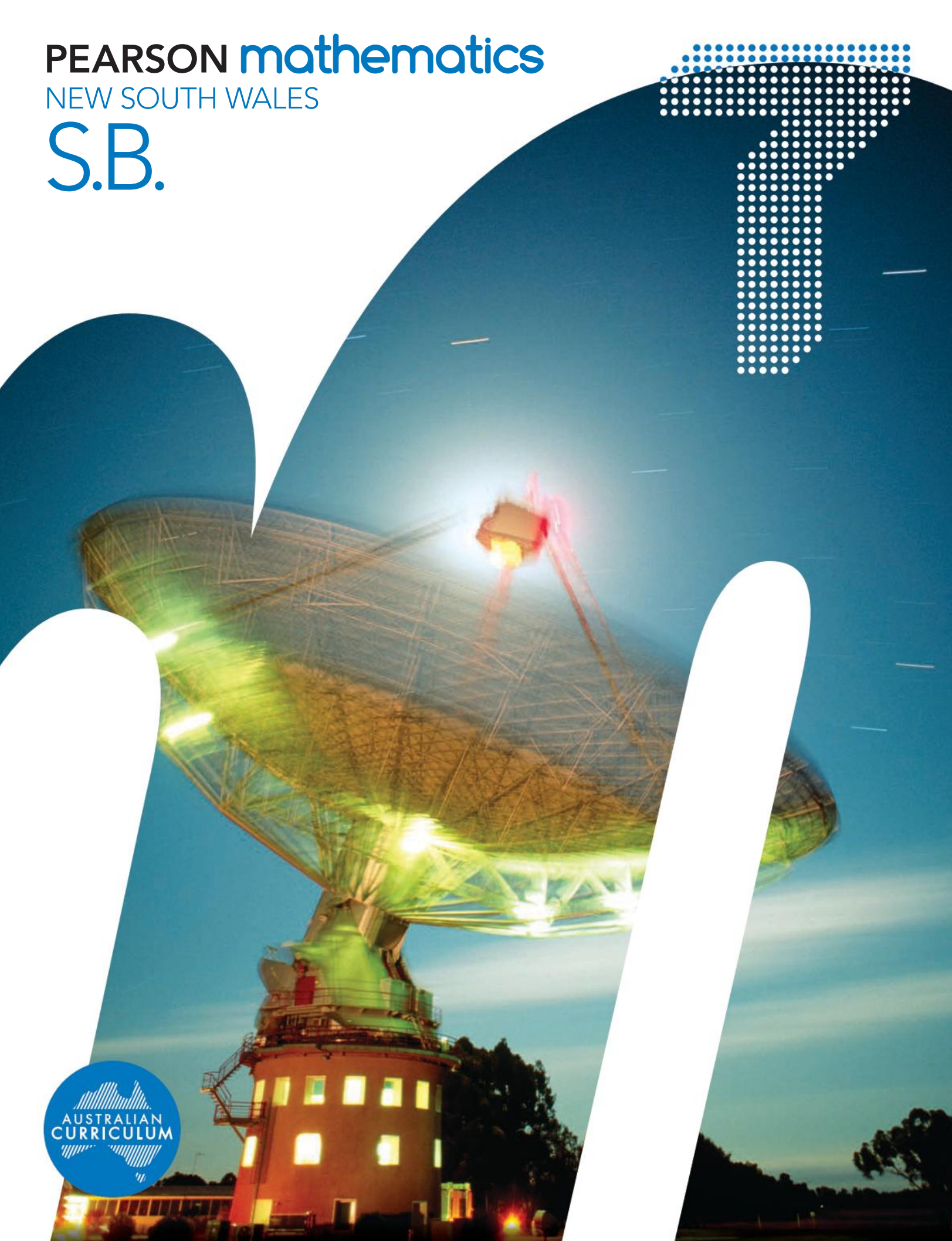
PEARSON mathematics

NEW SOUTH WALES

S.B.





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I am Maths
Calculating risk
Climbing to ever greater heights

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We would like to thank our authors for their extraordinary dedication and their contribution to the development of this project.



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Fibonacci poem

The poem on the inside front cover is called a Fib. The sequence of syllables in the lines follows a Fibonacci sequence: 1, 1, 2, 3, 5, 8.

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PEARSON mathematics
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Pearson Mathematics New South Wales provides you with a 'nuts and bolts', no-nonsense Mathematics series that saves you time and delivers differentiated pathways for students of all abilities, at an affordable price. We've built this series from the ground up specifically for the Australian Curriculum, developing it for the NSW syllabus in conjunction with the national edition. We've found it important to create our resources for the Australian Curriculum from scratch to ensure we can incorporate the latest research about how students learn Mathematics best.

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- worked solutions to questions
- over 150 Bridging Worksheets to support and scaffold students across Years 7 and 8 who struggle with secondary mathematics concepts, so that they are able to complete exercises
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ProductLink

Digital teacher resources including editable teaching programs, syllabus correlations, Recall worksheets and chapter tests are available for free when you purchase **Pearson Mathematics New South Wales**. To access, visit **www.pearsonplaces.com.au** and log in. Click on 'Toolkit', then select 'ProductLink' and browse to your title.

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Pearson Mathematics New South Wales provides a differentiated pathway for every student, enabling you to support the full range of student abilities.

Pearson Mathematics New South Wales differentiated pathway structure

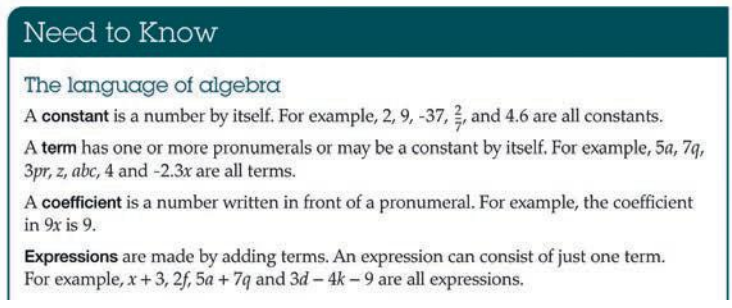
Recall	Each chapter has a Recall section that tests whether students have the prerequisite skills and understanding to begin a chapter. Every Recall question has a matching Recall Worksheet available free on ProductLink that teaches the relevant skill and allows students to practise it.	**Below Stage Level**
Bridging	Over 150 Bridging Worksheets are included with Pearson eBook 3.0 to support and scaffold students across Years 7 and 8 who struggle with secondary mathematics concepts, so that they are able to complete specific Student Book exercises.	
Foundation	A Foundation pathway through each exercise is shown in the Navigator feature to ensure students at this level are given sufficient skills practice as well as some exposure to the higher-order aspects of Working Mathematically.	**3 Navigator Pathways At Stage Level**
Standard	A Standard pathway through each exercise is shown in the Navigator feature to ensure students at this level are given a balance of skills practice and the higher-order aspects of Working Mathematically.	
Advanced	An Advanced pathway through each exercise is shown in the Navigator feature to ensure students at this level are given skills practice as well as an emphasis on the higher-order aspects of Working Mathematically.	
Challenge	Each chapter includes a Challenge page of high-level extension work for the most able students.	**Above Stage Level**

Pearson Mathematics New South Wales provides students with just the right amount of information in one no-frills book at each year level.

In Years 9 and 10, **Pearson Mathematics New South Wales** covers the full range of Stages 5.1, 5.2 and 5.3 (and Stage 4 revision where relevant), with content from all Stages explicitly labelled for maximum flexibility

Stage 4 Stage 5.1 Stage 5.2 Stage 5.3

Need to Know sections provide concise, accurate theory that promotes understanding.



Need to Know

The language of algebra

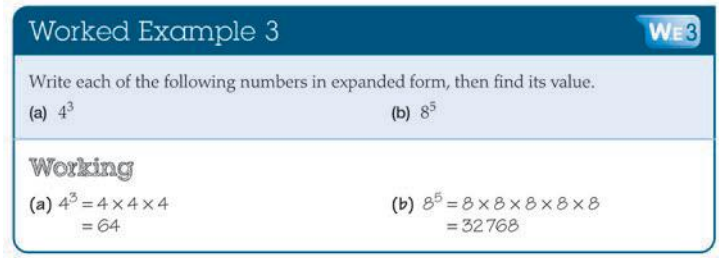
A **constant** is a number by itself. For example, 2, 9, -37, $\frac{2}{7}$, and 4.6 are all constants.

A **term** has one or more pronumerals or may be a constant by itself. For example, $5a$, $7q$, $3pr$, z, abc, 4 and $-2.3x$ are all terms.

A **coefficient** is a number written in front of a pronumeral. For example, the coefficient in $9x$ is 9.

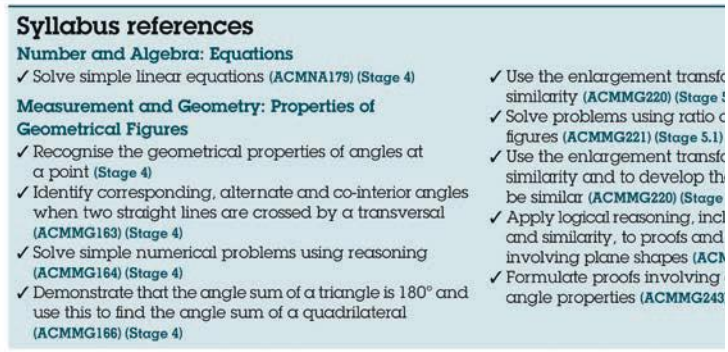
Expressions are made by adding terms. An expression can consist of just one term. For example, $x + 3$, $2f$, $5a + 7q$ and $3d - 4k - 9$ are all expressions.

Worked Examples clearly demonstrate the working that students will need to show to achieve full marks in a test.

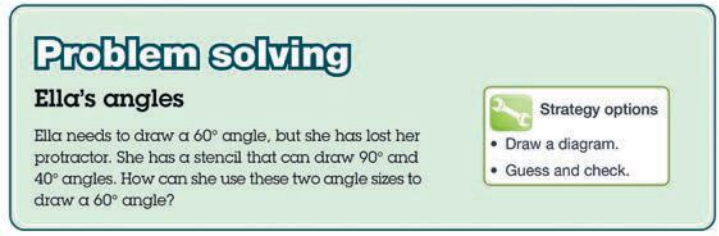


Pearson Mathematics New South Wales is a perfect match to the NSW Syllabus for the Australian Curriculum, including all the higher-order aspects of Working Mathematically and the technology requirements.

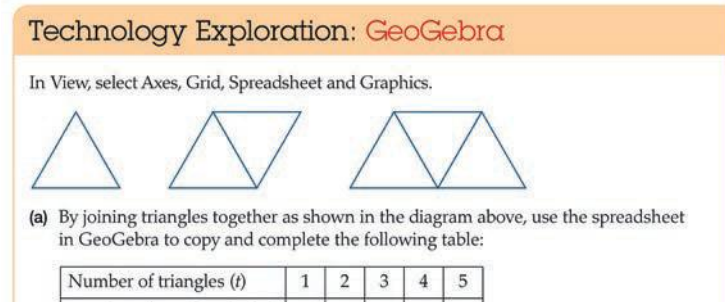
Syllabus references are provided at the start of every chapter, while a detailed **Syllabus Correlation** document (available free on ProductLink) shows where every element of syllabus content is covered.



Questions in every exercise are categorised into 'Fluency and Understanding' and 'Reasoning, Open-ended and Communicating'. High quality **Problem solving** tasks also appear regularly which refer to strategies from the Problem solving toolkit.



Technology Explorations appear regularly throughout the text, providing students with a deeper understanding of concepts covered and making it easy to integrate ICT into the classroom.



Pearson Mathematics New South Wales provides regular graded revision and reinforcement to help your students consolidate their learning.

Half-time—mid-chapter review



Chapter review—end-of-chapter review



Mixed review—cumulative multi-chapter review



NAPLAN practice—end-of-chapter section to help students prepare for NAPLAN tests



PEARSON mathematics
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Stage 4 Curriculum Grid for Year 7

This maps the NSW Mathematics Syllabus (Stage 4) for the Australian Curriculum content to the *Pearson Mathematics New South Wales 7* student book.

Detailed syllabus references are given at the start of each chapter. A full Stage 4 curriculum audit, including content elaborations, is available on ProductLink.

Stage 4	Year 7
Number and Algebra	
Computation with Integers	Ch. 1,Ch. 2
Apply the associative, commutative and distributive laws to aid mental and written computation **(ACMNA151)**	1.1, 1.3, 1.4
Compare, order, add and subtract integers **(ACMNA280)**	2.4, 2.5, 2.6, 2.7
Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies **(ACMNA183)**	1.1, 1.3, 1.5, 2.4, 2.5, 2.6, 2.7
Fractions, Decimals and Percentages	Ch. 3, Ch. 4
Compare fractions using equivalence; locate and represent positive and negative fractions and mixed numerals on a number line **(ACMNA152)**	3.1, 3.2, 3.3
Solve problems involving addition and subtraction of fractions, including those with unrelated denominators **(ACMNA153)**	3.4
Multiply and divide fractions and decimals using efficient written strategies and digital technologies **(ACMNA154)**	3.5, 3.6, 3.7, 4.5, 4.6
Express one quantity as a fraction of another, with and without the use of digital technologies **(ACMNA155)**	3.1, 3.3
Round decimals to a specified number of decimal places **(ACMNA156)**	4.2
Investigate terminating and recurring decimals **(ACMNA184)**	4.3
Connect fractions, decimals and percentages and carry out simple conversions **(ACMNA157)**	4.7
Find percentages of quantities and express one quantity as a percentage of another, with and without the use of digital technologies **(ACMNA158)**	4.7, 4.8
Financial Mathematics	Ch. 4
Investigate and calculate 'best buys', with and without the use of digital technologies **(ACMNA174)**	4.10
Ratio and Rates	Ch. 4, Ch. 5
Recognise and solve problems involving simple ratios **(ACMNA173)**	4.9
Solve a range of problems involving ratios and rates, with and without the use of digital technologies **(ACMNA188)**	4.9, 4.10
Investigate, interpret and analyse graphs from authentic data **(ACMNA180)**	5.9
Algebraic Techniques 1	Ch. 5
Introduce the concept of variables as a way of representing numbers using letters **(ACMNA175)**	5.1, 5.2, 5.4, 5.6
Extend and apply the laws and properties of arithmetic to algebraic terms and expressions **(ACMNA177)**	5.1, 5.2, 5.3, 5.6, 5.9
Simplify algebraic expressions involving the four operations **(ACMNA192)**	5.6
Algebraic Techniques 2	Ch. 5
Create algebraic expressions and evaluate them by substituting a given value for each variable **(ACMNA176)**	5.3, 5.4, 5.5
Indices	Ch. 1, Ch. 2
Investigate index notation and represent whole numbers as products of powers of prime numbers **(ACMNA149)**	1.2, 1.5, 2.1, 2.3
Investigate and use square roots of perfect square numbers **(ACMNA150)**	1.2

Equations	Ch. 5, Ch. 7
Solve simple linear equations **(ACMNA179)**	5.2, 7.1, 7.2, 7.3, 7.4, 7.5
Solve linear equations using algebraic techniques and verify solutions by substitution **(ACMNA194)**	7.4, 7.5
Linear Relationships	Ch. 5, Ch. 10
Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point **(ACMNA178)**	5.7
Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates **(ACMMG181)**	10.1, 10.2, 10.3, 10.4
Plot linear relationships on the Cartesian plane, with and without the use of digital technologies **(ACMNA193)**	5.5, 5.8

Measurement and Geometry	
Length	Ch. 6
Find perimeters of parallelograms, trapeziums, rhombuses and kites **(ACMMG196)**	6.2
Area	Ch. 6
Choose appropriate units of measurement for area and convert from one unit to another **(ACMMG195)**	6.3
Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving **(ACMMG159)**	6.3, 6.4, 6.5
Volume	Ch. 6, Ch. 10
Draw different views of prisms and solids formed from combinations of prisms **(ACMMG161)**	10.6, 10.7
Develop the formulas for the volumes of rectangular and triangular prisms and of prisms in general; use formulas to solve problems involving volume **(ACMMG198)**	6.6
Properties of Geometrical Figures 1	Ch. 8, Ch. 10
Classify triangles according to their side and angle properties and describe quadrilaterals **(ACMMG165)**	8.6, 8.7
Identify line and rotational symmetries **(ACMMG181)**	10.5
Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral **(ACMMG166)**	8.6
Use the properties of special triangles and quadrilaterals to solve simple numerical problems with appropriate reasoning	8.6, 8.7
Angle Relationships	Ch. 8
Use the language, notation and conventions of geometry	8.2
Recognise the geometrical properties of angles at a point	8.2, 8.7
Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal **(ACMMG163)**	8.4
Investigate conditions for two lines to be parallel **(ACMMG164)**	8.4
Solve simple numerical problems using reasoning **(ACMMG164)**	8.3, 8.4

Statistics and Probability	
Data Collection and Representation	Ch. 9
Investigate techniques for collecting data, including census, sampling and observation **(ACMSP284)**	9.1, 9.3
Construct and compare a range of data displays, including stem-and-leaf plots and dot plots **(ACMSP170)**	9.1, 9.3, 9.4, 9.5, 9.6
Single Variable Data Analysis	Ch. 9
Calculate mean, median, mode and range for sets of data and interpret these statistics in the context of data **(ACMSP171)**	9.2
Investigate the effect of individual data values, including outliers, on the mean and median **(ACMSP207)**	9.2
Describe and interpret data displays using mean, median and range **(ACMSP172)**	9.2, 9.6
Probability 1	Ch. 9
Construct sample spaces for single-step experiments with equally likely outcomes **(ACMSP167)**	9.7
Assign probabilities to the outcomes of events and determine probabilities for events **(ACMSP168)**	9.7
Identify complementary events and use the sum of probabilities to solve problems **(ACMSP204)**	9.7

1 Whole numbers

Recall 1

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

1 What is the place value of the red digit?

(a) 45 783

A eight **B** tens **C** eighty-three **D** hundreds

(b) 1 264 184

A two **B** hundreds **C** ten thousands **D** hundred thousands

2 **(a)** Rearrange the following numbers in ascending order (from smallest to largest).

567, 4500, 0, 74, 11 100, 6008, 12, 602

(b) Rearrange the following numbers in descending order (from largest to smallest).

1200, 204, 987, 2196, 240, 95, 2400, 1010

3 Find:

(a) 50 000 + 6000 + 800 + 90 + 5

(b) 7 000 000 + 20 000 + 5000 + 70 + 3

4 **(a)** Round 1245 off to the nearest:

(i) 10 **(ii)** 100 **(iii)** 1000

(b) Round 8983 off to the nearest:

(i) 10 **(ii)** 100 **(iii)** 1000

5 Find:

(a) $3 \times 2 \times 3$ **(b)** $5 \times 3 \times 3 \times 2$ **(c)** $2 \times 2 \times 2$ **(d)** $10 \times 10 \times 10 \times 10$

6 Set out these calculations in your preferred way and work out the answers.

(a) 456 + 56 **(b)** 16 + 2047 **(c)** 90 + 1267 + 341

7 Set out these calculations in your preferred way and work out the answers.

(a) 298 – 123 **(b)** 854 – 227 **(c)** 1406 – 249

8 Set out these calculations in your preferred way and work out the answers.

(a) 45×7 **(b)** 134×5 **(c)** 34×95

9 Set out these calculations in your preferred way and work out the answers.

(a) $844 \div 4$ **(b)** $3708 \div 9$ **(c)** $897 \div 7$

Syllabus references

Number and Algebra: Computation with Integers

- ✓ Apply the associative, commutative and distributive laws to aid mental and written computation **(ACMNA151)**
- ✓ Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies **(ACMNA183)**

Number and Algebra: Indices

- ✓ Investigate index notation and represent whole numbers as products of powers of prime numbers **(ACMNA149)**
- ✓ Investigate and use square roots of perfect square numbers **(ACMNA150)**

Mental strategies

1.1

Need to Know

Using the commutative law

The **commutative law** states that reversing the order in which any two numbers are added or multiplied does not affect the result. For example, $4 + 5 = 5 + 4$ and $2 \times 3 = 3 \times 2$.

Using the associative law

The **associative law** states that, when adding or multiplying three or more numbers, combining any two adjacent numbers does not affect the result.

For example, $(6 + 7) + 8 = 6 + (7 + 8)$ and $(2 \times 3) \times 5 = 2 \times (3 \times 5)$.

The commutative and associative laws do *not* apply to subtraction and division.

For example, $8 - 2 \neq 2 - 8$, $6 \div 2 \neq 2 \div 6$, $(9 - 5) - 4 \neq 9 - (5 - 4)$ and $(40 \div 10) \div 2 \neq 40 \div (10 \div 2)$.

Multiples of 10 are easy to add and multiply so the commutative and associative laws can be used to create multiples of 10.

For example, $43 + 34 + 16 = 43 + (34 + 16)$, $7 \times 15 \times 2 = 7 \times (15 \times 2)$, $7 + 9 + 3 = 7 + 3 + 9$ and $5 \times 37 \times 2 = 5 \times 2 \times 37$.

Using the distributive law

The **distributive law** allows a large number to be multiplied by another number by splitting it into parts with addition or subtraction before each part is multiplied by the other number and the **product** of the parts combined to find the total product.

Step 1 To multiply a large number, split it into 10s and 1s (or 100s, 10s and 1s).

For example: $7 \times 16 = 7 \times (10 + 6)$ and $7 \times 28 = 7 \times (30 - 2)$.

Step 2 Multiply each of these numbers separately.

7 lots of 10 plus 7 lots of 6	and	7 lots of 30 minus 7 lots of 2
$= 7 \times 10 + 7 \times 6$		$= 7 \times 30 - 7 \times 2$
$= 70 + 42$		$= 210 - 14$

Step 3 Add or subtract each of the products.

$= 112$	and	$= 196$

Worked Example 1

WE1

Calculate the following using the associative or commutative laws.

(a) $8 + 17 + 3$ **(b)** $17 \times 5 \times 20$ **(c)** $7 + 32 + 13$ **(d)** $2 \times 13 \times 5$

Working

(a) $8 + 17 + 3$
$= 8 + (17 + 3)$
$= 8 + 20$
$= 28$

(b) $17 \times 5 \times 20$
$= 17 \times (5 \times 20)$
$= 17 \times 100$
$= 1700$

(c) $7 + 32 + 13$
$= 7 + 13 + 32$
$= 20 + 32$
$= 52$

(d) $2 \times 13 \times 5$
$= 2 \times 5 \times 13$
$= 10 \times 13$
$= 130$

Worked Example 2

WE2

Evaluate the following using the distributive law.

(a) 7×22 **(b)** 15×9

Working

(a) 7×22
$= 7 \times (20 + 2)$
$= 7 \times 20 + 7 \times 2$
$= 140 + 14$
$= 154$

(b) 15×9
$= 15 \times (10 - 1)$
$= 15 \times 10 - 15 \times 1$
$= 150 - 15$
$= 135$

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1.1 Mental strategies

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Fluency and understanding

WE1

1 Calculate the following using the associative or commutative laws.

(a) $9 + 34 + 6$ **(b)** $17 + 42 + 18$ **(c)** $48 + 39 + 51$
(d) $19 \times 50 \times 2$ **(e)** $15 \times 25 \times 4$ **(f)** $23 \times 125 \times 8$
(g) $8 + 23 + 42$ **(h)** $15 + 57 + 35$ **(i)** $64 + 79 + 56$
(j) $5 \times 6 \times 2$ **(k)** $4 \times 6 \times 5$ **(l)** $2 \times 42 \times 5$

WE2

2 Evaluate the following using the distributive law.

(a) 17×9 **(b)** 19×8 **(c)** 49×6
(d) 6×31 **(e)** 7×52 **(f)** 5×43
(g) 99×9 **(h)** 77×3 **(i)** 57×8
(j) 101×8 **(k)** 113×5 **(l)** 124×11

3 Use any appropriate mental strategy to work out the following.

(a) $23 + 41 + 57$ **(b)** $347 + 156$ **(c)** $335 - 170$
(d) $37 + 128 + 63$ **(e)** $77 + 78 + 23$ **(f)** $89 + 116 + 11$
(g) 103×6 **(h)** 22×11 **(i)** 3×194
(j) $147 + 213$ **(k)** 19×14 **(l)** $4 \times 7 \times 15$

4 Choose the correct answer to the following question.
23×7 could be calculated by:

A multiplying 3 and 7, then adding 20
B multiplying 20 and 7, then adding 3
C multiplying 20 and 7, multiplying 3 and 7, then adding the products together
D multiplying 20, 3 and 7 altogether.

Hint

When two numbers are multiplied together, the answer is called the 'product'.

5 Bilal has completed the first three stages of a bike rally. He rode 87 km in Stage 1, 95 km in Stage 2, and 63 km in Stage 3. Use mental strategies to calculate:

(a) the total distance that Bilal has ridden so far

(b) how far Bilal still has to ride, if the total rally distance is 480 km.

6 The Year 7s at Mountain View Secondary College are undertaking a project to improve their environment. Each student will plant 5 seedlings of a native plant. Use a mental strategy to calculate how many seedlings 8 classes of 25 students will need.

7 Jason is saving $8 every week for some new cricket gear. Use mental strategies to calculate:

(a) how much Jason has saved after 17 weeks

(b) how much he still has to save if the cricket gear he wants costs $189.

8 Carlos is monitoring traffic on a busy road. Twelve cars go past him in 1 minute.

(a) Use a mental strategy to calculate how many cars Carlos can expect to go past in 1 hour, based on his 1 minute count.

(b) List two reasons why the actual number of cars might be less than your answer to **(a)**.

9 Jessica earns $5 every time she walks her neighbour's dog. If she walks the dog 3 times a week, how much will she earn in 6 months?

Hint

There are 26 weeks in 6 months.

10 Alicia is ordering stationery for her office cupboard. Use mental strategies to calculate the cost of each of the following, in dollars.

(a) 8 notepads at 98 cents each

(b) 3 gluesticks at 77 cents each

(c) 12 pens at 59 cents each

(d) 5 boxes of paperclips at 82 cents each

Reasoning, communicating and open-ended

11 Below are some mistakes made by students on a test, and their explanation of the method they used. Write what each student has done incorrectly and what the answer should be.

(a) $21 \times 7 = 161$. Kate: 'I multiplied 7 by 20, and this gave me one less lot of 21 than I needed. So, then I added 21.'

(b) $35 \times 3 = 140$. Sam: 'I doubled 35, then doubled my answer to get 140.'

(c) $256 - 65 = 209$. Leah: 'I first subtracted 56 to get back to 200, and then added the remaining 9.'

12 Four students were asked to write down a mental maths strategy for calculating 9×29. Here are their suggestions:

Alex: $10 \times 29 - 29$ Indrah: $10 \times 29 - 9$
Khalid: $9 \times 30 - 9$ Lucy: $9 \times 30 - 30$

(a) Who has written a correct strategy?

(b) Why do two different strategies give the same answer?

(c) Explain what is wrong with the other strategies.

13 Tranh is in the hardware shop buying some supplies. He has 5 picture hooks, which are 28 cents each, 12 curtain rings, which are 15 cents each, and a small hammer costing $6. As he walks to the checkout, Tranh wonders if he has enough money, as he only has $10 in his pocket. Describe the mental calculations Tranh could do to be sure he has enough money to pay for his items. Is $10 enough?

14 Describe how you could use your calculator to work out 11×23 if the '1' button was broken.

15 Brendan has conducted a survey of the number of pets owned by each member of his class. His results are: 2, 2, 2, 1, 1, 2, 3, 3, 2, 2, 1, 4, 1, 3, 5, 2, 4, 1, 2, 1, 3, 2, 1.

Brendan has been trying to use his calculator to add up his list of numbers, but he keeps losing his place in the list. Suggest a method that Brendan could use to organise his numbers and use his calculator more efficiently. Use your method to add up Brendan's list.

1.2 Indices

Need to Know

Indices

An **index**, **power** or **exponent** can be used to show repeated multiplication.

For example, $5 \times 5 \times 5 \times 5 = 5^4$ where 5 is the **base** and 4 is the **index**, **power** or **exponent**. The index shows how many times the base appears in the multiplication string.

A number written with a base and an index is in **index form**.

$$5^{4} \leftarrow \text{index}$$
$$\text{base} \rightarrow$$

When it is written showing all the multiplications, it is in **expanded form**. The result of the multiplication gives the value.

Index form	Expanded form	Value
5^4	$5 \times 5 \times 5 \times 5$	625

A **perfect square** is a number that can be represented by squares of different lengths. 1, 4, 9 and 16 are perfect squares as $1 = 1 \times 1$, $4 = 2 \times 2$, $9 = 3 \times 3$ and $16 = 4 \times 4$.

To 'square' a number, multiply it by itself. For example, 6 squared $= 6 \times 6$ and the whole number obtained by squaring is a perfect square. So 36 is a perfect square, is equal to 6 squared, and is written as 6^2.

A **perfect cube** is a number that can be represented by cubes of different lengths. 1, 8, 27 and 64 are perfect cubes as $1 = 1 \times 1 \times 1$, $8 = 2 \times 2 \times 2$, $27 = 3 \times 3 \times 3$ and $64 = 4 \times 4 \times 4$.

To 'cube' a number, multiply it by itself and itself again. For example, 4 cubed $= 4 \times 4 \times 4$ and the whole number obtained by cubing is a perfect cube. So 64 is a perfect cube and is written as 4^3.

Square roots and cube roots

Finding the **square root** or **cube root** of a number is the reverse of squaring or cubing a number.

$6^2 = 36$ so the square root of 36 = 6 and is written as $\sqrt{36} = 6$.

$4^3 = 64$ so the cube root of 64 = 4 and is written $\sqrt[3]{64} = 4$.

Technology Exploration: Calculator

Index form on a calculator

Most calculators have keys such as [^] or [x^y] to help you evaluate numbers in index form. Some calculators have an [x^2] key and an [x^3] key for calculating squares and cubes.

For example, 4^3 could be calculated [4] [^] [3] [=] or [4] [x^y] [3] [=] or [4] [x^3] [=].

Scientific calculators have a square root [$\sqrt{\ }$] key. A cube root can be found using a [$\sqrt[x]{\ }$] key.

For example, $\sqrt{256}$ could be calculated [$\sqrt{\ }$] [2] [5] [6] [=] and $\sqrt[3]{512}$ could be calculated [3] [$\sqrt[x]{\ }$] [5] [1] [2] [=].

Worked Example 3

WE3

Write each of the following numbers in expanded form, then find its value.

(a) 4^3 **(b)** 8^5

Working

(a) $4^3 = 4 \times 4 \times 4$
$= 64$

(b) $8^5 = 8 \times 8 \times 8 \times 8 \times 8$
$= 32\,768$

1.2 Indices

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Navigator

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Equipment required: Calculator for Questions **1(j)–(l), 3, 10, 12, 17, 19(a), 21(a)**

Fluency and understanding

1 Write each of the following numbers in expanded form, then find its value.

WE3

(a) 2^3 **(b)** 2^4 **(c)** 2^6
(d) 1^8 **(e)** 0^7 **(f)** 10^3
(g) 6^4 **(h)** 5^5 **(i)** 8^6
(j) 11^3 **(k)** 12^4 **(l)** 14^5

2 Write each of the following in index form.

(a) $8 \times 8 \times 8$ **(b)** $4 \times 4 \times 4 \times 4 \times 4 \times 4$
(c) $12 \times 12 \times 12 \times 12 \times 12$ **(d)** $16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16 \times 16$
(e) seventeen cubed **(f)** nineteen squared
(g) eight to the power of 4 **(h)** thirteen to the power of seven
(i) base 11, index 7 **(j)** base 9, index 6

3 Find the value of each of the following.

(a) 4^7 **(b)** 6^6 **(c)** 9^4
(d) 8^3 **(e)** 7^5 **(f)** 15^3
(g) seventeen squared **(h)** fifty cubed **(i)** the cube of 4
(j) 5×10^2 **(k)** 4×10^3 **(l)** $10^3 \times 2$
(m) 2 to the power of 10 **(n)** 3 to the power of 9 **(o)** base 1, index 8

4 Write in words how we could say each of the following.

(a) 5^2 **(b)** 31^2 **(c)** 3^3 **(d)** 27^3
(e) 4^5 **(f)** 9^6 **(g)** 1^4 **(h)** 7^7

5 If 1 is the first square number and 4 is the second, write the 5th, 6th and 7th square numbers.

6 If 1 is the first cube number and 8 is the second, write the 4th, 5th and 6th cube numbers.

7 Evaluate:

(a) $\sqrt{25}$ **(b)** $\sqrt{49}$ **(c)** $\sqrt{100}$ **(d)** $\sqrt{121}$

(e) $\sqrt{1}$ **(f)** $\sqrt{0}$ **(g)** $\sqrt{4900}$ **(h)** $\sqrt{400}$

(i) $\sqrt[3]{8}$ **(j)** $\sqrt[3]{64}$ **(k)** $\sqrt[3]{1000}$ **(l)** $\sqrt[3]{1}$

(m) $\sqrt[3]{0}$ **(n)** $\sqrt[3]{125}$ **(o)** $\sqrt[3]{8000}$ **(p)** $\sqrt[3]{27\,000}$

Hint

$\sqrt[3]{5}$ means the cube root of 5. It does not mean $3 \times \sqrt{5}$.

8 (a) In $2^3 = 8$ the base number is:

A 2 **B** 3 **C** 6 **D** 8

(b) The square of 5 is:

A 2.2 **B** 10 **C** 25 **D** 125

(c) The cube root of 729 is:

A 9 **B** 27 **C** 243 **D** 2187

9 Evaluate the following without using a calculator.

(a) $1^4 + 2^2$ **(b)** $2^3 - 1^6$ **(c)** $3^4 - 2^4$

(d) $2^3 \times 2^2$ **(e)** $2^2 \times 2^4$ **(f)** $3^3 \times 3^2$

(g) $2^5 + 5^2 - 6^2$ **(h)** $10^2 + 3^2 - 4^3$ **(i)** $5^2 - 1^6 + 3^3$

10 (a) Arrange these numbers in ascending order:

$4^5, 5^4, 1^{200}, 10^3, 4^6, 5^5$

(b) Arrange these numbers in descending order:

$100^2, 10^5, 1^{1000}, 0^{100}, 3^2, 2^3$

Hint

Ascending order means from smallest to largest. Descending order means from largest to smallest.

11 (a) Write down two numbers between 5 and 40 that are both even and square.

(b) Write down two numbers between 30 and 90 that are both odd and square.

12 Evaluate these with your calculator, using the most efficient method possible:

(a) $16^3 - 4096$ **(b)** $21^4 - 4481$ **(c)** $15^4 - 5625$

(d) $36^3 \times 53$ **(e)** $14^4 \times 14$ **(f)** $19^5 \times 21$

(g) $2^{12} + 2^{18}$ **(h)** $3^{10} + 12^4$ **(i)** $4^9 + 3^{11}$

13 Only the square roots of perfect squares are whole numbers. Square roots of other numbers lie in between the perfect square roots (they are decimal numbers). For example, $\sqrt{16} = 4$ and $\sqrt{25} = 5$, so $\sqrt{20}$ would lie in between 4 and 5. Find which two consecutive whole numbers the following lie between. (Consecutive numbers come one after the other; e.g. 8 and 9.)

(a) $\sqrt{10}$ **(b)** $\sqrt{5}$ **(c)** $\sqrt{20}$ **(d)** $\sqrt{62}$

(e) $\sqrt{99}$ **(f)** $\sqrt{2}$ **(g)** $\sqrt{70}$ **(h)** $\sqrt{108}$

14 Arrange the following in ascending order.

(a) $2^4, \sqrt[3]{64}, 5, \sqrt{49}$ **(b)** $7^2, \sqrt[3]{125}, 9, \sqrt{60}$

(c) $\sqrt[3]{8}, 10, \sqrt{90}, 4^2$ **(d)** $\sqrt{80}, 8, \sqrt{105}, 3^3$

15 **(a)** Complete the following table of the powers of 10.

Index form	Expanded form	Value
10^1	10	10
10^2	10×10	
10^3		1000
10^4		

(b) For each row of the table, compare the index numbers in the first column with the number of zeroes in the third column. Describe the pattern.

16 For each of the following, find the missing power of 10.

(a) $20\,000 = 2 \times 10^{\square}$ **(b)** $400 = 4 \times 10^{\square}$ **(c)** $5000 = 5 \times 10^{\square}$

(d) $300\,000 = 3 \times 10^{\square}$ **(e)** $80 = 8 \times 10^{\square}$ **(f)** $7\,000\,000 = 7 \times 10^{\square}$

(g) $150\,000 = 15 \times 10^{\square}$ **(h)** $91\,500 = 915 \times 10^{\square}$ **(i)** $2\,340\,000 = 234 \times 10^{\square}$

Reasoning, communicating and open-ended

17 **(a)** Use your calculator to work out $5^2 \times 5^3$ and $5 \times 5 \times 5 \times 5 \times 5$.

(b) Now, try it with $2^3 \times 2^6$ and $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$.

(c) What can you conclude?

18 How many whole numbers have the value of their square root in between:

(a) 2 and 3 **(b)** 5 and 6 **(c)** 8 and 9?

19 **(a)** Use your calculator to answer TRUE or FALSE to each of the following statements.

(i) 4^6 is bigger than 6^4 **(ii)** 2^{10} is bigger than 10^2

(iii) 3^9 is bigger than 9^3 **(iv)** 19^2 is bigger than 2^{19}

(b) Look at your answers for part **(a)** and, *without* using your calculator, answer TRUE or FALSE to each of the following statements.

(i) 9^8 is bigger than 8^9 **(ii)** 2^{100} is bigger than 100^2

20 **(a)** The number 10^{100} was called a googol by the mathematician Edward Kasner.

(i) How many 10s are multiplied together to give a googol?

(ii) How many zeroes would follow the 1 in a googol?

(b) If you raise the number 10 to the power of a googol, you get a number called a googolplex.

(i) How many 10s are multiplied together to give a googolplex?

(ii) How many zeroes would follow the 1 in a googolplex? How much time do you think you save by writing a googolplex in index form?

21 **(a)** Complete the following.

$11^2 =$ ____ $111^2 =$ ____ $1111^2 =$ ____

(b) Look at the pattern in part **(a)** and, *without* using a calculator, copy and complete the following.

$11\,111^2 =$ ____ $111\,111^2 =$ ____ $1\,111\,111^2 =$ ____

Hint

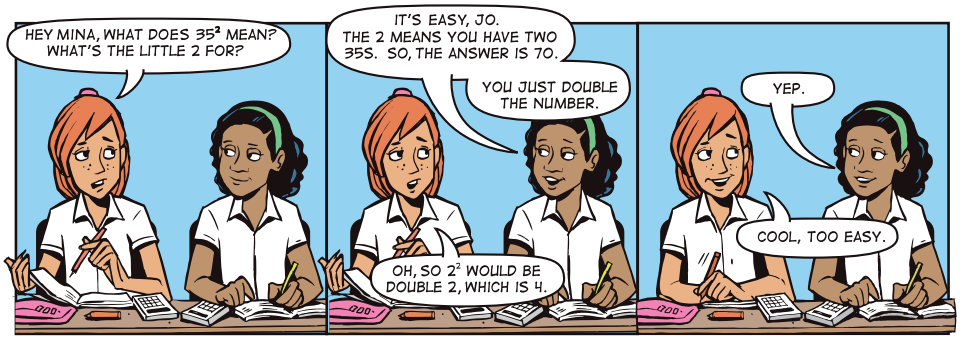
Use index numbers in Question **22**.

22 Use the digits 1, 2 and 3 to create at least three numbers that are greater than 500.

23 **(a)** A number has two digits when it is squared and three digits when it is cubed. What might be the number?

(b) A number has three digits when it is squared and four digits when it is cubed. What might be the number?

24



(a) What is wrong with Mina's explanation? Write down a better explanation that Mina could give to Jo.

(b) What question did Jo get right? Why was this the case?

Problem solving

Teacher's age

Cheryl, who is a teenager, was curious about her maths teacher's age.

'Well', said the teacher, 'Right now the second digit of my age is a square number and the product of my digits is the cube of a number. In 10 years' time my age will be a square number and 25 years from now my age will be a number that has been cubed.'

'I know your age,' Cheryl said with confidence. 'It's also interesting that the difference between our ages is one more than an odd number that has been squared.'

How old is the teacher?

How old is Cheryl?

Strategy options

- Guess and check.
- Make a table.
- Work backwards.
- Test all possible combinations.

1.3 More strategies for multiplication and division

Need to Know

The result of a multiplication is called a **product**. The numbers being multiplied are called **multipliers**.

The result of a division is called a **quotient**. The number being divided is called the **dividend** and the number it is divided by is called the **divisor**. Sometimes after division, there is a **remainder**.

18	×	27	=	486	
multiplier		multiplier		product	
32	÷	10	=	3	rem 2
dividend		divisor		quotient	remainder

Multiplication and division are opposite operations. If there is no remainder, the divisor is a **factor** of the dividend. For example, $6 \div 3 = 2$ so 3 is a factor of 6 and $2 \times 3 = 6$.

Multiplying or dividing using factors

Multiply or divide by splitting the multiplier or divisor into factors and performing a series of simpler operations. For example, to multiply by 4, multiply by 2 and then 2 again. To divide by 6, divide by 2 and then by 3.

Multiplying using an array

The distributive law is used in an array to break large numbers into parts
$43 \times 37 = (40 + 3) \times (30 + 7)$.

The parts are multiplied separately 40×30, 40×7, 3×30 and 3×7.

Then all the results of the multiplication are added together: $1200 + 280 + 90 + 21 = 1591$.

Multiplying and dividing by multiples of 10

To multiply whole numbers by numbers that are multiples of 10:

1. write each multiple of 10 as the product of a number and a power of 10
2. rewrite the multiplication, grouping the powers of 10 together
3. multiply the other numbers, using the total number of zeroes in the powers of 10 to write the correct number of zeroes in the answer.

To divide by whole numbers that are multiples of 10:

1. divide both numbers by 10 until one number is no longer a multiple of 10. Show this by cancelling zeroes
2. perform the simplified division.

Worked Example 4 WE4

Calculate the following using factors.

(a) 34×8 **(b)** $900 \div 15$

Working

(a) 34×8
$= 34 \times 4 \times 2$
$= 34 \times 2 \times 2 \times 2$
$= 68 \times 2 \times 2$
$= 136 \times 2$
$= 272$

(b) $900 \div 15$
$= 900 \div 3 \div 5$
$= 300 \div 5$
$= 60$

Worked Example 5 WE5

Calculate 23×47 using an array.

Working

$23 = 20 + 3$ $47 = 40 + 7$

	20	3
40		
7		

	20	3
40	800	120
7	140	21

	20	3	
40	800	120	920
7	140	21	161

$$\begin{array}{r} 920 \\ +\ 161 \\ \hline 1081 \end{array}$$

Worked Example 6 WE6

Calculate the following using multiples of 10.

(a) 7×900 **(b)** 6000×30 **(c)** $4500 \div 90$

Working

(a) 7×900
$= 7 \times 9 \times 100$
$= 63 \times 100$
$= 6300$

(b) 6000×30
$= 6 \times 1000 \times 3 \times 10$
$= 6 \times 3 \times 1000 \times 10$
$= 18 \times 10\,000$
$= 180\,000$

(c) $4500 \div 90$
$= 450 \div 9$
$= 50$

1.3 More strategies for multiplication and division

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Fluency and understanding

1 Calculate the following using factors. WE4

(a) 27×4	**(b)** 41×6	**(c)** 62×8
(d) $600 \div 4$	**(e)** $153 \div 9$	**(f)** $864 \div 16$
(g) 35×9	**(h)** $390 \div 6$	**(i)** 70×15
(j) 15×12	**(k)** $450 \div 25$	**(l)** $360 \div 24$

2 Calculate the following using an array. WE5

(a) 12×41	**(b)** 17×26	**(c)** 19×68
(d) 23×61	**(e)** 31×49	**(f)** 39×56
(g) 42×47	**(h)** 49×53	**(i)** 55×55
(j) 61×82	**(k)** 72×77	**(l)** 96×51

3 Calculate the following. WE6

(a) 700×3	**(b)** 400×5	**(c)** 2000×13
(d) $360 \div 20$	**(e)** $5400 \div 200$	**(f)** $8000 \div 500$
(g) 1200×400	**(h)** $400 \times 32\,000$	**(i)** $7000 \times 20\,000$
(j) $12\,000 \div 20$	**(k)** $350\,000 \div 700$	**(l)** $8\,400\,000 \div 4000$

4 Calculate the following using any suitable strategy.

(a) $180 \div 20$	**(b)** $450 \div 90$	**(c)** $800 \div 25$
(d) 40×19	**(e)** 53×20	**(f)** 102×18
(g) $2700 \div 18$	**(h)** $1075 \div 25$	**(i)** $2250 \div 15$
(j) 120×12	**(k)** 59×72	**(l)** 98×25

5 Charlotte is a distance runner in training. She runs 20 laps of a 400 metre race track 6 times a week. How many metres does Charlotte run in a week?

6 Year 7 students are practising for sports day. They need to be divided into five equal groups. There are 130 students in the year. Explain a quick way the teacher could use to work out how many students will be in each group, and find the answer.

7 Marie's monthly mobile phone bill is $52. If she continued to pay this amount every month, use a mental or written strategy to calculate how much Marie will have paid in phone bills by the end of her 2-year contract.

8 A group of eight friends have won $2000 and want to divide it equally between them. How much will each receive?

9 Rehan is paying back a bank loan that he used to buy a house. His repayments are set at $1800 per month for 30 years. How much money will Rehan have paid back at the end of this period of time?

10 Dharma has printed 3700 brochures advertising his business. He wants to place them in the letterboxes of houses in the local area. Dharma estimates that there are about 40 houses in each street. How many streets will he be able to cover with 3700 brochures?

Reasoning, communicating and open-ended

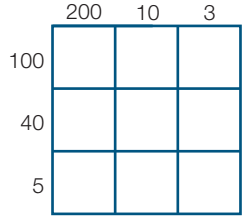
11 A travelling salesperson has lost his calculator. He needs to work out the individual cost of an item that is packaged into groups of 12 items, and costs $660 for the pack.

(a) He decides to approximate by dividing by 10. Find the answer he obtains.

(b) Calculate the actual cost of each individual item.

(c) He sells five items at his approximated cost. How much did he lose or gain?

12 Using an array, multiplication can be extended from two-digit to three-digit numbers by adding an extra row and an extra column to the grid. For example, 213×145 can be calculated using this array.



Use an extended grid to calculate the following.

(a) 162×246 **(b)** 356×412 **(c)** 107×560

13 Which of the following is an incorrect method for calculating $240 \div 4$?

A Halve 240, then halve the answer.

B Calculate $2 \div 4$, $4 \div 4$ and $0 \div 4$, then add the results together.

C Calculate $200 \div 4$ and $40 \div 4$, then add the quotients.

D Calculate $24 \div 4$, then multiply the answer by 10.

14 ◆ × ● = 1620

(a) What could ◆ and ● be? Find at least seven different combinations of whole numbers.

(b) Find at least three combinations where both ◆ and ● are two-digit numbers.

15 Here is part of Kim's maths homework. She had been learning about splitting up numbers to be multiplied, so she decided to use that strategy to complete the following.

Multiplication

(1) 9×24
$= 9 \times 2 + 9 \times 4$
$= 18 + 36$
$= 54$

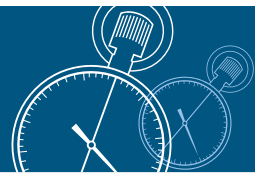
(2) 12×43
$= 12 \times 4 + 12 \times 3$
$= 48 + 36$
$= 84$

(a) Kim is a bit worried that her answers don't look big enough. What are the correct answers?

(b) Explain to Kim the mistake she has made in both questions.

(c) Give Kim some advice so that she can avoid similar mistakes in the future.

Half-time 1



1 Calculate the following by using an appropriate mental strategy. Ex 1.1, 1.3

(a) $4 \times 17 \times 5$ **(b)** $15 \times 23 \times 2$ **(c)** $33 + 45 + 107$

(d) 40×19 **(e)** $37 + 49 + 153$ **(f)** $127 + 65 + 69$

2 **(a)** Write the value of the following. Ex 1.2

(i) 6^3 **(ii)** 5^5 **(iii)** 10^1 **(iv)** 7^2

(v) $\sqrt{49}$ **(vi)** $\sqrt[3]{8}$ **(vii)** $\sqrt{100}$ **(viii)** $\sqrt[3]{64}$

(b) Which of your answers to **(a)** is:

(i) a perfect square **(ii)** a perfect cube?

3 Calculate the following. Use mental strategies if possible. Ex 1.1, 1.3

(a) 19×9 **(b)** 41×8 **(c)** 5×38 **(d)** 22×17

(e) 147×3 **(f)** 12×53 **(g)** 67×71 **(h)** 92×84

4 Perform the following divisions. Use mental strategies if possible. Ex 1.3

(a) $136 \div 4$ **(b)** $174 \div 6$ **(c)** $580 \div 5$ **(d)** $940 \div 20$

(e) $128 \div 8$ **(f)** $351 \div 9$ **(g)** $270 \div 15$ **(h)** $1344 \div 24$

5 Write the following in index form. Ex 1.2

(a) $3 \times 3 \times 3 \times 3 \times 3 \times 3$ **(b)** 6 to the power of 5 **(c)** base 4, index 7

(d) fifteen squared **(e)** twenty-one cubed **(f)** $41 \times 41 \times 41$

6 Calculate the following. Ex 1.3

(a) 70×60 **(b)** 150×20 **(c)** 400×300

(d) 250×800 **(e)** $3600 \div 40$ **(f)** $9000 \div 300$

(g) $7500 \div 1500$ **(h)** $124\,000 \div 200$ **(i)** $60\,000 \div 400$

7 13 000 tickets were sold to a music concert at a cost of $52 each. Use a combination of strategies to calculate the amount made by the promoters of the concert from ticket sales. Ex 1.1, 1.3

1.4 Estimating and rounding

Need to Know

An **estimate** is an approximate value for a calculation and is used when an exact answer is not required or to check that an exact answer is sensible.

To round a number to the first digit, look at the second digit of the number. If it is 0, 1, 2, 3 or 4, keep the first digit the same and replace the other digits with zeroes. If it is 5, 6, 7, 8 or 9, increase the first digit by 1, and replace the other digits with zeroes.

When **rounding**, we use the symbol ≈ which means 'approximately equal to'.

Estimating multiplication and division

Multiplication and division can be estimated by rounding to the first digit. This estimate can be compared to the exact answer. Estimates are calculated with and without a calculator.

Estimating using other ways of rounding

Instead of rounding to the first digit, round to a closer multiple of 5 or 10 to get a more accurate estimate of the actual answer.

Overestimating and underestimating

When multiplying, an overestimate will be obtained if both numbers are rounded up and an underestimate will be obtained if both numbers are rounded down. When dividing, an overestimate or underestimate will occur if one number is rounded up and the other rounded down.

To achieve a reasonable approximate answer:

- when multiplying, round one number up and the other number down
- when dividing, both numbers should be rounded up, or both rounded down.

Worked Example 7

WE 7

Round the following numbers to the first digit.

(a) 361 **(b)** 2050 **(c)** 8

Working

(a) *361 ≈ 400*

(b) *2050 ≈ 2000*

(c) *8 (This number is already rounded to the first digit.)*

Worked Example 8

WE 8

Calculate:

(i) an approximate answer to the following by first rounding to the first digit

(ii) the exact answer without a calculator using an appropriate strategy

(iii) the exact answer using a calculator.

(a) 402×52 **(b)** $77\,328 \div 432$

Working

(a) (i) $402 \times 52 \approx 400 \times 50$
$\approx 20\,000$

(ii) $402 \times 52 = 20\,904$

	50	2
400	20 000	800
2	100	4

(iii) calculator answer = 20 904

(b) (i) $77\,328 \div 432 \approx 80\,000 \div 400$
≈ 200

(ii) $77\,328 \div 432 = 77\,328 \div 3 \div 12 \div 12$
$= 25\,776 \div 12 \div 12$
$= 248 \div 12$
$= 179$

(iii) calculator answer = 179

Worked Example 9

WE 9

Calculate:

(i) an approximate answer to the following by first rounding to a multiple of 5 or 10

(ii) the exact answer using a calculator and compare this with your estimate.

(a) $4587 \div 49$ **(b)** 197×23

Working

(a) (i) $4587 \div 49 \approx 4600 \div 50$
$\approx 460 \div 5$
$\approx 450 \div 5$
≈ 90

(ii) Calculator answer = 93

The estimate was very close to the exact answer.

(b) (i) $197 \times 23 \approx 200 \times 25$
≈ 5000

(ii) Calculator answer = 4531

The estimate is much larger.

Answers page 446

1.4 Estimating and rounding

Navigator

Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3 Columns 1 & 2, Q4, Q5, Q8, Q10, Q12, Q13, Q14	Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Columns 2 & 3, Q4, Q5, Q6, Q7, Q8, Q9, Q12, Q13, Q14	Q1 Columns 3 & 4, Q2 Column 3, Q3 Columns 2 & 3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q13, Q14, Q15

Fluency and understanding

WE7

1 Round the following numbers to the first digit.

(a) 68 **(b)** 74 **(c)** 10 **(d)** 6

(e) 619 **(f)** 650 **(g)** 483 **(h)** 970

(i) 4846 **(j)** 3723 **(k)** 9510 **(l)** 9643

(m) 75 000 **(n)** 716 599 **(o)** 800 050 **(p)** 1 801 021

WE8

2 Calculate **(i)** an approximate answer to the following by first rounding each number to the first digit, **(ii)** the exact answer without a calculator using an appropriate strategy, and **(iii)** the exact answer using a calculator.

(a) 81×41 **(b)** 47×84 **(c)** 62×89

(d) $2970 \div 54$ **(e)** $3162 \div 62$ **(f)** $1950 \div 75$

WE9

3 Calculate **(i)** an approximate answer for each of the following by first rounding each number to a convenient multiple of 5, 10, 100 or 1000, and **(ii)** the exact answer and compare this with your estimate.

(a) 41×26 **(b)** 122×54 **(c)** 191×14

(d) $387 \div 43$ **(e)** $378 \div 21$ **(f)** $208 \div 52$

(g) 1587×23 **(h)** $33\,997 \times 21$ **(i)** 4036×253

(j) $3536 \div 52$ **(k)** $10\,961 \div 95$ **(l)** $44\,898 \div 14$

4 In each case, which of these estimates do you think is the closest? Don't try to count or measure.



(a) About how many people are there in the photo?

A 700 **B** 4000

C 30 000 **D** 100 000

(b) Approximately how far is it:

(i) from Alice Springs to the South Australian border, by road?

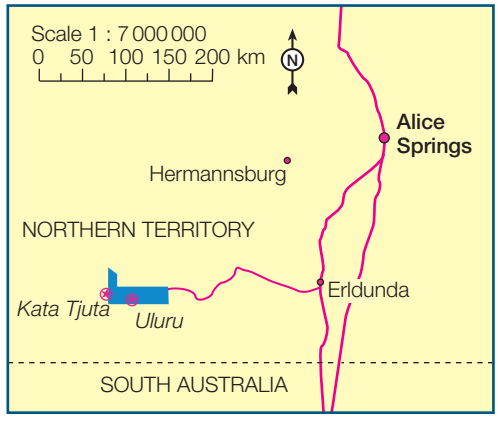
A 75 km **B** 150 km

C 300 km **D** 600 km

(ii) directly from Uluru to Alice Springs (i.e. in a straight line)?

A 40 km **B** 350 km

C 789 km **D** 1000 km



5 Estimate the following by rounding the smaller number to the first digit and the larger number to that same place value (e.g. if the smaller number is rounded to the nearest 10, round the larger number to the nearest 10 also).

(a) 1247 + 323 **(b)** 10 290 + 178 **(c)** 14 532 + 2788

(d) 823 – 79 **(e)** 1176 – 241 **(f)** 453 – 276

6 For each of the following, find an estimate for the answer by first rounding each number to the first digit. State whether the actual answer will be higher or lower than the estimate.

(a) 364×57 **(b)** 2398×426 **(c)** 153×16

(d) $1996 \div 24$ **(e)** $943 \div 86$ **(f)** $597 \div 136$

7 In each case, choose the best estimate from the alternatives given. Don't do the actual calculation.

(a) 321×73

A 210 **B** 2100 **C** 2163 **D** 21 000

(b) 405×950

A 360 000 **B** 400 000 **C** 450 000 **D** 3 600 000

(c) $7865 \div 24$

A 400 **B** 900 **C** 1600 **D** 210 000

(d) $999\,160 \div 527$

A 220 **B** 330 **C** 660 **D** 2000

8 Last weekend, 255 345 people attended NRL matches. They paid, on average, $22 to attend their game.

(a) Round the attendance to the first digit.

(b) Use your answer to **(a)**, and a rounded estimate for the ticket price, to find an estimate for the total entry cost.

9 **(a)** Tahnee earns $38 295 per year. Approximately how much is this per week? Calculate by rounding to convenient numbers first. (There are 52 weeks in a year.)

(b) Catriona earns $175 per week from her part-time job. Approximately how much is this per year? Calculate by rounding to convenient numbers first.

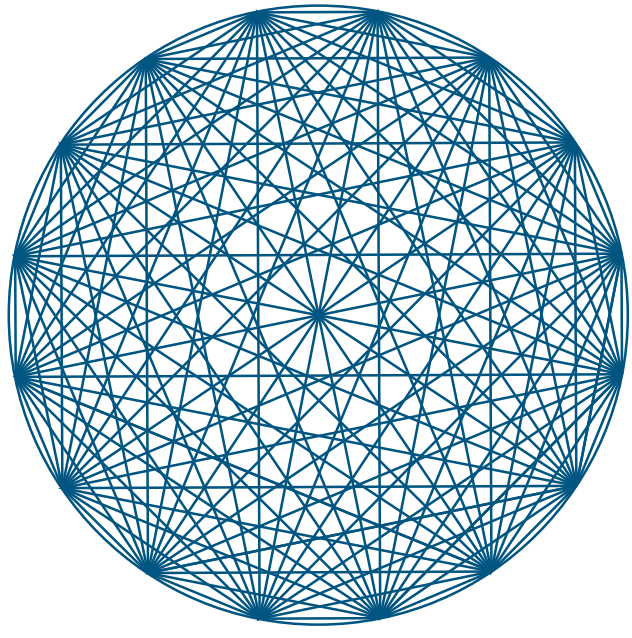
Reasoning, communicating and open-ended

10 **(a)** Approximately how many straight lines are there in this picture?

A 100 **B** 500

C 5000 **D** 10 000

(b) Explain a way of finding the exact number of lines without counting them all.



11 79×5003 and 76×5488 both give the same approximate answer of 400 000 when each number is rounded to the first digit. Which of the actual answers to the two calculations will be closer to 400 000? Explain how you arrived at your answer.

12 **(a)** Use rounding to the first digit to find an approximate answer for $1344 \div 21$.

(b) Find an approximate answer for $1344 \div 21$ by first rounding to convenient numbers, such as multiples of 10.

(c) Which method do you think has given a closer estimate to the actual answer? Give a reason for your prediction.

(d) Work out the actual answer and see if you were correct.

13 Ms Kelly, the assistant principal at Summerhill Secondary School, is organising buses to transport students to the pool for the swimming carnival. Each bus can seat 52 people, and she has 428 staff and students going (although she doesn't know how many might be sick or away on the day). How many buses should Ms Kelly order? Why is estimating by rounding not helpful in this situation?

14 Natalie is out shopping. She is heading towards the checkout when she suddenly wonders if she has enough money. She mentally rounds her purchases off in her head and adds them up.

Here is Natalie's rounded off, mental list:

T-shirt ≈ \$20
Earrings ≈ \$10
Necklace ≈ \$10
Lip gloss ≈ \$10
Total ≈ \$50

'Terrific!' thinks Natalie. 'I've got exactly enough!'

The actual prices of the items are: T-shirt \$17; earrings \$14; necklace \$13; lip gloss \$9.

(a) There is nothing wrong with Natalie's rounding. Why is she in trouble when she gets to the checkout?

(b) Describe to Natalie a 'safer' method for rounding money.

15 Write down three pairs of numbers that when rounded to the first digit, then multiplied together, give:

(a) 3200 **(b)** 200 **(c)** 100 000 **(d)** 42 000

Order of operations

1.5

Need to Know

Order of operations rules ensure that everyone will get the same answer for the same calculation.

For example: $1 + 2 \times 3 + 4 = 11$

$(1 + 2) \times 3 + 4 = 13$

$1 + 2 \times (3 + 4) = 15$

Order of operations rules

1. Always do calculations in brackets first.
2. Next, do any calculations with indices (powers).
3. Do the multiplication and division next, in order from left to right, as you come to them.
4. Finally, do the addition and subtraction, in order from left to right, as you come to them.

For example: $18 \div 3 \times (7 - 5 + 4)^2$

$= 18 \div 3 \times (2 + 4)^2$

$= 18 \div 3 \times 6^2$

$= 18 \div 3 \times 36$

$= 6 \times 36$

$= 216$

Worked Example 10

WE10

Calculate the following, showing all steps of working.

(a) $24 + 6 \div 2 - 1 \times 4$

(b) $14 - 9 + 6^2 \div (7 + 2) \times 3$

Working

(a) $24 + 6 \div 2 - 1 \times 4$
$= 24 + 3 - 1 \times 4$
$= 24 + 3 - 4$
$= 27 - 4$
$= 23$

(b) $14 - 9 + 6^2 \div (7 + 2) \times 3$
$= 14 - 9 + 6^2 \div 9 \times 3$
$= 14 - 9 + 36 \div 9 \times 3$
$= 14 - 9 + 4 \times 3$
$= 14 - 9 + 12$
$= 5 + 12$
$= 17$

Answers page 446

1.5 Order of operations

Navigator

Q1 Column 1, Q2, Q3, Q4, Q5 Column 1, Q7, Q9, Q11, Q12, Q13	Q1 Column 2, Q2, Q4, Q5 Column 2, Q6, Q7, Q8 Column 1, Q9, Q10, Q11, Q12, Q13	Q1 Column 2, Q4, Q5 Column 2, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13

Fluency and understanding

WE10

1 Calculate the following, showing all steps of working.

(a) $1 + 8 \times 3$

(b) $8 - 5 \div 5$

(c) $6 \times 2 - 1$

(d) $8 \div 4 - 2$

(e) $25 - 2 \times 11$

(f) $6 \div 3 + 3 \times 5$

(g) $8 - 24 \div 12 + 3$

(h) $8 \times 3 \div 4 \times 2$

(i) $9 \times (10 - 7) \div 3$

(j) $24 \div (7 + 5) \times 5^2$

(k) $88 \div 8 - 6 \times (5 - 4)$

(l) $12 \times 5 + 4 \times (10 - 4)$

(m) $38 - 7 \times 2^2 + 13 - 4 \div 2$

(n) $9 - 2 + 5 + 3 \times 4 \div 6$

(o) $23 - 5 + (17 - 2) \times 3 + 5^2$

(p) $28 \div 7 \times 3^2 + (5 - 1) \div 2 + 3$

2 State TRUE or FALSE for the following.

(a) For $2 + 6 \times 4$ we would calculate $2 + 6$ first.

(b) For $6 + 12 \div 3$ we would calculate $12 \div 3$ first.

(c) For $8 + 40 \div (3 + 5) \times 10^2$ we would calculate $8 + 40$ first.

(d) For $24 + 6 \div 2 - 1 \times 4$ we would calculate $6 \div 2$ first.

(e) $4 + 12 \div 2$ simplifies to 8.

(f) $20 \div 5 - 1$ simplifies to 3.

3 (a) $8 + 6 - 4 \div 2$ is equal to:

A 5 **B** 9 **C** 12 **D** 16

(b) $12 - 8 \div 4 + 2^2$ is equal to:

A 5 **B** 6 **C** 9 **D** 14

4 (a) Which calculation would you do first and which would you do second in each of these questions?

(i) $14 - 2 \times 6 + 2 \times 2$

(ii) $(15 \div 5) + 6 \times 3$

(b) Calculate the value of the expressions in **(a)**.

5 Put brackets into these statements, where necessary, to make them true.

(a) $6 + 6 \times 3 = 36$

(b) $10 - 4 \times 5 = 30$

(c) $9 - 8 \times 6 + 4 = 10$

(d) $12 + 6 \div 7 - 4 = 14$

(e) $6 \div 3 + 3 \times 5 = 5$

(f) $3 \times 6 \div 8 - 4 + 5 = 2$

(g) $3 \times 10 - 7 \div 9 + 12 = 13$

(h) $18 \div 3 \times 5 - 3 + 2 = 14$

(i) $7 + 3 \div 4 + 1 = 2$

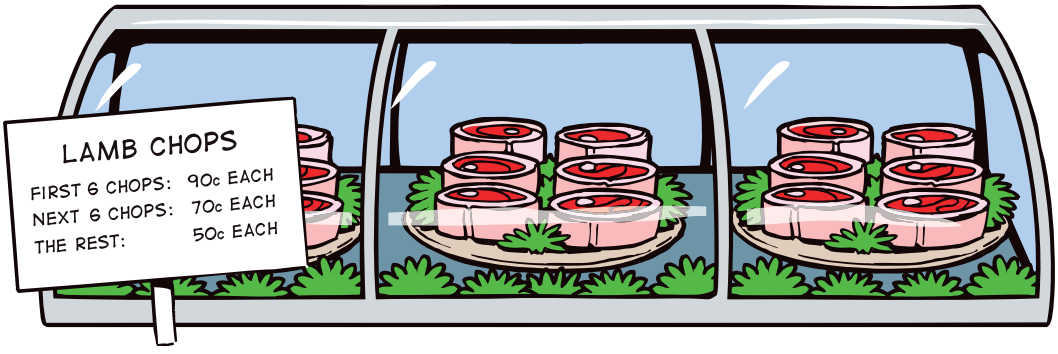
(j) $5 - 3 \times 8 - 6 \div 2 = 2$

6 During 'peak' time, Shania's mobile phone company charges 73c per minute, plus a 29c 'flagfall' (a single amount charged as soon as a call is connected). During 'off-peak' time, there is no flagfall, and a rate of 50c per minute is charged per call. Shania has $20 credit on her phone. She makes one 5-minute call during 'peak' time, and one 8-minute call during 'off-peak' time.

(a) Calculate the total cost of the two calls. Show your working clearly.

(b) Using the symbols +, –, ×, and at least one pair of brackets, write down a calculation that Shania could do to work out how much credit she had left on her phone after the two calls were made.

7 A butcher had an oversupply of lamb chops. He decided to tempt people to buy more of them by putting up the following sign.



Stuart bought 8 chops, Mahalia bought 11 chops, and Sofia bought 17 chops.

The amount Stuart paid can be worked out using the following calculation.

$6 \times 90 + 2 \times 70$. This gives an answer of 680 cents, which can then be written as $6.80.

(a) Write down a calculation similar to the one above to work out how much Mahalia paid, then do the calculation.

(b) Write down a calculation similar to the one above to work out how much Sofia paid, then do the calculation.

(c) How much more than Stuart did Sofia pay?

8 Replace each $*$ with one of the four symbols (+, –, ×, ÷) to make the statement true.

(a) $2 + 21 * 3 = 9$

(b) $15 - 6 * 2 = 12$

(c) $14 - 8 * 6 = 0$

(d) $7 * 5 * 6 = 29$

(e) $14 * 3 * 2 = 15$

(f) $(24 * 6) * 10 = 3$

(g) $8 * 5 * 2 - 6 = 12$

(h) $12 * 2 + 1 * 9 = 15$

9 Replace each $*$ with either $<$, $>$ or $=$ to make the statement true.

(a) $6 \times (4 \div 2) \times 3 * (6 \times 4) \div 2 \times 3$

(b) $(1 + 4) \times 20 \div 5 * 1 + (4 \times 20) \div 5$

(c) $100 + 10 \div 10 * (100 + 10) \div 10$

(d) $36 \div 6 \times (3 - 3) * 36 \div 6 \times 3 - 3$

Hint

Remember $<$ means 'is less than' and $>$ means 'is greater than' when read from left to right.

Reasoning, communicating and open-ended

10 Remove the unnecessary brackets from these statements.

(a) $(4 \times 3) + (6 - 2) = 16$

(b) $8 + (4 \div 2) + (6 \times 2) = 22$

(c) $4 + (20 \div 5) \times (3 + 7) = 44$

(d) $6 \times (3 + 2) - (12 \div 2) = 24$

11 Sean typed the following into his calculator: $8 + 6 \div 2 + 3 \times 5$. Shazhad typed the same into the 'calculator' application of his mobile phone. Sean's answer was 26. Shazhad's answer was 50.

(a) Do the calculation to decide whose answer was correct.

(b) How did the other calculator come up with a different answer?

12 By placing one pair of brackets in different positions, show all the possible answers for this question: $8 + 4 \times 6 \div 2 - 1$.

13 Jing has worked out that the answer to $30 \div 2 \times (2 + 3)$ is 3. Here is his working.

$30 \div 2 \times (2 + 3)$
$= 30 \div 2 \times 5$ Step 1
$= 30 \div 10$ Step 2
$= 3$ Step 3

Jing's teacher has marked it as incorrect. What is the correct answer? Explain to Jing which step of his working is incorrect, what he should have done instead, and why.

Problem solving

Four 4s = 100?

1 Using the number 4 exactly four times, together with any of the four operations (+, −, ×, ÷) and brackets if you need them, see if you can make the numbers 0 to 9. A way to get 5 has been done for you. There is more than one way in many cases.

0 =
1 =
2 =
3 =
4 =
5 = $(4 \times 4 + 4) \div 4$
6 =
7 =
8 =
9 =

2 See how many numbers up to 100 you can find with just four 4s. You might like to work with a partner or in a small group.

3 Using the digits of the year 2013 exactly once, together with brackets and any of the four operations, see how many of the numbers from 1 to 10 you can make.

e.g. $(3 + 1) \times 2 + 0 = 8$

Strategy options

- Guess and check.
- Test all possible combinations.

Mixed whole number problems

1.6

Need to Know

Strategies for solving word problems

- Read the information carefully. Look for words that are associated with the four operations +, −, ×, ÷. Some words are listed below.

+ **Sum**	− **Difference**	× **Product**	÷ **Quotient**
add	subtract	multiply	divide
total	less than	how much	how many
altogether	minus	lots of	shared between

- Decide which skills are needed. Many problems require more than one calculation.

Worked Example 11

WE11

Find the answers to the following questions.

(a) A rugby team scored 37 points in the first half and 28 points in the second half. How many points did they score in total for the match?

(b) A new caravan cost $45 650. Three years later it was worth $38 580. By how much had the value of the caravan decreased in that time?

(c) A horse-riding school charges $36 for each 1-hour lesson. How much would it cost for 15 lessons?

(d) Roula jogs at a pace of 9 kilometres per hour. How many hours did she jog in a week in which she ran a total of 171 kilometres?

Working

(a) 'total' indicates a sum, so 37 and 28 are added to give the answer
$37 + 28 = 65$
65 points were scored for the match.

(b) 'decrease' indicates a difference, so $38 580 is subtracted from $45 650
$45\,650 - 38\,580 = 7070$
The value of the caravan had decreased by $7070.

(c) 'how much' indicates a product, so $36 and $15 are multiplied
$36 \times 15 = 540$
The lessons would cost $540.

(d) 'how many' indicates a quotient, so 171 kilometres is divided by 9 km/h
$171 \div 9 = 19$
Roula jogged for 19 hours during that week.

Answers page 447

1.6 Mixed whole number problems

Navigator

Q1, Q2, Q3, Q6, Q7, Q8, Q9, Q11, Q12, Q15	Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12, Q13, Q15, Q16	Q4, Q5, Q6, Q7, Q10, Q11, Q12, Q13, Q14, Q15, Q16

Fluency and understanding

WE11

1 Find the answers to the following questions.
   (a) The Australian cricket team made 425 in their first innings and 299 in their second. How many runs did they make in total for the match?
   (b) A new car cost $28 500. Five years later it was worth $19 275. By how much had the value of the car decreased in that time?
   (c) A piano teacher charges $27 for each 1-hour lesson. How much would it cost for 13 lessons?
   (d) Sula has a job that pays her $12 per hour. How many hours did Sula work in a week for which her pay was $372?

2 Harvey 'Scoop' Roberts, a journalist with the *Monthly Farm News*, can type 60 words a minute. How long does it take him to type an article of 1800 words?

3 Wendy is training to be an Olympic swimmer. Every morning she swims 3600 m in a 50 m pool. How many laps is that?

4 Little Lucy is exactly 5 weeks old. How many minutes old is she?

5 The highest mountain in the world, measured from sea level, is the Himalayan peak of Mount Everest. It is 8848 m above sea level. If we measure mountains that start under the ocean, the tallest mountain in the world from base to peak is Mauna Kea on the island of Hawaii. It measures 10 203 m, of which 4205 m is above sea level.
   (a) How much of Mauna Kea is below sea level?
   (b) If we don't count the part of Mauna Kea that is under water, how much higher is Mount Everest?

6 Cathy is out shopping for bargains. She bought two pairs of $68 shoes in a store that said 'buy one pair, get the second pair half price'. She bought two $19 T-shirts where the special deal was 'buy one, get $10 off the second'. Cathy also bought a pair of earrings for $25. What was the total cost of her shopping spree?

7 Mick is a baker in a supermarket. He makes hot cross buns and packs them in bags of 6. He stacks 4 shelves with 8 bags on each shelf. At the end of the day, Mick has 3 bags of buns left. How many hot cross buns did he sell?

8 The two longest rivers in the world are the Amazon (6448 km) and the Nile (6670 km). The longest river in Australia is the Darling (2739 km).
   (a) How much longer is the Nile than the Amazon?
   (b) How much longer is the Nile than the Darling?
   (c) How much longer is the Amazon than the Darling?

9 The width of a painting including the frame is 85 cm. If the frame is 6 cm wide all the way around, what is the width of the unframed painting?

10 The Pizza Pit-Stop employs five people. The two cooks work 36 hours each per week for $18 an hour. The three waiters work 30 hours each per week for $15 an hour. What does the Pizza Pit-Stop pay its five employees in total per week?

Reasoning, communicating and open-ended

11 Sam has 28 model planes on a display shelf. Mark has double Sam's number of model planes. Harvey has half as many planes as Sam. What is the difference between the number of model planes that Harvey and Mark have?

12 Hiram is moving house. He has 13 boxes of possessions, but can only fit 4 at a time in his car.

(a) Beginning the day at his old house and ending it at his new house, how many times will Hiram need to travel between the two houses in order to shift all his boxes?

(b) If the distance between the two houses is 7 km, how far will he travel?

13 Jules Verne wrote about travelling around the world in 80 days.

(a) Write 80 days as a number of weeks plus a number of days.

(b) How many months, weeks and days is it, if we assume there are 30 days in a month?

(c) How many months, weeks and days is it, if we assume that there are 4 weeks in a month?

(d) Explain why your answers to **(b)** and **(c)** are different.

14 Weighing Jason's pet dog Scruffy at the vet's was a problem, as Scruffy kept jumping off the scales. The vet held Scruffy and stood on the scales. Then, Jason held Scruffy and stood on the scales. Finally, the vet, Jason and Scruffy all stood on the scales.

The scales read as follows:
The vet and Scruffy = 91 kg
Jason and Scruffy = 56 kg
The vet, Jason and Scruffy = 138 kg

Work out how much Scruffy weighs.

15 **(a)** Write two numbers that have a sum greater than 90 but a product less than 1800.

(b) Write two numbers that have a difference of less than 10 but have a sum greater than one-quarter of their product.

16 Simon owns a company that produces fruit juice. The juice comes in three different-sized bottles—300 mL, 600 mL and 1000 mL (1 L), weighing 300 g, 600 g and 1000 g (1 kg) respectively. Simon packs same-size bottles into boxes of 20, and delivers them by truck to shops. Simon's truck can hold a maximum load of 1000 kg.

(a) For each of the different-sized bottles, write down the mass in kg of a box of 20 bottles. (Remember, 1000 g = 1 kg.)

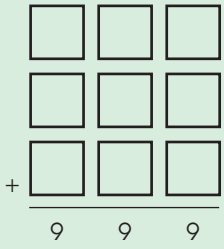
(b) How many boxes of each type of juice can Simon get on the truck to make a load of 1000 kg?

(c) Find at least two different combinations of boxes that would make a load of 1000 kg.

Problem solving

Nine hundred and ninety-nine

Copy the following boxes into your work book.



Using each of the digits from 1 to 9 once in each of the boxes, create three three-digit numbers whose sum is 999.

(It may help to write the digits 1 to 9 on small pieces of paper, so you can move them around the boxes easily.)

How many different combinations can you find?

Aim to find at least four combinations of different three-digit numbers.

Strategy options

- Test all possible combinations.
- Break problem into manageable parts.

Challenge 1



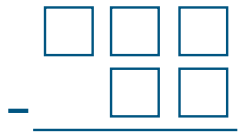
1 The digits 5, 6, 7, 8 and 9 can be arranged to form five-digit even numbers. The tens digit in the smallest of these numbers is:

A 6 **B** 7 **C** 8 **D** 9

2 How many different numbers can you make by multiplying two one-digit even numbers?

A 6 **B** 8 **C** 9 **D** 10

3 Each of the digits 3, 5, 6, 7 and 8 is placed once in a box in the diagram opposite. If the two-digit number is subtracted from the three-digit number, the smallest difference is:



A 261 **B** 269 **C** 271 **D** 278

4 Kim and Brian are given savings accounts by their grandmother, with the same amount of money in each. Kim is a good saver and puts an extra $5 into her account each week. Brian takes $1 out of his account every week. After 10 weeks, Kim has 3 times as much in her account as Brian has in his account. What was the starting amount in each account?

5 The product of the ages of two teenagers and their father is 15 181. How old is the father?

6 Look at the following number arrangement.

1

3 5

7 9 11

13 15 … …

What will be the sum of the seventh row? Look for a pattern in your row totals. What will be the sum of the one-hundredth row?

7 How many square numbers are there between 8 and 8008?

8 Find the next three terms in this number pattern 4, 7, 11, 18, 29, ...

9 The counting numbers are arranged in six columns as shown. In which column will 1001 appear?

A	B	C	D	E	F
1	2	3	4	5	6
12	11	10	9	8	7
13	14	15	16	17	18
…	…	…	…	20	19

10 **(a)** Write down the values of 0^2, 1^2, 2^2, 3^2, … , 9^2.

(b) Consider the last digit of each of the values found in **(a)**. What are the only digits that a square number can end in?

(c) What digits can't a perfect square end in?

(d) Which of the following numbers can't be perfect squares?

(i) 327 **(ii)** 324 **(iii)** 343 **(iv)** 289 **(v)** 732

(e) Without using a calculator, write 961 as a perfect square by first deciding what the square root could end in and then thinking about the factors of 900.

Chapter review 1

D.I.Y. Summary

Key Words

associative law	divisor	index form	power
base	estimate	indices	product
commutative law	expanded form	multipliers	quotient
cube root	exponent	order of operations	remainder
distributive law	factor	perfect cubes	rounding
dividend	index	perfect squares	square root

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 For the number 7^3, 7 is the _____ and 3 is the _____, or _____.

2 1, 9 and 25 are all examples of _____. 1, 8 and 27 are all examples of _____.

3 The _____ of 8 is 2.

4 In the statement $97 \div 3 = 32$ rem 1, 97 is the _____, 3 is the _____, 32 is the _____, and 1 is the _____.

5 One strategy for making multiplication easier is to use the _____; for example, $6 \times 17 = 6 \times (10 + 7)$.

To find the value of a statement such as $(9 + 5) \times 3 - 2$, we would use the _____.

6 The 'long way' of writing a number in index form is in _____.

7 The _____ of 16 is 4, because $4 \times 4 = 16$.

8 $7 \times 5 = 5 \times 7$ is an example of the _____.

9 When two or more numbers are multiplied together, the answer is called the _____.

10 We can obtain an _____ of the answer to a calculation by _____ to the first digit.

Fluency and understanding

1 Use an appropriate mental strategy to help calculate each of the following. Ex.1.1

(a) $89 + 53$ **(b)** $71 + 67$ **(c)** $93 + 125 + 7$ **(d)** $5 \times 12 \times 2$

(e) 21×15 **(f)** 19×23 **(g)** $4 \times 9 \times 5$ **(h)** 32×17

2 Find the value of the following. Ex.1.2

(a) 12^2 **(b)** 5^3 **(c)** 10^5

(d) $\sqrt{64}$ **(e)** $\sqrt{900}$ **(f)** $\sqrt[3]{27}$

(g) $5^3 + 2^2$ **(h)** $8^4 - 3^5$ **(i)** $2^3 \times 3^2$

3 Write each of the following numbers in index form. Ex.1.2

(a) $7 \times 7 \times 7 \times 7 \times 7$ **(b)** ten cubed

(c) five squared **(d)** twelve to the power of eight

4 Calculate the following using any suitable strategy. Ex 1.3

(a) 35×40 **(b)** 850×200 **(c)** 757×7 **(d)** 98×31

(e) $4400 \div 200$ **(f)** 27×59 **(g)** $612 \div 4$ **(h)** $756 \div 7$

5 Use rounding to the first digit to find approximate answers to the following. Ex 1.4

(a) 35×241 **(b)** 1763×27 **(c)** $3299 \div 64$ **(d)** $57\,008 \div 192$

(e) 159×286 **(f)** 3427×963 **(g)** $4398 \div 487$ **(h)** $86\,005 \div 334$

6 Find approximate answers to the following by first rounding each number to a convenient multiple of 10 or 100. Ex 1.4

(a) $554 \div 7$ **(b)** $2136 \div 19$ **(c)** $15\,624 \div 216$ **(d)** 28×604

(e) 157×419 **(f)** 523×1998 **(g)** $54\,327 \div 915$ **(h)** $12\,794 \div 183$

7 Calculate: Ex 1.5

(a) $9 \div (2 + 1) - 2$ **(b)** $(3 \times 8) \div 4 + 7$ **(c)** $12 - 6 \times 2 + 11$

(d) $7 + 12 \div 4 - 1 \times 2^2$ **(e)** $(13 - 5 \times 2) + (20 \div 10)$ **(f)** $[5 \times (9 + 1)] - 3^2$

8 Misha pays \$112 for two T-shirts and a pair of jeans. If the T-shirts were \$23 each, how much did the jeans cost? Ex 1.6

9 Cartons of milk are packed into boxes that are then stacked into larger crates for transport to supermarkets. If there are 20 cartons in a box, and 9 boxes in a crate, use mental strategies to calculate how many cartons of milk are delivered to a supermarket that ordered 4 crates.

10 Use rounding to the first digit to estimate the answers to the following, then state whether the actual answer will be higher, lower, or close to your estimate.

(a) 3741×22 **(b)** 265×341 **(c)** $25\,736 \div 49$ **(d)** $96\,001 \div 17$

11 Sasha runs a small free-range egg farm. On Tuesday, she collected 283 eggs from her hens.

(a) How many cartons will she be able to fill if each carton can hold 12 eggs?

(b) How many eggs will she have left over?

12 In the calculation of $2 \times [30 \div (4 - 1)] + 6$ the first operation to do is:

A $+$ **B** $-$ **C** $\times$ **D** $\div$

13 Put brackets into these statements, where necessary, to make them true.

(a) $4 \times 2 + 3 \div 5 - 1 = 3$ **(b)** $5 + 1 \div 6 + 4 + 2 = 7$

14 Melissa's salary is \$57 935 per year.

(a) Approximately how much is this per week? Calculate by rounding to the first digit first.

(b) Consider how you rounded the numbers in **(a)**. Will Melissa's actual weekly salary be more or less than the approximate answer you calculated?

15 Which two consecutive numbers do the following lie between?

(Hint: Consider the perfect squares on either side of the number being considered.)

(a) $\sqrt{12}$ **(b)** $\sqrt{30}$ **(c)** $\sqrt{88}$ **(d)** $\sqrt{150}$

16 The local restaurant pays its casual employees $15 per hour. The following table shows the number of hours worked by some of the casual staff. Work out the pay for each of these workers and the total of the wages bill.

Employee	Number of hours worked
Jessica	12
Amy	11
Aaron	9
Nikkita	6
Josef	14

17 Replace each * with one of the four operations (+, –, ×, ÷) to make the statement true.

(a) $9 * 7 * 3 = 30$ **(b)** $16 * 4 \times 2 * 2 = 12$

18 As Jared drove into a carpark, he noticed that the cost of parking was $6 for the first hour, then $3 for every hour after that. Jared's car was parked for 6 hours.

(a) Write down a calculation that Jared could use to work out how much his parking has cost him.

(b) As Jared drives out of the carpark, he gives the attendant $40. Use your answer to part **(a)** to calculate how much change he will receive.

19 Write these numbers in expanded form and then work out the answer.

(a) $5^3 + 2^2$ **(b)** $8^4 - 3^5$ **(c)** $(3^2 - 2^3) \times 6^2$

20 **(a)** Complete this sequence of the powers of 2:

$2^1 = 2$ $2^4 =$ _____

$2^2 = 4$ $2^5 =$ _____

$2^3 = 8$ $2^6 =$ _____

(b) Will 2^{10} be larger or smaller than 1000? Write the difference.

21 Put these numbers in ascending order: 2^4, $\sqrt{121}$, 10^2, 3^3, 4, $\sqrt{81}$

Reasoning and communicating

22 Michelle has $17 in her purse. Michelle's dad empties out the 'loose change' jar and divides the money equally between his 3 children. Michelle now has $24 in her purse. How much loose change was in the jar?

23 The train to Suntown has broken down and 224 passengers are stranded. Carly, the train controller, needs to order some buses for them. Each bus can seat 42 people. How many buses should Carly order? Explain why rounding down to the first digit is not useful in this situation.

24 How many whole numbers have the value of their square root in-between 4 and 5?

25 Hamish has a collection of 12 alien figurines. His friend Ben has half this number. Hamish's brother Sean has three times the number that Ben has. The difference between the number of aliens in Sean's collection and the number in Hamish's is:

A 6 **B** 12 **C** 18 **D** 24

26 Explain why 3^2 is not equal to 3×2.

27 Will the actual answer to 57×196 be greater or less than an approximate answer calculated by rounding each number to the first digit? Explain how you decided.

NAPLAN practice 1

Numeracy: Non-calculator

1 Another way of writing 7^3 is:

A $7 + 3$ **B** 7×3

C $7 \times 7 \times 7$ **D** $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

2 What is the best way to estimate the total cost of these items: skateboard, $87; knee pads, $31; helmet, $54?

A $80 + $30 + $50 **B** $80 + $30 + $60

C $90 + $30 + $50 **D** $90 + $40 + $60

3 The correct answer to $18 + 12 \div (5 + 1)$ is:

A 5 **B** 6 **C** 7 **D** 20

4 The answer to 24×379 would be closest to:

A 1000 **B** 2000 **C** 5000 **D** 10 000

5 Which one of these has the same value as 13×5?

A $10 + 3 + 5$ **B** $10 \times 3 + 5$ **C** $10 \times 5 + 3$ **D** $10 \times 5 + 15$

Numeracy: Calculator allowed

6 'Mini-Munches' chocolate bars can be bought individually for 85c each or in packs of 6 for $5. You need to buy 34 bars for a party. What is the least amount you can pay?

7 Tom is stacking a drinks fridge in the supermarket with cans of cola. The fridge has 3 shelves. On each shelf Tom can place 5 rows of 12 cans. How many cans will fit in the fridge?

8 The area of New Zealand is 268 676 square kilometres. What is this area rounded to the nearest thousand square kilometres?

A 268 700 **B** 268 000 **C** 269 000 **D** 270 000

9 The government of New South Wales asked everyone in the state to reduce their water usage to less than 155 L per person per day. Which of the following families has not achieved this target?

Family name	Number of people in the family	Total water used by the family in 1 day
Bennett		760 L
Shwarma		632 L
Nicolai		918 L
Zhang		459 L

Integers

2

Recall 2

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

1 Copy and complete these within 3 minutes.

(a)	$6 \times 7 =$	$6 \times 6 =$	$6 \times 4 =$	$6 \times 11 =$	$6 \times 8 =$
(b)	$7 \times 11 =$	$7 \times 7 =$	$7 \times 5 =$	$7 \times 2 =$	$7 \times 3 =$
(c)	$8 \times 7 =$	$8 \times 6 =$	$8 \times 4 =$	$8 \times 10 =$	$8 \times 8 =$
(d)	$9 \times 12 =$	$9 \times 3 =$	$9 \times 5 =$	$9 \times 11 =$	$9 \times 8 =$
(e)	$12 \times 7 =$	$12 \times 6 =$	$12 \times 12 =$	$12 \times 9 =$	$12 \times 11 =$

2 **(a)** List all the digits with which an even number can end.

(b) List all the digits with which an odd number can end.

3 Copy and complete each of the following by writing a < (less than) or > (greater than) sign between the given values.

(a) 10 _____ 7 **(b)** 3 _____ 6 **(c)** 2 _____ 0 **(d)** 0 _____ 5

4 Calculate:

(a) $3 + 8 + 12$ **(b)** $22 + 19 - 7$ **(c)** $22 - 9 + 87 - 35$

(d) $18 - 9 - 4$ **(e)** $72 - 39 + 14$ **(f)** $51 + 43 - 11 - 7$

5 Write the following temperatures in order from coldest to warmest.

(a) 15°C, 7°C, 0°C, -4°C, 21°C, -11°C

(b) 5°C, -3°C, 10°C, -25°C, 32°C, -14°C

6 Write the following in expanded form, then evaluate.

(a) 7^2 **(b)** 3^4 **(c)** 2^6 **(d)** 1^9

7 Calculate the following.

(a) $3^2 \times 5^2$ **(b)** $4^3 \div 2^3$ **(c)** $8^2 + 6^2$ **(d)** $9^2 - 7^2$

Syllabus references

Number and Algebra: Computation with Integers

✓ Compare, order, add and subtract integers **(ACMNA280)**

✓ Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies **(ACMNA183)**

Number and Algebra: Indices

✓ Investigate index notation and represent whole numbers as products of powers of prime numbers **(ACMNA149)**

2.1 Multiples, factors and divisibility

Need to Know

Multiples

Multiples of a whole number are found by multiplying it by another whole number. For example, the multiples of 7 are 7, 14, 21, 27, 35, …. Multiples are also created with repeated addition.

A **common multiple** of two numbers is a number that both of them divide into exactly. For example, 14, 28, 42 and 56 are common multiples of 7 and 2.

The **Lowest Common Multiple** (LCM) of two numbers is the smallest number that both of the numbers divide into exactly. For example, the LCM of 7 and 2 is 14.

Factors

A **factor** is a number that divides exactly into another number. 'Exactly' means that there is no remainder left after the division. For example, the factors of 28 are 1, 2, 4, 7, 14 and 28.

A **common factor** of two numbers is the number that divides exactly into both of them. For example, the common factors of 18 and 30 are 2, 3 and 6.

The **Highest Common Factor** (HCF) of two numbers is the largest number that divides exactly into both of the numbers. The highest common factor is also known as the **Greatest Common Divisor** (GCD). For example, the HCF of 18 and 30 is 6.

Divisibility

A larger number is **divisible** by a smaller number if dividing by the smaller number gives an exact whole number answer with no remainder. **Divisibility tests** can be used to find the factors of large integers.

A number is divisible by …	If it passes this divisibility test
2	The last digit is an even number (0, 2, 4, 6 or 8).
3	The sum of the digits is divisible by 3.
4	The number formed by the last two digits is divisible by 4.
5	The last digit is 0 or 5.
6	The number is even (divisible by 2) and also divisible by 3.
8	The number formed by the last 3 digits is divisible by 8.
9	The sum of the digits is divisible by 9.
10	The last digit is 0.

Worked Example 1 WE1

Find all the factors of each of the following numbers.

(a) 12 **(b)** 110

Working

(a) $1 \times 12 = 12$
$2 \times 6 = 12$
$3 \times 4 = 12$

Factors of 12: 1, 2, 3, 4, 6, 12.

(b) $1 \times 110 = 110$
$2 \times 55 = 110$
$5 \times 22 = 110$
$10 \times 11 = 110$

Factors of 110: 1, 2, 5, 10, 11, 22, 55, 110.

Worked Example 2 WE2

Determine which of the numbers 75, 98, 110 and 132 are divisible by each of the following.

(a) 3 **(b)** 4 **(c)** 5 **(d)** 6

Working

(a) 75: $7 + 5 = 12$ ✓
98: $9 + 8 = 17$ ✗
110: $1 + 1 + 0 = 2$ ✗
132: $1 + 3 + 2 = 6$ ✓

75 and 132 are divisible by 3.
98 and 110 are not divisible by 3.

(b) 75 ✗
98 ✗
110 ✗
132 ✓

132 is divisible by 4.
75, 98 and 110 are not divisible by 4.

(c) 75 ✓
98 ✗
110 ✓
132 ✗

75 and 110 are divisible by 5.
98 and 132 are not divisible by 5.

(d) Using the working from **(a)**:
75:12 ✗
98: 17 ✗
110: 2 ✗
132: 6 ✓

132 is divisible by 6.
75, 98 and 110 are not divisible by 6.

Worked Example 3 WE3

Find the lowest common multiple (LCM) of the following set of numbers, by first listing the multiples of each: 4 and 6.

Working

4: 4, 8, (12), 16, 20, 24, …

6: 6, (12), 18, 24, 30, 36, …

LCM of 4 and 6 is 12.

Worked Example 4 WE4

Find the highest common factor (HCF) of the following pairs of numbers, by first listing the factors of each number: 12 and 18.

Working

12: 1, 2, 3, 4, 6, 12

18: 1, 2, 3, 6, 9, 18

HCF of 12 and 18 is 6.

Answers page 448

2.1 Multiples, factors and divisibility

Navigator

Q1 Columns 1–3, Q2, Q3 Columns 1 & 2, Q4 Columns 1–3, Q5, Q6, Q7, Q9, Q10, Q12, Q13, Q14, Q15, Q18, Q23	Q1 Columns 2 & 3, Q2, Q3 Columns 2 & 3, Q4 Columns 2–4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q18, Q19, Q20(a), Q23, Q24	Q1 Columns 3 & 4, Q2, Q3 Column 3, Q4 Columns 3 & 4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q20, Q21, Q22, Q24, Q25

Fluency and understanding

WE1

1 Find all the factors of each of the following numbers.

(a) 18	**(b)** 16	**(c)** 23	**(d)** 24
(e) 20	**(f)** 35	**(g)** 36	**(h)** 42
(i) 53	**(j)** 60	**(k)** 77	**(l)** 84

WE2

2 Determine which of the numbers 92, 108, 245 and 3100 are divisible by each of the following.

(a) 3 **(b)** 4 **(c)** 5 **(d)** 8 **(e)** 9

WE3

3 Find the lowest common multiple (LCM) of the following sets of numbers, by first listing the multiples of each.

(a) 2 and 5	**(b)** 3 and 9	**(c)** 5 and 25
(d) 5 and 6	**(e)** 4 and 7	**(f)** 8 and 12
(g) 7 and 9	**(h)** 10 and 12	**(i)** 6 and 11
(j) 9 and 12	**(k)** 20 and 50	**(l)** 8 and 14
(m) 3, 4 and 5	**(n)** 2, 25 and 50	**(o)** 20, 50 and 60

WE4

4 Find the highest common factor (HCF) of the following pairs of numbers, by first listing the factors of each number.

(a) 10 and 15	**(b)** 8 and 24	**(c)** 5 and 12	**(d)** 26 and 36
(e) 11 and 33	**(f)** 28 and 70	**(g)** 44 and 22	**(h)** 10 and 30
(i) 40 and 70	**(j)** 32 and 60	**(k)** 35 and 70	**(l)** 42 and 48

Hint

A factor of a number can't be larger than the number itself.

5 **(a)** The lowest common multiple of 8 and 1 is:

A 8 **B** 16 **C** 24 **D** 80

(b) Which of the following is a factor of 34?

A 4 **B** 12 **C** 17 **D** 68

6 **(a)** A number divisible by 2, 3 and 5 is:

A 6 **B** 15 **C** 60 **D** 65

(b) Which pair of numbers are both divisible by 4?

A 38 and 42 **B** 38 and 52 **C** 38 and 60 **D** 52 and 60

7 **(a)** Which one of the following numbers is not a multiple of 8?

A 4 **B** 24 **C** 72 **D** 88

(b) Which of the following is not a factor of 42?

A 1 **B** 6 **C** 21 **D** 84

8 State TRUE or FALSE for the following.

(a) 346 is a multiple of 3.

(b) 872 is divisible by 6.

(c) 2 is a factor of 348.

(d) 52 is a multiple of 4.

(e) 854 is divisible by 9.

(f) 3 is a factor of 56 902.

9 For each group of numbers, find **(i)** the LCM and **(ii)** the HCF.

(a) 4, 6 and 10

(b) 6, 8 and 12

(c) 8, 12 and 16

(d) 10, 25 and 40

10 Complete the following sentences by using the words 'multiple', 'factor' or 'divisible'.

(a) 32 is a multiple of 8 because it is _____ by 8.

(b) 6 is a _____ of 54, so 54 is a multiple of 6.

(c) 72 is divisible by 9, so that makes it a _____ of 9.

(d) 4 is a factor of 60, so 60 is _____ by 4.

11 **(a)** If 24 lollies are placed into bags so that each bag contains the same number, how many lollies can be in each bag? List all possible answers.

(b) If 36 lollies are placed into bags so that each bag contains the same number, how many lollies can be in each bag? List all possible answers.

12 Ms Williams wants to arrange the seating in the hall for the Year 7s. There must be the same number of chairs in each row. She wants the students to take up all the seats in a row. There are 96 students.

(a) How many rows could there be, and how many seats are in each row? Give all possible combinations, including impractical ones.

(b) Ms Williams would like the arrangement to be as 'square' as possible. Which arrangement is best for this?

13 The smallest number divisible by 3, 4 and 5 is:

A 12 **B** 24 **C** 30 **D** 60

14 If two events occur at different time intervals, the lowest common multiple (LCM) of the two time intervals is the point when the two events coincide, or occur together. Use this information to answer the following question.

In a city lighting display, one set of lights flashes every 25 seconds and the other set flashes every minute. If they are turned on at the same time, write down the next three times when the two sets of lights flash together.

15 Mr Rasheed is putting his students into groups to work on a project. Students must be in groups of 3 or 4. He has 26 students in his class. Find the two different ways Mr Rasheed can divide up his class.

16 **(a)** Find the lowest number greater than 50 that is divisible by 7.

(b) Find the lowest number greater than 100 that is divisible by 11.

(c) Find the first common multiple of 2 and 7 that is greater than 100.

(d) Find the first common multiple of 2, 5 and 7 that is greater than 200.

Reasoning, communicating and open-ended

17 Peter power-walked around an oval while Mei-ling jogged. They started and finished at the same time. They started on the same spot and went in the same direction, keeping up a constant speed for 1 hour. Peter walked 8 laps and Mei-ling jogged 24 laps in the hour.

(a) How many times did Mei-ling pass Peter?

(b) How many times did Mei-ling pass Peter exactly on the spot where they started?

(c) At the beginning of which laps did Mei-ling pass Peter exactly on the spot where they started?

18 **(a)** Copy the table shown. Perform the divisibility tests on the numbers in the left column. Circle the number if the original number is divisible by it. The first one has been done for you.

100 000	(2)	3	(4)	(5)	6	(8)	9	(10)
202 008	2	3	4	5	6	8	9	10
12 121 212	2	3	4	5	6	8	9	10
300 300 300	2	3	4	5	6	8	9	10
7 500	2	3	4	5	6	8	9	10
900 090	2	3	4	5	6	8	9	10
123 456 789	2	3	4	5	6	8	9	10

(b) Complete the following.

(i) If a number is divisible by 4 it will also be divisible by _____.

(ii) If a number is divisible by 9 it will also be divisible by _____.

(c) Explain your answers to **(b)**.

19 The test to determine whether a number is divisible by 6 is to test whether it is divisible by 2 *and* 3. Explain why the test works.

20 A *perfect* number is a number for which the sum of its factors (excluding itself) equals the number. The first perfect number is 6, as $1 + 2 + 3 = 6$.

(a) What is the next perfect number? It is less than 40.

(b) The next perfect number is between 490 and 510. See if you can find it.

21 An *abundant* number is a number for which the sum of its factors is greater than two times the number itself. The first abundant number is 12, as $1 + 2 + 3 + 4 + 6 + 12 = 28$, which is greater than 2×12. Find the next 2 abundant numbers. (Both are less than 40.)

22 **(a)** How can you always find a common multiple of a pair of numbers?

(b) How can you check if this number is the lowest common multiple?

23 Darren is designing a box for 60 identical chocolates to be placed in rows.

(a) Draw three ways Darren could arrange the chocolates in the box.

(b) Which of your arrangements do you think is the most practical for a chocolate box? Explain your answer.

24 Zena is five years of age and Sam is less than 90 years old. Sam's age is a multiple of three and is also a multiple of Zena's age. Find three possible ages Sam could be.

25 Is it possible to find the highest common multiple of two or more numbers? Explain your answer.

Primes and composites 2.2

Need to Know

Prime numbers

A number that has exactly two factors, itself and 1, is a **prime number**.

For example, 7 is a prime number as its factors are 1 and 7.

Composite numbers

A number that has more than two factors is called a **composite number**.

For example, 8 is a composite number as its factors are 1, 2, 4, and 8.

The number 1 and coprimes

The number 1 is a special number. It is neither prime nor composite.

Two numbers are said to be **coprime** if their highest common factor is 1.

For example, 7 and 11 are coprime.

The sieve of Eratosthenes

Eratosthenes was a Greek mathematician who lived from 276 BCE to 195 BCE. He was famous for his 'sieve', a method for finding prime numbers.

See if you can reproduce what he did.

Copy the table and follow the instructions.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Step 1 Cross out the number 1.

Step 2 Go to the next number, which is 2, and circle it. Then, cross out all of the other multiples of 2.

Step 3 Go to the next number that isn't crossed out. This should be 3. Circle it. Then, cross out all of the other multiples of 3.

Step 4 Go to the next number that isn't crossed out, circle it, then cross out all of its multiples.

Step 5 Repeat for the next number that isn't crossed out. Keep repeating until there is no 'next number'.

Answers page 449

2.2 Primes and composites

Navigator

Q1, Q2, Q3, Q5, Q6, Q7, Q8, Q10, Q11, Q13, Q14, Q17	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q17, Q18	Q1, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q15, Q16, Q17, Q18

Use 'The sieve of Eratosthenes' on the previous page to help you answer Questions **1–6**.

Fluency and understanding

1 Write the prime numbers between 1 and 20.

2 How many single-digit prime numbers are there? List them.

3 List all the prime numbers between 20 and 60.

4 **(a)** The first prime number after 50 is:

A 51 **B** 53 **C** 55 **D** 57

(b) A number coprime with 18 is:

A 9 **B** 21 **C** 24 **D** 25

5 **(a)** What is the next prime number after 60?

(b) What is the next composite number after 60?

(c) What are the two odd composite numbers less than 20?

(d) What is the largest prime number less than 50?

6 Write TRUE or FALSE for each of the following statements.

(a) 21 is prime. **(b)** 38 is composite.

(c) 59 is prime. **(d)** 49 is prime.

(e) 5 and 7 are coprime. **(f)** 5 and 6 are coprime.

(g) All even numbers greater than 2 are composite.

(h) All odd numbers are primes.

7 Name a divisibility test that shows that the following numbers are composites.

(a) 410 **(b)** 621 **(c)** 9909

(d) 4 516 803 **(e)** 87 912 404 **(f)** 2 871 025

8 Are the following pairs of numbers coprime? Give reasons for your answer.

(a) 9 and 17 **(b)** 8 and 11 **(c)** 13 and 52 **(d)** 27 and 63

Reasoning, communicating and open-ended

9 Explain why any pair of prime numbers is coprime.

10 2 is the only even prime number. Explain why.

11 Explain why it is easy to tell that 4 567 278 is a composite number.

12 Explain why 2 and 3 are the only two consecutive prime numbers.

13 What is the smallest difference between any two consecutive composite numbers?

14 **(a)** Find the numbers closest to 100 that are coprime with 100.

(b) Find the numbers closest to 36 that are coprime with 36.

15 Will a prime number always be coprime with any other whole number? Explain your answer.

16 If one number is a multiple of another number and both numbers are greater than 1, explain why they cannot be coprime.

17 A conjecture is a mathematical statement that is believed to be true, but has not yet been proven. Goldbach's conjecture (named after the mathematician Christian Goldbach) states that 'every even number greater than 2 can be written as the sum of two primes'. Choose 10 even numbers, and use them to demonstrate Goldbach's conjecture.

18 A pair of 'Sophie Germain primes' (named after the mathematician) is a pair of prime numbers where one number is exactly one more than double the other number. For example, 11 and 23 are Sophie Germain primes, because $11 \times 2 + 1 = 23$. Find two more pairs of Sophie Germain primes.

2.3 Prime factors

Need to Know

A **prime factor** of a number is a factor that is also a prime number.

Every whole number can be written as the unique product of its prime factors.

Factor trees

A factor tree can be used to find the prime factors of a number. To construct a factor tree:

Step 1 Draw two lines (branches) from the number.

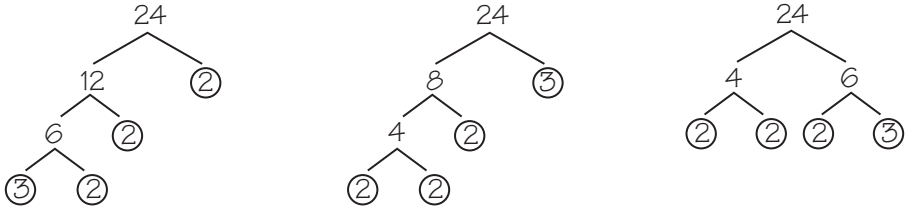
Step 2 Write two factors whose product is the number at the end of the branches.

Step 3 If either or both of the factors are prime, circle them. If either or both are composite, draw new branches and repeat Step 2.

Step 4 Repeat Step 3 until all the factors are circled prime factors.

There may be many different factor trees constructed for a number but the prime factors found will always be the same.

For example, factor trees for 24:



The number can then be written as the product of prime factors, writing any repeated factors in index form.

$24 = 2 \times 2 \times 2 \times 3$

$= 2^3 \times 3$

Prime factors can also be found using repeated division. Using divisibility rules to identify prime factors, divide these into the number until the last answer is 1.

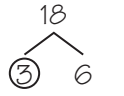
Using prime factors

If two numbers are written as the product of prime factors, the HCF is the product of all the common prime factors.

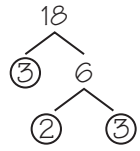
Worked Example 5 WE5

Draw a factor tree for the number 18, then express the number as a product of its prime factors in index form.

Working



$18 = 3 \times 6$



$18 = 2 \times 3 \times 3$
$= 2 \times 3^2$

Worked Example 6 WE6

Use repeated division to find the prime factors of 84. Express the number as a product of its prime factors in index form.

Working

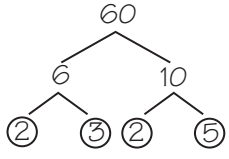
$2 \overline{)\,84}$
$2 \overline{)\,42}$
$3 \overline{)\,21}$
$7 \overline{)\,7}$
$\quad 1$

$84 = 2 \times 2 \times 3 \times 7$
$= 2^2 \times 3 \times 7$

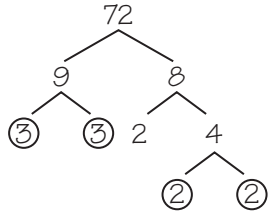
Worked Example 7 WE7

Use prime factors to find the HCF of the following pair of numbers: 60 and 72.

Working



$60 = (2 \times 2 \times 3) \times 5$



$72 = 2 \times (2 \times 2 \times 3) \times 3$

$HCF = 2 \times 2 \times 3$
$= 12$

Answers page 450

2.3 Prime factors

Navigator

Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3 Columns 1 & 2, Q4, Q5, Q9, Q10, Q11, Q14	Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Columns 2 & 3, Q4, Q5, Q6, Q9, Q10, Q11, Q12, Q13, Q14	Q1 Column 4, Q2 Columns 3 & 4, Q3 Column 3, Q4, Q5, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q15

Fluency and understanding

WE5

1 Draw a factor tree for each of the following numbers, then express the number as a product of its prime factors in index form.

(a) 8	**(b)** 12	**(c)** 20	**(d)** 48
(e) 14	**(f)** 26	**(g)** 68	**(h)** 44
(i) 64	**(j)** 72	**(k)** 108	**(l)** 144
(m) 200	**(n)** 750	**(o)** 1000	**(p)** 1236

WE6

2 Use repeated division to find the prime factors of the following numbers. Express each number as a product of its prime factors in index form.

(a) 8	**(b)** 12	**(c)** 28	**(d)** 36
(e) 39	**(f)** 77	**(g)** 51	**(h)** 38
(i) 30	**(j)** 63	**(k)** 96	**(l)** 132
(m) 168	**(n)** 198	**(o)** 288	**(p)** 212

WE7

3 Use prime factors to find the HCF of the following pairs of numbers.

(a) 8 and 20	**(b)** 12 and 18	**(c)** 12 and 36
(d) 36 and 48	**(e)** 24 and 56	**(f)** 28 and 84
(g) 64 and 96	**(h)** 70 and 98	**(i)** 60 and 105
(j) 66 and 110	**(k)** 80 and 128	**(l)** 130 and 156

4 **(a)** 120 written as the product of its prime factors is:

A $2 \times 4 \times 15$ **B** $3 \times 8 \times 5$ **C** $2 \times 2 \times 5 \times 6$ **D** $2 \times 2 \times 2 \times 3 \times 5$

(b) The prime factors of 550 are:

A 2, 5, 11 **B** 2, 10, 55 **C** 5, 10, 11 **D** 2, 11, 25

5 Which numbers are the product of these prime factorisations?

(a) $2^3 \times 3^2$	**(b)** $2^2 \times 3 \times 5^2$	**(c)** $2^4 \times 3^3 \times 5^3$
(d) $3^2 \times 5 \times 7^2$	**(e)** $2^3 \times 5^2 \times 7 \times 11$	**(f)** $2^6 \times 11^2$

6 $6 = 2 \times 3$ and $8 = 2^3$. Use these facts to write the following numbers as products of prime numbers.

(a) 48	**(b)** 96 (48×2)	**(c)** 144 (48×3)

7 $33 = 3 \times 11$ and $10 = 2 \times 5$. Use these facts to write the following numbers as products of prime numbers.

(a) 330	**(b)** 660	**(c)** 990

8 The prime factors of 192 are 2 and 3 and the prime factors of 42 are 2, 3 and 7. List the prime factors of 192×42.

9 **(a)** The HCF of a pair of numbers can also be used to find the lowest common multiple (LCM). Copy and complete the following method.

Prime factors of 36: $2 \times __ \times __ \times __$

Prime factors of 100: $2 \times __ \times __ \times __$

Highest common factor (HCF) of 36 and 100: $2 \times __ = __$

Factors of 36 not used to find the HCF: $__ \times 3 = __$

Factors of 100 not used to find the HCF: $5 \times __ = __$

HCF × all unused factors of 36 × all unused factors of 100: $__ \times 3 \times __ \times 5 \times __ = 900$

The result is the lowest common multiple of 36 and 100.

(b) Use this method to find the LCM for the following pairs of numbers.

(i) 8 and 18	**(ii)** 12 and 18	**(iii)** 8 and 20
(iv) 8 and 72	**(v)** 20 and 26	**(vi)** 20 and 800
(vii) 20 and 1000	**(viii)** 400 and 800	**(ix)** 800 and 1000

10 Marcus is making identical balloon arrangements for a party. He has 84 red balloons and 54 blue balloons. He would like to sort his balloons into groups, each group containing the same number of red and blue balloons, with no balloons left over.

(a) Use prime factors to find the highest common factor of 84 and 54.

(b) If Marcus split his balloons into this number of groups, how many red and blue balloons would be in each group?

Reasoning, communicating and open-ended

11 A number less than 550 is the product of four prime factors: 2, 3, 5 and a fourth prime factor between 15 and 20. What is the fourth factor, and what is the number?

12 **(a)** Find the smallest number that can be written in prime factor form with:

(i) exactly two different prime factors

(ii) exactly three different prime factors

(iii) exactly four different prime factors.

(b) Repeat **(a)**, but remove the condition that the factors are different.

13 140 tickets were sold to the school concert. Each ticket sold was numbered from 1 to 140 and a lucky door prize was awarded to each person who had a ticket with a number that was a multiple of 15, but not a multiple of 9, 10 or 25. Five friends had the ticket numbers 45, 80, 90, 105 and 135.

(a) Do any of them have a winning ticket? If so, what is their winning number?

(b) If 250 tickets were sold, how many winning tickets would there be? Find the winning numbers.

14 A number between 100 and 200 is the product of four primes and has three prime factors (one factor is repeated). Explain how you might go about finding this number and write down two such numbers.

15 To find the prime factors of a number, Ray uses the divisibility tests. He then writes down the number from each divisibility test that worked.

(a) Using this method, what would Ray say are the prime factors of 20?

(b) What is wrong with Ray's method?

(c) Suggest how Ray's method can be improved so that it is accurate.

2.4 Introduction to integers

Need to Know

Integers

The **integers** are all of the **positive** whole numbers (1, 2, 3, …), the **negative** equivalents of these (-1, -2, -3, …), and zero, which is neither positive nor negative.

If a number has no sign in front of it, we assume it is positive. 7 = +7

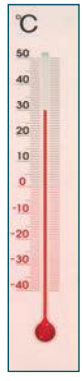
The number line below shows positive and negative integers and zero.



Comparing integers

The further to the right a number is on a horizontal number line, the greater its value. For example, +5 > -10.

'>' is the symbol for 'is greater than'.



The further to the left a number is on a horizontal number line, the smaller its value. For example, -10 < +5.

'<' is the symbol for 'is less than'.

Vertical number lines can also represent directed numbers, where the higher a number is on the line, the greater its value. A thermometer is an example of a vertical number line.

Indicating direction

A + (**positive**) sign or a – (**negative**) sign is also used to show direction. Below is a list of words usually associated with + (**positive**) or – (**negative**) direction.

+ (positive)	– (negative)
up	down
increase	decrease
gain	lose
deposit	withdrawal
profit	loss
above	below

The words '**deposit**' and '**withdrawal**' are banking words used to describe the movement of money into and out of a bank account. '**Profit**' and '**loss**' are words used to describe whether money has been made or lost by a business.

Worked Example 8

WE 8

Write down an integer suggested by each of the following.

(a) depositing $660 into your bank account

(b) diving 5 m below the surface of the ocean

Working

(a) +660 ('deposit' suggests an increase)

(b) -5 ('below' suggests a decrease)

Worked Example 9

WE 9

Write a > or < symbol between each pair of integers to make a correct statement.

(a) -9 _____ -4 **(b)** 0 _____ -10 **(c)** 7 _____ -48

Working

(a) -9 < -4

(-9 is to the left of -4 on a number line)

(b) 0 > -10

(0 is to the right of -10 on a number line)

(c) 7 > -48

(7 is to the right of -48 on a number line)

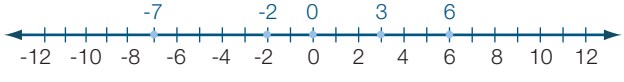
Worked Example 10

WE 10

By first placing the following numbers on a number line, arrange the following integers in ascending order (from smallest to largest).

-2, 3, -7, 0, 6

Working



-7, -2, 0, 3, 6

Answers page 451

2.4 Introduction to integers

Navigator

Q1 Column 1, Q2 Columns 1–3, Q3 Columns 1 & 2, Q4 Columns 1 & 2, Q5, Q6, Q7, Q8, Q11, Q12, Q13, Q14, Q16, Q18	Q1 Column 1, Q2 Columns 2–4, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5, Q7, Q8, Q9 Column 1, Q11, Q12, Q14, Q15, Q16, Q17, Q18	Q1 Column 2, Q2 Columns 3 & 4, Q3 Column 3, Q4 Column 3, Q5 Columns 2 & 3, Q7, Q8, Q9, Q10, Q11, Q12, Q14, Q15, Q16, Q17, Q18

Fluency and understanding

WE8

1 Write down an integer suggested by each of the following.

(a) a profit of $300
(b) a loss of $50 000
(c) 8 seconds before blast-off
(d) 4 hours after take-off
(e) 36 m above sea level
(f) a deposit of $45
(g) a withdrawal of $20
(h) a win by 12 points
(i) a 2-point loss
(j) down 7 floors
(k) up 28 floors
(l) I owe the bank $40 000.
(m) I have $225 in the bank.
(n) The water level dropped by 9 m.

WE9

2 Write a > or < symbol between each pair of integers to make a correct statement.

(a) +6 _____ -3
(b) +1 _____ -3
(c) -7 _____ +5
(d) -2 _____ +6
(e) -12 _____ -4
(f) -7 _____ -1
(g) -2 _____ -11
(h) -9 _____ 0
(i) 0 _____ -2
(j) -35 _____ +7
(k) -4 _____ +67
(l) -11 _____ +75

WE10

3 By first placing the following numbers on a number line, arrange the following integers in ascending order (from smallest to largest).

(a) -3, -12, 6, -4, -1
(b) -11, 10, 9, 2, -10
(c) 1, -6, -4, 9, 2
(d) -8, -5, 7, 1, -6
(e) -1, -4, -7, -8, -12
(f) -56, 23, -79, 0, 6

4 Arrange the following integers in descending order (from largest to smallest).

(a) -5, -6, 3, 7, 8
(b) -8, 7, 10, 1, -12
(c) 1, -8, -4, -9, -6
(d) -12, 0, -5, 7, -11
(e) -11, -5, -2, -12, -9
(f) 0, 79, -54, 4, -9

Hint

The closer a number is to the positive end of a number line, the larger its value.

5 State the opposite of:

(a) west 300 km
(b) 3 days late
(c) 7°C below zero
(d) add 5
(e) subtract 22
(f) go left 3 m
(g) -2
(h) +16
(i) -350

6 **(a)** The directed number +2 is suggested by:

A going up two flights of stairs
B going down two flights of stairs
C losing 2 kg in weight
D the temperature going from 19°C to 17°C.

(b) The directed number -4 is suggested by:

A going up four flights of stairs

B the rain gauge filling with 4 mm of rain

C gaining 4 kg in weight

D the temperature going from 22°C to 18°C.

7 Write TRUE or FALSE for each of the following.

(a) 58 > -60 **(b)** -44 < -50 **(c)** -29 > 30 **(d)** -31 < -29

(e) -75 > 70 **(f)** -92 > -11 **(g)** 19 > -100 **(h)** 82 > -99

8 Write down the integers that lie between:

(a) -4 and 3 **(b)** -2 and 2 **(c)** -9 and -4

(d) -5 and 0 **(e)** -37 and -42 **(f)** -120 and -115

9 Find the next three numbers in each pattern.

(a) -18, -16, -14, _____, _____, _____ **(b)** -20, -15, -10, _____, _____, _____

(c) 9, 6, 3, _____, _____, _____ **(d)** -6, -12, -18, _____, _____, _____

(e) 30, 20, 10, _____, _____, _____ **(f)** 12, 7, 2, _____, _____, _____

10 The balances of four traders' bank accounts show the following figures.

Ben	$428	Sue	-$23
Damien	-$260	Carol	$325

(a) Who has the largest account?

(b) Who owes their bank the most?

11 The minimum and maximum temperatures recorded at Uluru during one week were:

Day	Maximum (°C)	Minimum (°C)
Monday	19	-3
Tuesday	16	-7
Wednesday	22	0
Thursday	20	-1
Friday	23	2
Saturday	20	-2
Sunday	18	-4

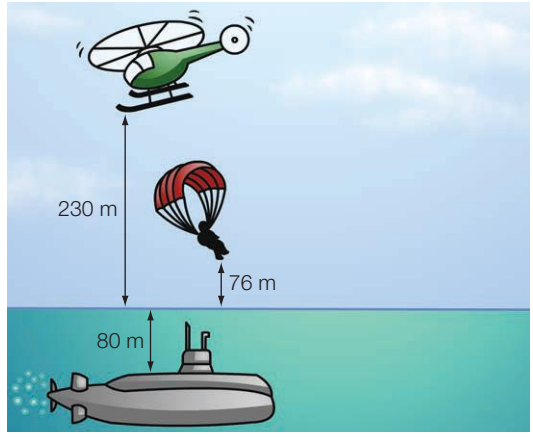
(a) On which day was the lowest minimum recorded?

(b) When was the highest minimum recorded?

(c) Which day had the greatest difference between the minimum and the maximum?

12 In a combined operation, the air force and navy have a helicopter and a submarine at the levels shown.

A parachutist has 76 m to fall before hitting the ocean.



(a) How far is the parachutist from the submarine?

(b) What distance from the helicopter is the parachutist?

(c) How far apart are the helicopter and the submarine?

Reasoning, communicating and open-ended

13 If you cycled 5 km west, then 3 km east, then 2 km west:

(a) Write a directed number sentence to represent this journey.

(b) What single journey could you have taken to get to the finishing point?

(c) What is the opposite of this single journey?

14 If you got in a lift on the ground floor, travelled 7 floors up, then 2 floors up, then 6 floors down and finally 1 floor up:

(a) Write a directed number sentence to represent this sentence.

(b) What single journey could you have taken to get to your final floor?

(c) What is the opposite of this single journey?

15 On Monday, Sam withdrew $80 from his bank account at an ATM. On Tuesday, his pay of $350 was deposited into the account, and he also deposited $40 that he had received for his birthday. On Thursday, Sam used the account to pay a $120 bill online, and on Friday he withdrew $50 from an ATM.

(a) Did Sam have more or less money at the end of the week than at the start?

(b) How much more or less did he have?

16 Write any three negative integers greater than -8.

17 The game of indoor cricket is similar to regular cricket in that batters can score 1, 2, 3, 4 or 6 runs off each ball that is bowled to them. It is different in that pairs of batters stay in for 4 overs (an 'over' is a group of 6 balls). If they lose their wicket, or 'get out', in these 4 overs, they do not leave the pitch. Instead, 5 runs are subtracted from their score, which is shown as the combined total of the runs scored by both batters.

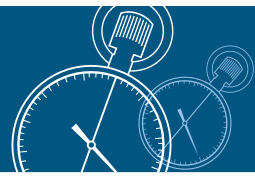
(a) In their first over, Angelo and Murray scored as follows: 2, -5 (wicket), 1, 2, 4, -5 (wicket). What was their overall score at the end of the over?

(b) At the end of their first over, Saul and Jeremy's score was 4. On each ball of the over, they had either scored runs or lost their wicket. List at least three ways they could have reached 4 in six balls.

18 Sharif is a computer technician. He works in an office building with 25 floors, helping to fix computer problems for other employees. Write a short story using this sequence of directed numbers, describing a day at work for Sharif.

0, +21, -13, +7, 0, +4, 0, -2, +5, 0, -1, -2, 0

Half-time 2



1 **(a)** Find the lowest common multiple (LCM) of 9 and 15. Ex 2.1

(b) Find the highest common factor (HCF) of 48 and 72.

2 The common factors of 18 and 30 are: Ex 2.1

A 1, 18, 23 **B** 1, 2, 3, 6 **C** 1, 9, 18, 30 **D** 2, 3, 9, 18

3 Write each of the following as either a positive or a negative integer. Ex 2.4

(a) a deposit of $85 into a bank account **(b)** a weight loss of 7 kg

(c) a win by 3 goals **(d)** a profit of $28 000

(e) 40 m below sea level **(f)** going up 11 floors in a building

4 List all the prime numbers between 30 and 50. Ex 2.2

5 Write a < (less than) or > (greater than) symbol between each of the following to make a true statement. Ex 2.4

(a) 6 _____ 10 **(b)** -4 _____ 7 **(c)** 3 _____ -12 **(d)** -8 _____ -15

6 Draw a factor tree for each of the following, then express each number as a product of its prime factors in index form. Ex 2.3

(a) 54 **(b)** 60 **(c)** 98 **(d)** 104

7 **(a)** A number is divisible by 15 if it is divisible by both 3 and 5. Use divisibility tests to decide if 76 905 is divisible by 15. Ex 2.1

(b) Show, using two divisibility tests, that 4734 is divisible by 6.

8 Write the following sets of integers in ascending order (smallest to largest). Ex 2.4

(a) -37, 7, -30, 0, -3, 3 **(b)** 54, -20, -1, -5, -40

9 Use prime factors to find the HCF of the following pair of numbers: 48 and 120. Ex 2.3

10 The sum of two prime numbers is 30. The numbers cannot be: Ex 2.2

A 7 and 23 **B** 11 and 19 **C** 13 and 17 **D** 14 and 16

11 Three racing cars, Honda, Holden and Ford, take 25 seconds, 30 seconds and 50 seconds, respectively, to complete one lap of a circuit. They start the race at the same time and from the same position. Ex 2.1

(a) How many seconds will pass before the cars are at the same position on the track?

(b) How many laps had each car completed by this time?

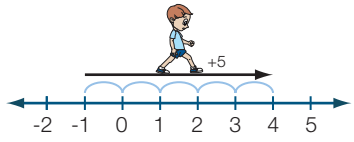
(c) How many laps did the Honda complete in the time taken by the Ford to complete 6 laps?

2.5 Adding and subtracting positive integers

Need to Know

Adding positive integers

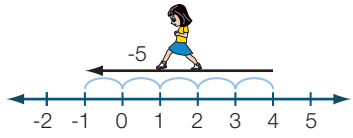
To add a positive integer, face the positive direction of the number line and move the required number of spaces along the number line. This will result in a move to the right. For example, to calculate -1 + 5, start at -1 on the number line and move forward 5 spaces to reach +4, so -1 + 5 = 4.



Addition
Move to the right
-1 + 5 = 4

Subtracting positive integers

As addition and subtraction are opposite operations, move in the opposite direction to subtract a positive integer. Face the negative direction on the number line and move the required number of spaces along the number line. This will result in a move to the left. For example, to calculate 4 – 5, start at +4 on the number line and move 5 spaces to the left to reach -1, so 4 – 5 = -1.

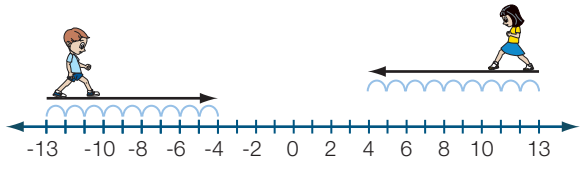


Subtraction
Move to the left
4 – 5 = -1

Using number line symmetry

The number line can be thought of as being a reflection, or mirror image, around 0.

This symmetry can be useful when working with integers. For example, the journey to calculate -13 + 9 (moving to the right) is the mirror image of the journey to calculate +13 – 9 (moving to the left), with answers of -4 and +4, respectively.



So, to calculate -85 + 23, simply calculate 85 – 23, then place a negative sign in front of the answer.

85 – 23 = 62

So -85 + 23 = -62.

Worked Example 11

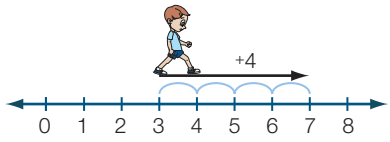
WE11

Use a number line to perform each of the following additions.

(a) $3 + 4$ **(b)** $-5 + 6$ **(c)** $-10 + 8$

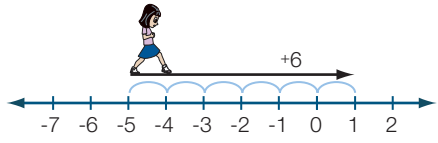
Working

(a)



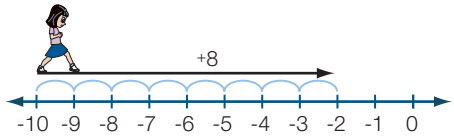
$3 + 4 = 7$

(b)



$-5 + 6 = 1$

(c)



$-10 + 8 = -2$

Worked Example 12

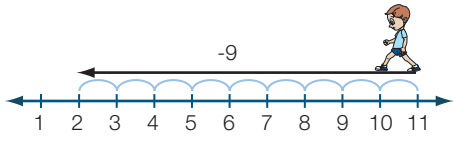
WE12

Use a number line to perform each of the following subtractions.

(a) $11 - 9$ **(b)** $8 - 10$ **(c)** $-4 - 5$

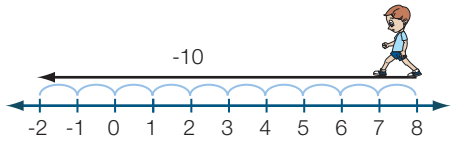
Working

(a)



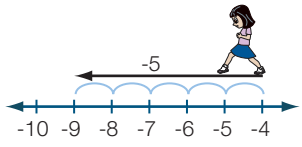
$11 - 9 = 2$

(b)



$8 - 10 = -2$

(c)



$-4 - 5 = -9$

Answers page 452

2.5 Adding and subtracting positive integers

Navigator

Q1 Columns 1–3, Q2 Columns 1 –3, Q3, Q4 Column 1, Q5, Q6, Q8, Q12, Q13	Q1 Columns 2–4, Q2 Columns 2–4, Q3, Q4 Column 2, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13	Q1 Columns 3 & 4, Q2, Columns 3 & 4, Q3, Q4 Columns 2 & 3, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14

Fluency and understanding

WE11

1 Use a number line to perform each of the following additions.

(a) 8 + 3 **(b)** 7 + 1 **(c)** 3 + 8 **(d)** 1 + 7

(e) -9 + 7 **(f)** -11 + 6 **(g)** -10 + 5 **(h)** -12 + 6

(i) -8 + 14 **(j)** -7 + 19 **(k)** -5 + 30 **(l)** -20 + 40

(m) -30 + 20 **(n)** -17 + 13 **(o)** -75 + 55 **(p)** -87 + 56

WE12

2 Use a number line to perform each of the following subtractions.

(a) 5 – 3 **(b)** 8 – 2 **(c)** 12 – 7 **(d)** 11 – 11

(e) 8 – 14 **(f)** 6 – 17 **(g)** 10 – 18 **(h)** 11 – 15

(i) 30 – 40 **(j)** -3 – 5 **(k)** 2 – 22 **(l)** -5 – 7

(m) -9 – 3 **(n)** -24 – 6 **(o)** -35 – 34 **(p)** -66 – 58

3 **(a)** -5 + 3 may be calculated by following which of these instructions?

A start at +5 and walk 3 steps to the right, arriving at 8

B start at -5 and walk 3 steps to the right, arriving at -2

C start at +5 and walk 3 steps to the left, arriving at 2

D start at -5 and walk 3 steps to the left, arriving at -8

(b) -15 – 10 may be described by which number line journey?

A start at -15 and walk 10 steps to the right, arriving at -5

B start at +10 and walk 15 steps to the left, arriving at -5

C start at +15 and walk 10 steps to the left, arriving at +5

D start at -15 and walk 10 steps to the left, arriving at -25

4 Calculate the following.

(a) 3 + 4 + 5 **(b)** 5 + 2 + 4 **(c)** 7 + 1 + 4

(d) -8 + 6 + 9 **(e)** -29 + 5 + 6 **(f)** -37 + 7 + 8

(g) 6 – 7 – 4 **(h)** 3 – 5 – 8 **(i)** 12 – 16 – 4

(j) 14 + 9 – 10 **(k)** 23 – 30 + 6 **(l)** 44 – 51 + 8

5 The temperature in the desert is -12°C, but rises by 20°C. What is the new temperature?

6 A lift is at the third basement floor (3 floors below ground level). It moves up 14 levels, then stops. On which floor does it stop?

7 A submarine 30 m below the surface of the ocean rises 16 m, then dives a further 20 m. What depth is it at now?

8 A diver jumps from a platform 12 m high and dives 17 m to the bottom of a pool. How deep is the pool?

9 You have $5 in your pocket, but you owe a friend $12 and someone else owes you $20. If everyone pays up, how much will you have at the end?

10 Write the following number sentences in words.

(a) $+5 - 7 = -2$ **(b)** $-8 + 9 = +1$ **(c)** $-4 - 3 = -7$

Reasoning, communicating and open-ended

11 (a) Show the following pairs of calculations on separate number lines.

(i) $-1 + 2$ and $+2 - 1$ **(ii)** $-7 + 4$ and $+4 - 7$ **(iii)** $-13 + 6$ and $+6 - 13$

(b) What do you notice? Comment on your observation.

(c) Use your observation from **(b)** to write the answers to the following without using a number line.

(i) $-37 + 86$ **(ii)** $-72 + 95$ **(iii)** $-104 + 119$

12 (a) Draw a number line from -12 to 12. Draw a number line journey showing $5 - 3 = 2$ and the 'mirror image' journey showing $-5 + 3 = -2$.

(b) Use the number line to help you write the 'mirror image' journeys for the journeys represented by these number sentences:

(i) $8 - 6 = 2$ **(ii)** $5 - 4 = 1$ **(iii)** $-10 + 6 = -4$ **(iv)** $-12 + 9 = -3$

(c) Calculate these additions on the negative side of the number line by considering their positive 'mirror image' journeys.

(i) $-23 + 17$ **(ii)** $-48 + 27$ **(iii)** $-59 + 34$ **(iv)** $-62 + 43$

13 The thermometer at a weather station in the desert near Alice Springs read -4°C at 6 a.m. The temperature then rose until it reached a maximum of 25°C at 12 p.m. What might the thermometer readings have been at 7, 8, 9, 10 and 11 a.m.?

14 Write down three different positive integers that when added to -13 give a negative answer.

Problem solving

The slippery snail

A snail is at the bottom of a low garden wall 40 cm high. If the snail manages to move 4 cm up the wall each day, but slips down 3 cm each night, how long does it take the snail to reach the top of the wall?



Strategy options

- Draw a diagram.
- Act it out.
- Look for a pattern.

2.6 Adding and subtracting negative integers

Need to Know

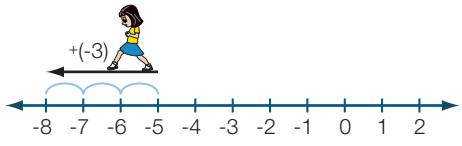
Brackets are used around a negative number to separate it from operation signs to avoid confusion.

Adding negative integers

To add a negative integer, face the positive direction of the number line and move backward the required number of spaces along the number line. The result becomes more negative and will lie further to the left on the number line. This will give the same result as subtracting a positive integer.

For example, to calculate -5 + (-3), start at -5, face the positive direction and move 3 spaces backwards to reach -8, so -5 + (-3) = -8.

This is the same result as -5 – 3 = -8.



To add a negative number, subtract its opposite.

-4 + (-7)
= -4 – 7
= -11

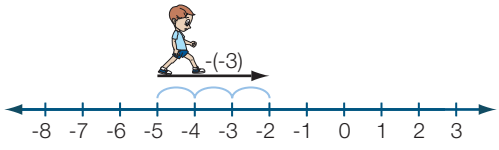
+2 + (-3)
= +2 – 3
= -1

Subtracting negative integers

To subtract a negative integer, face the negative direction of the number line and move backwards the required number of spaces along the number line. The result becomes more positive and will lie further to the right on the number line. This will give the same result as adding a positive integer.

For example, to calculate -5 – (-3), start at -5, face the negative direction and move 3 spaces backwards to reach -2, so -5 – (-3) = -2.

This is the same result as -5 + 3 = -2.



To subtract a negative number, add its opposite.

3 – (-1)
= 3 + 1
= 4

-7 – (-5)
= -7 + 5
= -2

Worked Example 13

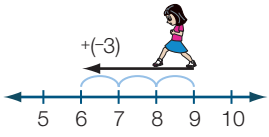
WE13

Use a number line to perform each of the following additions.

(a) 9 + (-3) **(b)** 2 + (-6) **(c)** -8 + (-2)

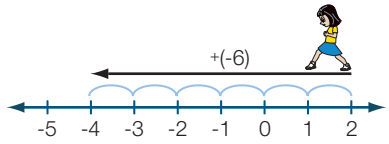
Working

(a)



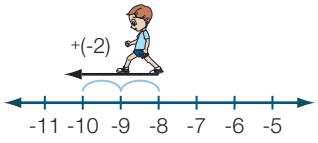
9 + (-3) = 6

(b)



2 + (-6) = -4

(c)



-8 + (-2) = -10

Worked Example 14

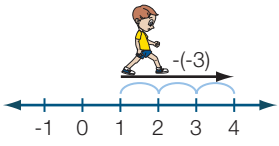
WE14

Use a number line to perform each of the following subtractions.

(a) 1 – (-3) **(b)** -2 – (-6) **(c)** -10 – (-5)

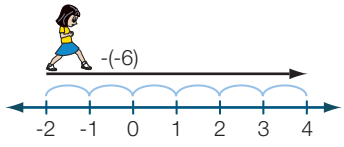
Working

(a)



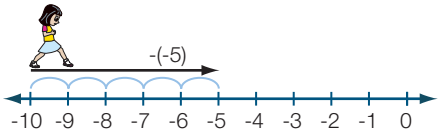
1 – (-3) = 4

(b)



-2 – (-6) = 4

(c)



-10 – (-5) = -5

Answers page 452

2.6 Adding and subtracting negative integers

Navigator

Q1 Columns 1–3, Q2 Columns 1–3, Q3, Q4 Column 1, Q5, Q6, Q9, Q10, Q11, Q12, Q15	Q1 Columns 2–4, Q2 Columns 2–4, Q3, Q4 Columns 2 & 3, Q7, Q8, Q9, Q10, Q11, Q12, Q15	Q1 Columns 3 & 4, Q2 Columns 3 & 4, Q3, Q4, Q7, Q8, Q10, Q11, Q12, Q13, Q14, Q15

Fluency and understanding

WE13

1 Use a number line to perform each of the following additions.

(a) 5 + (-4) **(b)** 3 + (-2) **(c)** 12 + (-3) **(d)** 8 + (-2)

(e) 2 + (-6) **(f)** 4 + (-9) **(g)** 10 + (-17) **(h)** 9 + (-20)

(i) -2 + (-7) **(j)** -3 + (-8) **(k)** -8 + (-12) **(l)** -10 + (-24)

(m) -5 + (-5) **(n)** -30 + (-60) **(o)** -17 + (-12) **(p)** -62 + (-71)

WE14

2 Use a number line to perform each of the following subtractions.

(a) 10 – (-1) **(b)** 4 – (-2) **(c)** 5 – (-5) **(d)** 6 – (-6)

(e) -3 – (-11) **(f)** -9 – (-12) **(g)** -4 – (-20) **(h)** -17 – (-11)

(i) -3 – (-15) **(j)** -9 – (-8) **(k)** -5 – (-13) **(l)** -19 – (-13)

(m) -30 – (-20) **(n)** -55 – (-45) **(o)** -67 – (-41) **(p)** -81 – (-53)

3 **(a)** The finishing point for the number line journey +6 + (-12) may be found by following which of these instructions?

A start at +6 and walk 12 steps to the left, finishing at -6

B start at +6 and walk 12 steps to the right, finishing at +18

C start at -12 and walk 6 steps to the left, finishing at -18

D start at -6 and walk 12 steps to the right, finishing at +6

(b) The answer to +3 – (-20) may be found by following which of these instructions?

A start at +3 and walk 20 steps to the left, arriving at -17

B start at +3 and walk 20 steps to the right, arriving at 23

C start at -20 and walk 3 steps to the right, arriving at -17

D start at -3 and walk 20 steps to the left, arriving at -23

4 Calculate the following.

(a) 9 + (-7) + (-2) **(b)** 6 + (-5) + (-7) **(c)** 9 + (-12) + (-4)

(d) -1 + (-4) + (-5) **(e)** 10 + (-8) + (-7) **(f)** 24 + (-12) + (-6)

(g) 2 – (-1) – (-2) **(h)** 1 – (-6) – (-2) **(i)** -4 – (-7) + (-6)

(j) -7 – (-10) + (-5) **(k)** 12 + (-16) + (-4) **(l)** 48 + (-32) – (-6)

5 The temperature at sunset was 12°C. During the night it fell by 16°C to the minimum temperature. What was the minimum temperature?

6 A worker steps into a lift on the 7th floor of an office building and travels down 10 levels. Which level does she finish at? Write your answer as an integer.

7 Your company is $2000 in debt. You then receive notice that you owe the bank $1500. What is your financial state now?

8 A farmer lowers a bucket from the top of a 1 m high well using a 10 m rope attached to the handle. If she lowers the whole length of rope, how far below ground does the bucket reach?

9 A bird flying 6 m above sea level suddenly spots a fish and dives 9 m straight down before catching the fish. How far below the surface was the fish?

10 A miner is sitting on a ledge 8 m below the top of a vertical mine shaft when he drops his torch. If the torch falls 23 m before hitting the bottom of the shaft, how deep is the shaft?

11 Write the following number sentences in words.

(a) $-5 + (-6) = -11$ **(b)** $+7 - (-8) = +15$ **(c)** $-9 - (-14) = +5$

Reasoning, communicating and open-ended

12 **(a)** Draw a number line from -12 to +12. Draw number line journeys showing $2 + 3 = 5$ and $-2 + (-3) = -5$.

(b) Use the number line to help you write the 'mirror image' journeys for the journeys represented by these number sentences.

(i) $1 + 6 = 7$ **(ii)** $4 + 7 = 11$ **(iii)** $-3 + (-8) = -11$ **(iv)** $-5 + (-4) = -9$

(c) Work out these calculations on the negative side of the number line by considering their positive 'mirror image' journeys.

(i) $-21 + (-15)$ **(ii)** $-28 + (-27)$ **(iii)** $-36 + (-41)$ **(iv)** $-65 + (-29)$

13 **(a)** Evaluate the following.

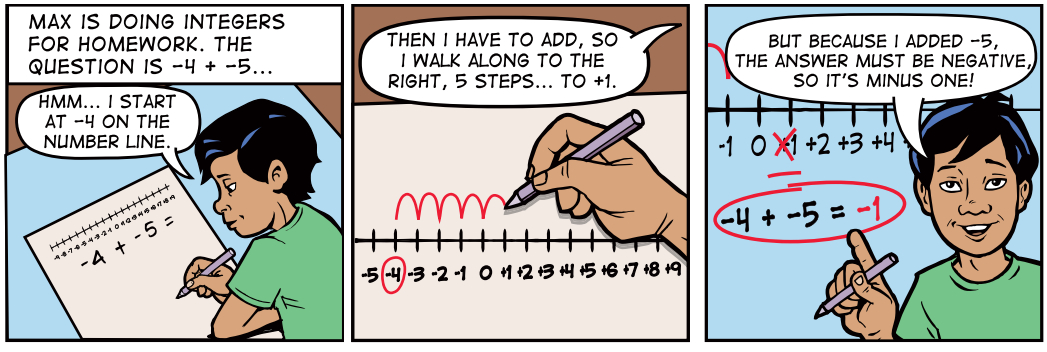
(i) $12 - (+6)$ **(ii)** $12 - (-6)$ **(iii)** $-3 - (+6)$ **(iv)** $-3 - (-6)$

(b) Copy and complete the following sentence.

Whether you start with a positive or a negative integer, if you subtract a positive number, then the result is _____ and if you subtract a negative number, then the result is _____.

14 Write down three integers that when subtracted from -21 give a positive answer.

15



(a) What is the correct answer?

(b) Explain to Max where he went wrong, and how to do these types of questions correctly in the future.

2.7 Simplifying addition and subtraction

Need to Know

Adding a negative number and subtracting a positive number give the same result, a move to the left on the number line; that is, subtraction.

+ (-) and – (+) can be replaced by –.

Subtracting a negative number and adding a positive number give the same result, a move to the right on the number line; that is, addition.

+ (+) and – (-) can be replaced by +.

Simplification rules

If the two signs are the same, the result is addition.
+ (+) = +
– (-) = +

If the two signs are opposite, the result is subtraction.
– (+) = –
+ (-) = –

To apply these rules the two signs must be *next* to each other when the calculation is written down. The rules do not apply otherwise.

Worked Example 15 WE15

Simplify each of the following by writing a single sign between the values, then calculate the answer. Use a number line if necessary.

(a) 4 + (+9) **(b)** -1 + (-3) **(c)** 10 – (+1) **(d)** -6 – (-5)

Working

(a) 4 + (+9)
= 4 + 9
= 13

(b) -1 + (-3)
= -1 – 3
= -4

(c) 10 – (+1)
= 10 – 1
= 9

(d) -6 – (-5)
= -6 + 5
= -1

2.7 Simplifying addition and subtraction

Answers page 453

Navigator

Q1 Columns 1 & 2, Q2, Q3 Columns 1–3, Q4, Q5 Columns 1 & 2, Q6, Q7, Q10, Q13	Q1 Columns 2 & 3, Q2, Q3 Columns 2 & 3, Q4, Q5 Columns 2 & 3, Q6, Q8, Q9, Q10, Q12, Q13	Q1 Column 4, Q2, Q3 Column 4, Q4, Q5 Columns 2–4, Q6, Q8, Q9, Q10, Q11, Q12, Q13

Fluency and understanding

1 Simplify each of the following by writing a single sign between the values, then calculate the answer. Use a number line if necessary.

WE15

(a) +5 + (+2)	**(b)** -1 + (+4)	**(c)** +5 + (+6)	**(d)** -9 + (+7)
(e) +5 – (+3)	**(f)** -8 – (+4)	**(g)** +1 – (+12)	**(h)** -3 – (+7)
(i) +12 + (-3)	**(j)** +10 + (-4)	**(k)** +7 + (-9)	**(l)** +4 + (-8)
(m) -7 – (-6)	**(n)** -3 – (-9)	**(o)** +2 – (-10)	**(p)** +1 – (-1)
(q) -7 + (-3)	**(r)** -9 + (-3)	**(s)** -4 – (-9)	**(t)** -2 – (-6)

2 State TRUE or FALSE for the following.

(a) 25 + (+5) = 25 – 5 **(b)** 30 + (-10) = 30 – 10

(c) 15 – (+20) = 15 – 20 **(d)** 40 – (-50) = 40 – 50

3 Calculate:

(a) 6 – 18	**(b)** 9 – 20	**(c)** -5 + 9	**(d)** -3 + 6
(e) -12 + 7	**(f)** -10 + 2	**(g)** -9 – 4	**(h)** -7 – 5
(i) -60 + 30	**(j)** -52 + 21	**(k)** -27 + 84	**(l)** -28 + 93

4 **(a)** -9 – (-6) – (+4) simplifies to:

A -9 – 6 – 4 **B** -9 + 6 + 4 **C** -9 + 6 – 4 **D** 9 + 6 + 4

(b) 14 – (+4) + (-3) is the same as:

A -14 – 4 – 3 **B** 14 – 4 – 3 **C** 14 – 4 + 3 **D** 14 + 4 + 3

5 Calculate:

(a) -6 + 7 + 5	**(b)** -4 + 6 + 5	**(c)** -12 + 3 + 2	**(d)** -11 + 3 + 4
(e) -8 + 5 – 3	**(f)** -6 + 2 – 4	**(g)** -4 + 12 – 4	**(h)** -3 + 10 – 6
(i) -5 – 1 – 6	**(j)** -2 – 5 – 4	**(k)** 36 – 40 – 5	**(l)** 43 – 50 – 2

6 Your monthly bank statement shows a balance of $260 at the start of a particular month, followed by the transactions for the month listed below.

Credit	$30
Debit	-$80
Credit	$200
Debit	-$60
Debit	-$500

What is your account balance at the end of the month?

Hint

A deposit into your account is a credit. A withdrawal from your account is a debit.

7 A maintenance worker in an office block starts work on the ground floor, and then travels as follows.

UP 10 floors
DOWN 7 floors
DOWN 5 floors
UP 4 floors
DOWN 6 floors
UP 12 floors

(a) What was the highest floor the worker reached?

(b) What was the lowest floor he travelled to?

(c) Which floor did he finish on?

8 Diana makes and sells soft toys. She keeps records of her profits and losses over a 10-week period. These records are shown in the table. Find Diana's overall profit or loss during this time.

Week	Result	
1	Profit	$120
2	Profit	$25
3	Loss	-$70
4	Profit	$210
5	Loss	-$150
6	Loss	-$180
7	Loss	-$90
8	Profit	$40
9	Loss	-$160
10	Loss	-$95

Reasoning, communicating and open-ended

9 Complete the following addition and subtraction grids. Begin with the number at the top of each column, and either add or subtract the numbers at the left of each row.

(a)

+	-6	+20		31
+4				
		+13		
	+2			
			-23	22

(b)

–	8	-11		
	5			
-7			26	
+5				-30
		-1		

10 A 'magic square' is a square grid of numbers where the sum of each of the rows, columns and diagonals is the same number.

(a) What is the magic sum for this 3×3 magic square?

(b) Complete the magic square in part **(a)**.

	6	-1
	2	
5		

11 You can make new magic squares by adding or subtracting the same number from every number in an existing magic square.

(a) Make a new 3×3 magic square by subtracting 4 from each number in the magic square in Question **10 (a)**.

(b) What is the new magic sum? Explain why the new magic sum is not 4 less than the old magic sum.

12 In the sport of golf, each 'hole' has a particular number of 'shots' it should take you to get the ball from the tee to the hole. This is known as the 'par' for the hole. If you take this number of shots to get the ball in the hole, your score for the hole is 0. If your number of shots for the hole is *less than par*, this is shown on the bottom row of the scorecard by a *negative number*. If your number of shots is *greater than par*, this is shown with a *positive number*.

(a) Carla played nine holes of golf. She scored under par on four holes, par on three holes and over par for the rest. If Carla took a total of 35 shots to complete her round, fill in what her scorecard may have looked like.

Hole number	1	2	3	4	5	6	7	8	9	Total
Par for the hole	4	4	5	3	4	5	4	3	4	36
Shots taken										35
Score compared to par										

(b) Carla played another round of nine holes. Her overall score was -1 (1 under par). What could her scorecard have looked like, if she did not score par (0) on any hole?

13 Find two numbers, one positive, one negative, that have a difference of 31. Find two more such pairs.

Challenge 2



1 What are the next three numbers in each pattern?

(a) 17, 13, 9, 5, _____, _____, _____

(b) -31, -25, -19, -13, _____, _____, _____

2 The product of three brothers' ages is 72 and their sum is 14. The youngest of the brothers are twins. What are the ages of the brothers?

3 Find three consecutive numbers (numbers that come one after another, such as 5, 6, 7) so that the sum of the first and third numbers is 172.

4 In a school there are 150 students in Year 7. For a performance in the school hall, chairs are arranged in rows, with the same number of chairs in each row, so that all 150 students are seated with no spare seats. If 10 more chairs were added to each row, everyone could be seated in 4 fewer rows, allowing the people in the back row to be closer to the stage. How many chairs were in each row in the original seating arrangement?

5 What is the sum of all the digits in the numbers from 1 to 100?

6 Shannon needs to know the total amount of money her friends have raised for the 'Save the Koala' fund. All they will tell her is this:

Patricia and Tim have collected $130 between them.
If Tim and Jenni pooled their money, they would have $150.
Jenni and Robyn have $100 altogether.
Robyn and Hilary have just $70 in total.
If Hilary and Patricia combined their money, they would have $90.

Tell Shannon what she needs to know.

7 A number less than 100 gives a remainder of 2 when divided by 4, a remainder of 3 when divided by 5, and a remainder of 4 when divided by 6. Find the number.

8 41 is a prime number. If the order of its digits is reversed it becomes 14, which is NOT a prime number. The number of two-digit prime numbers that DO give a prime number when their digits are reversed is:

A 8 **B** 9 **C** 10 **D** 11

9 **(a)** How many zeroes are at the end of the number given by

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 19$?

A 1 **B** 2 **C** 3 **D** 4

(b) How many zeroes are at the end of the number given by

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 29$?

A 3 **B** 4 **C** 5 **D** 6

(c) How many zeroes are at the end of the number given by

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 59$?

A 9 **B** 10 **C** 11 **D** 12

(d) How many zeroes are at the end of the number given by

$10 \times 11 \times 12 \times 13 \times 14 \times 15 \times 16 \times \ldots \times 99$?

A 18 **B** 19 **C** 20 **D** 21

Chapter review 2

D.I.Y. Summary

Key Words

common factor	factor	negative
common multiple	Greatest Common Divisor (GCD)	positive
composite number	Highest Common Factor (HCF)	prime factor
coprime	integers	prime number
deposit	loss	profit
divisibility	Lowest Common Multiple (LCM)	withdrawal
divisible	multiple	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 A _____ of 6 is 18. The _____ of 6 and 4 is 12.

2 The _____ are all of the positive and negative whole numbers, and zero, which is neither positive nor negative.

3 1, 2, 3, 6, 9 and 18 are the _____s of 18. The _____ of 18 and 27 is 9.

4 A number that is not _____ by any numbers other than 1 and itself is called a _____.

5 A number with more than two factors is called a _____.

6 When you put money into a bank account, you are making a _____.

7 The addition of two negative numbers will always give a _____ answer.

8 If you sell something for less money than you bought it for, you have made a _____.

9 Every whole number greater than 1 can be written as the unique product of its _____s.

10 You make a _____ when you take money out of your bank account.

11 If you sell something for more than you bought it for, you have made a _____.

12 Two numbers are _____ if their highest common factor is 1.

Fluency and understanding

1 Find the LCM of: Ex 2.1

(a) 9 and 6 **(b)** 9 and 12 **(c)** 10 and 15

2 List all the factors of: Ex 2.1

(a) 36 **(b)** 48 **(c)** 51 **(d)** 100

3 Find the HCF of: Ex 2.1

(a) 24 and 56 **(b)** 18 and 72 **(c)** 45 and 80

4 Copy this table and do the divisibility tests. Circle the number if the original number is divisible by it.

5301	2	3	4	5	6	9	10
10 000	2	3	4	5	6	9	10
333 333	2	3	4	5	6	9	10
31 700	2	3	4	5	6	9	10
43 521 820	2	3	4	5	6	9	10

Ex 2.1

5 State whether each of the following numbers is a prime number or a composite number, and explain why.

Ex 2.2

(a) 5 **(b)** 16 **(c)** 77 **(d)** 276 350

6 By drawing a factor tree or using the 'repeated division' method, express each number as a product of its prime factors.

Ex 2.3

(a) 24 **(b)** 30 **(c)** 88 **(d)** 200

7 Use prime factors to find the HCF of:

Ex 2.3

(a) 27 and 36 **(b)** 72 and 96 **(c)** 108 and 240

8 Write an integer to represent the following.

Ex 2.4

(a) 14 degrees below zero **(b)** an altitude of 200 metres

9 State the opposite of:

Ex 2.4

(a) north 5 km **(b)** adding 27

10 Write < or > between the following pairs of numbers to make a true statement.

Ex 2.4

(a) -52 _____ 25 **(b)** 19 _____ -20

11 Arrange the following numbers in ascending order.

Ex 2.4

(a) -7, 12, 0, -9, 7 **(b)** 4, -4000, 40, 400

12 Calculate:

Ex 2.5

(a) +16 + 2 **(b)** -3 + 18 **(c)** -15 + 5 **(d)** +9 – 3

(e) +1 – 5 **(f)** +16 – 8 **(g)** +7 – 12 **(h)** -14 – 18

13 Calculate:

Ex 2.6

(a) +7 + (-10) **(b)** +9 + (-6) **(c)** -11 – (-4) **(d)** -4 – (-4)

(e) -12 – (-5) **(f)** +5 + (-3) **(g)** -8 – (-5) **(h)** -5 + (-7)

14 Rewrite the following with a single sign between the integers, then evaluate.

Ex 2.7

(a) 9 – (+11) **(b)** -3 + (+10) **(c)** -10 – (-21) **(d)** 8 + (-12)

(e) -4 – (-41) **(f)** -14 + (+28) **(g)** -5 – (+8) – (-2) **(h)** 4 + (-9) + (+2)

15 Use the words 'multiple', factor' and 'divisible' to complete the following sentences.

(a) 45 is _____ by 9, so that makes it a _____ of 9.

(b) 8 is a _____ of 56, so 56 is a _____ of 8.

(c) 27 is _____ by 3, so that makes it a _____ of 3.

16 If 96 lollies are to be divided into packets so that each packet contains the same number, how many lollies can be in each packet? Give all possible combinations.

17 Use the symmetry of the number line to help you calculate the following.

(a) -31 + 19 **(b)** -54 – 27 **(c)** -22 + (-38) **(d)** -9 – (61)

18 Describe the number line journey you could follow to find the value of:

(a) $+3 + (-8)$ **(b)** $-6 - (+5)$

19 For each pair of numbers, state whether or not they are coprime. If not, explain why.

(a) 11 and 27 **(b)** 51 and 63 **(c)** 14 and 35 **(d)** 24 and 55

20 In the game of indoor cricket, 3 runs are subtracted from a team's score every time a wicket is taken. Here is what happened in the first over of a game.

Ball 1: 2 Runs
Ball 2: Wicket
Ball 3: 1 Run
Ball 4: Wicket
Ball 5: 1 Run
Ball 6: 4 Runs

What was the score at the end of the over?

21 There are 84 male and 108 female guests at an official dinner. The dinner organiser wants to have an equal number of male and female guests at each table.

(a) Use prime factors to find the HCF of 84 and 108, and so find the number of tables required.

(b) Use your answer from **(a)** to find the number of male and female guests at each table.

22 Find the first common multiple of 2, 7 and 9 that is greater than 500.

23 **(a)** What is the first prime number after 70?

(b) What is the first composite number after 70?

24 Michelle made deposits of \$210, \$25, \$45 and \$66 into her bank account during one month, and withdrawals of \$35, \$56, \$214 and \$102 during the same period.

(a) At the end of the period, had her balance increased or decreased?

(b) By how much had it increased or decreased?

25 Joanna and Petra are on two different Ferris wheels, both rotating clockwise. The first wheel takes 25 seconds to make a rotation and the other takes 30 seconds. If Joanna and Petra were both at eye level at the bottom of each of their Ferris wheels when they start turning, how many seconds will pass until they are again both at the bottom at eye level?

26 Miners in a copper mine are working 900 m underground. They get in a lift and travel a further 250 m down. What depth are they working at now? Write your working and answer using negative integers.

Reasoning and communicating

27 Copy the following and write $<$, $=$ or $>$ to make true statements.

(a) $4 - (-7)$ ____ $4 + 7$ **(b)** $-3 + 2$ ____ $3 - 2$

(c) $5 + (-3)$ ____ $5 - (+3)$ **(d)** $-8 - 9$ ____ $-8 + 9$

28 If you know that a number is divisible by 8, what other numbers do you also know it is divisible by?

29 A number between 900 and 1000 has four prime factors: 2, 5, 7 and one other factor. What is the number, and what is the missing factor?

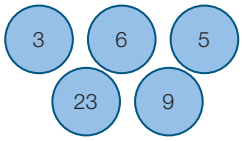
NAPLAN practice 2

Numeracy: Non-calculator

1 Alicia is standing at -4 on a large number line placed on the floor. She walks 11 steps in the positive direction. At which number is she standing now?

A -15 **B** -7 **C** 7 **D** 15

2



The sum of the composite numbers in the group shown above is:

A 8 **B** 15 **C** 18 **D** 31

3 A maintenance worker in a city office building gets in a lift in the 3rd basement level (3 floors below ground level) and goes up 11 levels. What floor does he get out on?

4 The number 42 written as a product of prime factors is:

A 21×2 **B** $2 \times 3 \times 7$ **C** 1×42 **D** 6×7

5 On a sunny winter's day in Moscow, the temperature at midday was 3°C. By midnight it had dropped to -9°C. The integer that represents this change is:

A -12 **B** -3 **C** 3 **D** 12

Numeracy: Calculator allowed

6 A train passes a town every week and every 10 days an aircraft flies overhead. If the train and the aircraft were observed to be at the town on a certain day, after how many more days will both appear again at the same time?

A 10 **B** 11 **C** 50 **D** 70

7 Vin has $260 in his bank account. During one month, he makes the following transactions.

Deposit: $55
Withdrawal: -$75
Withdrawal: -$33
Deposit: $85
Withdrawal: -$27

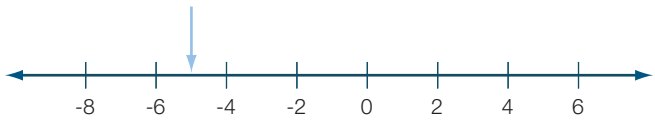
How much does Vin have in his account at the end of the month?

8 The ages in years of three people are 65, 39 and 52. The highest common factor of the three ages is:

A 1 **B** 13 **C** 39 **D** 165

9 The arrow is pointing to an integer on the number line.

What number is at this position? ____



Mixed review A

Fluency and understanding

1 Write these integers in ascending order. Ex.2.4

8, 17, -10, 0, -25, 32, -48

2 Write the following in index form. Ex.1.2

(a) 9 squared **(b)** $7 \times 7 \times 7 \times 7$ **(c)** 4 cubed

3 List all numbers divisible by both 8 and 6 that are less than 100. Ex.2.1

4 Write an integer to represent the following. Ex.2.4

(a) a bank withdrawal of $570 **(b)** a win by 5 points

5 Write < or > between the following pairs of numbers to make a true statement. Ex.2.4

(a) -27 ____ 14 **(b)** 0 ____ -35

6 Calculate: Ex.1.3

(a) $8000 \div 200$ **(b)** $1200 \div 4$ **(c)** $45\,000 \div 90$

(d) 30×120 **(e)** 400×1500 **(f)** 2000×5000

7 Use a mental strategy to calculate the following. Ex.1.1

(a) $4 \times 17 \times 5$ **(b)** $183 + 220$ **(c)** 42×19

(d) 36×11 **(e)** $169 + 71$ **(f)** $5 \times 24 \times 8$

8 Evaluate: Ex.1.5

(a) $6 \times 4 \div 2 \times 6$ **(b)** $5 + 6 \times 7$ **(c)** $18 + 12 - 7 + 6$

(d) $2 + 5 \times 9$ **(e)** $18 \div 6 - 3$ **(f)** $8 \times (15 - 5)$

9 Calculate the following. Ex.2.5, 2.6

(a) $-9 + 7$ **(b)** $5 + (-8)$ **(c)** $-3 - (-7)$ **(d)** $-6 - 11$

10 Estimate the answers to the following by rounding to the first digit. Ex.1.4

(a) 17×93 **(b)** 46×281 **(c)** 337×240

(d) $953 \div 11$ **(e)** $8195 \div 237$ **(f)** $12\,495 \div 5400$

11 Arrange the following numbers in ascending order (from smallest to largest). Ex.2.4

(a) 5, 0, -15, 10, -5 **(b)** -300, 3, 0, -30, 3000

12 Find the lowest common multiple (LCM) of: Ex.2.1

(a) 8 and 12 **(b)** 12 and 16

13 Find the highest common factor (HCF) of: Ex.2.1

(a) 36 and 27 **(b)** 64 and 72

14 List the factors of each of these numbers and state whether each number is prime or composite. Ex.2.2

(a) 18 **(b)** 23 **(c)** 44 **(d)** 79

15 Write each number as the product of its prime factors in index form. Ex.2.3

(a) 63 **(b)** 48 **(c)** 72 **(d)** 120

Ex 2.7

16 Simplify the following by writing a single sign between the values, then calculate the answer.

(a) $-6 + (-7)$ **(b)** $4 - (-11)$ **(c)** $-2 + (+7)$

(d) $22 + (-9)$ **(e)** $-5 - (-10)$ **(f)** $18 + (+3)$

17 A submarine 110 m below the surface of the water rises 80 m, then dives 150 m. What depth is it at now?

18 Which of the following numbers are:

(a) prime

(b) perfect squares

(c) powers of 2?

3, 7, 9, 24, 11, 16, 19, 43, 32, 28, 13, 8, 2, 25

19 What is the first perfect cube that is divisible by both 3 and 4?

20 Calculate an approximate answer for the following by rounding to the first digit, then state whether the actual answer will be higher or lower.

(a) 256×37 **(b)** 1379×24 **(c)** $5498 \div 46$

21 The Royal Easter Show runs for 7 days. The total attendance at this year's show was 62 982.

(a) Approximately how many people per day was this? Use rounding to a convenient multiple of 1000 to calculate your answer.

(b) If each person paid an average ticket price of $12, use your answer from **(a)** to calculate how much money the show organisers made from ticket sales. Use some mental or written strategies to calculate your answer.

Reasoning and communicating

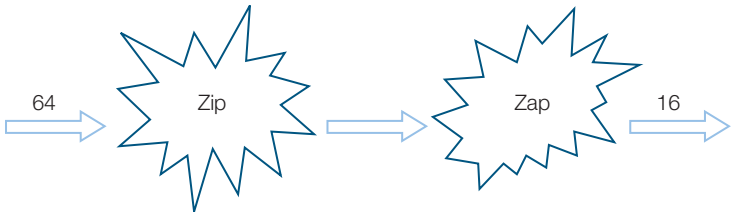
22 On Monday, Kiran withdrew $100 from his bank account at an ATM. On Tuesday, he used his account to pay his $85 phone bill online. On Wednesday, he deposited $250. On Friday, he withdrew another $60 from the account.

(a) By the end of the week, did Kiran have more or less money in his bank account?

(b) How much more or less?

23 The number 64 passes through two 'magic clouds ' Zip and Zap and emerges as the number 16.

Which of the following can describe what Zip and Zap did to the number passing through?



A Zip: square root, Zap: square

B Zip: cube root, Zap: square

C Zip: square root, Zap: nothing

D Zip: nothing, Zap: cube root

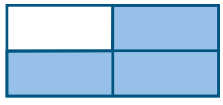
Fractions

3

Recall 3

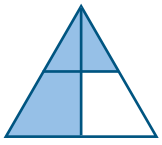
Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

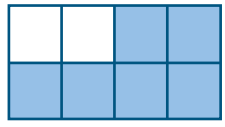
1 Which of these have three-quarters of the whole shape shaded in?

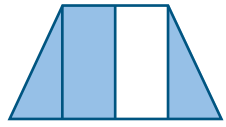
(a) 

(b) 

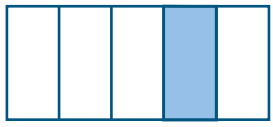
(c) 

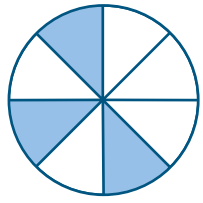
(d) 

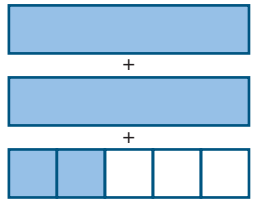
(e) 

(f) 

2 Write down a fraction to represent each of the following diagrams or descriptions.

(a) 

(b) 

(c) 

(d) One person's share of a birthday cake divided equally between 10 people.

(e) Joseph eats five out of eight equal-sized slices of a pizza.

(f) two-thirds **(g)** twelve-fifths **(h)** seven-sixteenths

3 **(a)** Write these in order from largest to smallest: $\frac{3}{4}, 1, 0, 2, 1\frac{1}{4}$

(b) Write these in order from smallest to largest: $\frac{3}{8}, \frac{1}{8}, 0, \frac{7}{8}, \frac{11}{8}, 1, \frac{9}{8}$

4 Calculate:

(a) $\frac{4}{7} + \frac{1}{7}$ **(b)** $\frac{8}{11} - \frac{2}{11}$ **(c)** $\frac{2}{3} + \frac{2}{3}$

5 **(a)** Write the first five multiples of 8. **(b)** Write the first five multiples of 12.

(c) Write the lowest common multiple (LCM) of 8 and 12.

(d) Write all the factors of 24. **(e)** Write all the factors of 36.

(f) Write the highest common factor (HCF) of 24 and 36.

Syllabus references

Number and Algebra: Fractions, Decimals and Percentages

- ✓ Compare fractions using equivalence; locate and represent positive and negative fractions and mixed numerals on a number line **(ACMNA152)**
- ✓ Solve problems involving addition and subtraction of fractions, including those with unrelated denominators **(ACMNA153)**
- ✓ Multiply and divide fractions and decimals using efficient written strategies and digital technologies **(ACMNA154)**
- ✓ Express one quantity as a fraction of another, with and without the use of digital technologies **(ACMNA155)**

3.1 Understanding fractions

Need to Know

A **fraction** is used to show parts of a whole. To write a fraction two numbers are required, one number divided by another. A fraction represents division.

$\frac{3}{5}$

The **numerator** tells us how many equal parts we have.

The **denominator** tells us how many equal parts one whole has been divided into.

Drawing the line between the numerator and the denominator is equivalent to writing the division sign, ÷.

Proper fractions

Proper fractions have a numerator that is less than the denominator. They have values less than 1.

For example: $\frac{1}{2}$ and $\frac{3}{5}$.

Improper fractions

Improper fractions have a numerator that is greater than or equal to the denominator. They have values greater than or equal to 1.

For example: $\frac{3}{2}$, $\frac{9}{9}$ and $\frac{17}{6}$.

Any whole number can be written as an improper fraction. Example: $3 = \frac{3}{1}, \frac{6}{2}, \frac{9}{3}, \ldots$.

Mixed numbers

Mixed numbers have whole number parts and fraction parts written separately.

For example: $1\frac{1}{2}$, $2\frac{5}{6}$ and $34\frac{2}{7}$.

Using a number line

A number line can be used to represent fractions. The size of fractions can be shown and they can then be compared.

If the distance between the integers is the same as the denominator of the fraction, it is easy to locate the fraction on the number line.

For example: If dividing a number line into thirds, make the distance between integers 3 cm. Then 1 cm would represent $\frac{1}{3}$.

Negative fractions

Fractions as well as integers can be negative. Negative fractions are found on the left of zero and get smaller as you move further to the left. They can be written as proper fractions, improper fractions and mixed numbers.

For example: $-\frac{3}{7}$, $-\frac{15}{4}$, $-3\frac{1}{2}$

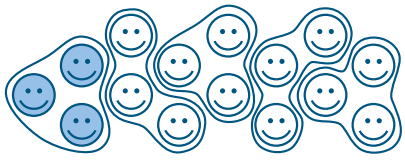
Worked Example 1 WE1

How many smileys are there in $\frac{4}{5}$ of this collection?



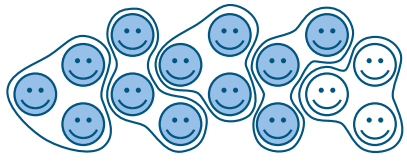
Working

$15 \div 5 = 3$, so $\frac{1}{5}$ of $15 = 3$



$\frac{4}{5}$ is 4 groups of 3 smileys,

so $\frac{4}{5}$ of $15 = 4 \times 3 = 12$



Worked Example 2 WE2

Copy this number line and show the positions of these fractions: $-\frac{3}{5}$, $\frac{4}{5}$ and $\frac{8}{5}$.

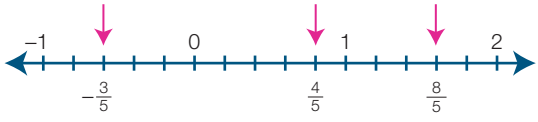


Working





(Divide the number line into fifths.)



Worked Example 3 WE3

Write the whole number 9 as an improper fraction with a denominator of 4.

Working

$$9 = \frac{\square}{4}$$

$$= \frac{9}{1} \times \frac{4}{4}$$

$$= \frac{36}{4}$$

Worked Example 4 WE4

For the following, write the amount each student receives as a fraction (or as a mixed number if appropriate).

5 blocks of chocolate are shared equally among 8 students.

Working

$5 \div 8$

$= \frac{5}{8}$

Answers page 455

3.1 Understanding fractions

Navigator

Q1, Q2, Q3 (a), Q4, Q5, Q6, Q7 Columns 1 & 2, Q8, Q11 (a), Q13, Q16, Q17, Q19 (a), Q20 (b), Q22	Q1, Q2, Q3 (b), Q4, Q5, Q6, Q7 Columns 2 & 3, Q8 (d)–(f), Q9, Q10, Q11, Q13, Q14, Q16, Q17, Q18, Q19 (b), Q20 (b), Q22	Q2 (b), Q3 (b), Q4, Q6, Q7 Column 3, Q8 (d)–(f), Q9, Q10, Q11 (b), Q12, Q13, Q14, Q15, Q17, Q18, Q19 (a), Q20, Q21, Q22

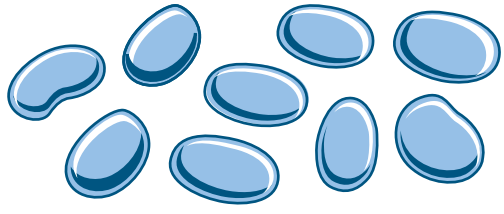
Fluency and understanding

WE1

1 (a) How many smileys are there in $\frac{3}{4}$ of this collection?



(b) How many jelly beans are there in $\frac{2}{3}$ of this collection?



WE2

2 (a) Copy this number line and show the positions of these fractions: $-\frac{1}{4}$, $\frac{1}{4}$, $\frac{5}{4}$ and $\frac{8}{4}$.



(b) Copy this number line and show the positions of these fractions: $-\frac{5}{6}$, $\frac{2}{6}$, $-\frac{1}{6}$ and $\frac{8}{6}$.



WE3

3 (a) Write the whole number 5 as an improper fraction with a denominator of:

(i) 2 **(ii)** 7 **(iii)** 11 **(iv)** 5 **(v)** 1

(b) Write the whole number 13 as an improper fraction with a denominator of:

(i) 2 **(ii)** 5 **(iii)** 8 **(iv)** 13 **(v)** 1

4 **(a)** For each of the following, write the amount each student receives as a fraction (or as a mixed number if appropriate). WE4

(i) 1 pizza is shared equally between 2 students.

(ii) 2 apples are shared equally between 3 students.

(iii) 6 packets of lollies are shared equally between 5 students.

(iv) 10 packets of biscuits are shared equally between 7 students.

(b) In which of the above situations does a student receive more than one whole?

5 Here are 16 lollies. How many will you eat if you eat these fractions of the total?

(a) $\frac{1}{4}$

(b) $\frac{3}{4}$

(c) $\frac{1}{8}$

(d) $\frac{5}{8}$



6 **(a)** What fraction of the bananas in this bunch are rotten?

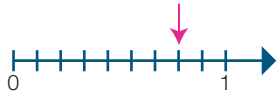


(b) What fraction of the flowers in this vase are red?

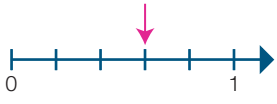


7 Write the value of the fraction indicated by the arrow on each of the number lines below.

(a)

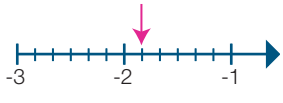

(b)


(c)


(d)


(e)


(f)


(g)


(h)


(i)


8 Write a fraction or a mixed number to show each of these:

(a) a numerator of 8 and a denominator of 17

(b) denominator of 4 and a numerator of 15

(c) nine chocolate biscuits in a packet of 20

(d) two wholes and two thirds

(e) 4 complete pairs of socks and one odd sock

(f) 3 whole 24-piece blocks of chocolate, with 7 extra pieces.

9 Write a fraction to show each of these:

(a) the weekend days as a fraction of a whole week

(b) 1 hour out of a whole day

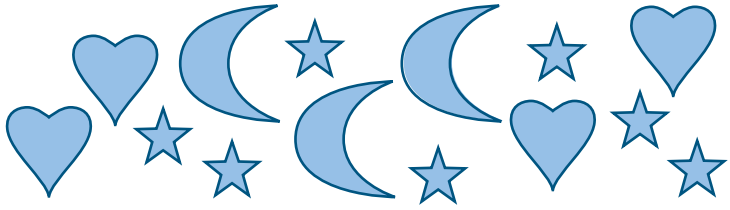
(c) 1 second out of a whole minute

(d) 17 minutes out of a whole hour

(e) 157 mL of cola drunk from a 375 mL can

(f) 421 L of water in a 500 L rainwater tank.

10 What fraction of this collection of shapes are:



(a) stars **(b)** stars or hearts **(c)** not hearts?

11 **(a)** Draw a diagram to show that if 5 pizzas are shared equally between 6 students, then each student receives $\frac{5}{6}$ of a pizza.

(b) Draw a diagram to show that if 5 blocks of chocolate are shared equally between 4 students, then each student receives 1 full block and $\frac{1}{4}$ of a second block.

12 Tim is 149 cm tall. His dad is 185 cm tall.

(a) Write Tim's height as a fraction of his dad's height.

(b) Write his dad's height as a fraction of Tim's height.

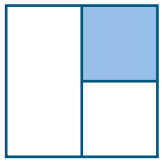
13 **(a)** After driving 8 km, the Johnson family are $\frac{1}{4}$ of the way to the zoo. How far away from the zoo do the Johnsons live?

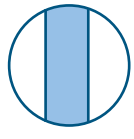
(b) Toby and his mum shared a bag of lollies: $\frac{2}{3}$ for Toby and $\frac{1}{3}$ for his mum. His mum had 6 lollies. How many did Toby have? How many lollies were in the bag?

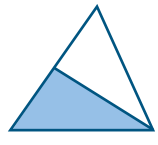
Reasoning, communicating and open-ended

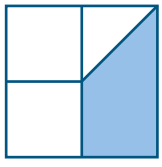
14 4 girls share 5 pizzas evenly between them. 5 boys share 4 of the same size pizzas evenly between them. Write the fraction of pizza that each boy and each girl receives. Who gets more pizza, a boy or a girl?

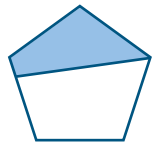
15 To show fractions, a whole must be divided into equal parts. For each of the shapes below, consider whether the shaded section represents one or more equal parts. State the fraction shown. If you don't think it is possible to state a definite fraction, explain why.

(a) 

(b) 

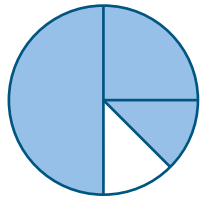
(c) 

(d) 

(e) 

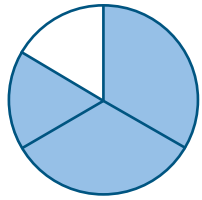
(f) 

16 The fraction of this circle that is shaded is:



A $\frac{3}{4}$ **B** $\frac{4}{5}$ **C** $\frac{5}{6}$ **D** $\frac{7}{8}$

17 The fraction of this circle that is shaded is:



A $\frac{3}{4}$ **B** $\frac{4}{5}$ **C** $\frac{5}{6}$ **D** $\frac{7}{8}$

18 How many whole numbers would appear on the section of the number line between $\frac{1}{2}$ and $\frac{11}{2}$?

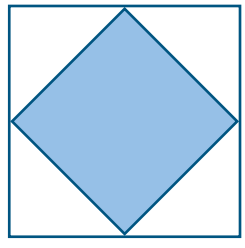
19 Draw a diagram that shows:

(a) three-eighths of an object **(b)** $\frac{5}{6}$ of a collection of objects **(c)** $\frac{9}{3}$

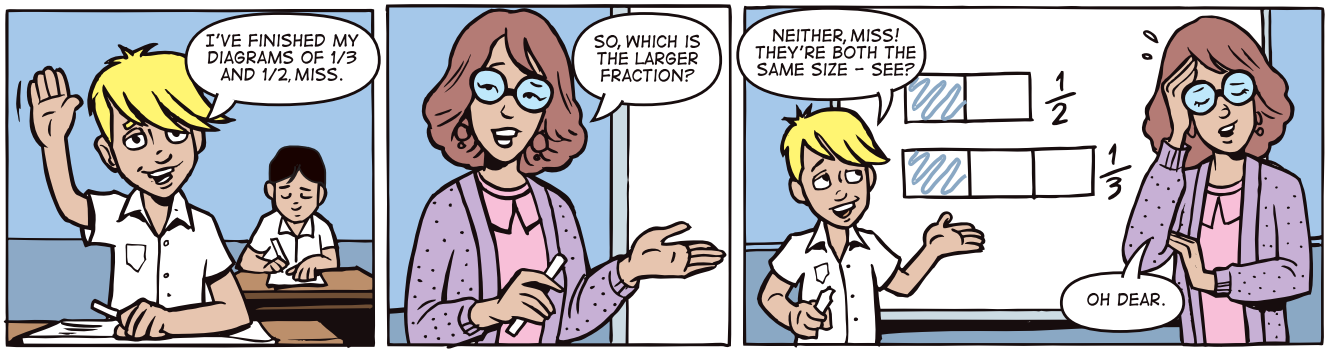
20 (a) This shape  represents $\frac{1}{5}$ of a whole. Draw two examples of what one whole could look like.

(b) This shape  represents $\frac{1}{4}$ of a whole. Draw two examples of what two wholes could look like.

21 Draw a diagram to help you explain why the fraction of this tile that is shaded is $\frac{1}{2}$.



22



Oliver's diagram is supposed to show which of the fractions $\frac{1}{2}$ or $\frac{1}{3}$ is the larger fraction.

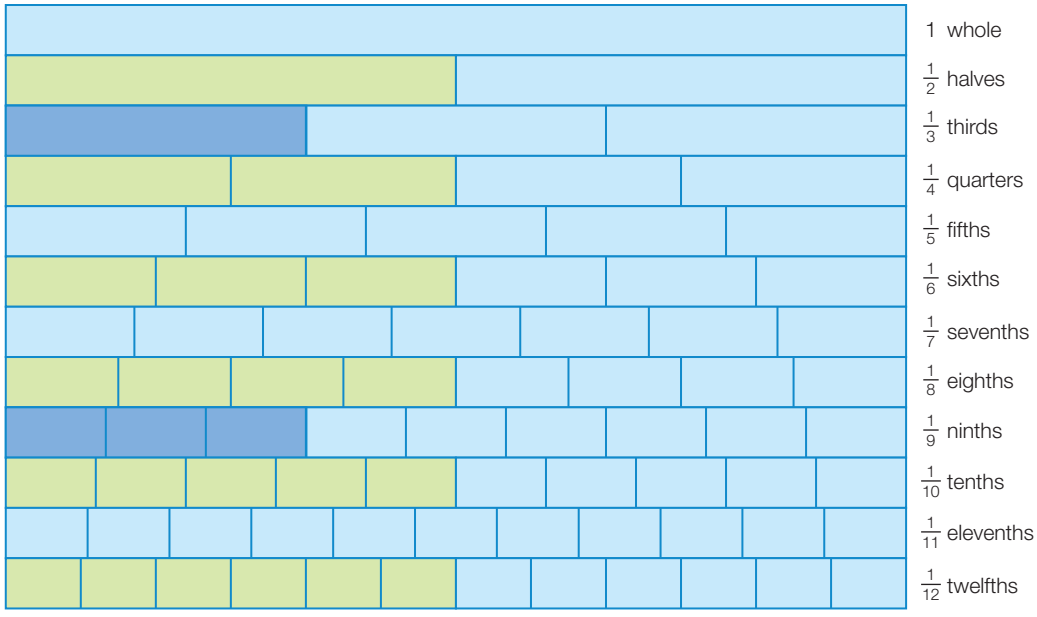
(a) Explain to Oliver what is wrong with his diagram.

(b) Draw a more accurate diagram to help you explain where he has gone wrong.

3.2 Working with fractions

Need to Know

A fraction wall



Equivalent fractions

A fraction wall shows **equivalent fractions**; fractions that represent the same amount but have different numerators and denominators. For example: $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$.

(These fractions are all shaded in green on the fraction wall. The dark blue sections are equal to $\frac{1}{3}$.)

Equivalent fractions are found either by multiplying both the numerator and the denominator by the same number, or by dividing both the numerator and the denominator by the same number.

Simplifying fractions

To write a fraction in its **simplest form**, or to **simplify** it, we divide the numerator and the denominator by their highest common factor (HCF). The process of simplifying fractions is often called '**cancelling**'.

Answers to fraction questions should always be written in simplest form.

Mixed numbers and improper fractions

To write an improper fraction as a mixed number, divide the numerator by the denominator to get the whole number part. We then write the remainder as the numerator of the fraction part.

To write a mixed number as an improper fraction, write the whole number part as an improper fraction with the same denominator. Then, add the proper fraction part.

Worked Example 5

WE 5

Write pairs of equivalent fractions by copying and completing the following.

(a) $\frac{7}{10} = \frac{\square}{40}$

(b) $\frac{27}{36} = \frac{3}{\square}$

Working

(a) $\frac{7}{10} = \frac{7}{10} \times \frac{4}{4}$

$= \frac{28}{40}$

(b) $\frac{27}{36} = \frac{3 \times 9^{1}}{4 \times 9^{1}}$

$= \frac{3}{4}$

Worked Example 6

WE 6

Write the following fraction and mixed number in their simplest form.

(a) $\frac{24}{60}$

(b) $3\frac{12}{28}$

Working

Method 1: Find the HCF

(a) HCF of 24 and 60 = 12

$\frac{{}^{2}\cancel{24}}{{}_{5}\cancel{60}}$

$= \frac{2}{5}$

(b) HCF of 12 and 28 = 4

$3\frac{{}^{3}\cancel{12}}{{}_{7}\cancel{28}}$

$= 3\frac{3}{7}$

Working

Method 2: Do several small divisions

(a) $\frac{{}^{12}\cancel{24}}{{}_{30}\cancel{60}}$

$= \frac{{}^{6}\cancel{12}}{{}_{15}\cancel{30}}$

$= \frac{{}^{2}\cancel{6}}{{}_{5}\cancel{15}}$

$= \frac{2}{5}$

(b) $3\frac{{}^{6}\cancel{12}}{{}_{14}\cancel{28}}$

$= 3\frac{{}^{3}\cancel{6}}{{}_{7}\cancel{14}}$

$= 3\frac{3}{7}$

Worked Example 7

WE 7

Write the improper fraction $\frac{34}{7}$ as a mixed number.

Working

$34 \div 7 = 4$ rem 6

$\frac{34}{7} = 4\frac{6}{7}$

Worked Example 8

WE8

Write the mixed number $7\frac{3}{5}$ as an improper fraction.

Working

$$7\frac{3}{5} = 7 + \frac{3}{5}$$
$$= \frac{35}{5} + \frac{3}{5}$$
$$= \frac{38}{5}$$

Answers page 456

3.2 Working with fractions

Navigator

Q1 Columns 1–3, Q2 Columns 1–3, Q3 Columns 1-3, Q4 Columns 1–3, Q5 (a) & (b), Q6, Q7, Q8, Q9, Q10, Q11, Q15, Q16, Q17, Q18 (a)–(c), Q19, Q22	Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q21, Q22	Q1 Column 4, Q2 Column 4, Q3 Column 4, Q4 Column 4, Q5, Q6, Q7, Q8, Q9, Q12, Q13, Q14, Q15, Q16, Q17, Q20, Q21, Q22

Fluency and understanding

WE5

1 Write pairs of equivalent fractions by copying and completing the following.

(a) $\frac{3}{4} = \frac{\square}{20}$ (b) $\frac{1}{3} = \frac{7}{\square}$ (c) $\frac{1}{2} = \frac{\square}{52}$ (d) $\frac{2}{5} = \frac{16}{\square}$

(e) $\frac{8}{100} = \frac{\square}{25}$ (f) $\frac{15}{100} = \frac{\square}{20}$ (g) $\frac{42}{70} = \frac{6}{\square}$ (h) $\frac{7}{11} = \frac{\square}{99}$

(i) $\frac{4}{3} = \frac{\square}{27}$ (j) $\frac{28}{21} = \frac{4}{\square}$ (k) $\frac{24}{16} = \frac{\square}{4}$ (l) $\frac{72}{36} = \frac{\square}{9}$

(m) $\frac{8}{\square} = \frac{88}{55}$ (n) $\frac{\square}{5} = \frac{42}{35}$ (o) $\frac{54}{\square} = \frac{9}{6}$ (p) $\frac{\square}{36} = \frac{4}{9}$

WE6

2 Write the following fractions and mixed numbers in their simplest form.

(a) $\frac{5}{10}$ (b) $\frac{3}{9}$ (c) $\frac{3}{12}$ (d) $\frac{7}{21}$

(e) $\frac{4}{10}$ (f) $\frac{8}{22}$ (g) $\frac{10}{16}$ (h) $\frac{36}{24}$

(i) $2\frac{14}{21}$ (j) $3\frac{5}{30}$ (k) $1\frac{6}{20}$ (l) $12\frac{24}{30}$

(m) $6\frac{20}{45}$ (n) $3\frac{25}{100}$ (o) $2\frac{35}{42}$ (p) $100\frac{12}{84}$

Hint

If you need to revise how to find the HCF, go to pages 34 and 36.

3 Write these improper fractions as mixed numbers. WE7

(a) $\frac{7}{5}$ **(b)** $\frac{13}{6}$ **(c)** $\frac{23}{4}$ **(d)** $\frac{15}{7}$

(e) $\frac{37}{10}$ **(f)** $\frac{48}{5}$ **(g)** $\frac{44}{7}$ **(h)** $\frac{59}{10}$

(i) $\frac{77}{9}$ **(j)** $\frac{91}{12}$ **(k)** $\frac{107}{100}$ **(l)** $\frac{97}{40}$

4 Write these mixed numbers as improper fractions. WE8

(a) $1\frac{3}{4}$ **(b)** $3\frac{1}{5}$ **(c)** $6\frac{2}{3}$ **(d)** $5\frac{9}{10}$

(e) $4\frac{3}{8}$ **(f)** $4\frac{7}{11}$ **(g)** $10\frac{2}{7}$ **(h)** $6\frac{8}{11}$

(i) $6\frac{5}{12}$ **(j)** $3\frac{9}{100}$ **(k)** $6\frac{8}{9}$ **(l)** $8\frac{7}{9}$

5 (a) Which one of the following is the simplest form of $3\frac{35}{45}$?

A $3\frac{5}{9}$ **B** $3\frac{3}{4}$ **C** $3\frac{7}{9}$ **D** $3\frac{70}{90}$

(b) Which of the following fractions is not equivalent to $\frac{10}{15}$?

A $\frac{2}{3}$ **B** $\frac{30}{45}$ **C** $\frac{40}{60}$ **D** $\frac{20}{25}$

(c) As a mixed number, $\frac{33}{4}$ equals:

A $3\frac{3}{4}$ **B** $7\frac{4}{5}$ **C** $8\frac{1}{4}$ **D** $8\frac{1}{3}$

(d) As an improper fraction, $2\frac{5}{8}$ equals:

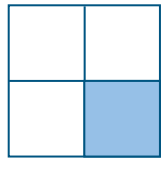
A $\frac{7}{8}$ **B** $\frac{21}{8}$ **C** $\frac{25}{8}$ **D** 21

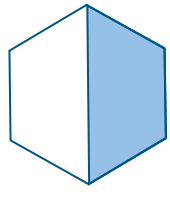
6 Use the fraction wall on page 78 to identify the fraction or fractions that are equivalent to:

(a) $\frac{3}{5}$ **(b)** $\frac{2}{8}$ **(c)** $\frac{5}{6}$ **(d)** $\frac{9}{12}$

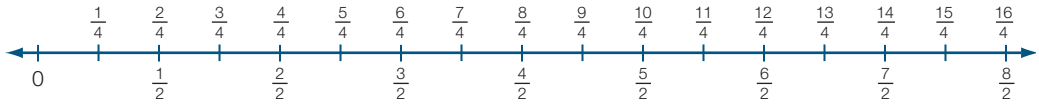
7 (a) Copy each of these figures. Write down the fraction that is shaded.

(b) In each figure, rule two diagonal lines joining the opposite vertices (corners). Name the equivalent fraction of the shaded part that you have created.





8 **(a)** The top side of this number line is divided into quarters, and the bottom side into halves.



What do you observe about the positions of the equivalent pairs of fractions $\frac{1}{2}$ and $\frac{2}{4}$, $\frac{2}{2}$ and $\frac{4}{4}$, $\frac{3}{2}$ and $\frac{6}{4}$, and $\frac{4}{2}$ and $\frac{8}{4}$?

(b) Copy the number line below. Divide the top side into thirds and the bottom side into ninths. Use your observation from **(a)** to identify pairs of equivalent fractions.



9 Monica, Michelle and Travis are sharing an apple pie that has been cut into 12 equal slices. Monica and Michelle decide to have $\frac{1}{4}$ of the apple pie each and give Travis the remainder. How many slices of pie did each person receive? Draw a diagram that shows this information.

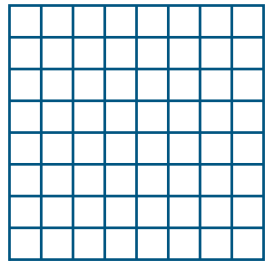
10 Six pizzas were each sliced into 8 equal pieces. If $4\frac{3}{8}$ of the pizzas were eaten, how many slices is that?

11 A block of chocolate has 24 individual squares. How many squares of the chocolate will be gone if $\frac{7}{12}$ of the chocolate is eaten?

12 Talia shaded a fraction of this long rectangle.



How many squares on this grid should Talia shade to show a fraction equivalent to the fraction she shaded in the rectangle above?



13 Write your answers in simplest form.

(a) What fraction of an hour has passed from 3:49 p.m. to 3:51 p.m.?

(b) What fraction of an hour has passed from 11:03 a.m. to 11:23 a.m.?

(c) What fraction of an hour has passed from 8:13 p.m. to 8:25 p.m.?

(d) What fraction of an hour has passed from 2:36 a.m. to 2:45 a.m.?

14 Imagine that the whole rectangle on the top of the fraction wall on page 78 represents 1 hour. How many minutes would each of the individual fractions on the wall represent? Write any answers that are not whole numbers as mixed numbers.

Reasoning, communicating and open-ended

15 How many whole numbers would appear on the section of the number line between $\frac{4}{3}$ and $\frac{51}{9}$? (Hint: Convert these improper fractions to mixed numbers.)

16 Which of the following fractions could not be the simplified form of an original fraction that had a denominator of 40?

A $\frac{3}{8}$ **B** $\frac{2}{5}$ **C** $\frac{3}{7}$ **D** $\frac{9}{10}$

17 Which rows of fractions in the fraction wall on page 78 have no equivalent fractions on the wall? Explain why.

18 For each of the following problems, it may be useful to draw and label a number line with the known fractions. Then, locate the position of the unknown fraction and identify it. Equivalent fractions can help you do this.

Which fraction lies exactly in-between the following fractions on the number line?

(a) $\frac{2}{5}$ and $\frac{3}{5}$ **(b)** $\frac{3}{7}$ and $\frac{4}{7}$ **(c)** $\frac{1}{4}$ and $\frac{2}{4}$

(d) $\frac{6}{10}$ and $\frac{7}{10}$ **(e)** $\frac{1}{9}$ and $\frac{6}{9}$ **(f)** $\frac{2}{11}$ and $\frac{7}{11}$

19 Write three fractions that are equivalent to $\frac{4}{5}$.

20 Write three improper fractions that simplify to $1\frac{2}{3}$.

21 For each of these pairs of fractions, draw a diagram that shows they are equivalent.

(a) $\frac{1}{3}, \frac{5}{15}$ **(b)** $\frac{3}{4}, \frac{12}{16}$

22 From the fraction wall on page 78 choose three fractions with three different denominators that have no equivalent fractions in the wall.

3.3 Estimating and comparing fractions

Need to Know

The **Lowest Common Denominator (LCD)** is the lowest common multiple of the denominators of fractions being compared.

For example, the LCD of $\frac{1}{3}$ and $\frac{2}{5}$ is 15.

Comparing fractions using the LCD

- Find the lowest common multiple of the denominators.
- Rewrite each fraction as an equivalent fraction with this multiple as the new denominator.
- Write '>' (greater than), '<' (less than) or '=' (equal to) between the fractions.

For example, to compare $\frac{1}{3}$ and $\frac{2}{5}$, write each with the same LCD. $\frac{1}{3} = \frac{5}{15}$ and $\frac{2}{5} = \frac{6}{15}$, so $\frac{2}{5} > \frac{1}{3}$.

If working with mixed numbers, write them as improper fractions first.

Comparing the denominators

If two fractions have the same numerator but different denominators, the *larger fraction* is the one with the *smaller number as the denominator.*

For example, $\frac{5}{9} > \frac{5}{12}$, $\frac{3}{7} > \frac{3}{8}$.

Comparing to a common fraction or integer

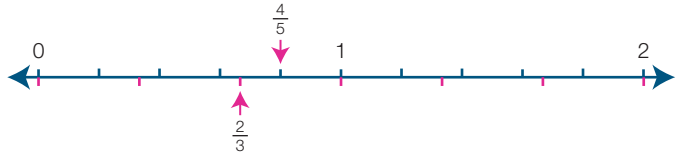
Visualise what a common number such as $\frac{1}{2}$, $\frac{1}{4}$, 1 or 2 'looks like' by imagining a shape, such as a circle or a rectangle, with that fraction shaded in. How would your fraction look in comparison? Would it be bigger or smaller?

For example, $\frac{2}{5} < \frac{1}{2}$ and $\frac{4}{7} > \frac{1}{2}$, so $\frac{1}{3} < \frac{4}{7}$.

Comparing using a number line

A number line can be used by dividing each side into different parts equal to the denominators of each fraction.

For example, to compare $\frac{4}{5}$ and $\frac{2}{3}$, divide one side of the number line into fifths and the other side into thirds. This shows that $\frac{4}{5}$ lies to the right of $\frac{2}{3}$, so $\frac{4}{5} > \frac{2}{3}$.



One number is larger than another if it lies to the right of it on the number line.

Worked Example 9

WE9

By finding the LCD, determine which fraction in each of the following pairs is larger.

(a) $\frac{3}{5}$ or $\frac{4}{7}$

(b) $\frac{15}{6}$ or $2\frac{7}{12}$

Working

(a) $\frac{3}{5}$ or $\frac{4}{7}$

Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, ...

Multiples of 7: 7, 14, 21, 28, 35, 42, 49, ...

LCD = 35

$\frac{3}{5} \times \frac{7}{7} = \frac{21}{35}$ $\frac{4}{7} \times \frac{5}{5} = \frac{20}{35}$

$\frac{21}{35} > \frac{20}{35}$

So, $\frac{3}{5} > \frac{4}{7}$

(b) $2\frac{7}{12} = \frac{31}{12}$

$\frac{15}{6}$ or $\frac{31}{12}$

LCD = 12

$\frac{15}{6} \times \frac{2}{2} = \frac{30}{12}$

$\frac{30}{12} < \frac{31}{12}$

So, $\frac{15}{6} < 2\frac{7}{12}$

3.3 Estimating and comparing fractions

Answers page 457

Navigator

Q1 Columns 1–3, Q2, Q3 Columns 1 & 2, Q5, Q7, Q8, Q9, Q10, Q12, Q13, Q15, Q17	Q1 Columns 2 & 3, Q2, Q3 Columns 1–3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q15, Q17	Q1 Column 4, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q17

Fluency and understanding

1 By finding the LCD, determine which fraction in each of the following pairs is larger.

WE9

(a) $\frac{3}{5}, \frac{7}{15}$ (b) $\frac{13}{12}, \frac{5}{4}$ (c) $\frac{7}{8}, \frac{5}{6}$ (d) $\frac{5}{9}, \frac{7}{12}$

(e) $\frac{2}{3}, \frac{5}{7}$ (f) $\frac{1}{6}, \frac{2}{9}$ (g) $\frac{3}{8}, \frac{5}{12}$ (h) $\frac{7}{4}, \frac{11}{6}$

(i) $1\frac{3}{4}, \frac{11}{8}$ (j) $\frac{5}{2}, 2\frac{1}{4}$ (k) $\frac{16}{7}, 2\frac{1}{5}$ (l) $2\frac{2}{7}, 2\frac{4}{9}$

Hint

Can't remember how to find the LCD? Turn to pages 34 and 35 to revise LCM first.

2 Use the fraction wall on page 78 to determine which fraction in each of the following pairs is larger.

(a) $\frac{3}{7}$ or $\frac{3}{10}$ **(b)** $\frac{7}{9}$ or $\frac{7}{8}$ **(c)** $\frac{4}{5}$ or $\frac{5}{6}$ **(d)** $\frac{1}{3}$ or $\frac{2}{5}$

(e) $\frac{9}{12}$ or $\frac{3}{4}$ **(f)** $\frac{5}{11}$ or $\frac{4}{9}$ **(g)** $\frac{6}{7}$ or $\frac{7}{8}$ **(h)** $\frac{3}{5}$ or $\frac{7}{11}$

3 Copy the following fraction pairs. Use any appropriate strategy to decide which fraction is larger, and place < or > between them. If the fractions are equivalent, write = between them.

(a) $\frac{2}{7}$ _____ $\frac{5}{7}$ **(b)** $\frac{4}{5}$ _____ $\frac{4}{7}$ **(c)** $\frac{1}{2}$ _____ $\frac{5}{8}$ **(d)** $\frac{8}{11}$ _____ $\frac{9}{10}$

(e) $\frac{2}{4}$ _____ $\frac{3}{6}$ **(f)** $\frac{6}{12}$ _____ $\frac{7}{10}$ **(g)** $\frac{3}{7}$ _____ $\frac{5}{8}$ **(h)** $\frac{5}{9}$ _____ $\frac{2}{5}$

(i) $\frac{5}{6}$ _____ $\frac{7}{8}$ **(j)** $\frac{7}{11}$ _____ $\frac{8}{10}$ **(k)** $\frac{6}{10}$ _____ $\frac{3}{5}$ **(l)** $\frac{4}{5}$ _____ $\frac{6}{9}$

(m) $\frac{2}{4}$ _____ $\frac{4}{2}$ **(n)** $\frac{24}{6}$ _____ $\frac{8}{2}$ **(o)** $\frac{18}{11}$ _____ $\frac{9}{5}$ **(p)** $\frac{9}{7}$ _____ $\frac{11}{9}$

4 (a) Draw a number line from 0 to 2. Indicate the approximate positions of the following fractions on the number line, then use the number line to write the fractions in ascending order (smallest to largest).

$\frac{3}{4}, \frac{2}{3}, \frac{6}{4}, 1\frac{1}{3}$

(b) Draw a number line from 0 to 3. Indicate the approximate positions of the following fractions on the number line, then use the number line to write the fractions in descending order (largest to smallest).

$\frac{5}{2}, \frac{1}{2}, \frac{4}{10}, 1\frac{7}{10}$

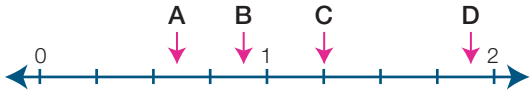
5 Ali gets 7 out of 8 shots at goal in the basket. Joseph gets 10 out of 12 shots in. By comparing fractions, determine who is the more accurate shooter.

6 In a box containing 16 chocolates, 5 of them contain nuts. A larger box with 64 of the same brand of chocolates contains 23 nut chocolates. Which box contains the greater fraction of chocolates with nuts?

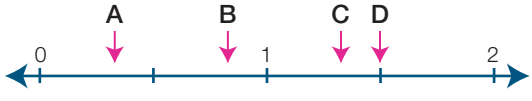
7 Use the fraction wall on page 78 to determine which fraction or fractions on the wall lie in-between the following (include all equivalent fractions).

(a) $\frac{9}{10}$ and $\frac{11}{12}$ **(b)** $\frac{5}{8}$ and $\frac{7}{10}$ **(c)** $\frac{3}{8}$ and $\frac{4}{9}$ **(d)** $\frac{2}{11}$ and $\frac{3}{11}$

8 (a) Which arrow is pointing closest to the location of $\frac{9}{10}$ on this number line?



(b) Which arrow is pointing closest to the location of $1\frac{1}{3}$?



9 **(a)** Which one of the following is closest to $\frac{5}{8}$?

A $\frac{2}{9}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{7}{8}$

(b) Which one of the following is closest to 2?

A $\frac{11}{12}$ **B** $\frac{3}{2}$ **C** $\frac{7}{4}$ **D** $\frac{7}{3}$

Reasoning, communicating and open-ended

10 Write each of these lists in ascending order (from smallest to largest). Explain using a series of brief points, or steps, how you decided the order.

(a) $\frac{2}{3}, \frac{5}{6}, \frac{7}{8}, \frac{1}{2}, \frac{3}{4}, \frac{10}{11}$ **(b)** $\frac{4}{7}, 1, \frac{7}{4}, \frac{6}{7}, \frac{7}{8}, 0$

11 Write each of these lists in descending order (from largest to smallest). Explain briefly how you decided the order.

(a) $\frac{4}{7}, \frac{2}{9}, \frac{2}{3}, 1, \frac{11}{21}, \frac{45}{63}$ **(b)** $\frac{21}{16}, 1\frac{3}{80}, \frac{5}{4}, 1\frac{7}{20}, \frac{39}{40}, 1\frac{3}{10}$

12 **(a)** Estimate the fraction of the Australian flag that is blue.



(b) Estimate the fraction of Australia that is taken up by the state of Queensland.



(c) Estimate the fraction of this jug that has cordial in it.



13 For each of the following pairs of identical shapes, state whether the fraction of the shape that is shaded is larger in **A** or in **B**. If the fraction shaded is the same in **A** and in **B**, write 'same'.

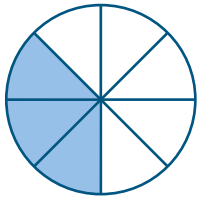
(a) **A**



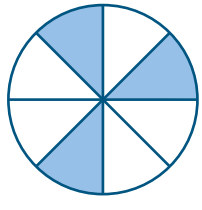
B



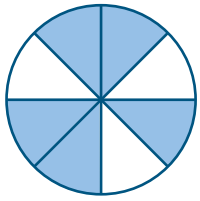
(b) **A**



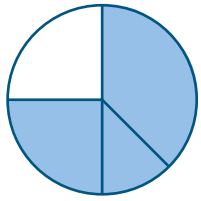
B



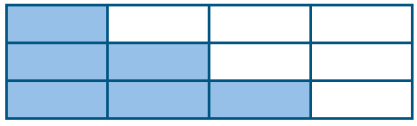
(c) A



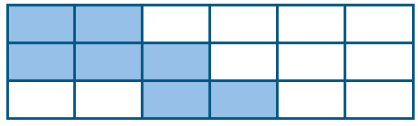
B



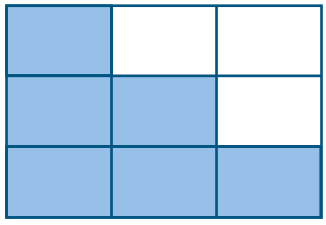
(d) A



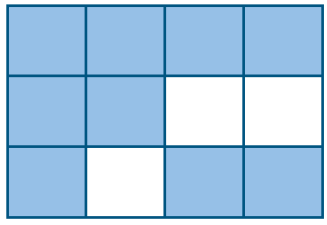
B



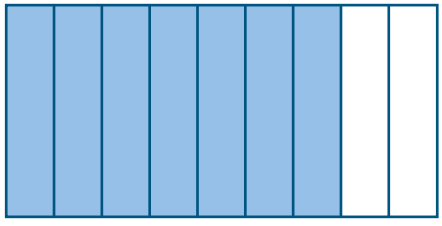
(e) A



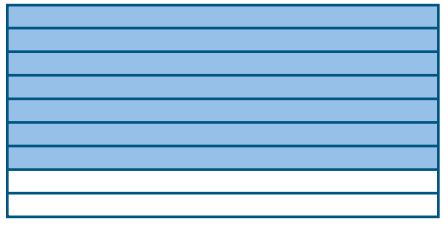
B



(f) A



B



14 Angus has 100 mL left of his 500 mL bottle of cola. Gianni has 50 mL left in his 300 mL bottle.

(a) Who has drunk more cola?

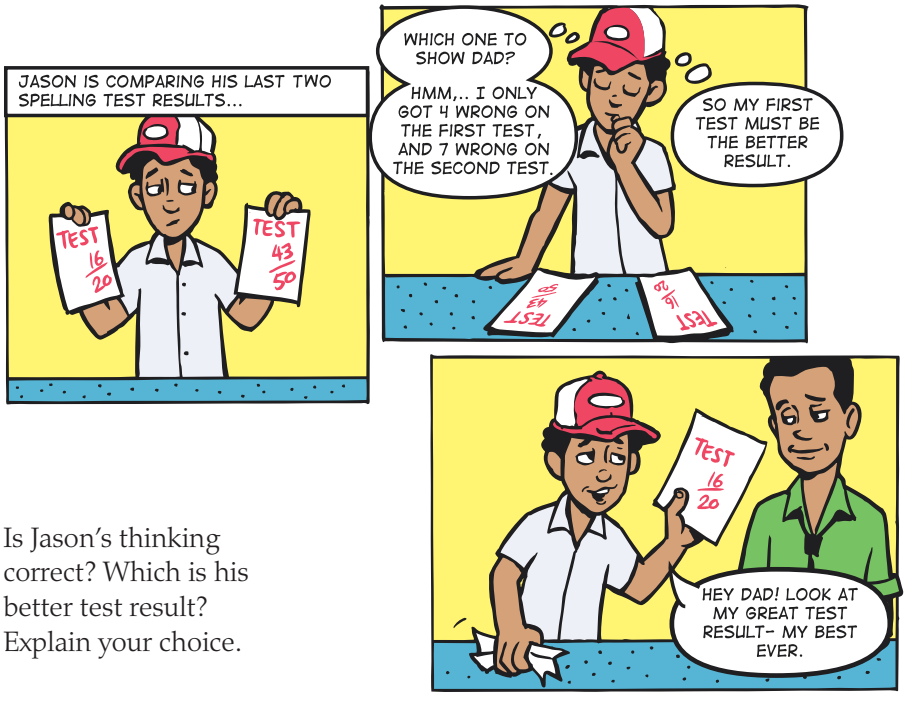
(b) Who has drunk a greater fraction of the cola in their bottle?

15 **(a)** A proper fraction is greater than $\frac{3}{4}$ but less than 1. What might the fraction be? Give two possibilities.

(b) An improper fraction is greater than 2 but less than 3. What might the fraction be? Give two possibilities.

16 Write three fractions that are greater than $\frac{3}{5}$ and less than $\frac{4}{5}$.

17



Is Jason's thinking correct? Which is his better test result? Explain your choice.

Adding and subtracting fractions

3.4

Need to Know

Adding fractions using the LCD

To add or subtract fractions, we must have fractions with the same denominator.
If the denominators are the same, the size of the fraction parts is the same, and we can simply count how many of each part we have.

For example, $\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$ $\quad\quad$ $\frac{3}{7} - \frac{2}{7} = \frac{1}{7}$.

If the denominators are different, each fraction must be written as an equivalent fraction with the same denominator, the LCD (Lowest Common Denominator).

For example, $\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15}$ and $\frac{2}{5} - \frac{1}{3} = \frac{6}{15} - \frac{5}{15}$.

$= \frac{11}{15}$ $\quad\quad$ $= \frac{1}{15}$

Estimating answers to fraction problems

To estimate an answer to adding or subtracting fractions, compare each fraction to another fraction. For example, $\frac{3}{5} + \frac{3}{7} \approx 1$ as $\frac{3}{5}$ is slightly $> \frac{1}{2}$ and $\frac{3}{7}$ is slightly $< \frac{1}{2}$. The actual answer is $1\frac{1}{35}$.

Adding fractions using the grid method

1. Draw two identical squares or rectangles.
2. Divide one horizontally using the denominator of the first fraction.
3. Divide the other vertically using the denominator of the second fraction.
4. Lay one square or rectangle on top of the other to form a grid.
5. Where the grid squares are shaded twice due to the overlap, shade an equal number of blank grid squares to remove the overlap.
6. The resultant grid represents the fraction sum.

Adding and subtracting mixed numbers

To add or subtract mixed numbers, we can either:

- add or subtract the whole number parts and fraction parts separately

or

- convert the mixed numbers to improper fractions, then add or subtract them in the usual way.

Adding and subtracting negative fractions

Negative fractions can be added and subtracted using the same processes as used for integers and positive fractions.

For example, $\frac{1}{3} + -\frac{2}{5} = \frac{1}{3} - \frac{2}{5}$

$= \frac{5}{15} - \frac{6}{15}$

$= -\frac{1}{15}$

Technology Exploration: Calculator

Working with fractions on a calculator

The key that can be used for entering fractions into your calculator will often look something like this $\square/\square$. To enter a fraction, press $\square/\square$, then key in the numerator and the denominator, using the cursor keys to move between them. To enter a mixed number, press SHIFT $\square/\square$.

On some calculators, the key looks like this: $a\frac{b}{c}$, or this $\frac{a}{b}$. To enter a fraction, such as $\frac{5}{6}$, type 5 $a\frac{b}{c}$ 6. You may need to press = for the fraction to be displayed on the screen. To enter a mixed number, such as $4\frac{3}{8}$, press 4 $a\frac{b}{c}$ 3 $a\frac{b}{c}$ 8.

The screen will use symbols such as ⌐, ⌙ or _ to separate the whole numbers, numerator and denominator.

Check that you can work with fractions correctly on your calculator by entering some additions and subtractions for which you already know the answers. For example, if you enter $\frac{3}{8} + 1\frac{2}{3}$ correctly, the answer of $2\frac{1}{24}$ will appear on the screen.

If you want to convert a mixed number that you have already entered to an improper fraction, press SHIFT and the $a\frac{b}{c}$ key, or SHIFT and the S⇔D key. (You might need to press = to see the result.)

You will also notice that pressing the S⇔D key, or pressing the $a\frac{b}{c}$ key twice, converts a fraction to its decimal form.

Worked Example 10

WE10

Calculate the following, giving your answers in simplest form. Use estimation to judge whether your answers are reasonable.

(a) $\frac{2}{3} + \frac{3}{4}$

(b) $\frac{7}{10} - \frac{1}{5}$

Working

(a) Estimate: *between 1 and 2*

$\frac{2}{3} + \frac{3}{4}$ LCD = 12

$= \frac{8}{12} + \frac{9}{12}$

$= \frac{17}{12}$

$= 1\frac{5}{12}$

Reasonable

(b) Estimate: *about* $\frac{1}{2}$

$\frac{7}{10} - \frac{1}{5}$ LCD = 10

$= \frac{7}{10} - \frac{2}{10}$

$= \frac{5}{10}$

$= \frac{1}{2}$

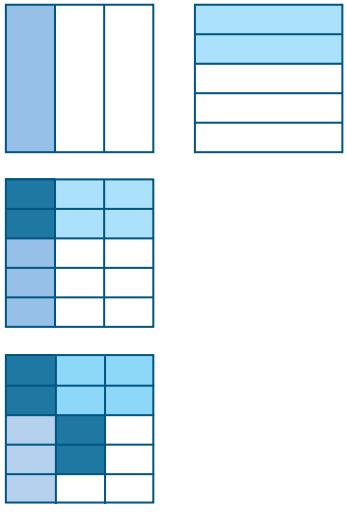
Exact

Worked Example 11

WE11

Add $\frac{1}{3}$ and $\frac{2}{5}$ by using the grid method.

Working



$\frac{1}{3} + \frac{2}{5}$

(Draw two identical squares or rectangles. Divide them up and shade in each fraction.)

(Lay one square or rectangle on top of the other to form a grid.)

(Where the grid squares are shaded twice due to overlap, shade an equal number of blank grid squares to 'remove' the overlap. The resultant shaded grid represents the fraction sum.)

$\frac{11}{15}$

Worked Example 12

WE12

Calculate $4\frac{5}{6} + 2\frac{3}{4}$. Write your answer as a mixed number in simplest form.

Working

Method 1: Add wholes and fractions separately

Estimate: between 7 and 8

$$4\frac{5}{6} + 2\frac{3}{4}$$
$$= 4 + 2 + \frac{5}{6} + \frac{3}{4}$$
$$= 6 + \frac{5}{6} + \frac{3}{4}$$
$$= 6 + \frac{10}{12} + \frac{9}{12}$$
$$= 6 + \frac{19}{12}$$
$$= 6 + 1\frac{7}{12}$$
$$= 7\frac{7}{12}$$

Reasonable

Working

Method 2: Use improper fractions

Estimate: between 7 and 8

$$4\frac{5}{6} + 2\frac{3}{4}$$
$$= \frac{29}{6} + \frac{11}{4}$$
$$= \frac{58}{12} + \frac{33}{12}$$
$$= \frac{91}{12}$$
$$= 7\frac{7}{12}$$

Reasonable

Worked Example 13 WE13

Calculate $6\frac{5}{8} - 2\frac{3}{4}$. Write your answer as a mixed number in simplest form.

Working

Method 1: Subtract wholes and fractions separately. Estimate: less than 4

$$6\frac{5}{8} - 2\frac{3}{4}$$
$$= 6 - 2 + \frac{5}{8} - \frac{3}{4}$$
$$= 4 + \frac{5}{8} - \frac{3}{4}$$
$$= 4 + \frac{5}{8} - \frac{6}{8}$$
$$= 3 + \frac{8}{8} + \frac{5}{8} - \frac{6}{8}$$
$$= 3 + \frac{13}{8} - \frac{6}{8}$$
$$= 3\frac{7}{8}$$

Working

Method 2: Use improper fractions
Estimate: less than 4

$$6\frac{5}{8} - 2\frac{3}{4}$$
$$= \frac{53}{8} - \frac{11}{4}$$
$$= \frac{53}{8} - \frac{22}{8} \qquad \text{LCD} = 8$$
$$= \frac{31}{8}$$
$$= 3\frac{7}{8}$$

Answers page 458

3.4 Adding and subtracting fractions

Navigator

Q1 Columns 1 & 2, Q2, Q3 Columns 1 & 2, Q4 Columns 1 & 2, Q5, Q6 Column 1, Q7, Q8 Column 1, Q9, Q10, Q11, Q13, Q16 Column 1, Q17, Q19, Q20	Q1 Columns 2 & 3, Q2, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5, Q6 Column 2, Q7, Q8 Column 2, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q20	Q1 Column 4, Q3 Columns 3 & 4, Q4 Columns 3 & 4, Q5, Q6 Column 3, Q7, Q8 Column 3, Q10, Q12, Q13, Q14, Q15, Q16, Q17, Q18, Q19, Q20

Fluency and understanding

WE10

1 Calculate the following, giving your answers in simplest form. Use estimation to judge whether your answers are reasonable.

Hint

Make sure you don't add or subtract fractions until you've rewritten them with the same denominator.

(a) $\frac{5}{14} + \frac{1}{7}$ (b) $\frac{2}{5} - \frac{7}{20}$ (c) $\frac{5}{8} - \frac{7}{40}$ (d) $\frac{4}{55} + \frac{2}{11}$

(e) $\frac{5}{6} + \frac{1}{3}$ (f) $\frac{15}{49} + \frac{5}{7}$ (g) $\frac{3}{5} + \frac{13}{20}$ (h) $\frac{3}{4} + \frac{1}{5}$

(i) $\frac{1}{4} + \left(-\frac{1}{6}\right)$ (j) $\frac{5}{9} - \left(-\frac{3}{4}\right)$ (k) $\frac{1}{6} - \left(-\frac{3}{7}\right)$ (l) $\frac{11}{15} - \left(-\frac{7}{20}\right)$

(m) $\frac{11}{12} - \left(-\frac{3}{10}\right)$ (n) $\frac{23}{30} + \left(-\frac{7}{20}\right)$ (o) $\frac{3}{5} - \left(-\frac{9}{14}\right)$ (p) $\frac{41}{45} + \left(-\frac{11}{18}\right)$

2 Add the following fractions by using the grid method. WE11

(a) $\frac{2}{3}+\frac{1}{2}$ (b) $\frac{3}{4}+\frac{1}{7}$

3 Calculate the following. Write your answers as mixed numbers in simplest form. WE12

(a) $3\frac{1}{7}+4\frac{2}{7}$ (b) $8\frac{1}{9}+5\frac{5}{9}$ (c) $2\frac{3}{8}+7\frac{1}{4}$ (d) $6\frac{7}{10}+1\frac{1}{5}$

(e) $1\frac{3}{4}+1\frac{1}{2}$ (f) $3\frac{1}{5}+2\frac{9}{10}$ (g) $4\frac{1}{2}+2\frac{3}{4}$ (h) $1\frac{7}{12}+7\frac{2}{3}$

(i) $6\frac{4}{5}+5\frac{1}{2}$ (j) $3\frac{8}{9}+7\frac{1}{6}$ (k) $1\frac{4}{11}+3\frac{2}{5}$ (l) $1\frac{4}{25}+2\frac{3}{20}$

4 Calculate the following. Write your answers as mixed numbers in simplest form. WE13

(a) $3\frac{5}{7}-2\frac{1}{7}$ (b) $3\frac{2}{9}-2\frac{5}{9}$ (c) $5\frac{7}{9}-1\frac{1}{3}$ (d) $7\frac{11}{12}-6\frac{1}{4}$

(e) $3\frac{1}{12}-1\frac{5}{6}$ (f) $4\frac{7}{10}-2\frac{1}{5}$ (g) $4\frac{1}{10}-2\frac{4}{5}$ (h) $6\frac{1}{18}-4\frac{1}{6}$

(i) $2\frac{1}{9}-1\frac{1}{6}$ (j) $2\frac{1}{4}-1\frac{2}{5}$ (k) $2\frac{1}{8}-1\frac{1}{6}$ (l) $5\frac{1}{8}-2\frac{2}{3}$

5 (a) $6-\frac{3}{8}$ (b) $4-\frac{2}{3}$ (c) $8-\frac{5}{9}$

(d) $3-\frac{4}{5}$ (e) $2-\frac{5}{6}$ (f) $5-\frac{4}{7}$

6 Find the following. You might like to use your calculator to check your answers.

(a) $\frac{3}{8}-\frac{1}{6}-\frac{1}{3}$ (b) $\frac{1}{2}-\frac{3}{4}-\frac{1}{3}$ (c) $\frac{7}{10}-\frac{4}{5}-\frac{1}{2}$

(d) $1\frac{1}{2}+\frac{2}{5}+2\frac{3}{10}$ (e) $2\frac{1}{3}+5\frac{3}{4}-1\frac{2}{5}$ (f) $3\frac{1}{4}-2\frac{1}{5}-3\frac{1}{3}$

7 (a) $\frac{5}{8}-\frac{1}{3}=$

A $\frac{4}{24}$ B $\frac{7}{24}$ C $\frac{4}{5}$ D $\frac{23}{24}$

(b) $1\frac{2}{3}+\frac{3}{20}=$

A $1\frac{1}{17}$ B $1\frac{5}{23}$ C $1\frac{5}{20}$ D $1\frac{49}{60}$

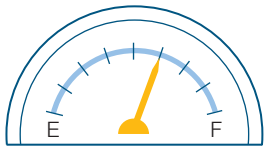
8 (a) $-\frac{3}{4}+\frac{4}{5}$ (b) $-\frac{3}{8}+\frac{2}{7}$ (c) $-\frac{4}{9}+\frac{2}{5}$

(d) $-\frac{2}{3}-\frac{1}{2}$ (e) $-\frac{6}{7}-\frac{3}{4}$ (f) $-\frac{6}{7}-\frac{2}{9}$

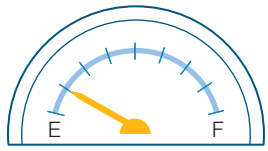
(g) $-2\frac{1}{2}+1\frac{1}{4}$ (h) $-7\frac{1}{3}+5\frac{1}{4}$ (i) $-6\frac{2}{3}+4\frac{1}{5}$

(j) $-1\frac{1}{3}-2\frac{1}{5}$ (k) $-4\frac{1}{2}-3\frac{3}{7}$ (l) $-8\frac{1}{9}-2\frac{4}{7}$

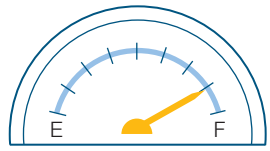
9 When Eric got in his car and started driving, his fuel gauge looked like this:



Eric stopped to get petrol when his fuel gauge looked like this:



After putting some petrol in his car, Eric's fuel gauge looked like this:



Write a fraction addition or a subtraction to answer the following.

(a) What fraction of the tank of fuel did Eric use between the first two gauge readings? Write your answer in simplest form.

(b) What fraction of the tank of petrol did Eric add when he was at the petrol station? Write your answer in simplest form.

10 What fraction of a large pizza was eaten if Steven had $\frac{5}{12}$ of the pizza and Lara had $\frac{1}{3}$ of it? Write your answer in simplest form.

11 Jaydeep, Corey and Isaac were sharing the driving on a trip along the coast. If Jaydeep drove $\frac{3}{5}$ of the distance and Isaac drove $\frac{3}{20}$, what fraction of the total distance did Corey drive?

12 Belinda worked part-time at a fast food restaurant. Her hours for the three days she worked one week were $2\frac{1}{4}$, $3\frac{1}{2}$ and $4\frac{1}{3}$. Calculate the total number of hours she worked for the week.

13 Five large packets of cereal were bought for a Year 7 camp. On the first day $1\frac{1}{3}$ packets were eaten, on the second day $\frac{3}{4}$ of a packet was eaten and on the third day $2\frac{1}{6}$ packets were eaten. How much cereal was available for the fourth day?

Reasoning, communicating and open-ended

14 A fraction is added to $\frac{3}{5}$. The lowest common denominator for the two fractions is 40. Which of the following fractions was added?

A $\frac{2}{6}$ **B** $\frac{3}{8}$ **C** $\frac{13}{20}$ **D** $\frac{11}{15}$

15 A fraction is added to $\frac{3}{10}$. Which of the following would give an improper fraction as the result?

A $\frac{7}{15}$ **B** $\frac{7}{12}$ **C** $\frac{3}{5}$ **D** $\frac{18}{25}$

16 **(a)** $11 - 2\frac{1}{3}$ **(b)** $8 - 4\frac{1}{4}$ **(c)** $11 - 5\frac{1}{8}$

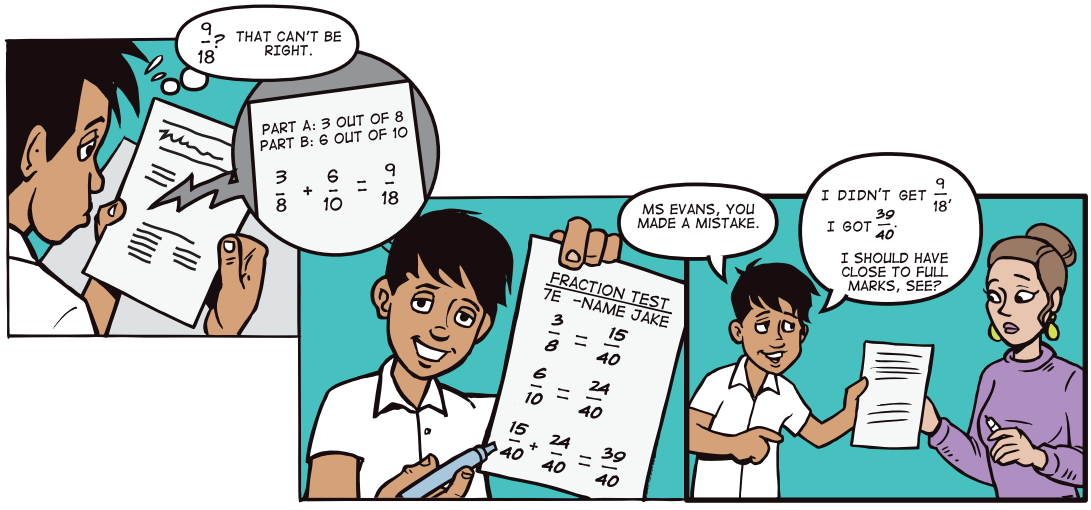
(d) $5 - 1\frac{2}{7}$ **(e)** $9 - 3\frac{2}{9}$ **(f)** $15 - 7\frac{5}{6}$

17 This question appeared on a Year 7 maths test: 'What whole number is $\frac{7}{8} + \frac{12}{13}$ approximately equal to?' Several students answered 19 or 21, both of which are incorrect. How might the students have arrived at their incorrect answers? Describe how you could use estimation to arrive at the correct answer.

18 Find two fractions that add to $\frac{3}{4}$ and do not have a denominator of 4.

19 Use a diagram to help you explain why $\frac{3}{10} + \frac{4}{10} = \frac{7}{10}$, not $\frac{7}{20}$. Why do we add the numerators, but not the denominators?

20



Who do you think is right, Jake, his teacher, or both? Explain why.

Problem solving

Fill in the blanks

1 Fill in the blanks with positive whole numbers to make mathematically true statements. Do not use the same number twice within a statement. See if you can come up with at least two different combinations.

(a) $\frac{\blacklozenge}{4} + \frac{1}{\blacklozenge} = \frac{\blacklozenge}{20}$

(b) $\frac{\bigstar}{\bigstar} - \frac{\bigstar}{6} = \frac{\bigstar}{12}$

2 Use four of the digits 1, 3, 4, 5, 6, 7 to make two fractions whose sum is close to but less than 1.

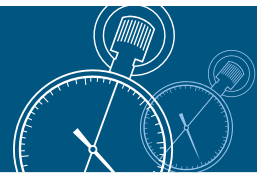
$\frac{\blacksquare}{\blacksquare} + \frac{\blacksquare}{\blacksquare} < 1$

Try to come up with at least three combinations. Which combination is the closest to 1?

Strategy options

- Guess and check.
- Test all possible combinations.

Half-time 3



Ex 3.1

1 There are 15 cookies in a bag.



(a) If I eat 7 cookies, what fraction of the total cookies is this?

(b) If I eat $\frac{2}{5}$ of the cookies in the bag, how many cookies do I eat?

Ex 3.2

2 Simplify the following.

(a) $\frac{18}{48}$ **(b)** $\frac{9}{36}$ **(c)** $3\frac{9}{12}$ **(d)** $7\frac{35}{50}$

Ex 3.2

3 Convert the following improper fractions to whole numbers or mixed numbers.

(a) $\frac{27}{4}$ **(b)** $\frac{40}{8}$ **(c)** $\frac{50}{10}$ **(d)** $\frac{37}{7}$

Ex 3.3

4 Determine which fraction in each of these pairs is the larger number, and write a < or > sign between them. If they are equivalent, write an '=' sign between them.

(a) $\frac{5}{8}$ ____ $\frac{2}{3}$ **(b)** $\frac{4}{9}$ ____ $\frac{15}{27}$ **(c)** $\frac{48}{56}$ ____ $\frac{6}{7}$ **(d)** $\frac{11}{5}$ ____ $\frac{9}{4}$

Ex 3.1, 3.2, 3.3

5 Draw a number line from 0 to 2. Divide the bottom side of the number line into quarters and the top side into eighths.

(a) Show the positions of these fractions with a labelled arrow: $\frac{3}{4}, \frac{5}{8}, \frac{5}{4}, \frac{9}{8}, \frac{7}{4}, \frac{14}{8}$.

(b) Identify five pairs of equivalent fractions that are shown on the number line.

(c) Use the number line to determine which fraction out of the following pairs is larger.

(i) $\frac{5}{4}$ or $\frac{9}{8}$ **(ii)** $\frac{11}{8}$ or $\frac{7}{4}$

Ex 3.4

6 Calculate the following, writing your answers in simplest form.

(a) $\frac{3}{8}+\frac{3}{4}$ **(b)** $\frac{7}{15}-\frac{3}{10}$ **(c)** $\frac{7}{9}+\frac{5}{6}$ **(d)** $\frac{11}{18}-\frac{5}{12}$

Ex 3.3

7 Place the following in ascending order by first locating them on a number line.

$\frac{1}{3}, \frac{6}{3}, \frac{9}{4}, \frac{3}{4}, 2\frac{2}{3}$

Ex 3.2

8 7 pizzas were each sliced into 6 equal pieces.

(a) If $4\frac{5}{6}$ of the pizzas were eaten, how many slices was that?

(b) If $3\frac{1}{2}$ of the pizzas were eaten, how many slices were left over?

Ex 3.4

9 Calculate the following.

(a) $2\frac{7}{12}+2\frac{1}{10}$ **(b)** $1\frac{16}{25}+2\frac{13}{20}$ **(c)** $2\frac{13}{20}-1\frac{3}{100}$ **(d)** $3\frac{9}{10}-2\frac{8}{25}$

Ex 3.1, 3.2

Ex 3.4

10 Dave is leading a group of 12 people on a bushwalk. He has brought 30 L of water to supply the group.

(a) How many litres does each group member receive? Write your answer as both an improper fraction and as a mixed number in simplest form.

(b) One of the group members has drunk $\frac{3}{4}$ of a litre by lunchtime. How many litres do they have left?

Multiplying fractions

3.5

Need to Know

The multiplication sign, ×, means '*lots of*', or simply '*of*'.

For example, 8 lots of \$5 = 8 × \$5 = \$40.

A **unit fraction** is a fraction with a numerator of 1.

Finding a fraction of a whole number

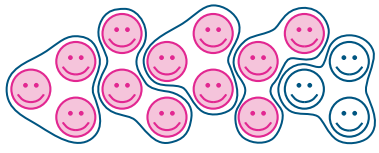
Either:

- find the unit fraction (by dividing by the denominator), then multiply by the numerator

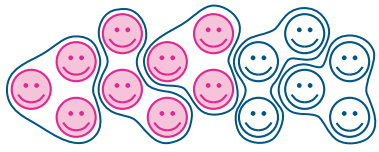
or

- write the whole number as an improper fraction and multiply the numerators together and the denominators together. Cancel any common factors between numerators and denominators before multiplying (this avoids having to work with large numbers).

Finding a fraction of a fraction

$\frac{4}{5} \times 15 = 12$  and $\frac{3}{4} \times 12 = 9$ 

so $\frac{3}{4} \times \frac{4}{5} \times 15 = 9$

However, $\frac{3}{5} \times 15 = 9$ 

so $\frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$.

Multiplying fractions

- Write mixed numbers as improper fractions before multiplying

  For example, $\frac{3}{16}$ of $5\frac{3}{5} = \frac{3}{16}$ of $\frac{28}{5}$

- Replace 'of' with a '×' symbol

  $\frac{3}{16}$ of $\frac{28}{5} = \frac{3}{16} \times \frac{28}{5}$

- Cancel any common factors

  $\frac{3}{\cancel{16}_{4}} \times \frac{\cancel{28}^{7}}{5} = \frac{3}{4} \times \frac{7}{5}$

- Multiply the numerators together and the denominators together

  $\frac{3}{4} \times \frac{7}{5} = \frac{21}{20}$

- Simplify the answer further if necessary.

  $\frac{21}{20} = 1\frac{1}{20}$

Worked Example 14

WE14

Find $\frac{2}{5}$ of 40.

Working

Method 1: Find the unit fraction, then multiply

$\frac{1}{5}$ of $40 = 8$

$\frac{2}{5}$ of $40 = 2 \times 8$

$= 16$

Working

Method 2: Multiply two fractions

$\frac{2}{5}$ of 40

$= \frac{2}{\cancel{5}_{1}} \times \frac{\cancel{40}^{8}}{1}$

$= \frac{16}{1}$

$= 16$

Worked Example 15

WE15

Find $\frac{3}{4}$ of $\frac{4}{5}$.

Working

$\frac{3}{4}$ of $\frac{4}{5}$

$= \frac{3}{\cancel{4}_{1}} \times \frac{\cancel{4}^{1}}{5}$

$= \frac{3}{5}$

Worked Example 16

WE16

Find $4 \times 2\frac{3}{20}$. Write your answer as a mixed number, if appropriate.

Working

$$4 \times 2\frac{3}{20}$$
$$= \frac{4}{1} \times \frac{43}{20}$$
$$= \frac{\cancel{4}^{1}}{1} \times \frac{43}{\cancel{20}_{5}}$$
$$= \frac{43}{5}$$
$$= 8\frac{3}{5}$$

3.5 Multiplying fractions

Answers page 459

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Fluency and understanding

1 Find the following.

WE14

(a) $\frac{1}{3}$ of 18 (b) $\frac{1}{4}$ of 12 (c) $\frac{1}{6}$ of 18 (d) $\frac{1}{9}$ of 36

(e) $\frac{3}{4}$ of 28 (f) $\frac{4}{5}$ of 30 (g) $\frac{6}{7}$ of 56 (h) $\frac{3}{8}$ of 72

(i) $\frac{3}{2}$ of 20 (j) $\frac{6}{5}$ of 100 (k) $\frac{11}{10}$ of 50 (l) $\frac{5}{4}$ of 60

2 Find the following.

WE15

(a) $\frac{1}{2}$ of $\frac{2}{13}$ (b) $\frac{2}{9} \times \frac{1}{2}$ (c) $\frac{4}{5} \times \frac{5}{9}$ (d) $\frac{3}{7} \times \frac{7}{13}$

(e) $\frac{5}{7} \times \frac{1}{10}$ (f) $\frac{3}{5}$ of $\frac{10}{11}$ (g) $\frac{5}{12}$ of $\frac{6}{7}$ (h) $\frac{5}{6}$ of $\frac{1}{11}$

(i) $\frac{6}{7} \times \frac{3}{8}$ (j) $\frac{9}{10} \times \frac{5}{6}$ (k) $\frac{4}{7}$ of $\frac{2}{3}$ (l) $\frac{3}{10} \times \frac{9}{25}$

WE16

Hint

When cancelling common factors, don't cancel out two things on the same line—cancel something on the bottom with something on the top.

3 Find the following. Write your answers as mixed numbers, if appropriate.

(a) $3 \times \frac{2}{9}$ **(b)** $\frac{3}{4} \times 8$ **(c)** $\frac{5}{8} \times 2$ **(d)** $3 \times \frac{5}{12}$

(e) $5 \times 2\frac{3}{10}$ **(f)** $8 \times 1\frac{3}{4}$ **(g)** $1\frac{4}{5} \times 15$ **(h)** $5\frac{1}{12} \times 8$

(i) $5\frac{1}{3} \times \frac{3}{8}$ **(j)** $3\frac{1}{2} \times 3\frac{1}{5}$ **(k)** $4\frac{1}{5} \times 4\frac{2}{7}$ **(l)** $1\frac{1}{8} \times 3\frac{5}{9}$

4 Calculate the following, giving your answers in simplest form.

(a) $\frac{3}{11} \times \frac{11}{5} \times \frac{1}{4}$ **(b)** $\frac{2}{7} \times \frac{7}{9} \times \frac{1}{5}$ **(c)** $\frac{6}{7} \times \frac{5}{8} \times \frac{2}{3}$

(d) $1\frac{2}{3} \times \frac{12}{13} \times \frac{1}{2}$ **(e)** $3\frac{2}{5} \times \frac{5}{6} \times \frac{7}{17}$ **(f)** $\frac{4}{9} \times 3\frac{1}{2} \times \frac{6}{7}$

5 **(a)** $\frac{2}{5}$ of $2\frac{1}{2}$ cups of sugar is:

A $\frac{1}{5}$ cup **B** $\frac{4}{10}$ cup **C** 1 cup **D** $2\frac{1}{5}$ cups

(b) $\frac{1}{5}$ of $\frac{1}{3}$ of 15 is:

A 1 **B** 2 **C** 3 **D** 5

6 A group of 7 workers shared third prize in a local raffle. The prize was worth $2170.

(a) What fraction of the prize money does each worker receive?

(b) How much money does each worker get?

7 The proceeds of a $48 000 inheritance were to be shared between four cousins. Each cousin was to receive a specific fraction of the inheritance. Michael was to receive $\frac{5}{24}$, Marcus $\frac{1}{12}$, Lily $\frac{3}{8}$ and Julie $\frac{1}{3}$. How much money did each cousin receive?

8 A chocolate bar was made up of 45 equal pieces. Ling had to share it equally with her brother and sister, Gao and Chen.

(a) How many pieces did each person receive? Copy this chocolate block into your book and divide it up to show each person's share.

(b) Ling then decided to divide her share of chocolate equally between herself and four of her friends.

(i) How many pieces did each person receive?

(ii) What fraction of the original block is this?

(iii) On your drawing of the chocolate bar, show the fraction that Ling and her friends receive.

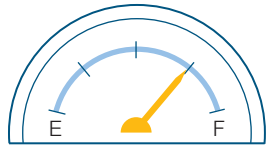
9 Annika cut one-quarter of an apple pie in halves to share with her brother Arjun. What fraction of the original whole pie do they each now have?

10 Draw a rectangle that is 5 cm long and 4 cm wide. Divide it lengthways into fifths. Shade in $\frac{1}{5}$. Now, take the shaded section and divide it into quarters. Shade in $\frac{3}{4}$ of this section.

(a) What fraction of the original rectangle have you shaded twice?

(b) Write the fraction multiplication that is shown by your diagram.

11 A petrol tank was filled to its capacity of 52 litres but now has the petrol gauge reading shown.



(a) How much petrol has been used?

(b) How much petrol is still in the tank?

12 Melissa surveyed 75 Year 7 students. $\frac{2}{3}$ said they enjoyed studying mathematics. $\frac{4}{5}$ of those who enjoyed mathematics also said they enjoyed studying science.

(a) How many students said they enjoyed mathematics?

(b) How many students said they enjoyed mathematics and science?

13 Angelina is making Custard Cream cookies. Below are the ingredients that make 24 cookies.

180 g butter $\frac{1}{2}$ cup icing sugar $1\frac{1}{4}$ cups self-raising flour $\frac{1}{3}$ cup custard powder

Rewrite the ingredients to make 60 cookies. (Hint: What do you need to multiply 24 by to get 60?)

Reasoning, communicating and open-ended

14 Without doing the calculation, which is larger: $1\frac{1}{2} \times 12$ or $\frac{1}{3} \times 9$?

15 Without doing any calculations, say which of the following would give the biggest answer.

A $\frac{1}{2} \times \frac{3}{4}$ **B** $\frac{2}{5} \times \frac{2}{3}$ **C** $\frac{3}{4} \times \frac{6}{5}$ **D** $1\frac{1}{2} \times 1\frac{2}{5}$

16 The missing number in the statement $\frac{5}{12} \times \frac{\square}{10} = \frac{5}{8}$ is:

A 4 **B** 5 **C** 10 **D** 15

17 Write two fractions that multiply to give $\frac{1}{6}$ as the simplified answer.

18 $\frac{1}{\square} \times 3\square = 1\square$

Fill each of the boxes with a single digit (0–9) to make the statement correct. There are six possible combinations. Try to find at least three of them.

19 Asher was asked to find $\frac{3}{5}$ of 60. His answer was 4. Here is his working:

$$\frac{\overset{1}{\cancel{3}}}{5} \times \frac{\overset{20}{\cancel{60}}}{1} = \frac{20}{5} = 4$$

Asher knows his answer is not correct, because he knows that $\frac{3}{5}$ is bigger than one half, and 4 is nowhere near half of 60! Explain where Asher has gone wrong. Include the correct working and answer to the problem.

3.6 Dividing fractions

Need to Know

Whole numbers divided by fractions

If 3 whole apples are cut into quarters, how many apple quarters will there be?

'How many quarters in 3' can be written as $3 \div \frac{1}{4} = ?$



There are 4 quarters in each apple, so there are 12 quarters in 3 apples: $3 \div \frac{1}{4} = 12$.

If the apples are cut into fifths, there would be fifteen fifths: $3 \div \frac{1}{5} = 15$.

If the apples are cut into sixths, there would be eighteen sixths: $3 \div \frac{1}{6} = 18$.

The answer is found by multiplying the whole number by the denominator of the fraction.

For example:

$3 \div \frac{1}{4}$	$3 \div \frac{1}{5}$	$3 \div \frac{1}{6}$
$= 3 \times \frac{4}{1}$	$= 3 \times \frac{5}{1}$	$= 3 \times \frac{6}{1}$
$= 12$	$= 15$	$= 18$

The fractions have been turned upside down or 'inverted', and the division sign changed to multiplication.

Inverting a fraction (turning it upside down) is also known as 'finding the **inverse**'.

The inverse of $\frac{1}{4}$ is $\frac{4}{1}$ or simply 4.

To divide a whole number by a fraction, multiply by the inverse of the fraction.

Fractions divided by fractions

The 'invert the fraction you are dividing by and multiply' method also applies when dividing one fraction by another.

For example, $\frac{1}{5} \div \frac{1}{3} = \frac{1}{5} \times \frac{3}{1}$

$= \frac{3}{5}$

Using equivalent fractions

Another method of dividing fractions is to convert both numbers to equivalent fractions with the same denominator (LCD). Then perform division on the numerators.

Convert mixed numbers to improper fractions before dividing.

Worked Example 17 WE17

Calculate $9 \div \frac{3}{8}$.

Working

Method 1: Multiply by the inverse

$$\frac{9}{1} \div \frac{3}{8}$$
$$= \frac{9}{1} \times \frac{8}{3}$$
$$= \frac{{}^{3}\cancel{9}}{1} \times \frac{8}{\cancel{3}_{1}}$$
$$= \frac{3 \times 8}{1 \times 1} = \frac{24}{1}$$
$$= 24$$

Working

Method 2: Use equivalent fractions with the same denominator

LCD = 8

$$9 = \frac{72}{8}$$

$$9 \div \frac{3}{8}$$
$$= \frac{72}{8} \div \frac{3}{8}$$
$$= 72 \div 3$$
$$= 24$$

Worked Example 18 WE18

Calculate $1\frac{1}{6} \div \frac{2}{3}$.

Working

Method 1: Multiply by the inverse

$$1\frac{1}{6} = \frac{7}{6}$$

$$\frac{7}{6} \div \frac{2}{3}$$
$$= \frac{7}{6} \times \frac{3}{2}$$
$$= \frac{7}{{}_{2}\cancel{6}} \times \frac{{}^{1}\cancel{3}}{2}$$
$$= \frac{7 \times 1}{2 \times 2}$$
$$= \frac{7}{4}$$
$$= 1\frac{3}{4}$$

Working

Method 2: Use equivalent fractions with the same denominator

LCD = 6

$$1\frac{1}{6} = \frac{7}{6}$$

$$\frac{2}{3} = \frac{4}{6}$$

$$1\frac{1}{6} \div \frac{2}{3}$$
$$= \frac{7}{6} \div \frac{4}{6}$$
$$= 7 \div 4$$
$$= 1\frac{3}{4}$$

Answers page 460

3.6 Dividing fractions

Navigator

Q1 Columns 1–3, Q2 Columns 1 & 2, Q4, Q5, Q6, Q8, Q9, Q12, Q15	Q1 Columns 3 & 4, Q2 Columns 3 & 4, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q15	Q1 Column 4, Q2 Column 4, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15

Fluency and understanding

WE17

1 Calculate the following.

(a) $6 \div \frac{1}{4}$ **(b)** $2 \div \frac{1}{6}$ **(c)** $3 \div \frac{1}{7}$ **(d)** $4 \div \frac{1}{9}$

(e) $4 \div \frac{2}{9}$ **(f)** $4 \div \frac{4}{5}$ **(g)** $5 \div \frac{3}{7}$ **(h)** $7 \div \frac{4}{9}$

(i) $6 \div 1\frac{1}{8}$ **(j)** $8 \div 2\frac{2}{3}$ **(k)** $6 \div 3\frac{1}{5}$ **(l)** $3 \div 2\frac{1}{4}$

WE18

2 Calculate the following.

(a) $\frac{1}{2} \div \frac{1}{4}$ **(b)** $\frac{1}{4} \div \frac{1}{6}$ **(c)** $\frac{3}{4} \div \frac{2}{3}$ **(d)** $\frac{7}{10} \div \frac{1}{2}$

(e) $\frac{1}{3} \div 2$ **(f)** $\frac{1}{5} \div 4$ **(g)** $\frac{3}{4} \div 3$ **(h)** $\frac{9}{10} \div 6$

(i) $\frac{9}{10} \div 1\frac{3}{5}$ **(j)** $\frac{7}{6} \div 2\frac{1}{12}$ **(k)** $\frac{4}{5} \div 1\frac{8}{9}$ **(l)** $\frac{9}{10} \div 2\frac{2}{3}$

(m) $2\frac{2}{3} \div 1\frac{1}{3}$ **(n)** $5\frac{1}{2} \div 5\frac{3}{8}$ **(o)** $4\frac{1}{2} \div 1\frac{1}{5}$ **(p)** $4\frac{5}{7} \div 2\frac{3}{4}$

3 **(a)** $5 \div \frac{15}{8} =$

A $\frac{8}{75}$ **B** $\frac{3}{8}$ **C** $2\frac{2}{3}$ **D** $\frac{75}{8}$

(b) $\frac{16}{7} \div \frac{20}{49} =$

A $\frac{5}{28}$ **B** $\frac{320}{343}$ **C** $\frac{343}{320}$ **D** $5\frac{3}{5}$

4 Copy and complete the following. Do all working in your head.

(a) There are _____ halves in one, so $1 \div \frac{1}{2} =$ _____.

(b) There are _____ halves in three, so $3 \div \frac{1}{2} =$ _____.

(c) There are _____ quarters in one, so $1 \div \frac{1}{4} =$ _____.

(d) There are _____ quarters in two, so $2 \div \frac{1}{4} =$ _____.

5 The inverse of $\frac{1}{3}$ is:

A $\frac{1}{6}$ **B** $\frac{2}{3}$ **C** 1 **D** 3

6 There is $\frac{5}{8}$ of a metre of ribbon in the textiles class cupboard. Students require $\frac{1}{10}$ of a metre each for their projects. How many students will be able to use the ribbon?

7 Scientists in a medical laboratory work in shifts of $3\frac{1}{2}$ hours each. How many whole shifts are to be worked during a period of 42 hours?

8 Hannah is a baker whose specialty is scones. She has a sack containing 80 cups of flour. Her scone recipe uses $2\frac{2}{3}$ of a cup of flour for each batch of scones. How many batches of scones will Hannah get from her sack of flour?

9 On a 15 km fun run course, drinks stations are placed every $2\frac{1}{2}$ km from the start. How many drinks stations will there be on the course? (There are no drinks stations at the start or finish lines.)

10 The hit movie 'Exterminator 5' runs for $1\frac{3}{4}$ hours. A local cinema complex plans to screen it back to back (alternating the screenings between 2 cinemas). The first screening will start at 1:30 p.m. The last screening must finish by 11 p.m. How many times can the movie be screened within this time period?

Reasoning, communicating and open-ended

11 The missing number from the statement $\frac{3}{4} \div \frac{\square}{8} = 2$ is:

A 1 **B** 2 **C** 3 **D** 4

12 Without doing a calculation, state which will give the larger number as a result:
$\frac{4}{5} \div \frac{1}{3}$, or $\frac{4}{5} \div \frac{1}{3}$?

13 Consider this sequence: $5 \div \frac{3}{4} = 6\frac{2}{3}$, $5 \div \frac{3}{5} = 8\frac{1}{3}$, $5 \div \frac{3}{6} = 10$, $5 \div \frac{3}{7} = 11\frac{2}{3}$.

(a) Why does the quotient (the answer to the division) increase as the denominator of the fractions in red is increased?

(b) Predict the answer to the next term in the sequence; i.e. $5 \div \frac{3}{8}$ will be ...

14 Use digits from 0 to 9 to fill in the boxes to make a correct statement. Find at least two different combinations.

$$4 \div \frac{2}{\square} = 1\square$$

15 Here is Gabi's working on a fraction division problem:

$$3 \div \frac{1}{4} = 3 \div 4 = \frac{3}{4}$$

Her friend Natalie has worked it out differently. This is what Natalie wrote:

$$3 \div \frac{1}{4} = \frac{3}{1} \times \frac{4}{1} = 12$$

'That can't be right,' said Gabi, '12 is way too big to be the answer!'

Which of the girls' working is correct? Explain what the other has done wrong.

3.7 Mixed fraction problems

Need to Know

The order of operations rules that apply to whole numbers also apply to fractions.

1. Simplify brackets first.
2. Perform any multiplication or division working from left to right.
3. Perform any addition or subtraction working from left to right.

Worked Example 19

WE19

Simplify the following using the correct order of operations.

(a) $3+\frac{2}{5}\times\frac{1}{2}$

(b) $\frac{6}{7}\times\left(1\frac{2}{3}+\frac{1}{4}\right)-\frac{3}{7}$

Working

(a) $3+\frac{2}{5}\times\frac{1}{2}$

$=3+\frac{{}^{1}\cancel{2}\times 1}{5\times{}^{1}\cancel{2}}$

$=3+\frac{1}{5}$

$=3\frac{1}{5}$

(b) $\frac{6}{7}\times\left(1\frac{2}{3}+\frac{1}{4}\right)-\frac{3}{7}$

$=\frac{6}{7}\times\left(\frac{5}{3}+\frac{1}{4}\right)-\frac{3}{7}$

$=\frac{6}{7}\times\left(\frac{20}{12}+\frac{3}{12}\right)-\frac{3}{7}$

$=\frac{6}{7}\times\frac{23}{12}-\frac{3}{7}$

$=\frac{\cancel{6}^{1}\times 23}{7\times{}^{2}\cancel{12}}-\frac{3}{7}$

$=\frac{23}{14}-\frac{3}{7}$

$=\frac{23}{14}-\frac{6}{14}$

$=\frac{17}{14}$

$=1\frac{3}{14}$

Technology Exploration: Calculator

Entering fractions on a calculator

Enter the required fractions using the fraction keys on your calculator and the operations as they occur in the question. Do not press enter until you have entered the entire question. Your calculator will evaluate using the correct order of operations.

For example, $\frac{3}{4} - \frac{3}{5} \div 1\frac{1}{2} + 1\frac{8}{9} \times 2\frac{1}{4} = 4\frac{3}{5}$

3.7 Mixed fraction problems

Answers page 460

Equipment required: a calculator for Questions **5** and **6**

Navigator

Q1 Column 1, Q2 (a), Q3 (a) & (b), Q4, Q5, Q6, Q7, Q8, Q9, Q14, Q17	Q1 Column 2, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q14, Q15, Q16, Q18	Q1 Columns 2 & 3, Q2, Q3, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q18

Fluency and understanding

1 Simplify the following using the correct order of operations.

WE19

(a) $2 + \frac{3}{4} \times \frac{1}{3}$ **(b)** $5 - \frac{7}{8} \times \frac{1}{7}$ **(c)** $\frac{5}{11} \div (5 + 4)$

(d) $\frac{3}{4} \times \left(\frac{1}{3} + \frac{1}{6}\right) \times \frac{1}{2}$ **(e)** $\frac{7}{8} \times \left(\frac{9}{14} - \frac{2}{7}\right)$ **(f)** $\frac{5}{6} \times \left(\frac{2}{5} + \frac{1}{3}\right)$

(g) $1\frac{5}{8} + \left(1\frac{3}{4} - 1\frac{1}{8}\right) \times 5$ **(h)** $2\frac{3}{4} + \left(4\frac{1}{2} - 3\frac{1}{4}\right) \times \frac{1}{2}$ **(i)** $1\frac{3}{5} + \left(3\frac{3}{10} - 1\frac{1}{5}\right) \div 2$

(j) $1\frac{1}{4} - \left(3\frac{1}{3} - 2\frac{5}{6}\right) \div 2$ **(k)** $3\frac{3}{8} + \left(8\frac{1}{4} - 6\frac{3}{8}\right) \div 3$ **(l)** $\frac{7}{9} \div \left(1\frac{5}{6} - 1\frac{3}{5}\right) \times \frac{1}{5}$

2 **(a)** $\frac{3}{7} \times \frac{1}{3} + 5 =$

A $5\frac{1}{7}$ **B** $5\frac{1}{3}$ **C** $5\frac{5}{21}$ **D** $5\frac{7}{9}$

(b) $6 - \frac{9}{13} \div \frac{3}{4} =$

A $5\frac{1}{13}$ **B** $5\frac{25}{52}$ **C** $5\frac{12}{13}$ **D** $7\frac{1}{13}$

3 Write out the following questions using fractions, the operations symbols (×, ÷, +, –) and brackets where necessary. Then, evaluate using the correct order of operations.

(a) Add $\frac{7}{8}$ and $\frac{5}{6}$, and then multiply the total by 2.

(b) Find the product of $\frac{4}{5}$ and $\frac{1}{3}$, then add $1\frac{1}{2}$.

(c) Find the difference between $2\frac{4}{5}$ and $1\frac{2}{3}$, then multiply by 3.

(d) Find out how many $\frac{2}{3}$s in 5, then multiply by 4.

4 A survey was taken of 120 people as they left Wally's Sandwich Bar. Write your answers to the following in simplest form.

(a) If 80 were male, what fraction were male?

(b) What fraction were female?

(c) If 45 were under 21 years of age, what fraction were under 21?

(d) What fraction were 21 or over?

(e) If $\frac{3}{4}$ had bought a drink, how many had bought a drink?

(f) Of those who bought a drink, $\frac{2}{5}$ had bought a salad roll. How many people had a drink and a salad roll?

5 Use a calculator to evaluate the following.

(a) $6\frac{7}{9} \times 5\frac{2}{11}$ **(b)** $14\frac{1}{7} \times 3\frac{1}{3} \times 5\frac{2}{5}$ **(c)** $11\frac{1}{4} \times 8\frac{4}{5} \div 6\frac{1}{9}$

(d) $39\frac{3}{4} \div 6\frac{2}{3}$ **(e)** $7\frac{3}{8} \times 4\frac{1}{9} \div 2\frac{1}{3}$ **(f)** $37\frac{4}{5} \div 12\frac{1}{4} \times 8\frac{3}{10}$

6 For each of the following:

(i) estimate your answer

(ii) calculate the answer

(iii) use a calculator to check your answer

(iv) compare your estimate to the exact answer.

(a) $\left(4\frac{1}{2} + 3\frac{2}{5}\right) \times 8\frac{1}{4}$

(b) $11\frac{2}{3} \div 2\frac{7}{8} \times \left(5\frac{1}{4} - 2\frac{1}{8}\right)$

(c) $6\frac{1}{4} - 3\frac{1}{3} \div 5\frac{1}{7}$

7 Evaluate:

(a) $\frac{3}{11}$ of $4\frac{5}{7}$ **(b)** $\frac{7}{8}$ of $11\frac{1}{5}$ **(c)** $\frac{5}{7}$ of $8\frac{1}{6}$

8 Grant is training in his backyard pool for the 1500 m freestyle at the next Olympics. Unfortunately, the pool is only 20 m in length.

(a) What fraction of the total 1500 m race would he have swum after completing just 1 lap?

(b) How many laps would he have to complete to swim 1500 m?

(c) What fraction of the total 1500 m race would he have swum after completing 6 laps?

(d) How far would he have swum if he had completed $\frac{1}{10}$ of the race distance?

(e) How far would he have swum if he had completed $\frac{3}{4}$ of the race distance?

9 Valerie is running laps of her school oval to train for Rigby High's Mini Marathon. She ran $4\frac{1}{2}$ laps before having to rest. After her rest, she ran another $3\frac{1}{4}$ laps before stopping again. Then, she struggled through another $\frac{1}{3}$ of a lap. How many laps did she complete altogether?

10 One weekend, Louisa and Andrew walk from Ferndale to Greenhill and back, a total of $2\frac{3}{4}$ km. The next weekend they walk from Ferndale through Greenhill to Highvale, which is $5\frac{1}{3}$ km.

How far (in km) is it from:

(a) Ferndale to Greenhill

(b) Greenhill to Highvale

(c) Ferndale to Highvale and back again?

11 Jeremy worked at an amusement park for 4 hours at the normal hourly rate, then another $2\frac{1}{2}$ hours at $1\frac{1}{2}$ times the normal rate. What was the equivalent number of normal hours he was to be paid?

Reasoning, communicating and open-ended

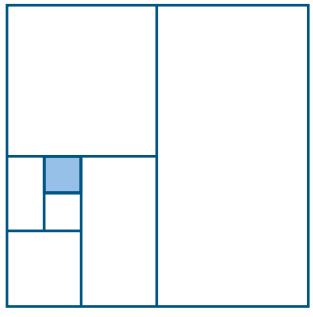
12 Theresa, Isabelle and Bryce worked on a mathematical problem and came up with the answers $8\frac{1}{2}$, $8\frac{2}{3}$ and $8\frac{3}{5}$, respectively. The correct answer was $8\frac{7}{12}$. Which of the three students was closest to the correct answer?

13 Despina's department store advertises a winter sale in which they claim everything is between $\frac{1}{3}$ and $\frac{1}{2}$ off. For which of the following items is this incorrect?

Item	Normal price	Sale price
Bath towels	\$22	\$12
Camera	\$900	\$600
Cutlery set	\$250	\$120
Microwave oven	\$540	\$350
Jeans	\$60	\$35

14 The Happy Valley Farming Cooperative has divided its square block of land in the way shown in the diagram. Each section of land is exactly half of the section next to it.

The May family has been given the shaded region to farm. If the total block of land is 1, what fraction of the total *doesn't* the May family farm?



15 Prashanth has agreed to sell badges for a charity. He receives a large box and places them, with permission, at the school reception. After several days, Prashanth took an estimate of the number of badges he had left. He emptied out the box, divided the pile of badges roughly into halves, and put half back in the box. He then halved the pile he had left, and returned half to the box. He did this one more time, after which he counted the number of badges in the pile, and found he had 13.

(a) What fraction of the total number of badges does 13 approximately represent?

(b) How many badges did Prashanth have in the box, approximately?

Hint

Draw a diagram or flowchart to show this process.

16 Write down three numbers in fraction form that are less than $\frac{7}{8}$ but greater than $\frac{1}{2}$.

17 A and B are two different numbers selected from the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, where A is always greater than B.

(a) What values can $\frac{A+B}{A-B}$ have? Give three possibilities.

(b) What is the largest value this fraction can have?

18 'Ideal fractions' are pairs of fractions whose sum and product are the same. (They give the same number when added as when multiplied.)

(a) Show that $\frac{7}{3}$ and $\frac{7}{4}$ are ideal fractions.

(b) Show that $\frac{5}{3}$ and $\frac{5}{2}$ are ideal fractions.

(c) Look carefully at the numerators and denominators of the fraction pairs in **(a)** and **(b)**. Try to find a pattern.

(d) Write down another pair of ideal fractions.

Problem solving

Mega-equivalents

1 Use four of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 to make a fraction equivalent to $\frac{1}{2}$ that has two digits in the numerator and in the denominator (e.g. $\frac{34}{68}$).

2 Now, use six digits to make another fraction equivalent to $\frac{1}{2}$ that has three digits in the numerator and denominator.

3 See if you can form another fraction equivalent to $\frac{1}{2}$ using eight digits (four in numerator and denominator).

4 Can you make a fraction equivalent to $\frac{1}{2}$ using all 10 digits?

Strategy options

- Guess and check.
- Test all possible combinations.

Challenge 3



1 If $\frac{1}{3}$ of a number is 16, then $\frac{3}{8}$ of this number is:

A 8 **B** 16 **C** 18 **D** 24

2 Kerry bought a new water tank for her garden. The tank is initially empty, but fills to $\frac{2}{3}$ full of capacity after rain. The tank now holds 480 litres of water. How many litres does the tank hold when full?

3 If $\frac{\nabla}{16}$ lies between $\frac{1}{4}$ and $\frac{3}{8}$, and ∇ is a whole number, then ∇ equals:

A 5 **B** 6 **C** 7 **D** 8

4 Two proper fractions (in simplest form) are subtracted, the first with a denominator of 10, and the second with a denominator of 6. If the answer is given in simplest form, the largest possible numerator that it could have is:

A 8 **B** 9 **C** 11 **D** 22

5 The number halfway between $\frac{1}{5}$ and $\frac{1}{20}$ is:

A $\frac{1}{10}$ **B** $\frac{1}{8}$ **C** $\frac{7}{40}$ **D** $\frac{1}{4}$

6 Two frogs live together in a backyard pond. One frog croaks every 4 minutes, the other frog croaks every $2\frac{1}{2}$ minutes. If they both croak together at 11 a.m., what is the first time after 1 p.m. that they will croak together?

7 $2 + \dfrac{6}{1 + \frac{1}{2}}$ equals:

A 2 **B** 4 **C** 6 **D** 8

8 Express this fraction in its simplest form:

$$\cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \frac{1}{3}}}}$$

9 If 60 is added to one-third of a number, the resulting value is double the number. What is the number?

10 Five numbers are put in a row from smallest to largest. The difference between each adjacent number (a number and the number next to it) is the same. If the first number is $\frac{1}{8}$ and the last is $\frac{1}{6}$, what are the three numbers in-between, in simplest form?

11 Tom was born into a strange mathematical family. His grandmother sent him a money voucher for his birthday. The card said:

Dear Tom,
I am giving you the following amount to spend as you wish:
$\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ of $1000.
Love, Grandma

How much birthday money did Tom receive?

Chapter review 3

D.I.Y. Summary

Key Words

cancelling	improper fraction	mixed number	simplest form
denominator	inverse	numerator	simplify
equivalent fractions	Lowest Common Denominator (LCD)	proper fraction	unit fraction
fraction			

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 In a fraction, the _____ tells us how many parts we have, and the _____ tells us the size of each part.

2 A _____ has a value between 0 and 1.

3 To _____ a fraction, divide the numerator and denominator by a common factor. This process is known as _____.

4 Dividing by a fraction is the same as multiplying by the _____ of the fraction.

5 An _____ has a numerator that is greater than or equal to the denominator.

6 _____ are located at exactly the same position on a number line.

7 A _____ is a fraction with a numerator of 1.

Fluency and understanding

1 Here are 3 dozen (36) eggs.

Ex 3.1, 3.2

(a) If $\frac{4}{9}$ of the eggs are slightly cracked, how many eggs are damaged?



(b) The farmer's 3 best hens laid 15 of the eggs pictured. What fraction of the total is this? Write your answer in simplest form.

2 Write the value of the fractions shown by the arrows on this number line:

Ex 3.1



3 Write **(a)** 6 and **(b)** 11 as improper fractions with denominators of:

Ex 3.1

(i) 3 **(ii)** 10 **(iii)** 1

4 **(a)** 3 pizzas are shared between 5 students. Write the amount of pizza each student gets as a fraction.

Ex 3.1

(b) 7 packets of lollies are shared between 4 students. Write the number of packets each student receives as a mixed number.

5 Find equivalent fractions by copying and completing the following. Ex.3.2

(a) $\frac{2}{7} = \frac{12}{\square}$ **(b)** $\frac{12}{18} = \frac{\square}{6}$ **(c)** $\frac{15}{30} = \frac{45}{\square}$ **(d)** $\frac{\square}{24} = \frac{5}{4}$

6 Write the following fractions in their simplest form. Ex.3.2

(a) $\frac{12}{15}$ **(b)** $\frac{48}{20}$ **(c)** $3\frac{60}{72}$ **(d)** $1\frac{8}{40}$

7 **(a)** Write $3\frac{2}{7}$ as an improper fraction. Ex.3.2

(b) Write $\frac{50}{9}$ as a mixed number.

8 Copy and complete the following by inserting <, > or =. Ex.3.3

(a) $\frac{5}{13} ____ \frac{1}{2}$ **(b)** $\frac{6}{16} ____ \frac{30}{80}$ **(c)** $\frac{5}{12} ____ \frac{3}{8}$ **(d)** $\frac{8}{9} ____ \frac{9}{10}$

9 Evaluate the following. Estimate first to check that your answers are reasonable. Ex.3.4

(a) $\frac{5}{12} + \frac{7}{8}$ **(b)** $\frac{7}{18} - \frac{2}{9}$ **(c)** $\frac{2}{3} - \frac{2}{7}$ **(d)** $\frac{1}{6} + \frac{7}{10}$

10 Evaluate the following. Write your answers as mixed numbers in simplest form. Ex.3.4

(a) $3\frac{2}{5} - 1\frac{3}{4}$ **(b)** $2\frac{1}{4} + 5\frac{11}{12}$ **(c)** $8 - 2\frac{5}{7}$ **(d)** $4\frac{5}{6} - 4\frac{2}{3}$

11 Calculate the following. Ex.3.5

(a) $\frac{3}{7}$ of \$28 **(b)** $\frac{2}{9}$ of $\frac{3}{4}$

12 Evaluate the following. Ex.3.5

(a) $\frac{6}{11} \times \frac{55}{18}$ **(b)** $2\frac{4}{9} \times 5$ **(c)** $\frac{2}{3} \times \frac{9}{16} \times 1\frac{1}{7}$

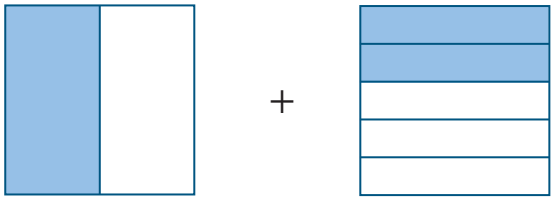
13 Evaluate the following. Ex.3.6

(a) $9 \div \frac{3}{4}$ **(b)** $21 \div 3\frac{2}{7}$ **(c)** $\frac{6}{5} \div \frac{32}{15}$ **(d)** $3\frac{1}{4} \div 3\frac{1}{3}$

14 Simplify by using the correct order of operations. Ex.3.7

(a) $4 + \frac{5}{6} \times 2\frac{1}{3}$ **(b)** $7\frac{2}{5} - \left(3\frac{1}{10} - \frac{3}{5}\right)$ **(c)** $\frac{7}{8} \times \left(\frac{3}{4} + 4 \times \frac{3}{4}\right)$

15 **(a)** What fraction sum is represented by the following two grids?



(b) Calculate the fraction sum using the grid method or any other method.

16 Write a fraction or a mixed number in simplest form to show each of these:

(a) eight chocolate biscuits in a packet of 24

(b) two complete '6-packs' of soft drink cans, with one can left over

(c) one packet of twelve colour pencils and four pencils in a second packet

(d) nine hours sleep in one day.

17 There are 12 boys and 15 girls in class 7A. 6 of the students have blond hair, and 2 have red hair. Write the following as fractions of the whole class, in simplest form.

(a) the number of girls

(b) the number of blond-haired students

(c) the number of students with neither blond nor red hair

18 Write each set of numbers in order from smallest to largest, by using strategies for comparing fractions.

(a) $\frac{3}{4}, \frac{1}{2}, 1, \frac{1}{3}, \frac{3}{5}$

(b) $2, \frac{13}{5}, \frac{5}{4}, \frac{30}{50}, 1\frac{2}{5}$

19 Jamal gets 7 out of 9 shots at goal in the netball ring, whereas Kayla gets 9 out of 12 shots in. Determine who is the more accurate shooter by comparing fractions.

20 Elise and Reece each have an identical block of chocolate. Elise eats $\frac{2}{5}$ and Reece eats $\frac{2}{3}$.

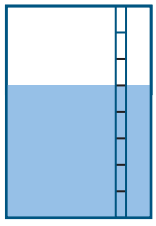
(a) How much more has Reece eaten, as a fraction of a block?

(b) Together, have Elise and Reece eaten more or less than a whole block of chocolate? Estimate, then calculate your answer.

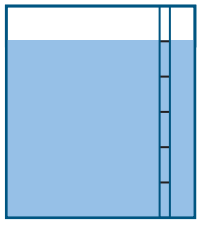
21 Gary had $1\frac{1}{2}$ bags of cement in his shed. He used $\frac{2}{3}$ of a bag to mix up some concrete. What fraction of a full bag does he have left?

22 Jarrod owns a petrol station that has three main fuel tanks. There is a dip stick in each tank that shows the level of fuel remaining in each tank. How many litres are left in each tank?

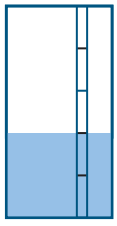
(a) Full tank = 48 000 L



(b) Full tank = 60 000 L



(c) Full tank = 30 000 L



23 Here are the ingredients for a choc-orange cake:

180 g butter	$\frac{2}{3}$ cup castor sugar	4 eggs
$1\frac{3}{4}$ cups self-raising flour	2 tablespoons cocoa	$\frac{1}{2}$ cup orange juice

Rewrite the ingredients to make a cake that is one-quarter of the size of this cake.

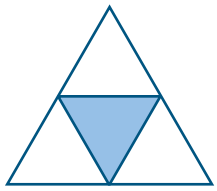
Reasoning and communicating

24 State the fraction of the area of these shapes that is shaded.

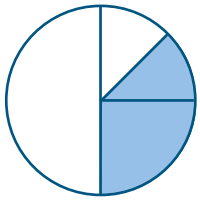
(a)



(b)



(c)



25 45 minutes into a fun run, Katya had completed 4 km of the 10 km course, whereas Mabok had completed 7 km of the 15 km course.

(a) Who had the least distance still to run?

(b) Who had completed a bigger fraction of their particular course?

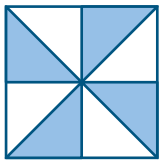
26 Which of the following will give an answer of 1?

A $\frac{1}{4} \div \frac{1}{4}$ **B** $\frac{1}{2} \div \frac{1}{4}$ **C** $1 \div \frac{1}{4}$ **D** $4 \div \frac{1}{4}$

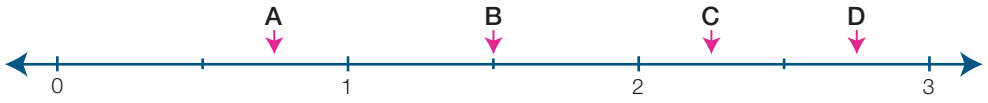
NAPLAN practice 3

Numeracy: Non-calculator

1 Which diagram does not have exactly $\frac{1}{2}$ of the area shaded?

A  **B**  **C**  **D** 

2 Which arrow is pointing closest to the location of $\frac{11}{8}$ on this number line?



3 Kevin is mixing sand, gravel and cement to make concrete. Sand makes up $\frac{1}{2}$ of the mixture, and gravel is $\frac{1}{3}$. What fraction of the mixture is cement?

A $\frac{1}{6}$ **B** $\frac{1}{5}$ **C** $\frac{1}{4}$ **D** $\frac{1}{3}$

4 Esther is making vanilla ice-cream using these ingredients that make 6 servings:

5 eggs
1 cup sugar
2 cans condensed milk
6 cups milk
$1\frac{1}{2}$ teaspoons vanilla

How many teaspoons of vanilla will Esther need to make 30 servings?

A $5\frac{1}{2}$ **B** $7\frac{1}{2}$ **C** $30\frac{1}{2}$ **D** 45

Numeracy: Calculator allowed

5 A school has 200 students. 48 of the students are involved in the school production. The fraction of students who are involved in the school production is closest to:

A one-fifth **B** one-quarter **C** one-third **D** one-half

6 Which of the following fractions is greater than 2 but less than 3?

A $\frac{3}{4}$ **B** $\frac{12}{9}$ **C** $\frac{17}{7}$ **D** $\frac{15}{5}$

7 Imran ran $\frac{1}{6}$ of a cross-country course. He ran 3 km. How long is the course?

A 6 km **B** 9 km **C** 12 km **D** 18 km

8 Which fraction lies exactly halfway between $\frac{3}{8}$ and $\frac{4}{8}$ on the number line?

A $\frac{3}{16}$ **B** $\frac{1}{4}$ **C** $\frac{1}{3}$ **D** $\frac{7}{16}$

4 Decimals, percentage and ratio

Recall 4

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

1 Copy and complete each of the following by writing < (less than), > (greater than), or = (equal to) between the given numbers.

(a) 0.1 _____ 0.01 **(b)** 2 _____ 2.2 **(c)** 0.3 _____ 0.1

(d) 3.2 _____ 4.1 **(e)** 0.008 _____ 0.09 **(f)** 0.7 _____ 0.07

2 The number 43 can be written in expanded form as 'four tens and three ones'. Write the following in expanded form.

(a) 72 **(b)** 603 **(c)** 9251 **(d)** 11 080

3 Write each of the following **(i)** in words and **(ii)** as decimals.

(a) $\frac{7}{10}$ **(b)** $\frac{8}{1000}$ **(c)** $\frac{3}{100}$ **(d)** $1\frac{9}{10}$

4 Calculate:

(a) $\begin{array}{r} 34 \\ +\ 76 \\ \hline \end{array}$ **(b)** $\begin{array}{r} 925 \\ +\ 610 \\ \hline \end{array}$ **(c)** $\begin{array}{r} 67 \\ 409 \\ +\ 3 \\ \hline \end{array}$ **(d)** $\begin{array}{r} 459 \\ 6013 \\ +\ 27 \\ \hline \end{array}$

5 Calculate:

(a) $\begin{array}{r} 74 \\ -\ 25 \\ \hline \end{array}$ **(b)** $\begin{array}{r} 823 \\ -\ 376 \\ \hline \end{array}$ **(c)** $\begin{array}{r} 8289 \\ -\ 384 \\ \hline \end{array}$ **(d)** $\begin{array}{r} 2000 \\ -\ 352 \\ \hline \end{array}$

6 Calculate:

(a) 2×17 **(b)** 25×96 **(c)** 3×48 **(d)** 51×800

7 Perform the following divisions.

(a) $362 \div 2$ **(b)** $9459 \div 9$ **(c)** $5600 \div 200$ **(d)** $4500 \div 30$

8 Calculate:

(a) 70×100 **(b)** $12 \times 10\,000$ **(c)** 9200×1000

(d) $80 \div 10$ **(e)** $3600 \div 100$ **(f)** $250\,000 \div 1000$

(g) 1.2×10 **(h)** 6.3×100 **(i)** 0.0427×1000

(j) $58 \div 10$ **(k)** $901 \div 100$ **(l)** $76.2 \div 1000$

9 Write these percentages as fractions in simplest form.

(a) 50% **(b)** 10% **(c)** 25% **(d)** 20% **(e)** 1% **(f)** 60%

Syllabus references

Number and Algebra: Fractions, Decimals and Percentages

- ✓ Multiply and divide fractions and decimals using efficient written strategies and digital technologies **(ACMNA154)**
- ✓ Round decimals to a specified number of decimal places **(ACMNA156)**
- ✓ Find percentages of quantities and express one quantity as a percentage of another, with and without the use of digital technologies **(ACMNA158)**
- ✓ Investigate terminating and recurring decimals **(ACMNA184)**
- ✓ Connect fractions, decimals and percentages and carry out simple conversions **(ACMNA157)**

Number and Algebra: Ratio and Rates

- ✓ Recognise and solve problems involving simple ratios **(ACMNA173)**

Number and Algebra: Financial Mathematics

- ✓ Investigate and calculate 'best buys', with and without digital technologies **(ACMNA174)**

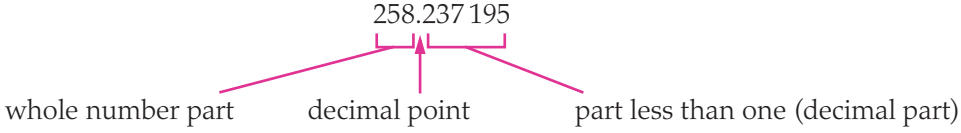
Place value and comparing decimals

4.1

Need to Know

Decimal place value

Like fractions, decimals are used to write numbers that have a part that is less than one. The **decimal point** is used to separate the whole number part from the part less than one.



The number of **digits** after the **decimal point** of a number show how many **decimal places** that number has.

Because the number above has six digits after the decimal point, we say it has six decimal places.

Place value table

Hundreds 100	Tens 10	Ones 1	•	Tenths $\frac{1}{10}$ (0.1)	Hundredths $\frac{1}{100}$ (0.01)	Thousandths $\frac{1}{1000}$ (0.001)	Ten-thousandths $\frac{1}{10000}$ (0.0001)	Hundred-thousandths $\frac{1}{10000}$ (0.00001)	Millionths $\frac{1}{100000}$ (0.000001)
2	5	8	•	2	3	7	1	9	5

If there is no digit for a particular **place value** in a decimal number, we use a zero to show this. For example, five and two tenths and three thousandths is written as 5.203. It is important to include the zero between the 2 and the 3. Leaving it out would mean 5.203 becomes 5.23, which is a different number (that is, five and two tenths and three hundredths).

Expressing decimals in different forms

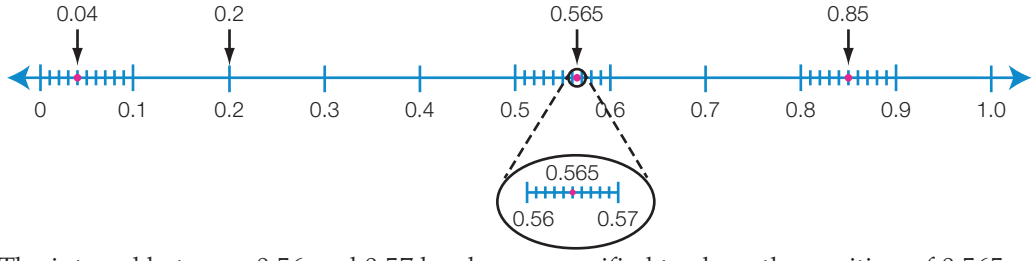
Decimal numbers can be written in several ways:

- Decimal form: 258.237 195
- Expanded fractional form: $258 + \frac{2}{10} + \frac{3}{100} + \frac{7}{1000} + \frac{1}{10\,000} + \frac{9}{100\,000} + \frac{5}{1\,000\,000}$
- Expanded word form: *two* hundreds, *five* tens, *eight* ones, *two* tenths, *three* hundredths, *seven* thousandths, *one* ten-thousandth, *nine* hundred-thousandths and *five* millionths.

Comparing decimals using a number line

If a unit on a number line is divided into 10, each division represents one tenth, $\frac{1}{10}$ or 0.1. If the one-tenth interval is divided into 10 again, each division represents one hundredth, $\frac{1}{100}$ or 0.01. Dividing one-hundredth into 10 gives divisions of one thousandth, $\frac{1}{1000}$ or 0.001.

The positions of the decimal numbers 0.04, 0.2, 0.565 and 0.85 are shown with arrows.



The interval between 0.56 and 0.57 has been magnified to show the position of 0.565 more clearly. It shows 1 hundredth divided up into 10 parts, each equal to 1 thousandth. $\frac{1}{100} + \frac{100}{1000}$

The decimal number with the most digits is not necessarily the largest; e.g. 0.565 < 0.85.

A number's size depends on the value and position of the digits; e.g. 0.2 > 0.04.

Worked Example 1

WE1

(a) Write $4 + \frac{3}{10} + \frac{6}{1000} + \frac{5}{10\,000}$ as a decimal.

(b) Write 6.2807 in expanded fractional form.

Working

(a) $4 + \frac{3}{10} + \frac{6}{1000} + \frac{5}{10\,000}$

$= 4.3065$

(b) 6.2807

$= 6 + \frac{2}{10} + \frac{8}{100} + \frac{7}{10\,000}$

Worked Example 2

WE2

(a) Write seven units, three tenths, six hundredths, seven thousandths and four hundred-thousandths as a decimal.

(b) Write 28.0045 in expanded word form.

Working

(a) 7.367 04

(b) 28.0045

= Two tens, eight ones, four thousandths and five ten-thousandths

Worked Example 3 WE3

(a) Write the value of the 6 in 4.368 as a fraction.

(b) Write the value of the 9 in 0.0109 in words.

Working

(a) $\frac{6}{100}$

(b) nine ten-thousandths

Worked Example 4 WE4

Write < or > between each of the following pairs of decimals to make a true statement.

(a) 5.173 _____ 5.0731 **(b)** 0.472 39 _____ 0.4731 **(c)** 3.5 _____ 3.51

Working

(a) 5.173 > 5.0731 **(b)** 0.472 39 < 0.4731 **(c)** 3.5 < 3.51

4.1 Place value and comparing decimals

Answers page 462

Navigator

Q1, Q2, Q3, Q4 Column 1, Q5 Column 1, Q7, Q8 (a) & (b), Q9 Column 1, Q10, Q11, Q13, Q14, Q15, Q17, Q19	Q1, Q2, Q3, Q4 Column 2, Q5 Column 2, Q6, Q8 (c) & (d), Q9 Column 2, Q10, Q11, Q12, Q14, Q15, Q16, Q18, Q19	Q1 (b), Q2 (b), Q3, Q4 Column 2, Q5 Column 2, Q6, Q7, Q8 (c) & (d), Q9 Column 2, Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q18, Q19

Fluency and understanding

1 **(a)** Write each of the following as a decimal. WE1

(i) $45 + \frac{4}{10} + \frac{6}{100} + \frac{2}{1000}$

(ii) $12 + \frac{5}{10} + \frac{1}{100} + \frac{9}{1000} + \frac{3}{10\,000} + \frac{7}{100\,000} + \frac{2}{1\,000\,000}$

(iii) $3 + \frac{7}{10} + \frac{9}{100} + \frac{8}{1000} + \frac{5}{10\,000}$

(iv) $1 + \frac{8}{1000} + \frac{3}{10\,000} + \frac{3}{100\,000} + \frac{2}{1\,000\,000}$

(v) $\frac{7}{10} + \frac{8}{100} + \frac{6}{1000} + \frac{6}{100\,000}$

(vi) $7 + \frac{3}{100} + \frac{3}{1000} + \frac{4}{100\,000} + \frac{7}{1\,000\,000}$

Hint

Don't forget to write a zero if you have no digits to go in a place value column.

(b) Write each of the following in expanded fractional form.

(i) 6.63 **(ii)** 0.921 **(iii)** 0.7345

(iv) 7.826 **(v)** 23.913 04 **(vi)** 45.004 589

WE2

2 (a) Write each of the following as a decimal.

(i) six ones and five tenths

(ii) nine tenths and seven hundredths

(iii) two tenths, seven hundredths and three thousandths

(iv) three tens, seven ones, four tenths, two hundredths and one ten-thousandth

(v) one ten, four ones, nine tenths, five hundredths, seven thousandths, six ten-thousandths, two hundred-thousandths and three millionths

(vi) seven hundreds, four thousandths, five ten-thousandths and nine hundred-thousandths

(b) Write each of the following in expanded word form.

(i) 5.2 **(ii)** 4.9 **(iii)** 34.17 **(iv)** 0.61

(v) 2.794 **(vi)** 7.5092 **(vii)** 35.865 43 **(viii)** 0.820 027

WE3

3 (a) Write the value of the 2 in each of the following as a fraction.

(i) 6.012 **(ii)** 0.004 52 **(iii)** 3.287 **(iv)** 2.034

(b) Write the value of the 7 in each of the following in words.

(i) 5.734 **(ii)** 0.0076 **(iii)** 1.2037 **(iv)** 8.130 037

WE4

4 Write < or > between each of the following pairs of decimals to make a true statement.

(a) 2.4 _____ 0.42 **(b)** 2.32 _____ 1.955

(c) 0.65 _____ 0.57 **(d)** 0.3003 _____ 0.333

(e) 4.7038 _____ 4.7312 **(f)** 8.251 _____ 8.2501

(g) 7.02 _____ 7.002 **(h)** 4.7367 _____ 4.7376

(i) 0.927 _____ 0.927 34 **(j)** 6.013 _____ 6.01

(k) 3.406 _____ 3.4063 **(l)** 0.9995 _____ 0.9986

5 Write TRUE or FALSE for each of the following.

(a) $7.5 < 5.77$ **(b)** $4.1 > 4.12$

(c) $6.08 > 6.8$ **(d)** $67.54 < 67.504$

(e) $3.023 < 3.203$ **(f)** $0.547 > 0.547\,08$

(g) $2.000\,012 < 2.0001$ **(h)** $4.145\,29 > 4.200\,01$

6 53.017 expressed in expanded word form is:

A five tens, three ones, one tenth and seven hundredths

B five tens, three ones, one hundredth and seven thousandths

C five tenths, three hundredths, one ten-thousandth and seven hundred-thousandths

D five tens, three ones, one tenth and seven thousandths

7 Nine hundredths, four thousandths and three ten-thousandths is equal to:

A 0.090 43 **B** 0.0943 **C** 0.943 **D** 9.43

8 For each of the following sets of numbers, copy the number line shown, then mark the positions of the numbers with a labelled arrow. (You may have to estimate the position of some.)



(a) 2.05, 2.09, 2.6, 2.12, 2.59

(b) 2.0, 2.8, 2.88, 2.9, 2.805

(c) 2.7, 2.4, 2.07, 2.04, 2.407

(d) 2.2, 2.4, 2.85, 2.35, 2.05

9 Write each set of decimals in order from smallest to largest.

(a) 2.3, 2.03, 2.13

(b) 8.7, 8.007, 8.67

(c) 6.646, 6.6403, 6.64

(d) 0.0095, 0.0905, 0.0509

(e) 5.3281, 5.38, 5.003821

(f) 3.616, 3.116, 3.661

(g) 0.92, 0.29, 0.092

(h) 0.85, 0.815, 0.086

10 Joel wrote down the five-digit number as he saw it on his electronic stopwatch: 37901. However, he forgot to put in the decimal point. Anna knew that the race was timed to a thousandth of a second and was able to put the decimal point in the correct place. Write the number with the decimal point in the correct place.

11 The difference in times between two skiers in a downhill race was 0.437 of a second. Write this:

(a) in expanded fraction form

(b) as a single fraction.

12 For each of the following sets of decimals, draw a section of the number line, marked in either tenths or hundredths, and indicate the position of each number in the set.

(a) 0.4, 0.72, 1.01

(b) 1.7, 1.25, 0.95

(c) 0.06, 0.045, 0.038

(d) 2.9, 3.2, 2.75, 3.06

13 Jane records her three best practice times for the 100 m sprint. They are as follows: 13.95 seconds, 13.08 seconds and 13.69 seconds.

(a) Which was Jane's fastest time?

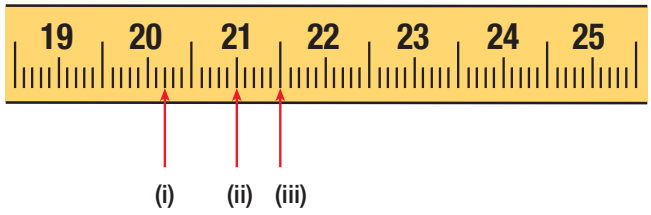
(b) Which was Jane's slowest time?

14 Egor throws a shot-put the following distances during a competition: 25.6 m, 25.56 m and 25.081 m. Which distance was Egor's longest throw?

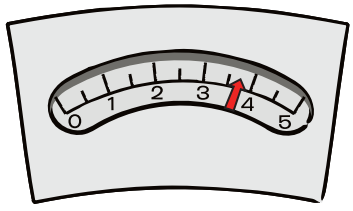
Reasoning, communicating and open-ended

15 Write the value of the decimals indicated by the arrows on the following scales.

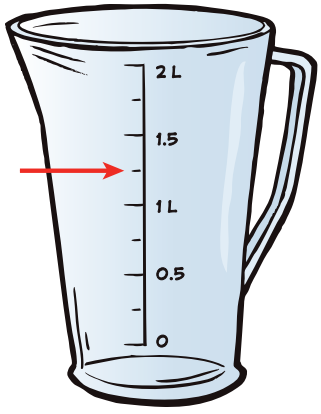
(a)



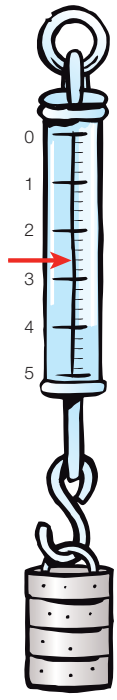
(b)



(c)



(d)



16 State the mistakes that the following students have made.

(a) Minh states that the decimal 34.162 has a 2 in the hundredths column.

(b) Al writes four hundreds, nine ones, six tenths and seven hundredths as 409.067.

(c) Max writes seven ones, eight hundredths and nine ten-thousandths as 0.7809.

(d) Polly writes 0.9056 as $\frac{9}{10} + \frac{5}{100} + \frac{6}{1000}$.

17 **(a)** Write down at least six numbers between 4.5 and 4.7.

(b) Put your numbers from **(a)** in ascending order.

18 **(a)** Use any digit from 0 to 9 in each box to make a correct statement. Digits can be repeated. Find at least three different answers.

$\square.\square\square < 5.\square$

(b) Explain any limitations on the other numbers that can be used when certain numbers are used in some boxes.

19 Donna has cut out the digits 2, 5 and 7, as well as a decimal point, from coloured paper to stick on a poster to show a decimal number. If she uses all three digits:

(a) how many different decimal numbers with two decimal places could she make

(b) how many decimal numbers greater than 5.4 could she make?

Rounding decimals

4.2

Need to Know

Rounding decimal numbers

Step 1 Determine the number of decimal places that you are required to **round** to. The digit in this place will either remain as it is, or will increase by one.

Step 2 Look at the next digit to the right of this digit. If this digit is:

- 0, 1, 2, 3 or 4, leave the digit considered in **Step 1** and delete all digits after it
- 5, 6, 7, 8 or 9, increase the digit considered in **Step 1** by one and delete all digits after it.

If the number in the place value you are rounding to is 9 make the 9 a 0, and add one to the digit in front of it. For example, 1.499 rounded to the nearest hundredth is 1.50. Write a 0 in the hundredths column (even though it makes no difference to the value of the number).

For example:

The number line below shows how the number 1.364 is rounded:

- To the nearest tenth (1 decimal place): 1.364 lies between the tenths values of 1.3 and 1.4.

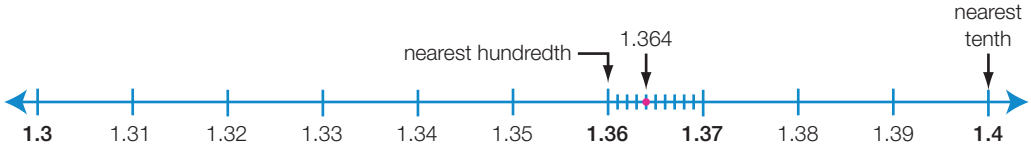
  The 6 in the hundredths place value column places it closer to 1.4.

  1.364 rounded to the nearest tenth is *rounded up* to 1.4.

- To the nearest hundredth (2 decimal places): 1.364 lies between the hundredths values of 1.36 and 1.37.

  The 4 in the thousandths column places it closer to 1.36.

  1.364 rounded to the nearest hundredth is *rounded down* to 1.36.



Rounding money

In Australia, the 5-cent coin is the smallest coin in use so, if money is being paid in cash, the following rules apply.

If the amount ends in…	*then it is rounded…*	*the last digit becomes…*
1 or 2 cents	down	0
3 or 4 cents	up	5
6 or 7 cents	down	5
8 or 9 cents	up	0

If the amount ends in 0 or 5 cents, then the exact amount is paid.

Rounding does not apply to electronic transactions.

Worked Example 5 WE5

Round the following decimals to the number of decimal places shown in brackets.

(a) 3.785 (2) **(b)** 0.958 34 (3) **(c)** 6.214 96 (4)

Working

(a) 3.785
3.785
3.79

(b) 0.958 34
0.958 34
0.958

(c) 6.214 96
6.214 96
6.2150

Worked Example 6 WE6

Round the following amounts of money to the nearest 5 cents.

(a) $8.53 **(b)** $7.02 **(c)** $4.99

Working

(a) $8.55 (Here, we round the 3 up to the nearest 5 cents.)

(b) $7.00 (Here, we round the 2 down to the nearest multiple of 10 cents, which is 0.)

(c) $5.00 (Here, we round the 9 up to the nearest multiple of 10 cents, which is 100.)

Answers page 463

4.2 Rounding decimals

Navigator

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Fluency and understanding

WE5

1 Round the following decimals to the number of decimal places shown in brackets.

(a) 4.88 (1) **(b)** 6.72 (1) **(c)** 7.635 (2)
(d) 4.552 (2) **(e)** 0.6416 (3) **(f)** 3.2772 (2)
(g) 0.314 42 (4) **(h)** 11.828 55 (2) **(i)** 2.917 96 (2)
(j) 18.499 95 (3) **(k)** 90.899 99 (4) **(l)** 18.999 999 (3)
(m) 23.4097 (2) **(n)** 57.0804 (3) **(o)** 259.899 99 (4)

Hint
Don't just cut off the digits, check the next digit first.

WE6

2 Round the following amounts of money to the nearest 5 cents.

(a) $4.52 **(b)** $2.76 **(c)** $11.03 **(d)** $23.88
(e) $176.35 **(f)** $542.06 **(g)** $789.54 **(h)** $56.75
(i) $6774.99 **(j)** $1149.97 **(k)** $8989.99 **(l)** $9999.99

3 0.837 256 rounded to the nearest ten-thousandth is:

A 0.837 **B** 0.8372 **C** 0.837 26 **D** 0.8373

4 At the 2009 World Athletics Championships, Usain Bolt set a new 100 m record time of 9.58 seconds. Round this time to the nearest tenth of a second.

5 The total shown on the receipt for Russell's grocery shopping is $53.67. If Russell is paying in cash, how much money will he have to hand over?

6 Sir Donald Bradman's cricket batting average was 99.943 (rounded to the nearest thousandth). What would be his average rounded to the nearest:

(a) tenth **(b)** hundredth?

7 The top temperature recorded one day in Albury was 41.6°C. What figure should the weather presenter read out in his weather report as the day's maximum if he rounds the temperature reading to the nearest whole number (i.e. no decimal places)?

8 The price of a box of chocolates is $11.97. If Beau buys the chocolates and pays with cash, how much will he pay?

9 A physics student calculates the speed of sound to be 352.153 metres per second. In case her equipment is not accurate enough for this result to be reliable, she decides to round it to the nearest tenth. What value does she write in her report, assuming she rounds off correctly?

Reasoning, communicating and open-ended

10 Ruth calculates the length of shelf required for a wall unit to be 1.266 66 m. As her tape measure is only accurate enough to measure in centimetres (cm), she needs to round off this figure to the nearest cm. If 1 cm = 0.01 m, what length should she measure?

11 In a cycling sprint race at a velodrome, the following results were recorded: 12.149 s, 12.143 s, 12.138 s, 12.152 s and 12.157 s.

(a) Write these in order from fastest to slowest.

(b) It was found that there was a small error in the timing equipment and the results were to be recorded to the nearest hundredth. What were the new times?

(c) Does this affect the race result?

12 **(a)** The total cost of Rani's shopping was $20.34. Round this amount to the nearest 5 cents.

The individual costs of the items Rani bought were:

Cereal	$6.93
Yoghurt	$4.78
Bananas	$2.51
Juice	$3.63
Pasta	$2.49

(b) How much more would Rani have paid if the individual prices of the items were each rounded to the nearest 5 cents?

13 Write at least three different decimal numbers that, when rounded, would give the number 3.79.

14 Write down two prices that would give $100 dollars when rounded to:

(a) the nearest 5 cents **(b)** the nearest dollar.

15 Taking 2.548 46 and rounding it to four decimal places gives 2.5485, then rounding this to three decimal places gives 2.549.

What answer do you get when rounding 2.548 46 to three decimal places?

Explain why this is different, and what is wrong with doing it the first way.

16 Why do you think 1 cent and 2 cent coins were taken out of circulation? List two or three possible reasons why a government might make this decision.

4.3 Decimals and fractions

Need to Know

Writing decimals as fractions

A decimal can be written in expanded fraction form which can be simplified to a single fraction or mixed number.

For example: $3.72 = 3 + \frac{7}{10} + \frac{2}{100}$

$= 3 + \frac{70}{100} + \frac{2}{100}$

$= 3 + \frac{72}{100}$

$= 3\frac{72}{100}$ or $\frac{372}{100}$

$= 3\frac{18}{25}$ or $\frac{93}{25}$

Writing fractions as decimals

There are two methods that can be used.

Method 1: Write the fraction as an equivalent fraction with a denominator of 10, 100, 1000, etc.

The digits in the numerator of this new fraction are the decimal values.

For example: $\frac{4}{5} = \frac{8}{10}$ $= 0.8$ and $\frac{13}{20} = \frac{65}{100}$ $= 0.65$

This method is only useful for fractions with denominators that have factors of 2, 5 or 10.

Method 2: Add a decimal point and as many zeroes as required to the number in the numerator and divide by the number in the denominator.

For example: $\frac{7}{8} = 7.000 \div 8$ $= 0.8$ and $\frac{37}{40} = 37 \div 40$ $= 3.70000 \div 4$ $= 0.925$

A number that has a finite number of decimals is called a **terminating decimal**.

For example: 1.4 or 0.375

Recurring decimals

When the division gives a repeating pattern of decimal digits, it is called a **recurring decimal**. If a recurring pattern appears, stop the division and write the answer with a dot above the repeating digit or pattern.

For example: $\frac{2}{3} = 0.\dot{6}$, $\frac{1}{12} = 0.08\dot{3}$, $\frac{4}{7} = 0.\overline{571\,428}$ or $0.\dot{5}71\,42\dot{8}$

Technology Exploration: Calculator

Fractions and decimals on a calculator

On a scientific calculator, the $a\frac{b}{c}$ or S⇔D key can be used to convert between the fraction and the decimal forms of a number. Pressing the key again will convert it from decimal form back to the fraction.

Recurring decimals are displayed as approximations with the last digit rounded up or down as appropriate.

For example: $\frac{2}{3}$ is displayed as 0.666 666 667 when the display is fixed to 9 decimal places. The last decimal place has been rounded up.

Worked Example 7

WE7

Convert each of the following decimals to fractions, giving your answers in simplest form.

(a) 6.28 **(b)** 0.3125

Working

(a) 6.28

$= 6\frac{28}{100}$ or $\frac{628}{100}$

$= 6\frac{7}{25}$ or $\frac{157}{25}$

(b) 0.3125

$= \frac{3125}{10\,000}$

$= \frac{625}{2000}$

$= \frac{25}{80}$

$= \frac{5}{16}$

Worked Example 8 WE8

Convert each of the following fractions to decimals.

(a) $\frac{7}{5}$ (b) $\frac{3}{8}$ (c) $\frac{4}{3}$

Working

(a) $\frac{7}{5}$

$= \frac{14}{10}$

$= 1\frac{4}{10}$

$= 1.4$

$\frac{7}{5} = 1.4$

(b) $\frac{3}{8}$

$= \frac{3.0}{8}$

$= \frac{3.000}{8}$

$= 0.375$

$$8\overline{)3.0}\quad 0.$$

$$8\overline{)3.0^{6}0^{4}0}\quad 0.375$$

(c) $\frac{4}{3}$

$= 4.000 \div 3$

$= 1.\dot{3}$

$$3\overline{)4^{1}0^{1}0^{1}0}\quad 1.333$$

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4.3 Decimals and fractions

Navigator

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Equipment required: Calculator may be used for Question **2 (i)–(p)**

Fluency and understanding

WE7

1 Convert each of the following decimals to fractions, giving your answers in simplest form.

(a) 4.1	**(b)** 6.5	**(c)** 8.6	**(d)** 9.4
(e) 4.71	**(f)** 2.37	**(g)** 0.35	**(h)** 2.48
(i) 5.009	**(j)** 0.884	**(k)** 6.128	**(l)** 3.172
(m) 7.045	**(n)** 0.088	**(o)** 6.0015	**(p)** 1.0075

Hint

Make sure you put the correct number of zeroes in the denominator.

2 Convert each of the following fractions to decimals.

WE8

(a) $\frac{7}{10}$ (b) $\frac{9}{100}$ (c) $\frac{17}{100}$ (d) $\frac{123}{1000}$

(e) $\frac{1}{5}$ (f) $\frac{3}{25}$ (g) $\frac{13}{50}$ (h) $\frac{7}{4}$

(i) $\frac{7}{8}$ (j) $\frac{11}{16}$ (k) $\frac{23}{40}$ (l) $\frac{47}{60}$

(m) $\frac{1}{9}$ (n) $\frac{7}{12}$ (o) $\frac{17}{6}$ (p) $\frac{16}{15}$

3 0.55 expressed as a fraction in simplest form is:

A $\frac{55}{1000}$ B $\frac{11}{200}$ C $\frac{11}{25}$ D $\frac{11}{20}$

4 2.000 47 is equal to:

A $2\frac{47}{100}$ B $2\frac{47}{1000}$ C $2\frac{47}{10\,000}$ D $2\frac{47}{100\,000}$

5 As part of a science experiment, Galina's reaction time was measured to be 0.48 seconds. What fraction of a second is this?

6 One inch is approximately equal to 2.54 centimetres. What is this when expressed as a fraction in simplest form?

7 Hair grows at an average rate of 1.35 cm per month. Write this as a mixed number in simplest form.

8 A virus particle has a diameter of 0.000 024 mm. Write this as a fraction in simplest form.

9 A red blood cell's diameter is $\frac{1}{200\,000}$ of a metre. Write this measurement in decimal form.

Reasoning, communicating and open-ended

10 A bank offers its customers an interest rate of 5.85 per cent. Write this rate in single fraction form. Is this rate higher than the $5\frac{3}{4}$ per cent offered by another bank?

11 Jeremy has five different containers and is trying to place them in ascending order of size. The volumes of all five containers are measured in litres but some are given using decimals and others using fractions. The tubs have the following volumes: $4\frac{1}{5}$, 4.375, 4.865, $4\frac{9}{10}$, $4\frac{1}{2}$.

Help Jeremy put them in order from the smallest to the largest.

12 (a) Write three different fractions that have decimal values between 3.4 and 3.5 and have a denominator of 100.

(b) Write three different fractions that have decimal values between 3.4 and 3.5 where each fraction has a different denominator that is not a power of 10 (such as 100 or 1000).

13 Two numbers are rounded to one decimal place to give 5.6. What could those two numbers be? Give your answers in fraction form.

4.4 Decimal addition and subtraction

Need to Know

To add or subtract decimals, line up the decimal points so that digits with the same place value are underneath each other.

If the numbers have different numbers of decimal places, zeroes can be written in the 'empty' place value columns to help them line up correctly.

Worked Example 9

WE9

Calculate: 12.45 + 6 + 0.3678

Working

$$\begin{array}{rr} & 12.^{1}4500 \\ & 6.0000 \\ + & 0.3678 \\ \hline & 18.8178 \end{array}$$

Worked Example 10

WE10

Calculate: 15 – 2.147

Working

$$\begin{array}{rr} & {}^{4}1{}^{9}5.{}^{9}0{}^{1}0 \\ - & 02.147 \\ \hline & 12.853 \end{array}$$

4.4 Decimal addition and subtraction

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Navigator

Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q13, Q14	Q1 Column 2, Q2 Column 2, Q3, Q4, Q5, Q6, Q8, Q10, Q11, Q12, Q13, Q15, Q17	Q1 Column 3, Q2 Column 3, Q4, Q5, Q6, Q8, Q11, Q12, Q13, Q15, Q16, Q17

Fluency and understanding

1 Calculate: WE9

(a) 8.3 + 7.9 **(b)** 4.85 + 8.09 **(c)** 0.237 + 0.677

(d) 25.61 + 0.038 **(e)** 5.098 + 21.32 **(f)** 0.025 + 39.786

(g) 9.703 + 5.624 + 7.5 **(h)** 7.35 + 0.609 + 2.3 **(i)** 0.648 + 7.31 + 0.9

(j) 6 + 5.017 + 12.9 **(k)** 2.59 + 15 + 0.005 **(l)** 0.0004 + 3.583 + 8

2 Calculate: WE10

(a) 5.7 – 3.8 **(b)** 7.05 – 2.93 **(c)** 23.982 – 11.735

(d) 9.663 – 2.7 **(e)** 4.984 – 1.8 **(f)** 7.238 – 3.4

(g) 9.5 – 2.24 **(h)** 16.2 – 8.75 **(i)** 3.7 – 0.931

(j) 8 – 7.44 **(k)** 3 – 2.92 **(l)** 93 – 0.698

3 Which of the following shows the correct way to set out the addition of 0.56, 15.092 and 2.7?

A
$$\begin{array}{r} 0.5\,6 \\ 1\,5.092 \\ +\ 2.7 \\ \hline \\ \end{array}$$

B
$$\begin{array}{r} 0.\,56 \\ 15.092 \\ +\ 2.7 \\ \hline \\ \end{array}$$

C
$$\begin{array}{r} 0.56 \\ 15.092 \\ +\quad 2.7 \\ \hline \\ \end{array}$$

D
$$\begin{array}{r} 0.56 \\ 15.092 \\ +\ \ 2.7 \\ \hline \\ \end{array}$$

4 Daily rainfall totals for three days over a long weekend were 3.78, 2.5 and 6.42 millimetres. What was the total rainfall over this three-day period?

5 John loads his shopping trolley with several items priced as shown.

1 frozen lasagna	$5.71
2 L orange juice	$4.25
1 tub of yoghurt	$3.77
1 packet of cereal	$6.14

(a) Find the total cost of John's purchases.

(b) Round the total to the nearest 5 cents.

6 Cristina's bank account balance was $335.96 just before she withdrew $40.45 to pay a bill. How much did she have left in her account after the withdrawal?

7 In a gymnastics competition, Adelia scores the following from the five judges: 7.5, 8.5, 7.9, 8 and 8.6. Find her total score.

8 A family drives to a holiday resort and records the car's odometer readings as shown.

Departure: 234.8 km Arrival: 502.7 km

How far did they travel to get to the resort?

9 During a car tour of Tasmania, Helena travelled distances of 25.64 km, 165.35 km and 5.97 km all in one day. How many kilometres did she travel altogether that day?

10 Georgina pays for $36.35 worth of groceries with a $100 note. How much change should she receive?

11 Nico pours 1.625 litres of milk from a full 2-litre container. How much milk is left in the container?

Reasoning, communicating and open-ended

12 Amanthi's credit card bill shows the following purchase amounts: $23.56, $40.15, $7.89, $18.48, $45.50, $21.73 and $8.59. The total at the bottom of her statement is $184.38, which she is sure is wrong.

(a) What should the total be?

(b) How do you think the error was made?

13 Tina sells flowers at a market stall. One day, she takes in $473.50 from the sale of flowers. If the flowers cost Tina $128.35, and she had to pay a fee of $14.50 to rent the stall for one day, what profit did she make for the day?

14 **(a)** Write two numbers that add up to 2.871.

(b) Write three numbers, each with three decimal places, that add up to 3.86.

(c) Explain how you chose the last digit of the numbers you used in **(b)**.

15
$$\begin{array}{r} 4.\square \\ +\ \square.8\square \\ \hline 7.\square 5 \end{array}$$

(a) Use one digit from 0 to 9 to fill each box and make a correct sum. Find at least three different combinations.

(b) Was your choice of digits restricted? Explain how.

16 **(a)** Write two decimal numbers, each with three decimal places, that have a difference of 8.712.

(b) Write two decimal numbers, each with three decimal places, that have a difference of 8.71.

(c) Explain how you chose the last digit of the numbers you used in **(b)**.

17 Two Australian swimmers finish first and second in the women's 100 m freestyle at the Olympics. The difference in their times is 0.04 seconds. The Olympic record was 53.52 seconds. Assuming that they both break this record by less than $\frac{2}{10}$ of a second, what could their times be?

Decimal multiplication 4.5

Need to Know

Multiplying a decimal by a whole number

Multiply as though both values are whole numbers. Then, place a decimal point in the answer so that it has the same number of decimal places as the decimal number in the question.

To check whether your decimal point is in the right place, estimate your answer before calculating it.

Multiplying a decimal by a power of 10

Multiplying a number by 10 makes each digit in the number ten times its previous value.

For example, $43.25 \times 10 = 432.5$.

To multiply by a power of 10, move the decimal point to the *right* the same number of place values as the zeroes in the power of 10. Write a zero in any empty place values.

$$0.45 \times 10^1 = 0.45 \times 10$$
$$= 4.5$$

$$0.45 \times 10^2 = 0.45 \times 100$$
$$= 45$$

$$0.45 \times 10^3 = 0.45 \times 1000$$
$$= 450$$

Dividing a decimal by a power of 10

Dividing a number by 10 makes each digit in the number one-tenth of its previous value.

For example, $65.1 \div 10 = 6.51$.

To divide by a power of 10, move the decimal point to the *left* the same number of place values as the zeroes in the power of 10. Write a zero in any empty place values.

$$78.9 \div 10^1 = 78.9 \div 10$$
$$= 7.89$$

$$78.9 \div 10^2 = 78.9 \div 100$$
$$= 0.789$$

$$78.9 \div 10^3 = 78.9 \div 1000$$
$$= 0.0789$$

Multiplying one decimal by another

Step 1 Write both numbers as the division of a whole number and a power of 10.

Step 2 Multiply as though both numbers are whole numbers.

Step 3 Divide the product by each of the powers of 10 identified in **Step 1**. Show this by moving the decimal place to the left.

Remember, while 'move the decimal point' is used, it is the digits themselves that actually move as their values increase or decrease.

Worked Example 11

WE11

Calculate:

(a) 4.172×3 **(b)** 52.63×14 **(c)** 0.0003×2

Working

(a) $4.172 \times 3 \approx 4 \times 3$
≈ 12 (estimate)

$$\begin{array}{r} 4.172 \\ \times \quad {}^{2}3 \\ \hline 12.516 \end{array}$$

$4.172 \times 3 = 12.516$

(b) $52.63 \times 14 \approx 50 \times 14$
$\approx 14 \times 5 \times 10$
≈ 700 (estimate)

$$\begin{array}{r} 52.63 \\ \times \quad {}^{1\,2\,1}14 \\ \hline 21\,052 \\ 52\,630 \\ \hline 736.82 \end{array}$$

$52.63 \times 14 = 736.82$

(c)
$$\begin{array}{r} 0.0003 \\ \times \quad 2 \\ \hline 0.0006 \end{array}$$

$0.0003 \times 2 = 0.0006$

Worked Example 12

WE12

Calculate:

(a) 0.6295×7000 **(b)** $9.81 \times 340\,000$

Working

(a) 0.6295×7000
$= 0.6295 \times 7 \times 1000$
4.4065×1000
$= 4406.5$

$$\begin{array}{r} 0.6295 \\ \times \quad {}^{4\ 2\ 6\ 3}7 \\ \hline 4.4065 \end{array}$$

(b) $9.81 \times 340\,000$
$= 9.81 \times 34 \times 10\,000$
$333.54 \times 10\,000$
$= 3\,335\,400$

$$\begin{array}{r} 9.81 \\ \times \quad {}^{2\,3}34 \\ \hline 3924 \\ {}_{1}2_{1}9430 \\ \hline 333.54 \end{array}$$

Worked Example 13 WE13

Calculate:

(a) 0.63×0.4 **(b)** 0.014×0.002

Working

(a) 0.63×0.4
$= 63 \div 100 \times 4 \div 10$
$= 252 \div 1000$
$= 0.252$

$$\begin{array}{r} 0.63 \\ \times \quad 0.4 \\ \hline 0.252 \end{array}$$

(b) 0.014×0.002
$= 14 \div 1000 \times 2 \div 1000$
$= 28 \div 1\,000\,000$
$= 0.000\,028$

$$\begin{array}{r} 0.014 \\ \times \quad 0.002 \\ \hline 0.000\,028 \end{array}$$

4.5 Decimal multiplication

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Fluency and understanding

1 Calculate: WE11

(a) 7.6×4 **(b)** 5.3×7 **(c)** 6.5×9
(d) 5×4.62 **(e)** 0.54×4 **(f)** 6×12.33
(g) 8.047×8 **(h)** 0.619×7 **(i)** 3×18.309
(j) 22.08×13 **(k)** 48.76×25 **(l)** 37.09×32
(m) 0.0004×2 **(n)** 0.0005×7 **(o)** 1.0006×3

2 Calculate: WE12

(a) 2.4×60 **(b)** 3.27×70 **(c)** 2.991×40
(d) 0.57×200 **(e)** 4.34×500 **(f)** 6.625×400
(g) 4.591×6000 **(h)** 6.38×4300 **(i)** 3.14×9300
(j) $0.571 \times 68\,000$ **(k)** $0.9465 \times 71\,000$ **(l)** $7.4832 \times 55\,000$

3 Calculate: WE13

(a) 0.6×0.8 **(b)** 0.4×0.6 **(c)** 0.9×0.4
(d) 9.32×0.7 **(e)** 4.12×0.3 **(f)** 5.26×0.8
(g) 7.74×0.31 **(h)** 9.46×0.22 **(i)** 3.76×0.18
(j) 0.004×0.6 **(k)** 0.0308×0.3 **(l)** 0.072×0.05

Hint

Remember, the number of decimal places in your answer should be the same as the number in the decimal being multiplied.

4 A reasonable estimate for the answer to 1.8×15 is:

A 3 **B** 16.8 **C** 30 **D** 300

5 How many decimal places will there be in the answer to 12.789×41?

A 1 **B** 2 **C** 3 **D** 4

6 How many decimal places will there be in the answer to 0.36×1.04?

A 2 **B** 3 **C** 4 **D** 5

7 Renée buys 52 components from an electronics shop for \$0.35 each. What is the total cost of the components?

8 A pill bottle contains 37 pills, each of mass 0.287 grams. What is the total mass of the bottle's contents?

9 Write the following as whole numbers.

(a) 3.5 million **(b)** 4.26 billion **(c)** 1.3 trillion (1 trillion = 1000 billion)

(d) 57.08 million **(e)** 10.2 trillion **(f)** 20.045 billion

10 A snail crawls 0.041 metres in an hour. If it kept up this pace, how far could it move in a day?

11 Amy orders 27.5 metres of timber priced at \$4.29 per metre from her local hardware store. How much will the timber cost her in total? (Round your answer to the nearest cent.)

12 Gavin sells 300 punnets of berries to a supermarket. If he is paid \$1.72 for each punnet, how much money does he receive?

13 In 2010, Mexican businessman Carlos Slim was declared the world's richest man, with a fortune worth 53.5 billion US dollars. Write this amount as a whole number.

14 **(a)** Marita fills her car's fuel tank with petrol at a cost of \$1.35 per litre. If she adds 70 litres, how much must she pay?

(b) Ali pumps 9.5 litres of petrol into his car's tank. How much did the fuel cost him if he paid the same price per litre as Marita? Round your answer to the nearest cent.

15 The instructions on a bag of fertiliser say to spread 0.75 kilograms per square metre of area to be fertilised. Raisa wants to fertilise 2600 square metres of property using this product. How many kilograms of fertiliser should she use?

16 Bryan has calculated that he needs 3.6 square metres of tiles for his bathroom. What will he have to pay if the tiles he wants cost \$22.39 per square metre? (Round your answer to the nearest cent.)

Reasoning, communicating and open-ended

17 Kim works in an ice-cream parlour, and one hot day sells 500 scoops of vanilla. If there is 0.185 litres of ice-cream in each scoop and the day started with 130 litres of vanilla in the freezer, how many litres of vanilla ice-cream are left at the end of the day?

18 Yoghurt contains 167 kilojoules per 100 grams. How many kilojoules would a 175 gram tub contain?

19 In April 2010, the Australian Bureau of Statistics calculated that the population of Australia was 22 296 411.

(a) Write this number as a decimal number, with the number of millions as the whole number part, and the rest of the number rounded to one decimal place.

(b) Why would your answer to **(a)** be the number you would expect to see reported in the news, instead of the 'exact' value?

20 **(a)** A woodwork teacher needs 50 pieces of pine, each 0.135 metres long, to give to students to make a particular model. What total length (in metres) of timber does the teacher need? (Ignore any lost length due to saw cuts.)

(b) If he can buy the pine in 3.5 m lengths, how many pieces does he need, and how much does he have left over?

21 A multi-bag of mini chocolate bars costs $3.95 and contains 12 bars. Each mini chocolate bar can also be purchased individually for 40 cents.

(a) How much would 12 mini chocolate bars cost if they were purchased individually?

(b) How much do you save by purchasing the multi-bag?

(c) Charlotte wants to buy 30 mini bars for a party. Should she buy them individually, or in multi-bags? Which way is cheaper?

22 Wayne is following his grandfather's recipe for tomato sauce, which requires 3.5 pounds of tomatoes. (A pound is an old imperial unit of mass.) Because he only has scales in kilograms, Wayne decides to convert the mass to kilograms. A website tells him that 1 pound equals 0.454 kilograms. What answer should he get when he converts the mass of the tomatoes to kilograms? Round your answer to one decimal place.

23 Francesca read her electricity meter one Saturday, and again at the same time the next Saturday. She recorded the following readings.

First Saturday: 7562.3 kilowatt-hours
Second Saturday: 7638.6 kilowatt-hours

(a) How many kilowatt-hours of electricity did Francesca's household use in the week?

(b) If electricity costs $0.13 per kilowatt-hour, find the total cost of the electricity supplied in the week.

24 Jayapal has been asked to work out the length if every student in Year 7 lay end to end. He has been told that there are 89 students in Year 7 and that their average height is 1.63 m.

This is his working out:

$89 \times 1 = 89$
$89 \times 6 = 534$
$89 \times 3 = 267$

$$\begin{array}{r} 89 \\ 534 \\ +\ 267 \\ \hline = 880 \end{array}$$

Answer: 8.80 m.

(a) Where has Jayapal made his error? Explain how he could use estimation to tell that he was incorrect.

(b) Explain how he could modify his method to get the correct answer.

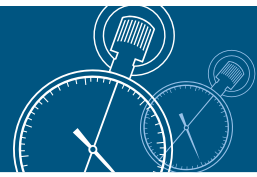
(c) Show how you would complete this problem.

25 Two numbers are multiplied together to give 68.4.

(a) Find at least two possible pairs of numbers with one number being a multiple of 10.

(b) Find at least two possible pairs of numbers with neither being a multiple of 10.

Half-time 4



Ex 4.3

1 Convert the following decimals into fractions (or mixed numbers where appropriate) in simplest form.

(a) 0.25 **(b)** 0.3 **(c)** 3.54 **(d)** 9.345

Ex 4.1

2 Put the following decimals in order from smallest to largest.

9.01, 19.12, 0.03, 0.321, 9.003 24

Ex 4.4

3 Calculate:

(a) 12.5 + 7.9 **(b)** 124.54 + 8.379 **(c)** 0.32 + 2.5

(d) 15 – 8.98 **(e)** 127.061 – 43.2 **(f)** 6 – 4.9

Ex 4.2

4 Round the following numbers to three decimal places.

(a) 4.8765 **(b)** 13.0037 **(c)** 2.3498 **(d)** 23.032 49

Ex 4.5

5 Calculate the following.

(a) 7.6×4 **(b)** 28.3×5 **(c)** 1.37×9

(d) 45.2×30 **(e)** 0.26×2000 **(f)** 6.78×500

(g) 4.3×1.7 **(h)** 39.82×15.4 **(i)** 1.2×0.0005

Ex 4.4

6 Franji is saving up to purchase a games console. She has saved the following amounts over the last four weeks: $25, $47.50, $18.70, $9.35.

(a) How much has she saved in total?

(b) If the console is $149, how much more does Franji have to save?

Ex 4.1

7 Write the following in decimal form.

(a) $23 + \frac{4}{10} + \frac{9}{100} + \frac{7}{10000}$ **(b)** $14 + \frac{235}{1000}$

(c) 2 hundreds, 9 tens, 3 ones, 4 tenths, 1 hundredth and 5 thousandths

(d) 9 tens, 0 ones, 6 tenths, 2 hundredths and 7 ten-thousandths

Ex 4.2

8 Round the following amounts of money to the nearest 5 cents.

(a) $1.93 **(b)** $35.08 **(c)** $57.44 **(d)** $189.98

Ex 4.3

9 Write the following fractions as decimals, using the correct notation for recurring decimals if necessary.

(a) $\frac{4}{5}$ **(b)** $\frac{17}{20}$ **(c)** $\frac{2}{3}$ **(d)** $\frac{7}{12}$

Ex 4.1

10 The times recorded by the finalists in a track cycling sprint race were: 59.023 s, 59.105 s, 59.23 s, 59.125 s. Write the times in order from fastest to slowest.

Ex 4.4, 4.5

11 Melinda is out buying some ingredients for a cooking class. She buys:

12 packets of pasta at $2.49 each
18 tins of tomatoes at $1.35 each
6 kg of mince at $8.99 per kg
3 packets of cheese at $4.78 per packet

Find the total cost of Melinda's shopping.

Decimal division

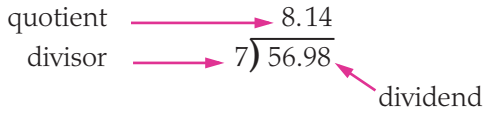
4.6

Need to Know

Dividing a decimal by a whole number

Place a decimal point in the quotient in line with the decimal point in the dividend, then perform the division.

For example:



Dividing a decimal by a multiple of 10

Write the divisor as a whole number multiplied by a power of 10. Perform two separate divisions. Divide by the whole number. Then divide by the power of 10 by moving the decimal point to the left the same number of places as zeroes in the power.

For example:

$23.4 \div 9000 = 23.4 \div (9 \times 1000)$

$= 23.4 \div 9 \div 1000$

$= 2.6 \div 1000$

$= 0.0026$

Dividing a decimal by another decimal

Multiply both numbers by a power of 10 or another number so that the divisor is now a whole number. Perform the division.

For example:

$13.52 \div 1.2$
$= 135.2 \div 12$ (multiplying both numbers by 10)
$= 11.27$

$0.75 \div 2.5$
$= 1.5 \div 5$ (multiplying by 2)
$= 0.3$

Note: While the words 'moving the decimal point' are often used, it is the digits that actually move.

Worked Example 14 WE14

Calculate the following. Round your answers to three decimal places if necessary.

(a) $5.28 \div 3$ **(b)** $0.41 \div 7$

Working

(a)
$$3\overline{)5.^{2}2^{1}8} \quad 1.\ 7\ 6$$

(b)
$$7\overline{)0.4^{4}1^{6}0^{4}0} \quad 0.0\ 5\ 8\ 5\ldots$$

0.059 (to 3 d.p.)

Worked Example 15 WE15

Calculate the following. Round your answers to three decimal places where necessary.

(a) $9.76 \div 400$ **(b)** $14.7 \div 80$

Working

(a)
$9.76 \div 400$
$= 9.76 \div 4 \div 100$
$= 2.44 \div 100$
$= 0.0244$
$= 0.024$ (to 3 d.p.)

$$4\overline{)9.^{1}7^{1}6} \quad 2.4\ 4$$

Working

(b)
$14.7 \div 80$
$= 14.7 \div 8 \div 10$
$= 1.8375 \div 10$
$= 0.183\,75$
$= 0.184$ (to 3 d.p.)

$$8\overline{)14.^{6}7^{3}0^{6}0^{4}0} \quad 1.\ 8\ 3\ 7\ 5$$

Worked Example 16 WE16

Calculate the following. Round your answer to three decimal places if necessary.

(a) $3.35 \div 0.005$ **(b)** $10.52 \div 0.9$

Working

(a)
$3.35 \div 0.005$
$= (3.35 \times 1000) \div (0.005 \times 1000)$
$= 3350 \div 5$
$= 670$

$$5\overline{)33^{3}50} \quad 6\ 70$$

Working

(b)
$10.52 \div 0.9$
$= (10.52 \times 10) \div (0.9 \times 10)$
$= 105.2 \div 9$
$= 11.689$ (to 3 d.p.)

$$9\overline{)10^{1}5.^{6}2^{8}0^{8}0^{8}0} \quad 1\ 1.\ 6\ 8\ 8\ 8$$

4.6 Decimal division

Answers page 465

Navigator

Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3 Column 1, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q17, Q19, Q23, Q24	Q1 Columns 2 & 3, Q2 Column 2, Q3 Columns 2 & 3, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q19, Q20, Q23, Q24	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q18, Q19, Q20, Q21, Q22, Q23, Q24

Fluency and understanding

1 Calculate the following. Round your answers to three decimal places if necessary.

WE14

(a) 8.48 ÷ 4 **(b)** 12.96 ÷ 3 **(c)** 14.22 ÷ 9

(d) 3.3 ÷ 6 **(e)** 2.4 ÷ 7 **(f)** 5.1 ÷ 9

(g) 39.42 ÷ 6 **(h)** 10.6 ÷ 7 **(i)** 0.47 ÷ 9

(j) 0.42 ÷ 6 **(k)** 1.22 ÷ 3 **(l)** 18.46 ÷ 7

(m) 154.33 ÷ 5 **(n)** 84.365 ÷ 8 **(o)** 347.507 ÷ 7

> **Hint**
>
> Don't forget to write a zero in the quotient if at any stage the divisor 'won't go'.

2 Calculate the following. Round your answers to three decimal places if necessary.

WE15

(a) 3.6 ÷ 40 **(b)** 8.1 ÷ 90 **(c)** 36.216 ÷ 60

(d) 96.58 ÷ 200 **(e)** 2.296 ÷ 400 **(f)** 23.4 ÷ 600

(g) 233.4 ÷ 3000 **(h)** 40.6 ÷ 8000 **(i)** 18.9 ÷ 7000

(j) 68.4 ÷ 120 **(k)** 3.19 ÷ 110 **(l)** 8.97 ÷ 1300

3 Calculate the following. Round your answers to three decimal places if necessary.

WE16

(a) 5.14 ÷ 0.2 **(b)** 4.12 ÷ 0.4 **(c)** 5.1 ÷ 0.6

(d) 15.48 ÷ 0.04 **(e)** 9.018 ÷ 0.09 **(f)** 1.736 ÷ 0.07

(g) 2.382 ÷ 0.006 **(h)** 0.3996 ÷ 0.009 **(i)** 0.196 ÷ 0.008

(j) 7.7 ÷ 0.011 **(k)** 3.78 ÷ 0.012 **(l)** 1.16 ÷ 0.008

(m) 1.74 ÷ 0.0002 **(n)** 8.05 ÷ 0.0007 **(o)** 13.36 ÷ 0.0008

4 How many decimal places will the answer to 298.12 ÷ 100 have?

A 1 **B** 2 **C** 3 **D** 4

5 Evaluating which of the following will give the answer to 3.2 ÷ 0.005?

A 0.0032 ÷ 5 **B** 320.0 ÷ 5 **C** 3200.0 ÷ 5 **D** 32 000.0 ÷ 5

6 A block of metal of mass 2.312 kg is to be divided equally into 8 pieces. What will the mass of each piece be?

7 A company makes a profit of 7.86 million dollars. If the profit is to be divided equally among the four company owners, how much would each owner get? (Answer in millions of dollars.)

8 The total bill for a wedding reception for 100 guests is $9398.55. How much does this work out to per guest? (Round your answer appropriately.)

9 Anthony needs to mix 0.07 litres of environmentally friendly pest repellent with water each time he fills the tank of his garden sprayer. If he has 0.84 litres of repellent left, how many sprayer tankfuls can he mix?

10 A jeweller calculated that a popular style of necklace contains 0.009 g of gold. How many necklaces could she make using the 2.332 g of gold in her workshop?

11 Eleanor's car travels 37.29 km on three litres of fuel. How far would it travel on one litre of fuel?

12 A 4000 millilitre container of orange juice costs $8.90. Determine the cost per 100 mL. (Round your answer appropriately.)

13 Simon buys 7 kg of spicy sausages for $32.95. How much is this per kilogram?

14 Regina has a tub containing 5.8 kilograms of worms which she uses for fishing. If the average worm has a mass of 0.003 kilograms, how many are in her tub?

15 Hai is making a long-distance call on his mobile. The call costs $0.90 per minute. For how many minutes can he continue his call if he has $5.85 credit left on his phone?

16 A 1.2 kilometre taxi trip costs Joan $5.70. How much is this per kilometre?

Reasoning, communicating and open-ended

17 The captains of two netball teams were discussing which of their teams was taller. The heights of Angie's team were 1.49 m, 1.54 m, 1.65 m, 1.67 m, 1.58 m, 1.73 m and 1.56 m. The heights of Jessica's team were 1.73 m, 1.58 m, 1.59 m, 1.62 m, 1.45 m, 1.51 m and 1.62 m. Add the seven heights and divide the result by seven to find the average (mean) height of each team. Which of the two teams was the taller, on average? Answer correct to two decimal places.

18 Monica and Celeste were trying to determine the best bargain of three different jars of coffee. The 200 g jar of BestCafe cost $5.95, the 500 g jar of Maxine House was $14.85 and the 300 g tin of Brazilian Roast was $6.75. Which is the best buy in terms of price per 100 grams?

19 Karen was counting the coins in her cash register drawer and found that she had $1.65 in 5-cent coins, $2.30 in 10-cent coins, $5.20 in 20-cent coins and $7.50 in 50-cent coins. How many of each type of coin did she have?

20 Jacob received a bill from a rent-a-car firm that gave the total cost as $310. This included the daily cost of $24.80 and $1.20 per kilometre travelled. If Jacob had hired the car for 2 days, how many kilometres did he travel?

21 Write three numbers, each with three decimal places, that when divided by 4 give an answer between 0.5 and 1.

22 For each of the following, write two equivalent divisions by multiplying the divisor by a number that is not a power of 10.

(a) $10.05 \div 0.5$ **(b)** $36.04 \div 0.02$

23 The area of a rectangle is calculated by multiplying the length and the width. If the area of a rectangle is 8.45 m^2, what might be the length and width of the sides? Give two possible values for the side dimensions. For each of the side dimensions, work out the related perimeter (distance along the boundary of the rectangle).

24 Tran divided 5.25 by 5 and wrote down an answer of 1.5.

(a) What error did Tran make?

(b) Explain to Tran what he needs to do to calculate decimal divisions accurately.

Percentages, fractions and decimals

4.7

Need to Know

The words '**per cent**' literally mean 'for every hundred' or 'out of one hundred'. The percentage symbol is %. Percentages can be thought of and written as fractions with a denominator of 100. Percentages can also be thought of as division by 100.

100% is equivalent to 1 whole, $\frac{1}{1}$ and 1.0.

Writing a percentage as a fraction

- Write the percentage value as the numerator of a fraction with a denominator of 100.
- Simplify the fraction if possible.

For example: $25\% = \frac{25}{100}$

$= \frac{1}{4}$

Writing a percentage as a decimal

- Divide the percentage value by 100.

For example: $25\% = 25 \div 100$

$= 0.25$

Writing a fraction as a percentage

We can use one of two methods.

Method 1: Write the fraction as an equivalent fraction with a denominator of 100. Use the numerator of this new fraction as the percentage.

For example: $\frac{1}{4} = \frac{25}{100}$

$= 25\%$

Method 2: Multiply the fraction by 100%.

For example: $\frac{1}{4} = \frac{1}{4} \times 100\%$

$= 25\%$

Writing decimals as percentages

Multiply the decimal by 100%.

For example: $0.25 = 0.25 \times 100\%$

$= 25\%$

Rational numbers

All fractions, terminating decimals, recurring decimals and percentages are known as **rational numbers** as they can all be written in the form of $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

Worked Example 17 WE17

Write 12% as a fraction in simplest form.

Working

$$12\% = \frac{12}{100}$$
$$= \frac{3}{25}$$

Worked Example 18 WE18

Write 45% as a decimal.

Working

$$45\% = 45 \div 100$$
$$= 0.45$$

Worked Example 19 WE19

Write each of the following fractions as percentages.

(a) $\frac{4}{5}$ **(b)** $\frac{27}{40}$

Working

(a) $\frac{4}{5} = \frac{4}{5} \times \frac{20}{20}$

$= \frac{80}{100}$

$= 80\%$

(b) $\frac{27}{40} = \frac{27}{40} \times \frac{100}{1}\%$

$= \frac{27}{\cancel{40}_{2}} \times \frac{\cancel{100}^{5}}{1}\%$

$= \frac{135}{2}\%$

$= 67.5\%$

Worked Example 20 WE20

Write each of the following decimals as percentages.

(a) 0.04 **(b)** 0.125

Working

(a) $0.04 = 0.04 \times 100\%$
$= 4\%$

(b) $0.125 = 0.125 \times 100\%$
$= 12.5\%$

4.7 Percentages, fractions and decimals

Answers page 465

Navigator

Q1, Q2, Q3 Columns 1 & 2, Q4, Q5, Q6, Q7, Q9, Q11, Q12, Q15, Q16, Q17	Q1, Q2, Q3 Columns 2 & 3, Q4, Q5, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q15, Q16, Q17	Q1, Q2, Q3 Columns 3 & 4, Q4, Q5, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q17

Fluency and understanding

1 Write each of the following percentages as fractions in simplest form. WE17

(a) 15% **(b)** 24% **(c)** 18% **(d)** 64%

(e) 110% **(f)** 122% **(g)** 140% **(h)** 198%

2 Write each of the following percentages as decimals. WE18

(a) 34% **(b)** 13% **(c)** 78% **(d)** 92%

(e) 120% **(f)** 123% **(g)** 254% **(h)** 320%

3 Write each of the following fractions as percentages. WE19

(a) $\frac{2}{5}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{3}{10}$ **(d)** $\frac{7}{20}$

(e) $\frac{11}{10}$ **(f)** $\frac{3}{2}$ **(g)** $\frac{28}{25}$ **(h)** $\frac{6}{5}$

(i) $\frac{3}{8}$ **(j)** $\frac{1}{3}$ **(k)** $\frac{5}{6}$ **(l)** $\frac{4}{9}$

(m) $\frac{13}{15}$ **(n)** $\frac{19}{30}$ **(o)** $\frac{33}{40}$ **(p)** $\frac{44}{60}$

4 Write each of the following decimals as percentages. WE20

(a) 0.87 **(b)** 0.34 **(c)** 0.65 **(d)** 0.96

(e) 1.34 **(f)** 3.58 **(g)** 5.32 **(h)** 1.21

5 Complete the following table.

Percentage	Fraction (simplest form)	Decimal
40%		
	$\frac{7}{10}$	
		0.05
	$\frac{1}{50}$	
75%		
	$\frac{5}{4}$	

6 $\frac{1}{4}$, $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{5}$ are all common fractions.

(a) Change each of these fractions into decimals.

(b) Change each of these decimals into percentages.

7 95% of all the animal species in the world are insects. Write this as a fraction in simplest form.

8 At Valley High School, $\frac{5}{8}$ of the students are studying Chinese. Write this fraction as a percentage.

9 Place the following in ascending order (smallest to largest).

(a) 82%, 0.03, $\frac{7}{50}$, $\frac{3}{5}$, 0.8, 57%

(b) $\frac{1}{2}$, 73%, 1.05, $\frac{3}{10}$, 43%, 0.05

10 The Petronas Towers in Kuala Lumpur are approximately 0.6 times the size of the tallest building in the world, the Burj Khalifa in Dubai.

(a) Write the height of the Petronas Towers as a fraction of the height of the Burj Khalifa.

(b) Now, write the equivalent percentage.

Reasoning, communicating and open-ended

11 A rugby stadium contains 43 500 people. 37% of the crowd supports England and the rest supports Australia. What percentage of the crowd supports Australia?

12 Allison is a journalist. She has surveyed 150 people and found that 6 believe that aliens exist.

(a) Write the survey result:

(i) as a fraction in simplest form

(ii) as a percentage

(iii) as a decimal.

(b) Which of the three forms of the result would you recommend that Allison uses as the headline for the article she is writing? Give a reason for your answer.

(c) Use your answer from **(b)** to write a potential headline.

13 $\frac{3}{4}$ is 3 lots of $\frac{1}{4}$, or $3 \times \frac{1}{4}$. How is this fact reflected in the percentage values for $\frac{1}{4}$ and $\frac{3}{4}$?

14 13% of a class have red hair, 62% of the class have black hair.

(a) What is the total percentage of the class that has red or black hair?

(b) What is this total as a decimal?

(c) What percentage of the class has a hair colour that is not red or black?

(d) Write your answer to **(c)** as a decimal.

15 Write down at least three fractions that have equivalent percentage values between 10 and 40 per cent.

16 Find at least three percentages that have decimal values between 0.34 and 0.35.

17 Estimate the percentage of one weekday that you would spend:

(a) sleeping **(b)** eating **(c)** studying **(d)** socialising.

Using percentages

4.8

Need to Know

Finding the percentage of a quantity

Two methods can be used.

Method 1

Step 1 Break the percentage into smaller amounts that are easy to calculate.

Step 2 Find each percentage.

Step 3 Add the parts together.

For example: 12% of 84 = (10 + 2)% of 84

= 10% of 84 + 2% of 84

= 8.4 + 1.68

= 10.08

Method 2

Step 1 Write the % as a fraction of 100 (simplifying the fraction can often help) or as a decimal.

Step 2 Multiply the amount by the fraction or decimal.

For example: 12% of 84 = $\frac{12}{100} \times \frac{84}{1}$ or 0.12×84

$= \frac{1008}{100}$

= 10.08

Writing one amount as a percentage of another

Step 1 Write the amount as the numerator of a fraction, with the other amount as the denominator.

Step 2 Convert this fraction to a percentage. If the percentage is not a whole number, then rounding to one decimal place gives an accurate enough answer in most cases.

Technology Exploration: Calculator

Percentages on a calculator

The [%] key on a calculator can be used to find the percentage of a quantity.

For example: To calculate 12% of 84, press [8] [4] [×] [1] [2] then press the [%] key. 10.08 will be displayed. You may need to use [SHIFT] to access the % function. Notice that the quantity is input first.

The [%] key can also be used to write one quantity as a percentage of another.

For example: To write 13 as a percentage of 65, press [1] [3] [÷] [6] [5], then press the [%] key. The display will give you the percentage of 20%.

Worked Example 21 WE21

Calculate the following percentage: 15% of 70.

Working

Method 1: Use basic percentages

(a) 15% = 10% + 5%

10% of 70

$= \frac{10^{1}}{100_{1}} \times 70$

$= 7$

5% of 70

= (10% of 70) ÷ 2

= 7 ÷ 2

= 3.5

15% of 70 = 7 + 3.5

= 10.5

Working

Method 2: Fraction or decimal multiplication

(b) 15% of 70

$= 15\% \times 70$

$= \frac{15}{100} \times 70$ or 0.15×70

$= \frac{{}^{3}\cancel{15}}{{}_{2}\cancel{100}} \times \frac{\cancel{70}}{1}$ or 1.5×7

$= \frac{3 \times 7}{2 \times 1}$ or 10.5

$= \frac{21}{2}$

= 10.5

Worked Example 22 WE22

Write the following as percentages. Round your answers to one decimal place, if necessary.

(a) Melissa got 21 out of 25 questions on her grammar test correct.

(b) 13 of the 24 students in class 7C have brown hair.

Working

(a) $\frac{21}{25} = \frac{21}{25} \times \frac{4}{4}$

$= \frac{84}{100}$

= 84%

Melissa got 84% of the test questions correct.

Working

(b) $\frac{13}{24}$

$= \frac{13}{{}_{6}\cancel{24}} \times \frac{{}^{25}\cancel{100}}{1}\%$

$= \frac{325}{6}\%$

= 54.2%

54.2% of the students in 7C have brown hair.

4.8 Using percentages

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Navigator

Q1, Q2, Q3 Columns 1 & 2, Q4, Q6, Q7, Q9, Q10, Q11, Q13, Q14, Q16, Q18, Q20	Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q15, Q16, Q18, Q20	Q1 Columns 2 & 3, Q2 (d)–(i), Q3, Q4, Q5, Q8, Q9, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q20

Equipment required: Calculator may be used for Questions **2**, **7–12**, **15–17**

Fluency and understanding

1 Calculate each of the following percentages. WE21

(a) 50% of $36 **(b)** 10% of 450 m **(c)** 20% of 840 L

(d) 80% of 120 m **(e)** 90% of 500 kg **(f)** 2% of $8400

(g) 13% of 72 kg **(h)** 27% of $354 **(i)** 21% of 876 m

(j) 28% of 231.25 m **(k)** 34% of 210.4 L **(l)** 4% of $32.65

2 Write the following as percentages. Round your answers to one decimal place, if necessary. WE22

(a) 17 out of 20 on a spelling quiz.

(b) 5 out of 22 students in the class have blond hair.

(c) There are 100 kittens at the animal shelter, 40 of which are female.

(d) 4 out of 5 students surveyed own an MP3 player.

(e) 50 pairs of shoes were sold, of which 38 pairs were black.

(f) 54 out of 70 cars counted on a freeway had only 1 person in them.

(g) 15 of the 27 cars in the staff carpark are white or silver.

(h) 3 out of every 4 people on the bus said they were travelling to work.

(i) 28 out of 34 people surveyed said they were non-smokers.

3 Calculate the following using the 'basic percentages method' or any other mental strategy.

(a) 50% of $25 **(b)** 10% of 75 kg **(c)** 25% of 60 km

(d) 20% of 40 L **(e)** 1% of 360 m **(f)** 5% of $120

(g) 60% of 350 kg **(h)** 75% of 800 mL **(i)** 35% of $400

4 Mehmet successfully shot 13 basketball goals out of 18 attempts. The calculation that converts this result to a percentage would be set out as follows.

A $\frac{13}{100} \times \frac{18}{1}$ **B** $\frac{13}{18} \times \frac{100}{1}$ **C** $\frac{18}{13} \times \frac{100}{1}$ **D** $\frac{100}{13} \times \frac{18}{1}$

5 20% of $420 is:

A $8.40 **B** $20 **C** $84 **D** $200

6 40% of the 25 students in class 7A have black hair. How many students is this?

7 16 out of the 28 biscuits in a packet have cream centres. What percentage of the biscuits in the packet are cream biscuits?

8 A 2 litre bottle of orange drink contains 35% orange juice. Work out how many litres of orange juice is contained in the bottle. (It may help to work in mL.)

9 A lion can sleep for up to 20 hours a day. What percentage of a whole day is this?

10 There are 11 boys and 15 girls in class 7B. Write the number of boys and the number of girls as percentages of the total number of students in the class.

11 Damian kicked 7 goals from 10 attempts. Franco kicked 11 goals from 16 attempts.

(a) Express the boys' results as percentages.

(b) Who is the more accurate goalkicker?

12 Australia scored 381 runs in a cricket match against India. If Shane Watson scored 126 of those runs, what percentage of the team's total did he score?

13 Shops often uses percentages to advertise savings during sales. The percentage is used to calculate the amount that is removed from the price of the goods. This amount is known as the 'discount'.

(a) Calculate the dollar value of the discount on the following.

(i) \$40 shirt, 20% discount

(ii) \$125 jacket, 25% discount

(iii) \$65 jeans, 50% discount

(iv) \$239 suitcase, 10% discount

(v) \$799 camera, 40% discount

(vi) \$1299 sofa, 5% discount

(b) Now, find the sale price of the above items by subtracting the discount amount from the original price.

Reasoning, communicating and open-ended

14 8% of the apples in a crate were rotten. What percentage of the apples were not rotten?

15 The 180 students at St John's College all completed an online survey. The results showed that:

22% of the students had blue eyes

34% spoke two or more languages fluently

7% of them were left handed.

(a) Calculate the number of students in each category using the above three percentages.

(b) Considering that we are dealing with numbers of students, what is strange about your answer to **(a)**?

(c) What might be the actual numbers of students represented by each of the percentages?

16 Chelsea and Sonia are netballers. In one game, Chelsea scored 17 goals from 25 attempts, and Sonia scored 15 goals from 20 attempts. In the next game, Chelsea scored 19 goals from 27 attempts, and Sonia scored 17 goals from 23 attempts. Use percentages to show which one of the girls improved the accuracy of her goal shooting. Round your figures to one decimal place, if necessary.

17 The energy in food is measured in kilojoules (kJ). The National Heart Foundation of Australia recommends that no more than 30% of our daily energy intake should come from eating fats.

(a) A healthy, active woman should eat up to 9000 kJ of food a day. Up to how many kJ in the form of fats can she eat in 1 day?

(b) 1 gram of fat gives the body 37 kJ of energy. How many grams of fat would provide a woman with the maximum daily amount of kilojoules allowed from fat? Round your answer to the nearest whole number.

18 Suggest some reasons as to why some stores use percentages to advertise savings, rather than dollar amounts.

19 Mark saved \$5.00 when a store had a sale. Determine at least three different percentage discounts and original prices for the item that Mark purchased.

20 Fedora and Tanya were asked to show their working to the following question.
'Calculate 15% of 80'.

Here is Tanya's working:

$$\frac{15}{80} \times \frac{100}{1}$$
$$= \frac{150}{8}$$
$$= 18.75$$

Here is Fedora's working:

$$15\% \text{ of } 80$$
$$= 0.15 \times 80$$
$$= 12$$

Which of the girls' working is correct? Give a piece of advice to the other girl to help her avoid similar mistakes in the future.

Problem solving

The great half-price sale

Tricia and Carly were out shopping. They headed towards their favourite clothes shop. On the shop window was a sign that read: 'Take 50% off all marked prices!' Stuck over the top of that sign was another sign, saying: 'Take another 50% off reduced prices!'

'Eh?' said Carly. '50% is one half. If I take half off the price, then take off another half, there won't be any price left! That means everything in the shop is free!'

'I don't think so', said Tricia. 'How can you run a business like that?'

'Well', said Carly, 'If I pick up a pair of $100 jeans and take two lots of 50% off the price, what do they cost now?'

'Errr...', said Tricia.

Can you work it out? How much would the jeans cost?

Strategy options

- Draw a diagram.
- Break problem into manageable parts.

4.9 Ratio

Need to Know

Ratios

A **ratio** is a comparison of two or more amounts of the same type of quantity, written using a ':' symbol.

For example: 2 lengths can be compared.

The order in which a ratio is stated must be made clear. If the ratio of teachers to students in a class is 1 : 20, then the ratio of students to teachers is 20 : 1.

A **part : part ratio** compares separate parts of a whole.

A **part : whole ratio** compares a part (or parts) to a whole.

Ratios are written without any units.

Ratios, fractions and percentages

Ratios can be written as fractions or percentages.

For example, if 11 : 20 is a part : whole ratio, it can be written as $\frac{11}{20}$ or 55%.

Equivalent ratios

Equivalent ratios are created by multiplying or dividing each part in the ratio by the same number.

For example, 1 : 4 = 50 : 200.

Simplifying ratios

Ratios can be simplified by dividing each part in the ratio by a common factor. Dividing by the highest common factor (HCF) will give the ratio in simplest form.

For example, 60 : 300 = 1 : 5.

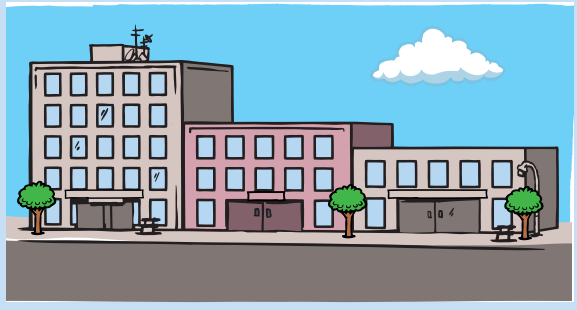
Worked Example 23

WE 23

Write a ratio for each of the following.

(a) the number of windows in the middle building compared to the number of windows in the smallest building

(b) the number of floors in the buildings, from left to right



Working

(a) Windows in middle building: 12
Windows in smallest building: 7
12 : 7

(b) 5 floors to 3 floors to 2 floors
5 : 3 : 2

Worked Example 24 WE24

A large enclosure at the zoo contains 5 giraffes and 12 zebras.

(a) Write a ratio comparing the number of zebras to the number of giraffes.

(b) Write a ratio comparing the number of giraffes to the total number of animals.

(c) Write the number of giraffes as a fraction of the total number of animals.

(d) Write the number of zebras as a percentage of the total number of animals, correct to the nearest whole number.

Working

(a) zebras : giraffes
= 12 : 5

(b) 5 + 12 = 17
giraffes : total animals
= 5 : 17

(c) $\frac{5}{17}$

(d) zebras : total animals
= 12 : 17

$\frac{12}{17} \times \frac{100}{1}$ %
= 71% (to nearest whole number)

4.9 Ratio

Answers page 466

Navigator

Q1, Q2, Q3 Columns 1 & 2, Q4 Columns 1 & 2, Q5, Q6, Q7, Q9, Q11, Q14, Q15, Q17, Q18, Q19	Q1 Column 2, Q2, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5, Q7, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q17, Q18, Q19	Q1 Column 2, Q2, Q3 Column 4, Q4 Column 4, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19

Fluency and understanding

1 Write a ratio for each of the following. WE23

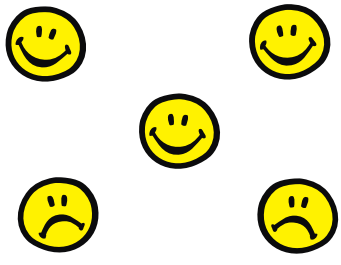
(a) circles to squares



(b) blue circles to red circles



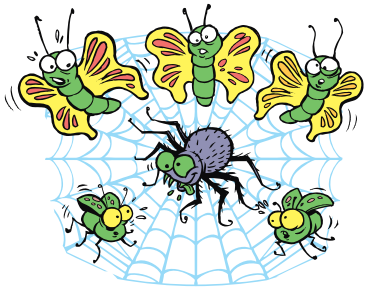
(c) happy faces to sad faces



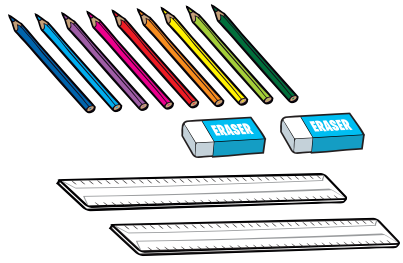
(d) ducks to swans



(e) spider to flies to butterflies



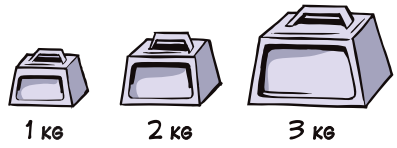
(f) rulers to pencils to erasers



(g) bees to flowers to leaves



(h) large mass to medium mass to small mass



WE24

2 Sumi has 3 canaries and 2 mice as pets.

(a) Write a ratio comparing the number of mice to the number of canaries.

(b) Write a ratio comparing the number of canaries to the total number of pets.

(c) Write the number of canaries as a fraction of the total number of pets.

(d) Write the number of mice as a percentage of the total number of pets.

3 Create pairs of equivalent ratios by multiplying or dividing each part in the ratio by the number given in brackets beside it.

(a) 1 : 3 (× 2)	**(b)** 1 : 8 (× 2)	**(c)** 1 : 11 (× 2)	**(d)** 1 : 33 (× 2)
(e) 4 : 5 (× 3)	**(f)** 3 : 7 (× 4)	**(g)** 6 : 9 (÷ 3)	**(h)** 8 : 12 (÷ 4)
(i) 10 : 8 (÷ 2)	**(j)** 18 : 9 (÷ 9)	**(k)** 8 : 5 (× 7)	**(l)** 12 : 11 (× 5)
(m) 48 : 18 (÷ 6)	**(n)** 20 : 6 (× 5)	**(o)** 60 : 54 (÷ 3)	**(p)** 15 : 13 (× 5)

4 Write the following ratios in simplest form, by dividing by the highest common factor (HCF).

(a) 2 : 10	**(b)** 3 : 12	**(c)** 4 : 18	**(d)** 6 : 21
(e) 21 : 14	**(f)** 24 : 15	**(g)** 32 : 20	**(h)** 48 : 36
(i) 54 : 64	**(j)** 56 : 68	**(k)** 72 : 81	**(l)** 38 : 95
(m) 75 : 50	**(n)** 120 : 80	**(o)** 105 : 60	**(p)** 240 : 66

5 There are 14 girls and 9 boys in a Year 7 maths class.

(a) The ratio of boys to girls is:

A 14 : 9 **B** 9 : 14 **C** 14 : 23 **D** 9 : 23

(b) The fraction of the whole class that are girls is:

A $\frac{9}{23}$ **B** $\frac{9}{14}$ **C** $\frac{14}{23}$ **D** $\frac{14}{9}$

6 A packet of M&M's® contains three colours: 12 red, 7 blue and 16 brown.

(a) Write the ratio of red to blue to brown M&M's.

(b) How many M&M's are in the packet?

(c) What fraction of the M&M's in the packet are red?

(d) What percentage of the M&M's are blue?

7 During a cricket season, Erika's team won 15 matches, drew 2 and lost 7.

(a) Write the won : lost : drew ratio for Erika's team for that season.

(b) What fraction of their games did Erika's team lose?

(c) Write your answer to **(b)** as a percentage, rounded to the nearest whole number.

8 The students at Hillview High School are voting for a school captain. The results show that for every 4 students who voted for Guy, 5 students voted for Dominic and 3 for Melissa.

(a) Write the ratio of the number who voted for Dominic to the number who voted for Guy, to the number who voted for Melissa.

(b) What fraction of the total vote did Dominic receive?

(c) Write your answer to **(b)** as a percentage, rounded to the nearest whole number.

9 There must be 1 instructor for every 6 students on a kayaking course.

(a) Write the comparison of students to instructors as a ratio.

(b) If 24 students want to do the course, use an equivalent ratio to determine how many instructors will be required.

(c) If 5 instructors are available, what is the maximum number of students that can do the course?

10 Tom's recipe for making concrete is to add gravel, sand and cement in the ratio 3 : 2 : 1 before adding water. Tom has 4 buckets of cement. Use an equivalent ratio to show how many buckets of sand and gravel he should add to get the correct concrete mixture.

11 'White gold' is commonly made by mixing gold and nickel in the ratio 9 : 1.

(a) Write the amount of nickel as a percentage of the total mixture.

(b) A jeweller wants to make 20 g of white gold. How much nickel will he require?

12 A survey found that 75% of Year 7 students were studying music.

(a) What percentage were not studying music?

(b) Use the two percentages to write a ratio that compares the group studying music to the group not studying music, in simplest form.

Reasoning, communicating and open-ended

13



Which point divides the above line so that the ratio shorter part : longer part is:

(a) 1 : 7 **(b)** 5 : 3 **(c)** 1 : 3 **(d)** 1 : 1

14 There must be at least 1 teacher for every 20 students on a school excursion.

(a) If 68 students are going on an excursion, how many teachers are required? Give reasons for your answer.

(b) Use your answer to **(a)** to write the actual ratio of teachers : students, in simplest form.

15 Chloe made a cordial drink by mixing 30 mL of concentrate with 150 mL of water. Her friend Hannah made a cordial drink by mixing 80 mL of concentrate with 200 mL of water. Hannah said her cordial drink was 'stronger' than Chloe's. Chloe could not tell by looking.

(a) Write the ratio of concentrate : water for each girl's drink, in simplest form.

(b) Use the ratios to state whether Hannah is correct, giving a reason for your answer.

16 Mia has three cats and five dogs as pets.

(a) What fraction of her pets are dogs?

(b) What fraction of her pets are cats?

(c) If Mia adopts another cat, what fraction of her pets are cats now?

(d) After adopting the new cat, Mia moves into a share house with Shona who also has some cats and dogs. If the final ratio of cats to dogs is now 1 : 1, what is the smallest number of cats and dogs that Shona brings to the house?

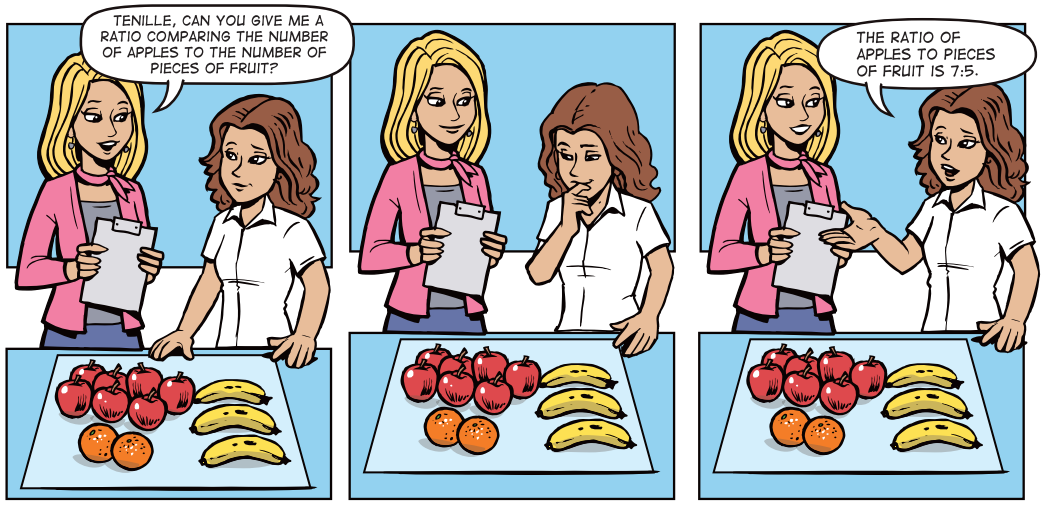
17 **(a)** What is the ratio of girls to boys in your class?

(b) Would the ratio be the same in all the Year 7 classes in your school? Explain your answer by giving examples.

(c) Would the ratio be the same for all school classes in Australia? Give some reasons for your answer.

18 Write three ratios that are equivalent to 4 : 7.

19



What mistake has Tenille made? What is the correct answer?

Rates

4.10

Need to Know

A **rate** is a way of comparing two amounts of different types of quantities. A rate is written as the amount of the first quantity for every 1 unit of the second quantity.

For example: Speed is the distance travelled over a unit of time.

Kilometres per hour (km/h) or metres per second (m/s) are the most common metric units used for speed, while miles per hour or feet per second are used in the imperial system.

The unitary method

Solving problems by first calculating a number (such as cost) per unit is called the **unitary method**.

The unitary method can be used to compare prices.

For example, \$7.21 for an 820 g box of cereal compared to \$5.99 for a 500 g box. Supermarkets are required to show the price per 100 g (or 100 mL or 1 kg or 1 L) for many products. This is called the **unit price**.

The unit price for the 820 g box $= 7.21 \div 820 \times 100$

$= \$0.88$ per 100 g

The unit price for the 500 g box $= 5.99 \div 500 \times 100$

$= \$1.20$ per 100 g

Worked Example 25

WE25

(a) If 8 kg of oranges costs \$24, find the cost of 6 kg of oranges.

(b) A machine in a soft drink factory produces 210 bottles every hour. How many bottles would be produced in 40 minutes?

Working

(a) \$24 for 8 kg
= \$3 for 1 kg (÷ 8)
= \$18 for 6 kg (× 6)

(b) 210 ÷ 60 = 3.5 bottles/min
3.5 bottles/min × 40 min
= 140 bottles in 40 min

Worked Example 26 WE26

Calculate the unit price (price per 100 g) of the following pair of products, and so determine which one is better value for money: A 500 g packet of pasta for $2.42 or a 750 g packet for $3.44.

Working

Number of units $= \frac{500}{100}$
$= 5$

Unit price $= \frac{\$2.42}{5}$
$= \$0.48/100\,g$

Number of units $= \frac{750}{100}$
$= 7.5$

Unit price $= \frac{\$3.44}{7.5}$
$= \$0.46/100\,g$

The 750 g packet is slightly better value for money (2c less per 100 g).

Answers page 467

4.10 Rates

Navigator

Q1, Q2, Q3, Q7, Q8, Q9, Q12, Q13, Q14, Q15, Q16, Q19	Q1, Q2, Q3, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q20	Q1 (d)–(f), Q2 (d)–(f), Q4, Q5, Q6, Q8, Q10, Q11, Q12, Q14, Q15, Q17, Q18, Q19, Q20

Fluency and understanding

WE25

1 **(a)** If 15 kg of potatoes costs $12, find the cost of 7 kg of potatoes.

(b) A machine in a food canning factory produces 250 cans every hour. How many cans would be produced in 25 minutes?

(c) If 14 pens cost $5.60, find the cost of 3 pens.

(d) Farmer Harry buys 8 cows for $6800. How much would 10 cows have cost him?

(e) If 21 bags of rice weigh 35 kg, how much do 15 bags of rice weigh?

(f) A printing machine prints 70 pages in 8 minutes. How many pages could it print in one hour?

WE26

2 Calculate the unit price (the price per 100 g) of the following pairs of products, and so determine which one is better value for money.

(a) a 600 g jar of jam for $4.09 or a 250 g jar for $2.68

(b) a 500 g packet of pasta for $2.81 or a 750 g packet for $3.45

(c) a 375 g jar of tomato paste for $1.28 or a 500 g jar for $3.25

(d) a 200 g jar of peanut butter for $3.21 or a 375 g jar for $4.93

(e) a 500 mL bottle of soy sauce for $2.71 or a 920 mL bottle for $4.29

(f) a 450 g bag of cat food for $3.78 or a 1 kg bag for $8.79

3 Nazeh has saved $2000 in 4 months. If he continues to save at this rate, how much will he have by the end of the year?

A $6000 **B** $8000 **C** $12 000 **D** $24 000

4 An entertainer receives $450 000 for a season of five performances, each of which lasts $1\frac{1}{2}$ hours. The rate of pay per minute is closest to:

A $60 000 **B** $5000 **C** $1500 **D** $1000

5 A caterer knows that 4 loaves of bread can make enough sandwiches for 25 people. How many loaves will he need to make enough sandwiches for 120 people?

6 **(a)** Which is better value for money, a 280 g jar of jam for $5.15, or a 400 g jar for $6.42?

(b) The 280 g jar goes on special for $4.49. Does this change your answer to **(a)**?

7 A mobile phone carrier charges 75c for 30 seconds. How much will a 4-minute phone call cost?

8 Rajid's cricket team required 85 runs from 12 overs to win the game. At what rate, in runs per over, must they score to win? (Answer to one decimal place.)

9 **(a)** Calculate the price per item in each pack of the following products, and so determine which pack is better value for money.

(i) a packet of 6 muesli bars for $4.83, or a packet of 10 for $7.51

(ii) a 12-pack of cans of soft drink for $14.20, an 18-pack for $20.21, or a 30-pack for $27.51

(iii) a box of 50 tea bags for $3.20, or a box of 200 for $7.99

(b) Describe another way you could compare the prices of the tea bags in part **(a) (iii)**.

Hint

200 is a multiple of 50.

10 Lucy has bought a 1-year subscription to *Australian Explorer* magazine. She paid $102 for 12 monthly magazines. The regular price at the newsagent for one copy of the magazine is $9.95. How much will Lucy save:

(a) per magazine **(b)** over the 1-year period?

11 Andrea has a leaky tap in her bathroom. She places a container underneath it and collects 1.2 litres in 8 hours. If Andrea does not fix the tap, use a mental or written method to calculate how much water would be lost in:

(a) 1 day **(b)** 1 week?

12 A 200 g serve of yoghurt contains 6.5 g of fat. Use a mental or written method to calculate how much fat there is in:

(a) a 100 g serve **(b)** a 400 g serve **(c)** a 500 g serve.

13 A 5-minute phone call costs $3.00. If the same call rate is charged, use a mental or written method to calculate the cost of:

(a) a 10-minute call **(b)** a 1-minute call **(c)** a 13-minute call.

Reasoning, communicating and open-ended

14 Charlie's bakery sells hot cross buns in packets of 6. One packet costs $7.20. A customer wants to buy exactly 10 buns. Charlie is happy to take 4 buns from a second packet, but how much should he charge the customer?

15 Imagine that you are standing in a supermarket, comparing the following pairs of products. Do some mental calculations to determine which product is the better value for money. Rounding prices slightly may help to simplify some calculations. Show how you arrived at your answer.

(a) A 1 kg box of washing powder for $5.49, or a 4 kg box for $15.97.

(b) Four 250 mL cartons of orange juice for $1.25 each, or a 1 L carton for $3.99.

(c) Buying 500 g of ham from the deli counter where the price is $13 per kg, or buying it in a packet for $5.83.

(d) A 400 g jar of honey for $7.99, or a 300 g jar for $6.54.

(e) Give some reasons why people might prefer to buy a product even though it is not the best value for money.

16 A mobile phone carrier charges a rate of 85 cents per minute for pre-paid calls. Calls made on a phone plan are charged at 75 cents per minute, plus a 35 cent connection fee. Which of the two types of charge is cheaper for a 5-minute phone call?

17 During the month of January, the water level in a Northern Territory dam increased by 78 000 kilolitres (1 kilolitre = 1000 litres).

(a) How many kilolitres per day was this? (Round your answer to two decimal places.)

(b) If the dam continued to fill at this rate, use your answer to **(a)** to calculate how much water will have flowed into the dam by the end of the year. (Assume there are 365 days in a year.)

(c) Would it be sensible to use your answer to **(b)** as a prediction for the amount of water that will flow into the dam over the year? Give some reasons for your answer.

18 'Trim and Tasty' cheese comes in three different-sized blocks:

250 g for $4.50
500 g for $7.42
750 g for $9.76

(a) How many times bigger than the 250 g block is the 500 g block?

(b) If the prices of the 500 g block and the 750 g block were in the same proportion to the price of the 250 g block as their sizes, what would you expect their prices to be?

(c) Often (but not always) larger sizes of a product have cheaper unit prices. Suggest a reason why this is the case.

19 List four supermarket products whose unit price would be usefully calculated per 100 mL, or per L.

20 Kumar needs 4.5 kg of rice for the students who are attending his cooking classes. It is available in the following sizes and prices.

500 g for $1.24 1 kg for $2.35 1.5 kg for $3.59 2 kg for $4.52

(a) List at least three different combinations of sizes that would give Kumar 4.5 kg of rice.

(b) Which combination is the cheapest?

Challenge 4



1 What percentage of whole numbers from 6 to 25 inclusive (i.e. including 6 and 25) are exact multiples of 6?

A 5 **B** 20 **C** 25 **D** 30

2 The numbers 5, 8, 7 and 3 are written on cards. Where should they be placed so that the following product has the greatest possible value?

$\square \times 0.\square\square\square$

3 Evaluate $0.1^2 - 0.1^3$.

4 Zinadene has $\frac{3}{4}$ of a dollar and Katrine has $\frac{3}{10}$ of a dollar. Together they have:

A \$0.95 **B** \$1.00 **C** \$1.05 **D** \$1.10

5 $200 \div 2.5 = 80$. Therefore, $20 \div 0.25 =$

A 0.80 **B** 8.0 **C** 80 **D** 800

6 When doing a series of additions using his calculator, Trent noticed that he added 49 095 instead of 49.95. In order to correct his error with a single entry, he should now:

A add 49.95 **B** subtract 49 045.05

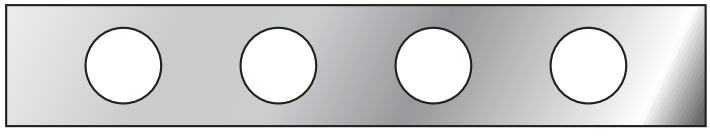
C add 49 045.05 **D** subtract 49 095

7 A decimal with 2 decimal places is multiplied by a decimal with 3 decimal places. Explain with number examples how the solution could have:

(a) 5 decimal places **(b)** 4 decimal places **(c)** 3 decimal places.

8 In Felicity's first six netball games she averaged 7.5 goals per game. If the least number of goals she scored was 4, what is the lowest possible value for the highest number of goals she scored?

9 Four holes are to be drilled along the centre line of a strip of metal so that their centres are 2.25 cm apart. The centres of the two end holes are to be 3.35 cm from their corresponding ends. What is the length of the strip of metal?



10 What is the quotient when 0.01 is divided by 0.002?

11 Mikaela had her salary reduced by 10%. She was later promoted and her salary was increased by 10%. If her original salary was \$30 000, her present salary is:

A \$24 300 **B** \$27 000 **C** \$29 700 **D** \$30 000

12 $\frac{2009}{20.09}$ equals

A 100 **B** 10 **C** 0.1 **D** 0.01

13 A town has 5000 residents, of whom 60% voted in a local council election. The result was that, of those who voted, 37% voted for *P*, 32% for *Q* and 31% for *R*. Under the voting system, *P* was elected. The number of residents who voted for *P* was:

A 1850 **B** 1110 **C** 960 **D** 930

Chapter review 4

D.I.Y. Summary

Key Words

decimal places	per cent	round
decimal point	place value	terminating decimal
digit	rate	unit price
equivalent ratios	ratio	unitary method
part : part ratio	rational numbers	
part : whole ratio	recurring decimal	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 4.87 has three _____s and two _____ .

2 When adding or subtracting decimals, it is important to line up the _____s.

3 _____ are created by multiplying each part in the ratio by the same number.

4 _____s compare two or more parts of a whole.

5 A _____ has a finite number of decimal places, a _____ has an infinite number of decimals, in a repeating pattern.

6 _____ literally means 'for every hundred'.

7 Calculating a _____ helps to compare the prices of different sizes and brands of products.

8 Explain the difference between 'round down' and 'round up'.

Fluency and understanding

1 Write in expanded fractional form. Ex 4.1

(a) 0.968 **(b)** 5.0702 **(c)** 6.005

2 Write as decimals: Ex 4.1

(a) six tenths, five hundredths, two thousandths and four ten-thousandths

(b) eight hundredths, nine hundred-thousandths and three millionths

3 Write as decimals: Ex 4.1

(a) $4 + \frac{3}{10} + \frac{8}{100} + \frac{6}{10000}$ **(b)** $50 + 7 + \frac{3}{100} + \frac{2}{1000} + \frac{9}{100\,000}$

4 Write each of the following in expanded word form. Ex 4.1

(a) 1.8531 **(b)** 0.070 06 **(c)** 61.0009

5 Write the value of the 9 in each decimal as **(i)** a fraction **(ii)** in words. Ex 4.1

(a) 5.091 **(b)** 0.0659 **(c)** 25.291

6 Write < or > in each of the following pairs of numbers to make a true statement. Ex 4.1

(a) 3.0427 _____ 3.0274 **(b)** 0.009 95 _____ 0.01

7 Write in order from smallest to largest. Ex 4.1

(a) 0.6055, 0.5506, 0.607 **(b)** 0.071, 0.701, 0.71

8 Copy this number line and show the positions of the following numbers: 0.25, 1.45, 0.09, 2.05. Ex.4.1



9 Round each of the following to the number of decimal places shown in the brackets. Ex.4.2

(a) 4.398 (1) **(b)** 34.6503 (2) **(c)** 23.098 (1) **(d)** 102.4728 (2)

10 Round the following to the nearest 5 cents. Ex.4.2

(a) \$41.21 **(b)** \$10.03 **(c)** \$79.97 **(d)** \$98.98

11 Write each of the following as a single fraction or mixed number in simplest form. Ex.4.3

(a) 3.9 **(b)** 0.62 **(c)** 2.45 **(d)** 0.0018

12 Write the following fractions as decimals, using the correct notation for recurring decimals if necessary. Ex.4.3

(a) $\frac{4}{5}$ **(b)** $\frac{19}{20}$ **(c)** $\frac{2}{9}$ **(d)** $\frac{10}{12}$

13 Calculate: Ex.4.4

(a) 23.6 + 4.79 **(b)** 3.768 + 10.9205 + 0.77 **(c)** 6 + 0.408 + 35.025

(d) 10.367 – 2.65 **(e)** 2.082 – 0.394 **(f)** 7 – 2.2198

14 Find the following products. Ex.4.5

(a) 3.376×4 **(b)** 2.99×35 **(c)** 6.09×33

(d) 23.6×2000 **(e)** 0.548×300 **(f)** 0.069×50

(g) 0.6×0.9 **(h)** 0.009×0.04 **(i)** 5.2×3.8

15 Calculate the following. Round answers to three decimal places where necessary. Ex.4.6

(a) 10.32 ÷ 4 **(b)** 7.028 ÷ 7 **(c)** 3.75 ÷ 8

(d) 480.6 ÷ 2000 **(e)** 8.23 ÷ 500 **(f)** 54.63 ÷ 300

(g) 0.48 ÷ 0.04 **(h)** 1.2 ÷ 0.5 **(i)** 11.9 ÷ 0.002

16 Write the following fractions as percentages. Ex.4.7

(a) $\frac{7}{10}$ **(b)** $\frac{5}{4}$ **(c)** $\frac{1}{8}$ **(d)** $\frac{2}{3}$

17 Convert the following decimals to percentages. Ex.4.7

(a) 0.38 **(b)** 0.96 **(c)** 3.55 **(d)** 0.09 **(e)** 3.234 **(f)** 0.5432

18 Calculate the following percentages. Ex.4.8

(a) 40% of \$60 **(b)** 15% of 85 L **(c)** 83% of 200 m **(d)** 12.5% of \$600

19 Write the following results as percentages, rounding answers to one decimal place if necessary. Ex.4.8

(a) 13 out of 20 **(b)** 35 out of 70

(c) 14 out of 40 **(d)** 28 out of 30

20 There are 56 girls and 64 boys in Year 7. Ex.4.9

(a) Write the ratio boys : girls in simplest form.

(b) Write the number of girls as a fraction of the total number of students, in simplest form.

(c) Write the number of boys as a percentage of the total number of students, to the nearest whole number.

Ex 4.10

21 **(a)** A 10 kg box of tomatoes costs $23.50. Use the unitary method to calculate the cost of a 3 kg box.

(b) If a 250 g block of cheese costs $7.85, find the unit price (the price per 100 g).

22 Write the populations of the following countries as whole numbers.

(a) China: 1.326 billion

(b) India: 1.14 billion

(c) Indonesia: 228.2 million

(d) New Zealand: 4.27 million

23 An employer wants to buy each of her staff a uniform. If each uniform costs $75.59, how much will the uniforms cost:

(a) for 40 staff

(b) for 100 staff?

24 Kayla bought the following items while out shopping: A $30 T-shirt discounted by 10%, and a $60 pair of jeans discounted by 40%.

(a) Calculate the dollar value of each of the discounts.

(b) Subtract the discount amount to find how much Kayla paid for each item.

25 **(a)** Which is better value for money: a 250 g jar of honey for $2.98, or a 400 g jar for $4.23?

(b) The 250 g jar goes on special for $1.98. Does this change your answer to **(a)**?

(c) Give one or two reasons why people might prefer to buy a certain size or product, even though it may not have the cheapest unit price.

26 A salad dressing is made by mixing oil and vinegar in the ratio 3 : 1.

(a) How much oil should be mixed with 25 mL of vinegar?

(b) Helen mixes 60 mL of oil with 40 mL of vinegar. Is this an equivalent ratio to the one given above? Demonstrate your answer by writing it in simplest form.

27 In a springboard diving competition, Claudia needed a score of 52 or more to take first place. For her final dive she chose a dive with a degree of difficulty of 2.7. The scores she received from the judges were 7.0, 8.0, 7.5, 8.5, 7.5, 8.0 and 8.0.

(a) Remove the highest and lowest scores and calculate the average of the remaining five scores (add them up and then divide by 5).

(b) Calculate the 'three-judge total' by multiplying this average by 3.

(c) Calculate her final score by multiplying the 'three-judge total' by the degree of difficulty. Did she do well enough to win?

Reasoning and communicating

28 Tim buys 6 individual cans of drink for $0.87 per can.

(a) If he pays in cash, find the total cost of the cans.

(b) A pack of 6 cans costs $4.80. How much could Tim have saved by buying the 6 pack?

(c) What is the individual cost of the cans in the 6 pack?

(d) Tim needs to buy 40 cans for a party. What is the cheapest way to buy them?

29 There must be at least one supervisor for every 12 children at a childcare facility. If there are 54 children booked in for a day, how many supervisors will be needed? Give a reason for your answer.

30 Sally is shopping for shoes. She has found the pair she wants in two different stores. Store A has them priced at $65, with a 'take 25% off' tag on them. Store B has them priced at $75, but the store is having a '30% off everything' sale. Where should Sally buy her shoes? Explain your choice by showing the relevant calculations.

NAPLAN practice 4

Numeracy: Non-calculator

1 Which number is greater than 0.06?

A 0.0069 **B** 0.007 **C** 0.05 **D** 0.1

2 Geraldine buys 400 snack bars for the school camp. If each bar costs $0.64, how much would Geraldine pay for the 400 bars?

A $25.60 **B** $256 **C** $464 **D** $25 600

3 What is the answer to 5.4 ÷ 0.9?

A 0.06 **B** 0.6 **C** 6.6 **D** 6

4 20% of a class have the flu. The class has 25 students. How many have the flu?

A 4 **B** 5 **C** 10 **D** 20

Numeracy: Calculator allowed

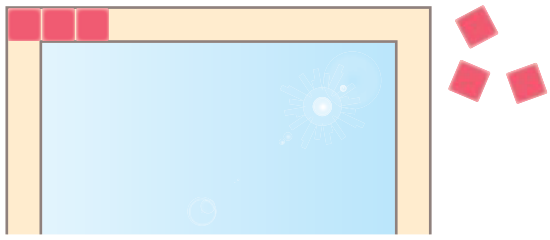
5 A pack of 10 lollipops costs $4.60.
A pack of 6 costs $3.20.
You need to buy 22 lollipops.

What is the least amount you can pay?

6 The price of grapes at the supermarket is $14 per kg. If Jack paid $3.50 for his grapes, how many grams did he buy?

A 0.25 g **B** 25 g **C** 100 g **D** 250 g

7 Lucy is adding a row of square tiles across the top of a mirror. Each tile is 4.5 cm wide. The mirror is 56 cm wide. Lucy wants to tile right across the top, with no gaps between the tiles. What calculation should she do to work out how many tiles she needs?



A 4.5 ÷ 56 **B** 5.6 ÷ 4.5 **C** 56 ÷ 4.5 **D** 4.5 × 56

8 The material for the curtains in the classroom costs $21.30 per metre. 8.4 metres is needed. How much will the material cost?

Mixed review B

Fluency and understanding

1 Write the following fractions in simplest form. Ex 3.2

(a) $\frac{12}{40}$ **(b)** $\frac{15}{50}$ **(c)** $\frac{48}{64}$ **(d)** $\frac{20}{65}$

2 Write a positive or a negative integer to describe the following. Ex 2.4

(a) a withdrawal from a bank account of $74 **(b)** a weight gain of 3 kg

(c) 215 m below sea level **(d)** an increase in length of 8 m

3 For each of the following numbers: Ex 2.3

(i) use a factor tree to find the prime factors

(ii) write the number as a product of its prime factors, using index notation.

(a) 76 **(b)** 108 **(c)** 250

4 Calculate approximate answers to the following by rounding to a convenient multiple of 5 or 10. Ex 1.4

(a) 238×21 **(b)** 107×31 **(c)** $478 \div 39$ **(d)** $604 \div 153$

5 For each pair of fractions, determine which one is larger, then write a > or < sign between them. Ex 3.3

(a) $\frac{2}{3}$ _____ $\frac{2}{5}$ **(b)** $\frac{5}{12}$ _____ $\frac{5}{9}$ **(c)** $\frac{11}{15}$ _____ $\frac{3}{4}$

6 Find: Ex 4.5, 4.6

(a) 0.45×0.6 **(b)** $45.9 \div 3$ **(c)** $43.5 \div 0.1$

7 Calculate the following by first writing a single sign between the integers. Ex 2.7

(a) $-1 + (-5)$ **(b)** $3 - (-11)$ **(c)** $-6 - (+4)$

(d) $9 + (-17)$ **(e)** $-8 - (-14)$ **(f)** $21 - (+30)$

8 Write each of the following as a fraction in simplest form. Ex 4.3

(a) 0.32 **(b)** 1.24 **(c)** 3.237

9 Write the following in index form. Ex 1.2

(a) $7 \times 7 \times 7 \times 7$ **(b)** 4 squared

(c) $2 \times 2 \times 3 \times 3 \times 3$ **(d)** 11 cubed

10 Find the lowest common multiples of the following numbers. Ex 2.1

(a) 5 and 6 **(b)** 3 and 6 **(c)** 2, 3 and 5

11 Write < or > between the following pairs of numbers. Ex 2.4

(a) -5 _____ 3 **(b)** -4 _____ -11 **(c)** -7 _____ 0

12 Find the HCF of the following numbers. Ex 2.1

(a) 12 and 30 **(b)** 16 and 40 **(c)** 24 and 60

13 Calculate the following. Write your answers in simplest form. Ex 3.4

(a) $\frac{1}{5} + \frac{3}{4}$ **(b)** $1\frac{2}{3} + 2\frac{3}{8}$ **(c)** $\frac{5}{6} - \frac{3}{8}$ **(d)** $3\frac{1}{4} - 2\frac{1}{2}$

14 Use an appropriate mental strategy to calculate the following.

(a) 29×7 **(b)** 44×11 **(c)** 13×15

(d) $260 \div 4$ **(e)** $2 \times 4 \times 7 \times 5$ **(f)** $440 \div 8$

15 Erin calculates that she needs 9 lots of 0.35 m of ribbon for her textiles project. She wants to round up to the nearest metre so that she makes sure she has enough.

(a) Calculate how many metres she should buy.

(b) If the ribbon costs \$0.75 per m, how much will she pay?

16 Write **(i)** a fraction in simplest form and **(ii)** a percentage to represent the following situations.

(a) 12 black jelly beans in a bag of 40

(b) 120 Year 7 students in a school of 520 students

17 A 250 g serve of tinned soup contains 7 g of fat. Calculate the amount of fat contained in:

(a) a 500 g serve **(b)** a 50 g serve **(c)** a 150 g serve.

18 7 pizzas were each sliced into 6 equal pieces.

(a) If $4\frac{5}{6}$ pizzas were eaten, how many slices is that?

(b) How many slices are left over?

19 Camilla is on an athlete's diet where $\frac{2}{3}$ of her daily energy must come from carbohydrates, $\frac{1}{4}$ from protein, and the rest from fats. Her daily energy allowance is 4800 kJ.

(a) Calculate how many kJ of carbohydrates she must have every day.

(b) Calculate how many kJ of protein she must have every day.

(c) What fraction of her daily kJ intake is fat?

20 Explain why $3^2 = 9$, but $3 \times 2 = 6$.

Reasoning and communicating

21 A lighting display in the city has a set of blue lights that flash every 6 minutes, and a set of red lights that flash every 8 minutes. How many times an hour will the lights flash together?

22 There is a large block of chocolate sitting in the Williams family pantry. Chloe breaks off and eats $\frac{1}{6}$, and returns the rest. Later, her brother Eamon breaks off and eats $\frac{2}{3}$ of the rest.

(a) What fraction of the original block is left?

(b) If the original block contained 54 squares of chocolate, who ate more, Chloe or Eamon?

23 Shania surveyed 130 Year 7 students to find out which charity they wanted to fundraise for. Shania reported back to the Student Council that 65% of the Year 7s wanted to support the Red Cross.

(a) What is 65% of 130?

(b) Considering that we are working with numbers of students, what is 'strange' about your answer to **(a)**?

(c) What might have been the actual number of students who said they wanted to support the Red Cross? Explain how Shania obtained her figure of 65%.

24 How many whole numbers lie on the number line between $\frac{4}{3}$ and $\frac{38}{4}$?

25 Jeffery has 46 footy cards in his collection. His little brother Joseph has half as many in his collection, whereas his friend Shahin's collection is twice the size of Jeffery's. How many more cards has Shahin than Joseph?

5 Algebra

Recall 5

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

1 Find the value of the ☐ in each of the following number sentences. If the ☐ appears more than once in a number sentence, the same number must be used.

(a) $3 + 7 = \square + 5$ **(b)** $5 - 1 = 2 \times \square$ **(c)** $3 + \square + 4 = \square \times 8$

(d) $5 - 4 + \square = 5$ **(e)** $10 \div \square = 5$ **(f)** $\square \times \square = 16$

2 **(a)** Multiply each of the following by 11 and subtract 3 from your answer.

(i) 4 **(ii)** 10 **(iii)** 200

(b) Multiply each of the following by itself and then take 6 from your answer.

(i) 4 **(ii)** 10 **(iii)** 8

3 Write the following sentences using only numbers, the four operation signs (+, –, ×, ÷) and the equals sign (=).

(a) The sum of eleven and seventeen is twenty-eight.

(b) The difference between nine and seven is two.

(c) The product of three and four is twelve.

(d) The quotient of sixteen and eight is two.

4 Write the following sentences using only numbers, the four operation signs and brackets. Then, use the order of operations to find the answer.

(a) Take the number six, multiply it by two, and then subtract four from it.

(b) Add five to the number ten, then divide what you get by five.

(c) Multiply the sum of six and five by three.

(d) The difference between twenty-three and twenty-one is multiplied by twelve.

5 Decode the following by writing the letters contained in each grid square.

(a) The fastest living creature
E2 E4 A1 E4 B3 A1 D3 C2 E4 A3 A4 A2 C4 D2 C2

(b) The largest land carnivore
E3 D2 D4 D3 A4 E3 B4 E4 A4 A1

	A	B	C	D	E
4	a	b	c	d	e
3	f	g	h	i	k
2	l	m	n	o	p
1	r	s	t	u	y

Syllabus references

Number and Algebra: Ratios and Rates

✓ Investigate, interpret and analyse graphs from authentic data **(ACMNA180)**

Number and Algebra: Algebraic Techniques

✓ Introduce the concept of variables as a way of representing numbers using letters **(ACMNA175)**

✓ Extend and apply the laws and properties of arithmetic to algebraic terms and expressions **(ACMNA177)**

✓ Simplify algebraic expressions involving the four operations **(ACMNA192)**

✓ Create algebraic expressions and evaluate them by substituting a given value for each variable **(ACMNA176)**

Number and Algebra: Equations

✓ Solve simple linear equations **(ACMNA179)**

Number and Algebra: Linear Relationships

✓ Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point **(ACMNA178)**

✓ Plot linear relationships on the Cartesian plane, with and without the use of digital technologies **(ACMNA193)**

Pronumerals and variables

5.1

Need to Know

Writing algebra

Pronumerals are letters or symbols we use for numbers we don't know.

An **unknown** is the actual number that the pronumeral represents.

A **variable** describes the unknown number if its value can change.

Modelling algebra

A packet of lollies may contain an unknown number of lollies.

The number of lollies can be represented by x.

x lollies



$x + 3$ lollies



$2x$ lollies



$2x + 3$ lollies



$3x + 9$ lollies



Algebra rules

- Addition and multiplication of pronumerals is commutative.

  For example, $a + b = b + a$ and $a \times b = b \times a$.

- Addition and multiplication of pronumerals is associative.

  For example, $(a + b) + c = a + (b + c)$ and $(a \times b) \times c = a \times (b \times c)$.

Using brackets

- Brackets show that addition or subtraction needs to be done before multiplication or division. They change the order of operations.

  For example, $2x + 3 \neq 2(x + 3)$

- Brackets can be left out when using a fraction bar.

  For example, $(a + b) \div 2$ can be written as $\frac{a + b}{2}$.

Other algebraic rules and conventions

- Leave out the '×' between a number and a pronumeral and between different pronumerals. For example, $2 \times n$ is written as $2n$ and $x \times y$ as xy.
- Replace ÷ with a fraction bar. For example, $n \div 2$ is written as $\frac{n}{2}$.
- When a multiplication sign is left out between a number and a pronumeral or between pronumerals, they act as if they were in brackets.

  For example, $a \div bc = a \div (b \times c) = \frac{a}{bc}$ but $a \div b \times c = \frac{ac}{b}$.
- Numbers are always written in front of the pronumeral.

  For example, $x \times 2$ is written as $2x$.
- When multiplying more than one pronumeral, write the letters in alphabetical order.

  For example, $f \times a \times c \times e$ is written as $acef$.
- When a pronumeral is raised to a power, use the same notation as for numbers.

  For example, $2 \times 2 \times 2 \times 2 = 2^4$ so write $c \times c \times c \times c = c^4$ and p squared as p^2.
- Numbers are usually written last. For example, $6a + 7$.

Worked Example 1

WE1

Write the following situations using algebra.

(a) Simon has x fish in his aquarium. He buys 10 more. How many fish does he have now?

(b) There are r cards in one pack. How many cards are there in three identical packs?

(c) Sonia has n dollars in her bank account. She withdraws $100. How much money is left in the account?

(d) There are d biscuits in a packet. Half of them are eaten. How many are left?

(e) A kitchen cupboard contains x plates and w bowls. What is the total number of plates and bowls in the cupboard?

Working

(a) Number of fish originally = x
Number of fish now = $x + 10$

(b) Number of cards in 1 pack = r
Number of cards in 3 packs = $3r$

(c) Amount of money in Sonia's bank account = $\$n$
Amount of money left in Sonia's account = $\$(n - 100)$

(d) Number of biscuits in a pack = d
Number of biscuits left = $\frac{d}{2}$

(e) Number of plates = x
Number of bowls = w
Number of plates and bowls = $x + w$

Worked Example 2

WE2

Write each of the following using algebra.

(a) The product of e, f and 7.

(b) c is divided by 4, then 1 is subtracted.

(c) The sum of x and 5 is multiplied by 3.

Working

(a) $7ef$

(b) $\frac{c}{4} - 1$

(c) $3(x + 5)$

5.1 Pronumerals and variables

Answers page 469

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Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q15	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q15, Q16	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16

Fluency and understanding

1 Write the following situations using algebra.

WE1

(a) There are p books in Jacob's locker. How many are left after Jacob takes out three for class?

(b) There are k people in the line for concert tickets. Nine more people join the end of the line. How many people are lined up now?

(c) There are v chocolates in one box. How many chocolates are there in 12 identical boxes?

(d) A group of four people won a prize of a dollars, which they shared equally. How much did each person receive?

(e) Kyle has d footy cards in his collection, while Khalid has e cards in his. If they combine their collections, how many cards do they have altogether?

WE2

2 Write each of the following using algebra:

(a) the sum of x and 2

(b) the product of 4 and y

(c) t is subtracted from 5

(d) the quotient when k is divided by 7

(e) the product of h, 6 and g

(f) d is multiplied by 10, then 8 is added

(g) 8 is added to d, and the result is multiplied by 10

(h) the sum of y and 9 is divided by 4

(i) 3 less than 5 lots of e

(j) the sum of r, s and t is divided by 3, then 11 is added

(k) y is multiplied by itself, then 20 is added

(l) x is squared, then multiplied by 8.

3 Nadine has 7 bags with x lollies in each bag, plus 9 extra lollies. The total number of lollies Nadine has altogether can be written using algebra as:

A $x + 7 + 9$ **B** $7x + 9$ **C** $9x + 7$ **D** $x^7 + 9$

4 The sum of z and 6 is divided by 2. When written using algebra it is:

A $z + 3$ **B** $\frac{z}{2} + 6$ **C** $z + 6 \div 2$ **D** $\frac{z + 6}{2}$

5 Write each of the following using algebra. Use brackets where necessary.

(a) 1 is added to a number, n, and the result is multiplied by 5.

(b) 2 is added to a number, d, and the result is divided by 9.

(c) The sum of two different numbers, a and b, is multiplied by 12.

(d) 3 is subtracted from the product of two different numbers, m and n.

6 Simplify the following expressions.

(a) $6a \times 2$ **(b)** $15y \div 3$ **(c)** $6cd \times 3c$

(d) $\frac{10m}{5}$ **(e)** $\frac{3p}{12}$ **(f)** $\frac{21k}{6}$

7 There are x people standing in line at the post office. Half of them leave, thinking they will come back later.

(a) How many people are in the line now?

(b) Three other people join the line. How many people are standing in the line now?

8 **(a)** What is the cost of d metres of timber, if it costs \$6 per metre?

(b) What is the cost of t kilograms of oranges, if they cost \$2.50 per kilogram?

9 What is the cost of hiring the community hall for x hours, if the rate is \$55 per hour, plus a one-off charge of \$300?

10 What is the cost of buying b movie tickets, at v dollars per ticket?

11 The cost of one desk is t dollars, and the cost of one chair is s dollars. If 12 new desks and 24 new chairs are bought for a classroom:

(a) what is the total cost of the desks

(b) what is the total cost of the chairs

(c) what is the total cost of the furniture?

Reasoning, communicating and open-ended

12 A purse contains n 5-cent coins and r 10-cent coins. There is no other money in it. Use algebra to write:

(a) the total value in cents of the 5-cent coins in the purse

(b) the total value in cents of the 10-cent coins in the purse

(c) the total value in cents of the money in the purse.

13 Emma makes and sells stationery gift packs. Her small pack has m pens, n pencils, q note pads and one eraser.

Her large pack has twice as many of each item. Use algebra to write:

(a) the total number of items in the small pack

(b) the total number of items in the large pack

(c) Emma needs to make up three small and two large packs. Use algebra to write the number of pens that she will need altogether.

14 (a) The length of a rectangle is 5 cm longer than its width. If w represents the width of the rectangle:

(i) write the length of the rectangle using w as the variable

(ii) draw a rectangle and label each side using w as the variable.

(b) Write, in terms of w, the total distance around the rectangle (the perimeter).

15 (a) Draw a diagram that could be represented by $3x + 5$.

(b) What kind of diagram could you draw to show $\frac{x}{4}$? Draw a diagram, adding any necessary descriptions or labels.

16 Rory's grandma says she is twice as old as the combined ages of Rory and his dad. Rory is r years old and his dad is d years old.

(a) Write Grandma's age using algebra.

(b) Write some possible ages for Rory, his dad and his grandma, if Rory is in primary school.

5.2 Terms, expressions and equations

Need to Know

The language of algebra

A **constant** is a number by itself. For example, 2, 9, -37, $\frac{2}{7}$, and 4.6 are all constants.

A **term** has one or more pronumerals or may be a constant by itself. For example, $5a$, $7q$, $3pr$, z, abc, 4 and $-2.3x$ are all terms.

A **coefficient** is a number written in front of a pronumeral. For example, the coefficient in $9x$ is 9.

Expressions are made by adding terms. An expression can consist of just one term. For example, $x + 3$, $2f$, $5a + 7q$ and $3d - 4k - 9$ are all expressions.

An **equation** contains an equal sign. It is made by writing two expressions equal to each other. The right-hand side (RHS) equals the left-hand side (LHS). For example, $9xy + z = 17$, $y = 6h$ and $2(p - 7) = p + 10$ are all examples of equations.

Consider $3xy - 7z^2 + 5 = 16 + x$.

- This is an equation because it has an equals sign.
- x, y and z are pronumerals that represent variables.
- 3 is the coefficient of xy and -7 is the coefficient of z^2.
- $3xy - 7z^2 + 5$ and $16 + x$ are expressions.
- $3xy$, $-7z^2$, 5, 16 and x are all terms.
- 5 and 16 are constants.

Worked Example 3

WE3

For $3p + 4q - 8pq = 7 + 2r$

(a) state whether it is an equation or an expression

(b) identify the coefficient of pq

(c) list all the variables

(d) write down any constants

(e) list all the terms used.

Working

(a) $3p + 4q - 8pq = 7 + 2r$ is an equation. It has an equals sign.

(b) Coefficient of pq is -8. (The negative sign is included.)

(c) p, q and r are the variables.

(d) The constant is 7.

(e) $3p$, $4q$, $-8pq$, 7 and $2r$ are the terms.

Worked Example 4 WE4

For the following situation:

(a) identify the variables

(b) define each variable using the given pronumerals

(c) write an equation to show the relationship between the variables.

A farmer has a number of sheep, x, and a number of ducks, y, in a paddock. There are five times as many sheep as ducks.

Working

(a) The variables are the number of sheep and the number of ducks.

(b) Let x = the number of sheep. Let y = the number of ducks.

(c) $x = 5y$.

5.2 Terms, expressions and equations

Answers page 470

Navigator

Q1, Q2, Q3, Q4 (a)–(d), Q5 (a) & (b), Q7	Q1, Q2, Q3, Q4 (b)–(e), Q5 (a)–(c), Q6, Q7	Q1, Q2, Q3, Q4 (c)–(f), Q5, Q6, Q7, Q8

Fluency and understanding

1 For each of the following: WE3

(i) state whether it is an equation or an expression

(ii) identify the coefficient of x

(iii) list all the variables

(iv) write down any constants

(v) list all the terms used.

(a) $2x + 3y - 7 + 4x^2$

(b) $32 - 4x = 3xy + 4z$

(c) $2xy - 3x^2 = 9 - 23x$

(d) $6 - x + 8y - 4xy$

2 For each of the following situations: WE4

(i) identify the variables

(ii) define each variable using the given pronumerals

(iii) write an equation to show the relationship between the variables.

(a) The number of boys, x, in the class is three times the number of girls, y.

(b) Fred is g years old. This is one less than three times James's age. James is h years old.

(c) At the market, an apple costs \$$m$ and a pear costs \$$n$. Two apples cost as much as three pears.

(d) A kilogram of potatoes costs \$$a$ and a watermelon costs \$$b$. Three kilograms of potatoes and a watermelon cost \$5.20.

(e) The distance, r, to Ranko's house is 5 km more than the distance, s, to Skye's house.

(f) You have v plants and w pavers. To make a path around a row of plants, you need to have twice as many pavers as you have plants plus six extra pavers.

3 Answer TRUE or FALSE for each of these statements.

(a) $6y$ is a term

(b) $7y - 9$ is an equation

(c) $y = 7x$ is a term

(d) ab is a term

(e) $r = 5t - 9$ is an expression

(f) $cd + 4$ is an expression

(g) $w = s - 5b$ is an equation

(h) $6g$ is an equation

(i) $rs = sr$

(j) $7 - 6xzy = 7 - 6yxz$

4 Which expression below matches the instruction given in each case?

(a) Choose a number and add any other number to it.

A $6 + a$ **B** $u + v$ **C** $c + 9$ **D** $a + b + 1$

(b) Choose a number and multiply it by any other number.

A $a - b$ **B** $7a$ **C** $4ab$ **D** ab

(c) Choose a number and multiply it by three, then subtract any other number.

A $3a - 1$ **B** $3n - y$ **C** $3m - 3$ **D** $3t$

(d) Choose a number and multiply it by two, add any other number, then subtract thirteen.

A $2f + 2d - 13$ **B** $2mn - 13$ **C** $2w + g - 13$ **D** $a + b - 13$

(e) Choose a number and add it to any other number, then multiply the answer by nine.

A $9xy$ **B** $9x + y$ **C** $9(x + y)$ **D** $9 + xy$

(f) Choose a number and multiply it by ten, choose any other number and multiply it by four, then add the two answers together.

A $w + v + 10 + 4$ **B** $14(v + w)$ **C** $10v + 4w$ **D** $40vw$

5 (a) A bottle of mass 120 g contains 25 tablets. The total mass of the bottle and tablets is 195 g.

(i) If the variable is the mass of a tablet, use a pronumeral to represent the variable.

(ii) Write an equation you would use to find the mass of a tablet.

(b) Half Mei Lee's age is the age she was eight years ago.

(i) If the variable is Mei Lee's age, use a pronumeral to represent the variable.

(ii) Write an equation you would use to find Mei Lee's age.

(c) In 10 years' time, Beth will be half her dad's age now.

(i) If one variable is Beth's age and the other other variable is her dad's age, use two pronumerals to represent these variables.

(ii) Write an equation you would use to find Beth's age now.

(d) In a ball game, a goal is worth 8 points and a penalty goal is worth 1 point. At a match, one team scored 100 points.

(i) Define the variables and represent them with pronumerals.

(ii) Using these pronumerals, write an equation you would use to show the team's score.

Reasoning, communicating and open-ended

6 A family on holiday travelled a certain distance on the first day. The next day they were able to travel twice as far as on the first day. On the third day, they travelled 480 km further than on the first day.

(a) Define the variable.

(b) Write an expression to show the distance travelled on the second day.

(c) Write an expression to show the distance travelled on the third day.

(d) If the distance travelled on the third day was three times the distance they travelled on the first day, write an equation to show this situation.

7 In Australian Rules football, a goal is worth 6 points and a behind is worth 1 point. A team has scored 114 points.

(a) Define any variables you need to use.

(b) Using these variables, write an equation for the number of points scored.

(c) Find at least three different ways that the 114 points could have been scored.

8 In Rugby Union, a try is worth 5 points and a conversion goal is worth 2 points. Penalty goals and drop goals are each worth 3 points. To be able to get a conversion goal you must first score a try.

(a) If the Waratahs win a match against the Brumbies by 23 points to 12, write down two different ways these points could have been scored.

(b) If P is the total number of points scored, t is the number of tries, g is the number of 2-point goals and p is the number of 3-point goals, write an equation to find the total number of points scored.

5.3 Using rules

Need to Know

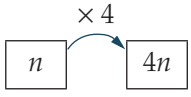
Relationships

A **relationship** describes a connection between variables so that changing the value of one variable changes the value of other variables in the expression. For example, there is a relationship between the number of pineapples bought and the total cost of the pineapples or the number of lollies in a packet and the number of lollies each person in a group would receive.

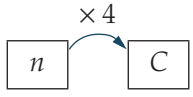
Rules and flowcharts

Relationships can be described by algebraic rules. For example: If a pineapple costs $4, the rule could be written $C = 4n$ where C is the total cost and n is the number of pineapples.

Flowcharts are used to show step-by-step instructions. For example,



or



are flowcharts that represent the rule $C = 4n$.

Using a table of values

A **table of values** is a grid that shows the relationship between two variables for particular values of one of the variables. We often use x and y as the variables in a table of values but any two pronumerals can be used.

A table of values to show the relationship $y = x + 2$ could look like this:

x	4	10	6	9	2
y	6	12	8	11	4

Using brackets

Brackets often are used in rules to change the order of operations.

For example: $y = 2x + 1$ and $y = 2(x + 1)$ are rules that represent different relationships between the variables x and y as shown in the table of values below.

In the rule $y = 2x + 1$, a number is multiplied by 2 and then 1 is added.

A table of values for $y = 2x + 1$ could look like this:

x	-3	5	2	0
y	-5	11	5	1

In the rule $y = 2(x + 1)$, 1 is added to a number before it is multiplied by 2.

A table of values for $y = 2(x + 1)$ could look like this:

x	-3	5	2	0
y	-4	12	6	2

Worked Example 5

WE5

For each of the following rules, draw a flowchart, write the rule using algebra, and complete the given table of values.

(a) y is equal to x divided by 4.

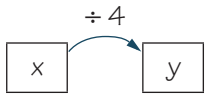
x	20	16	4	12	10
y					

(b) 2 is added to the product of x and 5 to get y.

x	2	5	4	3	0
y					

Working

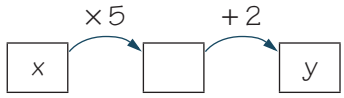
(a)



$y = \frac{x}{4}$

x	20	16	4	12	10
y	5	4	1	3	2.5

(b)



$y = 5x + 2$

x	2	5	4	3	0
y	12	27	22	17	2

5.3 Using rules

Answers page 470

Navigator

Q1 Column 1, Q2, Q3, Q4, Q5 (a) & (b), Q6, Q8, Q10, Q12	Q1 Column 1, Q2, Q3, Q4, Q5 (c) & (d), Q6, Q7, Q8, Q10, Q13, Q14	Q1 Column 2, Q2, Q3, Q4, Q5 (e) & (f), Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14

Fluency and understanding

1 For each of the following rules, draw a flowchart, write the rule using algebra, and complete the given table of values.

WE5

(a) y is equal to x plus 2.

x	13	11	7	28	1
y					

(b) To find y, subtract 5 from x.

x	6	18	9	85	5
y					

(c) To find y, double x.

x	2	3.4	10	11	101
y					

(d) y is equal to x divided by 3.

x	18	12	30	9	0
y					

(e) y is equal to multiplying x by 5, then subtracting 3.

x	4	2	0	5	20
y					

(f) To find y, add 5 to x, then divide by 10.

x	5	25	45	10	33
y					

Hint

Double x is the same as saying 'two times x'.

(g) To find y, divide x by 2, then subtract 1.

x	10	6	24	9	15
y					

(h) y is equal to x multiplied by itself.

x	3	11	7	6	10
y					

2 Choose the correct algebraic rule given in each case.

(a) y is equal to the sum of x and twelve.

A $y = x + 12$ **B** $y + 12 = x$ **C** $y = 12x$ **D** $y = \frac{x}{12}$

(b) To get y, subtract 50 from x.

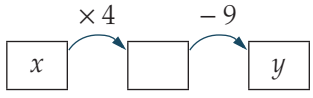
A $y = x - 50$ **B** $y = 50 - x$ **C** $y = x \times 50$ **D** $y = x + 50$

(c) To get y, add thirteen to x, then multiply by nine.

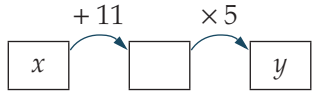
A $y = (x + 9) \times 13$ **B** $y = (x \times 13) + 9$ **C** $y = x + 13 \times 9$ **D** $y = 9(x + 13)$

3 Write the rule shown by each of these flowcharts. Make sure you use brackets where necessary.

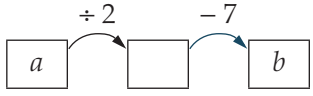
(a)



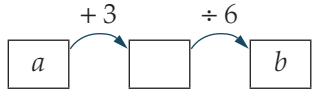
(b)



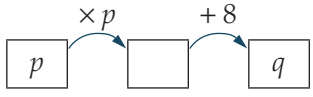
(c)



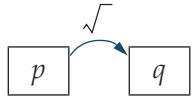
(d)



(e)



(f)



4 Rewrite each of these rules using algebra.

(a) To find y, subtract eighteen from x.

(b) y is equal to sixty multiplied by x.

(c) Divide x by seven to find y.

(d) To find y, add forty-three to x, then multiply by twenty.

(e) Multiply x by one hundred, then subtract fifty to find y.

(f) To find y, divide x by sixteen, then add thirteen.

(g) y is equal to x multiplied by itself.

(h) Subtract twelve from x, then divide by nine to find y.

(i) Multiply x by itself, then take away thirty-seven to find y.

5 For the following rules, draw a flowchart and describe the rule using words.

(a) $y = 5x + 11$

(b) $y = \frac{x}{7} - 5$

(c) $b = 7(a + 6)$

(d) $b = \frac{a - 7}{12}$

(e) $d = \frac{e + 9}{4} - 13$

(the flowchart for this rule should have three steps)

(f) $d = \frac{e^2 - 1}{5}$

(the flowchart for this rule should have three steps)

6 Joe has some lollies. He gives half to his little brother and then eats four of the rest.

(a) Draw a flowchart to show how many lollies he has left. Use x for the number of lollies he starts with and y for the final number.

(b) Write the rule to describe the situation.

(c) Use the rule to find how many lollies Joe has now if he had 20 to start with.

7 Anita has some money in her wallet. Her mum gives her $20 for doing some chores and she decides to spend half of the money she has now on a birthday present for her sister. She spends another $4 on a birthday card.

(a) Draw a flowchart you would use to find how much money she has spent. Use x for the initial amount in Anita's wallet and y for the amount she has spent.

(b) Write a rule to describe the situation.

(c) Use the rule to find how much money Anita spent if she had $12 in her wallet initially.

8 Your Uncle Harry is twice your age, plus five years.

(a) Show this information in a flowchart. Use m to represent your age, and H to represent Uncle Harry's age.

(b) Write a rule to show this situation.

(c) If you are 12 years old, how old is Uncle Harry?

9 To produce jeans it costs $245 to set up the machine, and then $7 for each pair of jeans. So, for one pair of jeans to be produced, it costs $\$245 + \$7 \times 1 = \$252$.

(a) Using this information, fill in the table.

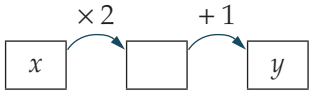
Number of pairs of jeans (n)	10	50	150	200
Cost to produce the jeans (C)				

(b) Write a rule for the cost of producing the jeans. Use C for cost, and n for the number of pairs of jeans.

Reasoning, communicating and open-ended

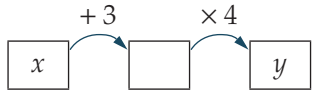
10 The following tables of values have only the y-values filled in. Work *backwards* along the flowchart to determine the values of x that were used, and complete the tables of values.

(a)



x					
y	5	11	15	9	21

(b)



x					
y	52	16	20	28	84

> **Hint**
>
> To work backwards along the flowchart, do the *opposite* operation to the one shown in the forwards direction.

11 For each of the following two rules:

(i) y is equal to the sum of x and 3, which is then multiplied by 2

(ii) to find y, multiply x by 2, then add 6.

(a) Draw a flowchart.

(b) Write the rule using algebra.

(c) Copy and complete the table of values below.

x	2	5	3	4	7
y					

(d) What do you notice about these two rules? Can you explain your observation?

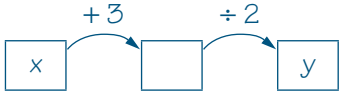
12 Construct your own table of values and complete it for the rule $y = \frac{x}{2} + 3$. Use at least five different values.

13 **(a)** Using two pronumerals (such as x and y), at least two of the following operations ($+, -, \times, \div$) and some arrows, create at least three different flowcharts that show a rule connecting x to y. Use a different combination of operations for each flowchart.

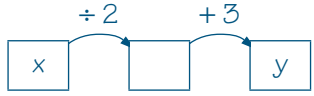
(b) Write the rules shown by your flowcharts using algebra.

14 Kim and Jai are working on the following question: 'Draw a flowchart that represents this rule: to obtain y, divide x by 2, then add 3'.

Kim's flowchart looks like this:



Jai's flowchart looks like this:



'Hey', said Jai. 'Yours looks different to mine. I must be wrong.'

'No, mate', said Kim. 'It's just a different way of writing the same thing.'

Is Kim's statement correct? Are they both right? If not, who has the correct flowchart, and why?

Problem solving

Cutting string

Equipment required: 1 brain, 2 lengths of string, scissors

1 Imagine that you have a piece of string. How many pieces will you have if you cut the string once? If you cut one of the pieces again, how many pieces will you now have? Check your answer using some string and scissors, then complete the following table.

Number of cuts	0	1	2	3	4	5	6	c
Total number of pieces								

2 This time, fold a piece of string in half each time before you cut it. How many pieces at each cut do you have now? *Remember*: Cut only one piece at each time.

Number of cuts	0	1	2	3	4	5	6	c
Total number of pieces								

Strategy options

- Act it out.
- Look for a pattern.

Formulas and substitution

5.4

Need to Know

A **formula** is a mathematical rule that uses two or more variables.

Substitution

For example: $V = IR$ and $F = \frac{9C}{5} + 32$ are formulas.

In mathematics, we can **substitute** a number for a variable. This allows us to find a value for, or **evaluate**, the formula or expression.

When substituting numbers into a formula, write in any 'hidden' multiplication signs.

For example: $b = 4a + 3$
$= 4 \times 5 + 3, a = 5$
$= 20 + 3$
$= 23$

Worked Example 6

WE 6

For each of the following formulas, evaluate y by substituting the given value of x.

(a) $y = 10x - 5, \quad x = 7$

(b) $y = \frac{x}{2} + 4, \quad x = 5$

Working

(a) $y = 10x - 5$
$y = 10 \times 7 - 5, x = 7$
$y = 70 - 5$
$= 65$

$y = 65, x = 7$

(b) $y = \frac{x}{2} + 4$

$y = \frac{5}{2} + 4, x = 5$

$= 2\frac{1}{2} + 4$

$= 6\frac{1}{2}$ or 6.5

$y = 6.5, x = 5$

Worked Example 7 WE7

(a) The cost of hiring a community hall is calculated using the formula $C = 65t + 200$, where C = cost (in dollars) and t = the hire period (in hours). Find the cost of hiring the hall for 4 hours.

(b) The area of a rectangle (in square metres) is given by the formula $A = lw$ where l = length (in metres) and w = width (in metres). Calculate the area of a rectangular room that is 4.5 m long and 3 m wide.

Working

(a) $C = 65t + 200$
$C = 65 \times 4 + 200$, $t = 4$
$= 260 + 200$
$= 460$
The cost of hiring the hall for 4 hours is $460.

(b) $A = lw$
$A = 4.5 \times 3$, $l = 4.5$, $w = 3$
$= 13.5$
The area is 13.5 square metres.

Answers page 472

5.4 Formulas and substitution

Navigator

Q1 Column 1, Q2, Q3, Q4 Column 1, Q5 (a) & (b), Q6, Q7, Q8, Q10, Q12	Q1 Column 2, Q2, Q3, Q4 Column 1, Q5, Q6, Q7, Q9, Q10, Q11, Q12	Q1 Column 2, Q2, Q3, Q4 Column 2, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12

Fluency and understanding

WE6

1 For each of the following formulas, evaluate y by substituting the given value of x.

(a) $y = x + 3$, $x = 4$
(b) $y = x + 5$, $x = \frac{1}{2}$
(c) $y = 2x - 7$, $x = 3$
(d) $y = 3x - 1$, $x = 8$
(e) $y = x - 1$, $x = 3.6$
(f) $y = x - 5$, $x = 30$
(g) $y = 25 - 3x$, $x = 7$
(h) $y = 30 - 2x$, $x = 1$
(i) $y = 4x$, $x = \frac{1}{2}$
(j) $y = 8x$, $x = 4$
(k) $y = 3(x + 5)$, $x = 1$
(l) $y = 6(x + 2)$, $x = 18$
(m) $y = \frac{x}{5}$, $x = 20$
(n) $y = \frac{x}{5}$, $x = 2.5$
(o) $y = 11(10 - x)$, $x = 4$
(p) $y = 12(10 - x)$, $x = 3$

WE7

2 **(a)** The cost of hiring power tools from the local hardware store is calculated using the formula $C = 35t + 60$, where C = cost (in dollars), and t = the hire period (in hours). Find the cost of hiring a sander for 6 hours.

(b) The average speed of a moving object (such as a car) can be calculated by the formula $s = \frac{d}{t}$, where s = speed (in kilometres per hour), d = distance (in kilometres) and t = time (in hours). Yoshi drives 210 km from Hobart to Launceston in 3 hours. Calculate her average speed.

3 Answer TRUE or FALSE for each of the following statements.

(a) If we substitute $a = 4$ into $b = 5a$, we get $b = 20$.

(b) If we substitute $a = 9$ into $b = a + 11$, we get $b = 20$.

(c) If we substitute $u = 4$ into $v = 6u + 1$, we get $v = 65$.

(d) If we substitute $q = 10$ into $k = 13q - 8$, we get $k = 122$.

(e) If we substitute $x = 6$ into $y = 4(x - 5)$, we get $y = 19$.

(f) If we substitute $x = 12$ into $y = 5(14 - x)$, we get $y = 10$.

4 Use each of the following rules to complete these tables of values.

(a) $b = 4a$

a	11	20	5	9	50
b					

(b) $y = 7x$

x	6	4	10	20	101
y					

(c) $n = 3m + 2$

m	1	2	10	6	5
n					

(d) $k = 4j + 7$

j	2	5	11	10	100
k					

(e) $q = 2p - 10$

p	11	15	10	20	100
q					

(f) $s = 8r - 2$

r	1	2	3	0	200
s					

(g) $v = 4(u - 1)$

u	5	3	1	201	6
v					

(h) $n = 3(m - 2)$

m	5	10	11	102	52
n					

5 (a) For the rule $m = 3x - 5$, when $x = 3$, m would be equal to:

A 1 **B** 2 **C** 3 **D** 4

(b) For the rule $y = \frac{x}{3} + 4$, when $x = 6$, y would be equal to:

A $\frac{10}{3}$ **B** $4\frac{1}{2}$ **C** 6 **D** 22

(c) For the rule $l = 3(n + 2) - 1$, when $n = 1$, l would be equal to:

A 5 **B** 6 **C** 8 **D** 9

6 Sam's new fuel-efficient car uses seven litres of petrol for every 100 km it travels.

(a) Write this as a formula to calculate the amount of petrol (p) that is needed to travel a distance (d) kilometres.

His father's 4WD uses 11 litres of petrol for every 100 km it travels.

(b) Write this as a formula to calculate the amount of petrol (f) that is needed to travel a distance of (d) kilometres.

(c) Use your formulas to calculate the number of litres each car uses to travel a distance of 400 km.

(d) Write a formula to calculate the cost (C), in dollars, of y litres of petrol if petrol costs \$1.30/litre.

(e) Calculate how much more Sam's father spent on petrol than Sam to travel 400 km.

Hint

Use the correct order of operations.

7 Jabbok, a mobile phone salesman gets $350 per week and $10 for every phone that he sells.

(a) Write this as a formula with n representing the number of phones sold and p representing the amount of money earned each week.

(b) If 15 phones are sold in the first week, use your formula to find how much Jabbok earns this week.

(c) If 20 phones are sold the next week, use your formula to find how much money Jabbok earns that week.

Reasoning, communicating and open-ended

8 I buy three loaves of bread and a $2.60 carton of milk. It costs me $\$D$.

(a) If l represents the cost of a loaf of bread, write a formula to find the cost of my shopping (D) in terms of l.

(b) Use the formula to evaluate D if $l = \$3.25$.

9 Jasmine is a caterer. She uses formulas to work out how much food she needs to feed different numbers of people.

(a) To calculate the number of sausages she needs at a barbecue, Jasmine uses the following formula: 'Allow 2 sausages for every person and have an extra 10 sausages'. Write Jasmine's formula using algebra using n to represent the number of people and s to represent the number of sausages.

(b) Another one of Jasmine's formulas for barbecues is: 'One bowl of salad will feed 8 people'. Write this formula using algebra. Use n to represent the number of people, and b for the number of bowls of salad.

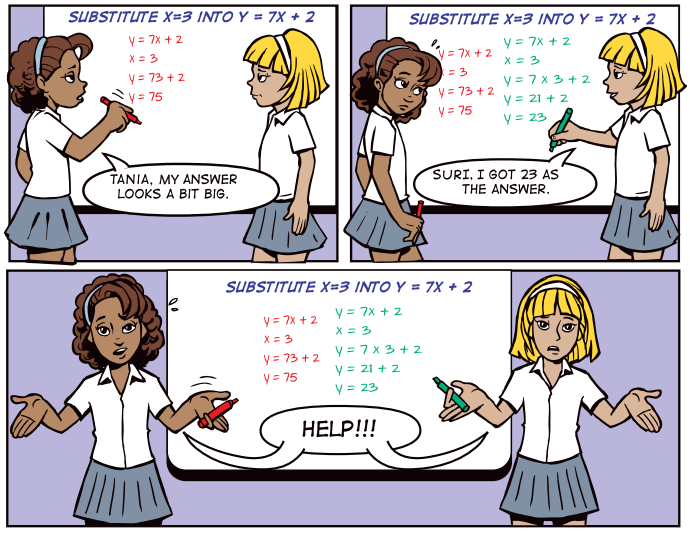
(c) Use the formulas you have written to determine the number of sausages and bowls of salad Jasmine will need to cater for 40 people.

10 Show that the formulas $b = 3(a + 5)$ and $b = 3a + 5$ are different by substituting $a = 4$ into each of them. Explain how the two are different in terms of the order of operations.

11 Make up two different formulas connecting x and y that use addition and multiplication. Use them to complete the following table for each one.

x	7	20	13	101
y				

12 Which of the two students has substituted correctly? What mistake has the other student made? Give them some advice so they can avoid a similar mistake in the future.



Patterns and rules

5.5

Need to Know

Algebra can be used to represent a pattern and can be written as a general rule or formula.

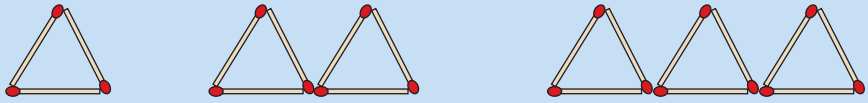
When trying to find a pattern, look for the number that is being added or subtracted every time. This will tell us what to multiply by in our rule.

Always try to understand how the rule works by looking at the pattern that has been formed.

Worked Example 8

WE 8

(a) Here is a matchstick pattern of triangles. Copy and complete this table of values by continuing the pattern.



Number of triangles (t)	1	2	3	4	5
Number of matches (m)					

(b) Find a general rule that connects the number of triangles in the pattern (t) to the number of matches used (m). Write the rule in words and in algebra.

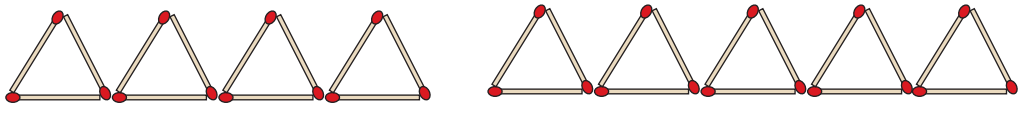
(c) Use your rule to find the number of matches required to make 100 triangles.

Working

(a)

Number of triangles (t)	1	2	3	4	5
Number of matches (m)	**3**	**6**	**9**		

Number of triangles (t)	1	2	3	4	5
Number of matches (m)	3	6	9	**12**	**15**



(b) $m = 3 \times t$

The number of matches is 3 times the number of triangles.

$m = 3t$

(c) $m = 3 \times 100$, $t = 100$

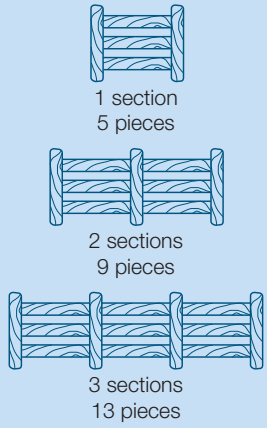
$m = 300$

300 matches are needed to make 100 triangles.

Worked Example 9

WE9

Frederico is building a fence around Farside Farm.
The fences are made up of pieces of timber as shown.



(a) Copy and complete this table of values by continuing the pattern.

Number of sections of fence (F)	1	2	3	4	5
Number of pieces of timber (P)	5	9			

(b) Find a general rule that connects the number of sections of fence (F) to the number of pieces of timber (P) needed. Write the rule in words and in algebra.

(c) Use your rule to find the number of pieces of timber required for a fence made up of 50 sections.

Working

(a)

Number of sections of fence (F)	1	2	3	4	5
Number of pieces of timber (P)	5	9	13	17	21

(b) 4 pieces are being added to make a new section. Therefore, $4F$ is in our rule.

$4F + 1$

The number of pieces is 4 times the number of sections plus 1 extra piece.

$P = 4F + 1$

(c) $P = 4F + 1$
$= 4 \times 50 + 1, \ F = 50$

$P = 200 + 1$
$= 201$

To make a fence of 50 sections, 201 pieces of timber are needed.

5.5 Patterns and rules

Answers page 472

Navigator

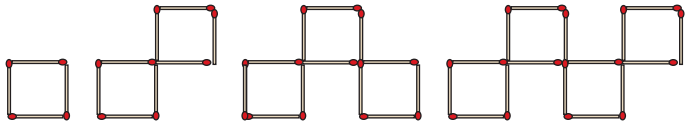
Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q10, Q12	Q1, Q2, Q3, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12	Q1, Q2, Q3, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13

Equipment required: Centimetre grid paper for Question **12**

Fluency and understanding

1 **(a)** Here is a matchstick pattern of squares. Copy and complete this table of values by continuing the pattern.

WE 8



Number of squares (s)	1	2	3	4	5	6	7	8
Number of matches (m)								

(b) Find a general rule that connects the number of squares in the pattern (s) to the number of matches used (m). Write the rule in words and in algebra.

(c) Use your rule to find the number of matches required to make 100 squares.

2 Larissa, a landscape gardener, uses sleepers to divide up gardens into separate beds as shown.

WE 9



1 garden bed
4 sleepers

2 garden beds
7 sleepers

3 garden beds
10 sleepers

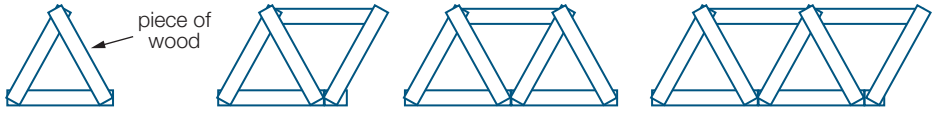
(a) Copy and complete this table of values by continuing the pattern.

Number of garden beds (B)	1	2	3	4	5
Number of sleepers (S)	4	7	10		

(b) Find a general rule that connects the number of garden beds (B) to the number of sleepers (S) needed. Write the rule in words and in algebra.

(c) Use your rule to find the number of sleepers required to divide the garden into a section made up of 21 beds.

3 Clarence the carpenter has been working on a new restaurant. The owners want a triangular woodwork design, like the one shown below, running across the walls.



1 triangle
3 pieces of wood

2 triangles
5 pieces of wood

3 triangles
7 pieces of wood

4 triangles
? pieces of wood

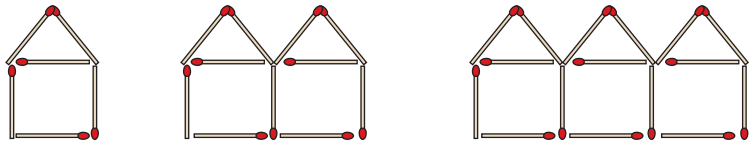
(a) Copy and complete this table of values by continuing the pattern.

Number of triangles (T)	1	2	3	4	5
Number of pieces of wood (P)	3	5	7		

(b) Find a general rule that connects the number of triangles to the number of pieces of wood required.

(c) Use your rule to find how many pieces of wood are required to make a total of 203 triangles.

4 Here is a matchstick pattern of houses.



1 house
6 matches

2 houses
11 matches

3 houses
? matches

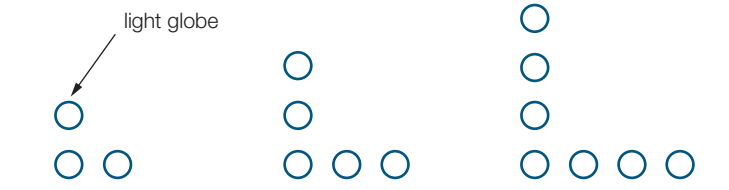
(a) Copy and complete the table below.

Number of houses (h)	1	2	3	4	5	6	7	8
Number of matches (m)	6	11						

(b) Find the general rule that connects the number of matches to the number of houses.

(c) Use your rule to find the number of matches required to build 20 houses.

5 Lightworks International Co., designers and manufacturers of large illuminated advertising signs, want to put a giant L made up of individual globes onto their largest building. They have already made some small Ls on some of their other buildings.



height of L = 2
number of globes = 3

height of L = 3
number of globes = 5

height of L = 4
number of globes = 7

(a) Construct and complete a table of values for this pattern. Use the three patterns given here to begin your table, then continue the pattern for the next two Ls in the pattern.

(b) Find the rule that connects the height of the L to the number of globes required. Let H = height of the L and G = number of globes.

(c) Use your rule to find how many globes you would need to make an L with a height of 120.

6 A rule is given as $L = 3n + 1$. Which statement is not true?

A When $n = 3$, $L = 10$.

B When $n = 4$, $L = 12$.

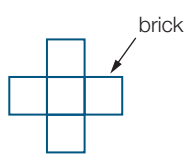
C When $n = 5$, $L = 16$.

D When $n = 6$, $L = 19$.

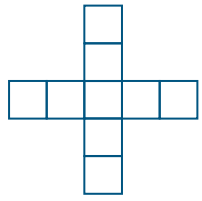
7 The White Cross charity organisation has a white cross as its emblem. Their buildings all have a white cross built into the brickwork using white bricks. Some smaller versions of the cross are shown.

(a) Draw the next two white crosses in the pattern.

(b) Construct and complete a table of values connecting the arm length (a) and the number of white bricks (b) for these four different-sized crosses.



arm length = 1
number of bricks = 5



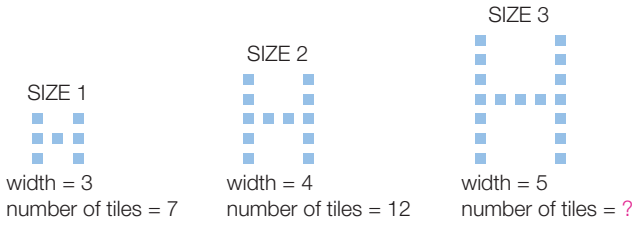
arm length = 2
number of bricks = 9

(c) Extend your table for arm lengths of five and six.

(d) Find the rule that connects the two pronumerals.

(e) Find how many white bricks would be needed to make a cross with an arm length of 52.

8 Mr Harrison has decided that he wants to use tiles to form large letter Hs, which are built into the brickwork of the walls of his company offices. He must work out how many tiles are needed to make different-sized Hs.



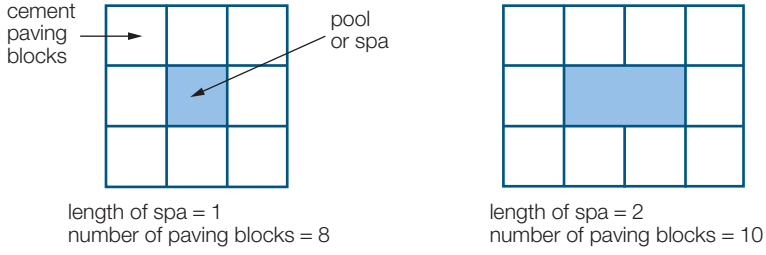
(a) Draw the next two Hs in the pattern.

(b) Construct and complete a table of values connecting the width of the H (w), and the total number of tiles (t).

(c) Find the rule for these five different sizes that connects the width of the H (w) to the total number of tiles (t).

(d) Find the number of tiles needed to make an H that is 12 tiles wide.

9 Plunge & Co., the Pool Paving People, specialises in large square cement paving blocks to surround swimming pools and outdoor spas, like the ones shown below.



(a) Draw the next two spas and pavers in the pattern.

(b) Construct and complete a table for values of up to four spa lengths, connecting the spa length (l) with the number of pavers used (p) for these four different spa sizes.

(c) Find a rule without brackets connecting the two pronumerals.

(d) Explain your rule by using diagrams of spas of different lengths. Which paving blocks does the constant represent? Where does the coefficient of the pronumeral representing spa length come from?

(e) Find a rule using brackets that also works.

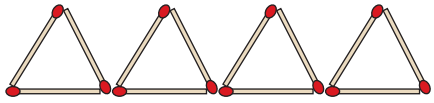
(f) Plunge & Co. has recently been asked by a swimming club to build a single-lane lap pool to help train its long-distance swimmers. It has been worked out that the lap pool is 345 paving blocks in length. Find out how many paving blocks will be needed to pave around the whole pool.

(g) For another job, Plunge & Co. uses 40 paving blocks to surround an outdoor spa. What is the length of this spa?

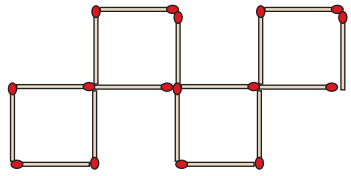
Reasoning, communicating and open-ended

10 In each of the following matchstick patterns, the diagrams show the pattern after the fourth set of matchsticks has been added.

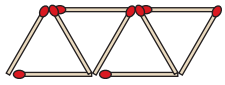
(i)



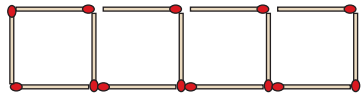
(ii)



(iii)



(iv)

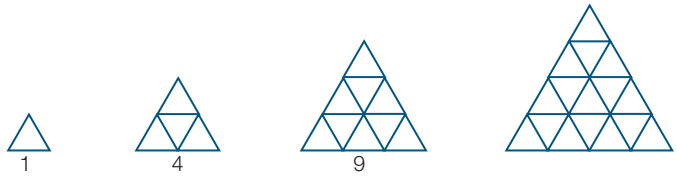


(a) Write a general rule for each of the patterns if one more set of matchsticks is added to the pattern each time. Use s = number of shapes and m = number of matchsticks as pronumerals.

(b) Patterns **(i)** and **(iii)** both add a new triangle every time. However, the rules are different. Patterns **(ii)** and **(iv)** both add a new square every time. However, the rules are also different. How are patterns **(i)** and **(ii)** different from the patterns in **(iii)** and **(iv)**? How is this difference shown in the rule for each pattern?

Not all patterns grow by a consistent amount each time as can be seen in the following question.

11 **(a)** Count *all* the small triangles in the fourth large triangle.



(b) How many small triangles do you think there will be in the fifth large triangle?

(c) Copy and complete the following table. Try to find the pattern to help you.

Large triangle	1	2	3	4	5	6
Number of triangles	1	4	9			

(d) Describe how the sequence of numbers formed in the bottom row of the table increases as extra elements of the pattern are added.

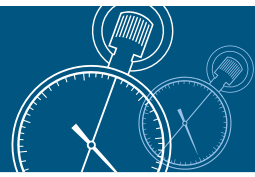
12 Choose one of the following letters: P, T, A, X, or Y. Create a design for the letter from small centimetre squares, and use it to create a pattern of letters that gradually increases in size. Try to find a rule that describes your pattern.

13 Shreyas has decided to make a closed matchstick shape with six matchsticks. He then adds matchsticks to make a pattern that has a rule $m = 5s + 1$ where m is the number of matchsticks and s is the number of repeated shapes.

(a) Construct a shape that Shreyas may have started with.

(b) Shreyas changes his shape and adds matchsticks to make a new pattern. He finds that the rule is now $m = 4s + 2$. Draw a second shape that he may have used.

Half-time 5



1 Which rule describes the given information? Ex.5.3

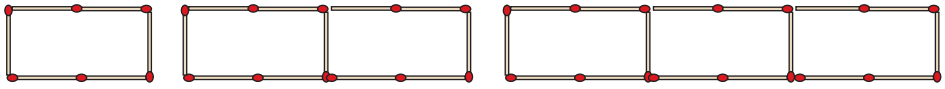
'Subtract 45 from x to get y.'

A $y = x + 45$ **B** $y = x - 45$ **C** $y = x \times 45$ **D** $y = x \div 45$

2 For each of the following rules, evaluate y by substituting the given values of x. Ex.5.4

(a) $y = x - 4$ when $x = 6$ **(b)** $y = 2(x - 7)$ when $x = -13$ **(c)** $y = 54 - 3x$ when $x = -9$

3 If s represents the number of shapes formed and M is the number of matchsticks used, which rule is correct for this pattern? Ex.5.5



A $M = 5s + 1$ **B** $M = 6s$ **C** $M = 5s - 1$ **D** $M = 4s + 2$

4 Write an equation to describe each of the following. Ex.5.2

(a) Tim has p footy cards. He collects 7 more. He now has 28 cards.

(b) 20 is obtained when 4 is subtracted from the product of x and 7.

(c) A bird aviary has w parrots and g canaries. When 4 of the canaries are sold, there are 14 birds left in the aviary. How many birds are in the aviary now?

5 If y is equal to 4 plus the quotient of x divided by 2: Ex.5.3

(a) draw a flowchart for this rule

(b) write the rule using algebra

(c) copy and complete this table of values for the rule.

x	2	8	10	5	13
y					

6 Nicola has six identical bags of lollies, plus an extra 11 loose lollies. If n represents the number of lollies in one bag, write an expression for the total number of lollies Nicola has. Ex.5.1

7 Given the rule $y = 4x - 3$, which statement is not true? Ex.5.4

A When $x = 6$, $y = 21$. **B** When $x = 5$, $y = 17$.

C When $x = 4$, $y = 13$. **D** When $x = 3$, $y = 0$.

8 The cost of an international phone call is 35 cents for every minute, plus a one-off 'flagfall' charge of 50 cents. Ex.5.4

(a) Write a rule to calculate the total cost of an international phone call. Use the pronumeral C for the cost and m for the length of the call, in minutes.

(b) Use your rule to calculate the cost of a 6-minute phone call. Write your answer in dollars.

(c) Ravi wants to call his parents in Singapore. He has $8. Will this be enough money for a 20-minute call?

5.6 Simplifying expressions with addition and subtraction

Need to Know

Like and unlike terms

Like terms:

- have the same pronumeral part
- can have different coefficients; $2x$ and $5x$ are like terms
- can have pronumerals raised to exactly the same power; $7x^3$ and $4x^3$ are like terms
- can have pronumerals in a different order; ab and ba are like terms.

Unlike terms:

- have different pronumeral parts; $2x$ and $2y$ are unlike terms
- can have the the same pronumerals raised to different powers; x^2 and x are unlike terms ($2^2 \neq 2$).

Using number laws

Because pronumerals represent numbers, number laws can be used in algebra to change the order of pronumerals.

$2 \times 3 = 3 \times 2$, so $ab = ba$ (communicative law)

$(3 \times 2) \times 5 = 3 \times (2 \times 5)$, so $(ab)c = a(bc)$ (associative law)

$abc = acb = bac = bca = cab = cba$

When terms have different pronumeral parts, pronumerals are written in alphabetical order. This helps to identify like terms.

For example, $6abc$, $3bca$, $4dab$, $8cad$, $11adb$, $7cda$ can be written as $6abc$, $3abc$, $4abd$, $8acd$, $11abd$ and $7acd$ showing that $6abc$ and $3bca$ are like terms, $4dab$ and $11adb$ are like terms and $8cad$ and $7cda$ are also like terms.

Adding and subtracting like terms

We can simplify an expression by adding or subtracting like terms. This is called 'collecting like terms'. We add or subtract like terms by adding or subtracting their coefficients.

- Only like terms can be added together or subtracted.
- The operation sign (+ or –) moves with the term that follows it.

Checking by substitution

We can replace a pronumeral with any number to check that our simplification is correct.

For example, we can check by substitution that $6x + 4x = 10x$ and $10x - 2x = 8x$ for all values of x.

If $x = 20$,

$6x + 4x = 6 \times 20 + 4 \times 20$
$= 120 + 80$
$= 200$

$10x = 10 \times 20$
$= 200$

and

$10x - 2x = 10 \times 20 - 2 \times 20$
$= 200 - 40$
$= 160$

$8x = 8 \times 20$
$= 160$

If $x = -7$,

$6x + 4x = 6 \times -7 + 4 \times -7$
$= -42 - 28$
$= -70$

$10x = 10 \times -7$
$= -70$

and

$10x - 2x = 10 \times -7 - 2 \times -7$
$= -70 + 14$
$= -56$

$8x = 8 \times -7$
$= -56$

If $x = \frac{1}{2}$,

$6x + 4x = 6 \times \frac{1}{2} + 4 \times \frac{1}{2}$
$= 3 + 2$
$= 5$

$10x = 10 \times \frac{1}{2}$
$= 5$

and

$10x - 2x = 10 \times \frac{1}{2} - 2 \times \frac{1}{2}$
$= 5 - 1$
$= 4$

$8x = 8 \times \frac{1}{2}$
$= 4$

Worked Example 10

WE10

Which of the following are like terms?

(a) $4x$ and $7x$
(b) $5ab$ and $9b$
(c) x and $12x$
(d) 8 and $8z$
(e) h^3 and $6h^3$
(f) xy and x^2y

Working

(a) $4x$ and $7x$ are like terms.
(b) $5ab$ and $9b$ are not like terms.
(c) x and $12x$ are like terms.
(d) 8 and $8z$ are not like terms.
(e) h^3 and $6h^3$ are like terms.
(f) xy and x^2y are not like terms.

Worked Example 11

WE11

Simplify each expression by collecting like terms. Check by substituting the values given in brackets for the pronumerals that your simplification is correct.

(a) $4y + 7y$; $(y = 2)$ **(b)** $2mn - 4mn$; $(m = 3, n = -1)$ **(c)** $8a + 4 - 3a$; $(a = 6)$

Working

(a) $4y + 7y$
$= 11y$

Check by substitution.
For $y = 2$,
$4y + 7y$
$= 4 \times 2 + 7 \times 2$
$= 8 + 14$
$= 22$
$11y = 1 \times 2$
$= 22$

(b) $2mn - 4mn$
$= -2mn$

Check by substitution.
For $m = 3$ and $n = -1$,
$2mn - 4mn$
$= 2 \times 3 \times -1 - 4 \times 3 \times -1$
$= -6 + 12$
$= 6$
$-2mn = -2 \times 3 \times -1$
$= 6$

(c) $8a + 4 - 3a$
$= 8a - 3a + 4$
$= 5a + 4$

Check by substitution.
For $a = 6$,
$8a + 4 - 3a$
$= 8 \times 6 + 4 - 3 \times 6$
$= 48 + 4 - 18$
$= 34$
$5a + 4 = 5 \times 6 + 4$
$= 34$

Worked Example 12

WE12

Simplify each expression where possible by collecting like terms.

(a) $4x + 3y - 9x + 2y$ **(b)** $6ab - 4a + 3ab - 7$

Working

(a) $4x + 3y - 9x + 2y$
$= 4x - 9x + 3y + 2y$
$= -5x + 5y$

(b) $6ab - 4a + 3ab - 7$
$= 6ab + 3ab - 4a - 7$
$= 9ab - 4a - 7$

Answers page 473

5.6 Simplifying expressions with addition and subtraction

Navigator

Q1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q9, Q11	Q1, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q8, Q9, Q11	Q1, Q2 Column 3, Q3 Column 3, Q4, Q5, Q7, Q8, Q9, Q10, Q11

Fluency and understanding

WE10

1 Which of the following are like terms?

(a) $3k$ and $5k$ **(b)** $4y$ and $5z$ **(c)** $11y$ and $12v$
(d) 17 and 8 **(e)** $6ab$ and $7bc$ **(f)** $2xy$ and $14yx$
(g) $4xyz$ and yzx **(h)** mnp and $3npm$ **(i)** 3, 5 and x

2 Simplify each expression by collecting like terms. Check by substituting the values given in brackets for the pronumerals that your simplification is correct. WE11

(a) $3x + 4x$ $(x = 3)$ **(b)** $12y + 4y$ $(y = 6)$ **(c)** $11a + 2a$ $(a = 11)$

(d) $13y - 9y$ $(y = -5)$ **(e)** $7m - 3m$ $(m = -3)$ **(f)** $-10x - x$ $(x = -5)$

(g) $3mn + 2nm$ $(m = 2, n = 4)$ **(h)** $8xy - 3xy$ $(x = 2, y = 8)$ **(i)** $-2z + 3z + 4$ $(z = 3)$

(j) $x + 4y + 6x$ $(x = 2, y = 9)$ **(k)** $10w + 3z + 4w$ $(w = 4, z = -2)$ **(l)** $17a + 2b - 5a$ $(a = -2, b = 3)$

3 Simplify each expression where possible by collecting like terms. WE12

(a) $3e + 4f + 7e - 6f$ **(b)** $12p + 8q - 4p - 12q$ **(c)** $8t + 15s - 16t - 3s$

(d) $6x - 4y + 3x + y^2$ **(e)** $12w - 2z + 3w - 3z^2$ **(f)** $10m + 9n - n - 5m^2$

(g) $3a + 17 + 4a - 8$ **(h)** $14x - 4 - 6x + 9$ **(i)** $21y - 36 - y - 17$

4 **(a)** A like term for $7x$ is:

A $12x$ **B** $3 + x$ **C** 7 **D** xy

(b) A like term for $6wxy$ is:

A $6w$ **B** $6x$ **C** $6y$ **D** $12xyw$

5 Simplify each expression where possible.

(a) $5e + 14f + 8e + 6$ **(b)** $18x + 6y - 4y + 2$ **(c)** $4 + 6x + 5y - 3$

(d) $2x - 3y + 4$ **(e)** $9m - 2n + 3mn$ **(f)** $6a + 11b - 5c - 7$

6 Write each of the following rules using algebra, substitute the numbers 2, 3 and 4 into each rule, then answer TRUE or FALSE for each of these statements.

(a) These two rules are the same:

Rule 1: 'Take any number and multiply it by four. Take the same number and multiply it by six. Then add the two answers together.'

Rule 2: 'Take a number and multiply it by ten.'

(b) These two rules are the same:

Rule 1: 'Take any number and multiply it by fifteen. Take the same number and multiply it by ten. Subtract the second answer from the first.'

Rule 2: 'Take a number and multiply it by five.'

Reasoning, communicating and open-ended

7 A number of coins are placed on a square of a chessboard. Twice as many coins are then placed on the next square. On the third square, there are twice as many coins stacked as there are on the second square. This process is continued for six squares.

(a) If x is the number of coins on the first square, how many are lying on the:

(i) second square **(ii)** third square **(iii)** sixth square?

(b) How many coins, in total, are there on the first six squares of the board?

(c) If each coin's value is $\$v$, what is the total value of the coins on the board?

(d) Find a number for x and a number for v that will make the total value of the coins \$378.

8 On each of her birthdays, Georgia is given as a present three times as much money as she was given for her previous birthday. On her first birthday she received d dollars.

(a) Write an expression for the amount of money Georgia received on her fourth birthday.

(b) How much money, in total, did Georgia receive for her first four birthdays? Write your answer in terms of d.

(c) How much money did Georgia receive altogether for her first four birthdays if $d = 8$?

(d) If Georgia receives \$567 on her fifth birthday, how much did she receive on her first birthday?

9 Kristian opens a bank account by depositing \$$y$. He then deposits \$$x$ every week and makes no withdrawals or other deposits.

The amount in his account after five weeks is:

A $x + 5y$ **B** $y + x + 5$ **C** $y + 5x$ **D** $5y + 5x$

10 This algebra pattern consists of the terms $x + y$, $2x + 3y$, $3x + 5y$, $4x + 7y$, … where the coefficient of x increases by 1 each time and the coefficient of y increases by 2 each time.

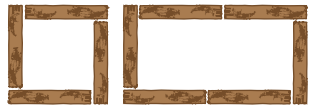
(a) Write an expression for the sixth term.

(b) Write an expression for the sum of the first five terms.

(c) Find two sets of values for x and y so that the sum of the first five terms is a multiple of 10.

(d) Find the sum for your chosen values.

11 A garden bed is made using rectangular timber planks. The planks are arranged to form square and rectangular garden beds as shown in the diagram. Let N be the total number of planks used to make the garden bed, L be the number of planks for the length of the bed and W be the number of planks for the width of the bed.



(a) Write down a rule that will find the number of planks needed for square garden beds of any side length.

(b) Write down a rule that will find the number of planks needed for a rectangular garden bed of length L and width W.

(c) Give examples of each type of garden bed using no more than 30 planks.

(d) If the length must always be double the width, rewrite the rule for the rectangular garden bed using only W as the pronumeral.

(e) Using this new rule, give examples of different types of rectangular garden beds using no more than 30 planks.

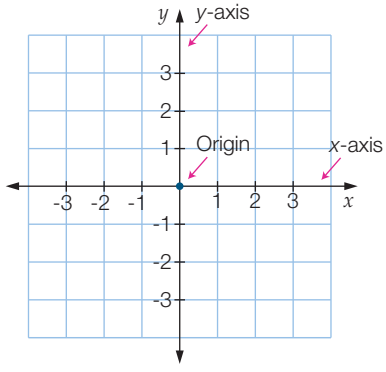
The Cartesian plane

5.7

Need to Know

The Cartesian plane

A **Cartesian plane** is constructed by drawing two lines at right angles to each other, one horizontal and the other vertical. The point at which they cross is called the **origin**. The horizontal line is called the x-axis and the vertical line is called the y-axis. Both **axes** (plural of axis) are number lines that extend infinitely in both directions.



Coordinates

The position of any point on a number plane is described by a pair of numbers called the **coordinates** of the point. Coordinates are always written in brackets as an **ordered pair** (x, y).

(x, y)

The x-coordinate shows the move to the left (–) or to the right (+) of the origin.

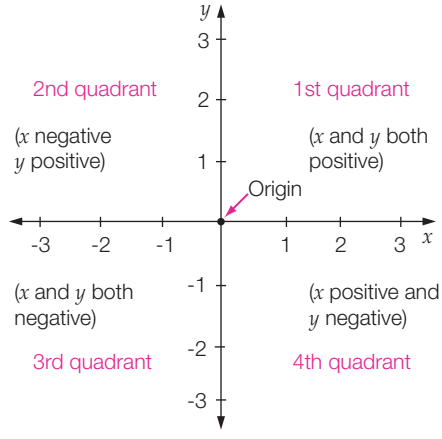
The y-coordinate shows the move up (+) or down (–) from the origin.

Quadrants

The x- and y-axes divide any plane into four **quadrants**.

The quadrants are numbered starting with the 1st quadrant, the quadrant in which both the x- and y-coordinates have positive values, then moving in an anticlockwise direction to the 2nd, 3rd and 4th quadrants.

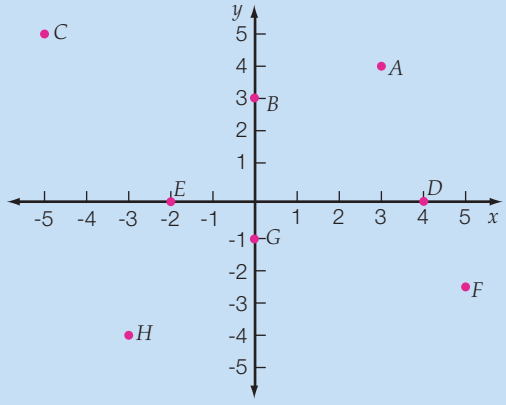
- We always move in the horizontal direction first and then in the vertical direction. This is a mathematical convention.
- The name 'ordered pair' tells you that order is important. The x-coordinate is always written first, and the y-coordinate is written second (x, y).
- The coordinates of the origin are $(0, 0)$.



Worked Example 13 WE13

(a) Write the coordinates of each of the points A to H shown on the Cartesian plane.

(b) State the quadrant in which each point is located.



Working

(a) $A = (3, 4)$
$B = (0, 3)$
$C = (-5, 5)$
$D = (4, 0)$
$E = (-2, 0)$
$F = (5, -2\frac{1}{2})$
$G = (0, -1)$
$H = (-3, -4)$

(b) A is in the 1st quadrant, C is in the 2nd quadrant, H is in the 3rd quadrant and F is in the 4th quadrant.

B, D, E and G lie on an axis, so they are not in any quadrant.

Answers page 474

5.7 The Cartesian plane

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q12	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q11, Q12, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q12, Q13

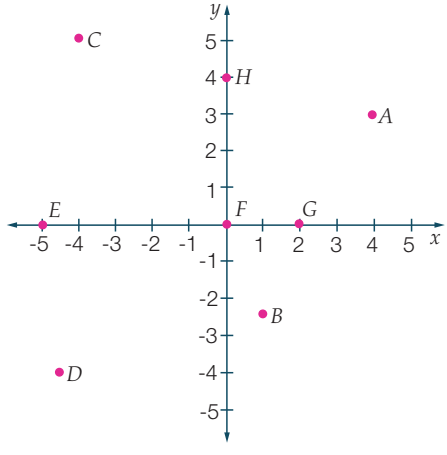
Equipment required: Graph paper for Questions **7**, **8** and **13**

Fluency and understanding

WE13

1 **(a)** Write the coordinates of each of the points A to H shown on the Cartesian plane.

(b) State the quadrant in which each point is located.



2 (a) State whether each of the following coordinates is in quadrant 1, quadrant 2, quadrant 3 or quadrant 4.

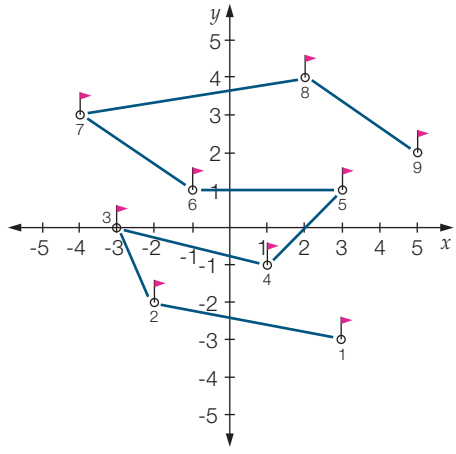
(i) (-2, 6) **(ii)** (5, -1) **(iii)** (-7, -4) **(iv)** (12, 10)

(v) (-32, 12) **(vi)** (51, -1) **(vii)** (-87, -90) **(viii)** (-2, 21)

(b) State whether each coordinate lies on the x-axis, the y-axis, or on both the x-axis and the y-axis.

(i) (0, 5) **(ii)** (0, -8) **(iii)** (0, 0) **(iv)** (-4, 0) **(v)** (15, 0)

3 The layout of a 9-hole golf course is shown on the number plane. Write in order the ordered pairs of the nine holes.



4 (a) A point is 1 unit right and 4 units up from the origin of a Cartesian plane. The coordinates of the point are:

A (4, 1) **B** (-4, 1) **C** (1, 4) **D** (-1, 4)

(b) A point is 5 units left and 2 units up from the origin of a number plane. The coordinates of the point are:

A (-5, 2) **B** (2, -5) **C** (5, -2) **D** (-2, 5)

(c) A point is 4 units down and 3 units left of the origin of a Cartesian plane. The coordinates of the point are:

A (-4, -3) **B** (-3, -4) **C** (3, -4) **D** (-3, 4)

5 (a) The origin has the coordinates:

A (1, 0) **B** (0, 1) **C** (1, 1) **D** (0, 0)

(b) One coordinate that lies on the x-axis is:

A (0, 4) **B** (1, 1) **C** (2, 4) **D** (1, 0)

(c) One coordinate that lies on the y-axis is:

A (3, 0) **B** (3, 3) **C** (0, -3) **D** (-3, 0)

6 (a) Which of the following points will give a vertical line passing through the x-axis when joined in a straight line to (10, -6)?

A (-6, -2) **B** (4, -6) **C** (10, 0) **D** (0, 10)

(b) Which of the following points will give a horizontal line passing through the y-axis when joined in a straight line to (8, 13)?

A (4, 2) **B** (8, -3) **C** (-5, 0) **D** (0, 13)

7 Rule a set of axes to form a Cartesian plane on a piece of grid or graph paper. Allow for a scale from -9 to 9 on the x-axis and -4 to 4 along the y-axis. Plot the following points and join them in the order given to form a picture. You may like to colour your picture when complete.

(-4, -1) (-5, -1) (-5, 0) (-9, 1) (-6, 3) (-1, 2) (-4, -1) (-4, -2) (-1, -4) (0, -3) (-1, 2) (0, 3) (1, 2) (6, 3) (9, 1) (5, 0) (5, -1) (4, -1) (1, 2) (0, -3) (1, -4) (4, -2) (4, -1) STOP

Now join (3, 4) to (0, 2.5) and join (-3, 4) to (0, 2.5).

8 Rule a set of axes to form a Cartesian plane on a piece of grid or graph paper. Allow for a scale from -8 to 9 along the x-axis and -11 to 11 along the y-axis. Join each of the following sets of points in the order given. When you reach the word STOP, lift your pencil and start again from the next pair of coordinates.

Join (3, 0) (3, 5) (2, 5) (2, 4) (1, 4) (1, 5) (0, 5) (0, 4) (-1, 4) (-1, 5) (-2, 5) (-2, 4) (-3, 4) (-3, 5) (-4, 5) (-4, -9) (3, -9) (3, 0) (4, 0) (4, -1) (5, -1) (5, 0) (6, 0) (6, -1) (7, -1) (7, 0) (8, 0) (8, -9) (3, -9) STOP

Join (5, 0) (5, 7) ($6\frac{1}{2}$, 10) ($6\frac{1}{2}$, 11) (9, 10) ($6\frac{1}{2}$, 10) (8, 7) (8, 0) STOP

Join (-7, -2) (-7, -9) (-4, -9) (-4, 3) (-5, 3) (-5, 2) (-6, 2) (-6, 3) (-7, 3) (-7, 6) ($-7\frac{1}{2}$, 8) (-8, 6) (-8, -1) (-7, -2) STOP

Join (-2, -6) (-1, -5) (0, -5) (1, -6) (-2, -6) (-2, -9) (-3, -11) (2, -11) (1, -9) (1, -6) STOP

Join (3, 5) (3, 7) ($3\frac{1}{2}$, 9) (4, 7) (4, 4) (3, 1) STOP Join (-4, 5) (-4, 9) (-2, 8) (-4, 8) STOP

Join (6, -5) (7, -5) (7, -2) (6, -2) (6, -5) STOP Join (6, 6) (7, 6) (7, 3) (6, 3) (6, 6) STOP

Join (2, -1) (2, 2) (1, 2) (1, -1) (2, -1) STOP Join (-3, -1) (-2, -1) (-2, -4) (-3, -4) (-3, -1) STOP

Join (-6, -3) (-5, -3) (-5, 1) (-6, 1) (-6, -3) STOP Join (3, 7) to (4, 7) STOP

Join (5, 7) to (8, 7) STOP Join (-8, 6) to (-7, 6) STOP

Join (-2, -6) to (-3, -1) STOP Join (1, -6) to (2, -11) STOP

9 **(a)** Which of the following coordinate pairs lies furthest up on the Cartesian plane?

A (-3, 2) **B** (2, 3) **C** (3, -2) **D** (1, -3)

(b) Which of the following coordinate pairs is the furthest to the left on the Cartesian plane?

A (-3, 2) **B** (2, 3) **C** (3, -2) **D** (1, -3)

10 The grid shown allows an archaeologist to reconstruct the layout of an old tomb. (Archaeologists call the point (0, 0) the datum point.) State the point(s) at which each of the following is found:

(a) the centre of the base of the treasure chest

(b) the middle of the rock face base

(c) the tip of the dagger

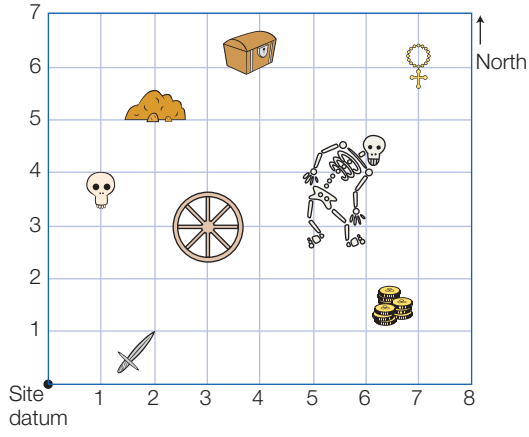
(d) the centre of the chariot wheel

(e) the right wrist and the left shoulder of the skeleton

(f) the four grid points that enclose all the coins

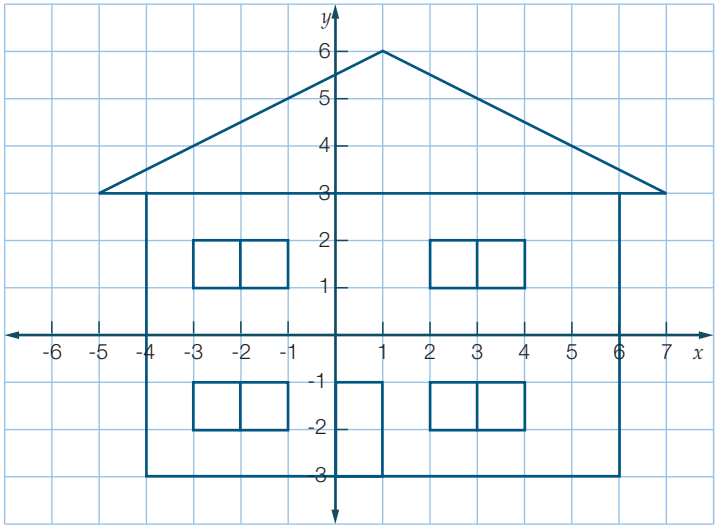
(g) the top of the skull lying by itself

(h) the chain at the point where the cross is attached.

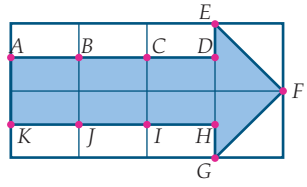


Reasoning, communicating and open-ended

11 Write instructions using coordinates so that this drawing could be reproduced by another student.



12 Draw this arrow shape on a Cartesian plane so that it lies in at least three quadrants, and give the coordinates of each of the points A to K.



13 On a piece of grid or graph paper, rule a set of axes to form a Cartesian plane. Allow for a scale from -5 to +5 on both axes.

(a) Using more than one quadrant on your Cartesian plane, use straight lines to draw a simple design of your own.

(b) Write instructions using coordinates so that your design could be reproduced by another student.

(c) Give your instructions to another student to see if they can draw your design exactly.

5.8 Patterns and plotting points

Need to Know

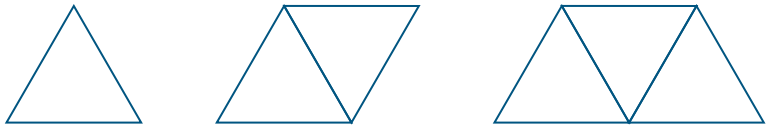
A graph in which a straight line can be drawn through all the points is called a **linear graph**.

The points given in the table of values can be plotted on a number plane and joined with a straight line. All points on a linear graph are related by a rule.

Technology Exploration: GeoGebra

A pattern of triangles

In GeoGebra, use the View menu to show the Spreadsheet view. In the Graphics view, right-click to show the Grid as well as the Axes.



(a) By joining triangles together as shown in the diagram above, use the Spreadsheet view in GeoGebra to copy and complete the following table:

Number of triangles (t)	1	2	3	4	5
Number of matchsticks (m)					

(b) Use GeoGebra to plot the points from the table. Choose any two points and use the 'Line through Two Points' tool  to draw a line through these points.

(c) Do all the other points lie on this line?

(d) Find the rule that connects t and m.

(e) Enter this rule in the input bar to draw the graph of this rule. (You will need to use x for t and y for m.) What do you notice?

(f) Repeat **(a)**–**(e)** using squares.

(g) Repeat **(a)**–**(e)** using pentagons.

Worked Example 14

WE14

(a) Plot the points (-2, -1), (-1, 0), (0, 1), (1, 2) and (2, 3) on a number plane.

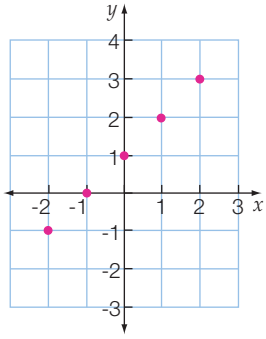
(b) Rule a straight line passing through all the points.

(c) Summarise the set of points in a table of values.

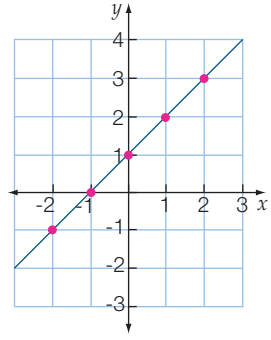
(d) Write down the rule linking the x- and y-values.

Working

(a)



(b)



(c)

x	-2	-1	0	1	2
y	-1	0	1	2	3

(d) The rule is $y = x + 1$.

Worked Example 15

WE15

(a) List the coordinates of the points given in the table of values opposite.

(b) Plot these points on a number plane.

(c) Rule a straight line passing through all the points.

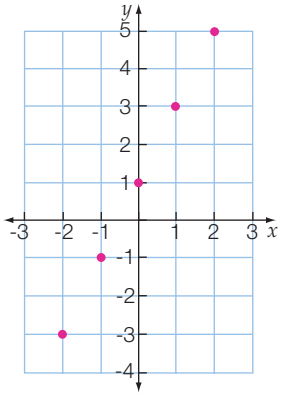
(d) Write down the rule linking the x- and y-values.

x	-2	-1	0	1	2
y	-3	-1	1	3	5

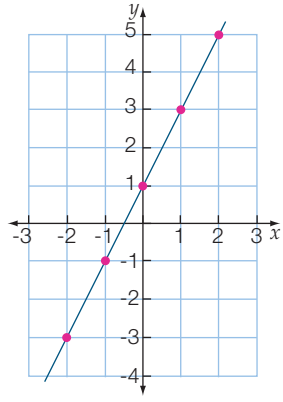
Working

(a) (-2, -3), (-1, -1), (0, 1), (1, 3), (2, 5)

(b)



(c)



(d) The rule is $y = 2x + 1$.

Answers page 474

5.8 Patterns and plotting points

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q10	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10

Equipment required: Graph paper

Fluency and understanding

WE14

1 **(a)** Plot the points (-2, 0), (-1, 1), (0, 2), (1, 3) and (2, 4) on a number plane.

(b) Rule a straight line passing through all the points.

(c) Summarise the set of points in a table of values.

(d) Write down the rule linking the x- and y-values.

WE15

2 **(a)** List the coordinates of the points given in the table of values below.

x	-1	0	1	2	3
y	-3	-1	1	3	5

(b) Plot these points on a number plane.

(c) Rule a straight line passing through all the points.

(d) Write down the rule linking the x- and y-values.

3 **(a)** Plot the points (1, 6), (2, 5), (3, 4), (4, 3), (5, 2) and (6, 1) on a number plane.

(b) Join the points in this order.

(c) Summarise the set of points in a table.

(d) Write down the rule linking the x- and y-values.

4 Which one of the points (-1, -1), (2, 2), (3, 4) and (5, 5) will not lie on the same straight line as the three other points?

A (-1, -1) **B** (2, 2) **C** (3, 4) **D** (5, 5)

5 **(a)** Plot the points (-2, 8), (0, 6) and (5, 1) on a number plane.

(b) Draw a straight line passing through all the points.

(c) What is the y-coordinate of a point on the line if its x-coordinate is -1?

(d) What is the x-coordinate of a point on the line if its y-coordinate is 5?

6 **(a)** Plot the points (1, 1), (1, 5), (5, 5) and (5, 1) on a number plane and join the points in order.

(b) Name the geometrical figure drawn in part **(a)**.

Reasoning, communicating and open-ended

7 **(a)** Plot the points (0, 0), (2, 2) and (-5, -5) on a number plane.

(b) Draw a straight line passing through all the points.

(c) Write down the coordinates of three other points through which the line passes.

(d) Summarise the set of points in a table.

(e) Write down the rule linking the x- and y-values.

(f) By substituting $x = -3$ into the rule, show that the point (-3, -3) lies on the line.

8 **(a)** Plot the points (-1, -5) and (5, 13) on a number plane and join them with a straight line.

(b) Write down the coordinates of three other points through which the line passes.

(c) Summarise the set of points in a table.

(d) Write down the rule linking the x- and y-values.

(e) Substitute $x = 0$ into the rule to show that (0, -3) does not lie on the line.

(f) Plot (0, -3) on the number plane to confirm it does not lie on the line.

9 **(a)** Plot the point (3, 3) on a number plane.

(b) Draw a straight line passing through this point and one other point.

(c) Write down the coordinates of two other points through which the line passes.

(d) Write down the rule linking the x- and y-values for this straight line.

(e) Repeat **(b)**, **(c)** and **(d)** using (3, 3) and a different point to find a different rule.

10 Using -4 to 4 on the x- and y-axes:

(a) choose a point and write down its coordinates

(b) choose two other points that will form a line that is not vertical or horizontal

(c) find the rule for that line.

(d) Give your instructions to another student to see if they can draw your design exactly.

5.9 Interpreting graphs

Need to Know

Sometimes, when plotting points showing information relating to two variables, the points should not be joined. This type of graph is called a **point graph**.

If we join points with straight lines, they may not form a linear graph, as the points may not all lie on the same straight line. These graphs are called **line graphs**. Examples of line graphs are travel graphs where time is plotted on the x-axis and distance is plotted on the y-axis. When lines are horizontal, the distance does not change. When lines are steepest, the speed is greatest.

Worked Example 16

WE16

This graph shows the age and height of Faye and Morgana.

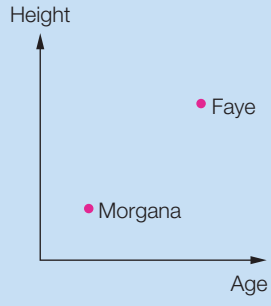
Answer TRUE or FALSE to each of these statements.

(a) Faye is taller than Morgana.

(b) Morgana is older than Faye.

(c) Faye is younger than Morgana.

(d) Morgana is shorter than Faye.



Working

(a) The point for Faye is higher, so she is taller than Morgana. TRUE

(b) The point for Faye is further to the right, so she is older than Morgana. FALSE

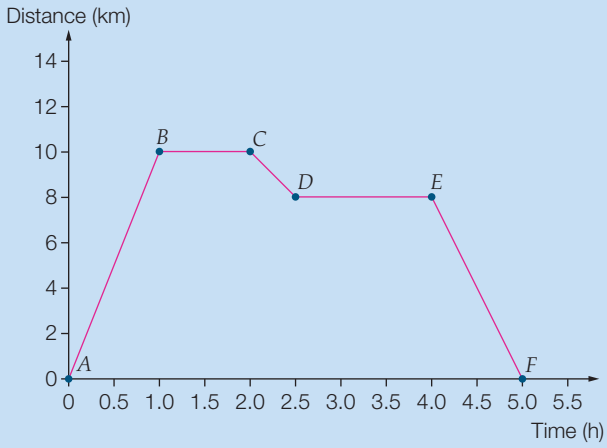
(c) The point for Morgana is further to the left, so she is younger than Faye. FALSE

(d) The point for Morgana is lower, so she is shorter than Faye. TRUE

Worked Example 17 WE17

May and Kim go jogging. They leave from Kim's house, jog to their friend Jenny's place, stay there for a swim, then jog to May's place. They stay there for a while and watch a DVD before jogging back to Kim's house.

Using the graph given on the right that shows their journey, answer the following questions.



(a) What are the graph coordinates of Kim's house?

(b) How far away from Kim's house does Jenny live?

(c) How long did it take to reach Jenny's house?

(d) How long did they stay at Jenny's house?

(e) How far from Kim's place does May live?

(f) How long did they stay at May's house?

(g) How long were they away from Kim's house?

(h) When were they jogging the fastest?

Working

(a) (0, 0)

(b) 10 km

(c) 1 h

(d) 1 h

(e) 8 km

(f) $1\frac{1}{2}$ h

(g) They were away for 5 hours.

(h) They jogged fastest from Kim's to Jenny's place.

5.9 Interpreting graphs

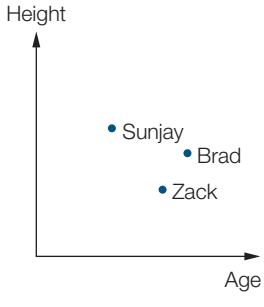
Answers page 476

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q11, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q12, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13

Fluency and understanding

1 This graph shows the age and height of Sunjay, Zack and Brad. WE16



Answer TRUE or FALSE to each of these statements.

(a) Brad is taller than Sunjay.

(b) Zack is the shortest.

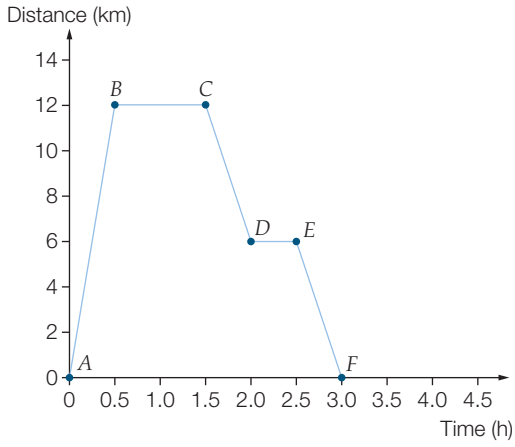
(c) Brad is older than Zack.

(d) Sunjay is the youngest.

WE17

2 Ray and Jim go for a bike ride. They leave from Jim's house, ride to their friend Terry's house, stay there for lunch, and then ride to Ray's house. After a short break, they ride back to Jim's house.

Using the graph given opposite that shows their journey, answer the following questions.



(a) What are the graph coordinates of Jim's house?

(b) How far away from Jim's house does Terry live?

(c) How long did it take to reach Terry's house?

(d) How long did they stay at Terry's house?

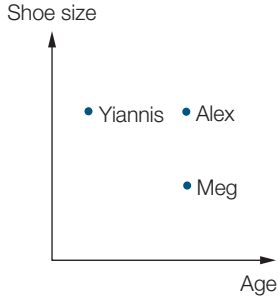
(e) How far from Jim's house does Ray live?

(f) How long did they stay at Ray's house?

(g) How long were they away from Jim's house?

(h) When were they riding the fastest?

3 Use the graph opposite to answer the following questions.



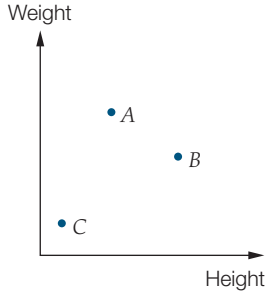
(a) Who is the youngest?

(b) Who takes the smallest shoe size?

(c) What can you say about the shoe sizes of Yiannis and Alex?

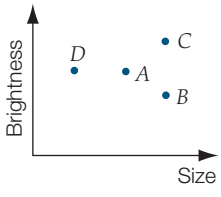
(d) What can you say about the ages of Meg and Alex?

4 Consider this graph showing the weight and height of an elephant, a giraffe and a kangaroo. Match each point with the animal it represents.

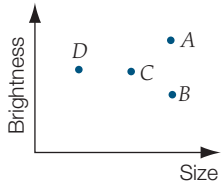


5 Four light globes, *A*, *B*, *C* and *D* have the properties that globe *A* and globe *B* are equally bright and globe *C* and globe *D* are of equal size. Which graph can represent this situation?

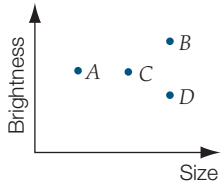
A



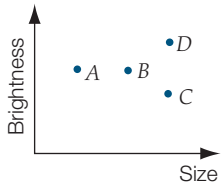
B



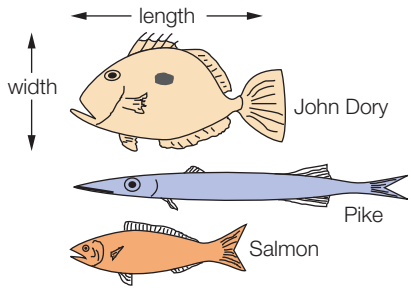
C

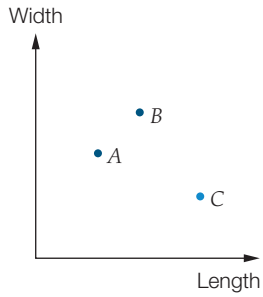


D



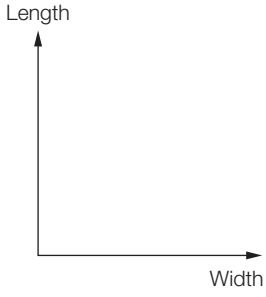
6 Consider this graph showing the widths and lengths of various fish.





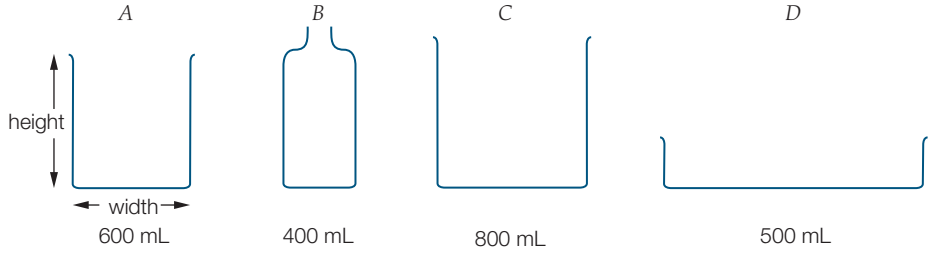
(a) Match each point with the fish it represents.

(b) Copy and complete this graph to show the positions of the points *A*, *B* and *C* for the three fish.



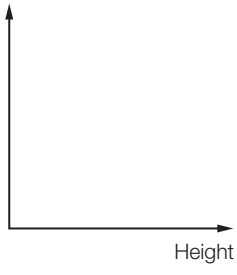
Reasoning, communicating and open-ended

7

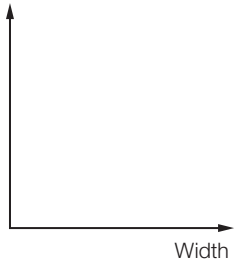


Copy and complete the following point graphs for the containers *A*, *B*, *C* and *D*.

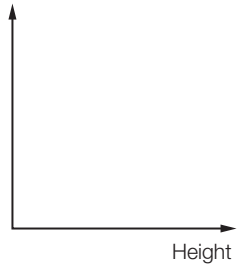
(a)



(b)

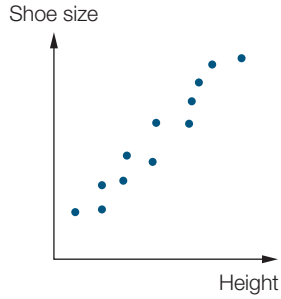


(c)

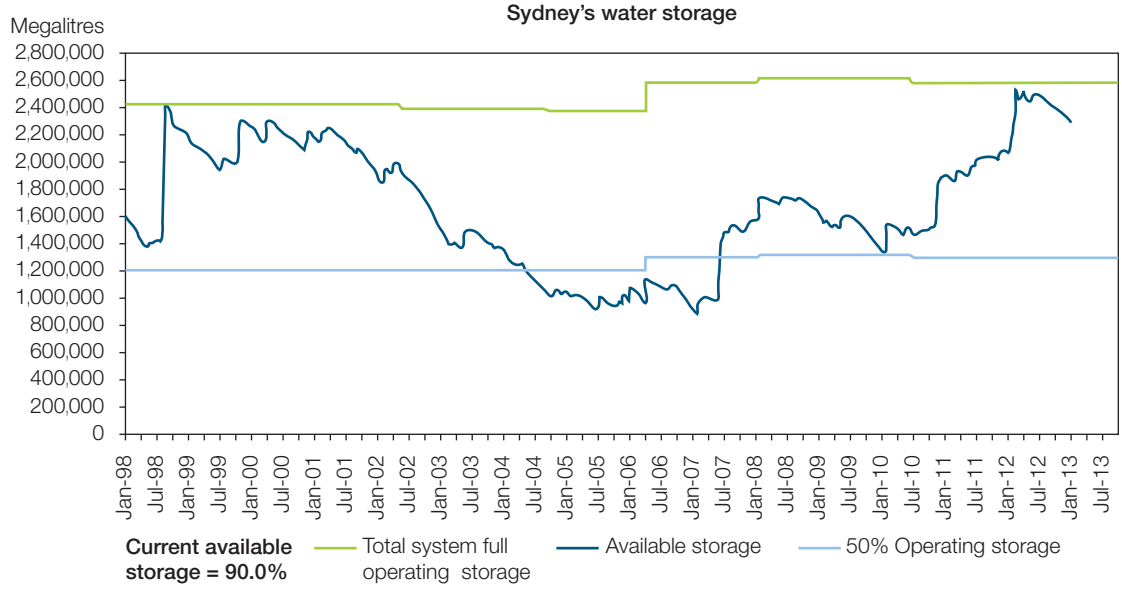


8 **(a)** What are the two variables shown in this graph?

(b) What happens to the shoe size as the height increases?



9 Large cities are usually surrounded by dams that collect rain to supply the city with water. The total amount of water stored in the dams is called the water storage. When this amount is calculated as a percentage of the total capacity of all the dams, it gives a measure of how full the dams are on average.



Source: http://www.sca.nsw.gov.au/dams-and-water/weekly-storage-and-supply-reports/2014/13-December-2012

Above is a graph of Sydney's water storage from the start of 1998 to the end of 2012. Use this graph to answer the following questions.

(a) What was the storage volume at the start of 1998?

(b) Convert this volume to a percentage of the total available water storage. Give your answer to the nearest whole percentage.

(c) When has the water storage been at maximum capacity?

(d) When has the water storage been at less than half of its maximum capacity?

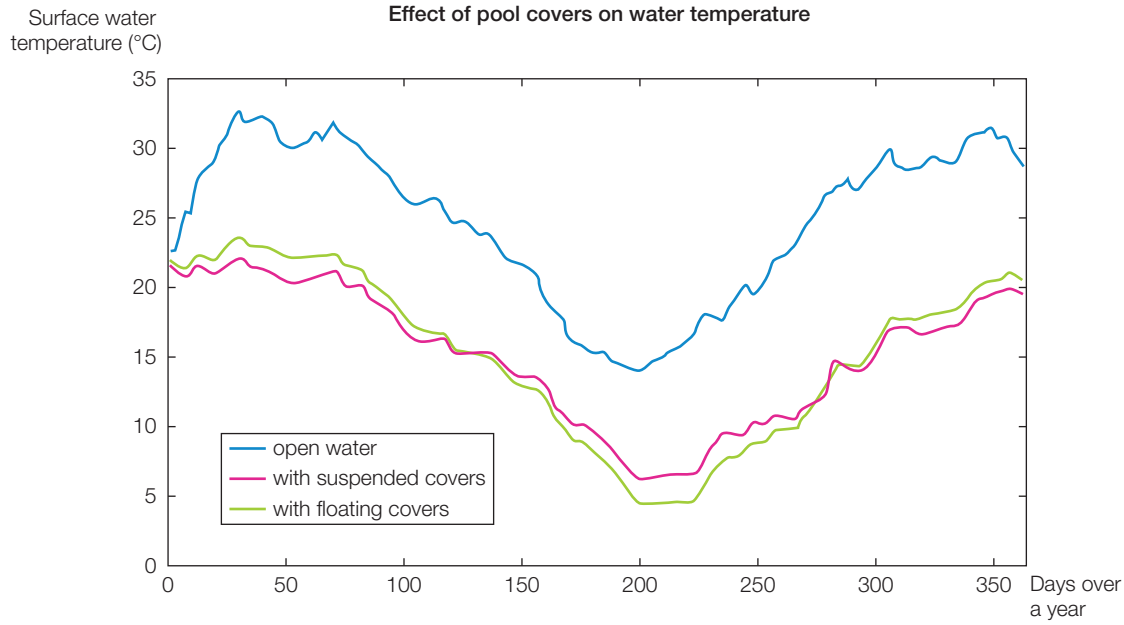
(e) Why has the water storage risen since January 2010?

(f) Why is there an increase in storage percentage in the second half of most years?

(g) What does the graph tell you about Sydney's rainfall from the start of 2001 to mid-2007?

(h) When was the water storage the lowest in the time period 1998–2012? What was the lowest volume reached?

10 Evaporation is a major source of water loss from storage facilities. The rate at which water evaporates increases as temperature increases. The lower the temperature of the water, the less water will be lost to evaporation.



Above is a graph showing the effect on water temperature of using covers of different types on a pool. Use this graph to answer the following questions.

(a) Do you think covers reduce evaporation? Give reasons for your answer.

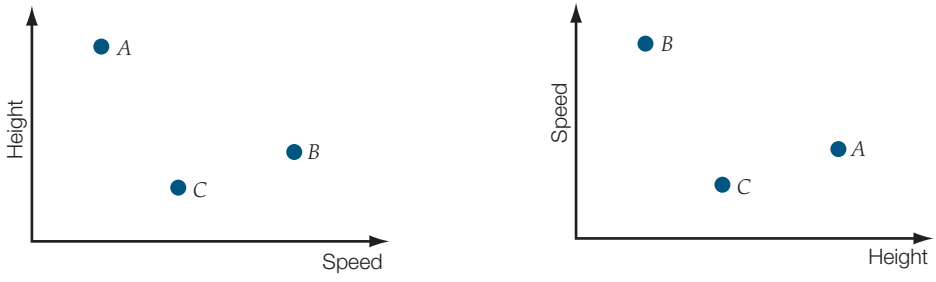
(b) Is one type of cover more effective than the other at reducing evaporation? Give reasons for your answer.

(c) When is a floating cover more effective than a suspended cover?

(d) What effect overall does the use of pool covers have on the water temperature?

11 Tom, Adam, Sheena and Toula play basketball. Sheena is shorter than Tom, and Toula scores more points than Adam. One point of view is that the taller the basketball player, the more points they score. Using the variables 'Height' and 'Points scored', draw two possible graphs that show the relationship.

12 Agnes transferred the points on the first graph to the graph with swapped axes. She left *C* in the same place but swapped the positions of *A* and *B*.



(a) Explain why Agnes' actions are not correct for these graphs. When would it be correct to do the simple swap with *A* and *B*?

(b) What would need to be the properties of a point if it is to stay in the same place?

(c) Draw what the second graph should look like.

13 Draw a travel graph of your own and write a few sentences to explain the journey.

Challenge 5



1 George has 15 coins, all 50c and 20c pieces. If the 20c pieces were 50c pieces and the 50c pieces were 20c pieces, he would have $2.10 more. How many 50c pieces does he actually have?

2 If $p \bigstar q$ means $3(p + q)$, what is the value of $5 \bigstar (2 \bigstar 8)$?

3 Steven and Claire live next door to each other. The product of their house numbers is 483. What are their house numbers?

4 If $x > 4$, put these terms in order from smallest to largest.

$$\frac{x}{4} \qquad \frac{4}{x} \qquad \frac{4}{x+1} \qquad \frac{x+1}{4}$$

5 Each letter represents one of the digits 1, 2, 3, 4 or 5.

$$\begin{array}{r} \text{K L} \\ \times \quad \text{M} \\ \hline \text{P N} \\ \hline \end{array}$$

The answer to this multiplication is correct.

Which statement is correct?

A K = 3 **B** L = 3 **C** M = 3 **D** N = 3

6 What are possible positive values for a and b if $(a \times b)^2 = a \times b^2$?

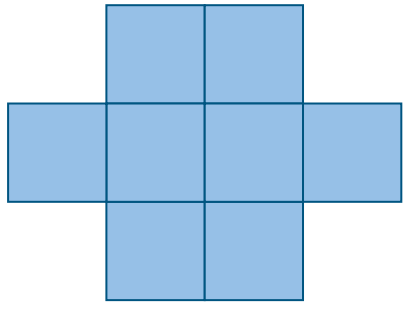
7 In a 30-question test you earn 9 marks for every correct answer and lose 5 marks for every wrong answer. You must attempt every question.

(a) How many marks do you get for 15 correct answers? (Assume you attempt all the questions.)

(b) What is the greatest number of marks you can get if you have more answers wrong than correct?

(c) Is it possible to score exactly zero on the test? Explain why or why not.

8 Place one of the numbers 1, 2, 3, 4, 5, 6, 7 and 8 in each square so that no consecutive numbers are beside each other; that is, 2 is not next to a 3 either above, below, to the left or to the right.



9 If half the number represented by x is 24, what is the value of $2x$?

10 The sum of three consecutive integers is 90. What is the smallest of the three integers?

A 28 **B** 29 **C** 30 **D** 31

11 Find two numbers which when multiplied together make 1 000 000 if neither of the numbers contains any zeroes.

Chapter review 5

D.I.Y. Summary

Key Words

axes	define	like terms	point graph	table of values
Cartesian plane	equation	linear graph	pronumeral	terms
coefficient	evaluate	line graph	quadrant	unknown
constant	expression	ordered pair	relationship	unlike terms
coordinates	formula	origin	substitute	variable

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 A _____ is a letter or a symbol that represents a number.

2 Completing a _____ is helpful when finding the _____ that describes a pattern.

3 To find the value of the expression $2x + 3$, when $x = 1$, we need to _____ 1 for x, and _____ the expression.

4 The _____ is the point (0, 0). It is the point where the two _____ of the Cartesian plane intersect.

5 A pronumeral represents an _____ or a _____.

6 A number written next to a pronumeral is the _____ of the pronumeral. The pronumeral is multiplied by that number.

7 To simplify the expression $3x + 4x + 5$ you collect _____.

8 The _____ (3, 5) can be plotted on the _____ by finding the point 3 across and 5 up. Draw a diagram clearly showing this point plotted in its correct location.

Fluency and understanding

1 Write the following situations using algebra. Ex.5.1

(a) There are 12 biscuits in a packet. How many are in n packets?

(b) Tarin has a packet of p lollies. He eats half, then gives 3 to his brother. How many lollies does he have left?

(c) Multiply p by 7, then subtract 9.

(d) Subtract g from 11, then divide by 5.

2 Write an equation for each of the following situations, taking care to define all the variables you have used. Ex.5.2

(a) Joe's age in years is four plus three times Beth's age.

(b) Three bottles of juice and five salad rolls cost $18.60.

3 For each of the following rules: Ex.5.3

(i) draw a flowchart that shows how to get to y from x

(ii) write the rule using algebra

(iii) copy and complete the table of values for that rule.

(a) Add three to x to get y.

x	57	34	12	4	1.1	64
y						

(b) To find y, multiply x by three, then subtract five.

x	4	3	$2\frac{1}{2}$	9	10	12
y						

4 Choose the correct algebraic notation for each rule. Ex.5.3

(a) y is equal to x plus 6.

A $y = x - 6$ **B** $y = x + 6$ **C** $y = x \times 6$ **D** $x = y + 6$

(b) Subtract five from x, then multiply by three to get y.

A $y = (x - 3) \times 5$ **B** $y = (x + 5) \times 3$ **C** $y = (x - 5) \times 3$ **D** $y = x - 5 \times 3$

5 For each of the following formulas, evaluate p by substituting the given values of a. Ex.5.4

(a) $p = 6 - 20a$ **(i)** $a = 3$ **(ii)** $a = -1$

(b) $p = 2(a - 4) + 11$ **(i)** $a = 2$ **(ii)** $a = -5$

6 If you substitute $x = 3$ into the formula $y = 5x - 2$, the value of y is: Ex.5.4

A $y = 3$ **B** $y = 5$ **C** $y = 13$ **D** $y = 17$

7 Use each of the following rules to complete the tables of values. Ex.5.4

(a) $y = x - 7$

x	7	9	12	20	8.1	107
y						

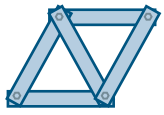
(b) $y = 10(x - 3)$

x	4	$5\frac{1}{2}$	13	7	78	54
y						

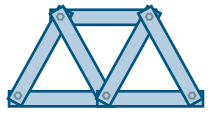
8 The frame of a bridge is made up of triangular sections. The triangles are made up of girders held together by bolts. Ex.5.5



1 triangle
3 girders
3 bolts



2 triangles
5 girders
4 bolts



3 triangles
7 girders
5 bolts

(a) Complete the table of values by continuing the pattern.

Number of triangles (t)	1	2	3	4	5
Number of girders (g)	3	5			
Number of bolts (b)	3	4			

(b) Write down the rule that relates the number of triangles, t, to the number of girders, g.

(c) Write down the rule that relates the number of triangles, t, to the number of bolts, b.

(d) **(i)** How many girders are needed to create each extra triangular section?

(ii) How is this reflected in the table of values and the rule?

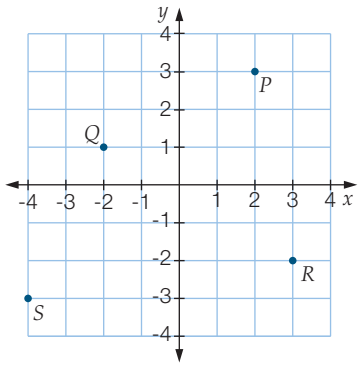
(e) **(i)** How many bolts are needed for each new triangular section?

(ii) How is this reflected in the table of values and the rule?

9 Simplify each expression by adding or subtracting like terms. Ex.5.6

(a) $12a - 7a$ **(b)** $6a + 12b - 7a + 11b$ **(c)** $x + y + 3x$

10 Write the coordinates of each of the following points shown on the Cartesian plane opposite. Ex.5.7



(a) P **(b)** Q

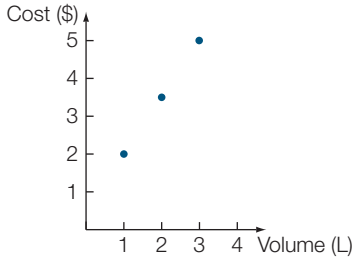
(c) R **(d)** S

11 **(a)** Plot the points (-1, -4), (1, 2), (2, 5) and (3, 8) on a number plane and draw a straight line passing through all the points. Ex.5.8

(b) Summarise the set of points in a table.

(c) Write down a rule linking the x- and y-values.

12 Juice is sold in three sizes: 1 L, 2 L and 3 L. Use the graph opposite to answer the following questions. Ex.5.9



(a) How much does a 2 L juice cost?

(b) Which size is the best value? Why?

(c) Why is this information given in a point graph?

13 Gayle, the kindergarten teacher, has eight packets of crayons with n crayons in each packet, plus an extra three loose crayons.

(a) Write an expression for the total number of crayons Gayle has.

(b) If all the crayons are to be shared equally between the 15 children in Gayle's class so that none are left over, write an expression to show how many crayons each child receives.

(c) Find the smallest possible value for n.

14 Draw a flowchart for each of the following rules.

(a) $h = 7g$ **(b)** $d = 9c - 2$ **(c)** $d = \frac{c}{10} + 13$ **(d)** $h = 5(g + 8)$

15 Tamsin is working out the number of bottles of soft drink she needs for a party. She is going to allow one bottle for every four people, plus five extra bottles.

(a) Write out Tamsin's rule using algebraic notation. Let b = number of bottles of drink required and n = number of people at the party.

(b) If Tamsin has 16 people at the party, how many bottles of drink will she need?

16 Simplify if possible:

(a) $2a - 7b + 3a + 2$ **(b)** $5a + 12b - 7 - 3b$ **(c)** $x + y + xy$

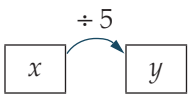
17 The points (0, 1), (2, 5) and (5, 11) are plotted on a number plane and a straight line drawn through the points. Which one of the following points also lies on the line?

A (1, 2) **B** (3, 7) **C** (3, 6) **D** (4, 10)

Reasoning and communicating

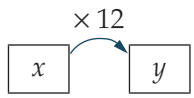
18 The following tables of values have only the y-values filled in. Work backwards along the flowchart to determine the values of x that were used, and complete the tables.

(a)



x						
y	2	-7	20	1	0	-6

(b)



x						
y	72	120	36	-12	0	60

19 For each of these tables, find the rule that is being used.

(a)

x	76	54	8	28	9	103
y	69	47	1	21	2	96

(b)

x	15	10	2	7.3	9	22
y	34	24	8	18.6	22	48

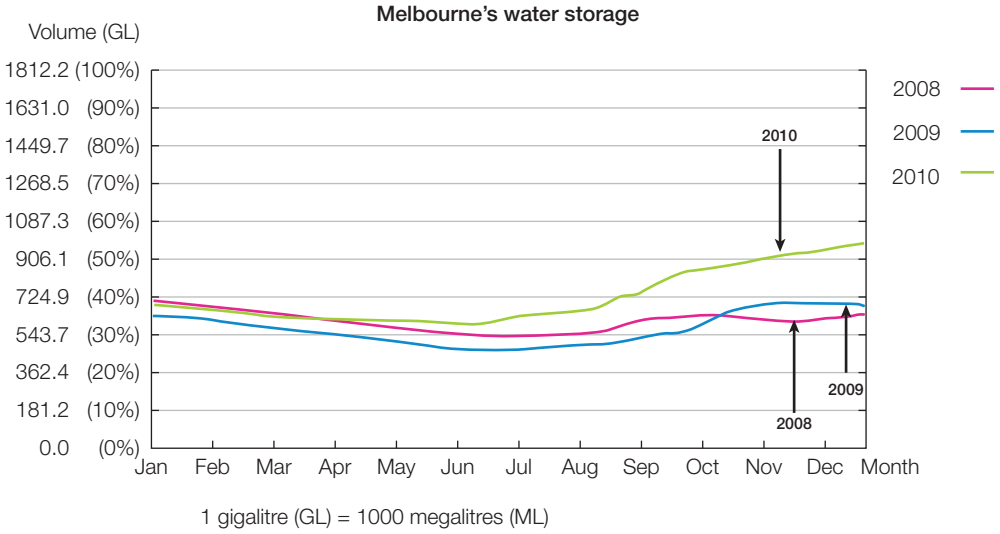
20 Jack has \$21.00 to buy some apples and oranges. An apple costs \$1.80 and an orange costs \$1.40.

(a) Using appropriate pronumerals to represent the number of apples and oranges purchased, write an equation to show the situation when Jack spends all of the \$21.00.

(b) How many apples and how many oranges could Jack buy using all the money?

(c) If Jack buys four apples and five oranges, how much change will he receive?

21



Above is a graph of Melbourne's water storage for the years 2008, 2009 and 2010.

Use this graph to answer the following questions.

(a) Compare the storage levels at the start of 2008, 2009 and 2010. Were the levels similar? Which year started with the lowest storage level? What was the difference between the highest and lowest storage percentage at the start of each year?

(b) Compare the storage levels at the end of 2009 and 2010. What is the difference in storage percentage?

(c) During which 3 months over the 3-year period was the storage level the highest?

(d) During which 3 months over the 3-year period was the storage level the lowest?

NAPLAN practice 5

Numeracy: Non-calculator

1 A number is multiplied by eight and then five is added. The answer is 61. What is the number?

2 x and y stand for numbers. x and y are connected by a rule.

x	2	3	5	8
y	8	13	23	38

What is the rule?

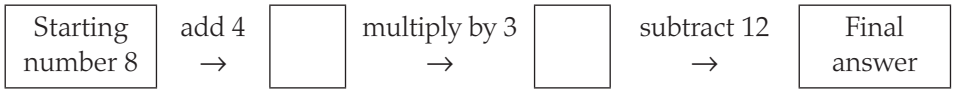
A $y = 4x + 1$

B $y = 5x - 2$

C $y = 4x$

D $y = 10x - 27$

3 Ting Li followed this rule. She started with 8.



(a) What is the correct final answer using Ting Li's starting number of 8?

(b) Alex followed the same rule using a different starting number. Alex's starting number is -2. What was his final answer?

Numeracy: Calculator allowed

4 Ethan followed a different rule to Ting Li. There were three steps. Ethan started with the number 21 and ended up with a final answer of 30.

Starting number 21	Step 1 →		Step 2 →		Step 3 →	Final answer

(a) Write the numbers from 1 to 3 on the lines below to show an order of steps that Ethan followed.

Step _____ add 8

Step _____ divide by 3

Step _____ double the number

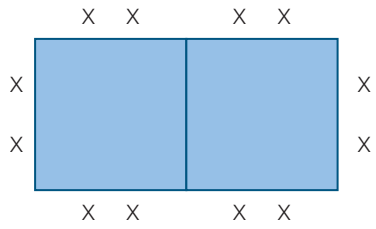
(b) Repeat the procedure in part **(a)** to get a final answer of 22.

5 A pack of 10 lollipops costs $4.60.
A pack of six costs $3.20.
You need to buy 22 lollipops.

What is the least amount you can pay?

6 Yusef investigated the seating plan for a restaurant using this table of values.

Number of tables	2	3	4	5
Number of people seated	12	16	20	?



The diagram shows the seating arrangement for 2 tables.

What rule did Yusef use to work out how many people can be seated at 5 tables?

A number of people seated = number of tables

B number of people seated = number of tables + 4

C number of people seated = number of tables × 6 − 1

D number of people seated = number of tables × 4 + 4

Measurement

6

Recall 6

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

1 Calculate the following.

(a) $3.4 + 6.8 + 4.2 + 2.1$ **(b)** $45.6 - 5.28$ **(c)** $54 + 11.3 - 26.04$

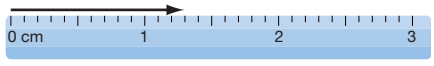
(d) 6×4.2 **(e)** 9.2×6.1 **(f)** 18.96×1000

(g) $2780 \div 100$ **(h)** $3 \div 10$ **(i)** $45.9 \div 1000$

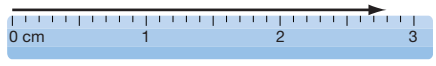
2 State the length of the arrow shown on each ruler:

(i) in millimetres **(ii)** in centimetres.

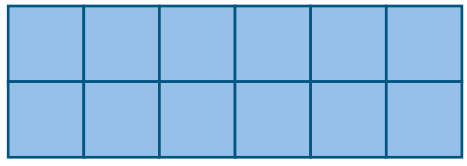
(a)



(b)



3 This rectangle is made up of centimetre squares. Calculate:



(a) the perimeter in cm

(b) the area in cm^2.

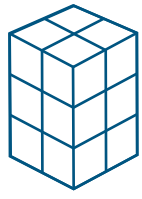
4 Copy and complete the following conversions.

(a) 2 km = _____ m **(b)** 30 mm = _____ cm **(c)** 5 m = _____ cm

(d) 1500 g = _____ kg **(e)** 4 L = _____ mL **(f)** 20 kg = _____ g

(g) 250 mL = _____ L **(h)** 1.75 m = _____ mm **(i)** 500 g = _____ kg

5 How many sugar cubes are in the stack shown here?



Syllabus references

Measurement and Geometry: Length

- ✓ Choose appropriate units of measurement for length **(ACMMG108) (Stage 3)**
- ✓ Calculate the perimeters of rectangles using familiar metric units **(ACMMG109) (Stage 3)**
- ✓ Convert between common metric units of length **(ACMMG136) (Stage 3)**
- ✓ Solve problems involving the comparison of lengths using appropriate units **(ACMMG137) (Stage 3)**
- ✓ Find perimeters of parallelograms, trapeziums, rhombuses and kites **(ACMMG196)**

Measurement and Geometry: Area

- ✓ Choose appropriate units of measurement for area and convert from one unit to another **(ACMMG195)**
- ✓ Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving **(ACMMG159)**

Measurement and Geometry: Volume

- ✓ Develop the formulas for the volumes of rectangular and triangular prisms and of prisms in general; use formulas to solve problems involving volume **(ACMMG198)**

6.1 Units of length

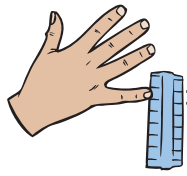
Stage 3

Need to Know

The metric system

The system of measurement used in Australia is known as the **metric** system. All lengths in the metric system are derived from dividing or multiplying a standard length, the **metre**, by powers of 10 (10, 100, 1000, …).

The metric units of **length** used most often are:

			
kilometre (km) 1 km = 1000 metres	**metre (m)** 1 m = 100 centimetres	**centimetre (cm)** 1 cm = 10 millimetres	**millimetre (mm)**

Estimating lengths

When estimating lengths, it helps to be able to picture the size of each unit of length, or to compare the object in question to a 'reference' measurement.

Converting units of length

larger units — km ⇄ m ⇄ cm ⇄ mm — smaller units

km → m: × 1000; m → km: ÷ 1000

m → cm: × 100; cm → m: ÷ 100

cm → mm: × 10; mm → cm: ÷ 10

To convert from a *larger* unit to a *smaller* unit, we *multiply*.
To convert from a *smaller* unit to a *larger* unit we *divide*.

Worked Example 1

WE1

Use the reference measurement to help you estimate:

(a) the length

(b) the height

of this motor home.



Working

(a) The motor home is approximately 5 m long.

(b) The motor home is about 3 m high.

Worked Example 2 WE2

Copy and complete the following conversions.

(a) 62.4 km = _____ m

(b) 87.5 cm = _____ m

Working

(a) 62.4 km
= 62.4 × 1000 m
= 62 400 m

(b) 87.5 cm
= 87.5 ÷ 100 m
= 0.875 m

Worked Example 3 WE3

Copy and complete the following conversions.

(a) 8.7 m = _____ mm

(b) 530 cm = _____ km

Working

(a) 8.7 m
= 8.7 × 100 cm
= 870 cm
= 870 × 10 mm
= 8700 mm

(b) 530 cm
= 530 ÷ 100 m
= 5.3 m
= 5.3 ÷ 1000 km
= 0.0053 km

6.1 Units of length

Answers page 478

Navigator

Stage 3

Q1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q13, Q15, Q16, Q18	Q1, Q2 Column 2, Q3 Column 2, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q15, Q16, Q18	Q1, Q2 Column 3, Q3 Column 3, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q18

Fluency and understanding

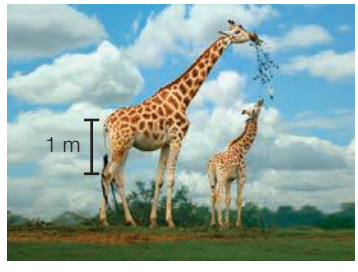
1 Use the reference measurement to help you estimate the following. WE1

(a)



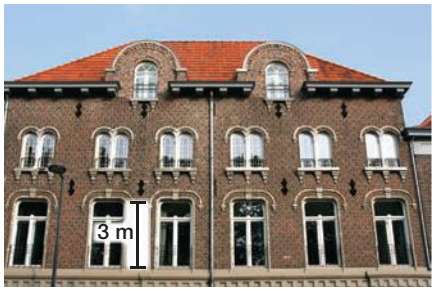
The length of this car.

(b)



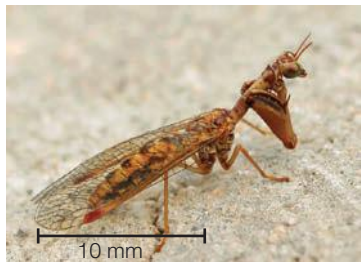
The adult giraffe's height.

(c)



The height of the tallest part of this building.

(d)



The length of this mantis fly.

WE2

2 Copy and complete the following conversions.

(a) 5 km = ____ m **(b)** 3.6 km = ____ m **(c)** 0.008 km = ____ m

(d) 65 m = ____ cm **(e)** 0.55 m = ____ cm **(f)** 1.2 m = ____ cm

(g) 2.9 cm = ____ mm **(h)** 6.1 cm = ____ mm **(i)** 0.35 cm = ____ mm

(j) 90 cm = ____ m **(k)** 0.3 cm = ____ m **(l)** 3750 cm = ____ m

(m) 4200 m = ____ km **(n)** 570 m = ____ km **(o)** 9.2 m = ____ km

(p) 80 mm = ____ cm **(q)** 255 mm = ____ cm **(r)** 1.8 mm = ____ cm

WE3

3 Copy and complete the following conversions.

(a) 3.2 m = ____ mm **(b)** 4.95 m = ____ mm **(c)** 9000 cm = ____ km

(d) 34 500 cm = ____ km **(e)** 0.003 km = ____ cm **(f)** 560 mm = ____ m

(g) 97 mm = ____ m **(h)** 0.342 km = ____ mm **(i)** 7800 mm = ____ km

(j) 1.92 km = ____ cm **(k)** 2400 cm = ____ km **(l)** 0.89 m = ____ mm

4 State which metric unit would be most appropriate for measuring:

(a) the length of your foot **(b)** the distance from Sydney to Melbourne

(c) the distance around an athletics track **(d)** the length of a shoelace

(e) the distance run in a marathon **(f)** the length of your bed.

5 **(a)** Which of the following gives the best estimate of the distance across a suburban street?

A 50 cm **B** 5 m **C** 50 m **D** 5 km

(b) Which of the following gives the best estimate of the length of a mouse's tail?

A 0.1 cm **B** 1 cm **C** 10 cm **D** 100 cm

(c) The diameter of a 20 cent coin is about:

A 2.8 mm **B** 2.8 cm **C** 2.8 m **D** 2.8 km

(d) The length of a ball-point pen could be:

A 15 mm **B** 50 mm **C** 15 cm **D** 15 m

6 Place the lengths below in order from smallest to largest, by first converting them to a common unit.

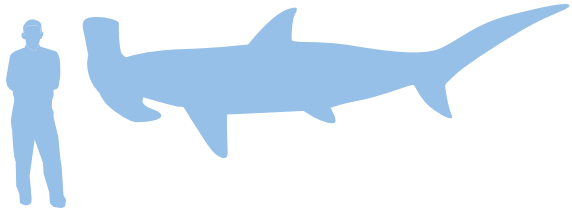
(a) 0.4 m, 4000 mm, 4 cm, 0.04 km **(b)** 3.2 km, 360 m, 290 000 cm, 3100 m

(c) 71.4 m, 0.64 km, 5600 cm, 820 000 mm **(d)** 0.9 cm, 90 mm, 0.095 m, 0.0089 km

7 At the Beijing Olympics in 2008, Australia's Steve Hooker won a gold medal and broke the Olympic record in the men's pole vault, clearing the bar at 596 centimetres. How many metres is this?

8 Situated on Heard Island, Big Ben is the bigger of Australia's two active volcanoes. Its summit is 2745 m above sea level. What is its height in kilometres?

9 The average man is 1.8 metres tall. Use the diagram to estimate the length of the hammerhead shark.



10 The bird-eating spider of South America has a body length of 89 mm and a leg span of 254 mm. Write these measurements in centimetres.

11 A giant jellyfish was once measured at 2290 mm across and 36 000 mm long. Convert these dimensions to metres.

12 Claire is building a desk and she wants it to be wide enough so that she can fit the length of two A4 sheets across it. If the length of an A4 sheet is 298 mm, how wide does Claire's desk have to be in millimetres? What is this length in centimetres?

Reasoning, communicating and open-ended

13 The direct distance 'as the crow flies' between Melbourne and Adelaide is about 800 km. Estimate the direct distance between:

(a) Canberra and Brisbane

(b) Adelaide and Perth

(c) Sydney and Darwin.



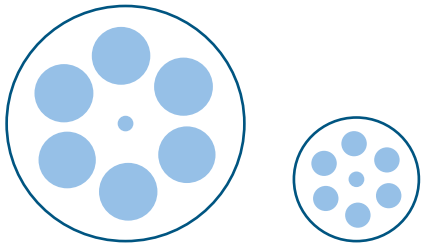
14 One of the fastest growing plants in the world is the bamboo plant. Its stem can grow 300 mm a day. If, on Monday morning, a bamboo plant was one metre tall, and it grew 300 mm every day, how tall will it be on Saturday morning? Write the answer in metres.

15 **(a)** Guess the answer to each of the following, then use your ruler to help you decide.

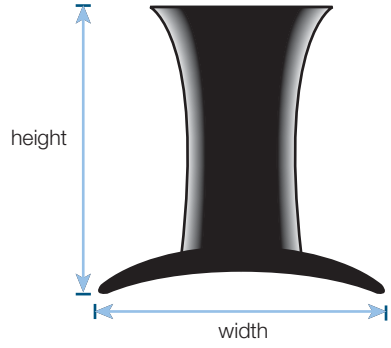
(i) Which horizontal line is longer?



(ii) Which diagram has the larger central circle?



(iii) Which is longer, the height of the hat, or its width?



(b) Explain each illusion or effect above and why you think it happens.

16 A builder jotted down the measurements needed for a door.

Why did the builder write these measurements using millimetres instead of cm or m?

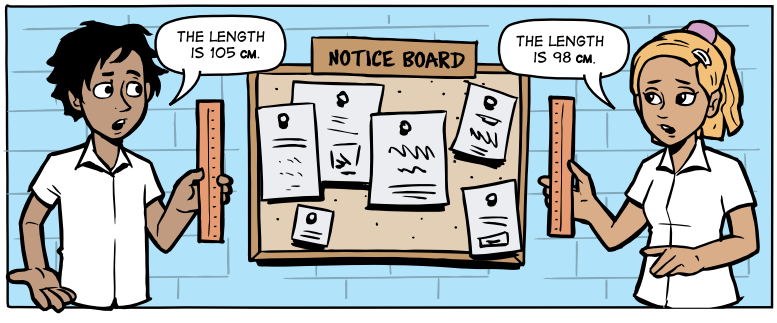
height of door	2032 mm
width of door	821 mm

17 The following are mistakes made by a student when converting units. For each, explain what mistake has been made and write the correct answer.

(a) 27 m = 0.27 cm ✗ (b) 765 mm = 7.65 cm ✗

(c) 3800 m = 3.08 km ✗ (d) 1356 cm = 1.356 m ✗

18 Two students have been asked to measure the length of the school notice board.



Suggest possible reasons for the difference in their answers.

Perimeter

6.2

Need to Know

Perimeter is the distance along the boundary of a shape.

To find the perimeter of a shape, simply add the lengths of all the sides, making sure they are all in the same units.

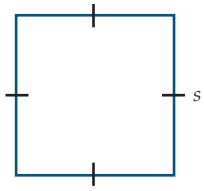
The perimeter of a square or a rectangle

Rules can be used to find the perimeter of regular shapes.

The pronumeral, l, is used to represent the length and the pronumeral, b, is used to represent the **breadth** (or width).

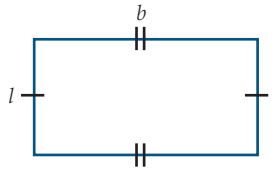
The pronumeral, s, is used to represent the length of the side of a square.

Perimeter of a square



$P = 4s$

Perimeter of a rectangle



$P = 2l + 2b$
or
$2(l + b)$

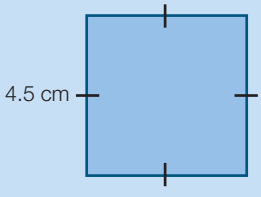
Sides that are marked in the same way (with the same number of small dashes) have the same length.

Worked Example 4

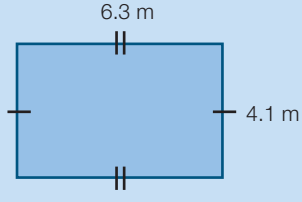
WE4

Find the perimeter of each of these shapes.

(a)



(b)



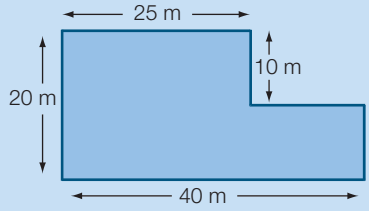
Working

(a) $P = 4s$
$= 4 \times 4.5$
$= 18$ cm

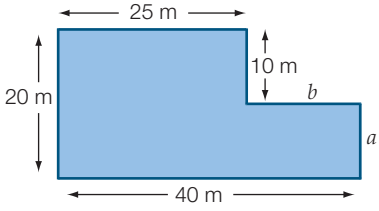
(b) $P = 2l + 2b$
$= 2 \times 6.3 + 2 \times 4.1$
$= 12.6 + 8.2$
$= 20.8$ m

Worked Example 5 WE5

Calculate the perimeter of this shape.



Working



$a = 20\text{ m} - 10\text{ m}$
$= 10\text{ m}$

$b = 40\text{ m} - 25\text{ m}$
$= 15\text{ m}$

$P = 10 + 25 + 20 + 40 + 10 + 15$
$= 120\text{ m}$

Answers page 479

6.2 Perimeter

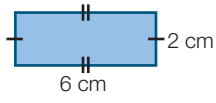
Navigator

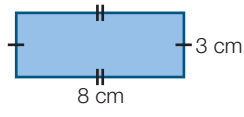
Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q12	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q11, Q12	Q1 (a)–(c), Q2 (a)–(c), Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12

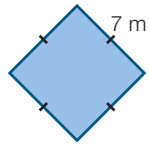
Fluency and understanding

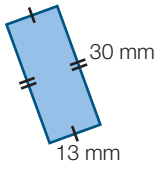
WE4

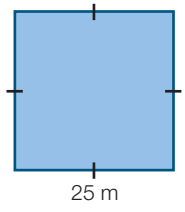
1 Find the perimeter of each of these shapes.

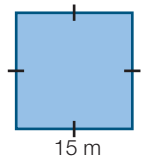
(a)


(b)


(c)


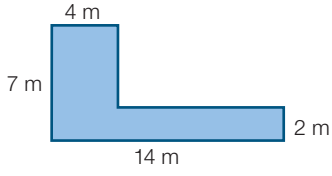
(d)


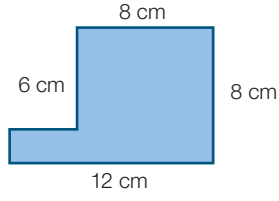
(e)


(f)


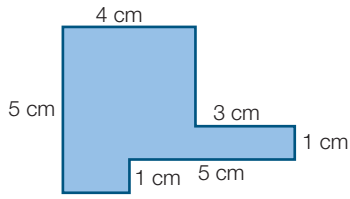
WE5

2 Calculate the perimeter of the shapes below.

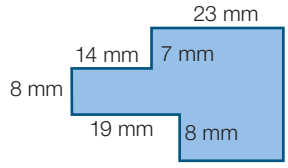
(a)


(b)


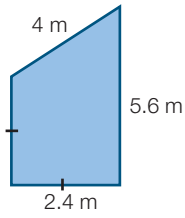
(c)



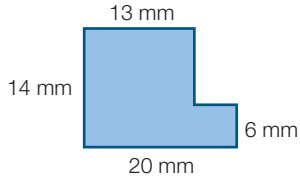
(d)



(e)



(f)



3 **(a)** The perimeter of a rectangle with a length of 25 cm and a width of 10 cm is:

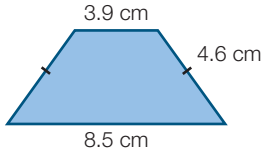
A 35 cm **B** 45 cm **C** 70 cm **D** 250 cm

(b) The perimeter of a square of side length 460 mm is:

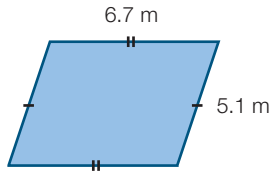
A 18.4 m **B** 9.2 m **C** 1.84 m **D** 0.92 m

4 Find the perimeter of these shapes.

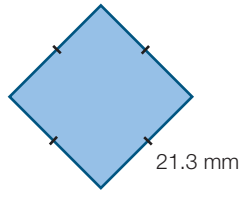
(a)



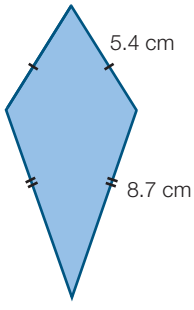
(b)



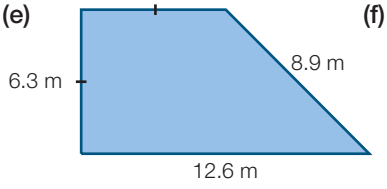
(c)



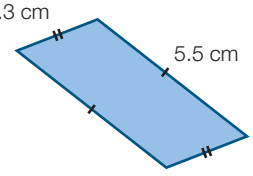
(d)



(e)

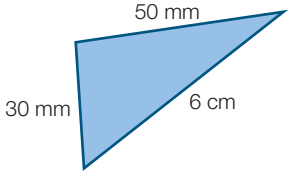


(f)

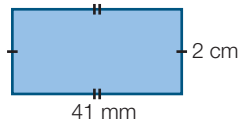


5 Calculate the perimeter of each of the following, first converting side lengths to the smaller unit when necessary.

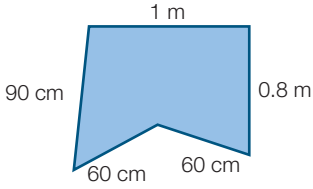
(a)



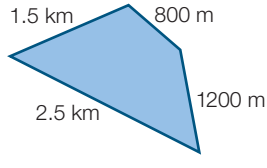
(b)



(c)

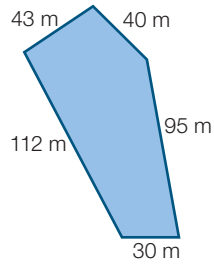


(d)



6 A group of PE students run around the perimeter of a court in their gymnasium, which is rectangular and has dimensions 38 m by 16 m. What distance will they run if they complete 6 laps?

7 Juan wishes to run two strands of wire around his property to mark its boundary. A plan of the property is shown on the right. If he can buy the necessary wire for 60 cents per metre, how much will the wire cost him?



8 Estimate the perimeter of each of the following objects.

(a) a 30 cm ruler **(b)** your desk

(c) a basketball court **(d)** your school's boundary fence.

Reasoning, communicating and open-ended

9 If the perimeter of a rectangle is 240 mm and its width is 20 mm, then its length is:

A 100 mm **B** 110 mm **C** 120 mm **D** 220 mm

10 The local swimming pool is rectangular, with dimensions of 25 m by 12 m. If it has a 1 m wide rectangular path around its perimeter, what is the distance along the outer edge of the path?

Hint

Drawing a diagram can help.

11 The length and width of a rectangle are both whole numbers in centimetres. Write down some possible dimensions if its perimeter is 84 cm.

12 'I am holding a picture of a shape with a perimeter of 16 cm,' the teacher said,'but it is not a rectangle or a triangle'. Draw two possible examples of the shape the teacher may be holding.

Problem solving

Ram's rulers

Ram is planning a woodwork model and needs to draw a line 6 cm long. He has been using as rulers two straight pieces of wood with no markings, but he knows their lengths are 10 cm and 8 cm. How can he use these pieces of wood to measure a 6 cm line?

Strategy options

- Guess and check.
- Work backwards.
- Make a model.

Area

6.3

Need to Know

Area is the amount of surface inside a plane (flat) shape. It is measured in 'square' units. When the area of a shape is calculated the number of squares that can fit inside the shape has been found.

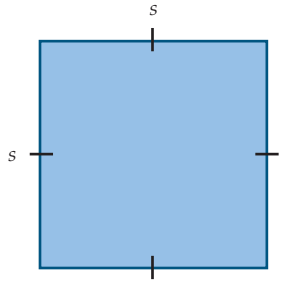
Units of area

A square millimetre	mm^2	1 mm length × 1 mm width = 1 mm^2
A square centimetre	cm^2	1 cm length × 1 cm width = 1 cm^2
A square metre	m^2	1 m length × 1 m width = 1 m^2
A square kilometre	km^2	1 km length × 1 km width = 1 km^2

The area of a square and a rectangle

Rules can be used to find the area of regular shapes.

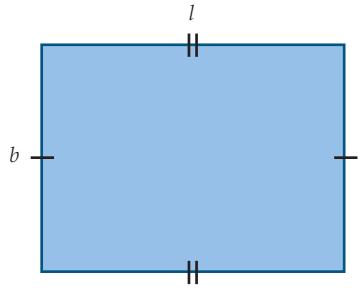
Area of a square



$A = s^2$

where s is the side length of the square

Area of a rectangle



$A = lb$

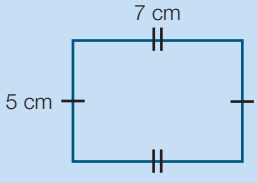
where l is the length and b is the breadth (or width) of the rectangle

Worked Example 6

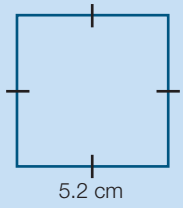
WE6

Calculate the area of the following shapes.

(a)



(b)



Working

(a) $A = lb$
$A = 7 \times 5$
$A = 35\ cm^2$

(b) $A = s^2$
$A = 5.2 \times 5.2$
$A = 27.04\ cm^2$

Answers page 479

6.3 Area

Navigator

Q1, Q2, Q3 (a), Q4, Q5, Q6, Q7, Q8, Q11, Q12, Q13, Q16, Q17, Q18, Q22, Q23	Q1 (a)–(f), Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13, Q15, Q16, Q17, Q18, Q19, Q20, Q21, Q22, Q23	Q1 (g)–(l), Q2, Q3 (b), Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12, Q14, Q15, Q16, Q17, Q19, Q20, Q21, Q22, Q23

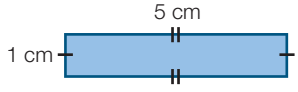
Equipment required: Calculator may be used for Questions **11–15**

Fluency and understanding

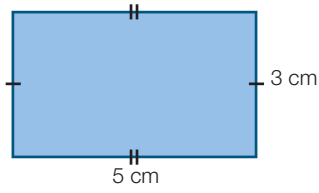
WE6

1 Calculate the area of the following shapes.

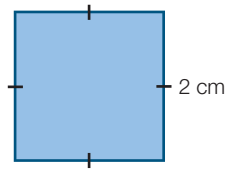
(a)



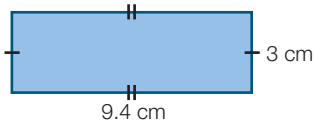
(b)



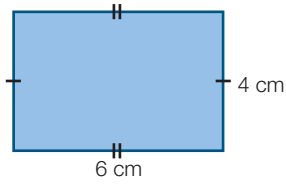
(c)



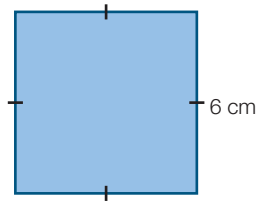
(d)



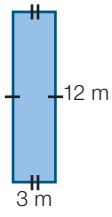
(e)



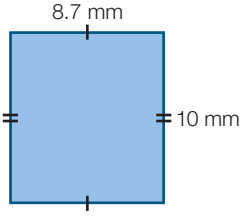
(f)



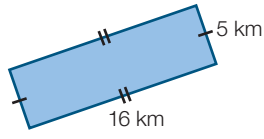
(g)



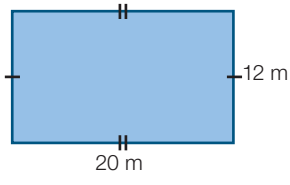
(h)



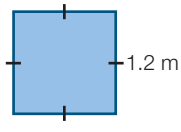
(i)



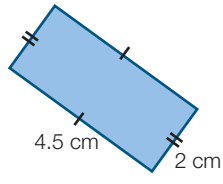
(j)



(k)



(l)



2 State which of the units of area—mm^2, cm^2, m^2 or km^2—would be most suitable for measuring the area of:

(a) a sheet of writing paper

(b) your smallest toenail

(c) a pizza

(d) a house block

(e) a football oval

(f) Tasmania

(g) a small watch face

(h) the floor of a classroom.

3 (a) A rectangle that has twelve rows of three square centimetres has an area of:

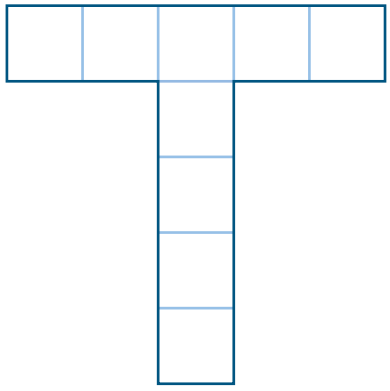
A 15 cm^2 **B** 30 cm^2 **C** 36 cm^2 **D** 72 cm^2

(b) If the area of a rectangle is 24 cm^2 and its width is 10 mm, then its length is:

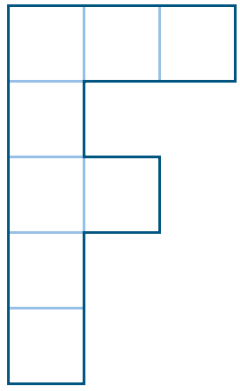
A 2.4 mm **B** 2.4 cm **C** 11 cm **D** 24 cm

4 The following shapes have been drawn on centimetre grid paper. Find **(i)** the perimeter and **(ii)** the area of each one.

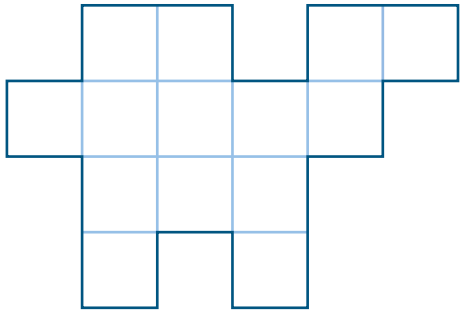
(a)



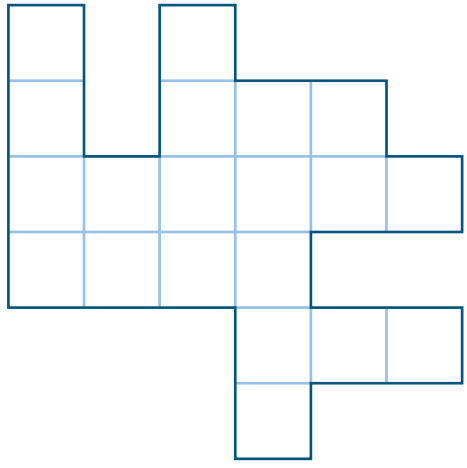
(b)



(c)

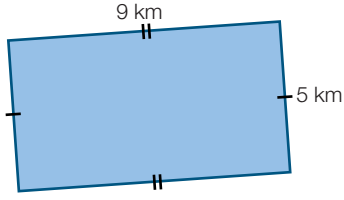


(d)

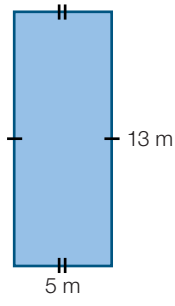


5 Find **(i)** the perimeter and **(ii)** the area of each shape below.

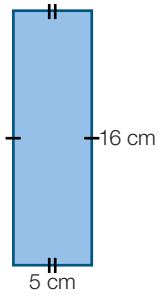
(a)



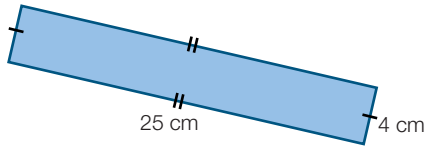
(b)



(c)



(d)



Hint

Convert all measurements to the same unit before finding the area.

6 Find the area in km^2 of a rectangle of length 3 km and width 800 m.

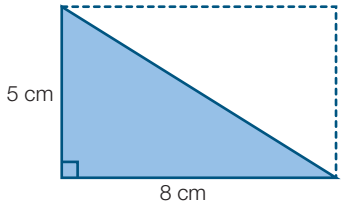
7 What is the area in cm^2 of a rectangle of length 450 mm and width 70 cm?

8 Find the width of a rectangle with length 7 cm and area 28 cm^2.

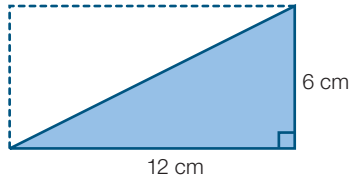
9 Find the length of a rectangle with width 2.6 m and area 13.52 m^2.

10 Find the area of each of the following triangles by first considering a rectangle.

(a)



(b)



11 The Ray family is installing solar power to their house. There will be 6 panels attached to their roof, each with dimensions of 2.4 m × 2.1 m.

(a) What is the area of each solar panel?

(b) What is the total area of the solar panels?

(c) If the solar panels generate a maximum of 6.5 kilowatt hours (kWh) of energy per square metre each day, how much energy could the Ray family's new solar system produce in a day?

12 Guy wishes to sow a lawn in a rectangular section of his yard, which is 8.5 m long and 6.9 m wide.

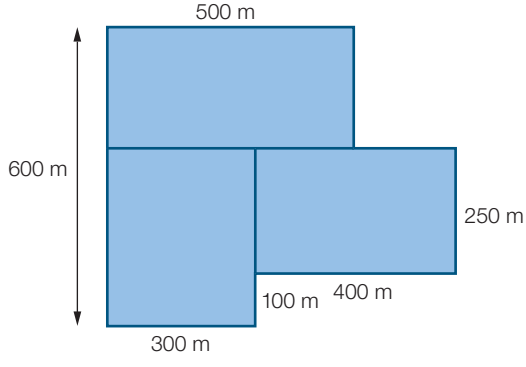
(a) How many grams of lawn seed will he need to use if the recommended seed coverage is 60 grams per square metre?

(b) What length of wood would Guy need to purchase to create a timber border for the section?

13 A builder wishes to estimate the number of bricks required to build a wall 4.2 m wide and 2.6 m high. If 1 square metre of brick wall contains 48 bricks, how many bricks are needed?

14 Jasmin decided to make a 45 cm square pillow for her study chair. One piece of fabric will be required for the front of the pillow, and one for the back. Each piece of fabric will need an extra 1 cm on all edges for the front and back to be sewn together. What area of fabric will Jasmin need to make the pillow?

15 A farmer wants to create three separate rectangular paddocks on his farm, as shown in the diagram.



(a) Find the area of the three paddocks using the plan provided.

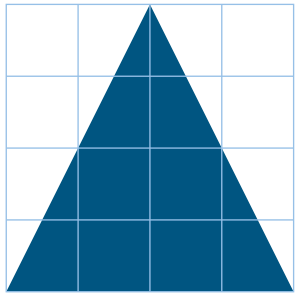
(b) To sow his paddocks, the farmer needs to spread 60 g of seed for every square metre. How many kilograms of seed will he need?

(c) What is the total length of fencing needed to surround each of the three paddocks?

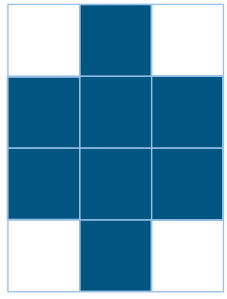
Reasoning, communicating and open-ended

16 The following shapes have been drawn on centimetre grid paper. Find the shaded area in each case.

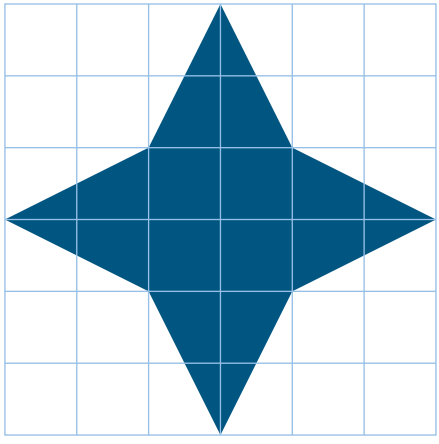
(a)



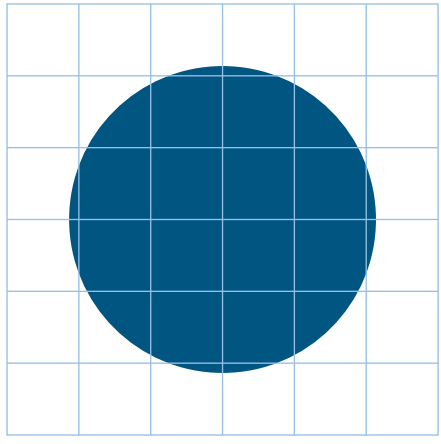
(b)



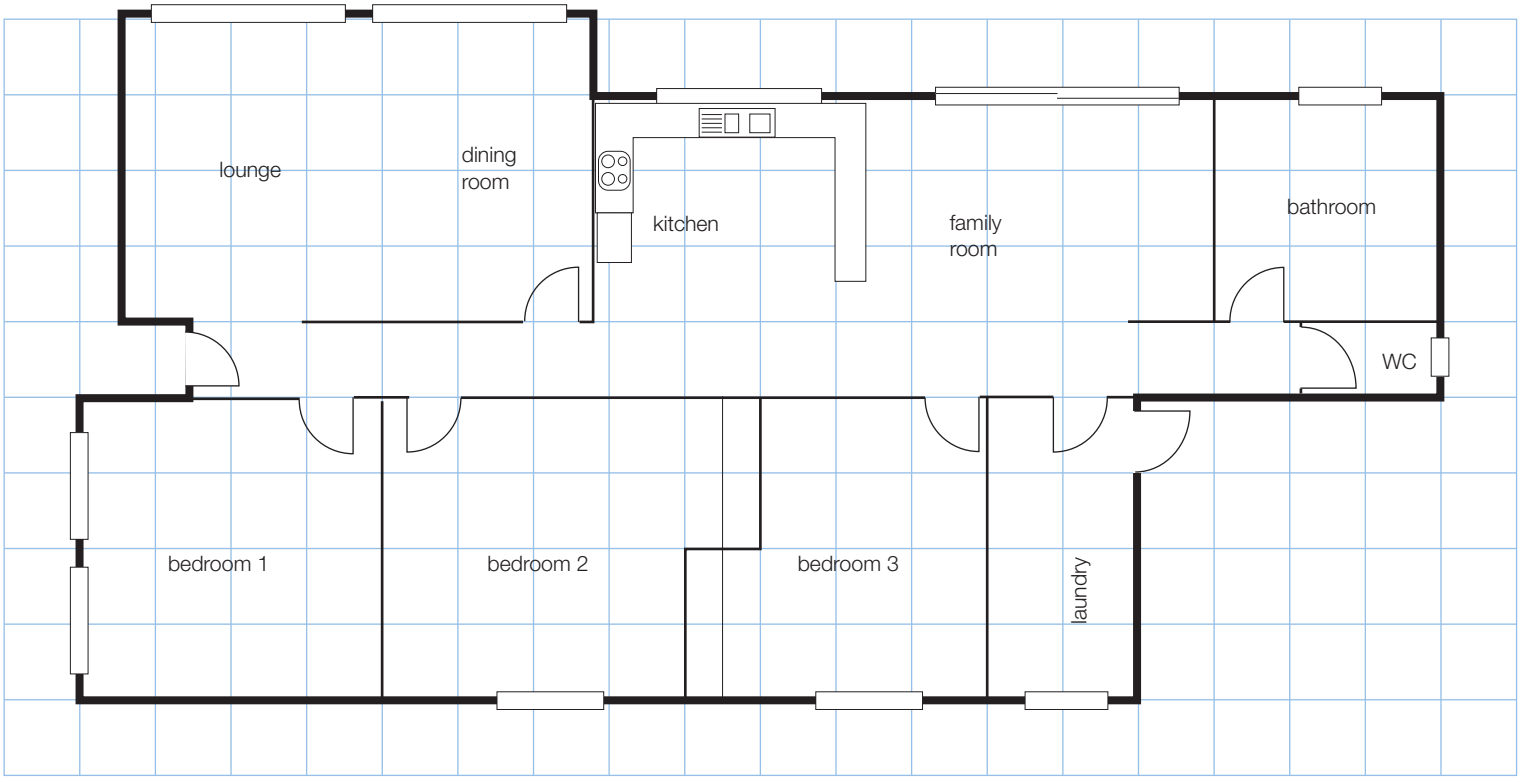
(c)



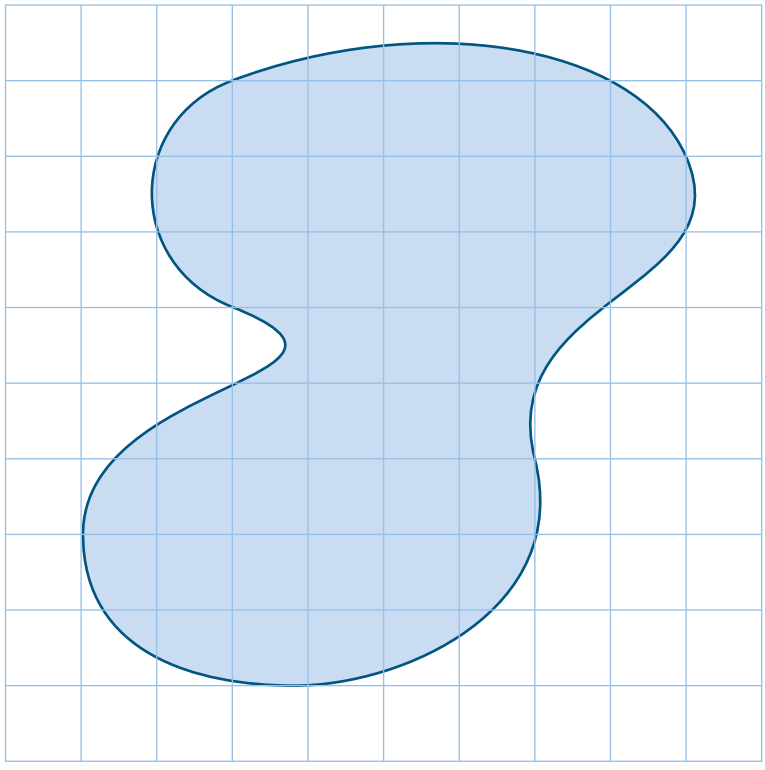
(d)



17 (a) Use the grid to estimate the area covered by the floor plan of this house. The squares on the plan below represent square metres.



(b) Use the grid to estimate the area of the lake. The squares on the grid represent square kilometres.



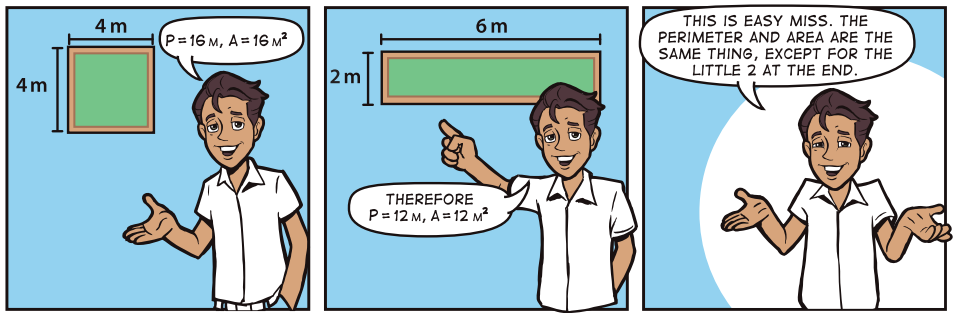
18 Write down all the possible whole number dimensions of a rectangle with an area of 20 cm^2.

19 Find the length and width of a rectangular room whose area is 60 m^2 and perimeter is 34 m.

20 Ethan and Magda were each asked to make a shape using four 1 cm squares, and then find the perimeter of their shapes. Ethan said the perimeter of his shape was 10 cm, whereas Magda said her shape had a perimeter of only 8 cm. Could they both be right? Draw what their shapes may have looked like.

21 Peta made a blanket for her newborn baby Mia's cot. The blanket has an area of 2.4 m^2. What might the perimeter of the blanket be?

22



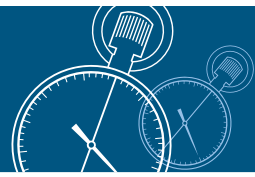
(a) One of Sam's calculations is not right. What has he done wrong?

(b) How would you explain the difference between area and perimeter to Sam?

(c) What is meant by the 'little 2' and why must it be written on the answers for area?

23 List three materials that are sold by the square metre.

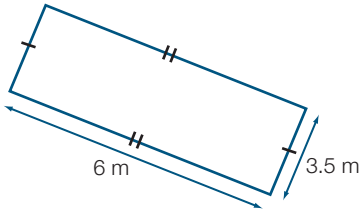
Half-time 6



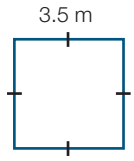
1 Find **(i)** the perimeter and **(ii)** the area of the following shapes.

Ex 6.2, 6.3

(a)



(b)



2 Complete the following conversions.

Ex 6.1

(a) 45 cm = ____ m **(b)** 0.015 km = ____ m **(c)** 307 m = ____ km

(d) 0.6 km = ____ cm **(e)** 196 000 m = ____ km **(f)** 2640 cm = ____ km

3 A garden bed is 5.2 m long and 3 m wide.

Ex 6.2, 6.3

(a) How much fertiliser will be required to cover the garden bed, if the recommended amount is 0.75 kg per square metre?

(b) What length of wooden fencing will be needed to surround the bed?

4 Which unit of area—mm^2, cm^2, m^2 or km^2—would be the most appropriate for measuring the area of:

Ex 6.3

(a) a soccer field **(b)** a postage stamp

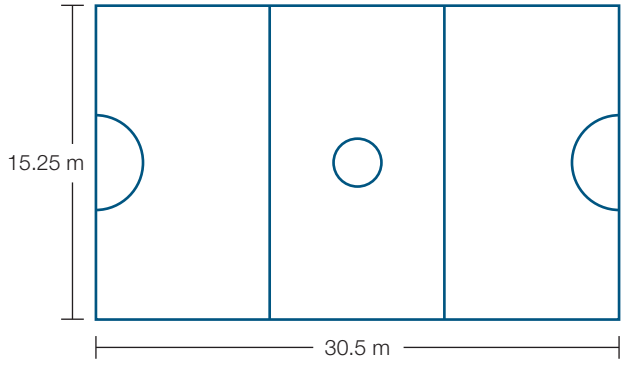
(c) a table placemat **(d)** a national park?

5 A rectangle has a perimeter of 26 cm and an area of 30 cm^2. What are its dimensions (length and width)?

Ex 6.3

6 A netball court is 30.5 m long and 15.25 m wide.

Ex 6.2, 6.3



(a) At training, players complete five laps around the boundary of the court. How many metres is this?

(b) Calculate the area of the court.

(c) The player in the position of 'goal attack' is allowed to move around in two-thirds of the court's area. Calculate this area, correct to one decimal place.

6.4 Area of a parallelogram

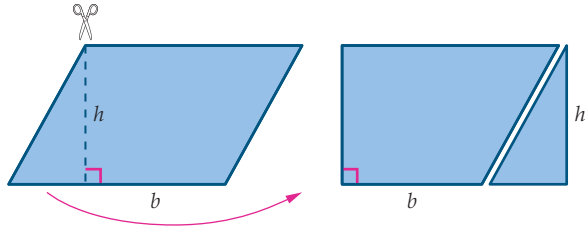
Need to Know

Parallelograms

A **parallelogram** is a four-sided shape that has two pairs of parallel sides. The pairs of parallel sides are marked by > and >> symbols. (Sides that are marked with the same number of > symbols are parallel.)

The parallelogram has a **base**, b, and a **perpendicular** height, h, which is drawn at right angles to the base. Perpendicular means 'at right angles to'. This is shown by the symbol for the right angle ⌞.

A parallelogram can be transformed into a rectangle by cutting down the dotted line marking the height and rearranging the two pieces formed into a rectangle, as shown.

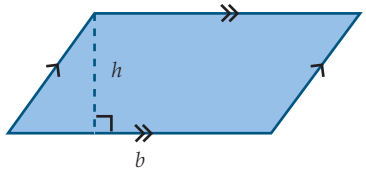


The base and height of the parallelogram become the length and width of a rectangle, so they can be multiplied together to find the area.

Area of a parallelogram

Multiply the base (b) by its perpendicular height (h).

$A = bh$

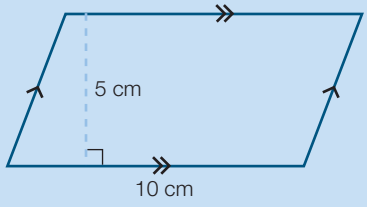


Worked Example 7

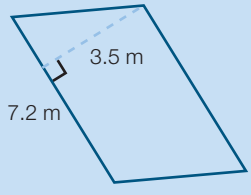
WE7

Find the area of each of the following parallelograms.

(a)



(b)



Working

(a) $A = bh$
$A = 10 \times 5$
$A = 50 \text{ cm}^2$

(b) $A = bh$
$A = 7.2 \times 3.5$
$A = 25.2 \text{ m}^2$

6.4 Area of a parallelogram

Answers page 480

Navigator

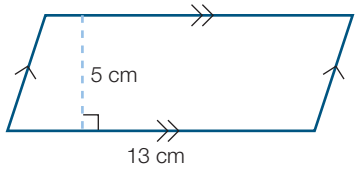
Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q10	Q1 (c)–(f), Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10	Q1 (e)–(f), Q2, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q11

Fluency and understanding

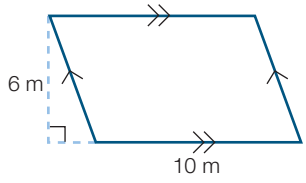
1 Find the area of each of the following parallelograms.

WE7

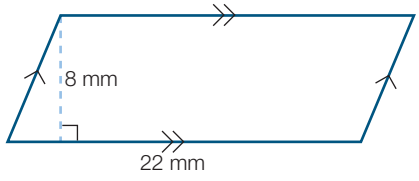
(a)



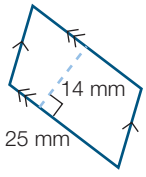
(b)



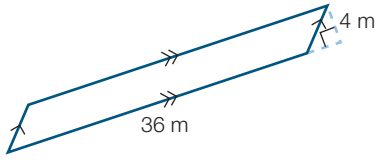
(c)



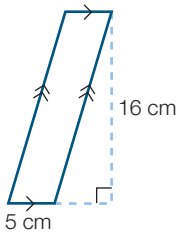
(d)



(e)

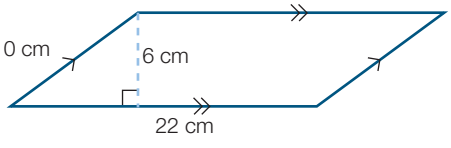


(f)



2 **(a)** Find the area of this parallelogram, in cm^2:

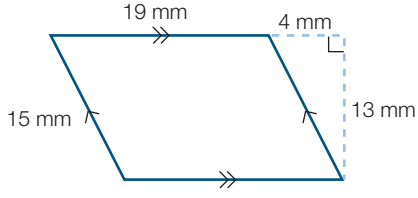
(b) Find the perimeter, in cm.



3 **(a)** The height of this parallelogram is:

A 4 mm **B** 13 mm

C 15 mm **D** 19 mm



(b) A parallelogram with a base length of 14 m, a sloping side length of 5 m and a height of 4 m has an area of:

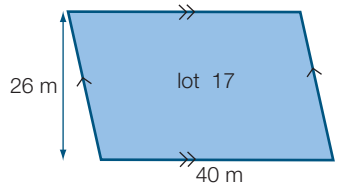
A $20\ m^2$ **B** $28\ m^2$ **C** $56\ m^2$ **D** $70\ m^2$

4 A parallelogram with a base length of 12 cm has an area of $90\ cm^2$. What is the height of the parallelogram?

5 The picture frame shown has a height of 15 cm and encloses an area of $300\ cm^2$. What is the base length of the picture frame?

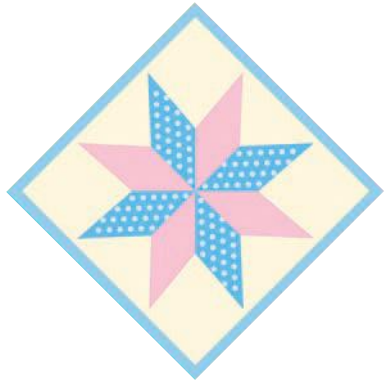


6 A block of land appears on a council map as shown. Calculate its area in square metres.



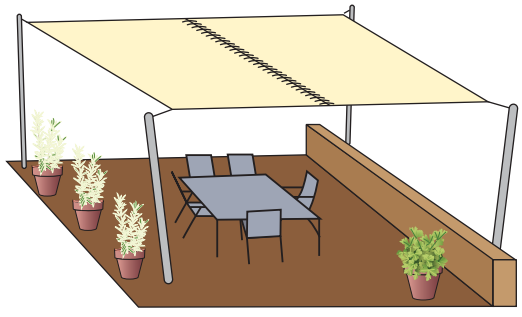
7 **(a)** How many identical parallelograms are shown in the quilt pattern opposite? (Ignore the different colours.)

(b) If a single parallelogram has a base length of 8.5 cm and a height of 6.5 cm, calculate the area of material required for all of the parallelograms on the quilt.



Reasoning, communicating and open-ended

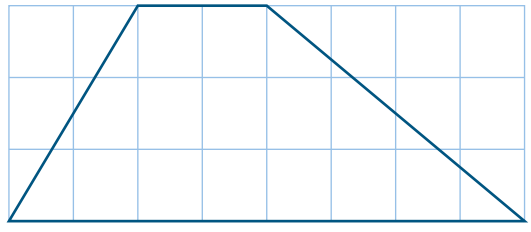
8 Two parallelogram-shaped shade sails are sewn together to provide shade for a courtyard, as shown. Each parallelogram has a base length of 4 m and a height of 2.5 m.



(a) At midday when the Sun is directly overhead, what area of shade will the shade sails provide?

(b) Show another way that the shade sails could be sewn together. Would this provide the same area of midday shade? Explain.

9 **(a)** Copy the trapezium and show the position of the cuts and the transformations required to form a rectangle.



(b) Calculate the area of the trapezium if one grid square = 1 cm^2.

10 State the dimensions (base and height) of three parallelograms that have an area of 32 cm^2.

11 Four identical parallelograms are placed together to form a larger parallelogram.

(a) Show how this can be done.

(b) State two possible dimensions of the smaller parallelogram, so that the larger parallelogram has an area of less than 100 cm^2.

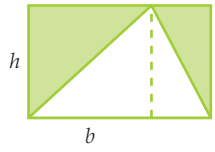
Area of triangles and composite shapes

6.5

Need to Know

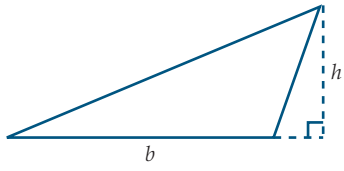
Area of a triangle

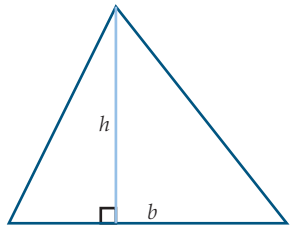
A triangle can be drawn inside a rectangle as shown and the perpendicular height marked with a dotted line. If the triangle is cut out and the two small triangles cut off are rearranged to cover the original triangle, these small triangles should cover the original exactly. This shows that the triangle area is half the area of the rectangle.



The area of a triangle is equal to half of the product of the base, b, and the perpendicular height, h.

$$A = \frac{bh}{2}$$

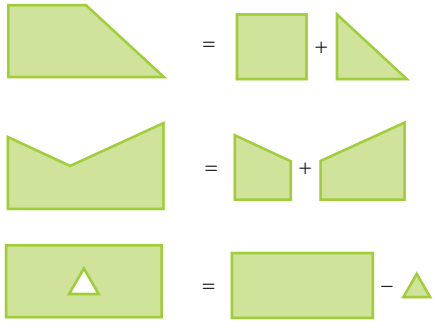




This rule works even if the perpendicular height lies outside the triangle.

Composite shapes

A **composite shape** is a shape that is made by combining two or more shapes, such as rectangles, squares, parallelograms and triangles. These are examples of composite shapes.



Area of composite shapes

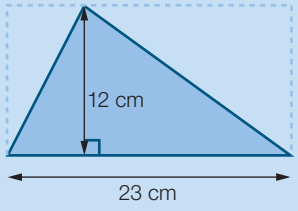
To find the area of a composite shape, divide it into known shapes and find the area of each. Next, add or subtract the area of each individual shape to find the total area.

Worked Example 8

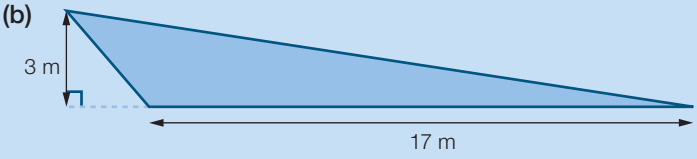
WE 8

Calculate the area of the following triangles.

(a)



(b)



Working

(a) $A = \frac{bh}{2}$

$A = \frac{23 \times 12}{2}$

$A = 138 \text{ cm}^2$

(b) $A = \frac{bh}{2}$

$A = \frac{17 \times 3}{2}$

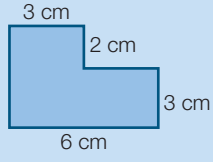
$A = 25.5 \text{ m}^2$

Worked Example 9

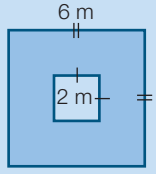
WE 9

Calculate the shaded area of the following composite shapes.

(a)

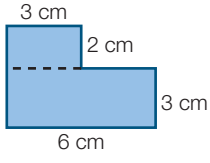


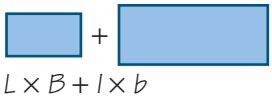
(b)



Working

(a)



Total area = 

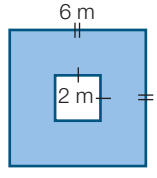
$= L \times B + l \times b$

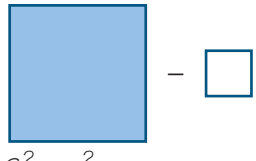
$= 3 \times 2 + 6 \times 3$

$= 6 + 18$

$= 24 \text{ cm}^2$

(b)



Total area = 

$= S^2 - s^2$

$= 6^2 - 2^2$

$= 36 - 4$

$= 32 \text{ m}^2$

6.5 Area of triangles and composite shapes

Answers page 480

Navigator

Q1, Q2 (a)–(f), Q3, Q4, Q6, Q7, Q9, Q11, Q13	Q1 Columns 2 & 3, Q2, Q3, Q5, Q6, Q7, Q8 (a) & (b), Q9, Q10, Q11, Q13	Q1 Column 3, Q2, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14

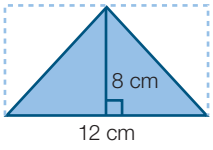
Equipment required: Calculator may be used for Questions **2**, **4**, **6**, **8** and **9**

Fluency and understanding

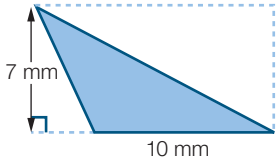
1 Calculate the area of the following triangles.

WE 8

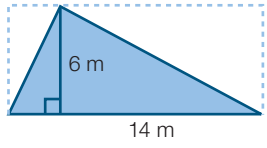
(a)



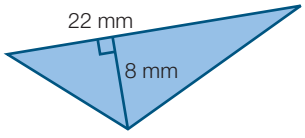
(b)



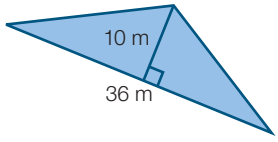
(c)



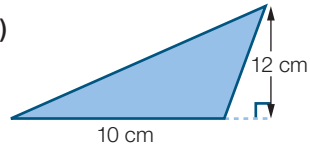
(d)



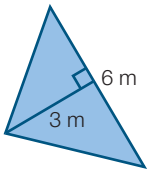
(e)



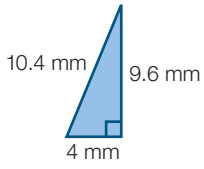
(f)



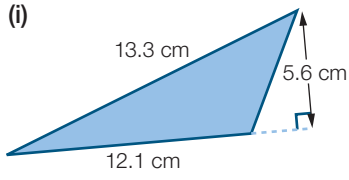
(g)



(h)



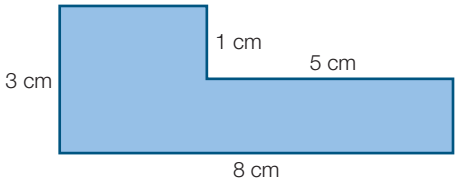
(i)



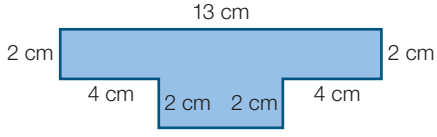
2 Calculate the shaded area of the following composite shapes.

WE 9

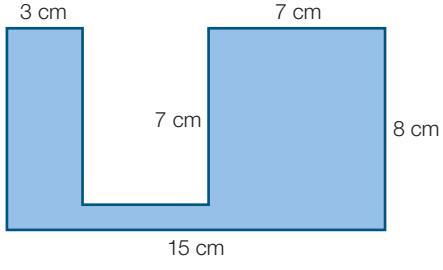
(a)



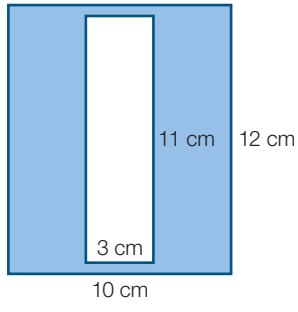
(b)



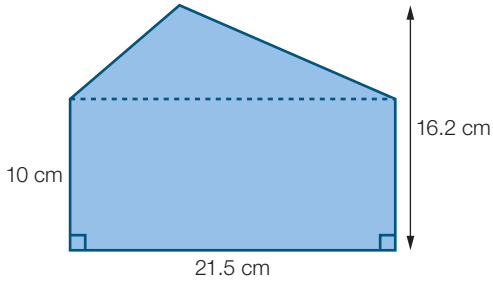
(c)



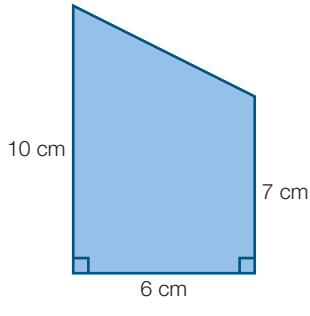
(d)



(e)



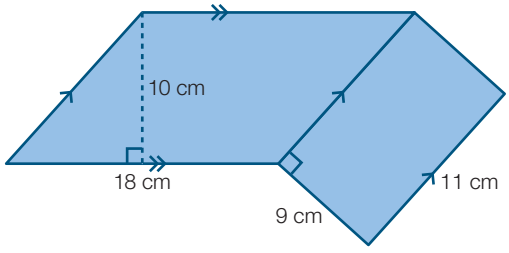
(f)



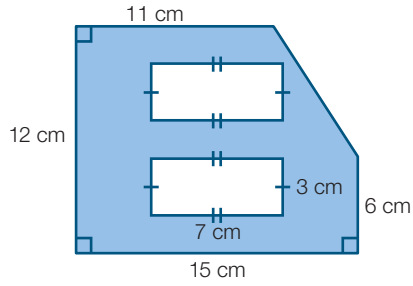
Hint

There is sometimes more than one way to split up a composite shape. Look for the easiest way.

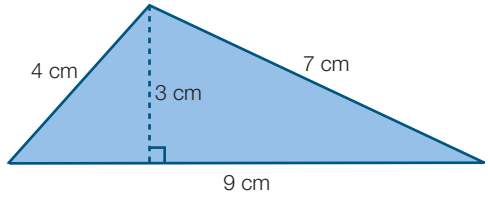
(g)



(h)

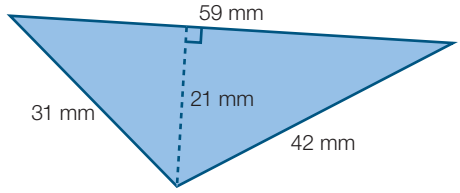


3 **(a)** In the diagram, the height of the triangle is:



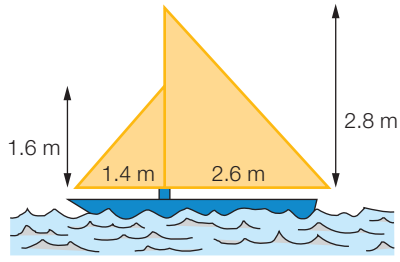
- **A** 3 cm
- **B** 4 cm
- **C** 7 cm
- **D** 23 cm

(b) The base length of the triangle is:

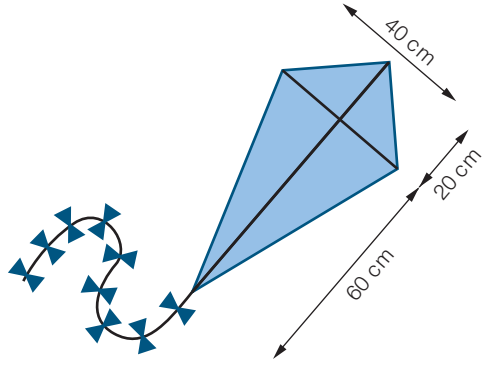


- **A** 21 mm
- **B** 31 mm
- **C** 42 mm
- **D** 59 mm

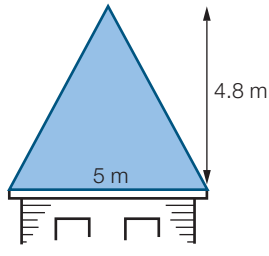
4 Find the total area of the sails on this yacht.



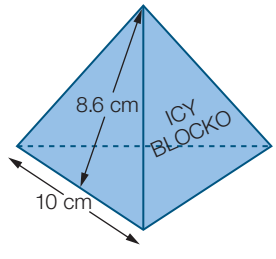
5 Find the area of paper required to build the kite shown.



6 The label on a tin of paint claims the contents will cover 4 square metres. How many tins would be needed to paint the triangular roof section pictured?

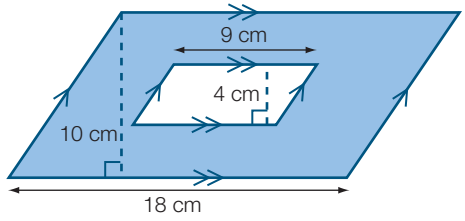


7 Find the total area of the four identical triangular faces of the container of frozen drink (a Tetrapak) shown.

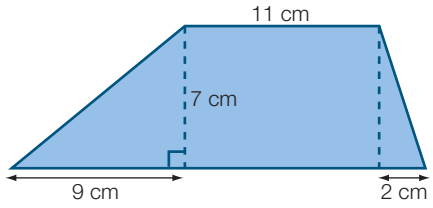


8 Find the area of each of the following composite shapes.

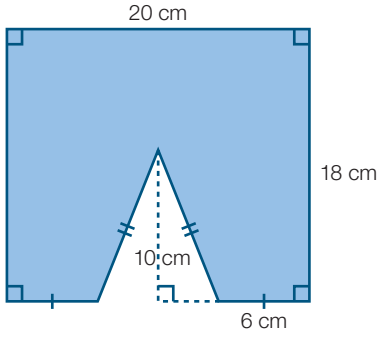
(a)



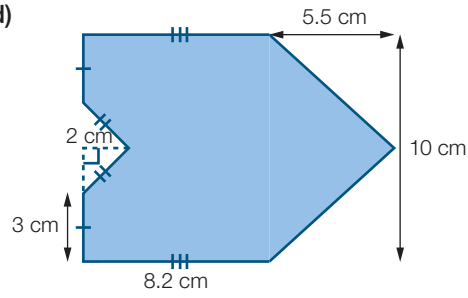
(b)



(c)



(d)



9 Terry is having a doorway put in one of the walls of his house.

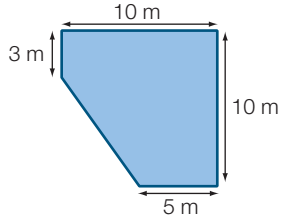
(a) If the wall is 5 m long and 2.4 m high, and the doorway measures 0.82 m by 2 m, what area of wall will remain after the doorway is completed?

(b) The architraves go along the sides and top of the doorway. What length of architraves is needed?

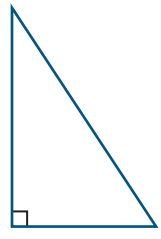
(c) Terry installs a door in the doorway, and paints two coats of varnish on both sides of the door. If 1 litre of varnish covers 4 square metres, how many litres of varnish did Terry use?

Reasoning, communicating and open-ended

10 Describe a method for finding the area of the composite shape shown without splitting it up into known shapes. Use your method to find the answer.



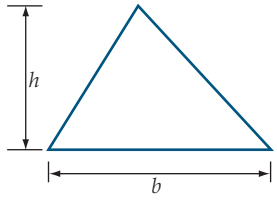
11 This triangle can be cut into two pieces to form a rectangle.



(a) Describe exactly where and how to make the cut.

(b) Use a diagram to show how the two pieces fit together to make a rectangle.

12 This triangle can be cut into four pieces to form a rectangle.



(a) Describe exactly where and how to make the cuts.

(b) Use a diagram to show how the four pieces fit together to make a rectangle.

13 A triangle has an area of 36 cm^2. Give two possible combinations for the triangle's base length and height.

14 The difference in area between a rectangle and a triangle is 12 cm^2. Draw a possible example of the pair, showing their dimensions.

6.6 Volume

Need to Know

Volume is the amount of space occupied by a 3D object. The volume of an object is measured by counting or calculating the number of cubes that would fit inside it. A cube with a length, width and height of 1 cm (a cubic centimetre) is commonly used to measure volume. The volume of large objects may be measured in cubic metres (m^3)—a cube with length, width and height of 1 m.

Units of volume

A cubic millimetre	mm^3	1 mm length × 1 mm width × 1 mm width = 1 mm^3
A cubic centimetre	cm^3	1 cm length × 1 cm width × 1 cm width = 1 cm^3
A cubic metre	m^3	1 m length × 1 m width × 1 m width = 1 m^3
A cubic kilometre	km^3	1 km length × 1 km width × 1 km width = 1 km^3

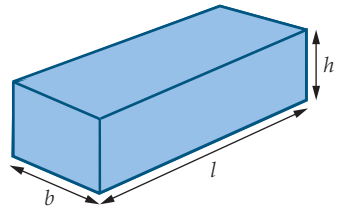
A **rectangular prism** is a solid where all faces are rectangles. It is made of layers of cubes. Each layer has the same rectangular shape so there is the same number of cubes in each layer.

Rules can be used to find the volume of regular shapes.

Volume of a rectangular prism

$V = l \times b \times h$

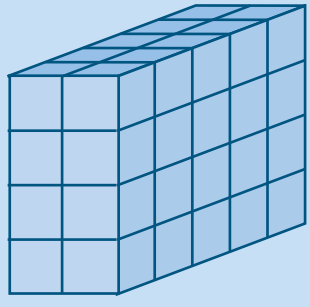
where l is the length, b is the width and h is the height.



Worked Example 10

WE10

Find how many cubic centimetres there are in the following rectangular prism by counting cubes. (Each small cube represents 1 cm^3.)



Working

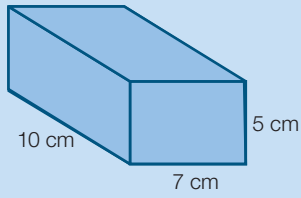
There are 10 cubes on the top layer.

There are 4 layers so it is 4 units high.

$10 \times 4 = 40$ cubes
$= 40\ cm^3$

Worked Example 11 WE11

Find the volume of the following rectangular prism in cm^3.



Working

$V = lbh$
$V = 10 \times 7 \times 5$
$= 350\ cm^3$

6.6 Volume

Answers page 480

Navigator

Q1, Q2, Q3, Q6 Column 1, Q7, Q9, Q10, Q12, Q13	Q1, Q2, Q4, Q5, Q6 Columns 1 & 2, Q7, Q9, Q10, Q11, Q12, Q13	Q1, Q2, Q4, Q5, Q6 Columns 2 & 3, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14

Equipment required: Calculator may be used for Questions **7–9**, **11** and **14**

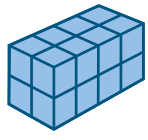
Fluency and understanding

1 How many cubic centimetres are there in the following rectangular prisms? (Each small cube represents 1 cm^3.) WE10

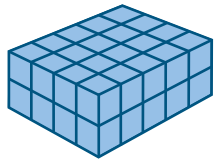
(a)



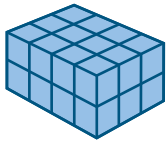
(b)



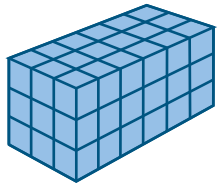
(c)



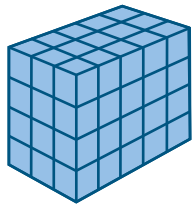
(d)



(e)

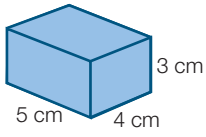


(f)

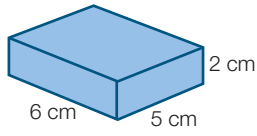


2 Find the volume of the following rectangular prisms in cm^3. WE11

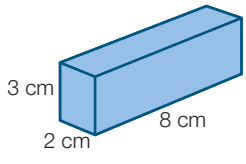
(a)



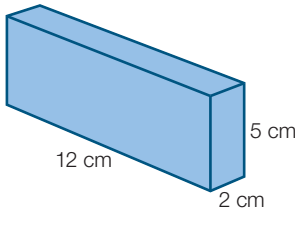
(b)



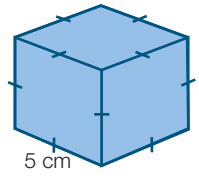
(c)



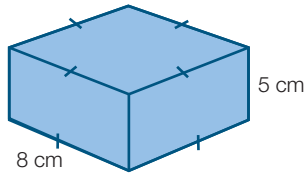
(d)



(e)



(f)



3 A room is 3 m long, 2 m wide and 2 m high. What is its volume in metres cubed?

4 A pocket dictionary has a cover 12 cm long and 9 cm wide and is 2 cm thick. Find the volume of this book.

5 Choose the correct answer for each of the following rectangular prisms.

(a) Length = 20 cm; width = 10 cm; height = 5 cm. The volume is:

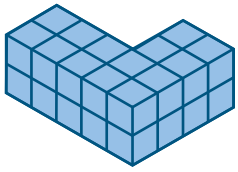
A 35 cm^3 **B** 100 cm^3 **C** 700 cm^3 **D** 1000 cm^3

(b) Length = 3 m; width = 75 cm; height = 50 cm. The volume is:

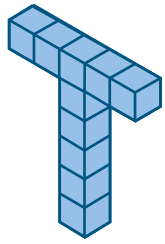
A 11.25 m^3 **B** 112.5 cm^3 **C** 11 250 cm^3 **D** 1 125 000 cm^3

6 Find the volume of each of these compound solids by counting the number of centimetre cubes.

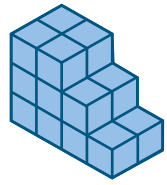
(a)



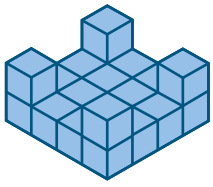
(b)



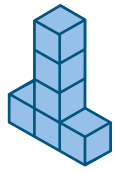
(c)



(d)



(e)



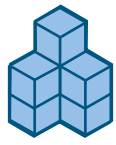
(f)



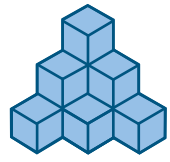
(g)



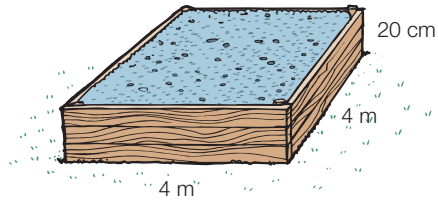
(h)



(i)

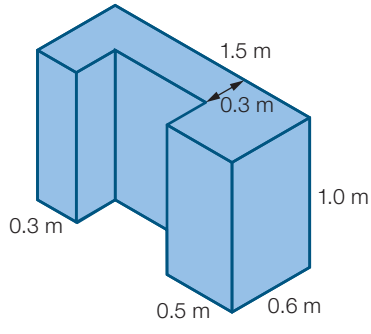


7 A water tank is to be placed on a wooden box filled with gravel to keep it level. How much gravel is needed, in m^3, to fill the base shown?



8 If a cake in the shape of a rectangular prism is 20 cm long, 15 cm wide and 6 cm thick when baked, what will its volume be after it is iced all over (including the bottom) with a layer of icing 0.5 cm thick?

9 Lachlan has finished the brickwork for his backyard barbecue, as shown opposite. Find the volume of bricks used in building the barbecue.



Hint

Divide the solid into rectangular prisms.

Reasoning, communicating and open-ended

10 A large toy box is completely full of identical wooden blocks that are 10 cm long, 6 cm wide and 4 cm in height. How many blocks can fit inside the toy box, if the dimensions of the toy box are: length 40 cm, width 30 cm and height 20 cm?

11 A rectangular prism has a volume of 216 cm^3. It is twice as long as it is wide, and its height is 3 cm. Find its length.

12 A rectangular prism is made from 30 one-centimetre cubes. Give possible values for the length, width and height of the prism.

13 At a school fair, a game involves guessing the number of jelly beans in a container. The person with the closest guess will win all the jelly beans.

(a) How many jelly beans do you think are in the jar?

(b) How did you make an accurate estimate of the number of jelly beans in the jar?

(c) If the jelly beans were the same volume but cube-shaped, would more or fewer jelly beans fit in the jar?

14 A small cube is placed inside a large cube of volume 64 cm^3. Give three possible side lengths of the smaller cube and calculate the volume of empty space inside the large cube.

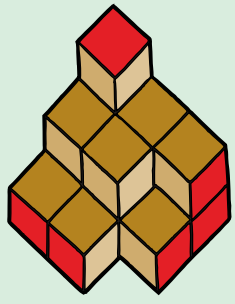


Problem solving

The red cube

A new water fountain is being constructed as a display for a shopping centre. The water will run smoothly over a large red cube. Construction has started, as shown on the right. How many of the remaining cubes will need one face painted? Two faces painted? Three faces painted?

In the completed water feature, how many cubes will have one, two and three faces painted?



Strategy options

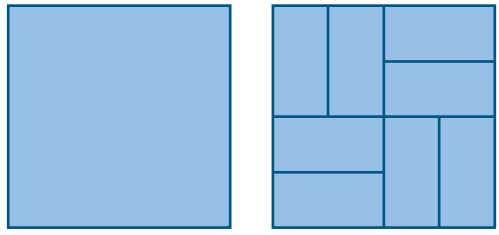
- Have I seen a similar problem?
- Make a model.

Challenge 6



1 If a $2 coin is about 2 mm thick, how much is a kilometre-high stack of $2 coins worth?

2 The perimeter of this square paddock is 800 m. It is divided into eight identical paddocks as shown.



The perimeter, in metres, of each of the eight smaller paddocks is:

A 150 **B** 200

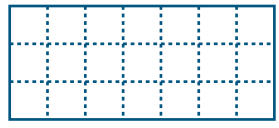
C 300 **D** 400

3 The weight limit for a lift is 1500 kilograms. The average weight of the people in the lift is 80 kilograms. If the combined weight of the people is 100 kilograms over the limit, how many people are in the lift?

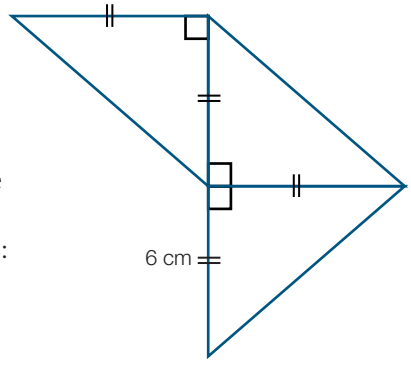
A 13 **B** 16 **C** 17 **D** 20

4 When a square piece of paper is folded in half vertically, the resulting rectangle has a perimeter of 39 cm. Find the area of the original sheet of paper, in square centimetres.

5 A rectangular area on a wall measuring 7 units by 3 units is covered with 7 rectangular tiles measuring 1 unit by 3 units. In how many ways can this be done?



6 The area of the entire figure shown is:



A 36 cm^2 **B** 54 cm^2

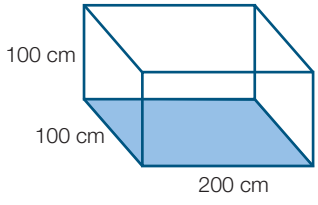
C 72 cm^2 **D** 108 cm^2

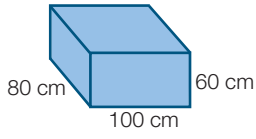
7 A rectangular sign has the dimensions 10 m by 16 m. If a square advertisement to be painted on it must have a border at least 1.5 m wide, the area of the largest square advertisement that can be painted on the sign is:

A 49 m^2 **B** 72.25 m^2

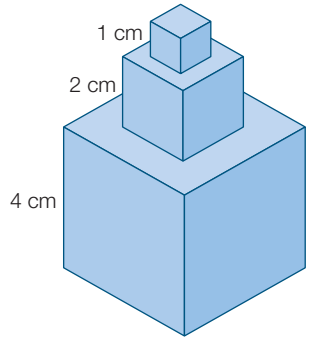
C 91 m^2 **D** 160 m^2

8 A fish tank has a base of 100 cm by 200 cm and a depth of 100 cm. A solid metal rectangular prism with dimensions 80 cm by 100 cm by 60 cm is placed in the tank and water is poured into the tank until it is level with the top of the metal prism. After the prism is carefully removed from the tank, the depth of the water remaining is measured. What are the three possible depths?





9 Three cubes are sitting on top of each other. The bottom cube has a side length of 4 cm, the cube on top of it has a side length of 2 cm and the very top cube has a side length of 1 cm. What is the visible area of the cube stack?



Chapter review 6

D.I.Y. Summary

Key Words

area	height	metric	perpendicular
base	kilometre	millimetre	rectangular prism
centimetre	length	parallelogram	volume
composite shape	metre	perimeter	width

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 _____ is the distance around the outside of a shape.

2 To calculate the _____ of a rectangle, you multiply the _____ by the width.

3 The area of a triangle is half the _____ multiplied by the perpendicular _____.

4 The _____ of a rectangular prism is found by multiplying the length, width and height together.

5 The system of measurement used in Australia is called the _____ system.

6 To find the area of a _____ we break it up into simpler shapes.

7 Multiplying the base length by the perpendicular height will find the area of a _____.

Fluency and understanding

1 **(a)** The height of an average house ceiling is approximately: Ex 6.1

A 260 mm **B** 2.6 m **C** 0.26 km **D** 2600 m

(b) The length of a classroom desk is approximately:

A 18 cm **B** 280 mm **C** 180 cm **D** 180 m

2 Copy and complete the following length conversions. Ex 6.1

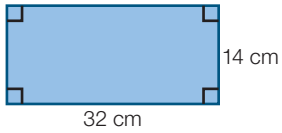
(a) 45.9 km = _____ m **(b)** 58 000 cm = _____ km

(c) 9.2 m = _____ mm **(d)** 42 m = _____ cm

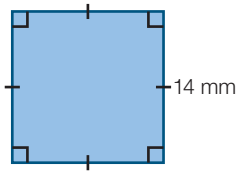
(e) 980 m = _____ km **(f)** 0.006 73 km = _____ mm

3 Find the **(i)** perimeter and **(ii)** area of each of the following shapes. Ex 6.2, 6.3, 6.5

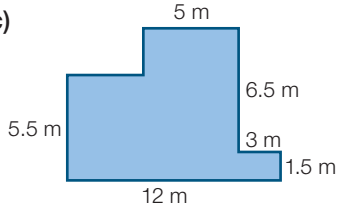
(a)



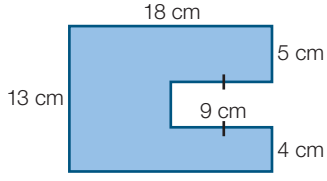
(b)



(c)



(d)



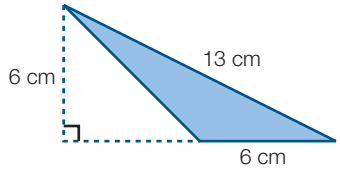
4 Find the perimeter and area of a squash court 9.75 m long and 6.4 m wide.

Ex. 6.2, 6.3

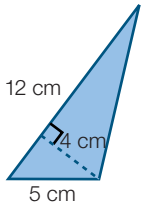
5 Find the area of each of the following shapes.

Ex. 6.3, 6.4

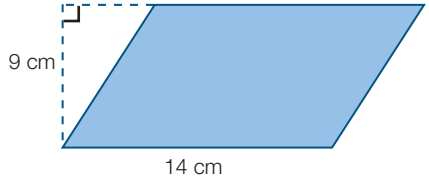
(a)



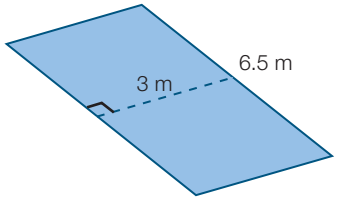
(b)



(c)



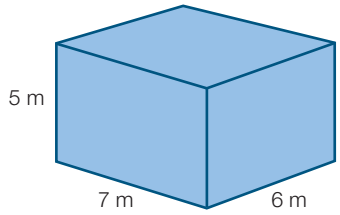
(d)



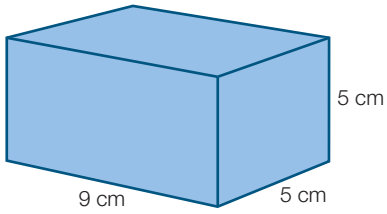
6 Determine the volume of the following solids.

Ex. 6.6

(a)

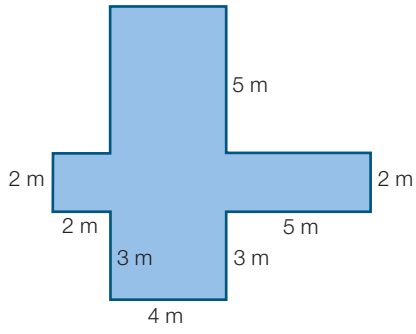


(b)

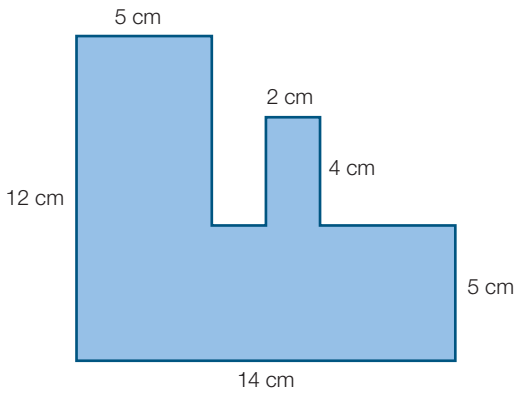


7 Find the area of each composite shape below.

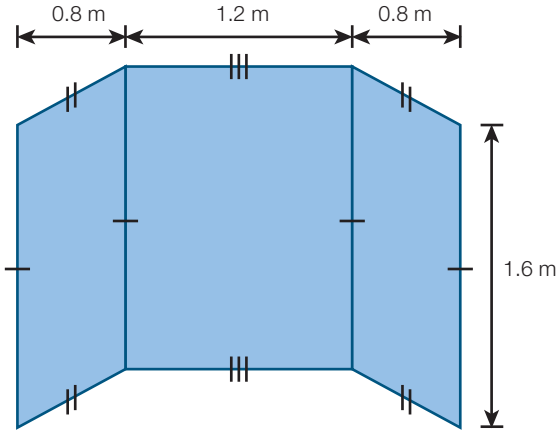
(a)



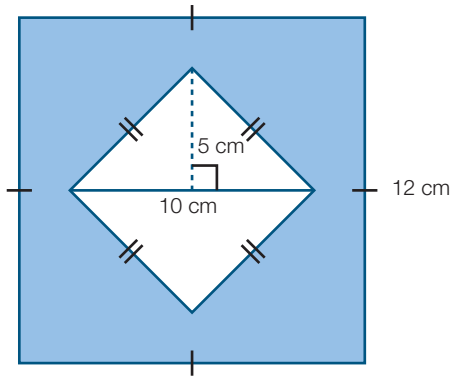
(b)



(c)



(d)



8 A carpenter calculates that the length of timber she requires to make a bookcase is 4850 mm.

(a) How many metres is this?

(b) If timber costs $6 per metre at the hardware store, how much will she pay?

9 Robbie is tiling the floor of his bathroom, which is a rectangle measuring 3 m by 2.5 m.

(a) Write these measurements in centimetres, then calculate the area of the floor in square centimetres.

(b) Robbie is using square tiles that have a side length of 25 cm. Calculate the number of tiles needed to cover the floor.

(c) If the tiles cost 80 cents each, calculate how much Robbie will pay for the required number of tiles.

10 Theresa is constructing a sand pit for her children. She has created a rectangular wooden box that is 2 m long, 1.5 m wide and 0.3 m high.

(a) Calculate the perimeter of the box.

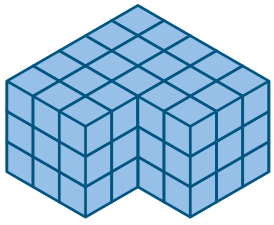
(b) Calculate the volume of the box.

(c) If one cubic metre of sand costs $15, calculate how much it will cost to fill the box to a depth of 25 cm with sand.

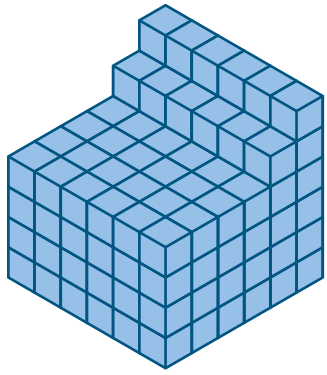
11 Tricia has obtained a quote of $21 per square metre for her driveway to be resurfaced. If her driveway is rectangular and measures 12 m long and 3.2 m wide, what will the job cost her?

12 Determine the volume of each of the following solids. (The cubes are centimetre cubes.)

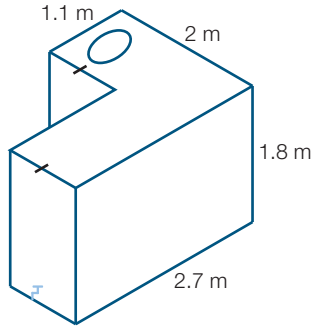
(a)



(b)



13 Cameron has had a water tank specially made to fit the side of his house. Calculate the volume of the tank, in cubic metres.

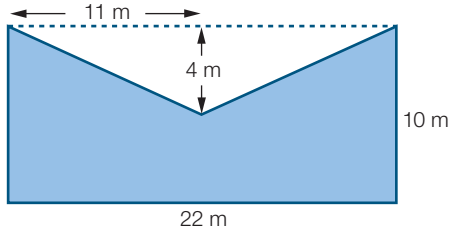


Reasoning and communicating

14 What is the perimeter of a square with an area of 25 cm^2?

15 What is the area of a rectangle with a perimeter of 32 cm and a length of 10 cm?

16 At right is a diagram of the available floor space (shaded) in an unoccupied car showroom. Calculate the area available for a display.



17 Five blocks of dimensions 3 cm × 5 cm × 6 cm are placed in a toy box of length 40 cm, width 20 cm and height 9 cm. Find the volume of space left in the box.

18 Draw a diagram to help you explain:

(a) how to turn a parallelogram into a rectangle

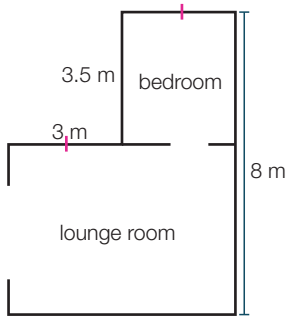
(b) how to turn a triangle into a rectangle.

NAPLAN practice 6

Numeracy: Non-calculator

1 Which has the larger area: a rectangle with a length of 20 cm and a width of 15 cm, or a triangle with a base of 20 cm and a height of 15 cm?

2 Li is having new carpet laid in the lounge room and bedroom of her apartment. What area is to be carpeted?



3 What is the perimeter of a square that has an area of 100 cm^2?

4 A rectangular picture frame has a perimeter of 80 centimetres. The frame has a length of 25 centimetres.

What is the width of the picture frame? _____ cm

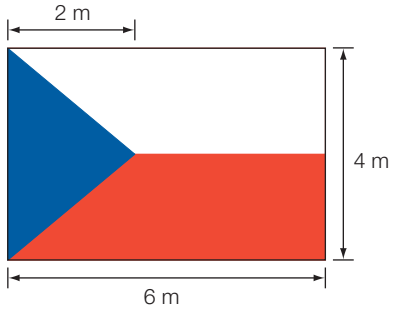
Numeracy: Calculator allowed

5 The flag of the Czech Republic is shown here.

The line where the red and white sections meet is halfway up the flag. If the blue triangle has a height of 2 m, the area of the flag that is red is:

A 8 m^2 **B** 9 m^2

C 10 m^2 **D** 16 m^2

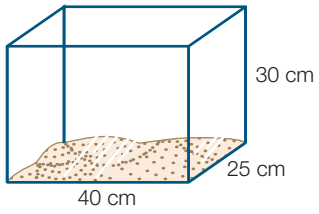


6 A pack of playing cards is 2.6 cm high. If there are 52 cards in the pack, what is the thickness of each card?

7 The Hudson family have bought a new fish tank, requiring a 20 mm deep layer of pebbles on the bottom. Which volume of pebbles should they buy?

A 1000 cm^3 **B** 2000 cm^3

C 20 000 cm^3 **D** 30 000 cm^3



8 Sanjay used identical cubes to build a rectangular prism. There were 18 cubes in its base. He used 54 cubes altogether. Which of these could be the dimensions of Sanjay's prism?

A 6 × 3 × 2 **B** 3 × 2 × 9 **C** 9 × 3 × 3 **D** 6 × 3 × 9

Mixed review C

Fluency and understanding

1 Write the following in algebra. Ex.5.1

(a) Akash has n pencils in his pencil case. He takes 8 out and lends them to a friend. How many pencils are in the case now?

(b) k dollars is divided evenly among 6 people. How much money does each person receive?

(c) There are 50 matches in one box. How many matches are there in h boxes?

2 Arrange the following decimals in order from smallest to largest. Ex.4.1

(a) 2.3, 2.003, 2.33, 2.323, 2.302 **(b)** 0.089, 0.129, 0.091, 0.0909, 0.0199

3 Write the following as fractions or mixed numbers in simplest form. Ex.3.1

(a) 6 out of 24 students in the class have black hair.

(b) 5 full dozen egg cartons and 4 eggs in another carton.

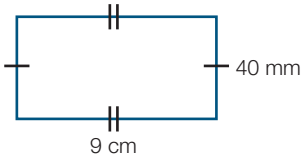
(c) 25 minutes out of an hour.

4 Write each of the following numbers as a product of their prime factors in index form. Ex.2.3

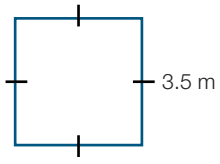
(a) 28 **(b)** 200 **(c)** 990

5 Find **(i)** the perimeter and **(ii)** the area of the following shapes, giving your answer in the larger unit where necessary. Ex.6.2, 6.3, 6.5

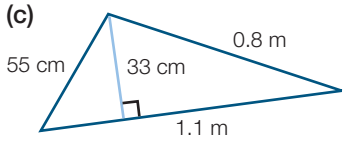
(a)



(b)



(c)



6 Calculate: Ex.1.5

(a) $7 + 32 \div 4 \times 2$ **(b)** $(33 + 12) \div 3 + 19$ **(c)** $6 \times 9 \div 2 + 4$

7 Calculate the following, writing your answers in simplest form and as a mixed number, if appropriate. Ex.3.5, 3.6

(a) $\frac{5}{6} \times \frac{4}{9}$ **(b)** $\frac{9}{10} \div \frac{7}{15}$ **(c)** $1\frac{3}{5} \times 2\frac{7}{8}$ **(d)** $4 \div \frac{2}{3}$

8 Copy and complete the following conversions. Ex.6.1

(a) 6.09 km = _____ m **(b)** 83 cm = _____ m **(c)** 5400 cm = _____ km

9 Calculate the following. Ex.2.5–2.7

(a) $-2 + 8$ **(b)** $14 - 17$ **(c)** $-15 + 3$ **(d)** $-5 - 6$

(e) $3 + (-9)$ **(f)** $-4 - (-1)$ **(g)** $11 - (-2)$ **(h)** $-7 - 10$

10 Use a mental or written strategy to calculate the following. Ex.1.1, 1.3

(a) 28×43 **(b)** $990 \div 6$ **(c)** 57×4

11 There are 8 red, 12 blue and 4 black jelly beans in a packet. Ex.4.9

(a) Write the ratio blue:red:black in simplest form.

(b) Write the number of red jelly beans as a percentage of the total, rounded to one decimal place.

12 Write the following decimals as fractions in simplest form.

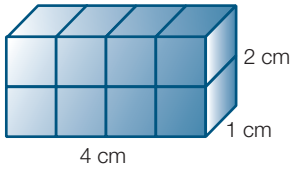
(a) 0.23 **(b)** 0.06 **(c)** 0.505

Ex.4.3

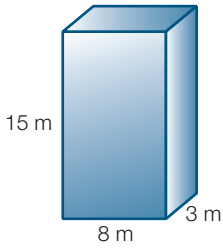
13 Find the volume of the following rectangular prisms.

Ex.6.6

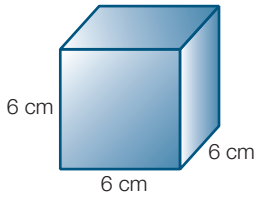
(a)



(b)



(c)



14 Copy and complete the table using the rule $y = 3(x + 2)$.

Ex.5.4

x	0	2	3	6	10
y					

15 Calculate:

Ex.4.4

(a) 4.67 + 8.8 **(b)** 8.93 + 0.006 35 **(c)** 19.87 + 21.075

16 Calculate:

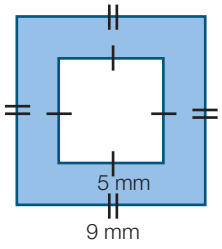
Ex.4.6

(a) 1.86 ÷ 600 **(b)** 0.42 ÷ 1000 **(c)** 10.605 ÷ 0.005

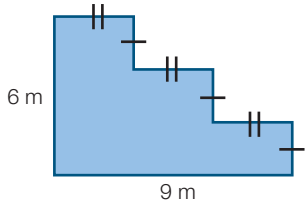
17 Joseph had $578.97 in his bank account. After his pay was deposited, there was $665.49. How much was Joseph's pay?

18 Find the area of the shaded regions.

(a)



(b)



19 Ben surveyed some Year 7 students to find out their favourite Australian animal. Here are his results:

Koala	8 students	Bilby	6 students
Wombat	4 students	Kangaroo	2 students

Write the preference for each animal **(a)** as a fraction in simplest terms, and **(b)** as a percentage of the total number of students.

Reasoning and communicating

20 Which is larger: 2^8 or 8^2?

21 The Hawksbury Magpies were playing in the U14 football grand final. All goals were scored by three boys. Johan scored $\frac{1}{3}$ of the goals, Rihbi scored $\frac{1}{2}$ of the goals and Kyle scored the rest. If the Magpies scored 24 goals, how many goals were scored by Kyle?

22 A rectangle has a length 5 cm greater than its width and a perimeter of 42 cm. What are the dimensions of the rectangle?

23 At a shop I can buy three cans of soft drink for the same price as a hamburger. How much is one can of drink if I can buy one can and a hamburger for $9?

Linear equations

7

Recall 7

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

1 Answer TRUE or FALSE to each of the following number sentences.

(a) $3 + 7 = 5 + 6$ **(b)** $8 \times 3 = 20 + 4$ **(c)** $\frac{24}{8} = 3 \times 5 - 14$

2 **(a)** The term $5x$ means:

A $5 + x$ **B** $5 - x$ **C** $5 \times x$ **D** $x \div 5$

(b) The term $\frac{x}{7}$ means:

A $x - 7$ **B** $x + 7$ **C** $x \times 7$ **D** $x \div 7$

(c) $4(x + 3)$ means:

A $4 + (x + 3)$ **B** $4 \times (x + 3)$ **C** $4 \times x + 3$ **D** $4 - (x + 3)$

3 Rewrite each of the following sentences using mathematical symbols and numbers only.

(a) Five added to seven is equal to twelve.

(b) Subtracting two from the product of four and six is equal to twenty-two.

(c) When the sum of eight and six is divided by seven, the answer is equal to two.

4 What is the final result after applying each of the following operations to the number in the box?

(a) $\boxed{8} \times 2$ then $- 4$ then $\div 3$ then $+ 5$

(b) $\boxed{10} \div 2$ then $\div 5$ then $+ 6$

5 **(a)** If $x = 2$, find the value of $4x - 1$.

(b) If $x = 7$, find the value of $\frac{x - 3}{2}$.

Syllabus references

Number and Algebra: Equations

✓ Solve simple linear equations **(ACMNA179)**

✓ Solve linear equations using algebraic techniques and verify solutions by substitution **(ACMNA194)**

7.1 Number sentences

Need to Know

A **true number sentence** is a number sentence in which the left-hand side (LHS) and the right-hand side (RHS) both have the same value.

For example: $2 \times 4 = 5 + 3$

LHS = RHS

A **false number sentence** is a number sentence in which the LHS and RHS have different values.

For example: $2 \times 4 = 5 + 4$

LHS ≠ RHS

≠ means 'not equal to', < means 'less than', > means 'greater than'

Worked Example 1

WE1

The following number sentences are not true. Rewrite each number sentence by changing the coloured number so that you have a true number sentence.

(a) $5 + 8 = 15$

(b) $2 \times 5 - 1 = 7$

Working

(a) $5 + 8 = 13$
$13 < 15$
$5 + 10 = 15$

(b) $2 \times 5 - 1 = 9$
$9 > 7$
$2 \times 4 - 1 = 7$

Worked Example 2

WE2

Decide which of the following are true number sentences.

(a) $8 \times 3 + 4 = 28$

(b) $\frac{6 \times 5}{3} = 7 + 2$

Working

(a) LHS $= 8 \times 3 + 4$
$= 28$
LHS = RHS
The sentence is true.

(b) LHS $= \frac{6 \times 5}{3}$
$= 10$
RHS $= 7 + 2$
$= 9$
LHS ≠ RHS
The sentence is false.

7.1 Number sentences

Answers page 482

Navigator

Q1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q8, Q10 (a), Q11 (a–d), Q12, Q14, Q16	Q1, Q2 Column 2, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q11 (a–f), Q12, Q14, Q16	Q1, Q2 Column 3, Q3, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15

Fluency and understanding

1 The following number sentences are not true. Rewrite each number sentence by changing the coloured number so that you have a true number sentence. WE1

(a) $3 + 4 = 10$ **(b)** $5 \times 6 = 25$ **(c)** $\frac{24}{4} = 4$

(d) $22 - 7 = 8 + 4$ **(e)** $3 \times 7 + 2 = 8 + 9$ **(f)** $1 + \frac{15}{5} = 6$

2 Decide which of the following are true number sentences. WE2

(a) $\frac{32}{4} + 7 = 16$ **(b)** $4 + 3 \times 2 = 14$ **(c)** $6 + 4 \times 5 = 25$

(d) $30 \div 6 = 4 + 1$ **(e)** $23 - 7 = 12 + 4$ **(f)** $8 + 4 = 6 \times 2$

(g) $7 - 5 = 9 \div 3$ **(h)** $10 \times 8 = 90 - 10$ **(i)** $\frac{6 + 10 + 12}{7} = \frac{20 + 4}{6}$

3 (a) The number missing from the true number sentence $3 \times ____ = 19 + 5$ is:

A 3 **B** 6 **C** 7 **D** 8

(b) The number missing from the true number sentence $4 \times 2 = ____ + 4$ is:

A 0 **B** 2 **C** 4 **D** 8

(c) The number missing from the true number sentence $\frac{24}{8} + ____ = 10 - 2$ is:

A 4 **B** 5 **C** 6 **D** 8

4 Rewrite each true number sentence using numbers and mathematical symbols only.

(a) Forty-three added to five is equal to forty-eight.

(b) Three multiplied by seven is equal to nineteen plus two.

(c) Fifty divided by five is equal to the product of five and two.

(d) When the sum of the numbers six and eight is subtracted from twenty, the answer is equal to twelve divided by two.

5 Write each of the following as a number sentence, then state whether it is TRUE or FALSE.

(a) Nineteen subtracted from thirteen is equal to six.

(b) Four divided by two is equal to eight.

(c) Eleven subtracted from eight is equal to six minus three.

(d) Twelve divided by four is equal to fifteen divided by five.

6 Jane is 5 years older than Todd. If Todd is 7 years old, then write a true number sentence to show Jane's age.

7 Six pens at 60c each cost the same as buying 4 exercise books at 90 cents each. Write this as a true number sentence.

8 The cost of a new shirt is $25 after a discount of $7 is given. Write a true number sentence to show the original price of the shirt.

9 A bus can carry 48 passengers. The bus is full, then 11 people leave while another 5 get on at the next stop. Write a true number sentence to show how many passengers are left on the bus.

10 Numbers are consecutive if one follows the other. The numbers 8 and 9 are consecutive numbers that add to 17.

(a) Write a true number sentence that shows two consecutive numbers that add to 33.

(b) Write a true number sentence that shows three consecutive numbers whose sum is 54.

Reasoning, communicating and open-ended

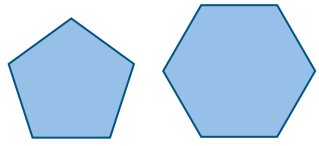
11 A piece of wire 60 cm long is used to make a triangle shape that has all three sides the same length. Write a true number sentence to answer each of the following.



(a) How long is each side of the triangle?

(b) The wire is now used to make a square. How long is each side?

(c) The wire is bent to make a regular pentagon. How long is each side?



(d) The wire is then used to make a regular hexagon. How long is each side?

(e) Write a number sentence you could use to determine the length of the side of a shape that has 20 sides of equal length made from the same piece of wire.

(f) Write a number sentence you could use to determine the length of the side of a shape that has 50 sides of equal length made from a wire 200 cm long.

(g) Use a 100 cm length of wire to make a shape whose sides are of equal length. Find three possible values for the number of sides the shape can have if the length of each side is to be a whole number and shorter than 25 cm.

12 Use the numbers 1, 2, 4, 8, the equals sign, =, and as many of the symbols +, −, ×, ÷ as you like to write three different true number sentences whose right-hand side is a number from 1 to 10.

Hint

Don't forget the order of operations!

13 A length of wire is used to make a shape that has eight sides of equal length. Find five different whole number values for the length of wire if the length of each side is to be greater than the number of sides.

14 $\square + \square \times \square - \square = 20$

What might the missing numbers be? Give three different answers.

15 Sam and Ruben have written the following number sentences. Sam wrote $3 + 4 \times 21 = 147$. Ruben wrote $3 + 4 \times 21 = 87$.

Who is correct? Why? Write a sentence so that the person who was incorrect can improve their skills.

16 Two different students were given the following number sentence.

$38 - 16 = \square - 12$

They presented the following solutions.

$38 - 16 = \square - 12$
$38 - 12 - 4 = \square - 12$
$38 - 4 - 12 = \square - 12$
$34 - 12 = \square - 12$
$\square = 34$

$38 - 16 = \square - 12$
$38 - (12 + 4) = \square - 12$
$38 + 4 - 12 = \square - 12$
$42 - 12 = \square - 12$
$\square = 42$

Which of the solutions is correct? Why? What is wrong with the incorrect solution?

7.2 Introduction to equations

Need to Know

When a value is unknown in a number sentence, a pronumeral is used to represent the unknown amount. This number sentence is called an **equation**. An equation is **solved** when a value is found for the unknown that makes the equation a true number sentence. This value is called the **solution** of the equation.

Solving simple equations

Solving by inspection can be used to solve an equation where only one operation is involved. These are called one-step equations and can be solved by looking at the equation.

For example:

$3x = 9$	$\frac{x}{2} = 7$	$x + 3 = 11$	$4 - x = 1$
$x = 3$	$x = 14$	$x = 8$	$x = 3$

Guess, check and improve can be used to solve equations that involve more than one operation. A value for the pronumeral is guessed and substituted into the equation. If the value makes the left-hand side (LHS) equal to the right-hand side (RHS), the solution has been found. If the LHS ≠ RHS, another value can be tried.

This method can work well if the solution is a small integer value.

Worked Example 3

WE3

Write each of the following equations in words.

(a) $5x - 3 = 1$

(b) $6 = \frac{x}{5} + 2$

Working

(a) When a number is multiplied by five and then three is subtracted, the result is equal to one.

or

Subtract three from the product of five and a number to get one as a result.

(b) When a number is divided by five and then two is added the result is equal to six.

or

Add two to one-fifth of a number to get six as a result.

Worked Example 4 WE4

For the following equation, check whether the value given in the brackets is the solution. (Does the value make the equation true?) Answer Yes or No.

$3x - 4 = 11 \quad (x = 5)$

Working

$\text{LHS} = 3x - 4$
$= 3 \times 5 - 4$
$= 11$

$\text{RHS} = 11$

Yes, $x = 5$ makes the equation true.

Worked Example 5 WE5

Find the solution to the following equation by using the guess, check and improve method.

$$\frac{6x - 1}{5} = 7$$

Working

Try $x = 5$

$$\frac{6 \times 5 - 1}{5} = \frac{30 - 1}{5}$$
$$= \frac{29}{5}$$
$$\neq \text{RHS}$$

Try $x = 6$

$$\frac{6 \times 6 - 1}{5} = \frac{36 - 1}{5}$$
$$= \frac{35}{5}$$
$$= 7$$
$$= \text{RHS}$$

$x = 6$ is the solution to the equation.

7.2 Introduction to equations

Answers page 482

Navigator

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q7, Q8, Q9, Q12	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12

Fluency and understanding

1 Write each of the following equations in words. WE3

(a) $n + 3 = 10$ **(b)** $4 + p = 7$ **(c)** $a + 5 = 6$

(d) $x - 2 = 4$ **(e)** $k - 7 = 2$ **(f)** $a - 8 = 6$

(g) $5k = 20$ **(h)** $7p = 14$ **(i)** $7a = 21$

(j) $\frac{x}{3} = 4$ **(k)** $\frac{f}{2} = 6$ **(l)** $\frac{y}{6} = 12$

(m) $2 + 3x = 7$ **(n)** $2 + 6x = 10$ **(o)** $1 + 3x = 8$

WE4

2 For the following equations, check whether the value given in the brackets is the solution. (Does the value make the equation true?) Answer Yes or No.

(a) $x + 6 = 11$ $(x = 5)$ **(b)** $y + 4 = 10$ $(y = 5)$ **(c)** $m + 2 = 11$ $(m = 9)$

(d) $d - 6 = 15$ $(d = 21)$ **(e)** $k - 3 = 8$ $(k = 5)$ **(f)** $a - 6 = 9$ $(a = 15)$

(g) $3x = 12$ $(x = 3)$ **(h)** $5a = 20$ $(a = 4)$ **(i)** $7m = 14$ $(m = 2)$

(j) $\frac{f}{2} = 16$ $(f = 8)$ **(k)** $\frac{m}{3} = 5$ $(m = 15)$ **(l)** $\frac{x}{7} = 14$ $(x = 2)$

(m) $5m + 1 = 9$ $(m = 2)$ **(n)** $2a - 2 = 8$ $(a = 3)$ **(o)** $5s - 4 = 6$ $(s = 2)$

(p) $\frac{x}{3} + 2 = 5$ $(x = 9)$ **(q)** $5 + \frac{b}{2} = 9$ $(b = 8)$ **(r)** $\frac{a}{15} + 2 = 5$ $(a = 30)$

WE5

3 Find the solution to each of the following equations by using the guess, check and improve method.

(a) $2x + 1 = 9$ **(b)** $6 + 5x = 41$ **(c)** $7x + 3 = 52$

(d) $\frac{x - 8}{3} = 7$ **(e)** $\frac{x - 5}{2} = 6$ **(f)** $\frac{x - 4}{5} = 12$

(g) $3(x - 11) = 21$ **(h)** $8(x - 2) = 32$ **(i)** $6(x + 13) = 54$

(j) $\frac{x}{2} - 5 = 33$ **(k)** $\frac{x}{6} - 8 = 14$ **(l)** $\frac{x}{3} + 7 = 61$

4 In words, the equation $2 = 5 - \frac{x}{6}$ can be written as:

A two is equal to five added to a number that is divided by six

B five subtracted from six times a number is equal to two

C five added to two is equal to a number that is divided by six

D a number is divided by six and the result is subtracted from five to give an answer of two.

5 (a) Which equation describes this sentence?

Four added to a number is equal to ten.

A $n + 10 = 4$ **B** $n + 4 = 10n$ **C** $4 + 10 = n$ **D** $n + 4 = 10$

(b) Which equation describes this sentence?

A number multiplied by three is equal to six.

A $x + 3 = 6$ **B** $6x = 3$ **C** $3x = 6$ **D** $\frac{x}{3} = 6$

(c) Which equation describes this sentence?

A number divided by four is equal to three.

A $4x = 3$ **B** $x + 4 = 3$ **C** $4 - x = 3$ **D** $\frac{x}{4} = 3$

6 (a) The solution to the equation $x + 3 = 18$ is:

A $x = 6$ **B** $x = 15$ **C** $x = 16$ **D** $x = 21$

(b) The solution to the equation $\frac{x}{2} = 4$ is:

A $x = 1$ **B** $x = 2$ **C** $x = 6$ **D** $x = 8$

(c) The solution to the equation $4 = x - 11$ is:

A -15 **B** 7 **C** 11 **D** 15

7 **(a)** Which equation describes this situation?

The cost, c, in dollars, of some grocery items plus a $3 delivery charge totals $55.

A $3c = 55$ **B** $c - 3 = 55$ **C** $c + 3 = 55$ **D** $55 + 3 = c$

(b) Which equation describes this situation?

Each of 4 people receive the same number of lollies, n, when a bag containing 36 lollies is shared among them.

A $4n = 36$ **B** $4 + n = 36$ **C** $\frac{n}{4} = 36$ **D** $n - 4 = 36$

(c) Which equation describes this situation?

A school minibus seats a total of 12 passengers when full. The number of passengers, p, on the bus decreases by 3 at the next stop so that the bus is now half full.

A $p - 3 = 12$ **B** $3p = 6$ **C** $p + 3 = 6$ **D** $p - 3 = 6$

8 Use guess, check and improve to find the solution to the following equations.

(a) $\frac{x}{2} = x - 5$ **(b)** $3x = 5x - 8$ **(c)** $\frac{3x - 1}{4} = \frac{2(x + 9)}{5}$

9 Daniel is three years older than his brother Christopher.

(a) Define a variable to represent Daniel's age.

(b) Use this variable to write an expression for Christopher's age.

(c) Write an expression to show how old Christopher will be in 15 years' time.

(d) If Christopher will be 32 in 15 years' time, write an equation using your answer to part **(c)** as the LHS of the equation and solve it to find Daniel's age now.

Reasoning, communicating and open-ended

10 Cameron has some money in his wallet. Libby has $5.

(a) Define a variable to represent the amount of money Cameron has in his wallet.

(b) If Cameron spends $14, write an expression to show how much money he has left.

(c) If Libby and Cameron now have $23 between them, write an equation and solve it to find how much money Cameron had to start with.

11 Pete is twice as old as Chee.

(a) Choose a value that is a multiple of 3 as the sum of their ages and write the equation that you would use to find Chee's age.

(b) Choose another value that is also a multiple of 3 as the sum of their ages and write a new equation you would use to find Chee's age.

(c) What age might Chee be if the sum of their ages is less than 30?

12 Write a short story that could be represented by the following equations.

(a) $14 - x = 11$ **(b)** $\frac{x}{5} = 12$ **(c)** $6x = 108$

7.3 Solving equations using backtracking

Need to Know

Some equations are difficult to solve by inspection or guess, check and improve because the solution is not a whole number, or more than one operation is involved.

For example, $x + 3\frac{1}{4} = 7\frac{1}{5}$, $3x = 157$, $x - 3.75 = 21.8$ or $\frac{x}{7} = 53\frac{1}{9}$ are all one-step equations but their solutions are not easy to see.

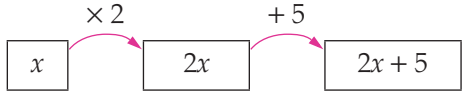
$2x + 5 = 11$, $3x - 4 = 31$, $\frac{5x}{4} = 5$, $\frac{x+6}{8} = 2$ are all two-step equations.

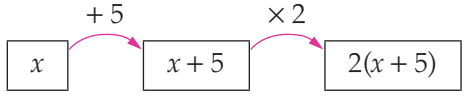
We need to use other methods to solve these equations.

Using flowcharts

Flowcharts have been used to build expressions. They show the order of operations that has been used. Applying the same operations in a different order produces different expressions.

For example:





$2x + 5 \neq 2(x + 5)$

Inverse operations

Flowcharts can be used to undo the equation by using the **inverse** (opposite) **operations** in the reverse order. The equation is solved by moving backwards along the flowchart.

$+$ and $-$ are inverse operations $\times$ and $\div$ are inverse operations

Hint

The order in which we apply the operation is important in building expressions.

Backtracking

Equations can be solved by **backtracking**. Make sure that the expression containing the variable is on the LHS and the constant is on the RHS.

Step 1 Draw a flowchart for the expression on the LHS of the equation using the correct order of operations. Write the operations on the arrow signs.

Step 2 Add boxes underneath the flowchart boxes and arrow signs in the opposite direction.

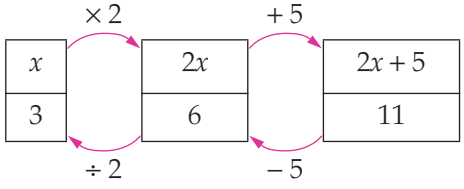
Step 3 Write the inverse operations on the arrow signs.

Step 4 Write the number on the RHS of the equation in the box on the right-hand side of the flowchart underneath the expression.

Step 5 Complete the flowchart using the inverse operations.

Step 6 The number under x is the solution.

For example, backtracking along this flowchart gives the solution $x = 3$ to the equation $2x + 5 = 11$.



Checking by substitution

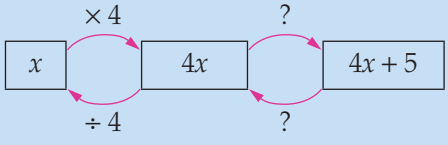
The answer can be checked by substituting it into the equation to replace the pronumeral. If the LHS = RHS, the answer is the solution.

Worked Example 6

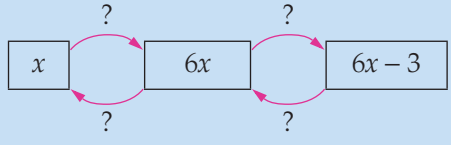
WE 6

Copy and complete the following simplified flowcharts to show the order of the operations needed to build and undo the expression.

(a)

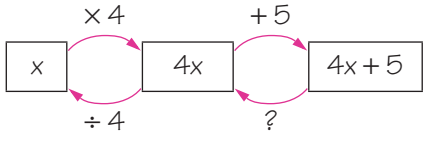


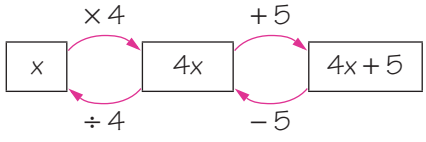
(b)



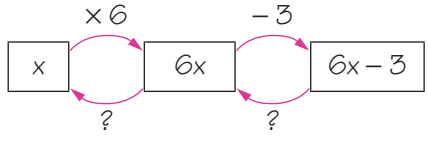
Working

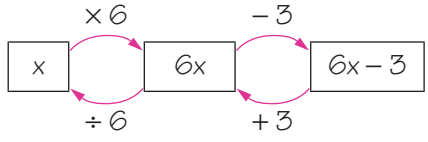
(a)





(b)



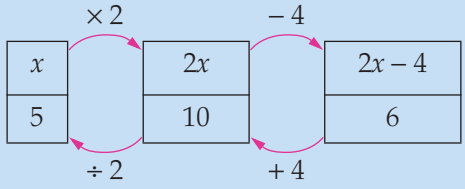


Worked Example 7

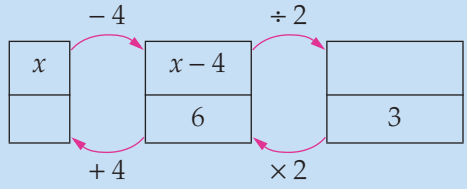
WE7

Write down **(i)** the equation to be solved and **(ii)** the solution to the equation shown in each of the following flowcharts. Complete the flowchart first if necessary.

(a)



(b)

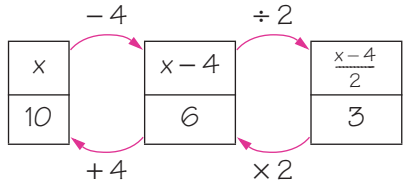


Working

(a) (i) Equation is $2x - 4 = 6$

(ii) Solution is $x = 5$

(b) (i)



$$\frac{x-4}{2} = 3$$

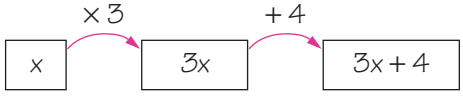
(ii) $x = 10$

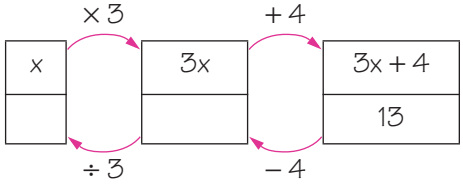
Worked Example 8

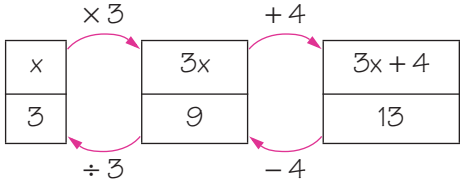
WE8

Draw a flowchart and use backtracking to solve the equation $3x + 4 = 13$.

Working







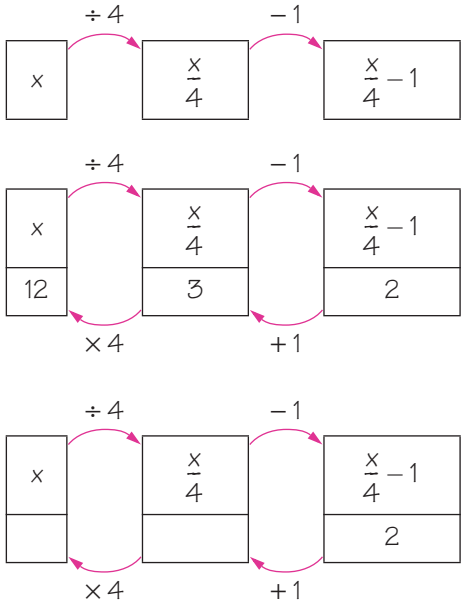
$x = 3$

Worked Example 9

WE9

Draw a flowchart and use backtracking to solve the equation $\frac{x}{4} - 1 = 2$. Check your solution by substitution.

Working



Check: LHS $= \frac{x}{4} - 1$

$= \frac{12}{4} - 1$

$= 3 - 1$

$= 2$

$= \text{RHS}$

$x = 12$

7.3 Solving equations using backtracking

Answers page 483

Navigator

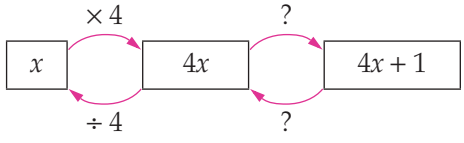
Q1 Column 1, Q2 Column 1, Q3 Columns 1 & 2, Q4 Columns 1 & 2, Q5 Column 1, Q6 Column 1, Q7 Column 1, Q8 Column 1, Q9, Q10 (a) & (b), Q13	Q1 Column 1, Q2 Column 1, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5 Column 2, Q6 Column 2, Q7 Column 2, Q8 Column 2, Q9, Q10 (a)–(c), Q11, Q12, Q13	Q1 Column 2, Q2 Column 2, Q3 Column 3, Q4 Column 3, Q5 Column 3, Q6 Column 3, Q7 Column 3, Q8 Column 3, Q9, Q10, Q11, Q12, Q13, Q14

Fluency and understanding

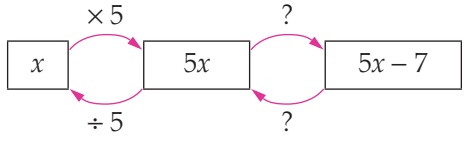
1 Copy and complete the following simplified flowcharts to show the order of the operations needed to build and undo the expression.

WE6

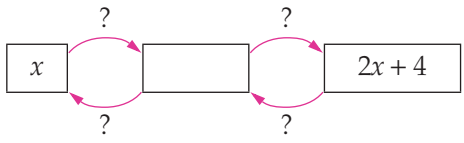
(a)



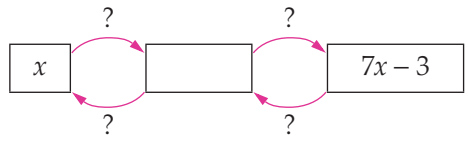
(b)



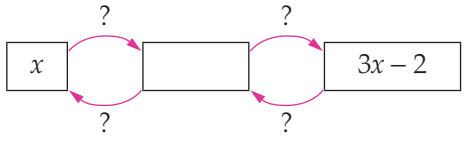
(c)



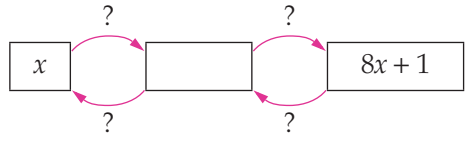
(d)



(e)



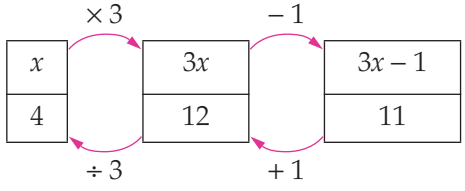
(f)



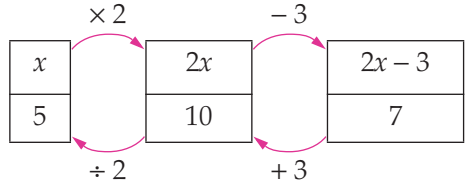
WE7

2 Write down **(i)** the equation to be solved and **(ii)** the solution to the equation shown in each of the following flowcharts. Complete the flowchart first if necessary.

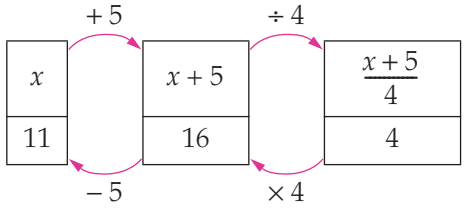
(a)



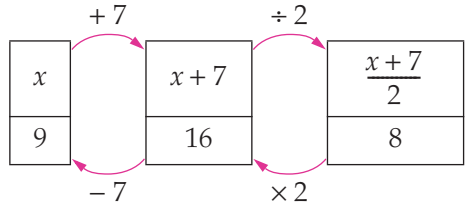
(b)



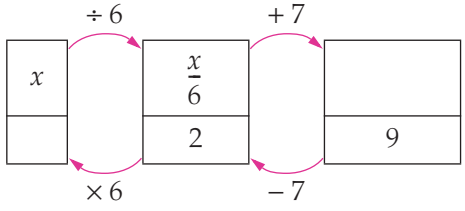
(c)



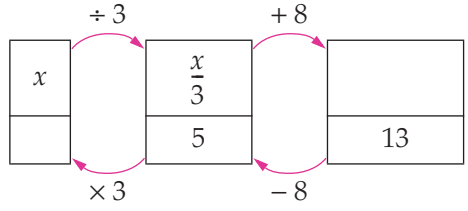
(d)



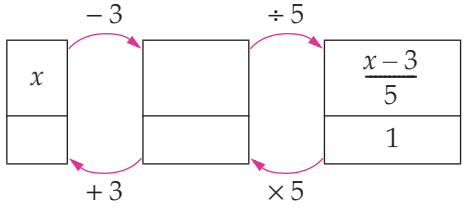
(e)



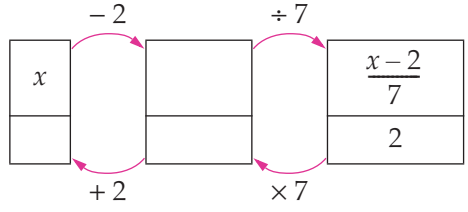
(f)



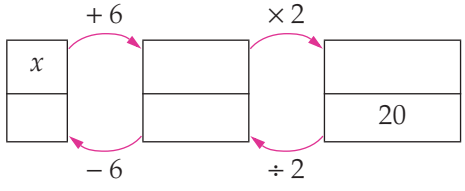
(g)



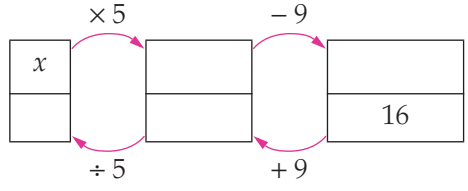
(h)



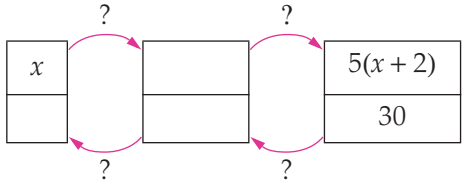
(i)



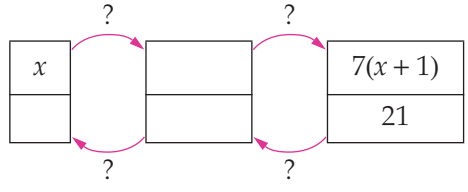
(j)



(k)



(l)



3 Draw a flowchart and use backtracking to solve each of the following equations. WE8

(a) $5x + 3 = 8$ **(b)** $7x + 3 = 24$ **(c)** $2x + 3 = 11$

(d) $2x - 7 = 3$ **(e)** $3x - 2 = 10$ **(f)** $8x - 11 = 13$

4 Draw a flowchart and use backtracking to solve each of the following equations. Check your solutions by substitution. WE9

(a) $\frac{x}{4} + 5 = 7$ **(b)** $\frac{x}{2} + 6 = 11$ **(c)** $\frac{x}{7} + 3 = 5$

(d) $\frac{x}{2} - 2 = 1$ **(e)** $\frac{x}{5} - 1 = 2$ **(f)** $\frac{x}{2} - 4 = 2$

5 Use backtracking to solve each of the following equations. Check your solutions by substitution.

(a) $3(x + 1) = 12$ **(b)** $2(x + 3) = 16$ **(c)** $2(x + 5) = 14$

(d) $5(x - 6) = 10$ **(e)** $6(x - 4) = 18$ **(f)** $3(x - 5) = 3$

6 Use backtracking to solve each of the following equations. Check your solutions by substitution.

(a) $\frac{x + 8}{3} = 3$ **(b)** $\frac{x + 5}{2} = 4$ **(c)** $\frac{x + 10}{7} = 2$

(d) $\frac{x - 2}{5} = 1$ **(e)** $\frac{x - 3}{4} = 2$ **(f)** $\frac{x - 5}{2} = 4$

7 Use backtracking to solve each of the following equations. Check your solutions by substitution.

(a) $2x + 3 = 13$ **(b)** $3x + 2 = 11$ **(c)** $5x + 6 = 11$

(d) $5x - 3 = 17$ **(e)** $4x - 1 = 7$ **(f)** $8x - 9 = 7$

(g) $2(x + 7) = 20$ **(h)** $4(x - 3) = 8$ **(i)** $6(x - 1) = 12$

(j) $\frac{x}{2} + 3 = 8$ **(k)** $\frac{x}{7} + 10 = 12$ **(l)** $\frac{x}{3} - 2 = 3$

(m) $\frac{x + 2}{4} = 2$ **(n)** $\frac{x - 4}{5} = 2$ **(o)** $\frac{x - 3}{2} = 2$

8 Solve each of the following three-step equations using backtracking. Check your solutions by substitution.

(a) $\frac{2x + 1}{3} = 3$ **(b)** $\frac{3x + 7}{2} = 11$ **(c)** $\frac{5x - 2}{4} = 7$

(d) $\frac{3x}{5} + 6 = 9$ **(e)** $\frac{2x}{7} + 8 = 12$ **(f)** $\frac{7x}{2} - 10 = 4$

(g) $\frac{2(x + 4)}{5} = 6$ **(h)** $\frac{3(x + 2)}{4} = 3$ **(i)** $\frac{2(x - 3)}{7} = 6$

9 The solution to $\frac{4x}{5} + 6 = 14$ is:

A $x = 1$ **B** $x = 10$ **C** $x = 16$ **D** $x = 25$

Hint

We need more than three boxes in the flowchart to solve these equations.

10 Write an equation using the following information, then solve each equation using backtracking. Use n to represent the unknown number.

(a) A number is doubled, then one is added to the result to give an answer of forty-three.

(b) When four is added to a number and the result is divided by five, the answer is four.

(c) A number is multiplied by seven and the result is divided by three to give an answer of twenty-one.

(d) A number is doubled, then six is added. The result is divided by ten to give an answer of two.

11 Belinda buys 3 packets of pencils. She gives five pencils to her friends. She sells the rest for 10 cents each and makes $6.70. How many pencils were in each packet? Using n as the number of pencils in the packet, write an equation and use backtracking to answer this question.

Reasoning, communicating and open-ended

12 Ying repairs televisions. The expression for the amount of money ($) she charges is $30x + 20$, where x is the number of hours she takes to repair a TV. Aldo also repairs televisions. The expression for the amount of money ($) he charges is $20x + 50$ where x is the number of hours he takes to repair a TV.

(a) Show the flowchart for the expression $30x + 20$.

(b) Show the flowchart for the expression $20x + 50$.

(c) Substitute the number $x = 1$ in both flowcharts and write down how much money Ying and Aldo charge for a repair that takes one hour.

(d) Ying charges a customer $95. How long did it take her to repair this TV?

(e) If Aldo charged $95, how long would he have taken?

(f) By trying different values for x, find the number of hours for which the amount charged by Ying and Aldo is the same.

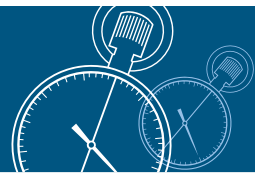
13 If x is the temperature in degrees Celsius, the expression $\frac{9x}{5} + 32$ gives the value of x in degrees Fahrenheit.

(a) Build and complete the flowchart for the expression.

(b) Use your flowchart to find two values for the temperature in degrees Celsius that give a whole number for degrees Fahrenheit.

14 After Ross performs four different operations on a number, he has a result of 10. Write an equation that fits this situation.

Half-time 7

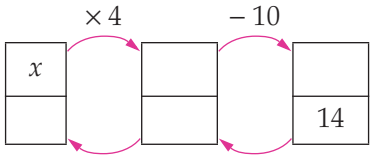


1 Solve the equation $\frac{x}{12} = 7$ by inspection. Ex.7.2

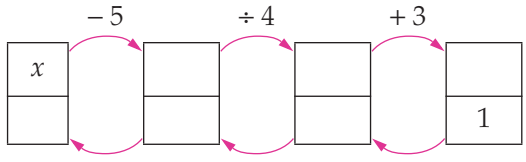
2 The perimeter of a rectangular room is 18 m. If the length of the room is 5 m, what is the width? Form an equation and solve it with backtracking. Ex.7.3

3 Copy and complete the following flowcharts. Write the equation and use the flowchart to find the solution for x. Ex.7.3

(a)



(b)



4 Write a true number sentence that shows two consecutive numbers whose sum is 43. Ex.7.1

5 **(a)** James is 5 years older than his brother Tom who is 7 years old. If James is j years old: Ex.7.2

(i) write an equation to represent this situation with Tom's age on the RHS

(ii) solve the equation by inspection to find James's age.

(b) Jude is 4 years younger than his sister Amanda who is 16 years old. If Jude is d years old:

(i) write an equation to represent this situation with Amanda's age on the RHS

(ii) solve the equation by inspection to find Jude's age.

6 Use backtracking to solve each of the following equations. Ex.7.3

(a) $3x + 4 = 11$ **(b)** $\frac{x}{2} + 5 = -3$ **(c)** $3(x - 8) = 15$

7 Write the equation $5x + 3 = 23$ in words. Ex.7.2

8 The following number sentences are not true. Rewrite them by changing the coloured number so that you have a true number sentence. Ex.7.1

(a) $3 + 6 = 11$ **(b)** $4 \times 3 = 26 - 6$ **(c)** $4 + \frac{28}{4} = 18$

9 Kate has a piece of cardboard that is x centimetres wide. She joins a 50-centimetre piece of cardboard of the same length to it and then divides the width into 7 strips so she can paint each strip a different colour of the rainbow. Each strip is 10 cm wide. Use backtracking to find the width of the original piece of cardboard. Ex.7.3

10 State whether the following number sentences are TRUE or FALSE. Ex.7.1

(a) $6 - 4 = 4 - 6$ **(b)** $2(3 + 5) = 2 \times 3 + 2 \times 5$ **(c)** $(8 \times 9) \times 2 = 8 \times (9 \times 2)$

11 Pete is five years older than his brother Sam. The sum of their ages is 23. If Pete is p years old: Ex.7.3

(a) write an expression to represent Sam's age

(b) write an equation to represent this situation with 23 on the RHS of the equation

(c) solve the equation using backtracking to find the ages of Pete and Sam.

7.4 Solving equations using the balance method

Need to Know

Equivalent equations

Equivalent equations have the same solution.

For example: $x = 2$, $2x = 4$ and $2x + 3 = 7$ are all equivalent equations.

Balance scales are kept in balance by performing the same operation on both sides of the balance, so the **balance method** is used to solve equations by creating equivalent equations. Do this by performing the same operation on both sides of the equation.

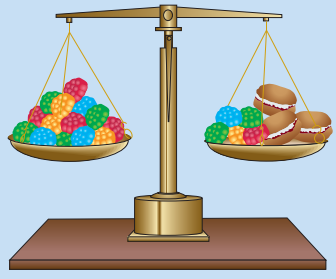
For example:

$$2x + 3 = 7$$
$$2x + 3 - 3 = 7 - 3 \quad \text{(subtracting 3 from both sides)}$$
$$2x = 4$$
$$\frac{2x}{2} = \frac{4}{2} \quad \text{(dividing both sides by 2)}$$
$$x = 2$$

Worked Example 10

WE10

This set of scales is balanced. The left-hand side has 14 lollies; and the right-hand side has three biscuits and five lollies.



(a) If one lolly is taken from the right-hand side, the scales become unbalanced. Which of the sides is now heavier?

(b) How can the scales be balanced without putting the lolly back?

(c) Starting again, if all of the lollies are taken from the right-hand side, what should be done to the left-hand side to balance the scales?

(d) How many lollies are equivalent to the three biscuits?

(e) How many lollies are equivalent to one biscuit?

Working

(a) The left-hand side is now heavier.

(b) Take a lolly from the left-hand side.

(c) Take five lollies from the left-hand side.

(d) Nine lollies = three biscuits.

(e) Three biscuits = nine lollies.
One biscuit = three lollies.

Worked Example 11 WE11

Form an equivalent equation to each of the following by performing the operation given in brackets to both sides of the equation.

(a) $x - 5 = 7 \ (+6)$ **(b)** $5x + 3 = 8 \ (-2)$ **(c)** $\frac{x}{2} = 4 \ (\times 2)$ **(d)** $8x = 24 \ (\div 4)$

Working

(a) $x - 5 = 7$
$x - 5 + 6 = 7 + 6$
$x + 1 = 13$

(b) $5x + 3 = 8$
$5x + 3 - 2 = 8 - 2$
$5x + 1 = 6$

(c) $\frac{x}{2} = 4$
$\frac{x}{2} \times 2 = 4 \times 2$
$x = 8$

(d) $8x = 24$
$\frac{8x}{4} = \frac{24}{4}$
$2x = 6$

Worked Example 12 WE12

Solve the equation $3x - 7 = 11$ using the balance method. Check your solution by substitution.

Working

$3x - 7 = 11$
$3x - 7 + 7 = 11 + 7$
$3x = 18$
$\frac{3x}{3} = \frac{18}{3}$
$x = 6$

Check: LHS $= 3x - 7$
$= 3 \times 6 - 7$
$= 18 - 7$
$= 11$
$=$ RHS

7.4 Solving equations using the balance method

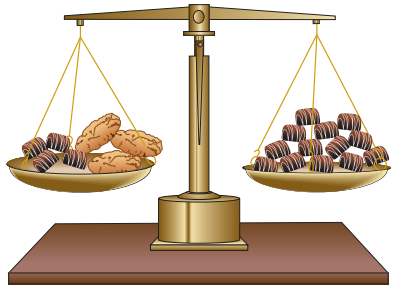
Answers page 485

Navigator

Q1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q7 Column 1, Q10, Q11, Q12	Q1, Q2 Column 2, Q3 Column 2, Q4, Q6, Q7 Columns 1 & 2, Q8, Q9, Q10, Q12	Q1, Q2 Column 2, Q3 Column 3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12

Fluency and understanding

1 This set of scales is balanced. The left-hand side has 4 chocolates and 3 biscuits; and the right-hand side has 13 chocolates.



WE10

(a) If one chocolate is taken from the left-hand side, the scales become unbalanced. Which of the sides is now heavier?

(b) How can the scales be balanced without putting the chocolate back?

(c) Starting again, if all of the chocolates are taken from the left-hand side, what should be done to the right-hand side to balance the scales?

(d) How many chocolates are equivalent to the three biscuits?

(e) How many chocolates are equivalent to one biscuit?

WE11

2 Form an equivalent equation to each of the following by performing the operation given in brackets to both sides of the equation.

(a) $x + 5 = 8$ $(+ 3)$

(b) $x + 4 = 12$ $(+ 5)$

(c) $x + 1 = 7$ $(+ 6)$

(d) $2x + 2 = 10$ $(- 1)$

(e) $3x + 5 = 10$ $(- 5)$

(f) $6x + 8 = 14$ $(- 5)$

WE12

3 Solve each of the following equations using the balance method. Check your solutions by subsitution.

(a) $3x + 5 = 8$

(b) $5x + 1 = 21$

(c) $7x + 3 = 17$

(d) $2x + 1 = 11$

(e) $2x + 5 = 19$

(f) $2x - 10 = 8$

(g) $3x - 4 = 20$

(h) $5x - 1 = 4$

(i) $6x - 2 = 16$

(j) $4x - 3 = 17$

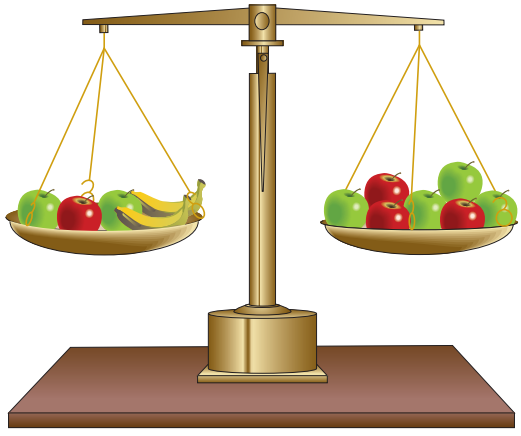
(k) $2x - 3 = 7$

(l) $4x - 1 = 31$

Hint

Inverse operations need to be applied in the correct order to find a solution.

4 This set of scales is balanced with 3 apples and 2 bananas on the left-hand side and 7 apples on the right-hand side.



If 4 apples are taken from the right-hand side, the scales can be balanced by:

A taking 4 apples from the left-hand side

B taking 3 apples and 1 banana from the left-hand side

C taking 2 apples and 2 bananas from the left-hand side

D taking 2 bananas from the left-hand side.

5 Trixie's father weighs twice as much as Trixie plus 12 kilograms. If her father weighs 104 kg:

(a) write an equation representing the situation using t to represent Trixie's weight

(b) solve the equation to find out Trixie's weight.

6 Naram has shot a number of 3-point goals as well as 18 other points in a game of basketball. If he shot a total of 30 points in the game:

(a) write this as an equation with p representing the number of 3-point goals shot by Naram

(b) solve your equation to find the number of 3-point goals that he shot.

7 Solve each of the following equations using the balance method. Check your solution by substitution.

(a) $\frac{2x}{3} = 8$

(b) $\frac{2x}{7} = 2$

(c) $\frac{5x}{2} = 20$

(d) $\frac{x+5}{3} = 5$

(e) $\frac{x+1}{3} = 7$

(f) $\frac{x+4}{5} = 3$

(g) $3x + 5 = 12$

(h) $3(x + 2) = 24$

(i) $3(x + 4) = -2$

(j) $2(x - 5) = 14$

(k) $7(x - 2) = 35$

(l) $8(x - 30) = 5$

Reasoning, communicating and open-ended

8 Fatima drives to and from work each day for the five-day working week, and drives an additional 85 kilometres on the weekend. She drives a total of 425 kilometres every week in her car. How far is it from her home to her workplace?

9 George has to take 5 litres of water with him on a boat trip. He has one 1.25 L container and three other containers of equal capacity. Together, the four containers will hold 5 litres. What is the capacity of the other containers?

10 Write down three different equations that are equivalent to $5x + 4 = 14$.

11 Vejay was given the equation $\frac{x+3}{7} = 3$ to solve and presented the following solution.

$$\frac{x+3}{7} = 3$$
$$\frac{x+3}{7} - 3 = 3 - 3$$
$$\frac{x}{7} = 0$$
$$\frac{x}{7} \times 7 = 0 \times 7$$
$$x = 0$$

(a) Explain to Vejay why his working out is incorrect.

(b) Show Vejay the process he should have used.

(c) How could Vejay avoid making the same mistake next time he's solving equations?

12 Aaron solved an equation and found it had a solution of $c = 12$.

Write two two-step equations that could have been the equation that he solved.

Problem solving

Paper mountain

Imagine a very, very large sheet of paper, 0.1 mm thick.

If you fold it in half, it is 0.2 mm thick.

How thick will it be if you fold it in half again?

Imagine the paper can be folded an unlimited number of times.

How many folds would be needed to create a paper stack that is taller than Mt Everest? (Mt Everest is 8848 m high.)

Strategy options

- Act it out.
- Look for a pattern.

7.5 Solving problems with equations

Need to Know

Equations can be written using information from word questions that represent everyday problems. The problem can be solved by finding the solution to the equation.

Worked Example 13

WE13

Phil has 70 cents. He buys a pear and has 25 cents left. If x represents the cost of a pear (in cents), form an equation and solve it to find the cost of the pear.

Working

Let x be the cost of a pear.

$$x + 25 = 70$$
$$x + 25 - 25 = 70 - 25$$
$$x = 45$$

$$\text{LHS} = 45 + 25$$
$$= 70$$
$$= \text{RHS}$$

The pear costs 45 cents.

Worked Example 14

WE14

Erica has bought two movie tickets and one bucket of popcorn for a total of $21. If the popcorn costs $5, how much does one movie ticket cost? Using t as the cost of a movie ticket, form an equation and solve it to find the value of t. (Assume both tickets are the same price.)

Working

Let t represent the cost in dollars of one movie ticket.

$$2t + 5 = 21$$
$$2t + 5 - 5 = 21 - 5$$
$$2t = 16$$
$$\frac{2t}{2} = \frac{16}{2}$$
$$t = 8$$

$$\text{LHS} = 2t + 5$$
$$= 2 \times 8 + 5$$
$$= 16 + 5$$
$$= 21$$
$$= \text{RHS}$$

One movie ticket costs $8.

7.5 Solving problems with equations

Answers page 485

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q11, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q12, Q13, Q14

Fluency and understanding

1 **(a)** Carmen has 95 cents. She buys an ice-cream and has 20 cents left. If x represents the cost of the ice-cream (in cents), form an equation and solve it to find the cost of the ice-cream. WE13

(b) Matthew can run 100 m in 12.8 seconds. In the school sports, Tony ran 1.5 seconds faster. If t represents Tony's time, form an equation and solve it to find Tony's time.

2 **(a)** Tran bought two sushi rolls and a juice for a total of \$8. If the juice costs \$1.60, how much does one sushi roll cost? Using d as the cost of a sushi roll, form an equation and solve it to find the value of d. (Assume both sushi rolls are the same price.) WE14

(b) Asif bought three DVDs and a CD for \$59. If the CD costs \$21.50, how much does one DVD cost? Using v as the cost of a DVD, form an equation and solve it to find the value of v. (Assume all DVDs are the same price.)

3 A bus has the capacity to seat 45 passengers. Eighteen passengers have boarded the bus and taken seats.

(a) If n represents the number of seats still available, the equation that can be formed is:

A $n - 18 = 45$ **B** $n + 45 = 18$ **C** $18n = 45$ **D** $n + 18 = 45$

(b) Solve the equation formed in part **(a)** to find the number of seats still available on the bus.

4 A roll of fabric is divided evenly among six people. Each person receives 2.5 m of fabric.

(a) If t represents the total length of fabric in the roll (in m), the equation that can be formed is:

A $t = 6 + 2.5$ **B** $t + 2.5 = 6$ **C** $\frac{t}{6} = 2.5$ **D** $2.5t = 6$

(b) Solve the equation formed in part **(a)** to find the total length of the roll of fabric.

5 The length of a rectangular vegetable garden is 4 m. The perimeter of wire fencing needed to enclose this garden is 13 m.

(a) If w represents the width (in m) of the vegetable garden, the equation that can be formed is:

A $w + 4 = 13$ **B** $w + 8 = 13$ **C** $2w - 8 = 13$ **D** $2w + 8 = 13$

(b) Solve the equation formed in part **(a)** to find the width of the vegetable garden.

6 Lucas bought a salad sandwich and a container of milk for his lunch.

(a) If the salad sandwich cost \$$x$ and the milk cost \$1.50, write an expression for the total cost of his lunch.

(b) Form an equation and solve it to find the cost of the salad sandwich if Lucas spent \$5.70 on his lunch.

7 **(a)** If x represents the length (in cm) of a square floor tile, write an expression for its perimeter.

(b) Form an equation and solve it to find the length of the floor tile if its perimeter is 84 cm.

8 A restaurant has several tables that seat four people, but only three tables that seat two people.

(a) If x represents the number of four-person tables available, write an expression for the total number of people that can be seated at the restaurant.

(b) A Saturday night booking for 62 people has been made. Six of these people are to be seated at the three two-person tables. Form an equation and solve it to find how many four-person tables will be needed.

9 At a particular rollerskating rink, it costs \$5 to hire skates and \$3 for each hour spent on the rink. If it costs Natalie \$14, form an equation and solve it to find how many hours she spent rollerskating. Let h represent the number of hours spent rollerskating.

10 The tallest man in recorded history was Robert Wadlow (1918–1940). At age 22, his height was 54 cm less than twice his height at age 5. If his height at age 22 was 272 cm, form an equation and solve it to find his height at age 5.

Hint

Let x represent his height at age 5.

11 A company called Tool Time hires out electrical tools, charging a non-refundable deposit of \$15 for each item plus an amount per day according to the type of tool.

Tool	Hire charge per day
Electric drill	\$8
Electric sander	\$13
Electric circular saw	\$24
Electric jackhammer	\$35
Concrete mixer	\$48

Let d = the number of days the tools are hired, then form an equation and solve it to answer each of the following.

(a) If Allie hires an electric drill from Tool Time and pays \$63, for how many days did she hire it?

(b) If Serena hires a concrete mixer from Tool Time and pays \$111, for how many days did she hire it?

(c) Kosta hires an electric sander and an electric saw from Tool Time. For how many days did he hire the tools if he paid \$178?

Reasoning, communicating and open-ended

12 The local council has hired a concreting firm to pave some footpaths. Each footpath is to be one metre wide. A wooden frame is needed before the concrete is poured. This frame is made up of one-metre lengths of wood.

The frames for some different lengths of footpath are shown in the diagrams below.



2 m long footpath 3 m long footpath 4 m long footpath

(a) How many pieces of wood are needed to construct the frame of a footpath of length:

(i) 2 metres **(ii)** 3 metres **(iii)** 4 metres?

(b) Write an expression for the number of pieces of wood needed to construct the frame for a footpath of length x metres.

(c) Form an equation and solve it to find how many pieces of wood are needed to construct the frame for a footpath of length:

(i) 6 metres **(ii)** 10 metres **(iii)** 13 metres.

(d) The concreting workers have only 36 pieces of wood. By using your equation and solving it, find the length of the longest footpath that could be poured at the one time.

13 Gary the gardener is keen to do some landscaping on a Saturday and has allowed for 7 hours work. It costs $30 per hour for him to hire a jackhammer and $22 per hour to hire a mulcher. He has budgeted to spend no more than $180. Keeping in mind that he can only use one piece of equipment at a time, what are three options for using the equipment and what would each option cost him?

14 A man is 5 times the age of his son. Given that the sum of their ages is no greater than 70, what are their possible ages? Assume that the answers are whole numbers.

Problem solving

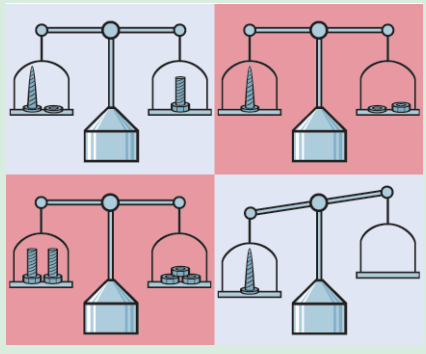
Algebraic puzzles

1 Four numbers are added together and the result is 255. One of the numbers is the square of one of the others. The largest number is double the square number, and the fourth number is 55. Find the numbers.

2 If a screw and a washer balance with a bolt, a screw balances with a washer and a nut, and two bolts balance with three nuts, find out how many washers will balance with a screw.

Strategy options

- Guess and check.
- Break problem into manageable parts.



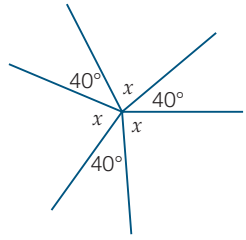
Challenge 7



1 When half of a number is increased by 15 the result is 49. The original number is:

A 68 **B** 34 **C** 32 **D** 17

2 Write an equation and solve it to find the value of x in the diagram.



3 Solve the equation: $\frac{3x-5}{4} = -6$.

4 If $59 + x = 73 + y$, then:

A $x = 14 - y$ **B** $x = 14 + y$

C $x = y - 14$ **D** $x = 14y$

5 Using only the digits 1, 2, 3 and 7 and the two mathematical symbols + and = only once, we can create a true number sentence $2^3 = 1 + 7$.

(a) Using the digits 2, 3, 4, 5 and the two mathematical symbols + and = only once and a number as a power, create a true number sentence.

(b) Using any different four digits from 0 to 9 and the two mathematical symbols + and = only once and one of the numbers as a power, create five different true number sentences. Do not use $2^3 = 1 + 7$ or your answer to **(a)**. Use 0 and 2 as a power only once.

(c) Use once only, any different four digits from 0 to 9 and the two mathematical symbols – and = only once and one of the numbers as a power to create five true number sentences. Use 0 and 1 as a power only once.

6 The inverse (reciprocal) of $\frac{2}{3}$ is $\frac{3}{2}$. If the inverse of $\frac{4x}{5}$ is $\frac{1}{20}$, then the value of x is:

A $\frac{1}{25}$ **B** $\frac{1}{16}$ **C** 16 **D** 25

7 Solve for y:

$$\frac{1}{4} + \frac{1}{5} + \frac{1}{y} = 1$$

8 Vladmir buys an equal number of 55-cent and $1.10 stamps and spends $80.85. How many stamps did he buy altogether? Form an equation and solve it.

9 A Lotto winner won $\frac{9}{10}$ of the total prize pool. Shortly after, she spent $\frac{3}{4}$ of her winnings, but still had $2700 left. What was the value of the total prize pool? Form an equation and solve it.

10 If $\frac{1}{3} > \frac{1}{4} > \frac{1}{m}$ and the difference between the first two fractions equals the difference between the last two fractions, find the value of m. (Hint: Write $\frac{1}{m}$ on one side of the equation before you take the inverse.)

11 Two integers, a and b, do not end in zero. If the product ab is a multiple of 10 and $a > b$, the last digit of $a - b$ cannot be:

A 1 **B** 3 **C** 5 **D** 7

Chapter review 7

D.I.Y. Summary

Key Words

backtracking	false number sentence	solution
balance method	flowchart	solve
checking by substitution	guess, check and improve	solving by inspection
equivalent equations	inverse operations	true number sentence

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 An equation can be solved by performing the _____ on both sides of the equation.

2 To _____ an equation is to find the value of the unknown that will make a true number sentence.

3 $4 + 5 = 10$ is a _____.

4 One way of solving an equation is by looking at the equation to see what number might be the answer. This is called _____.

5 Equations that have the same solution are said to be _____.

6 _____ is a method we can use to solve equations. It can be shown by moving backwards along a flowchart.

7 _____ is another method used to solve equations.

Fluency and understanding

1 The following number sentences are not true. Rewrite each number sentence by changing the coloured number so that you have a true number sentence. Ex.7.1

(a) $5 \times 4 = 30 - 2$ **(b)** $2 \times 7 + 6 = 9 + 5$

2 Find the unknown number in each of the following sentences. Ex.7.1

(a) Six subtracted from a number is equal to nine.

(b) A number divided by four is equal to seven.

3 Find the solution to each of the following equations by inspection. Ex.7.2

(a) $2x = 18$ **(b)** $x + 7 = 10$ **(c)** $\frac{x}{5} = 3$

(d) $x - 4 = 11$ **(e)** $x + 3 = 15$ **(f)** $x - 4 = 24$

4 Solve the following equations using guess, check and improve. Ex.7.2

(a) $3x + 25 = 16$ **(b)** $\frac{x}{7} + 51 = 63$ **(c)** $\frac{3x + 5}{4} = 8$

5 List the inverse (or opposite) operation to each of the following. Ex.7.3

(a) $+5$ **(b)** $\div 3$ **(c)** $\times 8$ **(d)** -4

6 Solve each of the following equations using a flowchart and backtracking. Ex 7.3

(a) $3x - 7 = 2$ **(b)** $\frac{x+6}{5} = 4$ **(c)** $4(x+1) = 20$ **(d)** $\frac{x}{8} - 2 = 3$

7 Use the balance method to solve the following equations. Ex 7.4

(a) $4x + 7 = 19$ **(b)** $5(x-2) = 20$ **(c)** $\frac{x}{3} + 5 = 10$

(d) $\frac{x-8}{6} = 3$ **(e)** $\frac{x+4}{5} = 5$ **(f)** $\frac{x-7}{3} = 10$

8 The equation $5x - 6 = 4$ has the solution: Ex 7.4

A $x = 1$ **B** $x = 2$ **C** $x = 3$ **D** $x = 4$

9 Danni bought 5 tickets to a show. She was given \$27.50 change from \$200. Use an equation to find how much each ticket cost. Let t be the cost of a ticket. Ex 7.5

10 Which equation describes this situation?

A restaurant seats a maximum of 64 patrons. On Saturday afternoon, it is fully booked for the evening meal service. At 5:30 p.m., a group booking for 16 is cancelled. If p represents the number of people who are still booked, the equation that can be formed is:

A $p - 16 = 64$ **B** $p = 64 + 16$ **C** $16p = 64$ **D** $p + 16 = 64$

11 Ross buys three dim sims and a bottle of juice.

(a) If each dim sim costs x cents and the bottle of juice costs \$1.40, write an expression for the total cost (in cents) of the food and juice.

(b) Use an equation to find the cost of one dim sim if Ross spends a total of \$2.75.

12 The air temperature at 4 p.m. is 28°C. Over the past hour, the temperature has risen by 5°C. What was the temperature at 3 p.m.? (Form an equation to solve by letting t represent the air temperature at 3 p.m.)

Reasoning and communicating

13 A triangle  made from matchsticks has three sides of equal length. If more matchsticks are added to make another triangle, we obtain this shape  .

(a) How many matchsticks of equal length does this shape have?

(b) A third triangle is added to the shape. How many matchsticks of equal length does this shape have?

(c) If n represents the number of triangles joined together in a row, write, using the pronumeral n, the number of matchsticks of equal length the shape has in total.

(d) **(i)** Use your answer to **(c)** to write down the equation for n triangles joined together that have 97 matchsticks of equal length.

(ii) Solve the equation obtained in part **(d) (i)**.

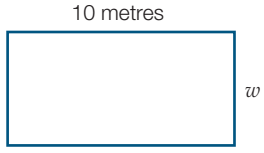
NAPLAN practice 7

Numeracy: Non-calculator

1 Jan is baking biscuits. She puts her biscuits in 3 equal rows. When they are baked, she eats one. She has 44 biscuits left. How many biscuits did she have in each row?

A 3 **B** 10 **C** 15 **D** 20

2 A rectangular garden bed has a perimeter of 30 metres and a length of 10 metres. If the width of the garden can be written as w, which equation best represents this situation?



A $w + 10 = 30$ **B** $w + 30 = 10$

C $2w + 10 = 30$ **D** $2w + 20 = 30$

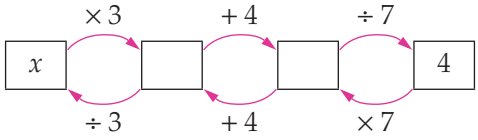
3 What is the missing number that makes this number sentence true?

$3 \times \square = 12 + 3 \times 6$

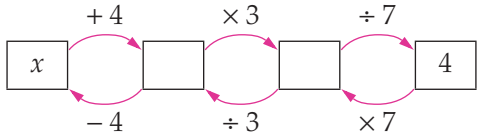
A 1 **B** 7 **C** 9 **D** 10

4 Voula thinks of a number and asks Renza to guess what it is. Voula states that after the number is tripled, four added and the result divided by seven, the answer is four. Which flowchart best represents how Renza might find Voula's mystery number? Let the mystery number = x.

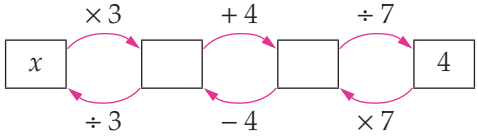
A



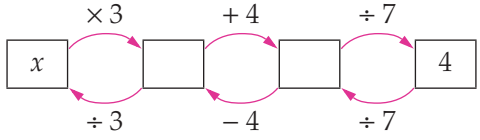
B



C



D



Numeracy: Calculator allowed

5 A pack of 4 doughnuts costs $3.60.
A pack of 6 doughnuts costs $4.00.
You need to buy 34 doughnuts.
What is the least amount you can pay?

A $23.60 **B** $26.40 **C** $26.50 **D** $29.90

6 The cooking time for a turkey is 30 minutes plus 20 minutes per kilogram. If M is the mass of the turkey in kilograms, the total cooking time in minutes is:

A $20 + 30M$ **B** $20M + 30$ **C** $20 \times 30M$ **D** $30 - 20M$

7 Julie, Nikki and Romina earned a total of $3400 in the last week. Julie earned twice as much as Nikki whereas Romina earned $200 more than Nikki. How much did Nikki earn in the last week?

A $600 **B** $800 **C** $1000 **D** $1600

8 The solution to $\frac{3(x-4)}{2} = 15$ is:

A $x = 6$ **B** $x = 10$ **C** $x = 14$ **D** $x = 58$

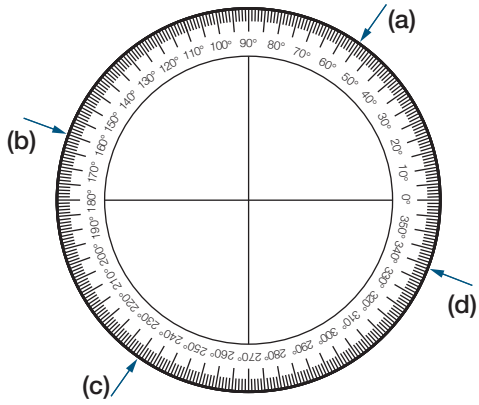
8 Angles and shapes

Recall 8

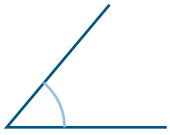
Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

1 State the value indicated by each arrow in this diagram.

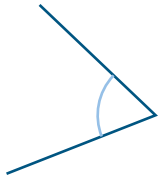
2 Arrange the following angles in order from smallest to largest.



A



B



C



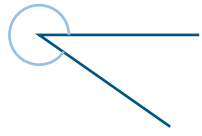
D



E

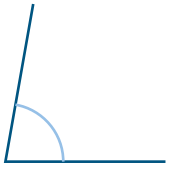


F



3 List the pairs of angles that look to be the same size.

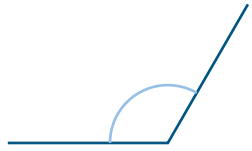
A



B



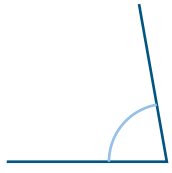
C



D



E



F



Syllabus references

Measurement and Geometry: Angles 1

✓ Estimate, measure and compare angles using degrees **(ACMMG112) (Stage 3)**

Measurement and Geometry: Angles 2

✓ Investigate, with and without the use of digital technologies, angles on a straight line, angles at a point, and vertically opposite angles; use the results to find unknown angles **(ACMMG141) (Stage 3)**

Measurement and Geometry: Angle Relationships

✓ Use the language, notation and conventions of geometry

✓ Recognise the geometrical properties of angles at a point

✓ Identify corresponding, alternate and co-interior angles when two straight lines are crossed by a transversal **(ACMMG163)**

✓ Investigate conditions for two lines to be parallel **(ACMMG164)**

✓ Solve simple numerical problems using reasoning **(ACMMG164)**

Measurement and Geometry: Properties of Geometrical Figures 1

✓ Classify triangles according to their side and angle properties and describe quadrilaterals **(ACMMG165)**

✓ Demonstrate that the angle sum of a triangle is 180° and use this to find the angle sum of a quadrilateral **(ACMMG166)**

✓ Use the properties of special triangles and quadrilaterals to solve simple numerical problems with appropriate reasoning

Measuring, estimating and drawing angles

8.1

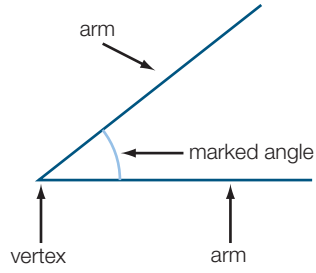
Stage 3

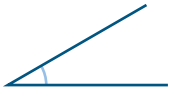
Need to Know

Angles

An **angle** is the space between two lines that start at the same point. This point is called the **vertex** of the angle and the lines form the **arms** of the angle.

Because two angles are formed between two arms, the angle required must be clearly marked.







Measuring angles

The size of an angle is the amount of turn from one arm to another. One method of measuring the amount of turn is by using **degrees** (°). When an arm is rotated completely around a circle for one full turn, this is called a **revolution**. A revolution is divided into 360 parts. Each part is called a degree, so a revolution equals 360°.

A **protractor** is a tool that shows degrees on a circular arc and can be used to draw and measure angles reasonably accurately. A full-circle protractor shows all 360°. A semicircular protractor shows only 180°.

Estimating and measuring angles

It is often important to be able to estimate angles by visualising how big or small they are, as it is not always possible or necessary to measure the angles exactly. A semicircular protractor or a full-circle protractor can be used to measure angles if a more exact measurement is required.

Using a semicircular protractor, place the centre point of the protractor over the vertex of the angle and align the baseline of the protractor with an arm of the angle. Use the scale on the protractor that starts with 0°. If the angle is greater than 180°, measure the smaller angle and subtract this measurement from 360° or use a full-circle protractor.

Drawing angles with a protractor

Draw a straight horizontal line with a ruler. Place the centre point of the baseline of the protractor on one end of the straight line. Locate the desired angle using the marked scale and mark a small dot on the outside of the protractor on the desired angle. (The baseline is not the edge of the protractor.) Remove the protractor and join the dot with the end of the line where the centre point was located. Mark the angle.

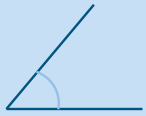
Worked Example 1

WE1

Estimate the size of each of the following angles, then use a protractor to measure the size of the angles. Write your answer, to the nearest degree, in a table using the headings as shown. Calculate the difference between your estimate and the measured size.

Estimated angle size	Measured angle size	Difference

(a)

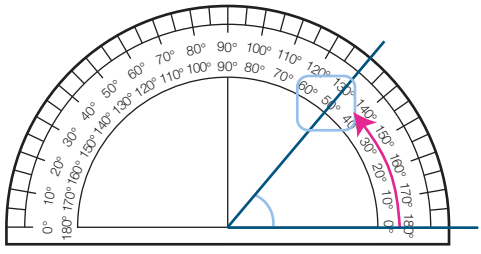


(b)



Working

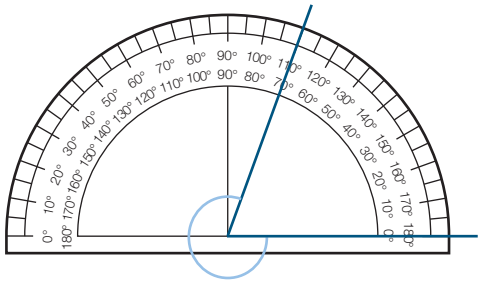
(a) Estimate: 45°



Estimated angle size	Measured angle size	Difference
45°	50°	5°

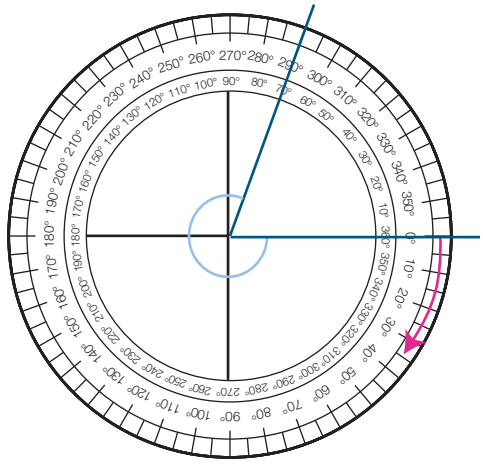
(b) Estimate: 300°

Method 1: Using a semicircular protractor



Estimated angle size	Measured angle size	Difference
300°	290°	10°

Method 2: Using a full-circle protractor



Estimated angle size	Measured angle size	Difference
300°	290°	10°

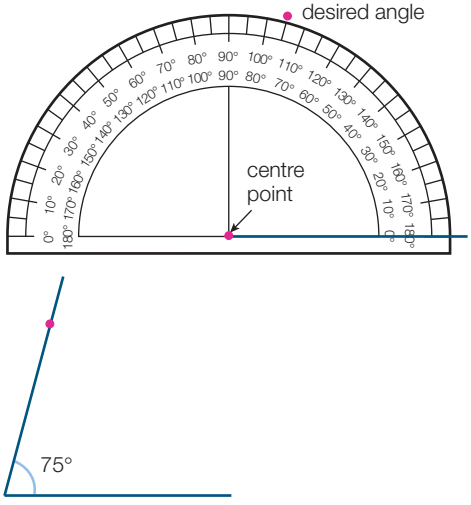
Worked Example 2 WE2

Use a protractor to draw the following angles.

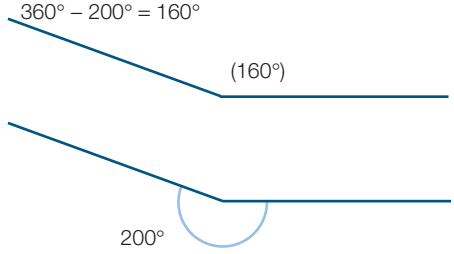
(a) 75° **(b)** 200°

Working

(a)



(b)



8.1 Measuring, estimating and drawing angles

Answers page 487

Stage 3

Navigator

Q1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11 Column 1, Q12, Q15	Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11 Column 2, Q12, Q13, Q16	Q1, Q2 Column 3, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11 Column 3, Q12, Q14, Q16

Equipment required: Protractor

Fluency and understanding

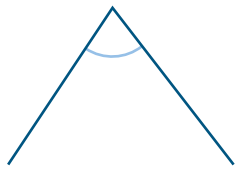
1 Estimate the size of each of the following angles, then use a protractor to measure the size of the angles. Write your answer, to the nearest degree, in a table using the headings as shown. Calculate the difference between your estimate and the measured size. WE1

Estimated angle size	Measured angle size	Difference

(a)



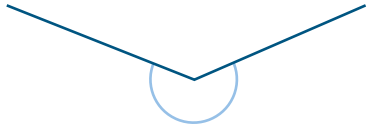
(b)

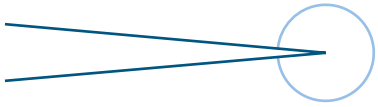


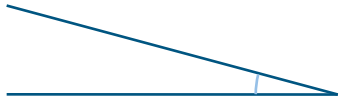
Hint

When one arm is too short, use a ruler to make it longer first.

(c) 

(d) 

(e) 

(f) 

WE2

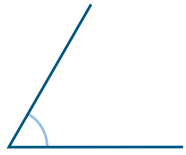
2 Use a protractor to draw the following angles.

(a) 10° **(b)** 70° **(c)** 55°

(d) 100° **(e)** 175° **(f)** 108°

(g) 290° **(h)** 265° **(i)** 318°

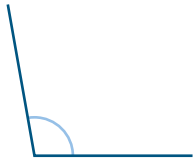
3 (a) The angle shown here is approximately:



A 20° **B** 45°

C 60° **D** 85°

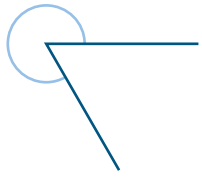
(b) The angle shown here is approximately:



A 80° **B** 100°

C 170° **D** 200°

(c) The angle shown here is approximately:



A 60° **B** 180°

C 240° **D** 300°

4 (a) Which of the angles below is about 170°?

A 

B 

C 

D 

(b) Which of the angles below is about 5°?

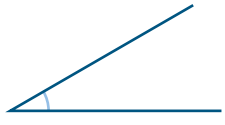
A 

B 

C 

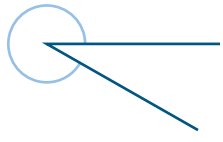
D 

(c) Which of the angles below is about 330°?

A 

B 

C 

D 

5 To draw a 250° angle with a semicircular protractor, which of the following would you do first?

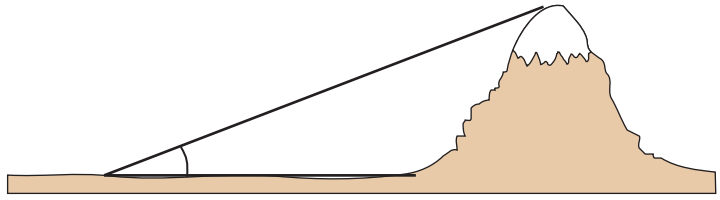
A Draw a 50° angle.

B Draw a 90° angle.

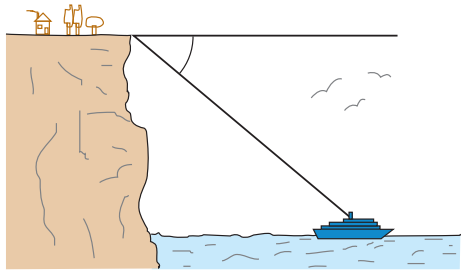
C Draw a 110° angle.

D Draw a 200° angle.

6 In the diagram, estimate the angle shown from ground level to the mountain top and then measure the angle with your protractor.



7 In the diagram, estimate the angle shown from the top of the cliff to the ship and then measure the angle with your protractor.

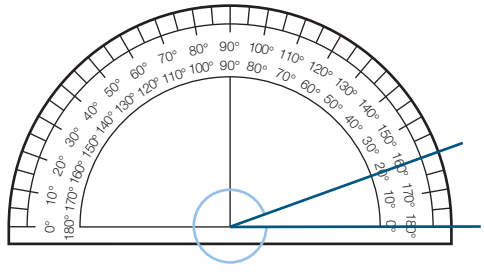


8 Explain how you would draw an angle of 320° with a semicircular protractor.

9 Liz uses a semicircular protractor to measure an angle larger than 180°.

Find the size of the actual angle (larger than 180°) if the smaller angle she has measured is:

(a) 30° **(b)** 115° **(c)** 160°



10 Draw the following using your protractor:

(a) a wire holding a vertical flagpole attached at 38° to the ground

(b) a ramp with a slope of 12°

(c) a ladder leaning against a vertical wall at 62° to the ground

(d) a round birthday cake sliced into 6 equal pieces.

Reasoning, communicating and open-ended

11 For each of the following angles, complete activities (i) to (vi):

(a) 35° **(b)** 65° **(c)** 83°

(d) 120° **(e)** 102° **(f)** 152°

(g) 200° **(h)** 265° **(i)** 325°

(i) Draw an estimate of the angle without the use of a protractor.

(ii) Use your protractor to measure the angles you have drawn to find the actual size of your angles.

(iii) Find the error (the difference between the given angle and your measurement).

(iv) Copy and complete the following table.

Estimated angle size	Measured angle size	Difference

(v) Which angles did you draw the most accurately?

(vi) Do you think it is harder to estimate angles greater than 180° than those less than 180°? Why?

12 Draw the following angles, estimate the size of the angle you have drawn and then check the size with a protractor.

(a) any angle smaller than 90°

(b) any angle between 90° and 180°

(c) any angle between 180° and 360°

13 Your friend Nick has just answered the following question.

> Use a protractor to measure the size of the following angle. Give your answer to the nearest degree.
>
> 

Nick says, 'The answers in the back of the book must be wrong! The book says that the answer is 135°, but my protractor says 45°.' Explain how Nick might have found his answer. Give Nick a hint, so that he can measure accurately all the time.

14 Without using a protractor, draw the following angles and estimate the size of each angle.

(a) two angles that add up to 90°

(b) three angles that add up to 180°

(c) four angles that add up to 360°

(d) Check your answers to **(a)**, **(b)** and **(c)** with a protractor.

15 **(a)** Choose any angle between 40° and 60°, draw it carefully with a protractor and mark the size of your angle on your diagram.

(b) Using a protractor, draw an angle 15° larger than the angle drawn in **(a)** and mark the size of your angle on your diagram.

(c) Using a protractor, draw an angle 195° larger than the angle drawn in **(a)** and mark the size of your angle on your diagram.

16 **(a)** Choose any angle less than 90°, draw it carefully with a protractor and mark the size of your angle on your diagram.

(b) Using a protractor, draw an angle twice the size of the angle you chose in **(a)** and mark the size of your angle on your diagram.

(c) Using a protractor, draw an angle twice the size of the angle in **(b)** and mark the size of your angle on your diagram.

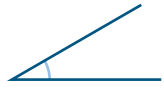
Classifying and naming angles

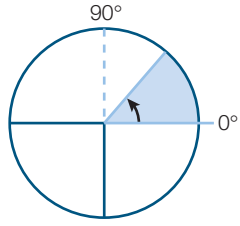
8.2

Need to Know

Classifying angles

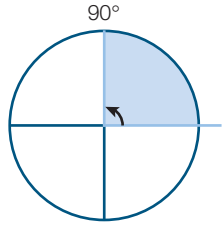
Acute angle: greater than 0° but less than 90°.





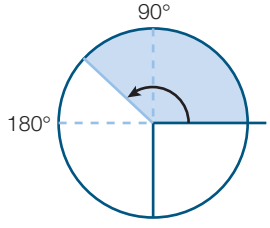
Right angle: exactly 90°, a $\frac{1}{4}$ turn. A small square drawn in the corner of an angle means it is a right angle.





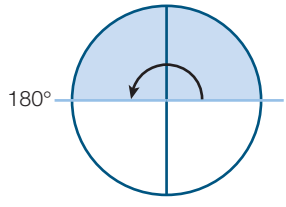
Obtuse angle: more than 90° but less than 180°.





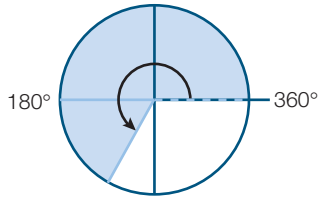
Straight angle: exactly 180°, a straight line angle, a half turn.





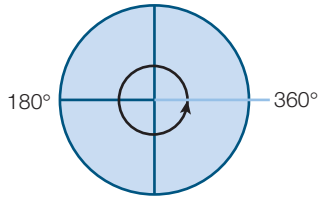
Reflex angle: greater than 180° but less than 360°.





Revolution: 360°, a full turn.





Points, lines and angles

A *point* defines a position in space but it has no size. It has no width or length. A point can be represented with a dot and labelled with upper case letters, such as A, B and P.



Similarly, a *line* has no width and has no beginning or end, extending infinitely in both directions. A drawn line represents a line. A line always means a straight line and does not include curves. A line can be drawn through any two points. Arrowheads on each end are used to show that the line goes on forever in both directions. A line that goes through points A and B is named $\overleftrightarrow{AB}$.



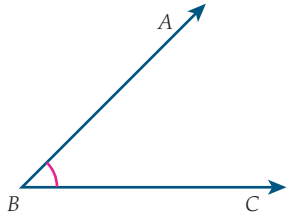
A **ray** is part of a line. Unlike a line, a ray has a beginning. It starts at a point called the vertex and extends infinitely in one direction. Draw a ray by starting it at a point and using an arrowhead on the other end. A ray that starts at point A and passes through B is named $\overrightarrow{AB}$.

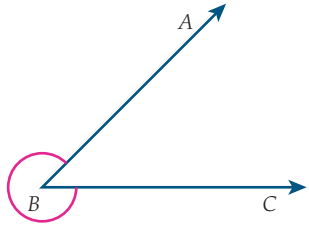


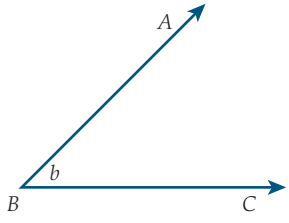
An **interval** is part of a line that has both a beginning and an end. It can also be called a **line segment**. We name an interval that starts at A and ends at B as $\overline{AB}$.



If two rays have a common vertex, two angles are formed. When an angle is drawn, mark clearly the angle required. Name the angles that are formed by $\overline{AB}$ and $\overline{BC}$ as $\angle ABC$ or as $\angle CBA$ with the vertex B named as the middle of the three points. Use the words acute, obtuse or reflex to indicate the chosen angle. An easier and clearer way to name an angle is to use a letter with units such as a or b in the angle space. Often the same letter as the letter on the vertex is used, though Greek letters such as θ (theta) are also used.

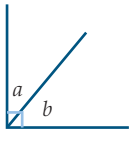




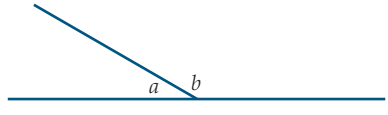


Complementary and supplementary angles

Complementary angles add to 90°:
$a + b = 90$

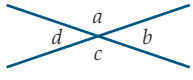


Supplementary angles add to 180°:
$a + b = 180$



Vertically opposite angles

Vertically opposite angles are equal.
$a = c$, $b = d$



8.2 Classifying and naming angles

Answers page 487

Navigator

Q1 Column 1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q13, Q15	Q1 Column 2, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q14, Q15	Q1 Column 3, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q15

Equipment required: Protractor for Questions **10(a)** and **12**

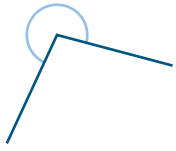
Fluency and understanding

1 State the type of angle shown in each case.

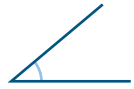
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

(g) 

(h) 

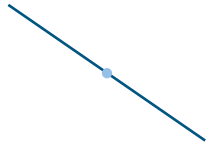
(i) 

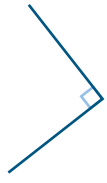
(j) 

(k) 

(l) 

(m) 

(n) 

(o) 

(p) 

(q) 

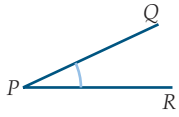
(r) 

2 Classify the following angles (acute, obtuse, right, etc.).

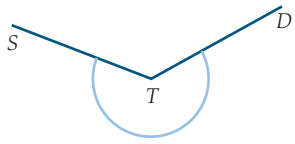
(a) 23°
(b) 117°
(c) 275°
(d) 360°
(e) 180°
(f) 75°
(g) 90°
(h) 165°
(i) 341°

3 Name the following angles.

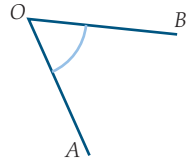
(a)



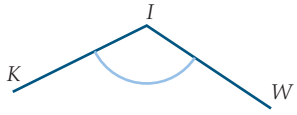
(b)



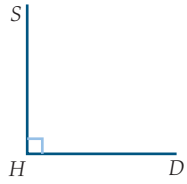
(c)



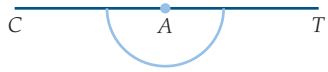
(d)



(e)



(f)



4 **(a)** Which one of the following is a pair of complementary angles?

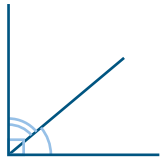
A 20° and 40° **B** 330° and 30° **C** 30° and 150° **D** 15° and 75°

(b) Which one of the following is a pair of supplementary angles?

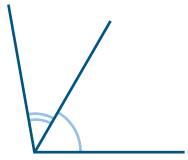
A 0° and 90° **B** 45° and 55° **C** 90° and 90° **D** 180° and 20°

(c) Which diagram shows supplementary angles?

A



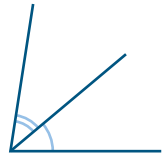
B



C

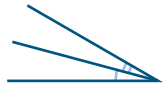


D

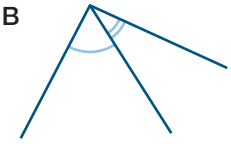


(d) Which diagram shows complementary angles?

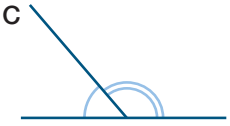
A



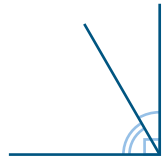
B



C



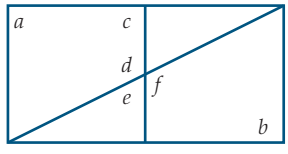
D



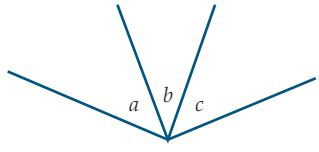
5 Which two angles are vertically opposite?

A a and b **B** a and c

C d and e **D** d and f



6 The angles a, b and c in this diagram have been drawn as adjacent angles. They have a common vertex and common arms.



For each of the following:

(a) 100°, 210°, 50° **(b)** 32°, 40°, 18° **(c)** 40°, 40°, 100°

(d) 32°, 161°, 85°, 82° **(e)** 44°, 46° **(f)** 39°, 141°

(i) draw the angles as adjacent angles

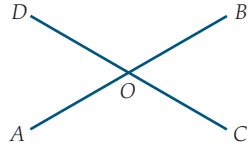
(ii) find the angle sum

(iii) describe the angle you have formed.

7 In the diagram opposite:

(a) name a pair of supplementary angles

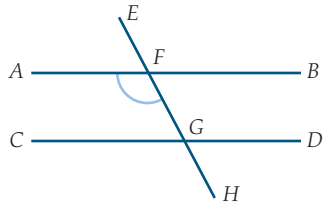
(b) name a pair of vertically opposite angles.



Hint

It might help you to remember that the 'c' of complementary stands for a corner (right angle) and the 's' of supplementary stands for a straight angle.

8 In the diagram, what could the marked angle be named?



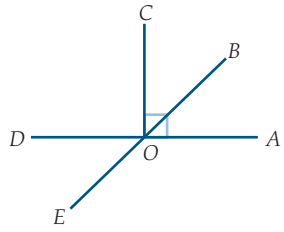
A $\angle F$

B $\angle AFE$

C $\angle AFG$

D $\angle BFG$

9 Using the diagram opposite, find, using letter names:



(a) an acute angle

(b) an obtuse angle

(c) a reflex angle

(d) a right angle

(e) a straight angle

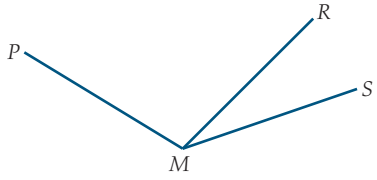
(f) one pair of complementary angles

(g) one pair of supplementary angles

(h) one pair of vertically opposite angles.

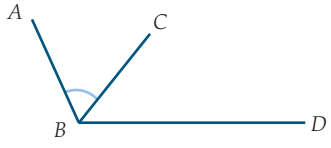
Reasoning, communicating and open-ended

10 **(a)** Measure the reflex angle *RMS* in the diagram.



(b) Why is it necessary to specify 'reflex' in part **(a)**?

11 **(a)** Why couldn't we refer to the marked angle as $\angle B$?



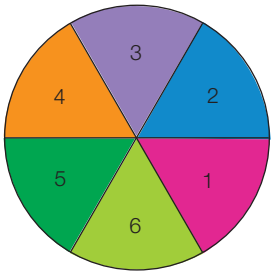
(b) Name two acute angles that have *B* as a vertex.

(c) Name an obtuse angle that has *B* as a vertex.

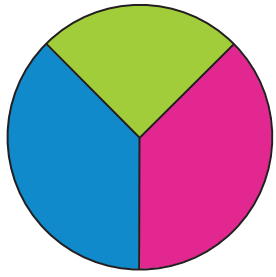
(d) Name a reflex angle that has *B* as a vertex.

12 Use a protractor to draw accurate copies of the following.

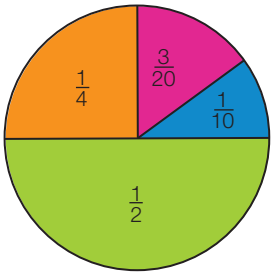
(a) A raffle wheel divided into six equal parts.



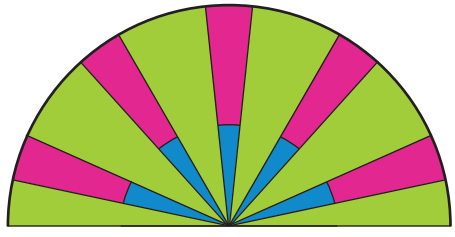
(b) This sign.



(c) This pie chart.



(d) This semicircular stained-glass window.



Hint

Divide your semicircle into 15 pieces first.

13 Draw two examples of:

(a) an acute angle **(b)** a reflex angle **(c)** an obtuse angle.

14 **(a)** Write two examples of pairs of complementary angles.

(b) Write two examples of pairs of supplementary angles.

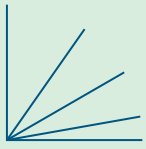
15 Write two examples of angles that add to make a revolution.

Problem solving

Count the angles

Equipment required: 1 brain, 1 protractor

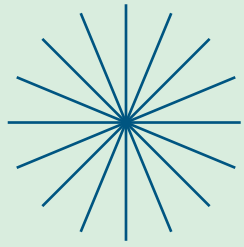
1 How many different acute angles can you count in the diagram? Measure each one.



2 How many different angles can you count above the horizontal line? Measure the size of each one.



3 How many different angles can you count in the diagram? Measure the size of each one.



Strategy options

- Test all possible combinations.
- Break problem into manageable parts.

Calculating angles

8.3

Need to Know

The sum of complementary angles = 90°.

The sum of supplementary angles = 180°.

The sum of angles in a revolution = 360°.

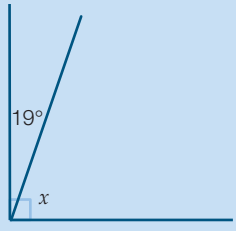
Vertically opposite angles are equal.

Worked Example 3

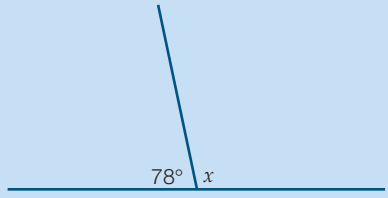
WE3

Find the size of angle x in each diagram.

(a)



(b)



Working

(a) *x and 19° are complementary angles.*

$x + 19 = 90$

$x + 19 - 19 = 90 - 19$

$x = 71°$

(b) *x and 78° are supplementary angles.*

$x + 78 = 180$

$x + 78 - 78 = 180 - 78$

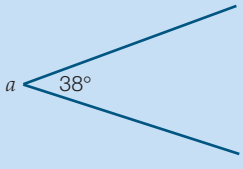
$x = 102°$

Worked Example 4

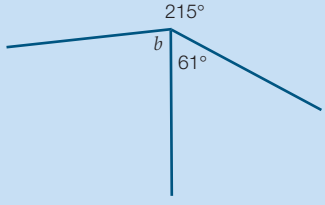
WE4

Find the value of the pronumeral in each diagram.

(a)



(b)



Working

(a) $a + 38 = 360$

$a + 38 - 38 = 360 - 38$

$a = 322°$

(b) $215 + 61 + b = 360$

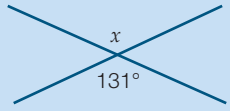
$276 + b = 360$

$276 - 276 + b = 360 - 276$

$b = 84°$

Worked Example 5 WE5

Find the value of x in the diagram.



Working

$x = 131°$ (*vertically opposite angles*)

Answers page 488

8.3 Calculating angles

Navigator

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q7 Column 1, Q8, Q10 Column 1, Q12	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q7 Column 2, Q10 Column 2, Q11, Q12, Q13	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q5, Q6, Q7 Column 3, Q9, Q10 Column 3, Q11, Q12, Q13, Q14

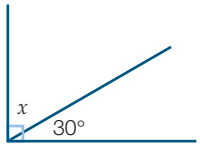
Equipment required: Protractor for Questions **11(a)** and **12**

Fluency and understanding

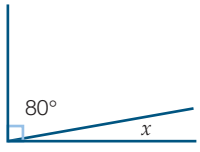
WE3

1 Find the size of angle x in each diagram.

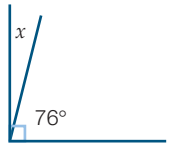
(a)



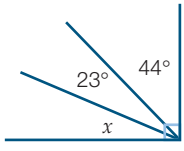
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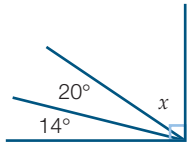
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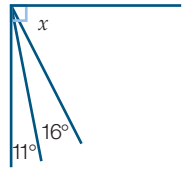
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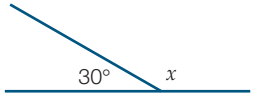
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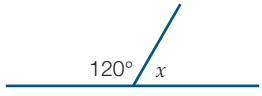
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(g)



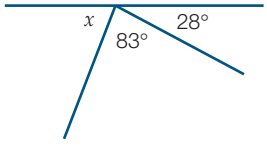
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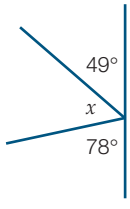
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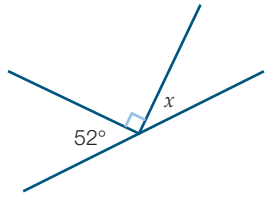
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(k)



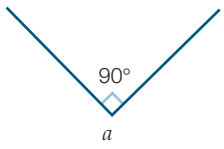
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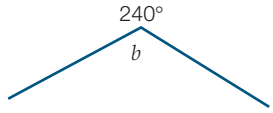
2 Find the value of the pronumeral in each diagram.

WE4

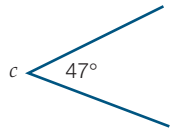
(a)



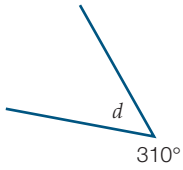
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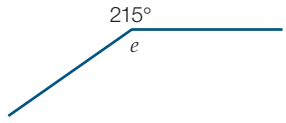
(c)



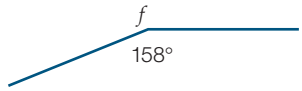
(d)



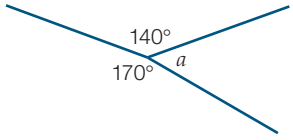
(e)



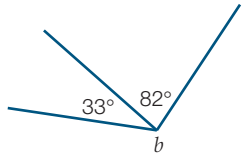
(f)



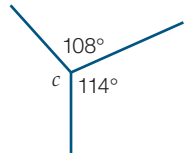
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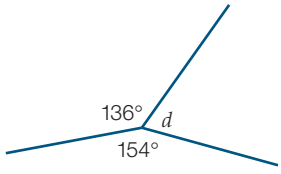
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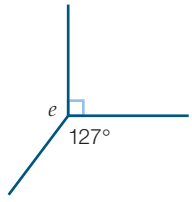
(i)



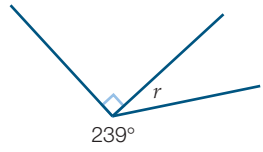
(j)



(k)



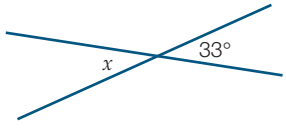
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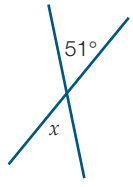
3 Find the value of x in each diagram.

WE5

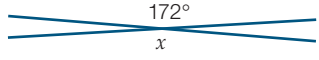
(a)



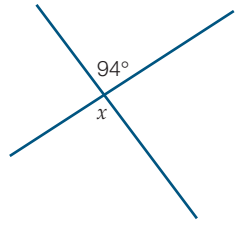
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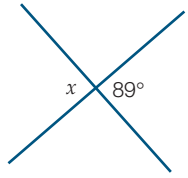
(c)



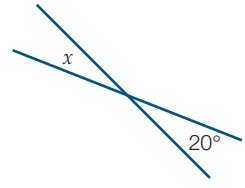
(d)



(e)



(f)



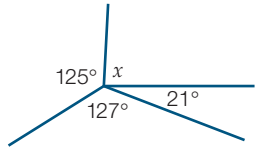
4 Find the complement of the following angles.

(a) 27° **(b)** 45° **(c)** 68° **(d)** 15°

5 Find the supplement of the following angles.

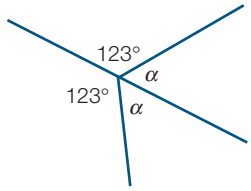
(a) 32° **(b)** 90° **(c)** 124° **(d)** 176°

6 Find the value of x in the diagram.

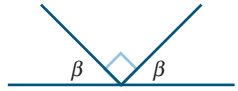


7 Find the value of the pronumerals in each diagram.

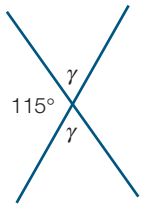
(a)



(b)



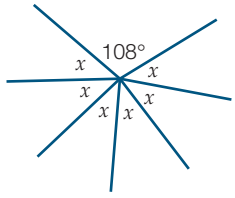
(c)



8 The value of x in the opposite diagram is:

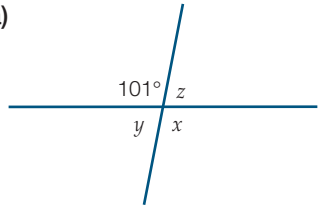
A 42° **B** 60°

C 108° **D** 360°

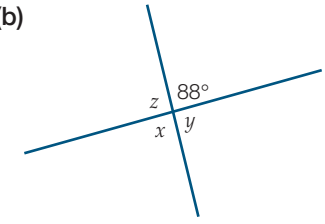


9 Find the value of the pronumerals in each diagram.

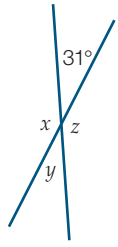
(a)



(b)



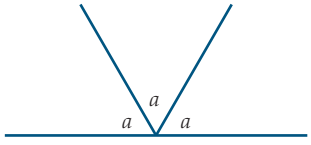
(c)



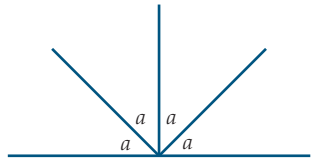
Reasoning, communicating and open-ended

10 Determine the size of angle a in each diagram. Give reasons for your answer.

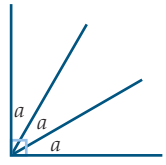
(a)



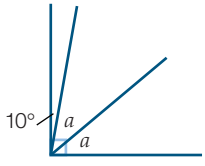
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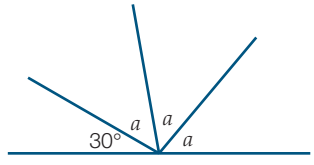
(c)



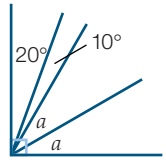
(d)



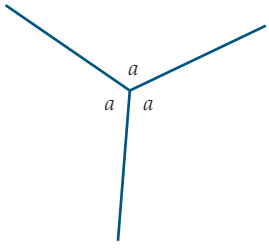
(e)



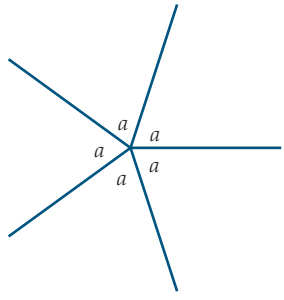
(f)



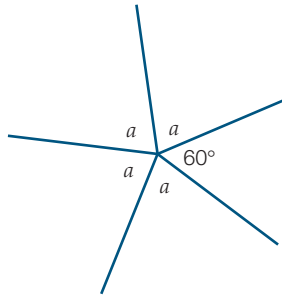
(g)



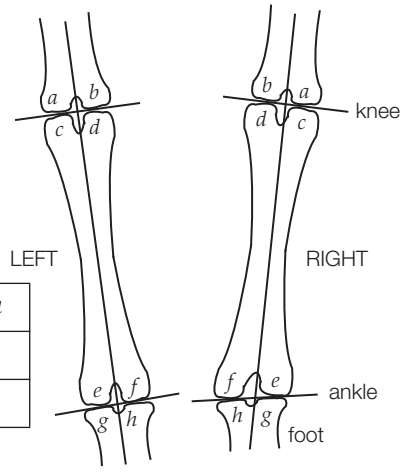
(h)



(i)



11 After breaking her leg in an accident, Kira had difficulty walking. She took her X-ray to a specialist, who marked in the following lines. He followed a procedure to decide whether she needed an operation. (You will need a protractor to measure some of the angles.)



(a) Measure all the angles and fill in the table below.

	a	b	c	d	e	f	g	h
Left leg								
Right leg								

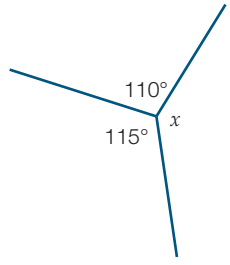
(b) All angles on Kira's knees must be approximately equal and all the angles on her ankles must also be approximately equal. Which leg do you think Kira broke?

(c) The differences between the angles b and f, and c and g, must be no more than 8°. If they are more, Kira needs an operation. Does she need an operation? Why?

12 Sabine was answering the question on the right.

Using her protractor, Sabine measured the angle x and found that it was 140°. This was incorrect. Explain where Sabine made a mistake.

Find the value of the pronumeral.



13 **(a)** Draw two lines that intersect each other.

(b) Measure the two supplementary angles you have created.

(c) Use your answers to **(b)** to work out the two angles you did not measure.

14 Draw two straight lines that intersect, forming two acute angles. What are the sizes of the other two angles?

Problem solving

Ella's angles

Ella needs to draw a 60° angle, but she has lost her protractor. She has a stencil that can draw 90° and 40° angles. How can she use these two angle sizes to draw a 60° angle?

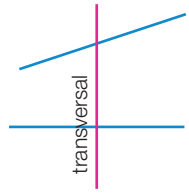
Strategy options

- Draw a diagram.
- Guess and check.

8.4 Angles and parallel lines

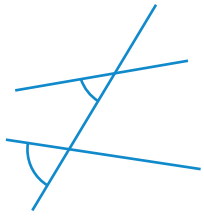
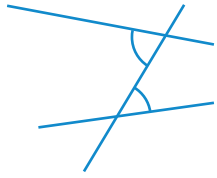
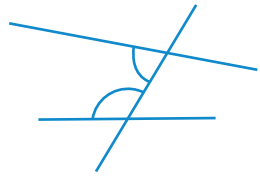
Need to Know

A **transversal** is a line that **intersects** (crosses or traverses) two or more other lines as shown.

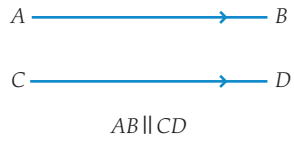


Angles formed by a transversal

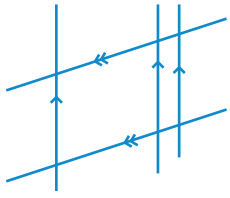
When a transversal intersects two other lines, pairs of angles are formed. These angles are given special names.

Corresponding angles	Alternate angles	Co-interior (allied) angles
		
These angles either lie above or below the two lines cut by the transversal and are on the same side of it. There are four pairs of corresponding angles formed when a transversal cuts two lines.	These angles lie between the two lines cut by the transversal but are on opposite sides. There are two pairs of alternate angles formed when a transversal cuts two lines.	These angles lie between the lines cut by the transversal and on the same side of it. There are two pairs of co-interior angles formed when a transversal cuts two lines.

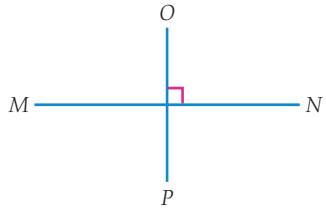
Parallel lines are lines that lie in the same **plane** (same flat surface) and are always the same distance apart. Lines that are parallel are each marked with an arrow pointing in the same direction. If more than one set of parallel lines appears in a diagram, use more than one arrow. $\parallel$ means 'is parallel to'.



$AB \parallel CD$



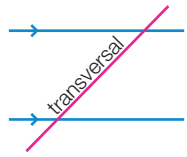
Perpendicular lines intersect at right angles. $\perp$ means 'is perpendicular to'.



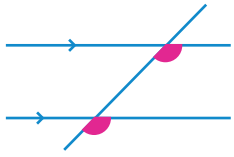
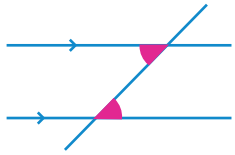
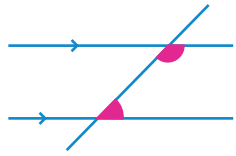
$OP \perp MN$

Properties of angles formed when a transversal cuts parallel lines

When parallel lines are crossed by a transversal, the pairs of angles on parallel lines described have special properties.



Corresponding angles | **Alternate angles** | **Co-interior (allied) angles**

Corresponding angles are *equal.*	Alternate angles are *equal.*	Corresponding angles are are *supplementary.*

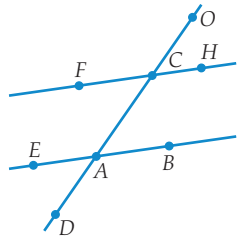
The opposite of the above is also true. If corresponding or alternate angles are equal or if co-interior angles are supplementary when two lines are cut by a transversal, the two lines are parallel.

Technology Exploration: GeoGebra

Investigating angles on parallel lines using technology

1. In GeoGebra, right-click in the Graphics view to hide the Axes.
2. From the menu, select Options > Labelling > New Points Only. Then, select Options > Rounding > 0 Decimal Places.

Creating a pair of parallel lines and a transversal



3. Construct a line using the 'Line through Two Points' tool,  by clicking on any two points in the Graphics view.
4. Select the 'Parallel Line' tool  . Create a parallel line by clicking on the existing line and then on anywhere above the line.
5. Create a transversal by selecting the 'Line through Two Points' tool, then clicking on point *A* and then on point *C* (following the diagram above right).
6. Select the 'New Point' tool. Use this tool to place points on the lines as shown in the diagram above.

Alternate angles and parallel lines

7 Now select the 'Angle' tool  . Click on point B, then A and then C to find $\angle BAC$.

Notice that the vertex of the angle is the middle letter. Find $\angle FCA$ in the same way. Note that points must be selected in a clockwise order; if you get the reflex angle, delete it and try selecting the points in the opposite order. You have just marked a pair of alternate angles.

8 Right-click inside one of the marked angles and select Object Properties… In the pop-up box that appears, go to the Colour tab to select a different colour. Next, right-click inside the other marked angle and select the same colour (the colour will be stored in the 'Recent:' box on the right-hand side of the Colour tab). Remember to close the pop-up box afterwards.

9 Select the 'Angle' tool. Mark $\angle CAE$ and $\angle ACH$ as described in step **7**. You have just marked another pair of alternate angles.

10 Select the 'Move' tool. Click on point B and drag it around.

(a) You have just marked two pairs of alternate angles between a pair of parallel lines. What do you notice as you move the lines?

(b) Copy and complete the statement: 'Alternate angles between parallel lines are …'

11 Right-click inside a marked angle and select Delete. Repeat for the other marked angles. (If some of the points have disappeared, move point B until they reappear.)

Corresponding angles and parallel lines

12 Select the 'Angle' tool, then mark $\angle HCG$ and $\angle BAC$ by clicking on the points. You have just marked a pair of corresponding angles.

13 Follow the process in step **8** to change the colour of both angles.

14 Repeat steps **12** and **13** for $\angle ACH$ and $\angle DAB$, for $\angle EAD$ and $\angle FCA$, and for $\angle CAE$ and $\angle GCF$. Make sure that each pair of angles has a different colour. You have just marked three more pairs of corresponding angles.

15 Select the 'Move' tool. Click on point B and drag it around. Take notice of the angle sizes.

(a) What do you notice about each pair of corresponding angles as you move the lines?

(b) Copy and complete the statement: 'Corresponding angles on parallel lines are ...'

16 Delete the marked angles as described in step **11**.

Co-interior angles and parallel lines

17 Select the 'Angle' tool, then mark $\angle ACH$ and $\angle BAC$ by clicking on the points. You have marked a pair of co-interior angles (also known as allied angles).

18 Follow the process in step **8** to change the colour of both angles.

19 Repeat steps **17** and **18** for $\angle CAE$ and $\angle FCA$ to mark another pair of co-interior angles.

20 Select the 'Move' tool, then click on point B and drag it around. Take note of the angle sizes.

(a) What do you notice about each pair of co-interior angles as you move the lines?

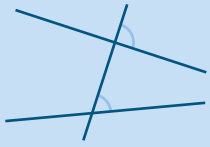
(b) Copy and complete the statement: 'Co-interior angles between parallel lines sum to …°.'

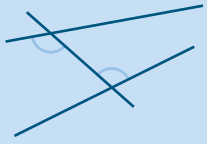
21 Delete the marked angles as described in step **11**.

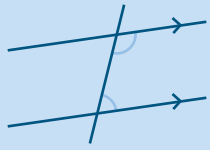
Worked Example 6

WE6

Identify each of the following pairs of angles as corresponding, alternate or co-interior angles.

(a) 

(b) 

(c) 

Working

(a) The angles are corresponding (on the same side of the transversal and above the lines).

(b) The angles are alternate (on opposite sides of the transversal and between the lines).

(c) The angles are co-interior (on the same side of the transversal and between the parallel lines).

8.4 Angles and parallel lines

Answers page 488

Navigator

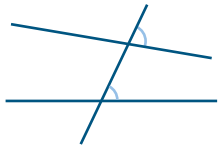
Q1, Q2, Q3, Q4, Q5, Q6 Columns 1 & 2, Q7, Q8 Column 1, Q9, Q10, Q13 Column 1, Q14 (a)	Q1, Q2, Q3, Q4, Q5, Q6 Columns 2 & 3, Q7, Q8 Column 2, Q9, Q11, Q12, Q13 Column 2, Q14 (a)	Q1, Q2, Q3, Q4, Q5, Q6 Columns 2 & 3, Q7, Q8 Column 3, Q11, Q12, Q13 Column 3, Q14, Q15

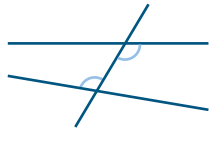
Equipment required: Protractor for Question **6 (a)**

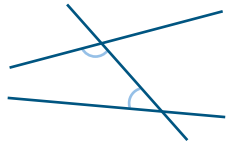
Fluency and understanding

1 Identify each of the following pairs of angles as corresponding, alternate or co-interior angles.

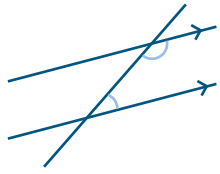
WE6

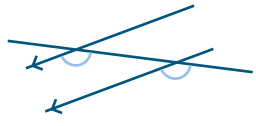
(a) 

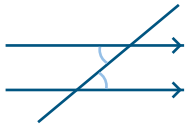
(b) 

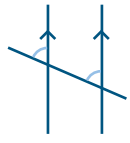
(c) 

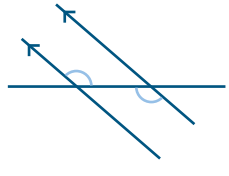
(d) 

(e) 

(f) 

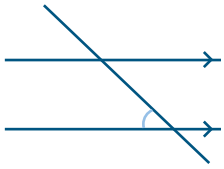
(g) 

(h) 

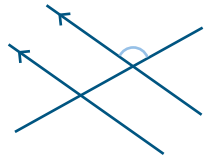
(i) 

2 Copy each of the following and mark in an angle corresponding to the one shown.

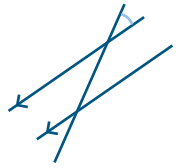
(a)



(b)

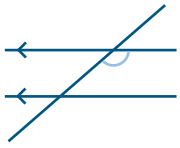


(c)

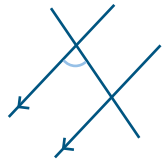


3 Copy each of the following and mark in an angle alternate to the one shown.

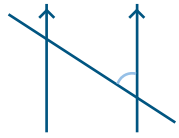
(a)



(b)



(c)

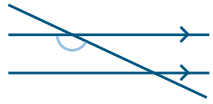


4 Copy each of the following and mark in an angle that is co-interior with the one shown.

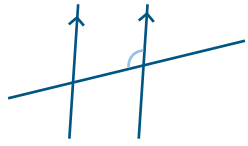
(a)



(b)



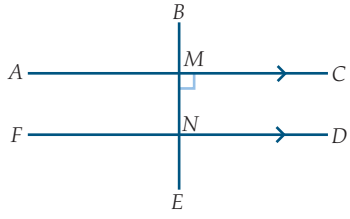
(c)



Hint

Remember that ⊥ means 'perpendicular to' and || means 'parallel to'.

5 Which of the following statements is not true about the diagram shown?



A $\angle AMB$ is corresponding to $\angle FNB$

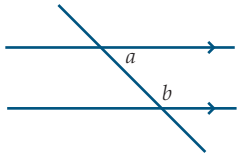
B $\angle BNF$ is alternate to $\angle EMC$

C $\angle AMN$ and $\angle CMN$ are co-interior angles

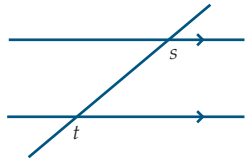
D BE is $\perp$ to FD

6 **(a)** Use a protractor to measure the labelled angles in each diagram below. State whether each pair of angles is corresponding, alternate or co-interior.

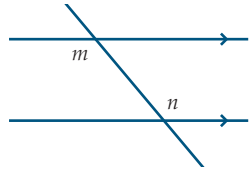
(i)



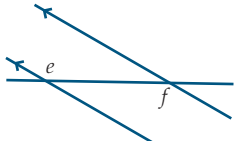
(ii)



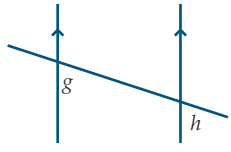
(iii)



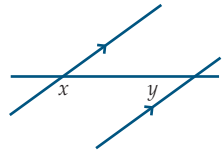
(iv)



(v)



(vi)



(b) State which pairs of angles in part **(a)** are equal. Which pairs of angles add to 180°?

7 State which angle in the diagram is:

(a) corresponding to u

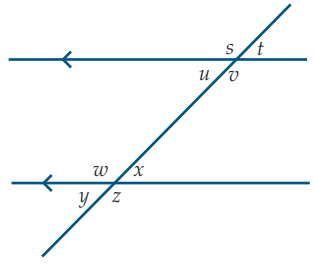
(b) alternate to v

(c) co-interior with w

(d) co-interior with x

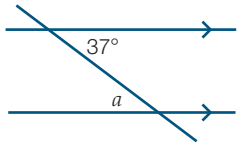
(e) corresponding to w

(f) alternate to u.

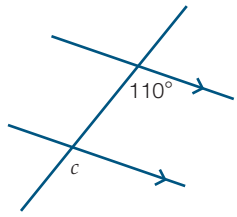


8 Find the value of the pronumerals in each case without the use of a protractor. Give a reason for your answer (e.g. corresponding angles on parallel lines).

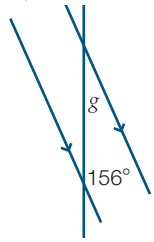
(a)



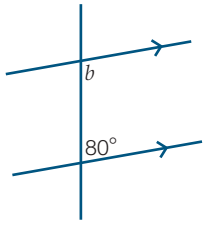
(b)



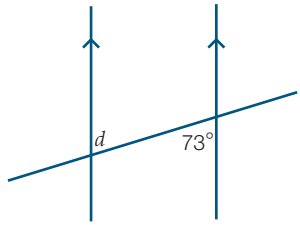
(c)



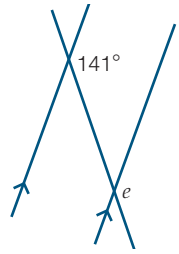
(d)



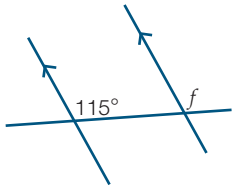
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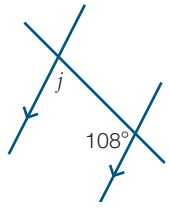
(f)



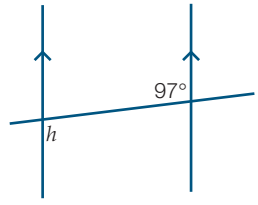
(g)



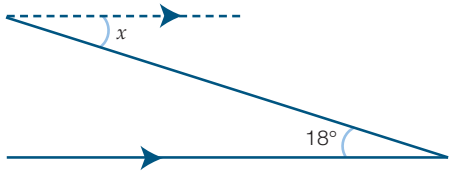
(h)



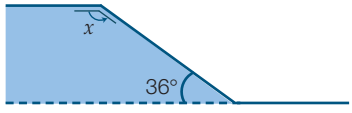
(i)



9 A pilot of a plane flying parallel to the ground can see the runway as it is coming in to land. The angle of approach to the runway is 18°. What angle must the pilot tilt the plane down to start the approach?

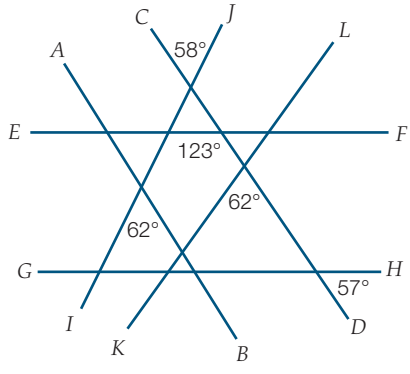


10 In an underground carpark a ramp is being constructed. The ramp will ascend at an angle of 36°. At what angle must the supports be positioned at the top of the ramp?



Reasoning, communicating and open-ended

11 Choose the pair of lines that are parallel and explain why you chose them.

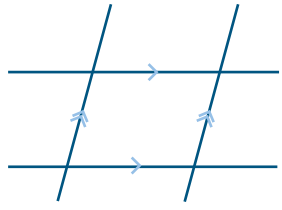


12 When a pair of parallel lines is cut by two parallel transversals, how many pairs of angles are formed that are:

(a) corresponding

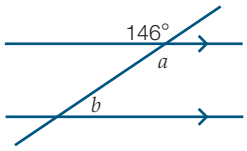
(b) alternate

(c) co-interior?

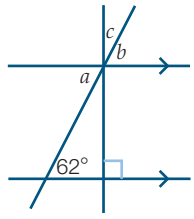


13 Find the value of each pronumeral below, giving a reason for your answer.

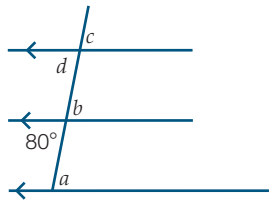
(a)



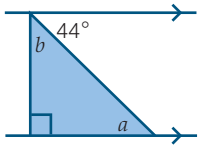
(b)



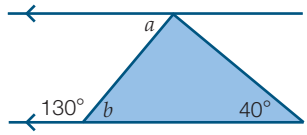
(c)



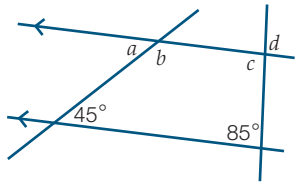
(d)



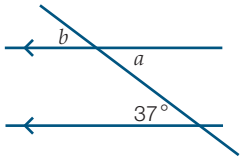
(e)



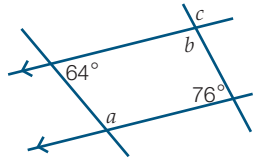
(f)



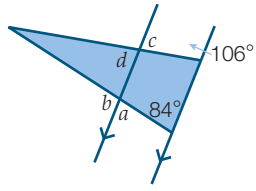
(g)



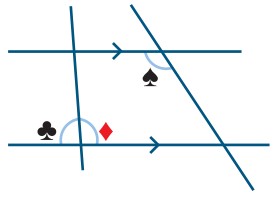
(h)



(i)



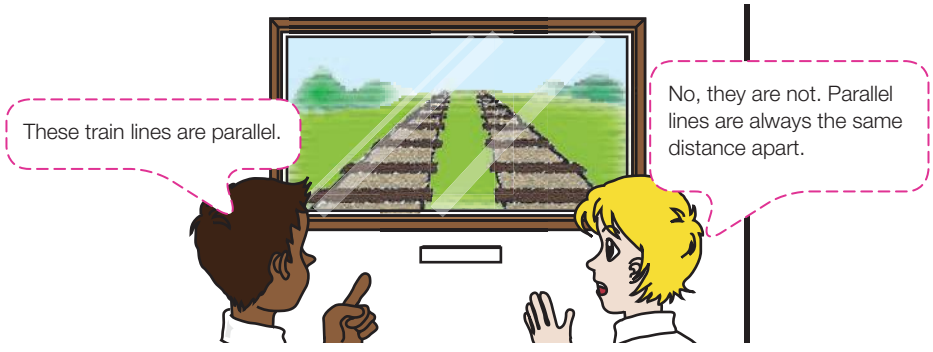
14 Aileen has drawn a pair of parallel lines with two transversals. She has marked three angles using different symbols.



(a) Copy the diagram into your book. Mark another three angles using the same symbols as Aileen so that there is a pair of corresponding angles, alternate angles and co-interior angles.

(b) In how many different ways can the three other angles be arranged, with no pair of angles sharing an angle with another pair? Draw a diagram for each combination.

15



How would you explain to the students whether the train lines are parallel or not?

Half-time 8

Equipment required: Protractor for Questions **3** and **5**

1 Draw the following angles by estimating their size. (Do not use a protractor.) Ex 8.1

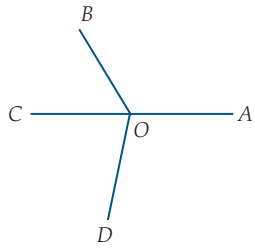
(a) 90° **(b)** 180° **(c)** 360°

(d) 34° **(e)** 75° **(f)** 120°

2 **(a)** Name the two acute angles in the diagram opposite. Ex 8.2

(b) Name the three obtuse angles in this diagram.

(c) Give another name for $\angle AOB$.



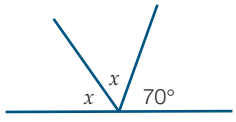
3 Draw the following angles using a protractor. Ex 8.1

(a) 90° **(b)** 180° **(c)** 360°

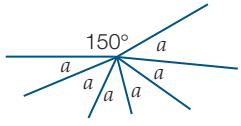
(d) 34° **(e)** 75° **(f)** 120°

4 Find the value of the pronumerals in each case and give reasons for your answer. Ex 8.3

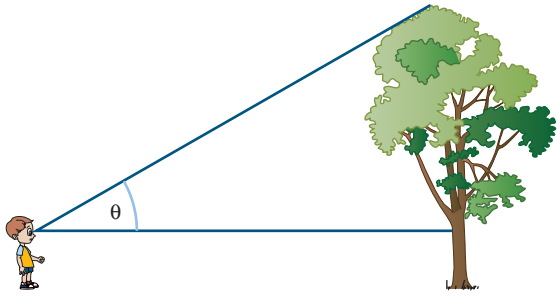
(a)



(b)

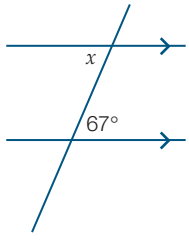


5 Measure the angle marked in the diagram opposite. Ex 8.1

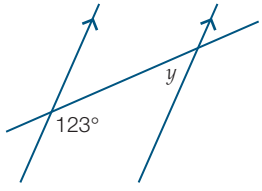


6 Find the value of the pronumerals in each case and give reasons for your answer. Ex 8.4

(a)



(b)



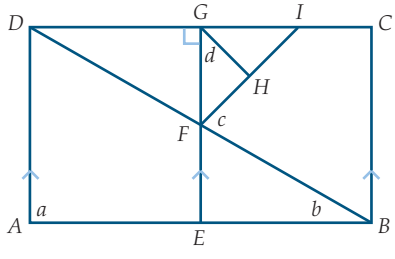
7 Using letter names such as $\angle ABC$: Ex 8.3, 8.4

(a) name an angle corresponding to a

(b) name an angle alternate to b

(c) name an angle supplementary to c

(d) name an angle complementary to d.



8.5 Polygons

Need to Know

Plane shapes are shapes that can be drawn on a flat surface called a plane.

Plane shapes whose sides are all straight lines are called **polygons**.

A **regular polygon** has all sides of equal length and all angles of equal size.

An **irregular** polygon has sides that are not all equal.

Convex polygons contain no interior angles greater than 180°.

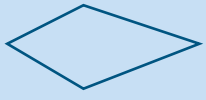
Concave polygons contain at least one interior angle greater than 180°.

Number of sides	Polygon name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
11	Undecagon
12	Dodecagon

Worked Example 7

WE7

Name each of the polygons below. State whether they are concave or convex.

(a) 

(b) 

(c) 

Working

(a) A convex quadrilateral
(b) A concave nonagon
(c) A concave octagon

8.5 Polygons

Answers page 490

Navigator

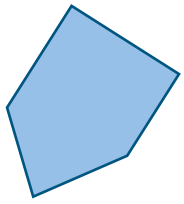
Q1 Column 1, Q2, Q3, Q4, Q5, Q6, Q7	Q1 Column 2, Q2, Q3, Q4, Q5, Q6, Q7	Q1 Column 3, Q2, Q3, Q4, Q5, Q6, Q7

Fluency and understanding

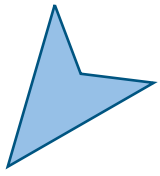
1 Name each of the polygons below. State whether each one is concave or convex.

WE7

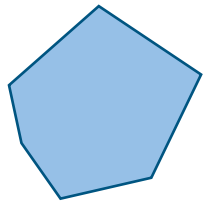
(a)



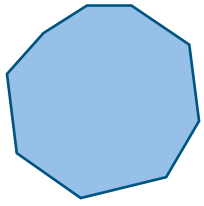
(b)



(c)



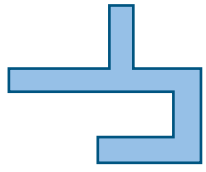
(d)



(e)



(f)



(g)



(h)

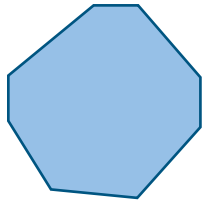


(i)



2 Choose the correct answer.

(a) The shape opposite is:



A a hexagon

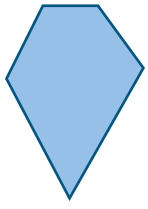
B a heptagon

C an octagon

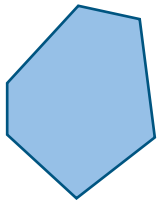
D a nonagon.

(b) Which of the shapes below is a heptagon?

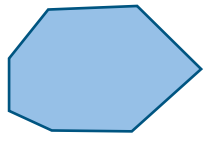
A



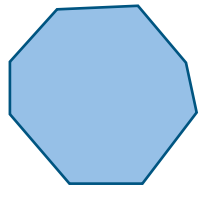
B



C



D



3 Name the type of polygon in each photograph.

(a)



(b)



(c)



(d)



Reasoning, communicating and open-ended

4 What is the greatest number of right angles that a pentagon can have? Demonstrate your answer.

5 **(a)** If a quadrilateral can have two sets of parallel sides, how many sets of parallel sides can a hexagon have?

(b) Based on your results from part **(a)**, how many pairs of parallel sides can an octagon and a decagon have? Explain any pattern you have found.

(c) Can this pattern be applied to a pentagon or to a heptagon? Explain why or why not.

6 Using a ruler, draw any:

(a) concave quadrilateral **(b)** concave hexagon **(c)** concave octagon.

7 Name some common objects that contain regular polygons.

Triangles

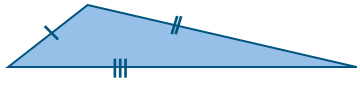
8.6

Need to Know

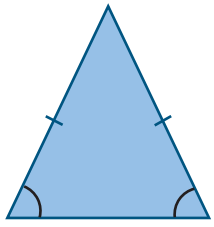
Classifying triangles

When the angles or lines of a shape are marked in a particular way, it means that all other angles or sides with that same marking are equal in size.

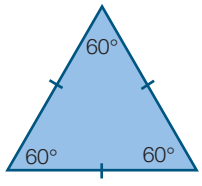
Scalene triangle—no equal sides and no equal angles.



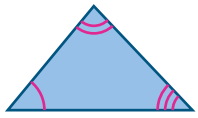
Isosceles triangle—two equal sides and two equal base angles, which are opposite the two equal sides.



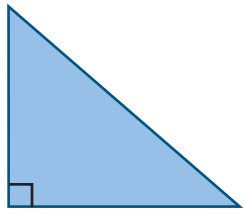
Equilateral triangle—three equal sides and three equal angles of 60° each.



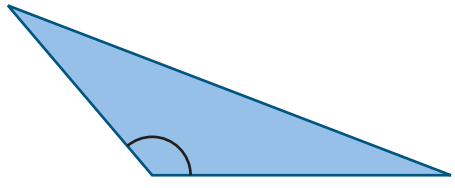
Acute-angled triangle—all three angles are acute (less than 90°).



Right-angled triangle—one angle is a right angle (equal to 90°).



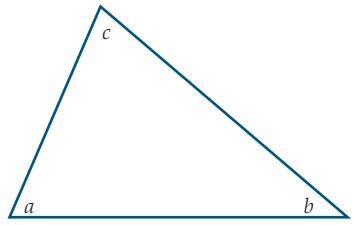
Obtuse-angled triangle—one angle is obtuse (greater than 90° but less than 180°).



Angle sum in a triangle

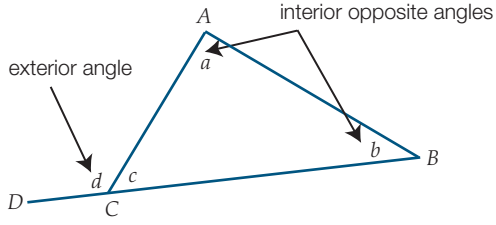
In any triangle, the three angles have a sum of 180°.

$a + b + c = 180$



Exterior angle of a triangle

The exterior angle of a triangle is equal to the sum of the two interior opposite angles in the triangle.



Technology Exploration: GeoGebra

Angles in a triangle using technology

In a new GeoGebra window, right-click in the Graphics view to hide the Axes.

To construct a triangle, select the 'Polygon' tool  . In the Graphics view, left-click to create the first point, move the cursor and left-click again for the second point, repeat for the third point and then click on the first point again to complete the triangle. Move in an anticlockwise direction so that the interior angles will be measured.

To measure the angles, select the 'Angle' tool and then click on 'poly 1' in the Algebra view (the panel on the LHS of the window) or click inside the triangle. All the interior angles will now be measured and displayed in the Algebra view.

In the input bar at the bottom of the window, enter Sum [{α,β,γ}]. To enter the Greek letters, click on the icon at the far right of the input bar. The sum will be displayed in the Algebra view as $\delta = 180°$.

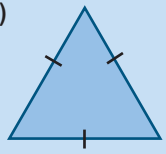
Use the 'Move' tool to select a vertex of the triangle. Click and drag to move the selected point around. You will see the angles change, but the sum remains constant at 180°. Select another vertex and move it around. Repeat this procedure with the third vertex.

Worked Example 8

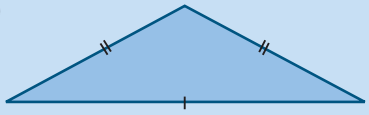
WE 8

Give each of the following triangles **(i)** its side name **(ii)** its angle name and **(iii)** describe the triangle with an angle and a side name.

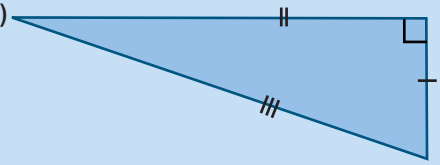
(a)



(b)



(c)



Working

(a) **(i)** equilateral triangle
(ii) acute-angled triangle
(iii) an acute-angled equilateral triangle

(b) **(i)** isosceles triangle
(ii) obtuse-angled triangle
(iii) an obtuse-angled isosceles triangle

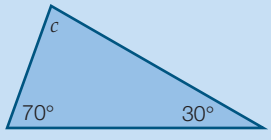
(c) **(i)** scalene triangle
(ii) right-angled triangle
(iii) a right-angled scalene triangle

Worked Example 9

WE 9

Find the size of the angle labelled c in each of the following triangles.

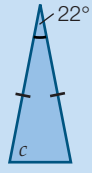
(a)



(b)



(c)



Working

(a) $c + 30 + 70 = 180$
$c + 100 = 180$
$= 80°$

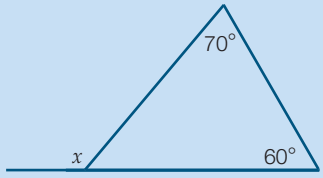
(b) $c + 90 + 16 = 180$
$c + 106 = 180$
$c = 74°$

(c) $c + c + 22 = 180$
$2c + 22 = 180$
$2c = 158$
$c = 79°$

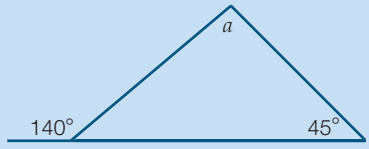
Worked Example 10 WE10

Calculate the value of the pronumeral in each of the following.

(a)



(b)



Working

(a) $x = 60 + 70$
$x = 130°$

(b) $140 = a + 45$
$a = 140 - 45$
$a = 95°$

Answers page 490

8.6 Triangles

Navigator

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4 Column 1, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q14, Q15, Q17, Q18, Q19	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4 Column 2, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q18, Q19	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4 Column 3, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q18, Q19

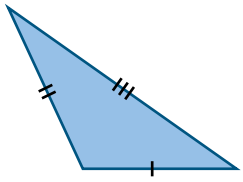
Equipment required: Protractor and ruler for Question **4**

Fluency and understanding

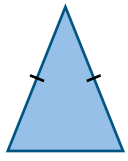
WE8

1 Give each of the following triangles **(i)** its side name **(ii)** its angle name and **(iii)** describe the triangle with an angle and a side name.

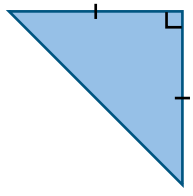
(a)



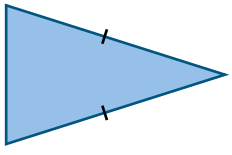
(b)



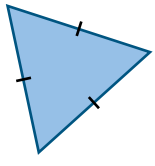
(c)



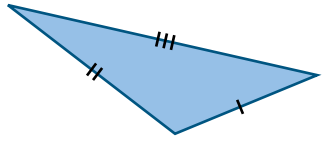
(d)



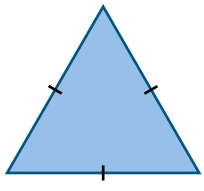
(e)



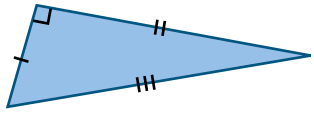
(f)



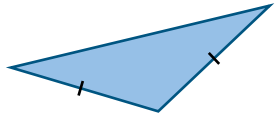
(g)



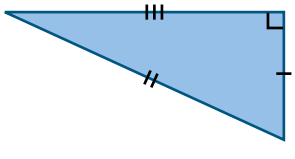
(h)



(i)



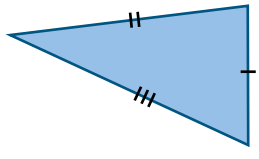
(j)



(k)



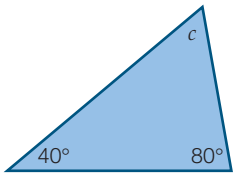
(l)



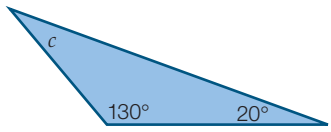
2 Find the size of the angle labelled c in each of the following triangles.

WE9

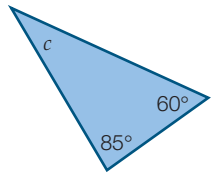
(a)



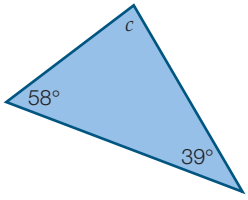
(b)



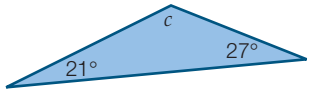
(c)



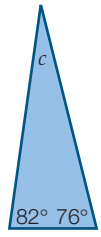
(d)



(e)



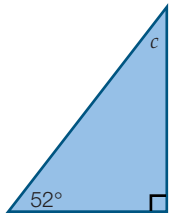
(f)



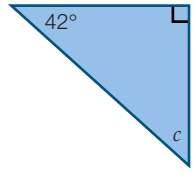
(g)



(h)



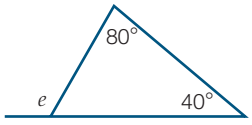
(i)



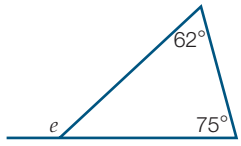
3 Calculate the value of the pronumeral in each of the following.

WE10

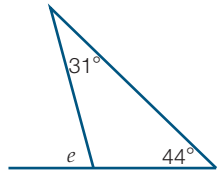
(a) **(i)**



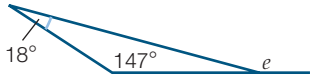
(ii)



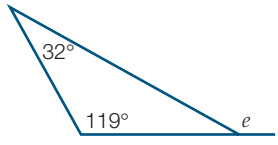
(iii)



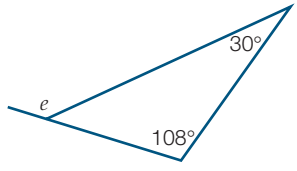
(iv)



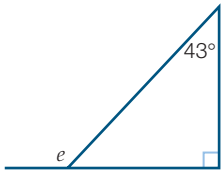
(v)



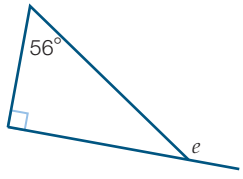
(vi)



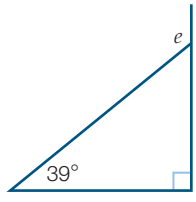
(vii)



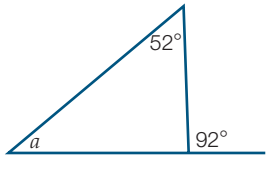
(viii)



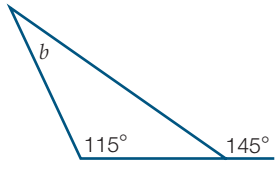
(ix)



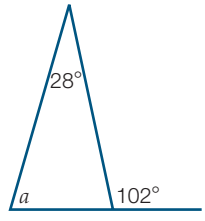
(b) **(i)**



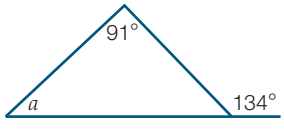
(ii)



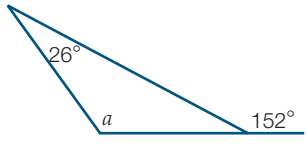
(iii)



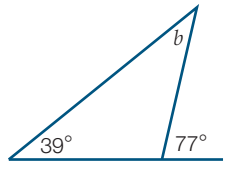
(iv)



(v)

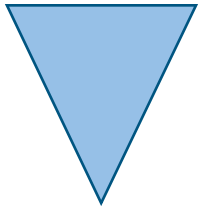


(vi)

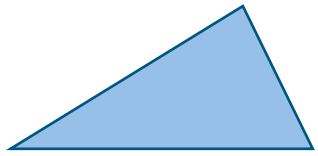


4 Use a ruler to measure the side lengths of the following triangles, and hence give a side name for each one. Give the angle names as well, using a protractor if necessary to check the angle types.

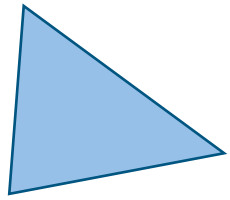
(a)



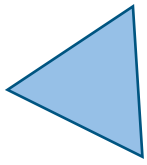
(b)



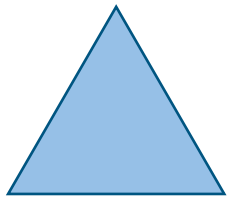
(c)



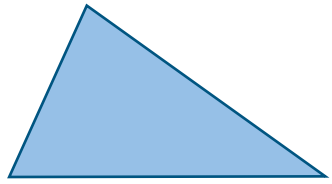
(d)



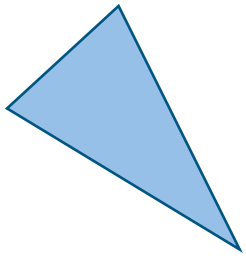
(e)



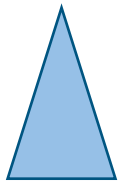
(f)



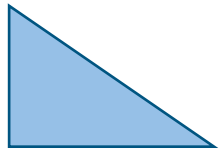
(g)



(h)



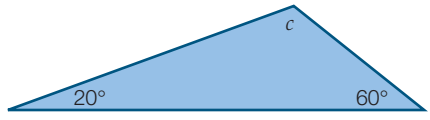
(i)



5 (a) The size of angle c is:

A 20° **B** 60°

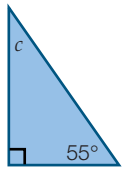
C 80° **D** 100°



(b) The size of angle c is:

A 35° **B** 45°

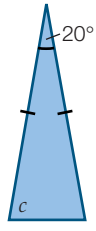
C 55° **D** 125°



(c) The size of angle c is:

A 20° **B** 80°

C 90° **D** 160°



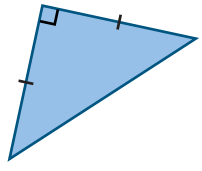
6 The triangle shown is best described as:

A isosceles, acute-angled

B isosceles, right-angled

C scalene, acute-angled

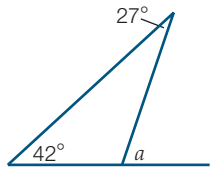
D scalene, right-angled.



7 **(a)** The value of the pronumeral is:

A 15° **B** 63°

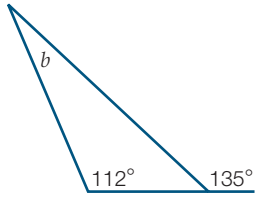
C 69° **D** 111°



(b) The value of the pronumeral is:

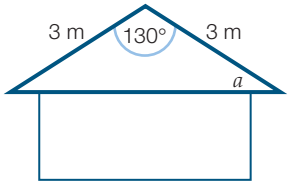
A 23° **B** 36°

C 42° **D** 68°

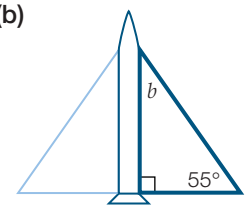


8 Name the type of triangle shown in bold in each diagram below. (Side or angle names may be used.) Find the value of any unknown angles marked with a pronumeral.

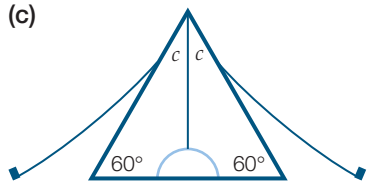
(a)



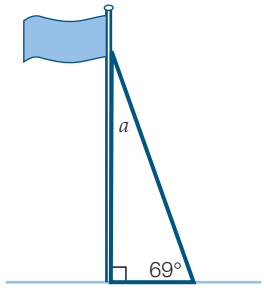
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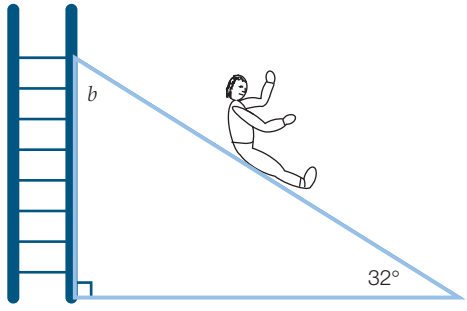
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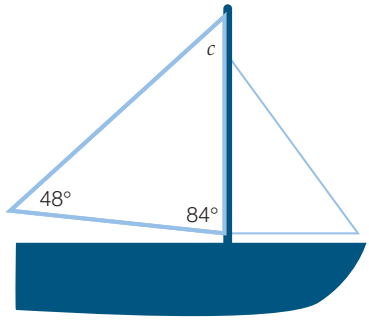
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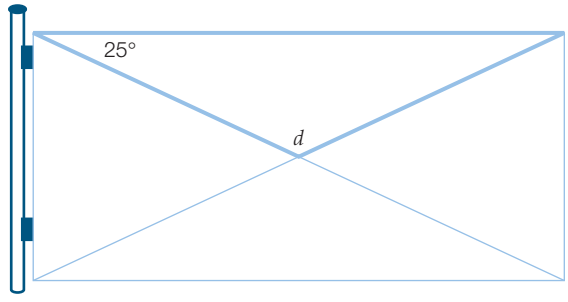
(e)



(f)



(g)



9 The diagonal of a square cuts the square into two triangles. Give the side and angle name of the triangles formed.

10 Two equilateral triangles are placed next to each other so that two sides are joined along their whole length to form a quadrilateral.

(a) What can you say about the side lengths of the quadrilateral (four-sided shape) formed?

(b) What will be the size of each of the angles in the quadrilateral?

Reasoning, communicating and open-ended

11 Is it possible to have a triangle with more than one obtuse angle? Explain by drawing a diagram.

12 What is the minimum number of acute angles a triangle must have?

13 Give the side and angle names of each of the following triangles, with reasons why.

(a) a triangle with one angle measuring 36° and another angle measuring 54°

(b) a triangle with one angle measuring 28° and another angle measuring 124°

(c) a triangle with two angles measuring 60° each

(d) a triangle with one angle measuring 12° and another angle measuring 12°

14 Explain why the following triangles are not possible.

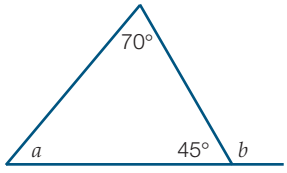
(a) a triangle with angles 55°, 65° and 70°

(b) an equilateral triangle with three 55° angles

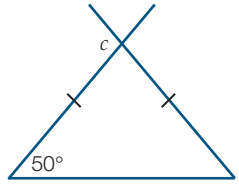
(c) isosceles triangle with 45°, 55° and 80° angles

15 Find the unknown angle or angles (represented by the pronumeral) in each of the following diagrams. You may need to use your knowledge of angle properties.

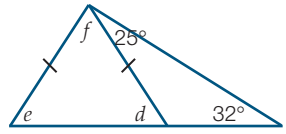
(a)



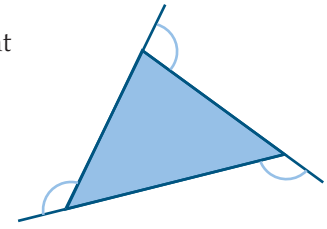
(b)



(c)



16 Find the sum of the marked angles in the diagram on the right without measuring the angles.



17 Draw three different obtuse-angled scalene triangles.

18 Draw an example of:

(a) an acute-angled scalene triangle

(b) an acute-angled isosceles triangle.

19 Draw three different triangles with an exterior angle between 80° and 100°.

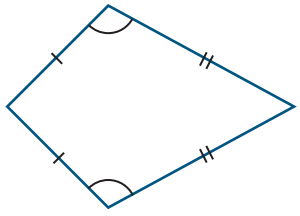
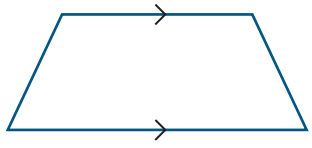
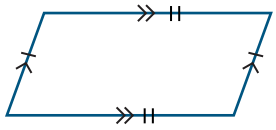
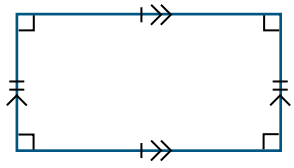
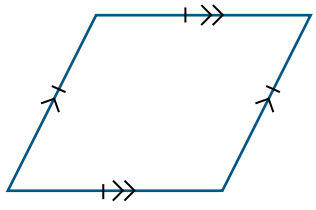
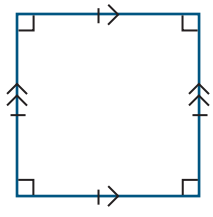
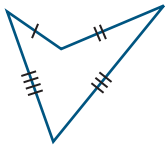
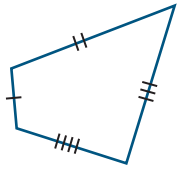
Quadrilaterals

8.7

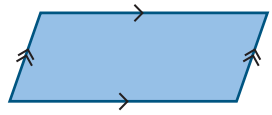
Need to Know

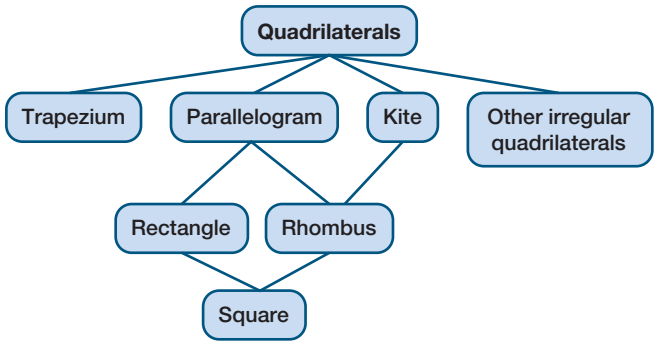
A plane shape with four straight sides is called a **quadrilateral**. Quadrilaterals are named by looking at the properties of their sides and angles.

Naming quadrilaterals

Quadrilateral	Description
Kite 	Two pairs of equal adjacent sides (sides next to each other) and one pair of equal opposite angles.
Trapezium 	Only one pair of parallel sides.
Parallelogram 	Two pairs of parallel sides. Opposite sides and angles are equal.
Rectangle 	Two pairs of opposite sides equal and all internal angles are right angles (equal to 90°).
Rhombus 	All four sides equal; opposite angles are equal. (This can be thought of as a parallelogram with all sides equal or a 'pushed over' square.)
Square 	All sides equal and all internal angles are right angles (equal to 90°).
Irregular quadrilaterals  concave  convex	A four-sided plane shape that has no special qualities. The diagonals of a convex quadrilateral lie inside the quadrilateral.

Parallel side pairs are indicated by marking the side with the same number of arrows in the same direction, as shown.





A quadrilateral may have more than one correct name. For example, a square may also be called a rectangle because it has all the angle and side properties of a rectangle (two pairs of parallel sides and 90° angles). It could also be called a rhombus, because its pairs of opposite sides are equal and its opposite angles are equal.

Angle sum of a quadrilateral

Every quadrilateral can be divided into two triangles.

The sum of the angles in a quadrilateral = 2 × sum of the angles in a triangle.

Therefore, the sum of the angles in a quadrilateral is 360°.

Angle properties of special quadrilaterals

The diagonally opposite angles of a parallelogram are equal.

The two interior opposite angles of a kite where unequal sides meet are equal.

Technology Exploration: GeoGebra

Angles in a quadrilateral using technology

1 In a new GeoGebra window, construct a quadrilateral using the 'Polygon' tool  as you did to construct a triangle (p. 316). Measure and sum the interior angles. This time, the angle sum will be 360°.

Once again, use the 'Move' tool to move all four points one by one. The angles will change but the angle sum stays constant at 360°.

2 Using GeoGebra's 'Polygon' tool, construct a parallelogram. Measure the size of all angles. Notice that the magnitude of each of the pairs of opposite angles is equal. Using the 'Move' tool, change the shape of the parallelogram. While the angle measurements change, the opposite angles remain equal in magnitude.

3 Using GeoGebra's 'Polygon' tool, construct a kite. Measure the size of all angles. Notice that the magnitude of one pair of opposite angles is equal. These are the angles between the two unequal sides.

8.7 Quadrilaterals

Answers page 491

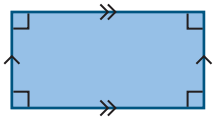
Navigator

Q1, Q2, Q3, Q4 Column 1, Q5, Q6 Column 1, Q7, Q8, Q9, Q11, Q12, Q14, Q15, Q16, Q17, Q20	Q1, Q2, Q3, Q4 Column 2, Q5, Q6 Column 2, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q19, Q20	Q1, Q2, Q3, Q4 Column 3, Q5, Q6 Column 3, Q7, Q8, Q10, Q11, Q12, Q13, Q14, Q15, Q17, Q18, Q19, Q20, Q21

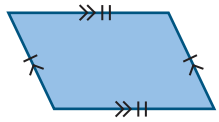
Fluency and understanding

1 Give the most accurate name for each shape below.

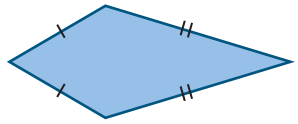
(a)



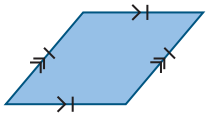
(b)



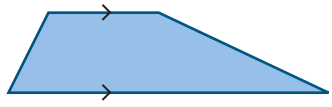
(c)



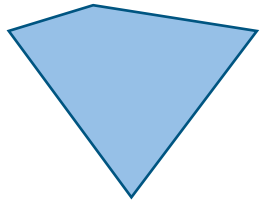
(d)



(e)



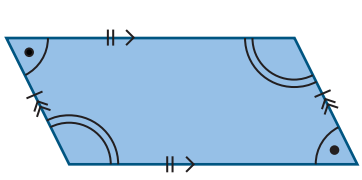
(f)

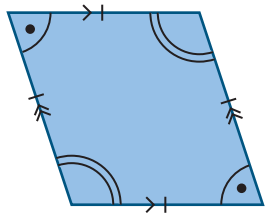


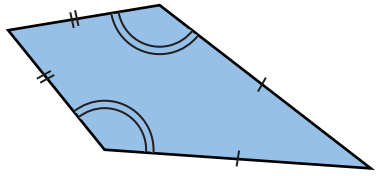
2 A quadrilateral has both pairs of opposite sides parallel and equal in length. We can be sure this quadrilateral is a:

A parallelogram **B** rhombus **C** rectangle **D** square.

3 State TRUE or FALSE for each of the following, referring to the three quadrilaterals below.







(a) The opposite angles in a parallelogram are equal.

(b) All four angles in a parallelogram are equal.

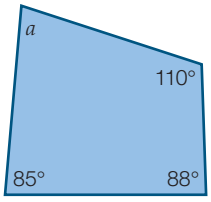
(c) The opposite angles in a rhombus are equal.

(d) All four angles in a rhombus are equal.

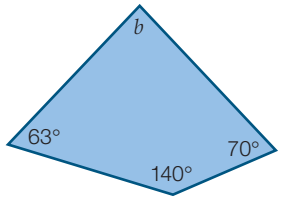
(e) One pair of opposite angles in a kite are equal.

4 Find the size of the unknown angle represented by a letter in each of the following quadrilaterals.

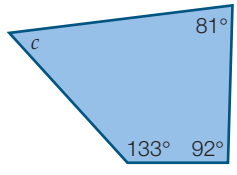
(a)



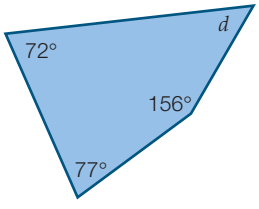
(b)



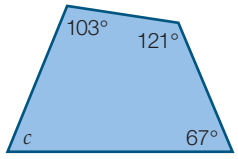
(c)



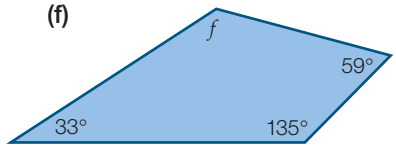
(d)

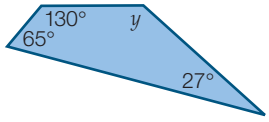


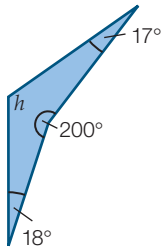
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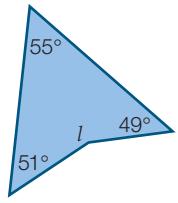


(f)



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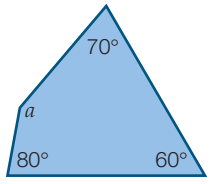
(h) 

(i) 

5 **(a)** The size of angle a shown is:

A 70° **B** 90°

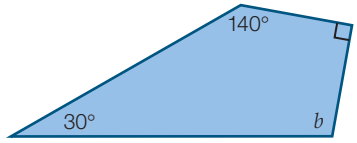
C 100° **D** 150°



(b) The size of angle b shown is:

A 10° **B** 90°

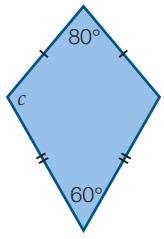
C 100° **D** 170°



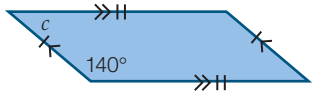
(c) The size of angle c shown is:

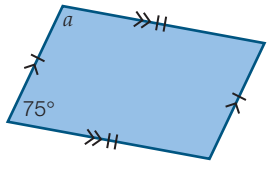
A 55° **B** 80°

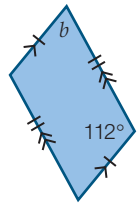
C 110° **D** 140°

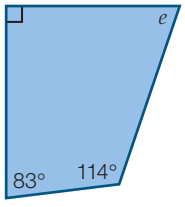


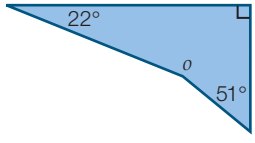
6 Find the size of the angle labelled with a letter in each quadrilateral.

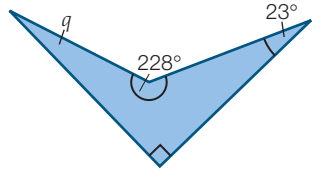
(a) 

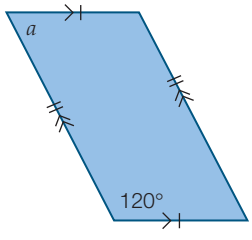
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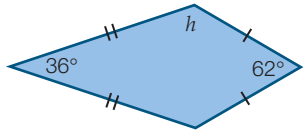
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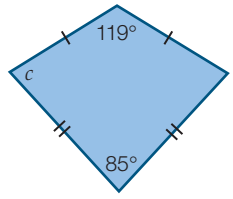
(d) 

(e) 

(f) 

(g) 

(h) 

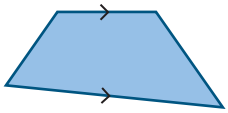
(i) 

Hint

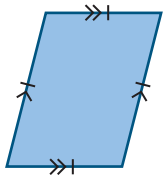
For Question **7**, make sure you do the markings for things such as equal sides, parallel sides and right angles.

7 The diagrams below were created by a tired illustrator. What type of shape do you think he meant to draw in each case? Draw a more accurate version of each shape, given that the markings on the lines are correct.

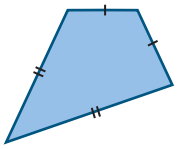
(a)



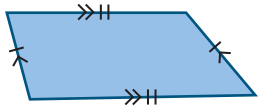
(b)



(c)

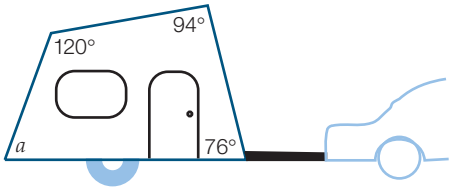


(d)

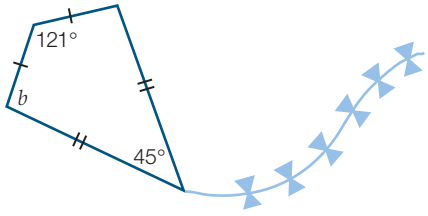


8 Find the angle represented by the pronumeral in each quadrilateral below. Give reasons to justify your answer.

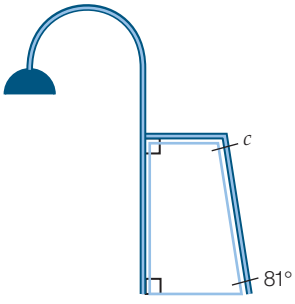
(a)



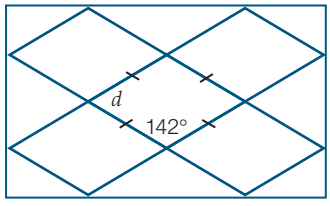
(b)



(c)



(d)



9 An angle in a parallelogram measures 40°. Draw a diagram and mark in the size of each of the other three angles.

10 A quadrilateral has one 45° angle and three unknown angles that are equal to each other. What is the size of each of the three unknown angles?

Reasoning, communicating and open-ended

11 All rhombuses are parallelograms, but not all parallelograms are rhombuses. Explain.

12 Is a parallelogram a type of trapezium? Explain.

13 All rectangles are parallelograms, but not all parallelograms are rectangles. Is this true? Explain why/why not.

14 What is the maximum number of:

(a) obtuse angles a quadrilateral can have? Confirm by drawing a diagram.

(b) reflex angles a quadrilateral can have? Confirm by drawing a diagram.

15 A quadrilateral has two angles of 70° and 34°. The other two angles are equal to each other.

(a) What is the size of the two unknown angles?

(b) What type of quadrilateral could this shape be?

16 Can a quadrilateral have the angles 35°, 85°, 130° and 120°? Explain your answer.

17 What properties of a square make it also classifiable as:

(a) a rectangle **(b)** a rhombus **(c)** a parallelogram?

18 Sam is looking at a kite, which has two given angles of 50° and 120°, but it is unclear which angle is which and where they are located on the kite. He thinks that the best way to solve a problem is to draw a diagram. Help Sam by drawing three different kites that could contain the two known angles of 50° and 120°.

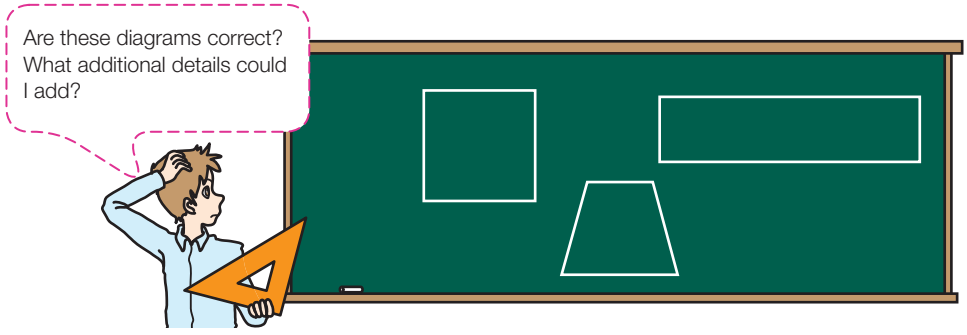
19 A quadrilateral has two angles equal to 80°.

(a) Name one type of quadrilateral that this could be and draw it, marking in the 80° angles.

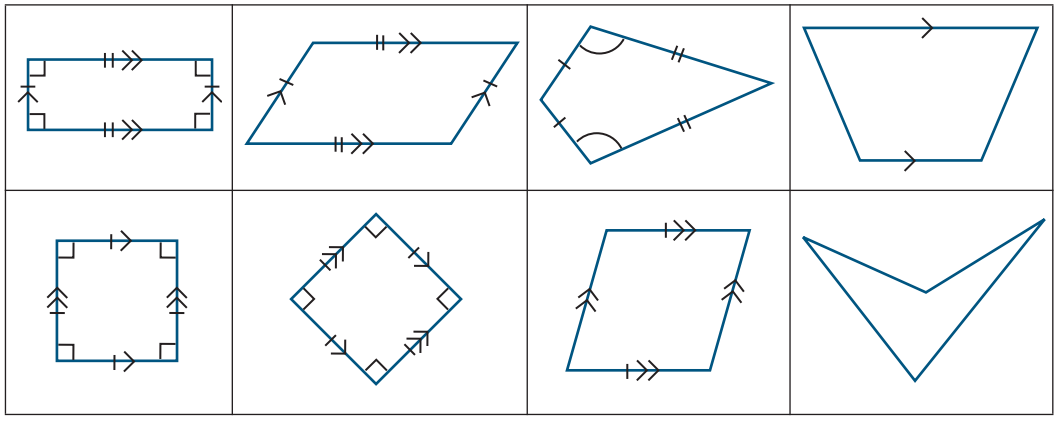
(b) How many different types of quadrilaterals can have two angles equal to 80°? List the different types.

(c) Draw an example of each, marking in the 80° angles.

20 Jack is asked by his teacher to draw a square, a rectangle and a trapezium. Do Jack's diagrams include enough information to clearly depict a square, a rectangle and a trapezium? Redraw his diagrams, adding any missing details.



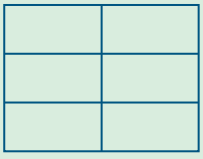
21 Divide these quadrilaterals into two groups. Explain the reason(s) you used to divide them into each group.



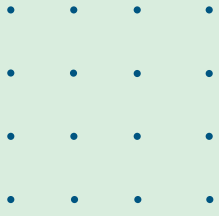
Problem solving

Quadrilateral quandaries

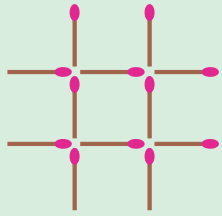
1 How many rectangles can you count here altogether? (Include rectangles that are the same size even if they are drawn in different positions.)



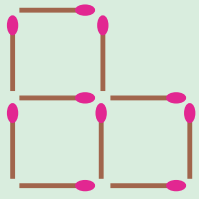
2 Find how many different size squares can be drawn on the grid shown.



3 Move three matchsticks in the arrangement shown to create three squares that touch each other.



4 Remove one match and re-position three others to form one square and two parallelograms.



Strategy options

- Draw a diagram.
- Look for a pattern.

Hint

If it seems like there are too many matches, try getting squares that meet at corners and don't share sides.

Challenge 8

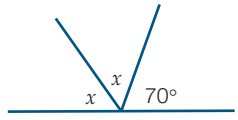


1 The smaller angle between the hands of a clock at half past two is:

A 90° **B** 105° **C** 120° **D** 135°

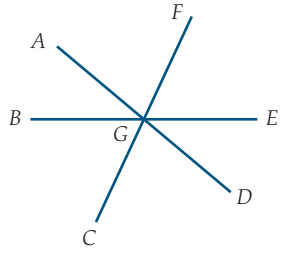
2 Two angles are complementary and one angle is 46° more than the other. What is the size of the larger angle?

3 Find the value of the pronumerals in the following.



4 In the diagram, $\angle BGF = 115°$ and $\angle AGE = 140°$.

What is the size of $\angle CGD$?



5 Two angles are supplementary and one angle is 54° more than the other. What is the size of the smaller angle?

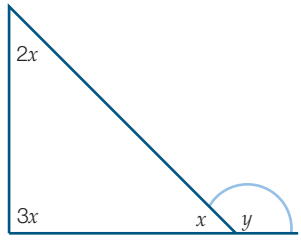
6 In the diagram, the value of y is:

A 30°

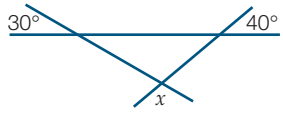
B 60°

C 90°

D 150°



7 In the diagram, the value of x is:



A 110°

B 70°

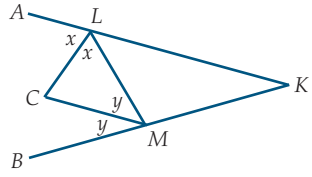
C 40°

D 30°

8 **(a)** Shortly after 12 o'clock, the hour and minute hands of a clock form an angle of 55°. What is the time now?

(b) Noor observes this and leaves, returning after 40 minutes. What is the acute angle between the hands now?

9 In the diagram, ALK and BMK are straight lines. The bisector of $\angle ALM$ meets the bisector of $\angle BML$ at C. If $\angle LCM$ is 70°, then $\angle LKM$, in degrees, is:



A 10°

B 20°

C 40°

D 110°

Chapter review 8

D.I.Y. Summary

Key Words

acute angle	intersect	plane	right-angled triangle
acute-angled triangle	interval	plane shapes	scalene triangle
alternate angles	irregular	polygon	square
angle	irregular quadrilaterals	protractor	straight angle
arms	isosceles triangle	quadrilateral	supplementary angles
co-interior (allied) angles	kite	ray	transversal
complementary angles	line segment	rectangle	trapezium
concave	obtuse angle	reflex angle	vertex
convex	obtuse-angled triangle	regular polygon	vertically opposite angles
corresponding angles	parallel lines	revolution	
degree	parallelogram	rhombus	
equilateral triangle	perpendicular lines	right angle	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 To measure an angle, the centre point of the _____ must match with the _____ of the angle. The lines must match with the _____ of the angle.

2 40° and 50° are _____.

3 _____ add to 180 _____s.

4 Lines drawn at right angles to each other are called _____.

5 The angle names listed in order from largest to smallest are _____, _____, _____, _____, _____, and _____.

6 The most general name for a four-sided figure is a _____.

7 Lines in the same plane that never meet are called _____.

8 An _____ has two sides equal in length and one unequal side.

9 A closed shape with only straight line sides is called a _____.

10 A square is a _____ with right angles.

11 A _____ has three sides all of different lengths.

12 A line crossing other lines is called a _____.

13 A _____ is a quadrilateral with one pair of parallel sides.

14 Angles that are between parallel lines and on the same side of the transversal are _____ and are _____.

Equipment required: Protractor for Questions **1**, **3** and **13**

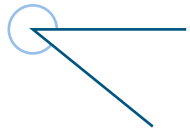
Fluency and understanding

1 Use a protractor to measure each of the following angles. Ex 8.1

(a) 

(b) 

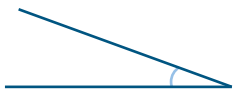
(c) 

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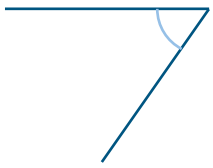
2 Match the following angle sizes with the drawn angles. Do not use a protractor. Ex 8.1

A 20° **B** 55° **C** 120° **D** 210° **E** 270° **F** 310°

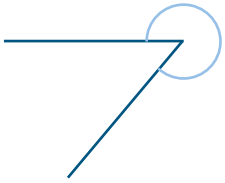
(a) 

(b) 

(c) 

(d) 

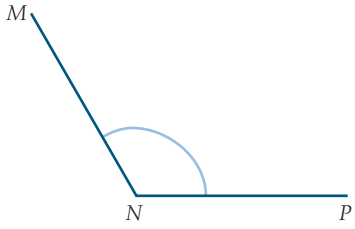
(e) 

(f) 

3 Use a protractor and a ruler to draw an angle of each size stated. Ex 8.1

(a) 56° **(b)** 162° **(c)** 257° **(d)** 304°

4 Which of the following is not true of the angle shown? Ex 8.2



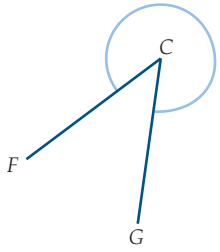
A It is named $\angle NPM$.

B It is named $\angle MNP$.

C It is named $\angle PNM$.

D It is an obtuse angle.

5 Which of the following is true of the angle shown? Ex 8.2



A It is obtuse.

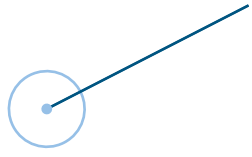
B It is reflex.

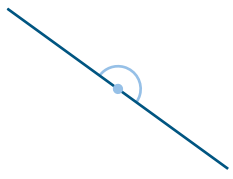
C It is less than 180°.

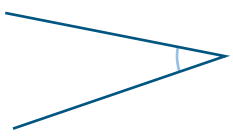
D It is called $\angle CFG$.

6 State the type of each angle below (i.e. acute, etc.). Ex 8.2

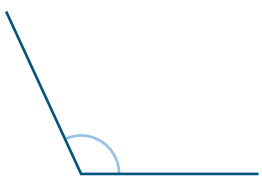
(a) 

(b) 

(c) 

(d) 

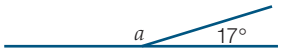
(e) 

(f) 

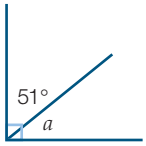
7 Find the value of a in each of the following diagrams.

Ex 8.3

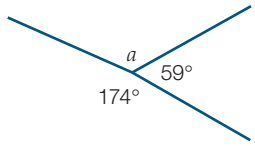
(a)



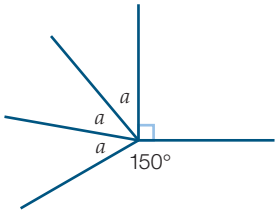
(b)



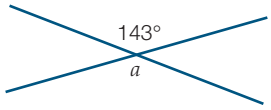
(c)



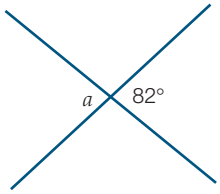
(d)



(e)



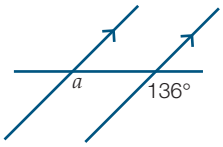
(f)



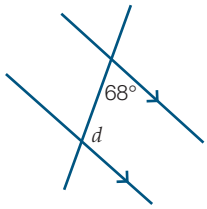
8 Find the value of the pronumeral in the following diagrams.

Ex 8.4

(a)



(b)



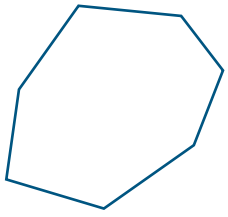
9 Name the following polygons, also stating whether they are regular or irregular, convex or concave.

Ex 8.5

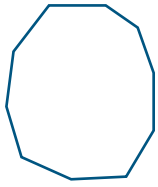
(a)



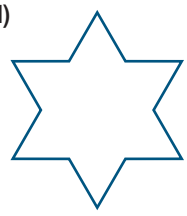
(b)



(c)

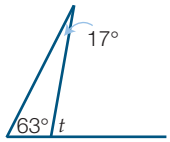


(d)



10 Find the value of the pronumeral in the diagram.

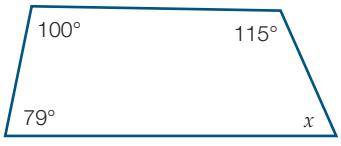
Ex 8.6



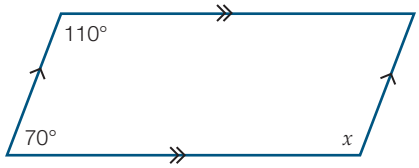
11 Find the unknown angle in each of the following quadrilaterals. Make sure you support your answer with reason(s).

Ex 8.7

(a)

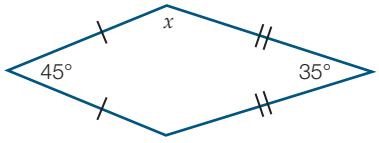


(b)

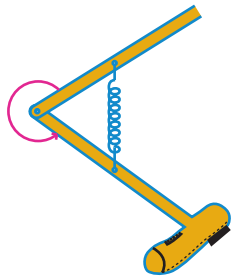


12 Find the value of x in the following quadrilateral. Make sure you support your answer with reason(s).

Ex 8.7



13 Eloise has constructed a mechanical leg to kick a football as part of her physical education investigation into kicking techniques. During one trial, the initial leg position was as shown. Find the size of the reflex angle of the leg.



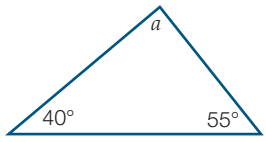
14 One angle in a right-angled triangle is 45°.

(a) What is the value of the other two angles?

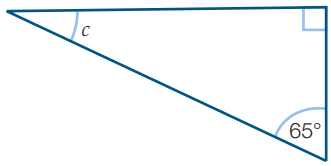
(b) The triangle described in **(a)** was named a right-angled triangle. Give another name for this triangle.

15 Find the missing angle in each of the following triangles. Make sure you support your answer with reason(s).

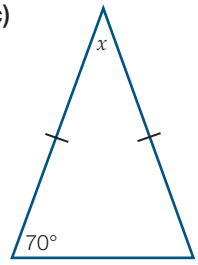
(a)



(b)

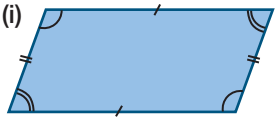


(c)

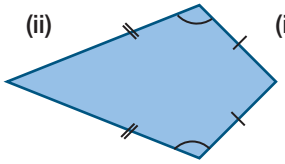


16 Using the shapes below, answer TRUE or FALSE to the following statements.

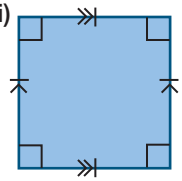
(i)



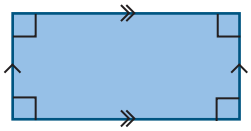
(ii)



(iii)



(iv)



(a) All four shapes are quadrilaterals.

(b) **(i)** and **(ii)** are kites.

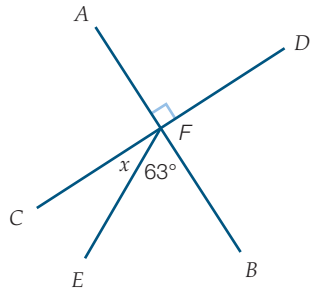
(c) **(i)**, **(iii)** and **(iv)** could be classified as parallelograms.

(d) **(i)** is a rhombus.

(e) **(iii)** is a rectangle.

Reasoning and communicating

17 Find the value of the angle x, giving reasons for each step.



18 What type of angle would result from adding an acute angle and a right angle?

19 What type of angle would result from adding a straight angle and an obtuse angle?

20 What types of angles could result from subtracting a reflex angle from a revolution?

NAPLAN practice 8

Numeracy: Non-calculator

1 Which is the largest angle?

A



B



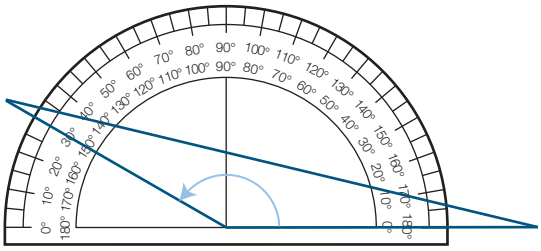
C



D



2 What is the size of the angle in the triangle marked by the arrow?



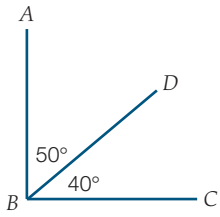
3 $\angle ABD$ and $\angle DBC$ are best described as:

A vertically opposite angles

B supplementary angles

C angles at a point

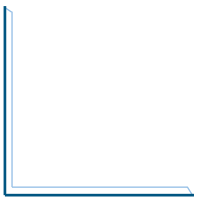
D complementary angles.



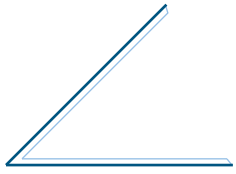
Numeracy: Calculator allowed

4 Which diagram shows a book opened to about 30°?

A

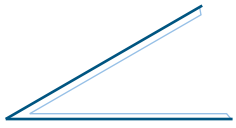


B



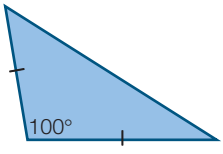
C

D

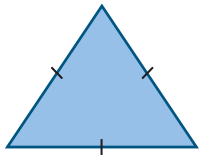


5 Which one of these is a right-angled isosceles triangle?

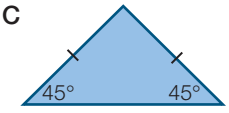
A



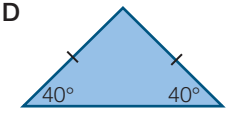
B



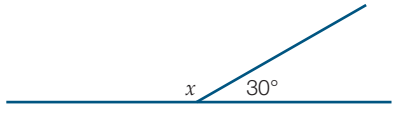
C



D



6 What is the value of x in this diagram?



A 30° **B** 60° **C** 120° **D** 150°

Mixed review D

Equipment required: Protractor for Question **15(c)**

Fluency and understanding

1 Simplify:

(a) $2a + 3b + 4a - 7b$ **(b)** $6a + 3b - 2c - b + 5c$

Ex 5.6

2 Write out factor trees for each of the following numbers, then express each number as the product of its prime factors in index form.

(a) 45 **(b)** 24 **(c)** 630

Ex 2.3

3 Write the following percentages as fractions in simplest form.

(a) 75% **(b)** 50% **(c)** 30% **(d)** 4%

Ex 4.7

4 Evaluate without using a calculator:

(a) 1.1×0.23 **(b)** $0.24 + 3.7 - 1.503$

Ex 4.4, 4.5

5 Solve the following equations using algebra.

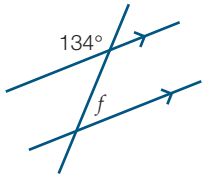
(a) $3x - 4 = -7$ **(b)** $\frac{x+5}{3} = -12$

Ex 7.4

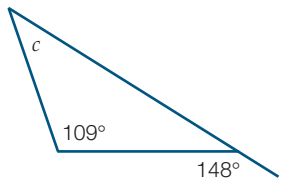
6 Find the value of the pronumeral in the following diagrams.

Ex 8.4, 8.6

(a)



(b)



7 Evaluate without a calculator:

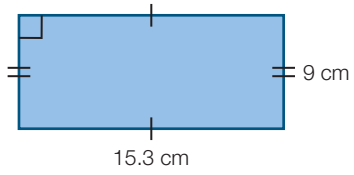
(a) $3\frac{1}{5} + 2\frac{1}{2} - 1\frac{3}{4}$ **(b)** $2\frac{1}{4} \times 1\frac{1}{3} \div 3\frac{1}{5}$

Ex 3.4–3.6

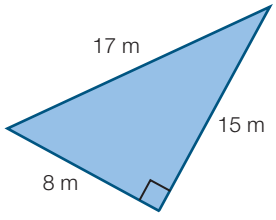
8 Find the area of the following shapes.

Ex 6.4, 6.5

(a)



(b)



9 The table on the right shows the minimum and maximum temperatures at Frosty Hollow for 1 week in winter.

(a) On which day was the lowest maximum recorded?

(b) On which day was the highest minimum recorded?

(c) Which day had the greatest difference between minimum and maximum?

Day	Minimum (°C)	Maximum (°C)
Monday	-4	5
Tuesday	-8	0
Wednesday	-2	1
Thursday	-5	-1
Friday	-6	6
Saturday	-4	4
Sunday	-9	2

10 Which of the following is the correct answer to $(4 + 3) \times 10 - 4 \div 2$?

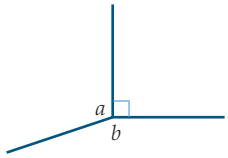
A 68 **B** 35 **C** 33 **D** 21

Reasoning and communicating

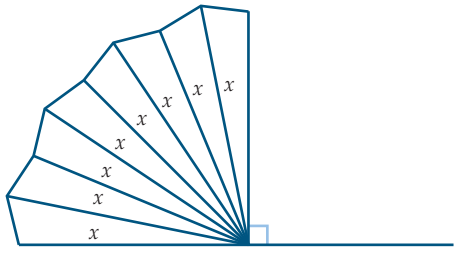
11 The angles in this diagram can be called:

A vertically opposite **B** complementary

C supplementary **D** angles at a point.

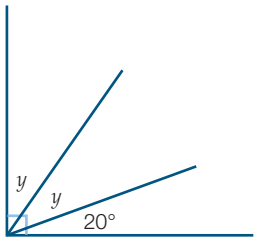


12 A fan, when opened out, looks like this. What is the size of the angle x between the folds of the fan?

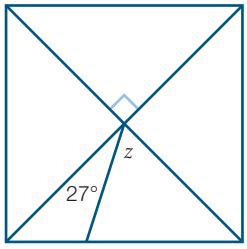


13 Find the value of the pronumeral.

(a)

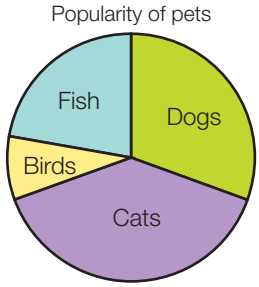


(b)



14 What two angles are formed by the two hands on a clock face at 5:00 p.m.?

15 John has drawn this pie chart, showing the popularity of pets.



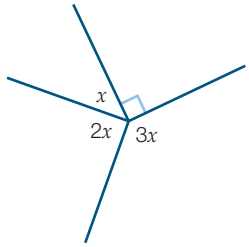
(a) Estimate the angle at the centre of each wedge.

(b) Without using a protractor, how could you check to see how reasonable the estimates you made in **(a)** are?

(c) Using a protractor, measure the size of each wedge.

16 You enter a lift on the fourth floor. Once inside, you are trapped and the lift takes you on an adventure! First, you go up five floors, then fall six floors, then fall another two floors before finally going up one floor and coming to rest. The doors open, much to your relief. What floor are you on?

17 Find the size of each angle in the diagram by first finding the value of x.

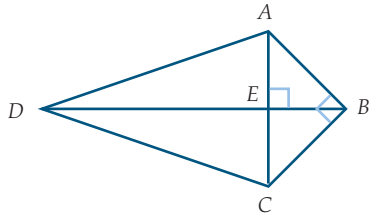


18 **(a)** Draw and label an acute angle $\angle ABC$.

(b) There is another angle in your diagram, apart from the acute angle. Classify it.

(c) Name the angle in **(b)**.

19 Name as many pairs of complementary angles, supplementary angles, vertically opposite angles and angles in a revolution as you can see in the figure.



9 Statistics and probability

Recall 9

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

1 **(a)** List the numbers you get if you count by fives from 0 to 30.

(b) List the numbers you get if you count by twos from 16 to 28.

(c) List the numbers you get if you count by hundreds from 1300 to 1700.

(d) List the numbers you get if you count by twenties from 60 to 200.

2 **(a)** List the multiples of 50 from 50 to 250.

(b) List the multiples of 25 from 100 to 200.

(c) List the multiples of 200 from 1400 to 2000.

3 Write the following lists of numbers in order from smallest to largest.

(a) 12, 22, 14, 20, 10, 16, 24, 6

(b) 15, 0, 50, 20, 55, 100, 25, 5

(c) $5, 1\frac{1}{2}, 6\frac{1}{2}, 1, \frac{1}{2}, 0, 3\frac{1}{2}, 5\frac{1}{2}, 2, 3, 2\frac{1}{2}, 4$

4 Copy and complete the pattern by adding the same number each time.

(a) 12, ____, ____, 21, ____, 27

(b) 120, ____, ____, ____, ____, 220

(c) 225, ____, ____, 300, ____, 350

5 Write the following fractions in simplest form.

(a) $\frac{2}{6}$ **(b)** $\frac{6}{8}$

Syllabus references

Statistics and Probability: Data Collection and Representation

- ✓ Investigate techniques for collecting data, including census, sampling and observation **(ACMSP284)**
- ✓ Construct and compare a range of data displays, including stem-and-leaf plots and dot plots **(ACMSP170)**

Statistics and Probability: Single Variable Data Analysis

- ✓ Calculate mean, median, mode and range for sets of data and interpret these statistics in the context of data **(ACMSP171)**
- ✓ Investigate the effect of individual data values, including outliers, on the mean and median **(ACMSP207)**
- ✓ Describe and interpret data displays using mean, median and range **(ACMSP172)**

Statistics and Probability: Probability 1

- ✓ Construct sample spaces for single-step experiments with equally likely outcomes **(ACMSP167)**
- ✓ Assign probabilities to the outcomes of events and determine probabilities for events **(ACMSP168)**
- ✓ Identify complementary events and use the sum of probabilities to solve problems **(ACMSP204)**

Collecting data

9.1

Need to Know

Statistical variables and data

In statistics, a **variable** is something that can be measured or observed and is expected to change over time or between individual observations.

Data is raw information that can be collected through observations, surveys or interviews. For example: Information gathered about the colour of the family car or the time Year 7 students spend on the internet each day.

Variables and the associated data can be divided into three types: categorical, discrete and continuous.

Non-numerical variables

A **categorical variable** is a variable that defines information that can be sorted into categories or groups of the same type.

For example: Eye colour is a categorical variable where the groups could be blue eyes, green eyes, brown eyes, etc.

The data would be the number of observations for each group.

The favourite fruit of a Year 7 class is another categorical variable where the groups could be apples, bananas, oranges, etc.

The data would be the survey responses to each group.

A categorical variable can appear to be numerical but the data has no numerical meaning.

For example: The number of a high-rise apartment is a categorical variable. The data collected, such as 256, means apartment 6 on the 25th floor and it makes no sense to find an average apartment number.

Numerical variables

A **discrete variable** is a variable whose values are taken from a countable set of possibilities.

Discrete data is information that has been counted.

For example: The number of emails received by a person each day for a year is a discrete variable.

Discrete data is the information that has been counted such as 24, 53, 17, 38,... giving 365 pieces of data, assuming 365 days in a year.

A **continuous variable** is a variable that can take any value within a specific interval. Continuous data is information that has been measured. In practice, these values are dependent upon the accuracy of the measuring device.

For example: The volume of water in a dam measured in gigalitres each day is a continuous variable.

Continuous data is information that has been measured such as 420, 423, 425, 421, 417,...

Recording data

Collected data can be recorded in a **frequency table**. Each time a piece of data occurs, it is recorded in a **tally** column. The **frequency** is the number of times an event occurs and is the total of each tally. The **range** = the largest data value – the smallest data value.

Grouped data

When there is a large amount of numerical data spread out over a wide range, it is often grouped. The preferred number of groups is five to ten. Although individual data values are lost, grouping data makes it a lot simpler to find statistical information without losing much accuracy.

Class intervals are a range of values within each group in a frequency table. All class intervals are the same width.

Class intervals for discrete data are presented in discrete groups.

For example: 0–9, 10–19, 20–29.

Class intervals for continuous data are presented in continuous groups.

For example: 0–<10, 10–<20, 20–<30.

Constructing a frequency table

1. Construct a table with three columns that shows the variable, the tally and the frequency.
2. Enter the variables into the variable column.

When grouping data, subtract the smallest data value from the highest to find the range. Then divide this number by 2, 5, 10, etc. until you get an interval size that gives between five and 10 groups. (If you get a decimal number, round to the nearest whole number.)

3. Enter the data into the tally column. Go through the data list systematically from the first piece of data to the last, tallying as you go. Notice $\cancel{||||}$ is used to represent 5.
4. Enter these tallies as numbers in the frequency column. Add the numbers in this column to check that you have tallied correctly.

Worked Example 1

WE1

For each of the following, state whether the data from a survey asking these questions is categorical, discrete or continuous.

(a) How many television sets are in your house?

(b) What is the height of the tallest person in your family?

(c) How far do you have to travel to get to school?

(d) What is your favourite dessert?

Working

(a) Discrete data (Here, we are counting the number of televisions.)

(b) Continuous data (Here, we are measuring height.)

(c) Continuous data (Here, we are measuring distance.)

(d) Categorical data (Here, we are putting things in categories.)

Worked Example 2

WE2

A Year 7 class of 25 students was surveyed about the number of brothers and sisters they have. The results were: 2, 3, 5, 0, 1, 1, 2, 0, 3, 1, 4, 0, 1, 1, 3, 2, 2, 2, 1, 1, 0, 1, 1, 2, 4

(a) Construct a frequency table showing this information.

(b) Is this data categorical, discrete or continuous?

Working

(a)

Number of brothers and sisters	Tally	Frequency
0	||||	4
1	𝍸 ||||	9
2	𝍸 |	6
3	|||	3
4	||	2
5	|	1
	Total	25

(b) It is discrete data.

Worked Example 3

WE3

The Exceptional Felines basketball team keeps track of the points they score on the way to the state championship. The point totals are:

73, 84, 68, 45, 52, 44, 45, 52, 66, 42, 43, 40, 53, 47, 82, 76, 42, 57, 65, 81, 80, 40, 56, 72, 74, 83, 41, 66, 76, 75, 68, 81, 82, 79, 58, 81, 78, 80, 78, 76

Group the results and then construct a frequency table showing this information.

Working

Range = 84 − 40
= 44

44 ÷ 5 = 8.8

Number of groups = $\frac{44}{5}$

= 8.8

So, we will group with an interval size of 5 and have 9 groups.

Number of points scored	Tally	Frequency
40–44	𝍸 ||	7
45–49	|||	3
50–54	|||	3
55–59	|||	3
60–64		0
65–69	𝍸	5
70–74	|||	3
75–79	𝍸 ||	7
80–84	𝍸 ||||	9
	Total	40

Answers page 493

9.1 Collecting data

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q12, Q14, Q16	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q14, Q16	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15

Fluency and understanding

WE1

1 For each of the following, state whether the data from a survey asking these questions is categorical, discrete or continuous.

(a) How do you travel to school?

(b) What colour hair do you have?

(c) How much time did you spend at the beach last summer?

(d) What is your height?

(e) How many cars passed by the school in a day?

(f) What is your favourite book?

(g) How many trees are in your front yard?

WE2

2 A Year 7 class of 25 students was surveyed about the number of pets they own. The results were:

0, 0, 2, 1, 1, 1, 0, 6, 9, 4, 3, 0, 1, 3, 2, 1, 1, 4, 5, 4, 3, 2, 1, 4, 7

(a) Construct a frequency table showing this information.

(b) Is this data categorical, discrete or continuous?

WE3

3 The number of calls per day to the emergency help department of an automobile association was recorded for the month of June. The results were:

120	100	75	60	66	100	110	197	55	80
155	179	154	43	75	98	172	130	25	55
110	99	72	90	113	65	87	94	112	18

Group the results and then construct a frequency table showing this information.

4 Students at a school noted the colour of teachers' cars in the car park. They found there were 13 black cars, 21 white cars, 25 silver cars, 7 red cars, 5 blue cars and 11 other colours.

(a) How many cars were in the car park?

(b) Construct a frequency table showing this information.

(c) Is this data categorical, discrete or continuous?

5 A Year 7 class was surveyed about how many hours of television they watched on a normal week night. The results were:

$3, 2, 1\frac{1}{2}, \frac{1}{2}, 1, 2\frac{1}{2}, 1, 0, 4\frac{1}{2}, 2\frac{1}{2}, 2, 2\frac{1}{2}, 2, 0, 4, 2, 1\frac{1}{2}, 2\frac{1}{2}, 3, \frac{1}{2}, 0, \frac{1}{2}, 1, 2\frac{1}{2}, 2, 2, \frac{1}{2}$

Construct a frequency table showing this information.

6 You and your classmates are completing a survey and one of the questions is as follows.

In Year 9, students studying Japanese will be offered the chance to go on a trip to Japan. Circle the number that best describes how you feel about this.

Very interested	Somewhat interested	Not sure	Not very interested	Not at all interested
5	4	3	2	1

What sort of data will arise from the responses to this question?

7 **(a)** Give an example of a discrete variable with values that are not whole numbers.

(b) If you and your classmates were asked to record your postcode, would the resulting data be categorical or discrete? Explain why you responded the way you did.

8 Look at the following data list.

2	8	9	15	22	24	17	16	28	31	40
15	52	16	51	42	39	27	26	42	55	46
17	22	18	5	47	46	33	21	19	21	17

Group the data using 0–9, 10–19, 20–29, etc. as the groups.

(a) The frequency for the 10–19 group is:

A 6 **B** 8 **C** 9 **D** 10

(b) The frequency for the last group is:

A 2 **B** 3 **C** 4 **D** 5

9 A class was surveyed about the number of hours spent exercising over a weekend. The results were as follows:

1, 3, 2, 1, 1, 2, 0, 2, 0, 1, 1, 2, 6, 3, 4, 2, 1, 0, 2, 8, 3, 5, 1, 2, 0, 2

(a) How many students were surveyed?

(b) Construct a frequency table of this data.

(c) How many students did not exercise at all?

(d) How many students spent more than 2 hours exercising?

(e) What was the most common number of hours spent exercising by members of the class?

(f) What is the greatest number of hours spent exercising by a student?

(g) One student hoped to qualify for the next Olympics, one student wanted to be drafted into the NRL and another student's netball team was in the finals. What data values are mostly likely to belong to these three students? Explain your answer.

10 The cast and crew of a school musical were to be given T-shirts advertising the production. There were five sizes available, XS (extra small), S (small), M (medium), L (large) and XL (extra large). The orders are recorded below:

S, M, S, XL, L, M, L, S, XS, M, L, S, M, L, L, M, XL, S, M, M, XL, L, M, S, XS, L, S, XL, M, L

Five students, all in Year 7, forgot to order a T-shirt.

(a) How many students were in the cast and crew of the musical?

(b) Construct a frequency table for this data.

(c) What type of data is this?

(d) What was the most popular T-shirt size ordered?

(e) How many T-shirts are smaller than the M size?

(f) The music teacher, Ms Campbell, could not decide whether to order one extra T-shirt in each size or five more M size T-shirts for the students who had not submitted an order. What would you advise her to do, explaining your advice.

Reasoning, communicating and open-ended

11 Look at the following data set.

20	32	42	53	63	21	33	42	53	65	22	34	43	57	61
24	34	48	50	61	24	39	40	50	62	29	31	41	52	62

(a) Construct a frequency table using class intervals of 20–29, 30–39, etc.

(b) Describe what you find in your frequency table.

(c) Construct a frequency table using class intervals of 20–24, 25–29, 30–34, etc.

(d) Describe what you find in your frequency table.

(e) What does this show?

12 Sixty 13-year-old students were tested to find their pulse rate (heart beat) when resting. The following beats per minute were measured.

70	68	68	76	79	68	76	55	55	60	60	94	72	65	64
93	71	62	67	82	76	65	77	82	81	59	74	74	67	68
78	76	63	82	81	82	74	70	66	63	84	81	69	84	79
71	70	54	68	64	78	58	84	61	75	72	73	71	91	66

(a) Construct a frequency table using class intervals of 51–60, 61–70, etc.

(b) Construct a frequency table using class intervals of 51–55, 56–60, 61–65, etc.

(c) Which table shows the spread of results more clearly? Give a brief reason to support your answer.

13 The following is a list of birth weights (g) of 30 babies.

2900	2805	2925	2010	2720	3125	2670	2555
2963	2972	3151	2515	3529	3098	2126	2417
3000	3254	2997	2986	2719	2842	3519	3509
3218	3002	2437	2222	2019	2113		

Construct a frequency table for this data using a suitable class interval. Briefly explain why you chose this class interval.

14 **(a)** A die is rolled 20 times. Construct a possible frequency table for the outcomes 1, 2, 3, 4, 5, 6 with no outcome having a frequency greater than 4.

(b) Fifty dogs were weighed. Their weights ranged from 1 kg to 25 kg. Construct a possible frequency table if the data is grouped using class intervals of 1–5, 6–10, 11–15, 16–20 and 21–25 with no data group having a frequency greater than 15.

15 Find out how many hours of television each of the students in your class watches on a normal week night by surveying your class. Construct a frequency table of your results. Round your answers to the nearest half hour.

(a) How many of your class members watch no television on a normal week night?

(b) How many of your class members watch less than 1 hour of television on a normal week night?

(c) How many of your class members watch 3 or more hours of television on a normal week night?

(d) How much television do the majority of the members of your class watch on a normal week night?

(e) Do you watch more or less television than the majority of the members of your class?

16 Find out the favourite school subject of each of the students in your class by surveying your class. Construct a frequency table of your results.

(a) What is the favourite subject of the members of your class?

(b) What is the second favourite subject of the members of your class?

(c) Did any student have no favourite subjects?

Measures of location and spread

9.2

Need to Know

Measures of location

The **mean** and **median** are both referred to as **measures of location** or **measures of centre**.

The mean, also known as the **average**, is a value used to represent the centre of a data set and is the sum of all the values in the data set divided by the number of values in that data set.

$$\text{Mean} = \frac{\text{sum of data values}}{\text{number of data values}}$$

For example: the mean of 6, 8, 14, 23 and 29 $= \frac{6 + 8 + 14 + 23 + 29}{5}$

$= 16$

The median, another value used to represent the centre of a data set, is the *middle* value of an ordered data set.

To calculate the median, arrange the data set in order, usually ascending, then count the number of data values.

If the number of values is odd, select the middle value.

For example: the median of 2, 3, 4, 5, 20 is 4.

If the number of values is even, select the two middle values. Add them together and divide by 2.

For example: the median of 2, 3, 4, 20 is 3.5.

The **mode** is the value that occurs *most often* in a data set. It is not a measure of location but is sometimes used for this purpose.

For example: the mode of 2, 3, 3, 5 is 3.

If there are two modes, the data is said to be **bimodal**.

For example: 2, 3, 3, 4, 5, 5, 9 is bimodal with a mode at 3 and another at 5.

Measure of spread

The **range** is a **measure of spread**. The range is the difference between the highest value and the lowest value in a data set.

Outliers

An **outlier** is a data value that is either much smaller or much bigger than the other members of the data set. The data need to be arranged in order before a decision about outliers can be made.

For example: If data on postal delivery times were being collected, a letter that was delivered a month after it was posted, when all other letters travelling the same route were delivered within five working days, could be considered an outlier.

Worked Example 4

WE4

Find the **(i)** mean, **(ii)** median, **(iii)** mode and **(iv)** range of each of the following sets of test results.

(a) 9, 4, 5, 7, 8, 7, 2

(b) 10, 7, 3, 4, 8, 2, 6, 5

Working

(a) (i) $\text{Mean} = \dfrac{\text{sum of data values}}{\text{number of data values}}$

$= \dfrac{9 + 4 + 5 + 7 + 8 + 7 + 2}{7}$

$= \dfrac{42}{7}$

$\text{Mean} = 6$

(ii) 2, 4, 5, (7), 7, 8, 9

7

$\text{Median} = 7$

(iii) $\text{Mode} = 7$

(iv) $\text{Range} = \text{Max} - \text{Min}$

$= 9 - 2$

$= 7$

(b) (i) $\text{Mean} = \dfrac{\text{sum of data values}}{\text{number of data values}}$

$= \dfrac{10 + 7 + 3 + 4 + 8 + 2 + 6 + 5}{8}$

$= \dfrac{45}{8}$

$= 5.625$

$\text{Mean} = 5.625$

(ii) 2, 3, 4, (5, 6,) 7, 8, 10

$\text{Median} = \dfrac{5 + 6}{2}$

$\text{Median} = 5.5$

(iii) There is no mode.

(iv) $\text{Range} = \text{Max} - \text{Min}$

$= 10 - 2$

$= 8$

Worked Example 5

WE5

Order the following data and identify any possible outliers in each of these data sets.

(a) 2, 4, 7, 3, 2, 1, 6, 9, 17, 7, 4, 1

(b) 1, 9, 13, 11, 8, 10, 5, 6, 4, 2, 9

Working

(a) Ordered data:

1, 1, 2, 2, 3, 4, 4, 6, 7, 7, 9, 17

The data value of 17 is a possible outlier as it is much larger than all the other data values.

(b) Ordered data:

1, 2, 4, 5, 6, 8, 9, 9, 10, 11, 13

No values in the data set are much larger or smaller than the other values, so there is probably no outlier.

Worked Example 6

WE 6

(a) For each of the following data sets calculate the: **(i)** mean, **(ii)** median, **(iii)** mode and **(iv)** range.

Set 1: 1 1 2 3 4 4 4 5 6 7 8 9 10 12 14 16 30

Set 2: 1 1 2 3 4 4 4 5 6 7 8 9 10 12 14 16 16

(b) What effect did the change in data values from Set 1 to Set 2 have on each of the statistics you have calculated?

Working

(a) (i) Set 1: Mean $= \frac{136}{17} = 8$

Set 2: Mean $= \frac{122}{17} = 7.18$

(ii) Set 1: The data are already in order; 17 values, so the median is the 9th value

Median = 6

Set 2: The data are already in order; 17 values, so the median is the 9th value

Median = 6

(iii) Set 1: Mode = 4

Set 2: Mode = 4

(iv) Set 1: Range = 30 − 1 = 29

Set 2: Range = 16 − 1 = 15

(b) The ranges are quite different.
The means are a little different.
There is no difference for either the median or the mode.

Technology Exploration: Calculator

Finding the mean on a calculator

Most calculators that students use will have some statistical functions built into them. One of the most useful of these is the mean. The problem is that every calculator will have its own special set of steps to follow to find these statistical values. The instructions that follow are for the Casio fx–82AU PLUS.

Put the calculator in the required single variable statistical mode: MODE 2 1

Enter the data, pressing = after each value. For this example we will use 1, 2, 3, 4, 5, 6. (We should already know the mean of these six values is $\frac{21}{6} = 3.5$.)

Then press AC (This clears the screen so you can start calculating values.)

Then press SHIFT STAT 4 2 = to obtain $\bar{x}$, which is the calculator shorthand for 'mean'.

You should get a result of 3.5 as before.

Technology Exploration: Excel

Simple statistics using a spreadsheet

A spreadsheet, such as Microsoft® Excel®, can be used to find the mean, median, mode and range of a data set. Excel has special function words that make these calculations simple.

AVERAGE(A1:A10)	finds the mean of the numbers in the stated cell range
MEDIAN(A1:A10)	finds the median of the numbers in the stated cell range
MODE(A1:A10)	finds the mode of the numbers in the stated cell range

There is not a function specifically for the range, but there is MAX and MIN which we can use to find the range.

You need to remember to put an = at the start of each cell entry when using these special functions.

Now, make up a list of 10 values and use the spreadsheet to calculate the various statistics. Also do the calculations by hand as a check that you got the technology approach correct.

Answers page 495

9.2 Measures of location and spread

Navigator

Q1 Column 1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q14	Q1 Column 2, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q14	Q1 Column 2, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q14

Equipment required: Calculator for Question **4**

Fluency and understanding

WE4

1 Find the **(i)** mean, **(ii)** median, **(iii)** mode and **(iv)** range of each of the following sets of test results.

(a) 7, 1, 7

(b) 5, 8, 5

(c) 6, 6, 6, 6, 6

(d) 4, 4, 4, 4, 4, 4, 4

(e) 7, 6, 7, 5, 7, 9, 7, 8

(f) 6, 5, 5, 5, 9, 9, 10

(g) 8, 7, 9, 5, 8, 2, 3

(h) 2, 8, 2, 6, 8, 6, 1, 2, 1, 4

(i) 4, 3, 6, 4, 8, 2, 5, 7, 6, 4

(j) 5, 3, 8, 7, 2, 5, 9, 5

(k) 1, 6, 2, 7, 9, 8, 9

(l) 6, 3, 2, 8, 9, 2, 5

WE5

2 Order the following data and identify any possible outliers.

(a) 2, 4, 2, 1, 4, 6, 7, 9, 11, 11, 7, 3

(b) 6, 7, 8, 9, 13, 2, 9, 12, 14, 13

(c) 12, 16, 17, 19, 21, 24, 25, 13, 15

(d) 17, 11, 18, 21, 22, 19, 20, 22, 17, 19

(e) 1.3, 1.4, 1.6, 1.7, 1.8, 3.1, 1.9, 2.0

(f) 1.4, 1.2, 2.6, 3.3, 2.9, 3.1, 1.7, 1.8, 2.2

3 **(a)** For each of the following data sets calculate the: **(i)** mean, **(ii)** median, **(iii)** mode and **(iv)** range.

WE6

Set 1: 8 9 9 10 10 10 10 10 14 15 16 18 19 20 22

Set 2: 1 9 9 10 10 10 10 10 14 15 16 18 19 20 22

(b) What effect did the change in data values from Set 1 to Set 2 have on each of the statistics you have calculated?

4 Find the **(i)** mean, **(ii)** median, **(iii)** mode and **(iv)** range of each of the following sets of data. Round your answers to two decimal places if necessary.

(a) 29, 36, 29, 23, 32, 25

(b) 67, 73, 72, 82, 50, 45, 76

(c) 56, 72, 39, 47, 89, 81, 63, 81

(d) 12.8, 6.5, 19.3, 14.2, 4.7, 6.8, 6.5, 11.3

(e) 16.3, 15.8, 11.4, 15.2, 12.3, 11.4, 13.5

(f) 11.01, 11.001, 10.1, 10.11, 10.01, 1.11, 11.11, 1.01, 11

5 *Without doing any calculations*, in each case, choose which one of the alternatives could be the mean.

(a) 9, 7, 8, 7, 7, 8, 6, 6, 9

A 3 **B** 5 **C** 7 **D** 10

(b) 6, 6, 6, 7, 6, 7, 4, 7, 2, 7

A 2 **B** 4 **C** 6 **D** 8

(c) 12, 19, 14, 18, 11, 17

A 10 **B** 12 **C** 16 **D** 19

(d) 86, 74, 85, 68, 91, 76, 80, 90

A 74 **B** 81 **C** 89 **D** 91

6 The Korea–Japan 2002 World Cup in soccer was played in more stadiums than ever before. This record was not beaten in Germany in 2006 (12 stadiums) or in South Africa in 2010 (10 stadiums). The following table shows the city and capacity of the different stadiums.

Korea	Capacity
Seoul	63 961
Incheon	52 179
Suwon	43 188
Daejeon	40 407
Daegu	68 014
Jeonju	42 391
Gwangju	42 880
Ulsan	43 550
Busan	55 982
Seogwipo	42 256

Japan	Capacity
Sapporo	42 000
Miyagi	49 000
Niigata	42 300
Ibaraki	42 000
Saitama	63 000
Shizuoka	50 600
Kobe	42 000
Osaka	50 000
Oita	43 000
Yokohama	70 000

(a) Find the **(i)** mean, **(ii)** median and **(iii)** range of the stadium capacities in Korea.

(b) Which stadium has closest to the mean capacity in Korea?

(c) Find the **(i)** mean, **(ii)** median and **(iii)** range of the stadium capacities in Japan.

(d) Which stadium has closest to the mean capacity in Japan?

(e) Use the mean and median to compare the sizes of the stadiums in the two countries.

(f) Find the **(i)** mean, **(ii)** median and **(iii)** range for all 20 stadium capacities.

(g) Which stadium is closest to the mean capacity for all 20 stadiums?

(h) Find the range of capacities for the stadiums in: **(i)** Korea and **(ii)** Japan.

Reasoning, communicating and open-ended

7 In the 2009 NRL season the Manly–Warringah team, in all games including the finals, scored the following points.

12 24 10 12 23 24 12 8 22 34 17 38 20

20 19 18 32 44 18 22 44 20 18 38 12

(a) Calculate the mean and median number of points scored for the season.

(b) Would you consider any of the scores to be outliers?

(c) Which of the mean and the median do you think better represents their scores for the season?

8 In each of the following situations state which of mean, median and mode would be the most suitable measure of centre to use. In each case give a brief reason for your choice.

(a) You have collected the maximum temperature recorded for each day of a particular month at the weather station nearest your school. You need a measure of centre to help describe the weather to a visitor from overseas.

(b) You have recorded the shirt size of each of the students in your class. The school shirts are sold in sizes XS, S, M, L and XL. You need this information as your parents are the shirt manufacturers.

(c) The sale price for every property sold by a real estate agency in a particular month has been recorded. You need to use a measure of centre in an article for the local newspaper.

9 Consider the following test results: 7, 5, 6, 8, 3

(a) Do you expect the mean to be an actual data value? Explain your answer.

(b) Find the mean of these results. Is the mean an actual data value?

(c) Do you expect the median to be an actual data value? Explain your answer.

(d) Find the median for the test results above.

10 **(a)** Explain what is meant by the statement 'The average family has 2.3 children'.

(b) Explain why the median of a group of whole-number values can only ever either end in .5 or be a whole number.

(c) If the data values are all whole numbers, can the mode ever be a decimal? Explain why or why not.

11 A small company has a manager, an assistant manager, two office workers and 10 factory workers. The manager is paid $125 000 per year, the assistant manager $85 000, the office workers $55 000 each, and the factory workers $50 000 each.

(a) Find the **(i)** mean, **(ii)** median, **(iii)** mode and **(iv)** range for the annual income of all the people in the company, rounding your answer to the nearest cent.

(b) Which measure of centre gives you the best idea about the salaries paid by this company?

(c) Omit the manager's salary and recalculate the **(i)** mean, **(ii)** median, **(iii)** mode and **(iv)** range.

(d) Comment on what you found.

12 The following data set appears to have two outliers, one at the start of the data and one at the end.

3 10 12 12 14 14 15 16 17 18 18 19 28

We have already seen that one outlier affects the mean and the range, but what happens here where there are two outliers?

(a) Calculate the mean, median and range for the data set.

(b) Recalculate these values after changing the 3 to 10 and the 28 to 19.

(c) What conclusions can you draw from your calculations?

13 As well as giving a set of results, make sure you include a short explanation of how you found your answers for each of the following.

(a) Find five test results for which the mean, median and mode are all equal to 6 if the test results are not all the same.

(b) Find five test results for which the mean and the median are 7 and the mode is 9.

(c) Find five test results for which the mean is 7 and the median and mode are 6.

14 Three students are finding the median of the following data values.

20, 17, 28, 23, 19, 21, 22, 20

Sam says 'The median is the fourth value, so is 23.'

Veran says 'No, you need to put the data values in order first, so the median is 20.'

Lakme says 'No, you are both wrong.'

Help Lakme explain to Sam and Veran why they are incorrect.

Problem solving

Running the race

1 In a sprint, the first three runners had times of 12.1, 12.1 and 10.9 seconds. Find the mean, median and mode for the three times.

2 In a different race, the first three runners had a mean time of 11.7 seconds, a median time of 10.8 seconds and a modal time of 10.8 seconds. Find the times for the three runners.

3 In a third race, the first three runners had a mean time of 11.8 seconds and a median time of 11.9 seconds. If all three times are different what can you say about the first and third placegetters?

Give at least three pairs of possible times for these two runners, using one decimal place accuracy in your solutions. Explain how you found your answers.

Strategy options

- Work backwards.
- Test all possible combinations.

9.3 Graphing single variable data

Need to Know

A **dot plot** is a statistical graph where, for each data value, a dot is placed on a scale. A dot plot is useful in deciding on outliers.

A **stem-and-leaf plot** consists of two parts, a stem and a leaf. Each piece of data must be divided into two parts so it can be plotted. For example, a score of 23 would be written as:

STEM	LEAF
2	3

All **column graphs** should include:

1. a title
2. categories or a numerical scale with equally spaced intervals (gaps) between the numbers on the horizontal axis
3. a numerical scale with equally spaced intervals (gaps) between the numbers on the vertical axis
4. a label for the horizontal axis (x-axis) that explains the variable being represented
5. a label for the vertical axis (y-axis) that shows the frequency
6. a small gap between each column (usually half a column width) to show that this is categorical or discrete (counted) data.

Bar graphs are the same as column graphs except the bars are horizontal.

Column or bar graphs should be used for categorical data and for ungrouped discrete data.

All **frequency histograms** should include:

1. a title
2. a continuous scale on the x-axis for the variable being represented
3. a numerical scale with equally spaced intervals between the numbers on the y-axis
4. a label for the horizontal axis that explains the variable that is being represented
5. a label for the vertical axis that shows the frequency
6. no gap between each column
7. a half-column width before the first column even if the first set of data starts at zero.

Histograms should be used for continuous data.

A **frequency polygon** is the graph made by joining the midpoints of the tops of the columns of a frequency histogram. The polygon is completed by joining the first point to the horizontal axis half a column width to the left of the first column and the last point to the horizontal axis half a column width to the right of the last column.

A frequency polygon is useful when comparing data sets as multiple frequency polygons can be drawn on the same axes.

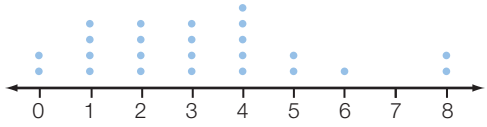
Worked Example 7

WE7

Draw a dot plot to illustrate the following data that represents the number of pets owned by each student in a class.

2, 5, 8, 6, 2, 4, 3, 4, 2, 1, 8, 3, 4, 0, 3, 1, 0, 3, 5, 1, 4, 2, 1, 4

Working



Worked Example 8

WE8

A class obtained the following percentage results (rounded to the nearest whole number) in a science test.

Express the following data as an ordered stem-and-leaf plot with a class interval of 10.

69, 54, 91, 42, 73, 75, 81, 96, 88, 76, 65, 83, 74, 76, 82, 61, 57, 64, 58, 79, 83, 90, 73, 60, 51

Working

STEM	LEAF
4	2
5	4 7 8 1
6	9 5 1 4 0
7	3 5 6 4 6 9 3
8	1 8 3 2 3
9	1 6 0

(unordered)

Key: 5 | 4 = 54

STEM	LEAF
4	2
5	1 4 7 8
6	0 1 4 5 9
7	3 3 4 5 6 6 9
8	1 2 3 3 8
9	0 1 6

(ordered)

Worked Example 9

WE 9

Calculate the median and range for each of the following data sets. Comment on whether there are any possible outliers in the data.

(a) The students in a class were asked to indicate the number of brothers and sisters they have. These are the results obtained:

1 1 2 5 2 3 2 0 0 4 2 3 9 1 0 2 1 4 1 5 3 2 0

Draw a dot plot and use it to help you answer the question.

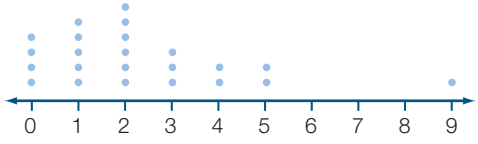
(b) A new display at a museum attracts varying numbers of people. A survey was conducted to count the number of people visiting the attraction each day. These are the results obtained for the first 20 days:

235 195 156 188 177 165 189 190 177 166
199 186 165 170 169 173 184 177 192 200

Draw an ordered stem-and-leaf plot and use it to help you answer the question.

Working

(a)



Median = 2

Range: 9 − 0 = 9

It would seem that 9 is an outlier for this data set.

(b)

STEM	LEAF
15	6
16	5 5 6 9
17	0 3 7 7 7
18	4 6 8 9
19	0 2 5 9
20	0
21	
22	
23	5

Key: 15 | 6 = 156

$$\text{Median} = \frac{177 + 184}{2}$$

$$= 180.5$$

Range: 235 − 156 = 79

It would seem that 235 is an outlier for this data set.

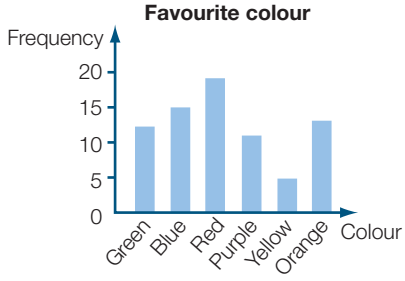
Worked Example 10

WE10

The frequency table below shows the responses from a group of children who were asked 'What is your favourite colour?' Only one response per student was recorded. Draw a column graph to represent this information.

Colour	Green	Blue	Red	Purple	Yellow	Orange
Frequency	12	15	19	11	5	13

Working



Worked Example 11

WE11

100 Year 7 students were surveyed about the number of hours they spent watching TV over the weekend. The data is presented below.

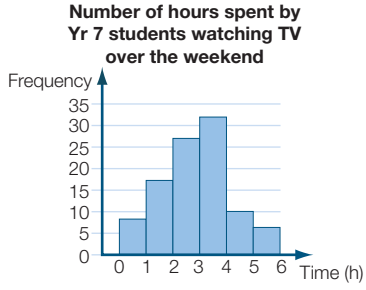
Hours spent watching TV over the weekend	0–<1	1–<2	2–<3	3–<4	4–<5	5 or more hours
No. of students	8	17	27	32	10	6

(a) Draw a frequency histogram to represent this data.

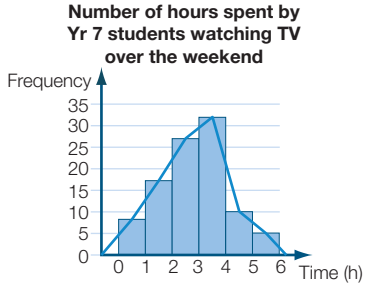
(b) Draw a frequency polygon over the top of your frequency histogram.

Working

(a)



(b)



Answers page 496

9.3 Graphing single variable data

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q14	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q13, Q14	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14

Fluency and understanding

WE7

1 Draw a dot plot to illustrate the following data that represents the number of birds counted by a bird watcher in the bush in 15-minute intervals over the course of 6 hours.

12, 18, 12, 16, 19, 13, 19, 20, 17, 15, 20, 21, 18, 15, 20, 18, 21, 21, 17, 12, 17, 19, 12, 21

WE8

2 A class obtained the following percentage results (rounded to the nearest whole number) in a science test.

Express each of the following data as an ordered stem-and-leaf plot with a class interval of 10.

(a) 57, 48, 42, 41, 31, 22, 37, 30, 23, 28, 34, 52

(b) 25, 44, 15, 29, 37, 26, 33, 35, 14, 8, 2, 27, 30, 42, 34, 21, 18, 3

WE9

3 Calculate the median and range for each of the following data sets. Comment on whether there are any possible outliers in the data.

(a) The students in a class were asked to indicate their shoe size. These are the results obtained:

5, 5.5, 7, 4.5, 4.5, 6, 10, 5.5, 4, 4, 5.5, 4.5, 6.5, 7.5, 6, 5.5, 4.5, 4, 6.5

Draw a dot plot and use it to help you answer the question.

(b) The number of points scored by each of the teams in the opening round of the local Australian Rules football competition was recorded. These are the results obtained:

110, 75, 88, 56, 59, 89, 112, 115, 121, 99, 67, 88, 77, 88, 100, 55

Draw an ordered stem-and-leaf plot and use it to help you answer the question.

WE10

4 The frequency table below shows the colour of cars passing the school in a particular timed period. Draw a column graph to represent this information.

Colour	White	Red	Green	Blue	Silver	Other
Frequency	42	6	5	12	18	17

WE11

5 The owner of a car dealership wants a graph of the number of cars sold plotted against the value of the cars sold in the last year. The data is presented below.

Value of car	Under $10 000	$10 000– < $20 000	$20 000– < $30 000	$30 000– < $40 000	$40 000– < $50 000	$50 000 or more
Number of cars sold	25	73	132	43	18	9

(a) Draw a frequency histogram to represent this data.

(b) Draw a frequency polygon over the top of your frequency histogram.

6 Express the following data as ordered stem-and-leaf plots with class intervals of 5.

(a) 23, 28, 34, 27, 38, 42, 31, 21, 22, 27, 20 (Use 2_2, 2_4, 3_2, 3_4 etc. as the stems.)

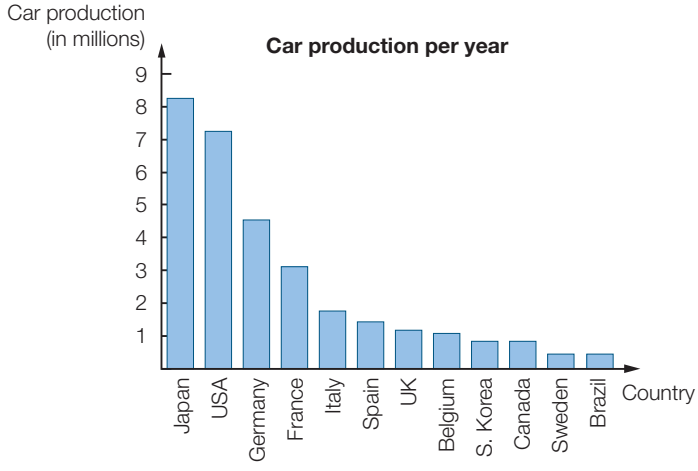
(b) 48, 57, 56, 53, 68, 64, 47, 58, 52, 51, 61, 42, 47 (Use 4_2, 4_4, 5_2, 5_4 etc. as the stems.)

7 The number of chocolate bars produced at a factory every hour over a 20-hour period is 146, 128, 133, 138, 140, 142, 125, 129, 136, 130, 133, 142, 126, 129, 150, 134, 126, 121, 128, 143.

(a) Using class intervals of 5, produce an ordered stem-and-leaf plot of the data.

(b) Draw an ordered stem-and-leaf plot of the data if it is expressed in intervals of 10.

8 This column graph shows the numbers of cars produced in a year by the world's major car-producing countries.



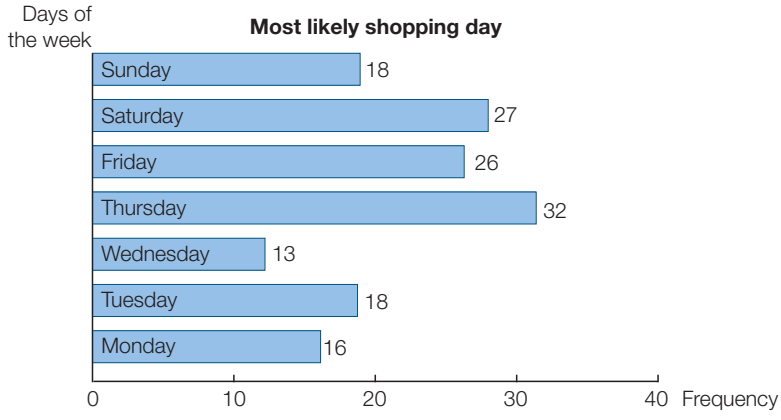
(a) Which country produces the most cars?

A USA **B** Germany **C** Japan **D** Brazil

(b) The number of cars produced by Germany is:

A 4.5 **B** 450 000 **C** 4 500 000 **D** 45 000 000

9 The bar graph at right shows the results for a survey of 150 people as to which day they were most likely to do their supermarket shopping. The actual numbers are included on the graph to make it easier for you to calculate your answers.

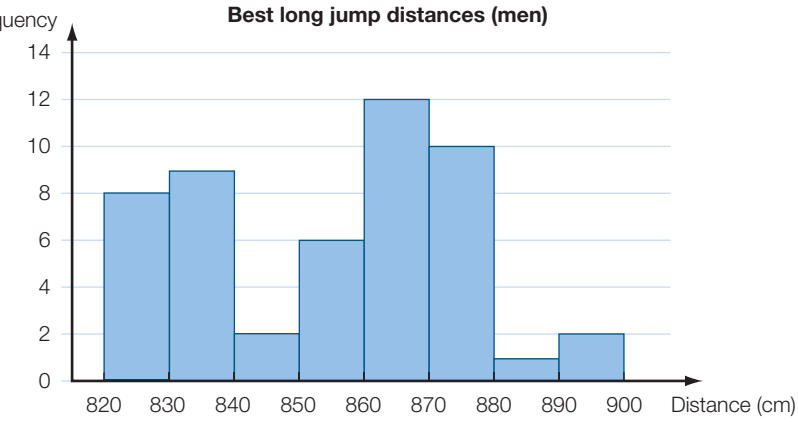


(a) Which day was most popular?

(b) How many people chose the least popular day?

(c) How many people chose the weekend?

10



The frequency histogram above shows the best long jump distances for men recorded from 1960 to 2009.

(a) What is the class interval?

(b) What is the range of the recorded distances?

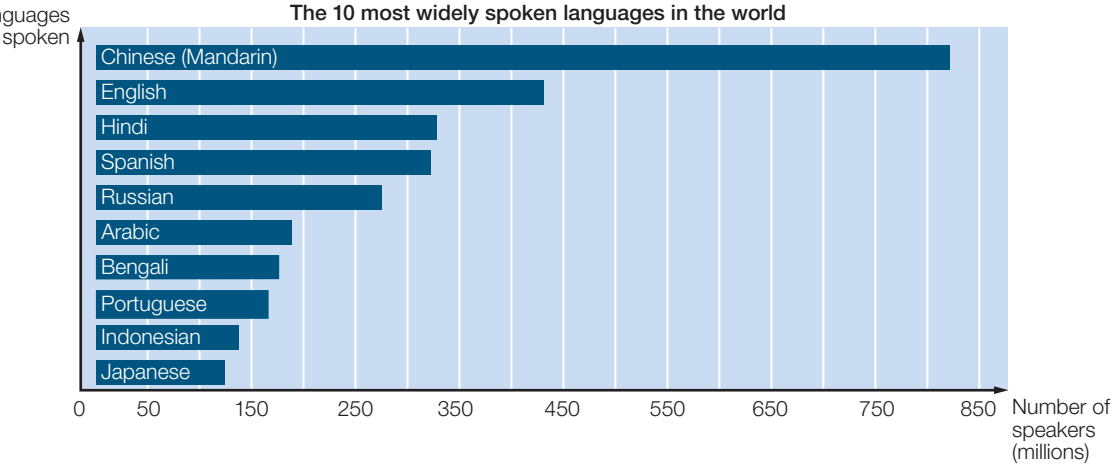
(c) Which class interval had six recorded jumps?

(d) How many jumps over 870 cm were recorded in this time period?

(e) How many jumps have been recorded? (This will be the sum of all the frequencies.)

Reasoning, communicating and open-ended

11



The horizontal bar graph above shows the 10 most widely spoken languages in the world.

(a) What are the differences between this and a column graph?

(b) Approximately how many people in the world speak Mandarin?

(c) Approximately how many people in the world speak English?

(d) What is the fourth most-spoken language in the world?

(e) What is unusual about the scale at the bottom of the graph?

(f) Could this have been drawn as a column graph?

12 A class of Year 7 students were asked how long it took them to get to school each day.

The times, to the nearest minute, are listed below:

34, 13, 26, 38, 23, 15, 19, 30, 28, 40, 36, 25, 14, 7, 10, 11, 8, 24, 32, 69, 29, 32, 44, 21, 18, 22

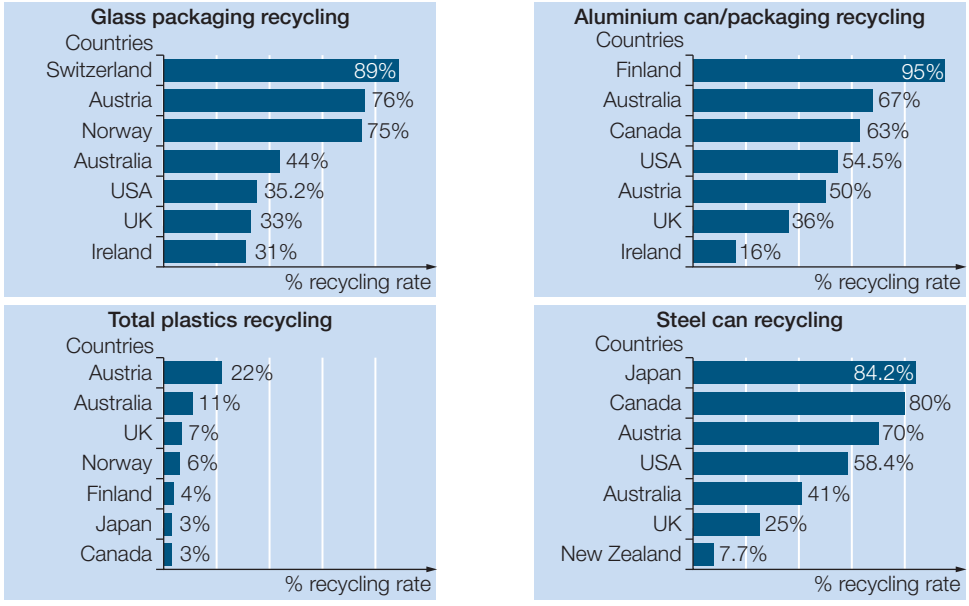
(a) Is this discrete or continuous data?

(b) Group the data with appropriate class intervals.

(c) Draw an appropriate graph to represent the data.

(d) Is there a possible outlier? Why or why not?

13 Compare the four recycling graphs to answer the following questions.



(a) Does Australia or Canada have a better recycling rate for aluminium?

(b) Does Australia or Canada have a better recycling rate for steel cans?

(c) Australia, the UK and Austria appear in all four graphs. Which country seems to be the worst recycler of the three?

(d) Which kind of recycling does Australia seem to be best at?

(e) Which kind of recycling does Australia seem to be worst at?

(f) Compare Australia's recycling rate in all four areas with the best recycling countries on the graphs given.

(g) Compare Australia's recycling rate in all four areas with the worst recycling countries on the graphs given.

14 Collect some **(a)** categorial, **(b)** discrete and **(c)** continuous data from the students in your class and draw an appropriate graph to represent each.

9.4 Comparing parts of a whole with graphs

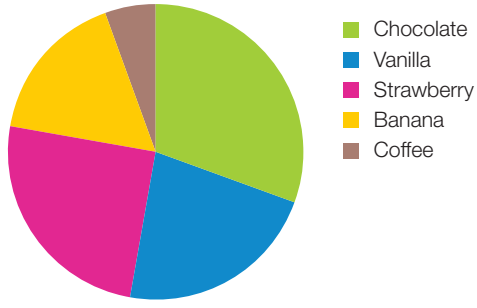
Need to Know

Sector graphs and divided bar or column graphs are used to show the various parts of a whole.

Sector graphs

Drawing sector graphs:

1. Add the various parts to find a total.
2. Divide each part by the total to obtain fractional parts of a whole.
3. Multiply each fractional part by 360° to find the sector angle.
4. Use a protractor to divide a circle into the various parts and draw your graph.
5. Sector graphs should have titles and a key that identifies each of the sectors.



Flavours of ice-cream bought at the beach café on Saturday

Divided bar or column graphs

Drawing divided bar or column graphs:

1. Draw a rectangle, making the length a multiple or a factor of the total.
2. Multiply each data value by the same factor or multiple to find the length of each section.



Flavours of ice-cream bought at the beach café on Saturday

Chocolate
Vanilla
Strawberry
Banana
Coffee

Technology Exploration: Excel

Drawing sector graphs and divided bar graphs

You can use a spreadsheet such as Excel to draw sector graphs and divided bar graphs. The process is illustrated using data from WE12.

Enter the data in the same way as it is shown in the worked example.

Highlight the data and select Insert > Pie and then choose the first of the options displayed.

This will produce a very simple sector graph quite like the one shown in Worked Example 12.

Now select Insert > Bar and choose the middle option on the top line (Stacked Bar) and then select Switch Row/Column from this same menu bar. You will now have a divided bar graph much like the one in the worked example.

Worked Example 12

WE12

All Year 7 students at a certain school were surveyed about their favourite type of pet and the responses were recorded opposite.

For the data in the table:

(a) draw a sector graph

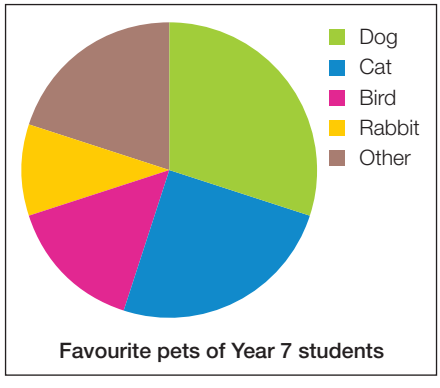
(b) draw a divided bar graph.

Type of pet	Number recorded
Dog	78
Cat	65
Bird	39
Rabbit	26
Other	52

Working

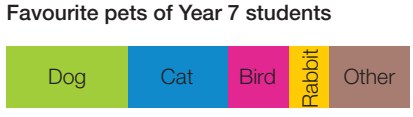
(a)

Type of pet	Number recorded	Fraction of a whole	Sector angle (°)
Dog	78	$\frac{78}{260}$	108
Cat	65	$\frac{65}{260}$	90
Bird	39	$\frac{39}{260}$	54
Rabbit	26	$\frac{26}{260}$	36
Other	52	$\frac{52}{260}$	72
Total	260	1	360



(b)

Type of pet	Number recorded	Length of line (mm)
Dog	78	39
Cat	65	32.5
Bird	39	19.5
Rabbit	26	13
Other	52	26
Total	260	130



Answers page 497

9.4 Comparing parts of a whole with graphs

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q12	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q11, Q12	Q1, Q2, Q3, Q4, Q6, Q8, Q9, Q10, Q11, Q12, Q13

Equipment required: Protractor for Questions **8**, **10** and **11**; computer with Microsoft® Excel® software or similar may be used for Question **13**

Fluency and understanding

WE12

1 For the data in each of the following tables:

(i) draw a sector graph **(ii)** draw a divided bar graph.

(a) Belinda picked some vegetables from her garden and recorded the number of each type of vegetable.

Vegetable	Number recorded
Tomatoes	20
Carrots	12
Cucumbers	8
Lettuce	10

(b) A group of tourists was asked to state their nationality and their responses were recorded.

Nationality	Number recorded
Italian	22
French	18
Japanese	12
Indonesian	8

(c) The number of pizzas purchased one evening at Abracadabra Pizza Parlour was recorded.

Pizzas	Number recorded
Super Supreme	6
Capricciosa	8
Hawaiian	4
Italiana	3
Australiana	2

(d) The number of each make of car in the staff car park at a small secondary school was recorded.

Car make	Number recorded
Ford	4
Holden	5
Mazda	4
Toyota	3
Mitsubishi	2
Other	5

2 For each of the following data tables:

(a) draw a sector graph

(b) draw a divided bar graph.

(i) Fifty people were asked their favourite music style.

Music style	Rock & Roll	Hip Hop	Techno	Rap	Punk	Pop
Number	10	9	6	3	5	17

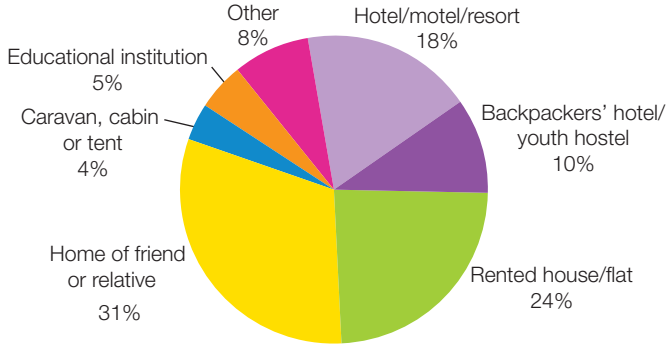
(ii) Seventy-five people were asked their favourite vegetable.

Vegetable	Potato	Carrot	Pumpkin	Cabbage	Capsicum	Cauliflower
Number	33	22	9	4	4	3

(iii) Justin has been working in a full-time job for 6 months. He was asked how he spent his money. He gave the following estimates.

Item	Rent	Food	Entertainment	Clothes	Other
Percentage	47	30	10	10	3

3 The sector graph below represents information about the type of accommodation used by overseas visitors to Australia.



(a) One of the sectors uses an angle of 64.8°. Which one is it?

A Hotel/motel/resort **B** Backpackers' hotel/youth hostel

C Rented house/flat **D** Other

(b) The Educational institution sector uses an angle closest to:

A 5° **B** 10° **C** 13° **D** 18°

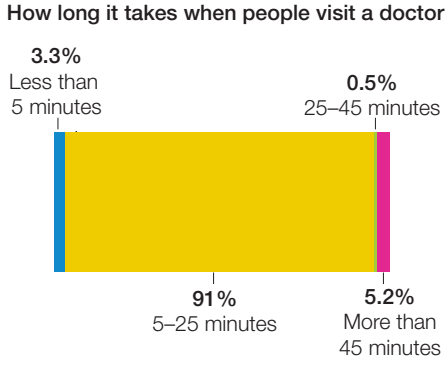
4 As part of a project to tackle global warming at a local level, a Victorian local council conducted a survey to see what percentage of greenhouse gas emissions was produced by each of its facilities and services. The following data was recorded: Buildings 28%, Streetlights 47%, Waste 1%, Vehicle fleet 24%.

(a) Create a sector graph to display the data.

(b) Create a divided bar graph to display the data.

(c) Which graph do you find easier to understand?--

5 The following graph shows how long people take when they visit a doctor.



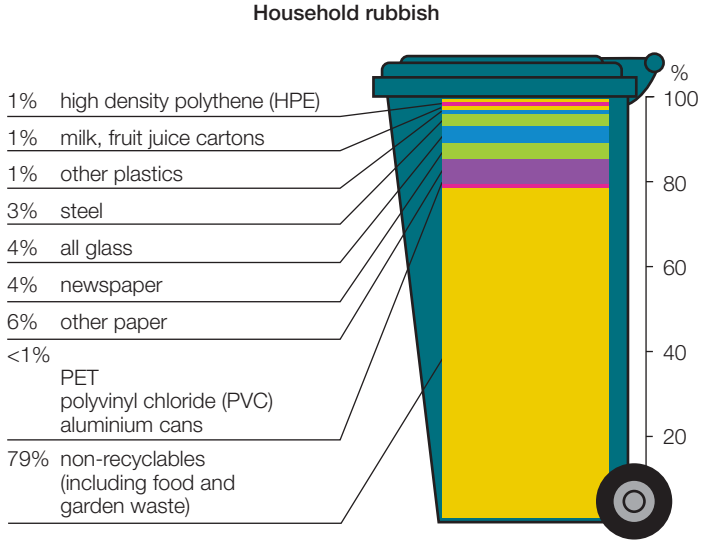
(a) What percentage of people spend less than 5 minutes when they visit a doctor?

(b) What percentage of people spend more than 45 minutes when they visit a doctor?

(c) How long do the majority of people take when they visit the doctor?

(d) What percentage of people spend less than 25 minutes when they visit the doctor?

6 Look at the following divided bar graph, which shows how much of different types of rubbish Australians throw out.



(a) What percentage of our rubbish is all glass?

(b) What percentage of our rubbish is milk and fruit juice cartons?

(c) What percentage of our rubbish is newspaper?

(d) **(i)** What is the largest category of rubbish?

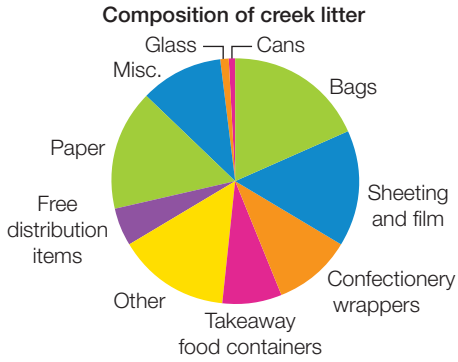
(ii) What percentage of rubbish is in this category?

(e) Do we throw out more high density polythene or more paper?

(f) Do we throw out more steel or more glass?

(g) Is the picture of the rubbish bin actually part of the graph?

7 Look at the following sector graph showing the type of litter in a particular creek.



Total number of litter items collected = 2231

(a) What was the most common type of litter?

(b) Were there more cans or more sheeting and film?

(c) Were there more takeaway food containers or more confectionery wrappers?

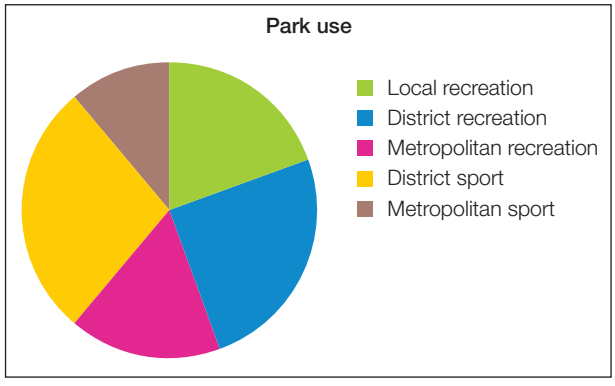
(d) When you compare this sector graph with the one in Question **3**, what is missing?

(e) Misc. stands for miscellaneous. What kinds of rubbish would be classified as miscellaneous?

(f) We are told how many litter items were collected in the survey. Is this important? Why or why not?

(g) Why are sector graphs like divided bar graphs?

8



Above is a sector graph showing park use in a local council area of an Australian city.

(a) Use a protractor to find the angle size of each sector.

(b) Use your answer from part **(a)** to find the percentage of the park that is used for metropolitan recreation.

(c) Use your answer from part **(a)** to find the percentage of the park that is used for district sport, to the nearest whole number.

(d) Use your answer from part **(a)** to find the percentage of the park that is used for sport, to the nearest whole number.

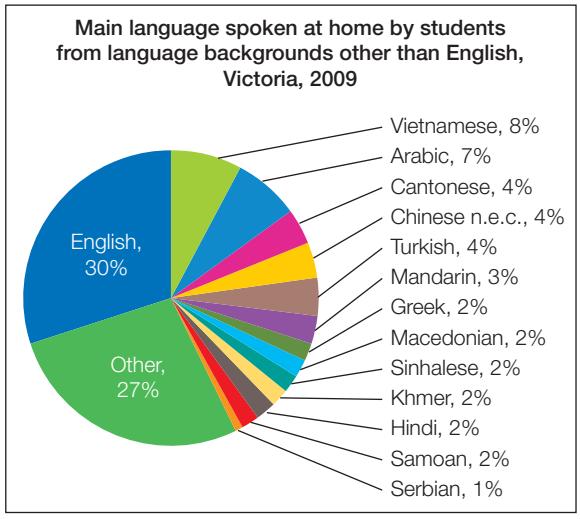
(e) Is more parkland used for sport or for recreation? State which is larger and by how much.

(f) What type of park use is represented by the smallest sector?

(g) Draw a divided bar graph of the data.

Reasoning, communicating and open-ended

9 Using the graph below, answer the following questions.



(a) What is the main language spoken at home by students from non-English-speaking backgrounds?

(b) What is the total percentage of students whose main language spoken at home is some form of Chinese (Cantonese, Mandarin or Chinese n.e.c. (not elsewhere classified))?

(c) List all the languages that are spoken in 2% of students' homes.

(d) Do more students speak Turkish or Arabic at home?

(e) Why is the 'Other' section so large?

(f) Suggest some of the languages in the 'Other' sector.

10 The following table shows vehicle sales for Australia in 2008, divided by state or territory.

State/Territory	Number
ACT	5 462
New South Wales	106 250
Northern Territory	3 756
Queensland	75 228

State/Territory	Number
South Australia	21 492
Tasmania	5 940
Victoria	89 826
Western Australia	39 560

(a) How many vehicles were sold in total in 2008?

(b) Find the sector angles that represent each state or territory (correct to the nearest degree).

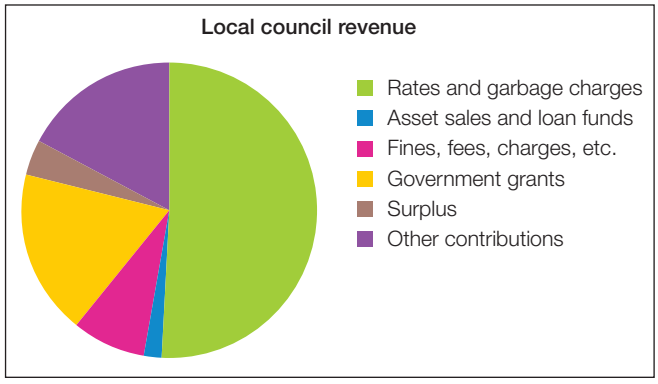
(c) Using a protractor, draw a sector graph for this information, clearly labelling each sector.

(d) If you were to draw a divided bar chart for this information, how long would you make your graph?

(e) Use this length to draw the divided bar chart and clearly show the length of each sector, correct to the nearest mm.

(f) Which graph do you think is easier to draw? Give a brief explanation to support your point of view.

11



The sector graph above shows the revenue (money) a local council expected to receive in the 2010/2011 financial year. Use this graph to answer the following questions.

(a) What is the major source of income for the council?

(b) Use a protractor to find the sector angle for the fines, fees, charges, etc. sector.

(c) Use the angle you found in part **(b)** to find the percentage of revenue (to the nearest %) raised through fines, fees, charges, etc.

(d) Does the council expect to receive more money from government grants or from other contributions? Explain your answer.

12 **(a)** Draw a sector graph to represent one characteristic of the students in your class.

(b) Draw a divided bar graph to represent a different characteristic of the students in your class.

13 Generate sector graphs for Question **10** using Excel (or similar software) if you have access to a computer. Compare your graphs with the computer-generated ones (you could use 3D graphs also) and comment on any differences.

Problem solving

Missing pieces of the pie

1 A sector graph has been drawn and labelled, but there is a 60° gap in it. This is to be divided between two pieces, *A* and *B*, so that *A* is twice the size of *B*. How big will the two pieces be? Give your answers correct to one decimal place if necessary.

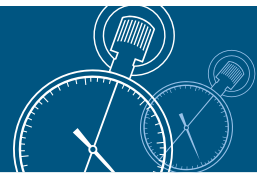
2 This time, the 60° piece is to be divided between three pieces, *A*, *B* and *C*, where *A* is twice as big as *B* and *B* is twice as big as *C*. How big will the three pieces be? Give your answers correct to one decimal place if necessary.

3 This time, the 60° piece is to be divided between four pieces, *A*, *B*, *C*, and *D*, where *A* is twice as big as *B*, *B* is twice as big as *C* and *C* is twice as big as *D*. How big will the four pieces be? Give your answers correct to one decimal place if necessary.

Strategy options

- Guess and check.
- Look for a pattern.

Half-time 9



Ex 9.2

1 Find the **(i)** mean (correct to one decimal place) **(ii)** median, **(iii)** mode and **(iv)** range for each of the following sets of test results.

(a) 3, 2, 4, 8, 6, 2, 4, 6, 3, 2, 1 **(b)** 4, 6, 3, 4, 3, 6, 5, 9, 8, 7, 6, 3, 2, 4, 5, 6, 4

Ex 9.4

2

Victoria's greenhouse gas emissions (end use allocation of emissions)	
Residential	17%
Commercial	16%
Manufacturing	32%
Passenger transport	14%
Primary production net	12%
Freight transport	5%
Waste	3%
Other	1%

Using the table given above:

(a) draw a sector graph **(b)** draw a divided bar graph.

Ex 9.1

3 The height of each of the 25 students in a class was recorded.

142	147	151	142	146	151	157	138	141
140	129	135	142	151	150	160	141	152
148	157	130	143	147	149	154		

(a) Why are class intervals of 125–129, 130–134, etc. appropriate?

(b) Using these class intervals, draw a frequency table showing this information.

Ex 9.1

4 The number of students absent from class was recorded for each of the 21 school days in a month.

3	0	1	2	4	1	0	1	0	2	0
1	0	0	2	1	3	0	1	2	0	

Construct a frequency table showing this information.

Ex 9.3

5 Draw a dot plot to illustrate the following data that represents the number of times the students in a class have travelled interstate.

6	4	0	1	2	3	1	2	2	0	0	0
1	2	4	5	3	2	5	3	0	1	1	2

Ex 9.3

6 The number of points scored by the top 12 basketball players are given below.

86 74 76 91 132 84 67 80 82 62 85 90

(a) Draw a stem-and-leaf plot to represent this data.

(b) Find the median for this data set.

(c) Can you identify any possible outliers? Justify your answer.

Line graphs

9.5

Need to Know

All **line graphs** should include:

1. a title
2. the independent variable always on the horizontal axis
3. a scale across the horizontal axis that uses equally spaced intervals and is clearly labelled, including any relevant unit of measurement
4. a scale on the vertical axis that also uses equally spaced intervals and is clearly labelled, including any relevant unit of measurement.

Line graphs link two variables, an independent variable and a dependent variable. The independent variable is often time.

Worked Example 13

WE13

The mean daily maximum temperature (°C) for each month in Hobart (Botanical Gardens) is shown in the table below:

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mean max. temp. (°C)	23.1	22.2	20.7	17.0	13.9	11.4	10.5	12.2	14.9	17.4	19.4	21.9

Bureau of Meteorology

(a) Draw a line graph to represent this data.

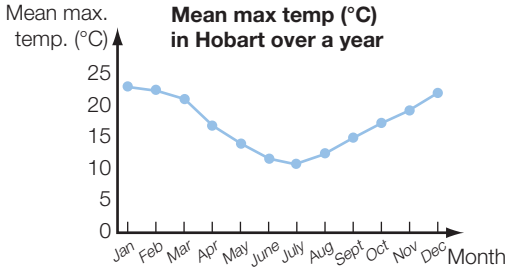
(b) For which months was the mean maximum temperature below 17°C?

Working

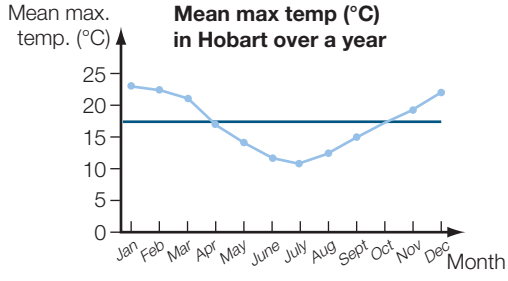
(a) Highest value = 23.1°

$$\text{Approximate scale} = \frac{23.1}{10} = 2.31$$

Scale 1 cm = 2°C



(b)



May, June, July, August, September

Technology Exploration: Excel

Drawing line graphs using a spreadsheet

You can use a spreadsheet such as Excel to draw line graphs. The process will be illustrated using the data from Worked Example 13.

Enter the data the way it is displayed in the worked example.

Highlight the data and select Insert > Line and then choose the first of the options displayed.

This will produce a very simple line graph quite like the one shown in Worked Example 13. You can smarten up the appearance of your graph to make it more 'mathematical' but we will not go into that here. You could explore these options for yourself.

If you wanted to draw the horizontal line on the graph, the easiest way would be to add another row to the table of values with 17 as each entry. Following the same process as above, you will then get a graph like the answer to (b) in the worked example.

Answers page 499

9.5 Line graphs

Navigator

Q1, Q2, Q3, Q4, Q5, Q8, Q9, Q10	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10

Fluency and understanding

WE13

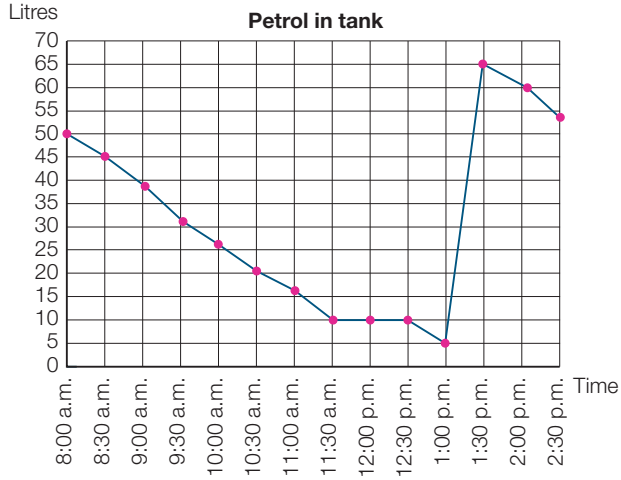
1 At an athletics competition, the wind speed is measured so that new records can be accepted. The following table shows the wind speed (metres per second) at 10-minute intervals for the hour over which the 100 m sprints were run.

Time	1:00 p.m.	1:10 p.m.	1:20 p.m.	1:30 p.m.	1:40 p.m.	1:50 p.m.	2:00 p.m.
Wind speed (metres per second)	0.9	0.95	1.3	1.2	1.15	1.1	0.95

(a) Draw a line graph to represent this data.

(b) When was the wind speed greater than 0.95 metres per second?

2 The graph shows the number of litres of petrol in a car at various times during the day.



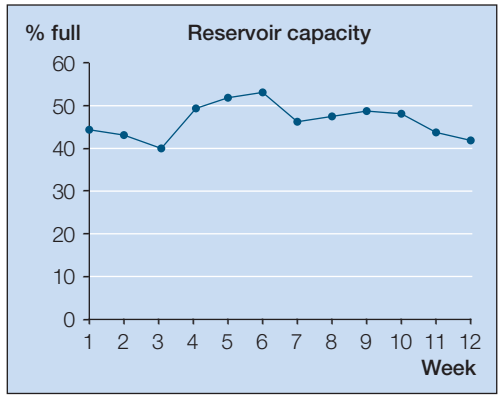
(a) How many litres were in the tank at the start of the day?

(b) When did the driver visit the petrol station?

(c) What was the minimum number of litres of petrol purchased?

(d) Between which times is it likely the driver had lunch?

3 Each week, the amount of water in a reservoir is recorded as a percentage of its maximum capacity. The graph on the right shows the results for a number of weeks.



(a) How full was the reservoir when measured in Week 4?

A 40% **B** 45%

C 50% **D** 55%

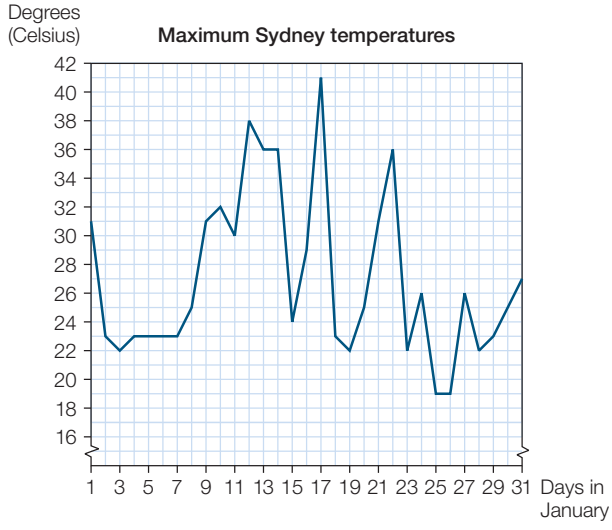
(b) There was heavy rain during a particular week. The measurement for which week shows the result of this rain?

A 2 **B** 4 **C** 5 **D** 6

(c) A large quantity of water was released from the reservoir to flush out the downstream areas. The measurement for which week most likely shows the result of this action?

A 2 **B** 3 **C** 7 **D** 10

4 The line graph opposite shows the maximum temperatures in Sydney for January of a particular year.



(a) **(i)** What was the highest maximum temperature reached?

(ii) On which date was this temperature reached?

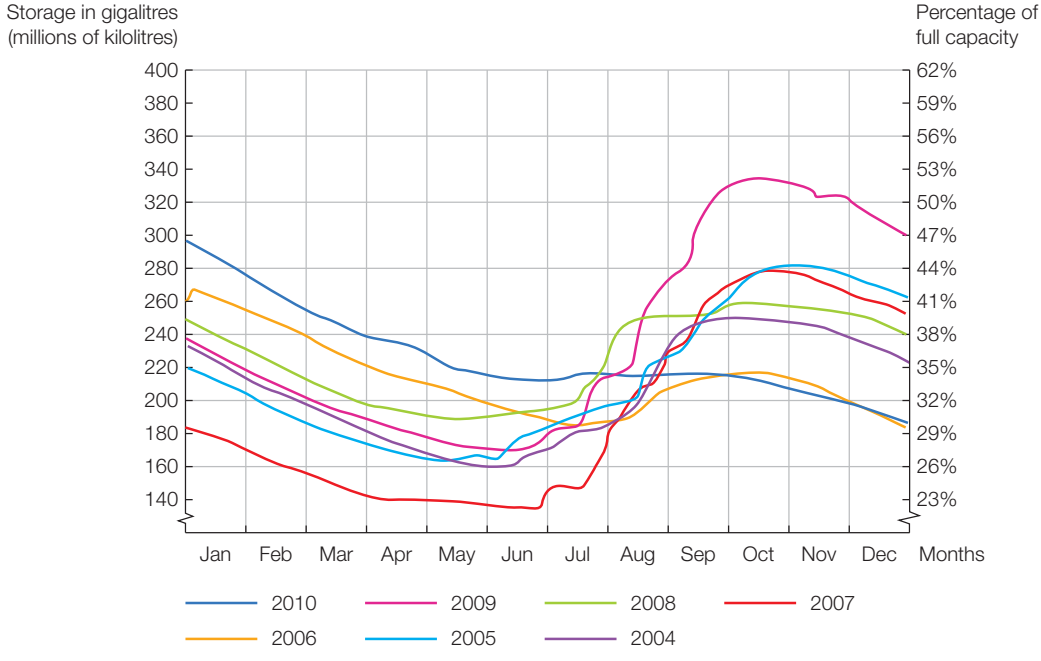
(b) **(i)** What was the lowest maximum temperature reached?

(ii) On which date(s) was this temperature reached?

(c) Up until 15 January, the maximum temperature was always between which two values?

(d) After 23 January, the maximum temperature was always between which two values?

5



Above is a graph of Perth's water storage from 2004 to 2010. Use this graph to answer the following questions.

(a) What was the storage percentage at the start and end of 2008?

(b) In which year was there almost no change in storage levels from the beginning of June to the end of September?

(c) Which 6-month period over the years 2004–2010 had the lowest water storage? What does this tell you about the rainfall in that period?

(d) By what percentage did the water storage levels decrease in December 2009? Check the percentage drop in water storage levels for other years in December. What does this tell you about the rainfall in December in Perth?

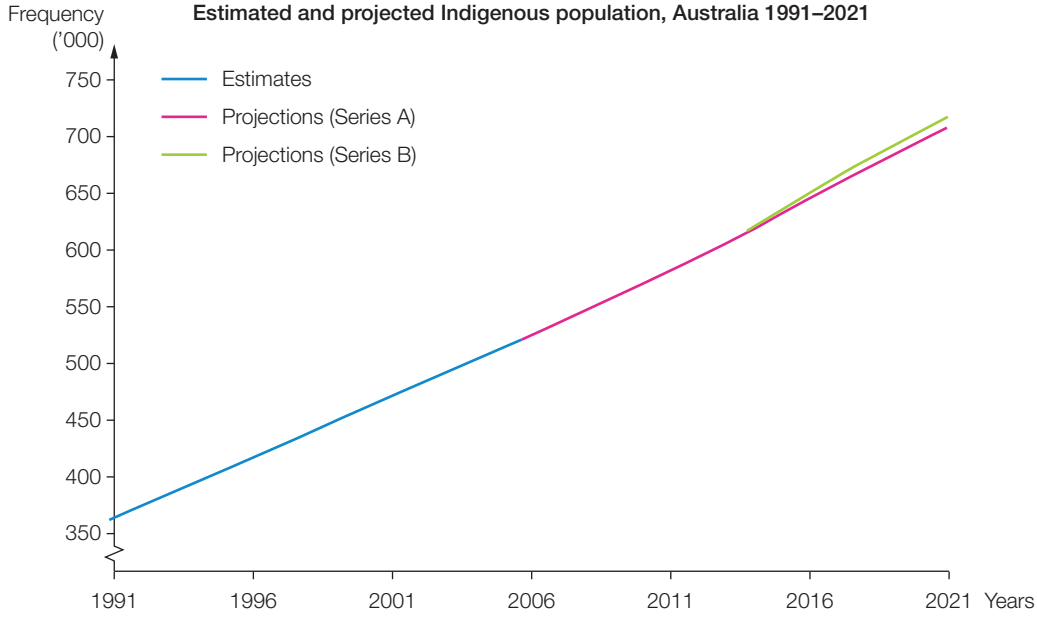
(e) Did the water storage levels increase or decrease over the year 2009? Find the increase or decrease percentage.

(f) From the data shown here, in which month is Perth likely to be the wettest?

(g) From the data shown here, in which month is Perth likely to be the driest?

Reasoning, communicating and open-ended

6 This graph shows the Indigenous population of Australia from 1991 to 2006 and the projected (estimated) population for 2006 to 2021. Different projections are made using different estimation methods for future populations.



(a) What was the estimated Indigenous population of Australia in 1991?

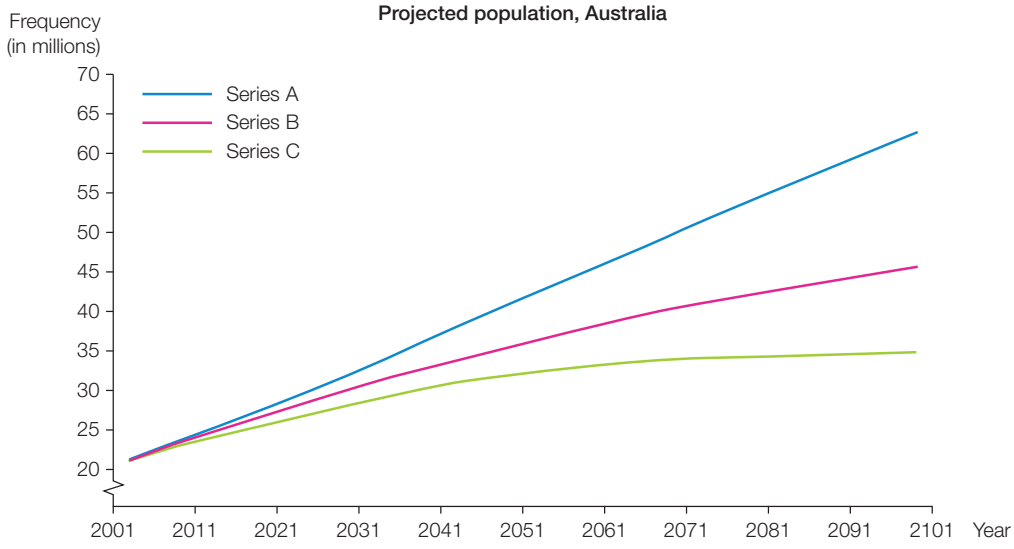
(b) What was the estimated Indigenous population of Australia 15 years later?

(c) Why do you think the graph from 1991 to 2006 is an estimated graph?

(d) What are the two projected values for the Indigenous population in 2021?

(e) Give two reasons why you think the Indigenous population has increased so much in the 30 years from 1991 to 2021.

7 Statisticians use data from the past to predict trends for the future. Below are three graphs that show different ways of predicting (or projecting) the future population of Australia.



(a) Use the graph to find the population of Australia in 2007.

(b) What is the projected population of Australia in 2041 on the Series A graph?

(c) Find the increase in population predicted over the time period shown on the Series A projection.

(d) What are the two main factors that would cause the population to increase according to the Series A projection?

(e) List at least two problems Australia would face if the Series A projection was correct.

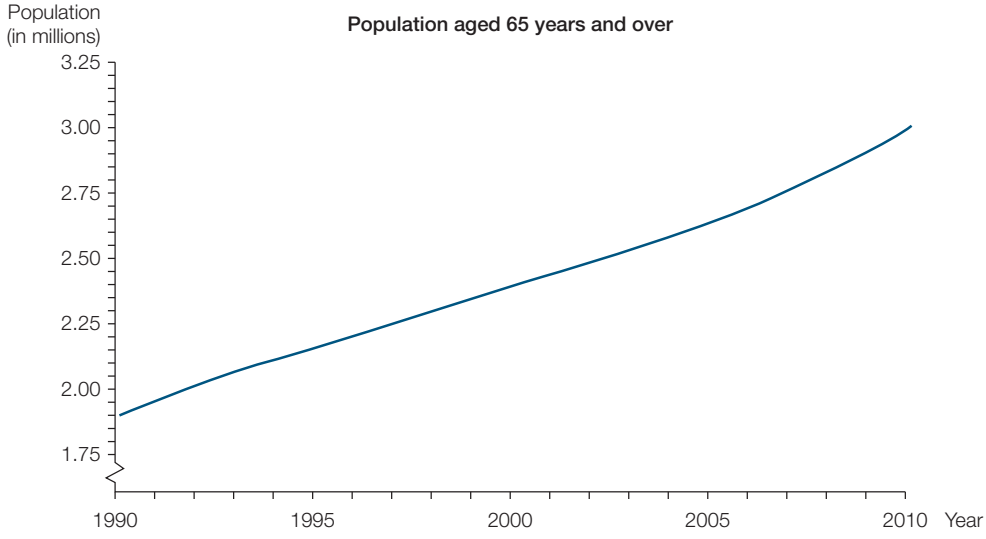
(f) What would be the projected population of Australia in 2061 on the Series B projection?

(g) If the number of births equals the number of deaths in the Series B projection, why would the population continue to grow?

(h) The Series C projection flattens out by 2081. What would be the projected population over the period 2081–2101?

(i) What needs to happen if the population of Australia is to stay stable (the same year after year)?

8



This line graph shows the number of Australians 65 years and older in the time period 1990–2010.

(a) In 2000, approximately how many Australians were 65 years or older?

(b) In which year did the number of Australians 65 years and older reach 2.75 million?

(c) Between which years does the graph follow an almost perfectly straight line? (Use a ruler to help you.)

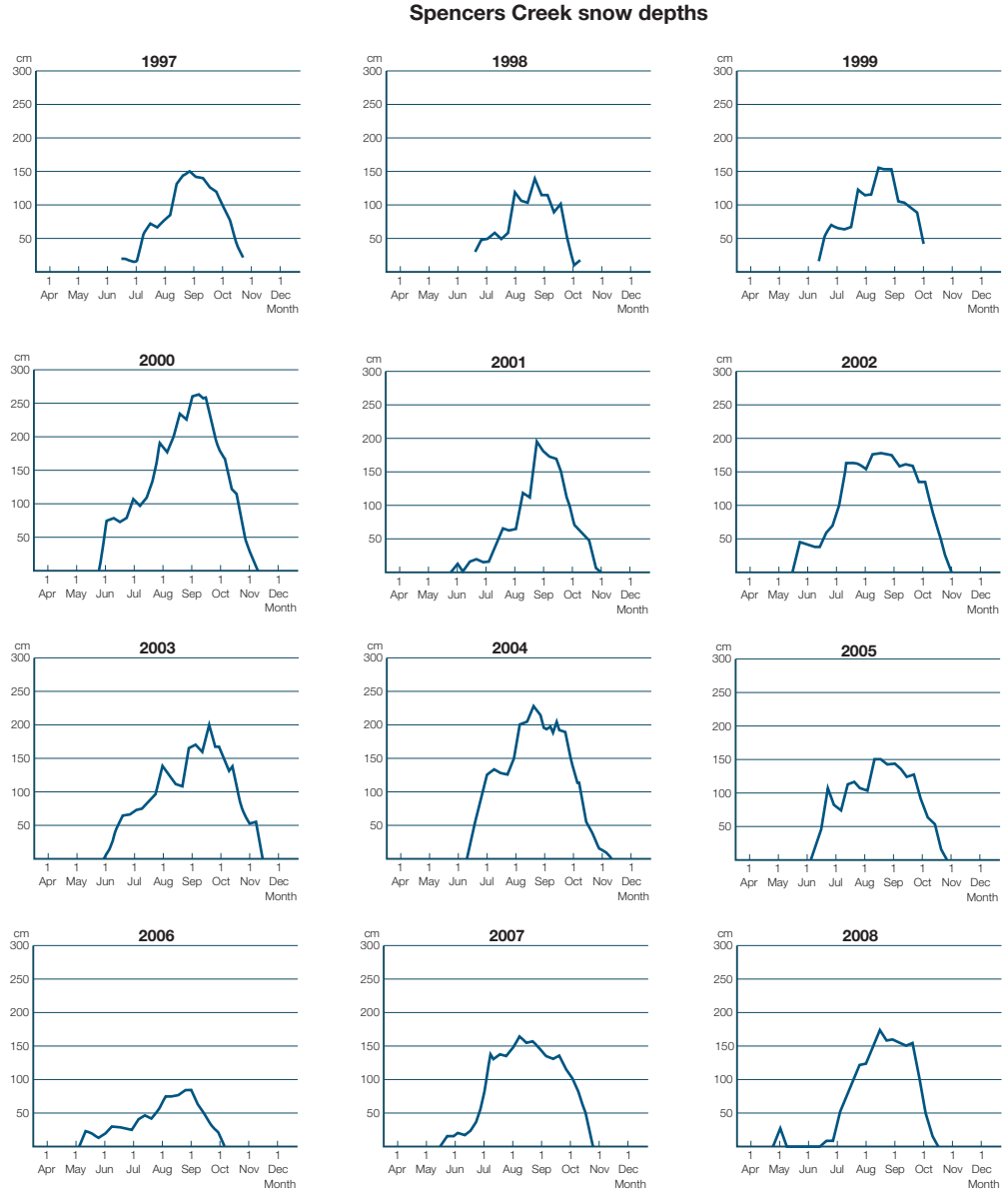
(d) Find the population aged 65 years and over in 2002, 2006 and 2010. Use these findings to describe the shape of the graph in the period 2002–2010.

(e) Use your answers to parts **(c)** and **(d)** to describe the shape of the graph overall.

(f) After World War II ended in 1945, Australia's birth rate increased dramatically. In addition, Australia's immigration policies have changed greatly since then, accepting immigrants from all around the world. Use these facts and your answer to part **(e)** to give two reasons why the shape of the graph has changed.

(g) What do you expect the population aged 65 years and over will be in 2015 if the present trend continues?

9 Look at the following line graphs that show the average snow depths (in centimetres) at Spencers Creek, Mt Kosciuszko, from 1997 to 2008. (The deeper the snow, the better it is for skiing.)



(a) Did the snow depth reach 2.5 metres at any time during the 12 years? If so, which year(s) did this happen?

(b) What was the greatest snow depth reached in 1997?

(c) What was the greatest snow depth reached in 2008?

(d) Which year had the most snow in June?

(e) Which year had the most snow through the whole of July?

(f) Which year had the worst October skiing?

(g) Which year had the best October skiing?

(h) Which year would have probably had the best November skiing?

(i) Which year had the worst skiing overall?

(j) Which month usually has the best skiing?

(k) Which month is usually the second best skiing month?

(l) It has been claimed that New South Wales' overall temperature has been getting higher due to climate change. Is there any evidence in these graphs to back this up?

(m) Why do we use line graphs rather than bar graphs to show this information?

10 When do you think you feel hungriest during the day? Draw a line graph that shows your level of hunger during the day from the time you wake up until the time you go to sleep. Do you think the shape of the graph would be similar for most people? Explain any differences you can think of.

9.6 Analysing data sets

Worked Example 14

WE14

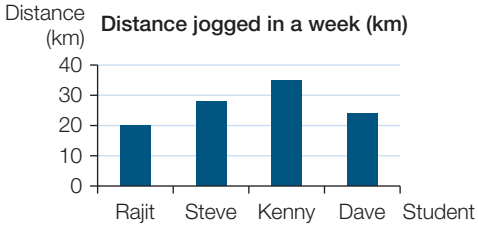
Rajit, Steve, Kenny and Dave are keen joggers. The tables below show some data on the four boys. Draw column graphs for each set of data and comment on the likelihood that there is a connection between any of the variables.

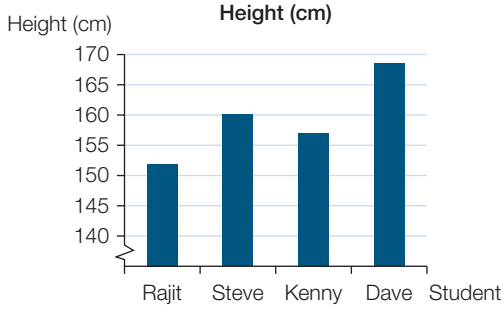
	Distance jogged in a week (km)
Rajit	20
Steve	28
Kenny	35
Dave	24

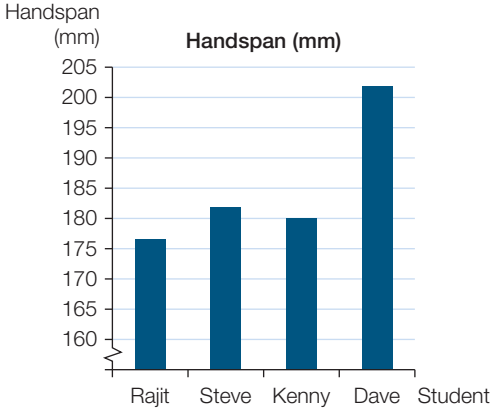
	Height (cm)
Rajit	152
Steve	160
Kenny	157
Dave	168

	Handspan (mm)
Rajit	177
Steve	182
Kenny	180
Dave	202

Working





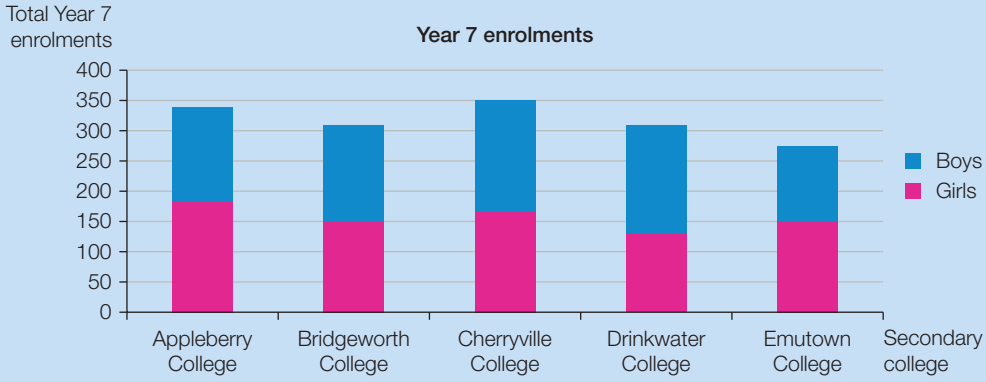


The shape of the graph of distance jogged each week is quite different from the shape of the other two graphs. There seems to be no connection between the distance they jog and either their height or their handspan.

The shapes of the graphs showing the height of the students and their handspan are very similar, so there could be a connection between these variables.

Worked Example 15

WE15



The graph above shows the Year 7 enrolment in five secondary colleges. Use the graph to answer the following questions.

(a) Which school has the highest Year 7 enrolment and what is its total Year 7 enrolment?

(b) Which part of the graph displays the number of girls and which school has the highest number of girls enrolled?

(c) Which school has the lowest number of boys enrolled?

(d) Which school has almost the same number of boys as girls?

(e) Which schools have about the same number of students enrolled in Year 7?

Working

(a) Cherryville College
350

(b) The lower part of each column corresponds to girls.
Appleberry College has the highest number of girls enrolled.

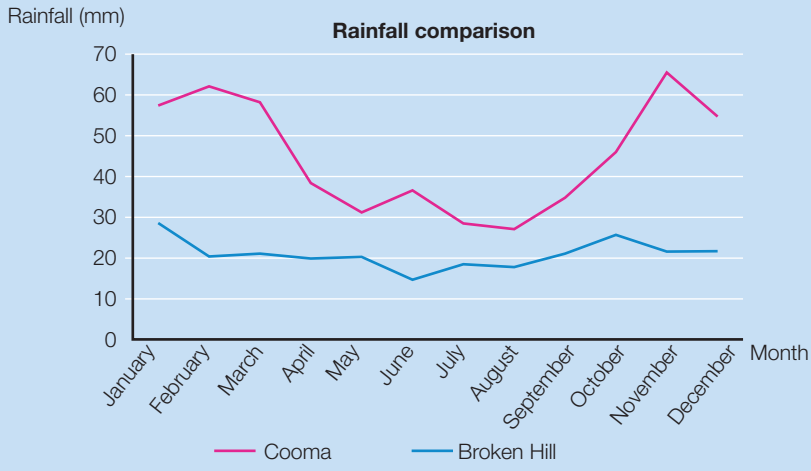
(c) Emutown College has the lowest number of boys enrolled.

(d) Bridgeworth College has about the same number of boys and girls enrolled.

(e) Bridgeworth College and Drinkwater College have the same number of students.

Worked Example 16 WE16

Use the graph to compare the monthly mean rainfall (mm) for two locations in NSW: Broken Hill Airport and Cooma Visitors Centre. Estimate the mean, median and range to assist your comparison.



Working

	Mean	Median	Range
Broken Hill	21	$\frac{20+21}{2}$ = 20.5	29 – 15 = 14
Cooma	45	$\frac{38+46}{2}$ = 42	66 – 27 = 39

Cooma is clearly much wetter than Broken Hill with both the mean and median for Cooma being slightly more than double the mean and median for Broken Hill. The range for Cooma is also much larger. Broken Hill has relatively even rainfall throughout the year, whereas Cooma is clearly much drier in the winter months.

Answers page 500

9.6 Analysing data sets

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11	Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q9, Q10, Q11, Q12

Fluency and understanding

WE14

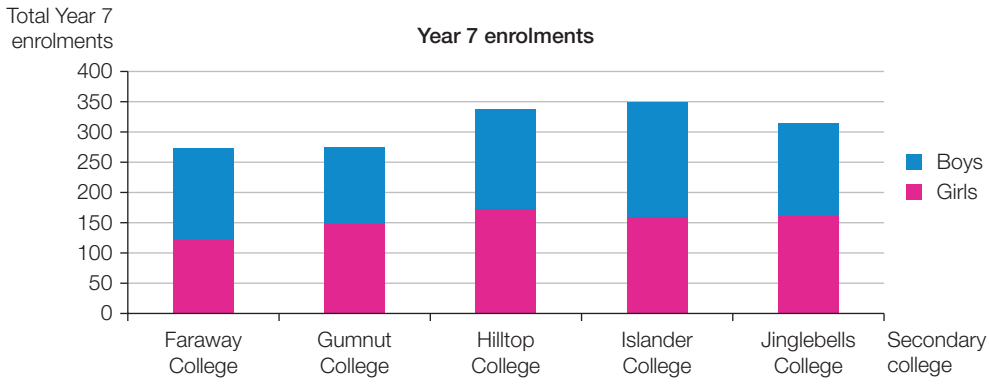
1 Veena, Fay, Kylie and Deanne are keen swimmers. The tables below show some data on the four girls. Draw column graphs for each set of data and comment on the likelihood that there is a connection between any of the variables.

	Hours spent training each week
Veena	15
Fay	20
Kylie	24
Deanne	18

	Height (cm)
Veena	147
Fay	153
Kylie	150
Deanne	162

	Foot length (mm)
Veena	217
Fay	236
Kylie	230
Deanne	242

2 WE15



The graph above shows the Year 7 enrolment in five secondary colleges. Use the graph to answer the following questions.

(a) Which school has the highest Year 7 enrolment and what is its total Year 7 enrolment?

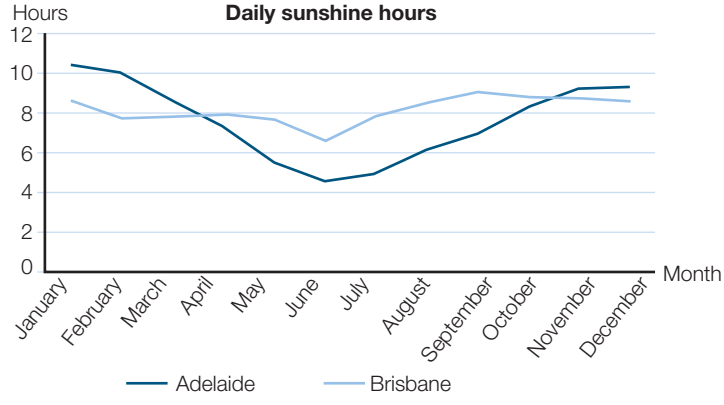
(b) Which part of the graph displays the number of girls and which school has the highest number of girls enrolled?

(c) Which school has the lowest number of boys enrolled?

(d) Which school has almost the same number of boys as girls?

(e) Which schools have about the same number of students enrolled in Year 7?

3 WE16 Use the following graph to compare the monthly mean sunshine (hours) for Adelaide (Airport) and Brisbane (Airport). Estimate the mean, median and range to assist your comparison.

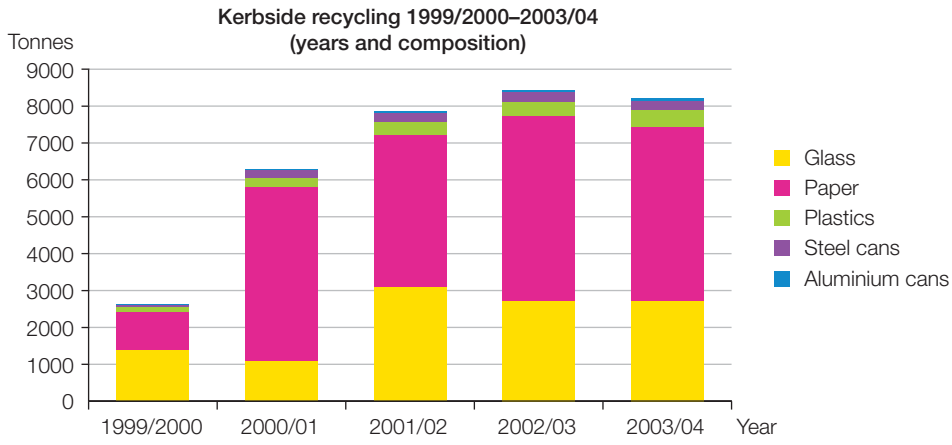


4 The following table gives the monthly mean minimum temperatures, measured in °C, for Adelaide (Kent Town) and Brisbane (Brisbane Aero).

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Adelaide	17.0	17.1	15.2	12.4	10.2	8.1	7.5	8.2	9.7	11.4	14.0	15.5
Brisbane	21.1	21.1	19.3	16.2	12.8	10.5	8.8	9.6	12.7	15.7	18.0	20.1

Compare the monthly mean minimum temperatures for the two cities using measures of centre and the range.

5



The graph above shows the types of material found in kerbside recycling by Shoalhaven Council over the time period 1999/2000–2003/04. Use the graph to answer the following questions.

(a) What material (by weight) was recycled the most in 1999/2000?

(b) What material (by weight) was recycled the most over the whole time period shown?

(c) Which time period was the best for recycling?

(d) Which was the best time period for glass recycling?

(e) Comment on the recycling of steel cans in the time period 2001–2004.

(f) Comment on the recycling of plastics over the time period shown.

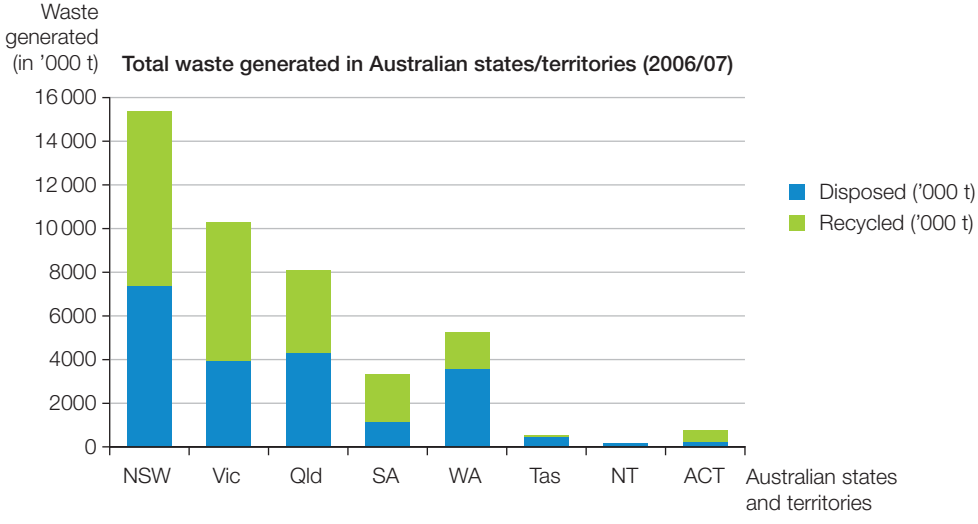
6 Compare the following data sets using as many statistics as possible.

(a) Set A: 1, 2, 4, 5, 6, 7, 7, 7, 8, 10 Set B: 2, 2, 2, 2, 4, 5, 8, 9, 9, 9, 9

(b) Set A: 2, 4, 1, 5, 6, 2, 3, 7, 8, 2, 4, 1 Set B: 5, 2, 7, 4, 7, 1, 8, 3, 9, 8, 9, 6

Reasoning, communicating and open-ended

7



Above is a graph of the estimated total waste generated in Australia 2006/07. The divisions show the amount of waste disposed of (sent to landfill) and the amount of waste recycled. Use the graph to answer the following questions.

(a) Which state/territory generated the most waste?

(b) How much waste did Queensland generate?

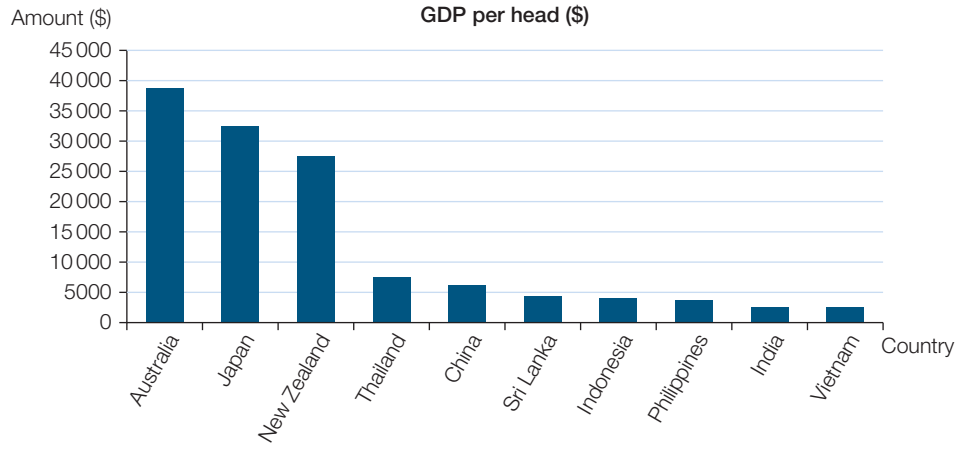
(c) Which states/territories recycled more than they disposed of?

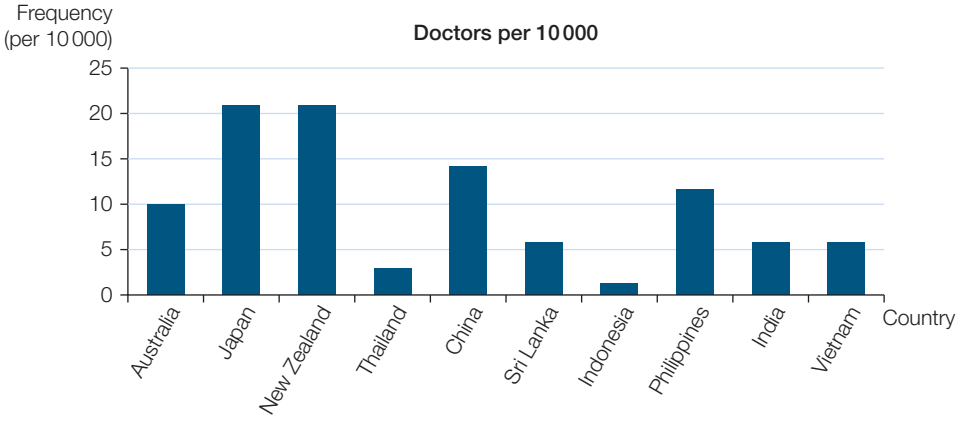
(d) Which state/territory recycled the largest proportion of its waste?

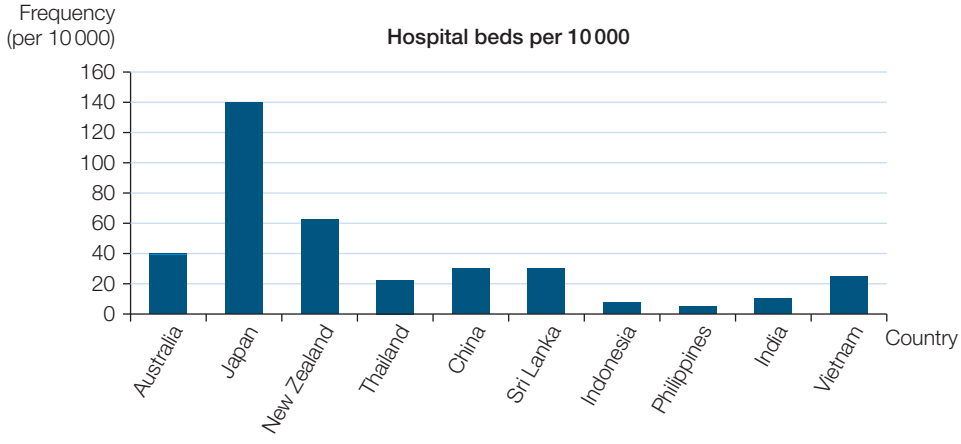
(e) Which state/territory recycled the smallest proportion of its waste?

(f) Did South Australia or Victoria recycle the larger proportion of its waste?

8







The Gross Domestic Product (GDP) of a country is one measure of a country's wealth. The number of doctors and hospital beds are measures of the health-care facilities of a country. Use the graphs above to answer the following questions.

(a) Do any of the graphs have a similar shape?

(b) Which country appears to have the best health-care facilities? Give a reason for your answer.

(c) Compare the health-care facilities of this country with its GDP per person.

(d) Which country appears to have the worst health-care facilities?

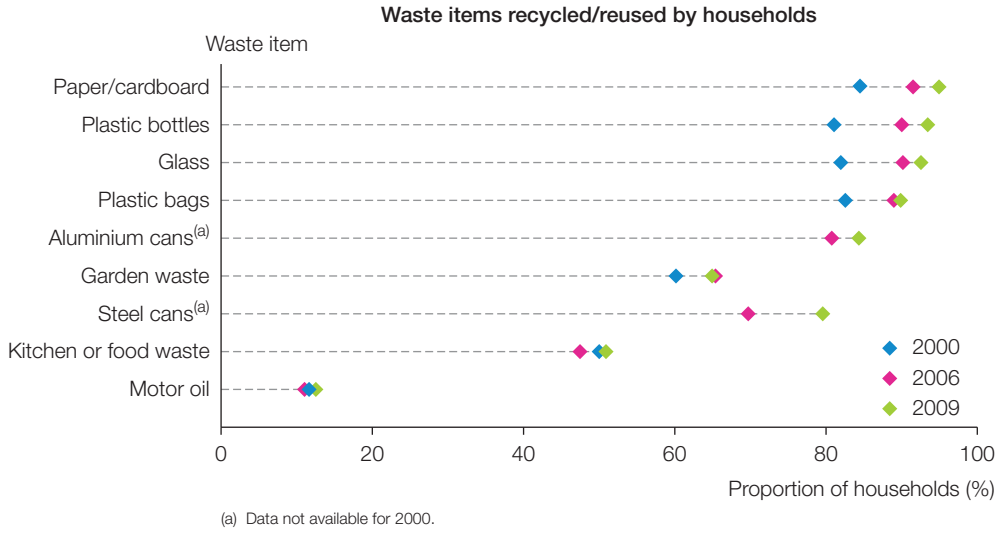
(e) Compare the health-care facilities of this country with its GDP per person.

(f) Comment on Austalia's health-care facilities compared with its GDP per person.

(g) Comment on Vietnam's health-care facilities compared with its GDP per person.

9 April went to the market and purchased 5 boxes of bananas at $40 a box, 3 boxes of apples at $55 a box and a box of mangoes at $65 a box. Her friend Izzy bought 3 boxes of bananas at $45 a box, 3 boxes of apples at $50 a box and a box of mangoes for $70. Compare the purchases of the two friends using as many statistics as possible.

10 This graph shows different types of waste items and the percentage of Australian households that recycle them.



(a) Data not available for 2000.

(a) **(i)** In 2009, what were the top three types of waste recycled by Australian households?

(ii) Suggest a reason for the answer to **(i)**.

(b) Which item had a large increase in the level of recycling between 2006 and 2009?

(c) Which three items had the most significant increase in the level of recycling between 2000 and 2009?

11 Sahrita measured the heights, in metres, of the 10 girls in her mathematics class. They were:

1.68 1.55 1.67 1.64 1.43 1.77 1.67 1.45 1.59 1.66

(a) Work out the mean, median and modal heights.

(b) Two new girls joined the class. The modal height did not change, but the mean changed by 0.15 m. Find two possible pairs of values for the heights of the new girls.

(c) Using the two pairs of values you used in **(b)** calculate the new median height.

12 Eric is comparing two sets of data. He has written that the mean of Set A is 1.5 greater than that for Set B, but the median for Set A is 2 less than the median for Set B. The mode and the range are the same for both sets. Construct two data sets that meet these descriptions, given that there are at least 15 results in each data set.

Probability and sample space

9.7

Need to Know

Sample space

The **sample space** is the complete list of all the possible **outcomes** of an event.

For example: all the outcomes of a probability chance experiment, such as rolling a die, tossing a coin, or selecting a card from a pack.

- Events that have no **chance** of happening (impossible) have a **probability** of 0 or 0%.
- Events that are certain to happen have a probability of 1 or 100%.
- Events that have a 50–50 chance of happening have a probability of $\frac{1}{2}$ or 0.5 or 50%.

Finding the probability of an event

$$\text{P(event)} = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

Complementary probabilities

All situations can be written so there are only two possible outcomes. An event may happen or it may not happen. These two events are called **complementary events** and the **complementary probabilities** add to 1.

P(event occurring) + P(event not occurring) = 1

A game is fair if each person participating has the same chance of winning.

Worked Example 17

WE17

Find the probability of:

(a) rolling an odd number with a normal die

(b) selecting a vowel (a, e, i, o, u) from a bag containing 26 identical pieces of paper, each containing a different letter of the alphabet.

Working

(a) *Sample space*: 1, 2, 3, 4, 5, 6

There are 6 outcomes in the sample space.

Favourable outcomes are: 1, 3 *and* 5.

There are 3 favourable outcomes.

$$\text{P(odd number)} = \frac{3}{6} = \frac{1}{2}$$

(b) *Sample space*: a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z

There are 26 outcomes in the sample space.

Favourable outcomes are: a, e, i, o, u

There are 5 favourable outcomes.

$$\text{P(vowel)} = \frac{5}{26}$$

Worked Example 18 WE18

If I roll an unbiased six-sided die, what is the probability of:

(a) not rolling a 6

(b) not rolling a 5 or a 6?

Working

(a) $P(\text{rolling a } 6) = \frac{1}{6}$

$P(\text{not rolling a } 6) = 1 - P(\text{rolling a } 6)$

$= 1 - \frac{1}{6}$

$= \frac{5}{6}$

(b) $P(\text{rolling a } 5) = \frac{1}{6}$ and $P(\text{rolling a } 6) = \frac{1}{6}$

$P(\text{rolling a 5 or a 6}) = \frac{1}{6} + \frac{1}{6}$

$= \frac{2}{6}$

$= \frac{1}{3}$

$P(\text{not rolling a 5 or a 6}) = 1 - \frac{1}{3}$

$= \frac{2}{3}$

Answers page 501

9.7 Probability and sample space

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q14, Q15	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13, Q15	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12, Q13, Q15

Fluency and understanding

WE17

1 Find the probability of:

(a) rolling an even number with a normal die

(b) choosing an 'a' from the letters of the word *mathematics*

(c) getting a 4 on a roll of a normal die

(d) drawing a card from a normal pack and getting a diamond

(e) drawing a card from a normal pack and getting a king

(f) drawing a card from a normal pack and getting a black 4

(g) drawing a card from a normal pack and getting a red 7

(h) drawing a card from a normal pack and getting the 9 of spades.

WE18

2 If I spin an equally divided five-sided spinner showing the numbers 1, 2, 3, 4 and 5, what is the probability of:

(a) not getting a 2

(b) not getting a 1 or a 2?

3 Decide whether you would describe the following events as 'impossible', 'not likely', 'likely' or 'certain'.

(a) Tomorrow will have a maximum temperature of -20°C.

(b) Christmas Day will be on 25 December.

(c) Sri Lanka will win the next cricket World Cup.

(d) The Sun will set tonight.

(e) You toss a coin and either a head or a tail turns up.

(f) You roll a die and a 1 turns up.

(g) You roll a die and a 7 turns up.

(h) You draw a card from a normal pack of 52 playing cards and it's the ace of clubs.

(i) You draw a card from a normal pack of 52 playing cards and it's either red or black.

4 Choose the value that you think is closest to the correct answer.

(a) The probability of winning first prize in Tattslotto is:

A 0 **B** 0.5 **C** 0.6 **D** 0.75

(b) The probability of Australia winning a gold medal in swimming at the next Olympics is:

A 0.3 **B** 0.4 **C** 0.9 **D** 1

(c) The probability of rain at some time during the next two months is:

A 0 **B** 0.2 **C** 0.5 **D** 1

(d) The probability of tossing a coin and getting a tail is:

A 0.2 **B** 0.3 **C** 0.5 **D** 0.7

(e) The probability of not drawing a heart from a pack of cards is:

A 0.0001 **B** 0.25 **C** 0.75 **D** 1

5 Write out the sample space for each of the following games.

(a) A ten-sided die, labelled 1 to 10, is rolled once.

(b) A letter is drawn at random from the letters of the word *probability*.

(c) Two coins are tossed at the same time.

6 A bag of jelly beans contains 4 blue, 5 red, 3 green, 2 purple and 1 black jelly beans.

(a) Rachel puts her hand in the bag and selects one jelly bean without looking. What is the probability of the jelly bean being:

(i) red **(ii)** green **(iii)** blue or purple?

(b) Rachel pulls out one red jelly bean and eats it. She selects another one from the bag without looking. Recalculate the three probabilities you calculated in **(a)**.

7 My fruit bowl contains five bananas, four apples, one orange and five peaches. If I choose one piece of fruit from the bowl at random, find:

(a) P(a banana) **(b)** P(a peach or an apple)

(c) P(not an apple) **(d)** P(not an orange nor a banana)

8 Steve and Nicky play a game based on the spinner shown. They each spin it once. Steve wins if he spins blue, and Nicky wins if she spins red.



(a) Is the game fair?

(b) Why is there a problem with the 'fairness' of this game?

9 Maria and Walter are playing a game that is based on the spinner shown. Are these games fair? Think about the sample space and the number of chances of winning.



(a) Maria wins if the pointer finishes on blue and Walter wins if the pointer finishes on red. If the pointer finishes on anything else, they spin again until one of them wins.

(b) Maria wins if the pointer finishes on blue or yellow and Walter wins if the pointer finishes on red. If the pointer finishes on green, they spin again until one of them wins.

(c) Maria wins if the pointer finishes on blue or green and Walter wins if the pointer finishes on red or yellow.

(d) Maria wins if the pointer finishes on blue, yellow or green, and Walter wins if the pointer finishes on red.

(e) Maria wins if the pointer finishes on yellow and Walter wins if the pointer finishes on red. If the pointer finishes on green or blue, they spin again until one of them wins. However, if green comes up three times in a row, Maria wins, and if blue comes up four times in a row, Walter wins.

10 Dominic rolls a standard six-sided die. Find:

(a) P(a 2 or a 4)

(b) P(a number larger than 2)

(c) P(a number less than 2)

(d) P(a multiple of 3)

(e) P(not a 5)

(f) P(not a number larger than 3)

Reasoning, communicating and open-ended

11 The following is a list of words you may have heard associated with probability.

even chance	very likely	virtually impossible	often
sure thing	most unlikely	almost always	quite often
likely	impossible	sometimes	usually
quite frequently	almost never	maybe	more often than not

Put these words in order from impossible to certain. Be prepared to defend your order in a class discussion.

12 A drawer contains six identical blue socks and two identical brown socks. How many socks do you have to take from the drawer to be certain of having:

(a) a pair of blue socks

(b) a pair of brown socks

(c) a pair of socks

(d) two odd socks?

Explain your answers in each case.

13 The following table shows the number of vehicles sold across Australia in March 2010 divided into a number of vehicle types.

Vehicle type	Number sold in March 2010
Light car	12 153
Small car	22 463
Medium car	7 256
Large car	8 816
Upper large car	272
People mover	1 234
Sports car	1 523
SUV Compact	9 651
SUV Medium	8 690
SUV Large	1 407
SUV Luxury	1 895

Source: VFacts (official results issued by the Federal Chamber of Automotive Industries)

Express your answers to the following questions as percentages, correct to one decimal place. Find the probability that a new vehicle buyer in March 2010, chosen at random from purchasers of vehicles in the table, had purchased a:

(a) light car

(b) small car

(c) medium car

(d) SUV of any type

(e) not a people mover

(f) not a sports car nor a people mover.

(g) Do you think that the probabilities would change very much from year to year? Why or why not?

(h) Is it more probable that a new vehicle buyer in March 2010 bought a small car or an SUV of any type? Justify your answer.

14 **(a)** Write down two events of your own that you think are impossible.

(b) Write down two events of your own that you think are not likely.

(c) Write down two events of your own that you think are likely.

(d) Write down two events of your own that you think are certain.

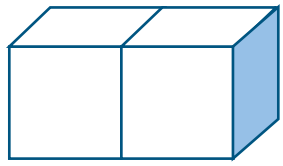
15 Make up a game between two players involving two dice that has a number of conditions for winning, but that you think is fair.

Challenge 9



1 The dots on opposite faces of a die total 7.

Two normal dice are placed next to each other on a table as shown.



Kenny walks around the table.

The greatest number of dots that Kenny can see is:

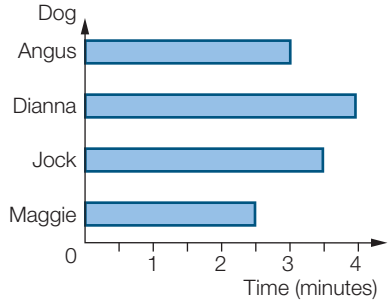
A 30 **B** 34 **C** 36 **D** 42

2 You meet three men at a meeting: Mr Black, Mr Green and Mr White. They are wearing black, green and white bow ties. Mr Black says, 'Did you notice that the colour of our ties are different from our names?' The person in the green bow tie says, 'Yes, you are correct!' What colour tie is each man wearing?

3 A die is constructed by labelling the faces of a wooden cube with the numbers 1, 1, 2, 3, 3 and 4. When it is tossed, each face has an equal chance of occurring. If this die is rolled once, the probability of rolling an odd number is:

A $\frac{1}{6}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$

4 Four Jack Russell terriers had a race swimming across a pond. The graph shows the time that each dog took to complete the race. Which dog won the race?



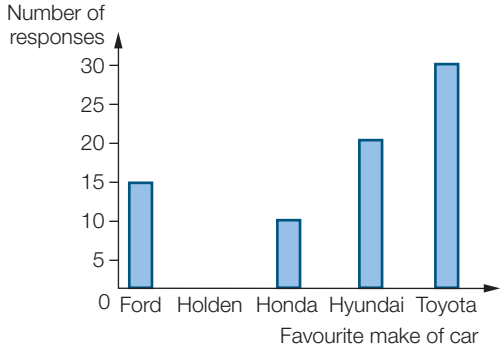
A Angus **B** Dianna

C Jock **D** Maggie

5 Vijay has a bag that contains 3 blue marbles, 6 green marbles, 2 red marbles and 6 yellow marbles. Vijay adds a number of white marbles to the bag and tells Samantha that if he now draws a marble from the bag, the probability of it being blue or green is $\frac{3}{7}$. The number of white marbles that Vijay added to the bag is:

A 3 **B** 4 **C** 5 **D** 6

6 In a survey, 1000 people were asked 'What is your favourite make of car?' Their responses were recorded and then graphed as percentages in a column graph. The column representing Holden has been omitted. What percentage of the people selected a Holden as their favourite make of car?



A 25 **B** 20

C 15 **D** 5

7 Jade has played 20 games of tennis and has won 95% of the games. If she does not lose any more games, how many more games in a row must she win in order to have then won 96% of the games?

A 1 **B** 4 **C** 5 **D** 10

Chapter review 9

D.I.Y. Summary

Key Words

average	continuous variable	mean	probability
bar graph	data	measures of centre	range
categorical variable	discrete variable	measures of location	sample space
chance	dot plot	measures of spread	stem-and-leaf plot
class interval	frequency histogram	median	tally
column graph	frequency polygon	mode	variable
complementary events	frequency table	outcomes	
complementary probabilities	line graph	outlier	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 The graphs in this chapter are all ways of presenting _____.

2 The listing of all the outcomes possible in a probability experiment is called the _____.

3 Of the three measures of central tendency considered, the one that represents the value occurring most frequently is called the _____.

4 The probability of an event occurring is another way of saying what _____ there is of the event happening.

5 For a frequency table for grouped data, the best graph is a _____.

6 The _____ is the sum of all the values in the data set divided by the number of values in that data set.

Fluency and understanding

Equipment required: Calculator for Question **2**

1 A Year 7 class was surveyed about how many television sets their families had at home. The results were: Ex 9.1

1, 1, 1, 1, 2, 1, 1, 3, 2, 5, 2, 1, 1, 0, 1, 2, 0, 4, 1, 2, 0, 3, 1

Construct a frequency table for this information.

2 Find the mean, median, mode and range for the following sets of data. Use a calculator when necessary, and give your answers correct to two decimal places. Ex 9.2

(a) 2, 4, 2, 7, 3, 5, 4, 2, 3, 6, 1

(b) 40, 20, 60, 30, 50, 10

(c) 3.3, 4.1, 3.9, 2.9, 4.4, 4.1, 3.8, 3.3, 4.4, 3.2, 3.9, 4.1, 2.9

3 Draw a bar graph to show the following data that represents the favourite style of TV program for a sample of 100 people. Ex 9.3

Style	Number of people
Comedy	35
Drama	21
Reality	17
Documentary	8
Game	19

4 **(a)** Draw a dot plot to represent the number of customers in a café recorded in 15-minute intervals over the course of 8 hours. Ex 9.3

4	4	7	3	6	4	5	9	4	8	8	5	7	9	10	12
11	8	9	8	6	2	4	4	5	5	2	5	3	0	1	1

(b) Calculate the median of the data set.

(c) Do there appear to be any outliers?

5 **(a)** Draw an ordered stem-and-leaf plot to represent the number of runs scored by each of the players at a local cricket club in one round of the season. Ex 9.3

46	64	31	0	50	42	11	7	4	0	5
21	17	115	0	42	61	15	18	5	3	1
40	0	52	56	77	50	41	20	7	2	2
88	17	58	63	14	0	18	19	17	6	2

(b) Calculate the median of the data set.

(c) Do there appear to be any outliers? If so, which values are they?

6 The sector graph below represents information about the money spent by overseas visitors to Australia. Ex 9.4

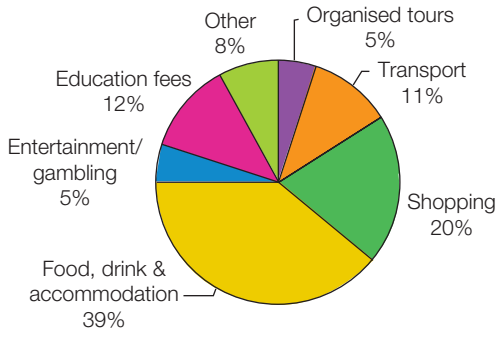
(a) One of the sectors uses an angle of 28.8°. Which one is it?

A Organised tours

B Transport

C Entertainment/gambling

D Other



(b) The Food, drink & accommodation sector uses an angle closest to:

A 95° **B** 110°

C 130° **D** 140°

7 The following are the sales figures for Beeper computer games from 2006 to 2011. Ex 9.5

Year	2006	2007	2008	2009	2010	2011
Units sold	10 000	25 000	35 000	30 000	40 000	45 000

Draw a line graph for this information.

8 The following table gives the mean rainfall, measured in mm, for Cairns (Aero) and Brisbane (Aero). Ex 9.6

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Cairns	396.8	452.5	426.1	198.2	91.4	46.1	29.3	27.0	33.4	39.7	93.4	178.5
Brisbane	109.3	121.6	76.1	63.5	122.8	70.6	27.1	35.2	32.7	65.2	102.0	120.9

Compare the mean rainfall for the two cities using as many statistics as you can.

9 The following table shows the handspan, measured in cm, of twenty 16-year-old students, divided into boys and girls. Ex 9.6

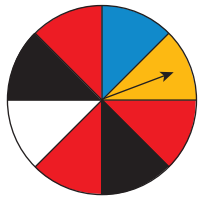
Boys	19.5	21.5	21	22.5	19	19.5	21	21.5	20.5	20
Girls	18	18.5	17.5	17	17	17.5	18	18.5	17	19.5

Draw a parallel dot plot and use it to help you describe the data set.

10 **(a)** When a normal die is rolled, the probability of getting a number less than 5 is equal to: Ex 9.7

A $\frac{1}{5}$ **B** $\frac{2}{3}$ **C** $\frac{5}{6}$ **D** 1

(b) If the spinner shown on the right is spun once, the probability of the pointer landing on red is:



A $\frac{1}{4}$ **B** $\frac{3}{8}$

C $\frac{1}{2}$ **D** $\frac{3}{5}$

11 A number of Year 7 students were surveyed about their shoe size. The results were:

$3, 5, 5\frac{1}{2}, 5, 4\frac{1}{2}, 4, 2\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}, 6, 2, 5\frac{1}{2}, 1\frac{1}{2}, 1, 3\frac{1}{2}, 4\frac{1}{2}, 3, 4, 5, 5\frac{1}{2}, 2\frac{1}{2}, 2, 3\frac{1}{2}, 4\frac{1}{2}, 3, 1, 3\frac{1}{2}, 1\frac{1}{2}, 4, 3\frac{1}{2}$

(a) Construct a frequency table to show this information.

(b) Which shoe size occurred most frequently?

(c) Which shoe size occurred least frequently?

12 Look at the following information.

How much Australians spend on pets each year ($ million)				
Expenditure type	**Dogs**	**Cats**	**Other**	**Total**
Food	560	431	90	1081
Vet charges and prescriptions	230	148	26	404
Pet care products/equipment	153	85	12	250
Pet services	116	35	–	151
Other expenses	109	20	12	141
Total	1168	719	140	2027

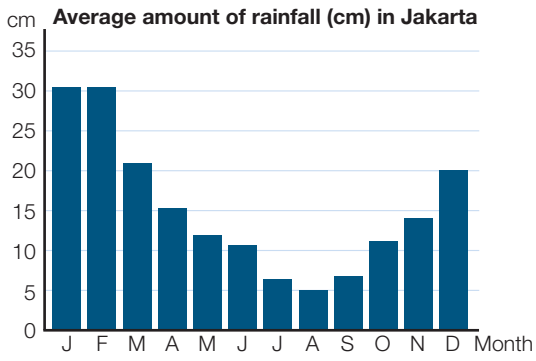
(a) Draw a bar chart showing the various expenditure types for dogs.

(b) Draw a divided bar chart to show the total expenditure for the three categories of pets.

(c) Draw a sector graph to show the various expenditure types for cats.

(d) Explain why a line graph could not be used for any of these sets of data.

13 Look at the following graph, which shows the average amount of rainfall (cm) in the city of Jakarta over 1 year.



(a) What type of graph is this?

(b) What is Jakarta's second driest month?

(c) What is the highest average monthly rainfall?

(d) Which 4 months form the rainy season?

14 Shelley and Douglas play a game in which they toss two coins. Shelley wins if two heads turn up, and Douglas wins if two tails turn up. If both a head and a tail turn up, they toss *one* of the coins again. If this comes up heads, Shelley wins. If it comes up tails, Douglas wins. Do you think this is a fair game?

15 **(a)** What value would the probability that you will be struck by lightning be close to?

(b) What value would the probability that it will rain at some time in the next six months be close to?

(c) If something has a probability of $\frac{1}{2}$ and something else has a probability of $\frac{3}{4}$, which one of the two is more likely to occur?

(d) If someone tells you something has a probability of 1.2 of happening, how do you know that they have made a mistake?

Reasoning and communicating

16 Twenty-five Year 7 boys and 25 Year 7 girls had their height measured to the nearest cm. The results are as follows.

Boys:	140	143	144	142	148	148	152	140	146	144
	154	151	145	140	141	147	153	150	149	140
	147	152	148	151	149					
Girls:	142	151	157	138	145	148	143	141	146	154
	156	154	150	143	150	144	148	144	157	156
	146	157	154	145	152					

(a) Construct an ungrouped frequency table for the boys.

(b) Find the **(i)** mode **(ii)** median and **(iii)** mean height for the boys.

(c) Construct an ungrouped frequency table for the girls.

(d) Find the **(i)** mode **(ii)** median and **(iii)** mean height for the girls.

(e) Describe the similarities and differences between the heights of the boys and the girls.

(f) Draw a combined ungrouped frequency table for boys and girls.

(g) Find the **(i)** mode **(ii)** median and **(iii)** mean height for the students.

(h) Describe the heights of the students.

17 In this game you have 12 counters and three dice. You place as many counters as you like on any number of squares, although you must place all 12. You roll the dice and add the numbers shown. If you have any counters on that number you remove them. You continue until the board is clear.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

(a) On which numbers should you never place any counters? Explain why.

(b) On which numbers should you place most counters? Explain why.

(c) Once you have decided on a spread of counters, will every game take the same number of turns to clear the board? Explain your answer.

NAPLAN practice 9

Numeracy: Non-calculator

1 Some students were surveyed about the number of hours spent using a computer over the last weekend. The following results were recorded: 4, 7, 3, 2, 4, 6, 8, 5, 9.

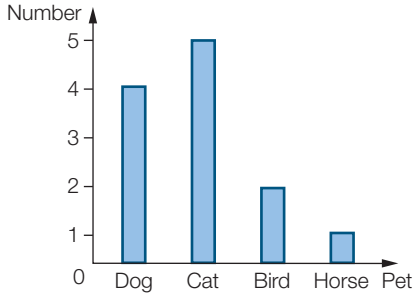
The median result is:

A 4 **B** 5 **C** 6 **D** 9

2 A jar contains 16 red, 12 black, 9 yellow and 8 green lollies.
Without looking, Jen takes one lolly from the jar.
What is the chance that the lolly is yellow?

A $\frac{1}{2}$ **B** $\frac{1}{3}$ **C** $\frac{1}{4}$ **D** $\frac{1}{5}$

3 Ricky surveyed the members of his cricket team to find out what pets they had. The results are shown in the graph on the right.

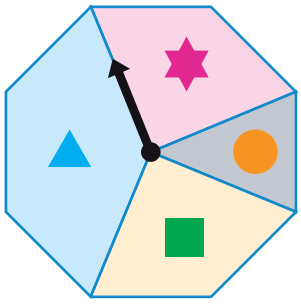


How many pets does the team have altogether?

A 4 **B** 5

C 12 **D** 15

4 Kenny spins the arrow 80 times.



Which table is *most* likely to show his results?

A

Shape section	Number of spins
★	10
●	30
■	20
▲	20

B

Shape section	Number of spins
★	20
●	30
■	10
▲	20

C

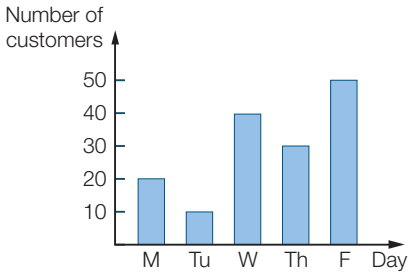
Shape section	Number of spins
★	20
●	10
■	20
▲	30

D

Shape section	Number of spins
★	20
●	20
■	10
▲	30

Numeracy: Calculator allowed

5 A business recorded the number of customers each day for a working week.



(a) Which day was the busiest?

(b) What was the mean number of customers per day?

(c) On which day was the number of customers closest to the mean number of customers per day?

6 The goal attack for the under 13 Panthers netball team scored the following number of goals in the first six games of the season: 14, 19, 23, 20, 28, 19.

The mean number of goals scored is closest to:

A 6 **B** 19.5 **C** 20.5 **D** 126

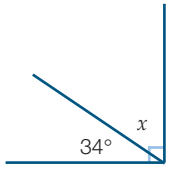
Transformation and visualisation 10

Recall 10

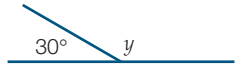
Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet.

1 Find the size of the unknown angle in each of the following.

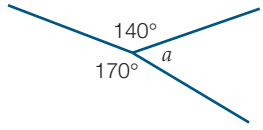
(a)



(b)

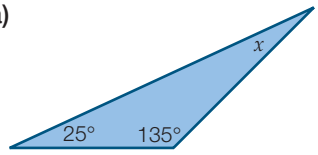


(c)

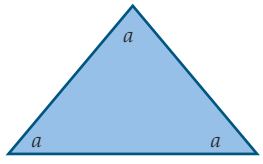


2 Find the size of the unknown angle in each of the following.

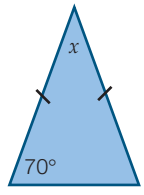
(a)



(b)

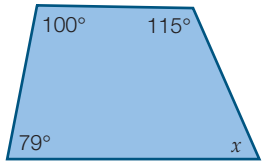


(c)

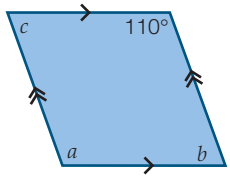


3 Find the size of the unknown angle in each of the following.

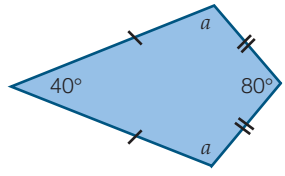
(a)



(b)



(c)



4 Draw an example of:

(a) a rhombus **(b)** a trapezium **(c)** an equilateral triangle

(d) a kite **(e)** an isosceles triangle **(f)** a parallelogram.

Mark all sides that are equal and/or parallel. Use letter names to name your figures.

Syllabus references

Number and Algebra: Linear Relationships

✓ Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates **(ACMMG181)**

Measurement and Geometry: Volume

✓ Draw different views of prisms and solids formed from combinations of prisms **(ACMMG161)**

Measurement and Geometry: Properties of Geometrical Figures 1

✓ Identify line and rotational symmetries **(ACCMMG181)**

10.1 Translations

Need to Know

Transformations

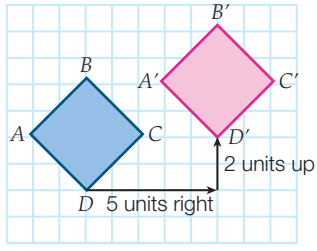
A **transformation** changes a particular shape (or figure) or set of points according to a defined rule. A transformation may slide, flip, turn or change the shape or size of a figure. The transformations dealt with here are those that move a shape but retain its original shape and size. These transformations are translations, reflections and rotations.

The first figure is called the **original** and the transformed shape is called the **image** of the transformation. Vertices of an image are denoted by a dash; e.g. A' is called 'A dash' or 'A prime'. The image of an original figure $ABCD$ is written $A'B'C'D'$.

Translations

A **translation** occurs when a figure is shifted or slid across the page to change its position.

The image is moved without turning. It is simply moved a number of units left or right, or a number of units up or down, or a combination of both. The example at right shows a figure that has been translated 5 units to the right and 2 units up. This translation can be written as [5 2].

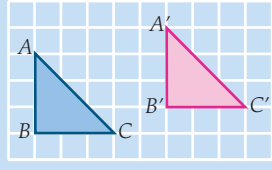


- To distinguish between the original figure and the translated image, you can use a different colour or broken lines.
- Translations can be reversed by doing the opposite horizontal and vertical moves. For example, the reverse of a translation of 4 units left and 3 units down is 4 units right and 3 units up.

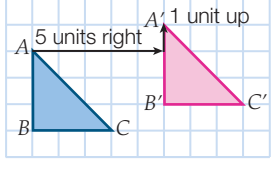
Worked Example 1

WE1

Describe the translation shown in the diagram.



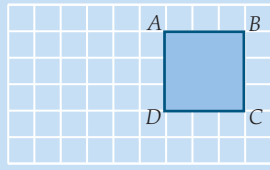
Working



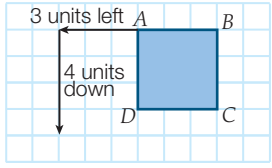
5 units right and 1 unit up or [5 1].

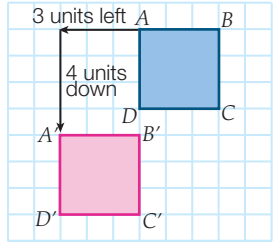
Worked Example 2 WE2

Copy the following onto grid paper and draw the resulting image after the translation: 3 units left and 4 units down.



Working





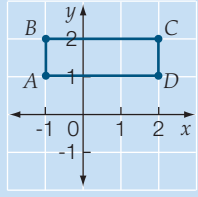
Worked Example 3 WE3

(a) Write the coordinates of each vertex on the given shape.

(b) Copy the shape onto grid paper and draw the resulting image after the translation of 2 units to the right and 3 units down.

(c) Write the coordinates of the translated vertices.

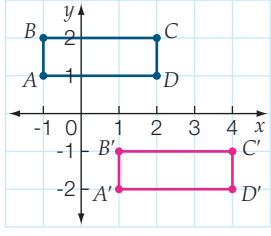
(d) Explain how the coordinates of the image could be found without drawing the shape.



Working

(a) $A = (-1, 1)$, $B = (-1, 2)$, $C = (2, 2)$, $D = (2, 1)$

(b)



(c) $A' = (1, -2)$, $B' = (1, -1)$, $C' = (4, -1)$, $D' = (4, -2)$

(d) Adding 2 to the x-coordinate of the original and subtracting 3 from the y-coordinate of the original will give the image coordinates.

Answers page 504

10.1 Translations

Navigator

Q1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q16, Q17	Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q17	Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17

Equipment required: Grid paper for Questions **2**, **3**, **9**, **13(b)**, **15(c)**, **16** and **17**

Fluency and understanding

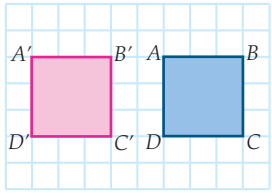
WE1

1 Describe the translation shown in each of the following diagrams.

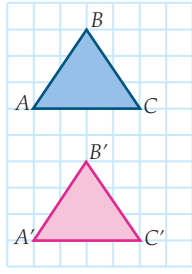
Hint

Translations can be written in square brackets.

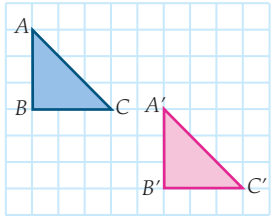
(a)



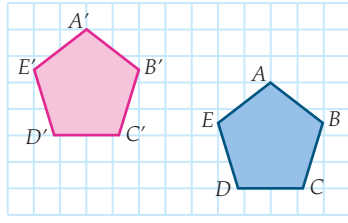
(b)



(c)



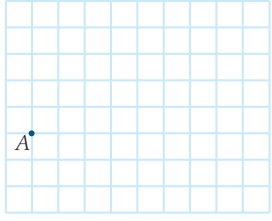
(d)



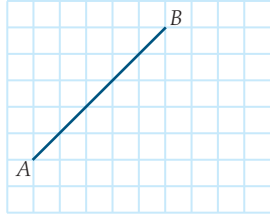
WE2

2 Copy each of the following onto grid paper and draw the resulting image after the translation.

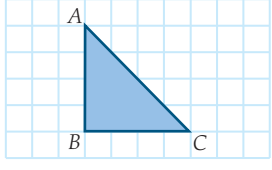
(a) 6 units right and 2 units up



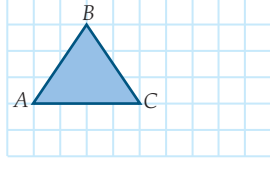
(b) 6 units left and 6 units down



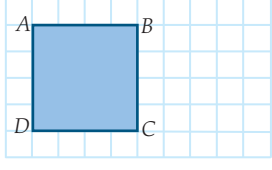
(c) 6 units left and 1 unit down



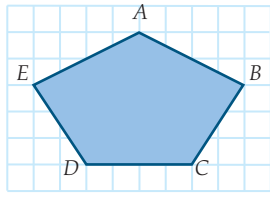
(d) 1 unit right and 6 units down



(e) [7 -5]

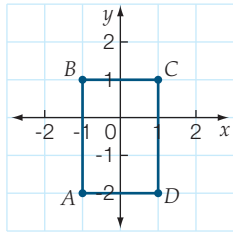


(f) [-6 4]



WE3

3 **(a)** Write the coordinates of each vertex on the given shape.

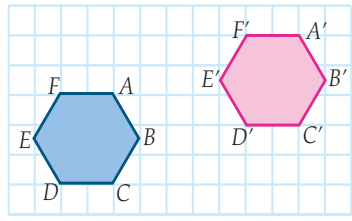


(b) Copy the shape onto grid paper and draw the resulting image after the translation of 1 unit to the left and 2 units up.

(c) Write the coordinates of the translated vertices.

(d) Explain how the coordinates of the image could be found without drawing the shape.

4 How has the hexagon in the diagram been translated?



A 7 units up and 2 units right

B 7 units right and 2 units up

C 7 units left and 2 units down

D 6 units right and 2 units up

5 What would be the reverse translation of 4 units left and 8 units down?

A 4 units up and 8 units right

B 4 units left and 8 units up

C 4 units left and 8 units down

D 4 units right and 8 units up

6 Write the translation [-2 6] using direction instructions (i.e. right, left, up, down).

7 Write the translation of 3 units left and 4 units down using square brackets.

8 Write the reverse translation of 10 units right and 4 units up using square brackets.

9 **(a)** Plot the points (-1, 4), (2, 3) and (0, 2) on a Cartesian plane and label them *A*, *B* and *C*.

(b) Join the points to form triangle *ABC*, then translate the shape 2 units right and 1 unit down, labelling the image with image notation.

(c) Write the coordinates of the translated points.

10 A figure is translated 4 units left, 3 units down, 6 units left and 7 units up. What would be the final position of the image compared to that of the original figure?

11 What single translation would be equivalent to 12 units down, 5 units right, 7 units left, 8 units up and 2 units right?

12 What single translation would be equivalent to a translation of [2 3] followed by a translation of [-2 6]?

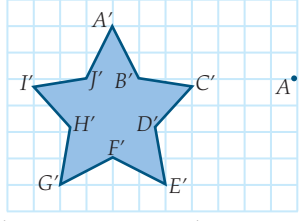
Reasoning, communicating and open-ended

13 **(a)** Without plotting points, write the coordinates of the image after the line joining *A*(4, 2) and *B*(6, -1) is translated 2 units left and 2 units up.

(b) Plot the line *AB* and its image *A'B'* on a Cartesian plane to check your answer to part **(a)**.

14 If a particular translation was 2 units right, what would be the single translation that would be equivalent to repeating 2 units right a hundred times?

15 The following diagram is of a translated image and a single point of the original figure.



(a) What translation has occurred to the original figure?

(b) What is the reverse translation?

(c) Use this to reverse the translation and draw the original figure.

16 Plot three points on a Cartesian plane and write their coordinates. Decide upon a particular translation and use this translation to translate your shape. Draw the image after the translation and write down the coordinates of the translated points using image notation.

17 On grid paper, draw a figure that has 4 vertices. Then, translate that figure in any direction and identify the reverse translation required to get the image back to the original figure.

10.2 Reflections

Need to Know

Reflections

A reflection is often called a mirror image. It is a reversed or 'flipped over' version of the original about a **line of reflection**. A reflected image has the same shape and size as the original but its position has changed and its orientation is reversed. Each point on a reflected image is the same distance from the line of reflection as the original, but on the opposite side. The distance used is the **perpendicular distance**, which is measured by a line that makes a right angle with the line of reflection.

- A **reflection** always flips the original, so the order in which the vertices are labelled in the image will be opposite to that in the original figure.
- A reflection is as far from one side of the line of reflection as the original figure is from the other side of it.
- When reflected, the properties of the shape will remain the same; for example, the size of the side lengths or angles will stay the same.

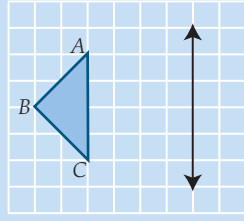
Reflecting a figure

1. Measure the perpendicular distance from each vertex on the original to the line of reflection.
2. Use this distance between the vertices and the line of reflection to plot the points on the opposite side of the line of reflection.
3. Join the points to produce the reflected image of the original figure.
4. Label the image vertices with image notation.

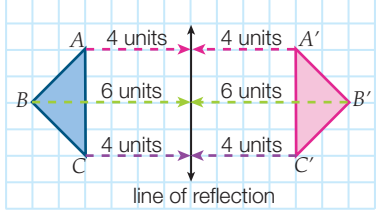
Worked Example 4

WE4

Copy this figure onto grid paper and draw the resulting image when the triangle is reflected in the line of reflection shown.



Working



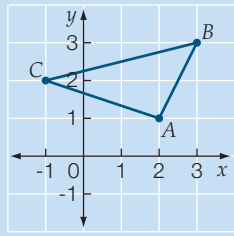
Hint

A point on the mirror line or line of reflection doesn't move when you reflect it.

Worked Example 5

WE 5

(a) **(i)** Write down the coordinates of each vertex on the given shape.

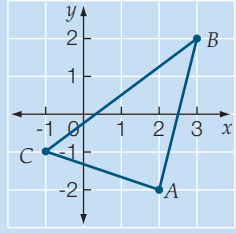


(ii) Copy the shape onto grid paper and draw the resulting image after a reflection in the x-axis, labelling each vertex with image notation.

(iii) Write the coordinates of the reflected vertices.

(iv) Explain how the coordinates of the image could be found without drawing the shape.

(b) **(i)** Write down the coordinates of each vertex on the given shape.



(ii) Copy the shape onto grid paper and draw the resulting image after a reflection in the y-axis, labelling each vertex with image notation.

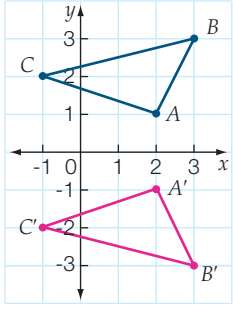
(iii) Write the coordinates of the reflected vertices.

(iv) Explain how the coordinates of the image could be found without drawing the shape.

Working

(a) **(i)** $A = (2, 1)$, $B = (3, 3)$, $C = (-1, 2)$

(ii)

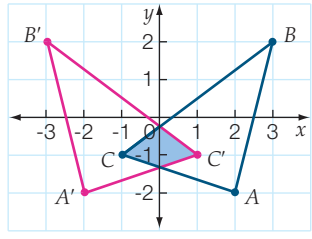


(iii) $A' = (2, -1)$, $B' = (3, -3)$, $C' = (-1, -2)$

(iv) The x-coordinate of the image stays the same as that of the original. The y-coordinate of the image is the negative of that of the original.

(b) **(i)** $A = (2, -2)$, $B = (3, 2)$, $C = (-1, -1)$

(ii)



(iii) $A' = (-2, -2)$, $B' = (-3, 2)$, $C' = (1, -1)$

(iv) The x-coordinate of the image is the negative of that of the original. The y-coordinate of the image stays the same as that of the original.

Answers page 505

10.2 Reflections

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q10, Q11, Q12	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q13

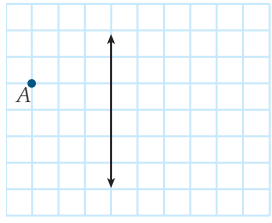
Equipment required: Grid paper for Questions **1–3**, **5–8** and **10–13**

Fluency and understanding

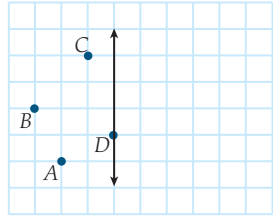
WE4

1 Copy the following figures onto grid paper and draw the resulting image when each is reflected in the line of reflection shown.

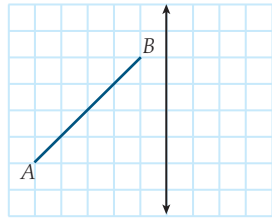
(a)



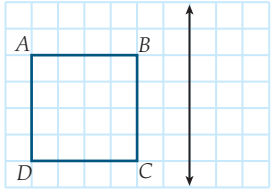
(b)



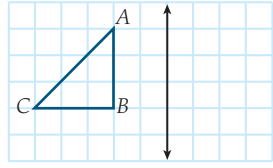
(c)



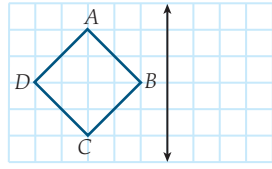
(d)



(e)

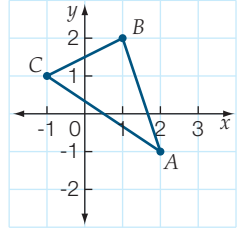


(f)



WE5

2 (a) **(i)** Write down the coordinates of each vertex on the given shape.

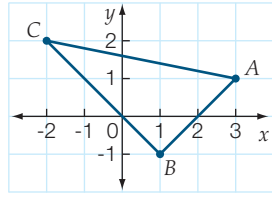


(ii) Copy the shape onto grid paper and draw the resulting image after a reflection in the x-axis, labelling each vertex with image notation.

(iii) Write the coordinates of the reflected vertices.

(iv) Explain how the coordinates of the image could be found without drawing the shape.

(b) **(i)** Write down the coordinates of each vertex on the given shape.



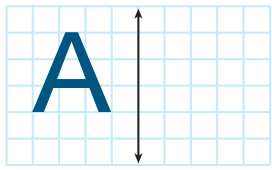
(ii) Copy the shape onto grid paper and draw the resulting image after a reflection in the y-axis, labelling each vertex with image notation.

(iii) Write the coordinates of the reflected vertices.

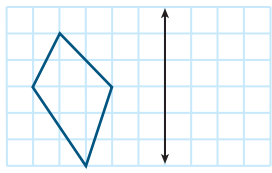
(iv) Explain how the coordinates of the image could be found without drawing the shape.

3 Copy each of the following figures onto grid paper, then draw the reflection of each in the line of reflection shown.

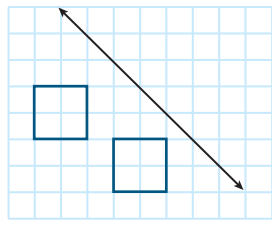
(a)



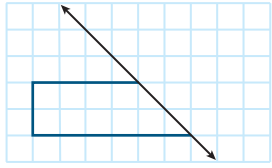
(b)



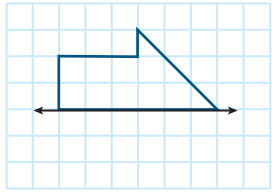
(c)



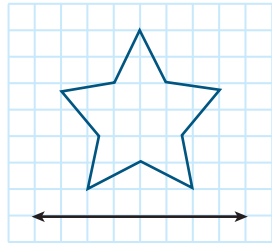
(d)



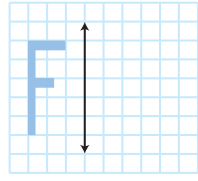
(e)



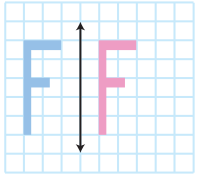
(f)



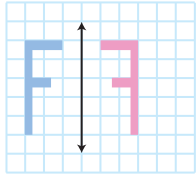
4 Which of the following options is the correct reflection of the capital F in the vertical line of reflection?



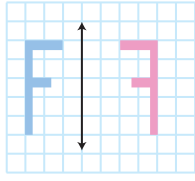
A



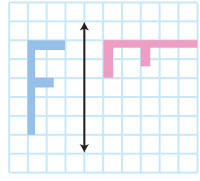
B



C



D



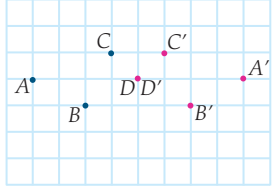
5 **(a)** Plot the points (-1, 5), (3, 1) and (2, -1) on a Cartesian plane and label them *A*, *B* and *C*.

(b) Join the points to form triangle *ABC*, then reflect the shape first in the *x*-axis and then in the *y*-axis.

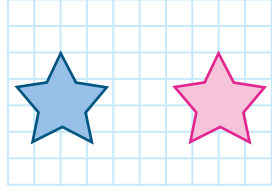
(c) Write the coordinates of the points after the second reflection.

6 Copy each of the following onto grid paper and indicate where the line of reflection should be placed to produce the given image.

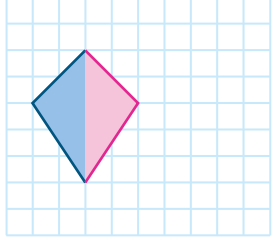
(a)



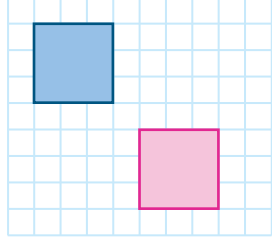
(b)



(c)

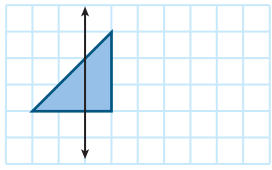


(d)

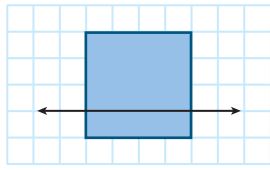


7 Copy each of the following onto grid paper and draw the reflected images. The image may be on both sides of the line of reflection.

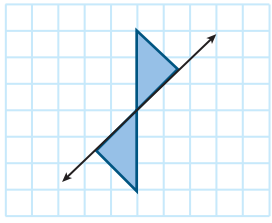
(a)



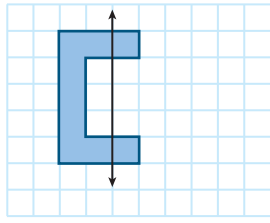
(b)



(c)



(d)



Reasoning, communicating and open-ended

8 **(a)** Without plotting points, write the coordinates of the image after the line joining $A(-2, 1)$ and $B(4, -3)$ is reflected in the x-axis.

(b) Plot the line AB and its image $A'B'$ on a Cartesian plane to check your answer to part **(a)**.

(c) Without plotting points, write the coordinates of the image after the line joining $A(-2, 1)$ and $B(4, -3)$ is reflected in the y-axis.

(d) Plot the line AB and its image $A'B'$ on a Cartesian plane to check your answer to part **(c)**.

9 Bruno has a digital clock on his glass bedside table. When he looked at the time, he noticed that at that particular instance, the time 8:30 was reflected perfectly on the table. Thinking this to be a little odd, he decided to find out the following.



(a) List the number of minutes past the hour when the minute part of the time reflects the correct time (include 00).

(b) In a 12-hour mode, list the hours when the hour part of the time reflects the correct time.

(c) In a 12-hour period, use your answers to parts **(a)** and **(b)** to find how many times the reflection on the table gives the correct time in a 12-hour mode.

(d) In a 24-hour period, how many times does the reflection in the table give the correct time in a 24-hour mode?

(e) Is your answer to part **(d)** double your answer to part **(c)**? Explain your answer.

10 Plot three points on a Cartesian plane and write their coordinates. Reflect this shape in either the x- or y-axis. Draw the image after the reflection and write down the coordinates of the reflected points using image notation.

11 On grid paper, draw a figure with four vertices. Reflect this figure in:

(a) a horizontal line of reflection

(b) a vertical line of reflection.

12 Repeat Question **11** with a figure with five, six, or seven vertices.

13 On grid paper, draw a figure with four vertices.

(a) Reflect this figure in a horizontal line of reflection and then in a vertical line of reflection.

(b) Reflect the same figure in the vertical line of reflection and then in the horizontal line of reflection.

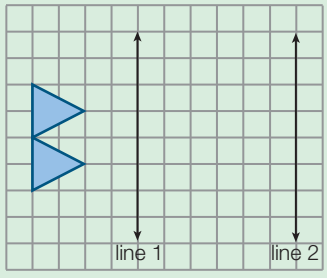
(c) Comment on the images found in parts **(a)** and **(b)**.

Problem solving

Mirror, mirror on the wall, does it matter where I stand at all?

Equipment required: 1 brain, grid paper

The diagram below consists of two blue triangles and two vertical lines of reflection.



To begin, reflect the two triangles in the line of reflection labelled 'line 1' and then in the line of reflection labelled 'line 2' and note the final location of the image. Next, switch the order in which the two triangles are reflected so that they are reflected in line 2 first, and then in line 1, and note the final location of the image.

Does the order in which these triangles are reflected in either line affect the final location and orientation of the final image? What did you notice about the final image? Can this transformation be described another way?

To justify your decision, experiment with different situations, such as drawing the two triangles originally on the right-hand-side of line 2 or in-between the lines. You can also alter the position of the lines of reflection. For example, line 1 and line 2 could be two horizontal lines or one could be horizontal and the other vertical.

Strategy options

- Draw a diagram.
- Test all possible combinations.

10.3 Rotations

Need to Know

Rotations

A figure is also transformed when it is **rotated** (turned) around a fixed point. This transformation is called a **rotation**. The fixed point is called the **centre of rotation** and is labelled with the letter *O*.

A centre of rotation may be inside or outside the original shape. This makes no difference to the method used to **rotate** the figure. The distance of each point from the centre of rotation remains the same after rotation.

When rotating any figure, these three things must be known:

- the location of the **centre of rotation**
- the size of the angle of rotation
- the direction of the rotation (clockwise ↷ or anticlockwise ↶).

The vertices of a rotated image are labelled in the same order as the original figures.

Rotations of 360°, 720° and other multiples of 360° result in the image in the same position as the original figure.

Two rotations that result in an image in the same position will always add to 360°. For example, 90° clockwise is equivalent to 270° anticlockwise as 90° + 270° = 360°.

A rotation in an anticlockwise direction is a positive rotation, e.g. 90°.

A rotation in a clockwise direction is a negative rotation, e.g. -90°.

Rotating a figure

1. Select a key point, such as a vertex, and join this point to the centre of rotation with a straight line.
2. Place the centre of a protractor on the centre of rotation, with the chosen vertex on the baseline.
3. Measure the given angle of rotation in the given direction from this vertex.
4. Plot the resulting point the same distance from the centre of rotation as the original point.
5. Repeat the process with the other vertices and connect the resulting points to produce the rotated image of the original figure.

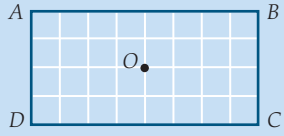
Hint

A clockwise rotation of 180° is the same as an anticlockwise rotation of 180°.

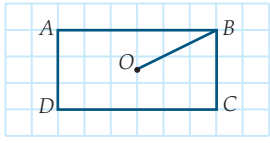
Worked Example 6

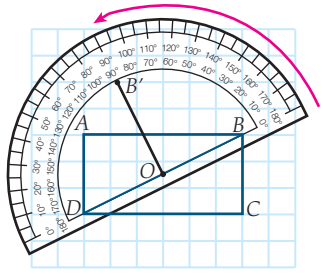
WE 6

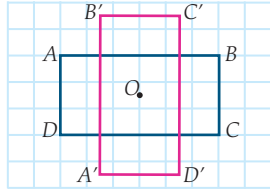
Copy this figure onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you. Rotate 90° in an anticlockwise direction (90°) about *O*.



Working



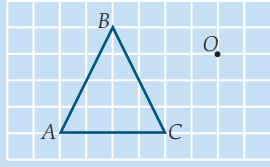




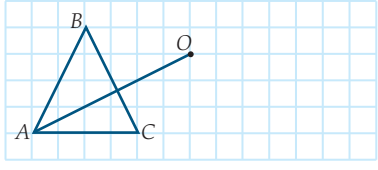
Worked Example 7

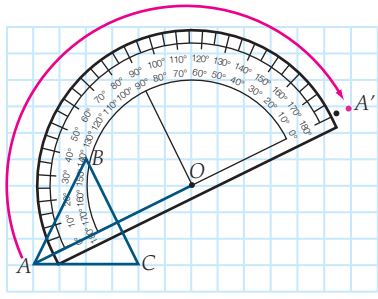
WE 7

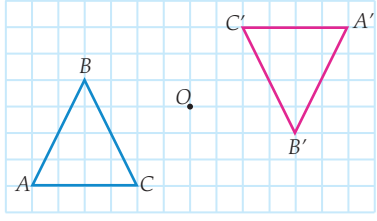
Copy the following figure onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you. Rotate 180° in a clockwise direction (-180°) about *O*.



Working



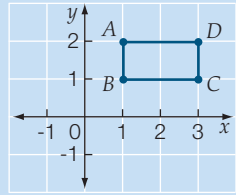




Worked Example 8

WE 8

(a) Write the coordinates of each vertex on the given shape and copy the shape onto grid paper.



(b) (i) Draw the resulting image after the original is rotated 90° in an anticlockwise direction about the origin.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

(c) (i) Draw the resulting image after the original is rotated 180° in an anticlockwise direction about the origin.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

(d) (i) Draw the resulting image after the original is rotated 270° in an anticlockwise direction.

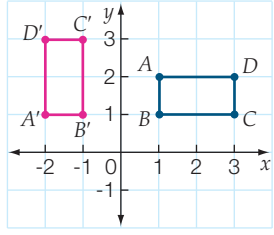
(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

Working

(a) $A = (1, 2)$, $B = (1, 1)$, $C = (3, 1)$, $D = (3, 2)$

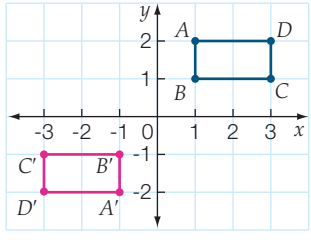
(b) (i)



(ii) $A' = (-2, 1)$, $B' = (-1, 1)$, $C' = (-1, 3)$, $D' = (-2, 3)$

(iii) The x-coordinates of the image vertices are the negative of the y-coordinates of the original vertices and the y-coordinates of the image vertices are the x-coordinates of the original vertices.

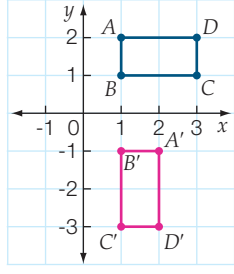
(c) (i)



(ii) $A' = (-1, -2)$, $B' = (-1, -1)$, $C' = (-3, -1)$, $D' = (-3, -2)$

(iii) All the coordinates of the image vertices are the negative of the original coordinates of the vertices.

(d) (i)



(ii) $A' = (2, -1)$, $B' = (1, -1)$, $C' = (1, -3)$, $D' = (2, -3)$

(iii) The x-coordinates of the image vertices are the y-coordinates of the original vertices and the y-coordinates of the image vertices are the negative of the x-coordinates of the original vertices.

10.3 Rotations

Answers page 508

Navigator

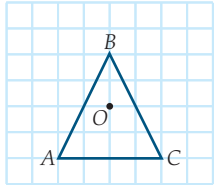
Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10 (a), Q11, Q14, Q15	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12 Column 1, Q14, Q15	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12 Column 2, Q13, Q14, Q15, Q16

Equipment required: Protractor and grid paper for Questions **1–3**, **6**, **9 (b)**, **10**, **14** and **16**

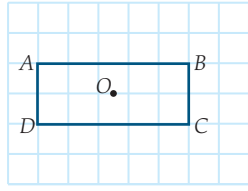
Fluency and understanding

1 Copy each of the following figures onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you. WE6

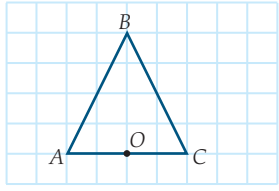
(a) Rotate 180° in an anticlockwise direction (180°) about *O*.



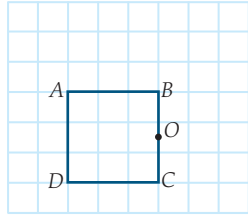
(b) Rotate 270° in an anticlockwise direction (270°) about *O*.



(c) Rotate 180° in a clockwise direction (-180°) about *O*.

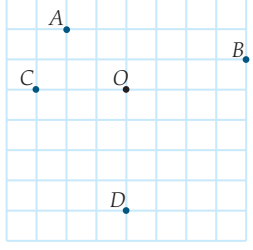


(d) Rotate 90° in a clockwise direction (-90°) about *O*.

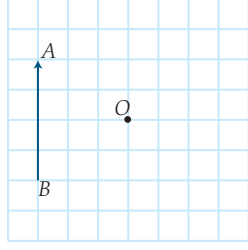


2 Copy each of the following onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you. WE7

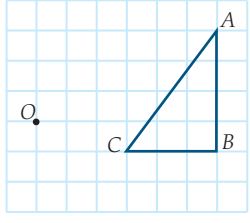
(a) Rotate each of the points 90° in a clockwise direction (-90°) about *O*.



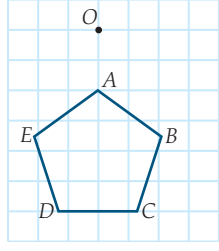
(b) Rotate 270° in an anticlockwise direction (270°) about *O*.



(c) Rotate 180° in a clockwise direction (-180°) about *O*.

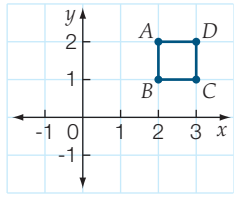


(d) Rotate 90° in an anticlockwise direction (90°) about *O*.



WE8

3 **(a)** Write the coordinates of each vertex on the given shape and copy the shape onto grid paper.



(b) **(i)** Draw the resulting image after the original is rotated 90° in an anticlockwise direction about the origin.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

(c) **(i)** Draw the resulting image after the original is rotated 180° in an anticlockwise direction about the origin.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

(d) **(i)** Draw the resulting image after the original is rotated 270° in an anticlockwise direction about the origin.

(ii) Write the coordinates of the rotated vertices.

(iii) Explain how the coordinates of the image could be found without drawing the shape.

4 Which of the following statements about rotation is incorrect?

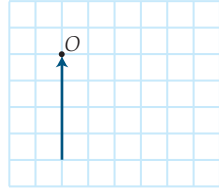
A Rotation can be in either a clockwise or an anticlockwise direction.

B A rotation of 360° will result in the image located in the same position as the original figure.

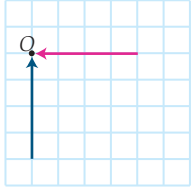
C A rotation will always produce an image that is a perfect replica and in the same orientation as the original figure.

D A rotation of 90° in a clockwise direction is equivalent to a rotation of 270° in an anticlockwise direction about the same centre of rotation.

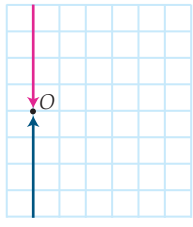
5 This arrow turns about the centre of rotation *O*. What does the shape look like after a turn of 180° anticlockwise?



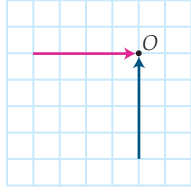
A



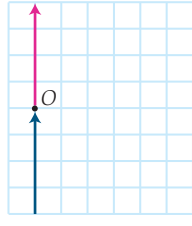
B



C



D



6 **(a)** Plot the points (1, 4), (2, 4), (2, 2) and (1, 2) on a Cartesian plane and label them *A*, *B*, *C* and *D*.

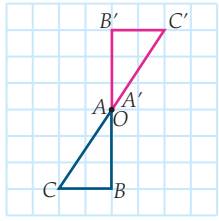
(b) Join the points to form a figure *ABCD*, then rotate the figure 90° in a clockwise direction about point *A*.

(c) Write the coordinates of the rotated points using image notation.

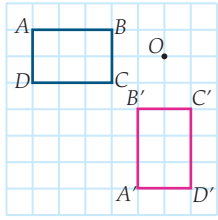
7 A clockwise rotation of 120° is the same as an anticlockwise rotation of how many degrees?

8 Find the rotation (the size of the rotation and the direction) that has taken place to produce the following images. Identify more than one rotation that will achieve the same result.

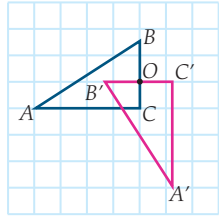
(a)



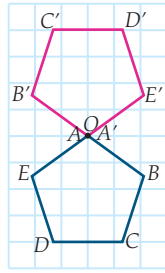
(b)



(c)



(d)



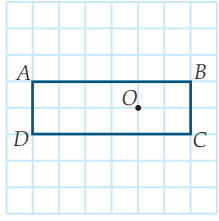
Reasoning, communicating and open-ended

9 **(a)** Without plotting points, write the coordinates of the image after the line joining $A(4, 2)$ and $B(3, -2)$ is rotated 180° clockwise about the origin.

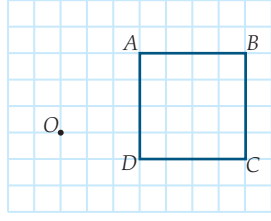
(b) Plot the line AB and its image $A'B'$ on a Cartesian plane to check your answer to part **(a)**.

10 Copy each of the following figures onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you.

(a) Rotate 90° in an anticlockwise direction about O.



(b) Rotate 90° in a clockwise direction about O.



11 The following are the capital letters of the alphabet.

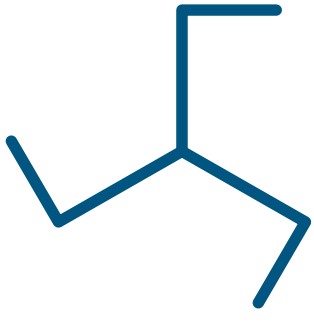
A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

(a) Which letters of the alphabet are able to be rotated about a centre of rotation in either direction and at an angle of less than 360° to produce an identical image in the same orientation as the original letter?

(b) Four of these letters have something else in common with each other. Can you identify these four letters? Describe the similarity you have noticed.

12 Estimate the smallest angle through which the following figures need to be rotated so the resulting image will match the original figure.

(a)



(b)



(c)



(d)



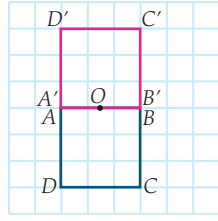
(e)



(f)



13 Is the following transformation a rotation of 180° around point O, a reflection along the line AB or both? Provide an explanation.



14 Plot four points on a Cartesian plane and write their coordinates. Decide upon a particular rotation and use this rotation to rotate your shape. Draw the image after the rotation and write down the coordinates of the rotated points using image notation.

15 Explain whether you agree or disagree with the following statement.

'A rotation of 180° in either direction and about any point of rotation can be equivalent to reflecting the same figure in a line of reflection through that point.'

16 On grid paper, draw two figures, a regular and an irregular polygon with the same number of sides. Decide upon a centre of rotation for each polygon that is on a vertex of that polygon.

(a) Rotate each of the figures by 90° about the centre of rotation in a clockwise direction and draw the image.

(b) Repeat this four times.

(c) Rotate the figures 45° about the centre of rotation.

(d) Repeat this eight times.

(e) Comment on the final images in parts **(b)** and **(d)**.

Combined transformations

10.4

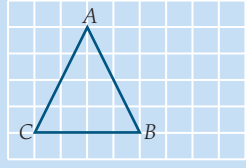
Need to Know

- Translations, reflections and rotations can be combined into a sequence of transformations.
- A side length can act as the line of reflection during a combined transformation.
- Different colours can be used to distinguish between several transformations.
- The properties of the three different types of transformations still hold true for combined transformations.
- After each transformation, the vertices are labelled with an additional dash. For example, the vertex A''' has been transformed by three different transformations.
- Label each image after each transformation.

Worked Example 9

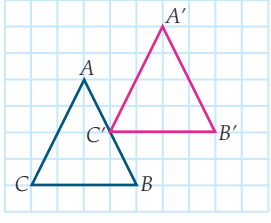
WE 9

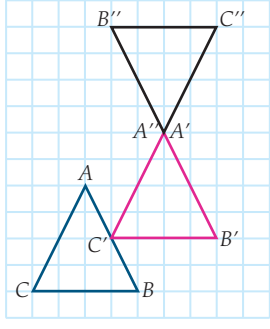
Copy the following figure onto grid paper and draw the resulting image after the combined transformation has been carried out. Use a protractor to help you. Label each vertex appropriately after each transformation.



A translation of 3 units right and 2 units up, followed by a rotation of 180° in a clockwise direction about the vertex A'.

Working

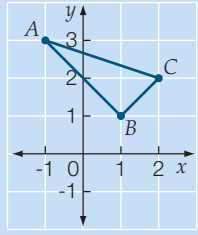




Worked Example 10

WE10

For this figure:



(a) write the coordinates of each vertex on the given shape and copy the shape onto grid paper

(b) reflect the shape in the x-axis

(c) write the coordinates of the reflected vertices

(d) translate this image 2 units right and 3 units up

(e) write the coordinates of the translated vertices

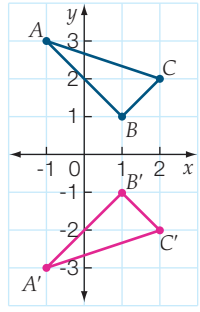
(f) rotate this translated image 90° in an anticlockwise direction about the origin

(g) write the coordinates of the vertices of the final image.

Working

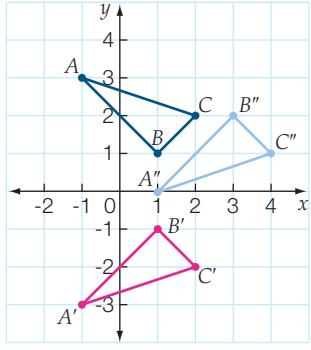
(a) $A = (-1, 3)$, $B = (1, 1)$, $C = (2, 2)$

(b)



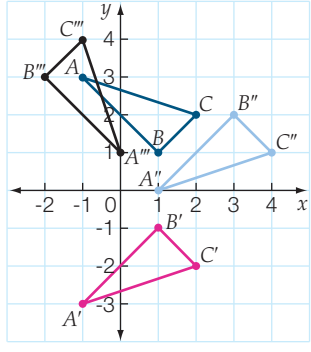
(c) $A' = (-1, -3)$, $B' = (1, -1)$, $C' = (2, -2)$

(d)



(e) $A'' = (1, 0)$, $B'' = (3, 2)$, $C'' = (4, 1)$

(f)



(g) $A''' = (0, 1)$, $B''' = (-2, 3)$, $C''' = (-1, 4)$

10.4 Combined transformations

Answers page 509

Navigator

Q1, Q2, Q3 Column 1, Q4, Q5, Q6, Q7, Q8 Column 1, Q9 Column 1, Q10, Q11, Q13, Q15	Q1, Q2, Q3 Column 2, Q4, Q5, Q6, Q7, Q8 Column 2, Q9 Column 2, Q10, Q11, Q12, Q13, Q15	Q1, Q2, Q3 Column 2, Q4, Q5, Q6, Q7, Q8 Column 2, Q9 Column 2, Q10, Q11, Q12, Q13, Q14, Q15

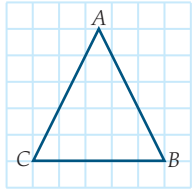
Equipment required: Protractor and grid paper for Questions **1–3**, **6**, **10 (b)** and **13–15**

Fluency and understanding

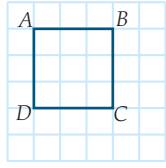
1 Copy each of the following figures onto grid paper and draw the resulting image after the combined transformation has been carried out. Use a protractor to help you. Label each vertex appropriately after each transformation.

WE9

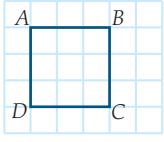
(a) A translation of 3 units left and 1 unit down, followed by a reflection in the line $B'C'$.



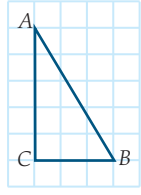
(b) A translation of 5 units right and 2 units down, followed by a reflection in the line $D'C'$.



(c) A translation of 5 units right, followed by a rotation of 90° about B' in a clockwise direction.

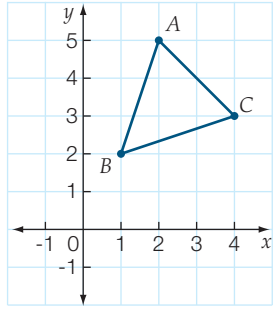


(d) A translation of 3 units left and 4 units down, followed by a rotation of 180° about C' in a clockwise direction.



2 For the figure given opposite:

WE10



(a) write the coordinates of each vertex on the given shape and copy the shape onto grid paper

(b) reflect the shape in the x-axis

(c) write the coordinates of the reflected vertices

(d) translate this image 5 units left and 3 units up

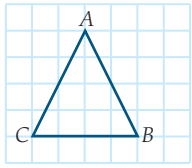
(e) write the coordinates of the translated vertices

(f) rotate this translated image 180° in a clockwise direction about the origin

(g) write the coordinates of the vertices of the final image.

3 Copy each of the following figures onto grid paper and draw the resulting image after the combined transformation has been carried out. Use a protractor to help you. Label each of the vertices appropriately after each transformation (for **(e)**–**(h)**, point O is a point on or inside the figure and is reflected with the shape).

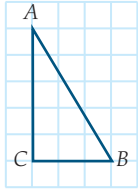
(a) Reflect along the line CB and translate 6 units right and 2 units up.



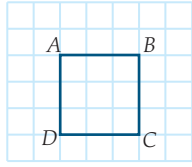
(b) Reflect along the line BC and translate 1 unit left and 3 units down.



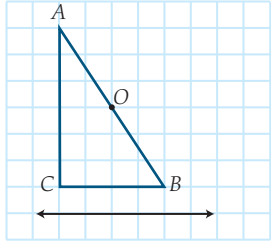
(c) Rotate 90° in a clockwise direction about vertex A and translate 8 units down.



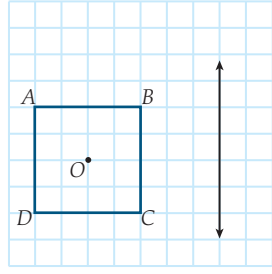
(d) Rotate 180° in an anticlockwise direction about vertex D and move 2 units left and 6 units up.



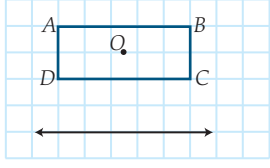
(e) Reflect along the line of reflection and then rotate 90° in an anticlockwise direction about the centre of rotation O'.



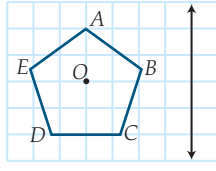
(f) Reflect along the line of reflection and then rotate 90° in a clockwise direction about the centre of rotation O'.



(g) Rotate 90° in an anticlockwise direction about the centre of rotation O and then reflect in the line of reflection.



(h) Rotate 180° in a clockwise direction about the centre of rotation O and then reflect in the line of reflection.



4 Which transformation reverses the order in which the vertices are labelled?

A rotation and translation
B reflection
C translation
D rotation

5 Which transformation can result in an image in the same position and orientation as the original figure?

A rotation and translation
B translation
C reflection
D rotation

6 **(a)** Plot the points (-1, 2), (2, 3) and (3, -1) on a Cartesian plane and label them *A*, *B* and *C*.

(b) Join the points to form a triangle *ABC*, then translate the shape 3 units right and 1 unit down.

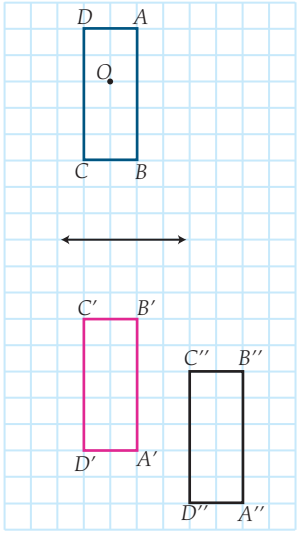
(c) Now, rotate the figure 90° in a clockwise direction about the origin.

(d) Write the coordinates of the vertices of the final image using image notation.

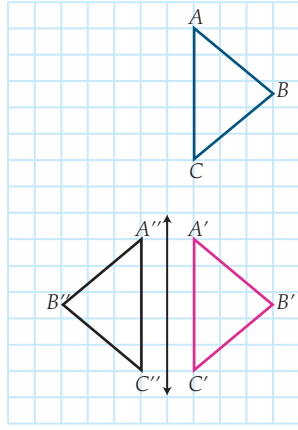
7 For the combined transformations in Question **1**, identify the necessary combined transformation that would move the image back to the position and orientation of the original figure.

8 Describe the transformations that have taken place to move the original figure in blue to the position in pink, then to its final position in black. List the relevant names of each transformation.

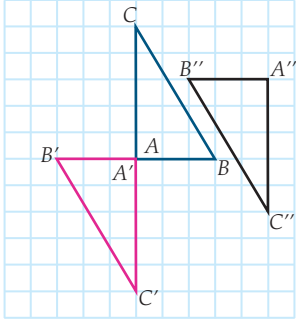
(a)



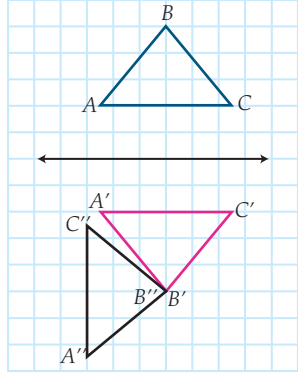
(b)



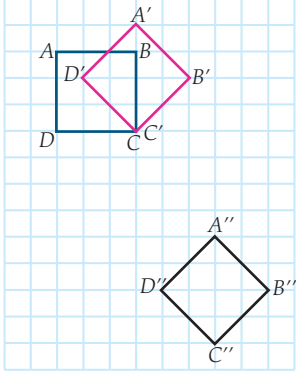
(c)



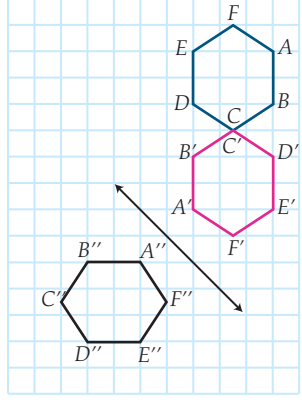
(d)



(e)



(f)



9 For the combined transformations stated in each part of Question **8**, identify the transformations required to move the final image back to the original position and orientation of the figure.

Reasoning, communicating and open-ended

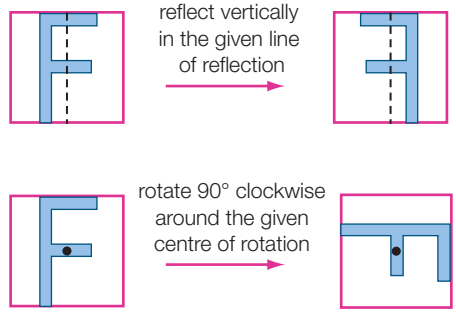
10 **(a)** Without plotting points, write the coordinates of the image after the line joining $A(-2, 3)$ and $B(4, -1)$ is translated 1 unit right and 5 units down, then reflected in the y-axis.

(b) Plot the line AB, the first image $A'B'$ and the final image $A''B''$ on a Cartesian plane to check your answer to part **(a)**.

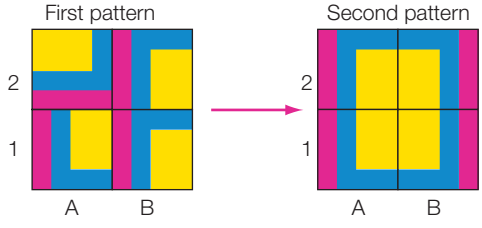
11 Sarah has written a computer program to transform pictures of tiles. There are only two instructions in her program.

1 Reflect vertically in the line of reflection through the centre of each tile.

2 Rotate 90° clockwise around the given centre of rotation.



Sarah wants to transform the first pattern to the second pattern.



Copy and complete the following instructions to transform the tiles B1 and B2. You must only reflect vertically and rotate 90° clockwise through the centre of each tile.

A1 – Tile is in the correct position.

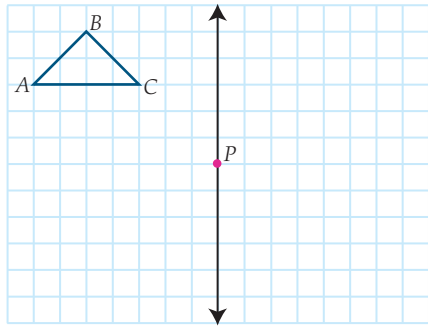
A2 – Reflect vertically, and then rotate 90° clockwise.

B1 – Rotate 90° clockwise and then …

B2 – …

12 The following combined transformation is performed on triangle ABC.

- A reflection in the given line of reflection.
- A rotation of 180° about point P.
- A reflection in the given line of reflection.



Find a single transformation that will move the image $A'''B'''C'''$ back to the original position of the figure. Draw a diagram to show each required transformation.

13 Plot three points on a Cartesian plane and write their coordinates. Decide upon two particular transformations and use them to transform your figure. Draw the image after both transformations and write down the coordinates of the transformed vertices using image notation.

14 Select one of the following capital letters of the alphabet: B, C, D, E, F, G.

(a) Perform a translation and then a reflection in a vertical line of reflection. Draw the resulting images on grid paper and label each image appropriately.

(b) Find a single transformation that will move the final image back to the original position. Show this transformation on your diagram.

(c) Use your answer for part **(b)** to find a single transformation that would produce the same result as the two transformations in part **(a)**.

15 A shape is rotated through 180° about a point O and then its image is reflected in a horizontal line of reflection passing through O.

(a) Choose any shape and, on grid paper, carry out the transformations described above.

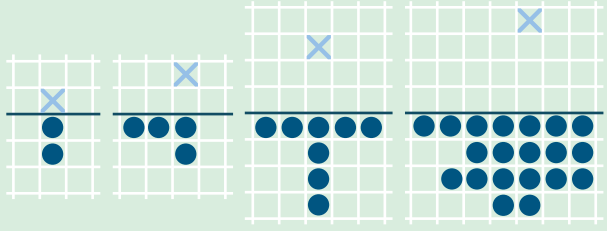
(b) Find a single transformation that would have the same results as the two transformations described above.

Problem solving

Conway's army

Equipment required: 1–2 brains, grid paper, approximately 40 small counters

Conway's army is a one-person puzzle that was created by mathematician John Horton Conway in 1961. It is a variant of the peg solitaire game and uses a checkerboard of any dimension. The board is divided by a horizontal line; above it are empty cells and below it are a random number of game pieces, which are referred to as the 'soldiers'. Examples of different boards and arrangements of soldiers are as follows:



How to play:

Play proceeds by jumping horizontally or vertically (not diagonally) over other pieces onto an empty space, where jumped-over pieces are then removed.

How to win:

The goal is to place a soldier as far above the horizontal line as possible. The cross in each of the diagrams at left is the maximum distance that a soldier can be placed from the horizontal line for the given number of original soldiers.

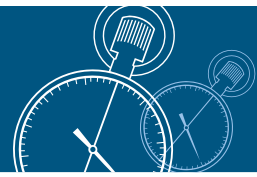
Strategy options

- Act it out.
- Test all possible combinations.

Questions

1. Create your own checkerboards with the horizontal line drawn. Begin by investigating how far four counters can be moved beyond the horizontal line. Try at least three different arrangements of the four counters. Is it possible to move a soldier past the second row?
2. Continue to investigate with 4, 5, 6, 7, 8, 9, 10, 15 and 20 counters to see how far past the horizontal line the counters can be moved.
3. Use your observations thus far to predict if it is possible to move a soldier to the fifth row above the horizontal line. Explain your answers.

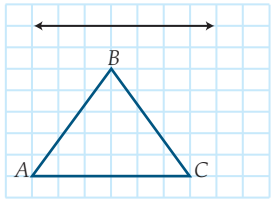
Half-time 10



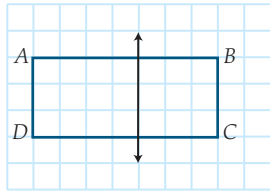
Ex.10.2

1 Reflect each of the following figures in the line of reflection shown.

(a)



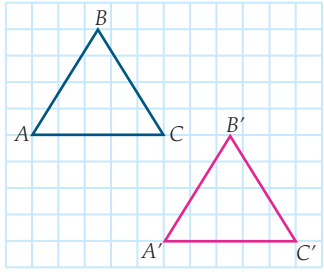
(b)



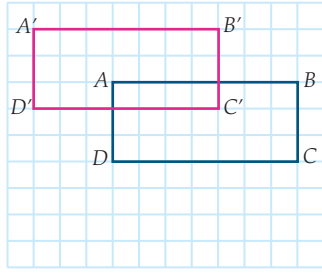
Ex.10.1

2 Describe the translation shown in each of the following diagrams.

(a)



(b)



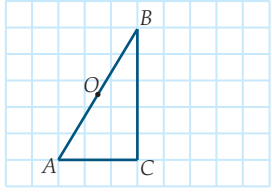
Ex.10.3

3 Show the image of each of the following figures after it is rotated about the given centre of rotation.

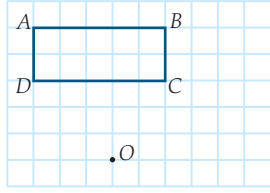
Rotate 180° in an anticlockwise direction about *O*.

Rotate 90° in a clockwise direction about *O*.

(a)

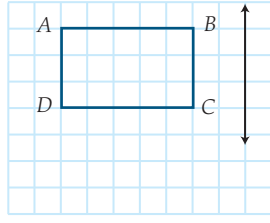


(b)



Ex.10.4

4 Reflect the following shape in the line of reflection shown, then translate it 2 units right and 1 unit down.



Ex.10.4

5 **(a)** Plot the points (1, 2), (3, 4) and (5, -2) on a Cartesian plane and join them to form a triangle.

(b) Draw the image after a translation of 6 units left and 1 unit up.

(c) Write the coordinates of the vertices of the translated points.

(d) Draw the image after the image drawn in part **(b)** is reflected in the *x*-axis.

(e) Write the coordinates of the vertices of this new image.

(f) Finally, rotate the image drawn in part **(d)** 180° clockwise about the origin.

(g) Write the coordinates of your final image.

Symmetry

10.5

Need to Know

Reflectional symmetry

Reflectional symmetry or mirror symmetry occurs when a line can be drawn through an object to divide it into two identical but reversed parts. Each part is the mirror image of the other. This line is called the **axis of symmetry**.

- An axis of symmetry is a mirror line that can be drawn through a shape to divide it into two identical but reversed halves.
- Some objects have multiple axes of reflectional symmetry.
- The **order of reflectional symmetry** is the number of axes of reflectional symmetry that can be drawn through a shape. Draw as many axes of symmetry as possible so that the object is divided into two identical but reversed halves. Count the number of axes of symmetry. This gives us the order of reflectional symmetry.
- An **asymmetrical** object has no **reflectional symmetry**.

Rotational symmetry

If a figure can be turned less than 360° about a point so that the image is identical to the original, that figure is said to have **rotational symmetry**. The point about which the figure is rotated is called the centre of rotation.

- The **order of rotational symmetry** is the number of times in a complete 360° rotation around the centre of rotation that an identical image of the original would be produced. Rotate the original around this point until an image identical to the original is reproduced. Repeat this process until the figure has rotated through 360°. Count the number of times in a complete 360° rotation that an identical image would be produced. This is the order of rotational symmetry.
- Dividing the order of symmetry into 360° gives the angle of rotation.

Worked Example 11

WE11

Copy each of the following figures and determine **(i)** whether it has reflectional symmetry by showing every possible axis of symmetry and **(ii)** the order of reflectional symmetry, if present.

(a)

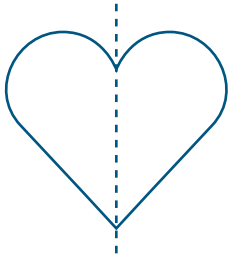


(b)



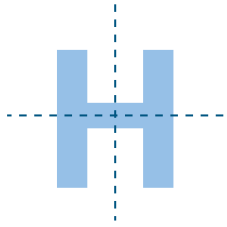
Working

(a) (i)



(ii) Order of reflectional symmetry = 1.

(b) (i)



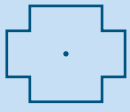
(ii) Order of reflectional symmetry = 2.

Worked Example 12

WE12

Determine **(i)** which of the following figures has rotational symmetry, **(ii)** the order of rotational symmetry and **(iii)** the size of the angle of rotation required to produce an identical image.

(a)



(b)



Working

(a) (i) Yes, it has rotational symmetry.

(ii) Order of rotational symmetry = 2.

(iii) $\frac{360}{2}$

$= 180°$

(b) (i) Yes, it has rotational symmetry.

(ii) Order of rotational symmetry = 5

(iii) $\frac{360}{5}$

$= 72°$

10.5 Symmetry

Answers page 513

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q10, Q11, Q12, Q13, Q15	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q10, Q11, Q12, Q13, Q14, Q16	Q1, Q2, Q3, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17

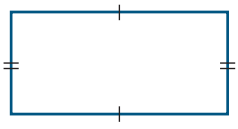
Equipment required: Grid paper for Question **12**

Fluency and understanding

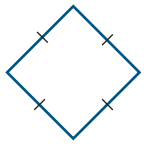
1 Copy each of the following figures and determine **(i)** whether it has reflectional symmetry by showing every possible axis of symmetry and **(ii)** the order of reflectional symmetry, if present.

WE11

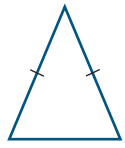
(a)



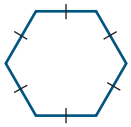
(b)



(c)



(d)



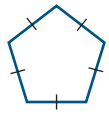
(e) W

(f) J

(g) F

(h) X

(i)



2 Determine **(i)** which of the figures from Question **1** has rotational symmetry, **(ii)** the order of rotational symmetry and **(iii)** the size of the angle of rotation required to produce an identical image.

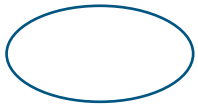
WE12

3 How many axes of symmetry do the following objects have?

(a)



(b)



(c)



For Questions **4**, **5** and **6** refer to the letters of the word M A T H S.

4 Which letter does not have an axis of symmetry?

A M **B** A **C** H **D** S

5 How many of the letters have one axis of symmetry?

A 5 **B** 2 **C** 3 **D** 4

6 Which letter has two axes of symmetry?

A H **B** M **C** A **D** S

7 Copy the lower case letters of the alphabet into your book.

a b c d e f g h i j k l m
n o p q r s t u v w x y z

(a) Which lower case letters are asymmetrical?

(b) Which lower case letters have one axis of symmetry?

(c) Which lower case letters have two or more axes of symmetry?

8 Copy the upper case letters of the alphabet into your book.

A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

(a) Which upper case letters are asymmetrical?

(b) Which upper case letters have one axis of symmetry?

(c) Which upper case letters have two or more axes of symmetry?

9 Do any of the capital letters from Question **8** have rotational symmetry? If so, which ones are they?

Reasoning, communicating and open-ended

10 Symmetry is present everywhere in nature. For each of the following images, determine whether it has almost perfect reflectional and/or rotational symmetry and state the order of each.

(a) 

(b) 

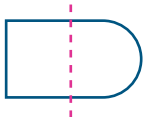
(c) 

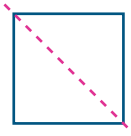
(d) 

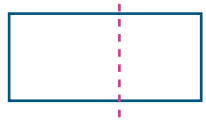
(e) 

(f) 

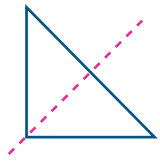
11 For each of the following shapes, state whether or not the dotted line is an axis of symmetry.

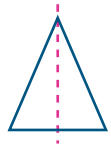
(a) 

(b) 

(c) 

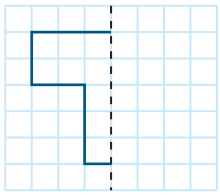
(d) 

(e) 

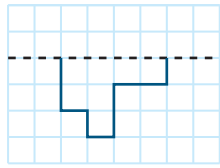
(f) 

12 For each of the following diagrams, use the dotted line as an axis of symmetry to complete the diagram using grid paper.

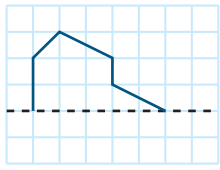
(a)



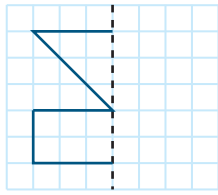
(b)



(c)



(d)



13 Which two quadrilaterals have exactly two lines of reflectional symmetry?

14 Which regular polygon has exactly three lines of reflectional symmetry?

15 WOW is a symmetrical word. Locate the axis of symmetry and find three more words that are also symmetrical.

16 The dates of some years have reflectional and/or rotational symmetry. An example of a year that has rotational symmetry is 1961 and a year that has reflectional symmetry is 1881.

Identify an example of a year that has:

(a) rotational symmetry

(b) reflectional symmetry

(c) both reflectional and rotational symmetry.

17 Describe when the following statement is true and when it is false.

'This parallelogram has reflectional symmetry.'

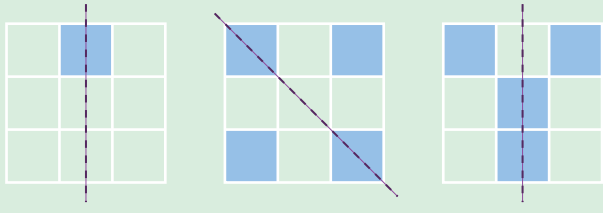
Problem solving

Symmetry challenge

How many symmetrical designs of shaded squares can you find in a blank 3 by 3 square grid?

Hint: Before you shade any squares, can you identify the lines of symmetry present in this 3 by 3 grid shown at left? What symmetries are possible if you shade one square only?

Here are three examples of possible symmetrical designs. Note the position of the line of symmetry.



Don't include any rotations;

e.g.  is considered the same as  .

Strategy options

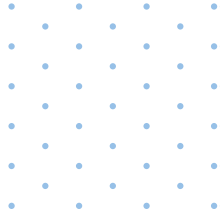
- Draw a diagram.
- Look for a pattern.
- Break problem into manageable parts.

10.6 Drawing and visualising 3D shapes

Need to Know

An **isometric** drawing is a two-dimensional (2D) drawing of a three-dimensional (3D) object. Isometric dot paper (triangular dot paper) is used to help draw 3D objects in 2D.

It is recommended that isometric dot paper be held in this orientation:



Worked Example 13

WE13

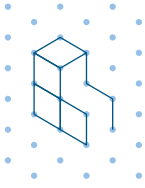
Use isometric dot paper to copy this shape.



Working



(Draw the top face and vertical lines from the corners of the face.)



(Draw lines from the bottom of the verticals you have just drawn, and then draw additional vertical lines from the ends of these lines.)



(Join the ends of the lines and shade in faces to create the illusion of 3D.)

10.6 Drawing and visualising 3D shapes

Answers page 514

Navigator

Q1, Q2 (a)–(c), Q3, Q4, Q5, Q6 (a)–(c), Q8, Q9 (a), Q10, Q11, Q13	Q1, Q2 (b), (d), (f), Q4, Q5, Q6 (c)–(e), Q7, Q8, Q9 (b), Q10, Q11, Q14	Q1, Q2 (d)–(f), Q4, Q6 (d)–(f), Q7, Q8, Q9 (c), Q10, Q11, Q12, Q14

Equipment required: Isometric dot paper for Questions **1**, **5**, **8**, **9** and **12–14**

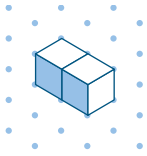
Fluency and understanding

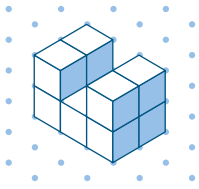
1 Use isometric dot paper to copy the following shapes.

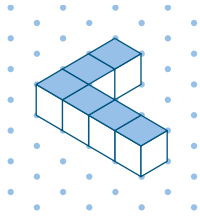
WE13

Hint

Is your isometric paper up the right way?

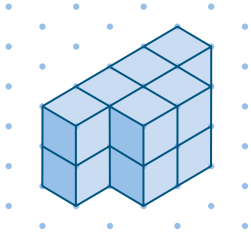
(a) 

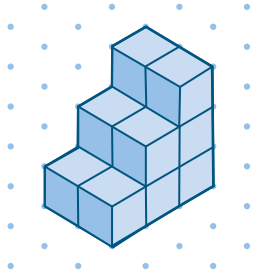
(b) 

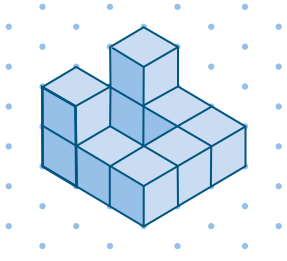
(c) 

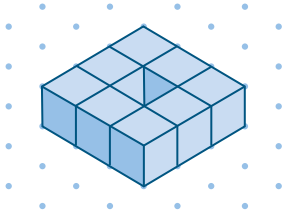
2 How many cubes would be required to build these solids? (Assume that there are no cubes missing at the back of the solids where you cannot see.)

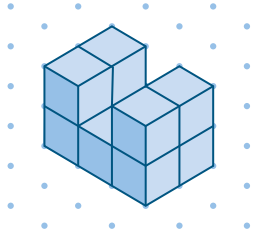
(a) 

(b) 

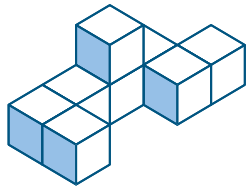
(c) 

(d) 

(e) 

(f) 

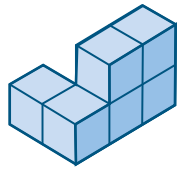
3 The number of cubes required to build this solid is:

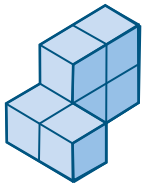


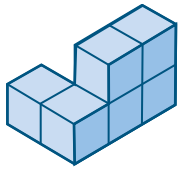
A 7 **B** 8

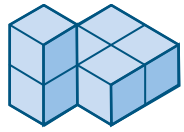
C 9 **D** 10

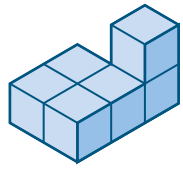
4 Which of the shapes below is *not* a rotation of the one shown?



A 

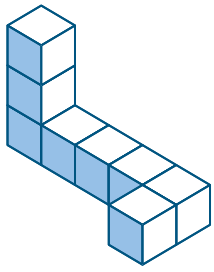
B 

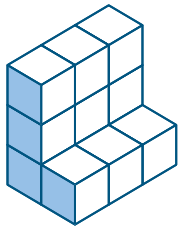
C 

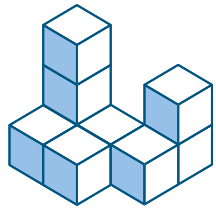
D 

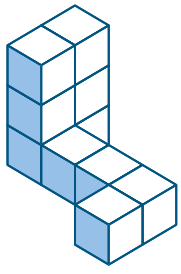
5 Using isometric dot paper, draw a 3D sketch of a Rubik's® cube. (A Rubik's cube is made of 27 cubes stacked three layers high, three layers wide and three layers deep.)

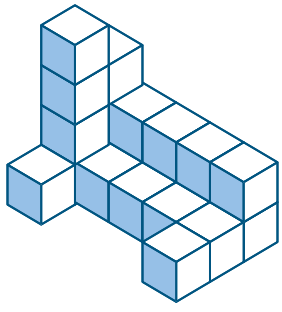
6 How many cubes are in each solid?

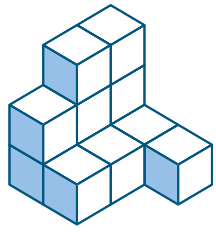
(a) 

(b) 

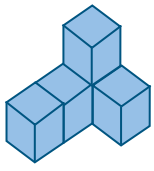
(c) 

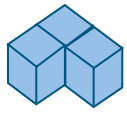
(d) 

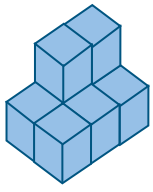
(e) 

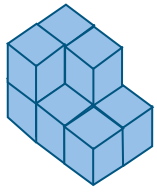
(f) 

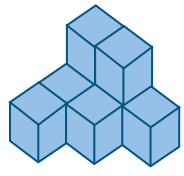
7 Which of the following shapes could result from joining the two shapes below?

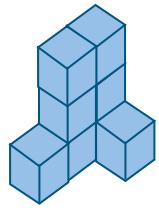




A 

B 

C 

D 

Reasoning, communicating and open-ended

8 Using this aerial photograph of the Pentagon in Washington, DC:



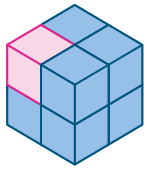
(a) draw a top view of the Pentagon

(b) draw a side view of the Pentagon.

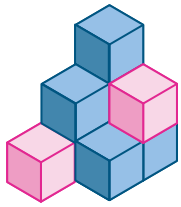
(c) Does it matter which side you choose? Explain.

9 Draw each solid after the pink shaded blocks have been removed.

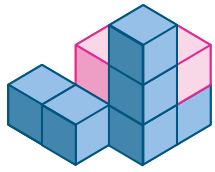
(a)



(b)



(c)



10 This shape is known as the impossible rectangle.



(a) Why is this shape known as the *impossible* rectangle? (You may like to try to make it with blocks.)

(b) Copy the impossible rectangle onto isometric dot paper.

(c) Draw a similar rectangle that is possible. Colour your drawing using three colours.

11 Draw the top view of this house.



12 A sculptor is designing a sculpture based on an arrangement of 24 cubes. Each cube has a width of 50 cm and the maximum height of the sculpture is to be 2.5 m. The sculptor wants to have a maximum of six cubes at the front and a maximum of four cubes in depth. Each cube must have at least one whole face touching another cube. Use isometric dot paper to design some sculptures that meet all these requirements.

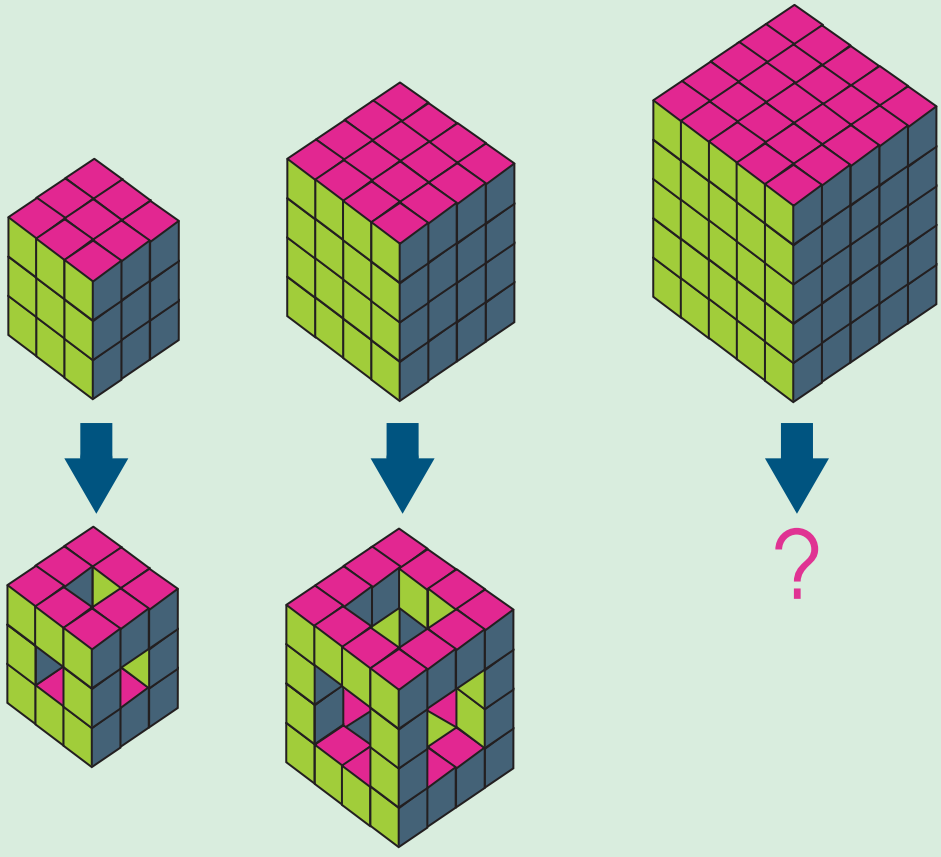
13 Using isometric dot paper, draw three different solids that can be built from 10 cubes that have whole faces touching.

14 Draw three shapes on isometric dot paper that, when joined together, form a $3 \times 3 \times 3$ cube.

Problem solving

What comes next?

Look at the pattern below.



A solid $3 \times 3 \times 3$ cube is made using 27 blocks. When one block is removed from the centre of each of the six faces and one block from the centre of the cube, a frame of the same cube is made using 20 cubes.

- Draw the next shape in the pattern.
- How many cubes are needed to make the frame of a $5 \times 5 \times 5$ cube?
- How many cubes are needed to make the frame of a $6 \times 6 \times 6$ cube?

Strategy options

- Make a table.
- Look for a pattern.

Plan views and elevations

10.7

Need to Know

Three-dimensional objects can be viewed from various positions or views.

Objects looked at from the top, front and side can be drawn in two dimensions, and are known as **plan views**, or orthogonal projections.

Drawings of houses and buildings viewed from a side are known as **elevations**.

Elevations may be referred to as front, rear and side elevations or by compass directions, such as east elevation.

Drawing elevations

- Highlight the face being viewed.
- Draw squares to make the shape you would see if you were looking at the face.

Worked Example 14

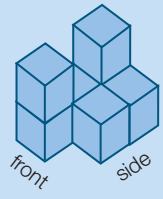
WE14

For the following solid, draw the:

(a) top view

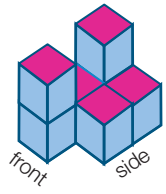
(b) front view

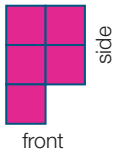
(c) side view.



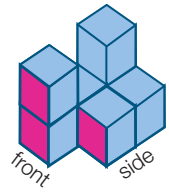
Working

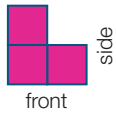
(a)



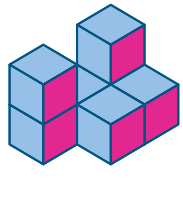


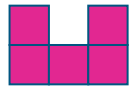
(b)





(c)





Answers page 514

10.7 Plan views and elevations

Navigator

Q1, Q2, Q3, Q5, Q6, Q7, Q9, Q10	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10

Equipment required: Isometric dot paper for Questions **5** and **7–10**

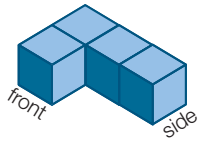
Fluency and understanding

WE14

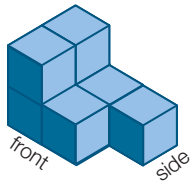
1 For each of the following solids, draw the:

(i) top view **(ii)** front view **(iii)** side view.

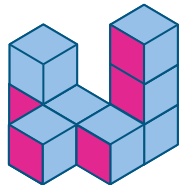
(a)



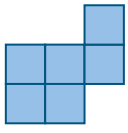
(b)



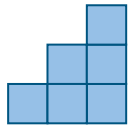
2 The front view of the solid shown has been shaded. The top view of this solid looks like:



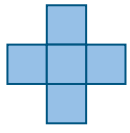
A



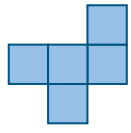
B



C

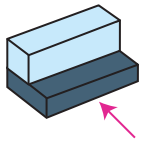


D

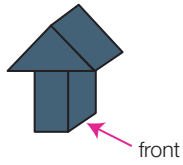


3 The following structures are made with coloured blocks. Draw the front elevation of each structure by imagining you are standing looking from the position of the arrow, then move 90° to the right and draw the side elevation.

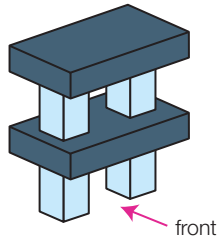
(a)



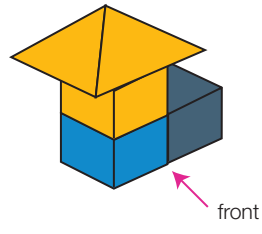
(b)



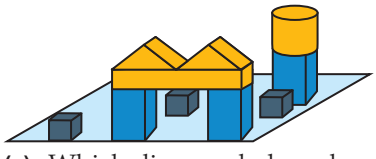
(c)



(d)

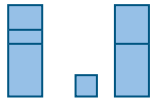


4

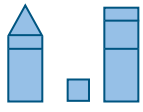


(a) Which diagram below shows a possible side elevation of the blocks above?

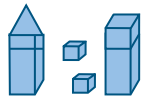
A



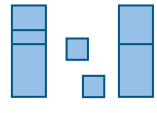
B



C



D



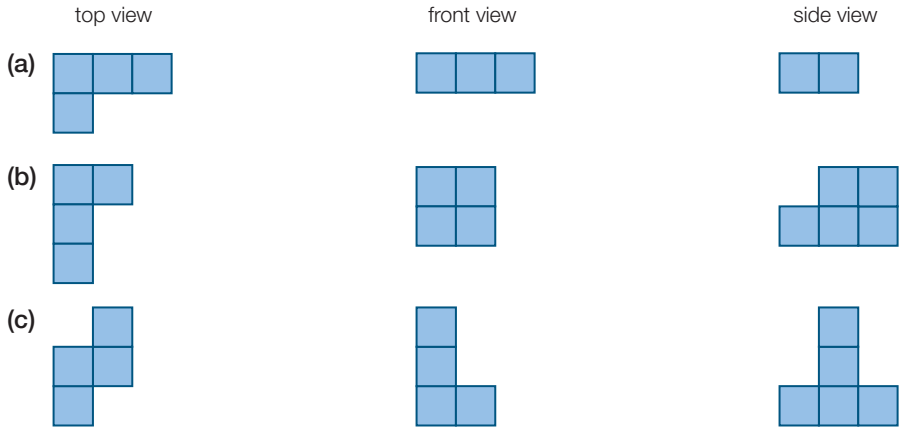
(b) Which elevation of the block structure above is shown opposite?



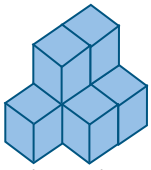
A front **B** rear

C side **D** top

5 For each set of the plan views below, draw the 3D solid on isometric dot paper.



6 A *mat plan* shows the top view of a shape and the number of cubes in each position.

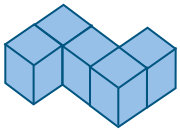

isometric drawing

2	1
2	1
1	

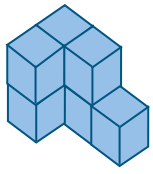
mat plan

Draw the mat plan for each of the shapes below.

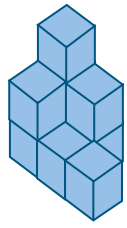
(a)



(b)



(c)



Reasoning, communicating and open-ended

7 Draw each of the following mat plans as 3D shapes on isometric dot paper.

(a)

1	1
1	

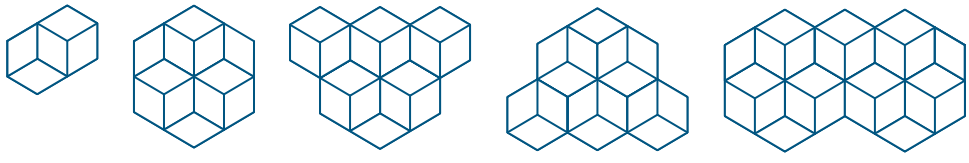
(b)

3	2
1	

(c)

3	3	3
2	2	
1		

8 (a) Copy the following diagrams onto isometric dot paper.



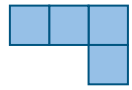
(b) On your drawings, use three colours to shade the different faces of the cubes.

(c) How many cubes are in each of the diagrams?

(d) There is more than one way to colour the diagrams. If the cubes were coloured differently, would each diagram still look like it has the same number of cubes?

(e) Draw the top view of each shape.

9 A shape made with small cubes looks like this from above.



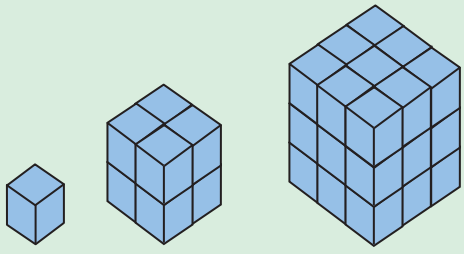
Draw three possible 3D drawings on isometric dot paper, stating the number of cubes in each diagram.

10 Select a three-dimensional object and draw the front, rear and side elevations.

Problem solving

Bird's-eye squares

Ezra glued some 1 cm cube blocks together to form different-sized cubes. He made four cubes of side length 2 cm, four cubes of side length 3 cm and had four 1 cm cubes left over.



Ezra made a solid with his different-sized cubes. The solid's top view was a square.

(a) What is the minimum number of cubes Ezra would need to build his solid?

(b) How many different solids can be made with the cubes he has made that have a top view of a square and are made with more than one cube? Draw the top view of each solid.

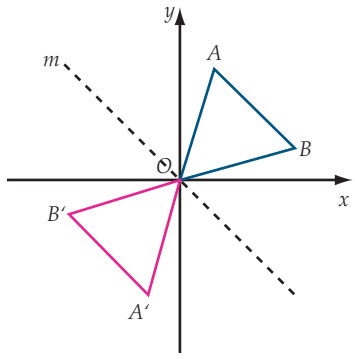
Strategy options

- Draw a diagram.
- Make a model.

Challenge 10



1 The equilateral triangle OAB has been transformed into the equilateral triangle $OA'B'$.



(a) Which single transformation changes OAB into $OA'B'$?

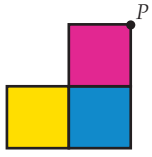
(b) Which two different transformations, applied one after the other, would change OAB into $OA'B'$?

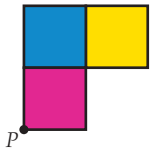
(c) OAB is reflected in the line m. What is the name of the new position of A?

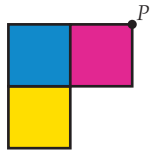
2 Jemima looks in a mirror and sees the clockface behind her. The hands appear to show 2:45. What will the time appear to be in 15 minutes?

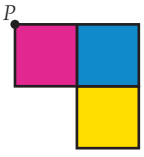
3 A horizontal line CD is copied three times, and each copy appears on top of the original. Using the terms 'rotation' and 'translation', describe how each copy must be moved in order to create a square. C ——— D

4 If the shape  P is rotated 90° clockwise about the point P, the resulting figure could be:

A 

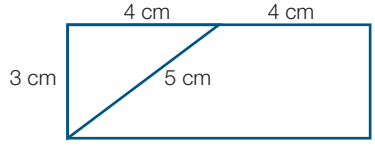
B 

C 

D 

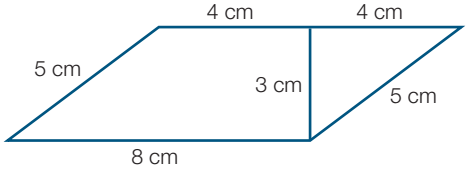
5 To be decoded, a message has to be read by looking in a mirror. Decode the following message: 'TAƎЯ OT ƆЯAH SI ƎGƎ ƎUIꟻƎD ꓤOIꟻOꓭ A'. To make sense of the message, what did you have to do?

6 A line is drawn on the 8 cm by 3 cm rectangle opposite to make a triangle and a trapezium.

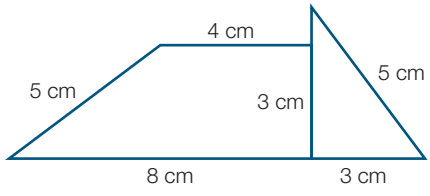


What transformation(s) need to be done to the triangle to create each of the following shapes?

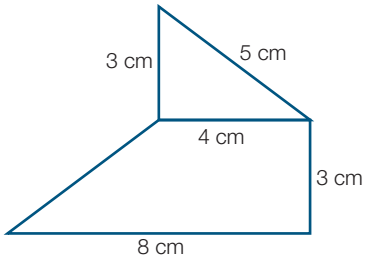
(a)



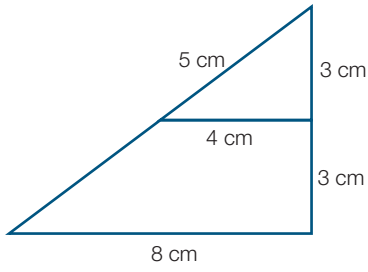
(b)



(c)



(d)



Chapter review 10

D.I.Y. Summary

Key Words

asymmetrical	isometric	perpendicular distance	rotation
axis of symmetry	line of reflection	plan views	rotational symmetry
centre of rotation	order of reflectional symmetry	reflection	transformation
elevations	order of rotational symmetry	reflectional symmetry	translation
image	original	rotate	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 Reflections, rotations and translations are three types of _____s.

2 When an object is turned about a point, a _____ has been performed.

3 When a figure is moved in a horizontal and vertical direction, a _____ has been performed.

4 When a figure is a reversed image of the original, a _____ has been performed.

5 The transformed version of a shape or diagram is called an _____.

6 A _____ acts as a mirror to reflect an object.

7 A point about which a figure is turned is called the _____.

8 The _____ indicates how many times the image is identical to the original in a complete 360° rotation.

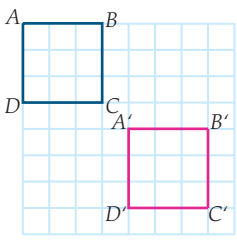
9 A 2D representation of a 3D solid can be drawn on _____ paper.

Fluency and understanding

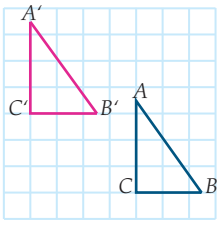
Equipment required: Grid paper for Questions **2**, **4–7**, **9**, **23** and **28**, isometric dot paper for Questions **14**, **16** and **17**, and a protractor for Questions **7**, **9**, **23** and **28**

1 Describe the translation shown in each of the following diagrams. Ex. 10.1

(a)

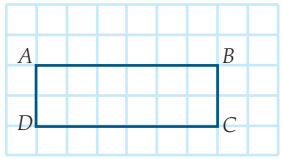


(b)

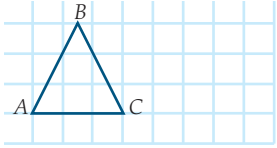


2 Copy each figure below and draw the resulting image after the translation required. Ex. 10.1

(a) 3 units right and 1 unit up



(b) 3 units left and 3 units up



3 What would be the reverse translation of 7 units right and 4 units down?

Ex. 10.1

A 7 units up and 4 units right **B** 7 units left and 4 units up

C 7 units left and 4 units down **D** 4 units left and 7 units up

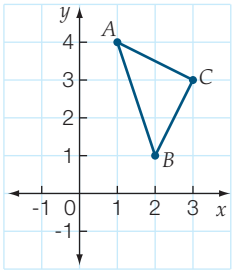
4 For the figure in the diagram opposite:

Ex. 10.1

(a) write the coordinates of the vertices

(b) copy the figure and translate it 4 units left and 3 units down, marking the image vertices with image notation

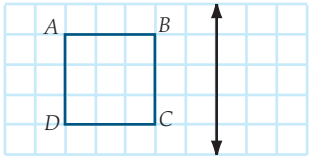
(c) write the coordinates of the vertices of the image.



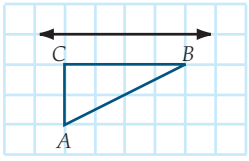
5 Copy each of the following figures onto grid paper and draw the resulting image when the figure is reflected in the line of reflection shown.

Ex. 10.2

(a)



(b)



6 For the figure in the diagram opposite:

Ex. 10.2

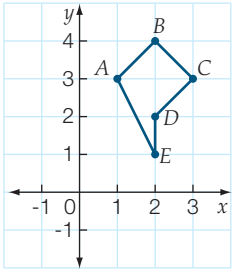
(a) write the coordinates of the vertices

(b) copy the figure and reflect it in the x-axis, marking the image vertices with image notation

(c) write the coordinates of the vertices of this image

(d) copy and reflect the original figure in the y-axis, marking the image vertices with image notation

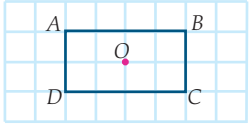
(e) write the coordinates of the vertices of this image.



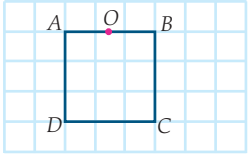
7 Copy each of the following figures onto grid paper and draw the resulting image after the given rotation. Use a protractor to help you.

Ex. 10.3

(a) Rotate 90° in a clockwise direction about O.



(b) Rotate 180° in an anticlockwise direction about O.



8 A clockwise rotation of 90° is equivalent to an anticlockwise rotation of how many degrees?

Ex. 10.3

A 180° **B** 360° **C** 90° **D** 270°

9 For the figure in this diagram:

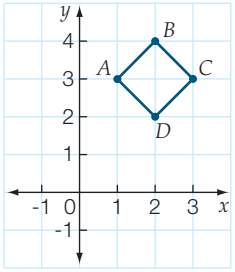
Ex. 10.3

(a) write the coordinates of the vertices

(b) copy the figure and rotate it 90° in an anticlockwise direction about the origin, marking the image vertices with image notation

(c) write the coordinates of the vertices of this image

(d) copy and rotate the original figure 180° in an anticlockwise direction about the origin, marking the image vertices with image notation



(e) write the coordinates of the vertices of this image

(f) copy and rotate the original figure 270° in an anticlockwise direction about the origin, marking the image vertices with image notation

(g) write the coordinates of the vertices of this image.

10 What are the reverse transformations for a translation of 5 units right and 8 units down followed by a clockwise rotation of 90°? Ex. 10.4

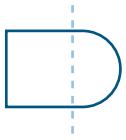
A An anticlockwise rotation of 90° and a translation of 5 units right and 8 units down

B A clockwise rotation of 180° and a translation of 5 units right and 8 units down

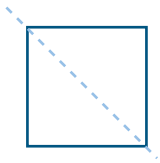
C An anticlockwise rotation of 90° and a translation of 5 units left and 8 units up

D A clockwise rotation of 90° and a translation of 5 units left and 8 units down

11 In which of the following shapes is the dotted line an axis of symmetry? Ex. 10.5

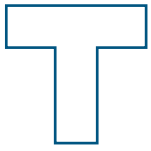
A 

B 

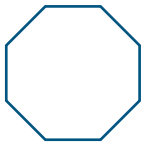
C 

D 

12 Copy the following shapes and draw lines of symmetry. Some shapes may have more than one line of symmetry. Ex. 10.5

(a) 

(b) 

(c) 

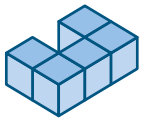
(d) 

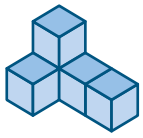
(e) 

(f) 

13 Identify the shapes from Question **12** that have rotational symmetry. Ex. 10.5

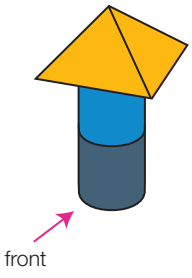
14 Two solids have been made using wooden cubes, as shown. Ex. 10.6

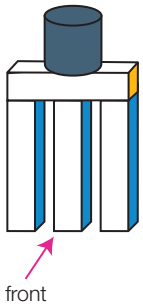
(a) 

(b) 

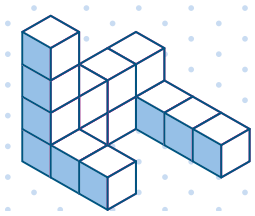
Use isometric dot paper to copy the shapes.

15 For each of the three-dimensional structures below, draw the front and side elevations. Ex. 10.7

(a) 

(b) 

16 Use isometric dot paper to draw the shape shown.

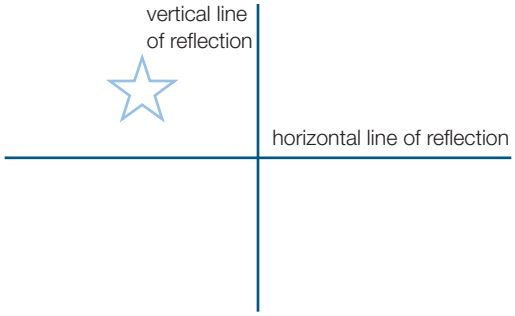


17 Use the shape above to draw:

(a) a front view **(b)** a top view **(c)** a side view.

18 What single translation would be equivalent to 6 units down, 7 units right, 2 units up, 8 units left, 4 units up and 5 units right?

19 **(a)** Using the diagram below, reflect the star in the vertical and then the horizontal lines of reflection.



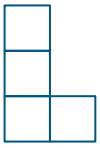
(b) Using the same starting position, reflect the star in the horizontal and then the vertical lines of reflection. Does it result in the same outcome as in **(a)**?

20 What rotation is equivalent to reflecting an object along horizontal, then vertical, lines of reflection?

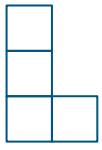
21 How many letters of the word MATHEMATICS do not have reflectional or rotational symmetry?

22 The plan views of a three-dimensional shape made with cubes are shown. Draw the solid on isometric dot paper.

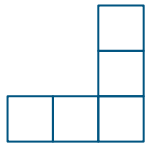
top view



front view



side view



23 On grid paper:

(a) draw the line joining the points $A(-1, 3)$ and $B(2, 5)$

(b) translate this line 3 units right and 2 units down

(c) reflect this image in the x-axis to form a new image

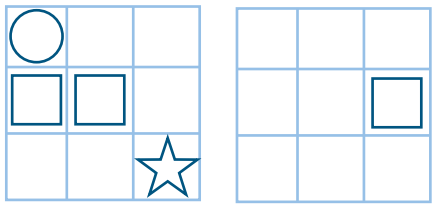
(d) rotate this image 90° in a clockwise direction (about the origin) to form a final image

(e) write the coordinates of the final image.

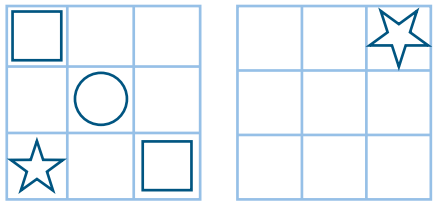
Reasoning and communicating

24 The following 3 by 3 squares comprise identical symbols, but each one has been reflected horizontally, vertically or along the diagonal and is missing three shapes. Your task is to draw the missing three shapes.

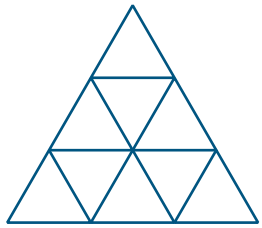
(a)



(b)



25 The following equilateral triangle comprises nine smaller equilateral triangles.

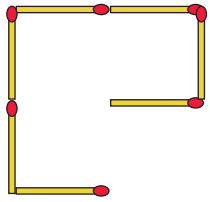


By shading the smaller equilateral triangles, create three different designs that have:

(a) one line of symmetry

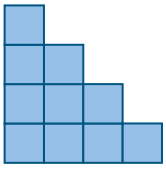
(b) three lines of symmetry.

26 The diagram at right shows a pattern made from sticking seven matchsticks of equal length to a piece of paper. What is the smallest number of matchsticks that need to be added so that the resulting image has at least one line of reflectional symmetry?

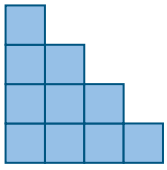


27 A solid is built using cubed blocks. The front, side and top views of the solid are shown. What is the minimum number of blocks this solid can have?

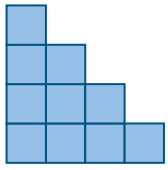
top view



front view



side view



28 Matt is asked to reflect a particular figure in the y-axis. This image is then rotated 180° in a clockwise direction.

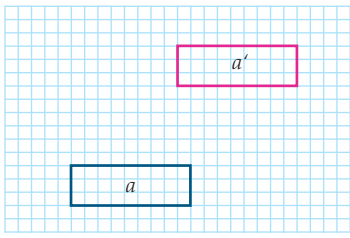
Matt believes he can replace these two combined transformations with one transformation.

Choose a shape of your own and perform the two transformations on it that Matt was required to perform. Is he correct? Can one transformation replace the two transformations? If so, write the required transformation.

NAPLAN practice 10

Numeracy: Non-calculator

1 The translation shown is:



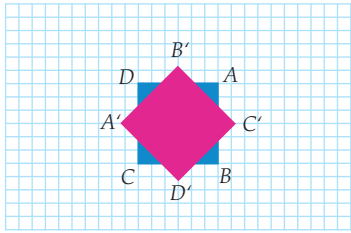
A 6 units right, 8 units up

B 8 units right, 9 units up

C 8 units down, 6 units left

D 6 units down, 8 units left

2 The rotation needed to transform the original shape (blue) to the new rotated image (pink) is:



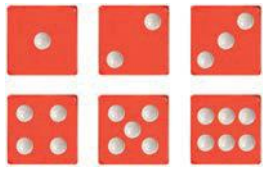
A 45° clockwise

B 135° anticlockwise

C 90° clockwise

D 90° anticlockwise

3 How many of the six faces of a die have fewer than three lines of symmetry?

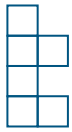


A 1 **B** 2

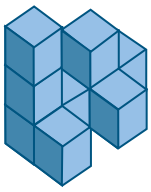
C 3 **D** 4

4 The top view of a shape is shown at right.

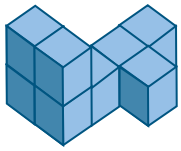
Which solid below has this top view?



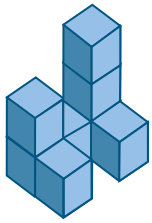
A



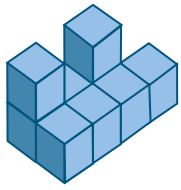
B



C



D

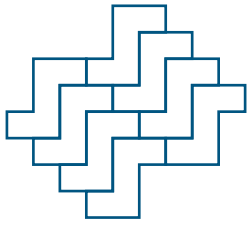


Numeracy: Calculator allowed

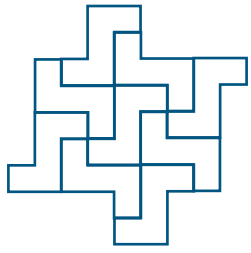
5 Mia has these S-shaped tiles.
They are blue on one side and red on the other side.
Mia wants to make a pattern with all the tiles blue side up.
Which one of the patterns can Mia not make?



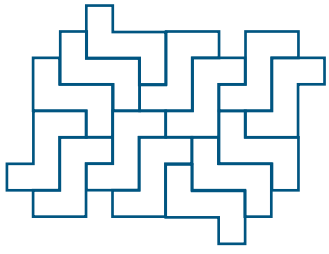
A



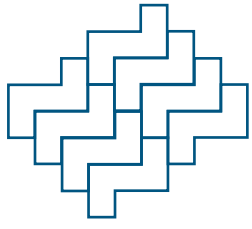
B



C



D

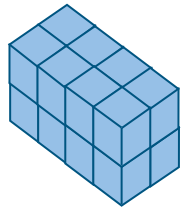


6 Which letter in the word SAME does not have an axis of reflectional symmetry?

A S **B** A **C** M **D** E

7 Janita glued small wooden cubes together to form the solid shown.
She then painted the solid on all surfaces.

How many wooden cubes are painted on just one face?



A 0 **B** 2 **C** 4 **D** 8

Mixed review E

Equipment required: Grid paper and protractor for Questions **11**, **17** and **18**

Fluency and understanding

1 Simplify: Ex 1.5

(a) $44 - 8 \times 4$ **(b)** $8 \times 6 \div 3 \times 4$ **(c)** $28 \div 4 + 3 \times 6$

2 Simplify: Ex 1.2

(a) $2^4 - 3^2$ **(b)** $3^3 \times 10^2$ **(c)** $10^5 - 10^3$

3 Calculate: Ex 4.4–4.6

(a) $15.006 + 2.45 + 0.059$ **(b)** 0.2×0.036 **(c)** $0.597 \div 0.3$

4 Use the following rules to complete the tables. Ex 5.3

(a) $n = 3m$

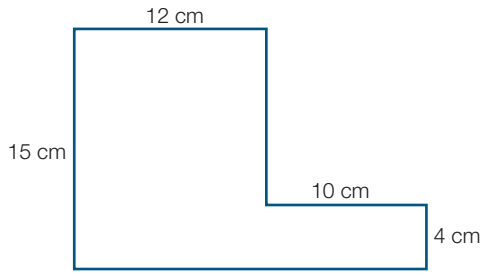
m	4	0	11	25	15
n					

(b) $t = s - 12$

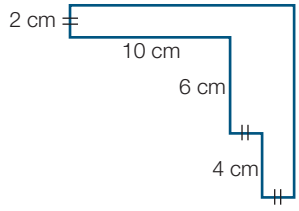
s	18	100	12	22	312
t					

5 Find the area of each of the following shapes. (All angles are right angles.) Ex 6.5

(a)



(b)



6 Find the following probabilities. Ex 9.7

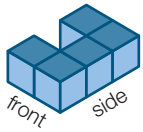
(a) getting a 6 on one roll of a normal die

(b) getting a red king on a single draw from a normal pack of cards

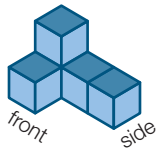
(c) getting two heads when tossing two coins

7 Two shapes have been made using wooden cubes, as shown. Ex 10.7

(a)



(b)



For each shape, draw: **(i)** the front elevation **(ii)** the top elevation.

8 Solve each of the following equations using the balance method. Check your answer by substitution. Ex 7.4

(a) $3(x - 1) = 6$ **(b)** $\frac{x}{4} + 1 = 7$ **(c)** $2(x + 3) = 12$

9 Find the supplementary angles for each of the following. Ex 8.3

(a) 35° **(b)** 127° **(c)** 90°

10 The following numbers represent the heights of students in a class, to the nearest centimetre. Ex. 9.1

148, 163, 158, 162, 151, 153, 170, 167, 165, 162, 172, 173, 157, 162, 158, 155, 163, 159, 160, 161

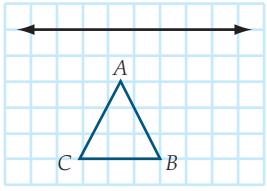
Draw the frequency table showing the information. You will need to group the results. Use 145–149, 150–154, 155–159 etc.

11 Show the image of each of the following figures after each of the following combined transformations. Use a protractor to help you. Ex. 10.4

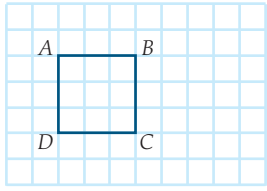
Reflect in the horizontal line of reflection and translate 4 units left and 3 units down.

Reflect in the line BC and rotate 90° in a clockwise direction about point C'.

(a)



(b)



12 For each of the following, identify the number of axes of symmetry. Ex. 10.5

(a) isosceles triangle **(b)** square **(c)** rhombus

13 Find the mean, mode, median and range of the following data, noting any possible outliers. Ex. 9.2

23, 34, 45, 26, 35, 42, 6, 34, 22, 36, 21, 29, 32, 34, 41, 39, 25, 27, 31, 30

14 A figure is translated 7 units left, 4 units down, 8 units right and 3 units up. What would be the final position of the image compared to that of the original figure?

15 **(a)** Find the prime factors of 24.

(b) Find the prime factors of 30.

(c) Use the prime factors to find the highest common factor of 24 and 30.

(d) Use the prime factors to find the lowest common multiple of 24 and 30.

16 **(a)** What is the size of each angle in an equilateral triangle?

(b) A parallelogram has one angle that measures 126°. What size is each of the three remaining angles?

17 Which capital letters of the alphabet that have both reflectional and rotational symmetry also have reflectional and rotational symmetry in lower case form?

Reasoning and communicating

18 A point (1, 1) is translated 2 units right and 1 unit up. It is then reflected in the y-axis and then reflected in the x-axis. It is then translated 2 units right and 1 unit up. Finally, it is rotated 180° in a clockwise direction about the origin. What are the coordinates of the final image?

19 **(a)** Find a data set with at least six different values that has a median of 6 and a mean of 8.

(b) Describe the process you followed to find these values.

(c) Why might it be easier to use an odd number of values in a question of this type?

20 A game is played in which three different coins are being tossed. It costs 50 cents to play this game. You win $1 (plus your original 50 cents) if more than one of the coins shows heads, otherwise you lose your money.

(a) Write down the set of results that are possible.

(b) Is the game fair? Explain your reasoning.

Answers

Worked solutions and answers to ***all*** activities appear on the eBook.

Chapter 1

Recall 1 (p. 2)

1 (a) B (b) D

2 (a) 0, 12, 74, 567, 602, 4500, 6008, 11 100

(b) 2400, 2196, 1200, 1010, 987, 240, 204, 95

3 (a) 56 895 (b) 7 025 073

4 (a) (i) 1250 (ii) 1200 (iii) 1000

(b) (i) 8980 (ii) 9000 (iii) 9000

5 (a) 18 (b) 90 (c) 8 (d) 10 000

6 (a) 512 (b) 2063 (c) 1698

7 (a) 175 (b) 627 (c) 1157

8 (a) 315 (b) 670 (c) 3230

9 (a) 211 (b) 412 (c) 128 rem 1

Exercise 1.1 (p. 4)

1 (a) 49 (b) 77 (c) 138 (d) 1900

(e) 1500 (f) 23 000 (g) 73 (h) 107

(i) 199 (j) 60 (k) 120 (l) 420

2 (a) 153 (b) 152 (c) 294 (d) 186

(e) 364 (f) 215 (g) 891 (h) 231

(i) 456 (j) 808 (k) 565 (l) 1364

3 (a) 121 (b) 503 (c) 165 (d) 228

(e) 178 (f) 216 (g) 618 (h) 242

(i) 582 (j) 360 (k) 266 (l) 420

4 C

5 (a) 245 km (b) 235 km

6 1000

7 (a) \$136 (b) \$53

8 (a) 720

(b) May be about to begin or end a busy time (e.g. peak hour). A bottleneck, accident or other traffic hold-up may occur.

9 \$390

10 (a) \$7.84 (b) \$2.31 (c) \$7.08 (d) \$4.10

11 (a) Multiplying 7 by 20 gives one lot less of 7 than needed, not one lot less of 21. Need to multiply 7 by 20, then add 7 to get 147.

(b) Doubling twice is the same as multiplying by 4, not by 3. You can double 35 to get 70, then add another 35 to get 105.

(c) The remaining 9 should have been subtracted to get 191, not added.

12 (a) Alex and Khalid's methods are correct.

(b) They both round off the multiplier (Alex rounds 9 to 10 and Khalid 29 to 30), multiply, then subtract 1 lot of the number they multiplied by (Alex subtracts 1 lot of 29, as he had 10 lots instead of 9, and Khalid subtracts 1 lot of 9, as he had 30 lots instead of 29).

(c) The other two students round off correctly, but then subtract the wrong number.

13 5 picture hooks at 28 cents each = \$1.40;
12 curtain rings at 15 cents each = $12 \times 10 + 12 \times 5 =$ \$1.80.
Total: \$(1.40 + 1.80 + 6) = \$9.20.
Tranh should have enough money.

14 Do the calculations $9 \times 23 + 2 \times 23$.

15 Group the numbers and then use multiplication and addition. He has 7 1s (worth $7 \times 1 = 7$), 9 2s (worth $9 \times 2 = 18$) and so on. When the multiplications are finished, add together the results. He should get 50.

Exercise 1.2 (p. 7)

1 (a) $2 \times 2 \times 2 = 8$

(b) $2 \times 2 \times 2 \times 2 = 16$

(c) $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$

(d) $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1$

(e) $0 \times 0 \times 0 \times 0 \times 0 \times 0 \times 0 = 0$

(f) $10 \times 10 \times 10 = 1000$

(g) $6 \times 6 \times 6 \times 6 = 1296$

(h) $5 \times 5 \times 5 \times 5 \times 5 = 3125$

(i) $8 \times 8 \times 8 \times 8 \times 8 \times 8 = 262\,144$

(j) $11 \times 11 \times 11 = 1331$

(k) $12 \times 12 \times 12 \times 12 = 20\,736$

(l) $14 \times 14 \times 14 \times 14 \times 14 = 537\,824$

2 (a) 8^3 (b) 4^6 (c) 12^5 (d) 16^9

(e) 17^3 (f) 19^2 (g) 8^4 (h) 13^7

(i) 11^7 (j) 9^6

3 (a) 16 384 (b) 46 656 (c) 6561 (d) 512

(e) 16 807 (f) 3375 (g) 289 (h) 125 000

(i) 64 (j) 500 (k) 4000 (l) 2000

(m) 1024 (n) 19 683 (o) 1

4 (a) five squared (b) thirty-one squared

(c) three cubed (d) twenty-seven cubed

(e) four to the power of five (f) nine to the power of six

(g) one to the power of four

(h) seven to the power of seven

5 25, 36, 49

6 64, 125, 216

7 **(a)** 5 **(b)** 7 **(c)** 10 **(d)** 11
(e) 1 **(f)** 0 **(g)** 70 **(h)** 20
(i) 2 **(j)** 4 **(k)** 10 **(l)** 1
(m) 0 **(n)** 5 **(o)** 20 **(p)** 30

8 **(a)** A **(b)** C **(c)** A

9 **(a)** 5 **(b)** 7 **(c)** 65 **(d)** 32
(e) 64 **(f)** 243 **(g)** 21 **(h)** 45
(i) 51

10 **(a)** 1^{200}, 5^4, 10^3, 4^5, 5^5, 4^6
(b) 10^5, 100^2, 3^2, 2^3, 1^{1000}, 0^{100}

11 **(a)** 16, 36 **(b)** 49, 81

12 **(a)** 0 **(b)** 190 000 **(c)** 45 000 **(d)** 2 472 768
(e) 537 824 **(f)** 51 998 079 **(g)** 266 240 **(h)** 79 785
(i) 439 291

13 **(a)** 3, 4 **(b)** 2, 3 **(c)** 4, 5 **(d)** 7, 8
(e) 9, 10 **(f)** 1, 2 **(g)** 8, 9 **(h)** 10, 11

14 **(a)** $\sqrt[3]{64}$, 5, $\sqrt{49}$, 2^4 **(b)** $\sqrt[3]{125}$, $\sqrt{60}$, 9, 7^2
(c) $\sqrt[3]{8}$, $\sqrt{90}$, 10, 4^2 **(d)** 8, $\sqrt{80}$, $\sqrt{105}$, 3^3

15 **(a)**

Index form	Expanded form	Value
10^1	10	10
10^2	10×10	100
10^3	$10 \times 10 \times 10$	1 000
10^4	$10 \times 10 \times 10 \times 10$	10 000
10^5	$10 \times 10 \times 10 \times 10 \times 10$	100 000

(b) The index number in the first column is the same as the number of zeroes after the 1 in the third column.

16 **(a)** 4 **(b)** 2 **(c)** 3 **(d)** 5
(e) 1 **(f)** 6 **(g)** 4 **(h)** 2
(i) 4

17 **(a)** $5^2 \times 5^3 = 25 \times 125 = 3125$. $5 \times 5 \times 5 \times 5 \times 5 = 3125$
(b) $2^3 \times 2^6 = 8 \times 64 = 512$. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 512$
(c) The sum of the powers is the number of times the base is multiplied with itself to obtain the answer.

18 **(a)** 4 **(b)** 10 **(c)** 16

19 **(a)** **(i)** True **(ii)** True **(iii)** True **(iv)** False
(b) **(i)** False **(ii)** True

20 **(a)** **(i)** one hundred **(ii)** one hundred
(b) **(i)** 10^{100} **(ii)** 10^{100}, more than a lifetime

21 **(a)** 121; 12 321; 1 234 321
(b) 123 454 321; 12 345 654 321; 1 234 567 654 321

22 $12^3 = 1728$; $31^2 = 961$; $3^{21} = 10\,460\,353\,203$

23 **(a)** 5, 6, 7, 8 or 9 **(b)** 10, 11, ... 21

24 **(a)** Mina is partly correct. The 'little 2' does mean two 35s, but they are multiplied together, not added. A better explanation for Mina to give Jo would be: 'The 2 tells you how many 35s you need to multiply together.'
(b) 2^2 is a bit of a special case. $2^2 = 2 \times 2$, which gives the same answer as $2 + 2$ or 'double 2'. However, this is not true for any other number, except 0.

Exercise 1.3 (p. 13)

1 **(a)** 108 **(b)** 246 **(c)** 496 **(d)** 150 **(e)** 17
(f) 54 **(g)** 315 **(h)** 65 **(i)** 1050 **(j)** 180
(k) 18 **(l)** 15

2 **(a)** 492 **(b)** 442 **(c)** 1292 **(d)** 1403 **(e)** 1519
(f) 2184 **(g)** 1974 **(h)** 2597 **(i)** 3025 **(j)** 5002
(k) 5544 **(l)** 4896

3 **(a)** 2100 **(b)** 2000 **(c)** 26 000 **(d)** 18 **(e)** 27
(f) 16 **(g)** 480 000 **(h)** 12 800 000
(i) 140 000 000 **(j)** 600 **(k)** 500 **(l)** 2100

4 **(a)** 9 **(b)** 5 **(c)** 32 **(d)** 760
(e) 1060 **(f)** 1836 **(g)** 150 **(h)** 43
(i) 150 **(j)** 1440 **(k)** 4248 **(l)** 2450

5 48 000 m

6 Divide by 10 and double the answer: 26.

7 $1248

8 $250

9 $648 000

10 92 streets, some left over

11 **(a)** $66 **(b)** $55 **(c)** He will gain $55.

12 **(a)**

×	100	60	2	+
200	20 000	12 000	400	= 32 400
40	4000	2400	80	= 6480
6	600	360	12	= 972
				39 852

(b)

×	300	50	6	+
400	120 000	20 000	2400	= 142 400
10	3000	500	60	= 3560
2	600	100	12	= 712
				146 672

(c)

×	100	0	7	+
500	50 000	0	3500	= 53 500
60	6000	0	420	= 6420
0	0	0	0	= 0
				59 920

13 B

14 **(a)** 2×810, 4×405, 5×324, 10×162, 20×81, 30×54, 6×270, 12×135, 18×90
(b) 18×90, 30×54, 20×81

15 **(a)** $9 \times 24 = 216$
$12 \times 43 = 516$

(b) Kim has forgotten that the first digit in the numbers she is splitting up (24 and 43) is in the 'tens' place value column. She is multiplying them as though they were units.

(c) Write out the number being split into tens and units:
$9 \times 24 = 9 \times (20 + 4)$
$12 \times 43 = 12 \times (40 + 3)$

Half-time 1 (p. 15)

1 (a) 340 (b) 690 (c) 185
(d) 760 (e) 239 (f) 261

2 (a) (i) 216 (ii) 3125 (iii) 10 (iv) 49
(v) 7 (vi) 2 (vii) 10 (viii) 4
(b) (i) 49 and 4 (ii) 216

3 (a) 171 (b) 328 (c) 190 (d) 374
(e) 441 (f) 636 (g) 4757 (h) 7728

4 (a) 34 (b) 29 (c) 116 (d) 47
(e) 16 (f) 39 (g) 18 (h) 56

5 (a) 3^6 (b) 6^5 (c) 4^7 (d) 15^2
(e) 21^3 (f) 41^3

6 (a) 4200 (b) 3000 (c) 120 000
(d) 200 000 (e) 90 (f) 30
(g) 5 (h) 620 (i) 150

7 $676 000

Exercise 1.4 (p. 18)

1 (a) 70 (b) 70 (c) 10 (d) 6
(e) 600 (f) 700 (g) 500 (h) 1000
(i) 5000 (j) 4000 (k) 10 000 (l) 10 000
(m) 80 000 (n) 700 000 (o) 800 000 (p) 2 000 000

2 (a) (i) $80 \times 40 = 3200$ (ii) $3200 + 80 + 40 + 1 = 3321$
(iii) 3321

(b) (i) $50 \times 80 = 4000$ (ii) $3200 + 160 + 560 + 28 = 3948$
(iii) 3948

(c) (i) $60 \times 90 = 5400$ (ii) $4800 + 160 + 540 + 18 = 5518$
(iii) 5518

(d) (i) 60 (ii) $\frac{2970}{54} = \frac{495}{9} = 55$ (iii) 55

(e) (i) 50 (ii) $\frac{3162}{62} = \frac{102}{2} = 51$ (iii) 51

(f) (i) 20 (ii) $\frac{1950}{75} = \frac{650}{25} = \frac{130}{5} = 26$ (iii) 26

3 (a) (i) $40 \times 25 = 1000$ (ii) 1066, fairly close
(b) (i) $120 \times 55 = 6600$ (ii) 6588, very close
(c) (i) $200 \times 15 = 3000$ (ii) 2674, not very close
(d) (i) $400 \div 40 = 10$ (ii) 9, very close
(e) (i) $400 \div 20 = 20$ (ii) 18, very close
(f) (i) $200 \div 50 = 4$ (ii) 4, exact
(g) (i) $1600 \times 25 = 40 000$ (ii) 36 501, fairly close
(h) (i) $34 000 \times 20 = 680 000$ (ii) 713 937, not very close
(i) (i) $4000 \times 250 = 1 000 000$ (ii) 1 021 108, fairly close
(j) (i) $3500 \div 50 = 70$ (ii) 68, very close
(k) (i) $11 000 \div 100 = 110$ (ii) 113, very close
(l) (i) $45 000 \div 15 = 3000$ (ii) 3207, fairly close

4 (a) C
(b) (i) C (ii) B

5 (a) $1200 + 300 = 1500$ (b) $10 300 + 200 = 10 500$
(c) $15 000 + 3000 = 18 000$ (d) $820 - 80 = 740$
(e) $1200 - 200 = 1000$ (f) $500 - 300 = 200$

6 (a) 24 000, actual answer will be lower
(b) 800 000, actual answer will be higher
(c) 4000, actual answer will be lower
(d) 100, actual answer will be lower
(e) 10, actual answer will be higher
(f) 6, actual answer will be lower

7 (a) D (b) B (c) A (d) D

8 (a) 300 000 (b) $6 million

9 (a) $760 (b) $9000

10 (a) A (b) $16 \times 15 \div 2 = 120$

11 79×5003 will give the closer answer because the numbers are much closer to the rounded ones (80 and 5000). 5488 is rounded a long way down and so gives an estimated answer that is a lot less than the actual one.

12 (a) $1344 \div 21 \approx 1000 \div 20 = 50$

(b) $1344 \div 21 \approx 1340 \div 20 = 67$

(c) Answer (b) will give a closer estimate to the answer because the initial numbers have been rounded far less.

(d) $1344 \div 21 = 64$—much closer to answer (b)

13 If she estimated by rounding down, she would run the risk of not having enough seats on the buses for 428 people; e.g. rounding to the first digit gives $400 \div 50 = 8$ buses, but eight 52–seater buses have 416 seats—way too few.

14 (a) Natalie's rounded down total outweighs her rounded up total. Natalie rounded the T-shirt up by $3 and the lip gloss up by $1 = $4. She rounded the earrings down by $4 and the necklace down by $3 = $7. She is $3 short.

(b) A 'safer' method is to round up all purchases to a convenient multiple of 5 or 10.

15 (a) 39 and 82 (b) 12 and 19
(c) 214 and 487 (d) 72 and 573

Exercise 1.5 (p. 22)

1 (a) 25 (b) 7 (c) 11 (d) 0
(e) 3 (f) 17 (g) 9 (h) 12
(i) 9 (j) 50 (k) 5 (l) 84
(m) 21 (n) 14 (o) 88 (p) 41

2 **(a)** False **(b)** True **(c)** False **(d)** True
(e) False **(f)** True

3 **(a)** C **(b)** D

4 **(a)** **(i)** 2×6, then 2×2 **(ii)** $15 \div 5$, then 6×3
(b) **(i)** 6 **(ii)** 21

5 **(a)** $(6 + 6) \times 3 = 36$ **(b)** $(10 - 4) \times 5 = 30$
(c) $(9 - 8) \times 6 + 4 = 10$ **(d)** $12 + 6 \div (7 - 4) = 14$
(e) $6 \div (3 + 3) \times 5 = 5$ **(f)** $3 \times 6 \div (8 - 4 + 5) = 2$
(g) $3 \times (10 - 7) \div 9 + 12 = 13$ **(h)** $18 \div 3 \times (5 - 3) + 2 = 14$
(i) $(7 + 3) \div (4 + 1) = 2$ **(j)** $(5 - 3) \times (8 - 6) \div 2 = 2$

6 **(a)** $73 \times 5 + 29 + 50 \times 8 = 794$, \$7.94
(b) $2000 - (5 \times 73 + 29 + 8 \times 50) = 1206$, \$12.06

7 **(a)** $6 \times 90 + 5 \times 70$
$= 540 + 350$
$= 890$ cents
$= \$8.90$

(b) $6 \times 90 + 6 \times 70 + 5 \times 50$
$= 540 + 420 + 250$
$= 1210$ cents
$= \$12.10$

(c) \$5.30

8 **(a)** $2 + 21 \div 3 = 9$ **(b)** $15 - 6 \div 2 = 12$
(c) $14 - 8 - 6 = 0$ **(d)** $7 \times 5 - 6 = 29$
(e) $14 + 3 - 2 = 15$ **(f)** $(24 + 6) \div 10 = 3$
(g) $8 + 5 \times 2 - 6 = 12$ **(h)** $12 \div 2 + 1 \times 9 = 15$

9 **(a)** $6 \times (4 \div 2) \times 3 = (6 \times 4) \div 2 \times 3$
(b) $(1 + 4) \times 20 \div 5 > 1 + (4 \times 20) \div 5$
(c) $100 + 10 \div 10 > (100 + 10) \div 10$
(d) $36 \div 6 \times (3 - 3) < 36 \div 6 \times 3 - 3$

10 **(a)** Both pairs of brackets are unnecessary.
(b) Both pairs of brackets are unnecessary.
(c) First pair is unnecessary.
(d) Second pair is unnecessary.

11 **(a)** 26 is the correct answer.
(b) The calculator 'does' order of operations, the phone does every operation as it is entered.

12 $(8 + 4) \times 6 \div 2 - 1 = 35$; $8 + (4 \times 6) \div 2 - 1 = 19$;
$8 + 4 \times (6 \div 2) - 1 = 19$; $8 + 4 \times 6 \div (2 - 1) = 32$;
$(8 + 4 \times 6) \div 2 - 1 = 15$; $8 + (4 \times 6 \div 2) - 1 = 19$;
$8 + 4 \times (6 \div 2 - 1) = 16$

13 Correct answer is 75. Step 2 is incorrect—he needed to do $30 \div 2$ first, then multiply by 5.

Exercise 1.6 (p. 26)

1 **(a)** 724 **(b)** \$9225 **(c)** \$351 **(d)** 31 hours

2 30 minutes

3 72 laps

4 50 400 min

5 **(a)** 5998 m **(b)** 4643 m

6 Shoes $68 + 34 = \$102$
T-shirts $19 + 9 = \$28$
Earrings = \$25
Total = \$155

7 $192 - 18 = 174$ buns sold

8 **(a)** 222 km **(b)** 3931 km **(c)** 3709 km

9 73 cm **10** \$2646 **11** 42

12 **(a)** 3 trips there and back (12 boxes), plus 1 more trip to new house (with last box), = 7 times
(b) 49 km

13 **(a)** 11 weeks, 3 days
(b) 2 months, 2 weeks, 6 days
(c) 2 months, 3 weeks, 3 days
(d) In **(b)**, 80 was divided by 30 to find the number of months. Dividing by a larger number gives a smaller remainder to split into weeks and days. In **(c)**, 80 was divided by 28 (4 weeks).

14 Vet + Jason + 2 Scruffys: $91 + 56 = 147$ kg
Scruffy's weight $= 147 - 138 = 9$ kg

15 **(a)** The two numbers need to be different by at least 30; e.g. 25 and 66.
(b) The two numbers need to be single-digit numbers; e.g. 4 and 7, 5 and 6, 9 and 3.

16 **(a)** 20×300 mL bottles: 6 kg
20×600 mL bottles: 12 kg
20×1000 mL bottles: 20 kg
(b) Numbers of each box that give a total mass of 1000 kg:
1 L: 50 boxes
600 mL: 83 boxes
300 mL: 166 boxes
(c) 23×1 L boxes + 30×600 mL boxes + 30×300 mL boxes
26×1 L boxes + 26×600 mL boxes + 28×300 mL boxes

Challenge 1 (p. 28)

1 D
Number must end in 6 or 8 to be even, smallest number needs biggest digit at end, so ends in 8. Largest remaining digit needs to be in the tens place.

2 C
List the possibilities, $2 \times 2 = 4$, $2 \times 4 = 8$, $2 \times 6 = 12$, $2 \times 8 = 16$, Note: 16 is repeated. $4 \times 4 = 16$, $4 \times 6 = 24$, $4 \times 8 = 32$, $6 \times 6 = 36$, $6 \times 8 = 48$, $8 \times 8 = 64$.

3 B
Three-digit number has to be as small as possible, two-digit number as large as possible: $356 - 87 = 269$.

4 $10 \times \$5 = \$50$, Kim has \$50 more, whereas Brian has \$10 less than the amount they each started with.

By trial and error: If they both start with \$50, Kim will now have \$100 and Brian will have \$40. But $\$40 \times 3 = \$120$. Too much, so try a smaller amount. If they both start with \$40, Kim will now have \$90 and Brian will have \$30. $\$30 \times 3 = \$90$. They both started with \$40.

5 Find the factors of 15 181. It helps to realise that the only pairs of numbers that multiply to give a number ending in 1 are: 1 and 1, 3 and 7, 9 and 9. Hence, possible teenage children could be 11, 13, 17 or 19. See which of these numbers are factors of 15 181, then work out the remaining age. The ages are 17, 19 and 47, so the father is 47 years old.

6 Numbers are consecutive odd numbers.

Sum of first row = 1, sum of second row = 3 + 5 = 8, sum of third row = 7 + 9 + 11 = 27

$1 = 1^3$, $8 = 2^3$, $27 = 3^3$. Sum of row is row number cubed.

Sum of 7th row = $7^3 = 343$.
Sum of 100th row = $100^3 = 1\,000\,000$.

7 $\sqrt{8} \approx 2.8$, $\sqrt{8008} \approx 89.5$, so the squares of 3, 4, 5, … 89 are between 8 and 8008. 89 – 2 = 87 numbers.

8 After the first two terms, each term is the sum of the previous two terms: 18 + 29 = 47, 29 + 47 = 76, 47 + 76 = 123. Terms are 47, 76, 123.

9 1001 ÷ 6 = 166.83, so 1001 will appear in the 167th row. The odd-numbered rows run from left to right and the row would start with 997, as row 166 ends with 996 (166 × 6) in column A. Counting across gives 1001 in column E.

10 (a) 0, 1, 4, 9, 16, 25, 36, 49, 64, 81

(b) 0, 1, 4, 5, 6, 9

(c) 2, 3, 7, 8

(d) (i), (iii), (v)

(e) 961: factor ends in 1 or 9. $900 = 30^2$, so try 31^2.

Chapter review 1 (p. 29)

1 (a) 142 (b) 138 (c) 225 (d) 120
(e) 315 (f) 437 (g) 180 (h) 544

2 (a) 144 (b) 125 (c) 100 000 (d) 8
(e) 30 (f) 3 (g) 129 (h) 3853
(i) 72

3 (a) 7^5 (b) 10^3 (c) 5^2 (d) 12^8

4 (a) 1400 (b) 170 000 (c) 5299 (d) 3038
(e) 22 (f) 1593 (g) 153 (h) 108

5 (a) 8000 (b) 60 000 (c) 50 (d) 300
(e) 60 000 (f) 3 000 000 (g) 8 (h) 300

6 (a) 550 ÷ 10 = 55 (b) 2000 ÷ 20 = 100
(c) 15 600 ÷ 200 = 78 (d) 30 × 600 = 18 000
(e) 160 × 400 = 64 000 (f) 500 × 2000 = 1 000 000
(g) 54 000 ÷ 900 = 60 (h) 12 800 ÷ 200 = 64

7 (a) 1 (b) 13 (c) 11 (d) 6 (e) 5 (f) 41

8 $66

9 720

10 (a) 80 000, close to (b) 90 000, close to
(c) 600, lower (d) 5000, higher

11 (a) 23 (b) 7

12 B

13 (a) 4 × (2 + 3) ÷ 5 – 1 = 3 (b) (5 + 1) ÷ 6 + 4 + 2 = 7

14 (a) $1200

(b) Melissa's weekly salary will be less than $1200.

15 (a) 3 and 4 (b) 5 and 6
(c) 9 and 10 (d) 12 and 13

16 Jessica: $180, Amy: $165, Aaron: $135, Nikkita: $90, Josef: $210; Total: $780

17 (a) 9 + 7 × 3 = 30 (b) 16 – 4 × 2 ÷ 2 = 12

18 (a) 6 + 3 × 5 (b) $19 change

19 (a) 129 (b) 3853 (c) 36

20 (a) $2^4 = 16$, $2^5 = 32$, $2^6 = 64$

(b) $2^{10} = 1024$; therefore, 2^{10} will be larger than 1000. The difference is 24.

21 4, $\sqrt{81}$, $\sqrt{121}$, 2^4, 3^3, 10^2

22 $21

23 224 ÷ 42 = 5 rem 14, so 6 buses will be needed. Rounding to the first digit gives 200 ÷ 40 = 5, meaning the 14 remaining passengers would be stranded—this is because 224 was rounded down.

24 8 numbers: 17, 18, 19, 20, 21, 22, 23, 24

25 A

26 $3^2 = 3 \times 3$, which is not equal to 3×2

27 Less than, because both of the numbers being rounded are being rounded up, and then multiplied.

NAPLAN practice 1

1 C 2 C 3 D 4 D
5 D 6 $28.40 7 180 8 C
9 The Shwarma family

Chapter 2

Recall 2 (p. 33)

1 (a) 42, 36, 24, 66, 48 (b) 77, 49, 35, 14, 21
(c) 56, 48, 32, 80, 64 (d) 108, 27, 45, 99, 72
(e) 84, 72, 144, 108, 132

2 (a) 0, 2, 4, 6, 8 (b) 1, 3, 5, 7, 9

3 (a) 10 > 7 (b) 3 < 6 (c) 2 > 0 (d) 0 < 5

4 (a) 23 (b) 34 (c) 65 (d) 5
(e) 47 (f) 76

5 (a) -11°C, -4°C, 0°C, 7°C, 15°C, 21°C

(b) -25°C, -14°C, -3°C, 5°C, 10°C, 32°C

6 (a) 49 (b) 81 (c) 64 (d) 1

7 (a) 225 (b) 8 (c) 100 (d) 32

Exercise 2.1 (p. 36)

1 (a) 1, 2, 3, 6, 9, 18 (b) 1, 2, 4, 8, 16
(c) 1, 23 (d) 1, 2, 3, 4, 6, 8, 12, 24

(e) 1, 2, 4, 5, 10, 20 (f) 1, 5, 7, 35

(g) 1, 2, 3, 4, 6, 9, 12, 18, 36 (h) 1, 2, 3, 6, 7, 14, 21, 42

(i) 1, 53

(j) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

(k) 1, 7, 11, 77

(l) 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84

2 (a) 108 (b) 92, 108, 3100

(c) 245, 3100 (d) None

(e) 108

3 (a) 10 (b) 9 (c) 25 (d) 30 (e) 28

(f) 24 (g) 63 (h) 60 (i) 66 (j) 36

(k) 100 (l) 56 (m) 60 (n) 50 (o) 300

4 (a) 10: 1, 2, 5, 10
15: 1, 3, 5, 15
HCF = 5

(b) 8: 1, 2, 4, 8
24: 1, 2, 3, 4, 6, 8, 12, 24
HCF = 8

(c) 5: 1, 5
12: 1, 2, 3, 4, 6, 12
HCF = 1

(d) 26: 1, 2, 13, 26
36: 1, 2, 3, 4, 6, 9, 12, 18, 36
HCF = 2

(e) 11: 1, 11
33: 1, 3, 11, 33
HCF = 11

(f) 28: 1, 2, 4, 7, 14, 28
70: 1, 2, 5, 7, 10, 14, 35, 70
HCF = 14

(g) 44: 1, 2, 4, 11, 22, 44
22: 1, 2, 11, 22
HCF = 22

(h) 10: 1, 2, 5, 10
30: 1, 2, 3, 5, 6, 10, 15, 30
HCF = 10

(i) 40: 1, 2, 4, 5, 8, 10, 20, 40
70: 1, 2, 5, 7, 10, 14, 35, 70
HCF = 10

(j) 32: 1, 2, 4, 8, 16, 32
60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
HCF = 4

(k) 35: 1, 5, 7, 35
70: 1, 2, 5, 7, 10, 14, 35, 70
HCF = 35

(l) 42: 1, 2, 3, 6, 7, 14, 21, 42
48: 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
HCF = 6

5 (a) A (b) C

6 (a) C (b) D

7 (a) A (b) D

8 (a) False (b) False (c) True (d) True

(e) False (f) False

9 (a) (i) 60 (ii) 2

(b) (i) 24 (ii) 2

(c) (i) 48 (ii) 4

(d) (i) 200 (ii) 5

10 (a) divisible (b) factor (c) multiple (d) divisible

11 (a) 2 bags with 12 lollies in each bag; 3 bags of 8; 4 bags of 6; 6 bags of 4; 8 bags of 3; 12 bags of 2; 24 bags of 1

(b) 2 bags with 18 lollies in each bag; 3 bags of 12; 4 bags of 9; 6 bags of 6; 9 bags of 4; 12 bags of 3; 18 bags of 2; 36 bags of 1

12 (a) 1 row of 96, 2 rows of 48, 3 rows of 32, 4 rows of 24, 6 rows of 16, 8 rows of 12, 12 rows of 8, 16 rows of 6, 24 rows of 4, 32 rows of 3, 48 rows of 2, 96 rows of 1

(b) 8 rows of 12 or 12 rows of 8

13 D

14 5 min, 10 min, 15 min

15 5 groups of 4 and 2 groups of 3; 2 groups of 4 and 6 groups of 3

16 (a) 56 (b) 110 (c) 112 (d) 210

17 (a) 15 times (b) 7 times (c) 4, 7, 10, 13, 16, 19, 22

18 (a)

202 008	2, 3, 4, 6, 8
12 121 212	2, 3, 4, 6
300 300 300	2, 3, 4, 5, 6, 9, 10
7 500	2, 3, 4, 5, 6, 10
900 090	2, 3, 5, 6, 9, 10
123 456 789	3, 9

(b) (i) 2 (ii) 3

(c) 2 is a factor of 4, so if a number is divisible by 4, it is also divisible by 2.

3 is a factor of 9, so if a number is divisible by 9, it is also divisible by 3.

19 6 is the LCM of 2 and 3.

20 (a) 28 (b) 496

21 18 and 20

22 (a) multiply the two numbers together

(b) If the HCF of the numbers is 1, this will be the LCM.

23 (a) 4 rows of 15; 5 rows of 12; 6 rows of 10

(b) Boxes that are squarer in shape are more practical and require less packaging to make, so 5×12 or 6×10 are more likely.

24 Sam's age must be a multiple of 3 and 5; i.e. a multiple of 15: 15, 30, 45, 60, 75.

25 No. Multiplying any common multiple by a whole number greater than 1 gives a higher common multiple.

Exercise 2.2 (p. 40)

1 2, 3, 5, 7, 11, 13, 17, 19

2 Four (2, 3, 5, 7)

3 23, 29, 31, 37, 41, 43, 47, 53, 59

4 (a) B (b) D

5 (a) 61 (b) 62 (c) 9, 15 (d) 47

6 (a) False (b) True (c) True (d) False

(e) True (f) True (g) True (h) False

7 (a) Divisible by 10, 5 and 2 (b) Divisible by 3 and 9

(c) Divisible by 3 and 9 (d) Divisible by 3 and 9

(e) Divisible by 2 and 4 (f) Divisible by 5

8 (a) Yes, HCF = 1 (b) Yes, HCF = 1

(c) No, HCF = 13 (d) No, HCF = 9

9 The only common factors of any two prime numbers is 1.

10 All other even numbers have 2 as a factor.

11 It is even, so 2 is a factor.

12 There are no more even prime numbers.

13 1

14 **(a)** 99 and 101 **(b)** 35 and 37

15 No, a prime number will only be coprime with any whole number that is not a multiple of the original prime number.

16 The smaller of the two numbers will be a common factor.

17 The first 10 even composite numbers:

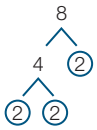
$4 = 2 + 2$
$6 = 3 + 3$
$8 = 3 + 5$
$10 = 5 + 5$
$12 = 5 + 7$
$14 = 7 + 7$
$16 = 11 + 5$
$18 = 11 + 7$
$20 = 13 + 7$
$22 = 11 + 11$

18 3 and 7 $(3 \times 2 + 1 = 7)$
2 and 5 $(2 \times 2 + 1 = 5)$
5 and 11 $(2 \times 5 + 1 = 11)$

Exercise 2.3 (p. 44)

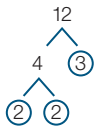
1 The factor trees can differ slightly from these. The factors at the end should be the same.

(a)



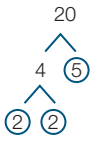
$8 = 2 \times 2 \times 2$
$= 2^3$

(b)



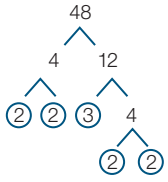
$12 = 2 \times 2 \times 3$
$= 2^2 \times 3$

(c)



$20 = 2 \times 2 \times 5$
$= 2^2 \times 5$

(d)



$48 = 2 \times 2 \times 2 \times 2 \times 3$
$= 2^4 \times 3$

(e)



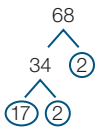
$14 = 2 \times 7$

(f)



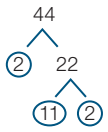
$26 = 2 \times 13$

(g)



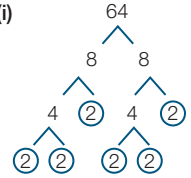
$68 = 2 \times 2 \times 17 = 2^2 \times 17$

(h)



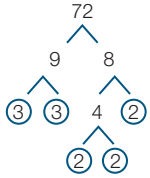
$44 = 2 \times 2 \times 11 = 2^2 \times 11$

(i)



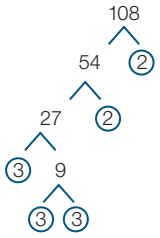
$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^6$

(j)



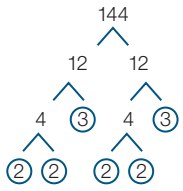
$72 = 2 \times 2 \times 2 \times 3 \times 3$
$= 2^3 \times 3^2$

(k)



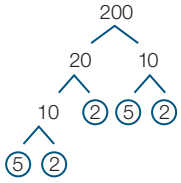
$108 = 2 \times 2 \times 3 \times 3 \times 3$
$= 2^2 \times 3^3$

(l)



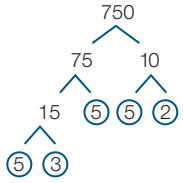
$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
$= 2^4 \times 3^2$

(m)



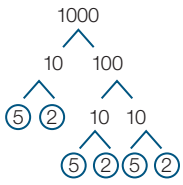
$200 = 2 \times 2 \times 2 \times 5 \times 5$
$= 2^3 \times 5^2$

(n)



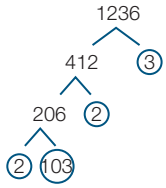
$750 = 2 \times 3 \times 5 \times 5 \times 5$
$= 2 \times 3 \times 5^3$

(o)



$1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$
$= 2^3 \times 5^3$

(p)



$1236 = 2 \times 2 \times 3 \times 103$
$= 2^2 \times 3 \times 103$

2 **(a)** $8 = 2^3$ **(b)** $12 = 2^2 \times 3$
(c) $28 = 2^2 \times 7$ **(d)** $36 = 2^2 \times 3^2$
(e) $39 = 3 \times 13$ **(f)** $77 = 7 \times 11$
(g) $51 = 3 \times 17$ **(h)** $38 = 2 \times 19$
(i) $30 = 2 \times 3 \times 5$ **(j)** $63 = 3^2 \times 7$
(k) $96 = 2^5 \times 3$ **(l)** $132 = 2^2 \times 3 \times 11$
(m) $168 = 2^3 \times 3 \times 7$ **(n)** $198 = 2 \times 3^2 \times 11$
(o) $288 = 2^5 \times 3^2$ **(p)** $212 = 2^2 \times 53$

3 **(a)**
$8 = 2 \times 2 \times 2$
$20 = 2 \times 2 \times 5$
HCF $= 2 \times 2$
$= 4$

(b)
$12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$
HCF $= 2 \times 3$
$= 6$

(c)
$12 = 2 \times 2 \times 3$
$36 = 2 \times 2 \times 3 \times 3$
HCF $= 2 \times 2 \times 3$
$= 12$

(d)
$36 = 2 \times 2 \times 3 \times 3$
$48 = 2 \times 2 \times 2 \times 2 \times 3$
HCF $= 2 \times 2 \times 3$
$= 12$

(e)
$24 = 2 \times 2 \times 2 \times 3$
$56 = 2 \times 2 \times 2 \times 7$
HCF $= 2 \times 2 \times 2$
$= 8$

(f)
$28 = 2 \times 2 \times 7$
$84 = 2 \times 2 \times 3 \times 7$
HCF $= 2 \times 2 \times 7$
$= 28$

(g)
$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$
HCF $= 2 \times 2 \times 2 \times 2 \times 2$
$= 32$

(h)
$70 = 2 \times 5 \times 7$
$98 = 2 \times 7 \times 7$
HCF $= 2 \times 7$
$= 14$

(i)
$60 = 2 \times 2 \times 3 \times 5$
$105 = 3 \times 5 \times 7$
HCF $= 3 \times 5$
$= 15$

(j)
$66 = 2 \times 3 \times 11$
$110 = 2 \times 5 \times 11$
HCF $= 2 \times 11$
$= 22$

(k) $80 = 2 \times 2 \times 2 \times 2 \times 5$
$128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
HCF $= 2 \times 2 \times 2 \times 2$
$= 16$

(l) $130 = 2 \times 5 \times 13$
$156 = 2 \times 2 \times 3 \times 13$
HCF $= 2 \times 13$
$= 26$

4 (a) D (b) A

5 (a) 72 (b) 300 (c) 54 000
(d) 2205 (e) 15 400 (f) 7744

6 (a) $2^4 \times 3$ (b) $2^5 \times 3$ (c) $2^4 \times 3^2$

7 (a) $2 \times 3 \times 5 \times 11$ (b) $2^2 \times 3 \times 5 \times 11$ (c) $2 \times 3^2 \times 5 \times 11$

8 2, 3, 7

9 (a) Prime factors of 36: $2 \times 2 \times 3 \times 3$
Prime factors of 100: $2 \times 2 \times 5 \times 5$
Highest common factor (HCF) of 36 and 100: $2 \times 2 = 4$
Factors of 36 not used to find the HCF: $3 \times 3 = 9$
Factors of 100 not used to find the HCF: $5 \times 5 = 25$
HCF × unused factors of 36 × unused factors of 100:
$4 \times 3 \times 3 \times 5 \times 5 = 900$

(b) (i) 72 (ii) 36 (iii) 40 (iv) 72 (v) 260
(vi) 800 (vii) 1000 (viii) 800 (ix) 4000

10 (a) 6 (b) 14 red balloons and 9 blue balloons

11 17; 510

12 (a) (i) 6 (ii) 30 (iii) 210
(b) (i) 4 (ii) 8 (iii) 16

13 (a) Yes, 105 is a winning number.

(b) There would be four winning tickets: 15, 105, 165 and 195.

14 Trial and error; e.g. try $2 \times 2 \times 3 \times 5$. If it is too small, try a bigger prime. $2 \times 2 \times 3 \times 11 = 132$; $2 \times 2 \times 5 \times 7 = 140$

15 (a) 2, 4, 5, 10 (and 20)

(b) A factor need not be prime. For example, 6 is a factor of 18, but 6 is a composite number.

(c) If a factor is composite, Ray should find its factors.

Exercise 2.4 (p. 48)

1 (a) +300 (b) -50 000 (c) -8 (d) +4
(e) +36 (f) +45 (g) -20 (h) +12
(i) -2 (j) -7 (k) +28 (l) -40000
(m) +225 (n) -9

2 (a) $+6 > -3$ (b) $+1 > -3$ (c) $-7 < +5$
(d) $-2 < +6$ (e) $-12 < -4$ (f) $-7 < -1$
(g) $-2 > -11$ (h) $-9 < 0$ (i) $0 > -2$
(j) $-35 < +7$ (k) $-4 < +67$ (l) $-11 < +75$

3 (a) -12, -4, -3, -1, 6 (b) -11, -10, 2, 9, 10
(c) -6, -4, 1, 2, 9 (d) -8, -6, -5, 1, 7
(e) -12, -8, -7, -4, -1 (f) -79, -56, 0, 6, 23

4 (a) 8, 7, 3, -5, -6 (b) 10, 7, 1, -8, -12
(c) 1, -4, -6, -8, -9 (d) 7, 0, -5, -11, -12
(e) -2, -5, -9, -11, -12 (f) 79, 4, 0, -9, -54

5 (a) east 300 km (b) 3 days early
(c) 7°C above zero (d) subtract 5
(e) add 22 (f) go right 3 m
(g) +2 (h) -16
(i) +350

6 (a) A (b) D

7 (a) True (b) False (c) False (d) True
(e) False (f) False (g) True (h) True

8 (a) -3, -2, -1, 0, 1, 2 (b) -1, 0, 1
(c) -8, -7, -6, -5 (d) -4, -3, -2, -1
(e) -38, -39, -40, -41 (f) -119, -118, -117, -116

9 (a) -12, -10, -8 (b) -5, 0, 5
(c) 0, -3, -6 (d) -24, -30, -36
(e) 0, -10, -20 (f) -3, -8, -13

10 (a) Ben (b) Damien

11 (a) Tuesday (b) Friday (c) Tuesday

12 (a) 156 m (b) 154 m (c) 310 m

13 (a) $-5 + 3 - 2 = -4$ (b) 4 km west (c) 4 km east

14 (a) $7 + 2 - 6 + 1 = 4$ (b) 4 floors up (c) 4 floors down

15 (a) Sam has more money at the end of the week.

(b) Withdrawals \$80 + \$120 + \$50 = \$250
Deposits \$350 + \$40 = \$390
\$140 more in the account

16 Any three of -7, -6, -5, -4, -3, -2, -1

17 (a) -1

(b) -5, 1, 6, 6, -5, 1 1, 1, -5, 2, 2, 3 4, 4, -5, 4, 2, -5

18 Students' own answers.

Half-time 2 (p. 51)

1 (a) 45 (b) 24

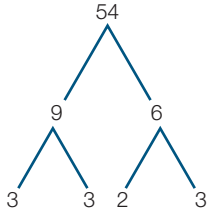
2 B

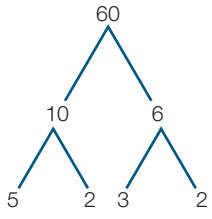
3 (a) +85 (b) -7 (c) +3 (d) +28 000
(e) -40 (f) +11

4 31, 37, 41, 43, 47

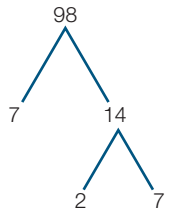
5 (a) $6 < 10$ (b) $-4 < 7$ (c) $3 > -12$ (d) $-8 > -15$

6 (a) $54 = 2 \times 3^3$ (b) $60 = 2^2 \times 3 \times 5$

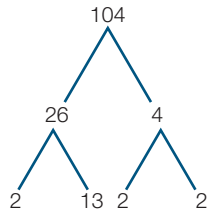




(c) $98 = 2 \times 7^2$



(d) $104 = 2^3 \times 13$



7 (a) 76 905 is divisible by both 3 and 5, so it is divisible by 15.

(b) 4734 is divisible by 2, as it is even. Sum of digits is 18, which is divisible by 3. Therefore, as 4734 is divisible by both 2 and 3, it is divisible by 6.

8 (a) -37, -30, -3, 0, 3, 7 (b) -40, -20, -5, -1, 54

9 24 **10** D

11 (a) 150 seconds (b) Honda 6 laps, Holden 5 laps, Ford 3 laps (c) 12 laps

Exercise 2.5 (p. 54)

1 (a) 11 (b) 8 (c) 11 (d) 8
(e) -2 (f) -5 (g) -5 (h) -6
(i) 6 (j) 12 (k) 25 (l) 20
(m) -10 (n) -4 (o) -20 (p) -31

2 (a) 2 (b) 6 (c) 5 (d) 0
(e) -6 (f) -11 (g) -8 (h) -4
(i) -10 (j) -8 (k) -20 (l) -12
(m) -12 (n) -30 (o) -69 (p) -124

3 (a) B (b) D

4 (a) 12 (b) 11 (c) 12 (d) 7
(e) -18 (f) -22 (g) -5 (h) -10
(i) -8 (j) 13 (k) -1 (l) 1

5 +8°C

6 the 11th floor

7 34 m below the surface

8 5 m

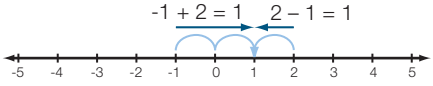
9 $13

10 (a) positive five subtract positive seven equals negative two

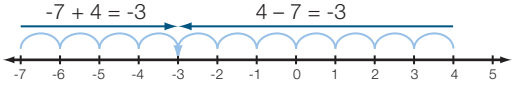
(b) negative eight add positive nine equals positive one

(c) negative four subtract positive three equals negative seven

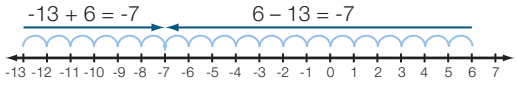
11 (a) (i)



(ii)



(iii)



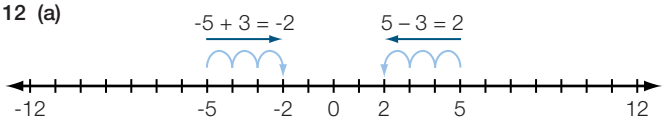
(b) The finishing point of the number line journeys for these pairs of additions and subtractions is the same. The pairs are equivalent—each addition can be rewritten as a subtraction. -1 + 2 = 2 – 1.

(c) (i) -37 + 86 = 86 – 37 = 49

(ii) -72 + 95 = 95 – 72 = 23

(iii) -104 + 119 = 119 – 104 = 15

12 (a)



(b) (i) -8 + 6 = -2 (ii) -5 + 4 = -1
(iii) 10 – 6 = 4 (iv) 12 – 9 = 3

(c) (i) -6 (ii) -21
(iii) -25 (iv) -19

13 Assuming that the temperature rises fairly gradually after sunrise, a possible answer might be:

6 a.m.	7 a.m.	8 a.m.	9 a.m.	10 a.m.	11 a.m.	12 p.m.
-4°C	0°C	7°C	13°C	17°C	21°C	25°C

14 Any three positive integers less than 13.

Exercise 2.6 (p. 58)

1 (a) 1 (b) 1 (c) 9 (d) 6
(e) -4 (f) -5 (g) -7 (h) -11
(i) -9 (j) -11 (k) -20 (l) -34
(m) -10 (n) -90 (o) -29 (p) -133

2 (a) 11 (b) 6 (c) 10 (d) 12
(e) 8 (f) 3 (g) 16 (h) -6
(i) 12 (j) -1 (k) 8 (l) -6
(m) -10 (n) -10 (o) -26 (p) -28

3 (a) A (b) B

4 (a) 0 (b) -6 (c) -7 (d) -10
(e) -5 (f) 6 (g) 5 (h) 9
(i) -3 (j) -2 (k) -8 (l) 22

5 -4°C

6 -3 (3 floors below ground level)

7 $3500 in debt

8 9 m

9 3 m

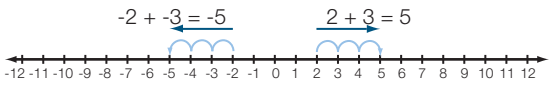
10 31 m

11 (a) negative five add negative six equals negative eleven

(b) positive seven subtract negative eight equals positive fifteen

(c) negative nine subtract negative fourteen equals positive five

12 (a)



(b) (i) $-1 - 6 = -7$ (ii) $-4 - 7 = -11$

(iii) $3 + 8 = 11$ (iv) $5 + 4 = 9$

(c) (i) -36 (ii) -55 (iii) -77 (iv) -94

13 (a) (i) 6 (ii) 18 (iii) -9 (iv) 3

(b) smaller; larger

14 Must be -22 or less; e.g. -25, -28, -35.

15 (a) The correct answer is -9, not -1.

(b) Max has moved in the wrong direction on the number line. To add a negative number, he needs to walk forwards, but in the negative direction.

Exercise 2.7 (p. 61)

1 (a) 7 **(b)** 3 **(c)** 11 **(d)** -2

(e) 2 **(f)** -12 **(g)** -11 **(h)** -10

(i) 9 **(j)** 6 **(k)** -2 **(l)** -4

(m) -1 **(n)** 6 **(o)** 12 **(p)** 2

(q) -10 **(r)** -12 **(s)** 5 **(t)** 4

2 (a) False **(b)** True **(c)** True **(d)** False

3 (a) -12 **(b)** -11 **(c)** 4 **(d)** 3

(e) -5 **(f)** -8 **(g)** -13 **(h)** -12

(i) -30 **(j)** -31 **(k)** 57 **(l)** 65

4 (a) C **(b)** B

5 (a) 6 **(b)** 7 **(c)** -7 **(d)** -4

(e) -6 **(f)** -8 **(g)** 4 **(h)** 1

(i) -12 **(j)** -11 **(k)** -9 **(l)** -9

6 -$150

7 (a) the 10th floor

(b) the 4th floor below the ground floor

(c) the 8th floor

8 -$350, a loss of $350

9 (a)

+	-6	+20	-14	31
+4	-2	+24	-10	35
-7	-13	+13	-21	24
+8	+2	+28	-6	39
-9	-15	+11	-23	22

(b)

–	8	-11	19	-25
3	5	-14	16	-28
-7	15	-4	26	-18
+5	3	-16	14	-30
-10	18	-1	29	-15

10 (a) 6

(b)

1	6	-1
0	2	4
5	-2	3

11 (a)

-3	2	-5
-4	-2	0
1	-6	-1

(b) -6

Each of the three digits in the row, column or diagonal has been reduced by 4. This means that the magic sum of the digits will be 12 less than the original magic sum.

12 (a)

Hole number	1	2	3	4	5	6	7	8	9	**Total**
Par for the hole	4	4	5	3	4	5	4	3	4	36
Shots taken	4	3	4	5	5	4	3	3	4	35
Score compared to par	0	-1	-1	+2	+1	-1	-1	0	0	-1

(b)

Hole number	1	2	3	4	5	6	7	8	9	**Total**
Par for the hole	4	4	5	3	4	5	4	3	4	36
Shots taken	3	6	4	4	5	3	3	4	3	35
Score compared to par	-1	+2	-1	+1	+1	-2	-1	+1	-1	-1

13 Ignoring the signs, the numbers will add to 31; e.g. -14 and 17, -10 and 21, 18 and -13

Challenge 2 (p. 64)

1 (a) 1, -3, -7 **(b)** -7, -1, 5

2 The twins are the youngest and have the same age. Look for three factors of 72 in which the smallest two are the same: $1 \times 1 \times 72$, $2 \times 2 \times 18$, $3 \times 3 \times 8$; $3 + 3 + 8 = 14$. ($6 \times 6 \times 2$ gives sum of 14, but twins are not the youngest.) The brothers are 3, 3 and 8 years old.

3 $172 \div 2 = 86$ (average of two numbers is the one in the middle) so the middle number is 86. Numbers are 85, 86, 87.

4 Consider factors of 150. $10 \times 15 = 150$,
$(10 - 4) \times (15 + 10) = 6 \times 25 = 150$, $(10 + 10) \times (15 - 4) = 220$;
$6 \times 25 = 150$, $(6 + 10) \times (25 - 4) = 336$, $(6 - 4) \times (25 + 10) = 70$.
Originally, there were 10 rows of 15 chairs.

5 11×0, $(10 + 10 + 1) \times 1$, $(10 + 10) \times$ each other digits.

$$\begin{aligned}\text{Sum} &= 0 + 21 + 20 \times (2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) \\ &= 21 + 20 \times 44 \\ &= 901\end{aligned}$$

6 The amount that each person raised has been counted twice.
$(\$130 + \$150 + \$100 + \$70 + \$90) \div 2 = \$540 \div 2 = \$270$

7 Numbers that give a remainder of 2 when divided by 4 are: 6, 10, 14, 18, 22, 26, 30, 34, 38, 42, 46, 50, 54, 58, 62, ...

Numbers that give a remainder of 3 when divided by 5 are: 8, 13, 18, 23, 28, 33, 38, 43, 48, 53, 58, 63, ...

Numbers that give a remainder of 4 when divided by 6 are: 10, 16, 22, 28, 34, 40, 46, 52, 58, 64, ...

Looking at the patterns, the first number to appear in all three is 58.

8 B

Prime: 11 → 11, 13 → 31, 17 → 71, 31 → 13, 37 → 73, 71 → 17, 73 → 37, 79 → 97, 97 → 79

Not prime: 19 → 91

Any primes in the 20s, 40s, 60s and 80s will give even numbers when reversed, so not prime.

Any primes in the 50s will give a number ending in 5 when reversed, so not prime.

9 **(a)** B $10, 15 \times 2 = 30$

(b) C $10, 15 \times 2 = 30, 20, 25 \times 4 = 100$

(c) D $10, 15 \times 2, 20, 25 \times 4 = 100, 30, 35 \times 2 = 70, 40, 45 \times 2 = 90, 50 \times 2 = 100, 55 \times 2 = 110$

(d) D $10, 15 \times 2, 20, 25 \times 4 = 100, 30, 35 \times 2, 40, 45 \times 2, 50 \times 2 = 100, 55 \times 2 = 110, 60, 65 \times 2 = 130, 70, 75 \times 4 = 300, 80, 85 \times 2 = 170, 90, 95 \times 2 = 190$

Chapter review 2 (p. 65)

1 **(a)** 18 **(b)** 36 **(c)** 30

2 **(a)** 1, 2, 3, 4, 6, 9, 12, 18, 36

(b) 1, 2, 3, 4, 6, 8, 12, 16, 24, 48

(c) 1, 3, 17, 51

(d) 1, 2, 4, 5, 10, 20, 25, 50, 100

3 **(a)** 8 **(b)** 18 **(c)** 5

4

5301	3, 9
10 000	2, 4, 5, 10
333 333	3, 9
31 700	2, 4, 5, 10
43 521 820	2, 4, 5, 10

5 **(a)** Prime, because it has exactly two factors, 1 and 5.

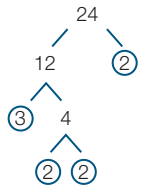
(b) Composite, because it has more than two factors; i.e. 1, 2, 4, 8, 16.

(c) Composite, because it has more than two factors; i.e. 1, 7, 11, 77.

(d) Composite, because it has more than two factors; e.g. 1, 10, 27 635, 276 350.

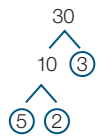
6 The factor trees can vary, but the factors on the last line should be the same, even though they may be in a different order.

(a)



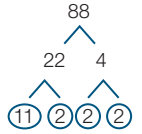
$24 = 2 \times 2 \times 2 \times 3$

(b)



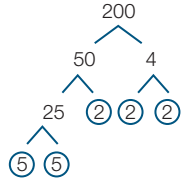
$30 = 2 \times 3 \times 5$

(c)



$88 = 2 \times 2 \times 2 \times 11$

(d)



$200 = 2 \times 2 \times 2 \times 5 \times 5$

7 **(a)** $27 = 3 \times 3 \times 3$
$36 = 2 \times 2 \times 3 \times 3$
HCF $= 3 \times 3$
$= 9$

(b) $72 = 2 \times 2 \times 2 \times 3 \times 3$
$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$
HCF $= 2 \times 2 \times 2 \times 3$
$= 24$

(c) $108 = 2 \times 2 \times 3 \times 3 \times 3$
$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$
HCF $= 2 \times 2 \times 3$
$= 12$

8 **(a)** -14 **(b)** +200

9 **(a)** south 5 km **(b)** subtracting 27

10 **(a)** $-52 < 25$ **(b)** $19 > -20$

11 **(a)** -9, -7, 0, 7, 12 **(b)** -4000, 4, 40, 400

12 **(a)** 18 **(b)** 15 **(c)** -10 **(d)** 6
(e) -4 **(f)** 8 **(g)** -5 **(h)** -32

13 **(a)** -3 **(b)** 3 **(c)** -7 **(d)** 0
(e) -7 **(f)** 2 **(g)** -3 **(h)** -12

14 **(a)** -2 **(b)** 7 **(c)** 11 **(d)** -4
(e) 37 **(f)** 14 **(g)** -11 **(h)** -3

15 **(a)** divisible, multiple **(b)** factor, multiple
(c) divisible, multiple

16 96 (1 packet), 48 (2 packets), 32 (3 packets), 24 (4 packets), 16 (6 packets), 12 (8 packets), 8 (12 packets), 6 (16 packets), 4 (24 packets), 3 (32 packets), 2 (48 packets), 1 (96 packets)

17 **(a)** -12 **(b)** -81 **(c)** -60 **(d)** -70

18 **(a)** Start at +3 and walk 8 steps to the left, finishing at -5.

(b) Start at -6 and walk 5 steps to the left, finishing at -11.

19 **(a)** coprime **(b)** not coprime, HCF = 3
(c) not coprime, HCF = 7 **(d)** coprime

20 2 runs

21 **(a)** HCF = 12, so 12 tables are required

(b) 9 females, 7 males

22 504

23 **(a)** 71 **(b)** 72

24 **(a)** decreased **(b)** $61

25 150 seconds

26 -900 + (-250) = -1150 m

27 **(a)** = **(b)** < **(c)** = **(d)** <

28 2 and 4

29 910 and 13

NAPLAN practice 2

1 C **2** B **3** 8th floor (-3 + 11 = 8)

4 B **5** A **6** D

7 $265 **8** B **9** -5

Mixed review A (p. 69)

1 -48, -25, -10, 0, 8, 17, 32

2 (a) 9^2 (b) 7^4 (c) 4^3

3 24, 48, 72, 96

4 (a) -570 (b) +5

5 (a) $-27 < 14$ (b) $0 > -35$

6 (a) 40 (b) 300 (c) 500
(d) 3600 (e) 600 000 (f) 10 000 000

7 (a) 340 (b) 403 (c) 798
(d) 396 (e) 240 (f) 960

8 (a) 72 (b) 47 (c) 29
(d) 47 (e) 0 (f) 80

9 (a) -2 (b) -3 (c) 4 (d) -17

10 (a) 1800 (b) 15 000 (c) 60 000 (d) 100
(e) 40 (f) 2

11 (a) -15, -5, 0, 5, 10 (b) -300, -30, 0, 3, 3000

12 (a) 24 (b) 48

13 (a) 9 (b) 8

14 (a) 1, 2, 3, 6, 9, 18 composite
(b) 1, 23 prime
(c) 1, 2, 4, 11, 22, 44 composite
(d) 1, 79 prime

15 (a) $63 = 3^2 \times 7$ (b) $48 = 2^4 \times 3$
(c) $72 = 2^3 \times 3^2$ (d) $120 = 2^3 \times 3 \times 5$

16 (a) $-6 - 7 = -13$ (b) $4 + 11 = 15$
(c) $-2 + 7 = 5$ (d) $22 - 9 = 13$
(e) $-5 + 10 = 5$ (f) $18 + 3 = 21$

17 -180 m

18 (a) 3, 7, 11, 19, 43, 13, 2 (b) 9, 16, 25
(c) 16, 32, 8, 2

19 216

20 (a) 12 000 (actual answer will be lower)
(b) 20 000 (actual answer will be higher)
(c) 100 (actual answer will be higher)

21 (a) 9000 (b) $108 000

22 (a) more (b) $5

23 B

Chapter 3

Recall 3 (p. 71)

1 (a), (b), (e)

2 (a) $\frac{1}{5}$ (b) $\frac{3}{8}$ (c) $2\frac{2}{5}$ or $\frac{12}{5}$ (d) $\frac{1}{10}$
(e) $\frac{5}{8}$ (f) $\frac{2}{3}$ (g) $\frac{12}{5}$ (h) $\frac{7}{16}$

3 (a) $2, 1\frac{1}{4}, 1, \frac{3}{4}, 0$ (b) $0, \frac{1}{8}, \frac{3}{8}, \frac{7}{8}, 1, \frac{9}{8}, \frac{11}{8}$

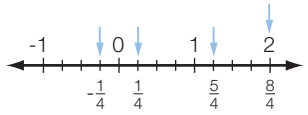
4 (a) $\frac{5}{7}$ (b) $\frac{6}{11}$ (c) $\frac{4}{3}$ or $1\frac{1}{3}$

5 (a) 8, 16, 24, 32, 40 (b) 12, 24, 36, 48, 60
(c) 24 (d) 1, 2, 3, 4, 6, 8, 12, 24
(e) 1, 2, 3, 4, 6, 9, 12, 18, 36 (f) 12

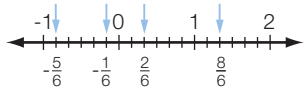
Exercise 3.1 (p. 74)

1 (a) 9 (b) 6

2 (a)



(b)



3 (a) (i) $\frac{10}{2}$ (ii) $\frac{35}{7}$ (iii) $\frac{55}{11}$ (iv) $\frac{25}{5}$ (v) $\frac{5}{1}$

(b) (i) $\frac{26}{2}$ (ii) $\frac{65}{5}$ (iii) $\frac{104}{8}$ (iv) $\frac{169}{13}$ (v) $\frac{13}{1}$

4 (a) (i) $\frac{1}{2}$ (ii) $\frac{2}{3}$ (iii) $\frac{6}{5}$ or $1\frac{1}{5}$ (iv) $\frac{10}{7}$ or $1\frac{3}{7}$

(b) (iii) and (iv)

5 (a) 4 (b) 12 (c) 2 (d) 10

6 (a) $\frac{5}{8}$ (b) $\frac{4}{9}$

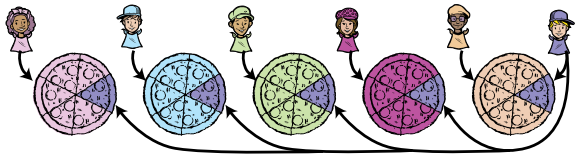
7 (a) $\frac{1}{2}$ (b) $\frac{7}{9}$ (c) $\frac{11}{4}$ (or $2\frac{3}{4}$)
(d) $\frac{3}{5}$ (e) $\frac{3}{14}$ (f) $\frac{25}{8}$ (or $3\frac{1}{8}$)
(g) $-1\frac{5}{6}$ (h) $-\frac{1}{3}$ (i) $-1\frac{2}{7}$

8 (a) $\frac{8}{17}$ (b) $\frac{15}{4}$ (c) $\frac{9}{20}$ (d) $2\frac{2}{3}$
(e) $4\frac{1}{2}$ (f) $3\frac{7}{24}$

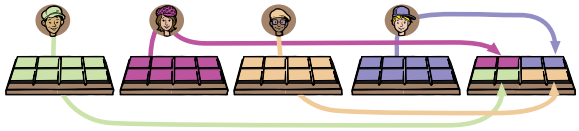
9 (a) $\frac{2}{7}$ (b) $\frac{1}{24}$ (c) $\frac{1}{60}$ (d) $\frac{17}{60}$
(e) $\frac{157}{375}$ (f) $\frac{421}{500}$

10 (a) $\frac{7}{14}$ or $\frac{1}{2}$ (b) $\frac{11}{14}$ (c) $\frac{10}{14}$ or $\frac{5}{7}$

11 (a)



(b)



12 (a) $\frac{149}{185}$ (b) $\frac{185}{149}$ or $1\frac{36}{149}$

13 (a) 32 km (b) Toby ate 12 lollies. 18 lollies in bag.

14 Boys: $\frac{4}{5}$ of a pizza each. Girls: $\frac{5}{4}$ of a pizza each ($1\frac{1}{4}$).
Girls each receive more pizza.

15 (a) $\frac{1}{4}$

(b) Not possible—not divided into equal parts and unable to create equal parts by further dividing shape up.

(c) $\frac{1}{2}$ (d) $\frac{3}{8}$

(e) Not possible—same reason as (b). (f) $\frac{1}{2}$

16 D 17 C

18 5 whole numbers: 1, 2, 3, 4, 5

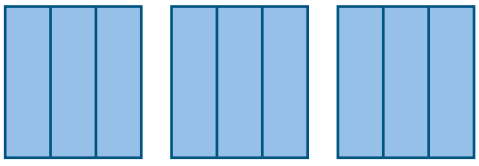
19 (a) $\frac{3}{8}$ of the shape is shaded.



(b) $\frac{5}{6}$ of the collection of stars is shaded.



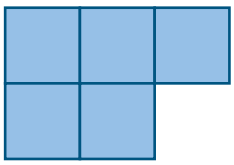
(c) $\frac{9}{3}$



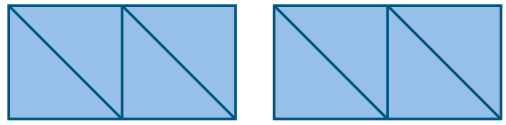
20 (a)



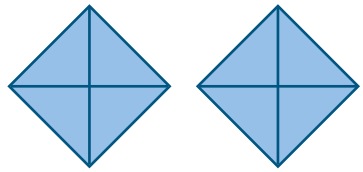
Or



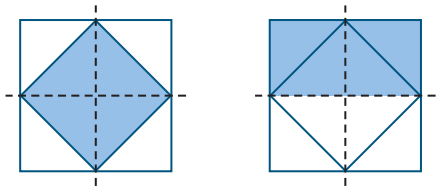
(b)



Or



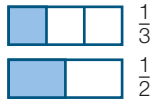
21



Drawing in a vertical and a horizontal line divides the square into quarters and shows that half of each quarter is shaded. We can rearrange things so that it is more obvious that half the square is shaded.

22 (a) Oliver's shaded areas (representing the fractions $\frac{1}{2}$ and $\frac{1}{3}$) look the same because he is not comparing fractions of the same-sized whole.

(b)



$\frac{1}{3}$

$\frac{1}{2}$

Exercise 3.2 (p. 80)

1 (a) 15 (b) 21 (c) 26 (d) 40
(e) 2 (f) 3 (g) 10 (h) 63
(i) 36 (j) 3 (k) 6 (l) 18
(m) 5 (n) 6 (o) 36 (p) 16

2 (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$
(d) $\frac{1}{3}$ (e) $\frac{2}{5}$ (f) $\frac{4}{11}$
(g) $\frac{5}{8}$ (h) $1\frac{1}{2}$ or $\frac{3}{2}$ (i) $2\frac{2}{3}$ or $\frac{8}{3}$
(j) $3\frac{1}{6}$ or $\frac{19}{6}$ (k) $1\frac{3}{10}$ or $\frac{13}{10}$ (l) $12\frac{4}{5}$ or $\frac{64}{5}$
(m) $6\frac{4}{9}$ or $\frac{58}{9}$ (n) $3\frac{1}{4}$ or $\frac{13}{4}$ (o) $2\frac{5}{6}$ or $\frac{17}{6}$
(p) $100\frac{1}{7}$ or $\frac{701}{7}$

3 (a) $1\frac{2}{5}$ (b) $2\frac{1}{6}$ (c) $5\frac{3}{4}$ (d) $2\frac{1}{7}$
(e) $3\frac{7}{10}$ (f) $9\frac{3}{5}$ (g) $6\frac{2}{7}$ (h) $5\frac{9}{10}$
(i) $8\frac{5}{9}$ (j) $7\frac{7}{12}$ (k) $1\frac{7}{100}$ (l) $2\frac{17}{40}$

4 (a) $\frac{7}{4}$ (b) $\frac{16}{5}$ (c) $\frac{20}{3}$ (d) $\frac{59}{10}$

(e) $\frac{35}{8}$ (f) $\frac{51}{11}$ (g) $\frac{72}{7}$ (h) $\frac{74}{11}$

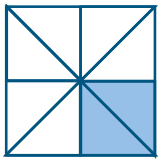
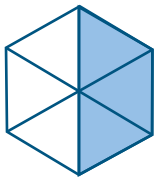
(i) $\frac{77}{12}$ (j) $\frac{309}{100}$ (k) $\frac{62}{9}$ (l) $\frac{79}{9}$

5 (a) C (b) D (c) C (d) B

6 (a) $\frac{6}{10}$ (b) $\frac{1}{4}$ and $\frac{3}{12}$

(c) $\frac{10}{12}$ (d) $\frac{3}{4}$ and $\frac{6}{8}$

7 (a) $\frac{1}{4}$, $\frac{1}{2}$

(b)  $\frac{1}{4} = \frac{2}{8}$ and  $\frac{1}{2} = \frac{3}{6}$

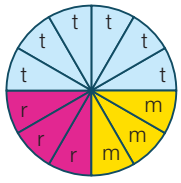
8 (a) They occupy the same position on the number line.

(b) 

$\frac{1}{3}$ and $\frac{3}{9}$, $\frac{2}{3}$ and $\frac{6}{9}$, $\frac{3}{3}$ and $\frac{9}{9}$, $\frac{4}{3}$ and $\frac{12}{9}$, $\frac{5}{3}$ and $\frac{15}{9}$,

$\frac{6}{3}$ and $\frac{18}{9}$, $\frac{7}{3}$ and $\frac{21}{9}$, $\frac{8}{3}$ and $\frac{24}{9}$, $\frac{9}{3}$ and $\frac{27}{9}$

9 Monica 3, Michelle 3, Travis 6



10 35 slices

11 14 squares

12 40 squares, $\frac{5}{8} = \frac{40}{64}$

13 (a) $\frac{1}{30}$ (b) $\frac{1}{3}$ (c) $\frac{1}{5}$ (d) $\frac{3}{20}$

14 $\frac{1}{2}$ = 30 minutes, $\frac{1}{3}$ = 20 minutes, $\frac{1}{4}$ = 15 minutes,

$\frac{1}{5}$ = 12 minutes, $\frac{1}{6}$ = 10 minutes, $\frac{1}{7} = 8\frac{4}{7}$ minutes,

$\frac{1}{8} = 7\frac{1}{2}$ minutes, $\frac{1}{9} = 6\frac{2}{3}$ minutes, $\frac{1}{10}$ = 6 minutes,

$\frac{1}{11} = 5\frac{5}{11}$ minutes, $\frac{1}{12}$ = 5 minutes

15 4 whole numbers: 2, 3, 4, 5

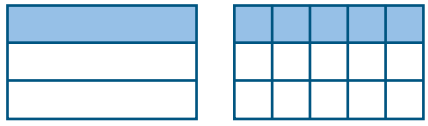
16 C

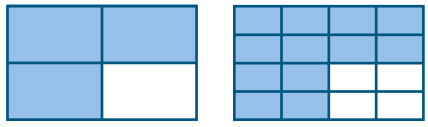
17 $\frac{1}{7}$ and $\frac{1}{11}$ have no equivalent fractions on the wall. This is because their denominators are prime numbers with no multiples in the other denominators on the fraction wall.

18 (a) $\frac{1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{3}{8}$ (d) $\frac{13}{20}$ (e) $\frac{7}{18}$ (f) $\frac{9}{22}$

19 $\frac{8}{10}$, $\frac{12}{15}$, $\frac{16}{20}$ etc.

20 $\frac{5}{3}$, $\frac{15}{9}$, $\frac{10}{6}$ etc.

21 (a) 

(b) 

22 $\frac{3}{7}$, $\frac{7}{9}$ and $\frac{5}{11}$

Exercise 3.3 (p. 85)

1 (a) $\frac{3}{5}$ (b) $\frac{5}{4}$ (c) $\frac{7}{8}$ (d) $\frac{7}{12}$

(e) $\frac{5}{7}$ (f) $\frac{2}{9}$ (g) $\frac{5}{12}$ (h) $\frac{11}{6}$

(i) $1\frac{3}{4}$ (j) $\frac{5}{2}$ (k) $\frac{16}{7}$ (l) $2\frac{4}{9}$

2 (a) $\frac{3}{7}$ (b) $\frac{7}{8}$ (c) $\frac{5}{6}$ (d) $\frac{2}{5}$

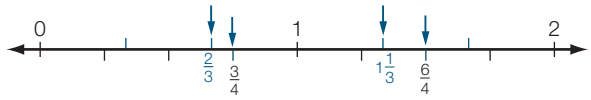
(e) equivalent fractions (f) $\frac{5}{11}$ (g) $\frac{7}{8}$ (h) $\frac{7}{11}$

3 (a) $\frac{2}{7} < \frac{5}{7}$ (b) $\frac{4}{5} > \frac{4}{7}$ (c) $\frac{1}{2} < \frac{5}{8}$ (d) $\frac{8}{11} < \frac{9}{10}$

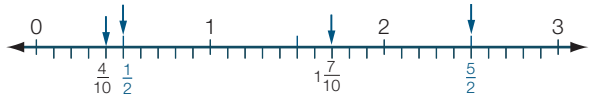
(e) $\frac{2}{4} = \frac{3}{6}$ (f) $\frac{6}{12} < \frac{7}{10}$ (g) $\frac{3}{7} < \frac{5}{8}$ (h) $\frac{5}{9} > \frac{2}{5}$

(i) $\frac{5}{6} < \frac{7}{8}$ (j) $\frac{7}{11} < \frac{8}{10}$ (k) $\frac{6}{10} = \frac{3}{5}$ (l) $\frac{4}{5} > \frac{6}{9}$

(m) $\frac{2}{4} < \frac{4}{2}$ (n) $\frac{24}{6} = \frac{8}{2}$ (o) $\frac{18}{11} < \frac{9}{5}$ (p) $\frac{9}{7} > \frac{11}{9}$

4 (a) 

In ascending order: $\frac{2}{3}$, $\frac{3}{4}$, $1\frac{1}{3}$, $\frac{6}{4}$

(b) 

In descending order: $\frac{5}{2}$, $1\frac{7}{10}$, $\frac{1}{2}$, $\frac{4}{10}$

5 $\frac{7}{8} > \frac{10}{12}$

Ali is the more accurate shooter.

6 $\frac{5}{16} < \frac{23}{64}$

The larger box has the greater fraction of nut chocolates.

7 **(a)** $\frac{10}{11}$ **(b)** $\frac{7}{11}, \frac{2}{3}, \frac{4}{6}, \frac{6}{9}$ and $\frac{8}{12}$

(c) $\frac{2}{5}, \frac{4}{10}, \frac{5}{12}$ and $\frac{3}{7}$ **(d)** $\frac{1}{5}, \frac{2}{10}, \frac{2}{9}, \frac{1}{4}, \frac{2}{8}$ and $\frac{3}{12}$

8 **(a)** Arrow B **(b)** Arrow C

9 **(a)** C **(b)** C

10 **(a)** $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}, \frac{10}{11}$

All of the fractions have a numerator that is 1 less than the denominator—they are '1 piece' less than 1 whole. Higher denominator numbers mean that the whole is being cut into more, smaller-sized pieces. This means that the 'missing pieces' that make the fraction less than 1 are getting smaller.

(b) $0, \frac{4}{7}, \frac{6}{7}, \frac{7}{8}, 1, \frac{7}{4}$

$\frac{7}{4}$ is the only improper fraction and so is greater than 1. All of the other fractions are less than 1, but greater than 0. $\frac{4}{7}$ and $\frac{6}{7}$ have the same denominator, so it is clear that $\frac{6}{7}$ is larger. $\frac{7}{8}$ is greater than $\frac{6}{7}$ for the same reasons described in **(a)**. (Alternatively, they can be converted to fractions with an LCD of 56: $\frac{49}{56} > \frac{48}{56}$.)

11 **(a)** $1, \frac{45}{63}, \frac{2}{3}, \frac{4}{7}, \frac{11}{21}, \frac{2}{9}$

All of the fractions are proper fractions, so they are all less than 1. They also all have an LCD of 63. Converting them to equivalent fractions with this LCD gives:

$1 > \frac{45}{63} > \frac{42}{63} > \frac{36}{63} > \frac{33}{63} > \frac{14}{63}$.

(b) $1\frac{7}{20}, \frac{21}{16}, 1\frac{3}{10}, \frac{5}{4}, 1\frac{3}{80}, \frac{39}{40}$

Converting all to equivalent fractions with an LCD of 80 gives:

$\frac{108}{80} > \frac{105}{80} > \frac{104}{80} > \frac{100}{80} > \frac{83}{80} > \frac{78}{80}$.

12 **(a)** approximately $\frac{3}{4}$

(b) Queensland takes up approximately $\frac{1}{5}$ of the area of Australia.

(c) approximately $\frac{2}{3}$

13 **(a)** same **(b)** same **(c)** B

(d) A **(e)** B **(f)** same

14 **(a)** Angus has drunk more cola.

(b) Gianni has drunk a greater fraction ($\frac{5}{6}$ to $\frac{4}{5}$).

15 **(a)** $\frac{4}{5}, \frac{5}{6}$ **(b)** $\frac{5}{2}, \frac{11}{4}$

16 $\frac{7}{10}, \frac{3}{4}, \frac{5}{7}$

17 Jason is comparing fractions of different wholes (different numbers of marks). To do this, he needs to rewrite them out of the same total by finding equivalent fractions with the LCD. $\frac{16}{20} = \frac{80}{100}$, $\frac{43}{50} = \frac{86}{100}$, so $\frac{43}{50}$ is his better result.

Exercise 3.4 (p. 92)

1 **(a)** $\frac{1}{2}$ **(b)** $\frac{1}{20}$ **(c)** $\frac{9}{20}$

(d) $\frac{14}{55}$ **(e)** $\frac{7}{6}$ or $1\frac{1}{6}$ **(f)** $\frac{50}{49}$ or $1\frac{1}{49}$

(g) $\frac{5}{4}$ or $1\frac{1}{4}$ **(h)** $\frac{19}{20}$ **(i)** $\frac{1}{12}$

(j) $\frac{47}{36}$ or $1\frac{11}{36}$ **(k)** $\frac{25}{42}$ **(l)** $\frac{13}{12}$ or $1\frac{1}{12}$

(m) $\frac{73}{60}$ or $1\frac{13}{60}$ **(n)** $\frac{5}{12}$ **(o)** $\frac{87}{70}$ or $1\frac{17}{70}$

(p) $\frac{3}{10}$

2 **(a)** $1\frac{1}{6}$ **(b)** $\frac{25}{28}$

3 **(a)** $7\frac{3}{7}$ **(b)** $13\frac{2}{3}$ **(c)** $9\frac{5}{8}$ **(d)** $7\frac{9}{10}$

(e) $3\frac{1}{4}$ **(f)** $6\frac{1}{10}$ **(g)** $7\frac{1}{4}$ **(h)** $9\frac{1}{4}$

(i) $12\frac{3}{10}$ **(j)** $11\frac{1}{18}$ **(k)** $4\frac{42}{55}$ **(l)** $3\frac{31}{100}$

4 **(a)** $1\frac{4}{7}$ **(b)** $\frac{2}{3}$ **(c)** $4\frac{4}{9}$ **(d)** $1\frac{2}{3}$

(e) $1\frac{1}{4}$ **(f)** $2\frac{1}{2}$ **(g)** $1\frac{3}{10}$ **(h)** $1\frac{8}{9}$

(i) $\frac{17}{18}$ **(j)** $\frac{17}{20}$ **(k)** $\frac{23}{24}$ **(l)** $2\frac{11}{24}$

5 **(a)** $5\frac{5}{8}$ **(b)** $3\frac{1}{3}$ **(c)** $7\frac{4}{9}$

(d) $2\frac{1}{5}$ **(e)** $1\frac{1}{6}$ **(f)** $4\frac{3}{7}$

6 **(a)** $-\frac{1}{8}$ **(b)** $-\frac{7}{12}$ **(c)** $-\frac{3}{5}$

(d) $\frac{21}{5}$ or $4\frac{1}{5}$ **(e)** $\frac{401}{60}$ or $6\frac{41}{60}$ **(f)** $-2\frac{23}{60}$

7 (a) B (b) D

8 (a) $-\frac{1}{20}$ (b) $-\frac{5}{56}$ (c) $-\frac{2}{45}$

(d) $-\frac{7}{6}$ or $-1\frac{1}{6}$ (e) $\frac{-45}{28}$ or $-1\frac{17}{28}$ (f) $-\frac{68}{63}$ or $-1\frac{5}{63}$

(g) $-1\frac{1}{4}$ (h) $-2\frac{1}{12}$ (i) $-2\frac{7}{15}$

(j) $-3\frac{8}{15}$ (k) $-7\frac{13}{14}$ (l) $-10\frac{43}{63}$

9 (a) $\frac{1}{2}$ (b) $\frac{3}{4}$

10 $\frac{3}{4}$ of the pizza

11 $\frac{1}{4}$

12 $10\frac{1}{12}$ hours

13 $\frac{3}{4}$ of a packet

14 B

15 D

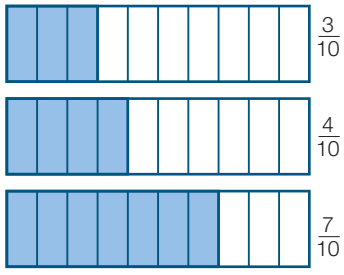
16 (a) $8\frac{2}{3}$ (b) $3\frac{3}{4}$ (c) $5\frac{7}{8}$

(d) $3\frac{5}{7}$ (e) $5\frac{7}{9}$ (f) $7\frac{1}{6}$

17 The correct answer is 2, as both fractions are slightly less than 1. Students answering 19 or 21 have just added the numerators or denominators.

18 $\frac{11}{40}$ and $\frac{19}{40}$

19 We add the numerators because we are 'counting up' how many equal-sized parts we have. Adding the denominators would change the part size.



20 Teachers use fraction notation just as a way of keeping track of marks for different sections of the test. The 'whole' is the total number of marks, and the number of correct answers is written as a fraction of that whole.

Half-time 3 (p. 96)

1 (a) $\frac{7}{15}$ (b) 6

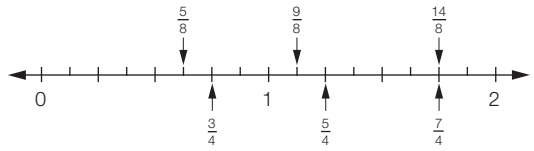
2 (a) $\frac{3}{8}$ (b) $\frac{1}{4}$

(c) $3\frac{3}{4}$ or $\frac{15}{4}$ (d) $7\frac{7}{10}$ or $\frac{77}{10}$

3 (a) $6\frac{3}{4}$ (b) 5 (c) 5 (d) $5\frac{2}{7}$

4 (a) $\frac{5}{8} < \frac{2}{3}$ (b) $\frac{4}{9} < \frac{15}{27}$ (c) $\frac{48}{56} = \frac{6}{7}$ (d) $\frac{11}{5} < \frac{9}{4}$

5 (a)

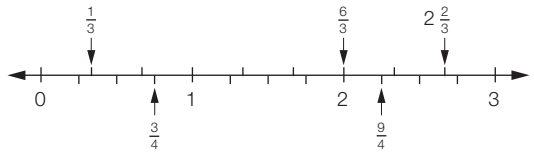


(b) $\frac{1}{4}$ and $\frac{2}{8}$, $\frac{2}{4}$ and $\frac{4}{8}$, $\frac{3}{4}$ and $\frac{6}{8}$, $\frac{5}{4}$ and $\frac{10}{8}$, $\frac{6}{4}$ and $\frac{12}{8}$

(c) (i) $\frac{5}{4}$ (ii) $\frac{7}{4}$

6 (a) $\frac{9}{8}$ or $1\frac{1}{8}$ (b) $\frac{1}{6}$ (c) $\frac{29}{18}$ or $1\frac{11}{18}$ (d) $\frac{7}{36}$

7



In ascending order: $\frac{1}{3}, \frac{3}{4}, \frac{6}{3}, \frac{9}{4}, 2\frac{2}{3}$

8 (a) 29 slices (b) 21 slices

9 (a) $\frac{281}{60}$ or $4\frac{41}{60}$ (b) $\frac{429}{100}$ or $4\frac{29}{100}$

(c) $\frac{81}{50}$ or $1\frac{31}{50}$ (d) $\frac{79}{50}$ or $1\frac{29}{50}$

10 (a) $\frac{5}{2}$ of $2\frac{1}{2}$ L (b) $1\frac{3}{4}$ L

Exercise 3.5 (p. 99)

1 (a) 6 (b) 3 (c) 3 (d) 4

(e) 21 (f) 24 (g) 48 (h) 27

(i) 30 (j) 120 (k) 55 (l) 75

2 (a) $\frac{1}{13}$ (b) $\frac{1}{9}$ (c) $\frac{4}{9}$ (d) $\frac{3}{13}$

(e) $\frac{1}{14}$ (f) $\frac{6}{11}$ (g) $\frac{5}{14}$ (h) $\frac{5}{66}$

(i) $\frac{9}{28}$ (j) $\frac{3}{4}$ (k) $\frac{8}{21}$ (l) $\frac{27}{250}$

3 (a) $\frac{2}{3}$ (b) 6 (c) $1\frac{1}{4}$ (d) $1\frac{1}{4}$

(e) $11\frac{1}{2}$ (f) 14 (g) 27 (h) $40\frac{2}{3}$

(i) 2 (j) $11\frac{1}{5}$ (k) 18 (l) 4

4 (a) $\frac{3}{20}$ (b) $\frac{2}{45}$ (c) $\frac{5}{14}$

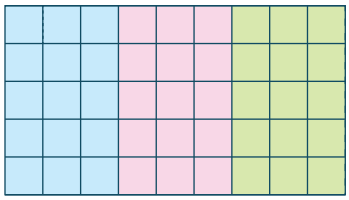
(d) $\frac{10}{13}$ (e) $\frac{7}{6}$ or $1\frac{1}{6}$ (f) $\frac{4}{3}$ or $1\frac{1}{3}$

5 (a) C (b) A

6 (a) $\frac{1}{7}$ (b) \$310

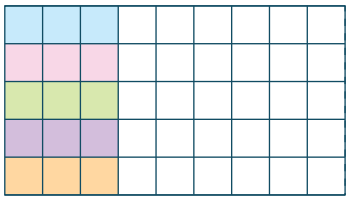
7 Michael \$10 000, Marcus \$4000, Lily \$18 000, Julie \$16 000

8 (a) 15



(b) (i) 3 (ii) $\frac{1}{15}$

(iii)



9 $\frac{1}{8}$ each

10 (a) $\frac{3}{20}$ (b) $\frac{1}{5}\times\frac{3}{4}$ or $\frac{3}{4}\times\frac{1}{5}$

11 (a) 13 L (b) 39 L

12 (a) 50 students (b) 40 students

13 450 g butter, $1\frac{1}{4}$ cups icing sugar, $3\frac{1}{8}$ cups self-raising flour, $\frac{5}{6}$ cup custard powder

14 $1\frac{1}{2}\times 12$ **15** D **16** D

17 $\frac{1}{2}\times\frac{1}{3}$

18 The six possible answers are $\frac{1}{2}\times 30=15$, $\frac{1}{2}\times 32=16$, $\frac{1}{3}\times 30=10$, $\frac{1}{3}\times 33=11$, $\frac{1}{3}\times 36=12$, $\frac{1}{3}\times 39=13$.

19 Asher has cancelled out common factors between the two numerators. Common factors can only be cancelled between numerators and denominators. Correct working:

$$\frac{3}{1\cancel{5}}\times\frac{\overset{12}{\cancel{60}}}{1}=\frac{36}{1}=36$$

Exercise 3.6 (p. 104)

1 (a) 24 (b) 12 (c) 21

(d) 36 (e) 18 (f) 5

(g) $\frac{35}{3}$ or $11\frac{2}{3}$ (h) $\frac{63}{4}$ or $15\frac{3}{4}$ (i) $\frac{16}{3}$ or $5\frac{1}{3}$

(j) 3 (k) $\frac{15}{8}$ or $1\frac{7}{8}$ (l) $\frac{4}{3}$ or $1\frac{1}{3}$

2 (a) 2 (b) $\frac{3}{2}$ or $1\frac{1}{2}$ (c) $\frac{9}{8}$ or $1\frac{1}{8}$

(d) $\frac{7}{5}$ or $1\frac{2}{5}$ (e) $\frac{1}{6}$ (f) $\frac{1}{20}$

(g) $\frac{1}{4}$ (h) $\frac{3}{20}$ (i) $\frac{9}{16}$

(j) $\frac{14}{25}$ (k) $\frac{36}{85}$ (l) $\frac{27}{80}$

(m) 2 (n) $\frac{44}{43}$ or $1\frac{1}{43}$ (o) $\frac{15}{4}$ or $3\frac{3}{4}$

(p) $1\frac{5}{7}$

3 (a) C (b) D

4 (a) 2, 2 (b) 6, 6 (c) 4, 4 (d) 8, 8

5 D **6** 6 students

7 12 whole shifts **8** 30 batches of scones

9 5 drinks stations **10** 5 complete screenings

11 C **12** $\frac{4}{5}\div\frac{1}{3}$

13 (a) As the denominator number gets larger, the part size gets smaller, so you can fit more of the fraction into the number being divided.

(b) $13\frac{1}{3}$

14 $4\div\frac{2}{5}=10$, $4\div\frac{2}{6}=12$, $4\div\frac{2}{7}=14$, $4\div\frac{2}{8}=16$, $4\div\frac{2}{9}=18$

15 Gabi has not multiplied by the inverse. 12 is a reasonable answer because she is dividing by a number less than 1.

Exercise 3.7 (p. 107)

1 (a) $\frac{9}{4}$ or $2\frac{1}{4}$ (b) $\frac{39}{8}$ or $4\frac{7}{8}$ (c) $\frac{5}{99}$

(d) $\frac{3}{16}$ (e) $\frac{5}{16}$ (f) $\frac{11}{18}$

(g) $4\frac{3}{4}$ or $\frac{19}{4}$ (h) $3\frac{3}{8}$ or $\frac{27}{8}$ (i) $2\frac{13}{20}$ or $\frac{53}{20}$

(j) 1 (k) 4 (l) $\frac{2}{3}$

2 (a) A (b) A

3 (a) $\frac{41}{12}$ or $3\frac{5}{12}$ (b) $\frac{53}{30}$ or $1\frac{23}{30}$ (c) $\frac{17}{5}$ or $3\frac{2}{5}$ (d) 30

4 (a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) $\frac{3}{8}$ (d) $\frac{5}{8}$

(e) 90 (f) 36

5 (a) $35\frac{4}{33}$ (b) $254\frac{4}{7}$ (c) $16\frac{1}{5}$

(d) $10\frac{3}{5}$ (e) $12\frac{167}{168}$ (f) $25\frac{107}{175}$

6 **(a)** **(i)** ≈ 64 **(ii)** $65\frac{7}{40}$ **(iii)** $65\frac{7}{40}$ **(iv)** close

(b) **(i)** ≈ 12 **(ii)** $12\frac{47}{69}$ **(iii)** $12\frac{47}{69}$ **(iv)** very close

(c) **(i)** $\approx 5\frac{2}{5}$ **(ii)** $5\frac{65}{108}$ **(iii)** $5\frac{65}{108}$ **(iv)** very close

7 **(a)** $1\frac{2}{7}$ **(b)** $9\frac{4}{5}$ **(c)** $5\frac{5}{6}$

8 **(a)** $\frac{1}{75}$ **(b)** 75 **(c)** $\frac{2}{25}$ **(d)** 150 m **(e)** 1125 m

9 $\frac{97}{12}$ or $8\frac{1}{12}$ laps

10 **(a)** $\frac{11}{8}$ or $1\frac{3}{8}$ km **(b)** $\frac{95}{24}$ or $3\frac{23}{24}$ km **(c)** $\frac{32}{3}$ or $10\frac{2}{3}$ km

11 $7\frac{3}{4}$ hours or $\frac{31}{4}$ hours

12 Bryce

13 cutlery set

14 $\frac{63}{64}$

15 **(a)** $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{8}$ **(b)** $13 \times 8 = 104$

Students' sample answers for Q16, 17 and 18.

16 $\frac{5}{8}, \frac{3}{4}, \frac{2}{3}$

17 **(a)** $\frac{2+1}{2-1} = \frac{3}{1} = 3$, $\frac{3+2}{3-2} = \frac{5}{1} = 5$, $\frac{4+1}{4-1} = \frac{5}{3}$ or $1\frac{2}{3}$ etc.

(b) 17 (when $A = 9$ and $B = 8$)

18 **(a)** $\frac{7}{3} + \frac{7}{4} = \frac{7}{3} \times \frac{7}{4} = \frac{49}{12}$ or $4\frac{1}{12}$

(b) $\frac{5}{3} + \frac{5}{2} = \frac{5}{3} \times \frac{5}{2} = \frac{25}{6}$ or $4\frac{1}{6}$

(c) The numerators are the sum of the denominators.

(d) Anything that obeys the statement in **(c)**;

e.g. $\frac{9}{4}$ and $\frac{9}{5}$, $\frac{11}{5}$ and $\frac{11}{6}$

Challenge 3 (p. 111)

1 C

Find number first, then find required fraction.

2 720 L

Fraction given equals volume given. Dividing by the fraction gives total volume.

3 A

Write fractions with lowest common denominator.

4 C

List all the possible fractions, remembering that they must already be in simplest form. Subtract the smallest fraction from the second list from the largest fraction from the first list and simplify.

5 B

Same method as in Q3 but remember to simplify fraction at the end.

6 1:20 p.m.

Divide each time into 60 minutes—realise that they both croak on the hour every hour. By listing or otherwise, find times after the hour that the frogs croak. This will give the first time they both croak together.

7 C

Rewrite the fraction doing the addition in the denominator. Then, do the division, then add.

8 $\frac{33}{109}$

Simplify each fraction step by step starting with the $3 + \frac{1}{3}$ fraction. Divide, then repeat the process of adding, then dividing until a single fraction remains.

9 36

Subtract $\frac{1}{3}$ of the number from 2 times the number.
This gives the fraction of the number equal to the amount.
Divide this fraction into the amount to find the whole.

10 $\frac{13}{96}, \frac{7}{48}, \frac{5}{32}$

Find the middle number using the method in Q3. Now, find the middle number between this number and the given numbers by finding equivalent fractions. Don't forget to write the fraction found in simplest form.

11 \$100

Cancelling common factors gives:

$$\frac{1}{\cancel{2}_1} \times \frac{\cancel{2}^1}{\cancel{3}_1} \times \frac{\cancel{3}^1}{\cancel{4}_1} \times \frac{\cancel{4}^1}{\cancel{5}_1} \times \frac{\cancel{5}^1}{\cancel{6}_1} \times \frac{\cancel{6}^1}{\cancel{7}_1} \times \frac{\cancel{7}^1}{\cancel{8}_1} \times \frac{\cancel{8}^1}{\cancel{9}_1} \times \frac{\cancel{9}^1}{10} \times \$1000$$

$$= \frac{1}{10} \times \$1000$$

$$= \$100$$

Chapter review 3 (p. 112)

1 **(a)** 16 **(b)** $\frac{5}{12}$

2 $\frac{3}{4}, \frac{5}{4}\left(\text{or } 1\frac{1}{4}\right)$

3 **(a)** **(i)** $\frac{18}{3}$ **(ii)** $\frac{60}{10}$ **(iii)** $\frac{6}{1}$

(b) **(i)** $\frac{33}{3}$ **(ii)** $\frac{110}{10}$ **(iii)** $\frac{11}{1}$

4 **(a)** $\frac{3}{5}$ **(b)** $1\frac{3}{4}$

5 **(a)** 42 **(b)** 4 **(c)** 90 **(d)** 30

6 **(a)** $\frac{4}{5}$ **(b)** $\frac{12}{5}$ or $2\frac{2}{5}$

(c) $3\frac{5}{6}$ or $\frac{23}{6}$ **(d)** $1\frac{1}{5}$ or $\frac{6}{5}$

7 (a) $\frac{23}{7}$ (b) $5\frac{5}{9}$

8 (a) < (b) = (c) > (d) <

9 (a) $\frac{31}{24}$ or $1\frac{7}{24}$ (b) $\frac{1}{6}$

(c) $\frac{8}{21}$ (d) $\frac{13}{15}$

10 (a) $1\frac{13}{20}$ (b) $8\frac{1}{6}$

(c) $\frac{37}{7}$ or $5\frac{2}{7}$ (d) $\frac{1}{6}$

11 (a) \$12 (b) $\frac{1}{6}$

12 (a) $\frac{5}{3}$ or $1\frac{2}{3}$ (b) $\frac{110}{9}$ or $12\frac{2}{9}$ (c) $\frac{3}{7}$

13 (a) 12 (b) $6\frac{9}{23}$ (c) $\frac{9}{16}$ (d) $\frac{39}{40}$

14 (a) $5\frac{17}{18}$ (b) $4\frac{9}{10}$ (c) $3\frac{9}{32}$

15 (a) $\frac{1}{2}+\frac{2}{5}$ (b) $\frac{9}{10}$

16 (a) $\frac{1}{3}$ (b) $2\frac{1}{6}$ or $\frac{13}{6}$ (c) $1\frac{1}{3}$ or $\frac{4}{3}$ (d) $\frac{3}{8}$

17 (a) $\frac{5}{9}$ (b) $\frac{2}{9}$ (c) $\frac{19}{27}$

18 (a) $\frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{3}{4}, 1$ (b) $\frac{30}{50}, \frac{5}{4}, 1\frac{2}{5}, 2, \frac{13}{5}$

19 Jamal

20 (a) $\frac{4}{15}$

(b) $\frac{16}{15}$ or $1\frac{1}{15}$ = more than a whole block of chocolate

21 $\frac{5}{6}$

22 (a) 30 000 L (b) 50 000 L (c) 12 000 L

23 45 g butter, $\frac{1}{6}$ cup castor sugar, 1 egg, $\frac{7}{16}$ cup self-raising flour, $\frac{1}{2}$ tablespoon cocoa, $\frac{1}{8}$ cup orange juice

24 (a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{3}{8}$

25 (a) Katya (6 km) (b) Mabok

26 A

NAPLAN practice 3

1 D **2** B **3** A **4** B

5 B **6** C **7** D **8** D

Chapter 4

Recall 4 (p. 116)

1 (a) 0.1 > 0.01 (b) 2 < 2.2 (c) 0.3 > 0.1

(d) 3.2 < 4.1 (e) 0.008 < 0.09 (f) 0.7 > 0.07

2 (a) seven tens and two ones

(b) six hundreds and three ones

(c) nine thousands, two hundreds, five tens and one one

(d) one ten-thousand, one thousand and eight tens

3 (a) (i) seven tenths (ii) 0.7

(b) (i) eight thousandths (ii) 0.008

(c) (i) three hundredths (ii) 0.03

(d) (i) one and nine tenths (ii) 1.9

4 (a) 110 (b) 1535 (c) 479 (d) 6499

5 (a) 49 (b) 447 (c) 7905 (d) 1648

6 (a) 34 (b) 2400 (c) 144 (d) 40 800

7 (a) 181 (b) 1051 (c) 28 (d) 150

8 (a) 7000 (b) 120 000 (c) 9 200 000 (d) 8

(e) 36 (f) 250 (g) 12 (h) 630

(i) 42.7 (j) 5.8 (k) 9.01 (l) 0.0762

9 (a) $\frac{1}{2}$ (b) $\frac{1}{10}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$ (e) $\frac{1}{100}$ (f) $\frac{3}{5}$

Exercise 4.1 (p. 119)

1 (a) (i) 45.462 (ii) 12.519 372 (iii) 3.7985

(iv) 1.008 332 (v) 0.786 06 (vi) 7.033 047

(b) (i) $6+\frac{6}{10}+\frac{3}{100}$

(ii) $\frac{9}{10}+\frac{2}{100}+\frac{1}{1000}$

(iii) $\frac{7}{10}+\frac{3}{100}+\frac{4}{1000}+\frac{5}{10\,000}$

(iv) $7+\frac{8}{10}+\frac{2}{100}+\frac{6}{1000}$

(v) $23+\frac{9}{10}+\frac{1}{100}+\frac{3}{1000}+\frac{4}{100\,000}$

(vi) $45+\frac{4}{1000}+\frac{5}{10\,000}+\frac{8}{100\,000}+\frac{9}{1\,000\,000}$

2 (a) (i) 6.5 (ii) 0.97 (iii) 0.273

(iv) 37.4201 (v) 14.957 623 (vi) 700.004 59

(b) (i) five ones and two tenths

(ii) four ones and nine tenths

(iii) three tens, four ones, one tenth and seven hundredths

(iv) six tenths and one hundredth

(v) two ones, seven tenths, nine hundredths and four thousandths

(vi) seven ones, five tenths, nine thousandths and two ten-thousandths

(vii) three tens, five ones, eight tenths, six hundredths, five thousandths, four ten-thousandths and three hundred-thousandths

(viii) eight tenths, two hundredths, two hundred-thousandths and seven millionths

3 **(a)** (i) $\frac{2}{1000}$ (ii) $\frac{2}{100\,000}$ (iii) $\frac{2}{10}$ (iv) 2

(b) (i) seven tenths (ii) seven thousandths

(iii) seven ten-thousandths (iv) seven millionths

4 **(a)** $2.4 > 0.42$ **(b)** $2.32 > 1.955$

(c) $0.65 > 0.57$ **(d)** $0.3003 < 0.333$

(e) $4.7038 < 4.7312$ **(f)** $8.251 > 8.2501$

(g) $7.02 > 7.002$ **(h)** $4.7367 < 4.7376$

(i) $0.927 < 0.927\,34$ **(j)** $6.013 > 6.01$

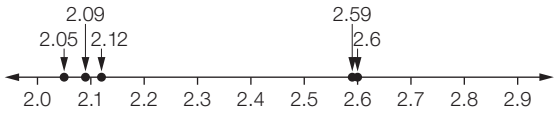
(k) $3.406 < 3.4063$ **(l)** $0.9995 > 0.9986$

5 **(a)** False **(b)** False **(c)** False **(d)** False

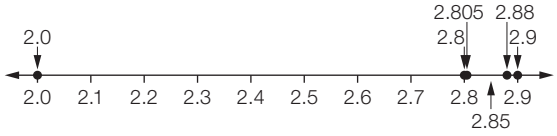
(e) True **(f)** False **(g)** True **(h)** False

6 B **7** B

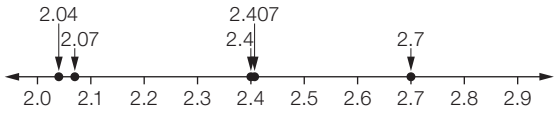
8 **(a)**



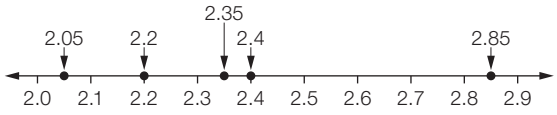
(b)



(c)



(d)



9 **(a)** 2.03, 2.13, 2.3 **(b)** 8.007, 8.67, 8.7

(c) 6.64, 6.6403, 6.646 **(d)** 0.0095, 0.0509, 0.0905

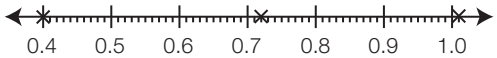
(e) 5.003 821, 5.3281, 5.38 **(f)** 3.116, 3.616, 3.661

(g) 0.092, 0.29, 0.92 **(h)** 0.086, 0.815, 0.85

10 37.901

11 **(a)** $\frac{4}{10} + \frac{3}{100} + \frac{7}{1000}$ **(b)** $\frac{437}{1000}$

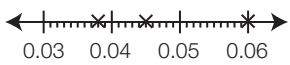
12 **(a)**



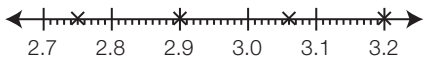
(b)



(c)



(d)



13 **(a)** 13.08 seconds **(b)** 13.95 seconds

14 25.6 m

15 **(a)** (i) 20.2 (ii) 21 (iii) 21.5

(b) 3.75 **(c)** 1.25 L **(d)** 2.60

16 **(a)** The 2 is in the thousandths place, not in the hundredths place.

(b) Should be 409.67. Al has written the 6 in the hundredths place and the 7 in the thousandths place.

(c) Max has put the 7 in the tenths place instead of in the ones place. Answer should be 7.0809.

(d) $\frac{9}{10} + \frac{5}{100} + \frac{6}{1000}$ means 0.956. Polly has not accounted for the 0 in the hundredths column. 0.9056 is the same as $\frac{9}{10} + \frac{5}{1000} + \frac{6}{10\,000}$.

17 **(a), (b)** 4.51, 4.52, 4.53, 4.6, 4.63, 4.67

18 **(a)** $2.13 < 5.6$, $3.52 < 5.1$, $4.69 < 5.3$

(b) 6, 7, 8, 9 could not be in the ones box.

19 **(a)** 2.57, 5.27, 7.25, 2.75, 5.72, 7.52, six different decimals

(b) 25.7, 27.5, 5.72, 52.7, 57.2, 7.25, 7.52, 72.5, 75.2, nine different decimals

Exercise 4.2 (p. 124)

1 **(a)** 4.9 **(b)** 6.7 **(c)** 7.64 **(d)** 4.55

(e) 0.642 **(f)** 3.28 **(g)** 0.3144 **(h)** 11.83

(i) 2.92 **(j)** 18.500 **(k)** 90.9000 **(l)** 19.000

(m) 23.41 **(n)** 57.080 **(o)** 259.9000

2 **(a)** \$4.50 **(b)** \$2.75 **(c)** \$11.05

(d) \$23.90 **(e)** \$176.35 **(f)** \$542.05

(g) \$789.55 **(h)** \$56.75 **(i)** \$6775.00

(j) \$1149.95 **(k)** \$8990.00 **(l)** \$10 000.00

3 D **4** 9.6 seconds

5 \$53.65

6 **(a)** 99.9 **(b)** 99.94

7 42°C **8** \$11.95

9 352.2 metres per second

10 1.27 m

11 **(a)** 12.138, 12.143, 12.149, 12.152, 12.157

(b) 12.14 s, 12.14 s, 12.15 s, 12.15 s, 12.16 s

(c) Yes—1st and 2nd, and 3rd and 4th, are now tied.

12 **(a)** \$20.35 **(b)** \$20.40—5 cents extra

13 3.792, 3.790 56, 3.787

14 **(a)** \$100.02, \$99.98 **(b)** \$99.60, \$100.48

15 2.548, rounding at the 1st step creates errors at the 2nd step.

16 The coins have no real 'purchasing' power anymore, as prices get higher and higher. They were not worth the cost of producing them.

Exercise 4.3 (p. 128)

1 **(a)** $4\frac{1}{10}$ **(b)** $6\frac{1}{2}$ **(c)** $8\frac{3}{5}$ **(d)** $9\frac{2}{5}$

(e) $4\frac{71}{100}$ **(f)** $2\frac{37}{100}$ **(g)** $\frac{7}{20}$ **(h)** $2\frac{12}{25}$

(i) $5\frac{9}{1000}$ **(j)** $\frac{221}{250}$ **(k)** $6\frac{16}{125}$ **(l)** $3\frac{43}{250}$

(m) $7\frac{9}{200}$ **(n)** $\frac{11}{125}$ **(o)** $6\frac{3}{2000}$ **(p)** $1\frac{3}{400}$

2 **(a)** 0.7 **(b)** 0.09 **(c)** 0.17 **(d)** 0.123

(e) 0.2 **(f)** 0.12 **(g)** 0.26 **(h)** 1.75

(i) 0.875 **(j)** 0.6875 **(k)** 0.575 **(l)** $0.78\dot{3}$

(m) $0.\dot{1}$ **(n)** $0.58\dot{3}$ **(o)** $2.8\dot{3}$ **(p)** $1.0\dot{6}$

3 D **4** D **5** $\frac{12}{25}$

6 $\frac{127}{50}$ (or $2\frac{27}{50}$) **7** $1\frac{7}{20}$

8 $\frac{3}{125\,000}$ **9** 0.000 005

10 $5\frac{17}{20}$ per cent, which is higher than $5\frac{3}{4}$ per cent ($5\frac{15}{20}$ per cent)

11 $4\frac{1}{5}$, 4.375, $4\frac{1}{2}$, 4.865, $4\frac{9}{10}$

12 **(a)** $3\frac{44}{100}$, $3\frac{45}{100}$, $3\frac{46}{100}$ **(b)** $3\frac{45}{99}$, $3\frac{44}{98}$, $3\frac{46}{97}$

13 $5\frac{62}{100}$, $5\frac{59}{100}$

Exercise 4.4 (p. 131)

1 **(a)** 16.2 **(b)** 12.94 **(c)** 0.914 **(d)** 25.648

(e) 26.418 **(f)** 39.811 **(g)** 22.827 **(h)** 10.259

(i) 8.858 **(j)** 23.917 **(k)** 17.595 **(l)** 11.5834

2 **(a)** 1.9 **(b)** 4.12 **(c)** 12.247 **(d)** 6.963

(e) 3.184 **(f)** 3.838 **(g)** 7.26 **(h)** 7.45

(i) 2.769 **(j)** 0.56 **(k)** 0.08 **(l)** 92.302

3 D **4** 12.70 mm

5 **(a)** $19.87 **(b)** $19.85

6 $295.51 **7** 40.5 **8** 267.9 km

9 196.96 km **10** $63.65 **11** 0.375 litres

12 **(a)** $165.90 **(b)** The fourth item ($18.48) was added twice.

13 $330.65

14 **(a)** 0.33 + 2.541 **(b)** 1.213 + 2.532 + 0.115

(c) The last digits of the numbers have to add up to 10 or 20.

15 **(a)** 4.1 + 3.85 = 7.95, 4.3 + 2.85 = 7.15, 4.6 + 2.85 = 7.45

(b) The last digit in the 2nd number must be a 5.

16 **(a)** 1.234 and 9.946 **(b)** 2.324 and 11.034

(c) The last digits must be the same.

17 53.47, 53.43

Exercise 4.5 (p. 135)

1 **(a)** 30.4 **(b)** 37.1 **(c)** 58.5 **(d)** 23.10

(e) 2.16 **(f)** 73.98 **(g)** 64.376 **(h)** 4.333

(i) 54.927 **(j)** 287.04 **(k)** 1219 **(l)** 1186.88

(m) 0.0008 **(n)** 0.0035 **(o)** 3.0018

2 **(a)** 144 **(b)** 228.9 **(c)** 119.64 **(d)** 114

(e) 2170 **(f)** 2650 **(g)** 27 546 **(h)** 27 434

(i) 29 202 **(j)** 38 828 **(k)** 67 201.5 **(l)** 411 576

3 **(a)** 0.48 **(b)** 0.24 **(c)** 0.36 **(d)** 6.524

(e) 1.236 **(f)** 4.208 **(g)** 2.3994 **(h)** 2.0812

(i) 0.6768 **(j)** 0.0024 **(k)** 0.009 24 **(l)** 0.0036

4 C **5** C **6** C

7 $18.20 **8** 10.619 grams

9 **(a)** 3 500 000 **(b)** 4 260 000 000

(c) 1 300 000 000 000 **(d)** 57 080 000

(e) 10 200 000 000 000 **(f)** 20 045 000 000

10 0.984 metres **11** $117.98

12 $516.00 **13** 53 500 000 000

14 **(a)** $94.50 **(b)** $12.83

15 1950 kilograms **16** $80.60

17 37.5 litres **18** 292.25 kilojoules

19 **(a)** 22.3 million

(b) The 'exact' value is not accurate, as births and deaths cause it to change daily. It is not necessary or helpful, for most purposes, to give a more precise number.

20 **(a)** 6.75 metres **(b)** 2, 0.25 m

21 **(a)** $4.80 **(b)** $0.85

(c) Purchasing 3 multi-bags ($11.85) is slightly cheaper than purchasing individual bars ($12).

22 1.6 kilograms

23 **(a)** 76.3 kilowatt-hours **(b)** $9.92

24 **(a)** He forgot the place value of the digits. Estimation: $2 \times 90 = 180$ m.

(b) He could have multiplied 89 by 1, 0.6 and 0.03 and added the answers together.

(c) Students' own answers.

25 **(a)** 20 and 3.42, 30 and 2.28

(b) 12.5 and 5.472, 15 and 4.56

Half-time 4 (p. 138)

1 (a) $\frac{1}{4}$ (b) $\frac{3}{10}$ (c) $3\frac{27}{50}$ (d) $9\frac{69}{200}$

2 0.03, 0.321, 9.003 24, 9.01, 19.12

3 (a) 20.4 (b) 132.919 (c) 2.82 (d) 6.02
(e) 83.861 (f) 1.1

4 (a) 4.877 (b) 13.004 (c) 2.350 (d) 23.032

5 (a) 30.4 (b) 141.5 (c) 12.33 (d) 1356
(e) 520 (f) 3390 (g) 7.31 (h) 613.228
(i) 0.0006

6 (a) \$100.55 (b) \$48.45

7 (a) 23.4907 (b) 14.235 (c) 293.415 (d) 90.6207

8 (a) \$1.95 (b) \$35.10 (c) \$57.45 (d) \$190.00

9 (a) 0.8 (b) 0.85 (c) $0.\dot{6}$ (d) $0.58\dot{3}$

10 59.023, 59.105, 59.125, 59.23 **11** \$122.46

Exercise 4.6 (p. 141)

1 (a) 2.12 (b) 4.32 (c) 1.58 (d) 0.55
(e) 0.343 (f) 0.567 (g) 6.57 (h) 1.514
(i) 0.052 (j) 0.07 (k) 0.407 (l) 2.637
(m) 30.866 (n) 10.546 (o) 49.644

2 (a) 0.09 (b) 0.09 (c) 0.604 (d) 0.483
(e) 0.006 (f) 0.039 (g) 0.078 (h) 0.005
(i) 0.003 (j) 0.57 (k) 0.029 (l) 0.007

3 (a) 25.7 (b) 10.3 (c) 8.5 (d) 387
(e) 100.2 (f) 24.8 (g) 397 (h) 44.4
(i) 24.5 (j) 700 (k) 315 (l) 145
(m) 8700 (n) 11 500 (o) 16 700

4 D **5** C

6 0.289 kg **7** \$1.965 million

8 \$93.99 or \$94 per guest **9** 12 tankfuls

10 259 necklaces **11** 12.43 km

12 \$0.22/100 mL **13** \$4.71

14 1933 worms **15** 6.5 minutes

16 \$4.75 per kilometre

17 Angie's team 1.60 m, Jessica's team 1.59 m. Therefore, Angie's team is taller on average by 0.01 m.

18 Brazilian Roast at \$2.25 per 100 grams

19 33 5-cent coins, 23 10-cent coins, 26 20-cent coins, 15 50-cent coins

20 217 km

21 2.002, 3.006, 3.666

22 (a) 20.1 ÷ 1 (multiply by 2), 40.2 ÷ 2 (multiply by 4)
(b) 180.2 ÷ 0.1 (multiply by 5), 720.8 ÷ 0.4 (multiply by 20)

23 10 m and 0.845 m, perimeter 21.69 m; 5 m and 1.69 m, perimeter 13.38 m

24 (a) He didn't consider the place value of the digits to the RHS of the decimal point.
(b) To do decimal division more accurately, Tran should perform the division, ignoring the decimal point, and then place a decimal point in the answer, so it lines up with the one in the dividend.

Exercise 4.7 (p. 145)

1 (a) $\frac{3}{20}$ (b) $\frac{6}{25}$ (c) $\frac{9}{50}$ (d) $\frac{16}{25}$
(e) $\frac{11}{10}$ (f) $\frac{61}{50}$ (g) $\frac{7}{5}$ (h) $\frac{99}{50}$

2 (a) 0.34 (b) 0.13 (c) 0.78 (d) 0.92
(e) 1.2 (f) 1.23 (g) 2.54 (h) 3.2

3 (a) 40% (b) 75% (c) 30% (d) 35%
(e) 110% (f) 150% (g) 112% (h) 120%
(i) 37.5% (j) $33.\dot{3}\%$ (k) $83.\dot{3}\%$ (l) $44.\dot{4}\%$
(m) $86.\dot{6}\%$ (n) $63.\dot{3}\%$ (o) 82.5% (p) $73.\dot{3}\%$

4 (a) 87% (b) 34% (c) 65% (d) 96%
(e) 134% (f) 358% (g) 532% (h) 121%

5

Percentage	Fraction	Decimal
40%	$\frac{2}{5}$	0.4
70%	$\frac{7}{10}$	0.7
5%	$\frac{1}{20}$	0.05
2%	$\frac{1}{50}$	0.02
75%	$\frac{3}{4}$	0.75
125%	$\frac{5}{4}$	1.25

6 (a) 0.25, 0.5, 0.1, 0.2 (b) 25%, 50%, 10%, 20%

7 $\frac{19}{20}$ **8** 62.5%

9 (a) 0.03, $\frac{7}{50}$, 57%, $\frac{3}{5}$, 0.8, 82%

(b) 0.05, $\frac{3}{10}$, 43%, $\frac{1}{2}$, 73%, 1.05

10 (a) $\frac{3}{5}$ (b) 60%

11 63%

12 (a) (i) $\frac{1}{25}$ (ii) 4% (iii) 0.04

(b) The percentage, as most members of the public would be able to comprehend and visualise the size of the number.

(c) '4% of us believe in aliens'

13 $\frac{1}{4} = 25\%$ and $\frac{3}{4} = 75\%$, $75 = 3 \times 25$

14 **(a)** 75% **(b)** 0.75 **(c)** 25% **(d)** 0.25

15 $\frac{1}{3}, \frac{1}{4}, \frac{3}{10}$

16 34.1%, 34.2%, 34.3%

17 Students' own answers.

Exercise 4.8 (p. 149)

1 **(a)** \$18 **(b)** 45 m **(c)** 168 L **(d)** 96 m
(e) 450 kg **(f)** \$168 **(g)** 9.36 kg **(h)** \$95.58
(i) 183.96 m **(j)** 64.75 m **(k)** 71.536 L **(l)** \$1.31

2 **(a)** 85% **(b)** 22.7% **(c)** 40% **(d)** 80%
(e) 76% **(f)** 77.1% **(g)** 55.6% **(h)** 75%
(i) 82.4%

3 **(a)** \$12.50 **(b)** 7.5 kg **(c)** 15 km **(d)** 8 L
(e) 3.6 m **(f)** \$6 **(g)** 210 kg **(h)** 600 mL
(i) \$140

4 B **5** C **6** 10 students

7 57.1% are cream biscuits **8** 0.7 L

9 $83.\dot{3}\%$ **10** Boys: 42.3%, Girls: 57.7%

11 **(a)** Damian 70%, Franco 68.75% **(b)** Damian

12 33.1% of the score

13 **(a)** **(i)** \$8 **(ii)** \$31.25 **(iii)** \$32.50
(iv) \$23.90 **(v)** \$319.60 **(vi)** \$64.95
(b) **(i)** \$32 **(ii)** \$93.75 **(iii)** \$32.50
(iv) \$215.10 **(v)** \$479.40 **(vi)** \$1234.05

14 92%

15 **(a)** 22% of 180 = 39.6, 34% of 180 = 61.2, 7% of 180 = 12.6

(b) Each of the answers is a decimal number, which is not possible when working with numbers of people (cannot have 0.6 of a student).

(c) Actual number with blue eyes could be either 39 or 40 (both give a percentage that rounds to 22 as the nearest whole number), actual number speaking two or more languages could be 61 or 62 (both give a percentage of 34 when rounded) and the actual number of left-handed students could be 12 or 13 (both give a percentage of 7 when rounded).

16 Chelsea—Game 1: 68%, Game 2: 70.4%
Sonia—Game 1: 75%, Game 2: 73.9%

Chelsea increased her percentage, so she improved her accuracy, whereas Sonia's accuracy dropped slightly.

17 **(a)** 2700 kJ **(b)** 73 g of fat

18 Savings look more attractive, and are more easily interpreted (e.g. 50% off is easily recognised as half price).

19 50%, \$10; 25%, \$20; 10%, \$50

20 Fedora's working is correct. Tanya has confused the method for finding a percentage with the method for writing one amount as a percentage of another. The question asked here has the percentage sign appearing in the question, which means that the first method (multiplying by the per cent) should be used. If the question contains the words 'out of', then the second method (writing one amount as a percentage of another) should be used.

Exercise 4.9 (p. 153)

1 **(a)** 5:3 **(b)** 4:7 **(c)** 3:2 **(d)** 8:3
(e) 1:2:3 **(f)** 2:9:2 **(g)** 3:4:7 **(h)** 3:2:1

2 **(a)** 2:3 **(b)** 3:5 **(c)** $\frac{3}{5}$ **(d)** 40%

3 **(a)** 2:6 **(b)** 2:16 **(c)** 2:22 **(d)** 2:66
(e) 12:15 **(f)** 12:28 **(g)** 2:3 **(h)** 2:3
(i) 5:4 **(j)** 2:1 **(k)** 56:35 **(l)** 60:55
(m) 8:3 **(n)** 100:30 **(o)** 20:18 **(p)** 75:65

4 **(a)** 1:5 **(b)** 1:4 **(c)** 2:9 **(d)** 2:7
(e) 3:2 **(f)** 8:5 **(g)** 8:5 **(h)** 4:3
(i) 27:32 **(j)** 14:17 **(k)** 8:9 **(l)** 2:5
(m) 3:2 **(n)** 3:2 **(o)** 7:4 **(p)** 40:11

5 **(a)** B **(b)** C

6 **(a)** 12:7:16 **(b)** 35 **(c)** $\frac{12}{35}$ **(d)** 20%

7 **(a)** 15:7:2 **(b)** $\frac{7}{24}$ **(c)** 29%

8 **(a)** 5:4:3 **(b)** $\frac{5}{12}$ **(c)** 42%

9 **(a)** 6:1 **(b)** 6:1 = 24:6, 6 instructors required
(c) 30

10 3:2:1 = 12:8:4, 8 buckets of sand, 12 buckets of gravel

11 **(a)** 10% **(b)** 2 g

12 **(a)** 25% **(b)** 75:25 = 3:1

13 **(a)** B **(b)** F **(c)** C **(d)** E

14 **(a)** 4 teachers are required—3 would only be enough for 60 students (1:20 = 3:60).

(b) 4:68 = 1:17

15 **(a)** Chloe: 1:5
Hannah: 2:5

(b) Hannah is correct. Looking at the simplified ratios, Hannah has 1 extra part of concentrate for the same number of parts of water.

16 **(a)** $\frac{5}{8}$ **(b)** $\frac{3}{8}$ **(c)** $\frac{4}{9}$

(d) 2 cats and 1 dog

17 **(a)** Answer will depend on the class.

(b) Unlikely, unless it is a single-sex school.

(c) No; there would be a big variation in all coeducational schools; some schools are mixed sex, others are single-sex.

18 8:14, 12:21, 16:28

19 The ratio should be 7:12; Tenille forgot to count the apples as fruit.

Exercise 4.10 (p. 158)

1 **(a)** $5.60 **(b)** 104 cans **(c)** $1.20
(d) $8500 **(e)** 25 kg **(f)** 525 pages

2 **(a)** 600 g: $0.68 per 100 g, 250 g: $1.07 per 100 g; 600 g is the better value

(b) 500 g: $0.56 per 100 g, 750 g: $0.46 per 100 g; 750 g is the better value

(c) 375 g: $0.34 per 100 g, 500 g: $0.65 per 100 g; 375 g is the better value

(d) 200 g: $1.61 per 100 g, 375 g: $1.31 per 100 g; 375 g is the better value

(e) 500 mL: $0.54 per 100 mL, 920 mL: $0.47 per 100 mL; 920 mL is the better value

(f) 450 g: $0.84 per 100 g, 1 kg: $0.88 per 100 g; 450 g is the better value

3 A **4** D **5** 19.2, or 20 loaves

6 **(a)** the 400 g jar

(b) Yes, the 280 g jar is now slightly better value for money, with both jars approximately $1.60 per 100 g.

7 $6 **8** 7.1 runs per over

9 **(a)** **(i)** packet of 6: $0.81 per bar, packet of 10: $0.75 per bar; packet of 10 is the better value

(ii) 12-pack: $1.18 per can, 18-pack: $1.12 per can, 30-pack: $0.92 per can; 30-pack is the better value

(iii) box of 50: $0.06 per bag, box of 200: $0.04 per bag, box of 200 is the better value

(b) Multiply the cost of the box of 50 by 4 to get the price of 200, or multiply the cost of the box of 50 by 2 and divide the cost of the box of 200 by 2, to get the prices per 100 bags.

10 **(a)** $1.45 **(b)** $17.40

11 **(a)** 3.6 L **(b)** 25.2 L

12 **(a)** 3.25 g **(b)** 13 g **(c)** 16.25 g

13 **(a)** $6.00 **(b)** $0.60 **(c)** $7.80

14 Individual cost of each bun in the packet = $1.20
$4 \times \$1.20 = \$4.80$
$7.20 (1 packet) + $4.80 (4 loose) = $12.00

15 **(a)** 4 kg box is better value:
$\$15.97 \approx \$16$
$\approx \$4/\text{kg}$

(b) 1 L carton is better:
1 L = 4 × 250 mL
$4 \times \$1.25 = 2 \times \$2.50 = \$5$

(c) packet ham is better value:
500 g of ham at $13/kg
= $6.50

(d) 400 g jar is better value:
$\$7.99 \approx \$8$
$\approx \$2$/100 g, or $6 for 300 g

(e) The product may be Australian made, the packaging may be appealing, or you may prefer a particular brand.

16 Plan: $4.10 is cheaper, Pre-paid: $4.25 is more expensive

17 **(a)** 2516.13 kL/day

(b) 918 387.45 kL

(c) No—cannot assume that the rainfall in January will be the same for the other months of the year. January is part of the wet season, so other months will definitely be drier.

18 **(a)** The 500 g block is two times bigger than the 250 g block.

(b) 500 g block: $9.00; 750 g block: $13.50

(c) The packaging of larger items is cheaper; 'bulk' buys are made more attractive so that consumers will purchase more of the product.

19 soft drink, milk, fruit juice, yoghurt

20 **(a)** 9×500 g; 4×1 kg $+ 1 \times 500$ g; 2×2 kg $+ 1 \times 500$ g

(b) 2×2 kg $+ 1 \times 500$ g is the cheapest ($10.28 compared to $11.16 and $10.64)

Challenge 4 (p. 161)

1 B

2 8×0.753. Try various combinations. Hopefully, the students will only need to investigate 8×0.753 and 7×0.853.

3 $0.01 - 0.001 = 0.009$

4 C $\$\frac{3}{4} = \$0.75$, $\$\frac{3}{10} = \$0.30$. Add the amounts.

5 C $200 \div 2.5 = 80$, so $20 \div 2.5 = 8$ and $20 \div 0.25 = 80$

6 B Amount is $49\,095 - 49.95 = 49\,045.05$ to be subtracted.

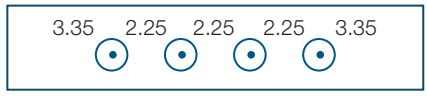
7 **(a)** The last digit of both of the decimal numbers must not multiply together to give a number ending in 0, e.g. $3.21 \times 4.547 = 14.595\,87$.

(b) The last digit of both of the decimal numbers must multiply together to give a number ending in 0, e.g. $1.25 \times 3.766 = 4.7075$.

(c) The last two digits of both of the decimal numbers must multiply together to give a number ending in 00, e.g. $7.25 \times 8.324 = 60.349$.

8 Scores a total of $6 \times 7.5 = 45$ goals. Least number of goals scored = 4, so she scores $45 - 4 = 41$ goals in 5 other games.
$41 = 4 \times 8 + 9$, so she must score at least 9 in one game.
$41 = 7 + 3 \times 8 + 10$, so she could score more than 9. The lowest value for the highest number of goals she scores is 9.

9 Length of strip of metal = $2 \times 3.35 + 3 \times 2.25 = 13.45$ cm



10 $0.01 \div 0.002 = 10 \div 2 = 5$

11 C $0.9 \times \$30\,000 = \$27\,000$; $1.1 \times \$27\,000 = \$29\,700$

12 A **13** B

Chapter review 4 (p. 162)

1 (a) $\frac{9}{10} + \frac{6}{100} + \frac{8}{1000}$ (b) $5 + \frac{7}{100} + \frac{2}{10\,000}$

(c) $6 + \frac{5}{1000}$

2 (a) 0.6524 (b) 0.080 093

3 (a) 4.3806 (b) 57.032 09

4 (a) one one, eight tenths, five hundredths, three thousandths and one ten-thousandth

(b) seven hundredths and six hundred-thousandths

(c) six tens, one one and nine ten-thousandths

5 (a) (i) $\frac{9}{100}$ (ii) nine hundredths

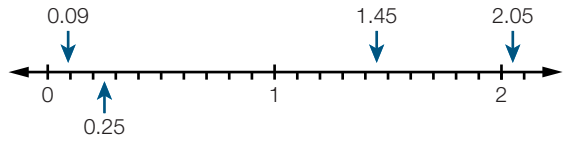
(b) (i) $\frac{9}{10\,000}$ (ii) nine ten-thousandths

(c) (i) $\frac{9}{100}$ (ii) nine hundredths

6 (a) $3.0427 > 3.0274$ (b) $0.009\,95 < 0.01$

7 (a) 0.5506, 0.6055, 0.607 (b) 0.071, 0.701, 0.71

8



9 (a) 4.4 (b) 34.65 (c) 23.1 (d) 102.47

10 (a) \$41.20 (b) \$10.05 (c) \$79.95 (d) \$99.00

11 (a) $3\frac{9}{10}$ (b) $\frac{31}{50}$ (c) $2\frac{9}{20}$ (d) $\frac{9}{5000}$

12 (a) 0.8 (b) 0.95 (c) $0.\dot{2}$ (d) $0.8\dot{3}$

13 (a) 28.39 (b) 15.4585 (c) 41.433
(d) 7.717 (e) 1.688 (f) 4.7802

14 (a) 13.504 (b) 104.65 (c) 200.97
(d) 47 200 (e) 164.4 (f) 3.45
(g) 0.54 (h) 0.000 36 (i) 19.76

15 (a) 2.58 (b) 1.004 (c) 0.469
(d) 0.240 (e) 0.016 (f) 0.182
(g) 12 (h) 2.4 (i) 5950

16 (a) 70% (b) 125% (c) 12.5% (d) $66.\dot{6}\%$

17 (a) 38% (b) 96% (c) 355% (d) 9%
(e) 323.4% (f) 54.32%

18 (a) \$24 (b) 12.75 L (c) 166 m (d) \$75

19 (a) 65% (b) 50% (c) 35% (d) 93.3%

20 (a) 8:7 (b) $\frac{7}{15}$ (c) 53%

21 (a) \$7.05 (b) \$3.14

22 (a) 1 326 000 000 (b) 1 140 000 000
(c) 228 200 000 (d) 4 270 000

23 (a) \$3023.60 (b) \$7559.00

24 (a) T-shirt discount \$3 Jeans discount \$24
(b) T-shirt \$27 Jeans \$36

25 (a) The 400 g jar is better value.

(b) Yes, the 250 g jar is now better value than the 400 g jar.

(c) People might only require a small amount of a product—a larger amount might be wasted, some amounts are in a more convenient size (e.g. travel packs)

26 (a) 75 mL (b) No; 60:40 = 3:2

27 (a) 7.8 (b) 23.4 (c) 63.18 Yes

28 (a) \$5.20 (b) \$0.40 (c) \$0.80

(d) Buy six 6-packs and four individual cans (total cost of \$32.28).

29 5 supervisors will be needed; 4 supervisors could only manage 48 children at the required ratio.

30 Store A discount = \$16.25, sale price = \$48.75

Store B discount = \$22.50, sale price = \$52.50

Sally should buy her shoes at Store A to pay the lowest price.

NAPLAN practice 4

1 D **2** B **3** D **4** B

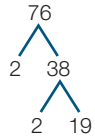
5 \$11 **6** D **7** C **8** \$178.92

Mixed review B (p. 166)

1 (a) $\frac{3}{10}$ (b) $\frac{3}{10}$ (c) $\frac{3}{4}$ (d) $\frac{4}{13}$

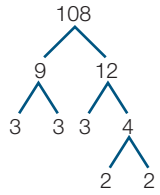
2 (a) -74 (b) +3 (c) -215 (d) +8

3 (a) (i)



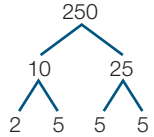
(ii) $2^2 \times 19$

(b) (i)



(ii) $2^2 \times 3^3$

(c) (i)



(ii) 2×5^3

4 (a) 4800 (b) 3300 (c) 12 (d) 4

5 (a) > (b) < (c) <

6 (a) 0.27 (b) 15.3 (c) 435

7 (a) -6 (b) 14 (c) -10 (d) -8
(e) 6 (f) -9

8 (a) $\frac{8}{25}$ (b) $1\frac{6}{25}$ (c) $3\frac{237}{1000}$

9 (a) 7^4 (b) 4^2 (c) $2^2 \times 3^3$ (d) 11^3

10 (a) 30 (b) 6 (c) 30

11 (a) < (b) > (c) <

12 (a) 6 (b) 8 (c) 12

13 (a) $\frac{19}{20}$ (b) $4\frac{1}{24}$ (c) $\frac{11}{24}$ (d) $\frac{3}{4}$

14 (a) 203 (b) 484 (c) 195 (d) 65
(e) 280 (f) 55

15 (a) 4 m (b) $3

16 (a) (i) $\frac{3}{10}$ (ii) 30%

(b) (i) $\frac{3}{13}$ (ii) 23.1%

17 (a) 14 g (b) 1.4 g (c) 4.2 g

18 (a) 29 slices (b) 13 slices

19 (a) 3200 (b) 1200 (c) $\frac{1}{12}$

20 $3^2 = 3 \times 3$, not 3×2

21 Twice—if the cycles start on the hour, they will flash at 24 and 48 past the hour.

22 (a) $\frac{5}{18}$

(b) Eamon (30 pieces compared to Chloe's 9 pieces).

23 (a) 84.5

(b) It is a decimal number, which is impossible when dealing with numbers of students.

(c) The actual number of students was either 84 or 85. Both numbers give a percentage of 65 when the answer is rounded to the nearest whole number.

24 8

25 69 cards

Chapter 5

Recall 5 (p. 168)

1 (a) 5 (b) 2 (c) 1 (d) 4 (e) 2 (f) ±4

2 (a) (i) 41 (ii) 107 (iii) 2197

(b) (i) 10 (ii) 94 (iii) 58

3 (a) $11 + 17 = 28$ (b) $9 - 7 = 2$
(c) $3 \times 4 = 12$ (d) $16 \div 8 = 2$

4 (a) $6 \times 2 - 4$; 8 (b) $(10 + 5) \div 5$; 3
(c) $(6 + 5) \times 3$; 33 (d) $(23 - 21) \times 12$; 24

5 (a) peregrine falcon (b) kodiak bear

Exercise 5.1 (p. 171)

1 (a) $p - 3$ (b) $k + 9$ (c) $12v$
(d) $\frac{a}{4}$ (e) $d + e$

2 (a) $x + 2$ (b) $4y$ (c) $5 - t$
(d) $\frac{k}{7}$ (e) $6gh$ (f) $10d + 8$
(g) $10(d + 8)$ (h) $\frac{y+9}{4}$ (i) $5e - 3$
(j) $\frac{r+s+t}{3} + 11$ (k) $y^2 + 20$ (l) $8x^2$

3 B **4** D

5 (a) $5(n + 1)$ (b) $\frac{2+d}{9}$ (c) $12(a + b)$ (d) $mn - 3$

6 (a) $12a$ (b) $5y$ (c) $18c^2d$
(d) $2m$ (e) $\frac{p}{4}$ (f) $\frac{7k}{2}$

7 (a) $\frac{x}{2}$ (b) $\frac{x}{2} + 3$

8 (a) $\$6d$ (b) $\$2.5t$

9 $\$(55x + 300)$ **10** $\$bv$

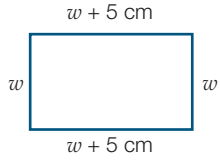
11 (a) $\$12t$ (b) $\$24s$ (c) $\$(12t + 24s)$

12 (a) $5n$ (b) $10r$ (c) $5n + 10r$

13 (a) $m + n + q + 1$ (b) $2m + 2n + 2q + 2$

(c) pens in large packs: $2m + 2m$,
pens in small packs: $m + m + m$
total pens = $3m + 4m$ or $7m$

14 (a) (i) $(w + 5)$ cm (ii)

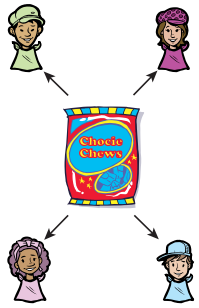


(b) $w + w + 5 + w + w + 5 = w + w + w + w + 10$ or $4w + 10$

15 (a)



(b) A packet of lollies shared between 4 people:



16 (a) Grandma's age is $2(r + d)$.

(b) Possible ages are: Rory 10, dad 30, grandma 80; Rory 5, dad 25, grandma 60.

Exercise 5.2 (p. 175)

1 **(a)** **(i)** expression **(ii)** 2
(iii) x and y **(iv)** -7
(v) $2x$, $3y$, -7 and $4x^2$

(b) **(i)** equation **(ii)** -4
(iii) x, y and z **(iv)** 32
(v) 32, $-4x$, $3xy$ and $4z$

(c) **(i)** equation **(ii)** -23
(iii) x and y **(iv)** 9
(v) $2xy$, $-3x^2$, 9 and $-23x$

(d) **(i)** expression **(ii)** -1
(iii) x and y **(iv)** 6
(v) 6, $-x$, $8y$ and $-4xy$

2 **(a)** **(i)** number of girls, number of boys
(ii) Let x = number of boys; let y = number of girls
(iii) $x = 3y$

(b) **(i)** Fred's age; James's age
(ii) Let g = Fred's age; let h = James's age
(iii) $g = 3h - 1$

(c) **(i)** cost of an apple, cost of a pear
(ii) Let m = cost of an apple; let n = cost of a pear
(iii) $2m = 3n$

(d) **(i)** cost of a kilogram of potatoes, cost of a watermelon
(ii) Let a = cost of a kg of potatoes; let b = cost of a watermelon
(iii) $3a + b = \$5.20$

(e) **(i)** distance to Ranko's house, distance to Skye's house
(ii) Let r = distance to Ranko's house; let s = distance to Skye's house
(iii) $s + 5 = r$

(f) **(i)** number of plants, number of pavers
(ii) Let v = number of plants, let w = number of pavers
(iii) $2v + 6 = w$

3 **(a)** True **(b)** False **(c)** False **(d)** True **(e)** False
(f) True **(g)** True **(h)** False **(i)** True **(j)** True

4 **(a)** B **(b)** D **(c)** B **(d)** C **(e)** C **(f)** C

5 **(a)** **(i)** Let m = mass of 1 tablet
(ii) $25m + 120 = 195$

(b) **(i)** Let a = Mei Lee's age now
(ii) $\frac{a}{2} = a - 8$

(c) **(i)** Let B = Beth's age; let D = dad's age
(ii) $B + 10 = \frac{D}{2}$

(d) **(i)** Let number of goals = g;
let number of penalty goals = p
(ii) $8g + p = 100$

6 **(a)** Let d = distance travelled on first day
(b) $2d$ **(c)** $d + 480$ **(d)** $d + 480 = 3d$

7 **(a)** Number of goals scored is g and number of behinds is b.
(b) $6g + b = 114$
(c) An example is 12 goals and 42 points.

8 **(a)** 4 tries, 1 penalty goal to 4 penalty goals. 2 converted tries, 3 penalty goals to 2 tries, 1 conversion goal.
(b) $P = 5t + 2g + 3p$

Exercise 5.3 (p. 179)

1 **(a)** $x \rightarrow \boxed{+2} \rightarrow y$

$y = x + 2$

x	13	11	7	28	1
y	15	13	9	30	3

(b) $x \rightarrow \boxed{-5} \rightarrow y$

$y = x - 5$

x	6	18	9	85	5
y	1	13	4	80	0

(c) $x \rightarrow \boxed{\times 2} \rightarrow y$

$y = 2x$

x	2	3.4	10	11	101
y	4	6.8	20	22	202

(d) $x \rightarrow \boxed{\div 3} \rightarrow y$

$y = \frac{x}{3}$

x	18	12	30	9	0
y	6	4	10	3	0

(e) $x \rightarrow \boxed{\times 5} \rightarrow \boxed{-3} \rightarrow y$

$y = 5x - 3$

x	4	2	0	5	20
y	17	7	-3	22	97

(f) $x \rightarrow \boxed{+5} \rightarrow \boxed{\div 10} \rightarrow y$

$y = \frac{x + 5}{10}$

x	5	25	45	10	33
y	1	3	5	1.5	3.8

(g) $x \rightarrow \boxed{\div 2} \rightarrow \boxed{-1} \rightarrow y$

$y = \frac{x}{2} - 1$

x	10	6	24	9	15
y	4	2	11	3.5	6.5

(h) $x \rightarrow \boxed{\times x} \rightarrow y$

$y = x^2$

x	3	11	7	6	10
y	9	121	49	36	100

2 (a) A (b) A (c) D

3 (a) $y = 4x - 9$ (b) $y = 5(x + 11)$ or $y = (x + 11) \times 5$

(c) $b = \frac{a}{2} - 7$ (d) $b = \frac{a + 3}{6}$

(e) $q = p^2 + 8$ (f) $q = \sqrt{p}$

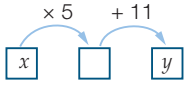
4 (a) $y = x - 18$ (b) $y = 60x$

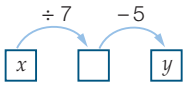
(c) $y = \frac{x}{7}$ (d) $y = 20(x + 43)$ or $y = (x + 43) \times 20$

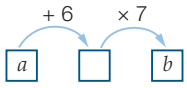
(e) $y = 100x - 50$ (f) $y = \frac{x}{16} + 13$

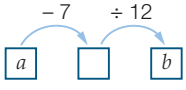
(g) $y = x^2$ (h) $y = \frac{x - 12}{9}$

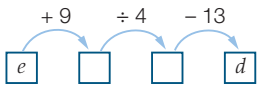
(i) $y = x^2 - 37$

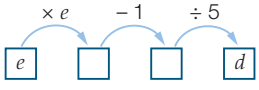
5 (a) 

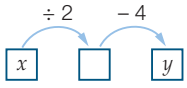
(b) 

(c) 

(d) 

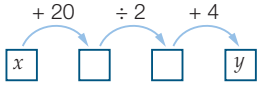
(e) 

(f) 

6 (a) 

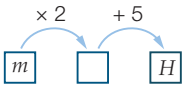
(b) $y = \frac{x}{2} - 4$

(c) $y = \frac{20}{2} - 4 = 6$ lollies left

7 (a) 

(b) $y = \frac{x + 20}{2} + 4$ (c) $y = \frac{12 + 20}{2} + 4 = 16 + 4 = 20$

Anita spent $20.

8 (a) 

(b) $H = 2m + 5$ (c) $H = 2 \times 12 + 5 = 24 + 5 = 29$

9 (a)

Number of pairs of jeans, n	10	50	150	200
Cost to produce the jeans, C	\$315	\$595	\$1295	\$1645

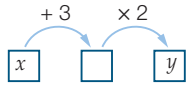
(b) $C = 7n + 245$

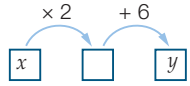
10 (a)

x	2	5	7	4	10
y	5	11	15	9	21

(b)

x	10	1	2	4	18
y	52	16	20	28	84

11 (a) (i) 

(ii) 

(b) (i) $y = 2(x + 3)$ or $y = (x + 3) \times 2$

(ii) $y = 2x + 6$

(c) (i)

x	2	5	3	4	7
y	10	16	12	14	20

(ii)

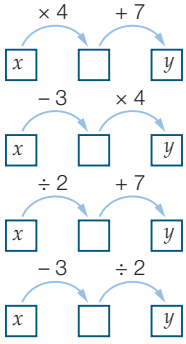
x	2	5	3	4	7
y	10	16	12	14	20

(d) The two tables of values are the same, which means the two rules are equivalent. In the first rule, both x and 3 are doubled in the second step, which gives $2x + 6$.

12 One possible table for the rule $y = \frac{x}{2} + 3$ is:

x	1	2	3	4	5
y	3.5	4	4.5	5	5.5

13 (a) Some possible two-step flowcharts are:



(b) The rules that correspond to the above flowcharts are:

$y = 4x + 7$

$y = 4(x - 3)$

$y = \frac{x}{2} + 7$

$y = \frac{x - 3}{2}$

14 Kim's statement that they are 'different ways of writing the same thing' is incorrect. Both boys have the correct operations of ÷ 2 and + 3 in their flowcharts, but the order in which they are used is important. The rule states that x is first divided by 2, then 3 is added, so Jai's flowchart is correct.

Exercise 5.4 (p. 184)

1 **(a)** 7 **(b)** $5\frac{1}{2}$ **(c)** -1 **(d)** 23

(e) 2.6 **(f)** 25 **(g)** 4 **(h)** 28

(i) 2 **(j)** 32 **(k)** 18 **(l)** 120

(m) 4 **(n)** 0.5 **(o)** 66 **(p)** 84

2 **(a)** \$270

(b) 70 kilometres per hour

3 **(a)** True **(b)** True **(c)** False

(d) True **(e)** False **(f)** True

4 **(a)**

a	11	20	5	9	50
b	44	80	20	36	200

(b)

x	6	4	10	20	101
y	42	28	70	140	707

(c)

m	1	2	10	6	5
n	5	8	32	20	17

(d)

j	2	5	11	10	100
k	15	27	51	47	407

(e)

p	11	15	10	20	100
q	12	20	10	30	190

(f)

r	1	2	3	0	200
s	6	14	22	-2	1598

(g)

u	5	3	1	201	6
v	16	8	0	800	20

(h)

m	5	10	11	102	52
n	9	24	27	300	150

5 **(a)** D **(b)** C **(c)** C

6 **(a)** $p = \frac{7}{100}d$ **(b)** $f = \frac{11}{100}d$

(c) $p = 28$ litres, $f = 44$ litres

(d) $C = 1.30y$ **(e)** \$20.80

7 **(a)** $\$350 + 10n = p$ **(b)** \$500

(c) \$550

8 **(a)** $D = 3l + 2.6$

(b) $D = 3 \times 3.25 + 2.6$
$= 9.75 + 2.60$
$= 12.35$

9 **(a)** $s = 2n + 10$ **(b)** $b = \frac{n}{8}$

(c) Jasmine will need 90 sausages and 5 bowls of salad.

10 Substituting $a = 4$ into $b = 3(a + 5)$ gives $b = 27$.

Substituting $a = 4$ into $b = 3a + 5$ gives $b = 17$.

In the first formula, the order of operations is add 5, then multiply by 3. In the second formula, the order of operations is multiply by 3, then add 5.

11 One possible formula is: $y = 3x + 5$

x	7	20	13	101
y	26	65	44	308

Another possible formula is: $y = 2(x + 4)$

x	7	20	13	101
y	22	48	34	210

12 Tania has substituted correctly. Suri has forgotten that $7x$ means $7 \times x$. In the future, Suri could include this in the first step of her working.

Exercise 5.5 (p. 189)

1 **(a)**

Number of squares (s)	1	2	3	4	5	6	7	8
Number of matches (m)	4	7	10	13	16	24	28	32

(b) Let s = number of squares, m = number of matches.
$m = 4s$

(c) 400 matches are needed to make 100 squares.

2 **(a)**

B	1	2	3	4	5
S	4	7	10	13	16

(b) $S = 3B + 1$

(c) $B = 21$ gives $S = 64$

3 **(a)**

T	1	2	3	4	5
P	3	5	7	9	11

(b) $P = 2T + 1$

(c) $T = 203$ gives $P = 407$
Clarence would need 407 pieces of wood.

4 **(a)**

Number of houses (h)	1	2	3	4	5	6	7	8
Number of matches (m)	6	11	16	21	26	31	36	41

(b) $m = 5h + 1$

(c) If $h = 20$

$m = 5 \times 20 + 1$
$= 101$

101 matches are needed to build 20 houses.

5 **(a)**

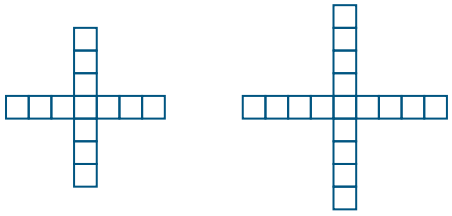
Height of L (H)	2	3	4	5	6
Number of globes (G)	3	5	7	9	11

(b) $G = 2H - 1$

(c) $H = 120$ gives $G = 239$. So, 239 globes would be needed.

6 B

7 **(a)**



(b)

a	1	2	3	4
b	5	9	13	17

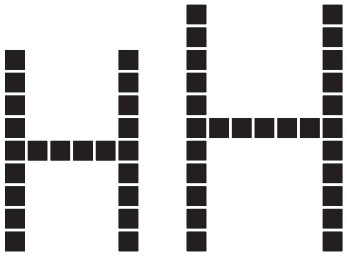
(c)

a	1	2	3	4	5	6
b	5	9	13	17	21	25

(d) $b = 4a + 1$

(e) $a = 52$ gives $b = 209$
209 bricks would be needed.

8 (a)

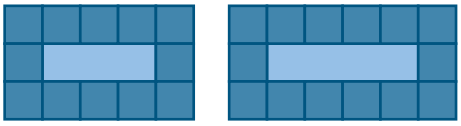


(b)

Width of the H (w)	3	4	5	6	7
Number of tiles (t)	7	12	17	22	27

(c) $t = 5w - 8$ **(d)** If $w = 12$, $t = 52$

9 (a)

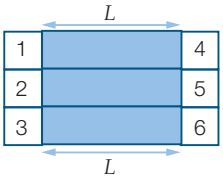


(b)

l	1	2	3	4
p	8	10	12	14

(c) $p = 2l + 6$

(d)



The constant is for the 3 pavers at each end of the pool. The coefficient of the pronumeral '2' is the number of pavers needed to increase the length each time.

(e) $p = 2(l + 3)$

(f) $l = 345$ gives $p = 696$
696 paving blocks are needed.

(g) The spa is 17 paving blocks long.

10 (a) **(i)** $m = 3s$ **(ii)** $m = 4s$
(iii) $m = 2s + 1$ **(iv)** $m = 3s + 1$

(b) In patterns **(i)** and **(ii)**, the number of matchsticks that are required to make each new shape is the same as the number of sides of the shape (3 for triangles and 4 for squares). In patterns **(iii)** and **(iv)**, 1 less matchstick is required, because the new triangle or square joins on to an existing shape. This is shown in the formula of $2s + 1$ and $3s + 1$ instead of $3s$ and $4s$.

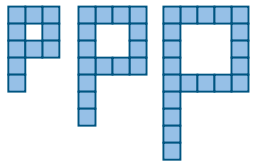
11 (a) 16 **(b)** 25

(c)

Large triangle	1	2	3	4	5	6
Number of triangles	1	4	9	16	25	36

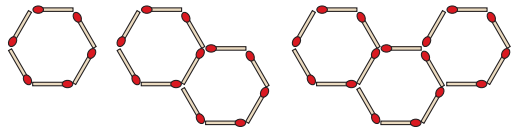
(d) The number of small triangles in each large triangle is the number of the large triangle squared.

12

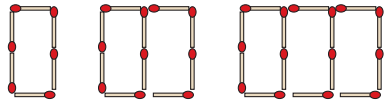


Width of P = w
Number of squares = s
$s = 5w - 5$

13 (a)



(b)



Half-time 5 (p. 193)

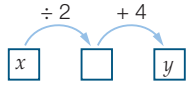
1 B

2 (a) 2 **(b)** -40 **(c)** 81

3 A

4 (a) $p + 7 = 28$ **(b)** $7x - 4 = 20$ **(c)** $w + g - 4 = 14$

5 (a)



(b) $y = 4 + \frac{x}{2}$

(c)

x	2	8	10	5	13
y	5	8	9	6.5	10.5

6 $6n + 11$ **7** D

8 (a) $C = 35m + 50$ (cost in cents)

(b) 260 cents = \$2.60

(c) When $n = 20$, $C = 750$ (\$7.50), so yes, he has enough money.

Exercise 5.6 (p. 196)

1 (a) Yes **(b)** No **(c)** No **(d)** Yes **(e)** No
(f) Yes **(g)** Yes **(h)** Yes **(i)** No

2 (a) $7x$ **(b)** $16y$ **(c)** $13a$ **(d)** $4y$
(e) $4m$ **(f)** $-11x$ **(g)** $5mn$ **(h)** $5xy$
(i) $z + 4$ **(j)** $7x + 4y$ **(k)** $14w + 3z$ **(l)** $12a + 2b$

3 (a) $10e - 2f$ **(b)** $8p - 4q$ **(c)** $-8t + 12s$
(d) $9x - 4y + y^2$ **(e)** $15w - 2z - 3z^2$ **(f)** $10m + 8n - 5m^2$
(g) $7a + 9$ **(h)** $8x + 5$ **(i)** $20y - 53$

4 (a) A **(b)** D

5 (a) $13e + 14f + 6$ **(b)** $18x + 2y + 2$
(c) $1 + 6x + 5y$ **(d)** $2x - 3y + 4$

(e) $9m - 2n + 3mn$ (f) $6a + 11b - 5c - 7$

6 (a) Rule 1: $4x + 6x$
Rule 2: $10x$
True

(b) Rule 1: $15y - 10y$
Rule 2: $5y$
True

7 (a) (i) $2x$ (ii) $4x$ (iii) $32x$

(b) $x + 2x + 4x + 8x + 16x + 32x = 63x$

(c) $\$63xv$

(d) $63 \times 6 = 378$. $x = 3$, $v = 2$

8 (a) $\$27d$ (b) $d + 3d + 9d + 27d = \$40d$

(c) \$320 (d) $81d = 567$, $d = 7$. She received \$7.

9 C

10 (a) $6x + 11y$ (b) $15x + 25y$

(c) x and y are either both even or both odd; e.g. $x = 1$, $y = 3$: $15x + 25y = 90$; $x = 2$, $y = 4$: $15x + 25y = 130$.

(d) Students' own answers.

11 (a) $N = 4L$ (b) $N = 2L + 2W$

(c) Square garden bed of length 4 sleepers. Rectangular garden bed of length 8 sleepers, width 7 sleepers.

(d) $L = 2W$, $N = 2 \times 2W + 2W = 6W$

(e) 10×5, 8×4, 6×3

Exercise 5.7 (p. 200)

1 (a) $A(4, 3)$, $B(1, -3)$, $C(-4, 5)$, $D(-4, -4)$, $E(-5, 0)$, $F(0, 0)$, $G(2, 0)$, $H(0, 4)$

(b) A quadrant 1, C quadrant 2, D quadrant 3, B quadrant 4, E–H not in a quadrant

2 (a) (i) quadrant 2 (ii) quadrant 4 (iii) quadrant 3
(iv) quadrant 1 (v) quadrant 2 (vi) quadrant 4
(vii) quadrant 3 (viii) quadrant 2

(b) (i) y-axis (ii) y-axis (iii) both (iv) x-axis (v) x-axis

3 (3, -3), (-2, -2), (-3, 0), (1, -1), (3, 1), (-1, 1), (-4, 3), (2, 4), (5, 2)

4 (a) C (b) A (c) B

5 (a) D (b) D (c) C

6 (a) C (b) D

7 The result is a butterfly.

8 The result is a castle.

9 (a) C (b) A

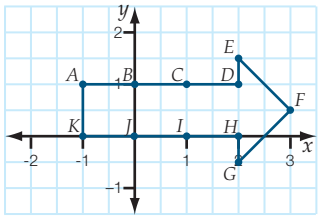
10 (a) (4, 6) (b) (2, 5) (c) (2, 1) (d) (3, 3)

(e) (5, 4) and (6, 4) (f) (6, 1), (6, 2), (7, 2), (7, 1)

(g) (1, 4) (h) (7, 6)

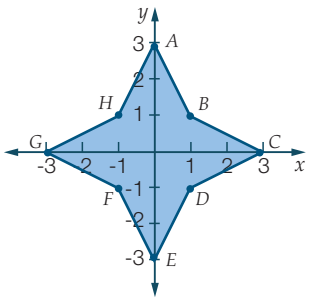
11 Join (7, 3) (1, 6) (-5, 3) (7, 3) STOP
Join (-4, 3) (-4, -3) (6, -3) (6, 3) STOP
Join (0, -3) (0, -1) (1, -1) (1, -3) STOP
Join (-3, 2) (-1, 2) (-1, 1) (-3, 1) (-3, 2) STOP
Join (-2, 1) (-2, 2) STOP
Join (4, 1) (4, 2) (2, 2) (2, 1) (4, 1) STOP
Join (3, 1) (3, 2) STOP
Join (4, -2) (4, -1) (2, -1) (2, -2) (4, -2) STOP
Join (3, -2) (3, -1) STOP
Join (-1, -2) (-1, -1) (-3, -1) (-3, -2) (-1, -2) STOP
Join (-2, -2) (-2, -1) STOP

12



A (-1, 1), B (0, 1), C (1, 1), D (2, 1), E (2, 1.5), F (3, 0.5), G (2, -0.5), H (2, 0), I (1, 0), J (0 ,0), K (-1, 0)

13 (a)

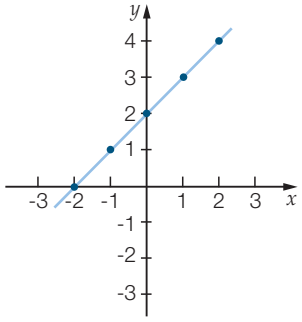


(b) Join the following points with straight lines.
$A(0, 3)$, B(1, 1), $C(3, 0)$, $D(1, -1)$, $E(0, -3)$, $F(-1, -1)$, $G(-3, 0)$, $H(-1, 1)$

(c) Colour the inside of the shape. Student to give to another student.

Exercise 5.8 (p. 206)

1 (a)–(b)



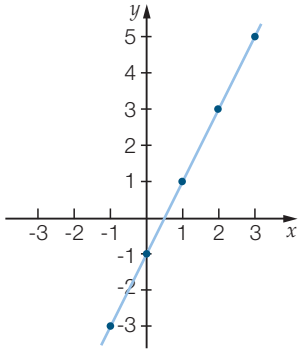
(c)

x	-2	-1	0	1	2
y	0	1	2	3	4

(d) $y = x + 2$

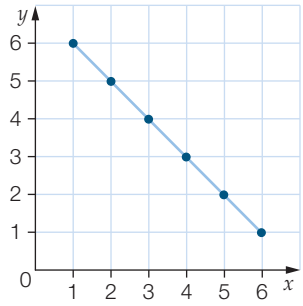
2 (a) (-1, -3), (0, -1), (1, 1), (2, 3), (3, 5)

(b)–(c)



(d) $y = 2x - 1$

3 (a)–(b)



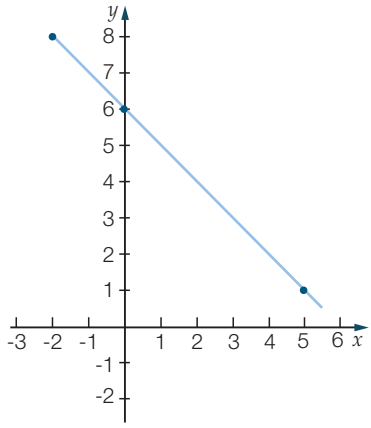
(c)

x	1	2	3	4	5	6
y	6	5	4	3	2	1

(d) $y = 7 - x$

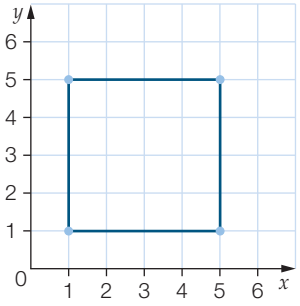
4 C

5 (a)–(b)



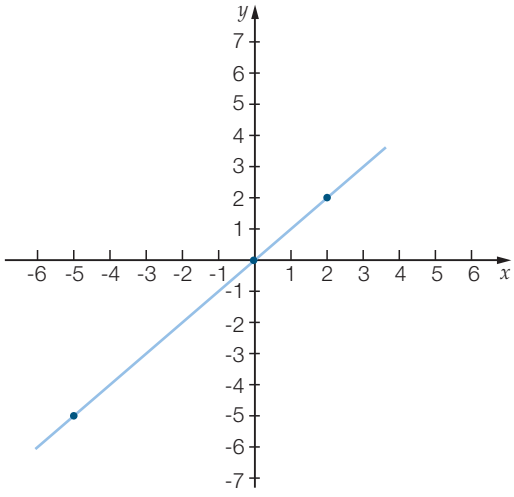
(c) $y = 7$ (d) $x = 1$

6 (a)



(b) square

7 (a)–(b)



(c) Any three of: (1, 1), (3, 3), (-1, -1), (-2, -2), (-3, -3), (-4, -4)

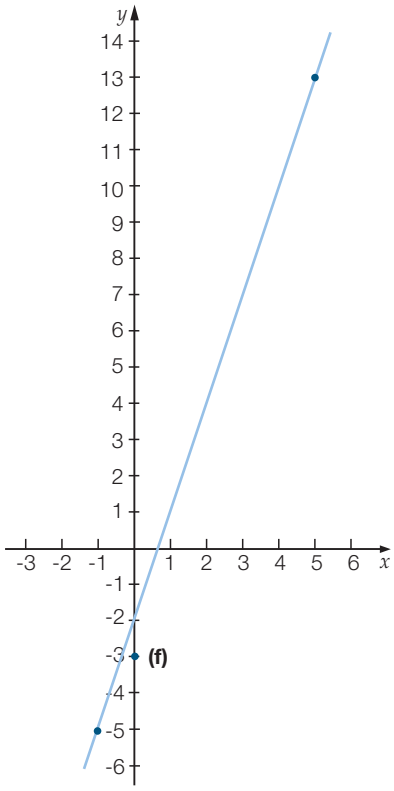
(d)

x	-5	-4	-3	-2	-1	0	1	2	3
y	-5	-4	-3	-2	-1	0	1	2	3

(e) $y = x$

(f) When $x = -3$, $y = -3$, therefore (-3, -3) does lie on the line.

8 (a)



(b) (2, 4), (3, 7), (4, 10)

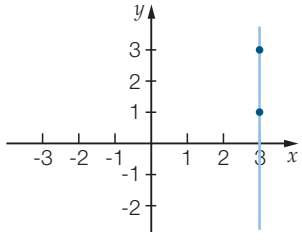
(c)

x	-1	2	3	4	5
y	-5	4	7	10	13

(d) $y = 3x - 2$

(e) When $x = 0$, $y = -2 \neq -3$, therefore (0, -3) does not lie on the line.

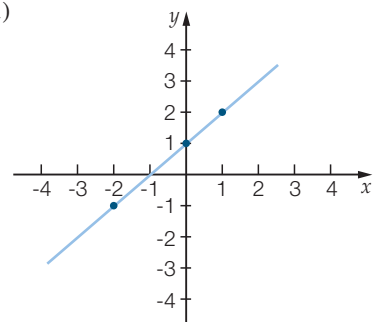
9 (a)–(b)



(c) (3, -1) and (3, 2) (d) $x = 3$

(e) (0, -3), (1, -1) and (2, 1); $y = 2x - 3$

10 (a) (0, 1)



(b) (-2, -1) and (1, 2) **(c)** $y = x + 1$

Exercise 5.9 (p. 209)

1 (a) False **(b)** True **(c)** True **(d)** True

2 (a) (0, 0) **(b)** 12 km **(c)** $\frac{1}{2}$ h **(d)** 1 h

(e) 6 km **(f)** $\frac{1}{2}$ h

(g) They were away for 3 hours.

(h) They rode the fastest from Jim's to Terry's house.

3 (a) Yiannis

(b) Meg

(c) Yiannis and Alex take the same shoe size.

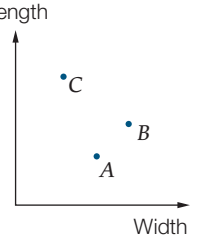
(d) Meg and Alex are the same age.

4 *A*: elephant; *B*: giraffe; *C*: kangaroo

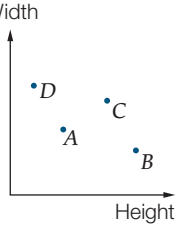
5 D

6 (a) *A*: salmon; *B*: John Dory; *C*: pike

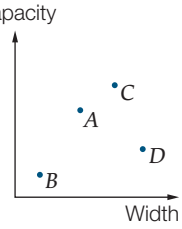
(b)



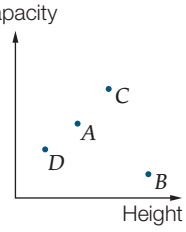
7 (a)



(b)



(c)



8 (a) height and shoe size

(b) The greater the height, the larger the shoe size.

9 (a) 1 600 000 megalitres

(b) 67%

(c) End of Sept to start Oct 1998

(d) From July 2004 to July 2007

(e) The drought broke and the storages received good rain.

(f) The winter snow melts and it usually rains a lot during winter.

(g) Sydney did not receive its average rainfall during those years so the storage level kept dropping.

(h) Jan 2007 ≈ 840 000 megalitres

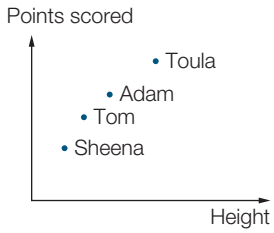
10 (a) Yes, the covers reduce water temperature, so evaporation would also be reduced.

(b) The suspended cover seems to be slightly better (≈ 200 days).

(c) in winter

(d) They reduce water temperature by ≈ 10°C.

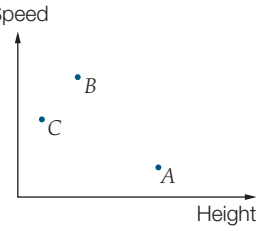
11 Students' own answers. For example:



12 (a) The first graph shows that *C* has moderate speed and low height, whereas the second graph shows *C* to have low speed and moderate height. A simple swap of *A* and *B* can only happen when the coordinates of *A* are opposite to the coordinates of *B*.

(b) Its height must be the same as its speed.

(c)



13 Student's own answers.

Challenge 5 (p. 214)

1 Trial and error. As 8 + 7 = 15, start with these numbers as they are nearly the same.

8 × 50c = $4, 7 × 20c = $1.40. Value = $5.40.
Reverse numbers: 7 × 50c = $3.50, 8 × 20c = $1.60.

Value = $5.10. This is 30c less, so, to have more, you need fewer 50c pieces.

6 × 50c = $3, 9 × 20c = $1.80. Value = $4.80.
Reverse numbers: 9 × 50c = $4.50, 6 × 20c = $1.20.

Value = $5.70. This is 90c more, need fewer 50c pieces.

4 × 50c = $2, 11 × 20c = $2.20. Value = $4.20.
Reverse numbers: 11 × 50c = $5.50, 4 × 20c = 80c.

Value = $6.30. This is $2.10 more, as required.

George has four 50c pieces.

2 $2 \bigstar 8 = 3(2 + 8) = 3 \times 10 = 30$

$5 \bigstar (2 \bigstar 8) = 5 \bigstar 30 = 3(5 + 30) = 3 \times 35 = 105$

3 As house numbers go up by 2 and are even on one side of the street and odd on the other, you have to find two numbers that differ by two and multiply to give 483.
As the product is odd, then each of the numbers is odd.
As $400 = 20 \times 20$, try the next two odd numbers.
$21 \times 23 = 483$. The house numbers are 21 and 23.

4 If $x > 4$, then $\frac{x}{4} > 1$ and $\frac{x+1}{4} > 1$

As $x + 1 > x$, then $1 < \frac{x}{4} < \frac{x+1}{4}$.

The reverse applies with x in the denominator. If $x > 4$, then $\frac{4}{x} < 1$ and $\frac{4}{x+1} < 1$. As $x + 1 > x$, then the fraction with denominator x is larger than the fraction with denominator $x + 1$, so $\frac{4}{x+1} < \frac{4}{x} < 1$.

Hence, $\frac{4}{x+1} < \frac{4}{x} < 1 < \frac{x}{4} < \frac{x+1}{4}$ and the order is $\frac{4}{x+1}, \frac{4}{x}, \frac{x}{4}, \frac{x+1}{4}$.

5 B If M = 1, then L = N and K = P, which is not possible as the letters are different numbers.

If L = 1, then M = N, not possible.

N can't be 1 as none of the products of the pairs of numbers ends in 1.

P > 1 as K × M > 1. Hence, K = 1.

L, M, and N can't equal 5. If L = 5 or M = 5, then N = 0 or 5. Hence, P = 5.

Using 2, 3 and 4 the only multiplication is $3 \times 4 = 12$.

Hence, N = 2 and P = 5 = 1 + 4, so M = 4, L = 3.

6 $a = 1$, and b can be any number.
(Note: zero is neither positive nor negative.)

7 **(a)** 15 correct means 15 wrong. Marks = $15 \times 9 - 15 \times 5 = 60$

(b) 14 correct means 16 wrong, Marks = $14 \times 9 - 16 \times 5 = 46$ is the most.

(c) By trial and error getting more wrong than correct.

12 correct, 18 wrong:
Marks = $12 \times 9 - 18 \times 5 = 108 - 90 = 18$

11 correct, 19 wrong:
Marks = $11 \times 9 - 19 \times 5 = 99 - 95 = 4$

10 correct, 20 wrong:
Marks = $10 \times 9 - 20 \times 5 = 90 - 100$ = less than zero (-10)

It is not possible to score zero on the test.

For better students you could discuss the following.

x correct means $(30 - x)$ wrong, and the
marks = $9 \times x - 5 \times (30 - x)$, = $14x - 150$

As 14 is not a factor of 150 you can never score 0.

8 The important thing to realise here is that the numbers 1 and 8 have only one number next to them, so they should be put in the middle two squares.

	5	3	
7	1	8	2
	4	6	

9 If half of x is 24, then x is 48, so twice x is 96.

10 B $90 \div 3 = 30$, take integers either side.

11 This is based on the result $2 \times 5 = 10$, so one of the numbers has only factors of 2 and the other only factors of 5. As there are six zeroes, the numbers are 2^6 and 5^6; that is, 64 and 15 625.

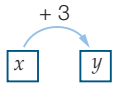
Chapter review 5 (p. 215)

1 **(a)** $12n$ **(b)** $\frac{p}{2} - 3$ **(c)** $7p - 9$ **(d)** $\frac{11 - g}{5}$

2 **(a)** Joe's age is represented as J; Beth's age is represented as B; $J = 3B + 4$

(b) Cost of a bottle of juice = j(\$); cost of a salad roll = r(\$).
$3j + 5r = 18.60$

3 **(a)** **(i)**

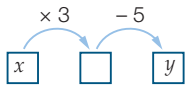


(ii) $y = x + 3$

(iii)

x	57	34	12	4	1.1	64
y	60	37	15	7	4.1	67

(b) **(i)**



(ii) $y = 3x - 5$

(iii)

x	4	3	$2\frac{1}{2}$	9	10	12
y	7	4	$2\frac{1}{2}$	22	25	31

4 **(a)** B **(b)** C

5 **(a)** **(i)** -54 **(ii)** 26

(b) **(i)** 7 **(ii)** -7

6 C

7 **(a)**

x	7	9	12	20	8.1	107
y	0	2	5	13	1.1	100

(b)

x	4	$5\frac{1}{2}$	13	7	78	54
y	10	25	100	40	750	510

8 **(a)**

No. triangles (t)	1	2	3	4	5
No. girders (g)	3	5	7	9	11
No. bolts (b)	3	4	5	6	7

(b) $g = 2t + 1$ **(c)** $b = t + 2$

(d) (i) 2

(ii) In the table of values, as the number of triangles increases by 1, the number of girders increases by 2. In the rule, 2 is the coefficient of t.

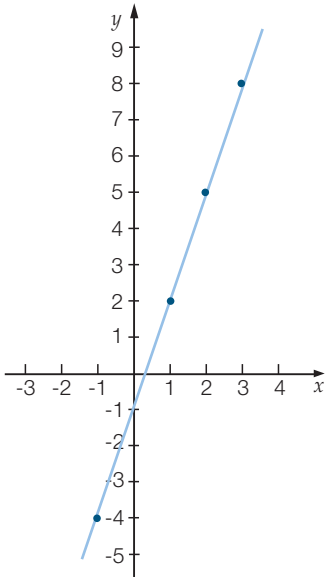
(e) (i) 1

(ii) In the table of values, as the number of triangles increases by 1, so does the number of bolts. In the rule, the coefficient of t is 1.

9 (a) $5a$ (b) $-a + 23b$ (c) $4x + y$

10 (a) (2, 3) (b) (-2, 1) (c) (3, -2) (d) (-4, -3)

11 (a)



(b)

x	-1	1	2	3
y	-4	2	5	8

(c) $y = 3x - 1$

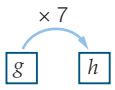
12 (a) \$3.50

(b) 3 L is the best value because it costs less per litre than the other sizes (\$1.67 compared to \$1.75 and \$2.00).

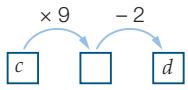
(c) This information is given as a point graph to enable comparison of prices of different sizes.

13 (a) $8n + 3$ (b) $\frac{8n+3}{15}$ (c) $n = 9$

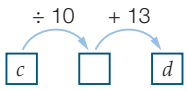
14 (a)



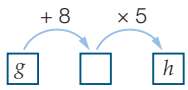
(b)



(c)



(d)



15 (a) $b = \frac{n}{4} + 5$ (b) 9

16 (a) $5a - 7b + 2$ (b) $5a + 9b - 7$ (c) $x + y + xy$

17 B

18 (a)

x	10	-35	100	5	0	-30
y	2	-7	20	1	0	-6

(b)

x	6	10	3	-1	0	5
y	72	120	36	-12	0	60

19 (a) $y = x - 7$ (b) $y = 2x + 4$

20 (a) $1.4c + 1.8d = 21$ (b) 7 apples and 6 oranges

(c) \$6.80

21 (a) Yes, although the start of 2009 was a bit lower than the others. Highest ≈ 39%, lowest ≈ 35%, so difference ≈ 4%.

(b) 54 – 38 = 16% increase

(c) Oct–Dec 2010

(d) Jun–Aug 2009

NAPLAN practice 5

1 7 2 B

3 (a) 24 (b) -6

4 (a) 2, 1, 3 (b) 3, 1, 2 or 3, 2, 1

5 \$11.00 6 D

Chapter 6

Recall 6 (p. 221)

1 (a) 16.5 (b) 40.32 (c) 39.26

(d) 25.2 (e) 56.12 (f) 18 960

(g) 27.8 (h) 0.3 (i) 0.0459

2 (a) (i) 13 mm (ii) 1.3 cm

(b) (i) 28 mm (ii) 2.8 cm

3 (a) 16 cm (b) 12 cm^2

4 (a) 2000 m (b) 3 cm (c) 500 cm

(d) 1.5 kg (e) 4000 mL (f) 20 000 g

(g) 0.25 L (h) 1750 mm (i) 0.5 kg

5 12 cubes

Exercise 6.1 (p. 223)

1 (a) 4 m (b) 5–6 m (c) 8–10 m (d) 20 mm

2 (a) 5000 m (b) 3600 m (c) 8 m

(d) 6500 cm (e) 55 cm (f) 120 cm

(g) 29 mm (h) 61 mm (i) 3.5 mm

(j) 0.9 m (k) 0.003 m (l) 37.5 m

(m) 4.2 km (n) 0.57 km (o) 0.0092 km

(p) 8 cm (q) 25.5 cm (r) 0.18 cm

3 (a) 3200 mm (b) 4950 mm (c) 0.09 km

(d) 0.345 km (e) 300 cm (f) 0.56 m

(g) 0.097 m (h) 342 000 mm (i) 0.0078 km

(j) 192 000 cm (k) 0.024 km (l) 890 mm

4 (a) cm (b) km (c) m
(d) cm (e) km (f) m

5 (a) B (b) C (c) B (d) C

6 (a) 4 cm, 0.4 m, 4000 mm, 0.04 km
(b) 360 m, 290 000 cm, 3100 m, 3.2 km
(c) 5600 cm, 71.4 m, 0.64 km, 820 000 mm
(d) 0.9 cm, 90 mm, 0.095 m, 0.0089 km

7 5.96 m

8 2.745 km

9 about 4.7 m

10 length = 8.9 cm, leg span = 25.4 cm

11 2.29 m across, 36 m long

12 596 mm, 59.6 cm

13 (a) 1100 km (b) 2500 km (c) 3700 km

14 2.5 m

15 (a) (i) The lines are the same length.
(ii) The central circles are the same size.
(iii) Yes, the hat is as wide as it is tall.
(b) (i) The external arrows make the lines appear longer.
(ii) Being surrounded by bigger dots makes the central dot appear smaller.
(iii) The vertical distance appears bigger by contrast.

16 Decimal points are not needed and conversions are not necessary.

17 (a) used division instead of multiplication; 2700 cm
(b) divided by 100 instead of 10; 76.5 cm
(c) zero added between 3 and 8; 3.8 km
(d) divided by 1000 instead of 100; 13.56 m

18 Inaccurate measuring with the ruler. The student with the longer length could be including the ends of the ruler before 0 and after 30.

Exercise 6.2 (p. 228)

1 (a) 16 cm (b) 22 cm (c) 28 m
(d) 86 mm (e) 100 m (f) 60 m

2 (a) 42 m (b) 40 cm (c) 24 cm
(d) 120 mm (e) 14.4 m (f) 68 mm

3 (a) C (b) C

4 (a) 21.6 cm (b) 23.6 m (c) 85.2 mm
(d) 28.2 cm (e) 34.1 m (f) 15.6 cm

5 (a) 140 mm (b) 122 mm (c) 390 cm (d) 6000 m

6 648 m **7** $384

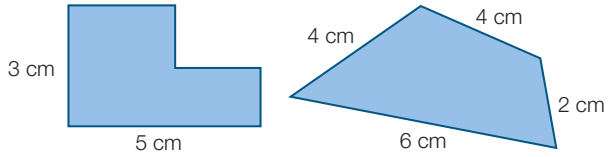
8 Students' own answers.

9 A

10 82 m

11 21 cm and 21 cm; 22 cm and 20 cm; 30 cm and 12 cm

12



Exercise 6.3 (p. 232)

1 (a) 5 cm^2 (b) 15 cm^2 (c) 4 cm^2
(d) 28.2 cm^2 (e) 24 cm^2 (f) 36 cm^2
(g) 36 m^2 (h) 87 mm^2 (i) 80 km^2
(j) 240 m^2 (k) 1.44 m^2 (l) 9 cm^2

2 (a) cm^2 (b) mm^2 (c) cm^2 (d) m^2
(e) m^2 (f) km^2 (g) mm^2 (h) m^2

3 (a) C (b) D

4 (a) (i) 20 cm (ii) 9 cm^2
(b) (i) 18 cm (ii) 8 cm^2
(c) (i) 24 cm (ii) 14 cm^2
(d) (i) 32 cm (ii) 20 cm^2

5 (a) (i) 28 km (ii) 45 km^2
(b) (i) 36 m (ii) 65 m^2
(c) (i) 42 cm (ii) 80 cm^2
(d) (i) 58 cm (ii) 100 cm^2

6 2.4 km^2 **7** 3150 cm^2 **8** 4 cm **9** 5.2 m

10 (a) 20 cm^2 (b) 36 cm^2

11 (a) 5.04 m^2 (b) 30.24 m^2 (c) 196.56 kW h

12 (a) 3519 grams (b) 30.8 m

13 525 bricks **14** 4418 cm^2

15 (a) 330 000 m^2 (b) 19 800 kg (c) 3350 m

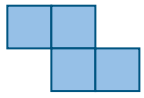
16 (a) 8 cm^2 (b) 8 cm^2
(c) 12 cm^2 (d) 14 cm^2 (approx.)

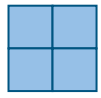
17 Approximate answers are:
(a) 131 m^2 (b) 45 km^2

18 Possible answers: 5 cm by 4 cm; 10 cm by 2 cm; 20 cm by 1 cm

19 12 m long, 5 m wide

20 Ethan's shape Magda's shape





21 Possible dimensions: 1 m by 2.4 m, perimeter = 6.8 m; 1.2 m by 2 m, perimeter = 6.4 m.

22 (a) Panel 2: rectangle's perimeter is 16 m.
(b) Perimeter is the length around a shape, whereas area is the space inside a flat shape.
(c) The 'little 2' means 'squared'. Area is measured in squares.

23 wood, carpet, turf (lawn)

Half-time 6 (p. 237)

1 **(a)** **(i)** 19 m **(ii)** 21 m^2

(b) **(i)** 14 m **(ii)** 12.25 m^2

2 **(a)** 0.45 m **(b)** 15 m **(c)** 0.307 km

(d) 60 000 cm **(e)** 196 km **(f)** 0.0264 km

3 **(a)** 11.7 kg **(b)** 16.4 m

4 **(a)** m^2 **(b)** mm^2 **(c)** cm^2 **(d)** km^2

5 length 10 cm, width 3 cm

6 **(a)** 457.5 m **(b)** 465.125 m^2 **(c)** 310.1 m^2

Exercise 6.4 (p. 239)

1 **(a)** 65 cm^2 **(b)** 60 m^2 **(c)** 176 mm^2

(d) 350 mm^2 **(e)** 144 m^2 **(f)** 80 cm^2

2 **(a)** 132 cm^2 **(b)** 64 cm

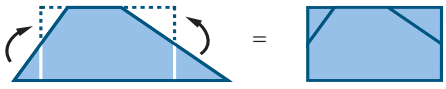
3 **(a)** B **(b)** C

4 7.5 cm **5** 20 cm **6** 1040 m^2

7 **(a)** 8 **(b)** 442 cm^2

8 **(a)** 20 m^2 **(b)** same amount of shade

9 **(a)**



(b) 15 cm^2

10 Possible dimensions: base 8 cm, height 4 cm; base 16 cm, height 2 cm; base 1 cm, height 32 cm

11 **(a)**



(b) base 4 cm, height 2 cm gives total area of 32 cm^2; base 7 cm, height 3 cm gives total area of 84 cm^2

Exercise 6.5 (p. 243)

1 **(a)** 48 cm^2 **(b)** 35 mm^2 **(c)** 42 m^2

(d) 88 mm^2 **(e)** 180 m^2 **(f)** 60 cm^2

(g) 9 m^2 **(h)** 19.2 mm^2 **(i)** 33.88 cm^2

2 **(a)** 19 cm^2 **(b)** 36 cm^2 **(c)** 85 cm^2

(d) 87 cm^2 **(e)** 281.65 cm^2 **(f)** 51 cm^2

(g) 279 cm^2 **(h)** 126 cm^2

3 **(a)** A **(b)** D

4 4.76 m^2 **5** 1600 cm^2 **6** 3 **7** 172 cm^2

8 **(a)** 144 cm^2 **(b)** 115.5 cm^2 **(c)** 320 cm^2 **(d)** 105.5 cm^2

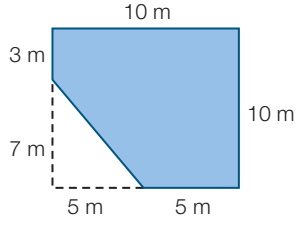
9 **(a)** 10.36 m^2 **(b)** 4.82 m **(c)** 1.64 L

10 Find the area of the big square and subtract the area of the small triangle.

$A = (10 \times 10) - (\frac{1}{2} \times 5 \times 7)$

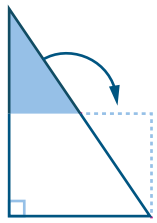
$= 100 - 17.5$

$= 82.5\ m^2$



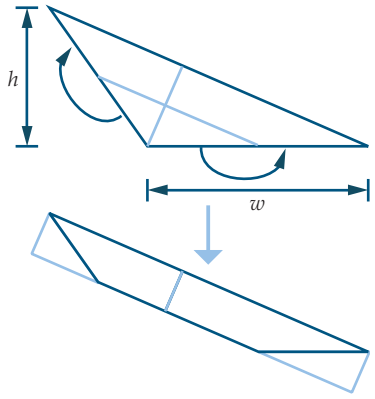
11 **(a)** The cut should be made exactly halfway up the height, parallel to the base or exactly halfway along the base, parallel to the height.

(b)



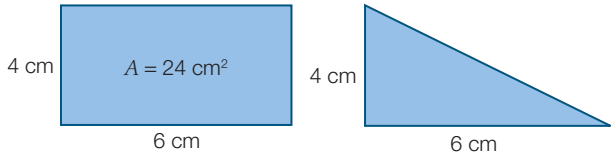
12 **(a)** Rule a line from the apex to the bottom side that is perpendicular (at right angles) to the bottom side. Halfway along this line, rule a second line parallel to the bottom line. Cut along these two lines.

(b)



13 9 cm and 8 cm; 6 cm and 12 cm

14



Exercise 6.6 (p. 247)

1 **(a)** 4 cm^3 **(b)** 16 cm^3 **(c)** 40 cm^3

(d) 24 cm^3 **(e)** 54 cm^3 **(f)** 60 cm^3

2 **(a)** 60 cm^3 **(b)** 60 cm^3 **(c)** 48 cm^3

(d) 120 cm^3 **(e)** 125 cm^3 **(f)** 320 cm^3

3 12 m^3 **4** 216 cm^3

5 **(a)** D **(b)** D

6 **(a)** 28 cm^3 **(b)** 10 cm^3 **(c)** 18 cm^3

(d) 20 cm^3 **(e)** 6 cm^3 **(f)** 4 cm^3

(g) 4 cm^3 **(h)** 7 cm^3 **(i)** 10 cm^3

7 3.2 m^3 **8** 2352 cm^3 **9** 0.69 m^3

10 100 **11** 12 cm

12 3 cm × 2 cm × 5 cm; 6 cm × 5 cm × 1 cm

13 **(a)** 900 jelly beans

(b) Calculate the dimensions of the jar (length, width and height) in terms of jelly beans, and then calculate the volume in terms of jelly beans.

(c) More jelly beans would be able to fit into the jar, as they would pack closely together with less space between them.

14 The side length of the large cube must be 4 cm, to give a volume of 64 cm^3. The 3 smaller cubes must have side lengths of 1 cm, 2 cm and 3 cm, giving volumes of 1 cm^3, 8 cm^3 and 27 cm^3 respectively.

Volume of smaller cube	Volume of empty space in large cube
1 cm^3	63 cm^3
8 cm^3	56 cm^3
27 cm^3	37 cm^3

Challenge 6 (p. 250)

1 2 mm ↔ \$2
2 m ↔ \$2000
2 km ↔ \$2 000 000
1 km ↔ \$1 000 000

2 C Sides of square are 200 m each. Each rectangle is 100 m by 50 m. Perimeter = $2 \times (100 + 50)$ m

3 D

4 Let square have sides of $2x$ cm. Rectangle has sides $2x$ cm and x cm. Perimeter of rectangle = $6x$ cm.

$6x = 39$, $x = 6.5$ cm. Area of paper = $13 \times 13 = 169$ cm^2

5 The colours are only used to make it easy to see the tiles.

The second pattern can occur 5 ways because the horizontal tiles can be in 5 different places.



The third pattern can occur in 3 ways because the vertical tiles can be in 3 places.



There are 9 different ways in total.

6 B **7** A

8 60 cm edge vertical,
volume of water = $200 \times 100 \times 60 - 100 \times 80 \times 60 = 720\,000$ cm^3

After block removed,
depth of water = $720\,000 \div (200 \times 100) = 36$ cm

80 cm edge vertical,
volume of water = $200 \times 100 \times 80 - 100 \times 80 \times 60 = 1\,120\,000$ cm^3

After block removed,
depth of water = $1\,120\,000 \div (200 \times 100) = 56$ cm

100 cm edge vertical,
volume of water = $200 \times 100 \times 100 - 100 \times 80 \times 60 = 1\,520\,000$ cm^3

After block removed,
depth of water = $1\,520\,000 \div (200 \times 100) = 76$ cm

9 Area of each face of bottom cube = $4 \times 4 = 16$ cm^2

Area of each face of middle cube = $2 \times 2 = 4$ cm^2

Area of each face of top cube = $1 \times 1 = 1$ cm^2

Visible area = $5 \times 16 - 4 + 5 \times 4 - 1 + 5 \times 1$
$= 76 + 19 + 5$
$= 100$ cm^2

Chapter review 6 (p. 251)

1 (a) B (b) C

2 (a) 45 900 m (b) 0.58 km (c) 9200 mm
(d) 4200 cm (e) 0.98 km (f) 6730 mm

3 (a) (i) 92 cm (ii) 448 cm^2
(b) (i) 56 mm (ii) 196 mm^2
(c) (i) 40 m (ii) 66.5 m^2
(d) (i) 80 cm (ii) 198 cm^2

4 Perimeter = 32.3 m, Area = 62.4 m^2

5 (a) 18 cm^2 (b) 24 cm^2 (c) 126 cm^2 (d) 19.5 m^2

6 (a) 210 m^3 (b) 225 cm^3

7 (a) 54 m^2 (b) 113 cm^2 (c) 4.48 m^2 (d) 94 cm^2

8 (a) 4.85 m (b) \$29.10

9 (a) 300 cm by 250 cm, area = 75 000 cm^2
(b) 120 tiles needed
(c) \$96

10 (a) 7 m (b) 0.9 m^3 (c) \$11.25

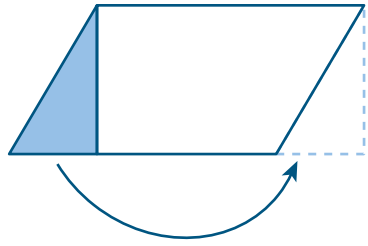
11 \$806.40

12 (a) 63 cm^3 (b) 162 cm^3

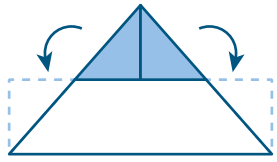
13 6.84 m^3 **14** 20 cm **15** 60 cm^2

16 176 m^2 **17** 6750 cm^3

18 (a)



(b)



NAPLAN practice 6

1 the rectangle **2** 37.5 m^2 **3** 40 cm **4** 15 cm

5 C **6** 0.05 cm **7** B **8** B

Mixed review C (p. 255)

1 (a) $n - 8$ (b) $\frac{k}{6}$ (c) $50h$

2 (a) 2.003, 2.3, 2.302, 2.323, 2.33
(b) 0.0199, 0.089, 0.0909, 0.091, 0.129

3 (a) $\frac{1}{4}$ (b) $5\frac{1}{3}$ (c) $\frac{5}{12}$

4 (a) $2^2 \times 7$ (b) $2^3 \times 5^2$ (c) $2 \times 3^2 \times 5 \times 11$

5 (a) (i) 26 cm (ii) 36 cm^2

(b) (i) 14 m (ii) 12.25 m^2

(c) (i) 2.45 m (ii) 0.1815 m^2

6 (a) 23 (b) 34 (c) 31

7 (a) $\frac{10}{27}$ (b) $1\frac{13}{14}$ (c) $4\frac{3}{5}$ (d) 6

8 (a) 6090 m (b) 0.83 m (c) 0.054 km

9 (a) 6 (b) -3 (c) -12 (d) -11

(e) -6 (f) -3 (g) 13 (h) -17

10 (a) 1204 (b) 165 (c) 228

11 (a) 3:2:1 (b) 33.3%

12 (a) $\frac{23}{100}$ (b) $\frac{3}{50}$ (c) $\frac{101}{200}$

13 (a) 8 cm^3 (b) 360 m^3 (c) 216 cm^3

14

x	0	2	3	6	10
y	6	12	15	24	36

15 (a) 13.47 (b) 8.936 35 (c) 40.945

16 (a) 0.0031 (b) 0.000 42 (c) 2121

17 $86.52

18 (a) 56 mm^2 (b) 36 m^2

19 Koala (a) $\frac{2}{5}$ (b) 40%

Bilby (a) $\frac{3}{10}$ (b) 30%

Wombat (a) $\frac{1}{5}$ (b) 20%

Kangaroo (a) $\frac{1}{10}$ (b) 10%

20 2^8 **21** 4 goals **22** 8 cm by 13 cm

23 $2.25

Chapter 7

Recall 7 (p. 257)

1 (a) False (b) True (c) False

2 (a) C (b) D (c) B

3 (a) $7 + 5 = 12$ (b) $4 \times 6 - 2 = 22$ (c) $\frac{8+6}{7} = 2$

4 (a) 9 (b) 7

5 (a) 7 (b) 2

Exercise 7.1 (p. 259)

1 (a) $3 + 7 = 10$ (b) $5 \times 5 = 25$

(c) $\frac{16}{4} = 4$ (d) $22 - 10 = 8 + 4$

(e) $3 \times 5 + 2 = 8 + 9$ (f) $1 + \frac{25}{5} = 6$

2 (a) False (b) False (c) False (d) True

(e) True (f) True (g) False (h) True

(i) True

3 (a) D (b) C (c) B

4 (a) $5 + 43 = 48$ (b) $3 \times 7 = 19 + 2$

(c) $\frac{50}{5} = 5 \times 2$ (d) $20 - (6 + 8) = \frac{12}{2}$

5 (a) $13 - 19 = 6$ False

(b) $\frac{4}{2} = 8$ False

(c) $8 - 11 = 6 - 3$ False

(d) $\frac{12}{4} = \frac{15}{5}$ True

6 $5 + 7 = 12$ **7** $6 \times 60 = 4 \times 90$

8 $25 + $7 = $32 **9** $48 - 11 + 5 = 42$

10 (a) $16 + 17 = 33$ (b) $17 + 18 + 19 = 54$

11 (a) $\frac{60}{3} = 20$ cm (b) $\frac{60}{4} = 15$ cm

(c) $\frac{60}{5} = 12$ cm (d) $\frac{60}{6} = 10$ cm

(e) Divide 60 by 20 (f) Divide 200 by 50

(g) Three from: 5, 10, 20, 25, 50 and 100

12 $8 + 1 + 2 - 4 = 7$, $8 \times 1 - 4 + 2 = 6$, $8 - \frac{4}{2} + 1 = 7$

13 72, 80, 88, 96, 104

14 $5 + 6 \times 3 - 3 = 20$; $1 + 3 \times 8 - 5 = 20$; $6 + 4 \times 5 - 6 = 20$

15 Ruben is correct. Sam added four before multiplying; which is incorrect. The order of operations should always be followed.

16 Differences need to be kept the same on both sides. Because 12 is 4 less than 16, the missing number must also be 4 less than 38. Therefore, the first solution is correct.

Exercise 7.2 (p. 263)

1 (a) Three added to a number is equal to ten.

(b) A number added to four is equal to seven.

(c) Five added to a number is equal to six.

(d) Two subtracted from a number is equal to four.

(e) Seven subtracted from a number is equal to two.

(f) Eight subtracted from a number is equal to six.

(g) A number multiplied by five is equal to twenty.

(h) A number multiplied by seven is equal to fourteen.

(i) A number multiplied by seven is equal to twenty-one.

(j) A number divided by three is equal to four.

(k) A number divided by two is equal to six.

(l) A number divided by six is equal to twelve.

(m) Three times a number added to two is equal to seven.

(n) Six times a number added to two is equal to ten.

(o) Three times a number added to one is equal to eight.

2 (a) Yes (b) No (c) Yes (d) Yes (e) No
(f) Yes (g) No (h) Yes (i) Yes (j) No
(k) Yes (l) No (m) No (n) No (o) Yes
(p) Yes (q) Yes (r) No

3 (a) $x = 4$ (b) $x = 7$ (c) $x = 7$ (d) $x = 29$
(e) $x = 17$ (f) $x = 64$ (g) $x = 18$ (h) $x = 6$
(i) $x = -4$ (j) $x = 76$ (k) $x = 120$ (l) $x = 162$

4 D

5 (a) D (b) C (c) D

6 (a) B (b) D (c) D

7 (a) C (b) A (c) D

8 (a) $x = 10$ (b) $x = 4$ (c) $x = 11$

9 (a) D is Daniel's age.
(b) $D - 3$
(c) $D + 12$
(d) $D + 12 = 32$, Daniel is 20 years old.

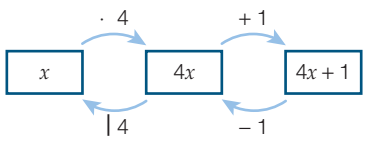
10 (a) C is the amount of money Cameron has in his wallet.
(b) $C - 14$
(c) $C - 14 + 5 = 23$, Cameron had \$32 to start with.

11 (a) $3c = 36$ (b) $3c = 27$ (c) 9 years old

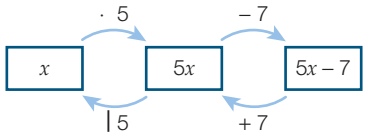
12 (a) Sean started with 14 marbles. He gave a number to his friend Sasha and was left with 11 marbles.
(b) Tali and 4 of her friends shared a packet of lollies equally between them. They received 12 lollies each.
(c) The number of leaves on a tree multiplied by 6 to give 108 leaves in the early days of spring.

Exercise 7.3 (p. 269)

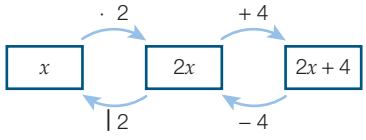
1 (a)



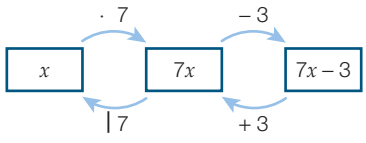
(b)



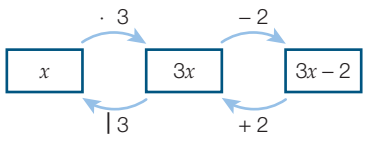
(c)



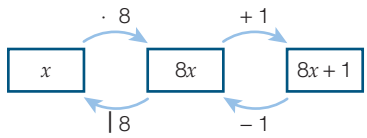
(d)



(e)

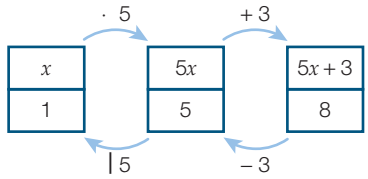


(f)



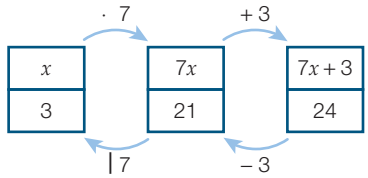
2 (a) (i) $3x - 1 = 11$ (ii) $x = 4$
(b) (i) $2x - 3 = 7$ (ii) $x = 5$
(c) (i) $\frac{x+5}{4} = 4$ (ii) $x = 11$
(d) (i) $\frac{x+7}{2} = 8$ (ii) $x = 9$
(e) (i) $\frac{x}{6} + 7 = 9$ (ii) $x = 12$
(f) (i) $\frac{x}{3} + 8 = 13$ (ii) $x = 15$
(g) (i) $\frac{x-3}{5} = 1$ (ii) $x = 8$
(h) (i) $\frac{x-2}{7} = 2$ (ii) $x = 16$
(i) (i) $2(x + 6) = 20$ (ii) $x = 4$
(j) (i) $5x - 9 = 16$ (ii) $x = 5$
(k) (i) $5(x + 2) = 30$ (ii) $x = 4$
(l) (i) $7(x + 1) = 21$ (ii) $x = 2$

3 (a)



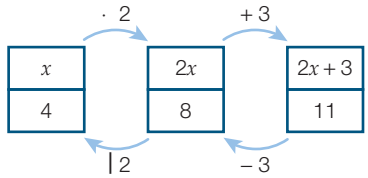
$x = 1$

(b)



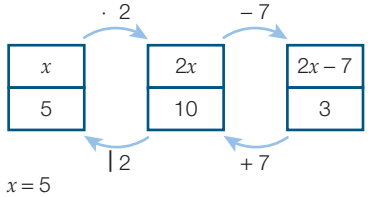
$x = 3$

(c)



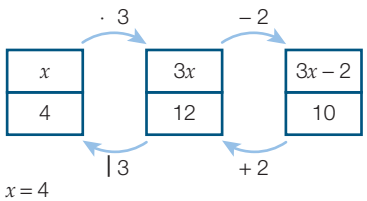
$x = 4$

(d)



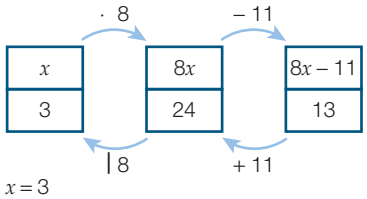
$x = 5$

(e)



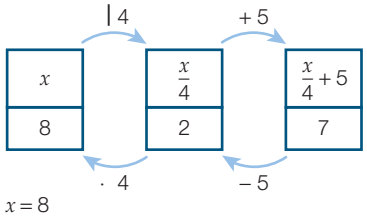
$x = 4$

(f)



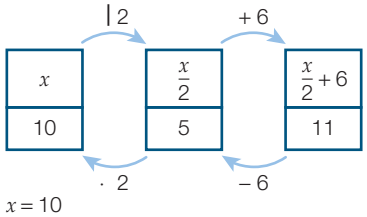
$x = 3$

4 (a)



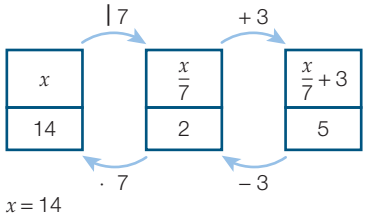
$x = 8$

(b)



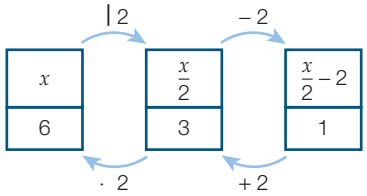
$x = 10$

(c)



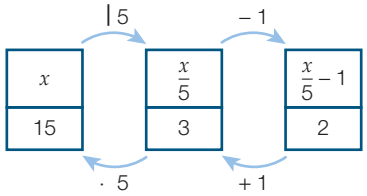
$x = 14$

(d)



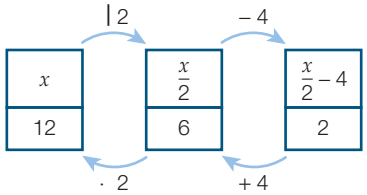
$x = 6$

(e)



$x = 15$

(f)



$x = 12$

5 (a) $x = 3$ (b) $x = 5$ (c) $x = 2$ (d) $x = 8$
(e) $x = 7$ (f) $x = 6$

6 (a) $x = 1$ (b) $x = 3$ (c) $x = 4$ (d) $x = 7$
(e) $x = 11$ (f) $x = 13$

7 (a) $x = 5$ (b) $x = 3$ (c) $x = 1$ (d) $x = 4$
(e) $x = 2$ (f) $x = 2$ (g) $x = 3$ (h) $x = 5$
(i) $x = 3$ (j) $x = 10$ (k) $x = 14$ (l) $x = 15$
(m) $x = 6$ (n) $x = 14$ (o) $x = 7$

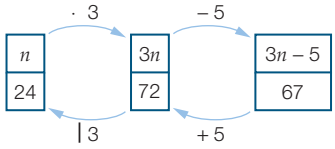
8 (a) $x = 4$ (b) $x = 5$ (c) $x = 6$ (d) $x = 5$
(e) $x = 14$ (f) $x = 4$ (g) $x = 11$ (h) $x = 2$
(i) $x = 24$

9 B

10 (a) $2n + 1 = 43$, $n = 21$ (b) $\frac{n+4}{5} = 4$, $n = 16$

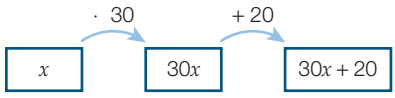
(c) $\frac{7n}{3} = 21$, $n = 9$ (d) $\frac{2n+6}{10} = 2$, $n = 7$

11 $3n - 5 = 67$

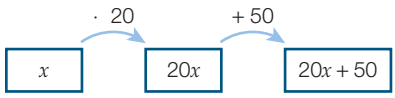


There were 24 pencils in each packet.

12 (a)



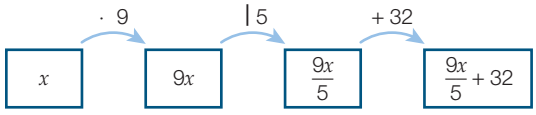
(b)



(c) Ying: \$50 Aldo: \$70 (d) $2\frac{1}{2}$ hours

(e) $2\frac{1}{4}$ hours (f) 3 hours

13 (a)



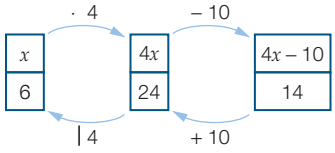
(b) 5, 10, 15, 20, ...

14 $\frac{2x+4}{6} + 6 = 10$

Half-time 7 (p. 273)

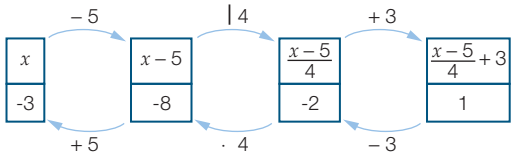
1 $x = 84$ **2** 4 metres

3 (a)



$x = 6$

(b)



$x = -3$

4 $21 + 22 = 43$

5 **(a)** **(i)** $j - 5 = 7$ **(ii)** 12

(b) **(i)** $d + 4 = 16$ **(ii)** 12

6 **(a)** $x = \frac{7}{3}$ **(b)** $x = -16$ **(c)** $x = 13$

7 A number is multiplied by 5 and has 3 added to it. The result is 23.

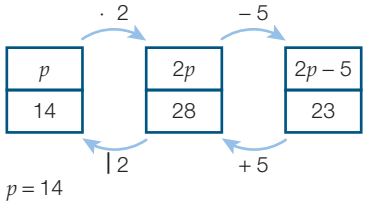
8 **(a)** $3 + 8 = 11$ **(b)** $4 \times 5 = 26 - 6$ **(c)** $4 + \frac{28}{2} = 18$

9 20 cm

10 **(a)** False **(b)** True **(c)** True

11 **(a)** $p - 5$ **(b)** $2p - 5 = 23$

(c)



$p = 14$

Pete's age = 14
Sam's age = 9

Exercise 7.4 (p. 275)

1 **(a)** right-hand side (RHS)

(b) Take one chocolate from the RHS.

(c) Take 4 chocolates from the RHS.

(d) 9 chocolates **(e)** 3 chocolates

2 **(a)** $x + 8 = 11$ **(b)** $x + 9 = 17$ **(c)** $x + 7 = 13$

(d) $2x + 1 = 9$ **(e)** $3x = 5$ **(f)** $6x + 3 = 9$

3 **(a)** $x = 1$ **(b)** $x = 4$ **(c)** $x = 2$ **(d)** $x = 5$

(e) $x = 7$ **(f)** $x = 9$ **(g)** $x = 8$ **(h)** $x = 1$

(i) $x = 3$ **(j)** $x = 5$ **(k)** $x = 5$ **(l)** $x = 8$

4 D

5 **(a)** $2t + 12 = 104$ **(b)** 46 kg

6 **(a)** $3p + 18 = 30$ **(b)** $p = 4$

7 **(a)** $x = 12$ **(b)** $x = 7$ **(c)** $x = 8$

(d) $x = 10$ **(e)** $x = 20$ **(f)** $x = 11$

(g) $\frac{7}{3}$ **(h)** $x = 6$ **(i)** $x = -4\frac{2}{3}$

(j) $x = 12$ **(k)** $x = 7$ **(l)** $x = 30\frac{5}{8}$

8 34 km **9** 1.25 L

10 $5x = 10$, $5x + 5 = 15$, $5x + 9 = 19$

11 **(a)** Vejay subtracted the 3 too early. He should have done it after multiplying by 7. The order was wrong.

(b) $\frac{x+3}{7} = 3$

$\frac{x+3}{7} \times 7 = 3 \times 7$

$x + 3 = 21$

$x + 3 - 3 = 21 - 3$

$x = 18$

(c) He could use a flowchart to work out the order of operations used to 'build up' the equation. He should then work backwards along the flowchart, using the inverse operations, to solve the equation.

12 Sample answers

$5(c - 9) = 15$

$\frac{c+8}{4} = 5$

Exercise 7.5 (p. 279)

1 **(a)** $x + 20 = 95$, the ice-cream costs 75 cents.

(b) $t + 1.5 = 12.8$, Tony ran the 100 m in 11.3 seconds.

2 **(a)** $2d + 1.6 = 8$, a sushi roll costs $3.20.

(b) $3v + 21.5 = 59$, a DVD costs $12.50.

3 **(a)** D **(b)** There are 27 seats available.

4 **(a)** C **(b)** The total length of the roll of fabric is 15 m.

5 **(a)** D **(b)** The width of the vegetable garden is 2.5 m.

6 **(a)** $x + 1.5$

(b) $x + 1.5 = 5.7$; $x = 4.2$; The salad sandwich costs $4.20.

7 **(a)** $4x$

(b) $4x = 84$; $x = 21$; The length of the floor tile is 21 cm.

8 **(a)** $4x + 6$

(b) $4x + 6 = 62$; $x = 14$; Fourteen tables that seat four people each are needed.

9 $3h + 5 = 14$; $h = 3$; Natalie rollerskated on the rink for three hours.

10 $2x - 54 = 272$; $x = 163$; at age 5, Robert Wadlow's height was 163 cm.

11 **(a)** $8d + 15 = 63$
$d = 6$ days

(b) $48d + 15 = 111$
$d = 2$ days

(c) $13d + 15 + 24d + 15 = 178$
$d = 4$ days

12

		Length of footpath	Number of pieces of wood	
(a)	**(i)**	2 m	6	$(2 \times 2 + 2)$
	(ii)	3 m	8	$(2 \times 3 + 2)$
	(iii)	4 m	10	$(2 \times 4 + 2)$

(b) A footpath of length x metres would need $(2 \times x + 2)$ or $(2x + 2)$ pieces of wood.

(c) (i) 14 (ii) 22 (iii) 28

(d) If x is the length of the footpath, then the equation to solve is $2x + 2 = 36$. The longest footpath that could be poured at the one time is 17 metres in length.

13 1 hour jackhammer ($30) plus 6 hours mulcher ($132) = $162

14 $x + 5x \leq 70$

The son is 10 and the father is 50.

Challenge 7 (p. 282)

1 A $49 - 15 = 34$

$34 \times 2 = 68$

2 $x + 40 + x + 40 + x + 40 = 360$, $3x + 120 = 360$, $3x = 240$, $x = 80$

3 $\frac{3x-5}{4} = -6$, $3x - 5 = -24$, $3x = -19$, $x = -\frac{19}{3}$

4 B $x = 73 + y - 59$, $x = 14 + y$

5 (a) $3^2 = 4 + 5$

(b) $4^2 = 7 + 9$, $2^4 = 7 + 9$, $2^3 + 1 = 9$, $3^2 = 1 + 8$, $3^2 = 0 + 9$

(c) $2^3 = 9 - 1$, $3^2 = 9 - 0$, $3^2 - 5 = 4$, $3^2 - 4 = 5$, $4^2 - 9 = 7$

6 D Reciprocal of $\frac{1}{20}$ is 20. $\frac{4x}{5} = 20$, $x = 25$

7 $\frac{5+4}{20} + \frac{1}{y} = 1$, $\frac{1}{y} = 1 - \frac{9}{20}$, $\frac{1}{y} = \frac{11}{20}$, $y = \frac{20}{11}$

8 Let Vladmir buy x of each stamp.

$0.55x + 1.1x = 80.85$

$1.65x = 80.85$, $x = 80.85 \div 1.65 = 49$

He bought 98 stamps.

9 $\frac{1}{4} \times \frac{9}{10}x = 2700$, $x = 300 \times 40 = \$12\,000$

10 $\frac{1}{3} - \frac{1}{4} = \frac{1}{4} - \frac{1}{m}$, $\frac{1}{m} = \frac{1}{4} + \frac{1}{4} - \frac{1}{3}$, $\frac{1}{m} = \frac{1}{2} - \frac{1}{3}$, $\frac{1}{m} = \frac{1}{6}$, $m = 6$

11 C The integers end in 2 and 5, 4 and 5, or 8 and 5. As $a > b$, consider $12 - 5 = 7$, $5 - 2 = 3$, $14 - 5 = 9$, $5 - 4 = 1$, $15 - 8 = 7$, $8 - 5 = 3$. $a - b$ ends in 1, 3, 7 or 9.

Chapter review 7 (p. 283)

1 (a) $7 \times 4 = 30 - 2$ (b) $2 \times 4 + 6 = 9 + 5$

2 (a) 15 (b) 28

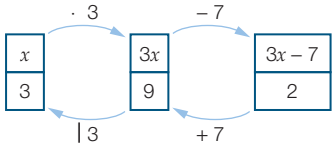
3 (a) $x = 9$ (b) $x = 3$ (c) $x = 15$ (d) $x = 15$

(e) $x = 12$ (f) $x = 28$

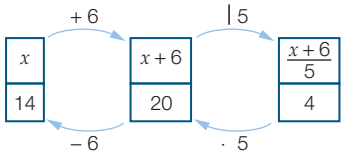
4 (a) $x = -3$ (b) $x = 84$ (c) $x = 9$

5 (a) -5 (b) $\times 3$ (c) $\div 8$ (d) $+4$

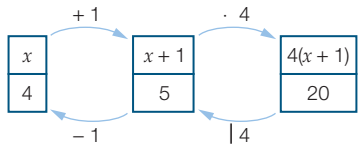
6 (a) $x = 3$



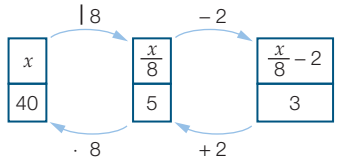
(b) $x = 14$



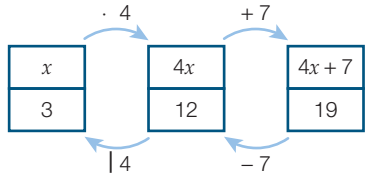
(c) $x = 4$



(d) $x = 40$

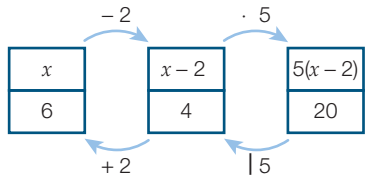


7 (a)



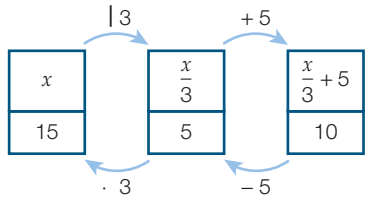
Solution is $x = 3$.

(b)



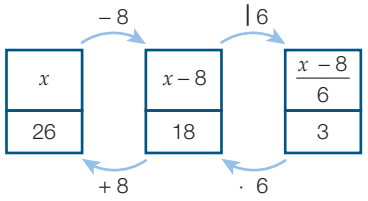
Solution is $x = 6$.

(c)



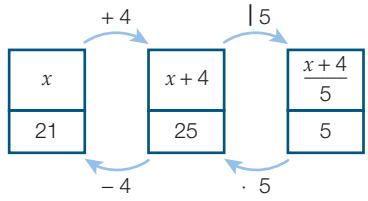
Solution is $x = 15$.

(d)



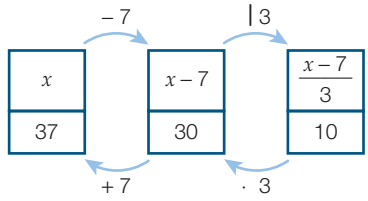
Solution is $x = 26$.

(e)



Solution is $x = 21$.

(f)



Solution is $x = 37$.

8 B

9 $5t + 27.50 = 200$, $t = \$34.50$

10 D

11 **(a)** $C = 3x + 140$

(b) $275 = 3x + 140$, $x = 45$ cents

Each dim sim costs 45 cents.

12 $t + 5 = 28$
$t = 23°C$

The temperature at 3 p.m. was 23°C.

13 **(a)** 5 **(b)** 7

(c) $2n + 1$ = number of matchsticks

(d) **(i)** $2n + 1 = 97$ **(ii)** $n = 48$

NAPLAN practice 7

1 C **2** D **3** D **4** C

5 A **6** B **7** B **8** C

Chapter 8

Recall 8 (p. 286)

1 **(a)** 55° **(b)** 160° **(c)** 235° **(d)** 339°

2 D, A, B, E, C, F

3 A and E, B and F, C and D

Exercise 8.1 (p. 289)

1 **(a)** 102° **(b)** 71° **(c)** 129° **(d)** 225°

(e) 350° **(f)** 15°

2 Students' own answers.

3 **(a)** C **(b)** B **(c)** D

4 **(a)** B **(b)** B **(c)** D

5 C **6** 21° **7** 40°

8 Draw a 40° angle and mark the reflex angle as 320°.

9 **(a)** 330° **(b)** 245° **(c)** 200°

10 Students' own answers.

11 Students' own answers.

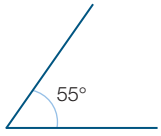
12 Students' own answers.

13 Nick has used the incorrect scale on his protractor.
He should start from 0 on one arm and follow around the scale to the second arm.

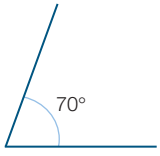
14 **(a)** 20°, 70° **(b)** 30°, 100°, 50°

(c) 110°, 120°, 80°, 50°

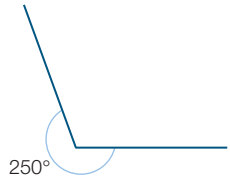
15 **(a)**



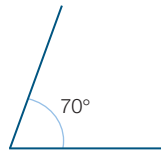
(b)



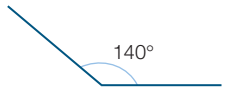
(c)



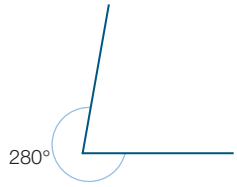
16 **(a)**



(b)



(c)



Exercise 8.2 (p. 295)

1 **(a)** straight **(b)** reflex **(c)** obtuse

(d) reflex **(e)** acute **(f)** acute

(g) acute **(h)** obtuse **(i)** revolution

(j) revolution **(k)** right **(l)** straight

(m) obtuse **(n)** straight **(o)** right

(p) right **(q)** revolution **(r)** reflex

2 **(a)** acute **(b)** obtuse

(c) reflex **(d)** revolution

(e) straight **(f)** acute

(g) right **(h)** obtuse

(i) reflex

3 (*Note:* Other forms of notation may be used.)

(a) $\angle QPR$ or $\angle RPQ$ **(b)** $\angle STD$ or reflex angle $\angle DTS$

(c) $\angle BOA$ or $\angle AOB$ **(d)** $\angle KIW$ or $\angle WIK$

(e) $\angle SHD$ or $\angle DHS$ **(f)** $\angle CAT$ or $\angle TAC$

4 **(a)** D **(b)** C **(c)** C **(d)** D

5 D

6 **(a)** **(i)** Students' own answers.

(ii) 360° **(iii)** revolution

(b) **(i)** Students' own answers.

(ii) 90° **(iii)** right angle

(c) **(i)** Students' own answers.

(ii) 180° **(iii)** straight angle

(d) **(i)** Students' own answers.

(ii) 360° **(iii)** revolution

(e) **(i)** Students' own answers.

(ii) 90° **(iii)** right angle

(f) **(i)** Students' own answers.

(ii) 180° **(iii)** straight angle

7 (a) $\angle AOD$ and $\angle AOC$ or $\angle AOC$ and $\angle BOC$ or $\angle AOD$ and $\angle BOD$ or $\angle BOC$ and $\angle BOD$

(b) $\angle AOD$ and $\angle COB$ or $\angle AOC$ and $\angle DOB$

8 C

9 (a) $\angle AOB$, $\angle BOC$, $\angle DOE$

(b) $\angle AOE$, $\angle BOD$, $\angle COE$

(c) $\angle AOB$ (reflex), $\angle AOC$ (reflex), $\angle AOE$ (reflex), $\angle BOC$ (reflex), $\angle BOD$ (reflex), $\angle COD$ (reflex), $\angle COE$ (reflex), $\angle DOE$ (reflex)

(d) $\angle AOC$, $\angle COD$

(e) $\angle AOD$, $\angle BOE$

(f) $\angle AOB$ & $\angle BOC$, $\angle BOC$ & $\angle DOE$

(g) $\angle AOB$ & $\angle BOD$, $\angle AOC$ & $\angle COD$, $\angle AOE$ & $\angle DOE$, $\angle BOC$ & $\angle COE$, $\angle BOD$ & $\angle DOE$

(h) $\angle AOB$ & $\angle DOE$, $\angle AOE$ & $\angle BOD$

10 (a) 334°

(b) $\angle RMS$ could be an acute angle.

11 (a) $\angle B$ could refer to a number of angles.

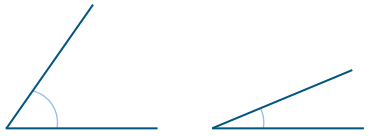
(b) $\angle ABC$, $\angle CBD$

(c) $\angle ABD$

(d) $\angle ABD$ (reflex) or $\angle ABC$ (reflex) or $\angle CBD$ (reflex)

12 Students to provide accurate copies of the figures in the textbook.

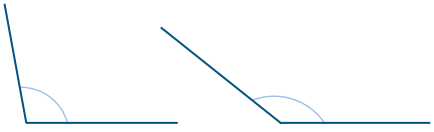
13 (a)



(b)



(c)



14 (a) 25° and 65°, 12° and 78°

(b) 112° and 68°, 92° and 88°

15 The angles given must add to 360°; 45°, 150° and 165°; 205°, 60° and 95°

Exercise 8.3 (p. 300)

1 (a) 60° (b) 10° (c) 14° (d) 23°
(e) 56° (f) 63° (g) 150° (h) 60°
(i) 175° (j) 69° (k) 53° (l) 38°

2 (a) 270° (b) 120° (c) 313° (d) 50°
(e) 145° (f) 202° (g) 50° (h) 245°
(i) 138° (j) 70° (k) 143° (l) 31°

3 (a) 33° (b) 51° (c) 172° (d) 94° (e) 89° (f) 20°

4 (a) 63° (b) 45° (c) 22° (d) 75°

5 (a) 148° (b) 90° (c) 56° (d) 4°

6 87°

7 (a) 57° (b) 45° (c) 65°

8 A

9 (a) $x = 101°$ (vertically opposite to 101°)
$y = z = 79°$ (supplementary to 101°)

(b) $x = 88°$ (vertically opposite to 88°)
$y = z = 92°$ (supplementary to 88°)

(c) $x = z = 149°$ (supplementary to 31°)
$y = 31°$ (vertically opposite to 31°)

10 (a) 60° (a straight line is 180°)

(b) 45° (a straight line is 180°)

(c) 30° (a right angle is 90°)

(d) 40° (a right angle is 90°)

(e) 50° (a straight line is 180°)

(f) 30° (a right angle is 90°)

(g) 120° (a revolution is 360°)

(h) 72° (a revolution is 360°)

(i) 75° (a revolution is 360°)

11 (a)

	Left leg	Right leg
a	90°	91°
b	90°	89°
c	90°	89°
d	90°	91°
e	92°	80°
f	88°	100°
g	88°	100°
h	92°	80°

(b) right leg (c) Yes, the differences are 11°.

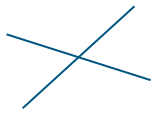
12 Sabine did not use the information given in the question. She should have used the property that angles at a point add to 360°, but instead used her protractor. The key is that the diagram is not drawn accurately.

13 (a)



(b) 45° and 135° (c) 45° and 135°

14

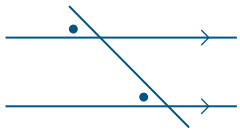
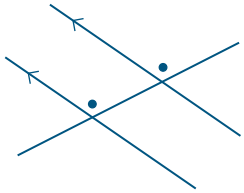
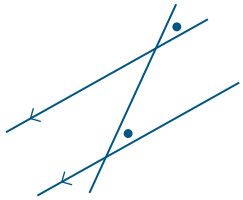


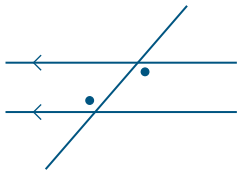
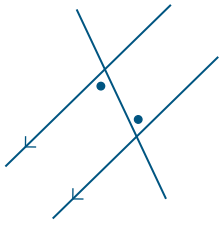
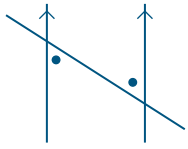
Both angles are 120°.

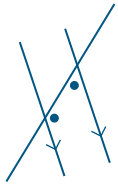
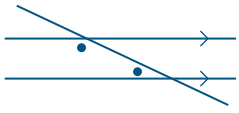
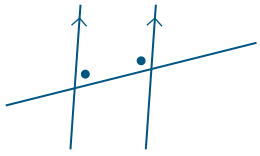
Exercise 8.4 (p. 307)

1 (a) corresponding (b) alternate
(c) co-interior (d) co-interior

(e) co-interior
(f) corresponding
(g) alternate
(h) corresponding
(i) alternate

2 (a)

(b)

(c)


3 (a)

(b)

(c)


4 (a)

(b)

(c)


5 C

6 (a) (i) $b = 135°$, $a = 45°$ (co-interior)
(ii) $s = 140°$, $t = 140°$ (corresponding)
(iii) $m = 130°$, $n = 130°$ (alternate)
(iv) $e = 151°$, $f = 151°$ (alternate)
(v) $g = 72°$, $h = 72°$ (corresponding)
(vi) $x = 144°$, $y = 36°$ (co-interior)

(b) (ii), (iii), (iv) and (v) have pairs that are equal; (i) and (vi) have pairs that add to 180°.

7 (a) y (b) w (c) u (d) v (e) s (f) x

8 (a) $a = 37°$ (alternate angles)
(b) $c = 110°$ (corresponding angles)
(c) $g = 24°$ (co-interior angles)
(d) $b = 100°$ (co-interior angles)
(e) $d = 73°$ (alternate angles)
(f) $e = 141°$ (corresponding angles)
(g) $f = 115°$ (corresponding angles)
(h) $j = 72°$ (co-interior angles)
(i) $h = 97°$ (alternate angles)

9 18°

10 144°

11 *EF* and *GH*. Co-interior angles are supplementary.

12 (a) 16 (b) 8 (c) 8

13 (a) $a = 146°$ (vertically opposite angle), $b = 34°$ (co-interior angle)

(b) $a = 62°$ (alternate angle), $b = 62°$ (vertically opposite angle), $c = 28°$ (angles in a right angle)

(c) $a = 80°$ (alternate angle), $b = 80°$ (corresponding angle), $c = 80°$ (corresponding angle), $d = 80°$ (vertically opposite angle)

(d) $a = 44°$ (alternate angle), $b = 46°$ (angles in a right angle)

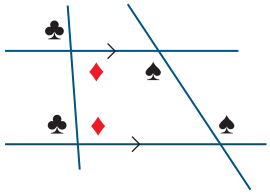
(e) $a = 50°$ (co-interior angle), $b = 50°$ (angles in a straight line)

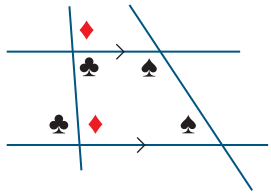
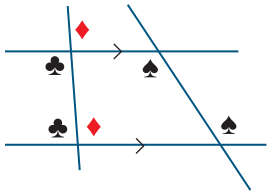
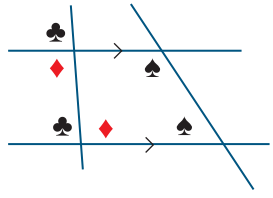
(f) $a = 45°$ (alternate angle), $b = 135°$ (co-interior angle), $c = 95°$ (co-interior angle), $d = 95°$ (vertically opposite angle)

(g) $a = 37°$ (alternate angle), $b = 37°$ (vertically opposite angle)

(h) $a = 116°$ (co-interior angle), $b = 104°$ (co-interior angle), $c = 104°$ (vertically opposite angle)

(i) $a = 84°$ (alternate angle), $b = 96°$ (angles in a straight line), $c = 74°$ (co-interior angle), $d = 74°$ (vertically opposite angle)

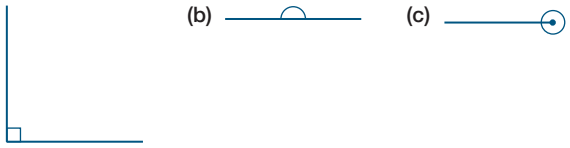
14 (a)


(b) Three:




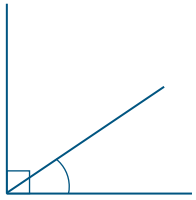
15 Perspective: in a 2D representation of a 3D situation, the parallel lines appear to converge on the horizon to give the impression of depth.

Half-time 8 (p. 311)

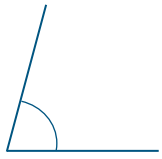
1 (a) (b) (c)



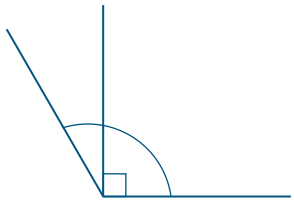
(d) About 30° is about one-third of a right angle.



(e) 75° is five-sixths of a right angle.



(f) 120° is 90° + 30°.



2 (a) $\angle BOC$ (or $\angle COB$), $\angle DOC$ (or $\angle COD$)

(b) $\angle BOA$ (or $\angle AOB$), $\angle AOD$ (or $\angle DOA$), $\angle BOD$ (or $\angle DOB$)

(c) $\angle BOA$

3 Students are required to draw angles for this question. The sizes match the diagrams in Question **1**.

4 (a) $x = 55°$ (Angles in a straight line add to 180°.)

(b) $x = 35°$ (Angles in a revolution add to 360°.)

5 30°

6 (a) $a = 67°$ (Alternate angles are equal.)

(b) $y = 57°$ (Co-interior angles add to 180°.)

7 (a) $\angle GEB$ (b) $\angle GDF$ (c) $\angle DFI$ (d) $\angle HGI$

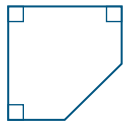
Exercise 8.5 (p. 313)

1 (a) pentagon (convex) (b) quadrilateral (concave)
(c) hexagon (convex) (d) nonagon (convex)
(e) octagon (concave) (f) dodecagon (concave)
(g) decagon (concave) (h) hexagon (concave)
(i) hexagon (concave)

2 (a) C (b) C

3 (a) octagon (b) dodecagon
(c) pentagon (d) hexagon

4 3



5 (a) 3

(b) 4 and 5. The pattern is: divide the number of sides by 2.

(c) No, a pentagon and a heptagon have an odd number of sides, so they cannot be paired.

6 (a) (b) (c)



7 50-cent piece (dodecagon), stop sign (octagon), soccer ball (pentagons & hexagons)

Exercise 8.6 (p. 318)

1 (a) (i) scalene (ii) obtuse
(iii) an obtuse-angled scalene triangle

(b) (i) isosceles (ii) acute
(iii) an acute-angled isosceles triangle

(c) (i) isosceles (ii) right
(iii) a right-angled isosceles triangle

(d) (i) isosceles (ii) acute
(iii) an acute-angled isosceles triangle

(e) (i) equilateral (ii) acute
(iii) an acute-angled equilateral triangle

(f) (i) scalene (ii) obtuse
(iii) an obtuse-angled scalene triangle

(g) (i) equilateral (ii) acute
(iii) an acute-angled equilateral triangle

(h) (i) scalene (ii) right
(iii) a right-angled scalene triangle

(i) (i) isosceles (ii) obtuse
(iii) an obtuse-angled isosceles triangle

(j) (i) scalene (ii) right
(iii) a right-angled scalene triangle

(k) (i) isosceles (ii) obtuse
(iii) an obtuse-angled isosceles triangle

(l) (i) scalene (ii) acute
(iii) an acute-angled scalene triangle

2 (a) 60° (b) 30° (c) 35° (d) 83° (e) 132°
(f) 22° (g) 14° (h) 38° (i) 48°

3 (a) (i) 120° (ii) 137° (iii) 75° (iv) 165° (v) 151°
(vi) 138° (vii) 133° (viii) 146° (ix) 129°

(b) (i) 40° (ii) 30° (iii) 74° (iv) 43° (v) 126°
(vi) 38°

4 (a) isosceles, acute-angled (b) scalene, acute-angled
(c) scalene, acute-angled (d) equilateral, acute-angled
(e) equilateral, acute-angled (f) scalene, acute-angled
(g) isosceles, acute-angled (h) isosceles, acute-angled
(i) scalene, right-angled

5 (a) D (b) A (c) B

6 B

7 (a) C (b) A

8 (a) isosceles; obtuse-angled, $a = 25°$
(b) scalene; right-angled, $b = 35°$
(c) equilateral, $c = 30°$
(d) scalene; right-angled, $a = 21°$
(e) scalene; right-angled, $b = 58°$
(f) isosceles; acute-angled, $c = 48°$
(g) isosceles; obtuse-angled, $d = 130°$

9 isosceles, right-angled

10 (a) all sides equal
(b) Two angles will be 60°, the other two will be 120°.

11 No, the three sides won't be able to meet.

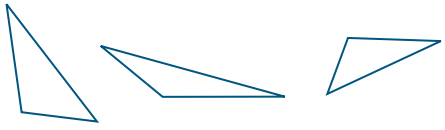
12 two

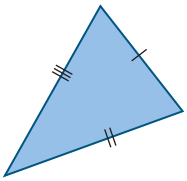
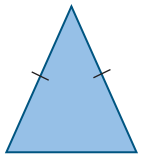
13 (a) right-angled, scalene (third angle 90°)
(b) obtuse-angled, isosceles (third angle 28°)
(c) acute-angled, equilateral (third angle 60°)
(d) obtuse-angled, isosceles (third angle 156°)

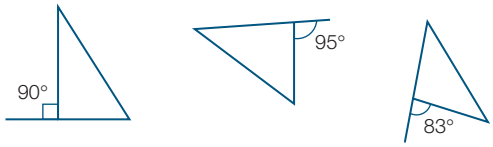
14 (a) Angles add up to more than 180° (i.e. 190°).
(b) Angles add up to 165° – must be 180°.
(c) Isosceles triangles must have 2 equal angles.

15 (a) $a = 65°$, $b = 135°$
(b) $c = 100°$
(c) $d = 57°$, $e = 57°$, $f = 66°$

16 360°

17 

18 (a)  (b) 

19 

Exercise 8.7 (p. 325)

1 (a) rectangle (b) parallelogram (c) kite
(d) rhombus (e) trapezium (f) quadrilateral

2 A

3 (a) True (b) False (c) True (d) False (e) True

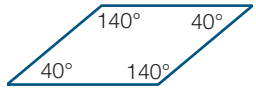
4 (a) 77° (b) 87° (c) 54° (d) 55° (e) 69°
(f) 133° (g) 138° (h) 125° (i) 205°

5 (a) D (b) C (c) C

6 (a) 40° (b) 105° (c) 68° (d) 73° (e) 197°
(f) 19° (g) 60° (h) 131° (i) 78°

7 (a) trapezium (b) rhombus
(c) kite (d) parallelogram

8 (a) $a = 70°$, angles in a quadrilateral add to 360°
(b) $b = 97°$, one pair of opposite angles in a kite are equal and angles add to 360°
(c) $c = 99°$, co-interior angle
(d) $d = 38°$, co-interior angle

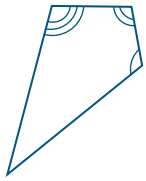
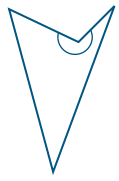
9 

10 105°

11 Parallelograms have opposite sides parallel and equal in length, which is also the case for a rhombus. However, a rhombus has all sides equal, and that is not necessarily true for all parallelograms.

12 No, a parallelogram must have two pairs of parallel sides whereas a trapezium must only have one pair of parallel sides.

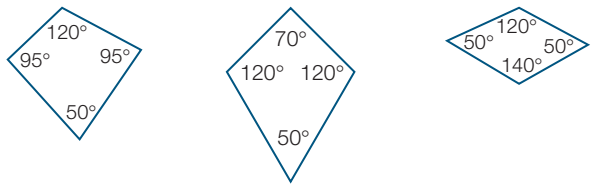
13 Yes, rectangles have all the properties of a parallelogram and more. Most parallelograms do not have 90° angles.

14 (a) three  (b) one 

15 (a) 128° (b) kite or irregular quadrilateral

16 No. Sum of angles in a quadrilateral = 360°, not 370°.

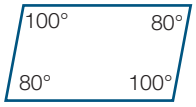
17 (a) 4 right angles
(b) 4 equal sides
(c) opposite sides are parallel

18 

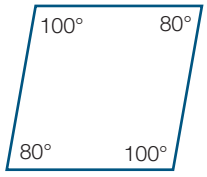
19 **(a)** parallelogram, rhombus, kite, trapezium, or irregular quadrilateral

(b) Five—**(i)** parallelogram, **(ii)** rhombus, **(iii)** kite, **(iv)** trapezium, or **(v)** irregular quadrilateral.

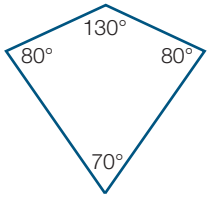
(c) **(i)**



(ii)



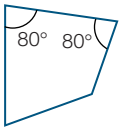
(iii)



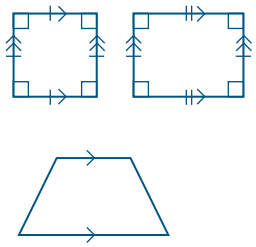
(iv)



(v)



20 Jack's diagrams lack important angle symbols, congruent and parallel line marking.



21 Parallelograms: rectangle, square, diamond, rhombus, parallelogram

Non-parallelograms: irregular quadrilateral, trapezium, kite

Challenge 8 (p. 330)

1 B At 2.30, the hour hand is halfway between 2 and 3. Total angle is $90° + (30° \div 2) = 105°$.

2 $90° - 46° = 44°$. Small angle $= 44° \div 2 = 22°$, large angle $= 46° + 22° = 68°$.

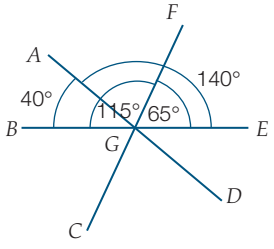
or: $90° \div 2 = 45°$, $46° \div 2 = 23°$. Small angle $= 45° - 23° = 22°$, large angle $= 45° + 23° = 68°$.

3 $180° - 70° = 110°$. $x + x = 110°$, so $x = 55°$.

4 Draw the diagram and mark the angles on it. Use this information to work out the other angles.

From the diagram, you can get $\angle AGF = 75°$

$\angle CGD = 75°$ (vertically opposite $\angle AGF$)



5 $180° - 54° = 126°$. Small angle $= 126° \div 2 = 63°$, large angle $= 54° + 63° = 117°$.

6 D $6x° = 180°$, $x = 30°$; $y = 180° - 30° = 150°$

7 A $x = 180° - (30° + 40°) = 180° - 70° = 110°$

8 **(a)** At 12.10, the minute hand makes an angle of 60° with the vertical and the hour hand makes an angle of $\frac{1}{6} \times 30°$ or 5°. The angle between them is $60 - 5° = 55°$.

(b) 40 minutes later, the time is 12.50. The minute hand makes an acute angle of 60° with the vertical and the hour hand makes an angle of $\frac{5}{6} \times 30°$ or 25°. The angle between them is $60° + 25° = 85°$.

9 C $x + y = 110°$, $\angle KLM = 180° - 2x$, $\angle KML = 180 - 2y$
$\angle KLM + \angle KML = 360° - 2(x + y) = 360° - 220° = 140°$,
$\angle LKM = 40°$

Chapter review 8 (p. 331)

1 **(a)** 42° **(b)** 196° **(c)** 157° **(d)** 321°

2 **(a)** E **(b)** A **(c)** D **(d)** B **(e)** C **(f)** F

3 Students are required to draw angles for this question.

4 A

5 B

6 **(a)** reflex **(b)** revolution **(c)** straight
(d) acute **(e)** right **(f)** obtuse

7 **(a)** 163° **(b)** 39° **(c)** 127°
(d) 40° **(e)** 143° **(f)** 82°

8 **(a)** 136° **(b)** 112°

9 **(a)** irregular concave pentagon
(b) irregular convex heptagon
(c) irregular convex decagon
(d) regular concave dodecagon

10 80°

11 **(a)** 66° (angles in a quadrilateral add to 360°)
(b) 110° (opposite angles in a parallelogram are equal)

12 140°, missing angles are the equal angles in a kite.

13 292°

14 **(a)** 45° and 90°
(b) isosceles triangle

15 **(a)** 85° **(b)** 25° **(c)** 40°

16 **(a)** True **(b)** False **(c)** True **(d)** False **(e)** True

17 $\angle CFB = 90°$, vertically opposite angles are equal.
$x = 27°$, complementary angles add to 90°.

18 an obtuse angle

19 a reflex angle

20 an acute angle, a right angle or an obtuse angle

NAPLAN practice 8

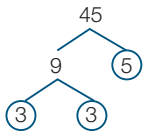
1 B **2** 150° **3** D **4** D

5 C **6** D

Mixed review D (p. 336)

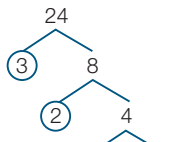
1 **(a)** $6a - 4b$ **(b)** $6a + 2b + 3c$

2 **(a)**



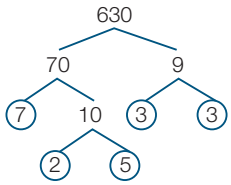
$3^2 \times 5$

(b)



$2^3 \times 3$

(c)



$2 \times 3^2 \times 5 \times 7$

3 **(a)** $\frac{3}{4}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{3}{10}$ **(d)** $\frac{1}{25}$

4 **(a)** 0.253 **(b)** 2.437

5 **(a)** $x = -1$ **(b)** $x = -41$

6 **(a)** 46° **(b)** 39°

7 **(a)** $3\frac{19}{20}$ **(b)** $\frac{15}{16}$

8 **(a)** 137.7 cm^2 **(b)** 60 m^2

9 **(a)** Thursday **(b)** Wednesday **(c)** Friday

10 A **11** D **12** 11.25°

13 **(a)** 35° **(b)** 63°

14 150°, 210°

15 **(a)** Students' own answers.

(b) If they add to 360°, then they are likely to be correct.

(c) Dogs 110°, Cats 140°, Birds 30° and Fish 80°

16 Second floor

17 $x = 45°$, $2x = 90°$, $3x = 135°$

18 **(a)** Students' own answers.

(b) reflex angle

(c) $\angle ABC$ (reflex)

19 Complementary angles: $\angle ABE$ & $\angle CBE$ (also $\angle ABE$ & $\angle BAE$, $\angle EBC$ & $\angle ECB$, $\angle BAE$ & $\angle BCE$, $\angle ECD$ & $\angle EDC$, $\angle EAD$ & $\angle EDA$).
Supplementary angles: $\angle AEB$ & $\angle CEB$, $\angle AED$ & $\angle CED$, $\angle AEB$ & $\angle AED$, $\angle CED$ & $\angle CEB$.
Vertically opposite angles: $\angle CEB$ & $\angle AED$, $\angle AEB$ & $\angle CED$.
Angles in a revolution: $\angle AEB$, $\angle CEB$, $\angle AED$ & $\angle CED$; $\angle BAE$, $\angle EAD$ and $\angle BAD$ (reflex); there are more around the edge of the kite.

Chapter 9

Recall 9 (p. 338)

1 **(a)** 0, 5, 10, 15, 20, 25, 30

(b) 16, 18, 20, 22, 24, 26, 28

(c) 1300, 1400, 1500, 1600, 1700

(d) 60, 80, 100, 120, 140, 160, 180, 200

2 **(a)** 50, 100, 150, 200, 250 **(b)** 100, 125, 150, 175, 200

(c) 1400, 1600, 1800, 2000

3 **(a)** 6, 10, 12, 14, 16, 20, 22, 24

(b) 0, 5, 15, 20, 25, 50, 55, 100

(c) $0, \frac{1}{2}, 1, 1\frac{1}{2}, 2, 2\frac{1}{2}, 3, 3\frac{1}{2}, 4, 5, 5\frac{1}{2}, 6\frac{1}{2}$

4 **(a)** 15, 18, 24 **(b)** 140, 160, 180, 200

(c) 250, 275, 325

5 **(a)** $\frac{1}{3}$ **(b)** $\frac{3}{4}$

Exercise 9.1 (p. 342)

1 **(a)** categorical

(b) categorical

(c) continuous

(d) continuous

(e) discrete

(f) categorical

(g) discrete

2 **(a)**

Number of pets owned	Tally	Frequency
0	||||	4
1	~~||||~~ ||	7
2	|||	3
3	|||	3
4	||||	4
5	|	1
6	|	1
7	|	1
8		0
9	|	1
		25

(b) discrete

3

Number of calls	Tally	Frequency
0–19	|	1
20–39	|	1
40–59	|||	3
60–79	~~||||~~ |	6
80–99	~~||||~~ |	6
100–119	~~||||~~ |	6
120–139	||	2
140–159	||	2
160–179	||	2
180–199	|	1
		30

4 **(a)** 82

(b)

Colour	Frequency
Black	13
White	21
Silver	25
Red	7
Blue	5
Other	11

(c) categorical

5

Hours of television watched	Tally	Frequency
0	| | |	3
$\frac{1}{2}$	| | | |	4
1	| | |	3
$1\frac{1}{2}$	| |	2
2	𝍸 |	6
$2\frac{1}{2}$	𝍸	5
3	| |	2
$3\frac{1}{2}$		0
4	|	1
$4\frac{1}{2}$	|	1
		27

6 Categorical

7 Students will provide their own answers, but these are samples.

(a) Shoe size, where the sizes are 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$,…. (This is not categorical as there is a fixed relationship between each consecutive pair of sizes.)

(b) Categorical. There is no sensible mathematics that you can do with the results. For example, you cannot find the mean postcode. The numerical postcodes work in exactly the same way as they would if they were all letters or a mix of numerals and letters.

8 **(a)** C **(b)** B

9 **(a)** 26

(b)

Number of hours spent exercising	Tally	Frequency
0	| | | |	4
1	𝍸 | |	7
2	𝍸 | | |	8
3	| | |	3
4	|	1
5	|	1
6	|	1
7		0
8	|	1
		26

(c) 4 **(d)** 7

(e) 2 hours **(f)** 8 hours

(g) 8, 6 and 5 hours. These students will be training for more hours compared to other students.

10 **(a)** 35

(b)

Size	Tally	Frequency
XS	| |	2
S	𝍸 | |	7
M	𝍸 | | | |	9
L	𝍸 | | |	8
XL	| | | |	4
Did not order	𝍸	5

(c) categorical **(d)** M **(e)** 9

(f) Because the students who forgot to order were all in Year 7, she should probably order 5 M size T-shirts, as the L and XL would be too big for most Year 7 students.

11 **(a)**

Class interval	Frequency
20–29	6
30–39	6
40–49	6
50–59	6
60–69	6

(b) Each of the intervals contains six values.

(c)

Class interval	Frequency
20–24	5
25–29	1
30–34	5
35–39	1
40–44	5
45–49	1
50–54	5
55–59	1
60–64	5
65–69	1

(d) The low-half class intervals have 5 values whereas the high-half class intervals have just 1 value. The values tend to appear at the lower end of the 10 groups.

(e) The size of the class interval can have an effect on the way we see the data.

12 **(a)**

Pulse rate (beats per minute)	Frequency
51–60	7
61–70	21
71–80	19
81–90	10
91–100	3

(b)

Pulse rate (beats per minute)	Frequency
51–55	3
56–60	4
61–65	8
66–70	13
71–75	10
76–80	9
81–85	10
86–90	0
91–95	3

(c) Probably (b). This table shows that the vast majority of students have a pulse rate between 61 and 85 beats per minute. When looking at table (a), the vast majority of students have a pulse rate between 61 and 90 beats per minute, but the high end of 90 might be a cause for medical concern.

13 The highest weight is 3529 g and the lowest is 2010 g. This is a difference of 1519 g. If we group in 250 g lots, we would have 7 groups. If we group in 200 g lots, we would have 8 groups. If we group in 125 g lots, we would have 14 groups—this is too many. We could choose either of the others. The table below is for lots of 250 g. It starts at 2000 g as this is a multiple of the difference being used.

Birth weight (g)	Frequency
2000–2249	5
2250–2499	2
2500–2749	5
2750–2999	8
3000–3249	6
3250–3499	1
3500–3749	3

14 (a)

Number rolled	Tally	Frequency
1	|||	3
2	||	2
3	||||	4
4	||||	4
5	|||	3
6	||||	4

(b)

Weight of dogs (kg)	Tally	Frequency
1–5	𝍸 𝍸 ||	12
6–10	𝍸 |||	8
11–15	𝍸 𝍸 ||||	14
16–20	𝍸 𝍸 |||	13
21–25	|||	3

15 Students' own answers.

16 Students' own answers.

Exercise 9.2 (p. 348)

1 (a) (i) 5 (ii) 7 (iii) 7 (iv) 6

(b) (i) 6 (ii) 5 (iii) 5 (iv) 3

(c) (i) 6 (ii) 6 (iii) 6 (iv) 0

(d) (i) 4 (ii) 4 (iii) 4 (iv) 0

(e) (i) 7 (ii) 7 (iii) 7 (iv) 4

(f) (i) 7 (ii) 6 (iii) 5 (iv) 5

(g) (i) 6 (ii) 7 (iii) 8 (iv) 7

(h) (i) 4 (ii) 3 (iii) 2 (iv) 7

(i) (i) 4.9 (ii) 4.5 (iii) 4 (iv) 6

(j) (i) 5.5 (ii) 5 (iii) 5 (iv) 6

(k) (i) 6 (ii) 7 (iii) 9 (iv) 8

(l) (i) 5 (ii) 5 (iii) 2 (iv) 7

2 (a) (i) 1, 2, 2, 3, 4, 4, 6, 7, 7, 9, 11, 11 (ii) No

(b) (i) 2, 6, 7, 8, 9, 9, 12, 13, 13, 14 (ii) Yes – 2

(c) (i) 12, 13, 15, 16, 17, 19, 21, 24, 25 (ii) No

(d) (i) 11, 17, 17, 18, 19, 19, 20, 21, 22, 22 (ii) Yes – 11

(e) (i) 1.3, 1.4, 1.6, 1.7, 1.8, 1.9, 2.0, 3.1 (ii) Yes – 3.1

(f) (i) 1.2, 1.4, 1.7, 1.8, 2.2, 2.6, 2.9, 3.1, 3.3 (ii) No

3 (a) (i) Set 1: Mean = 13.33 Set 2: Mean = 12.87

(ii) Set 1: Median = 10 Set 2: Median = 10

(iii) Set 1: Mode = 10 Set 2: Mode = 10

(iv) Set 1: Range = 14 Set 2: Range = 21

(b) Changing the first piece of data has an effect on both the mean and the range of the data, but has no effect on either the median or the mode.

4 (a) (i) 29 (ii) 29 (iii) 29 (iv) 13

(b) (i) 66.43 (ii) 72 (iii) No mode exists. (iv) 37

(c) (i) 66 (ii) 67.5 (iii) 81 (iv) 50

(d) (i) 10.26 (ii) 9.05 (iii) 6.5 (iv) 14.6

(e) (i) 13.7 (ii) 13.5 (iii) 11.4 (iv) 4.9

(f) (i) 8.50 (ii) 10.11 (iii) No mode exists. (iv) 10.1

5 (a) C; only one within the range of scores

(b) C; 8 too high and 4 too low

(c) C; only one of each score, so mean will be close to median

(d) B; close to median as single results relatively evenly spread

6 (a) (i) 49 480.8 (ii) 43 369 (iii) 27 607

(b) Incheon

(c) (i) 49 390 (ii) 46 000 (iii) 48 000

(d) Miyagi

(e) The mean capacity is slightly higher in Korea (49 480.8 compared to 49 390), but the median capacity is higher in Japan (46 000 compared to 43 369). Half of the stadiums in each country have a capacity less than 45 000 (i.e. smaller than the mean value).

(f) (i) 49 435.4 (ii) 43 369 (iii) 29 593

(g) Miyagi (again)
(It is interesting to note there are two stadiums in Japan quite close to the mean—Miyagi and Osaka—but none of the Korean stadiums is very close at all.)

(h) (i) 27 607 (ii) 28 000

7 (a) The mean score is 22.44 and the median is 20 points.

(b) No score stands out as an outlier.

(c) The median is probably a better predictor of scores as there are four vales of 38 or above, which will push the mean score out a bit too far.

8 (a) Mean; this is the classic 'average' situation

(b) Mode; the most frequently purchased size is the one that needs to be manufactured the most

(c) Median; very low or very high prices would have an impact on the mean

9 (a) No, because the mean is the sum of all the data values divided by the number of values and so it will not necessarily be a data value.

(b) 5.8, no it is not a data value.

(c) Yes, because there is an odd number of values and so the median will be the middle value.

(d) 6

10 (a) This statement means that the average number of children of all families is 2.3. Most families have 2 children, but because many families have more than two children, the average of all these numbers of children is 2.3.

(b) The middle number will always be a whole number. If there are two middle numbers, then the only decimal you can get by dividing them by 2 is one that ends in .5.

(c) No, as the mode is the value that occurs most often in a data set, and the data set only contains whole numbers.

11 (a) **(i)** \$58 571.43 **(ii)** \$50 000 **(iii)** \$50 000 **(iv)** \$75 000

(b) In this case, the mode does a good job as there are so many of this category of worker—well over half. However, the mean does take account of the other workers as well.

(c) **(i)** \$53 461.54 **(ii)** \$50 000 **(iii)** \$50 000 **(iv)** \$35 000

(d) The mean is the only measure of centre that changes, and it changes by a large amount (over \$5000). The range is reduced by a very large amount.

12 (a) Mean: 15.08 Median: 15 Range: 25

(b) Mean: 14.92 Median: 15 Range: 9

(c) Having an outlier at each end can balance out the mean, but there is a major impact on the range. The median is unaffected.

13 (a) 5, 6, 6, 6, 7. As the median is 6, this is the middle number and we need at least one more 6 to make it the mode. The total of the five results is 30, so we have 18 left, which we can break up any way we like as long as we don't move the median number.

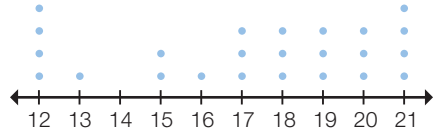
(b) 4, 6, 7, 9, 9. As the median is 7, this has to be the middle number and we need two 9s after that. This adds to 25 and the total required is 35, so we only have 10 left to play with.

(c) 4, 5, 6, 6, 14. The middle number is 6 and one more is 6. The total is 35, so we still have 23 to play with that can be allocated in many ways.

14 While Veran is correct to say the values must be ordered first when there is an even number of values, the median is found exactly midway between two of the values. The ordered list is: 17, 19, 20, 20, 21, 22, 23, 28, so the median is midway between 20 and 21 and so is 20.5.

Exercise 9.3 (p. 356)

1



2 (a)

STEM	LEAF
2	2 3 8
3	0 1 4 7
4	1 2 8
5	2 7

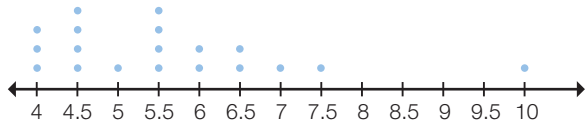
Key: 2 | 3 = 23

(b)

STEM	LEAF
0	2 3 8
1	4 5 8
2	1 5 6 7 9
3	0 3 4 5 7
4	2 4

Key: 1 | 4 = 14

3 (a)



The median is 5.5, the range is 6 and 10 seems to be an outlier.

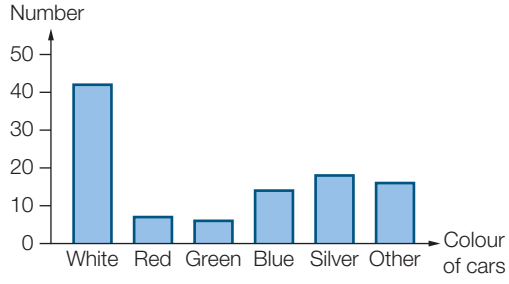
(b)

STEM	LEAF
5	5 6 9
6	7
7	5 7
8	8 8 8 9
9	9
10	0
11	0 2 5
12	1

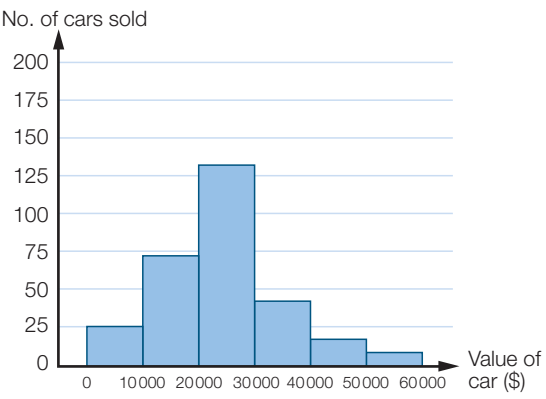
Key: 8 | 8 = 88

The median is 88, the range is 66 and there do not appear to be any outliers.

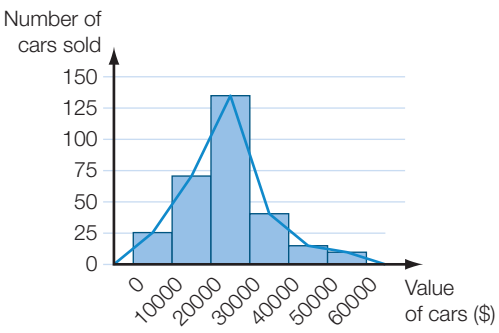
4



5 (a)



(b)



6 (a)

STEM	LEAF
2_L	0 1 2 3
2_U	7 7 8
3_L	1 4
3_U	8
4_L	2

Key: 2 | 1 = 21

(b)

STEM	LEAF
4_L	2
4_U	7 7 8
5_L	1 2 3
5_U	6 7 8
6_L	1 4
6_U	8

Key: 5 | 2 = 52

7 (a)

STEM	LEAF
12_L	1
12_U	5 6 6 8 8 9 9
13_L	0 3 3 4
13_U	6 8
14_L	0 2 2 3
14_U	6
15_L	0

Key: 12 | 6 = 126

(b)

STEM	LEAF
12	1 5 6 6 8 8 9 9
13	0 3 3 4 6 8
14	0 2 2 3 6
15	0

Key: 13 | 0 = 130

8 (a) C **(b)** C

9 (a) Thursday **(b)** 13 **(c)** 45

10 (a) 10 cm **(b)** 80 cm **(c)** 850–860

(d) 13 **(e)** 50

11 (a) The bars are horizontal; the scale is at the bottom.

(b) 820 million **(c)** 430 million **(d)** Spanish

(e) The first two numbers of the scale are given and then every second number is left off.

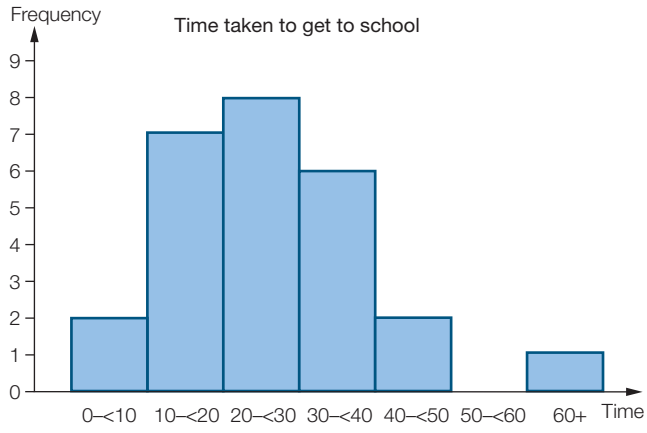
(f) Yes

12 (a) continuous

(b)

Distance	Tally	Frequency
0–<10	| |	2
10–<20	𝍸 | |	7
20–<30	𝍸 | | |	8
30–<40	𝍸 |	6
40–<50	| |	2
50–<60		0
60+	|	1

(c)



(d) Most students seem to live locally except for one student who travels 69 minutes. This could appear to be an outlier.

13 (a) Australia

(b) Canada

(c) UK

(d) Aluminium cans

(e) Plastics

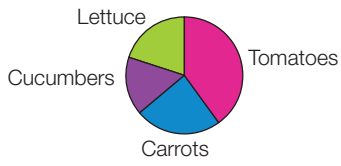
(f) Australia's recycling rate for glass is $\approx \frac{1}{2}$ of Switzerland's rate.
Australia's recycling rate for plastics is $\approx \frac{1}{2}$ of Austria's rate.
Australia's recycling rate for steel cans is a little under $\frac{1}{2}$ of Japan's rate.
Australia's recycling rate for aluminium cans/packaging is $\approx \frac{2}{3}$ of Finland's rate.

(g) Australia's recycling rate for glass is $\approx \frac{4}{3}$ Ireland's rate.
Australia's recycling rate for plastics is ≈ 4 times Canada's rate.
Australia's recycling rate for steel cans is over 5 times NZ's rate.
Australia's recycling rate for aluminium cans/packaging is over 4 times Ireland's rate.

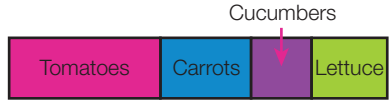
14 Students' own answers.

Exercise 9.4 (p. 362)

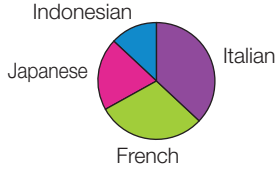
1 (a) (i)



(ii)



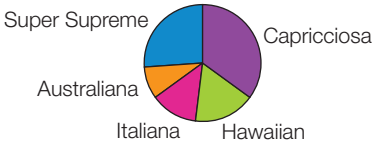
(b) (i)



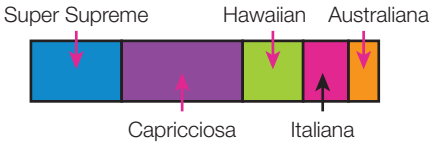
(ii)



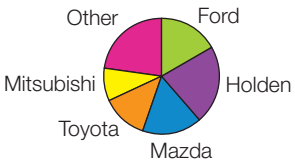
(c) (i)



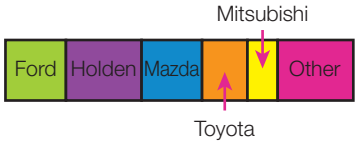
(ii)



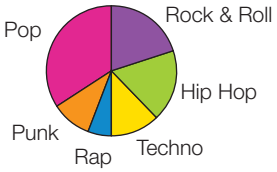
(d) (i)



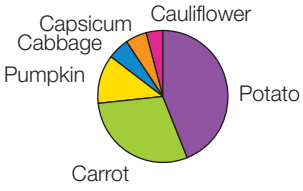
(ii)



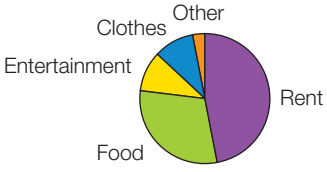
2 (a) (i)



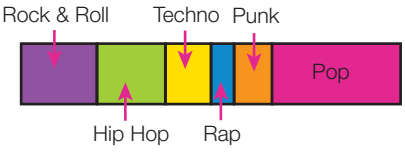
(ii)



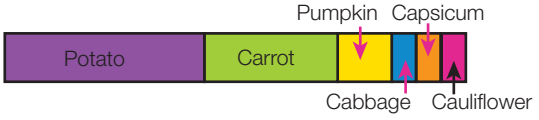
(iii)



(b) (i)



(ii)

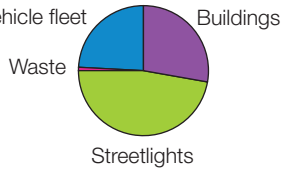


(iii)



3 (a) A (b) D

4 (a)



(b)



(c) Students' own answers.

5 (a) 3.3% (b) 5.2% (c) 5–25 minutes (d) 94.3%

6 (a) 4% (b) 1% (c) 4%

(d) (i) non-recyclables (ii) 79%

(e) paper (f) glass (g) No

7 (a) bags (b) sheeting and film

(c) confectionery wrappers (d) percentages

(e) all items not included in the other categories

(f) It is important because the survey would not be of much value if only a small number of items had been collected.

(g) They divide something into sections.

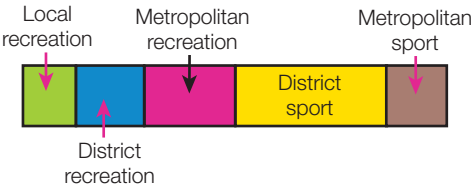
8 (a) Local recreation—70°, District recreation—90°, Metropolitan recreation—60°, District sport—100°, Metropolitan sport—40°

(b) 17% (c) 28% (d) 39%

(e) Recreation, 22% more

(f) Metropolitan sport

(g)



9 (a) English (b) 11%

(c) Hindi, Sinhalese, Samoan, Khmer, Macedonian, Greek

(d) Arabic

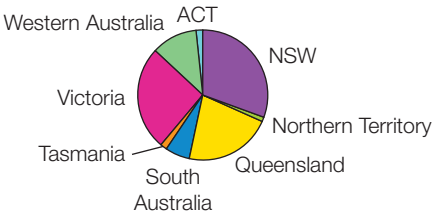
(e) There are many other languages spoken at home but each is less than 1% of the population.

(f) Students' own answers.

10 (a) 347 514

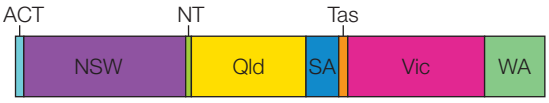
(b) ACT—6°, NSW—110°, NT—4°, Qld—78°, SA—22°, Tas—6°, Vic—93°, WA—41°

(c)



(d) A length of 70 mm might work reasonably well.

(e) Using this length, the pieces, in alphabetical order, would be: 1.1 mm, 21.4 mm, 0.8 mm, 15.2 mm, 4.3 mm, 1.2 mm, 18.1 mm, 8.0 mm. This is a total of 70.1 mm, which is about as good as we can do. Your graph should look something like the following, although its length will probably be different.



(f) It will probably be easier to draw the sector graph, although neither is very easy. Some rounding is certainly required with the divided bar chart.

11 (a) Rates and garbage charges

(b) 29–30°

(c) 8% (to nearest %)

(d) Government grants; this section is the larger of the two.

12 Students' own answers.

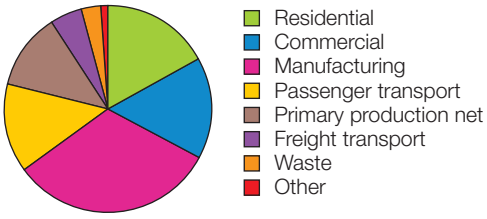
13 Students' own answers—graphs generated using Excel.

Half-time 9 (p. 368)

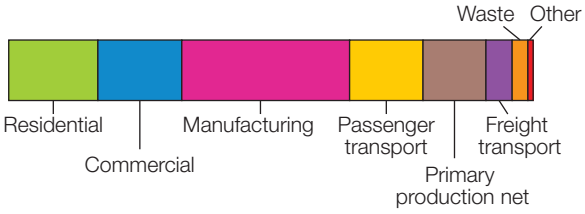
1 (a) (i) 3.7 (ii) 3 (iii) 2 (iv) 7

(b) (i) 5 (ii) 5 (iii) 4 and 6 (iv) 7

2 (a) **Victoria's greenhouse gas emissions**



(b)



3 (a) This gives eight groups, which is within the 5–10 range we prefer.

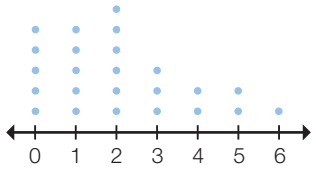
(b)

Height range	Frequency
125–129	1
130–134	1
135–139	2
140–144	7
145–149	5
150–154	6
155–159	2
160–164	1

4

Number absent	Frequency
0	8
1	6
2	4
3	2
4	1

5



6 (a)

STEM	LEAF
6	2 7
7	4 6
8	0 2 4 5 6
9	0 1
10	
11	
12	
13	2

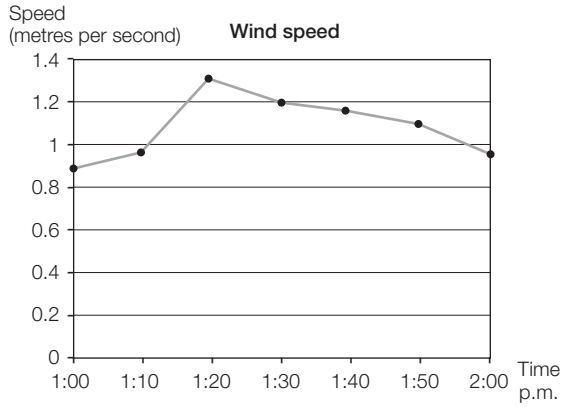
Key: 6 | 2 = 62

(b) 83

(c) 132 appears to be an outlier. The rest of the data in the set is bunched together.

Exercise 9.5 (p. 370)

1 (a)



(b) Between 1:10 p.m. and 2:00 p.m. (not inclusive)

2 (a) 50 (b) 1 p.m. (c) 60

(d) 11:30 a.m. and 12:30 p.m.

3 (a) C (b) B (c) C

4 (a) (i) 41° (ii) 17 January

(b) (i) 19° (ii) 25, 26 January

(c) 22° and 38° (d) 19° and 27°

5 (a) Start 2008 40%, end 2008 38%

(b) 2010 (c) Feb–July 2007

(d) 3% decrease. As the storage levels drop about 3% every year, there is not a lot of rain in December in Perth.

(e) Increase of 9.5%

(f) August (g) January

6 (a) 360 000 (b) 525 000

(c) Actual numbers were unknown because many Indigenous people live in isolated communities and many move from place to place.

(d) 705 000 and 715 000

(e) More Australians are willing to be recognised as of Indigenous descent. Better health care means less infant mortality and a longer life span.

7 (a) 22 million (b) 37 million (c) 41 million

(d) Number of births > number of deaths, immigration > migration

(e) Overcrowding of cities, water shortages, environmental damage, not enough food produced

(f) 37 million

(g) Immigration > migration

(h) 32–33 million

(i) Number of births + number of immigrants = number of deaths + number of migrants

8 (a) 2.4 million (b) 2007 (c) 1995–2005

(d) 2.45 million, 2.65 million, 3 million

(e) The population aged 65 years and over has been increasing steadily since 1990. The rate of increase in the period 2006–2010 has been even higher than in previous years.

(f) Babies born after the War are now aged 65 and older. Immigrants who arrived after 1945 are also now aged 65 and older.

(g) ≈ 3.75 million

9 (a) Yes, 2000 (b) 150 cm (c) 175 cm

(d) 2000 (e) 2002 (f) 1999

(g) 2000 or 2003 (h) 2003 (i) 2006

(j) August (k) September

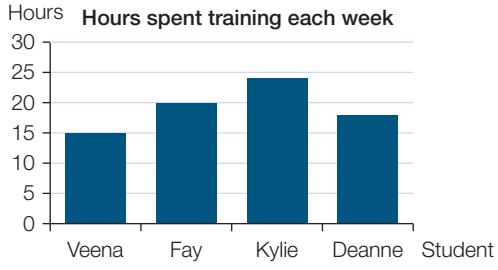
(l) Perhaps—the last 4 years have seen less snow than the previous 6 years, but it could simply be part of a longer cycle.

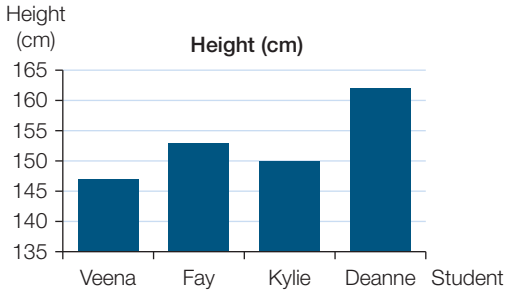
(m) Line graphs are more appropriate to show this information as the depth is changing according to time, and a large number of bars would be needed to show the daily change.

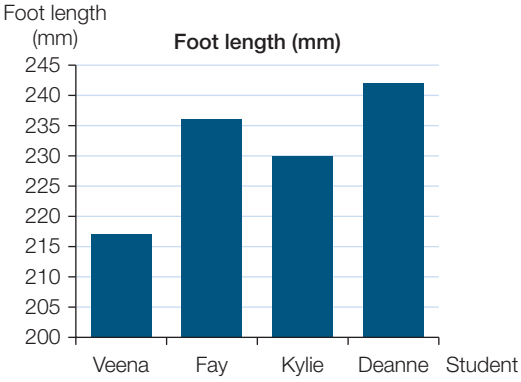
10 Students' own answers.

Exercise 9.6 (p. 378)

1







The shape of the graph of hours spent training each week is quite different from the shape of the other two graphs. There seems to be no connection between the hours spent training and either the height or the foot length.

The shapes of the height graph and the foot length graph are very similar, so there could be a connection between these two variables.

2 (a) Islander College, 350 students

(b) Bottom part; Hilltop College

(c) Gumnut College

(d) Jinglebells College

(e) Faraway and Gumnut Colleges

3

	Mean	Median	Range
Adelaide	7.5	7	6
Brisbane	8	8	3

Although Adelaide has the individual highest value, its lack of consistency means that Brisbane has a higher mean and median number of sunshine hours. Adelaide's range is about double that of Brisbane.

4

	Mean	Median	Range
Adelaide	12.1917	11.9	9.6
Brisbane	15.4917	15.95	12.3

Brisbane has a much greater range in temperature. However, both the mean and median indicate that Brisbane is, overall, a warmer place.

5 (a) Glass (b) Paper

(c) 2002/03 (d) 2001/02

(e) No apparent change in the amount recycled

(f) A steady increase in the amount recycled, but the amount is still very small

6 (a)

	Mean	Median	Mode	Range
Set A	5.7	6.5	7	9
Set B	5.55	5	2 & 9	7

The numbers in Set A are, on average, slightly larger and more spread out. The mode does not really give us any useful information.

(b)

	Mean	Median	Mode	Range
Set A	3.75	3.5	2	7
Set B	5.75	6.5	7, 8 & 9	8

Although the range of both sets is very close, the numbers in Set A appear, on average, to be quite a bit smaller than those in Set B. Again, the mode gives us no useful information.

7 (a) NSW (b) 8 000 000 t

(c) NSW, Vic, SA, ACT (d) ACT

(e) NT (f) SA

8 (a) No

(b) Japan, because they have many more hospital beds per 10 000 people than the other countries.

(c) Because Japan has one of the highest GDPs, the amount it spends on health facilities is to be expected.

(d) Indonesia

(e) Because Indonesia has one of the lowest GDPs, the amount it spends on health facilities is to be expected.

(f) Australia's health-care facilities do not seem to be as good as they should be given its GDP, which is the highest of all the countries shown.

(g) Vietnam's health-care facilities seem to be very good given its GDP, which is the lowest of all the countries shown.

9 April paid a mean price of $47.78 for her fruit, whereas Izzy paid $50.71. April bought 9 cases compared to 7 for Izzy with the range of prices paid by both friends being the same. Due to the small numbers, neither the median nor the mode will tell us anything useful.

10 (a) (i) Paper/cardboard, plastic bottles, glass

(ii) Easiest to collect, easiest to recycle

(b) Steel cans

(c) Paper/cardboard, plastic bottles and glass

11 (a) mean: 1.611 m median: 1.65 m mode: 1.67 m

(b), (c) Students will produce their own answers but will need to consider the following. The sum of the existing girls is 16.11, so the new total needs to be either 21.132 m or 17.532 m. The new girls cannot be the same height as any of the existing girls unless they are 1.65 m (the median).

12 Set A: 13, 10, 13, 21, 5, 12, 19, 12, 25, 20, 16, 3, 10, 10, 18, 17
Set B: 4, 2, 10, 19, 1, 23, 18, 17, 10, 3, 16, 18, 10, 14, 18, 17

Set A: median: 13, mode: 10, mean: 14, range: 22
Set B: median: 15, mode: 10, mean: 12.5, range: 22

Exercise 9.7 (p. 384)

1 (a) $\frac{1}{2}$ (b) $\frac{2}{11}$ (c) $\frac{1}{6}$ (d) $\frac{1}{4}$

(e) $\frac{1}{13}$ (f) $\frac{1}{26}$ (g) $\frac{1}{26}$ (h) $\frac{1}{52}$

2 (a) $\frac{4}{5}$ (b) $\frac{3}{5}$

3 (a) impossible (b) certain

(c) likely/not likely (d) certain

(e) certain (f) not likely

(g) impossible (h) not likely

(i) certain

4 (a) A (b) C (c) D (d) C (e) C

5 (a) {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

(b) {a, b, i, l, o, p, r, t, y}

(c) {two heads, one head and one tail, two tails}

6 (a) (i) $\frac{1}{3}$ (ii) $\frac{1}{5}$ (iii) $\frac{2}{5}$

(b) (i) $\frac{2}{7}$ (ii) $\frac{3}{14}$ (iii) $\frac{3}{7}$

7 (a) $\frac{1}{3}$ (b) $\frac{3}{5}$ (c) $\frac{11}{15}$ (d) $\frac{3}{5}$

8 (a) No

(b) Nicky's winning colour is two times the area of Steve's.

9 (a) Yes (b) No (c) Yes (d) No (e) No

10 (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$ (e) $\frac{5}{6}$ (f) $\frac{1}{2}$

11 Sample answer: impossible, virtually impossible, almost never, most unlikely, sometimes, maybe, even chance, quite frequently, more often than not, quite often, often, likely, usually, very likely, almost always, sure thing

12 (a) 4 (Possible: Brown, Brown, Blue, Blue)

(b) 8 (Possible: Blue, Blue, Blue, Blue, Blue, Blue, Brown, Brown)

(c) 3 (Possible: Blue, Brown, Blue)

(d) 7 (all 6 Blues could come out first, then Brown)

13 (a) 16.1% (b) 29.8% (c) 9.6% (d) 28.7% (e) 98.4%

(f) 96.3%

(g) Only very slightly as the types of cars people buy won't change much year to year.

(h) It is slightly more probable that a small car was bought as more were sold.

14 (a) Monday will follow Tuesday. Humans will run 100 m in under 5 seconds.

(b) It will reach 50°C in Australia next year. Australia will win the most gold medals at the next Olympics.

(c) It will rain in Sydney in the next month. Australia will win a swimming gold medal at the next Olympics.

(d) The Sun will rise tomorrow. Someone will be born in Australia today.

15 Students' own answers.

Challenge 9 (p. 388)

1 C If the dice are arranged so that the one-dot and the two-dot faces can't be seen, then the greatest number of dots will be viewable; that is, $42 - 6 = 36$.

Opposite dots on a die total 7
i.e. $1 + 6$, $2 + 5$, $3 + 4$, $4 + 3$, $5 + 2$, $6 + 1$,
there are 6 lots, $6 \times 7 = 42$.

Or: $2 \times (6 + 5 + 4 + 3) = 36$

2 The reason Mr Black is wearing the white tie and not the green tie is because in the passage, Mr Black is talking to the man wearing the green tie. Therefore, Mr Black must be wearing the white tie. Mr Green cannot be wearing the white tie so he must be wearing the black tie. Therefore, Mr White is wearing the green tie.

3 D 4 of the six faces will contain odd numbers.

4 D The shortest time wins the race, so the shortest bar is Maggie.

5 B $3B + 6G + 2R + 6Y = 17$ marbles. As the probability is $\frac{3}{7}$, the number of marbles in the bag must be a multiple of 7, so at least 21. There are 9 blue or green marbles, $9 = 3 \times 3$, so need 21 marbles in the bag. Vijay adds 4 white marbles to the bag.

6 A $15 + 10 + 20 + 30 = 75$. $100 - 75 = 25$

7 C Has won 19 of the 20 games, needs to win 24 out of 25 games to reach 96%, hence next 5 games.

Chapter review 9 (p. 389)

1

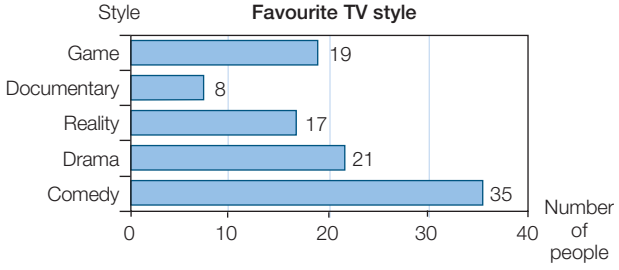
Number of television sets owned	Tally	Frequency
0	| | |	3
1	𝍸 𝍸 |	11
2	𝍸	5
3	| |	2
4	|	1
5	|	1
		23

2 **(a)** mean = 3.55, median = 3, mode = 2, range = 6

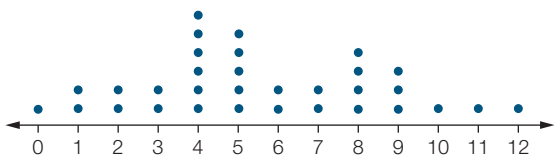
(b) mean = 35, median = 35, there is no mode, range = 50

(c) mean = 3.72, median = 3.9, mode = 4.1, range = 1.5

3



4 **(a)**



(b) 5

(c) No, the data set seems well spread out.

5 **(a)**

STEM	LEAF
0	0 0 0 0 0 1 2 2 2 3 4 5 5 6 7 7
1	1 4 5 7 7 7 8 8 9
2	0 1
3	1
4	0 1 2 2 6
5	0 0 2 6 8
6	1 3 4
7	7
8	8
9	
10	
11	5

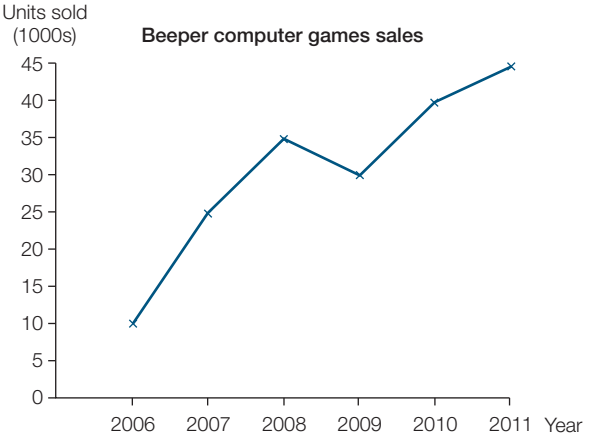
Key: 2 | 0 = 20

(b) 17.5

(c) The score of 115 appears to be an outlier.

6 **(a)** D **(b)** D

7

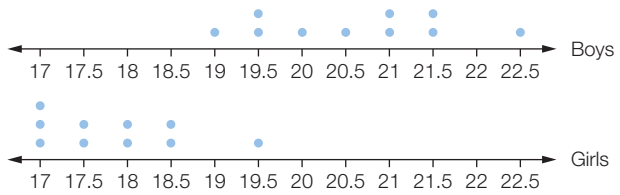


8

	Mean	Median	Range
Cairns	167.7	92.4	425.5
Brisbane	78.9	73.35	95.7

These two cities both receive a lot of rain, although Cairns clearly receives more rain than Brisbane. The wettest month for Cairns is far wetter than the wettest month for Brisbane, as reflected in the mean and range results. The driest months are almost identical. Cairns has a wet summer and a dry winter, whereas Brisbane follows a similar pattern but has a wetter spell in May and June. The median results are not that far apart which is, again, a reflection of the similar patterns, but still shows Cairns to be wetter.

9



The boys clearly have larger handspans. The median for boys is 20.75 and for girls it is 17.75. The range for boys is 3.5 compared to 2.5 for girls, so the data for boys are slightly more spread out. The most telling feature, however, is that only one girl has a handspan larger than the boy with the smallest handspan.

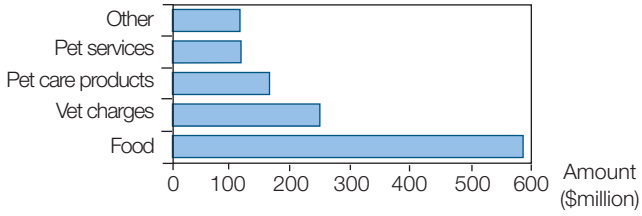
10 (a) B (b) B

11 (a)

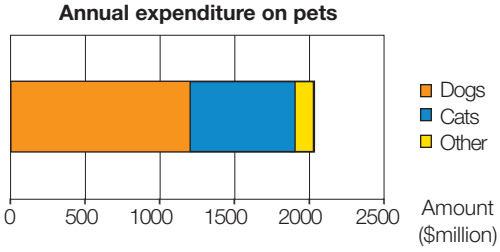
Shoe size	Frequency
1	2
$1\frac{1}{2}$	2
2	2
$2\frac{1}{2}$	3
3	3
$3\frac{1}{2}$	5
4	3
$4\frac{1}{2}$	3
5	3
$5\frac{1}{2}$	3
6	1

(b) $3\frac{1}{2}$ (c) 6

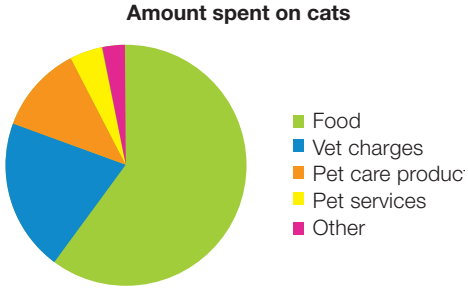
12 (a)



(b)



(c)



(d) The data sets were not compared over a period of time.

13 (a) column graph (b) July (c) 30 cm

(d) December, January, February, March

14 Yes, it is fair.

15 (a) 0 (b) 1 (c) the second

(d) Probabilities can't be greater than 1.

16 (a)

Height (cm)	Frequency
140	4
141	1
142	1
143	1
144	2
145	1
146	1
147	2
148	3
149	2
150	1
151	2
152	2
153	1
154	1

(b) (i) 140 (ii) 147 (iii) 146.6

(c)

Height (cm)	Frequency
138	1
141	1
142	1
143	2
144	2
145	2
146	2
148	2
150	2
151	1
152	1
154	3
156	2
157	3

(d) (i) 154, 157 (ii) 148 (iii) 148.8

(e) On all measures of centre, the girls are taller than the boys. The shortest student, however, is a girl, as is the tallest. The girls' heights are clustered more at the top of the range of values.

(f)

Height (cm)	Frequency
138	1
140	4
141	2
142	2
143	3
144	4
145	3
146	3
147	2
148	5
149	2
150	3
151	3
152	3
153	1
154	4
156	2
157	3

(g) (i) 148 (ii) 148 (iii) 147.7

(h) The three measures for the combined group are very close to each other. The average student is about 148 cm tall, the shortest is 138 cm and the tallest is 157 cm.

17 (a) Don't place any counters on 1, 2, 19 or 20 as these totals will never appear as the sum of the three dice.

(b) The scores 10 and 11 should appear more often than any other total.

(c) The results obtained on any particular game could be quite different from the next game.

NAPLAN practice 9

1 B **2** D **3** C **4** C

5 (a) Friday (b) 30 (c) Thursday

6 C

Chapter 10

Recall 10 (p. 395)

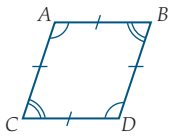
1 (a) 56° (b) 150° (c) 50°

2 (a) 20° (b) 60° (c) 40°

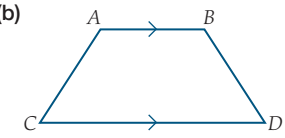
3 (a) 66° (b) $a° = 110°$, $b° = 70°$, $c° = 70°$

(c) $a° = 120°$

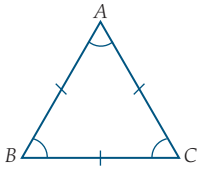
4 (a)



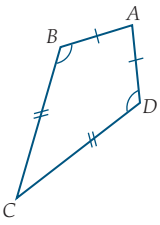
(b)



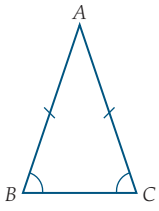
(c)



(d)



(e)



(f)



Exercise 10.1 (p. 398)

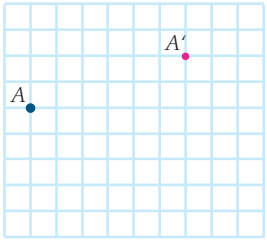
1 (a) 5 units left

(b) 5 units down

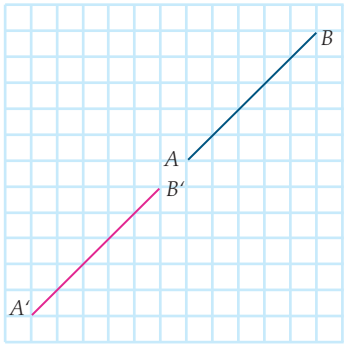
(c) 5 units right and 3 units down

(d) 7 units left and 2 units up

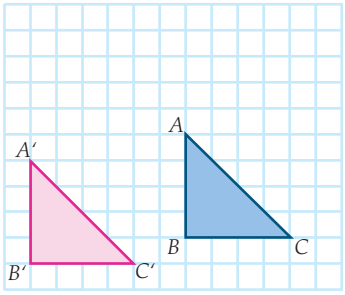
2 (a)



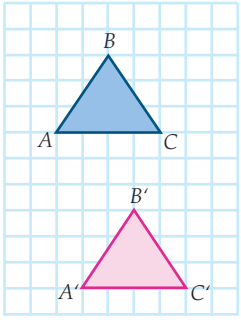
(b)



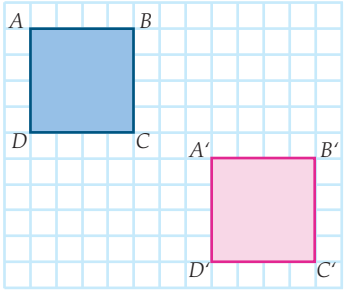
(c)



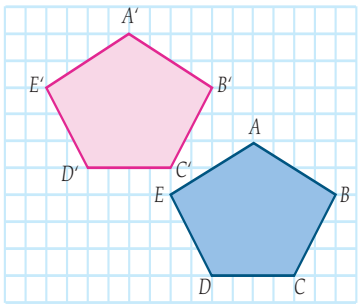
(d)



(e)

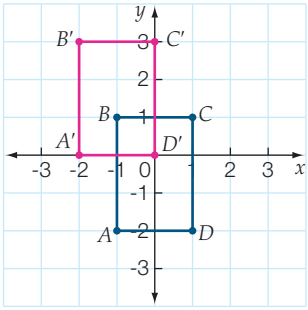


(f)



3 **(a)** $A(-1, -2)$, $B(-1, 1)$, $C(1, 1)$, $D(1, -2)$

(b)



(c) $A'(-2, 0)$, $B'(-2, 3)$, $C'(0, 3)$, $D'(0, 0)$

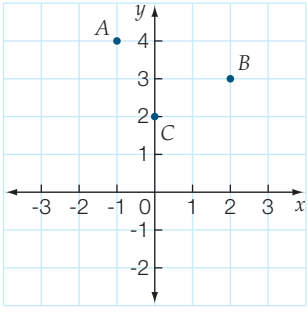
(d) Subtracting 1 from the x-coordinates of the original and adding 2 to the y-coordinates of the original will give the image coordinates.

4 B **5** D

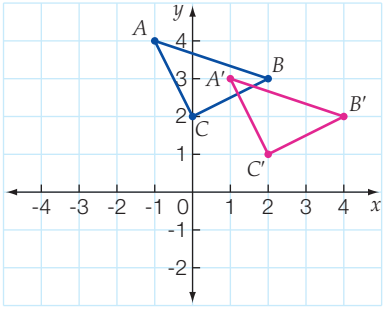
6 2 units left and 6 units up

7 [-3 -4] **8** [-10 -4]

9 **(a)**



(b)



(c) $A'(1, 3)$, $B'(4, 2)$, $C'(2, 1)$

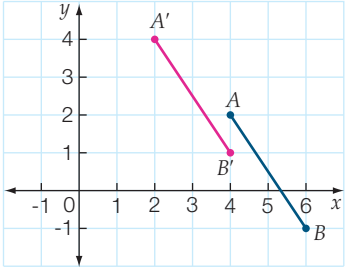
10 10 units left and 4 units up

11 4 units down

12 [0 9]

13 **(a)** $A'(2, 4)$, $B'(4, 1)$

(b)

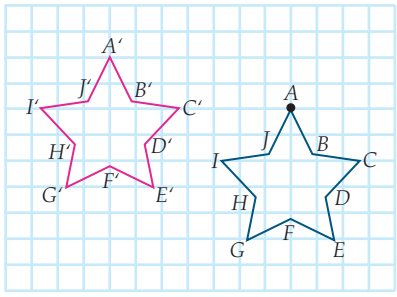


14 200 units right

15 **(a)** 7 units left and 2 units up

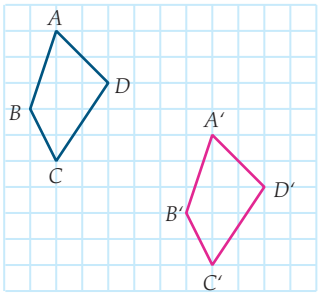
(b) 7 units right and 2 units down

(c)



16 Students' own answers.

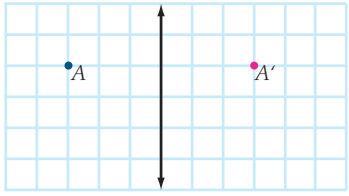
17



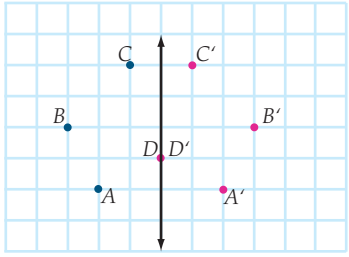
The reverse translation is 6 units left and 4 units up.

Exercise 10.2 (p. 402)

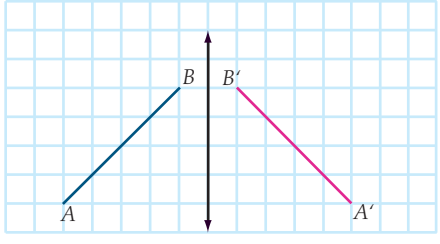
1 **(a)**



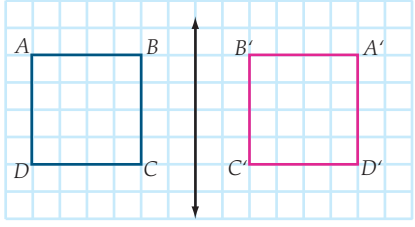
(b)



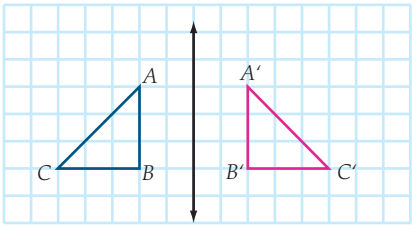
(c)



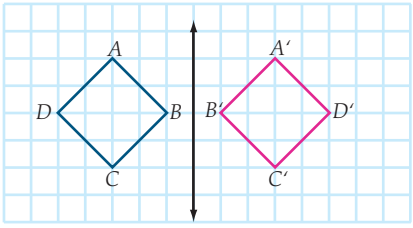
(d)



(e)

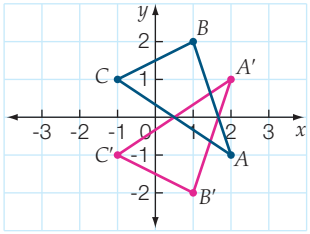


(f)



2 (a) (i) $A(2, -1)$, $B(1, 2)$, $C(-1, 1)$

(ii)

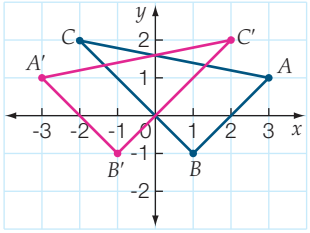


(iii) $A'(2, 1)$, $B'(1, -2)$, $C'(-1, -1)$

(iv) The x-coordinates of the image are the same as the x-coordinates of the original but the y-coordinates of the image are the negative of the y-coordinates of the original.

(b) (i) $A(3, 1)$, $B(1, -1)$, $C(-2, 2)$

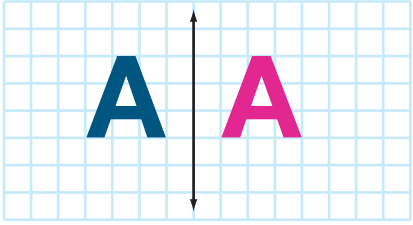
(ii)



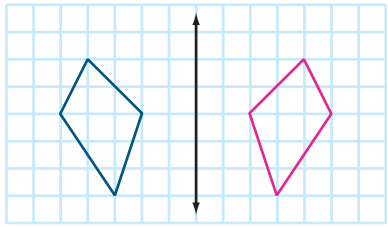
(iii) $A'(-3, 1)$, $B'(-1, -1)$, $C'(2, 2)$

(iv) The x-coordinates of the image are the negative of the x-coordinates of the original but the y-coordinates of the image are the same as the y-coordinates of the original.

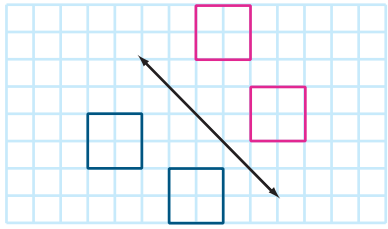
3 (a)



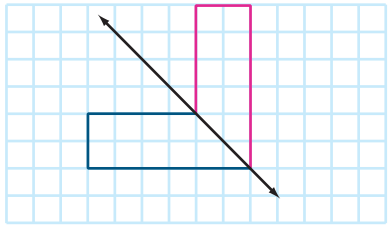
(b)



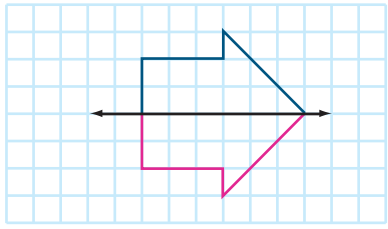
(c)



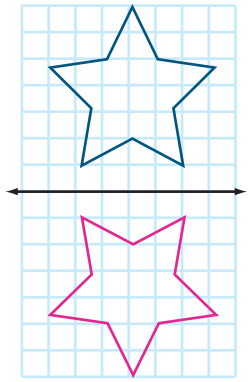
(d)



(e)

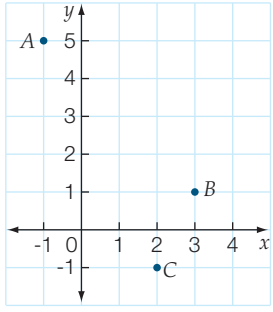


(f)

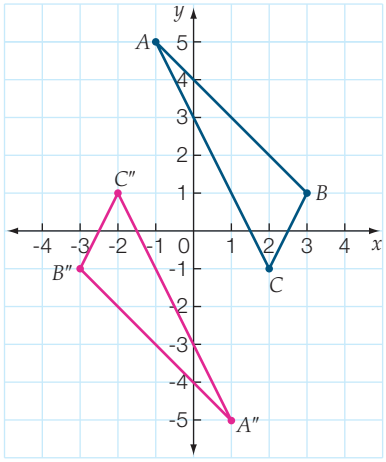


4 B

5 (a)

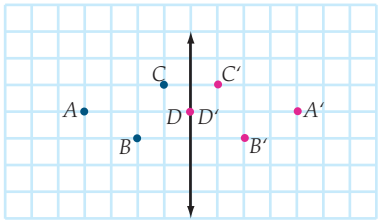


(b)

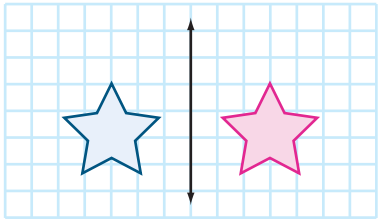


(c) $A''(1, -5)$, $B''(-3, -1)$, $C''(-2, 1)$

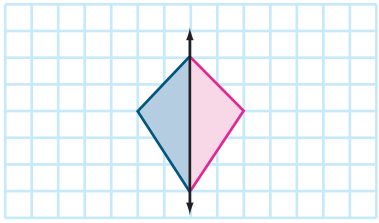
6 (a)



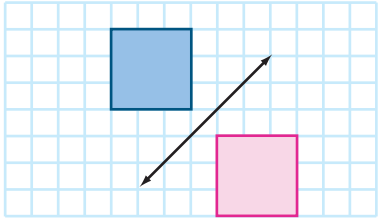
(b)



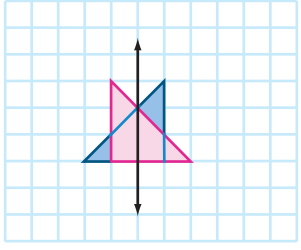
(c)



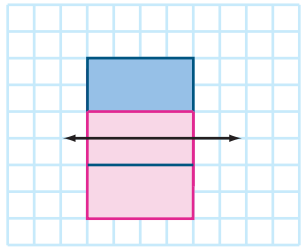
(d)



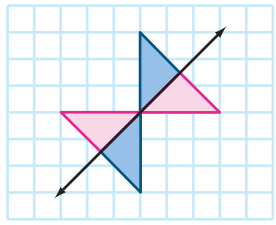
7 (a)



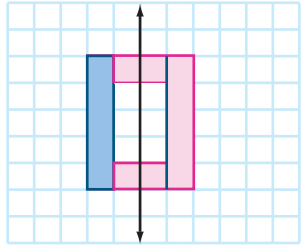
(b)



(c)

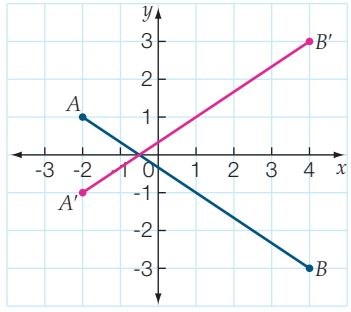


(d)



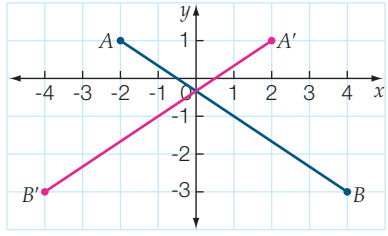
8 (a) $A'(-2, -1)$, $B'(4, 3)$

(b)



(c) $A'(2, 1)$, $B'(-4, -3)$

(d)



9 (a) 0, 1, 3, 8, 10, 11, 13, 18, 30, 31, 33, 38

(b) 0:00, 1:00, 3:00, 8:00, 10:00, 11:00

(c) 72

(d) 96

(e) No, in a 24-hour mode, there are only two more hours (13:00 and 18:00) than the 12-hour mode when the time is correctly reflected.

10 Students' own answers.

11 Students' own answers.

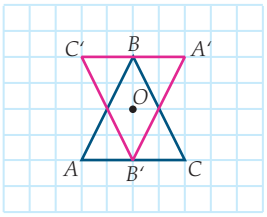
12 Students' own answers.

13 **(a), (b)** Students' own diagrams.

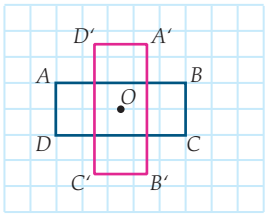
(c) The images found in parts **(a)** and **(b)** are the same.

Exercise 10.3 (p. 409)

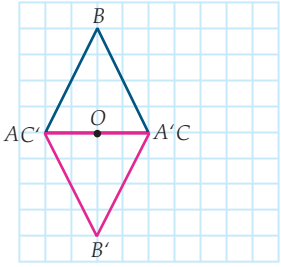
1 **(a)**



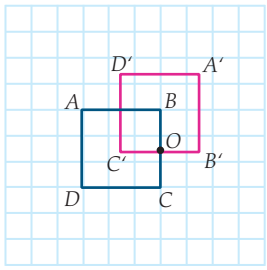
(b)



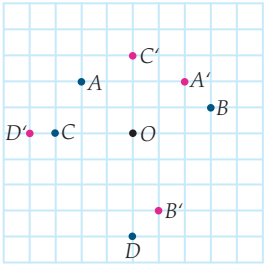
(c)



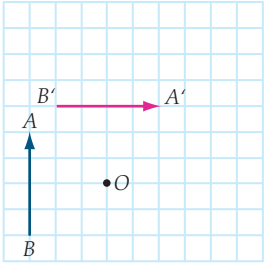
(d)



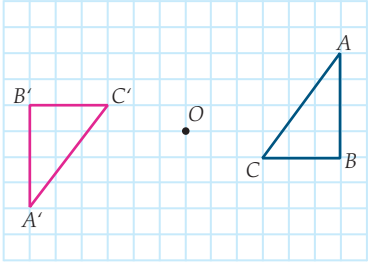
2 **(a)**



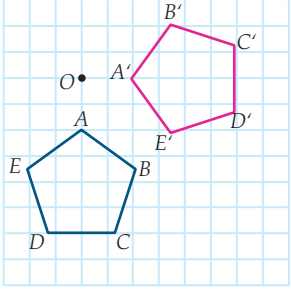
(b)



(c)

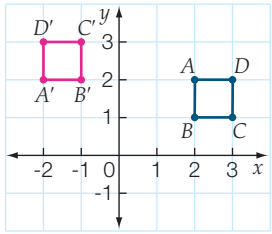


(d)



3 **(a)** $A(2, 2)$, $B(2, 1)$, $C(3, 1)$, $D(3, 2)$

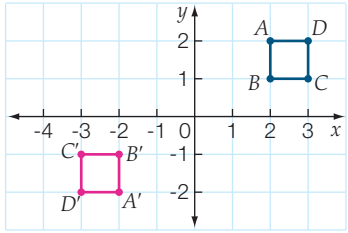
(b) **(i)**



(ii) $A'(-2, 2)$, $B'(-1, 2)$, $C'(-1, 3)$, $D'(-2, 3)$

(iii) The x-coordinates of the image are the negative of the y-coordinates of the original vertices and the y-coordinates of the image are the x-coordinates of the original vertices.

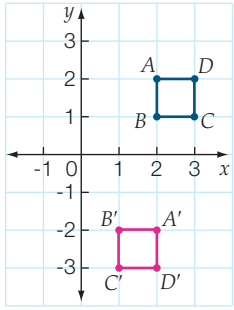
(c) **(i)**



(ii) $A'(-2, -2)$, $B'(-2, -1)$, $C'(-3, -1)$, $D'(-3, -2)$

(iii) All the coordinates of the image vertices are the negative of the original coordinates of the vertices.

(d) **(i)**



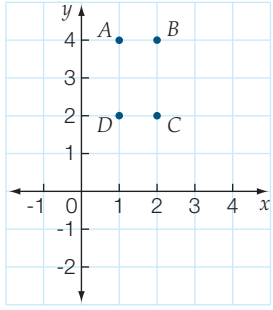
(ii) $A'(2, -2)$, $B'(1, -2)$, $C'(1, -3)$, $D'(2, -3)$

(iii) The x-coordinates of the image vertices are the y-coordinates of the original vertices and the y-coordinates of the image vertices are the negative of the x-coordinates of the original vertices.

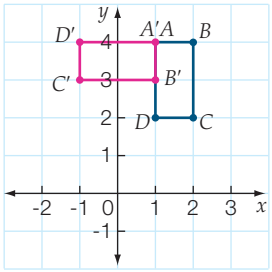
4 C

5 B

6 **(a)**



(b)



(c) $A'(1, 4)$, $B'(1, 3)$, $C'(-1, 3)$, $D'(-1, 4)$

7 240°

8 (a) Rotated 180° about O.

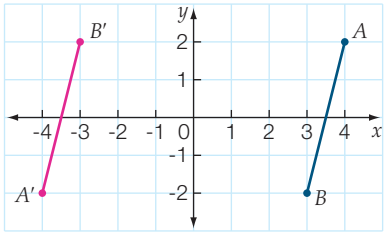
(b) Rotated 90° in an anticlockwise direction or 270° in a clockwise direction about O.

(c) Rotated 90° in an anticlockwise direction or 270° in a clockwise direction about O.

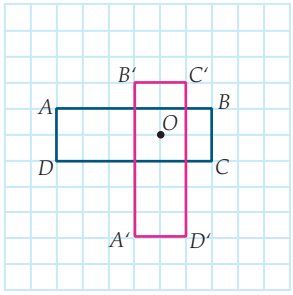
(d) Rotated 180° about O.

9 (a) $A'(-4, -2)$, $B'(-3, 2)$

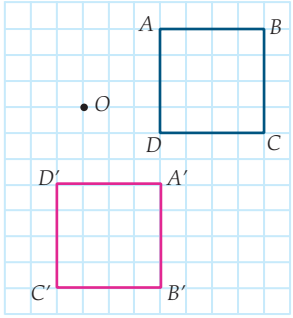
(b)



10 (a)



(b)



11 (a) H, I, N, O, S, X, Z

(b) All of these letters are perfectly symmetrical.
H, I, O, X

12 (a) 120° (b) 180° (c) 90°
(d) 72° (e) 22.5° (f) 36°

13 This is a reflection, because the orientation of the red square's vertices is completed in a manner that indicates a direct reflection. If it was rotated, the order of the vertices would also be rotated.

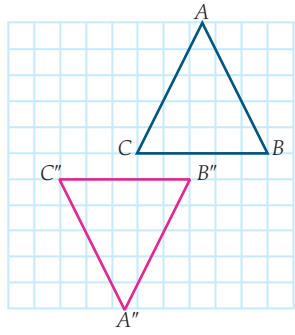
14 Students' own answers.

15 Disagree, as the order of the vertices of a figure is shifting in the same direction that the rotation is taking place. In a reflection, the vertices stay in a similar orientation, each an equivalent distance from the line of reflection.

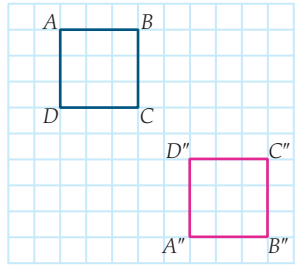
16 (e) The images in parts (b) and (d) are the same as the original polygons.

Exercise 10.4 (p. 415)

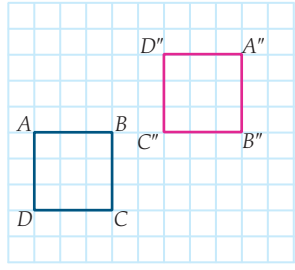
1 (a)



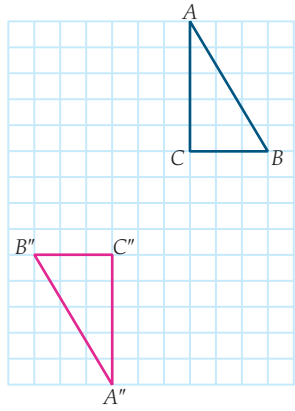
(b)



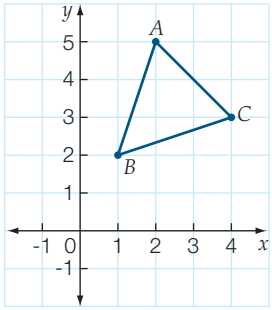
(c)



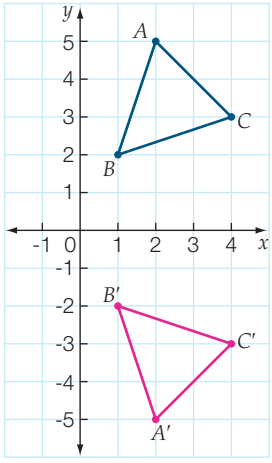
(d)



2 (a) $A(2, 5)$, $B(1, 2)$, $C(4, 3)$

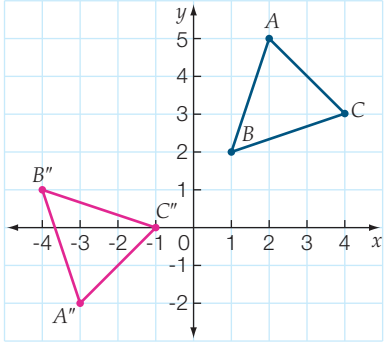


(b)



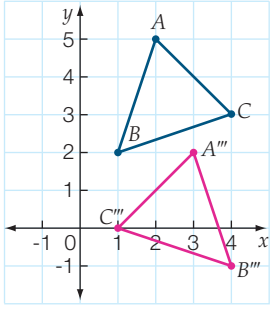
(c) $A'(2, -5)$, $B'(1, -2)$, $C'(4, -3)$

(d)



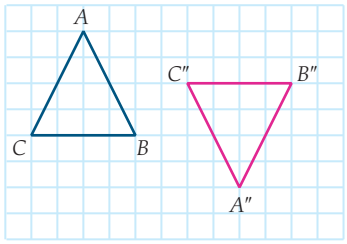
(e) $A''(-3, -2)$, $B''(-4, 1)$, $C''(-1, 0)$

(f)

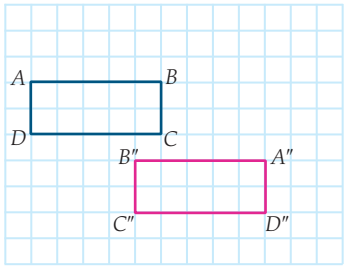


(g) $A'''(3, 2)$, $B'''(4, -1)$, $C'''(1, 0)$

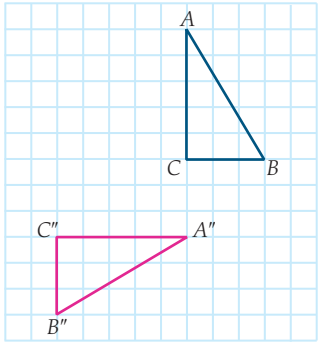
3 (a)



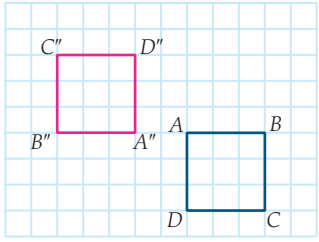
(b)



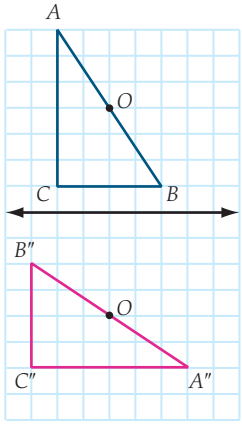
(c)



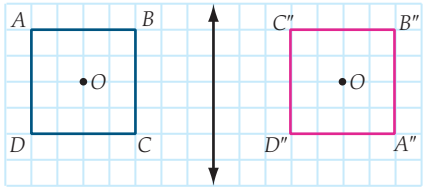
(d)



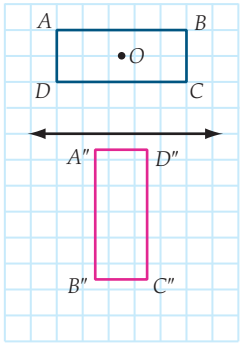
(e)



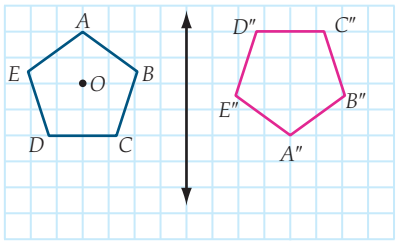
(f)



(g)



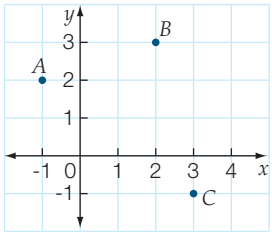
(h)



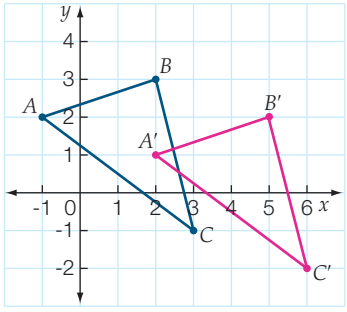
4 B

5 D

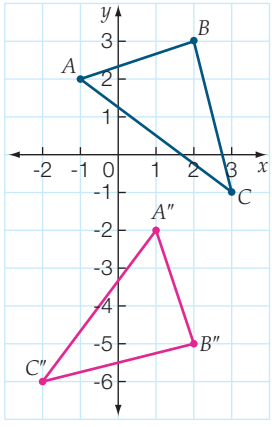
6 (a)



(b)



(c)



(d) $A''(1, -2)$, $B''(2, -5)$, $C''(-2, -6)$

7 (a) Reflect along the line $C''B''$ and translate 3 units right and 1 unit up.

(b) Reflect along the line $D''C''$ and translate 2 units up and 5 units left.

(c) Rotate 90° in an anticlockwise direction about B'' and translate 5 units left.

(d) Rotate 180° about C'' in either direction and translate 4 units up and 3 units right.

8 (a) Reflected along the axis of reflection and then translated 4 units right and 2 units down.

(b) Translated 8 units down and then reflected along the vertical line of reflection.

(c) Rotated 180° about point A and then translated 5 units right and 3 units up.

(d) Reflected along the axis of reflection and then rotated 90° in an anticlockwise direction about point B'.

(e) Rotated 45° in a clockwise direction about point C and then translated 8 units down and 3 units right.

(f) Rotated 180° about point C and reflected along the axis of reflection.

9 (a) Translate 4 units left and 2 units up and reflect along the axis of reflection.

(b) Reflect along the axis of reflection and translate 8 units up.

(c) Translate 3 units down and 5 units left and rotate 180° about point A'.

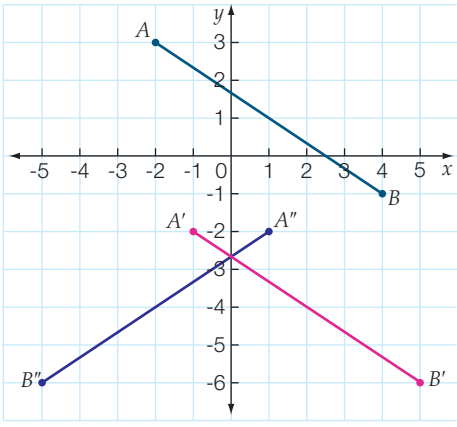
(d) Rotate 90° about point B'' in a clockwise direction and reflect in the axis of reflection.

(e) Translate 8 units up and 3 units left and rotate 45° in an anticlockwise direction about point C'.

(f) Reflect along the axis of reflection and rotate 180° about point C'.

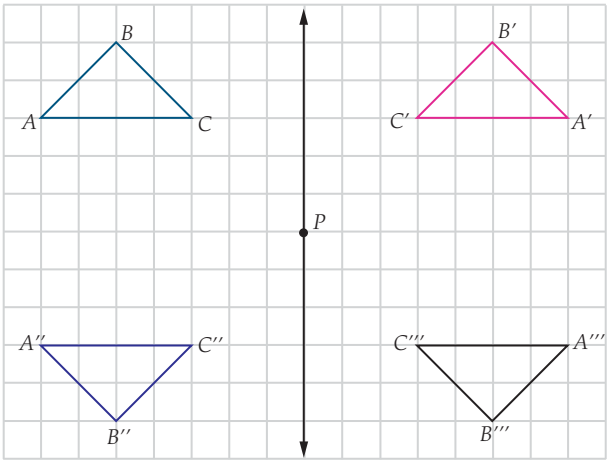
10 (a) $A''(1, -2)$, $B''(-5, -6)$

(b)



11 B1—Rotate 90° clockwise and then rotate 90° clockwise.
B2—Reflect vertically.

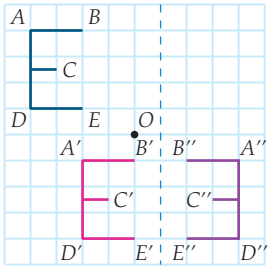
12



Rotation 180° about point *P*.

13 Students' own answers.

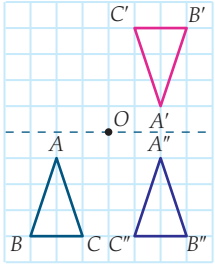
14 (a)



(b) Rotation 180° clockwise about *O*.

(c) Rotation 180° clockwise about *O*.

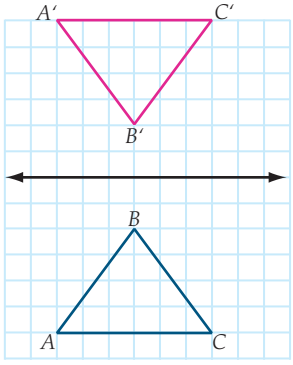
15 (a)



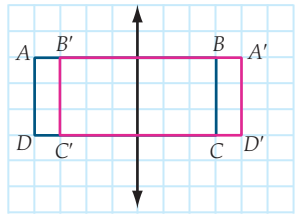
(b) A reflection in a vertical line of reflection passing through *O*.

Half-time 10 (p. 420)

1 (a)



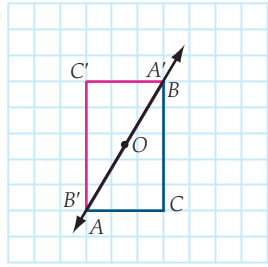
(b)



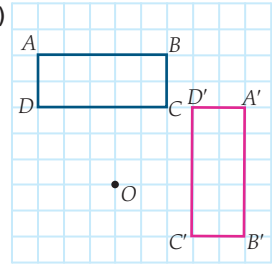
2 (a) Translated 5 units right and 4 units down.

(b) Translated 3 units left and 2 units up.

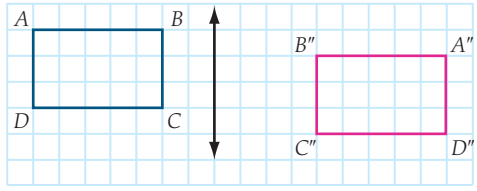
3 (a)



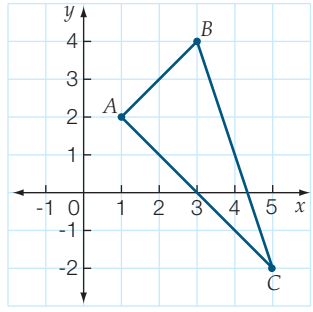
(b)



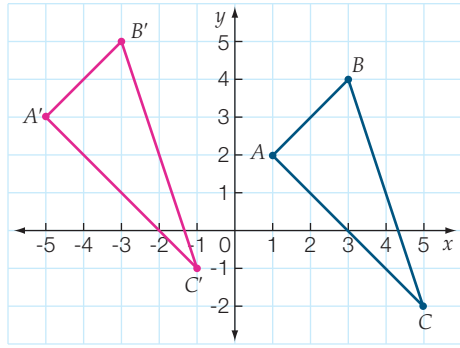
4



5 (a)

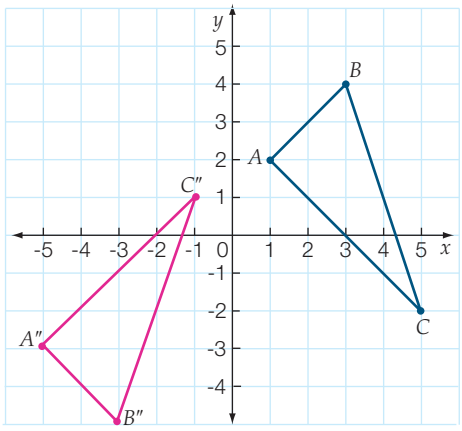


(b)



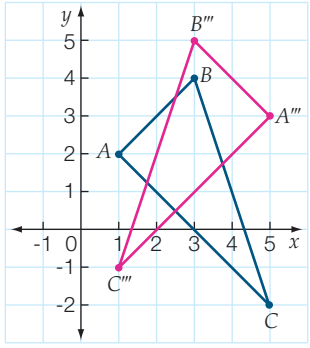
(c) *A*′(-5, 3), *B*′(-3, 5), *C*′(-1, -1)

(d)



(e) $A''(-5, -3)$, $B''(-3, -5)$, $C''(-1, 1)$

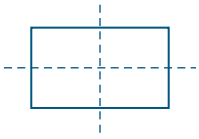
(f)



(g) $A'''(5, 3)$, $B'''(3, 5)$, $C'''(1, -1)$

Exercise 10.5 (p. 423)

1 (a) (i) Yes



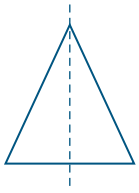
(ii) Order = 2

(b) (i) Yes



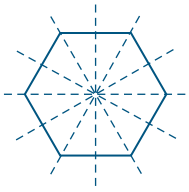
(ii) Order = 4

(c) (i) Yes



(ii) Order = 1

(d) (i) Yes



(ii) Order = 6

(e) (i) Yes



(ii) Order = 1

(f) No



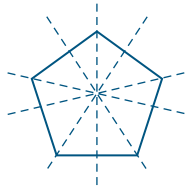
(g) No

(h) (i) Yes



(ii) Order = 2

(i) (i) Yes



(ii) Order = 5

2 (a) (i) Yes (ii) 2 (iii) 180°

(b) (i) Yes (ii) 4 (iii) 90°

(c) (i) No Does not have rotational symmetry.

(d) (i) Yes (ii) 6 (iii) 60°

(e) (i) No Does not have rotational symmetry.

(f) (i) No Does not have rotational symmetry.

(g) (i) No Does not have rotational symmetry.

(h) (i) Yes (ii) 2 (iii) 180°

(i) (i) Yes (ii) 5 (iii) 72°

3 (a) 4 (b) 2 (c) 1

4 D **5** C **6** A

7 (a) a, b, d, e, f, g, h, j, k, m, n, p, q, r, s, t, u, y, z

(b) c, i, v, w,

(c) l, o, x

8 (a) F, G, J, K, L, N, P, Q, R, S, Z

(b) A, B, C, D, E, M, T, U, V, W, Y

(c) H, I, O, X

9 Yes —H, I, N, O, S, X, Z

10 (a) Order 1 reflectional symmetry and no rotational symmetry

(b) Order 1 reflectional symmetry and no rotational symmetry

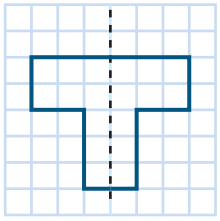
(c) Order 5 reflectional symmetry and order 5 rotational symmetry

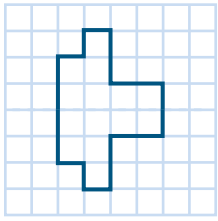
(d) Order 5 reflectional symmetry and order 5 rotational symmetry

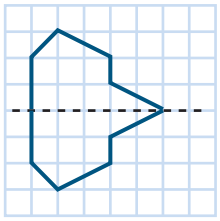
(e) Order 1 reflectional symmetry and no rotational symmetry

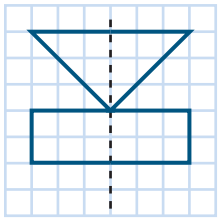
(f) Order 1 reflectional symmetry and no rotational symmetry

11 (a) No (b) Yes (c) No (d) No (e) Yes (f) Yes

12 (a) 

(b) 

(c) 

(d) 

13 rectangle and rhombus

14 equilateral triangle

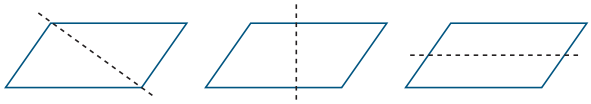
15 O is the axis of symmetry. Other words TOOT, MUM, MOM, HUH.

16 (a) Rotation: 1001, 1111, 1691, 1881, 1961

(b) Reflection: 1001, 1111, 1331, 1881

(c) Rotation and reflection: 1001, 1111, 1881

17 A parallelogram has reflectional symmetry if it is a rhombus, a rectangle or a square. Otherwise, it has no reflectional symmetry.

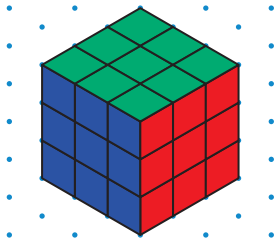


Exercise 10.6 (p. 427)

1 Check your shapes carefully against the ones in the question.

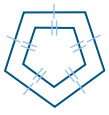
2 (a) 4 (b) 12 (c) 12 (d) 9 (e) 8 (f) 10

3 B 4 A

5 

6 (a) 8 (b) 12 (c) 9 (d) 10 (e) 21 (f) 12

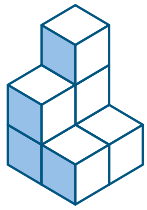
7 D

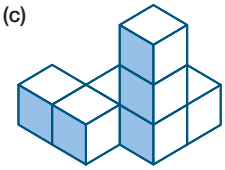
8 (a) 

(b) 

(c) No, each side is a rectangle with the same length and height.

9 (a) 

(b) 

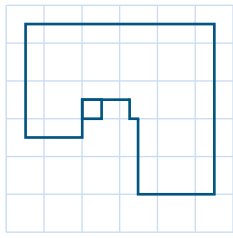
(c) 

10 (a) It is not a rectangle, as one side has 6 blocks and another has 4 blocks. It is not possible to make this shape.

(b) Students' own answers.

(c) Students' own answers.

11 Top view

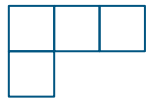


12 Students' own answers.

13 Students' own answers.

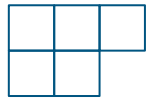
14 Students' own answers.

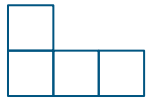
Exercise 10.7 (p. 432)

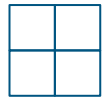
1 (a) (i) 

(ii) 

(iii) 

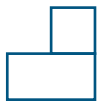
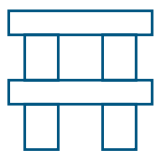
(b) (i) 

(ii) 

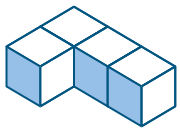
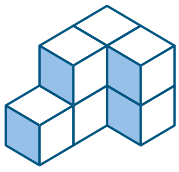
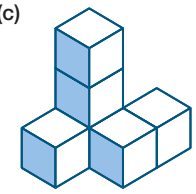
(iii) 

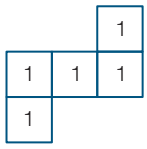
2 D

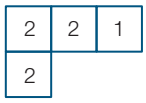
3

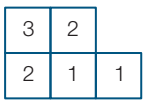
	Front	Side
(a)		
(b)		
(c)		
(d)		

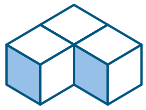
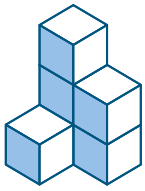
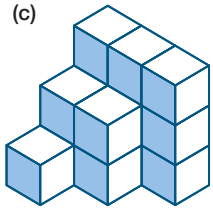
4 (a) A (b) B

5 (a)  (b)  (c) 

6 (a) 

(b) 

(c) 

7 (a)  (b)  (c) 

8 (a)–(b) Students' own drawings.

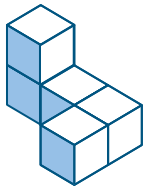
(c) 1; 3; 3 or 5; 3 or 5; 6 or 7

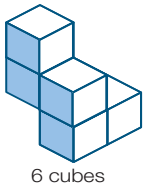
(d) Depending on the way they are shaded, the orientation of the cubes can appear differently. Some cubes may look like concave spaces. The answer to (c) gives the alternative numbers of cubes.

(e) Answers may vary.

9 Possible answers include:

 4 cubes

 5 cubes

 6 cubes

10 Students' own answers.

Challenge 10 (p. 435)

1 (a) Rotation through 180° in either direction.

(b) Reflection in the y-axis followed by reflection in the x-axis, or reflection in the x-axis followed by reflection in the y-axis.

(c) B'

2 2.30

3 One copy is fixed at C and rotated anticlockwise through 90°. Another copy is fixed at D and rotated clockwise through 90°. The third copy is translated upwards through a distance equal to the length of CD.

4 B

5 'A BOILED EGG IS HARD TO BEAT'. The whole message needs to be reflected in a line.

6 (a) Translate 8 cm to the right.

(b) Rotate 90° anticlockwise about the bottom corner, then translate 11 cm to the right.

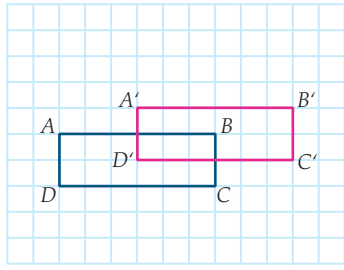
(c) Reflect in the 4 cm side, then translate 4 cm to the right.

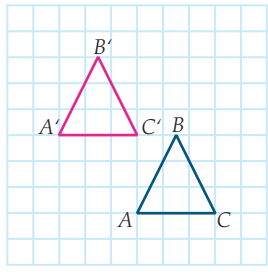
(d) Rotate 180° about the vertex located between the 4 cm and 5 cm sides.

Chapter review 10 (p. 436)

1 (a) Translated 4 units right and 4 units down.

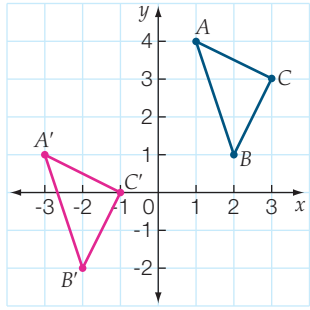
(b) Translated 4 units left and 3 units up.

2 (a) 

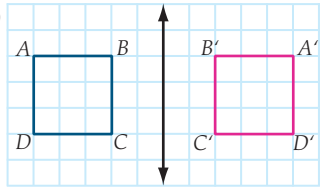
(b) 

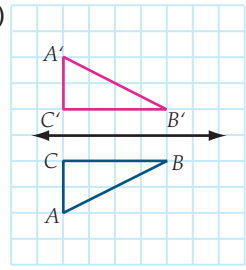
3 B

4 (a) $A(1, 4)$, $B(2, 1)$, $C(3, 3)$

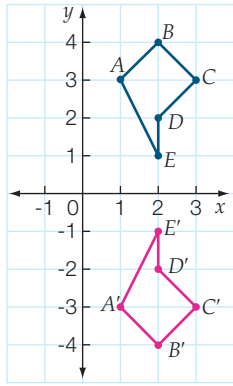
(b) 

(c) $A'(-3, 1)$, $B'(-2, -2)$, $C'(-1, 0)$

5 (a) 

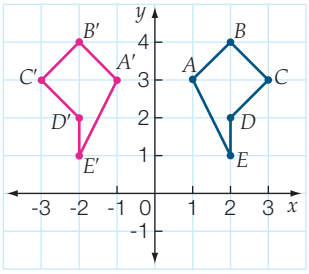
(b) 

6 (a) $A(1, 3)$, $B(2, 4)$, $C(3, 3)$, $D(2, 2)$, $E(2, 1)$

(b) 

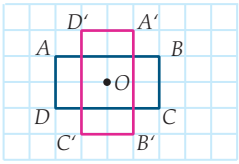
(c) $A'(1, -3)$, $B'(2, -4)$, $C'(3, -3)$, $D'(2, -2)$, $E'(2, -1)$

(d)

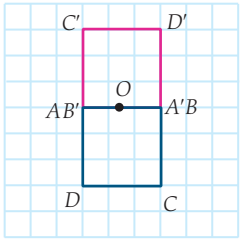


(e) $A'(-1, 3)$, $B'(-2, 4)$, $C'(-3, 3)$, $D'(-2, 2)$, $E'(-2, 1)$

7 (a)



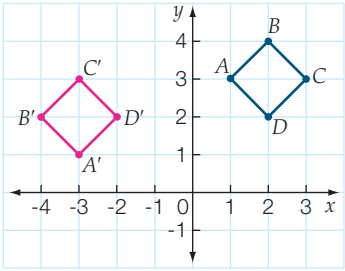
(b)



8 D

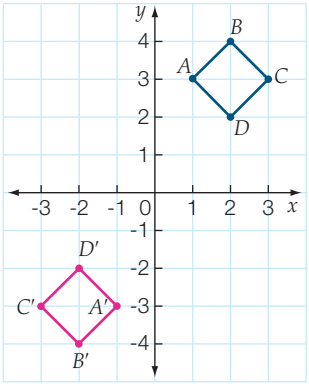
9 (a) $A(1, 3)$, $B(2, 4)$, $C(3, 3)$, $D(2, 2)$

(b)



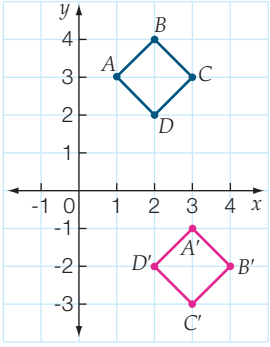
(c) $A'(-3, 1)$, $B'(-4, 2)$, $C'(-3, 3)$, $D'(-2, 2)$

(d)



(e) $A'(-1, -3)$, $B'(-2, -4)$, $C'(-3, -3)$, $D'(-2, -2)$

(f)



(g) $A'(3, -1)$, $B'(4, -2)$, $C'(3, -3)$, $D'(2, -2)$

10 C

11 C

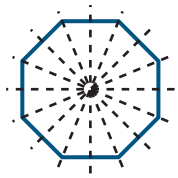
12 (a)



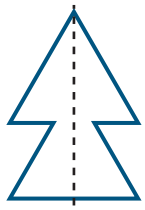
(b)



(c)



(d)



(e)



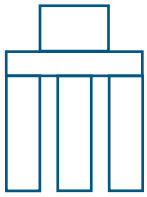
(f)



13 (c) and (e) have rotational symmetry.

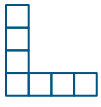
14 Check your drawing carefully against the question.

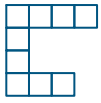
15

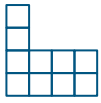
	Front elevation	Side elevation
(a)		
(b)		

16 Check your drawing carefully against the question.

17

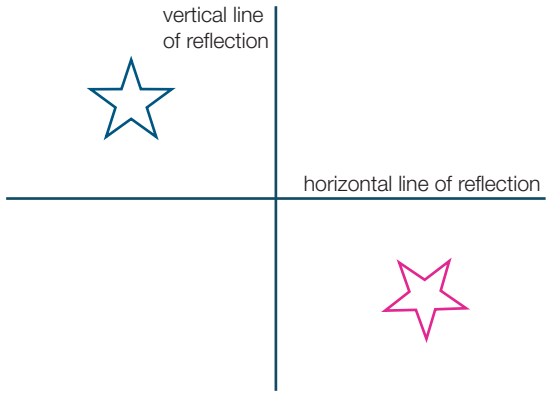

front


top


side

18 4 units right

19 (a)

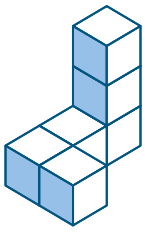


(b) Yes

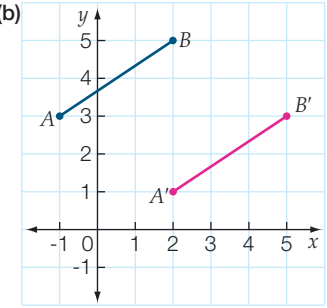
20 Rotating either 180° in a clockwise or anticlockwise direction about the intersection point of the axes.

21 None

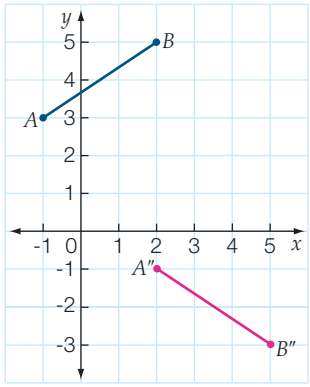
22



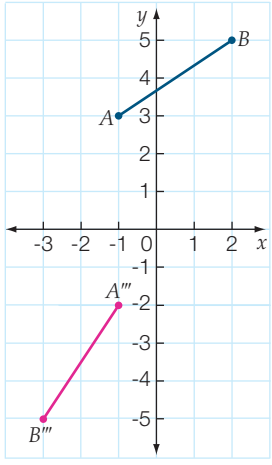
23 (a)–(b)



(c)

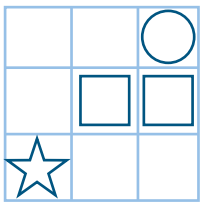


(d)

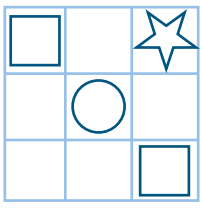


(e) $A'''(-1, -2)$, $B'''(-3, -5)$

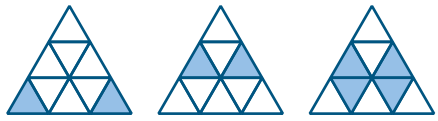
24 (a)



(b)



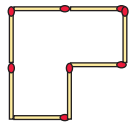
25 (a)



(b)

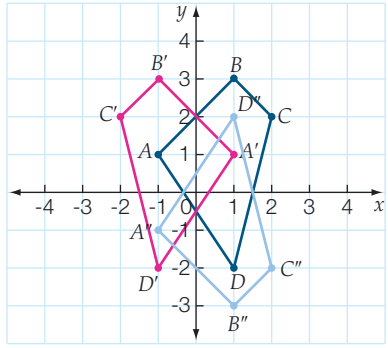


26 one match



27 19

28



Yes, Matt is correct. These two transformations could be replaced by a reflection in the x-axis.

NAPLAN practice 10

1 B **2** B **3** C **4** A

5 D **6** A **7** A

Mixed review E (p. 442)

1 (a) 12 (b) 64 (c) 25

2 (a) 7 (b) 2700 (c) 99 000

3 (a) 17.515 (b) 0.0072 (c) 1.99

4 (a)

m	4	0	11	25	15
n	12	0	33	75	45

(b)

s	18	100	12	22	312
t	6	88	0	10	300

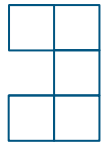
5 (a) 220 cm^2 (b) 60 cm^2

6 (a) $\frac{1}{6}$ (b) $\frac{1}{26}$ (c) $\frac{1}{4}$

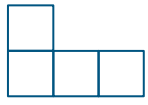
7 (a) (i)



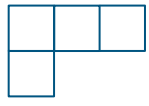
(ii)



(b) (i)



(ii)



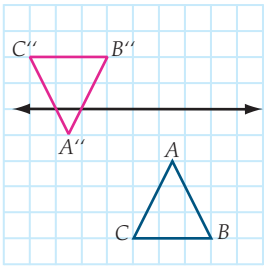
8 (a) $x = 3$ (b) $x = 24$ (c) $x = 3$

9 (a) 145° (b) 53° (c) 90°

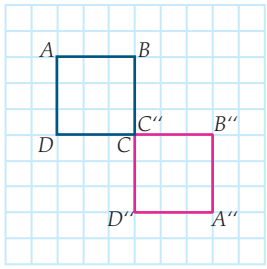
10

Height (cm)	Frequency
145–149	1
150–154	2
155–159	5
160–164	7
165–169	2
170–174	3
Total	20

11 (a)



(b)



12 (a) 1 **(b)** 4 **(c)** 2

13 mean = 30.6, median = 31.5, mode = 34, range = 39, outlier = 6

14 1 unit right and 1 unit down

15 (a) $24 = 2 \times 2 \times 2 \times 3$ **(b)** $30 = 2 \times 3 \times 5$

(c) 6 **(d)** 120

16 (a) 60° **(b)** 126°, 54°, 54°

17 O, X

18 (1, 1)

19 (a) Students' own answers.

(b) Work out what the numbers need to total using the mean, and then set out some boxes, putting 6 in place to make it the median. The other numbers can then be filled in around the set number(s).

(c) It is easier to use an odd number of values because the median is then only one single value.

20 (a) There are eight possible results: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT.

(b) You win with half of the results, and those wins are more than you lose on the other results. So, the game is not fair, but it is not fair to the operator!

Glossary and index

acute angle an angle greater than 0 degrees but less than 90 degrees (p. 294)

acute-angled triangle a triangle with all angles less than 90 degrees (p. 316)

alternate angles angles found between the two lines cut by a transversal, but on opposite sides of the transversal (p. 305)

angle the space between two lines that start at the same point (p. 288)

area the amount of space contained by a plane shape (p. 232)

arms (of an angle) two straight lines that meet at a vertex. The angle is the amount of turn measured between the two arms (p. 288)

associative law the order in which numbers are added or multiplied is not important; e.g. $33 \times 5 \times 2$ could be calculated as 165×2, or 33×10. Similarly, $29 + 13 + 17$ could be calculated as $42 + 17$, or $29 + 30$. This can be written in algebra as $a + (b + c) = (a + b) + c$ (p. 3)

asymmetrical having no symmetry (p. 422)

average the word used to collectively describe any of mean, median and mode (p. 346)

axes the number lines to which values of a variable are referenced. On the number plane, the x-axis runs in the horizontal direction and the y-axis runs in the vertical direction (p. 200)

axis of symmetry a line that divides a figure into two identical but reversed parts (p. 422)

backtracking a method of solving equations that uses a flowchart (p. 268)

balance method making equivalent equations by performing the same operation on both sides, keeping them balanced (p. 275)

bar graph a graph in which equally spaced bars show the frequency of data in different categories (p. 353)

base 1. the number that is raised to an index or power (p. 6); 2. the bottom of a shape (p. 239)

bimodal a data set in which exactly two of the data values have the highest frequency (p. 346)

breadth the distance from one side of a shape to the other, through the centre (p. 228)

cancelling the process of dividing the numbers in a fraction by a common factor to produce a simpler fraction (p. 78)

Cartesian plane (number plane) a grid made up of two axes that allows any point in the plane to be described by its coordinates (p. 200)

categorical data data that is collected according to a description; e.g. eye colour (p. 340)

centimetre one-hundredth of a metre (p. 223)

centre of rotation the point about which a figure is rotated (p. 407)

chance another word for probability (p. 384)

checking by substitution the process of checking an answer by substituting the solution into the equation (p. 268)

class interval the range of each class of data; used when converting large amounts of raw data into grouped data (p. 341)

coefficient a number written before a pronumeral; e.g. in $9x$, the coefficient of x is 9 (p. 175)

co-interior (allied) angles angles found between the two lines cut by a transversal and on the same side of the transversal. Co-interior angles are supplementary (p. 305)

column graph (or divided bar graph) a graph where the frequency of collected data is displayed in columns (pp. 353, 361)

common factor a number that is a factor of all numbers under consideration (p. 34)

common multiple a number that is a multiple of all numbers under consideration (p. 34)

commutative law the order in which two numbers are added or multiplied can be swapped without changing the result; e.g. $4 + 5 = 9$ and $5 + 4 = 9$, or $2 \times 3 = 6$ and $3 \times 2 = 6$. This can be written in algebra as $a + b = b + a$, and $a \times b$ $(ab) = b \times a$ (ba) (p. 3)

complementary angles two angles that add up to 90 degrees (p. 295)

complementary events an event may or may not happen (p. 384)

complementary probabilities add to 1 (p. 384)

composite number a number that has at least two factors other than 1 (p. 39)

composite shape a shape made by combining other shapes, such as rectangles and triangles (p. 242)

concave having a surface or boundary that curves inwards (p. 313)

constant a number by itself; e.g. 2, 7, 9.5 (p. 175)

continuous variable data that is collected by measuring; e.g. height (p. 340)

convex having a surface or boundary that curves outwards (p. 313)

coordinates a pair of numbers (x, y) that describes the position of a point on a number plane (p. 200)

coprime two or more numbers that have only 1 as common factors (p. 39)

corresponding angles angles found either both above or both below the two lines cut by a transversal and on the same side of the transversal (p. 305)

cube root a number that is cubed to produce a given number (p. 6)

data raw information (p. 340)

decimal places the number of digits after the decimal point of a number (p. 118)

decimal point a point that separates the whole number part of a number from the decimal part (p. 118)

define to identify the variable and represent it with a pronumeral

degree the unit that is used to measure the size of angles; a full turn is equal to 360 degrees (p. 288)

denominator the number below the line in a common fraction (p. 72)

deposit placing an amount of money into a bank account (p. 46)

digit any numeral from 0 to 9 (p. 118)

discrete variable data that is counted; e.g. the number of cans of soup sold by a supermarket over a period of time (p. 340)

distributive law a large number can be multiplied by splitting it into two smaller numbers, which can be multiplied individually, and the products added; e.g. $8 \times 17 = 8 \times (10 + 7) = 8 \times 10 + 8 \times 7 = 80 + 56 = 136$. This can be written in algebra as $a(b \pm c) = ab \pm ac$ (p. 3)

divided bar graph *see* column graph

dividend in a division calculation, the number being divided; e.g. for $169 \div 8 = 21$ rem 1, the dividend is 169 (p. 11)

divisibility tests can be used to find the factors of a particular number (p. 34)

divisible able to be divided by a number a whole number of times (there is no remainder) (p. 34)

divisor in a division calculation, the number being divided by; e.g. for $169 \div 8 = 21$ rem 1, the divisor is 8 (p. 11)

dot plot a graph showing the frequency of each piece of data as a dot (p. 353)

elevations 2D drawings of a structure from either the front, rear or side (p. 432)

equation two expressions either side of an equals sign (pp. 175, 263)

equilateral triangle a triangle with all sides equal (p. 316)

equivalent equations equations that have the same solution (p. 275)

equivalent fractions fractions that represent the same quantity but are written using different denominators; e.g. $\frac{1}{2} = \frac{2}{4}$ (p. 78)

equivalent ratios ratios that compare amounts in the same proportion, but have different numbers. One ratio is a multiple of the other (p. 153)

estimate a sensible guess that approximates an answer or value (p. 16)

evaluate to calculate the value of a formula or expression (p. 184)

expanded form a base raised to an index written as a multiplication of the base times itself (p. 6)

exponent *see* index (pp. 6, 34)

expression a collection of terms added together or subtracted (p. 175)

factor a number that divides exactly into another number, with no remainder (p. 11)

false number sentence a sentence written with numbers that is not true (p. 259)

flowchart shows the order of operations used to build an expression in algebra (p. 267)

formula a rule that is expressed with pronumerals and an equals sign (p. 184)

fraction a number that is not a whole number, and is represented using a numerator and a denominator (p. 72)

frequency the number of times a piece of data occurs in a data set (p. 340)

frequency polygon a graph made by joining the midpoints of the tops of the columns of a frequency histogram (p. 353)

frequency table a table of data showing a number of observations (p. 340)

guess, check and improve solving equations by choosing a number, substituting it into the equation and repeating this process until the LHS equals the RHS (p. 263)

height the measurement from the base to the top of a shape or a prism, perpendicular to the base (p. 239)

Highest Common Factor (HCF) the largest number that occurs in each of the sets of factors of a given group of numbers. Also known as the Greatest Common Divisor (GCD) (p. 34)

histogram a column graph where the width of the bars represents the various classes and the height of the bars represents their relative frequencies (p. 353)

image a figure produced by a transformation of the original (p. 397)

improper fraction a fraction with a numerator greater than or equal to the denominator (p. 72)

index (power) (*plural* **indices**) a number that tells us how many times the base number is multiplied by itself (p. 6)

index form a short way of writing a number multiplied by itself several times using a base raised to an index (p. 6)

integers all of the positive and negative whole numbers, along with zero (p. 46)

intersect the point at which two or more lines meet or cross (p. 305)

interval part of a line that has both a beginning and an end (p. 295)

inverse flipping the positions of the numerator and denominator gives the inverse of a fraction; e.g. the inverse of $\frac{2}{3}$ is $\frac{3}{2}$, the inverse of 4 is $\frac{1}{4}$ (p. 103)

inverse (opposite) **operation** a mathematical operation that undoes the effect of another mathematical operation; addition and subtraction, multiplication and division are the most common pairs of opposite operations (p. 267)

irregular an irregular shape has varying side lengths and angle sizes (pp. 313, 324)

isometric a triangular grid that allows 3D solids to be drawn on a 2D plane (p. 427)

isosceles triangle a triangle with two equal sides (p. 316)

kilometre 1000 metres (p. 223)

kite a quadrilateral with two pairs of adjacent sides equal in length (p. 324)

length distance from end to end (p. 223)

like terms algebraic terms that have exactly the same pronumeral part; e.g. $3ab$ and $5ba$ are like terms, but $2a$ and $3a^2$ are not (p. 195)

linear graph a graph in which a straight line can be drawn through all the points (p. 205)

line graph a straight line representing data, often collected over a period of time (pp. 209, 370)

line of reflection a line drawn through an object or image that divides it into two identical parts (p. 401)

line segment *see* interval

loss money lost by an individual or business selling goods for less than the amount for which they were purchased (p. 46)

Lowest Common Denominator (LCD) the lowest common multiple of a set of denominators (p. 84)

Lowest Common Multiple (LCM) the smallest number that occurs in each set of multiples of a given group of numbers (p. 34)

mean the average obtained by adding together all of the scores obtained and dividing by the number of scores (p. 346)

measure of centre one of three ways (mean, median, mode) used to explain what is happening around the middle of a data set (p. 346)

measure of location *see* measure of centre

measure of spread the difference between the highest and lowest value in a data set (p. 346)

median the middle value of a set of scores (p. 346)

metre the standard unit of length (p. 223)

metric the name for the system of measurement used in Australia (p. 223)

millimetre one-thousandth of a metre (p. 223)

mixed number a number that is made up of a whole number part and a proper fraction (p. 72)

mode the most frequently occurring score in a set of scores (p. 346)

multiple a number that may be divided exactly by a given number (with no remainder) is a multiple of that given number (p. 34)

multipliers the numbers being multiplied (p. 11)

negative used to describe numbers to the left of zero on the number line; also used to describe losses or decreases (p. 46)

numerator the number above the line in a common fraction (p. 72)

obtuse angle an angle greater than 90 degrees but less than 180 degrees (p. 294)

obtuse-angled triangle a triangle with one obtuse angle (p. 316)

order of operations a rule that states that when simplifying expressions you must do brackets first, then multiplication and division in the order they appear, then addition and subtraction in the order they appear (p. 21)

order of reflectional symmetry the number of axes of symmetry that can be drawn through a shape (p. 422)

order of rotational symmetry the number of times in a complete 360 degrees rotation that an identical image of the original would be produced (p. 422)

ordered pair the coordinates of any point on the number plane written with the x-coordinate first, followed by the y-coordinate; i.e. (x, y) (p. 200)

origin the point (0, 0) on the Cartesian plane (p. 200)

original the first version (p. 397)

outcomes the results of a probability experiment (p. 384)

outlier a piece of data that is quite different (much higher or much lower) from the rest of the data in a data set (p. 346)

parallel lines lines that lie in the same plane and are always the same distance apart (p. 305)

parallelogram a quadrilateral with both pairs of opposite sides parallel (p. 239)

part : part ratio a ratio that compares two or more parts of a whole (p. 153)

part : whole ratio a ratio that compares a part or parts to the whole (p. 153)

per cent a number expressed as an amount out of 100 (p. 144)

perfect cube found by multiplying a whole number by itself three times; e.g. $4 \times 4 \times 4$ (p. 6)

perfect square a whole number that is obtained by multiplying a number by itself; e.g. 25 is the 'square' of 5, as $5 \times 5 = 25$ (p. 6)

perimeter the distance along the boundary of a shape (p. 228)

perpendicular at right angles to (p. 239)

perpendicular distance is measured by a line that makes a right angle with the line of reflection (p. 401)

perpendicular lines lines that are at right angles to each other (p. 305)

place value the value of a digit in a number is given by its position (or place) in the number. In the decimal number system, the place value of a digit is 10 times the place value of the digit to the right (p. 118)

plan views technical drawings of an object from either the top, front or side (p. 432)

plane a flat two-dimensional surface (p. 305)

plane shapes enclosed figures drawn on a plane (p. 313)

point graph graph showing information relating to two variables where the points are not joined (p. 209)

polygon a many-sided (or many-angled) plane shape (p. 313)

positive used to describe numbers to the right of zero on the number line; also used to describe gains or increases (p. 46)

power *see* index (p. 6)

prime factor a factor that is a prime number (p. 42)

prime number a number that has only two factors, itself and 1 (p. 39)

probability the chance that some event will occur (p. 384)

product the result when two or more numbers are multiplied together (p. 3)

profit money made by an individual or business selling goods (once costs have been considered) (p. 46)

pronumeral a letter or symbol that represents a number (p. 170)

proper fraction a fraction in which the numerator is less than the denominator (p. 72)

protractor a device for measuring angles (p. 288)

quadrant the x- and y-axes divide the Cartesian plane into four parts, each of which is called a quadrant (p. 200)

quadrilateral a plane shape with four straight sides (p. 324)

quotient the result of a division calculation (p. 11)

range the difference between the highest and lowest value in a data set (p. 340)

rate a comparison of two amounts of different quantities, usually written as the number of the first for every one of the second (p. 158)

ratio a comparison of two or more amounts of the same quantity, written using the ':' symbol (p. 153)

rational numbers can be written in the form of $\frac{a}{b}$, where a and b are integers and $b \neq 0$ (p. 144)

ray part of a line which has a beginning and no end (p. 295)

rectangle a plane shape with two pairs of opposite sides equal and all internal angles 90 degrees (p. 324)

rectangular prism a prism with a rectangular end (p. 247)

recurring decimal a decimal number that has a repeating pattern of digits (p. 127)

reflection a transformation of a shape found after drawing the mirror image of it (p. 401)

reflectional symmetry if one or more lines of symmetry can be drawn through an object or image, that object or image is said to have reflectional symmetry (p. 422)

reflex angle an angle greater than 180 degrees and less than 360 degrees (p. 294)

regular polygon a polygon with all sides of equal length and all angles of equal size (p. 313)

relationship in algebra, the connection between two or more variables where changing the value of one variable affects the value of the other variable (p. 179)

remainder a number 'left over' at the end of a division calculation, usually indicated by 'rem'. (p. 11)

revolution a full 360 degree turn (pp. 288, 294)

rhombus a plane shape with all four sides equal and opposite angles of equal size (p. 324)

right angle an angle of exactly 90 degrees (p. 294)

right-angled triangle a triangle with one 90 degree angle (p. 316)

rotate turn (p. 407)

rotation a transformation in which a shape is turned on the flat surface about a point (p. 407)

rotational symmetry if an object or shape can be turned about a point less than 360 degrees and still look like the original object or shape, that object is said to have rotational symmetry (p. 422)

round to give an approximate value of a number to a specified degree (p. 124)

rounding approximating a decimal to a certain number of decimal places, or a number to a certain number of digits or to a certain place value (p. 16)

sample space a list of all the events that can occur in a game or experiment (p. 384)

scalene triangle all sides of different lengths; used to describe certain triangles (p. 316)

sector graph used to show the various parts of a whole (p. 361)

simplest form a fraction or ratio in which all common factors have been cancelled (p. 78)

simplify to write in simplest form by either cancelling common factors (for fractions and ratios) or by collecting like terms (p. 78)

solution the value of a pronumeral that makes the equation true; the answer (p. 263)

solve find the solution to (p. 263)

solving by inspection a method of solving equations that involves looking at the equation and working out the solution in your head (p. 263)

square a quadrilateral with all sides equal and all internal angles 90 degrees (p. 324)

square root a number that is squared to become a given number (p. 6)

stem-and-leaf plot (stem plot) a display of data in which the last digit in the data becomes a leaf and the rest of the data becomes a piece of the stem (p. 353)

straight angle an angle of exactly 180 degrees (p. 294)

substitute to replace a pronumeral with a given number (p. 184)

supplementary angles two angles that add up to 180 degrees (p. 295)

table of values a grid that displays the x- and y-values of a rule in algebra (p. 179)

tally the count of the frequency of different pieces of data (p. 340)

term those parts of an algebraic expression that are added or subtracted to make up the expression, consisting of one or more pronumerals multiplied together, possibly multiplied by a number (coefficient of the term) (p. 175)

terminating decimal a decimal number that has a finite number of digits (p. 127)

transformation a process (usually translation, rotation or reflection) carried out on a shape that changes its position and/or orientation (p. 397)

translation transformation of a shape by moving it in a straight line (p. 397)

transversal a line that cuts across each of two or more other lines (p. 305)

trapezium a quadrilateral with only one pair of opposite sides parallel (p. 324)

true number sentence a sentence that is written with numbers that is true (p. 259)

unit fraction a fraction that has 1 as the numerator (p. 98)

unit price the price per unit of quantity. The unit is often 100 mL or 1 L, 100 g or 1 kg (p. 158)

unitary method a problem-solving method that calculates an amount per unit as a step in the calculation (p. 158)

unknown in algebra, a number that is not known (p. 170)

unlike terms two or more algebraic terms that cannot be simplified as they have different pronumeral parts (p. 195)

variable in algebra, an unknown amount that can vary (p. 170)

vertex (*plural* **vertices**) the point where two or more lines meet (p. 288)

vertically opposite angles the pair of non-adjacent angles formed when two lines intersect (p. 295)

volume the amount of space occupied by a three-dimensional object (p. 247)

width *see* breadth

withdrawal taking an amount of money out of a bank account (p. 46)