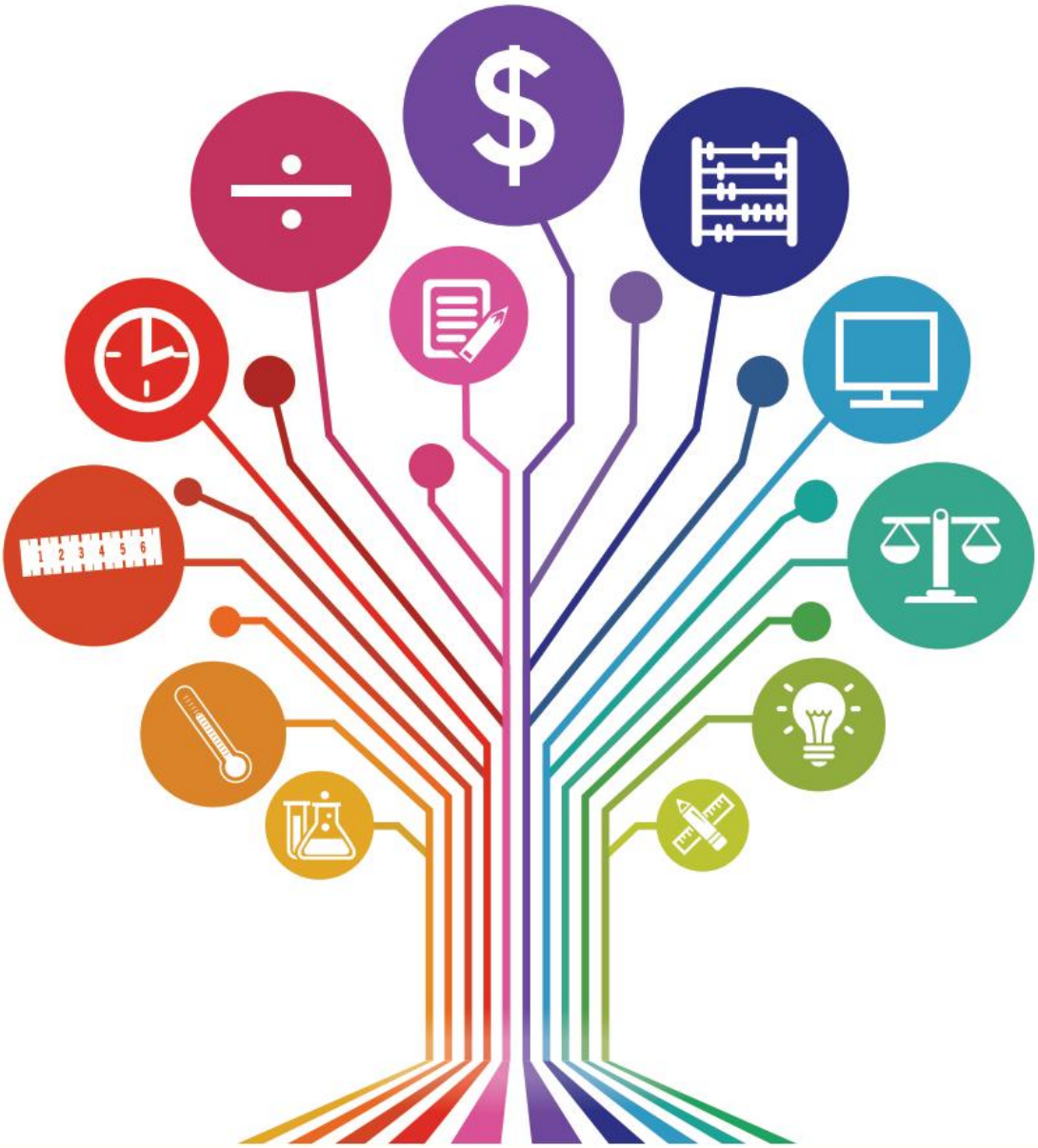
2-D and 3-D shapes and angles



Learner guide

Working with numbers

Aspire learning resources

Pre-employment skills

2-D and 3-D shapes and angles

Version 1.1

Aspire learning resources

Copyright Warning

2-D and 3-D shapes and angles

Level 1, 464 St Kilda Road
MELBOURNE VIC 3004 AUSTRALIA
Phone: (03) 9820 1300

First published July 2016

Cover design: Aspire Training & Consulting

e-ISBN 978-1-76031-886-4 (PDF version)
ISBN 978-1-76031-885-7

Contents

2-D and 3-D shapes

When we think about maths, the things that come to mind are calculations (sums) and working with numbers. Sometimes we forget that in maths we also measure and describe. In our world we are three dimensional (3-D). This means we have length, width and depth. Sometimes it helps to imagine and describe shapes as if they are in a two-dimensional (2-D) flat world. These shapes have length and width, but no depth, like a picture on a piece of paper.

In this learner guide we will learn about describing our world using the shapes we find in it. There are many shapes in our world and several systems for studying them.

Activity 1

Place some large sheets of paper (or newspaper) on the floor. Lie down and using a texta or pen trace around the outline of your body. If it is too hard, get someone to draw your outline for you. You are a 3-D object, but the outline you just drew is a 2-D image of you. Can you think of other everyday objects we refer to as 2-D?

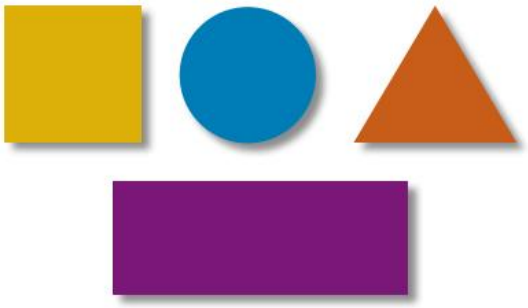
A photograph, poster or diagram are all 2-D illustrations of objects. The shapes in them are referred to as 2-D shapes. So, even though we live in a 3-D world we often use 2-D illustrations and 2-D shapes.

Story

Rudi is interested in all the objects and shapes of our world. He wants to train and work as a draftsperson, who draws plans and designs. Rudi also loves to draw and paint in his spare time. One day he is sketching his house when he notices that his drawing of the house contains the four most common 2-D shapes.

As Rudi discovered, the four most common shapes are:

- Square
- Circle
- Triangle
- Rectangle



Activity 2

Take a walk around your local community. Use this table to help you find objects that show these shapes. Some have already been done for you.

Triangle	Rectangle	Square	Circle
Some house roofs			Car tyres

Click to complete Activity 2

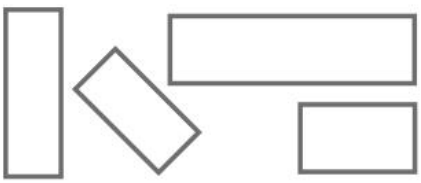
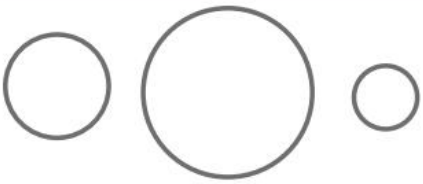
Story

Rudi and his daughter Lara go for a walk around their neighbourhood looking for the common shapes. Lara says, 'I always get these shapes wrong at school. All the triangles don't look the same. How do you know when something is a triangle?'

'I used to have problems at school too', says Rudi. 'I have a table I used that helped me. Maybe it can help you too.'

Activity 3

This is the table Rudi gave Lara. Using the four descriptions below, match each description to its shape.

Shape	Description
	
	
	
	

Descriptions

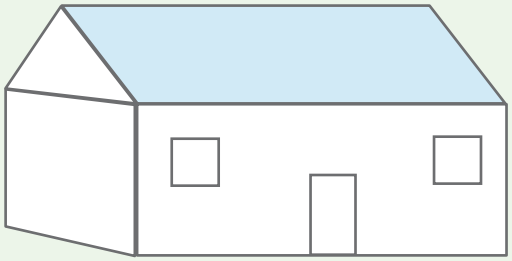
1. A shape made up of four sides and four corners. The four sides are all the same length.
2. A shape made up of three sides and three corners. The three sides are not always the same length.
3. A shape that has no straight sides or corners but has one curved or round side.
4. A shape made up of four sides and four corners. The two pairs of opposite sides are the same length.

Click to complete Activity 3

Angles in the world

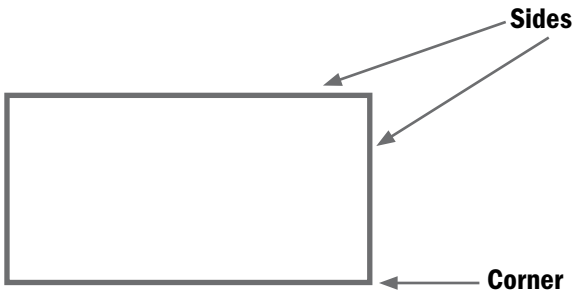
Story

Rudi is practising drawing houses when he notices that on his drawing, the houses have roofs that look like a rectangle pushed out of shape. He knows that on the real house the roof is a rectangle, but in the drawing it looks like another shape. Rudi remembers that he learnt about this shape in school. The shape is called a parallelogram. It is clear to Rudi now that the angles make this shape different from a rectangle.



What is an angle?

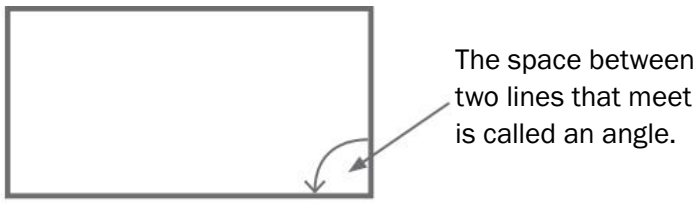
In maths, when we describe shapes we talk about sides and corners but we also use the word angles.



The lines that make the border of a shape are called sides.

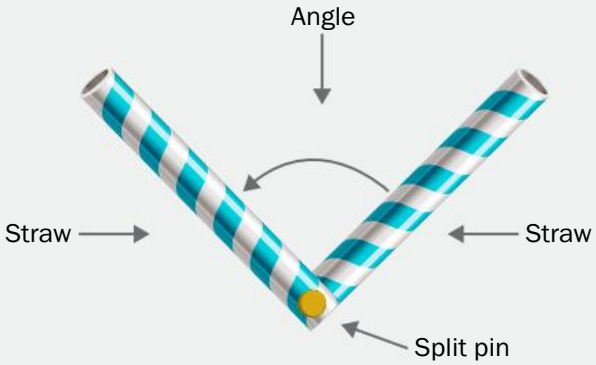
The point where two sides meet to help form a shape is called a corner.

So what is an angle? When two straight lines meet, the space formed between the two lines is an angle.



Activity 4

For this activity you need two straws and one split pin. Pin the two straws together as shown. Join them with the split pin.



Holding the two straws in your hands, move them apart and then closer together. Notice the space between the two straws. The space formed between the two straws is an angle.

1. What happens to the space between the two straws when you move them apart?

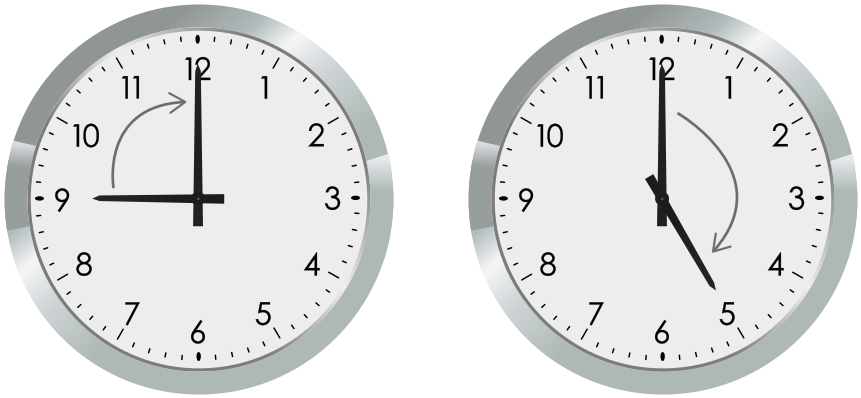
_____

2. What happens to the space between the two straws when you move them closer together?

_____

Click to complete Activity 4

Now look at a clock. Where the hour and minute hands of a clock meet, the space formed between the lines is an angle.

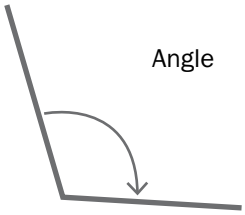


Looking at a clock we can also see another way of thinking about angles.

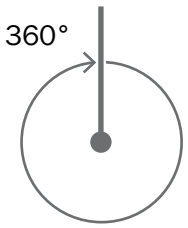
The angle between the clock hands changes when one hand turns away from the other. This turning is another way of describing an angle. So, as well as saying the space between the hands is an angle, we can also say the amount of turning one hand does is an angle.

Unit of measure for angles

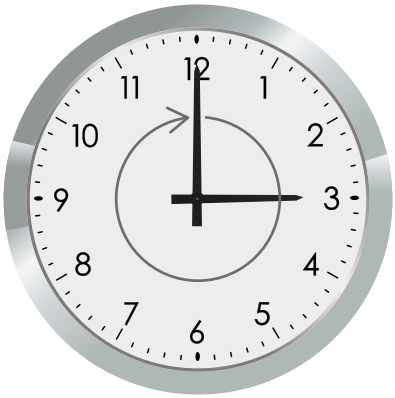
The amount of turn (or space) between two lines that meet is an angle.



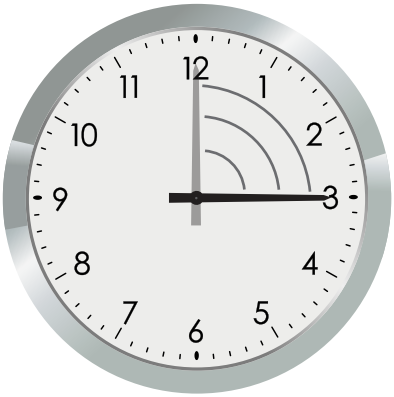
Angles are measured in degrees. A full circle turn is 360 degrees. This can also be written as 360˚. The little circle is the symbol for degrees.



So, as the minute hand on a clock goes around from 12 back to 12, we say it has turned an angle of 360˚.



So, if the minute hand goes from 12 to 3. How many degrees is this?



You can see that the hand has turned ¼ of the circle. To find the degree of the angle, divide the whole circle (360˚) by 4, to find ¼.

360˚ ÷ 4 = 90˚

So ¼ of a circle is 90˚.

Activity 5

Now think of a clock again. Remember, a minute hand turning around from 12 back to 12 makes a 360˚ turn.

Fill in the following table with the amount of circle and the size of the angle for each amount of time. The first row has been done for you. Ask your trainer if you need help.

Minute hand moves	Amount of time	Amount of circle	Angle size
From 12 to 12	One hour (60 minutes)	whole	360 ˚
From 12 to 3	¼ of an hour		
From 12 to 6	½ an hour		
From 12 to 9	¾ of an hour		

Click to complete Activity 5

So, you know that where the two hands of a clock meet, the space formed between the hands, or the amount of turn, is the angle. This is true with other things as well. For example, when two fingers meet, the space formed between the fingers is the angle.

Activity 6

Look around your classroom, home or local area to find some angles. Now estimate (guess) if you think the angles are closer to a whole circle, ½ a circle or ¼ of a circle. Write your findings below.

Click to complete Activity 6

Remember

- An angle is the amount of turn or the space between two lines that meet.
- Angles are measured in degrees.
- We can estimate the size of the angles we see around us.

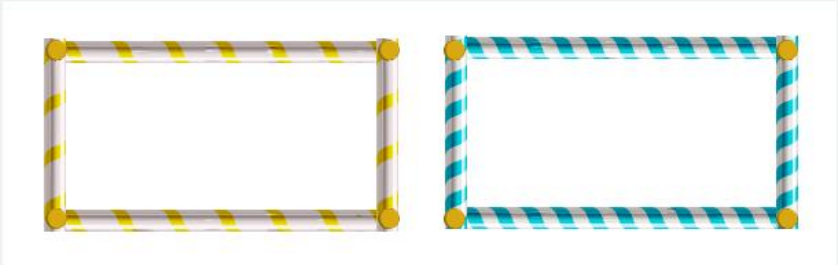
2-D shapes revisited

Now that we know about angles we can take another look at 2-D shapes.

Rudi's last drawing had a shape in it that looked like a rectangle pushed out of shape. This parallelogram has four sides like a rectangle, so how is it different from a rectangle? The next activity will teach you how.

Activity 7

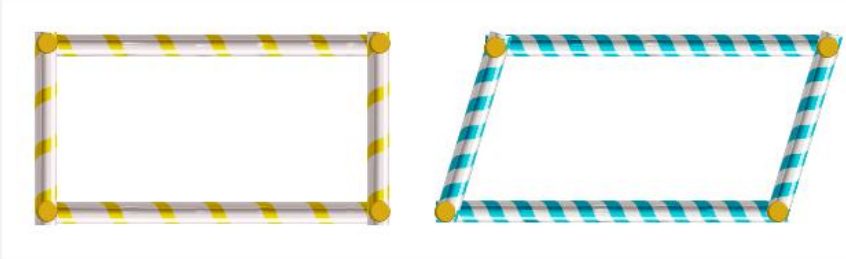
For this activity you need six drinking straws and eight split pins. First, cut two of the straws in half to get four smaller straws. Next, set up two rectangles using the eight straws as shown in the diagram.



Fasten each of the corners together with a split pin.

Leave one rectangle as is.

Then, move the short sides of the second rectangle as shown in the diagram below, to create a parallelogram.



1. Which sides are the same lengths in each shape?

_____

2. Which angles are the same size in the rectangle?

_____

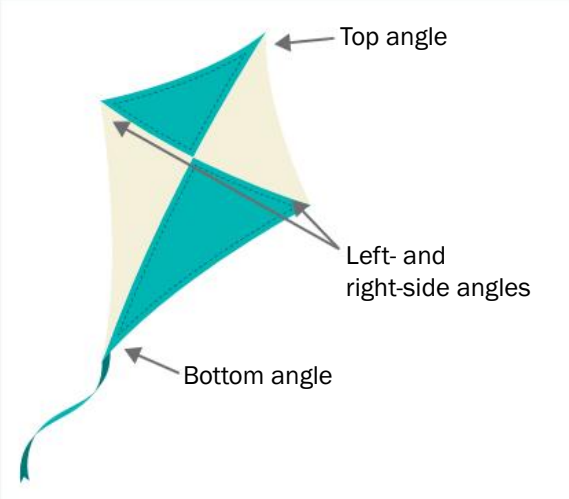
3. What happens to the angles in the parallelogram? Are they the same as the angles in the rectangle?

_____

Click to complete Activity 7

Activity 8

So, we know that a rectangle, a parallelogram and a square have four sides. Here is another four-sided shape. It is called a kite shape.



1. Which sides are the same length? (For example, top left and bottom right).

_____

2. Which sides are different lengths?

_____

3. Which angles do you think may be the same size?

_____

Click to complete Activity 8

Families of shapes

Story

Rudi is married and his family share the same surname (or family name). So Rudi, his wife and daughter all belong to the same family, but they are different people and each has their own first name.

We know that a square, a rectangle and a parallelogram are all four-sided shapes. However, they are not the same as each other. To help us, maths gives a family name to all 2-D shapes that have the same number of sides.

Activity 9

1. Here are some family names. Find out how many sides and angles each family has. Ask your trainer if you need help. The first is one done for you. See if you notice any patterns.

Shape family name	Number of sides	Number of corners	Number of angles
Triangles	3	3	3
Quadrilaterals			
Pentagons			
Hexagons			
Octagons			
Do you know any others?			

2. Did you notice any patterns?

_____

Click to complete Activity 9

The strength of shapes

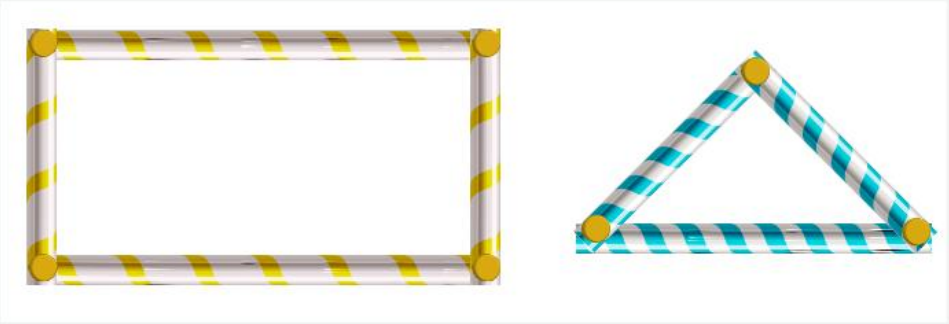
Some shapes are stronger than others. This means they hold their shape when they are under pressure or when force is applied to them. Can you guess what shapes are the strongest? The following activity can help you decide.

Activity 10

For this activity you need six drinking straws and seven split pins.

First, cut one of the straws in half to give you two smaller straws.

Set up one rectangle and one triangle as shown in the diagram below.



Fasten each of the corners in place with a split pin.

Now, move the rectangle's sides so you create a parallelogram again.

Now, do the same with the triangle.

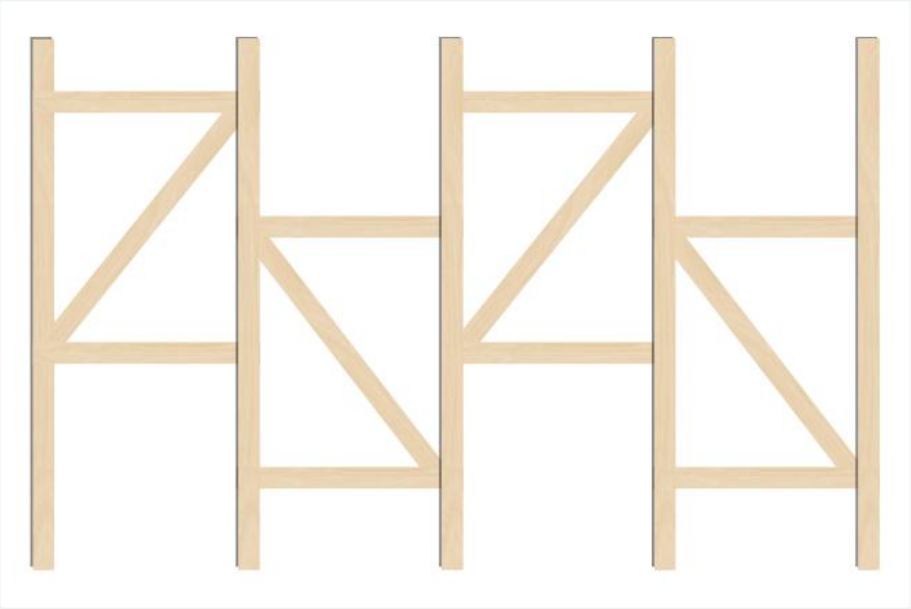
1. Does it shift like the rectangle did?

   _____

2. Do you think it is stronger or weaker than the rectangle?

   _____

3. Often when a house is built, the frame looks like the one below. Why do you think builders do this?



Click to complete Activity 10

A world with depth

When you draw a picture on a piece of paper, we say the shapes are 2-D. The real objects you draw are 3-D. In our world we are three dimensional (3-D) – we have height, width and depth.

A photograph of you is a 2-D representation (likeness) of a 3-D person. If Rudi sculpted a life-size statue of you, it would be a 3-D representation of you. To make an accurate statue of you, Rudi would need your measurements.

Activity 11

Fill in the following table of information with the measurements of your three dimensions – your height, width and depth. Use a measuring tape or ruler. Give rough measurements, you do not need to be exact. Your trainer can help you.

Name:	
Body part	**Measurement in centimetres (cm)**
Height	
Width across shoulders	
Width across waist	
Depth at your chest	
Depth at your waist	

Click to complete Activity 11

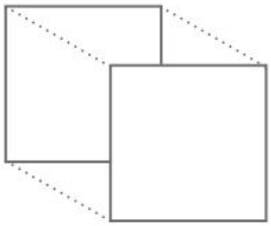
Story

An art class Rudi attends is called ‘Still life’. For this class he must gather a collection of objects, arrange them on a table and draw them. Rudi is sketching his chosen objects when he realises he has chosen the three most common shapes for containers.

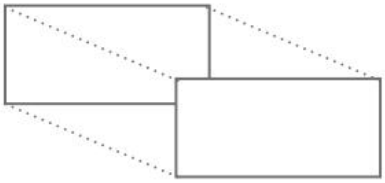
Activity 12

1. Rule straight lines along the dotted lines to connect the following 2-D shapes and turn them into drawings of 3-D objects. These are the three most common 3-D shapes that Rudi was sketching. What are they called?

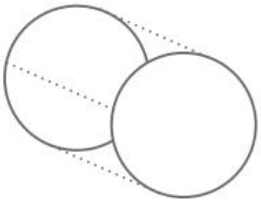
a.



b.



c.



a. _____

b. _____

c. _____

2. Can you think of any other 3-D shapes that are commonly used in our everyday lives?

_____

_____

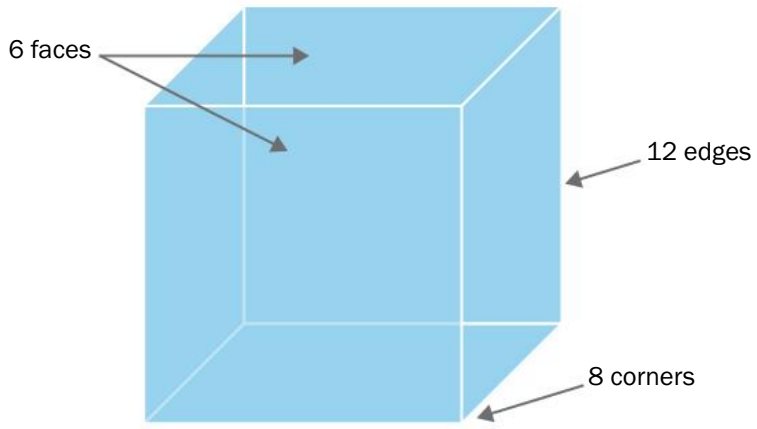
Click to complete Activity 12

Describing 3-D shapes

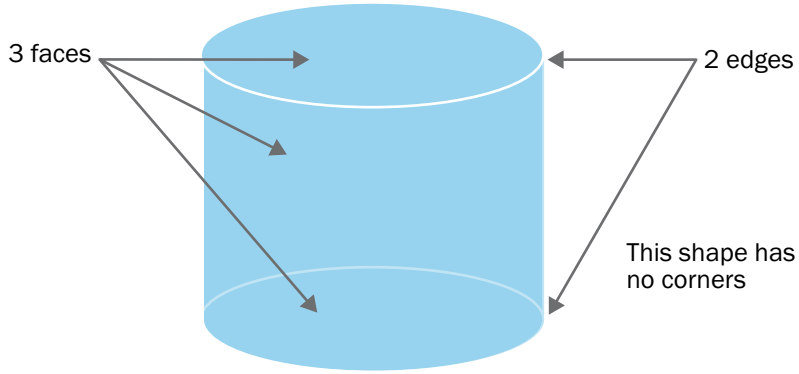
When we describe 2-D shapes we use words like sides and corners. Now that we are looking at 3-D shapes, we need to use different terms to avoid confusion.

3-D shapes have faces, edges and corners as shown in the following diagrams.

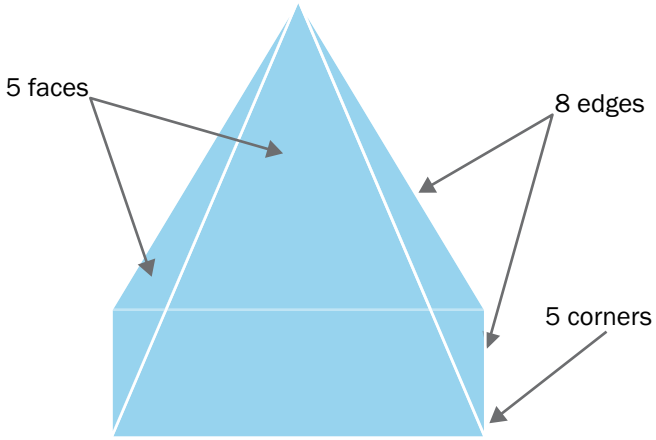
A cube



A cylinder



A square pyramid



A face is a surface of an object.

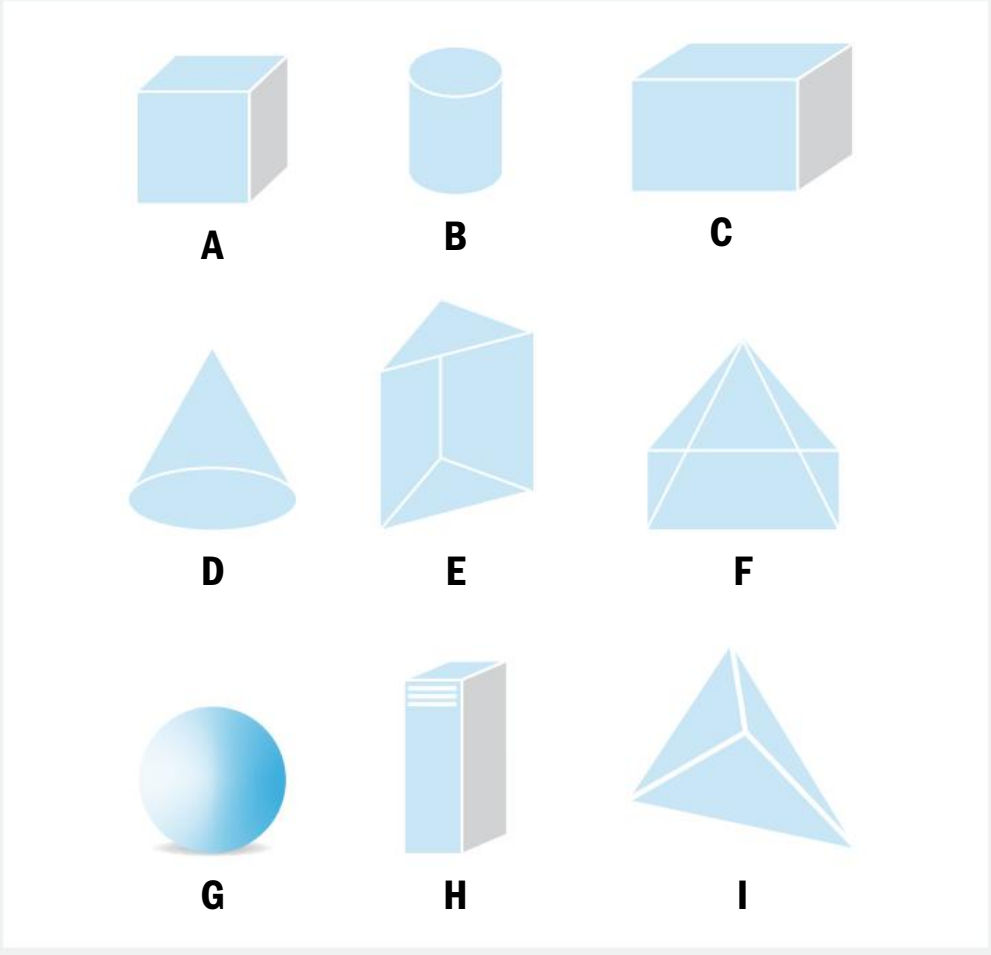
An edge is the line where two faces meet.

A corner is the point where two or more edges meet.

The bottom or base of a shape is a very important face as it is used to help identify and name the shape.

Activity 13

Look at the nine pictures shown below. Use the letters they are labelled with to complete the table. (Note: The shading can help you.)



The base is a triangle	The base is a square	The base is a rectangle	The base is a circle	No base

Click to complete Activity 13

Story

Rudi and his daughter Lara go for a walk around the neighbourhood to help him find some interesting shapes for his still-life drawing. Lara points out many objects, but she doesn't know what to call them. Rudi asks, 'Do you know how to talk about shapes and objects?'

'Not really,' says Lara, 'I get them mixed up most of the time and some of the names don't sound English.'

'You're right about the names. Some of them come from the Greek language. For example, 'penta' means five and 'gon' means corner, so a pentagon is a shape with five corners. When talking about 3-D shapes, we also use the word 'hedra', which means surface or face. So a 'pentahedra' is an object with five faces. Can you guess what an 'octahedra' is?' asks Rudi.

'An object with eight faces?' replies Lara.

'Exactly! I have another table that may help you, like the 2-D shape table I gave you', says Rudi.

Remember when we gave 2-D shapes family names to make it easier to talk about them? Well, we can do the same for 3-D shapes.

All 3-D shapes are polyhedra, which means shapes with many (poly) faces (hedra).

The three most common 3-D shape families are:

- prisms
- pyramids
- spheres.

When we name individual prisms or pyramids, we give them the name of the base (the shape on the bottom). For example, square prism, triangular prism, square pyramid or circular pyramid (or cone).

Activity 14

This is the table Rudi gave Lara. Using the three descriptions below, match each description to its shape.

Name	Diagram	Description
Prism		
Pyramid		
Sphere		

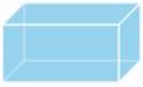
Descriptions

1. A shape with a bottom face but a point for its top and sides made up of slanting faces that are all the same height.
2. A shape with a curved surface.
3. A shape with a top and a bottom that are exactly the same size and shape, and sides made up of straight faces that are all the same height.

Click to complete Activity 14

Activity 15

Write in the missing information for each of the following 3-D shapes. The first one has been completed for you. Ask your trainer if you need help.

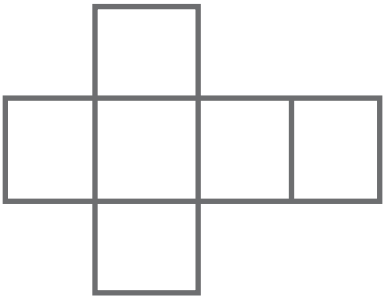
Shape	Name	The base is a ...	The top is a ...	Number of faces	Number of edges	Number of corners
	Sphere	No base	No top	1	0	0
						
						
						
						
						
						

Click to complete Activity 15

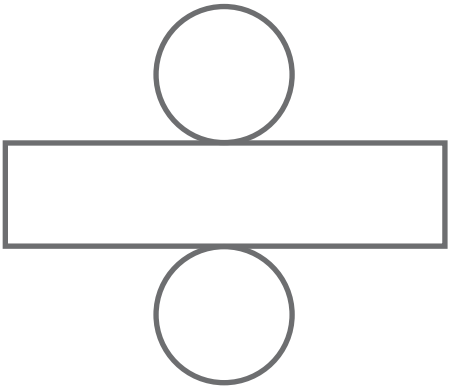
2-D shapes into 3-D objects

Three-dimensional shapes have length, width and depth. However, they can be turned into 2-D shapes if you open them out, or flatten them. Think of a tissue box. If you open it up along the folds and lay it out flat, you are left with a 2-D shape. This flat shape is called the net of the original 3-D shape/object. Here are some examples.

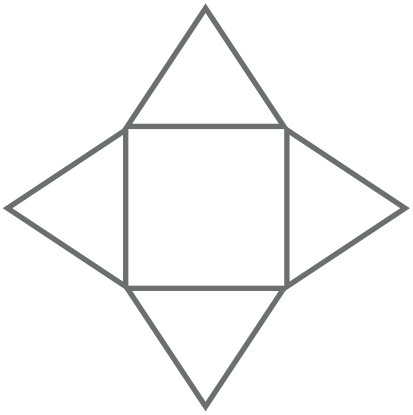
1.



2.



3.



Activity 16

Look at the three net diagrams. Can you work out the 3-D shape each net belongs to?

1. _____

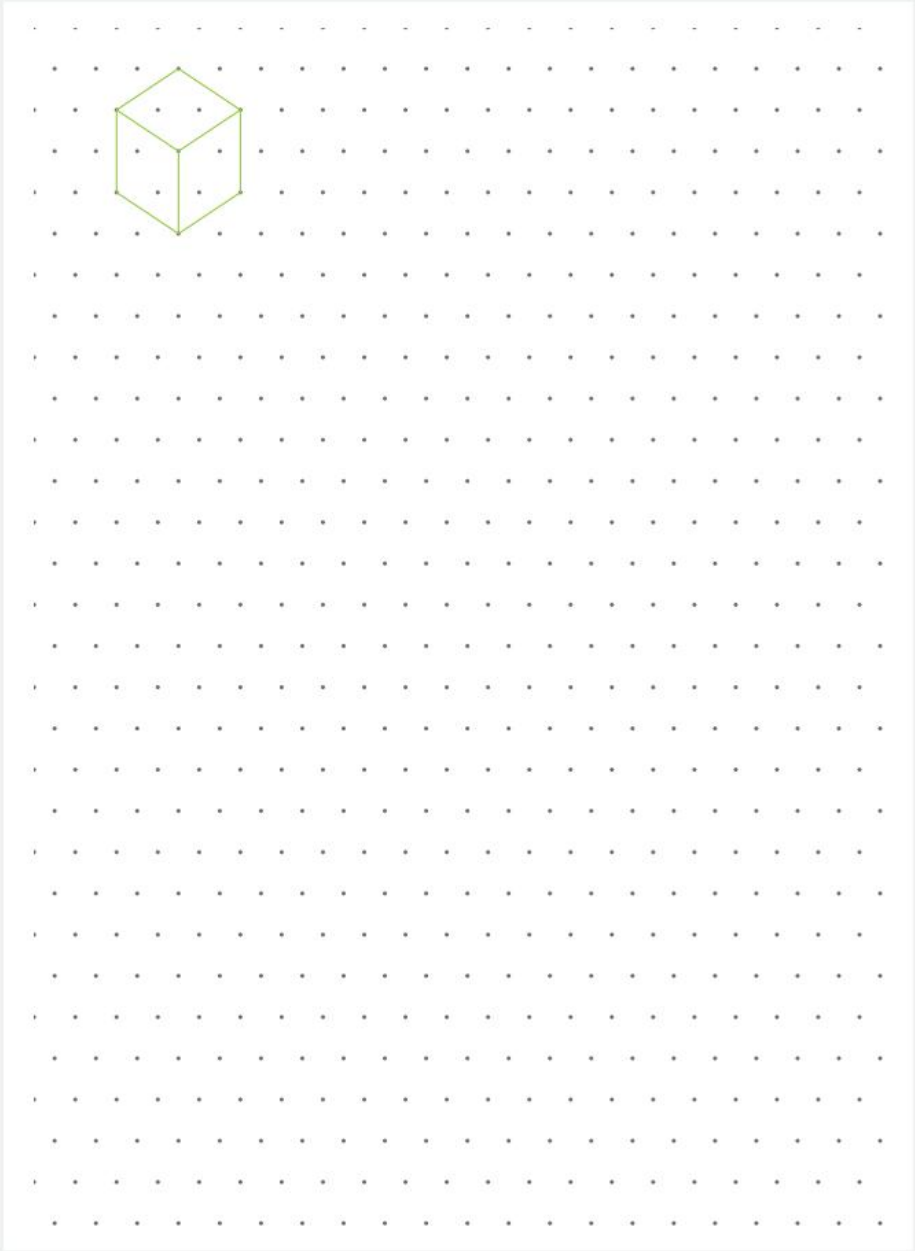
2. _____

3. _____

Click to complete Activity 16

Activity 17

Practise drawing 3-D shapes using this paper to help you. A cube has already been drawn.



Click to complete Activity 17

What you have learnt

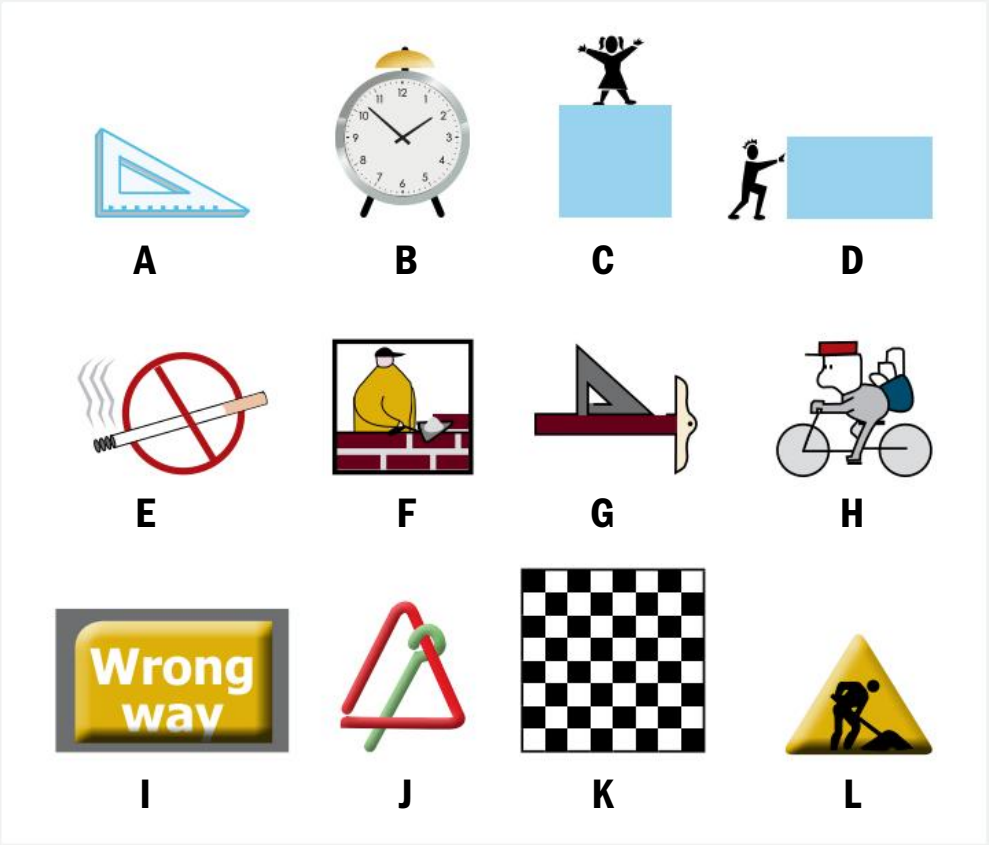
Put a ✓ in the box when you have learnt these things.

- ☐ Shapes in our world are three dimensional (3-D) because they have length, width and depth (height).
- ☐ Representations/drawings of 3-D shapes are two dimensional (2-D) as they lack the dimension of depth.
- ☐ The four basic 2-D shapes are the triangle, square, rectangle and circle.
- ☐ 2-D shapes are made up of lines called sides, points called corners and an angle at each corner.
- ☐ An angle is an amount of turn measured in degrees (˚).
- ☐ We can describe and estimate the size of angles by referring to fractions of a circle; for example, 90˚ is a quarter of a circle and 360˚ means you turn a whole circle.
- ☐ We can identify, describe and name 3-D shapes as a prism, pyramid or sphere.
- ☐ We name a specific prism or pyramid by the shape of its base. You can also describe and name 3-D shapes by the number of edges, corners and faces they have.

Check your learning

Answer the following questions.

1. Look at the 12 pictures below, then complete the following table to name which pictures have a triangle, square, rectangle or circle in them.



Triangle	Square	Rectangle	Circle

2. Stand up, face the front of the room and hold one arm out to point to the front. This position is your starting point each time.

   What angle do you turn through when you:

   a. Turn half a circle to point to the back of the room?

   _____

   b. Turn one quarter of a circle to point to the right side of the room?

   _____

   c. Turn three quarters of a circle to point to left side of the room?

   _____

   d. Turn a full circle to point to the front of the room again?

   _____

   e. Turn one third of a circle to point to the right side of the room?

   _____

   f. Turn two thirds of a circle to point to the left side of the room?

   _____

3. The following tables list some examples of families of shapes. Fill in the missing squares.

Triangle family	Sides	Angles	Name of individual shape
	3 all the same length		Equilateral triangle
	3 of all different lengths	3 all different	Scalene triangle
		2 are equal and the third is different	Isosceles triangle

Quadrilateral family	Sides	Angles	Name of individual shape
	4 – all the same length		
			
	4 – the 2 pairs of opposite sides are the same length	4 – the 2 pairs of opposite angles are the same size	Parallelogram
	4 – 2 top sides are the same length and the two bottom sides are the same	4 – 2 the same and the other 2 different	

4. Take a walk around your local community. Use the table below to help you find three objects that show the named shapes. Some have already been done for you.

Family name	Formal specific name	Common name	Example
Prism	Triangular prism	–	
	Rectangular prism	Box or cuboid	
	Square prism with all faces equal	Cube	
	Circular prism	Cylinder or pipe	
Pyramid	Triangular pyramid	Tetrahedron	House roof
	Rectangular pyramid	Pyramid	
	Square pyramid	Pyramid	
	Circular pyramid	Cone	
Sphere	Sphere	Ball	Globe of the world

Click to complete

Answers

Answers to activities

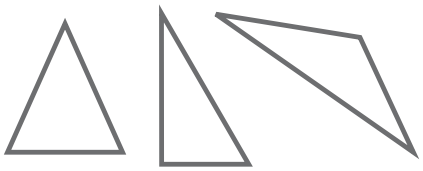
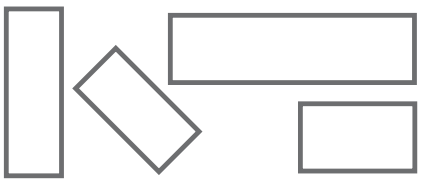
Activity 1

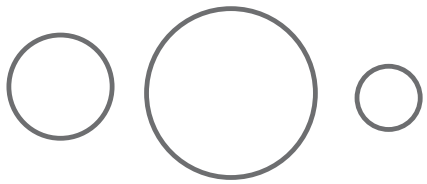
A variety of answers are appropriate; for example, postcards, notes, book pages, forms and bills all contain 2-D shapes.

Activity 2

Triangle	Rectangle	Square	Circle
Some house roofs Give-ways signs	Windows Street name signs Number plates Books	Windows Toilet paper	Car tyres Cups and plates

Activity 3

Shape	Description
	2. A shape made up of three sides and three corners. The three sides are not always the same length.
	1. A shape made up of four sides and four corners. The four sides are all the same length.
	4. A shape made up of four sides and four corners. The two pairs of opposite sides are the same length.

Shape	Description
	3. A shape that has no straight sides or corners but has one curved or round side.

Activity 4

Answer to Question 1

The space/angle gets bigger.

Answer to Question 2

The space/angle gets smaller.

Activity 5

Minute hand moves	Amount of time	Amount of circle	Angle size
From 12 to 12	One hour (60 minutes)	whole	360 ˚
From 12 to 3	¼ of an hour	¼ or one quarter	90 ˚
From 12 to 6	½ an hour	½ or one half	180 ˚
From 12 to 9	¾ of an hour	¾ or three quarters	270 ˚

Activity 6

Answers may vary. Things like a tree branch, the angles of car windows, or in the spokes on a bicycle may have been listed, containing various angles.

Activity 7

Answer to Question 1

In the rectangle the two long sides are the same length and the two short sides are the same length.

In the parallelogram the two long sides are the same length and the two short sides are the same length.

Answer to Question 2

In the rectangle all four angles are same size (¼ of a circle or 90˚ each).

Answer to Question 3

The angles in the parallelogram are different to the angles in the rectangle. The two pairs of opposite angles are the same size but one pair is not the same size as the other pair.

Activity 8

Answer to Question 1

The top left and top right sides are the same length and the bottom left and bottom right sides are the same length.

Answer to Question 2

The top and bottom sides are different lengths.

Answer to Question 3

The left- and right-side angles are the same size.

Activity 9

Shape family name	Number of sides	Number of corners	Number of angles
Triangles	3	3	3
Quadrilaterals	4	4	4
Pentagons	5	5	5
Hexagons	6	6	6
Octagons	8	8	8
Do you know any others?	Heptagon – 7 Decagon – 10	Heptagon – 7 Decagon – 10	Heptagon – 7 Decagon – 10

Activity 10

Answer to Question 1

The triangle does not shift.

Answer to Question 2

It is stronger than the rectangle.

Answer to Question 3

Builders use triangles with the rectangles of a frame to give it greater strength.

Activity 11

Answers will vary greatly depending on the person.

Activity 12

Answer to Question 1

The shapes created are:

- Cube or square box
- Rectangular box
- Tube or cylinder

Answer to Question 2

For example, a cone, witch's hat or funnel.

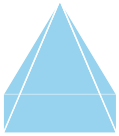
Activity 13

The base is a triangle	The base is a square	The base is a rectangle	The base is a circle	No base
E, I	A, H	C, F	B, D	G

Activity 14

Name	Diagram	Description
Prism		3. A shape with a top and a bottom that are exactly the same size and shape, and sides made up of straight faces that are all the same height.
Pyramid		1. A shape with a bottom face but a point for its top and sides made up of slanting faces that are all the same height.
Sphere		2. A shape with a curved surface.

Activity 15

Shape	Name	The base is a ...	The top is a ...	Number of faces	Number of edges	Number of corners
	Sphere	No base	No top	1	0	0
	Cube or square prism	Square	Square	6	12	8
	Triangular pyramid	Triangle	Point	4	6	4
	Cone or circular pyramid	Circle	Point	2	1	1
	Cylinder or circular prism	Circle	Circle	3	2	0
	Box or rectangular prism	Rectangle	Rectangle	6	12	8
	Square pyramid	Square	Point	5	8	5

Activity 16

1. Square prism or cube
2. Cylinder
3. Square pyramid

Activity 17

No written response is required here.

Answers to Check your learning

Answer to Question 1

Triangle	Square	Rectangle	Circle
A, G, J, L	C, F, K	D, F, I	B, E, H, L

Answer to Question 2

a. 180 ˚

b. 90 ˚

c. 270 ˚

d. 360 ˚

e. 120 ˚

f. 240 ˚

Answer to Question 3

Triangle family	Sides	Angles	Name of individual shape
	3 all the same length	3 all the same	Equilateral triangle
	3 of all different lengths	3 all different	Scalene triangle
	2 the same, one is different	2 are equal and the third is different	Isosceles triangle

Quadrilateral family	Sides	Angles	Name of individual shape
	4 – all the same length	4 – all the same	Square
	4 – 2 pairs of opposite sides are the same length	4 – all the same size	Rectangle
	4 – the 2 pairs of opposite sides are the same length	4 – the 2 pairs of opposite angles are the same size	Parallelogram
	4 – 2 top sides are the same length and the two bottom sides are the same	4 – 2 the same and the other 2 different	Kite

Answer to Question 4

Answers may vary. Examples have been provided.

Family name	Formal specific name	Common name	Example
Prism	Triangular prism	–	Toblerone chocolate bar
	Rectangular prism	Box or cuboid	Shoebox
	Square prism with all faces equal	Cube	Gift box
	Circular prism	Cylinder or pipe	Wrapping paper roll
Pyramid	Triangular pyramid	Tetrahedron	House roof
	Rectangular pyramid	Pyramid	
	Square pyramid	Pyramid	
	Circular pyramid	Cone	Ice-cream cone
Sphere	Sphere	Ball	Globe of the world Bubble

