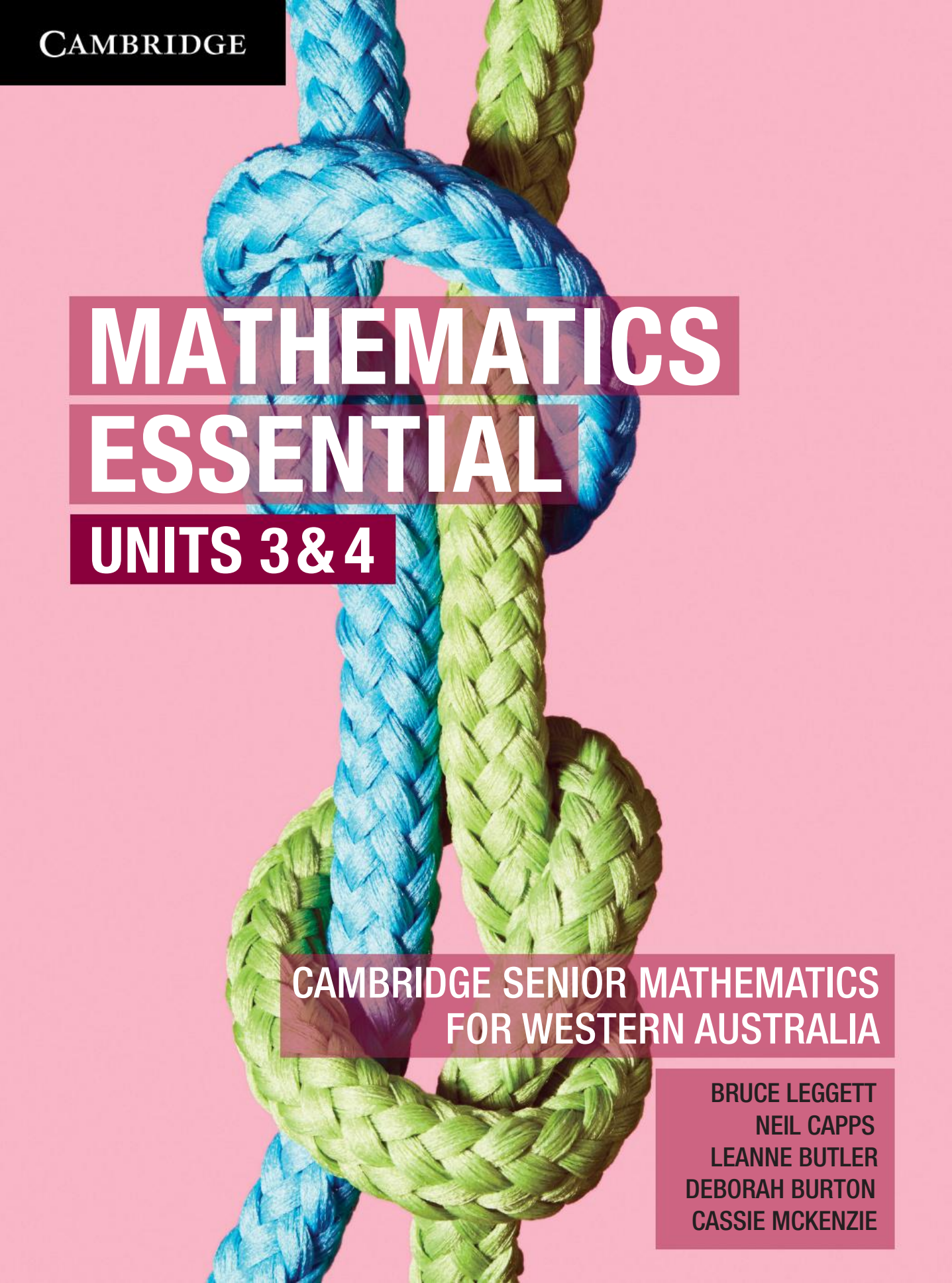




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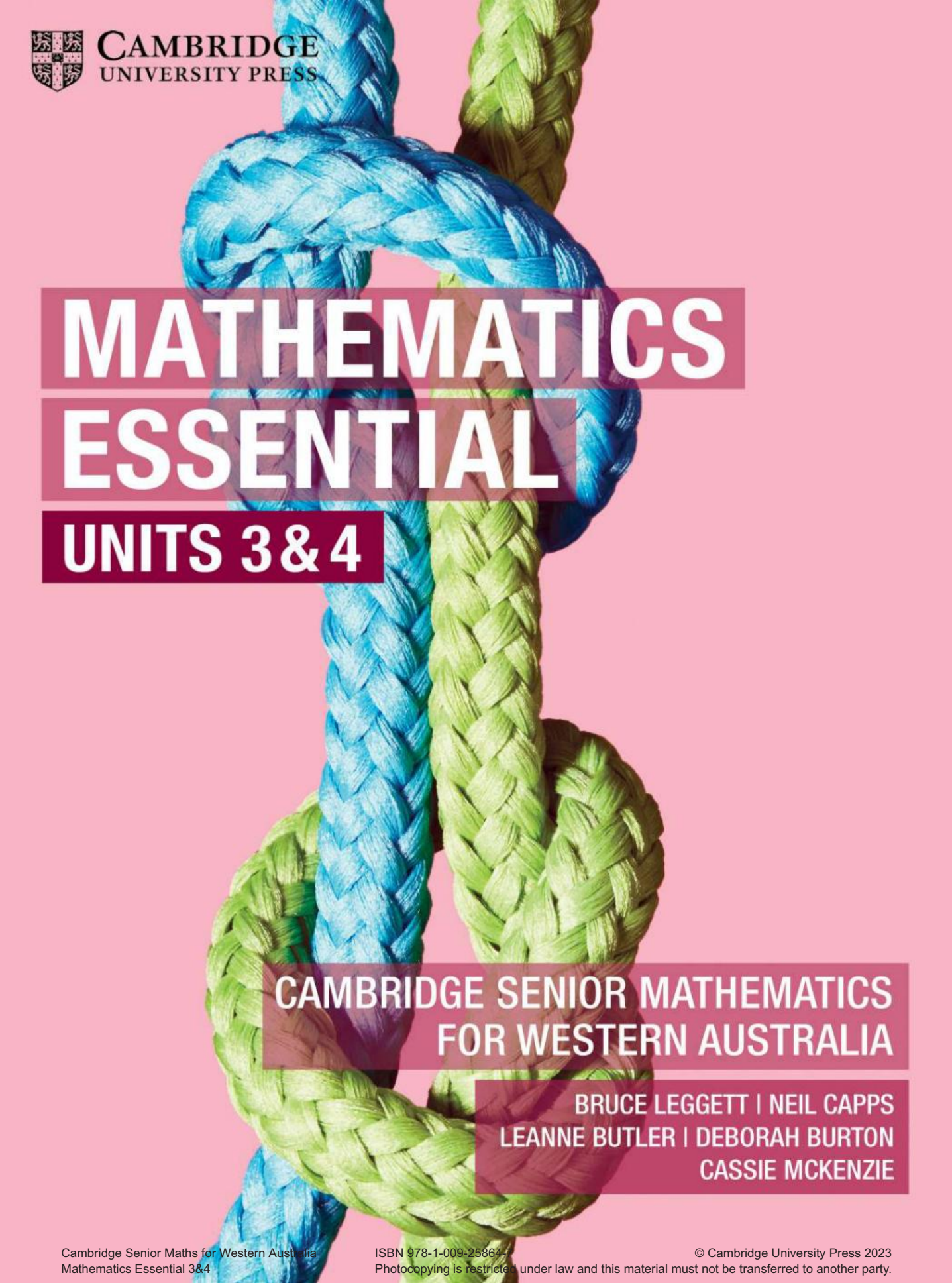
MATHEMATICS

ESSENTIAL

UNITS 3 & 4

**CAMBRIDGE SENIOR MATHEMATICS
FOR WESTERN AUSTRALIA**

**BRUCE LEGGETT
NEIL CAPPS
LEANNE BUTLER
DEBORAH BURTON
CASSIE MCKENZIE**



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Online appendices

Printable documents in the interactive textbook and online teaching suite:

A1.1 Glossary of terms

A1.2 Glossary of cognitive verbs

A2.1 Online guide to spreadsheets

A2.2 Online guide to the Desmos graphing calculator

A2.3 Links to online guides to using scientific calculators

Note: A printable copy of a formula sheet is available in the Interactive Textbook



Introduction

Cambridge Mathematics Essential for Western Australia Units 3 & 4 is a new resource aligned specifically to the Western Australian Mathematics Essential Year 12 syllabus. Covering both Units 1 and 2 in one resource, it has been written with practical contexts and worded questions as its priority alongside ample practice offered through worked examples and exercises. This course follows on from the Western Australian Mathematics Essentials Year 11 syllabus and expands upon the fundamental skills and knowledge developed in earlier levels.

The Unit 3 & 4 text has been carefully structured so that Unit 3 content can be reordered by teachers to fit the EST content for each year.

Unit 3 begins with extending the understanding of 2D and 3D shapes into complex shapes. The measurement chapters continue with the study of scales, plans and models, and the introduction of right-angled triangle mathematics. Throughout, the application of the mathematical thinking process is developed through the Mathematical Thinking Process Tasks at the end of chapters to support students in preparation for the EST. Graphs in practical situations and interpreting graphs is then introduced, and Unit 3 ends with chapters on data collection, including bivariate data, with ample practice at applying the statistical investigation process in preparation for Unit 4.

Unit 4 begins with probability and relative frequencies. Earth geometry and time zones are introduced in Chapter 8, and loans and compound interest and their application to the mathematical thinking process complete Unit 4.

Worked examples utilising appropriate technology are provided throughout the textbook, with screenshots and detailed user instructions for both scientific calculators and Excel spreadsheets.

The integration of the features of the textbook and the new digital components of the package, powered by Cambridge HOTmaths, are illustrated on pages x to xiv.

About Cambridge HOTmaths

Cambridge HOTmaths is a comprehensive, award-winning mathematics learning system – an interactive online maths learning, teaching and assessment resource for students and teachers, for individuals or whole classes, for school and at home. Its digital engine or platform is used to host and power the Interactive Textbook and the Online Teaching Suite, and selected topics from HOTmaths' own Years 9 and 10 courses area are available for revision of prior knowledge. All this is included in the price of the textbook.

Overview

Overview of the print textbook

- Syllabus references** are listed at the beginning of each chapter.
- Pre-tests** provide a check of requisite knowledge and skills. Links are provided to access HOTmaths lessons from earlier years as revision for those students who need it.
- Learning goals** based on the syllabus are given for each section.
- What you need to know** boxes list important concepts and principles in concise and accessible format.
- Worked examples** detail thinking and the solution in a logical sequence, and are linked to exercises. Video versions are provided in the Interactive Textbook to encourage independent learning.

Print book features — numbers refer to points above

Page 11D: Comparisons using rates and determining best value

11D Comparisons using rates and determining best value

LEARNING GOALS

- Compare rates of price per equal weight or volume to find the best-value buy:
 - with a calculator, and
 - without a calculator.
- Solve best-value rate problems in real-world situations.

When is it essential to compare rates?

Examples of when rates are compared include determining the best value for money, the most efficient production rates and the most economical rates of energy usage. Also, we can compare our pay rates to the Australian award pay rates, and we all need to compare our car's speed to the speed limit.

When a company makes decisions about large investments, it will consider and compare many types of rates. For example, a new building with a higher cost rate in \$/m² may be compared with alternative buildings that have lower cost rates. The company may consider whether the higher cost building will be longer lasting than one with a lower cost rate, which would require more frequent maintenance.

WHAT YOU NEED TO KNOW

- To find the **best buy**, convert each price option to the cost of equal amounts.
- For example, the option on the left is the best buy of these two brands of chainsaw oil.

$$+3 \text{ L } \$76.50/3 \text{ L} = \$25.50/2 \text{ L} > 3 \quad +4 \text{ L } \$106/4 \text{ L} = \$26.50/2 \text{ L} > 4$$
- The best value can be either the lowest rate or the highest rate, depending on the situation.

Page 11E: Chapter 11 Rates

- For example: The lowest rate is the best value for a car's fuel economy in litres/100 km.
- The highest rate of data download, in Mbps (megabits per second), is the best value.

Example 9 Comparing rates to find the best-value buy, without a calculator

Without a calculator, determine the cost of the following sugar brands per 100 grams, and hence, determine the best buy.

500 grams of Soft Sugar costs \$2.25, and 300 grams of Sun Sugar costs \$1.50.

WORKING	THINKING
Soft Sugar $\$2.25 \div 5 = \$0.45/100 \text{ grams}$	Since 500 grams is $5 \times 100 \text{ grams}$, divide \$2.25 by 5.
Sun Sugar $\$1.50 \div 3 = \$0.50/100 \text{ grams}$	Since 300 grams is $3 \times 100 \text{ grams}$, divide \$1.50 by 3.

It is the best value to buy the 500 g packet of Soft Sugar.

The lowest cost per 100 g is the better buy. Write the answer as a sentence.

Example 10 Comparing rates to determine the best-value buy, using a calculator

Determine the best value (i.e. the cheaper deal) between these products: 1.5 litres of Frank's Lemonade for \$1.80 and 2.5 litres of Lucy's Lemonade for \$2.80.

WORKING	THINKING
Frank's Lemonade $+1.5 \text{ L } \$1.80/1.5 \text{ litres} \div 1.5 = \$1.20/\text{L}$	Write the information as the rate \$/litre. $1.5 \text{ L} \div 1.5 = 1 \text{ L}$, so we also divide \$1.80 by 1.5.
Lucy's Lemonade $+2.5 \text{ L } \$2.80/2.5 \text{ litres} \div 2.5 = \$1.12/\text{L}$	$2.5 \text{ L} \div 2.5 = 1 \text{ L}$, so we also divide \$2.80 by 2.5.

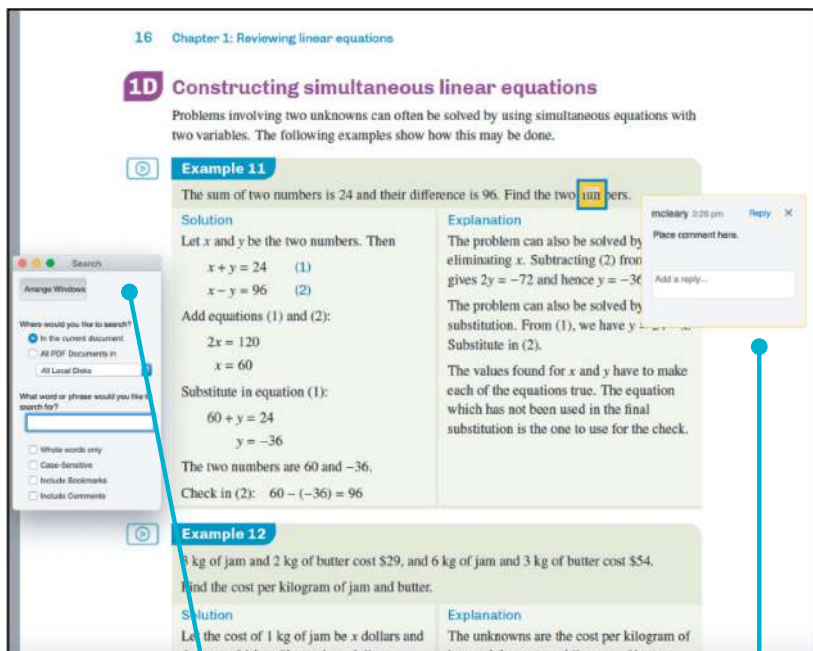
2.5 L of Lucy's Lemonade is the best buy.

The lowest cost per litre is the best value.

- 6 Exercises are divided into:
 - **Fundamentals** – integrating the Fundamental topic: Calculations throughout the topic, as required by the syllabus.
 - **Applications**: questions that model real-life contexts.
- 7 **Mathematical thinking process tasks** and **Statistical investigation process tasks** are provided at the end of chapters, and are, set out in stages with a flowchart based on the approach to problem-solving and mathematical thinking outlined by SCSA.
- 8 **Chapter checklists** comprise short questions assessing achievement of learning goals.
- 9 **Review** exercises contain assessment-style questions.
- 10 Examples and questions using **spreadsheets** are integrated throughout the text, with accompanying Excel files in the Interactive Textbook.
- 11 Additional linked resources in the Interactive Textbook and Online Teaching Suite such as practice assessment items are indicated by icons or notes in the text.

Overview of the downloadable PDF textbook

- 12 The convenience of a downloadable PDF textbook has been retained for times when users cannot go online.
- 13 PDF annotation and search features are enabled.



8

8

Overview of the Interactive Textbook

The **Interactive Textbook (ITB)** is an online HTML version of the print textbook powered by the HOTmaths platform, included with the print book or available as a separate purchase.

- 14** The material is formatted for on screen use with a convenient and easy-to-use navigation system and links to all resources.
- 15** **Definitions** pop up for key terms in the text and are also provided in a printable online **glossary**, while the HOTmaths dictionary is also accessible.
- 16** All worked examples have **video versions** to encourage independent learning.
- 17** The **Desmos scientific calculator**, **graphics calculator** and **geometry tool** are also available for students to use for their own calculations and exploration.
- 18** **Spreadsheets** are provided in Excel format.
- 19** **Quick quizzes** containing automarked multiple-choice questions enable students to check their understanding.
- 20** **Workspaces** for all questions, which can be enabled or disabled by the teacher, allow students to enter working and answers online and to save them. Input is by typing, with the help of a symbol palette, handwriting and drawing on tablets, or by uploading images of writing or drawing done on paper.
- 21** **Self-assessment tools** enable students to check answers, mark their own work, and rate their confidence level in their work. This helps develop responsibility for learning and communicates progress and performance to the teacher. Student accounts can be linked to the learning management system used by the teacher in the Online Teaching Suite, so that teachers can review student self-assessment and provide feedback or adjust marks.
- 22** **Worked solutions** are included and can be enabled or disabled in the student accounts by the teacher.
- 23** **Practice assessment** items are provided in downloadable PDF and Word files.
- 24** **Messages** from the teacher to students can be sent to assign tasks and tests.

INTERACTIVE TEXTBOOK POWERED BY THE HOTmaths PLATFORM

A selection of features is shown. Numbers refer to the descriptions on pages xii–xiv. HOTmaths platform features are updated regularly. Screenshots below show the Mathematics Methods title in the series - not all features of Mathematics Essential are shown.

14

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WORKSPACES AND SELF-ASSESSMENT

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21

Overview of the Online Teaching Suite powered by the HOTmaths platform

The Online Teaching Suite is automatically enabled with a teacher account and is integrated with the teacher's copy of the Interactive Textbook. All the teacher resources are in one place for easy access. The features include:

- 25** The HOTmaths learning management system with class and student analytics and reports, and communication tools.
- 26** Teacher's view of a student's working and self-assessment.
- 27** A HOTmaths-style test generator.
- 28** Chapter test worksheets as PDFs and editable Word documents.
- 29** Assessment practice items as PDFs and editable Word documents.
- 30** Editable curriculum grids and teaching programs.



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1 Measurement



In this chapter

- 1A** Recognising common 2D geometric shapes and 3D solids
 - 1B** Calculating perimeters of polygons and circles
 - 1C** Calculating perimeters of familiar composite shapes
 - 1D** Estimating and calculating areas of parallelograms and circles
 - 1E** Calculating the area of trapeziums, sectors and composite figures
 - 1F** Investigating nets of 3D solids
 - 1G** Calculating the surface areas of cubes, prisms and pyramids
 - 1H** Calculating the surface areas of spheres and cylinders
 - 1I** Calculating the surface areas of irregular solids
 - 1J** Using and converting between metric volume and capacity units
 - 1K** Estimating and calculating the volume and capacity of pyramids, cones, spheres and cylinders
- Mathematical thinking process task
- Chapter checklist
- Chapter review

Syllabus reference

Unit 3 Topic 1: Measurement (15 hours) and Topic 2: Scales, plans and models (2 hours)

Geometry

- recognise the properties of common two-dimensional geometric shapes and three-dimensional solids
- interpret different forms of two-dimensional representations of three-dimensional objects, including nets and perspective diagrams
- use terminology of geometric shapes; for example, point, line, angle, diagonal, edge, curve, face and vertex, parallel and perpendicular

Linear measure

- extend the calculation of perimeters to include polygons, circles and composites of familiar shapes

Area measure

- calculate areas of parallelograms, trapeziums, circles and semi-circles
- determine the area of composite figures by decomposition into familiar shapes
- determine the surface area of familiar solids, including, cubes, rectangular and triangular prisms, spheres and cylinders
- use addition of the area of the faces of solids to determine the surface area of composite solids

Volume and capacity

- recognise relations between volume and capacity, recognising that $1 \text{ cm}^3 = 1 \text{ mL}$ and $1 \text{ m}^3 = 1 \text{ kL}$
- calculate the volume and capacity of cylinders, pyramids and spheres

Pre-test

1 Determine the number of:

- a sides in a rectangle
- c edges in a cube

- b corners in a rectangle
- d corners in a cube

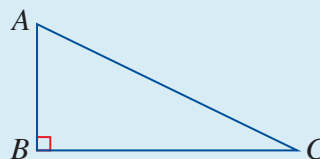
Hint Drawing a diagram may help you to determine the properties or features of a shape or solid.

2 Identify:

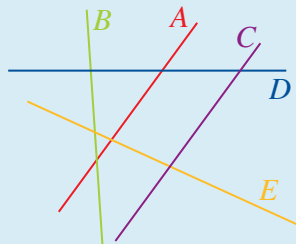
- a the unit of measure for angles
- b the number of angles in a triangle

3 Identify:

- a the right angle in the diagram shown
- b the size of a right angle in degrees

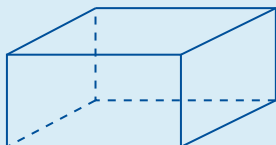


4 Determine which of the following lines are parallel lines.

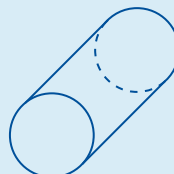


5 For each 3D solid:

- i count the number of faces
- a



- ii name the 2D shape of each face
- b



6 Evaluate:

a 1^2

b 9^2

c 10^2

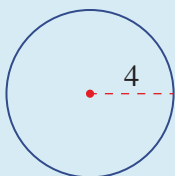
d 100^2

e 1000^2

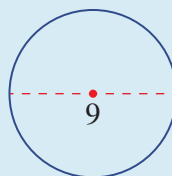
f 13^2

7 In the circles below, determine the size of the radius.

a



b



8 State the name of the symbol π and explain what it represents in relation to the lengths of the diameter of a circle and its circumference.

9 In this geometry diagram, identify the feature represented by these symbols:

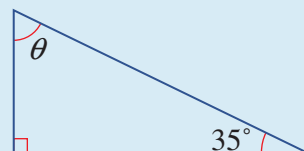
a



b

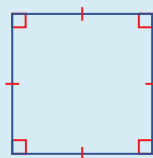


c

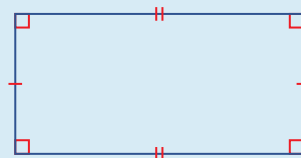


10 In these geometry diagrams explain what is meant by the marks on the sides.

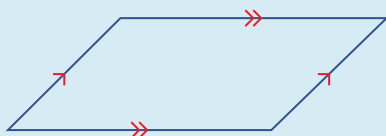
a



b



c



11 Explain the meaning of:

a 'milli-' as used in the unit 'millimetre'

b 'kilo-' as used in the unit 'kilometre'

12 How many millilitres are in a litre?

1A Recognising common 2D geometric shapes and 3D solids

LEARNING GOALS

- Identify the names and properties of 2D shapes including:
 - squares
 - rectangles
 - triangles
- Identify the names and properties of 3D solids including:
 - cubes
 - rectangular prisms
 - triangular prisms

Why is it essential to be able to identify the names and properties of various shapes?

Shapes and solids are used and seen everywhere. They can be found in manufactured items such as cardboard and jars. Shapes and solids can also be found in nature such as starfish and tree trunks. It is important to understand the properties of shapes when learning measurement as the properties can be used to describe and create shapes. The properties of shapes can be used in careers such as landscaping, building and engineering.



Modern machines can create extremely complex shapes.

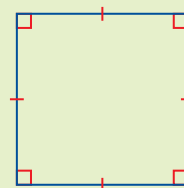
WHAT YOU NEED TO KNOW

- **Two-dimensional (2D)** shapes are flat. They have only two dimensions such as length and width. Examples of 2D shapes include square, rectangle, triangle, trapezium, parallelogram, rhombus, kite and hexagon.
- **Three-dimensional (3D)** solids include a third dimension of depth or height. Examples of 3D solids include sphere, cone, cylinder, cube, rectangular prism and triangular prism.

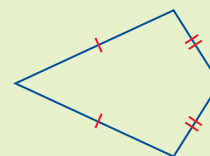
- **Properties** are the description of the shape or solid. The properties of 2D shapes include the number of **sides**, the number of **vertices**, whether the sides are equal, the size of **angles**, whether the angles are equal, and whether the sides are **parallel**. The properties of 3D solids include the number of **faces**, **edges** and vertices.

- Markings to show properties of 2D shapes

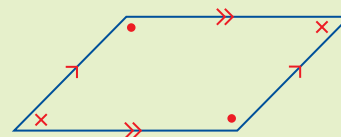
- Boxed corners represent an angle of 90° . Small ticks on the sides represent equal side lengths.



- Double markings show another set of equal sides. These sides are not the same length as the single marked sides.

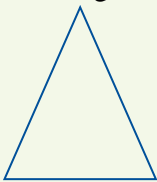
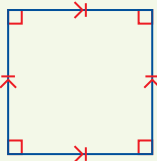


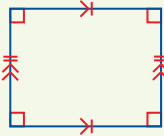
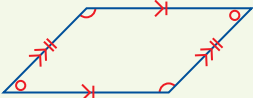
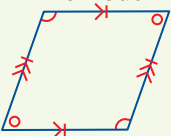
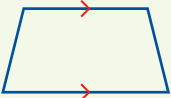
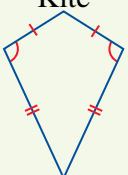
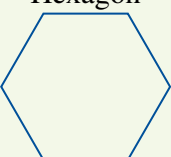
- Arrows represent parallel lines. Double arrows show another set of parallel lines. Small crosses, dots and arcs can be used to show equal angles.



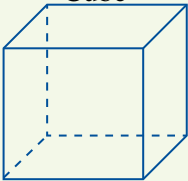
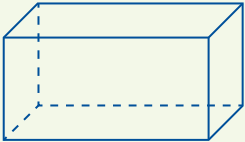
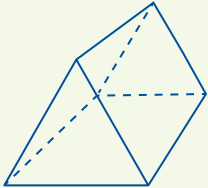
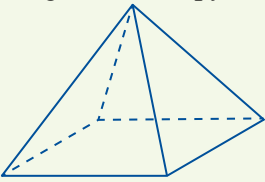
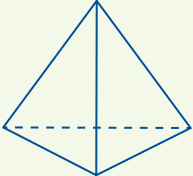
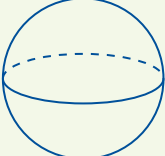
- A **vertex** (plural: vertices) is an angular point on a shape or solid. It is where two or more lines or edges meet.

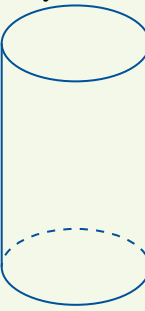
- 2D shape properties

Shape	Properties
Triangle 	3 sides 3 vertices All angles add to 180°
Square 	4 sides 4 vertices All sides are equal All angles are 90° All angles add to 360° Two pairs of parallel sides

Shape	Properties
Rectangle 	4 sides 4 vertices All angles are 90° All angles add to 360° Opposite sides are parallel Opposite sides are equal
Parallelogram 	4 sides 4 vertices All angles add to 360° Opposite sides are parallel Opposite sides are equal Opposite angles are equal
Rhombus 	4 sides 4 vertices All angles add to 360° All sides are equal Opposite angles are equal Opposite sides are parallel
Trapezium 	4 sides 4 vertices All angles add to 360° One pair of parallel sides
Kite 	4 sides 4 vertices All angles add to 360° Adjacent sides are equal One pair of opposite angles are equal
Hexagon 	6 sides 6 vertices All sides equal in a regular hexagon All angles equal in a regular hexagon

■ 3D solid properties

Solid	Properties
Cube 	6 faces 8 vertices 12 edges All faces are squares
Rectangular prism 	6 faces 8 vertices 12 edges All faces are rectangles
Triangular prism 	5 faces 6 vertices 9 edges Three faces are rectangles Two faces are triangles
Square-based and rectangular-based pyramid 	5 faces 8 edges 5 vertices Base is either a square or a rectangle Four faces are triangles
Triangular-based pyramid (tetrahedron) 	4 faces 4 vertices 6 edges Four faces are all triangles
Sphere 	0 faces No faces, vertices nor edges An evenly curved surface Perfectly symmetrical

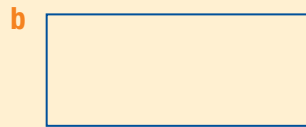
Solid	Properties
Cone 	0 faces No vertices nor straight edges One flat circular surface with no straight edge One curved surface around the circle
Cylinder 	0 faces No vertices nor straight edges Two flat, circular surfaces with no straight edges One curved surface that flattens to a rectangle



Example 1 Identifying the properties of 2D shapes

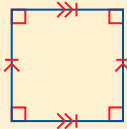
Complete the following for each of the shapes shown.

- Identify the name of the shape.
- Draw the shape.
- Mark in its properties.

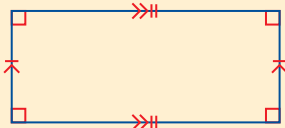


WORKING

- a i** Square
ii, iii



- b i** Rectangle
ii, iii



THINKING

← All angles are 90 degrees.
All sides are equal.
Two pairs of parallel sides.

← All angles are 90 degrees.
Two pairs of parallel sides.
Opposite sides are equal.

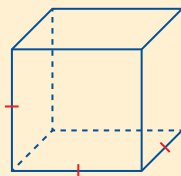


Example 2 Identifying the properties of 3D solids

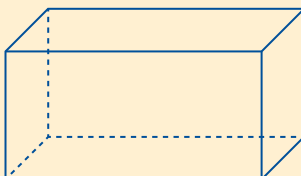
Complete the following for each of the solids shown.

- i Identify the name of the solid.
- ii Identify the number of faces, edges and vertices.

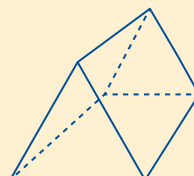
a



b



c



WORKING

- a i Cube
- ii 6 faces, 12 edges and 8 vertices
- b i Rectangular prism
- ii 6 faces, 12 edges and 8 vertices
- c i Triangular prism
- ii 5 faces, 9 edges and 6 vertices

THINKING

Faces – Count the number of flat sides on each solid.

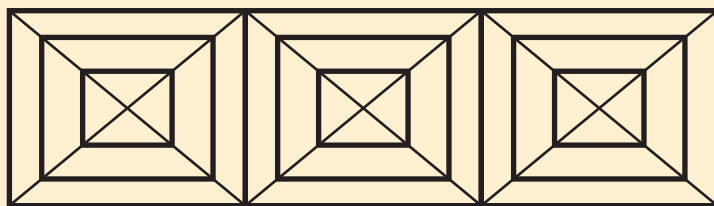
Edges – Count the number of edges (where two faces meet) on each solid.

Vertices – Count the number of angular points on each solid.



Example 3 Identifying 2D shapes in a real-world context

Valerie has designed a new fence that she would like to place along the front of her house. Identify the 2D shapes in each panel of the fence shown.



WORKING

Triangles

Trapeziums

Rectangles

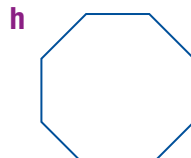
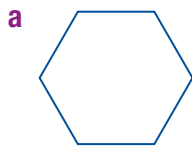
THINKING

Look for shapes that are common 2D shapes. Triangles are in the centre of the fence panels. Each panel is a rectangle and the trapeziums are between the triangles and rectangles in each frame.

Exercise 1A

FUNDAMENTALS

1 Identify each 2D shape **a–i** with the correct name from the list **A–I**.



A parallelogram

B hexagon

C triangle

D rectangle

E rhombus

F octagon

G trapezium

H pentagon

I square

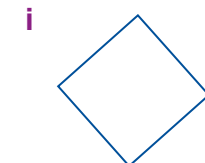
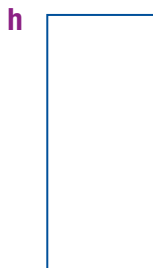
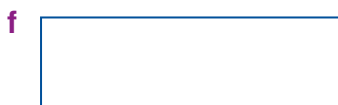
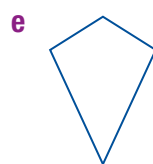
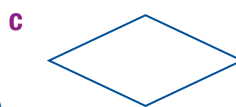
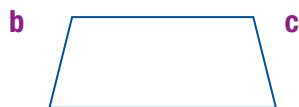
2 Copy the table and tick the boxes to show the correct properties for each of the following shapes.

Property	Square	Rectangle	Parallelogram	Trapezium	Triangle	Rhombus	Kite
Four sides							
Three sides							
All sides are equal length							
All angles measure 90°							
Not all angles measure 90°							
Two pairs of parallel sides only							
Two pairs of parallel sides of different lengths							

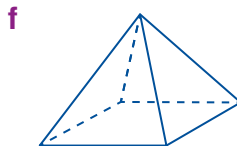
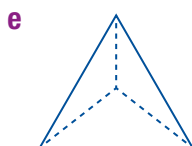
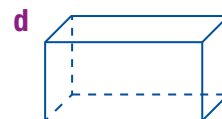
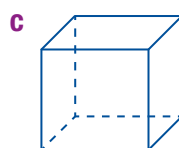
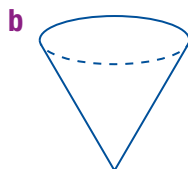
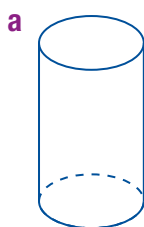
Example 1

3 Complete the following for each of the shapes shown.

- i** Identify the name of the shape.
- ii** Draw the shape and mark in its properties.
- iii** Describe the properties of each shape.



4 Identify each 3D solid **a–g** with the correct name from the list **A–G**.



- A** cube
- B** triangular prism
- C** rectangular prism
- D** triangular-based pyramid (tetrahedron)
- E** square-based pyramid
- F** cone
- G** cylinder

- 5 Determine the missing words in the following sentences.
- a The flat surface on a 3D solid is called the _____.
 - b The vertex or vertices describes the _____ of a 3D solid.
 - c A side of a 3D solid where two faces meet is called the _____.

Example 2

- 6 Using the properties of 3D solids, determine the blank spaces in the following table.

Name of solid	Faces	Edges	Vertices
	6	12	8
Triangular prism			
	5	8	5
Triangular-based pyramid			
Pentagonal prism			
Hexagonal prism			

APPLICATIONS

Example 3

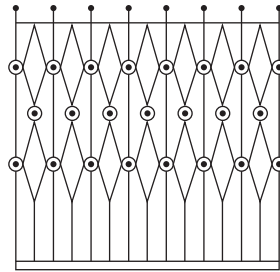
- 7 Cubism was a revolutionary style of art that was created by Pablo Picasso and Georges Braque around 1907. It is typically characterised by the use of geometric planes and shapes, similar to the image shown. Identify and create a list of the various 2D shapes within the image.



- 8 Geometric animal art is currently quite popular. It is often used in graphic design as well as in drawings and paintings. This involves using a variety of 2D shapes to create an animal. In the image shown, identify the two main shapes that are used to draw the flamingos.



- 11** Lyn has designed a new gate that she would like to have made. Identify the various 2D shapes that she has used.



- 12** Billy and Alana are looking to buy a block of land in a new housing estate. Each block has been numbered in the plan shown below. Identify which blocks are shaped like a:

- a** rectangle
- b** parallelogram
- c** trapezium



Source AV Jennings

13 Identify the name of the 3D solid that is most similar to the following real-life objects.



14 Identify the name of the 3D solid that is most similar to these landmarks.



Eiffel Tower in Paris



Pyramids in Egypt

1B Calculating perimeters of polygons and circles

LEARNING GOALS

- Calculate the perimeters of polygons and circles
- Apply the formula to calculate the circumference of a circle
- Apply the formula to calculate the arc length and perimeter of a sector

Why is it essential to understand how to calculate perimeters?

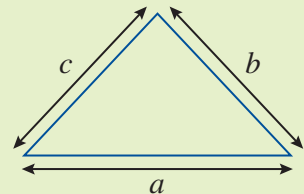
Calculating perimeters can be used in industries such as construction and design. In construction, perimeter can be used to calculate the total length around the property when building a fence. Perimeter is used to calculate the amount of timber required to frame a painting. Or it can also be used in fashion to work out the length of trimming required around the hem of a skirt.



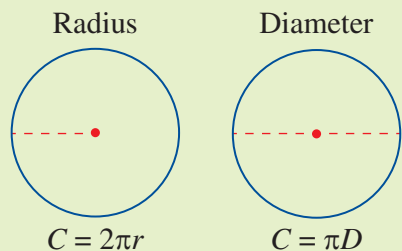
Perimeter is used to calculate the amount of fencing required around a property.

WHAT YOU NEED TO KNOW

- **Perimeter** is the distance around the outside of a shape. For example, the perimeter of the triangle in the image would be the total of $a + b + c$.



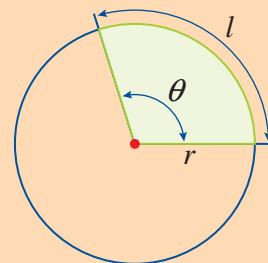
- **Circles** – The distance around the outside of a circle is called the **circumference**. The formula to calculate the circumference of a circle is $C = 2\pi r$ or $C = \pi D$, where r stands for radius and D stands for diameter. The radius extends from the centre to the circumference and the diameter extends all the way across the circle through the centre.



- **Arc length** – An **arc** is a curve that is part of the circumference of a circle. The arc length is the distance along the curved line that makes up a part of a circle.

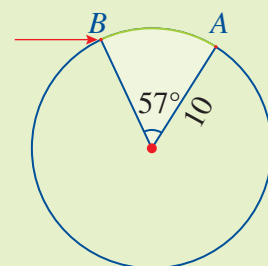
Formula for arc length:

$$l = \frac{\theta}{360} \times 2\pi r \text{ or } l = \frac{\theta}{360} \times \pi D$$



For example: For the arc length A to B in the diagram shown, the arc length is calculated using $l = \frac{\theta}{360} \times 2\pi r$, where θ is the angle at the centre.

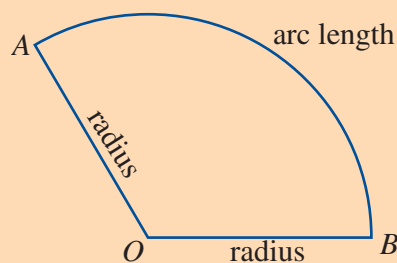
(so $\theta = 57^\circ$) and r is the length of the radius ($r = 10$).



- **Sector** – A sector is a section of the circle formed from two radii. We can identify the perimeter of a sector by calculating the arc length and then adding the two side lengths, which is the radius multiplied by 2.

Formula for perimeter of a sector:

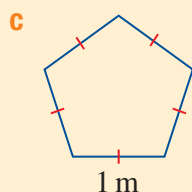
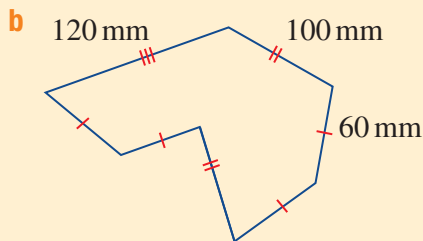
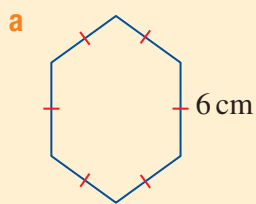
$$P = \frac{\theta}{360} \times 2\pi r + 2r$$



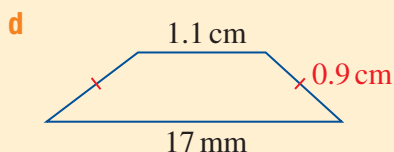


Example 4 Calculating the perimeter of 2D shapes

Calculate the perimeter of the following shapes.



For the following question, calculate the perimeter in the units highlighted in red.



WORKING

THINKING

a $6 + 6 + 6 + 6 + 6 + 6 = 36 \text{ cm}$ $\leftarrow$ Add all the side lengths together.

or

$6 \times 6 = 36 \text{ cm}$ $\leftarrow$ Alternatively, as there are 6 equal sides, multiply the side length by 6.

b $100 + 100 + 60 + 60 + 60 + 60 + 120 = 560 \text{ mm}$ $\leftarrow$ Add all the side lengths together.

or

$2 \times 100 + 4 \times 60 + 120 = 560 \text{ mm}$ $\leftarrow$ Alternatively, as there are 2 equal sides of 100 mm and 4 equal sides of 60 mm, multiply the equal side lengths by the number of them and then add the single side.

c $1 + 1 + 1 + 1 + 1 = 5 \text{ m}$ $\leftarrow$ Add all the sides together.

or

$5 \times 1 = 5 \text{ m}$ $\leftarrow$ Alternatively, as there are 5 equal sides, multiply the side length by 5.

d $17 \text{ mm} \div 10 = 1.7 \text{ cm}$ $\leftarrow$ Convert mm to cm first. There are 10 mm in every 1 cm so divide 17 by 10.

$0.9 + 0.9 + 1.1 + 1.7 = 4.6 \text{ cm}$ $\leftarrow$ Add all the side lengths together.

or

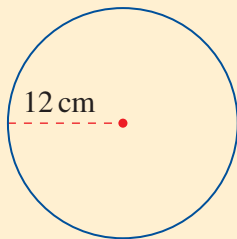
$2 \times 0.9 + 1.1 + 1.7 = 4.6 \text{ cm}$ $\leftarrow$ Alternatively, as there are 2 equal side lengths, multiply the equal side length by 2 and then add the single side lengths.



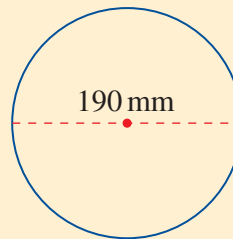
Example 5 Calculating the circumference of a circle

Calculate the circumference of the following circles using the appropriate formula. Round your answers to 2 decimal places.

a



b



WORKING

THINKING

a $C = 2\pi r$ $\leftarrow$ Given the radius so use $C = 2\pi r$ formula.

$C = 2 \times \pi \times 12$ $\leftarrow$ Substitute $r = 12 \text{ cm}$.

$C = 75.398\,223 \dots$ $\leftarrow$ Round your answer to 2 decimal places by looking at the 3rd decimal place. Round up as the 3rd number is greater than 5.
 $C \approx 75.40 \text{ cm}$

b $C = \pi D$ $\leftarrow$ Given the diameter so use $C = \pi D$ formula.

$C = \pi \times 190$ $\leftarrow$ Substitute $D = 190 \text{ mm}$.

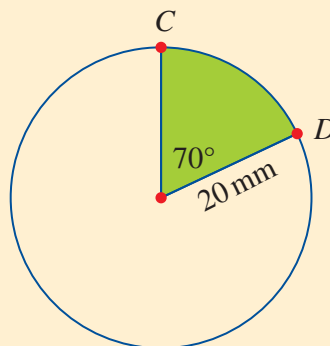
$C = 596.902\,604 \dots$ $\leftarrow$ Round your answer to 2 decimal places by looking at the 3rd decimal place. This number is less than 5 so we round down, which means we can keep the 2nd decimal place the same.
 $C \approx 596.90 \text{ mm}$



Example 6 Calculating the arc length and perimeter of a sector

Use the appropriate formula to calculate:

- a** the length of the arc CD , rounding your answer to 2 decimal places
- b** the perimeter of the sector



WORKING

THINKING

a $l = \frac{\theta}{360} \times 2 \times \pi \times r$ Use the formula $l = \frac{\theta}{360} \times 2\pi r$

$l = \frac{70}{360} \times 2 \times \pi \times 20$ Substitute $\theta = 70^\circ$ and $r = 20$ mm.

$l = 24.434609$ Round your answer to 2 decimal places by

$l = 24.43$ mm looking at the 3rd decimal place. This number

is less than 5 so we round down, which means we can keep the 2nd decimal place the same.

b Arc length = 24.43 mm Use the arc length found in part **a**.

$P = 24.43 + 2 \times 20$

$P = 64.43$ mm

The sides lengths of the sector are the radius of 20 mm. There are two side lengths so multiply by 2 and add to the length of the arc.



Example 7 Applying perimeter to practical problems

Griffin has 30 m of flexible fencing. He would like to use all of the fencing to make the largest circular pen for his pigs. Calculate the diameter of the largest pen that Griffin can make using all of the fencing. Round your answer to 2 decimal places.



WORKING

THINKING

$$C = \pi \times D \quad \leftarrow \text{Use the formula } C = \pi \times D$$

$$30 = \pi \times D \quad \leftarrow \text{Substitute into the formula the known measurements; that is, the circumference of 30 m.}$$

$$D = \frac{30}{\pi} \quad \leftarrow \text{To find the diameter, rearrange the equation by dividing the circumference by } \pi.$$

$$D = 9.549\,296 \quad \leftarrow \text{Round your answer to 2 decimal places.}$$

$$D = 9.55 \text{ m}$$

The diameter of the pen $\leftarrow$ Communicate your answer in a sentence.
will be about 9.55 m.

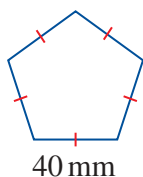
Exercise 1B

FUNDAMENTALS

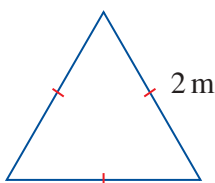
Example 4

1 Calculate the perimeter of the following shapes.

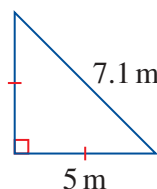
a



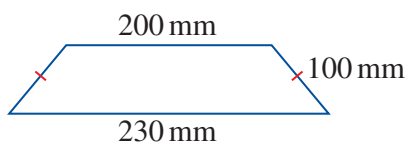
b



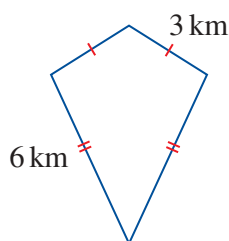
c



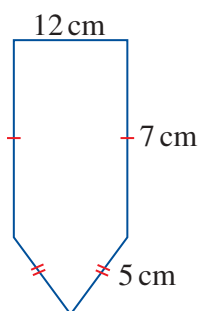
d



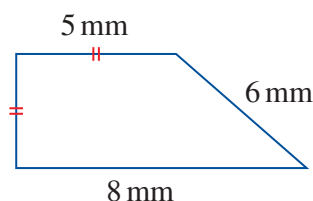
e



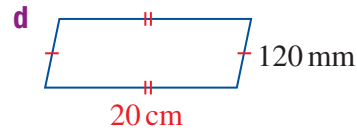
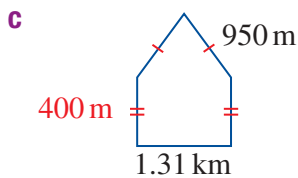
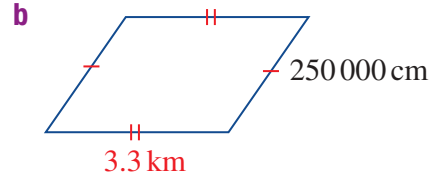
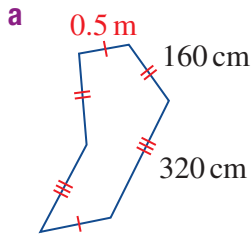
f



g



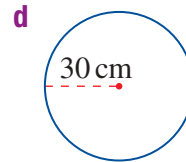
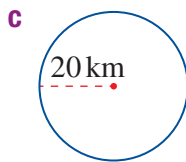
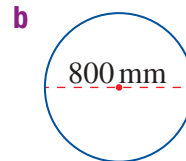
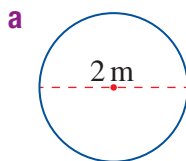
- 2 Calculate the perimeter for the following shapes using the units indicated in red.



Example 5

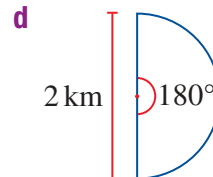
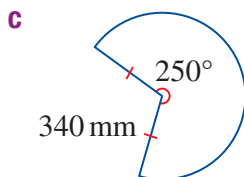
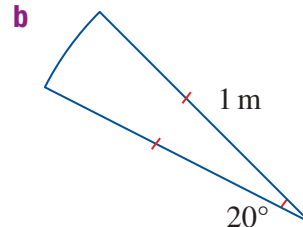
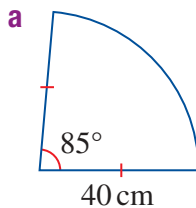
- 3 Calculate the circumference of the following circles. Round your answer to 2 decimal places.

Hint Use $C = 2\pi r$ or $C = \pi D$



Example 6

- 4 Calculate the arc length of the following sectors. Round your answer to 2 decimal places.



- 5 Calculate the perimeter for each of the sectors in question 4.

APPLICATIONS

For the following questions write your answers correct to 2 decimal places, where appropriate.

Example 7

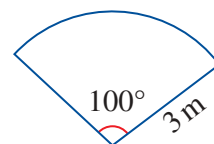
6 Quinn is baking her mother a cake for her birthday. Quinn uses a circular baking dish with a 20 cm diameter and plans on decorating the cake by wrapping a ribbon around it. Determine how much ribbon Quinn will require for the cake.

7 Lyn lives on a large property and has a new dog that has a reputation for running away. She has decided that she will put in an electric dog fence along the perimeter of her property. The shape of the property is a sector. It has a radius of 80 metres and an angle at the centre of 120° . Determine how many metres of electric dog fencing Lyn will need to purchase.

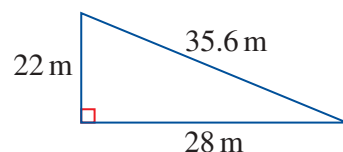
8 Bethany is planning on upcycling her old lampshade. It has a circular base that has a 38 cm diameter. She wishes to wrap a length of beading along the edge of the base. Determine the length of beading Bethany should purchase.

9 Josh is renovating his bathroom and has purchased a circular mirror. The measurement on the box states that the mirror has a circumference of 4 m. He would like to determine if the mirror will fit on a particular wall in the bathroom. Calculate the diameter of the mirror.

10 Stacey is installing a concrete pool in the shape of a sector at her new house. She needs to calculate the perimeter of the pool to work out how many pavers she will need to order. Use the image shown to calculate the perimeter of Stacey's pool.



11 Jarrah is building a fence for one of his yards to help keep his stock away from the road. The yard is triangular, as per the image shown. The fence design that he is planning has three railings of timber.



a Calculate the perimeter of the yard.

b Determine how many metres of timber that Jarrah requires to construct the fence.

12 Jana has made a circular cake with a radius of 15 cm for her daughter's fifth birthday party. After slicing the cake and serving it to the guests, she notices that there is $\frac{1}{4}$ (90°) of the cake remaining. She divides the cake four ways between her husband, daughter, son and herself. Calculate the arc length of each of these four slices.



1C Calculating perimeters of familiar composite shapes

LEARNING GOALS

- Identify composite shapes
- Calculate perimeters of composite shapes

Why is it essential to be able to calculate perimeters of composite shapes?

Often in life, shapes are more intricate in detail and sometimes form a composite shape. They are made by combining two or more familiar shapes. For example, the shape of a house plan can be L-shaped and is made up of two rectangles. The face of a building is another example as it may be a triangle and a rectangle combined.



In a quilt, simple shapes can be sewn together to make more complex composite shapes.

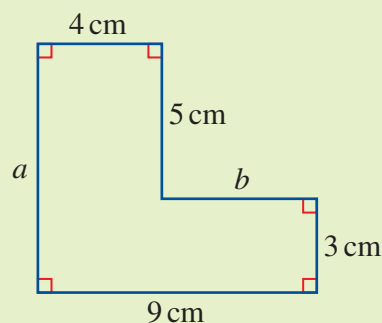
WHAT YOU NEED TO KNOW

- A **composite shape** is a shape that is made up of two or more basic shapes.
- Some composite shapes do not mark the length of every side as it is possible to calculate these lengths using either addition or subtraction.

- For example:

The length of the side marked a in the diagram is equal to the sum of the lengths of the opposite sides. As $5 + 3 = 8$ cm, $a = 8$ cm.

The side marked b is equal to the side length 9 cm minus the side length 4 cm. In length is equal. As $9 - 4 = 5$ cm, $b = 5$ cm.

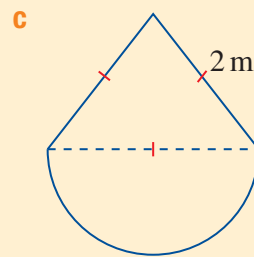
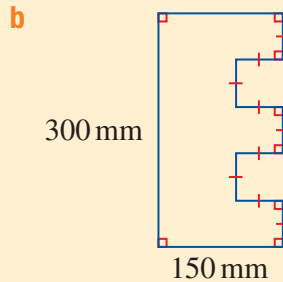
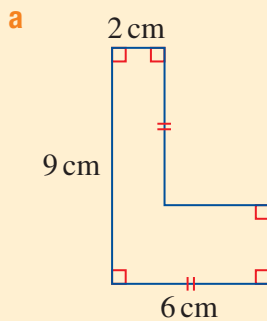


The perimeter of the composite shape is $8 + 4 + 5 + 5 + 3 + 9 = 34$ cm.



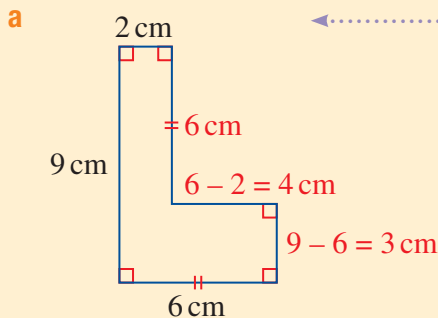
Example 8 Calculating the perimeter of composite shapes

Calculate the perimeter of the following composite shapes.



WORKING

THINKING

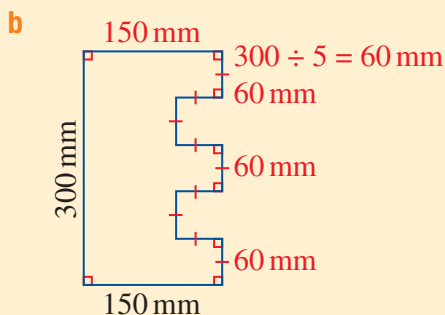


Two side lengths that are unmarked. These lengths are calculated by subtracting the opposite side lengths of the shape.

$$P = 2 + 6 + 4 + 3 + 6 + 9$$

$$P = 30 \text{ cm}$$

Add all the side lengths together to calculate the total perimeter.

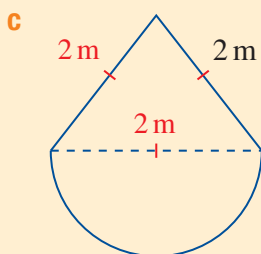


There are 5 small lengths that equal the total of the opposite side length. Divide 300 by 5 to calculate the length of one of these small lengths. There are 9 of these small lengths in total so multiply the answer by 9.

$$P = (9 \times 60) + (2 \times 150) + 300$$

$$P = 1140 \text{ mm}$$

Add the lengths of the remaining longer sides. Although not marked, we know that there are two side lengths of 150 mm because all the angles are right angled.



$$D = 2$$

$$\text{Length of arc} = \left(\frac{\pi \times 2}{2} \right) \\ \approx 3.14 \text{ m}$$

$$P = 3.14 + 2 + 2$$

$$P = 7.14 \text{ m}$$

NOTE: this shape almost looks like a sector, but it is not because the centre of the semi-circle is on the dotted line not at the point formed by the straight sides.

The diameter of the semi-circle is 2 m. As a semi-circle is half of a circle, calculate the circumference of the circle and then dividing by 2.

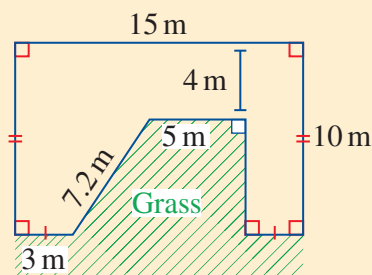
Add the remaining two sides of the triangle to calculate the total perimeter for the composite shape.



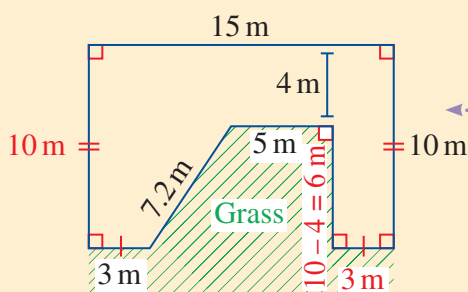
Example 9 Applying perimeter of composite shapes to practical problems

Noah has designed a garden bed for his new property, as per the image shown. He would like to place timber edging along the entire perimeter to hold the soil in place. Calculate how much timber Noah requires for his garden bed.

NOTE: The dimensions are measured around the 'outside' of the garden bed after the timber has been added. So you do not have to consider the thickness of the timber.



WORKING



$$P = 15 + 10 + 3 + 6 + 5 + 7.2 + 3 + 10$$

$$P = 59.2 \text{ m}$$

Noah has used 59.2 metres of timber to build his garden bed.

THINKING

There are two sides that are marked as 3 cm.

There are two sides that are marked as 10 cm.

There is only one length that is not given. We can calculate this length by subtracting 4 m from the opposite side length of 10 m. This tells us that the unmarked length is 6 m.

Add all the side lengths together to calculate the total perimeter.

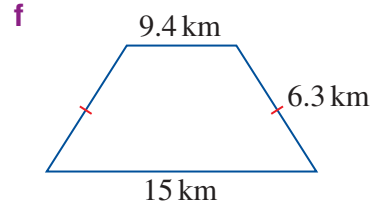
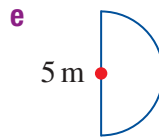
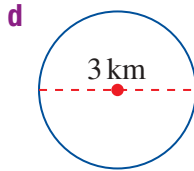
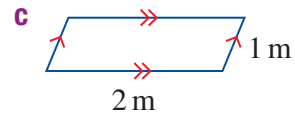
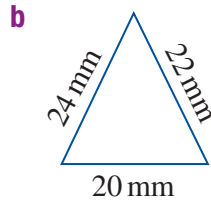
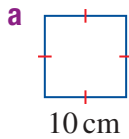
Communicate your solution in a sentence.

Exercise 1C

FUNDAMENTALS

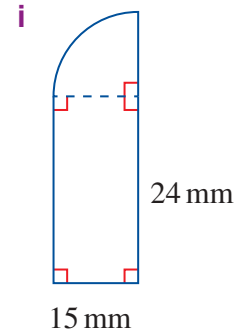
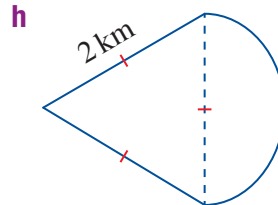
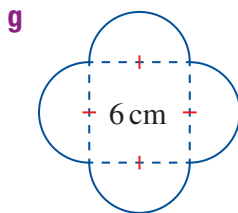
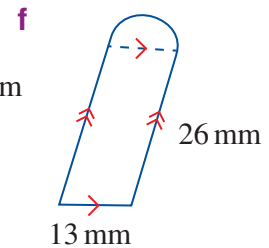
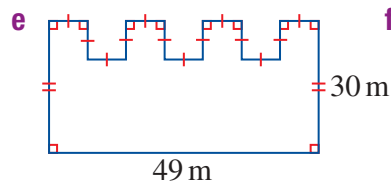
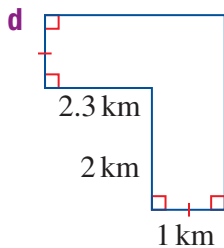
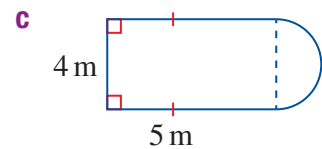
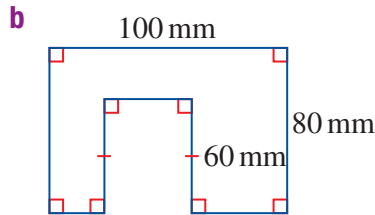
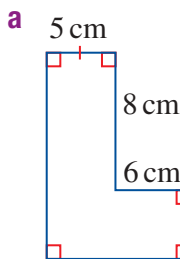
Where appropriate, write your answers to 2 decimal places.

- 1 Calculate the perimeter or circumference of the following common shapes.



Example 8

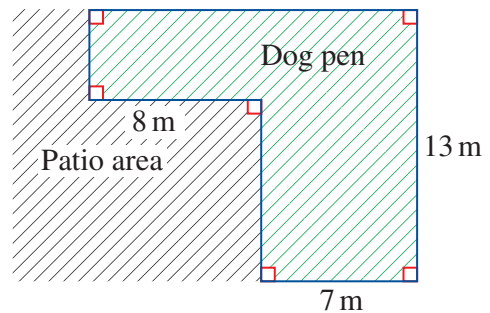
- 2 Calculate the perimeter for the following composite shapes.



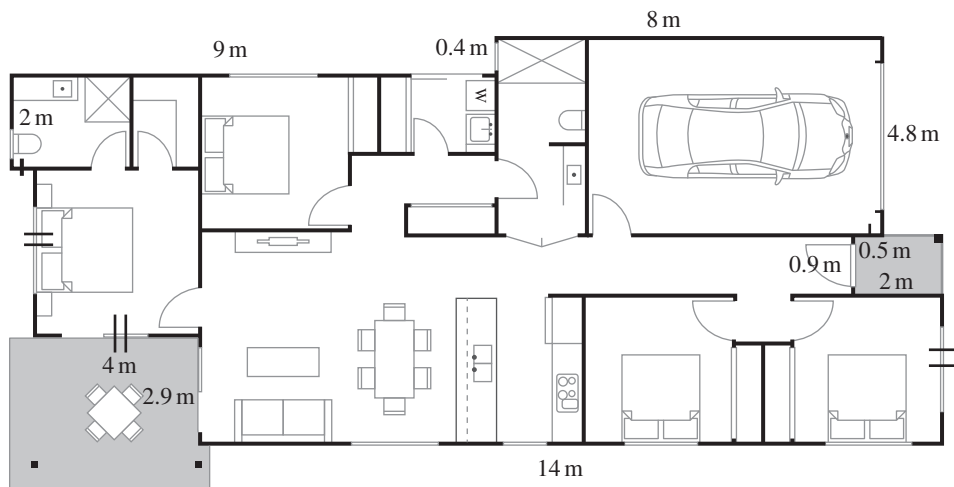
APPLICATIONS

Example 9

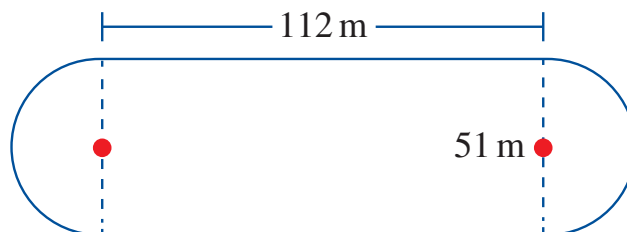
- 3 Russell needs to fence a large patch of grass area next to his patio to create a dog pen with dimensions shown in the diagram. Calculate the total length of fencing that Russell needs to purchase.



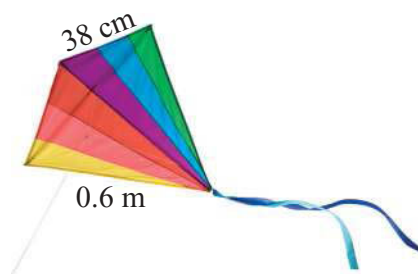
- 4 Valerie is renovating her house and she would like to purchase some paint to change the colour of the outside of her house. Her local hardware store has asked her for the perimeter and height of the external walls of her house. She knows the external walls are 3 metres high. Valerie is using her house plan to work out the perimeter. Calculate the perimeter of Valerie's house using the image shown.



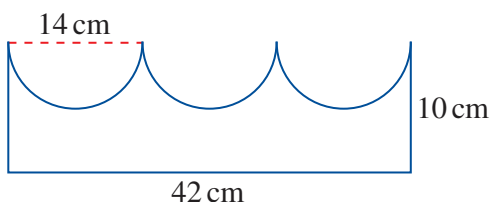
- 5 Mason is building a speed skating rink and he is required to place a railing around the outside of the rink for spectators. Use the diagram shown to calculate the length of the railing that will be required.



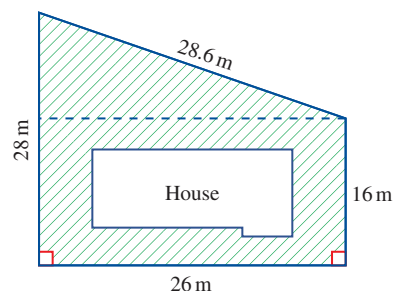
- 6 Mathew has cut and sewn a pattern for a kite in the shape of the image shown. He now needs to buy dowel (wooden rod) to insert along the edges of the kite before he can fly it. Calculate how many metres of dowel Mathew must purchase.



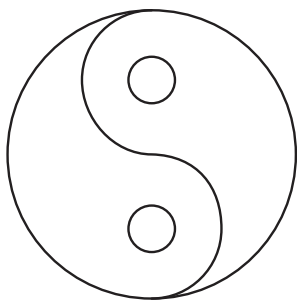
- 7 Anaya is making 8 crowns for her daughter's fifth birthday party. She plans to cut the crowns out of cardboard and stick decorative washi tape on the top and bottom of the crown to provide strength. The image shows an outline for the crown. Calculate how many centimetres of washi tape Anaya will need to purchase to complete the eight crowns.



- 8 Shekila's property contains her house situated in the front rectangular section of the property as shown in the diagram. She wishes to enclose the back triangular section of her yard with fencing. Calculate the length of fencing that Shekila will require.



- 9 Paul has been commissioned to paint a large yin-yang symbol, similar to the image below. The symbol is to have a 2 m diameter and the small dots are to have a 40 cm diameter. Paul needs to purchase black paint for the outline of the symbol. Calculate the perimeter of the black line so that Paul can purchase enough paint to complete the artwork.



Hint The inside swirl line can be calculated using two semi-circles.

1D Estimating and calculating areas of parallelograms and circles

LEARNING GOALS

- Calculate the area of various familiar shapes including:
 - parallelograms
 - circles
 - semi-circles
- Estimate the area of shapes

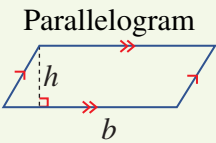
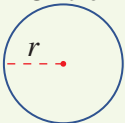
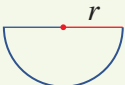
Why is it essential to understand how to calculate the area of a shape?

There are many real-life reasons where you would need to calculate the area of various shapes. This skill is used in careers such as fashion, construction, landscaping and painting. Calculating the area allows a landscaper to find the amount of turf required or a painter to work out how much paint is needed. It is also a good way to compare the sizes of properties when you are looking at buying or leasing (renting) some land.



WHAT YOU NEED TO KNOW

- The **formulas** for the areas of some common shapes are given in the table below.

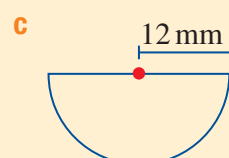
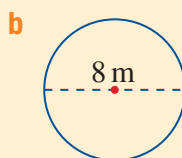
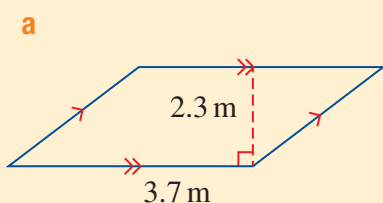
Shape	Area formula
 Parallelogram	$A = b \times h$
 Circle	$A = \pi r^2$ or $\pi \times r \times r$ * Note if given diameter then divide by 2 to calculate the radius.
 Semi-circle	$A = \frac{1}{2} \pi r^2$ or $\frac{1}{2} \times \pi \times r \times r$

- Abbreviations: A area, b breadth or base, h height, r radius, π (pi) = 3.142 approximately.
- Units of area include square millimetres, square centimetres, square metres, square kilometres and hectares, these can be abbreviated to mm^2 , cm^2 , m^2 , km^2 , ha.



Example 10 Calculating the area of a common 2D shape

Calculate the area of the following common 2D shapes.



Round your answer to 2 decimal places.

WORKING

THINKING

- a** $A = bh$ ← Calculate the area of the parallelogram using $A = bh$.
 $A = b \times h$
 $A = 3.7 \times 2.3$ ← Substitute the values for b and h .
 $A = 8.51 \text{ m}^2$

... Continued

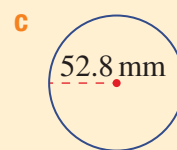
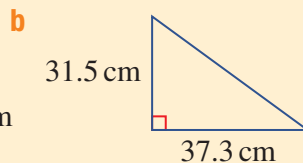
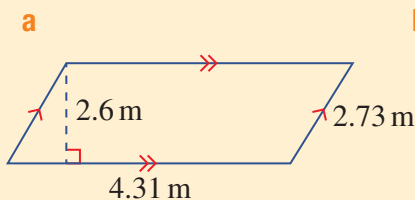
b $r = 8 \div 2 = 4$ ←..... Calculate the radius by dividing the diameter in half.
 $A = \pi r^2$ Calculate the area of the circle using $A = \pi r^2$.
 $A = \pi \times r^2$
 $A = \pi \times 4^2$ Substitute the values for r .
 $A = 50.27 \text{ m}^2$ Round your answer to 2 decimal places.

c $A = \frac{1}{2}\pi r^2$ ←..... Calculate the area of a semi-circle using $A = \frac{1}{2}\pi r^2$.
 $A = \frac{1}{2} \times \pi \times r \times r$
 $A = \frac{1}{2} \times \pi \times 12 \times 12$ Substitute the values of r .
 $A = \frac{1}{2} \times \pi \times 12^2$
 $A = 226.19 \text{ mm}^2$ Round your answer to 2 decimal places.



Example 11 Estimating the area of common 2D shapes

Estimate the area of the following common 2D shapes by rounding the dimensions to the nearest whole number.



Round your answer to 2 decimal places.

WORKING

a $2.6 \text{ m} \approx 3 \text{ m}$ ←.....
 $4.31 \text{ m} \approx 4 \text{ m}$

THINKING

Begin by rounding your dimensions to the nearest whole number. Look at the first decimal place, if it is 0.5 or more, then round your whole number up, if it is less than 0.5, then drop the decimal place.
 Note: The perpendicular height of the parallelogram is used (i.e. 2.6 m from this shape).

$A = b \times h$ ←..... Calculate the area of the rectangle using $A = b \times h$.

$A = 4 \times 3$ ←..... Substitute the values for b and h using the whole numbers to estimate the area of the parallelogram.
 $A = 12 \text{ m}^2$

- b** $37.3 \approx 37$ cm $\leftarrow \dots\dots\dots$ Begin by rounding your dimensions to the nearest whole number.
 $31.5 \approx 32$ cm

$$A = \frac{1}{2} (b \times h) \quad \text{Calculate the area of the triangle using } A = \frac{1}{2} (b \times h).$$

$$A = \frac{1}{2} (37 \times 32) \quad \text{Substitute the rounded values for } b \text{ and } h.$$

$$A = 592 \text{ cm}^2$$

- c** $52.8 \approx 53$ mm $\leftarrow \dots\dots\dots$ Begin by rounding the value to the nearest whole number.

$$A = \pi \times r^2 \quad \text{Calculate the area of the circle using } A = \pi \times r^2.$$

$$A = \pi \times 53^2 \quad \text{Substitute the value for } r.$$

$$A = 8824.73 \text{ mm}^2 \quad \text{Round your answer to 2 decimal places.}$$



Example 12 Applying the area of a shape to practical problems

Michelle is painting the top surface of a circular stage that has a diameter of 2.4 m. Calculate the area that Michelle will be painting.

WORKING

THINKING

$2.4 \div 2 = 1.2$ m $\leftarrow \dots\dots\dots$ Divide the diameter by 2 to calculate the radius of the stage.

$A = \pi \times r^2$ $\leftarrow \dots\dots\dots$ Given that the surface is circular, calculate the area using $A = \pi \times r^2$.

$A = \pi \times 1.2^2$ $\leftarrow \dots\dots\dots$ Substitute the value for r .
 $A = 4.52 \text{ m}^2$

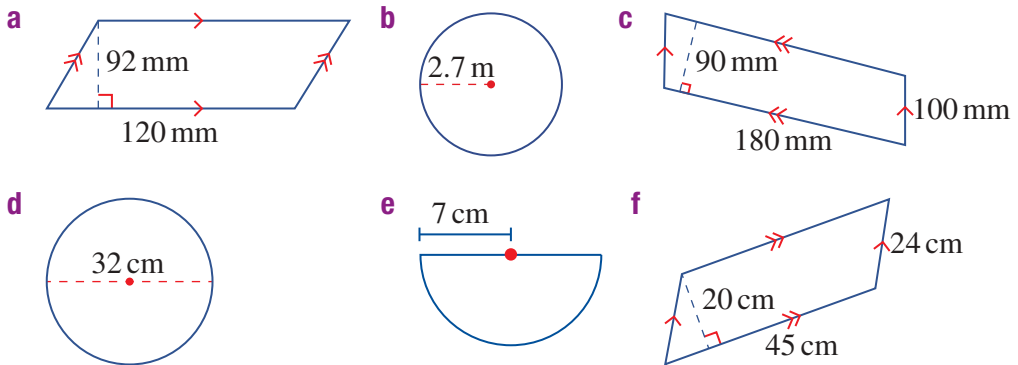
Michelle will be painting $\leftarrow \dots\dots$ an area that is 4.52 m^2 . Communicate your answer in a sentence.



Exercise 1D

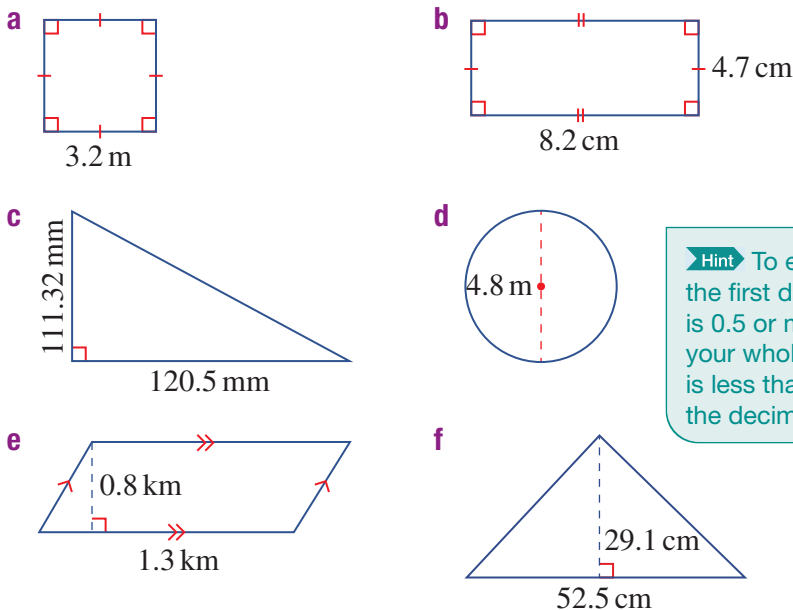
FUNDAMENTALS

- Example 10** 1 Calculate the area of the following common 2D shapes. Round to 2 decimal places where necessary.



Hint The height must be the perpendicular height for triangles and parallelograms.

- Example 11** 2 Estimate the area of the following common 2D shapes by rounding the dimensions to the nearest whole number.

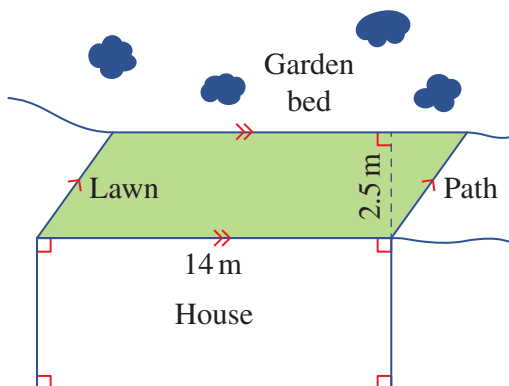


Hint To estimate look at the first decimal place. If it is 0.5 or more, then round your whole number up. If it is less than 0.5, then drop the decimal place.

APPLICATIONS

Example 12

- 3 Jarred is covering a display board with blue paper. The board is a parallelogram and it has a length of 3.5 m and perpendicular height of 1.5 m. Calculate the area that Jarred will be covering in blue paper.
- 4 Haley is tiling the floor of her circular bathroom, which has a radius of 2.8 metres. Calculate the area that Haley is tiling.
- 5 Ryan is laying an area of lawn that runs parallel between his house and the garden bed. The image below is the outline of the patch that he is turfing. Calculate the area of the new lawn.



- 6 Chelsea wants to paint the top and bottom of a semi-circular table top with a radius of 1.7 m. What area does she need to paint?
- 7 Nathan is constructing a circular sandpit for his children. The sandpit will have a radius of 900 mm. Calculate the area of the sandpit.
- 8 Dane has been commissioned to paint a mural on the side of a building. The mural will be a parallelogram and has a width of 5.7 m and a perpendicular height of 3 m. Calculate the area of the mural.
- 9 Graham has just purchased some parallelogram shaped land that is 220 m by 100 m. He knows that he can have 10 sheep per hectare, and he would like to have as many sheep as possible on his land.
- a Calculate the area of Graham's land in hectares.
- b Determine how many sheep Graham can keep on his new property.

Hint 1 hectare is 10 000 m².

1E Calculating the area of trapeziums, sectors and composite figures

LEARNING GOALS

- Apply the formula to calculate the area of trapeziums and sectors
- Apply knowledge of regular shapes to decompose composite shapes
- Calculate the area of composite shapes

Why is it essential to understand how to calculate the area of trapeziums, sectors and composite shapes?

In the previous chapter, there are many real-life reasons where you would need to calculate the area of these shapes. Trapeziums and sectors can be found in construction, painting, town planning, fashion and even cooking. The majority of shapes around the home are actually composite shapes.

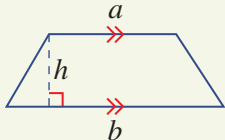
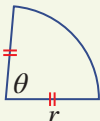


Understanding area and composite shapes can help with clothing design and manufacture.

WHAT YOU NEED TO KNOW

- A **trapezium** has four sides, two of which are parallel.
- A **sector** is a portion of a circle bounded by two **radii** and a segment of the **circumference**. The two radii form a **vertex** at the centre of a circle. The size of the sector can be specified by the length of the radii and the angle between the two radii at the vertex.

- The formulas for the areas of a sector and a trapezium are in the table below.

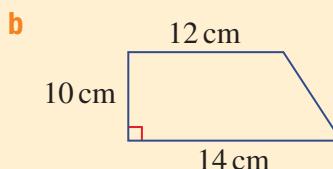
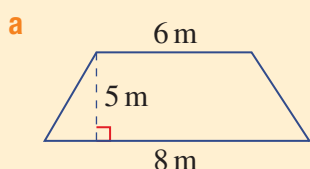
Shape	Area formula
Trapezium 	$A = \frac{1}{2} (a + b) \times h$ or $A = \frac{a + b}{2} \times h$
Sector 	$A = \frac{\theta}{360} \times \pi \times r^2$

- Abbreviations: a and b are lengths of sides, h height, r radius, π (pi) = 3.142 rounded, θ (theta) is unknown angle.
- Units of area include square millimetres, square centimetres, square metres, square kilometres and hectares. These can be abbreviated to mm^2 , cm^2 , m^2 , km^2 and ha.
- A **composite shape** is a shape that is made up of two or more common shapes. Addition and/or subtraction can be used to find the area of composite shapes.
- You will also need to recall the area formulas of the shapes from the previous section.



Example 13 Calculating the area of a trapezium

Calculate the area of the following trapeziums.



WORKING

a $A = \frac{1}{2} \times (a + b) \times h$

$$A = \frac{1}{2} \times (6 + 8) \times 5$$

$$A = 35 \text{ m}^2$$

b $A = \frac{1}{2} \times (a + b) \times h$

$$A = \frac{1}{2} \times (12 + 14) \times 10$$

$$A = 130 \text{ cm}^2$$

THINKING

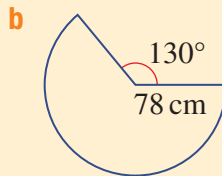
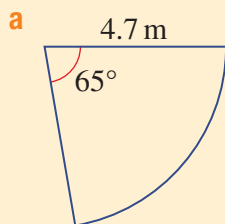
..... Calculate the area of the trapezium using $A = \frac{1}{2} \times (a + b) \times h$.
Substitute the values for a , b and h .

..... Calculate the area of the trapezium using $A = \frac{1}{2} \times (a + b) \times h$.
Substitute the values for a , b and h .



Example 14 Calculating the area of a sector

Calculate the area of the following sectors. Round your answer to 2 decimal places.



WORKING

THINKING

a $A = \frac{\theta}{360} \times \pi \times r^2$

←..... Calculate the area of the sector using
 $A = \frac{\theta}{360} \times \pi \times r^2$.

$A = \frac{65}{360} \times \pi \times 4.7^2$

←..... Substitute the values for the internal angle (θ) and the radius (r).

$A = 12.53 \text{ m}^2$

←..... Round your answer to 2 decimal places.

b $360^\circ - 130^\circ = 230^\circ$

←..... As the shape is a sector, calculate the internal angle first by subtracting the external angle from 360° .

$A = \frac{\theta}{360} \times \pi \times r^2$

Calculate the area of the sector using

$A = \frac{\theta}{360} \times \pi \times r^2$.

$A = \frac{230}{360} \times \pi \times 78^2$

Substitute the values for the internal angle (θ) and the radius (r).

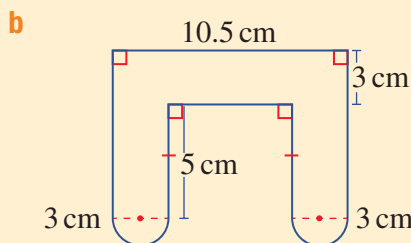
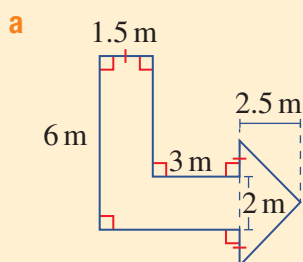
$A = 12\,211.37 \text{ cm}^2$

Round your answer to 2 decimal places.



Example 15 Calculating the area of composite shapes with addition

Calculate the area of the following composite shapes, correct to 2 decimal places if needed.

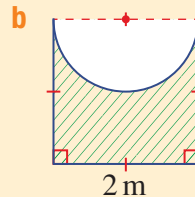
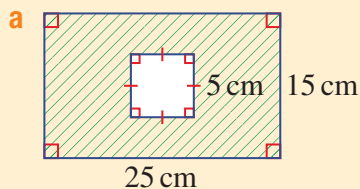


WORKING	THINKING
a There are 2 rectangles and one triangle. $A = l \times w$ ←..... ▢ $A = 6 \times 1.5 = 9 \text{ m}^2$ ←..... ▢ $A = 3 \times 2 = 6 \text{ m}^2$ ▶ $A = \frac{1}{2} \times b \times h$ ←..... $h = 2.5 \text{ m},$ ←... $b = 1.5 + 2 + 1.5 = 5 \text{ m}$ $A = \frac{1}{2} \times (5 \times 2.5) = 6.25 \text{ m}^2$ ←... $A = 9 + 6 + 6.25$ ←..... $A = 21.25 \text{ m}^2$	Decompose the composite shape by identifying the regular shapes. Calculate area of rectangles by using the formula. Substitute the values for l and w. Calculate area of triangle using the formula. The unlabelled parts of the base have tick marks indicating they are same as the side labelled 1.5 m. Substitute the values for b and h. Add all the shapes together to find the total area of the composite shape.
b There are 3 rectangles and two semi-circles. $A = l \times w$ ←..... ▢ $A = 10.5 \times 3 = 31.5 \text{ cm}^2$ ←... ▢ $A = 5 \times 3 = 15 \text{ cm}^2$ ▢ $A = 5 \times 3 = 15 \text{ cm}^2$ $r = 3 \div 2 = 1.5$ ←..... $A = \pi \times r^2$ ←..... ● $A = \pi \times 1.5^2$ ←..... $A \approx 7.07 \text{ cm}^2$ ←..... $A = 31.5 + 15 + 15 + 7.07$ ←... $A = 68.57 \text{ cm}^2$	Decompose the composite shape by identifying the regular shapes. Calculate area of all rectangles using the formula. Substitute the values for l and w. Both semi-circles have a diameter of 3 cm so they can make one whole circle when combined. The radius of the circle is half of the diameter. Calculate area of the circle using the formula. Substitute the value for r. Round your answer to 2 decimal places. Add all the shapes together to find the total area of the composite shape.



Example 16 Calculating the area of composite shapes with subtraction

For the following composite shapes, calculate the area of the shaded section, correct to 2 decimal places if needed.



WORKING

- a** There is one rectangle and one square that is cut out from the centre.

$$A = l \times w \quad \leftarrow \dots\dots\dots$$

$$\square A = 25 \times 15 = 375 \text{ cm}^2 \quad \leftarrow \dots\dots\dots$$

$$A = l \times l \quad \leftarrow \dots\dots\dots$$

$$\square A = 5 \times 5 = 25 \text{ cm}^2 \quad \leftarrow \dots\dots\dots$$

$$A = 375 - 25 = 350 \text{ cm}^2 \quad \leftarrow \dots\dots\dots$$

- b** There is one square and one semi-circle that is cut out from the top of the square.

$$A = l \times l \quad \leftarrow \dots\dots\dots$$

$$\square A = 2 \times 2 = 4 \text{ m}^2 \quad \leftarrow \dots\dots\dots$$

$$r = 2 \div 2 = 1 \text{ m} \quad \leftarrow \dots\dots\dots$$

$$A = \pi \times r^2 \quad \leftarrow \dots\dots\dots$$

$$A = \pi \times 1^2 = 3.14 \text{ m}^2 \quad \leftarrow \dots\dots\dots$$

THINKING

Decompose the composite shape by identifying the regular shapes.

Calculate the area of the rectangle using $A = l \times w$.

Substitute the values for l and w .

Calculate the area of the square using $A = l \times l$.

Substitute the value for l .

Subtract the square from the rectangle to find the shaded area.

Decompose the composite shape by identifying the regular shapes.

Calculate the area of the square using $A = l \times l$.

Substitute the value for l .

Find the radius by halving the diameter (divide by 2).

Calculate the area of the circle using $A = \pi \times r^2$.

Substitute the value for r .

WORKING

THINKING

 Area of semi-circle $\leftarrow \dots \dots \dots$ Divide the area of the circle by two, as the shape is half of a circle (semi-circle).

$$A = 3.14 \div 2$$

$$A = 1.57 \text{ m}^2$$

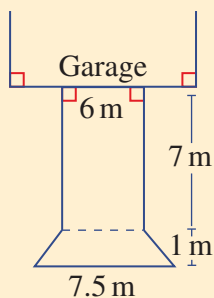
$A = 4 - 1.57$ $\leftarrow \dots \dots \dots$ Subtract the semi-circle from the square to find the shaded area.

$$A = 2.43 \text{ m}^2$$



Example 17 Applying the area of composite shapes to practical problems

A driveway, as shown in the diagram, is to be concreted. Calculate the area of the driveway, correct to 2 decimal places.



WORKING

THINKING

There is one rectangle and one trapezium.

Decompose the composite shape by identifying the regular shapes.

$$A = l \times w$$

$\leftarrow \dots \dots \dots$ Calculate the area of the rectangle using $A = l \times w$.

 $A = 7 \times 6 = 42 \text{ m}^2$

$\leftarrow \dots \dots \dots$ Substitute the values l and w .

$$A = \frac{1}{2} \times (a + b) \times h$$

$\leftarrow \dots \dots \dots$ Calculate the area of the trapezium using $A = \frac{1}{2} \times (a + b) \times h$.

 $A = \frac{1}{2} \times (6 + 7.5) \times 1 = 6.75 \text{ m}^2$

$\leftarrow \dots \dots \dots$ Substitute the values for a , b and h .

$$A = 42 + 6.75$$

$$A = 48.75 \text{ m}^2$$

$\leftarrow \dots \dots \dots$ Add the two areas together to calculate the total area of the composite shape.

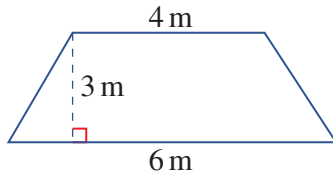
The area of the driveway is $\leftarrow \dots \dots \dots$ Communicate your answer using words.

Exercise 1E

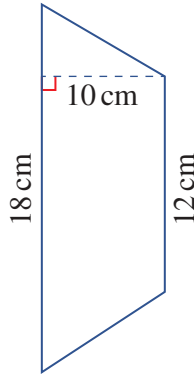
FUNDAMENTALS

Example 13 1 Calculate the area of the following trapeziums.

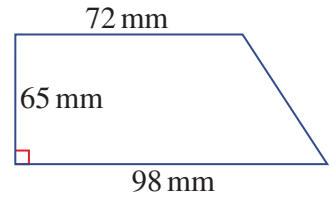
a



b



c

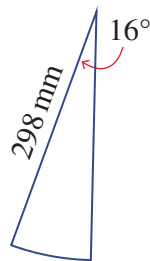


Example 14 2 Calculate the area of the following sectors.

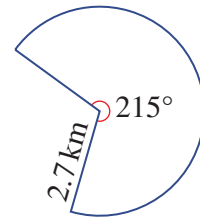
a



b

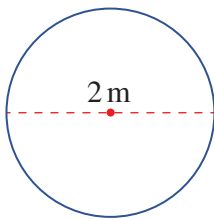


c

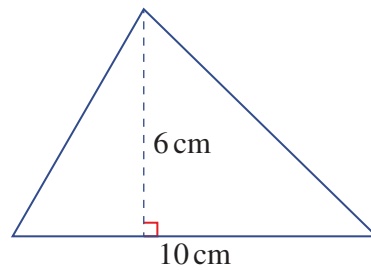


3 Calculate the area of the following common 2D shapes.

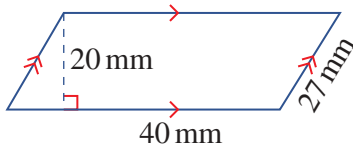
a



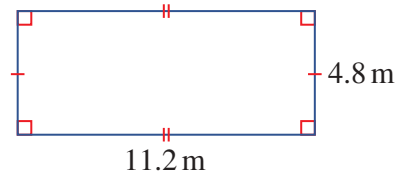
b



c

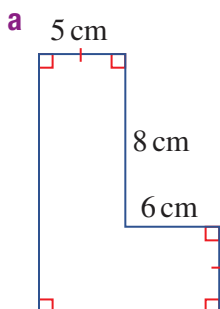


d

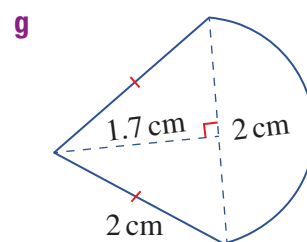
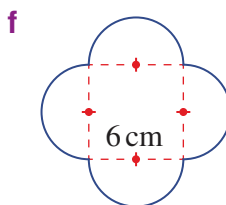
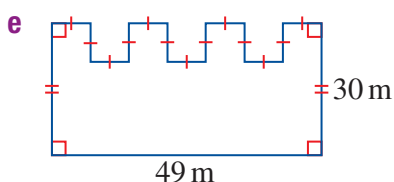
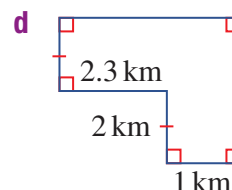
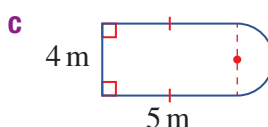
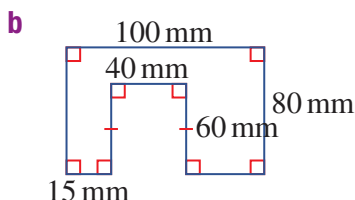


Example 15

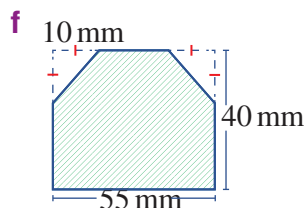
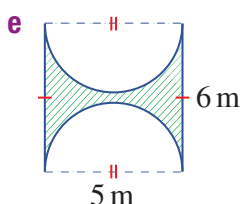
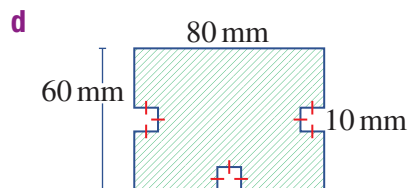
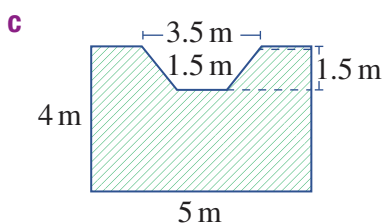
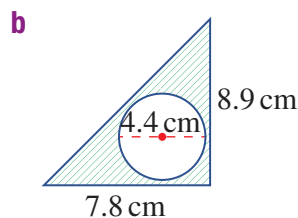
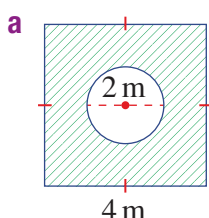
- 4 Determine the area of the following composite shapes by first decomposing the shapes into regular shapes and then adding the areas of the shapes.



Hint Redraw the shapes into the common shapes to help you to calculate the total area.


Example 16

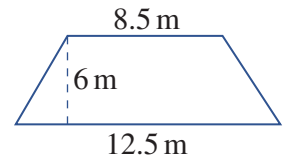
- 5 Determine the area of the shaded section in the following composite shapes by first decomposing the shape into regular shapes and then subtracting the sections that are not shaded. All angles that look like right angles are 90° .



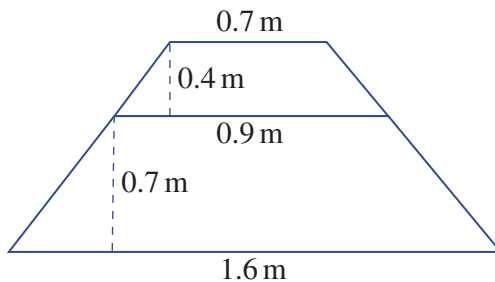
APPLICATIONS

Example 17

- 6** Ben is planning on making a new shade sail to run from the roof line of his house, across his deck and on to some posts that are near his pool. The design of the sail is shown right. Calculate the area of the material that Ben needs for the sail.

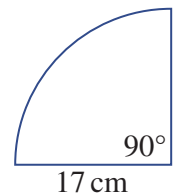


- 7** Hans has been commissioned to re-paint a fading sign of a pizza restaurant. He is calculating how much red paint he will require for the section of the sign shown. He has discovered that the sign makes two trapeziums and has measured the dimensions shown. Calculate the total area that will require red paint.

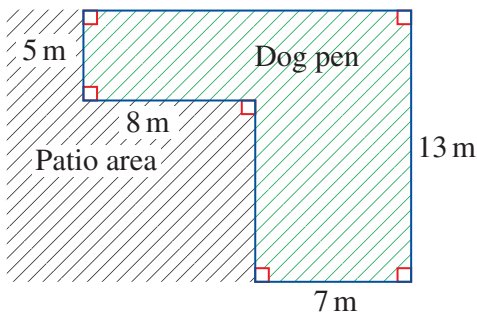


- 8** Leonie has ordered a pizza that has a diameter of 34 cm. She has eaten a quarter of the pizza.

- a** Use the diagram to calculate the area of pizza that Leonie has consumed.
- b** Determine the area of the pizza that will be left over for Leonie's friends to consume.

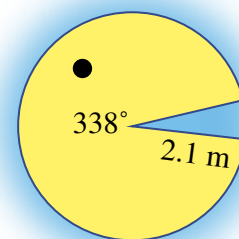


- 9** Ruby is laying a large patch of grass area next to her patio, as shown in the diagram, to create a dog pen. Calculate the total area of turf that Ruby needs to purchase.

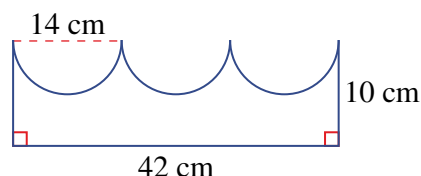


- 10** Randal has been commissioned to paint a large logo on a billboard. He needs to buy the correct amount of yellow paint that covers the area of the logo in the image below. Randal will need to multiply the total area by two as he will need to complete two coats of paint on the logo. If 1 litre of paint covers 15 m^2 , determine how many tins of paint Randal will need.

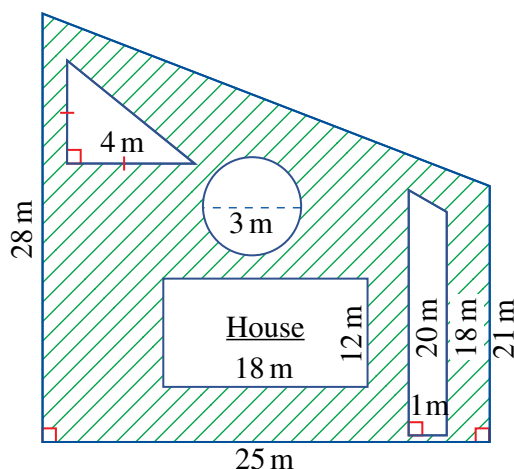
Note: Randal will start with yellow paint and will paint the eye of the icon over the top.



- 11** Anaya is making 8 crowns for her daughter's 5th birthday party. She plans on cutting the crowns out of cardboard and covering the outside surface in glitter. The image shown is an outline for the crowns, calculate the total area that Anaya will need to cover.



- 12** Below is an image of Shekila's property. She has outlined her house, water feature and two garden beds. Shekila would like to lay lawn everywhere else on her property. Calculate the total area of Shekila's lawn.



1F Investigating nets of 3D solids

LEARNING GOAL

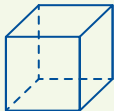
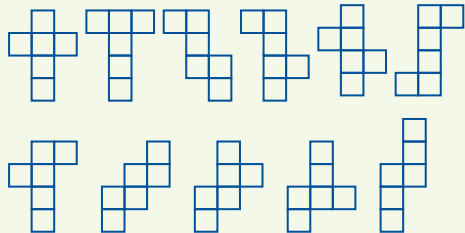
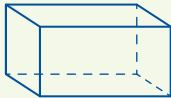
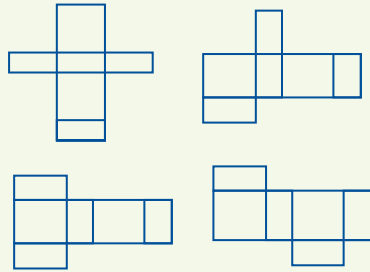
- Interpret various 2D representations of 3D solids in the form of a net, including:
 - cubes
 - rectangular prisms
 - triangular prisms

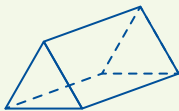
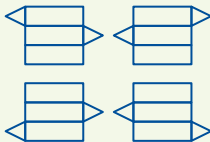
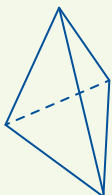
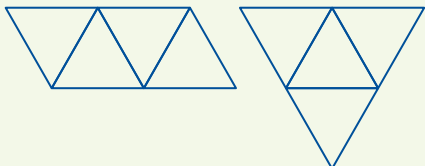
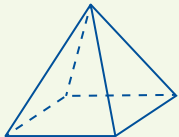
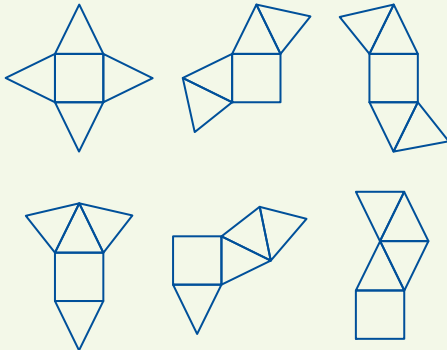
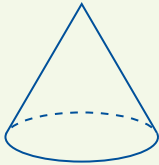
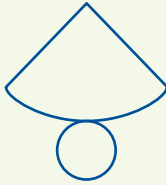
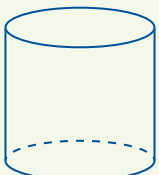
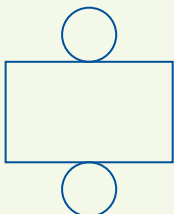
Why is it essential to be able to interpret 2D representations of 3D solids?

Creating a 2D form of a 3D solid helps us to communicate the exact dimensions and shapes required to create the solid. The concept of designing and creating an object in a solid state requires good communication to the manufacturer, and the best way for this to take place is in the form of a 2D plan. It particularly applies to careers in architecture, manufacturing and design. The 2D design of a 3D solid is known as a net.

WHAT YOU NEED TO KNOW

- A **net** is a flat shape (2D) that can be folded up into a 3D solid. A 3D solid can have more than one type of net.

3D solid	3D image	2D net representations
Cube		
Rectangular prism		

3D solid	3D image	2D net representations
Triangular prism		
Triangular-based pyramid		
Square-based pyramid		
Cone		
Cylinder		

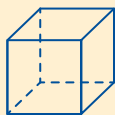




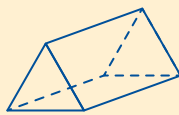
Example 18 Constructing a 2D net of a 3D solid

Construct a net that represents the following 3D solids.

a

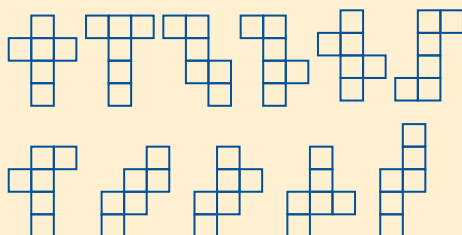


b

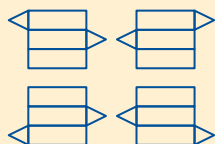


WORKING

a



b



THINKING

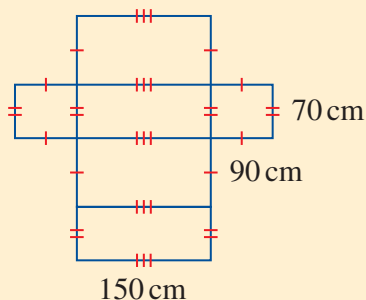
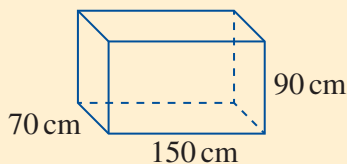
There are many possible solutions to the design of both nets, all of which are accurate. Check that you have the correct number of faces, edges and vertices to form the solid.



Example 19 Creating a 2D net from a 3D object

Kenji has purchased a rectangular television cabinet that is 90 cm high, 70 cm deep and 150 cm long. Construct the net of this cabinet, showing the measurements.

WORKING



THINKING

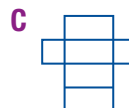
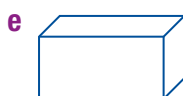
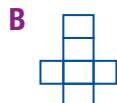
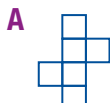
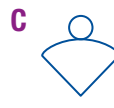
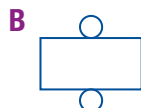
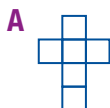
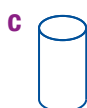
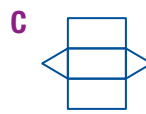
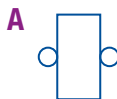
First draw a sketch of the 3D object with the dimensions.

Draw the net of a rectangular-based prism. The net includes 2 rectangles with length 150 cm and width 90 cm. The net includes 2 rectangles with length 150 cm and width 70 cm. The net also includes 2 rectangles with a length of 90 cm and a width of 70 cm.

Exercise 1F

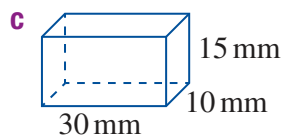
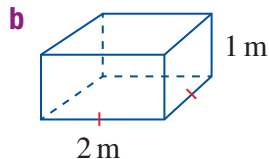
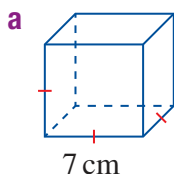
FUNDAMENTALS

1 Determine the correct net (A, B or C) for each of the following solids.

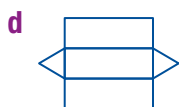
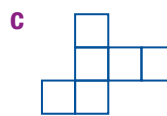
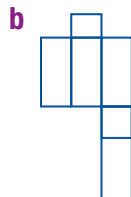
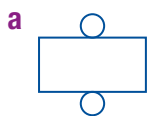


Example 18

2 Draw two different nets for each of the following 3D solids, ensuring that you include the measurements.



3 Match the following nets that are most similar to each of the real-life items in Exercise 1A Q13 (there may be more than one real-life object per net).



APPLICATIONS

Example 19

- 4** Sally has purchased a new fridge and it was delivered in a large box. Sally has cut the edges and flattened the box to form a net. Construct a diagram to identify what the net would look like. Ignore the extra flaps that cardboard boxes usually have.
- 5** Ahmed is planning to construct a large cube as part of a sculpture that he is creating as a display for the front of a hotel. He will be making the cube out of sheet metal. In order to make the cube, he must first cut out the net of the cube. He will then fold the sheet metal into a cube and weld it together.
- a** Design and draw a net of the cube.
- b** Identify how many folds Ahmed will need to make in order to construct the cube.
- 6** Josh has just purchased a storage box for his son's bedroom. The box has come in a flat pack. The first step on the assembly instructions is to lay out the timber pieces in the form of a net. Draw the net design, including the measurements of the box.



1G Calculating the surface areas of cubes, prisms and pyramids

LEARNING GOALS

- Draw 2D representations in the form of a net, to show all faces of a 3D solid
- Calculate the surface area of cubes, rectangular and triangular prisms
- Calculate the surface area of triangular-based pyramids
- Calculate the surface area of square-based and rectangular-based pyramids

Why is it essential to be able to calculate the surface area of 3D solids?

Calculating the surface area of 3D objects allows us to calculate how much material is needed to construct the surface of the object or to paint, cover or finish it. Applying yourself to learning this skill will also help you to develop your spatial awareness.

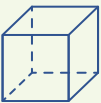
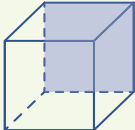
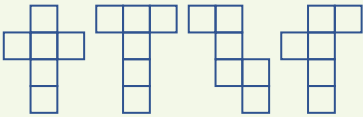
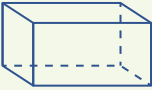
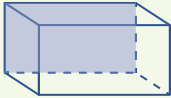
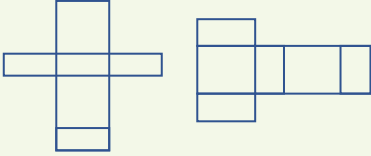
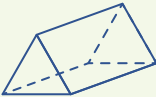
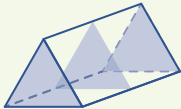
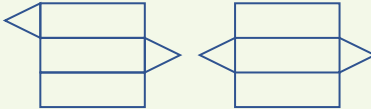


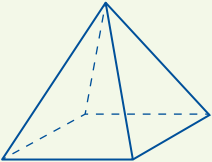
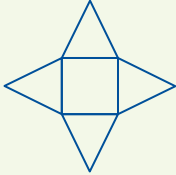
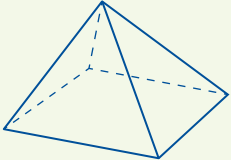
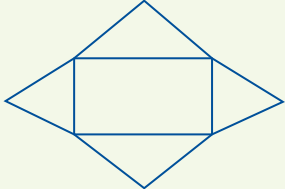
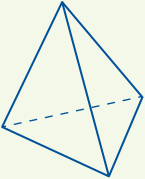
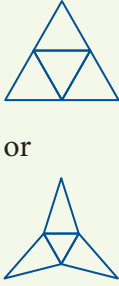
The amount of paint required to cover the surface area of a composite solid can be found by adding the surface areas of the visible regular shapes.

WHAT YOU NEED TO KNOW

- A **prism** is a 3D solid with flat faces, where each **cross-section** in a particular direction is the same shape and size. These cross-sections are shown in the middle column of the table below.
- A **pyramid** has a **polygon** for a base and all the other faces are triangular and meet at one **vertex** (a point). In this course, we only deal with pyramids that have a square, rectangular or triangular base. They are also **right pyramids** which means the vertex is vertically above the centre of the base. Also, the faces (other than the base) are of equal size in triangular and square-based right pyramids. Opposite faces are of equal size in rectangular-based right pyramids.
- When calculating the surface area of a prism or pyramid, start by drawing the **net** of the solid to identify all of its faces.

- A net is a flat shape (2D) made up of the faces that can be folded up into the 3D solid. To identify the net, imagine unfolding all of the faces of the 3D solid to make a flat shape. A prism will have different nets depending on which faces are left joined and which faces are separated.
- To calculate the surface area of a 3D solid, calculate the area of all of the **faces** (the 2D shapes) in the net and add them up.
- The formula for the area of a triangular face is $A = \frac{1}{2}(b \times h)$ where A is area, b is the length of the base, and h is the perpendicular height of the triangle.
- The formula for the area of a square face is $A = l^2$.
- The formula for the area of a rectangular face is $A = l \times w$.
- After drawing the net, you may find it helpful to group identical faces together and to draw each type, with dimensions. This is shown in Example 21 and 22.
- There are different types of triangular-based pyramids so be sure to use the correct base and height for each triangle when applying your formula.

Prism	Shaded cross-sections	Faces	2D net representation
Cube 	A square is the cross-section. 	6 square faces	
Rectangular prism 	A rectangle is the cross-section. 	6 rectangle faces	
Triangular prism 	A triangle is the cross-section. 	2 triangle faces that are the same size and shape 3 rectangle faces	

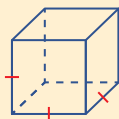
Pyramid	Faces	2D net representation
Square-based pyramid 	1 square face that is the base 4 triangle faces	
Rectangular-based pyramid 	1 rectangle face that is the base 4 triangle faces	
Triangular-based pyramid 	4 triangle faces	 or



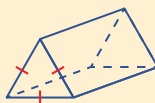
Example 20 Identifying and drawing the net of a solid

Draw a net to represent the following 3D solids.

a

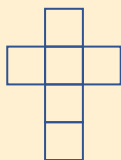


b

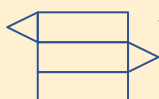


WORKING

a



b



THINKING

There are many possible solutions to the design of both nets, all of which are accurate. Check that you have the correct number of faces, edges and vertices to form the solid.

This 3D solid is a cube, so all six faces are squares.

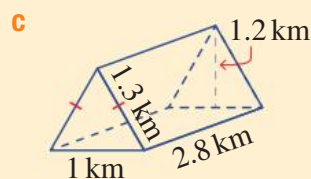
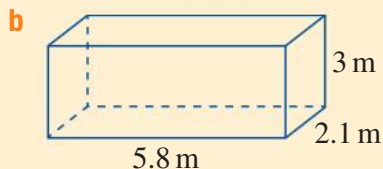
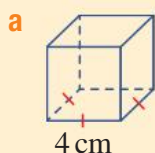
This 3D solid is a triangular prism, so there are 2 triangles that are the ends. The other 3 faces are rectangles.

Note: Other answers are possible.

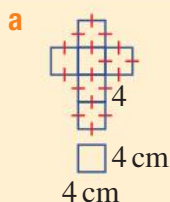


Example 21 Calculating the surface area of a cube, rectangular prism and triangular prism

Calculate the surface area of the following 3D solids.



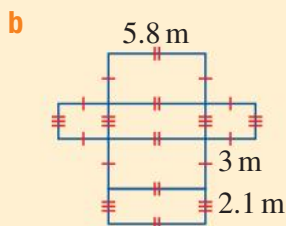
WORKING



$$A = l \times l$$

$$A = 4 \times 4 = 16 \text{ cm}^2$$

$$\begin{aligned} \text{Total surface area} &= 16 \times 6 \\ &= 96 \text{ cm}^2 \end{aligned}$$



A1

A2

A3

$$A = l \times w$$

THINKING

Draw a net of the cube, including the dimensions.

There are 6 square faces in a cube.

Calculate the area of one square using $A = l \times l$.

Substitute the value for l .

There are 6 identical square faces in the cube so calculate multiply the area of one square by 6.

Draw a net of the rectangular prism, including the dimensions.

There are 3 pairs of rectangles.

Two rectangles that are 5.8 m by 3 m.

Two rectangles that are 5.8 m by 2.1 m.

Two ends of the prism make two smaller rectangles that are 3 m by 2.1 m.

Calculate the area of each rectangle pair using $A = l \times w$.

WORKING

$$A1 = 5.8 \times 3 = 17.4 \text{ m}^2$$

$$A2 = 5.8 \times 2.1 = 12.18 \text{ m}^2$$

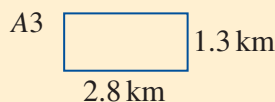
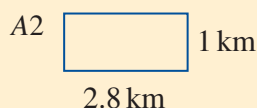
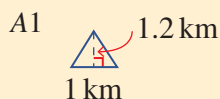
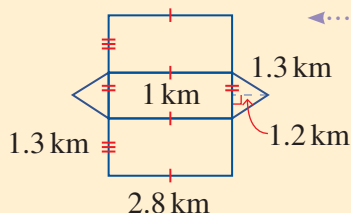
$$A3 = 2.1 \times 3 = 6.3 \text{ m}^2$$

Total surface area

$$= (17.4 \times 2) + (12.18 \times 2) + (6.3 \times 2)$$

$$= 34.8 + 24.36 + 12.6 = 71.76 \text{ m}^2$$

C



$$A = \frac{1}{2} \times b \times h$$

$$A1 = \frac{1}{2} \times (1 \times 1.2) = 0.6 \text{ km}^2$$

$$A = l \times w$$

$$A2 = 2.8 \times 1.3 = 3.64 \text{ km}^2$$

$$A3 = 2.8 \times 1 = 2.8 \text{ km}^2$$

Total surface area

$$= (0.6 \times 2) + (3.64 \times 2) + 2.8$$

$$= 1.2 + 7.28 + 2.8 = 11.28 \text{ km}^2$$

THINKING

Substitute the values for l and w in each rectangle pair.

Double each rectangle area and add all the areas together to find the total surface area of the rectangular prism.

Draw a net of the triangular prism, including the dimensions. There is one pair of triangles, one pair of identical rectangles and one other rectangle.

Two triangles have a base of 1 km and a height of 1.2 km.

Two identical rectangles are 2.8 km by 1 km.

One other rectangle is 2.8 km by 1.3 km.

Calculate the area of the triangles using $A = \frac{1}{2} \times b \times h$.

Substitute the values for b and h .

Calculate the area of each rectangle pair using $A = l \times w$.

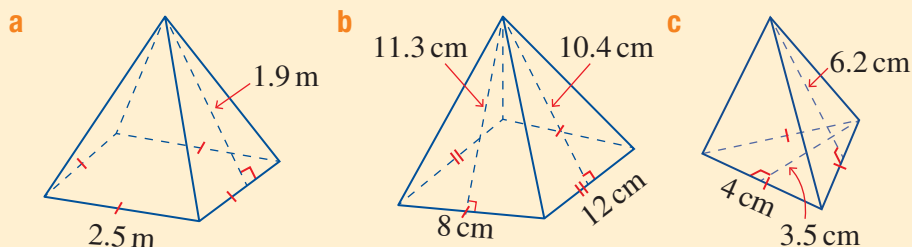
Substitute the values for l and w .

Double rectangle area ($A2$) and the triangle area ($A1$). Add all the areas together to calculate the total surface area of the triangular prism.



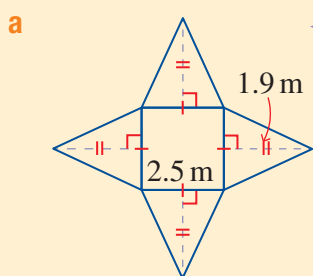
Example 22 Calculating the surface area of pyramids

Calculate the surface area for each of the following pyramids.

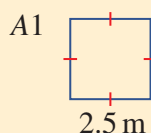


WORKING

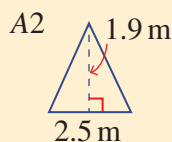
THINKING



← Draw a net of the square-based pyramid, including the dimensions.



← There is one square with side length of 2.5 m.



← There are four identical triangles with a base of 2.5 m and height of 1.9 m.

$$A = l \times l$$

← Calculate the area of the square (A1) using $A = l \times l$.

$$A1 = 2.5 \times 2.5 = 6.25 \text{ m}^2$$

← Substitute the values for l .

$$A = \frac{1}{2} \times (b \times h)$$

← Calculate the area of the identical triangles (A2) using $A = \frac{1}{2} \times (b \times h)$.

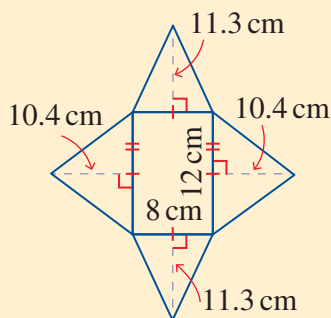
$$\begin{aligned} A2 &= \frac{1}{2} \times (2.5 \times 1.9) \\ &= 2.375 \text{ m}^2 \end{aligned}$$

← Substitute the values for b and h .

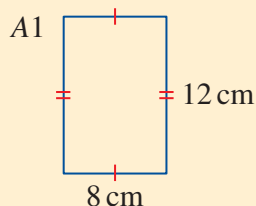
$$\begin{aligned} \text{Total surface area} &= 6.25 + (4 \times 2.375) \\ &= 6.25 + 9.5 \\ &= 15.75 \text{ m}^2 \end{aligned}$$

← Multiply the area for the identical triangles by 4. Add the area of the square base to the area of the 4 triangles to calculate the total surface area of the square-based pyramid.

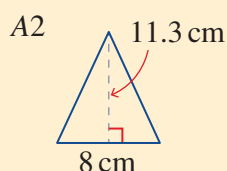
b



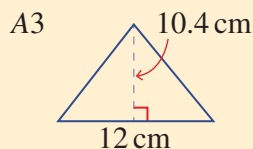
Draw a net of the rectangular-based pyramid, including the dimensions.



There is one rectangle with length 12 cm and width 8 cm.



There are two identical triangles with base 8 cm and height 11.3 cm.



There are two identical triangles with base 12 cm and height 10.4 cm.

$$A = l \times w$$

Calculate the area of the rectangle (A1) using $A = l \times w$.

$$A1 = 12 \times 8 = 96 \text{ cm}^2$$

Substitute the values for l and w .

$$A = \frac{1}{2} \times (b \times h)$$

Calculate the area of the identical triangles (A2) using $A = \frac{1}{2} \times (b \times h)$.

$$A2 = \frac{1}{2} \times (8 \times 11.3) = 45.2 \text{ cm}^2$$

Substitute the values for b and h .

$$A = \frac{1}{2} \times (b \times h)$$

Calculate the area of the identical triangles (A3) using $A = \frac{1}{2} \times (b \times h)$.

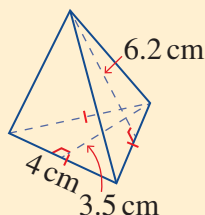
$$A3 = \frac{1}{2} \times (12 \times 10.4) = 62.4 \text{ cm}^2$$

Substitute the values for b and h .

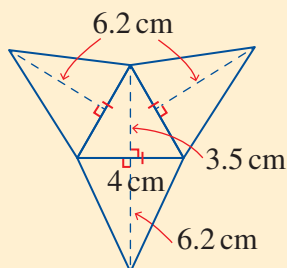
$$\begin{aligned} \text{Total surface area} &= (45.2 \times 2) + (62.4 \times 2) + 96 \\ &= 90.4 + 124.8 + 96 \\ &= 311.2 \text{ cm}^2 \end{aligned}$$

Multiply the areas of the triangles by 2 (for A2 and A3). Add the triangle areas to the rectangle base to calculate the total surface area of the rectangular-based pyramid.

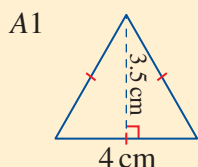
c



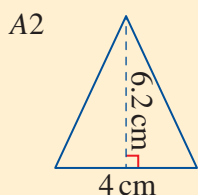
A copy of the diagram in the question is repeated here for convenience.



←..... Draw a net of the triangular-based pyramid, including the dimensions.



←..... There is one base triangle with base 4 cm and height 3.5 cm.



←..... There are three identical triangles with base 4 cm and height 6.2 cm.

$$A = \frac{1}{2} \times (b \times h)$$

←..... Calculate the area of the triangle (A1) using $A = \frac{1}{2} \times (b \times h)$.

$$A1 = \frac{1}{2} \times (4 \times 3.5) = 7 \text{ cm}^2$$

←... Substitute the values for b and h .

$$A = \frac{1}{2} \times (b \times h)$$

←..... Calculate the area of the identical triangles (A2) using $A = \frac{1}{2} \times (b \times h)$.

$$A2 = \frac{1}{2} \times (4 \times 6.2) = 12.4 \text{ cm}^2$$

←... Substitute the values for b and h .

$$\begin{aligned} \text{Total surface area} &= (12.4 \times 3) + 7 \\ &= 37.2 + 7 \\ &= 44.2 \text{ cm}^2 \end{aligned}$$

←..... Multiply the area of the identical triangle by 3 (A2). Add the base triangle area to the other triangle areas to calculate the total surface area of the triangular-based pyramid.



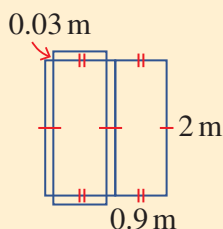
Example 23 Applying the surface area of 3D solids to practical problems

Cailin is painting the inside of her house. She has ten doors that she needs to remove from the hinges and paint. The doors are rectangular prisms in shape, as shown in the image. They are 0.9 m wide, 2 m high and 0.03 m thick. Determine the total surface area of all 10 doors that Cailin needs to paint.

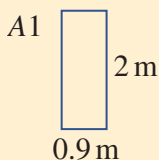


WORKING

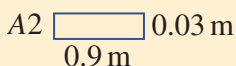
THINKING



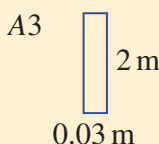
←..... Draw a net of the rectangular prism.



←..... There are three pairs of rectangles.
Two rectangles that are 0.9 m by 2 m.



←..... Two rectangles that are 0.9 m by 0.03 m.



←..... Two rectangles that are 0.03 m by 2 m.

$$A = l \times w$$

←..... Calculate the area of the rectangles using $A = l \times w$.

$$A1 = 0.9 \times 2 = 1.8 \text{ m}^2$$

$$1.8 \times 2 = 3.6 \text{ m}^2$$

←..... Multiply the area of each pair of rectangles by 2.

$$A2 = 0.9 \times 0.03 = 0.027 \text{ m}^2$$

$$0.027 \times 2 = 0.054 \text{ m}^2$$

$$A3 = 0.03 \times 2 = 0.06 \text{ m}^2$$

$$0.06 \times 2 = 0.12 \text{ m}^2$$

$$\begin{aligned} \text{Total surface area of one door} \\ = 3.6 + 0.054 + 0.12 = 3.774 \text{ m}^2 \end{aligned}$$

←..... Add together the areas of the three pairs of rectangles to find the total surface area of the rectangular prism for one door.

$$\begin{aligned} \text{Total surface area of 10 doors} \\ = 3.774 \times 10 = 37.74 \text{ m}^2 \end{aligned}$$

←..... Multiply your answer by 10 to calculate the total surface area for all 10 doors.

Cailin will need to paint a surface area of 37.74 m^2 .

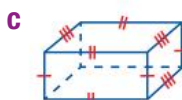
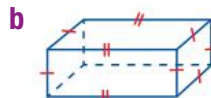
←..... Communicate your solution in a sentence.

Exercise 1G

FUNDAMENTALS

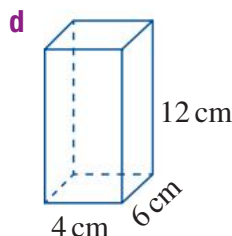
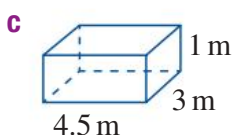
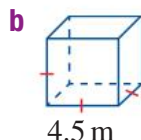
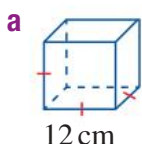
Example 20

- 1 Draw a net that represents each of the following 3D solids.



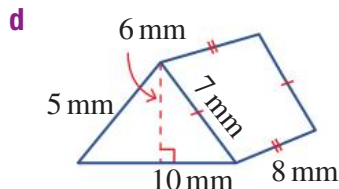
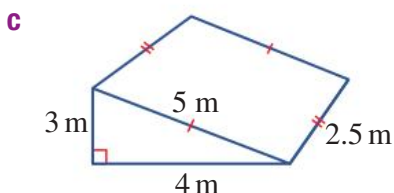
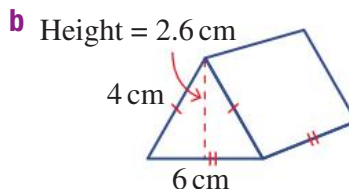
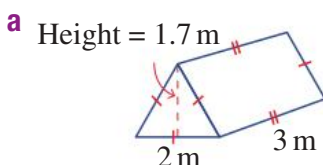
Example 21

- 2 Calculate the surface area of the following cubes and rectangular prisms.



Example 21

- 3 Calculate the surface area of the following triangular prisms.

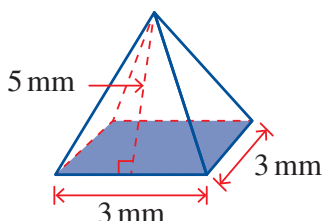


Hint Draw the net of each solid to help you work out the total surface area.

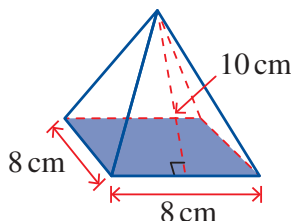
Example 22

- 4** Calculate the surface area of the following square-based pyramids.

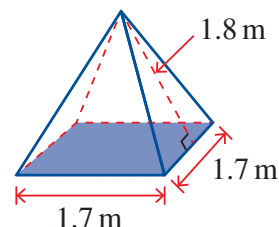
a



b



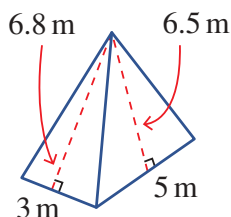
c



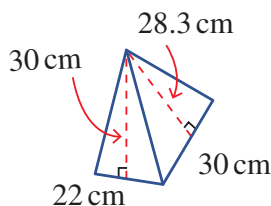
Example 22

- 5** Calculate the surface area of the following rectangular-based pyramids.

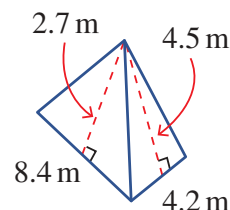
a



b

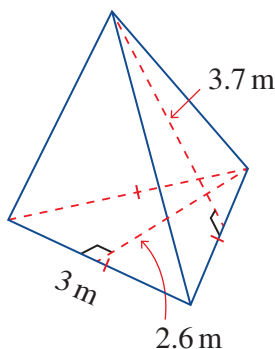


c

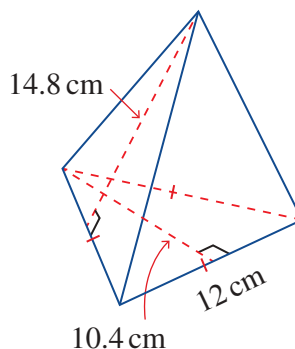


- 6** Calculate the surface area of the following triangular-based pyramids.

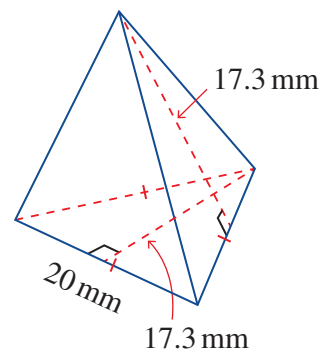
a



b



c



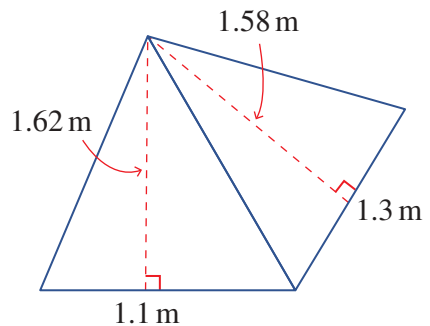
APPLICATIONS

Example 23

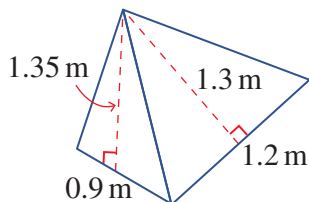
- 7** Scott is sewing together the outer cover of a foot stool that looks like a Rubik's cube. He would like the stool to be 60 cm square. Calculate the total area of material of all colours that Scott will require to cover the outside of the foot stool.



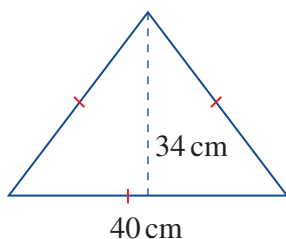
- 8** Michael is making a rectangular-based pyramid out of stainless steel to put on display in his garden. Each triangular face acts like a mirror, reflecting the plants around the pyramid. The rectangular base will be made out of stainless steel for strength. The plan for Michael's pyramid is shown in the diagram. Determine the total area of stainless steel required.



- 9** Anika makes children's play teepees and sells them at the markets. The design of her teepees are rectangular-based pyramids as shown in her sewing plan below. At her last market, she sold 8 teepees. Calculate the total area of material that Anika used to make the teepees that were sold at the market on that day.



- 10** Madison has won a giant Toblerone. She is planning on wrapping it in paper and giving it to her mum for Christmas. The Toblerone is 1.5 metres long and the measurements for the equilateral triangle at either end is shown in the diagram below. Determine the minimum total area of wrapping paper that Madison will require, ignoring the need to overlap the edges of the wrapping paper.



- 11** Renee is designing and sewing a tent for her husband. She plans on making it in the shape of a rectangular prism. It needs to be big enough that her husband can stand in it and it also needs to fit their double air mattress. Her husband is 1.8 metres tall and their air mattress is 1.6 metres wide and 2 metres long. Sketch a plan for Renee's tent and calculate the total area of material that she will require including the base (groundsheet).
- 12** The Eiffel Tower is a monument in Paris, France. It is similar in shape to a square-based pyramid with a base length of 99.9 metres and a height of 304 metres. Suppose a steel square-based pyramid was to be made to transport the Eiffel Tower. Its base length is 100 m and the perpendicular height of the triangles forming the four sides is 308 m. Determine the total surface area of steel required.



1H Calculating the surface areas of spheres and cylinders

LEARNING GOALS

- Apply a formula to calculate the surface area of spheres
- Apply a formula to calculate the surface area of cylinders

Why is it essential to be able to calculate the surface area of spheres and cylinders?

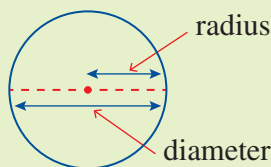
Spheres and cylinders are used in many industries such as manufacturing, sporting and farming equipment. Being able to calculate the surface area of these shapes allows us to calculate things such as the amount of leather needed for a ball or the amount of steel needed for a silo.



The area of glass needed for this building can be calculated using the methods in this section.

WHAT YOU NEED TO KNOW

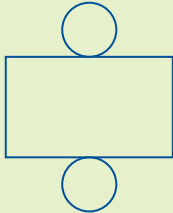
- A **sphere** is a perfectly round 3D solid.



- Note that spheres do not have a net.

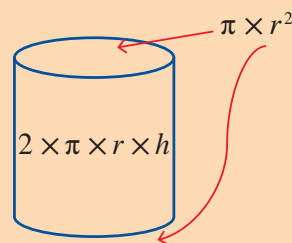
Formula for surface area of a sphere: $S = 4 \times \pi \times r^2$,
where r stands for radius.

- The **radius** of a sphere is half of the **diameter**. The diameter is the straight-line distance from the surface through the centre of the sphere to the surface on the other side, while the radius is the distance from the centre to the surface.

- A hemisphere is half a sphere, so the area of its curved surface is half that of the sphere (if you need the area to include the flat circular base, add the area of a circle with the same radius as the sphere).
- A **closed cylinder** has two identical parallel faces that are flat circles joined by a rectangle or square curved into a tube. 
- An **open cylinder** has either no circles at the ends, like a tube, or a circle at one end, like a container without a lid.
- The net of a closed cylinder, shown in the diagram, is made up of two circles joined by a rectangle or square. The length of the sides touching the circles is the same as the **circumference** of the circles. 
- To calculate the surface area of a closed cylinder, use the net (shown above) as a guide. The curved surface of the cylinder is a rectangle with a length that is the circumference of the circle, $2 \times \pi \times r$, and a width that is the height of the cylinder, h .
So the area of the curved surface is $A = 2 \times \pi \times r \times h$.
The ends of the cylinder are circles, so the $A = \pi \times r^2$. We have two circular ends, so we need to double the area of the circle.

$$S = \text{Area of curved rectangle} + 2 \times \text{Area of the circle}$$

$$S = 2 \times \pi \times r \times h + 2 \times \pi \times r^2$$



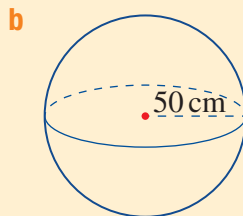
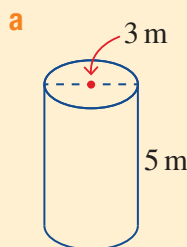
- If the cylinder is closed at one end only, like a tin without a lid, we only need one circular end.
So, $S = 2 \times \pi \times r \times h + \pi \times r^2$.
- If the cylinder is open at both ends, we do not need the two circular ends.
So, $S = 2 \times \pi \times r \times h$.





Example 24 Calculating the surface area of spheres and cylinders

Calculate the surface area of the following sphere and cylinder. Round your answers to 2 decimal places.



WORKING

a Radius = $3 \div 2 = 1.5$ m

Area of curved rectangular part

$$A = 2 \times \pi \times r \times h$$

$$A = 2 \times \pi \times 1.5 \times 5$$

$$A = 47.12 \text{ m}^2$$

Area of 2 circles

$$A = 2 \times \pi \times r^2$$

$$A = 2 \times \pi \times 1.5^2$$

$$A = 14.14 \text{ m}^2$$

$$S = 47.12 + 14.14$$

$$S = 61.26 \text{ m}^2$$

OR

$$S = 2 \times \pi \times r \times h + 2 \times \pi \times r^2$$

$$S = 2 \times \pi \times 1.5 \times 5 + 2 \times \pi \times 1.5^2$$

$$S = 47.12 + 14.14$$

$$S = 61.26 \text{ m}^2$$

b $S = 4 \times \pi \times r^2$

$$S = 4 \times \pi \times 50^2$$

$$S = 31\,415.93 \text{ cm}^2$$

THINKING

Determine the radius first by halving the diameter.

Calculate the surface area of the cylinder by separating the cylinder into the curved rectangle and two circles.

Area of curved rectangular part is

$$A = 2 \times \pi \times r \times h$$

$$\text{Area of circles is } A = 2 \times \pi \times r^2$$

Add together the parts of the surface area.

OR

By using the formula

$$S = 2 \times \pi \times r \times h + 2 \times \pi \times r^2$$

Substitute the values for radius (r) and height (h).

Round your answer to 2 decimal places.

Calculate the surface area of the sphere using the formula $S = 4 \times \pi \times r^2$.

Substitute the value for the radius (r).

Round your answer to 2 decimal places.



Example 25 Applying the surface area of a cylinder to a practical problem

Lachlan enjoys four-wheel driving and fishing at the beach. He is making a storage container for his fishing rods to attach to his four-wheel drive ute. He has purchased a 5-metre long PVC pipe and two end caps. The caps and pipe have a diameter of 25 cm. He needs to calculate the surface area of the storage container including the end caps, so that he can paint it black to match his vehicle. Calculate the surface area of the storage container.

WORKING

$$\text{Radius} = 25 \div 2 = 12.5 \text{ cm}$$

$$12.5 \div 100 = 0.125 \text{ m}$$

$$S = 2 \times \pi \times r \times h + 2 \times \pi \times r^2$$

$$S = 2 \times \pi \times 0.125 \times 5 + 2 \times \pi \times 0.125^2$$

$$S = 4.03 \text{ m}^2$$

Lachlan needs to paint an area of 4.03 m^2 .

THINKING

Determine the radius first by halving the diameter.

Convert centimetres to metres by dividing by 100. All units are now in metres.

The storage container creates a cylinder.

Calculate the surface area of the cylinder using the formula $S = 2 \times \pi \times r \times h + 2 \times \pi \times r^2$.

Substitute the values for r and h . Round your answer to 2 decimal places.

Communicate your solution in a sentence.



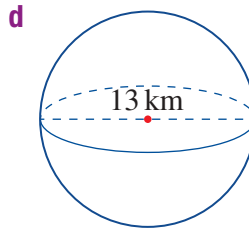
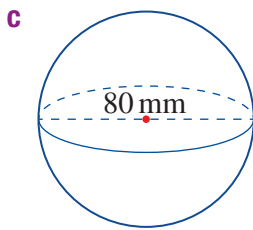
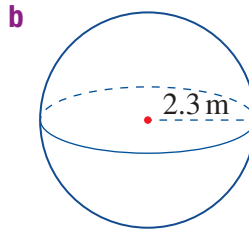
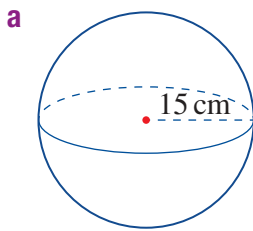
Exercise 1H

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a The formula to calculate the surface area of a sphere is _____.
 - b The formula to calculate the surface area of a cylinder is _____.
 - c $2 \times \pi \times r \times h$ calculates the area of the _____ of a cylinder.
 - d $2 \times \pi \times r^2$ calculates the area of the _____ of a cylinder.

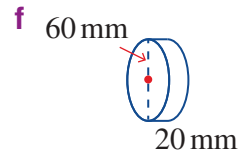
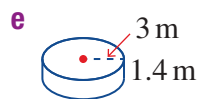
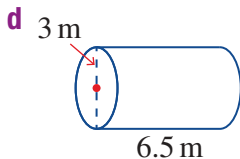
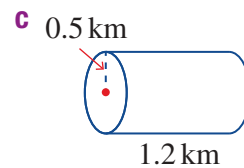
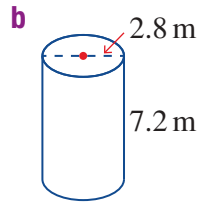
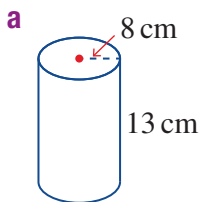
Example 24

- 2 Calculate the surface area of the following spheres. Round your answer to 2 decimal places where necessary.



Hint $A = 4 \times \pi \times r^2$

- 3 Calculate the surface area of the following closed cylinders. Round your answer to 2 decimal places where necessary.



Hint $A = 2 \times \pi \times r \times h + 2 \times \pi \times r^2$

APPLICATIONS

Example 25

- 4 Georgie has saved a Pringles container that her mum was going to throw out. It has a height of 30 cm and a diameter of 8 cm. She plans on decorating the container so that she can store some craft items in it. Georgie will be covering the container with contact. Calculate the total surface area of the container.



- 5 Tanya is powder coating a pole that will be holding up a sail. It has a height of 3 m and a radius of 0.08 m. Calculate the total surface area of the pole.
- 6 Asher works in a factory that makes tennis balls. His job is to apply the glue that holds the felt on the ball. An average tennis ball has a diameter of 6.6 cm. Determine the area of glue required for each ball.
- 7 In the heart of Adelaide, South Australia, lies Rundle Mall where a giant sculpture by Bert Flugelman resides called 'On Further Reflection'. It is two stainless steel spheres, one on top of the other, which were gifted to the city in 1977. The spheres have a diameter of 2.15 metres. Calculate the total surface area of both spheres.



- 8 Earth has a diameter of 12 742 km. Determine the total surface area of the Earth assuming it is a sphere.
- 9 Kanku has been given a digeridoo to paint. He will be painting the external curved surface black as a base for his artwork. The digeridoo is a cylinder with a length of 130 cm and a radius of 1.6 cm. Determine the area of the curved surface that Kanku will be painting.

11 Calculating the surface areas of irregular solids

LEARNING GOALS

- Calculate the surface area of cones
- Calculate the surface area of irregular solids

Why is it essential to be able to calculate the surface area of irregular solids?

Not all objects are formed with the use of familiar solids. Sometimes we need to identify the familiar shapes within an object so that we can calculate the surface area of an irregular solid. This helps in careers such as painting and construction, along with baking.



The main shape of this gingerbread house is made from a triangular prism on top of a rectangular prism.

WHAT YOU NEED TO KNOW

- For the purposes of this course, an **irregular solid** is either a **composite 3D solid** that consists of two or more of the common solids or it is part of a common solid, such as half a sphere or half a cylinder cut lengthways.
- We can calculate the surface area of irregular solids if their **nets** are formed from the 2D shapes.

Shape	Formula
Square	$A = l^2$
Rectangle	$A = l \times w$
Triangle	$A = \frac{1}{2} \times (b \times h)$
Parallelogram	$A = b \times h$
Trapezium	$A = \frac{1}{2} \times (a + b) \times h$
Circle	$A = \pi \times r^2$
Sector	$A = \frac{\theta}{360} \times \pi \times r^2$

- We may be able to calculate the surface area of irregular solids if they include spheres and cylinders (see the last section), and cones, which are introduced here.

Shape	Formula
Sphere	$S = 4 \times \pi \times r^2$
Closed cylinder	$S = 2 \times \pi \times r \times h + 2 \times \pi \times r^2$
Cone	$S = \pi \times r \times s + \pi \times r^2$

- Surface area of a cone

S = curved surface of cone + area of circle

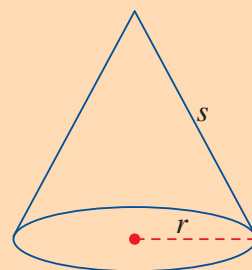
$$S = \pi \times r \times s + \pi \times r^2$$

S is the total surface area, which includes the area of the circular base, $\pi \times r^2$.

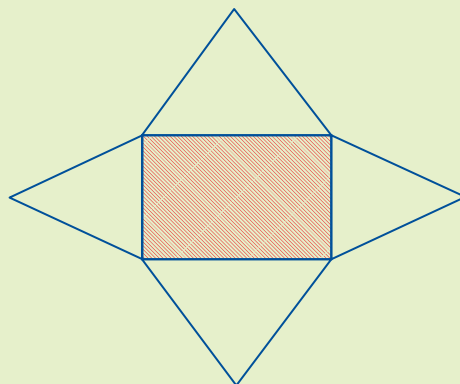
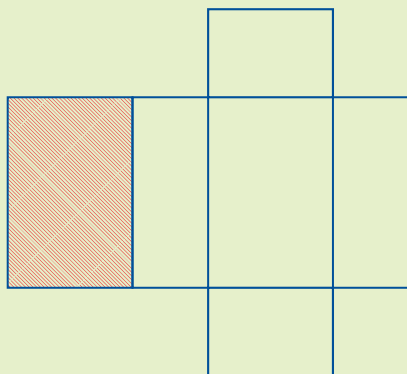
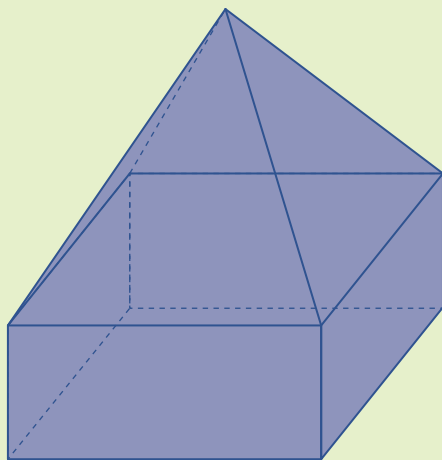
s is the 'slant height' – the shortest distance between the apex and the circumference of the base.

If you only need the curved surface of the cone use

$$A = \pi \times r \times s.$$



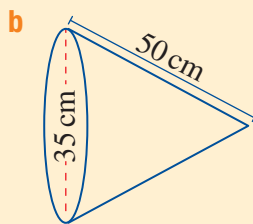
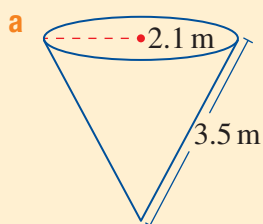
- For an irregular solid that includes a sphere, or a part of a sphere, calculate the surface area of the sphere separately. If the remaining part of the irregular solid has a net, calculate the surface area of the appropriate net. Add all of the surface areas together for the total surface area.
- For an irregular solid that includes a cylinder, or a part of a cylinder, calculate the area of the part of the net that is needed such as:
 - $S = 2 \times \pi \times r \times h + \pi \times r^2$ (a cylinder with only one closed end)
 - $S = 2 \times \pi \times r \times h$ (a cylinder with both ends open)
- For the total surface area of an irregular solid, you need to remember not to include any surfaces that are not visible from the outside of the solid.
 For example, shown is an irregular solid with a rectangular-based pyramid on top of a rectangular prism. Draw a sketch of the nets for each solid. Think about which parts are not visible in the solid:
 - In the rectangular prism, the top side is not included.
 - In the rectangular-based pyramid, the base rectangle is not included.





Example 26 Calculating the surface area of cones

Calculate the surface area of the following cones. Round your answer correct to 2 decimal places.



WORKING

a $S = \pi \times r \times s + \pi \times r^2$ ←.....

$$S = \pi \times 2.1 \times 3.5 + \pi \times 2.1^2$$

$$S = 36.95 \text{ m}^2$$

b $35 \div 2 = 17.5 \text{ cm}$ ←.....

$$S = \pi \times r \times s + \pi \times r^2$$

$$S = \pi \times 17.50 \times 50 + \pi \times 17.5^2$$

$$S = 3711.01 \text{ cm}^2$$

THINKING

Calculate the surface area of the cone using $S = \pi \times r \times s + \pi \times r^2$.

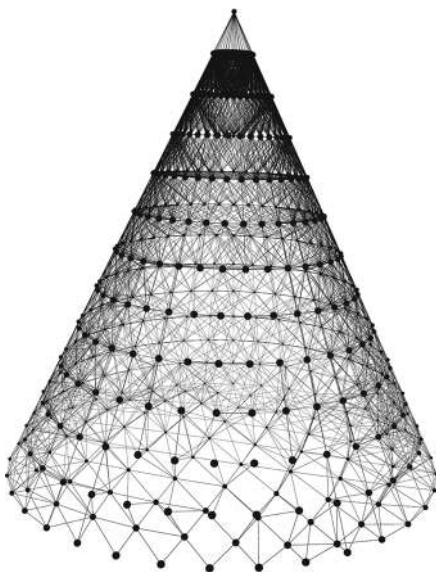
Substitute the values of r and s .
Round your answer to 2 decimal places.

Determine the radius first by halving the diameter.

Calculate the surface area of the cone using $S = \pi \times r \times s + \pi \times r^2$.

Substitute the values of r and s .

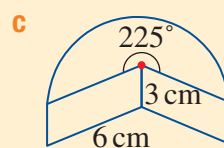
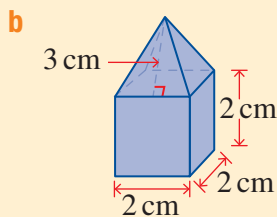
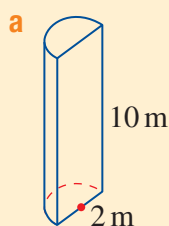
Round your answer to 2 decimal places.





Example 27 Calculating the surface area of irregular solids

Calculate the surface area for each of the following irregular solids.



WORKING

a Surface area for cylinder

$$S = 2 \times \pi \times r \times h + 2 \times \pi \times r^2$$

$$S = 2 \times \pi \times 1 \times 10 + 2 \times \pi \times 1^2$$

$$S = 69.12 \text{ m}^2$$

Surface area for half of cylinder

$$S1 = 69.12 \div 2$$

$$S1 = 34.56 \text{ m}^2$$

Area of rectangle face

$$A = lw$$

$$S2 = 10 \times 2$$

$$S2 = 20 \text{ m}^2$$

Total surface area

$$= S1 + S2$$

$$= 34.56 + 20$$

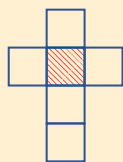
$$= 54.56 \text{ m}^2$$

b Area of square base

$$A = 2 \times 2 = 4 \text{ cm}^2$$

Surface area of bottom cube

$$S1 = 4 \times 5 = 20 \text{ cm}^2$$



THINKING

The formula for finding the surface area of a cylinder is

$$S = 2 \times \pi \times r \times h + 2 \times \pi \times r^2$$

The radius is half the diameter, so radius is 1 m.

Substitute $r = 1$ and $h = 10$ into formula.

This shape is only half a cylinder so divide your answer by 2.

Calculate the area of the rectangular face by using $A = lw$.

Substitute $l = 10$ and $w = 2$.

Calculate the total surface area by adding the surface for half the cylinder and the rectangle.

Calculate the area of the square base by using $A = lw$.

The top of the cube is not visible so instead of multiplying by 6, multiply by 5.

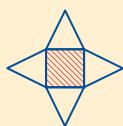
WORKING

Surface of the triangle faces

$$A = \frac{1}{2} \times (b \times h)$$

$$A = \frac{1}{2} \times (2 \times 3) = 3 \text{ cm}^2$$

$$S2 = 3 \times 4 = 12 \text{ cm}^2$$



Total surface area

$$= S1 + S2$$

$$= 20 + 12$$

$$= 32 \text{ cm}^2$$

c Area of the sector

$$A = \frac{\theta}{360} \times \pi \times r^2$$

$$A = \frac{225}{360} \times \pi \times 6^2 = 70.69 \text{ cm}^2$$

Surface area of two sectors

$$S1 = 70.69 \times 2$$

$$S1 = 141.37 \text{ cm}^2$$

Surface area of front two rectangles

$$A = lw$$

$$A = 6 \times 3 = 18 \text{ cm}^2$$

$$S2 = 18 \times 2$$

$$S2 = 36 \text{ cm}^2$$

Surface area of part of curved rectangle of a cylinder

$$A = \frac{\theta}{360} \times 2 \times \pi \times r \times h$$

$$S3 = \frac{225}{360} \times 2 \times \pi \times 6 \times 3$$

$$S3 = 70.69 \text{ cm}^2$$

$$S = S1 + S2 + S3$$

$$S = 141.37 + 36 + 70.69$$

$$S = 248.06 \text{ cm}^2$$

THINKING

Calculate the area of each triangle using the formula $A = \frac{1}{2} \times (b \times h)$.

There are 4 triangles the same size so multiply this by 4.

Add both sections of the irregular solid to calculate total surface area.

Calculate the area of a sector using the formula $A = \frac{\theta}{360} \times \pi \times r^2$.

The top and bottom of the irregular solid is the same sector so multiply this by 2.

Calculate the area of the two front panels, which are rectangles so use $A = lw$. As the rectangles are the same size multiply by 2.

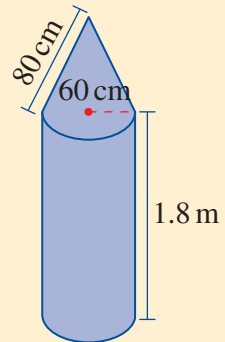
Calculate the area of the back panel, which is part of the curved rectangle of a cylinder so use $A = \frac{\theta}{360} \times 2 \times \pi \times r \times h$.

Add all surface area sections of the irregular solid to calculate total surface area.



Example 28 Applying the surface area of an irregular solid to practical problems

Melfred is painting a cardboard rocket that he has made with his son. Calculate the total surface area of the rocket. The ‘cap’ is a cone and the body is a cylinder. The base of the rocket is closed.



WORKING

Radius: $60 \div 100 = 0.6 \text{ m}$

Slant height of cone:

$80 \div 100 = 0.8 \text{ m}$

Surface area of cone without base

$$S1 = \pi \times 0.6 \times 0.8 \approx 1.51 \text{ m}^2$$

rounded to 2 d.p.

Surface area of the cylinder closed at one end.

$$S2 = 2 \times \pi \times 0.6 \times 1.8 + \pi \times 0.6^2$$

$$S2 \approx 7.92 \text{ m}^2$$

$$S = S1 + S2$$

$$S = 1.51 + 7.92$$

$$S = 9.43 \text{ m}^2$$

The rocket has a total surface area of 9.43 m^2 .

THINKING

Convert centimetres to metres by dividing by 100 as all measurements must be in the same units.

Calculate the surface area of the cone without the circular base by using the formula $A = \pi \times r \times s$.

Calculate the area of the cylinder by using the formula for a cylinder closed at one end. $A = 2 \times \pi \times r \times h + \pi \times r^2$

Add all the areas together to calculate the total surface area of the composite solid (if you use unrounded values you will get 9.42 m^2).

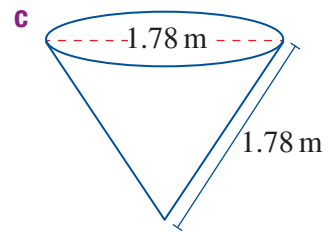
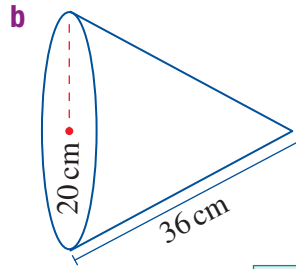
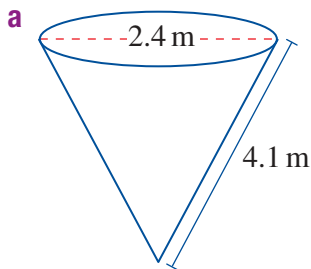
Communicate your solution using words.



Exercise 11

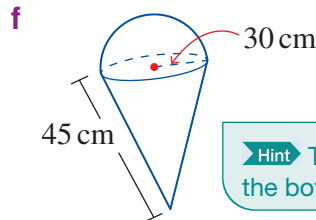
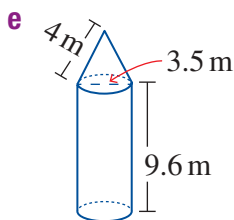
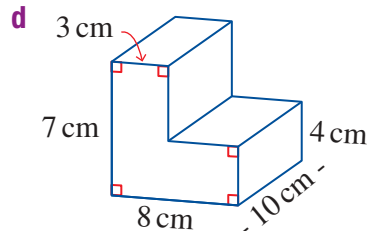
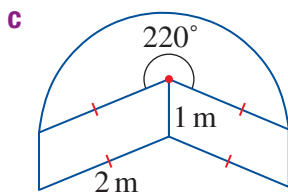
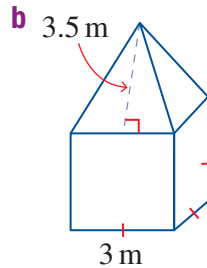
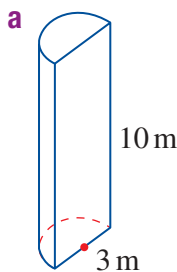
FUNDAMENTALS

Example 26 1 Calculate the surface area of the following cones.

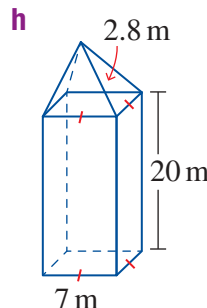
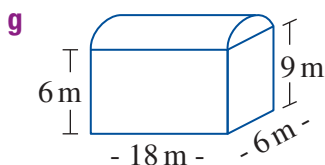


Hint $A = \pi \times r \times s + \pi \times r^2$

Example 27 2 Calculate the surface area of the following irregular solids.



Hint The top is a hemisphere
the bottom is a cone.

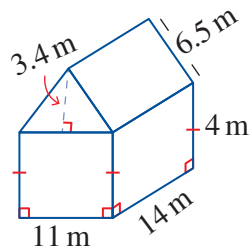


Hint Draw the shapes that make
all of the faces of the solid.

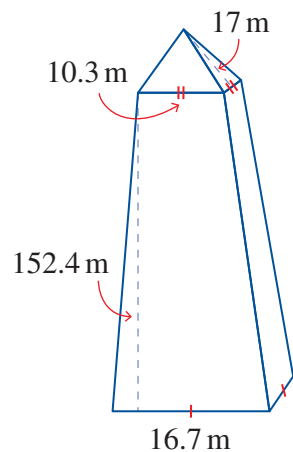
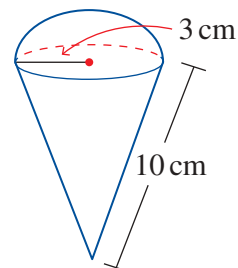
APPLICATIONS

Example 28

- 3 Millie is planning on painting the outside of her house. Use the diagram shown to calculate the total surface area.



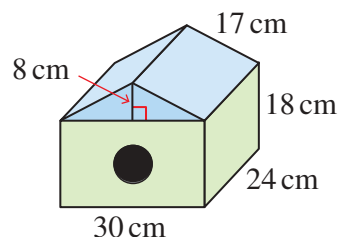
- 4 Tim has carved a wooden ice-cream for his son's play kitchen, as shown in the diagram. He plans on painting it for him. Determine the total area of paint that Tim will need.
- 5 Christine is making a layered sponge cake for her son's birthday. The rectangular tin she is using has a length of 30 cm, a width of 20 cm and a height of 5 cm. She is planning on stacking 3 cakes on top of each other. She will then ice the cake on all 4 sides and the top. Determine the total surface area for the icing.
- 6 A monument in Washington is 169.2 m tall in total. The main part has four equal-sized faces each in the shape of a trapezium, and there is a square-based pyramid on the top. Calculate the total surface area of the monument, using the approximate measurements in the diagram shown.



- 7 Jean has some camembert cheese in her fridge that has a radius of 7 cm and a height of 3.2 cm. She has eaten a large slice that is approximately 70° of the total cheese. Determine the remaining surface area of the cheese.



- 8 Mason has made a nesting box for his birds. The hole has a diameter of 9 cm. Calculate the external surface area of the nesting box after Mason cuts out the hole for the birds to enter.



1J Using and converting between metric volume and capacity units

LEARNING GOALS

- Understand the use and appropriate choice of the metric units of volume including cubic millimetres, cubic centimetres and cubic metres
- Use abbreviations such as mm^3 , cm^3 and m^3 to represent units of volume
- Convert between units of volume
- Understand the relationship between volume and capacity including the conversions between millilitres, litres, kilolitres and megalitres

Why is it essential to understand units of volume and capacity?

There are many careers, including pool installation, landscaping, cooking, storage hire or hairdressing, where volume capacity can be used. The production, storage, distribution and use of materials, whether gas, liquid or solid, and their containers, requires measurement and conversion of volume and capacity.



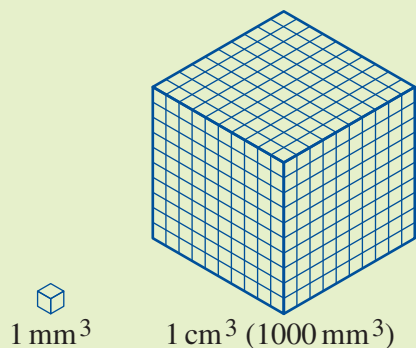
Measuring volume is a vital skill in laboratories.

It is an essential skill to be able to convert between the units of volume and capacity so that the capacity (or amount of gas, solid or liquid) can be calculated using the measurements of an object.

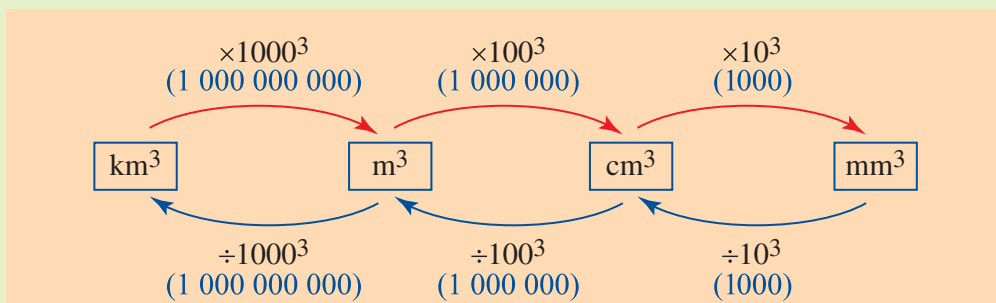
WHAT YOU NEED TO KNOW

- Calculating the **volume** of a solid is a measure of the amount of space taken up by a 3D shape.
- **Units of volume** include cubic millimetres, cubic centimetres, cubic metres and cubic kilometres, which can be abbreviated to mm^3 , cm^3 , m^3 and km^3 .

- To convert between units of volume:
 - use multiplication when converting from a larger unit to a smaller unit (e.g. from m^3 to cm^3)
 - use division when converting from a smaller unit to a larger unit (e.g. from cm^3 to m^3).
- If one linear unit is 10 times longer than another, then its cubic unit is 10^3 (i.e. 1000 times bigger).

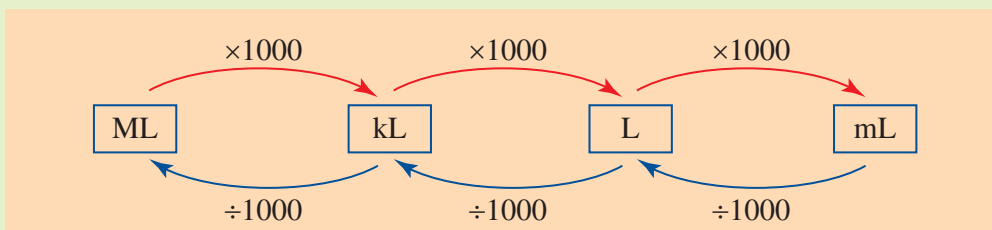


- Refer to the following conversion chart when solving problems that require conversion of units of volume.



- **Capacity** is the same as volume, but in everyday contexts we use capacity for the volume that a container (3D solid) can hold of a substance (solid, liquid or gas).
- Units of capacity include megalitres, kilolitres, litres, millilitres, which can be abbreviated to ML, kL, L and mL.

- Capacity is measured in units based on the litre. Use the following conversion chart for problems that require conversion of units of capacity.



- To convert between units of volume and capacity:
 - $1 \text{ cm}^3 = 1 \text{ mL}$
 - $1 \text{ m}^3 = 1000 \text{ L} = 1 \text{ kL}$
 - $1\,000\,000 \text{ L} = 1 \text{ ML}$
 - $1000 \text{ cm}^3 = 1000 \text{ mL} = 1 \text{ L}$
 - $1000 \text{ m}^3 = 1000 \text{ kL} = 1 \text{ ML}$



Example 29 Converting between units of volume

Convert these volume measurements into the units given in brackets.

- a** $0.72 \text{ m}^3 \text{ (mm}^3\text{)}$ **b** $10.5 \text{ cm}^3 \text{ (mm}^3\text{)}$
c $393 \text{ mm}^3 \text{ (cm}^3\text{)}$ **d** $800 \text{ cm}^3 \text{ (m}^3\text{)}$

WORKING

a $0.72 \times 1\,000\,000\,000$
 $= 720\,000\,000 \text{ mm}^3$

b $10.5 \times 10^3 = 10\,500 \text{ mm}^3$

c $393 \div 10^3 = 0.393 \text{ cm}^3$

d $800 \div 100^3 = 0.0008 \text{ m}^3$

THINKING

As there are 1000^3 cubic millimetres in a cubic metre and we are converting from m^3 to a smaller unit of mm^3 , we need to multiply 0.72 by 1000^3 , which is $1\,000\,000\,000$.

As there are 10^3 cubic millimetres in a cubic centimetre and we are converting from cm^3 to a smaller unit of mm^3 , we need to multiply 10.5 by 10^3 .

As there are 10^3 cubic millimetres in a cubic centimetre and we are converting from mm^3 to a larger unit of cm^3 , we need to divide 393 by 10^3 .

As there are 100^3 cubic centimetres in a cubic metre and we are converting from cm^3 to a larger unit of cubic metres, we need to divide 800 by 100^3 .



Example 30 Converting between units of capacity

Convert these capacity measurements into the units given in brackets.

a 2125 mL (L)

b 0.5 L (mL)

c 2.8 ML (L)

WORKING

THINKING

a $2125 \text{ mL} \div 1000 = 2.125 \text{ L}$

←..... To convert millilitres to litres, going from a smaller to a bigger unit, divide by 1000.

b $0.5 \times 1000 = 500 \text{ mL}$

←..... To convert litres to millilitres, going from a bigger to a smaller unit, multiply by 1000.

c $2.8 \times 1\,000\,000 = 2\,800\,000 \text{ L}$

←.... To convert megalitres to litres, going from a bigger to a smaller unit, multiply by 1 000 000.



Example 31 Converting between units of volume and units of capacity

Convert these volume and capacity measurements into the units given in brackets.

a 325 cm^3 (mL)

b 175 kL (m^3)

c 2 L (cm^3)

d 1825 cm^3 (L)

e 1.44 m^3 (kL)

f 1000 cm^3 (L)

g $356\,720 \text{ mm}^3$ (mL)

h $2\,035\,752.04 \text{ cm}^3$ (L)

WORKING

THINKING

a $325 \text{ cm}^3 = 325 \text{ mL}$

←..... There is 1 millilitre in 1 cubic centimetre.

b $175 \text{ kL} = 175 \text{ m}^3$

←..... There is 1 cubic metre in 1 kilolitre.

c $2 \text{ L} \times 1000 = 2000 \text{ mL}$
 $= 2000 \text{ cm}^3$

←..... As there are 1000 mL in a litre, multiply by 1000 to convert to mL, and then use $1 \text{ mL} = 1 \text{ cm}^3$.

d $1825 \text{ cm}^3 = 1825 \div 1000$
 $= 1.825 \text{ L}$

←..... As there are 1000 cubic centimetres in a litre, divide by 1000 to convert to litres.

e 1.44 kL

←..... Use $1 \text{ m}^3 = 1 \text{ kL}$ to convert to kL.

f $1000 \text{ cm}^3 \div 1000 = 1 \text{ L}$

←..... Given that $1000 \text{ cm}^3 = 1 \text{ L}$, divide by 1000 to convert to litres.

g $356\,720 \div 10^3 = 356.7 \text{ cm}^3$
 $= 356.7 \text{ mL}$

←..... Convert the answer to cm^3 by dividing by 10^3 .

Use $1 \text{ cm}^3 = 1 \text{ mL}$ to convert to mL.

h $2\,035\,752.04 \div 1000 = 2035.8 \text{ L}$

←... Given that $1000 \text{ cm}^3 = 1 \text{ L}$, divide the answer by 1000 to convert to litres.



The sizes of packaging cartons may be given in units of capacity or volume; both can easily be estimated from the dimensions of the carton.



Example 32 Applying unit conversion between volume and capacity

Mackenzie works at a plant nursery and has to fill 640 pots with soil. The pots have a capacity of 750 mL. The soil is measured in cubic metres. Determine the approximate amount of soil in cubic metres Mackenzie needs to fill the pots.

WORKING

$$750 \text{ mL} = 750 \text{ cm}^3$$

$$750 \text{ cm}^3 \times 640 = 480\,000 \text{ cm}^3$$

$$480\,000 \text{ cm}^3 \div 1\,000\,000 = 0.48 \text{ m}^3$$

Mackenzie needs 0.48 m^3 of soil.

THINKING

Convert 750 mL to cm^3 .

Multiply this by 640 pots.

There are 100^3 cubic centimetres in a cubic metre, and we are converting from cm^3 to a larger unit, so divide 480 000 by 100^3 , which is 1 000 000.

Communicate your answer by writing down the result.

Exercise 1J

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a Use multiplication when converting from a _____ unit to a _____ unit, for example, when converting from km^3 to m^3 .
 - b Use _____ when converting from a smaller unit to a larger unit, for example, when converting from _____ to m^3 .
 - c Capacity is a measure of a 3D shapes ability to _____ a substance, such as a _____, _____ or gas.
 - d A capacity of 1 _____ is equivalent to 1 cubic centimetre.
 - e One litre is equivalent to _____ cubic centimetres.
 - f One cubic metre is equivalent to one _____ (unit of capacity).
 - g One megalitre is equivalent to 1000 _____ (unit of capacity).

Hint A litre is equivalent to a cube with a side length of 10 cm (100 mm). You can work out the number of cubic centimetres or cubic millimetres in a litre by 'cubing' the side length (raising it to the power of 3).

Hint Ten one-litre cubes (side length 10 cm each) will fit exactly along each side of a one-cubic-metre cube (side length 100 cm).

- Example 29** 2 Convert these volume measurements to the units given in brackets.

- a 0.86 km^3 (m^3)
- b 18.4 cm^3 (mm^3)
- c 2.7 m^3 (cm^3)
- d 276 mm^3 (cm^3)
- e 1.38 km^3 (m^3)
- f 68 cm^3 (mm^3)
- g 129 cm^3 (m^3)
- h 2.7 m^3 (mm^3)

Hint If converting from a larger unit to a smaller unit, multiply. If converting from a smaller unit to a larger unit, divide.

- Example 30-31** 3 Convert these volume and capacity measurements to the units given in brackets.

a 265 cm^3 (mL)	b 412 kL (m^3)
c 6 L (cm^3)	d 2128 cm^3 (L)
e 27 mL (cm^3)	f 1247 mL (kL)
g 2.7 ML (kL)	h 27.5 m^3 (kL)
i 267 kL (ML)	j 0.8 L (mL)
k 628 mL (L)	l 3.2 ML (L)

APPLICATIONS

Example 32

- 4 Carlene has calculated that to spread mulch to a depth of 5 cm over a garden bed she needs to buy $800\,000\text{ cm}^3$ of mulch. She finds that mulch is sold in cubic metres. Calculate the number of cubic metres of mulch Carlene should order.
- 5 Tessa has 1089 m^3 of bitumen that she can use to pave the driveway on her farm. To work out how thick the bitumen layer can be she needs to convert this volume to cubic centimetres. Determine how many cm^3 are in 1089 m^3 .
- 6 Lee-Ann's pool holds 12 000 L of water. Unfortunately, the pool tiles are damaged so she has decided to fill the pool in with soil and make a big garden instead. Calculate the number of cubic metres of soil Lee-Ann will need in order to fill the pool.
- 7 Jarred works in a factory that makes sugar cubes. A sugar cube with a size of 1 cm^3 is made from 1 mL of liquid sugar. The factory produces 5000 cubes per day. Calculate the number of millilitres of liquid sugar that the factory requires each day.
- 8 An Olympic swimming pool has 2.5 megalitres of water. Calculate the number of litres in an Olympic swimming pool.
- 9 A large fuel truck carries around 10 kilolitres of fuel. Calculate how many litres of fuel a large truck can carry.



- 10 If all the water in Sydney Harbour was poured into a cubic tank with edges of 1 km, it would fill it to a depth of 562 m. Calculate the number of megalitres of water in Sydney Harbour.

1K Estimating and calculating the volume and capacity of pyramids, cones, spheres and cylinders

LEARNING GOALS

- Calculate the volume and capacity of the following regular objects:
 - square-based pyramids
 - rectangular-based pyramids
 - spheres
 - cones
 - cylinders
- Estimate the volume and capacity of pyramids, cones and spheres

Why is it essential to know how to calculate the volume of pyramids, cones, spheres and cylinders?

It is important to be able to measure volume and capacity of pyramids, cones, spheres and cylinders. Spheres are used in manufacturing balls used in sports and industry, and pyramids, cones and cylinders are used for ornaments, buildings and artwork as well as other applications.

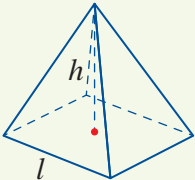
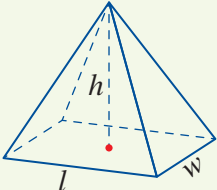
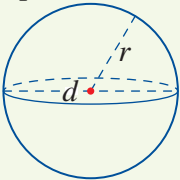
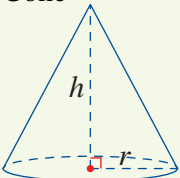
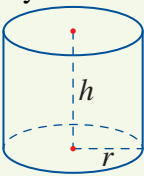


Volume is required to work out how much wax to use to make this candle.

WHAT YOU NEED TO KNOW

- You will need to recall the units and conversions in previous sections.
- Note that the pyramids dealt with are right pyramids, meaning that the point (apex) is above the centre of the base. The same applies to the cones, they are called right cones.

- Formulas for calculating the volume of pyramids, cones, spheres and cylinders are found in the table below.

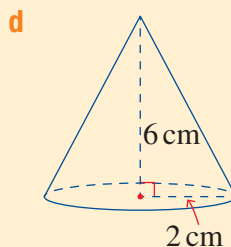
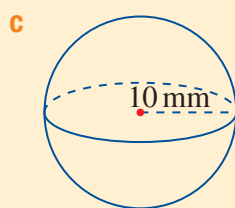
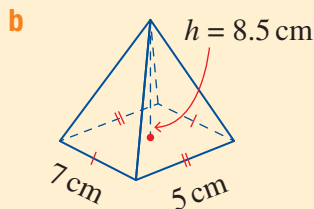
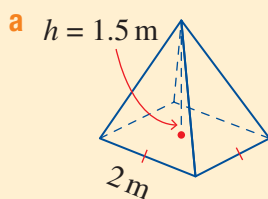
Shape	Rule
Square-based pyramid 	$V = \frac{1}{3} \times l^2 \times h$ (V: volume, l : length of side of base, h : height)
Rectangular-based pyramid 	$V = \frac{1}{3} \times l \times w \times h$ (V: volume, l : length of base, w : width of base, h : height)
Sphere 	$V = \frac{4}{3} \times \pi \times r^3$ (V: volume, r : radius, π : 3.142)
Cone 	$V = \frac{1}{3} \times \pi \times r^2 \times h$ (V: volume, r : radius, π : 3.142, h : height)
Cylinder 	$V = \pi \times r^2 \times h$ (V: volume, π : 3.142, r : radius of base or end, h : height)

- Linear measurements must be in the same units.
- The volume units will then be cubic units of the linear units used.
- When calculating capacity from linear dimensions, calculate the volume in cubic units and then convert to units of capacity.



Example 33 Calculating the volume of pyramids, spheres and cones

Calculate the volume of the following solids. Round your answer to 2 decimal places.



WORKING

THINKING

a $V = \frac{1}{3} \times l^2 \times h$ $\leftarrow \dots\dots$ As the shape is a square-based pyramid, apply the formula $V = \frac{1}{3} \times l^2 \times h$.
 $V = \frac{1}{3} \times 2^2 \times 1.5$
 $V = 2 \text{ m}^3$

Substitute the values for l and h .

b $V = \frac{1}{3} \times l \times w \times h$ $\leftarrow \dots\dots$ As the shape is a rectangular-based pyramid, apply the formula $V = \frac{1}{3} \times l \times w \times h$.
 $V = \frac{1}{3} \times 7 \times 5 \times 8.5$
 $V = 99.17 \text{ cm}^3$

Substitute the values for l , w and h .

c $V = \frac{4}{3} \times \pi \times r^3$ $\leftarrow \dots\dots$ As the shape is a sphere, apply the formula $V = \frac{4}{3} \times \pi \times r^3$.
 $V = \frac{4}{3} \times \pi \times 10^3$
 $V = 4188.79 \text{ mm}^3$

Substitute the value for r .

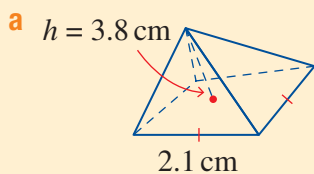
d $V = \frac{1}{3} \times \pi \times r^2 \times h$ $\leftarrow \dots\dots$ As the shape is a cone, apply the formula $V = \frac{1}{3} \times \pi \times r^2 \times h$.
 $V = \frac{1}{3} \times \pi \times 2^2 \times 6$
 $V = 25.13 \text{ cm}^3$

Substitute the value for r and h .

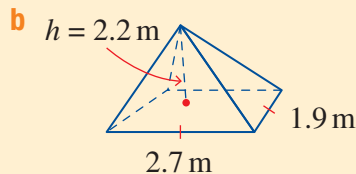


Example 34 Calculating the capacity of solids

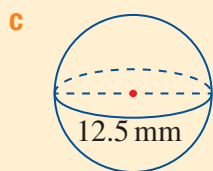
Calculate the capacity of the following solids. Round your answer to 2 decimal places.



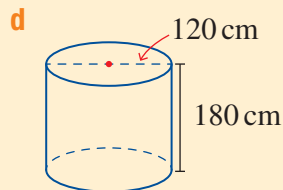
Convert to mL



Convert to kL



Convert to mL



WORKING

THINKING

a $V = \frac{1}{3} \times l^2 \times h$ As the shape is a square-based pyramid, apply the formula $V = \frac{1}{3} \times l^2 \times h$.
 $V = \frac{1}{3} \times 2.1^2 \times 3.8$
 Substitute the values for l and h .

$$V = 5.59 \text{ cm}^3$$

$V = 5.59 \text{ mL}$ Calculate the capacity using $1 \text{ cm}^3 = 1 \text{ mL}$.

b $V = \frac{1}{3} \times l \times w \times h$ As the shape is a rectangular-based pyramid, apply the formula $V = \frac{1}{3} \times l \times w \times h$.
 $V = \frac{1}{3} \times 2.7 \times 1.9 \times 2.2$
 Substitute the values for l , w and h .

$$V = 3.76 \text{ m}^3$$

$V = 3.76 \text{ kL}$ Calculate the capacity using $1 \text{ m}^3 = 1 \text{ kL}$.

c $r = 12.5 \div 2 = 6.25$ As the shape is a sphere, apply the formula
 $V = \frac{4}{3} \times \pi \times r^3$
 $V = \frac{4}{3} \times \pi \times 6.25^3$
 Substitute the value for r .

$$V = 1022.65 \text{ mm}^3$$

... Continued

$$1022.65 \div 10^3$$

$$= 1.02 \text{ cm}^3$$

$$= 1.02 \text{ mL}$$

←..... Calculate the capacity by converting to cubic centimetres. Divide by 10^3 to convert to cm^3 . Use $1 \text{ cm}^3 = 1 \text{ mL}$.

d $r = 120 \div 2 = 60$

←..... First determine the radius by dividing the diameter by 2.

$$V = \pi \times r^2 \times h$$

As the solid is a cylinder, apply the formula

$$V = \pi \times 60^2 \times 180$$

$$V = \pi \times r^2 \times h.$$

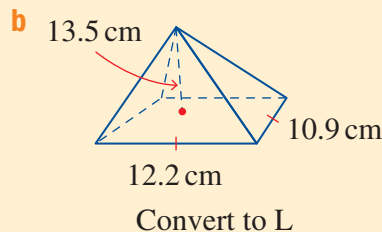
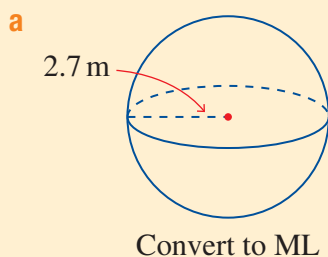
$$V = 2\,035\,752.04 \text{ cm}^3$$

Then substitute the values for $r = 60$ and $h = 180$.



Example 35 Estimating the volume and capacity of solids

Estimate the volume and capacity of the following solids by first rounding each measurement to the nearest whole number.



WORKING

a 2.7 rounds to 3

←..... Round the radius to the nearest whole number. As the first decimal place is greater than 5, round the whole number up to 3.

$$V = \frac{4}{3} \times \pi \times r^3$$

←..... As the shape is a sphere, apply the formula $V = \frac{4}{3} \times \pi \times r^3$. Substitute the estimated value for r .

$$V = \frac{4}{3} \times \pi \times 3^3$$

$$V = 113.10 \text{ m}^3$$

$$V = 113.10 \text{ kL}$$

←..... Calculate the capacity as $1 \text{ m}^3 = 1 \text{ kL}$.

$$113.10 \div 1000 = 0.1131 \text{ ML}$$

Divide by 1000 to convert to megalitres.

b 13.5 rounds to 14
12.2 rounds to 12
10.9 rounds to 11

Round each value to the nearest whole number. The height of the pyramid has a first decimal place of 5 so round up to 14. The length has a first decimal place of 2 so round down to the whole number 12. The width has a first decimal place of 9 so round the whole number up to 11.

$$V = \frac{1}{3} \times l \times w \times h$$

As the shape is a rectangular-based pyramid, apply the formula $V = \frac{1}{3} \times l \times w \times h$. Substitute the estimated values for l , w and h .

$$V = \frac{1}{3} \times 12 \times 11 \times 14$$

$$V = 616 \text{ cm}^3$$

$$V = 616 \text{ mL}$$

$616 \div 100 = 0.616 \text{ L}$ Calculate the capacity as $1 \text{ cm}^3 = 1 \text{ mL}$.



Example 36 Applying volume and capacity to practical problems

Shahida makes large spherical candles. She makes them by pouring melted wax into a spherical mould, which has an internal diameter of 24.3 cm. Calculate the capacity of the mould in litres.

WORKING

THINKING

$$V = \frac{4}{3} \times \pi \times r^3$$

As the shape is a sphere, apply the formula $V = \frac{4}{3} \times \pi \times r^3$.

$$r = 24.3 \div 2 = 12.15$$

Calculate the value of the radius.

$$V = \frac{4}{3} \times \pi \times 12.15^3$$

Substitute the value for r .

$$V = 7513.07 \text{ cm}^3$$

$$V = 7513.07 \text{ mL}$$

Calculate the capacity as $1 \text{ cm}^3 = 1 \text{ mL}$.

$$7513.07 \div 1000 = 7.5 \text{ L}$$

Divide by 1000 to answer in litres.

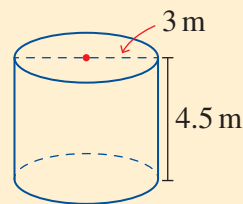
The mould has a capacity of 7.5 litres.

Communicate your solution by writing down the result.



Example 37 Applying volume and capacity to practical problems

Joanne has a rainwater tank on her property that has a diameter of 3 metres and a height of 4.5 metres. Calculate the amount of water in litres that her tank can hold.



WORKING

$$r = 3 \div 2 = 1.5$$

$$V = \pi \times r^2 \times h$$

$$V = \pi \times 1.5^2 \times 4.5$$

$$V = 31.808\,525\,62\,\text{m}^3$$

$$31.81 \times 100^3 = 31\,808\,525.62\,\text{cm}^3$$

$$31\,808\,525.62 \div 1000 \approx 31\,809\,\text{L}$$

Joanne's tank can hold 31 809 L of water.

THINKING

Determine the radius of the tank by dividing the diameter by 2.

Given that the solid is a cylinder, apply the formula $V = \pi \times r^2 \times h$.

Convert the answer to cubic centimetres by multiplying by 100^3 .

Given that $1000\,\text{cm}^3 = 1\,\text{L}$, divide the answer by 1000 to convert to litres, to the nearest litre.

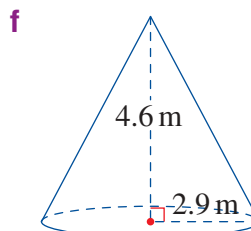
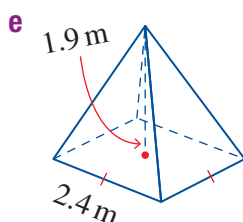
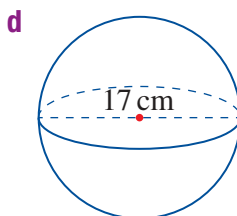
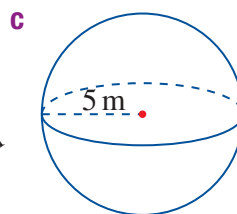
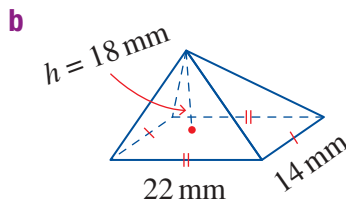
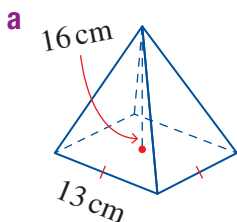
Communicate your solution by writing down the result.

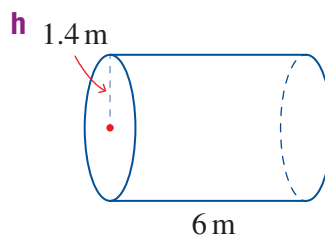
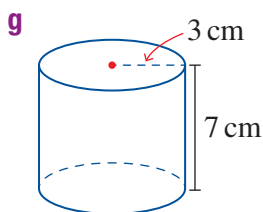
Exercise 1K

FUNDAMENTALS

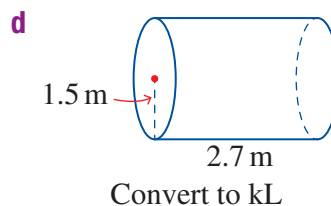
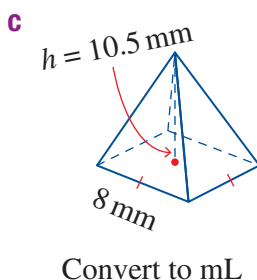
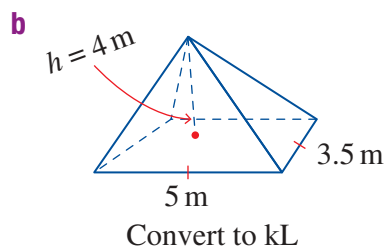
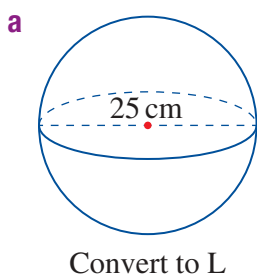
Example 33

1 Calculate the volume of the following solids.

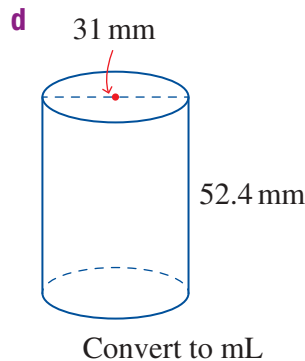
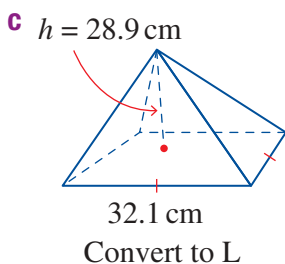
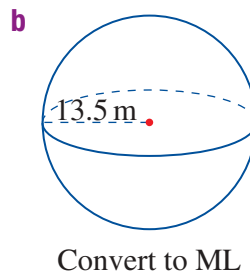
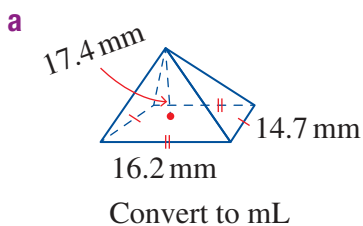




Example 34,37

2 Calculate the capacity of the following solids.

Example 35

3 Estimate the volume and capacity of the following solids.

APPLICATIONS

Example 36

- 4** Bronwyn has designed a rectangular-based pyramid made out of glass that can be used as a paperweight. Calculate the volume if the base has a length of 12 cm and a width of 9 cm, and its height is 8 cm.
- 5** In Richard's barber shop there is a spherical shaped gumball machine. It has an internal diameter of 0.6 metres. Calculate the volume of the gumball machine.
- 6** Phillip makes pyramid shaped snow globes that he then sells as souvenirs at his ski resort. The souvenirs have a 68 mm square base and they are 72 mm high. Calculate the volume of the pyramid snow globe.



- 7** Reuben's water tank is a cylinder and has a radius of 1.2 m and a height of 3 m. Calculate the volume of Reuben's water tank.
- 8** Michael's can of soft drink is 13 cm tall and has a diameter of 6 cm. Calculate the volume of the can and then the capacity of the can in mL, assuming it is a cylinder.
- 9** An industrial gas cylinder has a length of 457 cm and a diameter of 124.5 cm. Calculate the capacity of the cylinder in kilolitres, assuming the ends are flat circles, as with a standard geometric cylinder.



- 10** Ken is a truck driver and his petrol tanker has a length of 5.94 metres and a diameter of 2.2 metres. Calculate the capacity of the petrol tank in kL. Assume it is a cylinder.



- 11** Neil runs a factory that makes candles. One of the most popular designs is a square-based pyramid. The pyramids have an 8.4 cm square base and a height of 10.5 cm. Calculate the number of litres of wax that Neil would require if his factory made 150 candles in a day.

- 12** Susan makes freshly squeezed orange juice. She is planning on using 12 oranges that each have a radius of 3.2 cm when peeled and she does not remove the pulp from her juice. Estimate the total number of litres of juice that Susan should get from her oranges.



- 13** In 1483, Leonardo da Vinci designed a parachute that was shaped like an upturned square-based pyramid. He claimed that a piece of material with a square base of 7 metres wide and a height of 7 metres would enable a person to throw themselves from any height and not suffer any injuries from landing on the ground. Calculate the capacity of the air in litres in the upturned pyramid of da Vinci's design.



- 14** If Leonardo had used a cone with a base diameter of 7 m and a height of 7 m, calculate the capacity of the air in litres in his parachute from question **13**.

Mathematical thinking process task

Background: Prices for storage are often based on volume.

Task: Your task is to calculate the volume required for an average two-person household to store their items in a shipping container while they are overseas. There are three possible options for the shipping container, and the bigger the container, the more expensive it is to hire. The dimensions for each option are as follows:

Small – 3 m long, 2.4 m wide, 2.6 m high

Medium – 6 m long, 2.4 m wide, 2.6 m high

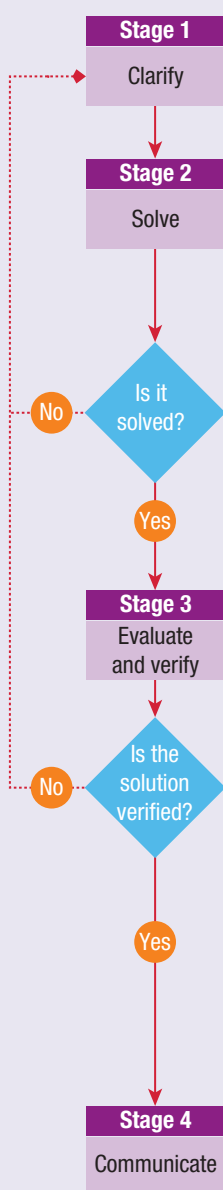
Large – 12 m long, 2.4 m wide, 2.6 m high

You will need to calculate the volume of each shipping container and then the volume of the furniture and household items that you believe an average two-person household will have. To do this you will need to research and estimate the volume of each household item that you believe would need to go into storage such as a couch, television, fridge, washing machine, beds, dining table, and various cardboard boxes. You can research and find websites of storage hire companies that provide calculators to help you estimate the required amount of volume for storage. For example, <https://mobistorage.com.au/space-calculator/>

Once you have calculated the required volume that you will need stored, you then must decide on which shipping container is the most appropriate size and create a report that shows your working and your findings.



Application of mathematical thinking process.

**Stage 1: Clarify**

Interpret the task and gather the key information and identify the mathematics which could help to complete the task

- 1 Research the sizes of various household items and find a storage space estimator website to use.

Stage 2: Solve

Apply existing mathematical knowledge and strategies to obtain a solution

- 2 Make a list of the items you need to be stored.
- 3 Calculate the volume of each of the shipping containers.
- 4 Calculate the estimated volume of each of the furniture items.
- 5 Estimate the number of packing boxes required and calculate the total volume for these.
- 6 Compare the total volume of the household items to the volume of the shipping containers, and decide which shipping container size would be required.

Stage 3: Evaluate and verify

Analyse information and data from a variety of sources and verify the reasonableness of the solution

- 7 Check that you have included all necessary household items from your original list.
- 8 Compare your total volume of your household items to an online calculator from a storage item calculator.
- 9 Check that you have shown all your working for your calculations.
- 10 Check that you have allowed enough excess volume to reasonably cater for composite-shaped furniture and also for spaces in between (your furniture is unlikely to fit together perfectly like a jigsaw puzzle so you are better to overestimate, of course keeping a budget in mind).

Stage 4: Communicate

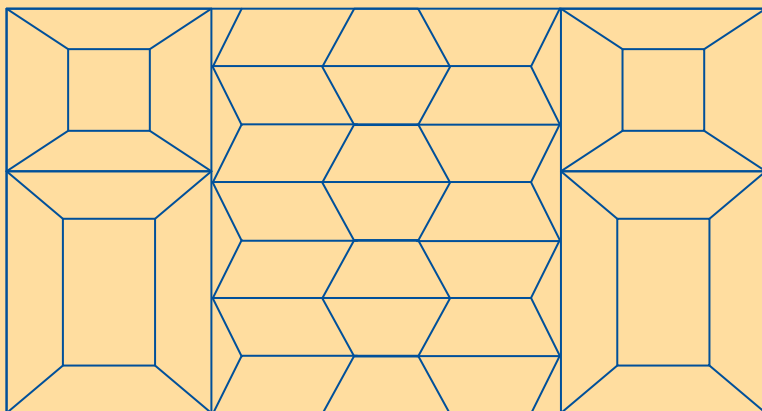
Communicate findings in a systematic and concise manner

- 11 Present all your working in the form of a report and justify your final decision including your reasoning for your solution.

Chapter checklist

☐ I can identify the names and properties of 2D shapes.

- 1 List the properties for a square, rectangle and triangle.
- 2 Identify as many 2D shapes as possible in the following image.

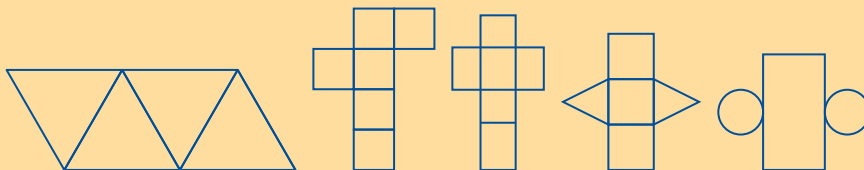


☐ I can identify the names and properties of 3D solids.

- 3 Construct a table that lists the number of faces, edges and vertices in the following solids: cube, triangular prism, rectangular-based pyramid, triangular-based pyramid, pentagonal prism, hexagonal prism.

☐ I can interpret 2D nets to identify 3D solids.

- 4 Identify each 3D solid that is formed from the following nets.



5 Draw nets that best represent each of the following 3D objects.

a



b



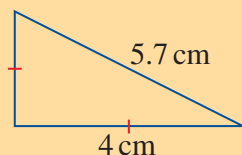
c



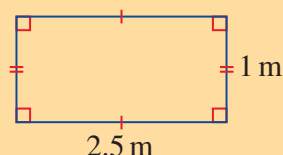
☐ I can calculate the perimeters of familiar shapes.

6 Calculate the perimeter of each of the following familiar 2D shapes.

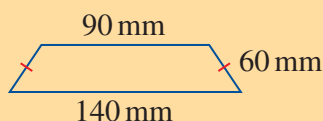
a



b



c



☐ I can calculate the circumference of a circle using the formula $2 \times \pi \times r$ or $\pi \times D$.

7 Calculate the circumference for the following circles, correct to 2 decimal places.

a 3.7 m radius

b 76 mm diameter

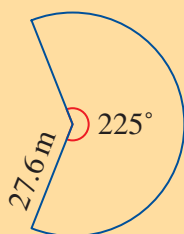
☐ I can calculate arc length and the perimeter of a sector.

8 Correct to 2 decimal places, calculate the:

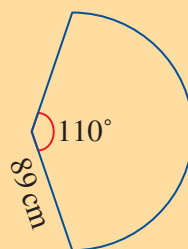
i arc length

ii perimeter of each sector

a



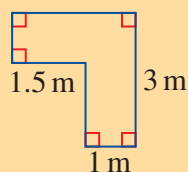
b



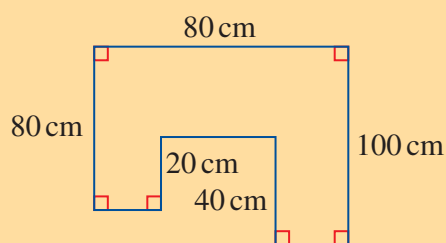
☐ I can calculate perimeters of composite shapes.

9 Calculate the perimeter of the following composite shapes. Where appropriate, write your answers correct to 2 decimal places.

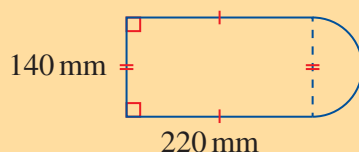
a



b



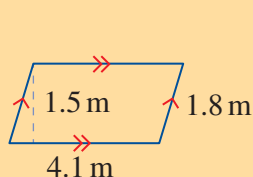
c



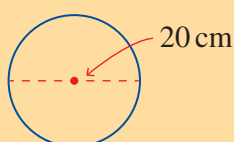
☐ I can calculate and estimate the area of various familiar shapes.

10 Calculate the area of the following common 2D shapes.

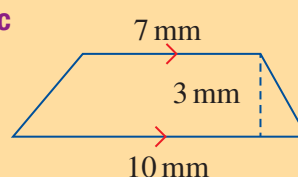
a



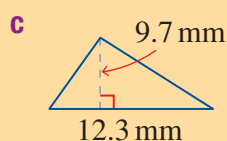
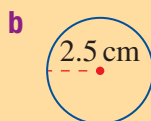
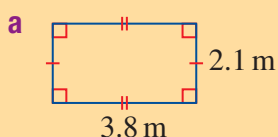
b



c

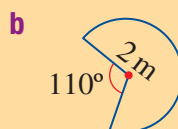
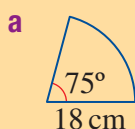


- 11** Estimate the area of the following common 2D shapes by rounding first to the nearest whole number.



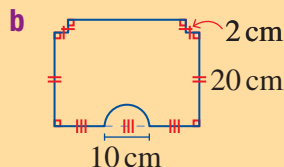
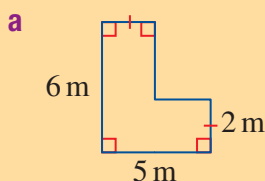
☐ I can calculate the area of sectors.

- 12** Calculate the area of the following sectors. Round your answers to 2 decimal places.



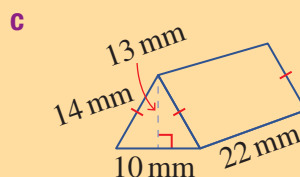
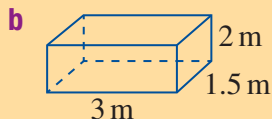
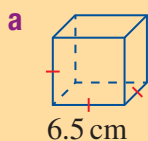
☐ I can calculate the area of composite shapes by decomposing into common shapes.

- 13** Calculate the area of the following composite shapes.



☐ I can calculate the surface area of cubes, rectangular and triangular prisms.

- 14** Calculate the surface area of the following solids.

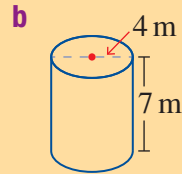
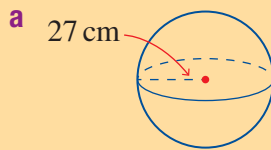


☐ I can calculate the surface area of spheres and cylinders.

15 The formula for the surface area of a sphere is _____.

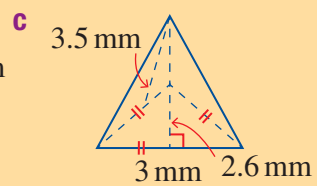
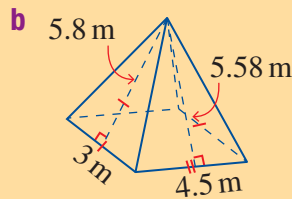
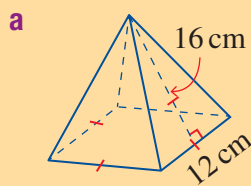
16 The formula for the surface area of a cylinder is _____.

17 Calculate the area of the following sphere and cylinder.



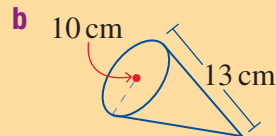
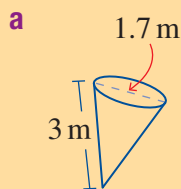
☐ I can calculate the surface area of square-based and rectangular-based pyramids.

18 Calculate the surface area of the following pyramids.



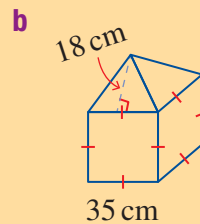
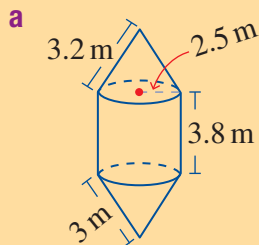
☐ I can calculate the surface area of cones.

19 Calculate the surface area of the following cones.



☐ I can calculate the surface area of irregular solids.

20 Calculate the surface area of the following irregular solids.



☐ I can use units of volume abbreviations.

21 Express the following metric units of volume to their abbreviated forms.

a Cubic millimetres

b Cubic centimetres

c Cubic metres

☐ I can convert between units of volume.

22 Convert the following measurements into the units given in brackets.

a 1.74 cm^3 (mm^3)

b 2.67 m^3 (cm^3)

c 2340 mm^3 (cm^3)

d 0.0001 km^3 (cm^3)

☐ I understand the relationship between volume and capacity including the conversions to millilitres, litres, kilolitres and megalitres.

23 Convert these volume and capacity measurements into the units given in brackets.

a 75 cm^3 (mL)

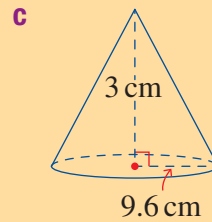
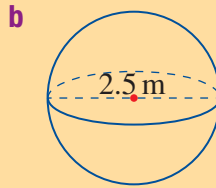
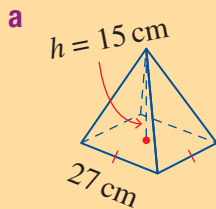
b 2.5 L (cm^3)

c 6172 cm^3 (L)

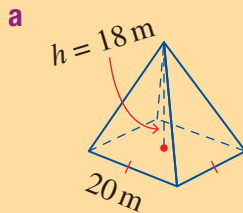
d 3.2 ML (kL)

☐ I can calculate the volume and capacity of pyramids, spheres, cones and cylinders.

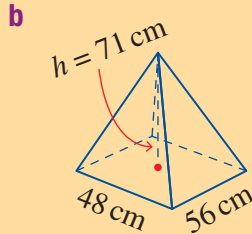
24 Calculate the volume of the following solids.



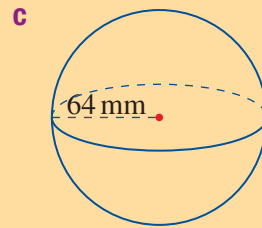
25 Calculate the capacity of the following solids.



Convert to ML



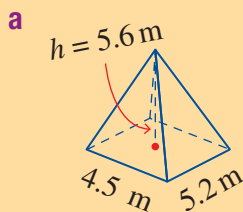
Convert to L



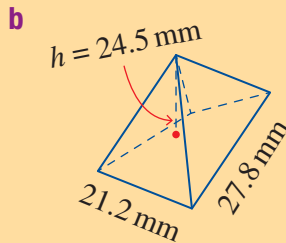
Convert to kL

☐ I can estimate the volume and capacity of pyramids, spheres, cones and cylinders.

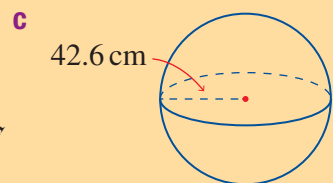
26 Estimate the volume and capacity of the following solids.



Convert to kL



Convert to mL



Convert to L

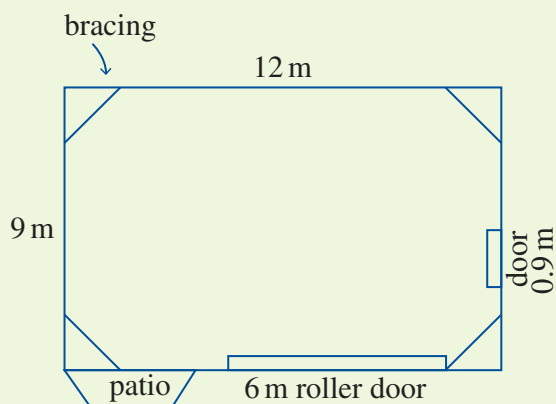
Chapter review

Where appropriate, write your answers correct to 2 decimal places.

Section 1A

- 1** Ashley is a cattle farmer in Busselton. He has a plan for a new hay shed, which is shown in the image.

- a** Identify and list the number of each 2D shape in the hay shed plan.
- b** List the properties of each of these 2D shapes.



- 2** Identify which 3D solid is most similar to the following real-life objects.

a



b



c



d

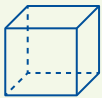
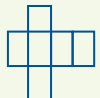
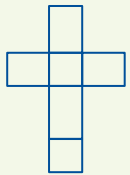
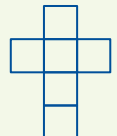
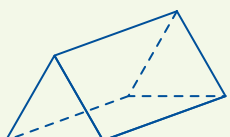
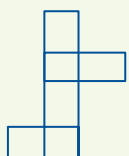
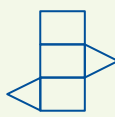
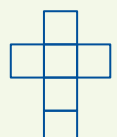
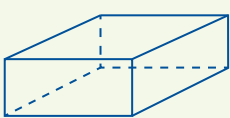
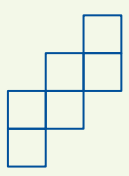
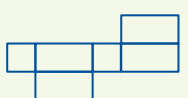
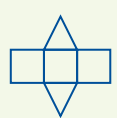
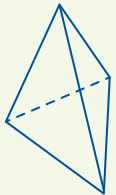
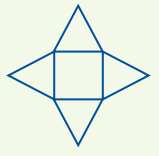
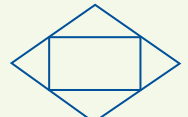


e



Section 1F

- 3** A workshop makes building blocks from metal sheets. Unfortunately, the diagrams of the building blocks and their nets have been mixed up. Decide which net matches with the 3D solid so that the workshop can continue making the building blocks.

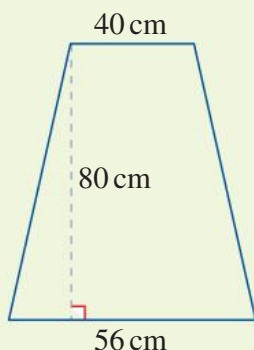
3D solid	Net
a 	i  ii  iii 
b 	i  ii  iii 
c 	i  ii  iii 
d 	i  ii  iii 

Section 1D

- 4** Lyla's horse paddock is $27\,190\text{ m}^2$, where her 3 horses roam. Determine the area of the horse paddock in hectares.
- 5** James has built a new circular training pen for his horses that has a radius of 2.2 metres. Calculate the area of the training pen.

Section 1E

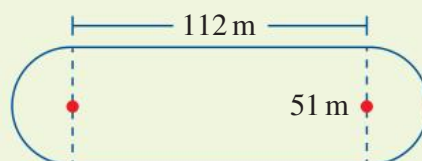
- 6 Emily has designed an A-line skirt that she is now planning on sewing. The image below shows the pattern for the front panel of the skirt. Calculate the area of material that Emily will need to make the front of the skirt.



- 7 Farmer Jo has slashed 310° from one of his circular planted hay crops that has a diameter of 800 metres. Determine the area of land that Jo has slashed for hay.



- 8 Mason is polishing a speed skating rink, as shown in the diagram. Calculate the area of the rink that Mason is polishing.



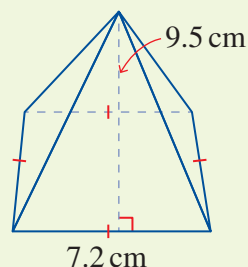
Section 1H

- 9 Mike is a coach for shot put. So that he can easily locate his equipment at sporting events, he paints each shot put yellow. The shot puts have a radius of 60 mm and he has 10 of them. Determine the total surface area that Mike has painted.

- 10** Alisha has just finished making a timber table. The table has 4 legs that are shaped like a cylinder and they have a height of 1.2 m and a diameter of 0.2 m. Before she attaches the legs to the table top, she wishes to varnish them first for protection. Determine the total area that Alisha will be varnishing.

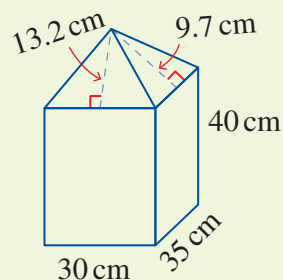
Section 1G

- 11** Haydn has bought his wife a candle in the shape of a square-based pyramid as shown. He has 160 cm^2 of wrapping paper left over at home. Determine if this will be enough wrapping paper to wrap the candle.



Section 1I

- 12** James has made a new letter box from steel that he plans on powder coating. Calculate the total surface area of the letter box as shown in the diagram right.

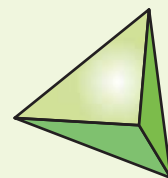


Section 1E

- 13** Cooper has ordered a pizza for dinner. There are 8 slices of pizza, so he has calculated that each piece of pizza is 45° of the whole pizza. Cooper eats 4 pieces of pizza and his wife eats 3 pieces. Determine the area of pizza that both Cooper and his wife have consumed, if the pizza has a diameter of 35 cm.

Section 1I

- 14** A Pyraminx is a puzzle that is similar to the Rubik's cube, but it is the shape of a triangular pyramid with a base that is the same as the sides. (This shape with all four triangular faces the same is called a tetrahedron). The base edge of the Pyraminx is 98 mm and the perpendicular height is 84.9 mm. The company that manufactures the product produces 400 per week and wraps them in plastic. Determine the minimum area of plastic sheet the company would be using each week. Do not include any overlap of the edges of the plastic.



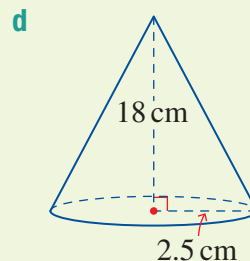
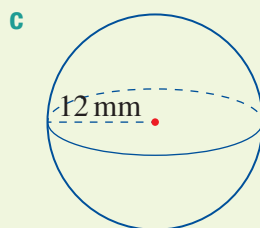
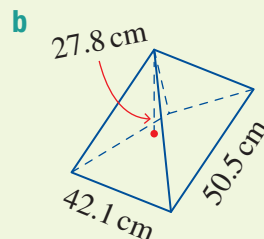
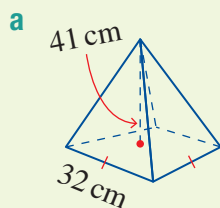
- 15** Jayden's largest silo on his farm is a cylinder with a cone for the cap. It has a diameter of 7.9 metres and the cylindrical section has a height of 19.7 metres. The slant height of the cone is 5.1 metres. Calculate the surface area of the silo.

Section 1J

- 16** Brian has spread 2987 cm^3 of soil. Calculate the volume of soil in cubic metres.
- 17** Ken's spa holds 1400 L of water. Calculate the capacity of the spa in cubic centimetres.
- 18** Daniel has a 50 cm cubic fish tank. Calculate the capacity of the tank in litres.

Section 1K

- 19** Jessica has had a new LPG tank installed in her car. It is a cylinder with a height of 65 cm and a diameter of 27 cm. Calculate the capacity of the tank in litres.
- 20** Jillian has been given a large beach ball that has a diameter of 900 cm. Calculate the capacity of the beach ball in megalitres.
- 21** Calculate the capacity of melted wax that is needed to make candles of these sizes and shapes, in litres for parts **a** and **b**, and in millilitres for parts **c** and **d**.



2 Scales, plans and 3D models





In this chapter

- 2A** Reviewing scales and interpreting scale symbols and abbreviations
 - 2B** Identifying and calculating measurements of length, perimeter and area from scale diagrams
 - 2C** Estimating and comparing quantities, materials and costs from scale diagrams
 - 2D** Understanding and applying drawing conventions of scale drawings
 - 2E** Constructing scale diagrams
 - 2F** Viewing and sketching 3D objects
- Mathematical thinking process task
Chapter checklist
Chapter review

Syllabus reference

Unit 3 Topic 2: Scales, plans and models (11 hours)

Interpret scale drawings

- interpret commonly used symbols and abbreviations in scale drawings
- determine actual measurements of angle, perimeters and areas from scale drawings
- estimate and compare quantities, materials and costs using actual measurements from scale drawings

Creating scale drawings

- understand and apply drawing conventions of scale drawings, such as scales in ratio, dimensions and labelling
- construct scale drawings by hand and by using appropriate software/technology

Three dimensional objects

- interpret plans and elevation views of models
- sketch elevation views of different models
- interpret diagrams of three-dimensional objects

Pre-test

1 Determine the following ratios in simplest form.

- a 12 : 144
- b 25 : 200
- c 50 : 750
- d 14 : 280
- e 11 : 330

Hint Find a common factor of the two numbers in the ratio. Divide both numbers by the common factor.

2 Convert the following measurements in millimetres to metres by dividing by 1000.

- a 2400 mm
- b 1800 mm
- c 14 000 mm
- d 36 000 mm

3 Convert the following measurements in metres to millimetres by multiplying by 1000.

- a 2.25 m
- b 1.4 m
- c 3.84 m
- d 0.279 m

4 Calculate the perimeters of rectangles with the following widths and lengths.

- a 10 m, 15 m
- b 8 cm, 20 cm
- c 7 m, 28 m
- d 120 mm, 200 mm

Hint Recall the definition of a perimeter.

5 Determine the cost of laying carpet in the following rooms, given the carpet costs \$62 per square metre laid.

- a 10 m^2
- b 24 m^2
- c 42 m^2
- d 60 m^2

Hint ' m^2 ' is the symbol for a square metre.

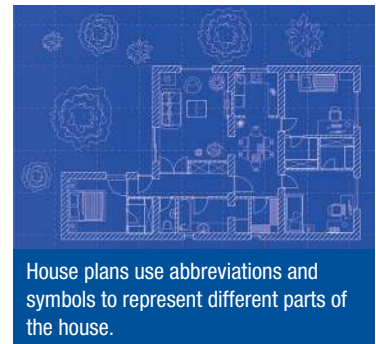
2A Reviewing scales and interpreting scale symbols and abbreviations

LEARNING GOALS

- Review the concept of a scale
- Convert between units of measure
- Simplify scales
- Identify and interpret common symbols and abbreviations in scale diagrams
- Use the internet to research common symbols and abbreviations

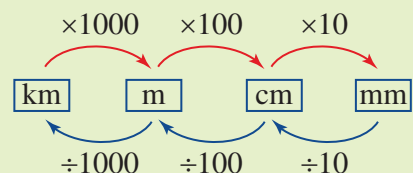
Why is it essential that we use symbols and abbreviations on a scale diagram?

Symbols and abbreviations are added to scale diagrams to help users interpret the information in the diagram and identify the use of each space or detail. Without symbols and abbreviations a detailed plan would be difficult to read as there would be too much information written on the plan.



WHAT YOU NEED TO KNOW

- A **scale** is a comparison of like quantities usually expressed as a **ratio** such as 1 : 100. The first number is the size on the diagram or plan, the second number is the actual size in real life. Remember this as Diagram : Actual.
- **Simplify** a scale by changing both the diagram and actual measurements to the same units, and then simplifying the ratio using common factors.
- Scales are used in construction and manufacturing industries.
- Scale drawings, scale diagrams, and plans are the same thing – the terms are used interchangeably.
- Plans are drawn to scale by designers, planners and architects prior to construction.
- Builders and manufacturers use **scale drawings** to build at full size.
- To convert between units of length use these calculations:
- **Symbols** and **abbreviations** are used on plans to avoid covering detail with text labels.
- Some special or unfamiliar terms such as **void** may be used on house plans.





Example 1 Simplifying scales in ratio

Simplify the following scales.

a 1 cm : 2 m

b 5 mm : 2 m

WORKING

a 1 cm : 2 m
1 cm : 200 cm

1 : 200

b 5 mm : 2 m
5 mm : 2000 mm

1 mm : 400 mm
1 : 400

THINKING

Convert both sides of the scale to the same units.
There are 100 cm in 1 m so multiply by 100.
 $2 \times 100 = 200$ cm
As both units are the same, remove the cm symbol.
Since the scale has 1 as the first number, it doesn't need to be simplified any more.

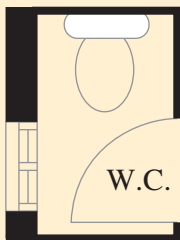
Convert both sides of the scale to the same units.
There are 1000 mm in 1 m so multiply by 1000.
 $2 \text{ m} \times 1000 = 2000 \text{ mm}$

Divide both sides by 5 to simplify.
As both units are the same, remove the mm symbol.



Example 2 Interpreting abbreviations used on scale drawings

Interpret the meaning of the abbreviation W.C. from the diagram.



WORKING

The abbreviation W.C. stands for
Water Closet, which is a toilet.

THINKING

Type 'W.C.' abbreviation into an internet search engine to discover its meaning.



Activity 2A Symbols, abbreviations and special terms used on house plans: See the Interactive Textbook for this activity to research and list abbreviations, symbols and special terms used on house plans.



Example 3 Interpreting symbols used on scale drawings

Interpret the meaning of the symbol  from the diagram.



WORKING

It is a cooktop.

THINKING

←..... As it is in the kitchen, we can use our general knowledge of kitchens to identify that a cooktop would look like this symbol.

Exercise 2A

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a A scale is a comparison of _____ quantities.
 - b _____ are used to represent objects on a scale diagram.
 - c We use _____ to simplify words on a scale diagram.
 - d To convert metres to kilometres we must _____ by 1000.
 - e To convert metres to centimetres we must _____ by 100.
- 2 Convert the following measurements into the unit indicated in brackets.
 - a 5 m (cm)
 - b 28 cm (mm)
 - c 3.73 m (cm)
 - d 2.75 cm (mm)
 - e 4 m (mm)
 - f 6.75 m (mm)
 - g 250 cm (m)
 - h 1200 mm (m)



Example 1 3 Simplify the following scales.

- a 1 cm : 4 m
- b 1 mm : 20 cm
- c 1 mm : 2 m
- d 1 mm : 3.75 m
- e 5 cm : 3 m
- f 7.5 cm : 3.6 m
- g 12 mm : 7.2 m
- h 15 mm : 4.5 m

Hint The scales must have the same units for both numbers.
Divide both numbers by a common factor.

Example 2 4 Interpret the meaning of each of the following abbreviations by first identifying its position in the scale diagram of the house.

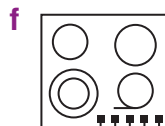
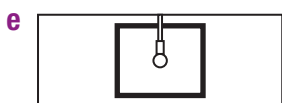
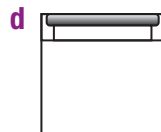
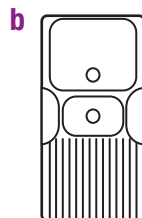
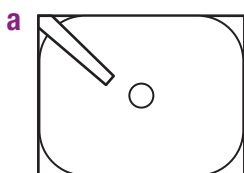


- a L'DRY
- b SRD
- c REF
- d ROBE
- e ASD
- f ASW

Hint Activity 4A will help set up to answer these questions.

Example 3

- 5** Interpret the meaning of each of the following symbols by first identifying its position in the scale diagram of the house in question 4.

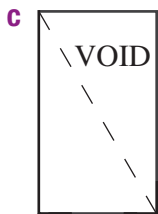
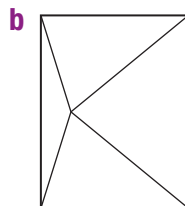
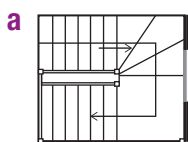


- 6** This plan is the top floor of a two-storey house. Interpret the meaning of each of the following by first identifying its position in the scale diagram of the house.

- a** W.I.R.
b PDR.
c W.I.L.



- 7** Interpret the meaning of each of the following symbols by first identifying its position in the scale diagram of the house in question **6**.



APPLICATIONS

- 8** Luther was looking over the internal plans for his new house from the builder and discovered some abbreviations that he did not understand. These were AS, U/G and ENS. Determine what the abbreviations represent on the plan by researching their meaning.
- 9** Lillian saw the abbreviations of FW, DP, HWS on her house plans and did not understand what they represented. Determine what the abbreviations represent on the plan by researching their meaning.
- 10** Lana wants to create a symbol to represent her grand piano on her new house plans. Create a symbol for her to use.

2B Identifying and calculating measurements of length, perimeter and area from scale diagrams

LEARNING GOALS

- Take precise measurements from drawings to create a scale
- Determine a scale from a plan
- Simplify scales
- Revise calculating perimeters and areas of shapes
- Use a scale to calculate actual measurements and perimeters from plans
- Determine costs in building by using perimeters

Why is it essential that we know how to read and interpret scale plans and diagrams?

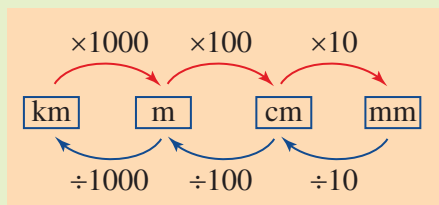
Many professions and occupations use scale diagrams and plans to complete projects such as designing a new car or constructing a building. These professions and occupations include planners, architects, landscapers, builders, designers and manufacturers. By knowing how to read and interpret scale plans and diagrams, a builder can accurately build the owner's/designer's house to the correct measurements and design.



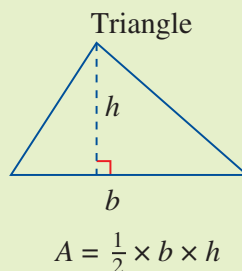
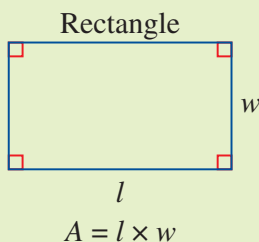
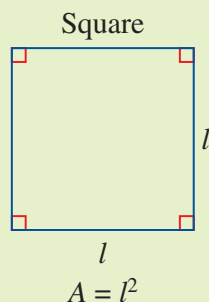
Understanding the symbols, abbreviations and scales of a house plan are important when interpreting the designer's details.

WHAT YOU NEED TO KNOW

- Use a ruler to measure to the nearest millimetre or tenth of a centimetre and ensure that you start at zero.
- Length is measured in metres or units based on the metre.
- Convert measurements of length by using these calculations:



- A scale is the ratio Diagram : Actual for a dimension.
- The **scale factor** is the second number in a scale ratio that starts with 1.
- To convert a diagram length to actual length, multiply by the scale factor.
- To convert an actual length to the diagram length, divide by the scale factor.
- Simplify** a scale by changing both the scale and actual measurements to the same units, and then simplifying the ratio using common factors.
- A perimeter is the outside distance of a 2D shape calculated by adding up the distances of all the sides.
- The area is the number of unit squares in a 2D shape.
- Substitute values into formulas to calculate areas.
- Common area formulas are:



A area
 l length
 w width
 h height
 b breadth

- A **composite shape** is a shape that is made up of two or more common shapes. Addition and/or subtraction can be used to find the area of composite shapes.
- You may also need to recall the area formulas of other shapes from sections 1D and 1E.



Example 4 Determining a scale from a plan

The actual length of the front of the house is 10 m.

- Determine the scale of the plan.
- Calculate the perimeter of the actual house.



WORKING

- The length of the front of the house on the diagram is 4 cm.

Diagram : Actual
4 cm : 10 m

4 cm : 1000 cm

1 : 250

- Perimeter
= length + width + length + width

Width = 31 mm

= 31 mm × 250
= 7750 mm

= 7.75 m

Perimeter
= 10 m + 7.75 m + 10 m + 7.75 m
= 35.5 m

THINKING

Measure the external length of the front of the house on the diagram.

Write the scale as the ratio of the length from the diagram to the actual length of the house.

There are 100 cm in 1 m.

Convert the actual length to cm by multiplying by 100.

Remove the units and simplify the scale by dividing both sides by 4.

We need the length and width to calculate the perimeter of the house. We know the length, now we need to calculate the width.

Measure the external width of the house on the scale diagram. (This time we have used millimetres)

Calculate its actual measurement by using the scale 1 : 250, which means multiply the diagram length by the scale factor which is 250.

Convert the measurement to metres. There are 1000 mm in 1 m, so we divide by 1000.

Calculate the perimeter.

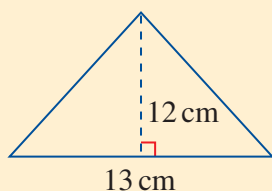
Note that in the example above we could have calculated the scale from the width, and used it to then calculate length. Any dimension can be used to calculate scales.



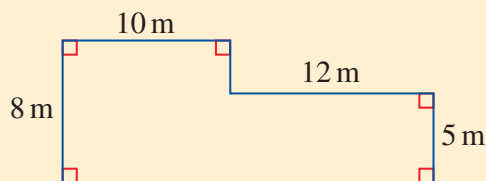
Example 5 Calculating the areas of 2D shapes

Calculate the area of the following shapes.

a



b



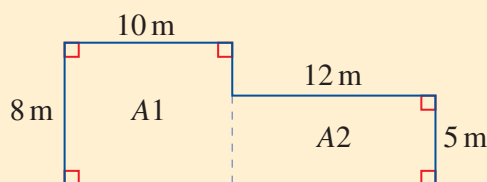
WORKING

a $A = \frac{1}{2} \times b \times h$

$A = \frac{1}{2} \times 13 \times 12$

$A = 78 \text{ cm}^2$

b



$A = l \times w$

$A1 = 10 \times 8$

$A1 = 80 \text{ m}^2$

$A = l \times w$

$A2 = 12 \times 5$

$A2 = 60 \text{ m}^2$

$A = 80 + 60$

$A = 140 \text{ m}^2$

THINKING

Write the area formula for a triangle.

Substitute the measurements into the formula.

Calculate the area of the triangle. Include the units.

Break the shape into two familiar shapes, which are rectangles. Call them A1 and A2.

Write the area formula for a rectangle. For the first rectangle, substitute the measurements into the formula.

Calculate the area of the first rectangle. Include the units.

For the second rectangle, substitute the measurements into the formula. Calculate the area of the second rectangle. Include the units.

Add the areas together to get the total area, A.



Example 6 Calculating the actual measurements from a scale diagram

The actual width of the toilet cubicle is 1 m.

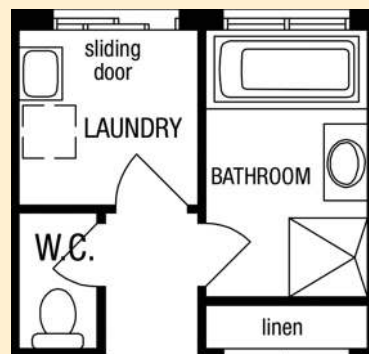
a Determine the scale for the diagram shown.

b Use the scale to determine the dimensions (in metres) of:

- i** the laundry
- ii** the bathroom

c Calculate the perimeter (in metres) of:

- i** the laundry
- ii** the bathroom



WORKING

a The width = 1 cm

Diagram: Actual

1 cm : 1 m

1 cm : 100 cm

1 : 100

b i Diagram dimensions are 2.2 cm by 2.2 cm

Actual dimensions are

220 cm by 220 cm

= 2.2 m by 2.2 m

ii Diagram dimensions are 2.1 cm by 3.5 cm

Actual dimensions are

210 cm by 350 cm

= 2.1 m by 3.5 m

c i $P = l + w + l + w$

$P = 2.2 + 2.2 + 2.2 + 2.2$

$P = 8.8$ m

ii $P = l + w + l + w$

$P = 2.1 + 3.5 + 2.1 + 3.5$

$P = 11.2$ m

THINKING

The toilet cubicle is labelled W.C.

Measure the internal width of the W.C.

Write as a scale of the W.C. width on the diagram to the actual W.C. width.

Convert the actual width to cm by multiplying by 100.

Remove the units.

Measure the internal length and internal width of the laundry.

Multiply both measurements by the scale factor of 100.

Convert to metres by dividing by 100.

Measure the internal length and internal width of the bathroom.

Multiply both measurements by the scale factor of 100.

Convert to metres by dividing by 100.

Use the length and width to calculate the perimeter of the laundry.

Use the length and width to calculate the perimeter of the bathroom.

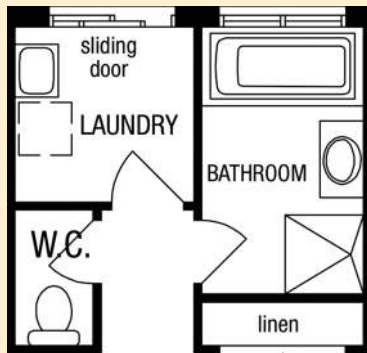


Example 7 Calculating areas using measurements from a scale diagram

The actual width of the toilet cubicle is 1 m.

a Determine the scale for the diagram below.

b Use the scale to calculate the area of the bathroom, in metres.



WORKING

a The width = 1 cm
Scale : Actual

$$\begin{aligned} 1 \text{ cm} &: 1 \text{ m} \\ 1 \text{ cm} &: 100 \text{ cm} \\ 1 &: 100 \end{aligned}$$

b Diagram dimensions are 2.1 cm by 3.5 cm

Actual dimensions are 210 cm by 350 cm
= 2.1 m by 3.5 m

$$A = l \times w$$

$$\begin{aligned} A &= 2.1 \times 3.5 \\ A &= 7.35 \text{ m}^2 \end{aligned}$$

THINKING

Measure the width of the toilet.
Write as a scale to the actual toilet width.

Convert the actual width to cm by multiplying by 100.
Remove the units.

Measure the length and width of the bathroom.

Multiply by the scale factor of 100.
Convert to metres by dividing by 100.

Write down the area formula for a rectangle.

Substitute the values to calculate the area of the bathroom and write the units.



Technology activity 2B: Using the measuring tool in Adobe Acrobat Reader to measure dimensions on a scale drawing in a PDF file. See the Interactive Textbook for this activity.

Exercise 2B

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a A _____ is the outside distance on a 2D shape.
 - b The _____ is the number of unit squares in a 2D shape.
 - c We _____ values into _____ to calculate areas.
 - d Area of a rectangle = _____ $\times$ _____
 - e Area of a triangle = $\frac{1}{2} \times$ _____ $\times$ _____

Example 4

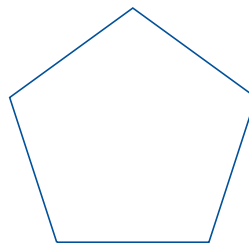
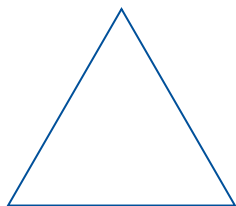
- 2 The actual length of the side of each shape is given in brackets. For each shape, measure the side lengths to:

i determine the scale

ii calculate the actual perimeter

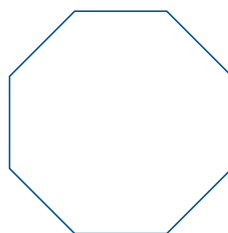
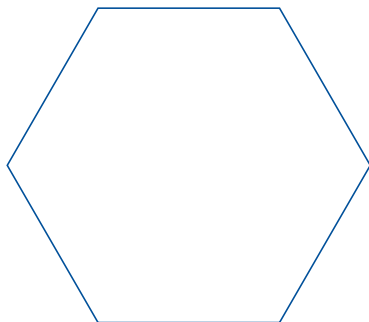
a (6 m)

b (2 m)



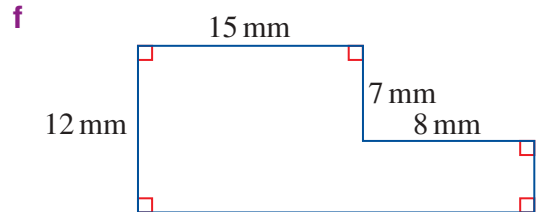
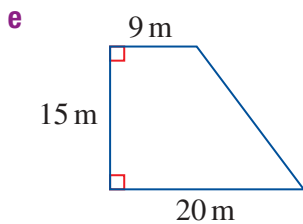
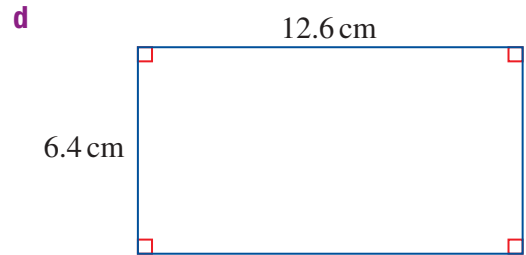
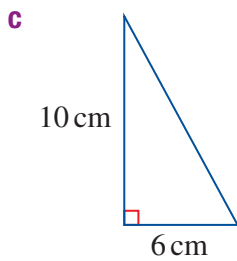
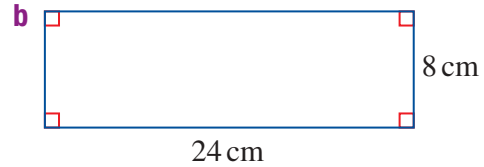
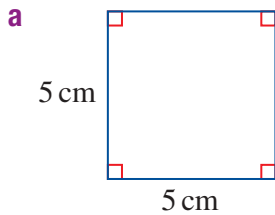
c (1.5 m)

d (7.5 m)

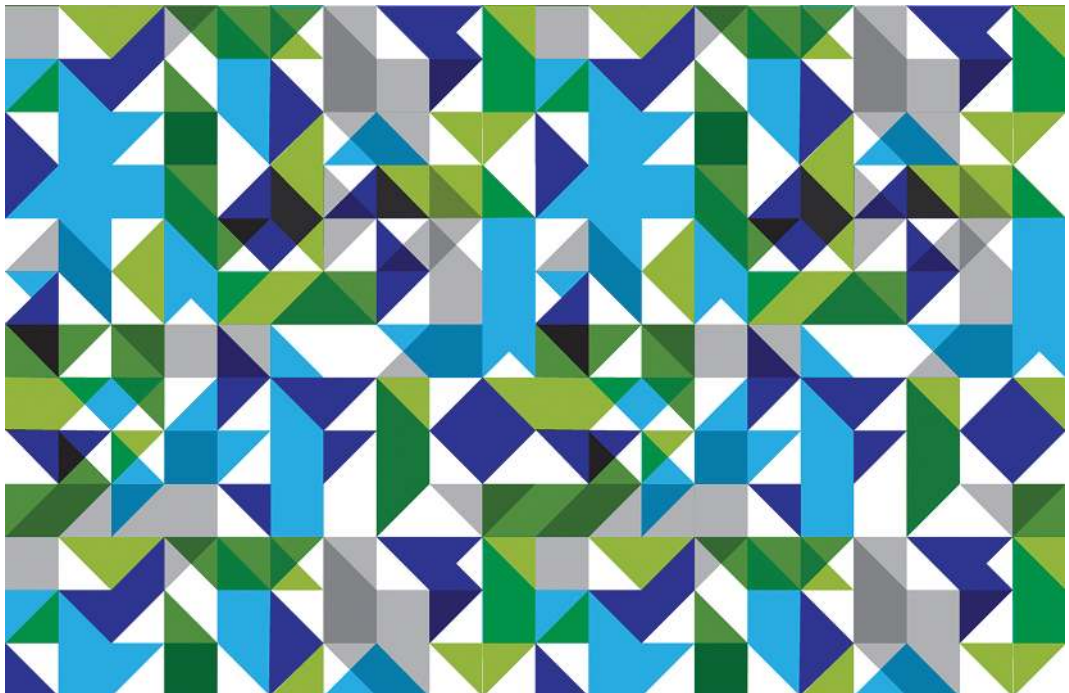


Hint A scale is a ratio of size in diagram to actual size.



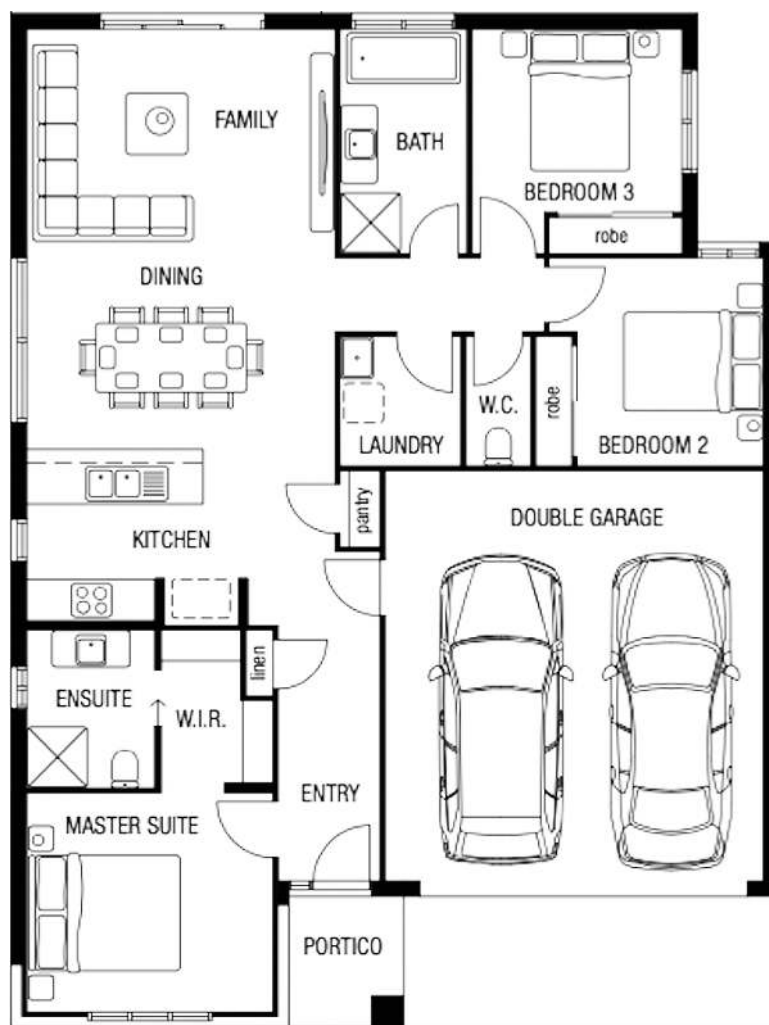
Example 5**3** Calculate the area of the following shapes to 1 decimal place where necessary.

Hint These last two are composite shapes that should be divided up.



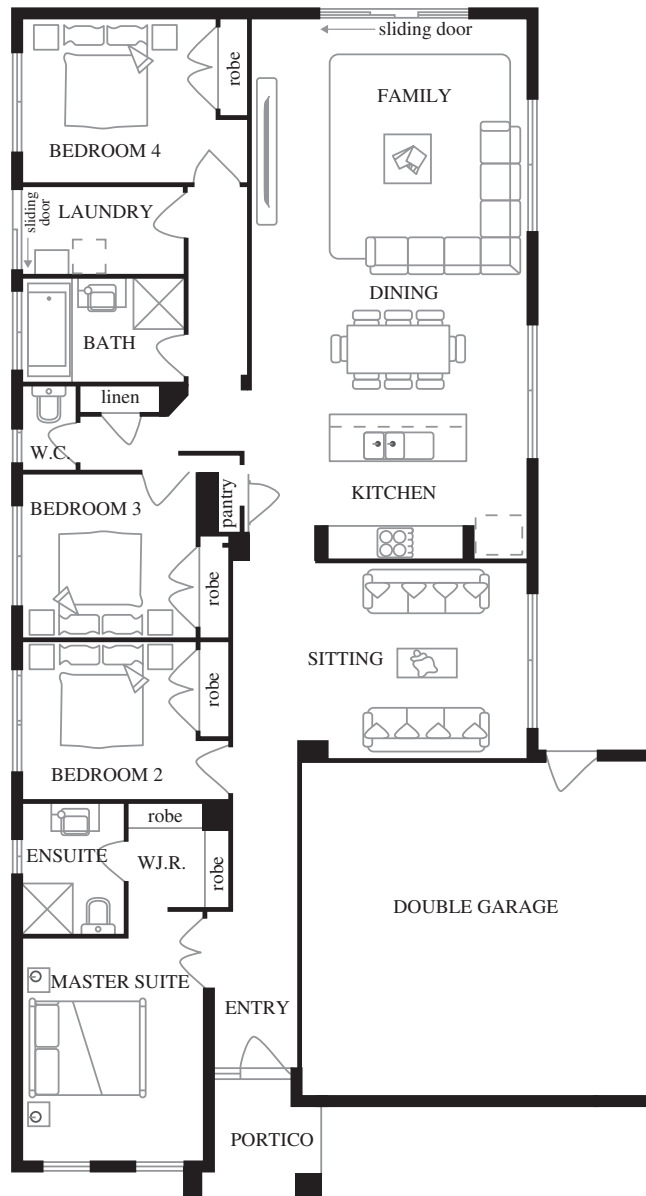
APPLICATIONS

- Example 6** 4 Refer to the plan below. The actual width of the garage is 6 m.
- a** Determine the scale of the plan.
 - b** Use the scale to determine the actual dimensions (in millimetres) of the:
 - i** bedroom 3 (including robe)
 - ii** W.C.
 - iii** laundry
 - iv** family/dining area
 - v** ensuite
 - vi** garage
 - c** Calculate the actual perimeter (in metres) of each part of the house in metres from part **b**.

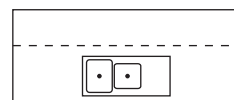


Example 7

- 5** Refer to the plan below. The actual width of the garage is 5 m.
- Determine the scale for the plan.
 - Use the scale to calculate the dimensions (in millimetres) of the:
 - master suite (not including the ensuite and W.I.R.)
 - garage
 - bathroom
 - ensuite
 - Calculate the area of the rooms listed in part **b** in square metres.



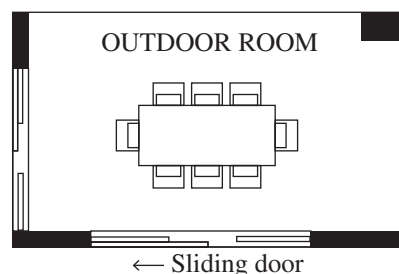
- 6 Monica measured the actual length of the island bench to be 3.6 m. Determine the scale of the plan.



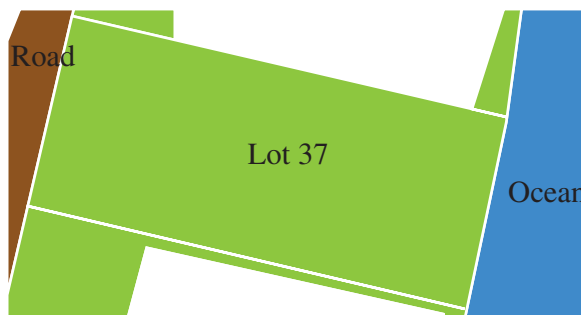
KITCHEN



- 7 Wayne measured the actual internal length of the outdoor room to be 4 m. Determine the scale of the plan.



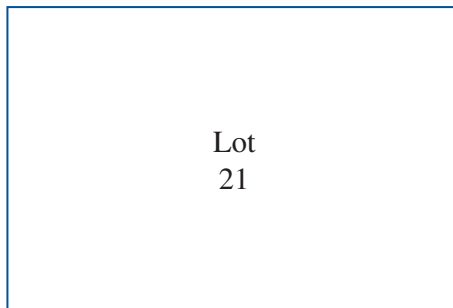
- 8 James measured the land frontage to the road of his new block (Lot 37) to be 20 m wide (This means the side of the plot next to the road.). He wants to determine the area of the block so that he can choose a house to fit on the block.



- a Determine the scale for the plan of the block of land.
b Use the scale to calculate the area of Lot 37.
Assume the block of land is a rectangle.

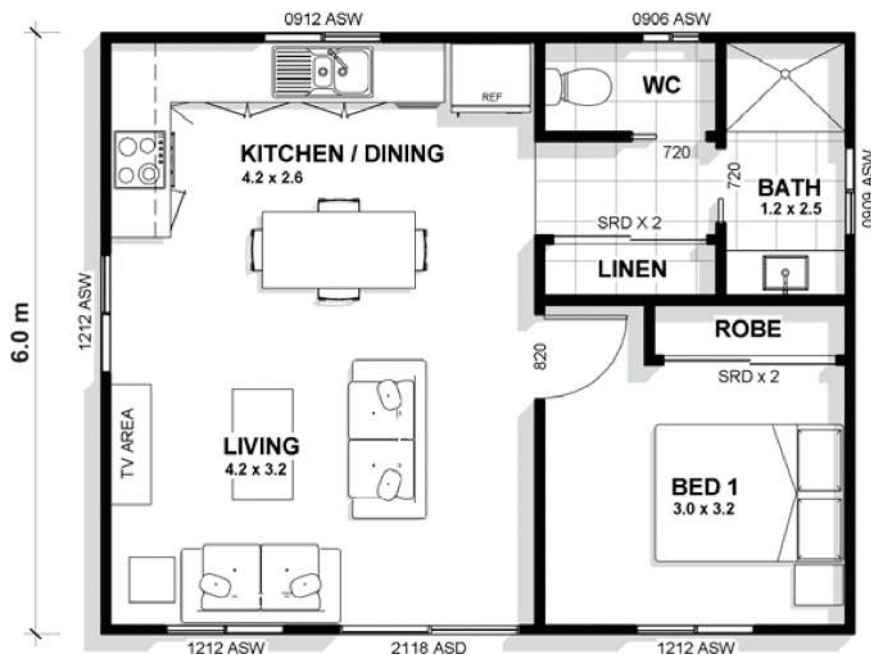


- 9** Wilma has bought a block of land named Lot 21. The actual length of the shortest side is 20 metres.
- Determine the scale for this diagram of the block of land.
 - Calculate the perimeter of her block of land.



- 10** Henry is building a granny flat in his backyard for his mother. The council allows a maximum area for a granny flat of 60 m^2 . One side length of the plan for the granny flat is marked as 6 m.
- Determine the scale for the plan of the granny flat.
 - Calculate the actual area of the Henry's granny flat.
 - Consider whether the granny flat meets council requirements.

Hint Measure on the diagram the side length that is labelled as 6 m, which will give you the scale.



2C Estimating and comparing quantities, materials and costs from scale diagrams

LEARNING GOALS

- Determine costs from dimensions and areas calculated from scale diagrams
- Estimate quantities of materials needed
- Compare costs of materials to determine best values

Why is it essential to be able to calculate dimensions and areas from plans?

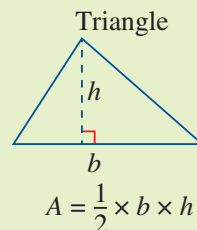
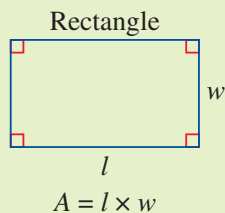
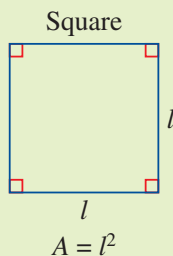
Homeowners need to budget for home renovations, building homes and landscaping. Many commonly used materials are sold by length or area. Skirting boards are sold by length. Stores sell floor coverings such as carpet and tiles by the square metre. Knowing how to calculate areas in square metres from a plan helps people to determine what tiles and carpet they can afford.



Tiles are priced in dollars per m^2 , and the tiler charges for labour in the same units.

WHAT YOU NEED TO KNOW

- Use a ruler to measure to the nearest millimetre or tenth of a centimetre, and ensure that you start at zero.
- Simplify a scale by changing both the scale and actual measurements to the same units, and then simplifying the ratio using common factors.
- The area is the number of unit squares in a 2D shape.
- Substitute values into formulas to calculate areas.
- Area formulas for some familiar 2D shapes.



- A 'linear' or 'lineal' metre is just a metre of length, the term is used to emphasise it is not a square metre.
- Cost is usually calculated as number of units multiplied by price in dollars per the same unit. For example, the cost of 3 m of material sold by length and priced at \$5.00 per metre is \$15.



Example 8 Determining costs using calculated areas

Ryan is tiling his home theatre, which is a rectangle with dimensions 5.5 m by 4 m. The tiles that Ryan has chosen cost \$29.95 per square metre and the tiler charges \$45 per square metre to lay the tiles.

- Calculate the area of Ryan's home theatre.
- Determine how much it will cost Ryan to tile his home theatre.
- After tiling the tiler offers to install a decorative skirting board around the perimeter of the room at a cost of \$3.60 per metre. Calculate the extra cost, ignoring the doorway.

WORKING

$$\begin{aligned} \text{a } A &= l \times w \\ A &= 5.5 \times 4 \\ A &= 22 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{b } \text{Add cost of tiles and tiler: } & \text{ } \\ \$29.95 + \$45 &= \$74.95 \text{ per m}^2 \\ \text{Total cost} &= 22 \text{ m}^2 \times \$74.95 \\ &= \$1648.90 \end{aligned}$$

$$\begin{aligned} \text{c } \text{Perimeter} &= \text{ } \\ \text{length} + \text{width} + \text{length} + \text{width} & \\ &= 5.5 + 4 + 5.5 + 4 \\ &= 19 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Cost of skirting} &= 19 \text{ m} \times \$3.60/\text{m} \\ &= \$68.40 \end{aligned}$$

THINKING

Write the formula for area of a rectangle.
Substitute the values of length and width into the formula.
Calculate the area.

Add up the cost per m^2 of the tiles and the labour of the tiler.
Calculate the total cost by multiplying the area by the cost per m^2 .

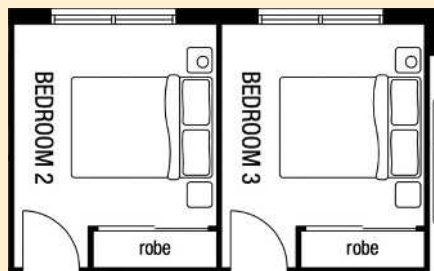
The perimeter is the distance around the edge of the room.
Add up the sides.

Calculate the cost by multiplying the perimeter by the cost per m.



Example 9 Determining costs using areas calculated from a scale diagram

Erin is replacing the carpet in two of her bedrooms as shown in the plan. The carpet that Erin has chosen costs \$43.75 per square metre fully laid. The actual robe length is 2 m.



- Determine the scale for the plan of the bedrooms.
- Determine the combined area of the bedrooms, including the robes, ignoring the joining wall and robe walls.
- Calculate the cost of the carpet.

WORKING

THINKING

- 16 mm ←..... Measure the length of the robe on the drawing.

16 mm : 2000 mm

1 : 125

Write the scale as a ratio of length in the drawing to the actual robe length in the same units.

Simplify the scale.
- The rooms are both ←..... Measure the length and width of the 2 rooms together, ignore the joining wall and robe walls in your calculations.

54 mm by 26 mm

54 mm × 125 by 26 mm × 125 ←..... Determine the actual dimensions of the rooms by multiplying by the scale factor of 125.

6750 mm by 3250 mm

6.75 m by 3.25 m ←..... Convert to metres.

$A = l \times w$ ←..... Use the area of rectangle formula.

$A = 6.75 \times 3.25$ Calculate the area of the rooms.

$A = 21.94 \text{ m}^2$
- Cost = $21.94 \text{ m}^2 \times \$43.75$ ←..... Calculate the cost by multiplying the area by the cost per m^2 .

= \$959.88

Exercise 2C

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a We _____ a scale by dividing both parts by the same number.
 - b A _____ is the second number in the scale ratio when the first number is 1.
 - c We calculate cost by multiplying the _____ per unit by the number of _____.

Example 8

- 2 Bella has just completed her new rumpus room that she built underneath her house. She wants to buy skirting boards to place around the edge of the rumpus room. Bella's rumpus room is 6 m by 4 m and the skirting boards cost \$20 per metre.
 - a Calculate the perimeter of the rumpus room.
 - b Calculate the cost of the skirting boards by multiplying the cost per metre by the perimeter.
- 3 Tom wants to carpet his bedroom. His bedroom is 3 m by 3.5 m in dimensions and the cost of carpet is \$52 fully laid per square metre.
 - a Calculate the area of the bedroom.
 - b Calculate the cost to carpet the room.

Hint Total cost is the price per unit times the number of units.

Example 9

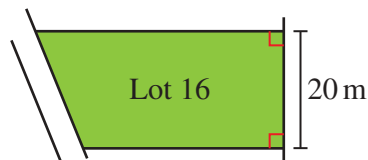
- 4 Tamika wants to tile her patio. The patio is 6 m by 5.5 m and the cost of tiling is \$65 fully laid per square metre.
 - a Calculate the area of the patio.
 - b Calculate the cost to tile the patio.

APPLICATIONS

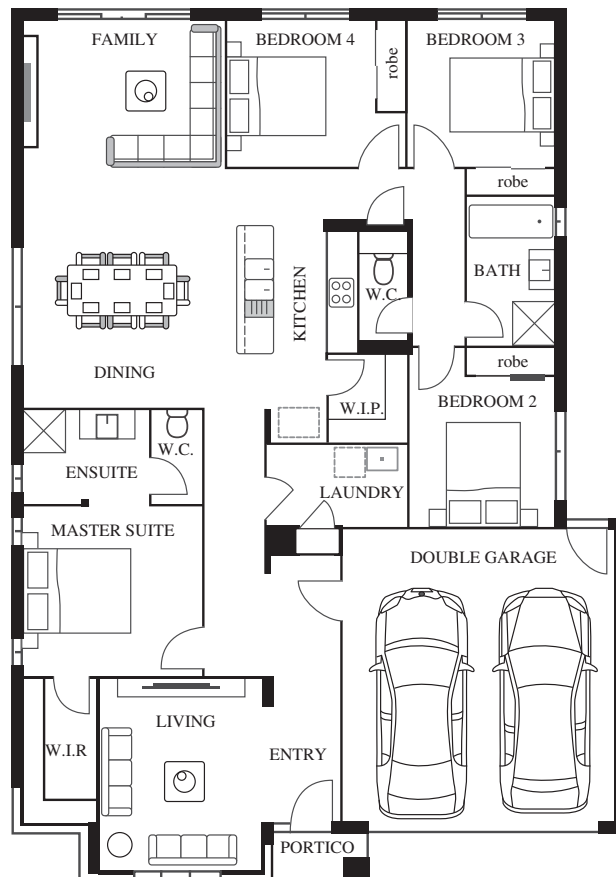
Henry is working on some extra features for the granny flat for his mother. Use the following scale drawing to answer questions 5 to 7.



- 5** Henry has decided that he wants to have a concrete mowing strip poured around the edge of the granny flat in his backyard.
- Determine the scale of the granny flat plan.
 - Use the scale to calculate the length of the granny flat.
 - Calculate the perimeter using the actual length and width of the granny flat. The concreting will cost \$20 per metre of the perimeter of the flat.
 - Calculate the total cost of the mowing strip by multiplying the cost per metre by the perimeter.
- 6** Henry's mum Dorethy has decided that she wants a special cornice (decorative strip to border the ceiling and the walls) in her bedroom. The cost of the cornice is \$4.75 per linear metre.
- Measure the length and width of the granny flat bedroom on the diagram, ignoring the robe and door.
 - Using the scale you have found in the previous question, determine the actual length and width of the bedroom.
 - Calculate the perimeter of the bedroom.
 - Calculate the cost of the cornice by multiplying the cost per linear metre by the perimeter.
- 7** Henry wants to lay wooden floorboards in the kitchen, dining and living space. The cost of the floorboards including laying is \$62 per square metre.
- Measure the length and width of the kitchen, dining and living area ignoring the cabinets and furniture.
 - Using the scale you have found in the previous question, determine the actual length and width of the kitchen, dining and living area.
 - Calculate the area of the kitchen, dining and living area.
 - Calculate the cost of laying the floorboards in the granny flat by multiplying the cost per square metre by the area.
- 8** Alaskah is buying a block of land as shown in the scale drawing. If she is fencing all 4 sides at a cost of \$47.50 a metre, calculate the total cost of fencing Alaskah's property.

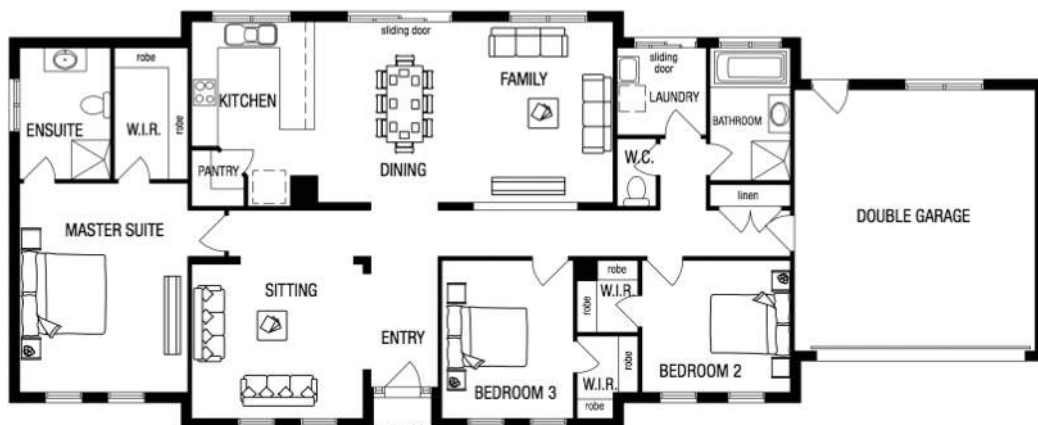


For questions 9 to 13 use the following house plan where the actual width of the garage is 6 metres.



- 9** Rae is a tiler who is quoting to tile the laundry floor. If Rae charges \$70 per square metre to lay the tiles and the tiles cost \$34 per square metre, determine the cost to tile the laundry floor.
- 10** Julie wants to carpet the living room, which is beside the entry.
- If the carpet costs \$53.75 per square metre, calculate the cost to carpet the living room.
 - Julie sees an advertisement for carpeting a lounge area for \$600, determine whether this would be a cheaper way for Julie to carpet her lounge area.

- 11** Aston wants to buy paving paint to paint the floor of his garage. One litre of paint covers 8 m^2 and costs \$54.
- Calculate how many tins of paving paint are needed to paint the floor of the garage.
 - Determine the total cost for the paint.
- 12** Jang has discovered that the garage does not have any ceiling insulation and she has decided to purchase the insulation and install it herself.
- If it costs \$11.50 per square metre, calculate the total cost for Jang to insulate the ceiling herself.
 - If the builder offers to insulate the garage for \$450, determine which will be the cheapest way for Jang to insulate her garage.
- 13** Jii built this house on a 30 metre and 20 metre block and he is going to lay turf at a cost of \$7.70 per square metre. He estimates the area of the house to the nearest square metre by measuring the greatest length by the greatest width. Determine the approximate cost to turf the block.
- Hint** The turf is laid on the area of the block minus the area of the house.
- 14** Gertie's Guttering is quoting for supplying gutters for the roof of the building shown. Unfortunately, the plan does not have a scale but the owner knows that the garage is 6.4 m wide. The guttering costs \$12 per metre.
- Determine the scale.
 - Calculate the perimeter of the building.
 - Calculate the overall cost of the guttering assuming that the perimeter of the roof is the same as the perimeter of the building.



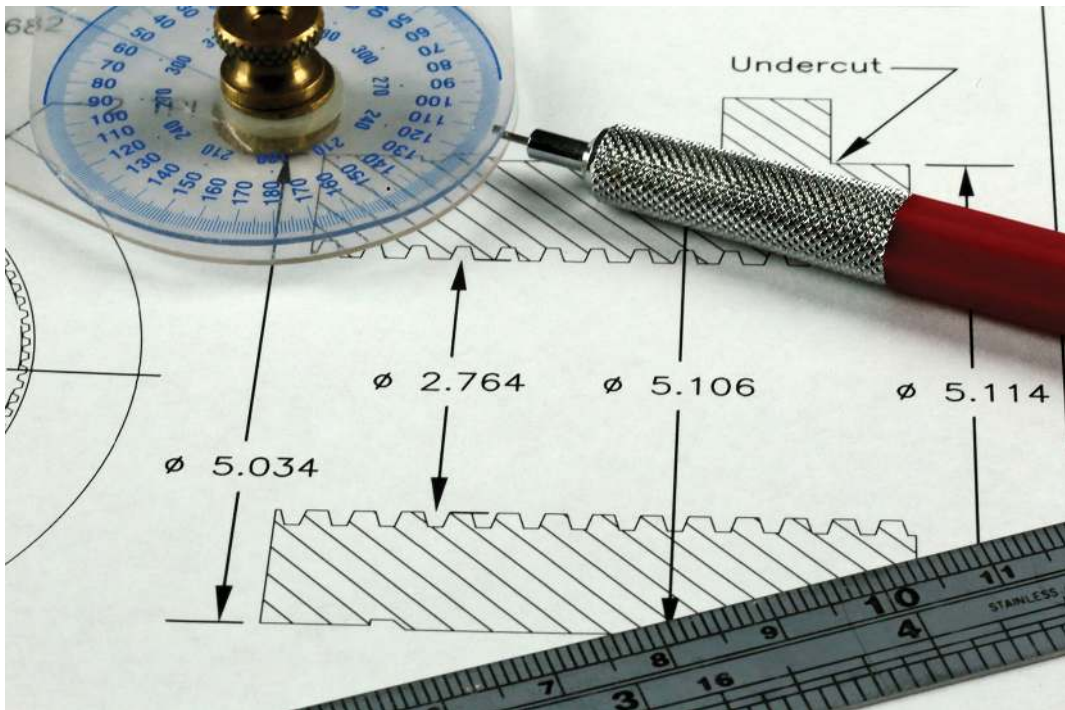
2D Understanding and applying drawing conventions of scale drawings

LEARNING GOALS

- Identify drawing conventions for scale diagrams
- Identify labelling techniques of ratio in scale diagrams
- Read measurements from scale diagrams
- Understand and apply drawing conventions of scale drawings

Why is it essential to be able to identify and apply drawing conventions for scale drawings?

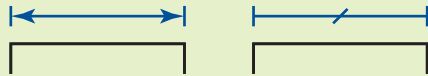
In order for plans to be used by multiple people in the process of any construction, a universally agreed convention must be used so that there is an understanding of what the plans say. Some people create plans and other people need to read the plans to build or construct so drawing conventions are vital.



Using drawing conventions to draw scale plans helps others interpret the plans for building or manufacturing.

WHAT YOU NEED TO KNOW

- Depending on what types of plans you are reading, the measurements are marked in slightly different ways, either with arrows or with lines marked on a parallel line.

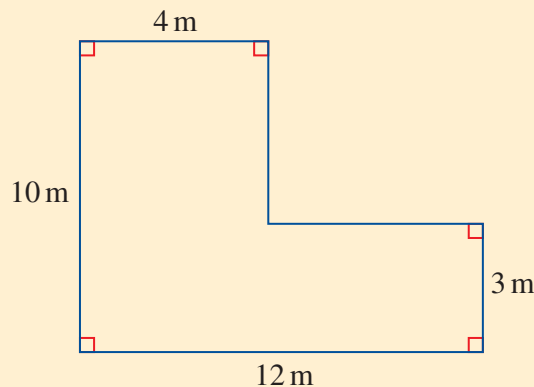


- A very common scale is 1 : 100 for building plans meaning that 1 centimetre on the plan indicates 1 metre in the actual building.
- An engineering scale where a small part is drawn enlarged on a plan would be 10 : 1 meaning that 10 millimetres (1 centimetre) on the plan would be 1 millimetre in reality on the part.
- Common symbols, as discussed in section 2A, are used in plans creating a uniformity such as the symbol for a shower. 
- In order not to crowd the plans, not all measurements are named; however, all the measurements can be found from other markings on the plans.



Example 10 Reading measurements from indicated dimensions

Determine the missing side measurements on the following diagram.



WORKING

Vertical measurement
 $= 10\text{ m} - 3\text{ m} = 7\text{ m}$

Horizontal measurement
 $= 12\text{ m} - 4\text{ m} = 8\text{ m}$

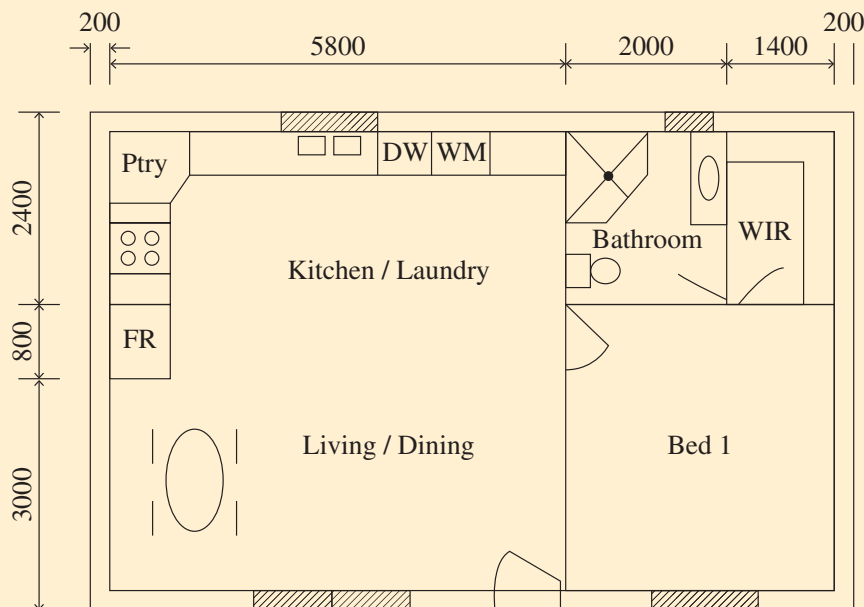
THINKING

◀..... Determine what is left when the 3 m side is subtracted from the 10 m side.

◀..... Determine what is left when the 4 m side is subtracted from the 12 m side.



Example 11 Understanding and applying drawing conventions of scale drawings



Use the plan shown to answer the following questions. Measurements are in mm.

- Identify the width of the external walls.
- Identify the internal dimensions of the kitchen/laundry/living/dining area.
- Determine what the measurement of 2000 mm found on the top side of the plan measures.
- Determine the internal measurements of bedroom 1.
- The council requires a granny flat to be less than 60 m^2 . Calculate the area of this plan to determine if it will fit council requirements.

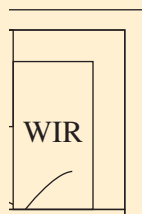
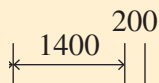
WORKING

- a** 200 mm



THINKING

The width of the external wall is shown as the first measurement along the length.



b

Length = 5800 mm
 Width = $3000 + 800 + 2400$
 $-200 - 200$
 $= 5800$ mm
 Dimensions are 5800 mm by 5800 mm.

Read the dimensions of the kitchen/laundry/living/dining area.

←... Length = 5800
 Width = $3000 + 800 + 2400 - 200 - 200$
 (subtracting the external wall from each end)

c The width of the bathroom.

←... Follow the lines on the measurement to identify the measured section.

d Width = $2000 + 1400 = 3400$ mm ←
 Length = $3000 + 800 - 200$
 $= 3600$ mm
 Dimensions are 3600 mm by 3400 mm.

Reading the dimensions from the plan.
 Width = $2000 + 1400$
 Length = $3000 + 800 - 200$
 (subtract the width of the outer wall)

e Length = $200 + 5800 + 2000$
 $+1400 + 200$
 $= 9600$ mm
 Width = $2400 + 800 + 3000$
 $= 6200$ mm

←... Read the outside dimensions of the building.

9600 mm = 9.6 m
 6200 mm = 6.2 m

Convert the measurements to metres.

$A = 9.6 \text{ m} \times 6.2 \text{ m}$
 $A = 59.52 \text{ m}^2$

Calculate the area by multiplying the length by the width.

The area will fit within the council requirements as it less than 60 m^2 .

Identify whether or not the granny flat fits the council requirements.



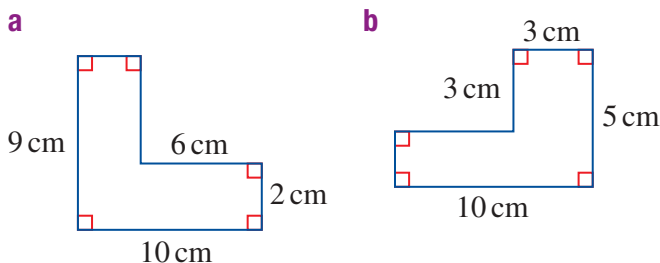
Exercise 2D

FUNDAMENTALS

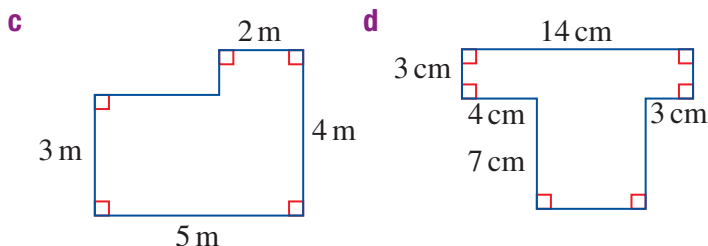
- Determine the missing words in the following sentences.
 - A common scale used for _____ is _____: 100.
 - Common _____ create uniformity in plans.
 - Not all _____ are always marked on a _____, however all _____ can be calculated from a plan.
 - Dimensions in plans are not always marked the same, but they usually have _____ or lines on parallel lines.

Example 10

- Determine the missing side measurements in the following diagrams.

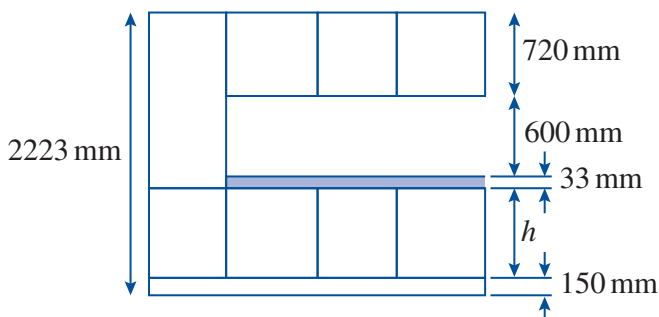


Hint The unknown side lengths are the difference between two or more known side lengths.



APPLICATIONS

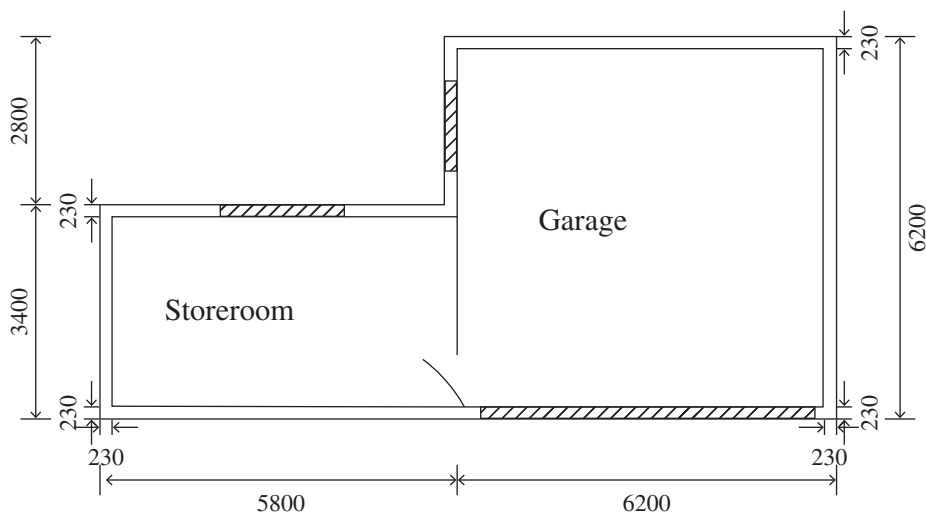
- Janice wants to work out the actual height of the base cabinets from this kitchen plan shown.



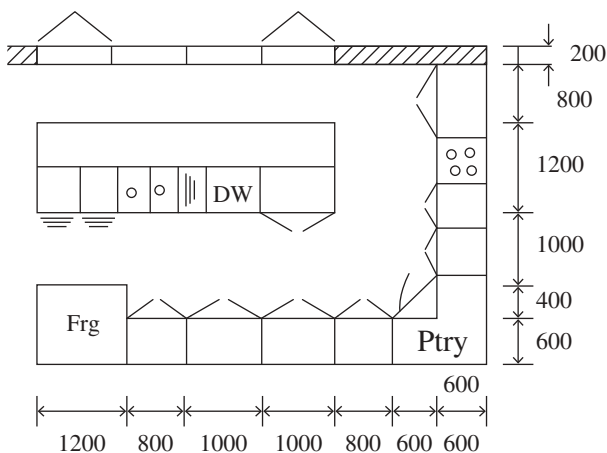
She realises that 33 mm is the thickness of the benchtop and 150 mm is the height of the kickboards under the bench. Use the measurements from the plan to calculate h , the height of the bottom cabinet doors for Janice.

Example 11

- 4** Use the plan shown to answer the following questions. The measurements are in millimetres.

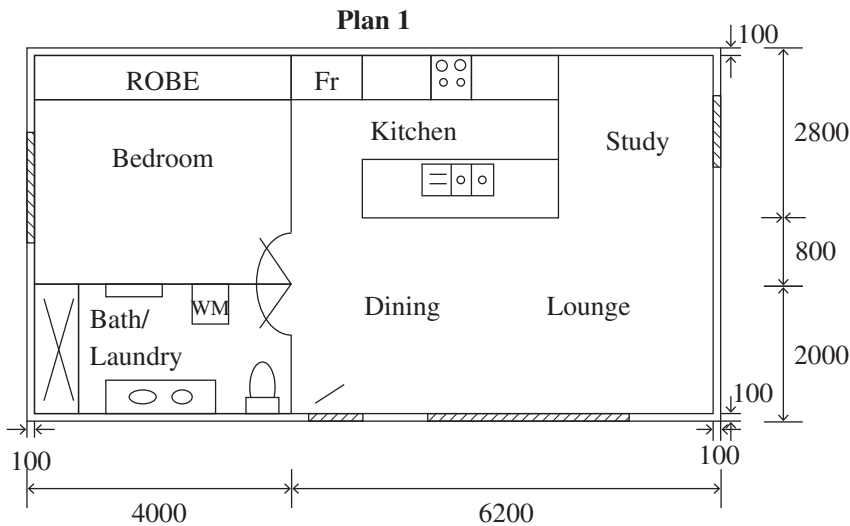


- Determine the actual internal dimensions of the garage, in metres.
 - Determine the actual external dimensions of the garage, in metres.
 - Determine the actual width of the external walls.
 - Identify the internal dimensions of the storeroom.
- 5** Use the plan shown to answer the following questions. The measurements are in millimetres.

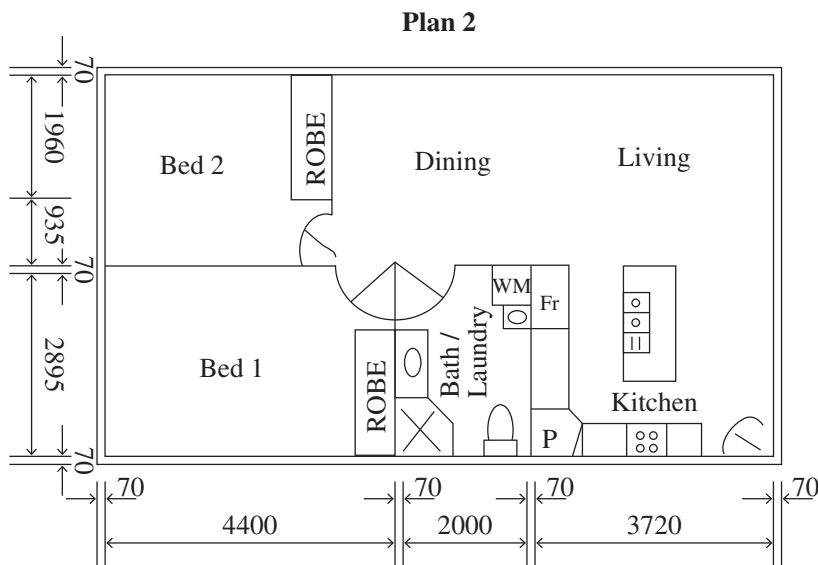


- Identify the window width in the kitchen plan.
- Identify the dimensions of the island bench in the kitchen.
- Determine the walk space distance between the island bench and the external wall/window.

- 6** Use plan 1 and plan 2 shown to answer the following questions. The measurements are in millimetres.



- Identify the width of the external walls in both plans.
- Identify the internal dimensions of the:
 - dining/lounge area in plan 1
 - bathroom/laundry in plan 2.
- Determine what these measurements indicate:
 - 2000 on the right-hand side of plan 1
 - 1960 on the left-hand side of plan 2.
- Determine the internal measurements including the robes of:
 - the bedroom in plan 1
 - bedroom 1 in plan 2.
- The council requires a granny flat to be less than 60 m^2 . Calculate the area of both plans to determine if they will fit council requirements.



2E Constructing scale diagrams

LEARNING GOALS

- Construct enlargement scale diagrams by hand
- Construct reduction scale diagrams by hand
- Construct scale diagrams using technology

Why is it essential to be able to construct scale diagrams?

Although there are many jobs that construct scale diagrams such as landscaper, builder, architect and cabinet makers, it is a useful skill for everyday people. It allows us to create an image to share our vision for renovating around our own homes and any landscaping we might do over the years. It is a useful process allowing us to share ideas, visions and inventions with other to help our dreams become a reality.



Scale drawings are necessary to renovate homes and kitchens.

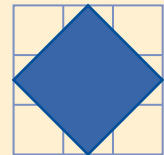
WHAT YOU NEED TO KNOW

- Recall the use of a scale to determine measurements in diagrams and how to indicate it with a ratio.
- Enlargement makes something bigger and reduction makes something smaller.
- Scales used so far have a small number first for a dimension in the diagram and a larger number second for the actual dimension in real life, such as 1 : 10. This indicates the diagram is a reduction of a large thing such as a building, a machine or a piece of furniture.
- Small objects such as machine parts are usually drawn with an enlargement, which is indicated by a scale where the first number is larger than the second, such as 10 : 1. This means 10 mm on the diagram is 1 mm of the actual object.
- To calculate by how many times the object is enlarged or reduced in the diagram, express the scale as a fraction with the first number on top and the second number on the bottom, and simplify if necessary. So 1 : 10 indicates the object in the diagram is $\frac{1}{10}$ of its actual size, and 10 : 1 indicates it is 10 times larger in the diagram.
- Grid paper is useful for drawing lines and shapes accurately with the help of a ruler.



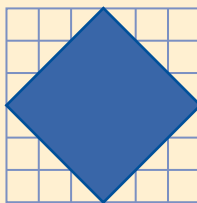
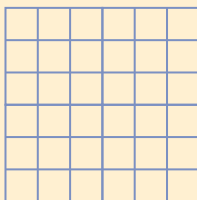
Example 12 Constructing a scale diagram by hand

Use an enlargement scale of 2 : 1 to construct a scale diagram of the following shape.



WORKING

The diagram has to be $\frac{2}{1} = 2$ times bigger than the original shape.



2 : 1

THINKING

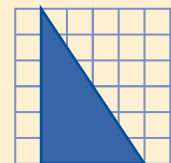
The scale of 2 : 1 has the first number larger than the second which confirms it means an enlargement. Express it as a fraction with the first number on top and the second number on the bottom; this is how much bigger the diagram has to be. The original shape is on a grid of 3 by 3 squares. $3 \times 2 = 6$ so to draw it two times bigger, construct a grid of 6 by 6 squares of the same size.

Draw the same shape on the new grid, and label it with the scale (2 : 1).



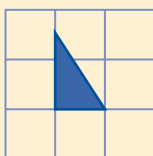
Example 13 Constructing a scale diagram by hand

Use a reduction scale of 1 : 4 to construct a scale diagram of the following shape.



WORKING

The diagram has to be $\frac{1}{4}$ the size of the original shape.



THINKING

Express the reduction scale as a fraction with the first number on top and the second number on the bottom; this is the fraction of the original shape that the diagram has to be. The triangle has a height of 6 squares and a base of 4 squares, dividing by 4 gives a height of 1.5 squares and a base of 1 square.

A grid of 3×3 squares of the same size will be enough for the diagram. Draw the reduced shape on the grid with the new dimensions.

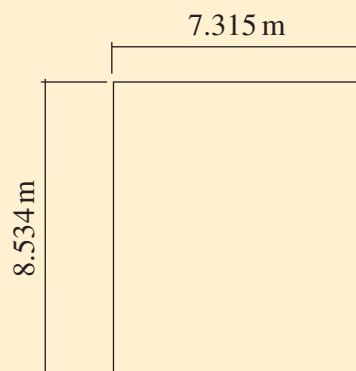


Example 14 Constructing a scale diagram using grid paper

Use the following steps to construct a scale diagram of the garage.

Use a scale of 1 : 100.

- Convert the measurements to mm, given the measurements are in metres.
- Apply the scale of 1 : 100 to convert the actual measurements.
- Draw a rectangle using the scale measurements on the grid paper.

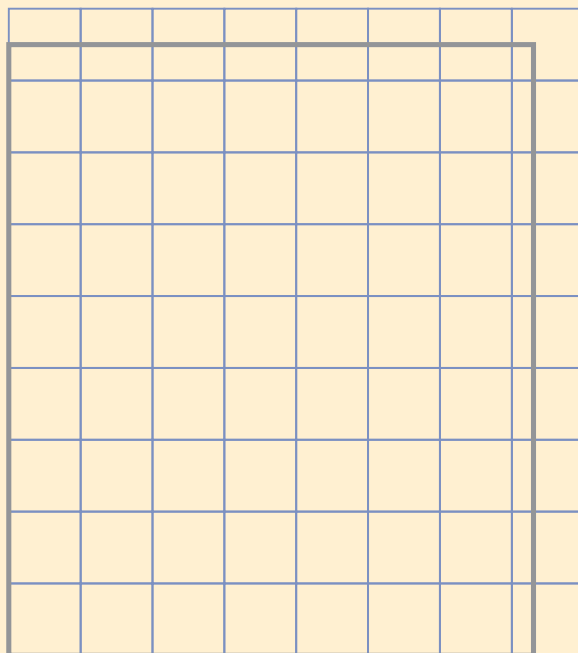


WORKING

a $7.315 \text{ m} \times 1000 = 7315 \text{ mm}$
 $8.534 \text{ m} \times 1000 = 8534 \text{ mm}$

b $7315 \text{ mm} \div 100 = 73.15 \text{ mm}$
 $8534 \text{ mm} \div 100 = 85.34 \text{ mm}$

c 1 : 100



THINKING

Multiply the measurements by 1000 to convert metres to millimetres.

Apply the scale by dividing the measurements by the scale factor, 100.

Draw a rectangle using the measurements on the grid paper. The longer side is about 8.5 squares and the shorter one is about 7.3 squares. To make it more accurate you could use grid paper with 1 mm squares, or use a ruler.

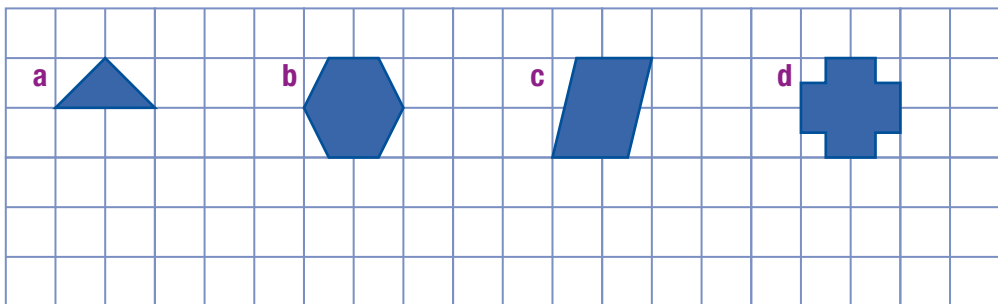
Exercise 2E

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a _____ of scale drawings enables people to _____ ideas.
 - b We can create scale diagrams by _____ and using _____.
 - c An _____ constructs scale _____ of a house and a _____ will build the house.

Example 12

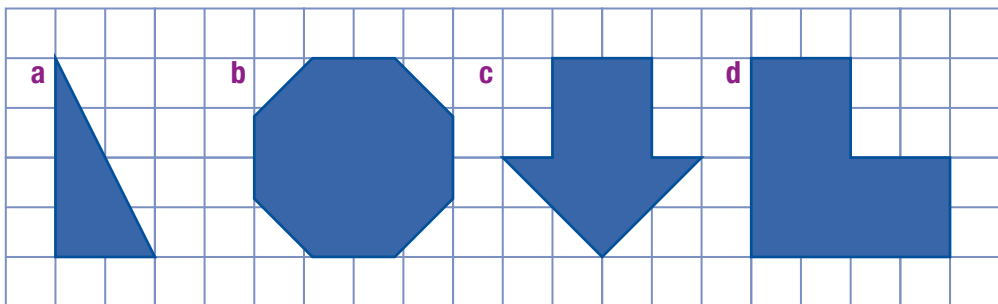
- 2 Use an enlargement scale of 3 : 1 to construct a scale diagram of the following shapes.



Hint Work out how many times bigger the scale diagram is to the original shape.

Example 13

- 3 Use a reduction scale of 1 : 4 to construct a scale diagram of the following shapes.



Hint Work out from the scale ratio what fraction of the original shape the scale diagram dimensions need to be.



APPLICATIONS

- 4** Construct a scale enlargement diagram using grid paper and a scale of 5 : 1 for an outline of the bolt shown. It has an overall length of 18 mm, a head with dimensions 10 mm by 4 mm and the threaded portion has a diameter of 4 mm. Do not show the threads, draw the outline of that part with straight not zig-zag lines. Indicate the scale on the diagram.



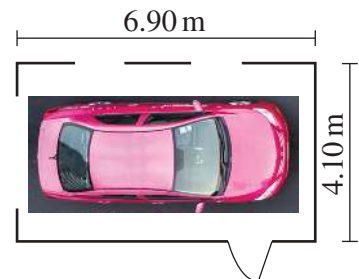
- 5** Construct a scale enlargement diagram using grid paper and a scale of 4 : 1 to show the outline of this drill bit. It has a length of 20 mm and a diameter of 2 mm. Do not show the cutting edges (called the *flutes*), use a straight line for the outline. Indicate the scale on the diagram.



Example 14

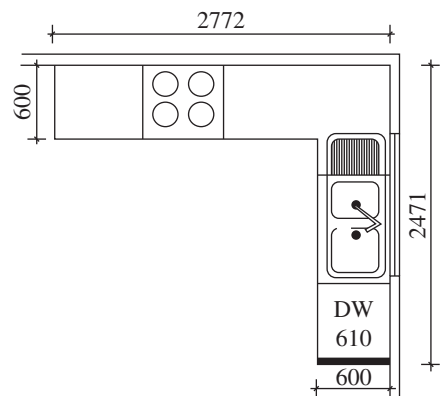
- 6** Use the following steps to construct a scaled diagram of the garage. Use a scale of 1 : 100.

- a** Convert the actual measurements to mm, given the measurements are in metres.
- b** Apply the scale of 1 : 100 to convert the actual measurements to diagram measurements.
- c** Draw a rectangle using the scale measurements on the grid paper.

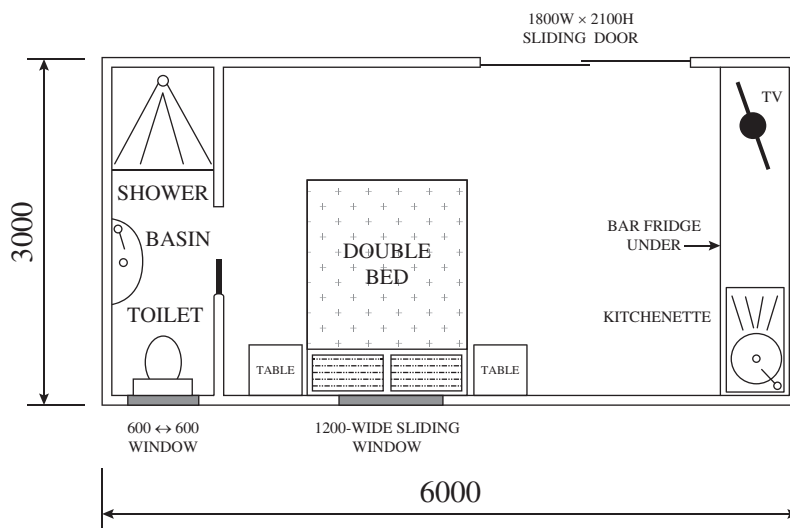


- 7** Use the following steps to construct a scale diagram of the outline of these kitchen cabinets using grid paper and a scale of 1 : 50.

- a** Apply the scale to the actual measurements to convert to diagram measurements.
- b** Draw the outline using the scale measurements on the grid paper. You do not have to draw the individual kitchen units, just the outline.



- 8 Construct a scale diagram of the exterior dimensions of this garage conversion using grid paper and a scale of 1 : 75.



- 9 Construct a 1 : 100 scale drawing showing the boundary of a rectangular block of land and the exterior walls of a rectangular shed placed in the centre of it. The block of land is $26\text{ m} \times 16.5\text{ m}$, and the shed is 12.5 m by 9 m . The longer side of the shed runs in the same direction as the longer side of the block. The yard is of equal width on opposite sides of the shed. Label the dimensions of the block of land in metres and of the shed in millimetres. Add the scale to the drawing.
- 10 Use technology to create a kitchen of your own design. Online kitchen programs are available through the internet and most schools will have a CAD (Computer Aided Design) program that can be useful.



- 11 Use technology to create a bathroom of your own design. Online bathroom programs are available through the internet and most school will have a CAD (Computer Aided Design) program that can be useful.

2F Viewing and sketching 3D objects

LEARNING GOALS

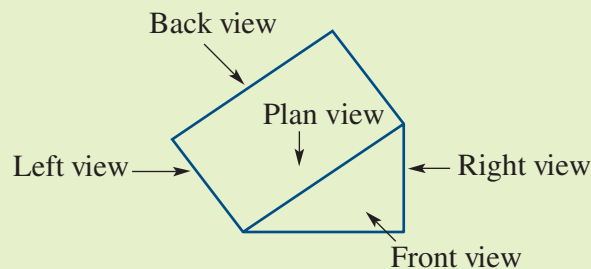
- Interpret elevation views of 3D models
- Sketch elevation views of 3D models
- Interpret diagrams of 3D objects

Why is it essential to be able to interpret and sketch views of 3D models?

As we have seen earlier, information involving 3D objects are often represented in 2D drawings. The ability to interpret the 2D drawing and visualise the 3D object is important in any design or building activities. The ability to take a 3D object and draw a 2D drawing that represents the 3D object, allows for the communication and collaboration on many design ideas, visions and inventions.

WHAT YOU NEED TO KNOW

- When viewing a 3D object the different views of the **vertical** surfaces are called the plan, front, left, right and back views.



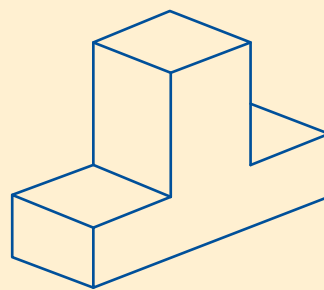
- The view from the top of the object is call a **plan**.
- Recall the use of a scale to determine measurements in diagrams and how to indicate it with a ratio.
- Enlargement makes something bigger and reduction makes something smaller.
- Scales used so far have a small number first for a dimension in the diagram and a larger number second for the actual dimension in real life, such as 1 : 10. This indicates the diagram is a reduction of a large thing such as a building, a machine or a piece of furniture.

- Small objects such as machine parts are usually drawn with an enlargement, which is indicated by a scale where the first number is larger than the second, such as 10 : 1. This means 10 mm on the diagram is 1 mm of the actual object.
- To calculate by how many times the object is enlarged or reduced in the diagram, express the scale as a fraction with the first number on top and the second number on the bottom, and simplify if necessary. So 1 : 10 indicates the object in the diagram is $\frac{1}{10}$ of its actual size, and 10 : 1 indicates it is 10 times larger in the diagram.
- Grid paper and isometric dot paper are useful for drawing lines and shapes accurately with the help of a ruler.



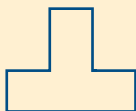
Example 15 Sketching elevation views of 3D models

Draw the plan, front, left, right and back views of this model with a scale of 1 : 1.

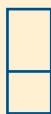


WORKING

Back and front view



Left and right view



Plan view



THINKING

Draw a 2D diagram of what the object looks like from the front view. This object is the same from the back view so only draw one diagram and label it with both front and back.

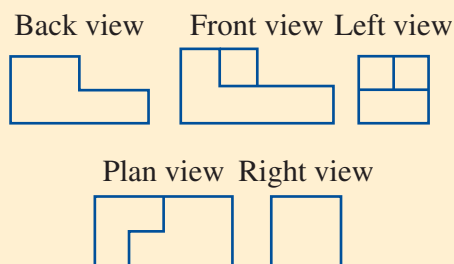
Draw a 2D diagram of what the object looks like from the left view. This object is the same from the right view so only draw one diagram and label it with both left and right.

Draw a 2D diagram of what the object looks like from the top and label it as the plan view.



Example 16 Interpreting elevation views of 3D models

Draw the 3D model that has the following views with a scale of 2 : 1.



WORKING

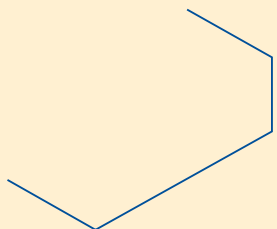
Measuring the plan view you find the base of the object measures 2 cm by 4 cm. To draw the scale diagram multiply the dimensions by 2.

$$2 \text{ cm} \times 2 = 4 \text{ cm} \quad 4 \text{ cm} \times 2 = 8 \text{ cm}$$



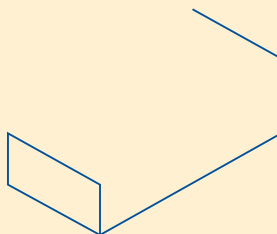
The right view is a 2 cm by 2 cm vertical square.

$$2 \text{ cm} \times 2 = 4 \text{ cm}$$



The left view has a 2 cm long by 1 cm wide rectangle at the base.

$$1 \text{ cm} \times 2 = 2 \text{ cm} \quad 2 \text{ cm} \times 2 = 4 \text{ cm}$$



THINKING

The scale is 2 : 1 so the scale drawing needs to be $\frac{2}{1}$ or $2 \div 1 = 2$ times larger.

The scale drawing needs a base of 4 cm by 8 cm.
Use 1 cm isometric dot paper to draw the front and side of the base.

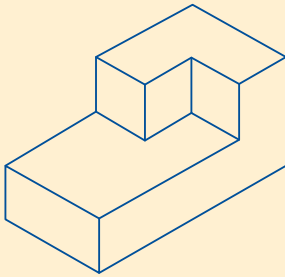
The drawing has a 4 cm by 4 cm vertical square from the right view.

... Continued

WORKING

The plan, front and back views show there is a 2 cm by 2 cm 'L' shapes $\frac{3}{4}$ square upper layer.

$$2 \text{ cm} \times 2 = 4 \text{ cm}$$

**THINKING**

The scale diagram has 4 cm by 4 cm 'L' shapes $\frac{3}{4}$ square upper layer.

**Example 17** Interpreting diagrams of 3D objects

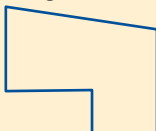
Draw the front and right views of this object.

**WORKING**

Front view



Right view

**THINKING**

Draw on grid paper the 2D view from the front.

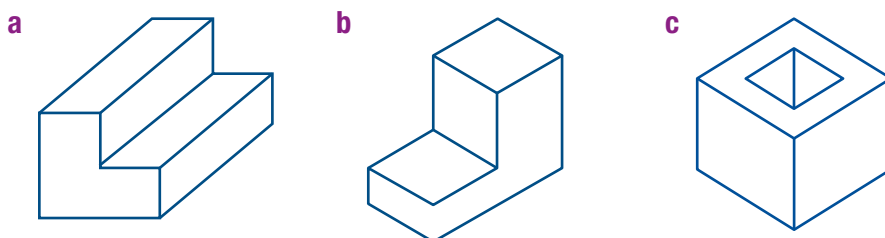
Draw on grid paper the 2D view from the right.

Exercise 2F

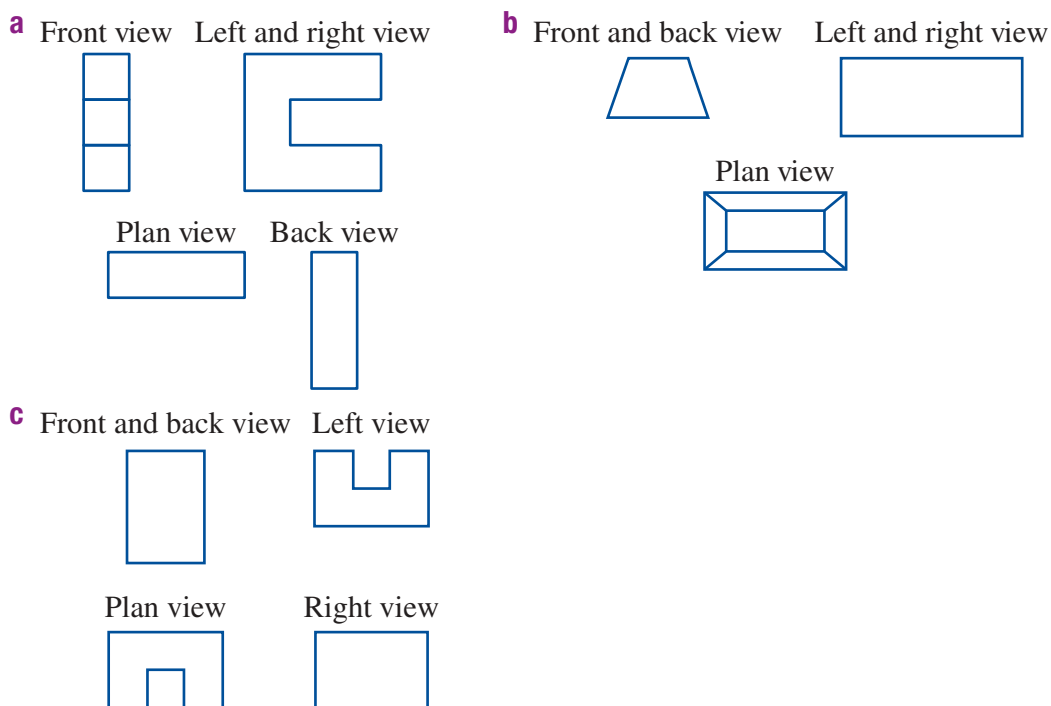
FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a A 3D object can be represented by 2D _____.
 - b The name of the five elevation views are _____, _____, _____, _____ and _____.
 - c Enlargement makes something _____ and _____ makes something smaller.

Example 15 2 Draw the five 2D elevation views of these models.



Example 16 3 Draw the 3D models represented by the 2D elevation views provided.

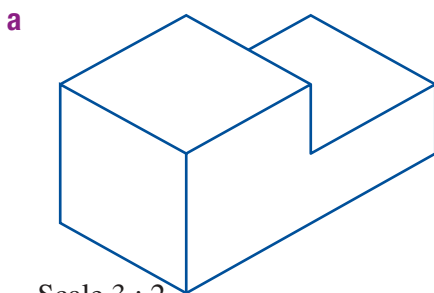


APPLICATIONS

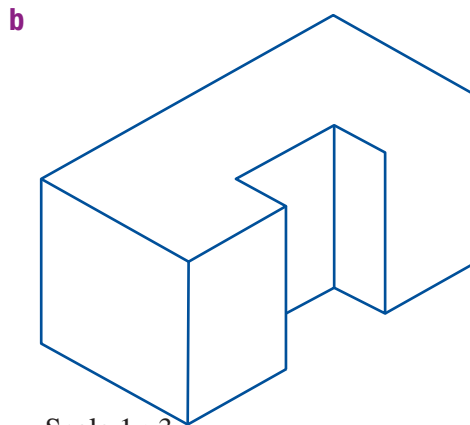
Example 17 4 Draw the most appropriate view of the objects below:



5 Using the scales given, draw a scale diagram of **i** the 3D model and **ii** the 2D elevation view from the front for each of the following:



Scale 3 : 2



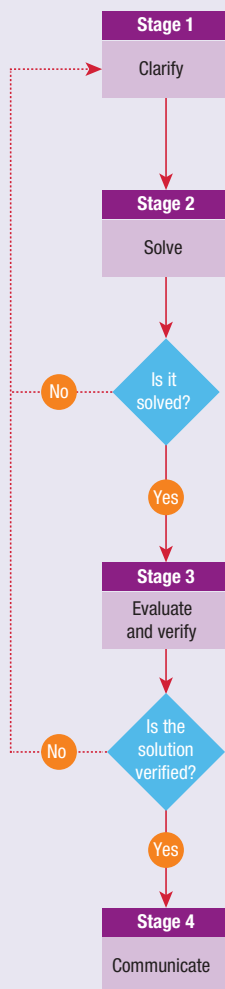
Scale 1 : 3

Mathematical thinking process task

Background: Floor plans of dwellings are the most common kind of scale drawing that most people will experience in their lives.

Task: The council have rezoned the rules for the construction of granny flats in the Geraldton area. They are now allowing an area of 90 m^2 . Your task is to design a granny flat that meets the council requirements. You can either use technology or draw by hand the granny flat plan. Your client wants to have a separate laundry and two toilets in the granny flat and a spare bedroom for guests and family to stay.

Application of mathematical thinking process.



Stage 1: Clarify

Interpret the task and gather the key information and identify the mathematics which could help to complete the task

- 1 Determine the elements you want to include in the granny flat.
- 2 Research minimum bedroom and bathroom sizing.

Stage 2: Solve

Apply existing mathematical knowledge and strategies to obtain a solution

- 3 Sketch the outline of your granny flat ensuring it is less than or equal to 90 m^2 .
- 4 Sketch the rooms into the outline of the granny flat.
- 5 Draw symbols onto the plan to identify fixtures and features.
- 6 Use abbreviations to label the smaller rooms on the plan.

Stage 3: Evaluate and verify

Analyse information and data from a variety of sources and verify the reasonableness of the solution

- 7 Show someone else your work for comment and check your area.

Stage 4: Communicate

Communicate findings in a systematic and concise manner

- 8 Submit your plan along with a paragraph identifying the positive traits of your design and the mathematical expression showing that it will meet council requirements.

Chapter checklist

☐ I understand and interpret scale symbols and abbreviations.

- 1 Convert 3.54 m into mm.
- 2 Simplify the scale 8 mm : 48 cm.
- 3 Identify the meaning of the abbreviation ROBE.
- 4 Identify the  symbol.

☐ I can identify and calculate measurements of length, perimeter and area from scale diagrams.

- 5 Calculate the area of a rectangle with a length of 6.4 m and a width of 5.3 m.
- 6 Jayden measured the actual internal length (wall above the word laundry) of the laundry to be 2.7 m long, determine the scale of the plan.



☐ I can estimate and compare quantities, materials and costs from scale diagrams.

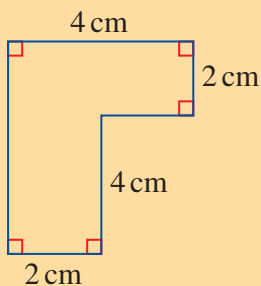
- 7 Tutu wants to carpet his bedroom. His bedroom has dimensions of 4 m by 4.2 m and the cost of carpet is \$48 fully laid per square metre.
 - a Calculate the area of the bedroom.
 - b Calculate the cost to carpet the room.

- 8 The internal length of the garage from the plan shown is 6 m. Jonah wants to tile the ensuite. If the tiles and tiling costs \$62 per square metre, calculate the cost to tile the ensuite.



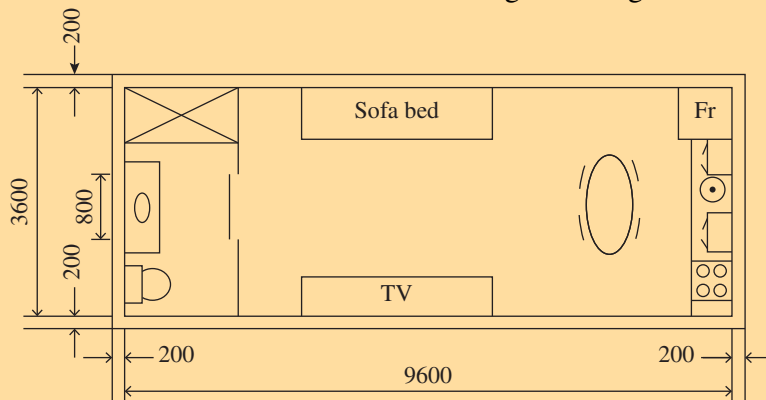
☐ I can understand and apply drawing conventions of scale drawings.

- 9 Determine the missing side measurements from the following diagram.



- 10 Use the following plan to complete the following.
- Identify the width of the external walls.
 - Identify the internal dimensions of the entire apartment.
 - Determine what the measurement of 800 mm found on the left-hand side of the plan measures.

d Determine the overall area of the building including external walls.



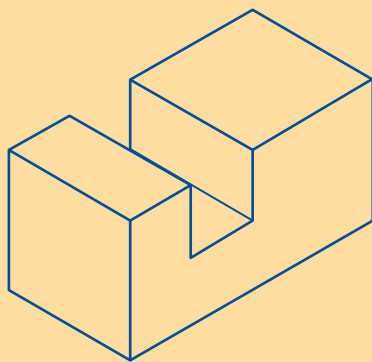
☐ I can construct a scale diagram.

11 Construct a scale enlargement diagram using grid paper and a scale of 4 : 1 for a bolt. It has a length of 15 mm, a head with dimensions 12.5 mm by 5 mm and the threaded portion has a diameter of 5 mm. Indicate the scale on the diagram.

12 Construct a scale diagram of a garage with dimensions of 5.8 m by 6 m using grid paper and a scale of 1 : 100.

☐ I can draw 2D elevation views of 3D models.

13 Draw the five elevation views of the model below.



Chapter review

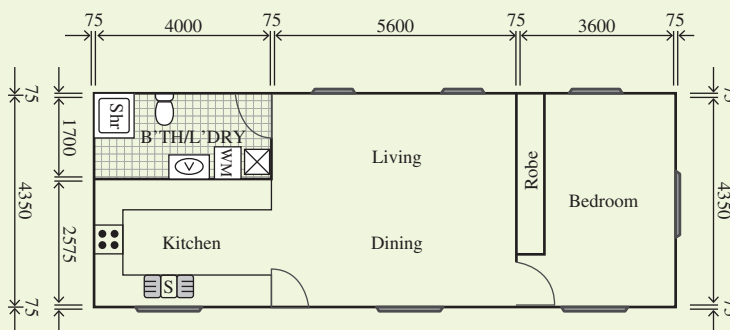
Refer to this diagram for Questions 1 to 5.



- Section 2A** 1 Simone is reading the plans of the house she would like to have built on her block of land. She does not know what the abbreviations W.C. and W.I.R. mean. Interpret these abbreviations for Simone.
- Section 2B** 2 Amanda knows that the internal width of the garage is 5.75 m.
- Determine the scale for the house plan.
 - Calculate the area of the sitting room.
 - Calculate the area the master suite (not including the W.I.R. and ensuite).
- Section 2C** 3 Aaron wants to retile the ensuite in his home. The costs of laying the tiles is \$35 per square metre and the tiles cost \$42.25 per square metre. Determine the area of the ensuite and calculate the total cost to have it retiled.
- 4 Gina wants to paint the floor of the garage with paving paint. If the paint covers 6 m^2 per litre and she needs to do 2 coats of paving paint, calculate the number of 4-litre tins Gina will need to purchase.
- 5 Dan is going to carpet bedroom 3 excluding the walk-in robe. Calculate the cost if the carpeting costs \$34 per square metre fully laid.

Section 2D

- 6 Use the plan shown below to answer the following questions, given that the measurements are in millimetres.
- Identify the width of the external walls.
 - Identify the dimensions of the bedroom including the robe.
 - Determine what the measurement of 4000 mm found at the top side of the plan measures.
 - Determine the internal measurements of the living/dining area.
 - The council requires a granny flat to be less than 60 m^2 . Calculate the area of this plan to determine if it will fit council requirements.



Section 2E

- 7 Construct a scale enlargement diagram using grid paper and a scale of 3 : 1 for the U-bolt shown. It has a height of 30 mm, a width of 15 mm and a thickness of 1.5 mm. Indicate the scale on the diagram and label its actual dimensions.



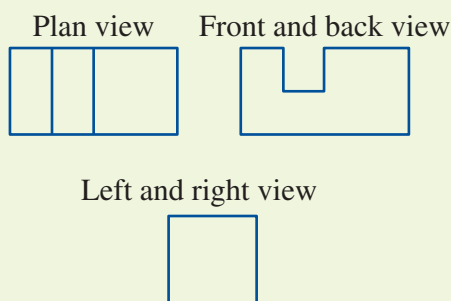
Section 2C

- 8 Construct a 1 : 100 scale diagram of a granny flat on grid paper with the following rectangular dimensions. Draw the first dimension in each pair horizontally on the plan, and the second dimension vertically on the plan.
- External dimensions 8950 mm by 5500 mm, with the longer side horizontal in the plan
 - Bedroom 1 located at the top left of the plan, internal dimensions 3000 mm by 3760 mm
 - Bathroom located at the top centre of plan, next to bedroom 1, internal dimensions 2400 mm by 2800
 - Study located at bottom left of plan, next to bedroom 1, internal dimensions 3000 mm by 1340 mm
 - The kitchen/dining and living areas make up the space left over, with the living area at the top right of the plan, and kitchen/dining in the bottom right of the plan.

Show the front door at the top of the plan, and doors to the enclosed rooms. Don't show windows or fixtures and features. Label the plan with the external dimensions, the names of the rooms and indicate their internal dimensions in a suitable way. Indicate the scale.



Section 2F 9 Draw the 3D model that has the following views with a scale of 1 : 2.



3 Right-angled triangles

In this chapter

- 3A** Calculating the hypotenuse by applying Pythagoras' theorem
 - 3B** Calculating a short side by applying Pythagoras' theorem
 - 3C** Determining unknown side lengths in problems by applying trigonometric rules
 - 3D** Determining unknown angles in problems by applying trigonometric rules
- Mathematical thinking process task
Chapter checklist
Chapter review

Syllabus reference

Unit 3 Topic 2: Scales, plans and models (4 hours)

Right-angled triangles

- apply Pythagoras' theorem to solve problems in practical two-dimensional views
- apply the tangent ratio to determine unknown angles and sides in right-angled triangles
- work with the concepts of angle of elevation and angle of depression
- apply the cosine and sine ratios to determine unknown angles and sides in right-angle triangles
- solve problems involving trigonometric ratios in practical two-dimensional views

Pre-test

- 1** Calculate the squares of the following numbers using your calculator.

- a** 6
- b** 8
- c** 11
- d** 24
- e** 33

- 2** Calculate the square roots of the following numbers using your calculator. Round to 2 decimal places where necessary.

- a** 169
- b** 289
- c** 320
- d** 3600
- e** 4336

Hint On most calculators, type in the number followed by the key with the square root sign.

- 3** Determine the value of the unknowns in the following problems using your calculator. Round to 2 decimal places where necessary.

- a** $c = \sqrt{12^2 + 16^2}$
- b** $c = \sqrt{10^2 + 24^2}$
- c** $c = \sqrt{21^2 + 43^2}$
- d** $a = \sqrt{39^2 - 15^2}$
- e** $b = \sqrt{23^2 - 17^2}$

Hint Follow 'order of operations'. Remember also that the root sign with a bar over the top of an expression means to treat the expression as if it is in brackets.

- 4** Determine the value the following problems using your calculator. Round to 2 decimal places where necessary.

- a** $15 \times \cos 20^\circ$
- b** $\sin 62^\circ \times 13$
- c** $\tan 45^\circ \div 21$
- d** $78 \div \cos 12^\circ$

Hint If your calculator has a mode for radians ('Rad'), make sure it is in degree mode ('Deg').

- 5** Construct a diagram using a right-angled triangle to show a firefighter who is looking to the top of a 60 m wall of a building and standing on the ground 100 m horizontally from the base of the wall of the building. Draw a line to show the firefighter's line of sight to the top of the wall.

3A Calculating the hypotenuse by applying Pythagoras' theorem

LEARNING GOALS

- Understand Pythagoras' theorem
- Identify the sides of a right-angle triangle
- Identify the hypotenuse of a right-angled triangle
- Calculate the hypotenuse given the other two sides
- Calculate the hypotenuse in a real-world context
- Verify square angles in construction by using Pythagoras' theorem

Why is it essential to be able to understand the use of Pythagoras' theorem?

Pythagoras' theorem is a very practical way to help builders verify right angles on building sites as it is vital on most constructions to have a perfect right angle on edges. This is a method of checking the edges of the walls are square. It is one of the most common mathematical theorems used in many occupations such as farming, landscaping, metalworking, and other manufacturing and construction industries.

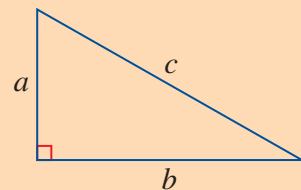


Pythagoras' theorem can be used to check that the corner of this soccer field is a right angle.

WHAT YOU NEED TO KNOW

- A **right angle** is a 90° angle, the same as is found at the corners of squares and rectangles, and it is also known as a square angle.
- The **hypotenuse** is the longest side of a triangle, which is found opposite the right angle.

The formula is known as $c^2 = a^2 + b^2$, where c is the length of the hypotenuse.

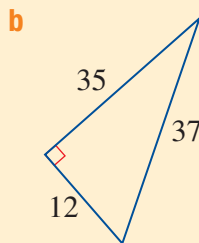
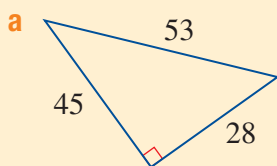


- **Pythagoras' theorem** states that the square of the longest side is equal to the sum of the other sides squared in a right-angled triangle.
- To find the value of c rearrange the formula to $c = \sqrt{a^2 + b^2}$.
- Rounding a decimal to a specific number of places.



Example 1 Identifying the hypotenuse on a right-angled triangle

Identify the length of the hypotenuse in each of the following triangles.



WORKING

- a** The hypotenuse is 53.

- b** The hypotenuse is 37.

THINKING

←..... The hypotenuse is the longest side and is also opposite the right angle.

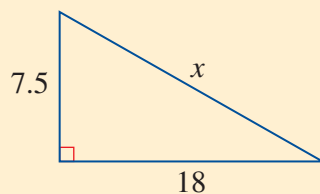
←..... The hypotenuse is the longest side and is also opposite the right angle.



Example 2 Determining the value of the hypotenuse given the other two sides

Complete the following steps to determine the value of the hypotenuse for the triangle shown.

- Identify the values of a , b and c .
- Substitute the values into the formula, $c^2 = a^2 + b^2$.
- Calculate the value of the hypotenuse using your calculator. Round to 1 decimal place when necessary.



WORKING

a $c = x, a = 7.5, b = 18$ ←.....

b $c^2 = a^2 + b^2$ ←.....
 $x^2 = 7.5^2 + 18^2$

c $x = \sqrt{7.5^2 + 18^2}$ ←.....
 $x = \sqrt{380.25}$
 $x = 19.5$

THINKING

Identify the hypotenuse, which is the longest side opposite the right angle. The other sides are a and b in any order.

Write the formula.

Substitute the values for a , b and c into the formula.

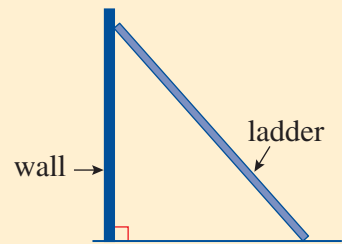
Calculate the value of the hypotenuse.

Round the final answer to 1 decimal place.

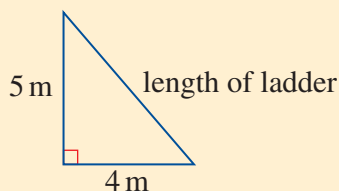


Example 3 Determining the value of the hypotenuse in a real-world context

A ladder is leaning against a vertical wall at a height of 5 metres up the wall. If the base of the ladder is 4 metres from the base of the wall, determine the length of the ladder, correct to 1 decimal place.

**WORKING**

$a = 5 \text{ m}, b = 4 \text{ m}, c = \text{length of ladder}$ ←.....



$c^2 = a^2 + b^2$ ←.....

$c^2 = 5^2 + 4^2$ ←.....

$c = \sqrt{4^2 + 5^2}$ ←.....

$c = \sqrt{41}$

$c = 6.403124237$

$c = 6.4 \text{ m}$ ←.....

The length of the ladder is 6.4 m. ←.....

THINKING

Identify the hypotenuse, which is the longest side opposite the right angle. The other sides are a and b in any order.

Draw a diagram with the known information.

Write the formula.

Substitute the values of a , b and c into the formula.

Calculate the value of the hypotenuse.

Round the answer to 1 decimal place.

Communicate your answer in a sentence.



Example 4 Determining if an angle is a right angle

Ziza is building a shed in her backyard. She wants to verify that the concrete slab she is having poured has been set out correct and is at right angles. She measured the two adjoining sides as 5 m and 6 m and the diagonal as 7.8 m. Verify the concrete slab will be set at right angles (within 0.2 m difference is allowable).

WORKING

$$a = 5 \text{ m}, b = 6 \text{ m}, c = 7.8 \text{ m}$$

$$c^2 = a^2 + b^2$$

$$c^2 = 7.8^2$$

$$c^2 = 60.84$$

$$a^2 + b^2 = 5^2 + 6^2$$

$$= 25 + 36$$

$$= 61$$

Each side of the equation is the same within 0.2 m.

THINKING

Identify the hypotenuse and the other two sides.
Write the formula.

Compare both sides of Pythagoras' theorem.

Communicate your answer in a sentence.

Exercise 3A

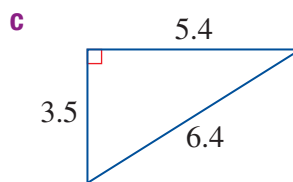
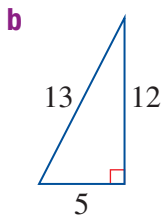
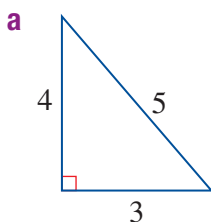
FUNDAMENTALS

- 1 Determine the missing words in the following sentences.

- a The _____ theorem is the _____ squared equals the _____ of the other two sides squared.
b The _____ theorem can only be applied to _____ triangles.
c The formula for Pythagoras' theorem is _____² = _____² + _____².

Example 1

- 2 Identify which side is the hypotenuse in these right-angled triangles.



- 3 Calculate the value of the following using your calculator. Round to 1 decimal place where necessary.

a $\sqrt{3^2 + 4^2}$

b $\sqrt{5^2 + 12^2}$

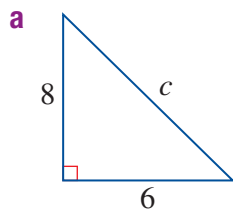
c $\sqrt{7^2 + 11^2}$

d $\sqrt{13^2 + 24^2}$

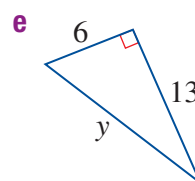
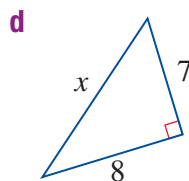
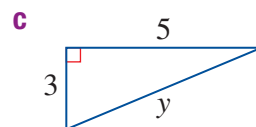
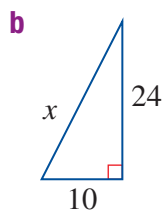
Example 2

4 Complete the following for each of the triangles shown.

- Identify the values of a , b and c .
- Substitute the values of a , b and c into the formula $c^2 = a^2 + b^2$.
- Calculate the value of the unknown side by using your calculator. Round your answer to 1 decimal place when necessary.



Hint The value of c is important to identify; a and b are just the other two short sides.

**Example 4**

5 Verify that the triangles with the following side lengths are right-angled triangles. An allowance of 0.1 difference may be applied due to rounding.

- 8, 15, 17
- 7, 24, 25
- 33, 56, 65
- 4.5, 6.3, 7.74
- 12.7, 18.6, 22.52

Hint The hypotenuse is the longest side.



APPLICATIONS

Example 3

- 6** The ladder rests against a tree 10 m above the ground. The base of the ladder is 3 m from the base of the tree.

- a** Substitute the values of a and b into the formula $c^2 = a^2 + b^2$.
b Determine the length of the ladder using your calculator. Round to 1 decimal place.

Hint The diagrams all involve right-angled triangles.



- 7** Khia is flying a kite that is at a vertical height of 80 metres and is 40 metres away from her measured horizontally along the ground. The string of the kite forms a straight line.
- a** Draw and label a diagram to show the situation.
b Identify a right-angled triangle in your diagram. Label the horizontal and vertical sides of the triangle with the values given. Label the hypotenuse as the unknown side length.
c Write the formula to find the hypotenuse. Substitute the values for a and b .
d Calculate the length of Khia's kite string, correct to 1 decimal place.
- 8** Chen is on top of a vertical cliff with a height of 120 m. He is about to ride a flying fox that lands at a horizontal distance of 175 m from the base of the cliff.
- a** Draw and label a diagram to show the situation.
b Identify a right-angled triangle in your diagram. Label the horizontal and vertical sides of the triangle with the values given. Label the hypotenuse as the unknown side length.
c Write the formula to find the hypotenuse. Substitute the values for a and b .
d Calculate the length the flying fox ride, correct to 1 decimal place.



- 9** Arthur is participating in a re-enactment and he needs to organise a siege on a castle that has a 10-metre wide moat around the castle. If the castle wall is 8 metres high, calculate the minimum ladder length needed to reach over the moat to the top of the castle wall. Round your answer correct to 1 decimal place.
- 10** Anmarie is designing a cable-stayed bridge. She needs to calculate the length of a cable that will attach to the bridge pylon (the column) with a vertical distance of 40 metres above the bridge deck. The other end of the cable will attach to the bridge deck at a horizontal distance of 20 metres from the middle of the bridge pylon. Calculate the required length of the cable, correct to 1 decimal place.



- 11** Holly is building a gate in the shape of a rectangle and she needs to add a diagonal brace (a piece of wood running along the diagonal of the rectangular face of the gate). If the gate is 1.2 metres high and 2.6 metres wide, calculate the length of the brace needed. Round your answer correct to 1 decimal place.
- 12** Hayden has designed a television that is 140 cm wide and 90 cm high. He is told that to market his television he needs to advertise the length of the screen diagonal, which is the measure commonly used for screen size. Calculate the length of the television screen diagonal, correct to 1 decimal place.
- Example 4** **13** Bitta the builder is checking the right angle between two walls in a rectangular room by measuring the lengths of the sides of the rectangle and its diagonal. She measures the sides of the room as 8 metres and 6 metres, and the diagonal as 10 metres. Verify that the walls are built with a right angle between the two walls.
- 14** Lance is wanting to renovate his kitchen. Before he orders new cabinets, he wants to confirm that the room is actually set at right angles. If one wall is 3 m and the adjoining wall is 3.6 m and the diagonal is approximately 4.69 m, verify the kitchen is set at right angles (within 0.1 m difference is allowable).

3B Calculating a short side by applying Pythagoras' theorem

LEARNING GOALS

- Understand Pythagoras' theorem
- Identify the hypotenuse on a right-angled triangle
- Calculate a short side given the hypotenuse and the other short side

Why is it essential to be able to calculate the side length in a right-angled triangle?

Right angles are everywhere in manufacturing and construction. Roof trusses are a good example. But they also occur in surveying, landscaping, sewing, graphic design, and other arts and crafts.

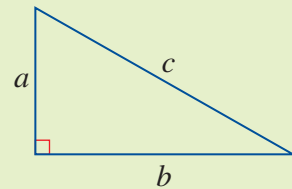
The importance of calculating side lengths is well illustrated by the image of the timber roof truss below. Its base length is determined by the width of the building, and its height is determined to give a practical and attractive slope to the roof. The length of every other piece of timber from which the truss is made can then be calculated using right-angled triangles and Pythagoras' theorem.



This roof truss is in the shape of two right-angled triangles joined along their shortest sides.

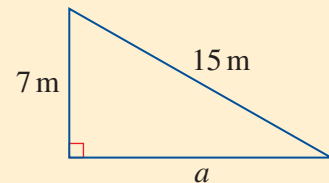
WHAT YOU NEED TO KNOW

- It does not matter which side is labelled a or b , as long as they are not the hypotenuse.
- Pythagoras' theorem states that the unknown side squared in a right-angled triangle is equal to square of the longest side minus the square of the other side.
- The formula is known as $c^2 = a^2 + b^2$, where c is the length of the hypotenuse.
- To find the value of a rearrange the formula to $a^2 = c^2 - b^2$.
- Rounding to a specific number of decimal places.


Example 5 Determining the unknown side given the hypotenuse and the other side

Complete the following steps to determine the unknown side for the triangle shown.

- Identify the values of a , b and c .
- Substitute the values of a , b and c into the formula $a^2 = c^2 - b^2$.
- Determine the value of the short side using your calculator. Round your answer to 1 decimal place if necessary.



WORKING

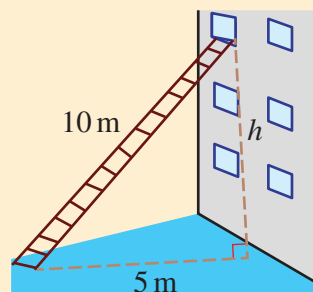
THINKING

a $c = 15 \text{ m}$, $a = ?$, $b = 7 \text{ m}$	Identify the hypotenuse, which is the longest side opposite the right angle. The other sides are a and b in any order.
b $a^2 = c^2 - b^2$ $a^2 = 15^2 - 7^2$	Write the formula. Substitute the values for a, b and c into the formula.
c $a = \sqrt{15^2 - 7^2}$ $a = \sqrt{176}$ $a = 13.266\ 499\ 16$ $a = 13.3$	Calculate the value of the short side. Round the final answer to 1 decimal place.



Example 6 Calculating the unknown side given the hypotenuse and the other side in a real-world context

A 10-metre ladder is leaning against a vertical wall at a height of h metres. If the base of the ladder is 5 metres from the base of the wall, calculate the height that the ladder reaches up the wall, correct to 1 decimal place.



WORKING

$$a = h \text{ m}, b = 5 \text{ m}, c = 10 \text{ m} \quad \leftarrow \dots\dots\dots$$

$$a^2 = c^2 - b^2$$

$$h^2 = 10^2 - 5^2$$

$$h^2 = 100 - 25$$

$$h^2 = 75$$

$$h = \sqrt{75}$$

$$h = 8.66 \text{ m}$$

$$h = 8.7 \text{ m} \quad \leftarrow \dots\dots\dots$$

The ladder reaches a height of 8.7 m. $\leftarrow \dots\dots\dots$

THINKING

Identify the hypotenuse, which is the longest side opposite the right angle. The other sides are a and b in any order. Write the formula. Substitute the values of a , b and c into the formula.

Calculate the value of the height of the ladder up the wall.

Round the answer to 1 decimal place.

Communicate your answer in a sentence.

Exercise 3B

FUNDAMENTALS

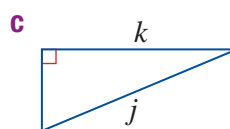
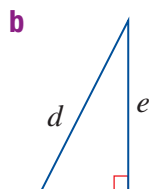
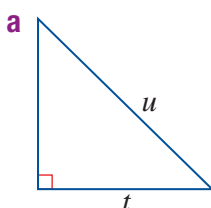
1 Determine the missing words in the following sentences.

a The _____ theorem is the _____ side squared equals the _____ squared minus the other _____ squared.

b A _____ is used to calculate the _____.

c The formula to calculate a short side on a right-angled triangle is _____² = _____² - _____².

- 2 The unknown side is 'a' in these right-angled triangles shown. Write the rule to find the value of a using the other values on the triangles.



- 3 Calculate the value of the following using your calculator. Round to 1 decimal place where necessary.

a $\sqrt{13^2 - 12^2}$

b $\sqrt{25^2 - 20^2}$

c $\sqrt{17^2 - 14^2}$

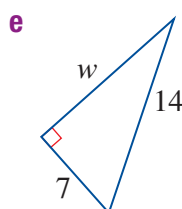
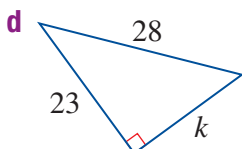
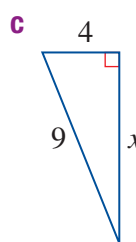
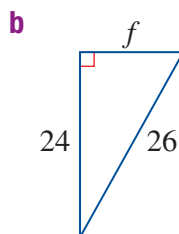
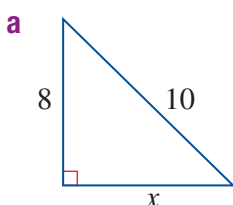
Example 5

- 4 Complete the following for each of the triangles shown.

i Identify the values of a , b and c .

ii Substitute the values of a , b and c into the formula $a^2 = c^2 - b^2$.

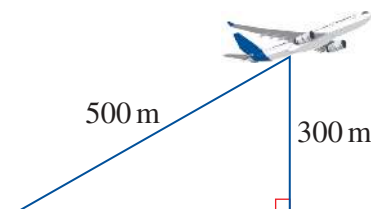
iii Calculate the value of the unknown side by using your calculator. Round your answer to 1 decimal place when necessary.



APPLICATIONS

- 5 A plane has just taken off and has travelled 500 m in a straight line sloping upwards. It is 300 m vertically above the ground.

- a** Substitute the values of c and b into the equation $a^2 = c^2 - b^2$.
- b** Determine the horizontal distance the plane has travelled, using your calculator. Round your answer to 1 decimal place.



- 6** Paul has been flying a kite but it is now stuck in the top of a tree. He had the full 50 metres of line out and it is pulled tight. The line forms a straight line from where he is standing, which is 29 metres horizontally from the base of the tree.
- Draw and label a diagram to show the situation.
 - Label the known sides with values given.
 - Write the formula to find a short side. Substitute the known values into the formula.
 - Calculate the height of the tree, correct to 1 decimal place.
- 7** Barbie has climbed up the ladder of a radio mast. She is level with the attachment point of a cable. The other end of the cable is attached to a point that is 40 metres horizontally from the base of the mast. The cable forms a straight line and is known to be 234 metres long.
- Draw and label a diagram to show the situation.
 - Label the known sides with values given.
 - Write the formula to find a short side. Substitute the known values into the formula.
 - Calculate the height that Barbie is above the ground, correct to 1 decimal place.
- Example 6** **8** Boyd has 5 metres of a ladder to climb onto a roof 3 metres high. Calculate how far from the base of the wall that the base of the ladder should be placed.
- 9** Julie wants to design a slippery slide to fit in her townhouse courtyard. She only wants it to take up 3 metres across the ground and has 3.5 metres of material for the actual slide. Determine the vertical height of the slide from the ground.
- 10** Jonah has a television with a 150 cm screen, which is measured as the length of the diagonal. If the screen is 75 cm high, calculate the width of the screen.
- 11** Melanie has bought an odd-shaped block of land in the shape of a right-angled triangle. If the two longest sides are 50 metres and 40 metres respectively, calculate the length of the third side of the triangle.

- 12** Sally wants to know what width of tarp material she needs to buy to make a two-person tent. She has a 1.5 metre pole for the middle of the tent and wants to allow 2.5 metres across the bottom of the tent for two people to fit inside the tent. Determine the width of the tarp that Sally needs to buy, rounded to 2 decimal places.



3C

Determining unknown side lengths in problems by applying trigonometric rules

LEARNING GOALS

- Understand the concept of a trigonometric ratio
- Determine the adjacent and opposite sides to an angle in a right-angled triangle
- Learn how to use the rules for a trigonometric ratio
- Use the rules to calculate the unknown sides of a right-angled triangle
- Calculate unknown sides in right-angled triangles in a real-world context

Why is it essential to understand other applications of calculating side lengths of a right-angled triangle?

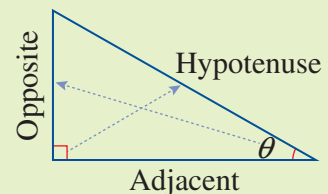
Several questions in the chapter so far have asked for calculations of vertical height and horizontal distance. These are examples of the use of right-angled triangles in calculating location and position, which is used in surveying, map-making, navigation and positioning systems.



Right-angled triangles have applications in navigation.

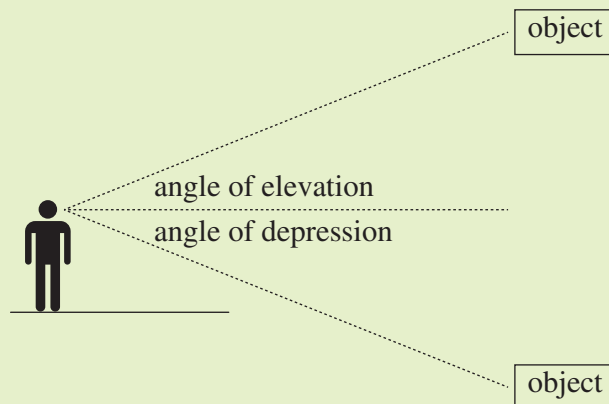
WHAT YOU NEED TO KNOW

- For a right-angled triangle with an angle labelled θ , the three sides should be named adjacent (next to the angle), opposite (not touching the angle) and the hypotenuse (the longest side opposite the right angle).
- We use trigonometric ratios to form a relationship between two sides and an angle in a right-angled triangle. The abbreviation SOH CAH TOA is used to help remember the trigonometric formulas shown in the diagram.



$$\begin{array}{c}
 \text{SOH CAH TOA} \\
 \swarrow \quad \downarrow \quad \searrow \\
 \sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}} \quad \cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}} \quad \tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}
 \end{array}$$

- Our calculators must be set to degree mode often seen by deg or D on the top of the calculator screen. Sin, cos and tan buttons can be found on all scientific calculators. To calculate the angle, shift or 2nd function sin, cos or tan is used.
- The angle of **elevation** is the angle from the horizontal going up. The angle of **depression** is the angle from the horizontal going down.



Example 7 Calculating the unknown side given the angle and one other side

Solve for the unknown value in the following. Round the answer to 2 decimal places.

a $\sin 48 = \frac{x}{15}$

b $\cos 22 = \frac{19}{y}$

WORKING

THINKING

a $\sin 48 = \frac{x}{15}$
 $15 \times \sin 48 = x$
 $x = 11.147$
 $x = 11.15$

b $\cos 22 = \frac{19}{y}$
 $y = 19 \div \cos 22$
 $y = 20.492$
 $y = 20.49$

←..... Rewrite the equation.
 Move divided by 15 to the other side as multiplied 15.
 (If the unknown value is on the top, then multiply.)
 Determine the unknown value using your calculator.
 As the third decimal place is a 5 or greater, round your answer up to 2 decimal places.

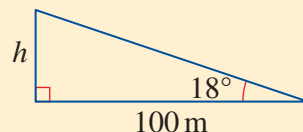
←..... Rewrite the equation.
 Swap the cos 22 with the y (If the unknown is on the bottom, then swap the values.)
 Determine the unknown value using your calculator.
 As the third decimal place is below 5, your answer stays the same at 2 decimal places.



Example 8 Calculating the unknown side given the angle and one other side, with the unknown as a numerator

Complete the following steps to calculate the unknown side in the triangle shown.

- Identify the known sides on the triangle.
- Determine which trigonometric ratio is required.
- Substitute the values into the ratio.
- Solve the ratio for the unknown value. Round answer to 1 decimal place.



WORKING

THINKING

a The angle is 18°. $\leftarrow$ 100 m is the adjacent side. h is the opposite side.	Identify the known sides in relation to the named angle in the triangle. The 100 m is next to the angle but is not the hypotenuse so it is the adjacent side. The h is opposite the named angle so it is the opposite side.
b $\tan \theta = \frac{\text{opp}}{\text{adj}}$ $\leftarrow$	The ratio that has the adjacent (A) and opposite (O) side named is tangent. (TOA)
c $\tan 18 = \frac{h}{100}$ $\leftarrow$	Substitute the values into the tangent ratio with $\theta = 18^\circ$, $O = h$, $A = 100\text{ m}$.
d $100 \times \tan 18 = h$ $\leftarrow$ $h = 32.491$ $h = 32.5\text{ m}$	Multiply $\tan 18$ by 100 to solve for h. Calculate the answer using your calculator. Round your answer to 1 decimal place.



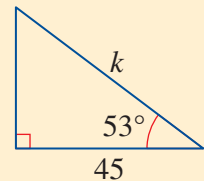
Land surveying makes extensive use of trigonometry.



Example 9 Calculating the unknown side given the angle and one other side with the unknown as a denominator

Complete the following steps to calculate the unknown side in the triangle shown.

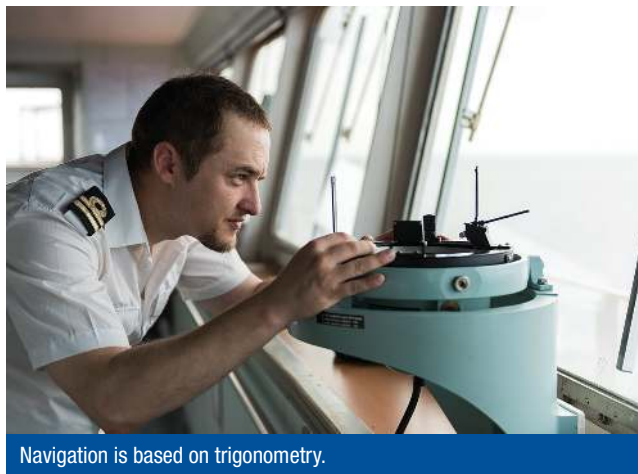
- Identify the known sides on the triangle.
- Determine which trigonometric ratio is required.
- Substitute the values into the ratio.
- Solve the ratio for the unknown value. Round your answer to 1 decimal place.



WORKING

THINKING

a The angle is 53°. 45 is the adjacent side. k is the hypotenuse.	Identify the known sides in relation to the named angle in the triangle. The 45 is next to the angle but is not the hypotenuse so it is the adjacent side. The k is opposite the right angle so it is the hypotenuse.
b $\cos \theta = \frac{\text{adj}}{\text{hyp}}$	The ratio that has the adjacent (A) and hypotenuse (H) side is cosine. (CAH)
c $\cos 53 = \frac{45}{k}$	Substitute the values into the cosine ratio with $\theta = 53^\circ$, $H = k$, $A = 45$.
d $k = \frac{45}{\cos 53}$ $k = 74.77$ $k = 74.8$	Swap the position of $\cos 53$ and the k within the ratio. Calculate the answer using your calculator. Round the answer to 1 decimal place.

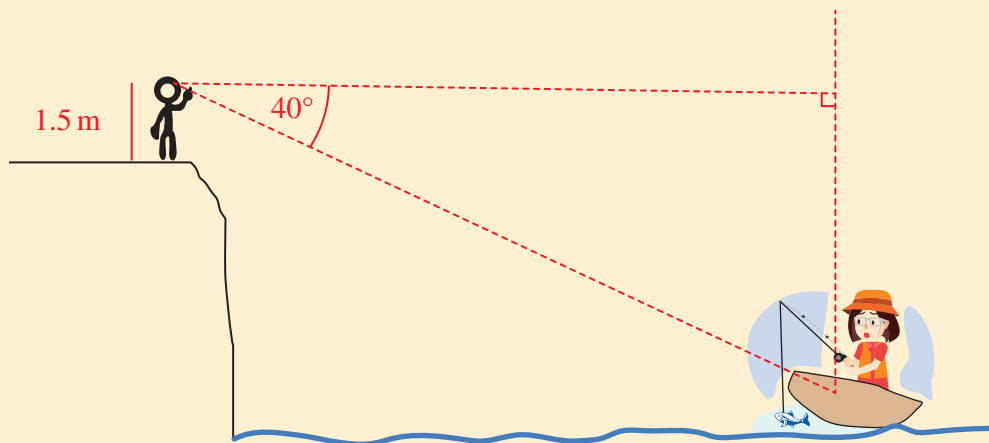


Navigation is based on trigonometry.



Example 10 Calculating the unknown side given the angle and one other side in a real-world context

Adel is fishing 100 metres from the base of a cliff. Her husband Rex waves to her to come home for dinner and looks down to her with an angle of 40° . Determine the height of the cliff given that Rex's eyeline is 1.5 metres above the cliff.



WORKING

$$\begin{aligned}\theta &= 40^\circ \\ \text{adj} &= 100 \text{ m} \\ \text{opp} &= x \\ \tan \theta &= \frac{\text{opp}}{\text{adj}}\end{aligned}$$

$$\tan 40^\circ = \frac{x}{100}$$

$$\begin{aligned}100 \times \tan 40^\circ &= x \\ x &= 83.9\end{aligned}$$

$$\text{The cliff is } 83.9 - 1.5 = 82.4 \text{ m}$$

The cliff is 82.4 metres high.

THINKING

Identify the values given for the right-angled triangle.

Identify the correct ratio required from SOH CAH TOA. The tangent ratio uses O and A.

Substitute the values into the tangent ratio.

Rearrange the ratio to determine the unknown value.

Calculate the height of the cliff by subtracting Rex's height from x .

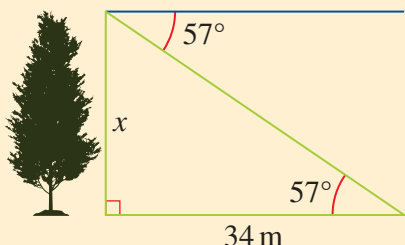
Communicate your answer in a sentence.



Example 11 Calculating the unknown side given the angle and one other side in a real-world context

Jane climbs to the top of a tree. She sees her friend standing 34 metres from the base of the tree with an angle of depression of 57 degrees. Determine the height of the tree that Jane has climbed.

WORKING



$$\begin{aligned} x &= \text{opposite} \\ 34 \text{ m} &= \text{adjacent} \\ \tan \theta &= \frac{\text{opp}}{\text{adj}} \end{aligned}$$

$$\tan 57 = \frac{x}{34}$$

$$\begin{aligned} 34 \times \tan 57 &= x \\ x &= 52.35 \\ x &= 52.4 \end{aligned}$$

The height of the tree is 52.4 metres.

THINKING

Draw a diagram to represent the information given.
Label the information on the diagram noting the angle of depression from Jane is equal to the angle of elevation from her friend 34 m away.

Identify the known sides in the diagram. Identify the correct ratio required from SOH CAH TOA. The tangent ratio uses O and A.

Substitute the known values into the tangent ratio.
Rearrange the ratio to determine the unknown value.

Communicate your answer in a sentence.

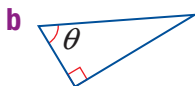
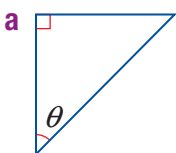
Exercise 3C

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a The tangent ratio equals the _____ side $\div$ the _____ side.
 - b The sine ratio equals the _____ side $\div$ the _____ side.
 - c The cosine ratio equals the _____ side $\div$ the _____ side.
 - d Drawing a _____ and labelling it will help solve trigonometric _____.

- 2 Label the sides of the triangles below with A (adjacent), O (opposite) and H (hypotenuse).

Hint The names relate to the angle θ .



- 3 Calculate the value of the following, rounding the answer to 1 decimal place where necessary.

a $24 \times \cos 50^\circ$

b $33 \times \sin 37^\circ$

c $48 \div \tan 82^\circ$

d $67 \div \cos 12^\circ$

Example 7

- 4 Solve for the unknown value in the following, rounding to 2 decimal places.

a $\sin 23 = \frac{x}{17}$

b $\cos 47 = \frac{y}{86}$

c $\tan 12 = \frac{a}{36}$

d $\tan 68 = \frac{23}{x}$

e $\sin 41 = \frac{16}{g}$

f $\cos 62 = \frac{118}{b}$

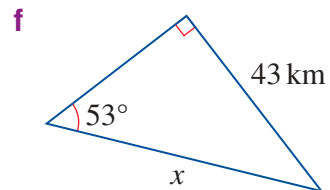
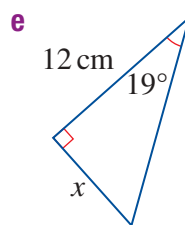
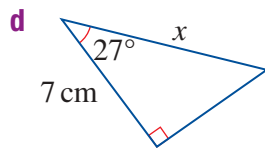
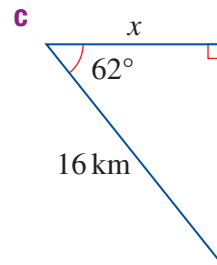
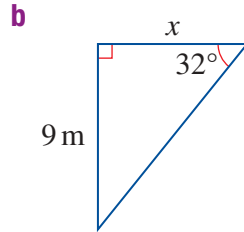
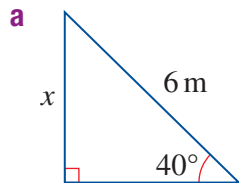


Estimating the height of tall objects uses angles of elevation and trigonometry.

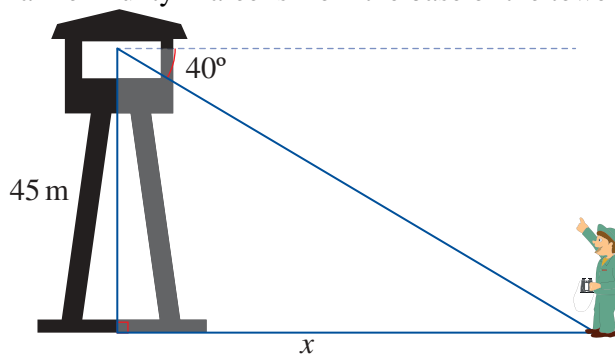
Example 8 & 9

5 Complete the following steps to calculate the unknown side in the triangle shown.

- Identify the known sides of the triangle.
- Determine which trigonometric ratio is required.
- Substitute the values into the ratio.
- Solve the ratio for the unknown value. Round your answer to 1 decimal place.

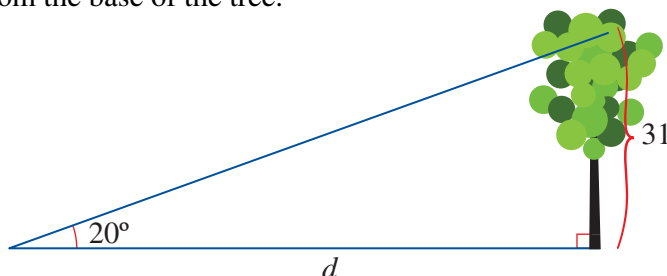
**APPLICATIONS****Example 10**

6 Bridget is in the tower at 45 m above the ground, and she can see her Aunt Maree on the ground at an angle of 40° . Complete the following steps to calculate how far her Aunt Maree is from the base of the tower.



- Determine which sides are the known sides (A, O, H).
- Determine the correct ratio to use from SOH CAH TOA.
- Substitute the values into the appropriate ratio.
- Calculate the correct value for the distance Aunt Maree is from the base of the tower. Round your answer to 1 decimal place.

- 7** Rove can see the top of a tree in the distance at an angle of 20° . The height of the tree is 31 metres. Complete the following steps to calculate how far he is standing from the base of the tree.



- Determine which sides are the known sides (A, O, H).
 - Determine the correct ratio to use from SOH CAH TOA.
 - Substitute the values into the appropriate ratio.
 - Calculate the correct value for the distance Rove is standing from the base of the tree. Round your answer to 1 decimal place.
- 8** Karen is looking up to the top of a building at an angle of 60 degrees. The base of the building is 36 metres from her on the ground. Complete the following steps to calculate the height of the building.
- Draw and label a diagram with the known sides and angle.
 - Determine which ratio is needed to calculate the unknown side.
 - Substitute the values into the appropriate ratio.
 - Calculate the height of the building.

Hint The diagram involves a right-angled triangle.

Example 11

- 9** Heidi is flying her rescue helicopter 1200 m above the ocean and sees a person in trouble at an angle of depression of 38 degrees. Complete the following steps to calculate the direct distance that the helicopter is from the person in the water.
- Draw and label a diagram with the known sides and angle.
 - Determine which ratio is needed to calculate the unknown side.
 - Substitute the values into the appropriate ratio.
 - Calculate the direct distance that the helicopter is from the person in the water.



- 10** Roland is sailing his boat and sees the top of the 120 m cliff at an angle of elevation of 15 degrees from the boat to the top of the cliff. Complete the following steps to calculate the distance that the boat is to the base of the cliff.
- a** Draw and label a diagram with the known sides and angle.
 - b** Determine which ratio is needed to calculate the unknown side.
 - c** Substitute the values into the appropriate ratio.
 - d** Calculate the distance that the boat is to the base of the cliff.
- 11** A cable is to be attached to the top of a large antenna that is 32 metres tall. If the cable is to have an angle from the ground to the top of the antenna of 65 degrees, calculate the length of the cable.
- 12** Pebbles is flying a kite and has let out the entire 55 metres of string. If the angle of the kite string is 44 degrees from the ground, calculate the horizontal distance to the kite.
- 13** Billy is running down a 322-metre slope on the hill. If he found the angle of depression of the hill to be 21 degrees, calculate how high the hill is above ground level.
- 14** A straight waterslide has an angle of elevation of 38 degrees and is 12 metres high. Calculate the actual length of the waterslide.



- 15** The school flagpole is casting a 4.5-metre shadow. If the angle of depression of the sunlight is 36 degrees, calculate the actual height of the flagpole.
- 16** Margret has seen a lighthouse 1350 metres horizontally from her boat. She has measured the angle of elevation from her boat to the top of the lighthouse as 52 degrees. Margret knows that the lighthouse is 50 metres tall on the top of the cliff. Determine the height of the cliff.

3D Determining unknown angles in problems by applying trigonometric rules

LEARNING GOALS

- Determine the named sides in relation to the unknown angle in a right-angled triangle
- Use the rules to calculate the unknown angle of a right-angled triangle
- Identify which trigonometric rules are required to solve real-world context questions, and hence calculate the unknown angle

Why is it essential to be able to calculate angles in right-angled triangles in a real-world context?

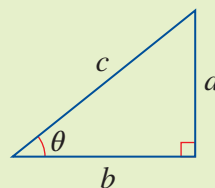
Wooden or metal supports and also cabling use angles within a specific range in order to maintain strength within a structure. Building inspectors use length and width measurements of a roof to check that the roof pitch meets building codes.



Many buildings use right-angled triangles in their construction.

WHAT YOU NEED TO KNOW

- Inverse sine ($\sin^{-1}$), inverse cosine ($\cos^{-1}$) and inverse tangent ($\tan^{-1}$) can be used to find angles in right-angled triangles.
 - $\sin \theta = \frac{a}{c}$ means $\theta = \sin^{-1} \left(\frac{a}{c} \right)$
 - $\cos \theta = \frac{b}{c}$ means $\theta = \cos^{-1} \left(\frac{b}{c} \right)$
 - $\tan \theta = \frac{a}{b}$ means $\theta = \tan^{-1} \left(\frac{a}{b} \right)$
- Note that $\sin^{-1}x$ does *not* mean $\frac{1}{\sin x}$.

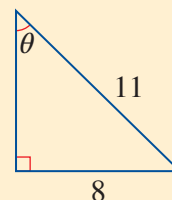




Example 12 Calculating the unknown angle given two other sides of the right-angled triangle

Complete the following steps to calculate the unknown side in the triangle shown.

- Identify the known sides of the triangle in relation to the angle you want to find.
- Determine which ratio uses the two sides given from SOH CAH TOA.
- Substitute the values into the ratio.
- Solve for the unknown angle in the ratio, rounding to 1 decimal place.



WORKING

THINKING

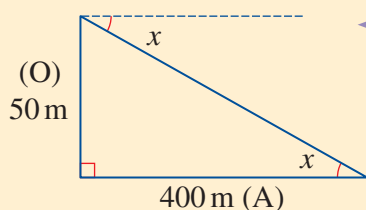
a 8 = opposite side 11 = hypotenuse	←..... The 8 is opposite the named angle and 11 is the hypotenuse.
b $\sin \theta = \frac{\text{opp}}{\text{hyp}}$	←..... Identify the correct ratio required from SOH CAH TOA. The sine ratio uses O and H.
c $\sin \theta = \frac{8}{11}$	←..... Substitute the values into sine ratio with θ°, O = 8 and H = 11.
d $\theta = \sin^{-1} \frac{8}{11}$ $\theta = 46.65^\circ$ $\theta = 46.7^\circ$	←..... Move sin to the other side of the equals sign as $\sin^{-1}$. Calculate the angle size using your calculator. Round the answer to 1 decimal place.



Example 13 Calculating the unknown angle given two other sides of the right-angled triangle in a real-world context

Ranger Yogi wants to build a flying fox (cable glider) to go from the top of the lookout to the ranger station on the ground. The lookout is 50 metres high and the base of the lookout is 400 metres from the ranger station. Determine the angle of depression of the cable connecting the top of the lookout to the ranger station.

WORKING



50 m = opposite

400 m = adjacent

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan x = \frac{50}{400}$$

$$x = \tan^{-1} \frac{50}{400}$$

$$x = 7.12^\circ$$

$$x = 7.1^\circ$$

THINKING

Draw a diagram.

Identify that the angle of depression equals the angle of elevation at the opposite side of the right-angled triangle.

Label the information on the diagram indicating A, O, H.

Identify the correct ratio required from SOH CAH TOA. The tangent ratio uses O and A.

Substitute the values into the tangent ratio with O = 50 and A = 400.

Move tan to the other side of the equal sign as $\tan^{-1}$.

Calculate the angle size using your calculator.

Round your answer to 1 decimal place.

Exercise 3D

FUNDAMENTALS

- Determine the missing words in the following sentences.
 - The _____ ratio equals the _____ side $\div$ the adjacent side.
 - The _____ ratio equals the adjacent side $\div$ the _____.
 - The sine _____ equals the _____ side $\div$ the hypotenuse.
 - To solve a word problem, we should always _____ a diagram and _____ it.
- Calculate the following, rounding to 2 decimal places where necessary.
 - $\cos^{-1} \frac{12}{13}$
 - $\sin^{-1} \frac{37}{61}$
 - $\tan^{-1} \frac{7}{38}$
 - $\tan^{-1} \frac{18}{16}$

Hint Remember that a fraction can be reduced to a single decimal number by dividing the top number by the bottom one on a calculator.

- 3** Solve for the unknown value in the following, rounding to 2 decimal places.

a $\sin \theta = \frac{12}{13}$

b $\cos \theta = \frac{5}{13}$

c $\tan \theta = \frac{14}{17}$

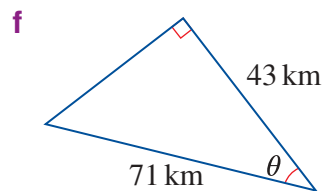
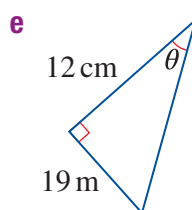
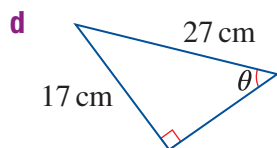
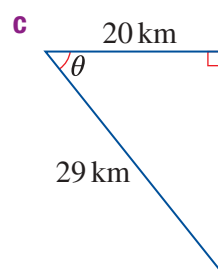
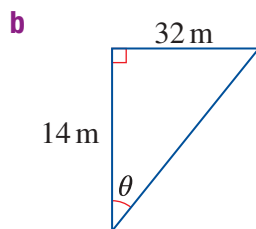
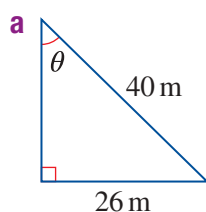
d $\sin \theta = \frac{29}{83}$

Hint You need the inverse trigonometric ratio buttons ($\sin^{-1}$, etc.) on your calculator to solve these equations. Such buttons are usually 'second function' buttons meaning you have to press a button labelled '2nd' before using or accessing them.

Example 12

- 4** Complete the following steps to calculate the unknown side in the triangle shown.

- Identify the known sides of the triangle.
- Determine which trigonometric ratio is required.
- Substitute the values into the ratio.
- Solve the ratio for the unknown value. Round your answer to 1 decimal place.

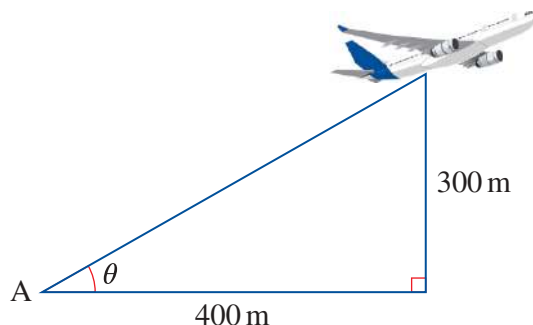


APPLICATIONS

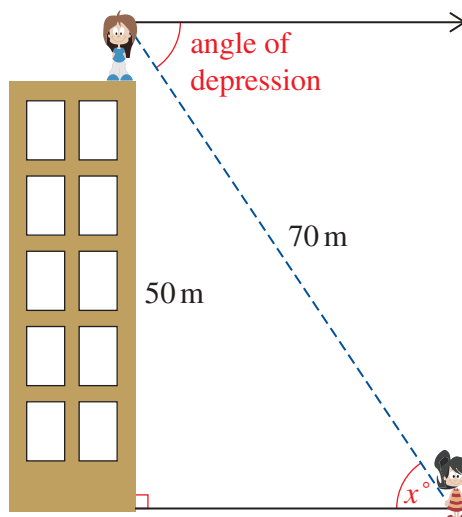
Example 13

- 5** Peta the pilot has just taken off from the runway from a point A as shown in the diagram. Complete the following steps to calculate the angle of take-off from the ground.

- Determine which sides are the known sides using A, O, H.
- Determine the correct ratio to use from SOH CAH TOA.
- Substitute the values into the appropriate ratio.
- Calculate the angle of take-off from the ground, correct to 1 decimal place.



- 6** Charlie is on top of a building that is 50 metres above the ground and she can see her friend Georgie outside the building on the ground. The direct distance between Charlie and Georgie is 70 metres. Complete the following steps to calculate the angle of depression from Charlie to Georgie.
- Determine which sides are the known sides using A, O, H.
 - Determine the correct ratio to use from SOH CAH TOA.
 - Substitute the values into the appropriate ratio.
 - Calculate the angle of depression from Charlie to Georgie, correct to 1 decimal place.



- 7** Douglass has climbed a big tree which is 20 metres high. His father leans a 30-metre ladder against the tree so that he can climb up the tree and help Douglass down to the ground. Complete the following steps to calculate the angle that the ladder makes with the tree.
- Draw and label a diagram with the known sides.
 - Determine which of the known sides are A, O, H.
 - Determine the correct ratio to use from SOH CAH TOA.
 - Substitute the values into the ratio and calculate the angle that the ladder makes with the tree, correct to 1 decimal place.
- 8** Holly has climbed to the top of Exmouth Cape Range, which is 208 metres high above sea level. She saw the ocean, which is 1100 metres horizontally from Holly's current position. Determine the angle that Holly is looking down towards the ocean.

- 9** A ski lift travels 324 metres to the top of a hill that is 212 metres high. Calculate the angle of elevation the ski lift is at the top of the hill from the start of the ski lift.



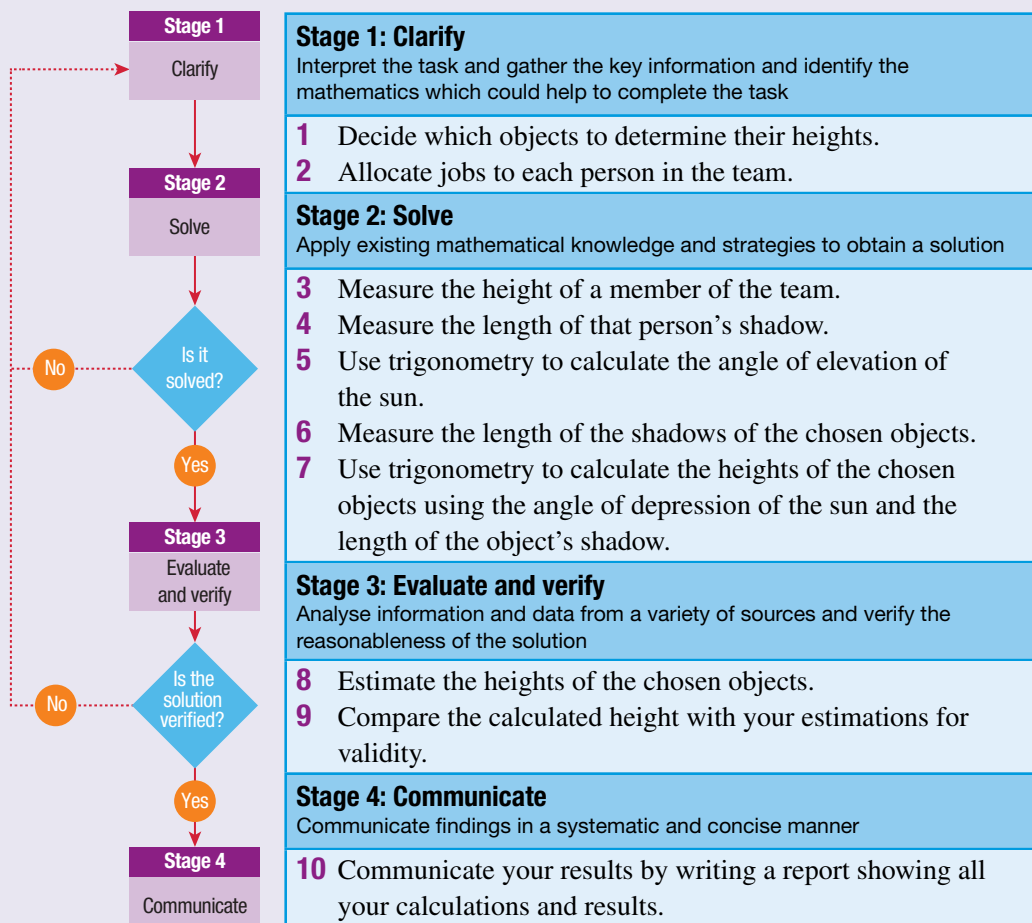
- 10** A castle has a 20-metre wall and a 40-metre wide moat. If an archer wanted to fire an arrow from the top of the wall at someone on the edge of the moat, determine the angle of depression that the arrow would need to be fired. (Assume the arrow will travel straight.)
- 11** A right-angled triangular roof is being built on a house that is 20 metres wide. The roof has a slope length of 28 metres. Calculate the angle of elevation of the roof.
- 12** A straight slippery slide has a height at the top of 2.5 metres and the slide itself is 5 metres long. Determine the angle of depression from the top of the slide.
- 13** Henry is 1.80 metres tall and sees a 24-metre flagpole while he is standing 20 metres away from the base of the flagpole. Determine the angle of elevation that Henry must raise his eyes to see the top of the flagpole. (Assume his eyes are 1.8 metres above the ground.)

Mathematical thinking process task

Background: If you don't have an electronic range-finder, how do you find the height of an object that you can't climb or reach the top of with a long tape measure? A common solution is to use shadow lengths. The sun's rays are for practical purposes parallel, which means the length of shadows of objects measured at the same time of day are proportional to their height.

Task: In a group, you are to use shadows to determine the heights of three different objects. An example may be the height of the football posts, the flagpole and a building. You are to use your knowledge of trigonometry and a ruler to determine the angle of depression of the sun by measuring a person's height and the length of their shadow. You are to then measure the length of the shadows on the various objects and use trigonometry to calculate the heights of the objects given your calculation of the angle of depression.

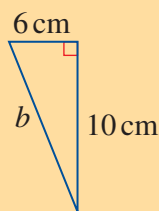
Application of mathematical thinking process.



Chapter checklist

☐ I can calculate the hypotenuse using Pythagoras' theorem.

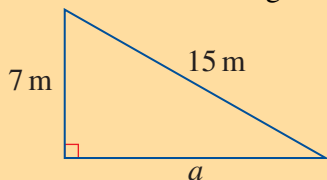
- 1 Calculate the value of b in the triangle shown, rounding to 1 decimal place.



- 2 Calculate the hypotenuse of a right-angled triangle given its base is 8 m and its height is 18 m. Round your answer to 1 decimal place.
- 3 Kristy is making a gate that is 2600 mm by 1800 mm and she needs to build a diagonal brace. Determine the length of the brace that Kristy will need to cut, rounding to the nearest whole number.

☐ I can calculate unknown short sides using Pythagoras' theorem.

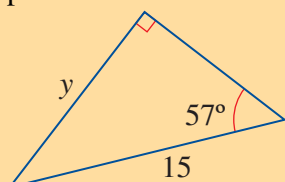
- 4 Solve for a in the diagram shown, rounding to 1 decimal place.



- 5 Calculate the base of a right-angled triangle given its hypotenuse is 26 metres and its height is 11 metres. Round your answer to 1 decimal place.
- 6 Brad ran 2.5 km due west and then ran due north for some distance. If he then ran 4.2 km back to his starting point, determine how far north Brad had run.

☐ I can apply the tangent, sine and cosine rule to determine unknown side lengths.

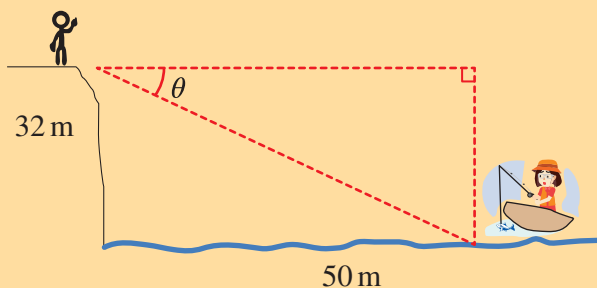
- 7 Calculate the value of y in the triangle shown, rounding to 1 decimal place.



- 8 A right-angled triangle has an angle of 30 degrees and an adjacent side of 25 metres.
- Calculate the length of the opposite side.
 - Calculate the length of the hypotenuse.
- 9 Terri is walking her puppy with a 2.5 m lead. The puppy is walking ahead and pulling the lead tight and it makes an angle of elevation from the dog to Terri's hand of 21° . Determine how much higher Terri's hand is than the puppies' collar.

☐ I can apply the tangent, sine and cosine rule to determine unknown angles.

- 10 Freda is standing on the top of a cliff and can see a person in a boat fishing in the ocean. The person in the boat is 50 m from the base of the 32 m-high cliff. Ignoring the height of Freda, calculate the angle of depression from Freda to the person in the boat.



- 11 A stunt ramp is 14 metres long horizontally and 5 metres high. Determine the angle of elevation of the ramp from the ground.

Chapter review

- Section 3A**
- 1 A castle drawbridge is 20 metres long and has chains connecting the end of the drawbridge to the top of the castle wall which is 25 metres high. Determine the length of the chains to 1 decimal place.
 - 2 An old tree needs a steel cable to help hold it upright. If the cable is attached 2 metres from the base of the tree and 5 metres high on the tree, determine the length of cable between the tree and the ground to the nearest whole number.
 - 3 Esmae walked due north on a 4 km hike and then turned 90 degrees and walked a further 2.5 km due west. Determine how far Esmae is from her starting point to 1 decimal place.
- Section 3B**
- 4 Tarek has a 3.5 metre ladder and leans it against a wall that is 2.5 metres high. Calculate the distance the base of the ladder is from the base of the wall, rounding to 1 decimal place.
 - 5 A tent pole that is 2.1 metres high has a rope attached that is 2.6 metres long. Determine how far the peg needs to be hammered into the ground from the base of the pole so that the rope will be tight.
- Section 3C**
- 6 River is standing 40 metres from the base of a waterfall. If she measured the angle to the top of the waterfall as 52 degrees, calculate the height of the waterfall.



- 7 Zek is flying a plane and approaching a runway at 1000 metres above the ground. He notices that the beginning of the runway is at an angle of 28 degrees from the plane. Calculate the distance the plane still needs to travel before reaching the beginning of the runway.

Section 3D

- 8 Nell slides down a straight slippery slide that is 6.2 metres long and 2.5 metres high. Calculate the angle of the slippery slide from the ground.
- 9 James hang glides from the top of a 368-metre cliff to land 823 metres away from the base of the cliff. Assuming James travelled in a straight line, determine the angle of depression that he flew.
- 10 The lighthouse keeper looks due east from the top of her lighthouse 182 metres above sea level and sees a ship at an angle of depression of 12 degrees. She sees another ship in the same direction at an angle of depression of 18 degrees. Calculate the distance between the two ships.



4 Graphs in practical situations



In this chapter

- 4A** Plotting coordinates on the Cartesian plane
 - 4B** Generating tables for linear functions including negative values for x
 - 4C** Graphing linear functions
 - 4D** Using and interpreting graphs in practical situations
 - 4E** Drawing graphs from data by hand and using technology
 - 4F** Interpreting linear graphs
- Statistical investigation process task
- Chapter checklist
- Chapter review

Syllabus reference

Unit 3 Topic 3: Graphs in practical situations (10 hours)

Cartesian plane

- demonstrate familiarity with Cartesian co-ordinates in two dimensions by plotting points on the Cartesian plane
- generate tables of values for linear functions drawn from practical contexts
- graph linear functions drawn from practical contexts with pencil and paper and with graphing software

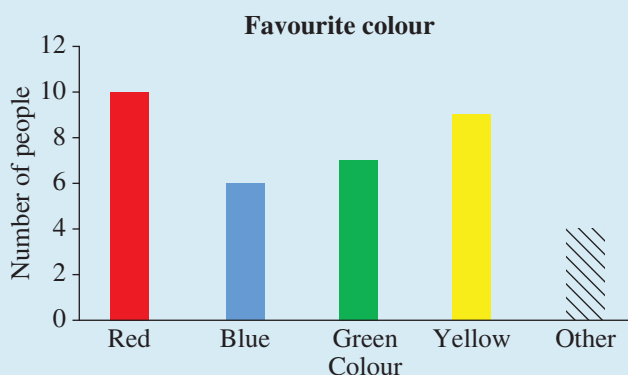
Using graphs

- interpret and use graphs in practical situations, including travel graphs, time series and conversion graphs
- draw graphs from given data to represent practical situations
- describe trend as increasing or decreasing for time series data
- identify the rate of change of the dependent variable, relating it to the difference pattern in a table and the slope of an associated line drawn from practical contexts
- determine and describe the significance of the vertical intercept in practical situations
- use the rate of change and the initial value to determine the linear relationship in practical situations
- interpret the point of intersection and other important features of given graphs of two linear functions drawn from practical contexts; for example, the 'break-even' point

Pre-test

- 1 Identify the x -coordinate in the following points.
 - a (3, 6)
 - b (12, -10)
 - c (-5, 3)
 - d (0, -2)
- 2 Identify the y -coordinate in the following points.
 - a (7, 2)
 - b (-5, 13)
 - c (-3, -6)
 - d (3, 0)
- 3 Carlos earns \$22 per hour, calculate how much he will earn for the following job times.
 - a 3 hours
 - b 8 hours
 - c 2.5 hours
 - d 6 hours 30 minutes
 - e 4.75 hours
- 4 Margareta earns \$40 per hour plus a \$50 call out fee, calculate how much she will earn for the following job times.
 - a 1 hour
 - b 3 hours
 - c 2.5 hours
 - d 4 hours 30 minutes
 - e 1.75 hours
- 5 Determine the next number in the following number patterns.
 - a 2, 4, 6, ...
 - b 3, 6, 9, ...
 - c 4, 7, 10, ...
 - d 2, 9, 16, ...

- 6** The following graph shows the results of a questionnaire asking people what is their favourite colour.

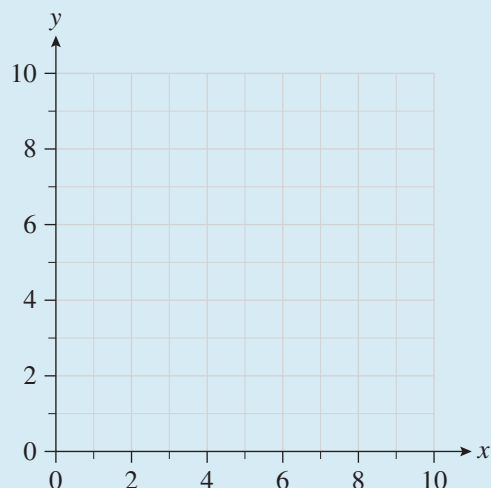


- a** How many survey participants selected blue?
- b** How many survey participants selected yellow?
- c** What is the most popular response?
- d** What is the difference between the number of people who chose green and the number of people who chose other?

- 7** Copy the grid to the right into your workbook.

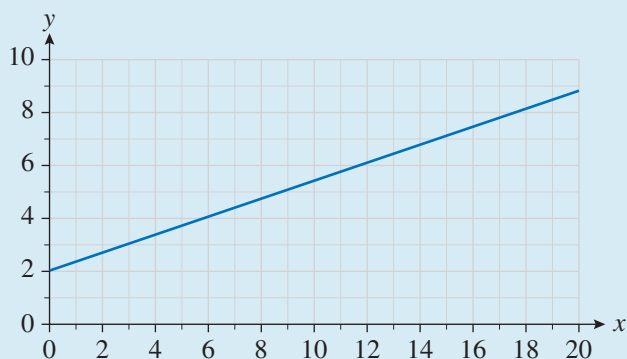
- a** Plot the following points onto your grid.

x	1	4	7	8	9
y	3	6	5	7	10



- b** Starting with the point where $x = 1$, join the points together with a ruler.
- c** What is the value of y when x is equal to 4?
- d** What is the value of x when y is equal to 7?

- 8** The graph shows the relationship between the variables x and y .



- a** What is the value of y when x is equal to 6?
- b** What is the value of y when x is equal to 15?
- c** What is the value of x when y is equal to 8?
- d** What is the value of x when y is equal to 5?

4A Plotting coordinates on the Cartesian plane

LEARNING GOALS

- Understand what a Cartesian plane is and the standard convention for naming coordinates
- Determine the x and y values of a coordinate from a graph
- Graph coordinates on a Cartesian plane
- Plot coordinates on a graph based on a Cartesian plane in a real-world context

Why is it essential to be able to plot coordinates?

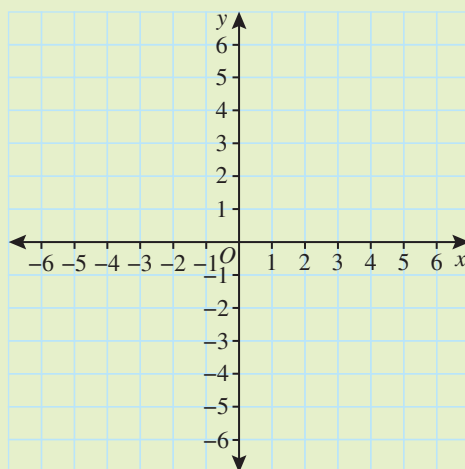
The ability to plot coordinates is an essential skill to being able to graph linear functions and bivariate data, especially scatterplots. And in business and finance, line graphs and scatterplots are the most widely used way of displaying data.



Linear graphs are used in a variety of areas particularly in finance and science fields.

WHAT YOU NEED TO KNOW

- A **Cartesian plane** is a graph with axes x and y labelled with whole number values including negative values. In its basic form it is arranged in a square with O at the centre.
- The **x -coordinate** is the horizontal distance on the x -axis.
- The **y -coordinate** is the vertical distance on the y -axis.
- A coordinate is written by convention as the x -coordinate first followed by the y -coordinate, in brackets: (x, y) .
- Line graphs and scatterplots in real-world contexts are usually based on a Cartesian plane displaying only the positive values of x and y .



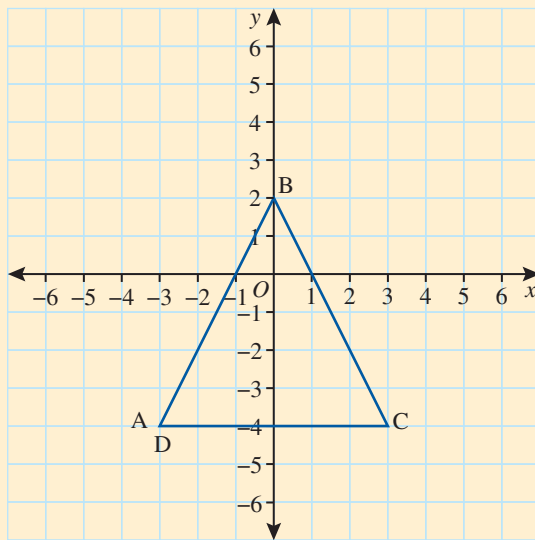


Example 1 Plotting coordinates on a Cartesian plane

Plot the following set of points connecting them in order and name the shape it makes.

A $(-3, -4)$, B $(0, 2)$, C $(3, -4)$, D $(-3, -4)$

WORKING



The coordinate points form a triangle.

THINKING

◀... To plot point A, go to -3 on the horizontal axis and then down 4 places to -4 on the vertical axis and mark point A.

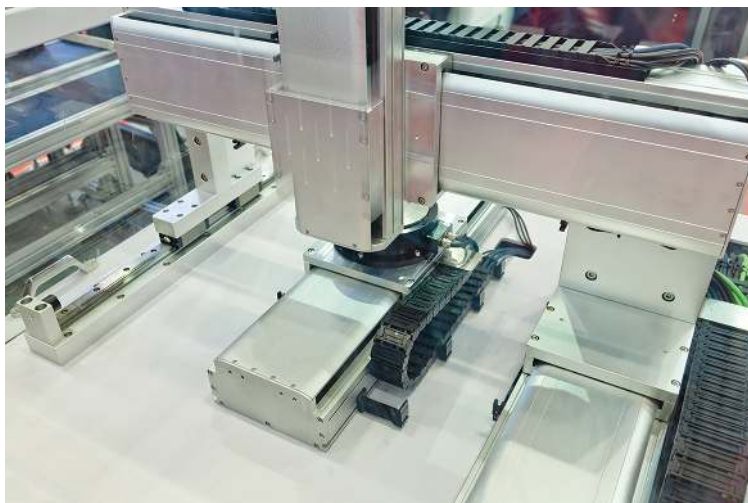
To plot point B, go to 0 on the horizontal axis and then up 2 places to 2 on the vertical axis and mark point B.

To plot point C, go to 3 on the horizontal axis and then down 4 places to -4 on the vertical axis and mark point C.

To plot point D, go to -3 on the horizontal axis and then down 4 places to -4 on the vertical axis and mark point D.

Connect points in order A to B to C to D with 3 straight lines.

Identify the shape formed.



This industrial robot positions itself using Cartesian coordinates to carry out a variety of functions.



Example 2 Plotting coordinates on a Cartesian plane in a real-world context

Ziah starts a gardening business to make some extra cash. In the first month he earned \$15, in the second month he earned \$45, in the third month he earned \$30 and in the fourth month he earned \$40.

- a** Complete the table using the values identified in the question.

Month	1	2	3	4
Earnings \$				

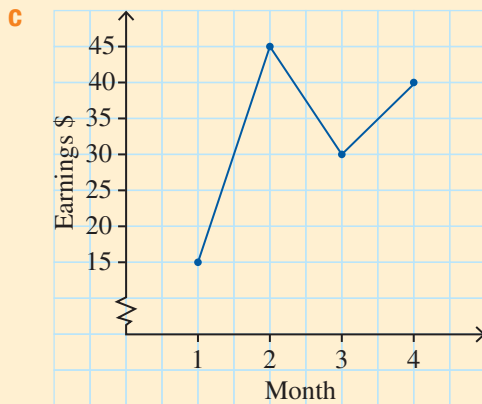
- b** Plot the coordinates (month, earning) on a graph.
c Join the coordinates in order of time.
d Determine the month that Ziah earned the most money.
e Calculate the total Ziah earned in the 4 months.

WORKING

a

Month	1	2	3	4
Earnings \$	15	45	30	40

- b** Identify that the graph needs to go



THINKING

Complete the table by entering the corresponding amount to the month.

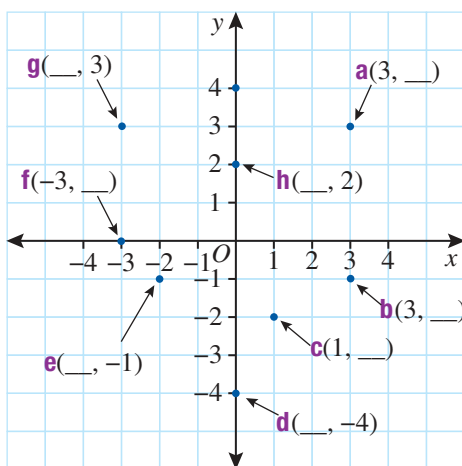
Identify that the graph needs to go to at least \$45 on the vertical axis and 4 on the horizontal axis. Draw the graph axes and label accordingly. Plot the points as per the table using (Months, Earnings). Join the points.

- d** Ziah earned the most in the 2nd month. Find the highest point in the graph.
e $\$15 + \$45 + \$30 + \$40 = \$130$ Calculate the total earnings by adding the monthly earnings.

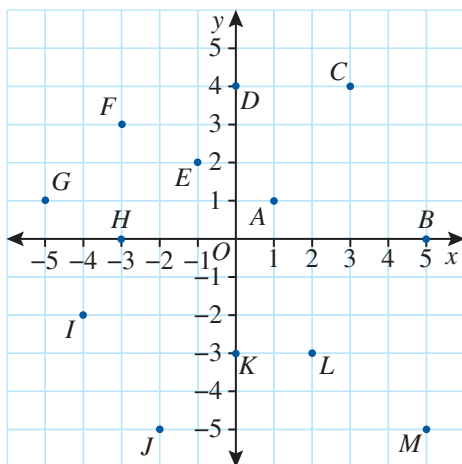
Exercise 4A

FUNDAMENTALS

- Determine the missing words in the following sentences.
 - The x -_____ is the distance along the _____ axis.
 - The y -coordinate is the _____ along the _____ axis.
 - The _____ plane contains both _____ and _____ numbers.
- Determine the missing coordinates labelled **a** to **h** below.



- Identify and write the coordinates of the points labelled **A** to **M** below.



Example 1

- Plot the following sets of points connecting them in order and name the shape it makes.
 - A (0, 2), B (2, 0), C (0, -4), D (-2, 0), E (0, 2)
 - A (-3, 1), B (-1, 2), C (1, 1), D (0, -1), E (-2, -1), F (-3, 1)
 - A (-4, -3), B (-4, 2), C (1, 2), D (1, -3), E (-4, -3)
 - A (-3, -1), B (-2, 1), C (3, 1), D (2, -1), E (-3, -1)

APPLICATIONS

- Example 2** 5 Dylan starts a pet walking business to make some extra cash. In the first month he earned \$28, in the second month he earned \$35, in the third month he earned \$49 and in the fourth month he earned \$42.



- a** Complete the table using the values identified in the question.

Month	1	2	3	4
Earning \$				

- b** Plot the coordinates (Month, Earning) on a graph.
c Join the coordinates in order of time.
d Determine the month that Dylan earned the most money.
e Calculate the total Dylan earned in the 4 months.
- 6 Darlene starts a mowing business to make some extra cash. In the first month she earned \$40, in the second month she earned \$60, in the third month she earned \$50 and in the fourth month she earned \$80.
- a** Complete the table using the values identified in the question.

Month	1	2	3	4
Earning \$				

- b** Plot the coordinates (Month, Earning) on a graph.
c Join the coordinates in order of time.
d Determine the month that Darlene earned the most money.
e Calculate the total Darlene earned in the 4 months.

- 7** Irene is making wraps for premature babies to help them stay warm. She made 10 in 2015, 14 in 2016, 18 in 2017 and 20 in 2018.
- a** Create a table of values for this data.
 - b** Create a graph to display this data.
 - c** Determine the year that Irene made the most wraps.
 - d** Calculate the total number of wraps that Irene knitted in the 4 years.



- 8** James has some stock and wants to buy a car by selling his stock to get money for a car. Given each share of the stock was worth \$26.50 in 2016, \$27.50 in 2017, \$27.00 in 2018 and \$26.50 in 2019. If James has 1000 shares, complete a table of values and then draw a graph to determine when he should have sold his stock to make the most money and how much he would receive.
- 9** Jayde has some stock and needs to buy her first car by selling her stock. Given each share of the stock was worth \$3.50 in 2016, \$4.75 in 2017, \$4.25 in 2018 and \$4.00 in 2019. If Jayde has 2000 shares, draw a graph and determine when she should have sold her stock to make the most money.

4B Generating tables for linear functions including negative values for x

LEARNING GOALS

- Substitute values into equations to calculate solutions
- Work with negative numbers
- Complete a table of values

Why is it essential to be able to use a table of values?

Tables of values are the starting point for graphing linear functions. As will be seen in Chapter 6, these graphs can be used to make predictions for the future. They are relevant to many occupations in business, finance, science, technology, health, economics and social studies.



Linear functions can be used in a variety of fields to predict future values.

WHAT YOU NEED TO KNOW

- A **linear function** is an equation of the form $y = mx + c$ where m and c are constants.
- Addition rules of **positive** and **negative** numbers.
- Multiplication rules of **positive** and **negative** numbers.

$$+ + = +$$

$$- + = -$$

$$+ - = +$$

$$+ - = -$$

$$+ \times + = +$$

$$- \times - = +$$

$$+ \times - = -$$

$$- \times + = -$$

The size of the circles indicates the size of the number.

- Solve an equation by substituting values for the variables into the equation.
For example: For $y = 2x + 3$, if $x = 4$ then $y = 2 \times 4 + 3 = 11$ (Calculate using BIDMAS)

- Complete a table of values for a linear function by substituting values for the variables into the equation.

For example: For $y = 2x + 3$, complete the table below by first substituting $x = -3$ and calculating the value of y :

$$y = (2 \times -3) + 3 = -6 + 3 = -3$$

x	-3	-2	-1	0	1	2	3
y	-3						

In the column under -3 , enter -3 in the row for y .
Continue for the remaining values of x .



Example 3 Generating tables of values for linear functions including negative values for x

Complete the table of values for the linear equation $y = -5x + 2$.

x	-2	0	2
y			

WORKING

When $x = -2$, $y = -5 \times -2 + 2$
 $y = 10 + 2 = 12$

When $x = 0$, $y = -5 \times 0 + 2$
 $y = 0 + 2 = 2$

When $x = 2$, $y = -5 \times 2 + 2$
 $y = -10 + 2 = -8$

x	-2	0	2
y	12	2	-8

THINKING

Substitute the value of -2 into the equation. Use order BIDMAS to simplify.

Substitute the value of 0 into the equation. Use order BIDMAS to simplify.

Substitute the value of 2 into the equation. Use order BIDMAS to simplify.

Complete the table using the values for y found above.



Example 4 Generating tables for linear functions in a real-world context

Erin is an electrician and she charges a \$60 call out fee and \$45 per hour. This can be mapped by the equation $y = 45x + 60$, where x is the number of hours worked and y is the cost for the client.

a Complete the table of values for Erin's television repair costs.

x	1	2	3
y			

b Calculate the cost of hiring Erin for $1\frac{1}{2}$ hours.

WORKING

$$\begin{aligned} \mathbf{a} \quad y &= 45 \times 1 + 60 = 105 \\ y &= 45 \times 2 + 60 = 150 \\ y &= 45 \times 3 + 60 = 195 \end{aligned}$$

x	1	2	3
y	105	150	195

$$\mathbf{b} \quad y = 45 \times 1\frac{1}{2} + 60 = \$127.50$$

THINKING

Substitute the value of 1 into the equation. Use order BIDMAS to simplify.

Substitute the value of 2 into the equation. Use order BIDMAS to simplify.

Substitute the value of 3 into the equation. Use order BIDMAS to simplify.

Use the values calculated to complete the table of values.

Substitute $x = 1\frac{1}{2}$ into the equation to calculate the cost of hiring Erin.

Exercise 4B

FUNDAMENTALS

- Determine the missing words in the following sentences.
 - To complete the _____ of values we must _____ values into the equation.
 - A _____ number multiplied by a negative number equals a positive _____.
 - A _____ number multiplied by a negative number _____ a negative number.
- Substitute $x = 2$ into the following equations to calculate the value of y .

a $y = 3x$	b $y = -5x$
c $y = x + 3$	d $y = x - 4$
e $y = 2x - 4$	f $y = 6 - 5x$

- 3** Substitute $x = -3$ into the following equations to calculate the value of y .

a $y = 4x$

b $y = -2x$

c $y = x + 4$

d $y = x - 3$

e $y = 2x + 3$

f $y = 3 - 4x$

Hint Use the addition and multiplication rules for positive and negative numbers.

- Example 3** **4** Complete the table of values for the following linear equations.

a $y = 3x$

x	-2	0	2
y			

b $y = -2x$

x	-3	0	3
y			

c $y = x + 3$

x	-2	0	2
y			

d $y = x - 2$

x	-3	0	3
y			

e $y = 4x + 1$

x	-2	0	2
y			

f $y = -2x - 2$

x	-3	0	3
y			

APPLICATIONS

- Example 4** **5** Lucinda is a television repair person and she charges a \$50 call out fee and \$60 per hour. This can be mapped by the equation $y = 60x + 50$, where x is the number of hours worked and y is the cost for the client.

- a** Complete the table of values for Lucinda's television repair costs.

x	1	2	3
y			

- b** Calculate the cost of hiring Lucinda for $2\frac{1}{2}$ hours.

- 6** Mario is a plumber and he charges a \$60 call out fee and \$45 per hour. This can be mapped by the equation $y = 45x + 60$, where x is the number of hours worked and y is the cost for the client.

a Complete the table of values for Mario's plumbing cost.

x	2	4	6
y			



b Calculate the cost of hiring Mario for $4\frac{1}{2}$ hours.

- 7** Francis has bought 15 kg of strawberries and unfortunately 2 kg of the strawberries go rotten each week. This can be mapped by the equation $y = 15 - 2x$, where x is the number of weeks and y is the total amount of non-rotten strawberries. Complete the table of values for Francis's strawberries.

x	0	2	4
y			

- 8** Lisa won \$1000 and spends \$150 per week from her winnings. This can be mapped by the equation $y = 1000 - 150x$, where x is the number of weeks and y is the amount Lisa has left. Complete the table of values for Lisa.

x	1	3	5
y			

- 9** In questions **7** and **8**, if the table was extended for more weeks there would come a point where all the strawberries would be rotten, and all the money would be gone. Determine approximately how many weeks it would take for:

- a** all Francis's strawberries to go rotten
b all Lisa's winnings to be gone.

- 10** Trev's Truck hire costs an initial \$120 plus \$50 per hour that the truck is rented. Create an equation to map this relationship and then create a table of values to show the cost to hire his truck for the first 5 hours.

Hint In the equation, the initial cost of \$120 stays the same, while separately, the time in hours has to be multiplied by the cost per hour.

4C Graphing linear functions

LEARNING GOALS

- Create graphs based on linear functions by hand
- Create graphs based on linear functions using technology
- Read values from the graphs created

Why is it essential to be able to graph linear functions?

The ability to graph linear functions is an essential skill for understanding the relationships between two variables contained in bivariate data. This skill enables people in finance, business and many other fields to use graphs to illustrate trends from data. In Chapter 6 this will lead to the ability to make predictions from the graphs. This is the most valuable application of graphs in everyday life.



WHAT YOU NEED TO KNOW

- A linear function is an equation of the form $y = mx + c$ where m and c are constants.
- Knowledge of Cartesian planes, substituting values into an equation and creating a table of values from these values are required for this section.
- The y -axis is the vertical axis and the x -axis is the horizontal axis.
- A scale for the x - and y -axis is created by reading the range of x values and y values in the data to be graphed.
- A plotted graph line is simply a series of points joined together.
- A straight line can be drawn using two coordinates, but we generally use three coordinates to ensure no mistakes have been made in determining these coordinates.
- Linear graphs can be plotted by hand and technology, such as spreadsheets, and online graphing calculators, such as Desmos, can also be used to create straight line graphs.



Example 5 Graphing linear equations by hand

Use the equation $y = 2x - 3$ to complete the following.

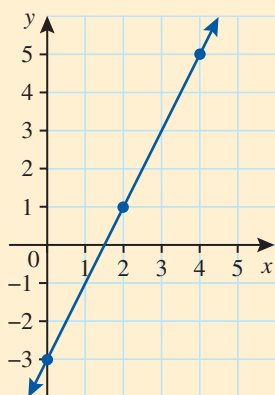
- Complete a table of values for $x = 0, 2, 4$.
- Determine the range of x and y values needed on the graph.
- Draw the graph axes and labels on graph paper.
- Plot the coordinates and draw a line through the points.

WORKING

a

x	0	2	4
y	-3	1	5

- b** x is from 0 to 4, y is from -3 to 5. **c** **d**



THINKING

..... Substitute the values of 0, 2 and 4 into the equation. Use order BIDMAS to simplify.

..... Identify the range of x and y required to be graphed.

Use the range of x and y to draw a graph large enough to display all the coordinates.

Plot the coordinates on the graph and draw a line passing through all three points.



Desmos activity 4C See the Interactive Textbook for this activity on how to graph a linear function using the Desmos online graphing calculator.



Spreadsheet activity 4C See the Interactive Textbook for this activity on how to graph a linear function using a spreadsheet.





Example 6 Graphing linear equations in a real-world context

Tamara bought a used car worth \$8000. The car depreciates (goes down in value) by \$1500 per year. The car's value can be described as $y = 8000 - 1500x$, where y is the car's value and x is the number of years.

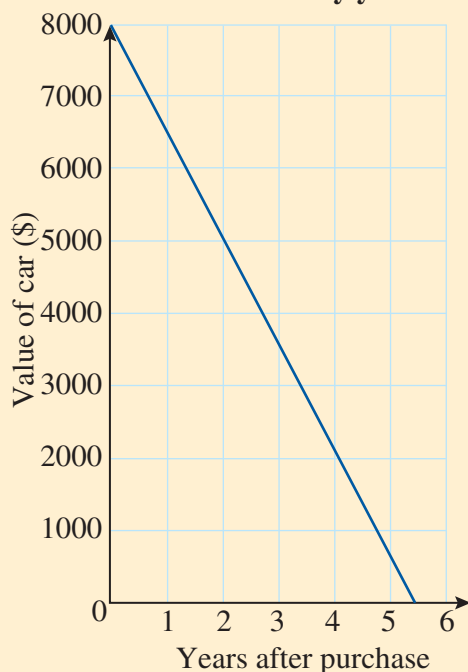
- Graph the equation of the value of her car by hand or with technology.
- Determine how many years it will take for the car to be worth under \$1000.

WORKING

THINKING

a

Value of car by year



←..... The range of y values was from 0 to 8000, so to show the car's value decrease with a step of 1000.

The range of x values was from 0 to 6 with a step of 1 as the car is worth nothing by the 6th year.

- b After 5 years the car will be less than \$1000.

←..... Once the line goes below the 1000 mark, the car is worth less than \$1000.



Exercise 4C

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a A _____ can be used to make future _____.
 - b To _____ a graph we need to find the _____ of values for the x -axis and y -axis.
 - c _____ programs allow us to create graphs _____ and _____.
- 2 For the following sets of points determine the vertical and horizontal range required for the graph axes. For example: The table shown has x values from 0 to 4 and y values from -2 to 6.

x	0	2	4
y	-2	2	6

a

x	1	2	3
y	-3	2	7

b

x	-2	0	2
y	-5	-1	3

c

x	-3	0	3
y	-12	2	16

d

x	0	3	6
y	0	13	26

- 3 Graph the following tables of values on the same graph.
 - i Plot the coordinates on the graph paper.
 - ii Connect the three coordinates by drawing a straight line through them.

a

x	-2	0	2
y	-1	3	7

b

x	0	2	4
y	4	1	-2

Example 5

- 4 For each of the following equations, complete the following.
 - i Complete a table of values for $x = 0, 2, 4$.
 - ii Determine the range of x and y values needed on the graph.
 - iii Draw the graph axes and labels on graph paper.
 - iv Plot the coordinates and draw the lines.
 - a $y = x + 3$
 - b $y = 2x$
 - c $y = 3x - 3$
 - d $y = -2x + 1$

- iv** $y = x$

Example 6

-
- A person is captured mid-air, performing a high takeoff on a surfboard. The surfboard is angled upwards, and the person is in a crouched position, ready to launch. A large, white, fan-shaped spray of water is erupting from the base of the takeoff, indicating the point of departure from the wave. The background shows a clear blue sky and the ocean surface.

- 9** Tom is a fisherman who has bought a boat worth \$38 000. The boat depreciates (goes down in value) by \$6000 per year and the value of the boat can be described as $y = 38\,000 - 6000x$, where y is the value of the boat and x is the number of years.
- a** Graph the equation of the value of the boat by hand or with technology.
- b** Determine how many years it will take for the value of the boat to be worth under \$15 000.
- 10** Mandeep is a taxi driver who charges a \$4.50 fee when picking up clients and then charges \$0.80 per km travelled. The taxi fare can be described as $y = 0.8x + 4.5$, where y is the total taxi fare and x is the number of km travelled. If Mandeep picks up a client at Perth Airport and takes them to the city which is 13 km away, create a graph to determine how much the total fare will be.
- 11** Taj is a limousine driver who charges a \$24 fee when picking up clients and then charges \$1.20 per km travelled. The taxi fare can be described as $y = 1.2x + 24$, where y is the total limousine fare and x is the number of km travelled. If Taj picks up a client at Perth Airport and takes them to Dalkeith which is 26 km away, create a graph to determine how much the total fare will be.



- 12** Kirra has a surfboard shop and it costs her \$800 a day in wages and rent to stay open. If she makes \$210 per surfboard she sells, write this as an equation and graph the equation to determine the amount of surfboards required to be sold to make a profit each day.

4D Using and interpreting graphs in practical situations

LEARNING GOALS

- Use graphs in practical situations
- Interpret and analyse data from graphs in practical situations

Why is it essential to know how to use and interpret graphs in practical situations?

There will be many times throughout your working career when you will encounter a graph. It could be when interpreting the relationship between voltage and current as an electrician; figuring out the tax to be paid on someone's income; estimating the value of a piece of machinery after a certain number of years; or reading off an expected height for a given weight. In any situation, being able to interpret graphs and knowing how to use them is an essential skill.



Understanding how to use and interpret graphs is in your hands.

WHAT YOU NEED TO KNOW

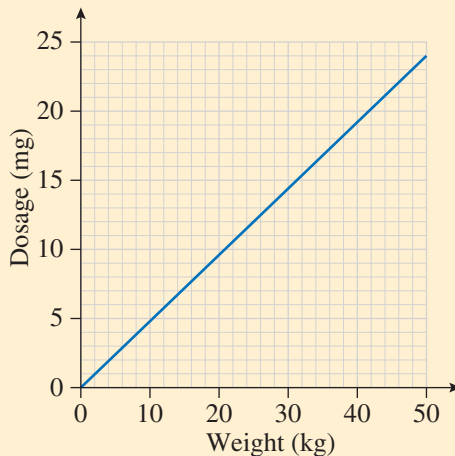
- How to read and interpret graphs from Chapter 4, sections 4A to 4C.



Example 7 Using a graph in a practical situation

Bruno works as a veterinarian assistant. In order to determine the amount of Valium required to give a dog suffering from anxiety, he uses the following graph.

- How much Valium should Bruno give a dog that weighs 10 kg?
- If Bruno gives 17 mg of Valium to a dog, determine the weight of the dog (assuming Bruno gave the correct dosage).



WORKING

a 5 mg



Find 10 kg on the horizontal axis and then trace a vertical line up until you reach the graph. Now trace a horizontal line across to the vertical axis to see the dosage required for a dog weighing 10 kg.

b 34 kg



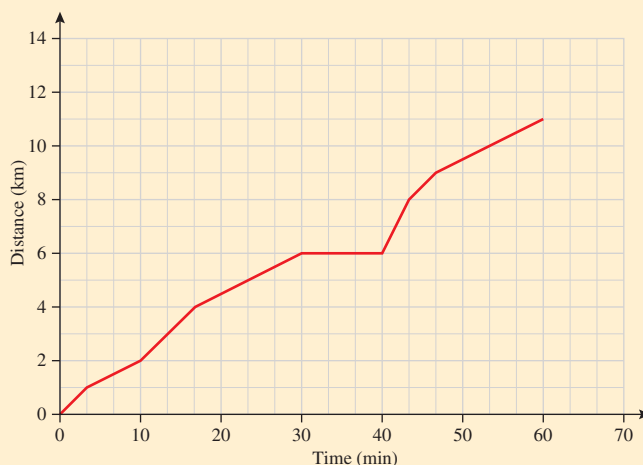
Find 17 mg on the vertical axis and then trace a horizontal line across until you reach the graph. Now trace a vertical line down to the horizontal axis to see the weight of a dog that requires 17 mg of Valium.

THINKING



Example 8 Interpreting and analysing data from a graph in a practical situation

Kim, a personal trainer, has tracked the progress of a client's recent marathon training session and displays it as a distance–time graph.



- How far had the client run after 30 minutes?
- What was the client doing between the time interval of 30 and 40 minutes?
- How long did it take the client to run 11 km?

WORKING

- a** 6 km



THINKING

Find 30 minutes on the horizontal axis and then trace a vertical line up until you reach the graph. Now trace a horizontal line across to the vertical axis to see the distance reached after 30 minutes

- b** The client was having a rest.



From the graph, it can be seen that there is a horizontal section of the graph between 30 and 40 minutes. This corresponds to no change in distance, so the client is not running anymore.

- c** 60 minutes

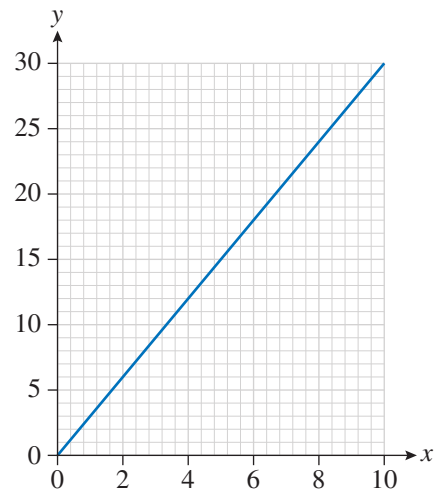


Find 11 km on the vertical axis and then trace a horizontal line across until you reach the graph. Now trace a vertical line down to the horizontal axis to see the time taken to run 11 km.

Exercise 4D

FUNDAMENTALS

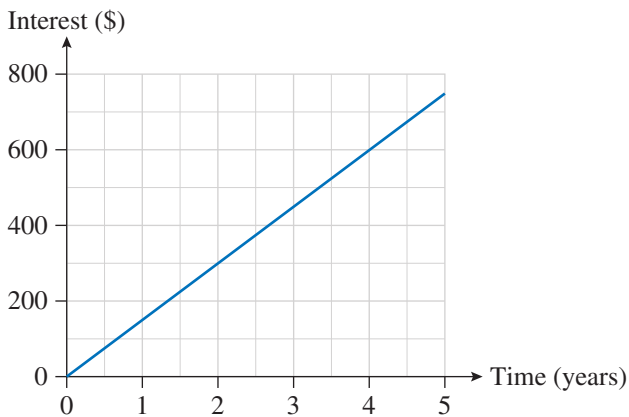
- 1** The graph shows the relationship between x and y .
- What is the value of y when x is equal to 2?
 - What is the value of y when x is equal to 7?
 - What is the value of x when y is equal to 15?
 - What is the value of x when y is equal to 27?



APPLICATIONS

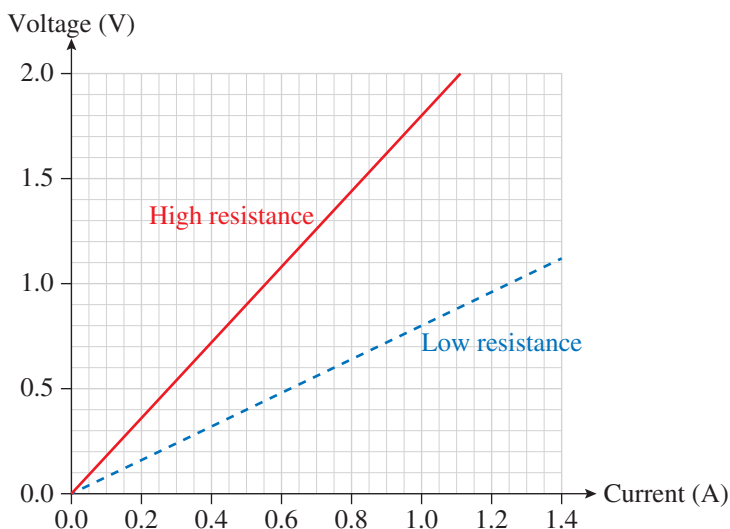
Example 7

- 2** The interest earned on an amount of money is the amount of money a financial institution gives you for investing it with them. The following graph shows the interest earned when \$5000 is invested in an investment bond.



- How much interest is earned after 2 years?
- How much interest is earned after 3 years?
- How long will it take to earn \$600 in interest?
- How long will it take to earn \$150 in interest?

- 3** An electrician uses a current–voltage graph to model the behaviour of resistors in an electrical circuit. The graph shows the relationship between current and the voltage of two resistors: one with high resistance and one with low resistance.

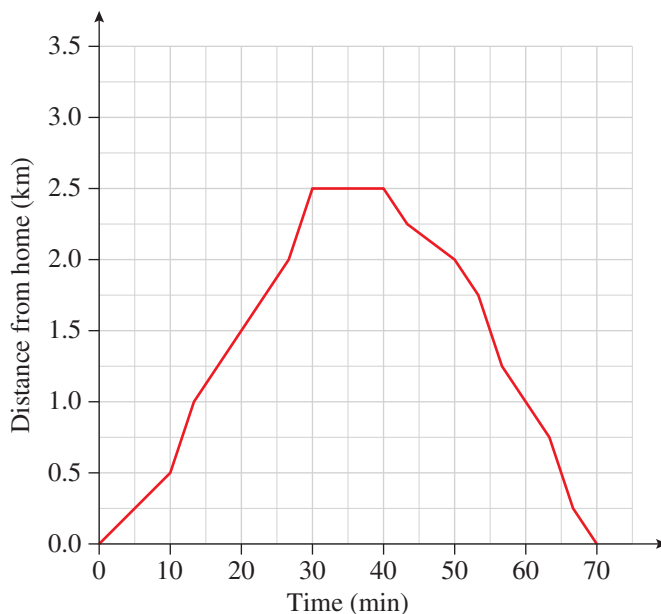


- Determine the voltage of the low-resistance resistor with a current of 0.5 A.
- Determine the voltage of the high-resistance resistor with a current of 0.95 A.
- Determine the current of the low-resistance resistor with a voltage of 0.8 V.
- Determine the current of the high-resistance resistor with a voltage of 0.9 V.

Example 8

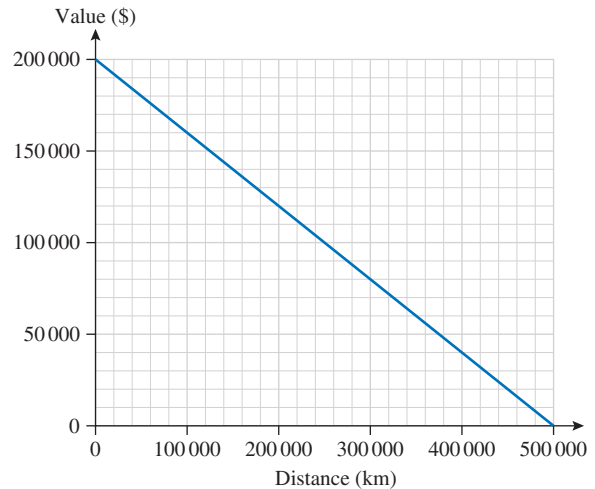
- 4** This is the distance–time graph of Daniel walking to his local library and then walking back home.

- How far does Daniel walk in 10 minutes?
- How long does it take Daniel to walk 2.5 km?
- After 60 minutes, how far is Daniel from his house?

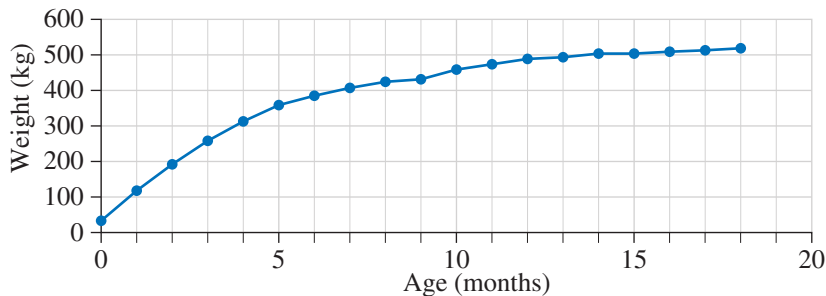


Hint 'Time' is the horizontal axis;
'Distance' is the vertical axis.

- 5** The owner of a fleet of school buses wishes to be able to predict the value of each vehicle according to how many kilometres it has travelled. A graph showing how the value of a school bus depreciates over its lifetime is given.



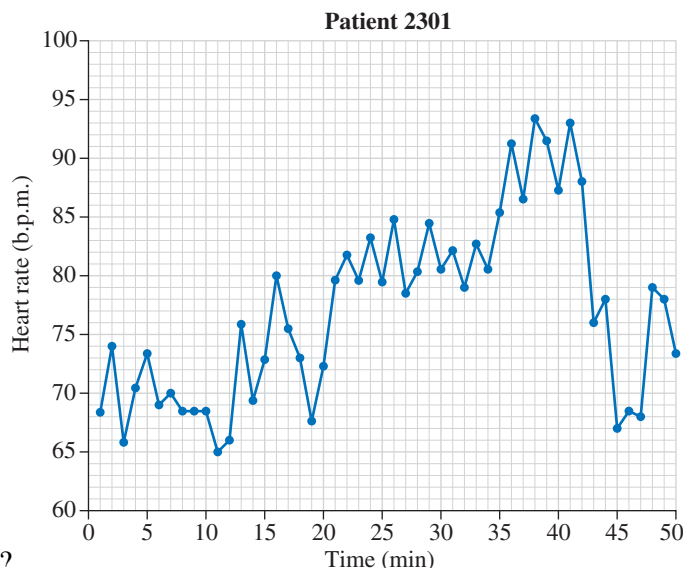
- Estimate the value of a school bus after it has travelled 100 000 km.
 - Estimate the value of a school bus after it has travelled 320 000 km.
 - After how many kilometres is a school bus worth \$150 000?
 - After how many kilometres is a school bus effectively worth nothing?
 - Give a reason why the answer to part **d** may not be realistic.
- 6** A farmer measures the weight of one of his cows every month to determine when it is ready to sell at market. A graph of the cow's weight is given.



- Approximately, what is the cow's weight after 2 months?
- Approximately, what is the cow's weight after 8 months?
- Approximately, when does the cow weigh 400 kg?
- In order to sell the cow for a respectful profit, the farmer prefers the cow to weigh at least 500 kg. After how many months should the farmer sell the cow at market?

- 7** James, a nurse, is assisting in a medical procedure during which he must monitor the patient's heart rate. The graph shows the patient's heart rate during the procedure.

- a** Estimate the patient's heart rate after 16 minutes.
- b** When does the patient's heart rate first get above 75 b.p.m.?

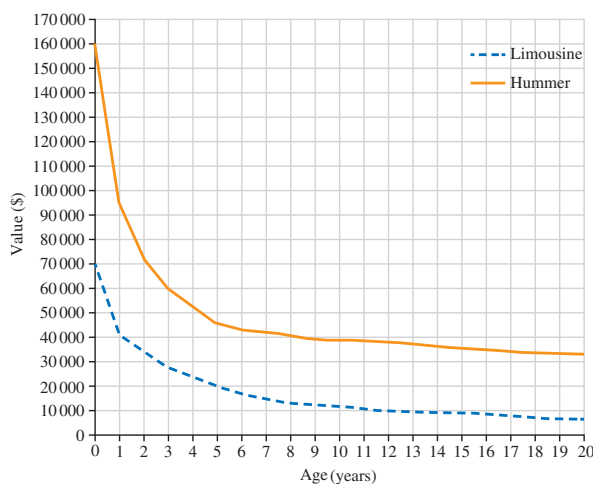


James is told to administer some medication to the patient when their heart rate goes above 90 b.p.m., in order to reduce their heart rate.

- c** At what point does James administer the medication?
- d** Estimate how long it takes for the medication to bring the patient's heart rate down to a stable level.
- e** Describe the trends of the patients heart rate.

- 8** Janette owns a limousine and stretch Hummer business. The line graph below shows the depreciation of both types of vehicles for the first 20 years of their lifetime.

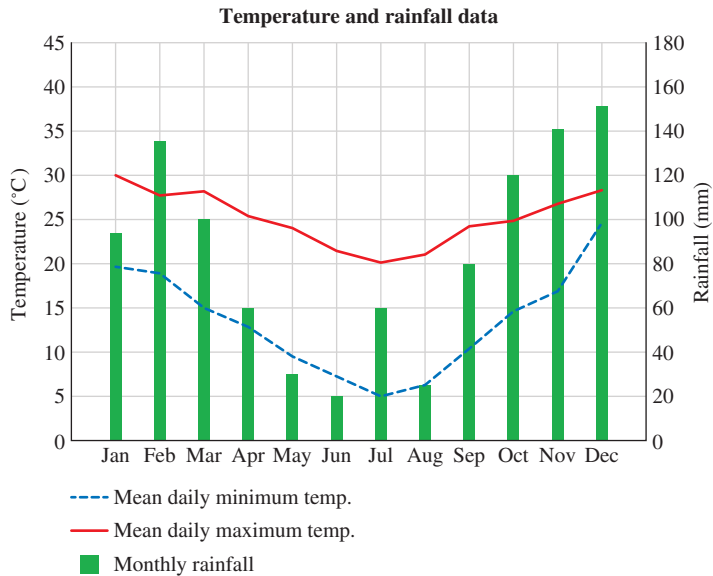
- a** What is the initial cost of each vehicle type?
- b** Approximately, what is the value of a limousine after 3 years?



- c** Approximately, what is the value of a Hummer after 4 years?
- d** Approximately, when is the limousine worth \$20 000?
- e** Approximately, when is the Hummer worth \$70 000?
- f** Using the graph, what would you estimate the value of each vehicle type to be after 25 years? Justify your response.

Hint Assume that the curves will continue their current trends. Where will they be at 25 years?

- 9 The graph below, which combines a line graph and a column graph on the same set of axes, shows the weather recorded at a resort island over a period of 1 year.



- a Which months had the highest/lowest rainfall?
 b Which months had the highest/lowest temperatures?
 c If you were to travel to this resort island for a holiday by the pool, when would be the best month? Justify your answer.

Hint What weather conditions are best suited to being by a pool?



4E Drawing graphs from data by hand and using technology

LEARNING GOALS

- Hand draw graphs from data representing practical situations
- Graph data from practical situations, using a spreadsheet

Why is it essential to draw graphs for practical situations by hand?

There are many situations in jobs where it is useful to draw a graph from a set of data; for example, determining the amount of tax to be paid based on income, or working out the cost of a construction depending on the amount of timber required. As it may be quite hard to interpret the relationship from the data in its raw form, drawing a graph from the data will make it much easier for the data to be interpreted and communicated.



Drawing a graph from the data makes it much easier to use and interpret the data.

WHAT YOU NEED TO KNOW

- How to draw column graphs
- How to draw line graphs



Example 9 Hand drawing a graph from data representing a practical situation

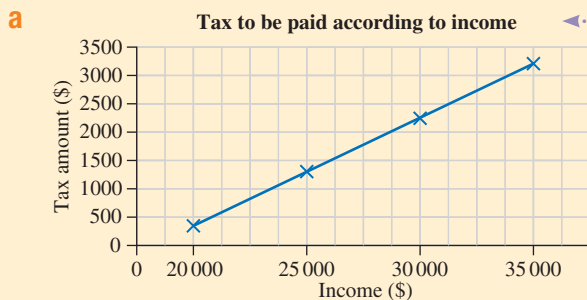
A small business owner is trying to determine how much income tax she must withhold from an employee's pay and pay to the Australian Taxation Office (ATO), depending on the income of employee. The table of values shows the amount of tax to be paid based on incomes between \$18 201 and \$37 000.

Note: this graph can only be used to calculate income between the amounts stated.

Income (\$)	20 000	25 000	30 000	35 000
Tax amount (\$)	342	1292	2242	3192

- Construct a line graph of the data given in the table by hand.
- Use the graph to estimate the amount of tax to be paid by someone who earns an income of \$29 000.

WORKING



- b** Approximately \$2000.

THINKING

Label the 'Income' on the horizontal axis and label the 'Tax amount' on the vertical axis.

Start the horizontal axis at 20 000 and go up in increments of 5000 to 35 000.

Start the vertical axis at zero and go up in 500s to 3500.

Mark a small cross at each data point.

Join up the data points with straight lines. Be sure to use a ruler when drawing straight lines.

Use your horizontal scale to find \$29 000. Trace a vertical line up to the graph line and then trace a horizontal line across to the vertical axis. Use this to estimate how much tax is paid.



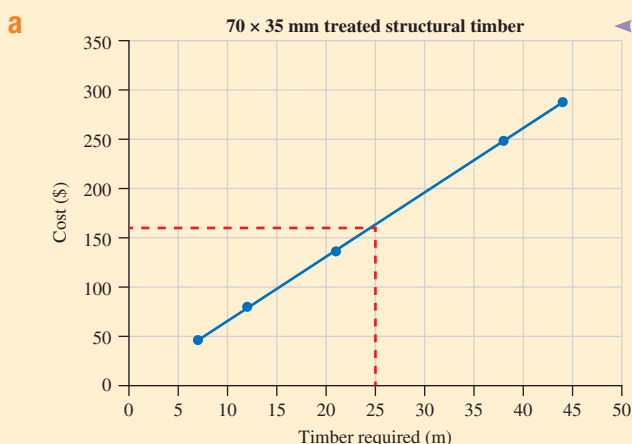
Example 10 Graphing data from a practical situation using a spreadsheet

Jordan works at a timber yard and has been asked to create a graph for customers to use so they can determine how much their order will cost, depending on how many metres of 70×35 mm treated structural timber they require. Jordan is given the following data.

Timber required (m)	7	12	21	38	44
Cost (\$)	45.50	78.00	136.50	247.00	286.00

- Using technology of your choice, create a line graph of the data.
- Using the line graph, estimate how much it will cost for an order of 25 m of 70×35 mm treated structural timber.

WORKING



- b** Approximately \$160.

THINKING

- Input the data into a graphing application such as a spreadsheet.
- Select the data and choose to insert a chart.
- Choose an appropriate chart from the given options.
- Add elements to the chart, such as a chart title and axes titles.
- Use your horizontal scale to find 25. Trace a vertical line up to the graph and then trace a horizontal line across to the vertical axis. Use this to estimate the cost of the order.

Exercise 4E

FUNDAMENTALS

- 1 The table shows the data between two variables, x and y .

x	2	4	6	8	10
y	4	8	12	16	20

- a Hand draw a line graph of the data presented, with x on the horizontal axis and y on the vertical axis.

Hint Remember to mark each data point as you are going along.

- b Use the line graph to find the value of y when x is 7.

- c Use the line graph to find the value of x when y is 10.



- 2 For each of the following, using technology of your choice, create a line graph of the data.

a

x	5	11	25	31
y	4	8.8	20	24.8

b

x	3	14	29	48
y	76.5	357	739.5	1224

APPLICATIONS

Example 9

- 3 To assist with the financial records, a shop owner wishes to create a graph of how much tax he must pay the ATO, depending on how much he pays himself for the year. The table shows the amount of tax to be paid based on incomes between \$37 001 and \$87 000.

Income (\$)	40 000	50 000	60 000	70 000
Tax amount (\$)	4547	7797	11 047	14 297

- a Hand draw a line graph of the data collected.
- b Use the graph to estimate the amount of tax to be paid if the shop owner pays himself \$55 000 for the year.

Hint Remember to use a ruler, as your graph will be more accurate.

Example 10

- 4** A plumber has a pricing structure where there is an initial call-out fee and then an hourly rate. The table shows the cost of hiring the plumber, depending on the number of hours spent working.

Hours	0	2	4	6	8
Cost (\$)	70	150	230	310	390

- a** Using technology of your choice, use a spreadsheet to create a line graph of this data.
- b** Using the line graph, estimate how much it will cost for a job that lasts 5 hours.



- 5** Studies have shown that people are better able to understand instructions for administering medicine when it includes a visual aid, such as a graph. A paediatric nurse would like to create a graph that shows what dosage of a particular medicine is required for babies, depending on their weight, to the nearest kilogram. The nurse gathers the data presented in the table.

Baby's weight (to the nearest kg)	Dosage of medicine (mL)
3	0.75
4	1.00
5	1.25
6	1.50
7	1.75
8	2.00
9	2.25

Using technology of your choice, create a column graph of this data.

- 6** A farmer notices that the yield of her cereal crop is dependent on how much fertiliser she uses. Over the course of four harvest seasons, the farmer collects data on crop yield (in kilograms per hectare) and amount of fertiliser used (in kilograms per hectare). The results are shown in the table.

Fertiliser used (kg/ha)	182	196	221	232
Crop yield (kg/ha)	2700	2840	3090	3200

- a** Hand draw a line graph of the data collected.
- b** Use the graph to estimate the crop yield if 200 kg/ha of fertiliser is used.
- c** Use the graph to estimate the amount of fertiliser required to get a crop yield of 2750 kg/ha.



- 7 An electrical engineer on a cruise ship is monitoring the temperature of a particular component according to the current running through it. The results of five measurements are given in the table.

Current (A)	1.0	1.5	2.0	2.5	3.0
Temperature ($^{\circ}\text{C}$)	20	36	52	68	84

- Using technology of your choice, use a spreadsheet to create a line graph of this data.
- Using the line graph, estimate the temperature of the component when there is a current of 1.8 A running through it.
- The electrical engineer knows that the component will fail when it reaches a temperature above 90°C . Use your graph to estimate the current that will cause this.



- 8 Francis works at a call centre that offers people the chance to sign up to a new mobile phone provider. To encourage workers to get as many people signed up as possible, the company offers cash bonuses to their employees. If a worker gets a certain number of people to sign up to the new mobile phone service in a 4-hour shift, they receive a cash bonus. A summary of the cash bonuses offered, depending on how many customers are signed up, is displayed in the table below.

Number of new customers	5	6	7	8	9	10	11	12
Cash bonus (\$)	20	25	30	36	42	50	57	65

- Hand draw a column graph of the data collected.
- Using your column graph from part **a**, create a poster designed to motivate employees to sign up as many new customers as possible in each 4-hour shift.

4F Interpreting linear graphs

LEARNING GOALS

- Identify and interpret the axes intercepts of linear graphs in practical contexts
- Identify and interpret the point of intersection of linear graphs in practical contexts

Why is it essential to be able to interpret linear graphs?

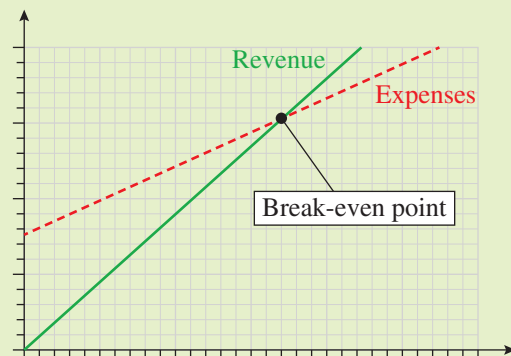
When interpreting linear functions, two key features are the axes intercepts and the point of intersection. The axes intercepts give us information about what the initial conditions are or how long it will take for something to occur. For example, the point of intersection could represent the point when a company will stop making a loss and start making a profit on the production of their product. Understanding these features is essential when trying to interpret the entire graph.

WHAT YOU NEED TO KNOW

- A **linear graph** is the graph of a linear function. A linear graph always takes the form of a straight line on a set of axes.
- The value where a linear graph crosses the vertical axis is called the **vertical axis intercept** (sometimes known as the **y-intercept**).
- The value where a linear graph crosses the horizontal axis is called the **horizontal axis intercept** (sometimes known as the **x-intercept**).
- The point where two linear graphs intersect is called the **point of intersection**.

In a business context:

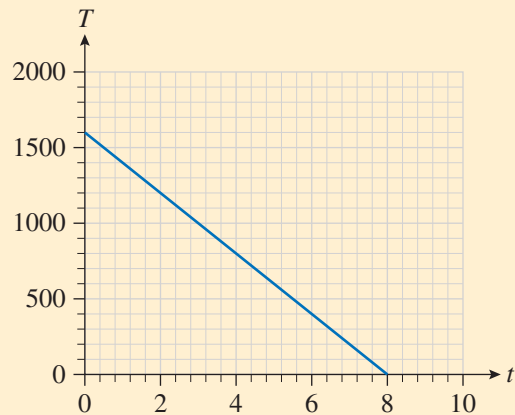
- Profit/Loss = revenue – expenses
 - If the result is positive, then this is a **profit**.
 - If the result is negative, then this is a **loss**.
- The point on a graph showing revenue and expenses where the revenue changes from being below the expenses to above the expenses is known as the **break-even point**, as shown here.





Example 11 Identifying and interpreting the axes intercepts of a linear graph in a practical context

A small business selling a trending toy has set up a stall at a toy market. The business owners are worried about stock levels throughout the day and so they have set up their computer to record how many toys they have left to sell. A graph of the number of toys remaining (T) against time (t), in hours, is presented.



- What is the number of toys remaining after 2 hours?
- How long does it take for the number of toys remaining to drop to 600?
- Identify and interpret the vertical axis intercept.
- Identify and interpret the horizontal axis intercept.

WORKING

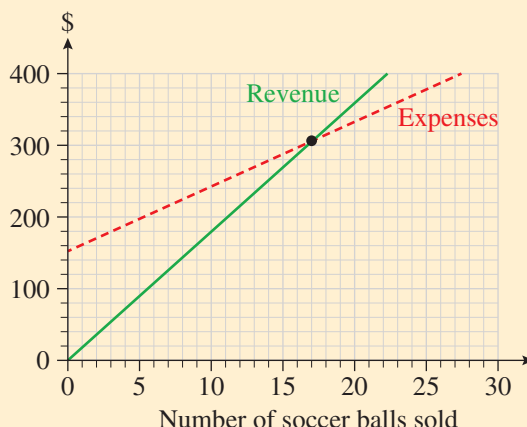
THINKING

a 1200 ←.....	Using your finger to trace the vertical line at 2 hours up to the graph, it can be seen that the number of toys remaining at this time is 1200.
b 5 hours ←.....	Using your finger to trace the horizontal line at 600 toys across to the graph, it can be seen that the time at this point is 5 hours.
c The initial number of toys available to sell is 1600. ←.....	The vertical axis intercept occurs when the graph crosses the vertical axis. This is when the time is 0 hours and the number of toys is 1600. This corresponds to the initial number of toys in stock.
d The business will run out of toys to sell after 8 hours. ←.....	The horizontal axis intercept occurs when the graph crosses the horizontal axis. This is when the time is 8 hours and the number of toys is zero. This corresponds to the amount of time taken for the business to run out of toys.



Example 12 Identifying and interpreting the point of intersection of a real-life linear graph

A vendor who sells soccer balls at sporting venues will have a varying amount of profit, depending on how many soccer balls she sells. The graph shows the revenue made (i.e. money made from selling the soccer balls) in green and the expenses (i.e. how much it costs to make the soccer balls) in red, against the number of soccer balls sold.



- If the vendor sells no soccer balls, will the vendor make a profit or a loss?
- Using the formula profit/loss = revenue – expenses, approximate the profit/loss when the vendor sells 10 soccer balls.
- Identify and interpret the point of intersection.

WORKING

- Loss
- Profit/Loss = $180 - 240 = -60$
Loss of approximately \$60.
- The point of intersection occurs at 17 soccer balls and approximately \$310. This means that when the vendor sells fewer than 17 soccer balls, she will make a loss. If the vendor sells more than 17 soccer balls, she will make a profit. This is the break-even point.

THINKING

If the vendor sells no soccer balls, then the revenue is \$0 and the expenses are \$150. This is a loss of \$150.

By reading the graph when 10 balls are sold, the revenue is approximately \$180 and the expenses are approximately \$240. Using the profit/loss formula gives -60 . This corresponds to a loss of approximately \$60.

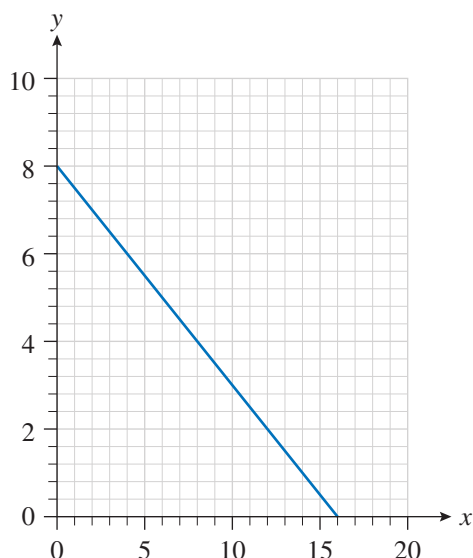
By looking at where the two linear graphs intersect, it can be seen how many soccer balls the vendor needs to sell for her to make a profit.

Exercise 4F

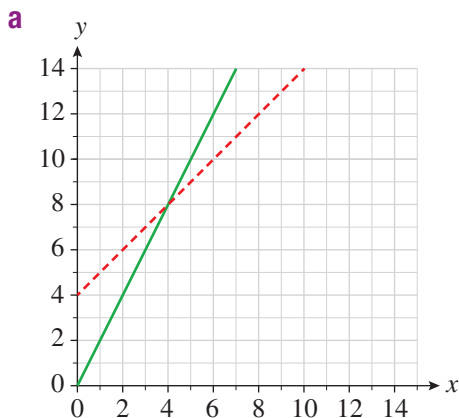
FUNDAMENTALS

- 1 The graph shows the relationship between the variables x and y .
 - a What is the value of y when x is equal to 10?
 - b What is the value of y when x is equal to 3?
 - c What is the value of x when y is equal to 5?
 - d What is the value of x when y is equal to 3.5?
 - e Identify the vertical axis intercept.

Hint At which value does the line intercept the axis?

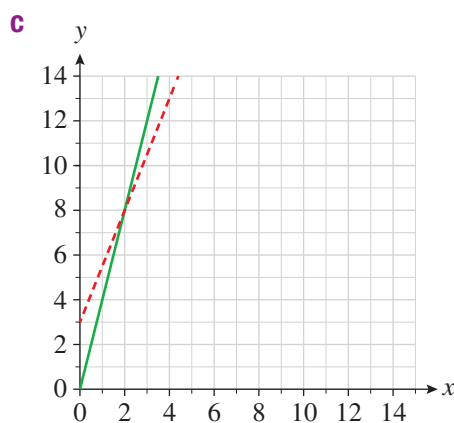
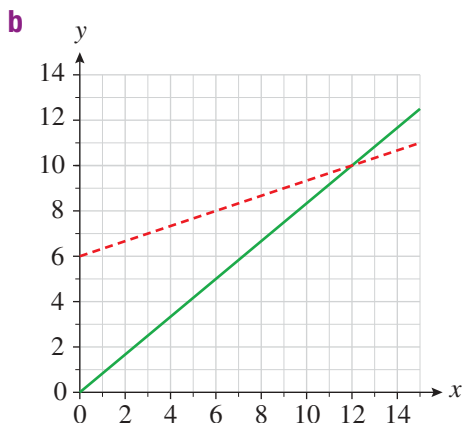


- f Identify the horizontal axis intercept.
- 2 For each of the following graphs, state the x and the y value for the point of intersection.



Hint At which point do the two lines intersect?

Hint When writing the values, give the x value then the y value.



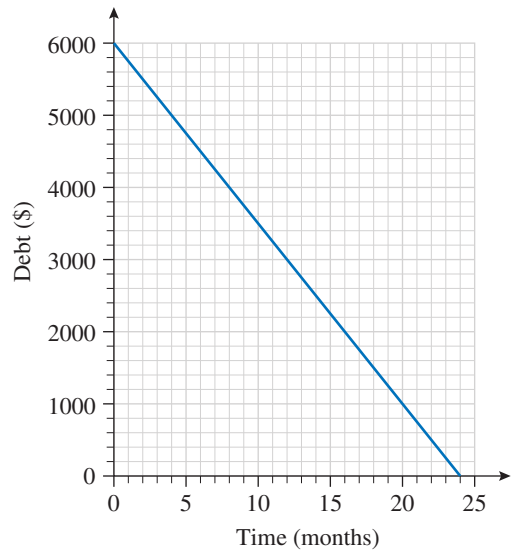
APPLICATIONS

Example 11

- 3** A small business has accumulated some debts. The owners of the business have made a graph to work out how long it will take them to be debt free.

- a** How much money will be owed after 4 months?
b After how many months will \$2000 still be owed?
c Identify and interpret the vertical axis intercept.

Hint Where will the line intercept the axis?



- d** Identify and interpret the horizontal axis intercept.

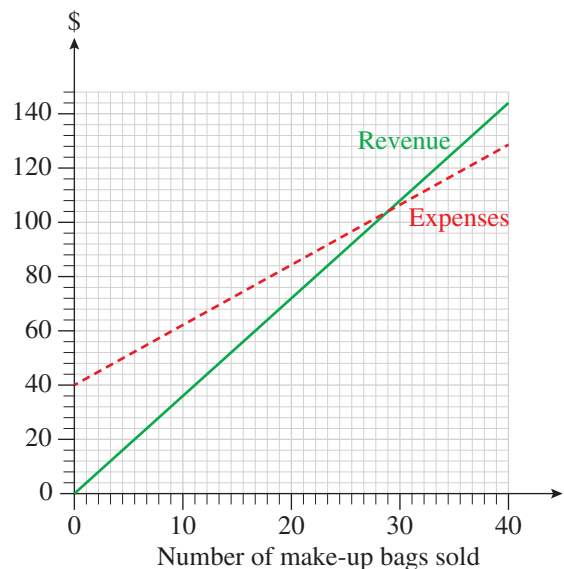
Example 12

- 4** Kacey owns a small business selling make-up bags. The amount of money that Kacey makes from selling the make-up bags at a market depends on how many she sells each month. The graph shows the revenue made (i.e. money made from selling the make-up bags) in green and the expenses (i.e. how much it costs to sell the make-up bags) in red, against the number of make-up bags sold in a month.

- a** If Kacey sells 10 make-up bags:
i approximately, what will be her revenue?
ii approximately, what will be her expenses?
iii approximately, how much money will Kacey lose from selling only 10 make-up bags?

- b** Approximately, find how much profit Kacey will make from selling 40 make-up bags.

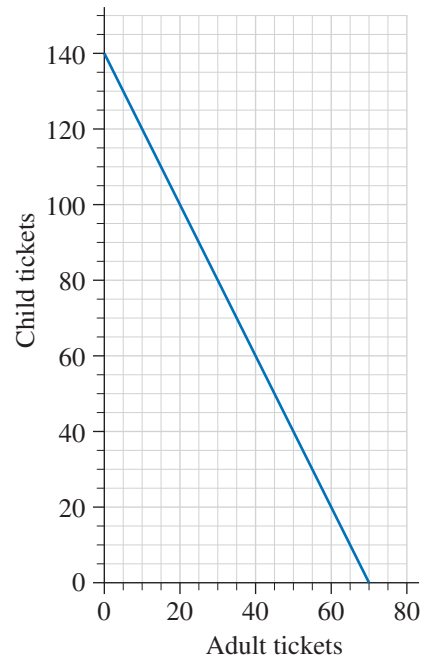
- c** Identify and interpret the point of intersection.



Hint Profit/Loss = revenue – expenses

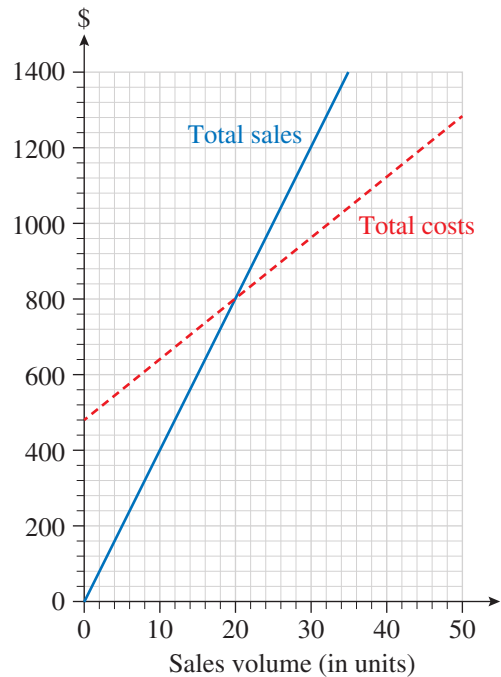
- 5** A small theatre company is putting on a production. It knows that it must sell at least \$350 worth of tickets to start making a profit. The graph shows the number of adult and child tickets that must be sold to make \$350.

- How many child tickets are required to be sold if 40 adult tickets are sold?
- How many adult tickets are required to be sold if 110 child tickets are sold?
- Identify and interpret the vertical axis intercept.
- Identify and interpret the horizontal axis intercept.

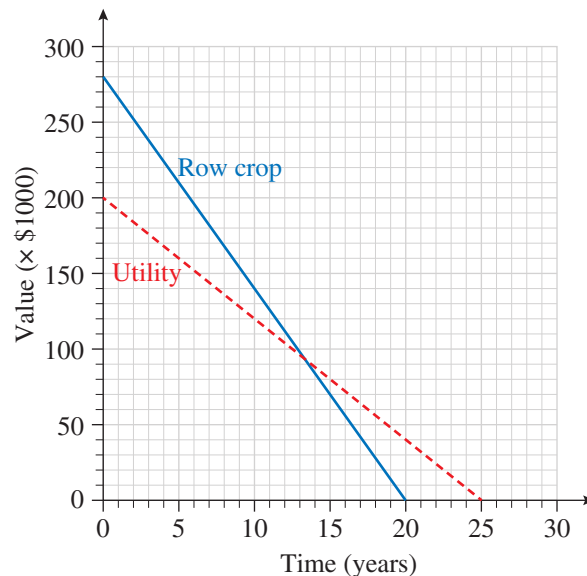


- 6** A small business sells rugby league training equipment sets for \$40 each, with variable costs of \$15 per unit and a total fixed cost. This information is shown in the graph.

- Identify and interpret the vertical axis intercept of the graph of total costs.
- Determine the profit/loss from selling 10 units.
- Determine the profit/loss from selling 26 units..
- Determine the company's break-even point, in units.
- Determine the company's break-even point, in dollars.



- 7** A farmer buys two new tractors for his farm: one utility tractor and one row crop tractor. He knows that each tractor's value will depreciate with each year that he owns them. To avoid selling them past a certain value, the farmer draws a graph of their value against time, as shown.



- Determine the value of the utility tractor after 10 years.
- Determine the value of the row crop tractor after 17 years.
- Identify and interpret the vertical axis intercept of the utility tractor.
- Identify and interpret the horizontal axis intercept of the utility tractor.
- Identify and interpret the vertical axis intercept of the row crop tractor.
- Identify and interpret the horizontal axis intercept of the row crop tractor.
- Identify and interpret the point of intersection.
- The farmer wishes to keep the tractors until they reach a value of \$50 000. How long does the farmer estimate to keep each tractor?



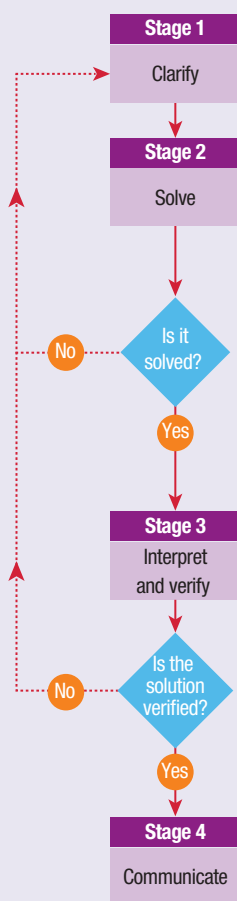
Statistical investigation process task

Background: Graphs are often the best tool for communicating complex information that aims to present comparisons for a decision or choice.

You have been asked by your club to gather and present information to help the members decide which destination to choose for a special celebration of the club's founding.

Task: Select five possible destinations and to draw five rainfall and temperature graphs to help choose the climate that suits the club's outdoor activities best. You must compare and contrast rainfall and temperature for each destination and recommend which place is the most suitable for the club, and what time of year to go.

Approach to statistical investigation process task.



Stage 1: Clarify

Clarify the problem and pose one or more questions that can be answered with data

- 1 Decide on five destinations.
- 2 Describe what kind of climate would suit the club best.

Stage 2: Solve

Design and implement a plan to collect or obtain appropriate data and select and apply appropriate graphical or numerical techniques to analyse the data

- 3 For each destination, research the monthly average rainfall and temperature for the past year and gather data to create the graphs.
- 4 For each destination, research the monthly average temperature for the past year.
- 5 Decide what type of graph to produce and create them using technology of your choice.
- 6 Describe how well each destination matches the required climate and decide which destination to recommend and when to go.

Stage 3: Interpret and verify

Interpret the results of this analysis and relate the interpretation to the original question

Once you have developed your graphs, consider:

- 7 Have you presented them in the most suitable way?
- 8 Have you decided on your recommendations and can you justify them?

Stage 4: Communicate

Communicate findings in a systematic and concise manner

- 9 Communicate your findings in a short report to the club that justifies your recommendations.
- 10 Include an Introduction (the kind of climate that will suit the club's activities), Body (presenting the destinations and graphs) and Conclusion (recommendations of destination and time of year).

Chapter checklist

☐ I can plot coordinates on the Cartesian plane in two dimensions.

- 1 Plot the set of points $(2, -2)$, $(1, 1)$, $(-1, 1)$, $(-2, -2)$, $(2, -2)$ and connect them in order. Name the shape that is made.
- 2 Luke starts a car cleaning business to make some extra cash. In the first month he earned \$80, in the second month he earned \$55, in the third month he earned \$100, and in the fourth month he earned \$90.
 - a Complete the table using the values identified in the question.

Month	1	2	3	4
Earning \$				

- b Plot the coordinates (Month, Earning) on a graph.
- c Join the coordinates in order of time.
- d Determine the month that Luke earned the most money.
- e Calculate the total Luke earned in the 4 months.

☐ I can generate tables for linear functions including negative values of x .

- 3 Complete the table of values for the linear equation $y = 3x - 2$.

x	-2	0	2
y			

- 4 Delta is a fridge mechanic and she charges a \$40 call out fee and \$70 per hour. This can be mapped by the equation $y = 70x + 40$, where x is the number of hours worked and y is the cost for the client.
 - a Complete the table of values for Delta's fridge repair costs.

x	1	2	3
y			

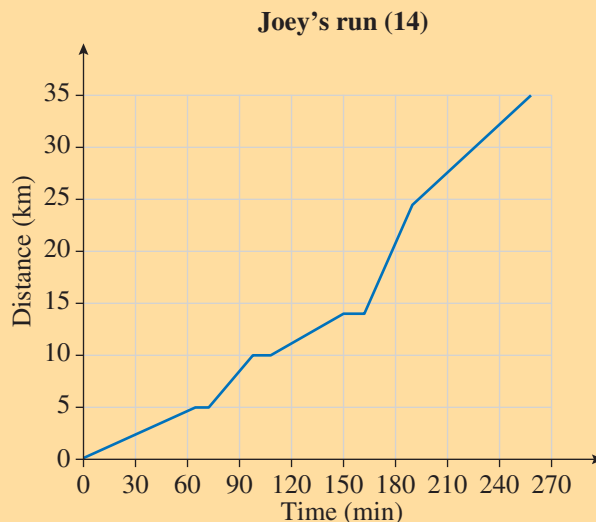
- b Calculate the cost of hiring Delta for $2\frac{1}{2}$ hours.

☐ I can graph linear functions by hand and using technology.

- 5 Use the equation $y = 2x + 3$ to complete the following.
 - a Complete a table of values for $x = -2, 0, 2$.
 - b Determine the range of x and y values needed on the graph.
 - c Draw the graph axes and labels on graph paper.
 - d Plot the coordinates and draw a line through the points.
- 6 Graph the equation $y = 2 - 3x$ using technology.
- 7 John is a helicopter pilot who charges a \$120 fee when picking up clients and then charges \$5 per km travelled. The helicopter costs can be described as $y = 5x + 120$, where y is the total helicopter cost and x is the number of kilometres travelled. If John picks up a client at Perth Airport and takes them to Lancelin, which is 138 km away, create a graph to determine how much the total fare will be.

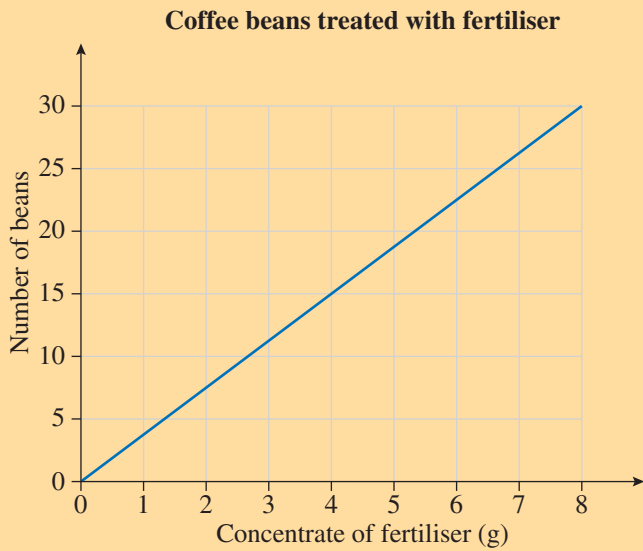
☐ I can interpret data from graphs of practical situations.

- 8 Given the distance–time graph shown, what distance and time did Joey achieve on her 14th run?



☐ I can analyse data from graphs of practical situations.

9 Referring to the line graph shown, how much fertiliser is required for a minimum of 15 beans to grow?



☐ I can hand draw graphs from given data to represent practical situations.

10 Hand draw a line graph of the data below.

	Current tax allocations			
Income (\$)	40 000	50 000	60 000	70 000
Tax amount (\$)	3798	6532	10 675	13 542

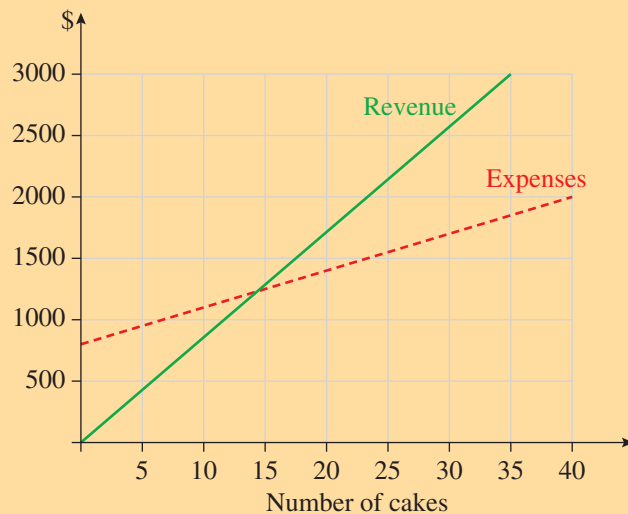
☐ I can draw graphs from given data to represent practical situations.

- 11** Using technology of your choice, create a line graph of the data below, using a spreadsheet.

	Cost of hiring a gardener				
No. of hours	0	2	4	6	8
Cost (\$)	50	150	250	350	450

☐ I can interpret the point of intersection on a linear graph.

- 12** Using the formula $\text{profit/loss} = \text{revenue} - \text{expenses}$, approximate the profit/loss when the business sells 30 cakes.



☐ I understand the terms vertical axis intercept (y-intercept) and horizontal axis intercept (x-intercept).

- 13** Identify and interpret the point of intersection from the previous graph.

Chapter review

Section 4A

- 1** Glen has started busking with his guitar to make some extra cash. In the first month he earned \$48, in the second month he earned \$41, in the third month he earned \$63, and in the fourth month he earned \$54.



- a** Complete the table using the values identified in the question.

Month	1	2	3	4
Earning				

- b** Plot the coordinates (Month, Earning) on a graph.
c Join the coordinates in order of time.
d Determine the month that Glen earned the most money.
e Calculate the total Glen earned in the 4 months.
- 2** Charlette sells jewellery at the local markets each week. In week 1 she made \$112, in week 2 she made \$146, in week 3 she made \$134, and in week 4 she made \$121. Enter Charlette's income into a table of values and draw the graph. Calculate the total Charlette made in the 4 weeks.

Section 4B

- 3** Jack is a pool cleaner and he charges a \$20 call out fee and \$38 per hour for cleaning the pool. This can be mapped by the equation $y = 38x + 20$, where x is the number of hours and y is Jack's total fee.

- a** Complete the table of values for Jack's fee.

x	0	1	2	3
y				

- b** Determine after how many hours Jack has earned more than \$100.

- 4 Fen won \$2000 and spends \$300 per week from her winnings until she has no money left. This can be mapped by the equation $y = 2000 - 300x$, where x is the number of weeks and y is the amount of money Fen has remaining. Complete the table of values for Fen's remaining winnings.

x	1	3	5
y			

Section 4C

- 5 Ian has purchased a mobile crane for \$90 000. The crane depreciates (goes down in value) by \$12 500 per year and its value can be described as $y = 90\,000 - 12\,500x$, where y is the crane's value and x is the number of years.
- a Graph the equation of his crane's value.
- b Determine after how many years the crane will be worth under \$16 000.



- 6 Zac is a taxi driver who charges a \$6.50 fee when picking up clients and then charges \$0.90 per km travelled. The taxi fare can be described as $y = 0.9x + 6.50$, where y is the total taxi fare and x is the number of kilometres travelled. If Zac picks up a client at Broome Airport and takes them to the Broome Port Jetty, which is 8 km away, create a graph and determine how much the total fare will be.

Section 4D

- 7 Emma, who works at a café, gets paid according to how many hours she works in a week. The table shows how much Emma is paid for a certain number of hours worked for that week.

Hours	19	23	27	35
Pay (\$)	342	414	486	630

- Hand draw a line graph of this data, joining all points with a straight line.
- Using the line graph, estimate how much Emma would get paid for working 30 hours in a week.
- Using the line graph, estimate how many hours Emma must work in order to earn \$300 in a week.

Section 4E

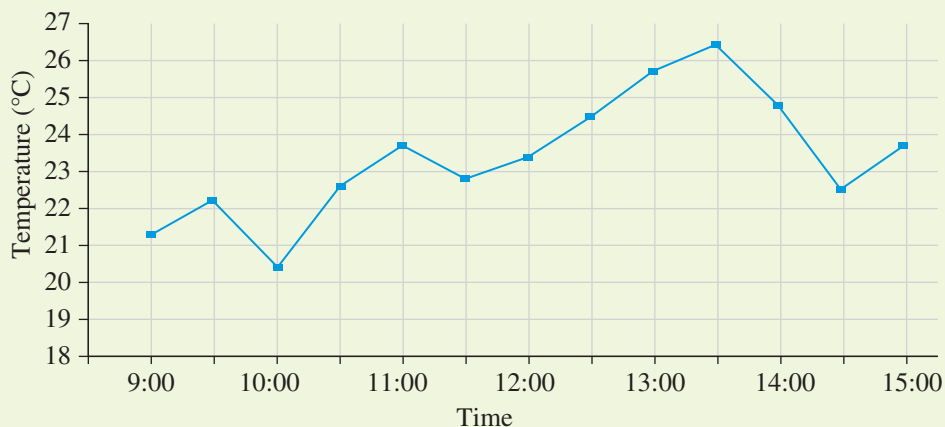


- 8 A veterinarian is putting together an informative chart for the dosage of medication required for cats, depending on their weight, when rounded to the nearest kilogram. The data that needs to be graphed is given below.

Cat's weight (to the nearest kg)	Dosage of medicine (mL)
1	1.2
2	1.4
3	1.7
4	2.0
5	2.4
6	3.0

Using technology of your choice, use a spreadsheet to create a column graph of the data.

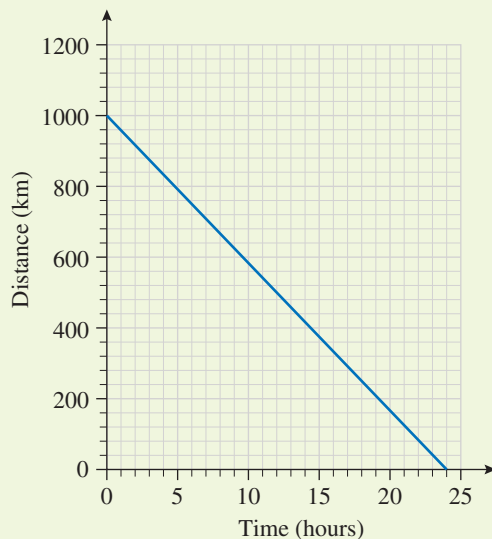
- 9 One of the responsibilities of Will, an aquaculture worker, is to monitor the condition of the fish in a particular tank. Will has been instructed to turn off a water heater when the water temperature rises above 25°C . To monitor the temperature, there is a graphical display showing the current temperature and a graph of how the temperature has varied over the past 6 hours, as shown below.



- Estimate the temperature of the water in the tank at 10 a.m.
- Estimate the temperature of the water in the tank at 11:30 a.m.
- At what time does the temperature of the water first rise above 25°C ?
- When does the graph suggest that Will has turned off the water heater? Justify your response.

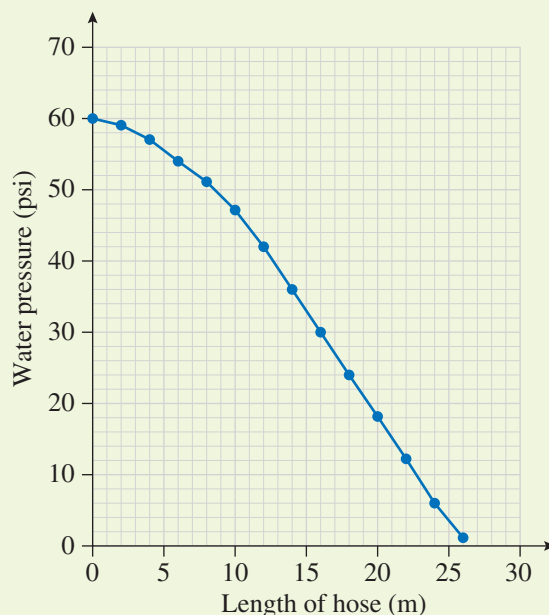
Section 4F

- 10 Frankie is on a direct train from Katherine to Alice Springs and is travelling at a constant speed. The distance from Alice Springs, in kilometres, against time, in hours, is shown in the graph.



- Approximately, how far from Alice Springs is Frankie after 6 hours?
- Approximately, how long does it take for Frankie to be 400 km from Alice Springs?
- Identify and interpret the vertical axis intercept.
- Identify and interpret the horizontal axis intercept.

- 11** A landscaper is installing an automated water system for a client. As the length of hose increases, the water pressure at each sprinkler decreases. The landscaper graphs this relationship, as shown right, in order to discuss it with the client.



- What is the water pressure when 12 m of hose is used?
- What is the water pressure when 8 m of hose is used?
- The client does not wish the water pressure to drop below 30 psi. Up to what length of hose does this allow the landscaper to install?

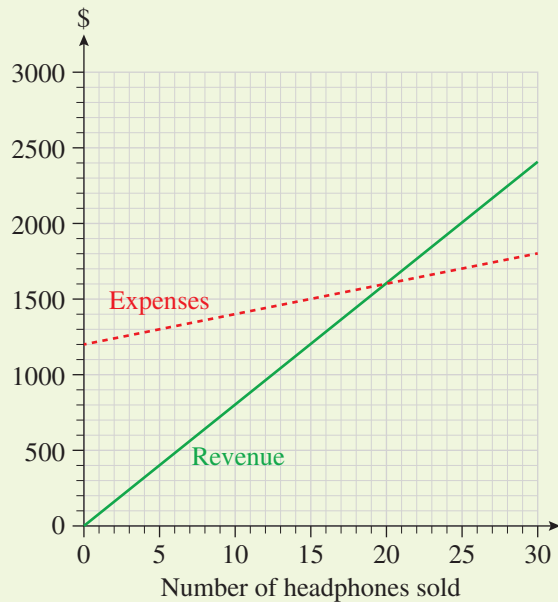


- 12** An accountant for a gym has been given the task of estimating the value of its range of motion machine, depending on its age. The accountant gathers data on how the value of the range of motion machine varies with age. The results are given in the table.

Age (years)	3	5	9	11
Value (\$)	12 750	11 250	8250	6750

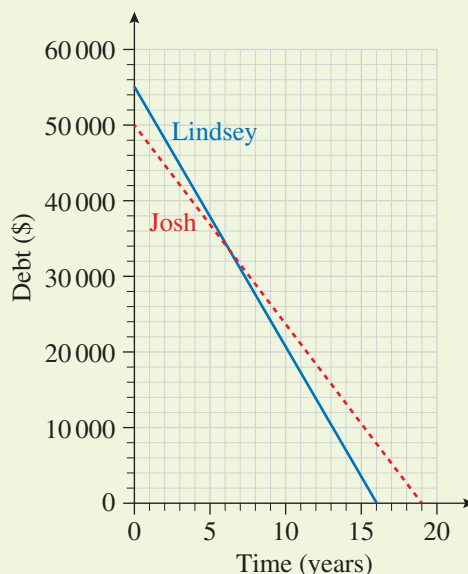
- Using technology of your choice, use a spreadsheet to create a line graph of the data. Display age on the horizontal axis, going from 0 to 25 years, and value on the vertical axis, going from \$0 to \$20 000.
- Using the line graph, estimate how much the range of motion machine is worth after 7 years.
- Using the line graph, estimate after how many years the range of motion machine will be worth \$10 000.
- State the value of the vertical axis intercept and interpret this value.
- State the value of the horizontal axis intercept and interpret this value.

- 13** A company that sells headphones will have a varying amount of profit, depending on how many headphones it sells. The graph shows the revenue made (i.e. money made from selling the headphones) in green and the expenses (i.e. how much it costs to make the headphones) in red, against the number of headphones sold.



- a** Approximate the profit/loss when the company sells 5 headphones.
- b** Approximate the profit/loss when the company sells 25 headphones.
- c** Identify and interpret the vertical axis intercept of the graph of expenses.
- d** Identify and interpret the point of intersection.

- 14** Both Lindsey and Josh have entered the workforce with a sizeable debt from paying for their education. They both pay off their debt in regular payments until it is paid off. In order to forecast how much they each have left to pay, Josh creates the graph shown of how much they each owe.



- a** Approximately, how much does Josh have left to pay after 10 years?
- b** Approximately, how much does Lindsey owe after 2 years?
- c** Approximately, after how many years will Josh have just \$10 000 owing?
- d** Approximately, after how many years will Lindsey have just \$16 000 left to pay?
- e** Identify and interpret the vertical axis intercept for the graph of Josh's debt.
- f** Identify and interpret the vertical axis intercept for the graph of Lindsey's debt.
- g** Identify and interpret the horizontal axis intercept for the graph of Josh's debt.
- h** Identify and interpret the horizontal axis intercept for the graph of Lindsey's debt.
- i** Identify and interpret the point of intersection.

5 Census and surveys





In this chapter

- 5A** Investigating a census
- 5B** Faults and errors in data collection
- 5C** Surveys and simple random sampling
- 5D** Systematic sampling
- 5E** Stratified sampling
- 5F** Identifying target populations
- 5G** Misrepresentation of the results
- 5H** Questionnaire design principles
- 5I** Misrepresentation in surveys
- Statistical investigation process task
- Chapter checklist
- Chapter review

Syllabus reference

Unit 3 Topic 4: Data collection (10 hours)

Census

- investigate the procedure for conducting a census
- investigate the advantages and disadvantages of conducting a census

Surveys

- understand the purpose of sampling to provide an estimate of population values when a census is not used
- investigate the different kinds of samples, for example, systematic samples, self-selected samples, simple random samples
- recognise the advantages and disadvantages of these kinds of samples; for example, comparing simple random samples with self-selected samples

Simple survey procedure

- identify the target population to be surveyed
- investigate questionnaire design principles; for example, simple language, unambiguous questions, consideration of number of choices, issues of privacy and ethics, freedom from bias

Sources of bias

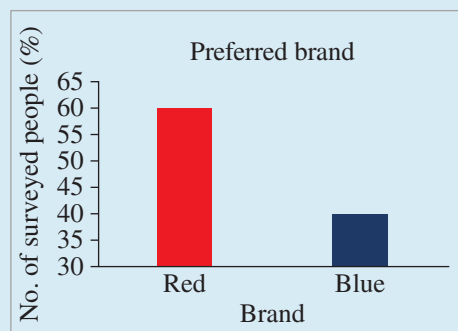
- describe the faults in the collection of data process
- describe sources of error in surveys; for example, sampling error and measurement error
- describe possible misrepresentation of the results of a survey due to the unreliability of generalising the survey findings to the entire population, for example, because of limited sample size or chance variation between samples
- describe errors and misrepresentation of the results of a survey, including examples of media misrepresentations of surveys and the manipulation of data to serve different purposes

Pre-test

- 1 Classify each of the following data as either categorical or numerical, and give two possible survey responses.

a money spent in canteen c age (years) e favourite music genre	b favourite sauce d preferred transport mode f number of monthly visits to cinema
- 2 Why might someone wish to compile each of the datasets in Question 1?
- 3 A high school with 100 students conducts a survey asking students what is their commute time from school to their home, in minutes. The survey receives five responses: 6, 10, 10, 15, 10.
 - a** Find an estimate for the average of the data.
 - b** Does the data represent the other 95 students' commute times? Explain why/why not. After a few days, the survey receives more responses: 20, 35, 25, 30, 28, 45, 38, 31, 25, 3, 27, 32, 17, 13, 36.
 - c i** Including the original data, find an estimate for the average of the new dataset.
ii Compared to part **a**, how has the average changed?
 - d** Would the extra responses be a better representation of the other students' commute times? Explain why/why not.
 - e** Group the responses in appropriate ranges and display them in a dot plot.
- 4 A survey is done to compile Year 11 students' test scores. Students can choose whether or not they wish to participate. The survey receives the following 10 responses.
 90 85 91 92 75 81 83 94 100 79
 - a** Display the results in a grouped frequency table.
 - b** Would a student with a lower test score willingly participate in this survey?
 - c** What problems does this survey have? (Don't forget to use your answer from part **b**.)
- 5 The number of people using a train station on weekdays is recorded for 20 days: 111, 150, 123, 143, 129, 136, 109, 112, 114, 113, 119, 128, 149, 146, 205, 195, 147, 131, 135, 125
 - a** What type of data is this?
 - b** Identify the lowest and highest number of commuters in a day.
 - c** Give an estimate of the average number of commuters on a weekday.
 - d** Display the data in any appropriate method (e.g. table, dot plot).
 - e** Are there any outliers in this data? If so, what are the possible explanations for them?
 - f** Suppose you wish to estimate the number of people using the train station on weekends. Is using the weekday data appropriate for this? Explain your reasoning.

- 6** Anita wishes to know her school's opinion on the newly built sports facility.
- What would be her target population?
 - What is a census, and is it practical for this situation?
 - Identify any potential problems in her sample choices, which are:
 - her year group
 - every third student entering the sports facility
 - everyone who completes her online survey
 - asking students randomly during recess and lunch break
 - the administrative staff
 - a mixture of students, where the students in each year group is in the same proportion as the actual number of students in the school.
 - Anita is now considering the questions she will be asking. State whether the following questions have any problems and, if so, identify them.
 - Do you usually go to the sports facility?
 - Most students like the new sports facility. What do you think of it?
 - How often do you use the basketball courts, badminton courts, gymnastic equipment, volleyball nets, toilets and change rooms in the new sports facility?
 - How satisfied are you with the facility? (with response options 'Satisfied' and 'Not satisfied').
 - Would you recommend other students to use the sports facility?
- 7** A bar graph showing a preferred soft drink brand is given.
- What is the percentage of respondents who prefer brand Red? What about brand Blue?
 - List anything that is wrong with the graph.
- 8** Gabriel wishes to know his electorate's political preference by asking 'Do you like political party A?' with response options 'Yes' and 'No'.
- What is wrong with the question?
 - Gabriel asks 10 people who are aged 60 years or older, and finds that 70% like political party A. Explain what is wrong with this sample.
 - Gabriel then asks 10 people aged 18 to 30 years. In this group, 30% like political party A. Using both results, what overall conclusion can be made about survey participants' opinion of political party A?
 - Are there any issues with Gabriel's overall survey?



5A Investigating a census

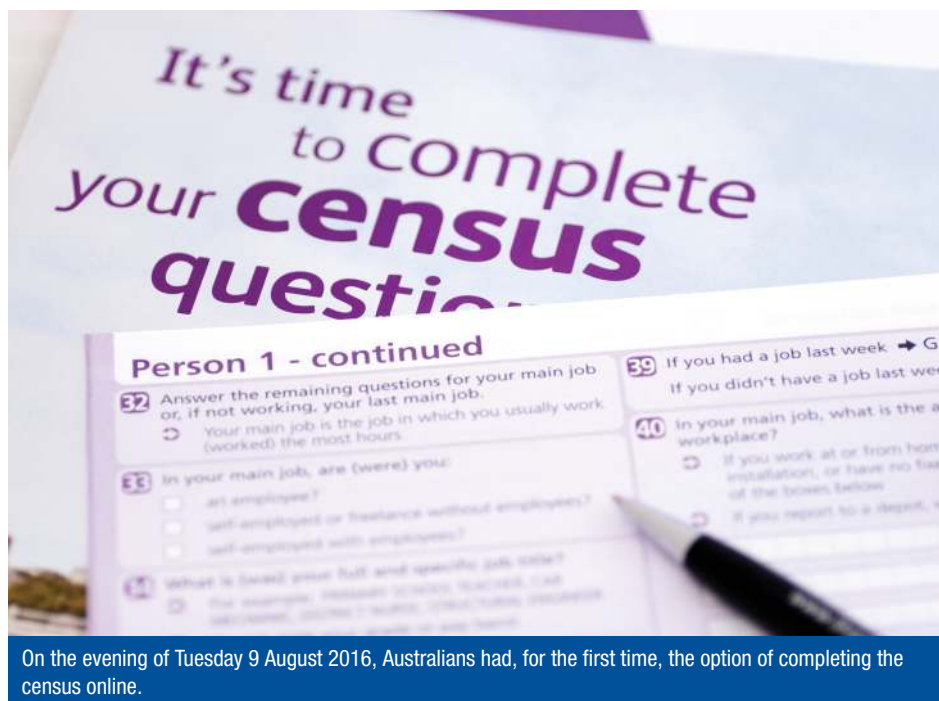
LEARNING GOALS

- Analyse how census data can be used by the government
- Investigate some aspects of the Australian national Census of Population and Housing including an outline of the procedure for conducting the census
- Investigate the advantages and disadvantages of conducting a census

Why is it essential to be able to conduct and analyse a census?

Censuses have been conducted by governments all over the world, dating back over 6000 years. A census determines the number of inhabitants in a country, various characteristics of the population, tells us about our way of life and helps us plan for the future.

Australia has conducted a national census every 5 years since 1911. According to the Australian Bureau of Statistics, census data ‘is used to distribute government funds and plan services for your community – housing, transport, education, industry, hospitals and the environment’.



WHAT YOU NEED TO KNOW

- A **census** can be conducted on any type of **population** (e.g. people, dwellings, vehicles, livestock). We will limit ourselves to a census of human populations.
- A census is an official count of the *entire* population of a country.
- A census also collects data about each member of the entire population.

The Australian national Census of Population and Housing

- The Australian Bureau of Statistics (ABS) is responsible for organising this census.
- All members of the population complete the census form on the same day.
- This census is conducted every 5 years and is compulsory. However, some of the questions are optional.
- Until 2016, only the paper version of the census form was used, which required about 40 000 people to personally deliver and collect it from every dwelling in Australia.
- The census can now be completed either online or using the paper form.
- It provides data about each person staying in the dwelling on that night.
- The information requested includes:
 - Your name, age, gender, marital status, ancestry and religion
 - Whether you are Aboriginal or Torres Strait Islander
 - Your usual place of residence; where you lived 1 year ago; and where you lived 5 years ago
 - Your place of birth and if you are an Australian citizen
 - If you need assistance with self-care, communication and mobility
 - Your education qualifications, employment status and annual income
 - Where you work, what you do, and how many hours you work per week
 - Details of unpaid work you may have done
- It also provides information about the dwelling such as: How many bedrooms? Do the occupants own it? Does it have internet? How many cars are present that night?



Example 1 Analysing how census data can be used by the government

- a** Explain how Australian population census statistics are used by the government for:
- i** electoral purposes
 - ii** government funding
- b** Identify a community need that is revealed by the following 2016 census statistics.
- i** The median age for Aboriginal and Torres Strait Islander population is 23 years compared to 38 years for the rest of the population.
 - ii** 47.4% of Australian families have both partners employed in either full-time or part-time work.

WORKING

THINKING

a i Population statistics are needed for the regular review of the electoral boundaries for House of Representative seats.	←.....	Politicians need to represent all regions fairly in our democracy. Each electorate should have close to an equal number of voters.
ii Population statistics are needed to determine the fair distribution of the Goods and Services Tax (GST) revenue to States and Territories.	←.....	States receive GST funds distributed by the federal government in proportion to the population of each State. These funds help to run our public schools, hospitals and other services.
b i Aboriginal and Torres Strait Islander population may require more medical assistance and also educational services.	←.....	The 15-year gap in median age indicates that the Aboriginal and Torres Strait Islander population is younger than Australia's overall population. This may mean a lower life expectancy and a higher proportion of children.
ii Working parents need relevant services for support, such as an increased need for childcare.	←.....	Politicians must keep up with the needs of our community when putting policy together.

Exercise 5A

FUNDAMENTALS

- 1 Demonstrate your understanding of terms by writing down the missing words in the following statements.
 - a A census is an official count of an _____ population.
 - b A census determines the _____ of inhabitants in a country, various characteristics of the population, tells us about our way of life and helps us _____ for the future.
 - c Census data is used to distribute government _____ and plan _____ for your community.

APPLICATIONS

Example 1

- 2 a Explain how Australian population census statistics are used by the government for:
 - i electoral purposes
 - ii government funding
- b Explain how the Australian government might use a census count of families with both parents working.
- 3 Investigate and describe the steps in the procedure for conducting the Australian national Census of Population and Housing.
- 4 Investigate and describe two advantages and two disadvantages of conducting a census.
- 5 Explain why it would be useful for the government to know the following statistics collected from 2016 Census data.
 - a Management and Commerce is the most popular field of study in Australia.
 - b 56% of women in Australia are employed, compared with 65% of men.
 - c Of the employed population aged 20–64 years, those with a non-school qualification (i.e. a degree, diploma or certificate) had a higher median personal income than those without a qualification.
 - d 35% of Queensland Aboriginal and Torres Strait Islander people are under 15 years of age.

Hint In a democracy, the electorates have close to equal numbers of voters.

- 6** A selection of questions from the 2016 Census are given in the box at right.
- a** Which questions collect numerical data?
 - b** What is the focus of Census questions 11–15?
 - c** Why is it useful for the ABS and the federal government to know the information collected in Census questions 11–15?
 - d** What is the focus of Census questions 16 and 17?
 - e** Why is it useful for the ABS and the federal government to know the information collected in Census questions 16 and 17?
 - f** Answering Census question 19 is optional. Why would this be an optional question?
 - g** What is the focus of Census questions 20–22?
 - h** Why is it useful for the ABS and federal government to know the information collected in Census questions 20–22?
 - i** What is the focus of Census questions 34–44?
 - j** Why is it useful for the ABS and the federal government to know the information collected in Census questions 34–44?
 - k** What is the focus of Census question 45?
 - l** Why is it useful for the ABS and the federal government to know the information collected in Census question 45?

ABS Census questions

- 11** Is the person an Australian citizen?
- 12** In which country was the person born?
- 13** In what year did the person first arrive in Australia to live here for one year or more?
- 14** In which country was the person's father born?
- 15** In which country was the person's mother born?
- 16** Does the person speak a language other than English at home?
- 17** How well does the person speak English?
- 18** What is the person's ancestry?
- 19** What is the person's religion?
- 20** Does the person ever need someone to help with, or be with them for, self care activities?
- 21** Does the person ever need someone to help with, or be with them for, body movement activities?
- 22** Does the person ever need someone to help with, or be with them for, communication activities?
- 34** Last week, did the person have a job of any kind?
- 35** In the main job held last week, was the person:
Working for an employer?
Working in own business?
- 36** Was the person's business:
Unincorporated?
Incorporated (e.g. Pty Ltd)?
- 37** Does the person's business employ people?
- 38** In the main job held last week, what was the person's occupation?
- 39** What are the main tasks that the person usually performs in that occupation?
- 40** For the main job held last week, what was the employer's business name?
- 41** For the main job held last week, what was the person's workplace address?
- 42** What best describes the industry or business of the employer at the location where the person works?
- 43** What are the main goods produced or main services provided by the employer's business?
- 44** Last week, how many hours did the person work in all jobs?
- 45** How did the person get to work on Tuesday, 9 August 2016?

5B Faults and errors in data collection

LEARNING GOALS

- Describe faults in the process of collecting data
- Identify bias in survey questions
- Identify bias in a sample selection
- Identify bias in measurements

Why is it essential to be able to identify bias in data collection?

The results obtained from a survey group are supposed to have the same characteristics as the population from which they are drawn. Bias is a tendency to favour one outcome over another. If bias is present in data collection it can influence the outcome of a survey, making it meaningless.



Bias in data makes it meaningless for the purpose it was collected.

WHAT YOU NEED TO KNOW

- Procedures for conducting a survey need to be followed carefully to avoid faults in the process of collecting data. Specifically:
 - **Processing errors** can occur when collating data.
 - ▶ For example: Collecting data in the incorrect units.
 - The **target population** needs to be considered when conducting a survey.
 - ▶ For example: If the target population is the school student population, then the sample must include students from every year level in the school.
 - Standard **protocols** need to be adhered to.
 - ▶ For example: Ask permission to use data collected, declare how the information is going to be used.
- **Bias** occurs when there is a difference between the sample result and the actual population value. Bias can be due to errors in either the **questioning**, **sampling** or **measurement** techniques applied.

- **Bias in questions** can happen when:
 - The question's wording leads the respondent to a particular answer.
 - ▶ For example: We think Papa's Pizza is the best. How great do you think Papa's Pizzas are?
This question leads the respondent to feel like they also must think that Papa's Pizzas are the best.
 - Questions are written in a positive or negative way.
 - ▶ For example: How annoying is it to have to wait for the doctor when you have made an appointment?
This implies that everyone thinks it is annoying when you must wait at the doctor's while they are attending to a very sick person. This may not be anyone's view except that of the person asking the questions.
 - There is no possibility of giving a particular response.
 - ▶ For example: Which sport do you play?
A netball **B** rugby league **C** volleyball
What would you answer if you played basketball? Or what if you played no sport at all?
- **Sampling error** can lead to bias when a difference exists between the survey result and the population value due to the sample selected. Sampling error can be attributed to:
 - a mistake
 - **variability** of samples, as different samples can give different results
 - sampling bias, in that a bad sample is chosen
 - small sample sizes have a greater chance of error.
- **Measurement error**
 - When you measure data, you should try to be as accurate and consistent as possible.
 - Accurate means you are as close to the actual value as possible.
 - Consistent means your measured values are close to each other when repeated.
 - ▶ For example, when we consider a game of darts:



consistent, not accurate



accurate, not consistent

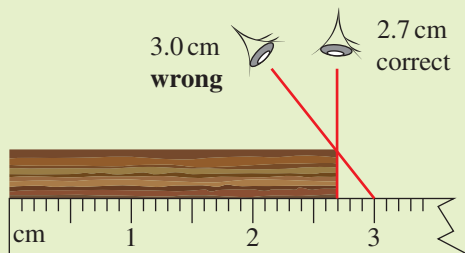


neither accurate nor consistent



accurate and consistent

- Errors and faults in measurement may be owing to:
 - systematic error caused by the user making the same error repeatedly or a faulty measuring device (e.g. scales not calibrated to zero)
 - random errors caused by user error. Parallax errors occur when your eye is not directly in line with the mark being measured and leads to variation in the measurements recorded at each trial.



Example 2 Identifying bias in survey questions

These three survey questions are about a stay in a hotel.

1 Rate the cleanliness of your hotel room:

☐ Spotless

☐ Immaculate

☐ Very clean

2 Everyone who stays at our hotel is satisfied. How did you enjoy your stay in our beautiful hotel?

3 How likely are you to recommend to your friends that they should stay at our beautiful hotel?

For each of the questions:

a Describe in what way each question is biased.

b Suggest a new question that does not contain bias.

WORKING

1 a No possibility of providing a neutral or negative response if you need to.

b Rate the cleanliness of your room:

Poor

Average

Excellent

2 a They are telling them what they want to hear back. They also indicate the hotel is 'beautiful'.

b How did you enjoy your stay in our hotel?

THINKING

There is no possibility of giving a particular response.

The question leads the respondent to a particular answer.

... Continued

- 3 a** By using beautiful' they are being very positive about the hotel and imply you should be too. ◀..... Question is written in a positive way.
- b** How likely are you to recommend to your friends that they should stay at our hotel?



Example 3 Identifying bias in the selection of samples

Jo wants to conduct a survey of students as part of a Student Representative Council initiative about the food offered at the canteen. She thinks of the following two ways of selecting a sample.

Sample 1: Ask 50 students who are in line at the canteen.

Sample 2: Ask 50 Year 11 students who are sitting in the yard during break.

- a** Determine if these samples are biased.
- b** Explain how Jo could ensure she obtains a representative sample of the student population.

WORKING

- a** Sample 1: This sample is biased. The students in line at the canteen are likely to be supportive of the current offerings.

Sample 2: This sample is biased. Asking only one year group is not ensuring the sample is representative of the whole school.

- b** To ensure a representative sample, Jo should conduct a random stratified sample.

THINKING

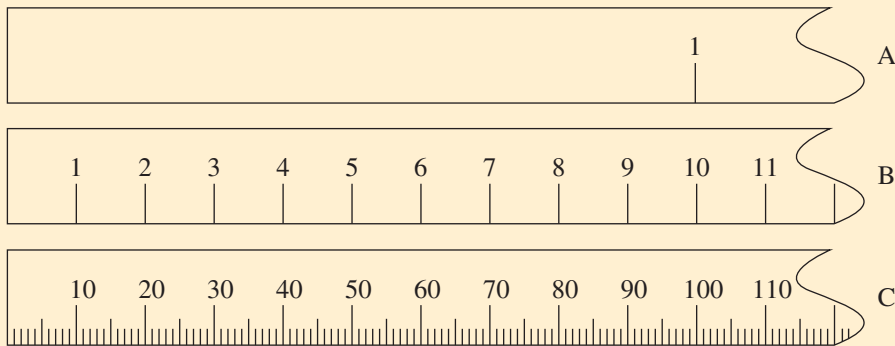
◀..... Students who do and do not use the canteen need to be included in the sample.

◀..... All year levels and genders need to be represented in the survey.





Example 4 Identifying bias in measurement



- a Explain which ruler will allow the most accurate measurements.
- b Explain what other things the accuracy of ruler measurements could depend on.

WORKING

- a Ruler C will allow the most accurate measurements.
- b Other things accuracy could depend on:
 - the position of the person measuring, as parallax error needs to be avoided.
 - Whether the ruler is faulty, e.g. if the zero end is broken off or markings have been rubbed off.

THINKING

- ←..... Ruler C has the most markings or divisions, and therefore the smallest distance between them. It can measure to the nearest millimetre. Ruler A is accurate to the nearest 10 cm and ruler B is accurate to the nearest centimetre.
- ←..... Parallax error can be avoided by always placing your eye directly in line with the measurement mark being read.
- ←..... Reasons that a ruler could be a faulty measuring device.

Exercise 5B

FUNDAMENTALS

- 1 Demonstrate your understanding of terms by writing down the missing words in the following statements.
 - a Bias occurs when there is a _____ between the survey result and the actual population value.
 - b Bias in a question can occur when:
 - i the question leads the respondent to a _____ answer.
 - ii the question is written in a _____ or negative way.
 - iii there is no _____ of giving a particular response.
 - c _____ error can lead to bias when a difference exists between the survey result and the population value due to the sample selected.
 - d When you measure data, you should try to be as _____ and _____ as possible to avoid measurement error.

APPLICATIONS

Example 3

- 2 For each of the following samples:
 - i identify in what way they may have a biased sample
 - ii suggest a sampling technique to use that would avoid bias.
 - a Asking people at a Dockers game to state their favourite sport.
 - b Asking people what is their favourite fast-food outlet while standing in front of a particular fast-food outlet in a food court.
 - c Surveying people in a retirement home about changes to the age pension.
 - d Asking people in a shopping centre at 10 a.m. on a Wednesday morning how many hours per week they work.
 - e Asking the parents what music should be played at a school dance.
 - f Surveying the price of a loaf of bread at two bakeries, three supermarkets and a local convenience store.
 - g Surveying 500 people in Perth about who they are going to vote for in the federal election



The time of day and the location can create sample selection bias.

Example 4

3 Determine which of the following problems could create measurement bias when taking measurements.

- a** You record 34.9 g rather than 39.4 g.
- b** You measure people's heights with their shoes on.
- c** Your ruler has 5 cm broken off the beginning.
- d** The scales read 150 g when nothing is on them.
- e** It takes you 0.2 s to click a stopwatch.

Example 2

4 a If you wished to conduct a survey about the favourite sports of Year 11 students, identify which of the following questions would you ask.

- A** Do you play sport?
- B** What sport do you play at school?
- C** What sports do you play?
- D** What sport do you like the most?

b Identify potential problems with the questions you rejected in part **a**.

5 For each of the following, rewrite the question so that it does not contain bias.

- a** I am not in favour of students using their mobile phones at school. What do you think?
- b** Social media obsession is turning people into antisocial beings. Do you agree?
- c** Do you prefer to visit the sunny Gold Coast or busy Sydney?
- d** Do you agree that we should have a sugar tax on those very unhealthy soft drinks?

6 An internet news website conducted a poll asking people to vote on whether we should have a death penalty in Australia for illegal drug importers. The poll was placed at the end of an article titled 'Drug epidemic in our suburbs'.

- a** Give reasons why there might be problems with the results of this poll.
- b** Explain how the poll could be better conducted.



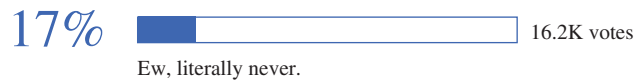
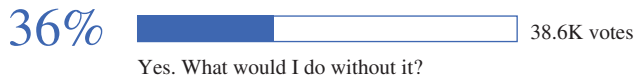
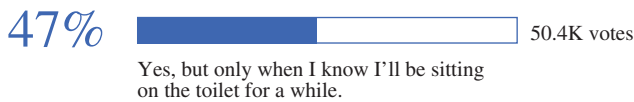
- 7** An online news provider aimed at young people ran an article about the amount of germs found in toilets and how they spread to your mobile phone if you use it while in the toilet. They ran the following poll at the end of the article.

So, do you bring your phone into the bathroom with you?

- ☐ *Yes. What would I do without it?*
☐ *Yes, but only when I know I'll be sitting on the toilet for a while.*
☐ *Ew, literally never.*

The survey yielded the following results.

So, do you bring your phone into the bathroom with you?



- a** Identify how many voters indicated they use their mobile phone while in the bathroom.
b Identify the percentage of respondents that indicated they use their mobile phone while in the bathroom.
c Determine the type of sampling method that has been used in this poll.

- 8** A marketing group conducted a survey on social media usage across 800 consumers and 1000 small business owners. Its survey yielded the results given in the table.

- a** Identify how many people participated in this survey.
b Identify the percentage of respondents that use their device to check social media when in the bathroom/toilet.

- c** Determine the type of sampling method that was likely used in this survey.
d This survey yielded very different results from that in Question 7 on the same topic. What reasons could have led to this difference?



Social media use in the home	
84%	Lounge/living room
59%	Bedroom
34%	Kitchen
20%	Study
14%	Bathroom
12%	Toilet

- 9 The degree of accuracy depends on the instrument you measure with. A set of digital kitchen scales that measures in whole grams has an error of ± 0.5 g. The kitchen scales shown in the image displays a weight of 47 g for the items being weighed.



- a Calculate the smallest weight the items could be.
- b Calculate the largest weight the items could be.
- c Identify which of the following might be the actual weight of the items.
47.6 g 46.7 g 46.4 g 47.3 g 47.0 g

- 10 Consider this blue line:

- a Determine the length of the line when you measure to the nearest millimetre.
- b Determine the length of the line when you measure to the nearest centimetre.
- c Determine the size of the measurement error when centimetres instead of millimetres are used to measure the line.
- d Calculate the percentage measurement error when centimetres instead of millimetres are used to measure the length of the line.

- 11 Three Science students measured the weight and volume of the same piece of metal to determine the density of the metal. The results of three trials are presented in the table.

	Harry	Craig	Patrick
Trial 1	23.14 g/mL	23.56 g/mL	23.03 g/mL
Trial 2	23.13 g/mL	23.56 g/mL	23.54 g/mL
Trial 3	23.17 g/mL	23.55 g/mL	23.25 g/mL
Average	23.15 g/mL	23.56 g/mL	23.27 g/mL

- a Explain which student you think is least accurate when measuring the metal.
- b Explain which student is the most consistent when measuring the metal.
- c Explain which student is most accurate and consistent when measuring the metal.

5C Surveys and simple random sampling

LEARNING GOALS

- Determine when a census or a survey is appropriate
- Understand the purpose of sampling
- Investigate and understand a simple random sampling procedure
- Investigate and understand self-selecting surveys
- Investigate the advantages and disadvantages of simple random samples and self-selected samples

Why is it essential to identify when to conduct a survey?

A census is very expensive and time consuming to conduct. A survey samples a smaller group than the entire population. A good quality survey can provide an accurate estimate of population characteristics and is a lot faster and cheaper to undertake.



A survey is the process of gathering information from a small group, known as a sample.

WHAT YOU NEED TO KNOW

- A **census** is an official count of an *entire* population.
- A **survey** is the process of gathering information from a small group, known as a **sample**, from within a population.
- The *purpose* of sampling is to gather data about the population. Information from the sample is used to estimate the characteristics of the population, without the need to survey the entire population.
- According to the ABS, the *advantages* of a survey are:
 - Costs would generally be lower than those for a census.
 - Results may be available in less time.
 - If good sampling techniques are used, the results can closely represent the features of the actual population.
- The ABS advise that the *disadvantages* of a survey are:
 - Data may not be representative of the total population, particularly when the sample size is small.
 - As data is collected from a subset of units (e.g. people) and inferences are made about the whole population, the data is subject to 'sampling' error.
 - A decreased number of units will reduce the detailed information available about subgroups within a population.

We will look at **four sampling techniques** in this chapter:

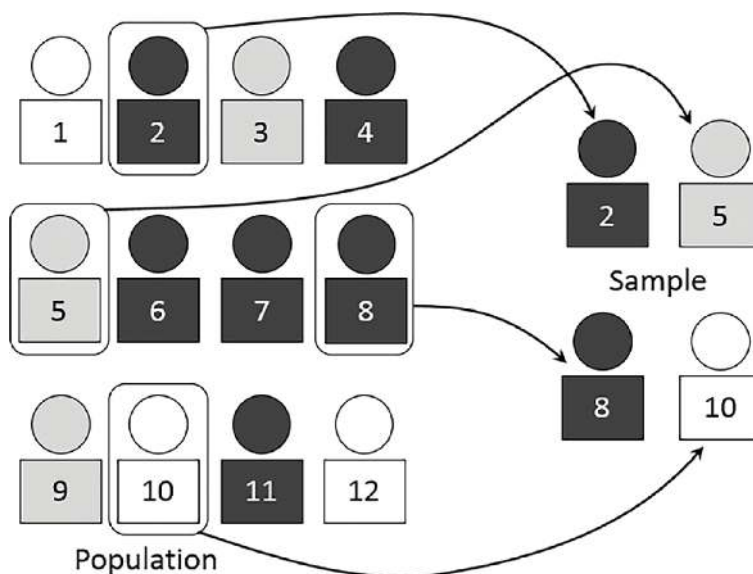
Simple random samples (Section 5C)

Self-selected samples (Section 5C)

Systematic samples (Section 5D)

Stratified samples (Section 5E)

- In a **simple random sample**, all members of a population have an equal chance of being selected for the survey.
 - Method: members are selected from a numbered list using random numbers until the desired sample size is achieved.
 - Advantages: easy to conduct; high chance of achieving a good representative sample.
 - Disadvantages: identifying all members of the population can be difficult; contacting members of the sample can be difficult.
- In a **self-selected sample**, anyone who wants to participate answers the question asked. For example, when a cricket app asks viewers to nominate which Ashes Test Match was their favourite for the summer.
 - Method: a question is posed by a TV program, app, social media page or newspaper and the sample consists of people who choose to respond.
 - Advantages: very easy to conduct and participants are willing to be involved.
 - Disadvantages: can be very biased as the people who respond may be from similar groups, and people can vote multiple times.



In a simple random sample, all population members have an equal chance of being selected.



Example 5 Choosing between a census and a survey

In the following situations for gathering information:

- i** determine whether it is better to use a census or a sample
- ii** explain the main reason for your choice.
- a** marking the roll in Maths class
- b** determining the length of time a brand of light bulb lasts
- c** testing the fire extinguishers at school
- d** determining the amount of time spent on social media by Year 11 students at your school

WORKING

THINKING

a i Census ii Every student needs to be checked that they are present or not.	Every student in the class must be counted.
b i Survey ii It is very expensive and time consuming to test all bulbs.	Testing each bulb until it fails means every bulb is destroyed by the testing process!
c i Census ii It is against the law to have a faulty fire extinguisher.	For safety reasons, all the fire extinguishers would need to be tested.
d i Survey ii A fair sample will provide a good estimate, and it is faster to organise and collate results.	It can be time consuming to survey the entire Year 11 cohort and the result may be the same as that from a sample.



Example 6 Selecting a simple random sample

Mrs Dunsdon wishes to select five students from her home room class to participate in a student satisfaction survey about the food at the tuckshop. The students in her class are:

Harrison	Sapphire	Kiara	Ainsleigh	Oscar
Saxon	Sophie	Dougal	Eileen	Theo
Dominic	Angus	Neve	Douglas	Izak
George	Sam	Lara	Tom	Michelle
Stuart	Laura	Darcy	Arun	Lily

- Assign the numbers 1–25 to the students in the class.
- Use Excel to generate five random numbers.
- Use your random numbers to select five students for the survey.
- Girls' names are written in orange. Calculate the fraction of the class that are girls.
- Calculate the fraction of your sample that are girls.
- Determine whether your sample is a suitable representation of the class.
- Identify some advantages and disadvantages of Mrs Dunsdon's random sampling method for discovering this school's student satisfaction level of tuckshop food.

WORKING

a

1 Harrison	6 Sapphire	11 Kiara	16 Ainsleigh	21 Oscar
2 Saxon	7 Sophie	12 Dougal	17 Eileen	22 Theo
3 Dominic	8 Angus	13 Neve	18 Douglas	23 Izak
4 George	9 Sam	14 Lara	19 Tom	24 Michelle
5 Stuart	10 Laura	15 Darcy	20 Arun	25 Lily

THINKING

Work your way down the class list and number students in order.

b

A1	B1	C1	D1	E1	F1
1	16				
2	18				
3	6				
4	8				
5	11				

Type
`=RANDBETWEEN(1, 25)`
 in your first cell to generate a random number between 1 and 25. Fill down to generate a total of five different random numbers.

... Continued

c Select the sample.

..... Match the random numbers to your class list.

1 Harrison	6 Sapphire	11 Kiara	16 Ainsleigh	21 Oscar
2 Saxon	7 Sophie	12 Dougal	17 Eileen	22 Theo
3 Dominic	8 Angus	13 Neve	18 Douglas	23 Izak
4 George	9 Sam	14 Lara	19 Tom	24 Michelle
5 Stuart	10 Laura	15 Darcy	20 Arun	25 Lily

Mrs Dunsdon has selected Ainsleigh, Douglas, Sapphire, Angus and Kiara for the survey.

d Fraction (class) girls = $\frac{10}{25} = \frac{2}{5}$ Fraction girls = $\frac{\text{number of girls}}{\text{total number of students}}$

e Fraction (sample) girls = $\frac{3}{5}$ Fraction girls = $\frac{\text{number of girls}}{\text{total number of students}}$

f This sample is not a good representation of the population. $\frac{2}{5}$ of the class are girls, so the sample should have approximately two girls but it has three girls. The class has $\frac{2}{5}$ of students that are girls. Ideally, the sample should have the same fraction of girls if the sample is to be a good representation of the population.

g Advantages: It is a simple procedure and fast to complete.
Disadvantages: It is a very small sample and might include students who don't use the tuckshop. Also, the sample is only from one class, not the whole student population.



Example 7 Analysing a self-selecting survey

A pizza shop puts a survey on its Facebook page. It says, 'Take our survey! pizzapi.com.au/survey For every 10 responses we get, we'll give away one FREE PIZZA! The survey should take you less than 10 minutes to complete. Your feedback will help make us even better!' A total of 261 people liked the post and 23 people shared it.

This is an example of a self-selecting survey.

- a** Who is likely to respond to the survey?
- b** Explain whether the responses are from a suitable cross-section of people who eat pizza.
- c** Consider whether the data gained was useful.
- d** Identify some advantages and disadvantages of this self-selecting survey.

WORKING

THINKING

- a** People who follow this pizza shop on Facebook will be the only people able to respond.
- b** No, as other pizza eaters and customers who do not follow the pizza shop on Facebook will not have an opportunity to participate. The responses will be only from people who follow the pizza shop on Facebook.
- c** Although biased in the responses it will get, the survey will be useful in that it did get some publicity for the pizza restaurant. 261 people liked their post and 23 people shared it, so it was likely that it was just part of an advertising campaign.
- d** Advantages: Easy to conduct, willing participants, it is also advertising.
Disadvantages: Biased as business is surveying only its own customers who are motivated to respond. Also, a person can vote multiple times.

Exercise 5C

FUNDAMENTALS

- 1 Demonstrate your understanding of terms by writing down the missing words in the following statements.
 - a A census is an official _____ of an *entire* population.
 - b A survey is the process of gathering _____ from a small group, known as a _____, within a population.
 - c The purpose of sampling is to gather _____ about the population. Information from the sample is used to estimate the _____ of the population, without the need to survey the entire population.
 - d In a simple random sample, all members of a population have an _____ chance of being selected for the survey.
 - e In a self-selected sample, anyone who _____ to participate answers the question(s) asked.

APPLICATIONS

Example 5

- 2 In the following situations for gathering information:
 - i determine whether it is better to use a census or a sample
 - ii explain the main reason for your choice.
 - a the number of new homes built in a suburb
 - b the popularity of the Prime Minister
 - c the amount of people preferring to buy their groceries online
 - d the amount of YouTube views for an artist
- 3 Determine whether a census or a survey has been conducted in the following scenarios.
 - a To find the most popular type of new car on the road in Western Australia, all registration details are checked for Western Australia in the past year.
 - b To find the most popular type of new car on the road in Western Australia, 200 new car owners are asked what type of car they purchased.
 - c To find the most popular fast food, 100 people in a shopping centre food court are asked what is their preferred fast food.
 - d The batting average of a cricket player is calculated.

Hint It can be time consuming and expensive to survey the entire population.

Example 6

- 4** Mr Jones is the Sports Master at Alpha SHS. He wishes to survey the Year 11 group about their satisfaction with the variety of sports offered in the co-curricular program. He has decided to ask 10 students to participate in the survey. Below is a list of the Year 11 students, with boys' names written in blue and girls' names written in orange.

1 Ellie	11 Brenda	21 Bonnie	31 Jack	41 Thomas	51 Brent	61 Jennifer
2 Erica	12 James	22 Will	32 Rhys	42 Cassius	52 William	62 Jessica
3 Connor	13 Damian	23 Ned	33 Nicholas	43 Caleb	53 Dale	63 Oliver
4 James	14 Molly	24 Christopher	34 John	44 Matthew	54 Jake	64 Hamish
5 Mark	15 Steven	25 Liam	35 Michael	45 Jack	55 Caitlin	65 Henry
6 Thomas	16 Alexander	26 Courtney	36 Reid	46 Eamon	56 Charles	66 Katelyn
7 Tennelle	17 Chloe	27 Ryan	37 Alakiir	47 Meg	57 LúC	67 Kobi
8 Samuel	18 Dominic	28 Charné	38 Angelo	48 Christopher	58 William	68 Jarrod
9 Amy	19 Thomas	29 Jacob	39 Jasper	49 Thomas	59 Clayton	69 Jack
10 Andrew	20 Kristen	30 Olivia	40 Breanna	50 Mark	60 Emma	70 Alexander

Mr Jones uses Excel to generate 10 random numbers. They are:

39, 57, 28, 15, 52, 58, 25, 10, 3, 5

- a** Identify the students that Mr Jones will ask to complete his survey.
 - b** Of the Year 11 students:
 - i** Determine how many are boys.
 - ii** Calculate the fraction that are boys.
 - c** Of the sample:
 - i** Determine how many are boys.
 - ii** Calculate the fraction that are boys.
 - d** How suitable is Mr Jones' sample as a representation of the Year 11 students?
- 5** Mr Jones repeats his survey from the previous question but uses a different selection of 10 random numbers. They are:
- 16, 10, 26, 54, 59, 14, 63, 68, 39, 32
- a** Identify the students that Mr Jones will ask to complete his survey.
 - b** Of the Year 11 students:
 - i** Determine how many are boys.
 - ii** Calculate the fraction that are boys.
 - c** Of the sample:
 - i** Determine how many are boys.
 - ii** Calculate the fraction that are boys.
 - d** How suitable is Mr Jones' sample as a representation of the Year 11 students?
 - e** Identify some advantages and disadvantages of Mr Jones' random sampling method for identifying the Year 11 students' satisfaction level with the sports offered.

- 6** Mr Jones is still not happy with his sample, so he decides to repeat his survey with a larger sample size of 20 students. He uses Excel to generate 20 random numbers between 1 and 70. They are:

52, 68, 4, 43, 3, 9, 57, 41, 45, 14, 42, 37, 21, 50, 27, 63, 13, 25, 26, 17

- a** Identify the students that Mr Jones will ask to complete his survey.
- b** Of the Year 11 students:
 - i** Determine how many are boys.
 - ii** Calculate the fraction that are boys.
- c** Of the sample:
 - i** Determine how many are boys.
 - ii** Calculate the fraction that are boys.
- d** How suitable is Mr Jones' sample as a representation of the Year 11 students?

Example 7

- 7** Voting in the KFC Classic Catches is an example of a self-selected survey. Participants in the competition are selecting their preferred catch; like they would select an option in a survey. Participants are placed into a draw to win a prize of \$2000 and \$50 worth of KFC.

- a** Who is likely to participate in this survey?
- b** What incentive is there to participate?
- c** In the survey's terms and conditions, it states that it costs 55 cents to vote and that people can enter the competition up to 90 times. What impact could this have on the outcome of the survey?
- d** Comment on the reliability of the survey method for determining the best Classic Catch of the summer.
- e** Identify some advantages and disadvantages of this self-selecting survey.

- 8** Via an app on their devices, cricket fans can offer their opinion on what should happen during a game or who is the best player. There is no cost to participate. An example of a question polled through the app is shown at right.

- a** Identify the type of sampling technique used to collect this data.
- b** This question was polled at a time when Australia was closing in on winning the Ashes series against England 4–0 and Steve Smith had scored centuries in most of the innings. Consider the reliability of this data.



5D Systematic sampling

LEARNING GOALS

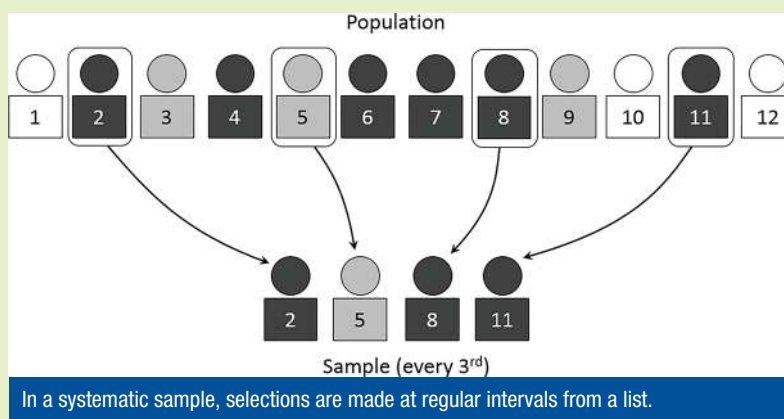
- Understand the procedure for selecting a systematic sample
- Identify advantages and disadvantages of systematic sample techniques

Why is it essential to identify when to use a systematic sample?

Systematic samples are used in an attempt to overcome bias and imbalance that can come from using a random sampling technique to select a sample from a population.

WHAT YOU NEED TO KNOW

- A **systematic sample** is made by selecting subjects at regular intervals.
 - For example: Asking every third student in the tuckshop line what is their preferred menu option for lunch.



- To determine how regularly a subject is selected from a list, we divide the population size by the required sample size.

$$\text{Interval} = \frac{\text{population size}}{\text{sample size}}$$

- For example: If we wish to have a sample size of 15 from a population of 600 students, we would select every $\frac{600}{15} = 40$ th student on the school roll.
- To avoid **bias**, generate a random starting place on the list to make the first selection.
- Advantages: very easy; the population is evenly sampled.

- Disadvantages: the sample can be biased in the following ways.
 - When the sample is not taken from a complete list of the population being investigated.
 - When some other trait of the population coincides with the frequency of selection or a trait occurs regularly in the population.
 - ▶ For example: If a faulty machine causes defects only in every 11th item manufactured, a sample taking every 10th item, out of a batch of 100, would include all good items and not be representative of the population.



Random samples are used to check the quality of production.



Example 8 Selecting a systematic sample

Mrs Dunsdon wants to use a systematic sample to select six students from her home room class to participate in a student satisfaction survey about the food at the tuckshop. The students in her class are:

1 Harrison	7 Sophie	13 Neve	19 Tom	25 Lily
2 Saxon	8 Angus	14 Lara	20 Arun	26 Tom
3 Dominic	9 Sam	15 Darcy	21 Oscar	27 Ryan
4 George	10 Laura	16 Ainsleigh	22 Theo	28 Libby
5 Stuart	11 Kiara	17 Eileen	23 Izak	29 Grace
6 Sapphire	12 Dougal	18 Douglas	24 Michelle	30 Kerri

- a Determine how regularly a student will need to be selected.
- b Use a random number generator to select a random starting point on the list.
- c Select the sample of students to be used in the survey.
- d Determine if the sample has the same mix of boys and girls as the home room class. Girls names are written in orange.
- e Consider whether this method is suitable in selecting a sample that represents the population.
- f Identify some advantages and disadvantages of Mrs Dunsdon's systematic sampling method for discovering this school's student satisfaction level of tuckshop food.

WORKING

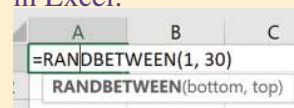
- a** Interval = $\frac{30}{6} = 5$
So choose every fifth student.

- b** Using Excel, start = 11.

THINKING

$$\text{Interval} = \frac{\text{population size}}{\text{sample size}}$$

Enter this formula in a cell in Excel:



- c** The students to select will be:

1 Harrison	7 Sophie	13 Neve	19 Tom	25 Lily
2 Saxon	8 Angus	14 Lara	20 Arun	26 Tom
3 Dominic	9 Sam	15 Darcy	21 Oscar	27 Ryan
4 George	10 Laura	16 Ainsleigh	22 Theo	28 Libby
5 Stuart	11 Kiara	17 Eileen	23 Izak	29 Grace
6 Sapphire	12 Dougal	18 Douglas	24 Michelle	30 Kerri

Select student 11 and then select every fifth student from the list after the 11th student. When you reach the end of the list, go back to the start.

Kiara, Ainsleigh, Oscar, Tom and Harrison.

- d** Fraction (class) girls = $\frac{13}{30}$ or 0.43
Fraction (sample) girls = $\frac{3}{6}$ or 0.5
The sample and class proportions of girls is very close, so the sample is suitable.

When dealing with small samples, we might not always get exactly the same proportion.

- e** The sample reflects the same characteristics as the population, therefore this has been a suitable method for determining the sample.

Give a reason for your decision.

- f** Advantages: It is easy and the class has been evenly sampled.

Disadvantages: It is a very small sample and might include students who don't use the tuckshop. Also, the results represent only Year 11 attitudes, not those of the whole student population.

The sample must be taken from a complete list of the population being investigated, otherwise the sample will be biased.

Exercise 5D

FUNDAMENTALS

- 1 Demonstrate your understanding of terms by writing down the missing words in the following statements.
 - a A systematic sample is made by selecting subjects from a list at _____ intervals.
 - b To determine how regularly a subject is selected from a list, _____ the population size by the required sample size.
 - c To avoid bias, start at a _____ place on the list to make the first selection.

Example 8a

- 2 For the following population sizes, determine the interval required between selections for the stated sample size.
 - a population = 750, sample size = 25
 - b population = 225, sample size = 15
 - c population = 480, sample size = 30
 - d population = 65, sample size = 20
 - e population = 1356, sample size = 75

Hint $\text{Interval} = \frac{\text{population size}}{\text{sample size}}$

APPLICATIONS

- 3 Describe the advantages and disadvantages of a systematic random sample.
- 4 Identify how bias has been introduced in the following systematic sampling procedures.
 - a Systematically selecting names from the electoral roll to determine Australian adults' attitudes to home ownership.
 - b Systematically selecting names from an alphabetical list of Years 11 and 12 students to determine Australian teenage attitudes to drug taking.

Hint The sample must be taken from a *complete* list of the population being investigated.



Example 8

- 5** Mr Jones is the Sports Master at Alpha SHS. He wants to survey the Year 11 students about their satisfaction with the variety of sports offered in the co-curricular program. He has decided to ask a sample size of 10 students to participate in the survey.

1 Ellie	11 Brenda	21 Bonnie	31 Jack	41 Thomas	51 Brent	61 Jennifer
2 Erica	12 James	22 Will	32 Rhys	42 Cassius	52 William	62 Jessica
3 Connor	13 Damian	23 Ned	33 Nicholas	43 Caleb	53 Dale	63 Oliver
4 James	14 Molly	24 Christopher	34 John	44 Matthew	54 Jake	64 Hamish
5 Mark	15 Steven	25 Liam	35 Michael	45 Jack	55 Caitlin	65 Henry
6 Thomas	16 Alexander	26 Courtney	36 Reid	46 Eamon	56 Charles	66 Katelyn
7 Tennelle	17 Chloe	27 Ryan	37 Alakiir	47 Meg	57 LúC	67 Kobi
8 Samuel	18 Dominic	28 Charné	38 Angelo	48 Christopher	58 William	68 Jarrod
9 Amy	19 Thomas	29 Jacob	39 Jasper	49 Thomas	59 Clayton	69 Jack
10 Andrew	20 Kristen	30 Olivia	40 Breanna	50 Mark	60 Emma	70 Alexander

Use this list of students to answer the following questions. The boys' names are shown in blue font and the girls' names are in orange font.

- Determine how regularly a student will need to be selected from the list.
- Starting at 50, select the sample of students to be used in the survey.
- Determine if the sample has the same mix of boys and girls as the Year 11 group.
- Explain whether this method has been suitable in selecting a sample that represents the population.
- Mr Jones is not happy with his sample.
 - Starting at 27, select the sample of 10 students to be used in the survey.
 - Determine if the sample has the same mix of boys and girls as the Year 11 group.
 - Explain whether this method has been suitable in selecting a sample that represents the population.
- The size of the sample can also affect the reliability of the sample that is selected. Mr Jones thinks that he might get a more reliable sample if he increases the sample size to 20.
 - Determine how regularly a student will need to be selected from the list.
 - Starting at 50, select the sample of 20 students to be used in the survey.
 - Determine if the sample has the same mix of boys and girls as the Year 11 group.
 - Explain whether this method has been suitable in selecting a sample that represents the population.
- Comment on the reliability of a systematic sample as a method for selecting a sample.

Hint When you reach the end of the list, go back to the start.

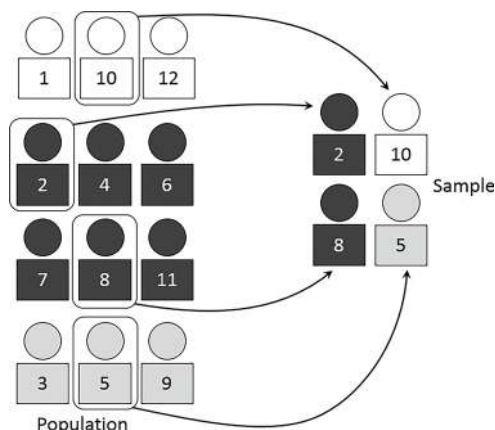
5E Stratified sampling

LEARNING GOALS

- Understand the procedure for selecting a stratified sample
- Identify advantages and disadvantages of stratified samples

Why is it essential to identify when to use a stratified sample?

As we have seen in the previous sections, random sampling can result in the characteristics of the sample not being the same as the characteristics of the population they are taken from. For example, if a higher or lower percentage of women are in the sample than in the population, this can lead to bias in the results obtained from a survey. A stratified sample tries to overcome any sampling imbalances by first dividing the population into groups, taking a random sample from each group and merging these to make one sample.



A stratified sample combines separate random samples from each subgroup of a population.

WHAT YOU NEED TO KNOW

- A **stratified sample** has the same characteristics as the population it is drawn from.
- To achieve the same characteristics and proportions in the sample, the population is divided up into groups that are called **strata**. The proportions of these groups are calculated, generally as a fraction or a percentage, and applied to the sample. Some examples of population characteristics that can be applied to the sample are:
 - geographic location
 - age
 - year level at school
 - ethnic background
- Once it is determined how many people should be surveyed from each strata group, a *random* sample is taken from each group to form the sample.
- The main *advantage* of a stratified sample is that it has the same characteristics of the population.
- The main *disadvantage* of a stratified sample is that it can be more time consuming to determine the sample.



Example 9 Selecting a stratified sample, keeping member proportions the same

Morgan is the president of the local hockey club. The club has 480 members of which are juniors, 150 are seniors and the remainder are life members. Morgan wishes to survey 10% of the members about changes to the club uniform.

- Calculate how many members she will survey.
- Calculate how many senior members should be included in the survey.
- Calculate how many juniors should be included in the survey.
- Calculate how many life members should be included in the survey.
- Verify that your stratified sample is appropriate.

WORKING

THINKING

a Sample size = $\frac{10}{100} \times 480$ $\leftarrow$ Calculate 10% of the total membership.
 $= 48$ On your calculator enter:
 $10 \div 100 \times 480 =$

b Seniors = $\frac{150}{480} \times 48$ $\leftarrow$ Seniors make up $\frac{150}{480}$ of the total membership, so they need to make up the same fraction of the sample.
 $= 15$

c Juniors = $\frac{280}{480} \times 48$ $\leftarrow$ Juniors make up $\frac{280}{480}$ of the total membership, so they need to make up the same fraction of the sample.
 $= 28$

d Life members = $480 - 280 - 150$ $\leftarrow$ First calculate how many life members there are.
 $= 50$
 Life members = $\frac{50}{480} \times 48$ Life members make up $\frac{50}{480}$ of the total membership. So they need to make up the same fraction of the sample.
 $= 5$

e Sample = $15 + 28 + 5$ $\leftarrow$ Check the final sample is 10% of the members.
 $= 48$
 This is the sample size calculated in part **a**.



Example 10 Selecting a stratified sample, keeping the same proportions of boys and girls as those in the class

Mrs Dunsdon wants to select 20% of students from her home room class to participate in a student satisfaction survey about the food at the tuckshop. Her class list shows boys' names written in blue and girls' names written in orange. She wants to use a stratified sample to ensure her sample has the sample proportions of girls/boys as those of her class.

Harrison	Sapphire	Kiara	Ainsleigh	Oscar
Saxon	Sophie	Dougal	Eileen	Theo
Dominic	Angus	Neve	Douglas	Izak
George	Sam	Lara	Tom	Michelle
Stuart	Laura	Darcy	Arun	Lily

- Calculate how many students are in the class.
- Calculate how many students in the class are boys.
- Calculate how many students in the class are girls.
- Calculate how many students will be included in the survey.
- Calculate how many students in the sample will be boys.
- Calculate how many students in the sample will be girls.
- Use Excel to generate random numbers to determine which boys will be used in the sample.
- Use Excel to generate random numbers to determine which girls will be used in the sample.

WORKING

- There are 25 students in the class. $\leftarrow$ This is the population.
- 15 students are boys. $\leftarrow$ This is the size of one of the strata groups.
- 10 students are girls. $\leftarrow$ This is the size of the other strata group.
- Sample size = $\frac{20}{100} \times 25 = 5$ $\leftarrow$ Calculate 20% of the class.
- Boys = $\frac{15}{25} \times 5 = 3$ $\leftarrow$ The sample must have three boys to have the same proportion of boys as the class.

THINKING

f Girls = $\frac{10}{25} \times 5 = 2$

..... The sample must have two girls to have the same proportion of girls as the class.

g Boys' sample:

..... There are 15 boys.

	A	B
1	13	
2	8	
3	6	
4		
5		

Type
`=RANDBETWEEN(1, 15)`
 into Excel and fill down to
 calculate three random
 numbers.

Select the 13th, 8th and 6th boys from the class list.

1 Harrison	1 Sapphire	4 Kiara	7 Ainsleigh	13 Oscar
2 Saxon	2 Sophie	8 Dougal	8 Eileen	14 Theo
3 Dominic	6 Angus	5 Neve	10 Douglas	15 Izak
4 George	7 Sam	6 Lara	11 Tom	9 Michelle
5 Stuart	3 Laura	9 Darcy	12 Arun	10 Lily

Angus, Dougal and Oscar will make up the boys' sample.

h Girls' sample:

..... There are 10 girls.

	A	B
1	13	
2	8	
3	6	
4		
5		

Type
`=RANDBETWEEN(1, 10)`
 into Excel and fill down to
 calculate two random
 numbers.

Select the 9th and 2nd girls on the class list.

1 Harrison	1 Sapphire	4 Kiara	7 Ainsleigh	13 Oscar
2 Saxon	2 Sophie	8 Dougal	8 Eileen	14 Theo
3 Dominic	6 Angus	5 Neve	10 Douglas	15 Izak
4 George	7 Sam	6 Lara	11 Tom	9 Michelle
5 Stuart	3 Laura	9 Darcy	12 Arun	10 Lily

Sophie and Michelle will make up the girls' sample.

Exercise 5E

FUNDAMENTALS

- 1 Demonstrate your understanding of terms by writing down the missing words in the following statements.
 - a A stratified sample has the same _____ as the population it is drawn from.
 - b To achieve the same characteristics and _____ in the sample, the population is divided up into _____ that are called strata.
 - c The proportions of the strata groups are calculated, generally as a _____ or a _____, and applied to the sample.

Example 9a

- 2 Evaluate the following.

a $\frac{15}{90} \times 54$

b $\frac{120}{350} \times 70$

c $\frac{37}{62} \times 5$

d $\frac{120}{320} \times 40$

APPLICATIONS

- 3 List the advantages and disadvantages of a stratified random sample.

Example 9

- 4 A local supermarket has 145 employees of which 50 work full-time, 35 work part-time and the rest are casual employees. The manager wishes to survey 20% of the staff about job satisfaction.
 - a Calculate how many casual employees there are.
 - b Calculate how many employees will be included in the survey.
 - c To get a true reflection of the staff in the sample, the manager would like to use a stratified sample. Calculate how many of each of the following she should include.
 - i full-time employees
 - ii part-time employees
 - iii casual employees

Hint First calculate the population fractions for each type of employee.



A good sample has the same proportion of subgroups as in the population.

Example 10

- 5** The Year 11 Coordinator would like to survey 15% of Year 11 students about the school formal next year. The Year 11 students are made up of 40 girls and 52 boys, and the Coordinator wishes to use a random stratified sampling technique. He has made up a list of all the students in Year 11.

Girls:

1 Gabrielle	6 Breanna	11 Mia	16 Claire	21 Julia	26 Chloe	31 Emily	36 Meg
2 Amy	7 Caitlin	12 Gabriella	17 Caitlin	22 Ellie	27 Kristen	32 Gabrielle	37 Cassie
3 Molly	8 Emma	13 Claudine	18 Katelyn	23 Erica	28 Meg	33 Tara	38 Shania
4 Bonnie	9 Jessica	14 Isabella	19 Caitlyn	24 Tenielle	29 Jennifer	34 Heidi	39 Nikita
5 Courtney	10 Ruby	15 Keely	20 Kate	25 Brenda	30 Bonnie	35 Georgia	40 Codey

Boys:

1 Christopher	8 William	15 Christopher	22 Jake	29 Riley	36 James	43 Will	50 Jasper
2 Thomas	9 Seamus	16 Liam	23 Charles	30 Thomas	37 Samuel	44 Ned	51 Caleb
3 Andrew	10 Thomas	17 Ryan	24 Oliver	31 Matthew	38 Damian	45 Jack	52 Jack
4 Jacob	11 Sean	18 Matthew	25 Henry	32 Elliot	39 Steven	46 Rhys	
5 John	12 Braden	19 Thomas	26 Jack	33 Matthew	40 Alexander	47 Nicholas	
6 Alexander	13 Mark	20 Brent	27 Jack	34 Thomas	41 Dominic	48 Michael	
7 Benjamin	14 James	21 William	28 Matthew	35 Connor	42 Thomas	49 Angelo	

- Calculate how many students will be required for the survey.
- Calculate how many students in the survey will be girls.
- Calculate how many students in the survey will be boys.
- Use this list of random numbers between 1 and 40 to select the sample of girls:
20, 26, 39, 25, 16, 18, 40
12, 21, 30, 6, 23, 1, 10
- Use this list of random numbers between 1 and 52 to select the sample of boys:
25, 26, 14, 43, 35, 32, 10, 8, 23, 6, 14, 30, 6, 39, 22

Hint Note: Work your way across the first row until you have the sample size required.

- 6** Geometry College is a co-educational boarding school. The Year 11 students comprise the following distribution.

	Day student	Boarder	Total
Girl	55	23	78
Boy	65	37	102
Total	120	60	180

The School Board would like to conduct a stratified survey of 20% of the Year 11 students to find their opinion about homework expectations.

- a** Calculate how many students will need to be surveyed.
b Calculate how many of each of the following students will be selected.
- i** girl day students
 - ii** girl boarders
 - iii** boy day students
 - iv** boy boarders

- 7** According to the Australian Bureau of Statistics, the Population of Australia in June 2017 was as shown in the table.

- a** There are 76 Senators in the Australian Senate, each representing the various States and Territories of Australia. If States and Territories are allocated numbers of seats in the Senate according to the principles of a stratified sample, calculate how many seats in the Senate each State or Territory should have?

Population at end of June quarter 2017

Preliminary data	'000
New South Wales	7 861.1
Victoria	6 323.6
Queensland	4 928.5
South Australia	1 723.5
Western Australia	2 580.4
Tasmania	520.9
Northern Territory	246.1
Australian Capital Territory	410.3
Australia	24 598.9

- b** The Senate is made up of 12 senators from each State and two each from the Australian Capital Territory and the Northern Territory. Compare this with your answers from part **a** and comment on the allocation of seats in the Australian Senate.

5F Identifying target populations

LEARNING GOALS

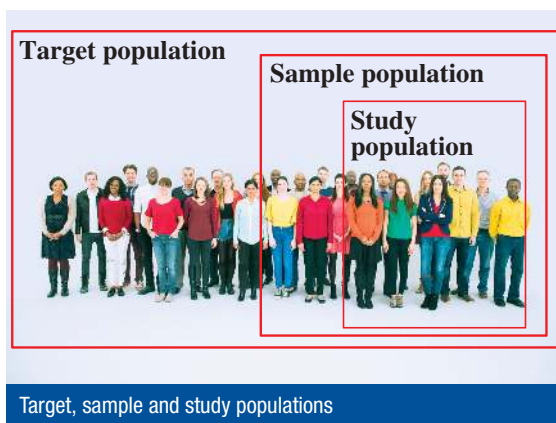
- Identify target populations for a survey
- Describe target, sample and study populations
- Understand the importance of the identification of a target population

Why is it essential to be able to identify a target population?

When designing a survey, you need to be aware of your target population. When you use a survey to gather information about a population, it is important that your survey participants are from a group within the overall population that you are trying to make generalisations about. Suppose you wished to find the average number of hours a learner driver spends with a driving school during their driver training; it would make sense to survey P-plate drivers about their experiences. It would not make sense to ask a group of Year 8 students about their driver training!

WHAT YOU NEED TO KNOW

- **Target population** refers to the *entire* group of individuals you are interested in for your survey.
 - For example: all P-plate drivers in Western Australia.
- **Sample population** is the group of individuals asked to participate in your survey.
 - For example: 10% of the P-platers at all the high schools in your suburb/town.
- **Study population** is the final group of people who agree to be involved in your survey. Being a smaller group, the data collected from the study group is easier to collect and manage. A survey can give accurate information about the target population only when information is taken in an unbiased way from the target.





Example 11 Identifying the target population for a customer survey

A chemist shop wishes to know if customers are happy with its service. The business emails customers who use its store loyalty card when making a purchase on a Saturday morning. Determine which one of the following is the target population:

- A** all the customers of the chemist shop
- B** the customers surveyed
- C** all the staff working in the chemist shop
- D** all the customers of all the shops in the shopping centre

WORKING

Option A; all the customers of the chemist shop

THINKING

Consider which population the customers belong to.



Example 12 Identifying the target population and why this is important

A local council wishes to know the opinions of residents of a suburb about the construction of a water detention basin, for storm water runoff, in a local park. It surveys 50 residents from the suburb.

- a** Determine which one of the following is the target population:
 - A** the 50 residents surveyed
 - B** the council workers
 - C** all the residents of the suburb
 - D** all the residents of the city
- b** Explain why it is important to identify this target population.

WORKING

- a** Option C

THINKING

Consider which population the sample is a part of.

- b** If survey participants are not suburb residents, you cannot make generalisations about the residents' views.

To make generalisations about a population, the survey participants must be only from that population.



Example 13 Describing the target population and why this is important

Tourism WA surveys people about their satisfaction with the Australind railway trip, which departs from Perth.

a Describe the target population.

b Explain why it is important to identify this target population.

WORKING

a People who have used the Australind railway.

b Conclusions would not be accurate if survey participants had not experienced the Australind railway.

THINKING

◀... The target population includes people using this railway, not just tourists to the Bunbury area.

◀... To make generalisations about a population, the survey participants must be only from that population.



To investigate hours of driving school training, the target population comprises all P-plate drivers

Exercise 5F

FUNDAMENTALS

- 1 Demonstrate your understanding of terms by writing down the missing words in the following statements.
 - a** Target population refers to the _____ group of individuals you are interested in for your survey.
 - b** _____ population is the group of individuals asked to participate in your survey.

- c** Study population is the final group of people who _____ to be involved in your survey.
- d** A survey can give _____ information about the target population only when information is taken in an unbiased way from the target population.

- 2** Match each sample (**A** to **D**) with its corresponding target population (**i** to **iv**).

Sample

- A** lettuce plants in a row
- B** P-plate drivers in Year 12
- C** parents at a P & F meeting
- D** customers who respond to an email survey

Target population

- i** all P-plate drivers in Western Australia
- ii** all the customers of a business
- iii** all the parents of the school
- iv** all the lettuce plants on the farm

APPLICATIONS

Example 11

- 3** This window popped up when a customer logged on to a photo printing website.

We welcome your feedback!

Thank you for visiting our site. You have been selected to participate in our customer survey to let us know how we can improve your experience. Please look at it at the end of your visit. The survey is conducted for our website by an independent company, SurveyPlus.

No, thanks

Yes, I will give feedback

The target population is:

- A** all people who use online providers to print photos
- B** all the members who use this website to print photos
- C** professional photographers who print photos
- D** all people who take photos
- 4** Shortly after doing the grocery shopping at a supermarket and using her customer loyalty card, Leanne received this message in an email from the supermarket.

Tell Gold supermarkets about it and get 100 bonus loyalty points!

Help us make your shopping experience even better by completing our survey. Click on the link below and tell us about your recent visit to our supermarket. To say thanks, we will give you 100 bonus loyalty points.

Start survey

The target population is:

- A** all people who shop at the supermarket
- B** all people who shopped at the supermarket that day
- C** all people who shopped in the shopping centre that day
- D** employees of the supermarket

Example 12

- 5** A school wishes to know the students' preference for home room classes.

a What is the target population?

- A** students from Year 11 only
- B** students who are new to the school
- C** students who have older brothers and sisters
- D** all students of the school

b Explain why it is important to identify this target population.

Hint To make generalisations about a population, the participants must be only from that population.

- 6** A business wishes to know if providing free access to a local gym during an employee's lunch break would be considered beneficial by the employees.

a What is the target population?

- A** employees who wish to be fitter
- B** employees who are overweight
- C** all employees of the company
- D** employees who do not move much during the workday

b Explain why it is important to identify this target population.

Example 13

- 7** Tourism Western Australia wishes to know the satisfaction levels of people taking boat trips to see the Ningaloo Reef.

a Describe the target population.

b Explain why it is important to identify this target population.

- 8** A vegetable grower wishes to check his plants for pests. He checks (i.e. surveys) a row of plants to see if he can find any. Describe the target population.

- 9** For quality assurance, a factory worker checks random items as they leave the production line. Describe the target population.

- 10** A local factory surveys its employees about the benefits of providing onsite childcare for the employees' children. Describe the target population.

- 11** A political party uses a consultation group to survey voters via a phone poll on their preferred Prime Minister. Describe the target population.

5G Misrepresentation of the results

LEARNING GOAL

- Identify issues that arise when applying the results of a survey to make generalisations about a population

Why is it essential to be aware of misrepresentation of results?

Statistics can be very useful for helping us to understand characteristics in a population. Not using good sampling and not questioning procedures can lead to biased results that do not represent the population we are trying to make a generalisation about. Care must be taken when interpreting statistical data and making generalisations about a population.

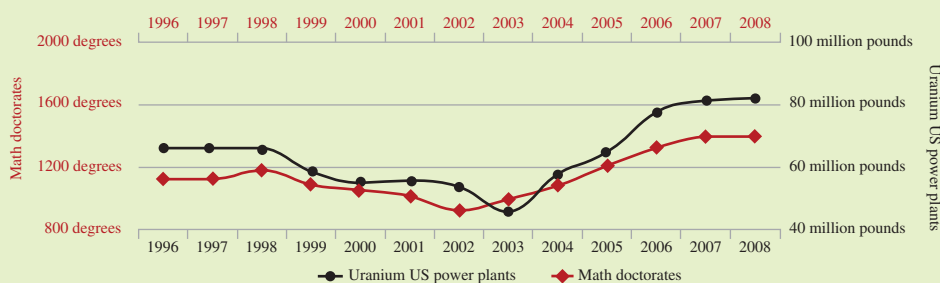


Care must be taken when applying the results of a sample to make generalisations about a population.

WHAT YOU NEED TO KNOW

- As discussed in Section 5B, **sampling techniques** must be applied carefully to avoid bias in the data collected. Common sources of bias in sampling can be:
 - Self-selection bias**, which results from voluntary survey respondents who are mostly interested in a particular outcome of a topic so one viewpoint can be over-represented.
 - Non-response bias**, which results from a difference between those who do and do not respond to the survey.
 - For example: In a study of newborn children, the parents who responded again after 1 year corresponded with the income level of the parents.
 - The sample is **not representative** of the population.

- **Procedures** for conducting a survey also need to be followed carefully to avoid bias. Specifically:
 - **Interviewer bias**, which can occur when the interviewer influences responses either deliberately or not.
 - **Questionnaire bias**, which can happen when the questions are not designed carefully.
 - **Processing errors**, which can occur when collating data.
 - When interpreting data, care must be given to avoid common errors of thinking.
 - **Don't assume small differences are significant.**
 - ▶ For example: A 1% change in people's preferred prime minister does not necessarily mean that a particular political party will win the next election. It could just be the result of a variation between samples.
 - **Ignore coincidence.** Sometimes completely unrelated statistics have common trends.
 - ▶ For example: The number of Mathematics doctorates (PhDs) awarded has no relationship with the amount of uranium stored at US nuclear power plants.
- But, according to the following graph, they do.



Source: www.tylervigen.com/spurious-correlations

- **Don't get cause and effect backwards.** When two things are related, care must be taken when identifying which is the cause and which is the effect.
 - ▶ For example: Consider depression and unemployment. Can depression lead to unemployment or is it the other way; that is, unemployment can lead to depression?
- **Don't forget to consider outside causes.** There can be a third factor causing a certain relationship.
 - ▶ For example: Students at a particular school may perform better than the State average on exams. One could think that attending that school leads to better results. It could be that the school is in a wealthy suburb and parents are able to afford tutors for the students, which leads to the above-average results.



Example 14 Misrepresentation of results

Penny surveyed 20 students in Year 11 about how long they spent on homework the night before. Fourteen students replied that they did more than 4 hours. Penny then reported to the Deputy Principal that ‘70% of students at the school have too much homework’.

- a** Identify any issues with the sampling technique.
- b** Identify any issues with the survey procedure.
- c** Identify any issues with the interpretation of results.

WORKING

THINKING

a The sample is not representative of the entire school population and consists of only 20 students from one year level.	◀..... The sample should cover all year levels and be larger.
b The question asked how long students had spent on homework the night before. It did not ask if the student felt they had too much homework or how many hours they usually do per night.	◀... Questions should ask what you want to know.
c It could have been that a Maths assessment was due that day for Year 11 and many students stayed up late the night before finishing it. The students might not usually do more than 4 hours of homework per night.	◀... Outside causes can have an influence on the results.

Exercise 5G

FUNDAMENTALS

- 1** Demonstrate your understanding of terms by writing down the missing words in the following statements.
 - a** Sampling techniques need to be carefully _____ to avoid _____.
 - b** Procedures for conducting a survey must be followed carefully to avoid bias. Specifically, _____ bias; _____ bias; and processing _____.
 - c** When interpreting data, care must be given to not _____ small differences are significant; not to ignore _____; don't get cause and effect _____; and don't forget to consider _____ causes.

Example 14

- 2 A company sends an email to all its customers following an online purchase, asking them to complete a survey.
 - a 46% of customers respond to the survey. Identify any issues with the sampling technique.
 - b For the survey question ‘How satisfied are you with our service?’ customers could respond either ‘Very satisfied’ or ‘Somewhat satisfied’. Identify any issues with the survey procedures.
 - c In a report to the shareholders, the company wrote that ‘100% of our customers are satisfied with our service’. Identify any issues with the interpretation of the results.

- 3 An online newsagency ran a poll asking readers ‘Which do you prefer: A summer barbeque or a Christmas party?’. 61% of respondents voted for a summer barbeque.
 - a Identify the type of sampling technique that has been used.
 - b Identify the issues that could arise from generalising the results of this survey to the population.

- 4 Joel asks 10 kids lined up at his club’s kiosk if they like the food being sold at the kiosk. All 10 students say they like the food that is sold at the kiosk. Joel reports to the Kiosk Manager that 100% of kids at the club like the food sold at the kiosk.
 - a Identify the issues that might arise from the sampling technique that Joel has used.
 - b Identify the issues that could arise from the reporting of the data.

- 5 Lachlan surveys his classmates, asking them ‘Are you a fan of the West Coast Eagles AFL team like me?’.
 - a Identify the issues that might be present in the survey procedure.
 - b Explain what Lachlan could do to improve his survey procedure.

- 6 When recording the number of hours classmates had completed as a Learner Driver, Libby accidentally recorded Grace’s entry of 18 hours as 81 hours.
 - a Identify the type of procedural error that has occurred.
 - b Explain what impact this will have on the survey’s results.



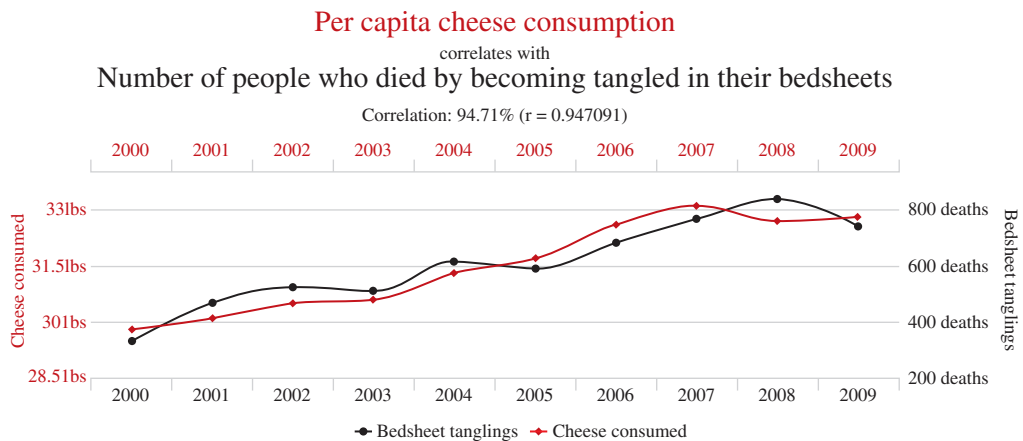
- 7** When following the ASX share market, Jeff noticed that the share price of a company had increased 15 cents in the past day (see table).

Date	Last
09/03/2018	21.760
08/03/2018	21.610

- a** When interpreting this data, what should Jeff consider?
Jeff decides to look at the last three months' share prices.



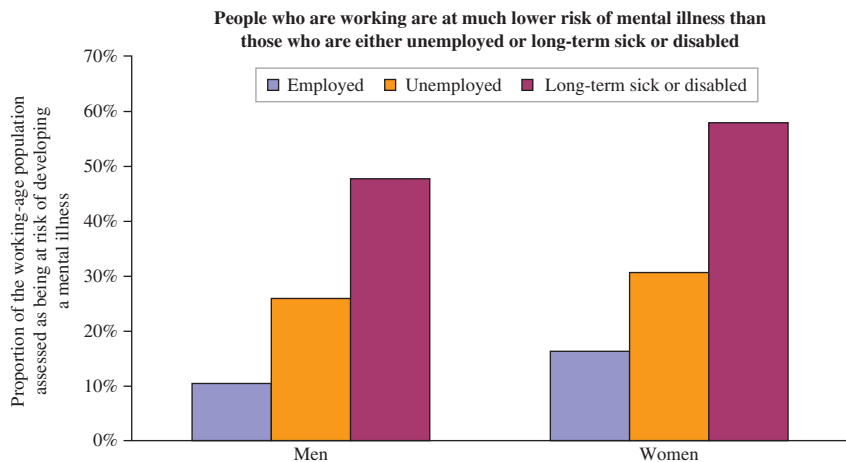
- b** By analysing the share prices over a longer period, explain whether or not the company seems as appealing to invest in.
- 8** The graph shows the relationship between cheese consumption and the number of people who died by becoming tangled in their bedsheets over the same time period.



Source: www.tylervigen.com/spurious-correlations

- a** Describe an error of thinking that could occur when interpreting this data.
b Give an example of a problematic conclusion from this graph.

- 9 Consider this graph, which shows a relationship between employment and mental illness.

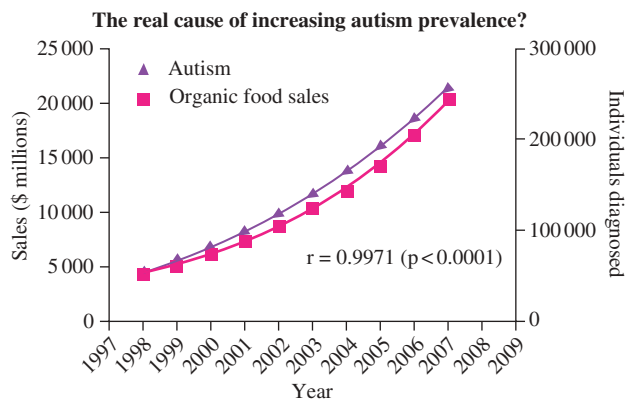


Source: British Household Panel Survey, University of Essex, Institute for Social and Economic Research; the data is the average for the five years to 2008/09; updated July 2010.

- The title gives a strong opinion on the causes of mental health. Determine the issue with the interpretation of the data.
- When interpreting the graph, explain if it is reasonable to assume:
 - mental illness affects employment; or
 - employment affects mental illness.
- Suggest a better title for the graph.

- 10 This graph was printed under the heading ‘The real cause of increasing autism prevalence?’.

- Identify any issues with the interpretation of this data as implied by the title.
- Identify an outside cause that might have influenced the growth in both statistics.



Source: www.cracked.com/article_20318_the-5-most-popular-ways-statistics-are-used-to-lie-to-you.html

- A study published in 2018 found that breastfed babies grew up to have a 3.7 point higher IQ, more years of education and earn 20% more than the average income by the time they are 30 years old. (<https://edition.cnn.com/2015/03/18/health/breastfeeding-iq-income/index.html>)
 - What the issue with the interpretation of this data?
 - Identify an outside cause that might have influenced the statistics.
 - Explain a possible danger of this interpretation of the data.

5H Questionnaire design principles

LEARNING GOAL

- Investigate and understand questionnaire design principles, including:
 - simple language
 - unambiguous questions
 - consideration of the number of response choices
 - issues of privacy and ethics
 - freedom from bias

Why is it essential to understand questionnaire design principles?

It is important to ask survey questions without ambiguity and bias. The questions must also be easy for the participant to understand and designed to allow you to be able to process the data effectively.



The available responses should accommodate all possible responses within a few choices.

WHAT YOU NEED TO KNOW

When designing a questionnaire, you need to consider the following principles.

- **Use simple language:** Do not use jargon or technical words in the survey questions that respondents may not understand. Be brief and keep sentences to fewer than 20 words.
- **Do not make questions ambiguous:** The meaning of your question must be very clear. Avoid words like rarely, sometimes, usually, few, some and most.
- **Consideration of the number of response choices:** Asking lots of questions or having lots of options to choose from when answering a question can lead to survey fatigue in the sample group. People are more likely to remember the first and the last choice when offered a lot of options.
- **Issues of privacy and ethics:** The information collected during a survey must be kept confidential and not shared with unauthorised people.
- **Freedom from bias:** Questions should not be leading the respondent to a particular answer.



Example 15 Designing responses with suitable choices

A local gym wishes to survey its clients about the amount of time per week they spend at the gym. The survey question is: How much time do you spend at this gym each week?

Here are two possible survey response options.

Option A: ☐ 2 hours ☐ 4 hours

Option B: ☐ 1 hour ☐ 2 hours ☐ 4 hours ☐ 5 hours ☐ 8 hours ☐ 15 hours

- For option A, identify one issue and describe why it is a problem.
- For option B, identify one issue and describe why it is a problem.
- Rewrite the answer options to cover a wide variety of hours with only four options to choose from.

WORKING

- Option A has too few possible answers, so the survey conclusion will not be accurate.
- Option B has too many possible answers, making it hard to collate the results.
- ☐ 1–2 hours ☐ 3–4 hours
☐ 5–6 hours ☐ more than 6 hours

THINKING

- ←..... Consider what is the smallest and largest number of hours that a person could exercise at the gym.
- ←..... It would be very time consuming to collate results from every possible answer.
- ←..... The available responses should accommodate all possible responses within a few choices.





Example 16 Simplifying the language of questions

Rewrite each of the following questions in a simpler form.

- a** Which teacher is the most knowledgeable and friendly?
- b** With what frequency has your child experienced a raised temperature in the past 30 days?
- c** Including yourself, any pets you own, but not including any relatives or friends that do not live in your household for at least 320 days a year, how many household occupants did you buy groceries, by groceries we mean any item of food bought at the grocery store, for in the last year?

WORKING

THINKING

a Which teacher is the most knowledgeable? Which teacher is the friendliest?	◀.....	A double-barrelled question makes it difficult to answer both parts accurately. The most knowledgeable teacher may not also be the friendliest.
b How many times has your child had a fever in the past month?	◀.....	Use simple words and keep sentences as short as possible.
c Including yourself, and any pets, how many regular household occupants did you buy groceries for in the past year?	◀.....	Keep sentences short and under 20 words. There is too much information in the original question.



Example 17 Wording questions to remove any ambiguities

Identify any ambiguous words in these questions and rewrite without any ambiguity.

- a** Do you usually purchase plane tickets online?
- b** Do you drink?

WORKING

THINKING

a Rewrite as: How often do you purchase plane tickets online?	◀.....	It is unclear how often is 'usually'. It is better to get an exact response from the respondent.
b Rewrite as: Do you drink sports drinks?	◀.....	It is unclear what type of drink you are referring to: water, soft drinks, caffeine drinks...?



Example 18 Designing questions free of bias

Identify any bias in the following questions and rewrite the question.

- a** Most of our clients prefer the look and feel of our renovated gym over the old one. How do you feel?
- b** The cost of concert tickets continues to go up to cover rising security costs. Do you think you should have to pay more when you buy your ticket or should the promoters be absorbing the rising costs?

WORKING

- a** Rewrite as: How do you feel about the renovations to the gym? ←.....
- b** Rewrite as: How much are you willing to pay for a concert ticket to cover additional security? ←.....

THINKING

The original question is leading the respondent to agree with 'most of the clients'.

The original question is loaded and is providing a reason for a problem.

Exercise 5H

FUNDAMENTALS

- 1** Demonstrate your understanding of terms by writing down the missing words in the following statements.
 - a** When writing a survey question, it is important to not use jargon or _____ words that respondents may not understand. Be brief and keep sentences to fewer than _____ words.
 - b** Questions should not be ambiguous. The _____ of your question must be very _____. Avoid words like rarely, sometimes, usually, few, some and most.
 - c** Consideration needs to be given to the _____ of response choices. Asking lots of questions or having lots of options to choose from when answering a question can lead to survey _____ in the sample group. People are more likely to remember the first and the last choice when offered a lot of options.
 - d** Issues of _____ and ethics need to be considered when conducting a survey. The information collected during a survey must be kept confidential and not _____ with unauthorised people.
 - e** Questions must be free from _____ and should not be _____ the respondent to a particular answer.

Example 15

- 2** The manager at a council swimming pool wishes to survey the people who use the pool about the amount of time they spend swimming per week.

The survey question is: How much time do you spend swimming at this swimming pool each week?

Here are two possible survey response options to include after this question.

Option A: ☐ 1 hour ☐ 5 hours

Option B: ☐ 1 hour ☐ 2 hours ☐ 3 hours ☐ 4 hours ☐ 5 hours
☐ 6 hours ☐ 7 hours

- a** Identify one issue in option A and describe why it is a problem.
- b** Identify one issue in option B and describe why it is a problem.
- c** Rewrite the answer options to cover a wide variety of possible hours with only four options to choose from.

Hint The available responses should accommodate all possible responses within a few choices.

APPLICATIONS

Example 16

- 3** Consider the survey question: Does it seem possible or impossible that the West Coast Eagles will win the AFL Western Derby, or do you feel they will always find a way to lose?

- a** Is this question too long?
- b** Besides bias, determine the key problem with this question.
- c** Identify the way in which this question contains bias.
- d** Rewrite the question so it is easier to respond to.

- 4** Consider the survey question: Students must take standardised NAPLAN tests in reading, writing and mathematics. Some teachers feel that students take too many tests. Others feel more should be required to test students in science and social studies. Do you think that your students take too many standardised tests?

- a** This question contains jargon commonly used by teachers when speaking with other teachers. Identify word(s) that could be considered jargon.
- b** The question is also rather long. Consider whether any of the question can be cut out.
- c** Rewrite the question so it is easier to understand.

Example 17

- 5** Consider the question: Where do you like to shop?

- a** Explain what makes this question ambiguous.
- b** Rewrite the question to find out where people like to do their grocery shopping.
- c** Rewrite the question to find out where people like to shop online.
- d** Rewrite the question to find out where people like to buy their runners from.

- 6** Consider the question: Where did you come from?
- a** Explain what makes this question ambiguous.
 - b** Rewrite the question to find out where the person was born.
 - c** Rewrite the question to find out where the person spent the morning.
- Example 18** **7** Identify which of the following questions do *not* contain any bias.
- a** Do you feel the TV news shows only the negative side of life?
 - b** Do you prefer the look of the plush new lounge?
 - c** Do you think school BYO computer policies advantage the wealthy?
 - d** Do you prefer this delicious flavoured milkshake or plain vanilla?
- 8** Identify any bias in these questions and rewrite without bias.
- a** Do you think the government should spend more money on the military to avoid a catastrophic invasion of the country?
 - b** Do you think the new policy is good?
 - c** Do you think the new recipe is better than the old recipe?
 - d** Do you think students should be allowed access to their own mobile phones while at school?
 - e** Government employees love their jobs. How do you feel about your job with the government?



- 9** Jenny has designed a questionnaire to survey students about the sports offered at school. Read her draft questionnaire and answer the questions that follow.

Question 1 How many times do you participate in sport and physical activity in a week?
☐ 1–2 ☐ 3–4 ☐ 5–6

Question 2 Please state what sport(s) you do at school. _____

Question 3 If you would like to do activities at school, when would you like to do them?

Question 4 Do you think the limited options of sports available at school are reasonable?

Question 5 People believe the school should offer more sports. If you agree, what sports should they offer?

Question 6 Personal details: Year group _____ Name: _____
 Gender _____ Date of birth: _____

Please place your completed survey in the box outside the Sports Staffroom.

- a** Question 1: This is a closed question because options are offered for the answer.
- i** Is it clear where the sport and physical activity occurs? Remember that Jenny is surveying students about the sports offered at school.
 - ii** Rewrite the question to remove any ambiguity.
 - iii** If a student does no sport, consider how they can answer.
 - iv** If a student does sport and exercise every day, what answer can they select?
 - v** Correct the available responses to accommodate all possible responses.
- b** Question 2: Is an example of an open-ended question, as a space is available to write a response. Jenny will get a range of answers to this question. It will be easier to process the data if options are available for the respondents to tick their response. Suggest a series of sports with tick boxes that Jenny could use.
- c** Question 3: A range of responses would be expected. Suggest a series of times, with tick boxes, that Jenny could use.
- d** Question 4 is an open-ended question.
- i** Identify the way in which the question is biased.
 - ii** Rewrite the question to remove any bias.
- e** Question 5 is an open-ended question.
- i** Identify the problem with this question.
 - ii** Rewrite the question to remove any issues with it.
- f** Question 6 collects personal information from respondents.
- i** Is the information collected necessary for the survey?
 - ii** Do you predict any issues with the way the survey responses are to be collected?

51 Misrepresentation in surveys

LEARNING GOAL

- Investigate errors and misrepresentations when reporting surveys

Why is it essential to be able to identify misrepresentation in surveys?

We have all heard the statement ‘There are lies, damned lies and statistics’. Sadly, statistics can be used to mislead from time to time. This can happen by manipulating results to give a false impression of facts. It is important to be aware of possible misrepresentation of results when applying the results of a survey to generalisations about a population.



WHAT YOU NEED TO KNOW

Survey results can be misrepresented to give a different impression of the characteristics of the population.

Common methods of misrepresentation to watch out for are:

- Misleading graphs**
- Omitting data** that does not suit the agenda of the author.
- Leaving survey questions out of a report can lead to **incorrect conclusions** being made about the data.
- Averages** can be influenced by a few small or large pieces of data.
 - For example: 1, 2, 3, 4, 5 has an average = 3; but 1, 2, 3, 4, 40 has an average = 10. One number was able to make a significant difference to the average calculated.
- Misuse of percentages.** When claiming a ‘fastest growing’ group, writers generally use percentages to support their claim.
 - For example: If a rare disease has four people affected by it and one more diagnosis is made, a 25% increase can be claimed. But, really, how significant is the increase?
- Exaggeration or misrepresentation of **cause and effect**, as discussed in Section 5G.



Example 19 Correlation vs causation

The number of people rescued in surf rips and the number of people who were involved in car accidents both rose in Western Australia during January. Identify which of the following could be a possible explanation.

- A** Surf rips cause car accidents.
- B** Car accidents cause surf rips.
- C** Surf rips and car accidents partly cause each other.
- D** Surf rips and car accidents are caused by a third factor.
- E** The correlation is purely by chance.

WORKING

We would expect both statistics to increase during the summer months when more people are at the beach and on the road for holidays. Therefore, option D, a third factor (i.e. summer), is a possible explanation for the correlation.

THINKING

Are the statistics related in any way?



Example 20 Omitting data

During an election campaign, some polls show Smith to be ahead by 5% and one shows Jones to be ahead by 2%. During a rally Jones tells his supporters, 'The polls show we are sure to win this election as I am ahead by 2%.'

How does this statement misrepresent the survey results?

WORKING

Jones has omitted data that does not suit his cause.

THINKING

He has used statistics that are only favourable to him.

Exercise 5I

FUNDAMENTALS

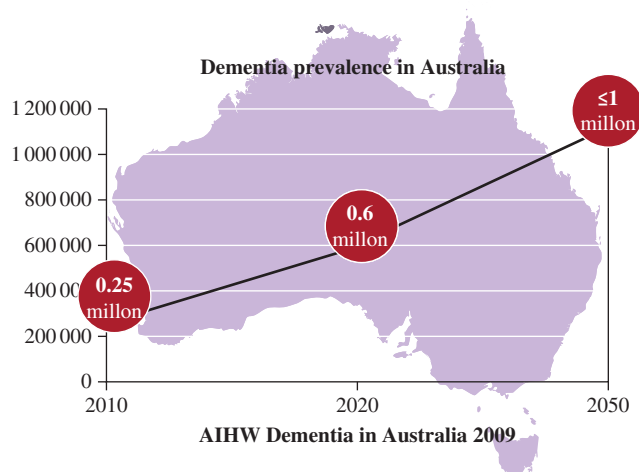
- 1** Demonstrate your understanding of terms by writing down the missing words in the following statements.
 - a** Survey results can be _____ when reporting results to give a different _____ of the characteristics of the population.
 - b** _____ graphs can give a false impression of data.
 - c** Omitting _____ that does not suit the agenda of the author is a technique designed to _____.

- d** Leaving survey questions out of a report can lead to _____ conclusions being made about the data.
- e** Averages can be influenced by a few _____ or _____ pieces of data.
- f** When claiming a 'fastest growing' group, writers are generally using _____ to support their claim.

APPLICATIONS

Example 19

- 2** Behind heart disease, dementia is the second leading cause of death in Australia today. This graph shows the anticipated prevalence of dementia in Australia, without any medical breakthrough to find a cure.



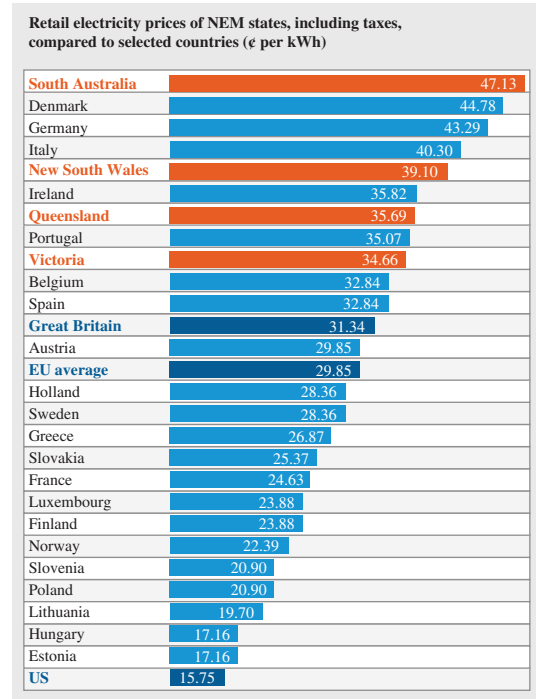
- a** Use the graph to determine the predicted number of dementia sufferers by:
 - i** 2020
 - ii** 2050 if you read the bubble
 - iii** 2050 if you read the line graph.
 - iv** Identify the mistake that has been made with the reading recorded in the bubble.
- b** In 2010 the population of Australia was 22.3 million people.
 - i** Use the graph to determine how many people suffered from dementia.
 - ii** Calculate the percentage of the population that suffered from dementia.
- c** In 2020 the population of Australia is expected to reach 24.0 million people. According to the graph:
 - i** Use the graph to determine how many people are expected to have dementia.
 - ii** Calculate the percentage of the population that is expected to be suffering from dementia.
- d** The population is expected to reach 35.9 million by 2050.
 - i** Use the graph to determine how many people are expected to have dementia.
 - ii** Calculate the percentage of the population that is expected to have been diagnosed with dementia in 2050.

- e** Give a possible cause for the expected increase in the number of people diagnosed with dementia.
- f** Describe two ways that the graph misrepresents the data.
- g** Construct another graph to represent this data with percentages on the vertical axis, and the same horizontal axis. Use the percentage of population calculations from parts **b ii**, **c ii** and **d ii**.
- 3** *The Financial Review* reported in August 2017 that ‘Australian households pay the highest power prices in the world.’ An excerpt from the article is given.

Australian residential customers are paying the highest electricity prices in the world – two to three times more than American households – but experts say they need more than information to navigate the thicket of discounts and offers.

South Australian households are paying the highest prices in the world at 47.13c per kilowatt hour, being more than Germany, Denmark and Italy, which heavily tax energy, after the huge increases on July 1, Carbon + Energy Markets’ MarkIntell data service says.

NSW households typically pay 39.1c per kWh – hard on Italy’s heels – while Queensland and Victoria’s typical retail charges of 34.7c to 35.7c per kWh exceed those in all but the four or five most expensive European countries, the MarkIntell data shows.



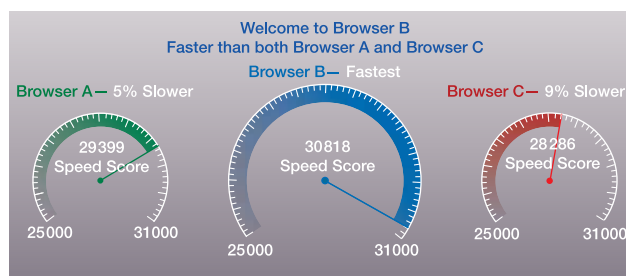
Source: www.afr.com/news/australian-households-pay-highest-powerprices-in-world-20170804-gxp58a

- a** Use the graph to determine how much consumers are paying per kWh in:
- i** the US
 - ii** South Australia
 - iii** Queensland
- b** Calculate the ratio of prices for:
- i** South Australia to the US
 - ii** Queensland to the US
- c** Use your answers to part **b** to verify the claim in the first paragraph that Australian residential customers are paying ‘two to three times more than American households’.

- d** Measure the length of the bar on the graph for:
- i** the US
 - ii** South Australia
 - iii** Queensland
- e** Calculate the ratio of bar lengths for:
- i** South Australia to the US
 - ii** Queensland to the US
- f** Use your answers to complete the following.
- i** Compare your ratios found in part **e** to those found in the article and in part **b**.
 - ii** Explain whether the claims in the article are consistent with the statistics and the graph.
 - iii** Identify a possible reason for any difference.
- g** Explain how the following factors could make a difference to the average prices quoted.
- i** sample size
 - ii** using the mean vs the median
 - iii** how the sample was collected
 - iv** how the electricity was generated – gas, coal, solar, hydroelectricity

Example 20

- 4** A local council collected data on the number of development applications it had approved. It showed that from 2011 to 2014 the number of development applications had increased, from 2014 to 2016 the number of applications had remained steady, and in 2017 the number of applications had dropped. In an advertising campaign designed to promote the region, the council stated, ‘Development applications have risen significantly from 2011 to 2017.’
- a** Identify the way in which the Council has misrepresented the survey’s results.
- b** Explain the purpose of misrepresenting the data.
- 5** The following graph appeared when installing computer software.



- a** Identify two ways in which the graph is misleading.
- b** Construct a histogram to represent the information.
- c** Check the calculated percentage change between Browser B and:
- i** Browser A
 - ii** Browser C.
- d** Is there any indication of the number of trials conducted in the survey? What factors about the sample might influence the results?
- e** Is there any indication how the data was calculated? Would you get the same result if an average or a median score were used?

Statistical investigation process task



Background: ABC news reported on its website in December 2017 that ‘Young workers most likely to use phones while driving’.

An extract from the article is given.

Distracted driving is a significant contributor to road accidents and fatalities. Mobile phone use while driving is a particularly important form of driver distraction.

It can increase the risk of traffic accidents by up to four times.

At any moment, roughly 1–2 per cent of Australian drivers are using their mobile phone while driving. In 2016 alone, police in New South Wales charged 39,000 people for doing so.

Our survey of 413 South Australians revealed that young working people were those most likely to use their phones while driving.

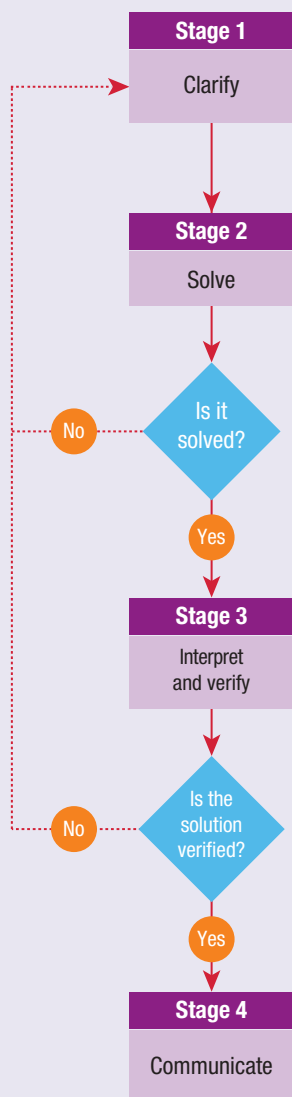
How prevalent is it?

One in three respondents in our survey reported never using their phones while driving; one in two reported rare or occasional use; and one in five reported frequent use.

Receiving incoming phone calls while driving was the most commonly reported behaviour: 61 per cent reported having received at least one call while driving in the past two weeks.

Task: The article makes a claim that ‘at any moment, roughly 1–2 per cent of Australian drivers are using their mobile phone’. You are to verify or refute (disprove) this statistic.

Approach to statistical investigation process task.



Stage 1: Clarify

Clarify the problem and pose one or more questions that can be answered with data

- 1 What are you required to do?
- 2 What information do you have?
- 3 What other information do you need?
- 4 How could you gather more information?
- 5 What assumptions will you make?

Stage 2: Solve

Design and implement a plan to collect or obtain appropriate data and select and apply appropriate graphical or numerical techniques to analyse the data

- 6 Gather more information through research or a survey.
- 7 Estimate the percentage of Australians driving and using a phone at any moment.
- 8 Produce graphs and/or tables required to help solve the problem.
- 9 Decide whether you can verify or refute the statistic quoted.

Stage 3: Interpret and verify

Interpret the results of this analysis and relate the interpretation to the original question

- 10 Have you verified or refuted the statistic?
- 11 Do you have the evidence to back up your decision?

Stage 4: Communicate

Communicate findings in a systematic and concise manner

- 12 Write a report with an introduction, body and conclusion.
 - State your main point.
 - Include the evidence in the form of statements, graphs and tables. Explain the evidence. Use a sentence starter like ‘This means...’.
 - In the conclusion, link back to your main point.

Chapter checklist

☐ I understand the procedures for conducting a census.

- 1 a What is a census?
- b How is a census conducted?

☐ I understand the advantages and disadvantages of a census.

- 2 a Explain three advantages of a census.
- b Explain three disadvantages of a census.

☐ I know the difference between a census and a survey.

- 3 a What is a survey?
- b Explain the advantages of a survey.
- c Explain the disadvantages of a survey.

☐ I understand the purpose of sampling.

- 4 Explain the purpose of sampling.

☐ I know about simple random samples.

- 5 a Describe how to select your sample using simple random samples.
- b Explain the advantages of a simple random sample.
- c Explain the disadvantages of a simple random sample.

☐ I know about self-selected samples.

- 6 a Describe how a self-selected sample works.
- b Explain the advantages of a self-selected sample.
- c Explain the disadvantages of a self-selected sample.

☐ I know about systematic samples.

- 7** **a** Describe how a systematic sample works.
b Explain the advantages of a systematic sample.
c Explain the disadvantages of a systematic sample.
d Write down the missing word in the following sentence.
 If we wish to systematically select 20 students from a school of 500 to participate in a survey, we select every _____th student on the roll.

☐ I know about stratified random samples.

- 8** **a** Describe how a stratified random sample works.
b Explain the advantages of a stratified random sample.
c Explain the disadvantages of a stratified random sample.

☐ I can identify a target population.

- 9** **a** Describe the target population.
b Describe the sample population.
c Describe the study population.

☐ I understand the principles of designing a questionnaire.

- 10** **a** Explain what it means to use simple language.
b Describe what is meant by ambiguous questions.
c Explain why we need to consider the number of response choices.
d Of any data obtained, explain why we need to consider privacy and ethics.
e Explain what is meant by the term bias.

☐ I can describe faults in the process of collecting data.

11 A government committee set up an online poll to assess the public's view on immigration. It distributed the link to the poll via social media sites and people were able to vote multiple times using different email addresses. People from outside Australia responded to the poll and promoted it widely on the internet.

- a** Give reasons why there might be problems with the results of this poll.
- b** Suggest how the poll could have been conducted better.

☐ I can describe sources of errors in surveys, including sampling error and measurement error.

12 A TV show asks viewers their opinions on an issue, through its Facebook page.

- a** Explain in what way the TV show might have a biased sample.
- b** Indicate a sampling technique to use that would avoid bias.

13 A set of bathroom scales always shows 0.2 kg when no one is standing on them. What type of error would be created when using these scales?

☐ I can identify misrepresentation of a survey's results due to misunderstanding the procedure or the reliability of generalising the survey findings to the entire population.

14 An online news agency ran a poll asking readers, 'Which do you prefer: free-to-air TV or on-demand-streaming?' 87% of respondents voted for streaming.

- a** What type of sampling technique has been used?
- b** What issues could arise from generalising the results of this survey to the population?

- ☐ I can identify errors and misrepresentation in surveys, including in the media.

- 15** A news article has the headline: **‘Alarming’ survey results show Australians are suffering from a ‘loneliness crisis’**. The story went on to say that *it’s ‘alarming’ 15% of men and 8% of women say they have no close relationships outside of their intimate relationship and 18% of Australians feel they can’t reach out to neighbours in a time of crisis. The article also said social media is playing a large role. ‘We know people can have 300 friends on Facebook and if you’re home alone on a Saturday night and you’re looking through your feed and everyone’s having a party out there, there is nothing more isolating.* (Source: www.4bc.com.au/alarming-survey-results-show-australians-are-suffering-from-a-loneliness-crisis/)
- a** In what way could the headline be misleading?
- b** In what way has the article misrepresented the statistics?

Chapter review

1 Match each term (A to I) with its definition (i to ix).

A census	i The group about which information is being sought
B random sample	ii A small group within a population
C sample	iii Subjects are selected from a list at regular intervals
D stratified sample	iv The entire group
E population	v An official count of an <i>entire</i> population
F survey	vi The sample has the same proportions as the population
G systematic sample	vii Random numbers are used to select the sample from a numbered list
H self-selecting sample	viii The process of gathering information from a small group
I target population	ix Anyone who wishes to participate answers the survey questions.

2 For the following, determine whether each is an example of a random sample, systematic sample, stratified sample or self-selecting sample.

- a** Viewers voting for their favourite celebrity in a reality TV show
- b** Selecting every fifth person on a list to participate in a survey
- c** Using random numbers to select the sample from a list
- d** Calculating the proportions of subgroups and ensuring the sample has the same proportions

3 Determine whether a census or a survey technique should be used for gathering information in the following situations.

- a** Determining the number of students in a school
- b** Finding the preferred menu for a school canteen
- c** The top 10 downloaded songs for the week
- d** The number of new cars sold in a week
- e** The all-time highest point scorer for the Perth Wildcats team
- f** The vaccination rates of children in a suburb

Section 5D

4 A touch football club has 460 members. The president wishes to use a systematic sample to survey the members to gauge their support for building a clubhouse. Determine what will be the interval on the list between the members he selects, if he wishes the sample size to be:

- a** 20 members
- b** 50 members
- c** 30 members

The following information applies to questions 5 to 9.

A day-care centre has 80 children. The director, Amy, wishes to conduct a survey of the children's parents to see if there is a need to extend the number of hours it offers for outside play. Amy creates an alphabetical list of all the 80 children registered at the centre. She colours the boys' names in blue and the girls' names in orange to be able to distinguish between the gender of the children.

1 Abigail	11 Aria	21 Chloe	31 Ethan	41 Jacob	51 Lily	61 Noah	71 Scarlett
2 Addison	12 Aubrey	22 Christopher	32 Evelyn	42 James	52 Logan	62 Nora	72 Sebastian
3 Aden	13 Audrey	23 Daniel	33 Gabriel	43 Jayden	53 Lucas	63 Oliver	73 Skylar
4 Alexa	14 Ava	24 David	34 Grace	44 John	54 Luke	64 Olivia	74 Sofia
5 Alexander	15 Aver	25 Dylan	35 Hannah	45 Joseph	55 Madison	65 Paisley	75 Sophia
6 Allison	16 Benjamin	26 Elijah	36 Harper	46 Joshua	56 Mason	66 Penelope	76 Victoria
7 Amelia	17 Brooklyn	27 Elizabeth	37 Henry	47 Layla	57 Matthew	67 Riley	77 William
8 Andrew	18 Camila	28 Ella	38 Isaac	48 Leah	58 Mia	68 Samantha	78 Wyatt
9 Anna	19 Carter	29 Emily	39 Isabella	49 Liam	59 Michael	69 Samuel	79 Zoe
10 Anthony	20 Charlotte	30 Emma	40 Jackson	50 Lillian	60 Natalie	70 Savannah	80 Zoey

Section 5C

- 5** Amy decides to use a random sample technique to select a sample of 15 children to ask their parents to be involved in the survey. Using Excel, she generates the following random numbers.

44 16 7 80 71 2 33 24
31 69 77 19 67 10 17

- Use the random numbers to select the sample to use in the survey.
- Calculate the fraction of children who attend the day-care centre who are boys.
- Calculate the fraction of the children in the sample that are boys.
- Determine whether the sample is a good representation of the population of children in the centre.

Section 5D

- 6** Not happy with her sample, Amy then decides to use a systematic sample technique to select the sample. She still wishes to use a sample size of 15.
- Determine how frequently she will select from the list.
 - Amy selects a random starting position using Excel. If she starts at 18, determine who will be the children selected for the sample.

- c** Calculate the fraction of children in the sample who are boys.
- d** Decide if the sample is a good representation of the population of children in the centre.

Section 5E **7** Still not happy with her sample, Amy finally decides to use a stratified sample technique to determine her sample. She reorganises her list to be a list of boys and girls as follows.

1 Aiden	11 Elijah	21 Joseph	31 Oliver	6 Anna	16 Elizabeth	26 Leah	36 Riley
2 Alexander	12 Ethan	22 Joshua	32 Samuel	7 Aria	17 Ella	27 Lillian	37 Samantha
3 Andrew	13 Gabriel	23 Liam	33 Sebastian	8 Aubrey	18 Emily	28 Lily	38 Savannah
4 Anthony	14 Henry	24 Logan	34 William	9 Audrey	19 Emma	29 Madison	39 Scarlett
5 Benjamin	15 Isaac	25 Lucas	35 Wyatt	10 Ava	20 Evelyn	30 Mia	40 Skylar
6 Carter	16 Jackson	26 Luke	1 Abigail	11 Avery	21 Grace	31 Natalie	41 Sofia
7 Christopher	17 Jacob	27 Mason	2 Addison	12 Brooklyn	22 Hannah	32 Nora	42 Sophia
8 Daniel	18 James	28 Matthew	3 Alexa	13 Camila	23 Harper	33 Olivia	43 Victoria
9 David	19 Jayden	29 Michael	4 Allison	14 Charlotte	24 Isabella	34 Paisley	44 Zoe
10 Dylan	20 John	30 Noah	5 Amelia	15 Chloe	25 Layla	35 Penelope	45 Zoey

- a** Calculate the fraction of the children attending the centre that are boys.
- b** If Amy still wishes to use a sample size of 15, determine how many boys will be in the sample.
- c** Use as many of the following random numbers, between 1 and 35, as required to determine the sample of boys.
25 28 20 19 20 4 8 24 32 31
- d** Calculate the fraction of children attending the centre who are girls.
- e** If Amy wishes to use a sample size of 15, determine how many girls will be in the sample.
- f** Use as many of the following random numbers, between 1 and 45, as required to determine the sample of girls.

15 37 19 3 16 20 28 17 44 29

- 8** Explain which survey technique provided the best sample for the director.

Section 5F **9** Identify the target population of the day-care centre director's survey.

Section 5H

10 Amy, the day-care centre director, drafts up the questionnaire that she would like to use for her survey of parents.

1	Child's name:	2	Age:	3	Gender:												
4	Encouraging children to spend time playing outdoors can help their motor, sensory, social and cognitive development. Are you happy with the amount of time your child gets to play outside while attending the day-care centre?																
5	What activities would you like to see offered during outside playtime? (Tick choices.) <table> <tr> <td>☐ bikes</td> <td>☐ rock climbing wall</td> <td>☐ cricket</td> </tr> <tr> <td>☐ painting</td> <td>☐ swings</td> <td>☐ touch football</td> </tr> <tr> <td>☐ sandpit</td> <td>☐ obstacle course</td> <td>☐ handball</td> </tr> <tr> <td>☐ monkey bars</td> <td>☐ laser skirmish</td> <td>☐ other</td> </tr> </table>					☐ bikes	☐ rock climbing wall	☐ cricket	☐ painting	☐ swings	☐ touch football	☐ sandpit	☐ obstacle course	☐ handball	☐ monkey bars	☐ laser skirmish	☐ other
☐ bikes	☐ rock climbing wall	☐ cricket															
☐ painting	☐ swings	☐ touch football															
☐ sandpit	☐ obstacle course	☐ handball															
☐ monkey bars	☐ laser skirmish	☐ other															
6	Does your child eat vegetables? ☐ Yes ☐ No ☐ Only when I cover them with tomato sauce.																
7	Are you and your partner married? ☐ Yes ☐ No																
<i>Thanks for your time. Please leave this form at the reception desk.</i>																	

a Consider question **4**.

- i** Questions should be fewer than 20 words long. Is the question too long?
- ii** Identify the words in the question that could be considered jargon.
- iii** Describe how the question is ambiguous.
- iv** Rewrite the question to shorten it and remove any jargon or ambiguity.

b Consider question **5**.

- i** Explain whether this is a reasonable number of choices.
- ii** Identify any the choices that are not relevant to a day-care centre.
- iii** Edit the list to make it a more reasonable set of possible responses.

c Consider questions **6** and **7**.

- i** Determine whether question **6** is a reasonable question to ask in this survey.
- ii** Determine whether question **7** is a reasonable question to ask in this survey.
- iii** Identify any problems there might be in asking these questions.

- iv Decide whether it is ethical to ask these questions. Explain your reasoning.
- v Explain whether these questions should be included in the survey.
- d Consider the method for collecting the responses. Describe whether this method is going to ensure that the privacy and confidentiality of the parents responding are guaranteed. Otherwise, propose how it could be improved.

Section 5I

- 11** Jim wishes to surprise his teenage son by purchasing him a new gaming console. He surveys his colleagues at work for advice on which platform to buy.
- a Describe a fault in Jim's method of collecting data.
 - b Explain how Jim could improve his survey.

Section 5B

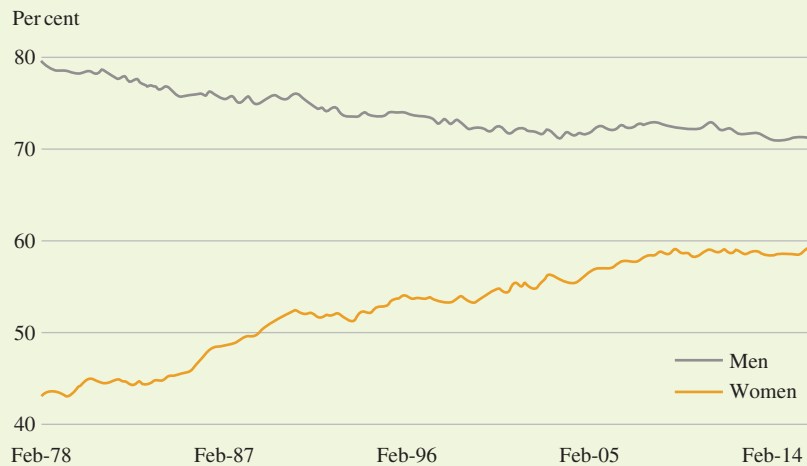
- 12** Customers at a local hardware store were asked in a survey 'How many times have you visited this hardware store in the past year?'.
- a Is this an example of questioning error, sampling error or measurement error?
 - b Identify in what way there is an error.
 - c Describe how the question could be improved.
- 13** When conducting a poll on how our taxes are spent, determine whether the following questions should be used and give a reason why for each.
- a Do you believe that you should be taxed so other people do not have to work?
 - b Do you think the government should help people who cannot find work?
 - c Do you support the government's assistance programs for the unemployed?
 - d What do you think about unemployment assistance?
- 14** When measuring the heights of students in her class, Mrs Tall used an old height chart that measured in inches (2.54 cm) and she measured to the nearest inch. She also asked students to keep their shoes on. Describe any faults in her method of collecting data.
- 15** When deciding where the staff Christmas party should be held for Hawking SHS, the Social Club Coordinator, who happens to be the Essential Maths teacher, Mr Evans, asked the six other Maths teachers for their preferred venue. The staff body consists of teachers, learning support assistants, cleaners, groundsmen and administration assistants. Identify any errors with Mr Evans's method of collecting data.

- 16** Mr Evans's question to the other Maths teachers was, 'I'd really like to hold the staff Christmas party this year at Trinomial Tavern. Do you think that is a good place?' Suggest a new question that does not contain bias.
- 17** Consider this line:
-
- a** Use a ruler to determine the length of the line when you measure to the nearest millimetre.
 - b** Use a ruler to determine the length of the line when you measure to the nearest centimetre.
 - c** Calculate the size of the measurement error when centimetres instead of millimetres are used to measure the line.
 - d** Calculate the percentage measurement error when centimetres instead of millimetres are used to measure the length of the line.
- 18** A mother weighs her child to check her child's growth.
- a** Identify the weight reading, in kilograms (kg).
 - b** Calculate the weight reading, in grams (g).
 - c** Choosing from 1 kg, 1 g, 10 g or 100 g, complete this statement: The scales are accurate to the nearest _____.
 - d** Determine the lightest possible weight of the child, in grams.
 - e** Determine the heaviest possible weight of the child, in grams.
 - f** Determine the error range for the scales.
 - g** The child's weight, to the nearest gram, is 11 168 g. Calculate the percentage error.
- 19** A cricket phone app polled its subscribers on who is the greatest batsman of all time. This happened as a very popular player was approaching a double century (200 runs) and the game was being televised live around the country. Describe any sources of error in this poll.
- 20** Regina surveyed eight parents, who were watching their child play volleyball in a school team, about how many hours a week their child spent playing sport. Six parents said their child played only volleyball and spent an hour training and 1 hour at the game each week. Regina reported to the sports master that '75% of parents felt the teams needed more training'.
- a** Identify any issue with the sampling technique.
 - b** Identify any issues with the survey procedure.
 - c** Identify any issues with the interpretation of the results.

- 21** A school tracked its students' results from Year 7 to Year 10 and found an improvement in the average academic performance of the students. The following statistics were also observed over the same time period.
- A** An increased number of students using BYO laptops at school.
 - B** A decrease in the number of students buying lunch from the tuckshop.
 - C** An increase in the number of students accessing help from teachers during lunch breaks and after school.
 - D** A decrease in the number of students using their phones at school, after the implementation of a policy banning mobile phones at school.
 - E** An improvement in the performance of the basketball team for that age group.

For each of the data trends **A** to **E**, decide if it could be an influencing cause on the students' improved academic performance. Give reasons to support your answer.

- 22** The following graph was part of a report on labour force statistics presented to the Parliament of Australia in 2015. The graph shows the participation rate in the workforce from 1978 to 2014, according to gender. Essentially, it is showing the percentage of men and women, aged 15 years and over, engaged in work over the time period.



Suggest a possible interpretation of the data shown in this graph.

Section 5G

- 23** Below is an excerpt from an article that appeared in the *Sydney Morning Herald* 16 March 2018.

Death traps: Push to phase out cars made before 2002

Vehicles older than 15 years — many lacking lifesaving airbags or electronic stability control — should be phased out to reduce road deaths, according to a leading non-profit safety ratings group.

The call by the Australasian New Car Assessment Program (ANCAP) came after a review of road fatalities over the past three years.

The rate of fatal crashes for vehicles 15 years and older was four times greater than for occupants in vehicles made in the past five years, crash data from 2014 to 2016 shows.

The research by ANCAP found that the average age of cars and sports utility vehicles involved in fatal crashes rose gradually from 12.5 years old in 2014 to 12.9 in 2015 and 13.1 in 2016. At the same time, the average age of Australian passenger cars stayed the same at 9.8 years old.

- a** Determine the increase in the age of cars and sports utilities involved in fatal car accidents between 2014 and 2016. Is this a large change? Identify factors that could influence the calculation of the average.
- b** The article states: ‘The rate of fatal crashes for vehicles 15 years and older was four times greater than for occupants in vehicles made in the past five years.’ Describe any other factors that could lead to the increase in car accidents in older cars.



6 Bivariate data

In this chapter

- 6A** Describing the association between variables of bivariate data
 - 6B** Identifying dependent and independent variables
 - 6C** Determining the line of best fit
 - 6D** Interpreting relationships between variables
 - 6E** Calculating the correlation coefficient using technology
 - 6F** Making predictions
 - 6G** Distinguishing between causality and correlation
- Statistical investigation process task
Chapter checklist
Chapter review

Syllabus reference

Unit 3 Topic 4: Data collection (5 hours)

Bivariate scatterplots

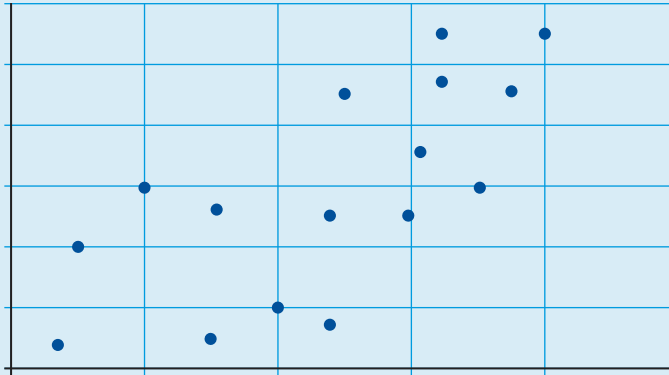
- describe the patterns and features of bivariate data
- describe the association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak)

Trend lines

- identify the dependent and independent variable
- fit a trend line by eye
- interpret relationships in terms of the variables, for example, describe trend as increasing or decreasing
- use the trend line to make predictions, both by interpolation and extrapolation
- recognise the dangers of extrapolation
- distinguish between causality and association through examples

Pre-test

- 1 Draw a line of best fit for the following distribution of points so that there are roughly as many points above the line as below.

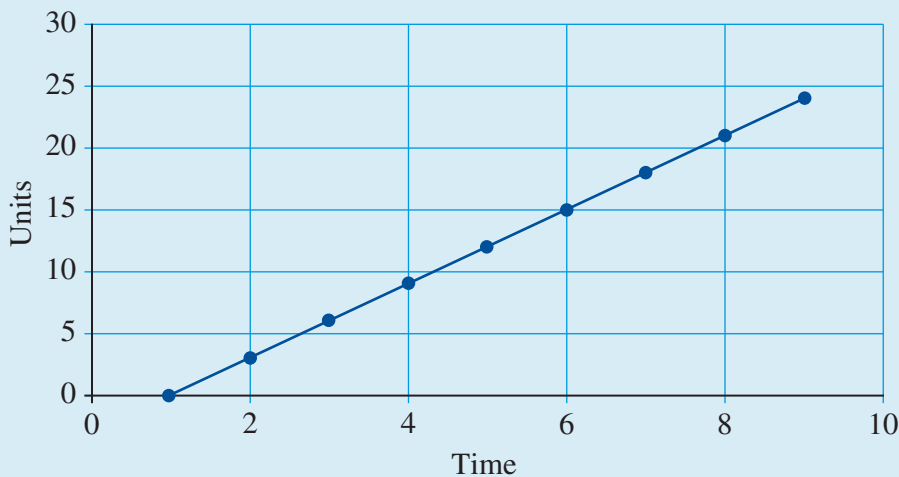


Hint Use a clear ruler so you can see when there are approximately the same number of dots either side of the line.

- 2 Create a scatterplot using the following dataset observing different students in a class rolling 2 dice.

Student	1	2	3	4	5	6	7	8	9	10
Roll 1	5	2	3	2	4	1	3	6	4	2
Roll 2	1	2	6	4	1	6	3	4	3	5

- 3 This is a graph of a quantity against time.



- a Determine the number of units at time 6.
 b For every increase in 1 unit of time, determine the increase in the number of units.

6A Describing the association between variables of bivariate data

LEARNING GOALS

- Understand definition of bivariate data
- Identify patterns and trends from bivariate data
- Determine positive/negative data correlations
- Determine linear/non-linear data correlations
- Determine strong/moderate/weak/no data correlations

Why is it essential to know about bivariate data and to identify associations between two variables?

The investigation of whether one thing affects another is at the heart of research and development across science, technology, health, social studies, economics and business. Data gathered when one thing affects another is bivariate data – there are two variables, and ‘bi’ means ‘two’ and ‘variate’ means varying. An important element of statistics is determining if any relationship exists between the variables in a set of bivariate data. These relationships can be used to make conclusions and even predictions.



Statistics can help determine whether there is a relationship between the diameter of the weather balloon and the weight of instruments it can lift.

WHAT YOU NEED TO KNOW

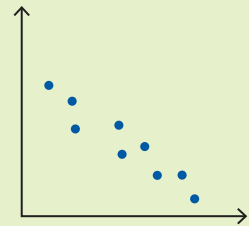
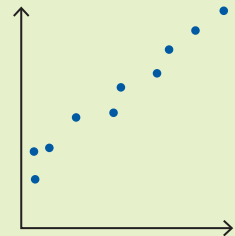
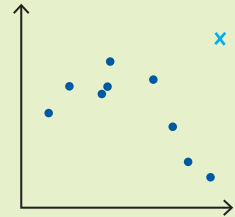
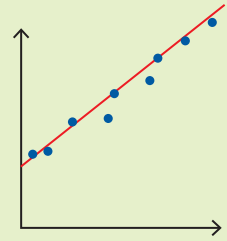
- **Bivariate data** is data for two **variables** that may have an **association**.

An example is hours of study in maths and maths test scores.

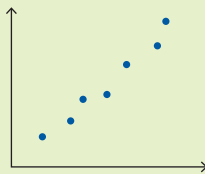
Hours of study	1	2	3	4	5
Test scores out of 20	3	6	11	13	19

- The association between them is that the hours of study affects the test scores. An association is also called a relationship or **correlation**.

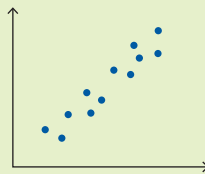
- A **scatterplot** is the most usual form of a graph of bivariate data, where the data are represented by individual dots on the graph.
- A **linear correlation** is when a graph of the data appears similar to a straight line with a slope, as shown here.
- A **non-linear correlation** is when a graph of the data produces a curve.
- An outlier is a data point that does not fit with the general pattern, like the data point marked 'x' in this scatterplot.
- A **positive correlation** is as one data variable increases so does the other data variable, so the graph slopes up to the right (e.g. comparing height to shoe size – the taller the person the larger the shoe size).
- A **negative correlation** is as one data variable increases the other data variable decreases, so the graph slopes down to the right (e.g. the larger number of days with sunny weather will mean the less water left in the dam due to evaporation).
- The correlations can be described as strong, moderate, weak or none depending on the scatter and how close to a straight line the data lies.



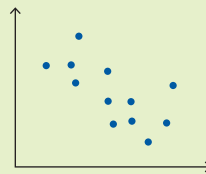
**Strong
correlation**



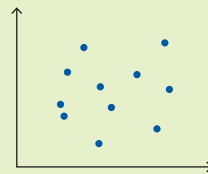
**Moderate
correlation**



**Weak
correlation**



**No
correlation**

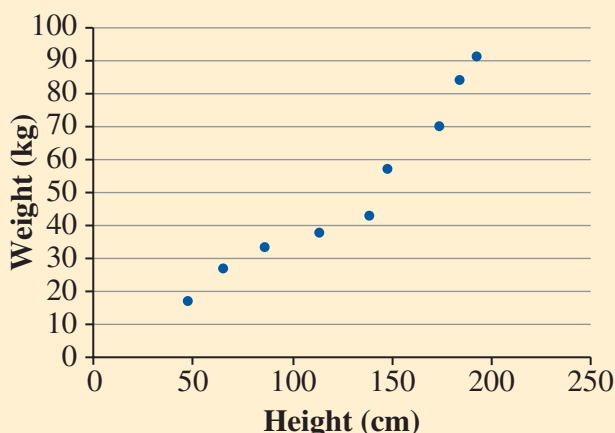




Example 1 Describing the association of variables of bivariate data from a scatterplot

The scatterplot shown maps the height and the weight of nine people.

- Describe any correlations in terms of positive or negative, linear or non-linear, and strong, moderate, weak or none.
- Determine what relationship if any exists between a person's height and weight.



WORKING

- The scatterplot has a positive, linear and strong correlation.
- As the person's height increases, the person's weight will also tend to increase.

THINKING

As the scatterplot goes up from left to right it is positive. The data is close to a straight line and is therefore linear, and as it is close to a line it is a strong correlation.

As there is a definite correlation, we can conclude a statement describing the relationship between a person's height and weight.





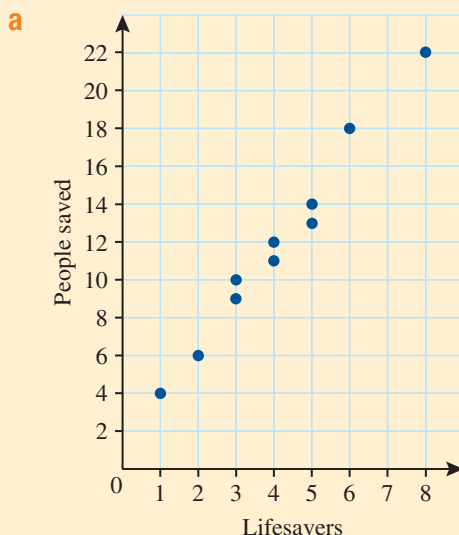
Example 2 Creating a scatterplot and describing the association of variables of bivariate data

The lifesaver association wants to show that the number of lifesavers patrolling a beach affects the number of people who are saved each year.

- Using the data in the table, create a scatterplot to determine any correlation between the number of lifesavers and the number of people saved that exists.
- Describe any correlations in terms of positive or negative, linear or non-linear, and strong, moderate, weak or none.
- Determine if the number of lifesavers have an effect on the number of people saved each year.

Number of lifesavers	4	6	2	1	5	8	4	3	6	5	3
Number of people saved	12	18	6	4	13	22	11	10	19	14	9

WORKING



THINKING

←.... Identify the range of 'Number of lifesavers' and 'Number of people saved' required to be graphed. Use the range of both to draw a graph large enough to display all the coordinates. Plot the coordinates on the graph from the table of data points to create a scatterplot.

- The correlation is positive, linear and strong. ←..... As the scatterplot goes up from left to right it is positive. The data is close to a straight line and is therefore linear, and as it is close to a line it is a strong correlation.
- It appears that the more lifesavers on patrol will result in more people being saved. ←..... As there is a definite correlation, we can conclude with a statement describing the relationship between the number of lifesavers and the number of people saved.

Exercise 6A

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a A _____ correlation displays data increasing from _____ to _____.
 - b A strong correlation describes a _____ that forms a pattern similar to a _____ line.
 - c Data with no _____ is said to have no correlation.
 - d A _____ correlation shows data that has some pattern but is not very strong.

- 2 Decide if the data variables have a relationship.
 - a weight of cars and fuel consumption
 - b temperature and the cost of a textbook
 - c number of flowers and number of bees
 - d height of a door and size of door handles
 - e amount of rain and the size of vegetables in the garden
 - f length of student's hands and the length of student's feet

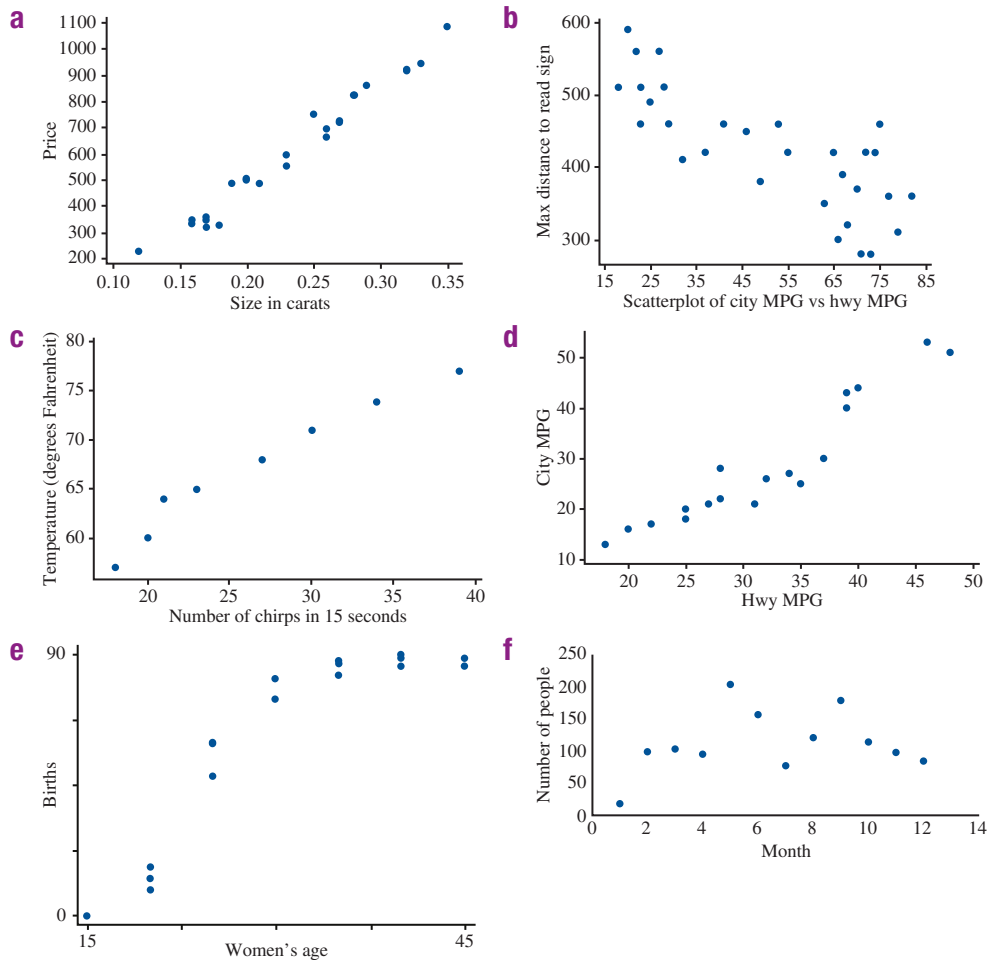
Hint If two variables are related, one of these should be true:

- when one of them changes, it will produce a change in the other; but not necessarily the other way around
- or when something changes, it causes both of them to change.



3 Describe the correlations between the variables in these graphs in terms of:

- i** direction (positive or negative)
- ii** shape (linear or non-linear)
- iii** strength (strong, moderate, weak or none)



4 Construct a scatterplot using technology (or by hand on graph paper) using the data below. From this scatterplot determine the correlation between the data variables that exists in terms of:

- a** direction (positive, negative, none)
- b** shape (linear or non-linear)
- c** strength (strong, moderate, weak or none)

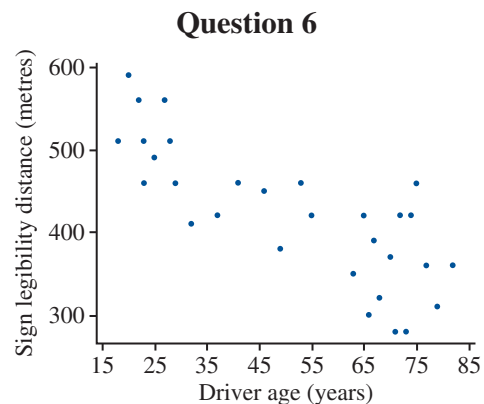
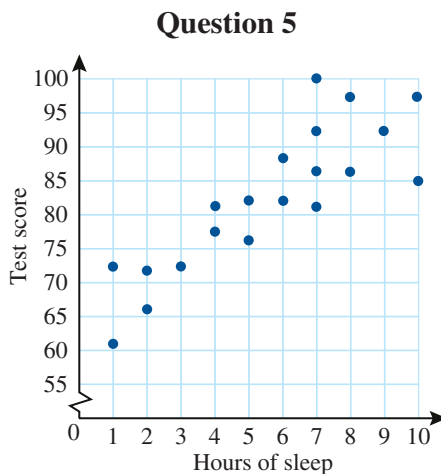
Hint An outlier can be disregarded when identifying a pattern.

Hours studied	9	1	5	4	3	5	0	1	2
Test score	90	86	84	92	91	100	76	82	85

APPLICATIONS

Example 1

- 5 The scatterplot shown below, left, maps student's test scores and the average hours of sleep per night. Describe any correlations in the terms of direction, shape and strength, and determine what effect the hours of sleep have on student's test scores, if any.



- 6 The scatterplot shown above, right, is mapping the age of a driver against their reading distance of street signs. Describe any correlations in the terms of direction, shape and strength, and determine what effect the age of the driver makes to the driver's reading distance of street signs, if any.
- 7 The police union are wanting to prove that the number of police serving in an area will lower the incidence of crime. Using the data in the table, create a scatterplot by hand or with technology to determine any correlation between the number of police and the incidence of crime that exists. Describe any correlations in terms of direction, shape and strength, and determine if the police union's belief is correct.

Number of police	15	21	8	14	19	31	17	12	18	9	12	14
Incidence of crime	28	16	36	24	21	19	21	26	22	31	24	26



- 8 The CSIRO are examining the effects of fertiliser on the yield per crop. Using the data in the table, create a scatterplot by hand or with technology to determine any correlations that exist between the amount of fertiliser used and the crop yield. Describe any correlations in terms of direction, shape and strength, and determine if the amount of fertiliser has an effect on the crop yield.

Fertiliser (kilograms)	100	125	180	80	250	140	276	112	211
Crop yield (tonnes)	7.2	7.7	8.4	6.4	12.4	8.0	13.0	7.3	10.9

- 9 It has been thought that the number of hours of playing video games per day by teenagers negatively affects their overall number of hours of sleep. Using the data in the table, create a scatterplot by hand or with technology to determine any correlations that exist between the hours per day of playing video games and the amount of sleep. Describe any correlations in the terms of direction, shape and strength, and determine if the number of hours playing video games affects the teenagers' sleep.

Number of hours playing video games per day	5	0	3	2.5	6	4	3.5	8	2	5	3	1.5
Number of hours of sleep per day	7	10	8	8.5	6	8	7.5	5	8.5	7.5	8.5	9

- 10 A survey was conducted with the school basketball team asking players how many hours per week they practised and the average points per game they scored. Using the data in the table, create a scatterplot by hand or with technology to determine any correlations that exist between the hours per week practising and the average points per week. Describe any correlations in terms of direction, shape and strength, and determine if the hours practising affects the points scored.



Hours practising per week	4	10	2	6	3.5	7	9	1	5.5	8
Average points per game	12	18	8	5	9	8	7	14	6	9

6B Identifying dependent and independent variables

LEARNING GOAL

- Identify dependent and independent variables

Why is it essential to be able to identify independent and dependent variables?

To understand bivariate data and the idea of an association between two variables and that they could be correlated we need to consider a vital difference between the variables, that a change in one variable (the independent variable) may cause a change in the other (the dependent variable). This is very important for research and development in science, health and technology, as well as in education, economics and social science.

WHAT YOU NEED TO KNOW

- A **variable** is a measure that may change.
- An **independent variable** is the variable in an experiment that we change or select in order to see what effect it has, or which changes naturally, like time or the weather. It is independent of what we are going to measure.
 - 'Independent' here does not mean independent of the researcher, because it is the thing that the researcher changes or allows to change.
 - An independent variable that changes naturally may need to be measured, e.g. the temperature of the environment or amount of rainfall. Often it is a characteristic of a location that we select, such as sea temperature. The fact that a variable is measured is therefore not the only thing that determines whether it is independent or dependent.
- A **dependent variable** 'depends' on the independent variable.
 - It is what we measure because we think it is changed by the independent variable. It could be something that is affected in an experiment or discovered in a survey, e.g. the number of organisms in locations that we select because those locations have differences in the independent variable.
 - The dependent variable reacts to the independent variable.
- The independent variable is the cause and the dependent variable is the effect.
- When graphing, the independent variable is placed on the horizontal (x -axis) and the dependent variable is placed on the vertical (y -axis).
- A line of best fit could be straight (the relationship is **linear**) or curved (the relationship is **non-linear**). In this course we will deal mainly with linear relationships.



Example 3 Identifying independent and dependent variables

Sigrid is interested in how different types of stress affects a patient's heart rate.

- a What is the independent variable?
- b What is the dependent variable?



WORKING

- a The independent variable is the type of stress, as this can be changed by the researcher.
- b The dependent variable is the heart rate, as it is dependent on the type of stress.

THINKING

- What can be changed by the researcher?
- What will be affected?
- What will be responding?
- What will be changed by the independent variable?

Exercise 6B

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a A variable is a measure that may _____.
 - b When graphing, the independent variable is placed on the _____ and the dependent variable is placed on the _____.
 - c The _____ is actually being measured and what is affected during the research.
 - d An _____ is the variable that is changed or allowed to change by the researcher.
 - e The values of both variables may _____; however, the difference is that the value of the independent variable is _____ by the researcher, while the value of the dependent variable only changes in _____ to the independent variable.
- 2 Define using everyday language:
 - a variable
 - b independent variable
 - c dependent variable

APPLICATIONS

Example 3

- 3 Alix is researching a new medication for asthma. She needs to alter the amount of medication given to see if the breathing rate changes.
 - a What could be the independent variable? Give a reason.
 - b What could be the dependent variable? Give a reason.



- 4 Michael is wanting to see how much fertiliser is required to help his plants produce more beans.
 - a What could be the independent variable? Give a reason.
 - b What could be the dependent variable? Give a reason.
- 5 Sonia is studying how rising ocean temperatures are impacting on the amount of coral in the Ningaloo Reef.
 - a What could be the independent variable? Give a reason.
 - b What could be the dependent variable? Give a reason.

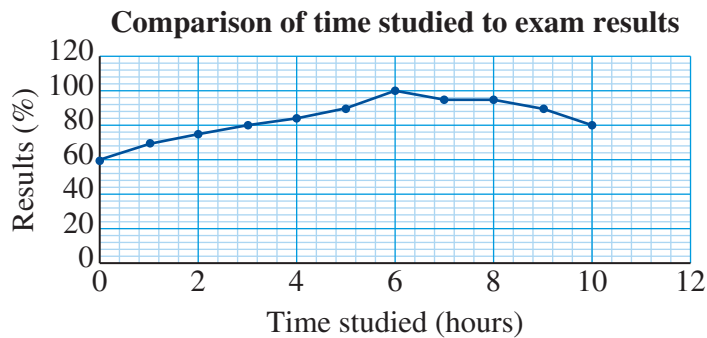
Hint The independent variable is the variable that is not affected by the other variable in the experiment.

- 6** Noah is researching how classical music can increase children's reading ability. He sets a variety of students to listen to classical music for different times and then collects their reading results.

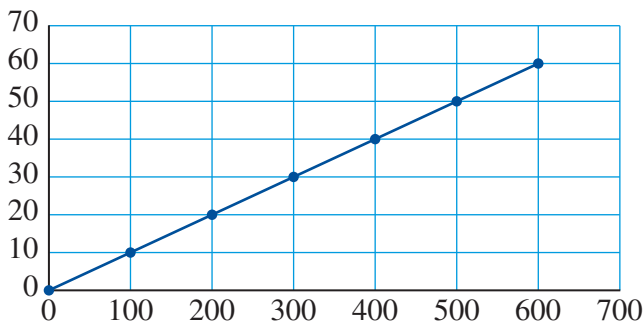
- a** What could be the independent variable? Give a reason.
b What could be the dependent variable? Give a reason.

- 7** The graph shows how the number of hours a group of students studied affected their exam results.

- a** What could be the independent variable? Give a reason.
b What could be the dependent variable? Give a reason.



- 8** Kaity has been delivering pamphlets to gain some extra pay. She graphed the number of pamphlets delivered and her pay. However, she has lost her titles and labels from her graph and cannot remember which is the dependent variable and which is the independent variable.



- a** What could be the independent variable? Give a label.
b What could be the dependent variable? Give a label.

6C Determining the line of best fit

LEARNING GOALS

- Determine the line of best fit by eye
- Use technology to determine and create the line of best fit

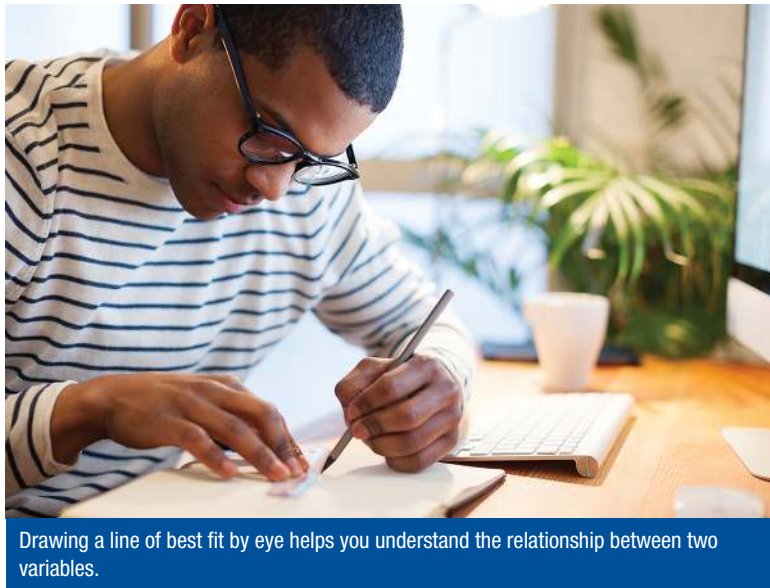
Why is it essential to be able to determine the line of best fit by eye?

The line of best fit is used to examine the relationship between two variables. It is a straight line drawn so there is an approximately equal number of points above and below the line on a scatterplot. The line of best fit will then tell us if the relationship between the variables is proportional; that is, if a change in the independent variable causes a proportional change in the dependent variable.

Learning how to find the line of best fit by eye, with the help only of a ruler, gives you an understanding of what a line of best fit is and what it means. The most accurate way to find the line of best fit is to enter the data into your calculator or computer and let the software do the work.

WHAT YOU NEED TO KNOW

- In surveys and experiments, most bivariate data collected results in a **scatterplot** rather than a neat line graph. This is because in the real world it can be hard to get accurate measurements and hard to prevent other factors from affecting your measurements.
- When graphing, the **independent** variable is placed on the x -axis and the **dependent** variable is placed on the y -axis.
- A **line of best fit** (or trend line or regression line) is a straight line that best represents the data on a scatterplot.
 - When drawing by eye, a straight line is drawn through the mean, balancing an approximately equal number of points above and below the line. It may pass through some of the points or none of the points on the graph.
 - The line of best fit shows the general direction of the change in the dependent variable in response to change in the independent variable. It is a way of removing the influence of all those other factors or random effects that also influence each measurement of the dependent variable.
 - When drawing a line of best fit by eye, start by placing your ruler on its edge in the position you think the line should be, so you can easily see the dots on either side. Mark the position of both ends of the line, then place the ruler flat to draw the complete line so that it is straight.



Drawing a line of best fit by eye helps you understand the relationship between two variables.



Example 4 Drawing the line of best fit by eye

Liv is working in a café and has suspected that there was a relationship between the outside temperature and her sales of hot chocolate. She has recorded the data in a table.

Outside temperature °C	20	18	10	11	13	10	11	8	9	4	2	1
Number of hot chocolates sold	0	1	8	6	7	7	10	13	14	16	18	20



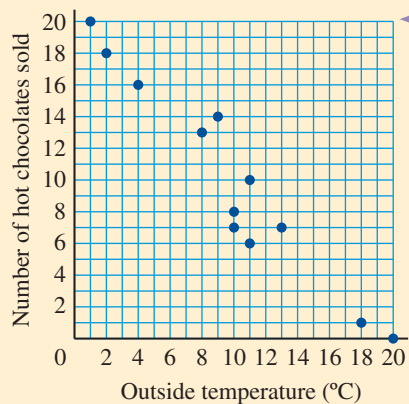
- a** Determine the independent and dependent variables.
- b** Draw a scatterplot by hand.
- c**
 - i** Calculate the mean ($\bar{x}$) of the x -axis.
 - ii** Calculate the mean ($\bar{y}$) of the y -axis.
- d** Determine the line of best fit by eye and draw in the line.

WORKING

- a** Independent variable is Outside temperature, as it is what is changing naturally in the environment and is not affected by sales of hot chocolate.
Dependent variable is Number of hot chocolates sold, as that is what is expected to change when the temperature changes.

THINKING

- ←... Determine the independent variable (thing that is controlled by researcher or environment) and dependent variable (thing that is predicted to change based on the independent variable).

b

Use grid paper.
Place the independent variable on the x -axis.
Place the dependent variable on the y -axis.
Plot the points.

c i

$$\bar{x} = \frac{20 + 18 + 10 + 11 + 13 + 10 + 11 + 8 + 9 + 4 + 2 + 1}{12}$$

$$\bar{x} = 9.75$$

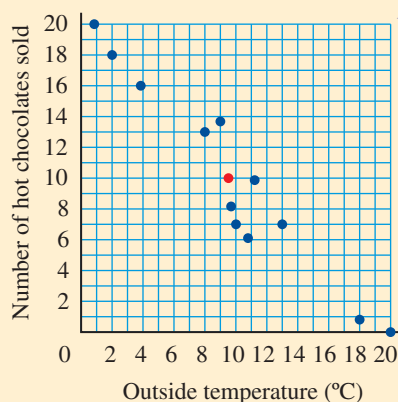
Find the mean ($\bar{x}$) of the x -axis (Outside temperature).

ii

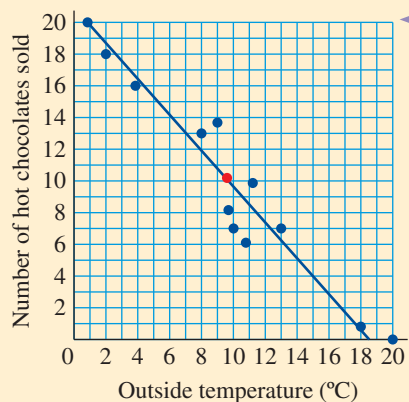
$$\bar{y} = \frac{0 + 1 + 8 + 6 + 7 + 7 + 10 + 13 + 14 + 16 + 18 + 20}{12}$$

$$\bar{y} = 10$$

Find the mean ($\bar{y}$) of the y -axis (Number of hot chocolates sold).



Plot the mean (9.75, 10).

d

Using your ruler, draw a line through the mean in the same directions as most of the data points. Ensure that as many data points are above the line as below it.



Example 5 Using a spreadsheet to create the line of best fit

Nate has noticed the relationship between the age of a person and the amount of emails they receive per day.

Age (years)	50	64	23	25	24	72	26	47	60	30	70	28
Number of emails per day	14	10	28	27	24	5	27	15	7	28	7	26

Use a spreadsheet to:

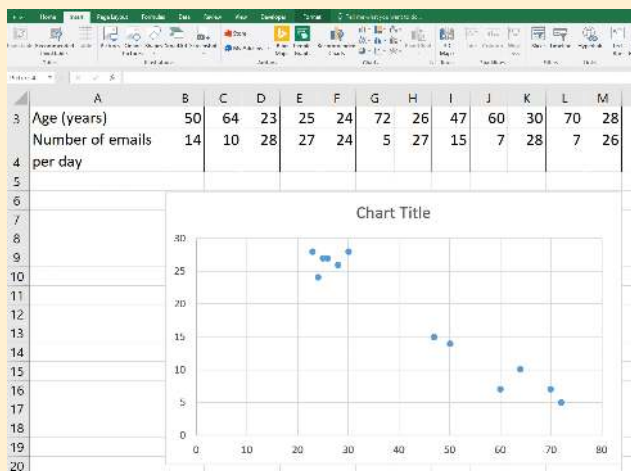
- graph the data in a scatterplot
- create the line of best fit.

WORKING

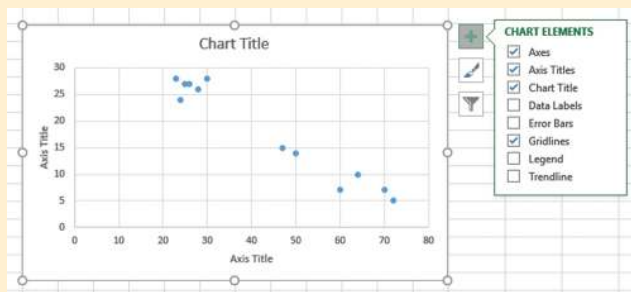
a

	A	B	C	D	E	F	G	H	I	J	K	L	M
2													
3	Age (years)	50	64	23	25	24	72	26	47	60	30	70	28
4	Number of emails per day	14	10	28	27	24	5	27	15	7	28	7	26
5													

Type data into spreadsheet cells.

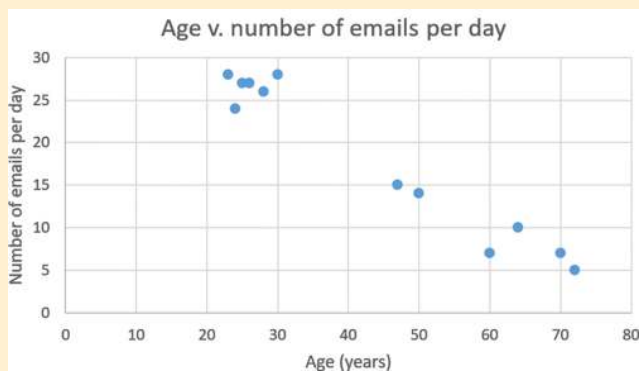


Select the data.
Click on the Insert tab > Charts > Scatter.



Click on chart to bring up the icon.
Click Axis Titles.

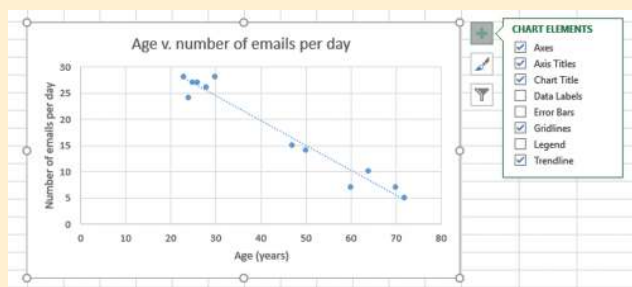
WORKING



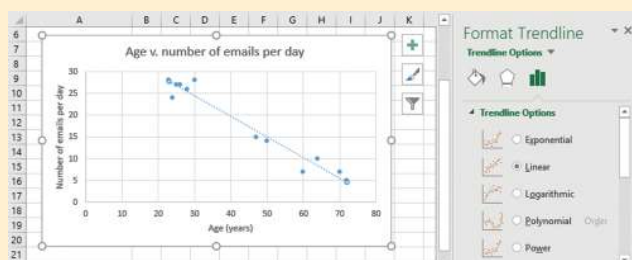
THINKING

←... Insert titles.

b



←... To find and create the line of best fit, click on chart to bring up the  icon. Click Trendline.



←... Right click the trendline to open Format Trendline. Adjust the settings in the Format Trendline pane to suit your requirements. Copy and Paste your graph to your report.



Desmos Activity: See the Interactive Textbook for an example and activity based on using the Desmos graphing calculator to fit a line to a scatterplot.

Exercise 6C

Note: It is suggested in questions 3 and 4 that the line of best fit be done by eye. The rest of the questions in the exercise could be done by eye, with a spreadsheet, with Desmos, or with another technology. Check with your teacher.

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a The line of best fit is used to examine the _____ between two variables.
 - b A line of best fit is a _____ line that best represents the data on a _____ plot.
 - c The line of best fit is also called the _____ line or the _____ line.
 - d The line of best fit is intended to _____ the _____ of the data.
 - e When graphing, the _____ variable is placed on the x -axis and the _____ variable is on the y -axis.
 - f The line of best fit is intended to highlight the _____ between the _____ and represent the _____ of the data.
- 2 Using everyday language for each of the following.
 - a Explain the process of drawing a scatterplot by hand.
 - b Explain the process of finding the line of best fit by eye.

APPLICATIONS

Example 4

- 3 Felicity is studying a type of soft coral that is often found in cooler waters, and she has collected the following data in an attempt to find out if the coral is more plentiful in cooler southern seas compared to tropical waters.

Ocean temp ($^{\circ}\text{C}$)	19	18	20	27	30	17	27	31	24
Number of corals per 100 m^2	140	160	122	40	12	154	19	2	80

- a Determine independent and dependent variables.
- b Create a scatterplot by hand.
- c Calculate the mean of the ocean temperature and the mean number of soft corals and plot the coordinates on the graph.
- d Draw the line of best fit by eye.



- 4 Sonia is training for a marathon and is recording her latest times.

Time (min)	25	40	35	37	90	40	115	35	95
Distance (km)	6	10	15	10	20	15	25	10	20

- Determine independent and dependent variables.
- Create a scatterplot by hand.
- Calculate the mean of time taken and the mean distance run and plot the coordinates on the graph.
- Draw the line of best fit by eye.



Spreadsheet Ex6CQ5–7 The data is available in a spreadsheet in the Interactive Textbook.

See the note at the start of this exercise about which method to use for these questions. The data is available in a spreadsheet in the Interactive Textbook.

- 5 David is wanting to see how much fertiliser helps his plants produce the most beans.

Amount of fertiliser (mL)	10	10	15	30	5	50	2	55	60
Number of beans per plant	6	8	9	18	5	28	3	32	40

- Determine independent and dependent variables.
- Create a scatterplot of the data.
- Calculate the mean of the amount of fertiliser and the mean of the number of beans and plot the coordinates.
- Draw the line of best fit.

Hint Remember to have approximately half of the points above and half of the points below the line of best fit.

Example 5

- 6 Deb is wanting to see if there is a relationship between a group of people's annual salary and how many years of work experience they had. She has collected the following data.

Years of experience	15	45	35	25	5	30	30	40	45	20	3	3	5	30	25
Annual salary ('000)	25	80	100	30	80	55	85	110	90	80	25	20	30	100	75

- Create a scatterplot of the data.
- Create the line of best fit.

- 7** Kay has been a nurse at a health centre for over 20 years and has noticed a relationship between children's height at 2 years and at 18 years. She has collected the following data.
- a** Create a scatterplot of the data.
 - b** Create the line of best fit.



2 yo height (cm)	80	83	87	81	83	88	84	85	86	83	85	80	88
18 yo height (cm)	157	167	171	160	162	172	167	170	170	164	165	159	171

- 8** Char volunteers as a surf life saver and has noticed a relationship between the average temperature and the amount of people on the beach. She collects the data below with the thought that it may help in planning how many lifesavers to have on duty according to the temperature forecast for each day.
- a** Create a scatterplot of the data.
 - b** Create the line of best fit.

Average temperature (°C)	23	28	25	34	29	35	27	37	26	29	28	30	36
Number of people on the beach	70	85	74	87	87	105	80	112	78	88	84	92	110



- 9 Mel is a personal trainer and has noticed a relationship between the hours of exercise people do and their resting heart rate (BPM).

a Create a scatterplot of the data.

b Create the line of best fit.

Exercise (hours per week)	0.2	0.4	2.3	9	0.5	0.7	6	8	4	1	1.2	1.5	2	8.5	5	3
Resting heart rate (BPM)	81	80	69	45	81	78	55	47	64	75	71	70	72	45	61	66

- 10 Leanne's writing a report about the relationship between a city's height (m) above sea level and their average yearly rainfall (mm).

a Create a scatterplot of the data.

b Create the line of best fit.

City	Average yearly rainfall (mm)	Height above sea level (m)
Armidale	1000	80
Orange	800	230
Bathurst	180	670
Goulburn	170	640
Toowoomba	275	600
Canberra	280	580
Alice Springs	300	580
Ballarat	690	450
Tamworth	680	400
Kalgoorlie	265	380

6D Interpreting relationships between variables

LEARNING GOAL

- Interpret relationships in terms of the variables

Why is it essential to be able to interpret relationships between variables?

Graphing bivariate data in scatterplot and fitting a line to it is just a display. Saying what it means, in other words interpreting the relationship between variables, is an integral part. This is how you draw conclusions from a bivariate dataset. Without an understanding of this, you will not be able to understand your research and could present the incorrect results from your data.

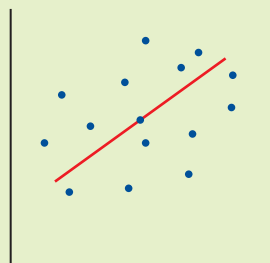
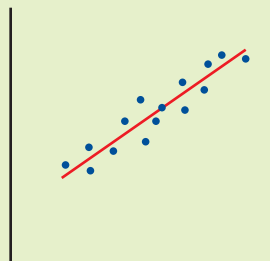


Interpreting the relationship between variables ensures that you present the correct results.

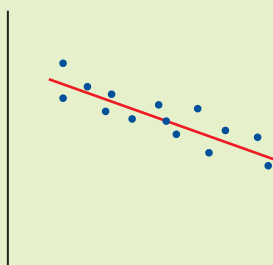
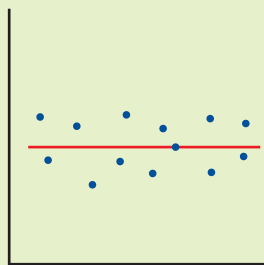
WHAT YOU NEED TO KNOW

- **Regression** analysis is the overall term for the process of identifying the relationships between an independent variable and a dependent variable using a scatterplot and line of best fit.
 - Regression analysis helps us understand how the typical value of the dependent variable changes when the independent variable is altered.

- A **correlation** is a measure of the strength of the linear relationship between two variables.
 - The correlations can be described as strong, moderate, weak or none depending on how close the points are to the line of best fit.
 - A positive correlation is a relationship between two variables that moves either positively or negatively together.
 - As one variable increases, the other variable increases.
 - As one variable decreases, the other variable decreases.
- When looking for a relationship, focus on:
 - the closer the points cluster to the line of best fit, the stronger the relationship that exists between the two variables, and there is most likely a correlation (i.e. **strong correlation**)
 - the further the points are from the line of best fit, the weaker the relationship that exists between the two variables, and therefore the less likely it is that there is a correlation (i.e. **weak correlation**).

**Positive correlation**

- When the points rise from lower left to upper right, as in the graphs above, there is a **positive correlation** (if x increases, y increases).
- When the points fall from upper left to lower right, there is a **negative correlation** (if x increases, y decreases).
- When the line of best fit is horizontal, there is **no correlation**.

**Negative correlation****No correlation**

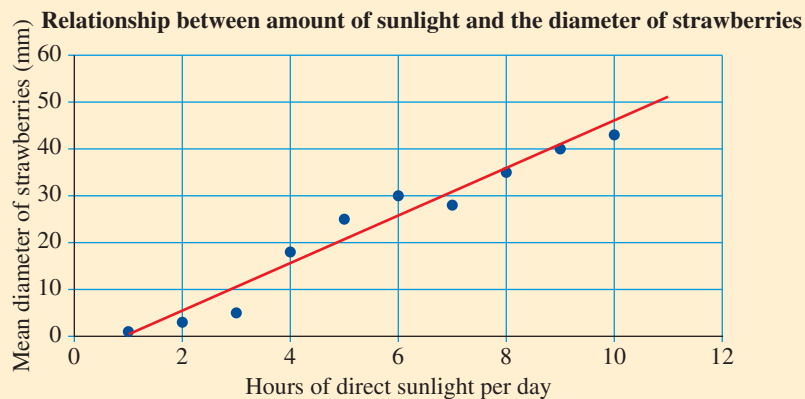


Example 6 Interpreting relationships in terms of the variables

Erica owns a strawberry farm. She is collecting data to see if there is a relationship between the amount of direct sunlight and the diameter of her strawberries. She has exposed different plants to different hours of direct sunlight. All plants are then placed in the same shade to make up to a total of 12 hours of daylight. She gathered the following data after measuring the strawberries and taking the mean diameter.

Amount of direct sunlight per day (hours)	1	2	3	4	5	6	7	8	9	10
Mean diameter of strawberries (mm)	1	3	5	18	25	30	28	35	40	43

She has created this scatterplot and line of best fit for the data.



- Determine which is the independent variable and which is the dependent variable.
- Describe any correlations in terms of positive or negative, linear or non-linear, and strong, moderate, weak or none.
- Interpret the relationship between the amount of sunlight and the diameter of the strawberries and explain what it means for the strawberry farm.

... Continued

WORKING

- a** The independent variable is the amount of direct sunlight, as Erica changes it for the experiment. The dependent variable is the diameter of the strawberries, as the amount of sunlight can change the growth of the strawberries.
- b** The correlation is positive, linear and strong.
- c** There is a strong relationship between the amount of sunlight and the diameter of the strawberries. As the amount of sunlight increases, the diameter of the strawberries tends to increase. More sunlight typically result in larger strawberries to grow. If Erica wants larger strawberries, she should provide more sunlight to her produce.

THINKING

Which variable possibly causes a change in the other variable?

The line of best fit goes up from left to right, therefore it is positive. The points follow a definite line, therefore it is linear. The points are close to the line of best fit, therefore there is a strong correlation.

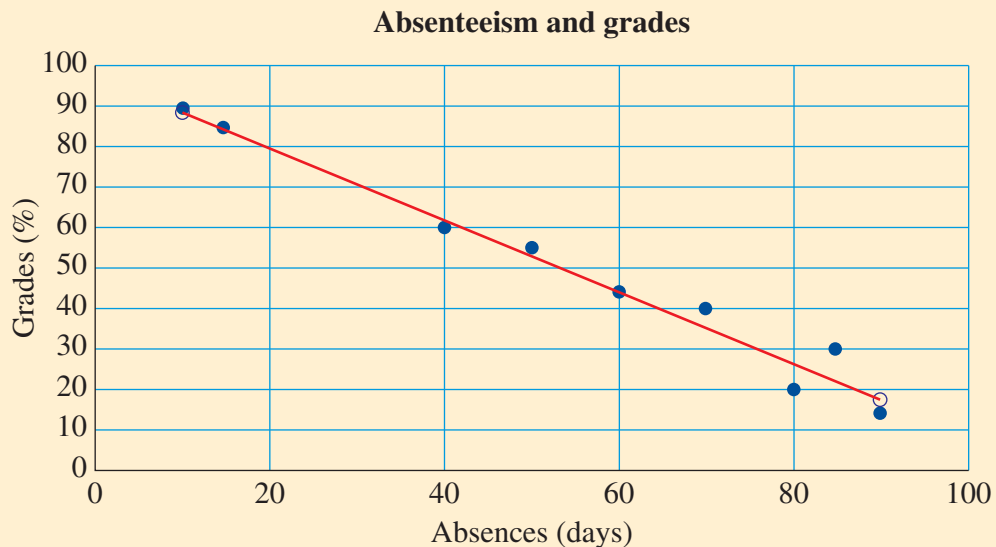
As there is a strong correlation, there is a relationship between the variables, which can be interpreted as direct sunlight causing larger strawberries to grow.





Example 7 Interpreting relationships in terms of the variables

Ty is a deputy principal of a school and is noticing a relationship between student absences and their grades. He has created the following scatterplot and line of best fit from the data.



- a** Determine which is the independent variable and which is the dependent variable.
- b** Describe any correlations in terms of positive or negative, linear or non-linear, and strong, moderate, weak or none.
- c** Interpret the relationship between the number of days absent and the grades achieved.
- d** Explain how the deputy principal might use this information.

WORKING

- a** The independent variable is the number of days absent, as it is not affected by the grades.

The dependent variable is the grades, as it is reasonable to expect that decreasing the time spent learning through absences will lead to a decrease in grades.

THINKING

◀.... What is the cause? See the x -axis as a hint.

◀..... What is the effect? See the y -axis as a hint.

- b** The correlation is negative, linear and strong. ◀.... The line of best fit goes down from left to right, therefore it is negative. The points follow a definite line, therefore it is linear. The points are close to the line of best fit, therefore there is a strong correlation.
- c** There is a strong relationship between the amount of days absent and the students' grades. The correlation is negative, as when days absent increase, the grades decrease. ◀..... As there is a strong correlation, there is a relationship between the variables.
- d** The deputy principal could use the information to show students and parents that absenteeism strongly affects grades. ◀.... How can this information assist the deputy principal?



Exercise 6D

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a _____ analysis is the process for identifying the _____ between a dependent variable and an independent variable.
 - b A correlation is a measure of the _____ of the linear relationship between two variables.
 - c A positive correlation is a relationship between two variables that moves either _____ or _____.
 - d A negative correlation is a relationship between two variables where one variable _____ as the other _____.

APPLICATIONS

Example 6, 7

- 2 Lee organises a rodeo once a year and is wondering if there is a relationship between the age of the rodeo riders and the amount of injuries at the rodeo.

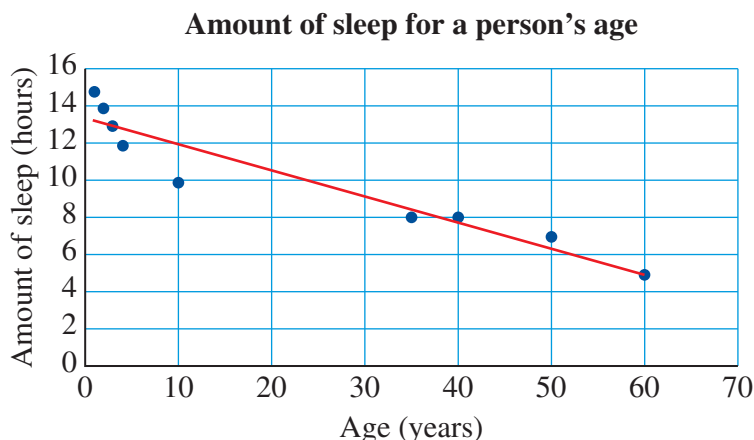


Age of riders (years)	35	30	25	15	20	23	32
Number of injuries	18	14	13	8	12	14	16

- a Determine which is the dependent variable and which is the independent variable.
- b Create a scatterplot.
- c Determine the line of best fit.
- d Describe any correlations in terms of positive or negative, linear or non-linear, and strong, moderate, weak or none.
- e Interpret relationships between the age of the rider and the injuries sustained.
- f Explain how a rodeo rider might use this data.

Hint When the points rise in a line going from lower left to upper right, there is a positive correlation (if x increases, y increases).

- 3** Eleanora has collected data to see if there is a relationship between a person's age and the number of hours of sleep they have each day. She has created this scatterplot and line of best fit from the data.



- a** Determine which is the dependent variable and which is the independent variable.

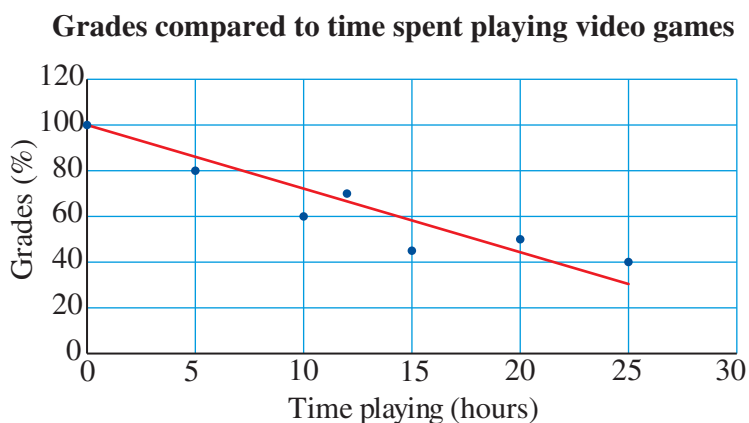
- b** Describe any correlations in terms of positive or negative, linear or non-linear, and strong, moderate, weak or none.

- c** Interpret the relationship between the age and amount of sleep.

- d** Explain how this information might be used.

Hint When the points fall in a line from upper left to lower right, there is a negative correlation (if x increases, y decreases).

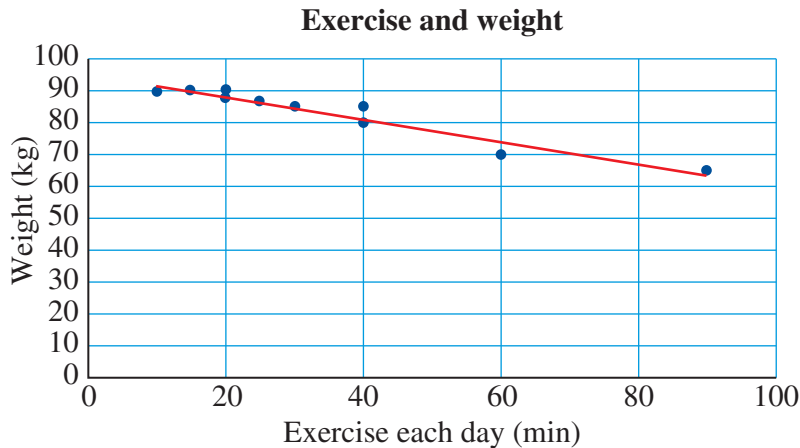
- 4** A teacher is trying to prove to his students that there is a relationship between hours spent playing video games and their grades. He has created this scatterplot and line of best fit from the data.



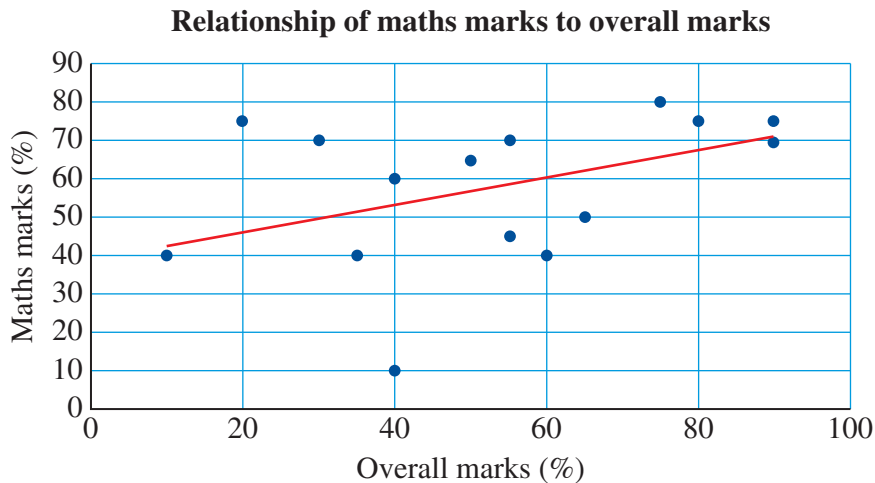
- a** Determine which is the dependent variable and which is the independent variable.

- b** Describe any correlations in terms of positive or negative, linear or non-linear, and strong, moderate, weak or none.

- c** Interpret relationships between time spent playing games and the grades achieved.
- d** Explain how the teacher could use this information.
- 5** A personal trainer is trying to persuade his clients to exercise more. He has created this scatterplot and line of best fit from the data he has gathered.

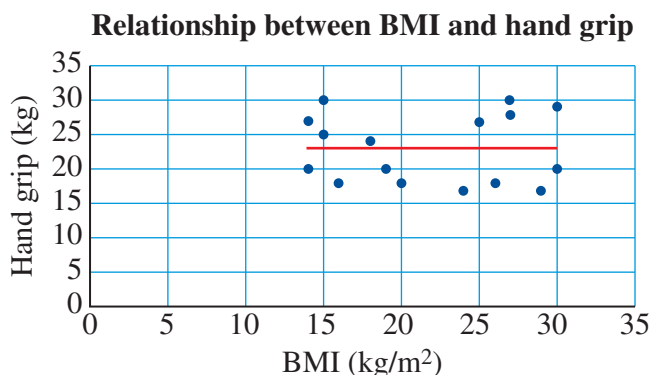


- a** Determine which is the dependent variable and which is the independent variable.
- b** Describe any correlations in terms of positive or negative, linear or non-linear, and strong, moderate, weak or none.
- c** Interpret the relationship between the hours spent exercising and weight.
- d** Explain how the personal trainer could use this information.
- 6** A maths teacher believes that her students' end-of-year marks in maths also relate to their overall marks in all subjects. She has created this scatterplot and line of best fit from data collected from her students.

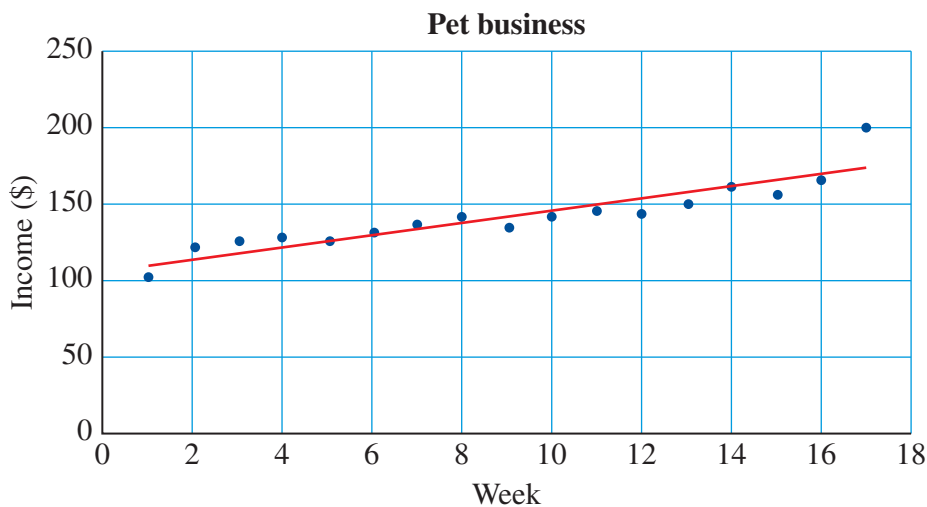


- a** Interpret the relationship between the overall marks and maths marks.
- b** Explain how the teacher could use this information.

- 7 A group of people were measured by a researcher to see if there is a relationship between hand grip strength and BMI. She has created this scatterplot and line of best fit from the data.

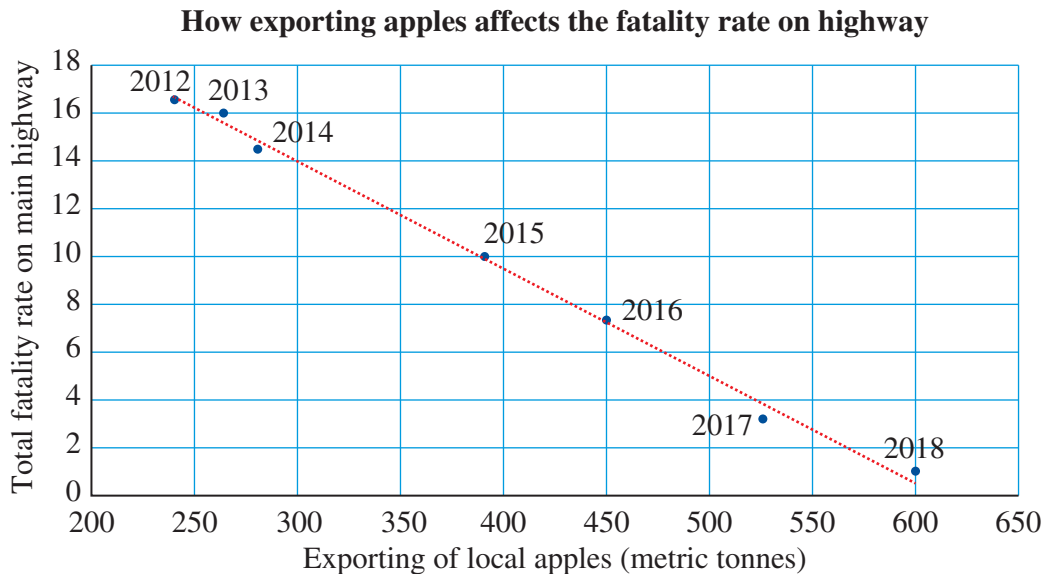


- a Interpret relationships between the BMI and hand grip.
- b Explain how this information could be used.
- 8 Will has started his own business, walking people's dogs. He is hoping to see strong positive relationship between each week of his business and his weekly income. He has created this scatterplot and line of best fit from data he has collected.



- a Interpret relationships between each week and income.
- b Explain how Will could use this information.

- 9 A Western Australian farmer has informed his local newspaper that he has discovered a relationship between exporting local apples and the fatality rate on their main highway each year. He has collected the following data to illustrate his theory.



- a Interpret relationships in terms of the two variables.
- b Explain what this means and justify whether or not you think the interpretation is valid.



6E Calculating the correlation coefficient using technology

LEARNING GOAL

- Use technology to determine the correlation coefficient (an indicator of the strength of linear association)

Why is it essential to be able to determine the correlation coefficient?

So far, we have determined whether two variables are correlated by the appearance of the line of best fit. Working mathematically and scientifically however requires working with numbers – in this case, a measurement of the size of the correlation. This is essential in research and development, in all subjects and fields.

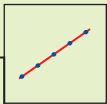
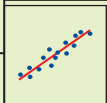
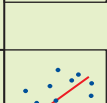
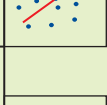
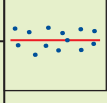
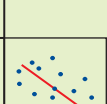
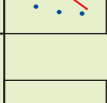
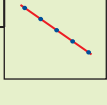


The correlation coefficient shows the degree to which points on a scatterplot lie on a line of best fit.

WHAT YOU NEED TO KNOW

- Correlation tells us about the strength of the linear relationship between two variables.
 - To measure correlation and the strength of the relationship we calculate the correlation coefficient.

- The **correlation coefficient** is based on the degree to which points on a scatterplot lie on a line of best fit.
 - The correlation coefficient has the symbol (r), which measures the strength and direction of a linear relationship between variables.
 - The value of r is always between $+1$ and -1 .
 - To interpret the correlation coefficient, use the following as a guide:

Value of correlation coefficient	Typical scatterplot	Interpretation
$+1$		1 : perfect positive correlation
0.75		0.75 to 0.99 : strong positive correlation
0.50		0.5 to 0.74 : moderate positive correlation
0.25		0.25 to 0.49 : moderate positive correlation
0		Above 0 up to 0.24 : weak or no positive correlation 0 : no correlation Below 0 down to -0.24 : weak or no negative correlation
-0.25		-0.25 to -0.49 : moderate negative correlation
-0.50		-0.5 to -0.74 : moderate negative correlation
-0.75		-0.75 to -0.99 : strong negative correlation
-1		-1 : perfect negative correlation

- To calculate the correlation coefficient, use a scientific calculator, a spreadsheet or other technology.



Example 8 Using technology (scientific calculator) to find the correlation coefficient

Dan is a used car salesman and needs to understand if there is a relationship between the value of cars and their age. He has collected the following data.

Age (years)	6	3	4	1	8	5	7	2	5
Value	13 000	21 000	18 000	29 800	9 000	15 700	11 000	25 000	15 300

- Use a scientific calculator to determine the correlation coefficient for the data showing the value of cars with respect to their age in years.
- Interpret the meaning of this value of the correlation coefficient (r).

WORKING

a



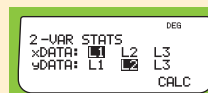
The correlation coefficient for this data is $r = -0.986$.

b $r = -0.986$

The correlation between the age of cars and their value has a very strong, negative relationship. This means that as the age of a car increases the value of a car decreases.

THINKING

Press to get to the tables to enter data.
 Press to clear away any old data.
 Type age of car in L1.
 Type value of car in L2.
 Press to get into 2-Variable Statistics:



Arrow down to CALC and press



Arrow down to find r (correlation).

Is the correlation positive/negative?
 Is the correlation strong/moderate/weak/none?
 The correlation is very strong, as it is very close to being -1 .



Calculator activity 6E Using a scientific calculator to find the correlation coefficient. See the Interactive Textbook for this activity.



Example 9 Using technology (spreadsheet) to determine the correlation coefficient

Sue is a nutritionist researching whether there is a relationship between eating chocolate and weight gain. The people in her trial ate the same basic healthy diet and followed the same exercise regime. They were given identical 200 g bars of chocolate to eat and their weight gain was recorded. She has collected the following data and calculated the mean of the weight gain.

No. of bars eaten per week	1	20	3	10	15	5	7	18	4
Mean weight gained per week (kg)	0.1	1.2	0.2	0.6	0.8	0.4	0.5	1	0.3

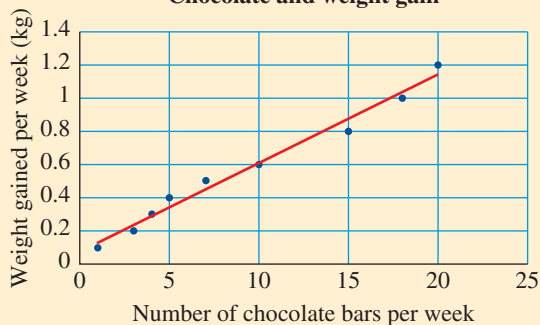
- Use a spreadsheet to create a scatterplot of the data, including a line of best fit.
- Use a spreadsheet to determine the correlation coefficient for the data showing relationship between the number of chocolate bars to weight gain.
- Interpret the meaning of this value of the correlation coefficient (r).



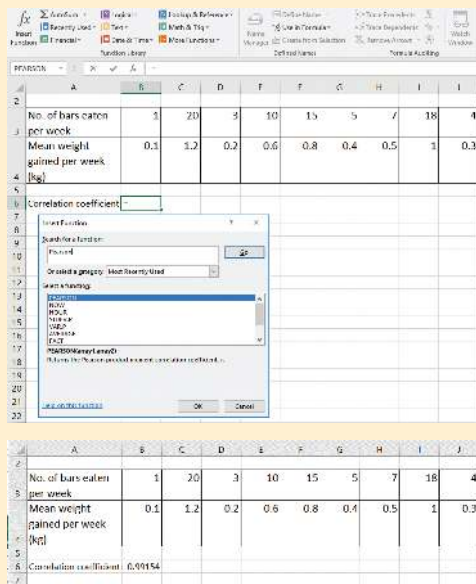
WORKING

a

Chocolate and weight gain



b



The correlation coefficient is 0.99.

c

$$r = +0.99$$

The correlation between the amount of chocolate bars eaten each week and weekly weight gain has a very strong, positive relationship. This means that as the number of chocolate bars increase, the amount of weight gain also increases.

THINKING

Draw a scatterplot to the data given in the table.

Insert a line of best fit to the scatterplot.

..... To find the correlation coefficient, select another cell.

Select Formulas

Insert Function

Write Pearson

(The correlation coefficient was originally called the Pearson Product Moment Correlation Coefficient, which is why we choose the formula PEARSON rather than correlation coefficient.)

Click OK.

Select the independent values into Array One.

Select the dependent values into Array Two.

Is the correlation positive/negative?

Is the correlation strong/moderate/weak/none?

The correlation is very strong, as it is very close to being +1.



Spreadsheet activity 6E Using a spreadsheet to find the correlation coefficient. See the Interactive Textbook for this activity.

Exercise 6E

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a Understanding how to find the correlation coefficient is essential as it enables the _____ in a relationship to be easily _____.
 - b To measure the _____ of the relationship, we calculate the correlation _____.
 - c The correlation coefficient has the symbol _____, which measures the _____ and _____ of a linear relationship between variables.
 - d The correlation coefficient can have values from _____ to _____:
 - i $+1 =$ _____
 - ii _____ = strong positive relationship
 - iii _____ = moderate positive relationship
 - iv 0.25 to $0.50 =$ _____
 - v _____ = no linear relationship
 - vi _____ = weak negative relationship
 - vii -0.50 to $-0.75 =$ _____
 - viii _____ = strong negative relationship
 - ix _____ = perfect negative relationship
- 2 Define using everyday language:
 - a correlation
 - b correlation coefficient

APPLICATIONS

Example 8

- 3 Charlie is wondering if there is a relationship between time spent studying for assessment and the amount of errors she makes in her assessments. She has collected the following data.

Hint A lot of people are confused by the word *negative*, (e.g. strong negative relationship) it just means that as one variable increases, the other decreases. It is not negative, as in a bad thing.

Assessment study time (min)	52	24	26	45	39	28	60
Assessment errors	3	16	15	7	10	14	1

- a Use a scientific calculator to determine the correlation coefficient for the data to 3 decimal places.
- b Interpret the meaning of this value of the correlation coefficient (r).

- 4 A tailor has been told that there is a relationship between the height and weight of men. He has collected the following data.

Hint Remember to always clear away any old data.

Men's height (m)	1.5	1.2	1.7	2	1.4	1.8	1.3	1.7	1.6
Men's weight (kg)	77	90	82	95	92	106	105	94	68

- a Use a scientific calculator to determine the correlation coefficient for the data.
b Interpret the meaning of this value of the correlation coefficient (r).



Example 9

- 5 A doctor has collected data on the age of her patients with a certain condition and number of sick days they have been forced to take in the past year.

Age of patients (years)	25	60	38	24	45	58	30	40	65	40
Number of days sick	20	5	15	20	10	7	5	12	8	18

- a Use a spreadsheet to create a scatterplot of the data, including a line of best fit.
b Use a spreadsheet to determine the correlation coefficient for the data.
c Interpret the meaning of this value of the correlation coefficient (r).

Example 9

- 6** Deb has noticed that whenever she goes in the sun, the following day she has a better sense of wellbeing. She has collected the following data to see if there is a relationship. Deb measured wellbeing by a scale of 1, with 10 being 'feeling excellent' and 1 being 'feeling low'.



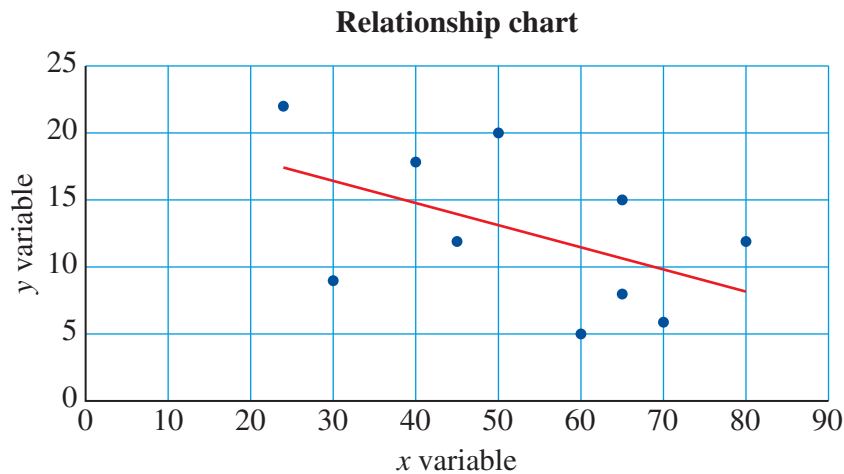
Feeling of wellbeing (1–10)	10	2	5	7	4	3	9	8	10	9
Time in the sun each day (min)	20	5	7	15	2	5	18	16	20	18

- a** Use a spreadsheet to create a scatterplot of the data, including a line of best fit.
- b** Use a spreadsheet to determine the correlation coefficient for the data showing whether there is a relationship between being in the sun and wellbeing.
- c** Interpret the meaning of this value of the correlation coefficient (r).

Hint Select the independent values into Array One. Select the dependent values into Array Two.



- 7** The following scatterplot and table has a correlation coefficient for the variables of -0.40 .



x variable	50	60	65	24	45	70	30	80	65	40
y variable	0	5	15	22	12	6	9	12	8	18

Using a spreadsheet, change some of the x and y variables to produce a set of data where the correlation coefficient is:

- a** strong positive correlation
- b** weak negative correlation
- c** moderate positive correlation

6F Making predictions

LEARNING GOALS

- Use the line of best fit to make predictions, both by interpolation and extrapolation
- Recognise the dangers of extrapolation

Why is it essential to know how to use the line of best fit to make predictions?

Being able to interpret the line of best fit and measure the correlation between two variables is only the starting point in research and development. The main purpose of the line is to determine its equation and to use it to predict values that are not in the dataset. It enables prediction of future values within the range of our data and outside the range. This is the real purpose of the topic covered in this chapter.



The purpose of creating a line of best fit for data and finding its equation is to be able to make predictions from it.

WHAT YOU NEED TO KNOW

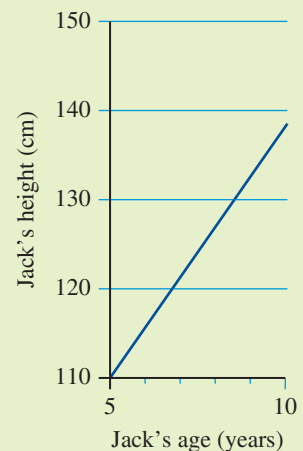
- A **prediction** in this context is a forecast of the value of the dependent variable based on a value of the independent variable, even though neither value was in the original dataset.
- A **line of best fit** enables a prediction about the data to be formed.
- The **correlation coefficient** is a measurement of the relationship between two variables, as well as the strength of that relationship. A coefficient of either $+1$ or -1 indicates the strongest relationship, and 0 indicates no relationship.
- If the line of best fit is a straight line, its equation has the form $y = mx + c$.
 - x represents the independent variable and y represents the dependent variable
 - m represents the gradient (slope of the line)
 - Gradient = $\frac{\text{rise}}{\text{run}} = \frac{\text{amount the line goes up}}{\text{amount the line goes across}}$
 - c represents the y -intercept (the point where the line crosses the y -axis).

- **Interpolation** is an estimation of a value on the line of best fit. The accuracy of prediction by interpolation generally depends on the strength of the correlation: stronger correlations mean more accurate predictions.

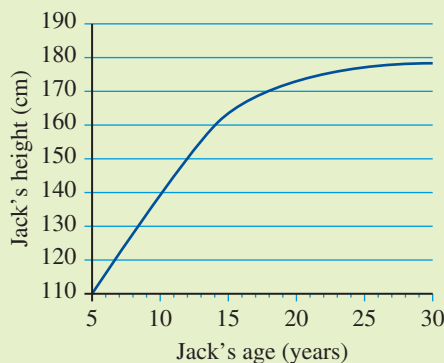
- **Extrapolation** is an estimation of a value based on extending the line of best fit.

- To extrapolate is to infer something that is not within the range of the particular dataset used to determine the line of best fit.

- The accuracy of a prediction made by extrapolation is also affected by the strength of the correlation, as with interpolation. In addition, the danger of extrapolation is that there may be natural reasons why the straight line of best fit is not valid beyond the range of the dataset. Consider this graph of Jack's height between the ages of 5 and 10 years old. Jack grew 30 cm in five years so if that is extrapolated to age 30, he would be 260 cm tall!



- The second graph shows the shape of Jack's age/height chart for ages 5 to 30 (not to the same scale), which explains why you cannot extrapolate from his growth between the ages of 5 and 10 years to predict his height at age 30. The line of best fit is only linear for part of the range.

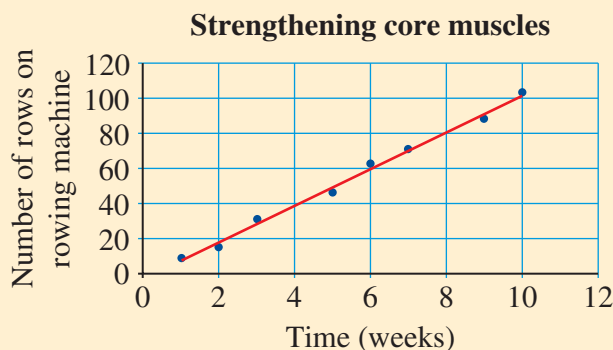


- Where the independent variable is time, for example the years when something had a particular monetary value, the main danger of extrapolating is that unknown future events could affect the future value.



Example 10 Using the line of best fit to make predictions, both by interpolation and extrapolation

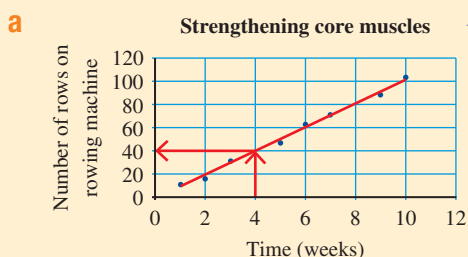
Bec has injured her core muscles. To gain strength she has been using a rowing machine. The line of best fit shows her progress after 10 weeks.



- Bec has lost her record of how many rows she completed at 4 weeks. Using interpolation, estimate how many rows she could have completed.
- Bec has also forgotten which week she was able to row 80 times. Using interpolation, estimate the week she rowed 80 times.
- Determine the equation of the line of best fit in the form $y = mx + c$.
- Using extrapolation with the equation of the line of best fit, predict how many rows Bec will be able to achieve by week 15.



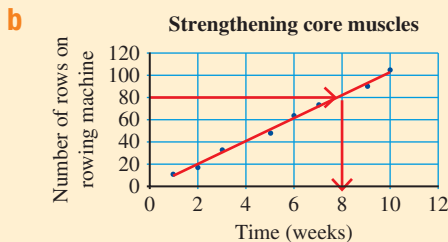
WORKING



Bec was able to row approximately 40 times in the fourth week.

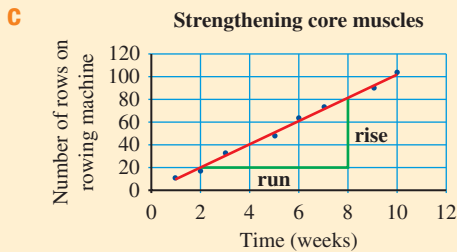
THINKING

Find 4 weeks on the x -axis, go up to the line of best fit, and across to the y -axis.



Bec was able to row 80 times around week 8.

Find 80 on the y-axis, go across to the line of best fit, and down to the x-axis.



The equation for the line of best fit is $y = mx + c$, where m is the gradient and c is the y-intercept.

Gradient:

$$\begin{aligned} \text{rise} &= 80 - 20 \\ &= 60 \end{aligned}$$

$$\begin{aligned} \text{run} &= 8 - 2 \\ &= 6 \end{aligned}$$

$$m = \frac{60}{6} = 10$$

$$y = 10x + c$$

y-intercept:

$$80 = 10 \times 8 + c$$

$$80 = 80 + c$$

$$c = 80 - 80$$

$$c = 0$$

Therefore, the equation for this line of best fit is:

$$y = 10x + 0$$

which can be shortened to $y = 10x$

Calculate the gradient:

$$\begin{aligned} \text{Gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{\text{amount the line goes up}}{\text{amount the line goes across}} \end{aligned}$$

Choose two sections on the line of best fit that have whole number coordinates.

Draw a right-angled triangle to measure the *rise* and the *run*.

Substitute 10 for m in the equation.

As we are unable to see where the line crosses the y-axis, we will need to substitute values of x , y and m to find the value of c . Then rearrange to make c the subject of the equation and evaluate for c .

d

$$\begin{aligned} y &= 10x \\ y &= 10 \times 15 \\ y &= 150 \end{aligned}$$

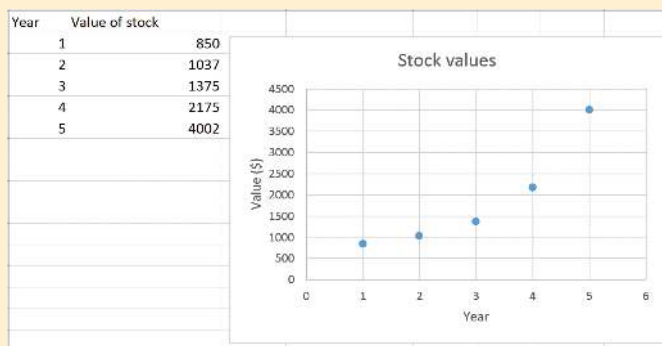
Bec will be able to row 150 times by week 15.

Substitute 15 for x in the equation from part c.



Example 11 Recognising the dangers of extrapolation

A stock broker was watching the increase of a particular company's stock. He put the past five year's values in a table and also created a graph.



He then predicted the stock values in the next two years. Determine the dangers of the predictions.

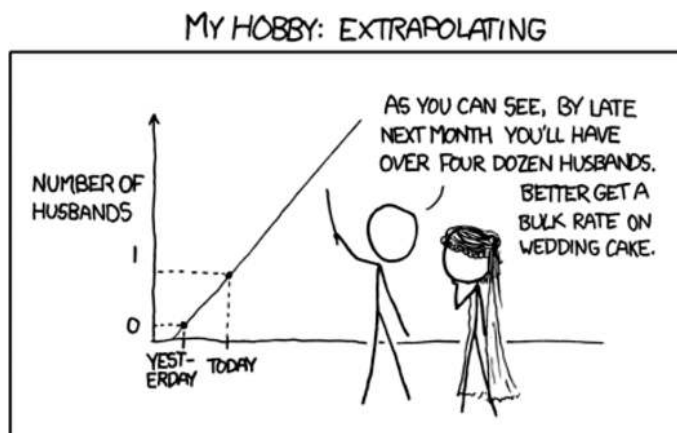
WORKING

There is no information to tell us why the stock was increasing in value over the past five years, so there is no way of knowing if the conditions for the increase will continue into the future. The danger with this prediction is that the stock market is not stable and future events are likely to be different from past history. Stock prices cannot be relied upon to continually increase.

THINKING

◀.... Is the stock market stable, i.e. could unknown future events affect the value of the stock in the future?

Note: The actual stock that this example was taken from did fall. Their 8th year value was \$2354; and their 9th year was \$1840.



Source: xkcd.com

Exercise 6F

FUNDAMENTALS

- Determine the missing words in the following sentences.
 - The equation for the line of best fit enables _____ to be made both _____ the plot and _____ of the plot.
 - The equation of a straight line is _____.
 - m represents the _____ (_____ of the line).
 - c represents the _____ (the point where the line crosses the _____).
 - _____ is an estimation of a value within two given points on the line of best fit.
 - _____ is an estimation of a value based on extending the line of best fit.
- Define using everyday language:
 - interpolation
 - extrapolation
- Determine which dataset A, B or C would be the most reliable based on the correlations stated. Explain your response.

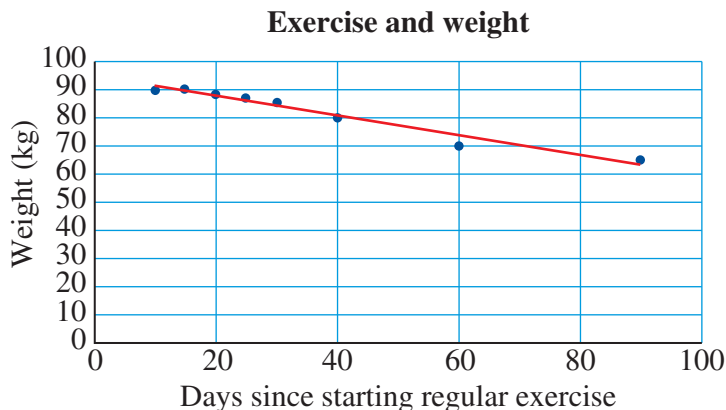
A $r = -0.94$ **B** moderate positive **C** $r = 0.12$
- Determine which dataset A, B or C would be the least reliable based on the correlations stated. Explain your response.

A weak linear negative **B** $r = 0.75$ **C** strong positive

APPLICATIONS

Example 10, 11

- Mindy is a 20-year-old who has been measuring her weight and the days since she started exercising regularly. She has recorded most of her measurements, but she has lost a couple of entries.



- a** Using interpolation, estimate what her weight was when she had been exercising for 50 days.

Hint Draw a right-angled triangle to measure the rise and the run.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{\text{amount the line goes up}}{\text{amount the line goes across}}$$

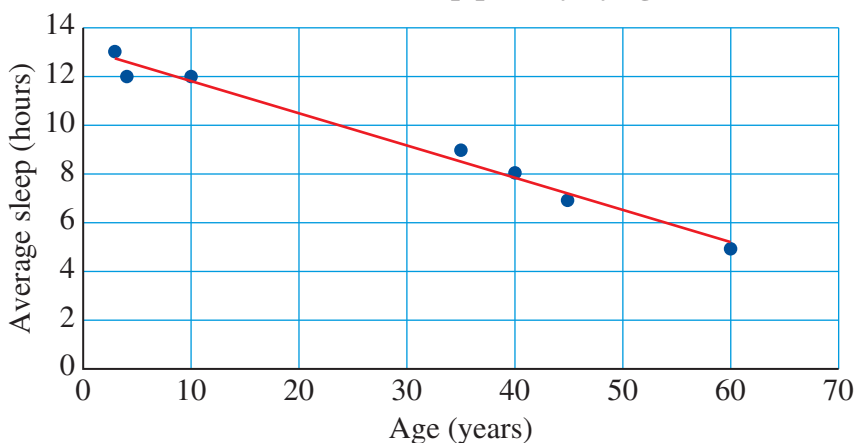
- b** Using interpolation, estimate how many days she had been exercising when she had reached 70 kg.
- c** Without technology, determine the equation of the line of best fit using $y = mc + c$.
- d** Use the equation of the line of best fit to predict what Mindy's weight will be after she has exercised for 120 days



- e** Explain how reliable is the prediction for part **d**.

- 6** Kent is researching the relationship between sleep and age, however he has lost a section of his data.

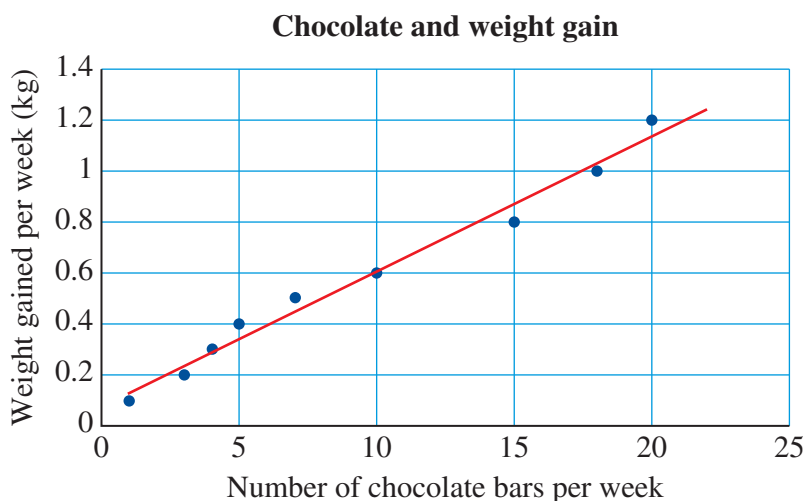
Amount of sleep per day by age



- a** Using interpolation, estimate the average amount of sleep required for someone who is 25 years old.
- b** Using interpolation, estimate on average the age of people who have 6 hours sleep per day.
- c** Without technology, determine the equation of the line of best fit using $y = mc + c$.
- d** Use the equation of the line of best fit to predict what how much sleep on average an eighty-year-old has.
- e** Explain how reliable is the prediction for part **d**.



- 7 In an experiment, a group of people have had the same diet and exercise but have been given different numbers of chocolate bars per week to eat. Their weight gain per week has been recorded and graphed against the number of chocolate bars eaten per week.



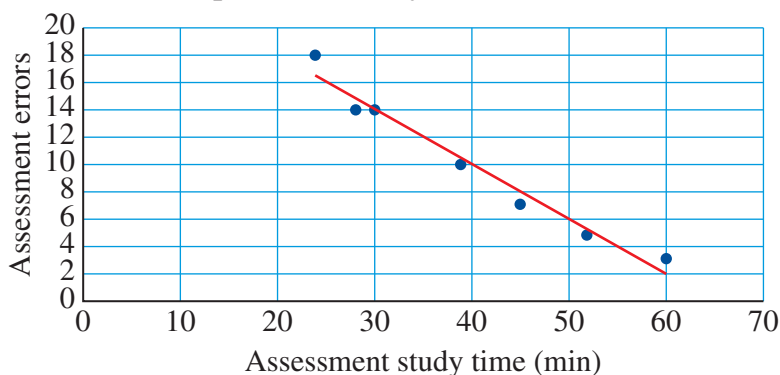
- Using interpolation, estimate what the weight gain would be for someone eating 12 bars of chocolate per week.
- Using interpolation, estimate how many chocolates would be eaten per week to gain 1.1 kg per week.
- Using technology, determine the equation of the line of best fit.
- Using a calculator, predict how many chocolates are required per week to gain 5 kg per week.
- Explain how reliable is the prediction for part d.

Hint When using a calculator, the equation is $y = ax + b$.



- 8 A teacher has collected data and produced this graph to try to convince his students that there is a relationship between the study time for an assessment and the number of errors they make in the assessment.

Relationship between study time and number of errors

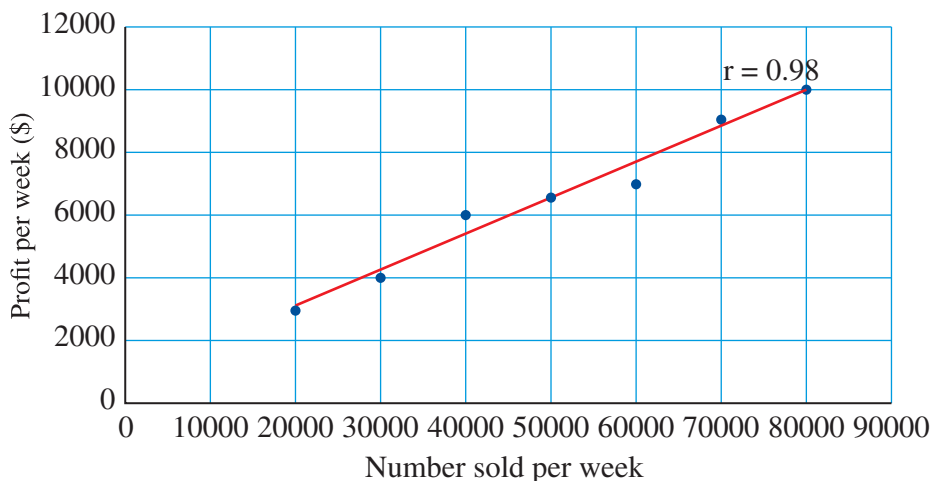


- Using interpolation, estimate how many errors there will be after 35 minutes of study.
- Using interpolation, estimate how much study time is required for only 4 errors.
- Using technology, determine the equation of the line of best fit.
- Using a calculator, predict how much study time results in there being 30 errors.
- Explain how reliable is the prediction for part d.



- 9 A business owner is wanting to sell a young couple part of his business and is using extrapolation above 80 000 sales per week from the following chart to enhance his sales pitch. Explain how extrapolation in this situation could be dangerous.

Sales and Profit



6G Distinguishing between causality and correlation

LEARNING GOAL

- Distinguish between causality and correlation through examples

Why is it essential to understand the difference between causality and correlation?

Two variables may have a relationship; however, it does not necessarily mean that one causes the other. Correlation is a specific number that describes the size and direction of a relationship between variables. Causation, however, reveals that the value of one variable is caused by the value of the other variable.

Typical reasons why correlation is not due to causation include:

- coincidence
- the value of both variables is the result of a third variable.



An association between two things does not necessarily mean one caused the other.

WHAT YOU NEED TO KNOW

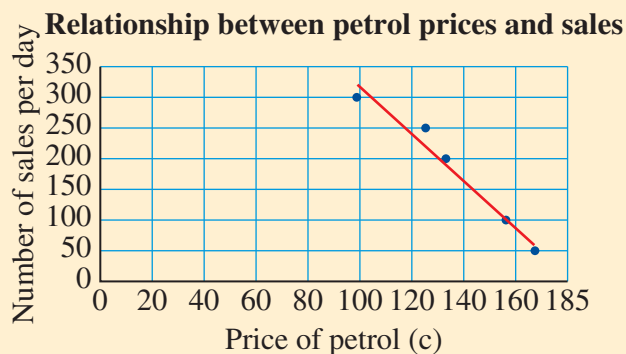
- **Causation**, or causality or a causal relationship, is the ability of one variable to impact another.
 - The first variable could bring the second variable into being or may cause the value of the second variable to change.
 - Only when the change in one variable causes the change in another parameter is there a causal relationship.
- **Correlation** is a measure of the strength of the relationship between two variables.

- Correlation does not imply causation.
 - Correlation does not explain why and how there is a relationship; it just reveals that a relationship may exist.
 - Causation states that any change in one variable will cause a change in another variable. This is also known as cause and effect. Distinguishing between a causal relationship and an association without causation is often a matter of common sense, but sometimes you cannot be certain of the answer and the data just shows that more research is needed.



Example 12 Distinguishing between causality and correlation (causality)

Guy works at a petrol station and has noticed a relationship between the price of petrol and the number of sales in a day.



- a Determine if there is a correlation between the two variables. Explain.
- b Determine if it is likely that there is causality between the two variables. Explain.

WORKING

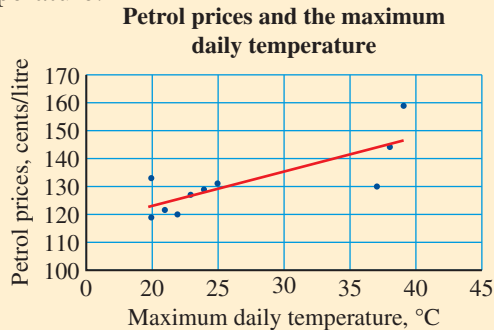
THINKING

- a There is a strong negative correlation $\leftarrow$ Does the graph show a correlation? between the price of petrol and the number of sales per day. As the price of petrol increases, the number of sales decreases.
- b It is likely that there is causality, as $\leftarrow$ Is there a cause and effect between the two variables? people will find another supplier or wait for the petrol prices to go down before they purchase petrol. The price affects the number of sales.



Example 13 Distinguishing between causality and correlation (no causality)

Kate is wondering if there is a relationship between petrol prices and the maximum daily temperature.



- a Determine if there is a correlation between the two variables. Explain.
- b Determine if there is causality between the two variables. Explain.

WORKING

- a The line of best fit has a slight positive slope, which means there is a relationship between petrol prices and the maximum daily temperature.
- b There does not appear to be any causality. There isn't a mechanism whereby a change in maximum daily temperature can cause the prices to change. There are many other variables that determine the change in petrol prices, for example supply and demand, location, and exchange rates. The correlation is likely to be a coincidence, and if Kate surveys prices and temperature over a longer period of time she may find there is no correlation.

THINKING

- ←.... Does the graph show a correlation?
- ←.... Is there a cause and effect between the two variables?



Exercise 6G

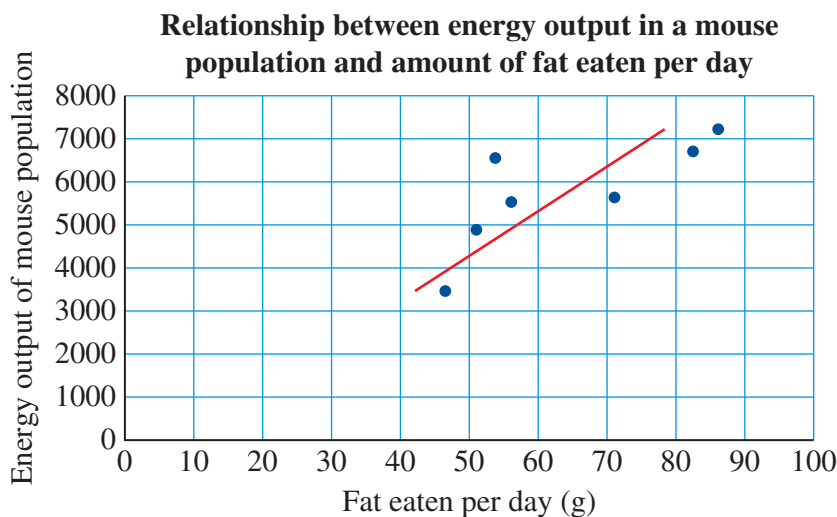
FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a Two variables may have a _____; however, it does not necessarily mean that one _____ the other.
 - b Causation is the ability of one variable to _____ another.
 - c Correlation is a measure of the _____ of the linear relationship between two _____.
 - d Correlation does not _____ causation.
- 2 Define using everyday language:
 - a correlation
 - b causation

APPLICATIONS

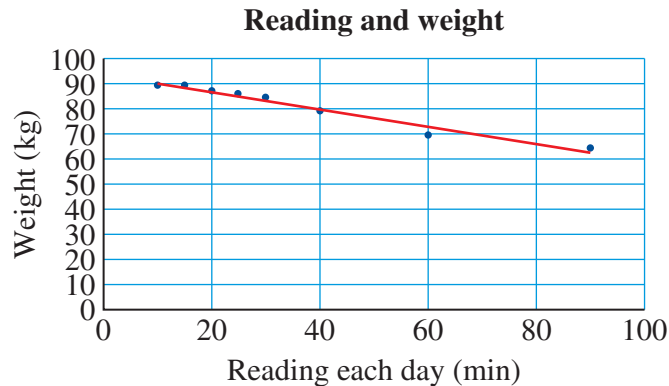
Example 12, 13

- 3 Ella is researching the relationship between fat in the diet of laboratory mice and their energy output. Using laboratory equipment, she has obtained the data shown in the graph.



- a Determine if there is a correlation between the two variables. Explain.
- b Determine if there is causality between the two variables. Explain.

- 4 Ebony is trying to lose weight. She thinks that perhaps when she reads books, she loses weight.

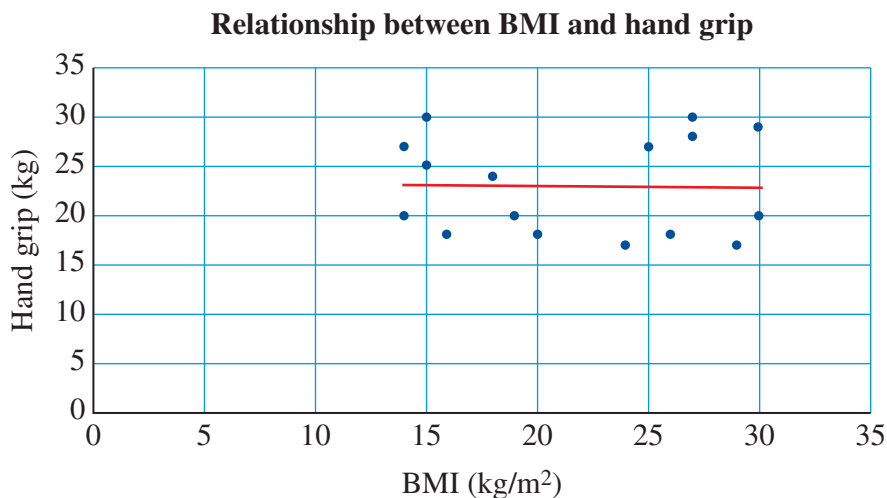


- a Determine if there is a correlation between the two variables. Explain.
- b Determine if there is causality between the two variables. Explain.

Hint Is there a cause and effect between the two variables?

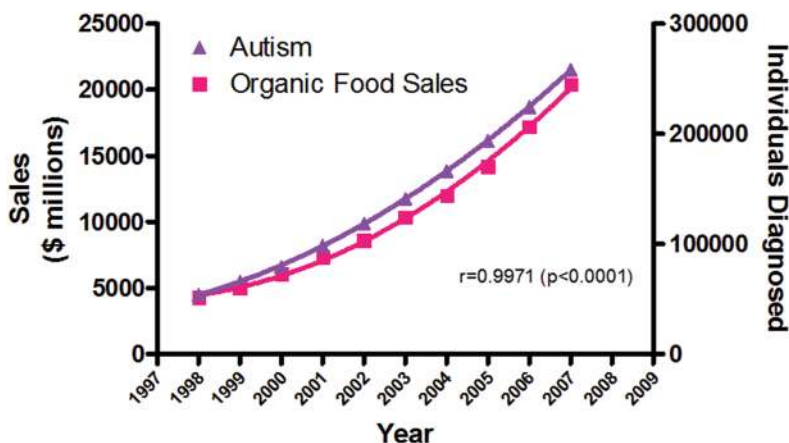


- 5 Finn is researching to see if there is a relationship between people's BMI and the strength of their hand grip. Her data (which was used in a previous question, in Exercise 6D) is shown below.



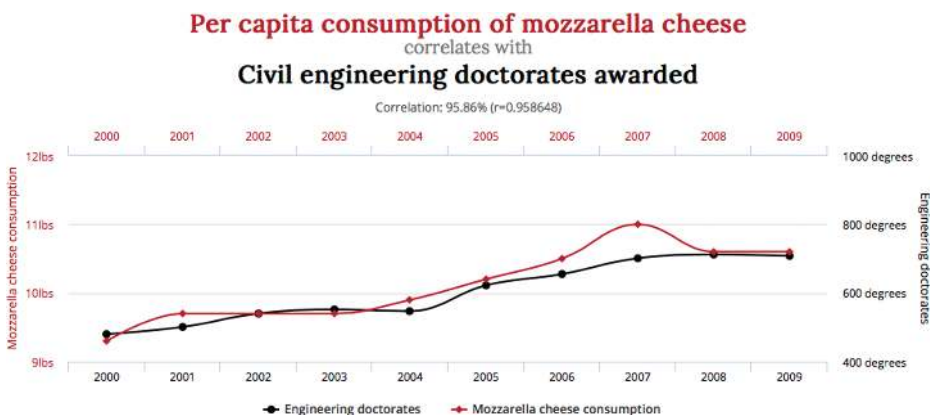
- a Determine if there is a correlation between the two variables. Explain.
- b Determine if there is causality between the two variables. Explain.

- 6 A researcher is studying whether there is a relationship between organic food sales and autism in children. The following graph was presented as evidence.



Sources: Organic Trade Association, 2011 Organic Industry Survey; U.S. Department of Education, Office of Special Education Programs, Data Analysis System (DANS), OMB# 1820-0043: "Children with Disabilities Receiving Special Education Under Part B of the Individuals with Disabilities Education Act"

- a Determine if there is a correlation between the two variables. Explain.
- b Determine if there is causality between the two variables. Explain.
- 7 A scientist is studying whether there is a relationship between civil engineering doctorates awarded and the consumption of mozzarella cheese. The following graph has been submitted as evidence.



- a Determine if there is a correlation between the two variables. Explain.
- b Determine if there is causality between the two variables. Explain.

Statistical investigation process task

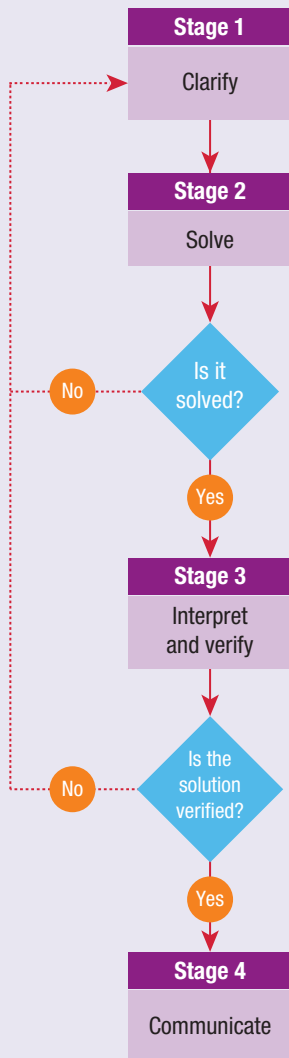
Background: $C = \pi D$

You have always been told that to find the circumference of a circle, you simply multiply its diameter by pi.

Task: Your task is to prove this formula by:

- 1 Find at least ten different circular objects (with different diameters).
- 2 Create a table with their diameter (cm) being the x and their circumference (cm) being the y .
- 3 From the table, create a scatterplot and find the line of best fit.

Approach to statistical investigation process task.



Stage 1: Clarify

Clarify the problem and pose one or more questions that can be answered with data

- 1 What does pi stand for?
- 2 What part of the circle is the circumference?
- 3 How do you find the diameter?
- 4 How can you find the data?

Stage 2: Solve

Design and implement a plan to collect or obtain appropriate data and select and apply appropriate graphical or numerical techniques to analyse the data

- 5 Collect the required amount of data.
- 6 Produce the table and scatterplot.
- 7 Complete the calculations to find the line of best fit.
- 8 Find the measurement of the slope.

Stage 3: Interpret and verify

Interpret the results of this analysis and relate the interpretation to the original question

- 9 Do you have enough variety in your measurements?
- 10 Have you presented them in the most suitable way?
- 11 What are the relationships between diameter and circumference?
- 12 What is the gradient similar to? Why?

Stage 4: Communicate

Communicate findings in a systematic and concise manner

- 13 Communicate your findings in a short report that justifies your findings.
- 14 Include:
 - Introduction* (What is the dependent and independent variable? Why?)
 - Body* (Include statements about the line of best fit and the relationship between variables)
 - Conclusion* (Distinguish between causality and correlation through your examples).

Chapter checklist

☐ I can identify the dependent variable and the independent variable.

- 1** Identify both the dependent variable and the independent variable from this data. Give reasons.

Number of cups of chai tea sold per hour in the winter recorded against outside air temperature.

Temperature °C	11	8	9	4	20	18	10	11	13	10	2	1
Number of chai teas sold	10	13	14	16	0	1	8	6	7	7	18	20

☐ I can determine the line of best fit by eye.

- 2** Draw a scatterplot from the previous data, then determine the line of best fit by eye.

☐ I can use technology to determine the line of best fit.

- 3** Determine the line of best fit from the previous data using a spreadsheet.

☐ I can interpret relationships in terms of the variables.

- 4** Interpret and explain the relationships in terms of the two variables from the previous data.

☐ I can use technology to calculate the correlation coefficient (an indicator of the strength of linear association).

- 5** Use a scientific calculator to determine the correlation coefficient from the previous data.

☐ I can use the line of best fit to make predictions, both by interpolation and extrapolation.

6 From the previous data make both an interpolation and an extrapolation prediction.

☐ I can recognise the dangers of extrapolation.

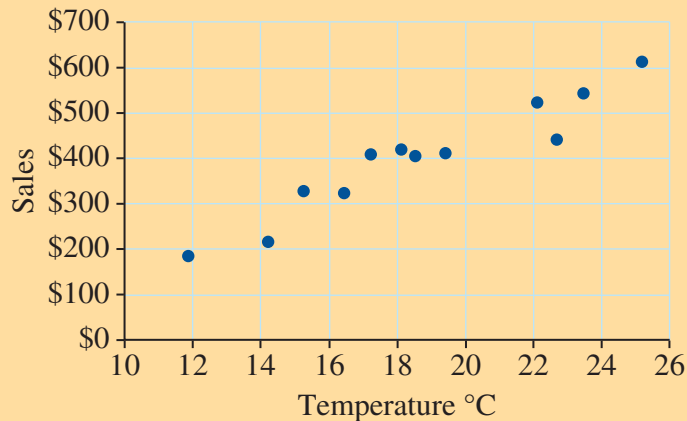
7 In relation to previous extrapolation prediction, state whether there are any dangers.

☐ I can distinguish between causality and correlation through examples.

8 In relation to previous data distinguish between causality and correlation.

☐ I can identify patterns in bivariate data in direction, form and strength.

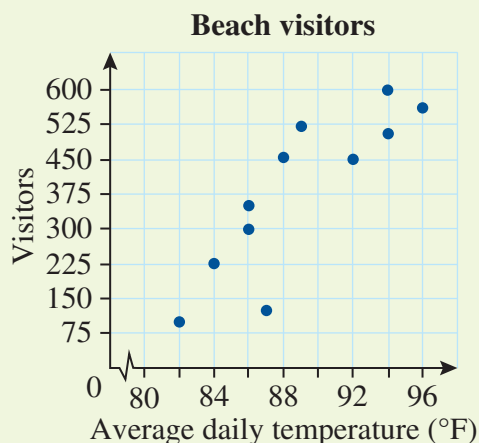
9 Describe the graph shown in terms of direction, form and strength.



Chapter review

Section 6A

- 1 The scatterplot shown maps the daily temperature at the beach and the number of visitors to the beach. Determine if any correlation exists in terms of direction, shape and strength. Comment on whether a relationship exists between the number of visitors attending the beach and the daily temperature.



- 2 Climate scientists have been mapping the temperatures at Antarctica, and they have recorded the average yearly temperature and the area of the ice. Determine if any correlation exists in terms of direction, shape and strength. Comment on whether a relationship exists between the average yearly temperature and the area of the ice.

Area of Antarctic ice ('000 000 km ²)	14	13.8	13.3	13.5	13.2	12.9	13.1	12.7	12.5
Average yearly temperature (°C)	8.3	8.2	8	7.8	7.9	7.5	8	7.8	7.3

Section 6B

- 3 Jon is studying how higher summer temperatures are impacting on the growth of his crops.

- What could be the dependent variable? Give a reason.
- What could be the independent variable? Give a reason.

Section 6C

- 4 Leanne is training for a bike ride and is recording her latest times. Draw a scatterplot and determine the line of best fit by hand.

Time (min)	25	40	35	37	90	40	115	35	95
Distance (km)	6	10	15	10	20	15	25	10	20

- 5 Zek is researching growth in snow peas. Identify the dependent variable and list three possible independent variables.

- 6 Lilly has noticed a relationship between the average temperature and the amount of sales of ice cones. Determine the line of best fit using technology.

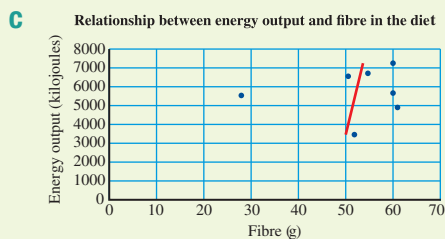
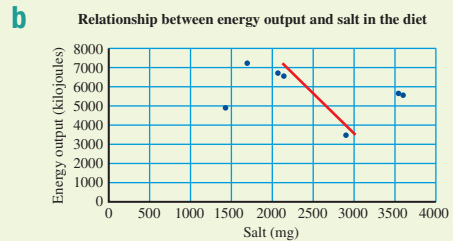
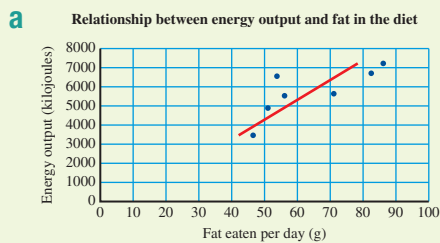
Average temperature (°C)	23	28	25	34	29	35	27	37	26	29	28	30	36
Number of ice cones sold	70	85	74	87	87	105	80	112	78	88	84	92	110

- 7 Glenn has been recording the value of his stock over the past five years. Determine the line of best fit using technology.

Year	Value of stock \$
2014	850
2015	1037
2016	1375
2017	2175
2018	4002

Section 6D

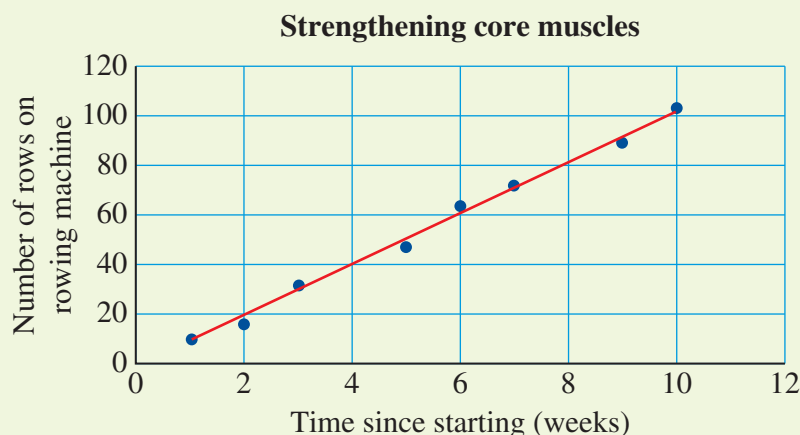
- 8 Interpret the relationship in terms of the variables of the following graphs.



- Section 6E** 9 Roxy is wondering if there is a relationship between time spent studying for an assessment and her grades for the assessment. Use a scientific calculator to determine the correlation coefficient for the data.

Assessment study time (min)	52	24	26	45	39	28	60
Assessment grades	87	23	25	42	40	27	90

- Section 6F** 10 Rachel is currently strengthening her core muscles by using a rowing machine. The data shows the number of rows (rowing strokes) she can do each week after she started.

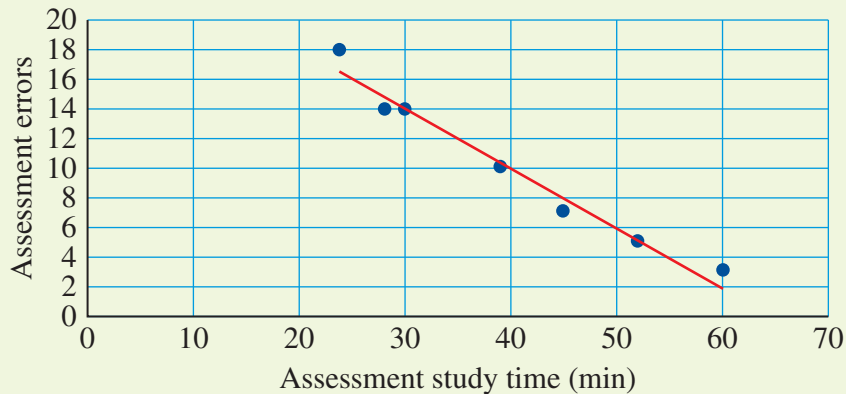


- Using interpolation, estimate the week that Rachel could do 40 rows.
- Without technology, use the equation of the line of best fit in the form $y = mx + c$ to predict how many rows Rachel will be able to achieve by week 16.
- Use a calculator to predict how many rows Rachel will be able to achieve by week 20.
- Explain the dangers of extrapolation with regards to this graph.

Section 6G

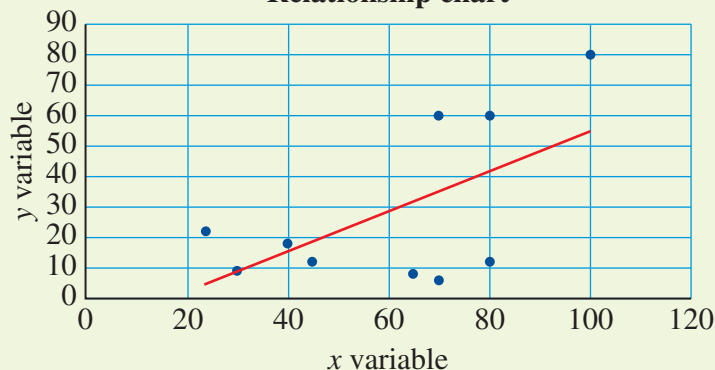
- 11** Scarlett has noticed that when she studies more for an assessment she makes fewer errors in the assessment.

Relationship between study time and number of errors



- a** Determine if there is a correlation between the two variables. Explain.
b Determine if there is causality between the two variables. Explain.
- 12** Your teacher wants to show examples of what the data and graphs of different correlation relationships look like, and she has produced the following scatterplot and table with a correlation coefficient for the variables of 0.60.

Relationship chart

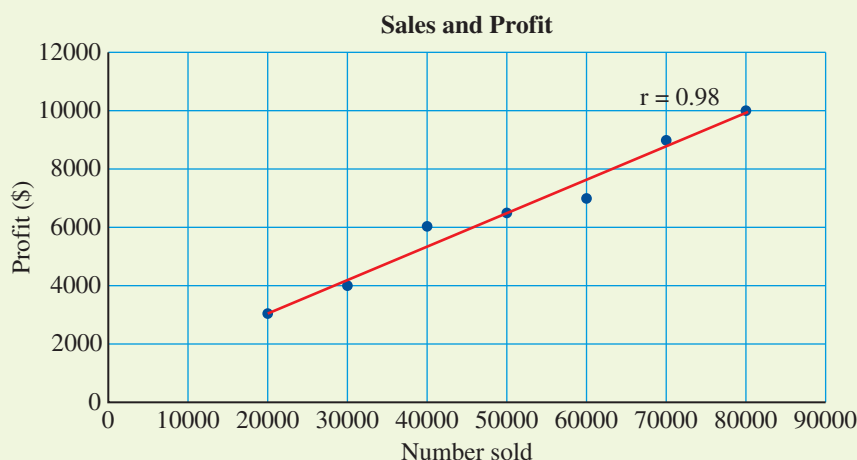


x variable	80	70	100	24	45	70	30	80	65	40
y variable	60	60	80	22	12	6	9	12	8	18

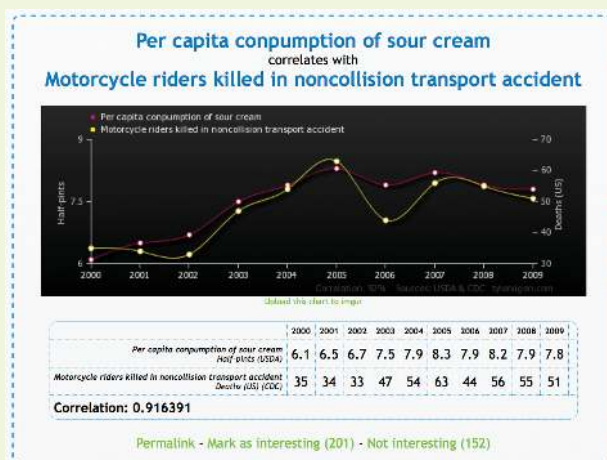
Using a spreadsheet, change some of the x and y variables to produce a set of data where the correlation coefficient is:

- a** weak positive correlation
b strong negative correlation
c moderate negative correlation

- 13** Use the following graph to devise an interpolation statement and an extrapolation statement. Explain the dangers of extrapolation.



- 14** Use the following graph to determine the correlation and causality statements.



Source: Tylervigen.com

- a** Determine if there is a correlation between the two variables. Explain.
- b** Determine if there is causality between the two variables. Explain.

A large, stylized blue number 7 is positioned on the left side of the page. To its right, the title 'Simple probabilities and simulations' is written in a bold, yellow, sans-serif font. The background is a white surface covered with numerous blue, semi-transparent spheres, each casting a soft shadow. A solid red vertical bar runs along the right edge of the page.

7

Simple probabilities and simulations

In this chapter

- 7A** Interpreting commonly used probability phrases and describing ways of expressing probabilities formally using fractions, decimals, ratios and percentages
 - 7B** Performing probability experiments using technology
 - 7C** Recognising the repetition of chance events
 - 7D** Identifying and calculating relative frequency
 - 7E** Identifying complication factors with real-life simulations
 - 7F** Constructing a sample space
 - 7G** Determining probabilities for an experiment
 - 7H** Using tree diagrams to determine probabilities
- Statistical investigation process task
- Chapter checklist
- Chapter review

Syllabus reference

Unit 4 Topic 1: Probability and relative frequencies (20 hours)

Probability expressions

- interpret commonly used probability statements, including ‘possible’, ‘probable’, ‘likely’, ‘certain’
- describe ways of expressing probabilities formally using fractions, decimals, ratios and percentages

Simulations

- perform simulations of experiments using technology
- recognise that the repetition of chance events is likely to produce different results
- recognise the law of large numbers and identify relative frequency as probability
- identify factors that may cause the simulation to no longer model the real world event

Simple probabilities in practical situations

- construct a sample space for an experiment which represents a practical situation
- use a sample space to determine the probability of outcomes for an experiment
- use arrays or tree diagrams to determine the outcomes and the probabilities for experiments

Probability applications

- identify situations in real-life contexts where probability is used for decision making
- determine and use probabilities (relative frequencies) from given data to predict proportions and the number of outcomes that are likely to occur

Pre-test

- 1 Write an appropriate probability value for each of the following probability phrases.
 - a uncertain
 - b impossible
 - c even chance
 - d likely

- 2 Round each of the following decimals correct to 2 decimal places.
 - a 0.454
 - b 0.438
 - c 0.595

- 3 Convert the following fractions to decimals correct to 2 decimal places.
 - a $\frac{17}{98}$
 - b $\frac{16}{52}$
 - c $\frac{2}{43}$

Hint Use division.

- 4 Express each fraction in its simplest form.
 - a $\frac{10}{15}$
 - b $\frac{6}{8}$
 - c $\frac{12}{14}$

- 5 Convert the following fractions to percentages correct to 2 decimal places.
 - a $\frac{16}{81}$
 - b $\frac{5}{36}$
 - c $\frac{4}{18}$

- 6 Calculate the following correct to 2 decimal places.
 - a $\frac{6}{8} \times 65$
 - b $\frac{12}{14} \times 50$
 - c $\frac{7}{12} \times 38$

- 7 Calculate the following.
 - a 10% of 45
 - b 18% of 460
 - c 1.5% of 120

8 A die is rolled and the number on the upper most face is noted.

a Identify the possible outcomes.

b Calculate the probability that the upper most face is:

i a 6

ii an even number

iii more than 2

iv less than 5

9 A list of random numbers are generated on a calculator to simulate the gender of 12 babies born in a week at a hospital. If the last digit is odd = boy, even = girl.

0.278 635 0.731 297 0.573 965 0.776 794 0.869 89 0.868 027
0.662 48 0.289 817 0.801 859 0.716 876 0.648 063 0.280 808

a Organise the results of the simulation into a frequency table.

b Calculate the relative frequency of boys based on the simulation.

c Calculate how many of the 12 babies you would expect to be boys.

d Reflect on any similarities or differences between your answers to parts **b** and **c**.

Outcome	Tally	Frequency
Odd/Boy		
Even/Girl		
Total		

7A Interpreting commonly used probability statements and describing ways of expressing probabilities formally using fractions, decimals, ratios and percentages

LEARNING GOALS

- Interpret commonly used probability statements
- Describe ways of expressing probabilities formally using fractions, decimals, ratios and percentages

Why is it essential to be able to interpret commonly used probability statements and describe ways of expressing probabilities formally using fractions, decimals, ratios and percentages?

Interpreting commonly used probability statements allows for the two-way communication of the chance of events occurring in an informal way. Describing ways of expressing probabilities formally using fractions, decimals, ratios and percentages allows for the two-way communication of the chance of events occurring in a quantitative way. The numerical values can then be used in calculations to identify the total number of possible outcomes based on experimental data and relative frequencies.

WHAT YOU NEED TO KNOW

- An **event** is an outcome or defined collection of outcomes of a random experiment.
- There are many terms that can be used in everyday life to describe the chance of an event occurring. Each of these can be related to a range of numerically quantified probabilities. See below:
- The probability of an event occurring can be between 0 (impossible) and 1 (certain)

Probability terms											
	Impossible	Unlikely	Uncertain	Improbable	Even chance	Probable	Possible	Likely	Certain		
Decimal	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Percentage	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Fraction	0	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{7}{10}$	$\frac{4}{5}$	$\frac{9}{10}$	1
Ratio		1 in 10	1 in 5	3 in 10	2 in 5	1 in 2	3 in 5	7 in 10	4 in 5	9 in 10	



Example 1 Interpreting commonly used probability statements

- a** If it is unlikely to rain tomorrow, would you take a raincoat when you go to the park tomorrow?
- b** Graeme said “It is impossible to fit that box onto my truck.” What would you need to do about transporting the box home?

WORKING

THINKING

- a** I would not take a rain coat. $\leftarrow \dots \dots \dots$ Unlikely suggests that there is a low probability of rain tomorrow.
- b** You would have to use a larger truck to transport the box home. $\leftarrow \dots \dots \dots$ If it is impossible to fit the box onto Graeme’s truck, then there is zero chance you can use Graeme’s truck.



Example 2 Describing ways to express probability statements

Match each probability value with the most appropriate term.

- a** 60% **b** 0.9 **c** $\frac{1}{3}$ **d** 4 in 5

Likely Impossible Probable Possible Improbable

WORKING

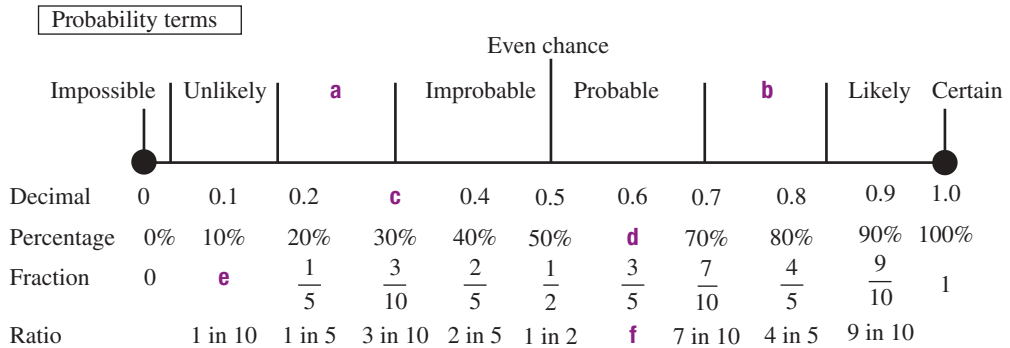
THINKING

- a** Probable $\leftarrow \dots \dots \dots$ Using the scale above, 60% is in the probable range.
- b** Likely $\leftarrow \dots \dots \dots$ Using the scale above, 0.9 is in the likely range.
- c** Improbable $\leftarrow \dots \dots \dots$ $\frac{1}{3} = 0.33$.
Using the scale above, 0.33 is in the improbable range.
- d** Possible $\leftarrow \dots \dots \dots$ Using the scale above, 4 in 5 is in the possible range.

Exercise 7A

FUNDAMENTALS

- 1 Determine the missing values or terms on the following scale.



Example 2

- 2 Match the follow probability value with the most appropriate term.

a 0.2 **b** $\frac{3}{4}$ **c** 35% **d** 1 in 15

A Unlikely **B** Certain **C** Improbable **D** Possible **E** Impossible

- 3 **a** In a group of 3–5 students, make a list of some other probability terms (at least 8 phrases).
b Write these terms and the ones above in order of likelihood from impossible to certain.
c Write next to each term a probability range that is appropriate for it.
- 4 Pick three terms from question 3 and write them into three probability statements that appropriately describes a real-world situation.
- 5 A raffle organiser claims that each ticket has a 1 in 25 chance of winning.
a Write the probability as a:
 i a fraction **ii** a decimal **iii** a percentage
b Does the probability mean that there are 25 tickets in the raffle?
c Write a phrase to describe the chance of having a winning ticket.
- 6 If it is half-time at a soccer match and Perth has a 65% chance of winning, Brisbane has a 25% chance of winning and there is a 10% chance of a draw:
a Which team do you think is winning at half time? Explain your choice.
b Write the probability that Perth will win as a decimal and a fraction.
c Find the sum of the probabilities for the soccer match. Explain your answer.

7B Performing probability experiments using technology

LEARNING GOALS

- Perform probability experiments
- Perform simulations of probability experiments using technology

Why is it essential to be able to perform simulations of real-life scenarios?

We all have said ‘what will happen if ...’ many times in our lives. When faced with a decision, having some idea of what outcome to expect can help with the process. Simulations are quick, cheap and effective ways to answer the ‘what will happen if ...’ question without the need to wait and see or running a probability experiment which can be time-consuming and expensive.



Engineers run simulations of traffic flow using computer software when designing new freeways to predict bottlenecks and find ways to eliminate them.

WHAT YOU NEED TO KNOW

- An **experiment** is a procedure undertaken to make a discovery. We can conduct probability experiments to simulate real-life problems (e.g. the probability of having a baby boy versus a baby girl).
 - Each repeat of an experiment is called a **trial**.
 - The results of the probability experiment need to be organised into a **frequency table**. Frequency is a count of how often an outcome appears.
- Physical techniques that can be used for simulation include:
 - Tossing a coin. For example: if using simulation to predict the gender of a baby before it is born, heads = girl, tails = boy
 - Rolling a die. For example: even result = girl, odd result = boy
 - Spinning a spinner. For example: even result = girl, odd result = boy
 - Drawing a selection 'from a hat'.
 - Drawing cards from a standard pack of playing cards which has four suits and two jokers. Red suits: hearts ♥ and diamonds ♦; Black suits: spades ♠ and clubs ♣. Each suit has 13 cards: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K



- Simple technology that can be used for simulation includes:
 - The random number feature on a scientific calculator returns a random decimal number between 0 and 1. For example: if the last digit is even = girl, odd = boy.

rand .23640104

Note: Most smart phones also have a scientific calculator app with a RAND function that works in the same way. On an iPhone, turn it sideways with the calculator app open to access the scientific calculator.
 - RandomBetween function in Excel. For example: entering =RANDBETWEEN(1,2) in a cell returns a random whole number between 1 and 2 that could be allocated to your selection. For instance, 1 = girl, 2 = boy. Pressing F9 in an Excel sheet with a probability simulation will repeat the probability experiment. The outcomes are never exactly the same.
 - Online calculators are useful when lots of trials are required. We will use online calculators in section 7C.



Example 3 Analysing physical probability experiments

Toss a coin to simulate the gender of 20 babies born at a hospital during a week. Use heads = girl, tails = boy.

- Organise the result of 20 coin tosses in a frequency table.
- Calculate the percentage of girls from your simulation.
- Decide if the simulation results varied from what you expected. Explain how they may differ.



WORKING

- a** Result of 20 coin tosses:

T H H H T
H H T T H
H H H H H
H T T H H

Result	Frequency
heads/girls	14
tails/boys	6

- b** Percentage girls = $\frac{14}{20} \times 100 = 70\%$

- c** The simulation did not produce the expected result. The percentage of girls in the simulation was 70%, you would expect it to be around 50%.

THINKING

You can use tally marks and record as you go or just record each result of the coin toss and count when done.

Percentage = $\frac{\text{amount}}{\text{total}} \times 100$

Simulations do not always give the results you would expect. Lots of trials need to be done to get a result that is close to what the expected value might be.

Note: In real life, the probability of a boy or a girl is not actually equal. 51.2% of babies born in industrialised countries are boys. For the purposes of this chapter, we will assume that it is equal.



Example 4 Conducting a simulation of a probability experiment using technology

Conduct a simulation to estimate how many students need to be chosen at random for each of the four school houses to be represented.

- Use Excel to generate data for which school house a student is from. Use the numbers 1, 2, 3, 4 to represent the four school houses.
- Copy and complete this frequency table to record which school house the simulation shows the students are from. Stop when all four school houses have been selected at least once.

House	1	2	3	4
Tally				
Frequency				

- Determine how many simulations it took to represent all four school houses.
- Identify which school house was represented more than other houses in the simulation.
- Explain a way the simulation experiment could be conducted using a physical technique.

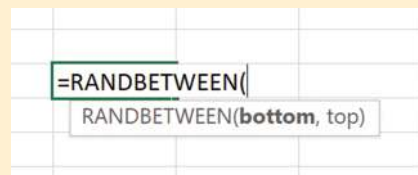
WORKING

- Data: houses of 12 students:

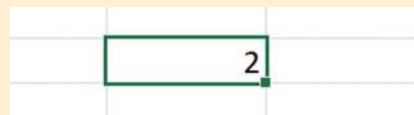
4 2 2 1
1 3 3 1
1 1 4 3

THINKING

`=RANDBETWEEN(1,4)` will randomly select a whole number between 1 and 4 to represent the four school houses.



Tip: Grabbing the little box in the bottom right corner and dragging down or across will give you more random numbers within the same range.



WORKING

b 4 2 2 1
 1 3 3 1
 1 1 4 3

Frequency table:

House	1	2	3	4
Tally				
Frequency	2	2	1	1

THINKING

←..... Cross off each simulated school house and record in the table. Stop once all the houses have been represented.

- c** It took 6 simulations for all four school houses to be represented. ←..... Count the number of houses crossed off or total up the frequencies.
- d** The first two school houses were both represented twice before the final school house, 3, was selected. ←..... Look for the highest frequency.
- e** Playing cards could be used where each suit represented a school house. The card would need to be replaced after each selection is noted. ←..... Any technique that has four possible choices would be suitable. Replacing the card each time keeps the chance of each being selected remaining the same.

Exercise 7B

FUNDAMENTALS

- 1** Determine the missing words in the following sentences.
- a** Physical techniques that can be used for _____ include tossing a, _____ rolling a _____ and spinning a _____.
- b** Simple _____ that can be used for simulation include the _____ number feature on our _____ that returns a random number between 0 and 1. The RANDBETWEEN function in _____ returns a random whole number _____ two _____ values that could be allocated to your selection.



Spreadsheet activity 7B: See the Interactive Textbook for this activity using the random number and counting functions in Excel to conduct a simulation for this exercise.

APPLICATIONS

Example 3

2 Complete the following questions based on this simulation.

- a** Toss two coins 20 times to simulate the gender of children in 20 two-child families. Use heads = girl, tails = boy. Copy and complete the following frequency table to record your results.

Two-child combination	Tally	Frequency
boy + boy		
boy + girl		
girl + girl		

- b** Calculate the percentage of two-child families with a boy and a girl combination in your simulation.
- c** Decide if the results of your simulation differ from what you might have expected. Explain how they may differ.



3 Complete the following questions based on this simulation.

- a** Toss three coins 20 times to simulate the gender of children in 20 three-child families. Use heads = girl, tails = boy. Record your results in a frequency table as follows:

Three-child combination	Tally	Frequency
3 × boys		
2 × boys + 1 × girl		
1 × boy + 2 × girls		
3 × girls		

- b** Calculate the percentage of three-child families with a boy/girl combination in your simulation.
- c** Decide if the results of your simulation differ from what you might have expected. Explain how they may differ.
- 4** A supermarket runs a promotion where a free collectable toy is given away with each \$30 spent in the store. There are 20 different toys to collect. Ashleigh runs a simulation using a spreadsheet to estimate how much money she would need to spend to collect all 20 toys. The simulation shows which toy she receives after spending \$30.

9	7	5	6	2	17	9	11	9	5
6	13	7	5	19	12	19	6	15	15
15	7	11	13	18	11	4	13	4	14
20	18	4	17	2	9	9	10	7	13
19	4	5	10	15	7	14	18	3	8
19	2	12	20	15	18	3	17	11	9
7	18	19	16	9	8	14	9	1	9

- a** Copy and complete this frequency table to record which toys the simulation shows she will receive. Stop when all 20 toys have been collected at least once.

Toy number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Tally																				
Frequency																				

- b** Determine how many simulations it took to collect all 20 toys.
- c** Calculate how much money she will need to spend to collect all 20 toys using the results of the simulation.
- d** Determine which toy was collected more than others in the simulation.
- e** Calculate the average number of each toy she collected so that she collected all 20 toys in the simulation.

Hint Look for the highest frequency.

Hint $\text{Average} = \frac{\text{total of all data values}}{\text{number of data values}}$

- 5 Ian has 7 different ties that he keeps in a box on his desk. He randomly selects a tie from the box each morning when he arrives at work. His workmate David uses a spreadsheet to conduct a simulation of which tie Ian will wear each day for the next month. The results are as follows:

Week	Monday	Tuesday	Wednesday	Thursday	Friday
One	5	7	1	1	4
Two	1	3	7	3	7
Three	1	5	7	5	7
Four	7	5	5	3	2

- a Copy and complete this frequency table for how many times each tie is worn over the four-week simulation.

Tie number	1	2	3	4	5	6	7
Tally							
Frequency							

- b Determine if any tie is worn more, or less, often than another.
- c Calculate how often the simulation shows that Ian wears the same tie more than once in a week.
- d Identify how often the simulation shows that Ian wears the same tie on two consecutive days.
- e Calculate the average number of times you would expect each tie to be worn over the four-week period. Determine if any of the results from the simulation differ to what you would expect.

Hint Expected number = $\text{trials} \div \text{number of possible outcomes}$

Example 4

- 6 Ben wants to run a simulation to estimate how many people he would need to ask before he finds two with the same birth month.
- a Explain why each of the following simulation techniques would or would not be suitable to use:
- Using a standard pack of cards and removing the Kings. The card drawn at random represents the month of birth. i.e. A = Jan, 2 = Feb, 3 = Mar J = Nov, Q = Dec. The card is not replaced after being drawn.
 - Using a deck of cards, as in part i, but the card is replaced after the selection is noted.

- iii Rolling two standard die and adding the two faces to simulate the month of birth.
- iv Using Excel and entering the formula =RANDBETWEEN(1,12).
- b Ben decides to use the deck of cards without the Kings and replace the card after each selection. He has the following results after 40 trials:

A	9	10	8	6	Q	J	5
3	5	5	5	3	8	A	7
7	A	A	2	2	3	8	4
J	8	7	3	Q	8	4	10
3	7	8	6	5	A	2	10

Organise the results of the simulation into a frequency table like the one following. STOP when one of the months has been selected twice:

Card drawn/month	A	2	3	4	5	6	7	8	9	10	J	Q
Tally												
Frequency												

- c Determine how many trials Ben needed to conduct to find two people with the same birth month.
- 7 From question 6b, Ben notices that not all the months seem to be represented evenly in his trials. He decides to see how big the differences are.
- a Organise the results of all 40 trials into a frequency table like the one following.

Card drawn/month	A	2	3	4	5	6	7	8	9	10	J	Q
Tally												
Frequency												

- b Decide if Ben was correct: were all the months represented the same amount of times in the 40 trials? Comment on the results.

Example 4

- 8 From question 6b, repeat Ben's experiment for 40 trials using Excel and the formula =RANDBETWEEN(1,12).
- a Determine how many trials it takes to find two people with the same birth month.
- b Decide if all the months were represented evenly over the 40 trials.

- 9** Faith wants to conduct a simulation to determine the average number of children a couple would need to have to ensure they have a combination of sons/daughters in their family. She decides to use the random number feature on her calculator. The last digit will determine the gender of a baby where even = girl, odd = boy. She starts recording her simulations in a table as follows.

Trial number	Outcome of trial	Family size
1	9(B), 4(G)	2
2	3(B), 9(B), 7(B), 8(G)	4
3	...	

- a** Conduct 20 trials of Faith's simulation. Organise your results in a table like the one above.
- b** Calculate the average size family based on the results of your simulation.
- 10** Gretel Bueta plays in the goal attack position for the Firebirds Netball team. In the 2018 season she scored 283 goals from 315 attempts.

- a** Show that her success rate for shooting a goal is 90%.
- b** Use the random number feature on your calculator to simulate 30 attempts at goal. If the last two digits are between 01–90, she successfully shoots the goal, if the last two digits are between 91–00, she misses the goal. Copy and complete the following frequency table to record your results.



Attempt at goal	Tally	Frequency
Successful		
Miss		
Total		

- c** Calculate the percentage success rate for the 30 trials.
- d** With a 90% success rate, determine how many of the 30 goals you would expect Gretel to successfully shoot.
- e** Compare your answer in part **d** to the number of successful attempts found in the simulation in part **b**. Comment on differences or similarities between the two results.

7C Recognising the repetition of chance events

LEARNING GOALS

- Use technology to perform simulations
- Recognise how repetition will affect results

Why is it essential to be able to understand repetition of chance?

Gambling is a popular pastime in Australia with many adults participating in a ‘flutter’ on the pokies, scratchies, sports results or racing. Unfortunately, gambling addiction is also a significant public health issue, with Australian Gambling Statistics reporting a total gambling expenditure of \$23.694 billion in 2017. This is an average of \$1251.39 per Australian adult. Problem gambling can cause emotional and financial distress. A common misconception that many people have in games of chance is that past results can affect future ones; this is not true and is a contributor to problem gambling.



In games of chance, like Lotteries, some numbers come out more often than others. This does not improve the chance of them appearing in the next draw.

Source: Queensland Government Statistician's Office, Queensland Treasury, *Australian Gambling Statistics 35th*, edition.

WHAT YOU NEED TO KNOW

- The **outcome** of previous trials has no impact on the chance of the outcome occurring in successive trials. As an example, say you tossed five heads in a row with a single coin. When lining up to toss the coin for a 6th time, even though tossing 6 heads in a row is very unlikely (around 1.56%), the chance of tossing a head remains at one in two, or 50%, for the 6th toss because chance has no memory.
- When conducting **trials** where all outcomes have an equal chance, we would expect the different outcomes to appear approximately the same number of times.
 - Expected number of occurrences of an outcome = trials $\div$ number of different outcomes
- When conducting **simulations**, we can get unusual results when only running a small number of trials. Increasing the number of trials can improve the reliability of our predictions.



Example 5 Conducting an experiment with repetition of trials

Using a single coin, conduct an experiment where the coin is tossed twice for ten trials.

a Organise the results into a table.

Trial	1	2	3	4	5	6	7	8	9	10
First toss										
Second toss										

- b** Determine how often the outcome in the second toss was the same as in the first toss.
- c** Calculate how many heads were recorded in the first toss.
- d** Calculate how many heads were recorded in the second toss.
- e** Calculate how many heads you would expect over ten trials.
- f** Determine the average number of heads observed per toss over the ten trials.
- g** Decide if the results were as expected.

WORKING

THINKING

a

Trial	1	2	3	4	5	6	7	8	9	10
First toss	T	T	H	T	T	H	T	T	T	H
Second toss	T	H	H	T	T	H	H	H	H	H

←..... Insert the results into a table.

- b** 6 out of the 10 trials had the same result in both tosses. ←..... Since the chance of a head or a tail is equally likely, we would expect about half the time to toss the same result as the previous toss. While we did not get exactly one half, it is very close.
- c** 3 heads were recorded in the first toss
- d** 7 heads were recorded in the second toss
- e** Expected number of heads = $10 \div 2 = 5$ ←..... The two outcomes should appear the same amount of times.
Therefore, we would expect to toss 5 heads from the ten trials.
- f** Average number of heads = $\frac{3+7}{2} = 5$ ←..... Average = $\frac{\text{sum}}{\text{how many}}$
- g** While the individual trials did not achieve the expected number of heads (i.e. 5), the average over the two trials was what was expected. ←... They were expected because we only did two trials. Repeating trials gives a better result.

Exercise 7C

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a The outcome of _____ trials has no impact on the _____ of the outcome occurring in successive trials.
 - b When conducting trials where all outcomes have an _____ chance, we would expect the _____ outcomes to appear approximately the _____ amount of times.
 - c When conducting simulations, we can get _____ results when only running a limited number of _____.
 - d Increasing the number of trials can improve the _____ of our predictions.

APPLICATIONS

Example 5

- 2 A coin is tossed 20 times. The results are shown below.

H	T	H	T	H	H	T	H	T	T
T	T	H	H	T	T	T	T	H	H

- a Determine how many different outcomes there are when a coin is tossed.
- b Calculate how many times you would expect to get the same result as the previous toss in the 20 trials.
- c Determine how many times the same result appears in consecutive tosses. Compare this answer with what you expected from part b.
- d Calculate how many times you would expect each outcome to appear in the 20 rolls.
- e Decide if any results appear more/less often than expected.



Regardless of previous results, each time a coin is tossed the chance remains the same for tossing a head.

- 3** A standard die is rolled 30 times. The results are shown below.

4	3	4	5	2	1	5	6	5	6
1	3	1	2	3	5	5	1	4	4
2	4	1	1	4	3	4	4	4	4

- Determine how many different outcomes there are when a die is rolled.
 - Calculate how many times you would expect to get the same result as the previous roll in the 30 trials.
 - Determine how many times the same result appears in consecutive rolls over the 30 trials. Compare this answer with what you expected from part **b**.
 - Calculate how many times you would expect each outcome to appear in the 30 rolls.
 - Decide if any results appear more/less often than expected.
- 4** A standard die is rolled 20 times and the results are recorded. The results of 10 trials are displayed.

	Dice Outcome Frequency						
Trial	1	2	3	4	5	6	Total
1	1	2	4	5	5	3	20
2	4	1	4	4	4	3	20
3	4	2	3	4	3	4	20
4	5	3	2	2	5	3	20
5	5	3	3	3	2	4	20
6	5	6	2	4	0	3	20
7	5	2	1	2	6	4	20
8	1	5	3	2	5	4	20
9	4	5	3	4	3	1	20
10	3	3	5	3	3	3	20
Total							



- Calculate how many times you would expect each face of the die to appear in 20 rolls.
- Determine if any outcome appeared more than another in:
 - the first trial
 - the sixth trial
- Calculate and complete the totals for each outcome in the bottom row of the table.
- Calculate how many times you would expect each face of the die to appear in 200 rolls, which is the grand total of rolls of the die from the last column of the table.
- Decide if any results from the 200 rolls differ from what you would expect.

- 5** A lottery website displays data of numbers drawn over time as a way of offering strategy tips for picking numbers to encourage people to participate in the lottery.

HOT NUMBERS statistics show these numbers have been drawn more often	6, 30, 37, 40, 36, 9
COLD NUMBERS statistics show these numbers have been drawn less often	16, 29, 4, 27, 12, 4
MOST PROFITABLE NUMBERS statistics show these numbers have resulted in bigger payouts	36, 10, 39, 25, 24, 21
LEAST PROFITABLE NUMBERS statistics show these numbers have resulted in smaller payouts	28, 37, 25, 6, 9, 15

- a** Decide if the ‘Hot Numbers’ are more likely to come up in the next draw than the ‘Cold Numbers’. Explain why or why not.
- b** Determine how many of the ‘Hot Numbers’ also appear in the:
- ‘Most Profitable Numbers’
 - ‘Least Profitable Numbers’
- c** Reflect on the results from part **b**. Explain why might this happen.
- d** The results from the last draw are shown on the right:
Determine how many of the numbers drawn were:
- Hot Numbers
 - Cold Numbers
 - Most Profitable Numbers
 - Least Profitable Numbers
- e** To win a division 6 prize of \$13.50, you need to have a minimum of 1 blue number and the two white numbers. If you played the numbers from the website, determine if any of the Hot, Cold, Most Profitable, Least Profitable number sets would have won a prize.

26

15

16

19

14

20

5

4

- 6** Use an online simulator to investigate tossing a coin with a very large number of repetitions. A suggested site is <http://www.shodor.org/interactivate/activities/Coin/>. Determine what happens to the distribution of Heads/Tails as the number of trials is increased.
- 7** Use an online simulator to investigate rolling a die with a very large number of repetitions. A suggested site is <https://academo.org/demos/dice-roll-statistics/>. Determine what happens to the distribution of the outcomes as the number of trials is increased for:
- one die
 - two dice



Spreadsheet activity 7C: See the Interactive Textbook for this activity using Excel to simulate the results of rolling a die 30 times with many repetitions.

7D Identifying and calculating relative frequency

LEARNING GOALS

- Calculate probability
- Identify relative frequency as a probability

Why is it essential to be able to calculate relative frequency when determining probability?

In many situations, it is difficult to determine the chance of an event occurring, for example, the chance of a football team winning their next match. We need to look at past events and records to estimate the chance of the team winning.



Past performance is used to estimate the chance of a team winning a match.

WHAT YOU NEED TO KNOW

- **Probability** is a numerical indicator of the chance of an event occurring.
 - Probability of an event = $\frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$
 - Probability ranges in value from 0 (will not happen) to 1 (will happen).
- **Relative frequency** can be used to estimate the probability from a simulation or survey.
 - Relative frequency of an event = $\frac{\text{number of favourable outcomes observed}}{\text{total number of trials}}$
 - Relative frequency ranges from 0 (did not happen) to 1 (happened in every trial).
 - Probability of a coin coming down heads: favourable outcomes 1 (heads) divided by total number of outcomes 2 (heads and tails) equals the probability of heads as 0.5.
 - In six coin tosses where 4 came down heads, the relative frequency of heads is the number of favourable outcomes observed (4) divided by the total number of trials (6), which equals the relative frequency of heads as 0.67.
 - When increasing the amount of trials to a very large amount, the relative frequency approaches the expected (calculated) probability.



Example 6 Calculating probabilities using relative frequency

A survey asked students how many times they had visited the dentist in the past year. The table below shows the findings:

Number of visits	0	1	2	3	4	5
Number of students	2	6	4	2	0	1

Determine the following based on selecting a student at random.

- Calculate the probability that the student has visited the dentist twice in the past year. Round your answer to 2 decimal places.
- Calculate the probability that the student has been to the dentist more than once. Round your answer to 2 decimal places.

WORKING

THINKING

- Total number of students $\leftarrow \dots\dots\dots$ Relative frequency of an event = $\frac{\text{number of favourable outcomes observed}}{\text{total number of trials}}$

$$= 2 + 6 + 4 + 2 + 0 + 1$$

$$= 15$$

Probability of 2 visits

$$= \frac{4}{15} = 4 \div 15 = 0.27$$
- Probability of more than 1 visit $\leftarrow \dots\dots$ More than once will mean they have visited 2, 3, 4 or more times.

$$= \frac{4 + 2 + 0 + 1}{15}$$

$$= \frac{7}{15}$$

$$= 7 \div 15 = 0.47$$





Example 7 Using relative frequency to make estimations

A deputy principal wants to look at ways of improving traffic congestion around the school by encouraging students to either walk, ride a bike, or catch a bus to school. He does a survey of a group of students and finds the following results:

Transport method	Car	Walk	Bike	Bus
Number of students	56	27	12	43

- Determine how many students were surveyed.
- Calculate the relative frequency of students who travel by car.
- The school has a total of 827 students. Use the relative frequency to estimate the number of students in the school who travel to school in a car.

WORKING

THINKING

- Number of students surveyed $\leftarrow \dots$ Total the number of students.
 $= 56 + 27 + 12 + 43 = 138$
- Relative frequency of car travel $\leftarrow \dots$ Relative frequency of an event =
 $= \frac{56}{138}$
 ≈ 0.41
number of favourable outcomes observed
total number of trials
- Estimate of students in the school who travel by car $\leftarrow \dots$ It is better to use the exact value and not a rounded value in subsequent calculations so use the fraction form of the relative frequency.
 $= \frac{56}{138} \times 827$
 $= 336$



Exercise 7D

FUNDAMENTALS

- Determine the missing words in the following sentences.
 - Probability is a numerical indicator of the _____ of an event occurring.
 - Probability of an event = $\frac{\text{number of _____ outcomes}}{\text{total number of _____}}$
 - Probability _____ in value from 0 (will _____ happen) to 1 (_____ happen).
 - Relative frequency can be used to _____ the probability from a simulation or _____.
 - Relative _____ of an event = $\frac{\text{number of favourable outcomes}}{\text{total number of trials}}$
- Convert the following fractions to decimals. Round your answer to 2 decimal places.

$\frac{14}{25}$ $\frac{54}{87}$	$\frac{125}{267}$ $\frac{17}{693}$

Hint top $\div$ bottom
- Calculate the following. Round your answer to the nearest whole number.
 - 0.28×568
 - 0.79×682
 - $\frac{24}{52} \times 467$
 - $\frac{39}{589} \times 1276$

APPLICATIONS

Example 6

- A class is surveyed about how many siblings each student has. The results are:

Number of siblings	0	1	2	3	4	5	6
Number of students	3	6	10	4	2	0	1

- Determine how many students were surveyed.
- Calculate the relative frequency of a student having 2 siblings. Express your answer as a decimal correct to 2 decimal places.
- Calculate the relative frequency of a student having more than 2 siblings.
- Calculate the relative frequency of a student having at least 2 siblings.
- Calculate the relative frequency of a student having fewer than 2 siblings.

5 The form guide for a horse race displays information about past performance of the horses. The numbers in the Career column show:

- how many races the horse has participated in
- how many times the horse has finished first
- how many times the horse has finished second
- how many times the horse has finished third

So, a Career of 10: 2–1–5 would mean the horse has participated in 10 races, won two races, came second once and third on five occasions.

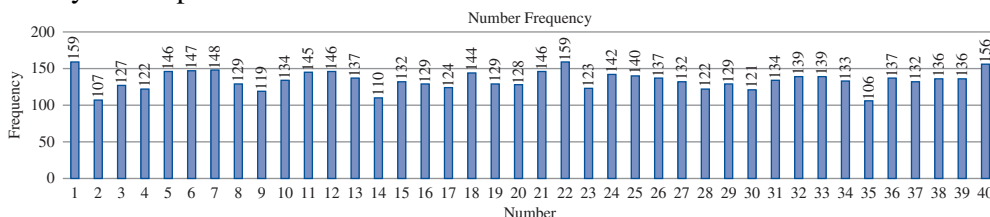
This table shows the field for a 1200 m race:

# Number, Horse (Barrier)	Career
1. Frequency Follie (6)	5 : 2–0–1
2. Statistics Sally (3)	4 : 1–0–0
3. Tally Marks (4)	4 : 2–0–1
4. Sum Total (2)	4 : 1–0–1
5. Decimal Diva (1)	6 : 1–1–1
6. Essential Ellie (7)	3 : 1–1–0
7. Relative Rita (5)	3 : 1–0–0

- For each horse in the race, calculate the relative frequency of them finishing in first place. Express your answer as a decimal correct to 2 decimal places.
- For each horse in the race, calculate the relative frequency of them finishing in either first, second or third place. Express your answer as a decimal correct to 2 decimal places.
- Based on your results from parts **a** and **b**, determine which horse has the best chance of:
 - winning this race
 - placing in this race



- 6 The following graph shows how many times each number has been drawn in a lottery over a period of time.



- Determine how many numbers have been drawn over the period of time.
 - Calculate the relative frequency of the number drawn being 13. Express your answer as a decimal correct to 2 decimal places.
 - Determine the relative frequency of the number drawn being a birthdate. Express your answer as a decimal correct to 2 decimal places.
 - Based on the past numbers being drawn, explain which numbers are more likely to be selected in future draws.
- 7 The age of migrants to Western Australia over a ten-year period is shown below:

Age (years)	0–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49	50–54	55–59	60–64	65 and over
Number	31 002	13 945	11 577	21 032	28 107	27 936	23 144	16 306	8392	4209	2640	4292

- Calculate how many migrants arrived in Western Australia over the period of study.
- Determine the relative frequency of migrant children who are aged 0–14 years.
- Calculate the relative frequency of migrants who are aged under 20 years.
- Calculate the relative frequency of migrants who are aged 60 years or older.

Example 7

- 8 Brittany wants to decorate a birthday cake with green chocolate coated candy and wants to know how many bags of mixed colour candy to buy. She knows from an internet search that there are approximately 105 pieces of candy per 100 grams and that a standard bag is 180 grams. She counted out a handful of candy and found that of the 38 pieces of candy, 7 were green. Brittany estimates that she will need 50 green candies for her project.
- Calculate how many pieces of candy would be in 20 grams.
 - Determine how many pieces of candy would be in a standard 180-gram bag.
 - Calculate the relative frequency of the green candy in her sample.
 - Use your answers from parts **b** and **c** to estimate how many pieces of green candy will be in one 180-gram packet.
 - Determine how many bags of candy Brittany will need to buy for her decorated cake project.



Technology Activity 7D: See the Interactive Textbook for this activity on using Excel to simulate rolling 2 dice and calculating relative frequency.

7E Identifying complication factors with real-life simulations

LEARNING GOAL

- Identify factors that cause unreliable results when conducting a simulation using techniques other than technology

Why is it essential to be able to identify complication factors in simulations?

When conducting a simulation, the outcomes need to be completely random in their selection. Poor techniques, such as not returning cards to a deck after selection or always tossing a coin with heads facing up, could alter the results achieved.



Poor techniques could complicate the results obtained from a simulation.

WHAT YOU NEED TO KNOW

- Poor techniques could complicate the results obtained from a simulation.
- If selecting from a hat or pack of cards:
 - always return the item to the hat/deck
 - mix/shuffle thoroughly before making the next selection
 - ensure there are no cards missing, but check there are no Jokers if the simulation doesn't involve them.
- When rolling a die, tossing a coin or spinning a spinner:
 - ensure that it is well tossed/spun so that the same selection does not continue to come up
 - do not always start with the same side up or at the same place
 - check the spinner rotates freely without sticking (e.g. that it does not stop in the same place each time).



Example 8 Identifying complicating factors with real-life simulations

Identify potential problems that could occur with the following simulation experiments.

- a** When simulating the gender of a baby, tossing a coin and always starting on heads.
- b** When simulating the month of birth for a person, rolling two dice and adding the faces together.
- c** When simulating the meal choice of chicken or beef for wedding guests, selecting a card from the top of a new unshuffled deck of cards, noting the colour of the card and then not returning the card to the deck before making the next selection.

WORKING

THINKING

- a** Tossing a coin and always starting on heads has the potential problem that depending on how high the coin is tossed and with how much spin, always starting on heads each time could bias the results towards the same result. ◀..... Always start at a different position.
- b** Simulating month of birth by rolling two dice has the potential problem that minimum sum is 2 so January would not be able to be achieved. February needs both dice to be 1, and December needs both dice to be 6, but June could come from either $3 + 3$, $2 + 4$ or $1 + 5$. ◀..... The simulation must be able to produce all results, and each outcome needs to be equally likely to occur.
- c** A new, unshuffled deck of cards is likely to have the cards grouped by suit, and so the first 13 drawn from the top will be the same colour – the deck should be shuffled, or the card drawn at random. If the card is not returned, then the number of options reduces, so if the first card is red, and not returned, then there are now more black cards than red cards to choose from. ◀..... Each outcome needs to be equally likely to occur.
A deck has two Jokers, which would not indicate either meal choice if drawn.

Exercise 7E

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a Poor _____ could complicate the results obtained from a _____.
 - b If _____ from a hat or pack of cards, always _____ the item to the hat/deck and mix/shuffle thoroughly before making the _____ selection.
 - c When rolling a die, tossing a coin or spinning a spinner, ensure that it is well tossed/spun so that the same _____ does not continue to come up and do not always start with the _____ side up or at the same place.

APPLICATIONS

Example 8

- 2 Use a set of standard playing cards to simulate which sport house each member of a group of 20 students is in, from a choice of four houses. Remove both Jokers from the deck and use these variations on simulation technique to compare results.
 - i Return the selected card to the deck after each selection and shuffle the deck between selections
 - ii Return the selected card to the deck after each selection but do not shuffle the deck between selections
 - iii Do not return the selected card to the deck after each selection but thoroughly shuffle between selections
 - iv Do not return the selected card to the deck after each selection and do not shuffle between selections
- a Copy and complete this frequency table to collate your results.

	Frequency			
Complicating factor	Hearts	Diamonds	Clubs	Spades
return/ shuffle				
return/ no shuffle				
no return/ shuffle				
no return/ no shuffle				

- b Evaluate which of the techniques could have possibly caused a problem with the results collected. Explain how.

- 3** Use a coin to simulate the gender of 20 babies born in a hospital in one week. Use these variations on simulation technique to compare results.
- i** Always start with heads
 - ii** Always start with the previous result up. For instance, if a head is tossed, start with a head facing up next turn
 - iii** High toss
 - iv** Low toss
 - v** Catch the coin off the toss and reveal the result on the back of your hand
 - vi** Allow the coin to fall to the ground from the toss
 - a** Copy and complete this frequency table to collate your results.

	Frequency	
Complicating factor	Heads	Tail
heads up		
previous result up		
high toss		
low toss		
catch and reveal		
fall to the ground		

- b** Evaluate which of the techniques could have possibly caused a problem with the results collected. Explain how.
- 4** Use a die to sample the following variations on simulation techniques over 20 trials and compare the results collected.
- i** Use a cup to shake the die before rolling
 - ii** Pick up the die and flick straight out of your hand without shaking
 - iii** Shake the die in your hands before rolling
 - a** Copy and complete this frequency table to collate your results.

	Frequency					
Complicating factor	1	2	3	4	5	6
use cup to shake						
no shaking of die						
shake in hands						

- b** Decide which of the techniques could have possibly caused a problem with the results collected. Explain how.

- 5** A club manager runs a ‘Happy Hour’ promotion each Sunday afternoon. At the start of ‘Happy Hour’, the bartender spins a wheel to determine the discount applied to all drinks sold during the hour. A discount of 50 cents, \$1, \$1.50 or ‘no discount’ is applied for the hour.



- a** Use a simulation technique (cards, spinner etc.) to estimate the relative frequency of each outcome over one year (trials = 52).
- b** Determine the average discount offered during ‘Happy Hour’ over the year.
- c** The manager notices that the bar staff re-spin the wheel if a ‘No discount’ comes up to keep the members happy.
 - i** In what way does this complicate the simulation that was conducted?
 - ii** Based on your results from part **a**, how often will the wheel need to be re-spun?
 - iii** Use a simulation technique to re-spin the wheel for the ‘No discount’ outcomes. Calculate the new relative frequency for each of the discounts.
 - iv** Calculate the new average discount offered during Happy Hour over the year.
- d** Whenever the ‘No discount’ comes up, the bar staff always re-spin the wheel. Suggest a better simulation method.

7F Constructing a sample space

LEARNING GOALS

- Determine the size of a sample space
- Use a table to construct a sample space
- Use a systematic list to construct a sample space

Why is it essential to use sample spaces?

A sample space is a list of all possible outcomes in a probability experiment, such as the six possible outcomes of rolling one die. Being able to construct a list of all possible outcomes of a probability experiment allows us to calculate the probability of an event occurring.



A sample space is a list of all possible outcomes. The image shows the sample space for throwing one die.

WHAT YOU NEED TO KNOW

- A **sample space** is a list of all possible outcomes. The list can be constructed by either using a table or a systematic list.
- A **table** is used when there are two stages in a probability experiment. The outcomes of one stage are listed across the top and the other stage is listed along the side. The table is then infilled with the combinations of the two trials.
- For example, when tossing a coin and rolling a die:

		Die outcomes					
		1	2	3	4	5	6
Coin outcomes	Head	Head, 1	Head, 2	Head, 3	Head, 4	Head, 5	Head, 6
	Tail	Tails, 1	Tails, 2	Tails, 3	Tails, 4	Tails, 5	Tails, 6

- The **size of the sample space** is determined by the product of the number of outcomes in each stage.
 - For example: A die has 6 outcomes {1, 2, 3, 4, 5, 6} and a coin has two outcomes {H, T}. Therefore, size of sample space = $6 \times 2 = 12$.

- A **systematic list** is constructed by systematically listing all the outcomes of the first stage with each of the outcomes of the second stage in turn.

- For example: A coin is tossed and a spinner is spun with the outcomes A, B, C on each selection. The sample space would be:

Head, A Head, B Head, C
Tail, A Tail, B Tail, C

- If selecting two items from a list, repeats cannot happen.
- For example: Colby, Sharlah, Bridget and Tom nominate for the school tennis team. Only two positions are available. The possible team combinations would be:

		First person selected			
		Colby	Sharlah	Bridget	Tom
Second person selected	Colby	X	S, C	B, C	T, C
	Sharlah	C, S	X	B, S	T, S
	Bridget	C, B	S, B	X	T, B
	Tom	C, T	S, T	B, T	X



Example 9 Using a table to construct a sample space

Nicole is packing for an overseas holiday. She plans on taking:

- 3 shirts that are blue, white and black
 - 2 pairs of trousers that are tan and black
- a If each combination of shirt/trousers is an outcome, calculate the size of the sample space.
- b Use a table to construct a sample space of the different outfit combinations.

WORKING

- a Sample space size = $3 \times 2 = 6$

THINKING

The product of the number of outcomes in each stage determines the size of the sample space.

- b Sample space:

Construct a table showing all outcomes.

		Shirts		
		blue	white	black
Trousers	tan	blue, tan	white, tan	black, tan
	black	blue, black	white, black	black, black



Example 10 Using a systematic list to construct a sample space

A pizza restaurant offers two types of base (thin or thick) and two types of sauce (BBQ or tomato). These can be ordered in any combination.

- Calculate the size of the sample space (the number of pizza combinations).
- Make a systematic list of the pizza combinations.

WORKING

- Sample space size = $2 \times 2 = 4$

- List of combinations:
Thin, BBQ; Thin, Tomato Thick, BBQ;
Thick, Tomato

THINKING

Sample space size = product of number of outcomes in each stage

Start with the first stage (bases) and add each of the elements from the second stage (sauces).



Example 11 Using a table without repetition to determine a sample space

Patrick is in the gym and wants to add weights to the bench press to increase the load. He can choose from 2 kg, 5 kg, 10 kg or 20 kg weights. There is only enough space to add two weights to the machine and only one of each of the weights to choose from.

- Calculate the size of the sample space.
- Use a table to construct a sample space of the different weight combinations.

WORKING

- Sample space size = $4 \times 3 = 12$

- Sample space:

THINKING

Once a weight has been selected, it cannot be selected again so the second selection only has 3 possible outcomes.

Construct a table without repetition of the outcomes.

		Weight No. 1			
		2	5	10	20
Weight No. 2	2	X	5,2	10,2	20,2
	5	2,5	X	10,5	20,5
	10	2,10	5,10	X	20,10
	20	2,20	5,20	10,20	X

Exercise 7F

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a A table is used when there are _____ stages in a probability experiment. The outcomes of one stage are listed across the top and the other stage is listed along the side. The table is then infilled with the _____ of the two trials.
 - b The size of the sample _____ is determined by the _____ of the number of outcomes in each stage.
 - c A systematic list is constructed by _____ listing all the outcomes of the first stage with each of the outcomes of the _____ stage in turn.
 - d If selecting two items from a list, repeats _____ happen.
- 2 For each of the following probability experiments:
 - i Rolling a die
 - ii Tossing a coin
 - iii Selecting a card from a standard deck and noting the colour
 - iv Selecting a card from a standard deck and noting the suit
 - a Determine how many elements are in the sample space.
 - b List the sample space.

APPLICATIONS

Example 9

- 3 When buying a car, Zara had the choice of body type (hatch, sedan, SUV) and colour (white, red, grey, blue).



- a Calculate the size of the sample space to determine how many different combinations of car are possible.
- b Use a table to determine the sample space for car choices.

- 4** A couple is expecting twin babies.
- a** Calculate the size of the sample space to determine how many different combinations of boy/girl are possible.
 - b** Use a table to determine the sample space for the gender of the two babies.

Example 10

- 5** Ryan is ordering a takeaway meal from the local Thai Restaurant. He has a choice of:
- Meat: Chicken, Beef, Prawns
Accompaniment: Steamed Rice, Fried Rice, Noodles
- a** Calculate the size of the sample space to determine how many different meal combinations are possible.
 - b** Use a systematic list to determine the sample space for meal choice combinations.
- 6** Grace is organising her 18th birthday celebrations at a local tavern. The menu has the following options for the set menu:
- Entrée: Salt & Pepper Calamari, Garlic Prawns, Chicken Satay
Main: Fillet Steak, Chicken Schnitzel, Barramundi, Chicken Parmigiana, Pasta Carbonara
- a** Calculate the size of the sample space to determine how many different combinations of entrée and main are possible.
 - b** Use a systematic list to determine the sample space for meal choice combinations.

Example 10

- 7** Greer has a messy sock drawer with two striped, two dotty (spotted), one orange and one black sock all thrown in together. She randomly selects two socks without checking if they are a matching pair.
- a** Calculate the sample size to determine how many different sock combinations are possible.
 - b** Use a table to determine the sample space for sock choices.

**Example 11**

- 8** Mike, Daniel, Brooke, Abby and Holly all nominate to be the student representative for their class. Only two positions are available on the Student Representative Council (SRC).
- a** Calculate the sample size to determine how many different SRC combinations are possible.
 - b** Use a table to determine the sample space for SRC combinations.

7G Determining probabilities for an experiment

LEARNING GOAL

- Use a sample space to determine the probability of outcomes for an experiment

Why is it essential to be able to calculate probabilities?

In the same way that relative frequency can be calculated based on simulations, theoretical probability can be determined by using a sample space; without the need to conduct probability experiments. There are many examples of fields where the ability to calculate probabilities is vital, such as weather and environmental forecasting, insurance (predicting the rate of losses and accidents), engineering (the chance of components failing), and business and finance (chance of future events). In science every time that results are reported it is necessary to calculate the chances that a particular score or measurement could have arisen by chance.



Scientists have to calculate the probability that their results arose by chance.

WHAT YOU NEED TO KNOW

- **Probability** is a numerical indicator of the chance of an event occurring.
 - Probability of an event = $\frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$
- The total number of **outcomes** is the size of the **sample space**.
- **Favourable outcomes** are the elements of the sample space that we are interested in, or which suit our needs.



Example 12 Calculating probabilities from a simple sample space

A standard die is rolled.

- Determine how many elements are in the sample space.
- List the sample space.
- Calculate the probability that a 4 is rolled.
- Calculate the probability that more than a 4 is rolled.
- Calculate the probability that at least 4 is rolled.
- Calculate the probability that less than 4 is rolled.
- Calculate the probability of rolling an odd number.

WORKING

THINKING

a Sample space size = 6	There are 6 faces on a standard die.
b Sample space = 1, 2, 3, 4, 5, 6	Include all results possible.
c 1, 2, 3, 4, 5, 6 Number of favourable outcomes = 1 Probability of 4 = $\frac{1}{6}$	Highlighting, or underlining, favourable outcomes in your sample space is a great exam technique.
d 1, 2, 3, 4, 5, 6 Number of favourable outcomes = 2 Probability of more than 4 = $\frac{2}{6}$ or $\frac{1}{3}$	'More than 4' does not include 4.
e 1, 2, 3, 4, 5, 6 Number of favourable outcomes = 3 Probability of at least 4 = $\frac{3}{6}$ or $\frac{1}{2}$	'At least 4' includes 4.
f 1, 2, 3, 4, 5, 6 Number of favourable outcomes = 3 Probability of less than 4 = $\frac{3}{6}$ or $\frac{1}{2}$	'Less than 4' does not include 4.
g 1, 2, 3, 4, 5, 6 Number of favourable outcomes = 3 Probability of odd = $\frac{3}{6} = \frac{1}{2}$	Odd numbers are 1, 3, 5, ...



Example 13 Calculating probabilities from a sample space in a table

Two standard dice are rolled, and the two faces are added together.

- Determine how many elements are in the sample space.
- Use a table to determine the sample space.
- Calculate the probability that a total of 4 is rolled.
- Calculate the probability that a total of more than a 4 is rolled.
- Calculate the probability that a total of at least 4 is rolled.
- Calculate the probability that a total less than 4 is rolled.
- Calculate the probability of rolling an even total.

WORKING

- a** Sample size = $6 \times 6 = 36$

- b** Sample space:

Die 1

Die 2	+	1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

- c** Number of favourable outcomes = 3

$$\text{Probability of a total of 4} = \frac{3}{36} = \frac{1}{12}$$

- d** Number of favourable outcomes = 30

$$\text{Probability of total more than 4} = \frac{30}{36} \text{ or } \frac{5}{6}$$

- e** Number of favourable outcomes = 33

$$\text{Probability of total of at least 4} = \frac{33}{36} \text{ or } \frac{11}{12}$$

THINKING

Sample size is the product of the number of elements in each stage of the experiment.

Construct a table to show all outcomes.

Highlighting, or underlining, favourable outcomes in your sample space is a great exam technique.

'More than 4' does not include 4.

'At least 4' includes 4.

... Continued

WORKING

THINKING

		Die 1					
	+	1	2	3	4	5	6
Die 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

←..... The sample space is copied here for convenience.

- f** Number of favourable outcomes = 3 ←..... 'Less than 4' does not include 4.
 Probability of total of less than 4 = $\frac{3}{36}$ or $\frac{1}{12}$
- g** Number of favourable outcomes = 18 ←..... Even numbers are 2, 4, 6, ...
 Probability of an even total = $\frac{18}{36}$ or $\frac{1}{2}$

Exercise 7G

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a** Probability is a numerical _____ of the chance of an _____ occurring.
 - b** Probability of an event = $\frac{\text{_____ of favourable outcomes}}{\text{total number of _____}}$
 - c** The _____ number of outcomes is the _____ of the sample space.
 - d** _____ outcomes are the elements of the sample space that suit our _____.

Example 12

- 2 A card is selected from a standard deck of 52 cards.

- a** If the colour of the card is noted:
 - i** determine the size of the sample space
 - ii** list the sample space
 - iii** calculate the probability of selecting a red card
- b** If the suit of the card is noted:
 - i** determine the size of the sample space
 - ii** list the sample space
 - iii** calculate the probability of selecting a Heart

Hint A standard pack of cards has 4 suits.
 Red suits: Hearts♥ and Diamonds♦
 Black suits: Spades♠ and Clubs♣
 Each suit has 13 cards:
 A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

- c** If the face of the card is noted:
 - i** determine the size of the sample space
 - ii** list the sample space
 - iii** calculate the probability of selecting an Ace

APPLICATIONS

Example 13

- 3** Two coins are tossed and the result noted.
 - a** Determine the size of the sample space.
 - b** Use a table to list the sample space.
 - c** Calculate the probability of tossing no tails.
 - d** Calculate the probability of tossing one tail.
 - e** Calculate the probability of tossing two tails.
 - f** Calculate the probability of tossing at least one tail.
- 4** An ice-cream shop offers two types of cones (Waffle and Sugar) and four different flavours of ice-cream (Chocolate, Vanilla, Strawberry and Salted Caramel).
 - a** Determine the size of the sample space.
 - b** Use a table to list the sample space.
 - c** Calculate the probability of a person ordering Salted Caramel in a Waffle cone.
 - d** Calculate the probability of a person ordering Salted Caramel ice-cream.
 - e** Calculate the probability of a person ordering ice-cream in a Waffle cone.
 - f** Calculate the probability of a person ordering Chocolate ice-cream or a Waffle cone.
- 5** A card is selected from a standard deck of 52 cards. Complete the following if the suit and face of the card is noted.
 - a** Determine the size of the sample space.
 - b** Use a table to list the sample space.
 - c** Calculate the probability of selecting the Ace of Hearts.
 - d** Calculate the probability of selecting a Picture Card (J, Q, K).
 - e** Calculate the probability of selecting an Ace or a Queen.
 - f** Calculate the probability of selecting a Spade or a Queen.
 - g** Calculate the probability of selecting a Picture Card or a Club.



- 6** Two four-sided dice are rolled, and the two faces are added together.

- a** Determine the size of the sample space.
- b** Use a table to list the sample space.
- c** Calculate the probability of a total of 5 being rolled.
- d** Calculate the probability of a total of at least 5 being rolled.
- e** Calculate the probability of a total that is more than 5 being rolled.
- f** Calculate the probability of a total that is less than 5 being rolled.

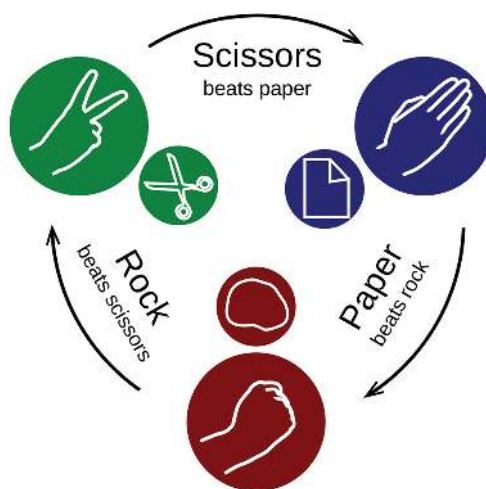


- 7** A student guesses the answer to two multiple choice questions in a test. Each question has four options, with only one being correct.

- a** Determine the size of the sample space.
- b** Use a table to list the sample space.
- c** Calculate the probability of getting both questions correct.
- d** Calculate the probability of getting one question correct.
- e** Calculate the probability of getting at least one question correct.
- f** Calculate the probability of getting no questions correct.

- 8** Hayley and Ben play 'Rock-Paper-Scissors' to decide who will pay for ice-cream. Each player randomly selects to be 'rock' or 'paper' or 'scissors'. The rules are:

- a** Determine the size of the sample space.
- b** Use a table to list the sample space.
- c** Calculate the probability of Ben playing 'Scissors'.
- d** Calculate the probability of both players playing 'Rock'.
- e** Calculate the probability of both players playing the same move.
- f** Calculate the probability of Hayley winning by playing 'Paper'.



Hint If two players play the same move, it is a draw and the game is played again.

7H Using tree diagrams to determine probabilities

LEARNING GOALS

- Construct a tree diagram for two or more trials
- Determine a sample space from a tree diagram
- Determine probabilities for experiments using a tree diagram

Why is it essential to be able to use a tree diagram to determine probabilities?

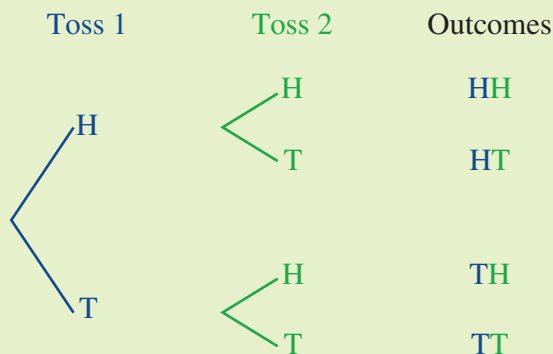
Tables and systematic lists are useful for determining sample spaces for probability experiments with up to two trials. If there are more than two trials, these methods are no longer useful, and we use a tree diagram instead. Tree diagrams help us determine sample spaces and probabilities for experiments with two or more trials.



When tossing two coins multiple times, it is useful to use a tree diagram to represent the outcomes.

WHAT YOU NEED TO KNOW

- Outcomes** for each **trial** are listed in columns. The sample space is a list of all possible outcomes written in a column at the end of the branches.
 - For example, if a coin is tossed two times, the **tree diagram** would be:



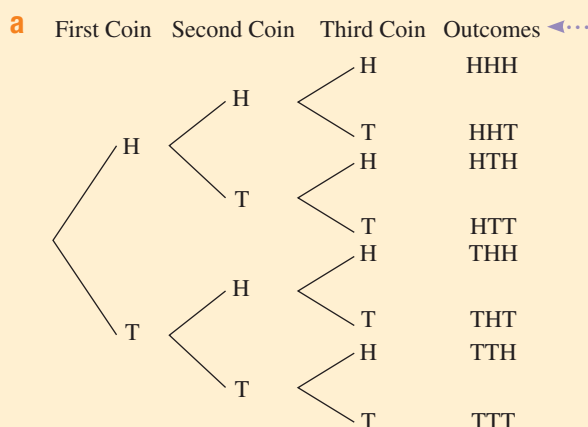


Example 14 Using a tree diagram to determine probabilities

Lilly tosses three coins for a probability experiment, noting the result from each coin as a Head (H) or a Tail (T).

- Construct a tree diagram showing all possible outcomes of the experiment.
- Determine the number of outcomes.
- Use the tree diagram to list all possible outcomes of the experiment.
- Calculate the probability of tossing three heads.
- Calculate the probability of tossing two heads.
- Calculate the probability of tossing one or two heads.
- Calculate the probability of tossing at least one head.

WORKING



THINKING

Construct a tree diagram ensuring that only the possible outcomes at each stage are included.

- There are 8 outcomes. Count how many outcomes are in the final column.
- The outcomes are HHH, HHT, HTH, HTT, THH, THT, TTH, TTT. Read across each branch set to create a list of the outcomes.
- Number of favourable outcomes = 1
Probability of HHH = $\frac{1}{8}$ There is only one outcome of HHH.
- Number of favourable outcomes = 3
Probability of two heads = $\frac{3}{8}$ HHT, HTH, THH are all favourable outcomes.
- Number of favourable outcomes = 6
Probability of one or two heads = $\frac{6}{8} = \frac{3}{4}$ HTT, THT, TTH, HHT, HTH, THH are all favourable outcomes.
- Number of favourable outcomes = 7
Probability of at least one head = $\frac{7}{8}$ Except for TTT, all other outcomes are favourable.



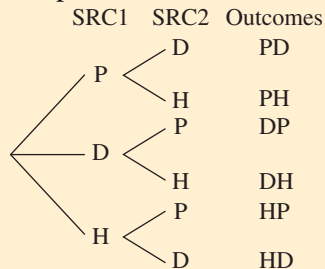
Example 15 Using a tree diagram without replacement to determine probabilities

Preston, Doug, and Hannah all nominate for the two positions as class Student Representative Council (SRC) representative.

- Construct a tree diagram showing all possible outcomes.
- Determine the number of outcomes.
- List all possible outcomes of the experiment from the tree diagram.
- Calculate the probability that Doug and Hannah are the SRC representatives.
- Calculate the probability that Preston is an SRC representative.

WORKING

- a** All possible outcomes:



THINKING

Each person can only be selected once. If selected in the first round, they cannot be selected in the second round.

- There are 6 outcomes. Count how many outcomes are in the final column.
- The outcomes are PD, PH, DP, DH, HP, HD. Read across each branch set to create a list of the outcomes.
- Number of favourable outcomes = 2
Probability that Doug and Hannah are the representatives = $\frac{2}{6}$ or $\frac{1}{3}$ DH and HD are both favourable outcomes.
- Number of favourable outcomes = 4
Probability that Preston is the representative = $\frac{4}{6}$ or $\frac{2}{3}$ Count the outcomes with P.

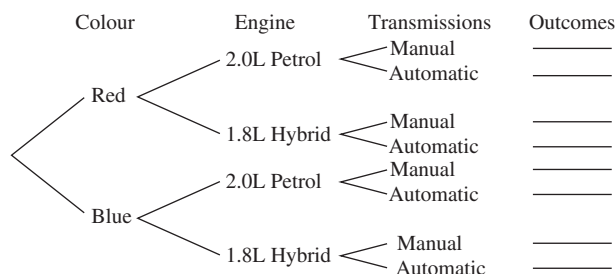
Exercise 7H

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a Tables and systematic lists are useful for _____ sample spaces for probability experiments with up to _____ trials.
 - b Tree diagrams help us determine sample spaces and _____ for experiments with 2 or _____ trials.
 - c Outcomes for each trial are listed in _____. The sample space is a list of all possible _____ written in a column at the end of the branches.

APPLICATIONS

- 2 Tenille is looking to buy herself a car. The car she has decided on comes with a choice of colour, engine type and transmission. A tree diagram of the possible choices is shown.



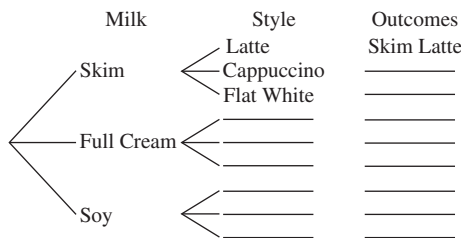
- a Complete the Outcomes column.
- b Determine how many car combinations are possible.
- c Calculate the probability that Tenille selects a Red Hybrid Automatic car.
- d Calculate the probability that Tenille selects a Red Automatic car.
- e Calculate the probability that Tenille selects an Automatic car.



3 When ordering a coffee, Jayde has a choice of:

- Milk: Skim, Full Cream, Soy
- Style: Latte, Cappuccino, Flat White

a Complete the tree diagram for her coffee order possible combinations:



- b** Determine how many order combinations are possible.
- c** List all the possible coffee combinations.
- d** Calculate the probability Jayde orders a Skim Milk Latte.
- e** Calculate the probability Jayde orders a Latte.
- f** Calculate the probability Jayde does not order a coffee made with Soy.

Example 14

4 Felix conducts a probability experiment by selecting a card at random from a standard deck of cards and noting the colour as Red (R) or Black (B). He conducts three trials of the experiment and returns the card to the deck after each selection.

- a** Construct a tree diagram showing all possible outcomes of the experiment.
- b** Determine the number of outcomes.
- c** Use your tree diagram to list all possible outcomes of the experiment.
- d** Calculate probability of Felix selecting exactly three red cards.
- e** Calculate probability of Felix selecting exactly two red cards.
- f** Calculate probability of Felix selecting less than two red cards.

5 Lochie is enjoying a meal at Murphy's Grill. The menu is shown:

- a** Construct a tree diagram showing all possible steak meal combinations of:
 - Steak: Eye (E), Rib (R) or Rump (P)
 - Potato: Chips (C) or Mash (M)
 - Side: Vegetables (V) or Salad (S)

- b** Determine the number of meal combinations.
- c** List all the meal combinations.
- d** Calculate the probability that Lochie has the Rump steak served with chips and salad.
- e** Calculate the probability that Lochie has the Rib Fillet steak served with chips.

Murphy's Grill	
200G EYE FILLET	34
250G RIB FILLET	35
350G RUMP	32
OUR GRILLS ARE SERVED WITH CHIPS OR MASH VEGETABLES OR SALAD	

- 6** Will has noticed that the school bus is equally likely to be either on Time (T) or Late (L) each morning. He conducts a probability experiment by rolling a die to determine the likelihood of being late for the next three mornings. If the result on the die is even then the bus is on Time (T); if the result is odd then the bus is Late (L).
- Construct a tree diagram showing all possible outcomes of the experiment.
 - Determine the number of outcomes.
 - List all the possible outcomes.
 - Calculate the probability that Will is on time for school all three days.
 - Calculate the probability that Will is late for school on one of the three days.
 - Calculate the probability that Will is late for school at least once over the three days.

Example 15

- 7** Chloe, Gabby, Libby and Michelle all nominate for the two positions in the school tennis team.
- Construct a tree diagram showing all possible outcomes.
 - Determine the number of outcomes.
 - List all the possible outcomes.
 - Calculate the probability that Libby and Gabby are in the tennis team.
 - Calculate the probability that Chloe is in the tennis team.
- 8** Cortay wants to have an ice-cream. He can have his choice of Waffle (W) or Sugar cone (S) and likes to have two scoops of ice-cream with *different* flavours. The flavour choices are: Chocolate (C), Wild Berry (B) and Vanilla (V).
- Construct a tree diagram showing all possible ice-cream outcomes.
 - Determine the number of outcomes.
 - List all the possible outcomes.
 - Calculate the probability that Cortay has a Waffle cone with Wild Berry and Chocolate ice-cream.
 - Calculate the probability that Cortay has no Chocolate ice-cream.



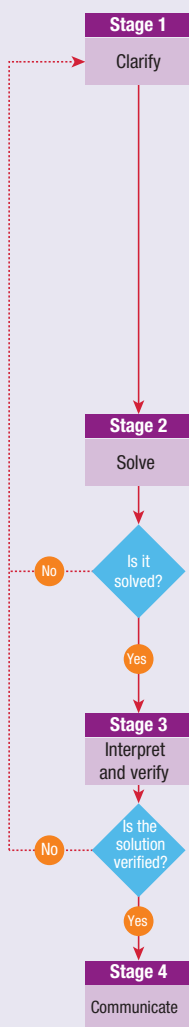
Statistical investigation process task

Background: During periods of extended drought, farmers reduce the size of their breeding flock due to a lack of available food.

Once the drought finally breaks, it takes several breeding seasons to return the herd to its original size through the breeding program.

Task: Assume a sheep farmer reduces his herd of 2400 female breeders to just 500 female breeders during a drought. He knows from previous breeding seasons that the chance of a female ewe having twins is $\frac{1}{6}$. You are to perform a simulation of several breeding seasons and estimate how many breeding seasons it will take for the farmer to return his female breeding herd to the original size.

Approach to statistical investigation process task.



Stage 1: Clarify

Clarify the problem and pose one or more questions that can be answered with data

- 1 What are you required to do?
- 2 What information do you have?
- 3 What other information do you need? E.g. For how many years is a breeding ewe able to produce lambs?
- 4 What assumptions will you make? E.g. What about the male lambs?
- 5 How could you conduct the simulation? E.g. Will you start with a smaller size sample?
- 6 How will you record the results of your simulation?

Stage 2: Solve

Design and implement a plan to collect or obtain appropriate data and select and apply appropriate graphical or numerical techniques to analyse the data

- 7 Gather information by conducting a simulation.
- 8 Use mathematics to calculate information required to solve the problem.
- 9 Produce graphs/tables required to solve the problem.
- 10 Have you been able to determine a time line for restoring the breeding flock to full capacity following the drought?

Stage 3: Interpret and verify

Interpret the results of this analysis and relate the interpretation to the original question

- 11 Have you answered the question?
- 12 Is the answer you have reasonable?

Stage 4: Communicate

Communicate findings in a systematic and concise manner

Write a report with an introduction, body and conclusion.

- 13 State your main point.
- 14 Include the evidence in the form of statements, graphs and tables.
- 15 Explain the evidence. Use a sentence starter like 'This means ...'
- 16 In the conclusion, link back to your main point.

Chapter checklist

☐ I can interpret commonly used probability terms.

- 1 If the Fremantle Dockers are unlikely to beat the West Coast Eagles.
 - a Which team is most likely to win the Western Derby?
 - b Give an appropriate probability value for the chance of a West Coast Eagles victory.

☐ I can perform simulations of probability experiments using technology.

- 2 Generate 20 random numbers with a calculator or spreadsheet.

☐ I can recognise that repetition of chance events is likely to produce different results.

- 3 If a die is rolled 300 times, calculate how many times you would expect each side to appear.

☐ I can identify relative frequency as probability.

- 4 A survey asked students how many sports they currently participate in. The table below shows the findings.

Number of sports	0	1	2	3
Number of students	42	57	24	2

If a student is selected at random:

- a calculate the probability that the student plays one sport. Round your answer to 2 decimal places.
- b calculate the probability that the student plays fewer than two sports. Round your answer to 2 decimal places.

☐ I can identify factors that could complicate the simulation of real-world events.

- 5 Explain the potential problems of doing a simulation by drawing cards from an unshuffled pack of cards.
- 6 In a probability simulation, explain if the coin should always be held the same way up before tossing.

☐ I can construct a sample space for an experiment.

- 7 A coin is tossed twice. Determine the sample space by using:
 - a table
 - b a systematic list
 - c a tree diagram

☐ I can use a sample space to determine the probability of outcomes of an experiment.

- 8 Use your sample space from the previous question to determine the probability of:
 - a two tails
 - b one tail
 - c at least one tail

☐ I can use a tree diagram to determine the outcomes and probabilities for experiments.

- 9 Sam has three shirts. They are blue, red and white. He is packing to go away for the weekend and needs to pack two shirts.
 - a Construct a tree diagram to show the possible combinations of the shirts he packs.
 - b Determine the probability that he packs the blue and red shirts.
 - c Determine the probability that he packs the white shirt.

Chapter review

Section 7B

- 1** Sam tosses a coin to simulate the gender of 30 job applicants at a fast food store during a week. Heads = girl, tails = boy. The results of the simulation are:

T	T	T	H	T	T	H	H	T	H
T	H	H	H	T	T	H	T	T	T
H	T	T	T	T	T	H	H	T	T

- a** Record the results of the simulation in a frequency table.

Result	Frequency
heads/girls	
tails/boys	

- b** Determine the percentage of boys from your simulation.
c Calculate how many boys you expect to find in the sample.
d Decide if the simulation varied from what you expected. Explain.
- 2** Celeste tosses two coins to simulate the gender of the children in 20 families with two children. Heads = girl, tails = boy. The results of the simulation are:

T	H	H	H	H	T	T	H	H	T
T	T	H	T	H	H	T	T	T	H
T	T	H	T	H	H	T	H	T	H
H	T	T	T	T	T	H	H	H	T

- a** Copy and complete the following frequency table to record your results.

Two-child combination	Tally	Frequency
boy + boy		
boy + girl		
girl + girl		

- b** Calculate the relative frequency of two-child families with a boy and a girl combination in your simulation.

Section 7C

- 3** In a probability experiment, Ayuen selects a card from a standard deck of cards and notes the suit. He repeats the experiment 20 times. The results are:

H	D	C	S	D	D	H	C	C	D
S	D	S	H	D	C	H	D	S	H

- Identify how many different outcomes there are when a card is selected at random from a deck and the suit noted.
- Determine how many times you would expect to get the same result as the previous selection in the 20 trials.
- Calculate how many times the same result appears in consecutive selections. Compare this answer with what you expected from part **b**.
- Determine how many times you would expect each outcome to appear in the 20 selections.

Section 7D

- 4** Ayuen thinks that some results appear more than others in his probability experiment.

- Using the data from question **3**, complete a frequency table for the experiment.
- Calculate the relative frequency for each outcome.
- Determine if any results appear more/less often than expected.
- Describe how Ayuen could change the experiment so that each outcome appears the same amount of times.

- 5** A survey asked shoppers how many re-usable bags they had with them when entering a supermarket. The table below shows the findings.

Number of bags	0	1	2	3	4	5
Number of shoppers	15	24	18	35	18	12

If a shopper is selected at random:

- calculate the probability that the shopper has 2 bags with them. Round your answer to 2 decimal places.
- determine the probability that the shopper has fewer than 4 bags with them. Round your answer to 2 decimal places.

Section 7E

6 Identify potential problems that could occur with the following simulation experiments.

- a** Not returning a selected card to the deck after each selection but thoroughly shuffling between selections.
- b** Always selecting the top card from a deck of cards.
- c** Always starting with heads when tossing a coin.
- d** Allowing the coin to fall to the ground from the toss.
- e** Picking up a die and flicking it straight out of your hand without shaking first.
- f** Using a cup to shake a die before rolling.

Section 7F

7 Two regular dice are rolled, and the two faces are added together.

- a** Determine the size of the sample space.
- b** Use a table to list the sample space.

8 Two coins are tossed, and the result noted.

- a** Determine the size of the sample space.
- b** Use a systematic list to determine the sample space.

Section 7G

9 Two regular dice are rolled and the two faces are added together. Use the sample space from question **7 b** to:

- a** Calculate the probability that a total of 8 is rolled.
- b** Calculate the probability that a total of at least 8 is rolled.
- c** Calculate the probability that a total more than 8 is rolled.
- d** Calculate the probability that a total less than 8 is rolled.

10 Two coins are tossed, and the result noted. Use the sample space from question **8 b** to:

- a** Calculate the probability of tossing two heads.
- b** Calculate the probability of tossing one head.
- c** Calculate the probability of tossing no heads.
- d** Calculate the probability of tossing at least one head.

Section 7H

11 A die is rolled three times and the outcome is noted as Odd (O) or Even (E)

- a** Draw a tree diagram showing all possible outcomes of the experiment.
- b** Determine the number of outcomes.
- c** From your tree diagram, list all possible outcomes of the experiment.

- d Calculate the probability of rolling two even numbers.
- e Calculate the probability of rolling three even numbers.
- f Calculate the probability of rolling less than two even numbers.

12 Connor estimates that his school bus is late once a week.

- a Show that $P(\text{bus is late})$ is 0.20.
- b Use the Random number feature on your calculator to simulate if the bus is late (0.001–0.200) or on time (0.201–0.999) for the next 4 school weeks.
- i Organise your results in a table.

- ii Determine many times is the bus 'late' in the simulation.
- c Connor estimated that the bus was late 20% of the time.
 - i In 4 weeks, determine how many times would you expect the bus to be late.
 - ii Describe any differences or similarities between the simulation in part b and the expected answer found in part c i.

13 Tanya wants to order Chinese takeaway for her family. They have four favourite dishes that she usually orders from: Lemon Chicken (C), Mongolian Lamb (L), Sweet and Sour Pork (P) and Satay Beef (B). Tanya needs to only order two *different* meals tonight.

- a Determine the size of the sample space.
- b Use a table to list the sample space.
- c Calculate the probability that Tanya orders Sweet and Sour Pork.
- d Calculate the probability that Tanya orders Lemon Chicken and Satay Beef.
- e Calculate the probability that Tanya orders Lemon Chicken and Satay Beef or Mongolian Lamb.
- f Calculate the probability that Tanya orders Lemon Chicken or Satay Beef.

- 14** As part of a History investigation into the one-child policy once adopted in rural communities in China, Anna wants to conduct a probability experiment to determine the average number of children a family would need to have in order to have at least one son. She decides to toss a coin, and say that heads = girl, tails = boy. The results of her tosses are below. She has indicated the number of children required for the first three families to have a boy child by grouping results together until a tail is tossed.

H	T	H	T	H	H	T	H	H	T
H	T	H	H	T	T	T	T	T	T
H	H	H	H	T	T	T	H	H	H
T	T	T	T	H	H	T	T	T	H

- a** Continue to identify the size of 20 families in Anna's simulation.
b Organise the results into a frequency table as follows.

Number of children	Tally	Frequency
1		
2		
3		
4		
5		
Total		

- c** Calculate the average family size based on Anna's simulation.
d Calculate the relative frequency for:
i 2 children
ii 1 child

8

Location and time zones



In this chapter

- 8A** Angle measurement and arc length
- 8B** Latitude and longitude
- 8C** Time zones and time differences
- Mathematical thinking process task
- Chapter checklist
- Chapter review

Syllabus reference

Unit 4 Topic 2: Earth geometry and time zones (15 hours)

Location

- locate positions on the earth's surface given latitude and longitude using a range of methods; for example, a global positioning system (GPS), a globe, an atlas and digital technologies
- use the arc length formula to calculate distances between two places on Earth on the same longitude
- determine distances between two places on Earth using appropriate technology

Time

- understand the link between longitude and time
- solve problems involving time zones in Australia and neighbouring nations making any necessary allowances for daylight saving
- solve problems involving Greenwich Mean Time and the International Date Line
- determine time differences between two places on Earth
- solve problems associated with time zones; for example, internet and phone usage
- solve problems relating to travelling east and west, incorporating time zone changes

Pre-test

- 1** Find the sum of these two angles.
 - a** 27.54° and 14.95°
 - b** 114.97° and 1.36°
 - c** 7.24° and 65.82°
- 2** Find the difference between these two angles.
 - a** 27.54° and 14.95°
 - b** 114.97° and 1.36°
 - c** 7.24° and 65.82°
- 3** Using the formula $d = \frac{\theta}{360} \times 2\pi r$, find the length of (to 2 decimal places):
 - i** minor
 - ii** major arcs of a circle with a central angle and radius of
 - a** 46° and $r = 25$ cm
 - b** 187° and $r = 100$ km
 - c** 98° and $r = 6371$ km
- 4** Multiply the decimal by 60 minutes and round to the nearest whole minute.
 - a** 0.25
 - b** 0.8
 - c** 0.56
 - d** 0.75
- 5** Divide your answers from questions **1** & **2** by 15° and round down to a whole number.

8A Angle measurement and arc length

LEARNING GOALS

- Convert angle measurements given in decimal form to degrees and minutes and vice versa
- Calculate the length of an arc

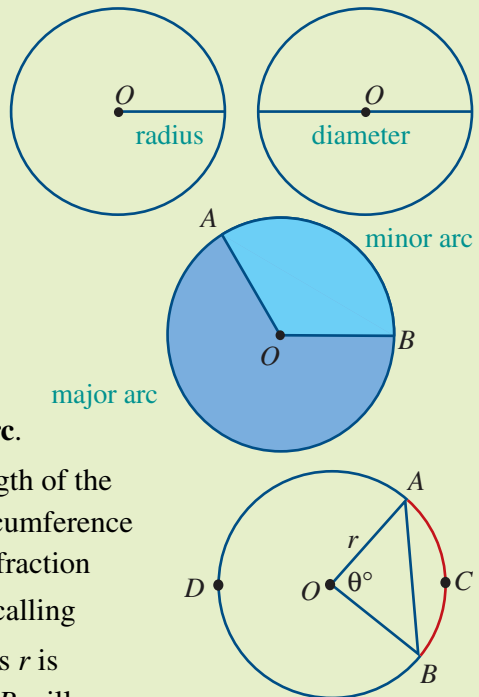
Why is it essential to be able to convert angle measurements given in decimal form to degrees and minutes and vice versa, and how to calculate the length of an arc?

This section will help you to understand and make calculations relating to Earth geometry. It will help you understand and make use of location information in the form of latitude and longitude to locate places on Earth and find the distances between them thus assisting with travel planning.

WHAT YOU NEED TO KNOW

- There are **60 minutes in a degree** where we write 56 minutes as $56'$. To get a feeling for the conversion consider the following.
 - $34.50^\circ = 34^\circ 30'$ as 0.50° is $\frac{1}{2}$ of a degree and so it is equivalent to $30'$ (30 minutes)
 - $34.25^\circ = 34^\circ 15'$ as 0.25° is $\frac{1}{4}$ of a degree and so it is equivalent to $15'$ (15 minutes)
 - $34.75^\circ = 34^\circ 45'$ as 0.75° is $\frac{3}{4}$ of a degree and so it is equivalent to $45'$ (45 minutes)
- **Changing from decimal form to degrees and minutes** is done by multiplying the decimal part of the number by 60. Consider the following.
 - For 34.7° , multiply 0.7° by 60. The result is 42 and we have 34.7° is equal to $34^\circ 42'$.
 - For 34.321° , multiply 0.321° by 60. The result is 19.26 and we have 34.321° is equal to $34^\circ 19'$ to the closest minute.
- **Changing from degrees and minutes to decimal form** is done by dividing the minutes by 60. Consider the following.
 - For $34^\circ 56'$ divide $56'$ by 60. The result is 0.9333 and we have $34^\circ 56'$ is equal to 34.93° , correct to 2 decimal places.
 - For $54^\circ 19'$ divide $19'$ by 60. The result is 0.31666 and we have $54^\circ 19'$ is equal to 54.32° correct to 2 decimal places.

- Any line segment drawn from the centre of a given circle to any point on the circle is called a **radius** (plural radii).
- Any line segment joining two points on the circle and passing through the centre is called the **diameter** of the circle.
- Any two points on a circle divide the circle into arcs. The shorter arc is called the **minor arc**, the longer is the **major arc**.
- Consider the diagram to the right, the length of the arc AB will be a fraction of the whole circumference of the circle which has a radius of r . The fraction of the circumference will be $\frac{\theta}{360}$, and recalling that the circumference of a circle of radius r is given by $2\pi r$. The **arc length** of the arc AB will be $\frac{\theta}{360} \times 2\pi r$.
- The formula for arc length is $d = \frac{\theta}{360} \times 2\pi r$
 - d = length of arc
 - θ = central angle of arc
 - r = radius of circle



Example 1 Converting angle measurements

- Change 32.45° to degrees and minutes.
- Change $44^\circ 32'$ to decimal form. Give your answer to 2 decimal places.

WORKING

- $0.45 \times 60 = 27'$
Therefore, $32.45^\circ = 32^\circ 27'$

- $32 \div 60 = 0.53$
Therefore, $44^\circ 32' = 44.53^\circ$
correct to 2 decimal places.

THINKING

◀..... Multiply the decimal part of the number by 60.

◀..... Divide the minutes part of the angle by 60.



Calculator activity 8A Converting angle measurements on a calculator

Solve the following using a calculator.

Casio fx82

- a. Change 32.45° to degrees and minutes.

Type 32.45 press $\boxed{\circ}'\boxed{''}$ $>$ $=$ to get the degrees and minutes form
 $32.45^\circ = 32^\circ 27'$

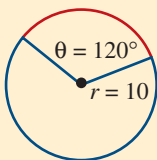
- b. Change $44^\circ 32'$ to decimal form.

Type 44 $\boxed{\circ}'\boxed{''}$ $>$ 32 $\boxed{\circ}'\boxed{''}$ $>$ $=$ $>$ $\boxed{\circ}'\boxed{''}$ to get the decimal form 44.533 ...



Example 2 Calculating the length of an arc

In this circle of radius length 10 cm, the angle at the centre is 120° .
 Find the small arc length correct to 1 decimal place.



WORKING

THINKING

$$d = \frac{\theta}{360} \times 2\pi r$$

←..... Write down the formula for the length of an arc.

$$d = \frac{120}{360} \times 2 \times \pi \times 10$$

←..... Substitute $\theta = 120^\circ$ and $r = 10$ into the equation.

$$d = \frac{20\pi}{3}$$

$$d = 20.9 \text{ cm to 1 decimal place}$$

Exercise 8A

Example 1

- 1 Convert each of the following angle measurements from decimal form to degrees and minutes.

a 32.45° b 43.20° c 122.46° d 91.12° e 0.75°

- 2 Convert each of the following angle measurements from degrees and minutes to decimal form.

a $32^\circ 45'$ b $15^\circ 35'$ c $7^\circ 22'$ d $142^\circ 44'$ e $67^\circ 15'$

3 What is the circumference of each circle? Answer correct to 2 decimal places.

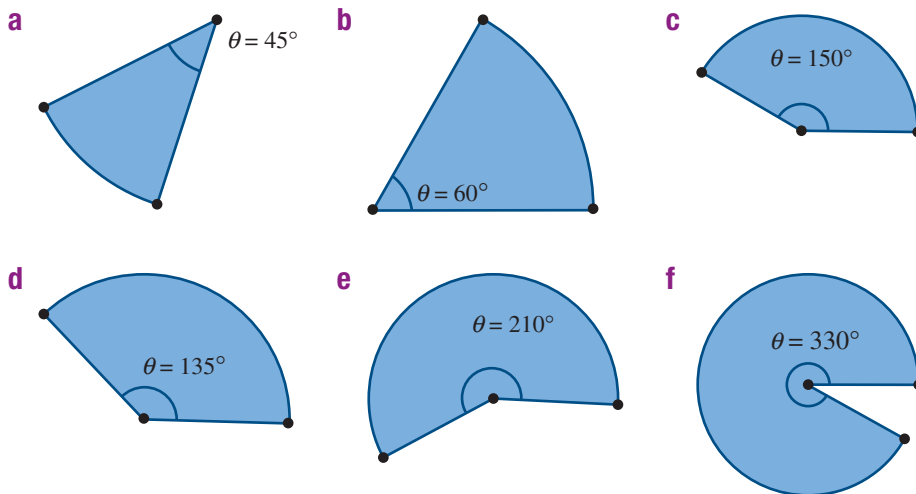
- a** Radius of 8 cm
- b** Radius of 14 m
- c** Radius of 45 mm
- d** Diameter of 12 mm
- e** Diameter of 14 m

4 What fraction of a circle is each sector?

- a** Angle at the centre is 90°
- b** Angle at the centre is 270°
- c** Angle at the centre is 30°
- d** Angle at the centre is 120°
- e** Angle at the centre is 60°
- f** Angle at the centre is 150°

Example 2

5 Find the arc length of each sector. The radius is 10 cm. Answer correct to 2 decimal places.



6 Find the arc length where the radius of the circle (given in cm) and the angle subtended at the centre are given. Give your answer correct to 2 decimal places.

- a** $r = 15, \theta = 50^\circ$
- b** $r = 20, \theta = 15^\circ$
- c** $r = 30, \theta = 150^\circ$
- d** $r = 16, \theta = 135^\circ$
- e** $r = 40, \theta = 175^\circ$
- f** $r = 30, \theta = 210^\circ$

8B Latitude and longitude

LEARNING GOALS

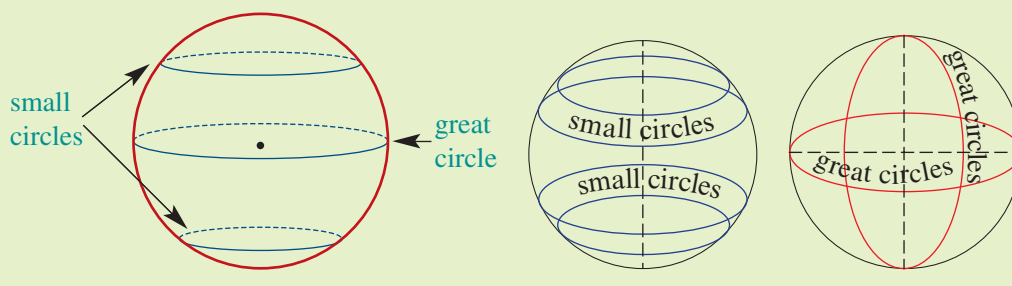
- Understand the grid system for locating places on Earth using latitude and longitude
- Recall the names and placement of the important lines of latitude and longitude
- Determine how to use latitude and longitude to locate places on Earth
- Calculate distances between locations on Earth using great circles

Why is it essential to understand and know how to use latitude and longitude?

Global Positioning Systems (GPS) and most of the maps we use include latitude and longitude. These lines allow you to quickly and accurately locate places and features on Earth's surface. This also allows you to calculate the distance between locations on Earth to assist in making travel plans.

WHAT YOU NEED TO KNOW

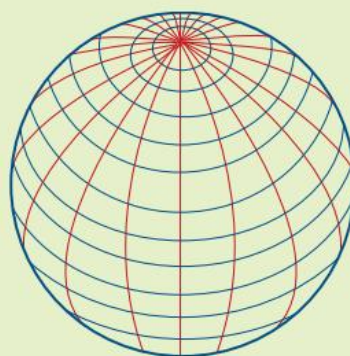
- A **great circle** is the circular boundary of a cross-section of a sphere that **contains a diameter** of the sphere. The cross-section contains the centre of the sphere.
- The circular boundary of cross-sections of the sphere that **do not contain a diameter** of the sphere are called **small circles**. The cross-section for a small circle does not contain the centre of the sphere.
- The diagrams below show great and small circles. The great circle in the left diagram is in the *plane of the equator*.



- The **shortest distance** between two points on the surface of Earth is the distance along the great circle that passes through those two points.
 - The great circle path from New Delhi to New York is shown in the figure to the right.
 - Below is a representation of the great circle route from Perth (PER) to London (LON).



- Similar to how we described points in the plane with Cartesian coordinates, to describe the location of points on the surface of Earth we use a grid of lines as shown here. These lines give *coordinates* called **latitude** and **longitude**. The red lines will be used for longitude and the blue lines for latitude.



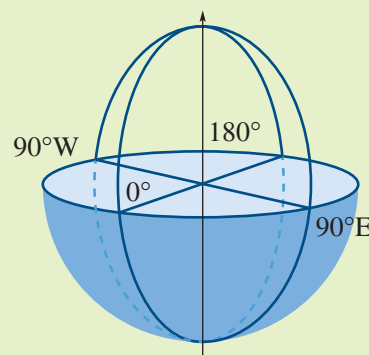
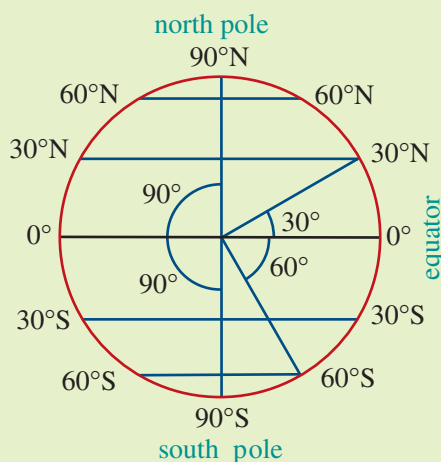
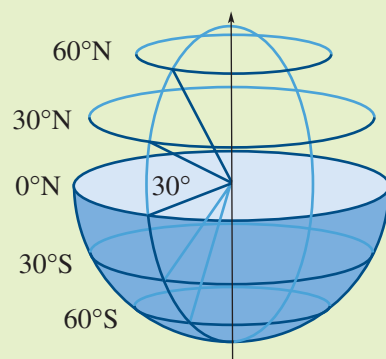
- **Meridians of longitude** are *semi-great circles* (an arc that goes from pole to pole) that pass through the north and south poles. The red lines on the sphere are meridians of longitude.
- **Parallels of latitude** are small circles whose planes are parallel to that of the equator. The blue lines in the sphere are parallels of latitude. The equator is the only latitude that is a great circle.

- The blue lines in the diagram opposite are parallels of latitude. The latitude of a point on a sphere is the elevation of the point from the plane of the equator.

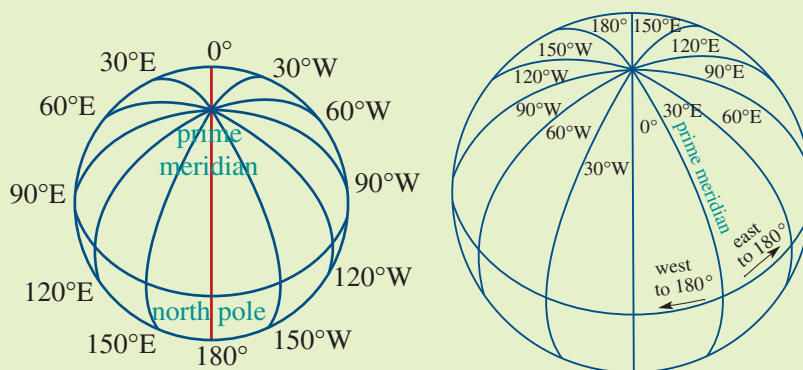
- The equator has latitude 0°N .
- The north pole has latitude 90°N .
- The south pole has latitude 90°S .
- The latitudes 60°N , 30°N , 60°S and 30°S are shown.

- In the diagram opposite, Earth has been sliced in half along a great circle. The vertical line through the poles is perpendicular, or at 90° , to the plane of the equator. At the surface of Earth, at a given latitude, draw a line from that location to the centre of Earth. The angle between this line and the equator is the latitude measurement. The diagram shows two examples: one for 30°N and one for 60°S .

- Lines of longitude are measured in degrees east or west of the **prime meridian** (0°).
 - The prime meridian is just the meridian of longitude chosen to measure 0° in longitude.
 - The lines of longitude shown in the diagram opposite are 0° , 90°E , 180° and 90°W . Note that you don't need to add E or W to the 0° or the 180° .
 - The prime meridian passes through Greenwich in England.
- The longitude of a location is the angle formed between the prime meridian and the line from the centre of Earth to the point where the meridian of longitude at that location meets the equator.

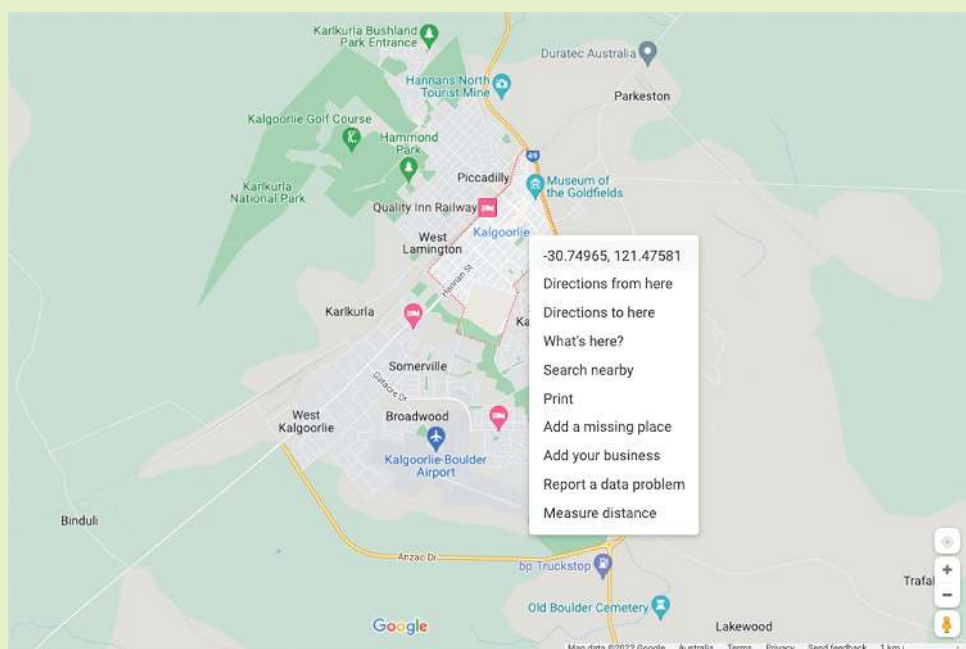


- The two diagrams below are diagrams of Earth looking down from the north pole, the evident plane in each diagram is the plane of the equator. They show different ways of looking at longitudes around Earth. From them it can be seen that meridians of longitude on the same great circle, but different sides of the prime meridian, are in opposite direction and add to 180° . For example:
 - The meridian 120°W is on the same great circle as the meridian 60°E .
 - The meridian 30°W is on the same great circle as the meridian 150°E .



- Any point on Earth's surface can be described by giving its latitude and longitude. For example, Perth has latitude 31.9523°S and longitude 115.8613°E . Albany has latitude 35.0268°S and longitude 117.8837°E . These are called the coordinates of the location.
- Hemispheres are used to describe different regions of Earth and are as follows.
 - **Northern hemisphere** is the half of Earth that lies north of the equator. Locations in the northern hemisphere, such as London, have their latitude described using $^\circ\text{N}$. For London this is 51.5°N .
 - **Southern hemisphere** is the half of Earth that lies south of the equator. Locations in the southern hemisphere, such as Perth, have their latitude described using $^\circ\text{S}$. For Perth this is 32.95°S .
 - **Western and eastern hemispheres** are defined through the prime meridian. Perth is in the eastern hemisphere and New York in the western hemisphere.
- Different technologies and tools can be used find the longitude and latitude of a location.

- Google Maps can be used to quickly find the longitude and latitude of anywhere on Earth. By clicking on a location in Google Maps, information is obtained including the latitude and longitude (the precise method will depend on your device and browser or app, check Google Maps Help if needed). Below is a map of a section of Inland Western Australia obtained from Google Maps and it shows the details for the town of Kalgoorlie. Here the coordinates of Kalgoorlie are given as $(-30.74965, 121.47581)$. Changing this form to the hemisphere notations is 30.7497°S , 121.4758°E and changing to degrees and minutes is $30^{\circ}44'\text{S}$, $121^{\circ}28'\text{E}$. This may be done with any location. Other methods of finding a location include using a Global Positioning System (GPS) or using an atlas.

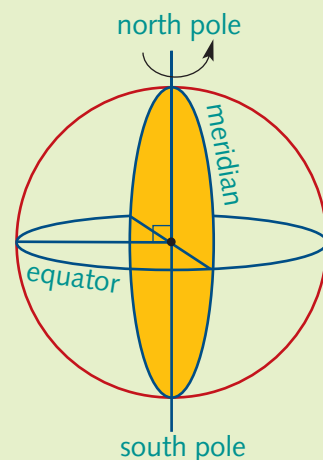


- **GPS** is a system used for worldwide navigation and surveying. It is commonly used for determining an exact location anywhere on Earth by obtaining the current time at a specific location. This is made possible by the network of 24 man-made satellites, called GPS satellites.
 - The system was used for military purposes; the GPS system became available for use by all people approximately 30 years ago. GPS provides latitudes and longitudes given to a high degree of accuracy. For example, through Google Maps you could enter an address and the GPS coordinates will be given for the place, or vice-versa.

- On a flat surface, the shortest distance between two points is a straight line. Since Earth's surface is curved, the shortest distance between A and B is the arc length AB of the great circle (the meridian) that passes through A and B. This is called the great circle distance.
- We can calculate the distance between two points on Earth using the difference in their latitudes, which we call the **angular distance** or $\varnothing$, and the arc length formula. Note that great circles of Earth have a radius of about 6371 km. The angular distance can be calculated as overpage.
 - Beijing (China) and Perth (Australia) have coordinates (40°N, 116°E) and (32°S, 116°E), respectively. These two cities have the same longitude to the nearest degree. As the cities are in different hemispheres, north and south, we need to add the latitudes to determine the angular distance. The angular distance = $(40 + 32) = 72^\circ$.
 - For Perth and Kununurra, the latitudes are 32°S and 16°S, respectively (to the nearest degree). As the cities are in the same hemisphere, south, we need to subtract the latitudes to determine the angular distance. The angular distance = $(32 - 16) = 16^\circ$. The cities are in the same hemisphere.
 - For locations on the same meridian in different hemispheres, we add the latitudes. For locations on the same meridian in the same hemisphere, we find the difference between the latitudes (always subtract smaller from larger).
- The formula for the distance between two points on the same meridian is given by:

$$d = \frac{\text{angular distance}}{360} \times 2 \times \pi \times 6371 \text{ km}$$

- All meridians pass through the poles and a point on the equator. Therefore, you can use the above formula to find the distance from any point on the surface of Earth to a pole or the equator if we know its latitude.
- The equator is also a great circle and therefore the distance between points on the equator are possible to find using the same arc length logic as above, but now angular distance will be the difference between longitudes.





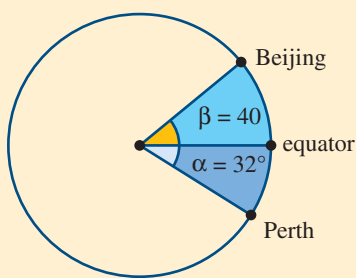
Example 3 Finding a distance along a meridian

Beijing (China) and Perth (Australia) have coordinates $(40^\circ\text{N}, 116^\circ\text{E})$ and $(32^\circ\text{S}, 116^\circ\text{E})$, respectively.

Calculate the shortest distance between Beijing and Perth, to the nearest kilometre.



WORKING



$$\theta = 40 + 32 = 72^\circ$$

$$d = \frac{\text{angular distance}}{360} \times 2 \times \pi \times 6371 \text{ km}$$

$$d = \frac{72}{360} \times 2 \times \pi \times 6371 \text{ km}$$

$$d = 8006 \text{ km}$$

NOTE: The calculated distance given on the internet is 7985 km, which is quite close to our approximate result.

THINKING

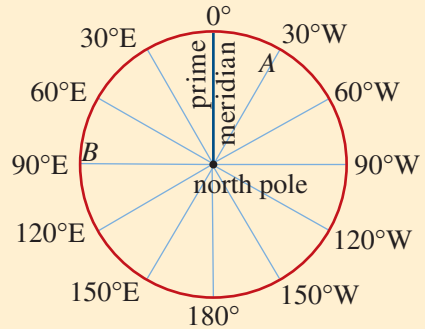
- ◀ The two cities have the same longitude correct to the nearest degree. Therefore they are on the great circle that is the meridian of longitude 116°E .
- ◀ Add the latitudes of each city to find the angle at the centre of the arc.
- ◀ Calculate the distance using the formula.
- ◀ Substitute 72° for the angular distance.
- ◀ Calculate the distance.



Example 4 Finding the distance between two points on the equator

Point A has longitude 30°W and latitude 0° .
 Point B has longitude 90°E and latitude 0° .
 Find the distance between the two points:

- a** if you fly east from A to B
b if you fly west from A to B



WORKING

a $\angle = 30 + 90 = 120^\circ$

$$d = \frac{\text{angular distance}}{360} \times 2 \times \pi \times 6371 \text{ km}$$

$$d = \frac{120}{360} \times 2 \times \pi \times 6371 \text{ km}$$

$$d = 13\,344 \text{ km}$$

Flying east the distance between A and B is 13 344 km.

b $\angle = 360 - 120 = 240$

$$d = \frac{\text{angular distance}}{360} \times 2 \times \pi \times 6371 \text{ km}$$

$$d = \frac{240}{360} \times 2 \times \pi \times 6371 \text{ km}$$

$$d = 26\,688 \text{ km}$$

Flying west the distance between A and B is 26 688 km.

THINKING

◀.. Flying east the angle between the locations is 120° .

◀.. Calculate the distance using the formula.

◀.. Substitute 120° for the angular distance.

◀.. Calculate the distance.
Write the answer in a sentence.

◀.. Flying west the angle is 240° found by subtracting 120° from 360° .

◀.. Calculate the distance using the formula.

◀.. Substitute 240° for the angular distance.

◀.. Calculate the distance.

◀.. Write the answer in a sentence.



Example 5 Finding a distance between two places on the equator

The cities of Pontianak (Indonesia) and Quito (Ecuador) are on the equator to the nearest degree. The longitude of Pontianak is 109°E and the longitude of Quito is 78°W . Find the distance between the two cities flying east from Pontianak to Quito.



WORKING

$$\angle = 78 + 109 = 187^{\circ}$$

$$\angle = 360 - 187 = 173^{\circ}$$

$$d = \frac{\text{angular distance}}{360} \times 2 \times \pi \times 6371 \text{ km}$$

$$d = \frac{173}{360} \times 2 \times \pi \times 6371 \text{ km}$$

$$d = 19\,237 \text{ km}$$

Flying east the distance between Pontianak and Quito is 19 237 km.

THINKING

- ◀◀ Add to find the total angle between the longitudes – we are considering the distance covered by flying west.
- ◀◀ Determine the angle between the two longitudes if flying east.
- ◀◀ Calculate the distance using the formula.
- ◀◀ Substitute 173° for the angular distance.
- ◀◀ Calculate the distance.
- ◀◀ Write the answer in a sentence.



Example 6 Finding the distance to the equator or a pole

Perth has latitude 32°S and longitude 115°E . Find the distance of Perth to:

- a** the equator **b** the south pole **c** the north pole

WORKING

a $\angle = 32 - 0 = 32^\circ$

$$d = \frac{\text{angular distance}}{360} \times 2 \times \pi \times 6371 \text{ km}$$

$$d = \frac{32}{360} \times 2 \times \pi \times 6371 \text{ km}$$

$$d = 3558 \text{ km}$$

Flying east the distance between Perth and the equator is 3558 km.

b $\angle = 90 - 32 = 58^\circ$

$$d = \frac{\text{angular distance}}{360} \times 2 \times \pi \times 6371 \text{ km}$$

$$d = \frac{58}{360} \times 2 \times \pi \times 6371 \text{ km}$$

$$d = 6449 \text{ km}$$

Flying east the distance between Perth and the south pole is 6449 km.

THINKING

◀◀ Along the plane of the meridian 115°E , the difference between the equator and Perth is $(32 - 0 = 32^\circ)$.

◀◀ Calculate the distance using the formula.

◀◀ Substitute 32° for the angular distance.

◀◀ Calculate the distance.

◀◀ Write the answer in a sentence.

◀◀ Along the plane of the meridian 115°E , the difference between the south pole and Perth is $(90 - 32 = 58^\circ)$.

◀◀ Calculate the distance using the formula.

◀◀ Substitute 58° for the angular distance.

◀◀ Calculate the distance.

◀◀ Write the answer in a sentence.

$$\text{c } \angle = 90 + 32 = 122^\circ$$

← Along the plane of the meridian 115°E , the difference between the north pole and Perth is $(90 + 32 = 122^\circ)$.

$$d = \frac{\text{angular distance}}{360} \times 2 \times \pi \times 6371 \text{ km} \quad \leftarrow \text{Calculate the distance using the formula.}$$

$$d = \frac{122}{360} \times 2 \times \pi \times 6371 \text{ km} \quad \leftarrow \text{Substitute } 122^\circ \text{ for the angular distance.}$$

$$d = 13\,566 \text{ km} \quad \leftarrow \text{Calculate the distance.}$$

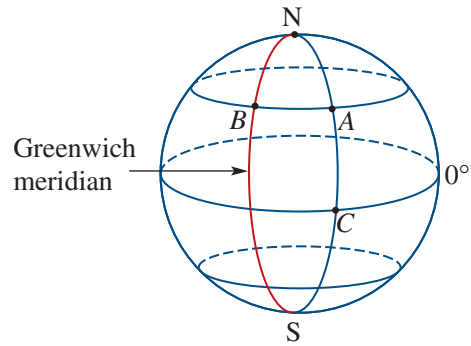
Flying east the distance between Perth and the north pole is 13 566 km. $\leftarrow$ Write the answer in a sentence.

Exercise 8B

- Use an atlas or Google Earth to name the place situated with the following coordinates.
 - $44^\circ 26'\text{N}$, $26^\circ 06'\text{E}$ (44.43°N , 26.10°E)
 - $59^\circ 54'\text{N}$, $10^\circ 45'\text{E}$ (59.91°N , 10.75°E)
 - $29^\circ 55'\text{N}$, $95^\circ 22'\text{W}$ (29.76°N , 95.36°W)
 - $1^\circ 21'\text{N}$, $103^\circ 49'\text{E}$ (1.35°N , 103.82°E)
 - $33^\circ 55'\text{S}$, $18^\circ 25'\text{E}$ (33.92°S , 18.42°E)
- Use an atlas, Google Earth or another method to state the coordinates of:

a Cairo	b Mexico City	c Buenos Aires
d Mumbai	e Lagos	f Harare
- Use Google Maps or an atlas or another method to find the coordinates of each of the following Western Australian locations.
 - Exmouth
 - Port Hedland
 - Albany
 - Collie
 - Geraldton
 - Esperance
 - The Western Australian Northern Territory border (Longitude only)

- 4 On the diagram to the right, the latitude and longitude of point A are $(65^\circ\text{N}, 75^\circ\text{E})$.
- a What are the coordinates of point B?
- b What are the coordinates of point C?



- 5 Athens has the coordinates $(38^\circ\text{N}, 24^\circ\text{E})$ and Sofia has the coordinates $(43^\circ\text{N}, 24^\circ\text{E})$.
- a What is the latitude and longitude of a point 20° due south of Athens?
- b What is the latitude and longitude of a point 20° due east of Athens?
- c What is the latitude and longitude of a point 60° due south of Sofia?
- d What is the latitude and longitude of a point 60° due west of Sofia?
- 6 The following table shows the latitude and longitude of cities around the world given to the nearest degree.

City	Latitude	Longitude
Brisbane	27°S	153°E
Melbourne	38°S	145°E
Cooktown	15°S	145°E
Townsville	19°S	147°E
Marseilles	43°N	5°E
Mexico City	19°N	99°W
Wellington	41°S	175°E
Zurich	47°N	8°E
London	52°N	0° (actually west of Greenwich)
Lima	12°S	77°W
Plymouth	50°N	4°W
Yangon	19°N	96°E

- a Which city (or cities) is closest to the following latitudes?
- i 15°N ii 28°S
- b Which city (or cities) is closest to the following longitudes?
- i 151°E ii 20°W
- iii Greenwich meridian iv Longitude of Sofia (from Question 5)

- c** Which cities are in the northern hemisphere?
- d** Which cities are in the western hemisphere?
- e** Which cities have the same latitude?
- f** Which cities have the same longitude?
- g** Which city is closest to the north pole?
- h** Which city is closest to the south pole?

Example 3, 4

- 7** Two places on the same meridian have latitudes 22°N and 35°S . Find the distance between the two places. Give your answer to the nearest kilometre.
- 8** Cairns 17°S and Melbourne 37.8°S are nearly on the same meridian. Assuming they are, find the distance between them. Give your answer to the nearest kilometre.
- 9** How far apart are Esperance (34°S , 122°E) and Broome (18°S , 122°E)? Give your answer to the nearest kilometre.
- 10** Perth and Onslow are 1920 km apart and both are on the same meridian. If the latitude of Onslow is 17°S , find the latitude of Perth. Perth is south of Onslow.
- 11** Calculate the shortest distance along a meridian in each of the following cases. Give your answer to the nearest kilometre.
 - a** Point X: latitude 10°N , longitude 18°W
Point Y: latitude 45°N , longitude 18°W
 - b** Point X: latitude 14°N , longitude 35°W
Point Y: latitude 13°S , longitude 35°W
 - c** Point X: latitude 23°S , longitude 140°W
Point Y: latitude 67°S , longitude 140°W
 - d** Point X: latitude 15°N , longitude 60°W
Point Y: latitude 25°S , longitude 60°W
 - e** Point X: latitude 15°N , longitude 70°W
Point Y: latitude 15°S , longitude 70°W

Example 4

- 12** The difference of longitude between two points on the equator is 32° . Find the distance between them in kilometres.

Example 5

- 13** There are places in Ecuador (South America) and Somalia (Africa) that are on the equator. In Ecuador, the longitude of a place X on the equator is 78°W and in Somalia the longitude of a place Y on the equator is 42°W . Find the distance between them in kilometres.

- 14** Calculate the shortest distance along the equator between places A and B having the following longitudes:
- a** A 137°E B 87°E
 - b** A 57°E B 13°W
 - c** A 57°W B 27°W
 - d** A 140°E B 160°W
 - e** A 95°W B 113°E

- 15** The distance between two points on the equator is 600 km. What is the difference in their longitudes?

Example 6

- 16** New Orleans has latitude 30°N and longitude 90°W . Find the distance of New Orleans to:
- a** the equator
 - b** the north pole
 - c** the south pole
- 17** Izmir has latitude 38°N and longitude 27°E . Find the distance of Izmir to:
- a** the equator
 - b** the north pole
 - c** the south pole
- 18** Ballarat has latitude 37.5500°S and longitude 143.8500°E . Find the distance of Ballarat to:
- a** the equator
 - b** the north pole
 - c** the south pole
- 19** Find the distance from the given point to the equator and to each of the poles.
- a** Latitude 42°N , longitude 134°E
 - b** Latitude 55°N , longitude 45°W
 - c** Latitude 15°S , longitude 35°E
 - d** Latitude 14°S , longitude 75°W

8C Time zones and time differences

LEARNING GOALS

- Find time differences between two places on Earth using time zones
- Solve problems in time planning associated with time differences between two places on Earth

Why is it essential to be able to find the times differences between places on Earth?

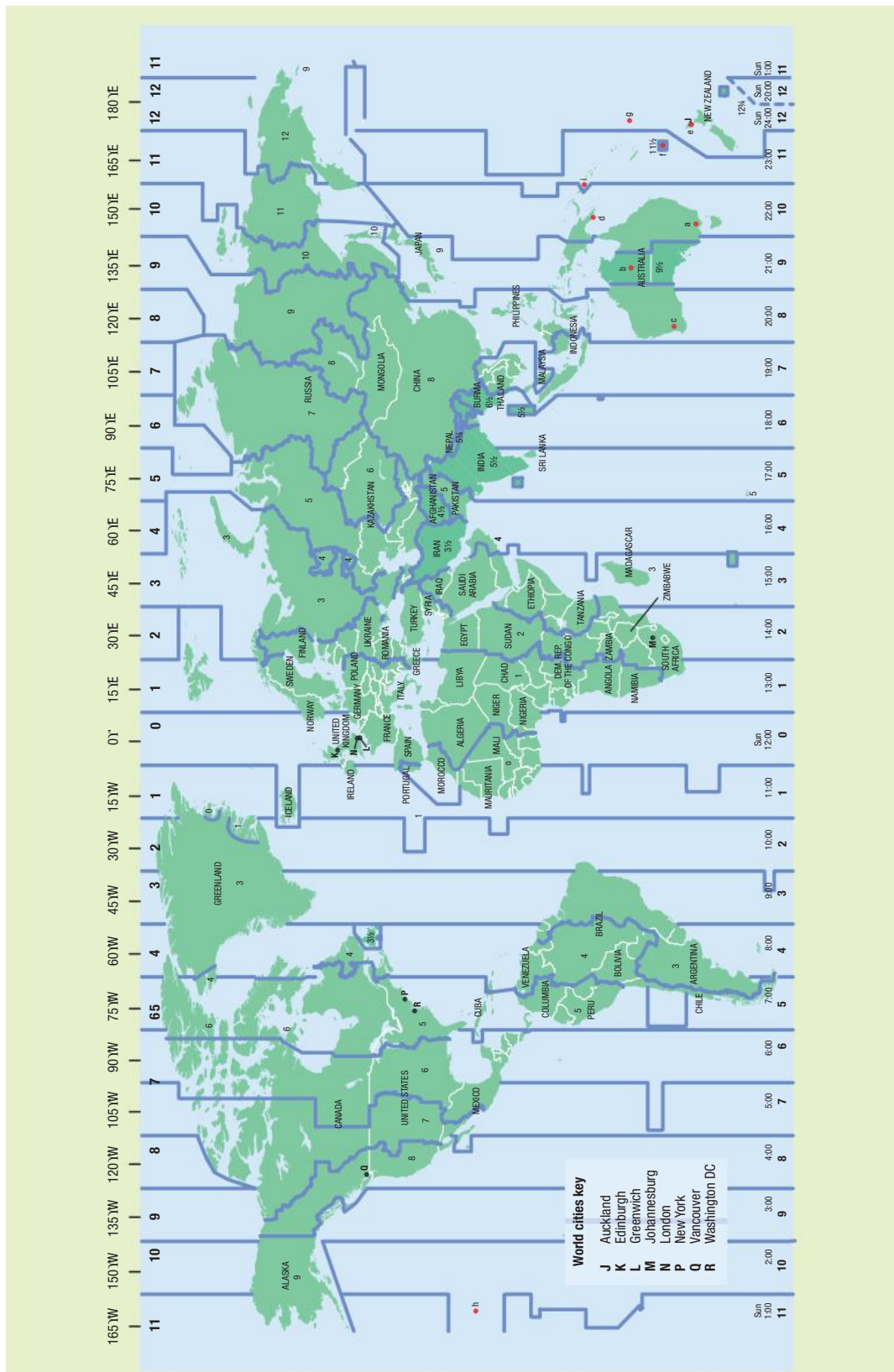
If we wish to communicate with other people in other places on Earth, we need to be able to know what time of the day it is in these other places. This is so that we can make contact during business hours or at a time that is socially acceptable. When we wish to travel to other places, we need to know what time it will be when we arrive so we can be prepared for that time of the day.



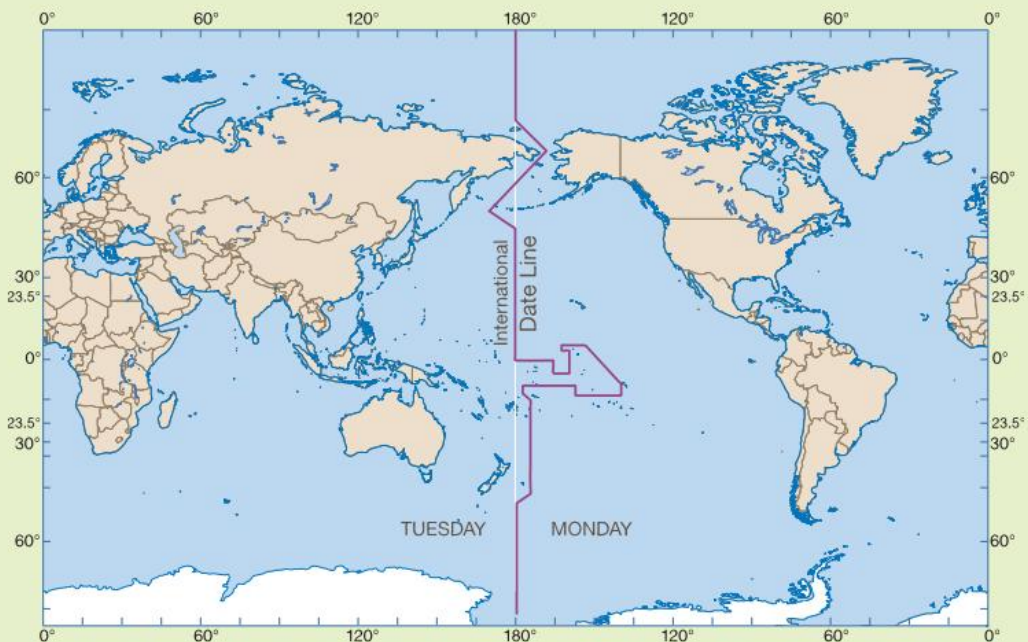
Knowing what time it is in other parts of the world is important when we want to communicate with family members, friends or colleagues in other countries.

WHAT YOU NEED TO KNOW

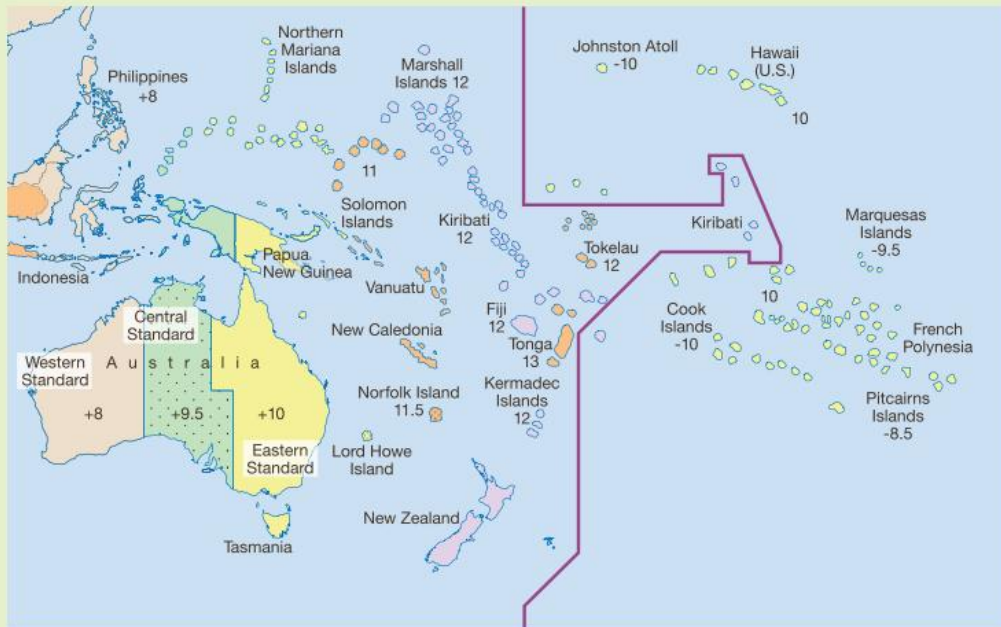
- The **time zones** are largely determined by the meridians of longitude. You can see from the map on page 478 that there are exceptions to this because of local requirements.
- If it is 12 noon along a meridian, then on the other side of the world along the meridian that makes up the other half of the great circle it is midnight. For example, when it is noon in Victoria on the 145°E meridian, it is midnight in the far east of Brazil on the 35°W meridian.
- Since Earth turns 360° in 24 hours, it turns 15° in 1 hour. So, for every 15° of longitude, the time difference is 1 hour, and for every 1° of longitude the time difference is 4 minutes.



- **Local times** around the world are given relative to the time along the prime meridian. The time at the prime meridian is taken as Coordinated Universal Time (UTC). This is not a time zone but is the primary time standard by which the world regulates clocks and time. For most purposes, UTC is considered interchangeable with Greenwich Mean Time (GMT), but GMT is no longer precisely defined by the scientific community. In the map on page 478, the time zones show the adjustments to UTC, which are taken in various locations in the world.
- Places east of the prime meridian are ahead of GMT (UTC), while places west are behind GMT (UTC).
- The International Date Line is an imaginary line on Earth's surface. The International Date Line is located halfway around the world from the prime meridian (0° longitude) at about the 180° meridian. The dateline runs from the north pole to the south pole. It is not straight but zigzags to avoid political and country borders and to not cut some countries in half. The location of the international dateline is shown in the map below. When you cross the International Date Line:
 - from west to east, you subtract a day
 - from east to west, you add a day.



- Australia is divided into three time zones: Eastern Standard, Central Standard and Western Standard. Some states change the clocks in summer to include daylight saving. Western Australian time is 8 hours ahead of GMT. New Zealand time is ahead of Western Standard time by 4 hours and is 12 hours ahead of GMT.



Example 7 Using time difference without time zones or summer time

Singapore is located at 1°N 104°E and Sydney is located at 34°S 151°E . What is the time difference between Singapore and Sydney?

WORKING

$$151 - 104 = 47^{\circ}$$

$$\frac{47}{15} = 3 \frac{2}{15}$$

The time difference in Sydney is 3 hours ahead.

THINKING

← Calculate the difference in longitude.

← Divide the difference by 15° (1 hour of difference per 15° of longitude).

← Write a sentence answer.

Exercise 8C

Use time difference without time zones or summer time.

1 Calculate the time differences between the following places.

- a** X longitude 150°E , Y longitude 120°E
- b** X longitude 0° , Y longitude 75°W
- c** X longitude 0° , Y longitude 75°E
- d** X longitude 123°E , Y longitude 150°E

Example 7

2 The following table shows the latitude and longitude of cities around the world.

City	Latitude	Longitude
Perth	32°S	115°E
Melbourne	38°S	145°E
Marseilles	43°N	5°E
Mexico City	19°N	99°W
Wellington	41°S	175°E
Zurich	47°N	8°E
London	52°N	0° (actually west of Greenwich)
Lima	12°S	77°W
Plymouth	50°N	4°W
Yangon	19°N	96°E

a Find the time difference between Perth and:

- i** Wellington
- ii** Marseilles
- iii** Yangon
- iv** London
- v** Mexico City

b If it is 6 a.m. in Perth, what time is it in:

- i** Wellington?
- ii** Marseilles?
- iii** Yangon?
- iv** London?
- v** Mexico City?

3 The time in a Pacific island is 10 hours behind GMT.

- a** What is the longitude of the island?
- b** What is the time in London when it is 4 a.m. on the island?
- c** What is the time on the island when it is 5:30 a.m. in London?

4 The longitude of Hanoi is 105°E , while the longitude of Cape Howe is 150°E .
When the time in Hanoi is 2:30 p.m., what is the time at:

- a** Cape Howe?
- b** London?

- 5** Kalgoorlie has longitude 121°E , while the Pacific island of Nauru has longitude 166°E .
- Calculate the difference in longitude between these two places.
 - Calculate the time difference between the two places.
 - What is the time in Nauru when it is 11:45 a.m. in Kalgoorlie?
- 6** You live in Jurien Bay (30.3576°S , 115.1676°E) and want to telephone a friend in one of the places listed below at 9 a.m. on a Saturday (their time). For each city, at what time (your time) should you call?
- Vancouver (123°W)
 - Helsinki (25°E)
 - Bologna (11°E)
 - San Francisco (122°W)
 - Vladivostok (132°E)
 - Fiji (178°E)
- 7** Dimitri is in Athens, which is two hours ahead of Greenwich Mean Time. Allan is in New York, which is five hours behind Greenwich Mean Time.
- Dimitri is going to ring Allan at 10 a.m. on Wednesday, Athens time. What day and time will it be in New York when he rings?
 - Allan is going to fly from New York to Athens. His flight will leave on Wednesday at 10 p.m., New York time, and will take 10 hours. What day and time will it be in Athens when he arrives?
- 8** Louise is in Dubai, which is three hours ahead of Greenwich Mean Time. David is in Sydney, which is ten hours ahead of Greenwich Mean Time.
- Louise is going to ring David at 10 a.m. on Wednesday, Sydney time. What day and time will it be in Dubai when she rings?
 - David is going to fly from Sydney to Dubai. His flight will leave on Wednesday at 10 p.m., Sydney time, and will take 14 hours. What day and time will it be in Dubai when he arrives?
- 9** Los Angeles is 8 hours behind GMT and Brisbane is 10 hours ahead of GMT.
- If it is 10 a.m. in Brisbane on Tuesday, what time is it in Los Angeles?
 - A plane leaves Brisbane at 10 a.m. on Tuesday and arrives in Los Angeles at 6 a.m. on Tuesday. What was the length of the flight? (Note that the plane crosses the International Date Line)
 - A plane leaves Los Angeles at 23:20 hours on Tuesday and arrives in Brisbane at 7:15 a.m. on Thursday. What was the length of the flight? (Note that the plane crosses the International Date Line)
- 10** Use the map on page 478 to answer these questions using the time zones indicated on the map. Ignore summer time. You may need to find the location in an atlas or using the internet. If it is 12 midday in Perth, what time and day is it in:
- Melbourne?
 - Alice Springs?
 - Brisbane?
 - Port Moresby?
 - Auckland?
 - Norfolk Island?
 - Suva?
 - Honolulu?
 - Solomon Islands?

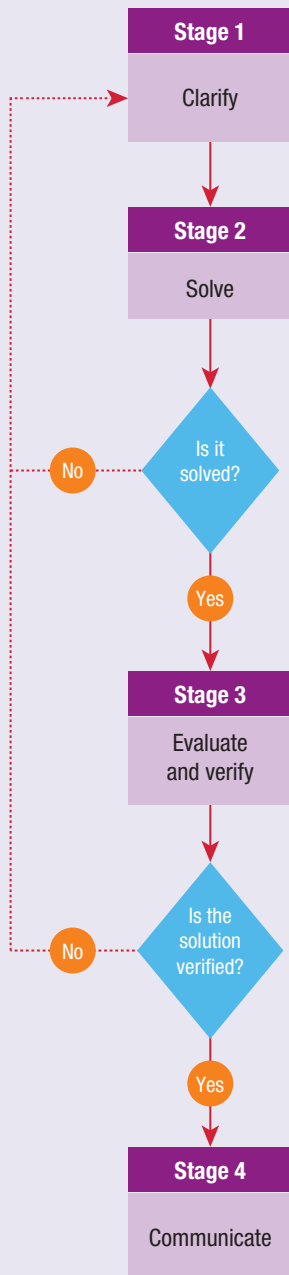
Mathematical thinking process task

Background: You have been called in to replace a sick skipper of a fishing charter boat. The sick skipper only left you partial information about the planned fishing trip. Here is what you have been given:

You are to leave Fremantle (32.05°S , 115.76°E) at 3 a.m., travel 82 km east to Five Fathom Bank (32.05°S , 114.87°E), fish for 2 hours, then travel to Josie's Spot (31.98°S , 115.76°E), fish, then travel 49 km east to Secret Spot (31.98°S , 114.37°E), fish, travel to a new spot at (31.66°S , 114.37°E), fish, travel to Last Chance Spot (31.80°S , 114.37°E), fish, then return to shore at Hillary's Marina (31.80°S , 115.76°E), travelling 111 km east by 9 p.m. The boat travels at 30 km/h and uses 60 L of diesel per hour of travelling, you have 300 L of diesel on board.

Task: Using the information above you need to plan the rest of the trip. To do this you need to decide on how long you will spend at each fishing spot and ensure you have enough diesel for the fishing trip, factoring in a 10% safety margin.

Application of mathematical thinking process.

**Stage 1: Clarify**

Interpret the task and gather the key information and identify the mathematics which could help to complete the task

- 1 Determine the total hours spent travelling.
- 2 Find the amount of hours left for fishing and allocate remainder to fishing spots.
- 3 Determine total fuel needed, and compare to fuel on board to determine if and how much more fuel is required.
- 4 Recall distance on great circle formula, distance time formula, applications of rates calculations and percentage increase calculations.

Stage 2: Solve

Apply existing mathematical knowledge and strategies to obtain a solution

- 5 Draw a fully labelled sketch of the fishing trip.
- 6 Calculate the missing distances and then the total distance travelled using distance on great circle formula.
- 7 Calculate total travel time using distance time formula.
- 8 Calculate total time, time available for fishing and allocate time to fishing spots.
- 9 Calculate diesel required using applications of rates calculations and percentage increase calculations.
- 10 Calculate how much more fuel is required.

Stage 3: Evaluate and verify

Analyse information and data from a variety of sources and verify the reasonableness of the solution

- 11 Use technology to confirm the missing distances calculated using distance on great circle formula. (Remember using $r_e = 6371$ km is an estimate).
- 12 Are the distances and diesel volumes reasonable?

Stage 4: Communicate

Communicate findings in a systematic and concise manner

- 13 Complete the plan for the fishing trip by drawing a fully labelled scaled diagram of the trip showing distances, locations, directions of travel and times.
- 14 Write a sentence about the diesel requirements.

Chapter checklist

☐ I can find the length of an arc of a circle.

- 1** Calculate the length of the small arc of a circle with radius 6371 km and a central angle of 75° .

☐ I can convert degrees expressed in decimal form to degrees and minutes, and vice versa.

- 2** Convert 53.45° to degrees and minutes.
3 Convert $68^\circ 48'$ to a decimal angle.

☐ I can use the meridians of longitude and parallels of latitude to describe locations on Earth.

- 4** What is the latitude and longitude of Perth WA?
5 What town is located at 23.20°S and 117.67°E ?

☐ I can find the distance around a great circle of two points with the same longitude.

- 6** What is the distance between Esperance (33.86°S 121.89°E) and Tom Price (22.70°S 121.89°E)?

☐ I can relate time zones with longitude.

- 7** What is the time difference between Geraldton (28.78°S 114.61°E) and Auckland (36.85°S 174.76°E)?

Chapter review

Section 8B

- 1 Two locations lie on the same meridian of longitude. One is 30° north of the other. What is the distance between the two locations, correct to the nearest kilometre?
- 2 Two locations lie on the same meridian of longitude. One is 35° south of the other. What is the distance between the two locations, correct to the nearest kilometre?

Section 8C

- 3 Dunedin has longitude 170°E , while Albany has longitude 118°E .
 - a Calculate the difference in longitude between these two places.
 - b Calculate the time difference between the two places (ignore time zones and daylight saving).
 - c What is the time in Dunedin when it is 2:45 a.m. in Albany? (Ignore time zones and daylight saving.)
- 4 The position of Rabaul is (4°S , 152°E). An island is 4° to the north of Rabaul and 48° east of Rabaul. Give the latitude and longitude of the island.
- 5 Pontianak has a longitude of 109°E , and Amazonas, a town in Brazil, has a longitude of 70°W . Both places lie on the equator. Find the shortest distance between these two places. Give your answer to the nearest kilometre.
- 6 Singapore has longitude 104°E and Sydney has longitude 151°E . What is the time difference between Singapore and Sydney? (Ignore time zones and daylight saving.)
- 7 Anthony lives in Albany and wants to phone his grandfather in London. It is 6 p.m. on Saturday in Western Australia. What time is it in London? (London GMT 0, Albany GMT 8)
- 8 Arlene lives in Geraldton and has a baby at 2 a.m. on Saturday. She wants to phone her mother who is on holiday in Samoa with the good news. What time is it in Samoa when she calls? (Geraldton GMT 8, Samoa GMT 13)
- 9 Arisa is from Japan and is studying in Perth. She plans to phone home on Sunday night at 8 p.m. What time is it in Tokyo? (Perth GMT 9, Tokyo GMT 9)

- 10** Terry is playing hockey at a tournament in Buenos Aires. After her team wins the semi-final at 5 p.m. on Friday, she phones her father in Bunbury to tell him the news. What time is it in Bunbury? (Bunbury GMT 8, Buenos Aires GMT 3)
- 11** Elizabeth is in Rome, which is one hour ahead of Greenwich Mean Time. Leslie is in Boston, which is five hours behind Greenwich Mean Time.
- a** Elizabeth is going to ring Leslie at 10 p.m. on Tuesday, Rome time. What day and time will it be in Boston when she rings?
 - b** Elizabeth is going to fly from Boston to Rome. Her flight will leave on Wednesday at 10 a.m., Boston time, and will take 10 hours. What day and time will it be in Rome when she arrives?
- 12** Osaka is at 34°N , 135°E , and Dallas is at 33°N , 97°W .
- a** Find the time difference between the two cities. (Ignore time zones.)
 - b** Rex lives in Dallas and wants to ring a friend in Osaka. In Dallas it is 8 p.m. Monday. What time and day is it in Osaka?
 - c** Rex's friend in Osaka sent him a text message, which happened to take 14 hours to reach him. It was sent at 10 a.m. Thursday, Osaka time. What was the time and day in Dallas when Rex received the text?



9 Simple and compound interest

In this chapter

- 9A** Understanding and calculating simple interest
 - 9B** Understanding and calculating compound interest
 - 9C** Applying compounding to practical problems
 - 9D** Using technology with investment problems
 - 9E** Investigating the effects of changing interest rates and compounding periods using technology
- Mathematical thinking process task
Chapter checklist
Chapter review

Syllabus reference

Unit 4 Topic 3: Loans and compound interest (14 hours)

Compound interest

- review the principles of simple interest
- understand the concept of compound interest as a recurrence relation
- consider similar problems involving compounding; for example, population growth
- use technology to calculate the future value of a compound interest loan or investment and the total interest paid or earned
- use technology to compare, numerically and graphically, the growth of simple interest and compound interest loans and investments
- use technology to investigate the effect of changing the interest rate and the number of compounding periods on the future value of a loan or investment

Pre-test

- 1** Convert the following percentages to fractions.
 - a** 15%
 - b** 6%
 - c** 2.4%
 - d** 11.25%
- 2** Calculate the following amounts correct to 2 decimal places.
 - a** 12% of \$45
 - b** 8% of \$1590
 - c** 1.8% of \$24 900
 - d** 3.1% of \$800
- 3** Change the following amounts by the given percentage.
 - a** Increase \$5750 by 12%
 - b** Increase \$600 by 3%
 - c** Decrease \$2120 by 4%
 - d** Decrease \$30 000 by 15%
- 4** Convert the following time periods.
 - a** 5 years to months
 - b** 3 years to weeks
 - c** 1 year to days
 - d** 7 years to fortnights
- 5** Use a calculator to determine the following. Express your answer correct to 2 decimal places.
 - a** 2.01^5
 - b** 1.025^3
 - c** 0.85^7
 - d** 0.55^3

9A Understanding and calculating simple interest

LEARNING GOALS

- Understand contexts where simple interest is appropriate
- Use the simple interest formula to calculate interest
- Apply simple interest to real-life applications
- Use a spreadsheet for simple interest calculations

Why is it essential to be able to calculate simple interest?

Interest is the cost of borrowing money, or the reward for investing money. It is quoted using percentages. When investigating loans, or investments, it is important to use the potential costs of interest on a loan, or the interest earned on an investment, to help make good decisions.



Interest is the reward for investing, or the cost of borrowing money.

WHAT YOU NEED TO KNOW

- **Simple interest** is calculated using an interest rate on the same amount of money each **time period**.
- The formula to calculate simple interest is $I = Pin$ (or $I = P \times i \times n$)
 - I = interest calculated
 - P = **principal** (the money borrowed or invested)
 - i = interest rate as a decimal per a time period, for example, 0.04 p.a. (p.a. means '**per annum**' which is 'per year'). However we are usually given the interest rate as a percentage, $r\%$, so it needs to be written as a fraction, $\frac{r}{100}$ when substituting into the formula.
 - n = number of time periods for which interest is calculated.
 - The time units of the time period and the interest rate **MUST** be the same, e.g. $i = \frac{4}{100}$ p.a. and $n = 3$ years; or $i = \frac{4}{100}$ per month and $n = 6$ months.
- Loans using simple interest require the borrowed amount and the interest to be repaid.
 - Total to repay = amount borrowed + interest
 - Regular repayment = total to repay $\div$ number of repayments



Example 1 Calculating simple interest for yearly time periods

Maxine deposits \$600 into her bank account. The account offers an interest rate of 3.2% p.a. simple interest. She leaves the money in the bank for four years.

- a** Calculate the total interest she would earn over four years.
- b** Determine how much money Maxine will have after the four years.

WORKING

a $P = \$600$ ←.....

$$i = \frac{3.2}{100}$$

$$n = 4$$

$$I = Pin$$
 ←.....

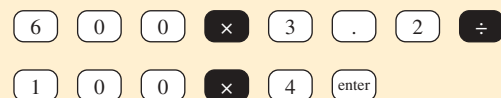
$$I = 600 \times \frac{3.2}{100} \times 4$$

$$I = \$76.80$$

WORKING

Identify the values that need to be substituted into the formula $I = Pin$.

Substitute the values into the formula.
Keystrokes on scientific calculator:



Money always has 2 decimal places because of the cents.

b Balance = $600 + 76.80$ ←..... Add the interest to the original amount.
= \$676.80



Example 2 Calculating simple interest for time periods other than a year

Maxine deposits \$600 into her bank account. The account offers an interest rate of 3.2% p.a. simple interest. Calculate the total interest she would earn if the money is invested for:

- a** 10 months
- b** 15 weeks
- c** 3 years and 6 months
- d** 60 days

WORKING

a $P = \$600$ ←.....

$$i = \frac{3.2}{100}$$

$$n = \frac{10}{12}$$

THINKING

Identify the values that need to be substituted into the formula $I = Pin$.
To change months to years, divide *number of months* by 12.

WORKING

THINKING

$$I = Pin \leftarrow \dots\dots\dots$$

$$I = 600 \times \frac{3.2}{100} \times \frac{10}{12}$$

$$I = \$16.00$$

Substitute the values into the formula.

Keystrokes on scientific calculator:

$$\begin{array}{ccccccc} 6 & 0 & 0 & \times & 3 & . & 2 & \div \\ 1 & 0 & 0 & \times & 1 & 0 & \div \\ 1 & 2 & \text{enter} \end{array}$$

b $P = \$600 \leftarrow \dots\dots\dots$

$$i = \frac{3.2}{100}$$

$$n = \frac{15}{52}$$

Identify the values that need to be substituted into the formula $I = Pin$.

To change weeks to years, divide *number of weeks* by 52

$$I = 600 \times \frac{3.2}{100} \times \frac{15}{52} \leftarrow \dots\dots\dots$$

$$I = \$5.54$$

Substitute the values into the formula.

c $P = \$600 \leftarrow \dots\dots\dots$

$$i = \frac{3.2}{100}$$

$$n = 3 + \frac{6}{12} = 3\frac{6}{12}$$

Identify the values that need to be substituted into the formula $I = Pin$.

To change months to years, divide *number of months* by 12.

$$I = 600 \times \frac{3.2}{100} \times 3\frac{6}{12} \leftarrow \dots\dots\dots$$

$$I = \$67.20$$

Substitute the values into the formula.

Keystrokes on scientific calculator:

$$\begin{array}{ccccccc} 6 & 0 & 0 & \times & 3 & . & 2 & \div \\ 1 & 0 & 0 & \times & 3 & \text{2nd} & \frac{n}{d} \\ 6 & \frac{n}{d} & 1 & 2 & \text{enter} \end{array}$$

d $P = \$600 \leftarrow \dots\dots\dots$

$$i = \frac{3.2}{100}$$

$$n = \frac{60}{365}$$

Identify the values to substitute into $I = Pin$.

To change days to years, divide *number of days* by 365.

$$I = 600 \times \frac{3.2}{100} \times \frac{60}{365} \leftarrow \dots\dots\dots$$

$$I = \$3.16$$

Substitute the values into the formula.



Example 3 Calculating simple interest loan amounts

John borrows \$1600 with a simple interest rate of 8.7% p.a over 9 months to buy a new laptop. He agrees to repay the loan in equal monthly repayments.

- Calculate the total amount of interest owed after 9 months.
- Calculate the total amount he will need to repay after 9 months.
- Calculate the monthly repayment.

WORKING

$$\begin{aligned}
 \text{a } P &= \$1600 \\
 i &= \frac{8.7}{100} \\
 n &= \frac{9}{12} \\
 I &= Pin \\
 I &= 1600 \times \frac{8.7}{100} \times \frac{9}{12} \\
 I &= \$104.40
 \end{aligned}$$

$$\begin{aligned}
 \text{b Total to repay} \\
 &= \text{amount borrowed} + \text{interest} \\
 &= 1600 + 104.40 \\
 &= \$1704.40
 \end{aligned}$$

$$\begin{aligned}
 \text{c Regular repayment} \\
 &= \text{total to repay} \div \text{number of repayments} \\
 &= 1704.40 \div 9 \\
 &= \$189.38
 \end{aligned}$$

THINKING

Identify the values that need to be substituted into the formula $I = Pin$.
To change months to years, divide *number of months* by 12.

Substitute the values into the formula.

Add the amount borrowed to the total interest.

Divide the total to repay by the number of repayments.
Money always has 2 decimal places because of the cents.



Exercise 9A

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a Interest is the cost of _____ money, or the reward for _____ money. It is calculated using _____.
 - b Simple interest is calculated using a percentage _____ rate on the _____ amount of money each time period.
 - c Loans using simple interest require the _____ amount and the _____ to be repaid.

- 2 Express the following time periods as a fraction of a year.
 - a 7 months = _____ year
 - b 31 weeks = _____ year
 - c 25 days = _____ year
 - d 18 months = _____ year
 - e 270 days = _____ year
 - f 9 weeks = _____ year

- 3 Calculate the value of the following percentages.
 - a 30% of \$23.70
 - b 10% of \$80.90
 - c 5% of \$126.45
 - d 3.45% of \$456.21
 - e 1.05% of \$364.85
 - f 0.65% of \$63.90

Hint 1 year = 12 months
 = 52 weeks
 = 26 fortnights
 = 365 days

Hint First divide the percentage by 100 to make it a decimal.

APPLICATIONS

Example 1

- 4 Calculate the simple interest earned on the following investments.
 - a \$780 is invested at 4.57% p.a. for 4 years
 - b \$4260 is invested at 6.32% p.a. for 3 years
 - c \$2030 is invested at 9.05% p.a. for 5 years
 - d \$625 is invested at 5.9% p.a. for 2 years
 - e \$1650 is invested at 4.2% p.a. for 5 years
 - f \$5050 is invested at 3.9% p.a. for 7 years
 - g \$8880 is invested at 5.79% p.a. for 6 years
 - h \$7280 is invested at 5.36% p.a. for 2 years

- 5** Angus invested \$10 000 in a fixed-term account for 4 years paying a simple interest rate of 3.65% p.a.
- Calculate the total amount of interest earned over the 4 years.
 - Calculate the total value of the investment after 4 years.

Example 2

- 6** Calculate the simple interest earned on the following investments.
- \$6200 is invested at 6.8% p.a. for 4 months
 - \$590 is invested at 5.11% p.a. for 20 weeks
 - \$8290 is invested at 4.48% p.a. for 15 days
 - \$6700 is invested at 2.26% p.a. for 2 weeks
 - \$7740 is invested at 5.35% p.a. for 14 days
 - \$858 is invested at 7.07% p.a. for 7 months
 - \$3700 is invested at 6.1% p.a. for 9 years and 6 months
 - \$6160 is invested at 5.3% p.a. for 8 years and 3 months

- 7** A company invested \$2 000 000 in the short-term money market at 8% p.a. simple interest for 30 days.
- Calculate the total amount of interest earned over the 30 days.
 - Calculate the total value of the investment after 30 days.

Hint The short-term money market is used for borrowing and investing money for up to one year.

Example 3

- 8** Caleb borrowed \$3000 with a simple interest rate of 9.7% p.a. over 18 months to buy a new lounge.



- Calculate the total amount of interest owed after 18 months.
- Calculate the total amount Caleb will need to repay after 18 months.
- Calculate the monthly repayment.

Hint Total to repay = amount borrowed + interest
Regular repayment = total to repay $\div$ number of repayments

- 9** Olivia borrowed \$5000 with a simple interest rate of 11.2% p.a. over 4 years to buy a car.

a Calculate the total amount of interest owed after 4 years.

b Calculate the total amount Olivia will need to repay after 4 years.

c Calculate the monthly repayment.



- 10** A bank offers the following simple interest rates on fixed-term deposits.

Term	Interest rate for \$10 000 to \$49 999	Interest rate for \$50 000 to \$1 999 999
60 months	2.65% p.a.	2.75% p.a.
24–33 months	2.60% p.a.	2.70% p.a.
12 months	2.20% p.a.	2.30% p.a.
6 months	2.05% p.a.	2.05% p.a.
3 months	2.00% p.a.	2.00% p.a.

- a** Ezikiel wins \$15 000 in a lottery. He decides to invest it for two years to use as a home loan deposit.

i Determine what interest rate the bank will offer.

ii Calculate how much interest he will earn in two years.

iii Determine how much his investment will be worth after the two years.

- b** Georgia inherits \$65 000. She decides to invest it for one year while she considers what to do with the money.

i Determine what interest rate the bank will offer.

ii Calculate how much interest will she earn in one year.

iii Determine how much her investment will be worth after one year.

- c** Jason has saved \$11 400 from his part-time job. He decides to invest it for six months while he hunts for a car to buy.

i Determine what interest rate the bank will offer.

ii Calculate how much interest he will earn.

iii Determine how much his investment will be worth after six months.

11 Cortay invests \$1500 into a bank account that pays 3.6% p.a.

a Set up a spreadsheet to model this investment over 10 years. A sample set-up is shown:

	A	B	C
1	Year	Simple Interest	Balance
2	1	$=1500*3.6/100*A2$	$=1500+B2$
3	2		
4	3		
5	4		
6	5		
7	6		

Hint Only complete the first row. Highlight cells, grab the little square in bottom right corner and drag down to auto fill the formulas.

Hint Highlight first two columns>Insert>Recommended Charts. Select a line graph.

Hint Highlight first and third columns while holding down Ctrl>Insert>Recommended Charts. Select a line graph.

b Use the spreadsheet table to determine the interest earned after:

- i** 5 years
- ii** 10 years.

c Use a table of values to determine the value of the investment after:

- i** 5 years
- ii** 10 years.

d Graph the Year vs Simple Interest using Excel.

e Graph the Year vs Balance using Excel.



Spreadsheet activity 9A: See the Interactive Textbook for this activity to set up a spreadsheet that will allow you to change the Principal and Interest Rate to investigate how this changes the interest earned.

9B Understanding and calculating compound interest

LEARNING GOALS

- Understand that compound interest is repeated interest calculations
- Apply the compound interest formula to annual compounding periods
- Apply the compound interest formula to various compounding periods
- Use a spreadsheet to calculate recurring interest calculations

Why is it essential to be able to calculate compound interest?

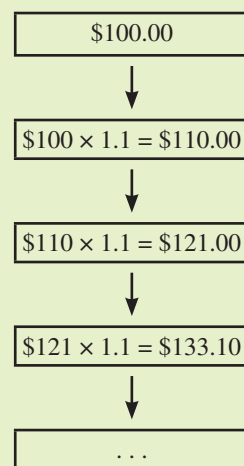
We investigated simple interest calculations in the previous section. More commonly though, interest is added to the account at the end of each time period. Future interest calculations are then made on the new balance, that also includes any previous interest payments. Interest then compounds, so we call this method of interest calculation *compound interest*.



With compound interest, interest is earned on the amount including the interest already earned and so the investment will grow more quickly.

WHAT YOU NEED TO KNOW

- At the end of each time period, the same interest calculation rule is applied to the amount of money in the account. The calculation ‘recurs’ (is repeated) each time. If no money is withdrawn or paid in, and the interest rate remains the same, this produces a sequence where each amount depends only on the previous amount. For example: \$100 is increased by 10% (which means it is multiplied by 1.1) and the result is multiplied by 1.1, and the result of that is multiplied by 1.1, and so on, as shown in the diagram.



- This is called a **recurrence relation**, of which compound interest is just one type. In this course, we won't use the mathematical form of a recurrence relation, we will use a formula which gives us the amounts in the sequence, which we will show as successive lines in a table or spreadsheet.
- Consider an investment of \$2000 placed in an account paying 4% p.a. for 3 years with interest added to the account at the end of each year. We can use a table to find the balance at the end of every year as follows:

Year	Principal	Interest	Balance
1	\$2000.00	$2000.00 \times \frac{4}{100} = \$80.00$	$2000.00 + \$80.00 = \$2080.00$
2	\$2080.00	$2080.00 \times \frac{4}{100} = \$83.20$	$2080.00 + \$83.20 = \$2163.20$
3	\$2163.20	$2163.20 \times \frac{4}{100} = \$86.53$	$2163.20 + \$86.53 = \$2249.73$

- Notice how the balance becomes the principal in the next line, and that the principal is multiplied by the same amount each time ($\frac{4}{100}$ in this case). This is what is meant by a recurrence relation.
- Another method is to consider that by adding 4% interest, at the end of the year we would now have 104% of the starting principal.

This can be expressed as $(100\% + 4\%) = \left(\frac{100}{100} + \frac{4}{100}\right) = \left(1 + \frac{4}{100}\right)$

After 1 year, Balance = $2000 \times \left(1 + \frac{4}{100}\right) = \$2080$

After 2 years, Balance = $2000 \times \left(1 + \frac{4}{100}\right) \times \left(1 + \frac{4}{100}\right) = \$2163.20$

After 3 years, Balance = $2000 \times \left(1 + \frac{4}{100}\right) \times \left(1 + \frac{4}{100}\right) \times \left(1 + \frac{4}{100}\right)$
 $= \$2249.73$

Here we are multiplying the starting principal by $(1 + \text{interest})$ and repeating the multiplication according to the number of time periods. For 3 years we multiply three times.

- We can use indices to write repeated, or recurring, multiplication. Hence after three years:

$$\text{Balance} = 2000 \times \left(1 + \frac{4}{100}\right)^3 = \$2249.73$$

- A formula for **compound interest** can now be developed.

$A = P(1 + i)^n$, where:

- A = the *accumulated* amount at the end of the investment
- P = the *principal* amount that you started with

- i = the *interest* rate as a decimal per a time period. Using the example with 4% above, $\frac{4}{100} = 0.04$, so $i = 0.04$ p.a. However we are usually given the interest rate, as a percentage $r\%$ p.a., it needs to be written as $\frac{r}{100}$ when put into the formula.
- n = the *number* of time periods (compounding periods)
- The time units of the compounding period and the interest rate **MUST** be the same, e.g. $i = \frac{4}{100}$ p.a and $n = 3$ years; or $i = \frac{4}{100}$ per month and $n = 6$ months.



Example 4 Calculating compound interest using a table

Titan invests \$900 into an account paying 3% p.a. interest for four years.

- a** Complete the following table to calculate Titan's interest and balance at the end of each year.

Year	Principal	Interest	Balance
1	\$900		
2			
3			
4			

- b** Determine how much interest he earned in four years.

WORKING

a

Year	Principal	Interest	Balance
1	\$900.00	$900.00 \times \frac{3}{100}$ = 27.00	$900.00 + 27.00$ = \$927.00
2	\$927.00	$927.00 \times \frac{3}{100}$ = 27.81	$927.00 + 27.81$ = \$954.81
3	\$954.81	$954.81 \times \frac{3}{100}$ = 28.64	$954.81 + 28.64$ = \$983.45
4	\$983.45	$983.45 \times \frac{3}{100}$ = 29.50	$983.45 + 29.50$ = \$1012.95

- b** Interest $1012.95 - 900.00 = \$112.95$

THINKING

◀ The interest is added to the principal at the end of each year.

The balance becomes the principal in the next year.

Money should always have 2 decimal places because of the cents.

◀ Subtract the starting principal from the final balance to calculate the interest earned.



Example 5 Calculating compound interest with yearly compounding periods using the formula

Riley invests \$2800 into an account paying 2.5% p.a. interest that compounds annually.

- Use the compound interest formula to find what Riley's investment will be worth after five years.
- Determine how much interest he earned in five years.

WORKING

a $P = \$2800$ ←.....

$$i = \frac{2.5}{100}$$

$$n = 5$$

$A = P(1 + i)^n$ ←.....

$$A = 2800 \left(1 + \frac{2.5}{100}\right)^5$$

$$A = \$3167.94$$

b Interest = $3167.94 - 2800$ ←.....
= \$367.94

THINKING

Identify the values that need to be substituted into the formula $A = P(1 + i)^n$.

Substitute the values into the formula.

To enter the power, use $\boxed{\wedge}$ the $\boxed{x\sqrt{}}$ or button on your calculator.

Keystrokes on scientific calculator:

$$\boxed{2} \boxed{8} \boxed{0} \boxed{0} \boxed{\times} \boxed{(} \boxed{1} \boxed{+} \boxed{)} \boxed{2} \boxed{.} \boxed{5} \boxed{\div} \boxed{1} \boxed{0} \boxed{0} \boxed{)} \boxed{\wedge} \boxed{5} \boxed{\text{enter}}$$

Subtract the starting principal from the final balance to calculate the interest earned.





Example 6 Calculating compound interest with various compounding periods using the formula

Joshua invests \$1600 into an account paying 2.5% p.a. interest for three years. Use the compound interest formula to find what his investment will be worth after three years if interest compounds:

- a** yearly **b** half-yearly **c** monthly
d weekly **e** daily

WORKING

THINKING

- a** $P = \$1600$ ← Identify the values that need to be substituted into the formula $A = P(1 + i)^n$.
 $i = \frac{2.5}{100}$
 $n = 3$
 $A = P(1 + i)^n$ ← Substitute into the formula and evaluate with a calculator. Money should always have 2 decimal places because of the cents.
 $A = 1600 \left(1 + \frac{2.5}{100}\right)^3$
 $A = \$1723.03$
- b** $P = \$1600$ ← The interest rate will need to be divided by 2 (and 100) since there are 2 half-years in a year. Leaving as a fraction will give a more exact answer.
 $i = \frac{2.5 \div 2}{100} = \frac{2.5}{200}$ per half-year
 $n = 3 \times 2 = 6$ half-years
 If interest compounds twice per year, then it will compound 6 times in three years.
 $A = P(1 + i)^n$ ← Substitute into the formula and evaluate with a calculator.
 $A = 1600 \left(1 + \frac{2.5}{200}\right)^6$
 $A = \$1723.81$



- c** $P = \$1600$ ←..... The interest rate will need to be divided by 12 (and 100) since there are 12 months in a year. Leaving as a fraction will give a more exact answer. If interest compounds every month, then it will compound 36 times in three years.
- $$i = \frac{2.5 \div 12}{100} = \frac{2.5}{1200} \text{ per month}$$
- $$n = 3 \times 12 = 36 \text{ months}$$
-
- $$A = P(1 + i)^n$$
- ←..... Substitute into the formula and evaluate with a calculator.
- $$A = 1600 \left(1 + \frac{2.5}{1200}\right)^{36}$$
- $$A = \$1724.28$$
-
- d** $P = \$1600$ ←..... The interest rate will need to be divided by 52 (and 100) since there are 52 weeks in a year. Leaving as a fraction will give a more exact answer. If interest compounds every week, then it will compound 156 times in three years.
- $$i = \frac{2.5 \div 52}{100} = \frac{2.5}{5200} \text{ per week}$$
- $$n = 3 \times 52 = 156 \text{ weeks}$$
-
- $$A = P(1 + i)^n$$
- ←..... Substitute into the formula and evaluate with a calculator.
- $$A = 1600 \left(1 + \frac{2.5}{5200}\right)^{156}$$
- $$A = \$1724.58$$
-
- e** $P = \$1600$ ←..... The interest rate will need to be divided by 365 (and 100) since there are 365 days in a year. Leaving as a fraction will give a more exact answer. If interest compounds every day, then it will compound 1095 times in three years.
- $$i = \frac{2.5 \div 365}{100} = \frac{2.5}{36\,500} \text{ per day}$$
- $$n = 3 \times 365 = 1095 \text{ days}$$
-
- $$A = P(1 + i)^n$$
- ←..... Substitute into the formula and evaluate with a calculator.
- $$A = 1600 \left(1 + \frac{2.5}{36\,500}\right)^{1095}$$
- $$A = \$1724.61$$

Exercise 9B

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a With compound interest, _____ is earned on the _____ already earned.
 - b The formula for interest is, $A = P(1 + i)^n$ where:
 - i A = the _____ amount at the end of the investment.
 - ii P = the _____ amount that you started with.
 - iii i = the _____ rate, as a percentage.
 - iv n = the _____ of compounding periods.
 - c i and n _____ be in the same time units as the compounding _____ (per month, per year etc).
- 2 Express the following percentages as fractions.
 - a 15%
 - b 6%
 - c 3.4%
 - d 2.03%
- 3 Convert one year to:
 - a months
 - b weeks
 - c half-years
 - d days
 - e quarters
 - f fortnights
- 4 A bank advertises interest rates of 2.4% p.a. Express this as a:
 - a half-yearly rate
 - b quarterly rate
 - c monthly rate
 - d fortnightly rate
 - e weekly rate
 - f daily rate

APPLICATIONS

Example 4

- 5 Kate invests \$2480 into an account that pays 3.2% p.a. compound interest for three years. Interest is compounded yearly.
 - a Copy and complete the following table to calculate her interest and balance at the end of each year.

Year	Principal	Interest	Balance
1	\$2480		
2			
3			

- b Determine how much interest she earned in three years.

- 6** Jasper invests \$3850 into an account that pays 2.10% p.a. compound interest for four years. Interest is compounded yearly.

a Copy and complete the following table to calculate his interest and balance at the end of each year.

Year	Principal	Interest	Balance
1			
2			
3			
4			

b Determine how much interest he earned in four years.

Example 5

- 7** Abby has saved \$9600 from her part-time job. She invests it for two years at 3.20% p.a. until she gets her P's and can buy a car. Interest is compounded yearly.

a Use the compound interest formula to calculate the balance of her account after two years.

b Determine how much interest she earned over the two years.

Hint To enter the power, use the $\square^{\wedge}$ or X^n button on your calculator.

- 8** Ben invests the \$4500 he earned from the sale of cattle for four years at 2.20% p.a. to save for a trip at the end of school. Interest is compounded yearly.

a Use the compound interest formula to calculate the balance of his account after four years.

b Determine how much interest he earned over the four years.

- 9** Calculate the final balance of the following investments.

a \$680 invested at 5% p.a. compounded annually for three years.

b \$275 invested at 3% p.a. compounded annually for six years.

c \$682 invested at 4% p.a. compounded annually for five years.

d \$1240 invested at 2.2% p.a. compounded annually for four years.

e \$5760 invested at 1.8% p.a. compounded annually for seven years.

f \$4030 invested at 2.15% p.a. compounded annually for five years.

Example 6

- 10** Natalie invests \$5000 in an account that earns interest at the rate of 3.6% p.a. compounding monthly. Use the compound interest formula to calculate how much is in her account after five years.

- 11** Georgia invests \$3800 in an account that earns interest at the rate of 2.4% p.a. compounding weekly. Use the compound interest formula to calculate how much is in her account after two years.

- 12** Calculate the final balance of the following investments.
- a** \$1370 invested at 8.85% p.a. compounded monthly for three years.
 - b** \$54 600 invested at 1.20% p.a. compounded half-yearly for eight years.
 - c** \$4300 invested at 5.07% p.a. compounded weekly for two years.
 - d** \$3350 invested at 6.1% p.a. compounded monthly for three years.
 - e** \$7960 invested at 4.92% p.a. compounded fortnightly for two years.
 - f** \$4220 invested at 4.66% p.a. compounded quarterly for four years.
- 13** Charlie invests \$3500 in an account that pays 4% p.a. interest compounded every year.
- a** Use a spreadsheet to calculate the interest and balance each year for five years. A sample set is shown:

Hint All formulas begin with =
 Multiplication is *
 Divide is /

	A	B	C	D
1	Year	Principal	Interest	Balance
2	1			
3	2			
4	3			
5	4			
6	5			

- i** Identify which cell you will need to enter the \$3500 principal.
 - ii** Determine the formula will you need to enter in cell C2.
 - iii** Determine the formula will you need to enter in cell D2.
 - iv** Determine the formula will you need to enter in cell B3.
- b i** Fill down to complete the spreadsheet.
- ii** Use your spreadsheet to calculate the value of Charlie's investment after five years.

9C Applying compounding to practical problems

LEARNING GOALS

- Apply the principles of compounding to real-life growth contexts such as inflation, population growth and investments
- Apply the principles of compounding to real-life decay contexts such as population decline and depreciation

Why is it essential to be able to apply compounding to real-live growth contexts?

The compound interest formula can be applied to many real-life situations other than banking. Any situation where there is growth that can be expressed as a recurrence relation can be modelled using the compound interest formula. While the percentage change used in the recurring calculations stays the same, the amount of increase gets bigger with each time period. Common examples are inflation, population growth, and appreciation in investments.

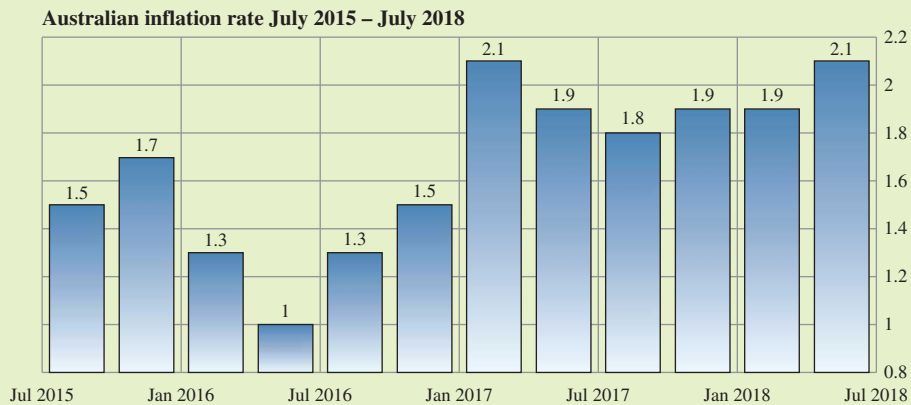
If the percentage change is a decrease (i.e. a negative percentage) rather than an increase, then the formula can model such things as depreciation on equipment and vehicles.



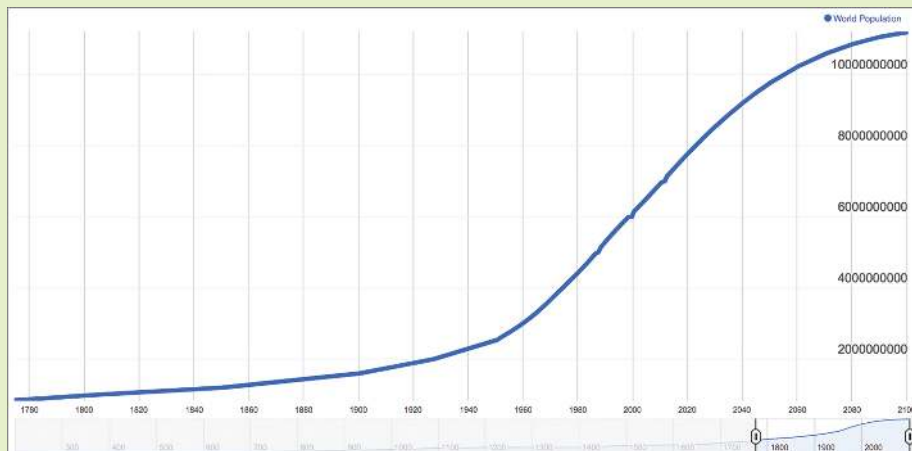
In the case of growth that can be modelled using the compound interest formula, the amount of growth increases with each time period.

WHAT YOU NEED TO KNOW

- Real-life applications, where there is a *percentage increase or decrease* over time, can be modelled using the compound interest formula, $A = P(1 + i)^n$. We use compound rather than simple interest because the increase or decrease is applied to previously increased or decreased values in a **recurrence relation**. Common applications include:
 - **Inflation:** a term that describes the increasing cost of items we need to buy. It is measured using the annual percentage change in the Consumer Price Index (CPI). We can use average percentage inflation over recent past time periods and the compound interest formula to predict the cost, or value, of items we need to purchase in the future. When we do this we assume that the inflation rate in the recent past will continue into the future, but the actual inflation rate varies somewhat randomly, as the following graph shows.



- **Population growth** is measured using percentage change on previous years. Australia's population growth in September 2018 had been 1.6% p.a. over the previous 12 months, reaching 25.1 million people. We can use population growth rates and the compound interest formula to predict the future size of populations.
- This graph shows the growth in the world population since the arrival First Fleet in Australia in 1788. The shape of the graph indicates a growth rate that follows the rules of the compound interest formula with a positive percentage rate.



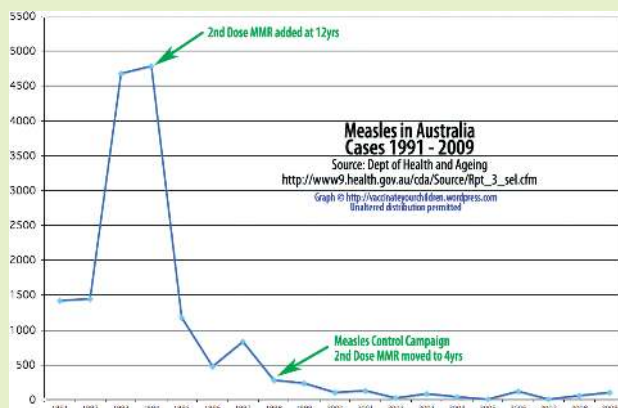
Source: <http://www.worldometers.info/world-population/>

- **Appreciation** is a measure of the increasing value of an investment over time. (The term is generally used for investments other than bank accounts.) Investments in real estate are common and when looking for a suburb to invest in, researching the annual growth of housing values in an area and applying the compound interest formula can give an indication of the future value of properties. For example:



Source: realestate.com.au

- Real-life applications where there is a *percentage decrease* over time can be modelled using the compound interest formula by using a negative percentage interest rate. That is, $A = P(1 - i)^n$. Common applications include:
 - **Depreciation** is the reducing value of items like cars, computers or machinery. For example, a new car generally loses 20% of its value in the first year and 15% of its value every year after that. We can use the compound interest formula, with a negative interest rate, to calculate the value of a car after several years.
 - **Population decline** can be applied to endangered species or instances of disease. The compound interest formula can be applied to predict the population after several years. For example, the graph below shows that the number of cases of the measles in Australia has declined since the introduction of a vaccine. The shape of the graph indicates a depreciation rate that between about 1993 and 2000 roughly followed the rules of the compound interest formula with a negative percentage rate.





Example 7 Applying compound interest to a growth (increasing) problem

A 3 L bottle of milk is \$3.30 at a supermarket in 2018. If the average rate of inflation is 2.1% p.a., estimate how much a 3 L bottle of milk will cost in:

a five years

b 2030

WORKING

a $P = \$3.30$ ←.....

$$i = \frac{2.1}{100}$$

$$n = 5$$

$$A = P(1 + i)^n$$
 ←.....

$$A = 3.30 \left(1 + \frac{2.1}{100}\right)^5$$

$$A = \$3.66$$

b $P = \$3.30$ ←.....

$$i = \frac{2.1}{100}$$

$$n = 2030 - 2018 = 12$$

$$A = P(1 + i)^n$$
 ←.....

$$A = 3.30 \left(1 + \frac{2.1}{100}\right)^{12}$$

$$A = \$4.23$$

THINKING

Identify the values that need to be substituted into the formula.

Substitute into the formula and evaluate with a calculator.

Money should always have 2 decimal places because of the cents.

Identify the values that need to be substituted into the formula.

Substitute into the formula and evaluate with a calculator.

Money should always have 2 decimal places because of the cents.





Example 8 Applying compound interest to a decreasing problem

A new car is worth \$29 990. If new cars depreciate at 15% p.a., estimate the value of the car in 6 years.

WORKING

THINKING

$$P = \$29\,990$$

$$i = \frac{15}{100}$$

$$n = 6$$

$$A = P(1 - i)^n$$

$$A = 29\,990 \left(1 - \frac{15}{100}\right)^6$$

$$A = \$11\,310.71$$

Identify the values that need to be substituted into the formula. Decreasing value, so a negative interest rate.

Substitute into the formula and evaluate with a calculator.

Money should always have 2 decimal places because of the cents.

Exercise 9C

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a Real-life applications, where there is a _____ increase over time, can be modelled using the _____ interest formula, $A = P(1 + i)^n$.
 - b Inflation is a term that describes the _____ cost of items we need to buy.
 - c Population growth can be _____ using percentage change on previous years.
 - d Appreciation is a measure of the increasing _____ of an investment over time.
 - e Real-life applications where there is a percentage _____ over time can be modelled using the compound interest formula by using a negative _____ rate.
 - f Depreciation is the _____ value of items like cars, computers or machinery.
- 2 Decide if the following real-life situations have an increasing or decreasing value.
 - a The cost of a loaf of bread in the future
 - b The value of a car a few years after purchase
 - c The population of an endangered animal in the future
 - d The value of an investment property in the future
 - e The population of Australia in the future

APPLICATIONS

Example 7

- 3** The average rate of inflation in Australia is 2.1% p.a.
- a** A burger combo meal costs \$10.65 at a fast food store. If the price increases with inflation, use the compound interest formula to estimate how much will it cost in 4 years.
 - b** A new car sells for \$20 490. If the price increases with inflation, use the compound interest formula to estimate how much will it cost in 20 years.
- 4** Bec receives a salary of \$93 000. Each year, her salary increases to match the rate of inflation. Calculate her salary in the next year if the rate of inflation is:
- a** 1.5% p.a.
 - b** 1.9% p.a.
- 5** The cost of petrol today is 145.9 cents per litre.
- a** Calculate the price of a litre of petrol in 20 years' time if inflation averages 1.9% p.a. over that time. Express your answer to 1 decimal place.
 - b** Calculate the price of a litre of petrol in 10 years' time if inflation averages 2.1% p.a. over that time. Express your answer to 1 decimal place.



- 6** The population of Australia was 24.8 million in 2018 with an annual growth rate of 1.32%. If the population continues to grow at the same rate, use the compound interest formula to estimate the population:
- a** 10 years after 2018
 - b** in 2100

- 7** Henry buys a home for \$447 500. If the annual growth rate in the suburb is 3.1% p.a., use the compound interest formula to estimate the value of the house in:
- a** 15 years
 - b** 20 years

**Example 8**

- 8** Rodney buys a new truck for his transport business for \$320 000. If the truck depreciates in value by 15% p.a., use the compound interest formula to estimate the value of the truck in:

- a** 5 years
- b** 10 years
- c** 15 years



- 9** Following the introduction of a vaccine for a disease, the number of people contracting the disease decreased by 20% each year.

- a** If 6500 people suffer from the disease when the vaccine was first introduced, use the compound interest formula to estimate how many people have the disease after:

- i** three years
- ii** five years

- b** Discuss why the disease is not fully eradicated after five years of 20% reduction each year.



- 10** Computers lose 45% of their value each year.

- a** Harrison buys a new laptop computer for \$1549.00. Determine the value of the laptop in two years' time.
- b** Lexi spends \$2099 on a new laptop computer for her daughter at the start of Year 10. Determine the value of the laptop when her daughter finishes Year 12.

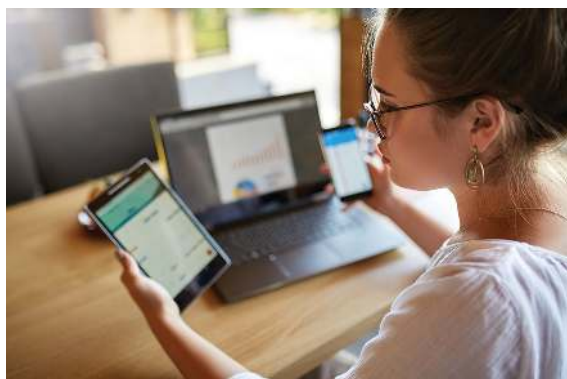
9D Using technology with investment problems

LEARNING GOALS

- Calculate the future value of a compound interest investment and the total interest earned using:
 - an online calculator
 - a spreadsheet
- Compare the growth of simple interest and compound interest investments using:
 - an online calculator
 - a spreadsheet

Why is it essential to be able to use technology when investigating investments?

We work hard to earn our money and we want to be able to save for things like holidays, cars, homes and our future. Having a plan for your savings and understanding the impact of your decisions will help you to be more successful in achieving your financial goals.



Understanding the impact of your decisions will help you to be more successful in achieving your financial goals.

WHAT YOU NEED TO KNOW

- There are many places online where you can go to calculate future value of investments. It is the core business of all the banks, so they all have calculators on their websites. The Australian Securities and Investment Commission (ASIC) Moneysmart website is free from bias, is a useful place to find information about savings and loans, and has lots of calculators to help you. (<https://www.moneysmart.gov.au/>)

- For example: With the Moneysmart compound interest calculator, you can easily find the total interest and savings if you save \$100 every month for 5 years at 4% p.a.

Compound interest calculator

Your strategy

Initial deposit:

\$0

Regular deposit:

\$100

Deposit frequency:

Monthly

Compound frequency:

Monthly

Number of years: (max 50)

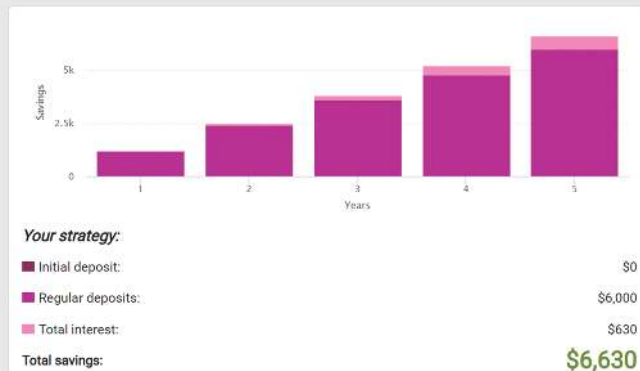
5 years

Interest rate: (max 20%)

4.00%

Effective interest rate: 4.07%

Results



- A spreadsheet can also be used to calculate this information:

	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
2	1	\$ -	0	100	\$ 100.00
3	2	\$ 100.00	0.333333	100	\$ 200.33
4	3	\$ 200.33	0.667778	100	\$ 301.00
5	4	\$ 301.00	1.003337	100	\$ 402.00

Scrolling down to the 60th month (5 years), we can see that the balance, total interest and regular deposits match the figures from the Moneysmart website:

	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
59	58	\$ 6,266.03	20.88676	100	\$ 6,386.91
60	59	\$ 6,386.91	21.28971	100	\$ 6,508.20
61	60	\$ 6,508.20	21.69401	100	\$ 6,629.90
62					
63			\$ 629.90	\$ 6,000.00	

We will develop this spreadsheet in Example 10.

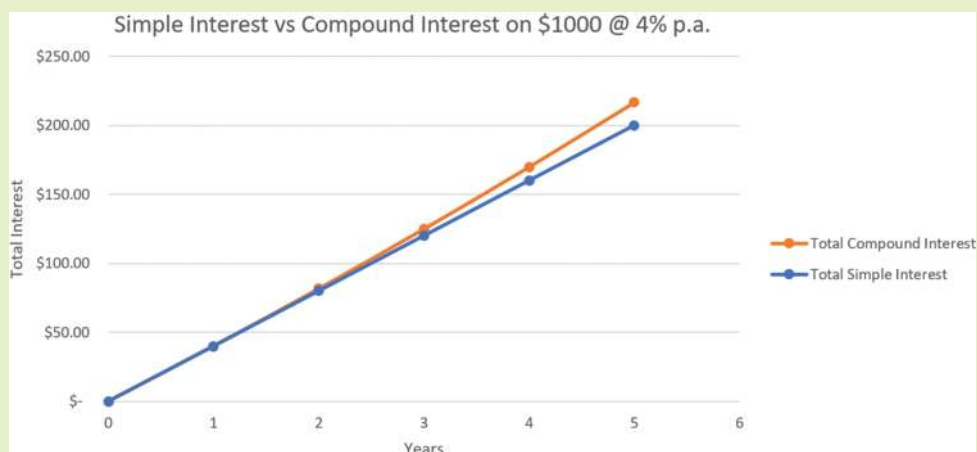
- Simple interest investments earn the same amount of interest every time period. Compound interest investments have the interest added after each time period and the amount of interest earned increases over time.

- For example: \$1000 is invested at 4% p.a. for 5 years.

Year	Simple interest	Total simple interest
1	\$40.00	\$40.00
2	\$40.00	\$80.00
3	\$40.00	\$120.00
4	\$40.00	\$160.00
5	\$40.00	\$200.00

Year	Compound interest	Total compound interest
1	\$40.00	\$40.00
2	\$41.60	\$81.60
3	\$43.26	\$124.86
4	\$44.99	\$169.86
5	\$46.79	\$216.65

Graphically, the difference between total interest earned is:



Notice how the simple interest generates a straight line and the compound interest generates a curved line. We will develop this table and graph as a technology activity in the Interactive Textbook for Exercise 9D.





Example 9 Using an online calculator to find future value and interest for a compound interest investment

Chris has saved \$1500 from his part-time job. He deposits it into an account that pays 2.6% p.a., compounding monthly. He continues to deposit \$100 into the account every month for the next 4 years. Use an online calculator to find:

- the future value of Chris's account after 4 years
- the total interest Chris earned with his account

WORKING

THINKING

a

Compound interest calculator

Your strategy

Initial deposit:

\$1,500

Regular deposit:

\$100

Deposit frequency:

Monthly

Compound frequency:

Monthly

Number of years: (max 50)

4 years

Interest rate: (max 20%)

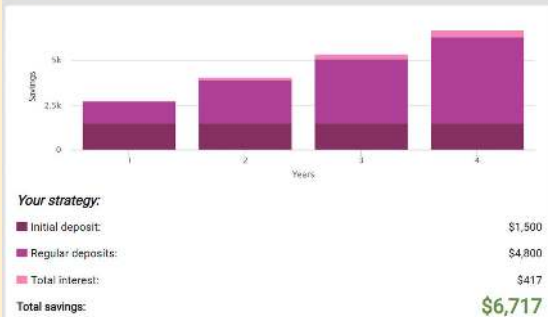
2.60%

Effective interest rate: 2.63%

Identify the values to enter in the Moneysmart compound interest calculator

<https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/compound-interest-calculator>

Results



Read the total savings from the online calculator.

Chris will have \$6717 in his account after 4 years.

Communicate your answer in a sentence.

- Chris will have earned \$417 interest over the 4 years.

Read the total interest from the online calculator.



Example 10 Using a spreadsheet to find future value and interest for a compound interest investment

Emma has already saved \$2500 from her online business selling handbags. She deposits it into an account that pays 2.15% p.a., compounding monthly. She continues to deposit \$200 into the account every month for the next 2 years.

Use a spreadsheet to find:

- a the future value of Emma's account after 2 years
- b the total interest Emma earned with her account

WORKING

- a Set up the spreadsheet with the following headings and formulas:

	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
2	1	2500	=B2*2.15/100*1/12	200	=B2+C2+D2
3	2	=E2	=B3*2.15/100*1/12	200	=B3+C3+D3
4	3				

Fill down for 2 years (24 months).

	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
22	21	\$6,659.95	\$ 11.93	200	\$6,871.88
23	22	\$6,871.88	\$ 12.31	200	\$7,084.19
24	23	\$7,084.19	\$ 12.69	200	\$7,296.88
25	24	\$7,296.88	\$ 13.07	200	\$7,509.96
26					

	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
23	22	\$6,871.88	\$ 12.31	200	\$7,084.19
24	23	\$7,084.19	\$ 12.69	200	\$7,296.88
25	24	\$7,296.88	\$ 13.07	200	\$7,509.96
26					
27			=SUM(C2:C25)		
28					

	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
23	22	\$6,871.88	\$ 12.31	200	\$7,084.19
24	23	\$7,084.19	\$ 12.69	200	\$7,296.88
25	24	\$7,296.88	\$ 13.07	200	\$7,509.96
26					
27			\$ 209.96	\$4,800.00	

Emma will have \$7509.96 after the two years.

- b Emma will have earned \$209.96 interest over the two years.

THINKING

Interest for one month = $P \times \frac{2.15}{100} \times \frac{1}{12}$

$$= P \times \frac{2.15}{100} \times \frac{1}{12}$$

The time period, n , needs to be in years to match the interest rate.

Balance = principal + interest + deposit

Principal next month = balance of last month

Make sure the deposit stays as 200 for every month.

Tip: Freeze the top row so the headings are still visible when you scroll down. See Excel Help.

- The formula =sum(cell range) will find the total interest and deposits made.

- Read the value in the final Balance cell.

- Communicate your answer in a sentence.

- Read the sum from the interest column.

Exercise 9D

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a Having a _____ for your savings and understanding the impact of your decisions will help you to be more _____ in achieving your financial goals.
 - b There are many places online that you can go to _____ future value of investments.
 - c A spreadsheet can be used to calculate the future _____ of investments and _____ earned.
 - d Simple interest investments earn the _____ amount of interest _____ time period.
 - e Compound interest investments have the interest _____ after each time period and the amount of interest earned _____ over time.



APPLICATIONS

Use the Moneysmart compound interest calculator to answer the following questions. <https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/compound-interest-calculator>

Example 9

- 2 Greg saves \$250 each fortnight into his superannuation account for twenty years. It earns 6% p.a. interest compounded monthly.
 - a Select the values that will need to be entered for the:
 - i initial deposit
 - ii regular deposit
 - iii deposit frequency
 - iv compound frequency
 - v number of years
 - vi interest rate
 - b Use the Moneysmart compound interest calculator to determine how much he will have in his superannuation account after 20 years.
 - c Use the Moneysmart compound interest calculator to determine how much he will have earned in interest.
- 3 Jeff wants to retire with a million dollars in his superannuation fund. He is currently 18 and has just finished school. He dreams of retiring when he is 55. His superannuation fund pays 5.6% p.a. compounded yearly and he plans on making a deposit every fortnight into the fund.
 - a Select the values that will need to be entered for the:
 - i initial deposit
 - ii deposit frequency
 - iii compound frequency
 - iv number of years
 - v interest rate
 - b Use the Moneysmart compound interest calculator to determine how much he will need to deposit each fortnight to reach his millionaire retirement dream.
 - c Use the Moneysmart compound interest calculator to determine how much he will need to earn in interest.



- 4** Shirley is president of her local sports club. The committee have decided that they need to plan for refurbishments of the club house that they think will need to be done in two years' time. They already have \$12 000 in the bank; they can save \$500 each month and the bank offers them 3.2% p.a. interest compounding monthly.



- a** Select the values that will need to be entered for the:

- i** initial deposit
- ii** regular deposit
- iii** deposit frequency
- iv** compound frequency
- v** number of years
- vi** interest rate

- b** Use the Moneysmart compound interest calculator to determine how much the committee will have in their account after two years.
- c** Use the Moneysmart compound interest calculator to determine how much they will have earned in interest.
- d** The committee estimate that they will need to have at least \$30 000 for the refurbishments. Use the online calculator to find how long it will take to reach their goal.
- e** The committee do not think that they can wait more than two years for the refurbishments. Use the Moneysmart compound interest calculator to determine how much they would need to save each month to reach the \$30 000 goal in two years.

- 5** Nicole is saving for a deposit on a house. She saves \$150 from her pay every fortnight into an account that pays 3.5% p.a. with interest compounding monthly.
- a** Use the Moneysmart compound interest calculator to calculate how much Nicole will have in her house deposit account after five years.
- b** Use the Moneysmart compound interest calculator to calculate how much Nicole will have in her house deposit account after 10 years.

- 6** Phill is Nicole's friend. After learning about how much Nicole had saved in five years, he decides he had better do the same thing to buy his own house. Since he is starting five years later, he doubles his regular payment, to catch up to Nicole. He also earns 3.5% p.a. interest that compounds monthly.



- a** Determine the size of Phill's regular fortnightly deposit.
- b** Use the Moneysmart compound interest calculator to calculate how much Phill will have in his house deposit account after five years.
- c** Decide whether Phill will be able to catch up to Nicole by doubling the size of his regular deposit. Explain why or why not.

Use a spreadsheet to answer the following questions.

Example 10

- 7** Allyson is planning an overseas holiday. She has just received her tax return which was \$1256 and she will use this towards her holiday. She budgets to save \$210 each month and can earn 3.15% p.a. compound interest that compounds each month.

- a** Set up a spreadsheet with the following headings:

	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
2					
3					
4					
-					

- b** Decide what will need to be entered in the following cells:
 - i** A2
 - ii** B2
 - iii** C2
 - iv** D2
 - v** E2
 - vi** A3
- c** Fill down to determine how much she has saved after 12 months.
- d** Sum the values in the interest column by using `=SUM(C2:C13)` in any cell to the right of the table. Determine how much interest Allyson earned in the 12 months.
- e** Allyson believes she will need to save \$5000 for her holiday. Continue to fill down your table to determine how many months it will take to reach her goal.

- 8** Bev is saving to buy a car. She already has \$3460 saved from her part-time job and can save \$500 per month. Her bank account pays 2.8% p.a. interest compounding monthly.

a Set up a spreadsheet with the following headings:

	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
2					
3					
4					

b Decide what will need to be entered in the following cells:

i A2

ii B2

iii C2

iv D2

v E2

vi A3

c Fill down to determine how much she has saved after 24 months.

d Sum the values in the interest column by using `=SUM(C2:C25)` in any cell to the right of the table. Determine how much interest Bev earned in the 24 months.

e Bev believes she will need to save \$20 000 for her car. Continue to fill down your table to determine how many months it will take to reach her goal.



Spreadsheet activity 9D: See the Interactive Textbook for this activity using a spreadsheet to compare the growth of simple interest and compound interest investments.



Desmos activity 9D: See the Interactive Textbook for this activity using the Desmos graphing calculator to compare the growth of simple interest and compound interest investments.

9E Investigating the effects of changing interest rates and compounding periods using technology

LEARNING GOAL

- Investigate the effect of the interest rate and the number of compounding periods on the future value of an investment using:
 - an online calculator
 - a spreadsheet

Why is it essential to understand the impact of interest rates and number of compounding periods?

We work hard for our money and knowing how to invest it wisely to earn the maximum possible amount of interest is essential to ensuring a comfortable financial future. Understanding the effect that interest rates and compounding periods have on the growth of your investments is essential.



The dollar amount of interest paid or earned can be affected by changes to interest rates and frequency of compounds.

WHAT YOU NEED TO KNOW

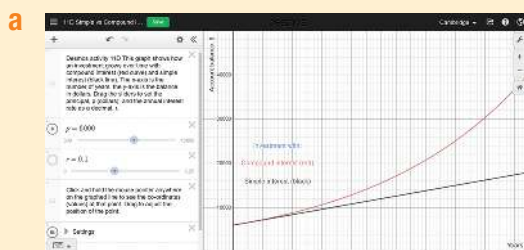
- Making changes to the interest rate and frequency of compounds will have an impact on the amount of interest earned.
- Compound interest was covered in Section 9B. The formula for compound interest is $A = P(1 + i)^n$.



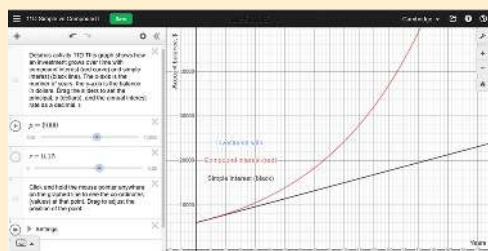
Example 11 Determining the impact of interest rate changes using an online calculator

- a** Use Desmos activity 9D from Exercise 9D in the Interactive Textbook to calculate the value of \$6000 after 15 years when it is invested at:
- 10% p.a. compounded yearly
 - 15% p.a. compounding yearly
- b** Calculate the difference in the amount of interest earned.
- c** Describe the changes to the shape of the compound interest curve when the interest rate increased.

WORKING



i After 15 years, investment = \$25 063.49.



ii After 15 years, investment = \$48 822.37.

b Difference = $48\,822.37 - 25\,063.49$
= \$23 758.88

c The curve for the compound interest investment (red line) became a lot steeper.

THINKING

Use the sliders for p and r to change to required value.

Track along the red line with the mouse, click and hold to find the value of the investment after 15 years.

The amount of interest earned almost doubled over the same time period when the interest rate was increased by half.

View the graph to describe the change when the interest increased.



Example 12 Using an online calculator to determine the impact of frequency of compounding periods

Baxter has \$15 000 to invest for two years. The bank offers a 2.5% p.a. interest rate.

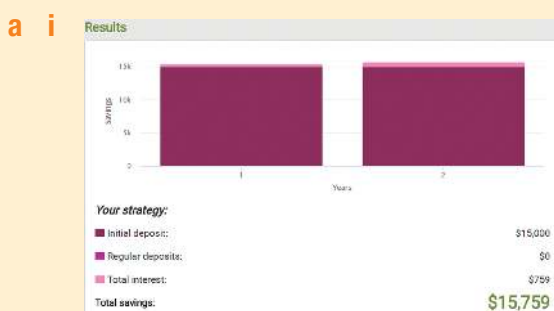
a Use the Moneysmart compound interest calculator <https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/compound-interest-calculator> to calculate the interest earned if interest compounds:

- i** annually **ii** monthly

b Describe the impact of increasing the frequency of the compounds on the amount of interest earned.

WORKING

THINKING



Total interest is \$759 when compounded annually.

Compound interest calculator

Your strategy

Initial deposit: \$15,000

Regular deposit: \$0

Deposit frequency: Monthly

Compound frequency: Annually

Number of years: (max 50)
2 years

Interest rate: (max 20%)
2.50%

Effective interest rate: 2.50%



Total interest is \$768 when compounded monthly.

b Extra interest was earned ($768 - 759 = \$9.00$) over the same time period with the same interest rate.

Use the *Alternate strategy* tab to change compounding frequency.

Alternative strategy

Delay start: 0 years

Change regular deposit: \$0 per month

Change compound frequency: Monthly

Subtract the total interest from each time period.



Example 13 Using a spreadsheet to calculate impact of changes to interest on investments

Set up a spreadsheet that will allow you to enter the variables for principal, interest rate, years and compounds per year, and then calculate the final amount when compounding at different time periods. A sample set up is:

	A	B
1	Principal, P	
2	Interest %p.a.	
3	Years	
4	Compounds per year	
5	Amount \$A	

- a** The formula for compound interest is $A = P(1 + i)^n$. Determine which cell has the value for:
- i** P **ii** i
- b i** Determine what is in cell B4.
- ii** Determine how to find the value of n required for the formula $A = P(1 + i)^n$.
- iii** Explain what will need to happen to the interest rate in the formula $A = P(1 + i)^n$ when compounding at intervals other than a year.
- c** Use the information above to construct a formula in cell B5 that will calculate the amount, \$A, of the investment.
- d** Use the spreadsheet to calculate the final amount in an account for \$5000 deposited at 7% p.a. for four years compounding:
- i** weekly **ii** monthly **iii** quarterly
- e** Describe the impact of increasing the number of compounding periods over the same time period.

WORKING

- a i** Principal = B1
- ii** Interest rate = B2

THINKING

←..... Cell referencing uses the column letter followed by the row number.

b i B4 is how many compounds occur per year.

ii $n = B4 * B3$

←..... Number of compounds = number of compounds per year $\times$ years

iii $i/B4/100$

Divide interest rate by number of compounds per year, and 100 because it is a percentage.

c $A = P(1 + i)^n$

P is in cell B1.

i is in cell B2 and needs to be divided by B4 and 100 (i.e. $B2/B4/100$).

n will be the product of B4 and B3.

So, the formula for

$A = P(1 + i)^n$ in cell B5 is
 $= B1 * (1 + B2/B4/100)^{(B3 * B4)}$

←... Number of compounds = number of compounds per year $\times$ years

The interest rate will need to be divided by the number of compounds per year and 100 because it is a percentage. Note that powers are entered in a spreadsheet using $^$.

d i

	A	B
1	Principal, P	5000
2	Interest %p.a.	7
3	Years	4
4	Compounds per year	52
5	Amount \$A	\$ 6,614.40
6		

←..... There are 52 weeks in a year.

ii

	A	B
1	Principal, P	5000
2	Interest %p.a.	7
3	Years	4
4	Compounds per year	12
5	Amount \$A	\$ 6,610.27

←..... There are 12 months in a year.

iii

	A	B
1	Principal, P	5000
2	Interest %p.a.	7
3	Years	4
4	Compounds per year	4
5	Amount \$A	\$ 6,599.65

←..... There are 4 quarters in a year.

e The more compounds the greater the amount of interest earned over the same time period.

←..... The highest final amount was with 52 compounds per year.

Exercise 9E

FUNDAMENTALS

- 1 Determine the missing words in the following sentences. Making _____ to the interest rate and frequency of compounds will have an _____ on the amount of _____ earned.
- 2 Complete the following conversions.
 - a 1 year = _____ months
 - b 1 year = _____ weeks
 - c 1 year = _____ days
 - d 1 year = _____ fortnights
 - e 1 year = _____ 6 months
 - f 1 year = _____ 4 weeks
 - g 1 year = _____ quarters

Use this table to answer questions in the exercise that follow on the next page.

Interest Rates on Term Deposits and Investment Accounts

Interest Rates current as at 28 September 2018 (Interest rates are subject to change at the Bank's discretion)

Term Deposit Interest Payment Options								
4 Weekly or Compound 4 weekly	Interest is paid every 28 days and/or at maturity.							
6 Monthly or Compound 6 Monthly	Interest is paid every 6 months and/or at maturity. For terms of 6 months or less, interest is paid at maturity.							
Annual or Compound Annually	Interest is paid every 12 months and/or at maturity. For terms of 12 months or less, interest is paid at maturity.							

Term Deposit Rates/Headline Rates*								
Interest Frequencies	\$1,000 – \$4,999	\$5,000 – \$9,999	\$10,000 – \$49,999			\$50,000 – \$1,999,999		
	Not available to new investments. All Interest Payment Options	All Interest Payment Options	4 Weekly	6 Monthly	Annual	4 Weekly	6 Monthly	Annual
Term in Months*	%pa	%pa	%pa	%pa	%pa	%pa	%pa	%pa
60	1.90	1.90	2.45	2.55	2.65	2.55	2.65	2.75
48-59	1.90	1.90	2.30	2.40	2.50	2.40	2.50	2.60
36-47	1.90	1.90	2.20	2.30	2.40	2.30	2.40	2.50
34-35	1.90	1.90	2.10	2.20	2.30	2.20	2.30	2.40
24-33	1.90	1.90	2.40	2.50	2.60	2.50	2.60	2.70
18-23	1.90	1.90	2.10	2.20	2.30	2.20	2.30	2.40
13-17	1.90	1.90	2.10	2.20	2.30	2.20	2.30	2.40
12	1.90	1.90	2.00	2.10	2.20	2.10	2.20	2.30
11	1.80	1.80	1.80	1.80	1.80	1.90	1.90	1.90
10	1.80	1.80	1.80	1.80	1.80	1.90	1.90	1.90
9	1.80	1.80	1.80	1.80	1.80	1.90	1.90	1.90
8	1.80	1.80	2.50	2.60	2.60	2.50	2.60	2.60
7	1.75	1.75	2.40	2.50	2.50	2.40	2.50	2.50
6	1.75	1.75	2.00	2.05	2.05	2.00	2.05	2.05
5	1.75	1.75	1.75	1.75	1.75	1.85	1.85	1.85
4	1.75	1.75	1.75	1.75	1.75	1.85	1.85	1.85
3	1.75	1.75	1.95	2.00	2.00	1.95	2.00	2.00
2	1.65	1.65	1.65	1.65	1.65	1.75	1.75	1.75
1	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50

Source: www.commbank.com.au

- 3** Harry has \$20 000 to invest for two years.
- a** Determine the number of months in two years.
 - b** Use the table on page 530 to determine the interest rate if interest is compounded:
 - i** annually **ii** 6 monthly **iii** 4 weekly
 - c** Determine how many times the interest will compound in a year if it compounds:
 - i** annually **ii** 6 monthly **iii** 4 weekly
 - d** Use the compound interest formula to calculate the value of the investment after two years if the interest is compounded:
 - i** annually **ii** 6 monthly **iii** 4 weekly
 - e** Decide which investment option is best for the \$20 000 over two years.

APPLICATIONS

Example 11

- 4** Use the Desmos activity 9D in the Interactive Textbook to answer the following questions.
- a** Calculate the value of \$8500 after 15 years when it is invested at:
 - i** 5% p.a. compounded yearly
 - ii** 10% p.a. compounding yearly
 - b** Calculate the difference in the amount of interest earned.
 - c** Comment on the difference in the amount of interest earned between the two rates.
 - d** Describe the changes to the shape of the compound interest curve when the interest rate increased.
- 5** Use the Desmos activity from Exercise 9D in the Interactive Textbook to answer the following questions
- a** Calculate the value of \$10 000 after 20 years when it is invested at:
 - i** 9% p.a. compounded yearly
 - ii** 12% p.a. compounding yearly
 - b** Calculate the difference in the amount of interest earned.
 - c** Comment on the difference in the amount of interest earned between the two rates.
 - d** Describe the changes to the shape of the compound interest curve when the interest rate increased.

Use the compound interest calculator on the Moneysmart website to answer the following questions. <https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/compound-interest-calculator>

Example 12

- 6** Baxter has \$8000 to invest for four years. The bank offers a 2.7% p.a. interest rate.
- Use the Moneysmart compound interest calculator to find the interest earned if interest compounds:
 - annually
 - monthly
 - Describe the impact of increasing the frequency of the compounds on the amount of interest earned.
- 7** Cherokee has \$5000 to invest for three years. The bank is offering two different types of accounts:
- 2.4% p.a. compounding annually
 - 2.2% p.a. compounding monthly
- Use the Moneysmart compound interest calculator to determine how much she will have in her account if she goes with the first option.
 - Use the Moneysmart compound interest calculator to determine how much she will have in her account if she goes with the second option.
 - Decide which account option Cherokee should go with.
- 8** Use a spreadsheet to answer the following question.

- a** Set up a spreadsheet that will allow you to enter the variables for principal, interest rate and number of compounds and calculate the final amount when compounding yearly, as shown above.

	A	B
1	Principal \$, P	1000
2	Interest %, i	5
3	Compounds, n	3
4	Amount \$ A	

- b** The formula for compound interest is $A = P(1 + i)^n$. Select the cell that has the value for:

i P **ii** i **iii** n

Hint Powers are entered using ^

- c** Construct the formula that will need to be entered in cell B4 to find the accumulated amount.
- d** Use the spreadsheet to calculate the amount in an account after \$1000 is deposited for three years at:
- 5% p.a. compounding annually
 - 10% p.a. compounding annually
 - 2% p.a. compounding annually
- e** Use the spreadsheet to calculate the amount in an account after \$65 000 is deposited for six years at:
- 3.25% p.a. compounding annually
 - 6.15% p.a. compounding annually
 - 2.01% p.a. compounding annually
- f** Describe the impact that changing the interest rate has on the amount of interest earned.

Example 13

9 Use a spreadsheet to answer the following question.

- a** Set up a spreadsheet that will allow you to enter the variables for principal, interest rate and compounds per year and then calculate the final amount when compounding at different time periods. A sample set is shown for \$1000 deposited at 6% p.a. for three years compounding monthly:

	A	B
1	Principal, P	1000
2	Interest %p.a. i	6
3	Years	3
4	Compounds per year	12
5	Amount \$A	=B1*(1+B2/B4/100)^(B3*B4)

- b** The formula for compound interest is $A = P(1 + i)^n$. Select the cell that has the value for:

i P **ii** i

c Use the spreadsheet to answer the following questions.

i Identify what is shown in cell B4.

ii Explain why the power needs to be $(B3*B4)$ for the formula to work in cell B5.

iii Explain why B2 is divided by B4 and 100 in cell B5.

d Use the spreadsheet to calculate the amount in an account after \$1000 is deposited at 6% p.a. for three years compounding:

i weekly **ii** monthly **iii** quarterly

e Use the spreadsheet to calculate the amount in an account after \$65 000 is deposited at 3.25% p.a. for six years compounding:

i weekly **ii** monthly **iii** quarterly

f Describe the impact of increasing how often interest is compounded on the amount of interest earned.

10 William is treasurer of his local sports club. The club has \$120 000 that they want to invest for two years.

a Use the table on page 540 to select the interest rate if interest is compounded:

i 4 weekly **ii** 6 monthly **iii** annually

b Use the spreadsheet from question 9 to calculate the amount in an account if interest is compounded:

i 4 weekly **ii** 6 monthly **iii** annually

c The bank offered different interest rates over different time periods. Describe what impact this may have on any advantage that is usually gained from more frequent compounds.

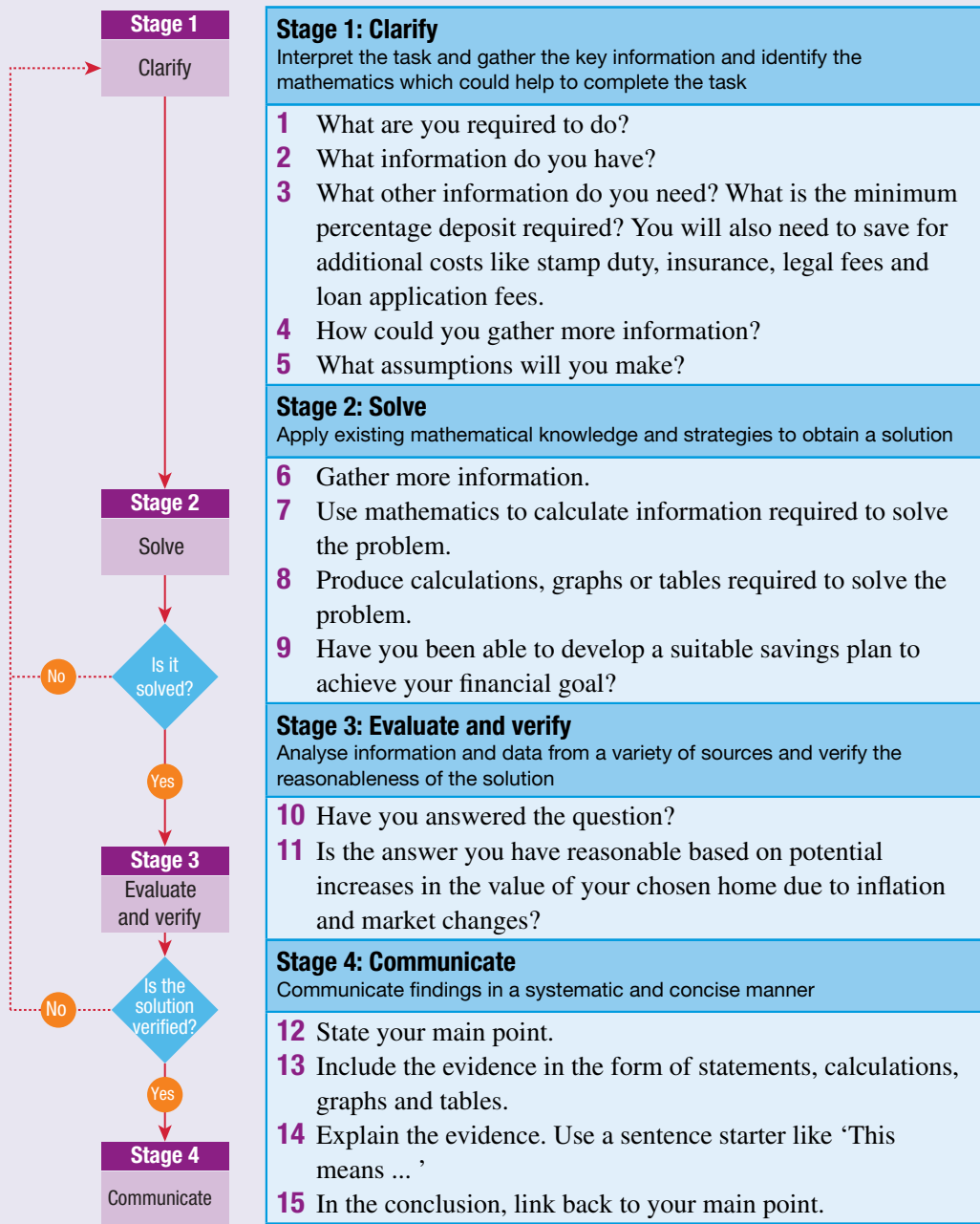
Mathematical thinking process task

Background: Buying your own home is a goal for most young people, but coming up with a deposit for a home loan is a challenge for many.

Task: You are to design a budget and savings plan for yourself with the goal of saving enough money for a deposit on a home loan. You will need to make assumptions about your income, living expenses and potential regular savings that can be made. Research savings options and interest rates with a bank that you could use for a potential future loan. Clearly identify and justify the size of the deposit required for your future home and the time you will take to achieve your goal.



Application of mathematical thinking process.



Chapter checklist

☐ I can calculate simple interest.

- 1 \$2682 invested at 4.2% p.a. for three years.
 - a Calculate the total interest earned over three years.
 - b Determine how much money will be in the account after the three years.

☐ I can calculate compound interest.

- 2 \$3540 invested at 2.7% p.a. compounded annually for four years.
 - a Determine how much money will be in the account after the four years.
 - b Calculate the total interest earned over four years.

☐ I can solve real-life problems involving compounding.

- 3 The population of Western Australia is 5.04 million people in 2018. The annual growth rate averages 1.6% p.a. Calculate the expected population of Western Australia in:
 - a 15 years
 - b 2050

☐ I can use the Moneysmart compound interest calculator to find the future value of an investment.

- 4 Anson is saving to buy a car. He already has \$8760 saved from his part-time job and can save \$400 per month. His bank account pays 2.7% p.a. interest compounding monthly. Use the Moneysmart compound interest calculator to calculate how much he has in his account after 2 years.

☐ I can use a spreadsheet to show growth of a compound interest investment.

- 5** Cody invests \$500 in an account that pays 4% p.a. interest compounded every year. A spreadsheet is used to calculate the interest and balance each year.

	A	B	C	D
1	Year	Principal	Interest	Balance
2	1			
3	2			
4	3			
5	4			
6	5			

- a** Identify the value that will be entered in cell B2.
- b** Decide which formula you will need to enter in cell C2.
- c** Determine the formula you will need to enter in cell D2.
- d** Determine the formula you will need to enter in cell B3.

☐ I can identify the effect of interest rate changes on investments.

- 6** \$7500 is deposited in an account for five years.
- a** Calculate the amount in the account after the five years if the interest rate is:
 - i** 3.75% p.a. compounding annually
 - ii** 5.15% p.a. compounding annually
 - b** Describe the effect of the interest rate change.

☐ I can identify the effect of the number of compounding periods on an investment.

- 7** \$7500 is deposited in an account for five years at 5% p.a. compounding interest.
- a** Calculate the amount in the account after five years if the interest is compounding:
 - i** annually
 - ii** fortnightly
 - b** Describe the effect of the change in compounding frequency.

Chapter review

Section 9A

- 1 Riley invested \$60 000 in a fixed term account for 3 years paying a simple interest rate of 2.65% p.a.
 - a Calculate the total amount of interest earned over the 3 years.
 - b Calculate the total value of the investment after 3 years.
- 2 A company invested \$5 000 000 in the short-term money market at 6.7% p.a. simple interest for 90 days.
 - a Calculate the total amount of interest earned over the 90 days.
 - b Calculate the total value of the investment after 90 days.

Section 9B

- 3 Electra invests \$1400 into an account that pays 3.8% p.a. compound interest for three years. Interest is compounded yearly.
 - a Copy and complete the following table to calculate her interest and balance at the end of each year.

Year	Principal	Interest	Balance
1	\$1400		
2			
3			

- b Determine how much interest she earned in three years.
- 4 Shae-Leah invests \$2000 in an account that earns interest at the rate of 4.6% p.a. compounding monthly. Use the compound interest formula to determine how much is in her account after four years.
- 5 Smithy has saved \$5200 from his part-time job. He invests it for two years at 3.20% p.a. so he can buy a car when he turns 18. Interest is compounded yearly.
 - a Use the compound interest formula to calculate the balance of his account after two years.
 - b Calculate how much interest he earned over the two years.

Section 9D

- 6** Tom is saving for a deposit on a house. He saves \$450 from his pay every fortnight into an account that pays 3.6% p.a. with interest compounding monthly. He uses the Moneysmart compound interest calculator to determine how much he will have saved after 5 years.

- a** Decide what he will need to enter on the online calculator at:
- initial deposit
 - regular deposit
 - deposit frequency
 - compound frequency
 - number of years
 - interest rate

Compound interest calculator

Your strategy

Initial deposit:

Regular deposit:

Deposit frequency:

Compound frequency:

Number of years: (max 50)

Interest rate: (max 20%)

- b** The calculator returns the following values:



- Determine how much he will have in his account after 5 years.
- Identify how much interest he has earned.

Section 9E

- 7** \$3500 is deposited for three years in an account.

- a** Calculate the amount in the account after three years at:
- 4.75% p.a. compounding annually
 - 5.25% p.a. compounding annually
- b** Describe the effect of the interest rate change.

- 8** \$3500 is deposited for five years in an account at 4.5% p.a. compounding interest.

a Calculate the amount in the account after five years if the interest compounds:

- i** every 6 months
- ii** weekly

b Describe the effect of the change in compounding frequency.

Section 9C

- 9** The population of Perth was 2.4 million in 2018 with an annual growth rate of 3.42% p.a. If the population continues to grow at the same rate, use the compound interest formula to estimate the population:

a 5 years after 2018 **b** in 2100

- 10** The average rate of inflation in Australia is 2.1% p.a. If a 3 L bottle of milk costs \$3.30 in 2018, use the compound interest formula to estimate how much will it cost:

a 15 years after 2018 **b** in 2050

- 11** The following table shows the interest and balance earned on a \$5000 investment earning 5% p.a. for five years when interest is calculated using simple interest vs compound interest.

	Simple interest			Compound interest		
Year	Principal	Interest	Balance	Principal	Interest	Balance
1	\$5000.00	\$250.00	\$5250.00	\$5000.00	\$250.00	\$5250.00
2	\$5000.00	\$250.00	\$5500.00	\$5250.00	\$262.50	\$5512.50
3	\$5000.00	\$250.00	\$5750.00	\$5512.50	\$275.63	\$5788.13
4	\$5000.00	\$250.00	\$6000.00	\$5788.13	\$289.41	\$6077.53
5	\$5000.00	\$250.00	\$6250.00	\$6077.53	\$303.88	\$6381.41

a Calculate the total interest earned in the:

- i** simple interest account
- ii** compound interest account

b Explain why the balance is the same at the end of the first year.

c Explain the key difference in how simple interest and compound interest is calculated.

- 12** Computers lose 45% of their value each year. Biar buys a new laptop computer for \$1899. Use the compound interest formula to estimate how much the laptop will be worth in two years' time.
- 13** Amy is planning an overseas holiday. She has just inherited \$3400 which she will use towards her holiday. She budgets to save \$360 each month and can earn 4.25% p.a. compound interest that compounds each month. She sets up a spreadsheet as follows.

	A	B	C	D	E
1	Month	Principal	Interest	Deposit	Balance
2					
3					
4					

Determine what needs to be entered in the following cells.

a A2

b B2

c C2

d D2

e E2

f B3

10 Reducing balance loans





In this chapter

- 10A** Understanding and modelling reducing balance loans
 - 10B** Modelling reducing balance loans using spreadsheets
 - 10C** Investigating the effect of changing interest rates, compounding periods and repayments
 - 10D** Investigating the effect of changing interest rates, compounding periods and repayments using spreadsheets
- Mathematical thinking process task
Chapter checklist
Chapter review

Syllabus reference

Unit 4 Topic 3: Loans and compound interest (6 hours)

Reducing balance loans (compound interest loans with periodic repayments)

- use technology and a recurrence relation to model a reducing balance loan
- investigate the effect of the interest rate and repayment amount on the time taken to repay a loan

Pre-test

1 Calculate the simple interest on the following investments.

- a** \$450 at 6% p.a. for one month
- b** \$980 at 5% p.a. for one week
- c** \$387 at 4% p.a. for one fortnight
- d** \$1345 at 5% p.a. for one quarter
- e** \$350 at 6% p.a. for 6 months

Hint 'p.a.' means per annum, i.e. per year.

2 \$4000 is borrowed at 6% p.a. simple interest for 2 years.

- a** Calculate the interest for the two years.
- b** Calculate the total amount repaid over the two years.
- c** Calculate the size of the monthly repayment.

3 Convert the following time periods to years and months.

- a** 48 months
- b** 260 weeks
- c** 78 fortnights
- d** 42 months
- e** 338 weeks
- f** 91 fortnights
- g** 200 months
- h** 200 weeks
- i** 200 fortnights



10A Understanding and modelling reducing balance loans

LEARNING GOALS

- Understand that reducing balance loans are compound interest loans with periodic repayments
- Model a reducing balance loan by using technology (online calculator)
- Calculate the future value of a compound interest loan and the total interest paid by using technology (online calculator)
- Compare the growth of simple interest and compound interest loans by using technology (online calculator)

Why is it essential to understand how reducing balance loans work?

We all need to borrow money at some stage of our lives. It may be to purchase a car or your first home. Most loans have interest calculated on the remaining balance. Understanding how the loan works and the savings to be made by paying more frequently, or just a little bit more, will help you to achieve your financial goals.

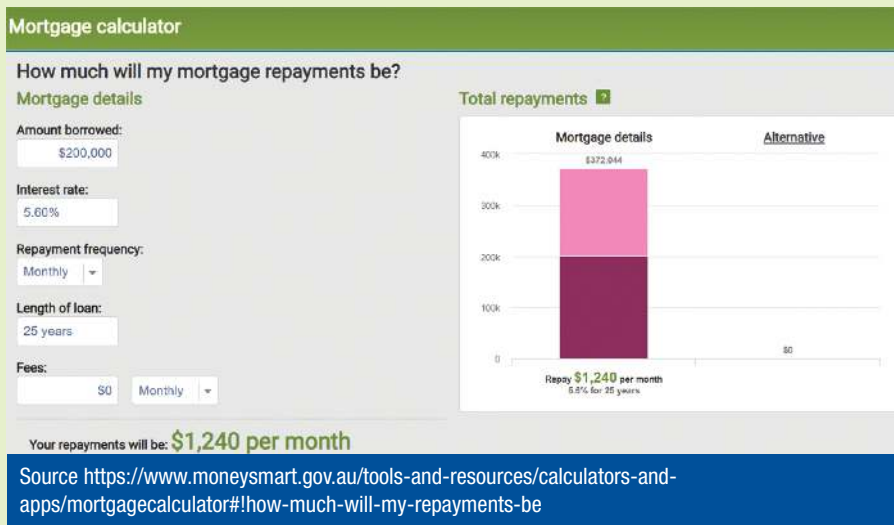


A home loan is the biggest expense you will ever have, so it is wise to look for ways to make savings.

WHAT YOU NEED TO KNOW

Simple interest loans were dealt with in Exercise 9A. They are not as common as reducing balance loans but are easy to calculate the interest and size of the repayments. **Reducing balance loans** have the interest calculated on the remaining balance each interest period.

- The mortgage calculator on the Moneysmart website allows you to enter how much you need to borrow and different variables such as the interest rate and the length of time you will take to repay the loan. The online calculator then generates for you the size of the regular repayments required for that loan. Moneysmart Mortgage Calculator: <https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/mortgage-calculator#!how-much-will-my-repayments-be>
 - For example: If you borrow \$200 000 over 25 years with an interest rate of 5.6% p.a. calculated on the reducing balance, the following calculations will be shown.



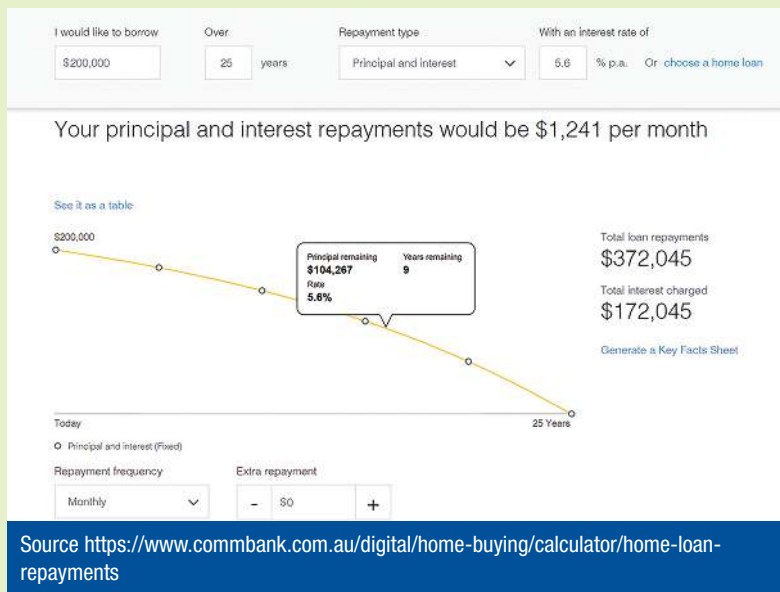
The monthly repayments would be \$1240.

The total amount repaid = $1240 \times 12 \times 25 = \$372\,000$

The total interest = total repaid – amount borrowed
 $= 372\,000 - 200\,000 = \$172\,000$

Note: The total interest is represented by the pink part of the bar graph, sitting on top of the principal.

- All banks provide loan calculators on their websites to help you with understanding the costs of a loan. We will use the Commonwealth Bank home loan calculator: <https://www.commbank.com.au/digital/home-buying/calculator/home-loan-repayments>
- For example: If \$200 000 was borrowed at 5.6% p.a. reducible interest over 25 years, the debt remaining would be modelled as:



Note: the difference in repayment amounts between the two online calculators is because one website has rounded down and the other has rounded up.

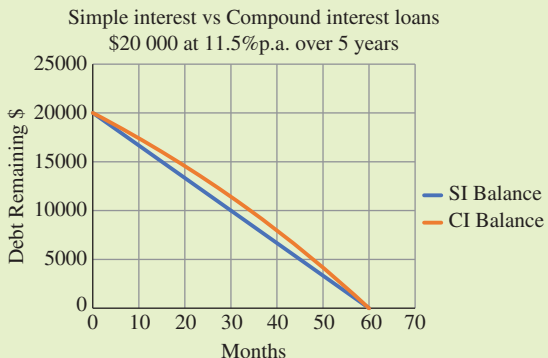
- By floating your cursor above the graph, we see that it takes more than 16 years to pay the first half of the debt, and less than 9 years to pay the second half of the debt. We could also see this information in a table by clicking on the link at the top left of the graph.

The shape of the graph shows that the debt remaining on the loan is slow to decrease at the start and decreases faster towards the end. This is because the *interest is calculated on the reducing balance of the loan* so will reduce over time; while the repayment remains the same. Repayments made towards the end of the loan will have more impact on the balance remaining.

- We can see this by looking at the first few months of this loan in a table:

Month	Principal	Interest	Repay	Balance
1	200 000.00	$I = 200\,000.00 \times \frac{5.6}{100} \times \frac{1}{12}$ $= 933.33$	1241	$200\,000.00 + 933.33$ $-1241 = 199\,692.33$
2	199 692.33	$I = 199\,692.33 \times \frac{5.6}{100} \times \frac{1}{12}$ $= 931.90$	1241	$199\,692.33 + 931.90$ $-1241 = 199\,383.23$
3	199 383.23	$I = 199\,383.23 \times \frac{5.6}{100} \times \frac{1}{12}$ $= 930.46$	1241	$199\,383.23 + 930.46$ $-1241 = 199\,072.69$

- Interest for each month is calculated using the formula $I = Pin$.
 - The balance at the end of each month is found by adding the interest and subtracting the repayment from the principal at the start of the month.
 - The balance from the previous month becomes the principal in the next month.
 - The interest reduces each month as it is calculated on the reducing balance.
- When comparing the graphs of debt remaining on simple interest and compound interest loans, their shape is different, as they were in Section 9D for investments. The simple interest graph is a straight line as the interest remains the same during the entire loan. The compound interest graph is curved as the interest decreases as the loan is paid off.



- The interest costs across the two loans is also significantly different:

Total interest (simple interest) = \$11 500

Total interest (compound interest) = \$6390.97

Note: This graph will be developed as a technology (spreadsheet) activity in Exercise 10B.



Example 1 Calculating a reducing balance loan

Tait has borrowed \$235 000 to buy a unit. The terms of his loan are 6.55% p.a. reducible interest over 20 years and his monthly repayments are \$1759 per month.

- a** Complete this table to determine how much Tait will owe on his loan after the first three months.

Month	Principal	Interest	Repayment	Balance
1				
2				
3				

- b** Determine the total of Tait's monthly repayments over the first three months.
c Calculate the total of the interest charges over the first three months.
d Calculate the difference between the amount repaid and the interest charges over the first three months.
e Determine how much Tait reduced his debt by in the first three months.

WORKING

- a** Month 1

$$\text{Principal} = 235\,000$$

$$\text{Interest} = 235\,000 \times \frac{6.55}{100} \times \frac{1}{12}$$

$$= \$1282.71$$

$$\text{Repayment} = \$1759$$

$$\text{Balance} = 235\,000 + 1282.71 - 1759$$

$$= \$234\,523.71$$

- Month 2

$$\text{Principal} = \$234\,523.71$$

$$\text{Interest} = 234\,523.71 \times \frac{6.55}{100} \times \frac{1}{12}$$

$$= \$1280.11$$

$$\text{Repayment} = \$1759$$

$$\text{Balance} = 234\,523.71 + 1280.11 - 1759$$

$$= \$234\,044.82$$

THINKING

Interest is calculated using the simple interest formula $I = Pin$. Put the percentage interest rate over 100 to convert to a decimal.

$$n = 1 \text{ month} = \frac{1}{12} \text{ year}$$

Balance = starting principal + interest – repayment made

Previous month's balance becomes this month's principal.

Interest formula $I = Pin$

WORKING

Month 3
Principal = \$234 044.82

$$\text{Interest} = 234\,044.82 \times \frac{6.55}{100} \times \frac{1}{12}$$

$$= \$1277.49$$

Repayment = \$1759

$$\text{Balance} = 234\,044.82 + 1277.49 - 1759$$

$$= \$233\,563.31$$

THINKING

Previous month's balance becomes this month's principal.

Interest formula $I = Pin$

Complete the values in the table.

Month	Principal	Interest	Repayment	Balance
1	\$235 000.00	\$1282.71	\$1759.00	\$234 523.71
2	\$234 523.71	\$1280.11	\$1759.00	\$234 044.82
3	\$234 044.82	\$1277.49	\$1759.00	\$233 563.31

b Total repayments = 1759×3
= \$5277

Three equal repayments were made.

c Total interest
= $1282.71 + 1280.11 + 1277.49$
= \$3840.31

The interest is reducing each month.

d Difference = $5277 - 3840.31$
= \$1436.69

Difference = total repaid – total interest

e Difference = $235\,000 - 233\,563.31$
= \$1436.69

Difference = original loan – debt remaining



Example 2 Using an online calculator for a compound interest loan

Alexander has borrowed \$20 000 over 5 years at 11.5% p.a. with reducible (compound) interest to buy a car. Use the Moneysmart personal loan calculator to determine:

- a the monthly repayment required
- b the total repaid
- c the total interest

WORKING

THINKING

a

Personal loan calculator

How much will my repayments be?

Personal loan details

Amount borrowed: \$20,000

Interest rate: 11.50%

Repayment frequency: Monthly

Length of loan: 5 years

Fees: \$0 Monthly

Your repayments will be: **\$440 per month**

..... <https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/personal-loan-calculator#!how-much-will-my-repayments-be>
Enter values into an online calculator.

The monthly repayment will be \$440.



..... This is different to the value on the graph (\$26 391) because the monthly repayment has been rounded to the nearest dollar.

$$\begin{aligned}\text{Total repaid} &= 440 \times 12 \times 5 \\ &= \$26\,400\end{aligned}$$

- c** Total interest = 26 400 – 20 000 = \$6400
- Total interest = total repaid – amount borrowed
You can also hover over the pink section of the graph to find the interest.



Example 3 Using an online calculator to model a reducing balance loan

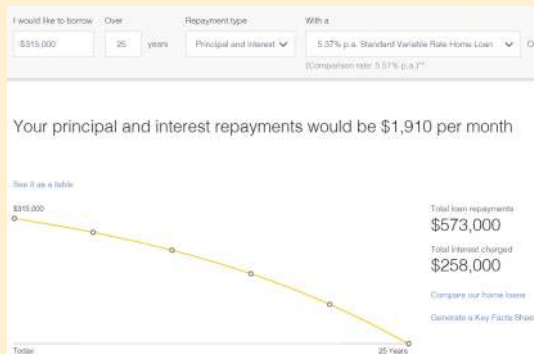
Ella has borrowed \$315 000 to buy a house. She will repay the loan monthly over 25 years with 5.37% p.a. reducible interest. Use the Commonwealth Bank home loan calculator to answer the following.

The Commonwealth Bank home loan calculator: <https://www.commbank.com.au/digital/home-buying/calculator/home-loan-repayments>

- Determine the size of the monthly repayments.
- Determine the total loan repayments.
- Determine the total interest charges.
- Determine the amount left owing after 20 years.
- Determine how many years it takes to pay off half the loan.

WORKING

THINKING



Input variables into online calculator. Read off required information.

- Monthly repayments = \$1910
- Total loan repayments = \$573 000
- Total interest charges = \$258 000
- 20 years: $25 - 20 = 5$ years remaining

Hover over graph to find required value.



There was \$98 847 owing after 20 years.

e Half the loan = $315\,000 \div 2$
 $= \$157\,500$

Need to find the first time
 the principal remaining is
 less than \$157 500.



With 8 years remaining, the principal falls
 below \$157 500.

Time taken = $25 - 8 = 17$ years

It takes 17 years to pay off the first half of
 the loan.

Exercise 10A

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a All loans require you to repay what you _____ plus _____.
 - b Reducing balance loans have the interest calculated on the _____ balance each _____ period.
 - c Interest for each month is calculated using the _____ $I = Pin$.
 - d The balance at the end of each month is found by _____ the interest and _____ the repayment from the principal at the start of the month.
 - e The balance from the previous month becomes the _____ in the next month.
 - f The interest _____ each month as it is calculated on the reducing balance.
 - g Repayments made towards the _____ of the loan will have more impact on the _____ remaining.
 - h Online _____ help you find the amount of interest you pay and the _____ cost of the loan.

APPLICATIONS

Example 1

- 2 Lana has borrowed \$2300 to purchase a new bed. She has agreed to pay off the loan in 12 months with reducible interest calculated at 12.65% p.a. Her monthly repayments are \$205. Lana is given a table of her loan schedule as follows.

Month	Principal	Interest	Repayment	Balance
1	\$ 2300.00	\$ 24.25	\$ 205.00	\$ 2119.25
2	\$ 2119.25	\$ 22.34	\$ 205.00	\$ 1936.59
3	\$ 1936.59	\$ 20.41	\$ 205.00	\$ 1752.00
4	\$ 1752.00	\$ 18.47	\$ 205.00	\$ 1565.47
5	\$ 1565.47	\$ 16.50	\$ 205.00	\$ 1376.97
6	\$ 1376.97	\$ 14.52	\$ 205.00	\$ 1186.49
7	\$ 1186.49	\$ 12.51	\$ 205.00	\$ 994.00
8	\$ 994.00	\$ 10.48	\$ 205.00	\$ 799.47
9	\$ 799.47	\$ 8.43	\$ 205.00	\$ 602.90
10	\$ 602.90	\$ 6.36	\$ 205.00	\$ 404.26
11	\$ 404.26	\$ 4.26	\$ 205.00	\$ 203.52
12	\$ 203.52	\$ 2.15	\$ 205.00	\$ 0.66

- Decide if Lana will completely pay off the loan in 12 months. Why will this happen?
- Explain how much Lana should pay in the last month to completely pay off the loan.
- Determine how much Lana will still owe on her loan after 6 months.
- Describe what happens to the interest each month.
- Calculate the total interest Lana pays on the loan.
- Calculate 12.65% of \$2300. Explain why this is different to your answer in part e.

Hint Consider the final balance.

- 3 Todd repays \$373 each month for four years on his \$15 000 loan to buy a motorbike.
- Determine how many repayments will he make.
 - Calculate how much he will repay in total.
 - Calculate how much he will pay in interest.

Example 1

- 4** Lochy has borrowed \$14 700 to buy a car. The terms of his loan are 11.45% p.a. reducible interest over four years and his monthly repayments are \$383 per month.

- a** Copy and complete this table to find how much Lochy will owe on his loan after the first three months.

Month	Principal	Interest	Repay	Balance
1	14 700	$14\,700 \times \frac{11.45}{100} \times \frac{1}{12}$ $= 140.26$	383	$14\,700 + 140.26 - 383$ $= 14\,457.26$
2	14 457.26			
3				

- b** Calculate the total of Lochy's monthly repayments over the first three months.
c Calculate the total of the interest charges over the first three months.
d Determine the difference between the amount repaid and the interest charges over the first three months.
e Determine how much Lochy will reduce his debt by in the first three months.
- 5** Stephanie has borrowed \$5600 to fund a trip overseas. The terms of her loan are 12.10% p.a. reducible interest over two years and her monthly repayments are \$264 per month.
- a** Copy and complete this table to find how much Stephanie will owe on her loan after the first four months.

Month	Principal	Interest	Repay	Balance
1				
2				
3				
4				

- b** Determine how much interest Stephanie will pay in the first month.
c Calculate how much Stephanie will reduce her debt by in the first month.
d Determine how much interest Stephanie will pay in the fourth month.
e Calculate how much Stephanie will reduce her debt by in the fourth month.
f Interpret the difference between your answers to parts **c** and **e**.

- 6** Gus has borrowed \$28 000 to start up a business. He has agreed to repay the loan in two years with 4.35% p.a. reducible interest. He will repay the loan with quarterly payments of \$3673.
- a** Determine how many repayments Gus will make.
 - b** Construct a loan schedule for Gus with the headings: Quarter, Principal, Interest, Repayment, Balance.
 - c i** Determine how much will be left owing after eight quarters.
 - ii** Explain why this will happen.
 - iii** Describe how can Gus fix this issue.
 - d** Calculate the total interest if this had been a simple interest loan.
 - e** Calculate how much interest Gus will pay on his reducible interest loan.

Use the calculators on the Moneysmart website <https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/loan-credit-and-debt-calculators> to answer the following questions:

Example 2

- 7** Lakeisha has borrowed \$18 000 to buy a car. Interest is calculated at 9.85% p.a. on the reducing balance (compound interest) and she will repay the loan over four years. Use the Moneysmart personal loan calculator to determine:
- a** the monthly repayment required
 - b** the total repaid
 - c** the total interest
- 8** Tilly has borrowed \$12 000 to pay for an overseas holiday. Interest is calculated at 10.85% p.a. on the reducing balance (compound interest) and she will repay the loan over three years. Use the Moneysmart personal loan calculator to determine:
- a** the monthly repayment required
 - b** the total repaid
 - c** the total interest
- 9** Charles has borrowed \$250 000 to buy a home. Interest is calculated at 5.85% p.a. on the reducing balance (compound interest) and he will repay the loan over 25 years.
- a** Use the Moneysmart Mortgage Calculator to determine:
 - i** the monthly repayment required
 - ii** the total repaid
 - iii** the total interest



- b** Charles thinks he could afford to repay \$1700 each month.
- Use the 'How can I repay my home loan sooner?' tab to determine how long it would now take Charles to repay the loan.
 - Determine how much Charles will now repay in total.
 - Determine how much Charles will now pay in interest.
 - Calculate the saving in interest by paying more each month.
 - Determine how much extra Charles will repay each month.
 - Explain what recommendations you would make about paying extra in repayments.

Use the Commonwealth Bank home loan calculator to answer the following questions: <https://www.commbank.com.au/digital/home-buying/calculator/home-loan-repayments>

Example 3

- 10** Kayla has borrowed \$260 000 to buy a home. She will repay the loan monthly over 25 years with 4.59% p.a. reducible interest. Use the Commonwealth Bank home loan calculator to determine the:

- size of the monthly repayments
- total loan repayments
- total interest charges
- amount left owing after 20 years
- number of years it will take to pay off half the loan



- 11** Declan has borrowed \$22 000 to buy a car. He will repay the loan monthly over 5 years with 10% p.a. reducible interest. Use the Commonwealth Bank home loan calculator to determine the:

- size of the monthly repayments
- total loan repayments
- total interest charges
- amount left owing after 3 years
- number of years it takes to pay off half the loan

- 12** A school has borrowed \$5 million to build a new sports centre. They will repay the loan in equal monthly payments over 5 years with 4.89% p.a. reducible interest. Use the Commonwealth Bank home loan calculator to determine the:

- size of the monthly repayments
- total loan repayments
- total interest charges
- amount left owing after 4 years
- number of years it takes to pay off half the loan

10B Modelling reducing balance loans using spreadsheets

LEARNING GOALS

- Model a reducing balance loan by using technology (spreadsheet)
- Calculate the future value of a compound interest loan and the total interest paid by using a spreadsheet
- Compare the growth of simple interest and compound interest loans by using a spreadsheet

Why is it essential to use technology for reducing balance loans?

We saw in the last section that the calculations required for a reducing balance loan schedule are repetitive and time consuming. Using a spreadsheet can make the process a lot quicker with less potential to make careless errors.



Using computer spreadsheets to do reducing balance loan calculations saves time.

WHAT YOU NEED TO KNOW

- We can use a spreadsheet to perform the table calculations we did in the last section.
- For example: If \$200 000 was borrowed at 5.6% p.a. reducible interest over 25 years, a suitable set up for the debt remaining would be:

	A	B	C	D	E
1	Month	Principal	Interest	Repayment	Balance
2	1	200000	$=B2*5.6/100*1/12$	1241	$=B2+C2-D2$
3	2	$=E2$	$=B3*5.6/100*1/12$	1241	$=B3+C3-D3$
4	3	$=E3$	$=B4*5.6/100*1/12$	1241	$=B4+C4-D4$

For a 25-year home loan, we would need to fill down rows to fully pay off the loan.

- Excel can calculate the size of a loan repayment using the formula:

$\text{=PMT}(\text{rate}, \text{nper}, \text{pv}, [\text{fv}], [\text{type}])$

where:

- rate The interest rate for the loan as a fraction or decimal
- nper The total number of payments to pay off the loan
- pv The present value of the principal; the amount of money borrowed
- fv The future value of the loan. As we will pay off our loans, this will be 0.
- type 0 means the payment is made at the end of the period, 1 means the payment is made at the beginning of the period. It is usual to pay at the end of the time period, so we will enter 0 for type.



Example 4 Using a spreadsheet for a compound interest loan

Note: the spreadsheet in this example can be accessed in the Interactive Textbook by clicking on the icon at left.

Georgina has borrowed \$335 000 over 20 years at 6.2% p.a. with reducible (compound) interest to buy a house. Use a spreadsheet to determine:

- the monthly repayment required
- the total repaid
- the total interest

WORKING

a

	A	B
1	Principal	335000
2	Rate	6.2
3	Years	20
4	Repayments/year	12
5		
6	Number of Payments	=B4*B3
7		
8	Repayment	$\text{=(PMT((B2/(100*B4)),B6,-B1,0,0))}$
9	Total Repaid	=B8*B6
10	Total Interest	=B9-B1

	A	B
1	Principal	335000
2	Rate	6.2
3	Years	20
4	Repayments/year	12
5		
6	Number of Payments	240
7		
8	Repayment	\$2,438.86

The monthly repayment is \$2439.

THINKING

The Excel formula needs to know rate, number of payments and principal.
 $\text{=PMT}(\text{rate}, \text{nper}, \text{pv}, [\text{fv}], [\text{type}])$

The rate will need to be divided by number of payments per year and 100 to make it a decimal.

Number of payments = payments per year $\times$ number of years
 As a formula this is =B4*B3

The principal is entered as a negative value in the formula since it is a debt. For [fv] and [type] we need to enter 0. Set up spreadsheet as shown. Round answer to nearest dollar amount.

... Continued

b For the total amount repaid:

	A	B
1	Principal	335000
2	Rate	6.2
3	Years	20
4	Repayments/year	12
5		
6	Number of Payments	=B4*B3
7		
8	Repayment	=PMT((B2/(100*B4)),B6,-B1,0,0))
9	Total Repaid	=B8*B6
10	Total Interest	=B9-B1

Total repaid = \$585 325.58

◀ Total repaid = repayment × number of payments

As a formula this is =B8*B6

c

	A	B
1	Principal	335000
2	Rate	6.2
3	Years	20
4	Repayments/yea	12
5		
6	Number of Paym	240
7		
8	Repayment	\$2,438.86
9	Total Repaid	\$585,325.58
10	Total Interest	\$250,325.58

Total interest = \$250 325.58

◀ Total interest = Total repaid – amount borrowed

As a formula this is =B9-B1



Example 5 Using a spreadsheet to model a reducing balance loan

Note: the spreadsheet in this example can be accessed in the Interactive Textbook by clicking on the icon at left.

Josh has borrowed \$25 000 to buy a car. He will repay the loan with monthly repayments over 4 years with 13.9% p.a. reducible interest.

- Use a spreadsheet to calculate the size of the monthly repayment to the nearest dollar.
- Set up a spreadsheet to model the loan schedule over 4 years.
- Determine the amount left owing after 48 months.
- Calculate the size of the last repayment to fix the overpayment.
- Determine how many months it takes to pay off half the loan.

WORKING

a Monthly repayment = \$682



Use the spreadsheet from Example 4.

	A	B
1	Principal	25000
2	Rate	13.9
3	Years	4
4	Repayments/year	12
5		
6	Number of Payments	48
7		
8	Repayment	\$681.91

	A	B
1	Principal	25000
2	Rate	13.9
3	Years	4
4	Repayments/year	12
5		
6	Number of Payments	=B4*B3
7		
8	Repayment	=PMT((B2/(100*B4)),B6,-B1,0,0))

b

	A	B	C	D	E
1	Month	Principal	Interest	Repayment	Balance
2	1	25000	=B2*13.9/100*1/12	682	=B2+C2-D2
3	2	=E2	=B3*13.9/100*1/12	682	=B3+C3-D3
4	3	=E3	=B4*13.9/100*1/12	682	=B4+C4-D4



All formulas need to start with =.
Use the same method as used in
Exercise 10A to calculate values
in cells.

Fill down for 48 months.

c -\$5.83 is left owing after 48 months.
The loan has been overpaid by \$5.83.



Scroll to the bottom to find answer.

	A	B	C	D	E
1	Month	Principal	Interest	Repayment	Balance
47	46	\$ 1,993.87	\$ 23.10	\$ 682.00	\$ 1,334.96
48	47	\$ 1,334.96	\$ 15.46	\$ 682.00	\$ 668.43
49	48	\$ 668.43	\$ 7.74	\$ 682.00	-\$ 5.83

Tip: freeze the top row so that the
headings stay visible when scrolling.

d Final payment = $682 - 5.83$
= \$676.17



Reduce final payment to finish with a
\$0 balance.

	A	B	C	D	E
1	Month	Principal	Interest	Repayment	Balance
47	46	\$ 1,993.87	\$ 23.10	\$ 682.00	\$ 1,334.96
48	47	\$ 1,334.96	\$ 15.46	\$ 682.00	\$ 668.43
49	48	\$ 668.43	\$ 7.74	\$ 676.17	-\$ 0.00

e Half the loan = $25\,000 \div 2$
= \$12 500



Scroll until you first find a balance
that is less than \$12 500.

It takes 28 months to repay half
the loan.

	A	B	C	D	E	F
1	Month	Principal	Interest	Repayment	Balance	
27	26	\$13,696.73	\$ 158.65	\$ 682.00	\$13,173.38	
28	27	\$13,173.38	\$ 152.59	\$ 682.00	\$12,643.98	
29	28	\$12,643.98	\$ 146.46	\$ 682.00	\$12,108.44	
30	29	\$12,108.44	\$ 140.26	\$ 682.00	\$11,566.69	
31	30	\$11,566.69	\$ 133.98	\$ 682.00	\$11,018.67	

Exercise 10B

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a Using _____ calculators and computer _____ to do reducing balance loan calculations saves time.
 - b A spreadsheet can help you understand the total _____ of a loan.
 - c A spreadsheet can do the repetitive table _____ when we enter in the formula.

APPLICATIONS



Spreadsheet Example 4 applies to question 2.

Example 4

- 2 Ayla has borrowed \$17 000 to buy a car. She will repay the loan with monthly repayments over 3 years with 6.9% p.a. reducible interest.
 - a Use a spreadsheet to calculate the size of each repayment to the nearest dollar.
 - b Calculate the total loan repayments.
 - c Calculate the total interest charges.



Spreadsheet Example 5 applies to questions 3–8.

Example 5

- 3 Use the information in question 2 to complete the following.
 - a Set up the spreadsheet to model the loan schedule over 3 years.
 - b Determine how much is left owing after 36 months.
 - c Calculate the size of the last repayment to fix the underpayment.
 - d Determine how many months it will take to pay off half the loan.
- 4 Clayton has borrowed \$6000 to fund an overseas holiday. He will repay the loan with fortnightly repayments over 2 years with 9.73% p.a. reducible interest.
 - a Use a spreadsheet to calculate the size of the repayment to the nearest dollar.
 - b Determine the total number of payments.
 - c Calculate the total loan repayments.
 - d Calculate the total interest charges.
- 5 Use the information in question 4 to complete the following.
 - a Set up a spreadsheet to model the loan schedule over 2 years.
 - b Calculate the interest for the first fortnight.
 - c Determine how much is left owing after 2 years.
 - d Calculate the size of the last repayment to fix the underpayment.
 - e Determine how many fortnights it will take to pay off half the loan.

Note: a spreadsheet for questions 6–8 can be accessed in the Interactive Textbook by clicking on the appropriate icon.

- 6** Lexi has borrowed \$20 000 to buy a car. Simple interest is charged at 11.5% p.a. and she will repay the loan over five years in equal monthly repayments.

a Calculate:

- i** the total interest.
 - ii** the total to repay
 - iii** the number of repayments
 - iv** how much she will need to repay each month
- b** Use a spreadsheet to show the amount still owing over the life of the loan. Use the following headings and formulas and fill down for 60 months.

	A	B	C	D
1			Simple Interest	
2	Month	Simple Interest	Repayment	SI Balance
3	0			20000
4	1	$=20000*11.5/100*1/12$	525	$=D3+B4-C4$
5	2	$=20000*11.5/100*1/12$	525	$=D4+B5-C5$

- i** Explain the formula used in cell B4.
 - ii** Explain the formula in cell D4.
 - iii** Explain why the formula in B5 the same as B4.
- c** In cell B65 enter the formula $=\text{SUM}(B4:B63)$. Decide if the answer matches your answer from part **a i**.
- d** In cell C65 enter the formula $=\text{SUM}(C4:C63)$. Decide if the answer matches your answer from part **a ii**.
- e** Describe the values in the Simple Interest column (Column B).

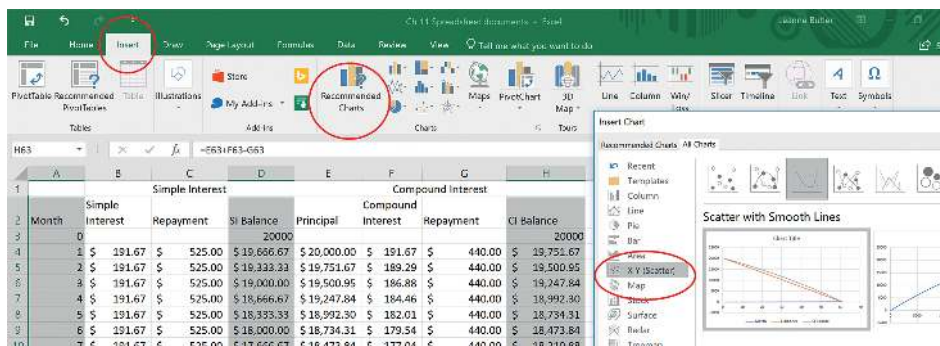
Hint Adding month 0 (start of the loan) will assist later when we want to graph the loans schedules.



- 7** Daniel is a friend of Lexi. He also has borrowed \$20 000 to buy a car. Reducing balance (compound) interest is charged at 11.5% p.a. and he also decides he will repay the loan over five years in equal monthly repayments.
- Use a spreadsheet to calculate:
 - the monthly repayment to the nearest dollar.
 - the total to repay.
 - the total interest.
 - Use the same spreadsheet as in question 6, and add the following headings and formulas and fill down for 60 months.

	E	F	G	H
1				Compound Interest
2	Principal	Compound Interest	Repayment	CI Balance
3				20000
4	20000	=E4*11.5/100*1/12	440	=E4+F4-G4
5	=H4	=E5*11.5/100*1/12	440	=E5+F5-G5

- Explain the formula used in cell F4.
 - Explain the formula in cell H4.
 - Explain the formula in cell E5.
- c** Describe what is happening to the values in the Compound Interest column (Column F).
- d** In cell F65 enter the formula =SUM(F4:F63).
- Decide if the answer matches your answer from part **a iii**.
 - Calculate the difference in the interest paid in the compound interest loan to the interest paid in the simple interest loan in question 6.
- e** In cell G65 enter the formula =SUM(G4:G63).
- Decide if the answer matches your answer from part **a ii**.
 - Calculate the difference in the total paid in the compound interest loan to the total paid in the simple interest loan in question 6.
- 8 a** Produce a graph to compare the balance owing on the loans from questions 6 and 7 over the five years. Hold down the Ctrl key and highlight the month, SI Balance and CI Balance columns. From the Insert Tab>Recommended Charts>All Charts>XY Scatter>Scatter with Smooth Lines as below.



- Add axis labels and a title to your graph.
- Comment on the difference between the graphs of the two loans.

10C Investigating the effect of changing interest rates, compounding periods and repayments

LEARNING GOALS

- Investigate the effect of the interest rate and the number of compounding periods on the future value of a loan using technology (online calculator)
- Investigate the effect of the interest rate and repayment amount on the time taken to repay a loan using technology (online calculator)

Why is it essential to understand interest rate changes?

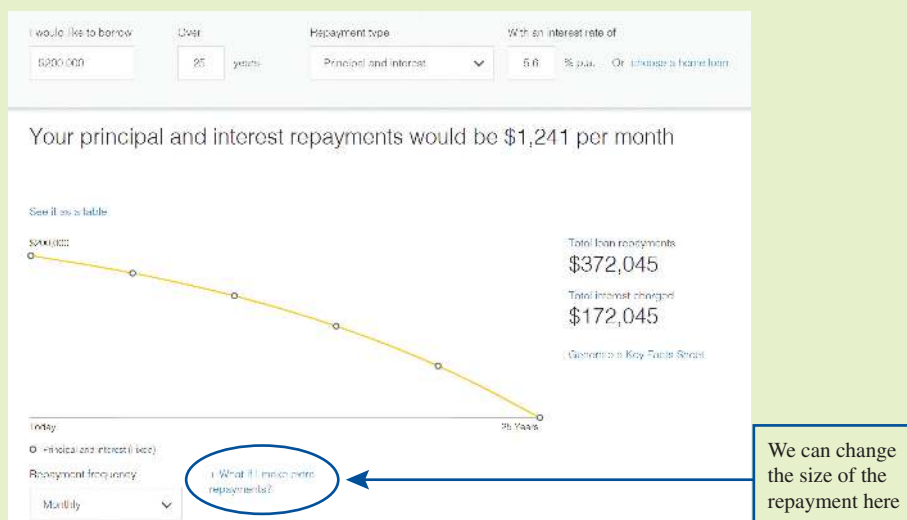
The Board of the Reserve Bank of Australia meets on the first Tuesday of every month to check on the state of the economy and set interest rates. At 2:30 p.m., they release a statement about their interest rate decision for that month. Their decisions have an impact on people around the country with variable interest rate home loans because banks can pass on the rates change to their customers. If rates go up, their monthly repayments will also go up. On the other hand, if rates go down, so will their repayments.



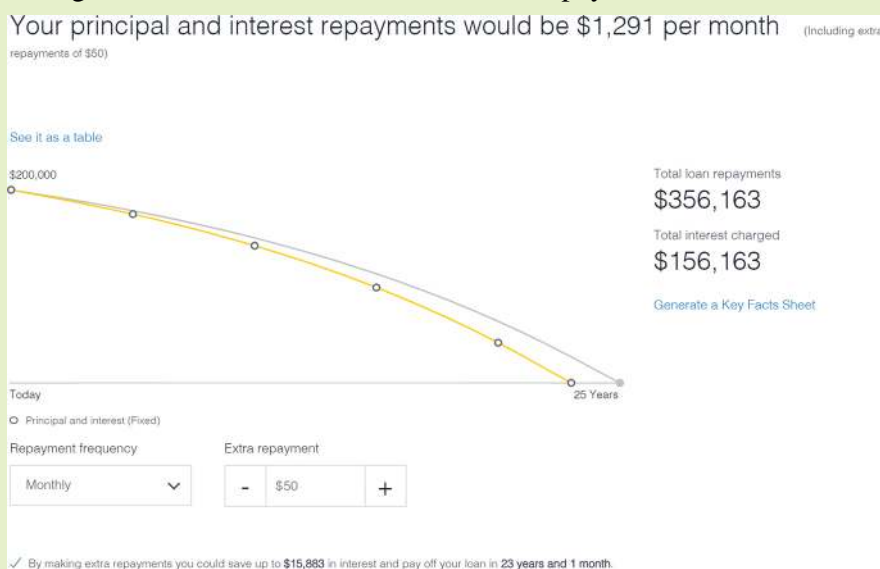
WHAT YOU NEED TO KNOW

- Changes to the size of repayments, frequency of repayments and interest rate changes all have an impact on the time taken to repay a loan and the amount of interest that will be paid. We will use the Commonwealth Bank home loan calculator and the Moneysmart Mortgage Calculator to further explore these effects:
 - https://www.commbank.com.au/digital/home-buying/calculator/home-loan-repayments?ei=tools_repayments
 - <https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/mortgage-calculator#!how-can-i-repay-my-loan-sooner>

- For example: A \$200 000 loan over 25 years at 5.6% p.a. reducible interest generated the following results for monthly repayments on the Commonwealth Bank online calculator.

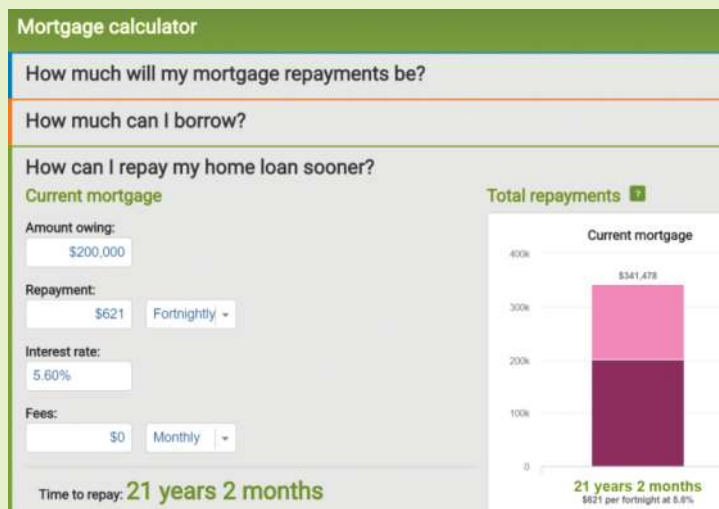


- If we pay an extra \$50 per month, we would pay off the loan faster and make savings on the amount of interest we need to pay.



- A common method used to save time and money on a home loan is to pay half the monthly repayment each fortnight. The Moneysmart Mortgage Calculator has a tab 'How can I repay my home loan sooner?' for this calculation.
- <https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/compound-interest-calculator>

- For example: If half of the \$1241 monthly repayment is paid each fortnight (Repayment = $1241 \div 2 \approx \$621$), we can see that the loan has been paid off in 21 years and 2 months at a total cost of \$341 478, or \$30 567 less than our original loan cost!



- Interest rates go up and down all the time. Rate changes can make a big difference to the size of the loan repayments and can affect the family budget. The Commonwealth Bank home loan calculator also covers this, so you can plan for worst case scenarios.

What if interest rates change?

What if interest rates change by % ?

New interest rate	New repayment amount	Change in repayment amount
6.10%	\$1,301 Per month	\$61 Per month



Example 6 Using an online calculator to investigate the effect of the interest rate, payment frequency and repayment amount

Clayton borrows \$200 000 to buy a home. Bank interest rates are 5.95% p.a. reducible interest and he wants to pay his loan off in 25 years.

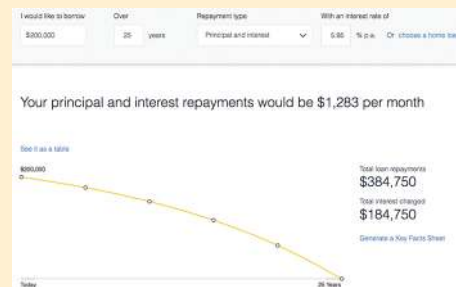
- a Use the Commonwealth Bank home loan calculator, https://www.commbank.com.au/digital/home-buying/calculator/home-loan-repayments?ei=tools_repayments, to determine:
 - i the monthly repayment
 - ii the interest charges if the loan is paid monthly
- b Clayton has heard that he can save money by paying fortnightly. Determine:
 - i the fortnightly repayment
 - ii the savings over 25 years
- c Clayton decides to pay an extra \$50 each fortnight. Calculate:
 - i the amount he would save in interest costs
 - ii the time he would save on his loan
- d Clayton is worried about what might happen if interest rates go up. Use the online calculator to determine how much more Clayton would need to pay each fortnight if rates go up 1% p.a.

WORKING

- a i Monthly repayment = \$1283
- ii Total interest charges = \$184 750

THINKING

Input values into the online calculator and read off the answers.

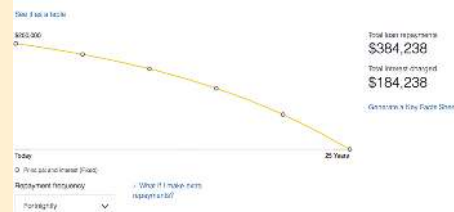


- b i** Fortnightly repayment \$592
Total interest \$184 238
- ii** Savings = $184\,750 - 184\,238$
= \$515

Note: still takes 25 years to repay the loan.

Change the Repayment frequency to fortnightly (see bottom left of screen).

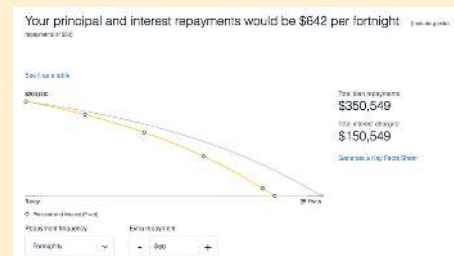
Your principal and interest repayments would be \$592 per fortnight



- c** By paying an extra \$50 per fortnight:
- i** Interest = \$150 549
Savings = $184\,750 - 150\,549$
= \$34 201

- ii** Time to repay 21 years 1 month
Savings = 25 years – 21 years 1 month
= 3 years 11 months

Add extra repayment below the graph.



- d** If rates increase by 1%, Clayton would need to find another \$58 each fortnight to repay his loan.

Scroll down the page and change the rate by 1% p.a.

What if interest rates change?

What if interest rates change by: - 1% + ?

Now interest rate: 6.95%
New repayment amount: \$700 fortnightly
Change in repayment amount: \$88 fortnightly





Example 7 Using an online calculator to investigate the effect of the interest rate, payment frequency and repayment amount

Hannah has borrowed \$275 000 to buy a home. Bank interest rates are 5.49% p.a. reducible interest and she wants to pay her loan off in 25 years.

Use the Moneysmart Mortgage Calculator, <https://www.moneysmart.gov.au/tools-and-resources/calculators-and-apps/mortgage-calculator#!how-much-will-my-repayments-be>.

- a Determine:
 - i the monthly repayment
 - ii the interest charges if the loan is paid monthly
- b Hannah has heard that she can save money by paying half the monthly repayment each fortnight.
 - i Calculate her fortnightly repayment to the nearest dollar.
 - ii Use the 'How can I repay my home loan sooner?' tab to determine the interest and time savings on her loan.
- c Hannah is worried about what might happen if interest rates go up. Use the online calculator to determine how much more Hannah would need to pay each fortnight if rates go up 0.5% p.a.

WORKING



- a
 - i Monthly repayment = \$1687
 - ii Total interest = \$231 130
- b
 - i Fortnightly payment = $1687 \div 2$
 $= 843.5 = \$844$
 - ii Interest = \$191 054
 Savings = $231\ 130 - 191\ 054$
 $= \$40\ 076$
 Time to repay = 21 years 3 months
 Saving = 25 years – 21 years 3 month
 $= 3\text{ years }9\text{ months}$

THINKING

Enter values in the Moneysmart Mortgage Calculator. Hover over the pink part of the bar to read off the total interest charges.

Round up and enter values in each tab.



- C** New rate = $5.49 + 0.5$
 $= 5.99\%$ p.a.
 New monthly repayment = \$1770
 New fortnightly payment = $1770 \div 2$
 $= \$885$
 Increase per fortnight = $885 - 844$
 $= \$41$
 Hannah would need to pay an extra
 \$41 per fortnight to repay her loan.

Go back to ‘How much will my mortgage payments be?’ and change the interest rate before calculating the new fortnightly repayment and any changes.

Mortgage calculator

How much will my mortgage repayments be?

Mortgage details

Amount borrowed: \$275,000

Interest rate: 5.99%

Repayment frequency: Monthly

Length of loan: 25 years

Term: 30

Your repayments will be: **\$1,770 per month**

Total repayments

Mortgage details

Amount borrowed: \$275,000

Interest rate: 5.99%

Repayment frequency: Monthly

Length of loan: 25 years

Term: 30

Your repayments will be: **\$1,770 per month**



Exercise 10C

FUNDAMENTALS

- 1** Determine the missing words in the following sentences.
Changes to the size of repayments, frequency of repayments and interest rate changes all have an _____ on the time taken to _____ a loan and the amount of _____ that will be paid.

APPLICATIONS

Do a search online for 'Commonwealth Bank home loan calculator' and use it to answer the following questions.

Example 6

- 2** Preston has borrowed \$287 000 to buy a home. Bank interest rates are 5.37% p.a. reducible interest and he wants to pay his loan off in 25 years.
- a** Use the Commonwealth Bank home loan calculator to determine:
 - i** the monthly repayment
 - ii** the interest charges if the loan is paid monthly
 - b** Preston has heard that he can save money by paying fortnightly. Determine:
 - i** his fortnightly repayment
 - ii** the savings over 25 years
 - c** Preston decides to pay an extra \$50 each fortnight. Calculate:
 - i** the amount he would save in interest costs
 - ii** the time he would save on his loan
 - d** Preston is worried about what might happen if interest rates go up. Use the online calculator to determine how much more Preston would need to pay each fortnight if rates go up 0.5% p.a.
- 3** Bella has borrowed \$185 000 to buy a townhouse. Bank interest rates are 6.24% p.a. reducible interest and she wants to pay her loan off in 20 years.
- a** Use the Commonwealth Bank home loan calculator to find the following:
 - i** the monthly repayment
 - ii** the interest charges if the loan is monthly
 - b** Bella has heard that she can save money by paying fortnightly. Determine:
 - i** her fortnightly repayment
 - ii** the savings over 20 years
 - c** Bella decides to pay an extra \$40 each fortnight. Calculate:
 - i** the amount she would save in interest costs
 - ii** the time she would save on her loan

- d** Bella has heard that interest rates may go down with a new government. Use the online calculator to determine how much less Bella would need to pay each fortnight if rates go down by 0.5% p.a.

Do a search online for 'Moneysmart mortgage calculator' and use it to answer the following questions.

Example 7

- 4** Sianta has borrowed \$180 000 to buy a home over 20 years with a reducible interest rate of 6.2% p.a.
- a** Use the Moneysmart Mortgage Calculator to determine the size of her monthly repayment.
 - b** Calculate the amount of interest will she pay.
 - c** Interest rates drop to 6.05% p.a., calculate her new monthly repayment.
 - d** Calculate the amount of interest she will save because of the rate drop.
- 5** Michael has borrowed \$40 000 to set up a small business over 5 years with a reducible interest rate of 8.2% p.a.
- a** Use the Moneysmart Mortgage Calculator to determine the size of his monthly repayment.
 - b** Calculate the amount of interest will he pay.
 - c** Interest rates increase to 8.5% p.a., calculate his new monthly repayment.
 - d** Discuss the impact the new interest rate might have on Michael's small business.
 - e** Calculate the amount of extra interest he will need to pay because of the rate increase.
- 6** Botrus is investigating borrowing \$265 000 to buy a home. The bank is offering 6.15% p.a. reducible rates and he thinks he will take 25 years to repay the loan.
- a** Use the Moneysmart Mortgage Calculator to determine the size of his monthly repayment.
 - b** Determine the total cost of the loan over 25 years.
 - c** When Botrus applies for the loan, he is advised to pay half the monthly repayment each fortnight. Determine his fortnightly repayment.
 - d** Use the 'How can I repay my home loan sooner?' tab to determine how long it will take Botrus to repay his loan by paying half the monthly repayment each fortnight.
 - e** Calculate the total cost of the loan if he pays half the monthly repayment each fortnight.
 - f** Discuss the advice you would give Botrus about his home loan.

- 7** Maddie borrows \$315 000 to buy a home. Bank interest rates are 6.19% p.a. reducible interest and she wants to pay her loan off in 25 years.

a Use the Moneysmart Mortgage Calculator to determine:

- i** the monthly repayment
- ii** the interest charges if the loan is paid monthly

b Maddie has heard that she can save money by paying half the monthly repayment each fortnight.

- i** Calculate her fortnightly repayment to the nearest dollar.
- ii** Use the ‘How can I repay my home loan sooner?’ tab to determine the interest and time savings on her loan.

c Maddie is worried about what might happen if interest rates go up. Use the online calculator to determine how much more Maddie would need to pay each fortnight if rates go up 0.5% p.a.



Hint The fortnightly payment will continue to be half of the monthly repayment.

- 8** Ben has borrowed \$286 000 to buy a home. Bank interest rates are 5.99% p.a. reducible interest and he wants to pay his loan off in 20 years.

a Use the Moneysmart Mortgage Calculator to determine:

- i** the monthly repayment
- ii** the interest charges if the loan is paid monthly

b Ben is not sure if he can afford to pay the monthly loan repayment over 20 years. If he opts to pay over 25 years, calculate:

- i** the monthly repayment
- ii** the interest charges
- iii** the additional interest charges by taking longer to repay the loan

c Ben wants to take advantage of the savings to be made by paying half the monthly repayment each fortnight.

- i** Calculate his fortnightly repayment to the nearest dollar.
- ii** Use the ‘How can I repay my home loan sooner?’ tab to determine the interest and time savings on his loan.

d Ben has heard that interest rates may go down with a new government. Use the online calculator to determine how much less Ben would need to pay each fortnight if rates go down 0.25% p.a.

10D Investigating the effect of changing interest rates, compounding periods and repayments using spreadsheets

LEARNING GOALS

- Investigate the effect of the interest rate and the number of compounding periods on the future value of a loan by using technology (spreadsheet)
- Investigate the effect of the interest rate and repayment amount on the time taken to repay a loan by using technology (spreadsheet)

Why is it essential to use technology to investigate changes to a loan?

As we saw in the previous section, when dealing with compound interest, changes to rates, compounding frequency and the amount regularly paid can influence the outcome of a loan. Using technology to investigate how you can make small changes to a loan has the potential to save you a lot of money.



Being well informed about loans has the potential to save you a lot of money in interest charges.

WHAT YOU NEED TO KNOW

- Making changes to the interest rate and frequency of compounds will have an impact on interest charges, the size of repayments required to pay off a loan and the total cost of a loan.
- Making changes to the size of the repayment will change the length of time taken to repay the loan and the amount of interest charged.
- We can use spreadsheets in a similar way to what we have done in previous sections to determine the impact of changes to interest rates, compounding frequency and repayments.



Example 8 Calculating the impact of changes to interest on loans using a spreadsheet

Ned has recently taken out a \$320 000 home loan. Interest rates are currently 5.7% p.a. and he is planning to repay the loan in 25 years.

- Use a spreadsheet (like the one used in section 10B) to determine the monthly repayment.
- Calculate the total amount he will repay.
- Calculate the total amount of interest he will pay.
- Interest rates are increased to 5.85% p.a., calculate his new monthly repayment.
- Determine how much extra will he pay each month.
- Determine how much extra will he now pay in total for the loan.
- Discuss the impact of the increased interest rate.

WORKING

a

	A	B
1	Principal	320000
2	Rate	5.7
3	Years	25
4	Repayments/year	12
5		
6	Number of Payments	300
7		
8	Repayment	\$2,003.48
9	Total Repaid	\$601,044.90
10	Total Interest	\$281,044.90

Monthly repayment = \$2003.48
(\$2003 to nearest dollar)

- Amount repaid = \$601 044.90
- Total interest = \$281 044.90

d

	A	B
1	Principal	320000
2	Rate	5.85
3	Years	25
4	Repayments/year	12
5		
6	Number of Payments	300
7		
8	Repayment	\$2,032.52
9	Total Repaid	\$609,756.91
10	Total Interest	\$289,756.91

Monthly repayment = \$2032.52
(\$2033 to nearest dollar)

THINKING

Set up spreadsheet as done in section 10B, example 4.

	A	B
1	Principal	320000
2	Rate	5.7
3	Years	25
4	Repayments/year	12
5		
6	Number of Payments	=B4*B3
7		
8	Repayment	=PMT((B2/(100*B4)),B6,-B1,0,0)
9	Total Repaid	=B8*B6
10	Total Interest	=B9-B1

- Read from spreadsheet.
- Read from spreadsheet.
- Adjust spreadsheet for new interest rate.

WORKING

- a The monthly repayment is \$1946. ◀◀

	A	B
1	Principal	315000
2	Rate	5.56
3	Years	25
4	Repayments/year	12
5		
6	Number of Payments	300
7		
8	Repayment	\$1,945.68

- b Set up a new spreadsheet as follows:

	A	B	C	D	E
1	Principal	315000			
2	Rate	5.56			
3	Years	25			
4	Repayment	1946			
5	Repayments/year	12			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
8	1	=B1	=B8*\$B2/100*1/\$B5	=\$B4	=B8+C8-D8
9	2	=B8	=B9*\$B2/100*1/\$B5	=\$B4	=B9+C9-D9
10	3	=B9	=B10*\$B2/100*1/\$B5	=\$B4	=B10+C10-D10

- c 25 years = $25 \times 12 = 300$ months ◀◀
Total interest = \$268 591.89

THINKING

Set up spreadsheet as in section 10B, example 5 to find the repayment.

	A	B
1	Principal	315000
2	Rate	5.56
3	Years	25
4	Repayments/year	12
5		
6	Number of Payments	=B4*B3
7		
8	Repayment	=PMT((B2/(100*B4)),(B4*B3),-B1,0,0)

◀◀ Interest = $P \cdot i \cdot n$

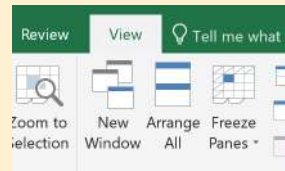
Using the dollar sign in the cell reference means we always use the interest value in cell B2 even when we fill down.

Cell B5 has the number of payments made per year. We want to use that to adjust time, n , in the interest calculation.

The repayment is in cell B4

Balance = principal + interest – repayment

Tip: to continue to show the table headings when you scroll down. Click in cell B8>View>Freeze Panes



Fill the table down for 300 months to model the entire loan.

Type =SUM(C8:C307) into a cell below the interest column.

	A	B	C	D	E
1	Principal	315000			
2	Rate	5.56			
3	Years	25			
4	Repayment	1946			
5	Repayments/year	12			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
304	297	\$ 7,490.36	\$ 34.71	\$ 1,946.00	\$ 5,579.07
305	298	\$ 5,579.07	\$ 25.85	\$ 1,946.00	\$ 3,658.92
306	299	\$ 3,658.92	\$ 16.95	\$ 1,946.00	\$ 1,729.87
307	300	\$ 1,729.87	\$ 8.02	\$ 1,946.00	\$ 208.11
308					
309			=sum(C8:C307)		
310					
311					

d Fortnightly repayment = $1946 \div 2$ ← Halve the monthly repayment.
= \$973

e Repayment = \$973
Repayments/year = 26(fortnights) ← Change values at the top only and fill down until the balance falls below \$0 for the first time.

Time to repay loan = 552 fortnights
= $552 \div 26$
= 21.23 years
= 21 years 3 months

	A	B	C	D	E
1	Principal	315000			
2	Rate	5.56			
3	Years	25			
4	Repayment	973			
5	Repayments/year	26			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
556	549	\$ 3,638.70	\$ 7.78	\$ 973.00	\$ 2,673.48
557	550	\$ 2,673.48	\$ 5.72	\$ 973.00	\$ 1,706.20
558	551	\$ 1,706.20	\$ 3.65	\$ 973.00	\$ 736.85
559	552	\$ 736.85	\$ 1.58	\$ 973.00	\$ -234.57

To change years from a decimal to months

$$21.23 - 21 = 0.23$$

$0.23 \times 12 = 2.76$ (or 3 months to nearest month) so

$$21.23 \text{ years} = 21 \text{ years } 3 \text{ months}$$

f Interest = \$221 861.43
Saving = $268\,591.89 - 221\,861.43$
= \$46 730.46

Time saving = 25 years
- 21 years 3 months
= 3 years 9 months

By paying half the monthly payment each fortnight, Daniel repays the loan 3 years and 9 months sooner and saves \$46 730.46 in interest.

←..... Type =SUM(C8:559) into a cell below the interest column to find total interest.

	A	B	C	D	E
1	Principal	315000			
2	Rate	5.56			
3	Years	25			
4	Repayment	973			
5	Repayments/year	26			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
556	549	\$ 3,638.70	\$ 7.78	\$ 973.00	\$ 2,673.48
557	550	\$ 2,673.48	\$ 5.72	\$ 973.00	\$ 1,706.20
558	551	\$ 1,706.20	\$ 3.65	\$ 973.00	\$ 736.85
559	552	\$ 736.85	\$ 1.58	\$ 973.00	\$ -234.57
560					
561			=SUM(C8:C559)		
562			SUM(number1, [number2], ...)		
563					

Exercise 10D

FUNDAMENTALS

- 1 Determine the missing words in the following sentences.
 - a Making changes to the _____ rate and _____ of compounds will have an _____ on the interest charges, size of repayments required to _____ off a loan and the total _____ of a loan.
 - b Making _____ to the size of the repayment will change the _____ of time taken to repay the loan and the amount of _____ charged.
- 2 Complete the following conversions.
 - a 1 year = _____ months
 - b 1 year = _____ weeks
 - c 1 year = _____ days
 - d 1 year = _____ fortnights
 - e 1 year = _____ 6 months
 - f 1 year = _____ 4 weeks
 - g 1 year = _____ quarters

APPLICATIONS



Spreadsheet Example 8 applies to question 3.

Example 8

- 3 Primrose borrows \$296 000 to buy a home. She agrees to repay the loan in monthly repayments over 25 years. Reducible interest is calculated at 6.35% p.a.
 - a Use a spreadsheet to calculate the:
 - i size of her monthly repayment
 - ii total amount repaid
 - iii total interest paid
 - b Another bank offers her a loan with only 6.25% p.a. interest. Adjust your spreadsheet to calculate:
 - i size of her monthly repayment
 - ii total amount repaid
 - iii total interest paid
 - c Calculate the savings to be made by taking the loan with the smaller interest rate.



Spreadsheet Example 9 applies to the following questions.

Example 9

- 4** Jewel borrows \$265 000 to buy a home. Bank interest rates are 5.45% p.a. reducible interest and she will repay the loan with monthly repayments over 20 years.
- Use a spreadsheet to determine the size of the monthly repayment. Round UP to the next dollar.
 - Set up a new spreadsheet to model her loan with the following headings:
- | | A | B | C | D | E |
|----|------------------------|------------------|--------------------------|------------------|----------------|
| 1 | Principal | 265000 | | | |
| 2 | Rate | 5.45 | | | |
| 3 | Years | 20 | | | |
| 4 | Repayments/year | 12 | | | |
| 5 | Repayment | 1816 | | | |
| 6 | | | | | |
| 7 | No. Repayments | Principal | Interest | Repayment | Balance |
| 8 | 1 | =B1 | =B8*\$B\$2/100*1/\$B\$4 | =B\$5 | =B8+C8-D8 |
| 9 | 2 | =E8 | =B9*\$B\$2/100*1/\$B\$4 | =B\$5 | =B9+C9-D9 |
| 10 | 3 | =E9 | =B10*\$B\$2/100*1/\$B\$4 | =B\$5 | =B10+C10-D10 |
- Fill down until the loan is repaid. Use the spreadsheet to calculate how much she will pay in interest.
 - Jewell wants to pay half her monthly repayment each fortnight to save interest. Calculate how much will she need to repay each fortnight.
 - Change the repayment amount and the number of repayments per year to determine how long it would take to repay the loan if paid fortnightly.
 - Use the spreadsheet to calculate the total amount of interest if she pays half the monthly repayment each fortnight.
 - Determine how much she will save in interest and time.
- 5** Lachlan borrows \$305 000 to buy a home. Bank interest rates are 5.16% p.a. reducible interest and he will repay the loan with monthly repayments for 20 years.
- Use a spreadsheet to calculate the size of the monthly repayment. Round UP to the next dollar.
 - Set up a new spreadsheet to model his loan with the same headings used in question **4 b**. Fill down until the loan is repaid.
 - Use the spreadsheet to calculate how much he will pay in interest.
 - Lachlan wants to pay \$40 extra each month. Determine how long it will take him now to pay off the loan.
 - Use the spreadsheet to calculate the total amount of interest if he repays an extra \$40 each month.
 - Determine how much he will save in interest and time.

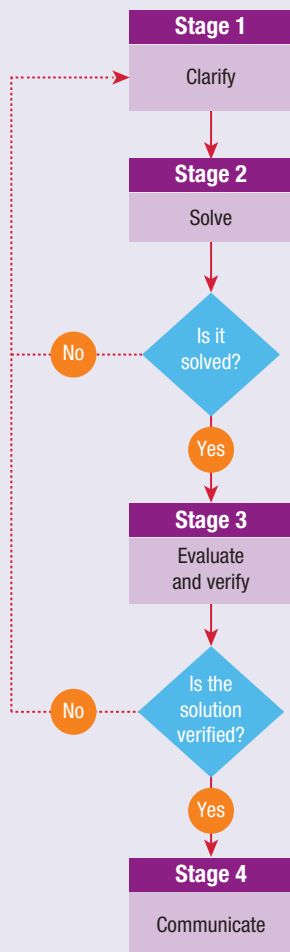
Mathematical thinking process task

Background: The great Australian dream is to own your own home. But is it really the best financial decision? Are you better off renting and saving the extra money you would have spent on rates, insurance and home maintenance?



Task: Compare the costs of buying and maintaining your own home over 10 years against the costs of renting a similar property over the same time and having a regular savings plan. After 10 years, the house will have increased in value but so will the renters' savings. Who is better off?

Application of mathematical thinking process.



Stage 1: Clarify

Interpret the task and gather the key information and identify the mathematics which could help to complete the task

- 1 What are you required to do?
- 2 What information do you have?
- 3 What other information do you need?
- 4 How could you gather more information?
- 5 What assumptions will you make?

Stage 2: Solve

Apply existing mathematical knowledge and strategies to obtain a solution

- 6 Gather more information.
- 7 Use mathematics from Chapters 9 and 10 to calculate information required to solve the problem
- 8 Produce graphs/tables required to solve the problem.
- 9 Have you been able to make a decision?

Stage 3: Evaluate and verify

Analyse information and data from a variety of sources and verify the reasonableness of the solution

- 10 Have you answered the question?
- 11 Is the answer you have reasonable?

Stage 4: Communicate

Communicate findings in a systematic and concise manner

- 12 State your main point.
- 13 Include the evidence in the form of statements, graphs and tables.
- 14 Explain the evidence. Use a sentence starter like 'This means...'
- 15 In the conclusion, link back to your main point.

Chapter checklist

☐ I understand that reducing balance loans are compound interest loans with periodic repayments.

- 1** Brooke borrowed \$16 000 to buy a car. The terms of her loan were 11.56% p.a. reducible interest over two years and her monthly repayments are \$750 per month.

a Complete this table to determine how much Brooke will owe on her loan after the first three months.

Month	Principal	Interest	Repay	Balance
1				
2				
3				

- b** Determine how much interest Brooke paid in the first month.
c Calculate how much Brooke reduced her debt by in the first month.
d Determine how much interest Brooke paid in the third month.
e Calculate how much Brooke reduced her debt by in the third month.
f Interpret your answers to parts **c** and **e**.

☐ I can use technology (spreadsheet) to model a reducing balance loan.

- 2** Lachlan borrows \$23 000 to buy a car. He repays the loan with monthly repayments of \$596 over four years with 11.15% p.a. reducible interest.
- a** Set up a spreadsheet to model the loan schedule over four years.
b Determine how much is left owing after 48 months.
c Calculate the size of the last repayment to fix the underpayment.
d Calculate the total loan repayments.
e Calculate the total interest charges.
f Determine how many months will it take to pay off half the loan.

☐ I can use technology (online calculator) to model a reducing balance loan.

- 3** Megan borrows \$24 000 to buy a car. Interest is calculated at 10.85% p.a. on the reducing balance (compound interest) and she will repay the loan over four years. Use an online calculator to calculate:
- a** the monthly repayment required
 - b** the total repaid
 - c** the total interest

☐ I can compare simple and compound interest loans.

- 4** Joe borrows \$24 000 to buy a car. Simple interest is calculated at 10.85% p.a. and he will repay the loan over four years. Calculate:
- a** the total interest
 - b** the total to repay
 - c** the number of repayments
 - d** how much he will need to repay each month
 - e** the difference in monthly repayments and total cost of Joe's simple interest loan and Megan's compound interest loan in question 3.

☐ I can use technology (online calculator) to investigate the effect of the interest rate and repayment amount on the time taken to repay a loan.

- 5** Isabelle borrows \$213 000 to buy a townhouse. Bank interest rates are 5.82% p.a. reducible interest and she wants to pay her loan off in 20 years.
- a** Use the Commonwealth Bank home loan Calculator to determine:
 - i** her monthly repayment
 - ii** interest charges if she pays her loan monthly
 - b** Isabelle decides to pay an extra \$50 each month:
 - i** determine how much she would save in interest costs
 - ii** calculate how much time will she save on her loan
 - c** Isabelle has heard that interest rates may go down with a new government. Use the online calculator to determine how much less she would need to pay each month if rates go down by 0.5% p.a.

- ☐ I can use technology (spreadsheet) to investigate the effect of the interest rate and repayment amount on the time taken to repay a loan.

- 6** Primrose borrows \$365 000 to buy a home. Bank interest rates are 5.02% p.a. reducible interest and her monthly repayments will be \$2138.
- a** Set up a spreadsheet to model her loan with the following headings:

	A	B	C	D	E
1	Principal	365000			
2	Rate	5.02			
3	Years				
4	Repayment	2138			
5	Repayments/year	12			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
8	1	=B1	=B8*\$B\$2/100*1/\$B\$5	=\$B\$4	=B8+C8-D8
9	2	=E8	=B9*\$B\$2/100*1/\$B\$5	=\$B\$4	=B9+C9-D9
10	3	=E9	=B10*\$B\$2/100*1/\$B\$5	=\$B\$4	=B10+C10-D10

- b** Fill the spreadsheet down to determine how long it will take Primrose to repay the loan.
- c** Use the spreadsheet to calculate how much she will pay in interest.
- d** Primrose wants to pay half her monthly repayment each fortnight to save interest. Calculate how much she will need to repay each fortnight.
- e** Change the repayment amount and the number of repayments per year to determine how long it would take to repay the loan if she paid half the monthly amount each fortnight.
- f** Use the spreadsheet to find the total amount of interest if she pays half the monthly repayment each fortnight. Calculate how much will she save in interest and time.

Chapter review

Section 10A

- 1** Betty borrowed \$3200 to purchase a new laptop. She agreed to pay off the loan in 12 months with reducible interest calculated at 9.85% p.a. Her monthly repayments are \$281.00. Betty is given a table of her loan schedule as follows.

Month	Principal	Interest	Repayment	Balance
1	\$3200.00	\$26.27	\$ 281.00	\$2945.27
2	\$2945.27	\$24.18	\$ 281.00	\$2688.44
3	\$2688.44	\$22.07	\$ 281.00	\$2429.51
4	\$2429.51	\$19.94	\$ 281.00	\$2168.45
5	\$2168.45	\$17.80	\$ 281.00	\$1905.25
6	\$1905.25	\$15.64	\$ 281.00	\$1639.89
7	\$1639.89	\$13.46	\$ 281.00	\$1372.35
8	\$1372.35	\$11.26	\$ 281.00	\$1102.62
9	\$1102.62	\$ 9.05	\$ 281.00	\$ 830.67
10	\$ 830.67	\$ 6.82	\$ 281.00	\$ 556.49
11	\$ 556.49	\$ 4.57	\$ 281.00	\$ 280.05
12	\$ 280.05	\$ 2.30	\$ 281.00	\$ 1.35

- a** Determine if Betty will completely pay off the loan in 12 months. Explain why this will happen.
- b** Calculate how much she should pay in the last month to completely pay off the loan.
- c** Determine how much Betty still owes on her loan after 6 months.
- d** Determine how much interest Betty paid in the first month.
- e** Calculate how much Betty reduced her debt by in the first month.
- f** Determine how much interest Betty paid in the fourth month.
- g** Calculate how much Betty reduced her debt by in the fourth month.
- h** Describe the difference in your answers to parts **e** and **g**.
- i** Explain why the interest reduces each month.
- j** Calculate how much interest Betty paid on the total loan.
- k** Calculate 9.85% of \$3200. Explain why this is different to your answer in part **j**.

- 2 Braydon repays \$512 each month for four years on his \$19 500 loan to buy a car.
 - a Determine how many repayments he will make.
 - b Calculate how much he repaid in total.
 - c Calculate how much he paid in interest.
- 3 Joseph repays \$1915 each month for twenty-five years on his \$285 000 home loan.
 - a Determine how many repayments he will make.
 - b Calculate how much he repays in total.
 - c Calculate how much he paid in interest.
- 4 Harry borrows \$2800 for a new computer. Simple interest is charged at 11.8% p.a. and he will repay the loan over 2 years in equal monthly repayments. Calculate how much he will need to repay each month.
- 5 Jasper borrowed \$18 500 to buy a car. The terms of his loan were 11.12% p.a. reducible interest over four years and his monthly repayments are \$479 per month.
 - a Complete this table to determine how much Jasper will owe on his loan after the first three months.

Month	Principal	Interest	Repay	Balance
1				
2				
3				

- b Calculate the total of Jasper's monthly repayments over the first three months.
- c Calculate the total of the interest charges over the first three months.
- d Determine the difference between the amount repaid and the interest charges over the first three months.
- e Determine how much Jasper reduced his debt by in the first three months.

Note: The following questions require technology beyond a scientific calculator and are not considered exam style questions. They do, however, cover the syllabus content for this topic.

- 6** Libby borrows \$318 000 to buy a home. She repays the loan monthly over 25 years with 5.58% p.a. reducible interest. Use an online calculator from a bank website to calculate the:
- a** size of the monthly repayments
 - b** total loan repayments
 - c** total interest charges
 - d** amount left owing after 20 years
 - e** number of years it takes to pay off half the loan

Section 10C

- 7** Nick borrows \$247 000 to buy a home. Bank interest rates are 5.63% p.a. reducible interest and he wants to pay his loan off in 25 years.
- a** Use the Commonwealth Bank home loan calculator to calculate:
 - i** his monthly repayment
 - ii** the interest charges if he pays his loan monthly
 - b** Nick has heard there are savings to be made by paying fortnightly, calculate his fortnightly repayment and savings over 25 years.
 - c** Nick decides to pay an extra \$50 each fortnight, calculate:
 - i** how much he would save in interest costs
 - ii** how much time he would save on his loan
 - d** Nick is worried about what might happen if interest rates go up. Use the online calculator to determine how much more Nick would need to pay each fortnight if rates go up 1% p.a.
- 8** Frank borrows \$386 000 to buy a home. Bank interest rates are 5.17% p.a. reducible interest and he wants to pay his loan off in 20 years.
- a** Use the Moneysmart Mortgage Calculator to determine:
 - i** his monthly repayment
 - ii** the interest charges if he pays his loan monthly
 - b** Frank is not sure he can afford to pay the monthly loan repayment over 20 years. If he opts to pay over 25 years, calculate:
 - i** his monthly repayment
 - ii** the interest charges
 - iii** his additional interest charges by taking longer to repay the loan
 - c** Frank wants to take advantage of the savings to be made by paying half the monthly repayment each fortnight.
 - i** Calculate his fortnightly repayment to the nearest dollar.
 - ii** Use the 'How can I repay my home loan sooner?' tab to determine interest and time savings on his loan.

- d** Frank has heard that interest rates may go down with a new government. Use the online calculator to determine how much less Frank would need to pay each fortnight if rates go down 0.15% p.a.

Section 10B

- 9** Ben borrows \$31 000 to buy a car. He repays the loan with monthly repayments of \$790 over 4 years with 10.25% p.a. reducible interest.
- Set up a spreadsheet to model the loan schedule over four years.
 - Determine how much is left owing after 48 months.
 - Calculate the size of the last repayment to fix the overpayment.
 - Calculate the total loan repayments.
 - Calculate the total interest charges.
 - Determine how many months it will take to pay off half the loan.

Section 10D

- 10** Harry borrows \$245 000 to buy a home. Bank interest rates are 6.15% p.a. reducible interest and his monthly repayments will be \$1601.
- Set up a spreadsheet to model his loan with the following headings:

	A	B	C	D	E
1	Principal	245000			
2	Rate	6.15			
3	Years				
4	Repayment	1601			
5	Repayments/year	12			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
8	1	=B1	=B8*\$B\$2/100*1/\$B\$5	=\$B\$4	=B8+C8-D8
9	2	=E8	=B9*\$B\$2/100*1/\$B\$5	=\$B\$4	=B9+C9-D9
10	3	=E9	=B10*\$B\$2/100*1/\$B\$5	=\$B\$4	=B10+C10-D10

- Use it to determine how long it will take Harry to repay the loan.
- Use the spreadsheet to calculate how much he will pay in interest.
- Harry wants to pay half his monthly repayment each fortnight to save interest. Calculate how much he will need to repay each fortnight.
- Use the spreadsheet to calculate how long it would take to repay the loan if paid fortnightly.
- Use the spreadsheet to calculate the total amount of interest if he pays half the monthly repayment each fortnight. Determine how much he will save in interest and time.

- 11** Tom borrows \$335 000 to buy a home. Bank interest rates are 6.20% p.a. reducible interest and his monthly repayments will be \$2200.
- a** Set up a spreadsheet to model his loan with the same headings used in question **10**.
 - b** Use the spreadsheet to determine how long it will take Tom to repay the loan.
 - c** Use the spreadsheet to calculate how much he will pay in interest.
 - d** Tom wants to pay \$50 extra each month. Use the spreadsheet to determine how long it will take him now to pay off the loan.
 - e** Use the spreadsheet to calculate the total amount of interest if he repays an extra \$50 each month. Determine how much he will save in interest and time.

Formula sheet

Please note: this is not an official formula sheet and it should not be used for assessment.

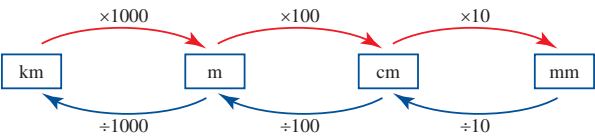
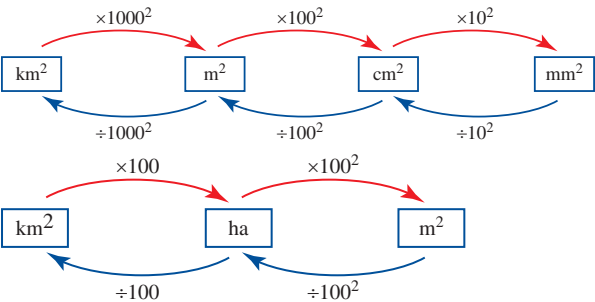
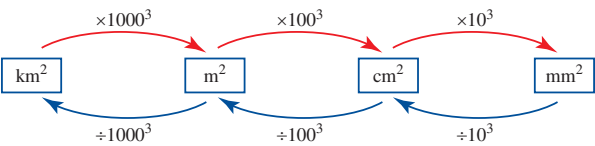
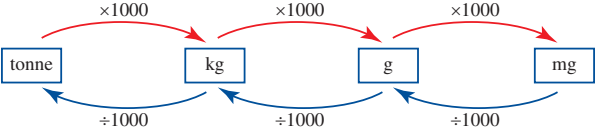
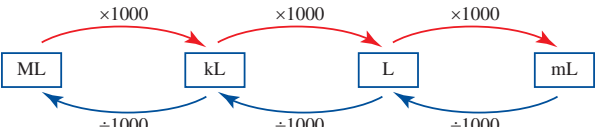
Mensuration			
circumference of a circle	$C = 2\pi r$	area of a circle	$A = \pi r^2$
area of a parallelogram	$A = bh$	area of a trapezium	$A = \frac{1}{2}(a + b)h$
area of a triangle	$A = \frac{1}{2}bh$	total surface area of a cone	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	surface area of a sphere	$S = 4\pi r^2$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	volume of a cylinder	$V = \pi r^2 h$
volume of a prism	$V = Ah$	volume of a pyramid	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$	arc length	$l = \frac{\theta}{180}\pi r$
area of a sector	$A = \frac{\theta}{360}\pi r^2$	distance on a great circle	$d = \frac{\theta}{360} \times 2\pi r$

Finance			
simple interest	$I = Pin$	compound interest	$A = P(1 + i)^n$

Trigonometry			
Pythagoras' theorem	$c^2 = a^2 + b^2$		
trigonometric ratios	$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$	$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$	$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

Location and time					
distance	$d = s \times t$	speed	$s = \frac{d}{t}$	time	$t = \frac{d}{s}$

Statistics	
mean	$\bar{x} = \frac{\text{sum of all data values}}{\text{number of data values}}$
range	range = highest score – lowest score
interquartile range (IQR)	$IQR = Q_3 - Q_1$

Conversions	
length unit conversion	
area unit conversion	
volume unit conversion	
mass unit conversion	
capacity unit conversion	

Glossary

A

Accuracy [p. 269] How close to the real value a measurement is

Ambiguity [p. 306] A phrase, word or question that is unclear and open to more than one interpretation

Angle [p. 7] Two rays starting from the same point; the measure of turn between two lines, measured in degrees; the difference in direction between two lines

Angle of depression [p. 182] When an observer looks at an object that is lower than ‘the eye of the observer’, the angle between the line of sight and the horizontal is called the angle of depression

Angle of elevation [p. 182] When an observer looks at an object that is higher than ‘the eye of the observer’, the angle between the line of sight and the horizontal is called the angle of elevation

Annual [p. 502] Occurring once every year

Appreciation [p. 510] An increase in value

Arc [p. 19] The part of a circle between two given points on the circle. The length of the arc of a circle is given by $l = \frac{\theta}{360} \times 2\pi r$, where r is the radius of the circle and θ is the angle in degrees subtended by the arc at the centre of the circle.

Area [p. 32] The measurement of the surface

Array An array is an ordered collection of objects or numbers.

Association [p. 335] A relationship where one variable can be used to predict the other

Australian Taxation Office (ATO) [p. 232] The government department responsible for collecting tax

Average [p. 258] A single number that best represents a set of values

Average rate [p. 511] The total change in one quantity divided by the total change in the second quantity

Axes [p. 206] The vertical and horizontal lines that define the number plane quadrants

Axes scale [p. 217] A system of ordered marks at fixed intervals referred to as a standard in measurement

B

Balance [p. 500] The balance of a loan or investment is the amount owed or accrued after a period of time

Bias [p. 265] The external influence of responses, which could potentially cause the results to be different from the true results

Bivariate data [p. 335] Data in which each observation involves recording information about two variables for the same person or thing. An example would be data recording the height and weight of the children in a preschool

Bivariate data scatter plot [p. 336]
See Scatterplot

Break-even point [p. 237] The point where revenue and expenses are equal

C

Capacity [p. 82] The quantity of fluid a container can hold

Cartesian plane [p. 206]

Two intersecting number lines are taken intersecting at right angles at their origins to form the axes of the coordinate system

Causality [p. 384] A relationship between an explanatory and a response variable is said to be causal if the change in the explanatory variable actually causes a change in the response variable. (Simply knowing that two variables are associated, no matter how strongly, is not sufficient evidence by itself to conclude that the two variables are causally related.)

Causation [p. 384] See causality

Census [p. 260] A survey of an entire population

Circumference [p. 38] The distance around the outside of a circle; the curved boundary

Closed cylinder [p. 67] A cylinder with both circular bases. See open cylinder.

Colon [p. 115] The symbol : which is used in ratios and operates similarly to the horizontal line of a fraction

Column graph [p. 230] A graphical representation of a single set of categorical or discrete data, where columns are used to show the frequency of scores

Composite 3D solid [p. 73] A solid which is made up of two or more solids

Composite shape [p. 26] A 2D shape made up of two or more 2D shapes

Compound interest [p. 500] The interest earned by investing a sum of money (the principal) is compound interest if each successive interest payment is added to the principal for the purpose of calculating the next interest payment.

Conversion [p. 459] Changing (converting) a value to the equivalent value expressed in a different form

Correlation [p. 335] A measure of the strength of the linear relationship between two variables

Correlation coefficient [p. 368] A statistical measure of the strength of the linear association between two numerical variables

Cosine ratio [p. 181] In any right-angled triangle,

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \text{ where } 0^\circ < \theta < 90^\circ$$

Cross-section [p. 53] The plane figure formed when you slice a solid figure parallel to one of its surfaces

D

Data [p. 206] Information gathered by observation, survey or measurement; e.g. in a probability experiment. Singular of data is 'datum'.

Dataset [p. 356] Information from different trials or sources

Denominator [p. 184] The number in a fraction below the horizontal line. It is the number of equal parts into which the whole is divided.

Dependent variable [p. 343]

The value that changes in response to changes made in the independent variable

Depreciation [p. 510] The reduction in value of an item over time

Depression [p. 182] See angle of depression

Diameter [p. 18] A line interval passing through a circle's centre with its end points on the circumference; the diameter is twice as long as the circle's radius

Discount [p. 430] An amount subtracted from a price, usually to sell goods faster

Distance [p. 459] The measure of how far apart two objects are. See length.

Distance-time graph [p. 225] A line graph showing the distance travelled on the vertical axis and the time on the horizontal axis

E

Edge [p. 7] The line interval where two faces of a solid meet

Elevation [p. 182] See angle of elevation

Employee [p. 292] A person who earns income working for someone else

Ethics [p. 306] The moral principles governing how people act and how things should be done

Experiment [p. 343] Try out or test new ideas or methods, especially in order to discover or prove something; undertake or perform a scientific procedure to test a hypothesis, make a discovery or demonstrate a known fact

Explanatory variable When investigating relationships in bivariate data, the explanatory variable (also known as the independent variable) is the variable used to explain or predict a difference in the response variable (also known as the dependent variable). For example, when investigating the relationship between the temperature of a loaf of bread and the time it has spent in a hot oven, temperature is the response variable and time is the explanatory variable.

Extrapolation [p. 376] In the context of fitting a linear relationship between two variables, extrapolation occurs when the fitted model is used to make predictions using values of the explanatory variable that are outside the range of the original data; extrapolation is a dangerous process as it can sometimes lead to quite erroneous predictions

F

Face [p. 7] A flat surface of a polyhedron. A face must be a polygon (have straight sides).

Factor [p. 115] A whole number that will divide into another number without a remainder

Formulas [p. 33] A special type of equation that shows the relationship between different variables

Fortnight [p. 521] A 2-week period; 14 days in a row

Frequency (distribution) table [p. 406] A table summarising data by showing all possible scores, from lowest to highest, in one column, and the frequency of each score in another column

Frequency [p. 284] The number of times a data value occurs

G

Goods and Services Tax (GST)

[p. 262] A government levy or tax included in the purchase price of merchandise. In Australia, this rate is 10%.

Graph [p. 206] The representation of data in a diagram

Great circle [p. 463] A circle on a sphere whose plane passes through the centre of the sphere. The shortest distance between two points on a sphere is along an arc of the great circle passing through the two points. See also small circle.

H

Hectare (ha) [p. 33] A unit of measurement for area equal to 10 000 m²

Horizontal axis [p. 217] Horizontal axis on the number plane

Horizontal axis intercept [p. 237] The value where the line of a graph crosses the horizontal axis

Hypotenuse [p. 169] The hypotenuse is the side opposite the right angle. It is also always the longest side in a right-angled triangle.

I

Income [p. 231] Money that 'comes in' (i.e. money you earn rather than money you spend). In most cases this will be the same as gross income.

Independent variable [p. 343]

The value that is deliberately controlled when collecting data. Sometimes referred to as the explanatory variable.

Inflation [p. 508] A rise in the price of goods and services or Consumer Price Index (CPI). It is often expressed as annual percentage.

Interest [p. 491] The amount of money paid (earned) for borrowing (lending) money over a period of time

Interpolation [p. 376] Using a regression line to make a prediction within the range of values of the explanatory variable.

Interviewer bias [p. 301] The influence of a response caused by the interviewer

Irregular solid [p. 73] The value that is deliberately controlled when collecting data

J

Jargon [p. 306] Special technical words used by a particular profession or group and are not well-known to the general population

K

Kilowatt hours (kWh) [p. 316] Unit of energy used to measure electricity usage

L

Latitude [p. 464] The angle or angular distance north or south of the equator

Law of large numbers The basic justification for statistical sampling is that the distribution which is obtained from a random sample tends to resemble the distribution of the population from which it was drawn. The tendency increases as the size of the sample increases. This tendency of distributions of random samples to resemble the distribution of their parent population more closely as sample size increases is called 'the law of large numbers'.

Length [p. 6] The measurement of the distance from end to end along a line; the longest dimension of a 2D shape or 3D object

Line graph [p. 206] A graph that shows the data as points joined with line segments

Line of best fit [p. 347] A straight line used to approximately model the linear relationship between two variables

Linear [p. 212] In a straight line

Linear graph [p. 217] A line on the graph showing the relationship between two variables

Longitude [p. 464] The angle or angular distance east or west of the prime meridian.

M

Mean [p. 347] the mean of a list of numbers is the sum of the data values divided by the number of values in the list; in everyday language, the mean is commonly called the average

Measurement error [p. 266] An error made during the process of collecting or recording of data

Misleading [p. 313] Giving the wrong idea or impression. Can be used to describe graphs if they make the data look better or worse than it actually is.

N

Net [p. 48] A diagram showing how the plane faces of a solid are joined to each other

Non-linear [p. 336] An equation or expression which when graphed does not produce a straight line

Non-response bias [p. 300] The effect on a survey's results caused by the difference between people who do and do not respond

O

Open cylinder [p. 67] A cylinder without a circular base. It is the curved part of the cylinder.

Outcome [p. 402] One of the possible results of a chance experiment

Outlier [p. 336] A value that is much larger or much smaller than the rest of the data

P

Parallax error [p. 267] An error caused by a person reading measurements from tools, such as rulers, at an angle. This causes results to be consistently too high or low.

Parallel [p. 7] Description of lines, surfaces, planes or objects that remain a constant distance from one another, and that never touch, cross or veer away from one another

Per [p. 134] Per means ‘for each’. It is used when the number of units of one quantity is given for each unit of another quantity. This is a rate. Per may be abbreviated to ‘p’ but, in units, it is usually shown as a forward slash ‘/’ (also known as a solidus). This is also a symbol for division because to calculate a rate you have to divide one quantity by another.

Per annum (p.a.) [p. 491] Each year

Perimeter [p. 18] The total distance (length) around the outside of a figure

Point of intersection [p. 237]

The point at which two lines cross each other and therefore have the same coordinates

Polygon [p. 53] A closed geometric shape constructed from straight line segments. In regular polygons, the sides are of equal lengths.

Population [p. 261] A population is the complete set of individuals, objects, places etc. that we want information about

Prediction [p. 375] Give an expected result of an upcoming action or event; suggest what may happen based on available information

Prime meridian [p. 465] The great circle located at 0° which passes through Greenwich, England and both the north and south pole

Principal [p. 491] An amount of money invested in a financial institution or loaned to a person or business

Prism [p. 53] A solid where each cross-section in a particular direction is exactly the same

Privacy [p. 53] The ability for someone to hide information about themselves and choose what they want to reveal

Probability [p. 420] The likelihood or chance of something; the relative frequency of the occurrence of an event as measured by the ratio of the number of cases or alternatives favourable to the event to the total number of cases or alternatives

Processing error [p. 265] Mistakes occurring when collecting data

Profit [p. 237] The amount of money made by selling something for more than its cost

Pyramid [p. 53] A solid where the base is a polygon and the other faces are formed by triangles with a common vertex (the apex)

Pythagoras’ theorem [p. 170]

The square of the hypotenuse of a right-angled triangle equals the sum of the squares of the lengths of the other two sides; in symbols, $a^2 + b^2 = c^2$

Q

Questionnaire bias [p. 301] When a response is influenced by the wording of the question itself

R

Radius [p. 18] The distance from the centre to any point on the circle (sphere). Half the diameter. *Plural:* radii

Random [p. 402] Done without conscious decision; all events have a chance of happening but there is no way to know which event will definitely happen.

Range [p. 376] The difference between the highest and lowest numbers in a set

Ratio [p. 115] A method used to compare two or more quantities measured in the same units

Recurrence relation [p. 500]

A relation that enables the value of the next term in a sequence to be obtained by one or more current terms. Examples include ‘to find the next term, add two to the current term’ and ‘to find the next term, multiply the current term by three and subtract five’.

Reducing balance loans [p. 546]

A loan that attracts compound interest, but where regular repayments are also made. In most instances the repayments are calculated so that the amount of the loan and the interest are eventually repaid in full.

Regression [p. 356] The process of fitting a mathematical model to data

Relative frequency [p. 420]

The number of items of a certain type divided by the number of all the items considered

Representative [p. 274] The sample reflects the entire population

Right angle [p. 169] An angle equal to 90°

S

Sample [p. 265] A small group randomly selected from the population

Sample population [p. 295] The group of individuals asked to participate in a survey

Sampling error [p. 266] Influence or error in results caused by a mistake or because the sample size was too small

Sampling technique [p. 283] Method of gathering a sample for a survey

Scale [p. 115] A ratio that compares a drawing or a model to the real object

Scale factor [p. 122] The number by which you multiply each side length to enlarge or reduce the size of a shape

Scatterplot [p. 336] A statistical graph used for displaying bivariate data. Data pairs are represented by points on a coordinate plane, the EV is plotted on the horizontal axis and the RV is plotted on the vertical axis.

Sector [p. 19] A region bounded by two radii of a circle and the arc between them

Self-selection bias [p. 300] Influence from voluntary survey respondents interested in a particular viewpoint

Side [p. 7] Any line segment forming part of the perimeter of a plane geometric figure

Simple interest [p. 491] Percentage rate per year paid on a loan or investment amount. Simple interest is generally charged on money borrowed or invested for a short period of time.

Formula:

$$\text{simple interest} = \text{principal} \times \text{rate} \times \text{time}$$

In the formula above, the principal is the amount borrowed or invested, rate is the percentage charged per time period (e.g. daily,

monthly or yearly) for interest, and time is the total number of time periods of the loan or investment. (Note that when using the formula, the units of time for the interest rate and total number of time periods must match. If the time is stated in years and the interest rate is given in a percentage per month, first convert the time in years to months.)

Simple random sample [p. 275]

A sample where everyone is randomly selected

Simulations [p. 405] Using technology (calculators or computers) to repeat a random process many times; e.g. random sampling

Sine ratio [p. 181] In any right-angled triangle,

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \text{ where } 0^\circ < \theta < 90^\circ$$

Small circles [p. 463] The circular boundary of a sphere cross-section that does not contain the diameter of the sphere.

Solid [p. 6] Having three dimensions (length, breadth and thickness) as a geometrical body or figure

Sphere [p. 66] A perfectly round object such as a ball

Spreadsheet [p. 350] A table comprised of rows and columns for entering data and headings. Computer spreadsheet programs allow you to enter formulas that will manipulate data electronically and update results immediately when data is changed.

Step graph A graph consisting of one or more non-overlapping horizontal line segments that follow a step-like pattern.

Strata [p. 288] A group within a population that reflects the characteristics of the entire population

Stratified sample [p. 288] A sample using categories or strata of a population. Members from each category are randomly selected. For example, one student is selected from each Year 7, 8, 9, 10, 11 and 12.

Study population [p. 295] Group of individuals who agree to participate in a survey

Superannuation [p. 521] An investment to which your employer must contribute by law and to which you may contribute part of your salary. It is compulsory in the hope that retirees will have enough funds for a comfortable retirement and will not have to rely on the age pension, which is funded by tax payers.

Survey [p. 274] A set of questions, which participants answer, designed to find out about something

Systematic error [p. 267] An error that repeatedly occurs, either from human error or a faulty measuring device

Systematic list [p. 431] A list that divides the population into a structured sample size. For example, sorting the names of people in alphabetical order.

Systematic sample [p. 283] A sample that divides the population into a structured sample size; for example, sorting the names of people in alphabetical order and selecting every fifth person.

T

Table [p. 212] An arrangement of data using rows and columns of boxes

Tangent ratio [p. 181] In any right-angled triangle,

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \text{ where } 0^\circ < \theta < 90^\circ$$

Target population [p. 265] The entire group of individuals the survey is interested in

Tax [p. 232] Money taken out of a person's income and given to the government for funding of services

Three-dimensional (3D) [p. 6] Having three dimensions, like a solid; requiring three coordinates to specify a point

Time [p. 477] Period of existence that is measured in seconds, minutes, hours, days, weeks, months, years and so on; this process considered as a whole

Time zone [p. 477] A region of the Earth that has a uniform standard time or local time. There are 24 time zones in total.

Time difference [p. 477] The length of a period of time between two times

Trapezium [p. 38] A trapezium is a quadrilateral with one pair of opposite sides parallel

Tree diagram [p. 442] A diagram used to show all the possibilities when several different options are available

Trends [p. 335] A general shape or direction created by data

Trial [p. 406] A single action in an experiment that randomly determines the event

Two-dimensional (2D) [p. 6] Having two dimensions, like a surface, but no depth; requiring two coordinates to specify a point

V

Vertical axis [p. 217] Vertical axis on the number plane

Vertical axis intercept [p. 237]

The value where the line of a graph crosses the vertical axis

Variability [p. 266] The amount by which data points in a dataset differ from the mean and from each other, which can also be measured by the spread or range

Variable [p. 335] A symbol, or the quantity it signifies, that may represent any one of a given set of number and other objects

Vertex [p. 7] In 2D shapes, a point where two straight lines meet to make an angle. In 3D objects, a point where three or more straight edges meet to make a corner. *Plural:* vertices

Volume [p. 81] The size, measure or amount of anything in three dimensions; the space occupied by a body or substance in cubic units; the SI unit of volume is the cubic metre (m³)

X

x-axis [p. 217] See horizontal axis

x-coordinate [p. 206] The horizontal distance on the x-axis

Y

y-axis [p. 217] See vertical axis

y-coordinate [p. 206] The vertical distance on the y-axis

Answers

Chapter 1

Pre-test

- 1 a** 4 sides **b** 4 corners
c 12 edges **d** 8 corners
2 a Degrees **b** 3
3 a Angle *B* **b** 90 degrees
4 Lines *A* and *C*
5 a i 6 faces **ii** 6 rectangles
b i 3 faces
ii 2 circles, 1 curved rectangle
6 a 1 **b** 81 **c** 100
d 10 000 **e** 1 000 000 **f** 169
7 a 4 units **b** 4.5 units
8 Pi is the symbol and it represents the ratio of the circumference of a circle to its diameter (e.g. for every 1 unit of diameter in a circle there will be 3.14 units of circumference).

- 9 a** Right angle **b** Angle measure
c Degrees
10 a All equal sides
b Opposite sides equal
c Parallel lines
11 a Milli represents one thousandth of a metre
b Kilo represents one thousand metres
12 1000 mL

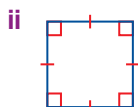
Exercise 1A

- 1 a** hexagon **b** square
c triangle **d** rhombus
e rectangle **f** trapezium
g pentagon **h** octagon
i parallelogram

2

Property	Square	Rectangle	Parallelogram	Trapezium	Triangle	Rhombus	Kite
Four sides	✓	✓	✓	✓		✓	✓
Three sides					✓		
All sides are equal length	✓					✓	
All angles measure 90°	✓	✓					
Not all angles measure 90°			✓	✓	✓	✓	✓
Two pairs of parallel sides only	✓	✓	✓			✓	
Two pairs of parallel sides of different lengths		✓	✓				

3 a i Square



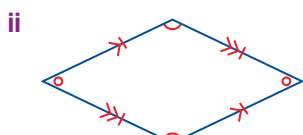
- iii** All 4 sides equal, all 4 angles 90° , opposite sides parallel

b i Trapezium



- iii** 4 sides, one pair of parallel sides

c i Rhombus



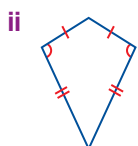
- iii** All 4 sides equal, opposite sides parallel, opposite angles equal

d i Parallelogram



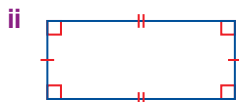
- iii** 4 sides, opposite sides parallel, opposite sides equal, opposite angles equal

e i Kite



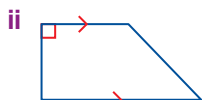
- iii** 4 sides, adjacent sides equal, one part of opposite angles equal

f i Rectangle



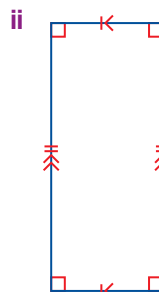
- iii** 4 sides, all 4 angles 90° , opposite sides parallel, opposite sides equal

g i Trapezium



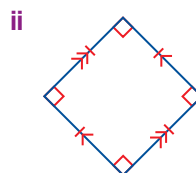
- iii** 4 sides, one pair of parallel sides
Note: the right angle in this case is not significant.

h i Rectangle



- iii** 4 sides, all 4 angles 90° , opposite sides parallel, opposite sides equal

i i Square



- iii** All 4 sides equal, all 4 angles 90° , opposite sides parallel

4 a cylinder

b cone

c cube

d rectangular prism

e triangular pyramid (tetrahedron)

f square-based pyramid

g triangular prism

5 a face **b** corners **c** edge

6

Name	Faces	Edges	Vertices
Rectangular prism or cube	6	12	8
Triangular prism	5	9	6
Square and rectangular-based pyramid	5	8	5
Triangular-based pyramid (tetrahedron)	4	6	4
Pentagonal prism	7	15	10
Hexagonal prism	8	18	12

7 Shapes in picture: circles, triangles, rectangles, trapeziums, squares and semi-circles

- 8** Triangles and trapeziums
9 Rectangle – living area, dining room, table, kitchen benches, master bedroom, bed, cinema and garage
 Circle – 3 gardens
 Trapezium – patio, driveway and linen
 Triangle – storage
 Other answers are possible.

- 10 a** Squares, rectangles and trapeziums
b Rectangles
c The left side of the roof is a square ($7\text{ m} \times 7\text{ m}$), while the other is a rectangle ($5\text{ m} \times 7\text{ m}$).

- 11** Circles, rhombuses, triangles, trapeziums and rectangles

- 12 a, b, c**

Shape	Rectangle	Parallelogram	Trapezium
House numbers	106–118, 135–137, 143–154	127–133, 138–142, 204–208, 211–247, 306–343	101–105, 119–126, 134, 201–203, 209, 210, 301–305

- 13 a** Cone and spheres
b Tetrahedron
c Cubes
d Triangular prism
e Rectangular prism
f Cube
g Rectangular prism
h Cylinder
i Triangular prism
14 Square-based pyramid

Exercise 1B

- 1 a** 200 mm **b** 6 m **c** 17.1 m
d 630 mm **e** 18 km **f** 36 cm
g 24 mm
2 a 19.6 m **b** 11.6 km
c 4010 m **d** 64 cm

- 3 a** 6.28 m **b** 2513.27 mm
c 125.66 km **d** 188.50 cm
4 a 59.34 cm **b** 0.35 m
c 1483.53 mm **d** 3.14 km
5 a 139.34 cm **b** 2.35 m
c 2163.53 mm **d** 5.14 km
6 62.83 cm
7 327.55 m
8 119.38 cm
9 1.27 m
10 11.24 m
11 a 85.6 m **b** 256.8 m
12 5.89 cm

Exercise 1C

- 1 a** 40 cm **b** 66 mm **c** 6 m
d 9.42 km **e** 12.85 m **f** 37 km
2 a 48 cm **b** 480 mm **c** 20.28 m
d 12.6 km **e** 200 m **f** 85.42 mm
g 37.70 cm **h** 7.14 km **i** 101.56 mm
3 56 m
4 57 m
5 384.22 m
6 1.96 m
7 1023.79 cm
8 66.6 m
9 11.94 m

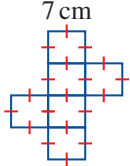
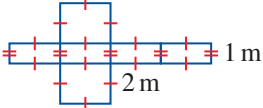
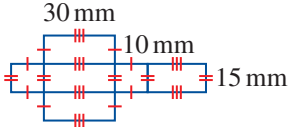
Exercise 1D

- 1 a** 11 040 mm² **b** 22.90 m²
c 16 200 mm² **d** 804.25 cm²
e 76.97 cm² **f** 900 cm²
2 a 9 cm² **b** 40 cm²
c 6716 mm² **d** 20 cm²
e 1 km² **f** 769 mm²
3 5.25 m²
4 24.63 m²
5 35 m²
6 4.54 m²
7 2 544 690.049 mm²
8 17.1 m²
9 a 2.2 ha **b** 22 sheep

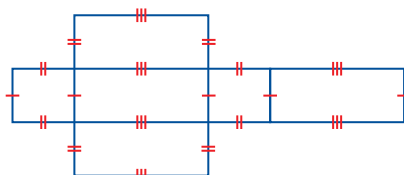
Exercise 1E

- 1 a 15 m^2 b 150 cm^2 c 5525 mm^2
 2 a 10.05 m^2 b $12\,399.38 \text{ mm}^2$
 c 13.68 km^2
 3 a 3.14 m^2 b 30 cm^2
 c 800 mm^2 d 53.76 m^2
 4 a 95 cm^2 b 5600 mm^2
 c 26.28 m^2 d 8.6 km^2
 e 1323 m^2 f 92.55 cm^2
 g 3.27 cm^2
 5 a 12.86 m^2 b 19.50 cm^2 c 16.25 m^2
 d 4500 mm^2 e 10.37 m^2 f 2100 mm^2
 6 63 m^2
 7 1.195 m^2
 8 a 226.98 cm^2 b 680.94 cm^2
 9 131 m^2
 10 2 tins
 11 1512.74 cm^2
 12 362.43 m^2

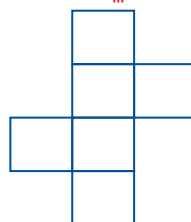
Exercise 1F

- 1 a B b C c B
 d A e B
 2 a  7 cm
 b  1 m
 2 m
 c  30 mm
 10 mm
 15 mm
 3 a Glass box, toilet roll
 b Phone, tool box
 c Ice cubes, dice
 d Chocolate, calendar
 e Ice cream cone
 f Four-sided dice

4

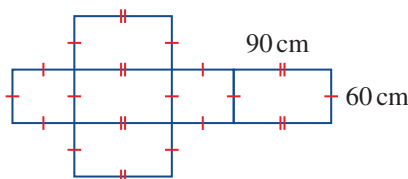


5 a



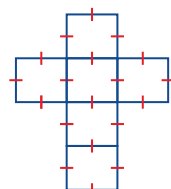
- b He would need to fold the metal net 5 times.

6

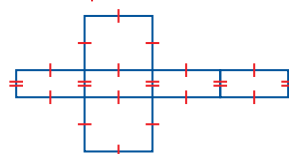


Exercise 1G

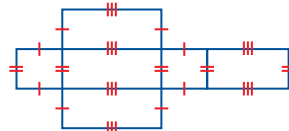
1 a



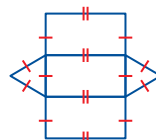
b



c



d



- 2 a** 864 cm^2 **b** 121.5 m^2
c 42 m^2 **d** 288 cm^2
3 a 21.4 m^2 **b** 99.6 cm^2
c 42 m^2 **d** 236 mm^2
4 a 39 mm^2 **b** 224 cm^2 **c** 9.01 m^2
5 a 67.9 m^2 **b** 2169 cm^2 **c** 76.86 m^2
6 a 20.55 m^2 **b** 328.8 cm^2 **c** 692 mm^2
7 $21\,600 \text{ cm}^2$
8 5.27 m^2
9 30.84 m^2
10 $19\,360 \text{ cm}^2$
11 Any sensible drawing will be accepted.
 19.36 m^2
12 $71\,600 \text{ m}^2$

Exercise 1H

- 1 a** $A = 4\pi r^2$ **b** $A = 2\pi rh + 2\pi r^2$
c curved surface
d two circles or two ends
2 a 2827.43 cm^2 **b** 66.48 m^2
c $20\,106.19 \text{ mm}^2$ **d** 530.93 km^2
3 a 1055.58 cm^2 **b** 75.65 m^2
c 5.34 km^2 **d** 75.40 m^2
e 82.94 m^2 **f** 9424.78 mm^2
4 854.51 cm^2
5 1.55 m^2
6 136.85 cm^2
7 29.04 m^2
8 $510\,064\,471.9 \text{ km}^2$
9 1306.90 cm^2

Exercise 1I

- 1 a** 19.98 m^2 **b** 3518.58 cm^2
c 7.47 m^2
2 a 84.19 m^2 **b** 66 m^2
c 27.04 m^2 **d** 382 cm^2
e 137.17 m^2 **f** 9896.02 cm^2
g 593.92 m^2 **h** 648.2 m^2
3 573.4 m^2
4 150.80 cm^2
5 2100 cm^2
6 8858.69 m^2

- 7** 406.19 cm^2
8 3656.38 cm^2

Exercise 1J

- 1 a** larger, smaller
b division, cm^3 or mm^3
c hold, solid, liquid
d millilitre
e 1000
f kilolitre
g kilolitres
2 a $860\,000\,000 \text{ m}^3$
b $18\,400 \text{ mm}^3$
c $2\,700\,000 \text{ cm}^3$
d 0.276 cm^3
e $1\,380\,000\,000 \text{ m}^3$
f $68\,000 \text{ mm}^3$
g $0.000\,129 \text{ m}^3$
h $2\,700\,000\,000 \text{ mm}^3$
3 a 265 mL **b** 412 m^3
c 6000 cm^3 **d** 2.128 L
e 27 cm^3 **f** $0.001\,247 \text{ kL}$
g 2700 kL **h** 27.5 kL
i 0.267 ML **j** 800 mL
k 0.628 L **l** $3\,200\,000 \text{ L}$
4 0.8 m^3
5 $1\,089\,000\,000 \text{ cm}^3$
6 12 m^3
7 5000 mL
8 $2\,500\,000 \text{ L}$
9 $10\,000 \text{ L}$
10 $562\,000 \text{ ML}$

Exercise 1K

- 1 a** 901.33 cm^3 **b** 1848 mm^3
c 523.60 m^3 **d** 2572.44 cm^3
e 3.648 m^3 **f** 40.5 m^3
g 197.92 cm^3 **h** 36.95 cm^3

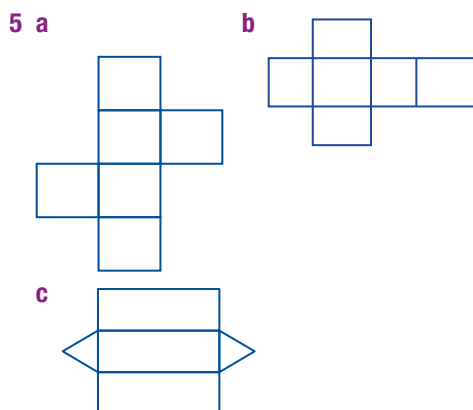
- 2 a 8.18 L b 23.33 kL
 c 0.224 mL d $19.09 \text{ m}^3 = 19.09 \text{ kL}$
 3 a $1381.21 \text{ mm}^3 = 1.38 \text{ mL}$
 b $10\,305.99 \text{ m}^3 = 10.31 \text{ ML}$
 c $9926.28 \text{ cm}^3 = 9.9 \text{ L}$
 d $39\,549.82 \text{ mm}^3 = 39.55 \text{ mL}$
 4 288 cm^3
 5 0.11 m^3
 6 $110\,976 \text{ mm}^3$
 7 13.57 m^3
 8 367.57 cm^3 , 367.57 mL
 9 5.56 kL
 10 22.58 kL
 11 37.04 L
 12 1.65 L
 13 114 333 L
 14 89 797 L

Chapter checklist

- 1 Square: All 4 sides equal, all 4 angles 90° , opposite sides parallel
 Rectangle: 4 sides, all 4 angles 90° , opposite sides parallel, opposite sides equal
 Triangle: 3 sides, 3 vertices
 2 Rectangles, squares, trapeziums, triangles, hexagons, parallelograms

Name	Faces	Edges	Vertices
Cube	6	12	8
Triangular prism	5	9	6
Square and rectangular-based pyramid	5	8	5
Triangular-based pyramid (tetrahedron)	4	6	4
Pentagonal prism	7	15	10
Hexagonal prism	8	18	12

- 4 Triangular-based pyramid, cube, square prism, triangular prism, cylinder



- 5 a b
 6 a 13.7 cm b 7 m c 350 mm
 7 a 23.25 m b 238.76 mm
 8 a i 108.38 m ii 163.58 m
 b i 170.87 cm ii 348.87 cm
 9 a 11 m b 400 cm c 799.91 mm
 10 a 6.15 m^2 b 314.16 cm^2
 c 25.5 mm^2
 11 a 8 m^2 b 28 cm^2 c 60 mm^2
 12 a 212.06 cm^2 b 8.73 m^2
 13 a 18 m^2 b 552.73 cm^2
 14 a 253.5 cm^2 b 27 m^2 c 966 mm^2
 15 $4\pi r^2$
 16 $2\pi rh + 2\pi r^2$
 17 a 9160.88 cm^2 b 113.10 m^2
 18 a 528 cm^2 b 56.01 m^2 c 19.65 mm^2
 19 a 10.28 m^2 b 722.57 cm^2
 20 a 108.38 m^2 b 7385 cm^2
 21 a mm^3 b cm^3 c m^3
 22 a 1740 mm^3 b $2\,670\,000 \text{ cm}^3$
 c 2.34 cm^3
 d $100\,000\,000\,000 \text{ cm}^3$
 23 a 75 mL b 2500 cm^3
 c 6.172 L d 3200 kL
 24 a 3645 cm^3 b 8.18 m^3
 c 289.5 cm^3
 25 a 2.4 ML b 63.62 L c 1.10 kL
 26 a $43.68 \text{ m}^3 = 43.68 \text{ kL}$
 b $4813.11 \text{ mm}^3 = 4.81 \text{ mL}$
 c $323\,830.24 \text{ cm}^3 = 323.83 \text{ L}$

Chapter review

- 1 a** Triangles – 4, rectangle – 3, trapezium – 1
b Triangles – bracing
 Properties: 3 sides, 3 vertices
 Rectangles – a door, roller door and the basic shape of the entire shed
 Properties: 4 sides, 4 vertices, all angles are 90° , two pairs of parallel sides, opposite sides are equal
 Trapezium – patio
 Properties: 4 sides, 4 vertices, one pair of parallel sides
- 2 a** Triangular prism
b Cube
c Rectangular prism
d Cylinder
e Cone
- 3 a** iii **b** ii
c ii **d** i
- 4** 2.719 ha
5 15.21 m²
6 3840 cm²
7 432 841.65 m²
8 7754.82 m²
9 452 389.34 mm²
10 3.27 m²
11 No
12 6985.5 cm²
13 Cooper = 481.06 cm², wife = 360.79 cm²
14 6 656 160 mm²
15 601.23 m²
16 0.002 987 m³
17 1 400 000 cm³
18 125 L
19 37.22 L
20 0.3817 ML
21 a 14 L **b** 19.70 L
c 7.24 mL **d** 117.8 mL

Chapter 2

Pre-test

- 1 a** 1: 12 **b** 1: 8 **c** 1: 15
d 1: 20 **e** 1: 30
- 2 a** 2.4 m **b** 1.8 m
c 14 m **d** 36 m
- 3 a** 2250 mm **b** 1400 mm
c 3840 mm **d** 279 mm
- 4 a** 50 m **b** 56 cm
c 70 m **d** 640 mm
- 5 a** \$620 **b** \$1488
c \$2604 **d** \$3720

Exercise 2A

- 1 a** like **b** Symbols **c** abbreviations
d divide **e** multiply
- 2 a** 500 cm **b** 280 mm **c** 373 cm
d 27.5 mm **e** 4000 mm **f** 6750 mm
g 2.5 m **h** 1.2 m
- 3 a** 1: 400 **b** 1: 200 **c** 1: 2000
d 1: 3750 **e** 1: 60 **f** 1: 48
g 1: 600 **h** 1: 300
- 4 a** Laundry
b Sliding robe door
c Refrigerator
d Wardrobe
e Aluminium sliding door
f Aluminium sliding window
- 5 a** Laundry sink
b Kitchen sink
c Bathtub
d Washing machine
e Bathroom vanity
f Kitchen stove
g Doorway
h Toilet

- 6 a** Walk-in robe (storage area for clothes)
b Powder room (a toilet area with a sink)
c Walk-in linen (storage area for linen)
- 7 a** Stairs (stairs that have a landing half way and continue up the other side)
b Shower (a rectangular shower area)
c Void (an area where the ground floor ceiling doesn't exist giving a sense of great space)
d Lounge chair (marks the area of a living room in a house)
e Coffee table (another indicator of a living room)
f A large window (it would show the window opens from the outside on both sides)
- 8** AS Australian Standards (designed to meet Australian standards)
 U/G Underground (marks underground power and water)
 ENS Ensuite (private bathroom linked to a bedroom)
- 9** FW Floor waste (a drain in the floor)
 DP Down pipe (pipes connecting the roof gutters to the storm water or tank pipes)
 HWS Hot water system (indicates where the hot water system is situated)



Exercise 2B

- 1 a** perimeter **b** area
c substitute, formulas **d** length, width
e base, height
- 2 a i** 1 : 200 **ii** 18 m
b i 1 : 80 **ii** 10 m
c i 1 : 50 **ii** 9 m
d i 1 : 500 **ii** 60 m

- 3 a** 25 cm² **b** 192 cm² **c** 30 cm²
d 80.6 cm² **e** 217.5 m² **f** 220 mm²
- 4 a** 40 mm : 6 m = 40 mm : 6000 mm
 = 1 : 150
- b i** 22 mm by 24 mm
 3300 mm by 3600 mm
ii 7 mm by 14 mm
 1050 mm by 1350 mm
iii 13 mm by 14 mm
 1950 mm by 1350 mm
iv 32 mm by 44 mm
 4800 mm by 6600 mm
v 13 mm by 16 mm
 1950 mm by 2400 mm
vi 40 mm by 45 mm
 6000 mm by 6750 mm
- c i** $P = 3.3 + 3.3 + 3.6 + 3.6 = 13.8$ m
ii $P = 1.05 + 1.05 + 1.35 + 1.35 = 4.8$ m
iii $P = 1.95 + 1.95 + 1.35 + 1.35 = 6.6$ m
iv $P = 4.8 + 4.8 + 6.6 + 6.6 = 22.8$ m
v $P = 1.95 + 1.95 + 2.4 + 2.4 = 8.7$ m
vi $P = 6 + 6 + 6.75 + 6.75 = 25.5$ m
- 5 a** 50 mm : 5 m = 50 mm : 5000 mm
 = 1 : 100
- b i** 26 mm by 37 mm
 2600 mm by 3700 mm
ii 50 mm by 49 mm
 5000 mm by 4900 mm
iii 24 mm by 15 mm
 2400 mm by 1500 mm
iv 15 mm by 24 mm
 1500 mm by 2400 mm
- c i** 9.62 m²
ii 24.5 m²
iii 3.6 m²
iv 3.6 m²
- 6** 30 mm : 3.6 m = 1 : 120
- 7** 50 mm : 4 m = 1 : 80

- 8 a** $25 \text{ mm} : 20 \text{ m} = 25 \text{ mm} : 20\,000 \text{ mm}$
 $= 1 : 800$
b Length 58 mm
 Actual length $58 \times 800/1000 = 46.4 \text{ m}$
 $A = 20 \text{ m} \times 46.4 \text{ m} = 928 \text{ m}^2$
- 9 a** $40 \text{ mm} : 20 \text{ m} = 1 : 500$
b Other side 60 mm = 30 m
 $P = 100 \text{ m}$
- 10 a** $80 \text{ mm} : 6000 \text{ mm} = 1 : 75$
b Other side measures 100 mm
 $A = 6 \text{ m} \times 7.5 \text{ m} = 45.0 \text{ m}^2$
c Yes, it meets council requirements.

Exercise 2C

- 1 a** simplify **b** scale factor
c price or cost; units
- 2 a** 20 m **b** \$400
- 3 a** 10.5 m^2 **b** \$546
- 4 a** 33 m^2 **b** \$2145
- 5 a** $80 \text{ mm} : 6000 \text{ mm} = 1 : 75$
b $100 \text{ mm} \times 75 = 7500 \text{ mm}$ or 7.5 m
c $P = 27 \text{ m}$
d \$540
- 6 a** 40 mm by 42 mm
b 3 m by 3.150 m
c 12.3 m
d \$58.43
- 7 a** 56 mm by 77 mm
b 4.20 m by 5.775 m
c 24.255 m^2
d \$1503.81
- 8** $15 \text{ mm} : 20 \text{ m} = 1 : 1333$
 Sides measure 25 mm, 57 mm,
 28 mm, 46 mm
 $P = 208 \text{ m}$
 Cost = \$9877.5
- 9** $40 \text{ mm} : 6000 \text{ mm} = 1 : 150$
 Laundry 20 mm by 12 mm
 $A = 3 \times 1.8 = 5.4 \text{ m}^2$
 Cost = \$561.60

- 10 a** 23 mm by 28 mm
 $A = 14.5 \text{ m}^2$
 Cost = \$778.84
b It would be cheaper to buy carpet for the flat at \$600.
- 11 a** Garage 40 mm by 43 mm
 $A = 6.0 \times 6.45 = 38.7 \text{ m}^2$
 5 tins of paint are needed.
b Cost = \$270
- 12 a** $\$11.50 \times 38.7 = \$445.05$
b It is cheaper for Jang to do it herself.
- 13** 90 mm by 126 mm
 Area is approximately 255 m^2
 Area to turf = $600 - 255 = 345 \text{ m}^2$
 Cost is approximately \$2657.50
- 14 a** $32 \text{ mm} : 6400 \text{ mm} = 1 : 200$
b Perimeter approx 382 mm on plan.
 Actual perimeter approx. = 76.4 m
c Cost = \$916.80

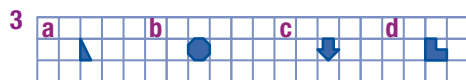
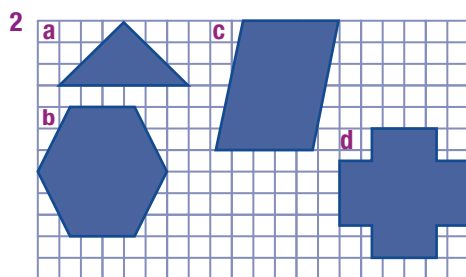
Exercise 2D

- 1 a** plans, 1
b symbols
c measurements, plan, measurements
d arrows
- 2 a** 4 cm, 7 cm **b** 7 cm, 2 cm
c 3 m, 1 m **d** 7 cm
- 3** 720 mm
- 4 a** 5740 mm by 5740 mm = 5.74 m by 5.74 m
b 6200 mm by 6200 mm = 6.2 m by 6.2 m
c 230 mm
d 2940 mm by 5570 mm or 2.94 m by 5.57 m
- 5 a** 4000 mm
b 4000 mm by 1200 mm
c 800 mm
- 6 a** Plan 1: 100 mm; plan 2: 70 mm
b i 2700 mm by 6100 mm
ii 2895 mm by 2000 mm

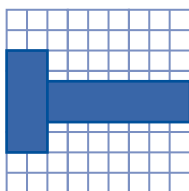
- c i** Width of bath/laundry including external wall
- ii** Length of the robe in bedroom 2
- d i** 3500 mm by 3900 mm
- ii** 4400 mm by 2895 mm
- e** Plan 1: 57.12 m² (meets council requirements); plan 2: 62.4 m² (does not meet council requirements).

Exercise 2E

- 1 a** Construction, share
- b** hand, technology
- c** architect, drawings, builder

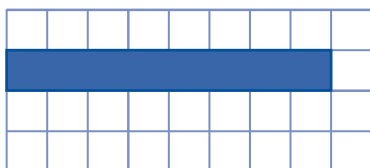


- 4** Squares 10 mm by 10 mm



Scale 5 : 1

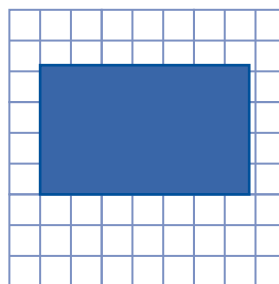
- 5** Squares 10 mm by 10 mm



Scale 4 : 1

- 6 a** 6.9 m by 4.1 m = 6900 mm by 4100 mm
- b** 69 mm by 41 mm

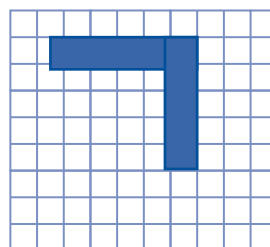
- c** Squares 10 mm by 10 mm



Scale 1 : 100

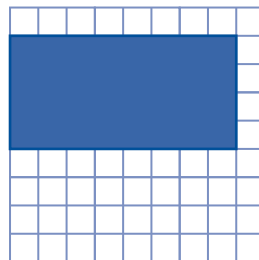
- 7 a** 55.4 mm by 12 mm and 49.4 mm by 12 mm

- b** Squares 10 mm by 10 mm



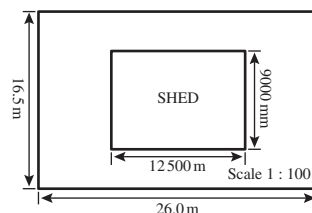
Scale 1 : 50

- 8** 3000 mm by 6000 mm
40 mm by 80 mm
Squares 10 mm by 10 mm



Scale 1 : 75

- 9** The drawing should be 260 mm by 165 mm, with details as shown in this thumbnail view:



- 10** Show your teacher what you have produced.
- 11** Show your teacher what you have produced.

Exercise 2F

1 a views

b plan, front, back, left, right

c bigger, reduction

2 a Front and back view



Plan view



Right view



Left view



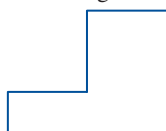
b Front view



Plan view



Left and right view



Back view



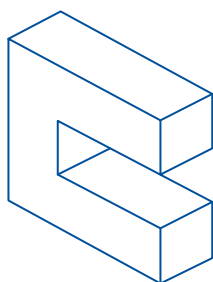
c Left and right
Front and back view



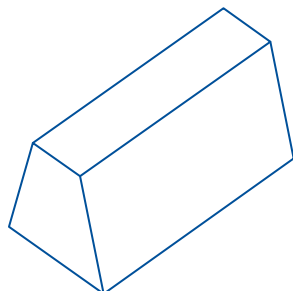
Plan view



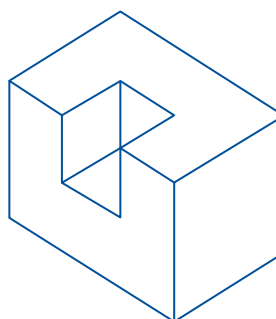
3 a



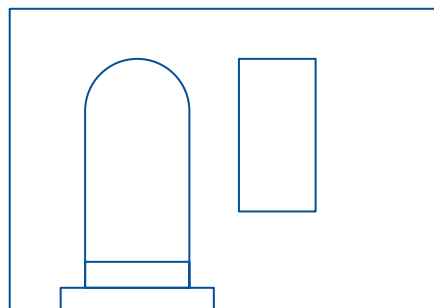
b



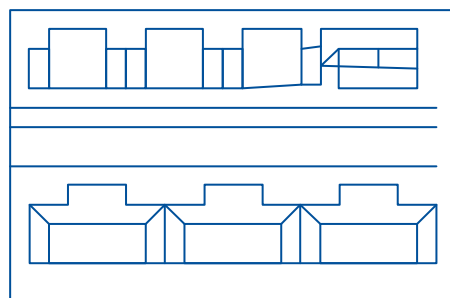
c



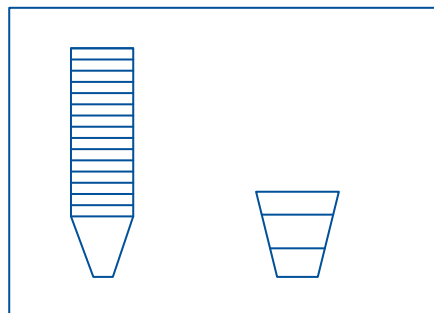
4 a



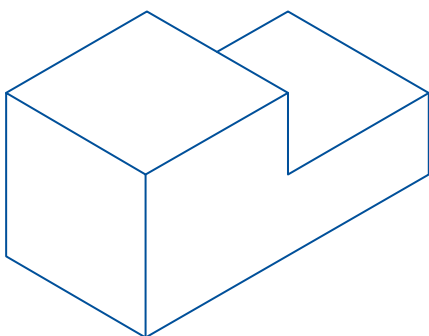
b



c



5 a i



ii

Front and back view



Left view



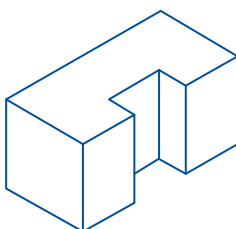
Right view



Plan view



b i

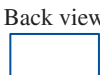


ii

Left and right view



Back view



Front view



Plan view



Chapter checklist

- 1 3540 mm
- 2 1 : 60
- 3 Wardrobe where clothes are stored

4 The laundry sink

5 33.92 m^2

6 1 : 100

7 a 16.8 m^2

b \$806.40

8 1 : 200

22 mm by 15 mm

$A = 2.64 \times 1.8 = 4.75 \text{ m}^2$

Cost = \$294.50

9 6 cm, 2 cm

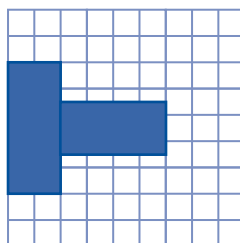
10 a 200 mm

b $3600 \text{ mm} \times 9600 \text{ mm}$

c The door width

d $4 \text{ m} \times 10 \text{ m} = 40 \text{ m}^2$

11 Squares 10 mm by 10 mm

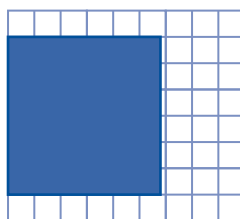


Scale 4 : 1

12 5800 mm by 6000 mm

58 mm by 60 mm

Squares 10 mm by 10 mm



13

Plan view



Front and back view

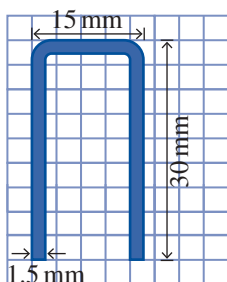


Left and right view



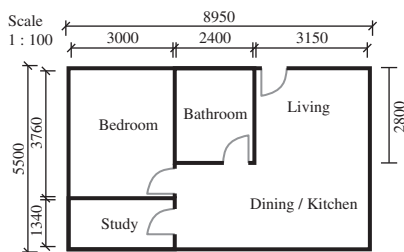
Chapter review

- 1 W.C. is the water closet or toilet
W.I.R. is the walk-in robe, a closet that you can actually walk into like a room.
- 2 a 1 : 250 b 15.0 m² c 20.0 m²
- 3 $A = 6.75 \text{ m}^2$
Cost = \$521.44
- 4 12 litres required so 3 tins of paint need to be purchased
- 5 Cost = \$328.50
- 6 a 75 mm
b 3600 mm by 4350 mm
c The length of the bathroom
d 5600 mm by 4350 mm
e $A = 60.75 \text{ m}^2$. It does not meet council requirements.
- 7 Squares 10 mm by 10 mm

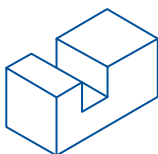


Scale 3 : 1

- 8 In the drawing the external dimensions should be 89.5 mm by 55 mm, with details as shown in this thumbnail view:



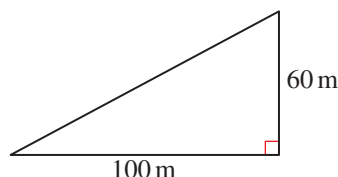
9



Chapter 3

Pre-test

- 1 a 36 b 64 c 121
d 576 e 1089
- 2 a 13 b 17 c 17.89
d 60 e 65.85
- 3 a 20 b 26 c 47.85
d 36 e 15.49
- 4 a 14.10 b 11.48
c 0.05 d 79.74
- 5

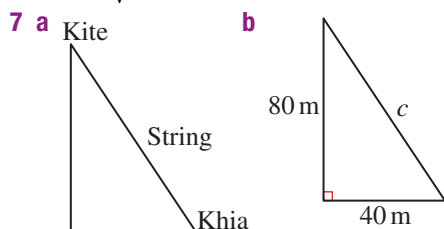


Exercise 3A

- 1 a Pythagoras, hypotenuse, sum
b Pythagoras, right-angled
c c, a, b
- 2 a 5 b 13 c 6.4
- 3 a 5 b 13
c 13.0 d 27.3
- 4 a i $a = 8, b = 6, c = c$
ii $c = \sqrt{8^2 + 6^2}$
iii $c = 10$
b i $a = 10, b = 24, c = x$
ii $x = \sqrt{10^2 + 24^2}$
iii $x = 26$
c i $a = 3, b = 5, c = y$
ii $y = \sqrt{3^2 + 5^2}$
iii $y = 5.8$
d i $a = 8, b = 7, c = x$
ii $x = \sqrt{8^2 + 7^2}$
iii $x = 10.6$
e i $a = 6, b = 13, c = y$
ii $y = \sqrt{6^2 + 13^2}$
iii $y = 14.3$

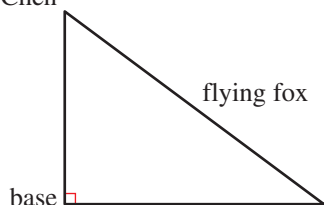
- 5 a** $8^2 + 15^2 = 17^2$, $289 = 289$ verified
b $7^2 + 24^2 = 25^2$, $625 = 625$ verified
c $33^2 + 56^2 = 65^2$, $4225 = 4225$ verified
d $4.5^2 + 6.3^2 = 7.74^2$, $59.94 = 59.91$
 approx. verified
e $12.7^2 + 18.6^2 = 22.52^2$, $507.25 = 507.15$
 approx. verified

6 a $c = \sqrt{3^2 + 10^2}$ **b** $c = 10.4$ m

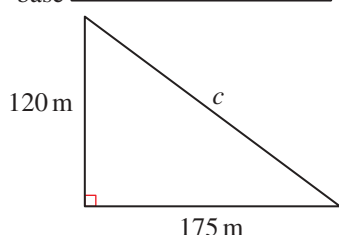


c $c = \sqrt{80^2 + 40^2}$ **d** $c = 89.4$ m

8 a Chen



b



c $c = \sqrt{120^2 + 175^2}$

d $c = 212.2$ m

9 12.8 m

10 44.7 m

11 2.9 m

12 166.4 cm

13 $6^2 + 8^2 = 10^2$

$36 + 64 = 100$

$100 = 100$ verified

14 $3^2 + 3.6^2 = 4.69$
 $9 + 12.96 = 21.96$
 $21.96 = 21.96$ verified

Exercise 3B

1 a Pythagoras, unknown, hypotenuse, side

b calculator, solution

c a, c, b

2 a $a = \sqrt{u^2 - t^2}$ **b** $a = \sqrt{d^2 - e^2}$

c $a = \sqrt{j^2 - k^2}$

3 a 5 **b** 15 **c** 9.6

4 a i $c = 10, a = x, b = 8$

ii $x = \sqrt{10^2 - 8^2}$

iii 6

b i $c = 26, a = f, b = 24$

ii $f = \sqrt{26^2 - 24^2}$

iii 10

c i $c = 9, a = x, b = 4$

ii $f = \sqrt{9^2 - 4^2}$

iii 8.1

d i $c = 28, a = k, b = 23$

ii $k = \sqrt{28^2 - 23^2}$

iii 16.0

e i $c = 14, a = w, b = 7$

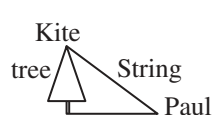
ii $w = \sqrt{14^2 - 7^2}$

iii 12.1

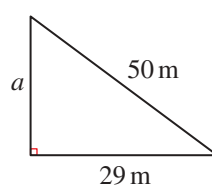
5 a $a = \sqrt{500^2 - 300^2}$

b $a = 400$ m

6 a

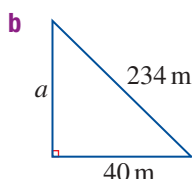
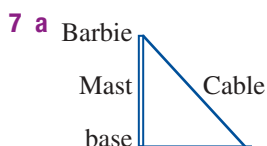


b



c $a = \sqrt{50^2 - 29^2}$

d $a = 40.7$ m



c $a = \sqrt{234^2 - 40^2}$

d $a = 230.6$ m

8 4 m

9 1.8 m

10 129.9 cm

11 30 m

12 3.91 m

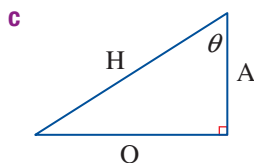
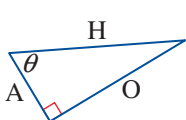
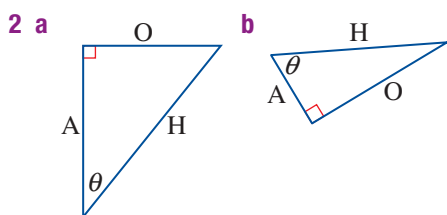
Exercise 3C

1 a opposite, adjacent

b opposite, hypotenuse

c adjacent, hypotenuse

d diagram, problems



3 a 15.4

b 19.9

c 6.7

d 68.5

4 a 6.64

b 58.65

c 7.65

d 9.29

e 24.39

f 251.35

5 a i $H = 6$ m, $O = x$

ii SOH

iii $\sin 40 = \frac{x}{6}$

iv 3.9 m

b i $A = x$, $O = 9$ m

ii TOA

iii $\tan 32 = \frac{9}{x}$

iv 14.4 m

c i $A = x$, $H = 16$ km

ii CAH

iii $\cos 62 = \frac{x}{16}$

iv 7.5 km

d i $A = 7$ cm, $H = x$

ii CAH

iii $\cos 27 = \frac{7}{x}$

iv 7.86 cm

e i $A = 12$ cm, $O = x$

ii TOA

iii $\tan 19 = \frac{x}{12}$

iv 4.1 cm

f i $H = x$, $O = 43$ km

ii SOH

iii $\sin 53 = \frac{43}{x}$

iv 53.8 km

6 a $A = x$, $O = 45$ m **b** TOA

c $\tan 40 = \frac{45}{x}$

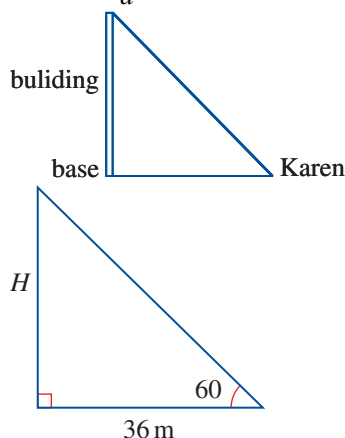
d $x = 53.6$ m

7 a $A = d$, $O = \frac{31}{d}$ m **b** TOA

c $\tan 20 = \frac{31}{d}$

d $d = 85.2$ m

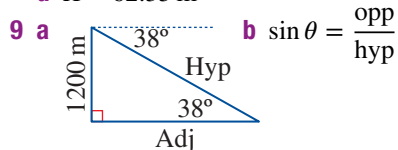
8 a



b TOA

c $\tan 60 = \frac{H}{36}$

d $H = 62.35$ m

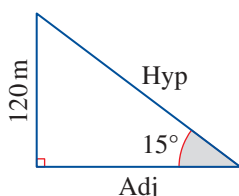


b $\sin \theta = \frac{\text{opp}}{\text{hyp}}$

c $\sin 38 = \frac{1200}{H}$

d $H = 1949.1$ m

10 a



b $\tan \theta = \frac{\text{opp}}{\text{adj}}$

c $\tan 15 = \frac{120}{A}$ d $A = 447.8 \text{ m}$

11 35.3 m

12 39.6 m

13 115.4 m

14 19.5 m

15 3.27 m

16 1677.9 m

Exercise 3D

1 a tangent, opposite

b cosine, hypotenuse

c ratio, opposite

d draw, label

2 a 22.62 b 37.34

c 10.44 d 48.37

3 a 67.38 b 67.38

c 39.47 d 20.45

4 a i $O = 26 \text{ m}$, $H = 40 \text{ m}$

ii SOH

iii $\sin \theta = \frac{26}{40}$

iv 40.5°

b i $O = 32 \text{ m}$, $A = 14 \text{ m}$

ii TOA

iii $\tan \theta = \frac{32}{14}$

iv 66.4°

c i $A = 20 \text{ km}$, $H = 29 \text{ km}$

ii CAH

iii $\cos \theta = \frac{20}{29}$

iv 46.4°

d i $O = 17 \text{ cm}$, $H = 27 \text{ cm}$

ii SOH

iii $\sin \theta = \frac{17}{27}$

iv 39.0°

e i $O = 19 \text{ m}$, $A = 12 \text{ m}$

ii TOA

iii $\tan \theta = \frac{19}{12}$

iv 57.7°

f i $A = 43 \text{ km}$, $H = 71 \text{ km}$

ii CAH

iii $\cos \theta = \frac{43}{71}$

iv 52.7°

5 a $O = 300$, $A = 400$

b TOA

c $\tan \theta = \frac{300}{400}$

d 36.9°

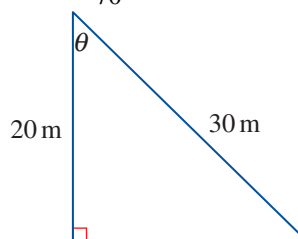
6 a $O = 50 \text{ m}$, $H = 70 \text{ m}$

b SOH

c $\sin \theta = \frac{50}{70}$

d 45.6°

7 a



b $O = 20 \text{ m}$, $H = 30 \text{ m}$

c $\cos \theta = \frac{20}{30}$

d 48.2°

8 10.7°

9 40.9°

10 26.6°

11 44.4°

12 30°

13 48°

Chapter checklist

1 11.7 cm

2 19.7 m

3 3162 mm

4 13.3 m

5 23.6 m

6 3.4 km

7 12.6

8 a 14.4 m

b 28.9 m

9 0.9 m

10 32.6°

11 19.7°

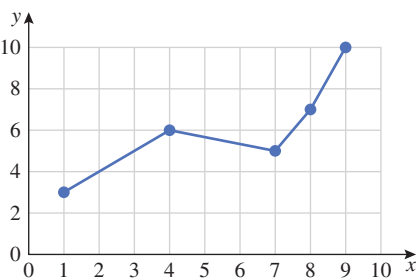
Chapter review

- 1 32.0 m
- 2 5 m
- 3 4.7 km
- 4 2.4 m
- 5 1.5 m
- 6 51.2 m
- 7 2130 m
- 8 23.8°
- 9 24.1°
- 10 296.1 m

Chapter 4

Pre-test

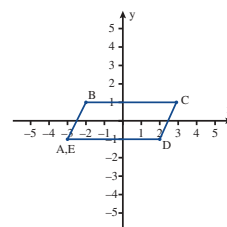
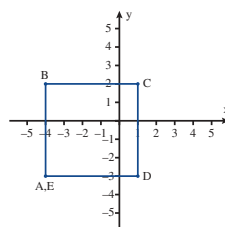
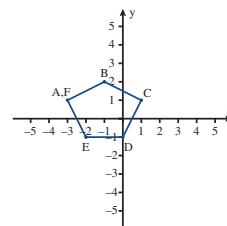
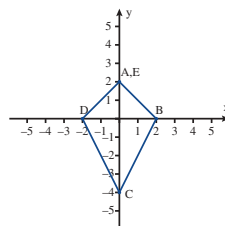
1 a 3	b 12
c -5	d 0
2 a 2	b 13
c -6	d 0
3 a \$66	b \$176
c \$55	d \$143
e \$104.50	
4 a \$90	b \$170
c \$150	d \$230
e \$120	
5 a 8	b 12
c 13	d 23
6 a 6	b 9
c Red	d 3
7 a, b	



c 6	d 8		
8 a 4	b 7	c 18	d 9

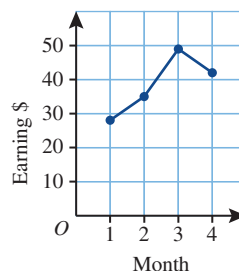
Exercise 4A

- 1 a coordinate, horizontal
b distance, vertical
c Cartesian, positive, negative
- 2 a 3 b -1 c -2
d 0 e -2 f 0
g -3 h 0
- 3 A (1, 1) B (5, 0) C (3, 4)
D (0, 4) E (-1, 2) F (-3, 3)
G (-5, 1) H (-3, 0) I (-4, -2)
J (-2, -5) K (0, -3) L (2, -3)
M (5, -5)
- 4 a It is a kite.
b It is a pentagon.
c It is a rectangle.
d It is a parallelogram.



5 a

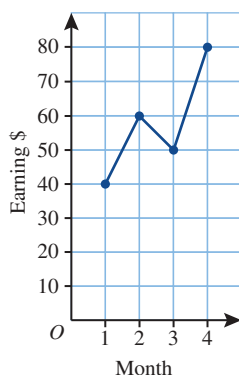
Month	1	2	3	4
Earning \$	28	35	49	42



- b** See graph
c See graph
d Month 3
e \$154

6 a

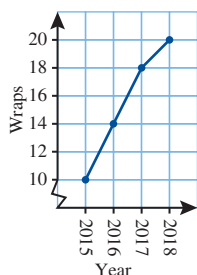
Month	1	2	3	4
Earning \$	40	60	50	80



- b** See graph
c See graph
d Month 4
e \$230

7 a

Year	2015	2016	2017	2018
Wraps	10	14	18	20

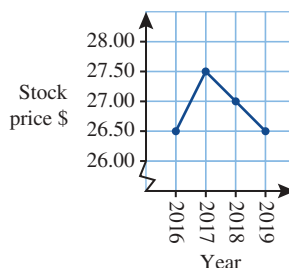


- b** See graph
c 2018
d 62

8

Year	2016	2017	2018	2019
Stock price \$	26.50	27.50	27.00	26.50

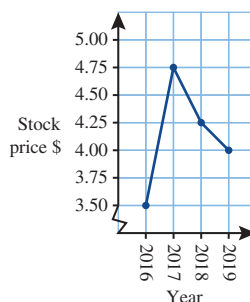
He should have sold his stock in 2017 to make \$27 500.



9

Year	2016	2017	2018	2019
Stock price \$	3.50	4.75	4.25	4.00

She should have sold her stock in 2017 to make \$9500.



Exercise 4B

- 1 a** table, substitute
b negative, number
c positive, equals
2 a 6 **b** -10 **c** 5
d -2 **e** 0 **f** -4
3 a -12 **b** 6 **c** 1
d -6 **e** -3 **f** 15

4 a

x	-2	0	2
y	-6	0	6

b

x	-3	0	3
y	6	0	-6

c

x	-2	0	2
y	1	3	5

d

x	-3	0	3
y	-5	-2	1

e

x	-2	0	2
y	-7	1	9

f

x	-3	0	3
y	4	-2	-8

5 a

x	1	2	3
y	110	170	230

b \$200

6 a

x	2	4	6
y	150	240	330

b \$262.50

7

x	0	2	4
y	15	11	7

8

x	1	3	5
y	850	550	250

9 a Approx. 8 weeks

b Approx. 7 weeks

10 $y = 50x + 120$

x	1	2	3	4	5
y	170	220	270	320	370

Exercise 4C

1 a graph, predictions

b create, range

c Computer, easily, accurately

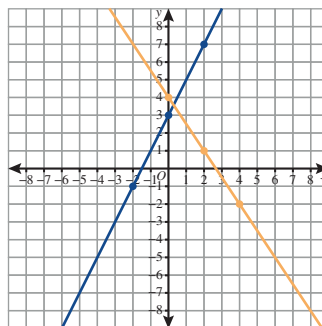
2 a x from 1 to 3, y from -3 to 7

b x from -2 to 2, y from -5 to 3

c x from -3 to 3, y from -12 to 16

d x from 0 to 6, y from 0 to 26

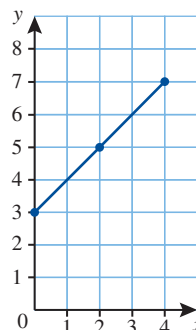
3 a and b



4 a i

x	0	2	4
y	3	5	7

ii x from 0 to 4, y from 3 to 7



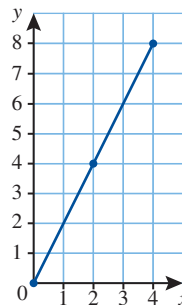
iii See graph

iv See graph

b i

x	0	2	4
y	0	4	8

ii x from 0 to 4, y from 0 to 8



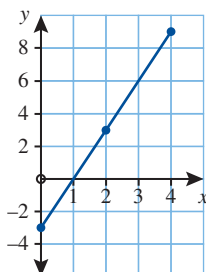
iii See graph

iv See graph

c i

x	0	2	4
y	-3	3	9

ii x from 0 to 4, y from -3 to 9



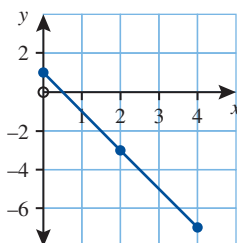
iii See graph

iv See graph

d i

x	0	2	4
y	1	-3	-7

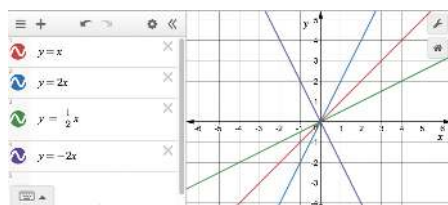
ii x from 0 to 4, y from -7 to 1



iii See graph

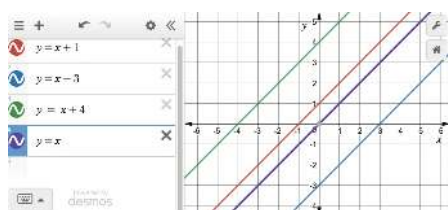
iv See graph

5 a



b The bigger number makes the slope steeper and the negative number makes the line go downhill.

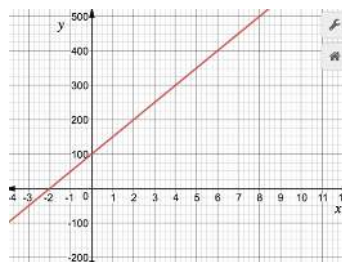
c



d The number at the end is where the line crosses the y-axis.

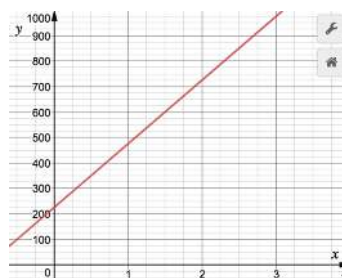
6 a See graph

b \$450



7 a See graph

b \$975



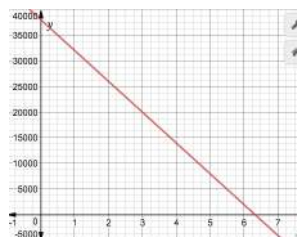
8 a See graph

b 5 years

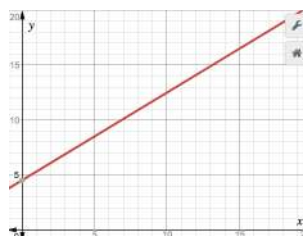


9 a See graph

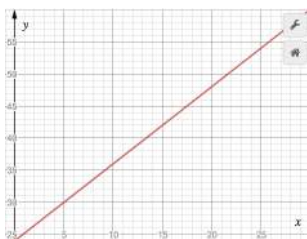
b 4 years



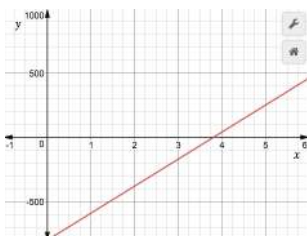
10 \$14.90



11 \$55.20



12 $y = 210x - 800$. At least 4 surfboards need to be sold each day to make a profit.



Exercise 4D

- 1 a 6 b 21 c 5 d 9
 2 a \$300 b \$450
 c 4 years d 1 year
 3 a 0.4 V b 1.7 V
 c 1 A d 0.5 A
 4 a 0.5 km b 30 min c 1 km
 5 a \$160 000 b \$70 000
 c 120 000 km d 500 000 km
 e Even if a school bus is broken down and cannot be driven, there would be value in its parts.
 6 a 200 kg b 420 kg
 c 6.5 months d 14 months
 7 a 80 b.p.m. b 13 min
 c 36 min d 7 min
 e Their heart rate is increasing until 40 min before decreasing.

8 a limousine \$70 000, Hummer \$160 000

b \$27 000

c \$53 000

d 5 years

e 2 years

f limousine \$5000, Hummer \$30 000.

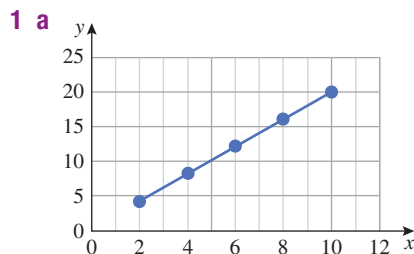
After 15 years, the value of each vehicle does not decrease by much each year, remaining around the stated values.

9 a The highest rainfall occurs during December and is lowest in June.

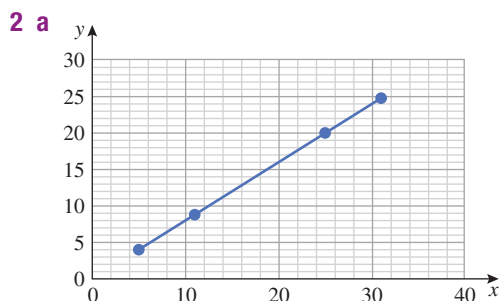
b Highest and lowest temperature occurs in January and July, respectively.

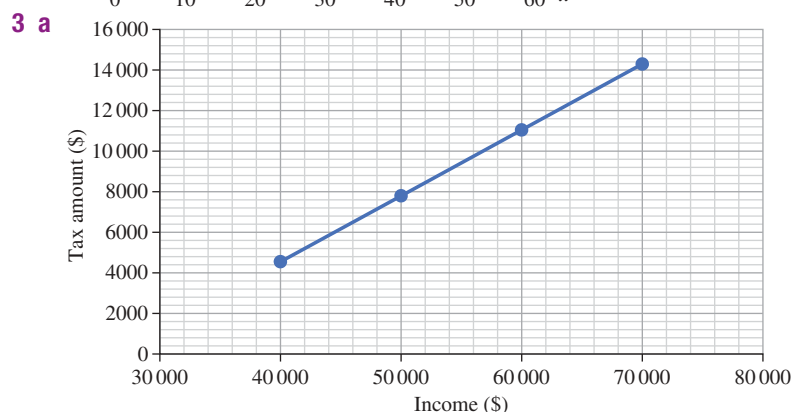
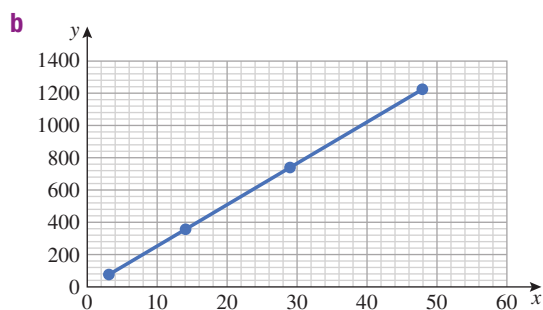
c Answers may vary. For example: If the resort is in the Northern Hemisphere, then travelling during Southern Hemisphere winter time will be best; i.e. June, July and August. Otherwise December, January and February are the best times during the year.

Exercise 4E

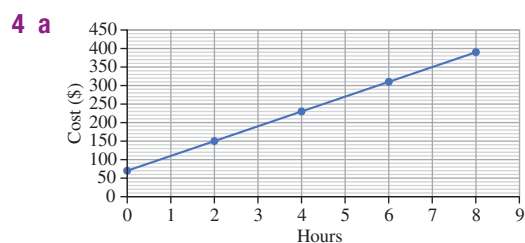


b 14 c 5

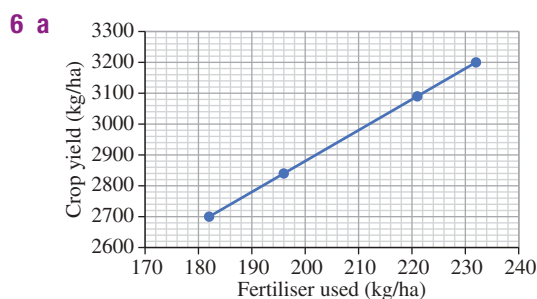
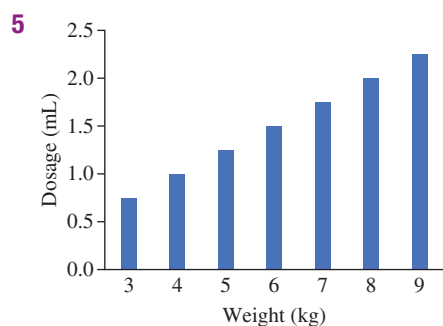




b \$9400

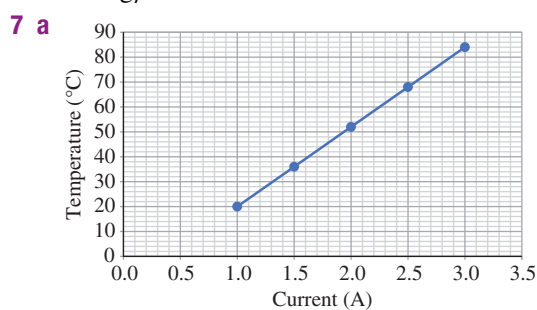


b \$270



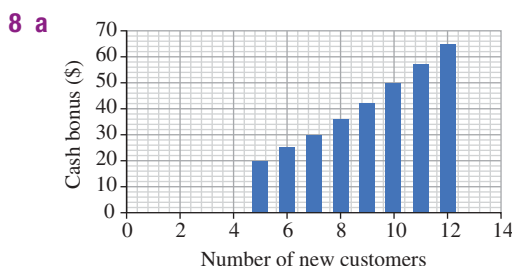
b 2880 kg/ha

c 187 kg/ha



b 46°C

c 3.2 A



b Answers will vary.

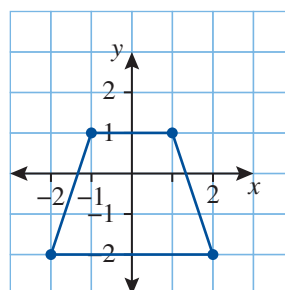
Exercise 4F

- 1 a** 3 **b** 6.5 **c** 6
d 9 **e** 8 **f** 16
- 2 a** $x = 4$, $y = 8$ **b** $x = 12$, $y = 10$
c $x = 2$, $y = 8$
- 3 a** \$5000 **b** 16
c \$6000; i.e. the initial amount of money that was owed.
d 24 months; i.e. the amount of time it will take to pay off the debt.
- 4 a i** \$35 **ii** \$62 **iii** \$27
b \$16
c Make-up bags = 30, revenue & expenses = \$105; this is the break-even point. Selling fewer than 30 bags will result in a loss and selling more than 30 bags will result in a profit.
- 5 a** 60 **b** 15
c Child tickets = 140; this represents how many child tickets are required to be sold to make \$350.
d Adult tickets = 70; this represents how many adult tickets are required to be sold to make \$350.
- 6 a** Total costs = \$480; this represents the total fixed cost.
b \$250 loss **c** \$150 profit
d Sales volume = 20 units
e Total sales = total costs = \$800
- 7 a** \$120 000 **b** \$40 000
c \$200 000; this represents the initial value of the utility tractor.

- d** 25 years; this represents after how many years the utility tractors will be effectively worth nothing.
e \$280 000; this represents the initial value of the row crop tractor.
f 20 years; this represents after how many years the row crop tractors will be effectively worth nothing.
g Time = 13.5 years, value = \$95 000; this represents when the two tractors are worth the same amount.
h Utility tractor 19 years, row crop tractor 16.5 years

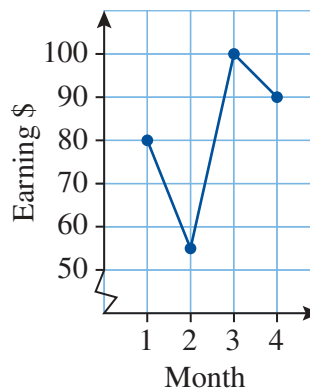
Chapter checklist

1 Trapezium



2 a

Month	1	2	3	4
Earning \$	80	55	100	90



- b** See graph **c** See graph
d 3rd month **e** \$325

3

x	-2	0	2
y	-8	-2	4

4 a

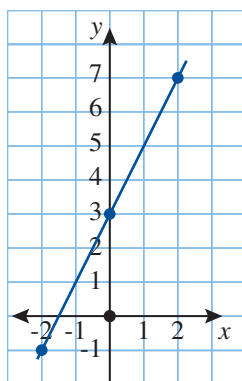
x	1	2	3
y	110	180	250

b \$215

5 a

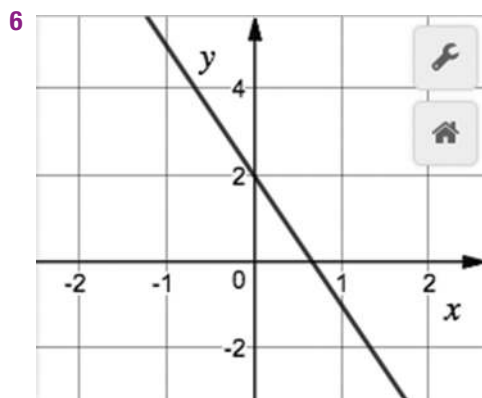
x	-2	0	2
y	-1	3	7

b x from -2 to 2, y from -1 to 7

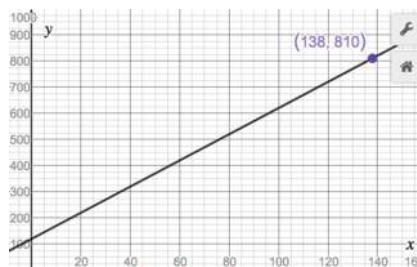


c See graph

d See graph

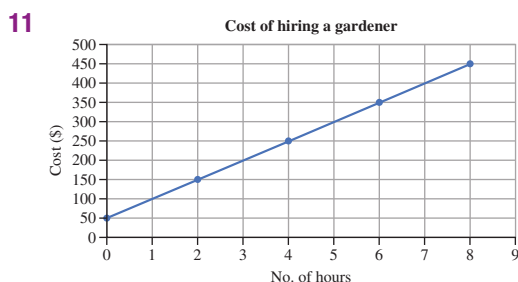
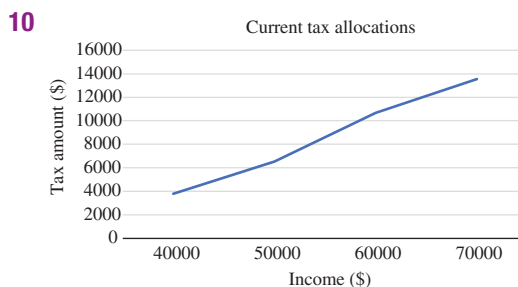


7 The total fare will be \$810.



8 Distance 35 km, time 255 min

9 4 g



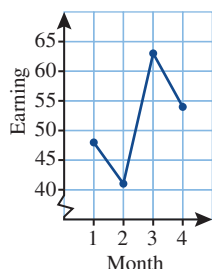
12 ~\$900

13 The point of intersection occurs at 15 cakes and is approximately \$1250. This means that if the company sells fewer than 15 cakes, it will make a loss. If it sells more than 15 cakes, it will make a profit. This is the break-even point.

Chapter review

1 a

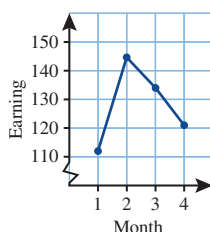
Month	1	2	3	4
Earning	\$48	\$41	\$63	\$54



- b See graph c See graph
d 3rd month e \$206

2

Week	1	2	3	4
Earning	\$112	\$146	\$134	\$121



\$513

3 a

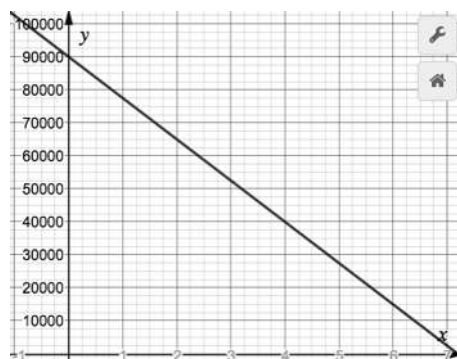
x	0	1	2	3
y	\$20	\$58	\$96	\$134

- b At 3 hours, he has earned more than \$100.

4

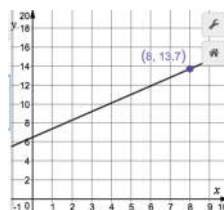
x	1	3	5
y	\$1700	\$1100	\$500

5 a See graph

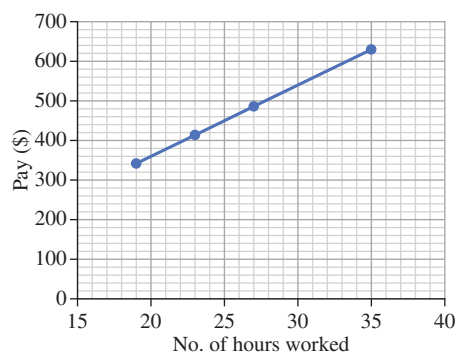


- b After 6 years

6 The total fare for 8 km is \$13.70.



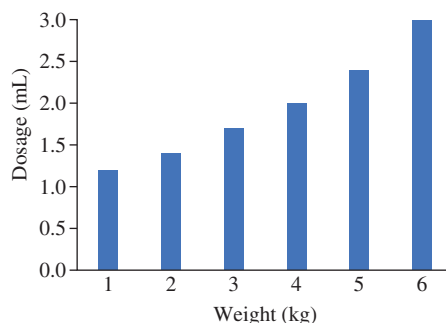
7 a



b \$540

c 17 h

8



9 a 20.5°C

b 22.8°C

c Between 12:30 hours and 13:00 hours

d Turned off at 13:30 hours since after this time the temperature of the water starts to decrease.

10 a 750 km

b 14 h

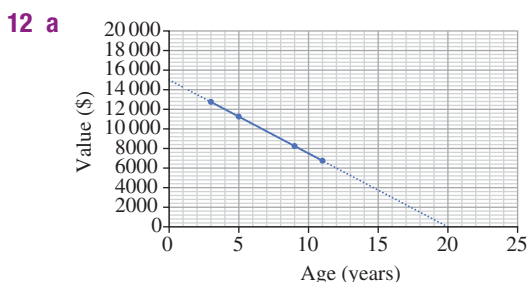
c Distance = 1000 km; this represents that, initially, Frankie was 1000 km from Alice Springs.

d Time = 24 h; this represents how long it took to reach Alice Springs on the train.

11 a 42 psi

b 51 psi

c 16 m



- b** \$9600
c 6.5 years
d Value = \$15 000; this represents the initial value of the range of motion machine.
e Age = 20 years; this represents after how long the range of motion machine is effectively worth nothing.
- 13 a** \$900 loss
b \$300 profit
c \$1200; this represents the initial cost of making the headphones, which is a one-off fixed cost.
d Headphones = 20, revenue & expenses = \$1600; this is the break-even point. Selling fewer than 20 headphones will result in a loss and selling more than 20 headphones will result in a profit.
- 14 a** \$24 000 **b** \$48 000
c 15 years **d** 11.5 years
e Debt = \$50 000; this represents the initial amount that Josh must pay.
f Debt = \$55 000; this represents the initial amount that Lindsey must pay.
g Time = 19 years; this represents how long it will take for Josh to pay off his debt.
h Time = 16 years; this represents how long it will take for Lindsey to pay off her debt.
i Time = 6 years, debt = \$34 000; this represents when both Josh and Lindsey have the same amount of debt to pay back.

Chapter 5

Pre-test

- 1 a** Numerical; e.g. \$2.50, \$4.60
b Categorical; e.g. garlic aioli, tomato
c Numerical; e.g. 14, 41
d Categorical; e.g. car, bike
e Categorical; e.g. rock, pop
f Numerical; e.g. 0, 4
- 2 a** Canteen staff want to know profits so they can change staffing and food choices.
b Fast-food restaurant might want to know which sauces to have more stock of.
c Advertising company might want to know which demographic to target.
d Government is planning future infrastructure based on preferred transport mode.
e A music store wants to know what kind of music it should stock more of.
f A cinema wants to know how many movies sessions they should have.
- 3 a** Actual average is 10.2 but any estimates near 10 will suffice.
b Other 95 students might have completely different commute times.
c i Average is 22.8; estimate is 20–25
ii The average has increased with new responses.
d Yes, would be better as there are more people responding, which means a better estimate of all students' commute times can be done.
e Group in 5 s; teacher to check dot plot

4 a

Range	Frequency
71–75	1
76–80	1
81–85	3
86–90	1
91–95	3
96–100	1
Total	10

- b** They are unlikely to participate as they might be embarrassed about their lower score and not wish others to know.
- c** This survey's participants might only have those students who wish to show their test score, as students with poor scores would prefer not to participate. This would result in the data appearing to show that students have a generally high score, when it may not be the case.
- 5 a** Numerical data
- b** Lowest 109, highest 205
- c** Average is 136; any estimate between 125 and 145 is acceptable.
- d** Data should be grouped. Histograms, dot plots, tables or stem-and-leaf plots can be used. Check with your teacher.
- e** Outliers are 205 and 195. Answers may vary. For example: There could be a special event (e.g. concert, gala, store opening) happening on that day, resulting in a higher number of commuters.
- f** Answers may vary. Must mention that people go to work on weekdays, whereas on weekends most people will relax and might not use trains, making it difficult to estimate.
- 6 a** Anita's main target population would be the students and teachers.
- b** A census is a count of the *entire* population. It is impractical as Anita would literally have to ask every single person in the school.
- c i** It does not properly represent the rest of the school such as other year groups.
- ii** Systematic survey, so may be slightly biased, as only those who use the facility are asked so does not represent the *entire* school.
- iii** Self-selected survey, which means only those who know about the survey and have a strong opinion on the facility will talk about it.
- iv** Randomised survey, so may be skewed and inaccurate, as identifying and contacting members can be difficult.
- v** Administrative staff are not the target population at all.
- vi** Stratified samples are more time consuming to determine.
- d i** 'Usually' is ambiguous.
- ii** Question is biased because it leads the respondent to a particular answer.
- iii** Question is too lengthy and wordy. 'Amenities in the sports facility' will suffice.
- iv** Question disregards people who don't have an opinion or are ambivalent.
- v** There are no problems with this question.
- 7 a** 60% prefer brand Red, 40% prefer brand Blue
- b** The axis begins at 30%, which makes it appear that brand Red is three times more popular than brand Blue, when in reality the difference is not that big. Colour is also an issue, as people will be drawn to the bright red colour compared with the dark blue.
- 8 a** Although the question wishes to know the overall political preference of the electorate, it is concerned only with whether participants like a certain political party. Also, there is some bias in using 'like' with 'party A'.
- b** The results only consider people above 60 years old, which means the sample is unrepresentative. % is also used in the results.
- c** Gabriel would conclude that people's opinion of political party A is characterised by age, when actually the amount of people who like party A and dislike party A are roughly the same.

- d** Gabriel asked a total of only 20 people, which means the results may be inaccurate. He also ignored those aged between 30 and 60 years, making the sample very unrepresentative of the electorate.

Exercise 5A

- 1 a** entire
b number, plan
c funds, services
- 2 a i** The data can be used for electoral purposes to group population into electorates with similar characteristics (e.g. families or no families) or when campaigning to the majority with the hope of winning an election.
ii The data can be used when assigning government funding to decide which target population it should allocate the most funding to or to decide what schemes/infrastructure should be given funding to support the general population.
- b** The government could use the information to determine how much money it needs to assign to childcare services, as well as to plan for future superannuation policies for both working parents.
- 3** The Australian Bureau of Statistics (ABS) organises the census. They send out paper forms, but it can also be completed online. The census is completed by everyone every 5 years, and everyone completes it on the same day. It works by sampling everyone who is present at the house on that night to get an idea of the current housing and population of Australia. Many things are asked about like age, gender, work and so on.

- 4** The **disadvantages of a Census** might **include**: might be difficult to enumerate (count) all units of the population within the available time; higher costs, both in staff and monetary terms, than those required for a sample; generally takes longer to collect, process, and release data than from a sample. The **advantages of a Census** include: provides a true measure of the population (no sampling error); benchmark data may be obtained for future studies; detailed information about small sub-groups within the population is more likely to be available.
- 5 a** Many students pay for their tertiary studies through HECS, which they have deducted from their income once they start earning a certain amount. Government needs to ensure that tertiary graduate students will end up in the workforce and repaying their debts, so it will encourage tertiary institutions to offer places in courses that will lead to high employability of graduates.
- b** From that statistic, the government can delve more into why fewer women than men are employed; e.g. taking leave to take care of children or pregnancy. Then it can begin planning to tackle those issues and help more women into the workforce.
- c** People with non-school qualifications will usually have more skills to offer, enabling them to have more technical and professional jobs that pay higher incomes. If there is clear evidence that having a qualification results in a higher income, the government has more incentive to give people more opportunities to undertake courses to obtain a qualification.

- d** Aboriginals and Torres Strait Islanders often live in rural areas, making it particularly hard to access many services and facilities, such as schools and hospitals. Knowing that many young people are having difficulty accessing education, the government can assign funding and promote programs to assist young people access these services.
- 6 a** Questions 13 and 44
- b** Immigration
- c** To be able to accurately describe the cultural diversity of the Australian population
- d** To determine how many people speak English as a second language
- e** To ensure that services are able to be provided to cater to non-English speaking Australians
- f** Some people are private about their religion
- g** To determine how many people require assistance with self-care, mobility and communication
- h** Some people need help with everyday activities. This can mean a family member might be unable to work full-time and might need monetary assistance. The government needs to plan for the funding of such services and programs.
- i** Employment
- j** By keeping track of which sectors are employing people, the government can use this information when promoting jobs and industries.
- k** To determine how people travel to work
- l** Rail, bus and road networks are all planned by governments. It is important to track how these services are being used and compare changes to previous censuses.

Exercise 5B

- 1 a** difference
b i specific **ii** positive **iii** possibility
c sampling
d accurate, consistent
- 2 a i** People at Dockers game will prefer AFL over other sports.
ii Asking people outside a sports store.
b i People near the store will prefer it over other fast-food outlets in the food court.
ii Stand near the entrance to the food court to ask people entering/leaving the food court.
c i People affected by any changes will have strong feelings for or against. It is better to have a wider selection of the population included in the survey.
ii Survey people in a shopping centre or by a phone poll etc.
d i Likely sample will not have full-time jobs since survey is being conducted at a time when most working people would be at work.
ii Survey people by phone.
e i Parents will have different preferences in music from those of their children/students attending the dance.
ii Ask students about what type(s) of music to be played at the dance.
f i The sample is too small.
ii Increase the size of the sample.
g i The sample is not representative of the entire population
ii Ask people in all States and Territories.
- 3 a** Processing fault: mistake
b Measurement fault: variability in thickness of soles makes it difficult to be accurate.
c Start measuring from a different starting point (e.g. 10 cm) to avoid a measurement fault.

- d** Measurement error: systematic error
e Measurement error: systematic error
- 4 a** D
b A rejected because the question does not collect the information required, as it is limited to yes/no responses.
 B and C rejected because there is no possibility of giving a particular response such as 'none'. Also, there is no way of indicating a favourite sport.
- 5 a** What do you think of the use of phones by students at school?
b What do you think the influence of social media is on people's socialness?
c Do you prefer to visit the beach or cities?
d Do you think that we should have a sugar tax on soft drinks?
- 6 a** The poll is a self-selecting poll that is limited to those people who read the news website. Having the poll at the end of the article might bias people's previously held beliefs about the issue.
b A phone poll of random people might give a more balanced response rate.
- 7 a** 89 000
b 83%
c Self-selecting sample
- 8 a** 1800
b 26%
c Possibly a self-selecting survey since it was posted on social media.
d Differences might be due to different sample size or demographics of participants, whereby an internet poll is likely to have lots of younger participants who use social media frequently and the marketing group poll is likely to have older participants.
- 9 a** 46.5 g
b 47.5 g
c 46.7 g, 47.3 g, 47.0 g

- 10 a** 74 mm
b 7 cm
c 4 mm
d 5.41%
- 11 a** Patrick is not accurate or precise. It is likely that the correct mass is around 23.15 grams.
b Craig is most consistent. There is little variation between his measurements although they are a lot different from the trials of the other students, indicating that Craig's accuracy is likely to be very low.
c Harry is accurate and consistent.

Exercise 5C

- 1 a** count
b information, sample
c data, characteristics
d equal
e wants
- 2 a** Census is better because you want to count the total number of new houses built.
b Survey is better because you want information from a population; that is, whether participants like the Prime Minister.
c Survey is better because you want to find out the population's preferences.
d Census is better because you want a total count of views.
- 3 a** Census
b Survey
c Survey
d Census
- 4 a** Jasper, LúC, Charné, Steven, William, William, Liam, Andrew, Connor, Mark
b i 49 boys in the year group
ii $\frac{49}{70}$ or $\frac{7}{10}$

- c i** 9 boys in the sample
ii $\frac{9}{10}$
- d** This is not a suitable sample. The proportion of boys in the sample is greater than the proportion in the population.
- 5 a** Alexander, Andrew, Courtney, Jake, Clayton, Molly, Oliver, Jarrod, Jasper, Rhys
- b i** 49 boys in the year group
ii $\frac{49}{70}$ or $\frac{7}{10}$
- c i** 8 boys in the sample
ii $\frac{8}{10}$
- d** This is not a suitable sample. The proportion of boys in the sample does not match the proportion in the population.
- e** Random sampling is fair and generally gives a good representative sample; however, it could be difficult in identifying all population members and resulting in the possibility of a skewed sample.
- 6 a** William, Jarrod, James, Caleb, Connor, Amy, Lúic, Thomas, Jack, Molly, Cassius, Alakiir, Bonnie, Mark, Ryan, Oliver, Damian, Liam, Courtney, Chloe
- b i** 49 boys in the year group
ii $\frac{49}{70}$ or $\frac{7}{10}$
- c i** 14 boys in the sample
ii $\frac{14}{20} = \frac{7}{10}$
- d** This is a suitable sample. The proportion of boys in the sample matches the proportion in the population.
- 7 a** Cricket fans who are willing to pay the cost to enter will participate in the survey.
- b** A single prize of \$2000 plus \$50 worth of KFC.
- c** Only those people who are willing to pay the 55c entry cost will participate.

Multiple votes means people could enhance their chance of winning by voting more often and, therefore, possibly affecting the results.

- d** This is not a reliable survey method as not all cricket fans have the ability to participate.
- e** The advantages of self-selected surveys are very easy to conduct, and participants wish to be involved. However, the disadvantages are that the results can be biased because people can vote multiple times, and only those people who are particularly interested will participate.
- 8 a** Self-selected survey
- b** The data is not reliable. There would be a lot of good feeling about Smith at the time, and some of the other players on the list might be unknown to some participants due to their age.

Exercise 5D

- 1 a** regular **b** divide **c** random
- 2 a** 30 **b** 15 **c** 16
d 3 **e** 18
- 3** Advantages of a systematic random sample include: very easy to conduct; and the population is evenly sampled.
 Disadvantages of a systematic random sample include: Can be biased if the sample is not taken from a complete list of the population; and can adversely affect results if some trait of the population coincides with the frequency of selection (e.g. every 12th item manufactured is a defect).
- 4 a** Not all Australian adults are on the electoral roll, as they might be new to the country.
- b** Not all teenagers will be at school in Year 11 or 12; some may have dropped out or graduated.

5 a $\frac{70}{10} = 7$

b

1 Ellie	11 Brenda	21 Bonnie	31 Jack	41 Thomas	51 Brent	61 Jennifer
2 Erica	12 James	22 Will	32 Rhys	42 Cassius	52 William	62 Jessica
3 Connor	13 Damian	23 Ned	33 Nicholas	43 Caleb	53 Dale	63 Oliver
4 James	14 Molly	24 Christopher	34 John	44 Matthew	54 Jake	64 Hamish
5 Mark	15 Steven	25 Liam	35 Michael	45 Jack	55 Caitlin	65 Henry
6 Thomas	16 Alexander	26 Courtney	36 Reid	46 Eamon	56 Charles	66 Katelyn
7 Tennelle	17 Chloe	27 Ryan	37 Alakiir	47 Meg	57 Lúic	67 Kobi
8 Samuel	18 Dominic	28 Charné	38 Angelo	48 Christopher	58 William	68 Jarrod
9 Amy	19 Thomas	29 Jacob	39 Jasper	49 Thomas	59 Clayton	69 Jack
10 Andrew	20 Kristen	30 Olivia	40 Breanna	50 Mark	60 Emma	70 Alexander

c Girls (population) = $\frac{21}{70} = \frac{3}{10}$ Girls (sample) = $\frac{1}{10}$

The sample and population are not in the same proportions.

d The method has not produced a sample that reflects the population this time. Using this sample could produce biased results.

e i

1 Ellie	11 Brenda	21 Bonnie	31 Jack	41 Thomas	51 Brent	61 Jennifer
2 Erica	12 James	22 Will	32 Rhys	42 Cassius	52 William	62 Jessica
3 Connor	13 Damian	23 Ned	33 Nicholas	43 Caleb	53 Dale	63 Oliver
4 James	14 Molly	24 Christopher	34 John	44 Matthew	54 Jake	64 Hamish
5 Mark	15 Steven	25 Liam	35 Michael	45 Jack	55 Caitlin	65 Henry
6 Thomas	16 Alexander	26 Courtney	36 Reid	46 Eamon	56 Charles	66 Katelyn
7 Tennelle	17 Chloe	27 Ryan	37 Alakiir	47 Meg	57 Lúic	67 Kobi
8 Samuel	18 Dominic	28 Charné	38 Angelo	48 Christopher	58 William	68 Jarrod
9 Amy	19 Thomas	29 Jacob	39 Jasper	49 Thomas	59 Clayton	69 Jack
10 Andrew	20 Kristen	30 Olivia	40 Breanna	50 Mark	60 Emma	70 Alexander

ii Girls (population) = $\frac{21}{70} = \frac{3}{10}$ Girls (sample) = $\frac{3}{10}$

Therefore, same mix in sample as in population.

iii This has been a suitable method for selecting a sample.

f i $\frac{70}{20} = 3.5$; therefore, we will pick every fourth name until we have the required sample size.

ii

1 Ellie	11 Brenda	21 Bonnie	31 Jack	41 Thomas	51 Brent	61 Jennifer
2 Erica	12 James	22 Will	32 Rhys	42 Cassius	52 William	62 Jessica
3 Connor	13 Damian	23 Ned	33 Nicholas	43 Caleb	53 Dale	63 Oliver
4 James	14 Molly	24 Christopher	34 John	44 Matthew	54 Jake	64 Hamish
5 Mark	15 Steven	25 Liam	35 Michael	45 Jack	55 Caitlin	65 Henry
6 Thomas	16 Alexander	26 Courtney	36 Reid	46 Eamon	56 Charles	66 Katelyn
7 Tennelle	17 Chloe	27 Ryan	37 Alakiir	47 Meg	57 Lúic	67 Kobi
8 Samuel	18 Dominic	28 Charné	38 Angelo	48 Christopher	58 William	68 Jarrod
9 Amy	19 Thomas	29 Jacob	39 Jasper	49 Thomas	59 Clayton	69 Jack
10 Andrew	20 Kristen	30 Olivia	40 Breanna	50 Mark	60 Emma	70 Alexander

$$\text{iii Girls (population)} = \frac{21}{70} = \frac{3}{10} = \frac{6}{20}$$

$$\text{Girls (sample)} = \frac{5}{20} = \frac{2.5}{10}$$

Although not the same proportions exactly, this sample is a reasonable reflection of the population.

iv This method has been suitable in selecting a sample that has characteristics similar to the population it is drawn from.

g A systematic sample does not always produce a sample that has the same characteristics as the population it is drawn from. Sample size and where you begin can result in a different outcome. Care must be taken to ensure that the sample is consistent with the population.

3 An advantage of a stratified sample is that it has the same characteristics as the population. A disadvantage is it can be more time consuming to determine the sample.

4 a 60 casual employees

b 29 employees

c i 10 full-time

ii 7 part-time

iii 12 casual

$$5 \text{ a Total sample} = \frac{15}{100} \times 92 = 13.8 \\ \approx 14 \text{ students}$$

$$\text{b Girls} = \frac{40}{92} \times 14 \approx 6$$

$$\text{c Boys} = \frac{52}{92} \times 14 \approx 8$$

d Girls

Exercise 5E

1 a characteristics

b proportions, groups

c fraction, percentage

2 a 9

b 24

c 3

d 15

1 Gabrielle	11 Mia	21 Julia	31 Emily
2 Amy	12 Gabriella	22 Ellie	32 Gabrielle
3 Molly	13 Claudine	23 Erica	33 Tara
4 Bonnie	14 Isabella	24 Tennelle	34 Heidi
5 Courtney	15 Keely	25 Brenda	35 Georgia
6 Breanna	16 Claire	26 Chloe	36 Meg
7 Caitlin	17 Caitlin	27 Kristen	37 Cassie
8 Emma	18 Katelyn	28 Meg	38 Shania
9 Jessica	19 Caitlyn	29 Jennifer	39 Nikita
10 Ruby	20 Kate	30 Bonnie	40 Codey

e Boys

1 Christopher	11 Sean	21 William	31 Matthew	42 Thomas
2 Thomas	12 Braden	22 Jake	32 Elliot	43 Will
3 Andrew	13 Mark	23 Charles	33 Matthew	44 Ned
4 Jacob	14 James	24 Oliver	34 Thomas	45 Jack
5 John	15 Christopher	25 Henry	35 Connor	46 Rhys
6 Alexander	16 Liam	26 Jack	36 James	47 Nicholas
7 Benjamin	17 Ryan	27 Jack	37 Samuel	48 Michael
8 William	18 Matthew	28 Matthew	38 Damian	49 Angelo
9 Seamus	19 Thomas	29 Riley	39 Steven	50 Jasper
10 Thomas	20 Brent	30 Thomas	40 Alexander	51 Caleb
			41 Dominic	52 Jack

6 a 36 students

b i Day girls = $\frac{55}{180} \times 36 = 11$

ii Boarder girls = $\frac{23}{180} \times 36 = 4.6 \approx 5$

iii Day boys = $\frac{65}{180} \times 36 = 13$

iv Boarder boys = $\frac{37}{180} \times 36 = 7.4 \approx 7$

7 a

State	'000s	Senate seats	
NSW	7861.10	24.287 41	24
Vic.	6323.60	19.5372	20
Qld	4928.50	15.226 94	15
SA	1723.50	5.324 872	5
WA	2580.40	7.972 324	8
Tas.	520.90	1.609 357	2
NT	246.10	0.760 343	1
ACT	410.30	1.267 65	1
Australia	24 598.90	75.986 10	76

b NSW, Qld and Vic. would be entitled to more Senators if population were to be used to determine the composition of the Senate. SA, WA, Tas., NT and ACT all would have less representation in the Senate if population size were to determine the composition of the Senate.

Exercise 5F

1 a entire

b sample

c agree

d accurate

2 A and iv, B and i, C and iii, D and ii

3 B

4 A

5 a D

b To ensure that the conclusion is not biased and takes into account everyone in the school.

6 a C

b They wish to know if all, not just a few, employees would like free access to the gym.

7 a Everyone taking boat trips to see the Ningaloo Reef

b To ensure that the conclusion is accurate, so that the satisfaction results comes only from those people who actually went on the boat trip.

8 All of the grower's plants

9 All items leaving the factory

10 All the employees of the factory

11 All voters

Exercise 5G

- 1 **a** applied, bias
b interviewer, questionnaire, errors
c assume, coincidence, backwards, outside
- 2 **a** Only 46% of customers responded and they are likely to be customers who are happy with the company. Unhappy customers may not have replied, thus resulting in non-response bias.
b The question is biased in that neutral or negative responses are not possible.
c Less than half the customers responded to the survey, which allowed only positive feedback to be given. It could be that the customers who did not respond are not happy with the service.
- 3 **a** Self-selecting survey
b Only people who follow the online newsagency will have voted in the poll. The sample is not representative of the entire population.
- 4 **a** The sample is not representative of the entire student body. It is likely that only those students who like the canteen food were invited to participate.
b By ignoring the opinions of students who do not use the canteen, changes that might need to be made to the menu are not made.
- 5 **a** There is interviewer bias in the question. This can influence the participant's response.
b Lachlan should remove any reference to which team is his favourite.
- 6 **a** This is a processing error.
b The average number of hours completed will be increased by the incorrect entry.
- 7 **a** Jeff needs to be wary of small changes and whether they can be applied to a longer time period.

- b** The share price has generally fallen over the 3 month period. It is not a good investment at this time.
- 8 **a** You need to ignore coincidence.
b You could assume that eating more cheese increases your chances of dying by becoming tangled in your bed sheets!
- 9 **a** It is suggesting that working lowers your risk of mental illness. The cause and effect might be the other way around.
b A; People with mental illness are likely to have difficulty maintaining employment.
c Proportion of population by employment status at risk of mental health illness
- 10 **a** It is implying a cause and effect but it is likely to be an outside cause that is influencing both sets of data.
b Advancements in science and medicine have made organic foods more accessible and increased the awareness and diagnosis of autism.
- 11 **a** Outside causes have been ignored.
b Home life, intellectual stimulation (e.g. parent reading to the child) and bonding between mother and child are overlooked as possible causes of success.
c The report could add undue pressure and stress to a mother who is unable to breastfeed for various reasons.

Exercise 5H

- 1 **a** technical, 20
b meaning, clear
c number, fatigue
d privacy, shared
e bias, leading
- 2 **a** There are only two options; many people don't fit into the two options provided in the survey.

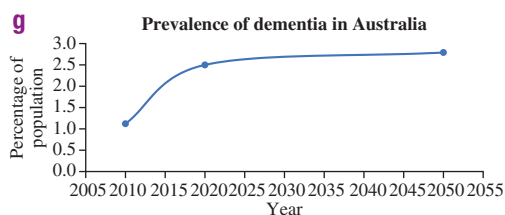
- b** There are too many options; can cause survey fatigue and people are more likely to remember only the two extreme options.
- c** Less than 1 hour, 1–2 hours, 3–4 hours, 5 or more hours
- 3 a** Yes; a question should have fewer than 20 words.
- b** It is a double-barrelled question. First question: Does it seem possible or impossible that the West Coast Eagles will win the AFL Western Derby. Second question: Do you feel they will always find a way to lose?
- c** It contains bias. It implies the West Coast Eagles will always find a way to lose the AFL Western Derby.
- d** ‘Do you think the West Coast Eagles has a chance of winning the AFL Western Derby?’
- 4 a** Standardised
- b** The middle two sentences offer opinion and can be deleted.
- c** ‘Students must take standardised NAPLAN tests in reading, writing, and mathematics. Do you think that your students take too many standardised tests?’
- 5 a** It is unclear what type of shopping is being done.
- b** Where do you like to do your grocery shopping?
- c** Which online shop do you like to make purchases?
- d** Where do you like to buy your runners?
- 6 a** Question does not set a time frame.
- b** Where were you born?
- c** Where did you spend the majority of your time this morning?
- 7 a** Bias **b** Bias
- c** Bias **d** Bias
- 8 a** ‘...avoid a catastrophic invasion...’ puts bias in the question. Rewrite as: ‘Do you think the government should spend more on the military?’
- b** ‘...good’ adds bias. Rewrite as: ‘What do you think of the new policy?’.
- c** ‘...new recipe better than the old recipe’ adds bias. Rewrite as: ‘What do you think of the new recipe?’
- d** There is no bias in the question but question could be rewritten to make it easier to understand, Rewrite as: ‘Do you think students should be allowed to use mobile phones at school?’
- e** Opening statement ‘...employees love their jobs’ adds bias. Rewrite as: ‘How do you feel about your job with the Government?’
- 9 a i** It is not clear that the survey is about sport at school.
- ii** On how many occasions do you participate in sport and physical activity at or with school in a week?
- iii** They are not accommodated
- iv** They are not accommodated
- v** Revised options:
- ☐ 0
- ☐ 1–2
- ☐ 3–4
- ☐ 5–6
- ☐ More often
- b** Samples could include:
- ☐ Netball
- ☐ Rugby
- ☐ Soccer
- ☐ Swimming
- ☐ Athletics
- ☐ Volleyball
- ☐ Touch
- c** Answers will vary:
- ☐ Before school
- ☐ After school
- ☐ Weekends
- ☐ Lunch breaks

- d i** The use of 'limited' implies that not enough sports options are offered.
- ii** Do you think the sports options available at school are reasonable?
- e i** The question is biased by offering an opinion before the question.
- ii** Do you think the school should offer more sports? If so, what sports should they offer?
- f i** No, there is no need to collect participants' date of birth or name.
- ii** The survey responses are not secure outside the Sports Staffroom and someone could access them who is not authorised to do so.

Exercise 5I

- 1 a** misrepresented, impression
- b** misleading
- c** data, mislead
- d** incorrect
- e** small, large
- f** percentages
- 2 a i** 0.6 million
- ii** ≤ 1.0 million
- iii** 1.1 million
- iv** The wrong inequality sign has been used. This reads as 'less than 1 million'.
- b i** 0.25 million
- ii** 1.12%
- c i** 0.6 million
- ii** 2.50%
- d i** 1.1 million
- ii** 3.06%
- e** People are living longer because of better treatment of other causes of death (e.g. cancer or heart disease etc.). As the population will grow between 2010 and 2050, so will the number of people with dementia naturally increase also.

- f** The horizontal axis does not have an even scale. The vertical axis should be a percentage of population. Having the line cross over a map of Australia might give the impression that almost the entire Australian population will have dementia by 2050.

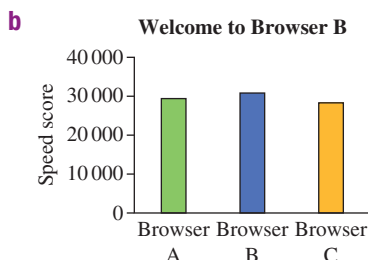


- 3 a i** 15.75 c
- ii** 47.13 c
- iii** 35.69 c
- b i** 2.99 : 1
- ii** 2.27 : 1
- c** The claim is valid; both ratios are between 2 and 3.
- d i** US 7 mm
- ii** SA 47 mm
- iii** Qld 32 mm
- e i** 6.71 : 1
- ii** 4.57 : 1
- f i** The ratios in part **e** are much higher.
- ii** The prices and the ratios stated are consistent with each other. The graph is not consistent with either.
- iii** To emphasise the high prices in Australia
- g i** Small samples are less reliable.
- ii** High/low pieces of data will affect the average.
- iii** Cities could be more expensive than areas closer to power stations etc.
- iv** Different forms of energy have different costs. There is no indication if this was considered.

4 a The Council has omitted data that did not suit its needs (e.g. the 2014–2016 and 2017 data).

b To give the impression of a booming economy in the region and to attract new investment.

5 a Gauge is bigger for Browser B. The meters do not start at zero.



c i 4.6%

ii 8.22%

d There is no indication of sample size. It might be a very small sample.

e There is no indication of how the results were calculated. An average result would be affected by high and/or low data, whereas a median not so much.

Chapter checklist

1 a A census is an official count of an *entire* population.

b All members of the population complete the census at the same time.

2 a Advantages: provides a true measure of the population (i.e. no sampling error); benchmark data may be obtained for future studies; and detailed information about small subgroups within the population is more likely to be available.

b Disadvantages: may be difficult to enumerate (i.e. count all units of the population within the available time); higher costs, both in staff and monetary terms, than for a sample; and generally takes longer to collect, process, and release data than from a sample.

3 a A survey is the process of gathering information from a small group (i.e. sample) from within a population.

b Advantages: costs would generally be lower than for a census; results may be available in less time; and if good sampling techniques are used, the results can be very representative of the actual population.

c Disadvantages: data may not be representative of the total population, particularly when the sample size is small; since data is collected from a subset of units and inferences are made about the whole population, the data is subject to ‘sampling’ error; and decreased number of units will reduce the detailed information available about subgroups within a population.

4 The purpose of sampling is to gather data about the population. Information from the sample is used to estimate the characteristics of the population, without the need to survey the entire population.

5 a Members are selected from a numbered list using random numbers until the desired sample size is achieved.

b Advantages: easy to conduct, high chance of achieving a good representative sample.

c Disadvantages: identifying all members of the population can be difficult, contacting members of the sample can be difficult.

6 a A self-selected sample is a sample comprising participants willing to answer the survey’s questions.

b Advantages: very easy to conduct and participants are willing to be involved.

c Disadvantage: can be very biased as people can vote multiple times.

- 7 a** A systematic sample is made by selecting subjects from a list at regular intervals.
- b** Advantages: very easy to conduct and the population is evenly sampled.
- c** Disadvantage: possibility that some other trait of the population coincides with the frequency of selection, creating a biased sample.
- d** $\frac{500}{20} = 25\text{th student}$
- 8 a** To achieve the same characteristics and proportions as those in the sample, the population is divided up into groups, called strata. The proportions of these groups are calculated, generally as a fraction or a percentage, and applied to the sample.
- b** An advantage is that it has the same characteristics as the population.
- c** A disadvantage is it can be more time consuming to determine the sample.
- 9 a** Target population refers to the entire group of individuals you are interested in including in your survey.
- b** Sample population is the group of individuals asked to participate in your survey.
- c** Study population is the final group of people who agree to be involved in your survey.
- 10 a** To use simple language means to not use jargon or technical words in the survey questions that respondents may not understand.
- b** Ambiguous questions are not clear in what they mean.
- c** Asking lots of questions or having lots of options to choose from when answering a question can lead to survey fatigue in the sample group.
- d** The information collected during a survey needs to be kept confidential/private and not shared with unauthorised people, as people may not want their information to be shared.
- e** Bias is when questions lead the respondent to the same answer.
- 11 a** People can vote more than once; only people with access to the internet can participate; people with strong views are more likely to participate and encourage others to do the same; self-selecting sample.
- b** Conduct a phone poll that covers all demographics in the community.
- 12 a** Only viewers with Facebook access can participate. Less likely to have older viewers participate.
- b** Phone poll; survey people that attend a live show.
- 13** Systematic error; all results will be wrong by the same amount.
- 14 a** Self-selecting survey; only people who wished to participate in the survey. Since poll is online, the participants are more likely to be heavy internet users who prefer on-demand streaming.
- b** The sample is biased and does not reflect the entire population. Broadcasters might put more resources into on-demand streaming and not meet the needs of free-to-air viewers.
- 15 a** The use of the word 'alarming' in the headline suggests a major problem with loneliness.
- b** Just because people say they have no close relationships does not mean they are lonely. The survey asked if they had close relationships, not if they were lonely.

Chapter review

1 A and v, B and vii, C and ii, D and vi, E and iv, F and viii, G and iii, H and ix, I and i

2 a Self-selecting

b Systematic

c Random

d Stratified

3 a Census **b** Survey **c** Census

d Census **e** Census **f** Survey

4 a 23 **b** 9 **c** 15

5 a

1 Abigail	11 Aria	21 Chloe	31 Ethan	41 Jacob	51 Lily	61 Noah	71 Scarlett
2 Addison	12 Aubrey	22 Christopher	32 Evelyn	42 James	52 Logan	62 Nora	72 Sebastian
3 Aiden	13 Audrey	23 Daniel	33 Gabriel	43 Jayden	53 Lucas	63 Oliver	73 Skylar
4 Alexa	14 Ava	24 David	34 Grace	44 John	54 Luke	64 Olivia	74 Sofia
5 Alexander	15 Avery	25 Dylan	35 Hannah	45 Joseph	55 Madison	65 Paisley	75 Sophia
6 Allison	16 Benjamin	26 Elijah	36 Harper	46 Joshua	56 Mason	66 Penelope	76 Victoria
7 Amelia	17 Brooklyn	27 Elizabeth	37 Henry	47 Layla	57 Matthew	67 Riley	77 William
8 Andrew	18 Camila	28 Ella	38 Isaac	48 Leah	58 Mia	68 Samantha	78 Wyatt
9 Anna	19 Carter	29 Emily	39 Isabella	49 Liam	59 Michael	69 Samuel	79 Zoe
10 Anthony	20 Charlotte	30 Emma	40 Jackson	50 Lillian	60 Natalie	70 Savannah	80 Zoey

b $\frac{35}{80} = 0.438$

c $\frac{9}{15} = 0.6$

d There is an over-representation of boys in the sample. There should be approximately $\frac{35}{80} \times 15 = 6.6$ boys in the sample.

6 a $\frac{80}{15} = 5.333$, so every fifth child on the list

b

1 Abigail	11 Aria	21 Chloe	31 Ethan	41 Jacob	51 Lily	61 Noah	71 Scarlett
2 Addison	12 Aubrey	22 Christopher	32 Evelyn	42 James	52 Logan	62 Nora	72 Sebastian
3 Aiden	13 Audrey	23 Daniel	33 Gabriel	43 Jayden	53 Lucas	63 Oliver	73 Skylar
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5 Alexander	15 Avery	25 Dylan	35 Hannah	45 Joseph	55 Madison	65 Paisley	75 Sophia
6 Allison	16 Benjamin	26 Elijah	36 Harper	46 Joshua	56 Mason	66 Penelope	76 Victoria
7 Amelia	17 Brooklyn	27 Elizabeth	37 Henry	47 Layla	57 Matthew	67 Riley	77 William
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c $\frac{9}{15} = 0.6$

d There is an over-representation of boys in the sample. There should be approximately $\frac{35}{80} \times 15 = 6.6$ boys in the sample.

7 a $\frac{35}{80} = 0.438$

b $\frac{35}{80} \times 15 = 6.5625$, which rounds up to seven boys

c Use the first seven random numbers without repeating.

1 Aiden	11 Elijah	21 Joseph	31 Oliver	6 Anna	16 Elizabeth	26 Leah	36 Riley
2 Alexander	12 Ethan	22 Joshua	32 Samuel	7 Aria	17 Ella	27 Lillian	37 Samantha
3 Andrew	13 Gabriel	23 Liam	33 Sebastian	8 Aubrey	18 Emily	28 Lily	38 Savannah
4 Anthony	14 Henry	24 Logan	34 William	9 Audrey	19 Emma	29 Madison	39 Scarlett
5 Benjamin	15 Isaac	25 Lucas	35 Wyatt	10 Ava	20 Evelyn	30 Mia	40 Skylar
6 Carter	16 Jackson	26 Luke	1 Abigail	11 Avery	21 Grace	31 Natalie	41 Sofia
7 Christopher	17 Jacob	27 Mason	2 Addison	12 Brooklyn	22 Hannah	32 Nora	42 Sophia
8 Daniel	18 James	28 Matthew	3 Alexa	13 Camila	23 Harper	33 Olivia	43 Victoria
9 David	19 Jayden	29 Michael	4 Allison	14 Charlotte	24 Isabella	34 Paisley	44 Zoe
10 Dylan	20 John	30 Noah	5 Amelia	15 Chloe	25 Layla	35 Penelope	45 Zoey

d $\frac{45}{80} = 0.563$

e $\frac{45}{80} \times 15 = 8.437$, which rounds down to eight girls

f As identified in part **c**

8 The stratified sample is the only sample that has the same proportions of boys and girls attending the day-care centre.

9 All the families who attend the day-care centre.

10 a i Too long

ii Sensory, cognitive

iii There is no scale for how happy the parents are. Some parents may be happy with 1 h of outside play and others may like 5 h.

iv On a scale of 1–5, how happy are you with the amount of outside play your child participates in while at the day-care centre?

Not happy				Very happy
1	2	3	4	5

- b i** There are too many choices.
- ii** Laser skirmish and contact sports like touch football are not appropriate.
- iii** Revised list:
- ☐ Bikes
 - ☐ Painting
 - ☐ Sandpit
 - ☐ Monkey bars
 - ☐ Swings
 - ☐ Obstacle course
 - ☐ Other
- c i** Not relevant or reasonable
- ii** Not relevant or reasonable
- iii** Information relating to survey questions 6 & 7 are a private matter for the families and have nothing to do with the day-care centre or the purpose of the survey.
- iv** Not ethical
- v** Not relevant and not needed in the survey
- d** If no one is at the reception, the response sheets could be easily accessed by someone who is not authorised to see the information.
- 11 a** The sample (i.e. work colleagues) is not representative of the target population (i.e. younger people).
- b** Survey his son's friends and/or friend's parents to find out what platforms they use.
- 12 a** Questioning error
- b** Survey relies on the respondent remembering how many times they have visited that store over a long period of time.
- c** Reduce the time frame. Ask if this is the main hardware store they visit.
- 13 a** No; this is a biased question that leads to a particular response.
- b** No; this is a biased question that leads to a particular response.
- c** This is a better question but using 'assistance' implies the unemployed are disadvantaged and require sympathy, so is somewhat leading.
- d** This is a good question having no bias and is not a leading question.
- 14** Mrs Tall will not be as accurate as she would have been had she measured in centimetres. Students' shoes will have varying sole thickness and this will affect the measurements taken.
- 15** The Mathematics teachers are not representative of the target population. Staff members from all areas should have been included in the survey.
- 16** Where do you think we should hold the staff Christmas party this year?
- 17 a** 57 mm
- b** 6 cm
- c** 3 mm
- d** 5.3%
- 18 a** 11.2 kg
- b** 11 200 g
- c** 100 g
- d** 11 150 g
- e** 11 249 g
- f** ± 50 g
- g** 0.29%
- 19** The timing of the poll would influence the way people vote, so there is procedural error where there is influence. Respondents are self-selecting and limited to those people who have the app on their phone, so the sample is biased.
- 20 a** Only parents from the one volleyball team were sampled. The survey should have included parents from other sports and teams to be more representative of the target population; that is, parents of children at the school playing sport.

- b** Questions should ask what you want to know. The parents were not asked if they felt the teams needed more training.

- c** The interpretation had nothing to do with the question asked.

- 21** A; possible cause and effect relationship.

Students could be able to work more productively with laptops.

B; unlikely to have an impact on academic performance. There is no obvious relationship between the two observations.

C; possible cause and effect relationship. Students could be able to understand more with extra help.

D; possible cause and effect relationship. Students could be able concentrate more without the distraction.

E and B; unlikely to have an impact on academic performance. There is no obvious relationship between the two observations.

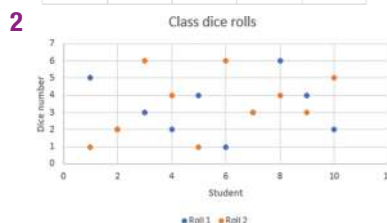
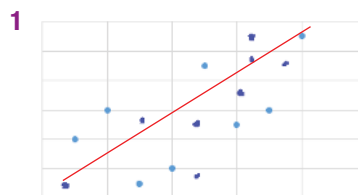
- 22** As men's participation has declined, women's has increased. This could be because more women are accessing higher education, better access to child care and improved flexibility in work arrangements.

- 23 a** 2014 12.5 years, 2016 13.1. This is an increase of 0.6 years. This is not a large change. The average can be influenced by a few pieces of data that are very high. If one or two cars were aged over 20 years, this could increase the average age of the cars involved in fatal accidents.

- b** Older cars are usually driven by younger drivers, who are inexperienced, or older drivers. Both groups are more likely to be involved in accidents regardless of the age of their vehicle.

Chapter 6

Pre-test

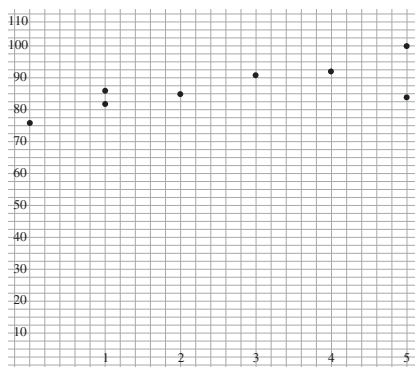


- 3 a** 15 **b** 3

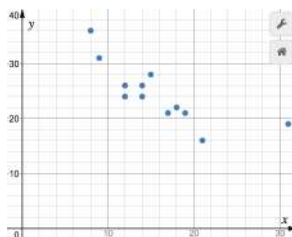
Exercise 6A

- 1 a** positive, left, right
b scatterplot, straight
c pattern
d weak
- 2 a** Yes **b** No **c** Yes
d No **e** Yes **f** Yes
- 3 a** Positive, linear, strong
b Negative, linear, weak
c Positive, linear, strong
d Positive, linear, moderate
e Positive, non-linear, strong
f None, non-linear, none
- 4 a** Positive
b Linear

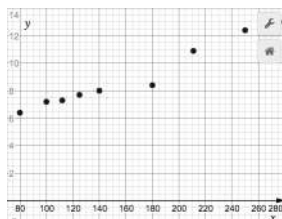
c Moderate



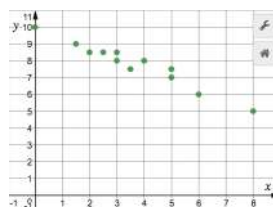
- 5** Positive, linear and moderate correlation exists, which indicates that students achieve higher if they get more sleep.
- 6** Negative, linear and weak correlation exists, which indicates that age definitely does affect the eyesight of drivers but at varying rates of decline.
- 7** There is a negative, linear, strong correlation between the number of police and the incidence of crime with only one outlier. The police union are correct with their assertions.



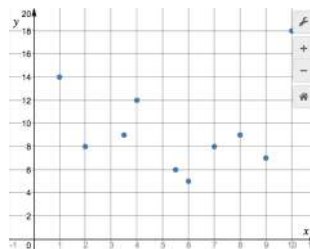
- 8** There is a positive, linear, strong correlation between the amount of fertiliser and the crop yield. The CSIRO can demonstrate the positive effect of fertiliser on crop yields.



- 9** There is a negative, linear, strong correlation that the more hours of video games being played on average results in less average sleep.



- 10** There is a negative, linear, weak correlation with an outlier. This would indicate that, although practice does generally improve the points scored by the players, some players improve at a greater rate than others.



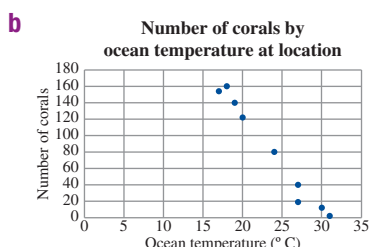
Exercise 6B

- 1**
 - a** change
 - b** horizontal (x-axis), vertical (y-axis)
 - c** dependent, variable
 - d** independent, variable
 - e** change, controlled, response
- 2** Answers may vary
 - a** A variable is something that can be measured. It can be basically anything, such as things, time, feelings, events, or age.
 - b** Independent variable can be changed by the researcher.
 - c** Dependent variable changes because of the independent variable.

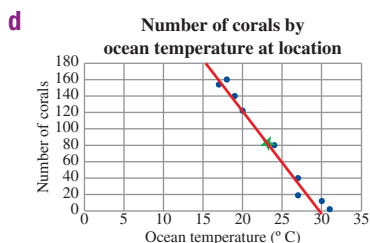
- 3 a** Independent variable: amount of medication, because it is the thing that Alix changes to see what effect it has.
- b** Dependent variable: breathing rate, because it is the thing that Alix measures to see if it is changed by the amount of medication.
- 4 a** Independent variable: amount of fertiliser. It is what Michael is changing.
- b** Dependent variable: amount (mass) of beans produced per plant. It is what we measure in the expectation that it will be changed by the independent variable.
- 5 a** Independent variable: ocean temperature. Although it has to be measured, Sonia 'changes' it by selecting different locations which have different temperatures.
- b** Dependent variable: amount of coral. It is what is being measured.
- 6 a** Independent variable: amount of time listening to classical music. It is what Noah sets.
- b** Dependent variable: reading test results each student received. It is what is being measured in the expectation that it may be changed by the time spent listening to classical music.
- 7 a** Independent variable: how long each student studies. It is what each student decides. It is also on the x -axis.
- b** Dependent variable: score each student received on their exam. It is what is being measured because we think it is affected by hours of study. It is also on the y -axis.
- 8 a** Independent variable: how many pamphlets were delivered. It is the cause. 'Number of pamphlets delivered'
- b** Dependent variable is the amount of pay received. It is what is being affected, the effect. 'Pay (\$)'

Exercise 6C

- 1 a** relationship
- b** straight, scatter
- c** trend, regression
- d** represent, trend
- e** independent, dependent
- f** relationship, variables, trend
- 2 a** To draw a scatterplot:
- Always use grid paper.
 - Determine which observations are dependent and which are independent.
 - Place the dependent variable on the y -axis.
 - Place the independent variable on the x -axis.
 - Use each paired value to plot the graph.
- b** To find the line of best fit by eye:
- Find the mean of the independent variables on the x -axis.
 - Find the mean of the dependent variable on the y -axis.
 - Plot the mean, using a different colour pen.
 - Put your ruler on its edge so you can see both sides of the plot easily.
 - Position the ruler on the mean to have approximately the same numbers of plots either side of it, keeping the line going through the mean.
 - Use your ruler to draw the line of best fit.
- 3 a** Independent variable: the ocean temperature. It is the temperature at the locations Felicity has chosen.
- Dependent variable: the amount of coral. It is what is being measured to see if it is affected by the ocean temperature.



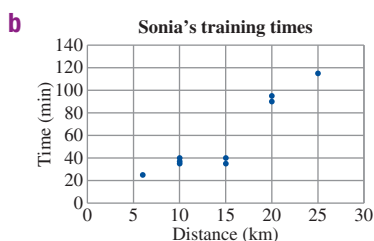
c 23.67, 81, shown on the graph below (★)



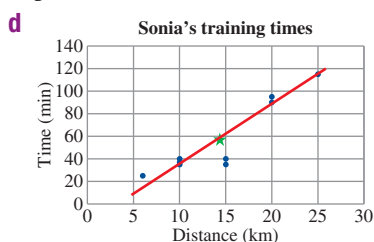
4 a Independent variable is the distance

Sonia ran.

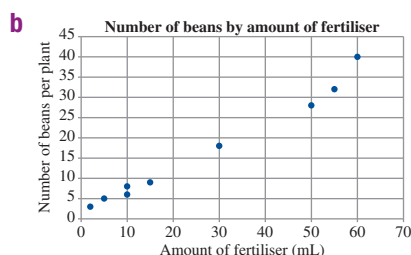
Dependent variable is the time each run took. It is what is being affected by the distance.



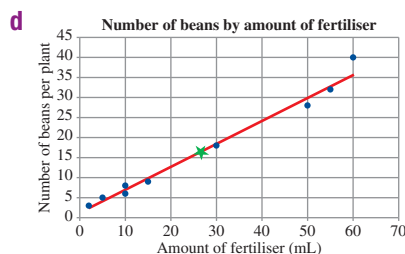
c 56.9, 14.6 shown on the graph in part d(★)



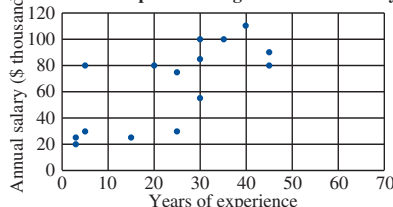
5 a Independent variable: the amount of fertiliser. It is what David chooses.
Dependent variable: amount of beans produced by each plant. It is what is being affected.



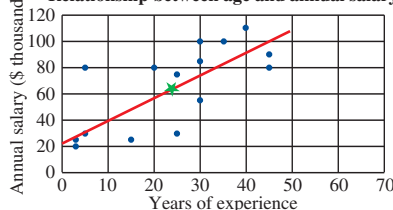
c 26.33, 16.56 (★)



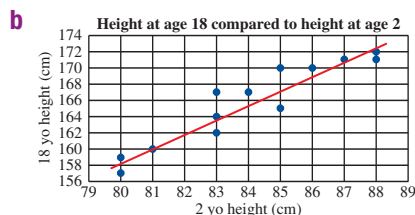
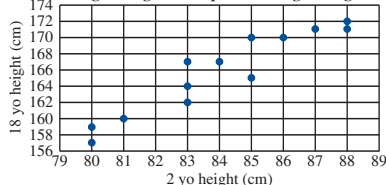
6 a Relationship between age and annual salary

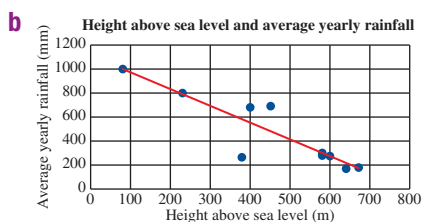
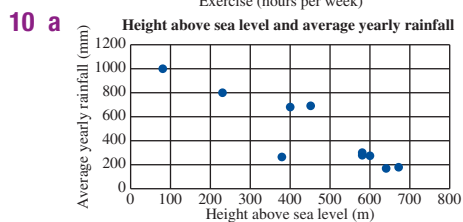
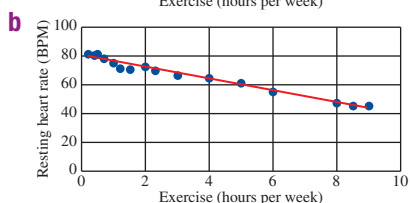
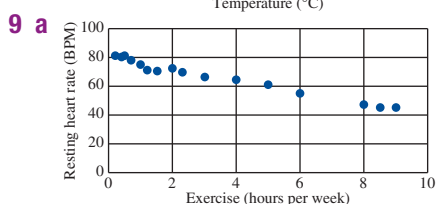
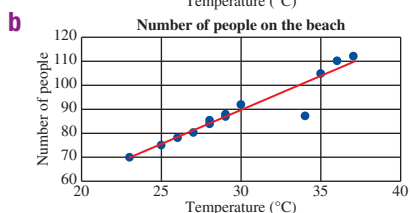
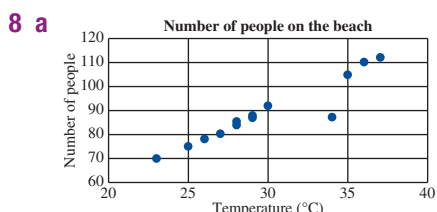


b Relationship between age and annual salary



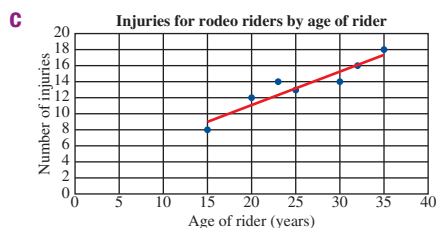
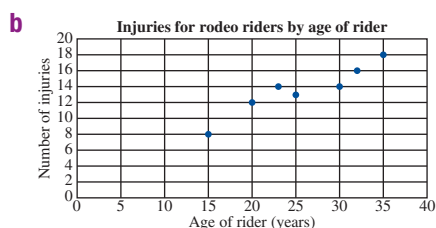
7 a Height at age 18 compared to height at age 2





Exercise 6D

- 1 a** Regression, relationships
- b** strength
- c** positively, negatively, together
- d** increases, decreases
- 2 a** Dependent variable: number of injuries;
independent variable: age of the riders.



- d** The correlation is positive, linear and strong.
- e** There is a strong relationship between the age of the rodeo riders and the number of injuries. The correlation is positive, as both variables are increasing with each other. As the age of the rider increases, the number of their injuries also increases.
- f** Answers may vary: As the rodeo rider ages they need to be more aware of the risks involved, to avoid injuries.
- 3 a** Dependent variable: amount of sleep;
independent variable: age of the person.
- b** Overall, correlation is negative, linear and moderate. However, the correlation looks to be different for children compared to adults. A line of best fit for ages 0 to 10 is negative, linear and strong. For older children and teenagers, the relationship may be non-linear.
- c** Over a lifetime, there is a moderate relationship between the amount of sleep someone requires and their age. The correlation is negative, as when one variable increases the other decreases. As the age of the person increases, the amount of sleep they have typically decreases. The correlation is stronger for young children and may be non-linear for older children and teenagers.

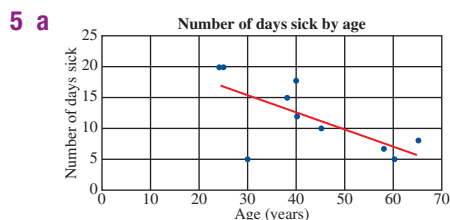
- d** Answers may vary: The data suggests that children sleep more than adults. People may need to adjust to having less sleep as they get older.
- 4 a** Independent variable: amount of time playing video games; dependent variable: their grades.
- b** The correlation is negative, linear and moderate.
- c** There is a moderate relationship between the amount of time spent playing video games and their grades. The correlation is negative, as the time spent playing increases, the grade score typically decreases.
- d** Answers may vary: If you want to receive higher grades, you need to lessen the amount of time playing video games.
- 5 a** Dependent variable: weight (kg); independent variable: amount of exercise done each day (min)
- b** The correlation is negative, linear and strong.
- c** There is a strong relationship between the amount of time spent exercising and people's weight. The correlation is negative, as when one variable increases the other decreases. It is strong, as most of the points are directly on the line of best fit. Individuals who spend more time exercising each day typically weigh less.
- d** Answers may vary: If you want to reduce your weight, you need to increase the amount of time you spend exercising.
- 6 a** The correlation is positive, linear and weak. There is a very weak relationship between the overall marks and the maths marks. The correlation is positive, i.e. an increase in the overall mark correlates with an increase in the maths mark. It is weak as most of the points are scattered from the line of best fit.
- b** There is a slight relationship between the two variables; however, the teacher would need to obtain more results each year to be able to determine if there is an actual relationship between his students' maths results and overall results.
- 7 a** There is no relationship between their BMI and hand grip. The BMI can vary and the hand grip can vary, but there is no correlation between them.
- b** According to this data there is no relationship between the two variables. BMI does not seem to change due to a person's strength in their hand grip.
- 8 a** The correlation is positive, linear and strong. There is a strong relationship between the number of weeks and the income. The relationship is positive, as the data shows when the number of weeks that have passed increases the income tends to also increase. The relationship is strong as the data is very close to the line of best fit.
- b** Will's business is doing well, and he should expect to increase his income as each week increases.
- 9 a** Looking at the data there does seem to be a strong relationship between exporting local apples and the fatality rate on the highway. The points show a strong, linear negative correlation, as the points are very close to the line of best fit. This also shows that as the exporting of local apples increases, the fatality rate on the main highway typically decreases.

- b** Answers may vary: Although there appears to be a relationship there is no common-sense explanation for it and we can't say that exporting more apples causes the road fatalities to reduce. The most likely explanation is that it is a coincidence that apple exports have gone up while fatalities have decreased. Another possibility is that improvements to the highway out of the area has made the road safer and also made the cost of road transport cheaper so increasing the apples exported.

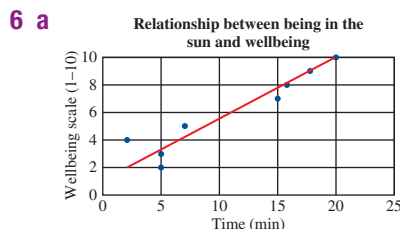
Exercise 6E

- 1 a** strength, measured
b strength, coefficient
c (r), strength, direction
d -1 , $+1$
i perfect positive relationship
ii $0.75 - 1$
iii $0.50 - 0.75$
iv weak positive relationship
v 0
vi $-0.25 - -0.50$
vii moderate negative relationship
viii $-0.75 - -1$
ix -1
- 2 a** Correlation is a measure of the strength of the linear relationship between two variables.
b The correlation coefficient (r) measures the strength and direction of a linear relationship between variables.
- 3 a** Correlation coefficient is -0.997 .
b The correlation coefficient is very close to being -1 ; therefore, the relationship is a very strong negative relationship. This means that as the study time increases the errors decrease.

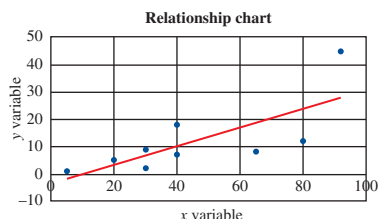
- 4 a** Correlation coefficient is 0.05 .
b The correlation coefficient is very close to being 0 ; therefore, there is no relationship. This means that according to this data there is no relationship between the men's heights and their weights.

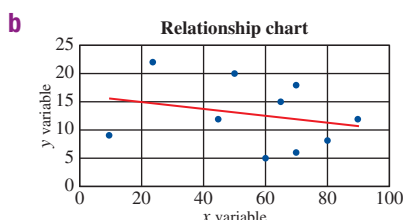


- b** Correlation coefficient is -0.68 .
c The correlation coefficient is between -0.5 and -0.75 . This means that the relationship between the age of the patient and their sick days is a moderate negative correlation. As age increases the number of sick days decrease. Suggesting the condition becomes less severe with age.

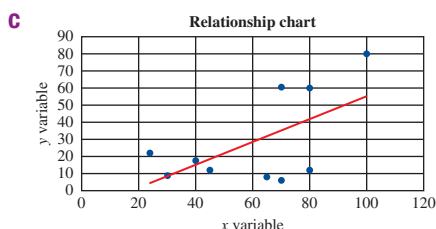


- b** Correlation coefficient is 0.96 .
c The correlation coefficient is nearly $+1$, which means that there is a very strong correlation between the amount of time in the sun and a sense of wellbeing. As the time in the sun increases, the sense of wellbeing should also increase.
- 7 a** Answers may vary





Answers may vary



Answers may vary

Exercise 6F

- 1 a** predictions, in, out
b $y = mx + c$
c gradient, slope
d y-intercept, y-axis
e Interpolation
f Extrapolation
- 2 a** Interpolation is predicting an outcome based on estimating a value on the line of best fit.
b Extrapolation is predicting an outcome based on extending the line of best fit, not within the range of the particular dataset.
- 3** A: $r = -0.94$, is the most reliable as it is the closest to either 1 or -1 .
- 4** A: weak linear negative is the least reliable as it has the weakest relationship between its variables.
- 5 a** Mindy's weight would have been about 75 kg when she had exercised for 50 days.
b According to the line of best fit Mindy had been exercising for 70 days when her weight reached 70 kg.
c The equation of the line of best fit is $y = -0.4x + 94$
d 46 kg

e The prediction in **d** is unreliable as we are extrapolating far beyond the domain of the given data. This example illustrates the dangers of extrapolating beyond the data: the line of best fit predicts Mindy would eventually reach the point of weighing nothing, which is impossible.

- 6 a** The average amount of sleep for someone who is 25 would be about 9 hours.
b 52 years old
c $y = -0.14x + 13.6$
d 2.4 hours
e The prediction is unreliable as we are extrapolating far beyond the domain of the given data. This is reflected in the result we achieve, common sense tells us that elderly people still sleep on average more than 3 hours a night.
- 7 a** 0.7 kg per week
b 19 chocolate bars per week
c $y = 0.05x + 0.08$
d 98.4 chocolates per day
e This prediction is not reliable since we are extrapolating far beyond the range of the given data. It is also unrealistic, as eating that number of bars per week would be difficult and likely to cause illness that would slow or stop consumption. In addition, weight gain has a lot more variables to consider than eating chocolates.
- 8 a** Approximately 12 errors
b About 55 minutes of study
c $y = -0.4x + 26$
d -10 hours
e This prediction is not just unreliable but impossible. It is not possible to study a negative number of hours, which demonstrates the dangers of extrapolation.

- 9** Extrapolation in business is fraught with danger. The correlation coefficient of 0.98 is very strong, but one must not let it lead to assuming that it is possible to increase sales beyond 80 000 per week. There are many possible barriers to this: there may simply not be enough customers or demand to sell more than this, or the business may not have the capacity to go beyond this, or the cost of selling and distributing greater numbers may be prohibitive. Extrapolation is only valid in situations that are realistically attainable.

Exercise 6G

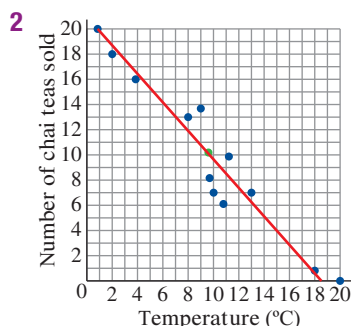
- 1 a** relationship, causes **b** impact
c strength, variables **d** imply
- 2 a** Correlation is a measure of the strength of the relationship between two variables.
b Causation states that any change in one variable will cause a change in another variable.
- 3 a** There is a strong positive correlation between the energy and fat. As the amount of fat increases, the amount of energy increases.
b There is causality, as fat in the diet can be used by the body for energy. The amount of fat affects the amount of energy.
- 4 a** There is a strong negative correlation between reading books and weight loss. As the time spent reading books increases, the weight decreases.
b There is no causality, as logically a change in book-reading habits should not affect weight. There may be other factors at play, for example, she may also be eating less or exercising more on days she reads more.
- 5 a** There is no clear trend at all in the scatter plot, and the points are scattered far from the line of best fit. Indicating that there is no relationship between people's BMI and the strength of their hand grip.

- b** There is no causality. The strength of a person's hand grip does not cause a change in their BMI. There are many other variables that determine the change in people's BMI (weight, exercise, and eating habits).

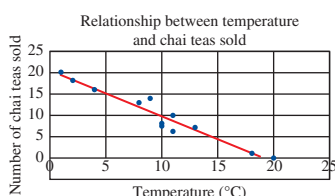
- 6 a** The data does show a strong positive correlation ($r = 0.9971$), as the correlation coefficient is nearly one.
b There is no obvious causality between the variables. For causality there would need to be something prevalent in organic food known to cause autism. This graph does not show a cause or effect between organic food sales and autism, and if different scales were used for the graph the curves would not match so well.
- 7 a** The data does show a positive correlation, as lines of best fit for both datasets would be closely matched.
b There is no causality between the variables, as there is no mechanism known for consuming mozzarella cheese leading to someone studying engineering.

Chapter checklist

- 1** The independent variable is the temperature, as it is independent of the number of chai teas sold. The dependent variable is the number of chai teas sold, as it is dependent on the temperature; the colder the day, the more people feel like having chai.



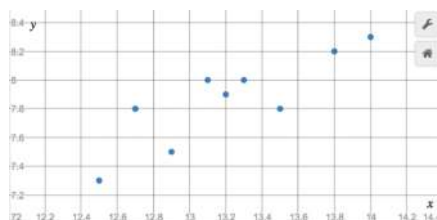
3



- 4 The correlation is strong negative. As the temperature decreases, the number of chai teas sold increases.
- 5 The correlation coefficient is -0.962647417 .
- 6 Answers may vary.
When the temperature is 5°C , 15 cups of chai tea will be sold.
When the temperature is 30°C , -12 cups of chai tea will be sold.
- 7 It is impossible to sell a negative number of cups of chai; therefore, the prediction by extrapolation in the previous question is not valid. The danger of extrapolation is extending it to impossible situations.
- 8 With this data there is a strong correlation of -0.96 , which being very close to 1, shows a very strong negative relationship. This data has some causality, as some people may be influenced by the temperature to cause them to choose to have chai tea. However, there are many other influences, for example tea/coffee preference, taste preferences, socioeconomic influences.
- 9 Positive, linear, strong correlation

Chapter review

- 1 There is a positive, linear, moderate correlation with an outlier, which would indicate that it is correct to believe that the warmer the temperature, the more visitors come to the beach.
- 2 The graph indicates that there is a positive, linear, weak correlation between the temperature and the area of ice at Antarctica.



This would say that temperature has a definite effect on the area of ice but there are probably other factors affecting it as well.

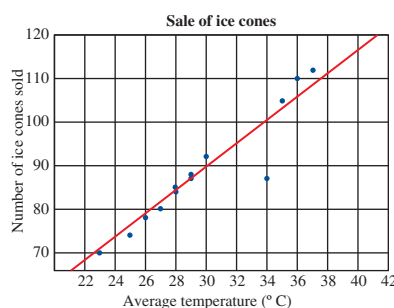
- 3 a The dependent variable is the growth of his crops, as it is affected by high summer temperatures, not the other way around.
- b The independent variable is the temperature, as it is not affected by what Jon does, but happens naturally.

4

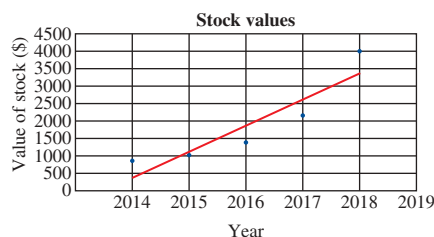


- 5 Dependent variable is growth of snowpeas; independent variables include sunlight, water, fertiliser, seed type, temperature

6



7



8 a There is a moderate relationship between energy output and fat.
The correlation is positive, as fat (g) in the diet increases, the energy output (kilojoules) increases.

b There is a weak relationship between energy output and salt in the diet.
The correlation is negative, as when one variable increases the other decreases.
As salt (mg) in the diet decreases, the energy output (kilojoules) increases.

c There is no relationship in this graph between energy output and fibre in the diet, as the line of best fit is nearly horizontal.

9 The correlation coefficient for the data is 0.939290.

10 a Rachel was able to row 40 times in week 4.

b 160 rows

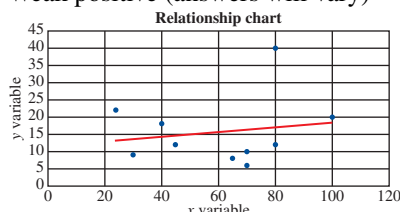
c 200 rows

d There is a risk, as Rachel's muscles may not be able to cope with the continuous increase. She may not be physically able to increase every week, showing the dangers of extrapolation. There is a limit to the maximum number of rows she can achieve, i.e. the graph can't keep going upwards.

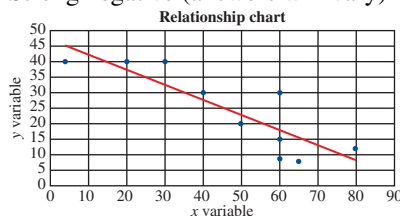
11 a There is a strong negative correlation between study time and number of errors.
As the amount of study time increases, the number of errors decreases.

b There is causality, as there is a reasonable mechanism to explain how more study time leads to fewer errors. The amount of study affects the number of errors.

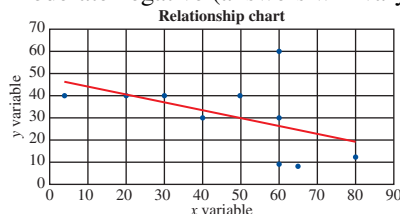
12 a Weak positive (answers will vary)



b Strong negative (answers will vary)



c Moderate negative (answers will vary)



13 Answers will vary.

Interpolation: After approximately 62 000 sales, the profit will be \$8000.

Extrapolation: After approximately 100 000 sales, the profit will be \$12 335.

The dangers of extrapolation in this graph is that sales and profit are very unpredictable and cannot be guaranteed to hold their trend. It may not be physically possible to sell more than 80 000.

14 a Lines of best fit for these data points would show a weak positive relationship between the consumption of sour cream and motorcycle accidents.

b There is no causality between the consumption of sour cream and motorcycle accidents, as there is no mechanism to explain how one variable could change the other.

Chapter 7

Pre-test

- 1 a 0.02–0.18 b 0
c 0.5 d 0.85–0.95
2 a 0.45 b 0.44 c 0.60
3 a 0.17 b 0.31 c 0.05
4 a $\frac{2}{3}$ b $\frac{3}{4}$ c $\frac{6}{7}$
5 a 19.75% b 13.89% c 22.22%
6 a 48.75 b 42.86 c 22.17
7 a 4.5 b 82.8 c 1.8

- 8 a {1,2,3,4,5,6}
b i $\frac{1}{6}$ ii $\frac{3}{6} = \frac{1}{2}$ iii $\frac{4}{6} = \frac{2}{3}$
iv $\frac{4}{6} = \frac{2}{3}$

9 a

Outcome	Tally	Frequency
Odd/Boy		8
Even/Girl		4
Total	12	12

- b Relative frequency of boys $\frac{8}{12}$
c Expected boys = $12 \div 2 = 6$
d The simulation resulted in more boys than would be expected. You would expect half the babies to be boys, which would be 6 in this size sample.

Exercise 7A

- 1 a Uncertain b Possible c 0.3
d 60% e $\frac{1}{10}$ f 3 in 5
2 a A b D c C
d A
3 See your Teacher

- 4 See your Teacher

- 5 a i $\frac{1}{25}$ ii 0.04 iii 4%
b No

- c See your Teacher

- 6 a Perth – Higher probability

- b $0.65, \frac{13}{20}$

- c 1–there are only three possible outcomes, so all probabilities sum to 1

Exercise 7B

- 1 a simulation, coin, die, spinner
b technology, random, calculator, Excel, between, decimal
2 a Answers will vary but should be around 5/10/5.
b Answers will vary but should be around 50%.
c The three rows of the table may have been deceptive. You could think that the three outcomes are equally likely, but BG is the same as GB so is more common than either of the single sex combinations.
3 a Answers will vary but should be around 3/7/7/3.
b Answers will vary but should be around 75%.
c The four rows of the table may have been deceptive. You could think that the four outcomes are equally likely but as an example, BBG is the same as GBB and BGB so is more common than either of the single sex combinations.

4 a

Toy number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Frequency	1	3	2	4	4	3	6	2	8	2	4	2	4	3	5	1	3	5	5	2

b 69 simulations

c $69 \times \$30 = \$2070$

d Toy number 9 was collected 8 times. Toy number 1 was the last one to complete the collection.

e Average = $\frac{69}{20} = 3.45$

5 a

Tie number	1	2	3	4	5	6	7
Frequency	4	1	3	1	5	0	6

b Tie number 7 is worn 6 times in 4 weeks. Tie number 6 is not worn at all.

c

Week	Monday	Tuesday	Wednesday	Thursday	Friday
One	5	7	1	1	4
Two	1	3	7	3	7
Three	1	5	7	5	7
Four	7	5	5	3	2

6 times the same tie is worn more than once in the same week.

d

Week	Monday	Tuesday	Wednesday	Thursday	Friday
One	5	7	1	1	4
Two	1	3	7	3	7
Three	1	5	7	5	7
Four	7	5	5	3	2

On two occasions, the simulation shows Ian wears the same tie on two consecutive days. Tie number one in week 1 and Tie number 5 in week 4.

e Expect each tie to be worn $\frac{20}{7} \approx 3$ times each. The results for ties 2, 4, 5, 6 and 7 are different to what you would expect.

6 a i Not suitable as the chance of selecting a particular card is lowered once it has been removed from the pack.

ii Suitable method. The chance of selecting a card remains the same for each trial.

iii Not suitable. The lowest sum is $1 + 1 = 2$ so impossible to get a January outcome in the simulation.

iv Suitable. All outcomes are equally likely.

b

Card drawn/month	A	2	3	4	5	6	7	8	9	10	J	Q
Tally												
Frequency	1		1		2	1		1	1	1	1	1

c It took 10 trials to repeat a result.

7 a

Card drawn/month	A	2	3	4	5	6	7	8	9	10	J	Q
Tally												
Frequency	5	3	5	2	5	2	4	6	1	3	2	2

b The months were not equally represented. Month 8, August, appeared 6 times in the simulations while month 9, September, only appeared once.

8 a Results will vary but will be at most 13 trials.

b Results will vary.

9 a Results will vary.

b Results will vary but will be around 2.5–3 children.

10 a $\% = \frac{283}{315} \times 100 = 89.8\% \approx 90\%$

b Results will vary but should be around 27 successes and 3 misses.

c Results will vary but should be approximately 90%.

d Expected goals = $\frac{90}{100} \times 30 = 27$

e Results will vary but should be similar if simulation is successful.

Exercise 7C

1 a previous, chance

b equal, different, same

c unusual, trials

d reliability

2 a 2

b Approximately $20 \div 2 = 10$

c

H	T	H	T	H	H	T	H	T	T
T	T	H	H	T	T	T	T	H	H

9 times. This is very close to what was expected.

d $20 \div 2 = 10$

e Heads = 9, Tails = 11. This is close to what was expected.

3 a b Approximately $30 \div 6 = 5$ times

c

4	3	4	5	2	1	5	6	5	6
1	3	1	2	3	5	5	1	4	4
2	4	1	1	4	3	4	4	4	4

6 times. This is close to what we expected

d $30 \div 6 = 5$ times

e There are more 4s and less 2s and 6s than expected.

Outcome	1	2	3	4	5	6
Frequency	6	3	4	10	5	2

4 a $20 \div 6 = 3.33$ So around 3 times each

b i More 4s and 5s than expected, less 1s

ii More 1s and 2s than expected, no 5s

c

	Dice Outcome Frequency						
	1	2	3	4	5	6	Total
Total	37	32	30	33	36	32	200

d $200 \div 6 = 33.33$ So around 33 times each

e There are more 1s and 5s than expected and less 3s.

5 a No. Each number has the same chance of being drawn each time.

b i 1 ii 3

c More people are choosing the 'Hot Numbers' so when they come up, they are not as profitable.

d i 0 ii 3

iii 0 iv 1

e Yes, the Cold Numbers would have won a division 6 prize.

6 Technology activity

As the number of trials increases, the two outcomes are evenly distributed.

- 7 a** As the number of trials increases, the six outcomes are evenly distributed.
b As the number of trials increases, the distribution becomes symmetrical with outcomes around 7 having the highest frequency.

Exercise 7D

- 1 a** chance
b favourable, outcomes
c ranges, not, will
d estimate, survey
e frequency, observed
- 2 a** 0.56 **b** 0.47 **c** 0.62 **d** 0.02
- 3 a** 159 **b** 539 **c** 216 **d** 84
- 4 a** 26
b $\frac{10}{26} \approx 0.38$
c $\frac{4+2+0+1}{26} = \frac{7}{26} \approx 0.27$
d $\frac{10+4+2+0+1}{26} = \frac{17}{26} \approx 0.65$
e $\frac{3+6}{26} = \frac{9}{26} \approx 0.35$
- 5 a** $\text{RF}(1) = \frac{2}{5} = 0.40$, $\text{RF}(2) = \frac{1}{4} = 0.25$,
 $\text{RF}(3) = \frac{2}{4} = 0.50$, $\text{RF}(4) = \frac{1}{4} = 0.25$,
 $\text{RF}(5) = \frac{1}{6} = 0.17$, $\text{RF}(6) = \frac{1}{3} = 0.33$,
 $\text{RF}(7) = \frac{1}{3} = 0.33$
b $\text{RF}(1) = \frac{3}{5} = 0.60$, $\text{RF}(2) = \frac{1}{4} = 0.25$,
 $\text{RF}(3) = \frac{3}{4} = 0.75$, $\text{RF}(4) = \frac{2}{4} = 0.50$,
 $\text{RF}(5) = \frac{3}{6} = 0.50$, $\text{RF}(6) = \frac{2}{3} = 0.67$,
 $\text{RF}(7) = \frac{1}{3} = 0.33$

c i Horse 3 has the best chance of winning based on past performances.

ii Horse 3 has the best chance of placing based on past performances.

6 a 5361

b $\frac{137}{5361} = 0.03$

c Numbers 1–31 are favourable,
 $\frac{4147}{5361} = 0.77$

d All numbers are equally likely. Past performance does not influence future outcomes.

7 a 192 582

b 0.16

c 0.23

d 0.04

8 a $105 \div 5 = 21$ pieces of candy per 20 grams

b $20 \text{ g} \times 9 = 180 \text{ g}$, $21 \times 9 = 189$ pieces of candy

c $\text{RF} = \frac{7}{38} \approx 0.18$

d $\frac{7}{38} \times 189 = 34.8$ so approximately 35 green candies per 180 g bag

e $50 \div 35 = 1.4$, so she will need $2 \times 180 \text{ g}$ bags

Exercise 7E

- 1 a** techniques, simulation
b selecting, return, next
c selection, same
- 2 a** Answers will vary.
b ii, iii, iv all have problems with maintaining consistently random results as they are either not returning selected cards or shuffling between selections.
- 3 a** Answers will vary.
b i: issue is always starting in the same place
 ii: starting with previous result may cause issues if not tossed properly
 iv: low toss may not get enough spin in to get a random result

- 4 a** Answers will vary.
b ii: unlikely to get enough spin to obtain a random result
5 a Answers will vary but should be fairly evenly distributed.
b Answers will vary but should be around

$$\frac{0 \times 13 + 0.5 \times 13 + 1.00 \times 13 + 1.5 \times 13}{52} = \frac{39}{52}$$

$$= \$0.75$$

c i \$0 discount is no longer an option
ii Answers will vary but should be around 13.

- iii** Answers will vary but should be around.

Discount	0	0.50	1.00	1.50
Relative frequency	$\frac{4}{52} \approx 0.08$	$\frac{16}{52} \approx 0.31$	$\frac{16}{52} \approx 0.31$	$\frac{16}{52} \approx 0.31$

- iv** Answers should be around

$$\frac{0 \times 4 + 0.5 \times 16 + 1.00 \times 16 + 1.5 \times 16}{52} = \frac{48}{52}$$

$$= \$0.92$$

- d** Only have three options available.

Exercise 7F

- 1 a** two, combinations
b space, product
c systematically, second
d cannot

- 2 a i** Size = 6 **ii** Size = 2
iii Size = 2 **iv** Size = 4
b i Sample Space = 1, 2, 3, 4, 5, 6
ii Sample Space = Heads, Tails
iii Sample Space = Red, Black
iv Sample Space = Hearts, Diamonds, Clubs, Spades

- 3 a** Size = $3 \times 4 = 12$

- b** Body type

Colour	Hatch	Sedan	SUV
White	White Hatch	White Sedan	White SUV
Red	Red Hatch	Red Sedan	Red SUV
Grey	Grey Hatch	Grey Sedan	Grey SUV
Blue	Blue Hatch	Blue Sedan	Blue SUV

- 4 a** Size = $2 \times 2 = 4$

- b** Baby 1

Boy 2	Boy	Girl
Boy	Boy, Boy	Girl, Boy
Girl	Boy, Girl	Girl, Girl

- 5 a** Size = $3 \times 3 = 9$

- b**

	Steamed Rice	Fried Rice	Noodles
Chicken	Chicken, Steamed Rice	Chicken, Fried Rice	Chicken, Noodles
Beef	Beef, Steamed Rice	Beef, Fried Rice	Beef, Noodles
Prawns	Prawns, Steamed Rice	Prawns, Fried Rice	Prawns, Noodles

6 a Size = $3 \times 5 = 15$

b

	S & P Calamari	Garlic Prawns	Satay
Steak	S & P Calamari, Steak	Garlic Prawns, Steak	Satay, Steak
Schnitzel	S & P Calamari, Schnitzel	Garlic Prawns, Schnitzel	Satay, Schnitzel
Barra	S & P Calamari, Barra	Garlic Prawns, Barra	Satay, Barra
Parmi	S & P Calamari, Parmi	Garlic Prawns, Parmi	Satay, Parmi
Pasta	S & P Calamari, Pasta	Garlic Prawns, Pasta	Satay, Pasta

7 a Size = $6 \times 5 = 30$

b

Sock 1		Stripe 1	Stripe 2	Dotty 1	Dotty 2	Orange	Black
Sock 2	Stripe 1	X	S2, S1	D1, S1	D2, S1	O, S1	B, S1
	Stripe 2	S1, S2	X	D1, S2	D2, S2	O, S2	B, S2
	Dotty 1	S1, D1	S2, D1	X	D2, D1	O, D1	B, D1
	Dotty 2	S1, D2	S2, D2	D1, D2	X	O, D2	B, D2
	Orange	S1, O	S2, O	D1, O	D2, O	X	B, O
	Black	S1, B	S2, B	D1, B	D2, B	O, B	X

8 a Size = $5 \times 4 = 20$

b

1st SRC		Mike	Daniel	Brooke	Abby	Holly
2nd SRC	Mike	X	D, M	B, M	A, M	H, M
	Daniel	M, D	X	B, D	A, D	H, D
	Brooke	M, B	D, B	X	A, B	H, B
	Abby	M, A	D, A	B, A	X	H, A
	Holly	M, H	D, H	B, H	A, H	X

Exercise 7G

1 a indicator, event

b number, outcomes

c total, size

d Favourable, needs

2 a i Size = 2

ii Red, Black

iii $P(\text{Red}) = \frac{1}{2}$

b i Size = 4

ii Hearts, Diamonds, Clubs, Spades

iii $P(\text{Heart}) = \frac{1}{4}$

c i Size = 13

ii A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K

iii $P(\text{Ace}) = \frac{1}{13}$

3 a Size = $2 \times 2 = 4$

b

		Coin 1	
		Head	Tail
Coin 2	Head	H, H	T, H
	Tail	H, T	T, T

c Number of favourable outcomes = 1,
 $P(\text{no tails}) = \frac{1}{4}$

d Number of favourable outcomes = 2,
 $P(\text{one tail}) = \frac{2}{4} = \frac{1}{2}$

e Number of favourable outcomes = 1,
 $P(\text{two tails}) = \frac{1}{4}$

f Number of favourable outcomes = 3,
 $P(\text{at least one tail}) = \frac{3}{4}$

4 a Size = $2 \times 4 = 8$

b

		Flavour			
		Chocolate	Vanilla	Strawberry	Salted Caramel
Cone	Waffle	Choc, Waffle	Van, Waffle	Straw, Waffle	Cara, Waffle
	Sugar	Choc, Sugar	Van, Sugar	Straw, Sugar	Cara, Sugar

c Number of favourable outcomes = 1, $P(\text{Salted Caramel in a Waffle cone}) = \frac{1}{8}$

d Number of favourable outcomes = 2, $P(\text{Salted Caramel}) = \frac{2}{8} = \frac{1}{4}$

e Number of favourable outcomes = 4, $P(\text{Waffle cone}) = \frac{4}{8} = \frac{1}{2}$

f Number of favourable outcomes = 5, $P(\text{Waffle cone or chocolate}) = \frac{5}{8}$

5 a Size = $4 \times 13 = 52$

b

		Face												
		A	2	3	4	5	6	7	8	9	10	J	Q	K
Suit	H	A, H	2, H	3, H	4, H	5, H	6, H	7, H	8, H	9, H	10, H	J, H	Q, H	K, H
	D	A, D	2, D	3, D	4, D	5, D	6, D	7, D	8, D	9, D	10, D	J, D	Q, D	K, D
	C	A, C	2, C	3, C	4, C	5, C	6, C	7, C	8, C	9, C	10, C	J, C	Q, C	K, C
	S	A, S	2, S	3, S	4, S	5, S	6, S	7, S	8, S	9, S	10, S	J, S	Q, S	K, S

c Number of favourable outcomes = 1, $P(\text{Ace Hearts}) = \frac{1}{52}$

d Number of favourable outcomes = 12, $P(\text{Picture Card}) = \frac{12}{52} = \frac{3}{13}$

e Number of favourable outcomes = 8, $P(\text{Ace or Queen}) = \frac{8}{52} = \frac{2}{13}$

f Number of favourable outcomes = 16, $P(\text{Spade or a Queen}) = \frac{16}{52} = \frac{4}{13}$

g Number of favourable outcomes = 22, $P(\text{Picture Card or a club}) = \frac{22}{52} = \frac{11}{26}$

6 a Size = $4 \times 4 = 16$

b

		Dice 1			
		1	2	3	4
Dice 2	1	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

- c** Number of favourable outcomes = 4, $P(\text{total of } 5) = \frac{4}{16} = \frac{1}{4}$
d Number of favourable outcomes = 10, $P(\text{total of at least } 5) = \frac{10}{16} = \frac{5}{8}$
e Number of favourable outcomes = 6, $P(\text{total more than } 5) = \frac{6}{16} = \frac{3}{8}$
f Number of favourable outcomes = 6, $P(\text{total less than } 5) = \frac{6}{16} = \frac{3}{8}$

7 a Size = $4 \times 4 = 16$

b

Question 1

Dice 2		Correct	Wrong	Wrong	Wrong
	Correct	✓✓	✗✓	✗✓	✗✓
	Wrong	✓✗	✗✗	✗✗	✗✗
	Wrong	✓✗	✗✗	✗✗	✗✗
	Wrong	✓✗	✗✗	✗✗	✗✗

- c** Number of favourable outcomes = 1, $P(\text{both correct}) = \frac{1}{16}$
d Number of favourable outcomes = 6, $P(\text{one correct}) = \frac{6}{16} = \frac{3}{8}$
e Number of favourable outcomes = 7, $P(\text{at least one correct}) = \frac{7}{16}$
f Number of favourable outcomes = 9, $P(\text{none correct}) = \frac{9}{16}$

8 a Size = $3 \times 3 = 9$

b

Hayley

Ben		Rock	Paper	Scissors
	Rock	R, R	P, R	S, R
	Paper	R, P	P, P	S, P
	Scissors	R, S	P, S	S, S

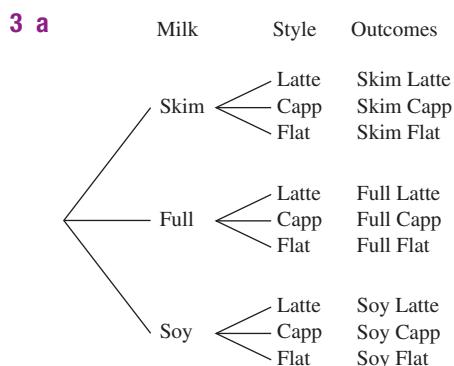
- c** Number of favourable outcomes = 3, $P(\text{Ben plays Scissors}) = \frac{3}{9} = \frac{1}{3}$
d Number of favourable outcomes = 1, $P(\text{both play rock}) = \frac{1}{9}$
e Number of favourable outcomes = 3, $P(\text{both play same move}) = \frac{3}{9} = \frac{1}{3}$
f Number of favourable outcomes = 1, $P(\text{Hayley wins with paper}) = \frac{1}{9}$

Exercise 7H

- 1 a** determining, two
b probabilities, more
c columns, outcomes
2 a Red, Petrol, Manual
 Red, Petrol, Auto
 Red, Hybrid, Manual
 Red, Hybrid, Auto
 Blue, Petrol, Manual

Blue, Petrol, Auto
 Blue, Hybrid, Manual
 Blue, Hybrid, Auto

- b** 8 combinations
c Number of favourable outcomes = 1,
 $P(\text{Red, Hybrid, Auto}) = \frac{1}{8}$
d Number of favourable outcomes = 2,
 $P(\text{Red, Auto}) = \frac{2}{8} = \frac{1}{4}$
e Number of favourable outcomes = 4,
 $P(\text{Auto}) = \frac{4}{8} = \frac{1}{2}$



b 9 combinations

c Skim Milk, Latte

Skim Milk, Cappuccino

Skim Milk, Flat White

Full Cream, Latte

Full Cream, Cappuccino

Full Cream, Flat White

Soy Milk, Latte

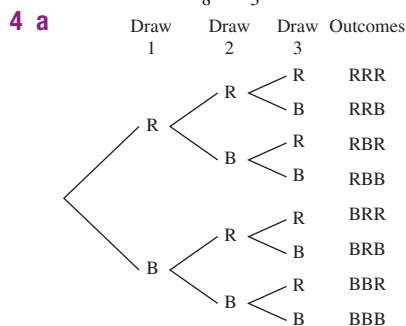
Soy Milk, Cappuccino

Soy Milk, Flat White

d Number of favourable outcomes = 1,
 $P(\text{Skim Milk, Latte}) = \frac{1}{9}$

e Number of favourable outcomes = 3,
 $P(\text{Latte}) = \frac{3}{9} = \frac{1}{3}$

f Number of favourable outcomes = 6,
 $P(\text{no Soy}) = \frac{6}{9} = \frac{2}{3}$



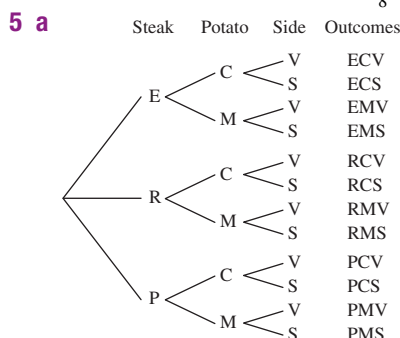
b 8 outcomes

c RRR, RRB, RBR, RBB, BRR, BRB, BBR, BBB

d Number of favourable outcomes = 1,
 $P(\text{RRR}) = \frac{1}{8}$

e Number of favourable outcomes = 3,
 $P(\text{two Red cards}) = \frac{3}{8}$

f Number of favourable outcomes = 4,
 $P(\text{less than two Red cards}) = \frac{4}{8} = \frac{1}{2}$



Key:

Steak: Eye (E), Rib (R), Rump (P)

Potato: Chips (C), Mash (M)

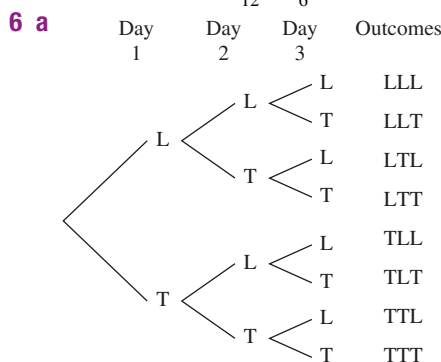
Side: Vegetables (V), Salad (S)

b 12 combinations

c ECV, ECS, EMV, EMS, RCV, RCS, RMV, RMS, PCV, PCS, PMV, PMS

d Number of favourable outcomes = 1,
 $P(\text{Rump, Chips, Salad}) = \frac{1}{12}$

e Number of favourable outcomes = 2,
 $P(\text{Rib, Chips}) = \frac{2}{12} = \frac{1}{6}$



b There are 8 outcomes.

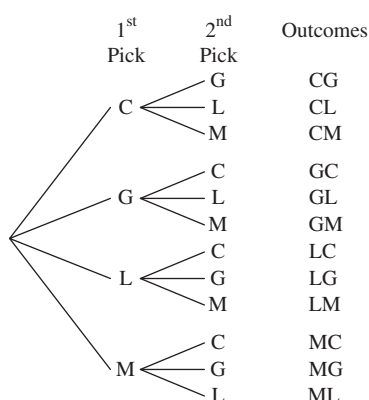
c LLL, LLT, LTL, LTT, TLL, TLT, TTL, TTT

d Number of favourable outcomes = 1,
 $P(\text{TTT}) = \frac{1}{8}$

e Number of favourable outcomes = 3,
 $P(\text{Late once}) = \frac{3}{8}$

f Number of favourable outcomes = 7,
 $P(\text{Late at least once}) = \frac{7}{8}$

7 a



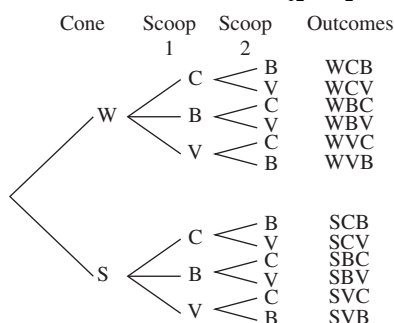
b There are 12 outcomes.

c CG, CL, CM, GC, GL, GM, LC, LG, LM, MC, MG, ML

d Number of favourable outcomes = 2,
 $P(\text{Libby and Gabby}) = \frac{2}{12} = \frac{1}{6}$

e Number of favourable outcomes = 6,
 $P(\text{Chloe is in the team}) = \frac{6}{12} = \frac{1}{2}$

8 a



b There are 12 combinations.

c WCB, WCV, WBC, WBV, WVC, WVB, SCB, SCV, SBC, SBV, SVC, SVB

d Number of favourable outcomes = 2,
 $P(\text{Waffle, Berry and Choc}) = \frac{2}{12} = \frac{1}{6}$

e Number of favourable outcomes = 4,
 $P(\text{no Choc}) = \frac{4}{12} = \frac{1}{3}$

Chapter checklist

1 a West Coast Eagles

b 0.2–0.95

2 Answers will vary.

3 Expected = $300 \div 6 = 50$ times each

4 Total outcomes = $42 + 57 + 24 + 2 = 125$

a $P(\text{plays one sport}) = \frac{57}{125} = 0.46$

b $P(\text{plays fewer than two sports}) = \frac{42+57}{125} = 0.79$

5 Adjacent cards may be of the same suit or in numerical order, so they may not be drawn randomly.

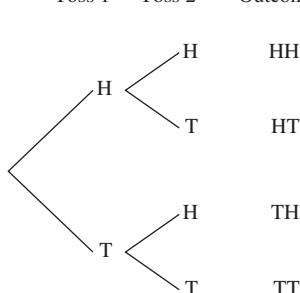
6 The coin should not always be held the same way up when tossing, because if the tossing action is very similar each time, the outcomes may not be random.

7 a

		Coin 1	
		Head	Tail
Coin 2	Head	H, H	T, H
	Tail	H, T	T, T

b HH, HT, TH, TT

c

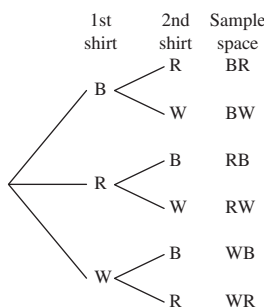


8 a $P(TT) = \frac{1}{4} = 0.25$

b $P(\text{one tail}) = \frac{2}{4} = 0.50$

c $P(\text{at least one tail}) = \frac{3}{4} = 0.75$

9 a



b $P(B + R) = \frac{2}{6} = \frac{1}{3}$

c $P(W) = \frac{4}{6} = \frac{2}{3}$

Chapter review

1 a

Result	Frequency
heads/girls	11
tails/boys	19

b $\% \text{ boys} = \frac{19}{30} \times 100 = 63\%$

c Expected = $30 \div 2 = 15$

d Yes, you would expect half the sample to be boys, but it was $\frac{19}{30}$ or 63%.

4 a, b

	Suit	Hearts	Diamonds	Clubs	Spades
a	Frequency	5	7	4	4
b	Relative freq	$\frac{5}{20} = 0.25$	$\frac{7}{20} = 0.35$	$\frac{4}{20} = 0.20$	$\frac{4}{20} = 0.20$

c Diamonds appear more than expected.

d Increasing the size of the sample to a large number would result in the outcomes being more evenly distributed.

5 a Total shoppers = 122,

$P(2 \text{ bags}) = \frac{18}{122} = 0.15$

b Number with less than 4 bags = $15 + 24 + 18 + 35 = 92$

$P(\text{less than 4 bags}) = \frac{92}{122} = 0.75$

6 a Not returning the card between selections changes the chance of that suit/colour etc. being selected in the next draw.

b If shuffling technique is poor, this could impact results.

c Depending on technique, starting on heads could result in the same outcome occurring more frequently.

d No issue

e If the dice does not get an opportunity to spin, the outcome could be the same as last time.

f No issue

2 a

Two-child combination	Tally	Frequency
boy + boy		5
boy + girl		11
girl + girl		4

b $RF(\text{boy/girl}) = \frac{11}{20} = 0.55$

3 a 4 outcomes: H, D, C, S

b Expected repeats = $20 \div 4 = 5$ times

c 2 times. This is less than expected but not too unusual.

d Expected outcomes = $20 \div 4 = 5$ times

7 a Size = $6 \times 6 = 36$

b

Dice 1		1	2	3	4	5	6
Dice 2	+	1	2	3	4	5	6
	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

8 a Sample size = $2 \times 2 = 4$

b HH, HT, TH, TT

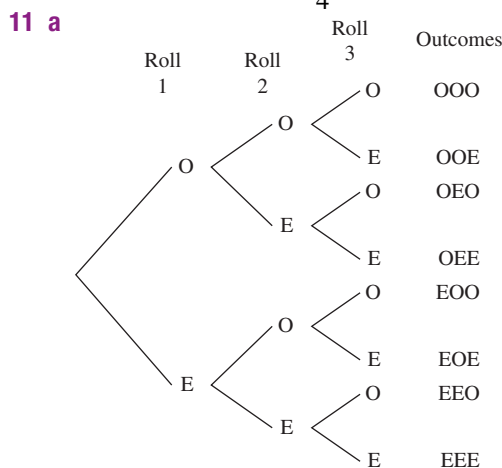
9 a Number of favourable outcomes = 5,
 $P(\text{total } 8) = \frac{5}{36}$

b Number of favourable outcomes = 15,
 $P(\text{total at least } 8) = \frac{15}{36} = \frac{5}{12}$

c Number of favourable outcomes = 10,
 $P(\text{total is more than } 8) = \frac{10}{36} = \frac{5}{18}$

d Number of favourable outcomes = 21,
 $P(\text{total is less than } 8) = \frac{21}{36} = \frac{7}{12}$

- 10 a** Number of favourable outcomes = 1,
 $P(HH) = \frac{1}{4}$
- b** Number of favourable outcomes = 2,
 $P(\text{one heads}) = \frac{2}{4} = \frac{1}{2}$
- c** Number of favourable outcomes = 1,
 $P(\text{no heads}) = \frac{1}{4}$
- d** Number of favourable outcomes = 3,
 $P(\text{at least one head}) = \frac{3}{4}$



- b** 8 outcomes
- c** OOO, OOE, OEO, OEE, EOO, EOE, EEO, EEE

- d** Number of favourable outcomes = 3,
 $P(\text{two even}) = \frac{3}{8}$
- e** Number of favourable outcomes = 1,
 $P(\text{three even}) = \frac{1}{8}$
- f** Number of favourable outcomes = 4,
 $P(\text{less than two even}) = \frac{4}{8} = \frac{1}{2}$

12 a $P(\text{bus is late}) = \frac{1}{5} = 0.20$

- b i** Answers will vary. Example response:

0.448	0.534	0.727	0.973	0.471
0.069	0.858	0.606	0.230	0.193
0.728	0.495	0.010	0.176	0.563
0.935	0.702	0.184	0.228	0.746

- ii** The bus is late on 5 occasions in the simulation shown.
- c i** $\frac{20}{100} \times 20 = 4$ times
- ii** Answers will vary. A simulation result of 3, 4, 5 would not be considered too unusual. If less than 3 or more than 5, the simulation result could be considered unusual.

13 a Sample size = $4 \times 3 = 12$

- b** Dish 1

	C	L	P	B
C	X	L, C	P, C	B, C
L	C, L	X	P, L	B, L
P	C, P	L, P	X	B, P
B	C, B	L, B	P, B	X

- c** Number of favourable outcomes = 6, $P(\text{orders Pork}) = \frac{6}{12} = \frac{1}{2}$
- d** Number of favourable outcomes = 2, $P(\text{Chicken and Beef}) = \frac{2}{12} = \frac{1}{6}$
- e** Number of favourable outcomes = 4, $P(\text{Chicken and Beef or Lamb}) = \frac{4}{12} = \frac{1}{3}$
- f** Number of favourable outcomes = 10, $P(\text{Chicken or Beef}) = \frac{10}{12} = \frac{5}{6}$

14 a

H	T	H	T	H	H	T	H	H	T
H	T	H	H	T	T	T	T	T	T
H	H	H	H	T	T	T	H	H	H
T	T	T	T	H	H	T	T	T	H

b

Number of children	Tally	Frequency
1		11
2		3
3		4
4		1
5		1
Total		20

c Average = $\frac{1 \times 11 + 2 \times 3 + 3 \times 4 + 4 \times 1 + 5 \times 1}{20} = \frac{38}{20} = 1.9$

d i RF(2 children) = $\frac{3}{20} = 0.15$

ii RF (1 child) = $\frac{11}{20} = 0.55$

Chapter 8

Pre-test

- 1 a** 42.49° **b** 116.33° **c** 73.06°
2 a 12.59° **b** 113.61° **c** 58.58°
3 a i 20.07 cm **ii** 137.01 cm
b i 301.94 km **ii** 326.38 km
c i 10 897.10 km **ii** 29 133.07 km
4 a 15' **b** 48' **c** 34' **d** 45'
5 2, 7, 4, 0, 7, 3

Exercise 8A

- 1 a** 32°27' **b** 43°12'
c 122°28' **d** 91°7'
e 45'
2 a 32.75° **b** 15.5833°
c 7.36667° **d** 142.733°
e 67.25°
3 a 50.27 cm **b** 87.96 m
c 282.74 mm **d** 37.70 mm
e 43.98 m

- 4 a** $\frac{1}{4}$ **b** $\frac{3}{4}$ **c** $\frac{1}{12}$
d $\frac{1}{3}$ **e** $\frac{1}{6}$ **f** $\frac{5}{12}$
5 a 7.85 cm **b** 10.47 cm
c 26.18 cm **d** 23.56 cm
e 36.65 cm **f** 57.60 cm
6 a 13.09 cm **b** 5.24 cm
c 78.54 cm **d** 37.70 cm
e 122.17 cm **f** 109.96 cm

Exercise 8B

- 1 a** Bucharest, Romania
b Oslo, Norway
c Houston, United States
d Singapore
e Cape Town, South Africa
2 a 30.04°N, 31.24°E
b 19.43°N, 99.13°W
c 34.60°S, 58.38°W
d 19.08°N, 72.88°E
e 6.5°N, 3.37°E
f 17.83°S, 31.03°E

- 3 a** 21.93°S, 114.1°E
b 20.31°S, 118.61°E
c 35.03°S, 117.89°E
d 33.36°S, 116.17°E
e 28.78°S, 114.61°E
f 33.86°S, 121.89°E
g 129°E
- 4 a** (65°N, 0°) **b** (0°, 75°E)
- 5 a** (18°N, 24°E) **b** (38°N, 44°E)
c (17°S, 24°E) **d** (43°N, 36°W)
- 6 a i** Mexico City, Yangon
ii Brisbane
b i Brisbane **ii** Plymouth
iii London **iv** Zurich
c Marseilles, Mexico City, Plymouth, Zurich, London, Yangon
d Plymouth, Mexico City, Lima, London (just)
e Mexico City, Yangon
f Cooktown, Melbourne
g London
h Wellington
- 7** 6338 km
8 2313 km
9 1779 km
10 34.27°S
- 11 a** 3892 km **b** 3002 km
c 4893 km **d** 4448 km
e 3336 km
- 12** 3558 km
13 4003 km
- 14 a** 5560 km **b** 7784 km
c 3336 km **d** 6672 km
e 16 902 km
- 15** 5.4°
- 16 a** 3336 km **b** 6672 km **c** 13 344 km
17 a 4226 km **b** 5782 km **c** 14 234 km
18 a 4176 km **b** 14 184 km **c** 5832 km

- 19 a** Equator 4670 km; North Pole 5338 km; South Pole 14 678 km
b Equator 6116 km; North Pole 3892 km; South Pole 16 124 km
c Equator 1668 km; North Pole 11 676 km; South Pole 8340 km
d Equator 1557 km; North Pole 11 565 km; South Pole 8451 km

Exercise 8C

- 1 a** X is 2 hours ahead of Y
b X is 5 hours ahead of Y
c Y is 5 hours ahead of X
d Y is 1 hour 48 minutes ahead of X
- 2 a i** 3 hours 28 minutes behind
ii 7 hours 52 minutes ahead
iii 1 hours 48 minutes ahead
iv 8 hours 12 minutes ahead
v 14 hours 48 minutes ahead
- b i** 7:28 a.m.
ii 6:08 p.m. the day before
iii 12:12 a.m.
iv 5:48 p.m. the day before
v 11:12 a.m. the day before
- 3 a** 150°W
b 2 p.m.
c 7:30 a.m.
- 4 a** 5:30 p.m. **b** 7:30 a.m.
- 5 a** 45° **b** 3 hours
c 2:45 p.m.
- 6 a** 6 p.m. Friday
b 3 a.m. Saturday
c 2 a.m. Saturday
d 6 p.m. Friday
e 11 a.m. Saturday
f 1 p.m. Saturday
- 7 a** 3 a.m. the same day
b 3 p.m. the next day

- 8 **a** 3 a.m. the same day
b 5 a.m. the next day
 9 **a** 4 p.m. Monday
b 14 hours
c 13 hours 55 minutes
 10 **a** 3 p.m. same day
b 1.30 p.m. same day
c 2 p.m. same day
d 2 p.m. same day
e 5 p.m. same day
f 4 p.m. same day
g 4 p.m. same day
h 6 p.m. the day before
i 3 p.m. same day

Chapter checklist

- 1 8340 km
 2 $53^{\circ}47'$
 3 68.80°
 4 $31.96^{\circ}\text{S } 115.86^{\circ}\text{E}$
 5 Paraburdoo
 6 1241 km
 7 +4 hours

Chapter review

- 1 3336 km
 2 3892 km
 3 **a** 52° **b** 3 hours 28 minutes
c 6:13 a.m.
 4 $(0^{\circ}, 160^{\circ}\text{W})$
 5 199.4 km
 6 3 hours 8 minutes
 7 10 a.m.
 8 7 a.m.
 9 9 p.m.
 10 4 a.m. Saturday
 11 **a** 4 p.m. Tuesday
b 2 p.m. Wednesday
 12 **a** 15 hours 28 minutes
b 11:28 a.m. Tuesday
c 8:32 a.m. Thursday

Chapter 9

Pre-test

- 1 **a** $\frac{15}{100}$ **b** $\frac{6}{100}$
c $\frac{2.4}{100}$ **d** $\frac{11.25}{100}$
 2 **a** $\frac{12}{100} \times 45 = \$5.40$
b $\frac{8}{100} \times 1590 = \$127.20$
c $\frac{1.8}{100} \times 24\,900 = \$448.20$
d $\frac{3.1}{100} \times 800 = \$24.80$
 3 **a** $\frac{12}{100} \times 5750 = \$690$; $5750 + 690 = \$6440$
b $\frac{3}{100} \times 600 = \$18$; $600 + 18 = \$618$
c $\frac{4}{100} \times 2120 = \$84.80$; $2120 - 84.80 = \$2035.20$
d $\frac{15}{100} \times 30\,000 = \$4500$; $30\,000 - 4500 = \$25\,500$
 4 **a** $5 \times 12 = 60$ months
b $3 \times 52 = 156$ weeks
c 365 days
d $7 \times 26 = 182$ fortnights
 5 **a** 32.81 **b** 1.08
c 0.32 **d** 0.17

Exercise 9A

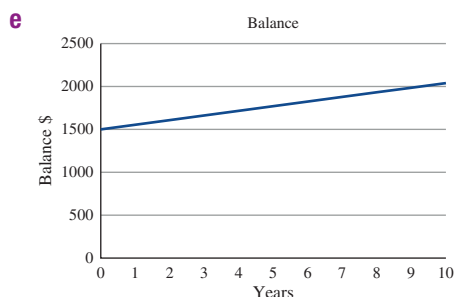
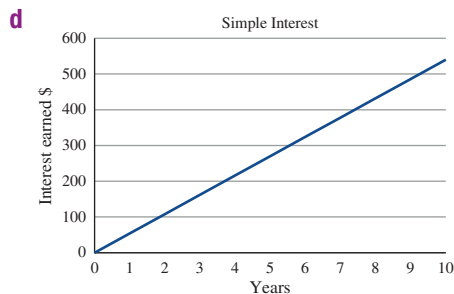
- 1 **a** borrowing, investing, percentages
b interest, same
c borrowed, interest
 2 **a** $\frac{7}{12}$ **b** $\frac{31}{52}$
c $\frac{25}{365}$ **d** $\frac{18}{12}$
e $\frac{270}{365}$ **f** $\frac{9}{52}$
 3 **a** $\frac{30}{100} \times 23.70 = \$7.11$
b $\frac{10}{100} \times 80.90 = \$8.09$
c $\frac{5}{100} \times 126.45 = \$6.32$
d $\frac{3.45}{100} \times 456.21 = \$15.74$
e $\frac{1.05}{100} \times 364.85 = \$3.83$
f $\frac{0.65}{100} \times 63.90 = \$0.42$

- 4 a** $I = 780 \times \frac{4.57}{100} \times 4 = \$142.58$
b $I = 4260 \times \frac{6.32}{100} \times 3 = \$807.70$
c $I = 2030 \times \frac{9.05}{100} \times 5 = \$918.58$
d $I = 625 \times \frac{5.9}{100} \times 2 = \$73.75$
e $I = 1650 \times \frac{4.2}{100} \times 5 = \$346.50$
f $I = 5050 \times \frac{3.9}{100} \times 7 = \$1378.65$
g $I = 8880 \times \frac{5.79}{100} \times 6 = \$3084.91$
h $I = 7280 \times \frac{5.36}{100} \times 2 = \$780.42$
5 a $I = 10\,000 \times \frac{3.65}{100} \times 4 = \$1460.00$
b Value = $10\,000 + 1460.00 = \$11\,460.00$
6 a $I = 6200 \times \frac{6.8}{100} \times \frac{4}{12} = \$140.53$
b $I = 590 \times \frac{5.11}{100} \times \frac{20}{52} = \$11.60$
c $I = 8290 \times \frac{4.48}{100} \times \frac{15}{365} = \$15.26$
d $I = 6700 \times \frac{2.26}{100} \times \frac{2}{52} = \$5.82$
e $I = 7740 \times \frac{5.35}{100} \times \frac{14}{365} = \$15.88$
f $I = 858 \times \frac{7.07}{100} \times \frac{7}{12} = \$35.39$
g $I = 3700 \times \frac{6.1}{100} \times 9 \frac{6}{12} = \$2144.15$
h $I = 6160 \times \frac{5.3}{100} \times 8 \frac{3}{12} = \$2693.46$
7 a $I = 2\,000\,000 \times \frac{8}{100} \times \frac{30}{365} = \$13\,150.68$
b Value = $2\,000\,000 + 13\,150.68 = \$2\,013\,150.68$
8 a $I = 3000 \times \frac{9.7}{100} \times \frac{18}{12} = \$436.50$
b Repay = $3000 + 436.50 = \$3436.50$
c Monthly repayment = $3436.50 \div 18 = \$190.92$
9 a $I = 5000 \times \frac{11.2}{100} \times 4 = \$2240.00$
b Repay = $5000 + 2240 = \$7240.00$
c Monthly repayment = $7240.00 \div 48 = \$150.83$
10 a i 2.60% p.a.
ii $I = 15\,000 \times \frac{2.6}{100} \times 2 = \$780$
iii \$15 780
b i 2.30% p.a.
ii $I = 65\,000 \times \frac{2.3}{100} \times 1 = \$1495$
iii \$66 495
c i 2.05% p.a.
ii $I = 11\,400 \times \frac{2.05}{100} \times \frac{6}{12} = \$116.85$
iii \$11 516.85

11 a

Year	Simple interest	Balance
1	54	1554
2	108	1608
3	162	1662
4	216	1716
5	270	1770
6	324	1824
7	378	1878
8	432	1932
9	486	1986
10	540	2040

- b i** \$270 **ii** \$540
c i \$1770 **ii** \$2040



Exercise 9B

- 1 a** interest, interest
b i accumulated **ii** principal
iii interest **iv** number
c must, frequency
2 a $\frac{15}{100}$ **b** $\frac{6}{100}$
c $\frac{3.4}{100}$ **d** $\frac{2.03}{100}$
3 a 1 year = 12 months
b 1 year = 52 weeks
c 1 year = 2 half-years

d 1 year = 365 days**e** 1 year = 4 quarters**f** 1 year = 26 fortnights**4 a** $\frac{2.4}{2}\% = 1.2\%$ per half year**b** $\frac{2.4}{4}\% = 0.6\%$ per quarter**c** $\frac{2.4}{12}\% = 0.2\%$ per month**d** $\frac{2.4}{26}\%$ per fortnight**e** $\frac{2.4}{52}\%$ per week**f** $\frac{2.4}{365}\%$ per day**5 a**

Year	Principal	Interest	Balance
1	\$2480	$2480 \times \frac{3.2}{100} = \$79.36$	$2480 + 79.36 = \$2559.36$
2	\$2559.36	$2559.36 \times \frac{3.2}{100} = \$81.90$	$2559.36 + 81.90 = \$2641.26$
3	\$2641.26	$2641.26 \times \frac{3.2}{100} = \$84.52$	$2641.26 + 84.52 = \$2725.78$

b $\$2725.78 - \$2480 = \$245.78$ **6 a**

Year	Principal	Interest	Balance
1	\$3850	$3850 \times \frac{2.1}{100} = \$80.85$	$3850 + 80.85 = \$3930.85$
2	\$3930.85	$3930.85 \times \frac{2.1}{100} = \$82.55$	$3930.85 + 82.55 = \$4013.40$
3	\$4013.40	$4013.40 \times \frac{2.1}{100} = \$84.28$	$4013.40 + 84.28 = \$4097.68$
4	\$4097.68	$4097.68 \times \frac{2.1}{100} = \$86.05$	$4097.68 + 86.05 = \$4183.73$

b $\$4183.73 - \$3850 = \$333.73$ **7 a** $P = \$9600, i = \frac{3.2}{100}, n = 2$

$$A = P(1 + i)^n$$

$$A = 9600 \left(1 + \frac{3.2}{100}\right)^2$$

$$A = \$10\,224.23$$

b $\$10\,224.23 - 9600 = \$624.23$ **8 a** $P = \$4500, i = \frac{2.2}{100}, n = 4$

$$A = P(1 + i)^n$$

$$A = 4500 \left(1 + \frac{2.2}{100}\right)^4$$

$$A = \$4909.26$$

b $4909.26 - 4500 = \$409.26$ **9 a** $P = \$680, i = \frac{5}{100}, n = 3,$

$$A = 680 \left(1 + \frac{5}{100}\right)^3 = \$787.19$$

b $P = \$275, i = \frac{3}{100}, n = 6,$

$$A = 275 \left(1 + \frac{3}{100}\right)^6 = \$328.36$$

c $P = \$682, i = \frac{4}{100}, n = 5,$

$$A = 682 \left(1 + \frac{4}{100}\right)^5 = \$829.76$$

d $P = \$1240, i = \frac{2.2}{100}, n = 4,$

$$A = 1240 \left(1 + \frac{2.2}{100}\right)^4 = \$1352.77$$

e $P = \$5760, i = \frac{1.8}{100}, n = 7,$

$$A = 5760 \left(1 + \frac{1.8}{100}\right)^7 = \$6526.15$$

f $P = \$4030, i = \frac{2.15}{100}, n = 5,$

$$A = 4030 \left(1 + \frac{2.15}{100}\right)^5 = \$4482.26$$

10 $P = \$5000, i = \frac{3.6}{1200}, n = 12 \times 5 = 60$

$$A = P(1 + i)^n$$

$$A = 5000 \left(1 + \frac{3.6}{1200}\right)^{60}$$

$$A = \$5984.47$$

11 $P = \$3800, i = \frac{2.4}{5200}, n = 2 \times 52 = 104$

$$A = P(1 + i)^n$$

$$A = 3800 \left(1 + \frac{2.4}{5200}\right)^{104}$$

$$A = \$3986.80$$

12 a $P = \$1370, i = \frac{8.85}{1200}, n = 3 \times 12 = 36,$

$$A = 1370 \left(1 + \frac{8.85}{1200}\right)^{36} = \$1784.85$$

$$\begin{aligned} \text{b } P &= \$54\,600, i = \frac{1.2}{200}, n = 8 \times 2 = 16, \\ A &= 54\,600 \left(1 + \frac{1.2}{200}\right)^{16} = \$60\,084.21 \\ \text{c } P &= \$4300, i = \frac{5.07}{5200}, n = 52 \times 2 = 104, \\ A &= 4300 \left(1 + \frac{5.07}{5200}\right)^{104} = \$4758.66 \\ \text{d } P &= \$3350, i = \frac{6.1}{1200}, n = 12 \times 3 = 36, \\ A &= 3350 \left(1 + \frac{6.01}{1200}\right)^{36} = \$4020.86 \\ \text{e } P &= \$7960, i = \frac{4.29}{2600}, n = 26 \times 2 = 52, \\ A &= 7960 \left(1 + \frac{4.92}{2600}\right)^{52} = \$8782.28 \\ \text{f } P &= \$4220, i = \frac{4.66}{400}, n = 4 \times 4 = 16, \\ A &= 4220 \left(1 + \frac{4.66}{400}\right)^{16} = \$5079.22 \end{aligned}$$

- 13 a i** B2
ii = B2*4/100
iii = B2 + C2
iv = D2
b ii Balance = \$4258.29

Exercise 9C

- 1 a** percentage, compound
b increasing
c measured
d value
e decrease, interest
f reducing
- 2 a** Increasing **b** Decreasing
c Decreasing **d** Increasing
e Increasing
- 3 a** $P = \$10.65, i = \frac{2.1}{100}, n = 4,$
 $A = 10.65 \left(1 + \frac{2.1}{100}\right)^4 = \$11.57$
b $P = \$20\,490, i = \frac{2.1}{100}, n = 20,$
 $A = 20\,490 \left(1 + \frac{2.1}{100}\right)^{20} = \$31\,049.66$
- 4 a** $P = 93\,000, i = \frac{1.5}{100}, n = 1,$
 $A = 93\,000 \left(1 + \frac{1.5}{100}\right)^1 = \$94\,395$
b $P = 93\,000, i = \frac{1.9}{100}, n = 1,$
 $A = 93\,000 \left(1 + \frac{1.9}{100}\right)^1 = \$94\,767$
- 5 a** $P = 145.9, i = \frac{1.9}{100}, n = 20,$
 $A = 145.9 \left(1 + \frac{1.9}{100}\right)^{20}$
 = 212.6 cents per litre

$$\begin{aligned} \text{b } P &= 145.9, i = \frac{2.1}{100}, n = 10, \\ A &= 145.9 \left(1 + \frac{2.1}{100}\right)^{10} \\ &= 179.6 \text{ cents per litre} \end{aligned}$$

- 6 a** $P = 24.8, i = \frac{1.32}{100}, n = 10,$
 $A = 24.8 \left(1 + \frac{1.32}{100}\right)^{10} = 28.28 \text{ million}$
b $P = 24.8, i = \frac{1.32}{100},$
 $n = (2100 - 2018) = 82,$
 $A = 24.8 \left(1 + \frac{1.32}{100}\right)^{82} = 72.69 \text{ million}$
- 7 a** $P = \$447\,500, i = \frac{3.1}{100}, n = 15,$
 $A = 447\,500 \left(1 + \frac{3.1}{100}\right)^{15} = \$707\,412.97$
b $P = \$447\,500, i = \frac{3.1}{100}, n = 20,$
 $A = 447\,500 \left(1 + \frac{3.1}{100}\right)^{20} = \$824\,074.25$
- 8 a** $P = \$320\,000, i = -\frac{15}{100}, n = 5,$
 $A = 320\,000 \left(1 - \frac{15}{100}\right)^5 = \$141\,985.70$
b $P = \$320\,000, i = -\frac{15}{100}, n = 10,$
 $A = 320\,000 \left(1 - \frac{15}{100}\right)^{10} = \$62\,999.81$
c $P = \$320\,000, i = -\frac{15}{100}, n = 15,$
 $A = 320\,000 \left(1 - \frac{15}{100}\right)^{15} = \$27\,953.35$
- 9 a i** $P = 6500, i = -\frac{20}{100}, n = 3,$
 $A = 6500 \left(1 - \frac{20}{100}\right)^3 = 3328$
ii $P = 6500, i = -\frac{20}{100}, n = 5,$
 $A = 6500 \left(1 - \frac{20}{100}\right)^5 = 2130$
b The disease is not fully eradicated because it is reducing by 20% of the current amount each year, not 20% of the original amount each year.
- 10 a** $P = \$1549, i = -\frac{45}{100}, n = 2,$
 $A = 1549 \left(1 - \frac{45}{100}\right)^2 = \$468.57$
b $P = \$2099, i = -\frac{45}{100}, n = 3,$
 $A = 2099 \left(1 - \frac{45}{100}\right)^3 = \$349.22$

Exercise 9D

- 1 a plan, successful
b calculate
c value, interest
d same, every
e added, increases
- 2 a i \$0 ii \$250
iii Fortnight iv Monthly
v 20 vi 6%
- b \$250 272
c \$120 272
- 3 a i \$0 ii Fortnight
iii Annually iv $55 - 18 = 37$
v 5.6%
- b \$331 (by guess and check)
c \$681 826
- 4 a i \$12 000 ii \$500
iii Monthly iv Monthly
v 2 vi 3.2%
- b \$25 167
c \$1167
d 3 years
e \$696 (by guess and check)
- 5 a \$21 276 b \$46 616
- 6 a \$300 b \$42 553
c No he was not able to catch up to Nicole. She has \$46 616 which is \$4063 more than he has. This is because she has been earning interest for a longer time.

7 b

	A	B	C	D	E
1 Month	Principal	Interest	Deposit	Balance	
1	1756	$=8 \times 3.15/100 \times 1/12$	210	$=B1+C1-D1$	
2	$=E2$	$=8 \times 3.15/100 \times 1/12$	210	$=B2+C2-D2$	
3	$=E3$	$=8 \times 3.15/100 \times 1/12$	210	$=B3+C3-D3$	
4	$=E4$	$=8 \times 3.15/100 \times 1/12$	210	$=B4+C4-D4$	
5	$=E5$	$=8 \times 3.15/100 \times 1/12$	210	$=B5+C5-D5$	
6	$=E6$	$=8 \times 3.15/100 \times 1/12$	210	$=B6+C6-D6$	

- c \$3852.84 d \$76.84
e 18 months
- 8 b
- | | A | B | C | D | E |
|---------|-----------|---------------------------------|---------|-------------|---|
| 1 Month | Principal | Interest | Deposit | Balance | |
| 1 | 2460 | $=8 \times 2.8/100 \times 1/12$ | 500 | $=B1+C1-D1$ | |
| 2 | $=E1$ | $=8 \times 2.8/100 \times 1/12$ | 500 | $=B2+C2-D2$ | |
| 3 | $=E2$ | $=8 \times 2.8/100 \times 1/12$ | 500 | $=B3+C3-D3$ | |
| 4 | $=E3$ | $=8 \times 2.8/100 \times 1/12$ | 500 | $=B4+C4-D4$ | |
| 5 | $=E4$ | $=8 \times 2.8/100 \times 1/12$ | 500 | $=B5+C5-D5$ | |
- c \$15 986.63 d \$526.63
e 32 months

Exercise 9E

- 1 changes, impact, interest
- 2 a 12 b 52 c 365 d 26
e 2 f 13 g 4
- 3 a $12 \times 2 = 24$ months
b i 2.6% p.a.
ii 2.5% p.a.
iii 2.4% p.a.
- c i Once
ii $12 \div 6 = 2$ times
iii $52 \div 4 = 13$ times
- d i $P = \$20\,000$, $i = \frac{2.6}{100}$, $n = 2$

$$A = 20\,000 \left(1 + \frac{2.6}{100} \right)^2 = \$21\,053.52$$

 ii $P = \$20\,000$, $i = \frac{2.5}{200}$, $n = 2 \times 2 = 4$,

$$A = 20\,000 \left(1 + \frac{2.5}{200} \right)^4 = \$21\,018.91$$

 iii $P = \$20\,000$, $i = \frac{2.4}{1300}$,

$$n = 2 \times 13 = 26,$$

$$A = 20\,000 \left(1 + \frac{2.4}{1300} \right)^{26} = \$20\,982.48$$
- e Best option is to invest the money with annual compounds at 2.6% p.a.
- 4 a i \$17 670.89 ii \$35 506.61
b Difference = $\$35\,506.61 - \$17\,670.89 = \$17\,835.72$
c Interest earned at 5% p.a. = \$9170.89
Interest earned at 10% p.a. = \$27 006.61
Doubling the interest rate led to approximately three times more interest being earned.
- d The curve increases a lot faster and is much steeper when the interest rate increases.
- 5 a i \$56 044.11 ii \$96 462.93
b Difference = $\$96\,462.93 - \$56\,044.11 = \$40\,418.82$

c Increasing the interest rate by 3% (or one third) led to an increase interest of approximately \$40 000, almost double, over the same period.

d The curve increases a lot faster and is much steeper when the interest rate increases.

- 6 a i** Annual compounds, interest = \$900
ii Monthly compounds, interest = \$911
b She earns more interest.

- 7 a** \$5369 **b** \$5341
c The first option, compounding annually at 2.4% p.a. gave the best return on her investment.

8 a

	A	B
1	Principal \$, P	1000
2	Interest %, i	5
3	Compounds, n	3
4	Amount \$ A	$=B1*(1+B2/100)^{B3}$

b i B1 **ii** B2 **iii** B3

c $B1*(1 + B2/100)^{B3}$

d i \$ 1157.63 **ii** \$1331.00
iii \$1061.21

e i \$78 750.57 **ii** \$92 989.38
iii \$73 243.63

f Higher rate = more money

- 9 b i** B1 **ii** B2
c i The number of times the interest compounds per year
ii The power in the equation, n, is the number of compounds over the duration of the investment. It will be the product of years (B3) and how many compounds per year (B4)
iii The interest rate needs to match the compounding periods (so divide by B4) and written as a fraction (so divide by 100)
d i \$1197.09 **ii** \$1196.68
iii \$1195.62
e i \$78 990.40 **ii** \$78 974.39
iii \$78 933.00

f More interest is earned when interest compounds more frequently.

- 10 a i** 2.5% p.a. **ii** 2.6% p.a.
iii 2.7% p.a.
b i \$126 146.47 (13 compounds per year)
ii \$126 362.74
iii \$126 567.48
c Normally you would expect to earn more interest from more frequent compounding periods, but the lower rate means that the yearly compounds result in the most amount of interest earned.

Chapter checklist

1 a $I = Pin = 2682 \times \frac{4.2}{100} \times 3 = \$337.93$
b Balance = $2682 + 337.93 = \$3019.93$

2 a $A = P(1 + i)^n$
 $A = 3540 \left(1 + \frac{2.7}{100}\right)^4 = \$3938.08$

b Interest = $3938.08 - 3540 = \$398.08$

3 a $A = P(1 + i)^n$
 $A = 5.04 \left(1 + \frac{1.6}{100}\right)^{15} = 6.39 \text{ million}$

b Time = $2050 - 2018 = 32 \text{ years}$,
 $A = P(1 + i)^n$

$A = 5.04 \left(1 + \frac{1.6}{100}\right)^{32} = 8.38 \text{ million}$

4 Anson has \$19 098 after 2 years.

5 a 500

b = $B2*4/100*1/12$ (simple interest formula)

c = $B2 + C2$ (principal plus interest)

d = D2 (the previous months balance becomes the principal in the next month)

6 a i $A = P(1 + i)^n$
 $A = 7500 \left(1 + \frac{3.75}{100}\right)^5 = \$9015.75$

ii $A = P(1 + i)^n$
 $A = 7500 \left(1 + \frac{5.15}{100}\right)^5 = \$9640.68$

b The higher rate generated more interest and the final balance is larger as a result.

- 7 a i** $A = P(1 + i)^n$
 $A = 7500 \left(1 + \frac{5}{100}\right)^5 = \$9572.11$
ii $A = P(1 + i)^n$
 $A = 7500 \left(1 + \frac{5}{2600}\right)^{130} = \$9627.88$
b The more frequent compounding periods generated more interest and the final balance is larger as a result.

Chapter review

- 1 a** $I = Pin = 60\,000 \times \frac{2.65}{100} \times 3 = \$4770$
b Total value = $60\,000 + 4770 = \$64\,770$
2 a $I = Pin = 5\,000\,000 \times \frac{6.7}{100} \times \frac{90}{365}$
 $= \$82\,602.74$
b Total value = $\$5\,082\,602.74$

3 a

Year	Principal	Interest	Balance
1	\$1400.00	\$53.20	\$1453.20
2	\$1453.20	\$55.22	\$1508.42
3	\$1508.42	\$57.32	\$1565.74

- b** Interest = $\$165.74$
4 $A = P(1 + i)^n$
 $A = 2000 \left(1 + \frac{4.6}{1200}\right)^{48} = \$2403.19$
5 a $A = P(1 + i)^n$
 $A = 5200 \left(1 + \frac{3.2}{100}\right)^2 = \$5538.12$
b Interest = $5538.12 - 5200 = \$338.12$
6 a i Initial deposit = $\$0$
ii Regular deposit = $\$450$
iii Deposit frequency = fortnightly
iv Compound frequency = monthly
v Number of years = 5
vi Interest rate = 3.6
b i Balance = $\$63\,991$
ii $\$5491$
7 a i $A = P(1 + i)^n$
 $A = 3500 \left(1 + \frac{4.75}{100}\right)^3 = \$4022.82$
ii $A = P(1 + i)^n$
 $A = 3500 \left(1 + \frac{5.25}{100}\right)^3 = \$4080.70$
b The larger interest rate earns more interest and the final balance is greater.

- 8 a i** $i = \frac{4.2 \div 2}{100} = \frac{4.5}{200}, n = 5 \times 2 = 10$
 $A = P(1 + i)^n$
 $A = 3500 \left(1 + \frac{4.5}{200}\right)^{10} = \$4372.21$
ii $i = \frac{4.5 \div 52}{100} = \frac{4.5}{5200},$
 $n = 5 \times 52 = 260$
 $A = P(1 + i)^n$
 $A = 3500 \left(1 + \frac{4.5}{5200}\right)^{260} = \$4382.70$
b The more frequent compounding period earns more interest and the final balance is greater.
9 a $A = P(1 + i)^n$
 $A = 2.4 \left(1 + \frac{3.42}{100}\right)^5 = 2.8 \text{ million}$
b Time = $2100 - 2018 = 82$ years
 $A = P(1 + i)^n$
 $A = 2.4 \left(1 + \frac{3.42}{100}\right)^{82} = 37.8 \text{ million}$
10 a $A = P(1 + i)^n$
 $A = 3.30 \left(1 + \frac{2.1}{100}\right)^{15} = \$4.51$
b Time = $2050 - 2018 = 32$ years
 $A = P(1 + i)^n$
 $A = 3.30 \left(1 + \frac{2.1}{100}\right)^{32} = \$6.42$
11 a i $\$1250.00$
ii $\$1381.41$
b They are both calculated on a principal of $\$5000$.
c Simple interest is always calculated on the original principal. Compound interest is calculated on the principal and any interest that has been added to the account.
12 $A = P(1 + i)^n$
 $A = 1899 \left(1 - \frac{45}{100}\right)^2 = \$574.45$
13 a A2: 1
b B2: 3400
c C2: $= B2 * 4.25 / 100 * 1 / 12$
d D2: 360
e E2: $= B2 + C2 + D2$
f B3: $= E2$

Chapter 10

Pre-test

- 1 a \$2.25 b \$0.94
 c \$0.60 d \$16.81
 e \$10.50
- 2 a $I = Pin = 4000 \times \frac{6}{100} \times 2 = \$480$
 b $4000 + 480 = \$4480$
 c $4480 \div 24 = \$186.67$
- 3 a $48 \div 12 = 4$ years
 b $260 \div 52 = 5$ years
 c $78 \div 26 = 3$ years
 d $42 \div 12 = 3.5$ years = 3 years 6 months
 e $338 \div 52 = 6.5$ years = 6 years 6 months
 f $91 \div 26 = 3.5$ years = 3 years 6 months
 g $200 \div 12 = 16.67$ years = 16 years + 0.67
 $\times 12$ months = 16 years 8 months
 h $200 \div 52 = 3.846$ years = 3 years + 0.846
 $\times 12$ months = 3 years 10 months
 i $200 \div 26 = 7.692$ years = 7 years + 0.692
 $\times 12$ months = 7 years 8 months

Exercise 10A

- 1 a borrow, interest
 b remaining, interest
 c formula
 d adding, subtracting
 e principal
 f reduces
 g end, balance
 h calculators, total
- 2 a No. The repayments were likely rounded to the nearest dollar and have not quite covered the debt.
 b $205 + 0.66 = \$205.66$
 c \$1186.49
 d The interest gets less each month.
 e Total interest = $24.25 + \dots + 2.15 = \$160.68$
 f $\frac{12.65}{100} \times 2300 = \$290.95$. Reducing balance loans have the interest calculated on the remaining debt. Simple interest loans have the same amount of interest every payment period.
- 3 a Number of repayments = $12 \times 4 = 48$
 b Total repaid = $373 \times 48 = \$17\,904$
 c Interest = $17\,904 - 15\,000 = \$2904$

4 a

Month	Principal	Interest	Repayment	Balance
1	\$14 700.00	\$140.26	\$383.00	\$14 457.26
2	\$14 457.26	\$137.95	\$383.00	\$14 212.21
3	\$14 212.21	\$135.61	\$383.00	\$13 964.82

- b $383 \times 3 = \$1149$
 c $140.26 + 137.95 + 135.91 = \$413.82$
 d $1149 - 413.82 = \$735.18$
 e $14\,700 - 13\,964.82 = \$735.18$

5 a

Month	Principal	Interest	Repayment	Balance
1	\$5600.00	\$56.47	\$264.00	\$5392.47
2	\$5392.47	\$54.37	\$264.00	\$5182.84
3	\$5182.84	\$52.26	\$264.00	\$4971.10
4	\$4971.10	\$50.13	\$264.00	\$4757.23

b \$56.47

c $5600 - 5392.47 = \$207.53$

d \$50.13

e $4971.1 - 4757.23 = \$213.87$

f As the loan progresses, interest reduces each month and Stephanie is able to reduce her loan by more each payment.

6 a $2 \times 4 = 8$ repayments

b

Quarter	Principal	Interest	Repayment	Balance
1	\$28 000.00	\$304.50	\$3673.00	\$24 631.50
2	\$24 631.50	\$267.87	\$3673.00	\$21 226.37
3	\$21 226.37	\$230.84	\$3673.00	\$17 784.20
4	\$17 784.20	\$193.40	\$3673.00	\$14 304.61
5	\$14 304.61	\$155.56	\$3673.00	\$10 787.17
6	\$10 787.17	\$117.31	\$3673.00	\$7231.48
7	\$7231.48	\$78.64	\$3673.00	\$3637.12
8	\$3637.12	\$39.55	\$3673.00	\$3.68

c i \$3.68 is left owing after 8 payments.

ii The repayment amount is likely rounded to the nearest dollar.

iii He will need to pay \$3.68 as well as the \$3673 to clear the debt with his last payment.

d $I = Pin = 28\,000 \times \frac{4.35}{100} \times 2 = \$2436$

e Interest = $304.50 + \dots 39.55 = \$1387.68$

7 a \$455

b Total repaid = \$21 840

c Interest = $21\,840 - 18\,000 = \$3840$

8 a \$392

b Total repaid = $392 \times 36 = \$14\,112$

c Interest = $14\,112 - 12\,000 = \$2112$

9 a i \$1588

ii Total repaid = $1588 \times 12 \times 25 = \$476\,400$

iii Interest = $476\,400 - 250\,000 = \$226\,400$

b i It would now take 21 years, 8 months to pay off the loan.

ii Payments = $21 \times 12 + 8 = 260$
Total repaid = $260 \times 1700 = \$442\,000$

iii Interest = $442\,000 - 250\,000 = \$192\,000$

iv Saving = $226\,400 - 192\,000 = \$34\,400$

v Extra payment/month = $1700 - 1588 = \$112$

vi If you can afford to pay a little more each month, it generates a large saving in interest costs and pays the loan off more quickly.

10 a Monthly repayment = \$1459

b \$437 544

c \$177 544

d Years remaining = $25 - 20 = 5$ years
Balance remaining = \$76 900

e Half loan = $260\,000 \div 2 = 130\,000$,
9 years remaining is first time below \$130 000

10	\$139,217
20	\$127,809
30	\$115,988

Time taken = $25 - 9 = 16$ years

11 a Monthly repayment = \$468

b \$28 047

c \$6047

- d** After 3 years, leaves 2 years remaining

3	\$14,139
2	\$9,746
1	\$4,893

Balance remaining = \$9746

- e** Half the loan = $22\,000 \div 2 = \$11\,000$

It takes 3 years for the balance to fall below \$11 000.

- 12 a** Monthly repayment = \$94 105

- b** \$5 646 264

- c** \$646 264

- d** After 4 years, leaves 1 year remaining

3	\$2,062,062
2	\$1,010,279
1	\$0

Balance remaining = \$1 010 279

- e** Half the loan = 5 million $\div 2$
= \$2.5 million. It takes 3 years
(2 years remaining) for the debt to fall
below \$2.5 million.

Exercise 10B

- 1 a** online, spreadsheets

- b** costs

- c** calculations

- 2 a** Monthly repayment = \$524

- b** Total repayments = \$18 868.81

- c** Total interest = \$1868.81

- 3 a**

1	Month	Principal	Interest	Repayment	Balance
2	1	17000	$=B2*6.9/100*1/12$	524	$=B2+C2-D2$
3	2	$=E2$	$=B3*6.9/100*1/12$	524	$=B3+C3-D3$
4	3	$=E3$	$=B4*6.9/100*1/12$	524	$=B4+C4-D4$

1	Month	Principal	Interest	Repayment	Balance
2	1	\$ 17,000.00	\$ 97.75	\$ 524.00	\$ 16,573.75
3	2	\$ 16,573.75	\$ 95.30	\$ 524.00	\$ 16,145.05
4	3	\$ 16,145.05	\$ 92.83	\$ 524.00	\$ 15,713.88

- b** \$5.33 still left owing

1	Month	Principal	Interest	Repayment	Balance
35	34	\$ 1,559.33	\$ 8.97	\$ 524.00	\$ 1,044.30
36	35	\$ 1,044.30	\$ 6.00	\$ 524.00	\$ 526.31
37	36	\$ 526.31	\$ 3.03	\$ 524.00	\$ 5.33

- c** Last payment = $524 + 5.33 = \$529.33$

- d** Half loan = $17\,000 \div 2 = \$8500$

It takes 19 months to pay the first half of the loan.

- 4 a** Fortnightly payment = \$127

- b** $2 \times 26 = 52$

- c** Total loan repayments = \$6613.91

- d** Total interest = \$613.91

- 5 a**

1	Month	Principal	Interest	Repayment	Balance
2	1	6000	$=B2*9.73/100*1/26$	127	$=B2+C2-D2$
3	2	$=E2$	$=B3*9.73/100*1/26$	127	$=B3+C3-D3$
4	3	$=E3$	$=B4*9.73/100*1/26$	127	$=B4+C4-D4$

- b** $I = Pin = 6000 \times \frac{9.73}{100} \times \frac{1}{26} = \$22.45$

- c** \$10.91

- d** $127 + 10.91 = \$137.91$

- e** Half loan = $6000 \div 2 = \$3000$. It takes
28 fortnights to repay half the loan.

- 6 a i** Interest = $Pin = 20\,000 \times \frac{11.5}{100} \times 5$
= \$11 500

- ii** Total to repay = $20\,000 + 11\,500$
= \$31 500

- iii** Number of repayments = $12 \times 5 = 60$

- iv** Monthly repayment = $31\,500 \div 60$
= \$525

- b i** Simple interest for one month

$$= Pin = 20\,000 \times \frac{11.5}{100} \times \frac{1}{12}$$

- ii** The balance owed at the end of the month is the previous amount owed plus interest and take away the repayment made.

- iii** Simple interest is the same every time period as it is always calculated on the original loan amount.

- c** Yes, the amount corresponds. Both are \$11 500.

- d** Yes, the amount corresponds. Both are \$31 500.

- e** They are always the same amount.

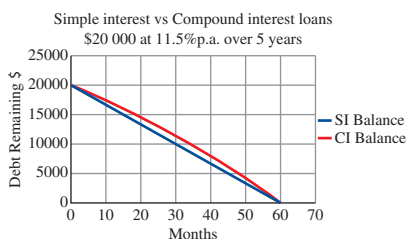
- 7 a i** Monthly repayment = \$440 to nearest dollar

- ii** Total to repay = \$26 391.13

- iii** Total interest = \$6391.13

- b i** Interest for one month
 $= Pin = \text{Principal} \times \frac{11.5}{100} \times \frac{1}{12}$
- ii** The balance owed at the end of the month is the previous amount owed plus interest and take away the repayment made.
- iii** The principal in the second month is the same as the closing balance of the previous month.
- c** The values are reducing.
- d i** It is not quite the same \$6388.09 vs \$6391.13. This is because of rounding.
- ii** Compound loan interest = \$6388.09, Simple loan interest = \$11 500, Difference in interest
 $= 11\,500 - 6388.09 = \$5111.91$
- e i** It is not quite the same \$26 400 vs \$26 391.13. This is because of rounding.
- ii** Compound loan total = \$26 400, Simple loan total = \$31 500, Difference in total paid
 $= 31\,500 - 26\,400 = \$5100.00$

8 a, b



- c** The simple interest loan produces a straight-line graph. The compound interest loan produces a curved-line graph.

Exercise 10C

- 1** impact, repay, interest
- 2 a i** \$1741 per month
- ii** Total interest = \$235 066
- b i** \$804 per fortnight, Interest = \$234 440
- ii** Savings = $235\,066 - 234\,440 = \$626$
- c i** Total interest = \$203 714, Savings = $235\,066 - 203\,714 = \$31\,352$
- ii** Repaid in 22 years 2 months. Time saved = 2 years 10 months
- d** An extra \$40 per fortnight
- 3 a i** \$1352 per month
- ii** Total interest = \$139 274
- b i** \$624 per fortnight.
- ii** Interest = \$138 822, Savings = $139\,274 - 138\,822 = \$452$
- c i** Interest = \$121 140, Savings = $139\,274 - 121\,140 = \$18\,134$
- ii** Repaid in 17 years 10 months, Time saved = 2 years 2 months
- d** Her new repayment would be \$24 less per fortnight.
- 4 a** \$1310 per month
- b** Interest = $(1310 \times 12 \times 20) - 180\,000 = \$134\,400$
- c** \$1295 per month
- d** Interest = $(1295 \times 12 \times 20) - 180\,000 = \$130\,800$, Saving = \$3600
- 5 a** \$815 per month
- b** Interest = $(815 \times 12 \times 5) - 40\,000 = \$8900$
- c** \$821 per month
- d** He needs to find an extra \$6 per month for his loan repayment. If he had a bigger loan, finding extra money could be difficult.
- e** Interest = $(821 \times 12 \times 5) - 40\,000 = \$9260$, Extra interest = \$360
- 6 a** \$1732 per month
- b** Total cost = $1732 \times 12 \times 25 = \$519\,600$
- c** $1732 \div 2 = \$866$ per fortnight
- d** 21 years
- e** Total cost = $866 \times 26 \times 21 = \$472\,836$
- f** He should pay his loan each fortnight. He pays it off 4 years faster and saves himself $(519\,600 - 472\,836) \$46\,764$ in interest charges.
- 7 a i** \$2066 per month
- ii** Total interest = \$304 887 (from website)

- b i** Fortnightly payment = $2066 \div 2$
= \$1033
- ii** Total interest = \$247 374 (from website) Savings = $304\,887 - 247\,374$
= \$57 513
Time to repay = 21 years, Time saved = 4 years
- c** New rate = $6.19 + 0.5 = 6.69\%$ p.a.
New monthly payment = \$2164
New fortnightly payment = \$1082
Additional payment = $1082 - 1033$
= \$49 per fortnight
- 8 a i** \$2047 per month
- ii** Total interest = \$205 362 (from website)
- b i** \$1841 per month
- ii** Total interest = \$266 286 (from website)
- iii** Extra interest = $266\,286 - 205\,362$
= \$60 924
- c i** Fortnightly payment = $1841 \div 2 = \$921$
- ii** Total interest = \$216 978 (from website) Savings = $266\,286 - 216\,978$
= \$49 308
Time = 21 years 1 month,
Time saving = 3 years 11 months
- d** New rate = $5.99 - 0.25 = 5.74\%$ p.a.
New monthly payment = \$1798
New fortnightly payment = \$899
Reduction in payment = $921 - 899$
= \$22 per fortnight

Exercise 10D

- 1 a** interest, frequency, impact, pay, cost
b changes, length, interest
- 2 a** 12
b 52
c 365
d 26
e 2
f 13
g 4

- 3 a i** \$1970.96
ii \$591 287.49
iii \$295 287.49
- b i** \$1952.62
ii \$585 786.41
iii \$289 786.41
- c** $\$295\,287.49 - \$289\,786.41 = \$5501.08$
- 4 a** Repayment = \$1816
- c** Total interest = \$170 591.37
- d** Repay = $1816 \div 2 = \$908$ per fortnight
- e** First balance less than \$0 at 452 fortnightly payments
Time = $452 \div 26 = 17.38$ years
= 17 years 5 months

No. Repayments	Principal	Interest	Repayment	Balance
451	\$ 1,667.63	\$ 3.50	\$ 908.00	\$ 763.12
452	\$ 763.12	\$ 1.60	\$ 908.00	\$ 143.28
453	\$ 143.28	\$ 0.30	\$ 908.00	\$ 1,051.58

- f i** Total interest = \$145 272.72,
ii Interest saved = $170\,591.37 - 145\,272.72 = \$25\,318.65$
Time saved = 20 years – 17 years 5 months
= 2 years 7 months
- 5 a** Monthly repayment = \$2040

	A	B	C	D	E
1	Principal	305000			
2	Rate	5.16			
3	Years	20			
4	Repayments/year	12			
5	Repayment	2040			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
8	1	=D1	=B8*5852/100*1/5054	=5655	=D8+C8-D8
9	2	=C8	=B9*5852/100*1/5054	=5655	=D9+C9-D9
10	3	=C9	=B10*5852/100*1/5054	=5655	=D10+C10-D10

	A	B	C	D	E
1	Principal	305000			
2	Rate	5.16			
3	Years	20			
4	Repayments/year	12			
5	Repayment	\$2,040.00			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
8	1	\$ 305,000.00	\$ 1,311.50	\$ 2,040.00	\$ 304,271.50
9	2	\$ 304,271.50	\$ 1,308.37	\$ 2,040.00	\$ 303,539.87
10	3	\$ 303,539.87	\$ 1,305.22	\$ 2,040.00	\$ 302,805.09

- c** Total interest = \$184 566.75
- d** New payment = $\$2040 + 40 = \$2080$
Time taken to repay loan is now
233 months = 19 years 5 months
- e i** Total interest = \$177 667.51
ii Interest saved = $184\,566.75 - 177\,667.51 = \$6\,899.24$
Time saved = $240 - 233 = 7$ months

Chapter checklist

1 a

Month	Principal	Interest	Repayment	Balance
1	\$16 000.00	\$154.13	\$750.00	\$15 404.13
2	\$15 404.13	\$148.39	\$750.00	\$14 802.52
3	\$14 802.52	\$142.60	\$750.00	\$14 195.12

b \$154.13

c $16\,000 - 15\,404.13 = \$595.87$

d \$142.60

e $14\,802.52 - 14\,195.12 = \$607.40$

f Brooke is increasing the amount she pays off her debt each month as the interest reduces.

2 a

Month	Principal	Interest	Repayment	Balance
1	\$23,000.00	\$213.71	\$ 596.00	\$22,617.71
2	\$22,617.71	\$210.16	\$ 596.00	\$22,231.86
3	\$22,231.86	\$206.57	\$ 596.00	\$21,842.44
4	\$21,842.44	\$202.95	\$ 596.00	\$21,449.39
5	\$21,449.39	\$199.30	\$ 596.00	\$21,052.69

b \$7.45 is left owing

c $\$596 + 7.45 = \$603.45$

Month	Principal	Interest	Repayment	Balance
47	\$ 1,182.90	\$ 10.99	\$ 596.00	\$ 597.89
48	\$ 597.89	\$ 5.56	\$ 603.45	\$ 0.00
		\$ 5,615.45	\$28,615.45	

d Total repaid = \$28 615.45

e Total interest = \$5615.45

f Half the loan = $23\,000 \div 2 = \$11\,500$
Balance fall below \$11 500 after
27 months

3 a Monthly repayment = \$619

b Total repaid = $619 \times 48 = \$29\,712$

c Interest = $29\,712 - 24\,000 = \$5712$

4 a $I = Pin = 24\,000 \times \frac{10.85}{100} \times 4 = \$10\,416$

b Total to repay = $24\,000 + 10\,416 = \$34\,416$

c No. = $4 \times 12 = 48$

d Repayment = $\$34\,416 \div 48 = \$717$

e Monthly repayments: Megan = \$619, Joe = \$717. Joe pays \$98 more per month.
Overall cost: Megan = \$29 712, Joe = \$34 416. Joe pays \$4704 more for his loan.

5 a i Repayment = \$1504 per month

ii Total interest \$147 951

b i Interest = \$138 046, Interest saving = $147\,951 - 138\,046 = \$9905$

ii Time to repay = 18 years 10 months
Time saved = 1 year 2 months

c She would save \$60 per month on her loan repayment.

6 a

	A	B	C	D	E
1	Principal	365000			
2	Rate	5.02			
3	Years				
4	Repayment	2138			
5	Repayments/year	12			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
8	1	\$ 365,000.00	\$ 1,526.92	\$ 2,138.00	\$ 364,388.92
9	2	\$ 364,388.92	\$ 1,524.36	\$ 2,138.00	\$ 363,775.28
10	3	\$ 363,775.28	\$ 1,521.79	\$ 2,138.00	\$ 363,159.07

b Loan repaid in 301 months = 25 years 1 month. (Only \$5.36 left owing after 300 months)

c Total interest = \$276 405.39

d Fortnightly payment = $2138 \div 2 = \$1069$

e Loan repaid in 559 fortnights = 21.5 years

	A	B	C	D	E
1	Principal	365000			
2	Rate	5.02			
3	Years				
4	Repayment	1069			
5	Repayments/year	26			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
564	557	\$ 2,272.55	\$ 4.39	\$ 1,069.00	\$ 1,207.93
565	558	\$ 1,207.93	\$ 2.33	\$ 1,069.00	\$ 141.27
566	559	\$ 141.27	\$ 0.27	\$ 1,069.00	\$ 927.46

- f** Total interest = \$231 643.54
 Saving in interest = 276 405.39 –
 231 643.54 = \$44 761.85
 Time saving = 25 years 1 month –
 21 years 6 months = 3 years 7 months

Chapter review

- 1 a** No. There is still \$1.35 left owing. This is because the monthly repayments must have been rounded down to the nearest dollar.
b $281 + 1.35 = \$282.35$
c She still owes \$1639.89 after 6 months.
d \$26.27 interest in the first month
e $3200 - 2945.27 = \$254.73$ reduced loan balance in first month
f \$19.94 interest in the fourth month
g $2429.51 - 2168.45 = \$261.06$ reduced loan balance in fourth month

- h** She is reducing the debt by a greater amount each month as the interest reduces.
i The interest gets less each month as it is calculated on the reducing balance.
j \$173.36 total interest
k $\frac{9.82}{100} \times 3200 = \$315.20$ This is simple interest and it is calculated on the original balance every month, unlike reducing balance interest which is calculated on a smaller amount each month.

- 2 a** Repayments = $4 \times 12 = 48$
b Repaid = $512 \times 48 = \$24\,576$
c Interest = $24\,576 - 19\,500 = \$5076$
3 a Repayments = $25 \times 12 = 300$
b Repaid = $1915 \times 300 = \$574\,500$
c Interest = $574\,500 - 285\,000 = \$289\,500$
4 $I = Pin = 2800 \times \frac{11.8}{100} \times 2 = \$660.80$
 Repay = $2800 + 660.8 = \$3460.80$
 No. Repayments = $2 \times 12 = 24$
 Repayment = $3460.80 \div 24 = \$144.20$

5 a

Month	Principal	Interest	Repayment	Balance
1	\$18 500.00	\$171.43	\$479.00	\$18 192.43
2	\$18 192.43	\$168.58	\$479.00	\$17 882.02
3	\$17 882.02	\$165.71	\$479.00	\$17 568.72

- b** Total repayments = \$1437.00
c Total interest = \$505.72
d Difference = $1437.00 - 505.72 = \$931.28$
e Debt reduced by = $18\,500 - 17\,568.72 = \$931.28$
6 a Monthly repayment = \$1968
b Total repaid = \$590 400
c Total interest = \$272 400
d Still owing = \$101 341
e Half the loan = \$159 000; With 8 years remaining she still owes \$152 119.80. Takes 17 years to pay the first half of the loan.
7 a i Repayment = \$1537 per month
ii Interest charges = \$213 810
b Repayment = \$709 per fortnight
 Interest = \$213 230
 Savings = $213\,810 - 213\,230 = \$580$
c i Interest = \$181 015,
 Savings = $213\,810 - 181\,015 = \$32\,795$
ii Time to repay = 21 years 9 months,
 Saving = 3 years 3 months
d Repayments would go up by \$71 per fortnight

- 8 a i** \$2584 per month
ii Interest = \$234 160
b i Repayment = \$2295 per month
ii Interest \$302 500
iii Additional interest = 302 500 – 234 160 = \$68 340
c i Fortnightly payment = $2295 \div 2$ = \$1148
ii Interest = \$252 245,
 Saving = 302 500 – 252 245 = \$50 255
 Time to repay = 21 years 5 months
 Time saving = 3 years 7 months
d New interest rate = $5.17 - 0.15$ = 5.02% p.a.
 New monthly repayment = \$2261
 New fortnightly repayment = \$1131 (nearest dollar)
 Savings per fortnight = 1148 – 1131 = \$17

9 a

Month	Principal	Interest	Repayment	Balance
1	\$31,000.00	\$ 264.79	\$ 790.00	\$30,474.79
2	\$30,474.79	\$ 260.31	\$ 790.00	\$29,945.10
3	\$29,945.10	\$ 255.78	\$ 790.00	\$29,410.88

- b** Overpaid loan by \$1.94
c $790.00 - 1.94 = \$788.06$
d Total loan repayments = \$37 918.06
e Total interest = \$6918.06
f Half the loan = $31\,000 \div 2 = \$15\,500$.
 The 27th month balance falls below \$15 500.

10 a

	A	B	C	D	E
1	Principal	245000			
2	Rate	6.15			
3	Years				
4	Repayment	1601			
5	Repayments/year	12			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
8	1	\$245,000.00	\$1,255.63	\$1,601.00	\$244,654.63
9	2	\$244,654.63	\$1,253.85	\$1,601.00	\$244,307.48
10	3	\$244,307.48	\$1,252.08	\$1,601.00	\$243,958.56

- b** Loan repaid after 301 months, or 25 years 1 month (only \$56.03 owing after 300 months)
c Interest = \$235 356.31
d $1601 \div 2 = \$801$ (to nearest dollar)
e Loan repaid after 545 fortnights, or 21 years
f Interest = \$190 837.47, Saving = $235\,356.31 - 190\,837.47 = \$44\,518.84$
 Time saving = 25 years 1 month – 21 years = 4 years 1 month

11 a

	A	B	C	D	E
1	Principal	335000			
2	Rate	6.7			
3	Years				
4	Repayment	2200			
5	Repayments/year	12			
6					
7	No. Repayments	Principal	Interest	Repayment	Balance
8	1	\$335,000.00	\$1,730.83	\$2,200.00	\$334,530.83
9	2	\$334,530.83	\$1,728.41	\$2,200.00	\$334,058.24
10	3	\$334,058.24	\$1,725.97	\$2,200.00	\$333,585.22

- b** Loan repaid after 300 months, or 25 years
c Total interest = \$324 678.39
d New payment = \$2250, Time to repay = 285 months or 23 years 9 months
e Interest = \$305 269.69, Saving = $324\,678.39 - 305\,269.69 = \$19\,408.70$
 Time saving = 25 years = 23 years 9 months = 1 year 3 months