(-4,0), B(0,2)	**18** A(-2,-1), B(-4,-2)	**19** A(5,-2), B(2,-3)

Gradient

The gradient, or slope, or m, is used to describe the steepness of lines.

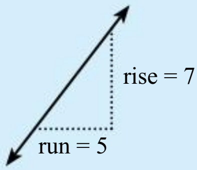
Gradient n. 1. the rate of ascent or descent. 2. the rate of rise or fall.

Exercise 16.6

Find the gradient of each of the following lines:

Slope up
Positive

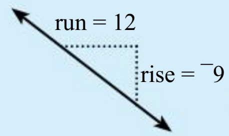
Slope down
Negative



Gradient = m = $\frac{\text{rise}}{\text{run}}$

m = $\frac{7}{5}$

m = 1.4



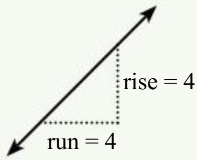
Gradient = m = $\frac{\text{rise}}{\text{run}}$

m = $\frac{^{-}9}{12}$

m = $^{-}0.75$

A rise of $^{-}9$ is actually a drop of 9.

1



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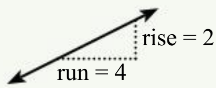

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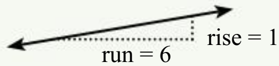

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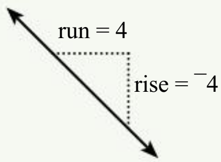

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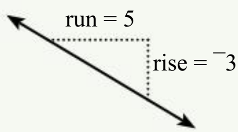

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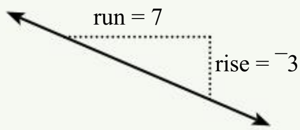
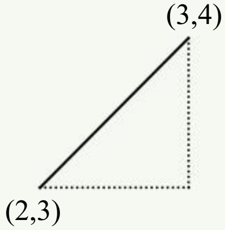


Can you calculate the run? (x coordinates)

Can you calculate the rise? (y coordinates)

7



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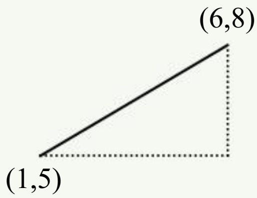

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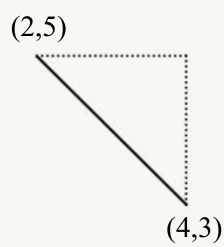


$$m = \text{gradient} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in y}}{\text{change in x}}$$

Lines are used to model millions of real-life problems.

Exercise 16.7

Find the gradient of each of the following line segments:

A(1,5) and B(4,2)

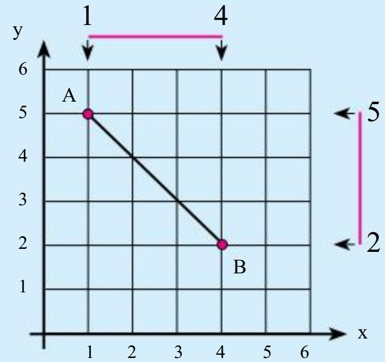
$$\text{Gradient} = m = \frac{\text{change in y}}{\text{change in x}}$$

$$m = \frac{(2-5)}{(4-1)}$$

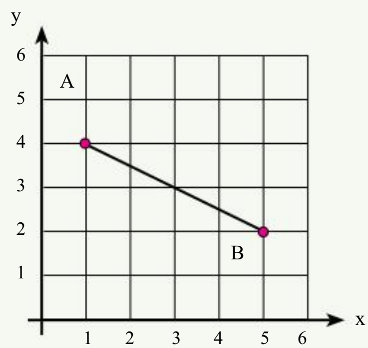
The brackets are important if using a calculator. See Technology 16.1

$$m = \frac{{}^{-}3}{3}$$

$$m = {}^{-}1$$



1



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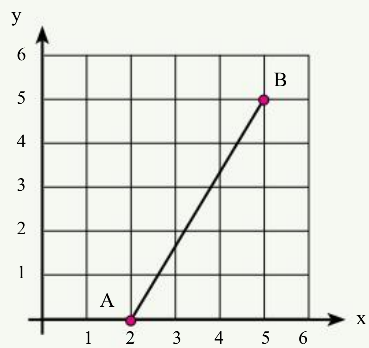

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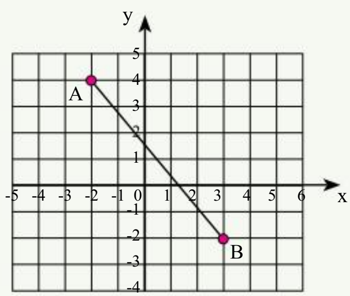

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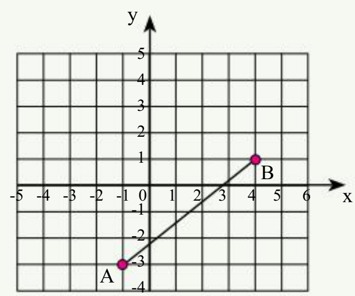


Plot the points first.

5 A(1,1), B(3,4)

6 A(2,5), B(6,1)

7 A(5,2), B(7,5)

8 A(2,3), B(4,1)

9 A(4,1), B(-2,3)

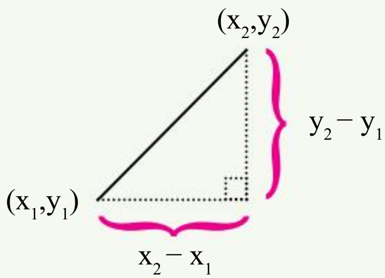
10 A(2,3), B(-5,-1)

11 A(3,0), B(-7,-8)

12 A(-3,1), B(-2,4)

13 A(-1,-2), B(3,1)

Formulas



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

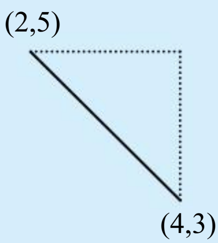
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$Mid = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Exercise 16.8

Find the gradient, length, and midpoint of each of the following line segments:

First complete: $x_1 = \ , y_1 =$
$x_2 = \ , y_2 =$



$x_1 = 2,\ y_1 = 5$
$x_2 = 4,\ y_2 = 3$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{(3-5)}{(4-2)}$$

$m = {}^{-}1$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(4-2)^2 + (3-5)^2}$$

$$d = \sqrt{2^2 + {}^{-}2^2}$$

Length = √8 or √4×2 or 2√2
or length = 2.83

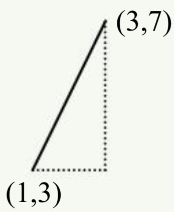
$$Mid = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$Mid = \left(\frac{2+4}{2}, \frac{5+3}{2}\right)$$

Midpoint is (3,4)

$x_1 = 1,\ y_1 = 3$
$x_2 = 3,\ y_2 = 7$

1



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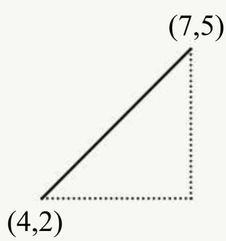
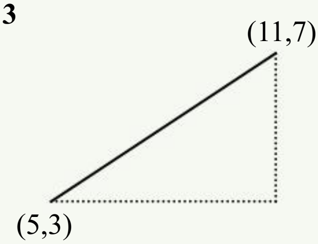


3



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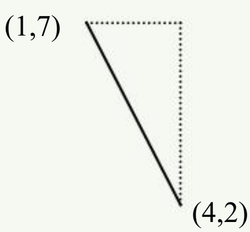

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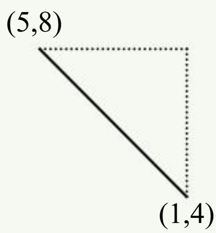

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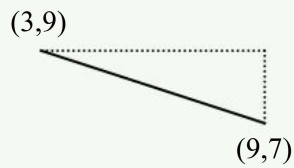
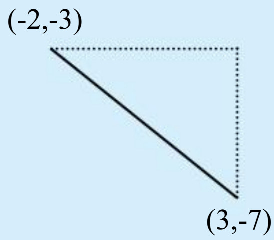


Exercise 16.9

Find the gradient, length, and midpoint of each of the following line segments:

First complete:	$x_1 =\ ,\quad y_1 =$ $x_2 =\ ,\quad y_2 =$



$x_1 = -2,\ y_1 = -3$
$x_2 = 3,\ y_2 = -7$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{^{-}7 - {^{-}3}}{3 - {^{-}2}}$$

$$m = \frac{^{-}4}{5}$$

$m = {^{-}0.8}$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(3 - {^{-}2})^2 + ({^{-}7} - {^{-}3})^2}$$

$$d = \sqrt{5^2 + {^{-}4}^2}$$

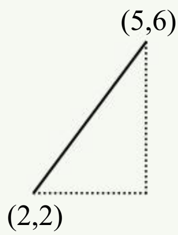
Length = $\sqrt{41}$ or 6.40

$$\text{Mid} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

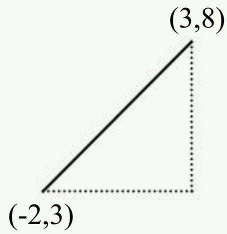
$$\text{Mid} = \left(\frac{^{-}2 + 3}{2}, \frac{^{-}3 + {^{-}7}}{2}\right)$$

Midpoint is (0.5, $^{-}5$)

1



2



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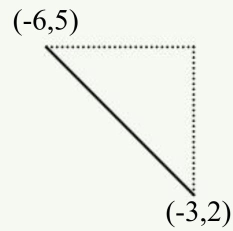

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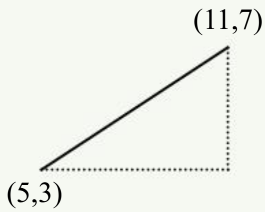

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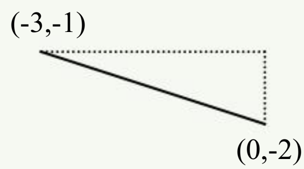

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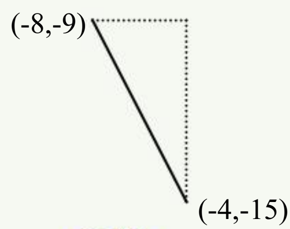


7 A triangle has the three endpoints A(2,1), B(4,5), C(7,2).

- **a)** Find the length of AB.
- **b)** Find the length of AC.
- **c)** Find the length of BC.

A sketch or even a plot will be a big help.

8 A parallelogram has the four endpoints A(8,7), B(1,4), C(-1,-1), D(6,2).

- **a)** Show that the diagonals bisect each other.
- **b)** Show that AB is parallel to CD (ie have the same gradient).
- **c)** Show that the opposite sides are equal in length.

Mental Computation

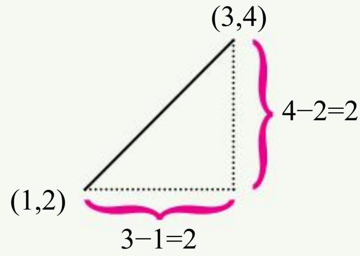
> You need to be a good mental athlete because many everyday problems are solved mentally

Exercise 16.10

1 Spell Gradient.
2 What is the gradient formula?
3 What is the distance formula?
4 What is the midpoint formula?

Given the points A(1,2), B(3,4)

5 What is the gradient of AB?
6 What is the length of AB?
7 What is the midpoint of AB?
8 A six-sided die is tossed, what is P(3)?
9 Area of a circle formula?
10 In the triangle, what is tan α?



$$\begin{aligned} \tan\alpha &= \text{opp/adj} \\ &= 2/2 \\ &= 1 \end{aligned}$$

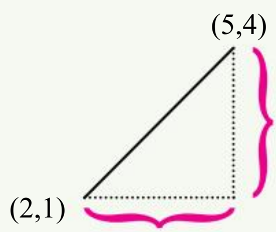
> A fine is a tax for doing wrong.
> A tax is a fine for doing well.

Exercise 16.11

1 Spell Hypotenuse.
2 What is the gradient formula?
3 What is the distance formula?
4 What is the midpoint formula?

Given the points A(2,1), B(5,4)

5 What is the gradient of AB?
6 What is the length of AB?
7 What is the midpoint of AB?
8 A six-sided die is tossed, what is P(5)?
9 Area of a circle formula?
10 In the triangle, what is tan α?



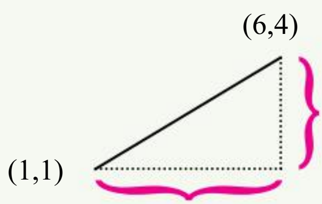
> The enemy diversion you have been ignoring will be the main attack - Murphy's Laws of Combat.

Exercise 16.12

1 Spell Segment.
2 What is the gradient formula?
3 What is the distance formula?
4 What is the midpoint formula?

Given the points A(1,1), B(6,4)

5 What is the gradient of AB?
6 What is the length of AB?
7 What is the midpoint of AB?
8 A six-sided die is tossed, what is P(<3)?
9 Area of a circle formula?
10 In the triangle, what is tan α?



Competition Questions

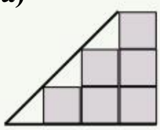
Prepare for mathematics competitions and build maths muscle at the same time.

$\sqrt{0.0009} = 0.03$

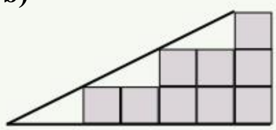
Exercise 16.13

1. What is the square root of 900?
2. What is the square root of 9?
3. What is the square root of 0.09?
4. What is the square root of 0.0009?
5. What is the square of 20?
6. What is the square of 2?
7. What is the square of 0.2?
8. What is the square of 0.02?
9. What is the gradient of the ramp, the thick line, in each of the following?

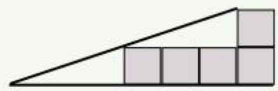
a)



b)



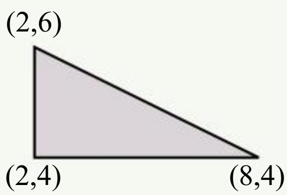
c)



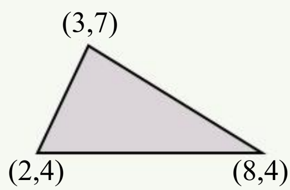
10. One angle in a right-angled triangle is 37°, what is the size of the other two angles?

11. What is the area of the triangle?

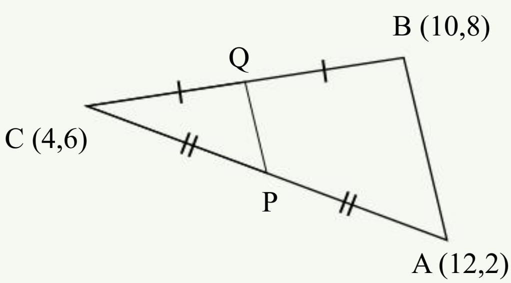
a)



b)



12. Show that PQ is parallel to AB (P and Q are midpoints).



13. The cube has 5 cm side lengths
What is the shortest distance between A and E?

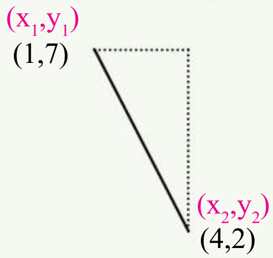
Investigations

Investigation 16.1 (x_1,y_1) and (x_2,y_2)?

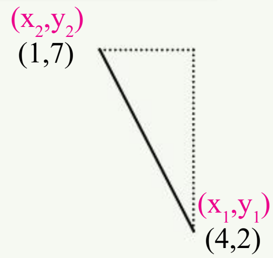
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Does it matter which point is (x_1,y_1) and which point is (x_2,y_2)?

Get the same answer anyway?



OR



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

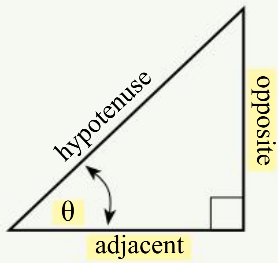
$$\text{Mid} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Investigation 16.2 **Gradient and Tan?**

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Is gradient and tan θ the same thing?



Investigation 16.3 **Ramp Gradient?**

Building regulations suggest that disability access ramps should have a maximum ramp gradient of 1 : 15

What does a ramp gradient of 1 : 15 mean?

A roof is considered pitched if its pitch is greater than 3.2 in 12.

What angle is 3.2 in 12?

Property Valuers calculate the value of property, art objects, and equipment. Valuations usually involve travelling and public contact.
- Relevant school subjects are English and Mathematics.
- Courses usually involve a Universtity Bachelor degree.

A Couple of Puzzles

Exercise 16.14

1 In alphanumerics, each letter represents a number. What is the value of each letter in each of the following alphanumerics:

a)

```
  ONE
  ONE
  ONE
 +ONE
 ----
 FOUR
```

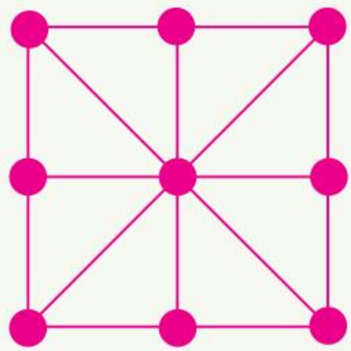
b)

```
  ONE
  ONE
  ONE
 +ONE
 ----
 FOUR
 +ONE
 ----
 FIVE
```

c)

```
  SEND
 +MORE
 -----
 MONEY
```

A Game



Ancient Egypt is an ancient game played by two, each player having three markers. The winner is the first person to get three markers in a row, column, or diagonal.

1 Draw a large version of the diagram shown on the right.

2 Take turns to place each of the three markers on the board. The centre spot is not to be used at this stage.

3 Continue to take turns sliding markers to empty spots until the game is won. The centre spot is now available.

Why shouldn't you tell secrets when there is a clock in the room?
Time will tell.

A Sweet Trick

1	Without looking, ask your audience to choose three different numbers from 1 to 9.	2 6 3
2	Have them write the three numbers in descending order.	6 3 2
3	Have them reverse the digits and find the difference.	$632 - 236 = 396$
4	Ask them to sum the digits.	$3 + 9 + 6 = 18$

The answer is always 18.

Brainstorm ways of presenting this trick. Eg Write 18 on a piece of paper and put in someone's pencil case beforehand.

Technology

Technology 16.1 Calculating gradient

1 Calculate $\frac{(2-5)}{(4-1)}$ $\boxed{(}\ \boxed{2}\ \boxed{-}\ \boxed{5}\ \boxed{)}\ \boxed{\div}\ \boxed{(}\ \boxed{4}\ \boxed{-}\ \boxed{1}\ \boxed{)}\ \boxed{=}$ to give $^{-}1$

The brackets are important in separating the numerator from the denominator.

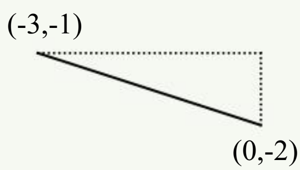
2 Simplify $m = \frac{^{-}7 - {}^{-}3}{3 - {}^{-}2}$

$\boxed{(}\ \boxed{+/-}\ \boxed{7}\ \boxed{-}\ \boxed{+/-}\ \boxed{3}\ \boxed{)}\ \boxed{\div}\ \boxed{(}\ \boxed{3}\ \boxed{-}\ \boxed{+/-}\ \boxed{2}\ \boxed{)}\ \boxed{=}$ to give $^{-}0.8$

Rounding to two decimal places, first look at the third decimal place:

56.231694 less than 5 thus **56.23**	27.01769 5 or more thus **27.02**	1.07276 less than 5 thus **1.07**	4.79634216 5 or more thus **4.80**

Technology 16.2 Spreadsheets



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

	a	b	c	d
1	x1	y1	x2	y2
2	-3	-1	0	-2
			Gradient	-0.33333
			Length	3.162277

Enter the formula:
=(d2−b2)/(c2−a2)

Enter the formula:
=sqrt((c2−a2)^2+(d2−b2)^2)

Technology 16.3 Applets

There are a very large number of interactive online gradient (slope), and distance between two points applets on the Internet.

Experiment with these activities.

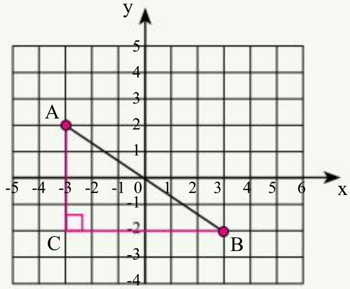
'May I have a large container of coffee?'
The letters in each word gives $\pi = 3.1415926$

Chapter Review 1

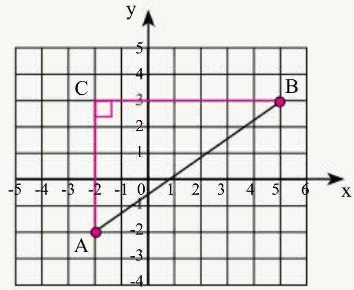
Exercise 16.15

1 Find the length of AB

a)



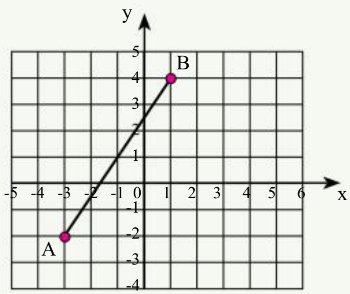
b)



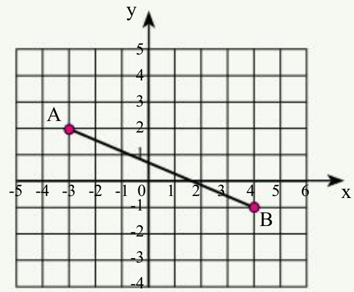
c) A(6,2), B(3,6) **d)** A(3,1), B(-1,-1) **e)** A(-1,-3), B(-2,-5)

2 Find the midpoint of the line segment AB.

a)



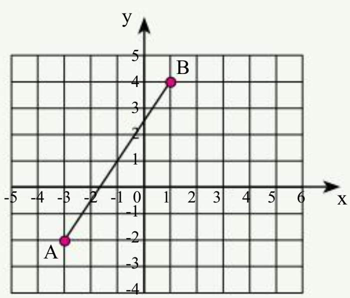
b)



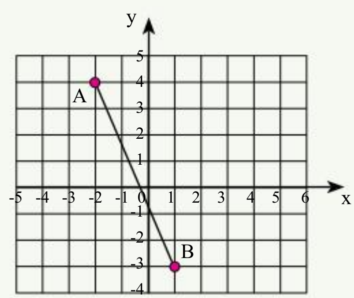
c) A(5,3), B(1,1) **d)** A(4,1), B(-2,3) **e)** A(4,2), B(-5,-1)

3 Find the gradient of each of the following lines.

a)



b)



c) A(1,3), B(4,6) **d)** A(5,-1), B(-1,3) **e)** A(-2,3), B(5,-1)

4 A parallelogram has the four endpoints A(1,7), B(4,11), C(-2,8), D(-5,4).

a) Show that AB is parallel to CD (ie have the same gradient).

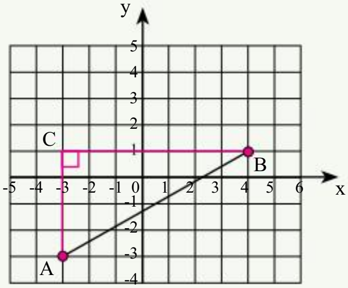
b) Show that the opposite sides are equal in length.

Chapter Review 2

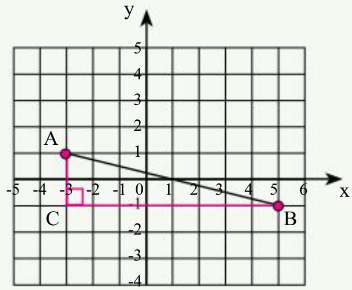
Exercise 16.16

1 Find the length of AB

a)



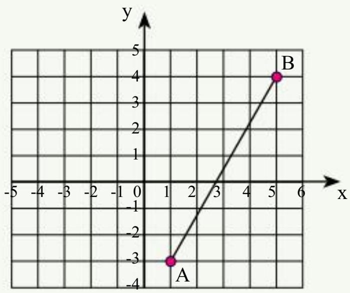
b)



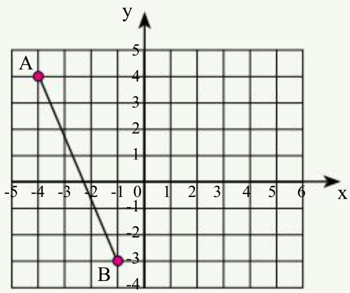
c) A(2,1), B(4,3) **d)** A(5,2), B(-3,-2) **e)** A(-2,-4), B(3,-4)

2 Find the midpoint of the line segment AB.

a)



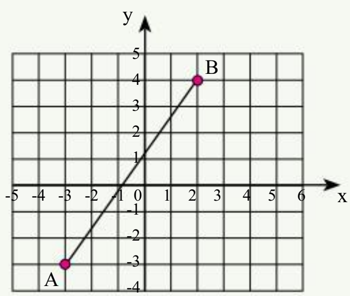
b)



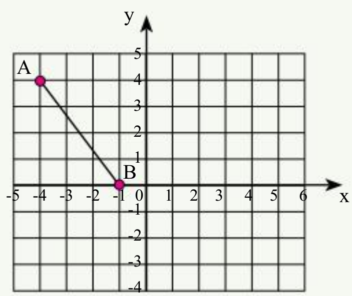
c) A(1,3), B(5,7) **d)** A(2,-1), B(-2,3) **e)** A(2,4), B(-4,-1)

3 Find the gradient of each of the following lines.

a)



b)



c) A(5,3), B(7,5) **d)** A(3,1), B(-2,4) **e)** A(2,6), B(-5,-2)

4 A parallelogram has the four endpoints A(-8,-2), B(-3,-1), C(-1,2), D(-6,1).

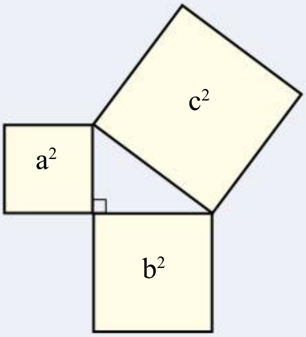
a) Show that AB is parallel to CD (ie have the same gradient).

b) Show that the opposite sides are equal in length.

Trigonometry 2 17

Measurement and Geometry ➡ Pythagoras and Trigonometry

★ Apply trigonometry to solve right-angled triangle problems.
★ Understand the terms 'adjacent' and 'opposite' sides in a right-angled triangle.
★ Select and accurately use the correct trigonometric ratio to find unknown sides and angles in right-angled triangles.



A TASK

- Demonstrate Phytagoras' theorem by showing that the areas a^2 and b^2 can be cut and arranged to exactly cover c^2.
- Make a powerpoint presentation of your demonstration (Insert a sequence of photos taken with a digital camera into Powerpoint - include photos of you and your group).
- Show the Powerpoint to the rest of the class, to a Year 8 class, to parents at the subject selections.

A LITTLE BIT OF HISTORY

Pythagoras (569 BC - 475 BC) founded a philosophical and religious school. Members, both men and women, were known as Pythagoreans. Several women Pythagoreans became famous philosophers. The mathematikoi, members of the inner circle, obeyed strict rules, had no possessions, and were vegetarians.

The Pythagorean belief that reality was mathematical in nature was fuelled by their observations of music, astronomy, and mathematics. For example, they noticed that strings produced harmonious tones when the ratios of the lengths of the strings were whole numbers.

Pythagoras teaching music.

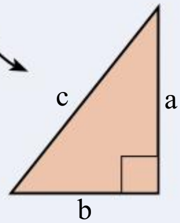
Pythagoras' Theorem

The hypotenuse is the longest side.
It is opposite the right-angle (90°).

In any right-angled triangle:
The square on the hypotenuse is equal to the sum of the squares on the other two sides.

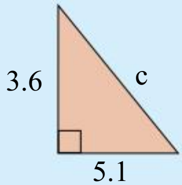
$c^2 = a^2 + b^2$

$a^2 + b^2 = c^2$

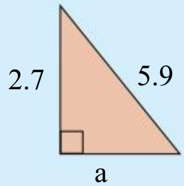


Exercise 17.1

Find the length of the unknown in each of the following (round to two decimal places):

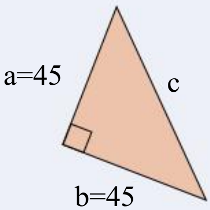


$$c^2 = a^2 + b^2$$
$$c^2 = 3.6^2 + 5.1^2$$
$$c^2 = 38.97$$
$$c = \sqrt{38.97}$$
$$\underline{c = 6.24}$$



$$a^2 + b^2 = c^2$$
$$a^2 + 2.7^2 = 5.9^2$$
$$a^2 = 5.9^2 - 2.7^2$$
$$a^2 = 27.52$$
$$a = \sqrt{3.87}$$
$$\underline{a = 5.25}$$

1



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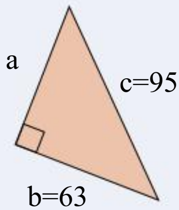

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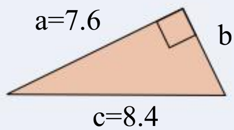
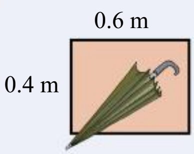


Rounding to two decimal places, first look at the third decimal place:

56.231694 ↑ less than 5 thus **56.23**	27.01769 ↑ 5 or more thus **27.02**	1.07276 ↑ less than 5 thus **1.07**	4.79634216 ↑ 5 or more thus **4.80**

4 What is the length of the diagonal of a 5 m square?

5 Will a 0.8 m umbrella fit in a 0.6 m by 0.4 m case?



6 A 2.8 m ladder is to be laid against a wall so that the top of the ladder is 2 m up the wall. How far out from the base of the wall should the ladder be placed?

7 A 45 m tower is supported by guy wires. The guy wires are attached to the top of the tower and anchored to the ground 18 m out from the base of the tower. How long are the guy wires?

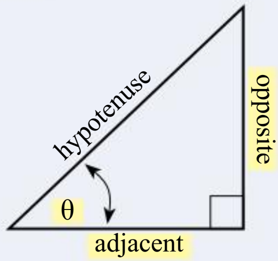
The Tan Ratio

Trigonometry was developed thousands of years ago to solve the many problems in surveying, engineering, architecture, astronomy, etc, etc, etc.

Trigonometry n. branch of mathematics dealing with the relationships between angles and sides of triangles.

Opposite is 'opposite' the angle.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



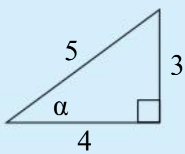
Adjacent means 'next to' the angle.

Greek letters:
α is alpha
θ is theta

Exercise 17.2
Find tan α and the size of the angle α (round to two decimal places):

3 is opposite α

4 is adjacent to α



$$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\alpha = \frac{3}{4} = 0.75$$

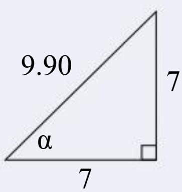
$\alpha = \tan^{-1} 0.75$
$\alpha = 36.87°$

Use your calculator:
2ndF | $\tan^{-1}$ | 0.75 | =

$\tan^{-1}$ means 'an angle whose tan is'.
Thus $\tan^{-1}$ 0.75 means 'an angle whose tan is 0.75' (which is 36.87°).

Make sure your calculator is on **degrees**.

1



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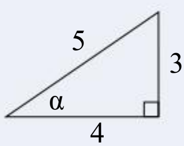

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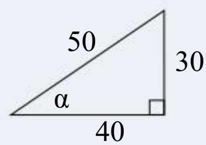

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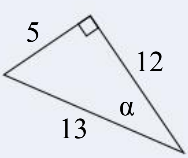

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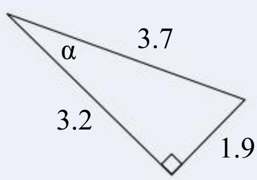

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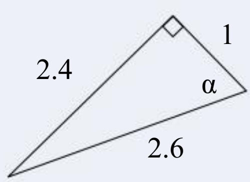
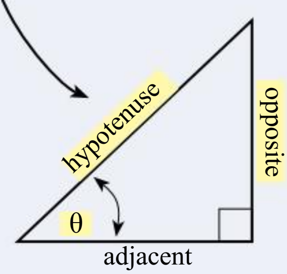


The Sine Ratio

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

The hypotenuse is the longest side.
It is opposite the right-angle (90°).

Opposite is 'opposite' the angle.

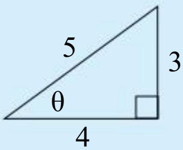


Exercise 17.3

Find sin θ and the size of the angle θ (round to two decimal places):

3 is opposite θ

5 is the hypotenuse



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin\theta = \frac{3}{5} = 0.6$$

$\theta = \sin^{-1} 0.6$

$\theta = 36.87°$

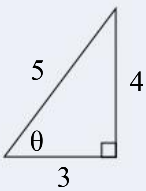
Use your calculator:

2ndF | $\sin^{-1}$ | 0.6 | =

$\sin^{-1}$ means 'an angle whose sine is'.
Thus $\sin^{-1}$ 0.6 means 'an angle whose sine is 0.6' (which is 36.87°).

Make sure your calculator is on **degrees**.

1



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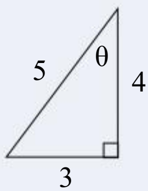

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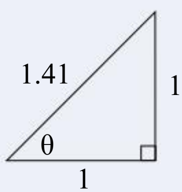

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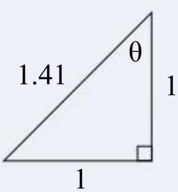

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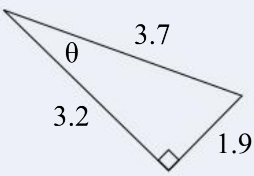

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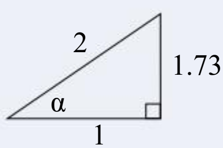
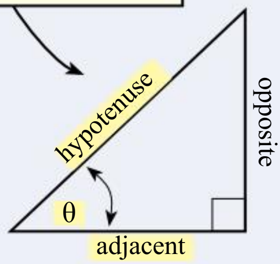


The Cos Ratio

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

The hypotenuse is the longest side.
It is opposite the right-angle (90°).



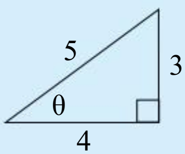
Adjacent means 'next to' the angle.

Exercise 17.4

Find cos θ and the size of the angle θ (round to two decimal places):

4 is adjacent θ

5 is the hypotenuse



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{4}{5} = 0.8$$

$\theta = \cos^{-1} 0.8$

$\theta = 36.87°$

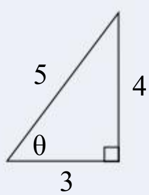
Use your calculator:

2ndF | $\cos^{-1}$ | 0.8 | =

$\cos^{-1}$ means 'an angle whose cosine is'.
Thus $\cos^{-1}$ 0.8 means 'an angle whose cos is 0.8' (which is 36.87°).

Make sure your calculator is on **degrees**.

1



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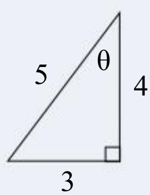

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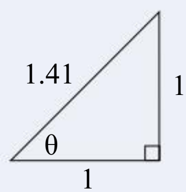

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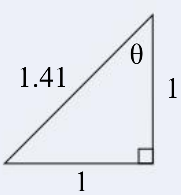

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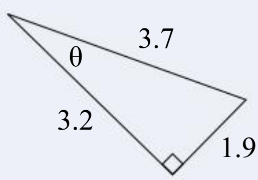

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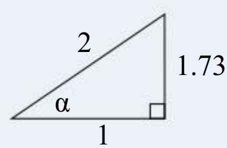


Trigonometry

Trigonometry can be used to find a side after knowing an angle and a side in a right-angled triangle.

The trick is which one?

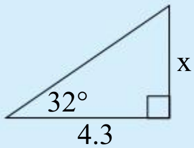
$\sin\alpha = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\cos\alpha = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\tan\alpha = \dfrac{\text{opposite}}{\text{adjacent}}$

Exercise 17.5

Find x in each of the following right-angled triangles:



$\alpha = 32$
adjacent = 4.3
opposite = x

$\sin\alpha = \dfrac{\text{opposite?}}{\text{hypotenuse}}$

$\cos\alpha = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\tan\alpha = \dfrac{\text{opposite?}}{\text{adjacent}}$ ← Use this one

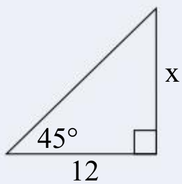
1 Write the knowns and unknown.
2 ✓ the knowns and ? the unknown.
3 The one to use is fully marked.

$\tan 32 = \dfrac{x}{4.3}$

$\tan 32 \times 4.3 = x$

$\underline{2.69 = x}$

1



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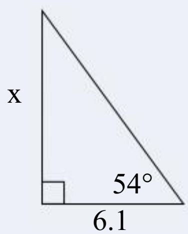

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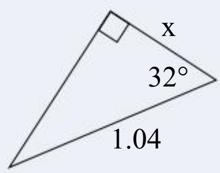
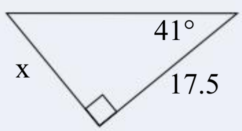


4



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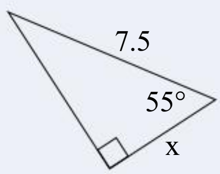

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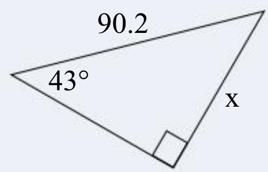
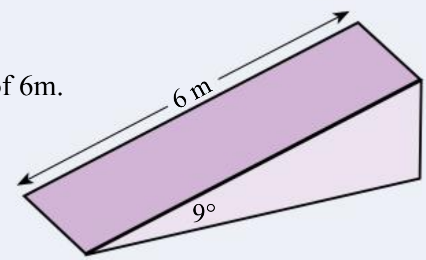


7 A ramp has an incline of 9° and a length of 6m.
What is the length of the horizontal base?



Trigonometry is used millions and millions of times every day.

The trick is which one?

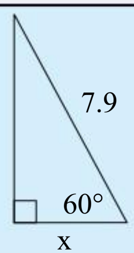
$\sin\alpha = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\cos\alpha = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

$\tan\alpha = \dfrac{\text{opposite}}{\text{adjacent}}$

Exercise 17.6

Find x in each of the following right-angled triangles:



α = 60
hypotenuse = 7.9
adjacent = x

$\sin\alpha = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\cos\alpha = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ ← Use this one

$\tan\alpha = \dfrac{\text{opposite}}{\text{adjacent}}$

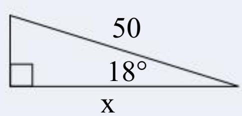
1 Write the knowns and unknown.
2 ✓ the knowns and ? the unknown.
3 The one to use is fully marked.

$$\cos 60 = \frac{x}{7.9}$$

$$\cos 60 \times 7.9 = x$$

$$\underline{3.95 = x}$$

1



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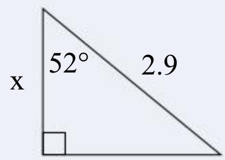

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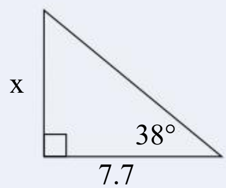
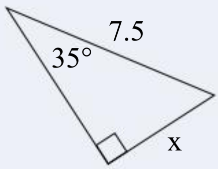


4



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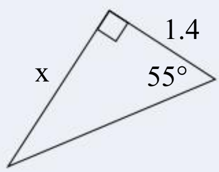

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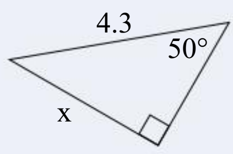


7 The beam of a see-saw is 3.6 m long.
When one end is resting on the ground,
the angle the beam makes with the ground is 17°.
How far above the ground is the other end?

Trigonometry

We have made use of the ratios of corresponding sides of right-angled triangles for thousands of years.

The trick is which one?

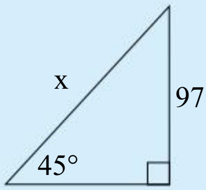
$$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$

Exercise 17.7

Find x in each of the following right-angled triangles:



$\alpha = 45$
opposite = 97
hypotenuse = x

$$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}}$$ Use this one

$$\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$

1 Write the knowns and unknown.
2 ✓ the knowns and ? the unknown.
3 The one to use is fully marked.

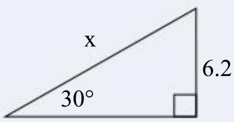
$$\sin 45 = \frac{97}{x}$$

$$\sin 45 \times x = 97$$

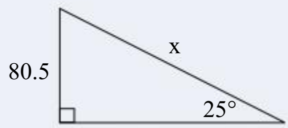
$$x = \frac{97}{\sin 45}$$

$$\underline{137.18 = x}$$

1



2



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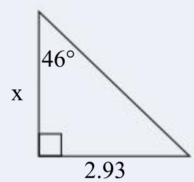

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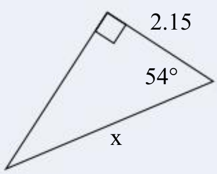

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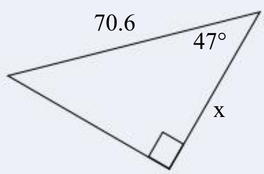

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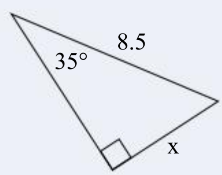
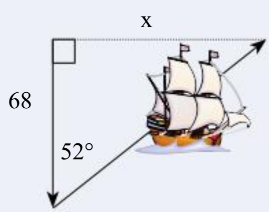


7 A sailing boat sails south for 68 km, then on a bearing of 52°T until it is due east of its starting point.
How far is the boat from its starting point?



Trigonometry can be used to find an **angle** after knowing two sides in a right-angled triangle.

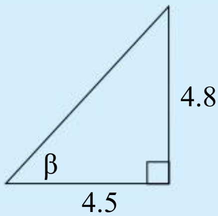
Pythagoras' Theorem can be used to find the third side after knowing two sides in a right-angled triangle.

Greek letters:
α is alpha
β is beta
θ is theta

The angles in a triangle sum to 180°

Exercise 17.8

Find β in each of the following right-angled triangles:



opposite = 4.8
adjacent = 4.5
angle = β

$$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$

Use this one

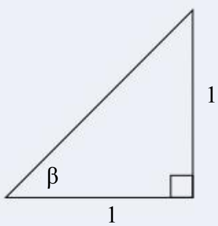
1 Write the knowns and unknown.
2 ✓ the knowns and ? the unknown.
3 The one to use is fully marked.

$$\tan\beta = \frac{4.8}{4.5}$$

$\tan\beta = 1.07$
$\beta = \tan{-1}(1.07)$
$\beta = 46.94°$

Use your calculator:
2ndF | $\tan^{-1}$ | 1.07 | =

1



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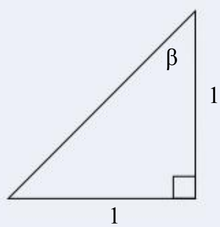
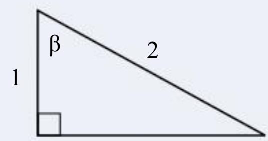


3



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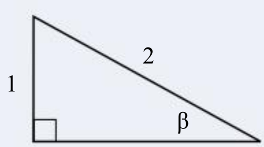

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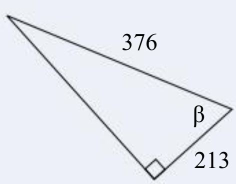

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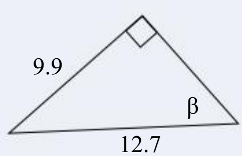
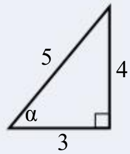


7 What is the pitch, θ, of the roof?

Mental Computation

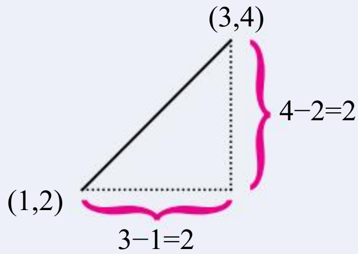
Mental computation gives you practice in thinking.

Exercise 17.9



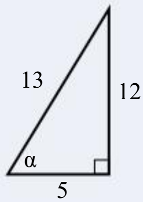
1 Spell Trigonometry.
2 In the triangle, what is sinα?
3 In the triangle, what is cosα?
4 In the triangle, what is tanα?
5 If one angle in a right-angled triangle is 60°, what are the other two angles?

Given the points A(1,2), B(3,4)



6 What is the gradient of AB?
7 What is the length of AB?
8 What is the midpoint of AB?
9 Write in scientific notation: 54 000
10 Circumference of a circle formula?

Exercise 17.10



Two wrongs don't make a right, but three lefts do.

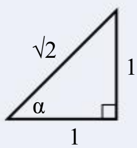
1 Spell Pythagoras.
2 In the triangle, what is sinα?
3 In the triangle, what is cosα?
4 In the triangle, what is tanα?
5 If one angle in a right-angled triangle is 30°, what are the other two angles?

Given the points A(2,2), B(3,4)

6 What is the gradient of AB?
7 What is the length of AB?
8 What is the midpoint of AB?
9 Write in scientific notation: 620 000
10 Area of a circle formula?

A full moon always rises at sunset.

Exercise 17.11



1 Spell Gradient.
2 In the triangle, what is sinα?
3 In the triangle, what is cosα?
4 In the triangle, what is tanα?
5 If one angle in a right-angled triangle is 45°, what are the other two angles?

Given the points A(1,2), B(3,4)

6 What is the gradient of AB?
7 What is the length of AB?
8 What is the midpoint of AB?
9 Write in scientific notation: 4 800 000
10 Circumference of a circle formula?

Hydrographers describe, measure, and map oceans, seas, rivers, and lakes.

- Relevant school subjects are English and Mathematics.
- Courses usually involve a surveying or engineering degree.

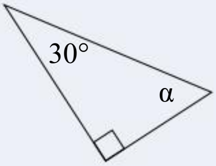
Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

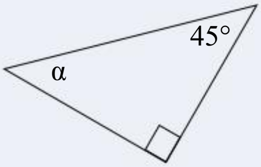
Exercise 17.12

1 What is the value of α in each of the following diagrams:

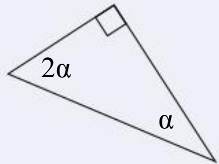
a)



b)

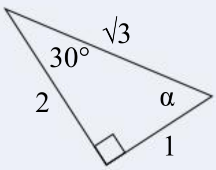


c)

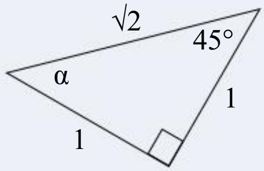


2 What is the value of sinα and cosα in each of the following diagrams:

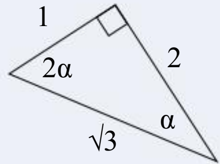
a)



b)

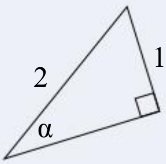


c)

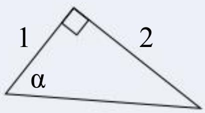


3 What is the value of α in each of the following diagrams:

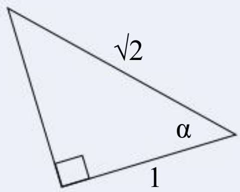
a)



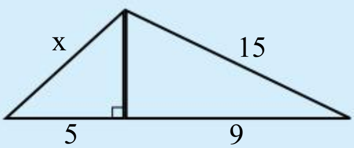
b)

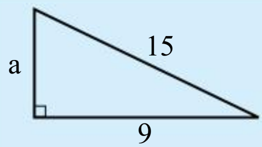


c)

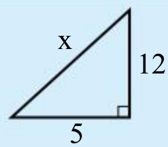


4 Find x in each of the following diagrams:



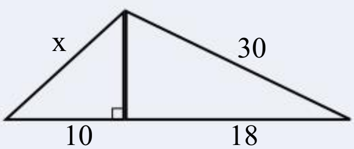


$$15^2 = a^2 + 9^2$$
$$225 - 81 = a^2$$
$$144 = a^2$$
$$12 = a$$

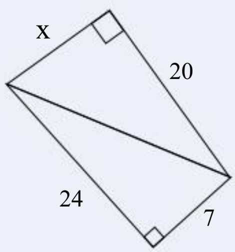


$$x^2 = 5^2 + 12^2$$
$$x^2 = 25 + 144$$
$$x^2 = 169$$
$$\underline{x = 13}$$

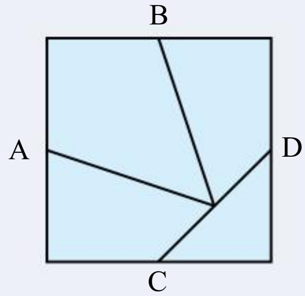
a)



b)



A Couple of Puzzles



Exercise 17.13

1 Can you say the number: 342 482 873 710?

2 If $10^{-3} = \frac{1}{10^3} = \frac{1}{1000} = 0.001$, what is 10^{-4}?

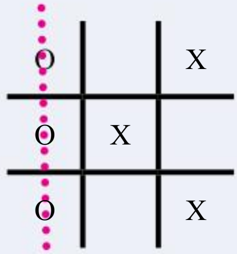
3 What are the next two numbers in the Fibonacci sequence:
1, 1, 2, 3, 5, 8, 13, ...

4 Cut the square into four pieces (A, B, C, D are midpoints).
Can you rearrange the four pieces into an equilateral triangle?

A Game

Nox is played like naughts and crosses.
The **difference** is that the loser is the person who gets three in a line.

Naughts has lost.



1 Decide who is naught and decide who is cross.

2 Take turns placing either one naught or one cross.

3 Keep playing until either there are no places left or someone has lost by having three naughts or three crosses in a line.

A Sweet Trick

1 Without looking, ask your audience to write a secret number between 1 and 100. 54

2 Using a calculator, have them multiply their age by 2, add 5, multiply by 50, and subtract 365. $(14\times2+5)\times50-365$
$= 1285$

Press = after each step.

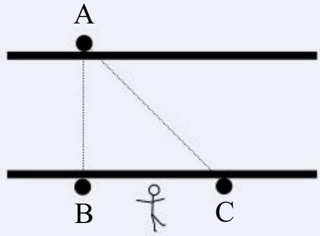
3 Then ask them to add their secret number, and then add 115. $1285+54+115$
$= 1454$

The first half is their age and the second half is their secret number.

How come?

Investigations

Investigation 17.1 Width of a River?



1 Visually find a point on the other side (A).
2 Mark a point on your side of the river (B).
3 Walk and count your paces along your side of the river until you estimate the angle ACB to be 45°.

The width of the river approximately = the number of your paces. Why?

Investigate

Other ways of finding the width of a river

Investigation 17.2 Height of Buildings?

Investigate

Similar ways of finding the height of a building

Hipparchus, around 100 BC, used a similar method above to measure the distance to the moon.

What is your estimate of the distance to the moon?

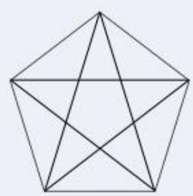
Investigation 17.3 Pythagoras Trees

Investigate

Pythagoras Trees

Investigation 17.4 Construct a Pentagram

Join the vertices of a regular pentagon.



Drawing a regular pentagon, all sides and all angles equal, may be the problem. The Internet suggests a variety of methods of constructing a regular pentagon.

The Pythagoreans used the pentagram as their symbol.
The pentagram is claimed to have magical properties.

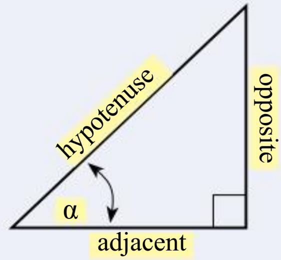
Technology

Use a spreadsheet to solve the previous exercises.

$\sin\alpha = \dfrac{\text{opposite}}{\text{hypotenuse}}$

$\cos\alpha = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

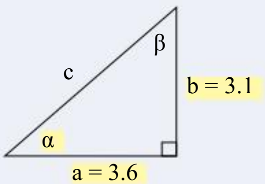
$\tan\alpha = \dfrac{\text{opposite}}{\text{adjacent}}$



Technology 17.1 The Spreadsheet

a) Given the opposite and adjacent, solve the triangle.

	a	b	c	d	e
1	Opposite	Adjacent	α	β	Hypotenuse
2	3.1	3.6	40.73	49.27	4.75



Enter the formula:
=atan(a2/b2)*180/pi()

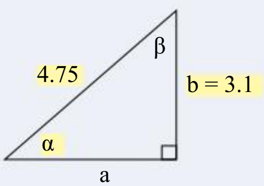
The *180/pi() is needed to convert radians to degrees.

Enter the formula:
=180−90−c2

Enter the formula:
=sqrt(a2*a2+b2*b2)

b) Given the opposite and hypotenuse, solve the triangle.

	a	b	c	d	e
1	Opposite	Hypotenuse	α	β	Adjacent
2	3.1	4.75	40.74	49.26	3.60



Enter the formula:
=asin(a2/b2)*180/pi()

The *180/pi() is needed to convert radians to degrees.

Enter the formula:
=180−90−c2

Enter the formula:
=sqrt(b2*b2−a2*a2)

Technology 17.2 The Internet

Use the Internet to find videos and lessons about trigonometry.

Trigonometry is a branch of mathematics that studies the relationships between the angles and sides of triangles.

What about triangles that are not right-angled?

Chapter Review 1

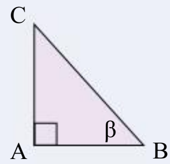
Exercise 17.14

1 For the adjacent triangle, name:

a) the hypotenuse.

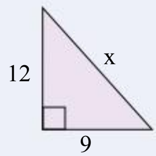
b) the side adjacent to the angle.

c) the side opposite the angle.

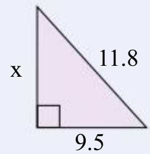


2 Use Pythagoras' Theorem to find the unknown:

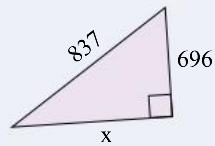
a)



b)

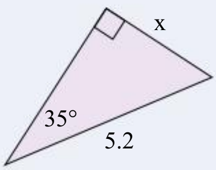


c)

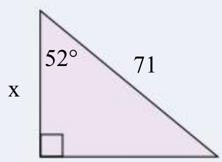


3 Find the unknown in each of the following triangles:

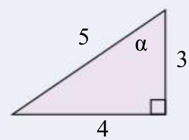
a)



b)



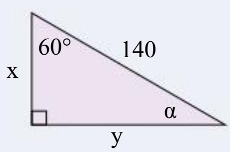
c)



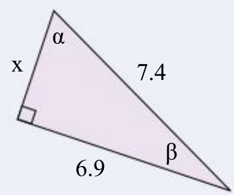
Solve means 'find **all** unknowns'.
(3 sides, 3 angles)

4 Solve the following triangles:

a)



b)



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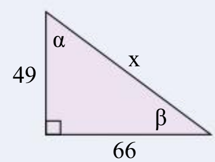
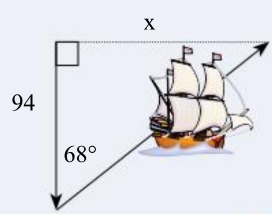
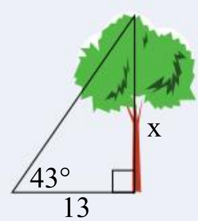


5 A sailing boat sails south for 94 km, then on a bearing of 68°T until it is due east of its starting point.
How far is the boat from its starting point?
How far has the boat travelled?



6 13 m out from the base of a tree, a clinometer measures the angle of elevation to the top of the tree as 43°. Find the height of the tree.

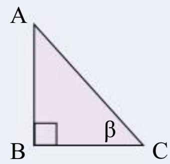


Chapter Review 2

Exercise 17.15

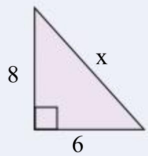
1 For the adjacent triangle, name:

a) the hypotenuse.
b) the side adjacent to the angle.
c) the side opposite the angle.



2 Use Pythagoras' Theorem to find the unknown:

a)



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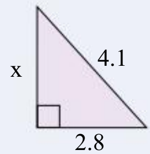

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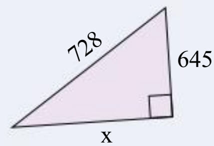
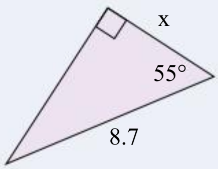
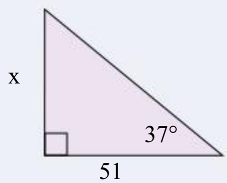


3 Find the unknown in each of the following triangles:

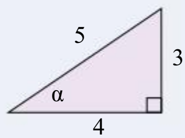
a)



b)



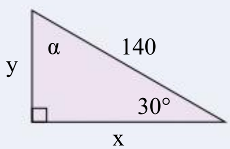
c)



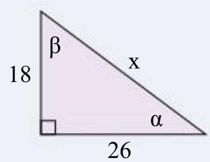
Solve means 'find **all** unknowns'.
(3 sides, 3 angles)

4 Solve the following triangles:

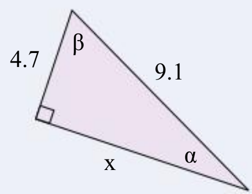
a)



b)

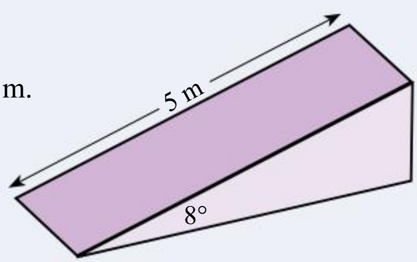


c)



5 The beam of a see-saw is 3.2 m long.
When one end is resting on the ground,
the angle the beam makes with the ground is 23°.
How far above the ground is the other end?

6 A ramp has an incline of 8° and a length of 5 m.
What is the length of the horizontal base?



Algebra 2

18

Number and Algebra → Patterns and algebra

- ★ Apply the distributive law to the expansion of algebraic expressions, including binomials, and collect like terms where appropriate.
- ★ Understand that the distributive law can be applied to algebraic expressions as well as numbers, and understand the inverse relationship between expansion and factorisation.
- ★ Extend and apply the index laws to variables, using positive integral indices.

A TASK

Give me a C: 262×2^{n}
n = 0: Middle C
n = 1: Octave above middle C
n = $^{-}1$: Octave below middle C

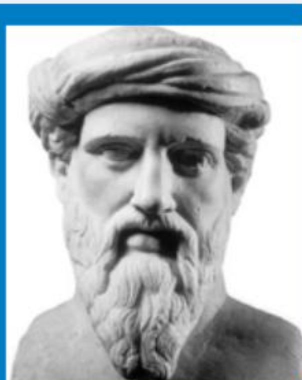
What is the mathematics of music?

- Use the Internet to find the frequencies of the various musical notes and their mathematical relationship.
- Make a poster for your classroom.
- You may wish to download a tone/frequency generator and demonstrate this to your class.
- You may also wish to download a frequency analyzer and use this to analyse the ability of your classmates to sing and hold musical notes (You will need a microphone connected to your computer's soundcard).

A LITTLE BIT OF HISTORY

Pythagoras (569 BC-475 BC) is believed to be the first to establish a mathematical relationship among the musical notes.

The Greek octave was made up of five notes.
Given a string length that produces an A
The next note is 4/5 the length
The next note is 3/4 the length
The next note is 2/3 the length
The next note is 3/5 the length
The next note is 1/2 the length and is again A, an octave above.



An octave above is the same note at a higher pitch.

Integer Warmup

Integers are positive and negative whole numbers.

Exercise 18.1

Calculate the following:

a) $3 - 4 = \underline{^{-}1}$	**b)** $^{-}6 + 2 = \underline{^{-}4}$
c) $^{-}7 - 2 = \underline{^{-}9}$	**d)** $8 - {^{-}1} = \underline{9}$

+	gain, above, positive, plus, etc.
–	loss, below, negative, minus, etc.

1 $7 - 3$	**2** $-2 + 5$	**3** $^{-}5 + 2$	**4** $4 - 7$
5 $^{-}1 - 3$	**6** $5 - {^{-}2}$	**7** $8 - 2$	**8** $1 - 4$
9 $7 - 0$	**10** $^{-}7 - 2$	**11** $^{-}3 + 2$	**12** $^{-}5 - {^{-}11}$
13 $^{-}6 - 5$	**14** $9 - 2$	**15** $14 - {^{-}11}$	**16** $^{-}20 - 3$
17 $6 - 7$	**18** $^{-}4 - 3$	**19** $12 - 8$	**20** $^{-}14 + 8$
21 $20 - 4 - 28 + 7$		**22** $^{-}37 - 23 + 13 - 6$	

Exercise 18.2

Calculate the following:

a) $8 \times 4 = \underline{32}$	**b)** $4 \div {^{-}2} = \underline{^{-}2}$
c) $^{-}5 \times 3 = \underline{^{-}15}$	**d)** $^{-}6 \div {^{-}2} = \underline{3}$

$+ \times + = +$	$+ \div + = +$
$- \times - = +$	$- \div - = +$
$+ \times - = -$	$+ \div - = -$
$- \times + = -$	$- \div + = -$

1 5×4	**2** $4 \times {^{-}7}$	**3** $^{-}10 \div 2$	**4** $^{-}18 \div {^{-}3}$
5 $^{-}3 \times 5$	**6** $^{-}3 \times {^{-}4}$	**7** $8 \div {^{-}2}$	**8** $^{-}16 \div {^{-}8}$
9 $7 \times {^{-}8}$	**10** $^{-}5 \times 8$	**11** $^{-}14 \div 2$	**12** $^{-}28 \div {^{-}7}$
13 $^{-}5 \times 7$	**14** $^{-}4 \times {^{-}9}$	**15** $32 \div {^{-}4}$	**16** $^{-}45 \div 5$
17 $6 \times {^{-}9}$	**18** $^{-}2 \times 7$	**19** $^{-}63 \div 7$	**20** $^{-}56 \div {^{-}7}$

Exercise 18.3

Calculate the following:

Order of Operations:
1 ( ) brackets first.
2 × and ÷ from left to right.
3 + and – from left to right.

a) $^{-}4 \times (3 - 5)$ $= {^{-}4} \times {^{-}2}$ $= \underline{8}$	**b)** $5 \times {^{-}2} - ({^{-}8} + 2) \div {^{-}3}$ $= 5 \times {^{-}2} - {^{-}6} \div {^{-}3}$ { ( ) } $= {^{-}10} - 2$ { × and ÷} $= \underline{^{-}12}$ { + and –}

1 $6 \times (5 - 2)$	**2** $9 + 2 \times {^{-}3}$
3 $^{-}6 \div 2 + 4$	**4** $(2 - 8) \times {^{-}5}$
5 $2 \times {^{-}4} + (1 \times {^{-}4} + 6)$	**6** $^{-}2 \times 4 + 6 \div {^{-}2}$
7 $({^{-}8} + 12) \div 4 - 2$	**8** $^{-}3 - 2 + {^{-}2} \times 6$
9 $9 - 4 - 7 + 5 \times {^{-}2}$	**10** $^{-}12 \div 6 + 4 - (2 + 5 \times {^{-}3})$
11 $2 \times (7 - 11 - 5) + 9$	**12** $^{-}3 \times {^{-}5} - 8 \div {^{-}1} - 2$
13 $16 \div {^{-}4} - 12 \times {^{-}3}$	**14** $^{-}15 - 5 \times {^{-}2} + 3({^{-}13} + 14)$
15 $3 \times {^{-}2} + 7({^{-}4} \div 2 + 1)$	**16** $^{-}4(4 + 5 \times {^{-}3})$
17 $5 - 3 \times {^{-}4} + 1 \times {^{-}2} + 5$	**18** $^{-}3 \times 2 + 2 \times {^{-}2} - (7 + 2 \times {^{-}3})$

Index Law Warmup

Law	Example 1	Example 2
Index Law 1 $a^m \times a^n = a^{m+n}$	$2^4 \times 2^2 = 2^{4+2} = 2^6$	$3x^4 \times 2x^2 = 6x^{4+2} = 6x^6$
Index Law 2 $a^m \div a^n = a^{m-n}$	$5^2 \div 5^5 = 5^{2-5} = 5^{-3}$	$6a^3 \div 3a^2 = 2a^{3-2} = 2a$
Index Law 3 $(a^m)^n = a^{m \times n}$	$(7^3)^2 = 7^{3\times 2} = 7^6$	$(x^3)^2 = x^{3\times 2} = x^6$
Index Law 4 $a^0 = 1$	$3^0 = 1$	$x^0 = 1$
Index Law 5 $a^{-m} = \frac{1}{a^m}$	$10^{-3} = \frac{1}{10^3}$	$x^{-5} = \frac{1}{x^5}$

Algebra is an essential tool in thousands of careers and is fundamental to solving millions of problems.

Exercise 18.4

Use the Index Laws to simplify each of the following:

1 $2^5 \times 2^3$	**2** $x^4 \times x^3$	**3** $3^4 \times 3^5$	**4** $a^7 \times b^3$
5 $x^4 \times x^2$	**6** $b^3 \times b^6$		
7 $4^6 \times 4^5$	**8** $10^2 \times 10^5$		
9 $5^{-3} \times 5^4$	**10** $8.1^2 \times 8.1^5$		
11 $2^4 \div 2^2$	**12** $10^9 \div 10^6$		
13 $c^5 \div c^2$	**14** $3^2 \div 3^4$	**15** $d^2 \div d^6$	**16** $e^3 \div x^6$
17 $3.2^5 \div 3.2^2$	**18** $x^2 \div x^8$	**19** $4^{-4} \div 4^4$	**20** $10^6 \div 10^{-3}$
21 $(5^2)^2$	**22** $(x^2)^3$	**23** $(6^2)^4$	**24** $(x^2)^5$
25 $(y^2)^5$	**26** $(s^{-3})^5$	**27** $(g^2)^{-4}$	**28** $(2^4)^2$
29 $(x^{-2})^{-3}$	**30** $(10^2)^3$	**31** $2a^2 \times 3a^3$	**32** $2b^4 \times 3b^2$
33 $2^5 \div 2^2$	**34** $8c^3 \div 2c^5$	**35** $10^6 \times 10^2 \times 10^4$	**36** $10^9 \times 10^6 \times 10^{-3}$

The bases aren't the same.
The answer is: $a^7 \times b^3$

Algebra Warmup

Algebra is fundamental to solving millions of real world problems.

Exercise 18.5

Simplify the following expressions:

$3x + 2x = \underline{5x}$	$5y - 7y = \underline{{}^{-}2y}$	$9b + 5b - b = \underline{13b}$

1 $4x + 3x$ **2** $7a - 3a$ **3** $4b + 2b$

4 $5x - 2x$ **5** $7c + 4c$ **6** $6y - 3y$

7 $24d + 3d$ **8** $6x - 5x$ **9** $5e + 7e$

10 $3x + x + 5x$ **11** $8f - 2f - 5f$ **12** ${}^{-}3x + 4x + 2x$

$7a + 5 - 4a = \underline{3a + 5}$	$5y^2 + 4y + y - 2y^2 = \underline{3y^2 + 5y}$

The key is to only join together the terms that are alike.

13 $5x + 6 + 2x$ **14** $8 + 3a + 4a$

15 $7 + 6x - 2x$ **16** $5a - 2a + 4b$

17 $4x + 6x + 3$ **18** $6a + 5b - 2a + 3b$

19 $2a^2 - 5 + 7a^2 + 7$ **20** $8x^4 + 7 - 3x^4 + 3$

Exercise 18.6

Multiply the numbers. Multiply the letters.

Simplify the following expressions:

$4 \times 2x = 4 \times 2 \times x = \underline{8x}$	$3d \times 5e = 3 \times d \times 5 \times e = \underline{15de}$	$10b \times \frac{1}{5} = 10 \times \frac{1}{5} \times b = \underline{2b}$

1 $2 \times 5x$ **2** $2 \times 7a$ **3** $5 \times 6e$

4 $6x \times 3$ **5** $2f \times 4n$ **6** $9c \times 3d$

7 $4a \times 2b$ **8** $8t \times 3d$ **9** $x \times 7y$

10 $\frac{m}{2} \times 4$ **11** $8 \times \frac{p}{2}$ **12** $12 \times \frac{k}{4}$

Exercise 18.7

$9a \div 6a$ and $\frac{9a}{6a}$ and $\frac{9}{6}$ and $\frac{3}{2}$ are the same thing.

Simplify the following expressions:

$8x \div 2 = \frac{8x}{2} = \underline{4x}$	$9a \div 6a = \frac{9a}{6a} = \underline{\frac{3}{2}}$	$18ay \div 4a = \frac{18ay}{4a} = \underline{\frac{9y}{2}}$

1 $6x \div 2$ **2** $16x \div 4$ **3** $8a \div 2$

4 $20a \div 5$ **5** $30y \div 3$ **6** $9k \div 3$

7 $18b \div 9$ **8** $15x \div 10$ **9** $12x \div 9$

10 $25x \div 5x$ **11** $15e \div 5e$ **12** $10y \div 4y$

Multiply the numbers.
Multiply the letters.

Index Law 1

$a^m \times a^n = a^{m+n}$

+ times − = −
− times + = −
− times − = +

Exercise 18.8

Simplify the following expressions:

$4a \times 3a = 4 \times 3 \times a \times a$ $= \underline{12a^2}$	$5mn \times {}^-2m^2n = 5 \times {}^-2 \times m \times m^2 \times n \times n$ $= \underline{{}^-10m^3n^2}$

m x m² = m x m x m = m³

1	$4x \times 3x$	**2**	$3d \times 4d$		
3	$5a \times 3a$	**4**	$7d \times 3d$		
5	$5x \times {}^-2x$	**6**	$2x \times {}^-3x$	**7**	${}^-3x \times 4x$
8	${}^-4a^2 \times {}^-3a$	**9**	$6p \times {}^-2p^3$	**10**	${}^-9w \times 3w^2$
11	$8s^3 \times 2s^2$	**12**	$4x^2 \times 5x \times 2x$	**13**	$3e \times e^2 \times 2d$
14	$5mn \times {}^-3m^2n$	**15**	$7pn \times {}^-4p^2n$	**16**	$4ab \times {}^-6a^2b$
17	${}^-3p^2d \times {}^-2pd$	**18**	${}^-4h^2 \times {}^-4h$	**19**	${}^-4a^2b^2c \times {}^-5a^2bc$

Divide the numbers.
Divide the letters

Index Law 2

$a^m \div a^n = a^{m-n}$

+ divided by − = −
− divided by + = −
− divided by − = +

$8x \div 4 = \underline{2x}$ $12x^5 \div 4x^2 = 3x^{5-2}$ $= \underline{3x^3}$	${}^-6x^5 \div 4x^2y = \dfrac{-6x^5}{4x^2y}$ $= \underline{\dfrac{-3x^3}{2y}}$

Calculators are very good at handling fractions:
See Technology 18.1

20	$10x \div 5$	**21**	$16a \div 4$		
22	$12x \div 3$	**23**	$14d \div 7$		
24	$14x^6 \div 2x^3$	**25**	$21x^7 \div 3x^4$	**26**	$8x^4 \div 4x^2$
27	${}^-8x \div 4$	**28**	$6x \div {}^-3$	**29**	${}^-10a \div {}^-2a$
30	${}^-4g^3 \div 2g^2$	**31**	${}^-12a^6 \div {}^-4a^2$	**32**	$8x^6 \div {}^-4x^4$
33	$16x^5y \div {}^-4x^3$	**34**	${}^-14x^4y \div {}^-7x^2$	**35**	${}^-20ab \div 4b$
36	${}^-16a^9c^2 \div 12a^6$	**37**	${}^-24e^5f^3 \div {}^-12e^2$	**38**	${}^-21a^5b^6c \div 28a^4b^3$

Distributive Law

$$a(b + c) = ab + ac$$

Multiply each inside term by the outside term.

Distribute - to spread out, to cover everything.

$+$ times $+ = +$
$+$ times $- = -$
$-$ times $+ = -$
$-$ times $- = +$

Exercise 18.9

Expand each of the following:

$4(a + 3) = \underline{4a + 12}$	$3(2b - 5) = \underline{6b - 15}$
$^{-}4(a + 3) = \underline{^{-}4a - 12}$	$^{-}3(2b - 5) = \underline{^{-}6b + 15}$
$5w(3w - 2m)$ $= \underline{15w^2 - 10mw}$	$^{-}5w(3w - 2m)$ $= \underline{^{-}15w^2 + 10mw}$

1 $4(x + 3)$ **2** $3(a + 2)$ **3** $8(b + 5)$

4 $^{-}4(x + 3)$ **5** $^{-}2(y + 5)$ **6** $^{-}3(d + 7)$

7 $2(3x - 4)$ **8** $6(2x - 3)$ **9** $5(4a - 3)$

10 $^{-}5(2x - 3)$ **11** $^{-}2(2y - 5)$ **12** $^{-}4(5n - 1)$

13 $x(2x + 3)$ **14** $4b(3b + 2)$ **15** $6f(5f - 3)$

16 $^{-}x(5 + 2x)$ **17** $^{-}8y(3x - 2y)$ **18** $^{-}5u(5u - 2v)$

Exercise 18.10

Simplify each of the following by expanding and then collecting like terms:

$8(2x + 3) + 5x + 7$ $= 16x + 24 + 5x + 7$ $= \underline{21x + 31}$	$3(5a - 2) + 3a - 9$ $= 15a - 6 + 3a - 9$ $= \underline{18a - 15}$

1 $5(x + 3) + 2x + 1$ **2** $3(x + 3) + 2x + 4$

3 $7(x - 2) + 4x + 19$ **4** $5(2a - 6) - 10a + 3$

$3(x + 2) + 2(x + 4)$ $= 3x + 6 + 2x + 8$ $= \underline{5x + 14}$	$^{-}5(x + 3) + 3(x - 1)$ $= ^{-}5x - 15 + 3x - 3$ $= \underline{^{-}2x - 18}$

5 $3(x + 3) + 2(x + 1)$ **6** $^{-}5(x + 3) + 3(x + 2)$

7 $4(a + 1) + 5(a + 3)$ **8** $^{-}2(c + 3) + 3(c + 2)$

9 $3(b + 2) + 2(b + 1)$ **10** $^{-}4(y + 5) + 2(y - 2)$

11 $5(m + 2) + 4(m + 2)$ **12** $8(x - 1) + 6(x - 1)$

13 $2(x - 2) + 3(x + 5)$ **14** $^{-}2(d - 3) + 3(d + 3)$

15 $^{-}x(3x + 1) + x(3x - 2)$ **16** $^{-}2y(3y - 2) + ^{-}4(y - 2)$

Factorisation

$$ab + ac = a(b + c)$$

The common term, a, is taken out and put at the front.

Factorisation is the inverse of distribution.

Exercise 18.11

Find the highest common factor of each of the following pairs of terms:

3a and 6a The factors of 3a are: 3, a The factors of 6a are: 2, 3, 6, a The highest common factor = 3a	4ef and 8fg The factors of 4ef are: 2, 4, e, f The factors of 6a are: 2, 4, 8, f, g The highest common factor = 4f

1	3b and 6b	**2**	4a and 8a	**3**	4b and 10a
4	8 and 12y	**5**	3x and 9x	**6**	5 and 10d
7	4ab and 8ac	**8**	4ab and 16ac	**9**	8a and 2b
10	21x and 6	**11**	15xy and 5y	**12**	12ef and 18fg

The highest common factor is taken out and put at the front.

Exercise 18.12

Factorise each of the following:

$2x + 6$ $= 2\times x + 2\times 3$ $= 2(x + 3)$	$8a + 4$ $= 4\times 2a + 4\times 1$ $= 4(2a + 1)$	$6x^2 + 9x$ $= 3x\times 2x + 3x\times 3$ $= 3x(2x + 3)$

1	$2a + 6$	**2**	$8x + 4$	**3**	$2c^2 + 10c$
4	$6x + 4$	**5**	$12x + 8$	**6**	$12b + 9$
7	$9y + 6$	**8**	$4c + 14$	**9**	$3d + 12$

$4x - 12$ $= 4\times x - 4\times 3$ $= 4(x - 3)$	$6ab - 8b$ $= 2b\times 3a - 2b\times 4$ $= 2b(3a - 4)$	$7t^2 - 5t$ $= t\times 7t - t\times 5$ $= t(7t - 5)$

10	$4g - 12$	**11**	$6xy - 8x$	**12**	$5x^2 - 3x$
13	$9t - 6$	**14**	$7xy - 14x$	**15**	$3d^2 - 9d$
16	$6a + 4$	**17**	$15m + 5$	**18**	$9r^2 + 3r$
19	$14p - 4$	**20**	$10st + 5t$	**21**	$12b^2 - 3b$
22	$10u + 15$	**23**	$18bc - 6c$	**24**	$14e^2 + 21e$
25	$10h - 12$	**26**	$21d + 15$	**27**	$24x^3 - 15x^2$

Distributive Law

The Distributive Law: $a(b + c) = ab + ac$

Exercise 18.13

Simplify each of the following by expanding and then collecting like terms:

$4(x + 3) - 3(x + 4)$	$^{-}4(x - 1) - 2(x - 4)$
$= 4x + 12 - 3x - 12$	$= {^{-}4x} + 4 - 2x + 8$
$= \underline{x}$	$= \underline{{^{-}6x} + 12}$

1 $3(x + 4) - 2(x + 2)$

2 $^{-}4(x - 1) - 2(x - 2)$

3 $2(x + 5) - 3(x + 4)$

4 $^{-}5(a + 2) - 2(a + 6)$

5 $4(b + 1) - 5(b - 2)$

6 $^{-}2(c + 2) - 2(c - 3)$

7 $5(n - 1) - 2(n - 4)$

8 $^{-}6(y - 1) - 5(y - 2)$

+ times + = +
+ times − = −
− times + = −
− times − = +

$$\mathbf{(a + b)(c + d) = a(c + d) + b(c + d)}$$

$(x + 5)(x + 4)$	$(x + 5)(x - 3)$
$= x(x + 4) + 5(x + 4)$	$= x(x - 3) + 5(x - 3)$
$= x^2 + 4x + 5x + 20$	$= x^2 - 3x + 5x - 15$
$= \underline{x^2 + 9x + 20}$	$= \underline{x^2 + 2x - 15}$

9 $(x + 2)(x + 1)$

10 $(x + 2)(x - 1)$

11 $(x + 3)(x + 1)$

12 $(x + 3)(x - 1)$

13 $(x + 2)(x + 3)$

14 $(x + 2)(x - 3)$

15 $(x + 5)(x + 2)$

16 $(x + 5)(x - 2)$

$(x + 3)^2 = (x + 3)(x + 3)$	$(x - 4)^2 = (x - 4)(x - 4)$
$= x(x + 3) + 3(x + 3)$	$= x(x - 4) - 4(x - 4)$
$= x^2 + 3x + 3x + 9$	$= x^2 - 4x - 4x + 16$
$= \underline{x^2 + 6x + 9}$	$= \underline{x^2 - 8x + 16}$

17 $(x + 1)^2$

18 $(x - 1)^2$

19 $(x + 2)^2$

20 $(x - 2)^2$

21 $(x + 3)^2$

22 $(x - 3)^2$

23 $(x + 4)^2$

24 $(x - 4)^2$

$1 \times 9 + 2 = 11$
$12 \times 9 + 3 = 111$
$123 \times 9 + 4 = 1111$

Factorisation

Distribute

a(b + c) → **ab + ac**

Factorise

Algebra is an essential tool in thousands of careers and is fundamental to solving millions of problems.

Factorisation is the inverse of distribution.

Exercise 18.14

Factorise each of the following:

$6x + 9$ $= 3\times 2x + 3\times 3$ $= 3(2x + 3)$	$4xy - 6x$ $= 2x\times 2y - 2x\times 3$ $= 2x(2y - 3)$	$10x^2 - 8x$ $= 2x\times 5x - 2x\times 4$ $= 2x(5x - 4)$

1 $6a + 9$	**2** $4ab - 6a$	**3** $10c^2 - 8c$
4 $14x + 10$	**5** $4ab - 6b$	**6** $8d^2 - 6d$
7 $8b + 4$	**8** $8xy - 10x$	**9** $9r^2 - 6r$
10 $9c + 12$	**11** $8xy + 10x$	**12** $16x^2 - 12x$
13 $6x + 10$	**14** $12st - 15t$	**15** $15p^5 - 36p^3$

$$\mathbf{c(a + b) + d(a + b) = (a + b)(c + d)}$$

$x(x + 5) + 4(x + 5)$ $= (x + 5)(x + 4)$	$x(x + 5) - 3(x + 5)$ $= (x + 5)(x - 3)$

16 $x(x + 5) + 3(x + 5)$	**17** $x(x + 5) - 4(x + 5)$
18 $x(x + 2) + 5(x + 2)$	**19** $x(x + 2) - 5(x + 2)$
20 $x(x + 9) + 4(x + 9)$	**21** $x(x + 3) - 2(x + 3)$

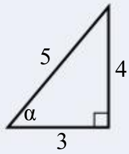
$x(x - 5) + 4(x - 5)$ $= (x - 5)(x + 4)$	$x(x - 5) - 3(x - 5)$ $= (x - 5)(x - 3)$

22 $x(x - 1) + 4(x - 1)$	**23** $x(x - 1) - 2(x - 1)$
24 $x(x - 6) + 3(x - 6)$	**25** $x(x - 5) - 4(x - 5)$
26 $x(x - 2) + 5(x - 2)$	**27** $x(x - 3) - 7(x - 3)$

Mental Computation

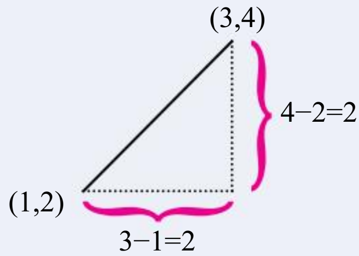
You need to be a good mental athlete because many everyday problems are solved mentally.

Exercise 18.15



1 Spell Distributive.
2 Simplify: $3x + 5x$
3 Expand: $3(x - 3)$
4 Factorise: $8x + 4$
5 In the triangle, what is $\sin\alpha$?

Given the points A(1,2), B(3,4)

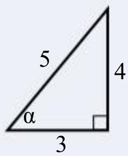


6 What is the gradient of AB?
7 What is the length of AB?
8 What is the midpoint of AB?
9 Write in scientific notation: 31 000
10 24×9

$$\begin{aligned}24\times9 &= 24\times10 - 24\\ &= 240 - 24\\ &= 216\end{aligned}$$

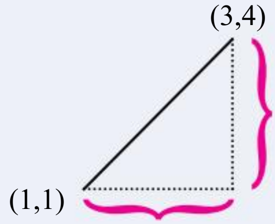
To multiply by 9, multiply by 10 and then subtract the number.

Exercise 18.16



1 Spell Factorise.
2 Simplify: $5a + 6a$
3 Expand: $2(x - 5)$
4 Factorise: $6x + 10$
5 In the triangle, what is $\cos\alpha$?

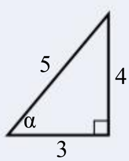
Given the points A(1,1), B(3,4)



6 What is the gradient of AB?
7 What is the length of AB?
8 What is the midpoint of AB?
9 Write in scientific notation: 540 000
10 33×9

It is better to light one small candle than to curse the darkness - Confucius.

Exercise 18.17



1 Spell Algebra.
2 Simplify: $4a + a$
3 Expand: $4(x - 7)$
4 Factorise: $2x + 8$
5 In the triangle, what is $\tan\alpha$?

Given the points A(2,3), B(4,6)

6 What is the gradient of AB?
7 What is the length of AB?
8 What is the midpoint of AB?
9 Write in scientific notation: 2 300 000
10 67×9

If you're in a vehicle going the speed of light, what happens when you turn on the headlights?

Competition Questions

Build maths muscle and prepare for mathematics competitions at the same time.

Order of Operations:
1 ( ) brackets first.
2 × and ÷ from left to right.
3 + and − from left to right.

Exercise 18.18

1 Evaluate each of the following:

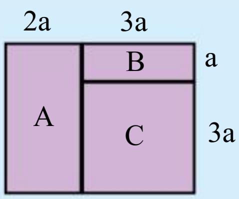
a) $1 + 2 \times 3 - 4$
b) $12 \div (3 \times 4) \times 5$
c) $6 \times 5 - 4 \times 3 \div 2 + 1$
d) $(6 - 5) - (4 - 3) - (2 - 1)$
e) $6 - (5 - 4) - (3 - 2) - 1$
f) $(6 - 5 - 4) - (3 - 2 - 1)$

2 Simplify each of the following:

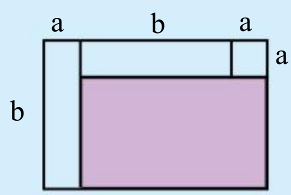
a) $10^3 \times 10^2$
b) $10^5 \div 10^3$
c) $10^2 \div 10^3 \times 10^4$
d) $10^7 \div 10^9 \times 10^2$

$10^7 \div 10^5$
$= 10^{7-5}$
$= \underline{10^2 \text{ or } 100}$

3 Find the shaded area of each of the following figures:

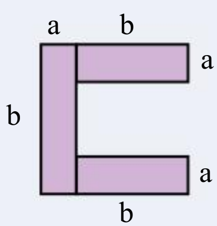


Area = A + B + C
$= 2a \times 4a + 3a \times a + 3a \times 3a$
$= 8a^2 + 3a^2 + 9a^2$
$= \underline{20a^2 \text{ units}^2}$

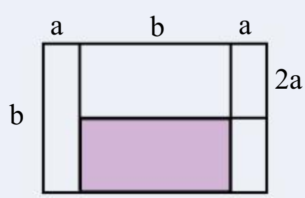


Area $= (a + b)\times(b - a)$
$= a(b - a) + b(b - a)$
$= ab - a^2 + b^2 - ab$
$= \underline{b^2 - a^2 \text{ units}^2}$

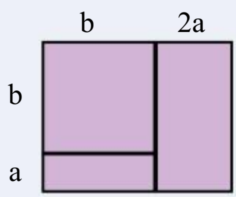
a)



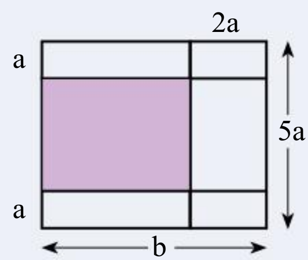
b)



c)

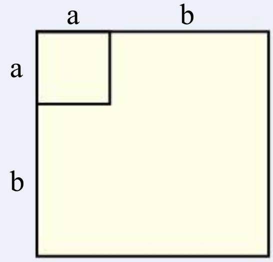


d)



Investigations

Investigation 18.1 $(a + b)(a + b) = a^2 + 2ab + b^2$
$(a + b)(a - b) = a^2 - b^2$



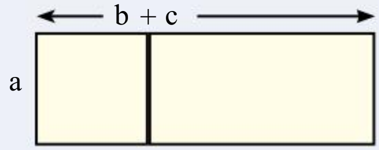
Use one of the below methods to prove either:

$$(a + b)(a + b) = a^2 + 2ab + b^2$$

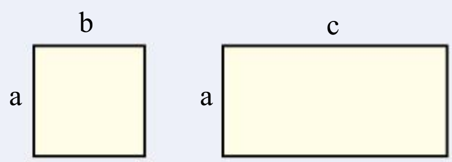
or $$(a + b)(a - b) = a^2 - b^2$$

Investigation18.2 **The Distributive law: $a(b + c) = ab + ac$**

1 Write an algebraic expression for the area of the rectangle shown on the right.



2 Write an algebraic expression for each of the two rectangles shown on the right and sum them together.



3 What do you notice?

Investigation 18.3 **The Distributive law: $a(b + c) = ab + ac$**
Use centicubes or counters to prove the distributive law for each of the following instances:

1 $2(3 + 1) = 2x3 + 2x1$

2 lots of (3 + 1) is the same as 2 lots of 3 and 2 lots of 1

2 3(2 + 1)= 3x2 + 3x1

3 2(4 + 2) = 2x4 + 2x2

4 4(3 + 2) = 4x3 + 4x2

Dental Hygientists provide preventative health care.
- Relevant school subjects are Mathematics and English.
- Courses usually involve a Universtity degree.

A Couple of Puzzles

Exercise 18.19

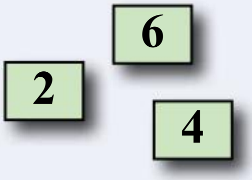
1 If a person breathes seven litres of air every minute,
How much air does a person breathe in a day?

2 What are the values of a, b, and c?

$a + b = 10$

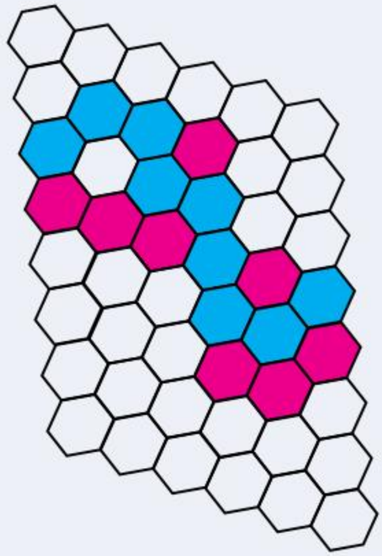
$a + c = 12$

$b + c = 14$



3 Use each of the digits 2, 4, and 6 to make as many three-digit numbers as you can.
How many three-digit numbers can you make?

A Game



Hex

1 Each player starts on opposite sides.

2 Two players take turns in marking a hex cell.

3 The winner is the player who makes a continuous line from one side to the other.

4 Hex boards are on the Internet.

A Sweet Trick

1	Ask your audience to write their age on a piece of paper.	14
2	Double the number.	$14 \times 2 = 28$
3	Add two.	$28 + 2 = 30$
4	Multiply by five.	$30 \times 5 = 150$
5	Add one.	$150 + 1 = 151$
6	Multiply by ten.	$151 \times 10 = 1510$
7	Add one.	$1510 + 1 = 1511$
8	Add the month of birth. (Jan=1, Feb=2, ... Dec=12).	$1511 + 5 = 1516$

Ask your audience for their four digit number. You tell them their age and month of birth.

Subtract 111 from the number:
$1516 - 111 = 1405$.
First two digits is age: 14
Last two digits is month: May

Technology

Technology 18.1 Simplifying Fractions
Scientific calculators are excellent in working with fractions:

1 Simplify $\frac{15}{35}$ $\boxed{15}$ $\boxed{a\frac{b}{c}}$ $\boxed{35}$ $\boxed{=}$ $\boxed{3r7}$ meaning $\frac{3}{7}$

2 Simplify $\frac{18}{4}$ $\boxed{18}$ $\boxed{a\frac{b}{c}}$ $\boxed{4}$ $\boxed{=}$ $\boxed{4r1r2}$ meaning $4\frac{1}{2}$

To change to a vulgar fraction: $\boxed{2ndF}$ $\boxed{a\frac{b}{c}}$ to give $\boxed{9r2}$ ie $\frac{9}{2}$

3 Use a scientific calculator to simplify the following ratios:
a) 3 : 9 **b)** 9 : 12 **c)** 16 : 24
d) 2.1 : 3.5 **e)** 14.4 : 12.6 **f)** 256 : 1024

Technology 18.2 Expanding and Factorising
Graphics calculators are capable of expanding and factorising:
1 Choose **expand** from the algebra menu.
2 Enter the algebraic expression: 3(4x − 5) to produce 12x − 15

1 Choose **factor** from the algebra menu.
2 Enter the algebraic expression: 12x − 15 to produce 3(4x − 5)

Technology 18.3 The Distributive Law and Factorising
There are a considerable number of resources about the Distributive Law and factorising on the Internet.

Try some of them.

Technology 18.4 Substitution
Use a spreadsheet to check your answers to previous exercises:

Use any substituting value other than 0

	A	B
1	Substituting value	3
2	2x + 6	12
3	2(x + 3)	12

Enter the first expression =2*B1 + 6

Enter the second expression =2*(B1 + 3)

Chapter Review 1

Exercise 18.20

$3x - 8x = {}^{-}5x$

1 Simplify the following expressions:

a) $3x + 2x$	**b)** $5x + 4x$	**c)** $3x - 6x$
d) $7x - 11x$	**e)** $5a - 4a + 2b$	**f)** $3r + 5x - 2r + 3x$
g) $4 \times 2x$	**h)** $5x \times 3$	**i)** ${}^{-}2b \times 6$
j) $4x \times {}^{-}5$	**k)** $4c^2 \times {}^{-}2c^3$	**l)** $5x^2 \times 2x \times 4x$
m) $16x \div 4$	**n)** $24m \div 8$	**o)** ${}^{-}8g \div 6$
p) ${}^{-}15x \div 3$	**q)** ${}^{-}10a \div {}^{-}5$	**r)** $14ab \div 4a$
s) ${}^{-}10t^4 \div {}^{-}4t^2$	**t)** ${}^{-}28a^7b^4c \div 21a^4b^2$	

${}^{-}4(3b - 2) = {}^{-}12b + 8$

2 Expand each of the following:

a) $5(x + 3)$	**b)** $3(a + 1)$	**c)** $6(b + 4)$
d) $4(3x - 1)$	**e)** $7(2c - 3)$	**f)** $5(6d - 2)$
g) ${}^{-}3(2e - 5)$	**h)** ${}^{-}4(x + 8)$	**i)** ${}^{-}9(3f + 5)$
j) ${}^{-}2x(3x + 2)$	**k)** ${}^{-}4x(5x - 6)$	**l)** ${}^{-}m(3m + 2)$
m) ${}^{-}4y(1 + 3y)$	**n)** ${}^{-}4x(2x - 3b)$	**o)** ${}^{-}5x(y - 2x)$

3 Simplify each of the following by expanding and then collecting like terms:

a) $3(x + 2) + 4(x + 1)$	**b)** $5(x + 2) + 3(x + 4)$
c) $5(a - 2) + 2(a + 4)$	**d)** ${}^{-}2(a - 2) + 3(a + 2)$
e) ${}^{-}2c(c + 1) + c(4c - 4)$	**f)** ${}^{-}2d(4d - 3) + {}^{-}4(d - 1)$
g) $(x + 3)(x + 2)$	**h)** $(x + 1)(x + 2)$
i) $(x + 3)(x - 1)$	**j)** $(x + 1)(x - 2)$
k) $(x + 2)(x - 2)$	**l)** $(x + 3)(x - 3)$
m) $(x + 1)^2$	**n)** $(x - 2)^2$

$15b - 9 = 3(5b - 3)$ $\quad$ $12ab - 8a = 4a(3b - 2)$

4 Factorise each of the following:

a) $4x + 8$	**b)** $3a + 12$	**c)** $8b + 10$
d) $15x + 6$	**e)** $16c + 12$	**f)** $15x + 12$
g) $6x - 3$	**h)** $2d - 6$	**i)** $4x - 16$
j) $10e - 8$	**k)** $16x - 6$	**l)** $15u - 9$
m) $6ab + 10a$	**n)** $12xy - 10x$	**o)** $8uv - 14u$
p) $14f - 12g$	**q)** $20x^2 - 14x$	**r)** $8n^2 - 18n$
s) $-3x - 15$	**t)** $-2p - 6$	**u)** $-2q + 8$
v) $-4x + 12$	**w)** $-4a^2 + 20a$	**x)** $-3x^2 - 9x$
y) $x(x + 3) + 2(x + 3)$	**z)** $x(x - 2) + 3(x - 2)$	

Chapter Review 2

Exercise 18.21

$3x - 8x = {}^{-}5x$

1 Simplify the following expressions:

a) $6x + 3x$	**b)** $2x + x$	**c)** $10x - 8x$
d) $5x - 9x$	**e)** $6a - 2a + 3b$	**f)** $4r + 5x - 3r + 2x$
g) $3 \times 2x$	**h)** $8x \times 2$	**i)** ${}^{-}3b \times 2$
j) $4x \times {}^{-}3$	**k)** $8d^3 \times {}^{-}4c^2$	**l)** $6x^2 \times 2x \times 3x$
m) $15x \div 3$	**n)** $21a \div 7$	**o)** ${}^{-}6h \div 6$
p) ${}^{-}16x \div 4$	**q)** ${}^{-}12a \div {}^{-}4$	**r)** $15cd \div 5d$
s) ${}^{-}18x^5 \div {}^{-}9x^2$	**t)** ${}^{-}30a^5b^4c \div 12a^4b^2$	

${}^{-}4(3b - 2) = {}^{-}12b + 8$

2 Expand each of the following:

a) $6(x + 4)$	**b)** $5(x + 2)$	**c)** $6(a + 3)$
d) $4(2x - 1)$	**e)** $3(2b - 1)$	**f)** $8(3c - 2)$
g) ${}^{-}5(4d - 2)$	**h)** ${}^{-}3(x + 5)$	**i)** ${}^{-}7(4e + 1)$
j) ${}^{-}x(2x + 5)$	**k)** ${}^{-}5x(x - 3)$	**l)** ${}^{-}p(5p + 3)$
m) ${}^{-}4d(1 + 3d)$	**n)** ${}^{-}7x(2x - 3a)$	**o)** ${}^{-}4x(y - 3x)$

3 Simplify each of the following by expanding and then collecting like terms:

a) $5(x + 1) + 4(x + 3)$	**b)** $4(x + 2) + 3(x + 5)$
c) $6(a - 3) + 2(a + 5)$	**d)** ${}^{-}3(x - 4) + 2(x + 2)$
e) ${}^{-}4x(x + 1) + x(3x - 2)$	**f)** ${}^{-}5d(2d - 1) + {}^{-}4(d - 5)$
g) $(x + 2)(x + 1)$	**h)** $(x + 1)(x + 2)$
i) $(x + 5)(x - 1)$	**j)** $(x + 3)(x - 2)$
k) $(x + 1)(x - 1)$	**l)** $(x + 2)(x - 2)$
m) $(x + 2)^2$	**n)** $(x - 1)^2$

$15b - 9 = 3(5b - 3)$ $12ab - 8a = 4a(3b - 2)$

4 Factorise each of the following:

a) $5x + 10$	**b)** $4x + 12$	**c)** $6x + 10$
d) $12a + 4$	**e)** $16b + 10$	**f)** $15c + 9$
g) $9x - 3$	**h)** $3d - 6$	**i)** $4e - 10$
j) $15x - 12$	**k)** $14x - 8$	**l)** $18x - 12$
m) $5xy + 10x$	**n)** $10xy - 12x$	**o)** $20uv - 15u$
p) $18x^2 - 20x$	**q)** $25u^2 - 15u$	**r)** $6t^2 - 12t$
s) $-4x - 6$	**t)** $-3n - 9$	**u)** $-2m + 8$
v) $-3x + 15$	**w)** $-6g^2 + 20g$	**x)** $-4x^2 - 12x$
y) $x(x + 2) + 5(x + 2)$	**z)** $x(x - 1) + 6(x - 1)$	

Data

19

Statistics and Probability ➞ Data Representation & Interpretation

- ★ Investigate techniques for collecting data, including census, sampling and observation.
- ★ Investigate reports of surveys in digital media and elsewhere for information on how data were obtained to estimate population means and medians.

What do you say to an annoying beetle? Stop bugging me.

A TASK

There are hundreds of examples of costly mistakes.

- Research some disasters caused by faulty data (how were the mistakes made?).
- What are common student mistakes?
- How best to reduce mistakes?
- Present your research.

A LITTLE BIT OF HISTORY

A computer bug is a mistake or error in computer software that produces faulty results.

1947 The first computer bug was a moth stuck in a relay of an early computer.

1962 Incorrect code produces incorrect trajectory for the Mariner I space probe. The probe is destroyed.

1985 A bug in the software sometimes results in lethal radiation doses from the Therac-25 medical accelerator. At least five people died.

The moth in Relay 70, Panel F causing the multiplier and adder to malfunction.

Data

Collecting data is the foundation of an intelligent world. The better the data the better our decisions.

Data

Quantitative
Data of this type is represented by numbers.

Height (1.47 m, 1.56 m, 1.49 m, etc.)
Temperature (21°C, 21°C, 21°C, etc.)
Cost of bread ($2.55, $3.22, $4.15, etc.)
Number of people (23, 18, 26, etc.)
Voltage (240v, 110v, 220v, etc)

Qualitative
Data of this type can be put into categories (no numbers).

Country (Australia, Thailand, Japan, etc.)
Gender (female, male).
Size (large, medium, small, etc.)
Breed (angus, hereford, shorthorn, etc.)
Colour (blue, red, green, etc.)

Exercise 19.1

Describe the following data as quantitative or qualitative:

The distance between towns The distance would be a number ∴ Quantitative data	The Year 9 subject list The subject list would be Maths, etc. ∴ Qualitative data (no numbers)

1 The heights of students in your class.
2 The occupation of people in the postcode area.
3 The weights of students in your class.
4 The country of birth of people in the postcode area.
5 The marital status of people in the postcode area.

Quantitative Data

Discrete
This type of data is counted.

Number of people (45, 3, 154, etc.)
Shoe size (6, 7, 8, 9, 10, etc.)
TVs in the house (2, 4, 3, etc.)
Text messages per day (13, 23, 8, etc)

Continuous
This type of data is measured.

Weight (2.14 kg, 3.09 kg, 1.16 kg, etc.)
Height (1.66 m, 1.57 m, 1,75 m, etc.)
Time to run 100 m (13.2 s, 14.6 s, etc.)
Area (2.6 cm^2, 34.7 cm^2, 3.65 cm^2, etc.)

Exercise 19.2

Describe the following quantitative data as discrete or continuous:

1 The time taken to walk 400 m.
2 The number of students in the room.
3 The daily maximum temperature.
4 The number of hens in each run.
5 The pulse rate before and after exercise.

Collecting Data

The objective is to collect quality data with a minimum of effort.

Census
A collection of data from the **whole** population.

Sample
A collection of data from **part** of the population.

Sample Bias:
The sample doesn't represent the population.

Sample Size:
Must be small enough to be economical but large enough to represent the population.

Example
What percentage of students have part-time jobs?
A census would collect data from **every** student.
A sample would collect data from **some** students.

A **Census** reflects the opinions of the whole population	
Advantages of samples ☺ A sample is more economical. ☺ Information can be gathered more quickly. ☺ A sample uses less resources.	**Disadvantages of samples** ☻ Tricky to get a good sample. ☻ The sample must be small but large enough to reflect the opinion of the population.

Exercise 19.3

1 What is the meaning of each of the following:

a) Census?
b) Sample?
c) Sample bias?
d) Sample size?
e) Random sample?
f) Stratified sample?

It would take 500 people at least 300 years to make a mistake as big as the super-computer made.

2 Comment on the following results
(4-major problem, 3-occasional, 2-minor, 1-no problem):

"Is litter in the playground a problem?

Major problem Occasional problem Minor problem No Problem

Every student:
Number = 452
Mean = 1.7
Litter is a minor problem at this school.

Sample:
Number = 15
Mean = 3.1
Litter can be a problem at this school.

Random sample:
Number = 50
Mean = 2.2
Litter is a minor problem at this school.

Sampling

The larger the sample size the better the population estimation.

The smaller the sample size the less resources used (in time, money etc).

A random sample helps avoid bias because each member of the population has an equal chance of being selected in the sample.

Exercise 19.4

1 Have each member of your class, to themselves, answer the following survey question (4-major problem, 3-occasional, 2-minor, 1-no problem):

"Is litter in the playground a problem?

Major problem | Occasional problem | Minor problem | No Problem

2 Number each member of your class from 1 to 25 (or 28 if there are 28 people in your class).

3 **a)** Generate 5 random numbers from 1 to 25 (See Technology 19.1).

b) Record the survey answers of the 5 class members whose numbers match the 5 random numbers.

c) Calculate the mean.

4 Repeat with random sample sizes of 10, 15, 20

5 Now record the survey answers of the entire class (ie a census of the population.)

Which sample size gives a good balance between error and effort?

Sample Size	Mean
5	
10	
15	
20	
Population	

More effort

More error

Some sources suggest that a good sample size is √n. (ie a sample size of 5 from a class of 25.)

What do you think?

Stratified Sampling

Stratum n. plural strata 1. One of a number of layers.

A stratified sample helps avoid bias because each member of a section of the population has an equal chance of being selected in the sample.

Exercise 19.5

How many students from each Year level of the following school should be randomly selected in a sample size of 30 students?

Year	Number
8	200
9	150
10	150

Total = 500

Yr 8 fraction = 200/500=0.4   No Yr 8s = 0.4×30 = 12

Yr 9 fraction = 150/500=0.3   No Yr 9s = 0.3×30 = 9

Yr 10 fraction = 200/500=0.3   No Yr 10s = 0.3×30 = 9

Total = 30

1 How many students from each Year level should be randomly selected in a sample size of 60 students?

Year	Number
8	200
9	150
10	150

2 How many students from each Year level should be randomly selected in a sample size of 50 students?

Year	Number
7	120
8	120
9	130
10	130

3 How many students of each gender should be randomly selected in a sample size of 10 students.

Gender	Number
Female	34
Male	27

4 How many people of each employee type should be randomly selected in a sample size of 150 employees.

Gender	Type	Number
Female	Full-time	263
Female	Part-time	348
Male	Full-time	176
Male	Part-time	392

Surprisingly, it appears that once the sampling size reaches around 800 there is little accuracy gain with larger sample sizes.

Random sampling technique is the crucial element for accuracy.

Questionnaires

Questions must be clear and unambiguous.
Use language that your intended respondents understand.
Don't lose trust by asking personal or embarrassing questions.
Each answer should be equally likely- no leading questions.
The answers should cover the whole range of possible responses.

Exercise 19.6

What is wrong with each of the following questionnaires?

How do you normally get to school?
(Please circle)
Car Bus Walk

Cyclists? Not all responses are covered.
Adding 'Other' as a response helps.

Maths is (Please ✓)
☐ Fantastic
☐ Wonderful
☐ Not too bad

Leading answers. There is not a full range of responses.

1 How many siblings do you have?

1 2 3 >4

68.237% of data is made up.

Analysis of data shows that people with the most birthdays live longer.

2 The Australian flag is the best.

YES NO

3 How much money do you have?
<$50
>$50

4 What is your favourite colour (Please ✓)?
☐ Red
☐ Orange
☐ Yellow
☐ Green
☐ Blue

It can be helpful to trial your questionnaire first.

Exercise 19.7

Design a questionnaire, survey your class, collate your data, and present your results for one of the following topics:

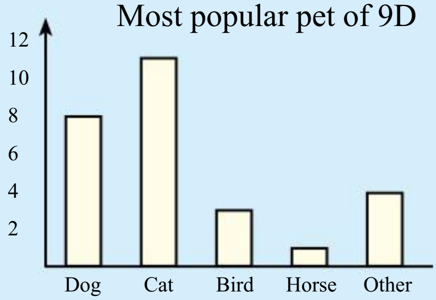
The most popular pet.

Which pet do you like best?

Dog	Cat	Bird	Horse	Other
				

Pet	Number
Dog	8
Cat	11
Bird	3
Horse	1
Other	4

Most popular pet of 9D



1 How do students get to school (car, bus, walk, etc)?
2 Number of brothers and sisters?
3 Favourite sport?
4 Sport lovers (watch more that 4 hrs sport per week)?
5 Favourite school subject?
6 How important is maths?
7 Exercise habits?
8 Study habits?
9 What music should be played at the school dance?
10 How many students have part-time jobs?

Did everyone understand your question?
Did your answers cover every response?
Was anyone offended by your question?
Did someone think your question was biased?
Did you think people answered truthfully?

Exercise 19.8

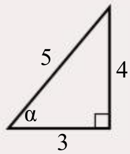
Now sample your school:

a) Use your above experience to redesign your questionnaire if necessary.
b) Decide a sample size based on a balance of effort and error.
c) Design your stratified sample.
d) Calculate the sizes of your stratified samples.
e) Randomly select your samples from each strata.
f) Collect data from your samples.
g) Collate your data and present your results.

Mental Computation

Exercise 19.9

1 Spell stratified.
2 What is a census?
3 What is a sample?
4 Why random sample?
5 Expand: $2(x - 1)$
6 Factorise: $8x + 6$
7 In the triangle, what is $\sin\alpha$?



Mental computation helps you learn how to think.

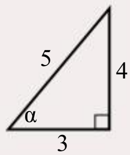
Given the points A(1,1), B(3,4)
8 What is the gradient of AB?
9 Write in scientific notation: 470 000
10 24×15

$$\begin{aligned}24\times15 &= 12\times2\times15\\ &= 12\times30\\ &= 360\end{aligned}$$

Those who live by the sword, get shot by those who don't - Murphy's Law of Combat

Exercise 19.10

1 Spell sampling.
2 What is a census?
3 What is a sample?
4 Why random sample?
5 Expand: $5(x - 2)$
6 Factorise: $10x + 4$
7 In the triangle, what is $\cos\alpha$?

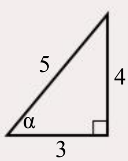


Given the points A(1,2), B(4,3)
8 What is the length of AB?
9 Write in scientific notation: 0.000 04
10 32×15

The enemy invariably attacks on two occasions:
A When you're ready for them.
B When you're not ready for them.
- Murphy's Laws of Combat

Exercise 19.11

1 Spell questionnaire.
2 What is a census?
3 What is a sample?
4 Why random sample?
5 Expand: $3(x - 4)$
6 Factorise: $8x + 4$
7 In the triangle, what is $\tan\alpha$?



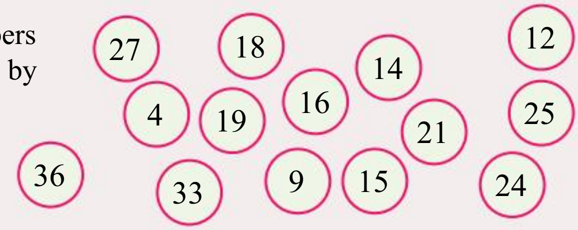
Given the points A(2,1), B(4,5)
8 What is the midpoint of AB?
9 Write in scientific notation: 21 000 000
10 14×15

Competition Questions

Prepare for mathematics competitions and build maths muscle at the same time.

Exercise 19.12

1 If all square numbers and numbers whose sum of digits is divisible by three are removed, what numbers are left?

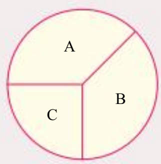


35 students in a random sample of 50 students have brown eyes. Estimate the number of students in a school of 400 with brown eyes.	fraction of brown eyes $= \frac{35}{50} = 0.7$ No students $= 0.7$ of $400 = 280$ 280 students have brown eyes.

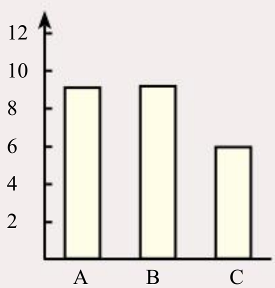
2 A random sample of 70 students was taken from a population of 2100 students. If 42 of the sampled students have brown eyes, estimate the total number of brown-eyed students.

3 A randomly selected sample of 800 respondents suggests that 240 of the respondents favour the policies of the political party. Estimate the number of people, from a population of 23 million people, who favour the policies of the political party.

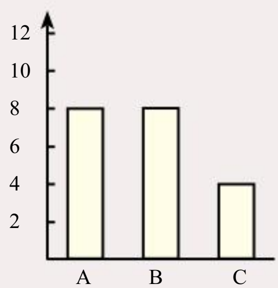
4 Which of the following column graphs represents the pie graph?



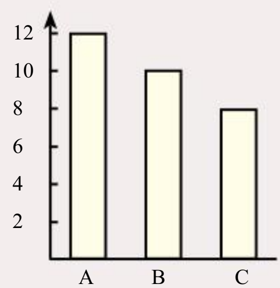
a)



b)



c)



5 Mark throws two dice.
The sum of their upper faces is 7.
Neither of the upper faces are prime.
What are the two numbers?

Investigations

Investigation 19.1 TV Ratings

☺ How are TV ratings made?
☺ Comment on the sample size.
☺ How is sampling bias avoided?
☺ Conduct your own TV Rating.

21% of Australian employees are disengaged in their workplace.

Sample of 1000 working people.
Telephone interviews.
Sampling error ±3%.

Investigation 19.2 Opinion Polls

☺ How are Opinion polls made?
☺ Comment on the sample size.
☺ How is sampling bias avoided?
☺ Find examples of Opinion polls that have been very wrong.
☺ Conduct your own Opinion poll.

Investigation 19.3 Media Bias

☺ Internet search "media bias examples".
☺ Find media articles that involve sampling.
☺ Is the sample size sufficient?
☺ Is the sample biased?

Two people were playing baseball when one of them was attacked by a savage dog. The other person hit the dog with the baseball bat killing the dog.
Newspaper report: Violent thug kills family pet.

Investigation 19.4 Undertake real-life research.

1 Form a team and brainstorm an appropriate problem or issue.

2 Plan the research
- Define the overall research question.
- Define subset research questions.
- Decide how to obtain data to answer the research questions.
- Consider ways of ensuring that the data collection is unbiased.
- Consider the equipment needed for the research.

3 Conduct the research
- Rehearse the data collection method.
- Collect the data.

4 Analyse the data
- Look for errors/outliers and decide what to do with the errors/outliers.
- Calculate the appropriate descriptive statistics.
- Choose an appropriate method of presentation (Histograms etc).

5 Report the conclusions
- Match the analysis with the research questions.
- Do the answers to the research questions indicate further research questions?

Technology

Technology 19.1 Methods of randomly selecting 5 class members

Assuming 26 people in the class.

a) Pen and Paper Method:

Give each member of your class a number from 1 to 26.
Write each number from 1 to 26 on a piece of paper (same sized pieces?).
Put the numbers in a container and randomly select 5 pieces of paper.

b) Calculator Method:

Give each member of your class a number from 1 to 26).
Find the random (Rand) button.
Repeat 5 times:

Rand | **×** | **26** | **+** | **1** | **=**

c) Spreadsheet Method:

Give each member of your class a number from 1 to 26.
Use the RandBetween to give random numbers from 1 to 26.

17	2	24	5	13

Enter the formula:
=RandBetween(1,26)

d) Graphics Calculator Method

Give each member of your class a number from 1 to 26.
Choose the random from the calculator (Math → Prob → randINT).

Enter: **randInt(1,26,5)**

This will give 5 random numbers from 1 to 26.

e) Internet Method

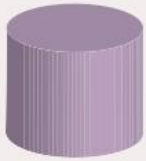
Give each member of your class a number from 1 to 26.
Find a "random number generator" on the Internet.
Follow the instructions.

Software Engineers design, write, and maintain computer software programs

- Relevant school subjects are English and Mathematics.
- Courses usually involve a software engineering degree.

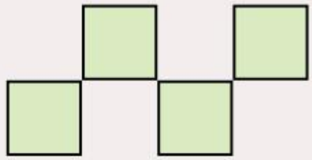
A Couple of Puzzles

Exercise 19.13 Give an estimate of 98 x 0.039



2 A drum is 120 kg when full
and 70 kg when half full.
How heavy is the empty drum?

3 The perimeter of the figure is 16 units.
Rearrange the four squares so that the
perimeter is 10 units, 12 units, and 14 units.



A Game

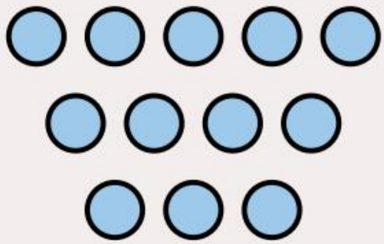
Nim is an ancient game for two players which looks simple but has subtle strategies.

It is possible to play Nim so that you win almost every time.

1 Draw circles in three rows as shown.

2 At each turn, select a row and cross out as many counters in that row as you wish.

3 The last person to take a turn is the winner.



A Sweet Trick

1	Ask your audience to choose any number (other than 0).	143
2	Double the number.	$143 \times 2 = 286$
3	Add 1.	$286 + 1 = 287$
4	Multiply by 5.	$287 \times 5 = 1435$
5	Keep only the last digit.	5
6	Multiply it by itself.	$5 \times 5 = 25$
7	Total the digits.	$2 + 5 = 7$

The answer is always 7.

Chapter Review 1

Qualitative
Data of this type can be put into categories (no numbers).

Quantitative
Data of this type is represented by numbers.

Exercise 19.14

1 Describe the following data as quantitative or qualitative:
- **a)** The heights of students in your class.
- **b)** The occupation of people in the postcode area.

Discrete
This type of data is counted.

Continuous
The data is measured.

2 Describe the following quantitative data as discrete or continuous:
- **a)** The time taken to walk 400 m.
- **b)** The number of students in the room.

3 What is the meaning of each of the following:
a) Census? **b)** Sample? **c)** Random sample?

4 How many students from each Year level should be randomly selected in a sample size of 50 students?

Year	Number
7	140
8	130
9	120
10	130

Questions must be clear and unambiguous.
Use language that your intended respondents understand.
Don't lose trust by asking personal or embarrassing questions.
Each answer should be equally likely- no leading questions.
The answers should cover the whole range of possible responses.

5 What is wrong with each of the following questionnaires?

a)

How do you normally get to school?
(Please circle)
Car Bus Walk

b)

What is your favourite colour (Please ✓)?
- ☐ Red
- ☐ Orange
- ☐ Yellow
- ☐ Green
- ☐ Blue

What grows larger the more you take away?
A hole.

Chapter Review 2

Qualitative
Data of this type can be put into categories (no numbers).

Quantitative
Data of this type is represented by numbers.

Exercise 19.15

1 Describe the following data as quantitative or qualitative:
- **a)** The weights of students in your class.
- **b)** The country of birth of people in the postcode area.

Discrete
This type of data is counted.

Continuous
The data is measured.

2 Describe the following quantitative data as discrete or continuous:
- **a)** The daily maximum temperature.
- **b)** The number of hens in each run.

3 What is the meaning of each of the following:
a) Census? **b)** Sample? **c)** Random sample?

4 How many people of each employee type should be randomly selected in a sample size of 125 employees.

Gender	Type	Number
Female	Full-time	365
Female	Part-time	286
Male	Full-time	297
Male	Part-time	451

Questions must be clear and unambiguous.
Use language that your intended respondents understand.
Don't lose trust by asking personal or embarrassing questions.
Each answer should be equally likely- no leading questions.
The answers should cover the whole range of possible responses.

5 What is wrong with each of the following questionnaires?

a)

Maths is (Please ✓)
☐ Fantastic
☐ Wonderful
☐ Not too bad

b)

The Australian flag is the best.

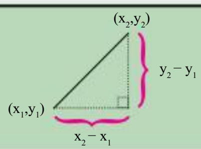
YES NO

From what number can you take half and leave nothing?
The number 8. Take away the top half and 0 is left.

Review 4

20

Chapter 16 Coordinate Geometry



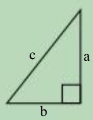
$$\text{Mid} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Chapter 17 Trigonometry 2

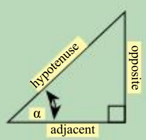
Pythagoras' Theorem



$c^2 = a^2 + b^2$

$a^2 + b^2 = c^2$

Trigonometry Ratios



$$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$

Chapter 18 Algebra 2

$({}^-2)^2 = {}^-2\times{}^-2 = 4$

$({}^-2)^3 = {}^-2\times{}^-2\times{}^-2 = {}^-8$

$({}^-2)^4 = {}^-2\times{}^-2\times{}^-2\times{}^-2 = 16$

Index Law 1

$a^m \times a^n = a^{m+n}$

Index Law 2

$a^m \div a^n = a^{m-n}$

Index Law 3

$(a^m)^n = a^{m\times n}$

Order of Operations:

1 ( ) brackets first.

2 × and ÷ from left to right.

3 + and − from left to right.

Zero Index

$a^0 = 1$

Negative Index

$a^{-m} = \frac{1}{a^m}$

Distributive Law: a(b + c) = ab + ac

$3(2b - 5)$
$= 6b - 15$

${}^-4(x - 1) - 2(x - 4)$
$= {}^-4x + 4 - 2x + 8$
$= {}^-6x + 12$

Factorisation: a(b + c) = ab + ac

$4x - 12$
$= 4(x - 3)$

${}^-6x^2 + 15x$
$= {}^-3x(2x - 5)$

Chapter 19 Data

Census
A collection of data from the **whole** population.

Sample
A collection of data from **part** of the population.

Sample Bias:
The sample doesn't represent the population.

A stratified sample helps avoid bias because each member of a section of the population has an equal chance of being selected in the sample.

Sample Size:
Must be small enough to be economical but large enough to represent the population.

Review 1

Exercise 20.1 Mental computation

1 Spell coordinate.

2 What is a census?

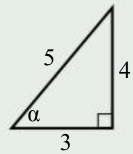
3 What is a sample?

4 Why random sample?

5 Expand: $5(x - 2)$

6 Factorise: $6x + 9$

7 In the triangle, what is $\tan\alpha$?



Given the points A(1,2), B(3,4)

8 What is the gradient of AB?

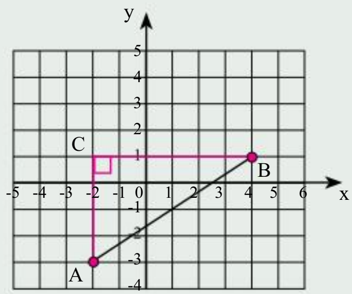
9 Write in scientific notation: 32 000

10 22×15

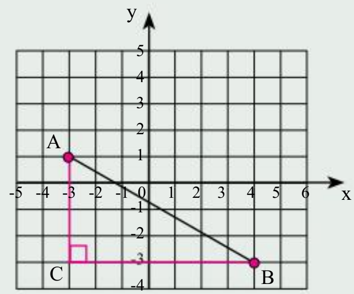
Exercise 20.2

1 Find the length of AB

a)



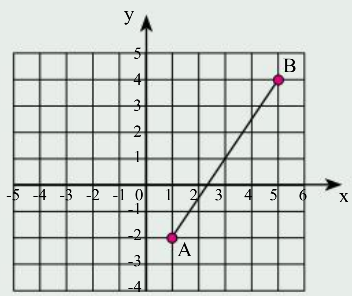
b)



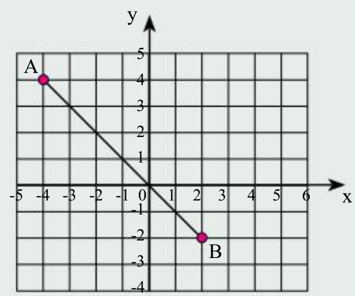
c) A(2,1), B(4,3) **d)** A(5,2), B(-2,-1) **e)** A(-1,-3), B(3,-4)

2 Find the midpoint of the line segment AB.

a)



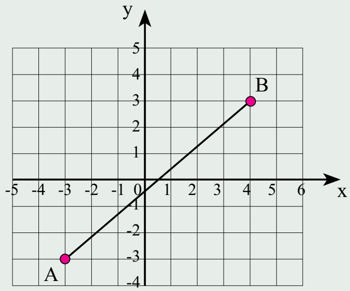
b)



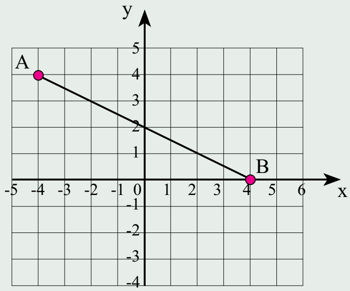
c) A(1,2), B(5,4) **d)** A(2,-3), B(-2,3) **e)** A(2,5), B(-4,-1)

3 Find the gradient of each of the following lines.

a)



b)



c) A(3,3), B(7,6) **d)** A(3,1), B(-1,4) **e)** A(2,1), B(-1,-2)

4 A parallelogram has the four corners A(-4,1), B(1,3), C(3,5), D(-2,3).

a) Show that AB is parallel to CD (ie have the same gradient).

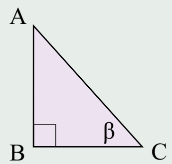
b) Show that the opposite sides are equal in length.

5 For the adjacent triangle, name:

a) the hypotenuse.

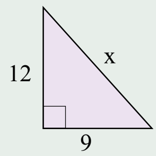
b) the side adjacent to the angle.

c) the side opposite the angle.

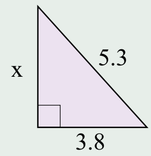


6 Use Pythagoras' Theorem to find the unknown:

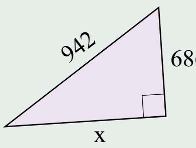
a)



b)



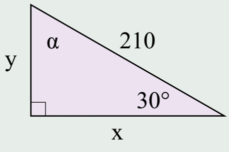
c)



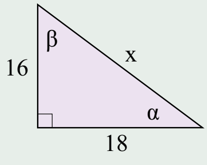
Solve means 'find **all** unknowns'. (3 sides, 3 angles)

7 Solve the following triangles:

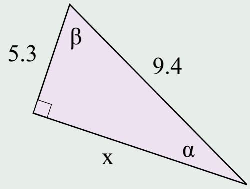
a)



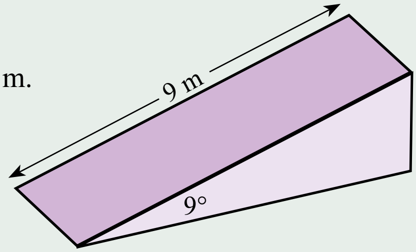
b)



c)



8 A ramp has an incline of 9° and a length of 9 m. What is the length of the horizontal base?



$\boxed{3x - 8x = {}^{-}5x}$

9 Simplify the following expressions:

a)	$2x + 3x$	**b)**	$5x + x$	**c)**	$9x - 7x$
d)	$2x - 7x$	**e)**	$5b - 2b + 3a$	**f)**	$5x + 2y - 2x + 3y$
g)	$4 \times 2x$	**h)**	$5x \times 2$	**i)**	${}^{-}4x \times 2$
j)	$5a \times {}^{-}3$	**k)**	$4b^3 \times {}^{-}4b^2$	**l)**	$2x^2 \times 2x \times 5x$
m)	$16x \div 8$	**n)**	$21d \div 3$	**o)**	${}^{-}8h \div 4$
p)	${}^{-}16x \div 2$	**q)**	${}^{-}15a \div {}^{-}3$	**r)**	$18cd \div 6d$
s)	${}^{-}24x^7 \div {}^{-}8x^2$	**t)**	${}^{-}36a^6b^3c \div 12a^5b^2$		

$\boxed{{}^{-}4(3b - 2) = {}^{-}12b + 8}$

10 Expand each of the following:

a)	$3(x + 2)$	**b)**	$4(x + 3)$	**c)**	$5(a + 2)$
d)	$6(2x - 3)$	**e)**	$3(3b - 1)$	**f)**	$4(3c - 4)$
g)	${}^{-}6(4d - 1)$	**h)**	${}^{-}7(x + 2)$	**i)**	${}^{-}5(4x + 3)$
j)	${}^{-}m(2m + 3)$	**k)**	${}^{-}5x(x - 5)$	**l)**	${}^{-}x(3x + 1)$
m)	${}^{-}2n(1 + 3b)$	**n)**	${}^{-}x(x - a)$	**o)**	${}^{-}5x(y - 2x)$

11 Simplify each of the following by expanding and then collecting like terms:

a)	$2(x + 1) + 3(x + 2)$	**b)**	$4(x + 3) + 2(x + 1)$
c)	$4(a - 3) + 5(a + 5)$	**d)**	${}^{-}2(x - 3) + 2(x + 3)$
e)	${}^{-}x(x + 1) + x(2x - 5)$	**f)**	${}^{-}4d(3d - 1) + {}^{-}2(d - 1)$
g)	$(x + 3)(x + 1)$	**h)**	$(x + 1)(x + 4)$
i)	$(x + 2)(x - 1)$	**j)**	$(x + 3)(x - 1)$
k)	$(x + 1)(x - 1)$	**l)**	$(x + 2)(x - 2)$
m)	$(x + 3)^2$	**n)**	$(x - 2)^2$

$\boxed{15b - 9 = 3(5b - 3)}$ $\boxed{12ab - 8a = 4a(3b - 2)}$

12 Factorise each of the following:

a)	$2x + 8$	**b)**	$5x + 15$	**c)**	$6x + 9$
d)	$12a + 3$	**e)**	$15b + 10$	**f)**	$12c + 9$
g)	$9d - 6$	**h)**	$4e - 8$	**i)**	$4f - 10$
j)	$16x - 12$	**k)**	$14x - 21$	**l)**	$20x - 16$
m)	$8xy + 12x$	**n)**	$15xy - 12x$	**o)**	$20mn - 15m$
p)	$20x^2 - 16x$	**q)**	$25x^2 - 20x$	**r)**	$9x^2 - 12x$
s)	$-6x - 10$	**t)**	$-3n - 12$	**u)**	$-2x + 10$
v)	$-6x + 15$	**w)**	$-9x^2 + 18x$	**x)**	$-5x^2 - 25x$
y)	$x(x + 3) + 3(x + 3)$	**z)**	$x(x - 2) + 5(x - 2)$		

13 Describe the following data as quantitative or qualitative:

a) The marital status of people in the postcode area.

b) The heights of students in your class.

14 Describe the following quantitative data as discrete or continuous:

a) The time to run 100m.

b) The pulse rate before and after exercise.

15 What is the meaning of each of the following:

a) Census? **b)** Sample? **c)** Random sample?

16 How many students from each Year level should be randomly selected in a sample size of 50 students?

Year	Number
7	150
8	180
9	140
10	130

17 What is wrong with each of the following questionnaires?

a)

How do you normally get to school?
(Please circle)
Car Bus Walk

b)

How much money do you have?
<$50
>$50

Review 2

Exercise 20.3 Mental computation

1 Spell trigonometry.

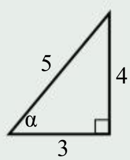
2 What is a census?

3 What is a sample?

4 Why random sample?

5 Expand: $6(x - 3)$

6 Factorise: $4x + 8$



7 In the triangle, what is $\sin\alpha$?

Given the points A(1,2), B(3,4)

8 What is the gradient of AB?

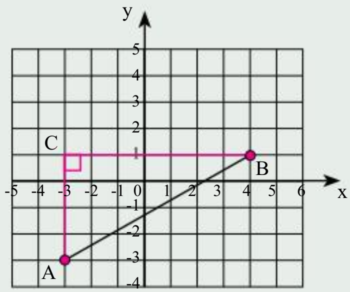
9 Write in scientific notation: 0.000 041

10 42×15

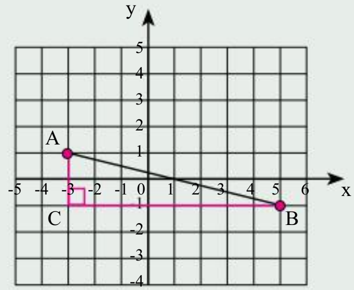
Exercise 20.4

1 Find the length of AB

a)



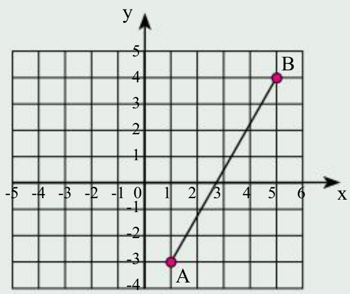
b)



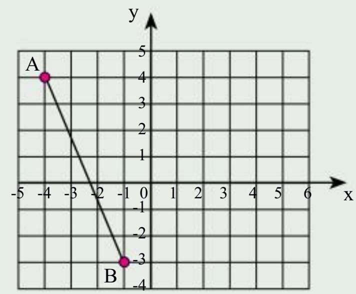
c) A(1,2), B(3,5) **d)** A(5,2), B(1,-2) **e)** A(-1,-4), B(3,-4)

2 Find the midpoint of the line segment AB.

a)



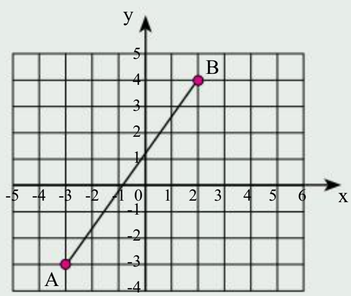
b)



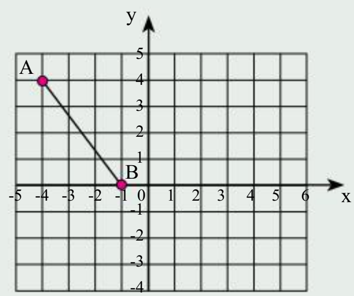
c) A(3,3), B(5,7) **d)** A(2,-1), B(-2,4) **e)** A(1,4), B(-4,-1)

3 Find the gradient of each of the following lines.

a)



b)



c) A(5,3), B(7,5) **d)** A(3,1), B(-2,4) **e)** A(2,6), B(-5,-2)

4 A parallelogram has the four endpoints A(-3,-4), B(-6,0), C(-4,1), D(-1,-3).

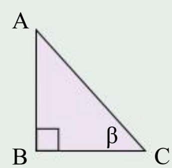
a) Show that AB is parallel to CD (ie have the same gradient).

b) Show that the opposite sides are equal in length.

5 For the adjacent triangle, name:

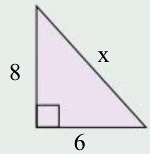
a) the hypotenuse.

b) the side adjacent to the angle.

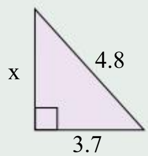


6 Use Pythagoras' Theorem to find the unknown:

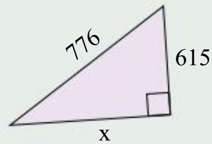
a)



b)



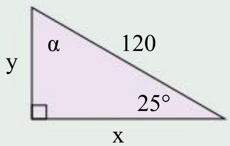
c)



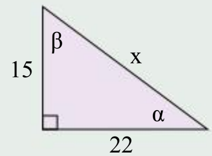
Solve means 'find **all** unknowns'. (3 sides, 3 angles)

7 Solve the following triangles:

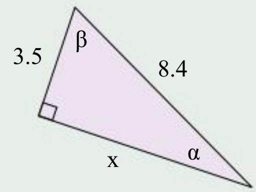
a)



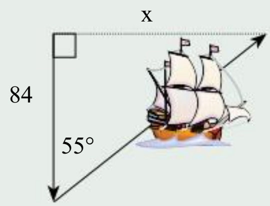
b)



c)



8 A sailing boat sails south for 84 km, then on a bearing of 55°T until it is due east of its starting point.
How far is the boat from its starting point?
How far has the boat travelled?



$3x - 8x = {}^{-}5x$

9 Simplify the following expressions:

a) $3x + 2x$ **b)** $6x + x$ **c)** $12x - 7x$
d) $3x - 7x$ **e)** $7x - 2x + 4y$ **f)** $5a + 4b - 2a + 2b$
g) $5 \times 6x$ **h)** $8x \times 2$ **i)** ${}^{-}3a \times 2$
j) $2x \times {}^{-}3$ **k)** $8b^4 \times {}^{-}4b^2$ **l)** $4x^2 \times 2x \times 3x$
m) $15x \div 3$ **n)** $14c \div 7$ **o)** ${}^{-}6d \div 3$
p) ${}^{-}16x \div 4$ **q)** ${}^{-}12e \div {}^{-}3$ **r)** $16fg \div 4f$
s) ${}^{-}12x^5 \div {}^{-}9x^2$ **t)** ${}^{-}30a^5b^3c \div 18a^3b^2$

${}^{-}4(3b - 2) = {}^{-}12b + 8$

10 Expand each of the following:

a) $3(x + 2)$ **b)** $4(x + 1)$ **c)** $5(a + 2)$
d) $4(2x - 1)$ **e)** $2(2b - 1)$ **f)** $8(2c - 3)$
g) ${}^{-}6(3d - 1)$ **h)** ${}^{-}5(x + 5)$ **i)** ${}^{-}2(4e + 1)$
j) ${}^{-}x(2x + 5)$ **k)** ${}^{-}5x(x - 3)$ **l)** ${}^{-}f(6f + 2)$
m) ${}^{-}5g(1 + g)$ **n)** ${}^{-}4x(5x - 3y)$ **o)** ${}^{-}5x(2x - y)$

11 Simplify each of the following by expanding and then collecting like terms:

a) $2(x + 1) + 3(x + 2)$ **b)** $5(x + 2) + 2(x + 3)$
c) $5(a - 3) + 2(a + 5)$ **d)** $^{-}3(x - 3) + 2(x + 2)$
e) $^{-}3x(x + 1) + x(x - 2)$ **f)** $^{-}2d(2d - 1) + {}^{-}4(d - 5)$
g) $(x + 2)(x + 1)$ **h)** $(x + 1)(x + 2)$
i) $(x + 4)(x - 1)$ **j)** $(x + 3)(x - 2)$
k) $(x + 1)(x - 1)$ **l)** $(x + 2)(x - 2)$
m) $(x + 1)^2$ **n)** $(x - 1)^2$

$\boxed{15b - 9 = 3(5b - 3)}$ $\boxed{12ab - 8a = 4a(3b - 2)}$

12 Factorise each of the following:

a) $5x + 15$ **b)** $3x + 12$ **c)** $4x + 10$
d) $12a + 4$ **e)** $8b + 10$ **f)** $15c + 9$
g) $9x - 6$ **h)** $3d - 6$ **i)** $5e - 10$
j) $15x - 12$ **k)** $12x - 8$ **l)** $14x - 12$
m) $8xy + 10x$ **n)** $10xy - 12x$ **o)** $25uv - 15u$
p) $15x^2 - 20x$ **q)** $30u^2 - 15u$ **r)** $4t^2 - 12t$
s) $-4x - 8$ **t)** $-3b - 9$ **u)** $-2c + 8$
v) $-3x + 15$ **w)** $-4g^2 + 20g$ **x)** $-2x^3 - 12x$
y) $x(x + 2) + 3(x + 2)$ **z)** $x(x - 1) + 5(x - 1)$

13 Describe the following data as quantitative or qualitative:
a) The weights of students in your class.
b) The country of birth of people in the postcode area.

14 Describe the following quantitative data as discrete or continuous:
a) The daily maximum temperature.
b) The number of hens in each run.

15 What is the meaning of each of the following:
a) Census? **b)** Sample? **c)** Random sample?

16 How many people of each employee type should be randomly selected in a sample size of 90 employees.

Gender	Type	Number
Female	Full-time	143
Female	Part-time	231
Male	Full-time	242
Male	Part-time	387

Paradoxes

Paradox comes from the Greek: 'para' = beyond and 'doxa' = opinion, belief.
A paradox is a statement that appears contradictory yet has an element of truth.

Eubulides, a sixth century B.C. Greek philosopher, invented the paradox of the liar.

All cretans are liars.
If a cretan is telling the truth, the cretan is lying.
If a cretan is lying, the cretan is telling the truth.

Consider the following liar paradox:

If the sentence is false then it is true.
If the sentence is true then it is false.

This sentence is false

Using paradoxes to prove that Santa Claus exists?

If this sentence is true then Santa Claus exists

This sentence is false and Santa Claus does not exist

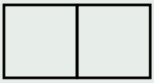
A paradoxical notice:

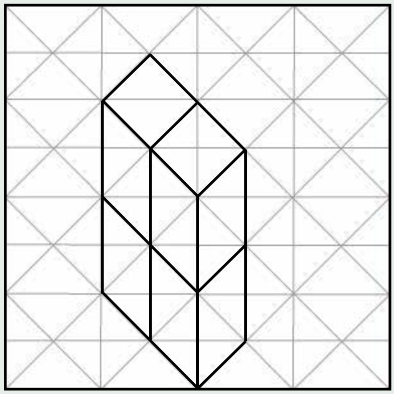
IGNORE this NOTICE

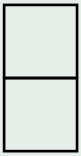
Isometric Drawing

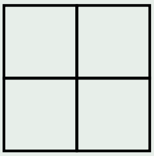
Example

Make an isometric drawing of the following:

Top 

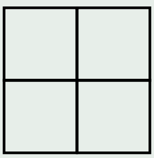
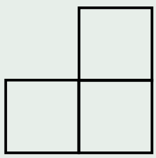
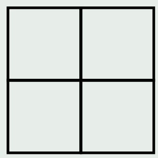


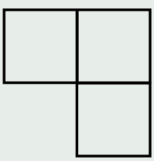
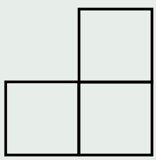
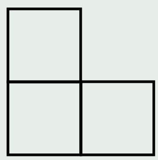
Side 

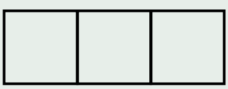
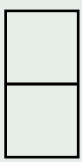
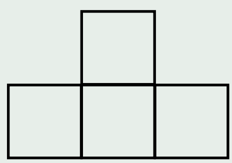
Front 

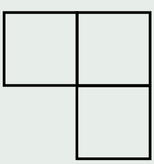
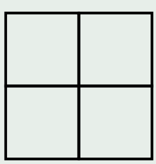
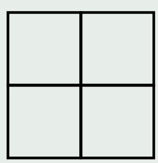
Make an isometric drawing of each of the following:

Top **Side** **Front**

a)   

b)   

c)   

d)   

Glossary

Acute − An acute angle is a sharp angle between 0° and 90°.

Angle − An angle is the measure of turn between two lines. Angles are measured in degrees from 0° to 360°, eg. 147°. In later studies other measures such as radians will be introduced.

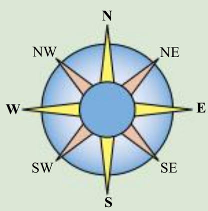
Angle sum of a polygon −
The sum of the interior angles of a triangle is 180°.
The sum of the interior angles of a quadrilateral is 360°.
The sum of the interior angles of a pentagon is 540°.
The general rule: Sum interior angles = (no sides − 2)x 180°

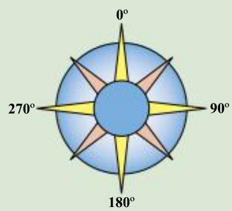
Ascending order of numbers is an order from smallest to largest.
Example: 2, 3, 5, 10 is in ascending order.

Average − An average is a central measure. Average and mean are the same.
(The mode, and median, although different, are also central measures).

Area − The area is the amount of surface.
Area of rectangle = length x breadth
Area of triangle = 1/2 x base x height
Area of circle = π x radius^2 = πr^2
Area of parallelogram = base x height

Bearing − The bearing is the angle measured clockwise from North.
The bearing of North is 0°.
The bearing of East is 90°.
The bearing of South is 180°.
The bearing of South West is 225°.
The bearing of West is 270°.





Bias − Unfair sampling. The sample does not represent the population.

Census − Collection of data from the entire population.

Centimetre − A centimetre is one-hundredth of a metre. 100 cm = 1 m.

Circumference − The circumference is the distance around the outside of a circle.
$C = 2\pi r$ or $C = \pi d$

Complementary angles are angles that sum to 90°.
Example: 40° and 50° are complementary angles.

Complementary events - The complement of any event (A) is the event (not A). The probabilities of complementary events add to 1.

Composite number has more than two factors.
Example: 8 has factors of 1, 2, 4, 8. 8 is a composite number.

Compound stem-and-leaf plot − A compound stem-and-leaf plot has two stem-and-leaf plots joined together.

Congruent objects have the same shape and the same size.
The symbol for congruence is ≡ or ≅
The tests of congruent triangles are:
SSS (side, side, side).
SAS (side, angle, side).
ASA (angle, side, angle).
RHS (right-angle, hypotenuse, side).

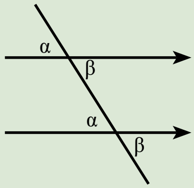
Consecutive numbers are numbers that follow one another.
Example: 3, 4, 5 are consecutive numbers.

Continuous numbers are numbers that can have any value. Weight is continuous because the weight of an object can be any number on the number line. Discrete numbers can have only certain values - the number of people in the class must be a whole number. There can't be 4.62 people in the class.

Compound Interest - Compound interest arises when interest is added to the principal. The interest that has been added also earns interest. This addition of interest to the principal is called compounding. Eg A bank account may have its interest compounded every year: in this case, an account with \$1000 initial principal and 20% interest per year would have a balance of \$1200 at the end of the first year, \$1440 at the end of the second year, and so on.

Coordinates − An ordered pair on numbers that fix a point in the cartesian plane.
Example: P(2,5). The point P is 2 units to the right and 5 units up from the origin (0,0).

Corresponding angles − matching angles when a line cuts a pair of lines.
If the lines are parallel, the corresponding angles are equal.



α is one pair of corresponding angles.
β is another pair of corresponding angles.

Cube − A cube is a three-dimensional object with all six faces congruent and each face having the shape of a square. A cube is one of the five platonic solids.

Cubed − A cubed number is the number multiplied by itself three times.
Example: Two cubed = $2^3 = 8$.

Cubic centimetre is the amount of space occupied by a cube with each side of length 1 cm. The unit is 1 cm^3 (1 L = 1 000 cm^3).

Cubic metre is the amount of space occupied by a cube with each side of length 1 m. The unit is 1 m^3 (1 m^3 = 1 000 L).

Data - Information collected for analysis or reference.

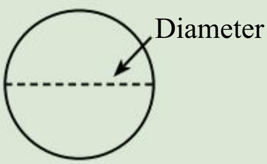
Decagon is a polygon with 10 sides and 10 angles.

Decimal place − The number of places after the decimal point.
Example: 5.281 has three decimal places.

Denominator in a fraction is the number at the bottom.

Descending order of numbers is an order from largest to smallest.
Example: 10, 5, 3, 2 is in descending order.

Diameter − The diameter of a circle is the length of the line joining two points on the circle and that passes through the centre of the circle.



Die − A die is a cube with each of the numbers 1, 2, 3, 4, 5, 6 on each of the six faces.
The opposite sides of a die sum to seven.
Die is singular, dice is plural.

Digit − A digit is a single number.
Example: The number 435 has the digits 4, 3, and 5.

Discrete numbers are numbers that can only have certain values, normally whole numbers.
Example: The number of people in the class is discrete (Can't be 4.62 people).

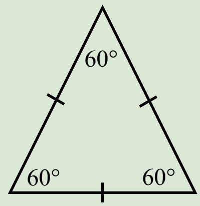
Distributive law − Each term in the brackets is multiplied by the term outside the brackets. Example: $3(a + 5) = 3a + 15$

Dividend − The dividend is the number being divided. In 45 ÷7, 45 is the dividend.

Divisor − The divisor is the number dividing. In 45 ÷7, 7 is the divisor.

Dodecahedron − A dodecahedron is a three-dimensional object with all twelve faces congruent and each face having the shape of a regular pentagon (5 sides of equal length). A dodecahedron is one of the five platonic solids.

Equation − An equation is a mathematical sentence with an equals sign.
Example: $2x + 5 = 9$ is an equation.



An **equilateral triangle** is a triangle with three equal sides.
Each of the three angles in an equilateral triangle are 60°.

Estimate − To make an approximate guess of the answer.
Example: An estimate of 43 x 26 is 40 x 30 = 1200

Evaluate − To evaluate an expression is to find the value of the expression.
Example: Evaluate $2x(3-1)$
$2x(3-1) = 2x2 = 4$

Even numbers are numbers that are exactly divisible by 2.
Example: 2, 4, 6, 8, 10 are even numbers.

Expand − Each term in the brackets is multiplied by the term outside the brackets.
Example: $3(a + 5) = 3a + 15$

Factors − The factors of a number are the numbers which divide exactly into the number.
Example: The factors of 6 are 1, 2, 3, 6.

Factorise − To make into a product.
Example: $3a + 15 = 3(a + 3)$

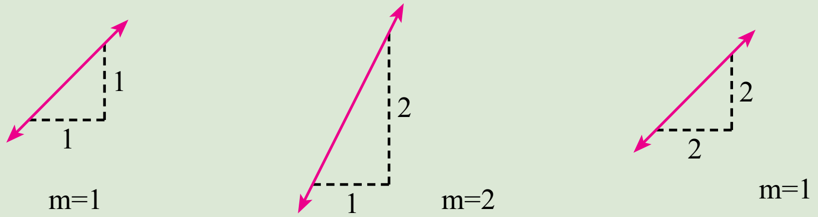
Finite − A definite number.
Example: {2, 3, 1, 6 } has a finite number of elements. It has 4 elements.
The opposite of finite is infinite.

Formula − A formula is an equation.
The formula for the perimeter of a circle is: $C = 2\pi r$

Frequency − The number of times a number occurs.
Example: 2, 3, 2, 3, 2, 2 The frequency of 2 is 4 {2 occurs 4 times}

Gram − A gram is a measure of mass and is one thousandth of a kilogram.
1 000 g = 1 kg

Gradient − A measure of the slope.
Example: The gradient of the line $y = 2x - 1$ is 2. $m = 2$



Greater than − The symbol for greater than is >
Example: 6 is greater than 4, $6 > 4$

Heptagon − A polygon with seven sides and seven angles.

Hexagon − A hexagon is a polygon with six sides and six angles.

Highest common factor − The largest factor that is common.
Example: The highest common factor of 12 {1,2,3,4,6,12} and 8 {1,2,4,8} is 4.

Hypotenuse - The longest side in a right-angled triangle.
The hypotenuse is opposite the right-angle.

Icosahedron − An icosahedron is a three-dimensional object with all twenty faces congruent and each face having the shape of an equilateral triangle.
A icosahedron is one of the five platonic solids.

Improper fraction − An improper fraction or vulgar fraction is a fraction with the numerator larger than the denominator.
Example: $\frac{5}{3}$ is an improper fraction.

Index − The power when a number is written in index form.
Example: $9 = 3^2$. The index is 2.

Infinite − Too large/many to be counted. Not finite.

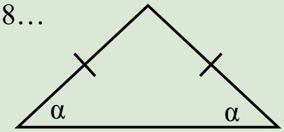
Integers are whole positive numbers and whole negative numbers.
Example: −2, 3, 4, −7 are integers. 3.4 is not an integer.

Intersection − The point where two lines cross each other. The common numbers in a set of two numbers.

Irrational number − An irrational number is a number that cannot be written as a common fraction or as a decimal fraction that terminates or recurs.

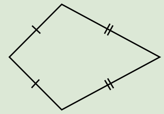
Example: π is irrational because it cannot be written as a decimal that terminates or recurs. $\pi = 3.14159265358\ldots$

$\sqrt{2}$ is irrational $= 1.41421356\ldots.$



Isosceles − A triangle with two sides of equal lengths.

The angles opposite the equal sides are equal.



Kite − A quadrilateral with two pairs of adjacent sides equal.

Kilogram − A kilogram is a measure of mass. 1 kg = 1 000 g. 1 tonne = 1 000 kg.

Kilometre − A kilometre is a measure of length. 1 km = 1 000 m.

Latitude − The latitude of a position on Earth is the angle North or South of the Equator.

Adelaide's latitude is 34.55°S, Adelaide's longitude is 138.35°E.
Brisbane's latitude is 27.28°S, Brisbane's longitude is 153.01°E.
Canberra's latitude is 35.27°S, Canberra's longitude is 149.12°E.
Darwin's latitude is 12.28°S, Darwin's longitude is 130.50°E.
Hobart's latitude is 42.53°S, Hobart's longitude is 147.19°E.
Melbourne's latitude is 37.82°S, Melbourne's longitude is 144.95°E.
Perth's latitude is 31.95°S, Brisbane's longitude is 115.83°E.
Sydney's latitude is 33.52°S, Sydney's longitude is 151.13°E.

Litre − A litre is a measure of volume. 1 kg = 1 000 mL.

Longitude − The longitude of a position on Earth is the angle East or West of the line of meridian through Greenwich. See Latitude for examples.

Mean − The mean of a set of numbers is the sum of the numbers divided by the number of numbers. The mean of 2, 4, 5, 7 =

Median − The median of a set of numbers is the middle number when the numbers have been put in order.

Example: Find the median of: 4, 5, 2, 3, 6, 7, 2
In order: 2, 2, 3, 4, 5, 6, 7
The median is 4
Find the median of: 1, 3, 1, 0, 4, 3
In order: 0, 1, 1, 3, 3, 4
The median is the mean of 1 and 3 = 2.

Metre − The metre, m, is the standard measure of length.

Millimetre − A millimetre is one thousandth of a metre. 1 m = 1 000 mm.

Mixed number − A mixed number consists of a whole number and a fraction.

Example: $2\frac{3}{5}$

Mode − The mode of a set of numbers is the number that occurs the most.
Example: 2, 4, 3, 3, 5, 3. The mode is 3 (3 occurs three times).
1, 5, 4, 1, 5, 3. The mode is 1 and 5 (bimodal).

Net − The net of a solid is the shape that can be folded to make the solid.

Numerator − The numerator is the top number in a fraction.

Obtuse angle − An obtuse angle is an angle between 90° and 180°.

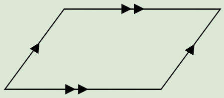
Octagon − An octagon is a polygon with eight sides and eight angles.

An **octahedron** is a three-dimensional object with all eight faces congruent and each face having the shape of an equilateral triangle. A octahedron is one of the five platonic solids.

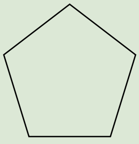
Odd numbers are numbers that are not exactly divisible by 2.
Example: 1, 3, 5, 7, 9 are odd numbers.

Obtuse angle − An angle greater than 90° and less than 180°.

Parallelogram − A parallelogram is a quadrilateral, four sided figure, in which the opposite sides are parallel.



Pentagon − A pentagon is a polygon with five sides and five angles.



Per annum − Per year.

Percentage − A percentage is a fraction of 100. $43\% = \frac{43}{100}$

Perimeter − The perimeter is the distance around the outside edge of a figure.

Perpendicular lines are lines that are at 90° to each other.

Pi, π, is the ratio of the circumference of a circle to the diameter.
$\pi = 3.14159265358\ldots$

Polygons are shapes made up of straight lines. Triangles (3 sides), quadrilaterals (4 sides), pentagons (5 sides), hexagons (6 sides) etc are polygons.

Polyhedron − A solid shape with flat sides. Cube, dodecahedron, icosahedron, etc

Power − The power of a number is the number of times the number is multiplied by itself.
Example: 3x3x3x3x3 = 3^5. {3 to the fifth power

Probability − The chance of an event happening.
Probability ranges from a low of 0 (no chance) to a high of 1 (certain).

$$P(event) = \frac{No\ of\ \ favourable\ outcomes}{Total\ no\ of\ \ outcomes}$$

Prime number − A prime number is a number with just two factors, 1 and itself.
2, 3, 5, 7, 11, 13 are prime numbers. 1 is not a prime number.

Prism − A prism is a three-dimensional shape in which the base shape is repeated from the bottom to the top.

Cylinder.
A circular based prism.

Probability is the chance of an event happening.
If a die is thrown, the chance of a 3 showing is

Quadratic − An equation in which the highest power of x is 2
Example: $y = 2x^2 - 5x + 3$

Quadrilateral − A quadrilateral is a figure with four straight lines.

Quartile − A value that divides the data in quarters.
Upper quartile, median, lower quartile.

Quotient − The quotient is the result of a division.
Example: The quotient of 10÷5 is 2.

Radius − The radius of a circle is the distance from the centre of a circle to a point on the circle.

Range − The difference between the highest and the lowest value.

Ratio − A ratio is a comparison of two quantities. A certain two stroke petrol is made by mixing one part of two stroke oil to 32 parts of unleaded petrol (1: 32).

Rational number − A rational number is a number that can be written as a common fraction or as a decimal fraction that either terminates or recurs.

Example: $\frac{1}{2}$ $\frac{3}{4}$ $\frac{-2}{3}$

Example: π is irrational because it cannot be written as a decimal fraction that terminates or recurs. $\pi = 3.14159265358\ldots$

Rectangle − A rectangle is a four sided figure in which the opposite sides are parallel and the internal angles are 90° (right-angles).

Rectangular prism − A rectangular prism is a prism in which the base is a rectangle.

Right-angle − A right-angle is 90°.

Rounding - Giving an approximation of a number using the nearest more convenient number is called rounding. When rounding to the nearest ten, 11, 12, 13 and 14, round to 10, whereas 15, 16, 17, 18 and 19 round to 20.

Similar figures have the same shape. Congruent figures have a the same shape and the same size.

Square − A square is figure with four equal sides and each internal angle of size 90°.

Square centimetre, cm^2, is the area occupied by a square with each side of length 1 cm.

Square metre, m^2, is the area occupied by a square with each side of length 1 m ($1\ m^2 = 10\ 000\ cm^2$).

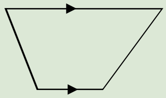
Symmetry - Property of regularity in shape by, for example, reflection or rotation. The letter T is symmetrical by reflection, the letter Z is symmetrical by rotation, the letter H is symmetrical by both reflection and rotation, the letter R is not symmetrical.

Surface area - The surface area of an object is the sum of the area of the various faces that make up the object.

Tonne − A tonne, t, is a measure of mass (1 t = 1 000 kg).

Transformation - A movement of figures and objects. The transformations translation (slide), rotation (turn) and reflection (flip) do not change the size or shape of the figure or object.

Trapezium − A trapezium is a four-sided figure with one pair of opposite sides parallel.



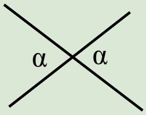
Triangle − A triangle is a figure with three sides.

Two-way table − A table that shows the sample space of a two-stage experiment.

Variables are letters used in equations, formulas, and expressions.
Example: x is a variable in the equation: $3x + 4 = 12$.

Vertex − The corner point of an angle.

Vertically opposite angles − A pair of non-adjacent angles formed when two lines intersect. Vertically opposite angles are equal.



Vinculum − The horizontal line separating the numerator from the denominator.

Volume of a figure is a measure of the amount of space occupied by the figure.
Example: The volume of a prism = area of base x height

Whole numbers are the positive counting numbers.
Example: 0, 1, 2, 3, 4, 5, etc.

x-intercept − The point where the graph cuts the x-axis.

y-intercept − The point where the graph cuts the y-axis.

Answers

Exercise 1.1 **1** 1 **2** 4 **3** 9 **4** 16 **5** 100 **6** 1 **7** 8 **8** 27 **9** 64 **10** 125 **11** 1000 **12** 16 **13** 81 **14** 10 000 **15** 625

16

	Square	Cube	Fourth	Fifth
1	1	1	1	1
2	4	8	16	32
3	9	27	81	243
4	16	64	256	1024
5	25	125	625	3125

17 100 **18** 1000 **19** 10 000 **20** 100 000 **21** 1 000 000 **22** 10 000 000

Exercise 1.2 **1** 64 **2** 78 125 **3** 1 **4** 1 000 000 **5** 256 **6** 256 **7** 256 **8** 65 536 **9** 19 683 **10** 19 683 **11** 19 683 **12** 7 625 597 484 987

Exercise 1.3 **1** 4^3 **2** 2^4 **3** a^3 **4** 10^3 **5** b^5 **6** h^3 **7** m^5 **8** 9^4 **9** 3^6

Exercise 1.4 **1** 4×4×4 **2** b×b×b×b **3** 5×5 **4** 2×2×2×2×2×2×2 **5** 6×6 **6** m×m×m×m×m **7** x×x×x×x **8** p×p×p×p×p **9** 1×1×1×1

Exercise 1.5 **1** $a^4 \times b^3$ **2** $3^3 \times 2^2$ **3** $a^5 \times b^4$ **4** $2^4 \times 3^3$ **5** $b^4 \times g^5$ **6** z^{11} **7** $p^5 \times q^3 \times r^4$ **8** $2^3 \times 3^2 \times 4^3$ **9** $4^4 \times g^6$

Exercise 1.6 **1** 2^5 **2** 3^5 **3** 2^6 **4** 4^7 **5** a^4 **6** b^5 **7** z^8 **8** w^7 **9** 2^4 **10** t^7 **11** 5^5 **12** 10^9 **13** c^7 **14** 10^6 **15** a^5 **16** b^6 **17** 2.4^7 **18** z^5 **19** 0.5^9 **20** 10^7 **21** 2^7 **22** 3^9 **23** a^8 **24** u^7

Exercise 1.7 **1** 2^2 **2** 4^3 **3** 2^2 **4** 4^1 or 4 **5** a^4 **6** b^5 **7** 10^2 **8** w^1 or w **9** 2^2 **10** t^3 **11** 5^2 **12** 10^3 **13** a^1 or a **14** 1.2^2 **15** 2^{15} **16** $p^0 = 1$ **17** m^2 **18** e^3 **19** c^3 **20** 10^2

Exercise 1.8 **1** 2^6 **2** 2^9 **3** 3^6 **4** 4^6 **5** a^4 **6** b^{12} **7** t^8 **8** n^{10} **9** 5^4 **10** s^{10} **11** m^6 **12** 3^8 **13** 5^9 **14** 10^6 **15** 10^3 **16** g^{10} **17** 10^6 **18** h^{12} **19** d^8 **20** 2^{15} **21** 2^8 **22** 2^{11} **23** 3^9 **24** 4^{10} **25** d^7 **26** b^{16} **27** t^{13} **28** n^{13} **29** 5^5 **30** s^{12} **31** m^7 **32** 3^9

Exercise 1.9 **1** 1 **2** 1 **3** 1 **4** 1 **5** 1 **6** 1 **7** 1 **8** 1 **9** 5 **10** 3 **11** 6 **12** 2 **13** 7 **14** 3 **15** 9 **16** 8 **17** 5 **18** s^2 **19** m **20** 3

Exercise 1.10 **1** 10^{-2} **2** 2^{-5} **3** a^{-4} **4** 10^{-4} **5** x^{-7} **6** x^{-1} **7** 10^{-6} **8** 3^{-4} **9** 4^{-1} or 2^{-2} **10** 9^{-1} or 3^{-2} **11** 27^{-1} or 3^{-3} **12** 16^{-1} or 2^{-4} **13** 1/4 **14** 1/4 **15** 1/16 **16** 1/10 **17** 1/100 **18** 1/1000 **19** 1/10000 **20** 1/100000 **21** 1/25 **22** 1/32 **23** 1/0.16 **24** 1/2.25 **25**

Power	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}
Value	16	8	4	2	1	0.5	0.25	0.125

26

Power	5^4	5^3	5^2	5^1	5^0	5^{-1}	5^{-2}	5^{-3}
Value	625	125	25	5	1	0.2	0.04	0.008

27

Power	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
Value	10000	1000	100	10	1	0.1	0.01	0.001

Exercise 1.11 **1** 2^{-3} **2** 5^{-4} **3** 3^3 **4** 10^{-1} **5** x^4 **6** $6x^{-1}$ **7** $5x^{-1}$ **8** $20x^{-2}$ **9** 4^2 **10** a^3 **11** $24x^{-3}$ **12** $10^0=1$ **13** 3^5 **14** 2^7 **15** 5^{-5} **16** 10^{-2} **17** x^6 **18** x^{-8} **19** $y^1=y$ **20** a^4 **21** $2x^6$ **22** $3b^6$ **23** 2^9 **24** 10^6 **25** 2^{-6} **26** 3^{-6} **27** 2^{-8} **28** 5^4 **29** x^6 **30** n^{-6} **31** a^{-12} **32** y^5 **33** $x^0=1$ **34** 3^{-2} **35** a^{-11} **36** 10^2

Exercise 1.12 **1** 3^7 **2** x^5 **3** 7^7 **4** x^4y^3 **5** a^4 **6** b^8 **7** z^5d^2 **8** 10^7 **9** 2^{-2} **10** 1.5^6 **11** $5^1=5$ **12** 10^3 **13** d^2 **14** 2^{-2} **15** a^{-4} **16** $e^5 \div x^6$ **17** 1.2^5 **18** z^{-5} **19** 4^{-9} **20** 10^5 **21** 3^4 **22** x^6 **23** 5^6 **24** x^{10} **25** a^{10} **26** s^{-10} **27** m^{-6} **28** 3^8 **29** x^8 **30** 10^6 **31** $12a^5$ **32** $10b^7$ **33** 4^3 **34** $5z^{-2}$ **35** 10^{11} **36** 10^3 **37** $10^0=1$ **38** $6x^5$ **39** $3x^{-1}$ **40** $1.5x^{-7}$

Exercise 1.13 **2** $^{-}2$ **3** 7 **4** 10^5 **5** z^{-4} **6** 2^6 **7** \$6.60 **8** \$3.85 **9** \$250 **10** \$3500

Exercise 1.14 **2** 2 **3** 3 **4** 10^7 **5** x^{-2} **6** 4^{-6} **7** \$8.80 **8** \$6.45 **9** \$250 **10** \$6000

Exercise 1.15 **2** $^{-}9$ **3** 6 **4** 10^3 **5** a^2 **6** 3^9 **7** \$9.90 **8** \$12.75 **9** \$450 **10** \$4000

Exercise 1.16 **1a** 235 **1b** 23 500 **1a** 23 500 000 **2** b **3** 24 **4** -3 **5** 10 **6** yes **7** yes **8** $803.84 cm^3$ **9** b **10** 3760 **11a** 4 **11b** 18 **11c** 48 **12** b **13** c **14** b **15a** b=4 **15b** b=6 **15c** b=4 **16** 900 000

Exercise 1.17 **1a** 0.01 **1b** 0.0001 **1c** 0.000 001 **1d** 0.001 **1e** 0.000 001 **1f** 0.000 000 001 **1g** 1 **1h** $^{-}1$ **1i** 1 **1j** $^{-}1$ **1k** 1 **1l** $^{-}1$ **1m** 1110 **1n** 30 **1o** 1 000 000 **1p** 3 **1q** 5 **1r** 7 **2a** 15 **2b** 75 **2c** 23 **3** 13 451 **4a** $2^0=1$ **4b** 3 **4c** 4 **5** 1 **6** 7 **7** 1 **8** 4 **9** 3

Exercise 1.18 **1** 150 **2** 10−1.1 many other answers **3** 7 **4** 1/8

Exercise 1.19 **1a** 1^4 **1b** 4^4 **1c** 10^6 **1d** g^7 **1e** 1.7^4 **1f** $(-5)^6$ **2a** $4^1=4$ **2b** 6^{-5} **2c** 2^2 **2d** $10^1=10$ **2e** x^3 **2f** $12a^{-2}$ **2g** $2x^{-1}$ **2h** $6x^{-3}$ **2i** 2^{-2} **2j** x **2k** $8x^{-4}$ **2l** 10 **2m** 2^{-1} **2n** 7 **2o** 3^{-5} **2p** 10^{-3} **2q** n^6 **2r** x^{-9} **2s** p **2t** a^5 **2u** $3x^7$ **2v** $2x^7$ **2w** 3^{10} **2x** 10^8 **3a** 3^{-6} **3b** 2^6 **3c** y^6 **3d** 10^3 **3e** 6^6 **3f** x^7 **3g** 10^{10} **3h** 2^{-4} **3i** 1.2^8 **3j** 4^3 **3k** 10^4 **3l** 8.1 **3m** 3^{-8} **3n** 10^{11} **3o** $8x^6$ **3p** $10x^8$ **3q** 10^4 **3r** 10^{-1} **3s** $10x^3$ **3t** $6x^0=6$ **3u** $x^0=1$ **3v** $2x^{-7}$

Exercise 1.20 **1a** 1^5 **1b** 2^4 **1c** 10^5 **1d** x^6 **1e** 6.9^4 **1f** $(-3)^6$ **2a** 5^{-1} **2b** d **2c** 2^3 **2d** 10^3 **2e** x^3 **2f** $6c^{-4}$ **2g** $5x^{-1}$ **2h** 14x **2i** $1^{-6}=1$ **2j** $x^0=1$ **2k** $12x^{-4}$ **2l** 10^4 **2m** 3^9 **2n** w^{13} **2o** 2^{-6} **2p** 10^{-5} **2q** m^7 **2r** $x^0=1$ **2s** 7 **2t** x^5 **2u** $2x^8$ **2v** $2x^9$ **2w** 2^{13} **2x** 10^{13} **3a** 2^{-9} **3b** 8^6 **3c** x^8 **3d** 10^6 **3e** 3^6 **3f** x^{12} **3g** 10^{11} **3h** 4^{-1} **3i** 6.7^{12} **3j** 5^2 **3k** 10 **3l** 2.1^{-2} **3m** 3^{-9} **3n** 10^{10} **3o** $12x^7$ **3p** $15x^{10}$ **3q** 10 **3r** 10^{-1} **3s** $14x^2$ **3t** $15x^{-1}$ **3u** 3^{-10} **3v** a^{16}

Exercise 2.1 **1** 6x **2** 2a **3** 9c **4** 5x **5** 19c **6** ¯2z **7** 27y **8** ¯2w **9** 8x **10** 7x **11** ¯5b **12** 4d **13** 13a **14** ¯2x **15** 9m **16** 9x **17** 3h **18** ¯x **19** 7x **20** a **21** 5x+4 **22** 6b+7 **23** 3x+8 **24** 6b−4a **25** 13x+7 **26** 3a+6b **27** x+4d **28** 4x+2 **29** $13b^2+5$ **30** $2x^3+8$ **31** 10xy+1 **32** $4as^2+7d^5$ **33** ¯x+4 **34** ¯5x−y

Exercise 2.2 **1** 15x **2** 14a **3** 18m **4** 12x **5** 15b **6** 4p **7** 27x **8** 8fn **9** 14xy **10** 10bh **11** 24dt **12** 7gk **13** 2mn **14** 5pr **15** 2km **16** $12x^2$ **17** $12d^2$ **18** $15a^2$ **19** $21d^2$ **20** ¯$10x^2$ **21** ¯$6x^2$ **22** ¯$12x^2$ **23** $12a^3$ **24** ¯$12p^4$ **25** ¯$27w^3$ **26** $16s^5$ **27** $40x^4$ **28** $6de^3$ **29** ¯$15m^3n^2$ **30** ¯$28n^2p^3$ **31** ¯$24a^3b^2$ **32** $6p^3d^2$ **33** $16h^3$ **34** $20a^4b^3c^2$

Exercise 2.3 **1** 4a **2** 4x **3** 3c **4** 3x **5** 2y **6** 2n **7** 4k **8** 3d **9** 3x/2 **10** 4/3 **11** 7/2 **12** 3/2 **13** 3y **14** 7d/2 **15** 9g/2 **16** ¯2x **17** ¯2p **18** 6 **19** ¯2 **20** $3a^3$ **21** ¯$2v^3$ **22** $3a^4y$ **23** ¯$2de^2$ **24** ¯9g/2 **25** ¯$2b^2c^2/3$ **26** $7a^5d/3$ **27** ¯$2d^2w^3z/3$

Exercise 2.4 **1** 4b+12 **2** 5c+10 **3** 2a+14 **4** 3g+3 **5** 6h+30 **6** 7n+28 **7** 4z−8 **8** 10s−20 **9** 12d−9 **10** 12f−28 **11** 16a−40 **12** 30h−36 **13** ¯2a−6 **14** ¯5r−10 **15** ¯2c−8 **16** ¯10m+20 **17** ¯3y−6 **18** ¯9w−54 **19** ¯6c+12 **20** ¯12e+16 **21** ¯20v−15 **22** $2t^2+3t$ **23** $12z^2+15z$ **24** $24g^2+30g$ **25** $6d^2-8d$ **26** $6n^2-12n$ **27** $7m^2-2m$ **28** ¯$2t^2-3t$ **29** ¯$12z^2+6z$ **30** ¯$24g^2-30g$ **31** ¯$3p-2p^2$ **32** ¯$12e^2+8ce$ **33** ¯$8bu+16u^2$

Exercise 2.5 **1** 5x+11 **2** 6x−13 **3** 12a+1 **4** 2b+39 **5** 2x−20 **6** 15x **7** 8y−42 **8** $14y^2+5y+2$ **9** $7t^2+9t$ **10** $27z^2+25z$

Exercise 2.6 **1** 5x+9 **2** ¯3x−2 **3** 5c+17 **4** ¯d−17 **5** 7h+11 **6** ¯18 **7** 5m+16 **8** 5y−12 **9** 8w+2 **10** 18 **11** 7e−1 **12** ¯7v−7 **13** t^2-7t **14** ¯$12z^2+4z+8$ **15** x+8 **16** ¯6x+8 **17** ¯x−2 **18** ¯7y−22 **19** ¯2a+16 **20** ¯4b+2 **21** ¯8n+17 **22** ¯12y+17

Exercise 2.7 **1** x^2+3x+2 **2** x^2+3x+2 **3** x^2+4x+3 **4** x^2+5x+4 **5** x^2+6x+8 **6** x^2+2x+1 **7** x^2+4x+4 **8** x^2+6x+9 **9** $2x^2+3x+1$ **10** $2x^2+5x+2$ **11** x^2+2x-3 **12** x^2+3x-4 **13** x^2-4 **14** x^2-16 **15** x^2+4x-5 **16** x^2-x-6 **17** x^2-2x-3 **18** x^2-x-6 **19** x^2-2x+1 **20** x^2-4x+4 **21** x^2-6x+9 **22** $x^2-10x+25$ **23** $2x^2+x-3$ **24** $6x^2-4x-2$ **25** $2x^2-2$ **26** $6x^2-8x+2$

Exercise 2.8 **1** 2(a+3) **2** 2(b+2) **3** 2(c+5) **4** 5(x+2) **5** 3(m+2) **6** 4(n+2) **7** 3(p+3) **8** 5(d+4) **9** 7(h+5) **10** 3(2a+1) **11** 5(2u+1) **12** 3(3r+1) **13** 3(5x+1) **14** 6(3g+1) **15** 7(3s+1) **16** 5(p+8) **17** 3(n+9) **18** 5(7x+1)

Exercise 2.9 **1** 3x **2** 4a **3** 2a **4** 2b **5** 3y **6** 5 **7** 3 **8** 4 **9** 4ab **10** 2p **11** 4 **12** 2 **13** 6 **14** 5y **15** 3f **16** 4 **17** 16h **18** 7t

Exercise 2.10 **1** 4(a−3) **2** 2(b−3) **3** 5(x−3) **4** 3(g−2) **5** 7(w−4) **6** 3(d−2) **7** 2(5a−2) **8** 5(2s−1) **9** 3c(5b−1) **10** 2(3u−4) **11** 2p(4q−3) **12** 4(3x−2) **13** 5u(2v−3) **14** 6(3b−1) **15** 7c(2c−3) **16** 4h(4h−3) **17** $7d(3d^2-2)$ **18** $6p^2(4p-2)$

Exercise 2.11 **1** ¯5(z+2) **2** ¯2(a+5) **3** ¯3(x+4) **4** ¯6(q+3) **5** ¯5(d+4) **6** ¯4(b+4) **7** ¯9(m+2) **8** ¯2(n+11) **9** ¯5(b+7) **10** ¯2(d−3) **11** ¯3(w−9) **12** ¯6(p−6) **13** ¯4x(x−3) **14** ¯5g(g−5) **15** ¯2e(e+13) **16** ¯2b(4bc−1) **17** ¯3q(2q−1) **18** ¯5o(3ao−1) **19** ¯$3a(2a^2-5)$ **20** ¯$3t^2(4t^2+5)$ **21** ¯$6p^3(p^2+6)$

Exercise 2.12 **2** 8a **3** 3x−6 **4** 2(3x+2) **5** ¯2 **6** 10^5 **7** 2^6 **8** 68 **9** 1474 **10** $15 000

Exercise 2.13 **2** 11x **3** 4x−12 **4** 3(3x+2) **5** ¯6 **6** 10^2 **7** x^{10} **8** 140 **9** 26576 **10** $18 000

Exercise 2.14 **2** 10y **3** ¯4x+8 **4** 2(3x+4) **5** ¯4 **6** 10^6 **7** x^9 **8** 126 **9** 68761 **10** $24 000

Exercise 2.15 **1** 4 **2** 2.5 **3** 9 **4** 75 **5** 5 **6** 30 250 **7** 6/5 or 1.2 **8** x=3 **9** x=4 **10** 23 **11** 2.2 **12** 18 **13** 10x+2 **14** ¯12a+3 **15** 13 **16** 20

Exercise 2.16 **1a** 3 **1b** 11 **1c** 7 **1d** 55 **1e** ¯13 **1f** 3 **2a** 32 **2b** 4 **2c** 8 **2d** 1 **3a** 2x+5y **3b** ¯2x−3y **3c** a+4b **3d** x−23 **3e** 2x−2 **3f** $11x^2-2x$ **4** 8^2 **5** 27^2 **6** x=2 **7** x=5/4 **8** a

Exercise 2.17 **1** 43 **2** in order from left: 1,4,2,1

Exercise 2.18 **1a** 3x **1b** 10x **1c** ¯2x **1d** 7x+6 **1e** 7b−4a **1f** $8x^5+rs^2$ **1g** 15x **1h** 15x **1i** ¯$6a^2$ **1j** $20x^3$ **1k** ¯$10y^4$ **1l** $12x^4$ **1m** 4x **1n** 3y **1o** ¯4 **1p** ¯4x **1q** 7 **1r** 9b/2 **1s** $5a^3/2$ **1t** ¯$2d^2e^3f/3$ **2a** 3x+6 **2b** 5a+20 **2c** 6y+42 **2d** 6d−8 **2e** 21h−28 **2f** 6x−15 **2g** ¯15t+20 **2h** ¯2y−6 **2i** ¯8p−32 **2j** ¯$2x^2-3x$ **2k** ¯$12x^2+6x$ **2l** ¯$24m^2-30m$ **2m** ¯$2g-5g^2$ **2n** ¯$12x^2+8xy$ **2o** ¯$8xy+16x^2$ **3a** 7x+26 **3b** 2x−6 **3c** 7y−2 **3d** ¯y+18 **3e** p^2-5p **3f** ¯$20b^2+8b+2$ **3g** x^2+5x+6 **3h** x^2+2x+1 **3i** x^2+4x-5 **3j** x^2-x-2 **3k** x^2-4 **3l** x^2-9 **3m** $2x^2-2$ **3n** $6x^2-11x+3$ **4a** 5(x+3) **4b** 3(y+2) **4c** 4(a+4) **4d** 3(6x+1) **4e** 6(3x+1) **4f** 7(2d+1) **4g** 4(n−2) **4h** 2(m−3) **4i** 5(x−3) **4j** 4(4y−1) **4k** 5(2x−1) **4l** 3a(5b−1) **4m** 2(3t−5) **4n** 4(3x−2) **4o** 2u(4v−3) **4p** 2f(7f−6) **4q** $7x(3x^2-2)$ **4r** $6p^2(4p-3)$ **4s** ¯5(a+3) **4t** ¯2(b+4) **4u** ¯2(c−3) **4v** ¯4x(x−3) **4w** ¯4g(g−6) **4x** ¯2x(x+10) **4y** ¯$3x(2x^2-5)$ **4z** ¯$3h^2(4h^3+5)$

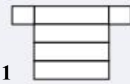
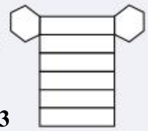
Exercise 2.19 **1a** 7x **1b** 9x **1c** ¯4x **1d** 3x+3 **1e** 11b−3a **1f** $6z^4+fg^2$ **1g** 12x **1h** 16x **1i** ¯$15a^2$ **1j** $6x^3$ **1k** ¯$8k^4$ **1l** $12x^4$ **1m** 4x **1n** 4o **1o** ¯3 **1p** ¯4x **1q** 2 **1r** 5n **1s** $2a^2$ **1t** ¯$6ae^3t/5$ **2a** 2x+10 **2b** 4z+12 **2c** 5y+15 **2d** 12w−24 **2e** 27c−45 **2f** 8x−4 **2g** ¯12v+8 **2h** ¯6r−6 **2i** ¯7u−35 **2j** ¯$4a^2-3a$ **2k** ¯$10x^2+15x$ **2l** ¯$12d^2-18d$ **2m** ¯$2p-3p^2$ **2n** $8y^2-12xy$ **2o** ¯$8y^2+16xy$ **3a** 5x+11 **3b** x+6 **3c** 8w−8 **3d** 3f+16 **3e** b^2-9b **3f** ¯$16g^2+17g+3$ **3g** x^2+5x+6 **3h** x^2+2x+1 **3i** x^2+3x-4 **3j** x^2+x-2 **3k** x^2-1 **3l** x^2-4 **3m** $2x^2+x-1$ **3n** $6x^2-5x+1$ **4a** 3(x+5) **4b** 2(a+4) **4c** 6(b+2) **4d** 3(5c+1) **4e** 9(2d+1) **4f** 6(2e+1) **4g** 3(f−3) **4h** 3(g−2) **4i** 5(h−3) **4j** 4(5m−1) **4k** 5(3x−1) **4l** 5b(3c−1) **4m** 2(6u−5) **4n** 2(7v−4) **4o** 2s(5t−3) **4p** 3r(4r−5) **4q** $6x(3x^3-2)$ **4r** $6y^2(2y^2-3)$ **4s** ¯5(b+4) **4t** ¯2(d+5) **4u** ¯3(c−2) **4v** ¯$6x(x^4-2)$ **4w** ¯6f(f−4) **4x** ¯5x(x+4) **4y** ¯$3x(4x^2-5)$ **4z** ¯$3h^2(6h^5+5)$

Exercise 3.1 **1** 13.69m^2 **2** 5.29cm^2 **3** 27m^2 **4** 432m^2 **5** 8.14km^2 **6** 7.685cm^2 **7** 4.06m^2 **8** 13 764m^2=1.38ha **9** 320cm^2 **10** 9.24m^2 **11** 148 068m^2= 14.81ha **12** 112m^2

Exercise 3.2 **1** 81m^2 **2** 486m^2 **3** 177cm^2 **4** 105m^2 **5** 200cm^2 **6** 3570cm^2 **7** 2138.5cm^2 **8** 57.75m^2 **9** 106cm^2

Exercise 3.3

Prism	Ends	Sides	Total faces
Triangular prism	2 triangles	3 rectangles	5
Rectangular prism	2 rectangles	4 rectangles	6
Pentagonal prism	2 pentagons	5 rectangles	7
Hexagonal prism	2 hexagons	6 rectangles	8
Heptagonal prism	2 heptagons	7 rectangles	9
Octagonal prism	2 octagons	8 rectangles	10

Exercise 3.4 1  **2**  **3** 

Exercise 3.5 **1** 232cm^2 **2** 302cm^2 **3** 145.68m^2 **4** 192.46cm^2 **5** 149.64m^2 **6** 934cm^2 **7** 672cm^2 **8** 600cm^2

Exercise 3.6 **1** 31.42mm **2** 38.33cm **3** 28.27cm **4** 26.08mm

Exercise 3.7 **1** 78.54mm^2 **2** 116.90cm^2 **3** 63.62cm^2 **4** 54.11mm^2

Exercise 3.8 **1** 408.41cm^2 **2** 1979.20cm^2 **3** 353.43m^2 **4** 318.87m^2 **5** 48.82m^2 **6** 12403.01cm^2

Exercise 3.9 **1a** 58.74m^2 **1b** 8 litres **1c** \$126 **2** estimate h=30m, r=15m, surface= 3534m^2, \$425 000 **3** 1 roll

Exercise 3.10 **2** A=½bh **3** 32m^2 **4** A=πr^2 **5** $^{-}2a$ **6** 3x−6 **7** 2(3x+2) **8** 10^5 **9** 10^4 **10** \$7.70

Exercise 3.11 **2** A=lb **3** 14m^2 **4** C=πD or 2πr **5** 12b^2 **6** 6x−2 **7** 3(2x+3) **8** a^6 **9** b^{-4} **10** \$5.20

Exercise 3.12 **2** A=½bh **3** 42m^2 **4** A=πr^2 **5** $^{-}3a$ **6** 10x−4 **7** 2(2x+5) **8** 10^{10} **9** a^6 **10** \$14.40

Exercise 3.13 **1a**  **1b**  **1c**  **1d**  **1e**  **1f**  **2** 600cm^2 **3** 128.68m^3 **4** 9m **5** 1:2 **6** 17a^2 **7** 175 m^2

Exercise 3.14 **1** 52cm **2** 8 **3a** 525cm^2 **3b** 257.08cm^2 **4** 18cm^2 **5** 6 times **6** tripled **7** 30 **8** 6a^2 **9** π:4

Exercise 3.15 **1** 0.001 **2** diameter=6

Exercise 3.16 **1a** 97m^2 **1b** 127.5m^2 **1c** 71.68m^2 **2a** 198cm^2 **2b** 2176cm^2 **2c** 133.52cm^2 **2d** 80+16π or 130.27cm^2 **3a** 105.53m^2 **3b** 14.07 L **3c** \$296

Exercise 3.17 **1a** 120m^2 **1b** 198m^2 **1c** 63.8m^2 **2a** 320cm^2 **2b** 1464cm^2 **2c** 416.20cm^2 **2d** 130+25π or 208.54cm^2 **3a** estimate h=32m, r=12m, SA=1320m^2, cost=\$145 200

Exercise 4.1 **1** m=2×steps+1 **2** m=2×steps+2 **3** m=3×steps+0 **4** m=3×steps−1 **5** d=2×steps+2

Exercise 4.2 **1** y=2x+2 **2** y=3x+1 **3** y=2x−7 **4** y=3x−10 **5** y=¯2x+17 **6** y=¯3x+8 **7** y=¯4x+7 **8** y=2x−9

Exercise 4.3 **1** y=2x+1 **2** y=2x+5 **3** y=¯2x+2 **4** y=¯2x−2

Exercise 4.4 **1**

x	-2	-1	0	1	2
y = x + 2	0	1	2	3	4

2

x	-2	-1	0	1	2
y = x + 3	1	2	3	4	5

3

x	-2	-1	0	1	2
y = 2x + 5	1	3	5	7	9

4

x	-2	-1	0	1	2
y = x − 1	¯3	¯2	¯1	0	1

5

x	-2	-1	0	1	2
y = 2x − 3	¯7	¯5	¯3	¯1	1

6

x	-2	-1	0	1	2
y = 3x + 1	¯5	¯2	1	4	7

7

x	-2	-1	0	1	2
y = −x + 2	4	3	2	1	0

8

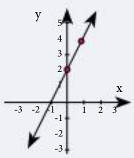
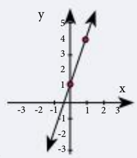
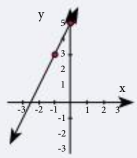
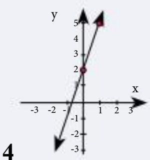
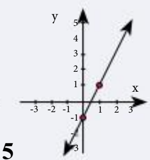
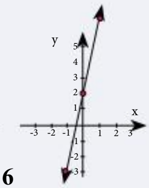
x	-2	-1	0	1	2
y = ¯3x + 4	10	7	4	1	¯2

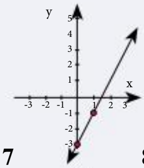
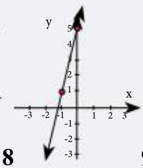
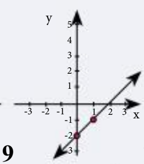
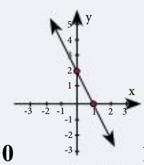
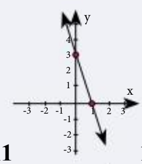
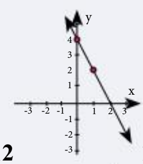
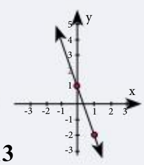
9

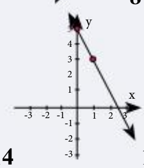
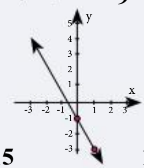
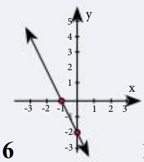
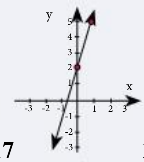
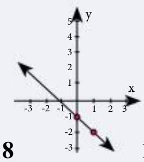
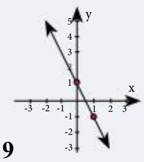
x	-2	-1	0	1	2
y = 40x + 60	¯20	20	60	100	140

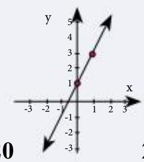
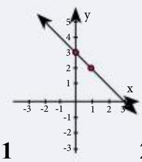
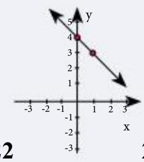
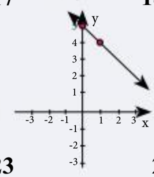
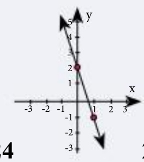
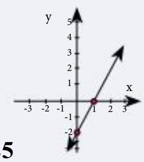
10

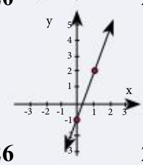
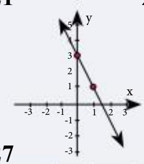
x	-2	-1	0	1	2
y = 25x − 10	¯60	¯15	¯10	15	40

Exercise 4.5 **1**  **2**  **3**  **4**  **5**  **6** 

7  **8**  **9**  **10**  **11**  **12**  **13** 

14  **15**  **16**  **17**  **18**  **19** 

20  **21**  **22**  **23**  **24**  **25** 

26  **27** 

Exercise 4.6 **1**

x	-2	-1	0	1	2
$y = x^2$	4	1	0	1	4

2

x	-2	-1	0	1	2
$y = x^2 + 1$	5	2	1	2	5

3

x	-2	-1	0	1	2
$y = x^2 + 2$	6	3	2	3	6

4

x	-2	-1	0	1	2
$y = x^2 + 3$	7	4	3	4	7

5

x	-2	-1	0	1	2
$y = x^2 - 1$	3	0	¯1	0	3

6

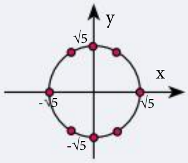
x	-2	-1	0	1	2
$y = x^2 - 2$	2	¯1	¯2	¯1	2

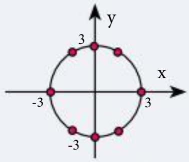
7

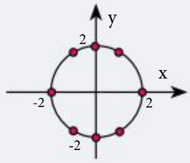
x	-2	-1	0	1	2
y = x(x + 1)	2	0	0	2	6

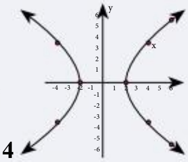
8

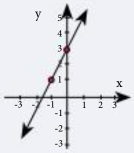
x	-2	-1	0	1	2
y = x(x − 1)	6	2	0	0	2

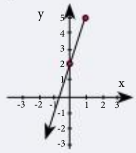
Exercise 4.7 1 

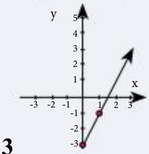
2 

3 

4 


Exercise 4.8 3


Exercise 4.9 3


Exercise 4.10 3

Exercise 4.8 2 y=2x−3 **4** A=½bh **5** 32m^2 **6** ¯4a **7** 2x−6 **8** 2(3x+2) **9** 10^5 **10** 210

Exercise 4.9 2 y=¯2x+5 **4** A=πr^2 **5** 21m^2 **6** 4m **7** 3x−6 **8** 3(x+3) **9** 10^7 **10** 270

Exercise 4.10 2 y=3x−2 **4** C=πD or C=2πr **5** 35m^2 **6** ¯3b **7** 5x+15 **8** 5(x+2) **9** 10^2 **10** 330

Exercise 4.11 1 m=3x+1 **2** 0.5 **3** 5 **4** $F=50h+40 **5** y=2x+1 **6** $C=40P/3+40 **7** b **8** 16 **9** 41

Exercise 4.12 1 20miles **2** yes **3** (50,1008) **4a** y=2x+1 **4b** y=¯2x−1 **5** C=1500b+7500 **6a** P=0.5m **6b** D

Exercise 4.13 1 24 **2** 6 **3** 108

Exercise 4.14 1 m=2step+1 **2a** y=2x+2 **2a** y=¯3x+8 **3a** y=2x+1 **3a** y=¯2x−2 **4a**

x	-2	-1	0	1	2
y = 2x − 3	¯7	¯5	¯3	¯1	1

4b

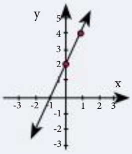
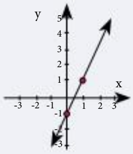
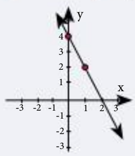
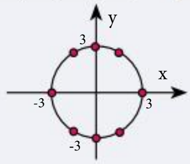
x	-2	-1	0	1	2
y = ¯3x + 4	10	7	4	1	¯2

4c

x	-2	-1	0	1	2
y = x^2 − 1	3	0	¯1	0	3

4d

x	-2	-1	0	1	2
y = x^2 − 2	2	¯1	¯2	¯1	2

5a 
5b 
5c 
6 

Exercise 4.15 1 m=3step+1 **2a** y=3x+2 **2a** y=¯4x+11 **3a** y=2x+5 **3a** y=¯2x+2

4a

x	-2	-1	0	1	2
y = 2x − 1	¯5	¯3	¯1	1	3

4b

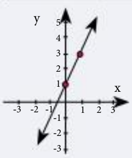
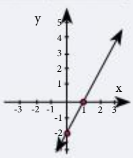
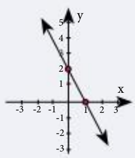
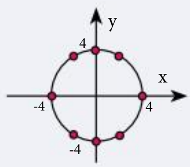
x	-2	-1	0	1	2
y = ¯3x + 2	8	5	2	¯1	¯4

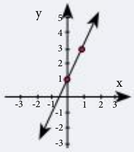
4c

x	-2	-1	0	1	2
y = x^2 − 1	3	0	¯1	0	3

4d

x	-2	-1	0	1	2
y = 2x^2 − 3	5	¯1	¯3	¯1	5

5a 
5b 
5c 
6 

Exercise 5.1 **2** $y=2x-2$ **3**  **4** $A=\frac{1}{2}bh$ **5** $12m^2$ **6** $^{-}2a$ **7** $3x-6$ **8** $2(3x+2)$ **9** 10^7 **10** 210

Exercise 5.2 **1a** 1^4 **1b** 4^6 **1c** 10^5 **1d** a^5 **1e** 1.3^4 **1f** $(^{-}2)^5$ **2a** 3^2 **2b** 4^{-3} **2c** 2 **2d** 10^2 **2e** x^4 **2f** $3x^{-2}$ **2g** $9x^{-1}$ **2h** $8x^{-1}$ **2i** 2^8 **2j** 9^9 **2k** 4^{-4} **2l** 10^{-2} **2m** x^{-8} **2n** a^{-1} **2o** b^5 **2p** $3x^8$ **3a** 2^{-6} **3b** 3^4 **3c** x^8 **3d** 10^6 **3e** 4^6 **3f** x^8 **3g** 10^9 **3h** 2^{-4} **3i** 1.3^8 **3j** 10^3 **3k** $10x^7$ **3l** $3x^{-7}$ **4a** $5x$ **4b** $7x$ **4c** $^{-}3x$ **4d** $6x+4$ **4e** $6d-4a$ **4f** $11a^5+2xy^2$ **4g** $6x$ **4h** $20x$ **4i** $^{-}10n^2$ **4j** $12x^3$ **4k** $^{-}6y^4$ **4l** $20x^4$ **4m** $5x$ **4n** $^{-}5x$ **4o** 5 **4p** $7b/2$ **4q** $5x^3/2$ **4r** $^{-}d^3e^2f/3$ **5a** $5x+15$ **5b** $2h+6$ **5c** $5d+10$ **5d** $8g-6$ **5e** $16x-24$ **5f** $14x-21$ **5g** $^{-}8p+12$ **5h** $^{-}6c-6$ **5i** $^{-}x^2-4x$ **5j** $^{-}12x^2+6x$ **5k** $^{-}6x^2+12xy$ **5l** $^{-}12xy+8x^2$ **6a** $8x+11$ **6b** $3x-1$ **6c** $6y+2$ **6d** $^{-}y+19$ **6e** $^{-}t^2-3t$ **6f** $^{-}15x^2+8x+2$ **6g** x^2+5x+6 **6h** x^2+2x+1 **6i** x^2+3x-4 **6j** x^2-1 **6k** x^2-4 **6l** $4x^2-8x+3$ **7a** $5(x+2)$ **7b** $3(x+2)$ **7c** $4(x+4)$ **7d** $3(6x+1)$ **7e** $6(4r+1)$ **7f** $7(2y+1)$ **7g** $4(x-3)$ **7h** $2(y-2)$ **7i** $5(x-3)$ **7j** $2(3p-5)$ **7k** $2(5w-4)$ **7l** $5a(2b-1)$ **7m** $^{-}5(x+3)$ **7n** $^{-}2(x+3)$ **7o** $^{-}2(x-4)$ **7p** $2b(5b-6)$ **7q** $7x(2x^2-3)$ **7r** $6x^2(4x-3)$ **8a** $119m^2$ **8b** $160m^2$ **8c** $49.3m^2$ **9a** $254cm^2$ **9b** $3160cm^2$ **9c** $375.04cm^2$ **9d** $238+28\pi$ or $325.96cm^2$ **10** \$296 **11** m=4×step+1 **12** $y={}^{-}5x+14$ **13a** $y=2x+3$ **13b** $y={}^{-}2x+2$

14a

x	-2	-1	0	1	2
$y = 2x - 2$	$^{-}6$	$^{-}4$	$^{-}2$	0	2

14b

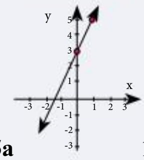
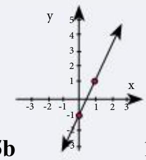
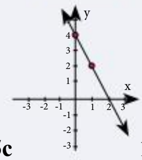
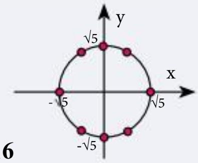
x	-2	-1	0	1	2
$y = {}^{-}2x + 4$	8	6	4	2	0

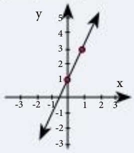
14c

x	-2	-1	0	1	2
$y = x^2 - 1$	3	0	$^{-}1$	0	3

14d

x	-2	-1	0	1	2
$y = x^2 - 2$	2	$^{-}1$	$^{-}2$	$^{-}1$	2

15a  **15b**  **15c**  **16** 

Exercise 5.3 **2** $y=3x-5$ **3**  **4** $A=\pi r^2$ **5** $20m^2$ **6** $2x$ **7** $3x-6$ **8** $2(3x+4)$ **9** 10^8 **10** 360

Exercise 5.4 **1a** 1^3 **1b** 6^5 **1c** 10^5 **1d** x^4 **1e** 9.2^4 **1f** $(^{-}4)^5$ **2a** 3^2 **2b** 5^{-2} **2c** 3^2 **2d** 10^3 **2e** x^4 **2f** $4x^2$ **2g** $8x^{-3}$ **2h** $12x^{-4}$ **2i** 3^9 **2j** 6^9 **2k** 2^{-7} **2l** 10^{-4} **2m** x^{-12} **2n** a **2o** b^5 **2p** $3x^7$ **3a** 2^{-6} **3b** 3^6 **3c** b^6 **3d** 10^4 **3e** 4^7 **3f** x^8 **3g** 10^5 **3h** 2^{-5} **3i** 3.1^7 **3j** 10^7 **3k** $15x^4$ **3l** $4x^{-5}$ **4a** x **4b** $11x$ **4c** $^{-}3x$ **4d** $6x+2$ **4e** $4b$ **4f** $5x^2+8y^5$ **4g** $27x$ **4h** $21x$ **4i** $^{-}6b^2$ **4j** $15x^3$ **4k** $^{-}8x^4$ **4l** $30x^4$ **4m** $3x$ **4n** $^{-}3x$ **4o** 6 **4p** $7y/4$ **4q** $3b^3/2$ **4r** $^{-}3x^2y^3z/2$ **5a** $4x+12$ **5b** $3a+6$ **5c** $6b+6$ **5d** $6c-3$ **5e** $15d-20$ **5f** $15e-10$ **5g** $^{-}6x+8$ **5h** $^{-}7x-28$ **5i** $^{-}4x^2-3x$ **5j** $^{-}6x^2+2x$ **5k** $^{-}12x^2+8xy$ **5l** $^{-}8xy+16x^2$ **6a** $7x+23$ **6b** $2x+18$ **6c** $8x-6$ **6d** $^{-}w+18$ **6e** a^2-5a **6f** $^{-}14b^2+8b+2$ **6g** x^2+4x+3 **6h** x^2+2x+1 **6i** x^2+3x-4 **6j** x^2-4 **6k** x^2-9 **6l** $6x^2+7x-3$ **7a** $5(x+2)$ **7b** $3(x+3)$ **7c** $4(x+2)$ **7d** $3(6p+1)$ **7e** $6(2v+1)$ **7f** $5(3w+1)$ **7g** $4(x-2)$ **7h** $2(n-3)$ **7i** $5(o-4)$ **7j** $2(3x-5)$ **7k** $4(3x-2)$ **7l** $2x(4y-3)$ **7m** $^{-}5(x+34$ **7n** $^{-}2(x+4)$ **7o** $^{-}2(x-43$ **7p** $2b(7b-6)$ **7q** $7x(3x^2-2)$ **7r** $12x^2(2x-1)$ **8a** $72m^2$ **8b** $147m^2$ **8c** $120.25m^2$ **9a** $222cm^2$ **9b** $2176cm^2$ **9c** $118.19cm^2$ **9d** $80+20\pi$ or $142.83cm^2$ **10** m=2×step+1 **11** $y=4x-11$ **12a** $y=3x-3$ **12b** $y={}^{-}2x+2$

13a

x	-2	-1	0	1	2
$y = 3x - 1$	$^{-}7$	$^{-}4$	$^{-}1$	2	5

13b

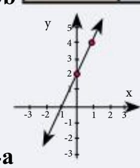
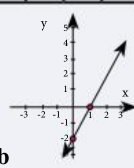
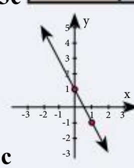
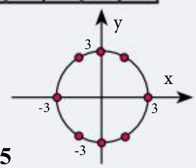
x	-2	-1	0	1	2
$y = {}^{-}2x + 3$	7	5	3	1	$^{-}1$

13c

x	-2	-1	0	1	2
$y = x^2 + 3$	7	4	3	4	7

13d

x	-2	-1	0	1	2
$y = x^2 - 1$	3	0	$^{-}1$	0	3

14a  **14b**  **14c**  **15** 

Exercise 6.1 **1a** 13:11 **1b** 11:13 **1c** 13:24 **1d** 11:24 **2a** 24:7 **2b** 7:24 **2c** 24:31 **2d** 7:31 **3a** 1/5,0.2,20% **3b** 1/2,0.5,50% **3c** 1/4,0.25,25% **3d** 3/5,0.6,60% **3e** 1/10,0.1,10% **3f** 2/5,0.4,40% **3g** 2/10,0.2,20% **3h** 4/5,0.8,80% **3i** 7/10,0.7,70% **3j** 3/10,0.3,30% **3k** 6/10,0.6,60% **3l** 5/10,0.5,50% **3m** 8/10,0.8,80% **3n** 1,1,100% **3o** 9/10,0.9,90% **4a** 1:2,0.5,50% **4b** 3:10,0.3,30% **4c** 4:5,0.8,80% **4d** 1:4,0.25,25% **4e** 3:4,0.75,75% **4f** 5:2,1.5,150% **4g** 31:10,3.1,310% **4h** 33:5,6.6,660%

Exercise 6.2 **1** 1:2 **2** 1:2 **3** 1:2 **4** 1:4 **5** 1:4 **6** 1:4 **7** 5:3 **8** 4:3 **9** 10:3 **10** 4:3 **11** 10:3 **12** 2:3 **13** 1:4 **14** 3:5 **15** 2:1

Exercise 6.3 **1** 1:5 **2** 1:5 **3** 1:6 **4** 20:1 **5** 3:1 **6** 5:3 **7** 1:2 **8** 1:3 **9** 5:12 **10** 6:1 **11** 3:8 **12** 9:5 **13** 1:2 **14** 5:1 **15** 1:5 **16** 3:1 **17** 2:3 **18** 3:1 **19** 2:3 **20** 5:8 **21** 3:4 **22** 1:2 **23** 5:1 **24** 2:1 **25** 1:3 **26** 1:5 **27** 5:1

Exercise 6.4 **1** in proportion **2** in proportion **3** in proportion **4** not in proportion **5** in proportion **6** not in proportion **7** in proportion **8** in proportion **9** in proportion **10** in proportion **11** in proportion **12** in proportion **13** not in proportion **14** in proportion **15** in proportion **16** in proportion **17** in proportion **18** in proportion **19** not in proportion **20** in proportion **21** in proportion **22** in proportion **23** in proportion **24** not in proportion

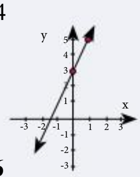
Exercise 6.5 **1** $A1207.55 **2** 4800revs **3** 22.75L **4** 765km **5** 15teeth **6** 5.75g**7** 5.8bags **8** 40mL **9** 20mins **10** $2 025 000

Exercise 6.6 **1** litres=0.2×km **2** Fuel=0.2×dist **3** Dose=1.5×Weight **4** Lime=15×ha **5** Cotton=1.5×ha **6** Price=450×Weight **7** Speed=0.34×time

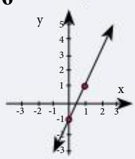
Exercise 6.7 **1** 66.5buckets **2** 416.67km **3** 18km **4** 46.67mins **5** 37.5mL **6** 12.5tbls **7** $11.70 **8** 325kWh

Exercise 6.8 **1** 5days **2** 18people **3** $180 **4** 30people **5** 24mins **6** 120km/h **7** 18h

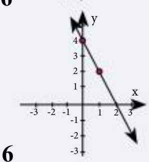
Exercise 6.9 **1a** $46.86 **1b** 0.74kg **2a** $197.89 **2a** 66kg **3a** $25.86 **3b** 3.71$m^2$ **4a** $NZ15.11 **4b** JPY66 188 **5a** 7824 **5b** $63.90 **6a** $SG1318.84 **6b** $AU758.24

Exercise 6.10 **2** 3:2 **3** 21L **4** 25people **5** y=3x−4 **6** 

7 C=πD **8** 2a **9** 3x+6 **10** 5(x+2)

Exercise 6.11 **2** 2:3 **3** 32L **4** 25days **5** y=¯2x+6 **6** 

7 $A=\pi r^2$ **8** ¯2a **9** 5x−10 **10** 2(3x+4)

Exercise 6.12 **2** 5:3 **3** 9L **4** 60people **5** y=4x−2 **6** 

7 C=πD **8** 5a **9** 6x+18 **10** 2(3x+5)

Exercise 6.13 **1** 1km **2** 4km **3** 222m **4** $9350 **5** $33 990 **6** 4800° **7** 7200° **8** 2h20mins **9** 2h16mins **10** 40min

Exercise 6.14 **1** $58 730 **2** one answer is 9/18

Exercise 6.15 **1a** 1:5 **1b** 3:4 **1c** 3:4 **1d** 2:3 **1e** 4:3 **1f** 10:3 **2a** in proportion **2b** in proportion **2c** in proportion **2d** in proportion **2e** in proportion **3a** Fuel=0.25×dist **3b** Dist=0.34×time **4a** $AU1185.19 **4b** 10km **4c** 765km **5a** 5days **5b** 25people **5c** 120km/h **6a** 16.67L/100km **6b** 10L/100km **6c** 5L/100km

Exercise 6.15 **1a** 2:1 **1b** 3:2 **1c** 3:2 **1d** 4:3 **1e** 4:5 **1f** 2:1 **2a** in proportion **2b** not in proportion **2c** in proportion **2d** in proportion **2e** not in proportion **3a** Fuel=0.2×dist **3b** tonnes=1.3×ha **4a** $AU842.70 **4b** 24km **4c** 877km **5a** 12days **5b** 35people **5c** 120km/h **6a** 20L/100km **6b** 6.67L/100km **6c** 2.5L/100km

Exercise 7.2 **1** 3,4,5-yes 4,5,6-no 6,8,10-yes 5,12,13-yes 7,15,16-no **2a** yes **2b** yes **2c** no **3** close enough to square **4** no **5** yes **6** no - longer

Exercise 7.3 **1a** 71.59 **1b** 27.86 **1c** 5.09 **2** 3.61 **3** 1445.08 **4a** 68cm **4b** 50cm **4c** 17inch **5** 5.98m **6** 309.23km

Exercise 7.4 **1a** 6.32 **1b** 4.73 **1c** 1313.40 **2** 1.96m **3** 3.65m **4** 24.68m **5** 71.70m

Exercise 7.5 **1** 7.62 **2** 7.62 **3** 8.60 **4** 9.49 **5** 4.24 **6** 5 **7** 8.06 **8** 11.18 **9** 11.31 **10** 8.54 **11** 5.66 **12** 5 **13** 8 **14** 7.07 **15** 5 **16** 3

Exercise 7.6 **1** rational-integral **2** irrational **3** rational-decimal **4** rational-decimal **5** rational-decimal **6** rational-decimal

Exercise 7.7 **2** $c^2=a^2+b^2$ **3** yes **4** 10 **5** 1:2 **6** 50mins **7** 40mins **8** 2a **9** 3x+6 **10** 230

Exercise 7.8 **2** $c^2=a^2+b^2$ **3** yes **4** 12 **5** 4:3 **6** 25mins **7** 32mins **8** 2x **9** 5x−10 **10** 320

Exercise 7.9 **2** $c^2=a^2+b^2$ **3** yes **4** 16 **5** 3:2 **6** 36mins **7** 20mins **8** 4x **9** 4x+20 **10** 180

Exercise 7.10 **1a** 5 **1b** 20 **5a** 7 **5b** $\sqrt{300}$ or $10\sqrt{3}$ **5c** 5 **5d** $\sqrt{85}-\sqrt{45}$ or $\sqrt{85}-3\sqrt{5}$

Exercise 7.11 **1** 48 **2** x=18°

Exercise 7.12 **1a** yes **1b** yes **1c** no **2** close enough to square **3** yes - close enough **4a** 15 **4b** 5.7 **4c** 4070.33 **5** 2.00m **6a** 7.07 **6b** 9.22 **6c** 3.16 **6d** 5.83 **6e** 10

Exercise 7.13 **1a** no **1b** no **1c** yes **2** close enough to square **3** no **4a** 5.29 **4b** 3.36 **4c** 2163.33 **5** 2.49m **6a** 7.28 **6b** 6.32 **6c** 3.61 **6d** 4.24 **6e** 7.62

Exercise 8.1 **1** △ABC≡△DEF **2** △ABC≡FDE **3** △ABC≡△FED **4** △ABC≡△EFD

Exercise 8.2 **1** △ABC≡△DFM {SAS} **2** △ABC≡△ ONM {SSS} **3** △ABC≡△KIG {SAS} **4** △ABC≡△TYZ {RHS} **5** △ABC≡△POR {AAS} **6** △ABC≡△GDF {SSS} **7** △ABC≡△QPR {AAS} **8** △ABC≡△LKH {RHS}

Exercise 8.5 **1** {AAA} **2** {AAA} **3** {AAA} **4** {AAA} **5** {SAS} **6** {SAS} **7** {SAS} **8** {RHS}

Exercise 8.6 **1** 20 **2** 32 **3** 6 **4** 75 **5** 12 **6** 55

Exercise 8.7 **1** 13m **2** 18.46m **3a** 3:1, 9:1 **3b** 3:1, 9:1

Exercise 8.8 **2** Three corresponding angles equal **3** ~ **4** 1:3 **5** yes **6** $\sqrt{10}$ **7** 5:4 **8** 25mins **9** 5x **10** $4.10

Exercise 8.9 **2** Ratios of three corresponding sides equal **3** ~ **4** 1:4 **5** yes **6** $\sqrt{5}$ **7** 4:3 **8** 18mins **9** $^{-}2x$ **10** $10.90

Exercise 8.10 **2** two corresponding angles equal and ratio of corresponding side equal **3** ~ **4** 2:1 **5** yes **6** $\sqrt{2}$ **7** 2:3 **8** 28mins **9** ^{-}x **10** $4.70

Exercise 8.11 **1** 22.5m **2** 80cm **3** 9:1 **4** 8:1 **5** 27:1 **6** $9\pi r^2$ **7** 2p **8** $9p^2/64$ **9a** $125s^3/8$ **9b** $75s^2/2$

Exercise 8.12 **1** 15 **2** Pour 8 l in to 5L then fill 3l from 5l, leaving 2l in 5l. Empty 3l into 8l, empty 5l into 3l. Fill 5l from 8l. Fill 3l from 5l, this leaves 4l in 5l. There are many other ways.

Exercise 8.13 **1** △ABC≡△VST {SSS} **3** {AAA} **4** {AAA} **5a** 20 **5b** 12 **6** 6.75m

Exercise 8.14 **1** △ABC≡△JKH {AAS} **3** {AAA} **4** {SAS} **5a** 32 **5b** 55 **6** 29.54m

Exercise 9.1 **1** 3.83 **2** 24.83 **3** 2.375 **4** 3.83 **5** 43.83 **6** 9.38 **7** 233.83 **8** $^{-}3.83$ **9** 0.58 **10** 0.42

Exercise 9.2 **1** 5 **2** 5 **3** 55 **4** 2.5 **5** 3.45 **6** 23.5 **7** 0 **8** 1

Exercise 9.3 **1** 29 **2** 5 **3** 2.4 **4** 6 **5** 5 **6** 5 **7** 11.9 **8** 1/2

Exercise 9.4 **1** mean=$492k, median=$480k,range=$80k **2** mean=$548k, median=$420k,range=$970k **3** mean=93s, median=94s,range=9s **4** mean=8.13, median=8,range=4 **5** mean=16.45, median=14,range=26

Exercise 9.5 **1**

Test scores (8|5 is 85)

6	6
7	6 8 8 9
8	2 3 3 4 5
9	4

mean=80.73, median=82,range=28

Temperatures (19|8 is 19.8)

17	0
18	6 7
19	8
20	2 3 6

2 mean=19.31, median=19.8,range=3.6

Ships (7|2 is 72)

6	3 4 5 5 9
7	0 1 2 2 3 4 7
8	0

3 mean=70.38, median=71,range=17

Units (8|2 is 8200)

7	5
8	2
9	3 4 5 6
10	0
11	0 0

4 mean=9500, median=9500,range=3500

Ages (5|7 is 57)

3	7
4	0 4 5 6 7 7 8 9
5	0 2 2 3 3 3 3 5 6 6 7 7 7 9 9
6	0 3 8

5 mean=52.44, median=53,range=31

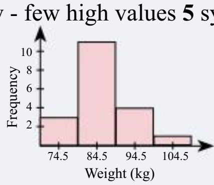
Rents (34|9 is 349)

34	5 8 8 9 9 9 9
35	0 0 1 1 1 2 2 6
36	2 3
37	4 5
38	5

6 mean=355.45, median=351,range=40

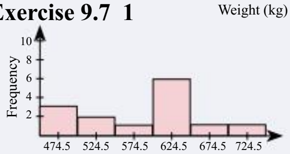
Exercise 9.6 **1** symmetrical **2** positive skew - few high values **3** negative skew - few low values **4** positive skew - few high values **5** symmetrical **6** negative skew - few low values

Exercise 9.7 **1**



mean=86.08, median=84.5,range=30

2



mean=585.21, median=624.5,range=250

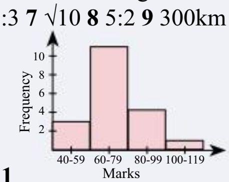
Exercise 9.8 **1** symmetrical **2** negative skew - few low values **3** positive skew - few high values **4** bimodal **5** positive skew - few high values **6** bimodal with negative skew - few low values

Exercise 9.11 **2** 14.5 **3** 5th **4** max−min **5** 3 corresponding angles equal **6** 1:4 **7** √5 **8** 3:2 **9** 300km **10** $7.80

Exercise 9.12 **2** 24.5 **3** 6th **4** total/no **5** ratio of 3 corresponding sides equal **6** 1:3 **7** √2 **8** 4:3 **9** 250km **10** $5.60

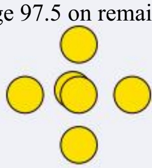
Exercise 9.13 **2** 34.5 **3** average of 5th&6th **4** 2 modes **5** ratio of 2 corresponding sides equal and included angle equal **6** 1:3 **7** √10 **8** 5:2 **9** 300km **10** $13.00

Exercise 9.14 **1**



2 0,0,0,0,0,4,6,6 **3** 3/8 **4** 3/16 **5** 30 **6** 46 **7** 105 **8** needs to average 97.5 on remaining 4 tests **9** 10

Exercise 9.15 **1**



Exercise 9.16 **1a** mean=17.5,median=15,range=28 **1b** mean=$49.2k,median=$48k,range=$8k **2a** symmetrical **2b** positive skew - few high values **2c** negative skew - few low values **2d** bimodal **3a** mean=41.23,median=42,range=25 **3b** mean=87.83,median=84.5,range=30

Exercise 9.17 **1a** mean=67,median=65,range=36 **1b** mean=4.32,median=4.2,range=1.3 **2a** bimodal **2b** positive skew - few high values **2c** negative skew - few low values **2d** symmetrical **3a** mean=85.5,median=86.5,range=21 **3b** mean=87,median=84.5,range=30

Exercise 10.1 **2** 14.5 **3** 5th **4** max−min **5** 3 corresponding sides equal **6** 1:3 **7** √5 **8** 3:4 **9** 150km **10** $13.00
Exercise 10.2 **1a** 1:3 **1b** 3:5 **1c** 3:5 **1d** 5:6 **1e** 2:3 **1f** 2:1 **2a** yes **2b** no **2c** yes **2d** yes **3a** Fuel=0.2×km
3b dist=0.34×time **4a** $AU125 **4b** 10km **5** 7.5days **6a** 16.67L/100km **6b** 10L/100km **6c** 5L/100km **7a** yes
7b yes **7c** no **8** not square **9a** 10.20 **9b** 5.33 **9c** 242.33 **10** 2.08 **11a** 7.07 **11b** 9.22 **11c** 3.61 **11d** 6.32 **11e** 6.32
12 △ABC≡△EFG{SSS} **14a** {AAA} **14a** {AAA} **15a** 15 **15b** 12 **16** 11.2m
17a mean=18.5,median=15,range=40 **17b** mean=54.6,median=54,range=15
18a positive skew - few high values **18b** negative skew - few low values
19a mean=79.9,median=81.5,range=30 **19b** mean=87.44,median=84.5,range=30
Exercise 10.3 **2** 24.5 **3** average of 5th and 6th **4** max−min **5** 3 corresponding angles equal **6** 1:3 **7** √2 **8** 5:4
9 250km **10** $10.40
Exercise 10.4 **1a** 3:1 **1b** 3:1 **1c** 2:1 **1d** 3:2 **1e** 6:5 **1f** 10:3 **2a** yes **2b** no **2c** yes **2d** yes **3a** Fuel=0.3×km
3b Tonnes=2×acres **4a** $AU900 **4b** 846km **5** 27people **6a** 20L/100km **6b** 6.67L/100km **6c** 2.5L/100km
7a yes **7b** no **7c** yes **8** not square **9a** 12 **9b** 3.04 **9c** 2163 **10** yes **11a** 8.60 **11b** 5 **11c** 3.16 **11d** 5.83 **11e** 10
12 △ABC≡△JKH{AAS} **14a** {AAA} **14a** {AAA} **15a** 32 **15b** 55 **16** 8.3m
17a mean=55.22,median=54,range=27 **17b** mean=165.33,median=166,range=19
18a positive skew - few high values **18b** symmetrical **19a** mean=22.8,median=25,range=21
19b mean=40.5,median=34.5,range=31

Exercise 11.1 **1** 2^3 **2** 8^4 **3** a^5 **4** 10^3 **5** b^5 **6** h^3 **7** m^5 **8** 10^4 **9** 3^6
Exercise 11.2 **1** 4×4×4 **2** b×b×b×b **3** 5×5 **4** 2×2×2×2×2×2×2 **5** 6×6 **6** m×m×m×m×m **7** x×x×x×x
8 p×p×p×p×p **9** 1×1×1×1
Exercise 11.3 **1** a^4b^3 **2** 2^23^3 **3** a^5b^4 **4** 2^410^3 **5** b^4g^5 **6** z^{11} **7** $p^5q^3r^4$ **8** $2^33^24^3$ **9** 10^4g^6
Exercise 11.4 **1** a^5 **2** 3^7 **3** a^6 **4** 10^6 **5** b^6 **6** 10^9 **7** 9×10^6 **8** a^310^6 **9** $3^4\times10^6$
Exercise 11.5 **1** 10^5 **2** 4^4 **3** 3^7 **4** 10^7 **5** x^6 **6** x^5 **7** x^7 **8** x^7 **9** 2^4 **10** a^9 **11** 3^5 **12** 10^9 **13** b^7 **14** 10^9 **15** c^6 **16** d^5 **17** 6.3^7
18 e^9 **19** 0.1^9 **20** 10^8 **21** 10^9 **22** z^9 **23** 2^{12} **24** y^7
Exercise 11.6 **1** 10^2 **2** 10^2 **3** 2^4 **4** 3^2 **5** x^3 **6** a^2 **7** 10^4 **8** b **9** 10^2 **10** n^4 **11** 2^2 **12** 10^4 **13** y^4 **14** 0.2^2 **15** 10^8 **16** $x^0=1$
17 10^2 **18** x^3 **19** a^5 **20** $10^0=1$
Exercise 11.7 **1** b^6 **2** b^6 **3** b^9 **4** 10^6 **5** 10^4 **6** a^{12} **7** 3^8 **8** 2^8 **9** 10^4 **10** x^{10} **11** y^6 **12** 5^{10} **13** 4^9 **14** 10^6 **15** 10^3 **16** a^{10}
17 10^9 **18** c^{12} **19** d^{10} **20** 0.3^{15} **21** 10^8 **22** 2^{11} **23** x^9 **24** z^{11} **25** 10^7 **26** b^{17} **27** 2^{13} **28** m^{18} **29** 3^5 **30** p^8 **31** 10^7 **32** 10^9
Exercise 11.8 **1** 1 **2** 1 **3** 1 **4** 1 **5** 1 **6** 1 **7** 1 **8** 1 **9** 3 **10** 4 **11** 9 **12** 3 **13** 8 **14** 2 **15** 5 **16** 6 **17** 2 **18** p^2 **19** 10 **20** 10
Exercise 11.9 **1** 10^{-2} **2** 10^{-5} **3** b^{-4} **4** 3^{-5} **5** x^{-3} **6** 10^{-1} **7** 10^{-4} **8** 2^{-6} **9** 10^{-2} **10** 10^{-3} **11** 2^{-3} **12** 3^{-3} **13** 10 **14** 10^3 **15** 10^8
16 1 **17** 3^{-1} **18** 10 **19** x^9 **20** 10^{-7} **21** x^2 **22** 10 **23** 2^4 **24** 3^2 **25** x **26** a^{-2} **27** 10^{-10} **28** b^{-1} **29** 10^{-6} **30** 5^{-9} **31** 2^{-8} **32** 10^4
33 x^6 **34** 10^{-6} **35** b^{-12} **36** y^6 **37** 3 **38** 4 **39** 9 **40** 3 **41** 6×10^5 **42** 18×10^3 **43** 3×10^4 **44** 2×10^7 **45** 12×10^9 **46** 6×10^{-6}
47 2 **48** $3x^{-3}$
Exercise 11.10 **1** 3.75×10^5 **2** 5.82×10^5 **3** 1.6×10^5 **4** 5.1×10^6 **5** 6.2×10^6 **6** 7.4×10^6 **7** 4.7×10^4 **8** 2.9×10^4
9 8.13×10^4 **10** 8.3×10^7 **11** 4.81×10^7 **12** 7.62×10^6 **13** 9.1×10^4 **14** 7.6×10^7 **15** 2.0×10^7 **16** 2.5×10^{11}
17 2.998×10^8 **18** 1.49×10^{11} **19** 1.27×10^7 **20** 1.0×10^{11}
Exercise 11.11 **1** 1000 **2** 300 000 **3** 80 000 **4** 400 000 **5** 330 **6** 810 000 000 **7** 63 000 000 **8** 71 100 000
9 1 000 000 000 **10** 387 900 **11** 9 324 000 **12** 7 821 400 000 **13** 2 101 200 000 000 **14** 550 000 000
15 23 000 **16** 320 000 000
Exercise 11.12 **1** 3.1×10^{-5} **2** 6.3×10^{-6} **3** 1×10^{-3} **4** 8×10^{-4} **5** 8.56×10^{-4} **6** 7.4×10^{-6} **7** 5×10^{-2} **8** 2.6×10^{-1}
9 4.91×10^{-1} **10** 1×10^{-3} **11** 1×10^{-4} **12** 1×10^{-5} **13** 9.1×10^{-3} **14** 6.6×10^{-5} **15** 9.39×10^{-8} **16** 2.5×10^{-8} **17** 3×10^{-9}
18 5.2×10^{-7} **19** 9×10^{-28} **20** 1.67×10^{-24}
Exercise 11.13 **1** 0.2 **2** 0.07 **3** 0.005 **4** 0.0001 **5** 0.000 067 **6** 0.000 003 2 **7** 0.000 835 **8** 0.000 000 009 91
9 0.000 000 0104 **10** 0.000 006 172 **11** 0.000 000 430 4 **12** 0.000 000 000 006 826 **13** 0.000 000 009 013
14 0.000 000 000 006 5 **15** 0.000 000 000 000 003 3 **16** 0.000 002 2
Exercise 11.14 **1a** 1.8×10^{10}m **1b** 1.08×10^{12}m **1c** 2.59×10^{13}m **2** 9.46×10^{15}m **3** 58.64h **4** 81.8years
5 3.3×10^{-7}s **6** 3.3×10^{-6}s **7** 1667km/h
Exercise 11.15 **1** 1.09×10^7m **2** 4.37×10^9m **3** 1.77×10^{-14}m **4** $2.21\times10^{19}m^3$ **5** $1.62\times10^{20}m^3$ **6** $1.41\times10^{27}m^3$
7a $5.5\times10^3kg/m^3$ **7b** $3.3\times10^3kg/m^3$ **7c** $4.0\times10^3kg/m^3$ Density of all 3 are of a similar order.
Exercise 11.16 **2** 10^7 **3** 5.7×10^5 **4** 4.9×10^{-4} **5** 3.2 **6** 2 **7** ratios of corresponding 3 sides are equal **8** √2
9 5:3 **10** 2min
Exercise 11.17 **2** 10 **3** 2.34×10^6 **4** 2.3×10^{-5} **5** 3.2 **6** 3 **7** ratios of 2 corresponding sides are equal and included angles are equal **8** √5 **9** 4:3 **10** 3min 20s
Exercise 11.18 **2** 10^{-7} **3** 5.54×10^9 **4** 6×10^{-7} **5** 3.2 **6** 3 **7** corresponding 3 angles are equal **8** √10 **9** 5:3 **10** 2min

Exercise 11.19 **1a** 0.01 **1b** 0.0001 **1c** 0.000 000 001 **1d** 0.000 000 01 **1e** 1 000 000 **1f** 10^{15} **1g** 2^{15} or 32 768 **1h** 1.0406 **2** b **3** 1.8×10^{10}m **4** 8 **5** 4 **6** 0 **7** 25 **8** 27
Exercise 11.20 **1** 31.7years **2** Sal-Math, Mal-Science,Al-Chess
Exercise 11.21 **1a** 10^{-3} **1b** 10^{-4} **1c** a^{-7} **2a** 10^2 **2b** 10^2 **2c** 10^{10} **2d** 10^{-7} **2e** 10^4 **2f** x^{-1} **2g** 10^{-6} **2h** 2^{-9} **2i** 5^{-6} **2j** 10^6 **2k** a^6 **2l** 8 **2m** 15×10^5 **2n** 6×10^2 **2o** 2×10^2 **2p** 15×10^{-7} **2q** 2x **2r** $4x^{-2}$ **3a** 9.5×10^4 **3b** 5.2×10^5 **3c** 1.6×10^7 **3d** 1×10^{-5} **3e** 5.2×10^{-6} **3f** 5×10^{-3} **3g** 6×10^{22} **3h** 4×10^{-8} **4a** 100 **4b** 62 000 000 **4c** 951 000 000 **4d** 0.002 **4e** 0.000 742 **4f** 0.000 004 55 **4g** 230 000 000 **4h** 0.000 000 000 000 000 000 000 000 9 **5** 2.14×10^7m **6** 1.65×10^{20}m^3 **7** 8
Exercise 11.22 **1a** 10^{-2} **1b** 10^{-5} **1c** x^{-9} **2a** 10 **2b** 10^{-2} **2c** 10^8 **2d** 10^{-8} **2e** 10^4 **2f** 1 **2g** 10^{-9} **2h** 5^{-12} **2i** 2^{-8} **2j** 10^9 **2k** y^{10} **2l** 9 **2m** 6×10^7 **2n** 15×10^4 **2o** 2×10^1 **2p** 8×10^{-6} **2q** 2 **2r** $4x^{-7}$ **3a** 4.3×10^5 **3b** 2.3×10^4 **3c** 2×10^8 **3d** 1×10^{-3} **3e** 7.2×10^{-5} **3f** 9×10^{-5} **3g** 1.4×10^8 **3h** 6×10^{-6} **4a** 10000 **4b** 5 500 000 **4c** 8 010 000 000 **4d** 0.004 **4e** 0.000 001 42 **4f** 0.000 000 000 003 05 **4g** 23 000 000 **4h** 0.000 000 9 **5** 4.40×10^8m **6** 1.44×10^{24}m^3 **7** 6

Exercise 12.1 **1** 16.55 **2** 47.42 **3** 109.20 **4** 44.42 **5** 4.05 **6** 113.28 **7** 3.31m **8** 14.14m
Exercise 12.2 **1** 4.73 **2** 2.63 **3** 1313.41 **4** 143.09m **5** 17.32cm
Exercise 12.3 **1** BC,AB,AC **2** BC,AC,AB **3** BC,AB,AC **4** AC,AB,BC **5** BC,AB,AC **6** AB,BC,AC
Exercise 12.5 **1** tanα=3/4,α=36.9° **2** tanα=3/4,α=36.9° **3** tanα=3/4,α=36.9° **4** tanα=8/15,α=28.1° **5** tanα=12/5,α=67.4° **6** tanα=5/12,α=22.6°
Exercise 12.6 **1** 8.09 **2** 37.69 **3** 6.52 **4** 5.25 **5** 2.00 **6** 7.45 **7** 27.53m **8** 12.15m
Exercise 12.7 **1** x=4.64,y=5.47 **2** x=8.02,y=12.74 **3** x=10.19,y=12.59 **4** x=12.83,y=13.22 **5** x=6.56,y=9.62 **6** x=9.99,y=22.00 **7** 4.6m **8** 4.18m **9** 4.3m **10** 49.1m
Exercise 12.8 **1** c=9.54,α=29.5°,β=60.5° **2** c=12.60,α=51.8°,β=38.2° **3** c=70.71,α=45°,β=45° **4** c=44.81,α=64.1°,β=25.9° **5** c=60.03,α=60.0°,β=30.0° **6** c=56.22,α=51.5°,β=38.5°
Exercise 12.9 **2** tan=opp/adj **3** 3 **4** 60° **5** √10 **6** 5.4×10^4 **7** 3.2×10^{-3} **8** 10^2 **9** 3.2 **10** 400
Exercise 12.10 **2** tan=opp/adj **3** 3/2 **4** 30° **5** √13 **6** 1.7×10^5 **7** 1.4×10^{-4} **8** 10^3 **9** 3 **10** 500
Exercise 12.11 **2** tan=opp/adj **3** 1 **4** 50° **5** √8 or 2√2 **6** 3×10^6 **7** 9×10^{-7} **8** 10^3 **9** 3 **10** 600
Exercise 12.12 **1** 20 **2** 2 **3** 0.2 **4** 0.02 **5a** 1 **5b** 0.5 **5c** 0.33 **6** 90°,53° **7** no **8** no **9** 10√2 **10** 8√2 **11** 13 **12a** 4√2 **12b** 6 **12c** 4√2
Exercise 12.13 **1** 9,16,25,36,49, ... 2500
Exercise 12.14 **1** BC,AB,AC **2a** 25 **2b** 4.55 **2c** 166.41 **3a** 36.9° **3b** 44.37 **3c** 11.81 **4a** c=83.63,α=41.1°,β=48.9° **4b** c=153.40,α=37.3°,β=52.7° **4c** c=85.91,α=27.1°,β=62.9° **5** 9.80m **6** 5.46km
Exercise 12.15 **1** AB,AC,BC **2a** 35 **2b** 5.18 **2c** 179.37 **3a** 36.9° **3b** 30.11 **3c** 2.42 **4a** c=122.92,α=39.4°,β=50.6° **4b** c=220.60,α=37.1°,β=52.9° **4c** c=91.66,α=28.8°,β=61.2° **5** 23.04m **6** 60km

Exercise 13.1 **1** 50.41m^2 **2** 24m^2 **3** 88.25cm^2 **4** 115.08km^2 **5** 31.85cm^2 **6** 459.96m^2 **7** 16 800m^2, 1.68ha **8** 42 550m^2,4.26ha **9** 63.62m^2 **10** 660.52cm^2 **11** 445
Exercise 13.2 **1** 79m^2 **2** 115.5cm^2 **3** 120.82mm^2 **4** 4056.64m^2 **5** 13.73m^2 **6** 32.48cm^2 **7** 293.79mm^2 **8** 18.85m^2
Exercise 13.3 **1a** 6.7cm^3 **1b** 0.9cm^3 **1c** 35m^3 **1d** 2.4m^3 **1e** 4 900 000cm^3 **1f** 1 900 000cm^3 **1g** 8300mm^3 **1h** 600mm^3 **2a** 4.2L **2b** 61 300 000L **2c** 5.8kL **2d** 0.72ML **2e** 7300L **2f** 610L
Exercise 13.4 **1** 471.24cm^3 **2** 283.27cm^3 **3** 525m^3 **4** 5.62m^3 **5** 157.5m^3 **6** 42612m^3
Exercise 13.5 **1** 216m^3 **2** 7776cm^3 **3** 110m^3 **4** 362.25m^3 **5** 5677.06cm^3 **6** 1178.10cm^3
Exercise 13.6 **1** \$1025 **2** \$210 **3** 5092m^3=5 092 000L=5.1ML **4** 18.82m^3=18 820L **5** 840m^3=840 000L
Exercise 13.7 **2** A=πr^2 **3** V=Areabase×h **4** 1 000 000L **5** 2 **6** √5 **7** 5×10^3 **8** 2×10^{-4} **9** 10^3 **10** 207
Exercise 13.8 **2** A=πr^2 **3** V=Areabase×h **4** 1kg **5** 3 **6** √10 **7** 8×10^4 **8** 3×10^{-5} **9** 10^{11} **10** 306
Exercise 13.9 **2** A=πr^2 **3** V=Areabase×h **4** 1000L **5** 1 **6** √2 **7** 9×10^6 **8** 1×10^{-7} **9** 10^{-2} **10** 414
Exercise 13.10 **1** 3cm **2** One answer: l=3cm,b=4cm,h=5cm **3** F **4** 13 **5** 24L **6** 21 **7** 100m^2 **8** 4.5m **9** 4.22m
Exercise 13.11 **1** 9 **2** 60L **3** 6
Exercise 13.12 **1a** 21m^2 **1b** 72.38cm^2 **1c** 153m^2 **2a** 7.2cm^3 **2b** 58m^3 **2c** 2 700 000cm^3 **2d** 900mm^3 **2e** 4.6L **2f** 410kL **2g** 1100L **2h** 250mL **3a** 452.39cm^3 **3b** 912m^3 **3c** 157.5m^3 **3d** 226.19cm^3 **4** \$760 **5** 787.5m^3
Exercise 13.13 **1a** 25.73cm^2 **1b** 128.68cm^2 **1c** 164.45mm^2 **2a** 92cm^3 **2b** 9.8m^3 **2c** 9 800 000cm^3 **2d** 600mm^3 **2e** 5.6L **2f** 350kL **2g** 4100L **2h** 490mL **3a** 48.45m^3 **3b** 157.5m^3 **3c** 591.6m^3 **3d** 527.79cm^3 **4** \$1080 **5** 990m^3

Exercise 14.1 **2a** 1/6 **2b** 1/2 **2c** 1/2 **2d** 1/2 **2e** 1/3 **2f** 2/3 **2g** 0 **2h** 1/3 **3a** 3/8 **3b** 5/8 **3c** 5/8 **3d** 3/8 **3e** 1 **3f** 0
Exercise 14.2 **1a** 1/4=0.25 **1b** 2/4=0.5 **1c** 1/4=0.25 **2a** 1/4=0.25 **2b** 2/4=0.5 **2c** 1/4=0.25 **3a** 1/8=0.125 **3b** 3/8=0.375 **3c** 3/8=0.375 **3d** 1/8=0.125 **4a** 1/16=0.0625 **4b** 4/16=0.25 **4c** 6/16=0.375 **4d** 4/16=0.25 **4e** 1/16=0.0625
Exercise 14.5 **1a** 1/36=0.03 **1b** 2/36=0.06 **1c** 3/36=0.08 **1d** 4/36=0.11 **1e** 5/36=0.14 **1f** 6/36=0.17 **1g** 5/36=0.14 **1h** 4/36=0.11 **1i** 3/36=0.08 **1j** 2/36=0.06 **1k** 1/36=0.03 **1l** 0 **1m** 3/36=0.08 **1n** 6/36=0.17 **1o** 15/36=0.42 **1p** 1/36=0.03 **1q** 6/36=0.17 **1r** 15/36=0.42 **1s** 18/36=0.5 **1t** 18/36=0.5 **1u** 12/36=0.33 **2a** 1/12=0.08 **2b** 1/12=0.08 **2c** 1/12=0.08 **2d** 3/12=0.25 **2e** 3/12=0.25 **2f** 6/12=0.5 **3a** 0 **3b** 1/24=0.04 **3c** 2/24=0.08 **3d** 3/24=0.13 **3e** 4/24=0.17 **3f** 4/24=0.17 **3g** 4/24=0.17 **3h** 3/24=0.13 **3i** 2/24=0.08 **3j** 1/24=0.04 **3k** 0 **3l** 0 **3m** 6/24=0.25 **3n** 6/24=0.25 **3o** 12/24=0.5
Exercise 14.6 **1a** 1/12=0.08 **1b** 7/12=0.58 **1c** 1/12=0.08 **1d** 7/12=0.58 **1e** 3/12=0.25 **1f** 9/12=0.75 **1g** 3/12=0.25 **1h** 9/12=0.75 **2a** 1/16=0.06 **2b** 9/16=0.56 **2c** 1/16=0.06 **2d** 9/16=0.56 **2e** 4/16=0.25 **2f** 12/16=0.75 **2g** 4/16=0.25 **2h** 12/16=0.75
Exercise 14.7 **1a** 18/36=0.5 **1b** 6/36=0.17 **1c** 18/36=0.5 **1d** 5/36=0.14 **1e** 19/36=0.53 **1f** 1/36=0.03 **1g** 24/36=0.67 **1h** 6/36=0.17 **1i** 24/36=0.67 **1j** 1 **1k** 0 **2a** 3/20=0.15 **2b** 4/20=0.2 **2c** 10/20=0.5 **2d** 10/20=0.5 **2e** 2/20=0.1 **2f** 10/20=0.5 **2g** 1/20=0.05 **2h** 10/20=0.5 **2i** 3/20=0.15 **2j** 14/20=0.7
Exercise 14.8 **1a** 4/25=0.16 **1b** 25/25=1 **1c** 13/25=0.52 **1d** 4/12=0.33 **2a** 4/23=0.17 **2b** 23/23=1 **2c** 11/23=0.48 **2d** 4/12=0.33 **3a** 21/29=0.72 **3b** 29/29=1 **3c** 2/29=0.07 **3d** 21/27=0.78
Exercise 14.9 **2** 1/6 **3** 1/3 **4** 1/36 **5** $A=\pi r^2$ **6** V=areabase×h **7** 2 **8** $\sqrt{5}$ **9** 5×10^3 **10** 210
Exercise 14.10 **2** 1/6 **3** 1/3 **4** 3/36=1/12 **5** $A=\pi r^2$ **6** V=areabase×h **7** 3 **8** $\sqrt{10}$ **9** 6.7×10^5 **10** 240
Exercise 14.11 **2** 1/6 **3** 1/2 **4** 6/36=1/6 **5** $A=\pi r^2$ **6** V=areabase×h **7** 4 **8** $\sqrt{17}$ **9** 9.1×10^{-5} **10** 270
Exercise 14.12 **1a** 14/80=0.18 **1b** 37/80=0.46 **2a** 6ways **2b** 6/20=0.3 **3a** 2/5 **3b** 3/10 **3c** 1/10
Exercise 14.13 **1** 65536 **2** 1-2-3-4-5 **3** The solid cylinder rolls downhill faster
Exercise 14.14 **1a** 1/4=0.25 **1b** 2/4=0.5 **1c** 1/4=0.25 **2a** 1/12=0.08 **2b** 7/12=0.58 **2c** 1/12=0.08 **2d** 7/12=0.08 **3a** 21/36=0.58 **3b** 0 **3c** 22/36=0.61 **3d** 0 **3e** 22/36=0.61 **3f** 2/36=0.06 **3g** 20/36=0.56 **3h** 2/36=0.06 **3i** 21/36=0.58 **3j** 1 **3k** 0 **4a** 15/26=0.58 **4b** 26/26=1 **4c** 4/26=0.15 **4d** 15/22=0.68 **5a** 5/15=0.33 **5b** 11/15=0.73 **6a** 18/90=0.2 **6a** 9/90=0.1
Exercise 14.15 **1a** 1/4=0.25 **1b** 2/4=0.5 **1c** 1/4=0.25 **2a** 1/16=0.06 **2b** 9/16=0.56 **2c** 4/16=0.25 **2d** 12/16=0.08 **3a** 18/36=0.5 **3b** 2/36=0.06 **3c** 20/36=0.0.56 **3d** 0 **3e** 19/36=0.53 **3f** 1/36=0.03 **3g** 24/36=0.67 **3h** 6/36=0.17 **3i** 24/36=0.67 **3j** 1 **3k** 0 **4a** 3/22=0.14 **4b** 22/22=1 **4c** 11/22=0.5 **4d** 3/11=0.27 **5a** 12/15=0.8 **5b** 8/15=0.53 **6a** 18/90=0.2 **6a** 18/90=0.2

Exercise 15.1 **2** 1/6 **3** 3/6=1/2 **4** 3/36=1/12 **5** $A=\pi r^2$ **6** V=areabase×h **7** 3/2 **8** $\sqrt{13}$ **9** 4.5×10^{-4} **10** 360
Exercise 15.2 **1a** 10^{-2} **1b** 10^{-5} **1c** x^{-4} **2a** 10^3 **2b** 10^8 **2c** 10^{-2} **2d** x^{-2} **2e** 10^8 **2f** 3 **2g** 12×10^3 **2h** 3×10^1 **2i** $2x^{-4}$ **3a** 4.3×10^5 **3b** 6.7×10^5 **3c** 5.5×10^7 **3d** 1×10^{-4} **3e** 3.1×10^{-5} **3f** 3×10^{-3} **3g** 6×10^{22} **4a** 10 000 **4b** 3 200 000 000 **4c** 40 100 000 **4d** 0.005 **4e** 0.000 082 **4f** 0.000 004 5 **4g** 0.000 000 000 000 000 000 000 000 9 **5** 4.02×10^7m **6** 1.10×10^{21}m **7** 8 **8a** BC **8b** AB **8c** AC **9a** 20 **9b** 4.36 **9c** 451.96 **10a** 36.9° **10b** 38.43 **10c** 4.47 **11a** c=112.61,α=41.8°,β=48.2° **11b** c=135.37,α=41.1°,β=48.9° **11c** c=76.00,α=29.6°,β=60.4° **12** 6.87m **13** 20m² **13b** 36.32cm² **13c** 272m² **14a** 7 300 000cm³ **14b** 600mm³ **14c** 7.6L **14d** 540kL **14e** 6100L **14f** 320mL **15a** 226.19cm³ **15b** 115.2m³ **16** $555 **17** 337.5m³=337 500L **18a** 1/4=0.25 **18b** 2/4=0.5 **18c** 1/4=0.25 **19a** 1/12=0.08 **19b** 7/12=0.58 **19c** 1/12=0.08 **19d** 7/12=0.58 **20a** 21/36=0.58 **20b** 0 **20c** 19/36=0.53 **20d** 1/36=0.03 **20e** 1 **20f** 24/36=0.67 **21a** 9/23=0.39 **21b** 23/23=1 **21c** 6/23=0.26 **21d** 9/17=0.53 **22a** 8/15=0.53 **22b** 3/15=0.2
Exercise 15.3 **2** 1/6 **3** 3/6=1/2 **4** 3/36=1/12 **5** $A=\pi r^2$ **6** V=areabase×h **7** 1 **8** $\sqrt{8}$ or $2\sqrt{2}$ **9** 7.4×10^{-4} **10** 510
Exercise 15.4 **1a** 10^{-5} **1b** 10^{-9} **1c** x^{-3} **2a** 10^{-2} **2b** 10^{10} **2c** 10^{-6} **2d** 1 **2e** 10^6 **2f** 9 **2g** 15×10^3 **2h** 2×10^2 **2i** $3x^{-5}$ **3a** 3×10^4 **3b** 4.8×10^6 **3c** 3.1×10^5 **3d** 3×10^{-3} **3e** 6.2×10^{-4} **3f** 6.3×10^{-7} **3g** 5.2×10^{-7} **4a** 1000 **4b** 4 200 000 **4c** 5 110 000 000 **4d** 0.0002 **4e** 0.000 000 71 **4f** 0.000 000 003 2 **4g** 149 000 000 000 **5** 3.80×10^7m **6** 9.28×10^{20}m **7** 4 **8a** AB **8b** AC **8c** BC **9a** 30 **9b** 4.73 **9c** 234.29 **10a** 36.9° **10b**548.55 **10c** 8.03 **11a** c=101.32,α=48.6°,β=41.4° **11b** c=134.54,α=48.0°,β=42.0° **11c** c=91.57,α=55.3°,β=34.7° **12** 65km **13** 27m² **13b** 88.25cm² **13c** 251m² **14a** 8 200 000cm³ **14b** 500mm³ **14c** 8.2L **14d** 630kL **14e** 9200L **14f** 420mL **15a** 15.30m³ **15b** 445.32cm³ **16** $555 **17** 270m³=270 000L **18a** 1/4=0.25 **18b** 2/4=0.5 **18c** 1/4=0.25 **19a** 1/12=0.08 **19b** 7/12=0.58 **19c** 1/12=0.08 **19d** 7/12=0.58 **20a** 18/36=0.5 **20b** 1/36=0.03 **20c** 34/36=0.94 **20d** 1/36=0.03 **20e** 1 **20f** 24/36=0.67 **21a** 10/21=0.48 **21b** 21/21=1 **21c** 5/21=0.24 **21d** 10/16=0.63 **22a** 5/21=0.24 **22b** 5/21=0.24

Exercise 16.1 **1** √61 or7.81 **2** √41 or 6.40 **3** √45 or 3√5 or 6.71 **4** √58 or 7.62
Exercise 16.2 **1** √5 or 2.24 **2** √5 or 2.24 **3** √5 or 2.24 **4** √13 or 3.61 **5** 3√2 or 4.24 **6** 3√2 or 4.24 **7** 4√2 or 5.66 **8** √26 or 5.10 **9** 6√2 or 8.49 **10** 3√2 or 4.24 **11** √5 or 2.24 **12** 2√5 or 4.47 **13** √13 or 3.61 **14** √5 or 2.24 **15** √10 or 3.16 **16** 2 **17** 2 **18** 4 **19** 1 **20** 4 **21** 1 **22** 3 **23** 2 **24** 1 **25** 5 **26** 8 **27** 2
Exercise 16.3 **1** 4 **2** 7 **3** 9.95 **4** ¯4 **5** 0.5 **6** 0.5
Exercise 16.4 **1** (3,3) **2** (3.5,2.5)
Exercise 16.5 **1** (0.5,1) **2** (1.5,-1) **3** (1.5,1) **4** (1.5,-1.5) **5** (1.5,3.5) **6** (2.5,2.5) **7** (3.5,3.5) **8** (3,4) **9** (4.5,3.5) **10** (4.5,0.5) **11** (0,0.5) **12** (1.5,2) **13** (-1.5,1) **14** (-2,-4) **15** (-2.5,2.5) **16** (1,-0.5) **17** (-2,1) **18** (-3,-1.5) **19** (3.5,-2.5)
Exercise 16.6 **1** m=1 **2** m=0.5 **3** m=0.17 **4** m=¯1 **5** m=¯0.6 **6** m=¯0.43 **7** m=1 **8** m=0.6 **9** m=¯1
Exercise 16.7 **1** m=¯0.5 **2** m=1.67 **3** m=¯1.2 **4** m=0.8 **5** m=1.5 **6** m=¯1 **7** m=1.5 **8** m=¯1 **9** m=¯0.33 **10** m=0.57 **11** m=0.8 **12** m=3 **13** m=0.75
Exercise 16.8 **1** m=2,d=√5 or 4.47,mid=(2,5) **2** m=1,d=3√2 or 4.24,mid=(5.5,3.5) **3** m=0.67,d=2√13 or 7.21,mid=(8,5) **4** m=¯1.67,d=5.83,mid=(2.5,4.5) **5** m=1,d=4√2 or 5.66,mid=(3,6) **6** m= ¯0.33,d=2√10 or 6.32,mid=(6,8)
Exercise 16.9 **1** m=1.33,d=5,mid=(3.5,4) **2** m=1,d=5√2 or 7.07,mid=(0.5,5.5) **3** m=¯1,d=3√2 or 4.24,mid=(¯4.5,3.5) **4** m=0.67,d=2√13 or 7.21,mid=(8,5) **5** m=¯0.33,d=3.16,mid=(¯1.5,¯1.5) **6** m=¯1.5,d=2√13 or 7.21,mid=(¯6,¯12) **7a** 2√5 or 4.47 **7b** 5.10 **7c** 3√2 or 4.24 **8a** Midpoint of AC and BD is same (3.5,3) **8b** m of AB and CD is same 0.43 thus parallel **8c** AB=CD=7.62 and AD=BC=5.39
Exercise 16.10 **2** $m=(y_2-y_1)/(x_2-x_1)$ **3** $d=\sqrt{((x_2-x_1)^2+(y_2-y_1)^2)}$ **4** $mid=((x_1+x_2)/2,(y_1+y_2)/2)$ **5** m=1 **6** d=√8 or 2√2 **7** mid=(2,3) **8** 1/6 **9** $A=\pi r^2$ **10** 1
Exercise 16.11 **2** $m=(y_2-y_1)/(x_2-x_1)$ **3** $d=\sqrt{((x_2-x_1)^2+(y_2-y_1)^2)}$ **4** $mid=((x_1+x_2)/2,(y_1+y_2)/2)$ **5** m=1 **6** d=√18 or 3√2 **7** mid=(3.5,2.5) **8** 1/6 **9** $A=\pi r^2$ **10** 1
Exercise 16.12 **2** $m=(y_2-y_1)/(x_2-x_1)$ **3** $d=\sqrt{((x_2-x_1)^2+(y_2-y_1)^2)}$ **4** $mid=((x_1+x_2)/2,(y_1+y_2)/2)$ **5** m=0.6 **6** d=√34 **7** mid=(3.5,2.5) **8** 2/6=1/3 **9** $A=\pi r^2$ **10** 0.6
Exercise 16.13 **1** 30 **2** 3 **3** 0.3 **4** 0.03 **5** 400 **6** 4 **7** 0.04 **8** 0.0004 **9a** m=1 **9b** m=0.5 **9c** m=0.33 **10** 53°,90° **11a** 6units^2 **11b** 9units^2 **12** Q(7,7),P(8,4),mQP=¯3, mAB=¯3, thus parallel **13** 5√3

Exercise 16.14 **1a** $\begin{array}{r}345\\345\\345\\\underline{345}\\1380\end{array}$ **1b** $\begin{array}{r}345\\345\\345\\\underline{345}\\1380\\\underline{345}\\1725\end{array}$ **1c** $\begin{array}{r}9567\\\underline{1085}\\10652\end{array}$
Exercise 16.15 **1a** 7.21 **1b** 8.60 **1c** 5 **1d** 4.47 **1e** 2.24 **2a** (-1,1) **2b** (0.5,0.5) **2c** (3,2) **2d** (1,2) **2e** (-0.5,0.5) **3a** 1.5 **3b** -2.33 **3c** 1 **3d** -0.67 **3e** -0.57 **4a** m of AB and CD=1.33 **4b** AB=CD=5, AD=BC=6.71
Exercise 16.16 **1a** 8.06 **1b** 8.25 **1c** 2.83 **1d** 8.94 **1e** 5 **2a** (3,0.5) **2b** (-2.5,0.5) **2c** (3,5) **2d** (0,1) **2e** (-1,1.5) **3a** 1.4 **3b** -1.33 **3c** 1 **3d** -0.6 **3e** 1.14 **4a** m of AB and CD=0.2 **4b** AB=CD=5.10, AD=BC=3.61

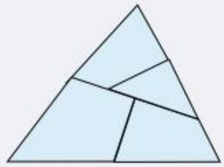
Exercise 17.1 **1** 45√2 or 63.64 **2** 71.11 **3** 3.58 **4** 5√2 or 7.07 **5** no **6** 1.96m **7** 48.47m
Exercise 17.2 **1** tanα=1, α=45° **2** tanα=0.75, α=36.87° **3** tanα=0.75, α=36.87° **4** tanα=0.42, α=22.78° **5** tanα=0.59, α=30.54° **6** tanα=2.4, α=67.38°
Exercise 17.3 **1** sinα=0.8,α=53.13° **2** sinα=0.6,α=36.87° **3** sinα=0.71,α=45.23° **4** sinα=0.71,α=45.23° **5** sinα=0.51,α=30.66° **6** sinα=0.87,α=60.46°
Exercise 17.4 **1** cosα=0.6,α=53.13° **2** cosα=0.8,α=36.87° **3** cosα=0.71,α=44.77° **4** cosα=0.71,α=44.77° **5** cosα=0.86,α=30.68° **6** cosα=0.5,α=60°
Exercise 17.5 **1** 12 **2** 8.40 **3** 0.88 **4** 15.21 **5** 4.30 **6** 61.52 **7** 5.93m
Exercise 17.6 **1** 47.55 **2** 1.79 **3** 6.02 **4** 4.30 **5** 2.00 **6** 3.29 **7** 1.05m
Exercise 17.7 **1** 12.4 **2** 190.48 **3** 2.83 **4** 3.66 **5** 48.15 **6** 4.88 **7** 87.04km
Exercise 17.8 **1** 45° **2** 45° **3** 60° **4** 30° **5** 55.49° **6** 51.22° **7** 42.21°
Exercise 17.9 **2** 0.8 **3** 0.6 **4** 1.33 **5** 30°,90° **6** 1 **7** √8 or 2√2 **8** (2,3) **9** 5.4×10^4 **10** $C=2\pi r$ or $C=\pi D$
Exercise 17.10 **2** 12/13 **3** 5/13 **4** 12/5 or 2.4 **5** 60°,90° **6** 2 **7** √5 **8** (2.5,3) **9** 6.2×10^5 **10** $A=\pi r^2$
Exercise 17.11 **2** 1/√2 **3** 1/√2 **4** 1 **5** 45°,90° **6** 1 **7** √8 or 2√2 **8** (2,3) **9** 4.8×10^6 **10** $C=2\pi r$ or $C=\pi D$

Exercise 17.12 **1a** 60° **1b** 45° **1c** 30° **2a** sinα=2/√3,cosα=1/√3 **2b** sinα=1/√2,cosα=1/√2 **2c** sinα=1/√3,cosα=2/√3 **3a** 30° **3b** 60° **3c** 45° **4a** 26 **4b** 15

Exercise 17.13 **1** Three hundred and forty-two billion four hundred and eighty-two million eight hundred and seventy-three thousand seven hundred and ten **2** 0.0001 **3** 21,34 **4**

Exercise 17.14 **1a** BC **1b** AB **1c** AC **2a** 15 **2b** 7.00 **2c** 464.92 **3a** 2.98 **3b** 43.71 **3c** 53.13° **4a** x=70,y=121.24,α=30° **4b** x=2.67,α=68.82°,β=21.18° **4c** x=82.20,α=53.41°,β=36.59° **5** 232.66km **6** 12.12m

Exercise 17.15 **1a** AC **1b** BC **1c** AB **2a** 10 **2b** 2.99 **2c** 337.58 **3a** 4.99 **3b** 38.43 **3c** 36.87° **4a** x=121.24,y=70,α=60° **4b** x=31.62,α=34.70°,β=55.30° **4c** x=7.79,α=31.10°,β=58.90° **5** 1.25m **6** 4.95m



Exercise 18.1 **1** 4 **2** 3 **3** −3 **4** −3 **5** −4 **6** 7 **7** 6 **8** −3 **9** 7 **10** −9 **11** −1 **12** 6 **13** −11 **14** 7 **15** 25 **16** −23 **17** −1 **18** −7 **19** 4 **20** −6 **21** −5 **22** −53

Exercise 18.2 **1** 20 **2** −28 **3** −5 **4** 6 **5** −15 **6** 12 **7** −4 **8** 2 **9** −56 **10** −40 **11** −7 **12** 4 **13** −35 **14** 36 **15** −8 **16** −9 **17** −54 **18** −14 **19** −9 **20** 8

Exercise 18.3 **1** 18 **2** 3 **3** 1 **4** 30 **5** −6 **6** −11 **7** −1 **8** −17 **9** −12 **10** 15 **11** −9 **12** 21 **13** 32 **14** −2 **15** −13 **16** 44 **17** 20 **18** −11

Exercise 18.4 **1** 2^8 **2** x^7 **3** 3^9 **4** a^7b^3 **5** x^6 **6** b^9 **7** 4^{11} **8** 10^7 **9** 5 **10** 8.1^7 **11** 2^2 **12** 10^3 **13** c^3 **14** 3^{-2} **15** d^{-4} **16** $e^3 \div x^6$ **17** 3.2^3 **18** x^{-6} **19** 4^{-8} **20** 10^9 **21** 5^4 **22** x^6 **23** 6^8 **24** x^{10} **25** y^{10} **26** s^{-15} **27** g^{-8} **28** 2^8 **29** x^6 **30** 10^6 **31** $6a^5$ **32** $6b^6$ **33** 2^3 **34** $4c^{-2}$ **35** 10^{12} **36** 10^{12}

Exercise 18.5 **1** 7x **2** 4a **3** 6b **4** 3x **5** 11c **6** 3y **7** 27d **8** x **9** 12e **10** 9x **11** f **12** 3x **13** 7x+6 **14** 7a+8 **15** 4x+7 **16** 3a+4b **17** 10x+3 **18** 4a+8b **19** $9a^2+2$ **20** $5x^4+10$

Exercise 18.6 **1** 10x **2** 14a **3** 30e **4** 18x **5** 8fn **6** 27cd **7** 8ab **8** 24dt **9** 7xy **10** 2m **11** 4p **12** 3k

Exercise 18.7 **1** 3x **2** 4x **3** 4a **4** 5a **5** 10y **6** 3k **7** 2b **8** 3x/2 **9** 4x/3 **10** 5 **11** 3 **12** 5/2

Exercise 18.8 **1** $12x^2$ **2** $12d^2$ **3** $15a^2$ **4** $21d^2$ **5** $-10x^2$ **6** $-6x^2$ **7** $-12x^2$ **8** $12a^3$ **9** $-12p^4$ **10** $-27w^3$ **11** $16s^5$ **12** $40x^4$ **13** $6de^3$ **14** $-15m^3n^2$ **15** $-28p^3n^2$ **16** $-24a^3b^2$ **17** $6p^3d^2$ **18** $16h^3$ **19** $20a^4b^3c^2$ **20** 2x **21** 4a **22** 4x **23** 2d **24** $7x^3$ **25** $7x^3$ **26** $2x^2$ **27** -2x **28** -2x **29** 5 **30** -2g **31** $3a^4$ **32** $-2x^2$ **33** $-4x^2y$ **34** $2x^2y$ **35** -5a **36** $-4a^3c^2/3$ **37** $2e^3f^3$ **38** $-3ab^3c/4$

Exercise 18.9 **1** 4x+12 **2** 3a+6 **3** 8b+40 **4** -4x-12 **5** -2y-10 **6** -3d-21 **7** 6x-8 **8** 12x-18 **9** 20a-15 **10** -10+15 **11** -4y+10 **12** -20n+4 **13** $2x^2+3x$ **14** $12b^2+8b$ **15** $30f^2-18f$ **16** $-5x-2x^2$ **17** $-24xy+16y^2$ **18** $-25u^2+10uv$

Exercise 18.10 **1** 7x+16 **2** 5x+13 **3** 11x+5 **4** -27 **5** 5x+11 **6** -2x-9 **7** 9a+19 **8** c **9** 5b+8 **10** -2y-24 **11** 9m+18 **12** 14x-14 **13** 5x+11 **14** d+15 **15** -3x **16** $-6y^2+8$

Exercise 18.11 **1** 3b **2** 4a **3** 2 **4** 4 **5** 3x **6** 5 **7** 4a **8** 4a **9** 2 **10** 3 **11** 5y **12** 6f

Exercise 18.12 **1** 2(a+3) **2** 4(2x+1) **3** 2c(c+5) **4** 2(3x+2) **5** 4(3x+2) **6** 3(4b+3) **7** 3(3y+2) **8** 2(2c+7) **9** 3(d+4) **10** 4(g-3) **11** 2x(3y-4) **12** x(5x-3) **13** 3(3t-2) **14** 7x(y-2) **15** 3d(d-3) **16** 2(3a+2) **17** 5(3m+1) **18** 3r(3r+1) **19** 2(7p-2) **20** 5t(2s+1) **21** 3b(4b-1) **22** 5(2u+3) **23** 6c(3b-1) **24** 7e(2e+3) **25** 2(5h-6) **26** 3(7d+5) **27** $3x^2(8x-5)$

Exercise 18.13 **1** x+8 **2** -6x+8 **3** -x-2 **4** -7a-22 **5** -b+14 **6** -4c+2 **7** 3n+3 **8** -11y+16 **9** x^2+3x+2 **10** x^2+x-2 **11** x^2+4x+3 **12** x^2+2x-3 **13** x^2+5x+6 **14** x^2-x-6 **15** $x^2+7x+10$ **16** $x^2+3x-10$ **17** x^2+2x+1 **18** x^2-2x+1 **19** x^2+4x+4 **20** x^2-4x+4 **21** x^2+6x+9 **22** x^2-6x+9 **23** $x^2+8x+16$ **24** $x^2-8x+16$

Exercise 18.14 **1** 3(2a+3) **2** 2a(2b-3) **3** 2c(5c-4) **4** 2(7x+5) **5** 2b(2a-3) **6** 2d(4d-3) **7** 4(2b+1) **8** 2x(4y-5) **9** 3r(3r-2) **10** 3(3c+4) **11** 2x(4y+5) **12** 4x(4x-3) **13** 2(3x+5) **14** 3t(4s-5) **15** $3p^3(5p^2-12)$ **16** (x+5)(x+3) **17** (x+5)(x-4) **18** (x+2)(x+5) **19** (x+2)(x-5) **20** (x+9)(x+4) **21** (x+3)(x-2) **22** (x-1)(x+4) **23** (x-1)(x-2) **24** (x-6)(x+3) **25** (x-5)(x-4) **26** (x-2)(x+5) **27** (x-3)(x-7)

Exercise 18.15 **2** 8x **3** 3x-9 **4** 4(x+2) **5** 0.8 **6** 1 **7** √8 or 2√2 **8** (2,3) **9** 3.1×10^4 **10** 216

Exercise 18.16 **2** 11a **3** 2x-10 **4** 2(3x+5) **5** 0.6 **6** 1.5 **7** √13 **8** (2,2.5) **9** 5.4×10^5 **10** 297

Exercise 18.17 **2** 5a **3** 4x-28 **4** 2(x+4) **5** 1.33 **6** 1.5 **7** √13 **8** (3,4.5) **9** 2.3×10^6 **10** 603

Exercise 18.18 **1a** 3 **1b** 5 **1c** 25 **1d** -1 **1e** 3 **1f** -3 **2a** 10^5 **2b** 10^2 **2c** 10^3 **2d** 1 **3a** 3ab units^2 **3b** b(b-2a) or b^2-2ab units^2 **3c** $2a^2+3ab+b^2$ units^2 **3d** $3ab-6a^2$ units^2

Exercise 18.19 **1** 10 080 L **2** a=4, b=6, c=8 **3** 6

Exercise 18.20 **1a** 5x **1b** 9x **1c** -3x **1d** -4x **1e** a+2b **1f** r+8x **1g** 8x **1h** 15x **1i** -12b **1j** -20x **1k** $-8c^5$ **1l** $40x^4$ **1m** 4x **1n** 3m **1o** -4g/3 **1p** -5x **1q** 2a **1r** 7b/2 **1s** $5t^2/2$ **1t** $-4a^3b^2c/3$ **2a** 5x+15 **2b** 3a+3 **2c** 6b+24 **2d** 12x-4 **2e** 14c-21 **2f** 30d-10 **2g** -6e+15 **2h** -4x-32 **2i** -27f-45 **2j** $-6x^2-4x$ **2k** $-20x^2+24x$ **2l** $-3m^2-2m$ **2m** $-4y-12y^2$ **2n** $-8x^2+12bx$ **2o** $-5xy+10x^2$ **3a** 7x+10 **3b** 8x+22 **3c** 7a-2 **3d** a+10 **3e** $2c^2-6c$ **3f** $-8d^2+2d+4$ **3g** x^2+5x+6 **3h** x^2+3x+2 **3i** x^2+2x-3 **3j** x^2-x-2 **3k** x^2-4 **3l** x^2-9 **3m** x^2+2x+1 **3n** x^2-4x+4 **4a** 4(x+2) **4b** 3(a+4) **4c** 2(4b+5) **4d** 3(5x+2) **4e** 4(4c+3) **4f** 3(5x+4) **4g** 3(2x-1) **4h** 2(d-3) **4i** 4(x-4) **4j** 2(5e-4) **4k** 2(8x-3) **4l** 3(5u-3) **4m** 2a(3b+5) **4n** 2x(6y-5) **4o** 2u(4v-7) **4p** 2(7f-6g) **4q** 2x(10x-7) **4r** 2n(4n-9) **4s** -3(x+5) **4t** -2(p+3) **4u** -2(q-4) **4v** -4(x-3) **4w** -4a(a-5) **4x** -3x(x+3) **4y** (x+3)(x+2) **4z** (x-2)(x+3)

Exercise 18.21 **1a** 9x **1b** 3x **1c** 2x **1d** -4x **1e** 4a+3b **1f** r+7x **1g** 6x **1h** 16x **1i** -6b **1j** -12x **1k** $-32d^3c^2$ **1l** $36x^4$ **1m** 5x **1n** 3a **1o** -h **1p** -4x **1q** 3a **1r** 3c **1s** $2x^3$ **1t** $-10ab^2/4$ **2a** 6x+24 **2b** 5x+10 **2c** 6a+18 **2d** 8x-4 **2e** 6b-3 **2f** 24c-16 **2g** -20d+10 **2h** -3x-15 **2i** -28e-7 **2j** $-2x^2-5x$ **2k** $-5x^2+15x$ **2l** $-5p^2-3p$ **2m** $-4d-12d^2$ **2n** $-14x^2+21ax$ **2o** $-4xy+12x^2$ **3a** 9x+17 **3b** 7x+23 **3c** 8a-8 **3d** -x+16 **3e** $-x^2-6x$ **3f** $-10d^2+d+20$ **3g** x^2+3x+2 **3h** x^2+3x+2 **3i** x^2+4x-5 **3j** x^2+x-6 **3k** x^2-1 **3l** x^2-4 **3m** x^2+4x+4 **3n** x^2-2x+1 **4a** 5(x+2) **4b** 4(x+3) **4c** 2(3x+5) **4d** 4(3a+1) **4e** 2(8b+5) **4f** 3(5c+3) **4g** 3(3x-1) **4h** 3(d-2) **4i** 2(2e-5) **4j** 3(5x-4) **4k** 2(7x-4) **4l** 2(9x-6) **4m** 5x(y+2) **4n** 2x(5y-6) **4o** 5u(4v-3) **4p** 2x(9x-10) **4q** 5u(5u-3) **4r** 6t(2t-2) **4s** -2(2x+3) **4t** -3(n+3) **4u** -2(m-4) **4v** -3(x-5) **4w** -2g(3g-10) **4x** -4x(x+3) **4y** (x+2)(x+5) **4z** (x-1)(x+6)

Exercise 19.1 **1** quantitative **2** qualitative **3** quantitative **4** qualitative **5** qualitative

Exercise 19.2 **1** continuous **2** discrete **3** continuous **4** discrete **5** discrete

Exercise 19.3 **1a** a collection of data from the whole population **1b** a collection of data from part of the popuation **1c** the sample doesn't represent the population **1d** must be small enough to be economical but large enough to represent the population **1e** a random sample helps avoid bias because each member of the population has an equal chance of being selected in the sample. **1f** a stratified sample helps avoid bias because each member of a section of the population has an equal chance of being selected in the sample **2** The census collected data from every student and gives strong confidence that its conclusion is correct. The sample size of 15 is probably too small and gives a conclusion with little confidence. The sample size of 50 appears, on this occasion, to have provided a correct conclusion.

Exercise 19.5 **1** yr8=24, yr9=18, yr10=18 **2** yr7=12, yr8=12, yr9=13, yr10=13 **3** female=6, male=4 **4** female_full=33,female_part=44, male_full=22, male_part=50

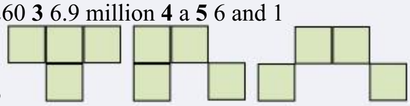
Exercise 19.6 **1** not all responses are covered (4,0?), what does sibling mean? **2** leading question **3** personal or embarrassing question. Question isn't clear **4** not all responses are covered - add other?

Exercise 19.9 **2** a collection of data from the whole population **3** a collection of data from part of the population **4** help avoid bias because each member of the population has an equal chance of being selected in the sample **5** 2x−2 **6** 2(4x+3) **7** 4/5 **8** m=1.5 **9** 4.7×10^5 **10** 360

Exercise 19.10 **2** a collection of data from the whole population **3** a collection of data from part of the population **4** help avoid bias because each member of the population has an equal chance of being selected in the sample **5** 5x−10 **6** 2(5x+2) **7** 3/5 **8** $\sqrt{10}$ **9** 4×10^{-5} **10** 480

Exercise 19.11 **2** a collection of data from the whole population **3** a collection of data from part of the population **4** help avoid bias because each member of the population has an equal chance of being selected in the sample **5** 3x−12 **6** 4(2x+1) **7** 4/3 **8** (3,3) **9** 2.1×10^7 **10** 210

Exercise 19.12 **1** 19,14 **2** 1260 **3** 6.9 million **4** a **5** 6 and 1

Exercise 19.13 **1** 4 **2** 20kg **3** 

Exercise 19.14 **1a** quantitative **1b** qualitative **2a** continuous **2b** discrete **3a** a collection of data from the whole population **3b** a collection of data from part of the population **3c** help avoid bias because each member of the population has an equal chance of being selected in the sample **4** yr7=13, yr8=13, yr9=12, yr10=13 **5a** not all responses covered - add other **5b** not all responses covered

Exercise 19.15 **1a** quantitative **1b** qualitative **2a** continuous **2b** discrete **3a** a collection of data from the whole population **3b** a collection of data from part of the population **3c** help avoid bias because each member of the population has an equal chance of being selected in the sample **4** yr7=32, yr8=26, yr9=27, yr10=40 **5a** leading answers **5b** leading question

Exercise 20.1 **2** a collection of data from the whole population **3** a collection of data from part of the population **4** helps avoid bias because each member of the population has an equal chance of being selected in the sample **5** $5x-10$ **6** $3(2x+3)$ **7** 4/3 **8** m=1 **9** 3.2×10^4 **10** 330

Exercise 20.2 **1a** $\sqrt{52}$ or $2\sqrt{13}$ or 7.21 **1b** $\sqrt{65}$ or 8.06 **1c** $2\sqrt{2}$ or 2.83 **1d** $\sqrt{58}$ or 7.62 **1e** $\sqrt{17}$ or 4.12 **2a** (3,1) **2b** (-1,1) **2c** (3,2) **2d** (1.5,0.5) **2e** (1,-3.5) **3a** 0.86 **3b** -0.5 **3c** 0.75 **3d** -0.75 **3e** 1 **4a** mAB=mCD=0.4 **4b** AB=CD=$\sqrt{29}$ or 5.39, AD=BC=$2\sqrt{2}$ or 2.83 **5a** AC **5b** BC **5c** AB **6a** 15 **6b** 3.70 **6c** 645.58 **7a** $x=181.87, y=105, \alpha=60°$ **7b** $x=24.08, \alpha=41.63°, \beta=48.37°$ **7c** $x=7.76, \alpha=34.32°, \beta=55.68°$ **8** 8.89m **9a** 5x **9b** 6x **9c** 2x **9d** -5x **9e** 3b+3a **9f** 3x+5y **9g** 8x **9h** 10x **9i** -8x **9j** -15a **9k** $-16b^5$ **9l** $20x^4$ **9m** 2x **9n** 7d **9o** -2h **9p** -8x **9q** 5a **9r** 6c/2 **9s** $3x^5$ **9t** -3abc **10a** 3x+6 **10b** 4x+12 **10c** 5a+10 **10d** 12x-18 **10e** 9b-3 **10f** 12c-16 **10g** -24d+6 **10h** -7x-14 **10i** -20x-15 **10j** $-2m^2-3m$ **10k** $-5x^2+25x$ **10l** $-3x^2-x$ **10m** -2n-6bn **10n** $-x^2+ax$ **10o** $-5xy+10x^2$ **11a** 5x+8 **11b** 6x+14 **11c** 9a+13 **11d** 12 **11e** x^2-6x **11f** $-12d^2+2d+2$ **11g** x^2+4x+3 **11h** x^2+5x+4 **11i** x^2+x-2 **11j** x^2+2x-3 **11k** x^2-1 **11l** x^2-4 **11m** x^2+6x+9 **11n** x^2-4x+4 **12a** 2(x+4) **12b** 5(x+3) **12c** 3(2x+3) **12d** 3(4a+1) **12e** 5(3b+2) **12f** 3(4c+3) **12g** 3(3d-2) **12h** 4(e-2) **12i** 2(2f-5) **12j** 4(4x-3) **12k** 7(2x-3) **12l** 4(5x-4) **12m** 4x(2y+3) **12n** 3x(5y-4) **12o** 5m(4n-3) **12p** 4x(5x-4) **12q** 5x(5x-4) **12r** 3x(3x-4) **12s** -2(3x+5) **12t** -3(n+4) **12u** -2(x-5) **12v** -3(2x-5) **12w** -9x(x-2) **12x** -5x(x+5) **12y** (x+3)(x+3) **12z** (x-2)(x+5) **13a** qualitative **13b** quantitative **14a** continuous **14b** discrete **15a** a collection of data from the whole population **15b** a collection of data from part of the population **15c** helps avoid bias because each member of the population has an equal chance of being selected in the sample **16** yr7=13, yr8=15, yr9=12, yr10=11 **17a** not all responses covered **17b** personal question and ambiguous question

Exercise 20.1 **2** a collection of data from the whole population **3** a collection of data from part of the population **4** helps avoid bias because each member of the population has an equal chance of being selected in the sample **5** $6x-18$ **6** $4(x+2)$ **7** 4/5 **8** m=1 **9** 4.1×10^{-5} **10** 630

Exercise 20.2 **1a** $\sqrt{65}$ or 8.06 **1b** $2\sqrt{17}$ or 8.25 **1c** $\sqrt{13}$ or 3.61 **1d** $4\sqrt{2}$ or 5.66 **1e** 4 **2a** (3,0.5) **2b** (-2.5,0.5) **2c** (4,5) **2d** (0,1.5) **2e** (-1.5,1.5) **3a** 1.4 **3b** -1.33 **3c** 1 **3d** -0.6 **3e** 1.14 **4a** mAB=mCD=-1.33 **4b** AB=CD=5, AD=BC=5 **5a** AC **5b** BC **5c** AB **6a** 10 **6b** 3.06 **6c** 473.23 **7a** $x=108.76, y=50.71, \alpha=65°$ **7b** $x=26.63, \alpha=34.29°, \beta=55.71°$ **7c** $x=7.64, \alpha=24.62°, \beta=65.38°$ **8** 119.96m **9a** 5x **9b** 7x **9c** 5x **9d** -4x **9e** 5x+4y **9f** 3a+6b **9g** 30x **9h** 16x **9i** -6a **9j** -6x **9k** $-32b^6$ **9l** $24x^4$ **9m** 5x **9n** 2c **9o** -2d **9p** -4x **9q** 4e **9r** 4g **9s** $4x^3/3$ **9t** $-5a^2bc/3$ **10a** 3x+6 **10b** 4x+4 **10c** 5a+10 **10d** 8x-4 **10e** 4b-2 **10f** 16c-24 **10g** -18d+6 **10h** -5x-25 **10i** -8e-2 **10j** $-2x^2-5x$ **10k** $-5x^2+15x$ **10l** $-6f^2-2f$ **10m** $-5g-5g^2$ **10n** $-20x^2+12xy$ **10o** $-10x^2+5xy$ **11a** 5x+8 **11b** 7x+16 **11c** 7a-5 **11d** -x+13 **11e** $-2x^2-5x$ **11f** $-4d^2-2d+20$ **11g** x^2+4x+3 **11h** x^2+3x+2 **11i** x^2+3x-4 **11j** x^2+x-6 **11k** x^2-1 **11l** x^2-4 **11m** x^2+2x+1 **11n** x^2-2x+1 **12a** 5(x+3) **12b** 3(x+4) **12c** 2(2x+5) **12d** 4(3a+1) **12e** 2(4b+5) **12f** 3(5c+3) **12g** 3(3x-2) **12h** 3(d-2) **12i** 5(e-2) **12j** 3(5x-4) **12k** 4(3x-2) **12l** 2(7x-6) **12m** 2x(4y+5) **12n** 2x(5y-6) **12o** 5u(5v-3) **12p** 5x(3x-4) **12q** 5u(6u-3) **12r** 4t(t-3) **12s** -4(x+2) **12t** -3(b+3) **12u** -2(c-4) **12v** -3(x-5) **12w** -4g(g-5) **12x** $-2x(x^2+6)$ **12y** (x+2)(x+3) **12z** (x-1)(x+5) **13a** quantitative **13b** qualitative **14a** continuous **14b** discrete **15a** a collection of data from the whole population **15b** a collection of data from part of the population **15c** helps avoid bias because each member of the population has an equal chance of being selected in the sample **16** female-full=13, female-part=21, male-full=22, male-part=35

Index

This text has been written for Year 9 students. The aim of the text is to assist students in investigating and understanding the exciting and very important world of Mathematics and to implement the intent of the Australian Mathematics Curriculum.

A literature review of learning from school textbooks was used to enhance the format of this textbook.

Each chapter, apart from Review, contains:

- ★ Numerous worked examples
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The author

The author is a mathematics head with 30 years experience, an honours masters in IT education, a PhD in mathematics education, a state award for mathematics education, and extensive publishing experience.

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