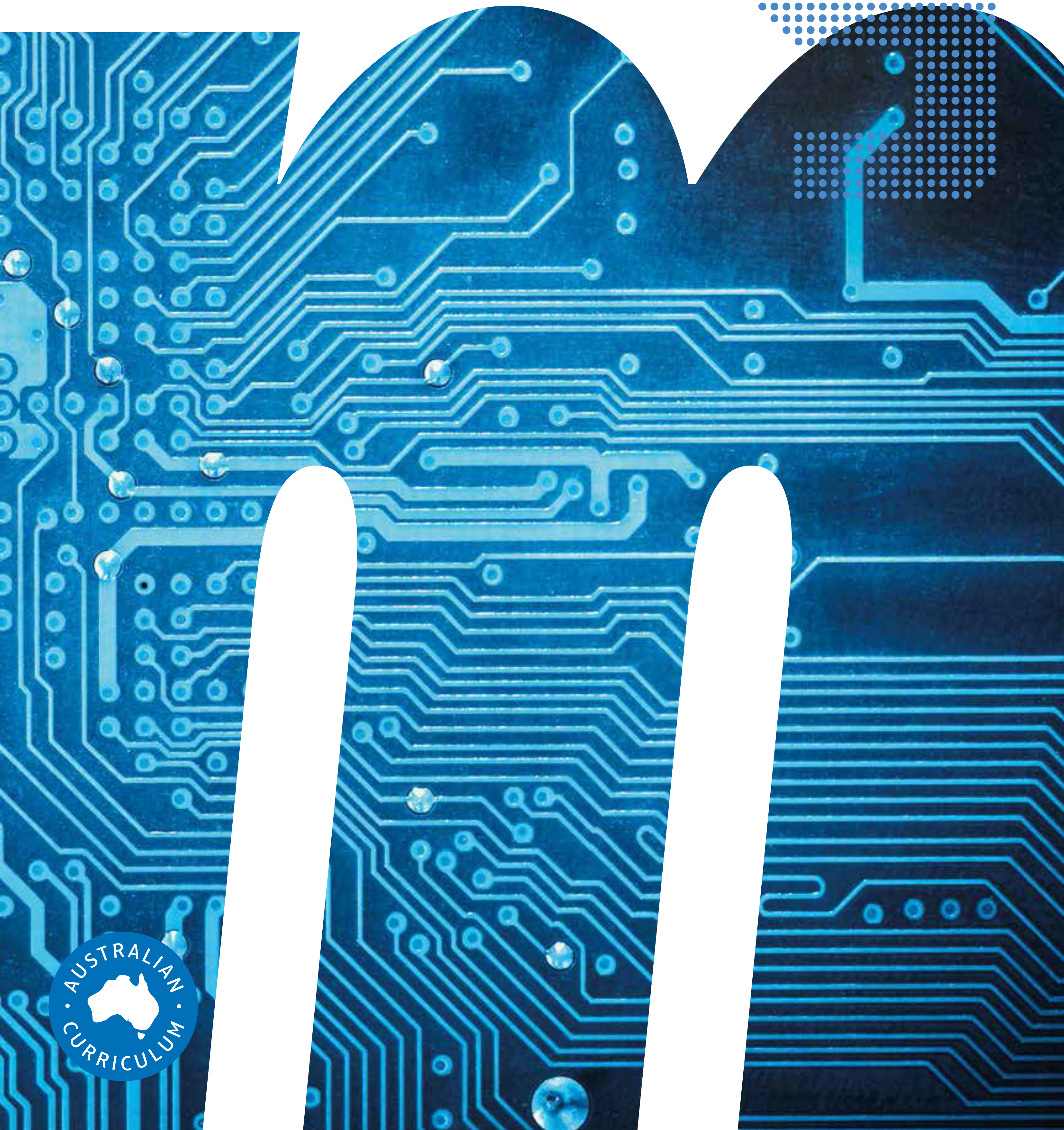
PEARSON mathematics

9

S.B.

2ND EDITION



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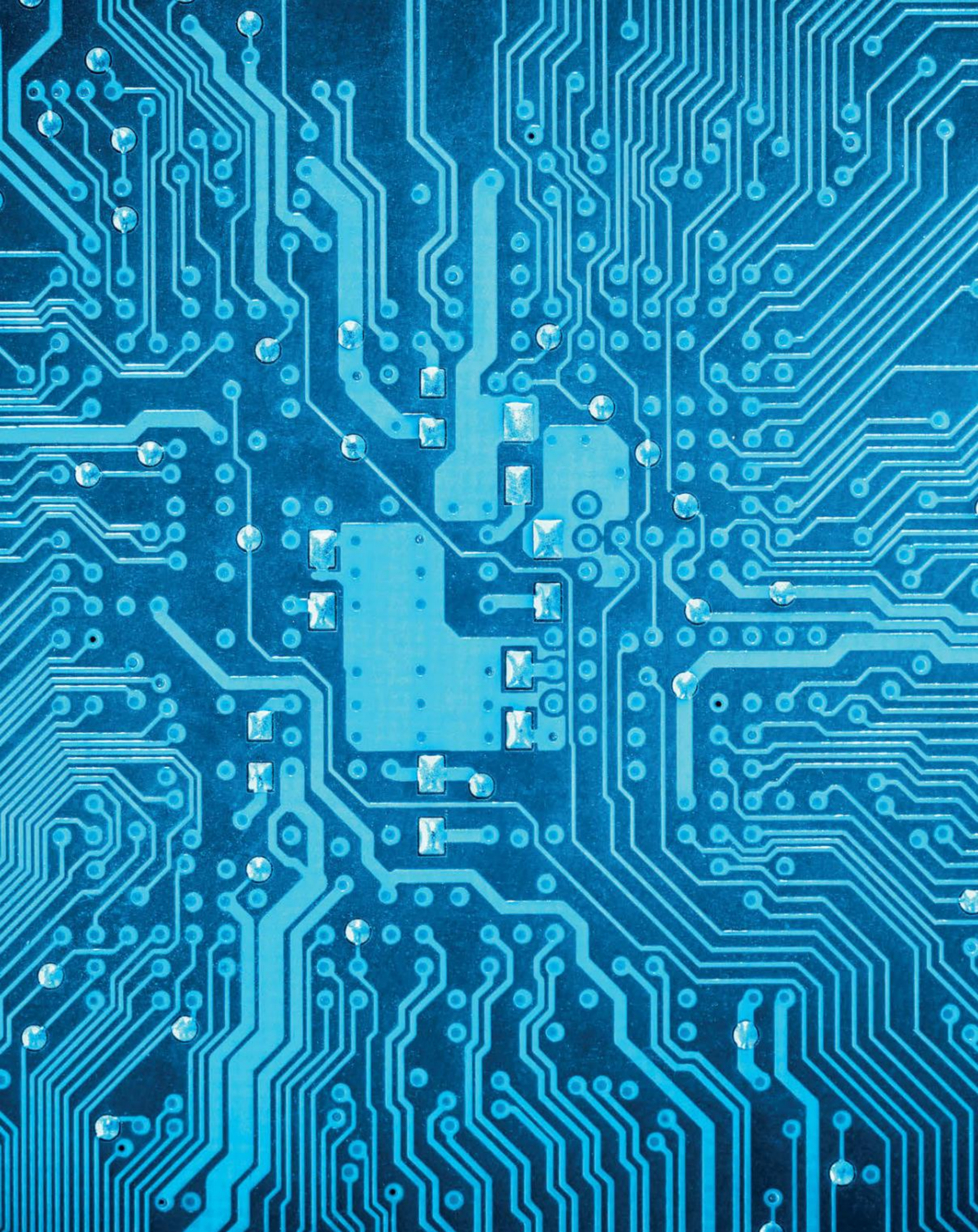

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PEARSON mathematics

Student Book

Homework Program

Teacher Companion 1

Teacher Companion 2

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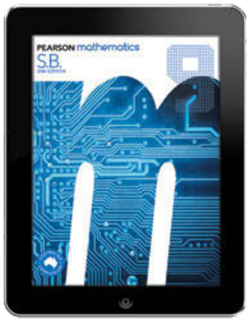
Lightbook Starter



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The Second Edition Student Book includes updated questions, activities and design, with full coverage of the Australian Curriculum: Mathematics as well as the Victorian Curriculum: Mathematics.

It incorporates the latest research as well as feedback from teachers and learners across Australia.

Content caters for students of all abilities, with improved differentiation of all exercise questions and more questions for students consolidating their skills.

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AUSTRALIAN CURRICULUM

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Lightbook Starter is an innovative digital resource powered by Pearson's award-winning Lightbook technology. It has been developed to help students learn key mathematical concepts, evaluate their understanding and track their progress. 'Before you begin' sections assess learner readiness before each chapter topic, while 'Check-in' questions can be used to evaluate learner understanding and practice after every chapter section.

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Using PEARSON mathematics

2ND EDITION

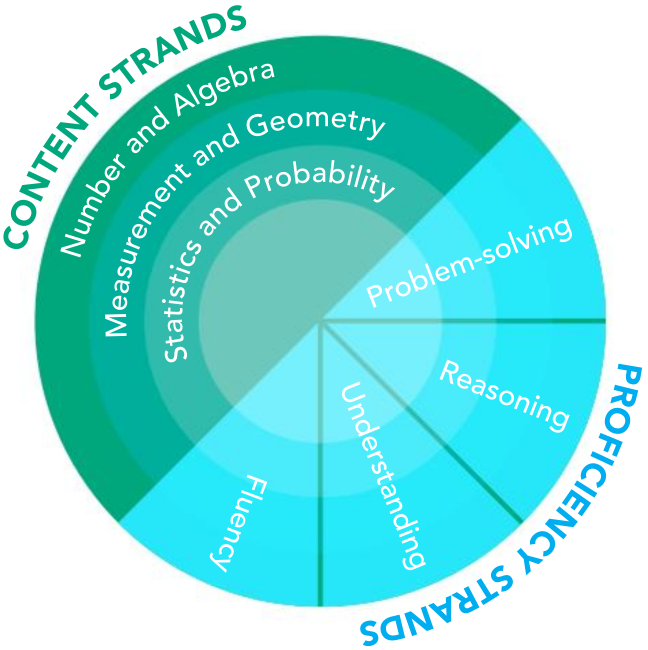
The Australian Curriculum: Mathematics aims to ensure that students:

- *are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and work lives and as active citizens*
- *develop an increasingly sophisticated understanding of mathematical concepts and fluency with processes, and are able to pose and solve problems and reason in number and algebra, measurement and geometry, and statistics and probability*
- *recognise connections between the areas of mathematics and other disciplines and appreciate mathematics as an accessible and enjoyable discipline to study.*

Australian CURRICULUM

Pearson Mathematics shares these aims.

Pearson Mathematics Second Edition is a compelling research-based series, written by experienced and practising Australian teachers with the support of Australia's leading mathematics education experts. It has been specifically designed to scaffold students' fluency development, conceptual understanding, reasoning and problem-solving skills.



Supporting all learners

Differentiation

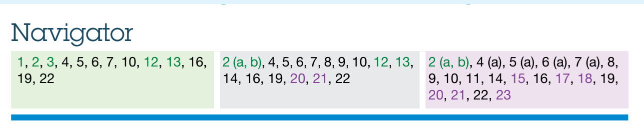
Pearson Mathematics Second Edition has been written and designed for the needs of the full ability spectrum of students in Australian classrooms.

1 Recall

Recall—Each chapter begins with a review of assumed and necessary knowledge for the chapter. For students needing extra revision, Recall Worksheets are available for each Recall question, with explanations to refresh understanding and exercises to practise skills.

3.1 Expanding brackets

Exercises—All Exercises include questions according to the Australian Curriculum Proficiency Strands: Fluency, Understanding and Reasoning. They have been carefully paced to help students build skills, develop deep conceptual understanding and apply learning. Every Exercise also has 'Open-ended' questions to encourage students' creative thinking and ability to communicate mathematics effectively.



Navigator

1, 2, 3, 4, 5, 6, 7, 10, 12, 13, 16, 19, 22	2 (a, b), 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 16, 19, 20, 21, 22	2 (a, b), 4 (a), 5 (a), 6 (a), 7 (a), 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23

Navigator—Three optional graded pathways through *every* Exercise, with every question rated as Foundation, Standard or Advanced level.

Challenge 4

Challenge—Every chapter includes a full page Challenge section for early finishers and advanced students.

Regular revision and reinforcement

Pearson Mathematics Second Edition has a broad range of cumulative and chapter-based revision.

Half-time 3

Half-time—A mid-chapter review of chapter content so far.

Chapter review 3

Chapter review—An end-of-chapter thorough review of chapter content.

Mixed review A

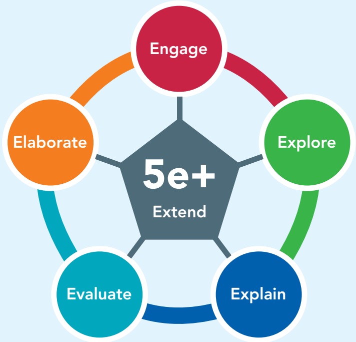
Mixed review—Cumulative revision that mixes content from previous chapters.

Encouraging inquiry and problem solving

Investigation

These are scaffolded to be accessible to all learners and structured following a 5e+ format.

Investigation



Exploration Spreadsheet and CAS

Pearson Mathematics Second Edition understands the importance places of students developing technological literacy. Explorations with CAS technology and spreadsheet software allow students to learn and practise their technical proficiency, while also helping to develop students' deeper understanding of the mathematical concepts covered.

Exploration Spreadsheet

Exploration CAS

Problem solving, Puzzles and Games

These are located throughout every chapter and give students fun opportunities to develop their problem-solving skills and logical reasoning.

Problem solving

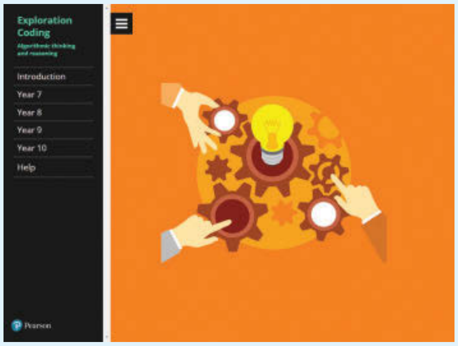
Puzzle

Game

Exploration Coding

Algorithmic thinking is an increasingly relevant component of mathematics education for the 21st century. Algorithmic thinking and coding tasks encourage learners to understand and develop reasoning skills with computational procedures, algorithms and logical problem-solving, exploring a coding environment within a mathematical context.

Exploration Coding



Real-life contexts and engagement

Exploration Task and STEM

Practical contexts provide useful learning opportunities and natural pathways into interesting, relevant mathematics.

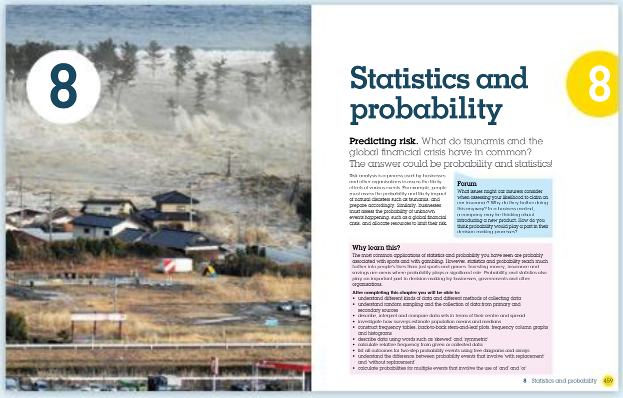
Exploration tasks are rich tasks that encourage student engagement, questioning and creative thinking. STEM activities allow students to explore topics related to Science, Technology, Engineering and Design with a Mathematics perspective.

Exploration Task

Exploration STEM

Home Page

Anticipate the 'Why learn this?' question and provide a motivating entry into every new chapter topic, including discussion-provoking Forum questions.



8

Statistics and probability

Predicting risk. What do tsunamis and the global financial crisis have in common? The answer could be probability and statistics!

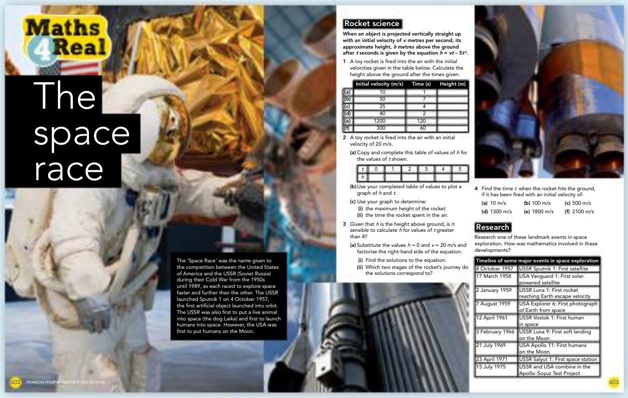
Forum

Why learn this?

Pearson Mathematics Second Edition has been designed to capture students' interest, with the incorporation of Maths 4 Real, Gamespace and more.

Maths 4 Real

Scenarios that help students make connections with mathematics in the real world.



Maths 4 Real

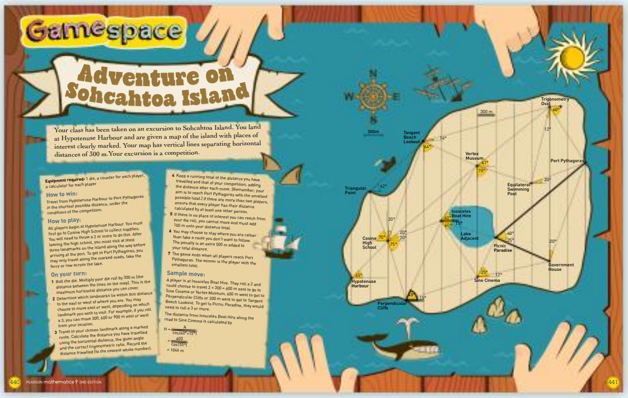
The space race

Rocket science

Research

Gamespace

Skill consolidation and reinforcement wrapped up in fun and quirky scenarios, including multi-player maths board games and solve-the-riddle, find-the-clue tasks and games.



Gamespace

Adventure on Sohcahtoa Island

1

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Financial mathematics

1

‘The best job in the world!’ Tourism Queensland once ran a world-wide advertising campaign with this phrase as the headline. They received 34 000 applications!

The position of ‘Island Caretaker’ was created to promote the islands of the Great Barrier Reef to the world. The list of ‘duties’ for the successful applicant to undertake included swimming, snorkelling, exploring and keeping an online photo diary and blog of their experiences. Anyone wishing to apply for the position had to make a 60-second video that showcased their skills and creativity. After sifting through the thousands of videos that were submitted, a shortlist of 16 applicants was formed. After the interview process, Englishman Ben Southall was announced as the succesful applicant of the 6-month contract worth $150 000. Ben’s enthusiastic blogging convinced many that the ad’s headline was not an exaggeration!.

Forum

What is your idea of ‘The best job in the world’? Give some reasons for your answer. Is it more important to have a well-paying job or to do something you enjoy? If you had to make a 60-second video as a job application, what would you include in your 60 seconds?

Why learn this?

How do you decide whether that discounted pair of jeans really is a bargain? How much money is the store making by selling them to you? How can you work out whether you have been paid the right amount of money for the work you have done? What is tax? How is it calculated? What is interest? How is it paid? What is credit? How does it work? The answers to these questions, and an understanding of the mathematics involved in earning, borrowing and spending money, are essential for all of us in our daily lives.

After completing this chapter you will be able to:

- perform calculations with percentages
- calculate profit, loss, discount, mark-up, selling price and cost price
- calculate incomes earned under a variety of payment systems
- understand how tax works and calculate basic income tax rates and deductions
- perform simple interest calculations
- understand different methods of payment (such as credit cards) and perform calculations associated with them
- calculate typical costs of using resources such as electricity, gas and water
- calculate typical costs of using communications devices such as mobile phones.

Recall 1

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 Convert the following fractions to decimals.

(a) $\frac{17}{100}$ **(b)** $\frac{57}{1000}$ **(c)** $\frac{321}{100\,000}$

2 Convert the following fractions to decimals. If the decimal is a recurring decimal, use the appropriate notation.

(a) $\frac{16}{25}$ **(b)** $\frac{9}{8}$ **(c)** $\frac{23}{40}$ **(d)** $\frac{1}{6}$ **(e)** $\frac{10}{11}$ **(f)** $\frac{5}{7}$

3 Convert the following decimals to fractions in simplest form.

(a) 0.79 **(b)** 0.003 **(c)** 0.014 **(d)** 1.065 **(e)** 10.140 **(f)** 0.0902

4 Evaluate the following.

(a) $5.8 + 2.9$ **(b)** $12.7 - 3.6$ **(c)** 5.2×1.5 **(d)** $6.2 \div 0.03$

5 Convert the following fractions and decimals to percentages. (Round your answers to 2 decimal places, if necessary.)

(a) $\frac{14}{20}$ **(b)** 0.91 **(c)** 0.032 **(d)** $\frac{23}{28}$

6 Write the following percentages as fractions in simplest form.

(a) 85% **(b)** 22% **(c)** 6.7% **(d)** $\frac{1}{2}\%$

7 Write the following percentages in decimal form.

(a) 49% **(b)** 30% **(c)** 2.5% **(d)** 370%

8 Calculate the following:

(a) $\frac{5}{6}$ of 90 m **(b)** $\frac{3}{4}$ of 105 kg **(c)** $\frac{11}{10}$ of \$25 600

9 Find:

(a) 5% of 20 m **(b)** 10% of 63.5 km **(c)** 22% of 18.1 cm

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Get rich quick?

In this activity, you will learn how to choose between investments with different compounding periods and dividends.

Percentages review

1.1

The ability to work with percentages, fractions and decimals is important for many daily calculations and estimations. It is helpful to know the fraction, decimal and percentage equivalents of these common values.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{20}$	$\frac{1}{100}$
Decimal	0.5	0.25	0.75	$0.\dot{3}$	$0.\dot{6}$	0.1	0.2	0.05	0.01
Percentage	50%	25%	75%	$33\frac{1}{3}\%$	$66\frac{2}{3}\%$	10%	20%	5%	1%

Finding a percentage of an amount

'Per cent' means 'out of 100' or 'for every hundred'. Percentages are just fractions with a denominator of 100; for example, $40\% = \frac{40}{100} = \frac{2}{5}$.

If you know some common fraction and percentage equivalents and how to find them, you will be able to perform many percentage calculations mentally (see Method 1 below). Method 2 involves writing the percentage as a fraction or decimal and multiplying by the amount. This is a good 'pen and paper' method.

Worked example 1

W.E. 1

Calculate the percentages of the given amounts:

(a) 35% of \$62

(b) 27.5% of \$300

Method 1: Use basic percentages

Thinking	Working
(a) **1** Break down the percentage amount into basic percentages that are easy to calculate, such as 25%, 10% or 5%.	(a) $35\% = 30\% + 5\%$ $= 3 \times 10\% + 5\%$
2 Calculate each of these basic percentages. (Here, 10% is found by dividing by 10, and 5% by halving 10%.)	10% of 62 $= 62 \div 10$ $= 6.2$ 5% of 62 $= (10\% \text{ of } 62) \div 2$ $= 6.2 \div 2$ $= 3.1$
3 Multiply and/or add the values of each of the basic percentages to find the required percentage value.	$3 \times 10\% + 5\%$ $= (3 \times 6.2) + 3.1$ $= 18.6 + 3.1$ $= 21.7$
4 Write the answer.	35% of \$62 = \$21.70

(b)	1	Break down the percentage amount into basic percentages that are easy to calculate.	(b) $27.5\% = 25\% + 2.5\%$
	2	Calculate each of these basic percentages. (Here, 25% is found by dividing by 4 and 2.5% by dividing 25% by 10.)	25% of 300 $= 300 \div 4$ $= 300 \div 2 \div 2$ $= 150 \div 2$ $= 75$ $2.5\% = 25\% \div 10$ $= 75 \div 10$ $= 7.5$
	3	Multiply and/or add the values of each of the basic percentages to find the required value.	$25\% + 2.5\%$ $= 75 + 7.5$ $= 82.5$
	4	Write the answer.	27.5% of $\$300 = \$82.50$

Method 2: Fraction or decimal multiplication

		Thinking	Working
(a)	1	Use the fact that 'of' is equivalent to '×' to rewrite the sentence as a mathematical one.	(a) 35% of 62 $= 35\% \times 62$
	2	Convert the percentage to a fraction by dividing by 100.	$= \frac{35}{100} \times 62$
	3	Do the multiplication. Simplifying the fractions first makes the multiplication easier. (Here, divide by common factors of 5 and then 2.)	$\frac{{}^{7}\cancel{35}}{{}_{20}\cancel{100}} \times \frac{62}{1}$ $= \frac{7}{{}_{10}\cancel{20}} \times \frac{{}^{31}\cancel{62}}{1}$ $= \frac{7}{10} \times \frac{31}{1}$ $= \frac{217}{10}$ $= 21.7$
	4	Write the answer.	35% of $\$62 = \$21.70$
(b)	1	Use the fact that 'of' is equivalent to '×' to rewrite the sentence as a mathematical one.	(b) 27.5% of 300 $= 27.5\% \times 300$
	2	Convert the percentage to a fraction by dividing by 100.	$= \frac{27.5}{100} \times \frac{300}{1}$
	3	Do the multiplication. Simplifying the fractions first makes the multiplication easier. (Here, divide by a common factor of 100.)	$= \frac{27.5}{{}_{1}\cancel{100}} \times \frac{{}^{3}\cancel{300}}{1}$ $= 27.5 \times 3$ $= 82.5$
	4	Write the answer.	27.5% of $\$300 = \$82.50$

Increasing or decreasing by a given percentage

A percentage can also represent a change in a quantity. You can calculate a percentage increase or decrease in a simple 'one-step' calculation if you use the fact that the original amount represents 100%.

Worked example 2

W.E. 2

Increase or decrease the following amounts by the given percentages. Round decimal answers to 2 decimal places.

(a) Increase \$420 by 65% **(b)** Decrease 87 kg by 13.6%

Thinking	Working
(a) **1** Add the percentage increase to 100%. Convert the new percentage to a decimal.	(a) $(100 + 65)\% = 165\%$ $= 1.65$
2 This decimal represents the scale factor. Multiply it by the original amount to find the new amount. (Use your calculator.)	$1.65 \times 420 = 693$
3 Write the answer.	\$420 increased by 65% is \$693.
(b) **1** Subtract the percentage decrease from 100%. Write this new percentage as a decimal.	(b) $(100 - 13.6)\% = 86.4\%$ $= 0.864$
2 This decimal represents the scale factor. Multiply it by the original amount to find the new amount. (Use your calculator.)	$0.864 \times 87 = 75.168$
3 Write the answer.	87 kg decreased by 13.6% is 75.17 kg.

Writing one amount as a percentage of another

To write one amount as a percentage of another:

1. Make sure both amounts are the same type, or measured in the same units. Convert units if necessary.
2. Write one amount as a fraction of the other.
3. Convert this fraction into a percentage by multiplying by 100%.

Worked example 3 W.E. 3

Round your answers to the following questions to 1 decimal place.

(a) Tyler kicked 13 goals from 18 shots at goal. Calculate his kicking accuracy as a percentage.

(b) A packet of 'Chocky' biscuits is on sale for \$2.35, a saving of 57c. Write the saving as a percentage of the original price.

Method 1: Fraction multiplication

Thinking	Working
(a) 1 Write the number as a fraction of the total. Multiply this fraction by 100%.	(a) $\frac{13}{18} \times \frac{100}{1}\%$
2 Simplify the multiplication by cancelling common factors first.	$= \frac{13}{{}^{9}\cancel{18}} \times \frac{{}^{50}\cancel{100}}{1}$ $= \frac{650}{9}$ $= 72\frac{2}{9}\%$ or $72.\dot{2}\%$
3 Write the answer, rounded to the specified number of decimal places, in a sentence.	Tyler's kicking accuracy is 72.2% (1 d.p.).
(b) 1 Find the original price by adding the saving back on to the sale price.	(b) $\$2.35 + \$0.57 = \$2.92$
2 Write the saving as a fraction of the original price, making sure both amounts are expressed in the same units (dollars). Multiply this fraction by 100%.	$\frac{0.57}{2.92} \times \frac{100}{1}\%$ $= \frac{57}{2.92}\%$ $= 19.520\,54\ldots\%$
3 Write the answer, rounded to the specified number of decimal places, in a sentence.	The saving is 19.5% of the original price (1 d.p.).

Method 2: Convert to a decimal, then multiply

Thinking	Working
(a) 1 Write the number as a fraction of the total. Convert this fraction to a decimal by dividing.	(a) $\frac{13}{18} = 13 \div 18 = 0.7222\ldots$
2 Multiply the decimal by 100%.	$0.7222\ldots \times 100\% = 72.2222\ldots\%$
3 Write your answer in a sentence (rounded).	Tyler's kicking accuracy is 72.2% (1 d.p.).

(b) **1**	Find the original price by adding the saving back on to the sale price.	(b) $\$2.35 + \$0.57 = \$2.92$
2	Write the saving as a fraction of the original price, making sure both amounts are expressed in the same units (dollars).	$\frac{0.57}{2.92}$
3	Convert this fraction to a decimal by dividing.	$\frac{0.57}{2.92} = 0.57 \div 2.92 = 0.1952\ldots$
4	Multiply the decimal by 100%.	$0.1952\ldots \times 100\% = 19.52\ldots\%$
5	Write your answer in a sentence (rounded).	The saving is 19.5% of the original price (1 d.p.).

Rounding errors

Many financial applications of percentages use decimals instead of fractions. For practical purposes, the rounded approximation of a recurring decimal is often used in a calculation. For example, $\frac{5}{6}$ expressed as a decimal is 0.833 333…. Rounded to 1 decimal place, it is 0.8. If this was rounded to 2 decimal places, it would be 0.83. A question will usually tell you how many decimal places to round your answer to.

As soon as you use a rounded value, you introduce error into your answer. To minimise this error you should round as late as possible in a calculation or use fractions for as long as you can before converting to a decimal and rounding. If you give a rounded answer, then you should state this (e.g. if you have rounded to 2 decimal places, write '2 d.p.').

Fractions and decimals on the calculator

If you are working with decimals on your calculator, keep all the decimal places in the calculator display and only round your final answer.

Most scientific calculators can convert between fractions, decimals and mixed numbers, using a key that is usually labelled **S⇔D** or $a\,{}^{b}/_{c}$.

1.1 Percentages review

Navigator

1 (a–f, j–l), 2 (a, b), 3 (a, c, e), 4 (a–d), 5, 6, 7, 9, 10, 11, 15, 17	1 (g–l, m–o), 2 (a, b, d, e), 3, 4 (a–d), 5, 6, 7, 8, 11, 12, 14, 15, 17, 18	1 (j–l, m–o), 2 (c, d, e), 4 (e-h), 7, 8, 12, 13, 14, 16, 17, 18

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Fluency

W.E. 1

1 Calculate the percentages of the given amounts. If necessary, round answers to 2 decimal places.

(a) 50% of \$200	**(b)** 25% of \$400	**(c)** 10% of \$75
(d) 20% of \$480	**(e)** 100% of \$55	**(f)** 10% of \$38
(g) 25% of \$40	**(h)** 45% of \$50	**(i)** 30% of \$150
(j) 11% of 60 L	**(k)** 66% of 400 m	**(l)** 77% of 35 kg
(m) 0.5% of \$3500	**(n)** 12.5% of 1500 m	**(o)** 7.2% of 15 L
(p) $5\frac{3}{4}\%$ of \$17 000	**(q)** $6\frac{1}{5}\%$ of \$90	**(r)** $\frac{2}{3}\%$ of \$200 000

W.E. 2

2 Increase or decrease the following amounts by the given percentages. If necessary, round answers to 2 decimal places.

- **(a)** Increase 50 by:
  - **(i)** 10% **(ii)** 50% **(iii)** 25%
- **(b)** Increase 240 by:
  - **(i)** 50% **(ii)** 35% **(iii)** 8% **(iv)** 140%
- **(c)** Decrease 5700 by:
  - **(i)** 20% **(ii)** 75% **(iii)** 46% **(iv)** 98%
- **(d)** Increase $64.15 by:
  - **(i)** 10% **(ii)** 260% **(iii)** 7.5% **(iv)** $6\frac{3}{4}\%$
- **(e)** Decrease 9.8 seconds by:
  - **(i)** 50% **(ii)** 13% **(iii)** 6.3% **(iv)** $3\frac{1}{3}\%$

3 Convert these fractions to percentages.

(a) $\frac{17}{20}$ **(b)** $\frac{8}{25}$ **(c)** $\frac{9}{20}$ **(d)** $\frac{17}{50}$ **(e)** $\frac{19}{50}$

W.E. 3

4 Round your answers to the following questions to 1 decimal place.

- **(a)** Ash kicked nine goals from 13 shots at goal. Calculate Ash's percentage kicking accuracy.
- **(b)** A jar of 'Nuttie's' peanut butter is on sale for $3.18, a saving of 64 cents. Write the saving as a percentage of the original price.
- **(c)** Maryam had 22 shots at the netball goals and got 14 in. Calculate Maryam's percentage shooting accuracy.
- **(d)** A box of 'Crunchy-Bix' cereal has been reduced by $1.02 to $4.37. Calculate the percentage saving based on the original cost price.
- **(e)** Raf got 76 of 114 serves into the tennis court. Calculate the percentage accuracy of Raf's serve.
- **(f)** A tin of 'Beanz' baked beans is on sale for $1.17, a saving of 32 cents. Write the saving as a percentage of the original price.
- **(g)** Brett bowled a cricket ball at a set of stumps 54 times. He hit the stumps 41 times. Calculate Brett's stump-hitting accuracy as a percentage of the number of balls bowled.
- **(h)** A tub of 'Yo-Cal' yoghurt has been reduced by 74 cents to $2.11. Calculate the percentage saving based on the original cost price.

Understanding

5 The label on a 2.5 L bottle of fruit juice says:

Pineapple juice:	78%
Orange juice:	15.5%
Passionfruit juice:	4.5%
Guava juice:	2%

Calculate the volume of each type of juice, rounding answers to the nearest mL if necessary.

6 Approximately 65% of the mass of an adult human is water. Akira weighs 58 kg. How much of Akira's mass is water, to the nearest 100 g?

7 This is the nutritional label from a packet of muesli bars. The 'serving size' is one bar, which has a mass of 20 g. Round your answers to the following questions to 1 decimal place.

INGREDIENTS

WHOLEGRAIN CEREALS (48%) [UNCLE TOBYS ROLLED **OATS** (37%), WHOLE **WHEAT** (11%)] RAW SUGAR, PUFFED RICE, VEGETABLE OIL [EMULSIFIER (**SOY** LECITHIN), ANTIOXIDANTS (304, 306)], MIXED NUTS (7%) [**CASHEWS, PECANS, ALMONDS, MACADAMIAS**], GLUCOSE (**WHEAT**), HONEY, COCONUT, DIETARY FIBRE (INULIN), TAPIOCA STARCH, SALT, SODIUM BICARBONATE, FLAVOUR, EMULSIFIER (**SOY** LECITHIN).

CONTAINS OATS, WHEAT, TREE NUTS AND SOY AS INDICATED IN BOLD TYPE. PRODUCT PROCESSED ON A LINE THAT ALSO PROCESSESS PRODUCTS CONTAINING MILK.

NUTRITION INFORMATION#

Servings per Package: 6 Serving Size: 20g (1 bar)

	Quantity per Serving	%Daily Intake* per Serving	Quantity per 100g
ENERGY	370kJ	4%	1840kJ
PROTEIN	1.7g	3%	8.3g
FAT, TOTAL	3.2g	5%	16.2g
- SATURATED	0.7g	3%	3.5g
CARBOHYDRATE	12.0g	4%	60.1g
- SUGARS	3.4g	4%	16.8g
DIETARY FIBRE	1.7g	6%	8.6g
SODIUM	55mg	2%	265mg

All specified values are averages

*Percentage Daily Intakes are based on the average adult diet of 8700kJ. Your daily intakes may be higher or lower depending on your energy needs.

(a) Use the information in the 'Quantity per serving' column to calculate the percentage mass of the muesli bar that is:

(i) carbohydrate

(ii) protein

(iii) fat (total).

(b) Sugar is a type of carbohydrate.

(i) What percentage of the mass of a muesli bar is sugar?

(ii) What percentage of the carbohydrate mass is sugar?

8 Charlotte is a runner whose personal best time for her favourite race is 3 minutes 34 seconds.

(a) Charlotte wants to decrease her time by 4%. How many seconds is this?

(b) At the state championship, Charlotte lowered her time by 6.9 seconds. What percentage improvement is this, correct to 2 decimal places?

9 There are two baby gorillas and seven adult gorillas in a zoo enclosure.

(a) What fraction of the gorillas are adult?

(b) What percentage of the gorillas are baby gorillas?

(c) One extra adult gorilla is added to the enclosure. Recalculate your answers to **(a)** and **(b)**.

10 Copy and complete the following table.

	Common fraction	Decimal fraction	Percentage
(a)	$\frac{1}{4}$		
(b)	$\frac{1}{2}$		
(c)	1		
(d)	$\frac{1}{5}$		
(e)	$\frac{3}{4}$		
(f)	$\frac{2}{5}$		

11 Copy and complete the following table.

	Common fraction	Decimal fraction	Percentage
(a)	$\frac{3}{10}$		
(b)		0.45	
(c)			72%
(d)	$\frac{5}{4}$		
(e)		0.56	
(f)			12.5%

12 Approximately $\frac{7}{8}$ of an iceberg lies under water. The mass of the iceberg is 150 000 tonnes.

(a) What mass of the iceberg is under water?

(b) What percentage of the whole mass is this?

13 At the end of the first week of trading shares in Jaydeep's company had increased their value by 17.5%. The shares were worth \$3.40 each at the beginning of the week.

(a) What is the value of a share at the end of the first week?

(b) At the end of the second week, the share price had decreased by 9% compared with its price at the beginning of that week. Calculate the value of one share at the end of the second week.

(c) Calculate the overall increase or decrease at the end of the 2 weeks as a percentage of the original price of \$3.40, correct to 2 decimal places.

Reasoning

14 The local newspaper had the following headline: 'Prime Minister's approval rating 67%'. The article stated that the newspaper had surveyed all 180 000 of its readers, of which 14% had responded. Of those who responded, 67% said that they thought the Prime Minister was doing a good job. How many people actually said this?

15 **(a)** **(i)** Increase 100 by 20%.

(ii) Now decrease the answer obtained by 20%.

(b) **(i)** Increase 300 by 30%.

(ii) Now decrease the answer obtained by 30%.

(c) Explain why you do not arrive back at the starting number after an increase and decrease by the same percentage.

16 In her Drama class, $\frac{3}{5}$ of Anita's overall mark is allocated to the practical exam and the remaining $\frac{2}{5}$ is allocated to the written exam. Anita scored 82% in her practical exam and 76% in the written exam. Calculate Anita's overall percentage for the subject, correct to 1 decimal place.

Open-ended

17 Write three whole numbers that when increased by 140% give an answer between 100 and 200.

18 Trent is looking to buy a new skateboard. In one store, the skateboard was on sale for $258, an advertised saving of $31. In another store, the same skateboard had a marked price of $299, but a sale sticker next to it said 'take 25% off the marked price'. Trent decided that $31 sounded like a better saving than 25% and bought the skateboard from the first store.

(a) Was this the right choice in terms of saving money? Explain why or why not.

(b) Describe a simple calculation Trent could have done while standing in the shop that could have helped him make a more effective decision.

Problem solving

Age-old dilemmas

1 My mother is three times my age.

My sister is 75% of my age and 15% of my grandmother's age.

My father is 40, 4 years older than my mother.

Find my age, my grandmother's age and my sister's age.

2 Harry, Larry and Garry are three brothers who all go to secondary school.

Harry and Larry are an exact number of years apart. They have their birthdays on the same day. Garry's birthday is on a different day.

Harry is 80% of Larry's age.

Garry is 110% of Larry's age.

How old are each of the brothers?

Strategy options

- Guess and check.
- Work backwards.
- Test all possible combinations.

1.2

Buying and selling

Discounts

A **discount** is an amount or percentage taken off the normal selling price of an item. Discounts can be offered for a variety of reasons:

- A store may be overstocked with certain items and might wish to clear them.
- Frequent customers are sometimes given special discounts.
- Reductions can be made for bulk purchases (e.g. 'Buy 9, get the 10th free!') to encourage high sales for a particular item.
- A store might have a loyalty system, where a discount is given for a certain number of purchases.

There are two different methods to find percentage discount in the worked example below. Use the method that is easiest for you.

Worked example 4

W.E. 4

A shop has a sale with 25% taken off all items. Calculate the sale price of a \$46 shirt.

Method 1: Find and subtract the discount amount

Thinking	Working
1 Calculate the discount in dollars. (Here, you can find 25% by halving twice, because 25% = $\frac{1}{4}$.)	$50\% \text{ of } \$46 = 46 \div 2$ $= 23$ $25\% \text{ of } \$46 = 23 \div 2$ $= \$11.50$
2 Subtract the discount from the original price to find the sale price.	$\text{Sale price} = 46 - 11.50$ $= \$34.50$

Method 2: Find the remaining percentage

Thinking	Working
1 Calculate the remaining percentage of the price by subtracting the discount percentage from 100%	$100\% - 25\% = 75\%$
2 Multiply the remaining percentage by the marked price to find the sale price.	$\text{Sale price} = 75\% \text{ of } \$46$ $= 0.75 \times 46$ $= \$34.50$

Profit and loss

Retail businesses manufacture products, or purchase them from suppliers. The cost of buying or manufacturing the goods (including things such as the cost of materials and transport) is called the cost price.

To make a **profit**, retailers need to sell their product to customers at a higher price than the cost price. This is the selling price. The difference between the selling price and the cost price is called the **mark-up**. If the selling price is less than the cost price, then a **loss** is made.

Profit or loss can be written as a percentage of the cost price or selling price.

$$\% \text{ Profit} = \frac{\text{profit}}{\text{CP}} \times 100\% \text{ or } \frac{\text{profit}}{\text{SP}} \times 100\%$$

where CP = cost price, SP = selling price

To find the % Loss, substitute ‘loss’ instead of ‘profit’ into the above formulas.

Worked example 5

W.E. 5

A home theatre system costs an electrical store $510. It sells the system for $965. Calculate, correct to 1 decimal place, the percentage profit made as a percentage of:

(a) the cost price

(b) the selling price.

Thinking	Working
(a) **1** Calculate the dollar value of the profit.	(a) $\text{Profit} = \text{SP} - \text{CP}$ $= \$(965 - 510)$ $= \$455$
2 Write the profit as a fraction of the cost price and multiply by 100%.	$\% \text{ Profit} = \frac{\text{profit}}{\text{CP}} \times 100\%$ $= \frac{455}{510} \times 100\%$ $= 89.2\%$ (1 d.p.)
(b) **1** Calculate the dollar value of the profit.	(b) $\text{Profit} = \$455$
2 Write the profit as a fraction of the selling price and multiply by 100%.	$\% \text{ Profit} = \frac{\text{profit}}{\text{SP}} \times 100\%$ $= \frac{455}{965} \times 100\%$ $= 47.2\%$ (1 d.p.)

When a profit is made, the percentage profit based on the cost price is always greater than the percentage profit based on the selling price. This is because the cost price is a smaller value than the selling price.

Profit and loss can also be applied to investments. People purchase land, property or goods (such as antiques or memorabilia) hoping that the value will increase. If the value of the investment is more than the cost of purchasing and maintaining it, then a profit is made.

Buying shares is a way of investing in a business. If the business is successful, investors receive some of the profits (based on how many shares they own). The value of the shares can also increase, which means they can be sold for profit.

Mark-up

The mark-up on a product is the amount added to the cost of the product, to make the selling price. It can vary from business to business and from product to product. A retailer may feel that they can sell more of a particular item at a lower mark-up. There may be a company policy that requires that certain products make a certain percentage profit and so a particular mark-up must be added. For example, a furniture store may require a 65% profit on all sofas sold.

You can use the mark-up to find the selling price from the cost price. The cost price represents 100%. If you want a profit of 65%, the selling price must be 100% + 65%, or 165% of the cost price. To find 165% of the cost price, you can multiply by 1.65.

Worked example 6 W.E. 6

A furniture store uses a mark-up of 65% on the cost price. What would a sofa that cost the store $300 be sold for?

Thinking	Working
1 Add the mark-up onto 100%. Convert to a decimal.	$65\% + 100\% = 165\%$ $= 1.65$
2 Multiply by the cost price to find the selling price.	$SP = 1.65 \times \$300$ $= \$495$

Calculating the original price

As a customer in a store you see only the final selling price of goods after the mark-up has been added. If you knew the value of the mark-up, you could calculate the cost price of the goods by reversing the process used to find the selling price.

In the above worked example, you *multiplied the cost price* of a sofa of $300 by 1.65 to find the selling price of $495. If you *divide the selling price* by 1.65, you find the cost price of $300.

$$\begin{tikzcd} \text{Cost price (CP)} \arrow[r, shift left, "\times (100\% + \text{mark-up } \%)"] & \text{Selling price (SP)} \arrow[l, shift left, "\div (100\% + \text{mark-up } \%)"] \end{tikzcd}$$

Worked example 7 W.E. 7

A jewellery store marks up the cost price of their goods by 75%. Calculate the cost price of a necklace that was sold for $56.

Thinking	Working
1 Add the mark-up percentage to 100%.	$100\% + 75\% = 175\%$
2 Write an equation.	$175\% \text{ of } CP = \$56$
3 Convert the percentage to a decimal and replace 'of' with a multiplication sign.	$1.75 \times CP = \$56$
4 Solve the equation by dividing both sides by the decimal value of the percentage.	$CP = \$56 \div 1.75$ $= \$32$

You can also use this method to determine the original price of discounted items. First subtract the discount percentage from 100% and convert this value to a decimal. Then, divide the discounted selling price (SP) to find the original price (OP).

For discounts:

$$\begin{tikzcd} \text{Original price (OP)} \arrow[r, shift left, "\times (100\% - \text{discount } \%)"] & \text{Selling price (SP)} \arrow[l, shift left, "\div (100\% - \text{discount } \%)"] \end{tikzcd}$$

Worked example 8

W.E. 8

A smartphone has a sale price of \$539 after a discount of 22.5%. What was the original price of the smartphone? Answer to the nearest dollar.

Thinking	Working
1 Subtract the discount from 100%.	$100\% - 22.5\% = 77.5\%$
2 Write an equation.	77.5% of $OP = \$539$
3 Convert the percentage to a decimal, and replace 'of' with a multiplication sign.	$0.775 \times OP = \$539$
4 Divide both sides by the decimal value of the percentage.	$OP = \$539 \div 0.775$ $= \$695$ (nearest \$)

1.2 Buying and selling

Navigator

1, 2, 3, 4, 5, 6, 7, 10, 12, 13, 16, 19, 22	2 (a, b), 4, 5, 6, 7, 8, 9, 10, 12, 13, 14, 16, 19, 20, 21, 22	2 (a, b), 4 (a), 5 (a), 6 (a), 7 (a), 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23

Answers p. 628

Fluency

1 Break the following percentages into percentages that are easier to find (for example 20% = 2 × 10%).

(a) 30% **(b)** 75% **(c)** 35% **(d)** 15% **(e)** 80% **(f)** 95%

2 Calculate the sale price of the following items:

W.E. 4

(a) a \$24 pair of shorts discounted by 15%

(b) a \$95 pair of shoes discounted by 25%

(c) a \$63.99 skirt discounted by 40%

(d) a \$419.95 suit discounted by 37.5%.

3 Find the remaining percentage when each percentage is subtracted from 100%.

(a) 30% **(b)** 15% **(c)** 37.5% **(d)** 25%

(e) 60% **(f)** 75% **(g)** 12% **(h)** 40%

W.E. 5

4 **(a)** A camera costs a camera shop \$360. The shop sells the camera for \$504, making a profit of \$144. Calculate, correct to 1 decimal place, the profit as a percentage of:

(i) the cost price **(ii)** the selling price.

(b) An old car costs a used car salesman \$4200. He sells the car for \$4050. Calculate the loss and use this to find, correct to 1 decimal place, the loss as a percentage of:

(i) the cost price **(ii)** the selling price.

W.E. 6

5 A clothing store has a policy of selling jeans and T-shirts with a mark-up on the cost price of 92%. Calculate the selling price of the following items, rounding your answer to the nearest 5 cents:

(a) a pair of jeans, cost price \$39.30 **(b)** a T-shirt, cost price \$8.75.

W.E. 7

6 A hardware store has a policy of marking up the cost price of their goods by 85%. Calculate the cost price of the following items, rounding your answer to the nearest cent:

(a) a packet of nails, selling price \$3.70 **(b)** a cordless drill, selling price \$51.00.

W.E. 8

7 **(a)** A tennis racquet has a sale price of \$149 after a discount of 15% has been applied. What was the original price of the racquet? Answer to the nearest dollar.

(b) A plasma TV has a sale price of \$2497 after a discount of 7.5% has been applied. What was the original price of the TV? Answer to the nearest dollar.

8 If a table costs \$486 and is later sold for \$350, the percentage profit or loss, based on the cost price, is:

A $\frac{(486-350)}{486} \times 100\%$ loss **B** $\frac{(486-350)}{350} \times 100\%$ loss

C $\frac{(486-350)}{350} \times 100\%$ profit **D** $\frac{(486-350)}{486} \times 100\%$ profit

9 If a laptop is sold for \$960 at a 114% profit, the cost of the laptop was:

A $\$960 \times 2.14 \times 960$ **B** $\$960 \times 2.14$

C $\$960 \times 1.14$ **D** $\frac{\$960}{2.14}$

10 If a desk is for sale for \$140 after a 25% discount, it means that its original price was:

A \$105 **B** \$175

C \$186.67 **D** \$245

Understanding

11 Calculate the selling price for the following cost prices and percentage profits (on cost price).

(a) $CP = \$48$, percentage profit = 45%

(b) $CP = \$60$, percentage profit = 60%

12 A builder buys $840 worth of timber to build an extension. If he is allowed a 12.5% discount, how much does he pay?

13 A hairdresser gets a discount of 24% on hair products bought in bulk. How much is saved on $370 worth of products?

14 A store purchases a bulk order of 120 watches for $960.

(a) Calculate the profit made if the watches are sold individually for $19.95 each.

(b) Calculate the total profit as a percentage of the cost price.

15 In Australia, a Goods and Services Tax (GST) of 10% is added to the price of all products and services purchased by consumers (with the exception of some basic grocery items). GST is added after the product has been marked up by the retailer. Mei is a clothing store owner who has just purchased a large number of T-shirts for $6.49 each. She adds a mark-up of 180%, then the 10% GST.

(a) What is the final selling price of the T-shirts? (Answer to the nearest cent.)

(b) What amount of the final price is GST?

16 Most of the time the marked prices you see on goods in shops have the 10% GST already added to the price. Find the original, pre-GST, price of:

(a) a DVD with a marked price of $29.90

(b) a vacuum cleaner with a marked price of $349.

17 A mechanic buys a car at an auction for $3120. The car is then repaired at a cost of $430. It is resprayed and has its panels straightened for $840. The car is then advertised and sold for $9800. Calculate the percentage profit on the selling price if the cost of advertising was $80.

18 Louise bought shares in the following companies through a sharebroker (a person who buys and sells shares on behalf of others). The sharebroker charges a fee of 2% of the value of the shares bought and sold.

150 shares in Oro Mining, bought for $39.05 each
200 shares in Beacon Bank, bought for $19.86 each
250 shares in JeansEast, bought for $6.14 each

(a) Calculate the sharebroker's fee.

(b) Including the fee, how much did Louise pay for her shares in total?

Louise instructs her sharebroker to sell her shares once they reach the following prices:

Oro Mining: $42
Beacon Bank: $21.50
JeansEast: $8

Her sharebroker does so and subtracts his fee.

(c) Calculate Louise's profit from the sale of the shares.

(d) Calculate Louise's profit as a percentage of the cost of her shares, including her sharebroker's fee. State your answer correct to 2 decimal places.

Reasoning

19 Evie's favourite shoe shop has a 'buy one pair, get the second pair half-price' offer. Another shoe shop nearby is offering 20% off everything. If the regular retail price of the shoes Evie wants in both stores is $49.95, which store is the cheaper place for Evie to buy two pairs? How much would she save, compared with the other store?

20 A furniture store purchases a dining table and chairs for $750. It marks up this price by 45% to obtain its selling price.

(a) What is the selling price of the dining set? State your answer correct to the nearest dollar.

(b) The furniture is not selling, so the selling price is discounted by 45%. Calculate the sale price to the nearest dollar.

(c) The mark-up on the cost price was 45%. Why doesn't a 45% discount on the selling price return the price to the original cost price?

(d) What percentage saving will they be able to advertise if they take the price back to the cost price? State your answer correct to the nearest whole number.

21 If you wanted to be able to advertise a discount of 30% on goods that cost you $5000 and still make a profit of 10%, at what price would you initially advertise them? Give your answer to the nearest dollar.

Open-ended

22 (a) An outdoor furniture setting has a marked price of \$999. State three different percentage discounts, correct to 1 decimal place, that would make the selling price between \$750 and \$800. Show your calculations.

(b) A lounge suite is sold for \$2450. State three different percentage discounts, correct to 1 decimal place, that could have been applied to the lounge suite if you know the marked price was somewhere between \$3000 and \$3200. Also state the marked price for your discount. Show the calculations that allow you to state your answer.

23 Using an example, explain why taking an additional 20% off an item already discounted by 30% is not the same as one single overall discount of 50%.

Problem solving

Patrick's prized painting

Patrick is an art lover who bought a painting for \$6000. A year later, Patrick has some financial difficulties and has to sell his painting for \$7000. As soon as he is able to, he buys the painting back again. Unfortunately he has to pay \$8000 to get it back. A year later Patrick moves house. He no longer has room for the painting, so he sells it for \$9000.

How much profit or loss does Patrick make?

Strategy options

- Act it out.
- Draw a diagram.
- Break problem down into manageable parts.

Gamespace

SHOPBAY

Greatest gain

'ShopBay' is an online auction site where people can sell goods. The seller has two options when listing an item for sale:

- put a 'starting bid' price and allow people to place higher bids for the item before a specified 'sale time' (say, 9 pm Thursday) or
- select a 'buy-it-now' option with a fixed price.

Penny decided to sell some things online on 'ShopBay'.

1 Which option, A or B, gives her more profit for each of the following items?

Item	Option A	Option B
Yoga mat	Starting price \$10. The price increases 50% by the sale time	Sell for a buy-it-now price of \$17
Guitar	Starting price \$200. The price increases 70% by the sale time	Sell for a buy-it-now price of \$332
Blue wig	Starting price \$24. The price increases 84% by the sale time	Sell for a buy-it-now price of \$45
Shoes	Starting price \$60. The price increases 20% by the sale time	Sell for a buy-it-now price of \$70
Antique teapot	Starting price \$330. The price increases 44% by the sale time	Sell for a buy-it-now price of \$450
Smartphone	Starting price \$50. The price increases 200% by the sale time	Sell for a buy-it-now price of \$150

True or false?

2 Which of the following statements are true and which are false?

(a) 'An easy way to calculate 30% is to calculate 10% and multiply it by 3.'

(b) 'To find 100% of something you multiply by 100.'

(c) 'An easy way to calculate 25% is to halve and then halve again.'

(d) 'To find 50% of something you multiply by $\frac{1}{50}$.'

(e) 'To find 150% you multiply by 2.5.'

(f) 'To find 1% you find 10% and divide by 5.'

(g) '20% is the same as a $\frac{1}{5}$.'

(h) 'To reduce something by 30% you multiply by 0.7.'

(i) 'To find 30% of something you divide by 3.'

3 For the statements that were false, rewrite them so they are correct.

Payment maze

At the end of your shopping on 'ShopBay' the total cost of the items in your online shopping cart is \$150.

There are a few different discounts and costs that must be added.

Find a way through the Payment Maze so that you pay the least amount at the checkout at the end.

1.3 Earning an income

The payment, or **income**, you receive for working in a particular job can be calculated in several different ways depending on the type of work done and the agreed working conditions.

Awards

An **award** is a document that sets out the minimum conditions of employment for employees in a certain occupation or industry. For example, there are awards for architects and airline pilots, or for people working in the fast food industry or hair and beauty industry.

Important information contained in awards includes:

- descriptions of the different types of jobs covered under the award
- the basic hourly, weekly or annual rate of pay for each of the different types of job
- standard hours of work—a full time job is usually around 38 hours per week
- extra amounts paid for extra or unusual hours worked (often called 'loading' or 'penalty rates')
- recreation leave—at least 4 weeks a year of paid recreation leave (i.e. time off work for a holiday) for full-time employees
- other types of paid and unpaid leave, such as sick leave, carer's leave or parental leave
- other working conditions, such as rest and meal breaks, allowances, training and dispute resolution.

Wage requirements for employers

Employees must be paid at least the minimum rate in the award for their industry—it can be more, but it can't be less. Many awards have special conditions for junior employees (those under 21 years of age).

Agreements and contracts

An agreement is like an award created specifically for the employees of a particular company. An agreement must be at least equal to the corresponding award.

Some people, often in a specialised or managerial position, sign an individual, negotiated contract before beginning their employment.

Wages

Wages are paid to people in many different job areas, including retail (shop assistants, packers), hospitality (waiting staff, kitchen staff) and trades (electricians, plumbers, hairdressers). Wages are based on a standard hourly rate of pay that applies to an agreed number of 'ordinary' hours, worked between set times. Extra work, or work outside of ordinary hours (such as evenings, weekends or public holidays), is paid as **overtime** at a higher hourly rate.

Worked example 9 W.E. 9

Sim is employed as a junior intern at a photographic studio. Sim's job is covered by an agreement that says he must be paid a standard hourly rate of $15.80 for a standard 38-hour Monday to Friday week. On Saturday he is paid time-and-a-half (1.5 times the standard hourly rate) for the first 3 hours and double time (twice the standard rate) for any additional hours thereafter. One week, Sim worked 4.5 hours on Saturday in addition to his standard hours. How much money did he earn?

Thinking	Working
1 Find the number of hours worked at each different rate. (The first 3 hours of a Saturday are paid time-and-a-half so the remaining 1.5 hours must be paid double time.)	Standard hours: 38 Time-and-a-half: 3 hours Double time: 1.5 hours
2 Multiply each number of hours by the appropriate hourly rate. (Multiply the standard rate by 1.5 for time-and-a-half and by 2 for double time.)	Standard hours: 38 × 15.8 = $600.40 Time-and-a-half: 3 × 15.8 × 1.5 = $71.10 Double time: 1.5 × 15.8 × 2 = $47.40
3 Add up the individual amounts to find the total wage.	Total = $718.90

Salary

People in professions, such as teachers, lawyers, engineers, scientists and accountants, are usually paid a **salary**—a set amount paid every fortnight or month. Other people who may also be on salaries are usually permanent employees of a company.

People on salaries usually do not get overtime payments for extra hours worked; occasionally, they may receive some time off instead.

Commission

A **commission** is usually paid to people working in sales, such as real estate agents, car salespeople, or representatives of manufacturing companies. The commission paid is either a percentage of the value of the sales achieved or a set amount for every product sold. Salespeople are also paid a **retainer** (a regular amount) per week or fortnight, to which any commission is added.

Worked example 10

W.E. 10

Lina works as a real estate agent. She receives a retainer of $450 a fortnight, plus a commission of 2.3% of the value of the property she sells. How much does Lina earn in a fortnight in which she sells a house for $340 000?

Thinking	Working
1 Let *C* be the value of the commission. Find *C* by calculating the stated percentage of the sale amount.	$C = 2.3\% \text{ of } \$340\,000$ $= 0.023 \times 340\,000$ $= \$7820$
2 Add the commission to the retainer to calculate Lina's total income.	$\text{Total income} = 7820 + 450$ $= \$8270$

Piece work

Piece work applies to workers in some manufacturing industries, such as the clothing, footwear and textiles industry, and other jobs, such as fruit picking and sheep shearing. Workers are paid per item produced (e.g. for each pair of jeans made or each bucket of grapes picked) rather than for the number of hours worked.

Worked example 11

W.E. 11

Emma is employed sewing dresses. She is paid at the rate of $7.50 per dress and she can complete 42 dresses each week. Calculate her weekly income.

Thinking	Working
Multiply the number of items produced (42) by the rate ($7.50).	$\text{Income} = 42 \times 7.5$ $= \$315$

Types of income

Wages:	Hourly rate of pay. Extra work may be paid as overtime at penalty rates (time-and-a-half, double time).
Salary:	Weekly, monthly or annual rate of pay. Overtime not usually paid; time off may be given instead.
Commission:	Retainer or flat fee plus percentage of the value of sales made, or a certain amount for every sale.
Piece work:	Payment per item produced.

1.3 Earning an income

Navigator

1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13 (a, b), 14, 17, 21	1 (a, b), 2 (a, b), 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 19, 21	3, 4, 5, 6, 8, 9, 11, 12, 13 (c, d), 14, 15, 16, 18, 19, 20, 21

Answers p. 629

Fluency

1 Calculate the weekly wage for the following if each person works a standard 38-hour week.

(a) A retail store worker earns $20.50 per hour.

(b) A plumber earns $28.80 per hour.

(c) An electrician earns $35.85 per hour.

(d) A supermarket worker earns $19.70 per hour.

2 Calculate the overtime wage each of the following people earns.

(a) A shop assistant on $18.70 per hour works 3 hours at time-and-a-half.

(b) A hairdresser on $19.85 per hour works 2.5 hours at time-and-a-half.

(c) A driver on $27.80 per hour works 3.5 hours at time-and-a-half.

(d) A cabinet maker earning $16.79 per hour, works 2 hours at time-and-a-half.

3 Teresa is paid time-and-a-half for the first 3 hours she works on the weekend, then paid double time for any more hours after that. Calculate the number of hours that she works at each rate if she works the following hours on a weekend.

(a) 6 hours total (b) 10.5 hours total (c) 4.5 hours total

4 Jake works in a clothing store. Under his award he is paid $14.70 per hour for a standard 38 hour week, time-and-a-half for the first 2 hours on Saturday and double time for any hours after that. How much will he receive if he works a standard week, and then 5 hours on Saturday? **W.E. 9**

5 Brian works as a secondhand car salesperson. He is paid a retainer of $180 per week plus a commission of 1.5% of the value of his sales. In a week in which he sells cars worth a total of $41 000, how much will Brian be paid? **W.E. 10**

6 A truck driver is paid $12.75 per tonne of timber delivered to a wood-chipping mill. If she delivers 120 tonnes to the mill, how much will she be paid? **W.E. 11**

7 A casual worker earns $10.75 per hour stacking shelves in a supermarket. If she works 18.5 hours in 5 days she will be paid:

A $18.5 \times \$10.75$ **B** $18.5 \times \$10.75 \times 5$ **C** $18.5 \times \dfrac{\$10.75}{5}$ **D** 18.5×5

8 If a retail assistant, paid at the rate of $9.80 per hour, works 7 hours standard plus 3 hours at time-and-a-half and 1 hour double time, the wages for that week will be:

A $\$(7 \times 9.8 + 3 \times 4.9 + 1 \times 19.6)$ **B** $\$(7 \times 9.8 + 3 \times 14.7 + 1 \times 19.6)$

C $\$(7 \times 9.8 + 3 \times 9.8 + 1 \times 9.8)$ **D** $\$(7 \times 9.8 + 3 \times 4.9 + 1 \times 4.9)$

9 A salesperson is paid a $130 retainer per week plus 2.5% of sales. If $20 000 worth of goods are sold during one week, the income will be:

A $\$2.5 \times 20\,000$ **B** $\$130 + (2.5 \times 20\,000)$

C $\$130 + 0.025 \times 20\,000$ **D** $\$0.025 \times 20\,000$

10 The following time sheet summarises the hours worked by four employees. Calculate the total income of each employee for the week if the standard rate is $16.40 per hour.

	Standard hours	Time-and-a-half	Double time
Adam	35	2	0
Britta	25	4	4
Con	30	0	4
Deng	20	2	2

Understanding

11 Aleesha is 18 and works part-time at a fast food outlet. The adult weekly wage for her position is $656 for a standard week consisting of 38 hours. As a trainee Aleesha receives 80% of the adult hourly rate.

(a) Calculate Aleesha's hourly rate.

Aleesha works from 4 pm to 8 pm, Wednesday to Sunday. Her award states that after 6 pm on weekdays, she must be paid an extra 10% of her hourly rate. An extra 25% is paid for all hours worked on Saturday, and an extra 75% for all hours worked on a Sunday.

(b) Calculate Aleesha's weekly wage.

Hint

Calculate the hourly rate of a full-time person first.

12 Marco works as a part-time apprentice hairdresser. His award states that a full-time person in his position must be paid a minimum wage of $650 for an ordinary 38-hour week, plus an extra 33% of the hourly rate for any hours worked on a Saturday. Calculate Marco's weekly wage if he works 8 hours a day for 4 weekdays and 5 hours on Saturday.

13 A real estate agent is paid commission on the sale of houses at the following rate:

2% on the first \$150 000 1.5% on the next \$100 000 1% on the rest

How much would the seller have to pay in commission if a house was sold for:

(a) \$280 000 **(b)** \$340 000 **(c)** \$495 000 **(d)** \$600 000?

14 Hayley is a telemarketer with an insurance company. She is paid a retainer of \$275 a week, plus \$50 commission for every customer she signs up on a new insurance policy. One week, Hayley sells five \$149 policies and three \$179 policies.

(a) Calculate Hayley's earnings for the week.

(b) Calculate Hayley's commission as a percentage of the value of the products she sold. Give your answer to 1 decimal place.

15 Erica sells cars and receives a weekly retainer of \$150 plus 0.75% of the value of the cars she sells. In a particular week Erica earns \$1275. What was the value of the cars Erica sold that week?

16 Patrick is paid \$28 per hour for the first 38 hours worked in a week. He is then paid time-and-a-half for the next 6 hours worked and double time for anything beyond that. How many hours did Patrick work in a week in which he earned:

(a) \$1190 **(b)** \$1372 **(c)** \$1652?

(d) How many hours overtime would need to be worked so that his total overtime payments were at least as much as his standard hours payment?

Reasoning

17 Another difference between earning wages and earning a salary is that some types of payments are additional to wage earnings, but are already incorporated into a salary.

(a) Hong gets paid a basic weekly wage of \$450. Her leave loading, added onto her pay for holidays, is 17.5% of her basic weekly wage.

(i) Calculate her leave loading per holiday week.

(ii) What pay will she receive during a 2-week holiday period?

(b) Barry's income is \$1250 per fortnight. If he receives leave loading of 17.5%, what pay will he receive for a 4-week holiday period?

18 When employed as a salesman, Nabil was given a choice as to how he would be paid.
Option A: A retainer of \$250 per week plus 2.5% of his sales.
Option B: No retainer but 3.5% of his sales.

(a) Nabil needs to earn at least \$1250 per week. What value of sales does he need to make under each payment option to meet this requirement? Give answers correct to the nearest dollar.

(b) What value of sales will give Nabil the same pay regardless of the option chosen?

(c) So, when would it be better for Nabil to choose Option A?

19 Lucy is the manager of a toy store. She earns a salary of \$48 700, which is paid fortnightly.

(a) If there are 26 fortnights in a year, calculate Lucy's fortnightly pay to the nearest cent.

(b) If Lucy works a 40-hour week, what is her equivalent hourly rate?

(c) During the mid-year stocktake, Lucy worked an extra 2 hours each weekday. She was not paid overtime. What was Lucy's equivalent hourly rate for that week?

Open-ended

20 Malik is an 18-year-old first-year apprentice electrician. His industry award states that first-year apprentices should be paid 40% of the adult hourly rate of $16.78, rising to 52% in the second year, 70% in the third year and 82% in the fourth and final year. Malik works 8 hours a day, 4 days a week. He goes to TAFE on the fifth day (he isn't paid for this). He also gets a tool allowance in his pay of $17 a week.

Malik thinks that he could earn more working the same number of hours in a fast food outlet. The award for that industry states that 18-year-old employees earn 70% of the minimum weekly adult wage of $600 for a 38-hour week, rising by 10% every year until they turn 21 and are on the full adult rate.

(a) Do some calculations to decide whether Malik would earn more at a fast food outlet than on his apprenticeship, either now or in the future.

(b) What would you advise Malik to do? (Some additional information to consider: fully qualified electricians can earn between $40 000 and $80 000 a year.)

21 Tegan has a casual position in a local supermarket. As a casual, Tegan is not entitled to leave and other benefits, but she is paid an extra 25% of the regular hourly rate of $15.80 an hour. She gets a further 10% per hour on Saturdays and Sundays.

Tegan has weekly living expenses (food, rent, transport etc.) of $300.

Give two or three different combinations of weekday and weekend hours that Tegan could work in order to earn $300 a week.

Problem solving

Careless Carly

Carly is given a savings account by her parents with some money already deposited into it. Carly is trying to avoid withdrawing money from the account but is not very good at sticking to her plan, so she withdraws $\$m$ every week. Soon she realises she has already withdrawn half the available money from the account. The next week Carly withdraws another $\$m$ and sees that the ratio of withdrawn money to money remaining is now 3 : 2. What is the ratio of withdrawn money to money remaining when Carly withdraws another $\$m$?

Strategy options

- Guess, check and improve.
- Test all possible combinations.

Investigation

Working out the roster

Equipment required: timetable (can be drawn according to the description given below; or, a template can be downloaded from the eBook or the Pearson Places website)

Marie is the supervisor of the toy department in a large store. She has to organise the weekly roster for her department. The store's opening hours are:

Mon–Thu: 9 am–6 pm
Fri: 9 am–9 pm
Sat: 9 am–5 pm
Sun: 10 am–4 pm

Employees must be at work half an hour either side of the opening and closing times, to set up and pack up the store.

Marie is the only full-time employee, working regular hours of 8:30 am–5 pm, Mon–Fri. Marie's job classification under the Retail Employees Award is Level 4.

Marie has one permanent part-time member on her team—Nicki, a Level 3 employee who works the regular hours of 8:30 am–12:30 pm, Tue–Sat.

Marie has four casual staff on her team. They are:

Name	Age	Job classification	Availability
Jian	20	Level 2 casual	Any time Tue–Thu, Sat and Sun
Hamish	19	Level 2 casual	After 12 pm on Mon, Wed and Fri, all day Sat
Sally	18	Level 1 casual	Any time Mon and Tue, and a maximum of 8 hours on the weekend
David	17	Level 1 casual	After 4 pm weekdays, after 12 pm on weekends

When Marie writes her roster, she needs to take into account the following conditions:

- two staff must be rostered on at all times, one of whom must be a supervisor
- if Marie is not working, the other supervisors are Nicki, Jian or Hamish
- no employee can be asked to work more than 9 hours at a time
- the minimum number of hours an employee must be paid for is 3 (even if they work less than this)
- neither Marie nor Nicki wish to work overtime (i.e. outside their regular hours)
- the retail award gives the minimum weekly wages for each of the levels of staff as follows:

Classification	Per week $
Retail Employee Level 1	721.50
Retail Employee Level 2	738.70
Retail Employee Level 3	750.20
Retail Employee Level 4	764.90

- staff under 21 are paid a percentage of the adult rate according to the following table:

Age	% of adult weekly rate of pay
17	60
18	70
19	80
20	90

The Big Question

Marie needs to design a roster that takes into account her staff's availability and the above conditions, and costs the company as little as possible in wages. What could Marie's roster for the week look like?

Engage

1 Use the tables above to calculate the standard hourly rate for Marie, Nicki, Jian, Hamish, Sally and David.

Explore

2 **(a)** Use the timetable template to allocate hours to each staff member.

(b) Once you have all the hours covered, see if you can shuffle or juggle the hours of each staff member to bring down the total wages that the store will need to pay.

Strategy options

- Draw up a table.
- Test all possible combinations.
- Guess and check.
- Break problem down into manageable parts.

Explain

3 **(a)** Does your roster take everyone's availability into account?

(b) Were any days harder to staff than others?

Elaborate

4 Is your roster the 'cheapest possible' roster? How confident are you of your answer?

5 Which staff member was most useful in 'plugging gaps' in the roster?

6 Which staff member was most difficult to place in the roster?

Evaluate

7 Summarise the main steps or stages you went through in completing your roster. Was there a particular order in which you placed your staff into the roster?

8 Which of Marie's list of conditions did you have the most difficulty remembering or ensuring it was met?

9 Why do you think there is a condition about the minimum number of hours for which a person can be paid?

Extend

10 Use the award information on minimum wages and junior rates to calculate the actual wages of each staff member and the total wage bill for the store for the week of your roster.

Tax and other pay deductions

1.4

What is tax?

Tax is money collected by the government from Australian residents. It is used to pay for public services, such as hospitals, schools, the police and defence forces and transport (road and train networks).

Income tax

The Federal Government collects **income tax** by having employers take a certain amount of income from each employee every time they are paid (usually fortnightly or monthly). This is known as the Pay As You Go (**PAYG**) system. The amount of tax depends on your income. The table below shows how income tax was calculated in the financial year 2015–16. The incomes given are annual incomes. Each row on the table shows the tax payable on incomes within a certain range. The ranges shown on each row of the table are often called 'tax brackets'.

Taxable income	Tax on this income
\$0–\$18 200	Nil
\$18 201–\$37 000	19c for each \$1 over \$18 200
\$37 001–\$80 000	\$3572 plus 32.5c for each \$1 over \$37 000
\$80 001–\$180 000	\$17 547 plus 37c for each \$1 over \$80 000
\$180 001 and over	\$54 547 plus 45c for each \$1 over \$180 000

Reference: https://www.ato.gov.au/rates/individual-income-tax-rates/

You can see from the table that no tax is paid if the annual income is \$18 200 or less. This is called the tax-free threshold. The amount you earn in total, before any tax is taken out, is called **gross income**. The amount left after tax and other deductions is called **net income** (or 'take home pay').

Superannuation

Superannuation (super) is a way of saving money for retirement. Employers are required to regularly pay a certain percentage of their employee's **earnings** into an account or fund. This payment is made in addition to the employee's salary. The money can then be used when a person retires (if they are over a certain age), or in an extreme situation such as death, illness or disability. Employees can also make voluntary contributions to their super in addition to those made by their employer.

Other pay deductions

Apart from income tax, other deductions from an employee's pay must be agreed to in writing by the employee. Unless it forms part of the employee's award or contract of employment, it is illegal for an employer to deduct money for things such as uniforms or tools without the employee's consent.

Net income = gross income – (tax + voluntary super contributions + any authorised deductions)

Worked example 12

W.E. 12

Calculate **(a)** the annual income tax and **(b)** the annual net income for an annual gross income of \$68 000. Assume no other pay deductions are made.

Thinking	Working
(a) **1** Use the tax table to find the tax bracket that this annual gross income falls into.	(a) Tax = \$3572 + 32.5c for each \$1 over \$37 000
2 Subtract the highest figure of the previous tax bracket (\$37 000 in this case) from the annual gross income.	$68\,000 - 37\,000 = 31\,000$
3 Calculate the stated proportion of the remaining amount (here it is 0.325, which is equivalent to 32.5c in every \$1).	$0.325 \times 31\,000 = 10\,075$
4 Add the amount calculated to the stated fixed amount (\$3572) to find the annual income tax.	Annual tax payable $= 10\,075 + 3572$ $= \$13\,647$
(b) Subtract the annual income tax from the annual gross income to find the annual net income.	(b) $68\,000 - 13\,647$ Annual net income $= \$54\,353$

Worked example 13

W.E. 13

Calculate **(a)** the tax that is taken each pay and **(b)** the net income each pay for a gross income of \$980 paid fortnightly. Assume no other pay deductions are made. (Assume there are 26 fortnights in a year.)

Thinking	Working
(a) **1** Calculate the annual gross income.	(a) $\$980 \times 26 = \$25\,480$
2 Use the tax table to find the tax bracket that this annual gross income falls into.	Tax = 19c for each \$1 over \$18 200
3 Subtract the tax-free threshold (\$18 200).	$25\,480 - \$18\,200 = 7280$
4 Calculate the stated proportion of the remaining amount (here it is 0.19, which is equivalent to 19c in every \$1) to find the tax payable.	Annual tax payable $= 0.19 \times 7280$ $= \$1383.20$
5 Divide the annual amount of tax by 26 to find the fortnightly tax. Round to the nearest cent.	Fortnightly tax $= 1383.2 \div 26$ $= \$53.20$
(b) Subtract the fortnightly tax from fortnightly gross income to find net income for the fortnight.	(b) Fortnightly net income $= 980 - 53.20$ $= \$926.80$

Tax deductions

Your **taxable income** is the part of your income that the government believes you should pay tax on. A **tax deduction** reduces the amount of income on which you have to pay tax. If you spend money on something that helps you to earn your income, you may be entitled to claim that cost as a tax deduction. These work-related expenses are different for each job, but may include tools, equipment and protective clothing necessary for your job, membership fees for professional associations or unions, or fees for attending training courses. If your work requires you to use your car or phone, you may be able to claim some of the costs of these. Other tax deductions include donations to recognised charities.

Taxable income = gross income − tax deductions

Tax returns

At the end of the financial year, most people complete a **tax return**. This is a document used to assess whether the total amount of tax paid over the year is correct. It takes into account all of a person's income (earned through work or investments) and deductions they can claim, such as work-related expenses.

Depending on their tax return, a person may receive a tax refund or may have to pay more tax.

Worked example 14

W.E. 14

Carrie is an engineer whose annual gross salary is $95 000. During the year she had work-related expenses totalling $1200, paid membership fees to Engineers Australia of $255 and donated $650 to the Red Cross.

(a) Calculate Carrie's taxable income.

(b) Calculate how much income tax Carrie should have paid using the table on p. 33.

Thinking	Working
(a) Subtract allowable deductions from gross income to find the taxable income.	(a) Taxable income = 95 000 − (1200 + 255 + 650) = 95 000 − 2105 = $92 895
(b) **1** Use the tax table to find the tax bracket that this income falls into.	(b) Tax payable is $17 547 + 37 cents for each $1 over $80 000.
2 Subtract the highest figure of the previous tax bracket ($80 000 in this case) from the taxable income.	92 895 − 80 000 = 12 895
3 Calculate the stated proportion of the remaining amount (here it is 0.37, which is equivalent to 37c in every $1).	0.37 of $12 895 = 0.37 × 12 895 = 4771.15
4 Add the amount calculated to the stated fixed amount ($17 547) to find the total tax payable.	Tax payable = 4771.15 + 17 547 = $22 318.15

1.4 Tax and other pay deductions

Navigator

Answers p. 629

1, 2 (a–e), 3 (a, b), 4, 5, 7, 8, 10, 13, 15	2 (a–e), 3 (a–d), 4, 6, 7, 8, 9, 10, 11, 13, 15	2 (d–h), 3 (c–f), 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

Fluency

1 Write the annual net income for the following situations.

- **(a)** Annual gross income = \$56 000, annual income tax = \$9747
- **(b)** Annual gross income = \$69 567, annual income tax = \$14 156.27
- **(c)** Annual gross income = \$71 457, annual income tax = \$14 770.52
- **(d)** Annual gross income = \$59 767, annual income tax = \$10 971

W.E. 12

2 Calculate **(i)** the annual income tax and **(ii)** the annual net income for the following annual gross incomes. Assume no other pay deductions are made.

(a) \$45 674 **(b)** \$77 365 **(c)** \$125 000 **(d)** \$29 576

(e) \$14 033 **(f)** \$200 000 **(g)** \$59 960 **(h)** \$5976

W.E. 13

3 Calculate **(i)** the tax deducted each pay and **(ii)** the net income each pay for the following gross incomes. Assume no other pay deductions are made and that there are 12 months, 26 fortnights and 52 weeks in a year.

(a) \$725, paid fortnightly **(b)** \$4630, paid monthly

(c) \$1262.39, paid fortnightly **(d)** \$176, paid weekly

(e) \$8635.24, paid monthly **(f)** \$537.04, paid weekly

W.E. 14

4 Rosanna is a nurse who receives an annual gross income of \$58 700.

- **(a)** What is her taxable income if she donates \$200 a year to Amnesty International, has work expenses totalling \$575 and pays \$160 for her Nurses Federation membership.
- **(b)** Calculate how much income tax Rosanna should pay annually using the tax table on p. 33.

5 Carol earns an annual gross income of \$146 730. She donates \$90 to charity and pays \$130 for union membership.

- **(a)** What are the total tax-deductible expenses?
- **(b)** Calculate the taxable income.
- **(c)** Calculate how much tax must be paid.

6 Tong has the following tax deductions: \$840 for charity, \$973 for work-related expenses, \$150 for professional membership fees. He earns an annual income of \$43 112.

- **(a)** What are the total tax-deductible expenses?
- **(b)** Calculate the taxable income.
- **(c)** Calculate how much tax must be paid.

7 The tax payable on \$56 740 is closest to:

A \$6416 **B** \$9988 **C** \$9999 **D** \$18 441

Understanding

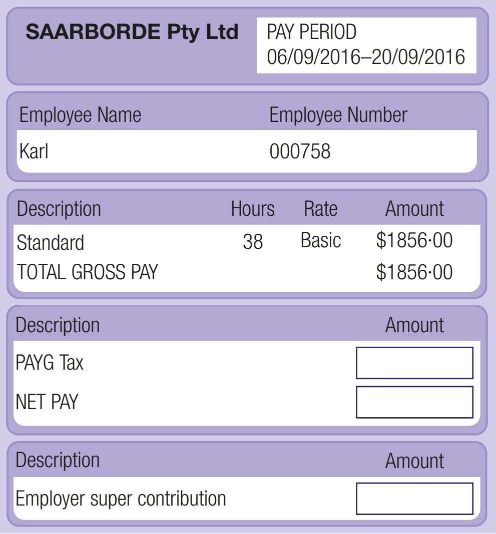
8 Karl's fortnightly gross income is $1856. There are 26 fortnights in a year.

(a) Calculate Karl's annual gross income.

(b) Use your answer to **(a)** to calculate Karl's fortnightly PAYG tax, using the tax table on p. 33.

(c) Find Karl's fortnightly 'net' pay (take-home pay).

(d) On top of his pay, Karl's employer must pay superannuation at a rate of 9.5% of his gross income. Calculate the employer's fortnightly super contribution.



SAARBORDE Pty Ltd

PAY PERIOD
06/09/2016–20/09/2016

Employee Name	Employee Number
Karl	000758

Description	Hours	Rate	Amount
Standard	38	Basic	$1856·00
TOTAL GROSS PAY			$1856·00

Description	Amount
PAYG Tax	
NET PAY	

Description	Amount
Employer super contribution	

9 Leanne earns a gross annual income of $67 200 and is paid monthly. Every month her employer also calculates 9.5% of her gross income for the month and pays it into her superannuation fund.

(a) How much is paid into Leanne's fund each month?

Leanne decides to make a voluntary super contribution of $80 each pay.

(b) Including Leanne's voluntary contribution, how much is paid into her super account by the end of the year?

(c) Calculate Leanne's monthly tax on her gross income (before her voluntary super of $80 is taken out).

(d) What is Leanne's net monthly income after tax and the voluntary super contribution are deducted?

10 Adila has two permanent part-time jobs. In her first job, she works 15 hours a week at an hourly rate of $18.29 per hour. In her second job, she works 12 hours a week at an hourly rate of $16.43 an hour. For her second job, Adila bought some protective clothing that cost $125. Her union fees cost her $97 a year. Assume there are 52 weeks in a year.

(a) Calculate Adila's annual gross income.

(b) Calculate Adila's annual taxable income.

(c) Calculate the tax payable on Adila's taxable income.

11 At the beginning of the financial year Justin started in a new position with an annual gross salary of $54 500. There are 52 weeks in a year.

(a) How much should Justin pay per year in tax?

(b) How much is Justin's weekly gross income?

(c) How much per week does Justin get withheld as PAYG tax?

Justin decides to leave his job after 34 weeks in the position, and travel around Australia without working for the rest of the financial year. Before leaving Justin had earned $35 635 and paid $6054 in PAYG tax.

(d) Use the table to calculate how much tax Justin should pay on the amount he earned in the financial year.

(e) How much tax should Justin be refunded? Round your answer to the nearest dollar.

Reasoning

12 Brad and Jennifer are married. Jennifer has a taxable income of $80 000 and Brad does not do paid work.

(a) How much tax does Jennifer pay?

(b) What percentage (correct to the nearest whole number) of the taxable income does this represent?

Brad and Jennifer decide they would be better off if they both worked part-time. They are able to find jobs that give each of them a taxable income of $40 000.

(c) What is their combined tax contribution?

(d) What percentage (correct to the nearest whole number) is this of their combined taxable income?

(e) How much better off are Brad and Jennifer now?

(f) Why is this different to Jennifer's contribution when she was the sole earner?

13 The five rows in the tax table on p. 33 are sometimes called tax brackets. If you change to a higher-paid job you may move from one tax bracket to the next. The following pairs of incomes represent the 'border' between two tax brackets. Calculate the difference in the tax paid on them.

(a) $37 000 and $37 001 **(b)** $80 000 and $80 001 **(c)** $180 000 and $180 001

(d) Use your results to explain why the amounts added to each of the tax rates given in the table are $3572, $17 547 and $54 547.

Open-ended

14 Do you think the tax table rates are fair? Give a reason for your answer.

15 Why do you think the government makes it compulsory for employers to pay superannuation for their employees?

Game

Master of the market

Equipment required: 2 dice

How to play: To get ready to play, each player should draw up the following table in their book:

Company	Number of shares bought	$ invested	Dice roll	$ profit/loss	Running total

Each player begins with $3000 to invest in the sharemarket. The profit or loss you make on that investment is determined by the roll of a die.

- On your turn, choose one of the three companies below, decide how many shares you will buy (between 10 and 100) and calculate the value of your investment (the cost of buying the shares).
- Fill in the first three columns of your table.
- Roll the dice to see how well your investment performs. Take the number rolled and use the investment outcomes table on the right to calculate your profit or loss.
- Fill in the fourth and fifth columns in the table above.
- Add or subtract the profit or loss to $3000 to keep a running total in the sixth column.
- You may invest money you won in a previous turn on your next turn if you choose.

How to win: The winner is the first player to double their money.

Investment outcomes:

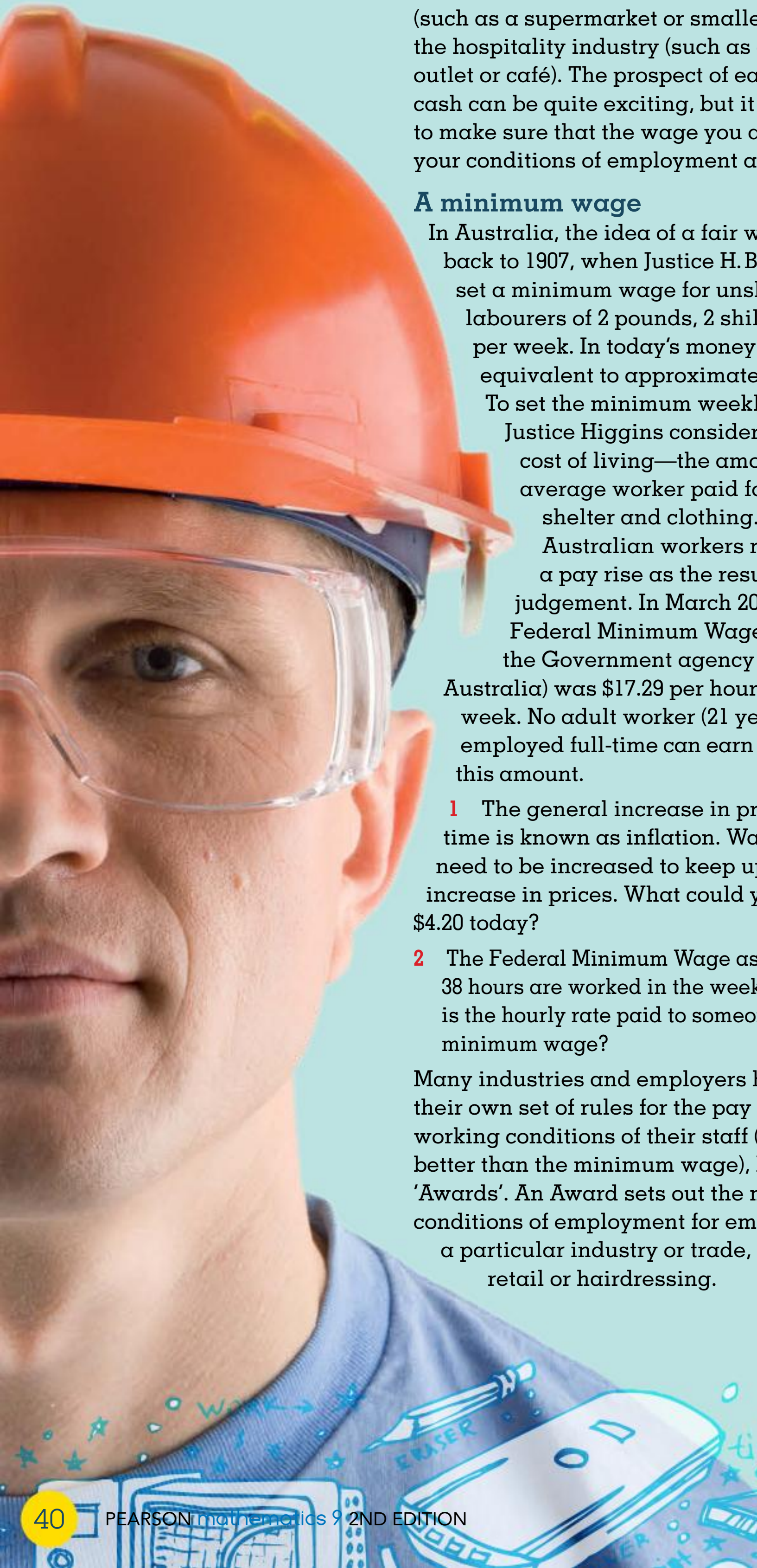
Dice roll	Profit or loss
2	100% loss
3	Lose everything except $50
4	75% loss
5	50% loss
6	25% loss
7	No profit or loss
8	25% profit
9	50% profit
10	75% profit
11	100% profit
12	200% profit

Companies:

- Company A $5 per share
- Company B $10 per share
- Company C $15 per share

Maths 4 Real

A FAIR



Many students seek a part time job while still at school—often in the retail sector (such as a supermarket or smaller shop) or the hospitality industry (such as a fast food outlet or café). The prospect of earning some cash can be quite exciting, but it is important to make sure that the wage you are paid and your conditions of employment are fair.

A minimum wage

In Australia, the idea of a fair wage goes back to 1907, when Justice H. B. Higgins set a minimum wage for unskilled labourers of 2 pounds, 2 shillings per week. In today's money this is equivalent to approximately $4.20. To set the minimum weekly wage, Justice Higgins considered the cost of living—the amount an average worker paid for food, shelter and clothing. Many Australian workers received a pay rise as the result of this judgement. In March 2016, the Federal Minimum Wage (set by the Government agency Fair Work Australia) was $17.29 per hour, or $656.90 a week. No adult worker (21 years or older) employed full-time can earn less than this amount.

1 The general increase in prices over time is known as inflation. Wages also need to be increased to keep up with this increase in prices. What could you buy for $4.20 today?

2 The Federal Minimum Wage assumes that 38 hours are worked in the week. What is the hourly rate paid to someone on the minimum wage?

Many industries and employers have their own set of rules for the pay and working conditions of their staff (often better than the minimum wage), known as 'Awards'. An Award sets out the minimum conditions of employment for employees in a particular industry or trade, for example retail or hairdressing.

Awards

Important information contained in Awards includes:

- descriptions of the different job classifications covered under the award
- the minimum weekly wage for each of the different job classifications
- the rates paid to junior employees (people under 21 years of age), apprentices and trainees
- extra amounts (often called 'loading' or 'penalty rates') paid for overtime
- special conditions for casual employees.

Here is some information taken from the General Industry Retail Award 2010. This award covers people who work in places such as supermarkets, department stores and clothing and furniture shops.

Minimum weekly wages (for a 38-hour week)

Classification	Per week
Retail Employee Level 1	$721.50
Retail Employee Level 2	$738.70
Retail Employee Level 3	$750.20
Retail Employee Level 4	$764.90

Junior rates

Junior employees are paid the following percentage of the minimum wages given above:

Age	% of weekly rate of pay
Under 16 years of age	45
16 years of age	50
17 years of age	60
18 years of age	70
19 years of age	80
20 years of age	90

WAGE

Overtime

Below are the overtime conditions for employees under the General Industrial Retail Award 2010:

Time worked	Conditions
Evening work Monday to Friday	An additional 25% will be paid for hours worked after 6 pm. This does not apply to casuals.
Saturday	An additional 25% will be paid for hours worked on a Saturday between 7 am and 9 pm. This does not apply to casuals.
Sunday	An additional 100% will be paid for all hours worked on a Sunday. For casuals, an additional 60% is paid for work between 7 am and 6 pm.
Public holidays	An additional 150% will be paid for all hours worked on a public holiday.

Casual employees

- Casual employees are paid an additional 25% of the hourly rate for full-time employees.
- Casual employees are not paid overtime for evening work or Saturday work, but must be paid an additional 60% of their hourly rate for work performed on Sunday between 7 am and 6 pm.

3 Sabrina is 15 and has just started work as a Retail Employee Level 1 on a casual basis.

(a) Calculate Sabrina's hourly rate by first calculating the casual hourly rate of an adult in her position.

(b) Sabrina works from 4 pm to 7 pm on Tuesday and Friday, and from noon to 5 pm on Saturday. When Sabrina works for 5 hours or more, she must take an unpaid meal break of 30 minutes. (Depending on the hours worked, she also gets one or two 10-minute rest breaks, but these are paid.)

Calculate Sabrina's pay for the week.

4 Adam is 19 and has a full-time job with the same company as a Retail Employee Level 2.

(a) Calculate Adam's hourly rate.

(b) Adam works from 8 am to 4 pm Monday to Friday, with a 30-minute unpaid meal break every day (plus two 10-minute paid breaks). Calculate Adam's weekly pay.

5 Here is Sabrina's work roster for the Christmas period:

24 Dec (Mon)	26 Dec (Wed)	27 Dec (Thurs)	28 Dec (Fri)	29 Dec (Sat)	30 Dec (Sun)
11 am –7 pm	10 am –4 pm	8 am –2 pm	9 am –5 pm	9 am –5 pm	9 am –5 pm

26 December is a public holiday, and the store is closed on Christmas Day (25 December).

Calculate Sabrina's pay for this time period, remembering that she must take a 30-minute unpaid meal break if she works 5 hours or more. (It may help to calculate her hourly rate for each day first.)

6 Imagine that you got a casual job at the same level as Sabrina. (Alternatively, find the award that covers a job you are interested in.) Write the days and times you would like to work, taking into account school, study time, sport and other commitments.

(a) Calculate your hourly rate.

(b) Calculate your weekly pay.

Research

Find out more about the rights of young people in the workplace. Areas you could include are safety, discrimination, unpaid 'trial work', joining a union and termination ('getting the sack'). Use your research to make a poster, presentation or short film.

Half-time 1

1.1 1 (a) Giaan had 15 shots at the basketball ring from the free throw line. She got 11 in. Calculate Giaan's percentage shooting accuracy, correct to 1 decimal place.

(b) A box of 100 'T42' tea bags is on sale for \$5.38, a saving of 91 cents. Calculate the value of the saving as a percentage of the original price, correct to 1 decimal place.

1.3 2 Ying is working as a grape picker in a vineyard. He is paid \$3.85 for each bucket of grapes picked. How much does Ying earn if he picks 17 buckets in one day?

1.2 3 Find the selling price of the following items:

(a) coffee table, cost price \$48, mark-up 65%

(b) recliner chair, cost price \$197, mark-up 40%

(c) bookcase, marked price \$67, discount 17.5%

(d) chest of drawers, marked price \$299, discount $33\frac{1}{3}\%$

1.1 4 Lucas wants to buy a new skateboard. In shop A it has a marked price of \$170 and a sticker that says 'take \$40 off'. Shop B has the same skateboard with a marked price of \$165 and a sticker that says 'take 25% off'. Decide which skateboard is cheaper.

1.3 5 Sheri is a salesperson for a publishing company. She is paid a \$365 weekly retainer as well as 1.7% of the value of the sales she makes. One week she made the following sales: \$10 980, \$4372, \$23 065, \$17 834 and \$31 178. Calculate Sheri's earnings for the week.

1.2 6 Mo bought an old desk for \$155. He bought some special polish for \$47.50, polished it up and then sold the desk for \$260.

(a) Calculate Mo's profit as a percentage of the cost price (taking into account all costs), correct to 1 decimal place.

(b) Calculate Mo's profit as a percentage of the selling price, correct to 1 decimal place.

1.4 7 Jasminder's annual taxable income is \$47 530, which is paid fortnightly. Use the tax table on p. 33 to calculate:

(a) her annual income tax

(b) her net fortnightly pay (assume there are 26 fortnights in the year).

1.2 8 Naomi has been out shopping for bargains. She has bought a pair of jeans for \$65 that had been reduced by 10%, a jacket for \$47 that had been reduced by 35% and a scarf for \$11 that had been reduced by 20%. Calculate the original price on each of the items and find out how much Naomi saved in total.

1.3 9 Franco works as a waiter in a restaurant. He is paid an hourly rate of \$18.95, with time-and-a-half for any hours worked on Saturday or after 10 pm on weekdays and double time for any hours worked on Sunday.

Here is Franco's time sheet for the week:

Tuesday	5 pm–10 pm
Wednesday	5 pm–10 pm
Thursday	5 pm–midnight
Friday	5 pm–midnight
Saturday	noon–4 pm
Sunday	noon–4 pm

Calculate Franco's wage for the week.

1.5

Simple interest

What is interest?

When money is borrowed, the borrower pays the lender for the use of that money, in addition to paying the money back. This additional amount is called **interest**. The original amount borrowed is called the **principal**. Interest is charged as a percentage of the principal, calculated at regular time periods.

Similarly, when money is invested (deposited into an account) with a bank, the bank pays the investor interest in addition to keeping the money secure, because it uses the investor's money to fund the bank's business.

Simple interest

There are different ways in which interest can be calculated. **Simple interest** is calculated on the initial amount borrowed or invested, no matter how that amount changes over the time of the loan.

The amount of interest paid depends on:

- the initial amount of money borrowed or invested, called the principal P
- the **interest rate** r that is charged or paid
- the time period T for which the money is borrowed or invested.

Simple interest formula

The amount of simple interest is calculated as:

$$I = PrT$$

Where I = simple interest (in dollars)
P = principal (in dollars)
r = interest rate (per year) as a decimal
T = time (in years)

An annual percentage rate r is often called the rate 'per annum' or 'p.a.', which means 'by the year' in Latin. It is usually given as a percentage, but you can convert this rate into decimal form by dividing the percentage amount by 100.

Worked example 15

W.E. 15

Calculate the simple interest paid on a loan of \$5000 at 16.4% p.a. interest over 4 years.

Thinking	Working
1 Write the formula for calculating simple interest.	$I = PrT$
2 Write the values for P (the amount borrowed), r (the interest rate in decimal form) and T (time in years).	$P = \$5000$ $r = 16.4\% = 0.164$ $T = 4$
3 Substitute the values into the formula and calculate the interest.	$I = 5000 \times 0.164 \times 4$ $= \$3280$

Interest is not always paid yearly

Interest can be calculated and paid quarterly, monthly or even daily. In these cases you must express the time period *T* as a fraction of a year when working with an annual rate.

In the following worked example, interest is paid quarterly, so *T* is expressed as a fraction with a denominator of 4. When interest is paid daily or monthly, then *T* is a fraction of 365 or 12.

Worked example 16

W.E. 16

Barbara borrows \$24 000 to set up a business. She repaid it over 5 years and 3 months, after paying 18.5% p.a. in simple interest, payable quarterly.

(a) How much interest did she pay quarterly?

(b) What was the total amount of money repaid?

Thinking	Working
(a) **1** Write the simple interest formula.	(a) $I = PrT$
2 Write *P*, *r*, and *T*. As interest is paid quarterly, *T* is expressed as $\frac{3}{12}$, or $\frac{1}{4}$ of a year.	$P = \$24\,000$ $r = 18.5\% = 0.185$ $T = \frac{3}{12} = \frac{1}{4}$
3 Substitute into the formula and calculate the simple interest.	$I = \$24\,000 \times 0.185 \times \frac{1}{4}$ $= \$1110$ per quarter
(b) **1** Identify the number of quarterly periods taken to repay the debt.	(b) $T = 5$ years 3 months $= 5\frac{1}{4}$ years
2 Convert the mixed number to an improper fraction to find the number of quarters.	$= \frac{21}{4}$ years $= 21$ quarters
3 Multiply the interest paid per quarter by the number of quarters.	$I = \$1110 \times 21$ $= \$23\,310$
4 Add the interest to the principal to find the total amount repaid.	Total repaid $= P + I$ $= \$24\,000 + \$23\,310$ $= \$47\,310$

When calculating daily interest, count the number of days the balance remains at a certain amount. Assume that interest is calculated on the balance at the end of the day. This means that any deposits or withdrawals made in a particular day are added or subtracted first, then the interest is calculated on the final balance.

Worked example 17

W.E. 17

Craig's bank statement for September is shown at right.

If the account pays 6% interest calculated daily, calculate the interest accrued in September.

Date	Transaction		Balance
Sept 1			\$490.15
Sept 6	Deposit	\$180	\$670.15
Sept 19	Withdrawal	\$400	\$270.15

Thinking	Working
1 Write the formula.	(a) $I = PrT$
2 Identify *P*, the daily balances.	$P = \$490.15, \$670.15, \$270.15$
3 Identify the rate *r*.	$r = 6\% = 0.06$
4 Find *T* for each balance by counting the number of days the balance remained at that amount, and writing it as a fraction of 365.	$T = \frac{5}{365}, \frac{13}{365}, \frac{12}{365}$
5 Substitute into the formula the respective values of *P*, *r* and *T* and calculate correct to 2 decimal places.	$I = \$490.15 \times 0.06 \times \frac{5}{365}$ $= \$0.40$ $I = \$670.15 \times 0.06 \times \frac{13}{365}$ $= \$1.43$ $I = \$270.15 \times 0.06 \times \frac{12}{365}$ $= \$0.53$
6 Calculate the total interest by adding the individual amounts.	$I = \$0.40 + \$1.43 + \$0.53$ $= \$2.36$

Finding *P*, *r* or *T*

If three of the four variables in the simple interest formula are known, then the fourth can be calculated.

Worked example 18

W.E. 18

Allan put \$12 500 into a cash management account. He received \$16 220.45 when he withdrew all of the money 3 years later. What was the average rate of return, p.a., on his principal (correct to 1 decimal place)?

Thinking	Working
1 Write the formula.	$I = PrT$
2 Calculate the interest earned by subtracting the principal from the total.	$I = \$16\,220.45 - \$12\,500$ $= \$3720.45$
3 Identify *P*, *r* (unknown) and *T*.	$P = 12\,500$ $r = ?$ $T = 3$ years

4 Substitute into the formula.	$3720.45 = 12\,500 \times r \times 3$
5 Simplify as much as possible.	$3720.45 = 37\,500r$
6 Solve the equation for *r* by dividing on both sides.	$\frac{3720.45}{37\,500} = r$
7 Convert to a percentage correct to 1 decimal place.	$r = 0.099\,212$ $= 9.9\%$

1.5 Simple interest

Navigator

Answers p. 630

1, 2, 4 (a–c), 5, 8, 9, 10, 11, 12, 13, 14, 19, 22 (a–c), 25	1 (c, d), 2 (a, b), 3, 4 (c, d), 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 20, 21, 22, 23, 25	1 (c, d), 3, 4 (c, d), 5, 6, 9, 10 (a), 11 (a), 12 (a), 14, 15, 16, 17, 18, 20, 21, 22, 24, 25

Fluency

W.E. 15

1 (a) Calculate the simple interest payable on a loan of \$45 000 at a rate of 12% p.a. for 3 years.

(b) Calculate the simple interest payable on a loan of \$3500 at an annual rate of 7% for 4 years.

(c) Calculate the simple interest payable on a loan of \$11 450 at 8.2% p.a. for 5 years.

(d) Calculate the simple interest payable on a loan of \$8500 at 5.5% p.a. for 6 months.

2 Find the number of time periods for the following.

(a) 2 years, how many quarters?

(b) 3 years, how many half-years?

(c) 6 years, how many quarters?

(d) 12 years, how many half-years?

3 Calculate the simple interest for the following:

(a) $P = \$440$, $r = 0.08$, $T = 5$ years

(b) $P = \$600$, $r = 0.09$, $T = 7$ years

(c) $P = \$369$, $r = 0.12$, $T = 3$ months

(d) $P = \$3800$, $r = 16.8\%$, $T = 20$ days

W.E. 16

4 Chloe borrowed \$15 000 to buy a new car. She repaid it over $3\frac{1}{2}$ years, at a simple interest rate of 12.5% p.a., payable every 6 months.

(a) How much interest did Chloe pay every 6 months?

(b) What was the total amount of money repaid?

5 Miette takes out a loan of \$5000 for a holiday. She repays it over the next 2 years and 3 months with a simple interest rate at 2.7% p.a.

(a) Calculate the interest added quarterly.

(b) Calculate the total amount that Miette repays.

W.E. 17

6 (a) A statement for an account offering 6.9% p.a. interest calculated daily is shown for the month of July.

How much interest is payable for July?

Date	Transaction	Balance
July 1		\$350
July 5	Withdrawal \$170	\$180
July 22	Deposit \$450	\$630

(b) A statement for an account offering 7.8% p.a. interest calculated daily is shown for the month of May.

How much interest is payable for May?

Date	Transaction	Balance
May 1		$65.20
May 16	Deposit $530	$595.20
May 22	Withdrawal $260	$335.20
May 27	Withdrawal $300	$35.20

W.E. 18

7 After investing $5000 into a fixed deposit account for 2 years for a trip to London, Inika received $5890. What percentage return p.a. did she get?

8 The simple interest on $870 for 4 years at 15% p.a. is:

A $870 \times 15 \times 4$

B $870 \times 0.15 \times 4$

C $870 \times 1.15 \times 4$

D $870 \times 0.15 \times 4 + 870$

9 What is the total amount which is paid back on a loan of $8150 over 3 years 6 months at 18.5% p.a.?

10 Calculate r, then write as a percentage (p.a.) correct to 1 decimal place for:

(a) $P = \$245$, $T = 5$ years, $I = \$110$

(b) $P = \$420$, $T = 2.25$ years, $I = \$132$

11 Calculate T (in years) correct to 1 decimal place for:

(a) $P = \$245$, $r = 0.06$, $I = \$116$

(b) $P = \$120$, $r = 0.07$, $I = \$32$

12 Calculate P (in dollars) for:

(a) $T = 3.5$ years, $r = 0.05$, $I = \$35$

(b) $T = 2$ years, $r = 0.08$, $I = \$216$

Understanding

13 Claudia, to save for a trip to Thailand, puts her tax return cheque of $950 into an account paying 8% p.a. simple interest. How long (in years, correct to 1 decimal place) will her money take to accumulate to $1300?

14 How much will be paid out to Sol, who puts $1400 into a bank account paying 6% simple interest p.a., if he withdraws the total amount after 240 days?

15 Victor agrees to lend Esme $1500 so that she can go on a holiday to Fiji. They agree that Esme will make quarterly payments, with simple interest to be paid at a rate of 12% p.a. As well as the principal, Esme will pay the debt in full in 5 years. Calculate Esme's quarterly payments.

16 Manny borrows $6100 from his mother to buy a car. He agrees to pay 11% simple interest each year without paying off the principal. After 2 years, in addition to the interest, he pays $2500 off his principal, reducing the principal for the rest of the loan. How much in total did he pay his mother if he pays off the loan in another 1.5 years?

17 Calculate the interest payable on an account in the month of April if it pays 5.8% interest p.a., calculated daily, and it has a balance of $155 for 3 days, $820 for 19 days, and $580 for the rest of the month.

18 After investing $6000 into a savings account paying 7% interest p.a., Ian received $6238. For how many days (correct to the nearest day) was the money invested?

19 What amount of money must be invested at 9% p.a. in order to earn $540 in interest in 3 months?

20 Tula repays her father a total of $350 in interest to cover a debt for a motorbike which she incurred 15 months ago, agreeing to pay 4% p.a. simple interest. How much did she borrow?

Reasoning

21 Dario opens a savings account that calculates interest on the daily balance. Interest is paid on the first of the following month. If at least $200 is deposited and there are no withdrawals, the interest rate for that month is 3% p.a. Otherwise the interest rate is 1.5% p.a.

Complete the table.

Date	Details	Amount	Balance
			$0
Jun 20	Deposit	$4000.00	
Jul 1	Interest		
Jul 30	Withdrawal	$500.00	
Aug 1	Interest		

22 Phillip borrows $15 000 from his mother to help pay for his new car. He needs to pay back the money within 3 years and pay 5% simple interest for the period of the loan. He intends to make monthly repayments to his mother.

(a) How much interest will Phillip need to pay?

(b) How much in total does Phillip owe his mother?

(c) How much is each monthly repayment?

(d) Assuming $\frac{1}{3}$ of the interest is repaid each year and $\frac{1}{3}$ of the principal is repaid each year, what percentage of each payment is interest?

(e) How much interest does Phillip still need to repay after the first 12 months?

(f) If the interest calculation was done again based on the principal still owing after 12 months, how much interest would be owing?

(g) Why is there a difference in the answers to parts **(e)** and **(f)**?

Open-ended

23 Tranh earns \$640 on an investment of \$8000. Give three possible simple interest rates as percentages per annum, and their corresponding time periods, that give interest of \$640 on an investment of \$8000.

24 Michael is sure that the bank has made a mistake on the interest it added to his account for the month.

Interest is calculated daily at a rate of 3.5% p.a. Here is Michael's statement:

Date	Transaction	Balance
1 June	Opening balance	\$578.12
9 June	\$250.03 deposit	\$828.15
21 June	\$185 withdrawal	\$643.15
30 June	Closing balance	\$643.15

Here are Michael's calculations:

$P = \$578.12, \$828.15, \$643.15$

$r = 0.035$

$T = \frac{9}{365}, \frac{13}{365}, \frac{10}{365}$

$$I = (\$578.12 \times 0.035 \times \frac{9}{365}) + (\$828.15 \times 0.035 \times \frac{13}{365}) + (\$643.15 \times 0.035 \times \frac{10}{365})$$

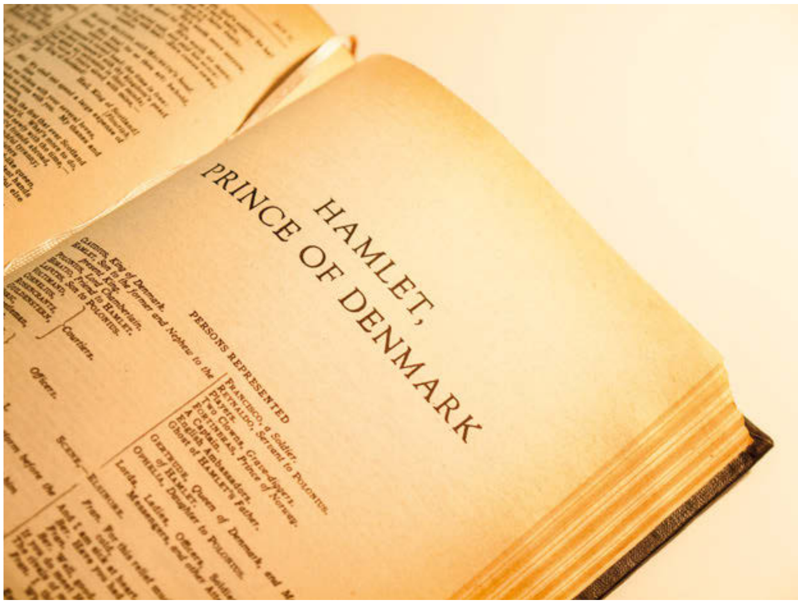
$$= \$0.4989 + 1.0324 + \$0.6167$$

$$= \$2.15$$

The amount of interest given on his bank statement is \$2.01.

Has the bank made a mistake? Or has Michael? Explain the difference in the two amounts of interest.

25 'Neither a borrower nor a lender be' is a line from Shakespeare's famous play *Hamlet*. Do you think this is good advice? Give some reasons for your answer.



Exploration Spreadsheet

Using iteration to investigate savings plans

Equipment required: 1 computer with Excel or similar spreadsheet software, or a spreadsheet on a CAS calculator

In mathematics, iteration is when a rule is applied to a value over and over again. The output from a mathematical rule becomes the next input to the rule. Iteration can be a very useful way to generate patterns or to model changes over time, such as changes to money or population.

For example, consider the iterative rule: double a number and take away 10.

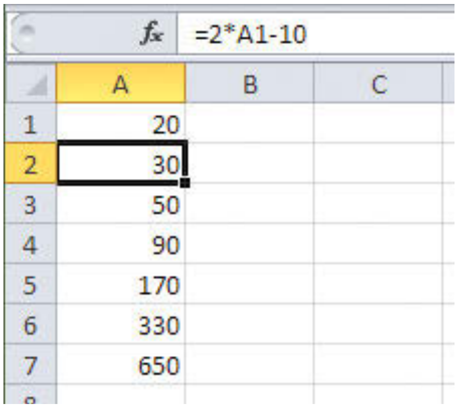
If the first input is 20, then this becomes $2 \times 20 - 10$ and the output is 30. Iterating this, the output now becomes the next input. Applying the rule to 30:

$2 \times 30 - 10 = 50$, so the next output is 50.

Each time the rule is applied, an iteration is being performed. The first iteration gave a result of 30 and the second iteration gave a result of 50.

Spreadsheets can be a very useful way to calculate iterations. You can enter the start value in cell **A1**, then the rule in **A2**, and then copy the formula down as far as you like.

The spreadsheet below shows the first six iterations of the rule given above. The value 30 is the result of the first iteration, but note it is in row 2.



fx =2*A1-10

	A	B	C
1	20		
2	30		
3	50		
4	90		
5	170		
6	330		
7	650		

1 Find the value of the fourth, tenth and twentieth iterations. You will have to extend the table beyond the size shown.

Holiday savings

Suppose you are able to save $1200 each year towards a major holiday in 10 years' time. You have to choose the best plan from the following options.

	Payment*	Interest rate
Plan A: Regular saver	$100 per month	9% p.a. paid monthly
Plan B: Periodical saver	$600 per 6 months	10% p.a. paid 6-monthly
Plan C: Yearly lump sum	$1200 per year	9% p.a. paid yearly

* All payments commence at the beginning of the payment period.

Investigating Plan A: Regular saver

Calculate the monthly rate of interest. Dividing the 9% annual rate by 12 gives the monthly rate of 0.75%, or 0.0075 as a decimal rate.

The amount saved at the beginning of the second month can be calculated:

Initial deposit = $100

Interest on deposit $= 0.0075 \times 100$
$= \$0.75$

Second deposit = $100

Total $= \$(100 + 0.75 + 100)$
$= \$200.75$

The amount saved at the beginning of the third month can be calculated:

Previous balance = $200.75

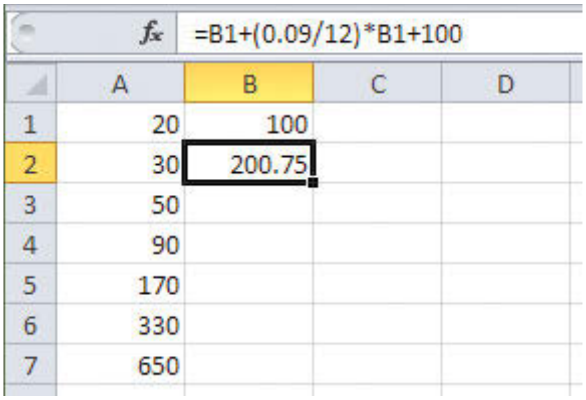
Interest on balance $= 0.0075 \times 200.75$
$= \$1.51$

Third deposit = $100

Total $= \$(200.75 + 1.51 + 100)$
$= \$302.26$

This is an iterative calculation for each month. You can calculate this in a spreadsheet by entering the initial amount of $100 into cell **B1**, then the formula **=B1+(0.09/12)*B1+100** into **B2**.

This should produce the result of 200.75 for the beginning of the second month, as shown.



fx =B1+(0.09/12)*B1+100

	A	B	C	D
1	20	100		
2	30	200.75		
3	50			
4	90			
5	170			
6	330			
7	650			

2 Explain what each part of the formula **=B1+(0.09/12)*B1+100** does.

3 **Fill Down** this formula into cell **B3**. This shows the amount in the account at the beginning of the third month. Does it match the value calculated earlier? What has happened to the formula?

4 Extend your calculations to find the amount saved at the beginning of each month for months 3–13 of the savings plan. (Hint: The end of the 12th month is the beginning of the 13th month.)

5 How much is saved at the end of 1 year (not including the payment of $100 at the beginning of month 13)?

Investigating Plan B: Periodical saver

To calculate the 6-monthly rate of interest, divide the 10% annual rate by 2. This gives a rate of 5% per 6 months, or 0.05 per month as a decimal rate.

At the beginning of the second 6-month period the amount saved can be calculated:

Initial deposit = $600

Interest on deposit = 0.05 × 600
= $30

Second deposit = $600

Total = $(600 + 30 + 600)
= $1230

Copy and complete the calculation below for the beginning of the third 6-month period:

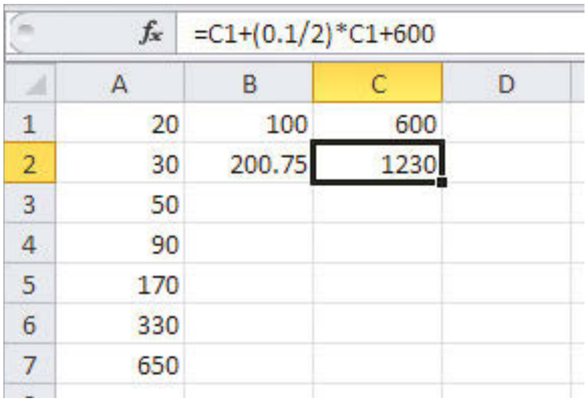
Previous balance = $_____

Interest on balance = 0.0_____ × 1230
= $_____.50

Third deposit = $600

Total = $(1230 + _____. 50 + 600)
= $_____.50

You can enter this plan in column **C** of the spreadsheet, similar to the plan already calculated in column **B**. Cell **C2** below shows this formula.



fx =C1+(0.1/2)*C1+600

	A	B	C	D
1	20	100	600	
2	30	200.75	1230	
3	50			
4	90			
5	170			
6	330			
7	650			

6 Explain what each part of the formula **=C1+(0.1/2)*C1+600** does.

7 **Fill Down** this formula to cell **C3**. Does this give you the same value you calculated earlier for the amount in the account at the beginning of the third 6-month period?

8 How much is saved at the end of one year if the payment of $600 (at the beginning of the second year) is not included?

Investigating Plan C: Yearly lump sum

9 Find the amount saved after one year. (Do not include the payment made at the start of the second year.)

Comparing the plans

10 Write a brief report to compare the plans and the amount saved in each plan after one year. Which plan would you recommend? How practical are the different plans?

1.6

Payment methods

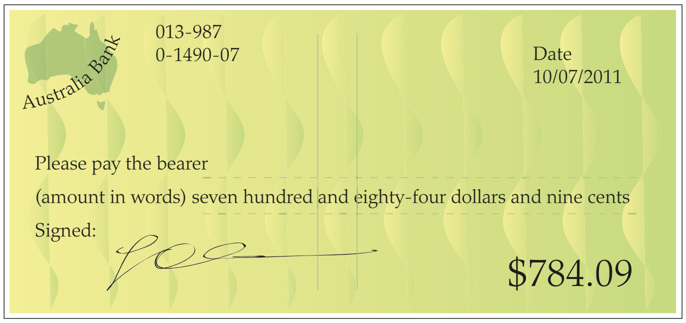
Cash money (notes and coins) is not the only way to pay for things.

- **Debit card**

  Debit cards allow money to be transferred electronically from your bank account to the store's account. This is done through the Electronic Funds Transfer at Point of Sale (**EFTPOS**) system. The money must be in your bank account for it to be transferred.

- **Credit card**

  Credit cards look like debit cards but there is one important difference. When you pay for something with a credit card, the card provider (usually a bank) pays for you. You now owe this money as a debt to the card provider. If you do not regularly pay back this debt, then a high rate of interest is charged, which increases the debt even more.



Australia Bank

013-987
0-1490-07

Date
10/07/2011

Please pay the bearer
(amount in words) seven hundred and eighty-four dollars and nine cents
Signed:

$784.09

- **Cheque**

  Cheques are used less and less as electronic banking becomes more common. A cheque is a special form that can be used to transfer money: on a cheque, you write the name of the person or company being paid, the amount in words, and your signature. The cheque must then be deposited at a bank. Many banks charge a cheque handling fee, and the money takes several days to become available.

- **Lay-by**

  When you put an item 'on lay-by', the store holds the item for you while you make payments. You receive the item after it has been completely paid for. No interest has to be paid for this form of payment.

- **Deferred payment**

  This describes the payment method for repaying university or college fees that have been accumulated by using HELP—the Higher Education Loan Program. Repayment begins only if the borrower (the student) earns more than a certain amount set by the government. Repayments are calculated as a percentage of income.

- **Buying on terms** ('buy now, pay later', sometimes called 'hire-purchase')

  Large purchases, such as furniture, computers or cars, can be bought in this way. You take the items home, then pay them off slowly over time. Many stores offer an 'interest-free period' where interest is not charged on the money that you haven't paid yet. However, if you have not repaid the loan by the end of this period, a very high interest rate may be charged.

Fees and charges

In addition to charging for EFTPOS transactions, banks often charge monthly account-keeping fees and/or fees for withdrawing cash from ATMs (automatic teller machines), particularly if the ATM belongs to another bank. There are also sometimes extra charges for paying with a credit card or for paying bills online.

Worked example 19

W.E. 19

Jemima's bank charges her a $2.00 account-keeping fee every month, plus the following charges:

- 50c for a withdrawal from the bank's ATMs
- $2.00 for a withdrawal from another bank's ATMs
- 75c for EFTPOS transactions
- 50c for transferring money online

In one month, Jemima makes two withdrawals from her own bank's ATM, three withdrawals from another bank's ATM, seven EFTPOS transactions and two online transfers.

Calculate the total amount of Jemima's bank charges for the month.

Thinking	Working
1 Go through the list and calculate each fee separately.	Bank ATM: $2 \times \$0.50 = \$1.00$ Other bank ATM: $3 \times \$2.00 = \$6.00$ EFTPOS: $7 \times \$0.75 = \$5.25$ Online transfers: $2 \times \$0.50 = \$1.00$ Account keeping fee: $2.00
2 Find the total and write your answer in a sentence.	Total fees = $15.25 Jemima's bank will charge her $15.25 for the month.

Using a credit card

Using a credit card is a popular method of payment. In 2015 there were approximately 16 million credit cards in use in Australia. When you use a credit card, the card provider (usually a bank) pays the cost, but you now owe that cost to the card provider.

Most credit cards have an interest-free period of up to 55 days. This period begins on the first day of the statement period and ends on the date when payment is due. This means that no interest is added to the money owed until after the 'interest free' period ends. If you pay the full amount by the due date, you will not be charged interest. However, if you do not pay in full by the due date you will be charged interest, which adds to the amount of money that you owe.

The interest rate for a credit card is usually stated as an annual percentage rate (APR). This varies from card to card, but is usually somewhere between 15% and 20%. Some credit cards do not have an interest-free period at all and begin charging interest from the day a purchase is made.

Calculating credit card interest

Every month, a 'statement' is produced by the credit card provider which calculates the interest and tells you how much money you owe. This is done as follows:

- The balance (the amount owing) at the end of each day of the statement period is found. The daily balances are then added up and divided by the number of days in the statement period to get an 'average daily balance', or ADB.
- A daily interest rate is calculated by dividing the annual rate by 365.
- The ADB is multiplied by the daily interest rate and by the number of days in the statement period to give the amount of interest for the period.
- This interest will then be charged on the next statement if the amount is not paid in full by the due date.

Worked example 20

W.E. 20

Ali has a credit card with an annual percentage rate (APR) of 18.4% interest and an interest-free period of 55 days. He has just received a statement covering the period from 24 May to 23 June. If Ali's average daily balance (ADB) for this period is $2184.36, calculate the interest that will be payable if he does not pay the account in full by the due date.

Thinking	Working
1 Find the daily interest rate by dividing the annual rate by 365. Express this as a decimal.	$r = 18.4\%\text{ APR} \div 365$ $= 0.0504\%$ $= 0.000\,504$
2 Calculate the number of days in the statement period, being sure to include the days at the beginning and end of the period.	$T = 24\text{ May to }23\text{ June}$ $= 31\text{ days}$
3 Define *P* as the average daily balance (ADB) for the period. Substitute the values for *P*, *r* and *T* into the interest formula and evaluate. Round your answers to the nearest cent.	$P = \$2184.36$ $I = PrT$ $= \$2184.36 \times 0.000\,504 \times 31$ $= \$34.13$

Buying on terms

Furniture and electrical stores often allow customers to take goods and pay for them over a period of time, instead of paying the total all at once. This is sometimes called 'buying on terms'.

An 'establishment fee' is usually paid at the beginning when the account is set up. 'Interest-free' terms give customers some time (usually 6–12 months) during which no interest is charged. During this period, customers only need to pay a small monthly repayment, plus a monthly account fee. If the remaining balance has not been paid by the end of the interest-free period, then interest will be charged from that point on.

'Buy now, pay later' terms are similar, except that no monthly repayments are needed during the interest-free period. The interest on any outstanding amount at the end of the interest-free period is calculated in the same way as credit card interest.

Worked example 21

W.E. 21

Ravi wants to purchase a new lounge suite. He has seen one advertised for \$2799, but he does not have the money. The store offers Ravi a 12-month interest-free payment plan where he must pay a minimum monthly repayment of 3% of the cost, a \$25 up-front application fee and a \$3.95 monthly account-keeping fee.

(a) How much will Ravi pay in fees over the 12 months?

(b) If he only makes the minimum monthly repayments, how much will Ravi have paid off the cost of the lounge suite after 12 months?

(c) If Ravi does not pay the remainder of the cost at the end of the 12 months, he will be charged 29% p.a. interest until it is all paid off. Calculate the interest that will be charged after the 13th month. (Assume there are 30 days in the month.)

Thinking	Working
(a) Multiply the monthly account-keeping fee by 12, then add the application fee.	(a) $(\$3.95 \times 12) + \$25 = \$47.40 + \$25$ $= \$72.40$
(b) Find the minimum monthly repayment, then multiply it by 12 months.	(b) 3% of $\$2799 = 0.03 \times 2799$ $= \$83.97$ per month $\$83.97 \times 12 = \$1007.64$ after 1 year
(c) **1** Find the outstanding balance—the amount still owing after 12 minimum monthly repayments. Define this as *P*.	(c) $P = \$2799 - \$1007.64$ $= \$1791.36$
2 Find the daily interest rate by dividing the annual rate by 365 then converting to a decimal.	$r = 29\% \div 365$ $= 0.079\,452\%$ $= 0.000\,795$
3 Define *T*.	$T = 30$ days
4 Find the interest by substituting the values into the interest formula, then evaluating.	$I = PrT$ $= \$1791.36 \times 0.000\,795 \times 30$ $= \$42.70$

1.6 Payment methods

Navigator

Answers p. 630			
	1, 2 (a, b), 3, 4, 5, 6, 9, 10, 12	1 (a, b), 2 (a, b), 3 (a, b), 4, 5, 6, 7, 9, 10, 12	1 (c, d), 2 (c, d), 4, 5, 7, 8, 9, 10, 11, 12

Fluency

W.E. 19

1 Federation Bank charges customers the following fees on their 'Easy Saver' account:

- a $3 monthly account-keeping fee
- 70c for withdrawals from the bank's ATMs
- $1.50 for withdrawals from other banks' ATMs
- 70c for EFTPOS transactions
- 50c for online transfers

Calculate how much the bank will charge the following customers in a month if the following transactions are made:

(a) five ATM withdrawals (one from a different bank), four EFTPOS payments, three online transfers

(b) six EFTPOS payments, four ATM withdrawals (two from the bank's ATMs), five bills paid online

(c) two withdrawals from the bank's ATMs, eight EFTPOS payments

(d) four withdrawals from other banks' machines, six online transfers.

2 Find the daily interest rate, as a percentage to 4 decimal places, for the following annual interest rates. (Assume that 1 year is 365 days.)

(a) 5.4% p.a. **(b)** 9.7% p.a. **(c)** 7.1% p.a. **(d)** 1.5% p.a.

3 Find the number of days between the following dates (including the start date and end date).

(a) 21 April to 20 July **(b)** 3 September to 2 December

(c) 19 March to 18 September **(d)** 5 December to 4 February

W.E. 20

4 Shannon has a credit card with an annual interest rate of 16.99% and an interest-free period of 55 days. She has just received a statement covering the period from 15 September to 14 October. If Shannon's average daily balance (ADB) for this period is $3205.48, calculate the interest that will be payable if she does not pay the account in full by the due date.

W.E. 21

5 John wants to buy a home entertainment system priced at $3299 but doesn't have the money. The store offers him a 6-month interest-free plan with a minimum monthly repayment of 5% of the cost, a $20 up-front application fee and a monthly account-keeping fee of $3.50.

(a) How much will John pay in fees over 6 months?

(b) How much will John have paid off the cost of the TV after 6 months if he only makes the minimum monthly repayments?

(c) If John does not pay the remainder of the cost at the end of 6 months, he will be charged 24% p.a. interest until it is paid off. Calculate the interest that will be charged after the seventh month. (Assume there are 30 days in the month.)

6 Jackson wanted to buy a new telescope for $109.95, but did not have enough money, so he placed it on lay-by. He paid a 10% deposit and needs to make payments of $15 a week.

(a) How long will it take Jackson to pay off his telescope?

(b) How much will his last repayment be?

Understanding

7 Farouk has just obtained a credit card that has an interest rate of 17.35% and an interest-free period of 55 days. His statement after his first month is shown.

(a) How many days are in the statement period?

(b) Calculate the average daily balance by finding the balance at the end of each day, adding them up and dividing by the number of days in the statement period. Remember to include the days at the beginning and end of the statement period (shown on the bill).

(c) Use your answer to **(b)** to calculate the interest that Farouk will be charged if he does not pay the balance in full by the due date.

Respectable Bank

Account Number: 8675 0032 6701 2130

Statement Period: 03/07/16 to 02/08/16

Date	Purchase	Amount	Balance
	Opening balance		$0.00
5 July	Shoes	$59.00	$59.00
11 July	Petrol	$72.30	$131.30
17 July	Concert ticket	$124.80	$256.10
22 July	Phone and internet bill	$89.95	$346.05
29 July	Groceries	$147.32	$493.37
1 August	Ski jacket and goggles	$239.86	$ 733.23
	Closing balance		$733.23

Hint

If no purchases have been made on a particular day, the closing balance is the same as the previous day's balance.

8 Yolanda has bought a new spa bath worth $4300 using a 'buy now, pay later' 24-month payment plan. The plan requires her to pay an up-front establishment fee of $25 and a monthly account-keeping fee of $2.95. Yolanda does not have to make any monthly repayments, although she can do so if she wants to. Yolanda must pay the full amount by the end of the 24-month period or she will pay 30% p.a. interest every month on whatever is owing until it is all paid off.

(a) Calculate Yolanda's fees for the 24 months.

(b) If Yolanda does not pay anything off the spa in the 24-month period, how much interest will be added to her account by the end of the 25th month? (Assume there are 30 days in the month.)

(c) Aside from the account-keeping fees, what voluntary monthly repayment amount should Yolanda make in order to have the spa paid off by the end of the 24 months?

9 Poh and Dzung are trying to decide which method they should use to pay their rent of $950 each month. They can use the bank's website to electronically transfer the money from their account into their landlord's account. Alternatively, they can post a cheque directly to their landlord's workplace. Poh's bank charges $6 to process a cheque.

(a) Online electronic transfers via the bank's website only allow transfers of up to $400 each time, at a cost of 70c per transfer. How much will it cost to transfer the $950 online?

(b) Which method of payment would you recommend to Poh and Dzung?

Reasoning

10 Edwin is leasing a work car using a hire-purchase agreement. He pays a monthly amount of $810 to the finance company for the use of the car over a 5-year period. This monthly amount includes the cost of the car and interest. At the end of the 5 years, Edwin is the owner of the car.

(a) Calculate the total amount Edwin pays for the car over 5 years.

(b) If the initial value of the car is $39 995, how much has Edwin paid in interest?

(c) Using the value of the car as the principal, calculate the equivalent rate p.a. of simple interest that Edwin has paid correct to 1 decimal place.

11 Shelby has used her credit card to buy furniture for her new house. This month, her credit card statement is for the period from 1 March to 31 March. The total owing on the credit card is $8691.57. Shelby is unable to pay her bill, so interest and a $9 late payment fee are added to the balance. The interest rate of the card is 20.99% p.a.

(a) Calculate the interest that is added if the ADB over the statement period was $6982.45.

(b) Calculate the opening balance on Shelby's account at the beginning of April.

(c) During April, Shelby makes two purchases: a plasma screen TV for $1299 on the 14th and a new lounge suite for $2499 on the 21st. Calculate the balance on her account on 30 April.

(d) If Shelby again doesn't pay her bill, how much interest will be calculated on April's balance? (You will need to calculate the ADB for April.)

(e) How much in interest and fees will have been charged to Shelby's account after 2 months of no payments?

(f) Use your answers from **(a)**–**(e)** to explain why credit card debt can become a problem for some people.

Open-ended

12 Kezia's bank charges her 50c for a withdrawal from the bank's ATMs, $2 for withdrawing from another bank's ATMs, 75c every time she uses EFTPOS and 50c every time she uses the internet to transfer money or pay bills online.

Kezia's bank statements show that she is losing between $10 and $20 a month in fees and charges.

(a) Give some combinations of fees that have a sum between $10 and $20.

(b) Give Kezia some advice to help her lower this amount.

The cost of resources

1.7

Electricity, gas and water are resources (utilities) that are supplied to households. You are billed according to your use of the resource.

Tariffs

A 'retail tariff' is the price charged per unit of electricity, gas or water used. Often, a more expensive tariff is applied after a certain number of units have been used. A cheaper tariff may be applied to low use or use during 'off peak' periods (say between 11 pm and 7 am, or from November until June). A 'network tariff' flat fee for the supply of the utility, often known as the service charge, also appears on the bill.

Electricity

Electricity is supplied to individual houses by a network of power stations, substations and cables, called the 'grid'. Each property connected to the grid has a meter that displays the amount of electrical energy consumed by that household. Companies that sell electricity take regular readings of the meter and bill the household for the energy that has been used.

To understand how electricity bills are calculated, consider how energy consumption is measured.

The basic unit for measuring energy is the joule (J). Different electrical devices use electrical energy at different rates. The rate at which something uses energy is called its **power**, which is measured in watts (W).

A 1-watt device uses 1 joule of energy in 1 second: 1 W = 1 J/s

1000 watts is 1 kilowatt (kW): 1000 W = 1 kW

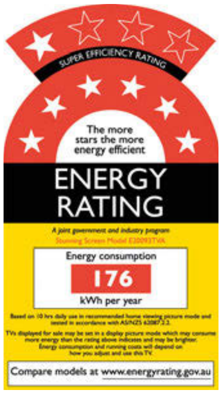
A 1000-watt device uses energy twice as fast as a 500-watt device.

The 'power rating' of a device tells you how many watts of power the device uses. It can usually be found on a label on the device itself. The label shown is from an electric kettle. The power rating of the kettle is 2.4 kW, or 2400 W.


Model 4100AU
Capacity 1.7 litre FILTER (F5)
230/240V. ~ ac.50Hz. 2.4kW
MADE IN UK
DO NOT IMMERSE
DO NOT OVERFILL
APPROVAL No. 11280
9127
437-143

Another label found on many devices is the 'Energy Rating' label. The label uses a 'star' rating system to rate how energy efficient the device is (1 star = not very efficient; 6 stars = extremely efficient). The label also tells you how much energy the device will consume in 1 year if used in the normal way.

To find the amount of electrical energy used by a device, multiply its power rating by the length of time (in hours, h) that the device has been in use. This gives the unit of energy used by the supplier to calculate your bill, the kilowatt-hour (kWh).



Energy consumption of a device (kWh) = power of the device (kW) × time (h)

The daily running cost of a device is calculated by multiplying its energy consumption by the electricity retail tariff:

Daily running cost ($) = energy consumption (kWh) × tariff ($/kWh)

Worked example 22

W.E. 22

For these household devices, calculate **(i)** the daily energy consumption and **(ii)** the daily running cost, given that the tariff is 18.5c/kWh.

(a) a 75 W light bulb switched on for 12 hours per day

(b) a 4300 W clothes dryer, in use for 45 minutes per day.

Thinking	Working
(a) (i) 1 Convert the power of the device to kW by dividing by 1000. Convert the time to hours.	$75\text{ W} = 0.075\text{ kW}$ $\text{Time} = 12\text{ h}$
2 Multiply the power by the number of hours the device is in use to find the daily energy consumption in kWh.	$0.075 \times 12 = 0.9\text{ kWh consumed daily}$
(ii) Multiply the daily consumption by the tariff to find the daily running cost. Convert to dollars and round to the nearest cent.	$0.9 \times 18.5 = 16.65\text{c}$ $= \$0.1665$ $= \$0.17\text{ per day}$
(b) (i) 1 Convert the power of the device to kW. Convert the time to hours.	$4300\text{ W} = 4.3\text{ kW}$ $\text{Time} = \frac{45}{60}\text{ h}$ $= 0.75\text{ h}$
2 Multiply the power by the number of hours the device is in use to find the daily energy consumption in kWh.	$4.3 \times 0.75 = 3.225\text{ kWh}$
(ii) Multiply the daily consumption by the tariff to find the daily running cost. Convert to dollars and round to the nearest cent.	$3.225 \times 18.5 = 59.6625\text{c}$ $= \$0.596\,625$ $= \$0.60\text{ per day}$

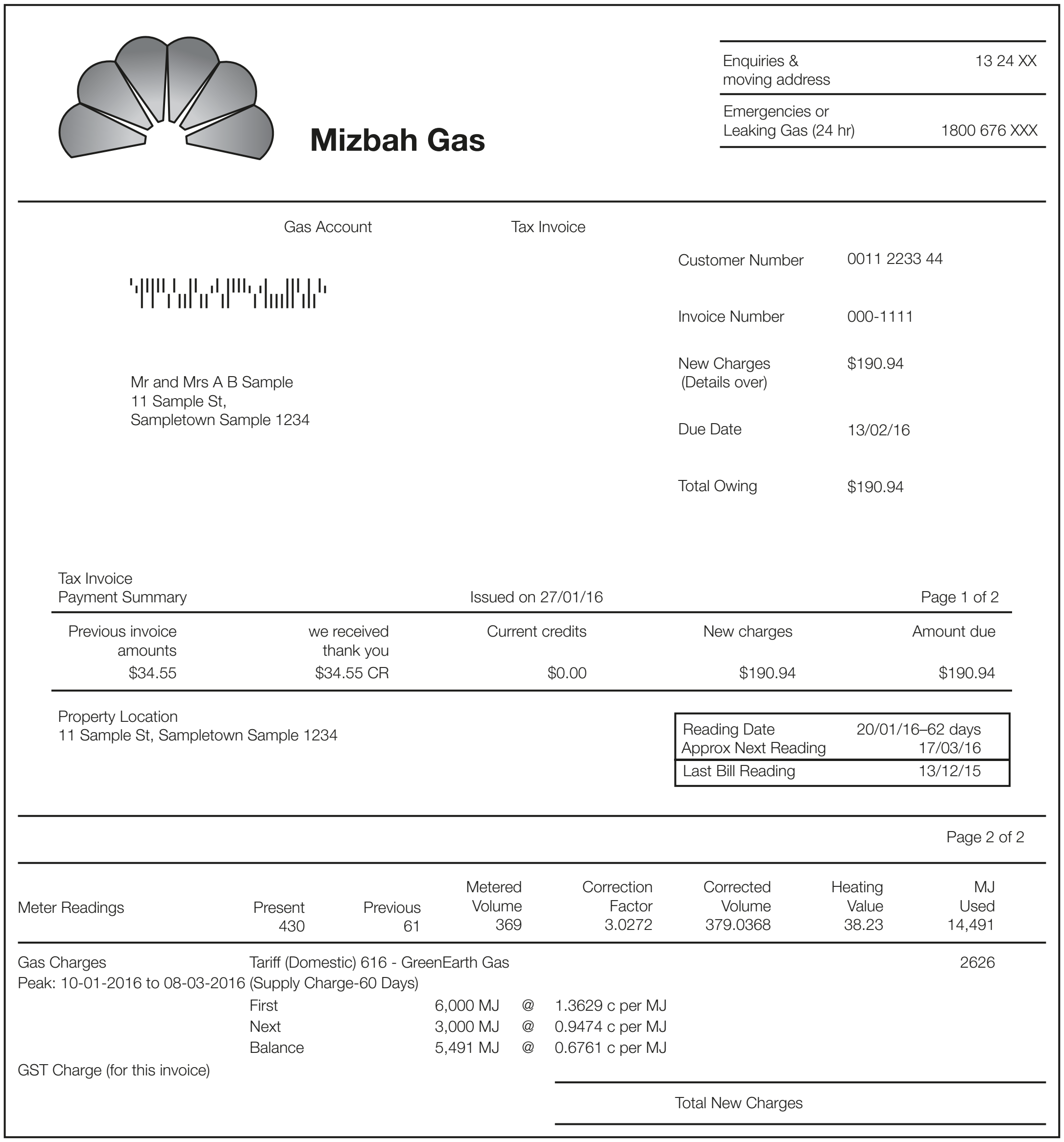
Gas

Most households in Australia are supplied with ‘natural gas’ (a mixture of methane and some other gases), which is usually just called ‘gas’.

Gas is commonly used for heating, cooking and hot water services. The volume of gas consumed (in cubic metres, m^3) is measured by a gas meter.

To calculate the cost of the energy produced by burning gas, the gas supplier converts the volume of gas burnt to an amount of energy in megajoules (MJ, where 1 MJ = 1 000 000 J) as follows:

Energy used (MJ) = volume of gas (m^3) × heating value (MJ released by burning 1 m^3 of gas) × correction factor (to adjust for differences in pressure)



Mizbah Gas

Enquiries & moving address	13 24 XX
Emergencies or Leaking Gas (24 hr)	1800 676 XXX

Gas Account Tax Invoice

Mr and Mrs A B Sample
11 Sample St,
Sampletown Sample 1234

Customer Number	0011 2233 44
Invoice Number	000-1111
New Charges (Details over)	$190.94
Due Date	13/02/16
Total Owing	$190.94

Tax Invoice
Payment Summary Issued on 27/01/16 Page 1 of 2

Previous invoice amounts	we received thank you	Current credits	New charges	Amount due
$34.55	$34.55 CR	$0.00	$190.94	$190.94

Property Location
11 Sample St, Sampletown Sample 1234

Reading Date	20/01/16–62 days
Approx Next Reading	17/03/16
Last Bill Reading	13/12/15

Page 2 of 2

Meter Readings	Present	Previous	Metered Volume	Correction Factor	Corrected Volume	Heating Value	MJ Used
	430	61	369	3.0272	379.0368	38.23	14,491

Gas Charges Tariff (Domestic) 616 - GreenEarth Gas 2626
Peak: 10-01-2016 to 08-03-2016 (Supply Charge-60 Days)

First	6,000 MJ	@	1.3629 c per MJ
Next	3,000 MJ	@	0.9474 c per MJ
Balance	5,491 MJ	@	0.6761 c per MJ

GST Charge (for this invoice)

Total New Charges

Worked example 23

W.E. 23

Look at the bill on the previous page for the gas supplied to a domestic household.

(a) What is the billing period (how many days of usage are covered by this bill)?

(b) What volume of gas was used in this period?

(c) How many megajoules (MJ) of energy was this gas volume equivalent to?

(d) Calculate the total cost of the gas used over this period.

(e) Calculate the total bill (including GST).

Thinking	Working
(a) Find the information on the bill. (Here the number of billing days is next to the Reading Date on the right-hand side.) Check the billing period is correct by finding the number of days from the Last Bill Reading date to the current Reading Date.	(a) Billing period = 62 days (19 April to 20 June)
(b) The volume of gas used is the difference between the present meter reading and the previous meter reading. (It is shown on this bill next to the Present and Previous readings as Metered Volume.)	(b) Volume of gas used = $369\ \text{m}^3$ (= 430 − 61)
(c) Find the information on the bill. (On this bill, the number of MJ is shown at the end of the Meter Readings row of numbers.) The number of MJ can be calculated by multiplying the Metered Volume, the Correction Factor and the Heating Value together.	(c) MJ used = 14 491 (= 369 × 1.0272 × 38.23)
(d) Multiply the tariffs by the number of MJs used. (Here, there are three tariffs, one for the first 6000 MJ used, one for the next 3000 MJ and one for the rest.)	(d) Cost of gas usage = (6000 × 1.3629) + (3000 × 0.9474) + (5491 × 0.6761) = 14 732.0651c = \$147.32
(e) Add the cost of gas usage calculated in **(d)** to the gas supply charge. Calculate 10% GST on this total and add it on.	(e) Total bill = gas usage + supply charge + GST = \$147.32 + \$26.26 + GST = \$173.58 + (10% of \$173.58) = \$173.58 + \$17.36 = \$190.94

Water

The water usage of households is also measured by a meter. Households are billed according to the number of kilolitres (1 kL = 1000 L) of water used. Water bills also contain charges for supplying water to the house and for piping sewage away from the house.

1.7 The cost of resources

Navigator

1, 2, 3, 4, 5, 10, 11, 12	1 (a–d), 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13	1 (e, f), 2, 4, 6, 7, 8, 9, 10, 11, 12, 13

Answers p. 631

Fluency

W.E. 22

1 For the following household devices, calculate **(i)** the daily energy consumption, and **(ii)** the daily running costs given that the tariff is 18.5c/kWh.

(a) A 270 W television switched on for 5 h a day

(b) A 950 W dishwasher, on for 1 cycle of 48 min a day

(c) A 1950 W electric kettle used for 13 min a day

(d) A 200 W computer switched on for $2\frac{1}{2}$ h a day

(e) A 2400 W heater, used for 8 h a day

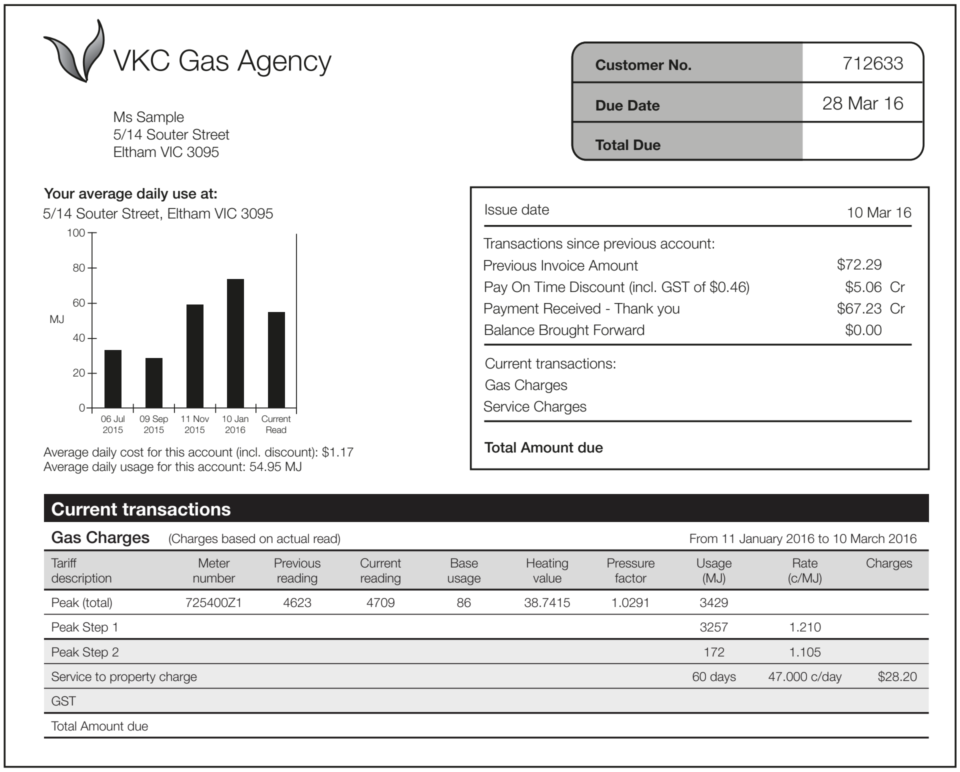
(f) A 1600 W refrigerator, on all day

(g) A 1700 W electric oven, on for 45 min a day

(h) A 1500 W air conditioner, on for $13\frac{1}{2}$ h a day

W.E. 23

2 From the gas bill shown, issued for the gas supplied to a domestic household (note that the total charges are not shown):



VKC Gas Agency

Ms Sample
5/14 Souter Street
Eltham VIC 3095

Customer No.	712633
Due Date	28 Mar 16
Total Due	

Your average daily use at:
5/14 Souter Street, Eltham VIC 3095

Average daily cost for this account (incl. discount): $1.17
Average daily usage for this account: 54.95 MJ

Issue date	10 Mar 16
Transactions since previous account:	
Previous Invoice Amount	$72.29
Pay On Time Discount (incl. GST of $0.46)	$5.06 Cr
Payment Received - Thank you	$67.23 Cr
Balance Brought Forward	$0.00
Current transactions:	
Gas Charges	
Service Charges	
Total Amount due	

Current transactions

Gas Charges (Charges based on actual read) From 11 January 2016 to 10 March 2016

Tariff description	Meter number	Previous reading	Current reading	Base usage	Heating value	Pressure factor	Usage (MJ)	Rate (c/MJ)	Charges
Peak (total)	725400Z1	4623	4709	86	38.7415	1.0291	3429		
Peak Step 1							3257	1.210	
Peak Step 2							172	1.105	
Service to property charge							60 days	47.000 c/day	$28.20
GST									
Total Amount due									

(a) What is the billing period (how many days of usage are covered by this bill)?

(b) From the current reading and previous reading, find the volume of gas used in this period. (The bill gives these figures in m^3.)

(c) How many megajoules (MJ) of energy was this volume equivalent to?

(d) What was the total cost of the gas used over this period? (Find the cost of the gas used for each tariff—Peak Step 1 and Peak Step 2—then add them together.)

(e) Calculate the total amount due, including GST.

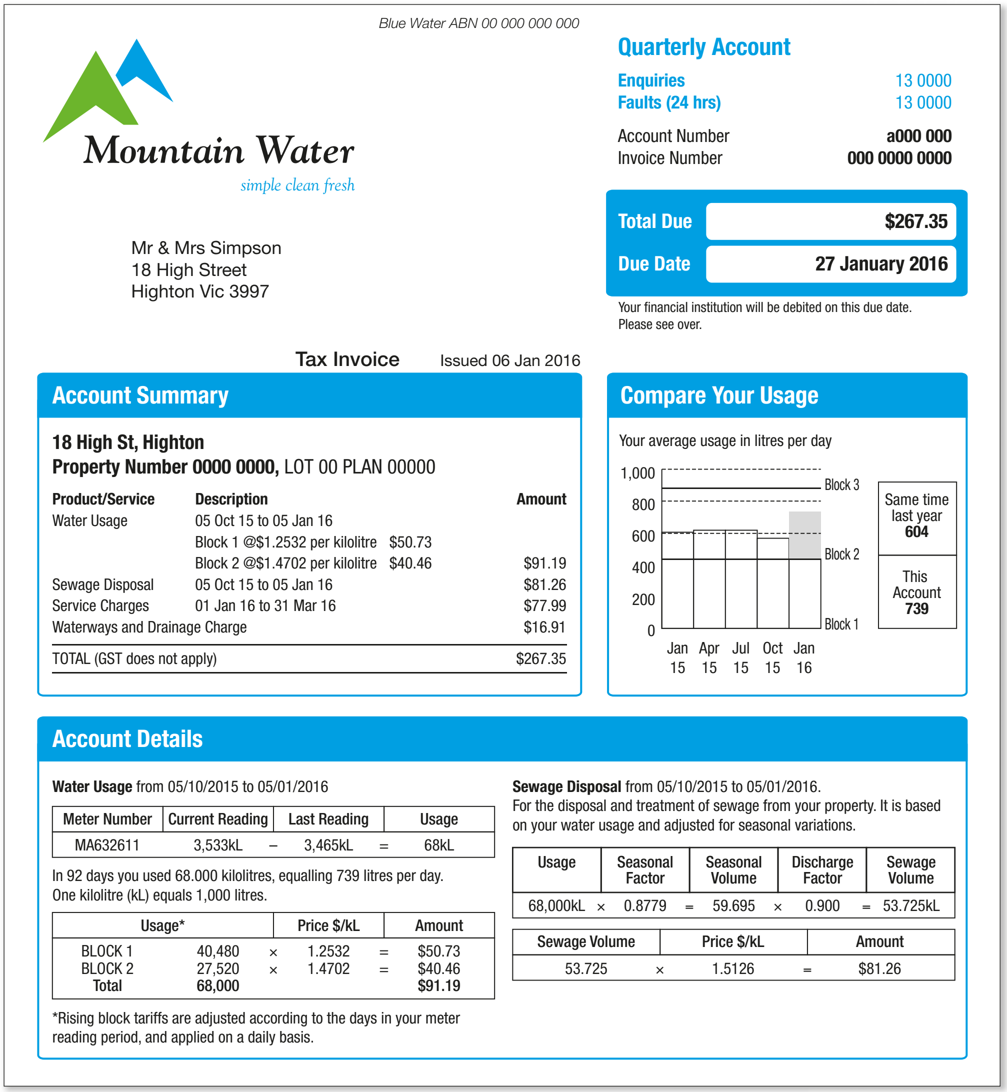
3 Estimate, then calculate, which of the following pairs of devices uses more kWh of energy:

(a) a 40 W games console in use for 80 min or a 180 W laptop in use for 20 min

(b) an 820 W dishwasher completing a 45 min cycle or a 1100 W washing machine (using hot water) completing a half-hour cycle.

Understanding

4 Here is a copy of the Simpson family's water bill.



Blue Water ABN 00 000 000 000

Mountain Water

simple clean fresh

Quarterly Account

Enquiries	13 0000
Faults (24 hrs)	13 0000
Account Number	**a000 000**
Invoice Number	**000 0000 0000**
Total Due	**$267.35**
Due Date	**27 January 2016**

Your financial institution will be debited on this due date.
Please see over.

Mr & Mrs Simpson
18 High Street
Highton Vic 3997

Tax Invoice Issued 06 Jan 2016

Account Summary

18 High St, Highton
Property Number 0000 0000, LOT 00 PLAN 00000

Product/Service	**Description**		**Amount**
Water Usage	05 Oct 15 to 05 Jan 16		
	Block 1 @$1.2532 per kilolitre	$50.73	
	Block 2 @$1.4702 per kilolitre	$40.46	$91.19
Sewage Disposal	05 Oct 15 to 05 Jan 16		$81.26
Service Charges	01 Jan 16 to 31 Mar 16		$77.99
Waterways and Drainage Charge			$16.91
TOTAL (GST does not apply)			$267.35

Compare Your Usage

Your average usage in litres per day

Same time last year **604**
This Account **739**

Account Details

Water Usage from 05/10/2015 to 05/01/2016

Meter Number	Current Reading		Last Reading		Usage
MA632611	3,533kL	–	3,465kL	=	68kL

In 92 days you used 68.000 kilolitres, equalling 739 litres per day.
One kilolitre (kL) equals 1,000 litres.

Usage*			Price $/kL		Amount
BLOCK 1	40,480	×	1.2532	=	$50.73
BLOCK 2	27,520	×	1.4702	=	$40.46
Total	**68,000**				**$91.19**

*Rising block tariffs are adjusted according to the days in your meter reading period, and applied on a daily basis.

Sewage Disposal from 05/10/2015 to 05/01/2016.
For the disposal and treatment of sewage from your property. It is based on your water usage and adjusted for seasonal variations.

Usage		Seasonal Factor		Seasonal Volume		Discharge Factor		Sewage Volume
68,000kL	×	0.8779	=	59.695	×	0.900	=	53.725kL

Sewage Volume		Price $/kL		Amount
53.725	×	1.5126	=	$81.26

Look closely at the bill to find the following:

(a) Besides water usage, what other charges appear on the bill?

(b) What amount has been charged for actual water usage?

(c) How many days in the billing period?

(d) How many kL were used in this period?

(e) There are four people in the Simpson family. On average, how many litres per day did each one use during this period? Give your answer to the nearest litre.

Hint

Look in the 'Account Details' section.

5 In the Patel household, five loads of washing are done each week. Each load is dried in a 4500 W drier. One cycle of the drier takes 50 minutes.

(a) How much energy (in kWh) does their dryer use each week?

(b) How much money would they save if they stopped using the dryer, and their electricity tariff was 19.8c/kWh?

6 Gianni has bought six 15 W energy-efficient light bulbs to replace six 75 W inefficient globes in his house. He estimates that each globe is on for approximately 3 hours a day. Calculate:

(a) how much energy Gianni will save each day

(b) how much money he will save in 1 year by replacing six light bulbs if his electricity retail tariff is 19c/kWh.

7 For this question you will need to refer back to the Simpson family's water bill in Question **4**. The family measure the flow rate of their showerhead and find it delivers 16 litres of water every minute (16 L/min). They now install a new efficient showerhead that delivers 9 L/min.

(a) How much water will they save per shower if their average shower time is 7 min?

(b) If each of the four members of the family has one shower a day, how many kL will the family save over the period of their water bill? Answer to the nearest kL.

8 When TVs and DVD players are switched on and off with a remote control they go into 'stand-by' mode. Typically, a TV in stand-by mode uses 10 W of power and a DVD player uses 7 W. Nerida estimates that her TV is in stand-by mode for 20 hours a day and the DVD player for 22 hours a day.

(a) Calculate the daily energy consumption (in kWh) of the TV and DVD player in stand-by mode.

(b) Use your answer from part **(a)** to calculate the daily cost of leaving the TV and DVD in stand-by mode if the tariff is 19c/kWh.

(c) Calculate the yearly saving Nerida could make on her energy bill if she turned both the TV and DVD completely off instead of turning them to stand-by mode.

Reasoning

9 In the water bill shown in Question **4**, two tariffs (Block 1 and Block 2) are used to calculate the water usage charge.

(a) What is the price difference between the two tariffs?

(b) Suggest a reason why the second tariff is applied after a certain number of kL have been used.

10 Utility companies often provide a graph on the bill that enables people to track their consumption over time. The graph that follows appeared on an electricity bill. It gives the average daily usage (in kWh) for each billing period of the last year as a bar graph. The line graph shows the total greenhouse gas emissions (in the form of carbon dioxide, CO_2) for each billing period. From the graph:

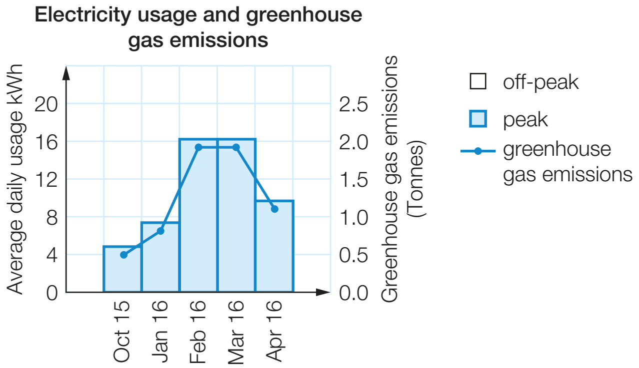
(a) Estimate the average daily use for the most recent billing period (Apr 16).

(b) Estimate the number of tonnes of CO_2 emitted during this period.

(c) If the billing period was 90 days, how many kg of CO_2 were emitted, on average, each day?

(d) What does the shape of the graph tell you about electricity consumption by this household over the year shown?

(e) Suggest a reason to explain the shape of the graph.



Average daily use (kWh)

This account: ☐

Same time last year: **5.55**

Average cost per day: **$2.35**

Total greenhouse gas emissions (Tonnes) this account:

Open-ended

11 From the list below, choose two of the following devices that you use on a daily basis. Estimate the number of hours a day that you use each of the devices. Calculate how much money you would save in 1 year if you halved the amount of time you spent using each of the devices. Assume a retail tariff of 19c/kWh.

Device	Typical power rating
Games console	25 W
Desktop computer	130 W
Laptop computer	60 W
Television (LCD)	270 W
Television (plasma)	311 W

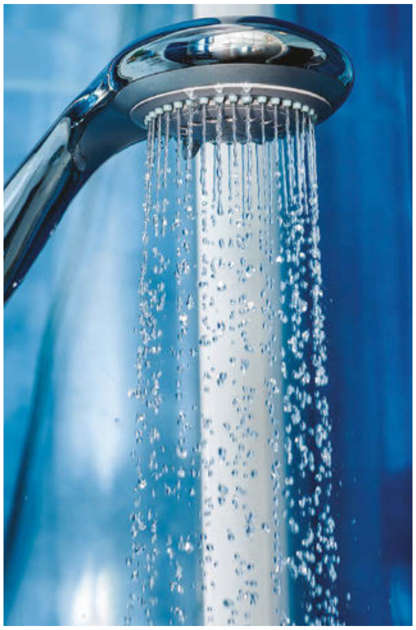
12 Make a realistic estimate of the average time it takes you to have a shower.

(a) If your showerhead delivers 15 L/min, how many litres are you using per shower?

(b) How many kL is this a year (assuming you have one shower every day)?

(c) If your water supplier charges $1.26/kL, how much do your showers cost in 1 year?

(d) Calculate how many kL and how much money you would save in 1 year if you reduced your daily shower time by 2 min.



13 **(a)** Make a list of the different ways in which gas can be used in a home.

(b) Considering your answer to part **(a)**, give a reason to explain the shape of the graph on the gas bill in Question **2**.

Communication costs

1.8

Mobile phones and the internet have become essential communication tools for almost everyone. Many Australian households also have a regular phone (known as a 'fixed line' or 'landline').

Calculating the cost of calling, messaging and downloading can be complicated, with many phone and internet companies offering hundreds of different arrangements for mobile and home phone use.

Payment for mobile phone use falls into one of two categories:

- 'Prepaid', where you purchase a certain amount of credit, say $20, and 'charge' your phone with this credit. When you make a call, the cost of the call is deducted from the credit. Once you run out of credit, you must purchase more credit before you can make more calls.
- 'Post-paid', where you enter into a contract with the phone company to use their network at certain call rates. You are billed every month for your phone usage. The length of the contract is usually 12 or 24 months. Most people with a post-paid arrangement have a 'cap plan' (see 'Different types of charges', below).

Different types of charges

- Call rate

  The amount you pay per minute to make a phone call. Different types of calls, such as local, national and international calls, as well as calls to mobiles, often have different call rates.

- Flagfall

  A one-off charge, sometimes called a connection fee, charged as soon as a call is connected.

- Call cost

  Calculated by multiplying the call rate by the number of minutes the call was made for (rounded up to the nearest minute), then adding the flagfall (if there is one).

- SMS

  An abbreviation of 'Short Message Service', known more commonly as a 'text message' or just a 'text'.

- MMS

  An abbreviation of ‘Multimedia Message Service’. This is similar to SMS, but is used for sending pictures, videos or music files to another mobile. These may be charged differently to SMS costs.

- Cap plan

  The user pays a set amount per month that covers all calls and messages (and often data) up to a certain value. For example, a $29 cap may give the user the equivalent of $150 worth of calls, messages and data. If this amount is exceeded, then the extra calls, messages and excess data are charged individually, sometimes at higher than usual rates. A plan like this means undertaking a contract. If you break the contract, extra fees will be charged (see ‘Early exit fee’).

- Data

  Many plans will have a given amount of data included in the cap, but will charge extra if you go over this limit in any particular month. Although computer science defines 1 GB = 1024 MB. For this chapter, assume 1 GB = 1000 MB, data costs will instead often define 1 GB = 1000 MB.

- Minimum cost/minimum spend

  The minimum amount that will need to be paid over the period of a contract (usually 12 or 24 months). Under a cap plan, the minimum spend is the monthly cap amount multiplied by the number of months in the contract. It does not include payment for the phone itself.

- Early exit fee

  The amount you will pay if you want to end a phone contract before the end date. Often it will mean at least paying the rest of the minimum spend. Frequently you will need to pay to ‘unlock’ your phone from the phone company’s network.

1.8 Communication costs

Navigator

1, 2, 3, 5, 8, 10, 11, 12	1 (a–d), 2, 3, 5, 6, 8, 9, 10, 11, 12	1 (a, b), 2, 4, 6, 7, 9, 10, 11, 12

Answers p. 631

Fluency

1 Calculate the minimum spend that would be required for the following. All plan costs are monthly.

(a) a $49 mobile phone plan over 12 months

(b) a $80 mobile phone plan over 24 months

(c) a $98 home phone and internet plan over 24 months

(d) a $19 mobile phone plan over 24 months

(e) a $40 mobile phone plan over 12 months

(f) a $198 home phone, internet and TV plan over 24 months.

2 Rafal has a 24-month phone plan with a minimum monthly spend of $19. His contract says that the early exit fee is the minimum monthly spend multiplied by the number of months left on his contract, plus an extra $250 if he leaves within the first 6 months. Calculate Rafal's exit fee if he exits:

(a) after 2 months

(b) after 5 months

(c) after 1 year

(d) after a year and a half.

3 Rhett has a prepaid plan for his mobile phone with a call rate of 72c/min and a flagfall of 30c. Text messages are free.

(a) Calculate the total cost of a 5-minute phone call.

(b) Rhett has $30 credit on his account. Calculate the remaining credit if he makes one 5-minute call and sends 10 text messages.

4 Alice puts $20 of prepaid credit onto her phone. The prepaid call rate of her phone company is 88c/min, plus a flagfall of 35c. Text messages are 28c each.

(a) Calculate the cost of a 3-minute phone call.

(b) Calculate how much credit Alice has left after making four 3-minute phone calls and sending six text messages.

Understanding

5 Jian has recently moved to Australia to study and wants to be able to call his family in China regularly on his mobile. One mobile phone network is offering the following international plan:

- international calls (landline or mobile): 40c/min, with flagfall of 20c
- domestic calls (within Australia, either landline or mobile): 3c/min, with flagfall of 25c
- SMS or MMS (national or international): 20c
- voicemail deposit: free
- voicemail retrieval: 40 c/min

Jian estimates that he will phone his family in China for half an hour, once a week. He thinks that he will make about six 5-minute national calls, send six SMS or MMS messages and check his voicemail for 1 minute, each day.

Calculate Jian's weekly phone bill.

6 A mobile phone carrier offers three different ways to buy the phone shown:

- 'over 24 months with a new plan commitment',
- 'over 12 months without a plan commitment', and
- 'upfront without a plan commitment'.

Refer to the advertisement to answer the following questions.

(a) What would be the total spend if you choose 'upfront without a plan commitment' and '\$1206 upfront', over 24 months?

(b) What is the minimum total cost if you choose 'over 24 months with a new plan commitment' and '\$60 per month'?

(c) Which of the options in parts **(a)** and **(b)** costs more over 24 months and by how much?

(d) The option 'over 24 months with a new plan commitment' has a condition that if the plan is exited early, you must pay the remainder of the phone cost. If you exit the '\$60 per month' plan after three months, how much will you need to pay to cover the remainder of the phone cost?

Be the first to write a review

CHOOSE A COLOUR:

- 5.5" Dual edge QHD Super AMOLED display with 2560 by 1440 resolution
- 12MP Dual Pixel fast auto-focus camera
- Expandable memory
- Water and dust resistant (IP68)*

*Samsung Galaxy S7 provides ingress protection against water and dust and to a maximum of 1.5 metres for up to 30 minutes, allowing it to be certified to an IP68 rating. See the Samsung Australia website to read more about IP68. Refer to Galaxy S7 user manual for more information regarding proper care and use of the device (including maintaining the water and dust resistant performance).

✓ AVAILABLE NOW Free Delivery: 1-4 business days

OVER 24 MONTHS WITH A NEW PLAN COMMITMENT ✓

OVER 12 MONTHS WITHOUT A PLAN COMMITMENT ○

UPFRONT

CHOOSE A PLAN

$55 PER MONTH
$30 PLAN + $25 PHONE COST
Min total cost over 24 months is $1,320.00

$60 PER MONTH
$40 PLAN + $20 PHONE COST
Min total cost over 24 months is $1,440.00

$65 PER MONTH
$50 PLAN + $15 PHONE COST
Min total cost over 24 months is $1,560.00

OVER 24 MONTHS WITH A NEW PLAN COMMITMENT ○

OVER 12 MONTHS WITHOUT A PLAN COMMITMENT ○

UPFRONT WITHOUT A PLAN COMMITMENT ✓

CHOOSE A PLAN

$1,206 UPFRONT
$30 PLAN + $1,176 UPFRONT

$1,216 UPFRONT
$40 PLAN + $1,176 UPFRONT

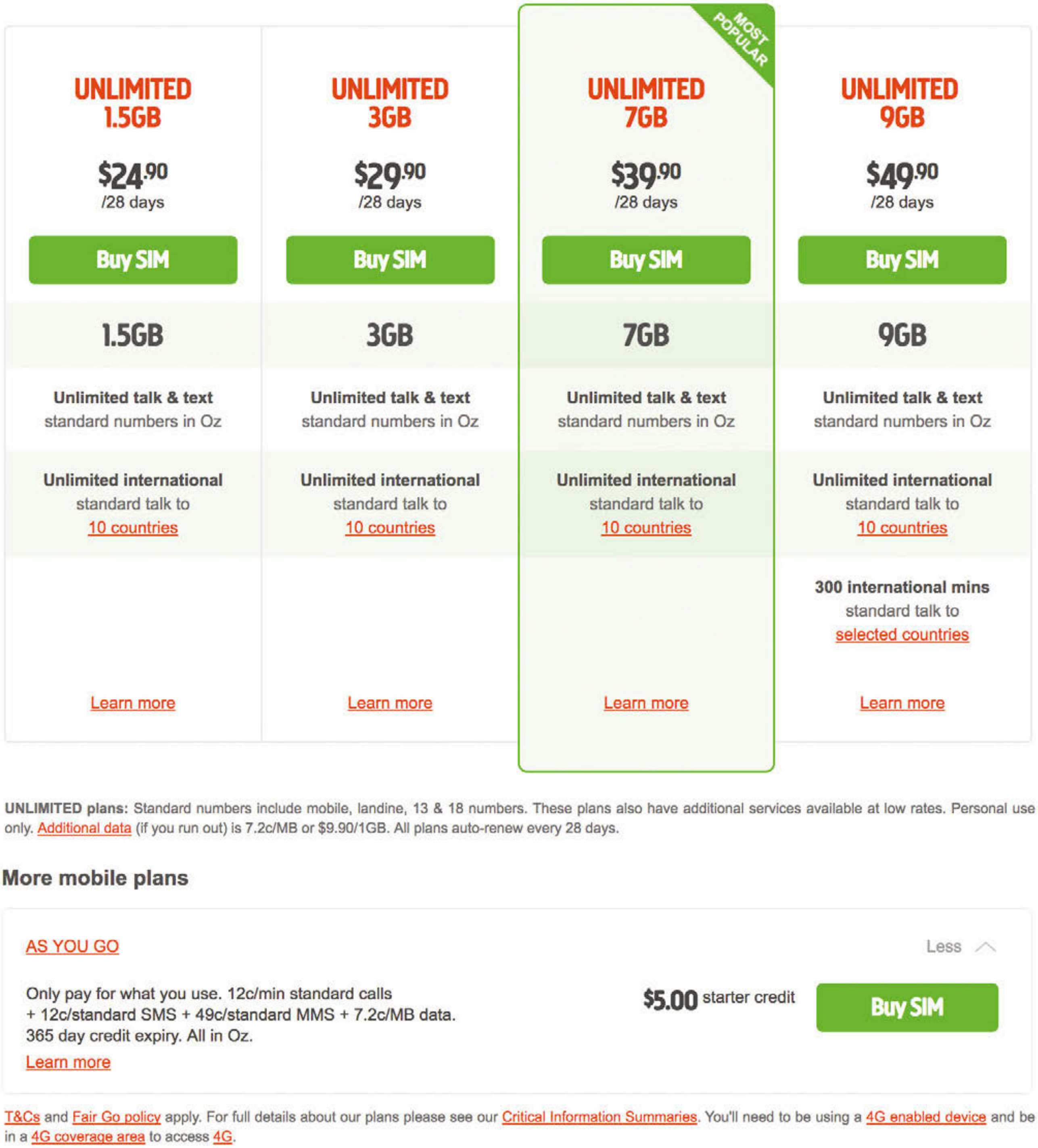
$1,226 UPFRONT
$50 PLAN + $1,176 UPFRONT

7 Henri is looking at the plans shown and trying to decide between Unlimited 1.5 GB and As You Go (shown at the bottom). He makes an average of five 1-minute calls, sends 10 SMSs, and uses 50 MB of data per day.

(a) Calculate the daily cost under the As You Go plan.

(b) How much would Henri's usage cost over 28 days under the As You Go plan?

(c) Which would be the cheaper option for Henri, Unlimited 1.5 GB or As You Go?



UNLIMITED 1.5GB	**UNLIMITED 3GB**	**UNLIMITED 7GB**	**UNLIMITED 9GB**
$24.90 /28 days	$29.90 /28 days	$39.90 /28 days	$49.90 /28 days
1.5GB	**3GB**	**7GB**	**9GB**
Unlimited talk & text standard numbers in Oz	**Unlimited talk & text** standard numbers in Oz	**Unlimited talk & text** standard numbers in Oz	**Unlimited talk & text** standard numbers in Oz
Unlimited international standard talk to 10 countries	**Unlimited international** standard talk to 10 countries	**Unlimited international** standard talk to 10 countries	**Unlimited international** standard talk to 10 countries
			300 international mins standard talk to selected countries

UNLIMITED plans: Standard numbers include mobile, landine, 13 & 18 numbers. These plans also have additional services available at low rates. Personal use only. Additional data (if you run out) is 7.2c/MB or $9.90/1GB. All plans auto-renew every 28 days.

More mobile plans

AS YOU GO

Only pay for what you use. 12c/min standard calls + 12c/standard SMS + 49c/standard MMS + 7.2c/MB data. 365 day credit expiry. All in Oz.

$5.00 starter credit

T&Cs and Fair Go policy apply. For full details about our plans please see our Critical Information Summaries. You'll need to be using a 4G enabled device and be in a 4G coverage area to access 4G.

Reasoning

8 Kwok has a mobile phone plan that charges 85 cents per minute plus a 29 cents flagfall for calls (to other mobiles and to landlines). At home, Kwok's family has a landline phone, but Kwok often uses his mobile instead. The call rates on the landline are as follows:

- calls to landline numbers: 30 cents (regardless of time)
- calls to mobiles: 80 cents per minute (no flagfall)

Calculate how much cheaper it is for Kwok to use the landline phone instead of his mobile to make a 10-minute call:

(a) to a local landline **(b)** to a mobile.

9 Johanna buys prepaid credit for her mobile. She is finding that she is constantly 'topping up' the credit every few days. Johanna is charged the following rates by her phone company:

- calls (to mobiles or landlines): 78c/min, plus a flagfall of 39c
- texts (SMS): 22c each
- data: $10 for 1 GB

Johanna estimates that, on average, she makes four phone calls (each call is approximately 5 minutes long), sends 10 text messages, and uses 200 MB of data every day.

(a) Calculate the cost of one of Johanna's 5-minute phone calls.

(b) Calculate Johanna's daily phone bill.

(c) Calculate Johanna's monthly phone bill (for 30 days in a month).

Johanna decides to go on a $79 cap plan for her mobile, which gives her $650 worth of calls, texts and data every month.

(d) If Johanna maintains her current phone habits and stays underneath the cap, how much money will she save **(i)** every month and **(ii)** every year?

Open-ended

10 Athena's phone company charges 82c/min per standard call, plus a flagfall of 30c. Ordinary text messages (SMS) cost 27c each, but picture text messages (MMS) cost 50c. Athena has $20 of credit on her phone.

How many calls and texts could Athena make before running completely out of credit?

Give three different combinations, making sure that you include calls of different time lengths.

11 Consider your own mobile phone use (if you don't have a mobile phone, consider someone you know).

(a) Approximately how many calls do you make per day, and how many minutes per call?

(b) How many messages would you send?

(c) If you know the call rates for your phone, use them to calculate the cost of your daily phone usage (not including internet data). If you don't know them, use a rate of 80 cents per minute and a flagfall of 35 cents for calls and 25 cents per message.

12 **(a)** Some people find it difficult to estimate how much data they consume when using their phone to browse the internet or download a song or app. Find out what a data allowance of 2 GB per month is equivalent to, in terms of how much time you can spend online or how many minutes of video you can watch.

(b) In recent years, many people have experienced 'bill shock', an unexpectedly high phone bill, often due to a high cost of data for internet usage. Should phone companies warn people when they are nearing their data limit for the month? Explain your answer.

Puzzle

Sudoku

Fill in the empty squares on the grid so that every row and column and each of the mini 3 × 3 grids contains the numbers 1 to 9 once each.

	2			3	6			
3						1	5	7
	8	9			5	6		
2	9		8	4				5
	3		9		1		7	
6				5	2		1	8
		6	7			4	9	
4	7	2						3
			5	8			2	

Challenge 1

1 Find five consecutive numbers such that the sum of the first and the fifth is 1708.

2 If 7^{2048} is divided by 100, what is the remainder?

3 Which numbers less than 100 have 5 as their smallest prime factor?

4 If m and n are different whole numbers, what is the smallest value of m for which $m^3 = n^4$ is possible?

5 A palindromic number reads the same backwards as it does forwards. There was one palindromic date in 2002, 20/02/2002 (20 February 2002). In the USA, dates are written with the month number first, then the day number, then the year. For example, 15 April 2017 is written as 04/15/2017.

(a) Write three palindromic dates in Australia for the 21st century.

(b) Write three palindromic dates in the USA for the 21st century.

6 Copy and complete the pattern below:

0, 3, 9, 18, 30, _____, _____, _____

7 Copy and fill in the missing numbers if the difference between each term in the sequence is the same:

$\frac{1}{12}$, _____, _____, $\frac{1}{7}$

8 If it takes 12 students 30 minutes to clean up 100 dishes after lunch on a school camp, how long should it take eight students to clean up 90 dishes?

9 What percentage is m of n?

A $\frac{n}{m}$
B $\frac{100n}{m}$
C $\frac{m}{n}$
D $\frac{100m}{n}$

10 When 1000^{2010} is written as a numeral, the number of digits written down is

A 2010
B 2011
C 6030
D 6031

11 Successive discounts of 10%, 20% and $33\frac{1}{3}$ % are equivalent to a single discount of

A 48%
B 50%
C 52%
D $63\frac{1}{3}$ %

Questions **12** and **13** may have more than one answer. List all the letters corresponding to correct answers.

12 Which of the following will simplify to 160?

A $\frac{40}{\frac{1}{4}}$
B $1000 \div 160\,000$
C 80×20
D $3.2 \div 0.2$

13 Which of the following will simplify to 5?

A $\sqrt{13^2 - 12^2}$
B 2.5^2
C $29 - 4 \times 6$
D $40 \div 4 \times 2$

Chapter review 1

Maths literacy

- award
- buying on terms
- cheque
- commission
- credit card
- debit card
- deferred payment
- discount
- earnings
- EFTPOS
- gross income
- income
- income tax
- interest
- lay-by
- loss
- net income
- overtime
- PAYG
- piece work
- principal
- profit
- rate of interest
- retainer
- salary
- simple interest
- superannuation
- tax
- tax deduction
- tax return
- taxable income
- wages

Copy and complete the following using the words and phrases from the list, where appropriate. A word or phrase may be used more than once.

1 Simple interest is calculated by multiplying the __________, the __________ (in decimal form) and the time period.

2 Before tax and other deductions are made, a person's earnings are referred to as __________.

3 When a business sells a product for more than it was bought or made for, a __________ has been made.

4 __________ is a payment based on a percentage of sales made.

5 Work-related expenses and donations to recognised charities are both examples of __________s.

6 When a person works __________ they should be paid at a rate higher than their normal rate of pay.

7 The set of rules outlining working conditions for employees, including wages, overtime and leave, is called an __________.

8 __________ is an amount of money paid into an account or fund to be accessed at retirement.

Fluency

1 Calculate the following: 1.1

(a) 130% of $47.85 (b) 0.5% of $52 000

2 (a) Ami is a hockey goalkeeper. In one game she stopped 13 of 16 shots on goal. Calculate Ami's 'stopping' percentage. 1.1

(b) A box of soft-centred chocolates is on sale for $7.34, a saving of $2.45. Calculate the dollar saving as a percentage saving, correct to 1 decimal place.

3 Calculate the selling price on the following items. Give answers correct to the nearest cent where necessary. 1.2

(a) A $75 shirt discounted by 30% (b) A $29.99 DVD discounted by $12\frac{1}{2}$%

4 A timber dining table cost $800 to construct and was sold for $1250. The percentage profit, based on cost price, is closest to: 1.2

A 36% **B** 43.75% **C** 56.25% **D** 64%

5 The Goods and Services Tax (GST) is a 10% tax added to the price of most retail items. 1.2

(a) Reprice the following so that the prices include GST:

(i) a $5.60 bottle of shampoo (ii) a $37.35 hair dryer

(b) The retail prices of the following items have GST included. Calculate the 'pre-GST' price of:

(i) a $129 calculator (ii) a $38.95 computer game

6 Andrea works as a casual in a supermarket. She earns $9.45 per hour for hours worked between 7 am and 7 pm, time-and-a-half for any hours worked after 7 pm or on Saturday and double time for any hours on Sunday. 1.3

Calculate Andrea's wage for the following week.

Monday	Wednesday	Friday	Saturday	Sunday
8 am–4 pm	8 am–4 pm	1 pm–10 pm	10 am–3 pm	10 am–3 pm

7 Nazim is picking grapes in a vineyard. He is paid $4.75 for every bucket of grapes picked. How much can Nazim earn in a 5-day week if he picks an average of 35 buckets in a day? 1.3

8 How much income tax is payable on the following taxable incomes? (Use the tax table on page 33.) 1.4

(a) $5900 (b) $47 890 (c) $62 780

9 Evan borrows $7000 to purchase a car. He will pay 8.5% simple interest over a period of 4 years. 1.5

(a) How much interest is payable?

(b) How much will Evan pay back altogether?

(c) If Evan makes monthly repayments, how much will these repayments need to be, to ensure he pays off both the principal and the interest in the 4 years?

10 Olivia has a credit card with an annual interest rate of 17.3%. The card has an interest-free period of 55 days. She has just received a statement covering the period from 12 June to 11 July. The average daily balance owing on the card over this period was $4509.82. Calculate the interest that will be payable if Olivia does not pay the full amount owing by the end of the interest-free period. 1.6

11 For the following household devices, calculate **(i)** the daily energy consumption, and **(ii)** the daily running cost, if the tariff is 19.8c/kWh. **1.7**

(a) A 215 W computer on for 9 hours a day.

(b) A 1750 W refrigerator on all day.

(c) A 900 W microwave oven used for 15 minutes a day.

12 Chen has a mobile phone plan that charges him 89c/min plus a flagfall of 30c to make calls. It costs 25c to send an SMS. Chen estimates that he makes three 5-minute calls per day, one 10-minute call and sends six SMS messages. Calculate the cost of Chen's daily phone usage. **1.8**

Understanding

13 Peter is a salesperson for an audiovisual company. He is paid a weekly retainer of $250 plus 3.5% of the value of his sales. One week, Peter sold products worth $4500, $7890, $2630 and $6100. Calculate his income for the week. **1.1**

14 Calculate the percentage rate of simple interest paid if a $5000 investment over $6\frac{1}{2}$ years gave a return of $7730. **1.5**

15 A shop purchases crystal vases for $75 each. **1.2**

(a) At what price should they be sold if there is an 85% mark-up?

(b) The vases are not selling well so they are discounted by 10%. What is their new selling price?

(c) What percentage mark-up does the discounted price represent on the cost price?

16 Effie works as a cleaner and is paid $19.50 per hour for a standard 38-hour week. If she works overtime she is paid time-and-a-half for the first 2 hours of overtime each day and double time for any hours after that. In one week Effie worked 3 hours overtime on Tuesday, 1 hour overtime on Wednesday and 4 hours overtime on Friday. Find her wage for the week. **1.3**

17 Jenny has a gross weekly income of $860. She donates $750 per year to registered charities, and has work-related expenses totalling $690 annually. **1.4**

(a) What is Jenny's annual gross income, assuming there are 52 weeks in a year?

(b) What is Jenny's taxable income?

(c) How much should Jenny pay in tax per year?

18 Natalie purchased an $80 necklace with lay-by. She had to pay a 20% deposit to begin the lay-by. **1.4**

(a) How much was the initial payment?

(b) If Natalie paid $15 per week, how many weeks did she take to pay off the necklace?

19 A monthly statement for a bank account offering 7% p.a. interest, calculated daily, has the entries shown for the month of June below. How much interest is payable for June? **1.6**

Date	Transaction	Balance
June 1	Opening balance	$100
June 14	Deposit $285	$385
June 21	Withdrawal $207	$178

20 Andrew has purchased a $7000 boat on terms, with an interest-free period of 24 months. He must pay a $25 establishment fee, a monthly account fee of $3.50 and a minimum monthly repayment of $60. **1.6**

(a) **(i)** How much will Andrew have paid off the cost of the boat after 24 months if he only makes the minimum repayments?

(ii) How much will still be owing?

(iii) How much will Andrew have paid in fees?

(b) If Andrew wants to have the boat paid in full by the end of the interest-free period, how much should his monthly repayments be?

(c) If Andrew has not paid the amount owing in full at the end of 24 months, an interest rate of 29% p.a. is charged. How much interest will be charged on the amount calculated in part **(a) (ii)** after the 25th month? (Assume 30 days in a month.)

Reasoning

21 Freya is 19 and has a part-time position at a supermarket. Her hourly rate for weekdays is $15.43, with time-and-a-half paid on Saturdays, and double time on Sundays. **1.3**

Tanya is also 19 and works in the same supermarket. Because she is a casual employee, her conditions are different to Freya's. Tanya gets an extra 25% on the hourly rate on weekdays. On weekends she gets a further 10% on top of her casual rate.

Both Freya and Tanya work for 18 hours during the week, 4 hours on Saturday and 4 hours on Sunday.

(a) Who earned more money?

(b) Why do casual workers receive an extra 25% on top of their hourly rate?

22 Annika works in a jewellery store. Her manager has asked her to put a mark-up of 78% on the cost price of all pairs of earrings. These earrings were later discounted by 20% during a mid-year sale. Find the overall profit or loss made by the store on a pair of earrings sold in the sale if the initial cost price of the earrings was $46.60. **1.2**

23 The Vu family have a 4500 W clothes dryer that takes 1 hour to complete a drying cycle. On average, the dryer completes three cycles a week. The Vu's electricity tariff is 19.5 c/kWh. Calculate the amount of money the family could save in a year if they stopped using their dryer. **1.7**

24 Rachel buys prepaid credit for her phone. She is finding that she is constantly 'topping up' the credit every few days. Rachel is charged the following rates by her phone company: **1.8**

- calls (to mobiles or landlines): 80c/min, plus a flagfall of 35c
- texts (SMS): 29c each
- data (using the internet): $10 per GB

Rachel estimates that in a typical day she makes three 5-minute calls, sends six SMS messages and spends approximately 2 hours browsing the internet on her phone, using about 100 MB of data (0.1 GB).

(a) Calculate the cost of Rachel's monthly phone bill, given her typical daily usage above, for 30 days in a month.

(b) Rachel is thinking of moving to a $59 cap plan for 24 months, which would give her $500 worth of calls, texts and data each month. If Rachel went on the plan and stayed under the monthly cap, how much would she save in the 24-month period compared with using prepaid credit?

Numeracy practice 1

Non-calculator

1 Julie bought 36 cans of soft drink for a party. 75% of the cans are empty at the end of the party. How many are still full?

A 4 **B** 9 **C** 16 **D** 27

2 Ali has 240 movies in his collection. Two-thirds of the movies are animated movies. Of the rest, 20% are action movies. How many action movies does Ali have in his collection?

A 16 **B** 48 **C** 80 **D** 160

3 The GST is a 10% tax added to the price of many retail items. A toaster costs \$33.50 before the addition of GST. Which calculation will give the new price of the toaster that includes the GST?

A \$33.50 + 10 **B** \$33.50 × 1.1 **C** \$33.50 + 0.1 **D** \$33.50 + 1.1

4 Emilio works part-time in a takeaway food shop. He gets \$15.80 an hour, except on Saturdays, when he gets one-and-a-half times his hourly rate for the first 3 hours and double his hourly rate for any hours worked after that.

One week, Emilio works 15 hours between Monday and Friday and 5 hours on Saturday. Which calculation will give Emilio's total wage for the week?

A \$15.8 × 20

B (\$15.8 × 15) + (\$15.8 × 1.5 × 5)

C (\$15.8 × 15) + (\$15.8 × 1.5 × 3) + (\$15.8 × 2 × 2)

D (\$15.8 × 15) + (\$17.3 × 3) + (\$17.8 × 2)

5 Raelene is going to invest \$5000 at 6% simple interest for 2 years. If she invested her money for twice as long, the amount of interest she would earn on her investment would:

A decrease slightly **B** double **C** halve **D** increase slightly.

Calculator allowed

6 Rohan bought a new phone at a 35% discount. If the phone was originally priced at \$99, how much did Rohan pay for the phone?

A \$64 **B** \$34.65 **C** \$64.35 **D** \$134

7 Of the 2153 books stored on the shelves of the local library; 847 are out on loan. What percentage of the library books are out on loan?

A 28.23% **B** 39.34% **C** 71.76% **D** 254.54%

8 Tyson purchases a \$1579 home theatre system on the following terms: 20% deposit up front, with monthly repayments of \$85 for the next 2 years. How much more money is paid using this method of payment than paying the full price up front?

9 Yang earns a yearly salary of \$57 300. The income tax rules that apply to Yang state that she must pay \$4650 plus 30c for every dollar over \$37 000. Calculate Yang's annual income tax bill.

10 A large chain of clothing stores has a policy of making 85% profit on all jeans and trousers sold. How much would a pair of jeans with a marked retail price of \$79.95 have cost the store? (Answer to the nearest cent.)

2

Pythagoras' theorem

2

Did Pythagoras 'steal' the theorem that bears his name? One of the world's most famous mathematical statements is not named after its discoverer.

Several ancient civilisations, including the Babylonians, Chinese and Egyptians, knew about the special relationship between the sides of a right-angled triangle. Pythagoras of Samos (believed to have lived 580–496 BCE) was a Greek philosopher who probably learnt about this theorem from Egyptian engineers known as the 'rope stretchers'. They used a circle of rope with 12 knots tied at regular intervals around the circle. Pegging the rope to the ground at intervals of 3, 4 and 5 knots produced a right-angled triangle, which was used to ensure that building foundations or walls could be constructed accurately.

So, why does Pythagoras get all the credit? Possibly because he led a secretive cult devoted to the study of mathematics, with their motto being 'all is number'.

Although Pythagoras did not discover the theorem, it is possible that he was the first to produce a proof of the theorem using geometry. The photograph on the opposite page is of a statue of Pythagoras on the island of Samos, in Greece.

Forum

Why do you think Pythagoras and his followers held secret meetings and kept their discoveries to themselves?

Think of some real-life objects that can be made of or be described by right-angled triangles.

Why learn this?

Pythagoras' theorem is a useful mathematical tool applied by builders, carpenters, engineers and surveyors to determine the straight-line distance between two points or to check whether an angle is a right angle. It is also used in the audio-visual industry to determine the height and width of TV and projection screens.

After completing this chapter you will be able to:

- use Pythagoras' theorem to determine whether a triangle is right-angled
- use Pythagoras' theorem to find side lengths in right-angled triangles
- apply Pythagoras' theorem to practical situations
- complete simple calculations using surds
- express irrational answers in exact (surd) form
- identify Pythagorean triples.

Recall 2

Equipment required: calculator for Questions **2–4, 6**

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 Round each of the following numbers to 2 decimal places.

(a) 45.789 **(b)** 12.2311 **(c)** 4.549 567 835 6

2 Use your calculator to find the following, correct to 2 decimal places where appropriate.

(a) 12^2 **(b)** 55^2 **(c)** 37.5^2

(d) $3^2 + 4^2$ **(e)** $9^2 + 13^2$ **(f)** $2.5^2 + 7.1^2$

3 Use your calculator to find the exact values of each of the following.

(a) $\sqrt{81}$ **(b)** $\sqrt{169}$ **(c)** $\sqrt{256}$

4 Use your calculator to find the following. Round your answers to 2 decimal places.

(a) $\sqrt{65}$ **(b)** $\sqrt{658}$ **(c)** $\sqrt{321.45}$

5 Solve the following equations.

(a) $x + 5 = 11$ **(b)** $144 + x = 225$ **(c)** $45 - x = 12$

6 Solve the following for positive values of x. Where necessary, express your answer as a decimal correct to 2 decimal places.

(a) $x^2 = 36$ **(b)** $x^2 = 39$ **(c)** $x^2 = 325$

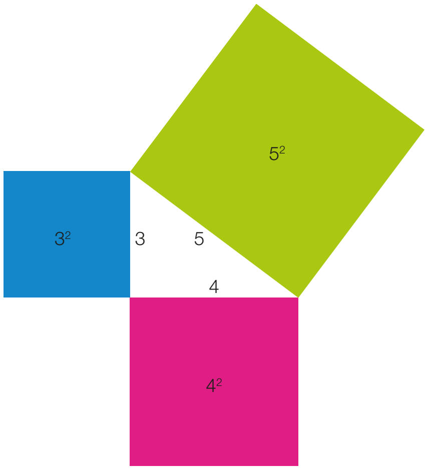
(d) $x^2 + 6 = 70$ **(e)** $x^2 - 20 = 101$ **(f)** $\frac{x^2}{4} = 325$

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Pythagorean triples

In this activity, you will explore the unusual pattern of Pythagoras' theorem solutions where the hypotenuse is a whole number.



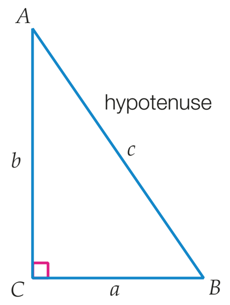
2.1

Pythagoras' theorem and right-angled triangles

A triangle that contains a 90° angle (a right angle) is called a **right-angled triangle**. The longest side of a right-angled triangle, which is opposite the right angle, is called the **hypotenuse**.

When labelling triangles, use capital letters for the vertices and use lower-case letters for the side lengths.

The triangle ABC at right, which can be written as ΔABC, has vertices A, B and C, with sides a, b and c opposite the vertices that have the same letter. Usually the hypotenuse is labelled as side c while the two shorter sides are a and b. (It does not matter which is a and which is b.)

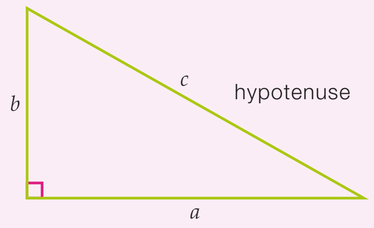


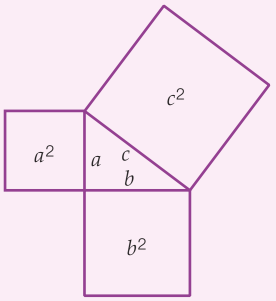
What is a theorem?

A **theorem** is a mathematical statement that can be shown to be true.
A **proof** is a step-by-step argument that shows the truth of a mathematical theorem.

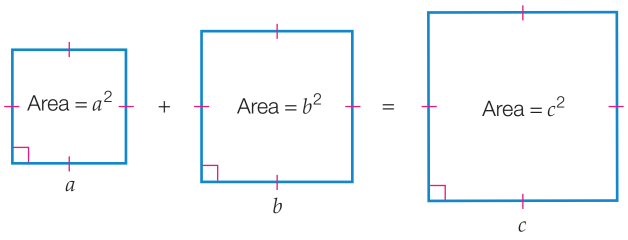
Pythagoras' theorem: For any right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two shorter sides.

$$c^2 = a^2 + b^2$$



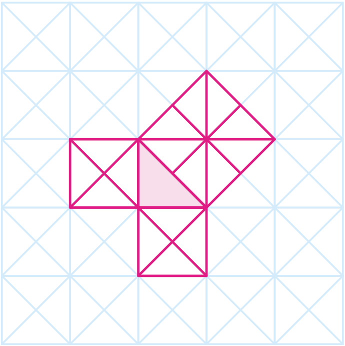


The area of a square of side a is a^2. In a right-angled triangle, the areas of the squares drawn onto the two shorter sides (see above) can be added to give the area of the square drawn onto the hypotenuse.



Demonstrating Pythagoras' theorem

The theorem can be visually demonstrated as shown. If you take an isosceles right-angled triangle and place it onto square grid paper that has been divided into small identical triangular tiles, the combined area of the squares on the two shorter sides is eight small triangles, which is the same number of small triangles covered by the square on the hypotenuse.



This shows only that the theorem works for one particular type of right-angled triangle. For the theorem to be shown as true, it must be proved to work for all right-angled triangles.

Proofs of Pythagoras' theorem by dissection

Mathematicians have established at least 85 different methods of proving that Pythagoras' theorem is true for every right-angled triangle. In 1830, a proof using areas was published by the mathematician Henry Perigal, who had the diagram of the proof carved on his gravestone!

Demonstrating Pythagoras' theorem using areas

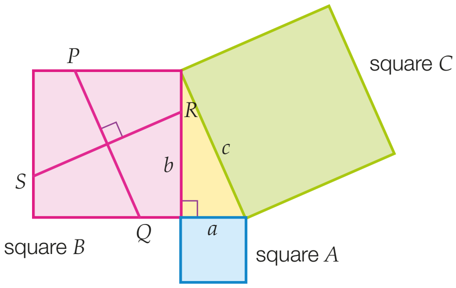
Step 1: On a piece of grid paper, draw any right-angled triangle, and label the sides a, b and c.

Step 2: Construct a square on each side of the triangle as shown below. Label the squares A, B and C to correspond to the sides a, b and c.

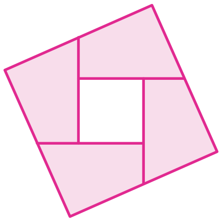
Step 3: Find the centre of square B. Rule a line PQ through the centre, so that it is parallel to the hypotenuse of the triangle.

Step 4: Rule a second line RS, perpendicular (at right angles) to PQ, that passes through the centre of the square.

Step 5: Cut square B into its four pieces. Then, cut out square A and square C.

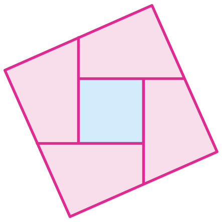


Step 6: Rearrange the four pieces of square B as shown.

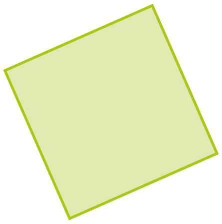


Step 7: Place square A in the centre. It should fit exactly. The total area of this square is equal to the sum of the area of square A and the area of square B:

Area = $a^2 + b^2$



Step 8: Compare the area of this square to the area of square C, c^2. The two squares should be identical in area, demonstrating that $c^2 = a^2 + b^2$.



The converse of Pythagoras' theorem

A **converse** is a reversal of a proved theorem.

Because Pythagoras' theorem is only true for right-angled triangles, the converse of Pythagoras' theorem can be used to check whether or not a triangle is right-angled. That is, if the sum of the squares of the two shorter sides is equal to the square of the longer side, then the triangle is right-angled.

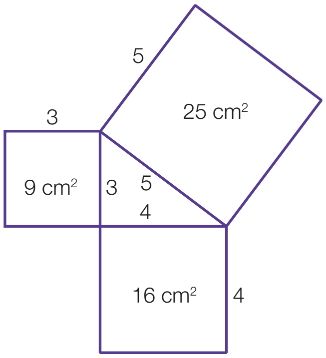
For example, consider a triangle of sides 3 cm, 4 cm, and 5 cm.

It can be shown that this is a right-angled triangle using the converse of Pythagoras' theorem.

Here, $a = 3$, $b = 4$ and $c = 5$.

Now: $c^2 = 5^2$
$= 25$

and: $a^2 + b^2 = 3^2 + 4^2$
$= 9 + 16$
$= 25$



As $c^2 = a^2 + b^2$ is true for this triangle, it is a right-angled triangle.

> If Pythagoras' theorem holds, then the triangle must be a right-angled triangle. This statement is known as the 'converse' of Pythagoras' theorem.

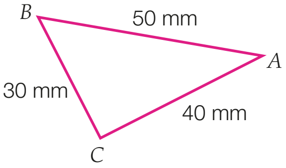
Builders use the converse of Pythagoras' theorem to make sure building frames and walls are right-angled. It is sometimes called 'the 3-4-5 principle'—can you see why?

Worked example 1

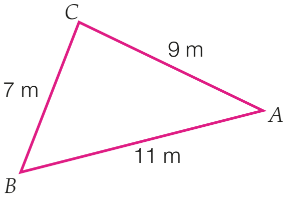
W.E. 1

Use the converse of Pythagoras' theorem to determine whether each of the following triangles is a right-angled triangle. If it is, state which angle is the right angle.

(a)



(b)



Thinking	Working
(a) 1 Identify the longest side as c and the other two sides as a and b.	(a) $a = 30$ mm, $b = 40$ mm, $c = 50$ mm
2 Evaluate the left-hand side of Pythagoras' theorem, c^2.	$c^2 = 50^2$ $= 2500$
3 Evaluate the right-hand side of Pythagoras' theorem, $a^2 + b^2$.	$a^2 + b^2 = 30^2 + 40^2$ $= 900 + 1600$ $= 2500$
4 Does $c^2 = a^2 + b^2$? If so, the triangle is right-angled.	$c^2 = a^2 + b^2$, so ΔABC is a right-angled triangle.
5 The right angle is the angle opposite the hypotenuse, c.	$\angle ACB$ is the right angle.

(b)	**1** Identify the longest side as c and the other two sides as a and b.	(b) $a = 7$ m, $b = 9$ m, $c = 11$ m
	2 Evaluate c^2.	$c^2 = 11^2$ $= 121$
	3 Evaluate $a^2 + b^2$.	$a^2 + b^2 = 7^2 + 9^2$ $= 49 + 81$ $= 130$
	4 Does $c^2 = a^2 + b^2$? If so, the triangle is right-angled. (Here, the triangle is not right-angled, so the symbol $\neq$ is used, which means 'is not equal to').	$c^2 \neq a^2 + b^2$, so ΔABC is not a right-angled triangle.

2.1 Pythagoras' theorem and right-angled triangles

Navigator

Answers p. 632

1, 2 (a, d, g), 3, 4, 6, 9, 11, 12	1, 2 (b, e, h), 3, 5, 6, 7, 8, 11, 12, 13	2 (c, f, i), 3 (c), 5, 6, 7, 8, 10, 11, 13, 14

Fluency

1 Match each statement with its diagram.

(a) The right angle is at X and the length of the hypotenuse is 26.

(b) The right angle is at Y and the length of the hypotenuse is 25.

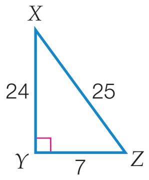
(c) The right angle is at X and the length of the hypotenuse is 13.

(d) The right angle is at X and the length of the hypotenuse is 25.

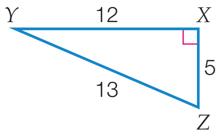
(e) The right angle is at Z and the length of the hypotenuse is 13.

(f) The right angle is at Y and the length of the hypotenuse is 20.

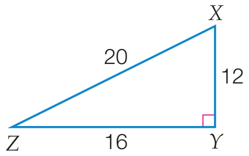
A



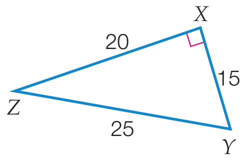
B



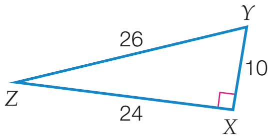
C



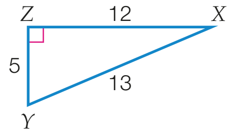
D



E



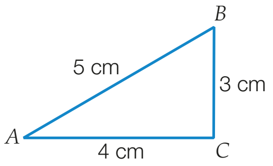
F



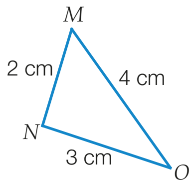
W.E. 1

2 Use the converse of Pythagoras' theorem to determine whether each of the following triangles is a right-angled triangle. If it is, state which angle is the right angle.

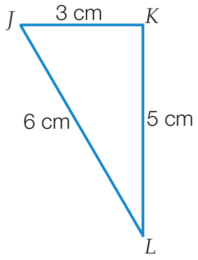
(a)



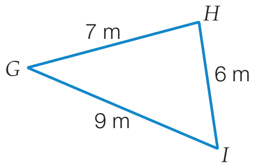
(b)



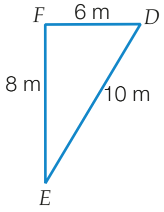
(c)



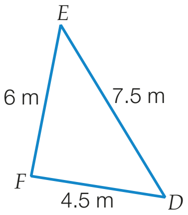
(d)



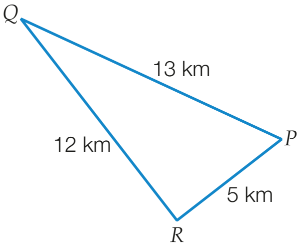
(e)



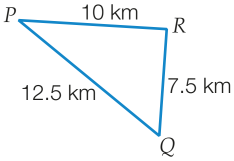
(f)



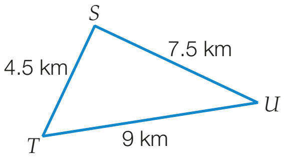
(g)



(h)

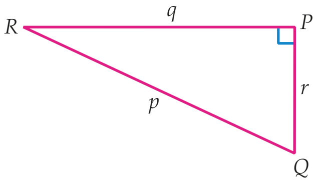


(i)



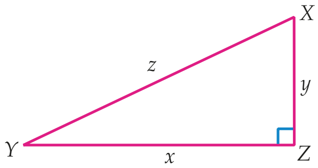
3 For each of the following triangles, select the correct statement of Pythagoras' theorem from those provided.

(a)



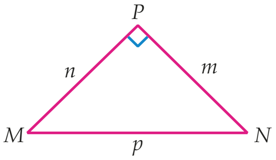
A $p^2 = q^2 + r^2$
B $q^2 = p^2 + r^2$
C $r^2 = p^2 + q^2$
D $q^2 - p^2 = r^2$

(b)



A $z^2 + y^2 = x^2$
B $x^2 + z^2 = y^2$
C $x^2 - y^2 = z^2$
D $z^2 = x^2 + y^2$

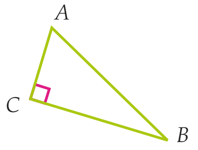
(c)



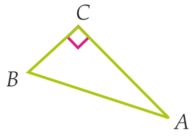
A $n^2 + p^2 = m^2$
B $p^2 + m^2 = n^2$
C $n^2 - m^2 = p^2$
D $n^2 + m^2 = p^2$

4 For each of the following right-angled triangles, identify which side length (*AB*, *BC* or *AC*) is the hypotenuse, and which angle is the right angle ($\angle ABC$, $\angle ACB$ or $\angle CAB$).

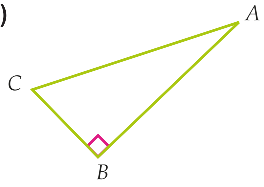
(a)



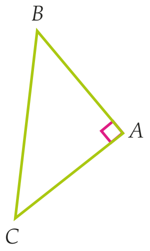
(b)



(c)

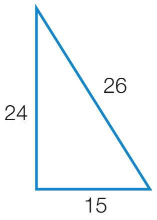


(d)

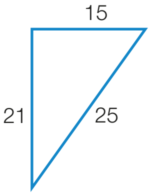


5 Which of these triangles is a right-angled triangle?

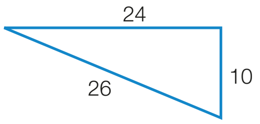
A



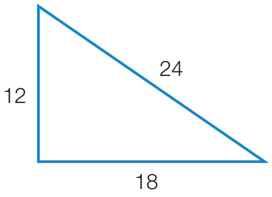
B



C



D



Understanding

6 **(a)** The longest side of a triangle is 15 cm long. The two shorter sides are 9 cm and 10 cm. Use the converse of Pythagoras' theorem to show that this triangle is not right-angled.

(b) The longest side of a triangle is 15 cm long. The two shorter sides are 9 cm and 12 cm. Use the converse of Pythagoras' theorem to show that this triangle is right-angled.

7 Find which of these triangles with the following side lengths are right-angled triangles.

(a) 9 mm, 12 mm, 15 mm

(b) 2 cm, 4 cm, 6 cm

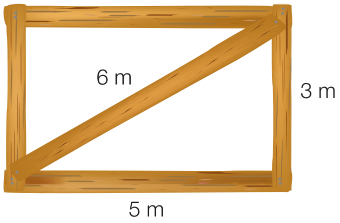
(c) 2 m, 2 m, 4 m

(d) 7 km, 24 km, 25 km

(e) 18 mm, 24 mm, 30 mm

(f) 5 cm, 12 cm, 13 cm

8 Jack has constructed part of a building frame. He needs to check that it is 'square'; that is, the corners are right angles. The frame is 5 m wide and 3 m high. Jack has measured the diagonal to be 6 m.



(a) Use the converse of Pythagoras' theorem to check whether the frame is 'square'.

(b) To make the frame 'square', should Jack increase or decrease the length of the diagonal?

9 For each of the following, construct right-angled triangles using the side lengths given in the table.

(a) Find the length of the hypotenuse *c* by measurement, then complete the rest of the table.

(b) Which columns of the table would you check, to see if Pythagoras' theorem was true?

(c) What would you look for in those columns?

a	b	c	a^2	b^2	$a^2 + b^2$	c^2
3 cm	4 cm					
12 cm	16 cm					
60 mm	80 mm					
5 cm	12 cm					

Reasoning

10 Pythagoras' theorem holds for the right-angled triangle whose sides are of length 5 cm, 12 cm and 13 cm.

(a) Triple each side length. Does Pythagoras' theorem still hold?

(b) Halve each original side length. Does Pythagoras' theorem still hold?

(c) What conclusion can you make from your answers to **(a)** and **(b)**?

11 Both a triangle whose sides are of length 3 units, 4 units and 5 units and a triangle whose sides are of length 5 units, 12 units and 13 units are right-angled triangles. Find another two right-angled triangles from the following list and justify your selection.

A 9 units, 12 units, 15 units

B 4 units, 5 units, 6 units

C 10 units, 24 units, 26 units

D 7 units, 14 units, 15 units

Open-ended

12 Draw a right-angled isosceles triangle, measure the three sides and show that Pythagoras' theorem holds.

13 You have a 40 m long piece of rope with which to create a large right-angled triangle on the school oval. Determine the lengths of the sides of three different triangles that you can create. You do not have to use the whole length of the rope.

14 Show how you would explain Pythagoras' theorem to a friend.

Problem solving

Olives and pots

Pythagoras loved the challenge of solving problems. According to legend, one of his favourite problems involved olives and clay pots.

If nine olives are placed in each pot, two pots will remain empty.

If six olives are placed in each pot, there will be three olives left over.

How many olives and clay pots are there?

Strategy options

- Draw a diagram.
- Make a model.
- Guess and check.
- Test all possible combinations.

2.2

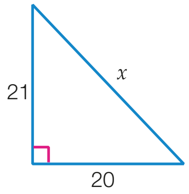
Finding the length of the hypotenuse

If you know the lengths of the two shorter sides of a right-angled triangle, you can use Pythagoras' theorem to find the length of the hypotenuse.

Worked example 2

W.E. 2

Calculate the length of the hypotenuse in the following triangle.



Thinking	Working
1 State Pythagoras' theorem and define the side lengths *a*, *b*, and *c*, letting *c* be the unknown length of the hypotenuse, *x*.	$c^2 = a^2 + b^2$ $a = 20,\ b = 21,\ c = x$
2 Substitute the values for *a*, *b* and *c* into Pythagoras' theorem.	$x^2 = 20^2 + 21^2$
3 Simplify the equation.	$x^2 = 400 + 441$ $x^2 = 841$
4 Solve for *x* by finding the square root of both sides of the equation.	$x = \sqrt{841}$ $x = 29$ units

Surds and decimal approximations

The length of the hypotenuse may not always be a whole number. It may have an exact decimal value, such as $c = \sqrt{6.25}$, which gives an exact answer of $c = 2.5$.

However, sometimes the length cannot be evaluated exactly by a decimal equivalent and is written as an approximation.

Consider a right-angled triangle with shorter side lengths of 1 m and 2 m:

$$\begin{aligned} c^2 &= a^2 + b^2 \\ c^2 &= 1^2 + 2^2 \\ c^2 &= 1 + 4 \\ c^2 &= 5 \\ c &= \sqrt{5}\text{ m} \end{aligned}$$

If you evaluate $\sqrt{5}$ on a calculator, it is shown as 2.236 067 977. This is a decimal approximation. There is no exact decimal equivalent because the number does not terminate (it has an infinite number of decimal places) and it has no repeating pattern.

Numbers that have an infinite number of decimal places with no repeating pattern are called **irrational numbers**. Irrational numbers such as $\sqrt{2}$, $\sqrt{5}$, $\sqrt{20}$ and $\sqrt{35}$ are known as **surds**. The only way to write these numbers exactly is in 'surd form', including the root sign, such as $\sqrt{2}$ or $\sqrt{5}$.

However, it is not always practical to write the side lengths of triangles as surds. Depending on the type of answer needed, you can use a calculator to find a **rational approximation** to an irrational number.

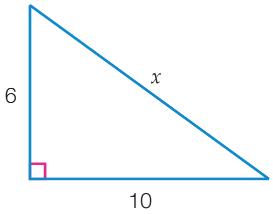
When you write an approximation, you should use the 'approximately equal to' symbol ≈ before writing the value as a decimal number correct to the appropriate number of decimal places. At other times you may leave the answer in surd form (also known as 'exact form').

For example: $c = \sqrt{5}$ m
$\approx 2.236\,067\,977$
$= 2.24$ m (correct to 2 decimal places)

Worked example 3

W.E. 3

Find the length of the hypotenuse in the following right-angled triangle. Give your answer in exact form (surd form).



Thinking	Working
1 State Pythagoras' theorem and define the side lengths.	$c^2 = a^2 + b^2$ $a = 6,\ b = 10,\ c = x$
2 Substitute the values for a, b and c into Pythagoras' theorem.	$x^2 = 6^2 + 10^2$
3 Simplify the equation.	$x^2 = 36 + 100$ $x^2 = 136$
4 Solve for x by finding the square root of both sides of the equation. Leave the answer as a square root (surd form).	$x = \sqrt{136}$

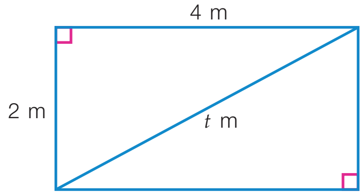
Pythagoras' theorem may be applied to other shapes provided they contain right-angled triangles. Sometimes, it is necessary to identify the required right-angled triangle by carefully adding a line or lines to the diagram.

Worked example 4

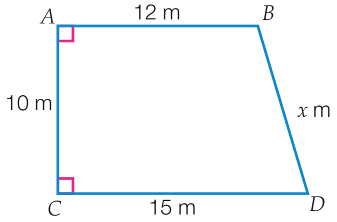
W.E. 4

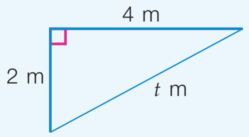
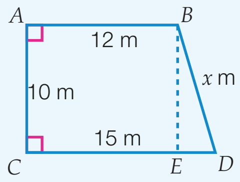
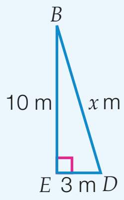
Use Pythagoras' theorem to find the value of the unknown length in each of the following diagrams. Give your answer as a rational approximation, correct to 2 decimal places, if necessary.

(a)



(b)

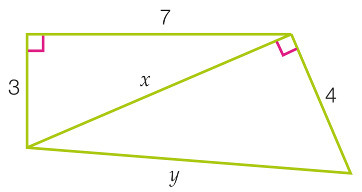


Thinking	**Working**
(a) **1** Identify a right-angled triangle and define the side lengths.	(a) Let $a = 2$, $b = 4$, $c = t$. 
2 State Pythagoras' theorem, then substitute in the values.	$c^2 = a^2 + b^2$ $t^2 = 2^2 + 4^2$
3 Simplify the equation.	$t^2 = 4 + 16$ $t^2 = 20$
4 Solve for *t* by finding the square root of both sides. Use a calculator to find its rational approximation and write it correct to 2 decimal places, with the correct units.	$t = \sqrt{20}$ $t \approx 4.472\,135\,955$ $t = 4.47$ m (2 d.p.)
(b) **1** Draw a line from point *B* down to the line *CD* to form a right-angled triangle *BED*.	(b) 
2 Redraw the right-angled triangle and define the side lengths.	$c = x$, $a = 10$ $b = ED$ $= 15 - 12$ $= 3$ 
3 State Pythagoras' theorem, then substitute in the values.	$c^2 = a^2 + b^2$ $x^2 = 10^2 + 3^2$
4 Simplify the equation.	$x^2 = 100 + 9$ $x^2 = 109$
5 Solve for *x* by finding the square root of both sides of the equation. Use a calculator to find its rational approximation and write it correct to 2 decimal places, with the correct units.	$x = \sqrt{109}$ $x \approx 10.440\,306\,51$ $x = 10.44$ m (2 d.p.)

Worked example 5

W.E. 5

Find the unknown lengths in the following diagram. Give your answers in exact form (surd form).



Thinking	Working
1 Identify the right-angled triangle where two of the three sides are known and define the side lengths.	$a = 3$ $b = 7$ $c = x$
2 State Pythagoras' theorem, then substitute in the values.	$c^2 = a^2 + b^2$ $x^2 = 3^2 + 7^2$
3 Simplify the equation.	$x^2 = 9 + 49$ $x^2 = 58$
4 Solve for *x* by finding the square root of both sides. Leave your answer as a square root (surd form).	$x = \sqrt{58}$
5 Identify the second right-angled triangle and define the side lengths using your answer for *x*.	$a = x = \sqrt{58}$ $b = 4$ $c = y$
6 State Pythagoras' theorem, then substitute in the values. Use the exact surd value determined previously.	$c^2 = a^2 + b^2$ $y^2 = (\sqrt{58})^2 + (4)^2$ $y^2 = 58 + 16$
7 Simplify the equation.	$y^2 = 74$
8 Solve for *y* by finding the square root of both sides. Leave your answer as a square root (surd form).	$y = \sqrt{74}$

2.2 Finding the length of the hypotenuse

Navigator

Answers p. 633

1, 2 (a–c), 3, 4, 5, 6 (a–f), 7, 8, 9, 10, 11, 18, 20	1 (a–c), 2 (a–c), 3, 4, 5, 6 (d–i), 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20	1 (d–f), 2 (d–f), 3 (a, c), 4, 5, 6 (g–l), 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

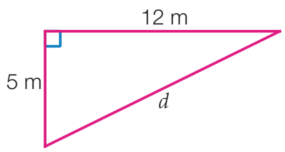
Equipment required: calculator

Fluency

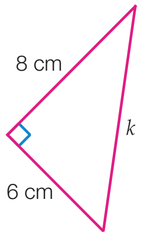
W.E. 2

1 Calculate the length of the hypotenuse in the following triangles.

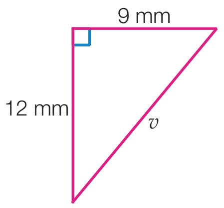
(a)



(b)



(c)



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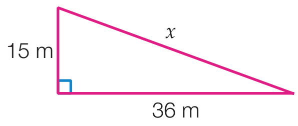

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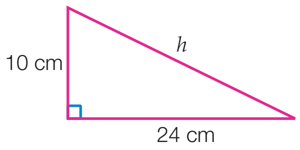
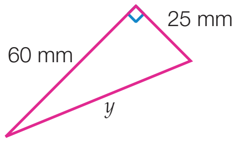


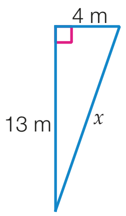
(f)



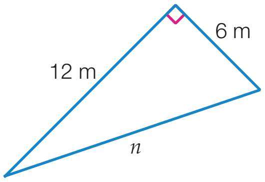
W.E. 3

2 Find the length of the hypotenuse in the following right-angled triangles. Give your answer in exact form (surd form).

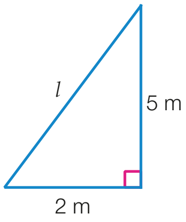
(a)



(b)



(c)



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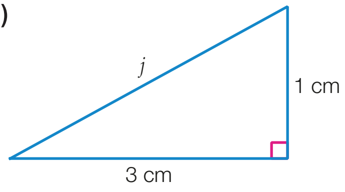

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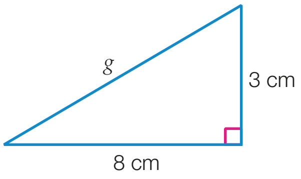
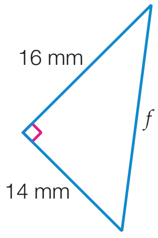


(f)

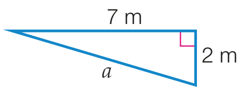


Hint

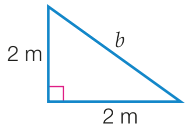
Don't forget to include the unit of measurement in your answer.

3 Find the length of the hypotenuse in each triangle. Write your answers as decimals, rounded to 2 decimal places.

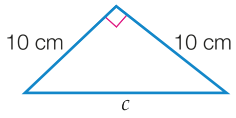
(a)



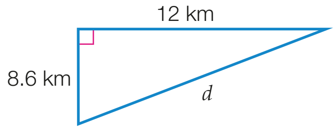
(b)



(c)

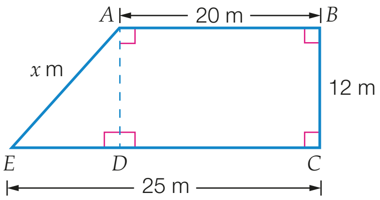


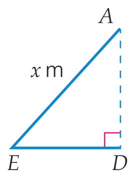
(d)



4 The diagrams below show a trapezium *ABCE* and the right-angled triangle *ADE*.

ΔADE is formed when a dotted line is drawn from *A* to *D*, with *AD* perpendicular to *CE*.

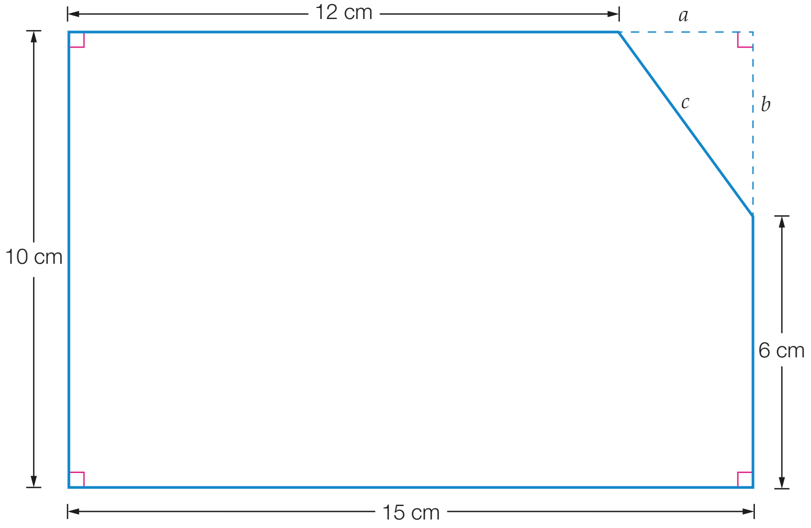




(a) In ΔADE, what are the lengths of the sides *AD* and *DE*?

(b) Use Pythagoras' theorem to find the length of *AE*.

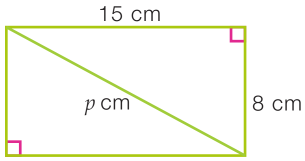
5 The diagram shows a 15 cm by 10 cm rectangle with a right-angled triangle cut from one corner. Find the triangle side lengths marked *a*, *b* and *c*.



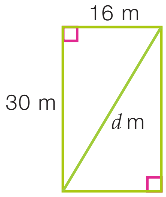
6 Use Pythagoras' theorem to find the value of the unknown length in each of the following diagrams. Give your answers as rational approximations, correct to 2 decimal places if necessary.

W.E. 4

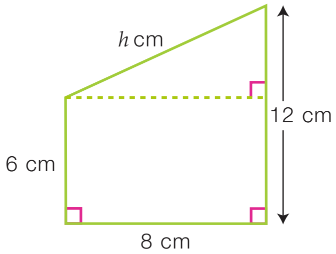
(a)



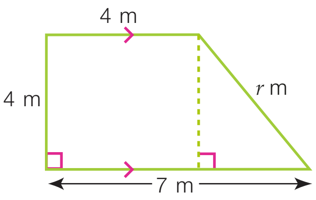
(b)



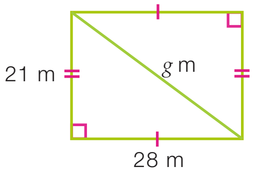
(c)



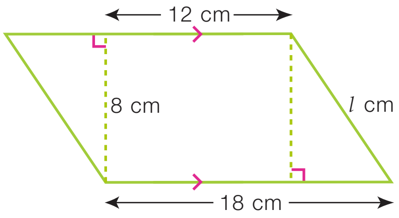
(d)



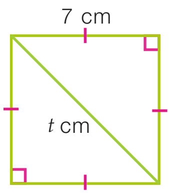
(e)



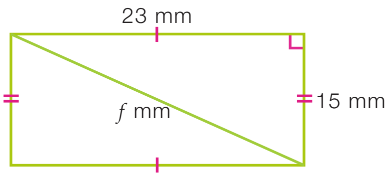
(f)



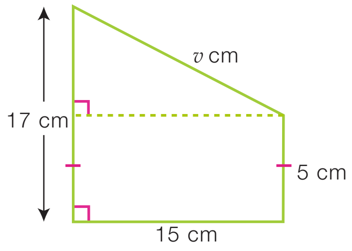
(g)



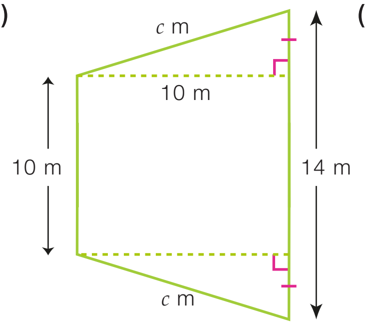
(h)



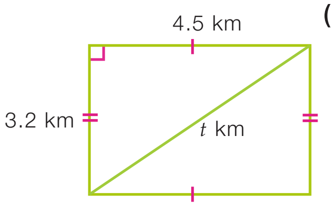
(i)



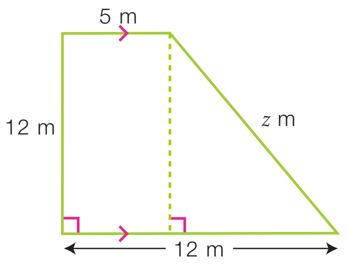
(j)



(k)



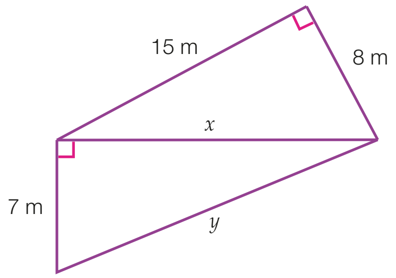
(l)



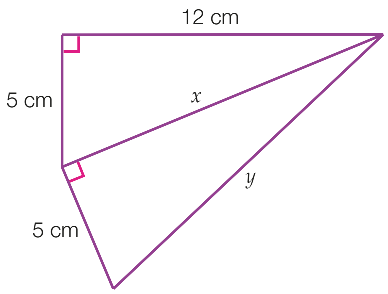
W.E. 5

7 Find the unknown lengths in the following diagrams. Give your answers in exact form (surd form).

(a)



(b)



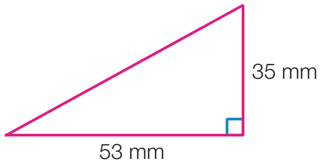
8 The length of the hypotenuse in this right-angled triangle, correct to 2 decimal places, is:

A 39.80 mm

B 63.51 mm

C 88.00 mm

D 4034.00 mm



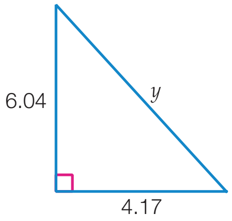
9 The value of y in the following diagram is closest to:

A 7.33

B 7.34

C 10.21

D 53.87



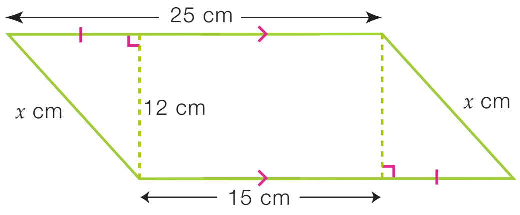
10 The value of x in the diagram is:

A $\sqrt{244}$ cm

B $\sqrt{288}$ cm

C $\sqrt{369}$ cm

D $\sqrt{769}$ cm



Understanding

11 A ladder is leaning against a framework and just reaches the top, a height of 4.27 m above the ground. The foot of the ladder is 1.41 m from the base of the framework.

(a) Draw a diagram of a right-angled triangle that represents the ladder, the ground and the framework. Label the diagram with the known measurements, and label the unknown side x.

(b) Use Pythagoras' theorem to calculate the length of the ladder, correct to 2 decimal places.

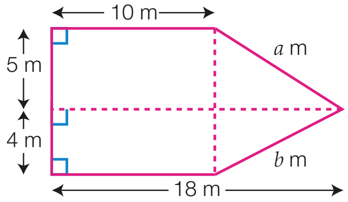
12 Corporal Vincent is in charge of designing a ropes course for an army training camp. In the final part of the course, recruits must slide down a rope connected from the top of a tower to a point on the ground 35 m from the base of the tower. The top of the tower is 14 m high and at right angles to the ground.

(a) Draw a diagram of a right-angled triangle that shows the tower, the rope and the ground. Label the diagram with the known measurements and label the unknown side x.

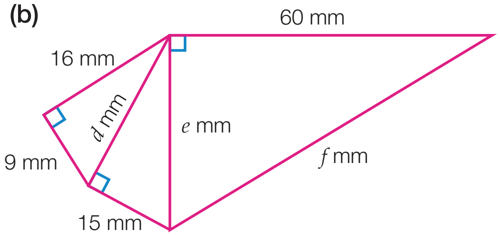
(b) Use Pythagoras' theorem to calculate the length of rope needed for this part of the course, correct to the nearest metre.

13 Find the exact value of the variables in the following diagrams. For any surd values, also give the answer correct to 2 decimal places.

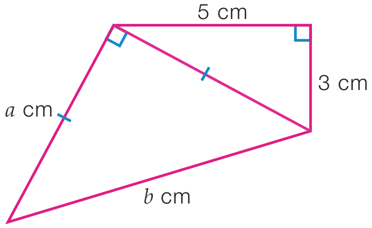
(a)



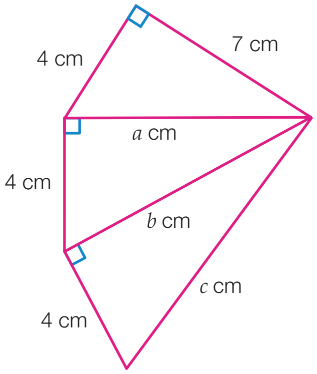
(b)



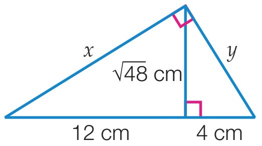
(c)



(d)



14 The values of x and y in the diagram are closest to:



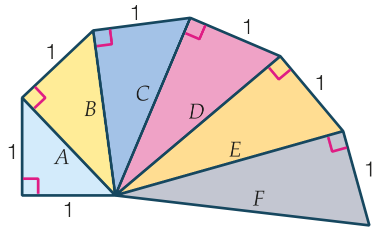
A $x = 9.80$ cm, $y = 8$ cm
B $x = 9.80$ cm, $y = 5.66$ cm
C $x = 13.86$ cm, $y = 8$ cm
D $x = 13.86$ cm, $y = 5.66$ cm

Reasoning

15 **(a)** Find the length of the hypotenuse of a right-angled isosceles triangle with a side length of 10 cm. Now divide the length of the hypotenuse by the length of the shorter side. Compare your answer to the value of $\sqrt{2}$, rounded to 3 decimal places.

(b) Find the length of the hypotenuse for several other right-angled isosceles triangles. Then, divide the length of the hypotenuse by the length of the shorter side. What do you notice about the result?

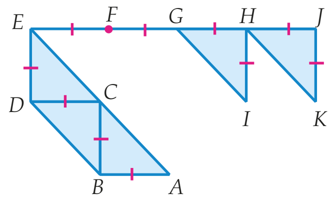
16 A Pythagorean spiral has the shape and side lengths shown.



(a) Use Pythagoras' theorem to calculate the values of the hypotenuses of A, B, C, D, E and F as exact values and correct to 2 decimal places where necessary.

(b) More triangles are added until the letter Z is used, so that the side lengths are $A, B, C, D, E, F, G, \ldots X, Y, Z$. How many side lengths cannot be written as exact decimal values?

17 **(a)** Use the letters shown to describe the shortest path possible to go from point A to point K if you can only follow the straight lines shown.



(b) If the shorter side of each triangle is 1 cm long, what is the total length of the path described in **(a)**? State your answer as an exact value.

18 Alex wants to buy a new TV. He has allocated a rectangular space on his lounge wall that is 98 cm in length and 54 cm in width. At the TV store, Alex learns that TVs come in the following screen sizes: 55 cm, 66 cm, 81 cm, 94 cm, 106 cm, 116 cm and 127 cm. He also learns that these 'screen sizes' are the diagonal distance from one corner of the screen to the other.

Which is the largest screen size that could fit in the space on Alex's wall?

Open-ended

19 A swimming pool is 50 m long and 25 m wide. Anastasia is trying to swim a length of the pool, starting from a corner at the shallow end. However, she has difficulty swimming in a straight line, so that instead she swims diagonally across the pool to a point at the other end. Draw a diagram and calculate four possible distances, correct to 2 decimal places, that Anastasia may actually swim when completing a lap of the 50 m pool.

20 The city council needs to build a wheelchair ramp from the ground to the top of a step that is 30 cm off the ground.

(a) Design three different ramps by first choosing the horizontal length, then calculating the sloping ramp length.

(b) Sketch your three ramps to scale.

(c) Which ramp design would you choose if the ramp were to be used every day?

Game

Perfect square root bingo

Equipment required: 2 dice

How to play:
Each player should draw a 3 × 3 grid and fill each grid space with any one of the following square roots (each used only once):

$\sqrt{4}$, $\sqrt{9}$, $\sqrt{16}$, $\sqrt{25}$, $\sqrt{36}$, $\sqrt{49}$, $\sqrt{64}$, $\sqrt{81}$, $\sqrt{100}$, $\sqrt{121}$, $\sqrt{144}$

Roll the dice. The sum of the dice is the number that both players can cross off their grid.

For example: if 2 and 3 are rolled, this gives $2 + 3 = 5$. You can cross $\sqrt{25} = 5$ off your grid.

The winner is the first player to cross off three numbers in a row on their grid. Play to win the best of three games.

Consider the following before your next game: Which square root is likely to come up more times than the others? Where would be a good location on your grid to place this number?

2.3

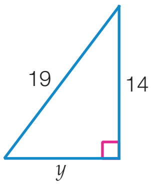
Finding the length of a shorter side

The length of one of the shorter sides of a right-angled triangle can be found if the lengths of the hypotenuse and the other side are known.

Worked example 6

W.E. 6

Find the value of the unknown shorter side in this right-angled triangle. Give your answer as an exact value.



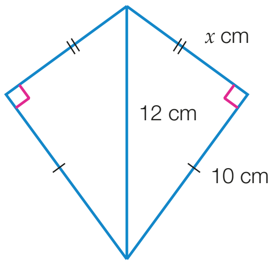
Thinking	Working
1 State Pythagoras' theorem and define the side lengths.	$c^2 = a^2 + b^2$ Let $a = y$, $b = 14$, $c = 19$
2 Substitute the values into Pythagoras' theorem.	$19^2 = y^2 + 14^2$
3 Rewrite so that the unknown is isolated on one side of the equation. (Here, 14^2 has been subtracted from both sides.)	$y^2 = 19^2 - 14^2$
4 Simplify the equation.	$y^2 = 361 - 196$ $y^2 = 165$
5 Solve for y by finding the square root of both sides of the equation. Leave your answer as a surd (exact form).	$y = \sqrt{165}$

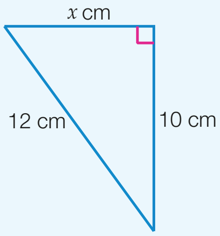
It may be necessary to draw a right-angled triangle from a diagram in order to apply Pythagoras' theorem.

Worked example 7

W.E. 7

Find the value of the variable, correct to 2 decimal places, in the diagram shown.



Thinking	Working
1 Draw a diagram of the appropriate right-angled triangle.	
2 State Pythagoras' theorem and define the side lengths.	$c^2 = a^2 + b^2$ Let $a = 10$, $b = x$, $c = 12$
3 Substitute the values into Pythagoras' theorem.	$12^2 = 10^2 + x^2$
4 Rewrite so that the unknown is isolated on one side of the equation. (Here, 10^2 has been subtracted from both sides).	$x^2 = 12^2 - 10^2$
5 Simplify the equation.	$x^2 = 144 - 100$ $x^2 = 44$
6 Solve for x by finding the square root of both sides of the equation. Use a calculator to find its rational approximation and write it correct to 2 decimal places.	$x = \sqrt{44}$ $x \approx 6.633\,2495$ $x = 6.63$ m (2 d.p.)

2.3 Finding the length of a shorter side

Navigator

1 (a–f), 2 (a, b, c), 3 (a, d, g, i), 4 (a), 5, 6, 7, 9, 11	1 (a–c, g–i), 2 (a, b, c, d), 3 (b, c, e, f, h, j), 4, 5, 6, 7, 9, 10, 11	1 (g–j), 2 (a, b, c, d), 3 (f–j), 4 (b), 5, 7, 8, 9, 10, 11

Answers p. 634

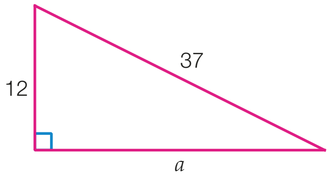
Equipment required: calculator

Fluency

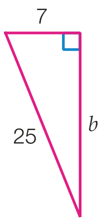
W.E. 6

1 Find the value of the unknown shorter side in these right-angled triangles. Give your answer as an exact value.

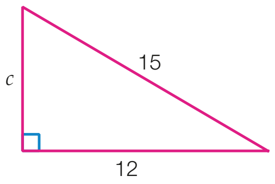
(a)



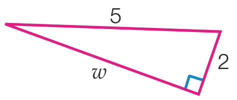
(b)



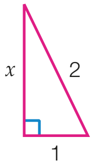
(c)



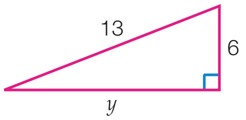
(d)



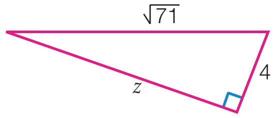
(e)



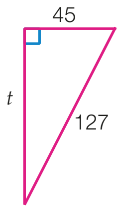
(f)



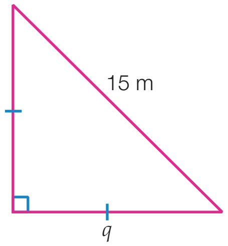
(g)



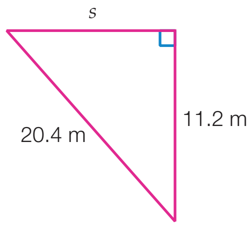
(h)



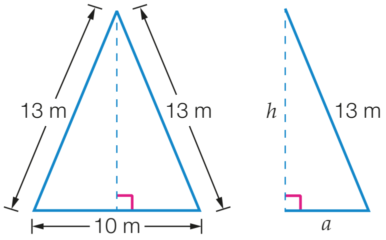
(i)



(j)



2 **(a)** An isosceles triangle with base length 10 m and equal sides of length 13 m is divided into two right-angled triangles, as shown. Fill in the missing numbers in the working below to find the value of h.



In the right-angled triangle:

a is half of 10 m, so $a =$ ______.

The length of the hypotenuse is ______.

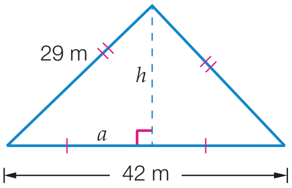
From Pythagoras' theorem:

$h^2 + a^2 = 13^2$
$h^2 + 5^2 = 13^2$
$h^2 + 25 =$ ______

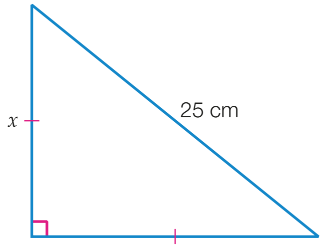
Subtract 25 from both sides of the equation:

$h^2 = 144$
$h =$ ______ $= 12$ m

(b) Find the values of a and h in this isosceles triangle.



(c) A right-angled isosceles triangle has a hypotenuse of 25 cm. Fill in the missing numbers in the working below to find the length of the equal shorter sides.



Let the length of the shorter sides be x cm.
The length of the hypotenuse is 25 m.

From Pythagoras' theorem:

$x^2 + x^2 =$ ______

Add like terms together:

$2x^2 = 625$

Divide both sides of the equation by 2:

$x^2 =$ ______

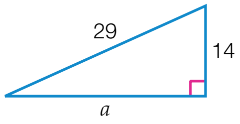
$x = \sqrt{312.5} \approx$ ______

(d) Find the length of the shorter sides of an isosceles right-angled triangle, given that the length of the hypotenuse is $\sqrt{392}$.

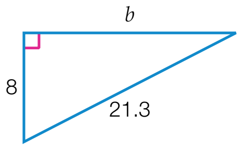
3 Find the missing values in these right-angled triangles. Give your answers as decimals, correct to 2 decimal places.

W.E. 7

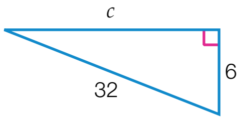
(a)



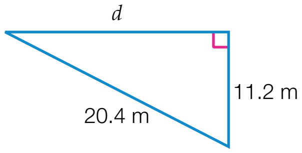
(b)



(c)



(d)



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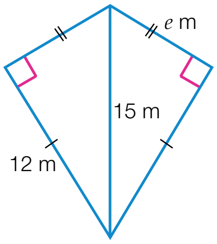
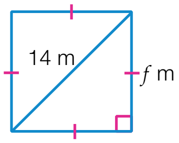
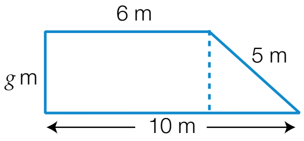


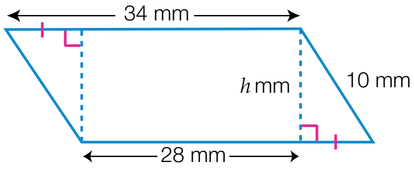
(f)



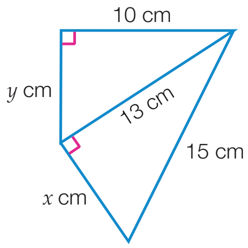
(g)



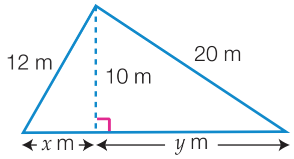
(h)



(i)



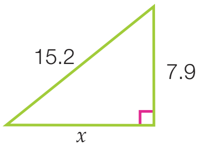
(j)



4 (a) The value of x in this right-angled triangle is closest to:

A 7.30 B 12.99

C 17.13 D 23.10



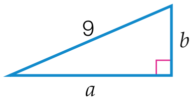
(b) The hypotenuse of a right-angled triangle has a length of 55 cm. One of the shorter sides has a length of 53 cm. The length of the third side is closest to:

A 2 cm B 14.6 cm C 14.7 cm D 216 cm

Understanding

5 Which statement in each set is *not* true?

(a)



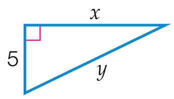
A a and b are both less than 9.

B $a^2 + b^2 = 9$

C $b^2 = 81 - a^2$

D $a + b > 9$

(b)



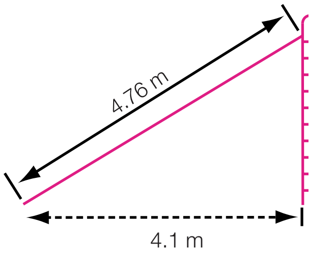
A $x^2 = y^2 - 25$

B y is more than 5.

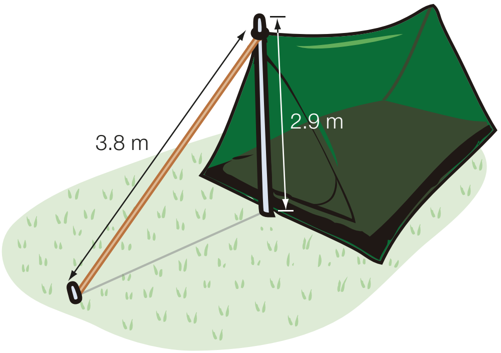
C $x^2 + 5^2 = y^2$

D $y = x + 5$

6 A playground slide has a length of 4.76 m, while it covers a horizontal distance of 4.1 m on the ground. What is the height of the slide, correct to 2 decimal places?



7 A 'guy rope' is attached to the top of a tent pole and pegged into the ground so that the rope is tight. If the rope is 3.8 m long, and the pole is 2.9 m long, how far from the base of the pole should the rope be pegged to make it tight?



Reasoning

8 If the hypotenuse of an isosceles right-angled triangle is 10 cm long, the length of the shorter sides is $\sqrt{50} \approx 7.07$ cm.

(a) Find the length of the shorter sides of an isosceles right-angled triangle with a hypotenuse of 20 cm, correct to 2 decimal places.

(b) Predict the length of the shorter sides of an isosceles right-angled triangle with a hypotenuse of 40 cm.

9 Matilda uses a piece of rope 12 m long to form an equilateral triangle.

(a) How long are the sides of the triangle?

(b) Find the height of the triangle, correct to 2 decimal places.

Open-ended

10 Safety regulations for ladders state that the distance that a ladder can be placed from a wall is no closer than one-quarter of its length and no further than half its length. If a ladder is 3.2 m long, give three heights that it can reach up the wall.

11 Find two sets of possible lengths of the shorter sides, correct to 2 decimal places, if the length of the hypotenuse of a right-angled scalene triangle is as given.

(a) 20 cm **(b)** 35 m

Puzzle

Roots but no plants

The square root of a number multiplied by itself gives the number. The cube root of a number is multiplied by itself twice to give the number. The fourth root is multiplied by itself three times.

"I know a number with a cube root that is ten times the fourth root", said Sue.

"Not possible!" said Alec.

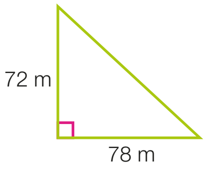
"Is too!" said Sue.

Who was right?

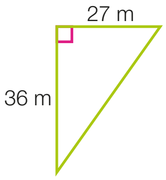
Half-time 2

1 For each of the following triangles, calculate the length of the hypotenuse. Round your answers to 2 decimal places where necessary. **2.2**

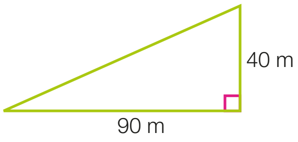
(a)



(b)

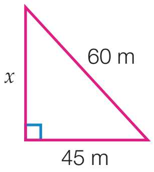


(c)

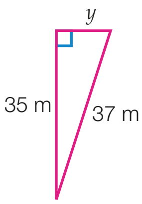


2 For each of the following triangles, calculate the length of the unknown side. Round to 2 decimal places where necessary. **2.3**

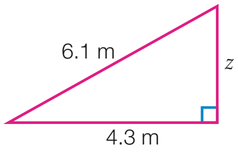
(a)



(b)



(c)



3 Use Pythagoras' theorem to show that a triangle with side lengths 10, 24 and 26 must be right-angled. **2.1**

4 Sharon is 20 metres due south of a target. She throws a ball which stops 16 metres due east of the target. **2.2**

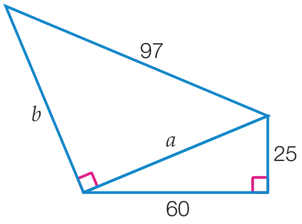
(a) Draw a diagram that shows this information.

(b) Calculate how far the ball travelled, correct to 2 decimal places.

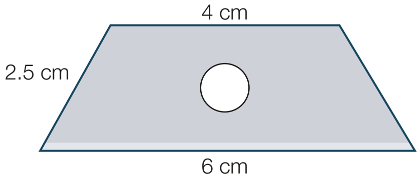
5 Decide if each triangle with the given side lengths is right-angled. **2.1**

(a) (16, 63, 65) **(b)** (48, 55, 73) **(c)** (13, 84, 86)

6 Calculate the exact values of *a* and *b*. **2.2**



7 A safety blade is a symmetrical shape with the dimensions shown. Calculate the height of the blade. Round your answer to 2 decimal places. **2.2**



8 A ladder 4 m long is placed against a wall so that the foot of the ladder is 1.5 m away from the wall. **2.3**

(a) Draw a diagram that shows this information.

(b) Calculate how far up the wall the ladder will reach. Give your answer correct to 2 decimal places.

Maths 4 Real

Pythagoras and plasma screens

Flat-screen televisions, such as plasma, LCD and LED screens, have been getting bigger and better every year. The high-definition picture, excellent colour and, more recently, 3D capabilities have had people racing to trade in their old TVs, with some even opting for complete home theatre systems. Manufacturers are now producing TVs with bigger and bigger screens as the demand grows.

Screen size

The screen sizes of televisions are traditionally given using the non-metric measurement unit of inches (which has the unit symbol ″). For example:

22″ 26″ 32″ 37″ 42″ 54″ 58″ 65″ 70″

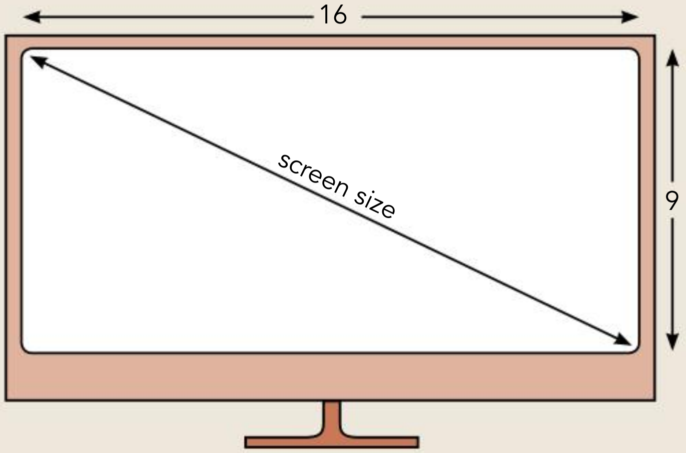
To convert these measurements in inches to centimetres, multiply by 2.54.

1 inch = 2.54 cm

Just what length does this 'screen size' measurement refer to? It is not the length or the width, but the diagonal distance across the screen from one corner to the other. This diagonal cuts the screen into two right-angled triangles, with the screen size measurement as the hypotenuse.

Aspect ratio

The aspect ratio of a screen is the ratio of the screen's length to its width. A common screen aspect ratio is 16 : 9. You can convert this ratio to a scale factor by dividing: 16 ÷ 9 = 1.78 (2 d.p.). This means that the length of the screen is 1.78 times the width. Before flatscreen TVs became common, most TV screens used to have an aspect ratio of 4 : 3, a scale factor of 1.33 (2 d.p.).



The customer's problem

1 Tom wants to buy a new TV. He has marked a space on his wall for it that measures 1.4 m in length and 1.1 m in width. When he arrives at the TV store, he learns that screen sizes are measured diagonally in inches and that the lengths and widths of the screens are always in a ratio of 16 : 9.

Tom wants the biggest screen possible, but he now has no idea how big that is!

Help Tom by following these steps:

(a) Write the ratio of the length to the width of the wall space in simplest form. Convert your ratio to a scale factor by dividing the length by the width.

(b) The ratio 16 : 9 written as a decimal scale factor is 1.78 (to 2 d.p.).

(i) Is the scale factor you calculated in **(a)** greater or less than 1.78?

(ii) To make your answer to **(a)** equal 1.78, should you decrease the length or the width of the wall space?

(c) Find a length and a width that give a ratio of 16 : 9 and make maximum use of Tom's wall space.

(d) Use Pythagoras' theorem to determine the size of the plasma screen with the length and width you determined in **(c)**. Convert your answer to inches. What size screen should Tom buy?

The technician's problem

2 Sam works for a company that installs home theatre systems. She has a customer who wants a TV with a 70″ screen. Sam needs to work out the length and width of the screen to know how much wall space is needed to hang it on. Sam uses the following method:

How to find the length and width of a TV screen

Step 1: The aspect ratio is 16 : 9, so imagine a TV that has a length of 16″ and a width of 9″. Use Pythagoras' theorem to find the diagonal screen size of this TV.

Step 2: Take the screen size of the TV the customer wants. Divide it by your answer to **(a)**. This gives a scale factor of the number of times the customer's TV is larger than the 16″ × 9″ TV you considered in **(a)**.

Step 3: To get the length of the customer's TV, multiply the scale factor calculated in **(b)** by 16. To get the width, multiply it by 9.

Step 4: Convert lengths in inches to centimetres by multiplying by 2.54.

(a) The wall space to hang the TV on is 1.3 m long and 1 m high. Follow the steps in Sam's method to determine whether a 70″ TV will fit in this space.

(b) Will a screen size of 42″ fit?

The biggest plasma screen in the world!

3 The biggest plasma screen ever produced was 150″. Find the length and width of the screen. Give your answer in metres.

Research

Find out about the aspect ratios that have been used to film movies and TV shows. What problems are encountered when a movie shot in one aspect ratio is shown on a screen with a different ratio? How are these problems solved?

Investigation

Estimating with Pythagoras

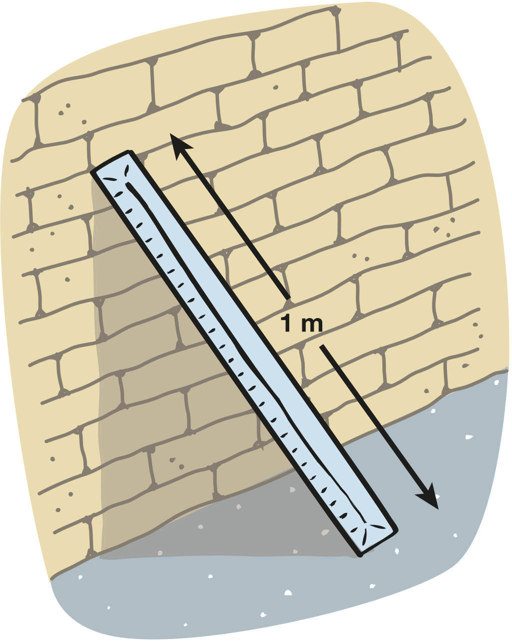
Equipment required: two 1-metre rulers, a wall

The Big Question

How does the height of a right-angled triangle change as you change the length of the base?

Engage

You can create a right-angled triangle by leaning a metre ruler up against a wall.



If you know the base length of this triangle (how far the end of the ruler is from the wall), you can use Pythagoras' theorem to calculate its height.

Rewriting $a^2 + b^2 = c^2$ as $a^2 = c^2 - b^2$ (subtracting b^2 from both sides of the equation) puts the theorem into a more useful form for these sorts of calculations.

1 Place the ruler against the wall so that the base length of the triangle is 20 cm. Write an estimation for the height of the triangle. Calculate the height of the triangle (remember, the length of the hypotenuse is 100 cm). Use the other metre ruler to measure the height. Compare your estimated, calculated and measured heights.

Explore

2 Create a series of five or six different triangles by placing the ruler lower down against the wall, lengthening the base. For each triangle, estimate, calculate and measure the height of the triangle. Make sure you know the length of the base of each triangle you create.

Strategy options

- Make a table.
- Act it out.
- Look for a pattern.

Explain

3 For each triangle, compare your estimated heights with the calculated heights. How close were they? Calculate your average error by finding the average of the difference between the estimated and the measured heights. Were you better at estimating taller or shorter heights?

4 Now, compare your measured heights with your calculations. Are they exactly the same? How could you explain any differences between them?

Elaborate

5 Can you see any pattern or connection between the increasing base length and the decreasing height? Were you expecting to see a pattern?

6 Draw a line graph of the height for each base length. What does your graph show?

7 Use your answers to **5** and **6** to answer the Big Question.

Evaluate

8 Consider how you went about solving this problem and the methods you used. Could you have approached it differently? How?

Extend

9 Using a tape measure or a piece of string for the hypotenuse, keep the base of the triangle constant (say 50 cm) and compare how the length of the hypotenuse changes as the height of the triangle is increased.

Applications of Pythagoras' theorem

2.4

Pythagoras' theorem is a powerful mathematical tool, useful in geometrical proofs as well as many everyday situations involving triangles.

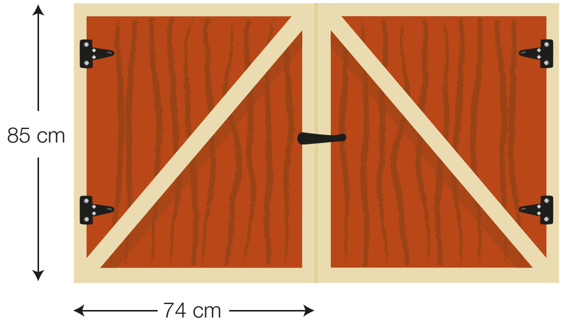
Follow these steps to solve a problem with Pythagoras' theorem:

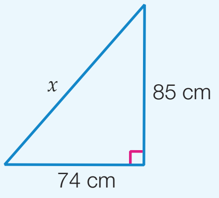
1 Draw a diagram with a right-angled triangle to show the situation.
2 Label the diagram with the right angle and the measurements given in the problem.
3 Identify the hypotenuse.
4 Label the unknown measurements that you need to find with variables.
5 If necessary, draw a simpler right-angled triangle diagram showing all the information.
6 Use Pythagoras' theorem to find the unknown length.
7 State your answer to the question in a sentence, including appropriate units.

Worked example 8

W.E. 8

What is the length of one of the diagonal supports of the frame shown? State your answer correct to 2 decimal places.



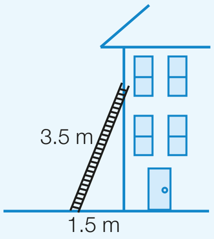
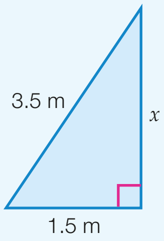
Thinking	Working
1 Draw a diagram of the appropriate triangle. Label the right angle, the given measurements and the unknown side.	

2 Use Pythagoras’ theorem to find the value of the variable.	$c^2 = a^2 + b^2$ $x^2 = 74^2 + 85^2$ $x^2 = 5476 + 7225$ $x^2 = 12\,701$ $x = \sqrt{12\,701}$ $x \approx 112.698\,7134$ $x = 112.70$ (2 d.p.)
3 State the answer correct to 2 decimal places.	The support is approximately 112.70 cm long.

Worked example 9

W.E. 9

A ladder 3.5 m long is leaning against a vertical wall with the base 1.5 m from the bottom of the wall on horizontal ground. How high up the wall does the ladder reach? State your answer correct to 2 decimal places.

Thinking	Working
1 Draw a labelled diagram with all measurements indicated.	
2 Draw a diagram of the appropriate triangle.	
3 Use Pythagoras’ theorem to find the value of the variable.	$c^2 = a^2 + b^2$ Let $a = 1.5$, $b = x$, $c = 3.5$ $3.5^2 = 1.5^2 + x^2$ $x^2 = 3.5^2 - 1.5^2$ $x^2 = 12.25 - 2.25$ $x^2 = 10$ $x = \sqrt{10}$ $x \approx 3.162\,277\,66\ldots$ $x = 3.16$ (2 d.p.)
4 State the answer.	The ladder will reach 3.16 m up the wall.

2.4 Applications of Pythagoras’ theorem

Navigator

1, 2, 4, 5, 7, 8, 11, 12, 14, 19, 20	1, 2, 3, 4, 6, 7, 9, 10, 11, 14, 16, 19, 20, 21	2, 3, 4, 6, 9, 10, 13, 14, 15, 16, 17, 18, 19, 20, 21

Answers p. 634

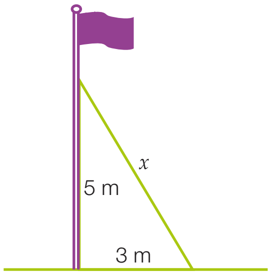
Equipment required: calculator

Where necessary, state your answers correct to 2 decimal places.

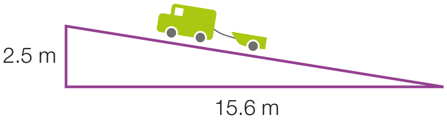
Fluency

W.E. 8

1 **(a)** A support wire is attached 5 m up a flagpole. The other end of the wire is attached to the ground 3 m from the base of the flagpole. What is the length of the wire?

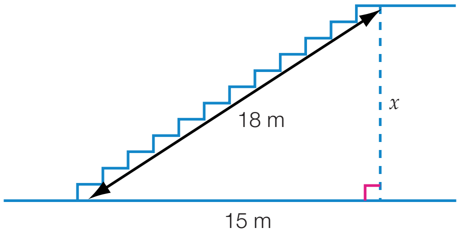


(b) A boat ramp is to be built that will cover a horizontal distance of 15.6 m while descending a vertical distance of 2.5 m. What is the length of the ramp?



W.E. 9

2 The escalator at a department store is 18 m long. When you ride on the escalator you move across 15 m. Calculate the height through which you travel from the lower level to the higher level.



3 **(a)** A leaf rake 1.8 m long is leaning against a vertical post. The distance from the bottom of the post to the bottom of the rake is 0.6 m. How high up the post will the top of the rake be?

(b) A rope is tied to a vertical tree trunk 2 m from the ground and is attached to the ground at a point 1.2 m from the base of the tree. How long is the segment of rope between the tree and the ground?

4 The top of a 6.5 m long ladder is resting against a wall at a point that is 4.2 m above the ground. What is the approximate distance of the base of the ladder from the base of the wall?

A 2.46 m **B** 2.99 m **C** 3.24 m **D** 4.96 m

5 A slide is 5 m long and the vertical ladder is 3.6 m. How far is the base of the ladder from the bottom of the slide?

6 Andrew is standing at the corner of a rectangular park, 35 m long and 28 m wide. He has decided to cross diagonally rather than go around the outside.

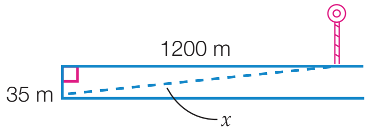
(a) How far does Andrew travel when crossing the park?

(b) How much distance does he save by not going around the outside?

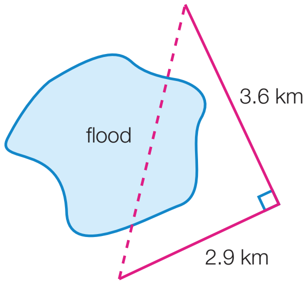
Hint

Draw a diagram for Questions **5** and **6** to represent the given information.

7 A racecourse has races that are run over a straight 1200 m course. One horse starts near the outside fence and runs in a straight line towards the finishing post on the inside fence. If the track is 35 m wide, how much further does this horse run than a horse that runs in a straight line along the inside fence?



8 During a heavy rainstorm, the road that Allie normally takes to travel home from school is flooded. She must take a detour as shown in the diagram. How much further does she now have to travel?



9 Edmond has attached a 17 m long support wire to a radio tower, 2 m from the top of the tower. If the other end of the wire is attached to the ground 7.5 m from the base of the tower, the total height of the tower is:

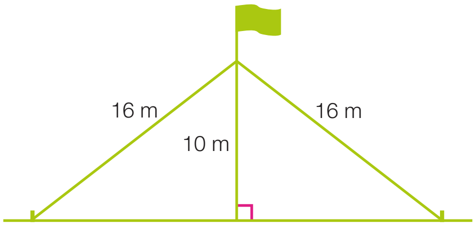
A 13.26 m **B** 15.00 m **C** 15.26 m **D** 17.26 m

Understanding

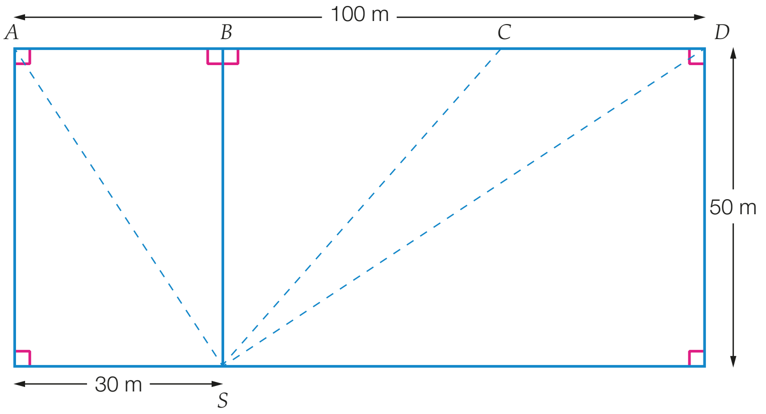
10 A flagpole, 10 m tall, is supported by two wires, each of length 16 m which are fastened to pegs in the ground. How far apart are the pegs?

A 6.25 m **B** 12.49 m

C 16.97 m **D** 24.98 m



11 Pythagoras Secondary School has a sports afternoon for the junior students. Lines are marked on a 100 m by 50 m grassed area for races. A diagram of the area is shown below.



All races will start at the point marked *S*. The Year 7 students will run straight across to *B* (50 m). Year 8 students will run to *A*, Year 9 to *C* and Year 10 to *D*.

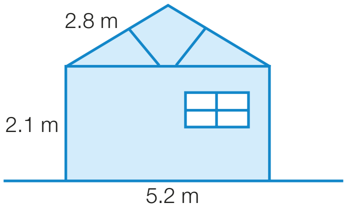
(a) Find the length of the Year 8 track (from *S* to *A*), rounded to the nearest metre. How much longer is this than the Year 7 track?

(b) How far do the Year 10 students have to run? Give your answer correct to the nearest metre.

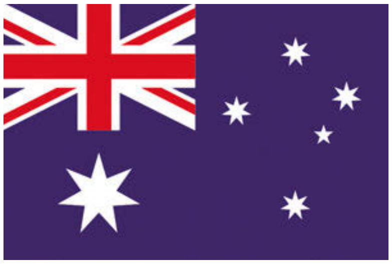
(c) The Year 9 track is 64 metres long. How far is point *C* from point *D*?

12 The length of the hypotenuse on an isosceles right-angled triangle is 16 cm. What is the length of the other two sides?

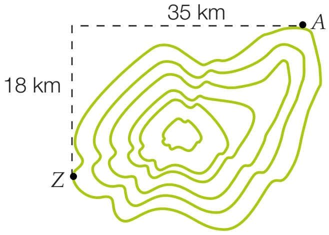
13 The end view of a garden shed is shown in the diagram. The shed is 5.2 m wide, the sides are 2.1 m high and the sloping part of the roof is 2.8 m long. Calculate the overall height of the shed.



14 The Australian flag flying at Parliament House in Canberra is 6.4 m wide and 12.8 metres long. It has been suggested that nylon tape be used in the lining along each diagonal and along each of its four sides to prevent creasing of the material. If the nylon tape costs $15 per metre, what total cost will be involved? State your answer to the nearest cent.



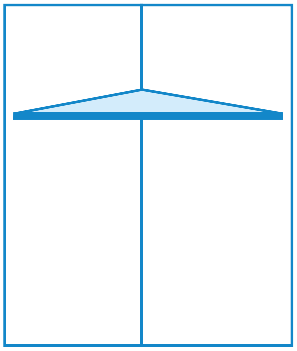
15 A new road must be built between the towns of Alderton and Zincton. Unfortunately, there is a mountain between the two towns. Building a right-angled road around the mountains will cost $2500 per kilometre. A road through a tunnel in the mountains will cost $4000 per kilometre. Which is the cheaper way to build the required road?



16 The rectangular frame for a wall of a new building is 7.5 m high and 18 m long. It has one diagonal support. If the frame, including the diagonal support, is constructed from steel girders that come in 10 m lengths, find the cost of the frame if the girders cost $230 each.

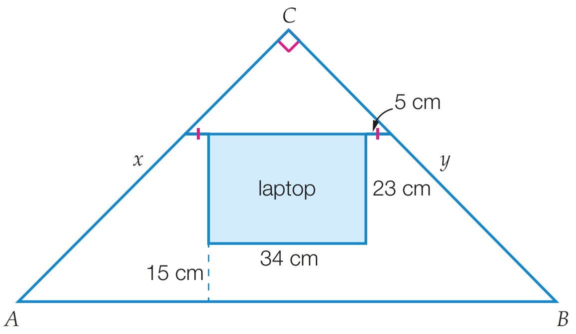
Reasoning

17 Tan is a student who has a very small bedroom. He wants to put a triangular shelf in the corner of his room to use as a desk. The triangle will be isosceles, with equal side lengths along the walls.

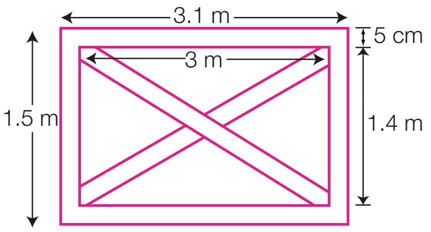


Tan draws a diagram of the top view of the smallest possible shelf, showing how he wants his laptop to fit on the shelf.

How long are the sides of the smallest possible triangular shelf? Give answers to the nearest centimetre, rounding up if any rounding is needed.



18 Angelo wants to build a rectangular gate from timber. The gate is to be 1.5 m high and 3.1 m wide. He has also decided to put in two diagonal cross supports to give the gate more strength. The timber he will use is 5 cm wide.

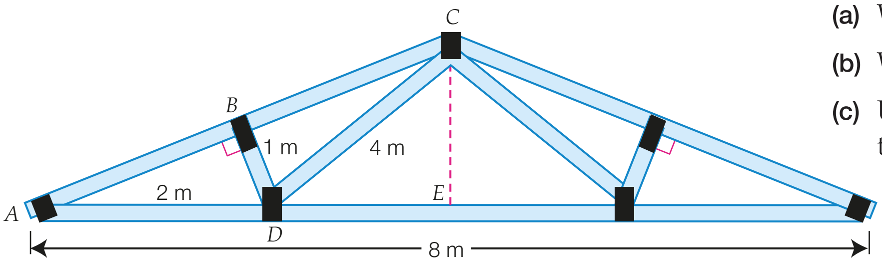


(a) What length of timber is required for the external rectangle?

(b) What length of timber is required for each of the diagonals?

(c) What is the total length of timber needed?

19 A symmetrical timber roof frame has been constructed with the dimensions shown below.



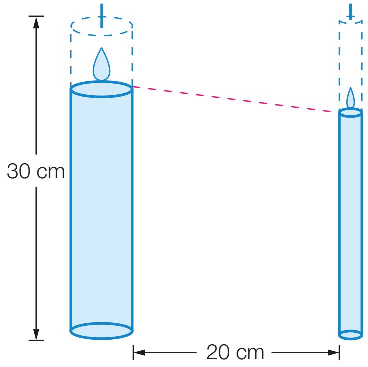
(a) What is the length of section AB?

(b) What is the length of section BC?

(c) Use your answers to **(a)** and **(b)** to find the total length of timber used.

Open-ended

20 Two candles, each initially of length 30 cm, are 20 cm apart. The thinner candle burns at a rate of 1 cm every 8 minutes and the other candle burns at a rate of 1 cm every 10 minutes. Find two possible lengths for the distance between the tops of the two candles, if the candles can burn for up to 3 hours.



21 Joel, a surveyor, is marking out the boundary of a block of land for a new house. From a peg in the ground he walks x metres north, $2x$ metres east, $4x$ metres south and $8x$ metres west. To finish, Joel then walks back to the peg.

(a) Choose a reasonable value for x and draw the shape of the block of land.

(b) How far did Joel walk altogether?

Puzzle

Pythagoras and the missing fig

Plato, a friend of Pythagoras, wanted to give him some of his 100 figs. This is how Plato did it.

Figs given to Pythagoras	Figs remaining
50	100 – 50 = 50
25	50 – 25 = 25
10	25 – 10 = 15
8	15 – 8 = 7
5	7 – 5 = 2
2	2 – 2 = 0

The total number of figs Pythagoras received was 50 + 25 + 10 + 8 + 5 + 2 = 100.

The total number of figs remaining is 50 + 25 + 15 + 7 + 2 + 0 = 99.

Can you explain why the two answers are not the same?

Pythagorean triples

2.5

A group of three positive whole numbers (*a*, *b*, *c*) that satisfy Pythagoras' theorem is called a **Pythagorean triple** or **Pythagorean triad**. Some common Pythagorean triples are shown in the table below.

(3, 4, 5)	(8, 15, 17)	(12, 35, 37)
(5, 12, 13)	(9, 40, 41)	(16, 63, 65)
(7, 24, 25)	(11, 60, 61)	(20, 21, 29)

Multiples of a Pythagorean triple must also satisfy Pythagoras' theorem. For example, if you multiply each number in (3, 4, 5) by 2, you have (6, 8, 10) and $6^2 + 8^2 = 10^2$. Dividing Pythagorean triples by a number also gives a set of numbers that satisfy Pythagoras' theorem. For example, dividing (7, 24, 25) by 2 gives (3.5, 12, 12.5) and $3.5^2 + 12^2 = 12.5^2$.

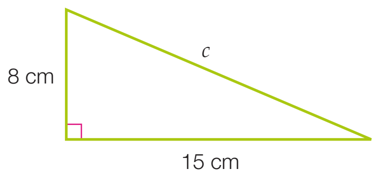
The numbers in a Pythagorean triple must satisfy Pythagoras' theorem exactly. Numbers that only work when they are approximate or rounded are not exact enough to be considered Pythagorean triples.

Worked example 10

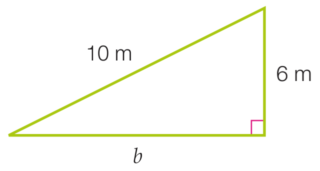
W.E. 10

Using your knowledge of common Pythagorean triples, state the value of the unknown side in each of the following triangles.

(a)



(b)



Thinking	Working
(a) **1** Check if the values of the shorter sides match a common triple, (*a*, *b*, *c*).	(a) $a = 8,\ b = 15,\ c = ?$
2 State the answer.	$c = 17$ cm
(b) **1** Check if the values of the hypotenuse and one of the shorter sides match a common triple, (*a*, *b*, *c*).	(b) $a = 6$ and $c = 10$ $= 3 \times 2$ $= 5 \times 2$ (6, ?, 10) is a multiple (× 2) of the common triple (3, 4, 5).
2 Find the missing number in the triple by multiplying the common triple number by the same multiple.	$b = 4 \times 2$
3 State the answer.	$b = 8$ m

2.5 Pythagorean triples

Navigator

Answers p. 634

1, 2 (columns 1, 2), 3 (a–c), 5, 6 (a–d), 7, 8, 13, 14	1, 2 (columns 2, 3), 5, 6 (a–d), 7, 8, 9, 10, 11, 13, 14	1, 2 (column 3), 4, 5, 6 (e–h), 7, 8, 9, 10, 11, 12, 13, 14

Fluency

1 **(a)** (3, 4, 5) is a Pythagorean triple. Fill in the missing numbers in these sentences about Pythagorean triples which are multiples of (3, 4, 5).

To make (9, 12, 15) you multiply each number by ____________.
To make (21, 28, 35) you multiply each number by ____________.
To make (60, 80, 100) you multiply each number by ____________.

(b) (7, 24, 25) is a Pythagorean triple. Write the new Pythagorean triples made when the following are applied.

The numbers are multiplied by 10.
The numbers are divided by 2.
The numbers are divided by 10.

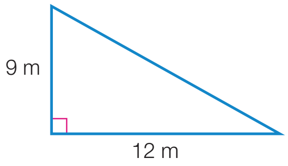
(c) (5, 12, 13) is a Pythagorean triple. Fill in the missing value in each Pythagorean triple made from (5, 12, 13).

(10, 24, ___), (___, 6, 6.5), (30, ___, 78)

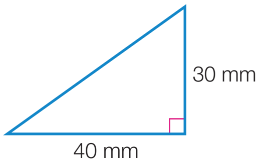
W.E. 10

2 Using your knowledge of Pythagorean triples, find the value of the missing sides in each of the following.

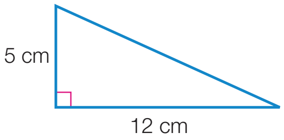
(a)



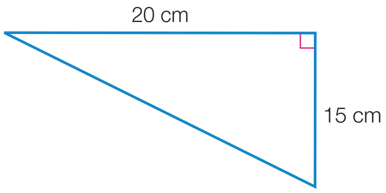
(b)



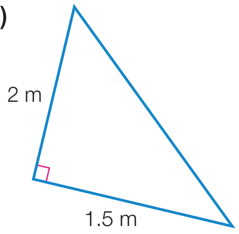
(c)



(d)



(e)



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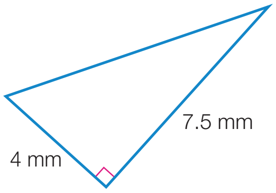

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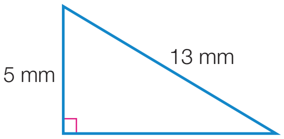
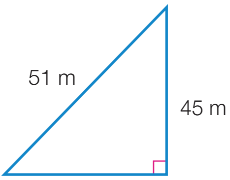
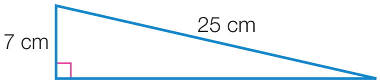


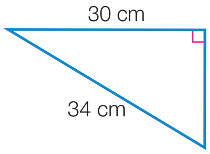
(h)



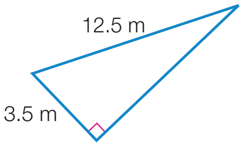
(i)



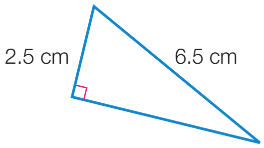
(j)



(k)



(l)



3 Find the value of c, using known Pythagorean triples.

(a) $a = 10, b = 24$	**(b)** $a = 15, b = 36$	**(c)** $a = 9, b = 12$
(d) $a = 27, b = 36$	**(e)** $a = 18, b = 80$	**(f)** $a = 50, b = 120$
(g) $a = 25, b = 60$	**(h)** $a = 500, b = 1200$	**(i)** $a = 80, b = 84$

4 If $a = 9$, $b = 40$ and $c = 41$ is a Pythagorean triple, which of the following is also a Pythagorean triple?

A (36, 160, 165) **B** (18, 80, 82) **C** (55, 240, 246) **D** (90, 405, 410)

5 If (5.6, 9, 10.6) satisfies Pythagoras' theorem, another set of numbers that also satisfies Pythagoras' theorem is:

A (65, 90, 106) **B** (10.2, 18, 1.06) **C** (14, 22.5, 26.5) **D** (14, 2.25, 20)

Understanding

6 Determine which of the following are Pythagorean triples.

(a) (6, 8, 10)	**(b)** (24, 45, 51)	**(c)** (14, 48, 50)	**(d)** (25, 60, 80)
(e) (10, 16, 28)	**(f)** (20, 48, 52)	**(g)** (10, 18, 22)	**(h)** (12, 50, 53)

7 Toula has just baked a cake in the shape of a rectangle. She wants to divide the cake into two right-angled triangles by cutting along a diagonal of the cake. Toula is then going to place a cake ribbon around the outside of each of the two triangular pieces of cake. The length of the cake is 32 cm and its width is 24 cm. Use a Pythagorean triple to determine exactly how much cake ribbon she will need.

8 **(a)** Complete the table, using only odd values for m.

(b) What do you notice about the numbers in each row of the table?

(c) Beginning with 'Take any odd number', describe the process used to generate the Pythagorean triples in the table.

m	$\frac{m^2-1}{2}$	$\frac{m^2+1}{2}$
a	b	c
3	4	5
5	12	13
7		
9		
11		

9 **(a)** Complete the table, using consecutive odd numbers for x and y.

(b) What do you notice about the numbers in the last three columns?

(c) Beginning with 'Take any two consecutive odd numbers', describe the process used to generate the Pythagorean triples in the table.

x	y	$x+y$	xy	$\sqrt{a^2+b^2}$
		a	b	c
1	3	4	3	5
3	5	8	15	17
5	7			
7	9			
9	11			

Reasoning

10 (48, 55, 73) is a Pythagorean triple.

If the numbers in the Pythagorean triple (7, 24, 25) are doubled, the new triple is (14, 48, 50).

If the numbers in the Pythagorean triple (16, 63, 65) are multiplied by 3, the new triple is (48, 189, 195).

Here are some Pythagorean triples with at least one number that is 24 or less. If a whole-number triple including the number 48 can be made, write the new triple. Write 'no' if it is not possible.

(a) (3, 4, 5)
(b) (5, 12, 13)
(c) (8, 15, 17)
(d) (9, 40, 41)
(e) (11, 60, 61)
(f) (12, 35, 37)
(g) (13, 84, 85)
(h) (24, 143, 145)

11 Explain how the Pythagorean triple (160, 384, 416) can be obtained from another Pythagorean triple.

12 Derek believes he has found a rule for Pythagorean triples. He says 'In each Pythagorean triple with no common factor, it appears that there is one even number and two odd numbers, one number with a factor of 5 and at least one prime number'.

3, 4, 5
5, 12, 13
7, 24, 25
9, 40, 41
11, 60, 61
13, 84, 85

8, 15, 17
12, 35, 37
16, 63, 65
20, 99, 101
24, 143, 145
28, 195, 197

20, 21, 29
28, 45, 53
36, 77, 85
44, 117, 125
52, 165, 173

From the given sets of Pythagorean triples:

(a) Find at least four triples that satisfy Derek's rule.

(b) Find at least one triple that does not satisfy Derek's rule.

(c) If you could modify Derek's rule to make it more accurate, how would you change it?

(d) Can you find the pattern in each column and then write the next triple in each column?

Open-ended

13 Choose one of the Pythagorean triples from the list on page 118 and list its multiples (a, b, c) that have a, b and c less than 300.

14 **(a)** Stefan told Gabriella that for a particular Pythagorean triple (a, b, c), the length a is a multiple of 4 and b is a multiple of 5. What is one Pythagorean triple that satisfies this information?

(b) Write two multiples of the answers to **(a)**.

Problem solving

How old was Pythagoras?

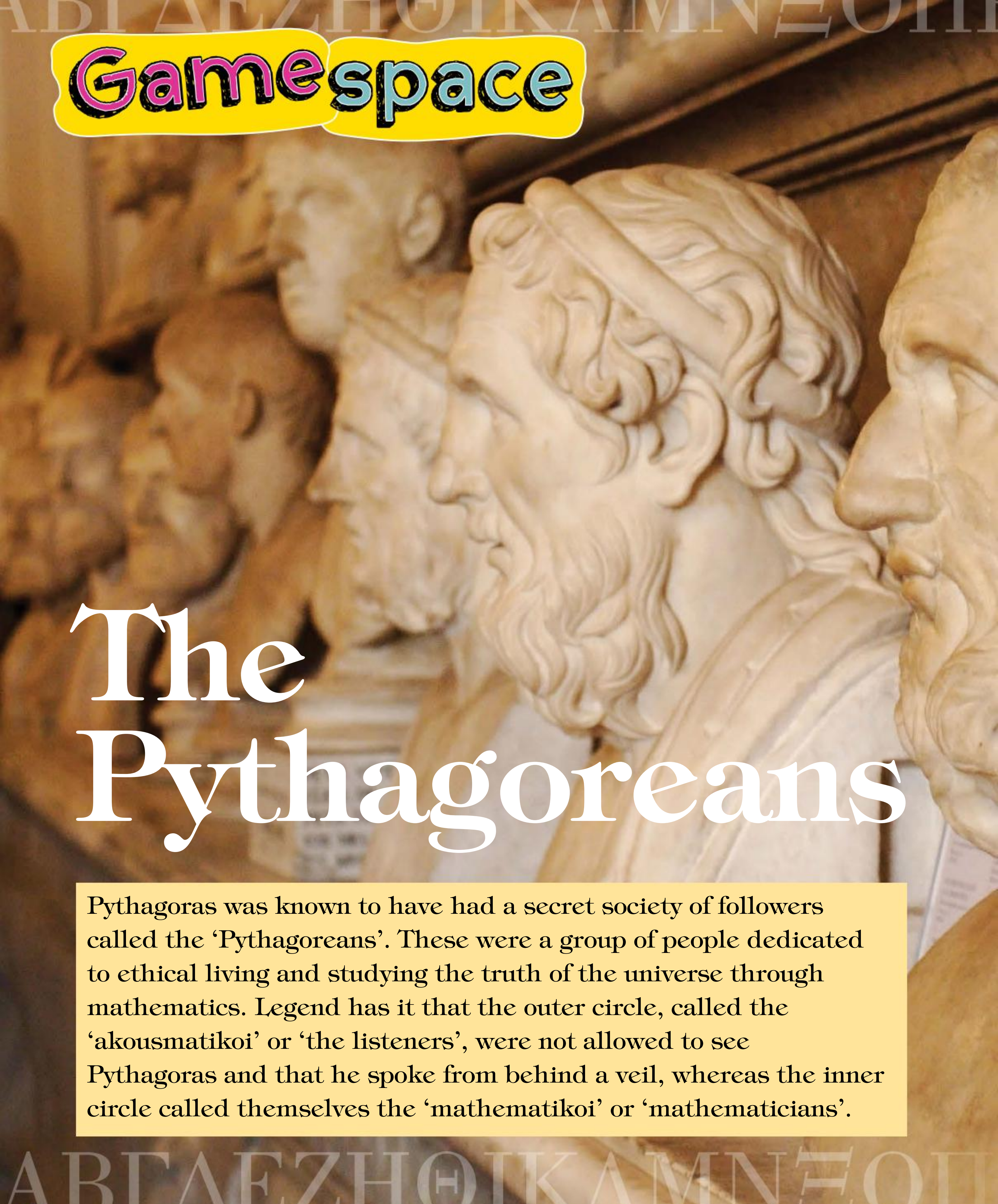
Use the clues about the Pythagorean triple (a, b, c), to find an estimate of how many years Pythagoras lived:

- c is Pythagoras' age, a value less than 100
- c is bigger than b and their difference is a cube number
- c is bigger than a and their difference is a square number
- a is a square number
- b is bigger than a and their difference is 41
- the digits of b are the same and are both odd numbers.

Strategy options

- Guess and check.
- Work backwards.
- Test all possible combinations.

Gamespace



The Pythagoreans

Pythagoras was known to have had a secret society of followers called the ‘Pythagoreans’. These were a group of people dedicated to ethical living and studying the truth of the universe through mathematics. Legend has it that the outer circle, called the ‘akousmatikoi’ or ‘the listeners’, were not allowed to see Pythagoras and that he spoke from behind a veil, whereas the inner circle called themselves the ‘mathematikoi’ or ‘mathematicians’.

Truth, or False?

There are many strange myths surrounding the Pythagoreans. To find which myths have been attributed to the Pythagoreans you need to complete the following tasks.

Once you have completed the tasks, your MYTH CODE numbers will tell you which of the myths in the 'Myth list' are actually true.

Myth Breaker #1

The Pythagoreans saw odd numbers as masculine and even numbers as feminine.

Write the following square roots as whole numbers and say whether, according to the Pythagoreans, the answers are masculine or feminine.

1 $\sqrt{9}$

2 $\sqrt{16}$

3 $\sqrt{25}$

4 $\sqrt{81}$

5 $\sqrt{36}$

6 $\sqrt{121}$

7 $\sqrt{64}$

8 $\sqrt{100}$

9 $\sqrt{144}$

10 $\sqrt{169}$

11 $\sqrt{225}$

MYTH CODE: The first three feminine numbers in your answers.

Myth Breaker #2

Pythagoras' theorem can be stated in several ways. Half of the equations below are correct statements of Pythagoras' theorem. Find which equations are correct.

1 $c^2 = a^2 + b^2$

2 $b^2 + a^2 = c^2$

3 $c^2 = b^2 - a^2$

4 $b^2 = c^2 - a^2$

5 $a^2 = b^2 - c^2$

6 $b^2 = a^2 - c^2$

7 $a^2 = c^2 - b^2$

8 $c^2 = a^2 - b^2$

MYTH CODE: The number of correct rearrangements.

Myth Breaker #3

Pythagorean triples are groups of three numbers (*a*, *b*, *c*) that satisfy Pythagoras' theorem. Which of the following groups of numbers are Pythagorean triples?

1 (1, 2, 3)

2 (3, 4, 5)

3 (5, 12, 13)

4 (5, 9, 11)

5 (7, 24, 25)

6 (15, 18, 22)

7 (8, 15, 17)

8 (16, 63, 65)

MYTH CODE: The number of groups of three that are not Pythagorean triples.

Myth Breaker #4

Solve the following equations.

$d = \sqrt{3^2 + 4^2}$

$c = \sqrt{3^2 + 5^2}$

$x^2 = 5^2 + 12^2$

$p^2 = 4^2 + 65$

$6 = q^2 + 4$

$5 = 41 - a^2$

$3 = 52 - b^2$

$1 = 101 - m^2$

MYTH CODE: The numerical value of *x*, *p* and *m*.

Myth list—which of these are true?

Myth 1 If you broke their rule of silence you were punished by death.

Myth 2 They avoided eating tomatoes.

Myth 3 Pythagoreans swore by the number 10 rather than by the gods.

Myth 4 They avoided eating beans.

Myth 5 One Pythagorean would only eat porridge in which an even number of oats had been used.

Myth 6 The inner circle lived at the school.

Myth 7 Each inner circle member was referred to by a number rather than a first name.

Myth 8 The inner circle owned no personal possessions.

Myth 9 They were mostly vegetarian.

Myth 10 Pythagoreans discovered square numbers.

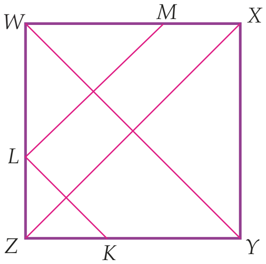
Myth 11 If you broke their rule of silence you were locked in a room and had to count whole numbers from 1 to 1 000 000 aloud.

Myth 12 They believed eating breakfast on an even numbered day was bad for men's health.

Myth 13 One Pythagorean discovered irrational numbers; the idea disturbed Pythagoras so much the man was executed.

Challenge 2

1 $WXYZ$ is a square of side 3 units. K is a point on YZ, L is a point on ZW and M is a point on WX. $KL \parallel YW$ and $LM \parallel ZX$. Let $ZK = a$ and $WL = b$ units.



(a) Write an equation for b in terms of a and rearrange to find $a + b$.

(b) What is the exact length of KL in terms of a?

(c) What is the exact length of LM in terms of b?

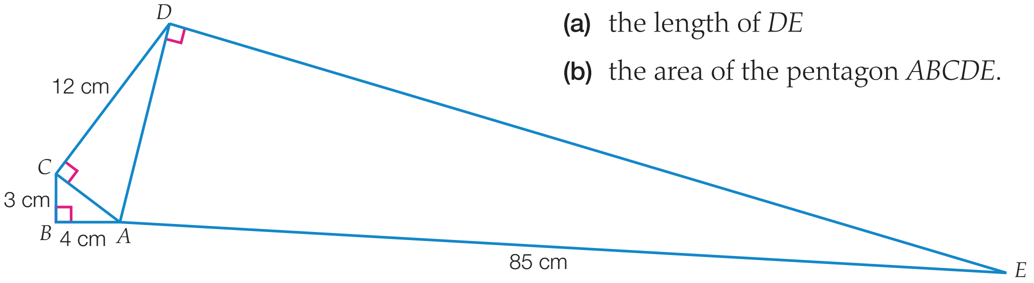
(d) What is the exact length of $KL + LM$?

2 Triangles ABC, ACD and ADE have a right angle at B, C and D, respectively. $AB = 4$ cm, $BC = 3$ cm and $CD = 12$ cm.

If $AE = 85$ cm, find:

(a) the length of DE

(b) the area of the pentagon $ABCDE$.



3 Write a whole number whose square root is between 7 and 8.

4 What is the smallest number that, when multiplied by 2016, gives a square number?

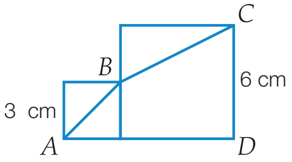
5 If m, n are integers and $m > n \geq 1$, then the numbers $m^2 - n^2$, $2mn$, $m^2 + n^2$ are Pythagorean triples (that is, a set of three numbers that obey Pythagoras' theorem).

(a) Show that $(m^2 + n^2)^2 = (m^2 - n^2)^2 + (2mn)^2$.

(b) Find the Pythagorean triples, with no common factors, whose smallest member is:

(i) 3 **(ii)** 5 **(iii)** 7 **(iv)** 8 **(v)** 9

6 A square of side length 3 cm is placed next to a square of side length 6 cm as shown. The perimeter of the quadrilateral $ABCD$ is:



A $3\sqrt{2}$ cm **B** $3\sqrt{5}$ cm

C $6\sqrt{2}$ cm **D** $3(5 + \sqrt{2} + \sqrt{5})$ cm

Chapter review 2

Maths literacy

converse
hypotenuse
irrational number
proof
Pythagoras' theorem
Pythagorean triple
rational approximation
right-angled triangle
surd
theorem

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 The longest side of a right-angled triangle is called the __________.

2 For any __________, the square of the length of the __________ is equal to the sum of the squares of the lengths of the two shorter sides.

3 The equation $c^2 = a^2 + b^2$ is known as __________.

4 A __________ is a logical, step-by-step argument that demonstrates the truth of a mathematical statement.

5 A mathematical statement that has been proven true is known as a __________.

6 When a number such as $\sqrt{84}$ is written as a rounded decimal, it's called a __________.

7 An irrational number written in exact form, such as $\sqrt{5}$, is known as a __________.

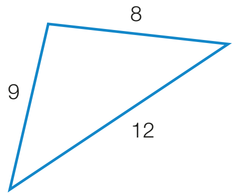
8 A group of three whole numbers that satisfy Pythagoras' theorem, such as (3, 4, 5), is called a __________.

Fluency

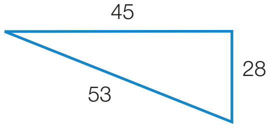
2.1

1 Use the converse of Pythagoras' theorem to determine whether the following triangles are right-angled.

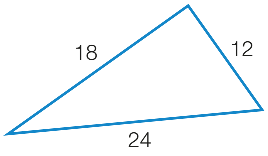
(a)



(b)



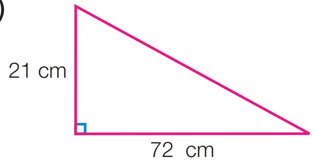
(c)



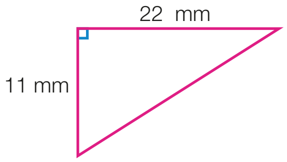
2.2

2 Find the length of the hypotenuse in the following right-angled triangles. Write your answers in exact form.

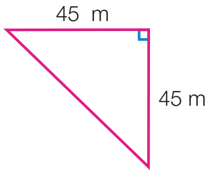
(a)



(b)



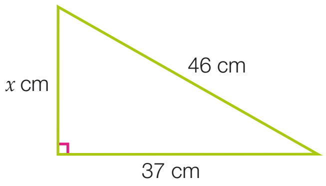
(c)



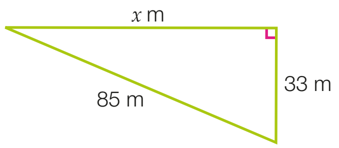
3 Use Pythagoras' theorem to find the value of x. Write your answer correct to 2 decimal places if necessary.

2.3

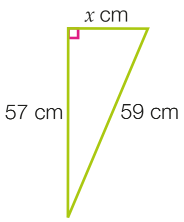
(a)



(b)



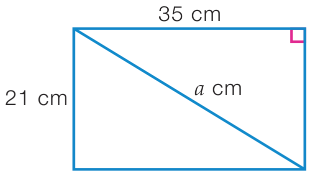
(c)



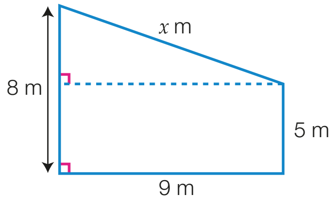
4 Use Pythagoras' theorem to find the value of the unknown in each of the following diagrams. Write your answer correct to 2 decimal places if necessary.

2.2, 2.3

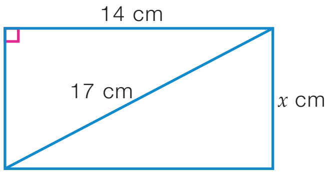
(a)



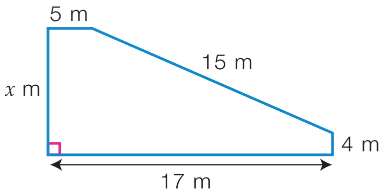
(b)



(c)



(d)



5 The Leaning Tower of Pisa was originally 150 m tall, but is leaning over so that a stone dropped from the top lands 5 m from the base. How far vertically does the stone fall?

2.4

A 149.92 m **B** 150.00 m **C** 159.98 m **D** 162.00 m

6 Find the value of c in each of the following Pythagorean triples.

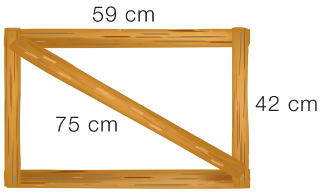
2.5

(a) $a = 12, b = 16, c = ?$ **(b)** $a = 45, b = 200, c = ?$ **(c)** $a = 132, b = 385, c = ?$

Understanding

7 Simon is making the base for a dog's kennel. He put in a diagonal brace for extra support.

2.1



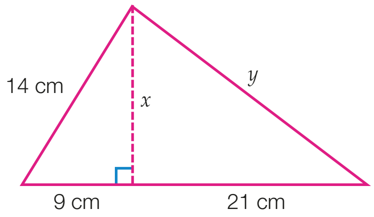
(a) From the measurements shown, has Simon built the base 'square'?

(b) What would be a reasonable measurement for the diagonal for Simon to consider the base to be 'square'?

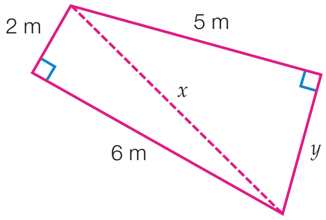
8 Find the exact value of the variables in the following diagrams.

2.2, 2.3

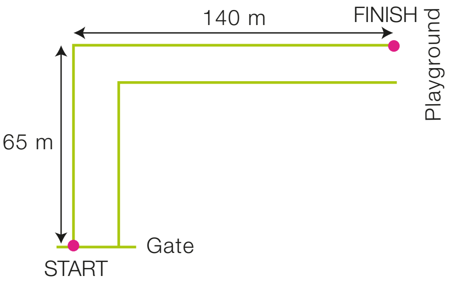
(a)



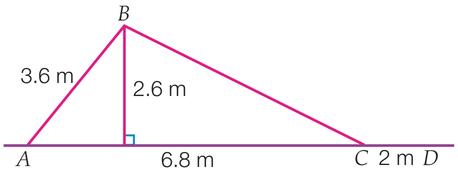
(b)



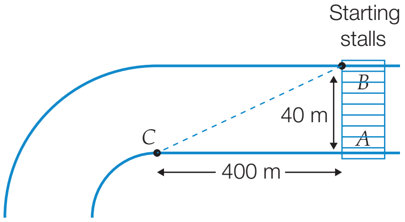
9 Ming and Tong decided to have a race from the gate of the park to the playground. Ming has decided to run along the left edge of the path and can do so at a speed of 7 metres per second. However, Tong decides to go directly from the gate to the playground, but this is very sandy and he can only run at 5 metres per second. Who wins the race to the playground? **2.2, 2.4**



10 A playground slide is 2.6 m high. The ladder is 3.6 m long. The distance from the bottom of the slide, *C*, to the base of the ladder, *A*, is 6.8 m. The end of the slide, *CD*, is 2 m. Find the total length of the slide from *B* to *D*. **2.2, 2.3**



11 Before each horse race, a barrier draw determines each horse's starting position. Closest to the rails is the best position and is represented by point *A* on the diagram shown. The least preferred position is near the outside edge of the track (point *B*). It can be assumed that all jockeys aim to take their horses in a direct line to the beginning of the first corner (point *C*). **2.2, 2.4**



(a) How much further does the horse at point *B* travel than the horse at point *A*?

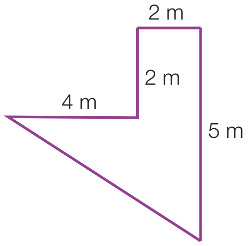
(b) Is this difference significant? How might this affect the race?

Reasoning

12 State whether or not each of the following is a Pythagorean triple. **2.5**

(a) (120, 350, 370) **(b)** (16, 30, 35) **(c)** (77, 420, 427)

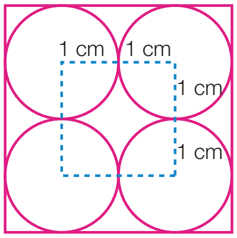
13 Daniel and Courtney are going to make a garden in the backyard of their new house. They have measured the space and it is shown in the diagram. **2.4**



(a) Calculate the perimeter of the garden and find the number of 2 m long sleepers they will need to form the perimeter of the garden.

(b) If 2 m long sleepers cost $35 each, how much will it cost to place sleepers all around the garden bed?

14 Four steel discs, each with radius 1 cm, are placed in a square frame as shown. **2.4**



(a) Find the length of the sides of this frame.

(b) In order to brace the frame, a diagonal support is to be added to the frame. Find the length of this support.

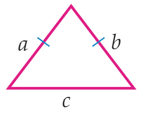
(c) Find the length of material needed to build the frame, including the diagonal brace.

Numeracy practice 2

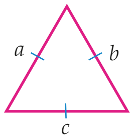
Non-calculator

1 Which triangle will satisfy Pythagoras' theorem?

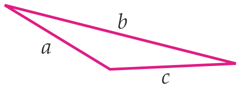
A



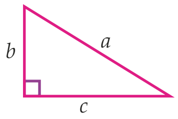
B



C



D



2 Choose the side lengths that form a right-angled triangle.

A (20, 21, 23) **B** (20, 48, 52) **C** (6, 7, 8) **D** (5, 7, 10)

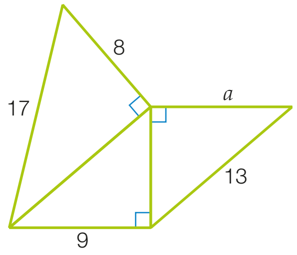
3 Which Pythagorean triple can be obtained from (36, 105, 111)?

A (12, 35, 37) **B** (36, 110, 333) **C** (24, 52.5, 55.5) **D** (72, 110, 222)

4 What is the value of the variable *a*?

A 5 **B** 7

C 9 **D** 11



Calculator allowed

5 The two shorter sides of a right-angled triangle are of length 14 mm and 48 mm, respectively. The length of the hypotenuse is:

A 25 mm **B** 34 mm **C** 46 mm **D** 50 mm

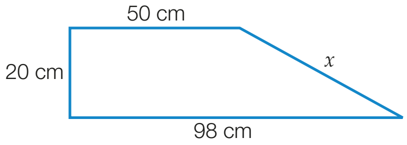
6 The hypotenuse of a right-angled triangle is 8.70 cm and one of its shorter sides is of length 6.30 cm. The length of the other shorter side is:

A 6.00 cm **B** 6.52 cm **C** 7.14 cm **D** 8.96 cm

7 What is the value of the variable in the diagram shown on the right?

A 42 cm **B** 48 cm

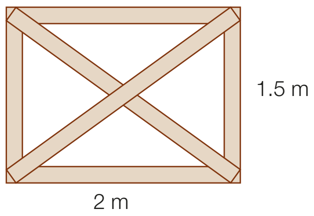
C 52 cm **D** 64 cm



8 Jemima wants to build a rectangular timber gate of width 2 m and height 1.5 m with two diagonal supports. What is the minimum total length of timber required to build the frame of the gate?

A 6 m **B** 7 m

C 9.5 m **D** 12 m



Mixed review A

Fluency

1 After investing $8000 into a savings account paying 6.5% interest p.a., Andrew had $8390 in his account. For how many months (correct to the nearest month) was it invested? **1.5**

2 Calculate the following: **1.1**

(a) 12.5% of 450 m **(b)** 38% of 208 kg **(c)** $5\frac{1}{2}\%$ of $3500

3 A triangle has side lengths of 51 cm, 140 cm and 141 cm. Determine if the triangle is right-angled. **2.1**

4 Convert the following to percentages, rounding answers to 1 decimal place: **1.1**

(a) 34 goals out of 47 shots

(b) four blond-haired students out of a class of 23

(c) nine rotten apples in a crate containing 140.

5 **(a)** A TV priced at $1649 is discounted by 15%. Find the new price of the TV. **1.2**

(b) A smartphone has been discounted down to a sale price of $699. If a 10% discount was applied, what was the original price of the phone? Answer to the nearest dollar.

6 Use the following table to calculate how much income tax should be paid on the taxable incomes given below. **1.4**

Taxable income	Tax on this income
$0–$18 200	Nil
$18 201–$37 000	19c for each $1 over $18 200
$37 001–$80 000	$3572 plus 32.5c for each $1 over $37 000
$80 001–$180 000	$17 547 plus 37c for each $1 over $80 000
$180 001 and over	$54 547 plus 45c for each $1 over $180 000

(a) $5500 **(b)** $35 000 **(c)** $54 000

7 Marc works part-time at the cinema. He receives a basic hourly rate of $16.82 for any hours worked between 9 am and 9 pm. He receives time-and-a-half for any hours worked after 9 pm and any hours on Saturday. He receives double time for any hours on a Sunday. Calculate Marc's wage for the hours worked in the following week.

1.3

Tuesday	Wednesday	Thursday	Saturday	Sunday
11 am–9 pm	2 pm–10 pm	3:30 pm–11:30 pm	12 pm–6 pm	2 pm–9 pm

8 For each of the following household items, calculate:

1.7

- **(i)** the weekly energy consumption, in kWh
- **(ii)** the weekly running cost, given an electricity tariff of 19.2 cents per kWh.

(a) A 100 W television, in use for 4.5 hours per day.

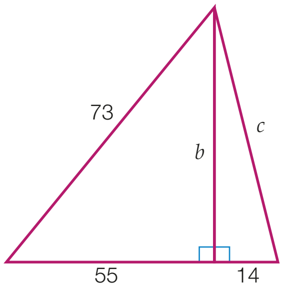
(b) A 75 W fridge, in use for the whole week.

9 Kazim is a real estate agent. He is paid a $650 retainer per week, plus 2.5% of the value of any houses he sells. Calculate Kazim's income for a week in which he sold a house for $415 000.

1.3

10 Find the values of b and c.

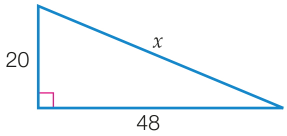
2.2, 2.3



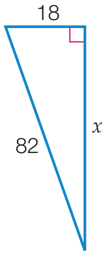
11 Find the value of the unknown side in each triangle. Give your answer in exact form.

2.2, 2.3

(a)



(b)



Understanding

12 Mustafa has 8 shots at the basketball ring and gets 7 in. His friend Adam has 11 shots and gets 9 in. Calculate the number of goals as a percentage of the successful shots for each boy to determine who is the more accurate shooter.

1.1

13 Richard inspects old buildings to see whether they need support to keep them standing straight up. He believes that an old house is beginning to lean, so he stands at a point on the ground 3 m from the base of a wall, which is 2.3 m high. He measures the distance from this point to the top of the wall and finds that it is 3.5 m.

2.5

(a) Draw a diagram of a triangle to represent the situation, labelling it with all known information.

(b) Use Pythagoras' theorem to determine whether the wall is at 90° to the ground.

14 Snooker Joe hits the cue ball. The ball travels 16 cm, strikes the side, rebounds at 90° to its original direction and travels a further 63 cm before coming to rest. What is the distance between the start and finish position of the ball?

9.4

15 After investing $5000 in a savings account for 8 months, Michael received $5347. What was the rate of interest, assuming a constant rate of simple interest?

1.5

16 An aircraft leaves Melbourne airport and flies 37 km due east. It then changes direction and flies 684 km due north to land at Waratah airport. What is the direct distance between the two airports?

2.4

Reasoning

17 A second-hand store buys a large crate of DVDs for $4.70 each. It places a 250% mark-up on them and puts them out for sale. Later on, the store discounts the marked price of the DVDs by 23%.

1.2

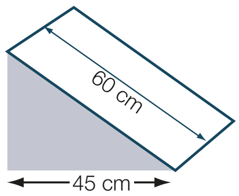
(a) What was the original selling price of the DVDs?

(b) What was the discounted price, rounded to the nearest 5 cents?

(c) What percentage mark-up does the new, discounted price represent, correct to the nearest whole number?

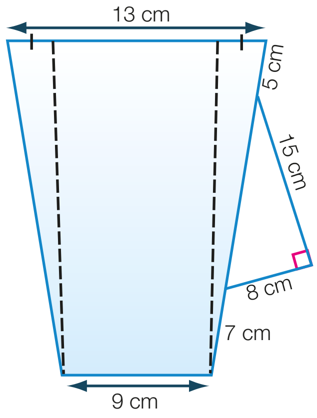
18 A ramp has the dimensions shown. It is placed against a raised platform that is 35 cm high. Does the ramp provide smooth access to the platform?

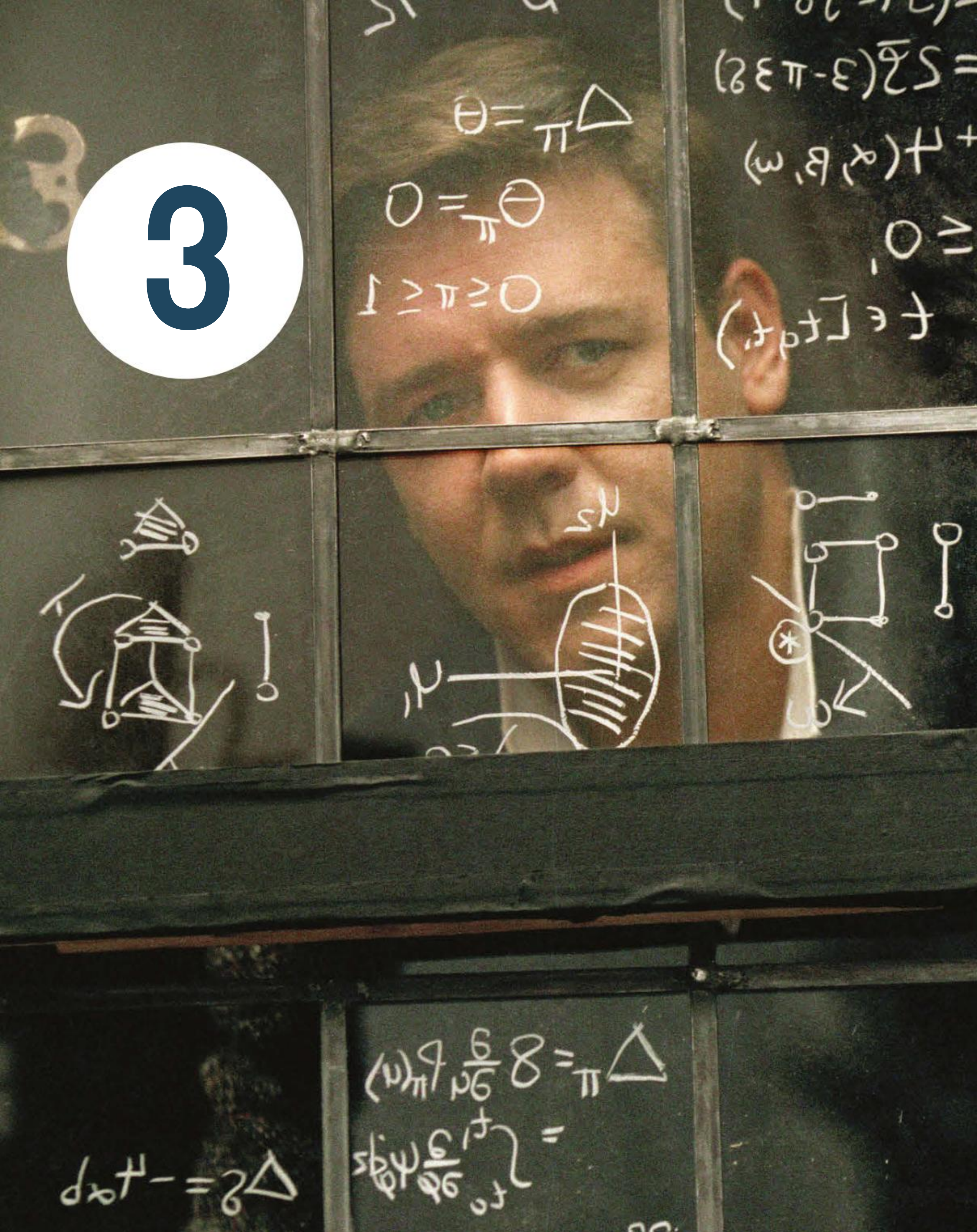
2.4



19 The cross-section of a water jug can be approximated by the shape shown. What is its height, correct to 2 decimal places?

2.4





3

Algebra

3

The game of algebra. What do strategy games, like noughts and crosses, have to do with winning the Nobel Prize in Economics?

Game theory is the mathematical analysis of strategies and interactions between people in competitive situations. One of the central components of game theory is the Nash equilibrium. This is named after John Nash, the mathematician who is the subject of the movie *A Beautiful Mind*, starring Russell Crowe. In a Nash equilibrium, no player in the 'game' can improve their strategy even if they know the strategies of the other players.

Mathematicians, and especially economists, analyse Nash equilibria using algebraic techniques to find solutions to complex equations. These are just slightly more advanced versions of the techniques you will study in this chapter.

At least eight recipients of the Nobel Prize in Economics, including John Nash, have been rewarded for their work involving game theory. (There is no Nobel Prize in Mathematics!)

Forum

If you go first in a game of noughts and crosses you should never lose. Do you know how to ensure this? What does this have to do with the Nash equilibrium?

Why learn this?

Algebra gives us a symbolic way to describe and generalise patterns that occur in the real world. Algebraic techniques allow you to manipulate algebraic expressions into different forms, to understand and to compare these patterns. This chapter on algebraic techniques will provide you with skills you can use later to graph and to solve equations. Understanding and interpreting algebraic equations can help you to investigate the past, explain the present and predict the future.

After completing this chapter you will be able to:

- use index laws and properties to simplify algebraic expressions, including expressions with negative powers
- understand how to use scientific notation and significant figures
- rearrange formulas to make different variables the subject
- use various techniques to expand algebraic expressions
- use various techniques to factorise algebraic expressions.

Recall 3

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 Simplify the following expressions.

(a) $3x + 4x + 7x$ **(b)** $5y + 7y - 4y$ **(c)** $3k^2 + 5k^2 - 7k^2$

2 Simplify the following expressions.

(a) $5y + 6h - 2h - 3y$ **(b)** $6x^2 + 5y^3 - 2x + 3y^3$

3 Simplify the following expressions.

(a) $3x \times 4y$ **(b)** $5k \times 4k$ **(c)** $-5g \times 6g \times 2g$

4 Simplify, writing your answer in index form.

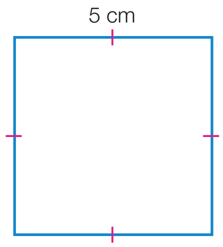
(a) $2^4 \times 2^9$ **(b)** $6^8 \div 6^2$ **(c)** $(5^2)^2$ **(d)** $(8.6325 \times 5.15)^0$

5 Expand:

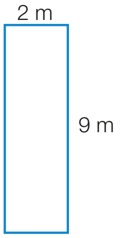
(a) $\left(\frac{4^2}{5^3}\right)^3$ **(b)** $(2^4 \times 5^2)^4$ **(c)** $\left(\frac{7^2 \times 3^4}{4^5}\right)^2$

6 Find **(i)** the perimeter and **(ii)** the area of each of the following.

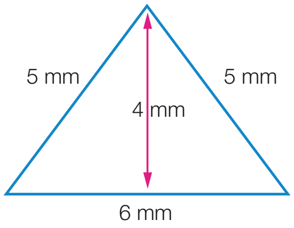
(a)



(b)



(c)



7 Expand the following expressions.

(a) $4(x + 2)$ **(b)** $6(x - 3)$ **(c)** $-2(x + 4)$

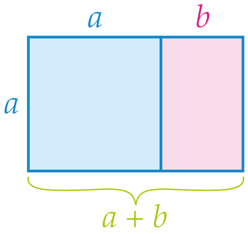
8 Factorise the following expressions.

(a) $6x - 12$ **(b)** $10x - 25$ **(c)** $-3x + 15$

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

How many different areas of the same rectangle?



In this activity, you will explore different ways that the area of a rectangle can be expressed using algebra.

Introducing index laws using variables

3.1

Index form

Index form is a way of expressing repeated factors in a concise way.

You can write a number as the product of its prime factors.

$81 = 3 \times 3 \times 3 \times 3$

This is called **expanded form**.

You can use indices to write the number in **index** form. An index is also called an exponent or power.

$$\text{number} \longrightarrow 81 = \underset{\underset{\text{base}}{\uparrow}}{3}^{4} \longleftarrow \text{index}$$

You can read this as '3 to the power of 4 is 81'.

81	=	$3 \times 3 \times 3 \times 3$	=	3^4
		Expanded form		Index form

Index laws

Recall the rules for calculating with numbers written in index form:

$2^4 \times 2^5 = 2^{(4+5)}$ $= 2^9$	When multiplying numbers in index form that have the same **base**, keep the base and add the indices.
$\frac{3^6}{3^4} = 3^{(6-4)}$ $= 3^2$	When dividing numbers in index form that have the same base, keep the base and subtract the indices.
$(4^2)^5 = 4^{(2 \times 5)}$ $= 4^{10}$	When raising a number in index form to a power, keep the base and multiply the indices.
$(2 \times 5)^3 = 2^3 \times 5^3$	When the product of factors in brackets is raised to a power, each factor in the brackets is raised to the same power.
$\left(\frac{7}{3}\right)^4 = \frac{7^4}{3^4}$	When the quotient of factors is raised to a power, each factor in the brackets is raised to the same power.

Also recall the following property:

$$\begin{aligned} 2^0 &= 1 \\ 4975^0 &= 1 \\ 6.347^0 &= 1 \\ \left(\frac{5}{7}\right)^0 &= 1 \end{aligned}$$

Any number, except 0, raised to the zero index is equal to 1 (0^0 is undefined).

These index laws can also be applied to any **variables** or pronumerals.

Multiplication with indices

If $a^2 = a \times a$ and $a^3 = a \times a \times a$, then $a^2 \times a^3 = a \times a \times a \times a \times a = a^5$.

Notice that $2 + 3 = 5$.

You can write:

$$\begin{aligned} a^2 \times a^3 &= a^{(2+3)} \\ &= a^5 \end{aligned}$$

Here is another example.

$$\begin{aligned} b^4 \times b^2 \times b &= (b \times b \times b \times b) \times (b \times b) \times (b) \\ &= b^7 \end{aligned}$$

Therefore, $b^4 \times b^2 \times b^1 = b^7$.

Notice that $4 + 2 + 1 = 7$.

Remember that $b = b^1$.

You can write:

$$\begin{aligned} b^4 \times b^2 \times b &= b^{(4+2+1)} \\ &= b^7 \end{aligned}$$

What do a^m and a^n mean?

$a^2 = a \times a$. There are two factors of base a in the multiplication.

$a^3 = a \times a \times a$. There are three factors of base a in the multiplication.

$a^m = \underbrace{a \times a \times \ldots \times a}_{m \text{ factors of } a}$. There are m factors of base a in the multiplication.

$$\begin{aligned} \text{So, } a^m \times a^n &= \underbrace{a \times a \times \ldots \times a}_{m \text{ factors of } a} \times \underbrace{a \times a \times \ldots \times a}_{n \text{ factors of } a} \\ &= \underbrace{a \times a \times \ldots \times a}_{m+n \text{ factors of } a} \\ &= a^{(m+n)} \end{aligned}$$

There are $m + n$ factors of base a in the multiplication string.

When multiplying variables in index form that have the same base, the base remains the same and the indices are added.

$a^m \times a^n = a^{(m+n)}$

Worked example 1

W.E. 1

Simplify each of the following, leaving your answers in index form.

(a) $7^3 \times 7^8$ **(b)** $c^3 \times c^6$ **(c)** $4h^5 \times 3h^9$

Thinking	Working
(a) 1 Are the bases the same? (Yes.)	(a) $7^3 \times 7^8$
2 Apply the index law for multiplication —keep the base and add the indices.	$= 7^{(3+8)}$ $= 7^{11}$
(b) 1 Are the bases the same? (Yes.)	(b) $c^3 \times c^6$
2 Apply the index law for multiplication—keep the base and add the indices.	$= c^{(3+6)}$ $= c^9$
(c) 1 Insert the multiplication signs. Group the coefficients (the numbers) and the variables.	(c) $4h^5 \times 3h^9$ $= 4 \times 3 \times h^5 \times h^9$
2 Simplify by writing the product of the coefficients and applying the index law for multiplication to the variables.	$= 12h^{(5+9)}$ $= 12h^{14}$

Division with indices

As with multiplication, there is a pattern for the division of numbers in index form. To calculate $a^5 \div a^3$, first write the division as a fraction: $\frac{a^5}{a^3}$

Then write each power in expanded form: $\frac{a \times a \times a \times a \times a}{a \times a \times a}$

Cancelling the common factor of a^3: $\frac{a \times a \times \overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}} \times \overset{1}{\cancel{a}}}{{}_{1}\cancel{a} \times {}_{1}\cancel{a} \times {}_{1}\cancel{a}}$

This simplifies to: $\frac{a^2 \times 1}{1} = a^2$

So $\frac{a^5}{a^3} = a^2$.

Notice that $5 - 3 = 2$.

You can write:

$$\begin{aligned}\frac{a^5}{a^3} &= a^{(5-3)} \\ &= a^2\end{aligned}$$

When dividing variables in index form that have the same base, the base stays the same and the indices are subtracted:

$$\begin{aligned}a^m \div a^n &= \frac{a^m}{a^n} \\ &= a^{(m-n)}\end{aligned}$$

Worked example 2

W.E. 2

Simplify each of the following, leaving your answers in index form.

(a) $\frac{11^9}{11^5}$ **(b)** $\frac{d^7}{d^3}$ **(c)** $20c^8e^7 \div 4c^3e^2$

Thinking	Working
(a) 1 Are the bases the same? (Yes.)	(a) $\frac{11^9}{11^5}$
2 Apply the index law for division: keep the base and subtract the indices.	$= 11^{(9-5)}$ $= 11^4$
(b) 1 Are the bases the same? (Yes.)	(b) $\frac{d^7}{d^3}$
2 Apply the index law for division: keep the base and subtract the indices.	$= d^{(7-3)}$ $= d^4$
(c) 1 Write the expression in fraction form.	(c) $\frac{20c^8e^7}{4c^3e^2}$
2 Simplify the coefficients (the numbers 20 and 4) by cancelling common factors.	$= \frac{{}^{5}\cancel{20}c^8e^7}{{}_{1}\cancel{4}c^3e^2}$
3 Apply the index law for division to powers that have the same base.	$= 5c^{(8-3)}e^{(7-2)}$
(Simplify $\frac{c^8}{c^3}$ and $\frac{e^7}{e^2}$.)	$= 5c^5e^5$

Power of a power

What does $(m^5)^4$ mean?

$$\begin{aligned}(m^5)^4 &= m^5 \times m^5 \times m^5 \times m^5\\ &= m^{(5+5+5+5)}\\ &= m^{(4\times 5)} \text{ or } m^{(5\times 4)}\\ &= m^{20}\end{aligned}$$

Alternatively,

$$\begin{aligned}(m^5)^4 &= (m \times m \times m \times m \times m) \times (m \times m \times m \times m \times m) \times (m \times m \times m \times m \times m)\\ &\quad \times (m \times m \times m \times m \times m)\\ &= m^{20}\end{aligned}$$

So:

$$\begin{aligned}(m^5)^4 &= m^{(5\times 4)}\\ &= m^{20}\end{aligned}$$

When raising a variable in index form to a power, keep the base and multiply the indices.

$$(a^m)^n = a^{(m\times n)}$$

This rule is the 'power of a power' law.

Worked example 3

W.E. 3

Simplify each of the following, leaving your answers in index form.

(a) $(6^3)^4$ **(b)** $(2^3)^2 \times (2^4)^3$ **(c)** $(5^3)^4 \div (5^5)^2$

(d) $(r^3)^5$ **(e)** $(p^3)^4 \times (p^5)^2$ **(f)** $(m^5)^3 \div (m^2)^6$

Thinking	Working
(a) Apply the power of a power law—keep the base and multiply the indices.	(a) $(6^3)^4$ $= 6^{(3 \times 4)}$ $= 6^{12}$
(b) 1 Apply the power of a power law to each term—keep the base and multiply the indices.	(b) $(2^3)^2 \times (2^4)^3$ $= 2^{(3 \times 2)} \times 2^{(4 \times 3)}$ $= 2^6 \times 2^{12}$
2 To multiply, apply the index law for multiplication—keep the base and add the indices.	$= 2^{(6 + 12)}$ $= 2^{18}$
(c) 1 Apply the power of a power law to each term—keep the base and multiply the indices.	(c) $(5^3)^4 \div (5^5)^2$ $= 5^{(3 \times 4)} \div 5^{(5 \times 2)}$ $= 5^{12} \div 5^{10}$
2 To divide, apply the index law for division—keep the base and subtract the indices.	$= 5^{(12 - 10)}$ $= 5^2$
(d) Apply the power of a power law—keep the base and multiply the indices.	(d) $(r^3)^5$ $= r^{(3 \times 5)}$ $= r^{15}$
(e) 1 Apply the power of a power law to each term—keep the base and multiply the indices.	(e) $(p^3)^4 \times (p^5)^2$ $= p^{(3 \times 4)} \times p^{(5 \times 2)}$ $= p^{12} \times p^{10}$
2 To multiply, apply the index law for multiplication—keep the base and add the indices.	$= p^{(12 + 10)}$ $= p^{22}$
(f) 1 Write the expression in fraction form.	(f) $\dfrac{(m^5)^3}{(m^2)^6}$
2 Apply the power of a power law to each term—keep the base and multiply the indices.	$= \dfrac{m^{(5 \times 3)}}{m^{(2 \times 6)}}$ $= \dfrac{m^{15}}{m^{12}}$
3 To divide, apply the index law for division—keep the base and subtract the indices.	$= m^{(15 - 12)}$ $= m^3$

3.1 Introducing index laws using variables

Navigator

Answers p. 636

1 (column 1), 2 (column 1), 3 (column 1), 5, 6 (columns 1, 2), 7 (a, b), 8, 9 (a), 10, 11, 12, 14, 16, 20	1 (column 2), 2 (column 2), 3 (column 2), 4, 5, 6 (columns 2, 3), 7, 8, 9, 10, 11, 12, 14, 17, 19, 20	1 (column 3), 2 (column 3), 3 (column 3), 4, 6 (column 3), 8, 9, 10, 11, 13, 14, 15, 17, 18, 19, 20

Fluency

W.E. 1

1 Simplify each of the following, leaving your answers in index form.

(a) $2^4 \times 2^5$
(b) $3^7 \times 3^4$
(c) $5^{11} \times 5^7$
(d) $a^3 \times a^4$
(e) $b^6 \times b^5$
(f) $5c^3 \times c^2$
(g) $3d^3 \times 2d^2$
(h) $4b^4 \times 10b^4$
(i) $7b^2 \times 4b^2$
(j) $4a^5 \times 3a$
(k) $7a^3 \times 2a$
(l) $20a^3 \times 10a^4$
(m) $6a^4b^2 \times 4a^5$
(n) $9a^3b^3 \times 3ab^2$
(o) $5b^4c^5 \times 6b^3c^2$
(p) $-5a^4b^6 \times 7b^2$
(q) $-9e^5f^5 \times -4ef$
(r) $3a^5b^8 \times -8ab^2$

W.E. 2

2 Simplify each of the following, leaving your answers in index form.

(a) $3^8 \div 3^5$
(b) $4^7 \div 4^2$
(c) $9^5 \div 9^3$
(d) $m^5 \div m^2$
(e) $4b^5 \div b^3$
(f) $5g^7 \div g^3$
(g) $\frac{n^6}{n^4}$
(h) $\frac{14b^7}{7b^5}$
(i) $\frac{22c^6}{11c^2}$
(j) $\frac{6a^5}{3a^4}$
(k) $\frac{12a^7}{8a^3}$
(l) $\frac{14e^4f^5}{10ef^2}$
(m) $10a^7b^7 \div 5a^4b^3$
(n) $-24a^5b^3 \div 21a^2b$
(o) $144a^6b^8 \div -12a^5b^7$

W.E. 3

3 Simplify each of the following, leaving your answers in index form.

(a) $(2^3)^4$
(b) $(3^2)^5$
(c) $(5^4)^2$
(d) $(3^2)^5 \times (3^4)^2$
(e) $(4^3)^2 \times (4^5)^3$
(f) $(7^2)^5 \times (7^4)^2$
(g) $(5^2)^8 \div (5^3)^2$
(h) $(8^5)^8 \div (8^2)^3$
(i) $(11^4)^8 \div (11^3)^9$
(j) $(b^7)^3$
(k) $(k^8)^4$
(l) $(a^6)^2$
(m) $y^5 \times (y^3)^4$
(n) $(k^3)^8 \times k^3$
(o) $(m^6)^3 \times m^2$
(p) $(g^2)^5 \times (g^4)^2$
(q) $(y^2)^4 \times (y^5)^3$
(r) $(t^3)^3 \times (t^2)^6$
(s) $(p^3)^4 \div (p^5)^2$
(t) $(n^7)^2 \div (n^2)^4$
(u) $(b^8)^3 \div (b^3)^7$

4 (a) $-4r^3 \times -r^2$ can be simplified to give:

A $4r^1$
B $-4r^5$
C $4r^5$
D $4r^6$

(b) The product of $-abc$, $-5a$ and $-2a$ is:

A $-10a^3bc$
B $-7a^2bc$
C $10abc$
D $10a^2bc$

Understanding

5 Copy and complete each of the following by writing the correct index number in the box.

(a) $\dfrac{x^7 \times x^9}{x^{12}}$

$= \dfrac{x^{\square}}{x^{12}}$ (Add indices)

$= x^{\square}$ (Subtract indices)

(b) $\dfrac{6y^4 \times y^6}{y^2 \times y^5}$

$= \dfrac{6y^{\square}}{y^{\square}}$ (Add indices)

$= 6y^{\square}$ (Subtract indices)

6 Simplify each of the following by completing any multiplication of variables first.

(a) $\dfrac{x^8 \times x^7}{x^6}$ **(b)** $\dfrac{t^7}{t^2 \times 3t^4}$ **(c)** $\dfrac{c^6 \times c^9}{c^5 \times c^3}$

(d) $\dfrac{j^4 \times j^8}{j^2 \times j^5}$ **(e)** $\dfrac{4a^7 \times 3a^4}{a^3}$ **(f)** $\dfrac{8q \times 3q^6}{12q^4}$

7 **(a)** $\dfrac{f^8 \times f^6}{f^{10}}$ simplifies to:

A f^4 **B** f^6 **C** f^{48} **D** f^{12}

(b) $\dfrac{15x^9 \times 3x^6}{9x^{10} \times x^4}$ simplifies to:

A $5x^9$ **B** $9x$ **C** $5x$ **D** $9x^9$

(c) $-36k^4n^3 \div 9n^2k^3$ can be simplified to:

A $4k^4n$ **B** $-4k^7n^6$ **C** $-4kn$ **D** $45k^{12}n^6$

(d) $\dfrac{42x^5y^7z^6}{14x^4y^2z^3}$ simplifies to:

A $3xy^5z^2$ **B** $28xy^5z^3$ **C** $3xy^5z^3$ **D** $3x^9y^9z^9$

8 **(a)** How many groups of $3a^2$ will divide into $15a^4$?

(b) Use your answer to **(a)** to rewrite this as a multiplication that has $15a^4$ as the product.

9 **(a)** $\dfrac{a^8 \times b^5}{a^2 \times a^6}$ simplifies to:

A $a^{16}b^5$ **B** $a^{10}b^{11}$ **C** b^5 **D** b^{11}

(b) $\dfrac{3c^7 \times 8d^4}{12c^3 \times 4c^4}$ simplifies to:

A $2d^4$ **B** $\dfrac{d^4}{2}$ **C** $\dfrac{c^4d^4}{24}$ **D** $\dfrac{d^4}{24}$

10 Simplify each of the following.

(a) $(m^5)^2 \div m^7$ **(b)** $(p^6)^4 \div p^{10}$ **(c)** $\dfrac{(k^4)^3}{k^9}$ **(d)** $\dfrac{(n^8)^2}{n^6}$

11 **(a)** $(k^2)^3 \times (k^5)^2$ simplifies to:

A k^{16} **B** k^{12} **C** k^{60} **D** k^{15}

(b) $(b^5)^4 \div (b^3)^5$ simplifies to:

A b^4 **B** b **C** b^5 **D** b^{12}

12 Simplify each of the following.

(a) $3a^2b^3 \times 4ab^2$ **(b)** $12m^4p^6 \div 3m^2p^2$ **(c)** $\dfrac{8d^3f^4}{2df^2}$

Reasoning

Hint

If the bases on both sides of the equation are the same, the expression for the powers must be equal.

13 Solve for x in the following equations.

(a) $5^{(x+2)} = 5^{(2x-1)}$ **(b)** $3^{(x+1)} = 3$ **(c)** $2^{(2x+4)} = 1$ **(d)** $4(7^x + 7^x) = 56$

14 On social media, the rapid spread of news, images or videos is sometimes called 'going viral'. One person might send a photo to 10 friends, who each forward it to 10 more friends, who each forward it to 10 more friends, and so on. The total number of people who see the photo depends on the number of times it is forwarded, known as the 'level of spread'.

Level of spread	0	1	2	3	4
Number of new people	1	10	100	1000	
Index form	10^0	10^1	10^2	10^3	

(a) Complete the last column of the table to show how many more people saw the photo after the fourth level of spread.

(b) In total, how many people had seen the photo after the third level of spread?

(c) If you define 'gone viral' as being seen by more than one million people, then how many levels of spread are needed before the photo has 'gone viral'?

15 When Josh started his part-time job, he decided to save some of his wages on a weekly basis. He was paid \$50.00 a week, so he decided to start by saving 5c the first week, 10c the next week, 20c the third week, continuing to double the amount he saved each week. He was shocked to find that, by the 11th week, he could not afford to keep up his plan.

(a) Copy the following table and complete the second column to show how much Josh saved each week for the first 5 weeks.

Week	Amount saved each week (c)	Amount saved each week (c) in index form
1		
2		
3		
4		
5		

(b) Complete the third column in this table by writing these amounts in index form.

(c) Write a rule in index form for finding the amount Josh saved in the nth week.

(d) Use your rule to calculate how much Josh was supposed to save in the 11th week.

(e) How much would Josh need to earn each week if he was to keep up his saving plan for 6 months (26 weeks)?

Hint

Look for a pattern. what values do you get if 1 is subtracted from a power of 2?

Open-ended

16 Find three different expressions for ■ and ▲ in terms of a and b to make this a true statement:

■ × ▲ $= -3a^2b^3$

17 Write three possible divisions involving indices that will give an answer of $5a^2b^3$.

18 Create three index number division expressions and simplify them using any of $24a^2b^3$, $3ab$, $48a^5b^6$, $22a^2b^2$ or 2×3^2a. Check your answers by giving the divisions to a friend to simplify.

19 Write three different sets of values for m and n so that $(a^m)^n$ simplifies to a^{24}.

20 The index laws are sometimes used incorrectly. Below is a list of equations that 'look' reasonable, but are actually incorrect.

In each case, explain the mistake that has been made and give the correct answer in its place. Explain why your answer is correct.

(a) $x^2x^4 = x^8$ **(b)** $\frac{x^9}{x^3} = x^3$ **(c)** $(x^2)^4 = x^{2^4} = x^{16}$

Problem solving

Square cubes

Equipment required: calculator

'I have been studying the cube numbers. You know, some of them are also square numbers!' It was smart Alec, bragging again.

'You're kidding!' said Sue. 'A number can't be a square and a cube.'

'Yes it can' said Alec. 'Like, 64 is both 8 squared and 4 cubed.'

If Alec is right there may be more.

1 Can you find any other 2-digit numbers that are a square and a cube?

2 With the help of your calculator, how many can you find under 1000?

Strategy options

- Test all possible combinations.
- Look for a pattern.
- Break into manageable parts.

Maths 4 Real

The digital world comes in byte-sized bits

You have probably heard of terms such as 'bit' and 'byte', or more likely 'kilobyte' (kB), 'megabyte' (MB) and 'gigabyte' (GB). These are measurement units for digital electronic data, which is used to encode documents, web pages, text messages, photos, music and videos in computers and across the internet.

1 Depending on which devices you can access right now, try to find out the following:

- **(a)** Approximately how big is a document file (such as Microsoft® Word) that you might create as part of your schoolwork (e.g. an essay)?
- **(b)** What is the file size of an average text message?
- **(c)** What is the file size of a 3-minute piece of music?
- **(d)** What is the capacity of the hard drive of your laptop or home computer?

What's a 'bit'?

The word '**bit**' is an abbreviation of the words '**b**inary dig**it**'. The binary digits are the digits 1 and 0, which can be used to make binary numbers.

Binary numbers are used by electronic devices to carry and store information, because each 1 and 0 can be represented as electricity, 'on' or 'off'.

In everyday life we are more familiar with decimal numbers, which use the digits 0–9. The decimal number system is called 'base-10', because the place values of the digits increase by a power of 10 as you move to the left.

You can take a decimal number such as 1101 and write it in a place value table as:

100 000 (10^5)	10 000 (10^4)	1000 (10^3)	100 (10^2)	10 (10^1)	1 (10^0)
		1	1	0	1

However, the binary number system is a 'base-2' system, which means that each place value is a power of 2 greater than the next place value to the right. The number 1101 in a base-2 place value table is this:

256 (2^8)	128 (2^7)	64 (2^6)	32 (2^5)	16 (2^4)	8 (2^3)	4 (2^2)	2 (2^1)	1 (2^0)
					1	1	0	1

To find the value of the binary number 1101 as a decimal number, you can write this in expanded form, then evaluate:

$$\begin{aligned} & 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &= 8 \quad + 4 \quad + 0 \quad + 1 \\ &= 13 \end{aligned}$$

2 Write the decimal number values of these binary numbers by writing them in expanded form first. (Note: the subscript $_2$ means 'base 2' and indicates a binary number.)

(a) 101_2 **(b)** 1011_2 **(c)** 1111_2

(d) 100111_2 **(e)** 101111_2 **(f)** 1001001_2

What's a 'byte'?

Bits are usually packaged into sets of 8. A set of 8 bits is called a byte, similar to the way that 12 eggs are called a dozen. With 8 bits in a byte, 1 byte can represent numbers in the decimal range 0–255.

	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
0 =	0	0	0	0	0	0	0	0
1 =	0	0	0	0	0	0	0	1
2 =	0	0	0	0	0	0	1	0
3 =	0	0	0	0	0	0	1	1
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
254 =	1	1	1	1	1	1	1	0
255 =	1	1	1	1	1	1	1	1

Note that empty place-value columns are filled with zeros so that the byte always has exactly 8 bits.

3 Write the following decimal numbers in 8-bit binary form.

(a) 3 **(b)** 7 **(c)** 19 **(d)** 34 **(e)** 154 **(g)** 204

So how do all these 1s and 0s make words and music?

Each number from 0 to 255, written in 8-digit binary 'byte form', can be used to correspond to a specific letter or symbol or formatting command (such as a space between words), which can be interpreted and then displayed by your computer.

To turn data in digital (binary) form into music, computers can use a Digital to Analogue Converter (DAC) to convert digital data into a varying electrical signal, which then makes different sounds as it passes through a speaker (or headphones).

I have 'eight gig' of data. What does this mean?

Prefixes such as 'kilo' and 'mega' are commonly used in front of measurement units such as 'litre'. 'Kilo' means 'one thousand' and 'mega' means one million. But the storage capacities of electronic data include so many bytes that bigger prefixes are needed, such as 'giga' (1 thousand million) and 'tera' (1 million million). In base-2 binary, these prefixes are usually not used for their decimal values (thousand, million, etc.), but instead they are used for the nearest base 2 index number:

Prefix	Symbol	Decimal value	Binary value
kilo	K	10^3 (1000)	2^{10} = 1 024
mega	M	10^6 (1 000 000)	2^{20} = 1 048 576
giga	G	10^9 (1 000 000 000)	2^{30} = 1 073 741 824
tera	T	10^{12} (1 000 000 000 000)	2^{40} = 1 099 511 627 776
peta	P	10^{15} (1 000 000 000 000 000)	2^{50} = 1 125 899 906 842 624
exa	E	10^{18} (1 000 000 000 000 000 000)	2^{60} = 1 152 921 504 606 846 976

The term 'eight gig' usually means 8 gigabytes (GB) of data, or 8 × 2^{30} (8 589 934 592) bytes.

4 The decimal values of the prefixes increase by a factor of 10^3 (1000) as you move down the table. What do the binary values of the prefixes increase by?

5 **(a)** The average size of a music file is 6 MB. How many bytes is that?

(b) How many 6 MB files could be stored on an 8 GB drive, assuming the entire 8 GB is available?

(c) If the average size of a video file is 700 MB, find two different combinations of music and video files that could be stored on a 16 GB storage drive (assuming the entire 16 GB is available)?

Research

6 Find out how to add or multiply binary numbers.

7 The sizes of music and video files can vary greatly. What different factors can affect file size, and in what ways?

8 Investigate digital photography. Find out the sizes of photo files taken with a digital camera and what can affect them. Find out how digital images can be stored and the numbers of photos that can be stored on different devices or in different formats.

3.2

More index laws and index properties

Expanding products

Consider expanding $(2\times a)^3$.

$$\begin{aligned}(2\times a)^3 &= (2\times a)\times(2\times a)\times(2\times a)\\ &= \underbrace{2\times 2\times 2}_{\text{3 factors of 2}}\times\underbrace{a\times a\times a}_{\text{3 factors of } a} &&\text{(grouping the coefficients together and the variables together)}\\ &= 2^3\times a^3\end{aligned}$$

So $(2\times a)^3 = 2^3\times a^3$

Each factor in the brackets has been raised to the power of 3.

What do you get when you expand $(kp)^4$?

$$\begin{aligned}(kp)^4 &= kp\times kp\times kp\times kp\\ &= \underbrace{k\times k\times k\times k}_{\text{4 factors of } k}\times\underbrace{p\times p\times p\times p}_{\text{4 factors of } p} &&\text{(grouping each type of variable together)}\\ &= k^4\times p^4\\ &= k^4p^4\end{aligned}$$

So $(kp)^4 = k^4p^4$

Each factor in the brackets has been raised to the power of 4.

> If a product of factors in brackets is to be raised to a power, each factor in the brackets is raised to that power.
>
> $$(ab)^m = a^mb^m$$

Worked example 4

W.E. 4

Use an index law to expand each of the following, leaving your answers in index form.

(a) $(3\times 5)^4$ **(b)** $(ab)^5$ **(c)** $(5p^4q)^3$

Thinking	Working
(a) Apply the index law for expanding products—raise both variables to the given power.	(a) $(3\times 5)^4$ $=3^4\times 5^4$
(b) **1** Apply the index law for expanding products—raise both variables to the given power.	(b) $(ab)^5$ $=a^5\times b^5$
2 Write the answer in the simplest way without the multiplication sign.	$=a^5b^5$

(c) **1** Apply an index law—raise each of the factors in the product (5, p^4 and q) to the power of 3.	(c) $(5p^4q)^3$ $= 5^3 \times (p^4)^3 \times q^3$
2 Apply other index laws to simplify the terms. (Here, $(p^4)^3$ simplifies to p^{12}.)	$= 5^3 \times p^{4 \times 3} \times q^3$ $= 5^3 \times p^{12} \times q^3$
3 Write the answer in the simplest way, remembering the order convention.	$= 5^3 p^{12} q^3$

Expanding quotients

How can you expand $\left(\frac{m}{n}\right)^5$?

$$\begin{aligned} \left(\frac{m}{n}\right)^5 &= \frac{m}{n} \times \frac{m}{n} \times \frac{m}{n} \times \frac{m}{n} \times \frac{m}{n} \\ &= \frac{m \times m \times m \times m \times m}{n \times n \times n \times n \times n} \begin{matrix} \longleftarrow \text{5 factors of } m \\ \longleftarrow \text{5 factors of } n \end{matrix} \\ &= \frac{m^5}{n^5} \end{aligned}$$

So $$\left(\frac{m}{n}\right)^5 = \frac{m^5}{n^5}$$

Each variable in the brackets has been raised to the power of 5. The rule is:

> If a quotient in brackets is to be raised to a power, each number or variable in the brackets is raised to that power.
>
> $$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

Worked example 5

W.E. 5

Use an index law to expand each of the following, leaving your answers in index form.

(a) $\left(\frac{4}{3}\right)^5$ **(b)** $\left(\frac{p}{q}\right)^7$ **(c)** $\left(\frac{2a}{c^4}\right)^3$

Thinking	Working
(a) Apply the index law for expanding quotients—raise both the numerator and the denominator to the power outside the brackets.	(a) $\left(\frac{4}{3}\right)^5$ $= \frac{4^5}{3^5}$
(b) Apply the index law for expanding quotients—raise both the numerator and the denominator to the power outside the brackets.	(b) $\left(\frac{p}{q}\right)^7$ $= \frac{p^7}{q^7}$

(c) **1** Apply the index law for expanding quotients—raise both the numerator and the denominator to the power outside the brackets.	(c) $\left(\frac{2a}{c^4}\right)^3$ $= \frac{(2a)^3}{(c^4)^3}$
2 Raise all parts of each term to the power. (Here, both 2 and *a* are raised to the power of 3.)	$= \frac{2^3 \times a^3}{c^{(4 \times 3)}}$
3 Apply any other index laws to simplify. (Here, the power of a power index law is used to simplify $c^{4 \times 3}$.)	$= \frac{2^3 a^3}{c^{12}}$

You can apply the inverse of these laws to simplify **expressions**.

Worked example 6

W.E. 6

Simplify each of the following, leaving your answers in index form.

(a) $2^3 \times 6^3$ **(b)** $5^4 \times h^4$ **(c)** $m^2 \times n^2$

Thinking	Working
(a) Apply the inverse of the index law for expanding products to simplify. Do not evaluate.	(a) $2^3 \times 6^3$ $= (2 \times 6)^3$ $= 12^3$
(b) Apply the inverse of the index law for expanding products to simplify.	(b) $5^4 \times h^4$ $= (5h)^4$
(c) Apply the inverse of the index law for expanding products to simplify.	(c) $m^2 \times n^2$ $= (mn)^2$

Worked example 7

W.E. 7

Simplify each of the following, leaving your answers in index form.

(a) $\frac{16^3}{4^3}$ **(b)** $\frac{t^4}{2^4}$ **(c)** $\frac{c^8}{d^8}$

Thinking	Working
(a) Apply the inverse of the index law for expanding quotients to simplify. Do not evaluate.	(a) $\frac{16^3}{4^3}$ $= \left(\frac{16}{4}\right)^3$ $= 4^3$
(b) Apply the inverse of the index law for expanding products to simplify.	(b) $\frac{t^4}{2^4}$ $= \left(\frac{t}{2}\right)^4$

(c) Apply the inverse of the index law for expanding products to simplify.	(c) $\frac{c^8}{d^8}$ $= \left(\frac{c}{d}\right)^8$

The zero index

If $2^0 = 1$, $45\,678^0 = 1$ and $0.000\,123^0 = 1$, does this work for any number a, so that $a^0 = 1$ also?

To simplify $a^m \div a^m$, you can subtract indices (second index law).

$$\begin{aligned} a^m \div a^m &= a^{(m-m)} \\ &= a^0 \end{aligned}$$

Alternatively, treating it as a fraction division:

$$\begin{aligned} a^m \div a^m &= \frac{a^m}{a^m} \\ &= \frac{a \times a \times a \times \ldots \times a}{a \times a \times a \times \ldots \times a} \\ &= 1 \end{aligned}$$

Therefore, $a^0 = 1$

> A variable or variable product raised to the power of zero equals 1.
> The only exception is 0^0, which is undefined.

Worked example 8

W.E. 8

Simplify each of the following.

(a) $4p^0$

(b) $\frac{2y^4 \times y^3}{y^5 \times y^2}$

(c) $\frac{m^3n^6}{m^3n}$

Thinking	Working
(a) Replace the variable to the zero index with 1 and simplify.	(a) $4p^0$ $= 4 \times p^0$ $= 4 \times 1 = 4$
(b) **1** First, simplify the numerator and the denominator of the fraction by applying the appropriate index law.	(b) $\frac{2y^4 \times y^3}{y^5 \times y^2}$ $= \frac{2y^{(4+3)}}{y^{(5+2)}}$ $= \frac{2y^7}{y^7}$
2 Apply the index law for dividing with indices.	$= 2y^{(7-7)}$ $= 2y^0$
3 Replace the variable to the zero index with 1 and simplify.	$= 2 \times 1$ $= 2$

(c) **1** When dividing numbers with the same base, subtract the indices.

(c) $\frac{m^3n^6}{m^3n}$

$= m^{(3-3)}n^{(6-1)}$

$= m^0n^5$

2 Replace the variable to the zero index with 1.

$= 1 \times n^5$

$= n^5$

Negative indices

What does 2^{-6} mean?

If you consider $2^6 = 64$, $2^5 = 32$, $2^4 = 16$, $2^3 = 8$, $2^2 = 4$, $2^1 = 2$ and $2^0 = 1$, you can see that the indices on the left-hand side of each equation are one less than the previous one and that the numbers on the right-hand side of the equations are half the previous one. To continue this pattern:

$$2^{-1} = \frac{1}{2},\quad 2^{-2} = \frac{1}{4} = \frac{1}{2^2},\quad 2^{-3} = \frac{1}{8} = \frac{1}{2^3}$$

So, $2^{-6} = \frac{1}{2^6}$

$8^{-5} = \frac{1}{8^5}$

Any number written with a negative power can be written as 1 divided by the number raised to the same power but positive.

Consider two ways to simplify $a^2 \div a^6$.

One way to do this is to subtract indices.

$$\begin{aligned} a^2 \div a^6 &= a^{(2-6)} \\ &= a^{-4} \end{aligned}$$

Alternatively, you can expand the numerator and denominator and cancel common factors.

$$\begin{aligned} \frac{a^2}{a^6} &= \frac{{}^1\cancel{a} \times {}^1\cancel{a}}{{}_1\cancel{a} \times {}_1\cancel{a} \times a \times a \times a \times a} && a \neq 0 \\ &= \frac{1}{a \times a \times a \times a} && a \neq 0 \\ &= \frac{1}{a^4} && a \neq 0 \end{aligned}$$

This shows that the expressions a^{-4} and $\frac{1}{a^4}$ are equivalent for all values of a, except $a = 0$, giving us the following index property:

$a^{-m} = \frac{1}{a^m}, a \neq 0$

A variable raised to a negative index is equal to 1 divided by the same variable raised to the positive index.

a^{-4} or $\frac{1}{a^4}$ is the **reciprocal** of a^4. Reciprocals always multiply to give a product of 1.

$$\begin{aligned} a^4 \times a^{-4} &= a^0 \\ &= 1 \end{aligned} \qquad \text{or} \qquad \begin{aligned} a^4 \times \frac{1}{a^4} &= \frac{a^4}{a^4} \\ &= 1 \end{aligned}$$

Worked example 9

W.E. 9

Simplify the following, writing your answer with positive indices.

(a) 3^{-2} **(b)** $4p^{-3}$ **(c)** $6x^{-5}y^2$

Thinking	Working
(a) Rewrite the expression using the negative index property. This changes the negative power to a positive power.	(a) $3^{-2} = \frac{1}{3^2}$
(b) Rewrite the expression using the negative index property. This changes the negative power to a positive power by using the reciprocal. The number (4) is not affected because the power is only applied to the variable (*p*), so it stays in the numerator.	(b) $4p^{-3} = 4 \times p^{-3}$ $= 4 \times \frac{1}{p^3}$ $= \frac{4}{p^3}$
(c) Rewrite the expression using the negative index property. This changes the negative index to a positive index. The number (6) and one of the variables (y^2) are not affected, so they stay in the numerator.	(c) $6x^{-5}y^2 = \frac{6y^2}{x^5}$

Index laws for powers with the same base

The following laws are true if $a, b \neq 0$ (*a* and *b* cannot be zero), and using integer values for *m* and *n*.

Multiplication of powers	$a^m \times a^n = a^{(m+n)}$	When multiplying numbers with the same base, keep the base and add the indices.
Division of powers	$a^m \div a^n = \frac{a^m}{a^n}$ $= a^{(m-n)}$	When dividing numbers with the same base, keep the base and subtract the indices.
Power of a power	$(a^m)^n = a^{mn}$	When raising a number in index form to a power, keep the base and multiply the indices.
Expanding products	$(ab)^m = a^m b^m$	When removing brackets around a product that has been raised to a power, raise each number to that power.
Expanding quotients	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$	When removing brackets around a quotient that has been raised to a power, raise each number to that power.

Index properties

The zero index	$a^0 = 1 \quad (a \neq 0)$	Any variable or product of variables raised to the power of 0 is equal to 1. Exception: 0^0 is undefined.
Negative indices	$a^{-m} = \dfrac{1}{a^m} \quad (a \neq 0)$	A variable raised to a negative power is equal to 1 divided by the same variable raised to the positive power.

3.2 More index laws and index properties

Navigator

Answers p. 637

1 (column 1), 2 (column 1), 3 (column 1), 4 (column 1), 5 (column 1), 6 (column 1), 7, 8 (a–c), 9 (a–c), 11 (column 1), 12 (column 1), 13 (column 1), 14 (a, c), 15, 21, 23	1 (column 2), 2 (column 2), 3 (column 2), 4 (column 2), 5 (column 2), 6 (column 2), 7, 8 (a–d), 9 (a–d), 10 (a, b), 11 (column 2), 12 (column 2), 13 (column 2), 14, 15, 18, 19, 20, 21, 23	1 (column 3), 2 (column 3), 3 (column 3), 4 (column 3), 5 (column 3), 6 (column 3), 7, 8 (d–f), 9 (d–f), 10 (c, d), 11 (column 3), 12 (column 3), 13 (column 3), 14, 15, 16, 18, 19, 20, 21, 22, 23

Fluency

W.E. 4

1 Use an index law to expand each of the following, leaving your answers in index form.

(a) $(6 \times 5)^2$ **(b)** $(11 \times 3)^4$ **(c)** $(12 \times 9)^5$

(d) $(4x)^2$ **(e)** $(p^3q)^2$ **(f)** $(a^4f)^3$

(g) $(2p)^5$ **(h)** $(p^3q^5)^7$ **(i)** $(c^2d^7)^4$

(j) $(bc)^3$ **(k)** $(6a^4)^3$ **(l)** $(9p^5q^4)^2$

(m) $(7m^3n^6)^2$ **(n)** $(2ab^2)^3$ **(o)** $(3ef^4)^4$

W.E. 5

2 Use an index law to expand each of the following, leaving your answers in index form.

(a) $\left(\frac{12}{5}\right)^2$ **(b)** $\left(\frac{7}{3}\right)^4$ **(c)** $\left(\frac{9}{2}\right)^8$

(d) $\left(\frac{y}{x}\right)^3$ **(e)** $\left(\frac{d}{e}\right)^5$ **(f)** $\left(\frac{m^3}{n}\right)^4$

(g) $\left(\frac{w^4}{z}\right)^2$ **(h)** $\left(\frac{r^5}{s}\right)^4$ **(i)** $\left(\frac{d^3}{g}\right)^2$

(j) $\left(\frac{3}{w^6}\right)^4$ **(k)** $\left(\frac{8}{m^2}\right)^3$ **(l)** $\left(\frac{2}{r^4}\right)^3$

(m) $\left(\frac{h^3}{g^2}\right)^5$ **(n)** $\left(\frac{a^3}{c^5}\right)^4$ **(o)** $\left(\frac{t^4}{v^3}\right)^3$

(p) $\left(\frac{5a}{b}\right)^3$ **(q)** $\left(\frac{c^4}{2x}\right)^6$ **(r)** $\left(\frac{f^6}{6g}\right)^3$

W.E. 6

3 Simplify each of the following, leaving your answers in index form.

(a) $3^2 \times 4^2$
(b) $4^6 \times 5^6$
(c) $8^4 \times 7^4$
(d) $4^8 \times h^8$
(e) $3^4 \times p^4$
(f) $7^9 \times r^9$
(g) $a^5 \times m^5$
(h) $k^7 \times t^7$
(i) $p^8 \times n^8$

W.E. 7

4 Simplify each of the following, leaving your answers in index form.

(a) $\frac{15^4}{3^4}$
(b) $\frac{24^3}{6^3}$
(c) $\frac{132^5}{11^5}$
(d) $\frac{r^5}{3^5}$
(e) $\frac{n^7}{6^7}$
(f) $\frac{g^9}{4^9}$
(g) $\frac{x^6}{y^6}$
(h) $\frac{k^{11}}{n^{11}}$
(i) $\frac{g^{10}}{h^{10}}$

W.E. 8

5 Simplify each of the following.

(a) k^0
(b) $7g^0$
(c) $7 + g^0$
(d) a^0b^2
(e) $3m^0n^0$
(f) $c^3d^0e^0$
(g) $-3a^4b^0$
(h) $-8g^{11}h^0$
(i) $-(4pr)^0$
(j) $y^4 \div y^4$
(k) $3x^7 \div x^7$
(l) $24m^6 \div 8m^6$
(m) $\frac{3x^4z}{z}$
(n) $\frac{h^5k^8}{h^5}$
(o) $\frac{8m^3n^4}{16m^3}$
(p) $\frac{a^5b^4}{a^2b^4}$
(q) $\frac{6h^4r^3}{2h^4r^3}$
(r) $\frac{9b^4d^3}{6b^4d^3}$
(s) $\frac{12e^3f^2}{16e^3}$
(t) $\frac{15r^2s^9}{27r^2}$
(u) $\frac{25k^4p^2}{15k^4}$

Hint

Remember that any number (except 0) divided by itself equals one.

W.E. 9

6 Simplify the following, writing your answers in index form.

(a) 4^{-2}
(b) 2^{-3}
(c) 5^{-4}
(d) 3^{-3}
(e) 5^{-2}
(f) 4^{-3}
(g) x^{-5}
(h) y^{-3}
(i) b^{-7}
(j) $3e^{-3}$
(k) $7x^{-5}$
(l) $3g^{-4}$
(m) $8p^{-5}q^{-2}$
(n) $2x^{-3}y^{-5}$
(o) $12a^{-3}b^{-9}$
(p) $3p^3q^{-1}$
(q) $5x^2y^{-7}$
(r) $7a^{-5}b^2$

Understanding

7 (a) $(2k)^6$ simplifies to:

A $2k^6$
B $12k$
C $64k^6$
D $12k^6$

(b) $(3a^4)^3$ simplifies to:

A $27a^{12}$
B $9a^{12}$
C $9a^7$
D $27a^7$

(c) $\left(\frac{3}{g}\right)^4$ simplifies to:

A $\frac{12}{g^4}$
B $\frac{81}{g^4}$
C $\frac{12}{g}$
D $\frac{81}{g}$

(d) $\left(\frac{k^8}{7}\right)^2$ simplifies to:

A $\frac{k^{10}}{7}$ **B** $\frac{k^{16}}{49}$ **C** $\frac{k^{16}}{7}$ **D** $\frac{k^{10}}{49}$

8 Simplify each of the following by first applying the index law for expanding products, then completing the multiplication, applying other index laws if necessary.

(a) $(xy)^2 \times (xy)^4$ **(b)** $(gh)^7 \times (gh)^2$ **(c)** $(3x)^3 \times (4x)^2$

(d) $(4y)^2 \times (5xy)^2$ **(e)** $(cd)^3 \times (c^3d^2)^3$ **(f)** $(x^2y^3)^2 \times (xy)^2$

9 Simplify each of the following by first applying the index law for expanding quotients, completing the multiplication, then simplifying if possible.

(a) $\left(\frac{f}{g}\right)^3 \times \left(\frac{f}{g}\right)^2$ **(b)** $\left(\frac{p}{q}\right)^6 \times \left(\frac{p}{q}\right)^4$ **(c)** $\left(\frac{d^6}{q^4}\right)^2 \times \left(\frac{d^5}{q^2}\right)^3$

(d) $\left(\frac{a^4}{b^2}\right)^2 \times \left(\frac{b}{a}\right)^6$ **(e)** $\left(\frac{2a}{b}\right)^2 \times \left(\frac{a}{b}\right)^3$ **(f)** $\left(\frac{3x^2}{y}\right)^3 \times \left(\frac{2x}{y}\right)^2$

10 Simplify each of the following by applying the expanding products and the powers of powers index laws, then the index law for division.

(a) $\frac{(w^4z^3)^3}{(w^5z^2)^2}$ **(b)** $\frac{(c^7e^5)^2}{(c^4e^2)^3}$ **(c)** $\frac{(pq)^{12}}{(p^2q^3)^3}$ **(d)** $\frac{(vw)^9}{(v^3w^2)^2}$

11 Simplify each of the following.

(a) $(x^3)^0$ **(b)** $(d^5)^0$ **(c)** $(k^0)^4$ **(d)** $(p^0)^2$

(e) $(m^3)^4 \times (m^2)^0$ **(f)** $(f^5)^4 \times (f^3)^0$ **(g)** $(t^0)^3 \times t^4$ **(h)** $((q^4)^5 \times (q^5)^4)^0$

(i) $\frac{x^6 \times x^5}{x^{11}}$ **(j)** $\frac{3k^2}{k \times 5k}$ **(k)** $\frac{4a^3}{a^2 \times 7a}$ **(l)** $\frac{n^4 \times n^3}{n^5 \times n^2}$

(m) $\frac{2b^6 \times 5c^8}{c^7 \times 4c}$ **(n)** $\frac{d^6 \times e^2}{3e \times 2e}$ **(o)** $\frac{c^4 \times d}{2c^2 \times 5c^2}$ **(p)** $\frac{p^7 \times q^5}{15p^2 \times p^5}$

12 Use the negative index property to write the following with positive indices, then evaluate.

(a) 4^{-2} **(b)** 3^{-3} **(c)** 5^{-4}

(d) $\frac{1}{2^{-1}}$ **(e)** $\frac{1}{4^{-3}}$ **(f)** $\frac{1}{3^{-2}}$

(g) 4×3^{-2} **(h)** 3×4^{-3} **(i)** 2×5^{-2}

(j) $-\frac{7 \times 3^{-2}}{3}$ **(k)** $-\frac{6 \times 2^{-4}}{2^{-3}}$ **(l)** $-\frac{3 \times 3^{-4}}{3^{-6}}$

13 Express each of the following with positive indices.

(a) $\frac{1}{d^{-2}}$ **(b)** $\frac{1}{k^{-5}}$ **(c)** $\frac{3}{a^{-1}}$

(d) $\frac{x^3}{y^{-6}}$ **(e)** $\frac{p^7}{q^{-2}}$ **(f)** $\frac{m^{-4}}{n^{-4}}$

(g) $\left(\frac{a^2}{b^3}\right)^{-4}$ **(h)** $\left(\frac{m^4}{n^5}\right)^{-2}$ **(i)** $\left(\frac{x^{-5}}{y^2}\right)^{-3}$

14 **(a)** a^{-4} is equal to:

A $-a^{4}$ **B** $-4a$ **C** $\frac{1}{a^{4}}$ **D** $\frac{1}{a^{-4}}$

(b) $\frac{1}{2^{-4}}$ is equal to:

A 16 **B** 8 **C** $\frac{1}{16}$ **D** -16

(c) $\frac{1}{y^{2}}$ is equal to:

A $-y^{2}$ **B** y^{-2} **C** $-y^{-2}$ **D** $2y$

(d) $\frac{1}{8}$ is equal to:

A 2^{3} **B** 2^{-3} **C** 3^{-2} **D** $\frac{1}{2^{-3}}$

Reasoning

15 Kellie and Eva are given the following question as part of their homework:

Simplify $\left(\frac{4x^{2}}{2x}\right)^{3}$.

Both had the correct answer, but Eva finished much faster. ‘How did you do it so fast?’ Kellie asked Eva.

Explain why Eva’s method took fewer steps.

Here is Kellie’s working:

$\left(\frac{4x^{2}}{2x}\right)^{3}$

$= \frac{4^{3} \times x^{(2 \times 3)}}{2^{3} \times x^{3}}$

$= \frac{64x^{6}}{8x^{3}}$

$= 8x^{3}$

Here is Eva’s working:

$\left(\frac{4x^{2}}{2x}\right)^{3}$

$= (2x)^{3}$

$= 8x^{3}$

16 By rewriting the expression if required and then using index laws, simplify each of the following using the smallest possible base.

(a) $\frac{9^{2n}}{9^{n}}$ **(b)** $2^{n} \times 4^{(n+1)}$ **(c)** $\frac{125}{5^{n}} \times \frac{5^{2n}}{25}$

17 **(a)** Using index notation, write the numbers 125, 25, 5 and 1, using base 5.

(b) Using index notation, write the next five numbers using the pattern in part **(a)**.

(c) Write the whole sequence of nine numbers in this pattern without using index notation.

18 **(a)** If I divide a number by its square, what is the result?

(b) If a room has one side length of $12a$ m and an area of 84 m^{2}, what is the length of the other side of the room?

19 A model for the number of planets N created in a section of a galaxy is given by the formula $N = \left(\frac{n^{2}T^{3}p}{b}\right)^{2}$. In the formula:

n is the number of suns
T is the temperature of the surrounding space
p is the number of planets already present
b is the number of black holes.

(a) Expand the formula.

(b) **(i)** How many new planets does the model predict if $T = 1$, $n = 1000$ and $b = p$?

(ii) If the temperature of the surrounding space could be a negative number, would the answer be positive or negative? Explain your reasoning.

Open-ended

20 **(a)** Write at least two expressions involving indices and multiplication that, when simplified, equal $36m^4n^8$.

(b) Write at least two expressions involving indices and division that can be simplified to $\frac{4m^4}{5n^6}$.

21 Write three different terms that $27x^3y^7$ could be divided by so that no x appears in the answer.

22 Write a question that could be simplified to $\frac{3x^2}{y^3}$ by using:

(a) any two index laws with an expression that has only positive indices

(b) two index laws with an expression that has at least one negative index

(c) any three index laws with an expression that has only positive indices

(d) three index laws with an expression that has at least one negative index.

Show, step by step, how your question can be simplified to obtain the final answer.

23 Jodie made four mistakes when simplifying the following index question. Find them, explain to Jodie what she did incorrectly and redo the question to get the correct answer. Jodie's working is shown below:

$$\frac{(3x^2y^4)^2}{2x^{-2}y^5} = \frac{6x^4y^6}{2x^{-2}y^5}$$

$$= 4x^2y$$

Puzzle

What is a 'googolplex'?

A 'googol' is 10^{100}. If you wrote out this number it would be a 1 followed by 100 zeros. A googol is more than the number of atoms in the observable universe, which is estimated to be between 10^{79} and 10^{81}.

So, how big is a googolplex?

To find out, complete the following.

1. Match the value of the number with its equivalent index form.
2. Replace the numbers in the code with its corresponding letter.

Answer key:
12-13-3 12-11 12-1-13 7-11-10-13-2 11-6
9-11-11-9-11-8

1	10
2	10 000 000 000
3	1
4	10 000 000
5	100 000 000 000
6	1 000 000 000
7	100
8	100 000
9	10 000
10	1 000 000
11	100 000 000
12	1 000 000 000 000
13	1000

E	10^3
F	10^9
G	10^4
H	10^1
L	10^5
N	10^0
O	10^8
P	10^2
R	10^{10}
T	10^{12}
U	10^{11}
W	10^6
D	10^7

3.3

Scientific notation and significant figures

Using powers of 10

It is often useful to write very large numbers or very small numbers as a product of a power of 10, such as when writing the distance from Earth to another planet or the size of an atom. (Remember that 'power of 10' is a common way of saying 10 raised to a power.)

When multiplying by powers of 10, people often describe this as 'moving the decimal point', but it is not actually the decimal point that moves. It is the digits that move to the left or right.

- When multiplying by a power of 10 with a positive index, move the decimal point to the right by the same number of places as the value of the index.

  This makes the number bigger. Zeros are inserted in places not filled by other digits.
- When multiplying by a power with a negative index, move the decimal point to the left by the same number of places as the value of the index (after the negative sign).

  This makes the number smaller. Zeros are inserted in places not filled by other digits.
- Multiplying by a negative power is the same as dividing by the reciprocal power:

$$\begin{aligned} 6.5 \times 10^{-4} &= \frac{6.5}{10^4} \\ &= \frac{6.5}{10\,000} \\ &= 0.000\,65 \end{aligned}$$

Worked example 10

W.E. 10

Find the value of each of the following.

(a) 4.35×10^6 **(b)** 1.78×10^{-3}

Thinking	**Working**
(a) Because the index of 10 is positive, move the decimal point to the right by the same number of places as the index of 10. This makes the number bigger. Insert zeros in places not filled by other digits. (Move the decimal six places to the right and insert four zeros.)	(a) $4.350\,000$ $4.35 \times 10^6 = 4\,350\,000$

(b) Because the index of 10 is negative, divide by the reciprocal power (10^3). Move the decimal point to the left by the same number of places as the index of 10 after the negative sign. This makes the number smaller. Insert zeros in places not filled by other digits. (Move the decimal point three places to the left and insert two zeros.)	(b) 0.000 78 $1.78 \times 10^{-3} = 0.001\,78$

Scientific notation

Scientists use **scientific notation** (also known as 'standard form') to make it easier to calculate with very large or very small numbers.

To write a number in scientific notation, put the decimal point after the first non-zero digit, then multiply by a power of 10 so that the product gives the original number. The power of 10 may have a positive index (e.g. 10^4) or a negative index (e.g. 10^{-3}).

A number in scientific notation is written as the product of a number a and a power of 10, where a is any number greater than or equal to 1 but less than 10:

$$a \times 10^b \text{ where } 1 \leq a < 10 \text{ and } b \text{ is an integer}$$

Consider the following examples.

The average distance from the Earth to the Sun is approximately 149 600 000 000 m. To write this in scientific notation, put a decimal point after the 1 and multiply by 10^{11} because the decimal point has moved 11 places to the left.

Therefore, $149\,600\,000\,000 = 1.496 \times 10^{11}$.

The mass of a particular atom may be 0.000 000 000 000 000 001 67 g. Put the decimal point after the 1 and multiply by 10^{-18} because the decimal point has moved 18 places to the right.

Therefore $0.000\,000\,000\,000\,000\,001\,67 = 1.67 \times 10^{-18}$.

Worked example 11

W.E. 11

Express each of the following in scientific notation.

(a) 5800

(b) 0.000 603 4

Thinking	Working
(a) Place the decimal point after the first non-zero digit and multiply by a power of 10 so that the product is the original number. Count the number of places right or left that the decimal has moved. Because it has moved to the left (three places to the left to get 5.8), multiply by a positive power (10^3). The zeros at the end do not need to be written.	(a) 5800 $5800 = 5.8 \times 10^3$

(b) Place the decimal point after the first non-zero digit and multiply by a power of 10 so that the product is the original number. Count the number of places right or left that the decimal has moved.

Because it has moved to the right (four places to the right to get 6.034), multiply by a negative power (10^{-4}). The zeros in front of the first digit do not need to be written.

Do not round off the decimal part of a number in scientific notation unless it is requested.

(b) 0.000 603 4

$0.000\,603\,4 = 6.034 \times 10^{-4}$

Scientific notation on a calculator

When the result of an arithmetic calculation is larger or smaller than the display on a calculator allows, it can be displayed in scientific notation.

For example:

$4\,589\,765\,125 \times 849\,235\,713 = 3.897\,792\,458 \times 10^{18}$

On a calculator, scientific notation is sometimes displayed using the letter E instead of the multiplication by 10.

The result above could be displayed as 3.897792458E18, where E18 represents $\times 10^{18}$.

Similarly, 3.7792311E-5 would represent $3.779\,231\,1 \times 10^{-5}$.

Significant figures

All measurements are only as accurate as the measuring devices allow.
Using **significant figures** is a way to represent how accurate the measurements are.

1. Write the number in scientific notation: $23\,467.8 = 2.346\,78 \times 10^4$
2. To write with a number of significant figures:

$$\begin{aligned} 2.346\,78 \times 10^4 &\approx 2.3 \times 10^4 \text{ to two significant figures} \\ &\approx 2.35 \times 10^4 \text{ to three significant figures} \\ &\approx 2.347 \times 10^4 \text{ to four significant figures} \end{aligned}$$

Remember to check the next digit after the one being rounded, to decide whether to round up or down.

Rules for counting and writing significant figures

Counting significant figures

- All non-zero digits are significant. The first significant figure in a number is the first non-zero digit:

  2.346 78 has six significant figures.
- Zeros between non-zero digits or at the end of a decimal number are significant:

  1**0** 203 has five significant figures.

  0.03**0** has two significant figures.
- Zeros at the end of an integer or at the beginning of a decimal number are not significant.

  1 350 000 has three significant figures.

  0.000 013 5 also has three significant figures.

Writing significant figures

- Zeros appearing at the end of a decimal number after rounding are significant figures and must be retained to give the required number of significant figures.

  7.999 written to three significant figures is 8.00.
- Zeros sometimes need to be added.

  7.2 written to three significant figures is 7.20.

Worked example 12

W.E. 12

Round each of the following to the number of significant figures specified.

(a) 50 842 to three significant figures

(b) 0.007 497 to three significant figures

Thinking	Working
(a) Write the number in standard form. Count from the left the number of significant figures required (three significant figures). Remember to check the fourth digit and round the third digit up if it is 5 or greater.	(a) $50\,842 = 5.084\,2 \times 10^4$ $\approx 5.08 \times 10^4$ or $50\,800$
(b) Write the number in standard form. Count from the left the number of significant figures required. Remember to round correctly. The zero at the end is a significant figure and must be retained.	(b) $0.007\,497 = 7.497 \times 10^{-3}$ $\approx 7.\,50 \times 10^{-3}$ or $0.007\,50$

3.3 Scientific notation and significant figures

Navigator

Answers p. 638

1 (columns 1, 2), 2 (column 1), 3 (a–c), 4 (a–c), 6 (a–d), 7 (a, c, e), 8, 9 (a, c, e), 10, 16, 18 (a), 20	1 (columns 2, 3), 2 (column 2), 3 (a–d), 4 (a–d), 5, 6 (a–d), 7 (a, c, e), 8, 9 (b, d, f), 10, 11, 13, 15, 17, 18, 20	1 (column 4), 2 (column 3), 3 (c–f), 4 (c–f), 5, 6 (b, d, g), 7 (b, d, f), 8, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21

Fluency

W.E. 10

1 Find the value of each of the following.

(a) 9.8×10^3 **(b)** 6.5×10^7 **(c)** 3.25×10^6 **(d)** 4.2071×10^5

(e) 2×10^{-3} **(f)** 5×10^{-5} **(g)** 7×10^{-8} **(h)** 5.032×10^{-2}

(i) 8.26×10^{-3} **(j)** 4.18×10^{-5} **(k)** 7.14×10^{-4} **(l)** 3.55×10^{-6}

W.E. 11

2 Express each of the following in scientific notation.

(a) 2560 **(b)** 90 020 **(c)** 614 000

(d) 190 000 **(e)** 8 050 000 **(f)** 56 000 000

(g) 0.305 **(h)** 0.03 **(i)** 0.0004

(j) 0.002 01 **(k)** 0.000 072 466 **(l)** 0.000 000 532 18

W.E. 12

3 Round each of the following to the number of significant figures indicated in the brackets.

(a) 394 024 (2) **(b)** 0.054 02 (3) **(c)** 674.5 (2)

(d) 0.003 (3) **(e)** 1 679 000 (2) **(f)** 0.899 5 (3)

4 To how many significant figures have each of the following been written?

(a) 825 **(b)** 63.73 **(c)** 145 000

(d) 0.0034 **(e)** 3.008 **(f)** 0.0040

5 **(a)** A fly's wing flaps once in 3 milliseconds. Express this time in seconds using scientific notation.

(b) The time for light to travel once around the equator is 134 milliseconds. Express this time in seconds using scientific notation.

(c) The average length of a blink is 300–400 milliseconds. Express this range of times in seconds using scientific notation.

6 Express each quantity described below using scientific notation.

(a) The distance around the Earth's equator is 40 075 km.

(b) The thickness of a soap bubble is about 0.000 000 1 m.

(c) A 10-cent coin is about 0.0015 m thick.

(d) Pluto is approximately 5900 million km from the Sun.

(e) The poliomyelitis virus has a diameter of 0.000 000 3 m.

(f) The melting point of gold is 1063 °C.

7 Express each quantity described below as a number.

(a) Radio waves travel at about 3×10^8 m/s.

(b) In a hydrogen atom, the average distance between the proton and the electron is 1.5×10^{-15} m.

(c) The thickness of fine paper is about 1.2×10^{-4} m.

(d) The aluminium in a soft drink can contains around 4×10^{24} atoms.

(e) This distance from the Sun to Jupiter is approximately 7.78×10^{8} km.

(f) The temperature at the centre of the Sun is about 1.5×10^{7} °C.

Understanding

8 Which number is the largest?

A 1.325×10^{5} **B** 2.853×10^{2} **C** 9.851×10^{-2} **D** 6.312×10^{4}

9 Use a calculator to evaluate each expression. Express your answers in scientific notation.

(a) $2.4 \times 10^{6} \times 5.9 \times 10^{4}$ **(b)** $7.34 \times 10^{3} \times 2.9 \times 10^{-6}$

(c) $7.86 \times 10^{3} \div (2.62 \times 10^{-8})$ **(d)** $4.38 \times 10^{9} \div (2.19 \times 10^{-3})$

(e) $6.2 \times 10^{-4} - 1.34 \times 10^{-5}$ **(f)** $4.6 \times 10^{-10} - 3.01 \times 10^{-11}$

(g) $2(4.3 \times 10^{3} - 6.9 \times 10^{7})$ **(h)** $5(9.1 \times 10^{5} - 2.7 \times 10^{9})$

(i) $\dfrac{3.21 \times 10^{3} \times 4.19 \times 10^{4}}{5 \times 10^{6}}$ **(j)** $\dfrac{5.92 \times 10^{4} \times 8.01 \times 10^{12}}{3 \times 10^{5}}$

(k) $\dfrac{3(2.5 \times 10^{6} + 4.9 \times 10^{7})}{6.5 \times 10^{-4}}$ **(b)** $\dfrac{4(7.3 \times 10^{9} + 2.1 \times 10^{4})}{8.6 \times 10^{-5}}$

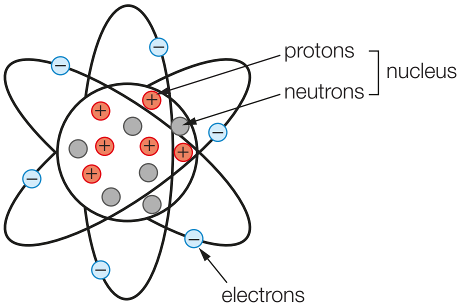
Hint

A negative power in the denominator can be written in the numerator with a positive power.

10 Express each quantity described below using scientific notation.

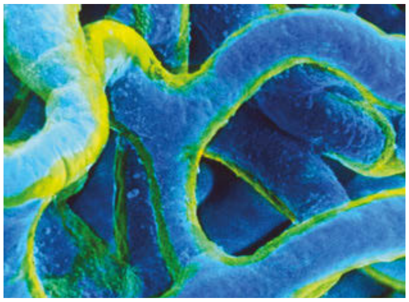
(a) The diameter of the nucleus of an atom is about 0.000 000 000 000 01 m.

(Note: The diagram shown is not to scale! Electrons are actually much further away from the nucleus.)



(b) The highest Sherpa village on the slopes of Mount Everest is about 4000 m above sea level.

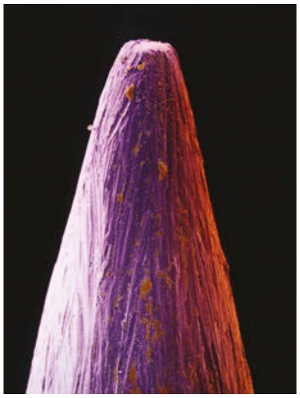
(c) This micrograph shows blood capillaries in skeletal muscle. These fine capillaries, which carry blood to the body's tissues, are about 0.000 02 m in diameter.



(d) The filament of an incandescent light bulb reaches a temperature of 2730 °C.

(e) 10 651 people attended the 2000 Summer Olympic Games in Sydney.

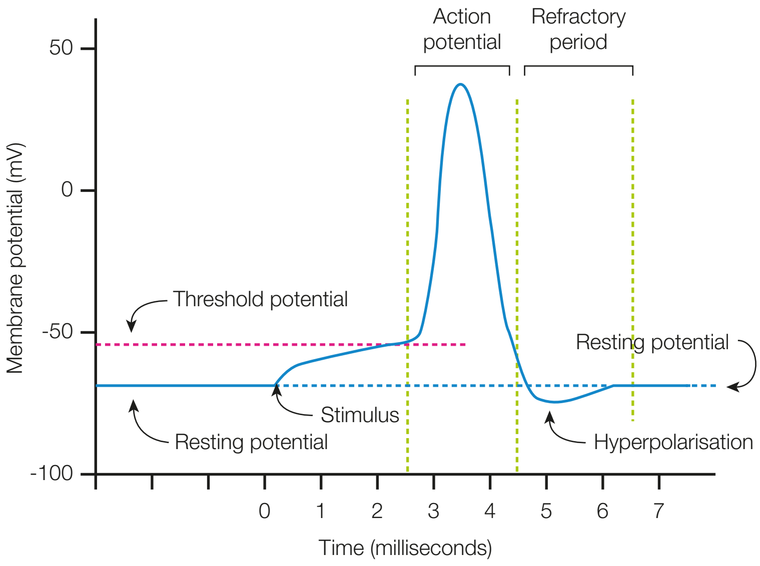
(f) This micrograph shows bacteria on the point of a household pin. About 100 million million atoms could fit on the point of a pin.



(g) The Sun is approximately 150 000 000 km from Earth. The temperature at the surface of the Sun is about 6000 °C.

(h) Australia has the highest rate of skin cancer in the world. More than 140 000 cases of skin cancer are reported each year in Australia.

11 The nervous system controls movement in the body. Electrical impulses travel through nerves to the muscles causing them to contract. Here is a graph showing the time it takes for a nerve to transmit an impulse. Use the graph to answer the following questions, expressing all times in seconds using scientific notation.



Events that characterise the transmission of a nerve impulse.

(a) How long does it takes for a nerve to return to resting potential after it has been stimulated?

(b) How long does it takes for a nerve to reach hyperpolarisation after it has reached the threshold potential?

(c) How long does the action potential last?

(d) How long does the refractory period last?

12 **(a)** A personal computer in 1982 required approximately 1000 nanoseconds (1 microsecond) to perform an operation. More recent computers require between 0.3 and 1 nanosecond to perform an operation. How much faster are more recent computers than the computers used in 1982? Express this range in seconds using scientific notation.

(b) The RAM of a personal computer in 1982 had a response time of 200 nanoseconds. The RAM of more recent computers has a response time of between 5 and 10 nanoseconds. How much faster is the response time of the RAM in more recent computers compared with computers in 1982? Express this range in seconds using scientific notation.

Hint

Express numbers in scientific notation before you compare them.

13

Time	Event
4570 mya	Planet Earth first forms
4440 mya	Earth cools enough to form oceans and the atmosphere
3800 mya	First life on Earth: single-celled bacteria (microbial mat)
2400 mya	First photosynthesis by plants to produce oxygen
1700–1200 mya	Multicellular plants and fungi (sea dwellers)
850–630 mya	Ice age 'snowball Earth'—extinction event
580 mya	Multicellular animals: *Charnia* (filter feeders like sponges; sea dwellers)
550 mya	First creatures capable of movement: trilobites (first arthropod)
530 mya	First predators with mouth, teeth and stomach: echinoderms (*Arkarua*)
490–476 mya	Plants and invertebrates on land: arthropods (insects, giant dragonfly)
370 mya	Vertebrate animals on land
251 mya	Extinction event
230 mya	Dinosaurs
150 mya	First birds
130–90 mya	Flowering plants and social insects (bees, termites, ants)
65 mya	Extinction of dinosaurs
6 mya	Early humans (hominins)
200 000 years ago	Modern humans

Note: mya = 'millions of years ago'.

Above is a time line of the development of life on Earth. Use this time line to answer the following questions. Write all answers in scientific notation.

(a) What is the age of the Earth?

(b) How long after the Earth cooled and the atmosphere developed did the first life on Earth appear?

(c) The ability of single-celled bacteria to create oxygen changed the Earth's atmosphere and allowed multicellular plants to develop. When did photosynthesis begin?

(d) For how long was the Earth completely covered in snow and ice (snowball Earth)?

(e) What was the time span over which multicellular plants and fungi developed?

(f) Remains of the first predator, an echinoderm, have been found in South Australia and named *Arkarua*. When did this animal first evolve?

(g) How long after invertebrates first moved out of the water and onto land did land vertebrates evolve?

(h) For how long was the Earth inhabited by dinosaurs?

(i) How many years before the first humans appeared on Earth did the first birds evolve?

(j) Extinction events are events that have extinguished almost all life on Earth. Recent extinction events are thought to have been due to volcanic eruptions or collisions with asteroids or comets, producing dust storms that blocked out the Sun, causing plants to die and disrupting the entire food chain. When were the last two extinction events?

(k) What is the time span from the formation of the Earth to the beginning of modern humans?

14 Light travels at a speed of approximately 3×10^8 m/s.

(a) Given that speed $= \frac{\text{distance}}{\text{time}}$, how far does light travel in:

(i) 2 minutes **(ii)** 6 weeks **(iii)** 1 year (365.25 days)?

Express your answers in scientific notation and in metres.

(b) If the average distance from the Earth to the Sun is 1.496×10^8 km, approximately how much time does it take for light from the Sun to reach Earth? Express your answer in minutes, correct to 1 decimal place.

Hint

speed $= \frac{\text{distance}}{\text{time}}$

15 Neil Armstrong sent messages back from the Moon on 20 July 1969. Radio waves travel at about 3.0×10^8 m/s. If the Moon is 384 000 km away from Earth, how long did it take, in seconds, for a message to travel from the Moon to Earth?

16 The thickness of a sheet of paper is approximately 2×10^{-4} m and the thickness of a sheet of cellophane is approximately 2×10^{-3} m.

(a) Which is thinner?

(b) Find the difference in thickness between the two sheets.

Reasoning

17 When processing olives to extract olive oil, the crushed olives are fed through a spinning centrifuge. This centrifuge contains two perforated drums that spin at different speeds. The outer drum spins at 1.83×10^5 revolutions per hour while the inner drum spins at 1.92×10^5 revolutions per hour.

(a) Express in scientific notation the difference in rotations, in revolutions per hour, between the inner and outer drums.

(b) Find the value of your answer to the previous question part (as an ordinary number not in scientific notation).

(c) The control panel of the olive-processing machine displays the difference in speeds between the two drums in revolutions per minute. What number should this display?

18 Two bacteria cultures, one containing 6.2×10^{11} cells and the other 3.5×10^9 cells, are combined in the one flask.

(a) How many cells are there in the flask?

(b) If the bacteria population doubles in size every 12 hours, how many cells will there be after 3 days?

(c) Find the value of your answer to **(b)**.

19 The planet Venus is approximately 108 million km from the Sun.

(a) Express this quantity in scientific notation.

(b) Write this distance in metres using scientific rotation.

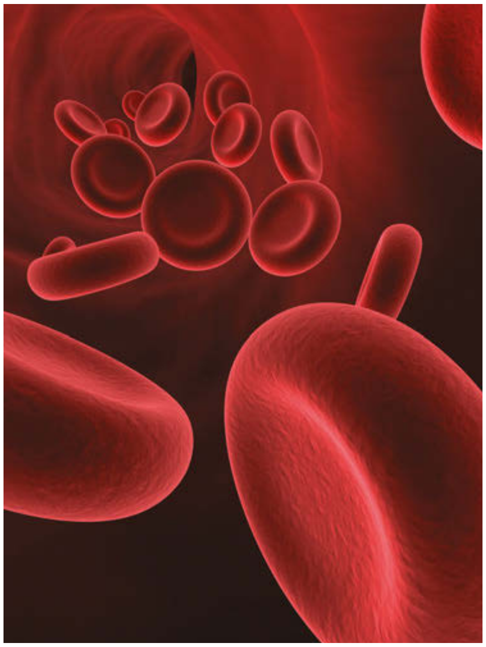
(c) If the orbit of Venus around the Sun is assumed to be circular, find the distance, in kilometres, covered by Venus in one orbit. Answer in scientific notation.

(d) Earth is about 1.5×10^{11} m from the Sun. Find the closest distance, in metres, between Venus and Earth. Answer in scientific notation.

Open-ended

20 Estimate the diameter of a hair from your head and write your answer in scientific notation.

21 Estimate the number of red blood cells in your body and write your answer in scientific notation. (Children typically have 3.8–5.5 million red blood cells per microlitre of blood. Children weighing 40–60 kg would have 2.7–4.5 L of blood.)



Problem solving

Stairway to the Moon

A standard piece of paper is approximately 0.1 mm thick.

If you cut it in half and place the two halves on top of each other, you now have a 'stack' of two pieces 0.2 mm thick. If you now cut the stack in half and place one half on top of the other, you have four pieces 0.4 mm thick.

The Moon is 363 100 km from the Earth at its closest point. How many 'cut 'n' stacks' will you need to do to have a pile of paper that reaches all the way to the Moon? (Assuming you are able to cut the stack as many times as you like.)

Strategy options

- Act it out.
- Make a table.
- Look for a pattern.

Gamespace

Equipment required: a handful of counters, calculator

Power maze

The aim is to work your way through the maze. It will test how well you know the index laws.

- You can only travel on squares that show correct index laws.
- You cannot move diagonally or onto squares that contain an incorrect equation. Use counters to keep track of your position.

On which letter (A–J) does your path through the maze finish?

START

$3 \times 3 = 3^2$	$5 \times 5 = 2^5$	$\frac{y^{12}}{y^3} = y^4$	$3^{22} \div 3^{11} = 3^2$	$5^0 = 5$	$6^3 = 6 \times 6 \times 6$
$6 \times 6 \times 6 = 6^3$	$3^{-3} = \frac{1}{27}$	$\frac{x^6}{x^3} = x^3$	$\frac{q^{50}}{q^{20}} = q^{30}$	when $x = 3$, $4x^0 = 12$	$t \times t \times t \times t = 4^t$
$4^3 = 3 \times 3 \times 3 \times 3$	$4^{-4} = -16$	$\frac{w^{10}}{w^2} = w^{12}$	when $x = 2$, $3x^0 = 3$	$(m \times h^2)^2 = mh^4$	$(y^0 + x^0)^2 = 1$
$2^{-3} = -6$	$2^{100} = 200$	$p^3 \times p^4 = p^{12}$	$k^7 \times k^3 = k^{10}$	$r^3 \times r^3 = r^6$	$(n^{12})^4 = n^{48}$
$2^0 = 0$	when $w = 2$, $-5w^3 = 40$	$r^3 \times w^2 = (rw)^5$	$p^3 \times p^3 = p^3$	$f^3 \times g^2 = (fg)^5$	$(7fd^3)^0 = 7fd^3$
$(tv^3)^2 = tv^5$	$r^5 \times r^4 = r^9$	$8s^4 - 5s^4 = 3s^4$	$\frac{c^3b^2}{b^2} = c^3$	$(x^2)^3 = x^5$	$(-2)^3 = -8$
$\frac{44n^{24}}{4n^6} = 11n^4$	$(2d^7 \times 5d)^0 = 1$	$2y^2 + 2y^3 = 4y^5$	$(t^2)^{-6} = \frac{1}{t^{12}}$	$(md^2)^0 = 1$	$(a^{13}z)^2 = a^{26}z^2$
$(tv^4)^5 = t^5v^{20}$	$\frac{h^9}{h^9} = 1$	$z^0 = 0$	$5e^4 - 2e^2 = 3e^2$	$(3t^2)^3 = 27t^5$	$(3j^3 \times 5k)^0 = 15j^3k$
$\frac{20n^7}{10n^2} = 2n^5$	$(m^3)^2 = m^5$	$(2g)^2 = 4g$	$3d^2 + 3d^2 = 6d^4$	$\frac{d^4g^3h^{10}}{d^4g^3h^{10}} = 0$	$(-5)^{-2} = \frac{1}{25}$
when $m = 3$, $-3m^2 = -27$	$-40j^5 \div 10j^4 = -4j$	$(x^5)^3 = x^{15}$	$2m^3 \times m^5 = 2m^8$	$(q^3d^4)^2 = q^6d^8$	$-7n^2 \times 6n^9 = -42n^{11}$

ER POWERS

	$\frac{mq^{26}}{q^{13}} = mq^{13}$	$(pq^3)^2 = p^3q^5$	$u^3 \div u^{-1} = u^{-3}$	$3 \times 3 \times 3 \times 3 = 4^3$	→	**A**
	$(3d^7 \times 4r)^0 = 1$	$(3e^2)^4 = 81e^8$	when $x = 5$, $-2x^2 = -50$	$(a^0)^3 = a^3$	→	**B**
	$(5m^4 + 3m^4)^0 = 8$	$8^{-1} = 0$	$f^3 \times g^2 = f^3g^2$	$\frac{h^9}{h^2} = h^7$	→	**C**
	$3s^4 + 3s^4 = 6s^4$	$\frac{y^5}{x^3} = y^2$	$4 \times s^2 = (4s)^2$	$\frac{v^{10}}{v^2} = v^5$	→	**D**
	$(k^3 \times m^2 \times 3)^0 = 1$	$2^6 = 6 \times 6$	$(-4r^3)^2 = 16r^6$	$(-2k)^3 = -8k^3$	→	**E**
	$(-3)^2 = 9$	$r^5 \times r^4 = r^{20}$	$-20j^4 \div -4j^4 = 5$	$6b^2 + 4a^2 = 10a^2b^2$	→	**F**
	$(2a^2 \times 3d)^3 = 18a^6d^3$	$(n^1)^1 = n^2$	$(2 + n^0)^3 = 27$	$(k^2 \times 2k^3)^2 = 4k^7$	→	**G**
	$(-y^3)^2 = -y^6$	$11^{-1} = \frac{1}{11}$	$(-2)^5 = -32$	$7q^4 - 7q^3 = 7q$	→	**H**
	$(3p)^2 = 9p^2$	$\frac{a^2b^3c^4}{ab^3c^4} = a$	$\left(\frac{1}{2}x\right)^2 = \frac{1}{2}x^2$	$(3b)^2 \times r^2 = 9br^2$	→	**I**
	$(a^2g^2)^3 = a^5g^5$	$d^2 \times d^2 = d^4$	$v^3 \times g^2 \times 3^0 = 1$	$4^0 = 1$	→	**J**

3.4

Rearranging formulas

A **formula** is an **equation** showing the relationship between two or more variables or unknowns. It is written with one variable on the left-hand side of the equation and all other variables on the right-hand side. The variable on the left-hand side is called the subject of the formula.

When the variable needed is not the subject of the formula, then you must rearrange the equation. For example, the formula for finding the velocity v of an object after accelerating over a distance d is $v^2 = u^2 + 2ad$. Rearrange this to make u, a or d the subject (that is, to the form $u = \ldots$, $a = \ldots$ or $d = \ldots$).

Use the correct order of operations to rearrange the formula correctly.

Worked example 13

W.E. 13

The formula for finding the velocity v of an object with an acceleration a and a starting velocity u, moving over a distance d, is $v^2 = u^2 + 2ad$.

(a) Rearrange the formula to make a the subject.

(b) Find a when $v = 5$, $u = 1$ and $d = 2$.

Thinking	Working
(a) 1 Identify the first operation that needs to be performed on the equation and perform that operation. (Subtract u^2 from both sides.)	(a) $v^2 = u^2 + 2ad$ $v^2 - u^2 = u^2 + 2ad - u^2$ $v^2 - u^2 = 2ad$
2 Identify the first operation that needs to be performed on the equation and perform that operation. (Divide both sides by $2d$.)	$\frac{v^2 - u^2}{2d} = \frac{2ad}{2d}$ $\frac{v^2 - u^2}{2d} = a$
3 Rewrite the formula with a as the subject. By convention, the subject is written on the left-hand side of the equals sign.	$a = \frac{v^2 - u^2}{2d}$
(b) 1 Substitute the known values into the rearranged formula.	(b) $a = \frac{v^2 - u^2}{2d}$ $a = \frac{5^2 - 1^2}{2 \times 2}$
2 Simplify to find the value of a.	$a = \frac{25 - 1}{4}$ $a = \frac{24}{4}$ $a = 6$

Alternatively, the values could be substituted into the original formula and the equation solved in the usual way.

Sometimes before the required variable can become the subject of the formula, the equations need to be undone with operations such as squaring both sides or taking the square root or reciprocal of both sides.

Worked example 14

W.E. 14

Rearrange the formula in each of the following to make the variable in brackets the subject of the formula.

(a) $A = \pi r^2 \quad (r)$ **(b)** $T = 2\pi\sqrt{\frac{l}{g}} \quad (l)$

Thinking	Working
(a) 1 Write the equation.	(a) $A = \pi r^2$
2 Identify the first inverse operation to perform on the equation and perform that operation (÷ π).	$\frac{A}{\pi} = r^2$
3 Identify the next inverse operation that needs to be performed so that the required variable is the subject of the formula (take the square root of both sides). Decide whether both the positive and negative values are appropriate. (In this case, the radius is a positive length, so reject the negative solution.)	$r = \sqrt{\frac{A}{\pi}}$
(b) 1 Write the equation.	(b) $T = 2\pi\sqrt{\frac{l}{g}}$
2 Identify the first inverse operation to perform on the equation and perform that operation (÷ 2π).	$\frac{T}{2\pi} = \sqrt{\frac{l}{g}}$
3 Identify the next inverse operation to perform on the equation and perform that operation (square both sides of the equation).	$\frac{T^2}{(2\pi)^2} = \frac{l}{g}$
4 Repeat this procedure until the required variable is the subject of the formula (× *g*).	$l = \frac{T^2 g}{4\pi^2}$

3.4 Rearranging formulas

Navigator

Answers p. 638

1 (a–c), 2 (a, d), 3 (a, d), 4 (column 1), 5, 6, 8, 10, 11, 15	1 (a, c, e), 2 (a–d), 3 (a, b, c, e), 4 (column 2), 5, 6, 7, 8, 9, 10, 11, 12, 13, 15	1 (b, d–f), 2 (b, c, e, f), 3 (b, c, e, f), 4 (column 3), 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16

Equipment required: calculator for Questions 5–7

Fluency

W.E. 13

1 **(a)** **(i)** Rearrange the formula to make k the subject if $P = mk$.

(ii) Find k when $P = 32$ and $m = -4$.

(b) **(i)** Rearrange the formula to make b the subject if $y = mx + b$.

(ii) Find b when $y = 9$, $m = 5$ and $x = 2$.

(c) **(i)** Rearrange the formula to make h the subject if $C = ah - g$.

(ii) Find h when $C = 19$, $a = -24$ and $g = -7$.

(d) **(i)** Rearrange the formula to make x the subject if $a = mx + c$.

(ii) Find x when $a = -21$, $m = 4$ and $c = 3$.

(e) **(i)** Rearrange the formula to make p the subject if $k = \frac{mp - 2}{n}$.

(ii) Find p when $k = 42$, $m = -4$ and $n = 5$.

(f) **(i)** Rearrange the formula to make w the subject if $u = \frac{w}{v} - xy$.

(ii) Find w when $u = 6$, $x = -3$, $y = 7$ and $v = -2$.

2 Rearrange the formula in each of the following to make the variable in brackets the subject.

(a) $v = u + at$ (u)

(b) $A = \frac{1}{2}bh$ (h)

(c) $P = \frac{W}{t}$ (W)

(d) $P = 2a - b$ (b)

(e) $P = 2a - b$ (a)

(f) $V = \frac{1}{3}\pi r^2 h$ (h)

W.E. 14

3 Rearrange the formula in each of the following to make the variable in brackets the subject.

(a) $V = \pi r^2 h$ (r)

(b) $E = \frac{1}{2}mv^2$ (v)

(c) $F = \frac{dh^2}{k}$ (h)

(d) $k = 7\sqrt{\frac{m}{n}}$ (m)

(e) $d = 2\sqrt{\frac{b}{c}}$ (b)

(f) $m = x\sqrt{\frac{y}{z}}$ (y)

4 Rearrange each of the following to make x the subject.

(a) $x + y = z$

(b) $x - 3a = 2b$

(c) $kx - m = n$

(d) $dx + c = f$

(e) $rt - x = p$

(f) $vy - x = -w$

(g) $\frac{2x + y}{p} = k$

(h) $\frac{t + rx}{w} = m$

(i) $\frac{mp + px}{m} = n$

(j) $cdx + e = gh$

(k) $axp - k = mn$

(l) $s - tux = vw$

(m) $b + \frac{xd}{y} = f$

(n) $\frac{mx}{n} + kp = t$

(o) $ab - \frac{dx}{c} - f = o$

5 The formula for finding the velocity v of a moving object is $v^2 = u^2 + 2as$. A correct rearrangement of the formula would be:

A $u^2 = v^2 + \frac{2as}{2}$

B $s = \frac{v^2 - u^2}{2a}$

C $a = \frac{2s}{v^2 - u^2}$

D $a = \frac{u^2 - v^2}{2s}$

Understanding

6 Sita is following an old recipe for a cake. The recipe gives the oven temperature in degrees Fahrenheit, but Sita needs to know the temperature in degrees Celsius. The formula for converting from Celsius to Fahrenheit is $F = \frac{9}{5}C + 32$.

(a) Rearrange the formula to make C the subject.

(b) The recipe calls for an oven temperature of 320 degrees Fahrenheit. Convert this temperature to degrees Celsius.

7 The formula for finding the velocity v of an object with a starting velocity of u and an acceleration of a over time t seconds is $v = u + at$.

(a) Rearrange the formula to make t the subject.

(b) Find the value of t when:

(i) $a = 14$ m/s^2, $u = 20$ m/s and $v = 76$ m/s

(ii) $a = 2.4$ m/s^2, $u = 5.7$ m/s and $v = 12.9$ m/s.

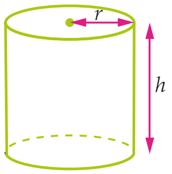
8 The circumference (C) of a circle is related to the radius (r) by the formula $C = 2\pi r$.

(a) Rearrange the formula to make r the subject.

(b) Using the calculator value for π, find the value of r when:

(i) $C = 628$ **(ii)** $C = 16.4$

9 The formula for the volume (V) of a cylinder is $V = \pi r^2 h$. Rearrange the formula to find h when $r = 5$ and $V = 157$. Use the calculator value for π and give your answer correct to 2 decimal places.



10 The perimeter of a rectangle is given by $P = 2(l + w)$, where l and w represent the length and width of the rectangle, respectively.

Rearrange the formula to make w the subject. Find the width of the rectangle if the length is 23 m and the perimeter is 84 m.

Reasoning

11 The kinetic energy (E) of an object, in joules, is found using the formula $E = \frac{1}{2}mv^2$, where m is the mass in kg and v is the velocity of the object in m/s.

(a) Determine E when $m = 2$ kg and $v = 1.2$ m/s.

(b) Determine m when $E = 162$ joules and $v = 9$ m/s.

(c) Determine v when $E = 1.96 \times 10^6$ joules and $m = 8$ kg.

(d) Explain why there are two answers for part **(c)** and what each answer means.

Hint

Find the total mass and the total volume.

12 The density (D) of a substance is related to its mass (m) and its volume (V) by the formula $D = \frac{m}{V}$.

(a) Find the difference in the mass of two objects that both have a density of 7 g/cm^3, if one object has a volume of 12 cm^3 and the other has a volume of 15 cm^3.

(b) A flotation device (buoy) is made of an outer material with a density of 1.8 g/cm^3 and a mass of 2.4 kg. It is filled with a material of density 0.9 g/cm^3 and a mass of 7.8 kg. To float on sea water, does this device have an overall density that is less than the density of sea water, 1.025 g/cm^3?

13 The area of a trapezium is given by the formula $A = \frac{(a+b)h}{2}$.

(a) Rearrange the formula to make b the subject.

(b) Use this rearranged formula to find the value of b if $a = 9$, $h = 6$ and $A = 66$.

(c) You could substitute these values for a, h and A into the original formula and solve the equation $66 = \frac{(9+b)\times 6}{2}$ to find the value of b. Explain why you would rearrange the formula to find the value of b instead of substituting the given values.

14 Consider this formula:

$$S = \frac{n}{2}(2a + (n-1)d)$$

Expand all the brackets in this expression and use this form to explain why it is difficult to make n the subject.

Open-ended

15 Ruth has been rearranging the formula for the area of a trapezium: $A = \frac{(a+b)h}{2}$. She has produced the following working while trying to make a the subject. Where has she made a mistake? Now rearrange the formula to make b the subject.

$$A = \frac{h(a+b)}{2}$$
$$2A = h(a+b)$$
$$\frac{2A}{h} = a+b$$
$$a = b - \frac{2A}{h}$$

Puzzle

In the prime of his life

Theo found that the product of his dog's age, his mother's age and his own age (all in years) is 8533. Theo is older than his dog. How old is Theo?

Half-time 3

1 Simplify each of the following, expressing your answer with positive indices. **3.1**

(a) $3x^3y \times 4x^4y$ (b) $\dfrac{8x^3y^5}{2xy^2}$ (c) $\dfrac{3m^4n^2 \times 2m^5n^3}{8m^3n^4}$

2 Rearrange the formula $z = 2xy - 3y$ to make x the subject and then find the value of x when $z = 7$ and $y = 5$. **3.4**

3 Simplify each of the following, expressing your answer with positive indices. **3.2**

(a) $(3x^4y^5)^2$ (b) $\left(\dfrac{a^4b^3}{6ab^4}\right)^2$ (c) $\dfrac{5^3x^4y^6}{3x^3y^2} \times \dfrac{x^6y}{x^7y^5}$

4 Express each quantity described below in scientific notation. **3.3**

(a) The Moon is 384 000 km away.

(b) 500 mL of water is lost through the skin daily.

5 To how many significant figures have each of the following been written? **3.3**

(a) 36 050 000 (b) 0.000 000 852 (c) 0.0830

6 Simplify the following, leaving your answer in index form. **3.1**

(a) $\dfrac{(2^3)^4 \times (5^6)^2}{(2^5) \times (5^2)^4}$ (b) $\dfrac{(3^2 \times 5^4)^2}{25 \times 27}$ (c) $\dfrac{8^3 \times 9^2}{6^4}$

7 Express each quantity described below as a number (not in scientific notation). **3.3**

(a) The number of bacteria that will fit on a pinhead is approximately 1.1×10^{14}.

(b) The nucleus of a leaf cell is about 4.4×10^{-6} m in diameter.

8 Simplify each of the following, expressing your answer with positive indices: **3.2**

(a) $\dfrac{(p^2q^3)^{-1} \times p^5q^{-2}}{p^3q^2}$ (b) $\dfrac{(c^{-3}d^{-4})^2 \times c^2d^6}{c^5d^3}$

(c) $\dfrac{4(a^2b)^3 \times 3a^{-5}b^{-2}}{6ab}$ (d) $\dfrac{6(mn^3)^2 \times 4m^{-4}n^{-1}}{3m^2n}$

9 Express each of the following with positive indices. **3.2**

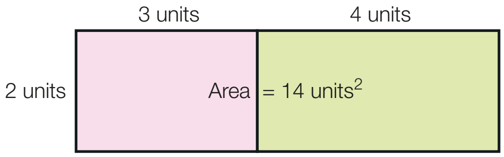
(a) x^{-5} (b) $3p^{-4}$ (c) $5m^{-3}n$ (d) $6p^{-4}q^{-1}r^2$

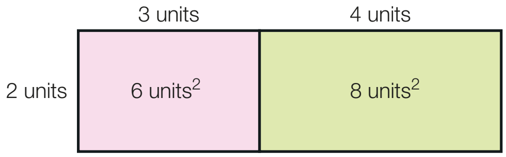
10 Write the following numbers correct to 3 significant figures: **3.3**

(a) 0.2999 (b) 80 (c) 62 078

3.5 Expanding the brackets

Brackets are used to change the order of operations, allowing addition or subtraction to be done before multiplication or division. You can rewrite the expression without brackets by multiplying every term inside the brackets by the number outside the brackets. This is called expanding the brackets.



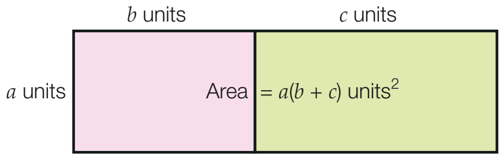


$$2(3+4) = 2 \times 7 = 14$$

and

$$\begin{aligned} 2(3+4) &= 2 \times 3 + 2 \times 4 \\ &= 6 + 8 \\ &= 14 \end{aligned}$$

If you replace numbers with variables, you can rewrite the expression without the brackets the same way.



$$a(b+c) \quad = \quad ab \quad + \quad ac$$

This is called the **distributive law**. Every term inside the brackets is multiplied by the term outside the brackets.

Distributive law

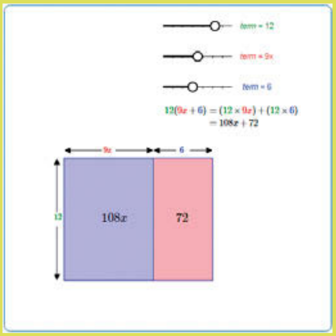
$$a(b+c) = ab + ac$$

Interactive

Distributive law for $a(b + c)$

Explore how the distributive law works for a simple algebraic expression with changing terms.

Go to the eBook or the Pearson Places website to access this interactive.



$12(9x+6) = (12 \times 9x) + (12 \times 6)$
$= 108x + 72$

Worked example 15

W.E. 15

Expand:

(a) $5(p + 4)$ **(b)** $2m(4m - 3n)$

Thinking	Working
(a) Multiply all terms inside the brackets by the term in front of the brackets. (Here $5(p + 4)$.)	(a) $5(p + 4) = 5 \times p + 5 \times 4$ $= 5p + 20$
(b) Multiply all terms inside the brackets by the term in front of the brackets. (Here $2m(4m - 3n)$.)	(b) $2m(4m - 3n) = 2m \times 4m + 2m \times (-3n)$ $= 8m^2 - 6mn$

Worked example 16

W.E. 16

Expand and simplify:

(a) $4a(a - 3) - 2(a + 5)$ **(b)** $3(2 - b) - (4 - b)$

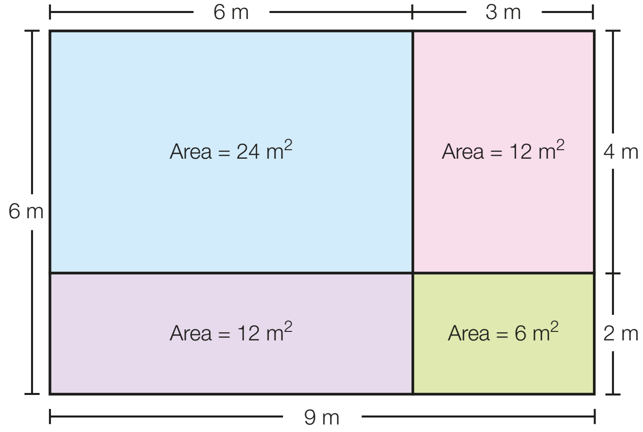
Thinking	Working
(a) **1** Expand both brackets by multiplying by the term in the front of the brackets. (Multiply the first bracket by $4a$ and the second bracket by -2.)	(a) $4a(a - 3) - 2(a + 5)$ $= 4a \times a + 4a \times (-3) + (-2) \times a + (-2) \times 5$ $= 4a^2 - 12a - 2a - 10$
2 Simplify by collecting like terms.	$= 4a^2 - 14a - 10$
(b) **1** Expand both brackets by multiplying by the term in the front of the brackets. (Multiply the first bracket by 3 and the second bracket by -1. Remember that the product of two negatives is positive: $-1 \times -b = b$.)	(b) $3(2 - b) - (4 - b)$ $= 3(2 - b) - 1(4 - b)$ $= 3 \times 2 + 3 \times (-b) + (-1) \times 4 + (-1) \times (-b)$ $= 6 - 3b - 4 + b$
2 Simplify by collecting like terms.	$= 2 - 2b$

Binomial products

A **binomial** expansion is the result of multiplying two terms in one set of brackets by two terms in another set of brackets.

$(a + b)(c + d) = ac + ad + bc + bd$.

Consider a room 6 metres long and 4 metres wide. If it is extended by 3 metres in length and 2 metres in width, find the new area, as in the diagram below.



The total area can be found in two ways:

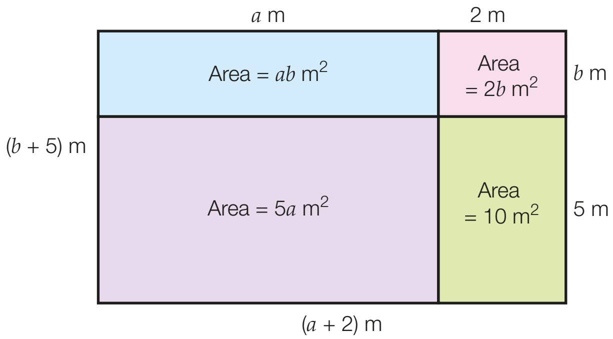
$(6 + 3)(4 + 2) = 9 \times 6$
$= 54\text{ m}^2$

or

$6 \times 4 + 6 \times 2 + 3 \times 4 + 3 \times 2 = 24 + 12 + 12 + 6$
$= 54\text{ m}^2$

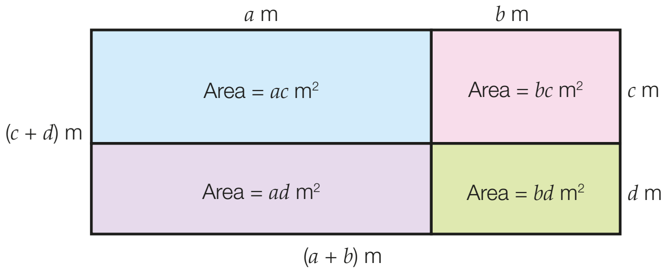
If a room a metres long and b metres wide has been extended by 2 metres in length and 5 metres in width, you can find the new area.

$$\begin{aligned}\text{Area } &= (a + 2)(b + 5)\\ &= a \times b + a \times 5 + 2 \times b + 2 \times 5\\ &= ab + 5a + 2b + 10\text{ m}^2\end{aligned}$$



If a room a metres long and c metres wide has been extended by b metres in length and d metres in width, you can find the new area using the following diagram:

$$\begin{aligned}\text{Area } &= (a + b)(c + d)\\ &= a \times c + a \times d + b \times c + b \times d\\ &= ac + ad + bc + bd\text{ m}^2\end{aligned}$$



Binomial expansion

$(a + b)(c + d) = ac + ad + bc + bd$

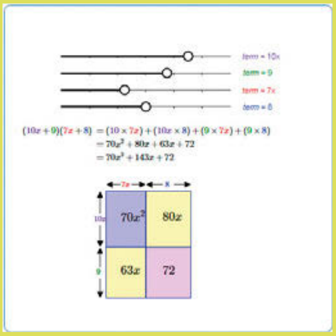
Every term in the second bracket is multiplied by every term in the first bracket.

Interactive

Distributive law for $(a + b)(c + d)$

Explore how the distributive law works for an algebraic expression of two sets of brackets with changing terms.

Go to the eBook or the Pearson Places website to access this interactive.



$$(10x + 9)(7x + 8) = (10 \times 7x) + (10x \times 8) + (9 \times 7x) + (9 \times 8)$$
$$= 70x^2 + 80x + 63x + 72$$
$$= 70x^2 + 143x + 72$$

Worked example 17

W.E. 17

Expand each of the following expressions.

(a) $(c + 4)(d + 2)$ **(b)** $(2m - 4)(3n - 5)$ **(c)** $(2p - 3r)(3t + 4)$

Thinking	Working
(a) 1 Expand the following by multiplying every term in the second bracket by every term in the first bracket. (Here $(c + 4)(d + 2)$.)	(a) $(c + 4)(d + 2)$ $= c \times (d + 2) + 4 \times (d + 2)$ $= c \times d + c \times 2 + 4 \times d + 4 \times 2$
2 Simplify.	$= cd + 2c + 4d + 8$
(b) 1 Expand the following by multiplying every term in the second bracket by every term in the first bracket. (Here $(2m - 4)(3n - 5)$.)	(b) $(2m - 4)(3n - 5)$ $= 2m \times (3n - 5) + (-4) \times (3n - 5)$ $= 2m \times 3n + 2m \times (-5)$ $+ (-4) \times 3n + (-4) \times (-5)$
2 Simplify.	$= 6mn - 10m - 12n + 20$
(c) 1 Expand the following by multiplying every term in the second bracket by every term in the first bracket. (Here $(2p - 3r)(3t + 4)$.)	(c) $(2p - 3r)(3t + 4)$ $= 2p \times (3t + 4) + (-3r) \times (3t + 4)$ $= 2p \times 3t + 2p \times 4$ $+ (-3r) \times 3t + (-3r) \times 4$
2 Simplify.	$= 6pt + 8p - 9rt - 12r$

Binomial expansion can be completed using FOIL.

First:	Multiply the first terms in each bracket.
Outside:	Multiply the outside terms in each bracket.
Inside:	Multiply the inside terms in each bracket.
Last:	Multiply the last terms in each bracket.

$$(a + b)(c + d)$$

After expansion, often like terms can be collected and the expression simplified.

Sometimes three terms are left, called a **quadratic trinomial**. This is demonstrated in the following example.

Worked example 18

W.E. 18

Expand each of the following expressions. Simplify where possible.

(a) $(y-2)(y+5)$ **(b)** $(t-3)(t-7)$ **(c)** $(4y+3)(5y-2)$

		Thinking	Working
(a)	**1**	Expand by multiplying every term in the second bracket by every term in the first bracket. (Here $(y-2)(y+5)$.)	(a) $(y-2)(y+5)$ $= y \times y + y \times 5 + (-2) \times y + (-2) \times 5$
	2	Write without multiplication signs and perform any possible calculations.	$= y^2 + 5y - 2y - 10$
	3	Simplify by collecting like terms.	$= y^2 + 3y - 10$
(b)	**1**	Expand by multiplying every term in the second bracket by every term in the first bracket. (Here $(t-3)(t-7)$.)	(b) $(t-3)(t-7)$ $= t \times t + t \times (-7) + (-3) \times t + (-3) \times (-7)$
	2	Write without multiplication signs and perform any possible calculations. Notice that the product of two negative numbers is a positive number ($-3 \times -7 = +21$).	$= t^2 - 7t - 3t + 21$
	3	Simplify by collecting like terms.	$= t^2 - 10t + 21$
(c)	**1**	Expand by multiplying every term in the second bracket by every term in the first bracket. (Here $(4y+3)(5y-2)$.)	(c) $(4y+3)(5y-2)$ $= 4y \times 5y + 4y \times (-2) + 3 \times 5y + 3 \times (-2)$
	2	Write without multiplication signs and perform any possible calculations. Notice that the product of a positive and negative number is a negative number ($+3 \times -2 = -6$).	$= 20y^2 - 8y + 15y - 6$
	3	Simplify by collecting like terms.	$= 20y^2 + 7y - 6$

3.5 Expanding the brackets

Navigator

1 (column 1), 2 (column 1), 3 (column 1), 4 (column 1), 5 (column 1), 6 (column 1), 7, 8, 9, 12, 13, 16	1 (column 2), 2 (column 2), 3 (column 2), 4 (column 2), 5 (column 2), 6 (column 2), 7, 9, 10, 11, 12, 13, 15, 16, 17	1 (column 3), 2 (column 3), 3 (column 3), 4 (column 3), 5 (column 3), 6 (column 3), 7, 9, 10, 11, 12, 13, 14, 15, 16, 18

Answers p. 639

Fluency

W.E. 15

1 Expand each of the following expressions.

(a) $2(x+5)$ (b) $4(a+9)$ (c) $8(b+3)$

(d) $8(x-2)$ (e) $9(x-2)$ (f) $3(k-4)$

(g) $-7(x+1)$ (h) $-4(d+8)$ (i) $-6(x+3)$

(j) $-9(c-6)$ (k) $-2(x-5)$ (l) $-5(x-6)$

(m) $x(x+3)$ (n) $m(m+5)$ (o) $x(x+9)$

(p) $-x(x-8)$ (q) $-x(x-4)$ (r) $-n(n-9)$

(s) $3x(x+2)$ (t) $2u(u-6)$ (u) $5x(x-1)$

(v) $-2x(x+7)$ (w) $-7q(q+1)$ (x) $-9b(b-6)$

Hint

Remember $-(x+2)$ really means $-1(x+2)$.

W.E. 16

2 Expand and simplify each expression.

(a) $3(x-2)+2(x+4)$ (b) $2(y+1)+3(y-2)$

(c) $4(2x-3)-(6x-1)$ (d) $3(4a+5)-(7a+2)$

(e) $a(2a+3)-2(3a-5)$ (f) $3x(x+2)-4(x-1)$

(g) $7y(y-3)-5(y+2)$ (h) $6k(2k+3)-3k(k-3)$

W.E. 17

3 Expand each of the following.

(a) $(x+6)(y+5)$ (b) $(x+4)(y+7)$ (c) $(c+d)(e+3)$

(d) $(r+5)(t-8)$ (e) $(a+2)(b-1)$ (f) $(p+3)(q-2)$

(g) $(a-3)(y-4)$ (h) $(m-2)(n-6)$ (i) $(k-5)(m-4)$

(j) $(4p+q)(m+n)$ (k) $(7x+y)(a+k)$ (l) $(3p+2r)(4s+t)$

(m) $(2a+1)(b-3)$ (n) $(3x-1)(x+2)$ (o) $(3m+2)(n-1)$

(p) $(2a-3b)(5m-3n)$ (q) $(5c-2d)(3x-4y)$ (r) $(7p-2q)(2r-3t)$

W.E. 18

4 Expand each of the following and simplify.

(a) $(a+3)(a+2)$ (b) $(c+5)(c+3)$ (c) $(y+7)(y+6)$

(d) $(x-3)(x+1)$ (e) $(y-2)(y+9)$ (f) $(a-7)(a+12)$

(g) $(b-2)(b-1)$ (h) $(x-4)(x-3)$ (i) $(p-8)(p-7)$

(j) $(3m+1)(2m+3)$ (k) $(4d+5)(2d+3)$ (l) $(3k+5)(7k+4)$

(m) $(4y-3)(2y+1)$ (n) $(5k+3)(2k-7)$ (o) $(2a+5)(4a-3)$

(p) $(6a-5)(2a-1)$ (q) $(4p-3)(3p-5)$ (r) $(3r-2)(8r-7)$

5 Expand each of the following and simplify.

(a) $(x+y)(x+y)$ (b) $(m+n)(m+n)$ (c) $(a+b)(a+b)$

(d) $(p-r)^2$ (e) $(c-d)^2$ (f) $(g-h)^2$

(g) $(2c+3d)(2c+3d)$ (h) $(3m+2n)(3m+2n)$ (i) $(5a+3b)(5a+3b)$

(j) $(5x-4y)(5x-4y)$ (k) $(2j-3k)(2j-3k)$ (l) $(7c-5d)^2$

6 Expand each of the following and simplify.

(a) $(m + n)(m - n)$ (b) $(p + q)(p - q)$ (c) $(a - b)(a + b)$

(d) $(3a + 2b)(3a - 2b)$ (e) $(4x - 3y)(4x + 3y)$ (f) $(7m - 3n)(7m + 3n)$

7 When simplified, $5m(-3 + 2n) - n(10m - 1)$ is:

A $n - 15m$ B $10n - 15m - 1$ C $-15m - 20mn + n$ D $15m + n$

8 $(3x - 2)(3x + 2)$ is equal to:

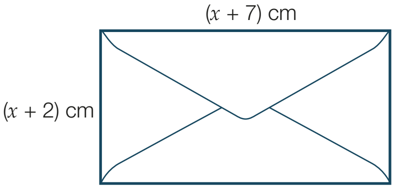
A $6x^2 + 4$ B $9x^2 - 12x - 4$ C $9x^2 + 6x - 4$ D $9x^2 - 4$

9 Expanding $(2x - 1)(x + 2)$ gives:

A $2x^2 - 2$ B $2x^2 + 3x + 2$ C $2x^2 + 3x - 2$ D $2x^2 + x + 2$

Understanding

10 Consider this envelope.



(a) Write the expression (in factorised form) for the area of this envelope.

(b) Use your answer to find the area when $x = 10$.

(c) What value would x represent if the area of the envelope is 104 cm^2? Try substituting different values for x until you find the appropriate one.

(d) Expand the expression you obtained for the area in part **(a)**.

(e) Check that the expanded expression is correct by calculating the area when $x = 10$ and comparing with the answer obtained in part **(b)**.

11 A vegetable bed, 15 m long and 3 m wide, is to be extended by x metres in both length and width.

(a) Write expressions for each of the following.

  (i) The new length of the garden bed.

  (ii) The new width of the garden bed.

  (iii) The new area of the garden bed as the product of factors.

  (iv) The new area of the garden bed as the expanded product of factors.

(b) Compare the area of the new garden bed with the area of the original garden bed. By how much has the area increased?

(c) Use your answer from part **(b)** to calculate how much extra area of garden bed is available if both the length and width are increased by 2 m.

12 The sides of a rectangle and a triangle have the lengths shown below.

(a) Write the expression for the area of the rectangle using the given lengths.

(b) Write the expression for the perimeter of the triangle using the given lengths.

(c) Find two values for x so that the area of the rectangle is equal to the perimeter of the triangle.

Reasoning

13 There are a number of ways to check that $x(x+2)=x^2+2x$ is a true statement. Choose two of these methods and show clearly how you are able to check that $x(x+2)=x^2+2x$ is true. Can you use these methods to show that $a(b+c)=ab+ac$? If so, show how.

14 Farmer Aathi needs to construct a holding paddock for his sheep. To allow for a 4-metre gate on one side, Aathi decides to make the length 4 metres longer than the width. Let x represent the width of Aathi's paddock.

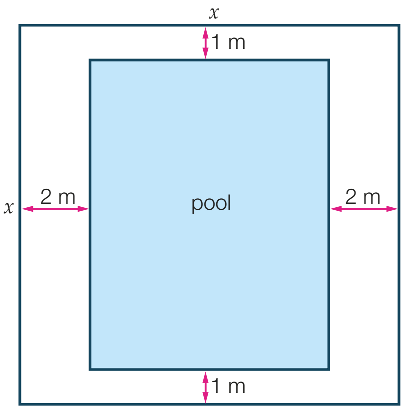
(a) Write an expression for the length of the paddock.

(b) Write an expression in factorised form for the area A of the paddock, then find the area in expanded form.

(c) Find the area of Aathi's paddock if the width is 16 metres.

(d) If Aathi is going to use all of a 100-metre roll of wire for his sheep paddock, find the length and width of the paddock. Aathi will not need to use fencing wire for the 4 metres where the gate will be.

15 A backyard, square in shape, is to have an in-ground rectangular swimming pool constructed in it. A path around the pool is to be laid as shown in the diagram.



(a) Write expressions for each of the following:

- **(i)** the total area of the backyard
- **(ii)** the width of the swimming pool
- **(iii)** the length of the swimming pool
- **(iv)** the area of the swimming pool.

(b) Using your answers to parts **(i)** and **(iv)**, write an expression for the area of the path. State your answer in simplest expanded form.

Open-ended

16 Choose two variables to represent the length and width of a rectangle. Increase each dimension by a fixed amount (e.g. increase the length by 2 units and the width by 3 units). Find the area of the new rectangle.

17 Three students gave the following incorrect answers when expanding $-3(5x-2)$:

(a) $-15x-2$ **(b)** $5x+6$ **(c)** $-8x+5$

Explain the errors made by each student. What is the correct answer?

18 Three students obtained the following incorrect answers when expanding $(x-3)(x+4)$:

(a) $x^2+7x+12$ **(b)** $2x-12$ **(c)** x^2+x-7

Explain what errors were made by each student. What is the correct answer?

Problem solving

Divisibility dilemma?

The number 123 456 is divisible by 2, 3 and 4. Find the smallest number that is divisible by all the digits from 2 to 9 (inclusive) if it begins with the digits 123 456.

Strategy options

- Test all possible combinations.
- Guess, check and improve.
- Break problem into manageable parts.

3.6

Expanding special products

It can be useful to look at the expansion of $(a + b)(c + d)$ for two special products.

Perfect squares

When you square a sum or a difference of two terms, this is called a **perfect square**. Perfect squares are expanded in exactly the same way as other binomial products.

Square of a sum

$$\begin{aligned}(a + b)^2 &= (a + b)(a + b)\\ &= a^2 + ab + ab + b^2\\ &= a^2 + 2ab + b^2\end{aligned}$$

Square of a difference

$$\begin{aligned}(a - b)^2 &= (a - b)(a - b)\\ &= a^2 - ab - ab + b^2\\ &= a^2 - 2ab + b^2\end{aligned}$$

This gives us a rule for expanding both types of perfect squares:

$(a$	$+$	$b)^2$	$=$	a^2	$+$	$2ab$	$+$	b^2
↑		↑		↑		↑		↑
First term (a)		Second term (b)		First term squared (a^2)		Twice the product of the two terms ($2 \times a \times b$)		Second term squared (b^2)
↓		↓		↓		↓		↓
$(a$	$-$	$b)^2$	$=$	a^2	$-$	$2ab$	$+$	b^2

Perfect square rules

$(a + b)^2 = a^2 + 2ab + b^2$

$(a - b)^2 = a^2 - 2ab + b^2$

The only difference in these two rules is the sign of the middle term, which is the same as the sign inside the brackets.

Worked example 19

W.E. 19

Expand each of the following expressions using a perfect square rule:

(a) $(p + 4)^2$ **(b)** $(d - 3)^2$ **(c)** $(2f + 5)^2$ **(d)** $(3m - 2n)^2$

Thinking	Working
(a) Use the rule $(a + b)^2 = a^2 + 2ab + b^2$. (Substitute $a = p$ and $b = 4$.)	(a) $(p + 4)^2 = p^2 + 2 \times p \times 4 + 16$ $= p^2 + 8p + 16$
(b) Use the rule $(a - b)^2 = a^2 - 2ab + b^2$. (Substitute $a = d$ and $b = 3$.)	(b) $(d - 3)^2 = d^2 - 2 \times d \times 3 + 9$ $= d^2 - 6d + 9$
(c) Use the rule $(a + b)^2 = a^2 + 2ab + b^2$. (Substitute $a = 2f$ and $b = 5$.)	(c) $(2f + 5)^2 = (2f)^2 + 2 \times 2f \times 5 + 25$ $= 4f^2 + 20f + 25$
(d) Use the rule $(a - b)^2 = a^2 - 2ab + b^2$. (Substitute $a = 3m$ and $b = 2n$.)	(d) $(3m - 2n)^2$ $= (3m)^2 - 2 \times 3m \times 2n + 2n \times 2n$ $= 9m^2 - 12mn + 4n^2$

Difference of two squares

If you multiply the sum of two terms by the difference of those terms, you can see that the terms inside the brackets are the same but the signs are opposite. This gives another useful rule.

$$\begin{aligned}(a+b)(a-b) &= a^2 - ab + ab + b^2\\ &= a^2 - b^2\end{aligned}$$

This rule is often referred to as DOTS (**difference of two squares**).

Difference of two squares rule

$(a+b)(a-b) = a^2 - b^2$

Worked example 20

W.E. 20

Use the difference of two squares rule to expand the following:

(a) $(x+3)(x-3)$ **(b)** $(5-k)(5+k)$ **(c)** $(3x+2)(3x-2)$ **(d)** $(4m-5n)(4m+5n)$

Thinking	Working
(a) Use the rule $(a+b)(a-b) = a^2 - b^2$. (Substitute $a = x$ and $b = 3$.)	(a) $(x+3)(x-3) = x^2 - 3^2 = x^2 - 9$
(b) Use the rule $(a+b)(a-b) = (a-b)(a+b) = a^2 - b^2$. (Substitute $a = 5$ and $b = k$.)	(b) $(5-k)(5+k) = 5^2 - k^2 = 25 - k^2$
(c) Use the rule $(a+b)(a-b) = a^2 - b^2$. (Substitute $a = 3x$ and $b = 2$.)	(c) $(3x+2)(3x-2) = (3x)^2 - (2)^2 = 9x^2 - 4$
(d) Use the rule $(a-b)(a+-b) = a^2 - b^2$. (Substitute $a = 4m$ and $b = 5n$.)	(d) $(4m-5n)(4m+5n) = (4m)^2 - (5n)^2$ $= 16m^2 - 25n^2$

3.6 Expanding special products

Navigator

1 (column 1), 2 (column 1), 3, 4, 5, 7, 8, 9, 10 (column 1), 13	1 (column 2), 2 (column 2), 3, 4, 5, 6, 7, 8, 9, 10 (column 2), 12, 13	1 (column 3), 2 (column 3), 3, 4, 5, 6, 7, 8, 9, 10 (column 3), 11, 12, 13

Answers p. 640

Fluency

W.E. 19

1 Expand each of the following expressions using a perfect square rule.

(a) $(d-1)^2$ **(b)** $(m+5)^2$ **(c)** $(k+2)^2$

(d) $(p-3)^2$ **(e)** $(x-8)^2$ **(f)** $(y-12)^2$

(g) $(m+n)^2$ **(h)** $(w-k)^2$ **(i)** $(6-c)^2$

(j) $(y+6)(y+6)$ **(k)** $(x+9)(x+9)$ **(l)** $(k-7)(k-7)$

(m) $(4y+3)^2$ **(n)** $(2w+9)^2$ **(o)** $(3x+5)^2$

(p) $(6k-1)^2$ **(q)** $(7-2a)^2$ **(r)** $(3-8d)^2$

(s) $(1-4c)(1-4c)$ **(t)** $(6+7y)(6+7y)$ **(u)** $(4g-h)(4g-h)$

(v) $(3a+d)^2$ **(w)** $(2a-3b)^2$ **(x)** $(5m+8n)^2$

W.E. 20

2 Use the difference of two squares rule to expand the following.

(a) $(c-5)(c+5)$ **(b)** $(a+4)(a-4)$ **(c)** $(k+8)(k-8)$

(d) $(9+x)(9-x)$ **(e)** $(3-y)(3+y)$ **(f)** $(1-k)(1+k)$

(g) $(m-n)(m+n)$ **(h)** $(x-y)(x+y)$ **(i)** $(c+d)(c-d)$

(j) $(v+w)(v-w)$ **(k)** $(p+q)(p-q)$ **(l)** $(h-g)(h+g)$

(m) $(2m-1)(2m+1)$ **(n)** $(3x+4)(3x-4)$ **(o)** $(6b-7)(6b+7)$

(p) $(1+5y)(1-5y)$ **(q)** $(8-5a)(8+5a)$ **(r)** $(3-2d)(3+2d)$

(s) $(4a+5c)(4a-5c)$ **(t)** $(3m-2n)(3m+2n)$ **(u)** $(2x-7y)(2x+7y)$

(v) $(4+ab)(4-ab)$ **(w)** $(3-cd)(3+cd)$ **(x)** $(5+jk)(5-jk)$

3 $(a+5)^2$ is equal to:

A $a^2+5a+10$ **B** $a^2+10a+25$ **C** $a^2+10a+10$ **D** a^2+25

4 $(2k-7)^2$ is equal to:

A $2k^2-28k+49$ **B** $4k^2+28k+49$ **C** $4k^2-49$ **D** $4k^2-28k+49$

5 When expanded, $(2-4x)(2+4x)$ is:

A $4-16x^2$ **B** $4+8x$ **C** $4-8x^2$ **D** $4-8x$

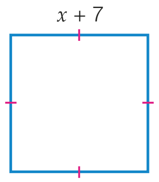
6 When expanded, $(ab+3cd)(ab-3cd)$ is:

A $ab-9cd$ **B** $a^2b^2+9c^2d^2$ **C** $a^2b^2-9c^2d^2$ **D** $a^2b^2+6c^2d^2$

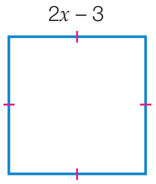
Understanding

7 **(a)** Find the area, in terms of x, of each of these squares. Expand your expressions.

(i)



(ii)



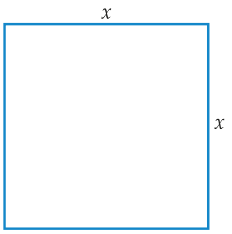
(b) Calculate the area of each where $x = 4$ cm.

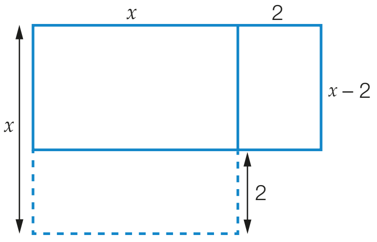
8 A square playing field is to be changed to a rectangular shape by adding 2 m to the length and subtracting 2 m from the width.

(a) Write an expression for the area of the square playing field.

(b) Write an expression for the area of the new rectangular playing field.

(c) Which playing field has the larger area? How much more area is available?





Reasoning

9 If a number n is a positive whole number, then n and $n+1$ are consecutive whole numbers. For example, if $n = 6$, then $n + 1 = 7$.

If you know the value of n^2, then you can use the expansion $(n+1)^2 = n^2 + 2n + 1$ to find the value of $(n+1)^2$. For example:

$$\begin{aligned} 7^2 &= (6+1)^2 \\ &= 6^2 + (2 \times 6) + 1 \\ &= 36 + 12 + 1 \\ &= 49 \end{aligned}$$

Given $20^2 = 400$, use the expansion $(n+1)^2 = n^2 + 2n + 1$ to find the value of 21^2.

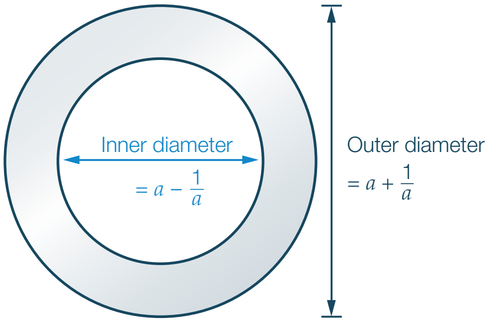
10 Because 16 can be written as $(10 + 6)$ or $(20 - 4)$, 16^2 can be written as $(10 + 6)^2$ or $(20 - 4)^2$. Evaluate the following, without a calculator, using your knowledge of special products.

(a) 23^2	**(b)** 49^2	**(c)** 112^2
(d) 3.8^2	**(e)** 6.01^2	**(f)** 11.96^2
(g) 102×98	**(h)** 2.3×1.7	**(i)** 6.04×5.96

11 A steel washer has the dimensions shown.

(a) Find the area of one side of the washer by using your knowledge of the expansion of special products.

(b) What can you say about the area of the washer for any value of a where $a > 0$?



12 **(a)** Find the shaded area using:

Area of a trapezium $= \frac{1}{2}(a + b)h$

(b) Find the shaded area using:

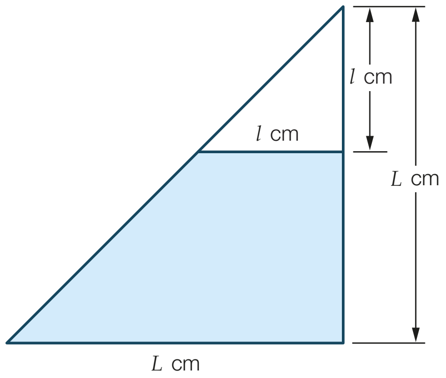
Area of a triangle $= \frac{1}{2}bh$

(c) Write an equation using part **(a)** and part **(b)**.

(d) If $L = 5l$, find the value of the shaded area:

- **(i)** using your answer to part **(a)**
- **(ii)** using your answer to part **(b)**.

(e) Show that the shaded area is 96% of the area of the large triangle when $L = 5l$.



Open-ended

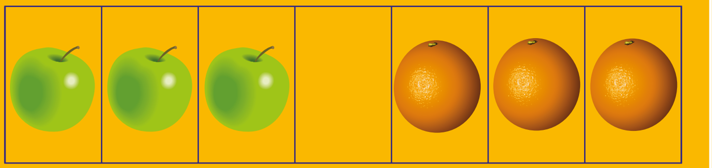
13 Find three values for a and b so that $a^2 - b^2$ produces a square number.

Problem solving

Oranges and apples

The aim is to swap the positions of the oranges and the apples. You can make two types of moves:

- You can slide a piece of fruit one space over into an empty space.
- You can hop a piece of fruit over a different type of fruit into a blank space on the other side.



1 What is the minimum number of moves needed so that the oranges are on the left and the apples are on the right?

2 If there were four or five pieces of each fruit, how many moves would it take?

Strategy options

- Solve a simpler problem.
- Act it out.
- Look for a pattern.

3.7

Factorising using common factors

The opposite (or inverse) of expanding is called factorising.

Expanding	Factorising
$a(b + c) = ab + ac$	$ab + ac = a(b + c)$

There are many different ways to **factorise** algebraic expressions. In this section you will look at the **highest common factor** (HCF). This method involves finding all the factors that are common to all terms in the expression.

Factorising using the highest common factor (HCF)

- Place the HCF outside the brackets.
- Divide each term by the HCF; this quotient goes inside the brackets.
- If the HCF is not taken out as a factor, the expression is not fully factorised (e.g. $4x + 8 = 2(2x + 4)$ is not fully factorised).
- A negative sign may be taken out as well as the HCF. Be careful to change signs inside the brackets (e.g. $-5x + 10 = -5(x - 2)$).

This procedure is called 'taking out the highest common factor'. The HCF may be a number, a variable or the product of a number and one or more variables. Once an expression is factorised, no matter what method has been used, it will have brackets in it.

Worked example 21

W.E. 21

Factorise each of the following:

(a) $5x + 20$ **(b)** $y^2 - y$ **(c)** $-9y - 6z + 3$

(d) $3a^2bc - acd$ **(e)** $-8mn^2 - 12m^2n$ **(f)** $a(b + 3) + 2(b + 3)$

Thinking	Working
(a) Find the HCF of both terms (HCF = 5). Write it down and follow with brackets. Divide each term by the HCF and write this quotient inside the brackets.	(a) $5x + 20 = 5(x + 4)$
(b) Find the HCF of both terms (HCF = y). Write it down and follow with brackets. Divide each term by the HCF and write this quotient inside the brackets.	(b) $y^2 - y = y(y - 1)$
(c) Find the HCF of both terms (HCF = 3). Write it down and follow with brackets. Take out the negative sign also. Divide each term by the factor outside the brackets and write the quotient inside the brackets. Remember to change the signs inside the brackets.	(c) $-9y - 6z + 3 = -3(3y + 2z - 1)$

(d) Find the HCF of both terms (HCF = ac). Write it down and follow with brackets. Divide each term by the HCF and write this quotient inside the brackets.	(d) $3a^2bc - acd = ac(3ab - d)$
(e) Find the HCF of both terms (HCF = $-4mn$). Write it down and follow with brackets. Divide each term by the HCF and write this quotient inside the brackets. Remember to change the signs inside the brackets.	(e) $-8mn^2 - 12m^2n = -4mn(2n + 3m)$
(f) Find the HCF of both terms (HCF = $(b + 3)$). Write it down and follow with brackets. Divide each term by the HCF and write this quotient inside the brackets.	(f) $a(b + 3) + 2(b + 3) = (b + 3)(a + 2)$

3.7 Factorising using common factors

Navigator

1 (column 1), 2 (column 1), 3 (column 1), 4 (column 1), 5, 7, 10, 12, 14	1 (column 2), 2 (column 2), 3 (column 2), 4 (column 2), 5, 6, 7, 10, 11, 12, 14	1 (column 3), 2 (column 3), 3 (column 3), 4 (column 3), 5, 6, 7, 8, 9, 10, 11, 12, 13, 14

Answers p. 640

Fluency

W.E. 21

1 Factorise each of the following.

(a) $3x + 9$
(b) $12y + 24$
(c) $6 + 42a$
(d) $2k - 6$
(e) $4m - 20$
(f) $8b - 4$
(g) $7x - xy$
(h) $4s - 9st$
(i) $3a - ab$
(j) $15y + 5w + 10x$
(k) $2a + 4b - 8c$
(l) $3x - 15y + 9$
(m) $3ac + 2abc$
(n) $5xy^2 + 2x^2y$
(o) $9f^2gh - 12fg^2$
(p) $6gh^2 + 18g^2h$
(q) $16x^2y - 40xy^2$
(r) $18ab - 42a^2b$
(s) $-4m - 20$
(t) $-7d - 49$
(u) $-12r - 60$
(v) $x(a + 3) + 5(a + 3)$
(w) $m(n - 2) + 9(n - 2)$
(x) $3(x + 5) + y(x + 5)$

2 Factorise each of the following.

(a) $10x + 25$
(b) $16y + 36$
(c) $14a - 35$
(d) $12d - 18$
(e) $18 - 27m$
(f) $36 - 15k$
(g) $24abc - 10b + 7bc$
(h) $5km - 3kn + 2ghk$
(i) $c + 4bc + 2ac$
(j) $x^2 + 3x$
(k) $y^2 + 6y$
(l) $5k + k^2$
(m) $8m - 3m^2$
(n) $4p^2 - 7p$
(o) $9a^2 - 5a$
(p) $6d - 9d^2$
(q) $22g - 14g^2$
(r) $24a - 32a^2$

3 Factorise each of the following by taking out the highest negative common factor.

(a) $-2m - 12$
(b) $-4k - 24$
(c) $-5h - 25$
(d) $-x^2 - x$
(e) $-y^2 - 3y$
(f) $-p^2 - 4p$
(g) $-16a + 4$
(h) $-21b + 7$
(i) $-30r + 6$
(j) $-24p + 40$
(k) $-18w + 16$
(l) $-45k - 63$
(m) $-15x^2 - 36x$
(n) $-14y^2 - 49y$
(o) $-20r^2 - 44r$
(p) $-8d - 4f - 6g$
(q) $-15a - 10b - 5c$
(r) $-14c - 28d - 7f$
(s) $-12x^2 - 16x + 20xy + 4xz$
(t) $-2km + 4km^2 - 6k - 8k^2$
(u) $-10b^2 + 4bc - 8bc^2 + 16ab$

4 Factorise each of the following.

(a) $4(y - 1) + w(y - 1)$
(b) $5(m - 3) + n(m - 3)$
(c) $5(p - 2) + r(p - 2)$
(d) $p(q + 5) - 2(q + 5)$
(e) $y(2x + 3) - (2x + 3)$
(f) $a(3p + 5) - (3p + 5)$
(g) $d(f - 1) - 6(f - 1)$
(h) $5d(d - 2) - 4(d - 2)$
(i) $3m(m - 6) - 7(m - 6)$

5 When factorised, the expression $3ab^2c - 6abc + 3bc$ is:

A $3bc(ab - 2a)$
B $3abc(b - 2 + 3a)$
C $3bc(ab - 2a + 1)$
D $6bc(ab - a + 2bc)$

6 When fully factorised, the expression $-2x(3m - 4) + 5y(3m - 4)$ is:

A $(3m - 4)(5y - 2x)$
B $(5y - 4)(3m - 2x)$
C $(-3m + 4)(-2x + 5y)$
D $(-2x - 4)(3m + 5y)$

7 What is the expression $2mnpq - 6mnp + 4npq$ when fully factorised?

A $2n(mpq - 3mp + 2pq)$
B $np(2mq - 6m + 4q)$
C $2np(mq - 3m + 2q)$
D $p(2mnq - 6mn + 4nq)$

Understanding

8 **(a)** Write the expression for 'eight times the product of two numbers added to twelve times one of the numbers'.

(b) Factorise the expression in **(a)**.

9 A rectangular pizza is of length d cm and width $(c + 2)$ cm. Another rectangular pizza of the same width has a length of 5 cm. The two pizzas are joined to make a longer pizza.

(a) Write the area of the new shape in factorised form.

(b) What is the relationship between c and d if the pizza is square?

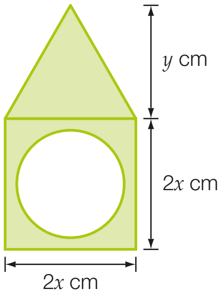
10 A floor-to-ceiling window of width x m is to be installed in a wall 5 m wide and $(y + 3)$ m high. The remaining part of the wall is to be plastered.

(a) Find the length of the wall to be plastered after the window is installed.

(b) Find the area of the wall to be plastered after the window is installed.

Reasoning

11 A wood template is to be made for the front of a letter box with the dimensions shown.



(a) Write an expression that would show the area of timber used in the construction before the circular opening is cut out.

(b) Express this answer in factorised form.

(c) If the diameter of the circular cut out needs to be 2 cm less than the width, find the area of the timber cut out for the opening.

(d) Write this in expanded form.

(e) Show that the total area of timber that will be used is $(4 - \pi)x^2 + (y + 2\pi)x - \pi$ cm^2.

(f) What area of timber would be wasted if the template was cut from a rectangular piece of timber with the smallest possible dimensions?

12 A lasagne is baked in a tray $(3p + 2)$ cm long and $(2p - 1)$ cm wide. A slice, the full width of the tray and $(r + 2)$ long is taken from the lasagne.

(a) Find the area of the lasagne, before the slice was removed, in factorised form.

(b) Find the area of the slice taken from the lasagne in factorised form.

(c) Use your answers to parts **(a)** and **(b)** to find the area remaining after the slice has been removed. Write your answer in simplest factorised form.

(d) How could you find the area in part **(c)** more simply? Show your working.

Open-ended

13 Make up three different expressions whose highest common factor is $3x$.

14 In a test, students were asked 'Factorise the expression $-5x + 10$'. Three students wrote these answers; one answer was correct, the other two answers were incorrect. Identify the correct answer and explain the mistakes the other two students made.

A $-5(x + 2)$ **B** $-5(x + 10)$ **C** $-5(x - 2)$

Game

Matching expressions

Equipment required: 20 pieces of paper cut into rectangles the size of playing cards

How to win: Match the most expressions and get more cards than your opponent.

How to play:

1. Each player writes five expressions in expanded form, one on each card, and their equivalent factored form on another five cards.
2. All 20 cards are mixed up and laid out face down. This is now a game of memory.
3. Player 1 turns two cards over. Players have 10 seconds to work out whether the cards are a match.
4. If they are a match, Player 1 takes the cards and has another go. If not, it is Player 2's turn.
5. After all the cards have been matched and claimed by a player, the game ends and the player with the most cards is the winner.

Investigation

Mr Gershwin's piano

Mr Gershwin wants to build a small theatre with a stage for piano recitals. The theatre cannot be longer than 16 metres, but must be able to seat at least 108 people in the main seating area, with extra space outside the seating area for wheelchairs.

The theatre must be rectangular with the following specifications:

- Its width W in metres correct to 1 decimal place is given by the expression $W = 4x - 4$.
- Its length L in metres correct to 1 decimal place, is twice the theatre's width.

The Big Question

Mr Gershwin wants to know the dimensions of the smallest and largest theatre he can build with the conditions given above. Can you help him?

Engage

1 **(a)** Find L and W for $x = 2$.

(b) Draw a diagram of this theatre showing the dimensions given in part **(a)**.

The stage is also rectangular and is to be built across the width of the theatre so that the length of the stage, in metres, a, is equal to the width of the theatre. The width of the stage, in metres, b, is half its length.

(c) Find the value of a and b for $x = 2$.

(d) Draw the stage in your diagram, showing the dimensions given in part **(c)**.

Explore

The audience is to be seated in rows in a rectangular space so that there is a 2-metre clearance from the front of the stage and from the back of the room and with a 1-metre aisle either side.

2 **(a)** Draw the seating area in your diagram showing the clearances.

(b) Find the length in metres, c, and width in metres, d, of the audience space for $x = 2$.

(c) Add the dimensions of your seating area to your diagram.

(d) If a space of 50 cm × 50 cm is allowed for each seat, how many seats could fit into this space?

(e) Choose three other values for x and calculate the number of people that could be seated for each of these values. Do not use integer values only.

Strategy options

- Make a table.
- Draw a diagram.
- Guess and check.
- Break the problem into manageable parts.

Explain

3 **(a)** Explain why values for $x \leq 1$ cannot be used for the length and width of the theatre.

(b) What did you find happened to the number of people that could be seated when you changed the values of x?

(c) **(i)** Explain why the smallest value for x is $1\frac{3}{4}$ m if there is to be any seating at all.

(ii) How many seats can fit if $x = 1\frac{3}{4}$ m?

Elaborate

4 **(a)** By substituting different values for x, find the dimensions of the smallest theatre that Mr Gershwin can build so that 108 people can be seated in the seating area. Use your results from **2 (e)** to help you.

(b) What is the largest value that x can be so that the length of the theatre does not exceed 16 metres?

(c) Hence, determine the range of values for x that Mr Gershwin could use to build his theatre.

Evaluate

5 **(a)** Is it sensible to change the size of the stage as the size of the theatre changes? Give a reason for your answer.

(b) Do you think the relationship used here is a good one? Give a reason for your answer.

Extend

Mr Gershwin also wants to know how much carpet he would need to cover the floor space not occupied by the stage or the seating area. You need to do the following calculations to find the area of floor to be covered with carpet.

6 **(a)** Find W and L in terms of x and factorise your answer.

(b) Find the area of the theatre in terms of x. Write your answer first in factorised form and then expand it.

(c) Find a and b in terms of x in factorised form.

(d) Find the area of the stage in terms of x. Write your answer first in factorised form and then expand it.

(e) Find c and d in terms of x.

(f) Find the seating area in terms of x. Write your answer first in factorised form and then expand it.

(g) Find the area in terms of x of the floor space of the theatre occupied by both the stage and the seating area.

(h) Use the area equations found in parts **6 (b)**, **(d)** and **(f)** to show that the area, in terms of x, of the remaining floor space in the theatre is equal to $(28x - 36)$ m^2.

(i) Show how this area could be found using an easier method.

3.8

Factorising by grouping in pairs

If there are four terms in an expression and there is no common factor for all four terms, you can sometimes group the expression in pairs of terms so that there is a common factor in each pair. If this gives you another common factor in brackets, you can factorise further. This technique is called **grouping in pairs**. Sometimes you may have to consider which two terms to pair together.

Worked example 22

W.E. 22

Factorise the following:

(a) $ab + 5b + 3a + 15$ **(b)** $3x + 12 - xy - 4y$

Thinking	Working
(a) 1 Factorise by taking a common factor from the first pair of terms (b) and a different common factor from the second pair of terms (3).	(a) $ab + 5b + 3a + 15$ $= b(a + 5) + 3(a + 5)$
2 Now you have a new common factor. (Here $(a + 5)$.) Write this factor in front of another set of brackets. Divide each product by this factor and write the quotient in the second set of brackets.	$= (a + 5)(b + 3)$
(b) 1 Factorise by taking a common factor from the first pair of terms (3) and a different common factor from the second pair of terms ($-y$). When taking out a negative common factor, change the signs inside the brackets.	(b) $3x + 12 - xy - 4y$ $= 3(x + 4) - y(x + 4)$
2 Now you have a new common factor. (Here $(x + 4)$.) Take out this common factor to complete the factorisation.	$= (x + 4)(3 - y)$

Worked example 23

W.E. 23

Factorise $df + 6g - 6f - dg$.

Thinking	Working
1 The first two terms (*df* and 6*g*) do not have a common factor. So, rewrite the expression to pair terms such that each pair of terms does have a common factor.	$df + 6g - 6f - dg$ $= df - 6f - dg + 6g$
2 Now factorise the first pair of terms by taking out a common factor (*f*) and take a different common factor from the second pair (-*g*). Change signs if necessary.	$= f(d - 6) - g(d - 6)$
3 Take out the remaining common factor to complete the factorisation. (Here $(d - 6)$.)	$= (d - 6)(f - g)$

Many quadratic expressions can be factorised into a product of two factors. The next example shows how quadratic expressions can be factorised using the grouping in pairs method.

Worked example 24

W.E. 24

For each of the following expressions, **(i)** factorise, **(ii)** simplify, **(iii)** write the relationship between your answers to parts **(i)** and **(ii)**, and **(iv)** state the relationship between the coefficients of the two middle terms and the last term.

(a) $x^2 + 5x + 6x + 30$ **(b)** $p^2 - 4p - 4p + 16$ **(c)** $a^2 - 12a + 12a - 144$

Thinking	Working
(a) **(i)** **1** Factorise by taking out a common factor from the first two terms and by taking out a different factor from the next two terms.	(a) (i) $x^2 + 5x + 6x + 30$ $= x(x + 5) + 6(x + 5)$
2 Factorise by taking out the new common factor.	$= (x + 5)(x + 6)$
(ii) Add the two middle terms because they are like terms.	(ii) $x^2 + 5x + 6x + 30$ $= x^2 + 11x + 30$
(iii) These answers must be equal to each other because they are both equal to the expression.	(iii) $x^2 + 11x + 30 = (x + 5)(x + 6)$

(iv)	**1**	Identify the coefficients of the two middle terms, remembering to include the signs, and look for a relationship with the last term (the product of these coefficients gives the last term).	(iv) $5 \times 6 = 30$
	2	State the relationship you have found.	The product of the coefficients of the two middle terms gives the last term.
(b) (i)	**1**	Factorise by taking out a common factor from the first two terms and by taking out a different factor from the next two terms.	(b) (i) $p^2 - 4p - 4p + 16$ $= p(p - 4) - 4(p - 4)$
	2	Factorise by taking out the new common factor.	$= (p - 4)(p - 4)$
	3	Write as a perfect square.	$= (p - 4)^2$
(ii)		Add the two middle terms because they are like terms.	(ii) $p^2 - 4p - 4p + 16$ $= p^2 - 8p + 16$
(iii)		These answers must be equal to each other because they are both equal to the expression.	(iii) $(p - 4)^2 = p^2 - 8p + 16$
(iv)	**1**	Identify the coefficients of the two middle terms, remembering to include the signs, and look for a relationship with the last term (the product of these coefficients gives the last term).	(iv) $-4 \times -4 = 16$
	2	State the relationship you have found.	The product of the coefficients of the two middle terms gives the last term.
(c) (i)	**1**	Factorise by taking out a common factor from the first two terms and by taking out a different factor from the next two terms.	(c) (i) $a^2 - 12a + 12a - 144$ $= a(a - 12) + 12(a - 12)$
	2	Factorise by taking out the new common factor.	$= (a - 12)(a + 12)$
(ii)		Add the two middle terms because they are like terms.	(ii) $a^2 - 12a + 12a - 144$ $= a^2 - 144$
(iii)		These answers must be equal to each other because they are both equal to the expression.	(iii) $a^2 - 144 = (a - 12)(a + 12)$

(c) **(i)** **1** Identify the coefficients of the two middle terms, remembering to include the signs, and look for a relationship with the last term (the product of these coefficients gives the last term).	(c) (i) $^{-}12 \times {}^{+}12 = {}^{-}144$
2 State the relationship you have found.	The product of the coefficients of the two middle terms gives the last term.

3.8 Factorising by grouping in pairs

Navigator

1 (column 1), 2 (column 1), 3 (column 1), 4, 5, 6, 8, 11, 12, 13	1 (column 2), 2 (column 2), 3 (column 2), 4, 5, 6, 7, 8, 9, 11, 12, 13	1 (column 1), 2 (column 1), 3 (column 2), 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

Answers p. 641

Fluency

W.E. 22

1 Factorise the following.

(a) $xy + 4y + 3x + 12$ **(b)** $pq + q + 5p + 5$ **(c)** $mn + 2n + 9m + 18$
(d) $ab + bf + ad + df$ **(e)** $pq + qt + pr + rt$ **(f)** $km + 3nk + 6m + 18n$
(g) $np - 7n + p - 7$ **(h)** $ab - 5b + a - 5$ **(i)** $mn - 2n + 7m - 14$
(j) $ab - 6b - 4a + 24$ **(k)** $x^2 + 2x - 8x - 16$ **(l)** $y^2 + 4y - 6y - 24$

W.E. 23

2 Factorise the following.

(a) $2p + 3k + kp + 6$ **(b)** $mn + pq + np + mq$ **(c)** $cd + 12 + 6c + 2d$
(d) $ad - hk - dk + ah$ **(e)** $xy - 8 - 4y + 2x$ **(f)** $eg - 3 - g + 3e$
(g) $bc - 1 + c - b$ **(h)** $xy - 3 + 3x - y$ **(i)** $mk - 3n + 3m - nk$

W.E. 24

3 For each of the following expressions **(i)** factorise, **(ii)** simplify, **(iii)** write the relationship between your answers to parts **(i)** and **(ii)**, and **(iv)** state the relationship between the coefficients of the two middle terms and the last term.

(a) $x^2 + 7x + 2x + 14$ **(b)** $j^2 + 3j + 5j + 15$ **(c)** $z^2 + z + 8z + 8$
(d) $x^2 + 2x - 8x - 16$ **(e)** $y^2 + 4y - 6y - 24$ **(f)** $x^2 + 3x - 4x - 12$
(g) $a^2 - 4a + 5a - 20$ **(h)** $k^2 - 2k + 5k - 10$ **(i)** $m^2 - 7m + 4m - 28$
(j) $y^2 - 3y - 7y + 21$ **(k)** $c^2 - 9c - 5c + 45$ **(l)** $d^2 - 8d - 12d + 96$
(m) $m^2 + 3m + 3m + 9$ **(n)** $r^2 + 6r + 6r + 36$ **(o)** $k^2 + 11k + 11k + 121$
(p) $p^2 - 5p - 5p + 25$ **(q)** $d^2 - 7d - 7d + 49$ **(r)** $n^2 - 8n - 8n + 64$
(s) $c^2 - 3c + 3c - 9$ **(t)** $t^2 - 2t + 2t - 4$ **(u)** $b^2 - 9b + 9b - 81$

4 Taking out the common factor from $k(j - 1) + (j - 1)$ results in:

A $(j - 1)(k + 1)$ **B** $(j + 1)(k - 1)$ **C** $(j - 1)(k)$ **D** $(j + 1)(k + 1)$

Understanding

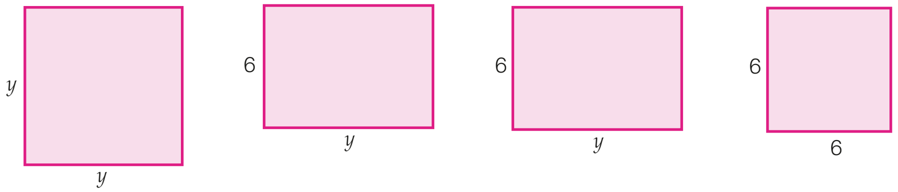
5 Square a number x, subtract four times x, add 12 and subtract three times x.

(a) Write the expression for the information given.

(b) Simply the expression.

(c) Factorise the expression.

6



(a) Write the expression for the total area of the four shapes above.

(b) Factorise the expression.

(c) Draw a diagram to show how the four shapes can fit together to make a square (include dimensions on your diagram).

Reasoning

7 (a) Factorise $ab + ac + ad + 7b + 7c + 7d$ by grouping in threes. That is, take out a common factor from three terms at a time.

(b) Show that, if $b = a$ and $c + d = 7$, the expression is a perfect square.

8 (a) Rewrite $7x$ using two terms so that the expression $x^2 + 7x + 10$ can be factorised by grouping in pairs.

(b) Now factorise your new expression.

9 (a) Factorise $35y^2 - 21y$.

(b) If this represents the area of a rectangle in cm^2 and the factors represent the width, write the dimensions of the rectangle.

(c) If an area of $15y - 9$ cm^2 is added to the area in part **(a)**, find the combined area in factorised form.

(d) Write the dimensions of the new rectangle if the factors in part **(c)** represent the length and width.

10 A trapezium has an area of $\frac{3x^2 + 2xy - 6x - 4y}{2}$ m^2 where x and y are both positive numbers and $x > 2$.

(a) Write the area in factorised form.

(b) If the smaller factor represents the height of the trapezium, what is the sum of the lengths of the two parallel non-equal sides?

11 A roll of plastic for blocking off areas of land comes in rolls that are $(x + 2)$ metres wide. Tess needs a length of plastic that is $(x + 20)$ metres long. Ying needs a length of plastic that is 22 metres long.

(a) Write expressions for the areas of plastic needed by Tess and by Ying.

(b) Write an expression to show the difference in area between the two pieces of plastic. Assume that Tess's piece is larger than Ying's piece.

(c) Factorise and simplify the expression from the previous question part, without expanding.

Open-ended

12 (a) Find two terms that, when added to $x^2 + ax$, can be factorised by grouping in pairs.

(b) Factorise your expression.

13 (a) Draw a rectangle of length l and width w. Extend the length by a certain amount and the width by a different amount. Use your diagram to write the area of the rectangle in two different ways.

(b) Draw a square of side s. Extend one pair of opposite sides by a certain amount and the pair of opposite sides by a different amount. Use your diagram to write the area of the new rectangle in two different ways.

(c) Draw a square of side s and extend the sides by the same amount to form a new square. Use your diagram to find the area of the new square in two different ways.

Puzzle

Trouble in variable paradise

Some variables were messaging online; the following is an excerpt of their conversation…

x: I am soooo smaller than you y

y: Lol, yeah but at least I'm not negative

x: *cries* you might be greater than me, but at least I'm not square like you

y: Aw, don't be cut x, together we make one, anyway although you're negative, you're a perfect cube.

x: Luv you y

What are the values of x and y?

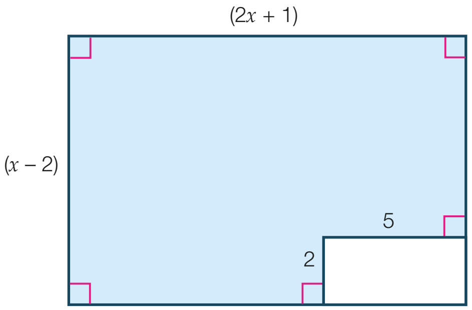
Challenge 3

1 Write two numbers that have a sum of 27 and a product of 72.

2 Bridget said, 'The day before yesterday I was 15 and next year I will be 18'.
When is Bridget's birthday and on what day did she say this?

3 How many perfect cubes are there between 2000 and 10 000?

4 Part of a rectangle is shaded as shown in the diagram.



(Diagram not to scale)

Which expression gives the area of the shaded part?

A $2x^2 - x - 8$
B $2x^2 - x - 12$
C $2x^2 - 3x - 8$
D $2x^2 - 3x - 12$

5 If ♦ represents a digit, find the value of ♦ where $[3(170 + \blacklozenge)]^2 = 272\,48\blacklozenge$.

6 a, b and c are positive whole numbers. Show that $(a - b)(b - c)(c - a)$ is divisible by 2.

7 Is the number $3^{404} + 4^{495}$ divisible by 5? Show why or why not.

8 Steven thinks of four different numbers. The sums Steven gets by pairing the numbers are 6, 7, 9, 9, 11 and 12. What are the four numbers?

9 A formula used in the science of optics is $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$.

(a) Make f the subject of the formula.

(b) Make v the subject of the formula.

10 **(a)** Show that the difference between any two-digit number and the number formed by reversing the digits is divisible by 9. (Hint: Let the two-digit number be $10a + b$.)

(b) The difference between any three-digit number and the number formed by reversing its digits is found. By what number is it divisible?

(c) The difference between any four-digit number and the number formed by reversing its digits is found. By what number is it divisible?

Chapter review 3

Maths literacy

base
binomial
difference of two squares
distributive law
equation
expand
expanded form
expression
factorise
grouping in pairs
highest common factor
index
index form
perfect square
quadratic trinomial
reciprocal
scientific notation
significant figures
variable

Copy and complete the following using the words and phrases from the list, where appropriate. A word or phrase may be used more than once.

1 An expression of the form $x^2 - y^2$ is called the ___________.

2 If an expression is factorised to obtain the product of two brackets that are exactly the same, it is called a ___________.

3 To ___________ an expression is to remove brackets. The reverse operation is to ___________ the expression and put it into brackets.

4 If you write a number in ___________, then it has a number between 1 and 10 multiplied by a positive or negative power of 10.

Fluency

1 Simplify each of the following. **3.1**

(a) $a^3b^5 \times a^2b^2$ (b) $m^4n^7 \times m^3n$ (c) $\dfrac{p^4q^6}{p^3q^9}$

(d) $\dfrac{(x^5)^2 \times x^8}{(x^4)^4}$ (e) $\dfrac{(x^2)^3 \times (y^4)^2}{xy^3 \times x^5}$ (f) $\dfrac{c^2d^4}{c^4d^2} \times \dfrac{c^6d}{(cd)^2}$

2 $(2x^3y^2)^3$ is the same as: **3.2**

A $2x^3y^6$ B $8x^3y^6$ C $2x^9y^6$ D $8x^9y^6$

3 Express each of the following with positive indices. **3.2**

(a) $-5m^{-1}$ (b) a^2b^{-6} (c) $x^{-2}y^{-3}$ (d) $a^{-4}b^{-9}$

4 Write each of the following in scientific notation. **3.3**

(a) 0.000 000 000 000 002 mm (b) 0.000 15 kg

(c) 320 000 000 L (d) 480 000 000 000 000 km

5 Write each of the following as a number (not using scientific notation). **3.3**

(a) 5.4×10^7 (b) 8.273×10^{-4}

(c) 1.4×10^{-5} (d) 7.162×10^3

6 Rearrange each of the following to make x the subject. **3.4**

(a) $z = \dfrac{3 - xy}{2}$ (b) $x + 2a = 4x$

(c) $s = xt + 8t^2$ (d) $\dfrac{2 + 4z}{3x} = 10$

7 Expanded and simplified, $2c(3b - 4) - 3b(-5 + 2c)$ is: 3.5

A $15b - 8c$ **B** $21b + 8c$ **C** $12bc - 12c$ **D** $7b + 14c$

8 Expand: 3.5

(a) $4(x + 7)$ **(b)** $x(x - 3)$ **(c)** $-5x(2x - 1)$

9 **(a)** Expand and simplify: 3.5

(i) $4(2x - 3) + 5(x + 1)$ **(ii)** $3x(6x + 5) - 2(3 - 2x)$

(b) Find the value of each expression when x has the value 4.

10 Expand and simplify: 3.5

(a) $(x + 8)(x + 2)$ **(b)** $(2a + 5)(3a - 4)$

11 Expand and simplify: 3.6

(a) $(x + 12)^2$ **(b)** $(5c - 2d)^2$ **(c)** $(4x - 3)(4x + 3)$

12 Factorise: 3.7

(a) $8a + 12$ **(b)** $-15k - 20k^2$

(c) $5p^2q - 3pq + 2pq^2 - pqr$ **(d)** $-24a - 6ab - 72$

13 Factorise by grouping in pairs. 3.8

(a) $ab + 4b + 3a + 12$ **(b)** $ab - 2b + a - 2$

(c) $ab - 6a - 5b + 30$ **(d)** $ab - 7 + 7b - a$

Understanding

14 Express the following in scientific notation. 3.3

(a) A 10-cent coin is about 0.0015 m thick.

(b) Pluto is approximately 5900 million km from the Sun.

15 Express each of the following as a numeral (not using scientific notation). 3.3

(a) The distance from the Sun to Jupiter is approximately 7.78×10^8 km.

(b) The temperature at the centre of the Sun is believed to be about 1.5×10^7 °C.

16 The length of a greeting card is 20 cm and the width is $(5 + x)$ cm. 3.4, 3.5, 3.6, 3.7

(a) Draw a diagram of the card.

(b) Write an expression for the perimeter.

(c) Factorise the expression obtained in part **(b)**.

(d) Write an expression for the area (in factorised form).

(e) Expand the expression obtained in part **(d)**.

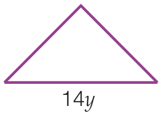
(f) Check that the two expressions obtained for the area are equivalent by substituting a value for x.

17 If the area of a rectangular postage stamp is $4x^2 + 12x$, what is the width of the stamp if the height is $2x$? 3.7

18 The mass of an object can be determined by finding the product of its density and its volume. If the density can be represented by $(m - 2)$ and the volume by $(k + 8)$, write an expression for the mass. 3.7

Suggest possible values that m and k could take to give a mass of 48.

19 Find the height of this triangular support structure if the base is $14y$ and the area enclosed by the beams is $35y^2 - 21y$. 3.4, 3.7



Reasoning

3.3

20 The heart pumps 7000 L of blood daily.

(a) Express this quantity in scientific notation.

(b) How many litres of blood are pumped by the heart:

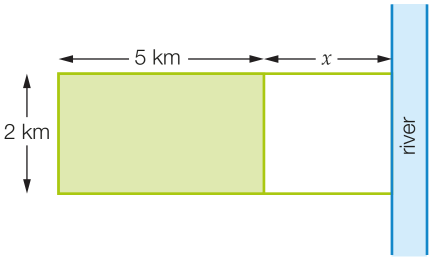
(i) each week

(ii) each second, to 1 decimal place

(iii) in 3 years (assuming no leap years)?

(c) Would this measurement be exact and the same for each person?

21 The Greenwood family had a rectangular block of land 2 km by 5 km for grazing sheep, as seen in the diagram. They then bought the land adjoining this leading down to the river. Depending on the time of year and the weather conditions, the width of the river can vary, so the land available for grazing sheep changes.



(a) Write an expression for the total area available for grazing sheep at any particular time. (Assume the land remains rectangular.)

3.5, 3.6, 3.7

(b) Suppose there are $\frac{1}{2}x$ sheep per km^2 of land. How many sheep are there altogether? Write your answer in expanded form.

(c) Fourteen sheep are moved to another location.

(i) Write the expression for the number of sheep remaining.

(ii) Expand $(x + 7)(x - 2)$. How is this related to the expression in part **(i)**?

(iii) Explain what happens when $x = 2$ km.

Numeracy practice 3

Calculator allowed

1 Which of the following numbers is the largest?

A 3.2×10^{18} **B** 9.6×10^{3} **C** 3.2×10^{-18} **D** 9.6×10^{-3}

2 What is the value of $7(x - 9)$, when $x = 15$?

3 Factorise the expression $14x + 21$.

Non-calculator

4 Which one of the following expressions is equivalent to $3(5x - 2)$?

A $15x - 1$ **B** $15x - 6$ **C** $15x + 1$ **D** $15x + 6$

5 A rule for y in terms of x is $y = 12 - 3x$. Write an equivalent rule for x in terms of y.

6 Which one of the following expressions is a fully factorised form of $9xy + 12y$?

A $3(3xy + 4y)$ **B** $9y(x + 2)$ **C** $y(9x + 12)$ **D** $3y(3x + 4)$

7 When the expression $-3x^2y + 6xy$ is fully factorised, the result is:

A $3xy(2 - x)$ **B** $-3xy(2 - x)$ **C** $-3xy(x - y)$ **D** $-3xy(x + y)$

Exploration STEM

You can download this activity from the eBook or the Pearson Places website.

Help find the best loan!

Australia's house prices have more than tripled in the past two decades. The median house price in Australia in 2015 was between $600 000 and $700 000, about eight times the median annual salary. At the same time, interest rates for home loans have remained low, and different banks, mortgage brokers and other companies have offered different loan packages with competitive interest rates.

A young couple are deciding to buy their first house and have asked you to advise them which home loan to apply for. Can you help them find the best home loan?

Exploration STEM

You can download this activity from the eBook or the Pearson Places website.

Grow your own Pythagoras tree

Pythagoras' theorem may be the most well-known mathematical theorem of all time. But did you know it is also an inspiration for art? Using squares and other shapes with side lengths that follow Pythagoras' theorem, amazing patterns and images can be created. Can you 'grow' your own 'Pythagoras tree' using only Pythagorean squares and triangles?

Exploration STEM

You can download this activity from the eBook or the Pearson Places website.

How healthy is your garden?

Plants need nutrients for steady and healthy growth, just as humans do. Plants also need the correct level of soil acidity (pH), because this affects how plants can absorb nutrients from the soil. To grow plants well, it is important to know the pH preferences of the different plants and the pH levels of the soil. Your task is to investigate the pH level and the distribution of elements nitrogen (N), phosphorus (P) and potassium (K) in different soils and gardens, so that you can recommend which plants should or should not be grown.

Exploration Coding

You can access this activity from the eBook or the Pearson Places website.

The bubble sort

What are the exact steps needed to order a list of numbers? Use the bubble sort algorithm to get smaller numbers to 'bubble up' to the start of the list.

4

Measurement

4

The 'Gimli Glider'. A commercial aircraft runs completely out of fuel mid-flight. How could this happen?

On 23 July 1983, Air Canada Flight 143 was about halfway through its journey from Montreal to Edmonton when alarms began to sound in the cockpit, indicating the fuel tanks were empty. No more fuel meant that both engines shut down and the pilots lost power to nearly all their instruments. Fortunately, the pilot, Captain Bob Pearson, was also an experienced glider pilot. He glided the plane to an old, disused airforce base at Gimli, where he managed to land it without causing any deaths or injuries to the 69 people aboard, or to the people on the ground racing go-karts on the old airstrip.

How did this near tragedy happen? At the time of the incident, Canada was in the middle of converting from the old imperial system of measurement to the new metric system. The amount of fuel required was calculated to be 22 300 *kilograms*. However, an error made by the ground and flight crews meant that incorrect conversion factors were used and the plane ended up with 22 300 *pounds* of fuel, or just over 10 000 kilograms, less than half the required amount.

Forum

When did Australia change from the imperial to the metric system of measurement?

Do any countries still use the imperial system?

Make a list of all the things (such as signs, labels or objects) that would need to be changed when the measurement system is changed.

Why learn this?

The ability to accurately estimate and calculate length, surface area and volume is a valuable skill both at home and in the workplace. In the kitchen, cooked food cools quicker when it has a larger surface area. In the bedroom, the volume of your wardrobe is the space available to store all your clothes. In the garden, you may make a path around the perimeter of a pool. Tradespeople such as painters, carpenters, hairdressers, chefs and mechanics require good measurement skills to be successful. Other professionals who need good measurement skills include surveyors, interior designers and engineers.

After completing this chapter you will be able to:

- calculate the perimeter of shapes with straight and curved edges
- calculate the areas of composite shapes made up of triangles, quadrilaterals, circles and parts of circles
- calculate the surface area of prisms and cylinders
- calculate the volume and capacity of prisms and cylinders
- convert units of length, area, volume and capacity.

Recall 4

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 Calculate the following.

(a) 7.21×10

(b) 18.21×1000

(c) $1571.23 \div 100$

(d) $0.74 \div 10\,000$

2 Copy and complete the following conversions.

(a) 4.2 cm = ______ mm

(b) 3.9 m = ______ cm

(c) 7.1 km = ______ m

(d) 450 mm = ______ m

(e) 18.6 cm = ______ m

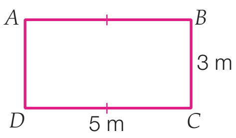
(f) 5060 m = ______ km

(g) 1100 cm = ______ km

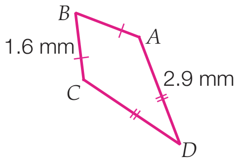
(h) 3720 mm = ______ km

3 What is the length of AB in each of the following diagrams? Where necessary, give your answers to 1 decimal place.

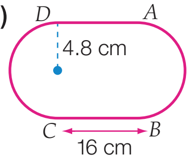
(a)



(b)

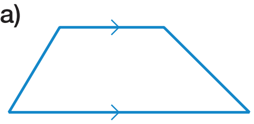


(c)

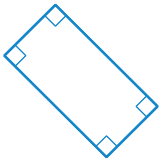


4 Match the shape to its name.

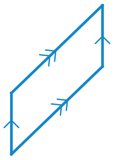
(a)



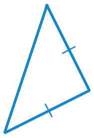
(b)



(c)



(d)



A rectangle

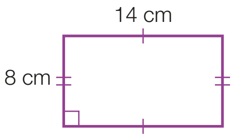
B isosceles triangle

C parallelogram

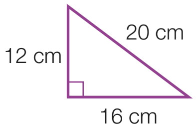
D trapezium

5 Calculate the perimeter and area of each of these shapes to the nearest whole number:

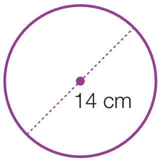
(a)



(b)



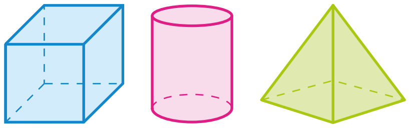
(c)



Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Seeing double



In this activity, you will explore the properties of certain 3D objects and their related nets.

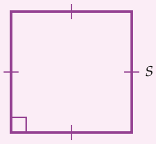
Perimeter

4.1

The **perimeter** of a shape is the total length of its boundary, measured in units such as cm or km. In practical situations you can measure the individual lengths and add them together. If you have a diagram or a well-defined shape, you can use shortcuts (or formulas) where you have some lengths that are equal. Equal lengths are shown by markings such as | and ||.

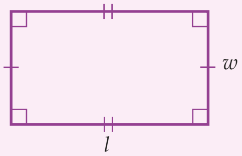
Perimeter formulas

Square



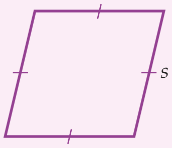
$P = 4s$

Rectangle



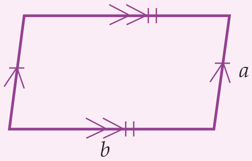
$P = 2l + 2w$ or $P = 2(l + w)$

Rhombus



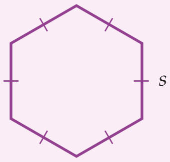
$P = 4s$

Parallelogram



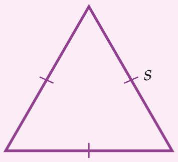
$P = 2a + 2b$ or $P = 2(a + b)$

Regular hexagon



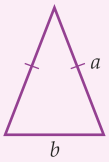
$P = 6s$

Equilateral triangle



$P = 3s$

Isosceles triangle



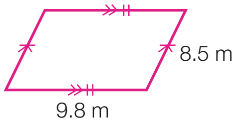
$P = 2a + b$

Worked example 1

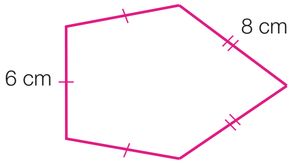
W.E. 1

Find the perimeter of these shapes.

(a)



(b)

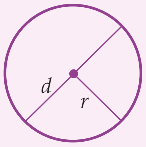


Thinking	Working
(a) **1** Identify the shape and write the formula.	(a) $P = 2(a + b)$
2 Substitute in the values and evaluate.	$P = 2(8.5 + 9.8)$ $= 2 \times 18.3$
3 Write the answer with the given unit of length.	$P = 36.6$ m

(b) 1 Write out the sum of the lengths.	(b) $P = 8 + 8 + 6 + 6 + 6$
2 Simplify if possible.	$= 2 \times 8 + 3 \times 6$
3 Write the answer with the given unit of length.	$= 34$ cm

Circumference of a circle

You should recall that the perimeter of a circle is called the **circumference**. The formula used to calculate the circumference of a circle is $C = 2\pi r$, where r is the radius, or $C = \pi d$, where d is the diameter. The plural of radius is radii.



Circumference of a circle

$C = \pi d$ or $C = 2\pi r$
where d = diameter, and r = radius.

If you know the circumference of a circle, you can use it to find the radius or diameter.

Worked example 2

W.E. 2

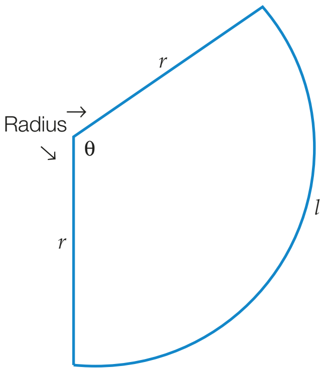
(a) Find the diameter of a circle with a circumference of 8.5 cm, correct to 1 decimal place.

(b) Find the radius of a circle with a circumference of 40 mm, correct to 1 decimal place.

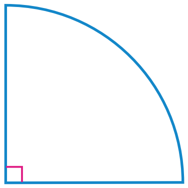
Thinking	Working
(a) 1 Write the circumference formula.	(a) $C = \pi d$
2 Rearrange the formula to make d the subject, by dividing both sides by π.	$\frac{C}{\pi} = d$
3 Substitute the value for C.	$\frac{8.5}{\pi} = d$
4 Calculate, rounding your answer to the specified number of decimal places. Remember to include the unit.	$d = 2.705\ldots$ $d = 2.7$ cm (1 d.p.)
(b) 1 Write the circumference formula.	(b) $C = 2\pi r$
2 Rearrange the formula to make r the subject, by dividing both sides by 2π.	$\frac{C}{2\pi} = r$
3 Substitute the value for C.	$\frac{40}{2\pi} = r$
4 Calculate, rounding your answer to the specified number of decimal places.	$r = 6.366\ldots$ $r = 6.4$ mm (1 d.p.)

Arc length and perimeter of a sector

A **sector** of a circle is the area contained within two radii and the arc that joins them. An **arc** is part of the circumference of a circle. The distance along the arc of a sector (l) is in proportion to the angle **subtended** by (made by) the arc at the centre of the circle.



To calculate the length of the arc, multiply the entire circumference by the fraction represented by the arc. For example, an arc subtending an angle of 90° has a length that is one-quarter the circumference of the circle, because $\frac{90}{360} = \frac{1}{4}$. The arc length is found by calculating $\frac{1}{4} \times 2\pi r$ (which can also be written as $\frac{2\pi r}{4}$ or $\frac{\pi r}{2}$).



Similarly, an arc subtending an angle of 60° has a length that is one-sixth the circumference, because $\frac{60}{360} = \frac{1}{6}$.

An arc subtending an angle of 130° has a length that is $\frac{130}{360}$ or $\frac{13}{36}$ of the circumference.

The arc length l of a sector can be found by using the formula:

$$l = \frac{\theta}{360} \times 2\pi r$$

where θ (the Greek letter theta) is the angle subtended by the arc at the centre of the circle.

To find the perimeter of a sector of a circle, add twice the radius to the length of the arc.

Arc length of a sector

$l = \frac{\theta}{360} \times 2\pi r$, where θ is the angle subtended at the centre.

Perimeter of a sector

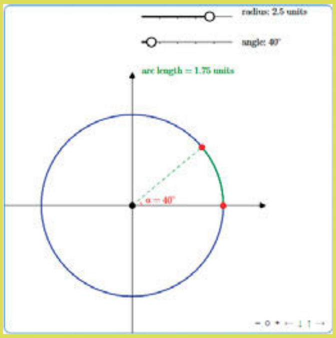
$P = l + 2r$

Interactive

Arc length of a sector

Calculate the arc length of a sector by moving the angle at the centre of the circle.

Go to the eBook or the Pearson Places website to access this interactive.

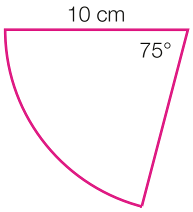


Worked example 3

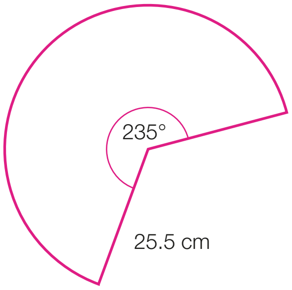
W.E. 3

Calculate **(i)** the arc length and **(ii)** the perimeter of each of the following sectors, to the nearest whole number.

(a)



(b)



Thinking	Working
(a) **(i)** **1** Write the appropriate formula.	(a) (i) $l = \frac{\theta}{360} \times 2\pi r$
2 Write the known values.	$r = 10$ cm, $\theta = 75°$
3 Substitute values into the formula.	$l = \frac{75}{360} \times 2 \times \pi \times 10$
4 Simplify the expression by cancelling common factors and/or multiplying. (Cancel common factors of 20 then 3.)	$= \frac{75}{{}_{18}\cancel{360}} \times {}^{1}\cancel{20} \times \pi$ $= \frac{{}^{25}\cancel{75}\pi}{{}_{6}\cancel{18}}$ $= \frac{25\pi}{6}$
5 Evaluate, writing your answer with the specified accuracy.	$= 13$ cm (nearest whole number)
(ii) **1** Write the appropriate formula.	(ii) $P = l + 2r$
2 Write the known values. (Use the exact value of *l*, rounding at the end of the calculation.)	$r = 10$ cm, $l = \frac{25\pi}{6}$ cm
3 Substitute values and calculate.	$P = 2 \times 10 + \frac{25\pi}{6}$ $= 33$ cm (nearest whole number)
(b) **(i)** **1** Write the appropriate formula.	(b) (i) $l = \frac{\theta}{360} \times 2\pi r$
2 Write the known values.	$r = 25.5$ cm, $\theta = 235°$
3 Substitute values into the formula.	$l = \frac{235}{360} \times 2 \times \pi \times 25.5$

4 Simplify the expression by cancelling obvious common factors and/or multiplying. (Cancel a common factor of 2.)	$= \frac{235}{\cancel{360}_{180}} \times \cancel{2}^{1} \times \pi \times 25.5$ $= \frac{5992.5\pi}{180}$
5 Evaluate, writing your answer with the specified accuracy.	$= 105$ cm (nearest whole number)
(ii) 1 Write the appropriate formula.	(ii) $P = l + 2r$
2 Write the known values. (Use the exact value of l, rounding at the end of the calculation.)	$r = 25.5$ cm, $l = \frac{5992.5\pi}{180}$ cm
3 Substitute values and calculate.	$P = 2 \times 25.5 + \frac{5992.5\pi}{180}$ $= 156$ cm (nearest whole number)

4.1 Perimeter

Navigator

1, 2, 3 (a–c), 4, 5 (a–d), 6, 7, 8 (a–c), 10, 12	1 (c–f), 2, 3 (d–f), 4, 5 (columns 1, 2), 6, 7, 8, 10, 12, 13	1 (e–f), 2, 3 (d–f), 5 (columns 2, 3), 6, 8, 9, 10, 11, 13, 14

Answers p. 644

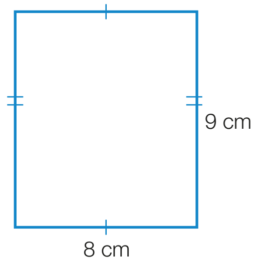
Equipment required: tape measure or ruler for Question **12**

Fluency

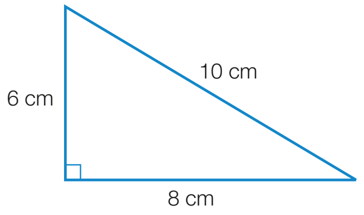
W.E. 1

1 Find the perimeters of the following shapes using the measurements given.

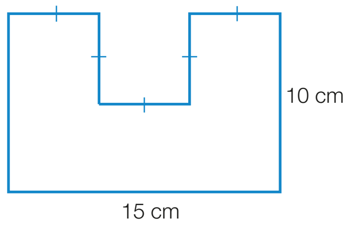
(a)



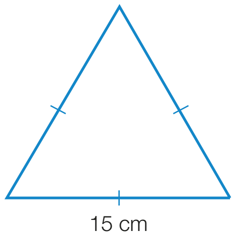
(b)



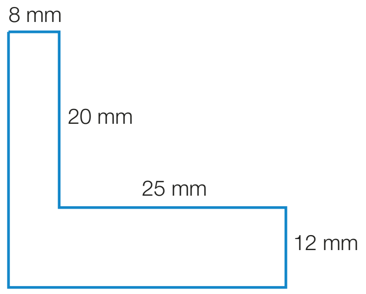
(c)



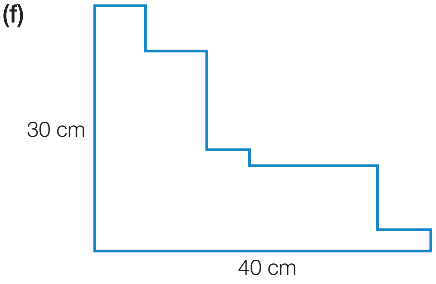
(d)



(e)



(f)



W.E. 2

2 **(a)** Find, correct to 1 decimal place, the diameter of a circle with a circumference of:

(i) 15 m **(ii)** 32 m **(iii)** 141.39 mm **(iv)** 188.52 mm

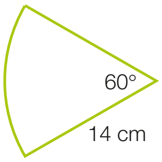
(b) Find, correct to 1 decimal place, the radius of a circle with a circumference of:

(i) 93 km **(ii)** 71 km **(iii)** 804.248 cm **(iv)** 1.2566 cm

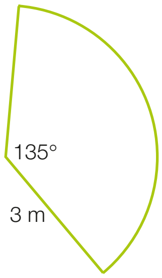
W.E. 3

3 Find **(i)** the arc length and **(ii)** the perimeter of each of the following sectors, to the nearest whole number.

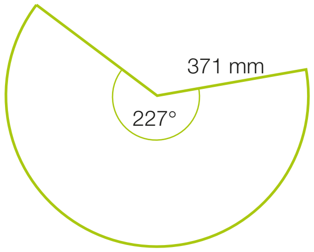
(a)



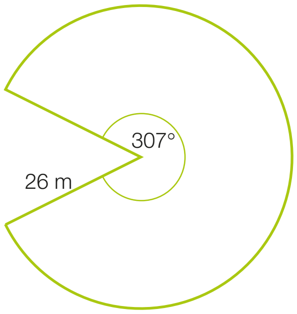
(b)



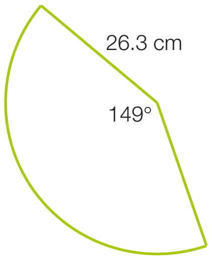
(c)



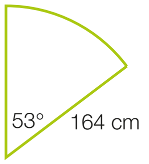
(d)



(e)



(f)



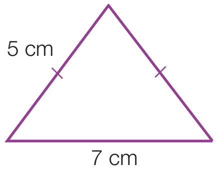
Understanding

4 Choose the correct answer.

The perimeter of the triangle at right is:

A 12 cm **B** 17 cm

C 19 cm **D** 35 cm

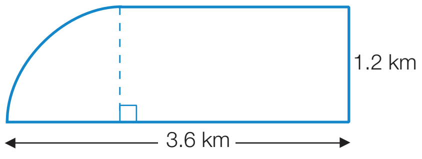


Warning

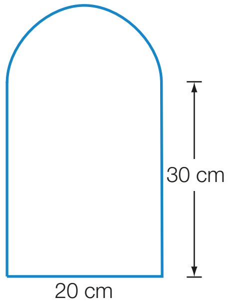
Dotted lines are there only to indicate measurements. Make sure you don't include them in your perimeter calculation.

5 Find the perimeters of these shapes, giving your answers to 1 decimal place where necessary.

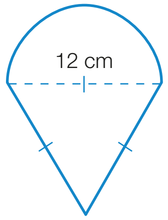
(a)



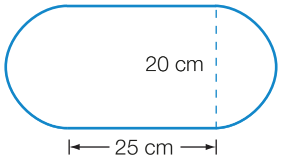
(b)



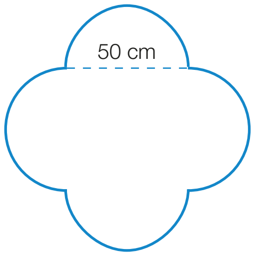
(c)



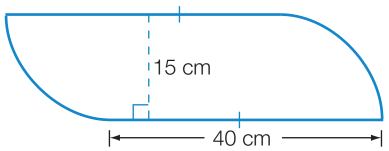
(d)



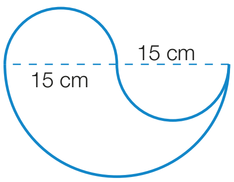
(e)



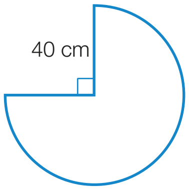
(f)



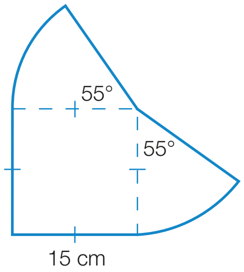
(g)



(h)



(i)



6 A round pizza has a circumference of 1.12 m.

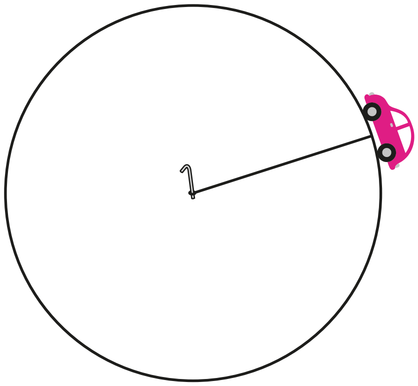
(a) Find the radius of this pizza. Give your answer in cm correct to 2 decimal places.

(b) The pizza is to be cut into 12 equal slices.

- **(i)** What is the angle subtended at the centre of each slice?
- **(ii)** Find the arc length of each slice, giving your answer as an exact value.
- **(iii)** Find the perimeter of each slice, giving your answer correct to 2 decimal places.

7 Find the circumference of Earth if the diameter, to the nearest 100 km, is 12 800 km.
Give your answer correct to the nearest 100 km.

8 Andrea attached her toy car to a peg using a string 36 cm long. The car would travel in a circular path when it is turned on as shown in the diagram.



(a) What is the radius of the car's circular path?

(b) Calculate the distance the car has travelled after completing one circular path. (Answer to the nearest cm.)

(c) If it is travelling at a speed of 20 cm per second, using your answer from part **(b)**, calculate the time it will take to complete this circular path.

(d) Andrea decided to extend the string so the car would have to travel a distance of 3.5 metres to complete one circular path.

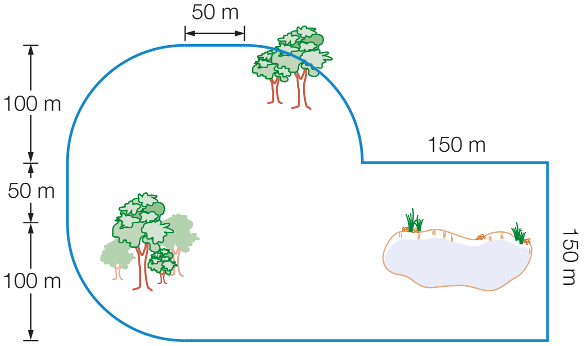
- **(i)** What is the circumference, in cm, of this path?
- **(ii)** Calculate the radius of the new circular path, giving your answer correct to 1 decimal place.
- **(iii)** How much longer is the string in cm?

Hint

Drawing a diagram can help you 'picture' a problem.

9 Every night Nicole walks her dog twice around the park. The plan of the park is shown below.

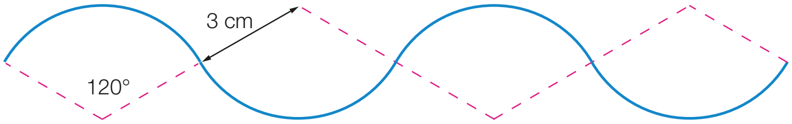
How far does she walk in km? (Answer to the nearest metre.)



Reasoning

10 If a plane travels halfway round the world at an altitude of 11 300 m, what extra distance, in km, will it fly compared with the distance measured at the surface of the Earth? (The radius of the Earth is approximately 6400 km.) Round your answer to 1 decimal place.

11 A sheet of corrugated iron has corrugations that are based on the arc of a circle of radius 3 cm and a central angle of 120°. Four corrugations are shown below.



How many corrugations are formed when a flat sheet of steel 1 m wide is bent to form a sheet of corrugated iron?

Open-ended

12 Estimate first, then use a tape measure or ruler to measure the perimeter of:

(a) the front cover of this book

(b) the top of your desk

(c) the whiteboard in your room

(d) the floor of the classroom.

13 Find whole number diameters of three different circles whose circumferences are between 50 cm and 90 cm.

14 Sketch and label a shape with a boundary containing straight as well as curved sections whose perimeter is between 30 and 40 cm.

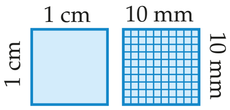
Area

4.2

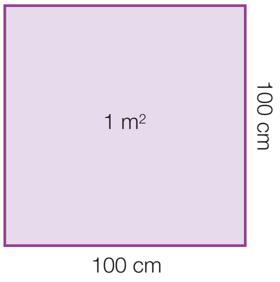
Converting units of area

Area is the amount of surface within a boundary of a two-dimensional shape. Area is measured in 'square' units; that is, square millimetres (mm^2), square centimetres (cm^2), square metres (m^2), hectares (ha) or square kilometres (km^2).

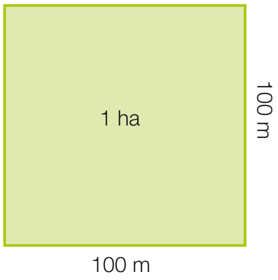
It is important to remember that converting between units of area is *not* the same as converting between units of length. Because the units are square units, the conversion factor is the *square* of the conversion factor for length. (The conversion factor is the number you multiply or divide by when converting one unit to another.)



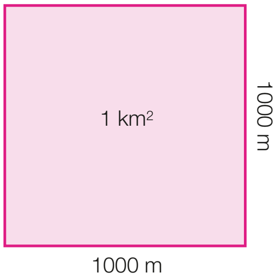
$$\begin{aligned} 1\ cm^2 &= 10\ mm \times 10\ mm \\ &= 100\ mm^2 \end{aligned}$$



$$\begin{aligned} 1\ m^2 &= 100\ cm \times 100\ cm \\ &= 10\,000\ cm^2 \end{aligned}$$

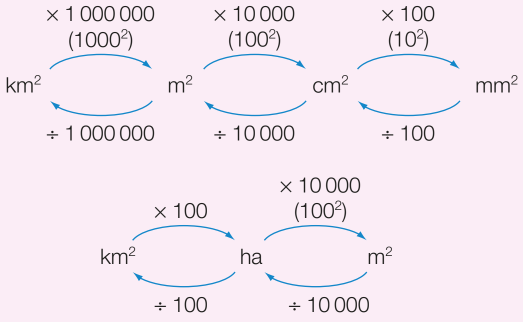


$$\begin{aligned} 1\ ha &= 100\ m \times 100\ m \\ &= 10\,000\ m^2 \end{aligned}$$



$$\begin{aligned} 1\ km^2 &= 1000\ m \times 1000\ m \\ &= 1\,000\,000\ m^2 \end{aligned}$$

Converting units of area:



A hectare (ha) is a square 100 metres by 100 metres ($10\,000\ m^2$). It is a handy unit for measuring land, such as the area of a farm or park.

Worked example 4

W.E. 4

Copy and complete the following conversions.

(a) 17.4 ha = _____ m^2 **(b)** 20 000 m^2 = _____ km^2

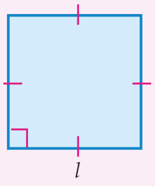
Thinking	Working
(a) You are converting from a larger unit to a smaller unit, so you need to multiply. There are 100 m^2 in every ha, so multiply by 100^2 to convert ha to m^2.	(a) 17.4×100^2 $= 174\,000\ m^2$
(b) You are converting from a smaller unit to a larger unit, so you need to divide. There are 1000 m^2 in every km^2, so divide by 1000^2.	(b) $20\,000 \div 1000^2$ $= 0.02\ km^2$

If you are asked to calculate the area of a shape in different units to the ones given, it is often easier to convert the dimensions of the shape into the required units of length first, then calculate the area.

Area formulas

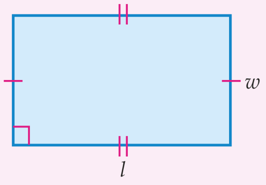
You should be familiar with the formulas for the following shapes.

Square



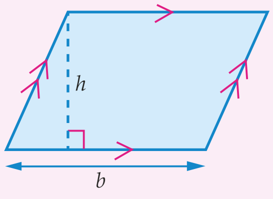
$A = l^2$

Rectangle



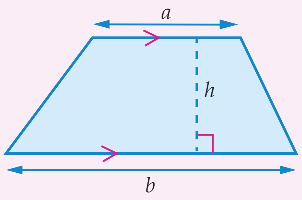
$A = lw$

Parallelogram



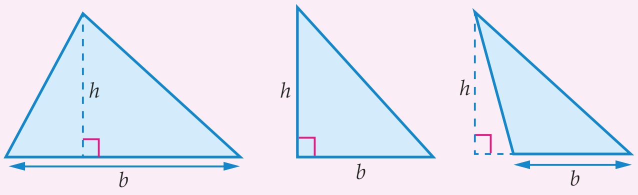
$A = bh$

Trapezium



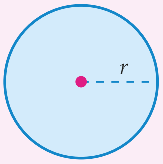
$A = \frac{(a + b)h}{2}$

Triangle



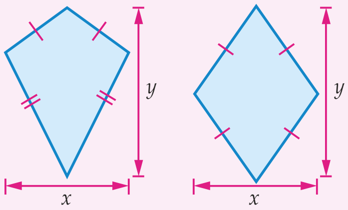
$A = \frac{bh}{2}$

Circle



$A = \pi r^2$

Kite and rhombus



$$A = \frac{xy}{2}$$

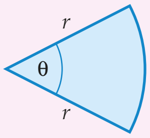
Area of a sector

The area of a sector is proportional to the angle of the sector at the centre of the circle. For example, a semicircle subtends an angle of 180° at the centre of the circle and its area is half the area of the full circle, because $\frac{180°}{360°} = \frac{1}{2}$.

Area of a sector

$$A = \frac{\theta}{360} \times \pi r^2$$

where θ is the angle subtended at the centre.

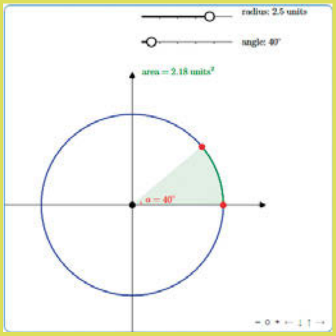


Interactive

Area of a sector

Calculate the area of a sector by moving the angle at the centre of the circle.

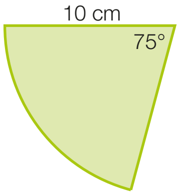
Go to the eBook or the Pearson Places website to access this interactive.



Worked example 5

W.E. 5

Find the area of this sector, correct to the nearest cm^2.



Thinking	Working
1 Write the appropriate formula.	$A = \frac{\theta}{360} \times \pi r^2$
2 Write the known values.	$r = 10$ cm, $\theta = 75°$

3 Substitute values.	$A = \frac{75}{360} \times \pi \times 10^2$
4 Simplify the expression by cancelling common factors and/or multiplying. (Divide by a common factor of 10, then 3.)	$= \frac{\overset{25}{\cancel{75}}}{\underset{12}{\cancel{360}}} \times \pi \times 10 \times \overset{1}{\cancel{10}}$ $= \frac{250\pi}{12}$
5 Calculate, rounding appropriately.	$= 65$ cm^2 (nearest whole number)

Composite shapes

Composite shapes are made up of different combinations of basic shapes. To find the area of composite shapes, break them up into simpler shapes, calculate the individual areas of the simpler shapes and add them together. For some shapes you can find the area remaining by subtracting the area of a piece that is not included from the area of the overall shape.

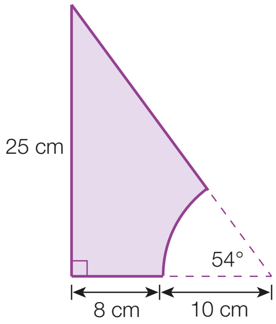
Worked example 6

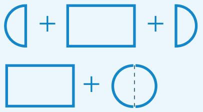
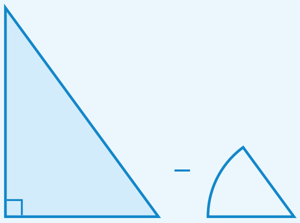
W.E. 6

Find the shaded area of each shape. Write your answers to the nearest whole number.

(a)

(b)



Thinking	Working
(a) 1 Break the composite shape into parts, combining parts to simplify if convenient.	(a) Area = 
2 Write the appropriate area formula for each shape and define the variables.	$A = lw + \pi r^2$, where $l = 15$ cm, $w = 8$ cm and $r = 4$ cm
3 Substitute the given measurements and simplify the expression.	$= 15 \times 8 + \pi \times 4^2$ $= 120 + 16\pi$
4 Evaluate, rounding your answer as necessary and writing it with the correct units.	$= 170$ cm^2 (nearest whole number)
(b) 1 Break the composite shape into parts.	(b) Area = 

2 Write the appropriate area formula for each shape and define the variables.	$A = \frac{bh}{2} - \left(\frac{\theta}{360} \times \pi r^2\right)$, where $b = 18$ cm, $r = 10$ cm, $\theta = 54°$ and $h = 25$ cm
3 Substitute the given measurements and simplify the expression by cancelling obvious factors. (Cancel common factors of 2 and 10.)	$= \frac{\overset{9}{\cancel{18}} \times 25}{\cancel{2}} - \left(\frac{\overset{27}{\cancel{54}}}{\underset{18}{\cancel{360}}} \times \pi \times 10^{\overset{1}{\cancel{2}}}\right)$ $= 225 - \left(\frac{270}{18}\pi\right)$
4 Evaluate, rounding your answer as necessary and writing it with the correct units.	$= 178$ cm^2 (nearest whole number)

Rounding and accuracy

You should note in the above examples that calculations are simplified before the actual calculation is performed. (Simplifying involves cancelling common factors and/or performing simple whole number multiplications.) But the answers are only rounded once, at the very end. By not rounding until the end, the most accurate final answer is found. If you use rounded answers as you work through a problem, then this makes a small amount of error each time, which makes an inaccurate final answer.

To use your calculator efficiently for these calculations, you will need to apply the correct order of operations using the bracket keys (**(** and **)**). For precise answers, you must enter the complete calculation as one equation, or else recall exact values of previous answers (usually by using the answer key **ANS**, but this may vary depending on the type of calculator).

4.2 Area

Navigator

1 (column 1), 2 (column 1), 3 (a–f), 4, 5, 6, 7, 8, 10, 12, 18, 19, 22	1 (column 2), 2 (column 2), 3 (d–i), 4, 5, 6, 7, 8, 10, 12, 13, 15, 17, 19, 21, 22	1 (column 2), 2 (column 3), 3 (g–l), 5, 6 (b, d), 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21

Answers p. 644

Fluency

W.E. 4

1 Copy and complete the following conversions.

(a) 3 m^2 = _____ cm^2

(b) 0.172 m^2 = _____ cm^2

(c) 72.5 cm^2 = _____ mm^2

(d) 0.6 cm^2 = _____ mm^2

(e) 9 ha = _____ m^2

(f) 4.07 ha = _____ m^2

(g) 35 m^2 = _____ ha

(h) 695 mm^2 = _____ cm^2

(i) 127 cm^2 = _____ m^2

(j) 835 cm^2 = _____ m^2

(k) 43.8 ha = _____ m^2

(l) 6850 m^2 = _____ ha

(m) 290 cm^2 = _____ m^2

(n) 39.6 cm^2 = _____ mm^2

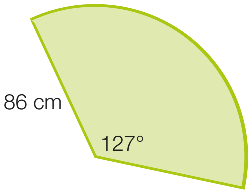
(o) 15 000 000 m^2 = _____ km^2

(p) 10 080 ha = _____ km^2

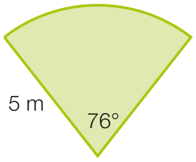
W.E. 5

2 Calculate the area of each of these sectors correct to 1 decimal place.

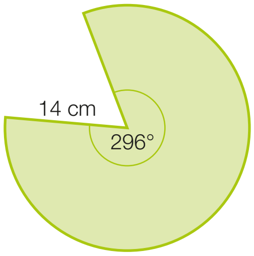
(a)



(b)



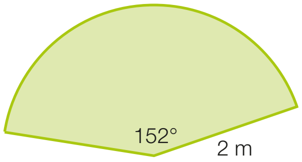
(c)



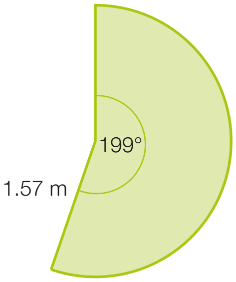
Warning

Do not round any answers until *after* you have added or subtracted the areas. Otherwise your result may be less accurate.

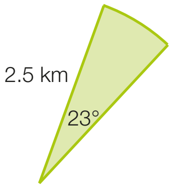
(d)



(e)



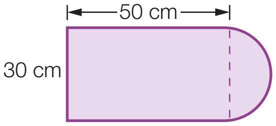
(f)



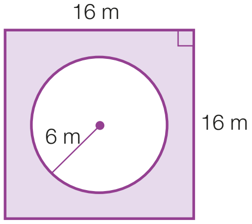
W.E. 6

3 Find the shaded area of each of the following shapes. Write your answers correct to the nearest whole number.

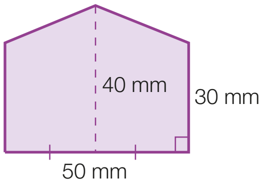
(a)



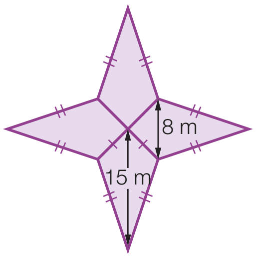
(b)



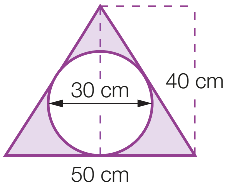
(c)



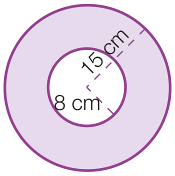
(d)



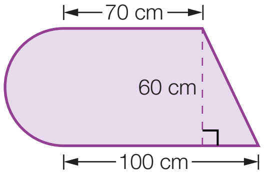
(e)



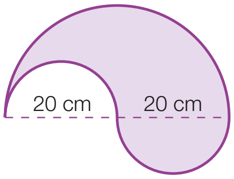
(f)



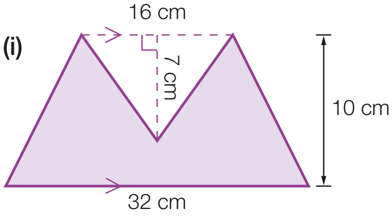
(g)



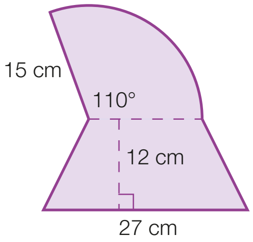
(h)



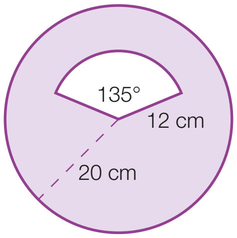
(i)



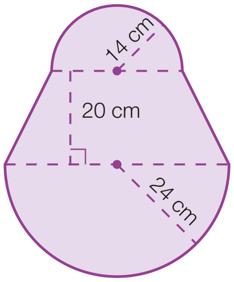
(j)



(k)

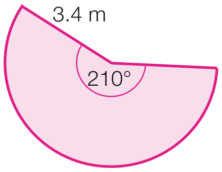


(l)



4 Choose the correct answer.

The area of the sector shown on the right is closest to:



A 12.5 m^2

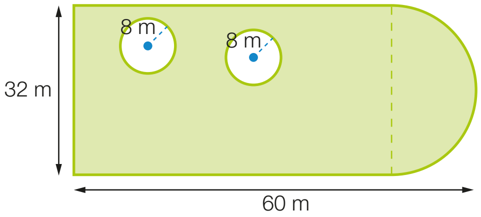
B 21.1 m^2

C 21.2 m^2

D 36.3 m^2

Understanding

5 A garden lawn has two ponds within its boundary, as shown.



(a) Find the exact area of each pond.

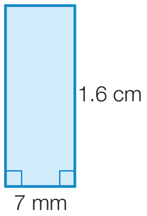
(b) Find the area of the lawn.

(c) How much seed is needed to grass the lawn if 1 kg of seed covers 8 m^2?

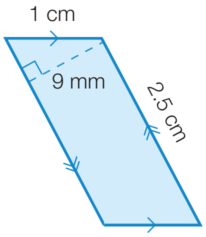
6 Find the area of each shape, correct to 2 decimal places where necessary, in:

(i) square millimetres **(ii)** square centimetres.

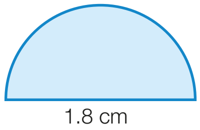
(a)



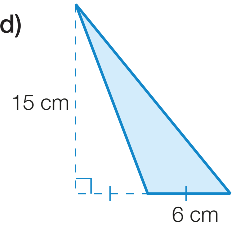
(b)



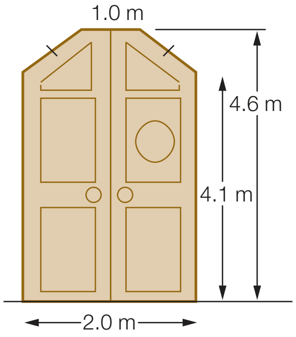
(c)



(d)



7 The doors at a local hall are shaped as shown.



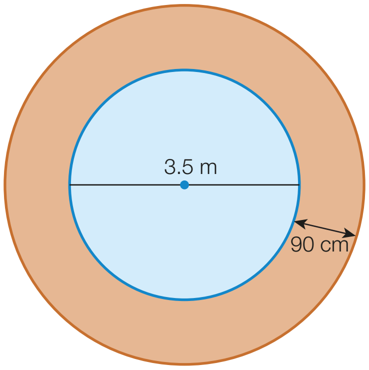
(a) What are the two shapes that can be used to calculate the area of the doors?

(b) Draw these two shapes, label all the necessary dimensions and hence calculate the area of each shape.

(c) What is the exact area of each pair of doors?

8 An annulus is the area between two circles that have the same centre but different radii.

A circular pond 3.5 m in diameter is surrounded by a path 90 cm wide. *Note*: The shape of the path is an annulus.



(a) Calculate the area, in cm^2, of the pond in terms of π.

(b) What is the radius of the outer circle?

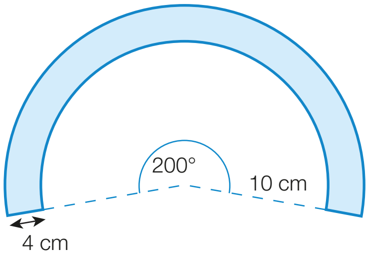
(c) Calculate the area of the path. Give your answer to the nearest cm^2.

The path is to be paved with small rectangular bricks that are 20 cm long and 10 cm wide.

(d) Calculate the area, in cm^2, of each rectangular brick.

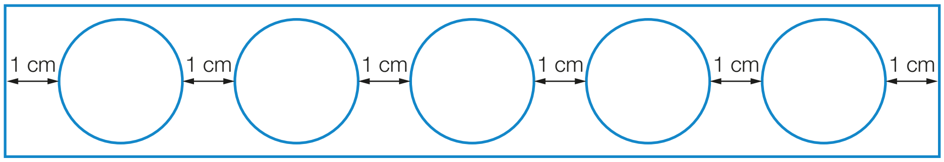
(e) Assuming that all the small bricks and their off-cuts can be used to pave the path, calculate the minimum number of bricks required to pave the path. Round your answer to the nearest number of bricks.

9 Nicola has made a collar from a sector of an annulus, as shown in the diagram. What area of fabric has been used for the collar? Give your answer correct to 1 decimal place.



10 Circular plastic drink coasters 8 cm in diameter are manufactured by feeding a rectangular strip of plastic 9 cm wide into a machine that cuts out the coasters. The machine starts 1 cm from one end of the strip, leaves 1 cm between coasters and 1 cm on the other end of the strip.

A strip which cuts out five coasters is shown below (not drawn to scale).



For this machine, answer the following questions:

(a) How long would a strip of plastic have to be to cut out 5 coasters?

(b) How long would a strip of plastic have to be to cut out 50 coasters?

(c) What would be the total area of 50 coasters? Give your answer in terms of π.

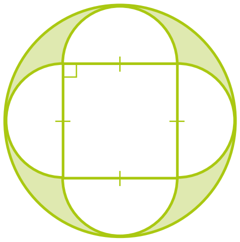
(d) What would be the total area of wasted plastic (to the nearest cm^2)?

(e) What percentage of the strip is used for making coasters? Give your answer to the nearest whole number.

Hint

How many of the radii of the smaller circle make up the side length of the square.

11 An architect is designing a decorative circular courtyard 22 m in diameter. The shaded area is to be a garden and the rest will be pavement.

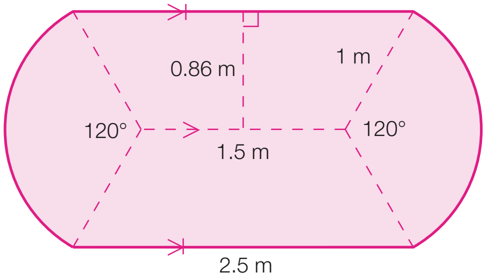


(a) Find the area of the pavement, correct to 1 decimal place.

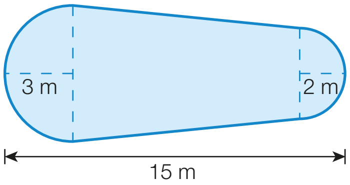
(b) Find the area of the garden, correct to 1 decimal place.

(c) What percentage of the courtyard will be pavement? Give your answer to the nearest whole number.

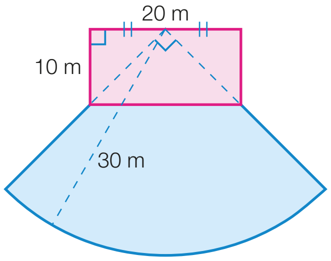
12 Norbert's dining table is the shape shown. Calculate its area, giving your answer correct to 2 decimal places.



13 Jennifer has a rectangular lawn at the back of her house that measures 20 m by 18 m. She is putting in a pond with the shape shown in the diagram. What area of lawn will she have left if the grass is to go all the way to the edge of the pond? (Give your answer correct to 1 decimal place.)



14 A new theatre is to be built. The design of the theatre includes a rectangular stage area 20 m × 10 m with a portion of a quadrant of a circle of radius 30 m for the audience to be seated in, as shown in the following diagram. Calculate the total area of the theatre to the nearest whole number.



Reasoning

15 How many rectangular house blocks 16 m × 20 m would fit in a 15 ha subdivision for a housing estate, assuming 20% of the area of the subdivision is used for roads?

16 A square has diagonals that are 20 cm long.

(a) Remembering that the diagonals of a square bisect each other at right angles, find the area of the square.

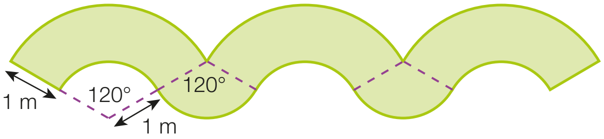
(b) Calculate the length of the sides of the square from the area you found, correct to 2 decimal places.

Hint

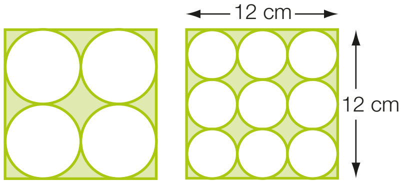
Use Pythagoras' theorem.

17 The area of a sector of a circle is 1000 cm^2. The radius of the sector is 30 cm. Calculate the size of the angle at the centre of the sector, correct to the nearest degree.

18 Part of the front of a shop awning consists of three sectors of an annulus and two sectors of a circle, as shown in the diagram. Calculate the total area of this section. (Give your answer correct to 1 decimal place.)



19 Touching circles are drawn inside two squares with side length 12 cm. Four identical circles (called a 4-Pak) just fit into the first square and nine (called a 9-Pak) fit into the second.



(a) What is the radius of each circle in:
(i) the 4-Pak **(ii)** the 9-Pak?

(b) Which of the shaded background areas is bigger?

20 A trapezium with an area of 900 cm^2 has parallel sides 40 cm apart. If one of these sides is twice as long as the other, the lengths of the parallel sides would be:

A 20 cm, 10 cm **B** 24 cm, 12 cm **C** 30 cm, 15 cm **D** 40 cm, 20 cm

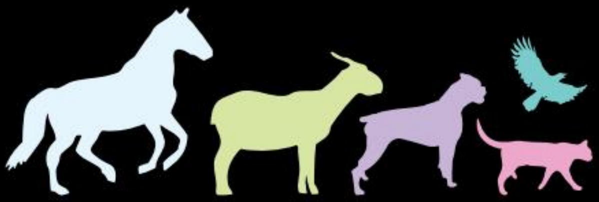
Open-ended

21 Steve's Pizza shop sells circular pizzas that are 14 cm in radius. Steve decides to start making rectangular instead of circular pizzas. He uses approximately the same amount of ingredients and makes the base the same thickness. Assuming that the length and width of the rectangular pizzas are whole number values in centimetres, write at least three sets of dimensions that are practical.

22 The perimeter of a rectangle is 60 cm. Calculate two possible areas for the rectangle.

Maths 4 Real

Veterinary maths



The job of a vet involves treating and performing surgery on sick animals. Desexing and vaccinations are the two most common procedures. Dogs are the most common animals dealt with, along with wombats, koalas, lizards, horses, cows, cats, sheep, birds, pigs and rats.

When an animal becomes dehydrated the amount of water to be replaced (the deficit) is estimated as a percentage of the body weight.

For example, a 35 kg calf with 10% dehydration has a water deficit of 3.5 kg (10% of 35 kg).

1 Find the deficit for an 80 kg Great Dane that is 5% dehydrated.

Each litre of water has a mass of 1 kg. For the calf, 3.5 kg of water is the same as 3.5 L, or 3500 mL.

2 Convert the deficit found in 1 to the number of mL needed for the Great Dane.

Animals that are severely dehydrated require additional fluid of 60 mL per day for each kilogram of body mass. For the calf, this would be: 60 mL × 35 = 2100 mL of additional fluid per day.

3 Find the additional fluid needed per day for the Great Dane.

The total amount of fluid in the first day of treatment is the total of the deficit and the additional fluid. For the calf, this would be 3500 mL + 2100 mL = 5600 mL.

4 Find the total amount of fluid needed in the first day for the Great Dane.

The fluid is given via an intravenous drip, for which 10 drops = 1 mL. For the calf, the number of drops needed in a day is 10 drops × 5600 = 56 000 drops.

5 Find the number of drops needed in a day for the Great Dane.

The drip needs to be set at a rate involving seconds.

The number of seconds in a day is:
$24 \times 60 \times 60 = 86\,400$ s.

The flow rate for the calf would be:

$$\frac{56\,000 \text{ drops}}{86\,400 \text{ s}} = 0.65 \text{ drops per second.}$$

6 Find the number of drops per second for the Great Dane.

7 Repeat 1 to 6 for a 12 kg Kelpie that is 10% dehydrated.

Research

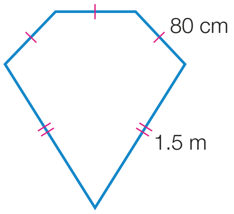
- Find out some of the common causes of dehydration in animals.
- When a dehydrated cat is brought into a surgery, water is sometimes injected directly under the skin, where it is quickly absorbed. Find out why a drip is used in some cases and not others.

Half-time 4

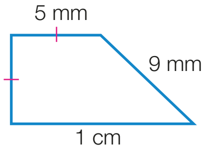
4.1

1 Find the perimeter of each shape, in the smaller units if two are given, rounding your answer to the nearest whole number.

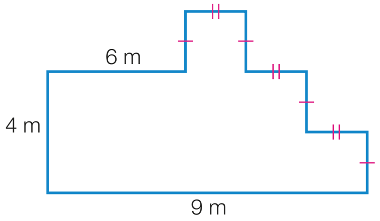
(a)



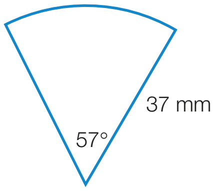
(b)



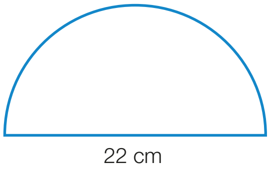
(c)



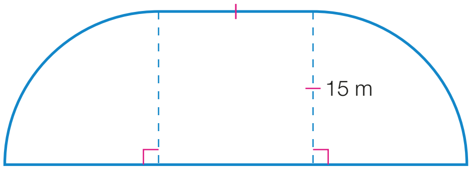
(d)



(e)



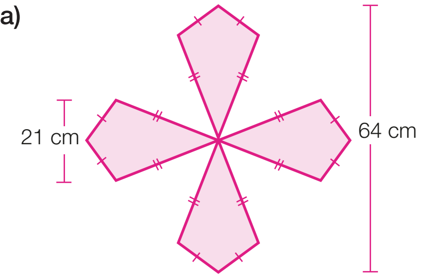
(f)



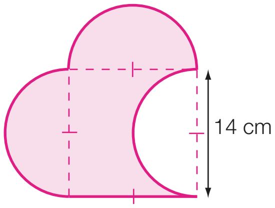
4.2

2 Find the shaded area, correct to the nearest whole number.

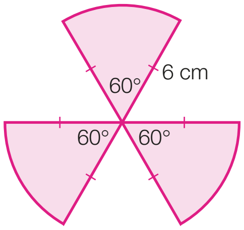
(a)



(b)



(c)



4.1

3 Calculate the distance along the surface of Earth (to the nearest 100 km) from the Equator to the South Pole given that the diameter of Earth is approximately 12 800 km.

4.2

4 A trapezium has an area of 600 cm^2. Its parallel sides are 15 cm and 25 cm long. Find the distance between the parallel sides.

4.2

5 Copy and complete the following conversions.

(a) 4 cm^2 = ______ mm^2

(b) 7.5 m^2 = ______ cm^2

(c) 0.26 km^2 = ______ m^2

(d) 19.8 ha = ______ m^2

(e) 3.05 km^2 = ______ ha

(f) 274 cm^2 = ______ m^2

4.2

6 The perimeter of a right-angled triangle is 90 cm. Two of the sides are 15 cm and 39 cm long. Find the area of the triangle.

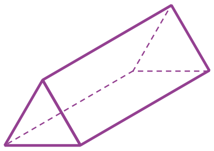
Surface area

4.3

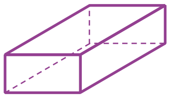
A **polyhedron** is a solid with flat faces and straight edges. The **surface area** (SA) of a polyhedron is the total area of its faces.

To find the surface area of a polyhedron, find the area of each face, then add them together.

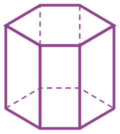
A **prism** is a three-dimensional object that has a uniform **cross-section** in the shape of a polygon. Prisms are named according to the cross-sectional shape. For example:



triangular prism



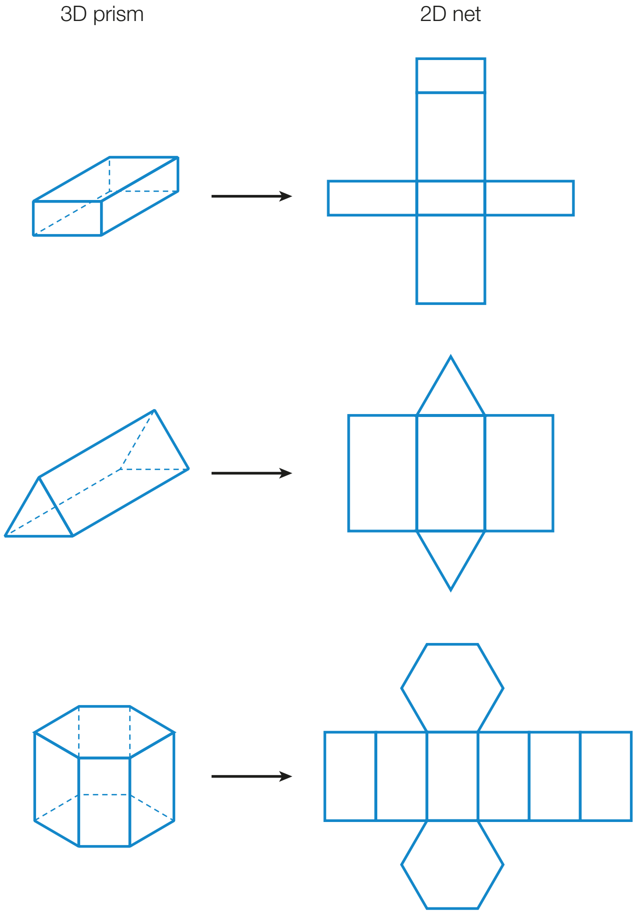
rectangular prism



hexagonal prism

Nets

To calculate surface area, it is helpful to consider the **net** of the object. The net is a two-dimensional (2D) representation of the three-dimensional (3D) prism. If the object is imagined as a box, the net can be created by cutting along the edges and laying the box out flat.



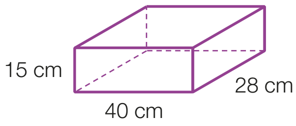
Drawing the net of a prism makes it easier to identify each of the faces and find each area. Nets of prisms are explored in more detail in section 6.7 'Solids and nets' (page 392).

Worked example 7

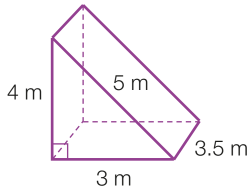
W.E. 7

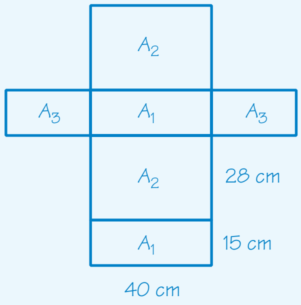
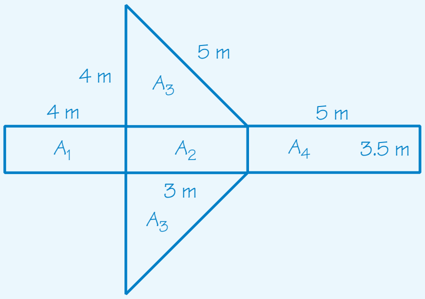
Draw the nets of the following polyhedra and use them to calculate their surface area.

(a)



(b)

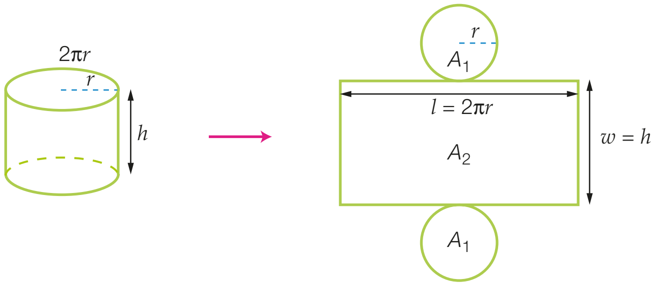


Thinking	Working
(a) **1** To draw the net, imagine cutting along the edges of the prism and 'flattening' it out. Draw each face as a flat shape. Check that your net has the same number of faces as the solid. Label each of the different areas A_1, A_2, A_3 … etc. Label identical faces in the same way.	(a) 
2 Write a formula that shows the surface area (SA) as the total of the individual areas.	$SA = 2A_1 + 2A_2 + 2A_3$ $SA = 2l_1w_1 + 2l_2w_2 + 2l_3w_3$
3 Substitute the values of the different dimensions and evaluate each different area.	$= 2 \times (40 \times 15) + 2 \times (40 \times 28) + 2 \times (28 \times 15)$ $= 1200 + 2240 + 840$
4 Find the total area, writing your answer with area units.	$= 4280 \text{ cm}^2$
(b) **1** To draw the net, imagine cutting along the edges of the prism and 'flattening' it out. Draw each face as a flat shape. Check that your net has the same number of faces as the solid. Label each of the different areas A_1, A_2, A_3 … etc. Label identical faces in the same way.	(b) 
2 Write a formula that shows the surface area (SA) as the total of the individual areas.	$SA = A_1 + A_2 + 2A_3 + A_4$ $= l_1w_1 + l_2w_2 + 2\frac{bh}{2} + l_3w_3$
3 Substitute the values of the different dimensions and evaluate each different area.	$= (3.5 \times 4) + (3 \times 3.5) + 2 \times \left(\frac{3 \times 4}{2}\right) + (5 \times 3.5)$ $= 14 + 10.5 + 12 + 17.5$
4 Find the total area, writing your answer with area units.	$= 54 \text{ cm}^2$

Note that in the above example, subscripts are used to indicate the dimensions of a shape if there is more than one of a particular type of shape. For example, the area of the rectangle labelled A_1 is written as l_1w_1, whereas that of rectangle A_2 is written as l_2w_2.

Cylinders

Although a cylinder is not strictly a prism (because its surfaces are not polygons), you can find the surface area of a cylinder in a similar way to a prism; that is, by drawing the net and calculating the area of each surface. A cylinder has three faces: a circle on each end and the curved surface in between them. If you cut this curved surface from top to bottom and flatten it out, you get a rectangle with a length equal to the circumference of the end circles and a width equal to the height of the cylinder.



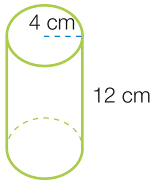
To find the surface area, first calculate and then add the areas of these three surfaces:

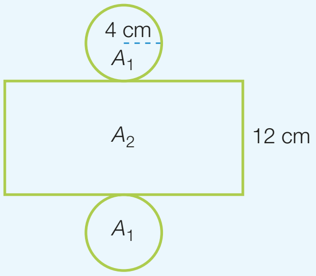
$$\begin{aligned} \text{SA} &= 2 \times A_1 + A_2 \\ &= 2 \times \pi r^2 + l \times w \\ &= 2\pi r^2 + 2\pi rh \end{aligned}$$

Worked example 8

W.E. 8

Find the surface area of this cylinder, correct to 1 decimal place.



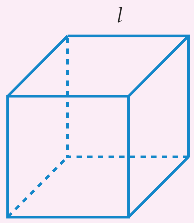
Thinking	Working
1 Draw the net of the cylinder. Label the two identical surfaces A_1 and the flattened curved surface A_2.	
2 Write a formula that shows the surface area (SA) as the total of the individual areas.	$\text{SA} = 2A_1 + A_2$ $= 2\pi r^2 + 2\pi rh$
3 Substitute the values of the dimensions and evaluate each different area.	$= 2 \times \pi \times 4^2 + 2 \times \pi \times 4 \times 12$ $= 32\pi + 96\pi$ $= 128\pi$
4 Find the total area, writing your answer to the specified number of decimal places, and with the correct units of area.	$= 402.1\ \text{cm}^2$ (1 d.p.)

Formulas for the surface area of prisms and regular solids

Although the surface area of a prism can be determined by drawing its net and calculating the area of each surface, it is sometimes useful to create and use a formula for some common types of prisms.

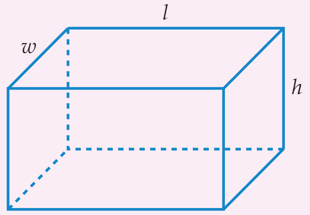
The net of a cube consists of six identical squares.

The surface area of a cube = $6l^2$, where l is the side length.



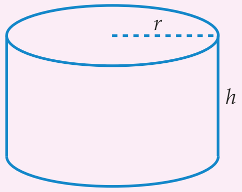
The net of a rectangular prism consists of three pairs of identical rectangles.

The surface area of a rectangular prism = $2lw + 2lh + 2wh$
= $2(lw + lh + wh)$,
where l, w and h are the three side lengths.



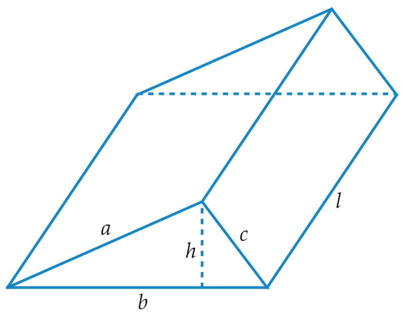
The net of a cylinder consists of two identical circles and a rectangle that has the circumference of the circular end as one of its dimensions.

The surface area of a cylinder = $2\pi r^2 + 2\pi rh$, where r is the radius of the circular face and h is the height of the cylinder.



For other prisms and regular solids, it is best to just use the net because there are too many variables to make a formula worthwhile.

For example, the formula for a triangular prism would require you to find $2 \times \frac{1}{2} \times bh + cl + al + bl$, which would be just as easy to do from the net itself.



4.3 Surface area

Navigator

1 (a–f), 2, 3, 4, 5, 6, 7, 8, 9, 11, 12	1 (columns 1, 2), 2 (a, c), 3, 4, 5, 6, 7, 8, 9, 11, 12, 13	1 (d–i), 2 (b, d), 3, 4 (b), 5 (c), 6, 7 (b), 8, 9, 10, 11, 12, 13

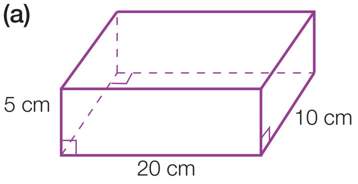
Answers p. 645

Fluency

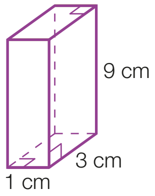
W.E. 7

1 Draw nets of the following polyhedra and use them to calculate their surface area.

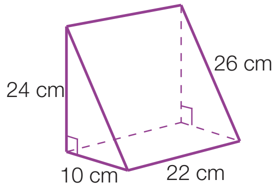
(a)



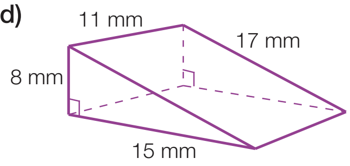
(b)



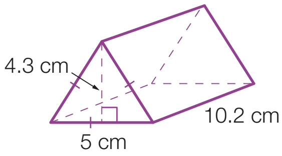
(c)



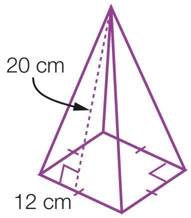
(d)



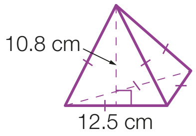
(e)



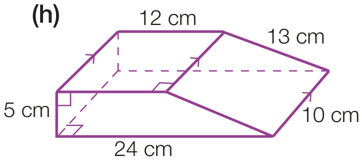
(f)



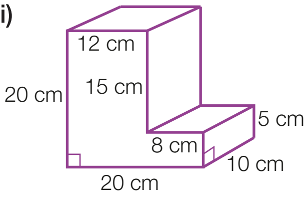
(g)



(h)



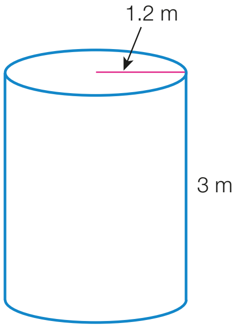
(i)



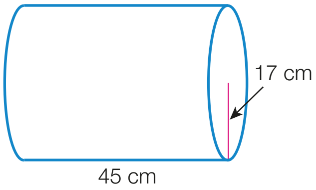
W.E. 8

2 Find the surface area of the following cylinders, correct to 1 decimal place.

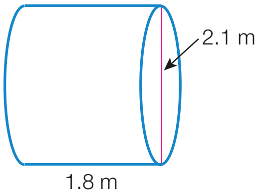
(a)



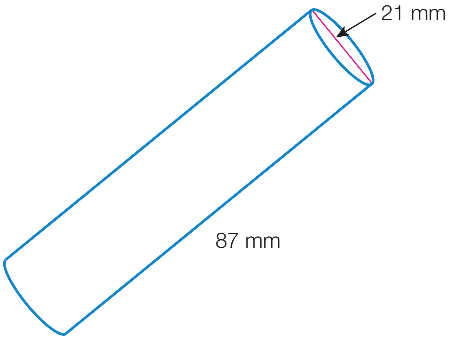
(b)



(c)



(d)



3 (a) The surface area of a cube with edge length 10 cm is:

A 200 cm^2 B 400 cm^2 C 600 cm^2 D 800 cm^2

(b) The surface area of a rectangular box with dimensions 40 cm, 30 cm and 20 cm is:

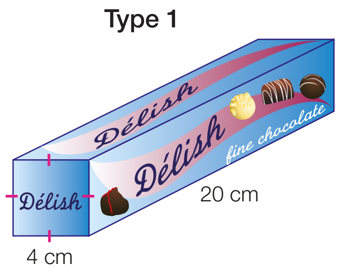
A 1800 cm^2 B 2400 cm^2 C 2600 cm^2 D 5200 cm^2

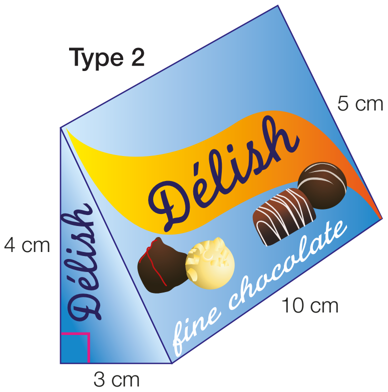
Understanding

4 Joshua has to design a box for a new brand of chocolate as shown.

(a) Find the surface area, in cm^2, of each box.

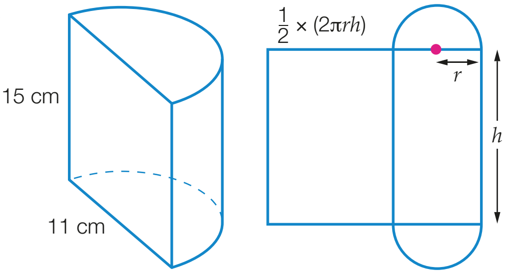
(b) Assuming there is no overlap, which of these boxes uses less cardboard?



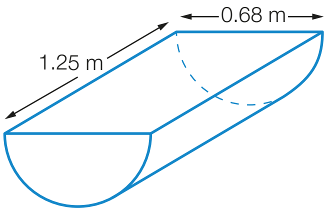


5 Draw nets of the following solids and use them to calculate their total surface area, correct to 2 decimal places. The net for part **(a)** is already shown.

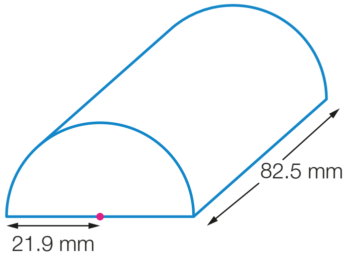
(a)



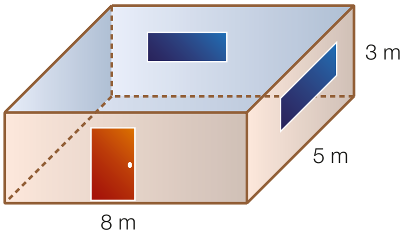
(b)



(c)



6 Leonardo has been hired to paint the walls and ceiling of a room, as shown in the diagram. The room has dimensions 8 m × 5 m × 3 m. It has two windows, each 2 m × 1.5 m, and a door 0.9 m × 2.3 m. Find the area to be painted (excluding the windows and door).



7 Paper labels are made to wrap around the curved surface of cans of food. A 1 cm overlap is allowed at the end so the label can be glued down. For each of the cans complete the following tasks.

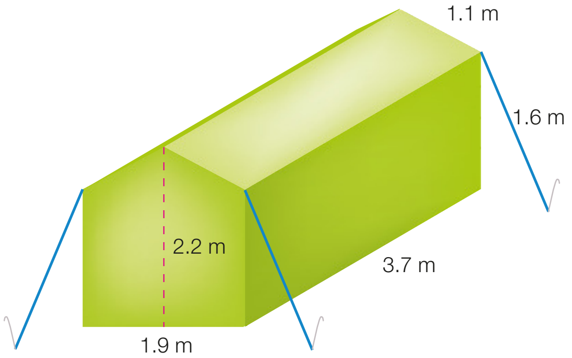
  (i) Draw a flattened curved surface of the paper label, showing all dimensions to 2 decimal places where necessary, including the 1 cm overlap.

  (ii) Calculate the area of paper required to label each of the following cans, to the nearest cm^2.

(a)

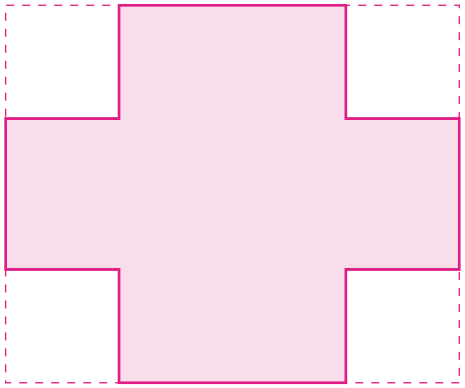
(b)

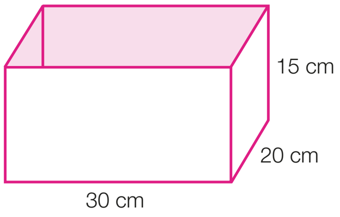
8 Calculate, to the nearest whole number, the area of the material used to make this tent, including the floor. (Assume that there is no overlap at the edges.)



Reasoning

9 The rectangular piece of cardboard on the left has four squares cut from the corners to make an open box whose dimensions are 30 cm × 20 cm × 15 cm.





(a) Calculate the total surface area of the outside of the box.

(b) Use the dimensions of the box (30 cm, 20 cm and 15 cm) to label the diagram of the rectangular piece of cardboard.

(c) What was the length and width of the original piece of cardboard?

(d) Calculate the area of the original piece of cardboard before the four squares were cut from the corners.

(e) Calculate the wasted area (the four squares cut from the corners) of the cardboard.

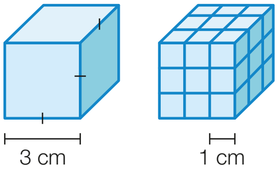
(f) What percentage of the original piece of cardboard is wasted in making the box? Give your answer to the nearest whole per cent.

10 Chewing your food is an important step in the digestive process. Chewing breaks down a large piece of food, making it easier to digest. Demonstrate your calculations using a cube to model a piece of food.

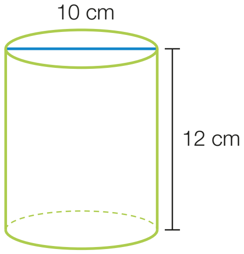
(a) Calculate the surface area of this 3 cm cube.

(b) Imagine this cube is chewed into smaller cubes each with a side length of 1 cm. Calculate the surface area of these smaller cubes.

(c) By how many times has the surface area of the original cube increased?



11 **(a)** Draw the net of this cylinder, labelling the measurements to the nearest whole number.



(b) The area of each circle on the ends of the cylinder can be found by the calculation:

A $\pi \times 10^2$ **B** $\pi \times 5^2$

C $2 \times \pi \times 10$ **D** $\pi \times 10$

(c) The width (shorter side) of the rectangular part of the net is:

A 5 cm **B** 10 cm **C** 12 cm **D** 24 cm

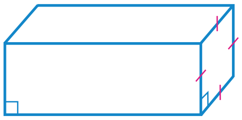
(d) The length of the rectangle can be found by the calculation:

A $\pi \times 10^2$ **B** $\pi \times 5^2$ **C** $\pi \times 10$ **D** $2 \times \pi \times 10$

Open-ended

12 What is the effect on the surface area of a cube if you double the length of its edges? Demonstrate your answer with two suitable cubes, one with an edge length that is double that of the other cube.

13 A rectangular box has a surface area of 960 cm^2. It has a square end, as shown. Each side length is a whole number of centimetres. Find two different sets of dimensions that satisfy these conditions.



Problem solving

Icosahedron

An icosahedron is one of the five Platonic solids, which are five 3D objects with special properties. Each face of a Platonic solid is a regular polygon which has all faces identical.

A particular icosahedron has a surface of 20 equilateral triangles of edge length 20 cm.

Find the surface area of this icosahedron. Round your answer to 1 decimal place.

Strategy options

- Draw a diagram.
- Have you seen a similar problem?
- Break the problem down into manageable parts.

Volume and capacity

4.4

The **volume** of a solid (a three-dimensional object) is the total amount of space that it occupies. This is measured in cubic units such as cubic millimetres (mm^3), cubic centimetres (cm^3) and cubic metres (m^3).

$$\begin{aligned} 1\text{ cm}^3 &= 1\text{ cm} \times 1\text{ cm} \times 1\text{ cm} = 10\text{ mm} \times 10\text{ mm} \times 10\text{ mm} = 10^3\text{ mm}^3 \\ &= 1000\text{ mm}^3 \\ 1\text{ m}^3 &= 1\text{ m} \times 1\text{ m} \times 1\text{ m} = 100\text{ cm} \times 100\text{ cm} \times 100\text{ cm} = 100^3\text{ cm}^3 \\ &= 1\,000\,000\text{ cm}^3 \end{aligned}$$

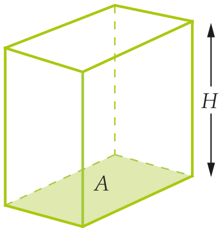
Because units of volume are cubic units, the conversion factor for converting from one unit to another is the cube of the conversion factor for length.

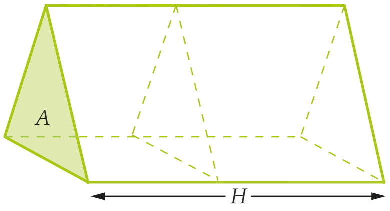
$$\begin{tikzcd} \text{km}^3 \arrow[r, bend left, "\times 1\,000\,000\,000 \; (1000^3)"] & \text{m}^3 \arrow[l, bend left, "\div 1000^3"] \arrow[r, bend left, "\times 1\,000\,000 \; (100^3)"] & \text{cm}^3 \arrow[l, bend left, "\div 100^3"] \arrow[r, bend left, "\times 1000 \; (10^3)"] & \text{mm}^3 \arrow[l, bend left, "\div 10^3"] \end{tikzcd}$$

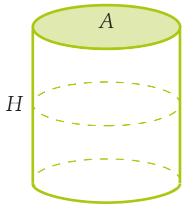
The formula $V = AH$ gives the volume of a prism, where A is the area of the base and H is the perpendicular height.

You have identified the base correctly if cutting parallel to it would leave the prism with a new 'base' identical in shape and size to the original base. In each of these diagrams, the base is shaded. (For the rectangular prism, all faces may be used as the base.)

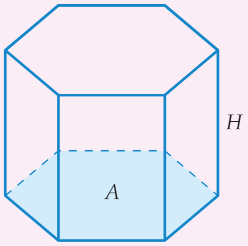
It is important to note that the base is not necessarily the face that the prism is 'sitting on'. It is the face that has a 'uniform cross-section'.





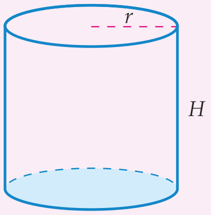


Note: Although a cylinder is not strictly a prism, it does have a uniform cross-section and so its volume can be found in the same way. The base is the circular face which has area $A = \pi r^2$. Multiplying the area of the base by the height of the cylinder gives $V = \pi r^2 H$.



Volume of prism

$V = AH$ where
A = area of the base
H = perpendicular height of the prism



Volume of cylinder

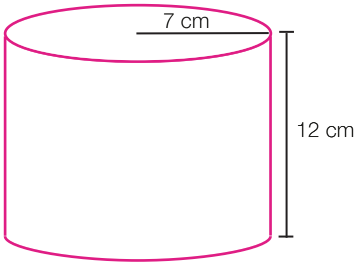
$V = \pi r^2 H$

Worked example 9

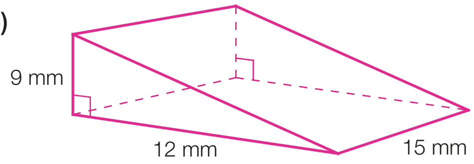
W.E. 9

Find the volume of these solids to the nearest whole number.

(a)



(b)



Thinking	Working
(a) **1** Write the general formula for volume.	(a) $V = AH$
2 Substitute the specific formula for the area of the base ($A = \pi r^2$).	$= \pi r^2 \times H$
3 Substitute the measurements.	$= \pi \times 7^2 \times 12$
4 Calculate the volume to the nearest cm^3.	$= 1847$ cm^3 (nearest whole number)
(b) **1** Write the general formula for volume.	(b) $V = AH$
2 Substitute the specific formula for the area of the base ($A = \frac{bh}{2}$).	$= \frac{bh}{2} \times H$
3 Substitute the measurements	$= \frac{12 \times 9}{2} \times 15$
4 Calculate the volume in mm^3.	$= 810$ mm^3

Capacity

Capacity describes the maximum amount that a container can hold, usually measured in millilitres (mL), litres (L), kilolitres (kL) or megalitres (ML). Volume is often used when describing the amount of liquid or gas a container could hold.

A 10 cm × 10 cm × 10 cm cube has a capacity of 1 L.

Converting units of capacity:

$1 \text{ mL} = 1 \text{ cm}^3$

$1 \text{ L} = 1000 \text{ mL} = 1000 \text{ cm}^3$

$1 \text{ kL} = 1000 \text{ L} = 1 \text{ m}^3$

$1 \text{ ML} = 1000 \text{ kL}$

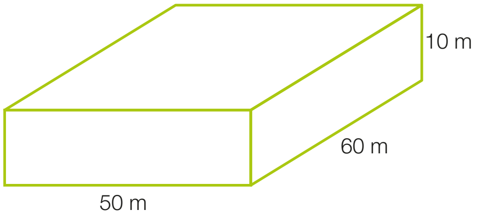
$$\begin{tikzcd}
\text{ML} \arrow[r, bend left, "\times 1000"] & \begin{matrix}\text{kL}\\(\text{m}^3)\end{matrix} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \text{L} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \begin{matrix}\text{mL}\\(\text{cm}^3)\end{matrix} \arrow[l, bend left, "\div 1000"]
\end{tikzcd}$$

Worked example 10

W.E. 10

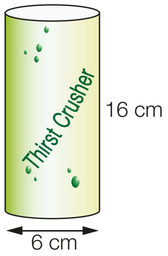
Find the capacity of each of the following in the units specified, rounded to the nearest whole number.

(a)



Water tank (in ML)

(b)



Can of soft drink (in mL)

Thinking	Working
(a) 1 Calculate the volume.	(a) $V = A \times H$ $= 60 \times 50 \times 10$ $= 30\,000 \text{ m}^3$
2 Convert to equivalent units of capacity.	$\therefore$ Capacity $= 30\,000$ kL
3 Convert to the required units.	$= 30$ ML
(b) 1 Calculate the volume.	(b) $V = \pi r^2 \times H$, where $r = \frac{6}{2} = 3$ cm $= \pi \times 3^2 \times 16$ $= 452 \text{ cm}^3$ (nearest whole number)
2 Convert to equivalent units of capacity.	$\therefore$ Capacity $= 452$ mL

Note: In each case you can assume that either the internal dimensions are given or that the material of the container is so thin that you can disregard the thickness.

4.4 Volume and capacity

Navigator

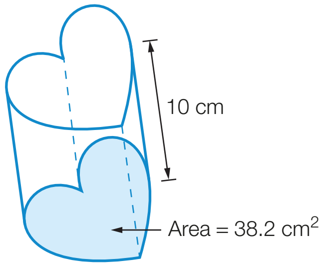
1 (a–c), 2 (a–d), 3, 4 (a, c), 5, 6 (a, b), 7 (a, d), 8, 9, 13, 14, 15	1 (a, c, d), 2 (a–d, e–g), 3, 4, 5, 6, 7, 8, 9, 10, 12 (a, b), 13, 14, 15	1 (c, d), 2 (a–d, e–h), 3 (b–e), 4 (b, d), 6, 7, 8, 9, 10, 11, 12, 13, 14, 15

Answers p. 646

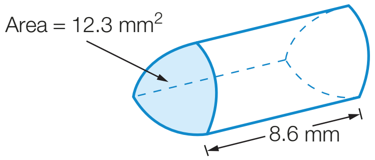
Fluency

1 Find the volume of these solids. The cross-sectional area of each solid has been given.

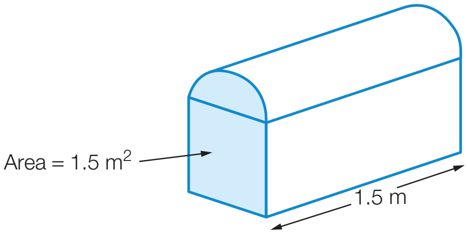
(a)



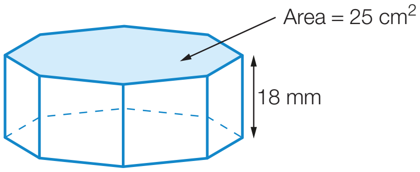
(b)



(c)



(d)



2 Convert each measurement into the units indicated in the brackets.

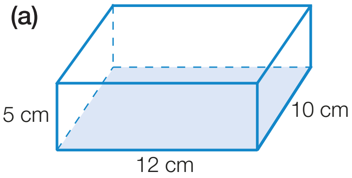
(a) 325 cm^3 (mL) **(b)** 2340 cm^3 (L) **(c)** 0.58 m^3 (L) **(d)** 63.5 m^3 (kL)

(e) $145\,000 \text{ cm}^3$ (kL) **(f)** 8500 m^3 (ML) **(g)** 1250 mm^3 (mL) **(h)** $750\,000 \text{ mm}^3$ (L)

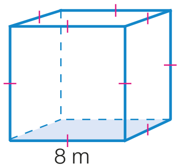
W.E. 9

3 Find the volume of these solids.

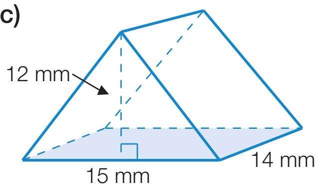
(a)



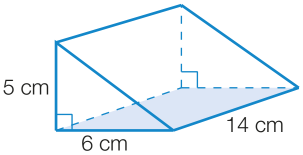
(b)



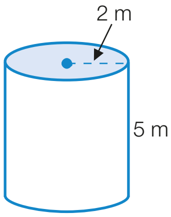
(c)



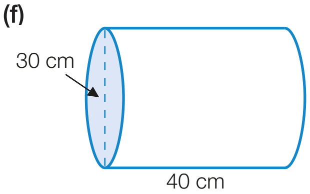
(d)



(e)



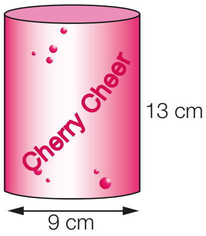
(f)



W.E. 10

4 Find the capacity of each of the following in the units specified.

(a)

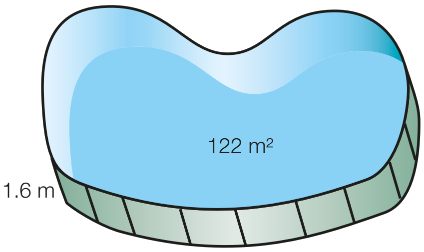


Cylindrical can of soft drink (in mL, to the nearest whole number)

(b)

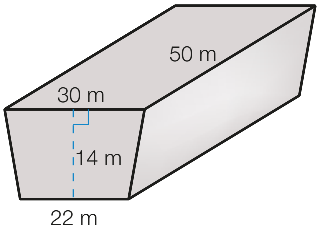
Square-based prism (in L, to 1 decimal place)

(c)



A kidney-shaped swimming pool (in kL, to the nearest whole number)

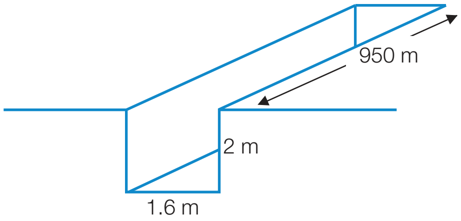
(d)



A storage tank on a ship (in ML, to 1 decimal place)

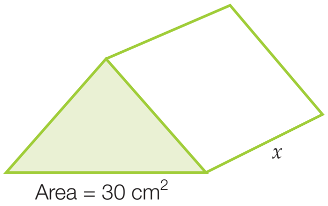
Understanding

5 The council is digging a trench to lay some new pipe. Find the volume of soil that will have to be removed.

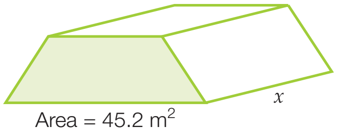


6 Find the height, labelled as x, of these prisms.

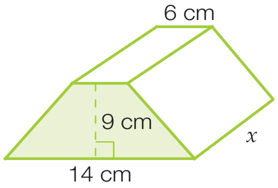
(a) Volume = 600 cm^3



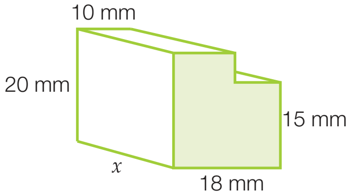
(b) Volume = 560.48 cm^3



(c) Volume = 1080 m^3

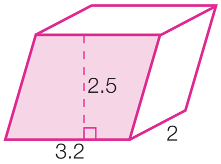


(d) Volume = 9600 mm^3



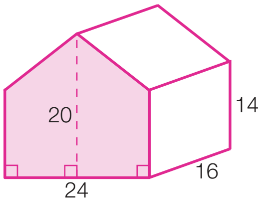
7 All the lengths in the diagrams below are in metres. The volume of each solid is:

(a)



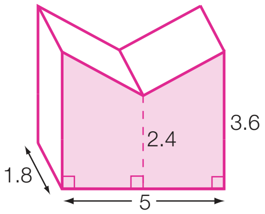
A 8 m^3
B 11.4 m^3
C 15.4 m^3
D 16 m^3

(b)



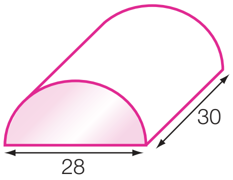
A 1320 m^3
B 1408 m^3
C 6528 m^3
D 7680 m^3

(c)



A 16 m^3
B 27 m^3
C 54 m^3
D 86.4 m^3

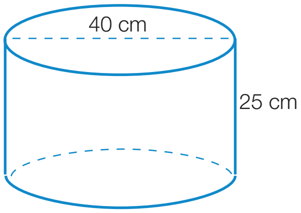
(d)



A 9236 m^3
B 11 760 m^3
C 36 945 m^3
D 79 168 m^3

Reasoning

8 Jenny is making a fruit punch for a party. She is using the cylindrical bowl shown and will fill it to 5 cm from the top.



(a) To what level will the fruit punch rise?

(b) Calculate the volume of the fruit punch in mL (to 1 decimal place).

(c) If each of the party cups holds 250 mL, find the number of drinks that can be served from the bowl.

9 A cylindrical water tank has a capacity of 8 kL and a height of 2 m.

(a) What is its volume in m^3?

(b) Calculate the base area of the tank.

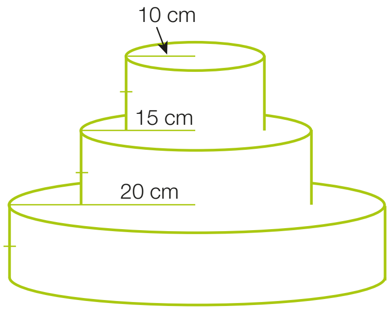
(c) Find its radius to the nearest cm.

(d) Water starts leaking out of a full tank at the rate of 25 L/day. How many days will it be before the tank is half empty?

10 Find the volume left after a wedge of 52° has been cut from a cheese wheel of diameter 30 cm and height 10 cm. Write your answer to the nearest cm^3.

11 **(a)** Find the volume of a three-tier cake if each tier is 7 cm high and the radii are 20 cm, 15 cm and 10 cm. Give your answer correct to the nearest cm^3.

(b) The baker decides to use the same amount of ingredients to make a cake with a square base of side length 40 cm. How high will it be, correct to the nearest cm?



12 A steel pipe has a diameter of 24 cm and is 1 cm thick. It is made in lengths of 2 m.

(a) What is the capacity of the pipe (to the nearest mL)?

(b) How many cubic centimetres of steel are in the length of pipe (to the nearest cm^3)?

(c) How many kilolitres of water, correct to 1 decimal place, would pass through one length of pipe in an hour if the water is flowing at a rate of 2 metres per second?

Open-ended

13 Give an example of a substance that would be measured in:

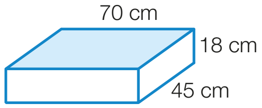
(a) mL **(b)** L **(c)** kL

14 Write the dimensions of any rectangular prism that has a capacity of each of the following:

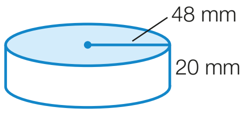
(a) 1 mL **(b)** 1 L **(c)** 1 kL **(d)** 1 ML

15 Sabanth was set the following two questions for homework.

(i) Calculate the volume of this prism, in m^3.



(ii) Calculate the volume of this cylinder, in cm^3.



Here is his working:

$$\begin{aligned} V &= lwh \\ &= 70 \times 45 \times 18 \\ &= 56\,700 \text{ cm}^3 \\ &= 567 \text{ m}^3 \end{aligned}$$

$$\begin{aligned} V &= \pi r^2 h \\ &= \pi \times 48^2 \times 20 \\ &= 144\,764.6 \text{ mm}^3 \text{ (1 d.p.)} \\ &= 14\,476.5 \text{ cm}^3 \text{ (1 d.p.)} \end{aligned}$$

(a) Identify the error that Sabanth has made.

(b) Give Sabanth a hint to help him avoid similar errors in the future.

Problem solving

Gold, set, match

The following statement has appeared on several 'weird facts' websites:

'A lump of pure gold that is only as big as a matchbox can be flattened into a sheet about the size of a tennis court.'

A matchbox is 5.5 cm long, 3.5 cm wide and 1.5 cm high.

A tennis court is 23.77 m long and 10.97 m wide.

1 If the 'fact' is true, and a matchbox-sized lump of gold was flattened out into a sheet to cover a tennis court, how thick would the sheet of gold be? Give your answer in mm.

2 Considering the size of your answer, do you think the 'fact' could actually be true?

Strategy options

- Draw a diagram.
- Make a model.
- Break the problem down into manageable parts.

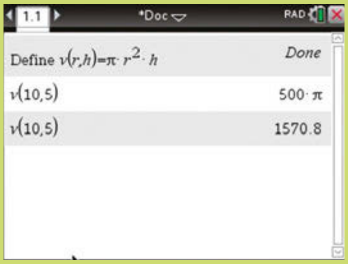
Exploration CAS

The properties of cylinders

A useful feature of CAS technology is that it enables you to define functions which can then be used to solve different but related mathematical problems. In this Exploration you will define a function to find the volume of a cylinder, then use this to find cylinder volumes with different values for the radius and the height.

Using TI-Nspire CAS

Select **menu** > **Actions** > **Define** and then enter the rest of the definition as shown in the first line below. After doing this, to find the volume for $r = 10$ and $h = 5$ you can simply enter ***v*(10,5)**. To get the approximate numerical answer press **ctrl** + **enter**.



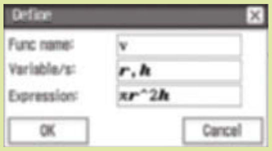
Define $v(r,h)=\pi\cdot r^2\cdot h$ Done

$v(10,5)$ $500\cdot\pi$

$v(10,5)$ 1570.8

Using Casio ClassPad CAS

From the menu select **Main**, and then select **Interactive** > **Define** and enter the rest of the definition as shown below. Use the **Keyboard**'s **ABC** tab to enter the **v**, but use the **Keyboard**'s **Var** tab to enter the ***r*** and ***h***.

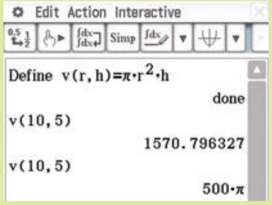


Func name: v

Variable/s: r, h

Expression: πr^2h

Now, to find the volume for $r = 10$ and $h = 5$ you can simply enter **v(10,5)**. To switch between answers in exact form and decimal form, you can switch between the **Standard** and **Decimal** modes by selecting the word at the bottom of the screen.



Define $v(r,h)=\pi\cdot r^2\cdot h$

done

$v(10,5)$

1570.796327

$v(10,5)$

$500\cdot\pi$

1 Copy the table shown below into your workbook. Use your CAS to find the volumes of the cylinders in the table.

Cylinder	Radius	Height	Volume
1	5	10	
2	10	10	
3	20	10	
4	2	10	
5	6	10	
6	18	10	
7	4	10	
8	16	10	
9	64	10	

For each of the following questions, round your answer to the nearest whole number.

2 In all the cylinders in **1**, the height has been kept constant at 10 units. In the first three cylinders the radius doubles, in the next three cylinders the radius triples, and in the last three cylinders the radius quadruples.

(a) What do the mathematical terms *double*, *triple* and *quadruple* mean?

(b) Use the values for volume you found using your CAS to calculate the ratios $\frac{V_3}{V_2}$ and $\frac{V_2}{V_1}$. Complete the statement: Doubling the radius of a cylinder increases its volume by a factor of ____________.

(c) Calculate the ratios $\frac{V_6}{V_5}$ and $\frac{V_5}{V_4}$. Complete the statement: Multiplying the radius of a cylinder by 3 increases its volume by a factor of ____________.

(d) Calculate the ratios $\dfrac{V_9}{V_8}$ and $\dfrac{V_8}{V_7}$. Complete the statement: Multiplying the radius of a cylinder by 4 increases its volume by a factor of ____________.

(e) Can you generalise the pattern above? That is, if you multiply the radius of a cylinder by a number k, where k is greater than 1, then how many times is the volume increased?

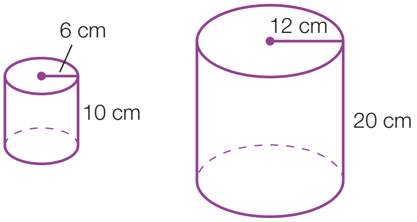
Taking it further

3 Use cardboard to make models of two cylinders with the same height, but one with a radius that is twice the other radius. Fill the smaller cylinder with sand and use this cylinder to fill the larger cylinder. How many fills of the smaller cylinder are needed to fill the larger cylinder? How does this compare with the answer to **2(b)**?

4 Use algebra to prove the statements you wrote to answer **2** parts **(b)** to **(e)**. Your teacher can help you with this.

5 Write and test a CAS function to calculate the volume of a rectangular prism.

6 Investigate similar cylinders. Similar cylinders have radii and heights that correspond in the same ratio. For example, consider the cylinders shown. The radii of the small and large cylinders are 6 cm and 12 cm, while the heights are 10 cm and 20 cm.



(a) Verify that the ratios of the radii and of the heights are the same (that is, show that $r_2 : r_1 = h_2 : h_1$).

(b) Calculate the ratio $V_2 : V_1$. What is the relationship between the ratios of the corresponding sides and the ratio of the volumes of the cylinders?

(c) Repeat parts **(a)** and **(b)** for other pairs of similar cylinders.

(d) Use algebra to find a general rule for this pattern.

Gamespace

ORIGAMI SPORTS SHIRT

How to make an origami shirt

Equipment required: half a sheet of A4 paper

1 Make a centre crease lengthwise by folding your paper in half and unfolding again.

Fold the edges towards the middle crease.

2 Fold out two of the corners, to approximately a 60° angle (this will make the sleeves).

3 Turn the shirt over and fold the bottom edge up 1 cm (this will make the collar).

4 Turn the shirt over again and fold the corners into the centre (this will make the neck of the collar).

5 Turn the shirt over and fold the lower edge up until it is under the collar.

6 The shirt is complete.

Questions

1 **(a)** Find the perimeter of your shirt (in cm).

(b) What shapes could you use to find the area of the shirt?

(c) Write the formulas for the area of the shapes you listed in part **(b)**.

(d) Find the area of your shirt (in cm^2).

(e) What percentage of the area of your original piece of paper does the T-shirt take up?

Footy facts: dehydration

Footballers lose a lot of liquid through sweat. This can cause dehydration, which can affect performance and, if severe enough, be life threatening.

2 If a footballer loses an average of 23 mL in 1 minute while training, how many litres per hour (L/h) is this?

3 Use your answer to **2** to calculate how many litres of fluid a player will have lost (to the nearest litre) if he trained for 3.5 hours?

4 Studies have shown that footballers should consume 130% of the fluid they lose. Using your answer to **3**, how many litres of water should the footballer consume to replace the fluid lost while training? Answer to 1 decimal place.

5 In order to consume that amount of water, how many L/h should the player drink, on average, when training over 3.5 hours?

6 Using your answer to **5**, how many mL per minute would this be?

7 If a player has a cylindrical drink bottle that is 9 cm in diameter and 15 cm high, what is its capacity in litres? (State your answer to the nearest litre.) Will this be enough to cover the player's hourly liquid needs?

Investigation

The humble milk carton

Equipment required: 1 empty milk carton, cardboard, scissors, glue, hot water (optional)

To save money and use fewer resources, manufacturers are always looking for the most efficient and economical ways of packaging food and drink.

The Big Question

Does it take more cardboard to make two separate 1 L milk cartons or one 2 L carton?

Engage

1 Make a 3D pencil diagram of your milk carton.

2 Find the dimensions (length, width and height) of the part of the milk carton that holds the milk (you may need to make some assumptions). Do these dimensions give a volume of 1 L? Label them on the diagram and use a calculation to show how these dimensions give a volume of 1 L.

Explore

3 Carefully open out the carton along its seams. (If you fill it with hot water, the glue will soften and make it easier to gently take apart.) Make a scale drawing of the net of the carton.

4 Find the area of cardboard required to make the carton.

5 Find the minimum area of cardboard required to make a 2 L carton. Show your calculations.

Strategy options

- Guess and check.
- Look for a pattern.
- Test all possible combinations.

Explain

6 Make a scale drawing of the net of your 2 L carton, labelling all the measurements necessary to make the carton. Show, with calculations, how these dimensions give a volume of 2 L.

Elaborate

7 (a) Use your answers to **4** and **5** to answer the Big Question.

(b) Does it take twice as much cardboard to make a carton with twice the volume?

(c) Which carton is the more efficient form of packaging?

Evaluate

8 Describe the process you used to find the area of cardboard needed to make a 2 L carton. Could you have gone about solving the problem in a different way?

9 Explain any assumptions you have made in your calculations of area and volume for both the 1 and 2 L cartons.

10 Is the design of your 2 L carton practical? Can the carton be handled easily? Could it fit inside a fridge door?

11 List the advantages and disadvantages of selling milk in 2 L instead of 1 L cartons.

Extend

12 Construct, out of cardboard, a prototype of your new 2 L carton.

13 Investigate whether it takes more aluminium to make two 375 mL soft drink cans or one 750 mL can.

Challenge 4

1 The volume of a cube is 19 683 cm^3. What is its surface area?

2 How many seconds will have you lived by the time you turn 15? (Assume three leap years and the same time of day as your birth.)

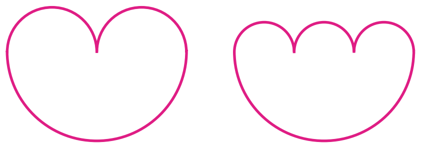
3 In a game of soccer, 11 players are on the field and two reserves are interchanged as needed. The game lasts 40 minutes. If each of the 13 members of the team had equal time on the field, how long was each of them off the field (to the nearest second)?

4 Bernard clicked his fingers once. One minute later he clicked them again. Two minutes later he clicked them again, then again 4 minutes later and 8 minutes later. If he continues this pattern, how many times will Bernard have clicked his fingers in one day?

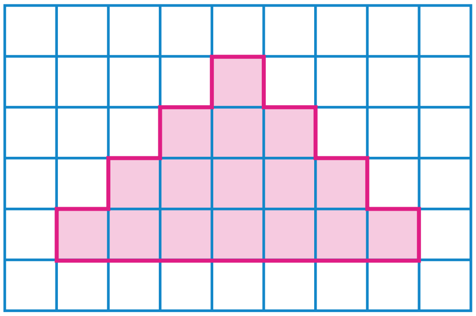
5 How can the contents of a 720 mL jug of cordial be divided evenly between two containers using three unmarked jugs that hold 150 mL, 330 mL and 390 mL?

6 A cube has its side lengths doubled. What happens to its volume? What happens to its surface area?

7 These diagrams are made up of semicircles, with the largest semicircle in each diagram having a diameter of 24 cm. Express the ratio 'perimeter of first figure : perimeter of second figure', in simplest form.

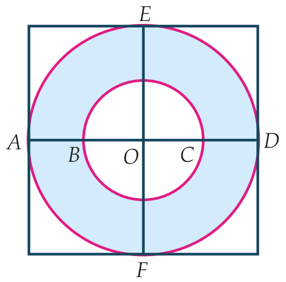


8 The area of the figure on the right is 64 cm^2. What is its perimeter?

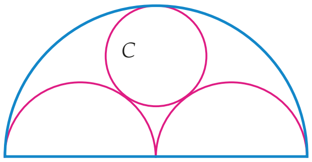


9 A, E, D and F are the midpoints of the sides of the square. $AD \perp EF$, AD and EF intersect at O and B is the midpoint of AO. The circles have radii OB and OA, respectively. The fraction of the square covered by the shaded area between the circles is:

A $\frac{3\pi}{16}$ **B** $\frac{\pi}{4}$ **C** $\frac{\pi}{16}$ **D** $\frac{1}{2}$



10 Two semicircles of radius 1 cm are drawn on the diameter of a semicircle of radius 2 cm. A circle C touches all three semicircles, as shown in the diagram. Calculate the radius of the circle C.



Chapter review 4

Maths literacy

arc
area
capacity
circumference
composite shape
cross-section
length
net
perimeter
polyhedron
prism
sector
subtended
surface area
volume

Copy and complete the following using the words and phrases from the list, where appropriate. A word or phrase may be used more than once.

1 The volume of a __________ or cylinder is found by multiplying the area of the base by the height.

2 The __________ of a container is usually measured in millilitres or litres, where $1\text{L} = 1000\text{ cm}^3$.

3 The word __________ refers to the total distance along the boundary of a figure.

4 The perimeter of a circle is called the __________.

5 __________ is the amount of surface within a boundary. To convert between units, square the conversion factor for length.

6 __________ is the amount of space occupied by a solid (three-dimensional object). To convert between units, cube the conversion factor for length.

7 An __________ is part of the circumference of a circle.

8 To find the __________ of a regular polyhedron, multiply the area of identical faces by the number of times each face appears.

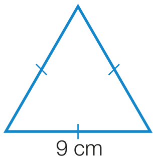
9 A __________ of a circle is the area between two radii and the arc that joins them.

10 A __________ is a solid in which every face is a polygon.

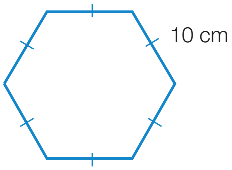
Fluency

1 Find the perimeters of these shapes. **4.1**

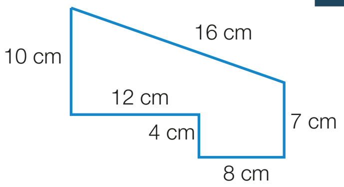
(a)



(b)

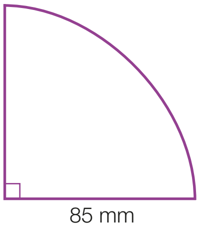


(c)

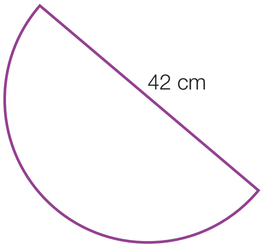


2 Calculate **(i)** the arc length and **(ii)** the perimeter of each sector to the same number of decimal places as the length in each question. **4.1**

(a)



(b)



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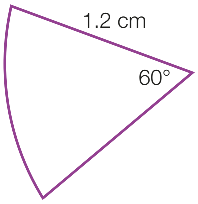

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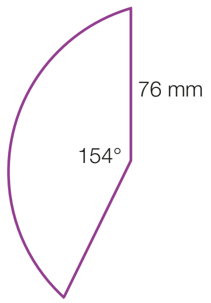
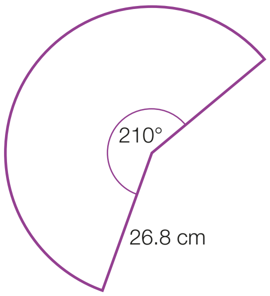
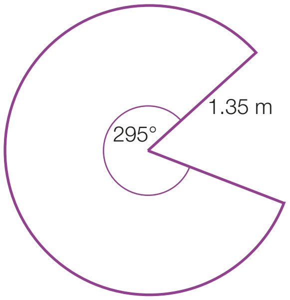


(e)



(f)



4.2

3 Copy and complete the following area conversions.

(a) 4.2 cm^2 = __________ mm^2 **(b)** 20 000 m^2 = __________ ha

(c) 682 500 cm^2 = __________ m^2 **(d)** 528 ha = __________ km^2

(e) 80.7 ha = __________ m^2 **(f)** 0.009 m^2 = __________ cm^2

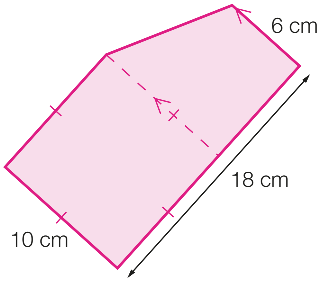
4.2

4 Calculate the area of each of the sectors in Question **2**, giving your answers to the same number of decimal places as given for the length.

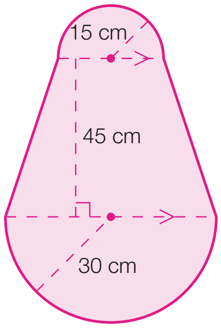
4.2

5 Find the shaded areas of the following composite shapes.

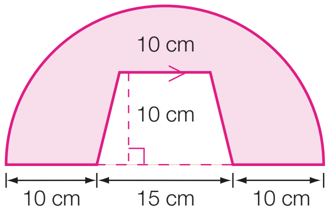
(a)



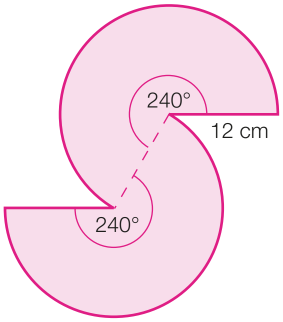
(b)



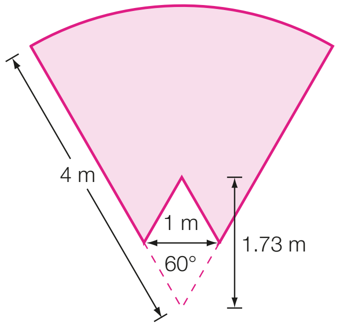
(c)



(d)



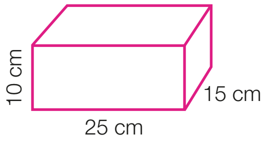
(e)



4.3

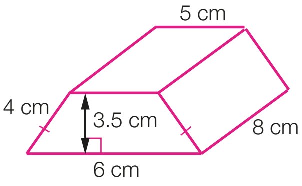
6 Find the surface area of each of these solids, correct to 2 decimal places where necessary.

(a)

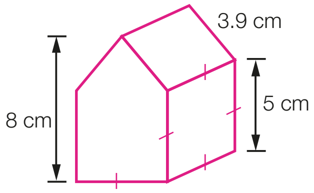


(b) (Give your answers in m^2)

(c)

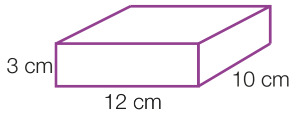


(d)

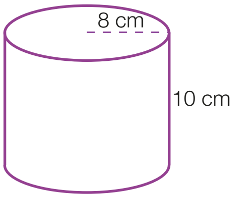


7 Find the volume of the following figures to the nearest whole number. **4.4**

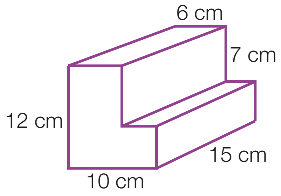
(a)



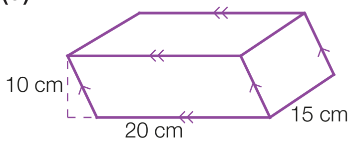
(b)



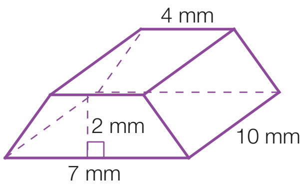
(c)



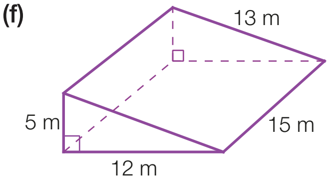
(d)



(e)



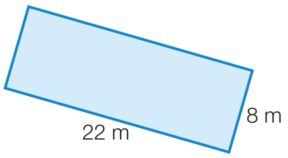
(f)



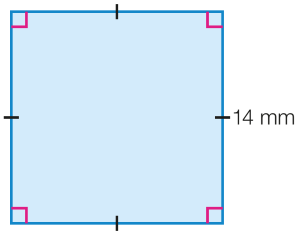
Understanding

8 Find the area of each figure below in the units indicated in brackets. **4.2**

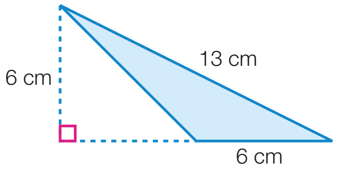
(a) (ha)



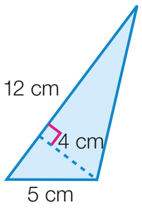
(b) (cm^2)



(c) (mm^2)

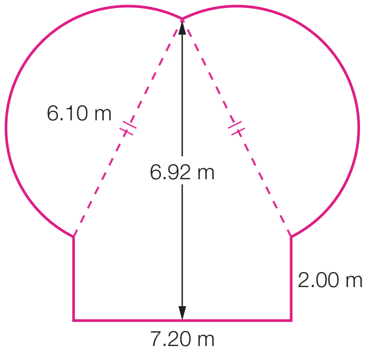


(d) (m^2)

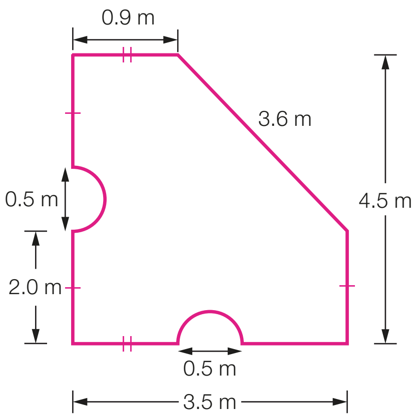


9 Find the area of the following shapes. Give your answers correct to 2 decimal places. **4.2**

(a)



(b)



10 A farmer planted a crop in a field 300 m long and 500 m wide. One month later, seven 'crop circles', each with a diameter of 21 m, were found one morning, flattening the crop in the circle. What area of crop remained? Give your answer to the nearest m^2. **4.2**

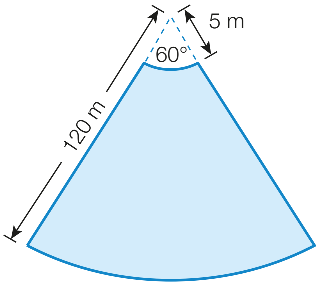
11 Find the capacity of a cylindrical soft drink can that is 5.5 cm wide and 13 cm high. Give your answer in mL, correct to 2 decimal places. **4.4**

12 A cube has a volume of 512 cm^3. Find its surface area. **4.3**

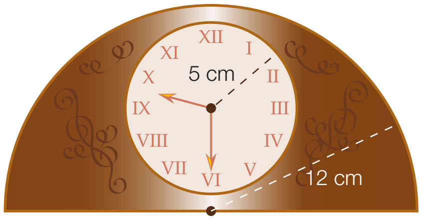
13 A snowball-throwing court has the measurements shown. **4.1, 4.2**

(a) Calculate the area that the thrower has to land the snowball in, to the nearest square metre.

(b) Find the perimeter of the shaded area, to the nearest metre.



14 An antique clock has a circular face of radius 5 cm on a semicircular background of radius 12 cm. The area around the clock face is decorated. Find the size of the decorated area to the nearest cm^2. **4.2**



15 A cylinder is to be made from a piece of sheet metal that is 30 cm square. If the height and diameter of the cylinder are to be 8 cm each, what area of metal is wasted? Write your answer to the nearest whole cm^2. **4.3**

Reasoning

16 **(a)** The inside surface of a rectangular swimming pool is to be tiled. If the pool is 12 m long, 5.5 m wide and 2.2 m deep, what is the tiled area? **4.3, 4.4**

(b) If the tiles are 12 cm long and 8 cm wide, how many will be needed (to the nearest 100)?

(c) How many litres will the pool hold when it is full?

17 A cylindrical paint tin has a radius of 11 cm and a capacity of 12.16 L. **4.4**

(a) What is the volume of paint in cm^3?

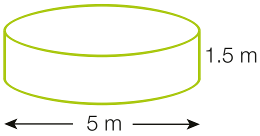
(b) What is the height of the tin to the nearest cm?

(c) If the paint just fills a rectangular paint tray that is 33 cm long and 22 cm wide, how deep is the tray? Write you answer correct to 2 decimal places.

18 A cylindrical container has a radius of 7 cm and a height of 10 cm. What must be the height of a rectangular container 7 cm long and 10 cm wide if the two containers are to have the same volume? Answer to the nearest centimetre. **4.3, 4.4**

19 **(a)** How many metric cups (250 mL) of water would it take to fill a water tank of diameter 5 m and depth 1.5 m? **4.4**

(b) How much time will the tank take to be filled, if a hose delivers 2 cups per second?



20 The hour hand of a clock is 4.5 cm long and the minute hand is 6 cm. **4.1**

(a) To the nearest cm, how much further does the tip of the minute hand travel in the course of the day (24 hours) than the hour hand?

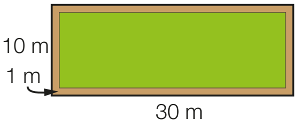
(b) What is the speed of each tip in centimetres per minute (cm/min)? Write your answer correct to 2 decimal places.

Numeracy practice 4

Non-calculator

1 A 30 m × 10 m garden is bordered on the inside by a 1 m wide path. The area of the path in m^2 is:

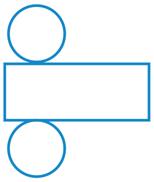
A 39 B 72 C 76 D 80



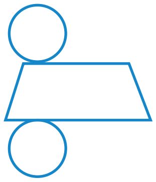
2 A regular tetrahedron is a solid with four equilateral triangles as faces. Find the surface area of a regular tetrahedron where the base of the triangular face is 6 cm and the height is 5.2 cm.

3 The net that could NOT form a cylinder is:

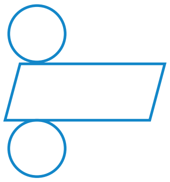
A



B



C



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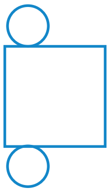
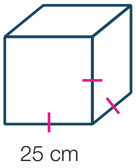


4 A cube has a surface area of 294 cm^2. How long is an edge of the cube (in cm)?

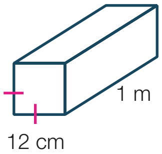
Calculator allowed

5 The shape with the biggest volume is:

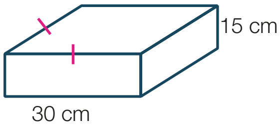
A



B



C



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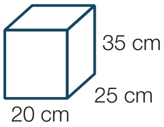
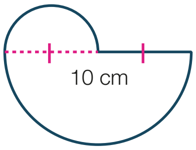


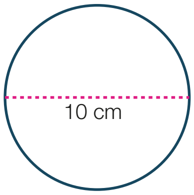
6 The perimeters of the shapes below, in order from smallest to largest, are:

A I, II, III B II, III, I C I, III, II D III, II, I

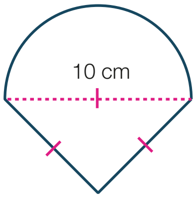
I



II



III



7 One full turn of the wheels on this car moves the car forward 195 cm. What is the diameter of the wheels? (Answer to the nearest cm.)

8 A cylindrical can full of water has a diameter of 50 cm and a height of 50 cm. How many smaller cylinders with a diameter and height of 5 cm can it fill?

A 10 B 100 C 1000 D 10 000

Mixed review B

Fluency

1 Calculate: 1.1

(a) 85% of 250 (b) 20% of \$0.40 (c) 90% of 80

2 Simplify: 3.1

(a) $10g \times 4h \times 2gh^2$ (b) $-12k \times 3k^3$ (c) $9ef \times 6e^2 \times 2g$

3 Expand these expressions and simplify where possible: 3.5

(a) $-8(x + 2)$ (b) $5y(6 - 11y)$ (c) $9(p + 7) + (3p - 5)$

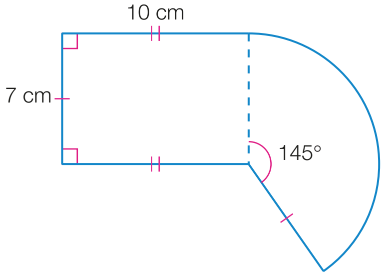
4 Factorise: 3.7

(a) $4m - 28$ (b) $7ed^2 + 35d$ (c) $8xy^2 - 2y$

5 Find the percentage that: 1.1

(a) 25 is of 175 (b) 90 is of 120 (c) 45 is of 360 (d) 100 is of 40

6 Find the area of the following composite shape.
Write your answer to the nearest whole number. 4.2



7 Complete these conversions. 4.2

(a) $20\text{ mm}^2 = \_\_\_\_\_\text{ cm}^2$ (b) $23.67\text{ km}^2 = \_\_\_\_\_\text{ m}^2$

(c) $0.007\text{ m}^2 = \_\_\_\_\_\text{ cm}^2$ (d) $14.5\text{ ha}^2 = \_\_\_\_\_\text{ m}^2$

8 Simplify the following expressions. 3.1

(a) $x^3 \times x^5$ (b) $x^8 \div x^4$ (c) $\frac{x^3 \times x^6}{x^5}$

9 A junior intern earns \$9.70 per hour for a standard 8-hour day. The intern is paid time-and-a-half for the first two hours of overtime on any day and double time for any hours after that. How much will the worker be paid for working the following hours in one day? 1.3

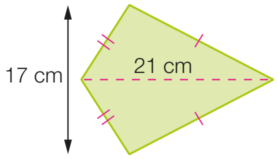
(a) 5 hours (b) 8 hours (c) 9 hours (d) 12 hours

10 Simplify each of the following: 3.2

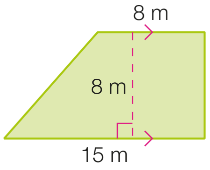
(a) z^0 (b) $5z^0$ (c) $(5z^0)$ (d) $3z^0 + 2z^0$

11 Find the shaded area of each of these shapes, correct to 1 decimal place where necessary. 4.2

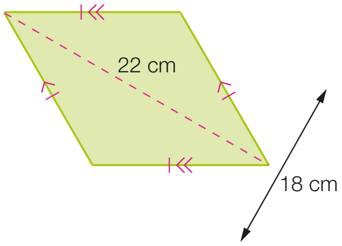
(a)



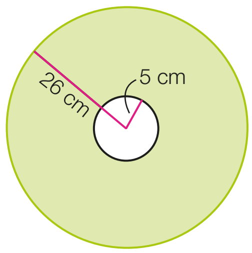
(b)



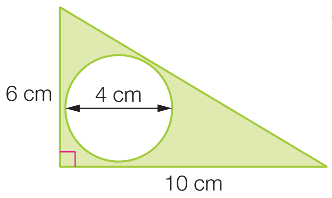
(c)



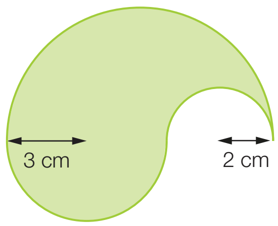
(d)



(e)



(f)



12 Use the following income tax table to find out how much income tax should be paid on the taxable incomes below. **1.4**

Taxable income	Tax on this income
\$0–\$18 200	Nil
\$18 201–\$37 000	19c for each \$1 over \$18 200
\$37 001–\$80 000	\$3572 plus 32.5c for each \$1 over \$37 000
\$80 001–\$180 000	\$17 547 plus 37c for each \$1 over \$80 000
\$180 001 and over	\$54 547 plus 45c for each \$1 over \$180 000

(a) \$6300 **(b)** \$18 600 **(c)** \$47 650 **(d)** \$82 700

13 Expand these expressions and simplify where possible. **3.5**

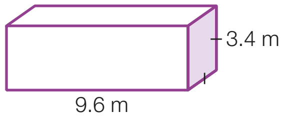
(a) $(x-3)(x+7)$ **(b)** $(x+6)^2$ **(c)** $(2x-5)^2$ **(d)** $(x+8)(x-8)$

14 Write the following in scientific notation. **3.3**

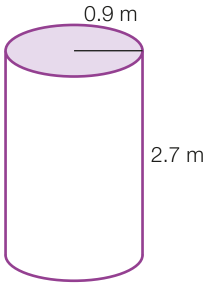
(a) 45 000 000 **(b)** 5 270 000 000 **(c)** 0.0003 **(d)** 0.000 000 298

15 Calculate the **(i)** surface area and **(ii)** volume of the following solids. State answers correct to 2 decimal places. **4.3, 4.4**

(a)



(b)



16 **(a)** Find the volume of a cylinder with a base diameter of 24 cm and a height of 16 cm, correct to 1 decimal place. **4.4**

(b) What is the capacity of this cylinder? Find the answer correct to 1 decimal place.

17 Simplify: **3.1**

(a) $4gh^4 \times 3g^3$ **(b)** $12a^2b^6 \times 2ab^3 \times 5a^4b^2$ **(c)** $\frac{5x^5y^6}{15x^4y}$

18 After investing \$8000 in a savings account paying 6.8% interest p.a., Justine received \$8321.93. For how many days, correct to the nearest day, was her money invested? **1.5**

Understanding

19 A department store has a policy that a profit of 87% must be achieved on all furniture sold. **1.2**

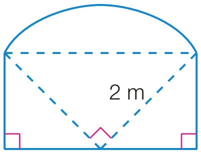
(a) What must a lounge suite, bought by the store for \$300, sell for? Answer to the nearest \$10.

(b) A dining table has a marked price of \$899. How much did the store purchase it for? Answer to the nearest dollar.

20 Simplify: 3.4

(a) $\dfrac{3m^5n^6 \times 4mn^2}{2m^2n^5 \times 3m^4n^3}$ **(b)** $\dfrac{5p^7q^4}{3p^3q} \div \dfrac{10p^5q^2}{9p^6q^6}$ **(c)** $\dfrac{5(x^3)^4y^6}{(x^2y)^2} \times \left(\dfrac{xy^3}{x^2y^2}\right)^3$

21 Find the area of the following shape. 4.2
Write your answer correct to 1 decimal place.



22 Kinetic energy is the energy (E) of a moving object, measured in joules (J). It is calculated 3.4

by the formula $E = \dfrac{mv^2}{2}$, where m is the mass in kg and v is the velocity of the object in m/s.

(a) Rearrange the formula for kinetic energy to make mass the subject.

(b) Use your rearranged formula to find the mass of a bowling ball that has a kinetic energy of 32.4 J and a velocity of 3.5 m/s. Round your answer to 1 decimal place.

23 Mario is paid a retainer of $250 per week plus a commission of 9.5% on the value of sales. 1.3
Calculate the income he would earn in one week if he sold goods to the value of:

(a) $1200 **(b)** $2500 **(c)** $3750 **(d)** $4210

24 **(a)** A cylindrical water tank has a capacity of 5000 L. If it has a height of 2.5 m, find its base radius. 4.4

(b) If it was emptied at a rate of 225 L per day, how many days would it take for the tank to be less than half full?

Reasoning

25 What happens to the total surface area of a rectangular prism if you multiply each dimension by two? 4.3

26 When employed as a sales assistant, Bettina was given a choice as to how she would be paid. 1.6

Option A: a retainer of $150 per week plus 5% of sales.

Option B: no retainer but 7.5% of sales.

(a) How much would Bettina need to sell to earn at least $950 per week under each of the options?

(b) What value of sales gives the same result regardless of the option chosen?

(c) When would it be better for Bettina to choose Option A?

27 Six circular discs of radius 5 cm fit snugly into the bottom of a rectangular box. Find the percentage of empty space at the bottom of the box, correct to 2 decimal places, if the discs are arranged in: 4.2

(a) one row of six **(b)** two rows of three.

(c) Which box is the more efficient container?

5

5

Linear relationships

Roads on the up and up. How steep can a road be, before cars and trucks can no longer travel along it safely?

When civil engineers design roads they need to ensure the roads are both safe to travel on and cost-effective to build. The Trollstigen (Troll Ladder), a mountain road in Norway, shows that the civil engineers decided to make a long, winding road to overcome the steepness of the mountain, thereby allowing a wide variety of vehicles to use the road. However, this road would have cost more money to build than a steep road that went straight up the slope. If civil engineers are designing a road for 4WD access only, the maximum gradient that the road can have is 1 : 5 or 20%. If they are designing a major road that will be used by all types of vehicles, the gradient must be less than 1 : 12 or 8%.

Forum

What effect would the road surface have on the maximum gradient?

What do you think a gradient of 1 : 5 means?

The safety of steep roads must be considered from both an 'uphill' and 'downhill' perspective. Why?

Why learn this?

Linear relationships are used to understand information in many different situations. Stockbrokers use linear relationships to model financial performance, to decide when to buy or sell. Doctors use linear relationships to decide whether someone is a healthy weight, based on their height (body mass index). Scientists use linear relationships to model the data collected from experiments, to investigate possible relationships between different variables.

After completing this chapter you will be able to:

- solve linear equations algebraically
- solve problems using linear equations
- find the distance between two points
- find the midpoint of two points
- represent linear relationships as algebraic expressions and graphs
- determine the gradient or slope of a linear relationship
- sketch linear graphs and find x-intercepts and y-intercepts.

Recall 5

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 If $P = 2k + 5$, find P where: **(a)** $k = 0$ **(b)** $k = 3$ **(c)** $k = -1$ **(d)** $k = -9$

If $C = 9 - 4d$, find C where: **(e)** $d = 1$ **(f)** $d = -2$ **(g)** $d = 0$ **(h)** $d = 4$

2 Plot these coordinate points on a Cartesian plane and label them.

(a) $A(1, 2)$ **(b)** $B(3, 2)$ **(c)** $C(0, 3)$ **(d)** $D(0, -4)$

(e) $E(-2, 0)$ **(f)** $F(3, 0)$ **(g)** $G(-3, -1)$ **(h)** $H(-4, 2)$

3 Complete a table of values for x and y to plot each of the following graphs.

x	-2	-1	0	1	2
y					

(a) $y = x + 4$ **(b)** $y = x - 2$ **(c)** $y = 2x$

4 Solve the following equations to find the value of the variable.

(a) $x + 1 = 7$ **(b)** $3 + y = 18$ **(c)** $a + 4 = -6$

(d) $b - 5 = 16$ **(e)** $c - 12 = 0$ **(f)** $d - 6 = -20$

5 Solve the following equations to find the value of the variable.

(a) $2g = 38$ **(b)** $0.5x = 21$ **(c)** $-9y = -54$ **(d)** $\frac{x}{7} = 5$ **(e)** $\frac{b}{4} = -3$ **(f)** $\frac{k}{-9} = -4$

6 Expand the brackets and simplify.

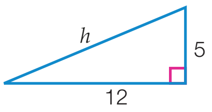
(a) $-3(5x + 2)$ **(b)** $2(7 - 3x) + 6$ **(c)** $3x + 1 + 6(2x + 7)$

7 Find the rule connecting x and y in the following table of values.

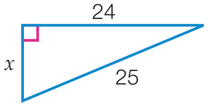
x	0	1	2	3	4	5
y	6	4	2	0	-2	-4

8 Use Pythagoras' theorem to find the unknown sides in the following triangles.

(a)



(b)

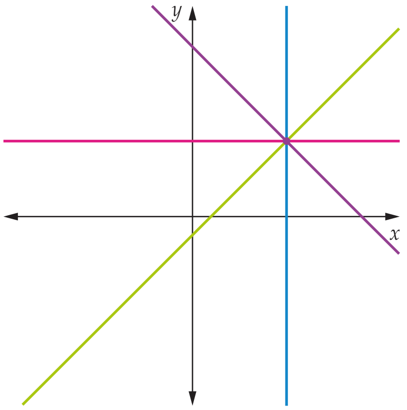


Exploration Task

You can download this activity from the eBook or the Pearson Places website.

Straight lines

In this activity, you will explore the gradients of straight lines passing through the same point.



Solving linear equations

5.1

Linear relationships are common in everyday life. A bath filling with water from a tap or the cost of electricity supplied to your home are examples of linear relationships.

A linear relationship between two variables exists if a uniform change in one variable, the **independent variable**, causes a uniform change in the other, the **dependent variable**.

A linear relationship produces a **linear graph** (straight line) and is represented by the equation $y = mx + b$. Graphing linear relationships will be covered later in this chapter and will require skills in solving linear relationships.

A **linear equation** is a linear relationship where the dependent variable has a specific value. Linear equations are solved to find the value of the independent variable. A linear equation has only one solution.

There are many methods that can be used to solve an equation. Each of these methods uses the mathematical property of **equivalence**. Equivalence means that the left-hand side of the equation is equal to the right-hand side. This means you must perform the same operation on both sides to maintain the equality or balance of the equation.

To solve an equation you can use **inverse operations** to change both sides of the equation until the unknown is on one side of the equation by itself. You need to use your knowledge of the order of operations, which you have used to build equations, to apply inverse operations in the correct order. Sometimes it helps to reorder the terms in the original equation to better identify the sequence of operations.

Worked example 1

W.E. 1

Solve the following linear equations. Check your solutions by substitution.

(a) $3x - 5 = -11$ **(b)** $7 - 2x = 19$

Thinking	Working
(a) 1 Make x the subject by performing the inverse operations in the opposite order on both sides of the equation. (Add 5 then divide by 3.)	(a) $3x - 5 = -11$ $3x - 5 + 5 = -11 + 5$ $3x = -6$ $\frac{3x}{3} = \frac{-6}{3}$ $x = -2$
2 Check your solution by substitution.	Check: $\text{LHS} = 3 \times -2 - 5$ $= -6 - 5$ $= -11$ $= \text{RHS}$
(b) 1 You can reorder the equation to make the operations easier to follow. Be careful to keep the sign in front of each term correct.	(b) $7 - 2x = 19$ $-2x + 7 = 19$

2 Make x the subject by performing the inverse operations in the opposite order on both sides of the equation. (Subtract 7 then divide by -2.)

$$\begin{aligned} -2x + 7 - 7 &= 19 - 7 \\ -2x &= 12 \\ \frac{-2x}{-2} &= \frac{12}{-2} \\ x &= -6 \end{aligned}$$

3 Check your solution by substitution.

$$\begin{aligned} \text{Check: LHS} &= 7 - 2 \times -6 \\ &= 7 + 12 \\ &= 19 \\ &= \text{RHS} \end{aligned}$$

Worked example 2

W.E. 2

Solve the following linear equations. Check your solutions by substitution.

(a) $\frac{2x}{5} - 1 = 7$ **(b)** $\frac{2x + 4}{3} = 4$

Thinking

Working

(a) 1 Make x the subject by performing the inverse operations in the opposite order on both sides of the equation. (Add 1, multiply by 5 and then divide by 2.)

(a)

$$\begin{aligned} \frac{2x}{5} - 1 &= 7 \\ \frac{2x}{5} - 1 + 1 &= 7 + 1 \\ \frac{2x}{5} &= 8 \\ \frac{2x}{5} \times 5 &= 8 \times 5 \\ 2x &= 40 \\ \frac{2x}{2} &= \frac{40}{2} \\ x &= 20 \end{aligned}$$

2 Check your solution by substitution.

$$\begin{aligned} \text{Check: LHS} &= \frac{2 \times 20}{5} - 1 \\ &= \frac{40}{5} - 1 \\ &= 8 - 1 \\ &= 7 \\ &= \text{RHS} \end{aligned}$$

(b) 1 Make x the subject by performing the inverse operations in the opposite order on both sides of the equation. (Multiply by 3, subtract 4 and then divide by 2.)

(b)

$$\begin{aligned} \frac{2x + 4}{3} &= 4 \\ \frac{3 \times (2x + 4)}{3} &= 4 \times 3 \\ 2x + 4 &= 12 \\ 2x + 4 - 4 &= 12 - 4 \\ 2x &= 8 \\ \frac{2x}{2} &= \frac{8}{2} \\ x &= 4 \end{aligned}$$

2 Check your solution by substitution.	Check: $\text{LHS} = \frac{2 \times 4 + 4}{3}$ $= \frac{8 + 4}{3}$ $= \frac{12}{3}$ $= 4$ $= \text{RHS}$

Worked example 3

W.E. 3

Solve the linear equation $5x - 2 = 3x + 8$. Check your solution by substitution.

Thinking	**Working**
1 Add or subtract terms containing the variable on both sides of the equation so that the variable term appears only on one side of the equation. (Subtract $3x$ from both sides in order to keep the coefficient of x positive.)	$5x - 2 = 3x + 8$ $5x - 3x - 2 = 3x - 3x + 8$ $2x - 2 = 8$
2 Make x the subject by performing the inverse operations in the opposite order on both sides of the equation. (Add 2 then divide by 2.)	$2x - 2 + 2 = 8 + 2$ $2x = 10$ $\frac{2x}{2} = \frac{10}{2}$ $x = 5$
3 Check your solution by substitution.	Check: $\text{LHS} = 5 \times 5 - 2$ $= 25 - 2$ $= 23$ $\text{RHS} = 3 \times 5 + 8$ $= 15 + 8$ $= 23$ $\text{LHS} = \text{RHS}$

Worked example 4

W.E. 4

Solve the linear equation $3(x + 4) = 2(1 - x)$. Check your solution by substitution.

Thinking	**Working**
1 Expand the brackets.	$3(x + 4) = 2(1 - x)$ $3x + 12 = 2 - 2x$
2 Add or subtract terms containing the variable on both sides of the equation so that the variable appears only on one side of the equation. (Add $2x$ to both sides in order to keep the coefficient of x positive.)	$3x + 2x + 12 = 2 - 2x + 2x$ $5x + 12 = 2$

3 Make *x* the subject by performing the inverse operations in the opposite order on both sides of the equation. (Subtract 12 then divide by 5.)	$5x + 12 - 12 = 2 - 12$ $5x = -10$ $\frac{5x}{5} = \frac{-10}{5}$ $x = -2$
4 Check your solution by substitution.	Check: $\text{LHS} = 3(-2 + 4)$ $= 6$ $\text{RHS} = 2(1 - -2)$ $= 6$ $\text{LHS} = \text{RHS}$

Equations containing fractions on both sides

To solve an equation containing a fraction on both sides:

- Express each term in the equation as a fraction with the same denominator.
- Multiply both sides of the equation by that denominator.

Worked example 5

W.E. 5

Solve the linear equation $\frac{2a - 1}{5} = \frac{a + 7}{4}$. Check your solution by substitution.

Thinking	Working
1 Eliminate the fractions by multiplying both sides by the denominators.	$\frac{2a - 1}{5} = \frac{a + 7}{4}$ $\frac{2a - 1}{5} \times 4 = \frac{a + 7}{4} \times 4$ $\frac{4(2a - 1)}{5} = a + 7$ $\frac{4(2a - 1)}{5} \times 5 = (a + 7) \times 5$ $4(2a - 1) = 5(a + 7)$
2 Expand the brackets.	$8a - 4 = 5a + 35$
3 Add or subtract terms containing the variable on both sides of the equation so that the variable appears only on one side of the equation. (Subtract 5*a* from both sides in order to keep the coefficient of *a* positive.)	$8a - 5a - 4 = 5a - 5a + 35$ $3a - 4 = 35$
4 Make *a* the subject by performing the inverse operations in the opposite order. (Add 4 then divide by 3.)	$3a - 4 + 4 = 35 + 4$ $3a = 39$ $\frac{3a}{3} = \frac{39}{3}$ $a = 13$

5 Check your solution by substitution.

Check: $\text{LHS} = \dfrac{2 \times 13 - 1}{5}$

$= \dfrac{26 - 1}{5}$

$= \dfrac{25}{5}$

$= 5$

$\text{RHS} = \dfrac{13 + 7}{4}$

$= \dfrac{20}{4}$

$= 5$

LHS = RHS

The steps shown in pink above do not need to be written out when the process is fully understood.

To solve an equation:

- If fractions are involved, express each term with a common denominator and then eliminate the fractions by multiplying each term on both sides by the common denominator.
- If brackets are involved, expand the brackets.
- If there is more than one term containing the unknown, collect those terms together.
- Identify the operations used to construct the equation and use inverse operations to deconstruct the equation in the opposite order to the way it was constructed.

5.1 Solving linear equations

Navigator

1 (column 1), 2 (column 1), 3 (column 1), 4 (column 1), 5 (column 1), 6 (column 1), 7 (a, b), 8 (a–c), 9, 10 (a, b), 11 (a–c), 12 (a–f), 13 (a–c), 14, 17 (a), 20	1 (column 2), 2 (column 2), 3 (column 2), 4 (column 2), 5 (column 2), 6 (column 2), 7 (b, c), 8, 9, 10, 11, 12 (d–i), 13 (d–i), 14, 15, 17 (a, b), 18 (a), 20, 21	1 (column 3), 2 (column 3), 3 (column 3), 4 (column 3), 5 (column 3), 6 (column 3), 7 (c, d), 8 (d–f), 9, 10, 11 (d–f), 12 (g–l), 13 (f–i), 14, 15, 16, 17, 18, 19, 20, 21

Answers p. 648

Fluency

W.E. 1

1 Solve the following linear equations. Check your solutions by substitution.

(a) $x + 5 = 9$
(b) $5b + 3 = 28$
(c) $6x + 7 = 1$
(d) $h - 2 = 5$
(e) $3f - 2 = -14$
(f) $5k - 3 = -1$
(g) $15 = a + 10$
(h) $-5 = 2x + 13$
(i) $-2 = 9y - 2$
(j) $-3b + 14 = -4$
(k) $-2a - 4 = 0$
(l) $-5b - 6 = 9$
(m) $-7 - p = 8$
(n) $15 - 4e = -1$
(o) $9 - 4g = -23$
(p) $4 = 17 - 5y$
(q) $-3 = 8 - 4p$
(r) $-5 = 11 - 8y$

W.E. 2

2 Solve these linear equations. Check your solutions by substitution.

(a) $\frac{x}{4} + 2 = 7$
(b) $\frac{3x}{7} + 5 = 8$
(c) $\frac{5x}{3} - 8 = 7$
(d) $3 - \frac{x}{3} = 8$
(e) $8 - \frac{3x}{4} = -1$
(f) $12 - \frac{3x}{2} = 3$
(g) $\frac{x+1}{4} = 2$
(h) $\frac{3x+1}{5} = -1$
(i) $\frac{7x+4}{6} = 3$
(j) $\frac{2-x}{3} = -1$
(k) $\frac{2-3x}{5} = 1$
(l) $\frac{9-4x}{3} = -5$
(m) $\frac{2x-3}{7} = 5$
(n) $\frac{7x-4}{5} = -12$
(o) $\frac{3x-1}{8} = -2$

W.E. 3

3 Solve the following linear equations. Check your solutions by substitution.

(a) $2c - 2 = c$
(b) $4b + 1 = 2b - 15$
(c) $9f + 5 = 5f - 3$
(d) $4d - 1 = d + 8$
(e) $5e - 3 = 2e + 9$
(f) $3a - 5 = a + 7$
(g) $3g + 10 = 4 + g$
(h) $2h - 7 = 13 + h$
(i) $4x - 3 = 6 + x$
(j) $k + 6 = 9 - k$
(k) $10j + 1 = 25 - 2j$
(l) $3y - 22 = -4 - 3y$
(m) $2m + 5 = 4m - 3$
(n) $5 + 4n = 7n - 1$
(o) $7x - 4 = 9x - 6$
(p) $2 - y = 3 - 2y$
(q) $19 - p = 16 - 2p$
(r) $6 + 3x = 21 - 2x$
(s) $15a - 17 = 16a - 19$
(t) $14 - 3x = 5 - 2x$
(u) $30 - p = 5p + 6$

W.E. 4

4 Solve the following linear equations. Check your solutions by substitution.

(a) $2(x + 1) = x + 4$
(b) $7(x + 4) = 3(x - 4)$
(c) $4(x - 5) = 5(3x + 7)$
(d) $3(x - 2) = 2(x + 3)$
(e) $5(2x + 3) = 2(3x + 1)$
(f) $4(3x - 2) = 3(2x + 9)$
(g) $2(3x + 1) = 4(3 - x)$
(h) $3(5x + 4) = 4(2 - 3x)$
(i) $4(3x - 11) = 5(29 - 3x)$

W.E. 5

5 Solve the following linear equations. Check your solutions by substitution.

(a) $b + 1 = \frac{b}{2}$
(b) $\frac{a-3}{2} = \frac{a+6}{3}$
(c) $\frac{c-4}{3} = c + 2$
(d) $\frac{d+5}{3} = \frac{d-1}{9}$
(e) $\frac{x-2}{3} = \frac{x+5}{4}$
(f) $\frac{3y-2}{7} = \frac{y+6}{4}$
(g) $\frac{2g-6}{6} = \frac{g+2}{4}$
(h) $\frac{7k-3}{4} = \frac{13-5k}{8}$
(i) $\frac{4-m}{5} = \frac{2-m}{6}$
(j) $\frac{2-3j}{5} = \frac{3-4j}{6}$
(k) $\frac{x+2}{3} = x - 1$
(l) $\frac{6p+4}{11} = p + 9$
(m) $\frac{2(x-1)}{2} = \frac{3x+2}{4}$
(n) $\frac{4(2y+3)}{5} = \frac{5(2-y)}{2}$

6 Solve these equations. Check your solutions by substitution.

(a) $\frac{x}{2} - 3 = 0$
(b) $\frac{x}{8} - 6 = -5$
(c) $\frac{x}{9} + 3 = 3$

(d) $3 - \frac{x}{4} = 12$　**(e)** $5 - \frac{x}{7} = 11$　**(f)** $-4 - \frac{x}{3} = -8$

(g) $\frac{4x}{5} = 3$　**(h)** $\frac{7x}{13} = 0$　**(i)** $\frac{2x}{7} = -4$

(j) $\frac{x+3}{2} = 9$　**(k)** $\frac{x+7}{3} = 5$　**(l)** $\frac{x+5}{9} = 3$

(m) $\frac{x-6}{2} = 8$　**(n)** $\frac{x-9}{8} = 6$　**(o)** $\frac{x-4}{3} = 17$

7 **(a)** To find a from $2a + 7$ you would need to:

A add 7 then divide by 2　**B** subtract 7 then divide by 2
C divide by 2 then subtract 7　**D** multiply by 2 then add 7

(b) To find p from $5p - 4$ you would need to:

A add 4 then multiply by 5　**B** add 4 then divide by 5
C divide by 5 then add 4　**D** multiply by 5 then add 4

(c) To find x from $-6x - 11$ you would need to:

A add 11 then multiply by -6　**B** multiply by -6 then add 11
C add 11 then divide by -6　**D** subtract 11 then divide by -6

(d) To find n from $1 - 4n$ you would need to:

A add 4 then multiply by 1　**B** add 1 then divide by -4
C subtract 1 then divide by -4　**D** subtract 1 then multiply by -4

8 **(a)** The equation $2z + 1 = 11$ has the solution:

A $z = 4$　**B** $z = 5$　**C** $z = 6$　**D** $z = -5$

(b) The equation $4x - 5 = 3$ has the solution:

A $x = 3$　**B** $x = -1$　**C** $x = 2$　**D** $x = 0$

(c) The equation $7 - 3q = 22$ has the solution:

A $q = 4$　**B** $q = -3$　**C** $q = 5$　**D** $q = -5$

(d) The equation $5 - 8n = -11$ has the solution:

A $n = 2$　**B** $n = -3$　**C** $n = 0$　**D** $n = -2$

(e) The equation $\frac{r}{3} - 4 = 2$ has the solution:

A $r = 10$　**B** $r = -6$　**C** $r = 9$　**D** $r = 18$

(f) The equation $\frac{a+3}{4} = 6$ has the solution:

A $a = 5$　**B** $a = 7$　**C** $a = 21$　**D** $a = 27$

9 **(a)** The linear equation $\frac{2a}{5} - 3 = 1$ has the solution:

A $a = -10$　**B** $a = -5$　**C** $a = 4$　**D** $a = 10$

(b) The linear equation $\frac{4y}{3} + 7 = -1$ has the solution:

A $y = -9$　**B** $y = -6$　**C** $y = 6$　**D** $y = 9$

10 **(a)** $9x + 4 = 2x - 3$ has the solution:

A $x = 1$　**B** $x = -1$　**C** $x = 2$　**D** $x = 4$

(b) $2y + 7 = -8 - 3y$ has the solution:

A $y = 2$　**B** $y = 1$　**C** $y = -2$　**D** $y = -3$

(c) $4(3k - 2) = 5(k + 4)$ has the solution:

A $k = 3$ **B** $k = -2$ **C** $k = 1$ **D** $k = 4$

(d) $\frac{p - 2}{2} = \frac{p + 1}{3}$ has the solution:

A $p = 8$ **B** $p = 5$ **C** $p = -4$ **D** $p = 0$

Understanding

11 Write an equation for each of the following and then solve the equation.

(a) 7 is added to x to give a result of 19.

(b) 5 is subtracted from y to give a result of 3.

(c) Three times a certain number is 14.

(d) One-quarter of a certain number is -5.

(e) Three more than two times a number is 13.

(f) 7 less than four times a number is -12.

12 Solve these equations. (First simplify each equation.)

(a) $3x + 2x + 7 = 32$ **(b)** $2x + x - 4 = -10$ **(c)** $6x + 3x - 4 = 14$

(d) $3(x + 5) + 8 = -4$ **(e)** $2(x + 3) + 1 = -5$ **(f)** $5(x - 2) - 3 = 7$

(g) $3(7 - x) - 6 = 0$ **(h)** $8(2 - x) + 1 = 9$ **(i)** $-4(3 - x) + 8 = -12$

(j) $-2(1 - 4x) - 3x = -2$ **(k)** $-7(5 - 2x) - 6x = 13$ **(l)** $-(8 - 5x) + 4x = -26$

13 Solve these linear equations.

(a) $\frac{4(2x + 3)}{7} = 1$ **(b)** $\frac{3(4x + 1)}{5} = -4$ **(c)** $\frac{5(3x + 2)}{2} = 1$

(d) $\frac{2(x - 1)}{3} = 5$ **(e)** $\frac{4(3x - 2)}{5} = -1$ **(f)** $\frac{7(2x + 1)}{2} = -2$

(g) $\frac{6(1 - 4x)}{15} = \frac{2}{3}$ **(h)** $\frac{3(2 - 5x)}{8} = -\frac{1}{2}$ **(i)** $\frac{3(7 - 2x)}{16} = \frac{1}{4}$

14 Adding 4 to twice a certain number gives the same result as subtracting 2 from the same number. Write an equation and solve it to find the unknown number.

15 A certain number when added to 36 gives the same result as adding 6 to this number and doubling the result. Write an equation and solve it to find the unknown number.

16 Half of the result of three times a certain number added to one is the same as one-fifth of the result when 4 is subtracted from this certain number. Write an equation and solve it to find the unknown number.

Reasoning

Hint

n does not have to be the first number. Three consecutive numbers could be $n - 1$, n and $n + 1$

17 Consecutive numbers, such as 6, 7 and 8, can be represented algebraically by n, $n + 1$ and $n + 2$, because to get from one number to the next consecutive number you simply add 1. Similarly, three consecutive odd numbers can be represented algebraically by n, $n + 2$ and $n + 4$ where n is an odd number. Using this information, write each of the following as an equation and solve to find the unknown numbers.

(a) Find the three consecutive odd numbers that sum to 195.

(b) Find the four consecutive even numbers that sum to 204.

(c) Find the six consecutive multiples of seven that sum to 609.

18 Some ancient Egyptians solved equations using a method of guessing and checking. This method is called the 'rule of false position'.

For example, consider the equation $x + \frac{x}{9} = 80$, which ancient Egyptians could have said as 'a number added to one-ninth of itself is 80'.

First, choose a possible value for x that is divisible by 9; for example, 9. Substitute this into the equation.

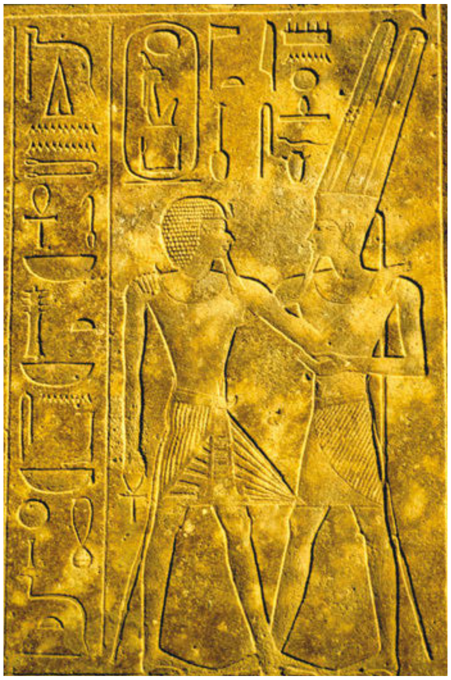
$$9 + \frac{9}{9} = 9 + 1 = 10$$

Because the answer you want is eight times what you got, try a number eight times the value of the number already tried.

$$9 \times 8 = 72$$

$$72 + \frac{72}{9} = 72 + 8 = 80$$

So the solution to the equation 'a number added to one-ninth of itself is 80' is 72.



For the following questions, form an equation, then use the rule of false position to solve it.

(a) A number added to one-sixth of itself is 28.

(b) A number added to one-quarter of itself is 45.

(c) A number added to one-eighth of itself is 99.

Open-ended

19 On a photocopied test, some numbers are not clear. One equation reads $2x + 7 =$ but the number on the right-hand side is unreadable. Given that x is a whole negative number, give three examples of what the number on the right-hand side could be.

20 Use at least three mathematical operations to make up three different equations that have a solution of 4.

21 Make up three equations similar to those in Question **3**, where the solution is a positive whole number.

Problem solving

Burning down

Two candles are lit. Each candle burns at a different rate and one is 3 cm taller than the other.

The taller candle is lit at 3:30 pm and the shorter one at 5 pm.

At 7:30 pm they are both the same height.

The taller candle burns out at 9:30 pm, while the shorter candle burns out at 9 pm.

How tall was each candle originally?

Strategy options

- Make a model.
- Draw a diagram.
- Guess, check and improve.
- Break into manageable parts.

Gamespace

a day out

You've got a fun day planned with your friends: movies, shopping, lunch and more. Will it turn out to be a good day or a bad day?

Equipment required: 1 die, a counter for each player

How to win: The aim of the game is to be the first to return home with the 'best day' and some points left, or to be the only one still left in the game.

START

You lose your wallet

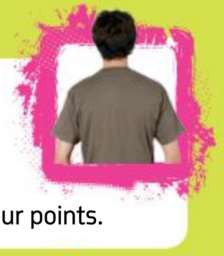
$2x - 1 = \bigstar$

Solve for x and deduct this value from your points.

You tap your 'friend' on the shoulder; the person turns around and it's a stranger

$\frac{x}{\bigstar} + 4 = 6$

Solve for x and deduct this value from your points.



You wave at a friend

$\frac{3 - x}{\bigstar} = 1$

Solve for x. If your answer is *positive*, your friend smiles and waves. Take the *positive* path. If your answer is *zero*, your friend smiles but doesn't wave. Wait for the next turn and roll again. If your answer is *negative*, your friend frowns at you. Take the *negative* path.

You can't see your friends' comments on social media because your smartphone is broken.

$\frac{2(x + \bigstar)}{3} = 10$

Solve for x and deduct this value from your points.

You spill your milkshake onto yourself

$\frac{12 + \bigstar \times x}{7} = x$

Solve for x and deduct this value from your points.

You receive a text from home asking why a $500 phone bill has arrived

$3y - 2y + \bigstar = 9$

Solve for y and deduct this value from your points.

You read a message received on your phone

$\frac{x + 5}{3} = \frac{9 - \bigstar}{3}$

Solve for x. If your answer is *positive* or *zero*, it's your phone. Take the *positive* path. If your answer is *negative*, it's someone else's phone. Take the *negative* path.

You lose your new smartphone

$-36r + 2 + 3 \times \bigstar = -33r - 4$

Solve for r and deduct this value from your points.

How to play: *Every player begins with 50 'best day' points; if you lose all your points you are out. Each player begins on START. Decide who will go first. You do not need to roll to move position; each player moves from event to event along the path.*

At each event, roll the die, substitute the value rolled for the star ★ in the equation and deduct the result from your reputation points.

At certain points where the path branches, your result will determine in which direction you must next move. Follow the instructions in the 'event' box. If you calculate a decimal answer, round it to the nearest whole number. When you are asked to roll a die twice, a represents the number rolled first and b the number rolled second.

You go to the hairdresser and end up with an 80s 'mullet'

$$\frac{5 \times \bigstar - z}{2} = \frac{7 \times \bigstar + 1}{4}$$

Solve for z and deduct this value from your points.

You have to babysit your little brother for the rest of the day

$9 - 5x = 4 \times \bigstar - 7y + 9$

Solve for y and deduct this value from your points.

You pass the person you have a crush on in the shopping centre, but realise too late you have snot hanging from your nose

$3p + 2 = 5p - 5 \times \bigstar$

Solve for p and deduct this value from your points.

Your favourite shop has closed down

$2 - q = 2 \times \bigstar - 2q$

Solve for q and deduct this value from your points.

Your dad says you can't stay at your friend's house because you haven't cleaned your room

$3(f - 2) = 3 \times \bigstar + 3$

Solve for f and deduct this value from your points.

Your mum comes up and kisses you on the cheek in front of your friends

$15n + 1 = 17n - \bigstar$

Solve for n and deduct this value from your points.

You watch a movie. Did you enjoy it?

Roll the die twice and substitute the values into the expression $(a - b)^2$. (a = first roll, b = second roll). If the total is more than 10, the movie was awful. Move left. If the total is less than 10, the movie was great! Move right.

LEFT

RIGHT

All the toilets are 'out of order'

$$\frac{5(x + 4)}{3} = \frac{14 \times \bigstar + 28}{3}$$

Solve for x and deduct this value from your points.

Waiting for the train

Roll the die twice and substitute the values into the expression $2b - a^2$. (a = first roll, b = second roll)

If the total is positive, your train has arrived. Go home.

If the total is zero or negative, you'll need to continue to wait for the train and try again on your next move.

HOME

5.2

Solving problems using linear equations

Many problems can be solved by setting up and solving an equation.

To solve a problem with an equation:

- Determine what the unknown(s) you need to find are and represent each with a variable.
- Form an equation, using the key words in the statement of the problem.
- Solve the equation using inverse operations.
- Answer the question in words.
- Check that your answers make sense and satisfy all conditions of the problem.

Worked example 6

W.E. 6

Mira is twice Sanjin's age. How old are Mira and Sanjin now if in 10 years the sum of their ages will be 89? Use *s* to represent Sanjin's age.

Thinking	Working
1 Represent one unknown with the prescribed variable.	Sanjin's present age $= s$
2 Identify the other unknown and decide what the relationship between the two unknown values is. Write the required expression.	Mira's present age $= 2s$
3 Decide what operation needs to be performed on each unknown value and write the required expression.	In 10 years: Sanjin's age $= s + 10$ Mira's age $= 2s + 10$
4 Use the information in the question to write an equation.	$(s + 10) + (2s + 10) = 89$
5 Simplify the equation.	$3s + 20 = 89$
6 Use inverse operations in the opposite order to solve the equation.	$3s = 69$ $s = 23$
7 Use the solution to the equation to find the values of all variables.	$2s = 2 \times 23 = 46$
8 Answer the question in words.	Sanjin is 23 years old and Mira is 46 years old.
9 Check that your answers make sense and satisfy all conditions of the problem.	In 10 years, they will be 33 and 56 years old, respectively; $33 + 56 = 89$.

5.2 Solving problems using linear equations

Navigator

1, 2, 3, 6, 7, 8, 9, 10, 11, 13, 16	1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16

Answers p. 649

Fluency

1 Jen's mum is 25 years older than Jen. Together their ages add up to 34. Write this as an equation using j to represent Jen's age. How old is Jen?

2 Kate's father is three times her age. How old are Kate and her father now if, in 4 years' time, the sum of their ages is 68? Use K to represent Kate's age.

W.E. 6

3 Andrew has x coins in his pocket. He tells Lori that she has five fewer coins than he has. If they have 17 coins between them, how many coins do they each have?

4 Huan is twice the age of Liu. If the difference of their ages is 8 years more than half of Liu's age, how old are Huan and Liu?

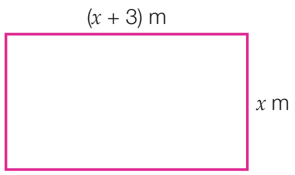
5 Choose the correct answer.

If $a = b + 2cx$, then:

A $x = \dfrac{a+b}{2c}$ B $x = \dfrac{a-b}{2c}$ C $x = \dfrac{a}{2c} - b$ D $x = a - \dfrac{2c}{b}$

Understanding

6 If the perimeter of this rectangular block of land is 46 m, find the length and width of the block.

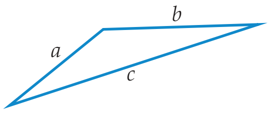


7 Three students take different routes home from school. Mark needs to walk twice as far as Annette, whereas Sue needs to walk four times as far as Annette. If the three cover a total distance of 2.1 km (or 2100 m), how far does each need to travel to get home from school?

8 Ben bought five ice-cream cones and received $1.75 change from his $10 note. How much did each ice-cream cone cost?

9 Ahn has 63 marbles. She has $3\frac{1}{2}$ times as many marbles as her friend Bianca. Write this as an equation using b to represent the number of Bianca's marbles. Use this equation to find the number of Bianca's marbles.

10 A scalene triangle is a triangle in which each side is of a different length (see diagram). The perimeter of a scalene triangle is 67 cm. The second side is 4 cm shorter than twice the length of the first side. The third side is 6 cm longer than two times the length of the first side. Calculate the lengths of the three sides of the triangle.



11 In 3 years' time Joanne will be twice as old as her daughter. If Joanne is 45 years old now, how old is her daughter?

Reasoning

12 A triangle has angles of $(2x + 11)°$, $(3x - 2)°$ and $4x°$. Find each angle.

13 An isosceles triangle has a perimeter of 45 cm. Its equal sides each measure 6 cm longer than the base. How long is the base of the triangle?

Hint

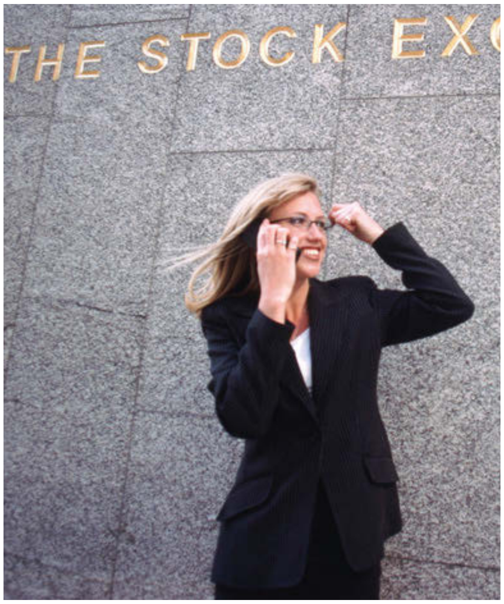
Let one of the ages be represented by a variable and express the other ages in terms of this variable.

14 A carton of iced coffee costs \$1 more than a large bottle of mineral water. If two iced coffees and three mineral waters cost \$7 more than the price of three iced coffees, find the cost of each type of drink.

15 A stock market analyst was investigating the trends of two independent stocks and was surprised to see that both stocks were at the same unit value after 12 months. She knew that both stocks had started at the same value, but had gone up and down differently during the past year.

One stock had risen to three times its original unit value, then lost \$8 before finishing at $\frac{3}{4}$ of this reduced unit value. The other stock added \$3 to its original unit value before crashing to $\frac{1}{5}$ of its improved unit value. It then showed a late improvement by increasing to seven times the reduced unit value.

Express this information as an equation, then solve it to find the original unit value of the stocks.



Open-ended

16 The sum of three consecutive whole numbers is a number between 70 and 100. Give three possible starting numbers for the group of three consecutive whole numbers.

Problem solving

Fractions in fractions in fractions

The fraction $\frac{2008}{1998}$ can be written in the form $a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d}}}$ where a, b, c and d are all positive integers.

Can you find the value of d?

Hint

- Write the fraction $\frac{2008}{1998}$ as a mixed number.

  This should help you find the value of a.

- $\frac{x}{y} = \frac{1}{\left(\frac{y}{x}\right)}$ may be useful.

Strategy options

- Break into manageable parts.
- Solve a simpler problem.
- Work backwards.

Coordinate geometry

5.3

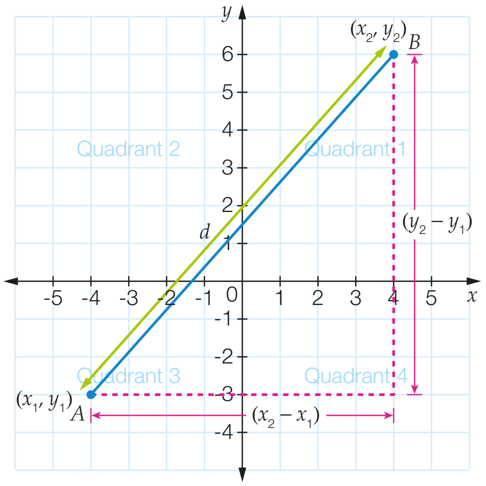
Coordinate geometry uses the axes on a Cartesian plane to locate points, lines, curves and shapes and find relationships between them.

- The horizontal axis is known as the x-axis and the vertical axis is known as the y-axis.
- The point of intersection of these axes is called the **origin**, which has the coordinates (0, 0).
- All points can be described by an ordered pair known as **coordinates** (x, y) as measured from the origin.
- An **interval** is the line segment formed when two points are joined.

Coordinate geometry is used to find relationships between points, such as the distance between two points, the midpoint of two points, the gradient and equation of lines and other curves, as well as the point of intersection of curves and lines. Coordinate geometry allows you to use algebraic and graphical techniques to solve geometric problems. In this section you will use coordinate geometry to explore linear relationships.

The distance formula

To find the distance between two points, use the Cartesian coordinate system because the horizontal and vertical axes are at right angles to each other.



In the diagram you have the points (x_1, y_1) and (x_2, y_2). To find the distance between these two points, connect them with a straight line. If the two points are $A(x_1, y_1)$ and $B(x_2, y_2)$, the horizontal distance is given by $(x_2 - x_1)$, the difference in the x-coordinates and the vertical distance is given by $(y_2 - y_1)$, the difference in the y-coordinates.

- The difference between the x-coordinates $(x_2 - x_1)$ gives the length of one side of a right-angled triangle.
- The difference between the y-coordinates $(y_2 - y_1)$ gives the length of another side.
- These two lines form two of the sides of a right-angled triangle, with the distance between the two points, d, the length of the hypotenuse.
- Pythagoras' theorem is used to find the distance between these two points.

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore\ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Worked example 7

W.E. 7

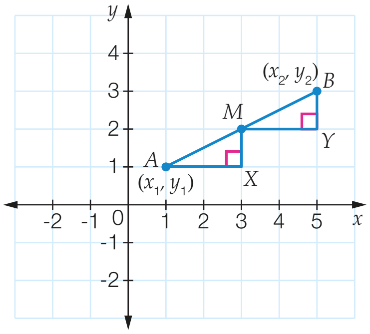
Find the distance between the following Cartesian coordinates, giving your answers in exact simplest form.

(a) (-4, 6) and (2, -2) **(b)** (-2, 1) and (2, -4) **(c)** (3, 5) and (5, 2)

Thinking	Working
(a) **1** Identify the two points.	(a) Let $(x_1, y_1) = (-4, 6)$ and $(x_2, y_2) = (2, -2)$
2 Write the distance formula and substitute in the relevant values. (Here $x_1 = -4$, $x_2 = 2$, $y_1 = 6$, $y_2 = -2$.)	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(2 - -4)^2 + (-2 - 6)^2}$ $= \sqrt{6^2 + (-8)^2}$ $= \sqrt{36 + 64}$ $= \sqrt{100}$
3 Evaluate and include units.	$= 10$ units
(b) **1** Identify the two points.	(b) Let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (2, -4)$
2 Write the distance formula and substitute in the relevant values. (Here $x_1 = -2$, $x_2 = 2$, $y_1 = 1$ and $y_2 = -4$.)	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(2 - -2)^2 + (-4 - 1)^2}$ $= \sqrt{4^2 + (-5)^2}$
3 Evaluate, leaving the answer in exact surd form and including units.	$= \sqrt{41}$ units
(c) **1** Identify the two points.	(c) Let $(x_1, y_1) = (3, 5)$ and $(x_2, y_2) = (5, 2)$
2 Write the distance formula and substitute in the relevant value. (Here $x_1 = 3$, $x_2 = 5$, $y_1 = 5$ and $y_2 = 1$.)	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $= \sqrt{(5 - 3)^2 + (2 - 5)^2}$ $= \sqrt{(2)^2 + (-3)^2}$ $= \sqrt{4 + 9}$
3 Evaluate by simplifying the surd value, leaving the answer in exact surd form and including units.	$= \sqrt{13}$ units

The midpoint formula

The midpoint of an interval is half-way between the two given coordinates that define that interval. The coordinates of the midpoint between two points on a Cartesian plane are found by using the average of each of the *x*- and *y*-coordinates. This formula can be derived using congruent triangle properties because ΔAXM and ΔMYB are congruent using SAS congruency.



The **midpoint**, $M(x, y)$, between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by:

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Worked example 8

W.E. 8

Find the coordinates of the midpoint between (-4, 6) and (2, -2).

Thinking	Working
1 Identify the two points.	Let $(x_1, y_1) = (-4, 6)$ and $(x_2, y_2) = (2, -2)$
2 Write the midpoint formula and substitute in the relevant values.	$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ $M(x, y) = \left(\frac{-4 + 2}{2}, \frac{6 + -2}{2}\right)$
3 Evaluate.	$M(x, y) = \left(\frac{-2}{2}, \frac{4}{2}\right)$ $M(x, y) = (-1, 2)$

The distance, d, between any two Cartesian coordinates (x_1, y_1) and (x_2, y_2) is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The midpoint $M(x, y)$ between two points (x_1, y_1) and (x_2, y_2) is given by:

$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

5.3 Coordinate geometry

Navigator

1 (column 1), 2 (column 1), 3, 4, 5, 6 (a, b), 7, 8, 9, 13, 15	1 (column 2), 2 (column 2), 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15	1 (column 3), 2 (column 3), 3, 4, 5, 6 (c), 7, 8, 10, 11, 12, 13, 14, 15, 16

Answers p. 649

Fluency

1 Find the distance between the following Cartesian coordinates, giving your answers in exact simplest form.

W.E. 7

(a) (0, 0) and (3, 4) **(b)** (-4, 2) and (8, -3) **(c)** (-4, 13) and (4, -2)

(d) (0, 0) and (8, 6) **(e)** (-4, 5) and (4, -1) **(f)** (-11, -4) and (13, 3)

(g) (1, 2) and (2, 5) **(h)** (-1, 3) and (5, 2) **(i)** (4, -3) and (-2, 2)

(j) (9, 13) and (16, 15) **(k)** (3, -4) and (-1, 2) **(l)** (2, -7) and (6, 1)

2 Find the coordinates of the midpoints between the following Cartesian points.

W.E. 8

(a) (0, 0) and (4, 8) **(b)** (-3, 5) and (1, 3) **(c)** (-9, -11) and (7, -3)

(d) (0, 0) and (15, 19) **(e)** (-5, 8) and (3, 7) **(f)** (-2, -4) and (5, 3)

(g) (1, 2) and (6, 7) **(h)** (-2, -5) and (7, 0) **(i)** (0, 0) and (9, 11)

3 The distance between (-4, -5) and (4, 1) is:

A 4	**B** 5	**C** 10	**D** 9

4 A line *CD* is drawn from the origin at *C* to point *D* at (8, 15). Find the length of *CD*.

5 A line *AB* is drawn from point *A* at (-2, 10) to point *B*(4, 1). Point *C* lies on the line *AB* half-way between points *A* and *B*. Find the coordinates of point *C*.

Understanding

6 **(a)** The midpoint of *CD* is (6, 5). If the coordinates of point *C* are (3, 8), find the coordinates of point *D*.

(b) The midpoint of *EF* is (-1, -2). If the coordinates of point *E* are (-7, 6), find the coordinates of point *F*.

(c) The midpoint of *AB* is (2, -4). If the coordinates of point *A* are (-5, -8), what are the coordinates of point *B*?

7 Use the distance formula and Pythagoras' theorem to show that the points *A*(-3, -2), *B*(1, 1) and *C*(-2, 5) form an isosceles right-angled triangle.

8 A triangle has the vertices (0, 5), (3, 3) and (1, 0).

(a) Find the length of each side, correct to 2 decimal places.

(b) What type of triangle is it?

Reasoning

Hint

Plot the known points on a Cartesian plane.

9 Use the distance formula to show that points *A*(-5, 24) and *B*(-5, 0) are the same distance from point *C*(0, 12).

10 The proposed course for a yacht race is mapped out with buoys positioned at *A*(-3, -4), *B*(-1, 3) and *C*(9, 8), but the position of the last buoy *D*(*x*, *y*) is not finalised. The race will begin at *O*(0, 0), proceed through buoys *A*, *B*, *C* and *D* and finish back at the starting point *O*.

(a) If the coordinates indicate distances measured in kilometres, what is the distance from the starting position *O* to the first buoy *A*?

(b) The judge's boat must be placed in the middle between buoys *A* and *C*. What are the coordinates of the judge's boat?

(c) If the judge's boat is also in the middle between buoys *B* and *D*, then what are the coordinates (*x*, *y*) of buoy *D*?

(d) What is the greatest distance that the yachts must travel between any two successive buoys?

(e) What geometric shape is formed by the buoys *ABCD*?

11 A quadrilateral has the vertices *A*(0, -2), *B*(-2, -6), *C*(-6, -8) and *D*(-4, -4).

(a) Find the midpoints of the diagonals.

(b) Find the length of each side.

(c) Find the length of each diagonal.

(d) What type of quadrilateral is it?

12 Prove and explain how the points *A*(3, 5), *B*(5, 1), *C*(-1, -2) and *D*(-3, 2) are the vertices of a rectangle.

13 If *E*(1, 6), *F*(-4, 3) and *G*(-1, -2) are the vertices of a square, find the missing vertex.

14 Points *A*(-6, 3), *B*(2, 5), *C*(0, -1) and *D*(-2, 1) are joined to form a quadrilateral. The midpoints of *AB*, *BC*, *CD*, and *DA* are joined to form another quadrilateral *PQRS*. Show that *PQRS* is a parallelogram.

Open-ended

15 Choose three points that would form an equilateral triangle when joined in order and justify your choice.

16 Choose four points that would form a kite when joined in order and justify your choice.

Puzzle

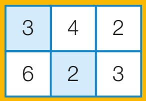
Hitori

Copy the following grid of numbers.

No number should appear more than once in any row or column. Your aim is to shade out numbers, to 'delete' them, so that this is true.

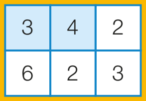
1	4	3	3	1	2
1	2	2	4	6	3
2	6	1	3	5	2
3	2	6	4	2	1
4	3	2	6	5	1
6	3	4	4	3	2

The cells that you cross out cannot be beside each other vertically or horizontally (diagonally is OK).



3	4	2
6	2	3

This shading is correct.



3	4	2
6	2	3

This shading is incorrect.

The cells that are left unshaded must form a single area. That is, you must be able to move from any unshaded cell to the next unshaded cell horizontally or vertically.

5.4 Plotting linear graphs

Linear relationships can be plotted on a Cartesian plane using x- and y-axes and a suitable scale. The relationship is used to construct a table of values, plot these points on axes and join them with a straight line.

Worked example 9

W.E. 9

For each of the following table of values:

(i) plot a graph to represent the information

(ii) write an equation to represent the linear relationship.

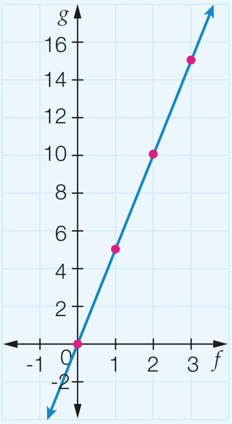
(a)

f	0	1	2	3
g	0	5	10	15

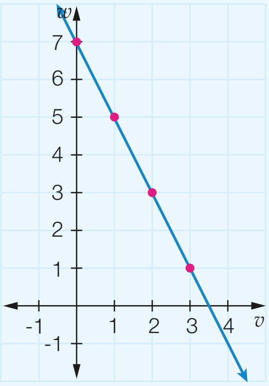
(b)

v	0	1	2	3
w	7	5	3	1

Thinking	**Working**
(a) **(i)** Rule up a set of Cartesian axes, choose a suitable scale for each axis, then number and label them. Plot the points from the table and join them with a ruler to form a straight line.	(a) (i)



(ii) Can the dependent variable be found from the independent variable with one operation (+, −, ×, ÷)? If the answer is yes, determine the operation and write the equation.	(ii) $g = 5f$
(b) **(i)** Rule up a set of Cartesian axes, choose a suitable scale for each axis, then number and label them. Plot the points from the table and join them with a ruler to form a straight line.	(b) (i)



(ii) **1** Can the dependent variable be found from the independent variable with one operation (+, –, ×, ÷)? If the answer is no, find the increase in the dependent variable (w) as the independent variable (v) increases by 1. A decrease is a negative increase (-2). This is the coefficient of our independent variable in the equation.

(ii) $-2v$ is in our equation

2 Decide what needs to be added or subtracted to find the equation.

7 needs to be added.

3 Write the equation.

$w = -2v + 7$

Worked example 10

W.E. 10

Use the equations given for each of the following to:

(i) copy and complete the table of values

(ii) plot the graph of each of the relationships.

(a) $M = 9n$

n	-3	0	3	6
M				

(b) $L = 5k - 8$

k	-4	0	4	8
L				

Thinking

Working

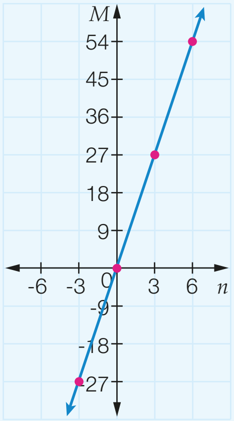
(a) **(i)** Use the rule to fill in the table.

(a) (i)

n	-3	0	3	6
M	-27	0	27	54

(ii) Draw and relabel the x- and y-axes as n and M. Choose a suitable scale for each axis, ensuring that equal increments are marked. Plot the points and join them with a straight line.

(ii)



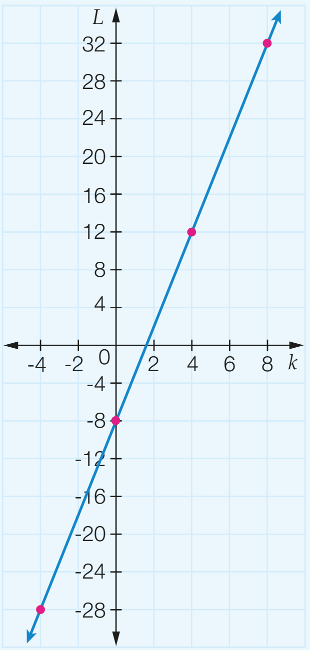
(b) **(i)** Use the rule to fill in the table.

(b) (i)

k	-4	0	4	8
L	-28	-8	12	32

(ii) Draw and relabel the x- and y-axes as k and L. Choose a suitable scale for each axis, ensuring that equal increments are marked. Plot the points and join them with a straight line.

(ii)



Worked example 11

W.E. 11

Lisa, a travel agent working for an airline, receives a base wage of \$160 each week and a bonus of \$40 for each flight ticket sold.

Number of tickets sold per week (n)	0	1	2	3	4	5
Weekly wage (W)						

(a) Copy and complete the table above.

(b) Plot a graph to show the information contained in the table.

(c) Write an equation to represent the information.

(d) If Lisa sold 12 tickets one week, how much did she earn that week?

Thinking

Working

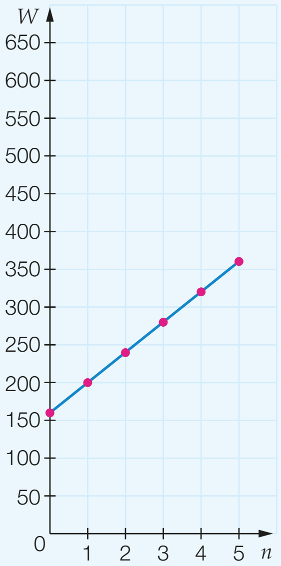
(a) Use the information to complete the table of values.

(a)

Number of tickets sold per week (n)	0	1	2	3	4	5
Weekly wage (W)	160	200	240	280	320	360

(b) Draw and relabel the x- and y-axes as n and W. Choose a suitable scale for each axis, ensuring that equal increments are marked. Plot the points and join them with a straight line.

(b)



(c) **1** By how much does the dependent variable (bonus) increase for each increase in the independent variable (number of tickets sold)? (Here, it is \$40.) This is the coefficient of the independent variable in the equation.

(c) $40n$ is in the equation.

2 What needs to be added or subtracted?

160 needs to be added to the equation.

3 Write the equation.

$W = 40n + 160$

(d) **1** Substitute the given value into the equation and evaluate. Alternatively, read off the value from the graph using 12 as the x-coordinate, then moving vertically to meet the line, and then horizontally to the y-axis to find the y-coordinate.

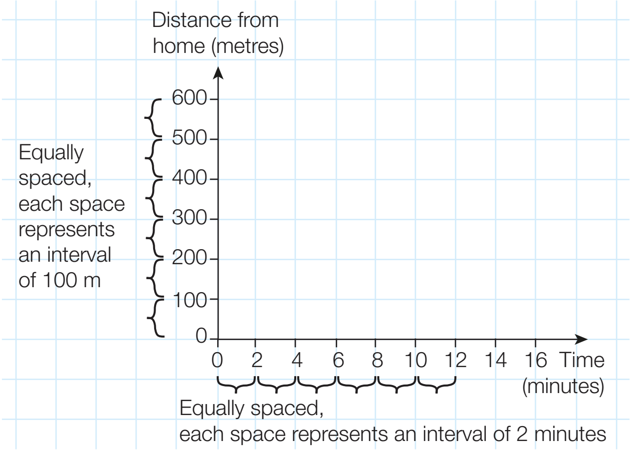
$W = 40n + 160$
$W = 40 \times 12 + 160$
$\quad = 480 + 160$
$\quad = 640$

2 Write your answer.

Lisa would earn \$640.

Plotting graphs accurately

Use graph paper. Each axis should have a clearly marked scale and a label. The scale may or may not be the same for each axis. On an axis, the scale divisions should be equally spaced with intervals representing equal amounts. An example is shown opposite.



Linear relationships can be represented both graphically (using graphs) and algebraically (using equations).
A linear relationship is a relationship between two variables that, when graphed, produces a straight line.

5.4 Plotting linear graphs

Navigator

Answers p. 650

1, 2 (column 1), 3, 4 (a, b), 5, 6, 7 (a–d), 8, 10, 11	1, 2 (column 2), 3, 4, 5, 6, 7, 9, 10, 11	1 (b), 2 (column 2), 3, 4 (d), 5, 6, 7, 8, 9, 10, 11

Equipment required: graph paper or grid paper

Fluency

W.E. 9

1 For each of the following tables of values:

(i) plot a graph to represent the information

(ii) write an equation to represent the linear relationship.

(a)

a	0	1	2	3
b	0	2	4	6

(b)

a	0	1	2	3
b	2	4	6	8

(c)

p	0	3	6	9
q	0	-3	-6	-9

(d)

c	0	3	6	9
d	5	2	-1	-4

W.E. 10

2 Use the equation given for each of the following to:

(i) copy and complete the table of values

(ii) plot the graph of each of the relationships.

(a) $A = 4L$

L	0	10	20	30
A				

(b) $Q = 5M$

M	0	2	4	6
Q				

(c) $J = 2G - 12$

G	0	4	8	12
J				

(d) $d = -6e + 5$

e	0	3	6	9
d				

(e) $B = \frac{3R}{5}$

R	-20	0	20	40
B				

(f) $C = \frac{5F}{6} - 4$

F	-6	0	6	12
C				

3 Hasif bought a wall clock on Sunday (day 0) and it showed the correct time. The next day (day 1) the clock had lost 3 seconds; on day 2, it lost another 3 seconds; on day 3, another 3 seconds; and so on.

W.E. 11

Day (*d*)	0	1	2	3	4	5
Total time lost (*t*)						

(a) Copy and complete the table above.

(b) Plot a graph to show the information contained in the table. Use the horizontal axis for the days and the vertical axis for the time lost (in seconds).

(c) Is the relationship linear?

(d) Write an equation to represent the information.

(e) How many seconds has the clock lost by the following Sunday?

4 Eddy, a television salesperson, receives a base wage of $80 each day and a bonus of $100 for each television he sells.

Number of televisions sold	0	1	2	3	4	5
Daily income ($)						

(a) Copy and complete the table above.

(b) Plot a graph to show the information contained in the table. Use the horizontal axis for the number of televisions sold and the vertical axis for daily income (dollars).

(c) Is the relationship linear?

(d) Write an equation to represent the information.

(e) If Eddy sold eight televisions in one day, how much did he earn that day?

5 **(a)** In the linear relationship $V = 3W - 14$, for $W = 7$ the value of V will be:

A -14 **B** -8 **C** -2 **D** 7

(b) In the linear relationship $P = \frac{3Q}{4} - 2$, for $Q = 8$ the value of P is equal to:

A 8 **B** 4 **C** 6 **D** -4

(c) In the linear relationship $C = \frac{5}{9}F - 32$, the value of C when F is equal to 63 is:

A -32 **B** -3 **C** 3 **D** 18

Understanding

6 A no-name brand detergent can be bought wholesale for 95c per litre. To supply information to customers quickly, the wholesaler has prepared a graph representing this information.

(a) Copy and complete this table.

Number of litres	0	1	2	3	4	5	6	7
Cost ($)								

(b) Plot the cost of the detergent against the number of litres of detergent bought. This is the wholesaler's graph. Use the horizontal axis for the number of litres and the vertical axis for the cost (dollars).

(c) Is the relationship linear? How can you tell?

(d) From your graph, how much would it cost to buy:

- **(i)** 3 L of detergent
- **(ii)** 2.4 L of detergent
- **(iii)** 5.7 L of detergent?

(e) From your graph, how many litres of detergent (to the nearest tenth of a litre) could be bought for:

- **(i)** $3.80
- **(ii)** $2.50
- **(iii)** $5.20?

(f) Write a relationship connecting the cost C (in dollars) to the amount of detergent d (in litres).

7 Steve is cycling at a constant speed of 185 metres per minute.

(a) Copy and complete the following table.

Time (minutes)	0	1	2	3	4	5	6	7
Distance cycled (metres)								

(b) Plot the distance against the time. Use the horizontal axis for time (minutes) and the vertical axis for distance cycled (metres).

(c) From the graph, how far does Steve cycle in 2.7 minutes?

(d) How long does it take Steve to cycle 700 metres?

(e) Extend your graph to find how long it takes Steve to cycle 2 km.

(f) Write the rule (or equation) connecting the distance cycled to the time taken.

(g) If Steve continued cycling, what distance would he have covered in 40 minutes?

Reasoning

8 Phuong works at the stock exchange buying and selling shares for different international and local companies. She needs to be able to convert quickly from one currency to another.

Hint

What is $A1.00 equivalent to in $US?

(a) If $20 in Australian currency (A$) converts to $21 in US currency (US$), draw a graph showing the relationship between the two currencies. Plot Australian currency on the horizontal axis and US currency on the vertical axis.

Use the graph to convert from one currency to the other for each of these amounts (rounding to the nearest 10c).

(b) A$7.00 to US$

(c) A$17.60 to US$

(d) US$5.50 to A$

(e) US$9.60 to A$

9 Wally, Sarah and Cleo are collecting money for an upcoming charity walk. Wally is asking for $1 for each kilometre he walks. Sarah, on the other hand, is asking for a $4 donation plus $0.50 per kilometre walked. Finally, Cleo is asking for a donation of $8 regardless of how many kilometres she walks.

(a) Draw up and complete a table of values to show the amount donated for each kilometre walked for Wally, Sarah and Cleo for distances (D) from 0 to 10 kilometres.

(b) Use your table of values to plot graphs showing the amount donated for each kilometre walked for Wally, Sarah and Cleo. Plot each graph on the same set of axes, with distance on the horizontal axis and amount donated on the vertical axis.

(c) From the graph and table of values, identify the distance for which all three collect the same amount of money.

(d) Identify the distances for which:

- **(i)** Wally raises the most money
- **(ii)** Sarah raises the most money
- **(iii)** Cleo raises the most money.

10 Frank and Bruno are two soccer players about to sign contracts with FC Linear. In the first year, Frank earns $120 000, which will increase by $60 000 for each successive year that he stays with the club. Bruno earns $80 000 in his first year, which increases by $80 000 for each successive year that he stays with the club. Explain in which years Frank will earn more than Bruno and in which years Bruno will earn more than Frank, assuming each stays with the club.

Open-ended

11 Draw three different linear relationships that show the same amount of distance travelled over different periods of time. Explain what happens to the graph as the time increases.

Problem solving

How many edges?

Equipment required: scissors, ruler and several pieces of A4 paper

Instructions: Divide an A4 sheet of paper into two pieces using a single straight cut. Then divide one of those pieces into two, also using a single straight cut. What is the maximum number of edges that the resulting three pieces can have in total? What would be the minimum number of edges of the three pieces?

Strategy options

- Act it out.
- Look for a pattern.
- Test all combinations.

Half-time 5

5.4

1 Decide which, if any, of the following tables represent linear relationships.

(a)

x	-1	0	1	2
y	-5	-3	-1	1

(b)

x	-2	0	2	4
y	5	2	-1	-3

(c)

x	0	1	2	3
y	0	1	1	2

(d)

x	-4	0	4	8
y	0	1	2	3

5.3

2 Find the midpoint of the line joining (6, 2) and (3, 8).

5.1

3 Solve the following linear equations.

(a) $2x - 5 = 13$ **(b)** $8 - 3y = -7$ **(c)** $\frac{3x}{4} - 2 = 10$

(d) $\frac{4x + 1}{5} = -7$ **(e)** $3x - 2 = 5x - 8$ **(f)** $5(x - 1) = 2(8 - x)$

(g) $4(3p - 5) = 5(7 - 2p)$ **(h)** $\frac{a + 11}{3} = \frac{3 - a}{4}$ **(i)** $\frac{3m - 5}{2} = \frac{2m + 25}{3}$

5.3

4 Find the distance between the points (-1, 7) and (4, -5).

5.2

5 Kiara and Rob are training to be weightlifters. Rob weighs 6 kg more than Kiara. Kiara can lift 6 kg less than one and a half times her weight. Rob can lift a quarter more than his weight. They find they can both lift the same weight. How much do Kiara and Rob each weigh?

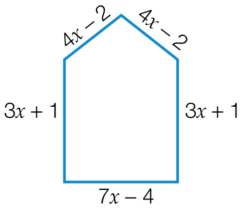
5.1

6 The equation $3x - 4 = 2(x - 5)$ has the solution:

A $x = -2$ **B** $x = -1$ **C** $x = -6$ **D** $x = 6$

5.2

7 The total perimeter of this shape is 36 m. Find x.



5.4

8 Use the equation $P = 3 - 4R$ to:

(a) copy and complete the following table of values

R	0	2	4	6
P				

(b) plot the graph of the relationship.

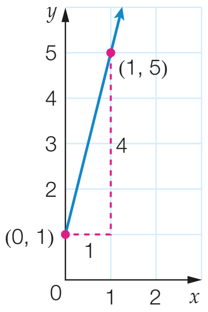
Gradient

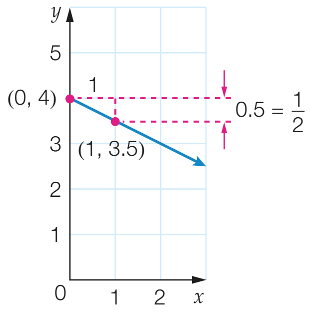
5.5

The measure of a line's steepness is called the **gradient** or **slope** of the line. The gradient of a line describes the change in the y-values as the x-value increases by 1.

For example:

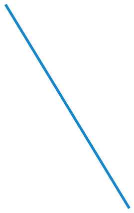
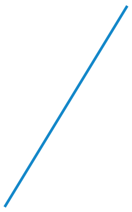
- a gradient of 4 means that as the x-value increases by 1, the y-value increases by 4.
- a gradient of $-\frac{1}{2}$ means that as the x-value increases by 1, the y-value decreases by $\frac{1}{2}$.





The size, or magnitude, of the gradient tells how steep the line is.

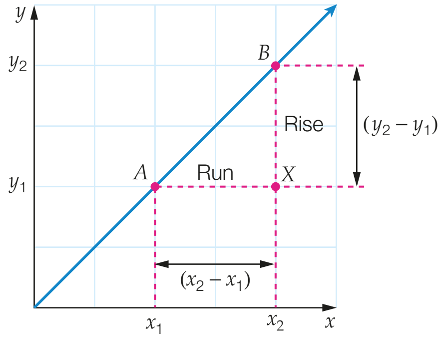
The sign of the gradient (positive or negative) tells whether the line slopes up or down as it goes from left to right.

Line				
Magnitude of gradient	Small	Large	Large	Small
Sign of gradient	Negative	Negative	Positive	Positive

The gradient of a line can be found by the ratio of the vertical **rise** to the horizontal **run** between any two points on the line.

To find the run, start at point A and move right horizontally (parallel to the x-axis) until you reach point X, which is in line vertically with point B. This change in x-value is the run.

To find the rise, start at point X and move straight up or down parallel to the y-axis until you reach point B. This change in the y-value is called the rise.



$$\text{Gradient } AB = \frac{\text{vertical rise}}{\text{horizontal run}}$$

The letter m is used to represent the gradient.

$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{\text{change in } y\text{-value}}{\text{change in } x\text{-value}}$$

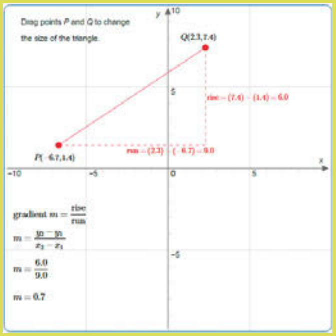
$$= \frac{y_2 - y_1}{x_2 - x_1}$$

Interactive

Gradient

Explore how the gradient of a line graph rises or falls as the values change.

Go to the eBook or the Pearson Places website to access this interactive.



Drag points P and Q to change the size of the triangle.

To keep the denominator always positive, choose (x_1, y_1) to be the point on the left and (x_2, y_2) to be the point on the right. If (x_2, y_2) is lower than the first, you move right first, then down. This means that the rise will be a negative number, which means the gradient of the line will also be negative.

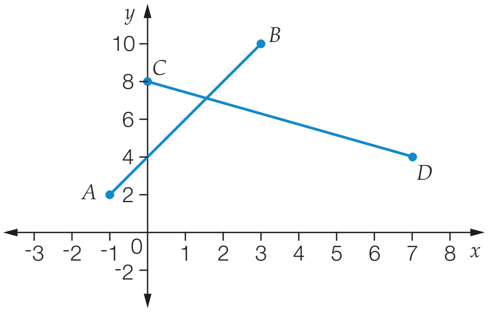
Worked example 12

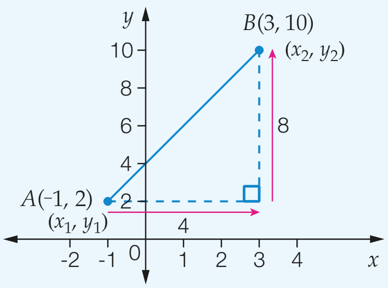
W.E. 12

Find the gradient of:

(a) the line segment AB

(b) the line segment CD.



Thinking	Working
(a) **1** Use the graph to identify the coordinates of (x_1, y_1) (Here $A = (-1, 2)$) and (x_2, y_2) (Here $B = (3, 10)$).	(a) 

2 Write the rule for finding the gradient.

$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

3 Substitute values into the rule and evaluate ($x_1, = -1$, $x_2 = 3$, $y_1 = 2$ and $y_2 = 10$).

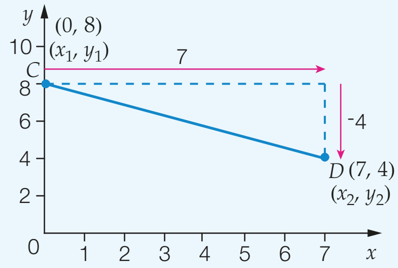
$$= \frac{10 - 2}{3 - -1}$$

$$= \frac{8}{4}$$

$$= 2$$

(b) 1 Use the graph to identify the coordinates of (x_1, y_1) (Here $C = (0, 8)$) and (x_2, y_2) (Here $D = (7, 4)$).

(b)



2 Write the rule for the gradient.

$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

3 Substitute values into the rule and evaluate ($x_1, = 0$, $x_2 = 7$, $y_1 = 8$ and $y_2 = 4$).

$$= \frac{4 - 8}{7 - 0}$$

$$= \frac{-4}{7}$$

$$= -\frac{4}{7}$$

Special cases for the gradient

There are two special cases for the gradient to consider: lines that are completely horizontal (flat) and lines that are completely vertical (straight up and down).

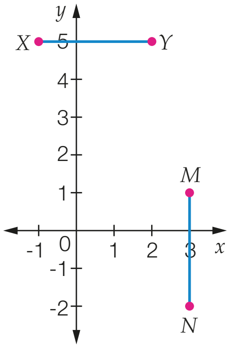
Worked example 13

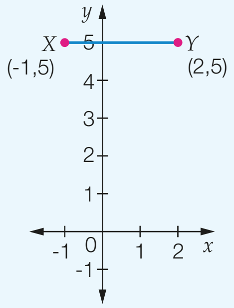
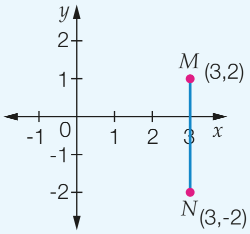
W.E. 13

Find the gradients of:

(a) the line segment *XY*

(b) the line segment *MN*.



Thinking	Working
(a) **1** Use the graph to identify the coordinates of (x_1, y_1) (Here $X = (-1, 5)$) and (x_2, y_2) (Here $Y = (2, 5)$).	(a) 
2 Write the rule to find the gradient.	$m = \frac{\text{rise}}{\text{run}}$ $= \frac{y_2 - y_1}{x_2 - x_1}$
3 Substitute values into the rule and evaluate ($x_1 = -1$, $x_2 = 2$, $y_1 = 5$ and $y_2 = 5$).	$= \frac{5 - 5}{2 - 1}$ $= \frac{0}{1}$ $= 0$ The gradient is equal to zero.
(b) **1** Use the graph to identify the coordinates of (x_1, y_1) (Here $M = (3, 1)$) and (x_2, y_2) (Here $N = (3, -2)$).	(b) 
2 Write the rule to find the gradient.	$m = \frac{\text{rise}}{\text{run}}$ $= \frac{y_2 - y_1}{x_2 - x_1}$
3 Substitute values into the rule and evaluate ($x_1 = 3$, $x_2 = 3$, $y_1 = 1$ and $y_2 = -2$).	$= \frac{-2 - 1}{3 - 3}$ $= \frac{-3}{0}$ This cannot be evaluated, so the gradient is undefined.

From the example **13 (a)** above, you can see that a horizontal line has a gradient of zero. In this worked example, you used the rule for finding the gradient and the points (1, 5) and (2, 5). If you used any two points that had the same y-coordinate, the line segment would be horizontal, the numerator in the gradient rule would always be 0 and the gradient would be zero.

From example **13 (b)**, you can see that a vertical line has an undefined gradient. In this worked example, you used the rule for finding the gradient and the points (3, 2) and (3, -1). The division cannot be evaluated, so it is undefined. If you used any two points that had the same x-coordinate, the line segment would be vertical, the denominator in the gradient rule must be zero and the gradient is **undefined**.

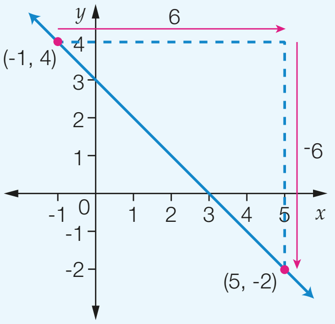
The gradient of any horizontal line is zero.

The gradient of any vertical line is undefined.

Worked example 14

W.E. 14

Find the gradient of the line joining (5, -2) and (-1, 4).

Thinking	Working
1 Draw a graph showing the line joining the points. Identify (x_1, y_1) and (x_2, y_2). (Here, $(x_1, y_1) = (-1, 4)$ and $(x_2, y_2) = (5, -2)$.)	
2 Write out the rule for finding the gradient.	$m = \frac{\text{rise}}{\text{run}}$ $= \frac{y_2 - y_1}{x_2 - x_1}$
3 Identify the values for x_1, y_1, x_2 and y_2 and substitute them into the rule and evaluate ($x_1 = 5$, $y_1 = -2$, $x_2 = -1$ and $y_2 = 4$).	$m = \frac{-2 - 4}{5 - -1}$ $= \frac{-6}{6}$ $= -1$

5.5 Gradient

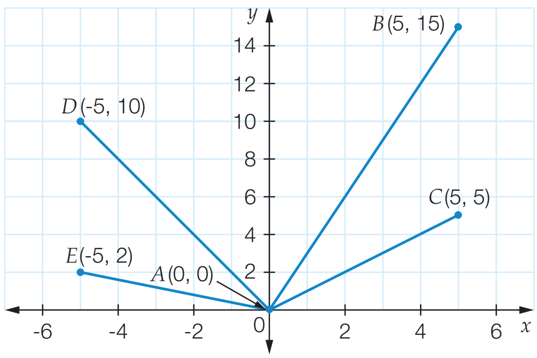
Navigator

1, 2, 3, 4 (column 1), 5, 6, 7, 8, 9, 10 (a, b, f), 12, 15, 17, 18	1, 2, 3, 4 (column 1), 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18	1, 2, 3, 4 (column 2), 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18

Answers p. 652

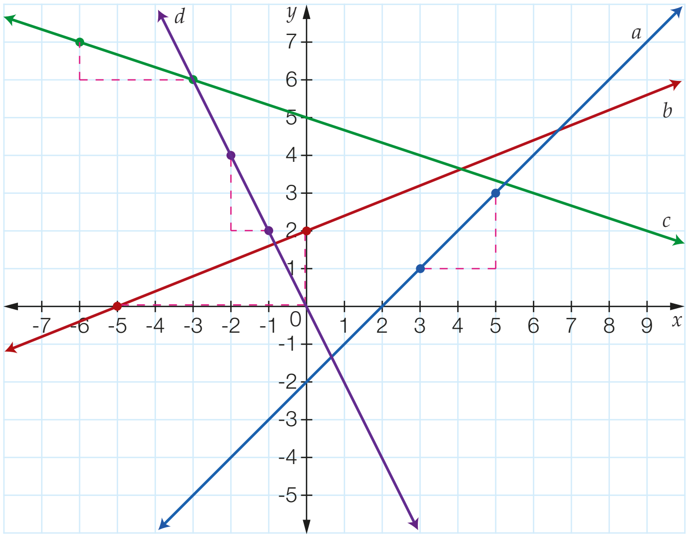
Fluency

1 Find the gradient of each of the line segments *AB*, *AC*, *AD* and *AE* shown in the graph.



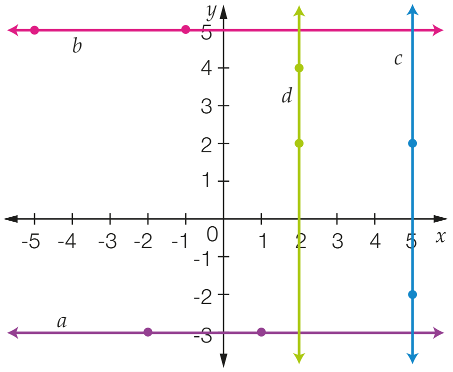
W.E. 12

2 Find the gradient of each of the lines *a*–*d* using the points and triangles indicated.



W.E. 13

3 Find the gradient of each of the lines *a*–*d* using the points on the lines as indicated.



W.E. 14

4 Find the gradient of the line joining each of the following pairs of points.

(a) (2, 3) and (4, 7)

(b) (1, 5) and (3, 11)

(c) (5, 8) and (6, 4)

(d) (3, 2) and (4, 8)

(e) (0, -2) and (3, 4)

(f) (1, -4) and (3, -2)

(g) (1, 6) and (7, 12)

(h) (-6, 2) and (-2, -2)

(i) (-1, -3) and (-4, -2)

(j) (5, 5) and (8, 20)

(k) (4, 2) and (-3, 2)

(l) (-3, -5) and (1, -5)

(m) (0, 7) and (3, 28)

(n) (3, -4) and (3, 8)

(o) (-7, 2) and (-7, 5)

5 Use your answers to Questions **1**, **2** and **3** to answer the following.

(a) If a line is sloping up to the right, does it have a positive or a negative gradient?

(b) If a line is sloping down to the right, does it have a positive or a negative gradient?

(c) What is the gradient of a horizontal line?

(d) What is the gradient of a vertical line?

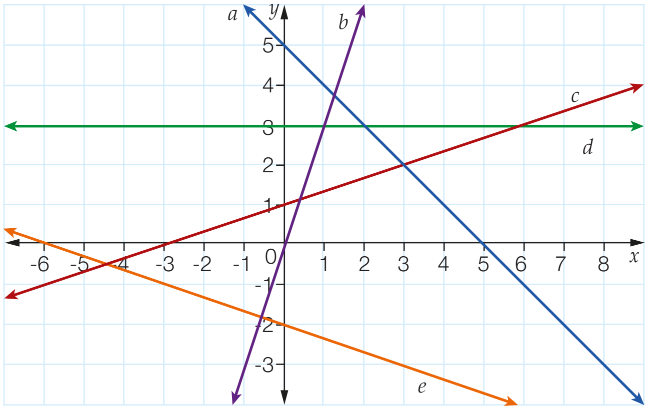
6 For each line *a*–*e* decide whether the gradient is:

A positive

B negative

C zero

D undefined



7 **(a)** Find the gradient of each of the lines in Question **6**.

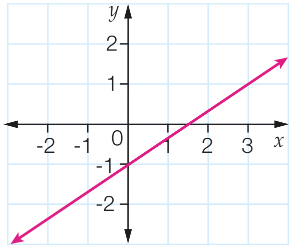
(b) Which line has the steepest positive gradient?

(c) Which line has the steepest negative gradient?

8 **(a)** The gradient of this line is:

A $-\frac{3}{2}$ **B** $-\frac{2}{3}$

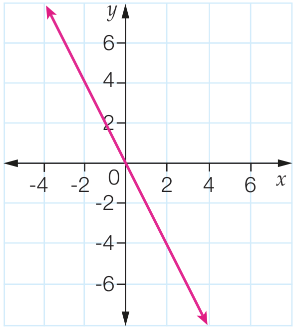
C $\frac{2}{3}$ **D** $\frac{3}{2}$



(b) The gradient of this line is:

A -2 **B** $-\frac{1}{2}$

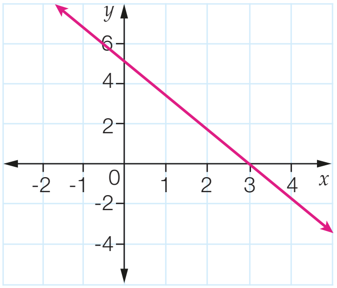
C $\frac{1}{2}$ **D** 2



(c) The gradient of this line is:

A $-\frac{5}{3}$ **B** $-\frac{3}{5}$

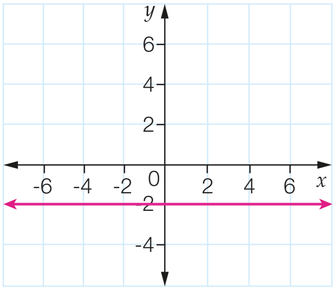
C $\frac{3}{5}$ **D** $\frac{5}{3}$



(d) The gradient of this line is:

A -2 **B** -1

C 0 **D** 2



Understanding

9 Sia rides her bike from her home at $A(0, 0)$ to the shops at B, a distance of 100 metres. The trip takes 10 seconds.

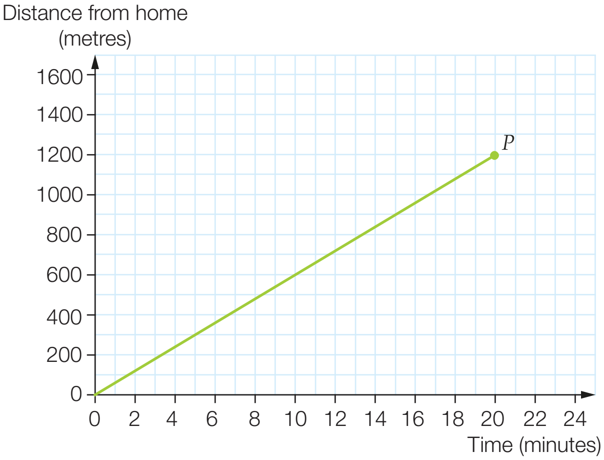
(a) Draw a set of Cartesian axes, choose a suitable scale for each axis, then number and label them. Use the horizontal axis for the time and the vertical axis for the distance. Plot points A and B and join them with a ruler to form a straight line. Write the coordinates of A and B on your graph.

(b) Find the gradient of the graph. Write the units of the gradient.

(c) What does the gradient of the graph mean in relation to Sia's bike ride?

(d) Write the equation of the graph.

10 Thanh decided to walk to school rather than take the bus. This graph shows his distance from home at any particular time on the walk. The school is located at point P.



(a) How far is the school from Thanh's home?

(b) How long did it take Thanh to walk to school?

(c) How far did Thanh walk every minute?

(d) What is the gradient of this graph?

(e) What was Thanh's speed in m/min on the walk?

(f) How far from home was Thanh after walking for:

(i) 6 minutes **(ii)** 12 minutes **(iii)** 18 minutes **(iv)** t minutes?

(g) Write the equation to find the distance from home D (in metres) after Thanh has walked t minutes.

11 Anna drove her car at a constant speed for 30 seconds, covering a distance of 450 m.

(a) Represent this information on a distance–time graph. (The distance travelled should be shown on the vertical axis and the time taken on the horizontal axis.)

(b) At what speed was Anna travelling?

(c) What is the gradient of your graph?

(d) Write an equation connecting the distance travelled to the time taken.

(e) Using your equation, calculate how far Anna would have travelled after 53 seconds if she maintained the same speed.

Reasoning

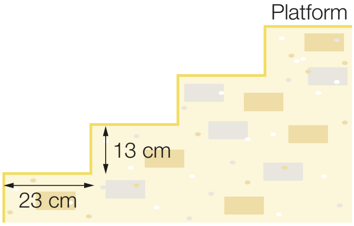
12 Line segment AB has coordinates $A(0, 0)$ and $B(2, 5)$. CD has coordinates $C(-1, -6)$ and $D(1, -1)$. EF has coordinates $E(-3, -2)$ and $F(-1, 3)$. GH has coordinates $G(-1, 7)$ and $H(1, 12)$.

(a) Using a suitable scale, sketch the four line segments on one set of Cartesian axes. Write the coordinates of A, B, C, D, E, F, G and H on your graph.

(b) Find the gradients of the four line segments.

(c) Show that the four line segments have the same gradient.

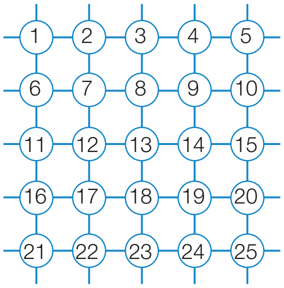
13 A standard step is 13 cm high and has a length of 23 cm. A wheelchair ramp is constructed to reach a platform that is four steps above ground level. If wheelchair access ramps have a gradient of 1 : 12 (this means a rise of 1 for a run of 12), find how far from the bottom of the stairs the ramp must start, for it to reach the top of the platform.



Hint

The gradients can be whole numbers, fractions, decimals positive and negative.

14 For a 5 × 5 grid labelled with the numbers 1–25, how many lines with different gradients can be drawn between any two of the grid numbers?



15 Sylvia is draining a bathtub of water. The graph shows the volume of water (in litres) in the bath after a number of minutes.

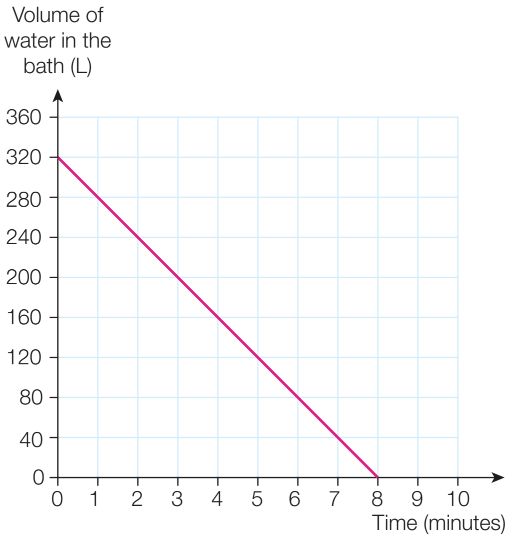
(a) How much water did the bath contain originally?

(b) How long did it take for the water to drain from the bath completely?

(c) What is the gradient of this graph?

(d) At what rate was the water being drained away? (That is, how many litres of water drained away each minute?)

(e) What does the negative represent in your answer to part **(c)**?



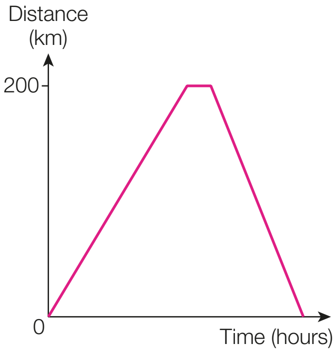
(f) How much water was left in the bath after:

(i) 4 minutes **(ii)** 6 minutes **(iii)** 2 minutes **(iv)** t minutes?

(g) Write the equation for the volume of water left in the bath, V (in litres), after t minutes.

(h) Why is this graph only drawn in the first quadrant?

16 Monique works for a courier company and has to make a long-distance delivery one very rainy day. The following graph represents her journey. The rain clears for Monique's return journey, so she can travel twice as fast as she did on her way to make the delivery. It takes her 4 hours and 10 minutes to make the delivery.



(a) At what speed is Monique travelling on her way to make the delivery?

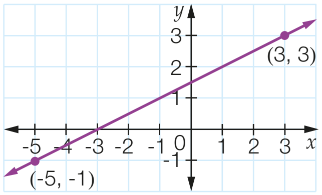
(b) At what speed is Monique travelling on her return journey?

(c) How much time does the entire trip take if Monique stops for 45 minutes for lunch?

Open-ended

17 The gradient of the line joining two points A and B is $-\frac{2}{3}$. If A is the point (1, 6), write three possible coordinates for B.

18 Kate and Sia are looking at a diagram the teacher has drawn on the whiteboard and the question written underneath, 'What is the gradient of this line?'



Kate writes
$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{3 - -1}{3 - -5} \\ &= \frac{4}{8} \\ &= \frac{1}{2} \end{aligned}$$

Sia writes
$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{3 - 1}{3 - 5} \\ &= \frac{2}{-2} \\ &= -1 \end{aligned}$$

The teacher tells Kate she is correct. Explain Sia's mistake.

Problem solving

Lost in the desert

An aircraft crashes in the desert at a point that could be labelled (-15, 12) on a Cartesian plane. A search party sets out by 4WD from the point (18, 3) and travels in a straight line, passing through the point (3, 9). Another aircraft also sets out from point (3, -15) and travels in a straight line, passing over the point (-3, -6).

Strategy options

- Draw a diagram.
- Break into manageable parts.

1 Will either of the search parties pass the point where the aircraft has crashed, if they continue on these paths? Explain.

2 If either (or both) of the original routes do not pass the crash location, give a different point (but with the same x-coordinate) that the searchers should pass through (or over) to be on course to find it.

Exploration CAS

Linear relationships

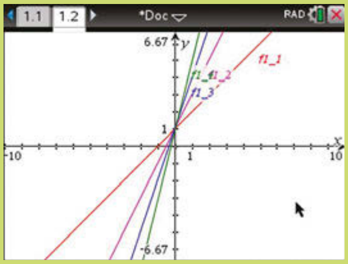
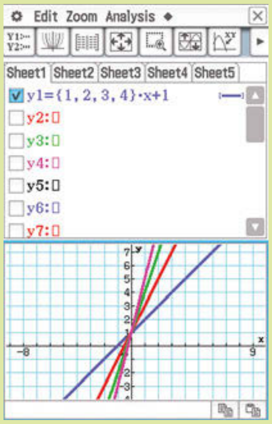
All linear (straight line) graphs can be written in the form $y = mx + b$ where m and b are numbers (e.g. $y = 3x + 5$ and $y = 2x - 7$). What happens to these graphs when m is changed or b is changed? Are there any patterns that you can find?

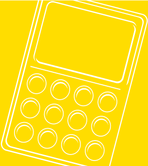
You can use a CAS calculator to explore the effects that m and b have on a linear graph.

Exploring *b* on a linear graph

Draw graphs of the following linear equations, changing the value of m while keeping the value of b constant (at $b = 1$):

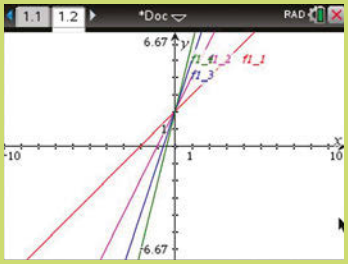
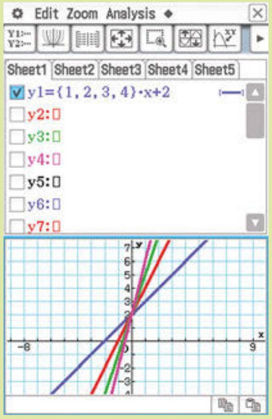
$y = x + 1$ $y = 2x + 1$ $y = 3x + 1$ $y = 4x + 1$

Using TI-Nspire CAS	Using Casio ClassPad CAS
Add **Graphs** to your document and enter the equation ***f1*(*x*) = {1,2,3,4}**×***x*+1** to be graphed. You can adjust the graph window settings by selecting **menu** > **Window/Zoom** > **Zoom – Standard**. The lines are labelled ***f1_1*** to ***f1_4***, where the last number is the value that was originally entered in the curly brackets. You can use the arrow keys to select the different lines and display their equations.  The lines have different gradients, but all pass through the y-axis at $x = 1$.	From the menu select **Graph & Table**. Enter the equation **y1={1,2,3,4}x+1** and tick the box. Select the graph icon to view the graphs. You can adjust the graph window settings by selecting **Zoom** > **Auto** or **Zoom** > **Zoom Out** until the lines are visible. The lines have different gradients, but all pass through the y-axis at 1.  ☑ y1={1, 2, 3, 4}·x+1 ☐ y2:□ ☐ y3:□ ☐ y4:□ ☐ y5:□ ☐ y6:□ ☐ y7:□



What happens to these graphs when the value of b is changed to 2?

$y = x + 2$ $y = 2x + 2$ $y = 3x + 2$ $y = 4x + 2$

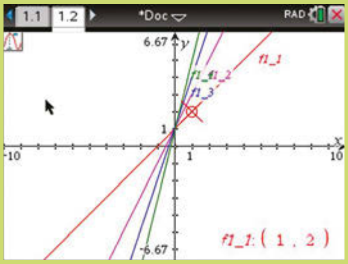
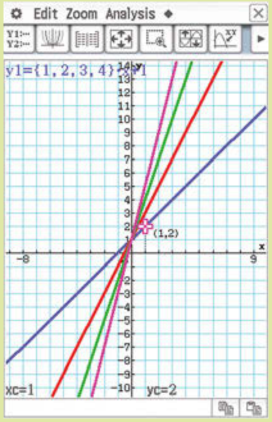
Using TI-Nspire CAS	Using Casio ClassPad CAS
Edit the equation to change the value of b to 2. To do this, you can use the arrow keys to select and display the equation.  	Edit the equation to change the value of b to 2. To do this, select the **1** at the end of the **y1** equation to highlight it, then make it a **2**. Select the graph icon ⊞ to view the new graphs.  ☑ y1={1, 2, 3, 4}·x+2 ☐ y2:□ ☐ y3:□ ☐ y4:□ ☐ y5:□ ☐ y6:□ ☐ y7:□

These lines appear to be the same as the previous lines, except that they pass through the y-axis at 2. The value of b seems to be the value of the y-intercept.

1 Test this by changing the value of b to several other numbers: positive and negative, integers and fractions. Record your results. Does the value of b always match the y-intercept?

Exploring *m* on a linear graph

To explore how changing the value of m changes the graph, return to the original graphs (where $b = 1$).

Using TI-Nspire CAS	Using Casio ClassPad CAS
Select **menu** > **Trace** > **Graph Trace**. Then, to indicate you want to know the coordinates for the point where $x = 1$, enter **1**.  f1_1: (1 , 2) In the bottom right-hand corner you can see the coordinates given for the point where $x = 1$ on the first line. You can use the arrow keys to find the coordinate pairs for $x = 1$ on the other lines. You should find that the steepest line contains the point with the coordinates (1, 5).	Now change b back to 1 in the **y1** equation and redraw the graphs. Select **Analysis** > **Trace**, then enter **1** to trace the point where $x = 1$. This should trace and label the point (1, 2). You can see that this is a point on the shallowest line of the four lines shown. Now use the arrow keys to select the steepest line to show that this line contains the point with the coordinates (1, 5).  y1={1, 2, 3, 4}·x+1 xc=1 yc=2

Go through this process again to find another two coordinate pairs of points on the lines, for example (3, 4) for the shallowest line and (3, 13) for the steepest line. Now find the gradients of the two lines.

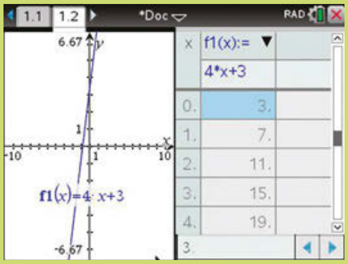
	Points	**Gradient**	**Equation $y = mx + b$**
Shallowest line:	(1, 2), (3, 4)	$\text{Gradient} = \frac{\text{rise}}{\text{run}}$ $= \frac{4-2}{3-1}$ $= \frac{2}{2}$ $= 1$	$y = x + 1$ $m = 1$ $b = 1$
Steepest line:	(1, 5), (3, 13)	$\text{Gradient} = \frac{\text{rise}}{\text{run}}$ $= \frac{13-5}{3-1}$ $= \frac{8}{2}$ $= 4$	$y = 4x + 1$ $m = 4$ $b = 1$

The value of m is the same as the gradient for both lines. The value of m seems to represent the gradient.

2 Test this by investigating $y = 4x + 3$. If b is the y-intercept and m is the gradient, then the graph should pass through the y-axis at $y = 3$ and have a gradient of 4. You can check this by finding two coordinate pairs of points on this line, calculating the gradient and then checking the y-value where $x = 0$.

Using TI-Nspire CAS

First, delete the existing equations by selecting **menu** > **Actions** > **Delete all**. Then enter the new equation **f1(x) = 4*x+3**. To see the table of values for this equation, select **menu** > **Table** > **Split-screen Table**.



x	f1(x):=
	4*x+3
0.	3.
1.	7.
2.	11.
3.	15.
4.	19.

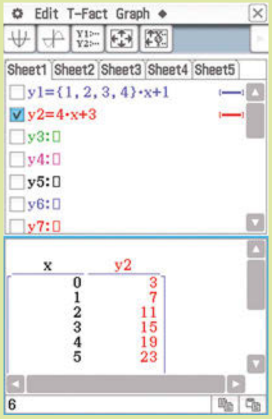
3.

Using Casio ClassPad CAS

Untick the equation **y1**. Enter a new equation **y2=4x+3** and tick its box instead.

Now select the table input icon and enter the values so that **Start** is 0, **End** is 10 and **Step** is 1.

Then select the draw table icon to display the table of values for this equation.



☐ y1={1, 2, 3, 4}·x+1
☑ y2=4·x+3
☐ y3:□
☐ y4:□
☐ y5:□
☐ y6:□
☐ y7:□

x	y2
0	3
1	7
2	11
3	15
4	19
5	23

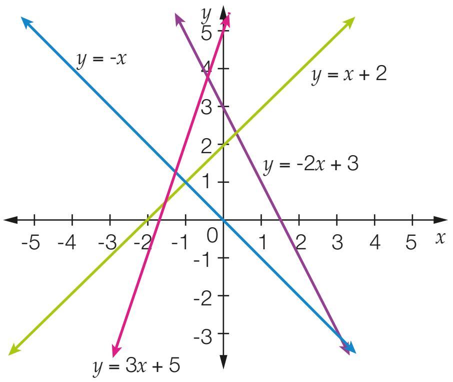
6

For example, consider the coordinate pairs (1, 7) and (4, 19). These give a gradient of $\frac{19-7}{4-1} = \frac{12}{3} = 4$, as predicted. Also, (0, 3) shows that the y-intercept is the b-value.

3 Now try the same tests with at least six other linear equations, including some with negative values for m and b. Do the graphs and tables always confirm that m is the gradient and b is the y-intercept?

5.6

Sketching linear graphs using the gradient and y-intercept



When you plot points for equations in the form $y = mx + b$, the points are in a straight line. The graphs of these equations are linear.

If the magnitude of m increases while b stays constant in $y = mx + b$, then the gradient of this line increases. If the magnitude of m decreases while b stays constant in $y = mx + b$, then the gradient of this line decreases. Using the formula gradient $= \frac{\text{rise}}{\text{run}}$, it can be shown that the coefficient of x is the gradient m.

If the value of b increases while m stays constant in $y = mx + b$ then the gradient stays the same but the y-**intercept** increases.

If the value of b decreases while m stays constant in $y = mx + b$ then the y-intercept decreases.

It can be shown that the value of the constant term b is the value of the y-intercept.

A linear graph shows a linear relationship between two variables. These linear relationships can be written as linear equations in two different forms, as described below.

The **gradient–intercept form**:

$y = mx + b$, where m is the gradient and b is the y-intercept

The **general form**:

$ax + by + c = 0$, where a, b and c are constants and $a \neq 0$, $b \neq 0$

In this equation, the gradient $m = \frac{-a}{b}$ and the y-intercept $= \frac{-c}{b}$ can be found by rearranging the equation to make y the subject.

Finding the y-intercept and gradient

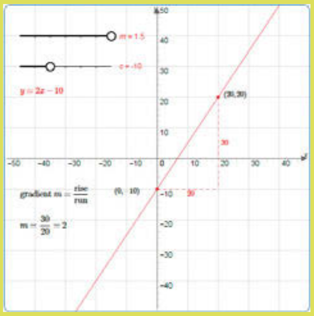
Before you can use linear equations to sketch a graph, you need to be able to identify the gradient and the y-intercept from the equation.

Interactive

Sketching linear graphs

Explore how the y-intercept and gradient affect the position and shape of linear graphs.

Go to the eBook or the Pearson Places website to access this interactive.



Worked example 15

W.E. 15

State the gradient and coordinates of the y-intercept of the graphs of each of the following linear equations:

(a) $y = -3x + 2$ **(b)** $y = x - 1$ **(c)** $2y = 5x - 4$ **(d)** $y + 2x = 6$

Thinking	Working
(a) 1 Ensure that the equation is written in the form $y = mx + b$.	(a) $y = -3x + 2$
2 The coefficient of x is the gradient m.	gradient = -3
3 The constant term is the y-value of the y-intercept.	y-intercept: (0, 2)
(b) 1 Ensure that the equation is written in the form $y = mx + b$.	(b) $y = x - 1$ $= 1x + (-1)$
2 The coefficient of x is the gradient m.	gradient = 1
3 The constant term is the y-value of the y-intercept.	y-intercept: (0, -1)
(c) 1 Ensure that the equation is written in the form $y = mx + b$.	(c) $2y = 5x - 4$ $y = \frac{5}{2}x - 2$
2 The coefficient of x is the gradient m.	gradient = $\frac{5}{2}$
3 The constant term is the y-value of the y-intercept.	y-intercept: (0, -2)
(d) 1 Ensure that the equation is written in the form $y = mx + b$.	(d) $y + 2x = 6$ $y = 6 - 2x$ $= -2x + 6$
2 The coefficient of x is the gradient m.	gradient = -2
3 The constant term is the y-value of the y-intercept.	y-intercept: (0, 6)

Sketching linear graphs through the origin, $y = mx$

Graphs that pass through the origin are in the form $y = mx$. The value of b is zero so that the origin is the y-intercept. The origin (0, 0) is one point on the graph and the gradient can be used to find a second point.

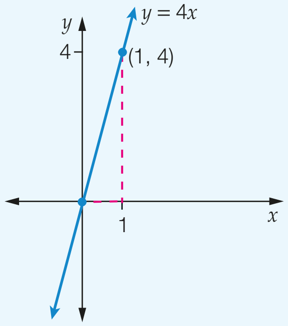
The gradient m is the coefficient of x: $m = \frac{\text{rise}}{\text{run}}$.

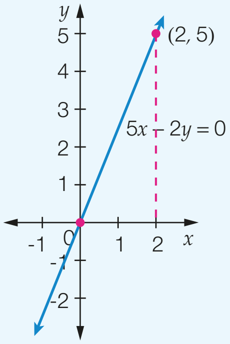
Worked example 16

W.E. 16

Sketch the graph of the relationships.

(a) $y = 4x$ **(b)** $5x - 2y = 0$

Thinking	Working
(a) 1 Find the y-intercept.	(a) y-intercept: When $x = 0$, $y = 4 \times 0$ $y = 0$ y-intercept is (0, 0)
2 State the gradient as a fraction so that the rise and run can be identified.	$m = 4$ $= \frac{4}{1}$ rise = 4; run = 1
3 Draw and label a set of axes.	
4 Mark the origin as the first point on the graph.	
5 Use the gradient to mark a second point. (Here there is a rise of 4 and a run of 1 to reach the point (1, 4).	
6 Rule a line through these two points.	
7 Label the graph.	
(b) 1 Find the y-intercept.	(b) y-intercept: When $x = 0$, $-2y = 0$ $y = 0$ y-intercept is (0, 0)
2 Rearrange the equation so that it is in the form $y = mx$.	$5x - 2y = 0$ $5x - 5x - 2y = 0 - 5x$ $-2y = -5x$ $\frac{-2y}{-2} = \frac{-5x}{-2}$ $y = \frac{5}{2}x$

3 State the gradient and use it to identify the rise and the run.	$m = \frac{5}{2}$ rise = 5; run = 2
4 Draw and label a set of axes. **5** Mark the origin as the first point on the graph. **6** Use the gradient to mark a second point. (Here there is a rise of 5 for a run of 2 to reach the point (2, 5)). **7** Rule a line through these two points. **8** Label the graph.	

The equations $y = 4x$ and $5x - 2y = 0$ are linear graphs for which the y-intercept is the origin.

Graphing using the y-intercept and gradient

There is no need to plot a number of points to graph linear relationships. Because only two points are needed to define a line, use the y-intercept as one point on the graph and the gradient, m, to locate a second point. Once two points are known, a straight line can be drawn through them.

Sketching other linear graphs, $y = mx + b$, $b \neq 0$

If the y-intercept is not the origin, then first identify the y-intercept from the equation before using the gradient to find a second point.

Using the equation $y = mx + b$ and substituting $x = 0$, you get $y = b$.

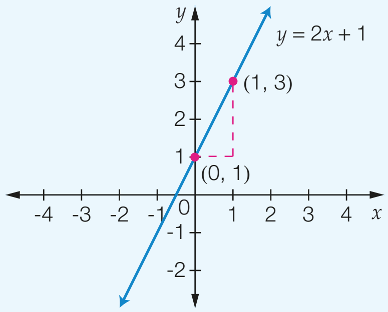
$x = 0$ is the x-coordinate of a point on the y-axis, so b is the y-value at the point at which the graph crosses the y-axis. This is the y-intercept.

Worked example 17

W.E. 17

Use the y-intercept and the gradient to sketch the graph of $y = 2x + 1$.

Thinking	Working
1 State the y-intercept	y-intercept: (0, 1)
2 State the gradient as a fraction so that the rise and run can be easily identified.	$\text{Gradient} = 2$ $= \frac{2}{1}$ $= \frac{\text{rise}}{\text{run}}$



3 Draw and label a set of axes.

4 Mark the y-intercept.

5 Start at the y-intercept (here (0, 1))and use the gradient to mark a second point (move horizontally the value of the run and move vertically the value of the rise). (Here the rise is 2 and the run is 1 to reach the point (1, 3).)

6 Rule a line through the two points continuing in both directions.

7 Ensure that the coordinates of the two points used to sketch the line are clear and that the line is labelled with its equation.

When the equation is not in a form where you can read off the gradient and y-intercept, rearrange the equation first to make y the subject.

Worked example 18

W.E. 18

Use the y-intercept and the gradient to sketch the graph of $2x + 3y = 6$.

Thinking	Working
1 Rearrange the equation so that it is in the form $y = mx + b$.	$2x + 3y = 6$ $3y = 6 - 2x$ $y = \frac{6 - 2x}{3}$ $= 2 - \frac{2x}{3}$ $= -\frac{2x}{3} + 2$
2 State the y-intercept.	y-intercept: (0, 2)
3 State the gradient as a fraction so that the rise and run can be easily identified.	Gradient $= -\frac{2}{3}$ $= \frac{\text{rise}}{\text{run}}$

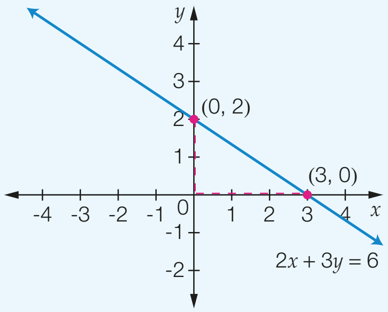
4 Draw and label a set of axes.

5 Mark the y-intercept.

6 Start at the y-intercept (here (0, 2)) and use the gradient to mark a second point (move horizontally the value of the run and move vertically the value of the rise (Here the rise is -2 and the run is 3 to reach the point (3, 0)).

7 Rule a line through the two points continuing in both directions.

8 Ensure that the coordinates of the two points used to sketch the line are clear and that the line is labelled with its equation.



Parallel and perpendicular lines

Two lines in the same plane with the same gradient are **parallel** lines. The converse of this is also true: parallel lines have the same gradient.

$y = 2x - 4$ and $y = 2x + 5$ are parallel because they both have a gradient of 2.

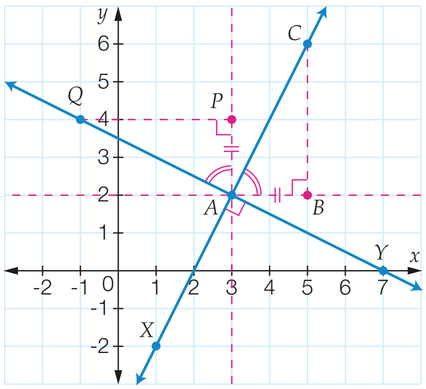
Perpendicular lines

The gradients of **perpendicular lines**, which are lines that intersect at right angles (90°), also have an important relationship.

Consider the two line segments shown, XC and YQ drawn at right angles and intersecting at A.

Draw lines parallel to the axes through A.

Draw two right-angled triangles ABC and APQ such that $AB = AP$. Now:



$\angle QAP = \angle CAB$ (both equal to $90^\circ - \angle CAP$)
$\angle QPA = \angle CBA = 90^\circ$ (given)
$\therefore \Delta APQ \equiv \Delta ABC$ (ASA congruency)
$\therefore PQ = BC$ (equal in length but opposite in direction)

Let m_1 = gradient of AC

$$= \frac{BC}{AB}$$

and m_2 = gradient of QA

$$= \frac{PA}{QP}$$

$$= \frac{AP}{PQ}$$

Then: $m_1 \times m_2 = \frac{BC}{AB} \times \frac{AP}{PQ}$

$$= \frac{{}^{1}\cancel{BC}}{{}_{1}\cancel{AB}} \times \frac{{}^{1}\cancel{AB}}{{}_{-1}\cancel{-BC}}$$

$$= -1$$

Hint

Direction is important: $QP = -PQ$

This shows that the product of the gradients of two perpendicular lines is equal to -1. Conversely it can be shown that if the product of the gradients of two lines $m_1m_2 = -1$, then the lines are perpendicular, so $m_2 = -\frac{1}{m_1}$.

Worked example 19

W.E. 19

Find the gradient of a line parallel to each of the following:

(a) $y = 2x - 5$

(b) $2x + 3y = 6$

Thinking	Working
(a) 1 Because y is the subject of the equation, identify the coefficient of x as the gradient in the given equation.	(a) $y = 2x - 5$ $m = 2$
2 Because parallel lines have the same gradient, this is the gradient of any parallel line.	Any parallel line will have a gradient of 2.
(b) 1 Rearrange the equation to make y the subject.	(b) $2x + 3y = 6$ $2x - 2x + 3y = 6 - 2x$ $\frac{3y}{3} = -\frac{2}{3}x + \frac{6}{3}$ $y = -\frac{2}{3}x + 2$
2 Because y is the subject of the equation, identify the coefficient of x as the gradient in the given equation.	$m_1 = -\frac{2}{3}$
3 Because parallel lines have the same gradient, this is the gradient of any parallel line.	Any parallel line will have a gradient of $-\frac{2}{3}$.

Worked example 20

W.E. 20

Find the gradient of a line perpendicular to each of the following:

(a) $y = 4x + 3$

(b) $3x - 5y = 10$

Thinking	Working
(a) 1 Identify the gradient in the given equation.	(a) $y = 4x + 3$
2 Because the product of the gradients of perpendicular lines = -1, the gradient of a perpendicular line = $-\frac{1}{m_1}$.	$m_1 = 4$ $m_2 = -\frac{1}{m_1}$ $= -\frac{1}{4}$

(b) 1 Rearrange the equation to make y the subject.

(b)
$$3x - 5y = 10$$
$$3x - 3x - 5y = 10 - 3x$$
$$-5y = -3x + 10$$
$$\frac{-5y}{-5} = \frac{-3x}{-5} + \frac{10}{-5}$$
$$y = \frac{3}{5}x - 2$$

2 Identify the gradient in the given equation.

$$m_1 = \frac{3}{5}$$

3 Because the product of the gradients of perpendicular lines = -1, the gradient of a perpendicular line = $-\frac{1}{m_1}$.

$$m_2 = -\frac{1}{m_1}$$
$$= -\frac{5}{3}$$

5.6 Sketching linear graphs using the gradient and *y*-intercept

Navigator

1 (a–f), 2 (a, c, e), 3, 4 (column 1), 5 (a–c), 6 (a–c), 7 (column 1), 8 (a), 9, 10 (a), 11 (a, c), 14, 16, 20, 22	1 (d–i), 2 (b–e, g), 3, 4 (column 2), 5 (a–c), 6 (a–c), 7 (column 2), 8, 9, 10 (a, b), 11 (a–c), 13, 14, 16, 17, 20, 22	1 (d–i), 2 (c, f–i), 3, 4 (column 3), 5 (d–f), 6 (d–f), 7 (column 3), 8, 9, 10, 11 (b, d), 12, 13, 15, 16, 17, 18, 19, 20, 21

Answers p. 653

Fluency

1 State the gradient and the coordinates of the y-intercept of the graphs for each of the following linear equations. W.E. 15

(a) $y = x + 3$ (b) $y = 2x + 1$ (c) $y = 7x + 3$

(d) $y = 3x - 2$ (e) $y = x - 4$ (f) $y = -4x + 3$

(g) $y = -x + 4$ (h) $y = 6 - 2x$ (i) $y = -5 + 2x$

2 Sketch the graphs of the following. W.E. 16

(a) $y = 2x$ (b) $y = 3x$ (c) $y = -x$

(d) $y = -2x$ (e) $y = \frac{2}{3}x$ (f) $y = \frac{4}{5}x$

(g) $4x + y = 0$ (h) $6x + y = 0$ (i) $4x + 3y = 0$

3 Use the y-intercept and gradient to sketch the graph of each of the equations in Question **1**. W.E. 17

W.E. 18

4 Use the y-intercept and gradient to sketch each of the following by first rearranging into the form $y = mx + c$.

(a) $2x + y = 4$
(b) $-x + y = -3$
(c) $-3x + y = -2$
(d) $4x - y = 1$
(e) $2x - y = -3$
(f) $3x + 2y = 4$
(g) $2x + 3y = 3$
(h) $2x - 5y = 10$
(i) $7x + 3y - 6 = 0$

W.E. 19

5 Find the gradient of lines parallel to each of the following.

(a) $y = 3x + 5$
(b) $y = 7 - 4x$
(c) $2y = 6x - 5$
(d) $6x - 2y = 3$
(e) $2x + 5y = 10$
(f) $4x - 3y = 8$

W.E. 20

6 Find the gradient of lines perpendicular to each of the equations given in Question **5**.

7 Write the equation of the straight line given:

(a) $m = 3$ and $b = 4$
(b) $m = 1$ and $b = 5$
(c) $m = 2$ and $b = 0$
(d) gradient is 4 and y-intercept is 0
(e) gradient is -5 and y-intercept is 1
(f) gradient is 3 and y-intercept is -4

8 (a) The equation of a linear graph with y-intercept 3 and gradient of -2 is:

A $y = 2x + 3$
B $2x - y = 3$
C $3x + 6 = 2$
D $2x + y = 3$

(b) The equation of a linear graph with gradient $\frac{1}{2}$ and y-intercept 1 is:

A $y = 2x + 1$
B $y = x + 2$
C $x - 2y = -2$
D $x + 2y = -2$

9 When using the y-intercept and gradient method of graphing, first mark in the y-intercept and then use the gradient to locate a second coordinate point.

(a) A line has a gradient of $\frac{3}{4}$. From the y-intercept, you move:

A 3 units right and 4 units up
B 4 units right and 3 units up
C 3 units right and 4 units down
D 5 units right and 3 units down

(b) A line has a gradient of $-\frac{2}{5}$. From the y-intercept, you move:

A 2 units right and 5 units down
B 5 units right and 2 units down
C 2 units left and 5 units up
D 5 units left and 2 units down

(c) A line has a gradient of 5. From the y-intercept, you move:

A 1 unit right and 5 units up
B 5 units right and 5 units up
C 1 unit right and 5 units down
D 5 units right and 5 units down

(d) A line has a gradient of -3. From the y-intercept, you move:

A 3 units right and 3 units up
B 1 unit right and 3 units down
C 0 units right and 3 units down
D 1 unit right and 3 units up

10 (a) A coordinate point that lies on the line $y = x + 7$ is:

A (2, 10)
B (-3, -4)
C (0, -7)
D (-4, 3)

(b) A coordinate point that lies on the line $3x + 2y = 11$ is:

A (5, -2)
B (2, 3)
C (-3, -1)
D (4, 0)

(c) A coordinate point that lies on the line $3y - x + 6 = 0$ is:

A (0, 2)
B (2, 0)
C (3, -1)
D (-3, 1)

11 Find the gradient of the line perpendicular to:

(a) a line with gradient 2

(b) a line with gradient 0

(c) the line passing through (4, 5) and (3, -2)

(d) the line passing through (-2, 6) and (3, 8).

12 Match the following.

(a) A line parallel to the line joining the points (2, 3) and (4, 7)

(b) A line perpendicular to the line joining the points (2, 3) and (4, 7)

(c) A line parallel to the line joining the points (7, 0) and (-1, 8)

(d) A line perpendicular to the line joining the points (7, 0) and (-1, 8)

(e) A line perpendicular to the line joining the points (-1, -3) and (-4, -2)

(f) A line parallel to the line joining the points (4, 2) and (-3, 2)

(g) A line parallel to the line joining the points (3, -4) and (3, 8)

A A line joining (-6, 2) and (-2, -2)

B A line joining (0, -2) and (3, 4)

C A line joining (-3, -5) and (1, -5)

D A line joining (7, 3) and (5, 4)

E A line joining (-1, -5) and (-1, 5)

F A line joining (1, -4) and (3, -2)

G A line joining (1, 5) and (3, 11)

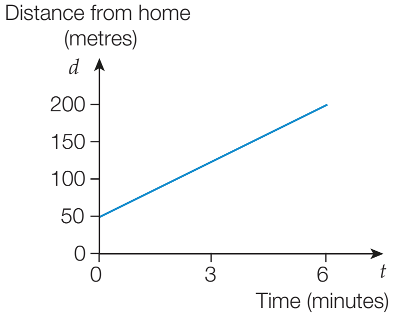
13 What is special about the following lines?

(a) Lines parallel to the line joining (4, 2) and (-3, 2).

(b) Lines parallel to the line joining (3, -4) and (3, 8).

Understanding

14 Consider this distance–time graph for Cassia as she walks to the shops.



(a) Write the intercept with the vertical axis.

(b) Find the gradient of this graph.

(c) Use this information to write an equation linking the distance from home (d) to the time (t).

(d) At what speed was she walking? (Refer to the equation.)

(e) If Cassia continued walking at the same speed, how far from home would she be after 20 minutes?

15 A distance–time graph is represented by the equation $d = 30t + 20$, where d is the distance from Geelong in kilometres and t is the time in hours.

(a) Write the gradient of the line.

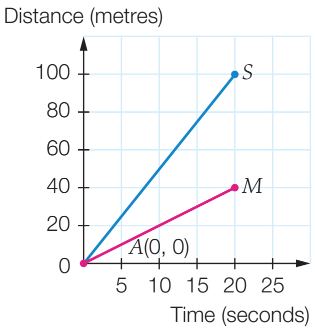
(b) What is the speed of travel?

(c) How far from Geelong did the journey start?

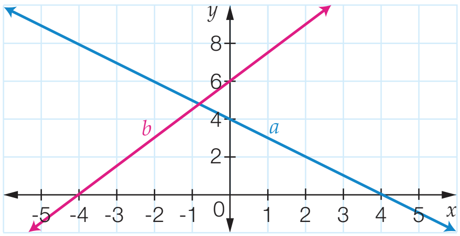
(d) If the journey had started 90 km from Geelong and a constant speed of 100 km/h was maintained, what equation could now be written to represent the relationship between d and t?

Reasoning

16 Jill and her brother Jack go to the shops from their home at point A, as shown on the graph. Jill goes to the milk bar (M) while Jack goes to the supermarket (S). Compare and contrast their journeys, considering the distances travelled, time taken, and the speeds at which they move.



17 Consider the graph at right. Show that the lines a and b are not perpendicular.



18 An oil tanker is being filled with crude oil. The oil tanker initially contained 10 000 tonnes of oil and, over an 8-hour period, it received a further 96 000 tonnes of crude oil. Three full storage tanks, each of 48 000 tonnes capacity, were being used to transfer the crude oil to the tanker. Each tank was being emptied simultaneously at the same rate of 4000 tonnes per hour.

(a) Sketch a graph showing the amount of crude oil (in tonnes) in the oil tanker during the 8-hour period.

(b) Find the gradient of this graph.

(c) Use this information to write an equation relating the amount of crude oil (m) in the tanker to the time (t).

(d) Sketch a graph showing the amount of crude oil (m) left in one of the storage tanks over the 8-hour period as oil is pumped out of it.

(e) Write an equation relating the amount of crude oil (m) left in one of the storage tanks to the time (t).

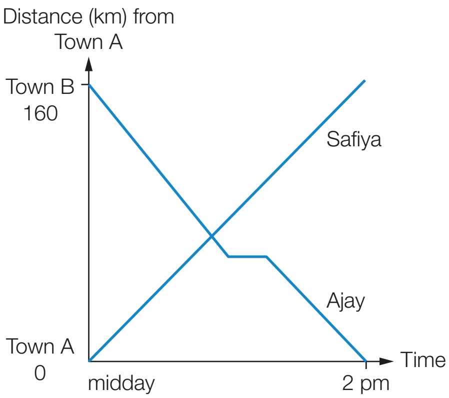
(f) How much crude oil was left in each of the storage tanks after the 8-hour period?

(g) If the pumps from one of the storage tanks were under repair, how long would it take to transfer the same amount of crude oil from the remaining storage tanks?

19 This graph shows the journeys two friends made between Town A and Town B, leaving at midday and arriving at their respective destinations 2 hours later.

Safiya travels at a constant speed straight from Town A to Town B. Ajay travels at a constant speed for the first hour. He rests for 15 minutes then continues to Town A at the same speed that Safiya is travelling.

Find how far each of them was from Town A after travelling for half an hour.



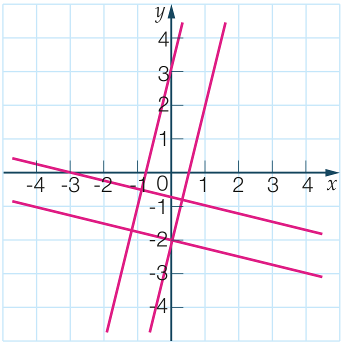
Open-ended

20 State at least three different coordinate points that lie on each of the following lines.

(a) $y = x - 4$ **(b)** $2x - 5y = -10$ **(c)** $4y - x + 2 = 0$

21 When the equations $y = 4x + 3$, $y = 4x - 2$, $y = -\frac{1}{4}x - 2$ and $y = -\frac{1}{4}x - \frac{3}{4}$ are plotted on a Cartesian plane, the area enclosed by the points of their intersection is a square.

Can you find another set of sloping lines that enclose a square?



22 **(a)** Write an equation for a line parallel to $y = 4x - 3$.

(b) Write an equation for a line perpendicular to $y = 4x - 3$.

Hint

Remember the properties of parallel and perpendicular lines.

Game

Equation battleships

Equipment required: different-coloured pens, grid paper, ruler

How to win: Be the first to sink your opponent's entire fleet of battleships.

Setting up:

1 On grid paper, draw a Cartesian plane with *x*- and *y*-values from -10 to 10.

2 Using different-coloured pens, players take turns to place four dots (ships) each, on any point (with whole-number coordinates) on the plane. Ships may be placed anywhere in the plane except along the *y*-axis.

3 Once all the ships are in place, players take turns to place five cannons at points along the *y*-axis. Each player marks their five cannons with a 'C' using their colour.

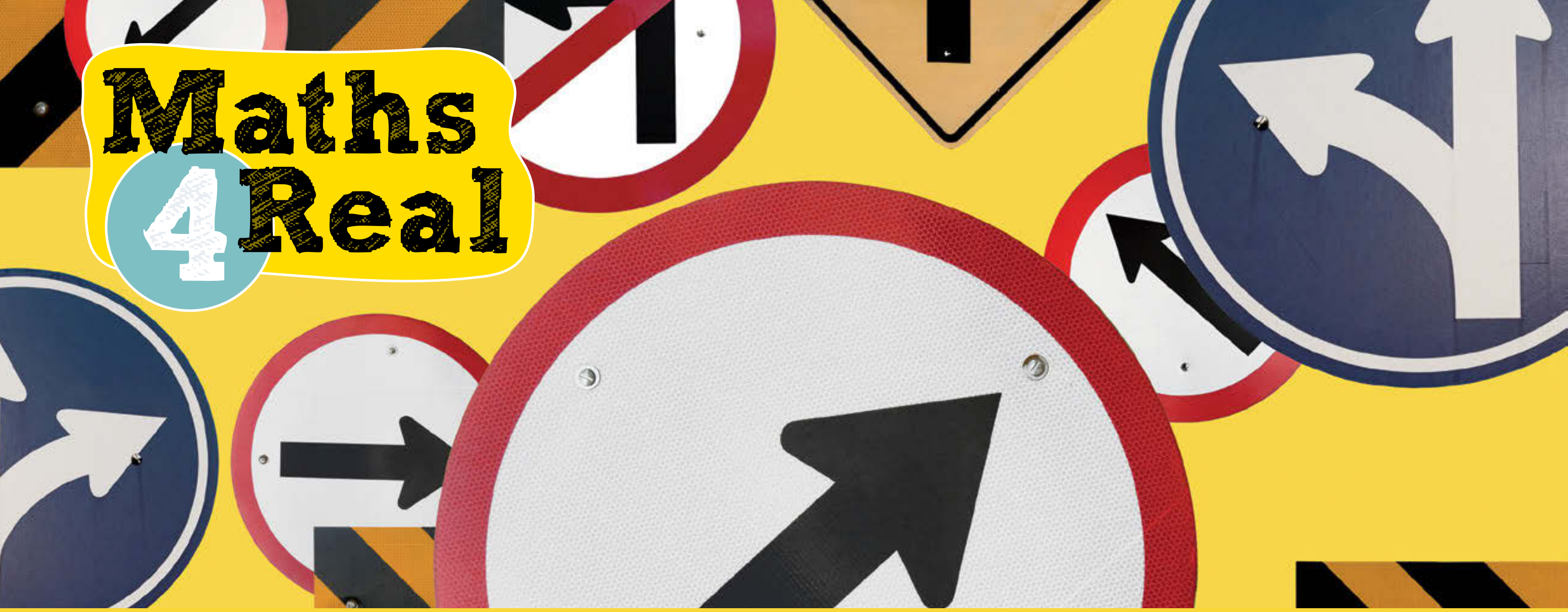
How to play: Players graph a linear equation to make a line from their cannon that intersects with at least one of the other player's ships. Be careful not to sink your own ships!

1 Player 1 states a linear equation that Player 2 then draws on the plane (using Player 1's colour).

The value of *m* in the equation ($y = mx + b$) cannot be zero (so $y = 2$, $y = -3$ etc. cannot be used as equations).

If the line passes through any ships, those ships are sunk (even if they are your own ships)!

2 Players take alternate turns until one player sinks all four of their opponent's ships and is declared the winner!



Engineering and linear relationships

Linear relationships are used in many different engineering disciplines. They are used by civil engineers to work out the gradient of new roads. Engineers use linear relationships to work out the total number of beams required to build a bridge. They can also be used to determine how long the traffic lights need to stay green to ensure minimal traffic congestion for a certain number of cars travelling through an intersection.

Traffic congestion

Civil engineers use algebra to devise ways to improve the flow of vehicles at busy intersections and on highways.

To try to decrease traffic gridlock, it is important to know the maximum number of cars that could pass through the intersection in a given amount of time if the traffic moved freely. This number can be represented by the variable n, which is dependent on the velocity of the cars (v), the length of each car (l) and the distance between cars (b).

To develop a mathematical model, mathematicians and engineers often simplify the situation by making assumptions. In this case it is best to assume all cars are the same length and that all cars are the same distance apart. Also, assume that there is only one lane of traffic moving forward in each direction and that no right turns are allowed.

1 Draw a diagram to show a set of cars waiting at traffic lights, marking b and l carefully on your diagram.

2 Write an equation that models the length of a queue of cars waiting at traffic lights in terms of n, l and b. Use d to represent the length of the queue that starts at the bumper of the first car and ends with the rear bumper of the last car.

3 Write an equation that models the velocity (v) of the queue of cars entering an intersection in terms of n, l and b. (Remember that velocity = $\frac{\text{distance}}{\text{time}}$ and that the distance will be the length of the queue d that will get through the lights while they are on green, starting with the front bumper of the first car and ending with the back bumper of the last car.)

4 Using v = 16 m/s, l = 5 m and b = 3 m, find n in terms of t, where t represents time.

5 If the lights are on green for 15 seconds (i.e. $t = 15$), what is the maximum number of cars that could enter the intersection in one direction?

6 **(a)** Traffic does not usually travel through lights at the speed limit. If cars have been stopped at a set of traffic lights, it takes a few seconds for them to accelerate to the speed limit, so a slower average speed is more realistic. Using $v = 10$ m/s (36 km/h), $l = 5$ m and $b = 3$ m, find n in terms of t.

(b) If the lights are on green for 15 seconds, recalculate the maximum number of cars that could enter the intersection in one direction using this slower speed.

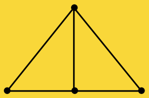
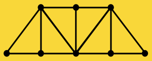
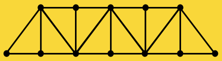
7 Not all traffic lights stay green for the same amount of time. On a major road the green light stays on longer than it does on a minor road. If the lights were on green for a further 5 seconds, how many more cars would enter the intersection? ($v = 10$ m/s, $l = 5$ m and $b = 3$ m.)

8 There is a traffic jam at an intersection and the velocity through the intersection has slowed. What is the maximum number of cars that can pass through the lights in one direction in 15 seconds given that the velocity is now only 5 m/s and $b = 2$ m?

Beams on a bridge

Beams of steel on a bridge meet at a joint. A truss is a type of structure constructed by beams and joints to form small to large triangles. Because triangles are the strongest type of polygon, trusses are used in many constructions, such as bridges and roofs.

Examine the table on the right, which outlines the number of members (m) and joints (j) for particular truss designs. A member is the part of a beam between joints.

Diagram	Number of members (m)	Number of joints (j)
	5	4
	13	8
		

1 **(a)** Copy and complete the table above.

(b) Use the information given in the table to draw a graph of the number of joints for an increasing number of members.

(c) Use the gradient and y-intercept of the graph to write an equation that shows the relationship between the number of members (m) and the corresponding number of joints (j).

2 Use your equation to find how many joints are needed in a truss with 29 members.

3 Use your equation to find how many members are in a truss with 29 joints.

4 What restriction needs to be placed on the values of m so that the trusses can be made correctly? Can m have any value (i.e. $m = 6$)?

5.7 Sketching linear graphs using intercepts

When sketching a linear graph, two points on the line are needed to draw the correct graph. In the previous section you saw how the y-intercept can be used as one point and how a second point can be found using the gradient.

When equations are given in the general form $ax + by + c = 0$, it is often easier to use another method for sketching the graph that does not involve rearranging the equation.

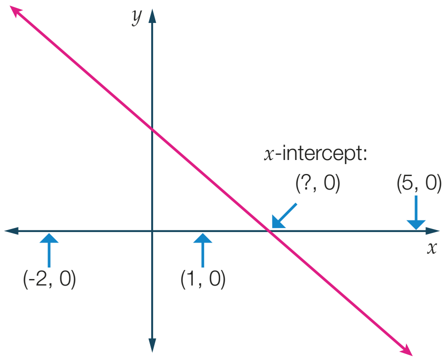
Graphing using x- and y-intercepts

The x-intercept

The x-intercept is the point where the line crosses the x-axis.

The y-coordinate for any point on the x-axis is always zero.

The x-coordinate of the x-intercept can be found by substituting $y = 0$ into the equation.

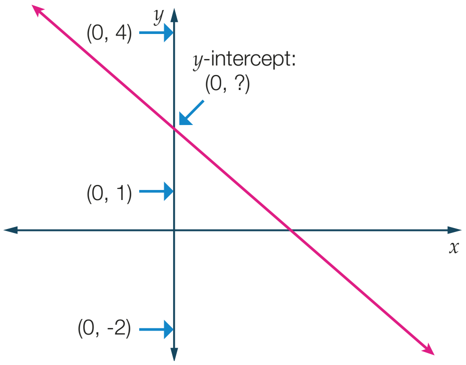


The y-intercept

The y-intercept is the point where the line crosses the y-axis.

The x-coordinate for any point on the y-axis is always zero.

The y-coordinate of the y-intercept can be found by substituting $x = 0$ into the equation.



To find the x-intercept, substitute $y = 0$ into the equation.

To find the y-intercept, substitute $x = 0$ into the equation.

Worked example 21

W.E. 21

Use the x- and y-intercepts to sketch the graph of $2x + y = 4$.

Thinking

1 Calculate the x-intercept by substituting $y = 0$ and solving the equation.

Working

$$2x + y = 4$$

Where $y = 0$:

$$\begin{aligned} 2x + 0 &= 4 \\ 2x &= 4 \\ x &= 2 \end{aligned}$$

$\therefore$ x-intercept $= (2, 0)$

2 Calculate the y-intercept by substituting $x = 0$.

Where $x = 0$:

$$\begin{aligned} 2 \times 0 + y &= 4 \\ y &= 4 \end{aligned}$$

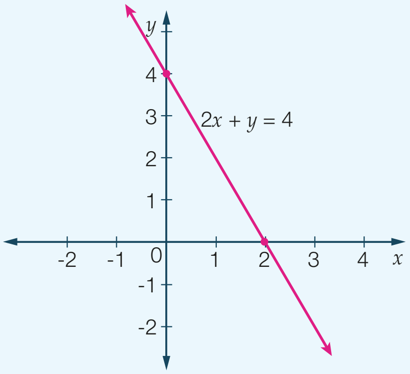
$\therefore$ y-intercept $= (0, 4)$

3 Draw and label a set of axes.

4 Mark the x- and y-intercepts.

5 Rule a line through the two points continuing in both directions.

6 Ensure that the intercepts are clearly labelled and that the line is labelled with its equation.



You can also use this method with linear equations in the form of $y = mx + b$.

Worked example 22

W.E. 22

Use the x- and y-intercepts to sketch the graph of $y = 6x - 2$.

Thinking

1 Calculate the x-intercept by substituting $y = 0$ and solving the equation.

Working

$$y = 6x - 2$$

Where $y = 0$:

$$\begin{aligned} 0 &= 6x - 2 \\ 6x &= 2 \\ x &= \frac{2}{6} \\ &= \frac{1}{3} \end{aligned}$$

$\therefore$ x-intercept $= \left(\frac{1}{3}, 0\right)$

2 Calculate the y-intercept by substituting $x = 0$.

Where $x = 0$:

$$y = 6 \times 0 - 2$$
$$= -2$$

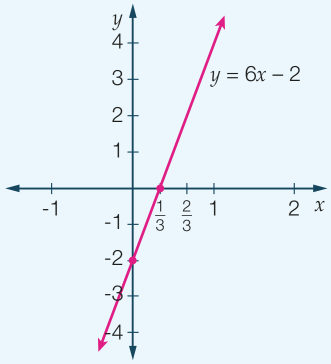
y-intercept = (0, -2)

3 Draw and label a set of axes.

4 Mark the x- and y-intercepts.

5 Rule a line through the two points continuing in both directions.

6 Ensure that the intercepts are clearly labelled and that the line is labelled with its equation.



Note: If the y-intercept is at the origin (e.g. $y = 4x$), this method cannot be used.

5.7 Sketching linear graphs using intercepts

Navigator

Answers p. 657

1 (a–f), 2 (column 1), 3, 4, 5 (a, b), 7, 8	1 (a–f), 2 (column 2), 3, 4, 5, 6, 7, 8	1 (d–i), 2 (column 3), 3, 4, 5, 6, 7, 8

Fluency

W.E. 21

1 Use the x- and y-intercepts to sketch each of the following.

(a) $x + 2y = 4$ **(b)** $3x + y = 6$ **(c)** $3x - y = 6$

(d) $x - y = 3$ **(e)** $4x - 2y = 12$ **(f)** $2x - 5y = 20$

(g) $2x + 5y = 10$ **(h)** $4x + 3y = 12$ **(i)** $3x + 4y = 2$

W.E. 22

2 Use the x- and y-intercepts to sketch each of the following.

(a) $y = 2x - 8$ **(b)** $y = 3x - 9$ **(c)** $y = x + 5$

(d) $y = x + 2$ **(e)** $y = 6 - 3x$ **(f)** $y = 2 - 4x$

(g) $2y = 3x + 4$ **(h)** $5y = 2x - 5$ **(i)** $3y = 6 - 2x$

3 **(a)** The coordinates of two points that lie on the line $2x + 3y = 6$ are:

A (2, 0) and (0, 3) **B** (-2, 0) and (0, -3)

C (3, 0) and (0, 2) **D** (-3, 0) and (0, -2)

(b) The coordinates of two points that lie on the line $3y = 2x - 6$ are:

A (-3, 0) and (0, 2) **B** (3, 0) and (0, -2)

C (-3, 0) and (0, -2) **D** (2, 0) and (0, -3)

Understanding

4 Let x and y represent two numbers. The difference between x and y is 3.

(a) Use the information above to write an equation.

(b) Use the x- and y-intercepts to sketch the graph of your equation in **(a)** above. Let x be the x-axis and y be the y-axis.

(c) Plot the point on your graph that represents x and y when x is equal to 5.

5 A milk vat is being drained for cleaning and maintenance. The volume, V, of milk remaining in the vat after a given time, t, is given by the relationship $V = 630 - 15t$, where V is in litres and t is in minutes.

(a) How much milk is left in the vat after 5 minutes?

(b) How much milk did the vat originally contain?

(c) How long will it take for the vat to be completely empty?

(d) Use this information to graph this relationship on a number plane.

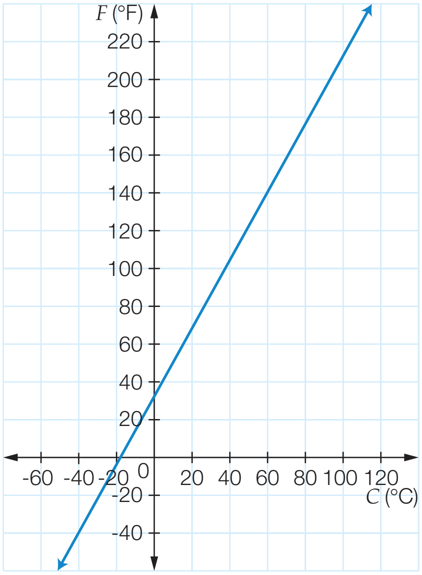
Reasoning

6 The graph shows the linear relationship between the Fahrenheit (F) and Celsius (C) temperature scales. It is known that water freezes at 0 °C, which is equivalent to 32 °F.

(a) Show that the equation showing the relationship between the Fahrenheit and Celsius scales is $F = \frac{9}{5}C + 32$.

(b) Water boils at 100 °C. Is 100 °F hotter or colder than this? Convert 100 °F to °C.

(c) Daniel Fahrenheit (1686–1736) observed that a water/ice/salt mixture freezes at 0 °F. What is this temperature in °C? Write your answer as a mixed number, e.g. $1\frac{3}{4}$.



7 Chantal has borrowed \$7200 from her parents to buy a car. She agrees to repay the amount, plus an extra \$300 in interest, to her parents at the rate of \$300 per month.

(a) What is the total amount she owes to her parents?

(b) How long will it take to pay off the loan?

(c) Sketch a graph of this relationship. (Put amount owing (\$$A$) on the vertical axis and time (t) in months on the horizontal axis.)

(d) Write an equation describing the amount Chantal owes her parents (\$$A$) after t months.

(e) How much money does Chantal owe after 6 months?

(f) How much money has Chantal repaid after 14 months?

Open-ended

8 Find three different pairs of x- and y-intercepts when the gradient of the graph is equal to 3.

5.8 Vertical and horizontal graphs

Other linear relationships can be of the form:

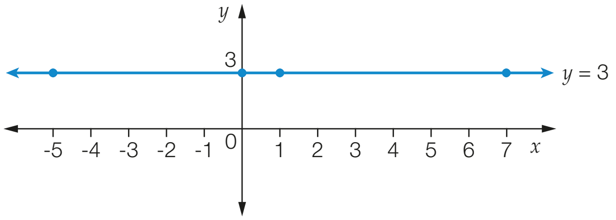
$y = b$ where b is a constant
or $x = a$ where a is a constant

Graphing equations in the form of $y = b$

Consider the equation $y = 3$. All coordinate points on the line will have a y-value of 3, no matter what value of x is given. Some coordinate points that lie on this line are (1, 3), (-5, 3), (0, 3) and (7, 3).

The sketch of the equation $y = 3$ is a straight line parallel to the x-axis.

$y = 3$ can be written in gradient–intercept form as $y = 0x + 3$, a straight line with zero gradient and a y-intercept of 3.

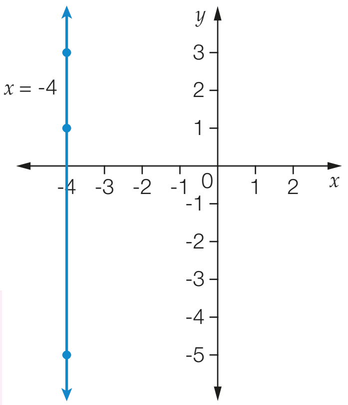


Graphing equations in the form of $x = a$

Similarly, in the equation $x = -4$, x will always have the value -4, regardless of the value of y used. Some coordinate points that lie on this line are (-4, 3), (-4, 0), (-4, -5) and (-4, 1).

The sketch of the equation $x = -4$ is a straight line parallel to the y-axis.

$x = 4$ cannot be written in gradient–intercept form because the gradient is undefined.



Linear graphs of the form:

$y = b$ are straight lines parallel to the x-axis (horizontal lines)

$x = a$ are straight lines parallel to the y-axis (vertical lines)

Worked example 23

W.E. 23

Sketch the graphs.

(a) $y = -2$ **(b)** $x = 5$

Hint

The scales on the x- and y-axes do not need to be the same.

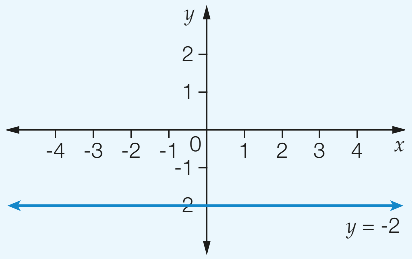
Thinking

(a) 1 The equation is of the form $y = b$, so it is a horizontal line passing through b on the y-axis ($b = -2$).

2 Ensure that the axes and any intercepts are labelled and that the line is labelled with its equation.

Working

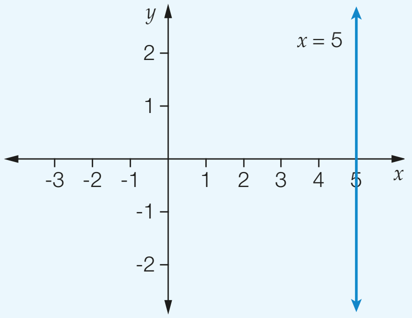
(a)



(b) **1** The equation is of the form $x = a$, so it is a vertical line passing through a on the x-axis ($a = 5$).

2 Ensure that the axes and any intercepts are labelled and that the line is labelled with its equation.

(b)



5.8 Vertical and horizontal graphs

Navigator

1 (a–f), 2, 3, 4, 5 (column 1), 6, 7, 8, 9	1 (d)–(i), 2, 3, 4, 5 (column 2), 6, 7, 8, 9, 10	1 (g)–(l), 2, 3, 4, 5 (column 3), 6, 7, 8, 9, 10

Answers p. 659

Fluency

W.E. 23

1 Sketch the graphs of the following, showing any intercepts on the axes.

(a) $y = 2$

(b) $y = 4$

(c) $x = -3$

(d) $x = -1$

(e) $x = 6$

(f) $x = \frac{5}{2}$

(g) $y = -3$

(h) $x = -2$

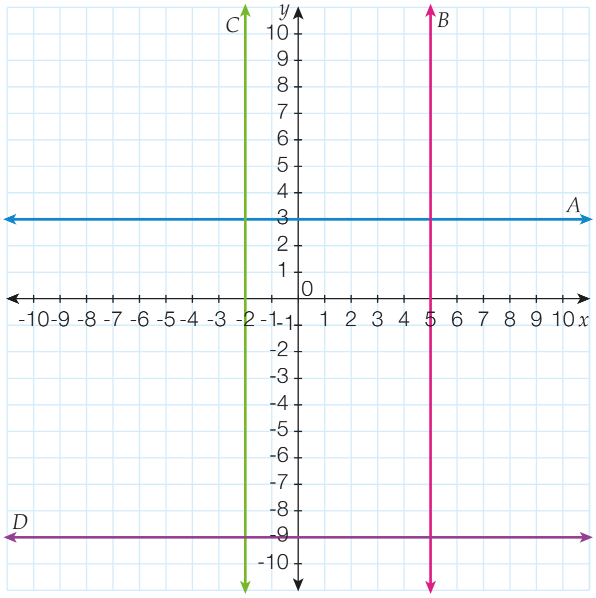
(i) $y = -\frac{3}{4}$

(j) $x = 4$

(k) $y = -\frac{7}{2}$

(l) $y = 5$

2 Write the equation of each of the lines A–D shown below.



3 Write the equation of the straight line given:

(a) $m = 0$ and $b = -6$

(b) $m = 0$ and $b = 2$

(c) gradient 0 and y-intercept $\frac{2}{3}$

(d) gradient 0 and y-intercept $\frac{3}{4}$

4 **(a)** Which coordinates represent a point on the line $y = 5$?

A (5, 0) **B** (5, 5) **C** (-5, 0) **D** (-5, -5)

(b) Which coordinates *do not* represent a point on the line $x = 1$?

A (1, 0) **B** (-1, 1) **C** (1, -1) **D** (1, 11)

(c) Which coordinates *do not* represent a point on the line $y = -8$?

A (8, -8) **B** (-8, -8) **C** (-8, 0) **D** (0, -8)

(d) Which coordinates represent a point on the line $x = -3$?

A (3, 0) **B** (3, 3) **C** (3, -3) **D** (-3, -3)

Understanding

5 Sketch the graphs of the following linear relationships using the most appropriate method, showing any relevant points.

(a) $2y = -6$
(b) $y = -6$
(c) $2x = 8$
(d) $x + 1 = 0$
(e) $x + 4 = 0$
(f) $y - 3 = 0$
(g) $5x = 4$
(h) $3y = 2$
(i) $2y = 5$
(j) $2y = 7$
(k) $3x = -5$
(l) $3y = -4$
(m) $5x = 0$
(n) $6x = 0$
(o) $7y = 0$
(p) $3y - 6 = 0$
(q) $2y - 8 = 0$
(r) $4x + 12 = 0$
(s) $4y - 3 = 0$
(t) $3x - 8 = 0$
(u) $2x + 7 = 0$

Reasoning

6 Five streets form the boundary of an undeveloped outer suburban block of land. Taking the southernmost point of the block as the coordinate point (0, 0), each of the roads can be plotted on a number plane. The graphs that represent the roads are $y = 2x$, $x = 3$, $y = 10$, $x + 2 = 0$ and $2y + 3x = 0$.

The local council plans to subdivide this land by putting in new roads along the lines $x = 0$, $y = -5x$ and $3y - 10x = 0$.

(a) Sketch each of the linear graphs that represent the boundary roads on the same number plane.

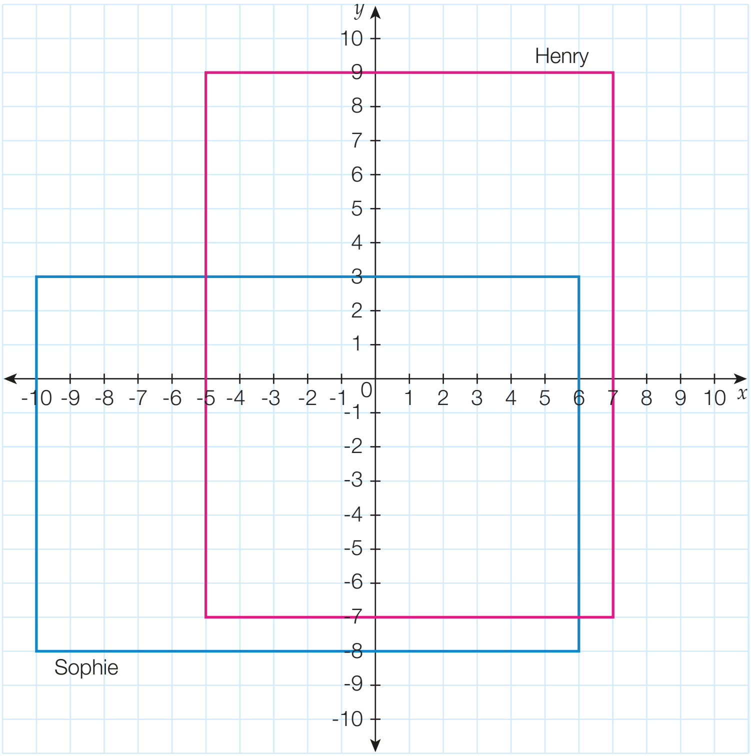
(b) From your sketch, determine the coordinate points that are common to each adjacent pair of graphs.

(c) Sketch each of the linear graphs that represent the new roads.

(d) Will an old shed located at the coordinate point (-1, 5) be affected by the new road?

Open-ended

7 Henry and Sophie were asked by their teacher to construct a rectangle of area 176 square units using only horizontal and vertical lines. Their different results are shown.



(a) Whose rectangle is correct and whose is not? Explain.

(b) The person with the correct rectangle must explain to the other person how to fix their answer. Write this explanation.

8 Write three equations whose graphs are parallel to the x-axis. Sketch the graphs for each.

9 Write three equations whose graphs are parallel to the y-axis. Sketch the graphs for each.

10 Draw three different pairs of linear graphs on the same number plane. Each pair of graphs must have a graph parallel to the x-axis and the other parallel to the y-axis. From these graphs, determine the coordinate point which is common to each pair.

Problem solving

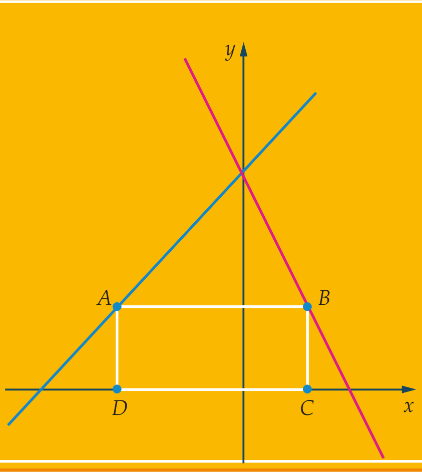
Area of a rectangle

In the diagram, $ABCD$ is a rectangle with A on the line $y = x + 10$, B on the line $y = -2x + 10$, and C and D on the x-axis.

If $AD = 4$, what is the area of rectangle $ABCD$?

Strategy options

- Solve a simpler problem.
- Break into manageable parts.



Investigation

A fun park dilemma

Equipment required: calculator

Jo and Adam are on holiday and have decided to visit a Fun Park attraction. They find that they have two options:

Option 1: A $50 entry fee, which includes unlimited rides.

Option 2: A $5 entry fee, which includes one free ride, and then pay $6 per additional ride.

The park has 10 different rides available.

Adam started to write the costs of going on an increasing number of rides: $5, $11, $17, $23, $29… and then stopped because he saw a pattern emerging. He thought for a while and then wrote down an equation. 'If you tell me how many rides you want to go on, I can tell you straight away which is the better option and what it will cost,' he said to Jo.

The Big Question

What equations did Adam use to choose the best payment option—the all-inclusive 'flat' $50 fee or the $5 + one free ride plus $6 for every extra ride?

Engage

The numbers 5, 11, 17, 23 form a pattern called an 'arithmetic sequence' because there is a common difference between each term.

1 What is the common difference (d) for the terms of the above pattern?

2 It is easy to continue number patterns such as the one above because the difference between two successive terms is always the same. Use the common difference to continue the pattern to find the:

(a) 5th term **(b)** 10th term **(c)** 15th term.

3 Use this pattern to write how much it would cost if Joanne paid the $5 entry fee and went on:

(a) one ride **(b)** five rides **(c)** ten rides.

Explore

4 An arithmetic sequence is dependent on the first term, a, and the common difference, d. Copy and complete the following table for an arithmetic sequence: T_1 is used to denote the first term in the sequence.

T_1 (term 1)	T_2 (term 2)	T_3	T_4	T_5
$= T_1$	$= T_1 + d$	$= T_2 + d$ $= T_1 + 2d$		
$= a$	$= a + d$	=	=	=

5 Using the completed table, write an equation for the 5th term in the sequence in terms of a and d.

$T_5 =$

6 Check this equation by substituting $a = 5$ and the value for d that you obtained in **1** to find the value of the 5th term and compare it with your answer to **2 (a)**.

7 Write similar equations for the 10th and 15th terms.

8 Use those equations to find the 10th and 15th terms.

Strategy options

- Break the problem into manageable parts.
- Look for a pattern.
- Test all possible combinations.

Explain

9 Use your equation to calculate how much money Joanne would save if:

(a) She paid the $50 entry fee and went on 10 rides instead of paying the $5 entry and $6 per additional ride.

(b) She paid the $5 entry fee and went on 6 rides instead of paying the $50 entry fee with unlimited rides.

10 By using equations or graphs, show Joanne how she can work out the best entry fee to pay for the number of rides she wants to go on.

11 What did you notice about the coefficient of d for each term in the table?

12 Explain how you could use the pattern you have found to find a rule for T_n, the nth term.

Elaborate

13 Write an equation to find the nth term of an arithmetic sequence.

14 Use your equation for the nth term of an arithmetic sequence to find:

(a) the 50th term of the sequence 5, 7, 9, 11…

(b) the 100th term of the sequence 2, 7, 12, 17…

(c) the 1000th term of the sequence -4, -2, 0, 2…

(d) the 1 000 000th term of the sequence 8, 14, 20, 26…

15 Find the first term of the sequence whose:

(a) 101st term is 209 and the common difference is 2

(b) 500th term is 1496 and the common difference is 5

(c) 768th term is 932 and the common difference is -4.

Evaluate

16 Comparing the methods used in **Engage** and **Explore**, which was the easier to use to find the 5th term?

17 Comparing the methods used in **Engage** and **Explore**, which was the easier to use to find the 15th term?

18 Which method would you use to find a term in an arithmetic sequence?

19 Where would knowledge of arithmetic sequences be used?

Extension

20 In 2004, the young mathematicians Ben Green and Terrence Tao proved that within lists of prime numbers there are special groups of numbers that form an arithmetic progression. These arithmetic progressions can be short or very long!

An example of a short sequence is the prime numbers 3, 7, 11. This has a common difference of 4, but the sequence ends at 11 because $11 + 4 = 15$, which is not a prime number.

Try to identify at least three more different arithmetic sequences in the following list of prime numbers. The smallest number of terms in a sequence is three and the sequences can be as long as you want:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Challenge 5

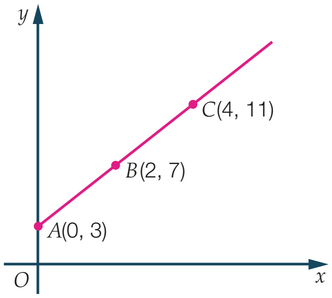
1 It takes Vinny 30 minutes to walk around the block, but Jesse can ride around on his bike in 15 minutes. If Vinny starts walking at 5:35 pm and Jesse follows at 5:48 pm, when will Jesse catch up to Vinny?

2 Jakki is travelling through country Victoria and is trying to decide on her mode of travel for the next 5 days. If she hires a bicycle for 2 days and a car for the other 3 days, then she will pay $150. If she hires a bicycle for 4 days and a car for the remaining 1 day, then she will pay $90. What is the cost of hiring a bicycle for one day?

3 Nina uses the following table to produce a straight line graph.

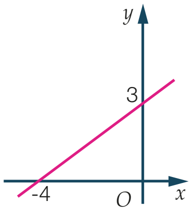
x	0	2	4
y	3	7	11

The point $D(7, w)$ is also on the line. Find the value of w.

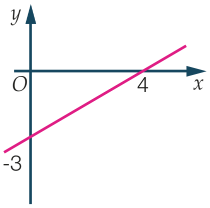


4 Which line has a gradient of $-\frac{3}{4}$? Diagrams are not to scale.

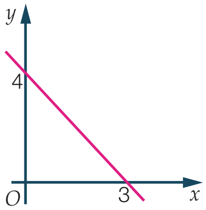
A



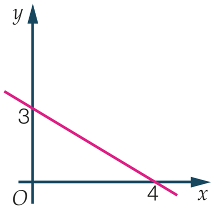
B



C



D



5 **(a)** The square $ABCD$ is plotted on a number plane. Given $A(1, 1)$, $B(3, 2)$ and $C(2, 4)$, find the coordinates of D.

(b) Write the gradients of AB and BC. What do you notice about your answers?

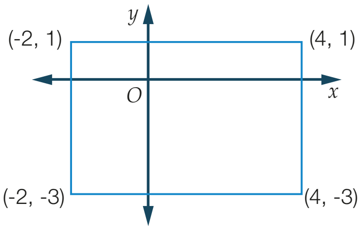
(c) Write the gradients of AD and DC. What do you notice about your answers?

(d) What can you say about the gradients of the parallel sides of the square?

(e) What can you say about the gradients of the perpendicular sides of the square?

6 The square $KLMN$ is to be constructed on the side KL. Given $K(-2, -1)$ and $L(1, 3)$, find two pairs of values for the coordinates of M and N.

7 The diagonals of this rectangle intersect at a point. What are the coordinates of this point?



8 Solve each equation for x.

(a) $\frac{x-a}{x-b} = 1 - \frac{x}{x-b}; x \neq b$

(b) $\frac{a}{bx} - \frac{b}{ax} = a^2 - b^2; a \neq 0, b \neq 0$

(c) $\frac{a-b}{x-c} = \frac{a+b}{x+c}; x \neq \pm c$

Chapter review 5

Maths literacy

coordinates
dependent variable
distance
equivalence
general form
gradient
gradient–intercept form
independent variable
intercept
interval
inverse operation
linear equation
linear graph
linear relationship
midpoint
origin
parallel
perpendicular
rise
run
slope
solve
straight line
undefined

Copy and complete the following using the words and phrases from the list, where appropriate. A word or phrase may be used more than once.

1 The ___________ or ___________ is a measure of a line's steepness.

2 The ___________ can be found by dividing the vertical rise by the horizontal ___________ between two points.

3 The graph of a linear relationship is called a ___________.

4 To solve an equation, perform the ___________ in the reverse order.

5 A ___________ has only one solution.

6 The point where a linear graph cuts the y-axis is called the y-___________.

Fluency

1 Solve the following equations. **5.1**

(a) $3x - 7 = -22$ **(b)** $7x + 4 = -3$ **(c)** $\frac{x}{5} + 9 = 0$

(d) $5 - \frac{x}{6} = -7$ **(e)** $6(x - 2) + 7 = -5$ **(f)** $4(2x - 5) = 0$

2 Solve these equations. **5.1**

(a) $\frac{4x}{3} - 20 = -8$ **(b)** $\frac{2x - 3}{5} = 7$

(c) $\frac{3 - 9x}{2} = -4$ **(d)** $\frac{4(7x + 1)}{11} = 8$

3 Solve these equations. **5.1**

(a) $4x - 5 = 8 - 2x$ **(b)** $8x + 5 = 3(2x - 11)$ **(c)** $5x + 18 = 3x$

(d) $2x = 9 - 7x$ **(e)** $\frac{1 - 4x}{8} = \frac{3x - 7}{4}$ **(f)** $\frac{2x - 5}{3} = \frac{x + 7}{4}$

4 **(a)** The linear equation $\frac{x}{8} - 5 = 4$ has the solution: **5.1**

A $x = 8$ **B** $x = 72$ **C** $x = 37$ **D** $x = 13$

(b) The linear equation $6x + 21 = 4x + 1$ has the solution:

A $x = 2$ **B** $x = 10$ **C** $x = -10$ **D** $x = -2$

5 Pete keeps pet mice. He has a lot of mice. He has just been given four more. His friend Mal also keeps pet mice, but he has 15 fewer than Pete has now. **5.2**

(a) Write an expression to show how many pet mice Pete has now.

(b) Write an expression to show how many pet mice Mal has now.

(c) If Mal has 10 mice now, use an equation to find out how many mice Pete had originally.

6 For the points $P(-3, 5)$ and $Q(6, 11)$: **5.3**

(a) find the exact length PQ

(b) find the midpoint of PQ.

7 A television is on sale for \$1850. Lea arranges to pay \$500 as a deposit and then weekly payments of \$90. **5.4**

(a) Copy and complete this table.

Number of weeks	0	1	2	3	4	5	6
Amount paid ($)							

(b) Plot the amount paid for the television set (A) against the number of weeks (n). Extend your graph to show the amount paid up to 20 weeks. (Plot number of weeks along the horizontal axis.)

(c) From your graph, how much would Lea have paid after 11 weeks?

(d) How long would it take Lea to pay in full?

(e) Write a relationship connecting the amount paid in dollars (A) to the number of weeks (n).

(f) Use this relationship to calculate the amount paid after 9 weeks. Check to see whether you obtained the same result from your graph.

8 Find the gradient of the line joining (3, 2) and (4, 7). **5.5**

9 **(a)** The gradient of the line joining the points (-5, 3) and (2, -4) is: **5.5**

A -1 **B** $\frac{1}{7}$ **C** 1 **D** $\frac{7}{3}$

(b) The gradient of the line joining the points (-2, 3) and (3, 3) is:

A 0 **B** $\frac{1}{5}$ **C** 1 **D** 5

10 Plot the graph of $y = 2x + 4$ by first completing a table of values for x and y. Find the gradient, x-intercept and y-intercept for this line. **5.6**

11 Write four points that lie on each of the following lines and draw the graph. **5.6**

(a) $y = 0$ **(b)** $x = -2$ **(c)** $y = 4$ **(d)** $x = 0$

12 Find the y-intercept and gradient for each of the following, and sketch the graph of each. **5.7**

(a) $y = 2x - 4$ **(b)** $2y = 3x - 4$ **(c)** $x + y = 5$

(d) $3x + 2y = 4$ **(e)** $y = 3x$ **(f)** $y = -2x$

13 Find the x- and y-intercepts for each of the following and hence sketch graphs of each, showing the intercepts on the axes. **5.7**

(a) $4x - y = 4$ **(b)** $2x + 5y = 20$ **(c)** $y - 3x = 6$ **(d)** $2x + y = 5$

14 **(a)** The y-intercept of the graph of the linear equation $y + 2x = 3$ is: **5.6, 5.7**

A -3 **B** -2 **C** 2 **D** 3

(b) The x-intercept of the graph of the linear equation $3x + 5y = 45$ is:

A 3 **B** 5 **C** 15 **D** 45

(c) The gradient of the graph of the linear equation $7x - 4y = 28$ is:

A $-\frac{7}{4}$ **B** $-\frac{4}{7}$ **C** $\frac{4}{7}$ **D** $\frac{7}{4}$

15 Sketch these graphs on the one number plane: **5.8**

(a) $y = -5$ **(b)** $y = 3$ **(c)** $x = -2$

Understanding

16 Polly walks at a constant speed for 5 minutes, travelling 350 metres. **5.2**

(a) To construct a distance–time graph, what two points can you use? (Do not draw the graph.)

(b) What is the gradient between these two points?

(c) What is Polly's average speed during these 5 minutes?

(d) Write an equation relating distance covered, d metres, to time taken, t minutes.

(e) Use your equation to find the distance Polly will travel in 8 minutes if she maintains her walking speed.

17 **(a)** The sum of three consecutive whole numbers is 66. What are the numbers? **5.2**

(b) The sum of four consecutive even whole numbers is 100. What are the numbers?

18 The length of a rectangular paddock is 7 m less than twice the width. If the perimeter is 58 m, write an equation and solve it to find the length and width of the paddock. **5.2**

Reasoning

19 Two lines are given by $kx + 6y - 5 = 0$ and $3x + 2y + 4 = 0$. Find the value of k such that the lines are parallel. **5.6**

20 A bicycle manufacturing company needs to start making a certain component to increase its range of products. $3000 needs to be spent on set-up costs and each component that is produced then costs an additional $5.75. **5.2, 5.6**

(a) Write a cost equation that describes the total cost C of producing x components.

(b) Find the cost of producing 500 components.

(c) How many components could be produced on a budget of $9000?

(d) Draw a graph of the cost equation to find the cost of 1000 components.

21 An electrician charges a fixed fee plus an hourly rate. A job that takes 2 hours costs $280 and a job that takes 3.5 hours costs $437.50. **5.2**

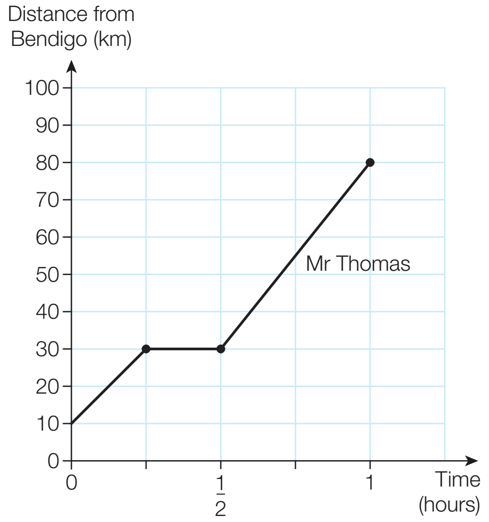
(a) Write the linear equation to find the charge $C for t hours of work.

(b) Find the amount she earns if she spends 8 hours on one job.

(c) Find the time taken to complete the job if the charge is $306.25.

22 Ms Thomas, who works in Bendigo, has arranged to meet her husband at home in exactly 1 hour. Mr Thomas works closer to home but on the same route. He leaves work at the same time as Ms Thomas but needs to make a stop along the way, as seen in the following, but he still arrives home on time.

5.2, 5.4
5.5, 5.8



(a) Copy this graph onto graph paper.

(b) How far from Bendigo did they live?

(c) How fast was Mr Thomas travelling in the first 15 minutes?

(d) How long did he stop on his trip home?

(e) How fast did he travel in the last leg of his trip?

(f) How far from Bendigo did Mr Thomas work?

(g) If Ms Thomas drives at a constant speed from Bendigo, draw the line on your graph that represents her travel.

(h) At what speed does Ms Thomas need to travel, to arrive home 1 hour later?

(i) Write a linear equation that represents her distance from Bendigo at any particular time.

(j) At what time does Ms Thomas pass Mr Thomas on the way home? (Use your graph.)

(k) At what distance from home does she pass him?

Numeracy practice 5

Non-calculator

1 Which expression is equivalent to $5 - 7x$?

A $7x - 5$	**B** $-7x - 5$	**C** $-7x + 5$	**D** $7x + 5$

2 What is the gradient of the straight line connecting the points (-3, -4) and (-1, 8)?

A 8	**B** 4	**C** 2	**D** 6

3 What is the value of b in the equation $5b - 4 = 2b + 17$?

4 If $y = -x + 4$ and $y = 2x - 8$, which point lies on both lines?

A (-3, 7)	**B** (4, 0)	**C** (1, -6)	**D** (3, 1)

5 Which of these points lies on the straight line joining (4, 4) and (20, 12)?

A (5, 5)	**B** (6, 6)	**C** (10, 7)	**D** (14, 8)

Calculator allowed

6 What number comes next in the following sequence?

25, 50, 75, 100…

7 $y = 2x - 1$
$y = 3x + 2$

Which value of x satisfies both these equations?

A $x = -3$	**B** $x = -1$	**C** $x = 1$	**D** $x = 3$

8 Find the value of x that makes the equation $\frac{3x - 5}{2} = 5$ true.

9 The sum of two consecutive numbers is 55. What are the numbers?

A 33 and 22	**B** 22 and 23	**C** 27 and 28	**D** 30 and 25

10 An isosceles triangle has two equal sides and a third side that is 15 cm longer. Find the length of the longest side if the perimeter is 84 cm.

6

Geometric reasoning

6

Rendered polygons to realistic animation.

The characters in animated movies look almost lifelike in their movements and expressions. It is hard to believe that they are created using only straight lines.

Today, most animated films are computer generated, with from 24 to 60 or more still images shown every second to create the illusion of movement. Three-dimensional graphics are created using a transparent polygon mesh mapped onto a two-dimensional surface for the final image. Each polygon is broken into small triangles and 'rendered', a process that adds colour, shading and lighting, creating smooth curves and circular shapes. Once rendered, movement points called avars (animation variables) can be placed on the image, allowing a character's features to move slightly from frame to frame. It is hard to believe that the characters, the scenery and all the images in animated movies are made with the simplest polygon of all, the triangle.

Forum

Do you think animated movies have become as good as they can be? What improvements would you like to see in animation? What are the advantages of animating, compared to filming live action with a camera? If animations become more and more realistic, will there be any need for 'real' actors?

Why learn this?

Understanding geometry improves your ability to produce new and more complex shapes, create innovative designs and build amazing, unusual and original inventions. Technology, engineering, architecture and the building industry all rely heavily on good knowledge of geometric principles to visualise, design and build things that are strong, elegant, efficient and practical.

After completing this chapter you will be able to:

- apply known properties of geometric figures, including angle properties associated with parallel lines, triangles and quadrilaterals, to find other angles
- use congruence rules for triangles to prove results for other figures
- use geometric reasoning to prove results about geometric figures
- calculate the angle sum of polygons and find angles in regular polygons
- know and use the properties of different types of quadrilaterals
- enlarge and reduce two-dimensional figures using projections and scale factors
- identify similar triangles and use them to solve mathematical and practical problems
- draw nets for different solids including polyhedra.

Recall 6

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

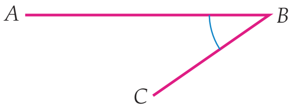
1 Draw angles of the following sizes, using a protractor and a ruler.

(a) 48° **(b)** 127° **(c)** 235° **(d)** 315°

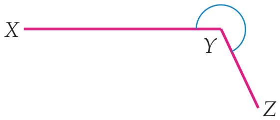
2 Name the type of each angle drawn in Question **1** using a word from this list: acute; right; obtuse; straight; reflex.

3 Use a protractor to measure the following angles.

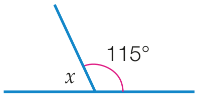
(a)



(b)



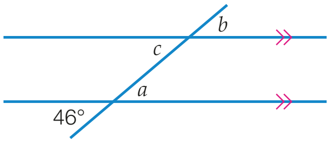
4 Choose the correct value for x.



A 45° **B** 245°

C 115° **D** 65°

5 Find the value of the variables in this diagram.



6 Solve for x.

(a) $x + 7 = 180$ **(b)** $2x + 40 + 3x = 360$ **(c)** $x + 20 + 5x - 80 = 360$

7 Use the scale and measure this picture to determine the actual measurements, in metres, of:

(a) the length of the car

(b) the height of the car.

Scale 100 : 1

8 Write the following as unit ratios. Round answers to 2 decimal places if necessary.

(a) 300 : 50 **(b)** 14 : 4 **(c)** 1400 : 64 **(d)** 258 000 : 376

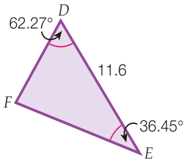
9 A baker's biscuit recipe requires 2.5 times as much flour as sugar. If 10 cups of flour are used in one batch of biscuits, how much sugar should be used?

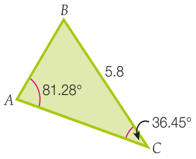
Exploration Task

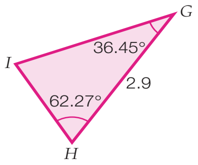
You can download this activity from the eBook or the Pearson Places website.

What makes two triangles similar to each other?

In the activity, you will explore the meaning of similarity and congruence using different triangles.



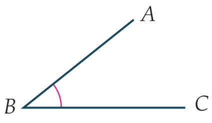
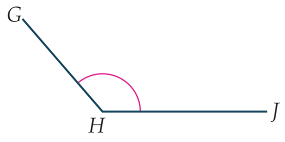
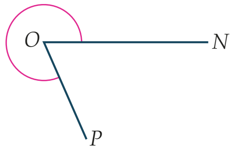
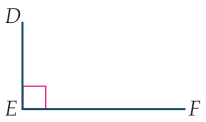
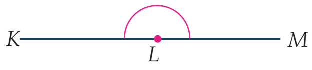




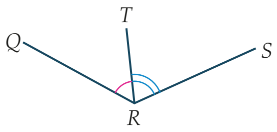
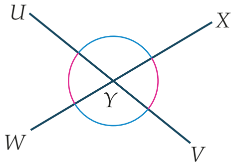
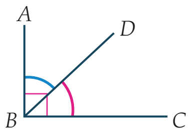
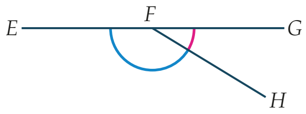
Angle review

6.1

Angles at a point

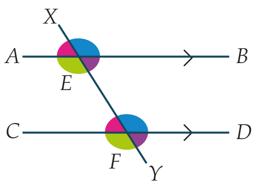
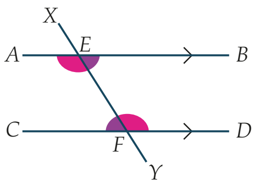
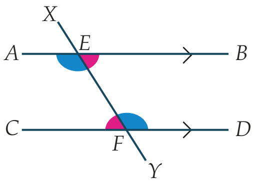
Acute angle:  Less than 90° $0 < \angle ABC < 90°$	Obtuse angle:  Greater than 90° but less than 180° $90° < \angle GHJ < 180°$	Reflex angle:  Greater than 180° but less than 360° $180° < \angle NOP < 360°$
Right angle:  Exactly 90° $\angle DEF = 90°$	Straight angle:  Exactly 180° $\angle KLM = 180°$	Revolution or angle of revolution:  Exactly 360°

Pairs of angles

Adjacent angles:  Angles that have a common vertex and a common arm. *R* is a common vertex. *TR* is a side common to both $\angle QRT$ and $\angle TRS$. $\angle QRT$ and $\angle TRS$ are a pair of adjacent angles.	Vertically opposite angles:  A pair of angles formed when two straight lines intersect. They are on opposite sides of the vertex. Vertically opposite angles are equal. The line segments *UV* and *WX* intersect at *Y*. $\angle UYW$ and $\angle XYV$ are a pair of vertically opposite angles. $\angle UYW = \angle XYV$ $\angle UYX$ and $\angle WYV$ are another pair of vertically opposite angles. $\angle UYX = \angle WYV$
Complementary angles:  A pair of angles whose sum is 90°. $\angle ABD$ and $\angle DBC$ are a pair of complementary angles. $\angle ABD + \angle DBC = 90°$	Supplementary angles:  A pair of angles whose sum is 180°. $\angle EFH$ and $\angle GFH$ are a pair of supplementary angles. $\angle EFH + \angle GFH = 180°$

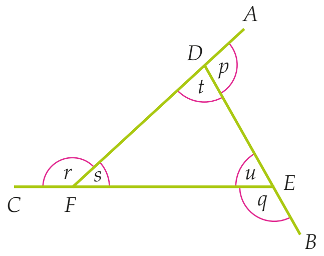
Angles on parallel lines

AB and CD are parallel ($AB \parallel CD$), the transversal XY cuts AB at E and CD at F.

Corresponding angles:	Alternate angles:	Co-interior (allied) angles:
 When a transversal cuts a pair of parallel lines, four pairs of equal **corresponding angles** are formed. The pairs of equal corresponding angles are: $\angle AEX = \angle CFE$ $\angle BEX = \angle DFE$ $\angle AEF = \angle CFY$ $\angle BEF = \angle DFY$	 When a transversal cuts a pair of parallel lines, two pairs of equal **alternate angles** are formed. The pairs of equal alternate angles are: $\angle AEF = \angle DFE$ $\angle BEF = \angle CFE$	 When a transversal cuts a pair of parallel lines, two pairs of supplementary **co-interior angles** are formed. The pairs of supplementary co-interior angles are: $\angle AEF + \angle CFE = 180°$ $\angle BEF + \angle DFE = 180°$

Exterior angle of a triangle

This is the angle formed when any side is extended. The **exterior angle** of a triangle is equal to the sum of the opposite **interior angles**.



In this diagram,

FD is extended to A to form the exterior angle $\angle ADE$

$\angle ADE = \angle DFE + \angle FED$

$p = s + u$

DE is extended to B to form the exterior angle $\angle BEF$

$\angle BEF = \angle EFD + \angle FDE$

$q = s + t$

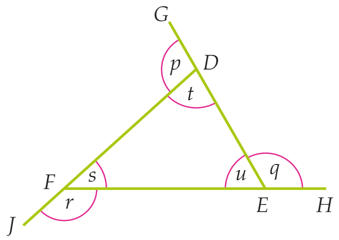
EF is extended to C to form the exterior angle $\angle CFA$

$\angle CFA = \angle FDE + \angle DEF$

$r = t + u$

Note that:

- DF can be extended to J to form the exterior angle $\angle JFE$, where $\angle JFE = \angle CFD$
- ED can be extended to G to form the exterior angle $\angle GDF$, where $\angle GDF = \angle ADE$
- FE can be extended to H to form the exterior angle $\angle HED$, where $\angle HED = \angle BEF$.

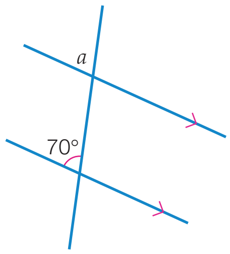


Worked example 1

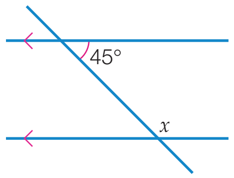
W.E. 1

Find the value of the variables in each diagram. Give a reason for your answer.

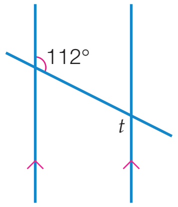
(a)



(b)



(c)



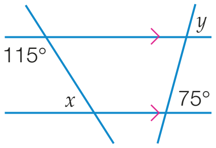
Thinking	Working
(a) Identify the type of angle relationship to find the value of the variable. Write the value of the variable. Write the type of angle relationship as the reason for your answer.	(a) 70° and *a* are corresponding angles on parallel lines, so $a = 70^\circ$.
(b) Identify the type of angle relationship to find the value of the variable. Find the value of the variable. Write the type of angle relationship as the reason for your answer.	(b) 45° and *x* are co-interior angles between parallel lines, so they add up to 180°. $x + 45^\circ = 180^\circ$ $x = 180^\circ - 45^\circ$ $= 135^\circ$
(c) Identify the type of angle relationship to find the value of the variable. Write the value of the variable. Write the type of angle relationship as the reason for your answer.	(c) 112° and *t* are alternate angles on parallel lines, so $t = 112^\circ$.

Worked example 2

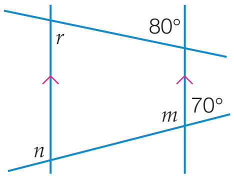
W.E. 2

Find the value of the variables in each diagram. Give reasons for your answer.

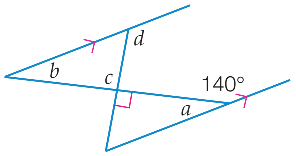
(a)



(b)



(c)



Thinking	Working
(a) Identify all the relationships in the diagram to find the value for each variable. Write the reason in brackets after your answer.	(a) $x = 180^\circ - 115^\circ$ $= 65^\circ$ (co-interior angles between parallel lines) $y = 75^\circ$ (corresponding angles on parallel lines)

(b) Identify all the relationships in the diagram to find the value for each variable. Write the reason in brackets after each answer.	(b) $m = 180° - 70°$ (straight angle) $= 110°$ $n = m$ $= 110°$ (corresponding angles on parallel lines) $r = 80°$ (alternate angles on parallel lines)
(c) Identify all the relationships in the diagram to find the value for each variable. Write the reason in brackets after each answer.	(c) $a = 180° - 140°$ (straight angle) $= 40°$ $b = a$ $= 40°$ (alternate angles on parallel lines) $c = 90°$ (vertically opposite) $d = b + c$ $d = 40° + 90°$ $= 130°$ (exterior angle of a triangle)

6.1 Angle review

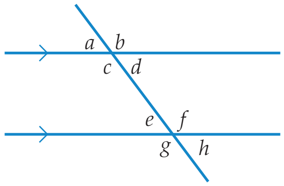
Navigator

Answers p. 667

1, 2, 3, 4 (a–c), 6 (a–e), 7, 8, 10, 13	1, 2, 3 (columns 1, 2), 4, 5, 6, 7, 8, 10, 11, 12, 13	1, 2, 3 (column 3), 4 (d–f), 5, 6, 7, 8, 9, 11, 12, 14

Fluency

1 Complete each statement by referring to the diagram and choosing from the following list of terms: co-interior, corresponding, alternate, vertically opposite, supplementary.



(a) The angles a and d are ____________ angles.

(b) The angles d and f are ____________ angles between parallel lines.

(c) The angles d and e are ____________ angles on parallel lines.

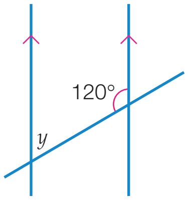
(d) The angles g and h are ____________ angles.

(e) The angles d and h are ____________ angles on parallel lines.

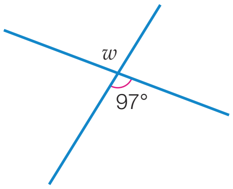
W.E. 1

2 Find the value of the variables in each diagram. Give reasons for your answer.

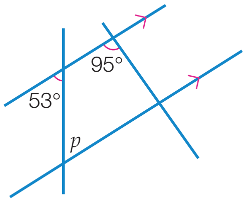
(a)



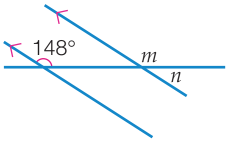
(b)



(c)

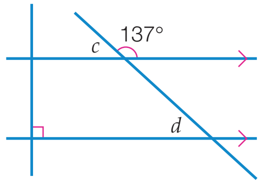


(d)

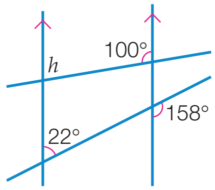


3 Find the value of the variables in each diagram. Give reasons for your answer.

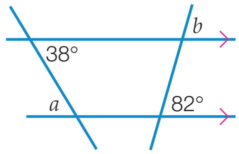
(a)



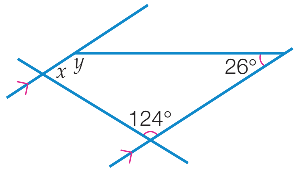
(b)



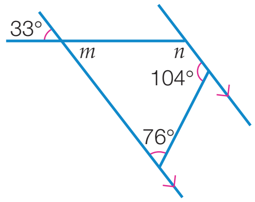
(c)



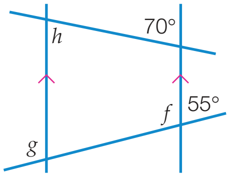
(d)



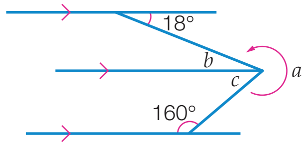
(e)



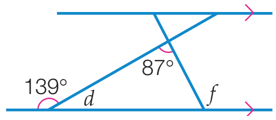
(f)



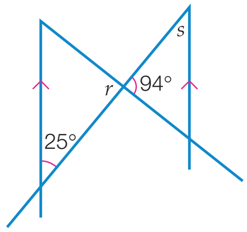
(g)



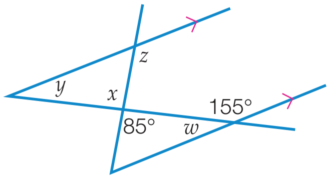
(h)



(i)

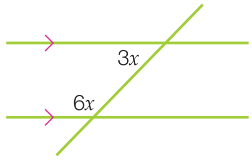


(j)

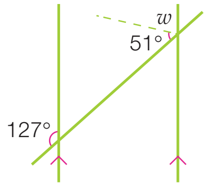


4 Find the value of the variables in each diagram. Give reasons for your answer.

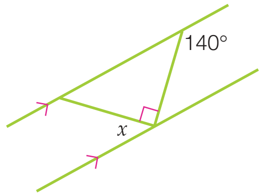
(a)



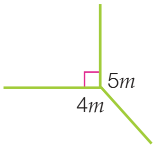
(b)



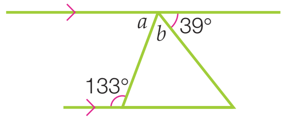
(c)



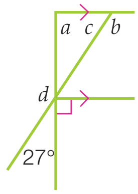
(d)



(e)

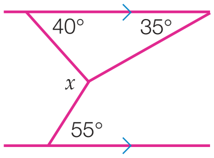


(f)



5 In the diagram, the value of x is:

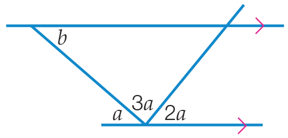
A 75 B 95 C 125 D 140



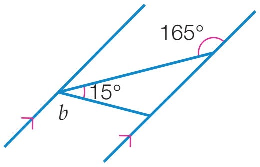
Understanding

6 Find the value of the variables in each diagram. Give reasons for your answer.

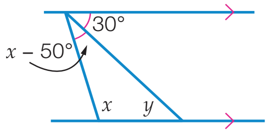
(a)



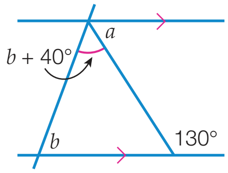
(b)



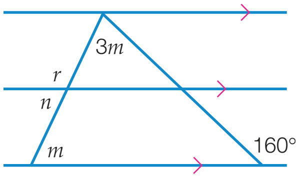
(c)



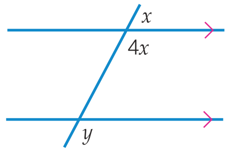
(d)



(e)



(f)



7 Write true (T) or false (F) for each of the following statements:

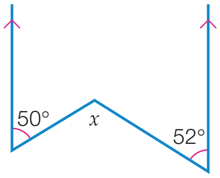
(a) Alternate angles are always equal.

(b) Co-interior angles are always supplementary.

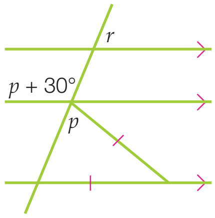
(c) If the sum of a pair of adjacent angles is greater than 180°, then one of the angles must be an obtuse angle.

(d) The exterior angle of a triangle is always an obtuse angle.

8 Find the value of x.



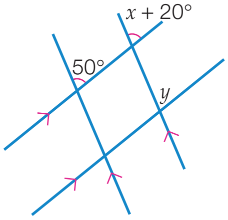
9 Find the value of the variables in the diagram. Give reasons for your answer.



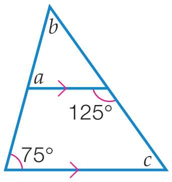
Reasoning

10 Find the value of the variables in each diagram.

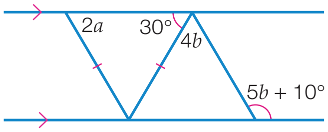
(a)



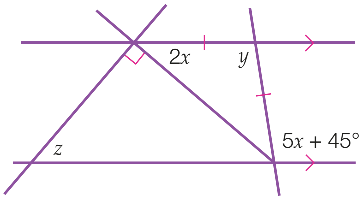
(b)



(c)



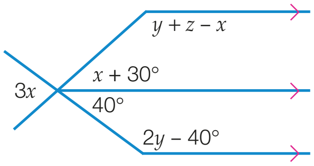
11 Find the values of x, y and z, giving reasons for your answer.



12 **(a)** Refer to the diagram on the right. Form an equation and solve it to find x. Give reasons for your answer.

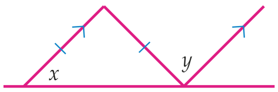
(b) Form an equation and solve it to find y. Give reasons for your answer.

(c) Hence, form an equation and solve it to find z.



Open-ended

13 Choose three possible values for x. Show that $2x + y = 180°$ and use this to find y for each of your x-values.



14 **(a)** Write a possible pair of values for x and y.

(b) Write a possible pair of values for m and n.

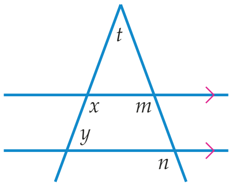
(c) Write an equation to find t in terms of n and y.

(d) Rewrite this equation to find t in terms of m and x.

(e) Use the equation in part **(c)** to find the value of t for your values of n and y.

(f) Use the equation from part **(d)** to find the value of t for your values of x and m.

(g) Compare your answers for parts **(e)** and **(f)**. Were they the same?



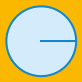
Game

Space invaders

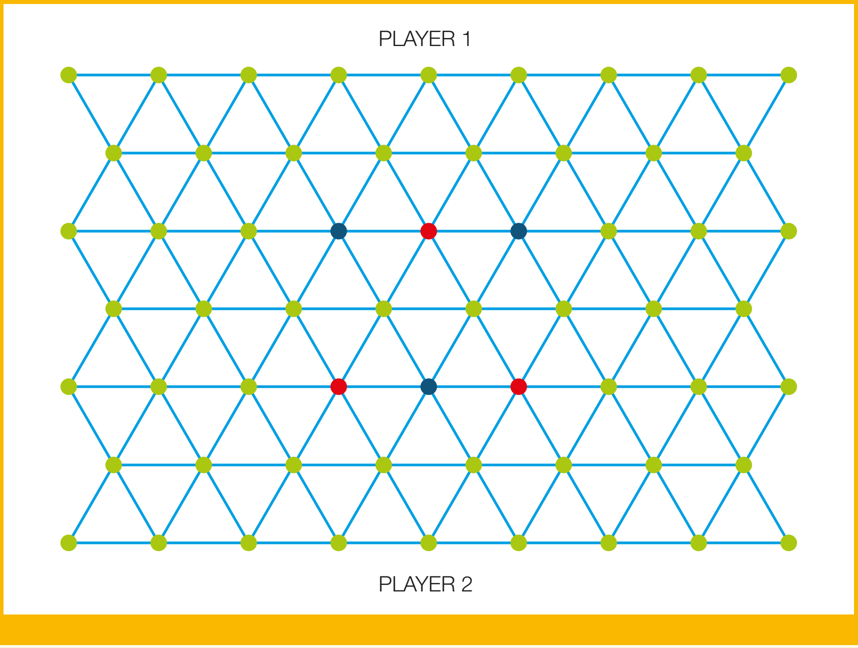
Equipment required: 3 counters per player, 1 die

How to win: The first player to place all their 'satellites' in a line is the winner.

How to play:



1. Players sit on opposite sides of the board. Each player puts three counters (satellites) on each of the red or blue starting points, facing to the right. Each counter must have a mark on its edge, which indicates which way the satellite is facing.
2. Players take turns rolling the die. The number rolled is multiplied by 60° to find the angle that a satellite can rotate, either clockwise or anticlockwise. Once rotated, a satellite can move any number of spaces in the direction that it is now facing, up to the edge of the board. Only one satellite can be rotated and moved each turn.
3. A player can bump an opponent's satellite and send it flying to the far end of the galaxy (the edge of the game board in the direction of the 'bump'). The player who bumps an opponent takes the position of the opponent's satellite before it was bumped. A satellite cannot be bumped if it is already on the edge of the board.
4. No two satellites can occupy the same point on the board at any one time.
5. Play continues until one player has all their satellites in the same line on the board (they do not have to be next to each other).



You can download a copy of this game board from the eBook or the Pearson Places website.

Gamespace

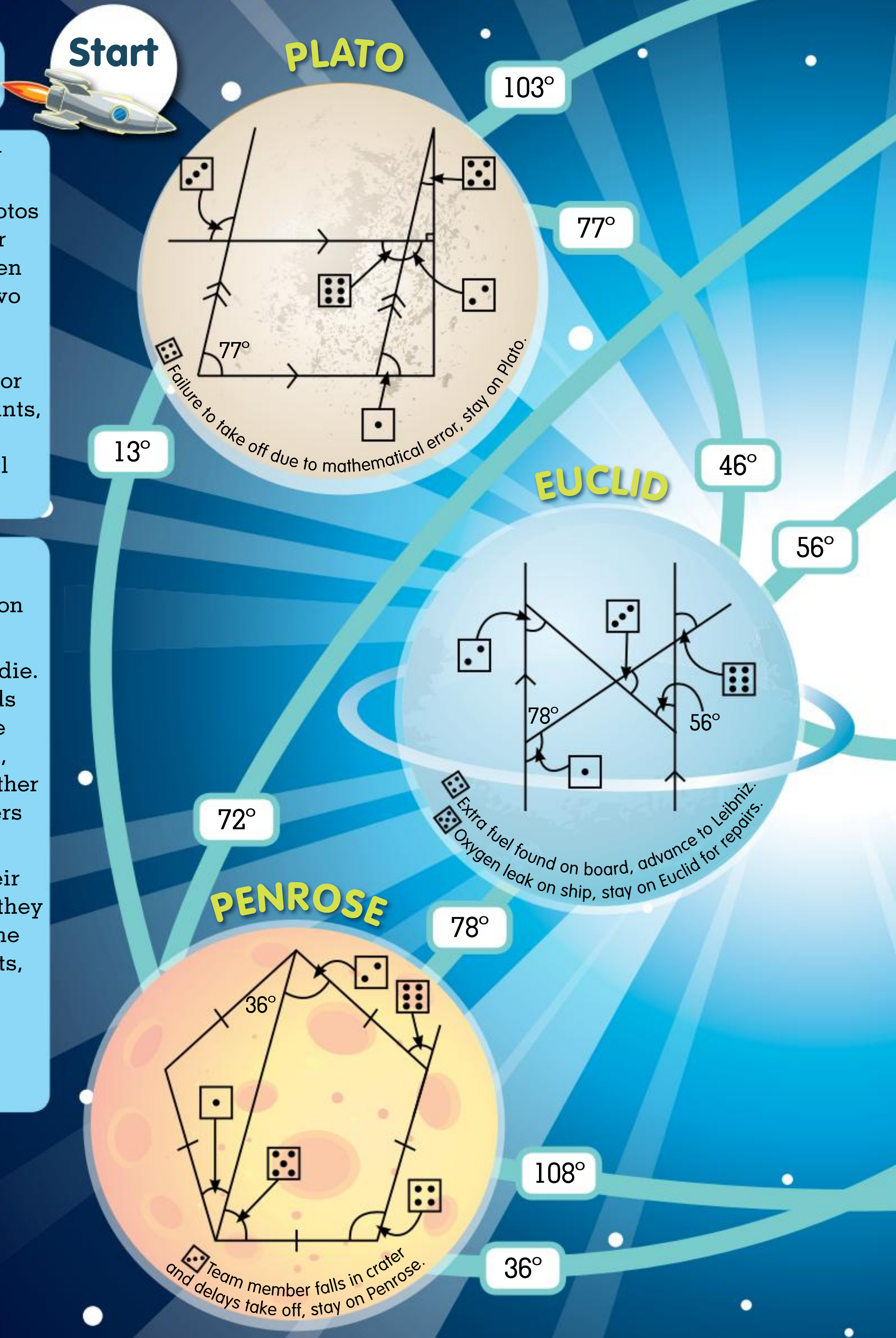
Space race home

Equipment required:
1 counter per player, 1 die

You and your friends are leading separate Australian space explorations. You have taken photos of nebulas, black holes and other space events that have never been seen before. But there are only two ways to reach Earth to get home again—by navigating your way through the galaxy step by step, or by collecting 500 or more fuel points, which will give you enough fuel to burst through the gravitational limits and head straight home.

How to play

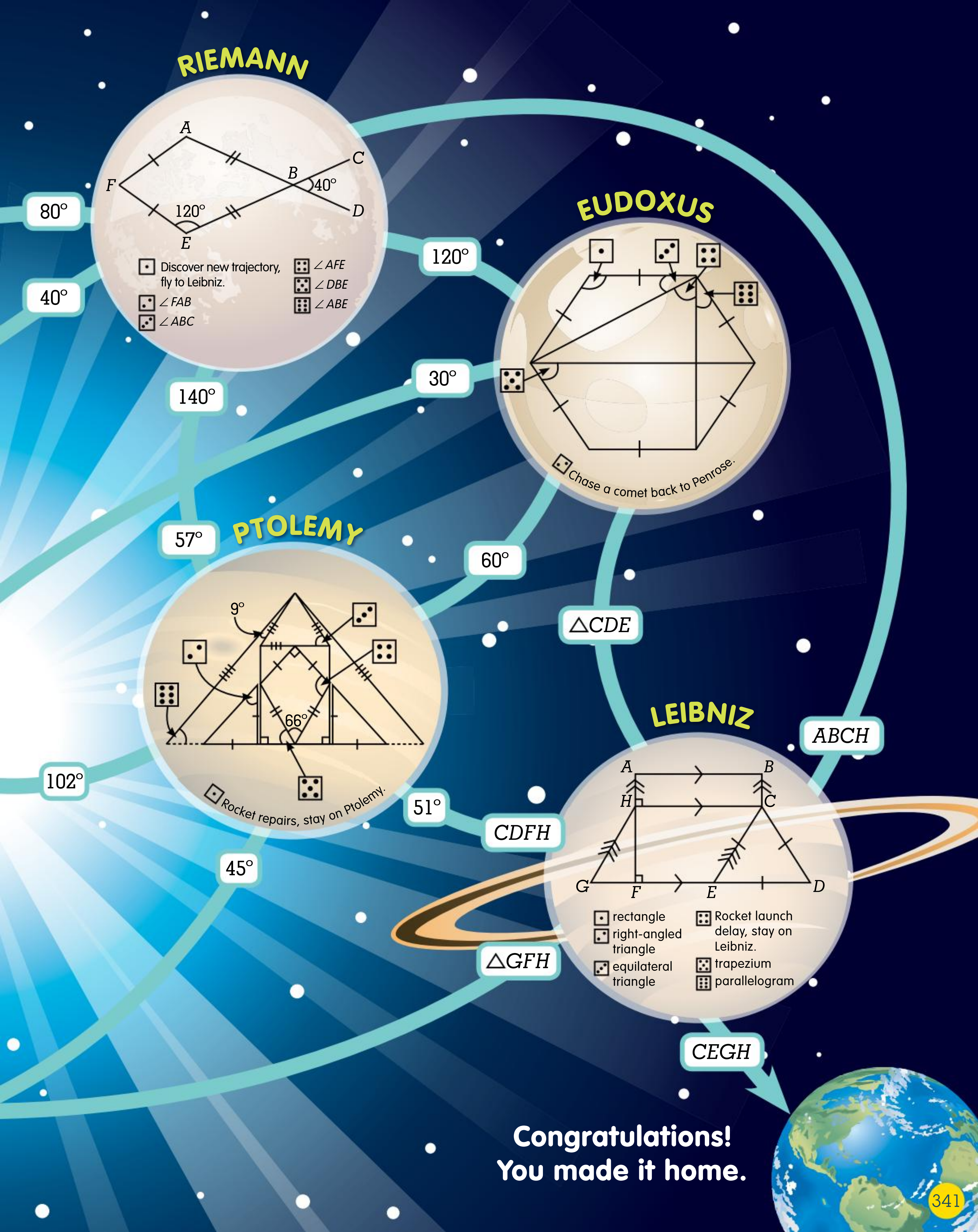
1. Each player places a counter on the starting planet, Plato.
2. Players take turns rolling the die. The number rolled corresponds to an unknown angle or shape name, which, when answered, will direct the player onto another planet. At intersections, players may choose which way to go.
3. Players must keep track of their answers and add them up as they go because the answers are the fuel points. With 500 fuel points, a player can blast their way immediately back to Earth.
4. The first player to reach Earth will be the winner. Good luck.



⚃ Failure to take off due to mathematical error, stay on Plato.

⚃ Extra fuel found on board, advance to Leibniz.
⚄ Oxygen leak on ship, stay on Euclid for repairs.

⚂ Team member falls in crater and delays take off, stay on Penrose.



RIEMANN

⚀ Discover new trajectory, fly to Leibniz.
⚁ ∠FAB
⚂ ∠ABC
⚃ ∠AFE
⚄ ∠DBE
⚅ ∠ABE

EUDOXUS

⚁ Chase a comet back to Penrose.

PTOLEMY

⚀ Rocket repairs, stay on Ptolemy.

LEIBNIZ

⚀ rectangle
⚁ right-angled triangle
⚂ equilateral triangle
⚃ Rocket launch delay, stay on Leibniz.
⚄ trapezium
⚅ parallelogram

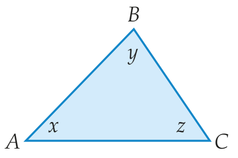
Congratulations!
You made it home.

6.2

Triangles and congruency

Triangles

Triangle:



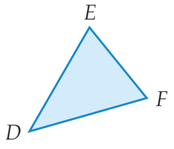
The sum of the internal angles of a triangle is 180°.

$x + y + z = 180^\circ$

$\angle CAB + \angle ABC + \angle BCA = 180^\circ$

Angle-named triangles

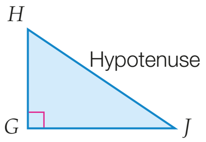
Acute-angled triangle:



All angles are acute.

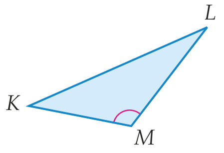
$\angle DEF < 90^\circ$
$\angle EFD < 90^\circ$
$\angle FDE < 90^\circ$

Right-angled triangle:



One angle equals 90°.
$\angle HGJ = 90^\circ$
The side opposite the right angle is called the hypotenuse.
HJ is the hypotenuse.

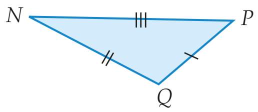
Obtuse-angled triangle:



One angle is obtuse.
$\angle KML > 90^\circ$
The other two angles are acute.
$\angle LKM < 90^\circ$
$\angle KML < 90^\circ$

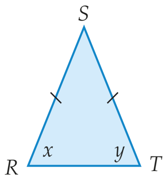
Side-named triangles

Scalene triangle:



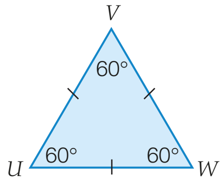
All sides and angles are unequal. NP, PQ and QN are all different lengths.

Isosceles triangle:



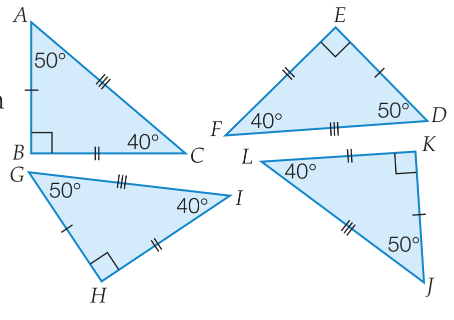
One pair of sides is equal in length. The angles opposite the equal sides are also equal in size. $RS = TS$
$\angle SRT = \angle STR$ or $x = y$

Equilateral triangle:



All sides are equal in length.
$UV = VW = WU$
All angles are equal to 60°.
$\angle UVW = \angle VWU =$
$\angle WUV = 60^\circ$

Congruent triangles



Two triangles are **congruent** if they are identical. This means that the corresponding sides are the same length and the corresponding angles are the same size.

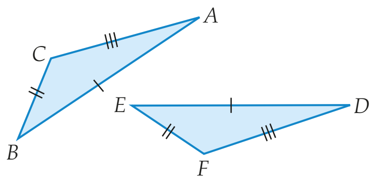
The triangles shown at right are all identical (congruent). Each triangle has sides marked with the same-length marking, while the angles opposite the sides of equal length are equal.

You can write $\Delta ABC \equiv \Delta DEF \equiv \Delta GHI \equiv \Delta JKL$, where the symbol ≡ means 'is congruent to'.

There are four tests for **congruent triangles**.

Tests for congruent triangles

1 Side Side Side (SSS):

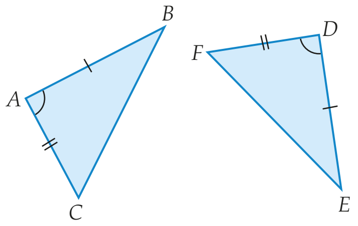


Two triangles are congruent if the corresponding sides have equal length.

$AB = DE$ (side)
$BC = EF$ (side)
$CA = FD$ (side)

$\Delta ABC \equiv \Delta DEF$ (SSS)

2 Side Angle Side (SAS):

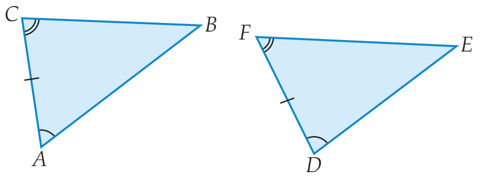


Two triangles are congruent if two corresponding sides have equal lengths and the angles included by these sides are equal.

$AB = DE$ (side)
$\angle CAB = \angle FDE$ (angle)
$AC = DF$ (side)

$\Delta ABC \equiv \Delta DEF$ (SAS)

3 Angle Side Angle (ASA):



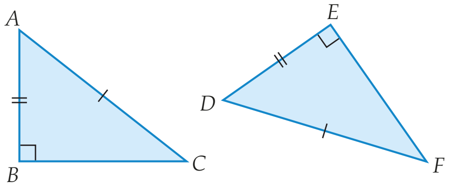
Two triangles are congruent if two corresponding angles are equal and a corresponding side is equal in length.

$\angle CAB = \angle DFE$ (angle)
$AC = DF$ (side)
$\angle CBA = \angle DEF$ (angle)

$\Delta ABC \equiv \Delta FED$ (ASA)

Note that the corresponding side does not need to be included between the two equal angles.

4 Right angle Hypotenuse Side (RHS):



Two right-angled triangles are congruent if the hypotenuse and one pair of corresponding sides are equal in length.

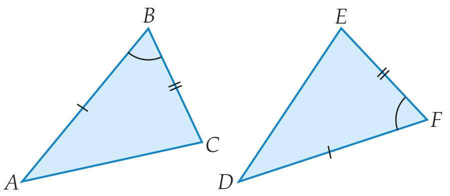
$\angle ABC = \angle DEF = 90°$ (right angle)
$AC = DF$ (hypotenuse)
$AB = DE$ (side)

$\Delta ABC \equiv \Delta DEF$ (RHS)

Worked example 3

W.E. 3

Show that the triangles *ABC* and *DEF* are congruent, stating the congruency test used.



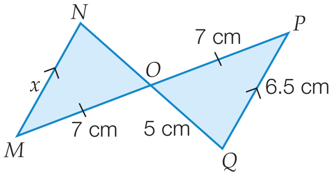
Thinking	Working
1 List the three equalities given and identify each as an Angle or a Side equality.	$AB = DF$ (given, Side) $\angle ABC = \angle DFE$ (given, Angle) $BC = FE$ (given, Side)
2 Give your answer.	$\Delta ABC \equiv \Delta DFE$ (SAS)

Worked example 4

W.E. 4

(a) Prove that $\Delta MNO \equiv \Delta PQO$.

(b) Find the value of x.



Thinking	Working
(a) **1** Name the two triangles to be used.	(a) ΔMNO and ΔPQO
2 List three equalities about the triangles, with reasons.	$\angle MNO = \angle PQO$ ($MN \parallel QP$ so $\angle MNO$ and $\angle PQO$ are alternate angles and are therefore equal). $\angle MON = \angle POQ$ ($\angle MON$ and $\angle POQ$ are vertically opposite so they are equal). $MO = PO = 7$ cm (given)
3 Identify each equality as an Angle or a Side equality.	$\angle MNO = \angle PQO$ (Angle) $MO = PO$ (Side) $\angle MON = \angle POQ$ (Angle)
4 Write the pair of congruent triangles and the test used.	$\Delta MNO \equiv \Delta PQO$ (ASA)
(b) **1** Identify pairs of corresponding sides.	(b) $MN = PQ$ and $NO = OQ$ (corresponding sides in congruent triangles).
2 Find x.	$x = MN = PQ$ $x = 6.5$ cm

Worked example 5

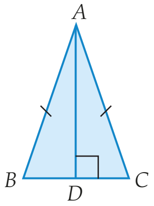
W.E. 5

(a) ΔABC is a triangle with $AB = AC$.
AD is perpendicular to BC.
Prove that $\angle ABC = \angle ACB$.

(b) What type of triangle is ΔABC? Explain your answer.

(c) What triangle property have you just proved?

(d) Show that $BD = DC$.



Thinking	Working
(a) **1** Name the two triangles that you will try to prove congruent.	(a) ΔABD and ΔACD.
2 Identify three sets of equalities, with reasons, that apply to the two triangles.	$\angle ADB = \angle ADC = 90°$ (given, Right angle) $AB = AC$ (given, Hypotenuse) AD is a common side (Side).
3 Write the pair of congruent triangles and the test used.	$\Delta ABD \equiv \Delta ACD$ (RHS)

4 State the result that you are proving.	∠ABC = ∠ACB (corresponding angles in congruent triangles).
(b) Use your knowledge of types of triangles or congruency.	(b) ΔABC is an isosceles triangle because AB = AC.
(c) State the answer as a sentence.	(c) The base angles (angles opposite equal sides) of an isosceles triangle are equal.
(d) Use congruency to explain why the sides are equal.	(d) Corresponding sides of the congruent triangles ABD and ACD are equal. ∴ BD = CD

6.2 Triangles and congruency

Navigator

1 (column 1), 2, 3, 4, 5, 6, 8 (column 1), 9, 11 (a, b), 16	1 (column 2), 2, 3, 4, 5, 6, 7, 8 (column 2), 9, 11 (a–c), 12, 14, 16	1 (column 3), 2, 3, 4, 5, 6, 7, 8 (column 2), 9, 10, 11 (c–f), 12, 13, 14, 15, 16

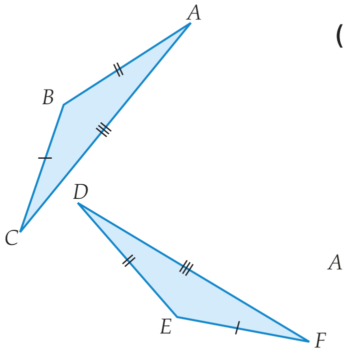
Answers p. 667

Fluency

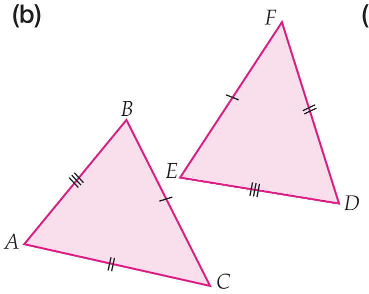
W.E. 3

1 Show that each of the following pairs of triangles are congruent, stating the congruency test used in each.

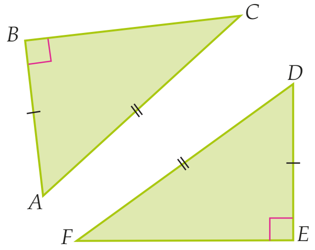
(a)



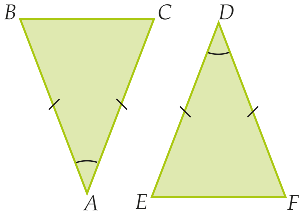
(b)



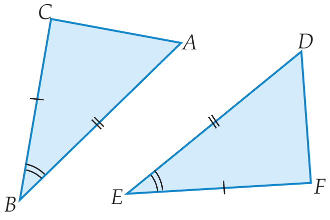
(c)



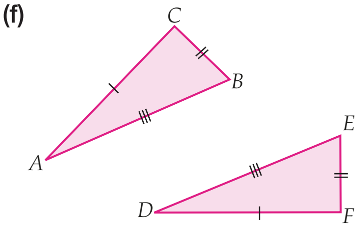
(d)



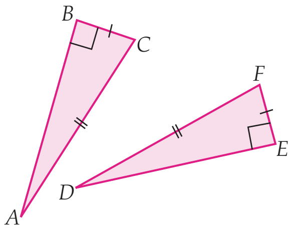
(e)



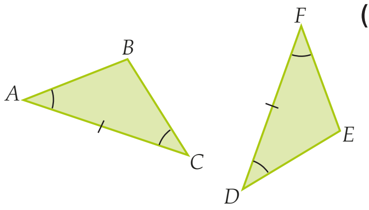
(f)



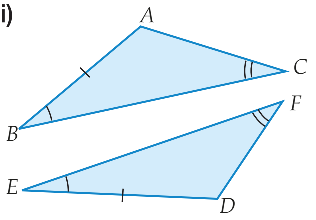
(g)



(h)



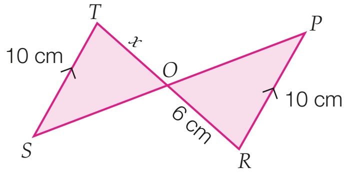
(i)



W.E. 4

2 **(a)** Prove that $\Delta STO \equiv \Delta POR$.

(b) Find the value of x.



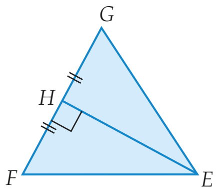
W.E. 5

3 **(a)** In ΔEFG, $FH = HG$ and EH is perpendicular to FG.
Prove that $\angle EFH = \angle EGH$.

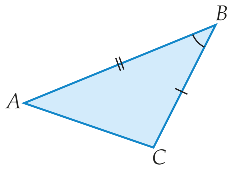
(b) What type of triangle is ΔEFG? Explain your answer.

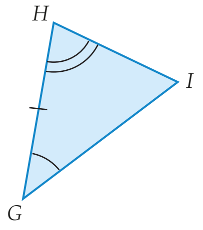
(c) What triangle property have you just proved?

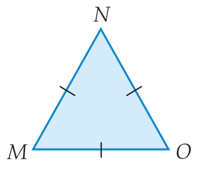
(d) Show that $EF = EG$.

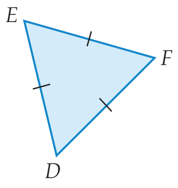


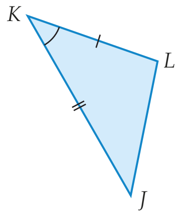
4 Find pairs of congruent triangles among the following diagrams. State the test used to show that they are congruent. Letter names must be in the correct order.

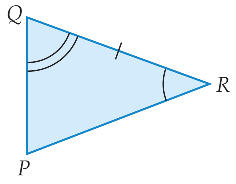






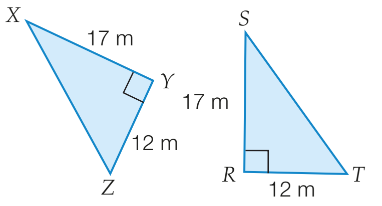




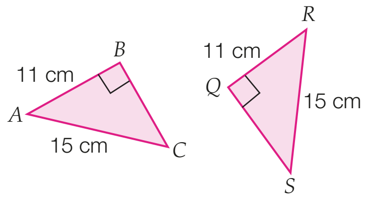


5 Prove that each of the following pairs of triangles is congruent, giving reasons for your answer.

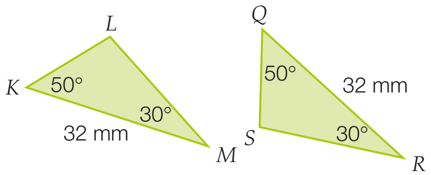
(a)



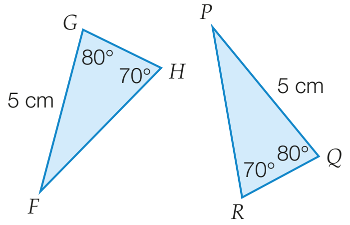
(b)



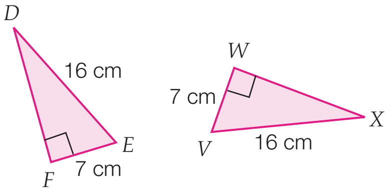
(c)



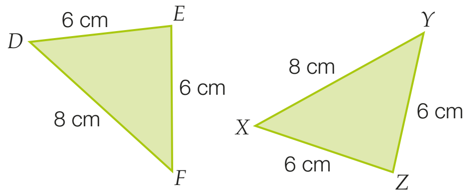
(d)



(e)



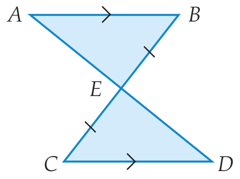
(f)



Understanding

6 Explain why two triangles with three equal angles may not necessarily be congruent. Show this by drawing diagrams.

7 Complete the following proof by copying it and writing in the missing steps.



AB || *CD*. *AD* and *BC* intersect at *E* so that *BE* = *EC*.
Prove that *AB* = *CD*.

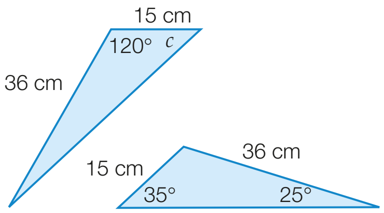
Proof:

In Δ*ABE* and Δ*DCE*,
∠________ = ∠________ (alternate angles, *AB* || *CD*, Angle)
________ = ________ (given, Side)
∠________ = ∠________ (vertically opposite angles, Angle)
∴ Δ________ ≡ Δ________ (ASA congruency)
∴ ________ = ________ (corresponding sides of congruent triangles)

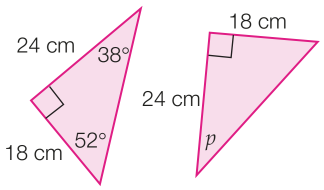
8 **(a)** State the test used to prove the following pairs of triangles are congruent.

(b) Use congruency to find the value of the unknown in each pair of triangles.

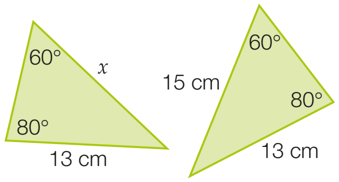
(i)



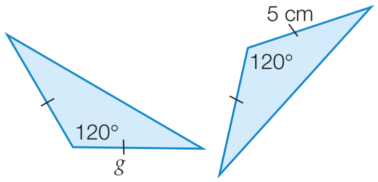
(ii)



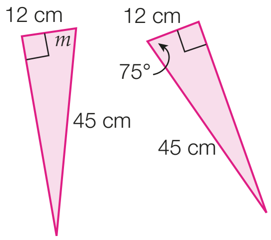
(iii)



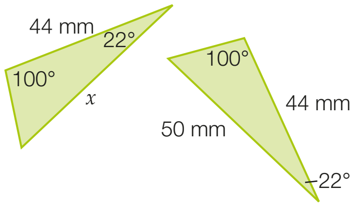
(iv)



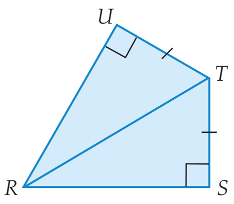
(v)



(vi)

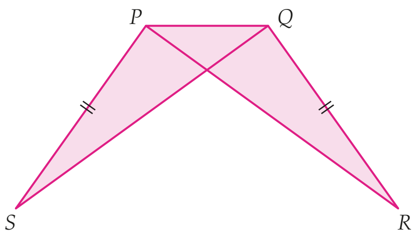


9 Prove Δ*RUT* ≡ Δ*RST*.



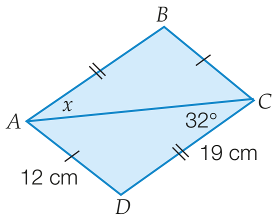
Reasoning

10 If *SP* = *RQ* and ∠*SPQ* = ∠*RQP*, prove that Δ*SPQ* ≡ Δ*RQP*.

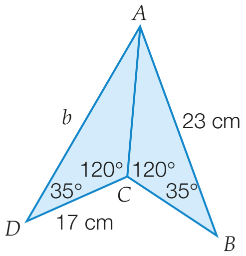


11 Prove that the following pairs of triangles are congruent and hence find the value of the variables.

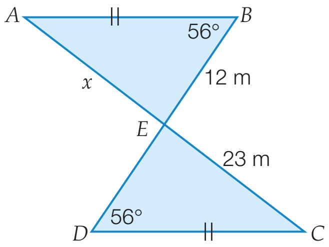
(a)



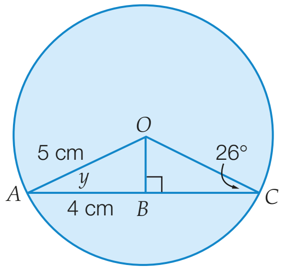
(b)



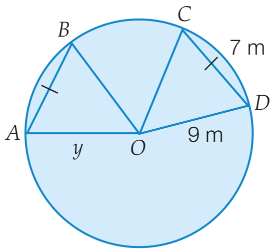
(c)



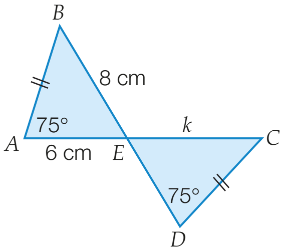
(d)



(e)



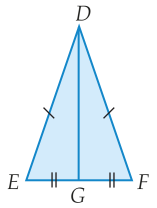
(f)



12 ΔDEF is an isosceles triangle with $DE = DF$. G is the midpoint of EF.

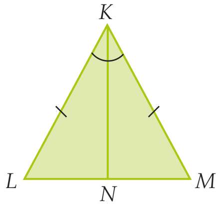
(a) Show that $DG \perp EF$.

(b) If ΔDEF is equilateral, prove that $\angle EDG = \angle FDG = 30°$.

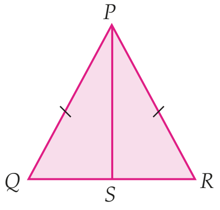


13 ΔKLM is an equilateral triangle with $KL = LM = MK$. N is a point on LM such that $\angle LKN = \angle MKN$.

Show that KN is the perpendicular bisector of LN (i.e. $LN = NM$ and $KN \perp LM$).



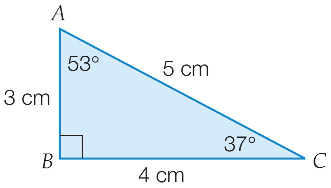
14 ΔPQR is an equilateral triangle with $PQ = QR = RP$. S is a point on QR that is joined to P. Explain why this is not enough information to prove that $QS = SR$.



Open-ended

15 Draw four pairs of congruent triangles and put just enough information on each diagram to show congruence. Each pair should represent a different test. Write the name of the test that applies to each pair of triangles.

16 ΔABC is right-angled at B. The lengths of its sides and the size of its angles are shown on the figure. Draw four triangles with enough information marked on them so that each one is congruent with ΔABC using each of the congruence tests.



Puzzle

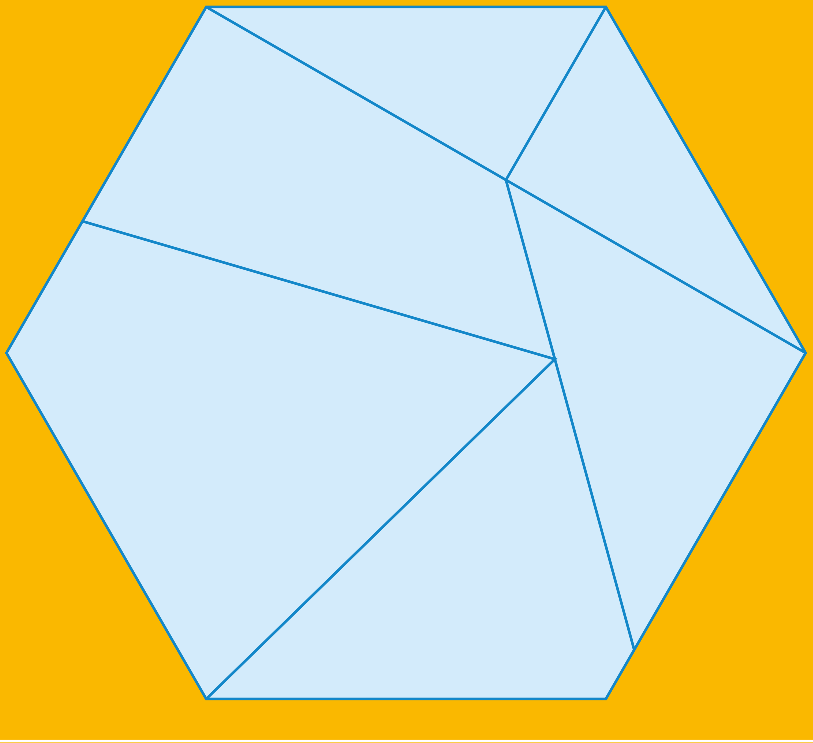
Hexagon jigsaw

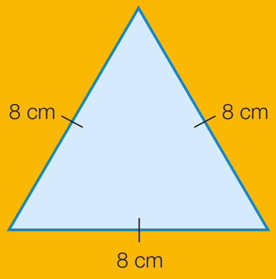
Equipment required: paper, scissors, hexagon jigsaw (draw your own copy of the diagram shown, or download a template from the eBook or the Pearson Places website)

By cutting a regular hexagon into pieces, it can be transformed into other polygons. Photocopy or trace the hexagon jigsaw template and cut it out. Divide the hexagon into pieces by cutting along the lines.

Rearrange the pieces into an equilateral triangle with side lengths of 8 cm. Some pieces may need to be reflected to make the transformation.

Can a hexagon be cut in such a way that it can be rearranged into other polygons? Try transforming a hexagon into a rectangle, square, parallelogram or irregular quadrilateral.





6.3

Special quadrilaterals and their properties

Polygons

A **polygon** is a two-dimensional shape with only straight sides. A shape with curved sides (like a circle) is not a polygon.

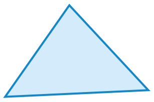
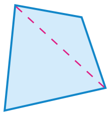
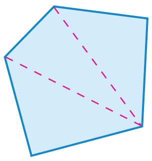
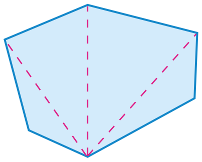
A **regular polygon** has all its sides and interior angles equal.

An equilateral triangle is the only regular three-sided polygon and a square is the only regular four-sided polygon. No other regular polygons have special names (e.g. a six-sided polygon with all sides equal is simply called a regular hexagon).

Angle sum of a polygon

Polygons can be divided into triangles by using straight lines to connect the vertices. By dividing a polygon into the minimum number of triangles, it is possible to calculate the sum of the internal angles. The minimum number of triangles will always be 2 less than the number of sides of the polygon. This gives a useful rule, based on the angle sum of a triangle:

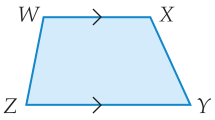
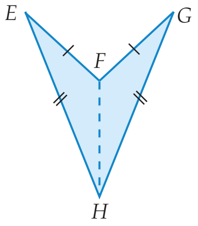
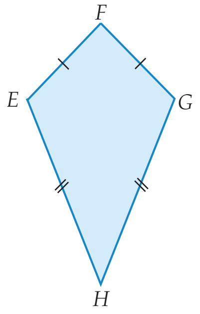
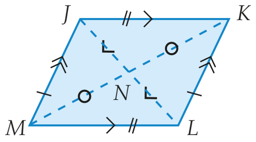
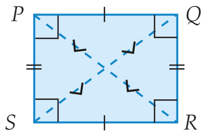
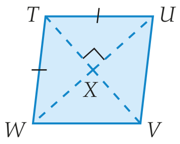
> A polygon with n sides can be divided into a minimum number of triangles $(n - 2)$.
> The sum of the interior angles of a polygon with n sides $= (n - 2) \times 180°$.

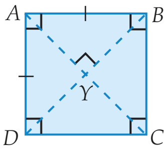
Polygon	***n***	**Angle sum using triangles**	**Angle sum using $(n - 2) \times 180°$**
Triangle 	3	$1 \times 180° = 180°$	$(n - 2) \times 180°$ $= (3 - 2) \times 180°$ $= 1 \times 180°$ $= 180°$
Quadrilateral 	4	$2 \times 180° = 360°$	$(n - 2) \times 180°$ $= (4 - 2) \times 180°$ $= 2 \times 180°$ $= 360°$
Pentagon 	5	$3 \times 180° = 540°$	$(n - 2) \times 180°$ $= (5 - 2) \times 180°$ $= 3 \times 180°$ $= 540°$
Hexagon 	6	$4 \times 180° = 720°$	$(n - 2) \times 180°$ $= (6 - 2) \times 180°$ $= 4 \times 180°$ $= 720°$

Definitions and properties

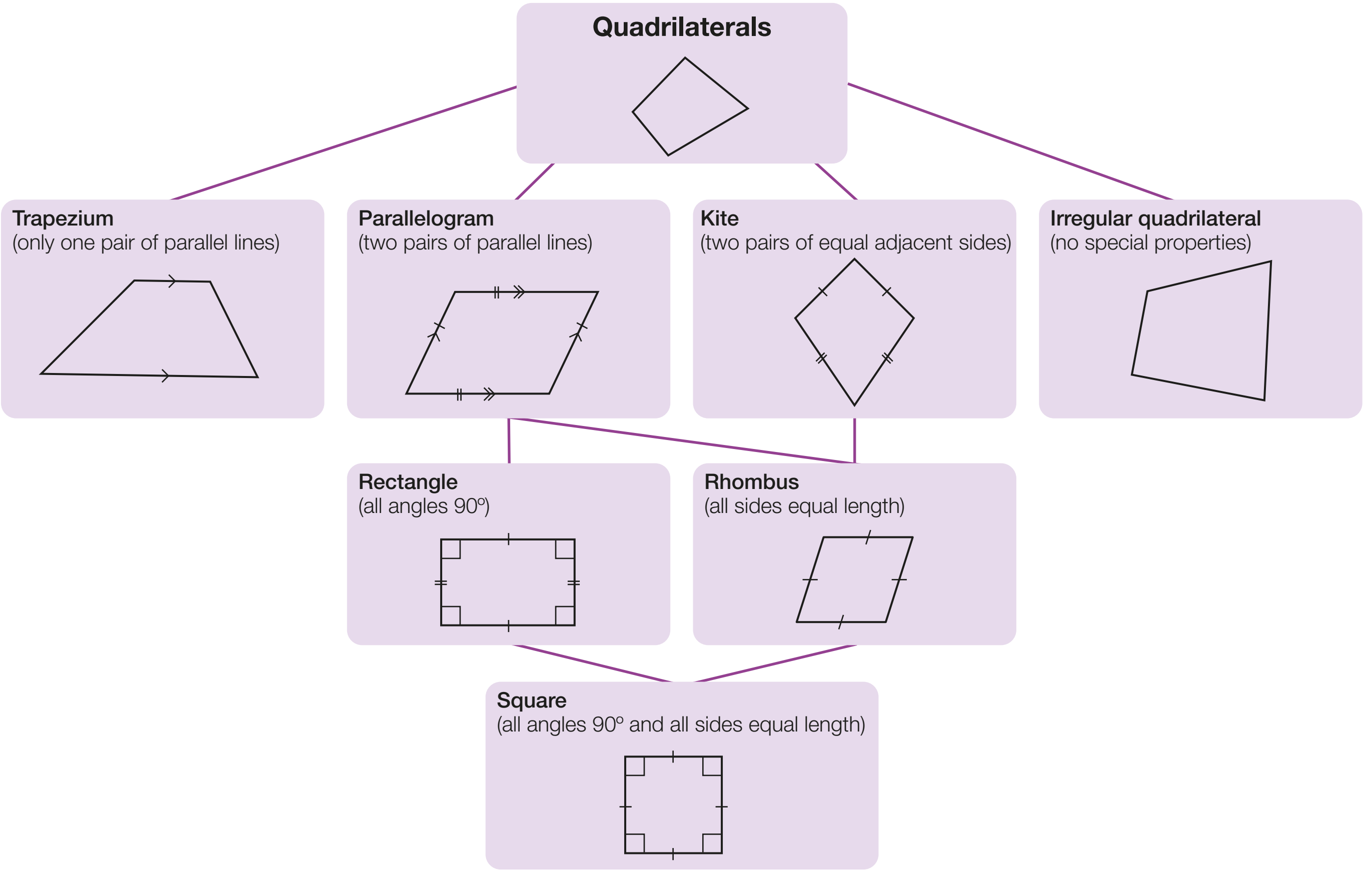
A **definition** is the minimum amount of information needed to describe a figure exactly.
A **property** is an attribute or characteristic of a figure than can be proved to be always true after it has been defined.

The congruence of shapes can be used to prove the properties of special quadrilaterals.

Shapes and definitions	**Properties**
Trapezium: A trapezium is a quadrilateral with only one pair of opposite sides parallel.  $WXYZ$ is a trapezium for which $WX \parallel ZY$.	A trapezium has no other special properties.
Kite: A kite is a quadrilateral with two pairs of adjacent sides equal. $EFGH$ is a kite in which $EF = FG$, $EH = HG$.  Concave kite  Convex kite	• The pair of angles between the unequal sides are equal: $\angle FEH = \angle FGH$ • The angles between the equal sides are bisected by the axis of symmetry through them. • There is one axis of symmetry, FH. • The diagonals are perpendicular: $FH \perp EG$ • The diagonal EG that passes through the equal angles is bisected by the axis of symmetry FH.
Parallelogram: A parallelogram is a quadrilateral with both pairs of opposite sides parallel.  $JKLM$ is a parallelogram in which $JK \parallel ML$, $JM \parallel KL$.	• Both pairs of opposite sides are equal: $JK = ML$, $JM = KL$ • Both pairs of opposite angles are equal: $\angle MJK = \angle MLK$, $\angle JML = \angle JKL$ • The diagonals bisect each other: $JN = NL$, $MN = NK$
Rectangle: A rectangle is a parallelogram with a right angle.  $PQRS$ is a rectangle in which $\angle PSR = 90°$.	A rectangle has all the properties of a parallelogram, plus: • each angle is 90°: $\angle PSR = \angle SRQ = \angle RQP = \angle QPS = 90°$ • the diagonals are equal in length: $PR = QS$
Rhombus: A rhombus is a parallelogram with two adjacent sides that are equal.  $TUVW$ is a rhombus in which $TU = TW$ (or a kite with all sides equal).	A rhombus has all the properties of a parallelogram, plus: • all the sides are equal: $TU = TW = WV = UV$ • the diagonals bisect the angles of the rhombus: $\angle WTV = \angle UTV$, $\angle WVT = \angle UVT$, $\angle TWU = \angle VWU$, $\angle TUW = \angle VUW$ • the diagonals bisect each other at right angles: $TX = XV$, $WX = XU$, $\angle WXT = \angle TXU = \angle UXV = \angle VXW = 90°$

Shapes and definitions	Properties
Square:  A square is a rectangle with one pair of adjacent sides equal. *ABCD* is a square in which $AB = AD$. or A square is a rhombus with a right angle. *ABCD* is a square in which $\angle ADC = 90^\circ$.	A square has all the properties of a rectangle, and all the properties of a rhombus: • all the sides are equal: $AB = AD = DC = BC$ • the diagonals bisect the angles of the square: $\angle DAC = \angle CAB$, $\angle DCA = \angle ACB$, $\angle ABD = \angle DBC$, $\angle ADB = \angle BDC$ • the diagonals bisect each other at right angles: $\angle AYD = \angle DYC = \angle CYB = \angle BYA = 90^\circ$ • the two diagonals and the two lines joining the midpoints are axes of symmetry • the diagonals are equal: $AC = BD$

The quadrilateral family

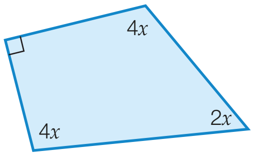


Worked example 6

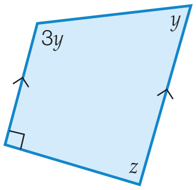
W.E. 6

Find the value of the variables in each figure.

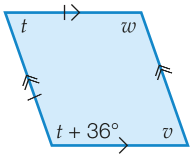
(a)



(b)



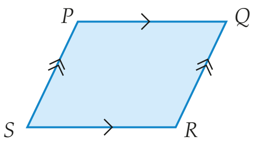
(c)



Thinking	Working
(a) 1 Identify the shape.	(a) This is a quadrilateral.
2 Form an equation with a reason.	$2x + 4x + 4x + 90° = 360°$
3 Solve the equation.	$10x + 90° = 360°$ $10x = 270°$ $x = 27°$
(b) 1 Identify the shape.	(b) This is a trapezium.
2 Form an equation with a reason.	$y + 3y = 180°$ (co-interior angles between parallel lines)
3 Solve the equation.	$4y = 180°$ $y = 45°$ $z + 90° = 180°$ (co-interior angles between parallel lines) $z = 90°$
(c) 1 Identify the shape.	(c) The figure is a parallelogram with one pair of adjacent sides equal; hence, it is a rhombus.
2 Form an equation with a reason.	$t + t + 36° = 180°$ (co-interior angles add to 180°)
3 Solve the equation.	$2t + 36° = 180°$ $2t = 144°$ $t = 72°$
4 Repeat for the other angles.	$v = 72°$ (opposite angles of a rhombus are equal) $w = 108°$ (interior angles between parallel lines)
5 List all answers.	$t = 72°$, $v = 72°$, $w = 108°$

Worked example 7

W.E. 7



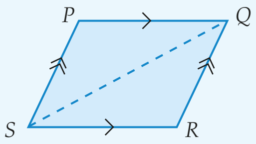
PQRS is a parallelogram is which $PQ \parallel SR$ and $PS \parallel QR$.

(a) Use congruence to prove that $\angle SPQ = \angle QRS$.

(b) Prove that $PQ = SR$ and $PS = QR$.

(c) Prove that $\angle PSR = \angle RQP$.

(d) What properties of a parallelogram have you proved?

Thinking	Working
(a) **1** Copy the diagram and join *SQ*.	(a) 
2 Name the two triangles that you will try to prove congruent.	ΔSPQ and ΔQRS
3 List three sets of equalities, with reasons, that apply to the two triangles.	$\angle PQS = \angle QSR$ (alternate angles, $PQ \parallel SR$) $\angle PSQ = \angle SQR$ (alternate angles, $PS \parallel QR$) QS is a common side.
4 Write the pair of congruent triangles and the test used.	$\Delta SPQ \equiv \Delta QRS$ (ASA)
5 Identify appropriate pairs of corresponding angles for the required result.	$\angle SPQ = \angle QRS$ (corresponding angles in congruent triangles)
(b) Identify appropriate pairs of corresponding sides.	(b) $PQ = SR$ (corresponding sides of congruent triangles) $PS = QR$ (corresponding sides of congruent triangles)
(c) Explain that the same method used in part **(a)** can prove congruency if the other diagonal is drawn.	If the diagonal PR is drawn, then the same steps as in (a) can be used to prove that $\Delta PSR \equiv \Delta RQP$. $\angle PSR = \angle RQP$ (corresponding angles in congruent triangles)
(d) State the properties.	(d) The opposite angles of a parallelogram are equal. The opposite sides of a parallelogram are equal.

Worked example 8

W.E. 8

ABCD is a rhombus in which $AB \parallel DC$ and $AD \parallel BC$. The diagonals *AC* and *BD* intersect at *E*.

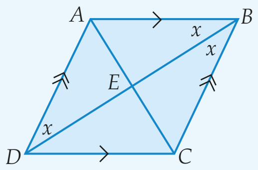
(a) Prove that $\angle ABD = \angle CBD$.

(b) Prove that $\Delta ABE \equiv \Delta CBE$.

(c) Prove that $\angle AEB = \angle CEB = 90°$.

(d) Prove that $AE = CE$.

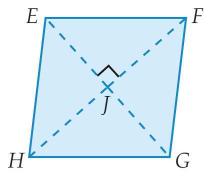
(e) What properties of a rhombus have you proved?

Thinking	Working
(a) 1 Draw a diagram and let $\angle ABE = x$.	(a) 
2 Explain what you marked on the diagram.	$\angle CDB = \angle ABD = x$ (alternate angles, $AB \parallel DC$) $BC = CD$ (adjacent sides of a rhombus) ΔBCD is isosceles $\angle CBD = \angle CDB = x$ (base angles of an isosceles triangle)
3 State the result you have shown.	$\angle ABD = \angle CBD$ (both equal x)
(b) 1 Name the two triangles you will try to prove congruent.	(b) Consider ΔABE and ΔCBE.
2 List three sets of equalities, with reasons, that apply to the two triangles.	$AB = BC$ (adjacent sides of a rhombus) $\angle ABE = \angle CBE$ (proved in (a)) *BE* is a common side.
4 Write the pair of congruent triangles and the test used.	$\Delta ABE \equiv \Delta CBE$ (SAS)
(c) 1 Identify appropriate pairs of corresponding angles for the required result.	(c) $\angle AEB = \angle CEB$ (corresponding angles in congruent triangles)
2 State what else you know about these angles.	$\angle AEB + \angle CEB = 180°$ ($\angle AEC = 180°$)
3 Make a conclusion.	$\angle AEB = 180°$ $\angle AEB = 90°$ $\angle AEB = \angle CEB = 90°$
(d) Identify a pair of corresponding sides.	(d) $AE = CE$ (corresponding sides in congruent triangles)
(e) Use the results from parts **(a)**, **(c)** and **(d)** to state the properties of a rhombus.	(e) The diagonals bisect the angles of a rhombus. The diagonals of a rhombus intersect at right angles. One diagonal bisects the other diagonal.

Worked example 9

W.E. 9

The diagonals of the quadrilateral *EFGH* bisect each other at right angles at *J*. Prove that *EFGH* is a rhombus.



Thinking	Working
1 List the information given in the question.	$EJ = GJ$, $HJ = FJ$, $\angle FJG = \angle HJG = \angle HJE = \angle FJE = 90°$
2 Use the definition of the shape to identify what needs to be proved. (A rhombus is a parallelogram with adjacent sides equal.)	In the quadrilateral the pairs of opposite sides must be parallel and a pair of adjacent sides must be equal.
3 Identify the triangles you will use to prove congruency.	Consider ΔEJF and ΔGJH
4 List three equalities, with reasons, from these triangles.	$EJ = GJ$ (given) $\angle EJF = \angle GJH$ (both 90°) $FJ = HJ$ (given)
5 Write the pair of congruent triangles and the test used.	$\Delta EJF \equiv \Delta GJH$ (SAS)
6 Identify a pair of corresponding angles.	$\angle EFJ = \angle GHJ$ (corresponding angles in congruent triangles)
7 Use you knowledge of angles on parallel lines to identify a pair of parallel lines (alternate angles).	$\angle EFH$ and $\angle GHF$ are a pair of equal alternate angles contained between EF and HG. $\therefore EF \parallel HG$
8 Identify another pair of triangles to be proved congruent and, using the same methods as in Steps **4**–**7**, identify another pair of parallel lines (alternate angles).	Similarly, $\Delta EJH \equiv \Delta GJF$ $\angle EHF = \angle HFG$. $\angle EHF$ and $\angle HFG$ are a pair of equal alternate angles contained between EF and HG. $\therefore EH \parallel FG$
9 Identify a third pair of triangles to be proved congruent.	Consider ΔEJF and ΔGJF.
10 List three equalities, with reasons, from these triangles.	$EJ = GJ$ (given) $\angle EJF = \angle GJF$ (both 90°) JF is a common side.
11 Write the pair of congruent triangles and the test used.	$\Delta EJF \equiv \Delta GJF$ (SAS)
12 Identify a pair of corresponding sides.	$EF = GF$ (corresponding sides in congruent triangles)

13	Write the pair of congruent triangles and the test used.	$\Delta EJF \equiv \Delta GJH$ (SAS)
14	Summarise what has been shown.	$EF = GF$ are a pair of equal adjacent sides of $EFGH$ and $EF \parallel HG$, $EH \parallel FG$ gives both pairs of opposite sides parallel. Hence, $EFGH$ is a parallelogram with a pair of adjacent sides equal, so $EFGH$ is a rhombus.

Use the definitions of each type of quadrilateral in your proofs.

6.3 Special quadrilaterals and their properties

Navigator

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13 (a), 15 (a), 18, 20, 26	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 17, 19, 20, 21, 22, 26	1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 22, 23, 24, 25

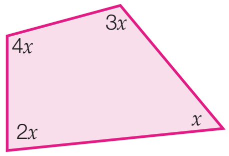
Answers p. 669

Fluency

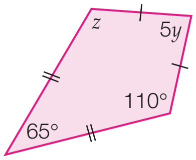
1 Find the value of the variables in each figure.

W.E. 6

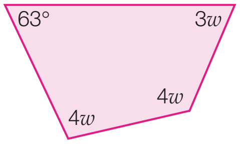
(a)



(b)

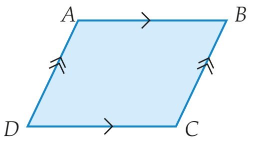


(c)



2 $ABCD$ is a parallelogram in which $AB \parallel DC$ and $AD \parallel BC$.

W.E. 7



(a) Use congruence to prove that $\angle DAB = \angle BCD$.

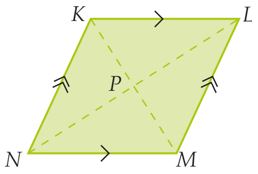
(b) Prove that $AB = DC$ and that $AD = BC$.

(c) Prove that $\angle ADC = \angle CBA$.

(d) What properties of a parallelogram have you proved?

3 $KLMN$ is a rhombus in which $KL \parallel NM$ and $KN \parallel LM$. The diagonals KM and LN intersect at P.

W.E. 8



(a) Prove that $\angle KLN = \angle MLN$.

(b) Prove that $\Delta KLP \equiv \Delta MLP$.

(c) Prove that $\angle KPL = \angle MPL = 90°$.

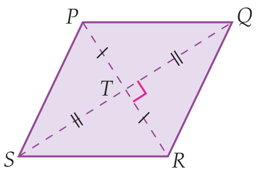
(d) Prove that $KP = MP$.

(e) What properties of a rhombus have you proved?

4 The diagonals of the quadrilateral $PQRS$ bisect each other at right angles at T.

W.E. 9

Prove that $PQRS$ is a rhombus.

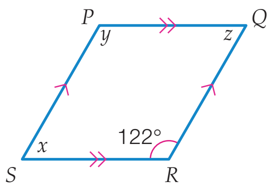


5 The size of each of the angles in a regular pentagon is:

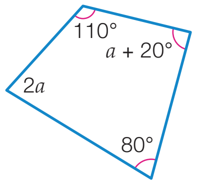
A 60° **B** 90° **C** 108° **D** 120°

6 For each figure, **(i)** identify the shape and **(ii)** find the value of the variables.

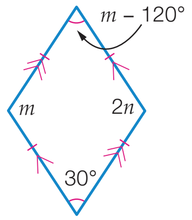
(a)



(b)

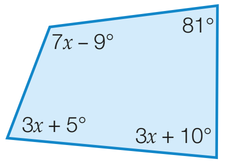


(c)

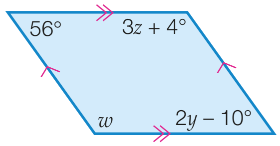


7 Find the value of the variables in each figure.

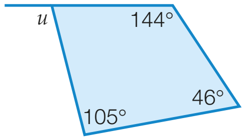
(a)



(b)



(c)



8 Write true (T) or false (F) for each statement.

(a) A trapezium has one pair of opposite sides equal and parallel.

(b) A square is a special rectangle.

(c) The diagonals of a kite are equal in length.

(d) The diagonals of a rhombus bisect each other at right angles.

9 In which regular polygon is each interior angle equal to 120°?

10 Which statement is true?

A A kite has one pair of opposite angles equal.

B A rectangle is a special kite.

C A rhombus is a rectangle.

D A rhombus is a special trapezium.

11 Which statement is false?

A All parallelograms have two pairs of parallel sides.

B All trapeziums have one pair of parallel sides of equal length.

C All kites have a pair of equal angles.

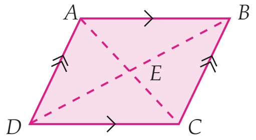
D All rectangles have a right angle.

12 **(a)** What is the sum of the angles of an octagon?

(b) What is the size of each interior angle of a regular octagon?

Understanding

13 *ABCD* is a parallelogram whose diagonals intersect at *E*. Using triangles *ADC* and *CBA*, prove that $\angle ADC = \angle CBA$.



(a) Copy and complete this proof by putting in the missing reasons.

Proof:

In ΔADC and ΔCBA,
$\angle ACD = \angle CAB$ (________ $AB \parallel DC$)
$\angle DAC = \angle BCA$ (________ AD____BC)
AC is a ________
$\therefore \Delta ADC \equiv \Delta CBA$ (________)
$\therefore \angle ADC = \angle CBA$ (________)

(b) Show that $\angle DAB = \angle BCD$.

14 In Question **13** you have proved that Δ*ADC* ≡ Δ*CBA*. Using this result:

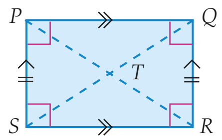
(a) show that the opposite angles of a parallelogram are equal

(b) show that the opposite sides of a parallelogram are equal

(c) if the diagonals intersect at point E, prove that Δ*AEB* ≡ Δ*CED*

(d) show that the diagonals of a parallelogram bisect each other.

15 *PQRS* is a rectangle in which *PS* = *QR* and *PR* and *QS* intersect at *T*. Prove that Δ*PRS* ≡ Δ*QSR*.



(a) Copy and complete this proof by filling in the missing steps.

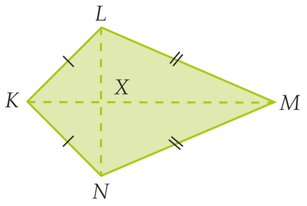
Proof:

In Δ*PRS* and Δ*QSR*,
∠_______ = ∠_______ = 90° (given)
_______ = _______ (given)
_______ = _______ (given)
∴ Δ_______ ≡ Δ_______ (SAS)

(b) Hence prove that *PR* = *QS*.

(c) What special property of a rectangle did you prove in **(b)**?

16 *KLMN* is a kite. *KM* and *LN* intersect at *X*.



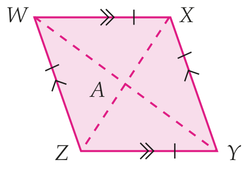
(a) Use triangles *KLM* and *KNM* to prove that ∠*KLM* = ∠*KNM*.

(b) Prove that Δ*LKX* ≡ Δ*NKX*.

(c) Show that ∠*LXK* = ∠*NXK* = 90°.

(d) What properties of a kite have you proved?

17 *WXYZ* is a rhombus as shown in the diagram. *WY* and *XZ* intersect at *A*.

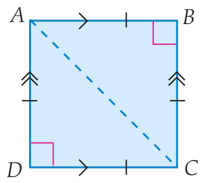


(a) Prove that ∠*WYX* = ∠*WYZ*.

(b) Explain why ∠*WZX* = ∠*YZX*.

(c) What property of a rhombus have you proved?

18 *ABCD* is a square as shown in the diagram. *AC* is a diagonal.

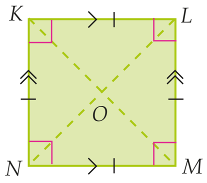


(a) Prove that ∠*ACD* = ∠*ACB*.

(b) Prove that ∠*ACD* = 45°.

(c) What properties of a square have you proved?

19 *KLMN* is a square as shown. *KM* and *LN* intersect at *O*.



(a) Prove that *KM* = *NL*.

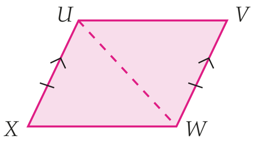
(b) Prove that *KO* = *OM* and that *NO* = *OL*.

(c) Prove that ∠*KOL* = 90°.

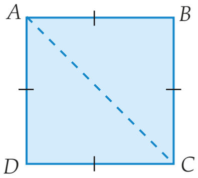
(d) What properties of a square have you proved?

Reasoning

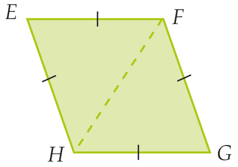
20 *UVWX* is a quadrilateral in which *UX* || *VW* and *UX* = *VW*. Prove that *UVWX* is a parallelogram.



21 *ABCD* is a quadrilateral in which all the sides are equal and $\angle ABC = 90°$. Prove that *ABCD* is a square.



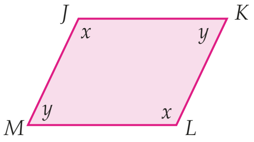
22 *EFGH* is a quadrilateral in which all the sides are equal. Prove that *EFGH* is a rhombus.



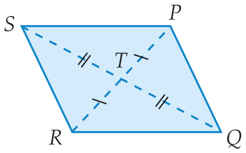
23 *JKLM* is a quadrilateral in which both pairs of opposite angles are equal; that is, $\angle MJK = \angle MLK$ and $\angle JML = \angle JKL$.

(a) Prove that $\angle MJK + \angle JML = 180°$.

(b) Hence prove that *JKLM* is a parallelogram.



24 *SPQR* is a quadrilateral in which the diagonals bisect each other at *T*, as shown. Prove that *SPQR* is a parallelogram.



Open-ended

25 **(a)** Simon knows that the statement 'All squares have equal sides' is true. He wonders whether the converse statement, 'All shapes with four equal sides are squares,' is true. Explain whether it is true or not.

(b) Select another true statement about a special quadrilateral. Write the converse statement and explain whether it is true or not.

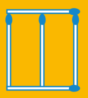
26 Explain why a square is a rectangle but a rectangle is not necessarily a square.

Puzzle

Matchstick quadrilaterals

Equipment required: matches (optional)

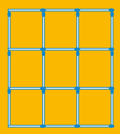
1 With five matchsticks, three quadrilaterals can be made, as shown.



(a) With six matchsticks, make six quadrilaterals.

(b) With six matches, make nine quadrilaterals.

2 Which six matches can you remove, without changing the position of the others, so that only three squares are left?

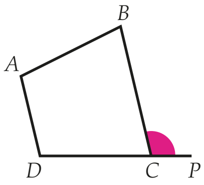


Investigation

Exterior angle sum of polygons

Equipment required: ruler, protractor

An exterior angle is formed if you extend one side of a polygon out from a vertex (corner). An exterior angle will always be less than 180°. In the diagram, side *DC* has been extended from *C* out to *P*. The angle *PCB* is one of the exterior angles of the polygon.

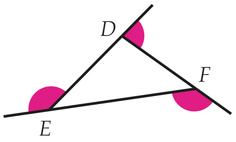


The Big Question

Is the sum of the exterior angles of a polygon always the same for all polygons? If so, why?

Engage

1 Draw a triangle and extend each of the sides to form exterior angles in a 'Catherine wheel' arrangement as shown in the diagram. Your diagram should be at least half a page in size.

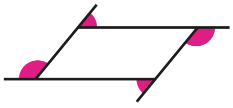


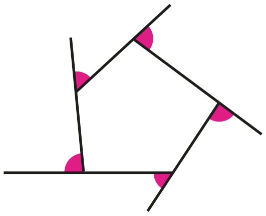
Measure each of the exterior angles and add them up.

2 Draw a different triangle. Make an exterior angle on each vertex and measure them using a protractor. What is the total sum of the exterior angles?

Explore

3 Find the sum of the exterior angles of a parallelogram, kite, pentagon and hexagon by drawing the shape and extending each side past the vertex, as shown. Measure the exterior angles formed in each shape and add them together.





Strategy options

- Draw a diagram.
- Look for a pattern.

Explain

4 On a loose sheet of paper, draw any convex polygon (a polygon with all interior angles less than 180°). Mark the exterior angles of this polygon and cut the angles out. Place all vertices together. What do you notice?

5 Write a sentence describing what you have found regarding the sum of exterior angles of convex polygons.

6 Use a diagram to explain why this property does not apply to concave polygons (polygons with one or more interior angles greater than 180°).

Elaborate

7 By exploring the sum of the exterior angles of triangles, quadrilaterals, pentagons and hexagons, find a rule for calculating the sum of the exterior angles of convex polygons.

Sum of the exterior angles of a convex polygon
= ______________________

8 Use your rule to find the sum of the exterior angles of a regular icosagon (20-sided polygon)? (A regular polygon has all sides of equal length.)

9 Answer the Big Question by summarising your findings, clearly identifying the types of polygons for which this rule works.

Evaluate

10 Do you believe your findings are correct for polygons of any number of sides? Justify your answer.

11 Do you believe that the number and variety of shapes you studied is enough to give a good indication that the rule you discovered is correct? How could you make your investigation of the sum of exterior angles in polygons more thorough?

Extend

12 (a) Find a rule to calculate the size of each exterior angle of any regular polygon where *n* is the number of sides of the polygon.

Hint

Remember that an exterior angle of a triangle is equal to the sum of the two opposite interior angles.

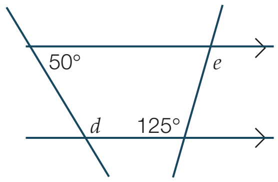
(b) Use this rule to show that the size of each interior angle of any regular polygon is $\frac{180(n-2)}{n}$, where *n* is the number of sides of the polygon.

Half-time 6

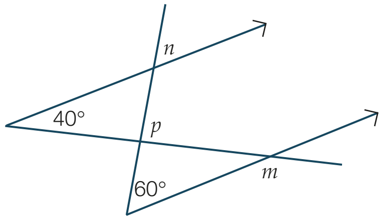
6.1

1 Find the value of the variables in each figure.

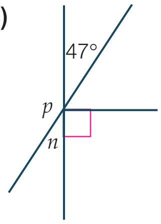
(a)



(b)



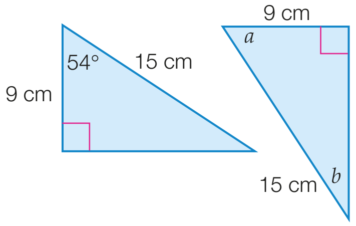
(c)



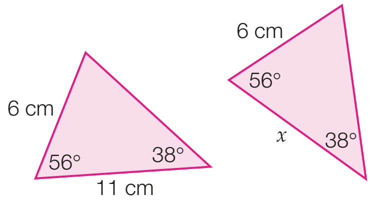
6.2

2 Find the value of the variables in each pair of congruent triangles. (State the congruency test used.)

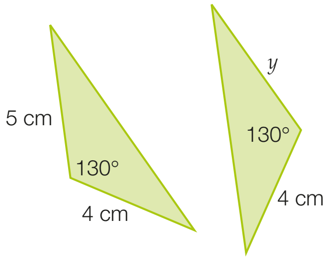
(a)



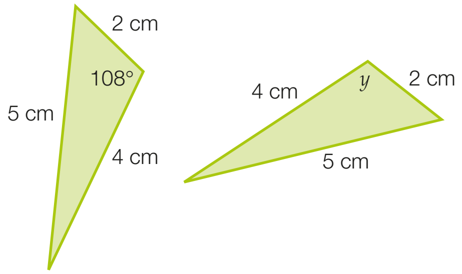
(b)



(c)



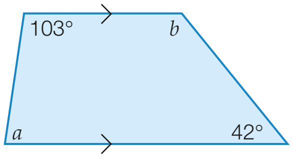
(d)



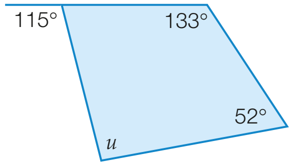
6.3

3 Find the value of the variables in each figure.

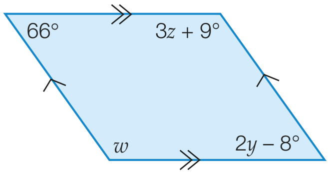
(a)



(b)



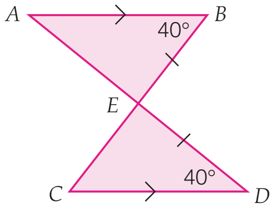
(c)



6.2

4 Complete the following proof by copying it and writing in the missing steps.

AD and *BC* intersect at *E* so that *BE* = *ED*. If ∠*ABC* = ∠*CDA* = 40°, prove that *AB* = *CD*.



Proof:

In Δ*ABE* and Δ*CDE*,

________ = ________ = 40° (given)
________ = ________ (given)
________ = ________ (vertically opposite angles)
∴ Δ________ ≡ Δ________ (ASA)
∴ ________ = ________ (corresponding sides of congruent triangles)

5 The value of x in the diagram is:

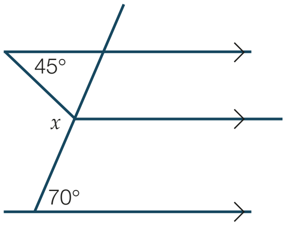
6.1

A 45°

B 65°

C 70°

D 115°



6 Which statements are true and which statements are false? Justify your answer.

6.3

(a) All rectangles are squares.

(b) All parallelograms are quadrilaterals.

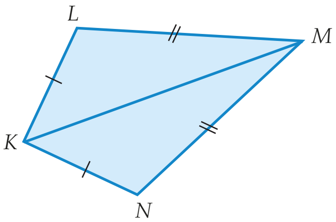
(c) All squares have equal diagonals.

(d) The diagonals of a parallelogram bisect each other at right angles.

7 Prove that $\Delta KLM \equiv \Delta KNM$.

6.2

List all the pairs of equal angles.



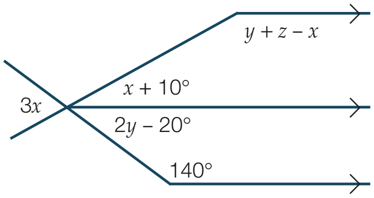
8 The diagonals of a quadrilateral are equal. Draw all the possible different quadrilaterals, naming each type.

6.3

9 Find the value of the variables in the diagram.

6.1

Give reasons for your answers.

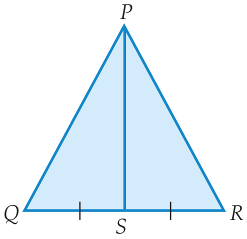


10 In ΔPQR, S is a point on QR such that $QS = SR$.

6.2

Explain why this is not enough information to prove that ΔQPS is congruent to ΔRPS.

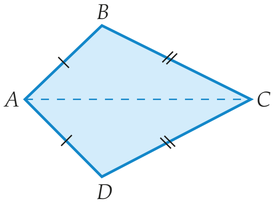
Write what additional information you would need to prove $\Delta QPS \equiv \Delta RPS$.



11 (a) $ABCD$ is a kite with $AB = AD$ and $BC = DC$. Construct the diagonal AC and then use congruency to show that $\angle ABC = \angle ADC$.

6.2, 6.3

(b) What property of a kite have you proved?



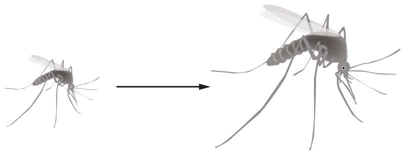
6.4

Enlarging and reducing

Maps, building plans and scale models are all based on scales. When drawing a map or constructing a model to scale, all measurements must be increased or decreased by the same multiplier; called the **scale factor**.

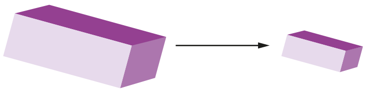
When you have both the original shape and the enlarged (or reduced) shape, you can easily find the scale factor by measuring the length of the corresponding sides of each shape.

Enlargement



A scale factor of 3 means the drawing is three times as big as the original. Every length on the enlarged shape is three times the length of the corresponding side on the original shape.

Reduction



A scale factor of $\frac{1}{2}$ means that the drawing is half the size of the original. Every length on the reduced shape is half the length of the corresponding side on the original shape.

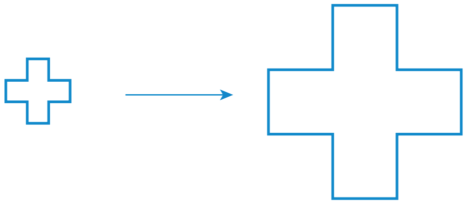
If the scale factor is greater than 1, the resulting figure is an enlargement.

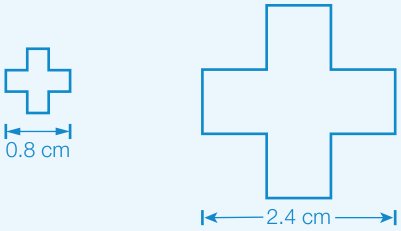
If the scale factor is less than 1, the resulting figure is a reduction.

Worked example 10

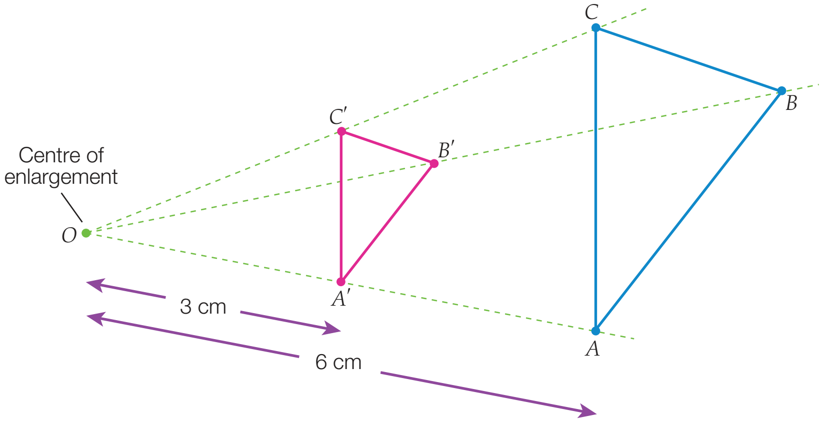
W.E. 10

What scale factor is used in this enlargement?



Thinking	Working
1 Choose corresponding sides on both shapes and measure each length.	
2 Determine the ratio of the enlarged measured length to the original measured length. The new length is always written first.	Enlarged length : original length 2.4 : 0.8 = 3 : 1
3 Calculate the scale factor.	Scale factor = 3

To enlarge or reduce a shape, you first need to choose a point of reference, known as the **centre of enlargement** (called O). Measure the distance from O to each of the points on the original shape and multiply that distance by the scale factor to determine the distance from O to the corresponding corners of the enlarged or reduced shape.



The original triangle ABC has been reduced by a scale factor of $\frac{1}{2}$ to produce triangle $A'B'C'$.

Note that the points on the new enlarged or reduced shape are labelled with the same letter as the corresponding point on the original shape, but with a mark (′) after the point. For example, point A on the original triangle ABC becomes point A' on the reduced triangle $A'B'C'$. A' is called A dash or A prime.

Scale factor and ratio

If you measure the side lengths of the reduced triangle and compare them with the corresponding side lengths of the original triangle ABC, you will find that each pair of sides is in the same ratio:

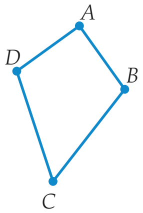
$A'B' : AB$	$A'C' : AC$	$B'C' : BC$
$= 2 : 4$	$= 2 : 4$	$= 1.3 : 2.6$
$= 1 : 2$	$= 1 : 2$	$= 2 : 4$
$= \frac{1}{2} : 1$	$= \frac{1}{2} : 1$	$= \frac{1}{2} : 1$

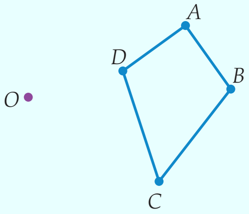
Written as a fraction, this ratio is equivalent to the scale factor, $\frac{1}{2}$.

Worked example 11

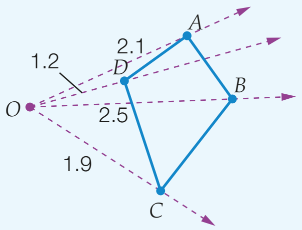
W.E. 11

Enlarge the following shape by a scale factor of 2.

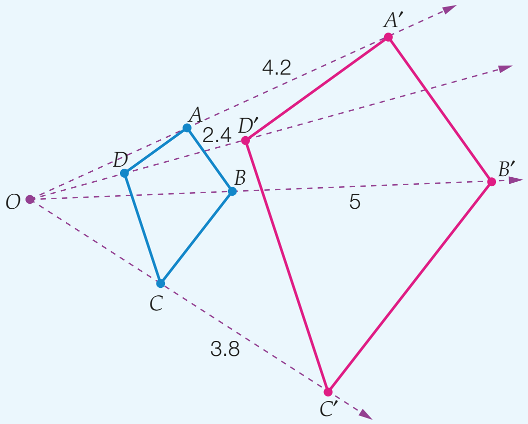


Thinking	Working
1 Select a centre of enlargement and label this point O.	

2 Rule lines from point O through each vertex of the shape. Measure the distance from O to each of the vertices.
(Here $OA = 2.1$ cm, $OB = 2.5$ cm, $OC = 1.9$ cm and $OD = 1.2$ cm.)



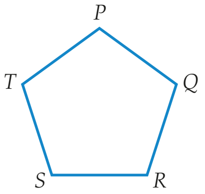
3 Multiply each distance you measured by the scale factor. Use a ruler to locate each of these new distances on the rays drawn from point O and mark them as points. Rule lines to join these new points, which form the enlarged shape.
(Here $OA' = 4.2$ cm, $OB' = 5$ cm, $OC' = 3.8$ cm and $OD' = 2.4$ cm.)



Worked example 12

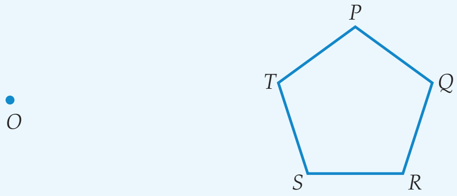
W.E. 12

Reduce the following shape by a scale factor of $\frac{1}{2}$.

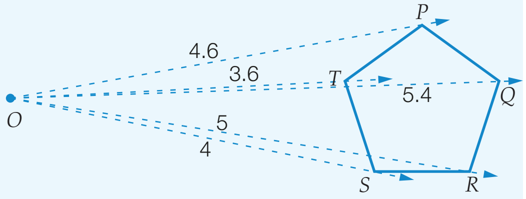


Thinking **Working**

1 Select a centre of enlargement and label this point O.

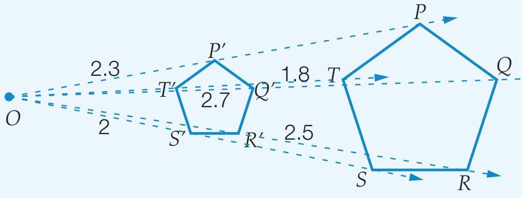


2 Rule lines from point O through each vertex of the shape. Measure the distance from O to each of the vertices.



3 Multiply each distance by the scale factor. Locate each of these new distances on the rays drawn from point O and mark them as points.

Rule lines to join these new points, which form the reduced shape.



6.4 Enlarging and reducing

Navigator

1, 2 (a), 3 (a), 4, 5, 7, 9 (a, b), 10, 12, 14

1, 2, 3 (a), 4, 5, 6, 7, 9, 10, 12, 13, 14

1, 2 (b), 3 (b), 4, 5, 6, 8, 9, 10, 11, 13, 14, 15

Answers p. 672

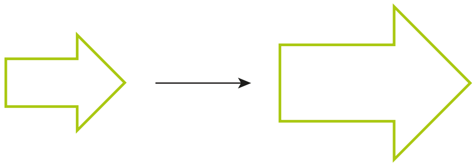
Equipment required: protractor for Question **8**, ruler for all

Fluency

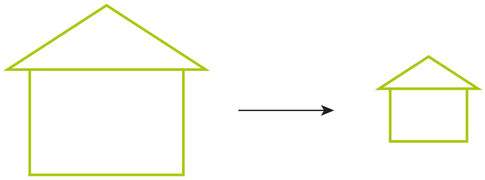
1 What scale factors have been used in each of the following enlargements or reductions?

W.E. 10

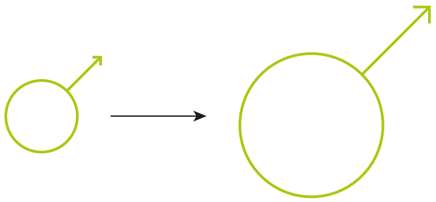
(a)



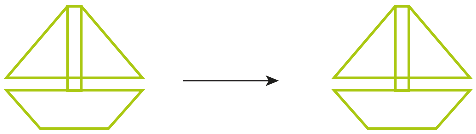
(b)



(c)



(d)

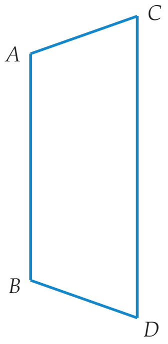


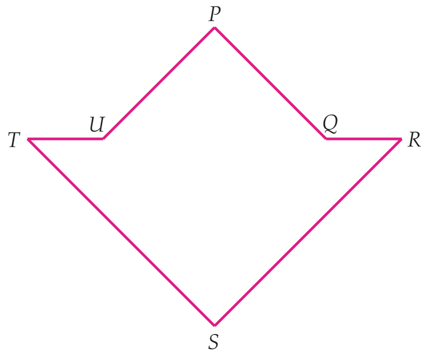
2 Enlarge each shape by the scale factors shown:

(a) **(i)** 3 **(ii)** 1.5

(b) **(i)** 4 **(ii)** $2\frac{1}{4}$

W.E. 11



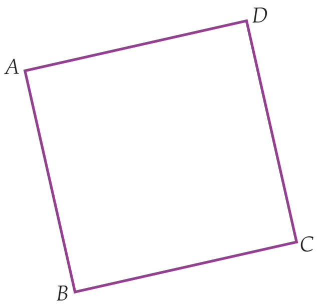


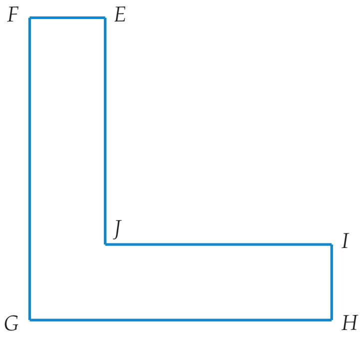
3 Reduce these shapes by the scale factor shown:

(a) **(i)** $\frac{1}{2}$ **(ii)** $\frac{1}{3}$

(b) **(i)** $\frac{3}{4}$ **(ii)** 0.6

W.E. 12

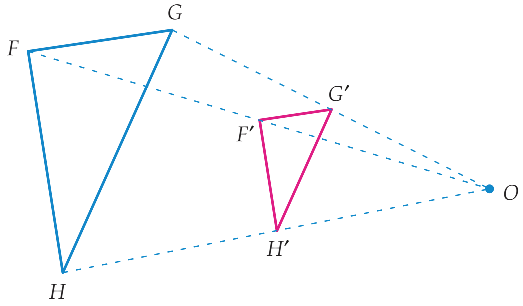




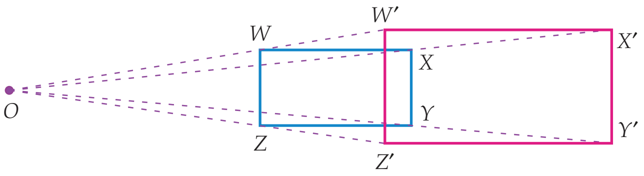
4 **(a)** ΔFGH has been reduced to $\Delta F'G'H'$. Measure each side of both triangles and find the following ratios.

- **(i)** $F'G' : FG$
- **(ii)** $F'H' : FH$
- **(iii)** $G'H' : GH$

(b) What is the scale factor?



5 **(a)** Rectangle $WXYZ$ has been enlarged to rectangle $W'X'Y'Z'$. Measure each side of both rectangles and find the following ratios.

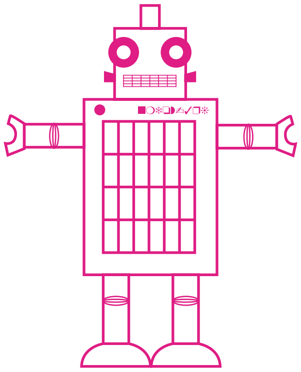


- **(i)** $W'X' : WX$
- **(ii)** $X'Y' : XY$
- **(iii)** $Y'Z' : YZ$
- **(iv)** $Z'W' : ZW$

(b) What is the scale factor?

6 A scale factor of $\frac{1}{4}$ has been used to draw this toy robot. The actual height of the robot is:

- **A** 1.2 cm
- **B** 18 cm
- **C** 19.2 cm
- **D** 20 cm



Understanding

7 Rectangle $ABCD$ has dimensions of 2 cm × 3 cm and is to be enlarged by a scale factor of 3. Draw this rectangle and use a centre of enlargement to produce the enlarged shape.

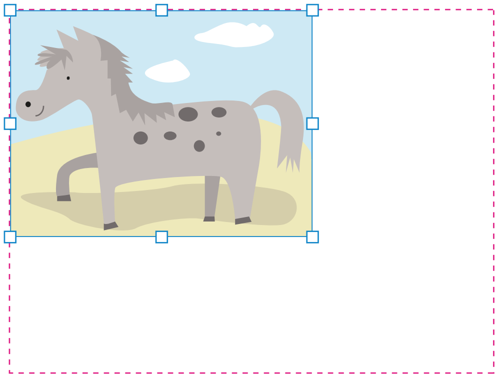
Hint

The internal angle of a regular pentagon is 108°.

8 A regular pentagon $JKLMN$ has a side length of 8 cm. Accurately construct this pentagon using a ruler and a protractor and enlarge it by a scale factor of $1\frac{1}{4}$ using a centre of enlargement, O. (Choose any reasonable point as point O.)

9 Mel is using her laptop to collect some images for a project. She has one rectangular image that measures 4.8 cm by 6.2 cm. By clicking and dragging her mouse on a corner of the image, Mel enlarges it so that it now measures 16.8 cm by 21.7 cm.

(a) What is the scale factor of this enlargement?



Later, Mel finds that the image is now too big. She reduces the image size to 7.2 cm by 9.3 cm.

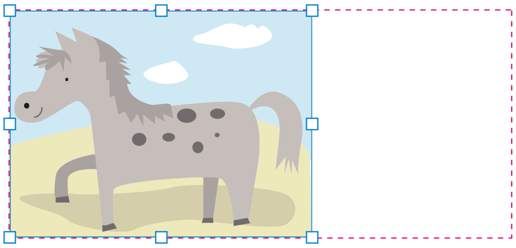
(b) What is the scale factor of this reduction of the enlarged image used in part **(a)**?

(c) Overall, has the original image been enlarged or reduced? Find the scale factor for this overall change.

(d) Mel takes a copy of her original image and clicks and drags the middle of one of the sides instead of the corner.

She finds that the length of the image changes from 4.8 cm to 10.2 cm, but the width remains the same as before.

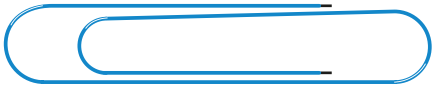
Is this image a true enlargement of the original image? Explain why or why not.



Reasoning

10 The area of a square is 36 cm^2. What is the area of the square after the dimensions of the square have been reduced by a scale factor of $\frac{1}{3}$?

11 The picture on the right shows a scale drawing of a paperclip after it has been enlarged by a scale factor of 2. To make a scale drawing of the paperclip that is 4.9 cm long, what scale factor would need to be applied to the real paperclip?

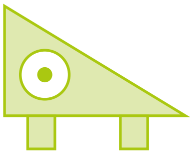


Open-ended

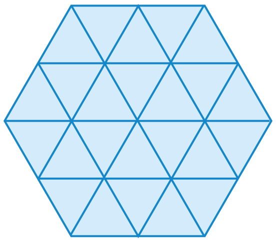
12 Draw a polygon with more than three sides. Reduce the shape by a scale factor of $\frac{1}{2}$. Enlarge the original shape by a scale factor of 3.

13 The area of a rectangle is 72 cm^2 after its length and width are enlarged by a scale factor of 3. What could the dimensions of the original rectangle be?

14 For the picture on the right, draw three scale drawings, stating the scale factor used each time.



15 Gina and Conan are playing a game using this hexagonal board. Gina notices that three different-sized equilateral triangles could be found on the board. She then comments, 'There are four small triangles in the medium sized triangle, so the scale factor must be 4, and there are nine small triangles in the large triangle so the scale factor must be 9.' Conan doesn't agree.



(a) Draw the three different sized triangles.

(b) Complete scale drawings of the small triangle with a scale factor of 4 and 9.

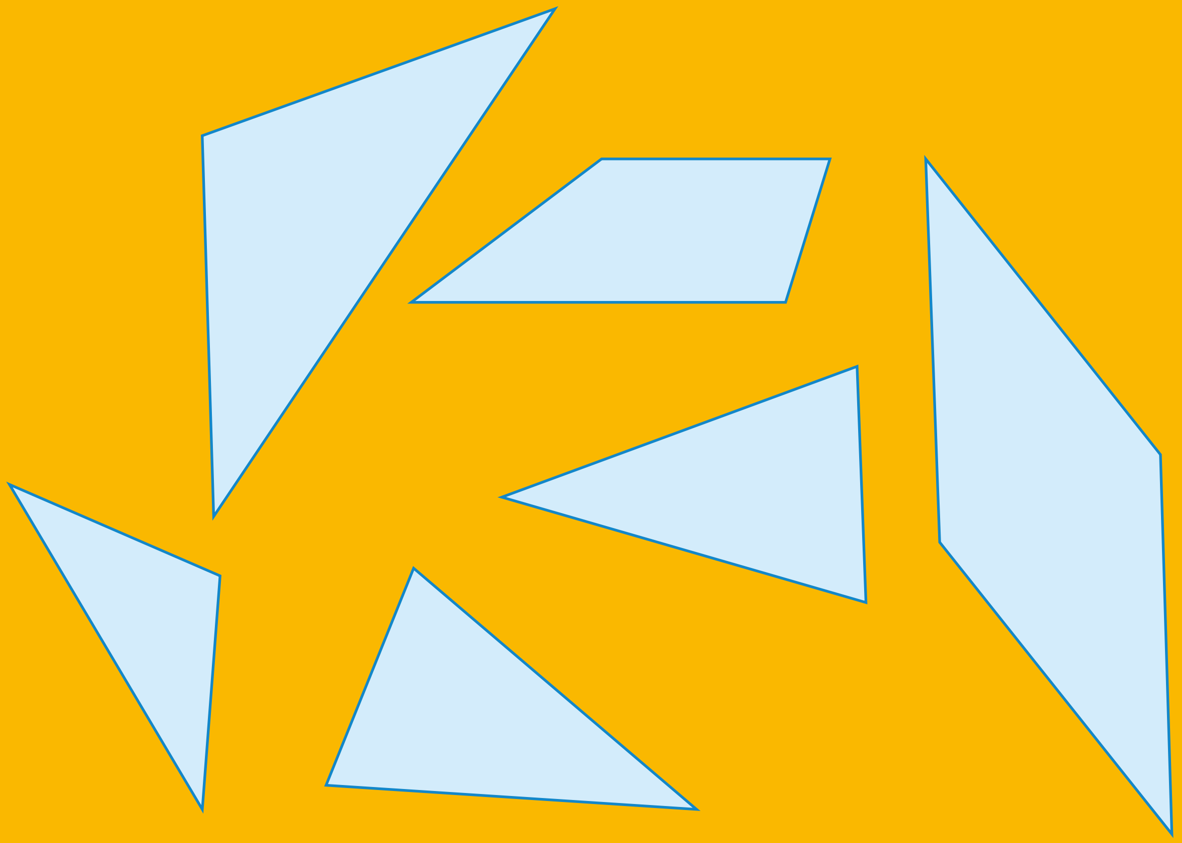
(c) What may Conan's answer be? How could he explain this answer to Gina?

Puzzle

What a star!

Equipment required: paper, scissors, shapes (draw your own copy of the shapes shown, or download a template from the eBook or the Pearson Places website)

Copy these shapes, cut them out and rearrange them to form a perfect five-pointed star.



Exploration CAS

Equipment required: TI-Nspire CAS or Casio ClassPad CAS

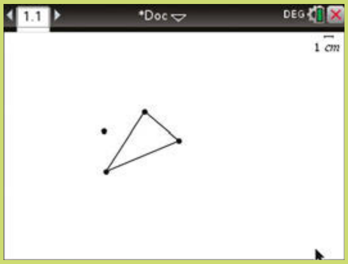
Enlarging or reducing a figure

Changing the size of a figure by a given scale factor

Using TI-Nspire CAS

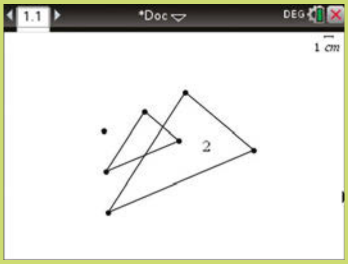
Add **Geometry** to your document. Select **menu** > **Shapes** > **Triangle**, then move the cursor to select the points where you want each vertex of the triangle to appear. Press **esc** to stop drawing.

Then select **menu** > **Points & Lines** > **Point** to draw a point outside the triangle, but not too far from it.



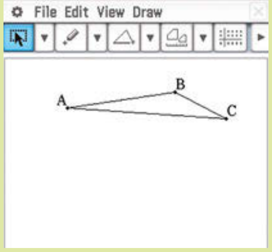
Select **menu** > **Transformation** > **Dilation**. Select the point, then select the triangle, then enter 2. A second triangle will appear with the dilation factor 2 as shown.

You can select the dilation factor number to move it around and to change its value. Change it from 2 to 0.5.

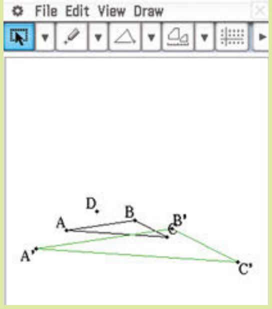


Using Casio ClassPad CAS

From the menu select **Geometry**. Select the scalene triangle icon △ and move the cursor to choose where you want the shape to appear. Don't make your triangle too small.



Select all 3 sides of the triangle, and then select **Draw** > **Construct** > **Dilation**. Select the dilation centre point and set the **Scale** to 2. The second triangle will appear. Use **View** > **Zoom to Fit** if the screen does not show both triangles completely.



You can delete the dilated triangle and repeat the steps above with a different **Scale** value to see what happens with a different dilation factor, such as 0.5.

1 How does the dilated triangle compare to the original triangle when the dilation factor is greater than 1?

2 How does the dilated triangle compare to the original triangle when the dilation factor is less than 1?

Using TI-Nspire CAS

Change the dilation factor back to 2. Then select **menu** > **Points & Lines** > **Line** to draw lines through the centre point of dilation and the vertices of the dilated triangle. What other points do these lines pass through?

Select the centre point of dilation and move the point around the screen.

Using Casio ClassPad CAS

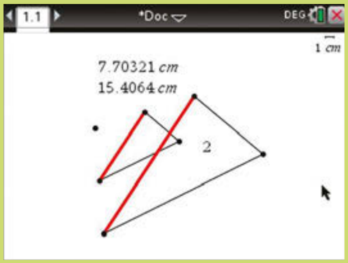
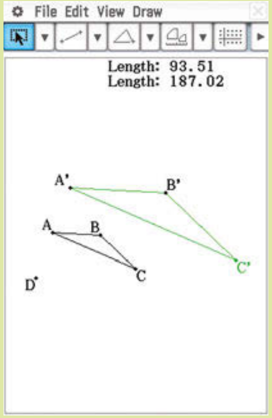
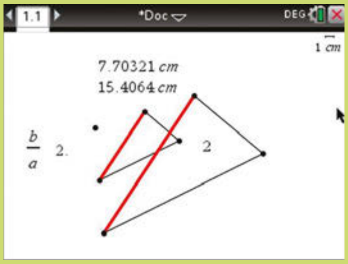
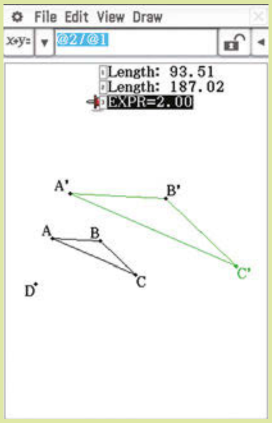
Redraw the triangle with a dilation factor of 2. Then select **Draw** > **Basic Object** > **Ray** to draw lines through the centre of dilation and the vertices of the dilated triangle. What other points do these lines pass through?

Select the centre point of dilation and move the point around the screen.

3 What happens when the point lies on one of the sides of the triangle?

4 Describe in general what happens as you move the point around the screen.

Investigating the lengths associated with enlargements

Using TI-Nspire CAS	Using Casio ClassPad CAS
First, select and delete the lines you drew on your triangles. Now measure the length of one pair of corresponding sides in the two triangles. To measure the length of a side, select **menu** > **Measurement** > **Length** and then select the two vertices that form the side. Make sure you select the vertices for the corresponding side in the other triangle. To show the measurements with more decimal places, select **menu** > **Settings…** and change **Display Digits** to **Float 6**.  7.70321 *cm* 15.4064 *cm* 	First, select and delete the lines you drew on your triangles. Now measure the length of one pair of corresponding sides in the triangles. To measure the length of a side, select the side and then select **Draw** > **Measurement** > **Length**. Make sure you select the corresponding side in the other triangle.  Length: 93.51 Length: 187.02
To calculate the ratio of the two side lengths, first select **menu** > **Actions** > **Text** and enter the expression ***b*/*a***. Then, select **menu** > **Actions** > **Calculate** and select this expression. You will be prompted to choose the values for *a* and then for *b* (the two side lengths) which you can do by selecting their labels on the screen. After you do this, the calculated value will appear.  7.70321 *cm* 15.4064 *cm* $\frac{b}{a}$ 2. 	To calculate the ratio of the two side lengths, select **Draw** > **Expression**, then select the right arrow in the top right corner of the screen. This will display the box where you can enter a calculation. Each of your side length measurements should now have a number label attached. To enter a measurement into the ratio expression, select the measurement's attached number label. This will appear in the expression as an @ symbol with the number label. Use this to calculate the ratio of the side lengths.  @2/@1 Length: 93.51 Length: 187.02 EXPR=2.00

Select one of the vertices of the original triangle and move it around.

5 What happens to the value of the ratio?

6 Change the dilation factor to 3 and move a vertex around. What happens to the value of the ratio?

7 Change the dilation factor to 0.5 and move a vertex around. What happens to the value of the ratio?

8 What does this tell you about the side lengths of corresponding sides of a dilated shape?

Now, similarly measure the length from the centre point of dilation to one of the vertices of the original triangle, and then from the centre point of dilation to the corresponding vertex of the other triangle.

9 What is the ratio of these two measurements?

10 Are the two different measurements you have looked at in this section in the same ratio for the other sides and ratios of the two triangles?

Investigating the areas of similar figures

Using TI-Nspire CAS	Using Casio ClassPad CAS
Change the dilation factor to 2. To find the area of a shape, you can select **menu** > **Geometry** > **Measurement** > **Area** and then select the shape.	Change the dilation factor to 2. To find the area of a shape, you can select all three sides of the shape and then select **Draw** > **Measurement** > **Area**.

Repeat your earlier steps to find the ratio of the areas of the two triangles.

11 What value do you find for the ratio $\frac{\text{area of large triangle}}{\text{area of small triangle}}$?

12 Move one of the vertices of the original triangle around the screen and observe how this ratio changes. What did you observe?

13 Change the dilation factor, for example to 3, and repeat this process. What is the value of the ratio $\frac{\text{area of large triangle}}{\text{area of small triangle}}$ now?

14 Change the dilation factor to 0.5 and repeat the process. What is the value of the ratio $\frac{\text{area of large triangle}}{\text{area of small triangle}}$ now?

15 What general conclusion can you reach about the connection between the areas of two similar triangles, given the dilation factor?

Taking it further

16 Repeat this exploration for a shape other than a triangle. Write some conclusions about the side lengths, the relationship to the dilation factor and the areas of the two shapes.

6.5

Similarity and similar triangles

Similar figures have the same shape, but not necessarily the same size. It is important to distinguish between similar figures and congruent figures. Congruent figures have the same shape and are *exactly* the same size.

The photos below are similar. (You can see that the length and width of the smaller frame have both been multiplied by the same scale factor of 1.5 to produce the length and width of the larger frame.)

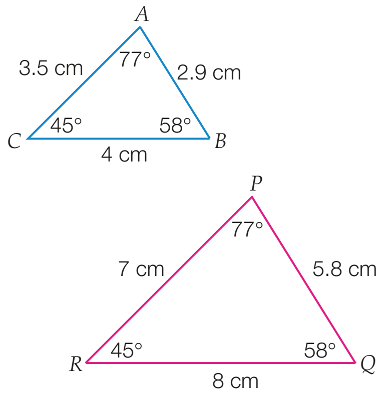
The triangles ΔABC and ΔPQR shown are similar.

The ratio of corresponding sides can be shown as:

$$\frac{PQ}{AB} = \frac{5.8}{2.9}$$
$$= 2$$

$$\frac{QR}{BC} = \frac{8}{4}$$
$$= 2$$

$$\frac{PR}{AC} = \frac{7}{3.5}$$
$$= 2$$



Corresponding angles are equal and the ratios of corresponding sides are equal.

The above information shows that ΔABC is *similar* to ΔPQR.

In similar figures:

- all pairs of corresponding sides are in the same ratio
- all pairs of corresponding angles are equal.

To show that two shapes are similar, use the ~ symbol or the ||| symbol.

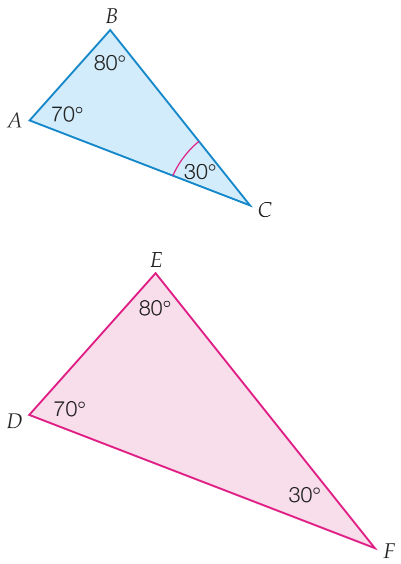
For example, the statement 'triangle *ABC* is similar to triangle *PQR*' is written as:

$\Delta ABC \sim \Delta PQR$ or $\Delta ABC \,|||\, \Delta PQR$

It is not necessary to know about all the sides and angles to determine that two triangles are **similar triangles**. There are several tests you can apply that give enough information to classify triangles as similar. The following tests can be used to show that two triangles are similar.

Tests for similar triangles

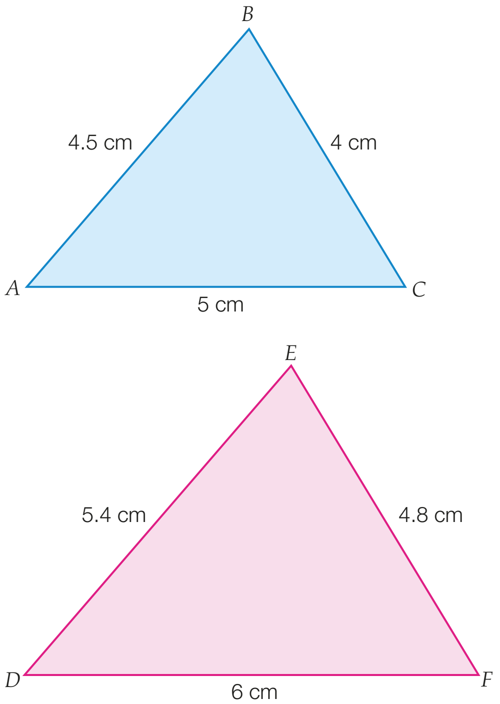
1 Angle, Angle, Angle (AAA):



$\Delta ABC \sim \Delta DEF$ (AAA)

Three pairs of angles are the same magnitude. (You only need to know the values of two of the pairs because the angle sum of a triangle is 180°, so you can calculate the value of the third pair.)

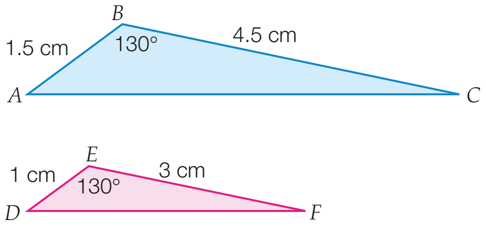
2 Side, Side, Side (SSS):



$\Delta ABC \sim \Delta DEF$ (SSS)

Three pairs of corresponding sides are in the same ratio.

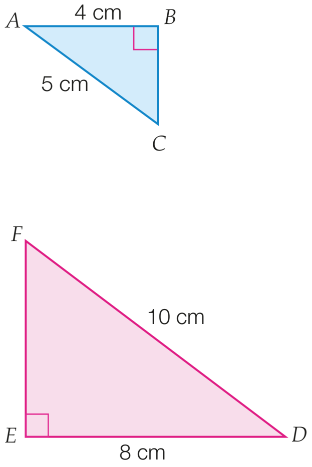
3 Side, Angle, Side (SAS):



$\Delta ABC \sim \Delta DEF$ (SAS)

Two pairs of corresponding sides have lengths in the same ratio and the included angles are equal.

4 Right angle, Hypotenuse, Side (RHS):



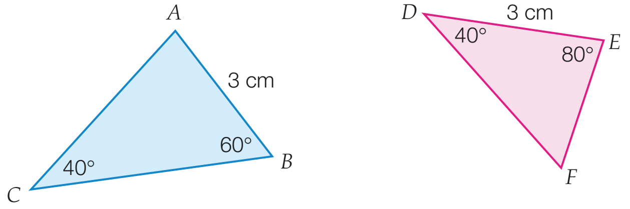
$\Delta ABC \sim \Delta DEF$ (RHS)

The hypotenuse and one side of a right-angled triangle are in the same ratio as the hypotenuse and one side of another right-angled triangle.

Worked example 13

W.E. 13

Use a similarity test to show that triangles *ABC* and *DEF* are similar.



Thinking	Working
1 State the sides and angles that can be used to justify similarity.	$\angle ACB = \angle EDF = 40°$ (Angle) $\angle ABC = \angle EFD = 60°$ (Angle) $\angle CAB = \angle DEF = 80°$ (Angle)
2 Formally state that the triangles are similar, taking care to state corresponding vertices in the same order. Write the test that was used to show similarity in brackets as a justification of your answer.	$\therefore \Delta ABC \sim \Delta EFD$ (AAA)

Similarity and scale factor

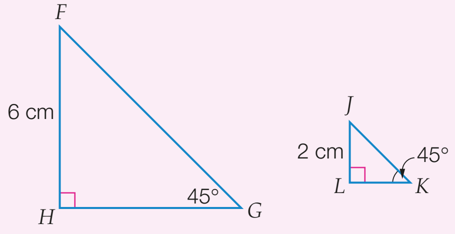
For two similar figures, the scale factor is the ratio of corresponding sides.

$$\frac{JL}{FH} = \frac{2}{6} = \frac{1}{3}$$

or

$$\frac{FH}{JL} = \frac{6}{2} = 3$$

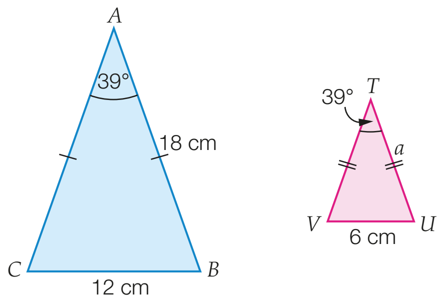
$\therefore$ ΔJKL is one-third the size of ΔFGH or ΔFGH is three times the size of ΔJKL.



Worked example 14

W.E. 14

Given that the following triangles are similar, find the value of a.



Method 1: Use the scale factor

Thinking	Working
1 Identify the scale factor using the corresponding sides for which values are given.	$\text{Scale factor} = \frac{UV}{BC}$ $= \frac{6}{12}$ $= \frac{1}{2}$
2 Find the unknown side length by multiplying its corresponding side length by the scale factor.	$a = 18 \times \frac{1}{2}$ $a = \frac{18}{1} \times \frac{1}{2}$ $a = 9 \text{ cm}$

Method 2: Set up an equation

Thinking	Working
1 Use the fact that the ratios of the corresponding sides are equal to form an equation.	$\frac{TU}{AB} = \frac{UV}{BC}$ $\frac{a}{18} = \frac{6}{12}$
2 Solve the equation.	$a = \frac{6}{12} \times 18$ $a = \frac{1}{2} \times \frac{18}{1}$ $a = \frac{18}{2}$ $a = 9 \text{ cm}$

6.5 Similarity and similar triangles

Navigator

Answers p. 673

1 (a, b), 2 (a–c), 3, 4, 5, 6 (a, b), 7 (a, c), 8, 9, 12, 13	1 (a, c), 2 (b–d), 3, 4, 5, 6, 7 (a, b), 8, 9, 10, 12, 13	1 (b, d), 2 (b, d), 3, 4, 5, 6, 7 (a, b), 8, 9, 10, 11, 12, 14

Equipment required: protractor for Question **10**

Fluency

W.E. 13

Hint

Remember to use the correct mathematical symbols:

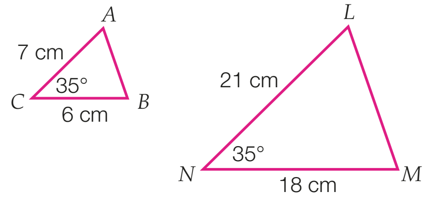
∠ means 'angle'

Δ means 'triangle'

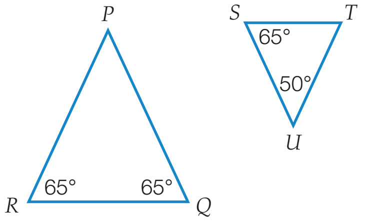
~ means 'is similar to'.

1 Use a similarity test to show why the following pairs of triangles are similar.

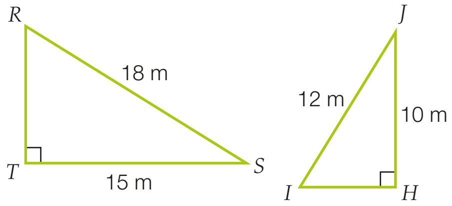
(a)



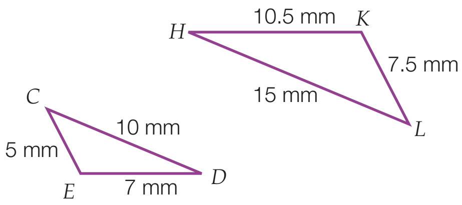
(b)



(c)



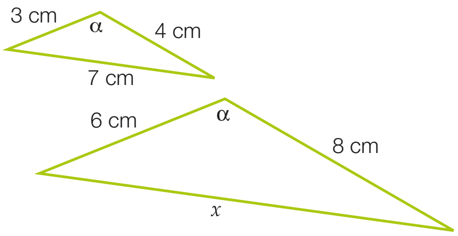
(d)



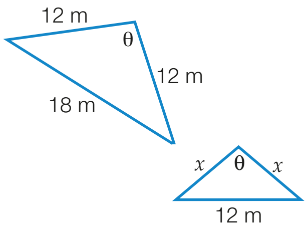
W.E. 14

2 Find the value of x in each of the following pairs of similar triangles.

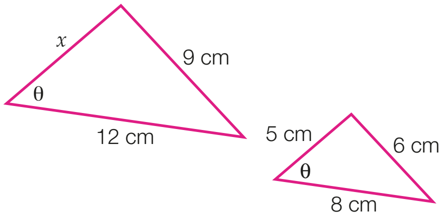
(a)



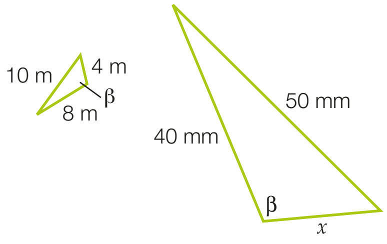
(b)



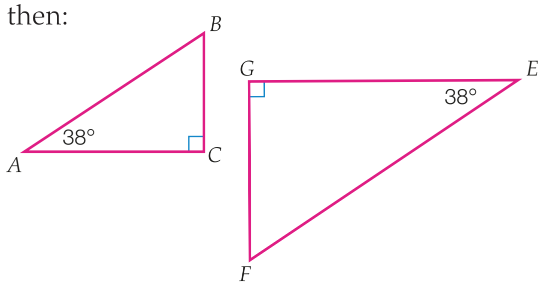
(c)



(d)



3 Given triangle ABC is similar to triangle EFG, then:



(a) the corresponding side to AB is

A FG **B** EF

C BC **D** EG

(b) the corresponding angle to $\angle EFG$ is

A $\angle CAB$ **B** $\angle ACB$

C $\angle ABC$ **D** $\angle BAC$

4 Given triangle *HJK* is similar to triangle *LMN*, then:

(a) the corresponding side to *LM* is

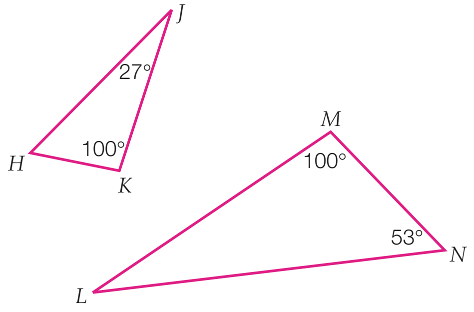
A *HJ* **B** *MN*

C *JK* **D** *LN*

(b) the corresponding angle to ∠*JHK* is

A ∠*LMN* **B** ∠*NLM*

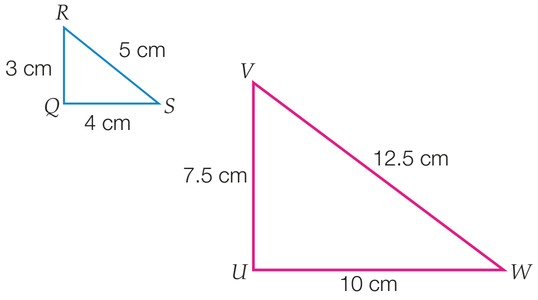
C ∠*LNM* **D** ∠*MLN*



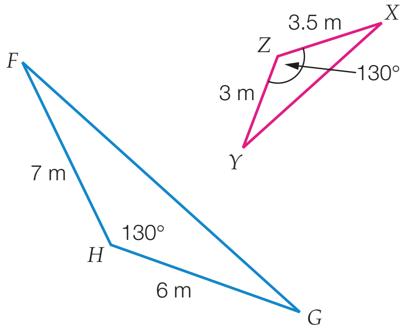
Understanding

5 Find the scale factor for the following pairs of similar triangles. (The blue triangles are the originals.)

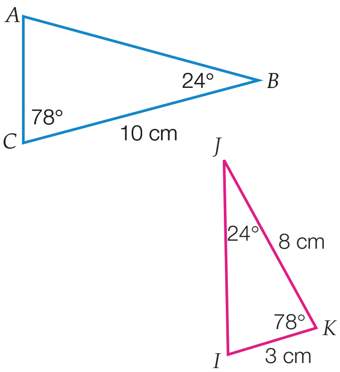
(a)



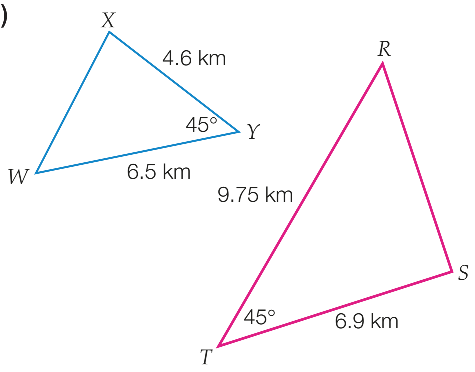
(b)



(c)

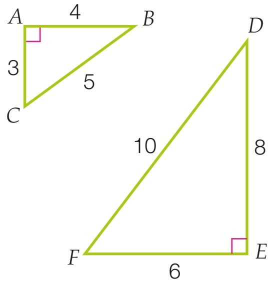


(d)

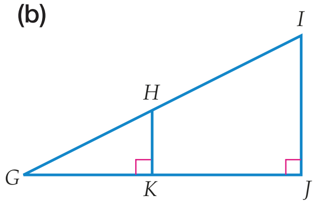


6 Show that these pairs of triangles are similar.

(a)



(b)



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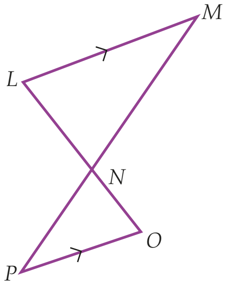
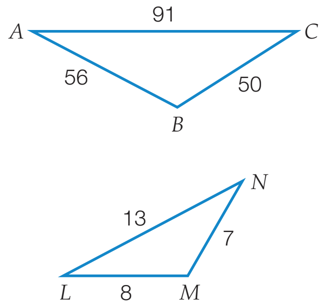
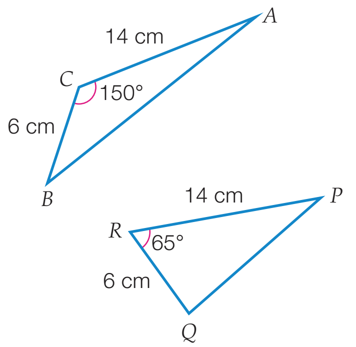


7 Show that these pairs of triangles are not similar.

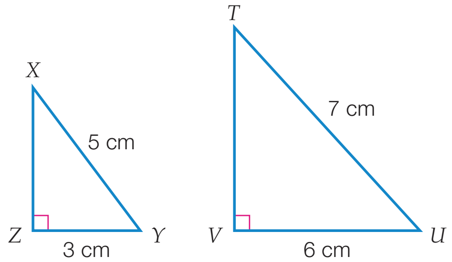
(a)



(b)



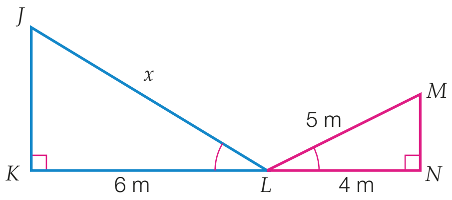
(c)



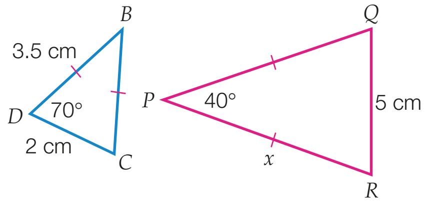
8 The following pairs of triangles are similar. (The blue triangles are the originals.) For each pair:

(i) state the scale factor **(ii)** find the value of the variable.

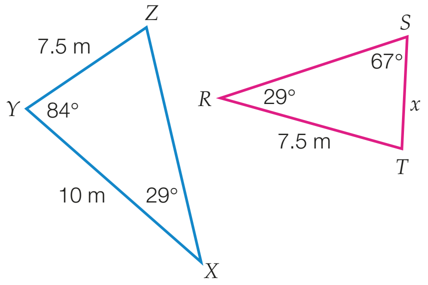
(a)



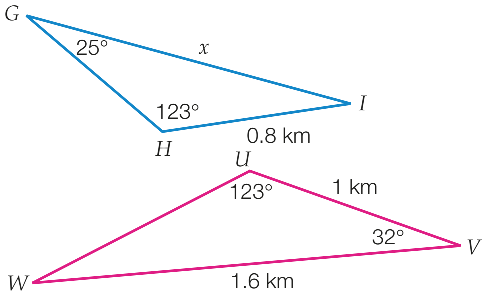
(b)



(c)



(d)



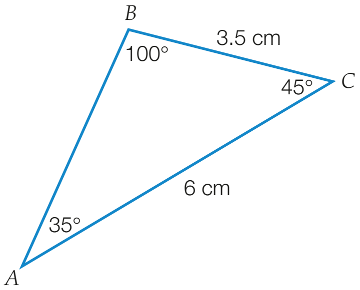
9 A 4 m ladder is leaning against a wall. The base of the ladder is 2 m from the wall. A second 5 m ladder is leaning against a wall. The base of this ladder is 3 m from the wall.

(a) Draw a diagram showing the triangle formed by the wall, the first ladder and the ground.

(b) Draw a diagram showing the triangle formed by the wall, the second ladder and the ground.

(c) Are the two triangles drawn in **(a)** and **(b)** similar? Justify your answer.

Reasoning

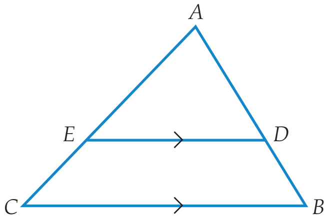


10 ΔABC and ΔPQR are similar triangles. ΔABC is shown on the right. $\angle PRQ = 45°$, $\angle RQP = 100°$ and $PR = 9$ cm. Use this information to:

(a) determine the scale factor

(b) construct ΔPQR using a protractor and ruler using the same scale.

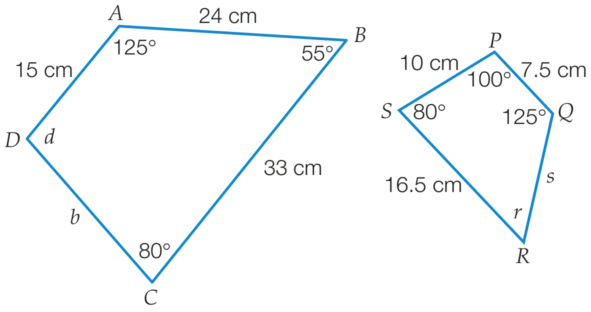
11 ΔABC and ΔADE are similar. Explain why any triangle divided with a line parallel to any side will produce two similar triangles.



12 Other two-dimensional shapes (and three-dimensional shapes) can be classed as similar. The quadrilaterals shown are similar.

(a) How can you prove this?

(b) Find the size of any unknown sides or angles in either quadrilateral.



Open-ended

13 Draw a pair of similar triangles that have angles of 30°, 60° and 90°. State the scale factor.

14 On a maths test, one of Geordie's justifications for the similarity of two triangles was:

$$\angle GHI = \angle UVT = 38°$$

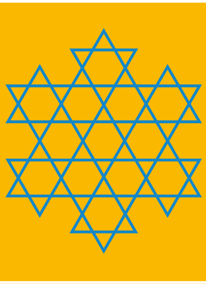
(a) Draw two similar triangles that match this justification.

(b) On the triangles drawn in part **(a)**, state the minimum amount of information necessary (angle magnitudes and/or side lengths) to be able to justify that the two triangles are similar.

Puzzle

How many triangles?

What is the total number of triangles in this diagram?



6.6 Solving problems using similar triangles

Many questions and problems can be solved by first identifying that two triangles are similar and then using the scale factor to find the length of any unknown sides. Unknown angles can be found by identifying corresponding angles.

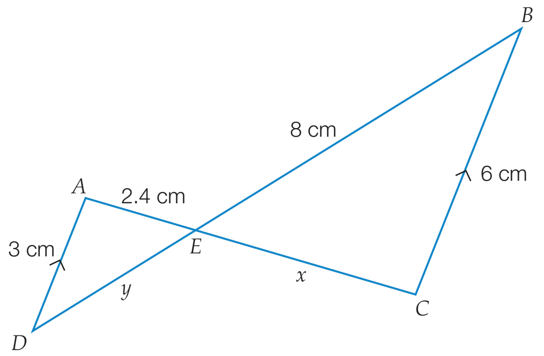
In order to identify similar triangles in a problem, you may need to recall angle facts relating to parallel lines being crossed by a transversal. This can help to identify angle values.

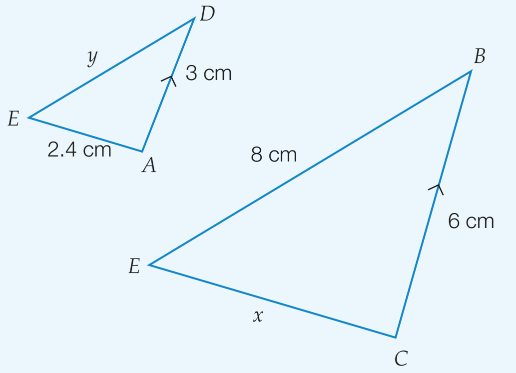
If two triangles are connected or overlapping, it is often useful to draw the two triangles separately and change the orientation of one so they both have the same orientation.

Worked example 15

W.E. 15

Show that the two triangles are similar and then find the values of x and y.



Thinking	Working
1 Redraw the triangles separately. Keep the orientation of one triangle the same and change the orientation of the other so they both have the same orientation. Identifying known equal angles and sides helps to draw the diagrams correctly.	
2 Use a similarity test to show that the triangles are similar. Write the abbreviation for the test used (AAA in this case) after the similarity statement.	$\angle AED = \angle BEC$ (vertically opposite angles) $\angle DAE = \angle BCE$ (alternate angles) $\angle ADE = \angle CBE$ (alternate angles) $\therefore\ \Delta ADE \sim \Delta BCE$ (AAA)

3 Identify the scale factor using the two values of the corresponding sides.

$$\text{Scale factor} = \frac{BC}{DA} = \frac{6}{3} = 2$$

4 Use the scale factor to determine the lengths of the unknown sides.

$$\frac{x}{EA} = 2$$
$$x = EA \times 2 = 2.4 \times 2 = 4.8 \text{ cm}$$

$$\frac{EB}{y} = 2$$
$$y = \frac{EB}{2} = \frac{8}{2} = 4 \text{ cm}$$

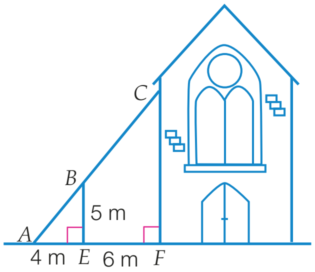
5 State the answer.

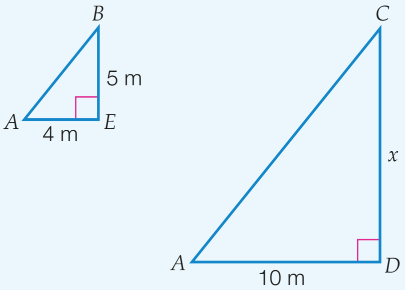
$x = 4.8$ cm and $y = 4$ cm

Worked example 16

W.E. 16

The wall of the local hall needs a support to keep it in place (as shown). The support is positioned on the ground 10 m from the base of the wall. There is a 5 m vertical prop for this support placed 6 m from the wall. How far up the wall does the support reach?



Thinking	Working
1 Draw the two triangles separately. Label each of the points on each triangle. (Here, the point *A* is common to both triangles.) Label the required length *x*.	
2 Show that the triangles are similar.	$\angle BAE = \angle CAD$ (common angle) $\angle BEA = \angle CDA = 90°$ (given in diagram) $\angle ABE = \angle ACD$ (angles in triangles $= 180°$) $\therefore \Delta ABE \sim \Delta ACD$ (AAA)
3 Use the fact that the ratio of the corresponding sides is equal to form an equation.	$\frac{CD}{BE} = \frac{AD}{AE}$

4 Solve the equation.

$$\frac{x}{5} = \frac{10}{4}$$
$$x = \frac{10}{4} \times 5$$
$$= \frac{5}{2} \times \frac{5}{1}$$
$$= \frac{25}{2}$$
$$= 12.5 \text{ m}$$

5 Answer the question (including units). ∴ The support reaches 12.5 m up the wall.

6.6 Solving problems using similar triangles

Navigator

Answers p. 673

1, 2, 3, 4, 5, 7, 9

1 (b, c), 2, 3, 4, 5, 6, 7, 9, 10

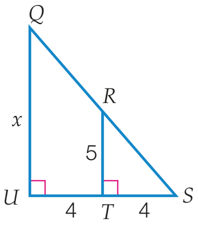
1 (c, d), 2, 3, 4, 5, 6, 7, 8, 9, 10

Fluency

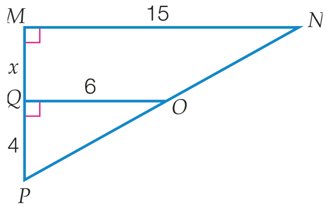
W.E. 15

1 In each of the following diagrams, show that the two triangles are similar and then find the value of x.

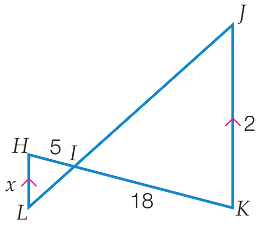
(a)



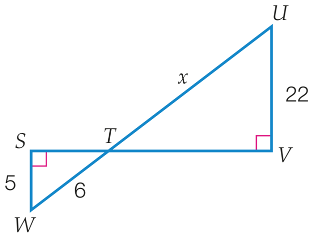
(b)



(c)



(d)

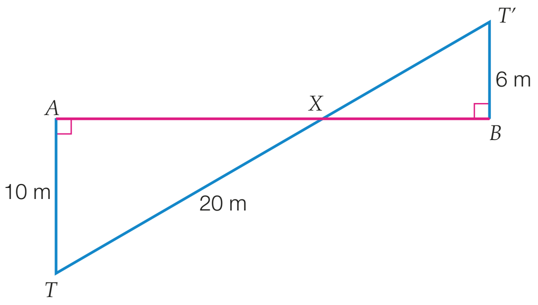


W.E. 16

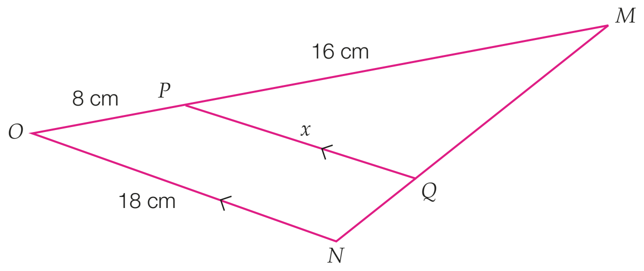
2 A tap T is placed on one end of a 10 m wall. A 20 m water pipe runs from the tap to a point X as shown on the diagram. It is to be extended to another tap at T' on another wall BT' 6 m long.

(a) Show that the triangles ATX and $BT'X$ in the diagram are similar.

(b) Calculate the total length of new pipe XT' required.



3 The diagram below shows two similar triangles.



(a) Which of the following equations could be used to determine the length of x?

A $\frac{8}{18} = \frac{x}{16}$ **B** $\frac{x}{18} = \frac{8}{16}$ **C** $\frac{x}{18} = \frac{16}{24}$ **D** $\frac{x}{16} = \frac{8}{24}$

(b) The scale factor that produces the larger triangle from the smaller is:

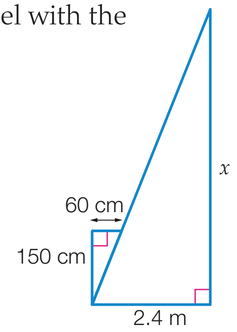
A 0.5 **B** $0.\dot{6}$ **C** 1.5 **D** 2

Understanding

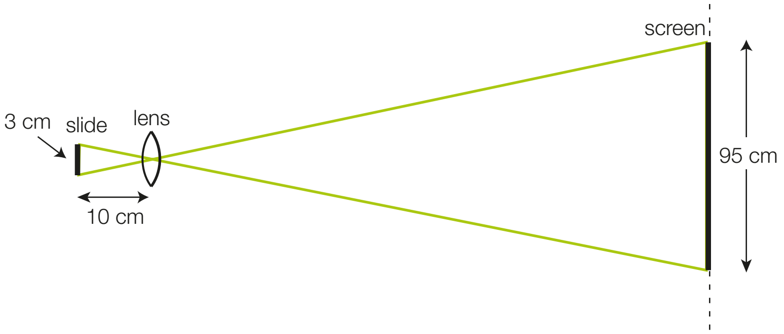
4 Noah is standing on a long ladder leaning against a wall. The base of the ladder is 2.4 m from the base of the wall. His friend Sean is holding the ladder to prevent it wobbling. Sean is standing with his shoulders 150 cm directly above the base of the ladder. His arms, 60 cm in length, are stretched horizontally, parallel with the ground, to hold the ladder.

A diagram of the information is shown.

How far up the wall does the ladder reach?



5 A light source projects the image of a photographic slide onto a screen, as shown in the diagram.

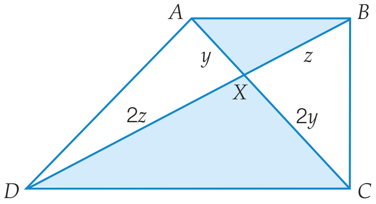


The slide is 3 cm high and positioned 10 cm behind a lens. Calculate the distance between the lens and the screen if the image on the screen is 95 cm high.

6 Martin plays tennis at night under floodlights. When he stands 4.5 m from the base of a floodlight, he casts a shadow 1.4 m long. If Martin is 1.7 m tall, how high is the floodlight?

Reasoning

7 A quadrilateral *ABCD* is drawn as shown with diagonals intersecting at point *X*.



(a) Show that the shaded triangles are similar.

(b) Hence show that *ABCD* is a trapezium.

(c) Will the non-shaded triangles also be similar? Explain why or why not.

8 A teacher was explaining how to use a stick to estimate the height of a tree.

Step 1: Have someone hold the stick vertically on the ground.

Step 2: Lie on the ground in a position so that, when looking at the tree on a line of sight that goes through the stick, the top of the tree and the top of the stick are in line.

Step 3: Unfortunately the teacher had forgotten Step 3.

(a) Draw a diagram of the situation and use your knowledge of similar triangles to complete the instructions for Step 3.

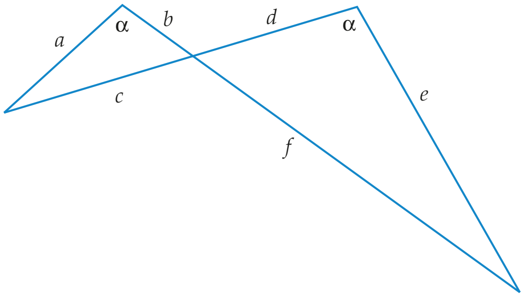
(b) List any practical problems you can see with this method.

Open-ended

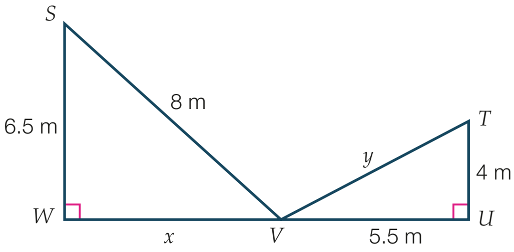
Hint

Be sure that $a + b > c$.

9 State a value for each of the variables in the diagram so that the two triangles are similar.



10 The teacher drew the following diagram on the board and asked the students to calculate the lengths of *x* and *y*. Santino's answers are shown.



Santino's answers:

To calculate x:

$$\frac{x}{5.5} = \frac{6.5}{4}$$

$$x = \frac{6.5}{4} \times 5.5$$

$$x = 8.94 \text{ m}$$

To calculate y:

$$\frac{y}{8} = \frac{4}{6.5}$$

$$y = \frac{4}{6.5} \times 8$$

$$y = 4.92 \text{ m}$$

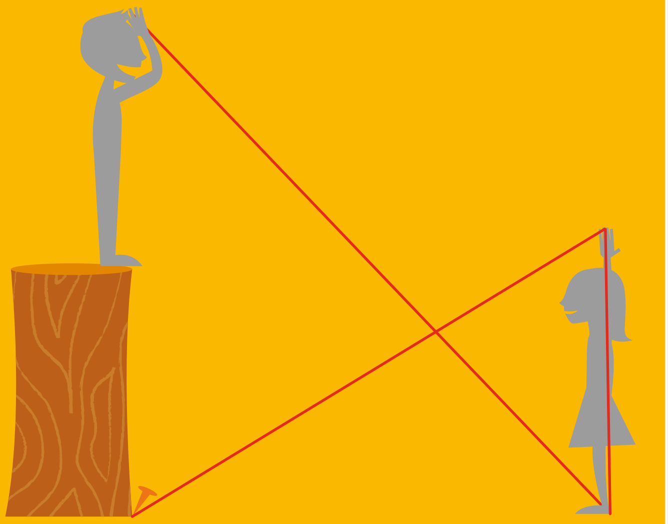
(a) Explain why the values Santino calculated for *x* and *y* are not correct.

(b) What method could Santino use to calculate the values of *x* and *y*?

Problem solving

Curiously crossed

Gina and Gerard are twins who are both exactly 1.5 metres tall. Gina has a very long rope. Gerard attaches one end of it to the bottom of a 1.5-metre tree stump. Gina picks up the rope and holds it taut above her head so it is 2 m above the ground. She lets the rest of the rope fall down behind her back and stands on it. Gerard then picks up the other end of the rope and stands on the tree stump so the rope crosses itself. This end of the rope just reaches the top of his head. How high off the ground do the ropes cross?



Strategy options

- Draw a diagram.
- Test all possible combinations.
- Break problem into manageable parts.
- Look for a pattern.

Investigation

Measuring shadows to find heights

Equipment required: measuring tape (or metre ruler), chalk

Thales of Miletus was an ancient Greek philosopher and mathematician who lived from around 625 to 547 BCE. With his knowledge of similar triangles, he was able to determine the heights of the Egyptian pyramids. He realised that when the length of his shadow was the same as his height, then the same must be true for all objects nearby.

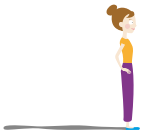
The Big Question

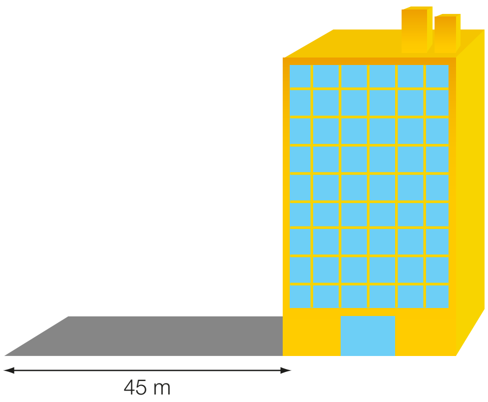
How can knowledge of similar triangle properties be used to measure tall objects?

Engage

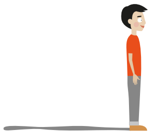
1 Based on Thales' observation, determine the height of:

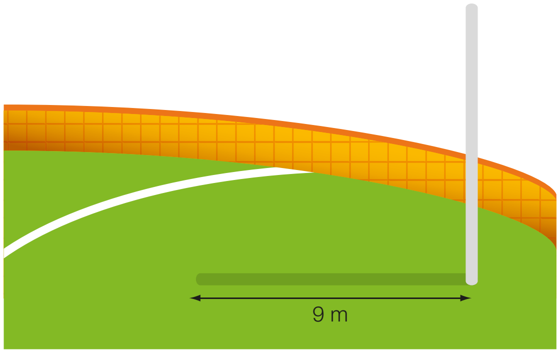
- **(a)** the building when the girl's shadow is the same length as her height





- **(b)** the goal post when the boy's shadow is the same length as his height.





- **(c)** A zookeeper was feeding a giraffe when she noticed that the giraffe's shadow was three times as long as her own shadow. This was observed when the zookeeper's shadow length was equal to her height. If the zookeeper's height is 1.8 m, what is the height of the giraffe?

Thales' observation is useful, but what if you want to know the height of an object when your shadow does not equal your height?

Explore

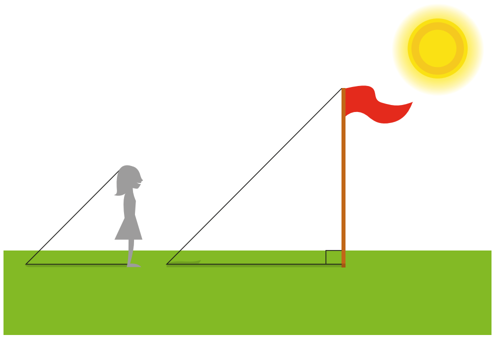
2 Choose an object that is too tall to measure with measuring tape, such as a flagpole, goal post or building.

3 Follow the steps below to calculate its height.

Step 1: Record your height, in metres, to the nearest cm (e.g. 1.54 m).

Step 2: Estimate the height of the tall object, so you can compare your estimate with the height calculated later.

Step 3: Measure the length of your shadow and the length of the shadow of the object of unknown height.



Step 4: Draw a diagram showing the two similar triangles, stating all measurements. Label the unknown height x.

Step 5: Using the knowledge that corresponding sides of similar triangles have the same ratio, calculate the height of the tall object. Record the calculated height.

Strategy options

- Draw a diagram.
- Make a table.

Explain

4 How do you know that the triangle formed by the object and its shadow and the triangle formed by you and your shadow are similar triangles?

Elaborate

5 What was the height of your chosen object? Choose another tall object and find its height using the same method.

Evaluate

6 What problems did you encounter while performing this investigation? How did/could you overcome these?

7 Did you find any shortcuts for calculating the heights of the tall objects you investigated?

8 How accurate do you believe this method is for determining the heights of tall objects? Explain.

9 What changes, if any, would you make to this investigation to make the measurements and findings more accurate?

10 List three things that could not be measured in this way.

Extend

11 Are two people with shadows the same length at the same time of day the same height? Explain.

12 Record your shadow length at hourly intervals.

(a) At what time is the length of your shadow equal to your height?

(b) At what time is the length of your shadow zero?

(c) Is there a relationship between time of day and shadow length?

13 Thales of Miletus is one of the earliest known Greek philosophers and mathematicians. Find out more about him and his discoveries.

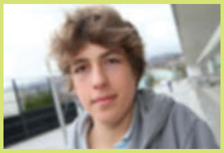
Maths 4 Real

Aspect ratio

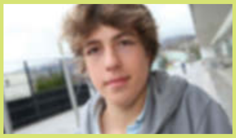
The 'aspect ratio' of an image produced by a camera, computer screen or projector is the ratio of the image's length **L** (horizontally) to its width **W** (vertically).

Traditional cameras and television images have an aspect ratio **L** : **W** of 4 : 3. Other cameras and films also produce images with aspect ratios of 3 : 2 and 16 : 9.

4 : 3 (Digital)



3 : 2 (35 mm film)



16 : 9

If you calculate $\frac{\mathbf{L}}{\mathbf{W}}$, you will get a scale factor that tells you how many times the length is longer than the width. Multiply the width by this scale factor to get the length.

1 (a) Write the ratios of 4 : 3, 3 : 2 and 16 : 9 as scale factors. (Write recurring decimals in exact decimal form.)

(b) (i) What would the aspect ratio and scale factor be for a square image?

(ii) Which of the three aspect ratios given here gives a 'squarer' image?

If you print images on photographic paper, there is a wide variety of photo paper sizes available. Common sizes are shown in the table below. The imperial measurement unit of inches (which has the unit symbol") is used to describe these lengths and widths.

Photo paper sizes

Length (inches)	6	7	8	10	14	15	15	18
Width (inches)	4	5	6	8	11	8	10	12

An image ratio of 3 : 2 is useful because the traditional standard photo size of 6" × 4" is in the same ratio.

2 (a) Which paper sizes in the table give the best 'fit' for an image with an aspect ratio of 3 : 2?

(b) Which paper sizes give the best 'fit' for an image with an aspect ratio of 4 : 3?

Give reasons for your choices.

Problems arise when you want to print an image shot in one aspect ratio onto paper with a different aspect ratio. Imagine that you want to print a 4 : 3 photo onto 6" × 4" photographic paper.

To increase the length of the smaller image from 4 to 6, you need to multiply by 1.5:

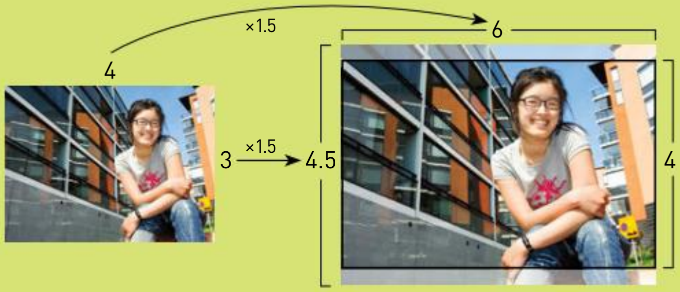
4 × 1.5 = 6

However, multiplying the width of the smaller image by 1.5 does not give the width of the larger one: 3 × 1.5 = 4.5, not 4.

There are two solutions to this problem.

Cropping

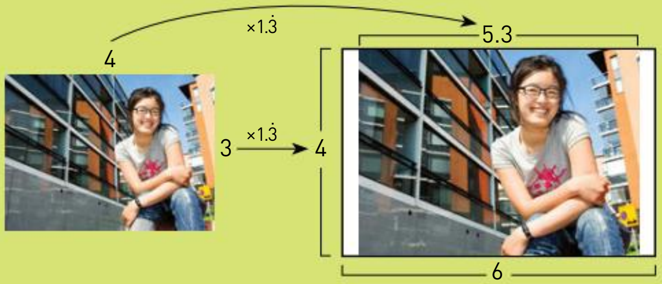
You can 'crop' (cut off) the extra 0.5" of width, meaning that some of the image is actually lost from the top and bottom of the shot.



OR

Enlarging

You can enlarge the image to fill the paper. This means multiplying by $1.\dot{3}$, instead of 1.5. $3 \times 1.\dot{3} = 4$. This leaves the length a little bit short: $4 \times 1.\dot{3} = 5.\dot{3}$, not 6. This results in two 'empty' bars appearing on either side of the image.



3 Imagine that you work in a photographic printing business.

(a) A customer has brought in some images taken in a 3 : 2 format and asked for them to be enlarged onto 15" × 8" paper. The customer would prefer that the photos be cropped rather than have bars appear around the edges. How much of the image will be lost in the cropping process?

(b) Another customer has brought in some images taken in a 4 : 3 format and asked for them to be enlarged onto 18" × 12" paper. She does not want the images cropped.

(i) How wide will the 'empty' bars be on the sides of the image?

(ii) Recommend a very similar photo paper size to the customer that requires no cropping of the image.

How big can I make my photos?

Digital images are composed of 'pixels', tiny squares that are put together like a mosaic to make up the image. A pixel is the smallest element of a digital image.

The resolution of a digital camera is measured in terms of how many pixels it can use per image. One megapixel is equivalent to one million pixels. A camera with more megapixels can create images with a higher resolution, because each image is more detailed.

More resolution also means that bigger enlargements can be made before the image loses detail and starts to look 'pixelated', the effect where individual pixels become visible.

A good resolution for printed photographs is 300 ppi (pixels per inch). This is the number of pixels that fit into a length of one inch of the image.

To find the number of megapixels required for an enlargement:

- multiply the length of the desired enlargement by 300 ppi
- multiply the width of the desired enlargement by 300 ppi
- multiply these two answers together
- divide by 1 million.

4 Ellie wants to buy a new camera. She thinks that the largest print size she will make is 12"× 8".

How many megapixels will she need for her camera to print with a resolution of 200 ppi?

Research

Investigate the history of the different aspect ratios used for film and television. Describe what can be done to show a film shot in one aspect ratio on a screen with a different aspect ratio.

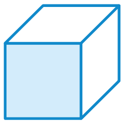
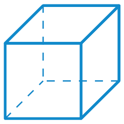
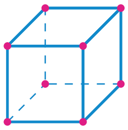
6.7

Solids and nets

A **solid** is a three-dimensional (3D) shape or object. Solids are made up of two-dimensional (2D) shapes called **faces**. Two faces meet at a line called an **edge**. Three or more faces meet at a **vertex**. The plural of vertex is vertices.

A solid where all faces are polygons is known as a **polyhedron**. The plural of polyhedron is polyhedra. All polyhedron edges are straight lines.

A polyhedron with faces that are all identical regular polygons is called a **regular polyhedron**.

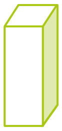
Face	Edge	Vertex
The flat part of a solid.  A cube has 6 faces.	The line segment where two faces meet.  A cube has 12 edges.	A corner where several edges and faces meet.  A cube has 8 vertices.

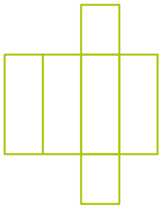
Nets

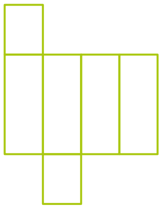
Boxes and containers come in many different shapes and sizes. Boxes and containers are made by drawing an outline on a flat surface and then folding the outline into the three-dimensional solid.

A **net** is the 2D outline of a solid that shows all the faces. A net can be folded to make the 3D solid.

Solids can have more than one net.







Special types of polyhedra

- **Prisms** have a polygon base and a regular cross-section (so that any cut parallel to the base gives an identical shape to the base).
- Pyramids have a polygon base but all other faces are triangles, so the shape tapers to a single vertex at the top.
- Platonic solids have identical regular polygon faces that meet at identical vertices. There are only five Platonic solids.
- Archimedean solids have faces made from two or more regular polygons that meet at identical vertices. There are 13 Archimedean solids.

6.7 Solids and nets

Navigator

1, 2 (a), 3, 4, 5, 6 (a, c), 7, 8, 9, 11, 12, 14

1, 2 (a), 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14

1, 2 (b), 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14

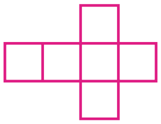
Answers p. 674

Equipment required: scissors, nets, thread, glue and a ruler for Questions **7** and **8**

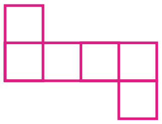
Fluency

1 Which of the following could not be a net for a cube?

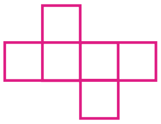
A



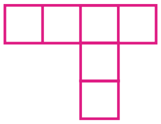
B



C



D



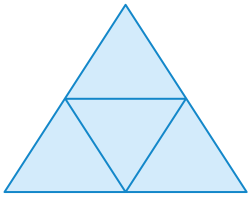
2 Draw a possible net for each of the following solids.

(a) a rectangular prism with dimensions of length 5 cm, width 2 cm and height 3 cm

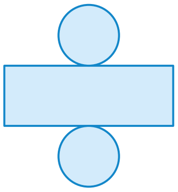
(b) a pentagonal prism of side length 2 cm and height 5 cm

3 Name the solids that the nets below would fold into.

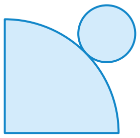
(a)



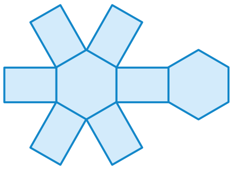
(b)



(c)

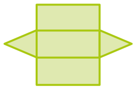


(d)

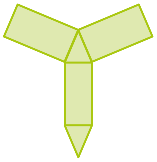


4 Which of the following nets could not be folded to make a triangular prism?

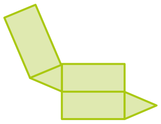
A



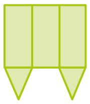
B



C



D



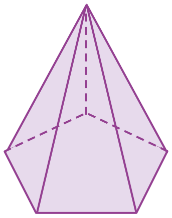
5 Which description matches the shape shown?

A 5 faces, 10 edges, 5 vertices

B 6 faces, 10 edges, 6 vertices

C 10 faces, 10 edges, 6 vertices

D 6 faces, 9 edges, 6 vertices



Understanding

6 Draw the net of the external packaging for the following objects (e.g. no folding flaps).

(a)

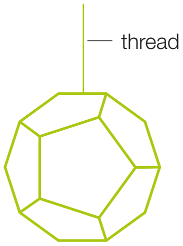
(b)

(c)

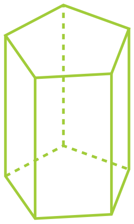
7 Draw a net of a cube with side lengths of 3 cm on cardboard, cut it out and construct a solid cube.

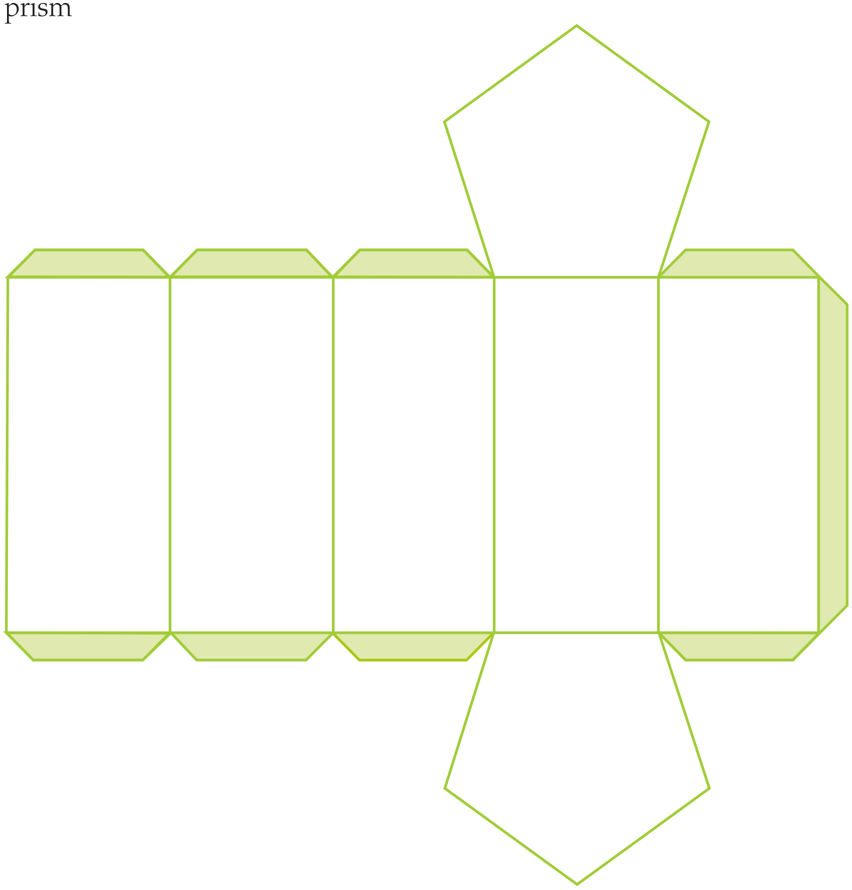
8 Follow these steps to construct solids from the nets shown. (You can download copies of these nets from the eBook or the Pearson Places website.)

- **(i)** Copy and cut out each net, including the tabs, and then crease along the fold lines so that a sharp line is formed.
- **(ii)** Fold the nets so the marked fold lines are on the inside of the model.
- **(iii)** Insert a length of thread in your models before attaching the last tab. The thread may be used for hanging the models in a display later on.

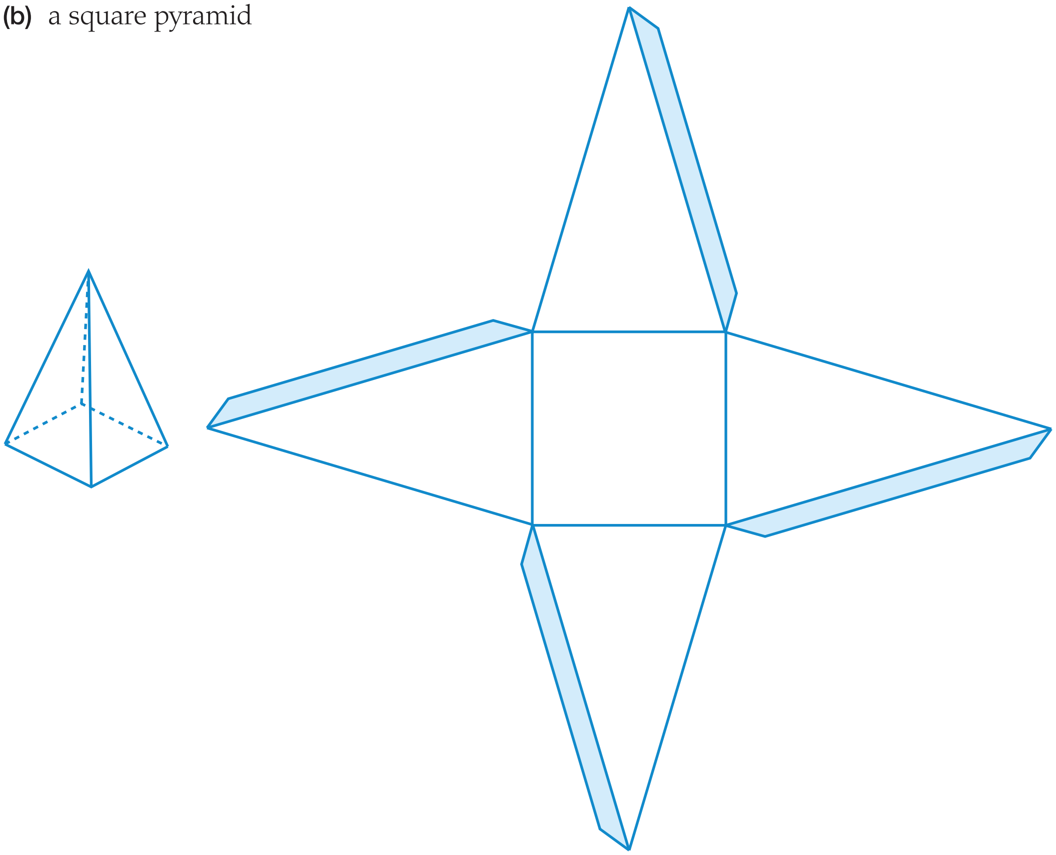


(a) a pentagonal prism



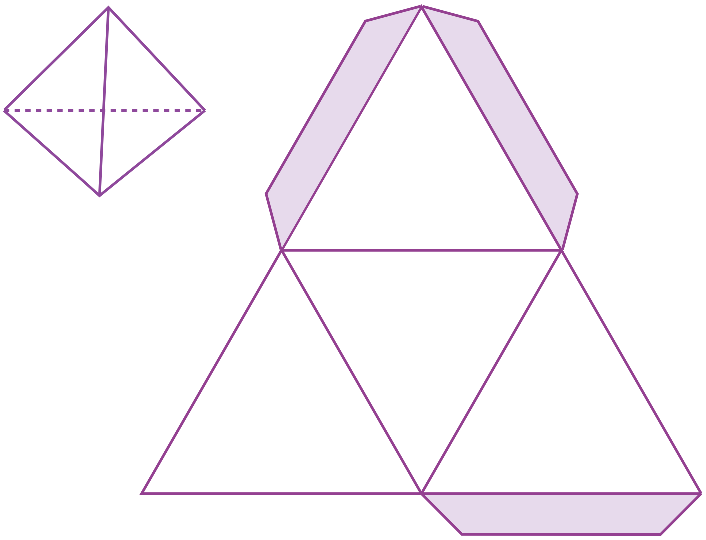


(b) a square pyramid



(c) a tetrahedron:

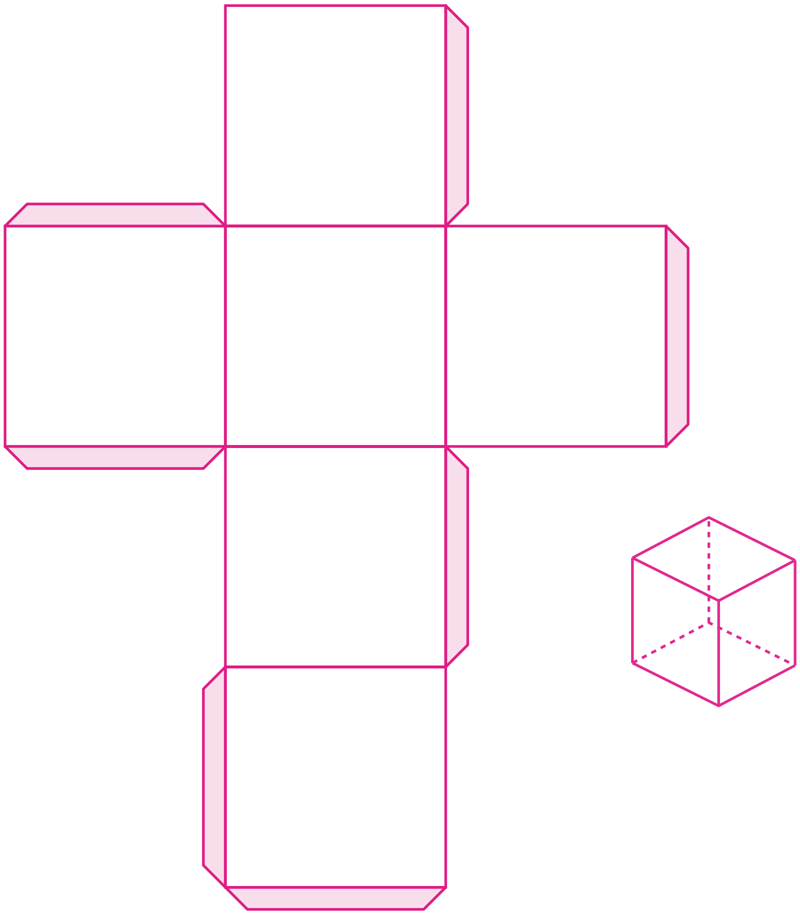
- has four identical triangular faces
- is a rectangular polyhedron, as each face is an equilateral triangle
- is a triangular pyramid.



Note

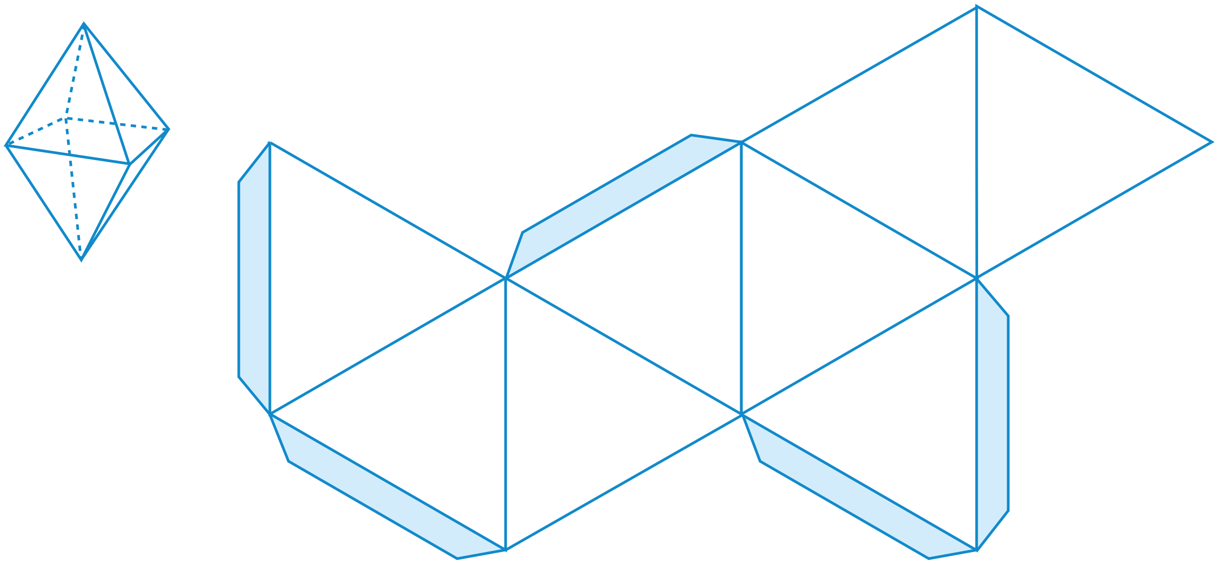
A tetrahedron is very strong. The atoms in diamonds are arranged in a lattice of tetrahedrons. Diamond is one of the hardest substances known.

(d) a cube



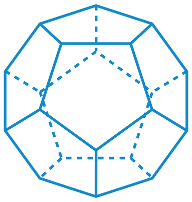
(e) an octahedron:

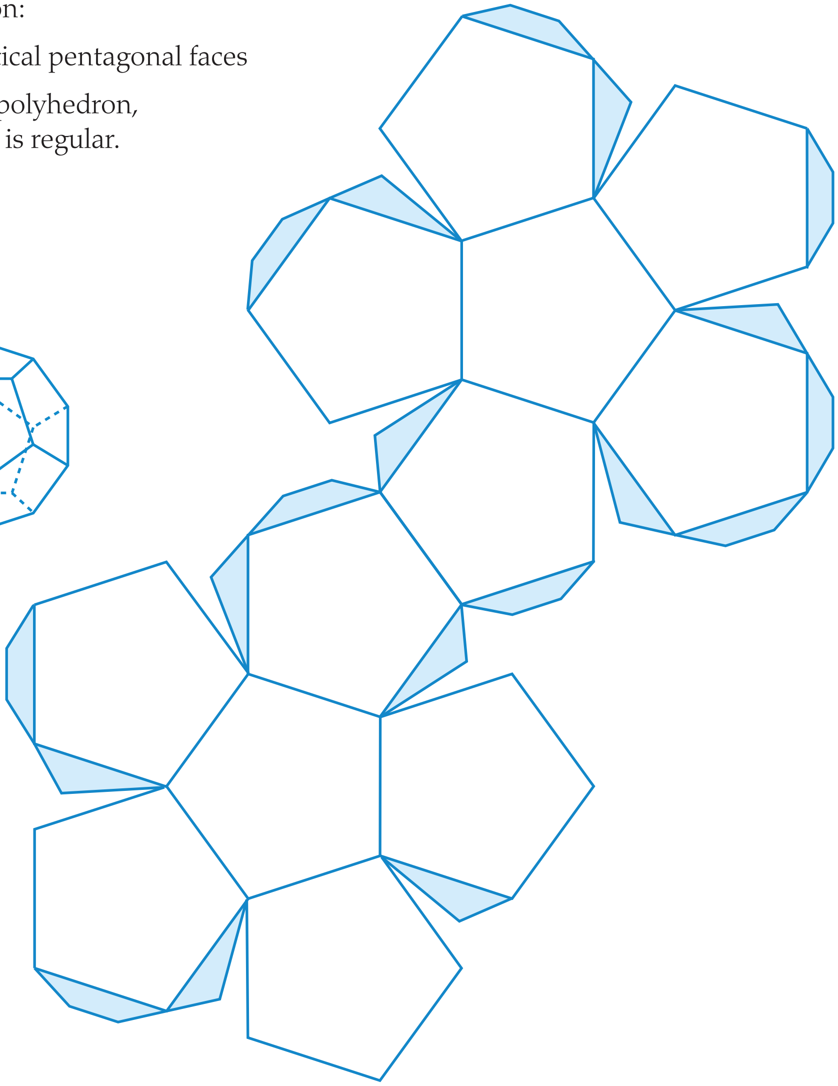
- has eight identical triangular faces
- is a regular polyhedron, as each face is a equilateral triangle
- can be thought of as two square-based pyramids joined together at the bases.



(f) a dodecahedron:

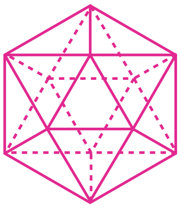
- has 12 identical pentagonal faces
- is a regular polyhedron,
  as each face is regular.





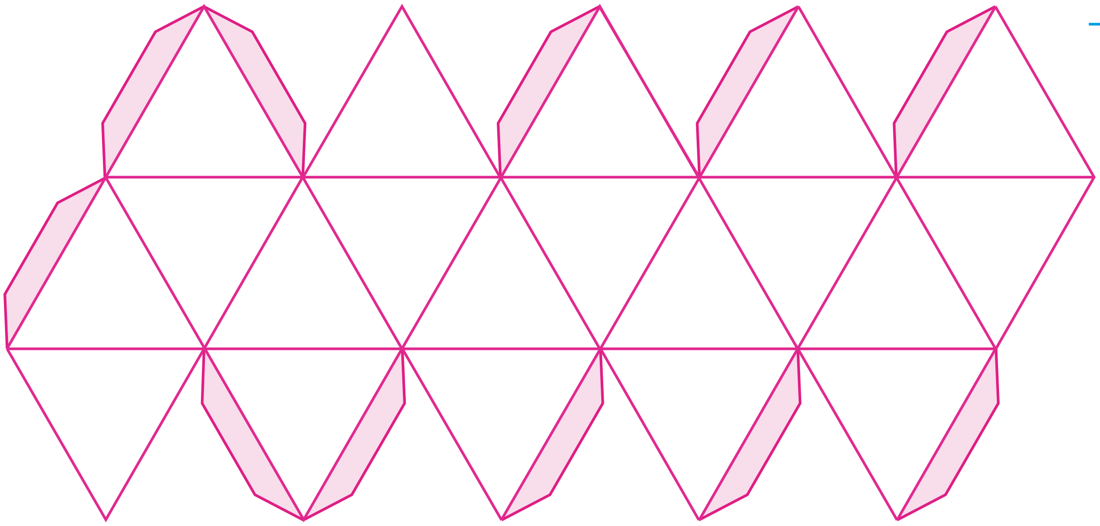
(g) an icosahedron:

- has 20 equilateral triangle faces
- is a regular polyhedron.



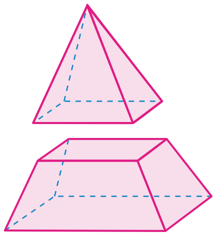
Note

The chicken pox virus has the shape of an icosahedron, a solid with 20 triangular faces.

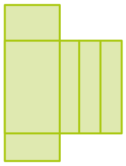


Reasoning

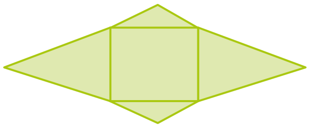
9 A square pyramid has been cut in two, as shown. How many faces, vertices and edges does each part have?



10 Roland drew nets of two solids shown below.



Net of a rectangular prism



Net of a rectangular pyramid

(a) Explain why the two nets above will not work.

(b) Correct Roland's drawings.

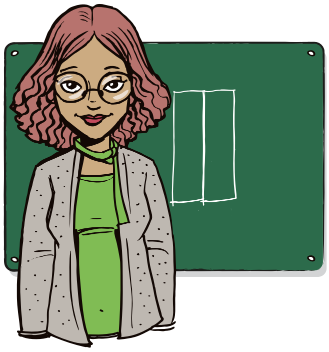
Open-ended

11 Draw a net of a solid that has nine faces.

12 State three solids whose nets have no rectangles.

13 Can a solid have only three faces? Can a polyhedron have only three faces? Explain your answers.

14 Saxon can only see part of the net his teacher is talking about. He can see two quadrilaterals. Draw a possible net and the solid it can fold into.



Puzzle

Matching cubes

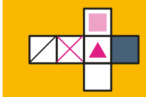
Can you match each cube with its net?

Cube 1

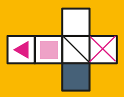
Cube 2

Cube 3

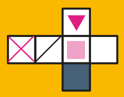
Cube 4



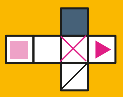
Net 1



Net 2



Net 3



Net 4

Challenge 6

Note: In this section the diagrams are not drawn to scale.

1 In ΔABC, sides AC and BC are equal and side AB is shorter than AC.

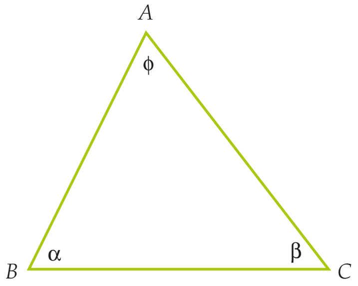
Which statement is true?

A α, β and ϕ are all equal

B $\alpha = \beta$

C $\beta = \phi$

D $\alpha = \phi$



2 How can you use six matchsticks to form four equilateral triangles, each with a side length of one matchstick?

3 In the diagram, JLM and KLN are straight lines. $JK = KL = LJ$.

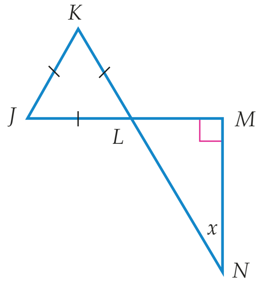
$\angle LMN = 90°$. What is the size of x?

A 15°

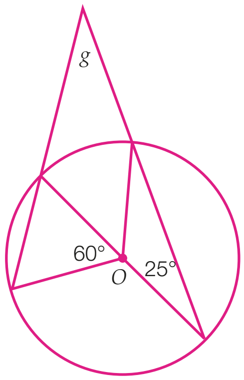
B 30°

C 45°

D 60°



4 Find the value of the angle g in this diagram.



5 In the quadrilateral $PQRS$, $PS = PR = PQ$.

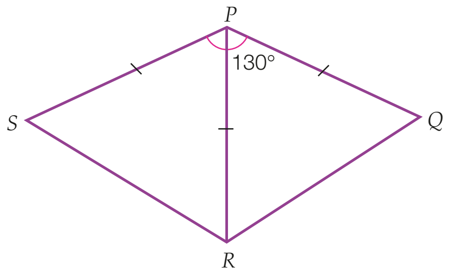
If $\angle SPQ = 130°$, then the size of $\angle SRQ$ is:

A 25°

B 115°

C 125°

D 135°



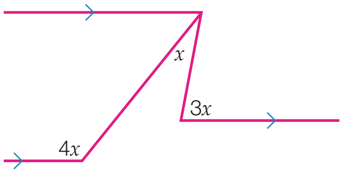
6 $\angle ABC$, $\angle CBD$ and $\angle DBA$ together make up an angle of revolution. If $\angle ABC$ is acute and $\angle CBD$ is obtuse, which of the following are possible for $\angle DBA$? There may be more than one answer.

A acute **B** obtuse **C** reflex **D** straight

7 What is the value of x?

A 22.5 **B** 30

C 36 **D** 45



Chapter review 6

Maths literacy

- alternate angles
- centre of enlargement
- co-interior angles
- congruent
- congruent triangles
- corresponding angles
- edge
- enlargement
- exterior angle
- faces
- interior angle
- net
- polygon
- polyhedron
- prism
- reduction
- regular polyhedron
- scale factor
- similar
- similar triangles
- solid
- vertex

Copy and complete the following using the words and phrases from the list, where appropriate. A word or phrase may be used more than once.

1 When a transversal cuts a pair of parallel lines, the pairs of __________ are equal, the pairs of __________ are equal, and pairs of __________ are supplementary.

2 __________ triangles are exactly the same shape and size.

3 The __________ of a triangle is equal to the sum of the interior opposite angles.

4 Multiplying the sides of a shape by a __________ greater than one will produce an __________ of the shape.

5 Which of the tests for similar triangles states that pairs of corresponding sides must be in the same ratio? __________

6 Which of the tests for similar triangles states that the corresponding hypotenuses and one other pair of sides of right-angled triangles must be in the same ratio? __________

7 Which of the tests for similar triangles states that one pair of corresponding angles must be equal and the pairs of corresponding sides on either side of the angle must be in the same ratio? __________

8 __________ have the same shape, but not necessarily the same size.

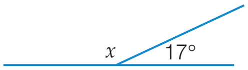
9 A __________ can be folded to make a three-dimensional solid.

Fluency

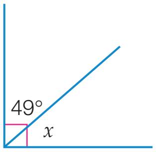
6.1

1 Find the value of x in the following diagrams:

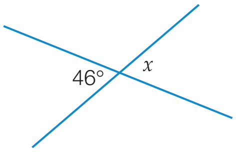
(a)



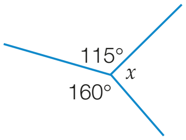
(b)



(c)



(d)



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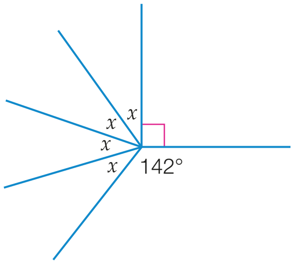

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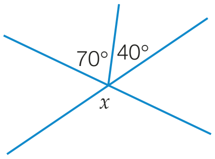
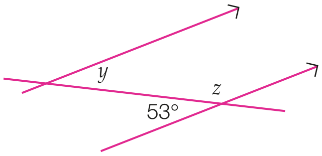


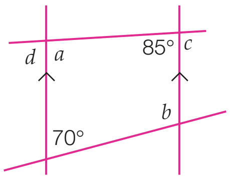
2 Find the value of the variables in the following diagrams:

6.1

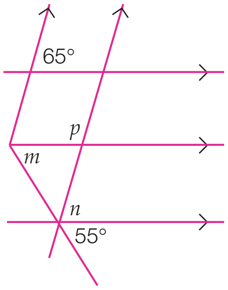
(a)



(b)



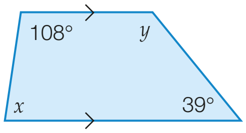
(c)



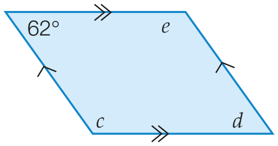
3 Find the value of the variables in the following diagrams:

6.3

(a)



(b)



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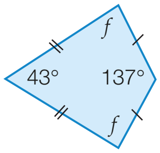


4 Match the name of the shape with its correct description.

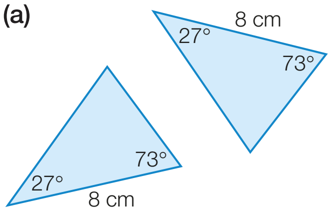
6.3

(a)	adjacent sides of equal length, no parallel sides	**A**	rectangle
(b)	one pair of opposite sides parallel	**B**	kite
(c)	four right angles, opposite sides equal in length	**C**	isosceles triangle
(d)	two sides equal in length	**D**	trapezium

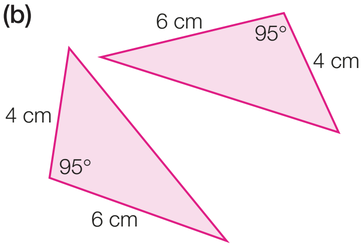
5 Explain why each of the following pairs of triangles are congruent.

6.2

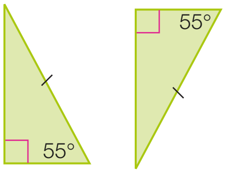
(a)



(b)



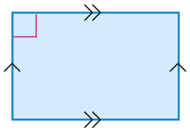
(c)



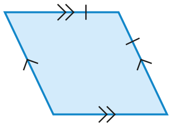
6 Identify each of the following shapes using the name that has all the properties shown.

6.3

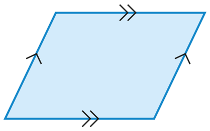
(a)



(b)



(c)

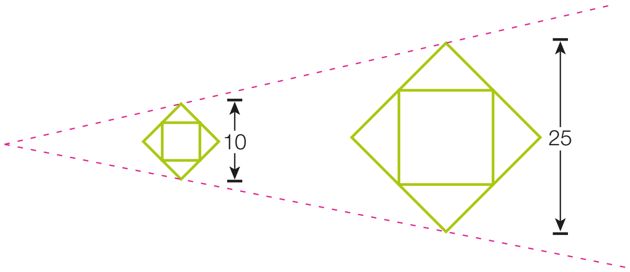


7 In the figure shown, what is the scale factor for:

6.4

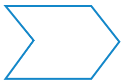
(a) the reduction

(b) the enlargement?



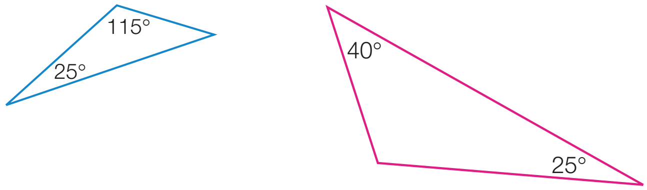
8 Enlarge the following shape by a factor of 2.4.

6.4



9 Show that the following pair of triangles are similar.

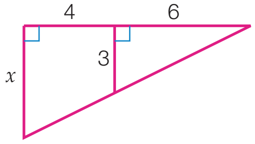
6.5



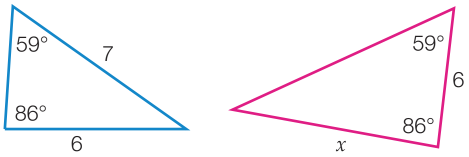
10 Find the value of x in the following diagrams.

6.5

(a)

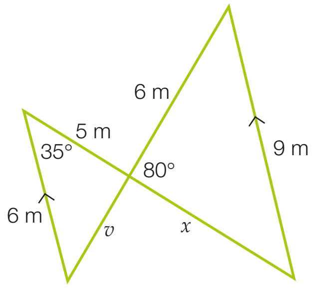


(b)



11 Show that the following pair of triangles are similar, then find the lengths of the unknown sides.

6.6



12 Draw the net for a regular tetrahedron.

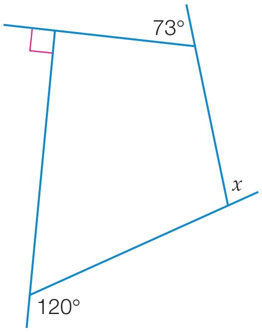
6.7

Understanding

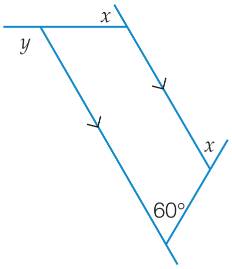
13 Find the value of the variables in each of these quadrilaterals.

6.1

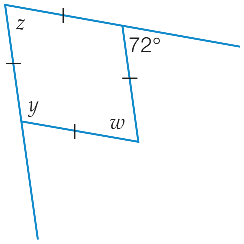
(a)



(b)



(c)



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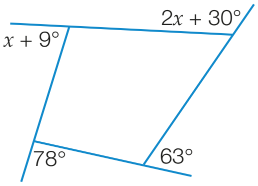
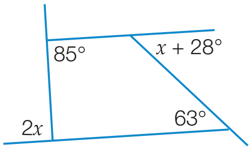
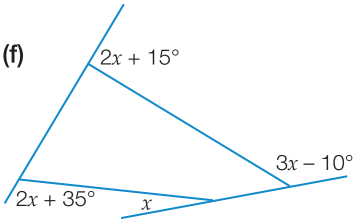


(e)



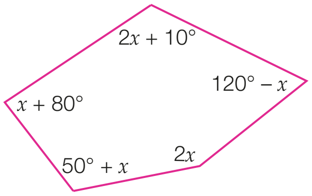
(f)



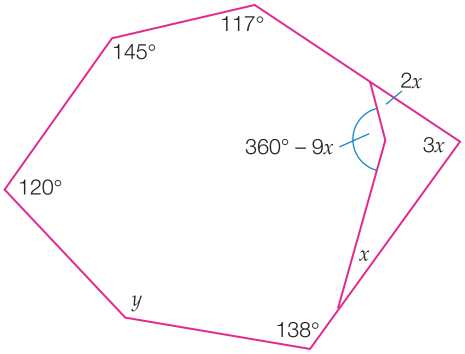
14 Find the values of the variables in each of these shapes.

6.3

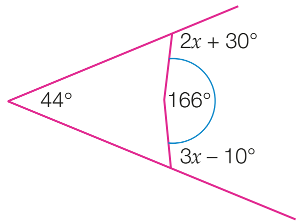
(a)



(b)



(c)



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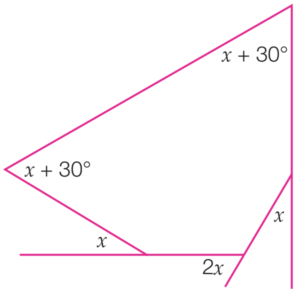

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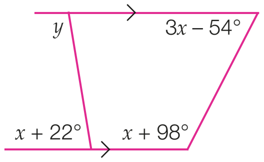
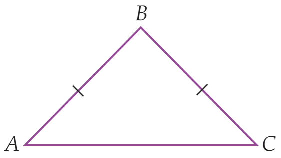


15 In ΔABC, $BA = BC$

6.2

(a) Prove that $\angle BAC = \angle BCA$ by first drawing the line BD perpendicular to AC, meeting it at D.

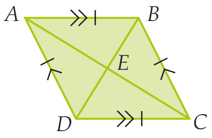
(b) What property of isosceles triangles have you just proved?



16 **(a)** For the diagram shown, prove $AC \perp BD$.

6.2, 6.3

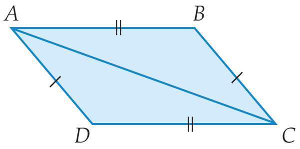
(b) What special property of a rhombus have you just proved?



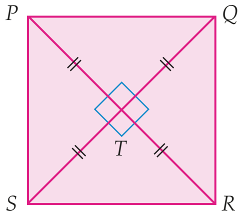
17 Show the following.

6.2, 6.3

(a) If both pairs of opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.

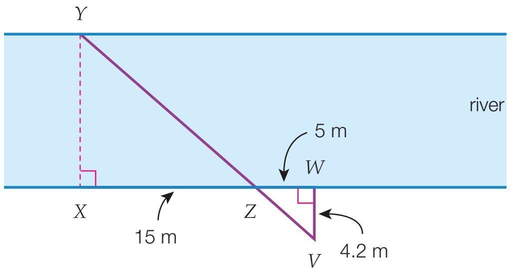


(b) If the diagonals of a quadrilateral are equal and bisect each other at right angles, then the quadrilateral is a square.



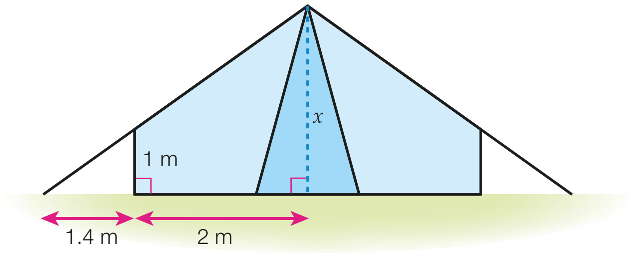
18 Angela is walking through the forest when she comes to a river. She wants to know the distance across the river. She can only measure the distances shown, so she draws the following diagram. Find the distance across the river.

6.6



19 Use your knowledge of similar triangles to calculate the height of the tent shown.

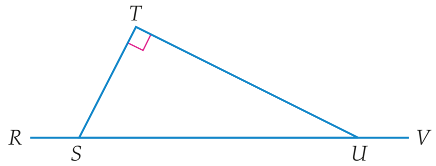
6.6



Reasoning

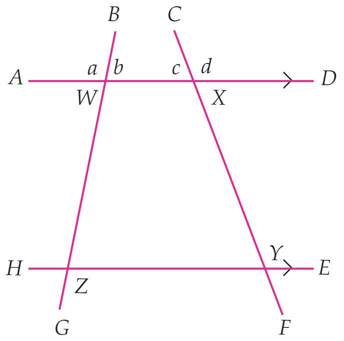
20 Prove that $\angle TSR + \angle TUV = 270°$

6.1, 6.2



21 Use the diagram to prove that the sum of the interior angles of a trapezium is 360°.

6.1, 6.2



22 A postcard measuring 11 cm × 8.5 cm is enlarged so that its new length and width are 120% the size of the original dimensions.

6.4

(a) What is the scale factor of the enlargement?

(b) What are the dimensions of the enlarged postcard?

(c) Find the area of **(i)** the original postcard and **(ii)** the enlarged image.

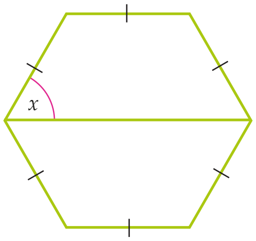
(d) Find the value of $\frac{\text{area of photocopied image}}{\text{area of original postcard}}$. Explain why this ratio is not the same as the scale factor you found in **(a)**.

(e) Photocopiers enlarge images according to area, not length. What percentage enlargement would you need to enter into the photocopier to achieve the same enlarged postcard?

Numeracy practice 6

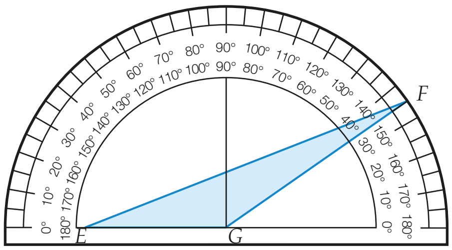
Numeracy: Non-calculator

1 Two isosceles trapeziums fit together to make the regular hexagon shown.



What is the value of x?

2 Δ*EFG* is shaded below. What is the value of ∠*FGE*?



A 35°

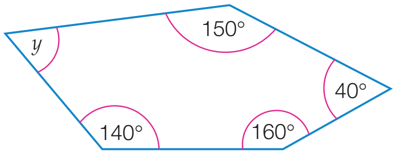
B 45°

C 145°

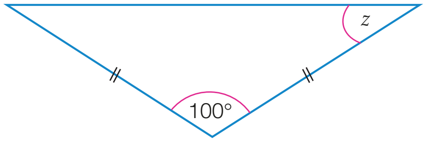
D 155°

Numeracy: Calculator allowed

3 What is the value of *y*?

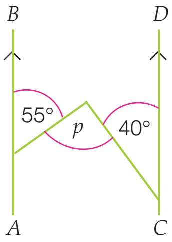


4 What is the value of *z*?



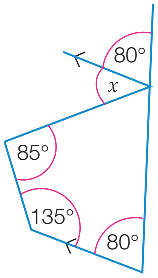
5 In the diagram, *AB* and *CD* are parallel.

What is the value of *p*?



6 In the diagram, a pair of parallel lines are shown.

What is the value of *x*?



7 Liam is 1.5 m tall. His shadow is 2 m long. A tree nearby has an 8 m shadow. How tall is the tree?

A 7.5 m

B 7 m

C 6 m

D 4 m

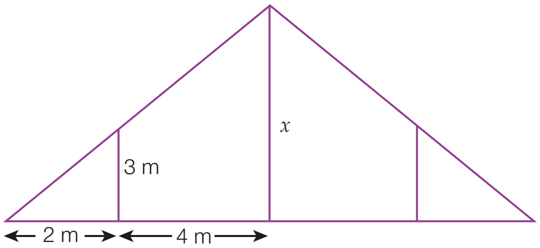
8 The support beams for a triangular barn are being put in place. How long will the support beam in the centre of the barn need to be?

A 6 m

B 9 m

C 12 m

D 15 m



9 A poster measuring 1.5 m × 0.9 m is enlarged by a scale factor of 3. What is the area of the enlarged poster?

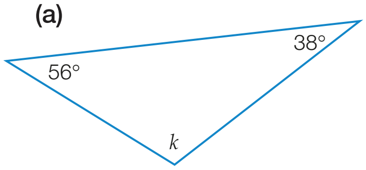
Mixed review C

Fluency

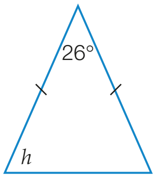
1 Find the perimeter of a semicircle of radius 12 m. Write your answer correct to 2 decimal places. **4.1**

2 Find the value of the variables in each of the following. **6.1**

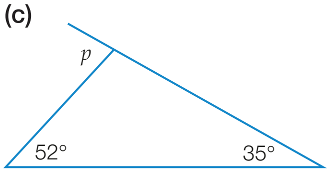
(a)



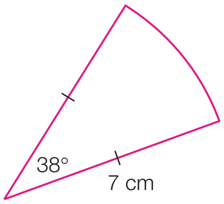
(b)



(c)

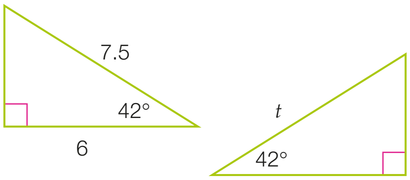


3 Find the area of the sector shown in the diagram. Give your answer correct to 2 decimal places. **4.2**

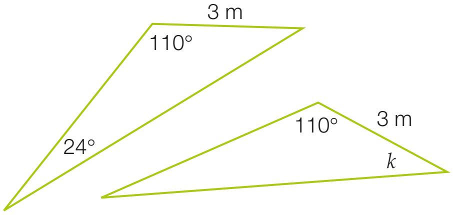


4 Find the values of the variables in the following congruent triangles. **6.2**

(a)



(b)



5 Find the surface area of a cylinder of height 10 cm and a base radius of 4 cm. Give your answer correct to 2 decimal places. **4.3**

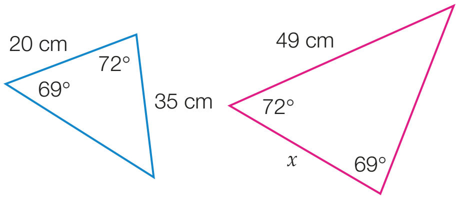
6 Calculate the simple interest for the following cases. **1.5**

(a) $P = \$5000$, $r = 4.7\%$ p.a., $T = 3$ years

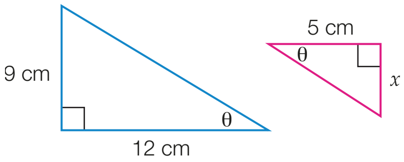
(b) $P = \$690$, $r = 8.7\%$ p.a., $T = 1.5$ years

7 Given that the following pairs of triangles are similar, state each scale factor and find the value each unknown x. (The blue triangles are the original.) **6.5**

(a)



(b)



8 Write the rules for the area of the following: **4.2**

(a) kite

(b) rhombus

(c) trapezium

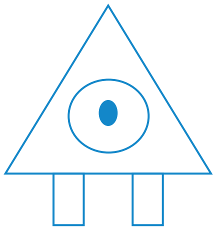
9 Factorise the following expressions: **3.8**

(a) $y^2 + 7y + 3y + 21$ **(b)** $cd + 3c + 4d + 12$ **(c)** $t^2 - 4t - 5t + 20$

10 For the illustration shown: **6.4**

(a) Enlarge by a scale factor of 3.

(b) Reduce by a scale factor of $\frac{1}{2}$.



11 Find the gradient of the line joining each of the following pairs of points. **5.5**

(a) (0, 4) and (5, 14) **(b)** (12, 8) and (5, 15) **(c)** (-8, -11) and (6, -4)

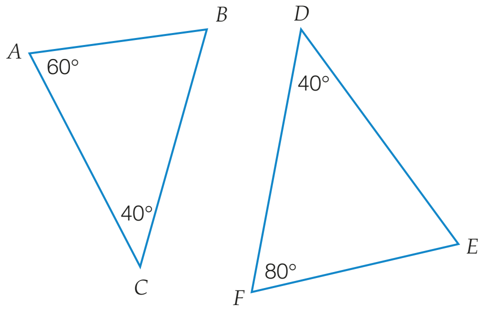
12 Solve the following equations: **5.1**

(a) $\frac{3x}{5} - 7 = 4$ **(b)** $\frac{4x + 7}{4} = 5$ **(c)** $\frac{2(3x + 8)}{5} = -2$

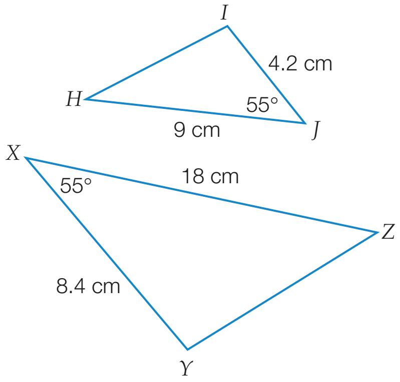
Understanding

13 Show that the following pairs of triangles are similar. **6.5**

(a)

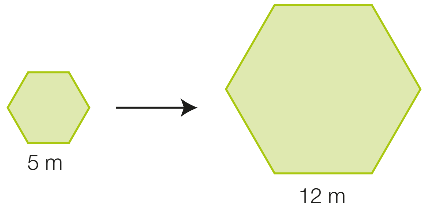


(b)

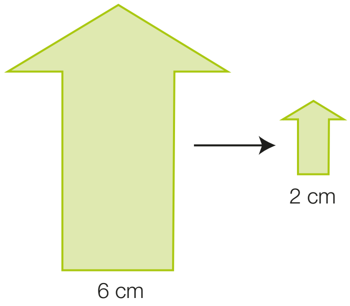


14 What scale factors have been used in the following? **6.4**

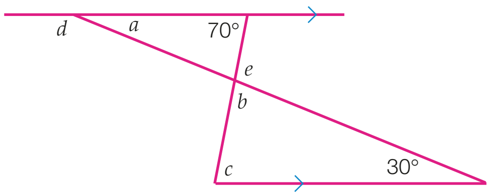
(a)



(b)



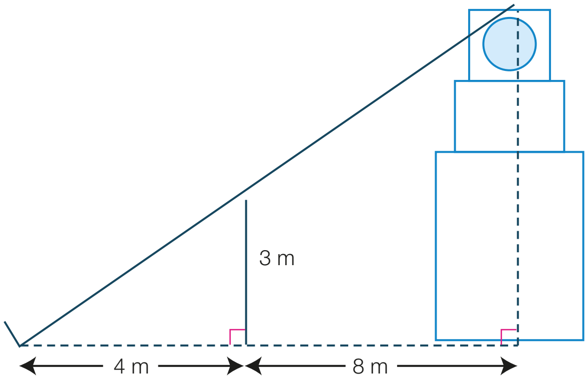
15 Find the values of the unknown angles in the diagram below. **6.1**



Reasoning

16 The clock on a town hall is being repaired. A safety wire has been attached to the top of the town hall, as shown. How tall is the town hall?

6.6, 6.7



17 **(a)** Expand the brackets in the expression $x(x + 4) - 21$.

6.5, 6.7

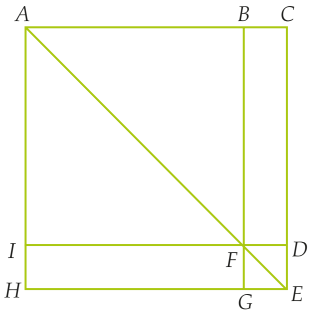
(b) Factorise this new expression as the product of two brackets.

(c) What value of x in this new expression gives 110×100?

(d) Use the last answer to show that $103 \times 107 - 21 = 110 \times 100$.

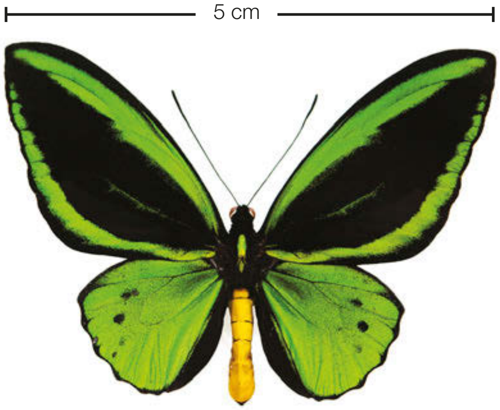
18 The diagram below has three different sized triangles. Name one example of each triangle and show that all three are similar.

6.2, 6.5



19 What scale factor is needed to enlarge this image of a butterfly to a width of 27 cm?

6.4



20 A standard size photo is 10 cm × 15 cm.

6.4

(a) If the photo is enlarged to 30 cm × 45 cm, what is the scale factor of the enlargement?

(b) Calculate the area of **(i)** the standard size photo and **(ii)** the enlargement.

(c) Calculate the value of $\frac{\text{area of enlargement}}{\text{area of standard size}}$.

(d) Compare your answers to **(a)** and **(c)**. What do you notice?

Exploration STEM

You can download this activity from the eBook or the Pearson Places website.

What colours do you see?

What colour is that dress? Some said it was blue and black, some said it was white and gold, but how is it possible to confuse this? Colours can appear different depending on the brightness of light and other colours around them.

Your task is to test your colour vision in different ways and to learn as much as possible about the phenomenon of colour perception.

Exploration STEM

You can download this activity from the eBook or the Pearson Places website.

How many fish in the sea?

To conserve natural resources such as fish and wildlife, you need to know and understand the way that populations change in the wild. How can you estimate how many fish (or other species) there are? Typically you must survey a sample of an area, then estimate the totals from this. What is the best way to do this?

Your task is to explore a simulated method for estimating populations in the wild.

Exploration Coding

You can access this activity from the eBook or the Pearson Places website.

What might I earn?

Explore a program that can help analyse the average weekly earnings of teenagers.

7

Trigonometry

7

Operating with triangles? Advances in digital imagery use mathematics to help save patients' lives.

A CAT or CT (Computed Tomography) scan uses a combination of X-rays and computer technology to produce an internal 'slice' image of the body, showing bones, muscles, blood vessels and internal organs. Trigonometry can be used to analyse images and locate the exact position of tumours, infections or other abnormalities. Doctors are able to not only know what is wrong before operating on a patient, but they can also calculate the exact position where they must operate.

Five hundred years ago, Leonardo da Vinci was dissecting dead bodies to view inside the human body. Now you need only push a button.

Forum

Explain why CT scans are said to have revolutionised the medical world. Why is accuracy important when calculating distances inside the human body? A CT scan can be used to find the location of an abnormality inside the body. What other information should a doctor know about an abnormality before operating?

Why learn this?

Trigonometry is the study of the relationships between the angles and sides of triangles and is essential in many fields. Trigonometry enables sailors to use the stars to navigate through open seas. It allows engineers and architects to calculate lengths and angles in their designs for buildings and structures. Surveyors and cartographers use trigonometry to calculate distances between landmarks and create accurate maps; oceanographers use trigonometry to study the movement of waves and tides; and optometrists use trigonometry to help them determine which lenses will help correct someone's vision.

After completing this chapter you will be able to:

- identify and find values for the three trigonometric ratios, given a reference angle
- use trigonometry to find side lengths in right-angled triangles
- use trigonometry to find angles in right-angled triangles
- use trigonometry to solve problems involving angles of elevation and depression
- use trigonometry to solve problems involving bearings.

Recall 7

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

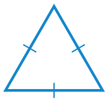
1 Solve the following equations. Where necessary, write your answer as a decimal correct to 4 decimal places.

(a) $2x + 16 = 41$ **(b)** $38 - x = 25$ **(c)** $x^2 + 10 = 74$

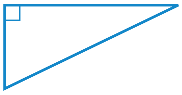
(d) $2x^2 = 98$ **(e)** $\frac{x}{15} = 0.356$ **(f)** $\frac{30}{x} = 0.4783$

2 State whether the following triangles are **(i)** equilateral, scalene or isosceles and **(ii)** acute-angled, obtuse-angled or right-angled.

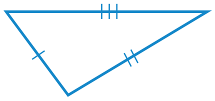
(a)



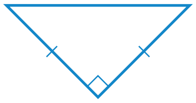
(b)



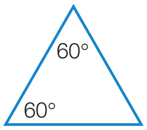
(c)



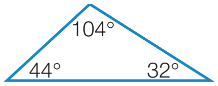
(d)



(e)



(f)



3 **(a)** For the formula $F = ma$, find F if $m = 6$ and $a = 7$.

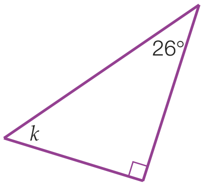
(b) Find S if $a = 21$, $b = 24$ and $c = 35$ in the formula $S = \frac{a + b + c}{2}$.

(c) For the formula $E = \frac{1}{2}mv^2$, find E if $m = 3$ and $v = 6$.

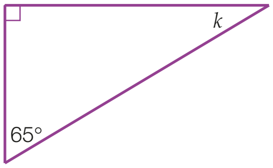
(d) For the formula $E = mx^2$, find m if $E = 100$ and $x = 5$.

4 Find the value of k in each of these triangles.

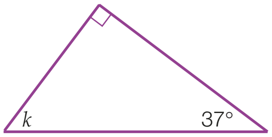
(a)



(b)



(c)



5 Simplify the following ratios and then express the answer as a fraction.

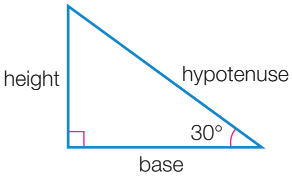
(a) $8 : 10$ **(b)** $36 : 24$ **(c)** $\frac{5}{6} : \frac{2}{3}$ **(d)** $3\frac{1}{3} : 2\frac{1}{2}$

(e) 54 cm : 72 cm **(f)** 3.2 m : 400 cm

Exploration Task

What's so special about right-angled triangles?

In this activity, you will explore the relationship between the ratio of the height and hypotenuse of a right angle and its internal angles.



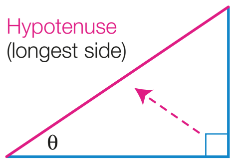
Introduction to trigonometry

7.1

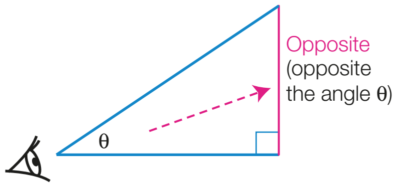
Trigonometry is the study of the relationship between the side lengths and angles in right-angled triangles. There are three ratios that compare the different lengths of the sides of a right-angled triangle. These ratios are called **trigonometric ratios**.

To define these trigonometric ratios you need to name the sides of the right-angled triangle. When naming the sides, you need to specify which of the other two angles you are referring to, because that will change which ratio is involved. In the diagrams below, the Greek letter theta (symbol θ), is used to identify the **reference angle**. Other commonly used angle symbols are α, β and ϕ (alpha, beta and phi). Angle sizes are commonly measured using **degrees**.

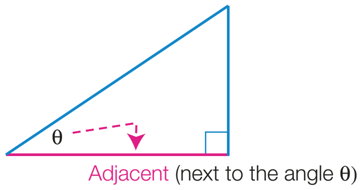
- The **hypotenuse** is always the longest side. It is always opposite the right angle.



- The **opposite side** is the side opposite the reference angle. To help you identify it, imagine you are standing looking at the angle θ. From this view, the opposite side is behind θ.



- The **adjacent side** is the side next to the reference angle. It is between the right angle and θ.



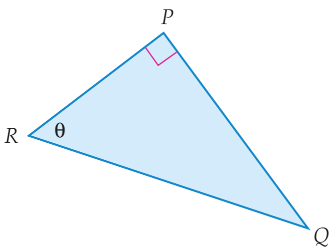
The side names can be abbreviated to H, O and A. The opposite and adjacent sides of a triangle depend on which angle is the reference angle.

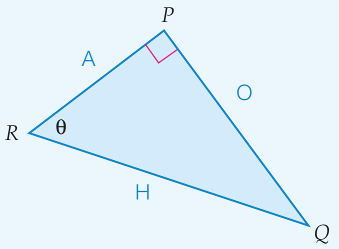
Each of the three trigonometric ratios uses two of the three sides of the triangle.

Worked example 1

W.E. 1

In the following triangle, label the hypotenuse (H), the opposite (O) and the adjacent (A) sides.



Thinking	Working
1 The hypotenuse is opposite the right angle. (*RQ* is the hypotenuse.) **2** The adjacent side is next to the reference angle θ. (*PR* is the adjacent side.) **3** Find the side opposite to the reference angle. (*PQ* is the opposite side.) Label the triangle accordingly.	

Worked example 2

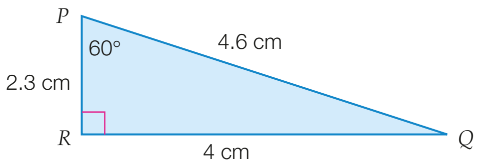
W.E. 2

For the triangle pictured, with a reference angle of 60˚, what is the length of:

(a) the hypotenuse

(b) the opposite side

(c) the adjacent side?



Thinking	Working
(a) Identify the longest side. This is the hypotenuse.	(a) Length of hypotenuse = 4.6 cm.
(b) Identify the side opposite the reference angle. This is the opposite side.	(b) Length of opposite side = 4 cm.
(c) Identify the side next to the reference angle. This is the adjacent side.	(c) Length of adjacent side = 2.3 cm.

7.1 Introduction to trigonometry

Navigator

Answers p. 676

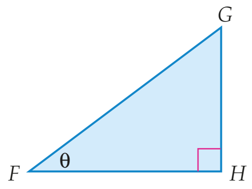
1 (a, c, e), 2 (a–c), 3, 4, 5 (a, b), 6, 7, 8, 9, 11, 12, 13	1 (b, d, f), 2 (d–f), 3, 4, 5, 6, 7, 8, 10, 11, 12, 13	1 (c, f), 2 (c, f), 3, 4, 5, 6, 10, 11, 12, 13

Fluency

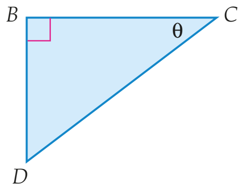
W.E. 1

1 Copy each of the following triangles and use the reference angle to identify the hypotenuse, the opposite side and the adjacent side. Label these sides with H, O and A, respectively.

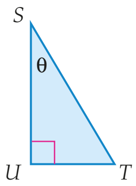
(a)



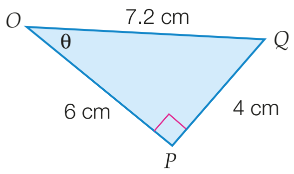
(b)



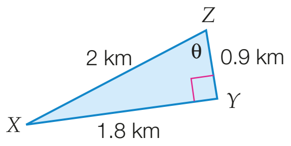
(c)



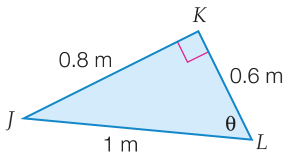
(d)



(e)



(f)

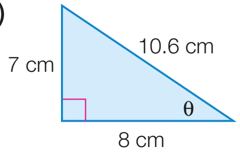


W.E. 2

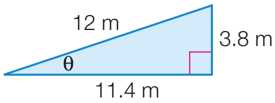
2 For each of the following triangles, state the length of the:

(i) hypotenuse **(ii)** opposite side **(iii)** adjacent side.

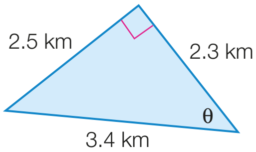
(a)



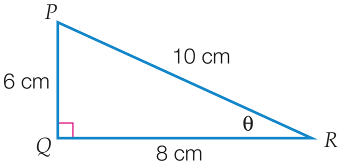
(b)



(c)

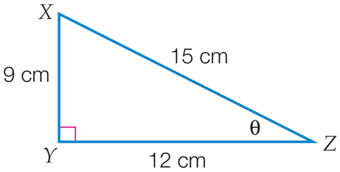


3 In this triangle, determine which of the following represents the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$ for the angle θ:



A $\frac{6}{10}$ **B** $\frac{6}{8}$ **C** $\frac{8}{10}$ **D** $\frac{8}{6}$

4 In this triangle determine which of the following represents the ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ for the angle θ:

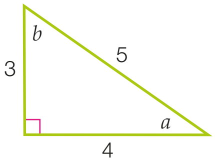


A $\frac{9}{15}$ **B** $\frac{9}{12}$ **C** $\frac{12}{15}$ **D** $\frac{12}{9}$

5 **(a)** Which of the following statements is incorrect?

- **A** The hypotenuse is the longest side in a right-angled triangle.
- **B** The hypotenuse is opposite the right angle.
- **C** The shorter side next to the reference angle is called the adjacent side.
- **D** A right-angled triangle always has two sides equal.

(b) From this triangle determine which of the following represents the ratio $\frac{\text{opposite}}{\text{adjacent}}$ for the angle a:



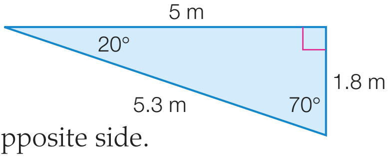
A $\frac{3}{5}$ **B** $\frac{3}{4}$ **C** $\frac{4}{5}$ **D** $\frac{4}{3}$

(c) Using the diagram from part **(b)**, what would be the ratio $\frac{\text{opposite}}{\text{adjacent}}$ for angle b?

A $\frac{3}{5}$ **B** $\frac{4}{5}$ **C** $\frac{4}{3}$ **D** $\frac{3}{4}$

Understanding

6 Consider the following triangle:

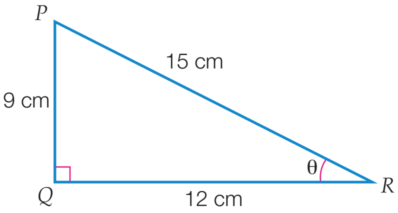


(a) State the length of the hypotenuse.

(b) If 20° is the reference angle, state the length of the opposite side.

(c) If the adjacent side is 1.8 m long, what is the size of the reference angle?

7 Consider the following triangle.



(a) Which side is the opposite side in the triangle?

A QR **B** PQ **C** PR

(b) What is the length of the adjacent side?

A 12 **B** 15 **C** 9

(c) State the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$.

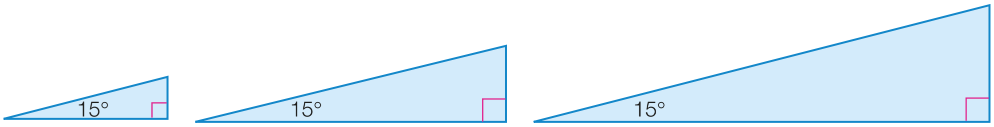
A $\frac{9}{12}$ **B** $\frac{12}{15}$ **C** $\frac{9}{15}$

8 A right-angled triangle has three sides of 5, 12 and 13.

(a) Which of these sides would be the hypotenuse?

(b) Draw a diagram of the triangle with the vertices labelled *A*, *B* and *C*, with the right angle at *B*. Mark the lengths on the sides.

9 (a) In the following right-angled triangles, measure the length of each side.



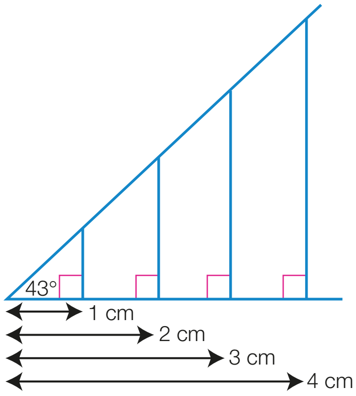
(b) For each triangle in **(a)** find the value of $\frac{\text{opposite}}{\text{hypotenuse}}$ and $\frac{\text{adjacent}}{\text{hypotenuse}}$.

Reasoning

10 (a) Use a protractor to draw an angle of 43°. Mark distances of 1 cm, 2 cm, 3 cm and 4 cm and rule a perpendicular line at each point, like in the diagram shown.

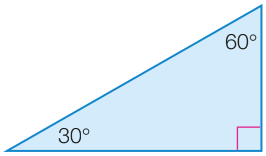
(b) Measure the heights of the four triangles created and, using 43° as the reference angle, calculate the ratio $\frac{\text{opposite}}{\text{adjacent}}$ in each triangle. Write your ratios in decimal form, rounded to 1 decimal place.

(c) The four triangles that you have drawn are similar triangles because they all have equal angles (the AAA similarity test). How does this help to explain your answer to part **(b)**?



11 (a) Use a ruler and protractor to accurately draw three differently sized triangles with angles of 30°, 60° and 90°. One triangle is shown here.

(b) Use your triangles from part **(a)** to copy and complete the table below. Write measurements to the nearest mm.



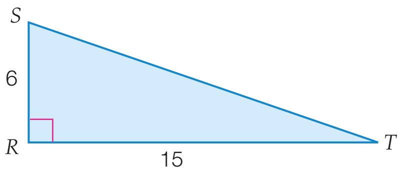
Triangle	Reference angle	Hypotenuse length	Opposite side length	Adjacent side length	$\frac{\text{Opposite}}{\text{Adjacent}}$	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$
1	30°						
2	30°						
3	30°						
1	60°						
2	60°						
3	60°						

(c) What do you notice about the values calculated in the last three columns?

(d) Because the three triangles drawn in part **(a)** are of different sizes but have the same angles, they are similar triangles. What do you know about the side lengths in similar triangles?

12 A teacher asked students to complete the triangle diagram shown by labelling the hypotenuse, opposite and adjacent sides.

Nash said, ‘The opposite side is 15’. Aisha said, ‘The opposite side is 6’. They are both right.

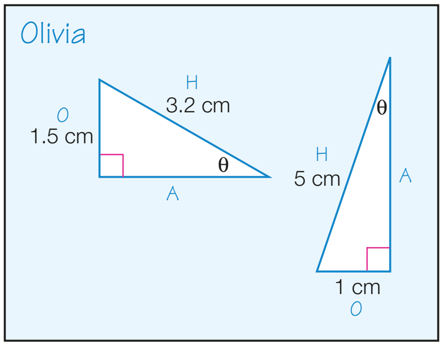


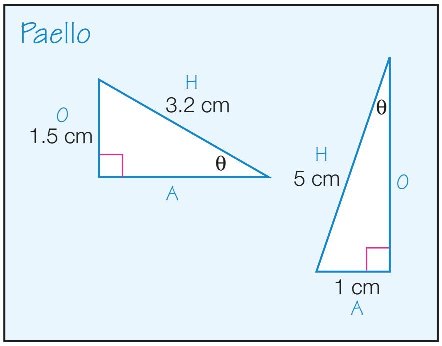
(a) What piece of information did the teacher forget to add to the triangle that makes it possible for both students to be correct?

(b) Draw the completed triangles of Nash and Aisha.

Open-ended

13 A class is given two right-angled triangles and asked to label the hypotenuse, opposite and adjacent sides, relative to the reference angle θ. The answers of two students, Olivia and Paello, are shown below:





Paello says to Olivia: ‘Your answer is different to mine. I think it’s wrong, because the adjacent side is always the one on the bottom of a right-angled triangle’.

Write what you would say to Paello to explain and correct his misunderstanding.

Puzzle

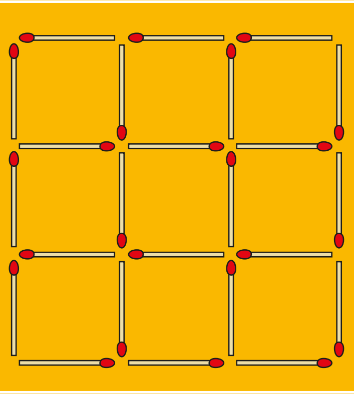
Matchstick madness

Consider the arrangement of matchsticks shown.

(a) Which four matches can you remove to leave five identical squares?

(b) Which eight matches can you remove to leave four identical squares?

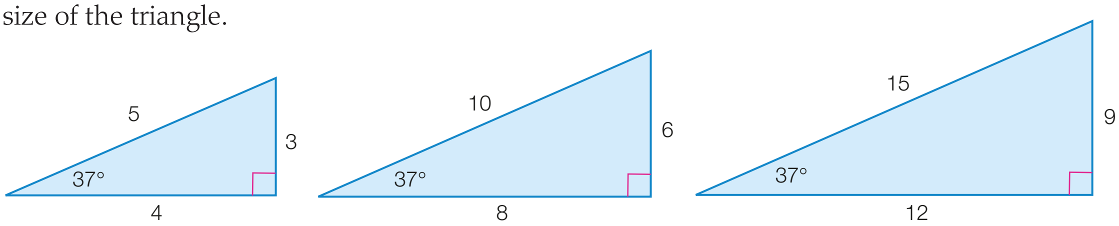
(c) Which six matches can you remove to leave three different squares?



7.2

Trigonometric ratios

All triangles with three angles the same are similar. One of the properties of similar triangles is that the ratios between corresponding side lengths are constant, regardless of the size of the triangle.



The ratio $\frac{\text{opposite}}{\text{adjacent}}$ is the same for each of the above triangles: $\frac{3}{4} = \frac{6}{8} = \frac{9}{12}$.

The same can be said for the ratios $\frac{\text{opposite}}{\text{hypotenuse}}$ and $\frac{\text{adjacent}}{\text{hypotenuse}}$. These three trigonometric ratios are called **sine** (sin), **cosine** (cos) and **tangent** (tan). Each ratio uses two of the three sides of a right-angled triangle. For a specific reference angle, the value of a particular ratio is always the same, regardless of the size of the triangle. For example:

The sine of 37° is 0.60 (correct to 2 decimal places).

The cosine of 37° is 0.80 (correct to 2 decimal places).

The tangent of 37° is 0.75 (correct to 2 decimal places).

The sine of θ is given by:	The cosine of θ is given by:	The tangent of θ is given by:
$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$	$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$

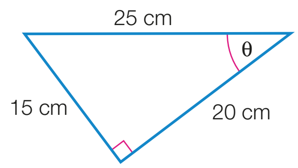
A common acronym to help with remembering the trigonometric ratios is:

SOH CAH TOA

Worked example 3

W.E. 3

Find the exact value of tan (θ).

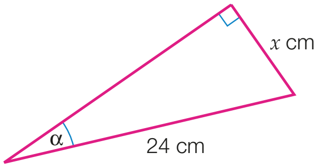


Thinking	Working
1 Write the rule for the tangent ratio.	$\tan(\theta) = \frac{O}{A}$
2 Identify the opposite and adjacent sides and substitute the lengths.	$\tan(\theta) = \frac{15}{20}$
3 Simplify if possible.	$= \frac{3}{4}$

Worked example 4

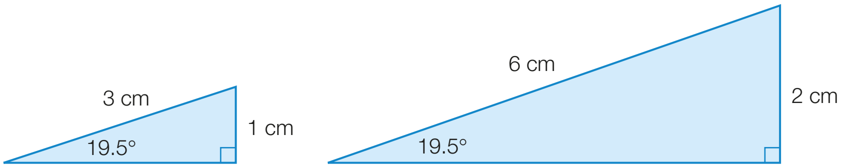
W.E. 4

If $\sin(\alpha) = \frac{1}{3}$, use the sine ratio to find the value of x in the following triangle.



Thinking	Working
1 Write the rule for the sine ratio.	$\sin(\alpha) = \frac{O}{H}$
2 Identify the hypotenuse and the opposite side and substitute the known values.	$\frac{1}{3} = \frac{x}{24}$
3 Solve the equation for x.	$\frac{1}{3} \times 24 = x$ $x = 8$ cm

The trigonometric ratio for a particular reference angle can be found by drawing the triangle and measuring the required side lengths. The right-angled triangle below shows that the sine ratio for 19.5° is approximately $\frac{1}{3}$.



This means that for right-angled triangles with a reference angle of 19.5°, the opposite side length will always be very close to one-third the length of the hypotenuse.

Warning

This measuring technique shows how the trigonometric ratios are found, but it requires an accurately drawn scale diagram. In the next section you will learn how to find these values using your calculator.

You should never assume that a diagram in a textbook or on a test is drawn to scale.

Worked example 5

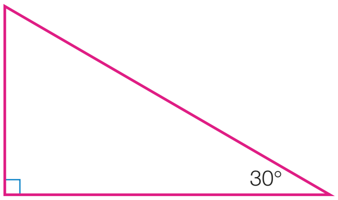
W.E. 5

Measure the side lengths of this triangle to the nearest millimetre and hence calculate the value (to 2 decimal places) for:

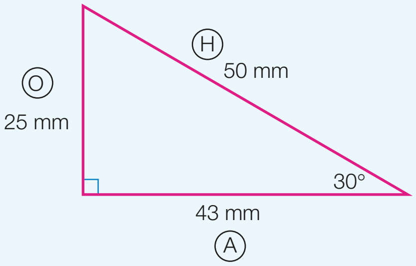
(a) sin (30°)

(b) cos (30°)

(c) tan (30°)



Thinking	Working
1 Measure the sides, putting measurements on the triangle.	
2 Label the hypotenuse (H), opposite (O) and adjacent (A).	



(a) Use SOH to calculate sin (30°).

(a) $\sin(30°) = \frac{O}{H}$

$= \frac{25}{50}$

$= \frac{1}{2}$

$= 0.50$

(b) Use CAH to calculate cos (30°).

(b) $\cos(30°) = \frac{A}{H}$

$= \frac{43}{50}$

$= 0.86$

(c) Use TOA to calculate tan (30°).

(c) $\tan(30°) = \frac{O}{A}$

$= \frac{25}{43}$

$= 0.58$

7.2 Trigonometric ratios

Navigator

Answers p. 677

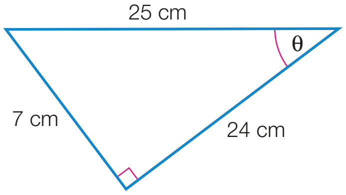
1 (a–c)(i), 2 (a–c)(i), 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 15, 16, 17, 20	1 (a–c)(ii), 2 (a–c)(ii), 3, 4, 5, 7, 9, 10, 11, 12, 13, 15, 16, 17, 19, 20	1 (a–c)(iii), 2(a–c)(iii), 6, 8, 9, 10, 12, 13, 14, 16, 17, 18, 19, 20

Fluency

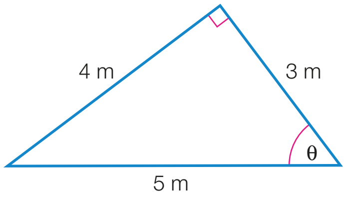
W.E. 3

1 **(a)** Find the exact value of tan (θ) in each triangle.

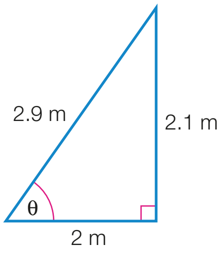
(i)



(ii)

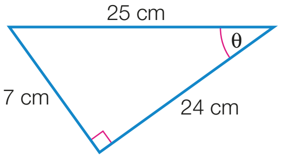


(iii)

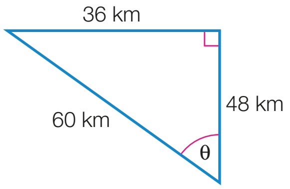


(b) Find the exact value of sin (θ) in each triangle.

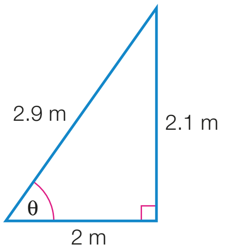
(i)



(ii)

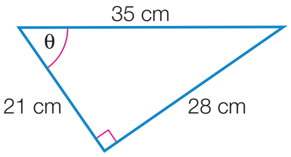


(iii)

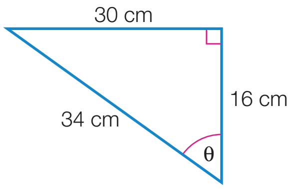


(c) Find the exact value of cos (θ) in each triangle.

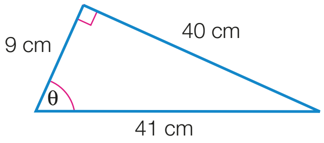
(i)



(ii)



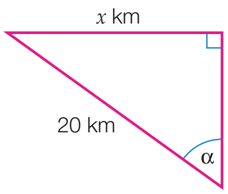
(iii)



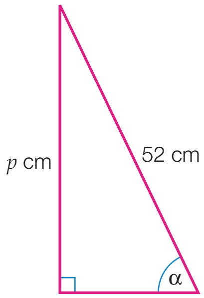
W.E. 4

2 **(a)** Find the value of the variable, correct to 2 decimal places, in the following triangles, where sin (α) is equal to the following values:

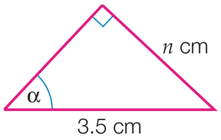
(i) $\frac{1}{3}$



(ii) $\frac{3}{4}$

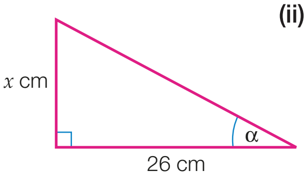


(iii) $\frac{1}{5}$

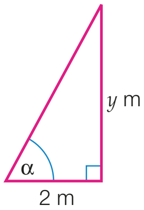


(b) Find the value of the variable, correct to 2 decimal places, in the following triangles, where tan (α) is equal to the following values:

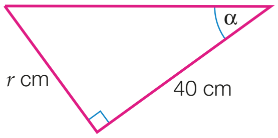
(i) $\frac{1}{2}$



(ii) 3

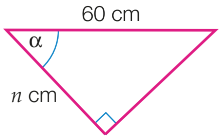


(iii) 0.52

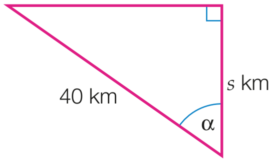


(c) Find the value of the variable, correct to 2 decimal places, in the following triangles, where cos (α) is equal to the following values:

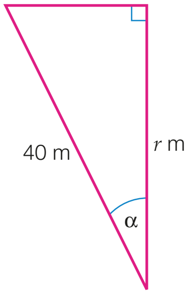
(i) $\frac{4}{5}$



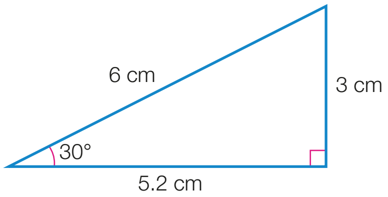
(ii) 0.3



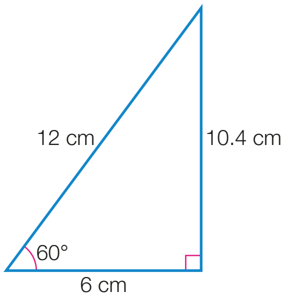
(iii) 0.56



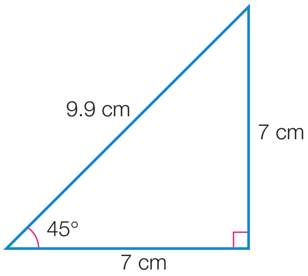
3 Find the value of sin (30°) from the diagram.



4 Find the value of cos (60°) from the figure.



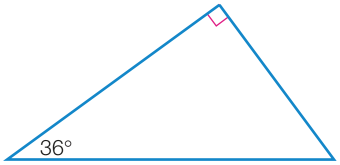
5 Find the value of tan (45°) from the figure.



W.E. 5

6 Measure the side lengths of this triangle to the nearest millimetre and hence calculate the value (to 2 decimal places) for:

(a) sin (36°) **(b)** cos (36°) **(c)** tan (36°)



Hint

What is the value of the unlabelled angle in the diagram?

7 **(a)** Measure the side lengths of this triangle to the nearest millimetre and hence calculate the value (to 2 decimal places) for:

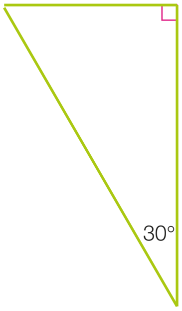
(i) tan (30°) **(ii)** cos (30°)

(iii) sin (30°) **(iv)** sin (60°)

(v) tan (60°) **(vi)** cos (60°)

(b) What do you notice about the values of cos (30°) and sin (60°)?

(c) What do you notice about the values of cos (60°) and sin (30°)?



Hint

What is the value of the third angle in this triangle?

8 **(a)** Measure the side lengths of this triangle to the nearest millimetre and hence calculate the value (to 2 decimal places) for:

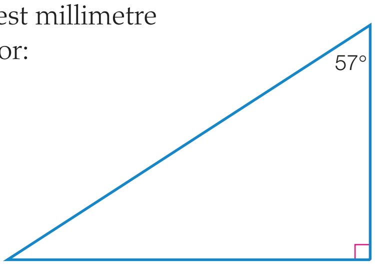
(i) cos (57°) **(ii)** tan (57°)

(iii) sin (57°) **(iv)** sin (33°)

(v) cos (33°) **(vi)** tan (33°)

(b) What do you notice about the values for sin (57°) and cos (33°)?

(c) What do you notice about the values for sin (33°) and cos (57°)?

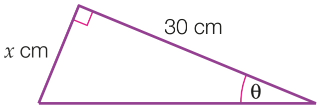


Understanding

9 **(a)** If tan (θ) = $\frac{2}{3}$, then the value of x in this triangle is:

A 15 **B** 20

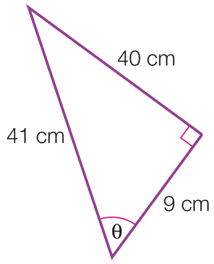
C 40 **D** 45



(b) In this diagram, cos (θ) is:

A $\frac{9}{41}$ **B** $\frac{40}{41}$

C $\frac{40}{9}$ **D** $\frac{41}{9}$



(c) The value for sin (71°) would be the same as:

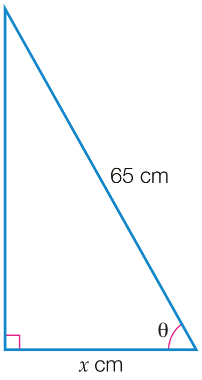
A cos (71°) **B** cos (19°)

C sin (19°) **D** cos (29°)

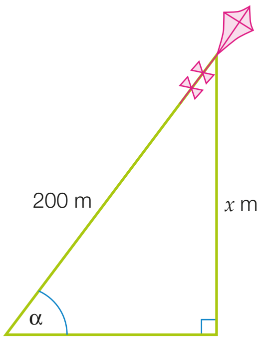
10 If cos (θ) = $\frac{5}{13}$, then the value of x in this triangle is:

A 65 **B** 25

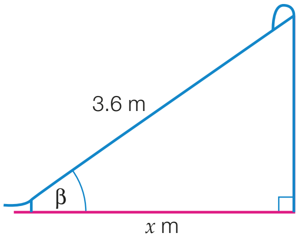
C 60 **D** 13



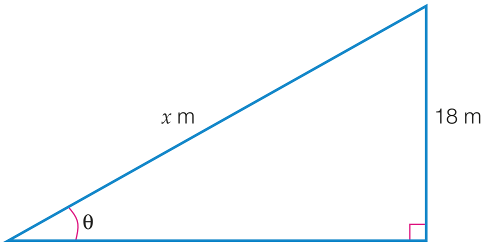
11 One windy day, Nam is flying a kite. Nam's kite string is 200 m long. If $\sin(\alpha) = \frac{1}{2}$, how high above the ground is the kite?



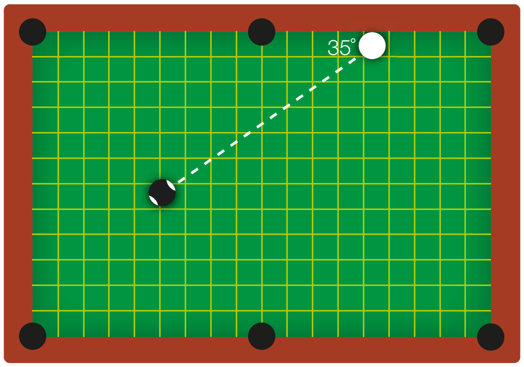
12 A slide is 3.6 m long. If $\cos(\beta) = \frac{7}{9}$, find the distance between the bottom of the slide and the bottom of the supporting tower.



13 An escalator inclined at an angle of θ from the ground reaches a height of 18 metres. If $\sin(\theta) = \frac{2}{3}$, what is the length of the escalator?



14 In a game of pool, the cue ball is at the edge of the table and travels 152 cm, at an angle of 35° with the side of the table, to hit the black ball. Before being hit, the black ball is 125 cm further down the table (horizontally) from the cue ball. Calculate the value of cos (35°), correct to 2 decimal places.



Reasoning

15 A 7.5 m ladder is leaning against a wall at an angle of 45°. The bottom of the ladder is 5.3 m from the base of the wall.

(a) Draw a diagram to scale (1 cm = 1 m) showing the information provided.

(b) What type of triangle is formed by the ladder, the wall and the ground?

(c) How far up the wall does the ladder reach?

(d) Explain how you can find the answer to part **(c)** without doing any calculations.

16 A slide that is 5 m long is built in a playground. The recommended angle of the slide with the ground is 37°. Parents are concerned that the slide may be too high for the children.

(a) If $\sin(37°) = \frac{3}{5}$, what is the height of the slide with this recommended slide angle?

(b) The parents are only happy if the height at the top of the slide is no greater than 2.5 m. What changes could be made so that this can happen?

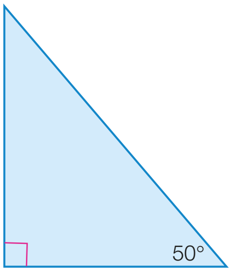
17 The sine of an angle is $\frac{12}{13}$.

(a) Draw the triangle and label the sides.

(b) Calculate the length of the other side.

(c) Determine the cosine ratio.

18 A right-angled triangle with a reference angle of 50° is shown.



(a) Measure the side lengths (in mm) and then calculate the values of sin (50°), cos (50°) and tan (50°), correct to 3 decimal places.

(b) In what way is the value of tan (50°) different to sin (50°) and cos (50°)? Explain why this happens.

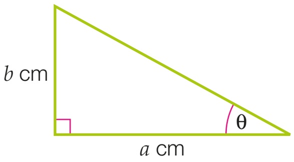
(c) Draw a right-angled triangle that has:

- **(i)** a tan (θ) value of less than 1
- **(ii)** a tan (θ) value of more than 1.

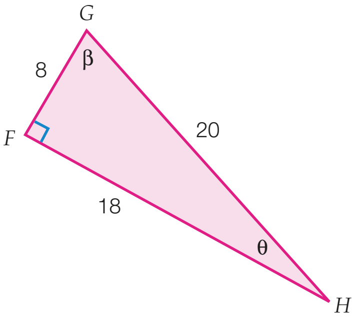
(d) Can the values of sin (θ) and cos (θ) be greater than 1? Why/why not?

Open-ended

19 In the following triangle, $\tan(\theta) = \frac{7}{10}$. Write at least three possible pairs of values for the lengths of sides a and b.



20 The teacher of a Year 9 class drew the following triangle on the board and asked the students to find the sin and cos ratios for the angles at β and θ.



After completing the task, Rashira looked at her answers and noticed that she had written $\sin(\theta) = \frac{8}{20}$ and $\cos(\beta) = \frac{8}{20}$. She said to her friend, 'They're the same! That can't be right, can it?' How would you explain to Rashira that she has the correct answers?

Using trigonometry to find side lengths

7.3

Recall that…

The sine of θ is given by:

$$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

SOH

The cosine of θ is given by:

$$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

CAH

The tangent of θ is given by:

$$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$$

TOA

To use trigonometry to find side lengths in right-angled triangles, you first need to identify which trigonometric ratio to use. To do this, identify which side lengths are involved in the problem. For example, if the adjacent side and the hypotenuse are the lengths of interest, then the cosine ratio will be used.

To calculate an unknown side length of a right-angled triangle using trigonometry:

1. Draw the right-angled triangle, showing all known information.
2. Label the sides with O, A and H in relation to the given angle.
3. Use SOH–CAH–TOA to decide which trigonometric ratio to use.
4. Form an equation and solve for the unknown.

Using technology to find a trigonometric ratio

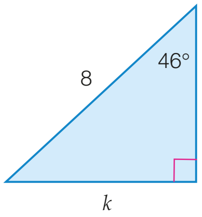
The value of a particular trigonometric ratio is always the same for a particular angle. You can use technology to find the value of a ratio, usually by entering functions or pressing keys labelled **sin** **cos** **tan**.

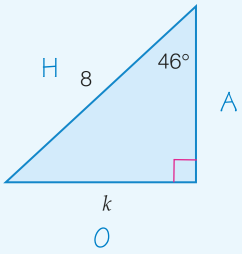
For example, to find the sine of 67°, you would enter **sin** **6** **7** **=**. Many trigonometric ratio values are long decimals. You should use the full value in your calculations (not rounded) and only round your final answer. Make sure your technology is set to 'degrees' mode (not 'radians' mode).

Worked example 6

W.E. 6

Find the value of k, correct to 2 decimal places.

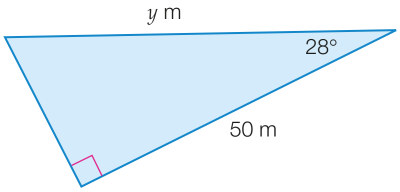


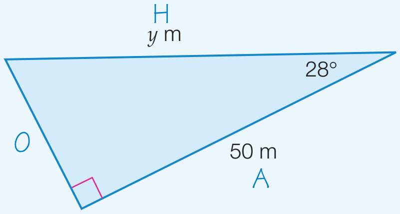
Thinking	Working
1 Use the reference angle to label the sides of the triangle H, O and A.	
2 List the information given.	$\theta = 46°$ $H = 8$ $O = k$
3 Identify the sides involved in the problem (H and O) and hence the trigonometric ratio that will need to be used (sine). Write out the rule for the required ratio.	$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$
4 Substitute the values.	$\sin(46°) = \frac{k}{8}$
5 Rearrange to make the unknown the subject of the equation (multiply both sides by 8).	$k = 8 \times \sin(46°)$
6 Use a calculator to determine *k*, correct to 2 decimal places.	≈ 5.75 (2 d.p.)

Worked example 7

W.E. 7

Find the value of *y*, correct to 2 decimal places.



Thinking	Working
1 Use the reference angle to label the sides of the triangle H, O and A.	
2 List the information given.	$\theta = 28°$ $A = 50$ m $H = y$ m
3 Identify the sides involved in the problem (H and A) and hence the trigonometric ratio that will need to be used (cosine). Write the rule for the required ratio.	$\cos(\theta) = \frac{A}{H}$

4 Substitute the values.

$$\cos(28°) = \frac{50}{y}$$

5 Rearrange to make the unknown the subject of the equation (multiply both sides by y, then divide by cos (28°)).

$$y \times \cos(28°) = 50$$

$$y = \frac{50}{\cos(28°)}$$

6 Use a calculator to determine y, correct to 2 decimal places.

$$\approx 56.63 \text{ m } (2 \text{ d.p.})$$

7.3 Using trigonometry to find side lengths

Navigator

1 (a–c), 2 (a–c), 3, 4, 6, 7, 8, 9, 10, 11, 12, 16

1 (a, c, e), 2 (d–f), 3, 4, 5, 6, 8, 9, 11, 12, 14, 15, 16

1 (d–f), 2 (g–i), 5, 6, 9, 11, 12, 13, 14, 15, 16

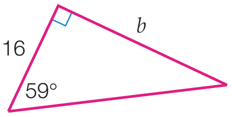
Answers p. 677

Fluency

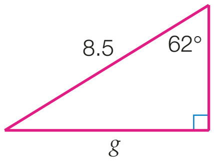
1 Find the value of each variable correct to 2 decimal places.

W.E. 6

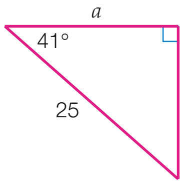
(a)



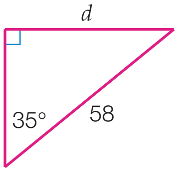
(b)



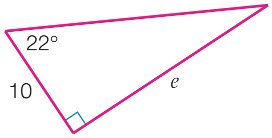
(c)



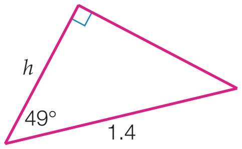
(d)



(e)



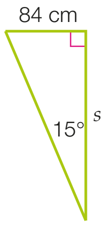
(f)



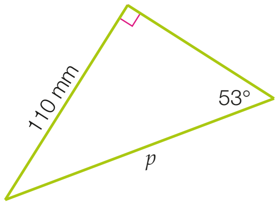
2 Find the value of the variable in each of the following triangles, correct to 2 decimal places.

W.E. 7

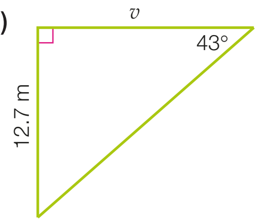
(a)



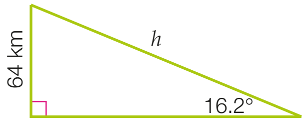
(b)



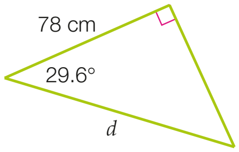
(c)



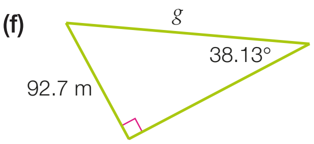
(d)



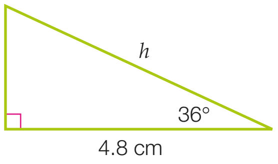
(e)



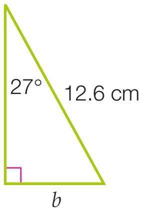
(f)



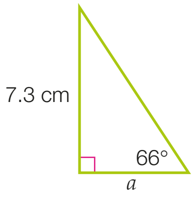
(g)



(h)

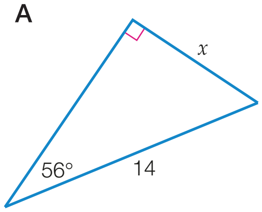


(i)

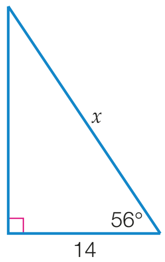


3 **(a)** For which of the following triangles can $\sin(56°) = \frac{14}{x}$ be solved to give x?

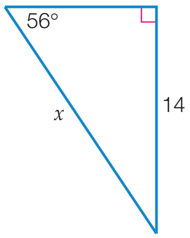
A



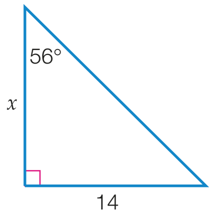
B



C

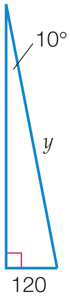


D

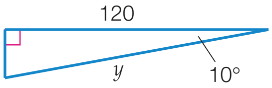


(b) For which of the following triangles can $\cos(10°) = \frac{y}{120}$ be solved to give y?

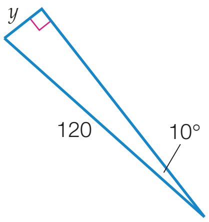
A



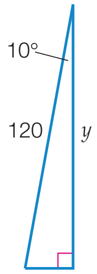
B



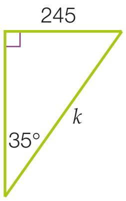
C



D



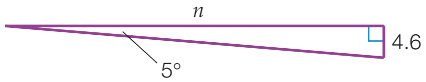
4 **(a)** The value of k could be found by using the equation:



A $\sin(35°) = \frac{k}{245}$ **B** $\tan(35°) = \frac{245}{k}$

C $\sin(35°) = \frac{245}{k}$ **D** $\cos(35°) = \frac{245}{k}$

(b) The value of n could be found by using the equation:



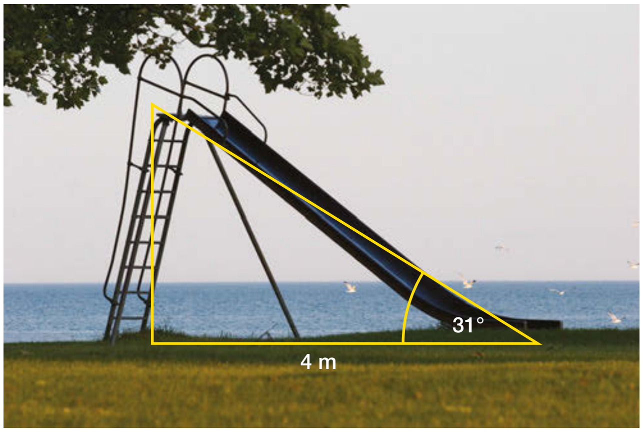
A $\sin(5°) = \frac{4.6}{n}$ **B** $\cos(5°) = \frac{4.6}{n}$

C $\tan(5°) = \frac{n}{4.6}$ **D** $\tan(5°) = \frac{4.6}{n}$

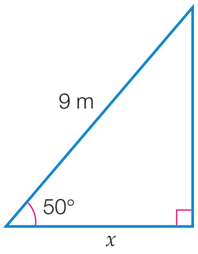
Understanding

For the following questions, express all answers correct to 2 decimal places.

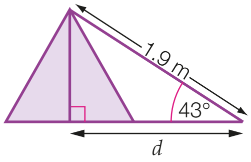
5 Use the information given in the photograph to find the height of the slide.



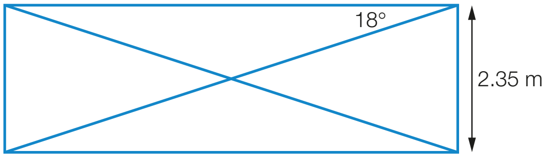
6 A fire ladder, 9 metres long, leans against a building and makes an angle of 50° with the ground. How far from the building is the bottom of the ladder? Give your answer correct to 2 decimal places.



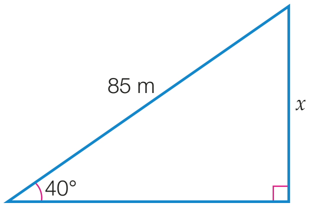
7 Calculate how far a tent peg is from a tent pole, if the 1.9 m support rope is pegged to make an angle of 43° with the ground.



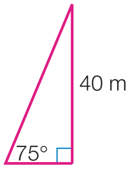
8 A rectangular gate has diagonal wooden supports that make an angle of 18° to the horizontal sections of the frame. If the gate is 2.35 m high, how long is one of the diagonal supports?



9 A kite is flying with a string 85 m long. The string is held tight at an angle of 40° to the ground. How high is the kite flying? Give your answer correct to 2 decimal places.



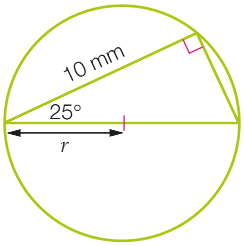
10 Members of a stunt team are to be catapulted into the air at an angle of 75° to the horizontal. Assuming they travel along a straight path, what horizontal distance will they have covered at the height of 40 m?



Reasoning

11 A road sign is in the shape of an equilateral triangle with a vertical measurement of 41.2 cm. How long is each side of the road sign?

12 Find the radius of this circle.



Open-ended

13 An angle in a right-angled triangle measures 63°. Give two possible pairs of lengths of the adjacent side and the hypotenuse.

14 Caillan's working for a trigonometry question is shown.

(a) What was the question? Draw the triangle.

(b) Calculate the length of the third side of the triangle.

$$\cos(15°) = \frac{20}{y}$$

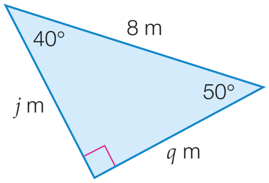
$$y \times \cos(15°) = 20$$

$$y = \frac{20}{\cos(15°)}$$

$$y = 20.7 \text{ cm}$$

15 The following answers were given by a student in a trigonometry test.

(i) Find the value of q.



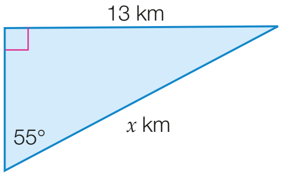
$$\cos(\theta) = \frac{A}{H}$$

$$\cos(40°) = \frac{q}{8}$$

$$8 \times \cos(40°) = q$$

$$q = 6.13 \text{ m}$$

(ii) Find the value of x.



$$\sin(\theta) = \frac{O}{H}$$

$$\sin(55°) = \frac{13}{x}$$

$$13 \times \sin(55°) = x$$

$$x = 10.65 \text{ km}$$

(a) Explain the mistake the student has made in each question.

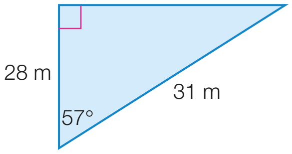
(b) Show the correct calculations and answers.

16 Explain the differences between using Pythagoras' theorem to find an unknown side of a right-angled triangle and using trigonometric ratios. For each method, state the minimum information needed to apply the method successfully.

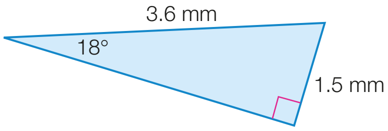
Half-time 7

1 Write the letter labels (O, A or H) of the unknown side in each triangle. **7.2**

(a)

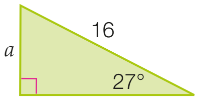


(b)

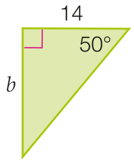


2 Find the value of each variable, correct to 2 decimal places. **7.3**

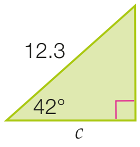
(a)



(b)

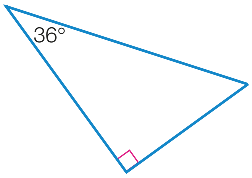


(c)



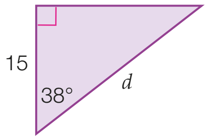
3 Measure the side lengths of this triangle to the nearest millimetre and hence calculate the value (to 2 decimal places) for: **7.2**

(a) sin (36°) **(b)** cos (36°) **(c)** tan (36°)

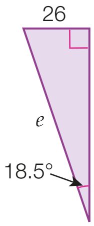


4 Find the value of each variable, correct to 2 decimal places. **7.3**

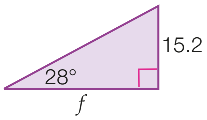
(a)



(b)

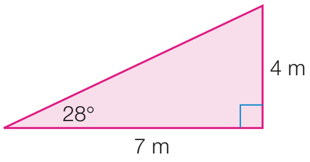


(c)

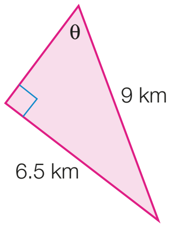


5 Label the sides of the following right-angled triangles as hypotenuse (H), opposite (O) and adjacent (A). **7.1**

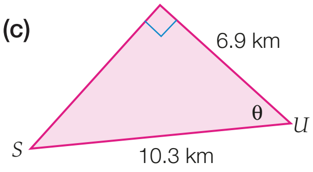
(a)



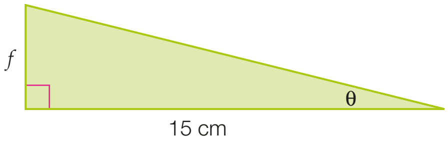
(b)



(c)



6 If $\tan(\theta) = \frac{2}{5}$, what is the value of f? **7.2**



7.4 Using trigonometry to find angles

To solve an equation such as $3x - 5 = 22$, you could use the 'inverse operations' of + 5 and ÷ 3. These inverse operations 'undo' the effect of the original operations (× 3, − 5) that form the equation.

You can similarly use special inverse operations to 'undo' the trigonometric ratios. For the inverse of using an angle to find a trigonometric ratio, you use a ratio to find the corresponding angles. For example, you could find the value of θ given $\sin(\theta) = 0.45$.

To use a trigonometric ratio in a right-angled triangle to find an angle, you must know two of the side lengths (or their ratio). This is because the trigonometric ratios sine, cosine and tangent are the ratios of two specific side lengths of a right-angled triangle.

If you know two side lengths, they will form one of the trigonometric ratios. You can then find the angle corresponding to this ratio by rearranging the trigonometric equation to make the angle the subject. This requires the trigonometric inverse.

Trigonometric ratio	Inverse
Sine of θ $\sin(\theta) = \frac{O}{H}$	$\theta = \sin^{-1}\left(\frac{O}{H}\right)$
Cosine of θ $\cos(\theta) = \frac{A}{H}$	$\theta = \cos^{-1}\left(\frac{A}{H}\right)$
Tangent of θ $\tan(\theta) = \frac{O}{A}$	$\theta = \tan^{-1}\left(\frac{O}{A}\right)$

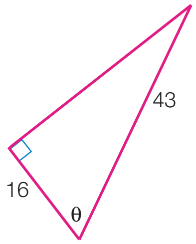
Note that the $^{-1}$ here does *not* mean 'to the power of -1'. Instead it means 'inverse'. When talking about $\sin^{-1}$ you must always say 'inverse sine', not 'sine to the power of negative 1'.

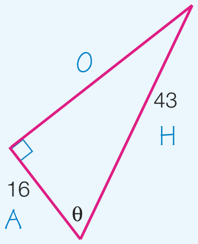
On most calculators, the inverse trigonometric functions are found by pressing **2nd F**, **SHIFT** or **INV** before pressing **sin**, **cos** or **tan**.

Worked example 8

W.E. 8

Find the value of θ correct to the nearest degree.

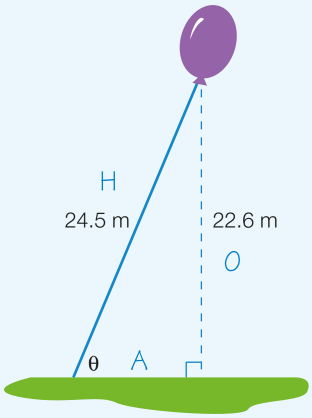


Thinking	Working
1 Label the sides of the triangle O, A and H.	
2 Identify the sides involved in the problem (adjacent and hypotenuse) and hence the trigonometric ratio that will need to be used (cosine). Write the rule for the required ratio.	$\cos(\theta) = \frac{A}{H}$
3 Substitute the values.	$\cos(\theta) = \frac{16}{43}$
4 Rearrange to make θ the subject.	$\theta = \cos^{-1}\left(\frac{16}{43}\right)$
5 Use a calculator to determine θ, correct to the nearest degree.	$\approx 68°$

Worked example 9

W.E. 9

A helium-filled balloon is attached to the ground by a 24.5 m string. A light breeze is blowing so that the base of the balloon is at a height of 22.6 m above the ground. What is the smaller of the two angles, to the nearest degree, that the string makes with the ground?

Thinking	Working
1 Draw a diagram that shows a right-angled triangle with all the given information. Label the sides O, A and H.	
2 Identify the sides involved in the problem (opposite and hypotenuse) and hence the trigonometric ratio that will need to be used (sine). Write the rule for the required ratio.	$\sin(\theta) = \frac{O}{H}$

3 Substitute the values.	$\sin(\theta) = \frac{22.6}{24.5}$
4 Rearrange to make θ the subject.	$\theta = \sin^{-1}\left(\frac{22.6}{24.5}\right)$
5 Use a calculator to determine θ.	$\approx 67.287°$
6 Answer the question, giving your answer correct to the nearest degree.	The string makes an angle of 67° with the ground.

7.4 Using trigonometry to find angles

Navigator

Answers p. 678

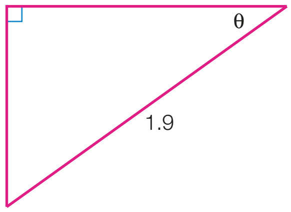
1, 2 (a–c), 3, 4, 6, 7, 8, 9, 10, 12, 13	1 (a, c), 2 (a, c), 3, 4, 5, 6, 8, 9, 10, 11, 12, 13	1 (b, d), 2 (b, d), 3, 4, 5, 6, 8, 9, 10, 11, 12, 13

Fluency

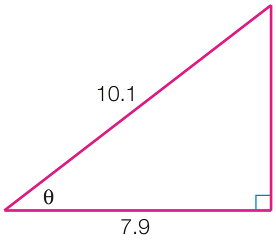
W.E. 8

1 Find the value of θ, correct to the nearest degree, in each of the following.

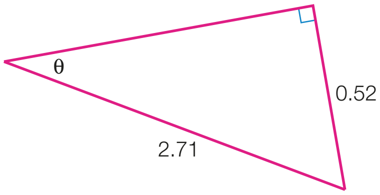
(a)



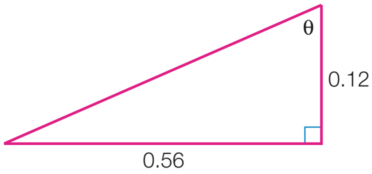
(b)



(c)

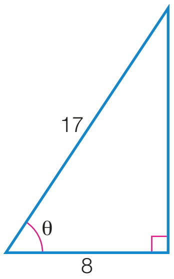


(d)

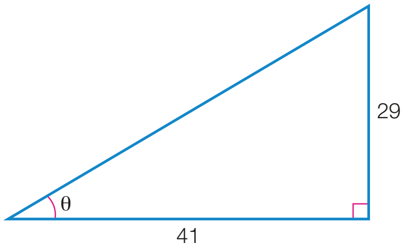


2 Find the value of each angle shown, to the nearest degree.

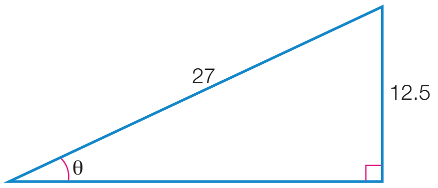
(a)



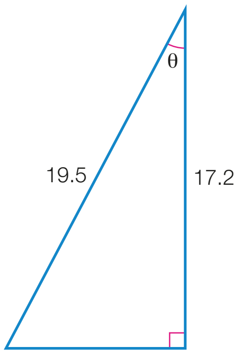
(b)



(c)

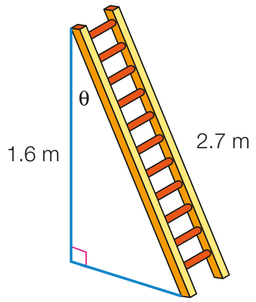


(d)

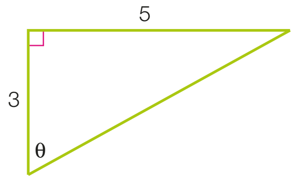


3 The top of a 2.7 m ladder, leaning against a wall, reaches a height of 1.6 m. What angle, to the nearest degree, does the ladder make with the wall?

W.E. 9



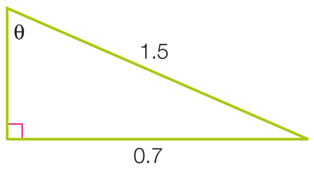
4 **(a)** The value of θ can be found by using the equation:



A $\cos(\theta) = \frac{3}{5}$ **B** $\tan(\theta) = \frac{5}{3}$

C $\sin(\theta) = \frac{3}{5}$ **D** $\tan(\theta) = \frac{3}{5}$

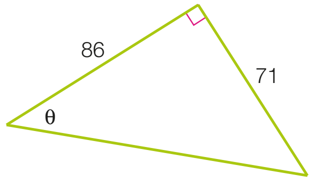
(b) The value of θ in this triangle would be equal to:



A $\tan^{-1}\left(\frac{0.7}{1.5}\right)$ **B** $\cos^{-1}\left(\frac{1.5}{0.7}\right)$

C $\sin^{-1}\left(\frac{0.7}{1.5}\right)$ **D** $\cos^{-1}\left(\frac{0.7}{1.5}\right)$

(c) The value of θ in this triangle would be equal to:

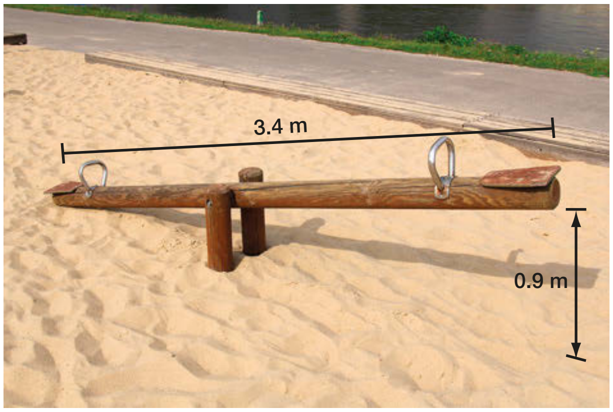


A $\cos^{-1}\left(\frac{71}{86}\right)$ **B** $\tan^{-1}\left(\frac{86}{71}\right)$

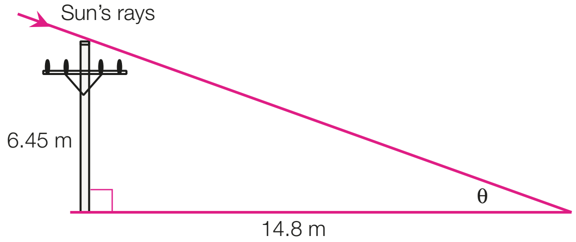
C $\sin^{-1}\left(\frac{71}{86}\right)$ **D** $\tan^{-1}\left(\frac{71}{86}\right)$

Understanding

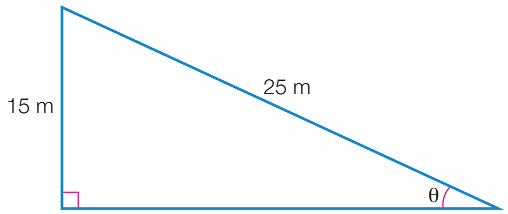
5 A seesaw is made from a plank of wood 3.4 m long supported in the middle, as shown. If the greatest height reached by one end of the seesaw is 0.9 m, what angle does the seesaw make with the ground? Round your answer to the nearest degree.



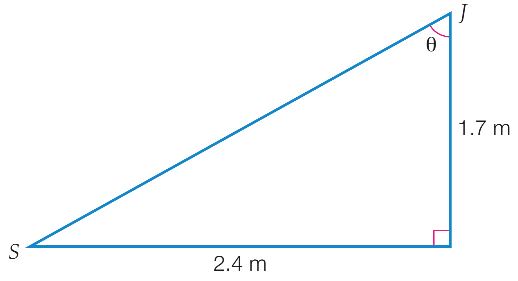
6 Find (to the nearest degree) the angle θ of the Sun's rays when a 6.45 m high power pole casts a shadow 14.8 m long.



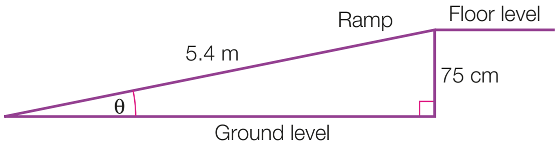
7 In an adventure park, a straight water slide is 25 m long and 15 m high. What angle will the water slide make with the surface of water? Give your answer to the nearest degree.



8 A spider on the ground is 2.4 metres away from John. John is 1.7 metres tall. At what angle, to the nearest degree, is John to the spider?



9 A builder uses a ramp to bring sand in a wheelbarrow from ground level up to floor level inside a building under construction. Floor level is 75 cm above ground level. If the ramp is 5.4 m long, what angle, to the nearest degree, does the ramp make with the ground?



Reasoning

10 **(a)** When the opposite side and hypotenuse are similar in length, sin (θ) is large. Draw a diagram to show this. Write lengths on the hypotenuse and the opposite side and calculate the value of sin (θ).

(b) When the opposite side is small and the hypotenuse is large, sin (θ) is very small. Draw a diagram to show this. Write lengths on the hypotenuse and the opposite side and calculate the value of sin (θ).

(c) Complete the following sentences:

(i) When the adjacent side and hypotenuse are similar in length, cos (θ) is ________.

(ii) When the adjacent side is small and the hypotenuse is large, cos (θ) is ________.

11 One angle between 0° and 90° gives a trigonometric ratio of 1.

(a) What does a ratio of 1 tell you about the lengths of the two sides involved?

(b) Which ratio is it? Explain how you know.

(c) What type of right-angled triangle gives this ratio a value of 1?

(d) What is the value of the angle involved?

Open-ended

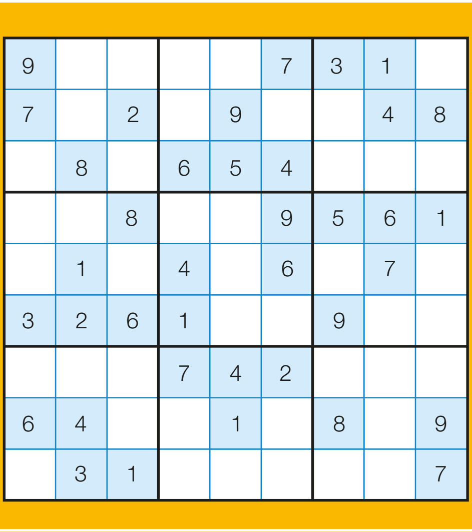
12 Two of the side lengths of a right-angled triangle are 9 and 15. What could the reference angle be?

13 An access ramp for a motorised wheelchair must make an angle with the horizontal that is between 6° and 9°. Find three different ramp lengths and the angles they must be placed at to give smooth access to a platform that is 65 cm high.

Puzzle

Sudoku

Copy the following Sudoku puzzle and complete the blank squares such that each row, column and smaller 3 × 3 square contains each of the digits 1–9 only once.



9					7	3	1	
7		2		9			4	8
	8		6	5	4			
		8			9	5	6	1
	1		4		6		7	
3	2	6	1			9		
			7	4	2			
6	4			1		8		9
	3	1						7

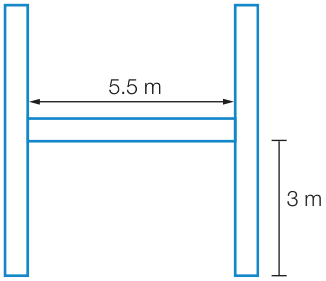
Investigation

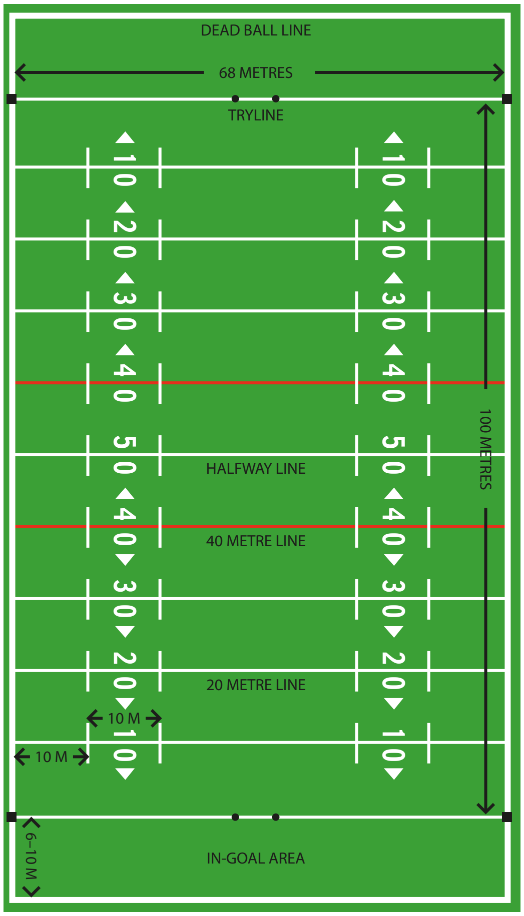
Kicking goals in rugby

Equipment required: calculator

To score in the game of Rugby League, teams need to move the ball down the field and place it over the try line. This is called a 'try' and is worth 4 points. After a try has been scored, the team can gain an extra 2 points by kicking a goal, known as a 'conversion'. A goal kicker must attempt to kick the goal from a position on the field anywhere on a line perpendicular to the try line at the point where the try was scored.

The goal consists of two posts 5.5 m apart, joined by a horizontal bar 3 m off the ground to make an 'H' shape. To score a goal, the ball must be kicked between the vertical posts and over the horizontal bar.





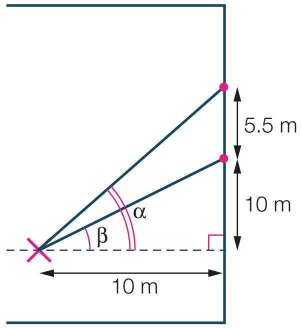
The Big Question

Where is the best position to kick from in order to maximise the chances of kicking a goal?

Engage

1 Imagine that a try is scored 10 m from the near goal post. Draw a diagram of the rugby field showing the goal posts, goal line and a line perpendicular to the goal line, 10 m from the near goal post.

Mark the goal kicker's position with an X, 10 m from the try line. Rule two lines from this point, one to the near goal post and one to the far post. Mark the angles formed by these lines with the line perpendicular to the goal line as α and β.



2 Separately draw the two right-angled triangles formed in the diagram that would be used to calculate the angles α and β. Calculate the kicking angle when standing 10 m from the try line.

3 Describe how the kicking angle can be calculated using the two angles.

Explore

4 Calculate the kicking angle for distances 0 and 20 m from the goal line after a try has been scored 10 m from the closer goal post. Record your results in a table.

Strategy options

- Draw a picture.
- Draw a table.
- Break the problem into manageable parts.

Explain

5 Do your results show a significant difference in the kicking angle at different positions? Would it make a significant difference to a player's ability to kick a goal?

Elaborate

6 Where is the best position for the goal kicker to kick from in order to maximise the angle between the goals (when a try has been scored 10 m from the closer goal post)?

7 **(a)** How far can a good rugby player kick? How would kicking distance affect where the kicker positions the ball?

(b) What other factors would the goal kicker have to take into account when choosing the position to kick from?

8 How might a rugby player use the information you have found in this Investigation? What other sports or situations might this type of investigation be useful for?

Evaluate

9 Explain the method of calculating the optimal kicking angle.

10 Did you develop any shortcuts when calculating each of the goal-kicking positions? If so, what were they?

11 What other methods could be used to find the optimal kicking angle, without using trigonometry?

12 Do you believe you have found the optimal goal-kicking position? How could you attain a more accurate result?

Extend

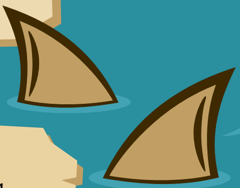
13 Investigate other try positions along the try line, such as closer to or further away from a goal post.

14 Is there a relationship between the optimal kicking angle and the placement of the try? Find out by drawing a rugby field and marking the best position to kick from for various try placements.

Gamespace

Adventure on Sohcahtoa Island

Your class has been taken on an excursion to Sohcahtoa Island. You land at Hypotenuse Harbour and are given a map of the island with places of interest clearly marked. Your map has vertical lines separating horizontal distances of 300 m. Your excursion is a competition.



Equipment required: 1 die, a counter for each player, a calculator for each player

How to win:

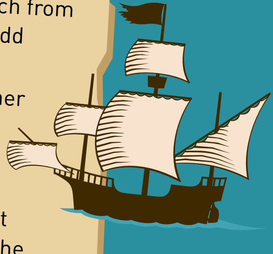
Travel from Hypotenuse Harbour to Port Pythagoras in the shortest possible distance, under the conditions of the competition.

How to play:

All players begin at Hypotenuse Harbour. You must first go to Cosine High School to collect supplies. You will need to throw a 2 or more to do this. After leaving the high school, you must visit at least three landmarks on the island along the way before arriving at the port. To get to Port Pythagoras, you may only travel along the marked roads, take the ferry or row across the lake.

On your turn:

1. Roll the die. Multiply your die roll by 300 m (the distance between the lines on the map). This is the maximum horizontal distance you can cover.
2. Determine which landmarks lie within this distance to the east or west of where you are. You may choose to move east or west, depending on which landmark you wish to visit. For example, if you roll a 3, you can move 300, 600 or 900 m east or west from your location.
3. Travel to your chosen landmark along a marked route. Calculate the distance you have travelled using the horizontal distance, the given angle and the correct trigonometric ratio. Record the distance travelled (to the nearest whole number).
4. Keep a running total of the distance you have travelled and that of your competition, adding the distance after each move. (Remember, your aim is to reach Port Pythagoras with the smallest possible total.) If there are more than two players, ensure that every player has their distance calculated by at least one other person.
5. If there is no place of interest you can reach from your die roll, you cannot move and must add 100 m onto your distance total.
6. You may choose to stay where you are rather than take a route you don't want to follow. The penalty is an extra 500 m added to your total distance.
7. The game ends when all players reach Port Pythagoras. The winner is the player with the smallest total.



Sample move:

A player is at Isosceles Boat Hire. They roll a 2 and could choose to travel 2 × 300 = 600 m east to go to Sine Cinema or Vertex Museum, 600 m west to get to Perpendicular Cliffs or 300 m west to get to Tangent Beach Lookout. To get to Picnic Paradise, they would need to roll a 3 or more.

The distance from Isosceles Boat Hire along the road to Sine Cinema is calculated by

$$H = \frac{A}{\cos(40°+15°)}$$

$$= \frac{600}{\cos(55°)}$$

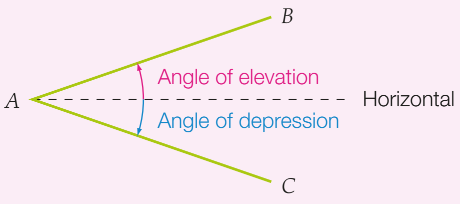
$$= 1046 \text{ m}$$


N
W
E
S
300m
300 m
Trigonometry Oval
60°
12°
Tangent Beach Lookout
14°
64°
Vertex Museum
41°
70°
Port Pythagoras
20°
Equilateral Swimming Pool
Triangular Point
42°
Isosceles Boat Hire
60°
40°
15°
Lake Adjacent
40°
25°
Picnic Paradise
20°
Cosine High School
70°
20°
20°
20°
75°
Government House
45°
Hypotenuse Harbour
25°
12°
Sine Cinema
48°
15°
Perpendicular Cliffs

7.5 Applications of trigonometry

The **angle of elevation** from A to B is the angle the upward-sloping line AB makes with the horizontal.

The **angle of depression** from A to C is the angle the downward-sloping line AC makes with the horizontal.



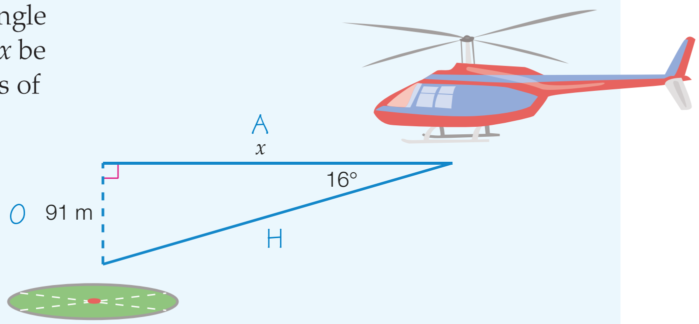
If you were positioned at A, you would need to *raise* your eyes from a horizontal line of sight to view an object at B.

You would need to *lower* your eyes from a horizontal line of sight to view an object at C.

Worked example 10

W.E. 10

A helicopter, flying at a constant altitude of 91 m, sees the middle of a landing pad at an angle of depression of 16°. How far, in metres, must the helicopter travel to be directly over the landing spot at an altitude of 91 m?

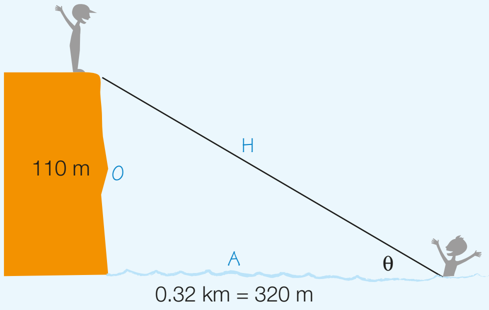
Thinking	**Working**
1 Draw a diagram that shows a right angle with all the known information. Let x be the required distance. Label the sides of the triangle O, A and H.	
2 Write all the given information.	$\theta = 16°$ $O = 91$ m $A = x$ m
3 Identify the sides involved in the problem (opposite and adjacent) and hence the trigonometric ratio that will need to be used (tangent). Write out the rule for the required ratio.	$\tan(\theta) = \frac{O}{A}$
4 Substitute the values.	$\tan(16°) = \frac{91}{x}$

5 Rearrange to make x the subject.	$x\tan(16°) = 91$ $x = \dfrac{91}{\tan(16°)}$
6 Use a calculator to determine x.	≈ 317.35 (2 d.p.)
7 Answer the question, giving your answer correct to the nearest metre.	The helicopter must travel 317 m before it is directly over the landing pad.

Worked example 11

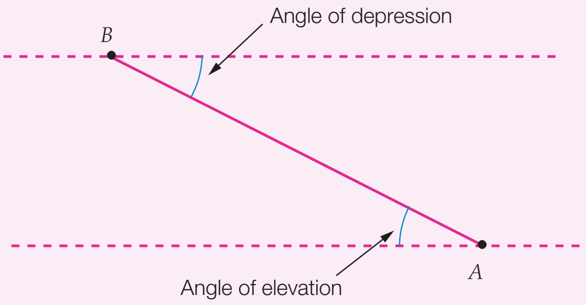
W.E. 11

A swimmer, 0.32 km from the base of a cliff, looks up and waves to her friend who is at the top of the 110 m cliff. Find the angle of elevation from the swimmer to the top of the cliff. Give your answer to the nearest degree.

Thinking	Working
1 Draw a diagram. Let θ be the unknown angle. Convert measurements to the same units. Label your diagram with O, A and H.	
2 Identify the sides involved in the problem (opposite and adjacent) and hence the trigonometric ratio that will need to be used (tangent). Write out the rule for the required ratio.	$\tan(\theta) = \dfrac{O}{A}$
3 Substitute the values.	$\tan(\theta) = \dfrac{110}{320}$
4 Rearrange to make θ the subject.	$\theta = \tan^{-1}\left(\dfrac{110}{320}\right)$
5 Use a calculator to determine θ.	≈ 18.97 (2 d.p.)
6 Answer the question, giving your answer correct to the nearest degree.	The angle of elevation from the swimmer to the top of the cliff is 19° (to the nearest degree).

Angles of elevation and depression are always measured from the horizontal.

The angle of elevation from Object A to Object B is the same as the angle of depression from Object B to Object A. The line AB crosses between two parallel lines, so the angle of elevation and the angle of depression are alternate angles (which are equal).

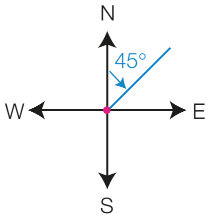


Bearings

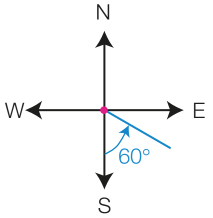
A **bearing** indicates used to indicate the direction of one object from another object or point. Bearings are used extensively in navigation, orienteering and map reading.

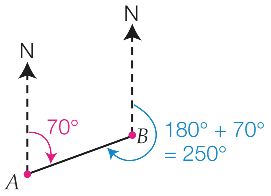
There are two common types of bearings:

True bearing: A direction measured as an angle clockwise from north. True bearings are always written using three digits. For example, an angle of 45° measured from north would be written as 045°T.



Compass bearing: A direction measured as an angle either side of north or south. For example, a bearing of S60°E is found by the angle that is 60° from south towards east.
This is sometimes called a conventional bearing.





Note that the bearing from one point *A* to another point *B* is not the same as the bearing from *B* to *A*.

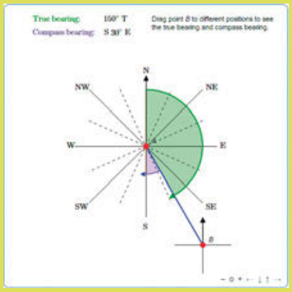
Bearing of *B* from *A* = 070°T or N70°E
Bearing of *A* from *B* = 250°T or S70°W

Interactive

Compass bearings

Compare how the compass bearing values and true bearing values change around the circle.

Go to the eBook or the Pearson Places website to access this interactive.



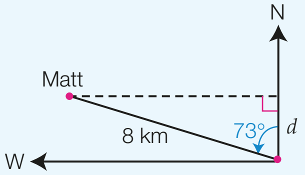
True bearing: 150° T
Compass bearing: S 30° E

Drag point B to different positions to see the true bearing and compass bearing.

Worked example 12

W.E. 12

Matt hikes for 8 km from his base camp on a bearing of N73°W. How far north is Matt from his base camp? Give your answer correct to 1 decimal place.

Thinking	Working
1 Draw a diagram showing all the known information. Let *d* be the required distance.	

2	Identify the sides involved in the problem (adjacent and hypotenuse) and hence the trigonometric ratio that will need to be used (cosine). Write out the rule for the required ratio.	$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$
3	Substitute the values.	$\cos(73°) = \frac{d}{8}$
4	Rearrange to make *d* the subject.	$d = 8 \times \cos(73°)$
5	Use a calculator to determine *d*.	≈ 2.339 km
6	Answer the question, giving your answer correct to 1 decimal place.	Matt is 2.3 km north of the base camp.

7.5 Applications of trigonometry

Navigator

1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 16, 18	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18	1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

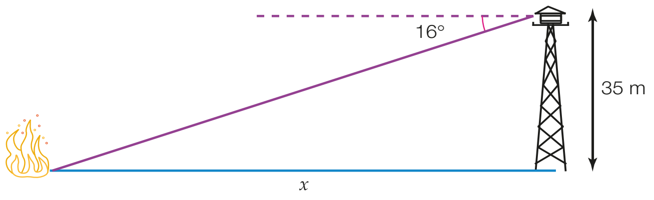
Answers p. 678

Fluency

Find all angles to the nearest degree and all lengths to 2 decimal places, unless told otherwise.

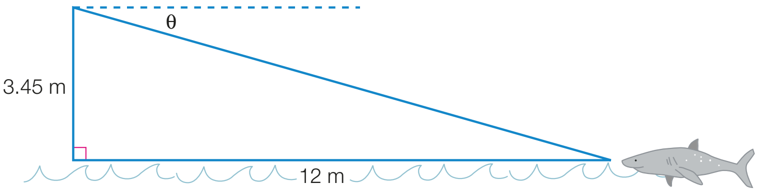
W.E. 10

1 Michelle, a forestry officer on watch from a lookout 35 m above the ground, notices a small fire at an angle of depression of 16°. How many metres (x) would a fire-fighting aircraft need to fly horizontally, from directly over the lookout, to drop its load on the fire?



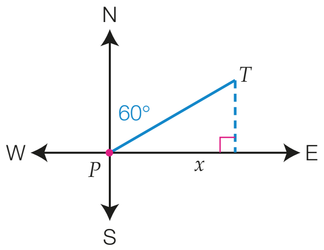
W.E. 11

2 The crew on a fishing boat spot a shark near the surface of the water nearby. They estimate that the horizontal distance between themselves and the shark is 12 m. If the crew are 3.45 m above sea level, find the angle of depression to the shark.

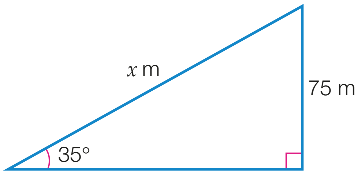


W.E. 12

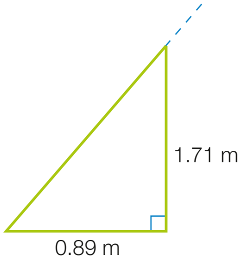
3 A train leaves Paddistone station and travels 40 km on a bearing of 060°T to Tarrigal station. How far east of Paddistone station is Tarrigal station?



4 Sam is flying a kite at a height of 75 m attached to a tight string inclined at 35° to the ground. What is the length of the string? Give your answer correct to 2 decimal places.



5 Find the angle of elevation of the Sun if Phillip is 1.71 m tall and he casts a 0.89 m shadow. Give your answer correct to 2 decimal places.



6 A 5.8 m tree casts a shadow across the ground. If the angle of elevation of the Sun is 55°, the length of the shadow is:

A 4.1 m **B** 4.8 m

C 7.1 m **D** 10.1 m

Understanding

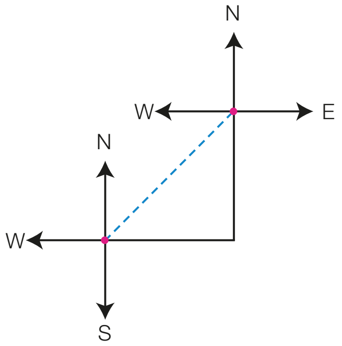
7 A horse gallops 4 km due south before turning and travelling 6.5 km due west. The bearing that the horse must travel on to return directly to its starting point is:

A N31.6°E

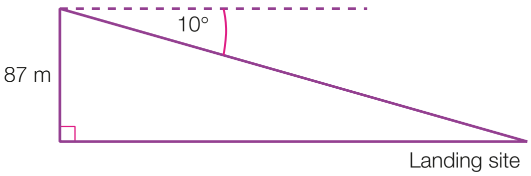
B N58.4°E

C S31.6°W

D S58.4°W



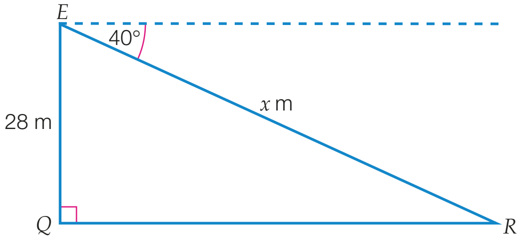
8 From a hot air balloon, the angle of depression to a marked landing site is 10°. If the observation deck in the balloon is 87 m above the ground, the horizontal distance the balloon needs to travel to reach the landing site is:



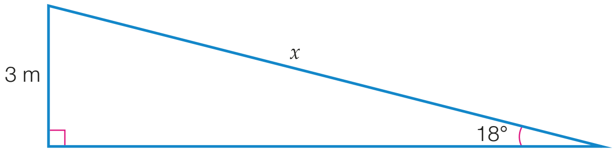
A 86 m **B** 88 m **C** 493 m **D** 501 m

9 A lifeguard spots a swimmer in distress from a lifeguard station 2.8 m high. If the angle of depression from the lifeguard to the swimmer is 17°, find the distance to the swimmer along the lifeguard's line of sight.

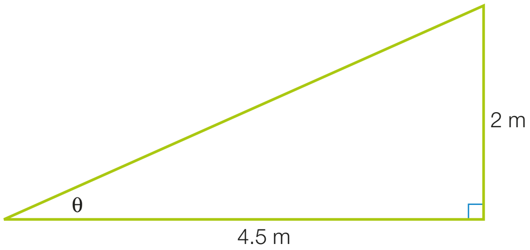
10 An eagle looks at a mouse on the ground at an angle of depression of 40°. If the eagle is 28 m above the ground, how far is the mouse from the eagle?



11 A clown in a circus is climbing on a tightly stretched rope tied from the top of a vertical pole to the ground. If the rope is inclined at an angle of 18° with the ground and the height of the pole is 3 metres, calculate, correct to 2 decimal places, the distance covered by the clown via the rope.



12 Martin has stopped his car at an intersection. He is 4.5 m from the base of a traffic signal, which is red, and judges the position of the green light to be 2 m higher than his horizontal line of sight. Through what angle of elevation must he raise his eyes to view the green light signal?



Reasoning

13 A ship travels 45 km on a bearing of N42°W.

(a) How far north of its starting point is the ship?

(b) How far west of its starting point is the ship?

14 A hiker walks 2.4 km east, then walks S75°W until she is due south of her starting position.

(a) Draw a diagram of the scenario.

(b) How far is she from her starting point?

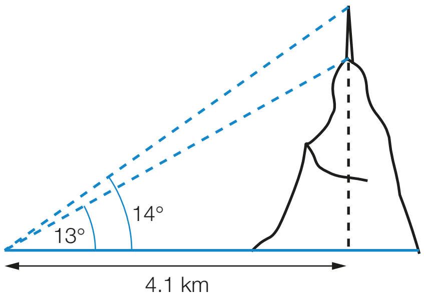
(c) How far did she walk on the bearing of S75°W before she was due south of her starting position?

15 Sarah is standing 100 m due south of a tower. Dougal is standing 140 m due west of the same tower. Using both compass bearings and true bearings, find the bearing of:

(a) Dougal from Sarah

(b) Sarah from Dougal.

16 A communications tower is on top of a nearby mountain. From a position 4.1 km away from the base of the mountain, the angle of elevation to the bottom of the tower is 13°, whereas the angle of elevation to the top of the tower is 14°. Find the height of the tower to the nearest metre.



17 From her vantage point on a cliff, Maria sights two swimmers in a direct line in front of her at angles of depression of 38.6° and 53.9°. If Maria is 50 m above the water level, find the distance between the two swimmers.

Open-ended

18 The angle of elevation from Ned's car to the top of a nearby tree is 27°. Give two pairs of possible values for the height of the tree and the horizontal distance of the tree from the car.

19 Draw a diagram that shows the angle of elevation and the angle of depression between two objects. Explain why these two angles are equal.

20 Germaine's maths test contains the following question:

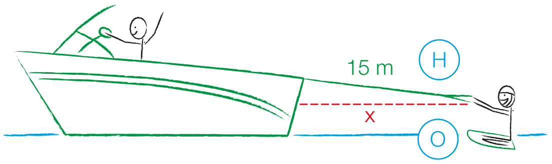
'Bianca is kneeboarding behind a boat on a 15 m rope. If the rope has an angle of depression of 4°, how far is Bianca from the back of the boat?'

Here is Germaine's answer. The teacher has marked it as incorrect.

(a) Identify the mistake and calculate the correct answer.

(b) How would you explain to Germaine the mistake she made?

Germaine's answer:



$\sin(4°) = \frac{x}{15 \text{ m}}$

$x = \sin(4°) \times 15$

$x = 1.05 \text{ m}$

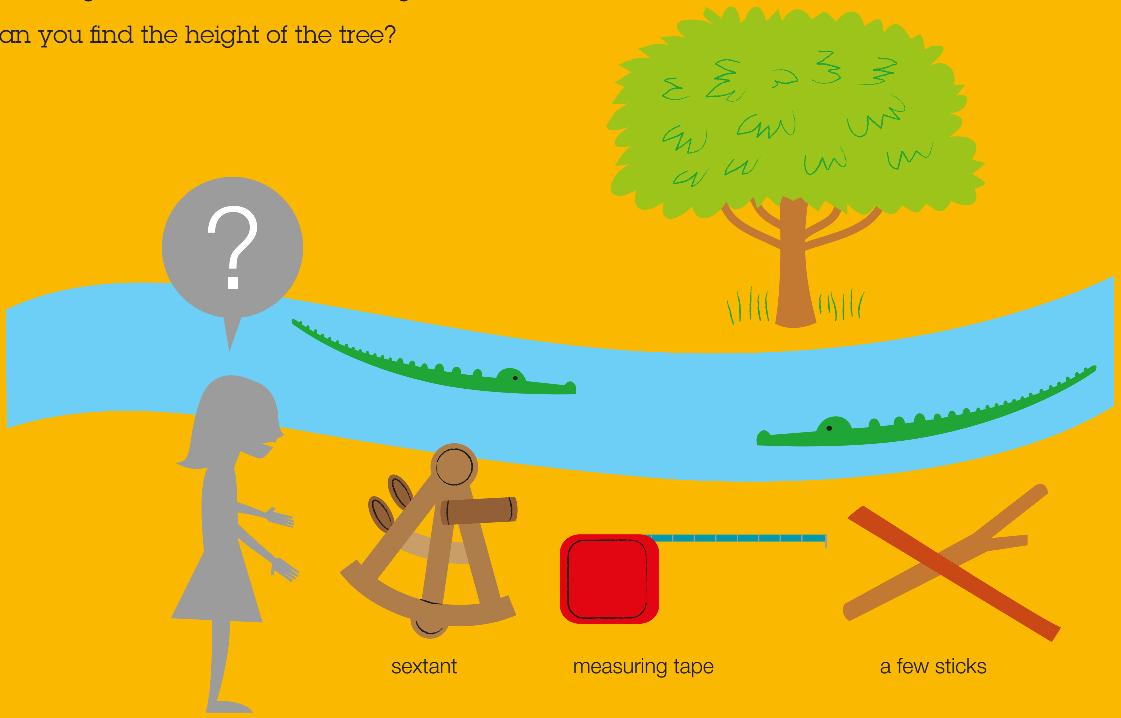
Problem solving

Finding the height of a tree

Your task is to find the height of a tree on the other side of a river that cannot be crossed. You have been given a 100 m tape measure and a sextant.

A sextant is an instrument for measuring the angle from your horizontal line of sight to the top of another object. This object may be higher or lower than your horizontal line of sight. You do not have to reach the object to be able to find the angle, but it must be within sight.

How can you find the height of the tree?



Strategy options

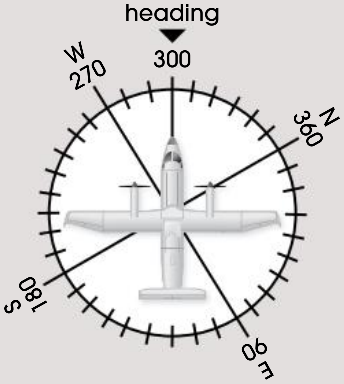
- Draw a picture.
- Break the problem into manageable parts.

Maths 4 Real

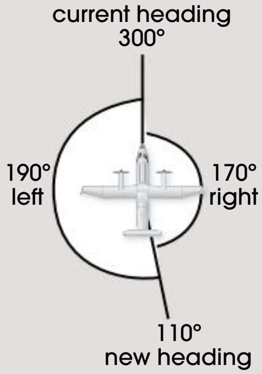
Air traffic controller

The airspace above Australia and all the aircraft that fly in it are controlled by air traffic controllers on the ground. The air traffic controllers must keep all aircraft from colliding, particularly when they are coming in to land. This is done by telling the aircraft to slow down, speed up or fly in a holding pattern.

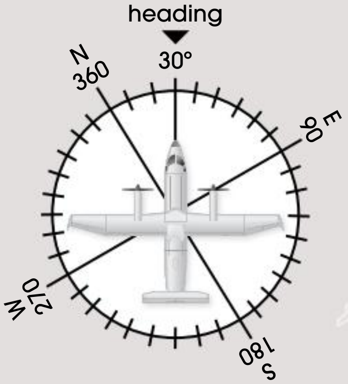
When an aircraft is approaching an airport or landing strip, an air traffic controller will tell the pilot which direction to approach from. All directions are given in degrees measured by a heading indicator. In the heading indicator, 360° is north, 90° is east and so on, in a clockwise direction around from north.



When an aircraft needs to change direction, the aircraft is told to turn left or right, usually through the shorter angle. For example, if an aircraft is on a heading of 300° and needs to change to 110°, it can turn 170° to the right or 190° to the left. Because 170° is the shorter angle, the aircraft is told to turn right by 170°.



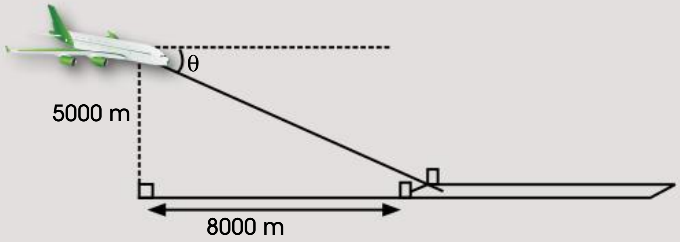
1 Refer to the heading indicator on the right.



(a) In which direction is the plane currently heading?

(b) The plane needs to change its heading to 200°. Find the turning needed to do this to the left or to the right. Would you tell the pilot to turn to the left or the right? Explain your answer.

2 An aircraft is making its approach to land. It is 8000 m from the end of the runway and 5000 m high, as shown below. At what angle would you tell the pilot to make the approach?



The air traffic controller's problem

A passenger jet bound for Canberra is flying on a heading of 325° at an altitude of 30 000 feet. (For aircraft altitude, the imperial unit of feet is still typically used instead of metres.) Canberra airport is closed due to fog, so the passenger jet must be diverted to another airport. The jet must turn to a heading of 150° and then descend 30 000 feet at an angle of 30°.

3 Should you tell the jet to turn to the left or the right, and by what angle? Draw a diagram to help you answer this.

4 **(a)** What distance along the ground (in feet) has the jet travelled from the beginning to the end of its descent of 30 000 feet? Draw a diagram.

(b) Using the conversion factor of 1 foot = 0.3048 m, calculate how many metres the jet has travelled (to the nearest metre).

Research

Investigate a career as an air traffic controller. What is their role? Why is this job important? What equipment do they use? What qualifications are required? What other mathematical skills are required to perform the job?

Challenge 7

1 If m, n are integers, where $m > n$ and $n \geq 1$, then $m^2 - n^2$, $2mn$, $m^2 + n^2$ form a Pythagorean triple (a set of three numbers that obey Pythagoras' theorem).

(a) Show that $(m^2 + n^2)^2 = (m^2 - n^2)^2 + (2mn)^2$.

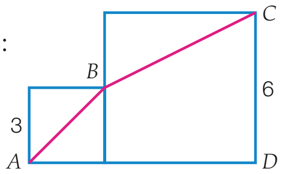
(b) Find the Pythagorean triples, with no common factors, whose smallest number is:

(i) 3 **(ii)** 5 **(iii)** 7 **(iv)** 8 **(v)** 9

2 A square of side length 3 cm is placed next to a square of side length 6 cm as shown. The perimeter of the quadrilateral $ABCD$ is:

A $3\sqrt{2}$ cm **B** $3\sqrt{5}$ cm

C $6\sqrt{2}$ cm **D** $3(5 + \sqrt{2} + \sqrt{5})$ cm

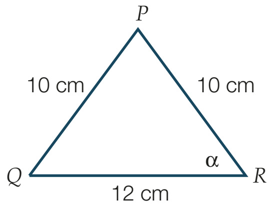


3 In ΔPQR, $PQ = PR = 10$ cm, $QR = 12$ cm.

(a) What type of triangle is PQR?

(b) The perpendicular from P to QR meets QR at S. Calculate the length of PS.

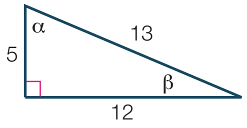
(c) The value of $\sin(\alpha)$ is:



A $\frac{4}{5}$ **B** $\frac{2}{3}$ **C** $\frac{3}{5}$ **D** $\frac{3}{10}$

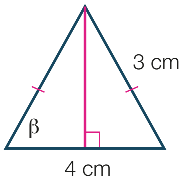
4 Which of the following is equivalent to $\cos(\alpha)$?

A $\sin(\alpha)$ **B** $\cos(\beta)$ **C** $\tan(\beta)$ **D** $\sin(\beta)$



5 For the triangle shown, which statement is correct?

A $\sin(\beta) = \frac{3}{4}$ **B** $\cos(\beta) = \frac{4}{3}$ **C** $\cos(\beta) = \frac{2}{3}$ **D** $\tan(\beta) = \frac{5}{2}$



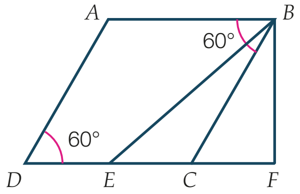
6 $ABCD$ is a rhombus in which the opposite angles B and D are each 60°. CD is bisected at E and BE is drawn. BF is perpendicular to DC produced. Let the sides of the rhombus be $2x$ units.

(a) Write the length of EC.

(b) Use trigonometry to find the length of CF.

(c) Calculate the length of BF as a surd.

(d) Calculate the size of $\angle EBF$.

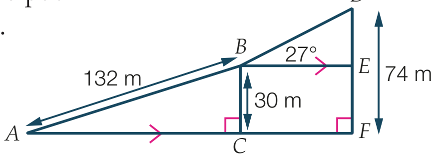


(e) Hence write the size of $\angle EBC$, correct to the nearest minute.

7 A mountain bike rider rides up a hill. The slope of the path he follows is shown in the diagram from A to B to D.

(a) The first stage of the track from A to B is 132 m long. The track rises 30 m over this distance (so $BC = 30$ m).

Find the angle of elevation (θ), to the nearest degree, of the track AB.



(b) If the second stage of the track is inclined at 27° to the horizontal, find the length of the track BD to the nearest metre.

(c) The winning time for a rider to climb the hill from A to D was 1 minute and 30 seconds. Calculate the average speed of the bike in metres per second (m/s). Give your answer correct to 1 decimal place.

Chapter review 7

Maths literacy

adjacent side
angle of depression
angle of elevation
bearing
compass bearing
cosine
degree
hypotenuse
opposite side
reference angle
sine
tangent
trigonometric ratio
trigonometry
true bearing

Copy and complete the following using the words and phrases from this list, where appropriate. A word or phrase may be used more than once.

1 When using the trigonometric ratios, you need to know which angle is to be used. This angle is called the __________.

2 The __________ of an angle is found by dividing the opposite side length by the length of the adjacent side.

3 The only trigonometric ratio that can have a value greater than 1 is __________.

4 A __________ is measured from the north or from the south.

5 The __________ is measured from the horizontal up to the line of sight, whereas the __________ is measured from the horizontal down to the line of sight.

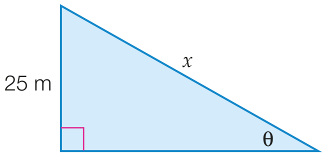
6 A __________ is measured clockwise from north.

Fluency

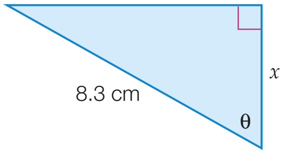
1 Name the two labelled sides of the following triangles as opposite, adjacent or hypotenuse and give the trigonometric ratio that would be used to calculate the unknown side.

7.1

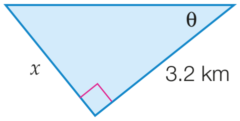
(a)



(b)



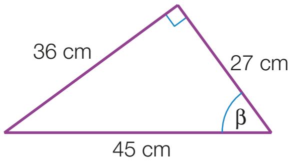
(c)



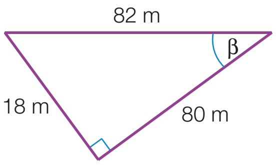
2 Find the value of tan (β) for each triangle, correct to 4 decimal places where appropriate.

7.2

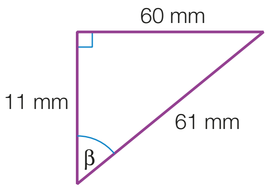
(a)



(b)

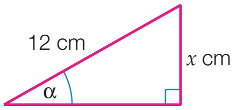


(c)

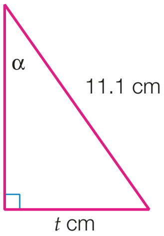


3 If $\sin(\alpha) = \frac{1}{3}$, find the value of the variable for each triangle, correct to 2 decimal places. **7.2**

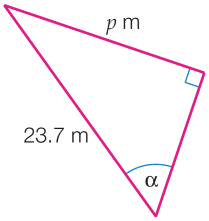
(a)



(b)

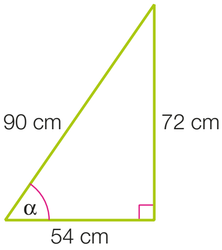


(c)

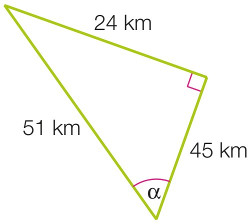


4 Find the exact value of cos (α) for each triangle. **7.2**

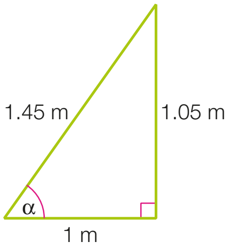
(a)



(b)

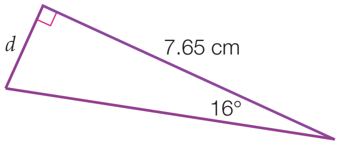


(c)

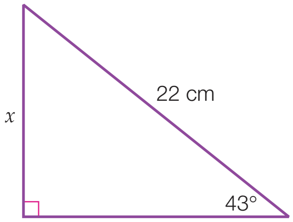


5 Find the value of the unknowns in the following triangles. Give your answers correct to 2 decimal places for side lengths and the nearest degree for angles. **7.3, 7.4**

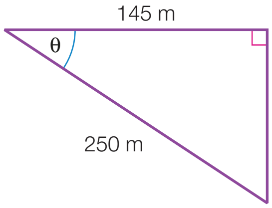
(a)



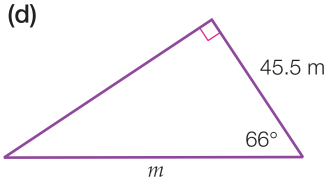
(b)



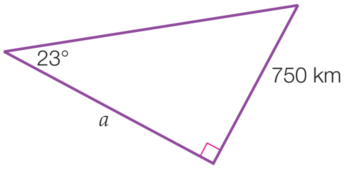
(c)



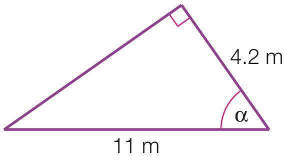
(d)



(e)



(f)



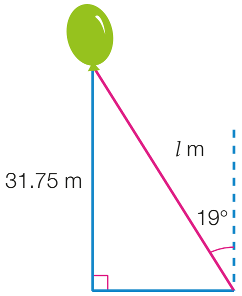
6 A kite is attached to a string that is 15 m long. If the string makes an angle of 26° to the horizontal, the height of the kite is: **7.5**

A 6.6 m **B** 7.4 m **C** 13.5 m **D** 34.2 m

7 A hiker travels 13.2 km on a bearing of 125°T. How far south and east is the hiker from the starting position? Express your answer correct to 2 decimal places. **7.5**

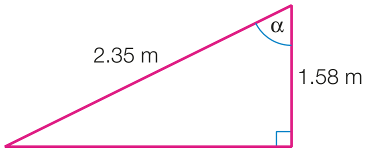
Understanding

8 A balloon on the end of a string is being blown by the wind. If the balloon is 31.75 m above the ground and the string makes an angle of 19° to the vertical, find the length of the string.



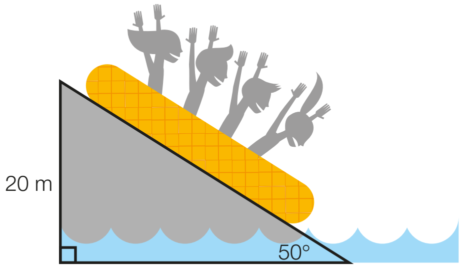
7.3

9 A children's slide has a vertical ladder 1.58 m high. Find the angle the slide makes with this ladder if it is 2.35 m long.



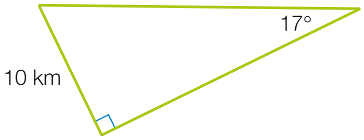
7.4

10 At an amusement park, a scary water raft ride descends a slide at an angle of 50° to the horizontal. If the slide starts 20 m above the ground, how long is the slide?



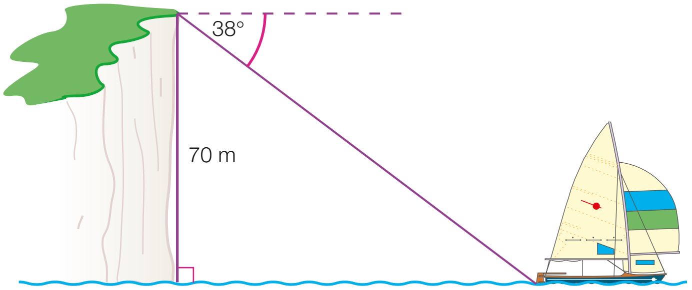
7.4

11 An orienteering trail follows a triangular course as shown. Find the total length of the course, correct to 1 decimal place.



7.3

12 The angle of depression from the top of a 70 m cliff to a boat out at sea is 38°. How far away is the boat from the base of the cliff?



7.5

13 A submarine travelled 56 km on a bearing of 025°T from point *A* to point *B*. How far to the north has the submarine travelled from its starting point?

7.5

14 A four-wheel drive solar car is being tested in the Simpson Desert. It travels north for 25 km, then turns and travels 14 km due west. What bearing must the car travel to return directly to its starting point, to the nearest degree?

7.5

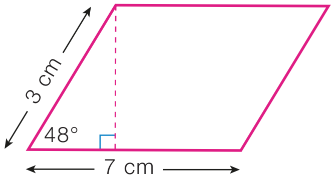
Reasoning

15 A 365 cm ladder leaning against a wall makes an angle of 68.2° with the ground. If the ladder slips 76 cm down the wall, what angle (correct to 1 decimal place) does it now make with the ground?

7.4

16 **(a)** Find the height of this parallelogram. Give your answer correct to 2 decimal places.

(b) Find the area of this parallelogram.



7.3

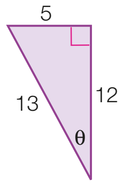
17 When Colin is 85 m away from a building he notices a window washer at an angle of elevation of 62° climbing up the side of the building. What will be the angle of elevation when the window washer has climbed up a further 10 m? Give your answer correct to 1 decimal place.

7.4

Numeracy practice 7

Non-calculator

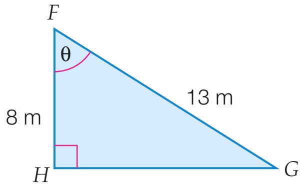
1 In the diagram on the right, the exact value of sin (θ) is:



A $\frac{5}{12}$ B $\frac{5}{13}$

C $\frac{12}{13}$ D $\frac{13}{12}$

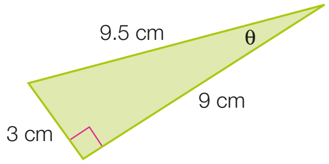
2 Which trigonometric ratio would be used to calculate angle θ?



A sin B cos

C tan D Pythagoras' theorem

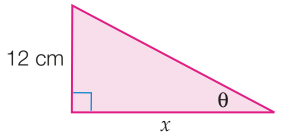
3 What is the tan ratio for θ in the triangle shown?



A $\frac{1}{3}$ B $\frac{9}{9.5}$

C $\frac{3}{9.5}$ D $\frac{9.5}{9}$

4 If tan (θ) = $\frac{3}{4}$, then the value of the variable in the diagram on the right is:

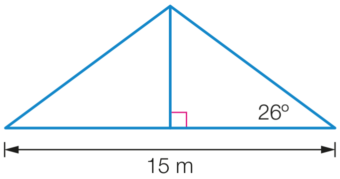


A 3 cm B 4 cm

C 9 cm D 16 cm

Calculator allowed

5 Part of a roof frame is shown. The frame has two symmetrical sides. What is the minimum length of timber needed for the diagonal parts of the frame?



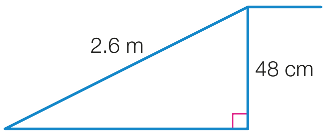
A 8 m B 9 m

C 16 m D 17 m

6 An aircraft is a horizontal distance of 4000 m from the airport, flying at 2500 m, when it is instructed to descend for landing. To the nearest degree, at what angle will it need to descend?

A 32° B 39° C 51° D 58°

7 A wheelchair ramp stretches from the ground to the top of a step 48 cm high. The ramp is 2.6 m long. To the nearest degree, what angle does the ramp make with the ground?



A 10° B 11°

C 32° D 33°

8 A rescue helicopter hovering 95 m above the ocean spots a swimmer at an angle of depression of 32°. What horizontal distance will the helicopter need to travel to be directly above the swimmer?

A 95 m B 112 m C 152 m D 179 m

8

Statistics and probability

8

Predicting risk. What do tsunamis and the global financial crisis have in common? The answer could be probability and statistics!

Risk analysis is a process used by businesses and other organisations to assess the likely effects of various events. For example, people must assess the probability and likely impact of natural disasters such as tsunamis, and prepare accordingly. Similarly, businesses must assess the probability of unknown events happening, such as a global financial crisis, and allocate resources to limit their risk.

Forum

What issues might car insurers consider when assessing your likelihood to claim on car insurance? Why do they bother doing this anyway? In a business context, a company may be thinking about introducing a new product. How do you think probability would play a part in their decision-making processes?

Why learn this?

The most common applications of statistics and probability you have seen are probably associated with sports and with gambling. However, statistics and probability reach much further into people's lives than just sports and games. Investing money, insurance and savings are areas where probability plays a significant role. Probability and statistics also play an important part in decision-making by businesses, governments and other organisations.

After completing this chapter you will be able to:

- understand different kinds of data and different methods of collecting data
- understand random sampling and the collection of data from primary and secondary sources
- describe, interpret and compare data sets in terms of their centre and spread
- investigate how surveys estimate population means and medians
- construct frequency tables, back-to-back stem-and-leaf plots, frequency column graphs and histograms
- describe data using words such as 'skewed' and 'symmetric'
- calculate relative frequency from given or collected data
- list all outcomes for two-step probability events using tree diagrams and arrays
- understand the difference between probability events that involve 'with replacement' and 'without replacement'
- calculate probabilities for multiple events that involve the use of 'and' and 'or'.

Recall 8

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

1 Calculate the mean, median and range of the following data (correct to 1 decimal place):

20, 17, 24, 28, 32, 14, 21, 18, 14, 22, 16, 25

2 A standard six-sided die is rolled once. Calculate the following probabilities as fractions:

(a) Pr(rolling a 2) **(b)** Pr(not rolling a multiple of 3) **(c)** Pr(rolling 4 or 5)

3 Find the mean (correct to 2 decimal places), and the median of the following data set:

Number of siblings	0	1	2	3	4	5	6
Frequency	12	17	23	20	18	4	2

4 List the sample space for drawing a particular suit from a standard pack of 52 playing cards.

5 For the following stem-and-leaf plots, calculate the **(i)** mean and **(ii)** median, and **(iii)** comment on the skewness of the data. Give your answer correct to 2 decimal places.

(a)

Stem	Leaf
4	8
5	2 3 4 5 6 7 7
6	4 5 5 6 7
7	3 4 5
8	2
9	1
10	9

Key: 4 | 8 = 48

(b)

Stem	Leaf
11	5 7
12	4 5
13	2 3 4 9
14	0 0 2 3 4
15	6 7 8 9 9 9
16	1 3 3 4 5 6
17	0 5 6 7 8

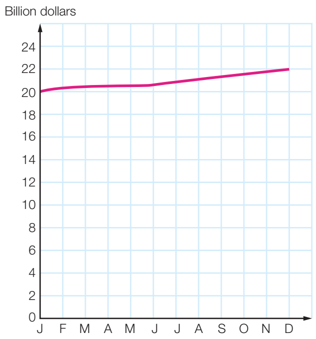
Key: 11 | 5 = 11.5

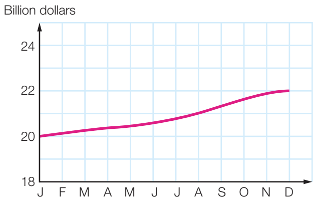
Exploration Task

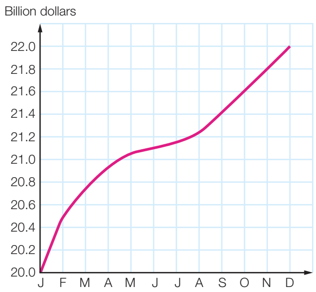
You can download this activity from the eBook or the Pearson Places website.

From data to information

In this activity, you will explore how presentation can change the interpretation of data.







Investigating data

8.1

Data types

Data can be divided into two basic types:

- numerical data sets, where the data consist of numerical values (also called quantitative data)
- categorical data sets, where the data consist of categories or descriptive terms (also called qualitative data).

Numerical data

When the number of students in a classroom is counted, the answer is always a whole number. For example, it is not possible for there to be 23.6 students in a class. When you ask your friends their shoe size you get answers like 5 or $6\frac{1}{2}$. The sizes are either a whole number or a half size.

It is not possible to buy shoes of size $5\frac{2}{3}$. In cases like these, where there are only clearly distinct values that can be obtained, this is **discrete data**.

The heights of students can be any values between the lowest and highest measurements. In practice, measured values are rounded, so that 165 cm might mean any actual height between 164.5 cm and 165.5 cm. As a person grows from 150 cm to 155 cm, their true height must go through a continuous spread of values, including 152 cm, 152.000 000 01 cm, 153.4017 cm and so on. Data sets of continuous numbers like this are described as **continuous data** sets.

To distinguish between these two numerical data types, you can ask, ‘Is the data measured or counted?’ If it is counted, it is discrete; if it is measured, it will usually be continuous.

Worked example 1

W.E. 1

Decide whether the following data sets are discrete or continuous.

(a) The number of goals scored by each team in the World Cup.

(b) The crowd size at each of the games in the World Cup.

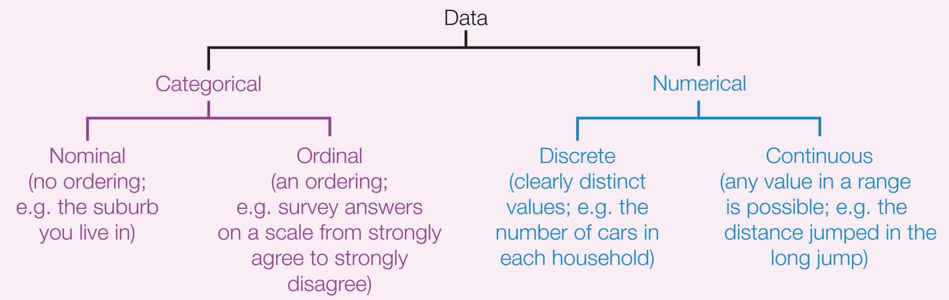
(c) The height of each of the players at the World Cup.

Thinking	**Working**
(a) Is the data set measured or counted? Measured data sets are continuous and counted data sets are discrete. This is a counted value.	(a) Discrete data
(b) Is the data set measured or counted? Measured data sets are continuous and counted data sets are discrete. This is a counted value.	(b) Discrete data
(c) Is the data set measured or counted? Measured data sets are continuous and counted data sets are discrete. This is a measured value.	(c) Continuous data

Categorical data

Categorical data are data that are not numerical. Just as numerical data can be split into two types, so can categorical data. When you record information such as the colour of the cars in the car park, the data simply 'names' the information, so this is called **nominal data**. When there is some order associated with the categories, then you have **ordinal data**. For example, the order might involve ratings on a scale from 'strongly agree' to 'strongly disagree'.

The different types of data can be summarised:



Investigating data-related questions

Some data-related questions can be answered by looking for and analysing existing data. The internet is a good source of data, but it is sometimes hard to find exactly what you want when you are not collecting the data yourself.

Data collected by someone else is called **secondary data**. Data you collect yourself is called **primary data**. When collecting primary data you can ask your own particular questions to find specific information. For example, you might conduct a survey to find the level of support for a change to a school's uniform.

'Which Australian state or territory has the fewest traffic accidents?' is an example where you can analyse secondary data. An internet search may identify government data related to road safety.

For example, consider the number of road deaths for each state and territory in one year, summarised in the following table.

State/territory	NSW	Vic	Qld	SA	WA	Tas	NT	ACT
Road deaths	397	303	327	99	209	40	75	14

This seems to indicate that New South Wales is the most dangerous state. But is this a fair conclusion? There are more people in New South Wales, so the total number may not be a fair way to compare with smaller states.

This next table shows the road deaths per 100 000 people in each of the states and territories.

State/territory	NSW	Vic	Qld	SA	WA	Tas	NT	ACT
Road deaths	397	303	327	99	209	40	75	14
Road deaths per 100 000 people	5.70	5.72	7.64	6.18	9.66	8.03	34.10	4.07

This shows that New South Wales has fewer road deaths per 100 000 people than other states and territories except the Australian Capital Territory, and that the Northern Territory has the most road deaths per 100 000 people.

Having more complete data allows you to analyse and understand it better.

Now consider the data of road deaths per 10 000 registered vehicles.

State/Territory	NSW	Vic	Qld	SA	WA	Tas	NT	ACT
Road deaths	397	303	327	99	209	40	75	14
Road deaths per 100 000 people	5.70	5.72	7.64	6.18	9.66	8.03	34.10	4.07
Road deaths per 10 000 vehicles	0.88	0.77	1.03	0.84	1.20	1.02	6.10	0.58

Again, this shows that the Northern Territory has the worst result while the Australian Capital Territory and Victoria have the best results.

The data seem to support the view that for this year of data, the Northern Territory was the most dangerous place to be on the roads and that the Australian Capital Territory was the safest.

Conducting a survey

People are surveyed about a wide variety of things, from voting preference to the TV programs they watch or the radio station they listen to. Surveys may be conducted by printed or online questionnaires, by phone or by personal interview. Before you can conduct a survey, there are many things to be decided to ensure your survey gives you usable and accurate information.

Designing a survey

First, a survey needs to be designed and its methods need to be decided on. Suppose that your school is thinking about changing the colours of the school uniform. It would be sensible for a committee to come up with a small range of uniform colour options, which can then be shown to the school community.

The design of your survey might be to simply have pictures of each uniform colour option, for people to rank in order of preference. The current uniform should be included as one of the options too.

With any survey, the questions you ask should have definite answers that can be recorded and tallied easily. You must decide what type of survey to give (for example, an interview or a printed questionnaire).

The next thing to consider is who should be surveyed. In the school uniform example there are three different interest groups to consider: students, parents, teachers.

Running a survey

You need to decide how many of each group should be surveyed. To survey an entire school population may be feasible in small schools (with fewer than 500 students), but in schools with more than 1500 students this would be slow and difficult. A representative sample of the school population would normally be used instead.

When an entire population is surveyed, this is called a **census**. Most surveys only involve a smaller sample of the population, so they are not a census.

Analysing survey results

After you get your survey results, you need to analyse them to decide what they mean. How do you organise and display your data? What do you do if there is no clear preference? Do you use a preferential voting system to allocate second preferences?

Preferential voting is when the options are ranked using numbers from 1 (first preference) to last. If no option has a simple majority after counting first preferences, then the least popular option is eliminated and its next preferences are added to the other totals. The least popular options are eliminated in order until one option has a majority.

Preferential voting is used in Australian state and federal elections. Because voting is compulsory in Australia for all eligible voters, an election here is a kind of census. However, in many other countries voting is optional, so the voters are a (self-selected) sample.

Observational data

Another way of collecting primary data is to simply observe it and record it yourself. For example, you could go outside and count the number of vehicles that pass the school during ten 3-minute periods. Data collected in this way is called **observational data**. However, you must be careful when drawing up the observation charts. For example, think about observing and recording the colour of the vehicles passing your school. How many colours should you be looking for? (Is 'dark green' different to 'green'?) You may need to have a category called 'other' to include unexpected categories.

8.1 Investigating data

Navigator

Answers p. 679

1, 2, 3, 4, 5, 6, 7, 9, 11	1, 2, 3, 4, 5, 6, 7, 8, 10, 12	1, 2, 3, 4, 5, 6, 7, 8, 10, 12

Fluency

W.E. 1

1 For each numerical data set, state whether it is discrete or continuous.

- **(a)** The number of apples in each 2 kg bag at the supermarket.
- **(b)** The weight of each bag of apples individually selected by customers in a fruit shop.
- **(c)** The head circumference of each player in a basketball team.
- **(d)** The number of matches in each of 30 boxes of matches checked during quality control.
- **(e)** The time taken by each competitor to complete the 100 m freestyle at the most recent Olympic Games.
- **(f)** The greatest recorded height cleared by each competitor in the pole vault at the most recent Commonwealth Games.
- **(g)** The number of children in each family on your street.
- **(h)** The average time it takes each student in your class to get to school each day.

2 For each of these data sets, state whether it is numerical or categorical.

(a) The colour of each car in the car park at the supermarket.

(b) The eye colour of each student in your class.

(c) The amount of pocket money received each week by each member of your class.

(d) The number of grains of sand in each of 10 separate matchboxes filled at the beach.

(e) The mass of each car in the carpark at your school.

(f) The mass of each student in your class, recorded correct to the nearest kg.

(g) The response of each person surveyed to a question with possible answers: agree, unsure, disagree.

(h) The party of choice for each person voting in a state election.

3 (i) For each of the data sets in Question **2** that you identified as numerical, state whether it is discrete or continuous.

(ii) For each of the data sets in Question **2** that you identified as categorical, state whether it is nominal or ordinal.

4 (a) Which one of these data sets could not be obtained by observation?

A the number of traffic lights along a road

B the proportion of people wearing coats

C the number of Holdens in a carpark

D the proportion of secondary students in Queensland born in Brisbane

(b) Which one of these numbers could be obtained using a survey?

A the proportion of blue cars parked in a street

B the number of farms in the Mallee

C the proportion of students in a school who use a bus to get to school

D the number of National Trust houses in Australia

5 Some road accident data related to motorcyclists are included in the following table.

	NSW	Vic	Qld	SA	WA	Tas	NT	ACT
Number of deaths	55	43	72	17	36	8	10	4
Deaths per 10 000 registered motorcyclists	3.71	3.16	5.17	4.30	4.67	6.51	20.28	3.94

Copy and complete the analysis of safe and dangerous places for motorcyclists by choosing the correct options in the sentences below.

The safest state or territory, with the lowest registered deaths per 10 000 registered motorcyclists, is (ACT *or* Vic). Although the most motorcyclists are killed in (Qld *or* NSW), the rate is much worse in (WA *or* NT). The deaths per 10 000 motorcyclists gives a (more *or* less) accurate picture than the raw figures.

Understanding

6 Use the data sets below to answer the questions that follow.

A Year 9 students rate a proposed new school bag as: awesome; good; satisfactory; poor

B maximum daily temperatures for Canberra this month

C number of siblings of each of the students in your class

D the main method of transport used by each student in your school

(a) Which data sets are categorical?

(b) Which of the categorical data sets from part **(a)** is nominal?

(c) Which data sets are numerical?

(d) Which of the numerical data sets from part **(c)** are discrete?

(e) Which data set is most likely to involve secondary sources?

7 Choose the most precise data type for each of the following.

(a) height of players in the Australian netball team

A discrete **B** continuous **C** categorical **D** categorical

(b) shoe size of players in the National Basketball League

A discrete **B** continuous **C** categorical **D** ordinal

(c) eyesight test results of umpires on a cricket umpires panel

A discrete **B** categorical **C** categorical **D** ordinal

8 Chad is conducting a survey related to the school canteen. The first few questions from Chad's survey are shown below.

Question 1: How often do you buy something from the school canteen?

Question 2: What is the best thing about the school canteen?

Question 3: What items should be available at the school canteen that are not presently available?

His friend Narelle tells Chad that his questions are too open and he would be better off including some suggested answers to his questions that can be chosen by the people surveyed.

Chad agrees with this suggestion. Give some options that Chad may include for each of these questions.

Reasoning

9 Sophie records the number of days each week that the maximum temperature is greater than 22 °C. Explain why Sophie's set of data is discrete and not continuous.

10 One of the questions on a statistics test was 'Is height continuous data or discrete data?'

Joe's answer was as follows:

Height is discrete because when I was measured in Physical Education class I was told by Mr Keeley that I was 172.2 cm. This is a discrete value.

Ngarla's answer was as follows:

Height is continuous because every value between two numbers is possible. At some stage I was 145 cm tall and sometime later I was 146 cm tall. There must have been a time when I was 145.6 cm tall, or 145.68 cm tall and even 145.687 cm tall. It is all to do with how precisely we measure the height.

Make some comments about these two answers. Are either of them correct? How would you answer the question?

Open-ended

11 **(a)** Write two data sets that are discrete. **(b)** Write two data sets that are continuous.

(c) Write two data sets that are categorical. **(d)** Write two data sets that are ordinal.

12 Your teacher has directed you to construct a recording sheet related to the type of vehicles that pass your school in a certain period of time.

(a) How many different types of vehicle do you think should be included on your sheet?

(b) If left to design the recording sheet individually, do you think other students will come up with the same list as you?

(c) Would it be a problem if different students had different sheets and were recording data on different streets that bordered your school?

Puzzle

Equation squares

Copy and complete the following puzzle by filling in all the blank spaces (not the blue squares) so that all the equations are true. A number may be repeated in a row or column, but all numbers must be positive whole numbers.

Now that you have completed that task, see if you can construct another puzzle of your own. But do not put zero in the bottom right-hand corner! Why would that make the puzzle easier to complete?

2	×		+		=	5
×	🟦	×	🟦	×	🟦	×
	×	4	−		=	
+	🟦	+	🟦	−	🟦	−
4	−		+	1	=	
=	🟦	=	🟦	=	🟦	=
8	+		+		=	23

8.2

Interpreting data

A census is a data set for an entire population, while a sample is a data set for only a portion of the population. Samples are taken so that predictions about the whole population can be made based on the results. For the predictions to be valid, you need to make sure that the sample is representative of the whole population and is not biased.

This means you need to make sure that no part of a population is over-represented in the sample. For example, if you wanted to find out the average number of children per family in your local area, so you stood outside a childcare centre and surveyed people there, then you would probably get a value that was too high, because people with no children would probably not be outside a childcare centre.

Random samples

Random sampling is usually a good way to avoid bias. A **random sample** is a sample for which every member of the population has the same chance of being selected. For example, you could achieve this in a small group by writing the name of each person on a piece of paper and selecting names from a hat. For bigger populations, you might assign a number to each person and use random numbers to identify those people selected.

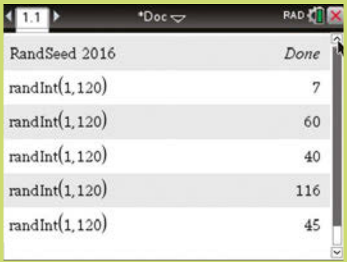
Random numbers

A random number generator will produce sequences of digits that can be used to match identification numbers for a population. Many scientific calculators and all CAS technology can produce random integers that are convenient for this purpose. Any repeated numbers can be used only once.

Using TI-Nspire CAS

For your CAS to generate random numbers, it should be 'seeded' first. To do this, select **menu** > **Probability** > **Random** > **Seed** and enter any 4-digit number as the 'seed'. You only need to do this once.

To generate a random number, select **menu** > **Probability** > **Random** > **Integer**, then enter the lowest possible and highest possible values separated by a comma as shown.

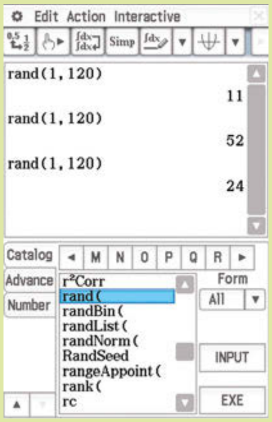


```
RandSeed 2016          Done
randInt(1,120)            7
randInt(1,120)           60
randInt(1,120)           40
randInt(1,120)          116
randInt(1,120)           45
```

Using Casio ClassPad CAS

From the menu select **Main**. Open the **Keyboard** and go to the **Catalog** tab to find the command **rand(**.

To generate a random number, enter **rand(** followed by the lowest possible and highest possible values, separated by a comma as shown.



```
rand(1,120)
                 11
rand(1,120)
                 52
rand(1,120)
                 24
```

Interpreting survey data

After collecting data and calculating statistics of the data, the next step is interpretation, to understand the meaning of the data. This is an important skill that is also useful when trying to interpret statistics presented in daily life, as in news reports.

A useful way to consider a data set is by using a single value to represent key information about the data set. This value is called a **measure of centre** or a **measure of location**.

You should be familiar with three important statistical measures like this: mean, median and mode.

- To find the **mean** of data, the total of the values is divided by the number of values. If the data values are x and the number of values is n, then the formula for the mean is $\bar{x} = \frac{\Sigma x}{n}$ where the Greek letter sigma Σ means 'the sum of'.
- The word **average** usually refers to the mean. However, in everyday use, 'average' does not always refer precisely to the mean, so you should be careful.
- The **median** is the middle data value when the values are in order. If there is an even number of data values, so that there are two middle values, then the median is the average of the two middle values.
- The **mode** is the most common (repeated) data value. There may be several modal values.

Worked example 2

W.E. 2

For the following data set, find **(a)** the mean and **(b)** the median.

2.6, 5.1, 3.7, 9.4, 3.1, 7.7, 8.5, 3.6

Round the mean value correct to 2 decimal places.

Thinking	Working
(a) **1** Write the formula for the mean.	(a) $\bar{x} = \frac{\Sigma x}{n}$
2 Find the total of all values.	$\Sigma x = 43.7$
3 Divide by the number of values.	$\bar{x} = \frac{43.7}{8}$ $= 5.4625$
4 Write the answer, correctly rounded.	Mean ≈ 5.46
(b) **1** Rewrite the data in ascending order.	(b) 2.6 3.1 3.6 3.7 5.1 7.7 8.5 9.4
2 Circle the middle value(s).	2.6 3.1 3.6 $\boxed{3.7\ \ 5.1}$ 7.7 8.5 9.4
3 Average the middle values if necessary.	$\frac{3.7 + 5.1}{2} = 4.4$
4 Write the answer.	Median $= 4.4$

Both the mean and median are often used in statistical reports. The mean can be distorted by extreme outlier data values, so in some cases it may be better to use the median for better comparison. If the data set is relatively symmetrical in its distribution, then both the mean and median will provide useful information about the data.

In most situations, when dealing with continuous data sets, the mode is not used very often. This is because there may be no data value that is present more than once, and so the 'most common' data value is not significant.

8.2 Interpreting data

Navigator

Answers p. 680

1, 2, 3, 4, 5, 6, 9, 10	1, 2, 3, 4, 5, 6, 7, 9, 10	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

Equipment required: scientific calculator or CAS for Questions **2**, **6**, **15**

Fluency

W.E. 2

1 Find **(i)** the mean and **(ii)** the median for each data set.

Round the mean correct to the number of decimal places given in brackets.

(a) 3 5 6 2 0 7 3 4 (1 d.p.)

(b) 21 28 25 24 29 (1 d.p.)

(c) 65.8 49.7 72.6 44.5 61.2 86.4 51.0 44.9 73.6 (2 d.p.)

(d) 0.13 0.51 0.14 0.75 0.68 0.42 (3 d.p.)

2 **(a)** Select a random sample of 10 out of a population of whole numbers from 1 to 100 by:

(i) using a random number generator (calculator or CAS)

(ii) putting numbers into a container and choosing them one at a time.

Summarise the class results to see if all numbers were equally represented.

(b) Select a random sample of 5 out of a population of 25 by:

(i) using a random number generator (calculator or CAS)

(ii) putting numbers into a container and choosing them one at a time.

Summarise the class results to see if all numbers were equally represented.

3 When choosing the random samples in Question **2**, if you get two identical numbers you should:

A ignore one of them and select another

B keep the two identical numbers

C discard both

D choose another selection process.

Understanding

4 A random number generator can be used to simulate the rolling of a die. The following are the results for a set of 30 rolls:

5, 5, 5, 3, 5, 2, 5, 6, 3, 1, 3, 3, 2, 2, 5, 6, 1, 1, 6, 1, 4, 6, 3, 3, 5, 5, 4, 1, 1, 5

(a) Which number appeared the most often?

(b) Do you think every set of 30 rolls would produce the same results?

(c) Use a random number generator to produce three sets of 30 rolls and comment on your results.

5 The two data sets A and B are almost identical.

A	25	25	26	26	27	27	27	27
B	25	25	26	26	27	27	27	35

(a) Find the mean of each data set.

(b) Find the median of each data set.

(c) Find the mode of each data set.

(d) Which measure is affected by the different data value?

6 In November of a particular year there was rainfall on 8 days and no rainfall on the other days. The rainfall (in mm) for the days that rain fell is given below.

15 23 16 2 37 54 12 49

Find the mean and median daily rainfall for November. Round the mean value correct to 1 decimal place.

Reasoning

7 May-li wants to achieve a mean score of at least 8 (out of 10) in her Maths tests. So far her scores are: 9 6 8 7.

(a) What minimum score does May-li have to get in the next test to get the required mean?

(b) If there are actually two tests remaining, what scores does May-li need to achieve at least a mean of 8? Give all the possibilities for the two tests.

8 There are 28 students in a class. Three are absent during a test. The remaining students achieve an average (mean) score of 53.2. When the three absent students return to do the test later, their scores are: 43 49 50.

Find the average score for all 28 students. Round correct to 1 decimal place.

9 A business employs the following staff:

Executives:	11	Managers:	26
Salespeople:	34	Office staff:	89

The owners of the business want to find out whether the employees would like a company gymnasium to be set up in the building, what facilities they would want the gym to have and when they would like it to be open. The owners decide to survey the employees.

(a) What is the population in this case?

(b) Would it be fair to sample four employees from each category of workers? Whose opinion would this favour?

(c) Given the relatively small total population, what other option should be examined?

Open-ended

10 Below is a population of 10 data values.

5.6 2.3 4.7 8.6 5.9 3.7 4.6 2.8 5.0 3.5

(a) Find the mean and median of this population.

(b) Use a random number generator to select a sample of three different data values from the population. Find the mean and median of your sample. Round the mean value correct to 2 decimal places where necessary.

(c) Repeat the previous question part for five more samples of the population.

(d) Compare your sample means and medians with the population mean and median.

11 Each year the company J.D. Power and Associates releases the results of a survey related to the reliability of cars and other vehicles over a 3-year period. The graph following shows the results for more than 34 000 original owners of vehicles after three years of ownership. The survey is designed to predict the reliability of vehicles.

(a) Which brand did the best in this survey?

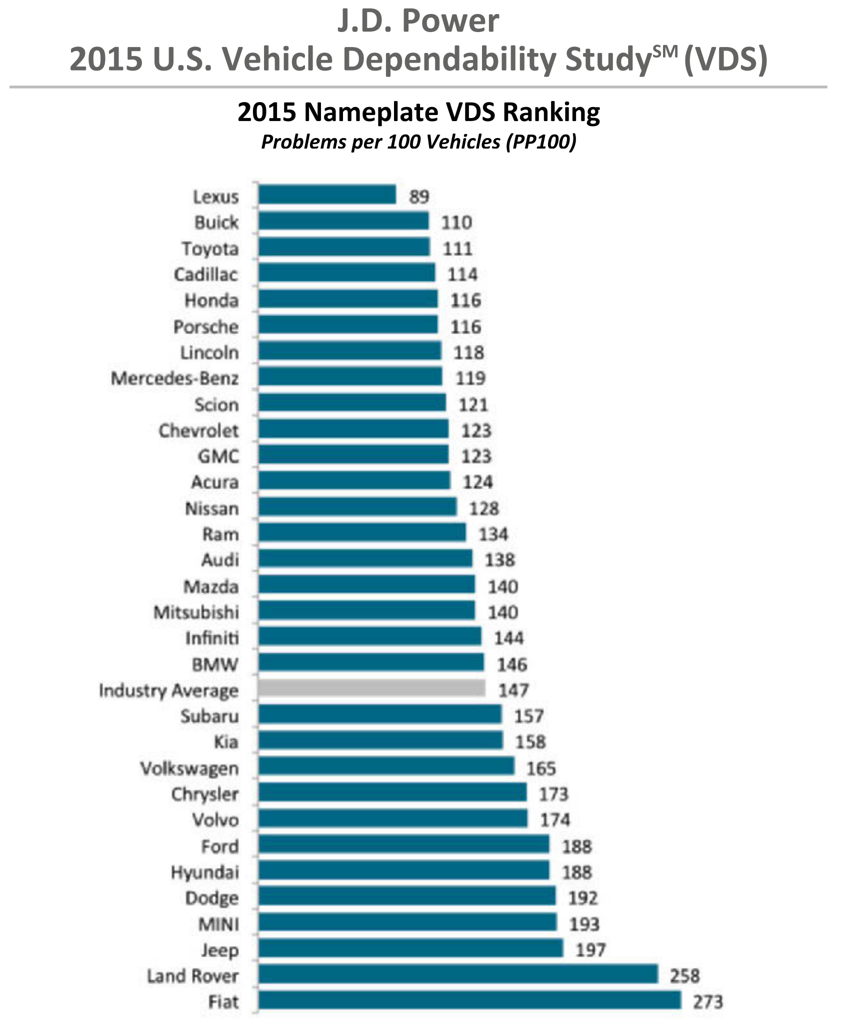
(b) What is the median number of problems per hundred vehicles in the survey?

(c) Which measure of centre does the 'industry average' refer to?

(d) Take a random sample of five brands and find the mean and median number of problems per 100 vehicles.

(e) Take a random sample of 10 brands and find the mean and median for each.

(f) Compare the mean and median values for the sample of 5 and the sample of 10 with the population mean and median values. Use the statistics from others in your class before making your evaluation.



Note: Included in the study but not ranked due to small sample size is Jaguar.

Source: J.D. Power 2015 U.S. Vehicle Dependability Study^{SM} (VDS)

Charts and graphs extracted from this press release for use by the media must be accompanied by a statement identifying J.D. Power as the publisher and the study from which it originated as the source. Rankings are based on numerical scores, and not necessarily on statistical significance. No advertising or other promotional use can be made of the information in this release or J.D. Power survey results without the express prior written consent of J.D. Power.

Puzzle

Code breaking

To solve these puzzles, use the clues to find each number matching each letter. Each number is a whole number from 1 to 6. The same whole number may be repeated in a particular code.

Code 1

A	B	C	D	E	F

A + B = 8 D × E = 15 B + F = 8 C × E = 25 D − A = 1

Code 2

A	B	C	D	E	F

D × E = 2 A × F = 9 C − E = 0 B ÷ E = 2 B + E = 6

Statistics from grouped data

8.3

Data in frequency tables

Organising data as lists in tables can be useful to help calculate statistics more easily. A **frequency table** can be used to find the mean of data as follows.

Step 1: Write each unique data value in a table column x, in order.

Step 2: In the next table column, write the number of times that each data value x appears in the data set. This is the frequency f of each data value.

Step 3: Write the total sum of the frequency column as Σf. This gives the total number of data values, so $\Sigma f = n$.

Step 4: Multiply each x-value by its frequency f and enter each product xf into a new column.

Step 5: Write the sum of the xf column values as Σxf. This gives the sum of all of the data.

Step 6: You can divide Σxf by Σf to find the mean: $\bar{x} = \dfrac{\Sigma xf}{\Sigma f}$

You can also use the frequency table to find the median. Count down the frequency f column, adding frequencies, until the 'middle' position (half of the total n) is reached. This is the median value.

If n is odd, the position of this median value can be calculated as $\dfrac{n+1}{2}$.

If n is even, the position of this median value can be calculated as $\dfrac{n}{2}$.

Worked example 3

W.E. 3

Test scores for Ms Statt's class of 26 students are collated into a frequency table as shown. Find the average test score for the class using **(a)** mean, **(b)** median and **(c)** mode.

Round the mean value correct to 1 decimal place.

Test scores	Number of students
5	2
6	8
7	5
8	9
9	2

Thinking

(a) **1** Call the data values x and the number of students f.

2 Extend the table to form a column for xf values and a row for totals.

3 Multiply each data value by its frequency.

4 Total the f and xf columns (but not the x column as there is not just one of each value).

Working

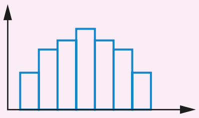
(a)

x	f	xf
5	2	10
6	8	48
7	5	35
8	9	72
9	2	18
Total	$\Sigma f = 26$	$\Sigma xf = 183$

	5	Use the formula to find the mean.	$\bar{x} = \frac{\Sigma xf}{\Sigma f}$ $= \frac{183}{26}$ $= 7.03\ldots$
	6	Write the answer, correctly rounded to 1 decimal place.	Mean = 7.0
(b)	1	Calculate the position of the middle score using $\frac{n+1}{2}$ if n is odd or $\frac{n}{2}$ and the next position when n is even.	(b) $n = 26$ $\frac{n}{2} = 13$ The middle is between the 13th and 14th scores.
	2	Move down the frequency column adding frequencies until the middle position is located.	$2 + 8 = 10$ $10 + 5 = 15$
	3	State the value of the corresponding score or average the values either side of the middle.	Positions 11–15 have scores of 7. Hence the 13th and 14th scores are both 7. Median = 7
(c)		Locate the highest value of the frequency, then write the corresponding data value.	The highest frequency is 9. $f = 9 \leftrightarrow x = 8$ Mode = 8

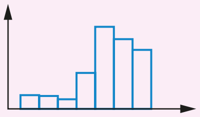
Symmetry and skew

When you look at statistical graphs, such as **frequency column graphs** and **histograms**, you need to describe what you can see. A **symmetrical graph** has data values with frequency evenly distributed about the median or middle value, whereas a **skewed graph** has more high-frequency values on one side of the median and a 'tail' of more lower-frequency values on the other side.



Symmetrical

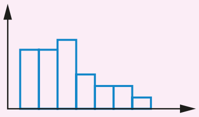
- If a graph is symmetrical, or close to symmetrical, then the mean, median and mode will be of very similar (central) values.



Negative skew

- If there are higher frequencies to the right and lower frequencies as a 'tail' off to the left, this is **negative skew**. The mean is to the left (negative) of the median.

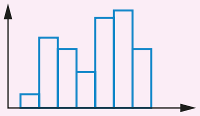
  median > mean



Positive skew

- If there are higher frequencies to the left and lower frequencies as a 'tail' off to the right, this is **positive skew**. The mean is to the right (positive) of the median.

  median < mean



Bi-modal

- If the graph is not skewed to one side or another and appears to have more than one peak, then the data may be **bi-modal** (two most frequent values) or multi-modal.

Worked example 4

W.E. 4

Use the following statistics for each data set to decide whether the data sets are symmetrical, bi-modal or skewed. If the data is skewed, state whether the skew is positive or negative.

(a) mode = 87
mean = 87.5
median = 87.2

(b) mode = 16
mean = 22.5
median = 19.3

Thinking	Working
(a) 1 Consider whether the mean, median and mode are close together.	(a) The values of the mean, median and mode are close together.
2 If so, the data set is symmetrical.	The data set is symmetrical.
(b) 1 Consider whether the mean, median and mode are close together.	(b) The values of the mean, median and mode are not close together. The mean is on the right of the median and the mode.
2 If not, look at which side of the mode and median the mean is located.	Therefore, the data are positively skewed.

Grouped data

When you are dealing with a large data set there are two choices: deal with the raw data, or group the data. If data is grouped, then the calculation of the statistics may be less accurate, but the calculations will be easier and faster.

When choosing the size of the group (also called the 'class' or the 'bin') aim to have between 5 and 10 groups. Group sizes of 2, 5, 10, 20, 50 etc. (convenient round numbers for calculations) are usually the best to work with.

When you have grouped data, you need a value to represent each group of values. This value is called the class centre or class midpoint. It is found by adding the endpoints together and dividing by 2.

Finding statistics of grouped data

To estimate the mean, you can use the class centre or midpoint of each group as the x-values in your calculations. The estimated mean is then the mean of the class centres.

Instead of finding the median, you can find the **median class interval**, which is the class that contains the median.

Instead of finding the mode, you can find the modal class, which is the class that is most common (highest frequency).

Worked example 5

W.E. 5

A sheep farmer wants to keep some lambs for breeding. For this to happen the lambs must reach a certain weight. The farmer decides to use statistics to plan a breeding program. The data in the frequency table gives the weight (in kg), of the lambs in the flock.

Weight (kg)	Frequency
25–<30	5
30–<35	12
35–<40	14
40–<45	16
45–<50	13
50–<55	10
55–<60	2

(a) Calculate the mean weight of the lambs. Give your answer correct to 1 decimal place.

(b) Calculate the median class interval for the weights of the lambs.

(c) State the modal class.

(d) Draw a histogram of the data.

(e) Describe the spread of the data in terms of its skewness.

Thinking | Working

(a) **1** To calculate the mean you need to find the class centre and multiply this value by the frequency.

(a)

Weight (kg)	x	Frequency	xf
25–<30	27.5	5	137.5
30–<35	32.5	12	390
35–<40	37.5	14	525
40–<45	42.5	16	680
45–<50	47.5	13	617.5
50–<55	52.5	10	525
55–<60	57.5	2	115
		$\Sigma f = 72$	$\Sigma xf = 2990$

2 The mean is then found by calculating: $\dfrac{\Sigma xf}{\Sigma f}$

$$\text{Mean} = \frac{\Sigma xf}{\Sigma f}$$
$$= \frac{2990}{72}$$
$$= 41.5 \text{ kg (1 d.p.)}$$

(b) Find the median class by counting down the frequency column until the middle is found. (Here, there are 72 values, so the median is between the 36th and 37th values.)

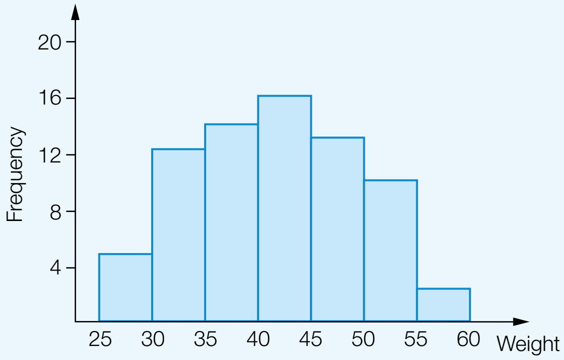
(b) Both the 36th and 37th values are in the 40–<45 class, so the median class interval is between 40 and 45 kg.

(c) Write which class corresponds to the highest frequency.

(c) The highest frequency 16 is for the class 40–<45, so the modal class is (40–<45) kg.

(d) Draw a histogram in the usual way, remembering to leave half a column width before starting the first column.

(d)



(e) Describe the shape of the histogram. Compare the mean value with the median class.

(e) *The graph is appropriately symmetrical.*

The estimated mean (41.5) is within the median class (40–<45).

The data is fairly symmetrical.

Using technology

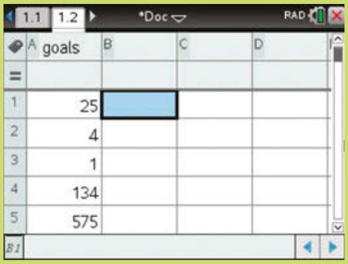
If you have a large list of raw data spread out over a large range, then it can take time to sort this into class intervals and draw a frequency graph to analyse its symmetry or skewness. However, technology makes this much easier, because it means you can use the same quick calculation process even for very large data sets.

Consider the following data set, the number of goals scored by each player on an AFL team list:

25	4	1	134	575	198	27	2	106	14	73	0	0
84	49	423	66	0	55	153	17	1	3	19	0	0
11	1	6	15	17	57	0	40	19	0	0	26	0
2	0	0	0	0	0	0						

Using TI-Nspire CAS

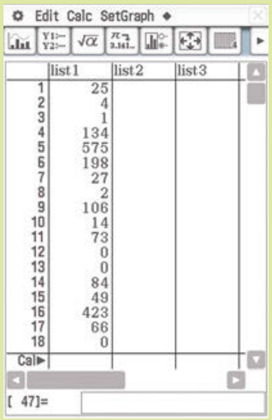
1 Add **Lists & Spreadsheet** to your document. Enter the name of column **A** as **goals**. Enter all the data from above into this column, starting in the first row.



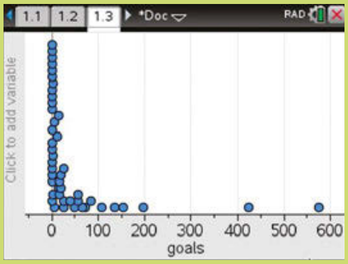
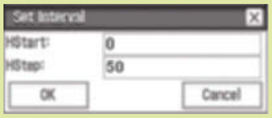
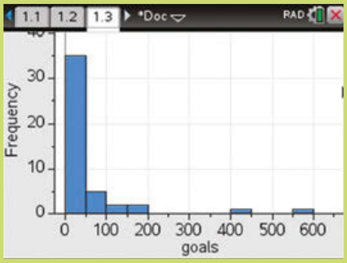
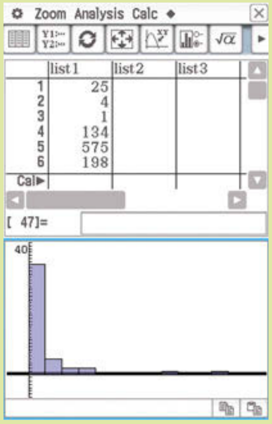
	A goals	B	C	D
=				
1	25			
2	4			
3	1			
4	134			
5	575			

Using Casio ClassPad CAS

1 From the menu select **Statistics**. Enter all the data from above into **list1**, starting in the first row.



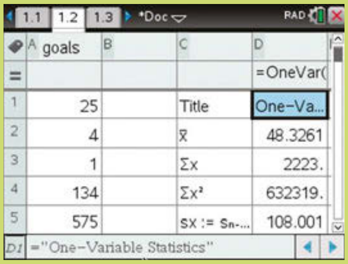
	list1	list2	list3
1	25		
2	4		
3	1		
4	134		
5	575		
6	198		
7	27		
8	2		
9	106		
10	14		
11	73		
12	0		
13	0		
14	84		
15	49		
16	423		
17	66		
18	0		

Using TI-Nspire CAS	Using Casio ClassPad CAS
2 Add **Data & Statistics** to your document. This will show a scattering of data points. Use the cursor to select the bottom of the screen, where the text shows **Click to add variable**, and select the variable **goals**. This will then display a dot plot.  	2 To create a frequency graph (histogram), select **SetGraph** > **Setting…** and set the **Type** to **Histogram** and the **XList** to **list1** as shown.  Draw: ● On ○ Off Type: Histogram XList: list1 Freq: 1 Then select the draw graph symbol 📊 and set the histogram start point **HStart** at 0 and the step value **HStep** at 50 as shown.  HStart: 0 HStep: 50
3 To create the graph, select **menu** > **Plot Type** > **Histogram**.  By default, the class interval has been set as 50. To change the class interval, you can select **menu** > **Plot Properties** > **Histogram Properties** > **Bin Settings** > **Equal Bin Width**.	3 The frequency graph will appear in the lower screen.  1 25 2 4 3 1 4 134 5 575 6 198 To change the class interval, you can go back to the data list and then do this again with a different **HStep** value.

The frequency graph shows that the data are positively skewed and that there are probably two outliers, 423 and 575. Before you started, you may have suspected these would be outliers!

Using TI-Nspire CAS

4 To find the key statistical values, go back to the list of data by selecting the tab **1.1** at the top of the window. Then select **menu** > **Statistics** > **Stat Calculations** > **One-Variable Statistics…**. Set **Num of Lists** to 1 and then select **goals** as the **X1 List**. You may have to scroll to see all the results.



	A goals	B	C	D
=				=OneVar(
1	25		Title	One-Va…
2	4		x̄	48.3261
3	1		Σx	2223.
4	134		Σx²	632319.
5	575		sx := sn-…	108.001

D1 ="One-Variable Statistics"

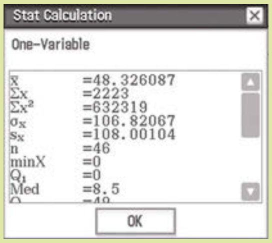
The statistics values should be given as follows:

Mean ($\bar{x}$) = 48.3261
Median (**MedianX**) = 8.5
IQR (**Q_3X** – **Q_1X**) = 49
Range (**MaxX** – **MinX**) = 575

Note that the range and IQR are not displayed, so they must be calculated from the given five-figure summary.

Using Casio ClassPad CAS

4 To find the key statistical values, select **Calc** > **One-Variable** and set the **XList** to **list1**. You may have to scroll to see all the results.



```
One-Variable

x̄      =48.326087
Σx     =2223
Σx²    =632319
σx     =106.82067
sx     =108.00104
n      =46
minX   =0
Q1     =0
Med    =8.5
```

The statistics values should be given as follows:

Mean ($\bar{x}$) = 48.3261
Median (**Med**) = 8.5
Mode = 0
IQR (**Q_3** – **Q_1**) = 49
Range (**maxX** – **minX**) = 575

Note that the range and IQR are not displayed, so they must be calculated from the given five-figure summary.

If you are working with grouped data, then you need to use the midpoint of each class interval instead of the individual (ungrouped) data values. You can then enter the frequencies into the second column and use the CAS similarly to find the statistics.

8.3 Statistics from grouped data

Navigator

1, 2, 3 (a), 4 (a), 5, 6, 7, 9, 12 (a, b), 13

1, 2, 3 (a), 4 (a), 5, 6, 7, 8 (a, b), 9, 12, 13, 14

1, 2, 3 (b), 4 (b), 5, 6, 7, 8 (c–f), 10, 11, 12, 13, 14

Answers p. 680

Fluency

1 The number of siblings for each student in a class is summarised in the table.

Analyse the average number of siblings per student by finding the following measures:

(a) mean, correct to 1 decimal place

(b) median

(c) mode

Number of siblings	Number of students
0	4
1	12
2	2
3	3
4	1

W.E. 3

2 Decide, using the following statistics for each data set, whether the data set is symmetrical or skewed. If the data are skewed, state whether the skew is positive or negative.

(a) mode = 68
mean = 60
median = 63.5

(b) mode = 132
mean = 158
median = 144

(c) mode = 235
mean = 233
median = 234

W.E. 4

3 **(a)** At a train station, each train is reported as delayed if it is 60 seconds or more late. The station records the number of seconds for each delay reported over a 60-day period. The number of seconds per delay are summarised in the table.

Seconds per delay	Frequency (f)
60–69	2
70–79	2
80–89	3
90–99	6
100–109	12
110–119	13
120–129	15
130–139	7

Use the grouped data to:

- **(i)** calculate the mean number of seconds per delay (give your answer correct to 2 decimal places).
- **(ii)** calculate the median class interval for the number of seconds per delay
- **(iii)** draw a frequency column graph of the data
- **(iv)** describe the spread of the data.

(b) The number of cars in a council car park was recorded every hour over a 60-hour period starting from midnight. The data are summarised below.

Number of cars	Frequency (f)
0–49	9
50–99	8
100–149	17
150–199	5
200–249	13
250–299	8

Use the grouped data to:

- **(i)** calculate the mean number of cars (give your answer correct to 2 decimal places).
- **(ii)** calculate the median class interval for the number of cars
- **(iii)** draw a frequency column graph of the cars
- **(iv)** describe the spread of the data.

W.E. 5

4 To ensure the correct amount of feed is ordered, a horse trainer with a large stable of horses keeps records of each horse's weight. The weights (in kg) of each horse are recorded in a frequency table so that the trainer can calculate useful statistics on the horses.

Weight (kg)	Frequency
250–<300	3
300–<350	13
350–<400	17
400–<450	20
450–<500	16
500–<550	14
550–<600	4

(a) Calculate the mean weight of the horses. Give your answer correct to the nearest kilogram.

(b) Calculate the median class interval for the weights of the horses.

(c) Draw a histogram of the data.

(d) Describe the spread of the data in terms of its skewness.

Hint

To find the class centre, add the endpoints and divide by 2.

5 **(a)** The class centre for a class interval of 0–19 is:

A 9 **B** 9.5 **C** 10 **D** 10.5

(b) The class centre for a class interval of 10–20 is:

A 14.5 **B** 15 **C** 15.25 **D** 15.5

6 The symmetrical set of data A has had some values altered to form the new set of data B, which is now skewed.

A

x	*f*
5	1
6	6
7	8
8	6
9	1

B

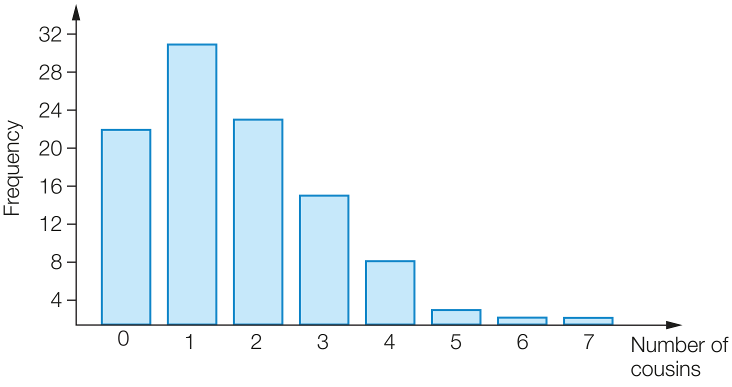
x	*f*
5	2
6	3
7	6
8	7
9	4

(a) Find the mean and median for the symmetrical set of data and for the skewed set of data. Round the mean correct to 1 decimal place where necessary.

(b) Complete the summary by filling in the words 'mean' and 'median':
For negatively skewed data, with a tail to the left, ____________ < ____________.

Understanding

7 The following graph shows the number of cousins for each of the students in Year 8.



(a) Describe the data in terms of its skewness.

(b) Estimate the mean number of cousins.

(c) Now, calculate the mean number of cousins. How good was your estimate?

(d) Do you think this is a realistic spread of cousins? What could you do to check?

8 Find the class intervals for each of the following lists of class centres for discrete data.

(a) 5.5, 15.5, 25.5, 35.5, 45.5

(b) 2, 7, 12, 17, 22

(c) 24.5, 74.5, 124.5, 174.5, 224.5

(d) 25.5, 75.5, 125.5, 175.5, 225.5

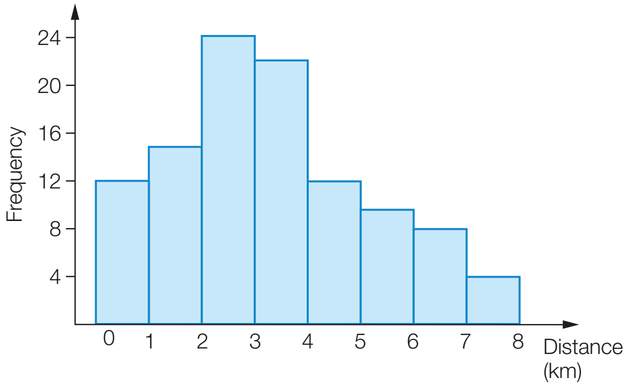
9 The following frequency table shows the time taken, in minutes, by the students at a particular school to complete a cross-country race.

Time (min)	Frequency
20–<22	2
22–<24	12
24–<26	27
26–<28	15
28–<30	8
30–<32	5
32–<34	1

(a) Calculate the mean time taken by the students to complete the cross-country race.

(b) Draw a histogram and use it to describe the data set.

10 The following histogram shows the distance travelled to school, measured in km, by each of the students in Year 9 at Westbow High School.



(a) Describe the graph in terms of symmetry or skewness.

(b) Make an estimate of the mean distance travelled to school.

Reasoning

11 The table shows the number of appearances by the 27 members of the Brisbane Roar soccer squad, five games into the 2015–2016 season.

Number of appearances	Frequency
0–19	12
20–39	3
40–59	5
60–79	1
80–99	1
100–119	4
120–139	0
140–159	0
160–169	1

(a) Calculate the estimated mean number of appearances for the members of the squad.

(b) State the interval that represents the median class.

(c) Which of these statistical measures do you think is better for describing the number of appearances for the squad?

(d) Matt McKay is the player with the highest number of appearances for the team. If Matt was replaced by a new player at game six, calculate the effect on the mean and median.

(e) How well do these new measures represent the number of appearances by the squad?

12 Answer questions for the stem-and-leaf plots below.

(i)

Stem	Leaf
3	7
4	1 3 5 6 8 9 9
5	2 4 6 6 9
6	4 7 7
7	3
8	5
9	2

Key: 3 | 7 = 3.7

(ii)

Stem	Leaf
14	5 8
15	1 6 7
16	1 3 6 6
17	2 5 7 9 9
18	2 5 5
19	4 5 9
20	0 6

Key: 14 | 5 = 145

(a) State the shape of the set of data in terms of symmetry or skewness.

(b) Calculate the mean from the raw data. Round correct to 1 decimal place where necessary.

(c) Convert the stem-and-leaf plot to a frequency table of grouped data. (For example, 14 | 5 8 becomes 140–149 with a frequency of 2.) Use the frequency table to calculate the estimated mean for each data set. Round the mean value correct to 1 decimal place.

(d) Compare the estimated mean (from the grouped data) with the actual mean.

(e) Due to a data error, the values in **(ii)** given as 145 and 148 were supposed to be 195 and 198 instead. Describe the shape of the stem-and-leaf plot if it is redrawn using the correct data.

Open-ended

13 **(a)** Construct a stem-and-leaf plot with at least 20 values so that the median is bigger than the mean.

(b) Construct a stem-and-leaf plot with at least 20 values so that the median is smaller than the mean.

14 A set of data is grouped to give the following frequency table.

Class	Frequency
0–<10	3
10–<20	6
20–<30	6
30–<40	2
40–<50	1

(a) Calculate the estimated mean from the grouped data. Round your answer correct to 1 decimal place.

(b) Make your own set of raw data to match the frequency table. Write each data value correct to 1 decimal place.

(c) Find the mean of your raw data. Round correct to 1 decimal place.

(d) Compare the estimated mean with your actual mean.

Puzzle

Complete the pattern

In each case you need to find the missing numbers in the pattern.

1

2	5	3	6	4	

2

11	13	16	18	21	

3

57		38			17	-27

4

-6	-7		-12	-16		-27

5 Now, see if you can make up another four number patterns of your own and get another student to try to solve them. What makes each pattern difficult or easy to find?

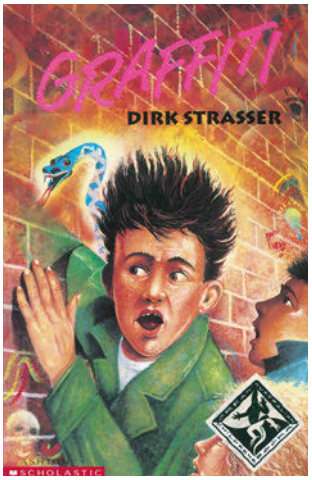
Investigation

Reading matters!

Equipment required: a couple of novels, some newspapers

Book reviewers often consider the ‘readability’ level of a novel. The number of long words used can give an impression of readability, but a more precise indication can be found by applying the test developed by Edward Fry.

The Fry readability test



The Fry readability test:

Step 1: Select a section at random from the text and count 100 words.

Step 2: Count the number of sentences (to the nearest sentence) in the 100 words.

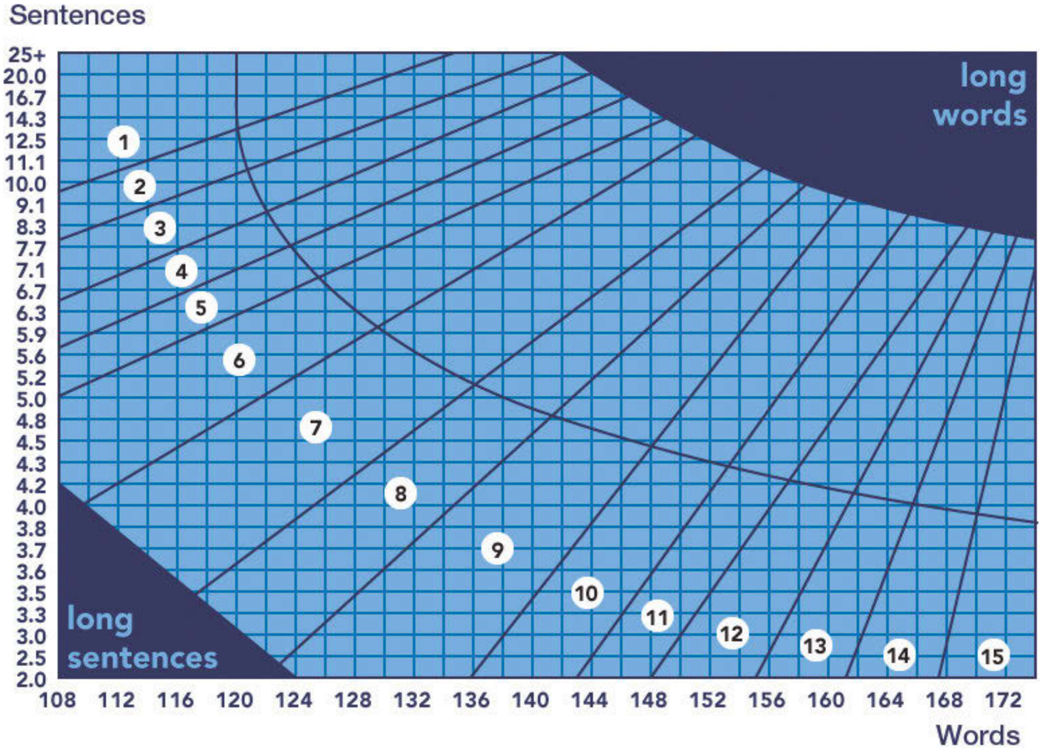
Step 3: Count the number of syllables in the 100 words (e.g. ‘graffiti’ has three syllables: gra-ffi-ti).

Step 4: Repeat steps 1–3 for two other 100-word sections.

Step 5: Find the mean number of sentences and syllables.

Section of text	Number of sentences in 100 words	Number of syllables in 100 words
1		
2		
3		
Mean values		

Step 6: Use the mean values and the chart shown below to determine the readability level.



The Big Question

Are the novels that you read written to a suitable readability level?

Engage

1 A 100-word section from the novel *Graffiti* appears below.

> I moved to the side and checked my shirt pocket. People behind me streamed past. Where did I put my ticket? I know I put it somewhere where I wouldn't lose it. Was that a policeman over there? Why was he staring at me? Stay calm, I told myself. Is it in my backpack? In my jacket pocket? No. I started to sweat like I do during a football match. I could see myself in gaol, eating bread and water. Aunt Vivien would keep promising to visit me, but she wouldn't turn up. I'd never get the chance to have my tooth fixed. Serving a life sentence with a gap in my mouth—I couldn't think of anything worse. Help!
>
> D. Strasser, *Graffiti*, Ashton Scholastic, p. 11

- **(a)** How many sentences does this contain?
- **(b)** How many syllables are there?
- **(c)** Use the chart and your answers to parts **(a)** and **(b)** above to determine a readability level based on just this one section. Why is it better to average results for three sections?

Explore

2 Check the readability level of a text you are reading now. Randomly choose a page, identify a section that is 100 words long and count the number of sentences and syllables. Repeat this for two other randomly selected sections and find the mean number of sentences and the mean number of syllables.

(To randomly choose a page in a book, you can just open the book wherever you like, pick the start of any paragraph you like and count 100 words from there. Then, open the book to another page and choose the start of any paragraph, which will be the starting point for your second lot of 100 words. Then, open the book to another page and follow the same process.)

3 What is the readability level of the text you chose?

Strategy options

- Break problem into manageable parts.
- Make a table.

Explain

4 Do you think if you chose another three sections from the same text that you would get a similar result? Check your theory by randomly choosing another three sections and performing the calculations.

5 Summarise your findings and make a statement about the suitability of the text for a person of your age, based on readability level.

For the purposes of this investigation, how close would you consider to be 'close enough'? For example, if you are 15 and the calculated reading age is 14, would you think this suitable? What about if the calculated reading age is 10?

Elaborate

6 Choose another text you have read recently and check its readability level.

Evaluate

7 Are the texts you are reading at a 'suitable' readability level?

8 Do you think you would ever go through this process before reading a text and then decide on the basis of the readability level whether to read it or not?

9 Does understanding about readability levels help you understand why you may find some texts too difficult to read, while other texts are too simple?

10 Do you think the readability level of a text is the only thing that should be considered when choosing a text to read?

Extend

11 Perform readability tests on different sections within a newspaper (e.g. main news section, sports, reviews). Is there a difference in readability levels within the newspaper? Why?

12 Compare readability levels within a different newspaper or magazine and comment on your findings. Work in small teams to investigate more aspects of each different text.

Gamespace

Rating the ratings

Who decides which shows remain on TV and which are cancelled?

Why is your favourite show suddenly moved from 8:30 pm Friday to 7 pm Tuesday? If the show can be streamed at any time later over the internet, does it matter when it is on first?

TV networks spend millions of dollars on programs that they hope will be successful. Most TV networks make money by selling advertising space during shows (the commercials, or 'ads', that break up programs) to other companies. To get the best value from advertising, companies need to ensure that their ad is shown to an audience as large as possible. And not just a large audience, but preferably their 'target audience', the people most likely to buy their product.

How do we know who's watching what?

To collect data on who is watching which show, a small electronic device, known as a 'peoplemeter', is installed on the TVs in selected households around Australia (with the permission of the people living in the house). The peoplemeter records minute-by-minute information on when the TV is being viewed and what is being viewed. This information is transmitted from the peoplemeter to the central database of a company called OzTAM (Australian Television Audience Management). OzTAM collates and organises all the viewing and makes it available for networks and advertisers to use. This information is known as 'ratings' because it can be used to tell which programs are the most and least popular.

Sampling the population

For 2017, 5250 households across the five major Australian cities (Sydney, Melbourne, Brisbane, Perth and Adelaide) had a peoplemeter installed. More than 3000 peoplemeters were also installed in regional areas, while a further 2120 peoplemeters were used to sample households with subscription television. The households with the peoplemeters installed make up what is called the 'Ratings Panel'.

1 (a) How many households in total make up the Ratings Panel?

(b) Given that the decisions made using the information collected from the Ratings Panel will affect all TV-watching Australians, do you think the number of households with a peoplemeter is a big enough sample ?

How is the sample chosen?

To decide in which households to place the peoplemeters, a large face-to-face survey is conducted. Although not a census, the number of households surveyed is five to ten times the size of the final sample that makes up the Ratings Panel. The survey collects some general information that is used to define the characteristics of the overall population, such as the gender, ages, occupations and education of the people in the household, the number and type of TVs in the household and who has the responsibility of buying the groceries for the household. The households surveyed are then classified into groups, based on household size, location and TV ownership. Households are then invited from each group to form the Ratings Panel by having a peoplemeter installed.

2 (a) Use your answer to Question 1(a) to determine the number of households that would have been surveyed originally. Give your answer as a range.

(b) Why do you think one of the questions is about who is responsible for buying the groceries for the household?

(c) Why do you think the households are classified into the groups described?

Producing the ratings

Imagine that the grand final of a talent show 'The Y Factor' has just been shown. Data from the Ratings Panel peoplemeters shows that 3998 of the Panel's households viewed some or all of the show.

Use the following information to help you answer the questions below:

- There are approximately 8.5 million households in Australia.
- The average number of people in a household is 2.5.

3 (a) What percentage of Ratings Panel households tuned in (correct to 1 decimal place)?

(b) Assuming that the panel's viewing is representative of the rest of Australia, how many Australian households tuned in?

(c) How many people does this represent?

4 Are there any programs that you think would produce quite different results in different states, or areas within states?

Skewing the data?

As well as detecting and sending program data from the TV itself, the peoplemeter has a special remote control that can be used by members of the household to register the number and ages of people watching a particular show.

5 How could this capability be used to distort, or 'skew', the data generated about the household? What would be the reasons for doing so?

Using the data

The viewing data collected from the Ratings Panel can be grouped into categories, such as the gender, age group and occupation of the viewer. This means that advertisers can determine what their 'target audience' is most likely to be watching (i.e. the type of person most likely to purchase the type of product being advertised). OzTAM's data can be split into the following age groups for both men and women:

0–15	13–24	18+	25+	30–49	40–64
5–12	16+	18–29	25–44	35–54	55+
10–15	16–24	18–49	25–54	40+	55–64
13–17	16–54	18–54	25–54*	40–54	65+

*with children

6 Imagine that you were advertising each of the products listed below. Select the viewer age groups you believe would represent your target market(s).

(a) a new model of ute or 4WD car

(b) a new type of hair colour that covers grey hair

(c) a new computer program that helps children learn to read

(d) a new 'energy' drink

Research

7 In the music industry, the Australian Recording Industry Association (ARIA) produces weekly charts of music, such as the Top 50 Singles Chart.

(a) How are charts such as this compiled?

(b) How do you think the information these sorts of charts provides compares with the OzTam information?

8.4 Comparing data sets

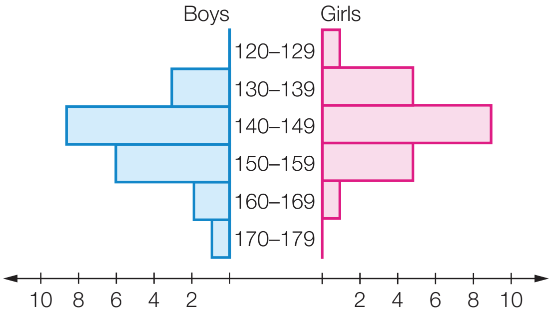
It is often useful to compare data sets that are similar. For instance, you might want to see whether boys and girls of the same age are the same height. To compare this, you can draw a back-to-back stem-and-leaf plot or a back-to-back histogram.

Back-to-back stem-and-leaf plot

Boys	Stem	Girls
	12	9
9 8 4	13	4 4 6 7 9
9 8 6 6 5 5 3 3 2	14	0 1 1 3 4 5 5 5 7
8 3 2 2 0 0	15	0 1 4 5 6
6 4	16	3
3	17	

Key: 12 | 9 = 129 cm

Back-to-back histogram



For the back-to-back stem-and-leaf plot, the right-hand side of the plot is displayed like a normal stem-and-leaf plot. The left-hand plot is displayed as a mirror image of a normal stem-and-leaf plot.

For a back-to-back histogram, you can draw the bars horizontally rather than vertically as usual.

Worked example 6

W.E. 6

The weights (in kg) of the players on the 2016 playing lists for the two AFL teams based in South Australia are as follows.

Adelaide Crows:	79	73	89	72	90	84	78	98	74	94
	86	81	88	94	91	101	82	104	84	72
	77	85	102	87	82	80	84	83	98	91
	86	87	90	81	82	97	86	82	100	75
Port Adelaide:	84	71	85	83	87	94	70	98	89	75
	105	88	89	84	76	80	89	92	78	89
	77	97	74	78	67	91	86	83	81	92
	94	90	79	94	94	82	94	82	81	

(a) Find the range for the complete data set.

(b) Draw a back-to-back stem-and-leaf plot to display the data. Make a statement about which team seems to be the heavier.

Thinking	Working
(a) Find the range by subtracting the smallest value from the largest value.	(a) $105 - 67 = 38$

(b) **1** Construct the back-to-back stem-and-leaf plot.

(b)

Adelaide Crows	Stem	Port Adelaide
	6	7
9 8 7 5 4 3 2 2	7	0 1 4 5 6 7 8 8 9
9 8 7 7 6 6 6 5 4 4 4 3 2 2 2 2 1 1 0	8	0 1 1 2 2 3 3 4 4 5 6 7 8 9 9 9 9
8 8 7 4 4 1 1 0 0	9	0 1 2 2 4 4 4 4 4 7 8
4 2 1 0	10	5

Key: 10|4 = 104 kg

2 Make a statement comparing the data sets.

It would seem that the Adelaide Crows are heavier, with more players who weigh 100 kg or more and no players who weigh less than 72 kg. A check of the median values confirms that the Crows average 85.5 kg while Port Adelaide averages 84 kg.

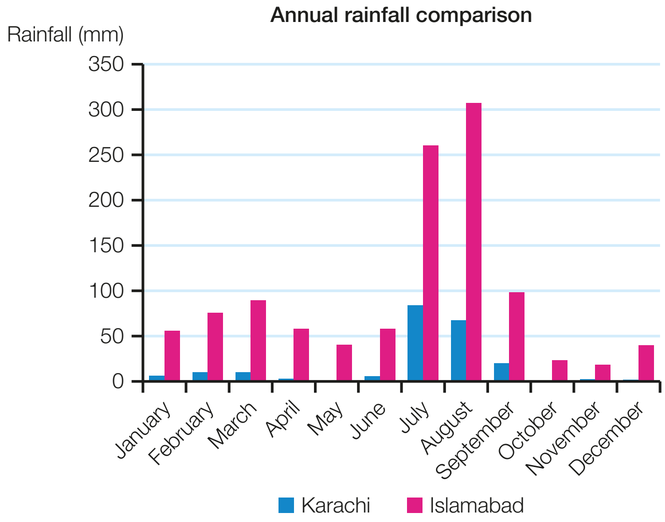
Comparative graphs

You can also compare data sets by looking at side-by-side column or bar graphs. This usually involves some interpretation of what is shown in the graph.

Worked example 7

W.E. 7

Pakistan has very diverse weather. The average monthly rainfall, in mm, for two of its largest cities is shown below.



	Rainfall (mm)	
Month	**Karachi**	**Islamabad**
Jan	6	56
Feb	10	74
Mar	12	89
Apr	4	62
May	0	39
Jun	6	62
Jul	86	267
Aug	67	310
Sep	20	98
Oct	1	29
Nov	2	18
Dec	4	37

(a) Compare the average monthly rainfall for the two cities using just what you can see on the graph.

(b) Use the table to calculate the mean, median and range of monthly rainfall for each city. Then compare the rainfall using these values. Round the mean value to 1 decimal place.

Thinking	Working
(a) Look at the graph and make sure you know which colour represents which city. Describe the graph as best you can.	(a) Islamabad is much wetter than Karachi, but both follow the same general pattern of having the most rain in July, August and September.
(b) 1 Find the mean for each city.	(b) $\bar{x} = \frac{\Sigma x}{n}$ Karachi: $\text{mean} = \frac{218}{12}$ $= 18.166 \ldots$ ≈ 18.2 mm Islamabad: $\text{mean} = \frac{1141}{12}$ $= 95.083 \ldots$ ≈ 95.1 mm
2 Find the median for each city.	Karachi: 0 1 2 4 4 6 6 10 12 20 67 86 median = 6 mm Islamabad: 18 29 37 39 56 62 62 74 89 98 267 310 median = 62 mm
3 Find the range for each city.	Karachi range: 86 – 0 = 86 mm Islamabad range: 310 – 18 = 292 mm
4 Write a sentence of comparison that refers to each of the three statistics.	The monthly rainfall in Islamabad is, on average, many times higher than in Karachi for both mean and median. The variation in average monthly rainfall in Islamabad is many times higher than in Karachi.

8.4 Comparing data sets

Navigator

1, 2, 3, 4, 5, 6 (a), 10

1, 2, 3, 4, 5, 6 (a, b), 7, 9, 10

1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Answers p. 681

Fluency

1 The heights (in cm) of the players on the 2016 playing lists for the two AFL teams based in Western Australia are as follows.

W.E. 6

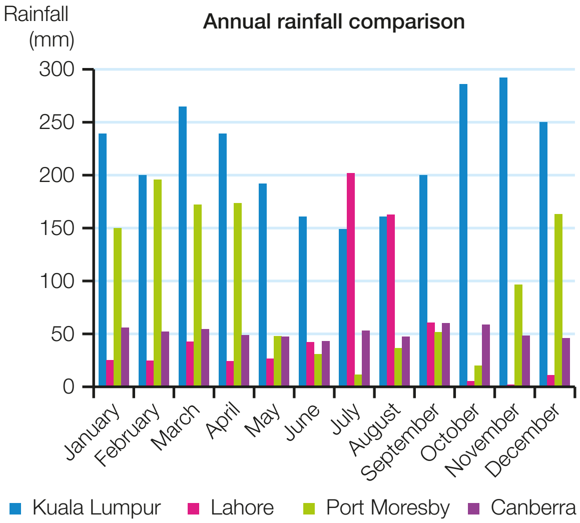
Fremantle Dockers:	197	174	189	185	188	203	185	197	186	190
	179	201	201	182	187	193	186	195	181	188
	189	192	182	176	192	199	182	178	211	187
	192	196	182	180	186	197	177	181		
West Coast Eagles:	185	194	179	190	195	187	184	183	191	183
	187	181	201	185	187	181	191	176	196	193
	183	188	203	196	179	183	195	197	201	186
	185	190	196	185	186	183	185	190		

(a) Find the range for each team.

(b) Draw a back-to-back stem-and-leaf plot to display the data and make a statement about which team seems to be the taller.

W.E. 7

2 The annual monthly rainfall, in mm, for Kuala Lumpur (Malaysia), Lahore (Pakistan), Port Moresby (Papua New Guinea) and Canberra (Australia) is shown below.



Month	**Rainfall (mm)**			
	KL	**Lahore**	**PM**	**Canberra**
Jan	239	23	150	60
Feb	200	29	194	51
Mar	260	41	170	56
Apr	240	20	173	49
May	190	22	49	48
Jun	160	36	31	38
Jul	150	202	12	52
Aug	160	164	37	48
Sep	200	61	53	65
Oct	280	12	20	62
Nov	290	4	97	59
Dec	250	14	164	46

(a) Compare the average monthly rainfall for Kuala Lumpur and Port Moresby using just what you see in the graph.

(b) Use the table to calculate the mean, median and range of monthly rainfall for Kuala Lumpur and Port Moresby. Compare the rainfall using these values. Round the mean value correct to 1 decimal place.

(c) Compare the average monthly rainfall for Lahore and Canberra using just what you see in the graph.

(d) Use the table to calculate the mean, median and range of monthly rainfall for Lahore and Canberra. Compare the rainfall using these values. Round the mean value correct to 1 decimal place.

3 The PGA Tour is one of the major events in the world of golf. The table below shows a summary of the scoring in the first two rounds of the 2015 event.

Score	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89
Frequency Day 1	0	0	0	1	1	7	4	9	15	16	16	15	16	20	9	8	9	1	1	1	1	1	0	0	1	0	1
Frequency Day 2	1	0	2	2	5	7	6	22	18	16	16	10	14	8	5	8	2	1	5	4	1	0	0	0	0	0	0

(a) Group the data in 4s, with the first group being 63–66, and draw a back-to-back histogram (frequency graph) of the data.

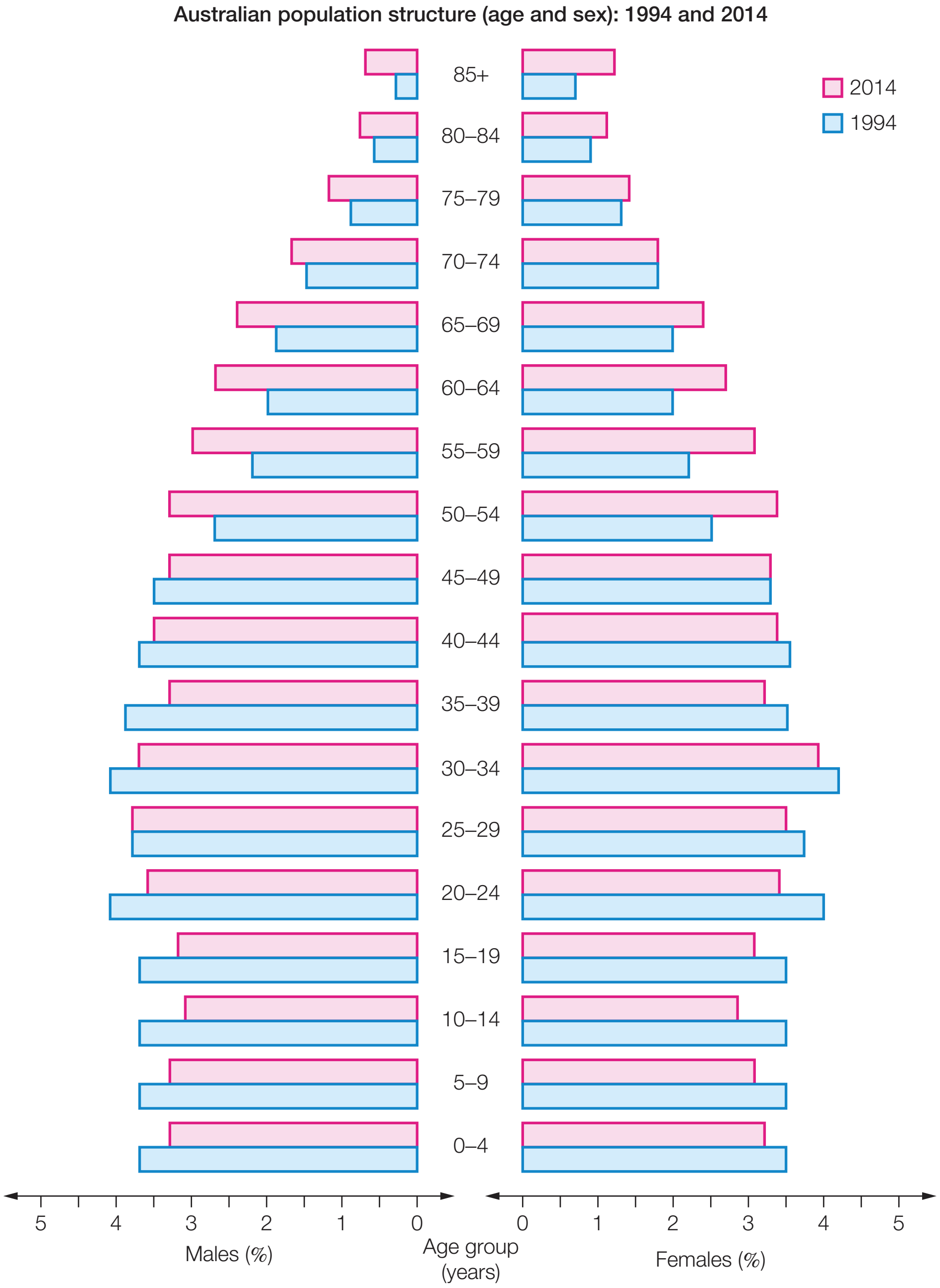
(b) Why was it appropriate to group the data in the way done in part **(a)**?

(c) Use the back-to-back graph to help you decide on which day scoring was easier. In golf, the lower the score, the better it is.

(d) Use the raw data in the frequency table to calculate the mean and median scores for each day. Do these values confirm what the graph shows?

Understanding

4



(a) What is the total value of all of the pink bars representing 2014?

(b) How is it possible for the 85+ bar for women in 2014 to be longer than the 80–84 bar?

(c) Describe what the graph tells you about the changes to the age of the population over the 20 years. Be sure to mention a key element that is missing from the information; how the numbers of males and females compare; and what has happened to the proportion of the young and to the proportion of the elderly.

5 Are Year 9 girls taller or shorter than Year 9 boys? One way to investigate this is to sample some Year 9 students. However, the Australian Bureau of Statistics has organised this for you already through the Census at Schools program. One of the features of the program is that you can sample Australian students based on many different criteria.

From the ABS website, the following data about a sample of 50 Year 9 Victorian students can be obtained.

Male heights (cm):	176	181	154	177	191	190	180	175	164	166
	147	188	175	159	157	183	173	174	177	185
Female heights (cm):	145	162	160	167	153	155	162	178	163	167
	164	161	174	165	171	164	160	170	169	162
	174	167	154	153	168	144	165	160	168	172

(a) Draw a back-to-back stem-and-leaf plot of the data.

(b) What conclusion would you draw from the stem-and-leaf plot drawn?

6 The average monthly rainfall, in mm, for four places in Australia is shown below.

	Jan	**Feb**	**Mar**	**Apr**	**May**	**Jun**	**Jul**	**Aug**	**Sep**	**Oct**	**Nov**	**Dec**
A	276.5	307.6	179.0	66.3	32.5	20.5	13.8	16.4	10.9	24.9	60.2	131.7
B	17.5	11.6	21.6	18.3	41.7	61.4	56.1	49.6	40.6	27.4	18.4	18.4
C	24.5	24.5	20.3	17.4	22.5	22.0	18.9	18.5	20.4	24.5	20.7	21.7
D	87.3	86.6	111.4	154.6	197.1	211.2	225.6	234.9	200.1	173.7	139.3	113.9

(a) Calculate the mean, median and range for each of these places.

(b) Describe the rainfall pattern for each place, including some mention of the seasons of the year.

(c) The four places, in no particular order, are Broken Hill (in outback NSW), Port Lincoln (a coastal town in SA), Weeaproinah (in southern Vic) and Townsville (in tropical north Qld). From the data, decide which place is which and justify your decision.

Reasoning

7 In golf events which are played over four rounds, approximately half the players are eliminated after the first two rounds and do not compete in the last two rounds. The following table shows a golf event's total scores for each player, for the first two rounds and for the third and fourth rounds (if they were still competing).

Rounds 1 and 2:

Score	136	137	138	139	140	141	142	143	144	145	146	147	148	149
Frequency	1	1	2	10	5	7	13	9	9	14	15	11	10	10

Score	150	151	152	153	154	155	156	157	158	159	160	161	162	
Frequency	7	5	5	4	2	4	5	0	1	2	0	1	1	

Rounds 3 and 4:

Score	136	137	138	139	140	141	142	143	144	145	146	147	148	149
Frequency	0	2	3	4	6	2	5	5	4	9	8	9	2	4

Score	150	151	152	153	154	155	156	157	158	159	160	161	162	
Frequency	2	3	2	1	0	0	0	0	0	0	0	0	0	

(a) Explain why grouping in 5s, with a first group of e.g. 136–140, would be the best way to group the data for a back-to-back graph.

(b) What is a problem with this grouping for the rounds 3 and 4 data?

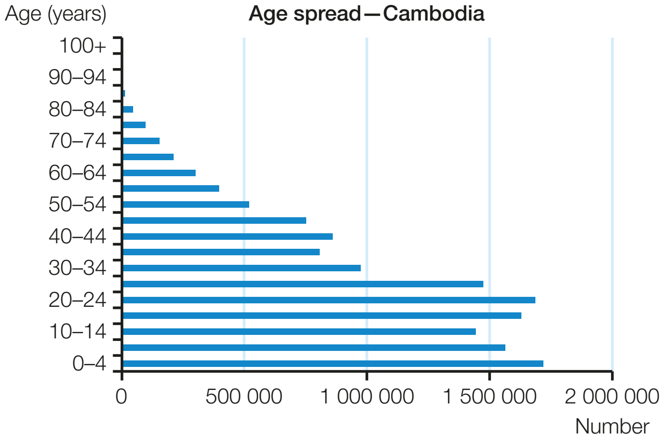
(c) Draw a back-to-back graph using this grouping.

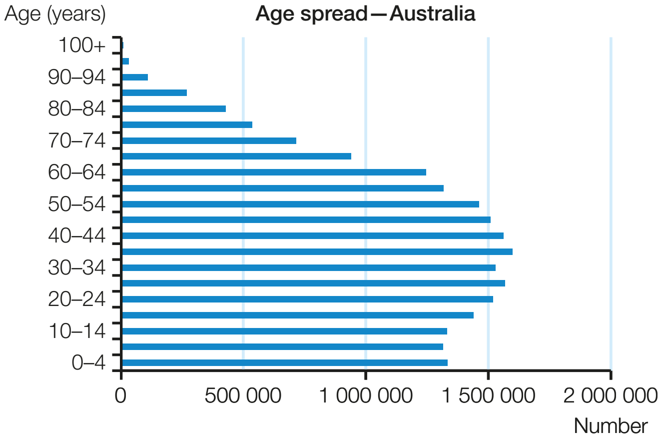
(d) Why do you think the two sides of the graph seem so different?

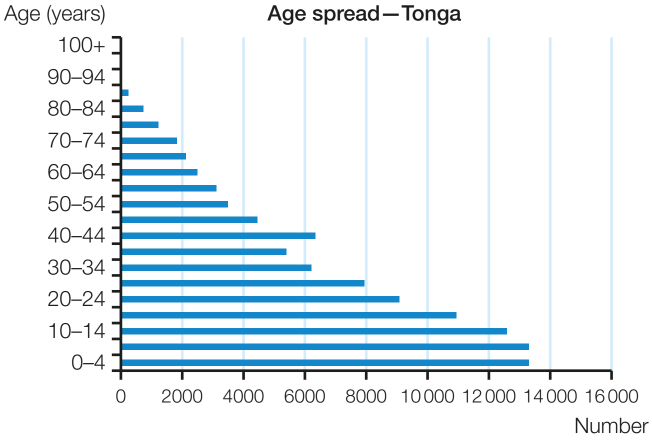
(e) Find the mean and median for each data set and use these to explain the differences.

8 The following graphs show the age spread for people in three countries.

Compare the distribution of ages of people in the three countries. In your answer mention 'scale on the Number axis', 'life span' and 'children' specifically.







Open-ended

9 Construct a back-to-back stem-and-leaf plot where the mean of the right-hand side is greater than its median while the mean of the left-hand side is less than its median. What might this data represent?

10 The website of the Australian Bureau of Statistics provides animated population pyramids of the Australian population that can be viewed in various ways to provide more information about different categories and groups. Visit the website and investigate population in your state (or territory) and compare this with the population 10 years ago, 20 years ago and 50 years ago. Compare this with the population of another state 10 years ago, 20 years ago and 50 years ago. Investigate and compare other topics you are interested in.

Maths 4 Real

COUNTING THE COUNTRY

The Australian Census is completed every 5 years. A genuine attempt is made to collect data about every person in Australia at the time of the census.

1 Why do you think the Government goes to the expense and effort of conducting a census every 5 years? Wouldn't it be easier to just collect some sample data?

The census is only one of the services provided by the Australian Bureau of Statistics (ABS). The ABS collects data and distributes information about many aspects of Australian life.

On 4 May 2011, the ABS estimated the Australian population to be 22 599 195 with a population growth rate of one new person every 1 min 37 s.

2 At this rate, what was the Australian population on 1 July 2011?

The following table shows some recent data related to the Australian population.

Year	Population ($\times 10^3$)	Median age (years)	% born in Europe	% born in Asia
1994	17855	33.4	13.5	4.7
1995	18072	33.7	13.3	4.9
1996	18311	34.0	13.2	5.1
1997	18518	34.4	13.0	5.3
1998	18711	34.8	12.8	5.3
1999	18926	35.1	12.5	5.3
2000	19153	35.4	12.3	5.4
2002	19413	35.7	12.0	5.5
2002	19641	36.0	11.9	5.7
2003	19873	36.2	11.7	5.7
2004	20111	36.4	11.7	5.7

Note: These questions can be answered using technology such as a spreadsheet or CAS.

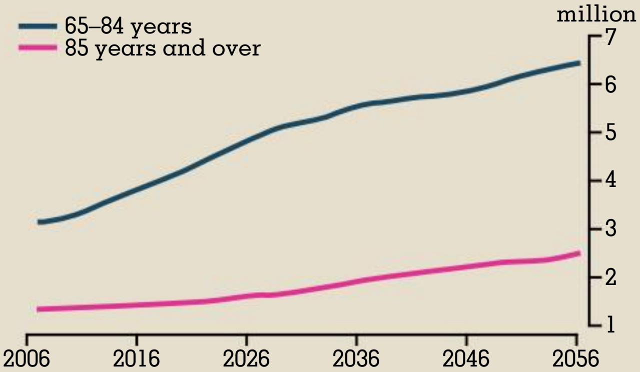
3 Use the data in the table to draw a line graph of the population over the decade. Extend your graph to predict the population in 2017. How does this estimate compare with the figure given earlier in this task?

4 Draw a line graph showing both the percentage of the population born in Europe and the percentage born in Asia. Do you think the percentage born in Asia will become greater than the percentage born in Europe? If so, when do you think this will occur?

5 The median age increased by 3 years in the period shown in the table. What does this tell you about Australia's population?

The following graph shows projected figures related to older people in the Australian population.

Projected aged populations: 2007[(a)]–2056, Series B



(a) Data for 2007 is estimated, all other years are projections

Source: *Population Projections, Australia. 2006 to 2101* (ABS cat no. 3222.0)

6 How might a graph like this be used by local, state and federal members of government when making policy decisions?

Half-time 8

8.2 **1** For each of the following stem-and-leaf plots, calculate the **(i)** mean and **(ii)** median, and **(iii)** comment on the skewness of the data. Where necessary, give your answers correct to 2 decimal places.

(a)

Stem	Leaf
3	0 1 9 9
4	4 5 6 7 9
5	3 8 9 9
6	4 5 6
7	7
8	0 1

Key: 7 | 7 = 77

(b)

Stem	Leaf
12	1 4 7
13	2 5 6 9
14	3 4 7 8 9
15	0 1 2 3 4
16	4 6 7 7
17	1 3 4

Key: 13 | 2 = 13.2

8.1 **2** Choose the most precise data type for each of the following.

(a) Rating on a scale of 1–5, given by members of the audience at a movie premiere.

A discrete **B** ordinal **C** categorical **D** categorical

(b) Time taken by competitors to complete the marathon at the 2016 Olympic Games.

A discrete **B** continuous **C** categorical **D** categorical

(c) Number of buckets of apples picked by each of the pickers at an orchard.

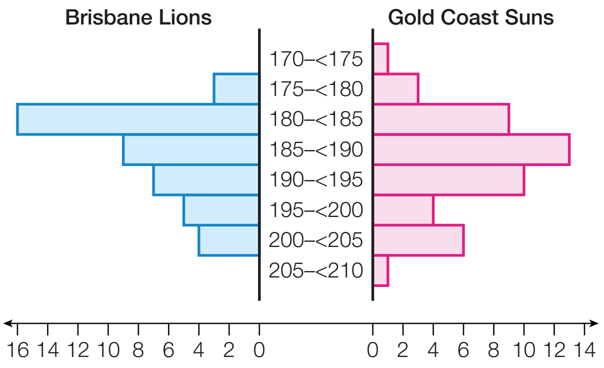
A discrete **B** continuous **C** categorical **D** ordinal

8.1 **3** A local company thinks it needs to change its corporate image and has hired a graphic artist to produce a new company logo. The graphic artist has come up with three possible logos. You are responsible for finding out which logo should be adopted by the company. Explain how you would decide this.

8.4 **4** The following table shows the heights of 40 students sampled from the Year 9 students at a particular school. The measurement has been taken to the nearest cm. The teacher knows that one group is boys and the other is girls, but has forgotten which is which. Draw a back-to-back stem-and-leaf plot and use this to help you decide which is which.

Group 1:	164	153	149	167	181	175	154	167	177	189
	155	166	180	170	173	155	156	183	177	160
Group 2:	172	166	155	161	172	155	151	158	167	171
	166	171	173	155	160	160	179	166	154	153

8.4 **5** The following back-to-back histogram shows the height (in cm) of the players on the 2011 playing lists of two AFL teams.



(a) Which team appears to be taller?

(b) What other information would you like to more fully answer this question?

(c) The mean and median heights for the Brisbane Lions are 188 cm and 186 cm, respectively, while for the Gold Coast Suns the values are 189 cm and 188 cm, respectively. Does this information make it easier for you to compare the teams?

Understanding probability

8.5

The **probability** or **chance** of any event is defined as a number within the range of 0 to 1.

If the probability is 0, the event is said to be **impossible**. For example, the probability of rolling a number greater than 6 with a normal die is 0.

If the probability is 1, the event is said to be **certain**. For example, the probability that the Prime Minister of Australia is an Australian citizen is 1. By defining event A as 'Prime Minister of Australia is an Australian citizen', this probability can be written as $\Pr(A) = 1$.

In probability, success is defined as a favourable outcome. If all outcomes are equally likely, then you can use the following rule:

$$\Pr(\text{success}) = \frac{\text{number of successful equally likely outcomes}}{\text{total number of possible equally likely outcomes}}$$

Complementary probabilities

Situations in probability can be considered so that there are only two possible outcomes: an event may happen ('success') or it may not happen ('failure'). These two possibilities are **complementary events** because Pr(success) + Pr(failure) = 1.

Consider a standard pack of cards that has four suits: hearts, diamonds, clubs and spades. When drawing a card from the pack, you may be interested in Pr(getting a heart) or Pr(not getting a heart). The outcome 'not getting a heart' is the complement of the outcome 'getting a heart'. The sum of the probability of an outcome and its complement is always 1.

To make things easier, letters can be used to represent the outcomes. In this example, H can stand for the outcome 'drawing a heart from the pack'. You can then write $\Pr(H) = \frac{1}{4}$. Using this style of notation 'not H' is the complement of H. So, $\Pr(\text{not } H) = 1 - \frac{1}{4} = \frac{3}{4}$.

$\Pr(A) + \Pr(\text{not } A) = 1$, where '$A$' and 'not A' are complementary .

This can also be expressed as:

$$\Pr(\text{not } A) = 1 - \Pr(A)$$

This rule makes sure that the two events are **mutually exclusive**. This is a necessary condition for complementary events.

Worked example 8

W.E. 8

In a box containing 24 chocolates, there are 10 cream-centred chocolates and 8 hard-centred chocolates, and the rest are filled with nuts. One chocolate is chosen at random. Find the probability of choosing:

(a) a cream-centred chocolate

(b) a chocolate that is not filled with nuts.

Thinking	Working
Assign letters to stand for each of the basic events. By 'choosing at random', each chocolate has the same chance of being selected, so the outcomes are equally likely.	Let C = choosing a cream-centred chocolate H = choosing a hard-centred chocolate N = choosing a chocolate filled with nuts
(a) $\Pr(C) = \frac{\text{no. of cream-centred chocolates}}{\text{total no. of chocolates}}$	(a) $\Pr(C) = \frac{10}{24}$ $= \frac{5}{12}$
(b) not N = choosing a chocolate that is *not* filled with nuts $\Pr(\text{not } N) = \frac{\text{no. of chocolates not filled with nuts}}{\text{total no. of chocolates}}$	(b) $\Pr(\text{not } N) = 1 - \Pr(N)$ $= 1 - \frac{1}{4}$ $= \frac{3}{4}$

Relative frequency

In many situations, although the exact probability of an event is not known, it is still possible to do an experiment or use existing data to see whether the event has occurred in the past. In this case, the **relative frequency** of the event occurring in the past gives an *estimate* of the probability of the event occurring in the future.

$$\text{Relative frequency} = \frac{\text{number of successful trials}}{\text{number of trials}}$$

Worked example 9

W.E. 9

Narelle is having some trouble with her car. Over the past 14 days it failed to start on eight occasions.

(a) Find the relative frequency of the failure to start.

(b) Estimate the probability that Narelle's car will fail to start on the fifteenth day. Write your answer as a percentage, correct to the nearest whole number.

Thinking	Working
(a) **1** Write the relative frequency as a fraction.	(a) Relative frequency $= \frac{\text{number of successful trials}}{\text{number of trials}}$ $= \frac{8}{14}$
2 Cancel down if possible.	$= \frac{4}{7}$
(b) **1** Convert the fraction to a decimal correct to 2 decimal places.	(a) $\Pr(\text{car fails to start}) \approx \frac{4}{7}$ $\approx 0.5714\ldots$
2 Write the decimal as a percentage, correct to the nearest whole number.	$= 57\%$

Theoretical probability is usually written as an exact fraction in simplest form.

Probability that is an estimate from data is usually written as a decimal or percentage, appropriately rounded.

$0 \leq \text{Pr(event)} \leq 1$

$0\% \leq \text{Pr(event)} \leq 100\%$

Expected frequency

Theoretical probability and relative frequency from existing data are both useful in estimating future occurrences of an event.

If an ordinary coin is tossed 100 times, you would estimate that heads will appear 50 times. Although getting exactly 50 heads may not happen, it is the best estimate based on the probability of 50%. This is the **expected frequency** of heads.

Expected frequency = Pr(event) × number of trials

Expected frequency = relative frequency × number of trials

Worked example 10

W.E. 10

For two of the first 9 days of school, Malcolm's bus was so late that he missed the start of the school day. Based on these results, with 38 days of school remaining, how many more times do you expect Malcolm to be late?

Thinking	Working
1 Write the relative frequency as a fraction	Relative frequency = $\frac{2}{9}$
2 Multiply the relative frequency by the number of days remaining.	Expected frequency = $\frac{2}{9} \times 38$ $= 8.44 \ldots$
3 Write the answer in words, rounding to fit the context.	Malcolm is likely to be late 8 more times this term.

When conducting probability experiments you often have different numbers of trials. To compare the outcomes for such different experiments you can use the relative frequency of the results. This is usually expressed as a fraction, decimal or percentage and is found by dividing the number of outcomes for a particular result by the total number of outcomes.

8.5 Understanding probability

Navigator

Answers p. 683

1, 2, 3, 4, 5, 6, 8, 10, 11, 12 (a, b), 13, 14, 15 (b), 17, 19	1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12 (a, b), 13, 14, 15 (a), 16, 17, 19, 20	1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20

Fluency

W.E. 8

1 To decide which lane he would swim in for the butterfly swimming event, Rohan drew a numbered token by random selection from a bag. The bag contained eight different tokens numbered from 1 to 8 to represent the eight lanes of the pool. Find the probability that Rohan:

(a) swims in lane 1

(b) swims in lane 8

(c) does not swim in lane 8

(d) swims in an even-numbered lane

(e) swims in one of lanes 2, 3 or 4.

W.E. 9

2 Madelene and Pieter enjoy playing against each other in ten-pin bowling. Over the past few years Madelene has won 26 times and Pieter has won 28 times. Estimate the probability that Madelene will win their next game.

W.E. 10

3 A game involves rolling two dice. A double (rolling the same number on both dice) is theoretically expected to be rolled every 1 in 6 rolls. How many times is a double likely to be rolled in a game of 250 rolls?

4 Your class collects the following data from observations of the cars passing your school.

Car	Holden	Ford	Toyota	Mazda	Honda	Hyundai	Other
Frequency	35	27	31	15	7	10	42

Assuming that these figures are a reliable estimate of all cars on the road, find the probability, written as a decimal correct to 2 decimal places, that the next car observed is:

(a) a Ford (b) a Holden or a Mazda (c) not a Toyota.

5 Elliot, Kris and Austin play card games every day. So far this year Elliot has won 45 games, Kris has won 38 games and Austin has won 67 games. Calculate the estimated probability of each to win the next game. Write your answers as decimals rounded to 2 decimal places where necessary.

6 **(a)** The probability of choosing a black jelly bean by random selection from the black and pink jelly beans shown is:

A $\frac{1}{4}$ **B** $\frac{2}{5}$

C $\frac{3}{5}$ **D** $\frac{2}{3}$

(b) The probability of choosing a blue ball by random selection from a bag containing only six blue balls is:

A 0 **B** $\frac{1}{5}$ **C** $\frac{1}{6}$ **D** 1

(c) Aaron has passed eight out of the last 10 maths tests. Based on these data, the probability that he will *not* pass his next maths test is:

A 0 **B** $\frac{1}{5}$ **C** $\frac{4}{5}$ **D** 1

7 Bill has beaten Dario in 15 of their last 25 badminton matches. The expected number of games that Dario will win out of the next 10 matches is:

A 3 **B** 4

C 5 **D** 6

8 A basketballer has scored the following points in the last 20 games:

22 15 19 27 16 31 40 29 19 26 14 22 33 41 44 32 31 20 24 30

Using these past results as a guide, estimate the following probabilities.

(a) The probability that the basketballer scores less than 20 points in the next game is:

A 20% **B** 25% **C** 30% **D** 33%

(b) The probability that the basketballer scores more than 35 points in the next game is:

A 10% **B** 15% **C** 20% **D** 25%

(c) The probability that the basketballer scores between 21 and 34 points in the next game is:

A 45% **B** 55% **C** 67% **D** 82%

9 A die with faces numbered 1–6 is biased so that $\Pr(1) = \Pr(2) = \Pr(3) = \frac{1}{4}$ and $\Pr(4) = \Pr(5) = \Pr(6)$.

(a) What is the fractional value for Pr(4)?

(b) Find Pr(even number).

(c) Find Pr(not a multiple of 3).

10 If the probability of choosing by random selection a red jelly bean from a packet is $\frac{4}{9}$, what is the probability of not selecting a red jelly bean?

Understanding

11 A jar contains 25 black marbles, 34 blue marbles and 11 white marbles.

If a sample of 10 marbles is taken out of the jar, how many of each colour would you expect?

12 The table below gives the life expectancy for men at various ages.

Current age (years)	20	25	30	35	40	45	50
Further life expectancy (years)	52.1	47.4	42.8	38.2	33.8	29.6	25.5

Current age (years)	55	60	65	70	75	80	85
Further life expectancy (years)	21.7	18.2	15.0	12.1	9.6	7.5	5.7

(a) What is the expected age of death for a man currently:

- **(i)** 20 years of age
- **(ii)** 30 years of age
- **(iii)** 40 years of age
- **(iv)** 55 years of age
- **(v)** 65 years of age
- **(vi)** 75 years of age?

(b) What do you notice about the figures? Is the figure constant?

Half of all men of a particular age are expected to live to at least their current life expectancy figure. So, half of all men currently 25 years of age are expected to be alive at the age of 72.4. The further life expectancy for a 72-year-old is 11.0 years.

(c) What fraction of men currently 25 years of age would you expect to live to the age of 83 years?

(d) Who would be interested in these figures, and why?

13 Find:

(a) Pr(being born on 17 June). Ignore leap years.

(b) Pr(being born in June).

(c) Pr(being born on the 17th of any month).

(d) Pr(being born in a month containing exactly 30 days).

(e) Pr(not being born in February).

14 A standard six-sided die is rolled. Find the probability of obtaining:

(a) a 2

(b) a 7

(c) either a 1 or a 2

(d) an odd number

(e) a number larger than 2

(f) a number between 0 and 7.

15 For each of the following experiments, complete the following steps.

- **(i)** Predict the number of times each of the outcomes will be obtained.
- **(ii)** Conduct the experiment and record the outcomes.
- **(iii)** Draw up a table of results.
- **(iv)** Use the table to calculate the probabilities for each of the outcomes achieved in the experiment.
- **(v)** Compare your predictions with the actual results.

(a) Toss two coins together 60 times. Record how many times the outcomes are two heads, or two tails, or one head one tail.

(b) Drop a drawing pin onto your page 100 times.

The two possible outcomes are point up:  and point down: 

(c) Use the random number key on your calculator. Push the button 100 times and note how many times each digit appears in the first place after the decimal point.

(d) Open to any page in a paperback novel. Take the first paragraph and record how often each letter occurs.

Reasoning

16 In a sample of 50 seedlings, nine had white flowers, 12 had mauve flowers, 18 had crimson flowers, two had pink flowers, eight had magenta flowers, and one had purple flowers. How many seedlings would a garden designer purchase to end up with each of the following expected numbers of flowers:

(a) 32 magenta

(b) 10 pink

(c) 120 mauve?

17 Over the whole basketball season Eloise scored with two-thirds of her free throws. In the Grand Final she is lining up for her third free throw. The opposition coach believes Eloise is sure to miss because she has scored with her first two. Explain why you think the coach is right or wrong about this.

18 A biased die, marked 1 to 6, is such that:

$$\Pr(6) = \Pr(2) = \Pr(1) = \frac{1}{12} \qquad \Pr(\text{not } 6) = 5 \times \Pr(5) + \Pr(1) \qquad \Pr(3) = \Pr(4) + \Pr(2)$$

(a) Find the probability of rolling each number.

(b) Find the expected number of times each number will appear in 50 rolls of the die.

Open-ended

19 You have an unbiased eight-sided die, with faces numbered 1 to 8. Write an event for each of the following probabilities.

(a) $\Pr(A) = \frac{1}{4}$ **(b)** $\Pr(B) = \frac{1}{2}$ **(c)** $\Pr(C) = \frac{5}{8}$

20 In some board games you need to roll a double 6 on a standard six-sided die before you can begin.

(a) Is there something special about double 6? Is it harder or easier to roll than double 3?

(b) **(i)** Roll two dice and record the number of trials needed to get to each of the double results. Complete the table below to help you.

Event	Trial no.
1, 1	
2, 2	
3, 3	
4, 4	
5, 5	
6, 6	

You do not have to get 1, 1 before 2, 2. Just keep counting. For example, 2, 2 may occur as trial 12 and 5, 5 as trial 30 etc.

(ii) Repeat this experiment 10 times and find the mean number of trials for each of the six doubles to occur.

(iii) Answer part **(a)** of this question again.

How many heads?

Equipment required: Microsoft® Excel or similar spreadsheet software. (For Casio ClassPad CAS or TI-Nspire CAS, you can download instructions from the eBook or the Pearson Places website.)

You can use a spreadsheet to model tossing coins and rolling dice, and also to organise and analyse the results.

1 In a new spreadsheet, enter the formula **=RANDBETWEEN(0,1)** into cell **A1**. This formula will randomly select an integer in the range between 0 and 1. This can be used to simulate a random coin toss, where 0 represents a result of 'tails' and 1 represents a result of 'heads'.

2 Now **Fill Down** to copy this formula from cell **A1** down the column to cell **A10**, to simulate the coin being tossed 10 times. Then **Fill Right** to copy this column across from column **A** to column **E**, to simulate 5 coins being tossed 10 times each.

3 To count the total number of 'heads' results, enter the formula **=SUM(A1:E10)** into cell **A12**. Then select **Calculate Now** (under the **Formulas** tab in Excel) to recalculate the random numbers again and again. What happens?

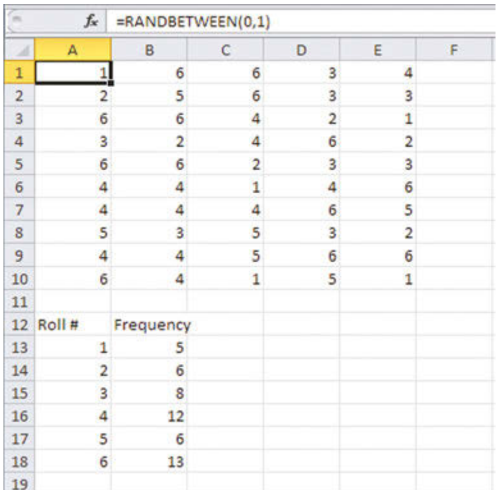
Record the results for 30 separate sets of 50 coin tosses. What was the smallest number of heads observed? What was the highest number of heads observed? Find the mean number of heads observed.

4 Rolling dice can be simulated in a similar way. In a new spreadsheet, enter the formula **=RANDBETWEEN(1,6)** into cell **A1**. This formula will randomly select integers between 1 and 6, to simulate a roll of a die. Use **Fill Down** and **Fill Right** as before, to copy this formula so that the spreadsheet will record 50 dice rolls (across from cell **A1** to cell **E10**).

5 To count the total number of each result, first enter **Face** into cell **A12** and enter **Frequency** into cell **B12**. Then enter the numbers 1–6 in cells **A13–A18** to label the different faces of the die.

In cell **B13**, enter **=COUNTIF(A$1:E$10, A13)**. This formula will look in all the cells from **A1** to **E10** and COUNT them IF they are the same as the value in cell **A13**. Now **Fill Down** to copy this formula down the column to cell **B18**, so that all 6 faces are counted.

(The **$** in **A$1:E$10** means that these cell references will not change if the formula is copied into another cell.)



=RANDBETWEEN(0,1)

	A	B	C	D	E	F
1	1	6	6	3	4	
2	2	5	6	3	3	
3	6	6	4	2	1	
4	3	2	4	6	2	
5	6	6	2	3	3	
6	4	4	1	4	6	
7	4	4	4	6	5	
8	5	3	5	3	2	
9	4	4	5	6	6	
10	6	4	1	5	1	
11						
12	Roll #	Frequency				
13	1	5				
14	2	6				
15	3	8				
16	4	12				
17	5	6				
18	6	13				
19						

You can again select **Calculate Now** (under the **Formulas** tab in Excel) to recalculate the random dice roll numbers again and again. In this way you can quickly make many hundreds of results, to investigate the long-term probability of any particular result.

Record the number of 4s rolled in 30 separate sets of 50 rolls. An example set of results is given below:

9	5	3	8	8	8	8	9	9	10
6	8	11	13	9	6	6	14	14	15
12	3	8	5	5	10	7	12	9	9

Theoretical probability predicts that $\frac{1}{6}$ of the rolls on a normal die should be 4. The results above show a total of 259 fours from 1500 results. This represents 0.1727 of the results. The decimal equivalent of $\frac{1}{6}$ is 0.1667, so the result is quite close.

How do your own results compare?

6 Collate all your class's results and check how close this is to the theoretical probability prediction. What does this mean? Explain.

Taking it further

7 How can you simulate rolling two dice simultaneously, recording the total and keeping a record of the results?

In a new spreadsheet, use column **A** to simulate rolling a die 50 times (using cells **A1–A50**) and the same again in column **B**. Find the sum of **A1** and **B1** (both dice) by writing an appropriate formula in **C1** and copy this down the column. Then set up the frequency table to find the frequency of the different dice sum values, using **=COUNTIF** formulas similar to before.

8 What would you also need to consider if you were interested in the difference between the two dice, rather than the sum?

Investigation

Game simulator

Equipment required: 1 standard six-sided die, calculator or CAS (optional)

The Big Question

Can you simulate results for a variety of sports using a die?

Have you ever wondered why some computer games are different every time you play? Inside the program is a random number generator that is used to make decisions about the path a game will follow each time. The random numbers will tend to generate different decisions each time. To explore this, you can look at some popular games that can be simulated using random numbers. The instructions below say to use dice to get the numbers, but you can use technology to generate the random numbers if you like.

Engage

1 To simulate a game of Aussie Rules football, play a game of 'Dice AFL'.

Rules

Each team rolls two dice. The first represents the number of goals that are kicked in a quarter and the second represents the number of behinds kicked.

Repeat this another three times and record your results in the table below.

Total the points for each team, giving 6 points for each goal and 1 point for each behind.

	Team A		Team B	
	Goals	Behinds	Goals	Behinds
1st quarter				
2nd quarter				
3rd quarter				
4th quarter				
Total				

Explore

2 Now simulate Rugby League by playing a game of 'Dice NRL'.

Scoring in Rugby League is 4 points for a 'try', which then leads to a conversion kick at goal that may gain a further 2 points. For this simulation, assume that the conversion kick success rate is 50%. Teams can also kick a field goal, which is worth 1 point, at any time.

Rules

Each team has 20 rolls of a single die (not counting bonus rolls), taken in turns.

A roll of 1 represents a field goal: score 1 point.

A roll of 4 represents a try: score 4 points.

Each try is followed by a bonus roll, where an even number represents a successful conversion kick and scores 2 additional points.

Any other number (a roll of 2, 3, 5 or 6) scores zero points.

Strategy options

- Make a table.

Explain

3 For the 'Dice AFL' game you played:

- **(a)** What is the lowest possible score for a quarter?
- **(b)** What is the lowest possible score for the game?
- **(c)** What is the highest possible score for a quarter?
- **(d)** What is the highest possible score for the game?

4 For the 'Dice NRL' game you played:

- **(a)** What is the lowest possible score for the game?
- **(b)** What is the highest possible score for the game?

5 Using your answers to **3** and **4**, how realistic do you think the scores were?

6 Are there limitations obvious to you with this simulation?

7 What assumptions have been made about the game to make it relatively simple to play?

8 Overall, comment on how well this simulates the real game.

Elaborate

9 Now try 'Dice Cricket' to simulate a game of cricket.

Rules

Each player has 10 cricketers.

Score runs by rolling the die and scoring each roll as a number of runs until you roll a 5, at which point the cricketer is out and the next cricketer is in. When all 10 cricketers have been rolled 'out', their runs are added to give the total score for the team. The second team then has their turn to bat. The winner is the team with the higher number of runs.

10 What is the highest score recorded for a single cricketer in either team?

11 How many cricketers scored a duck (no score)?

12 Do you think Dice Cricket simulates cricket better than the games of Dice AFL and Dice NRL played earlier? Why or why not?

Evaluate

13 Could you use something other than a die to improve your game and make it more realistic?

14 Are all the situations that occur in the game covered by the simulation?

15 Should the number allocations be changed?

16 How is the dice roll number allocation related to probability?

Extend

17 **(a)** Create your own simulation of a well-known sport.

(b) Write the rules for your simulation so that another person could play it with out needing to ask any questions.

(c) Explain how you decided upon the numbers you allocated to simulate events in the game.

(d) How did probability help with this?

(e) Do you think it is important to play the game before you finalise the design?

8.6

Probability events

Two-way tables

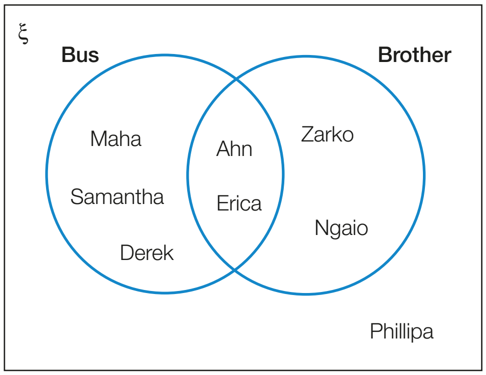
Single events can sometimes be split into categories in two different ways, to display the **sample space** (the full list of outcomes) as a **two-way table**. For example, consider cards from a standard pack of 52 playing cards. The two-way table below shows the four suits on the left and the 13 different cards for each suit across the top. There is a single outcome (card) in each position of the table.

	Ace	2	3	4	5	6	7	8	9	10	Jack	Queen	King
Hearts ♥	A	2	3	4	5	6	7	8	9	10	J	Q	K
Diamonds ♦	A	2	3	4	5	6	7	8	9	10	J	Q	K
Spades ♠	A	2	3	4	5	6	7	8	9	10	J	Q	K
Clubs ♣	A	2	3	4	5	6	7	8	9	10	J	Q	K

Venn diagrams

You should already be familiar with Venn diagrams. There are three situations for which Venn diagrams are really useful: when dealing with probability problems that involve the words ‘and’, ‘or’ and ‘not’.

For example, imagine you have asked your friends some questions. The results are summarised in a Venn diagram. The Venn diagram can be used to visualise and answer questions about the information.



Who travelled to school today by bus?

Maha, Samantha, Derek, Ahn, Erica

Who has a brother?

Ahn, Erica, Zarko, Ngaio

Who travelled to school today by bus and has a brother?

Ahn, Erica

Who travelled to school today by bus or has a brother?

Maha, Samantha, Derek, Ahn, Erica, Zarko, Ngaio

Who did not travel to school by bus today?

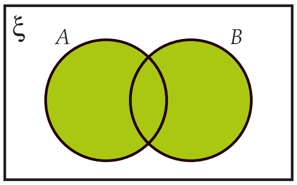
Zarko, Ngaio, Phillipa

How would you most precisely describe Phillipa?

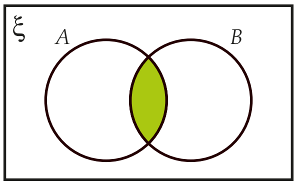
She did not travel to school by bus today and she does not have a brother.

Shading can be used on Venn diagrams to represent the three key situations:

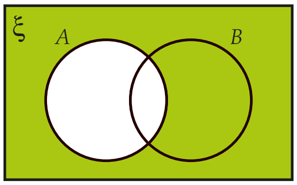
A or B



A and B



A' (not A)



Because of the overlap representing A and B:

$\Pr(A \text{ or } B) = \Pr(A) + \Pr(B) - \Pr(A \text{ and } B)$

Venn diagrams with two overlapping regions are easily summarised in a table displaying the number of elements in each region. The previous 'bus and brother' example can be shown as follows:

	Bus	**Not bus**	**Total**
Brother	2	2	4
No brother	3	1	4
Total	5	3	8

Notice that the two people in the overlap of the Venn diagram are in the 'Bus' column and 'Brother' row of the table, while the one person outside of the diagram regions is in the 'Not bus' column and in the 'No brother' row. The total at the end of the 'Brother' row matches the five people in the 'Brother' region, and the grand total of 8 is the number of 'elements' in the complete set, $n(\xi)$.

$$\Pr(A) = \frac{n(A)}{n(\xi)}$$

Mutually exclusive events

When you roll a standard six-sided die there are many different events that can occur. For example:

$\Pr(\text{rolling a } 1) = \frac{1}{6}$

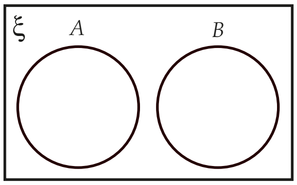
$\Pr(\text{rolling a } 3) = \frac{1}{6}$

$\Pr(\text{rolling an even number}) = \frac{1}{2}$

$\Pr(\text{rolling a multiple of } 3) = \frac{1}{3}$

It is not possible to roll a 1 and a 3 at the same time, so these events are **mutually exclusive**. Conversely, the events 'rolling a 3' and 'rolling a multiple of 3' are not mutually exclusive, because they can occur at the same time (if you roll a 3, which is also a multiple of 3).

Mutually exclusive events can be represented by a Venn diagram as shown, with no overlap. If an element belongs to set A it cannot belong to set B also.



For mutually exclusive events:

$\Pr(A \text{ or } B) = \Pr(A) + \Pr(B)$

Further applications of the basic probability rule

You can also encounter the words 'and', 'or' and 'not' in situations where you can just count the number of successful outcomes and apply the basic rule of probability that applies when dealing with equally likely outcomes:

$$\Pr(\text{event}) = \frac{\text{number of favourable equally likely outcomes}}{\text{total number of equally likely outcomes}}$$

Worked example 11

W.E. 11

The spinner shown in the diagram on the right is spun once.
Find the following probabilities.

- **(a)** Pr(2)
- **(b)** Pr(4)
- **(c)** Pr(2 or 4)
- **(d)** Pr(even number)
- **(e)** Pr(not 2)
- **(f)** Pr(even number)
- **(g)** Pr(odd number or factor of 6)

Thinking	Working
(a) Find the probability using the basic probability rule.	(a) $\Pr(2) = \frac{2}{8} = \frac{1}{4}$
(b) Find the probability using the rule.	(b) $\Pr(4) = \frac{1}{8}$
(c) Find the probability using the rule.	(c) $\Pr(2 \text{ or } 4) = \frac{3}{8}$
(d) Find the probability using the rule.	(d) $\Pr(\text{even number}) = \frac{3}{8}$
(e) Find the probability using the rule.	(e) $\Pr(\text{not } 2) = \frac{6}{8} = \frac{3}{4}$
(f) **1** Write as a sum of probabilities of individual numbers, since these are mutually exclusive.	(f) $\Pr(\text{even}) = \Pr(2) + \Pr(4)$
2 Substitute the individual values.	$= \frac{2}{8} + \frac{1}{8}$
3 Calculate and simplify if necessary.	$= \frac{3}{8}$
(g) **1** List the numbers for each event.	(g) Odd numbers: 1, 3, 5 Factors of 6: 1, 2, 3
2 Identify the numbers in the overlap.	Overlap: 1, 3
3 Write the probability formula that deals with overlap of two events.	$\Pr(A \text{ or } B) = \Pr(A) + \Pr(B) - \Pr(A \text{ and } B)$
4 Substitute and calculate.	$\Pr(\text{odd number or factor of } 6)$ $= \Pr(\text{odd number}) + \Pr(\text{factor of } 6) - \Pr(1, 3)$ $= \frac{5}{8} + \frac{6}{8} - \frac{4}{8}$ $= \frac{7}{8}$

8.6 Probability events

Navigator

1, 2, 3, 4, 5, 7, 8, 10, 13	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13	1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 12, 13

Answers p. 684

Fluency

W.E. 11

1 The spinner shown at right is spun once. Find the following probabilities.

(a) Pr(3)

(b) Pr(5)

(c) Pr(3 or 5)

(d) Pr(odd number)

(e) Pr(5 or even number)

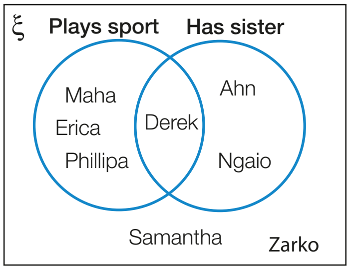
(f) Pr(5 or odd number)

(g) Pr(not 3 or 4)

(h) Pr(not even)

2 You have asked your friends some questions and summarised their answers in the Venn diagram shown. The questions were:

- Who has a sister?
- Who regularly plays sport?



Use the Venn diagram to answer the following questions.

(a) Who has a sister?

(b) Who does not regularly play sport?

(c) Who has a sister and regularly plays sport?

(d) Who has a sister or regularly plays sport?

(e) Complete the table to match the Venn diagram by recording the number of people for each category.

	Sport	No sport
Sister		
No sister		

3 A basket contains six black, eight brown and four white socks. Find the probability that a sock drawn at random from the basket will be:

(a) black or brown

(b) black or white

(c) brown or white.

4 A spinning lottery machine contains 30 lottery balls, numbered from 1 to 30. A ball is drawn from the machine. Find:

(a) Pr(2 or 10)

(b) Pr(even or 7)

(c) Pr(odd or multiple of 8)

(d) Pr(multiple of 7 or multiple of 8)

(e) Pr(multiple of 4 or multiple of 9)

(f) Pr(4 or multiple of 5).

5 Which of the following pairs of events are not mutually exclusive?

A obtaining 'heads' on the toss of a coin / obtaining 'tails' on the toss of a coin

B choosing an odd-numbered horse in a race / choosing horse number 7 in a race

C winning first prize in a raffle with a blue ticket / winning first prize in a raffle with a yellow ticket

D playing for England in the World Cup / playing for Brazil in the World Cup

6 A bowl contains 20 identical counters with a different one of the numbers from 1 to 20 written on it. The following sets have been defined.

A = 'multiples of 4' B = 'multiples of 3' C = '1, 3, 5, 7, 9' D = '1, 2, 3, 5, 8, 13'

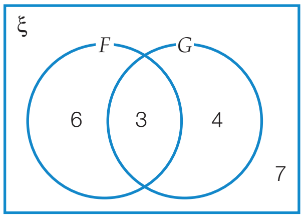
(a) State whether or not each of the following pairs of sets are mutually exclusive.

(i) A and B **(ii)** A and C **(iii)** A and D

(iv) B and C **(v)** B and D **(vi)** C and D

(b) Find the probability of each of A, B, C and D if a single counter is drawn from the bowl.

7 This Venn diagram shows the number of elements in each region. Find the following probabilities based on this data.



(a) Pr(F and G)

A $\frac{3}{20}$ B $\frac{3}{7}$

C $\frac{3}{13}$ D $\frac{13}{20}$

(b) Pr(F or G)

A $\frac{3}{20}$ B $\frac{1}{2}$ C 1 D $\frac{13}{20}$

(c) Pr(F)

A $\frac{3}{10}$ B $\frac{9}{20}$ C $\frac{9}{13}$ D $\frac{2}{3}$

Understanding

8 A standard pack of playing cards consists of 52 cards, half red and half black. The red cards are divided into two suits: hearts and diamonds. The black cards are divided into two suits: clubs and spades. Each suit is made up of 13 cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen and King.

If a card is drawn at random from the pack, what is the probability that the card is:

(a) black **(b)** a diamond

(c) black or a diamond **(d)** a diamond or the King of spades

(e) a King or an Ace or a 6

(f) the King of spades or the Queen of diamonds or the Jack of hearts

(g) the King of diamonds or a spade or the Ace of hearts

(h) not a King **(i)** not a diamond

(j) neither a King nor a Queen **(k)** a spade or a King

(l) a King or a Queen or a red card?

9 Consider the letters of the word *left* and the letters of the word *right*. A bag contains 26 pieces of card, each with a different one of the letters of the alphabet written on it. A letter is drawn at random from the bag. Find the following probabilities.

(a) The letter is in the word *left*. **(b)** The letter is in the word *right*.

(c) The letter is in either word.

(d) Explain in two different ways how you can get the answer to part **(c)**.

(e) The letter is in both words. **(f)** The letter is not in *left*.

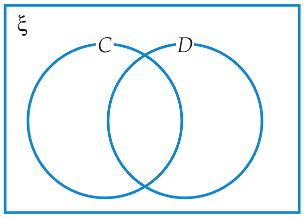
(g) The letter is not in *right*. **(h)** The letter is in neither word.

Reasoning

10 Using the Venn diagram shown, fill in the blanks to complete the probability equations.

(a) Pr(not C and not D) = 1 – Pr(___)

(b) Pr(D) = 1 – Pr(C) – Pr(not C and not D) + Pr(___)



11 If two events, A and B, are mutually exclusive then Pr(A) + Pr(B) = Pr(A or B). The opposite of this is also true: If Pr(A) + Pr(B) ≠ Pr(A or B) then the events are not mutually exclusive.

Consider the following situation where a red die and a blue die are rolled at the same time.

Let A = the red die shows 3
B = the blue die shows 4
C = the sum of the two numbers showing is 6
D = the difference between the two numbers showing is 1

(a) Copy and complete the following table to show the sample space.

		Red die					
		1	**2**	**3**	**4**	**5**	**6**
Blue die	**1**	1, 1	2, 1				
	2	1, 2					
	3						
	4						
	5						
	6						

(b) Calculate:

(i) Pr(A) **(ii)** Pr(B) **(iii)** Pr(C) **(iv)** Pr(D)

(c) Calculate:

(i) Pr(A or B) **(ii)** Pr(A or C) **(iii)** Pr(A or D)

(iv) Pr(B or C) **(v)** Pr(B or D) **(vi)** Pr(C or D)

(d) Are any of the pairs of events mutually exclusive?

Open-ended

12 A 12-sided die with faces numbered 1–12 is rolled.

Let set $A = \{1, 2, 5, 8\}$ (where the numbers represent possible die roll outcomes).

(a) Define two new sets B and C so that each new set is mutually exclusive with set A, but not mutually exclusive with the other new set.

(b) Define set D so that D is not mutually exclusive with any of the sets A, B or C.

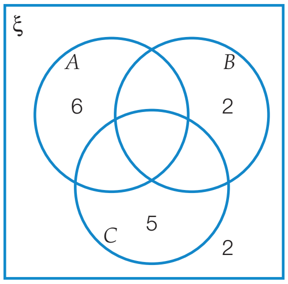
(c) Draw a Venn diagram showing all four sets.

(d) For the sets you have defined, find:

(i) Pr(A or D) **(ii)** Pr(not A and not D)

(iii) Pr(B or C) **(iv)** Pr(not B and not C).

13 **(a)** Copy the Venn diagram, filling in possible values for the number of elements in the regions that have been left blank.



(b) For the values you used, find:

(i) Pr(A) **(ii)** Pr(B) **(iii)** Pr(C).

(c) For the values you used, find:

(i) Pr(not A) **(ii)** Pr(not B) **(iii)** Pr(not C).

(d) For the values you used, find:

(i) Pr(A and B) **(ii)** Pr(B or C) **(iii)** Pr(not A and not C).

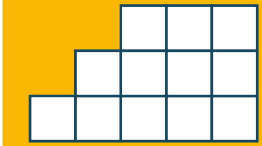
Game

Bigger is better

Equipment required: 10 small pieces of paper or small counters

Preparation: Write the digits 0 1 2 3 4 5 6 7 8 9 on separate pieces of paper or counters, turn them over and shuffle them around.

Each player should copy the following grid.



How to win: The aim of the game is to form the largest total possible when the three numbers in the grid are added.

How to play: Randomly select one of the small pieces of paper to reveal a digit. Each player must then write this digit in one of the squares in their grid.

The digit is then returned to the pile and re-shuffled before another digit is drawn. Each player then writes this digit in a square on their grid.

The process is repeated until all the squares in the grid are filled, so 12 digits will need to be drawn in order to fill the 12 squares of each player's grid.

Each player then adds the three-, four- and five-digit numbers formed.

You should play a number of times, each time aiming to improve on your highest score.

Representing probability

8.7

Multiple events

There are many situations that involve more than one outcome. Finding probabilities in these situations is easier if all the possible outcomes are shown. The full list of outcomes is called the **sample space.** It may be useful to show the sample space as a list, as a table or array, as a Venn diagram, or in some other way.

Worked example 12

W.E. 12

Ann-Marie rolls two dice.

(a) Show the sample space as an array.

(b) Using the sample space, find:

(i) Pr(total of 7)

(ii) Pr(total is less than 6)

Thinking

Working

(a) Write the number for the first die at the left, and the second die across the top. List the sample space as ordered pairs.

(a)

		Second die					
		1	2	3	4	5	6
First die	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 2)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

(b) **(i)** **1** Identify the elements of the successful event.

(b) Total of 7: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)

2 Calculate the number of equally likely possible outcomes.

$n(\xi) = 6 \times 6 = 36$

3 Write the probability as a fraction, simplifying if necessary.

$$\begin{aligned}\text{Pr(total of 7)} &= \frac{6}{36}\\ &= \frac{1}{6}\end{aligned}$$

(ii)	**1** Identify the elements of the successful event.	Total of 2: (1, 1) Total of 3: (1, 2), (2, 1) Total of 4: (1, 3), (2, 2), (3, 1) Total of 5: (1, 4), (2, 3), (3, 2), (4, 1)
	2 Write the probability as a fraction, simplifying if necessary.	$\Pr(\text{total of } 6) = \frac{10}{36}$ $= \frac{5}{18}$

For multiple events:

1 Show the sample space (e.g. as a table or array).

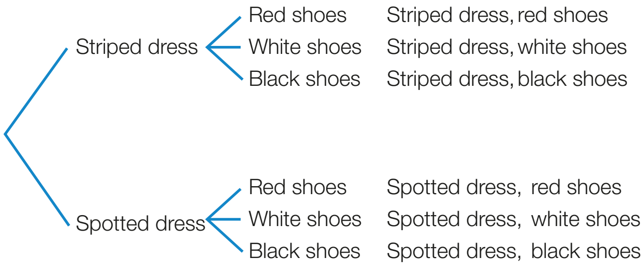
2 Count the number of successful outcomes.

3 Use the formula: $\Pr(\text{success}) = \frac{\text{number of equally likely successful outcomes}}{\text{total number of equally likely possible outcomes}}$

Tree diagrams

Another practical method of representing the sample space is to use a **tree diagram**, which can be useful for situations with multiple choices or multi-stage events.

For example, consider someone who can wear a striped or spotted dress with red, white or black shoes. What is the probability of wearing red shoes and a spotted dress, given these equally likely options?



The tree diagram clearly shows six outcomes. As each outcome is equally likely, the probability of wearing the spotted dress with red shoes is $\frac{1}{6}$.

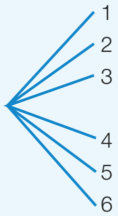
Worked example 13

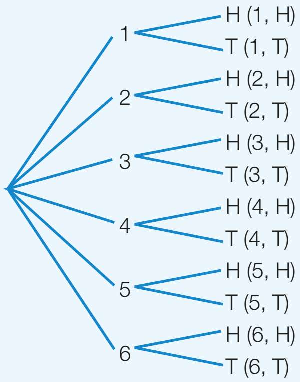
W.E. 13

A die is rolled and then a coin is tossed.

(a) Use a tree diagram to list the sample space.

(b) Find Pr(an even number followed by heads).

Thinking		Working
(a)	**1** Draw the first part of the tree with the number of branches to match the number of outcomes for the first event. In this case there are six: 1, 2, 3, 4, 5 or 6.	(a) 

	2	To each of the branches add the number of branches to match the number of outcomes for the second event. In this case there are two outcomes, heads (H) or tails (T). Write the resulting outcome at the end of each branch.	
	3	List the sample space.	(1, H), (1, T), (2, H), (2, T), (3, H), (3, T), (4, H), (4, T), (5, H), (5, T), (6, H), (6, T)
(b)	1	Identify the equally likely successful outcomes.	Even and head: (2, H), (4, H), (6, H)
	2	Calculate the total number of outcomes.	n(sample space) $= 6 \times 2 = 12$
	3	Find the probability using the formula: Pr(success) $= \frac{\text{number of equally likely successful outcomes}}{\text{total number of equally likely outcomes}}$ Simplify if possible.	Pr(even number and head) $= \frac{3}{12}$ $= \frac{1}{4}$

Sampling with and without replacement

Drawing cards from a pack and replacing the card before the next draw is called **sampling with replacement**. In situations like this, the probability stays the same for each draw. However, if you do not replace the card, then the probability changes after each draw. This is called **sampling without replacement**. It is best to write the changed sample space and work out the probabilities from there.

Worked example 14

W.E. 14

A bowl contains three apples and two pears. Joshua wants two pieces of fruit and doesn't care what he has, so he just randomly chooses two pieces of fruit from the bowl.

(a) Write the sample space, making sure you distinguish between different pieces of the same fruit.

(b) Find the probability that Joshua chooses:

(i) two apples **(ii)** two pears **(iii)** two different pieces of fruit.

Thinking			Working
(a)	1	Decide on the symbols to use for the sample space.	(a) Use A_1, A_2, A_3, P_1 and P_2 to represent the fruit.

2 Write the sample space as an array. Notice that you cannot get the same piece of fruit twice.

	A_1	A_2	A_3	P_1	P_2
A_1	—	(A_1, A_2)	(A_1, A_3)	(A_1, P_1)	(A_1, P_2)
A_2	(A_2, A_1)	—	(A_2, A_3)	(A_2, P_1)	(A_2, P_2)
A_3	(A_3, A_1)	(A_3, A_2)	—	(A_3, P_1)	(A_3, P_2)
P_1	(P_1, A_1)	(P_1, A_2)	(P_1, A_3)	—	(P_1, P_2)
P_2	(P_2, A_1)	(P_2, A_2)	(P_2, A_3)	(P_2, P_1)	—

(b) **(i)** Count the number of successful outcomes and divide by the total number of outcomes.	(b) (i) $\Pr(\text{two apples}) = \frac{6}{20} = \frac{3}{10}$
(ii) Count the number of successful outcomes and divide by the total number of outcomes.	(ii) $\Pr(\text{two pears}) = \frac{2}{20} = \frac{1}{10}$
(iii) Count the number of successful outcomes and divide by the total number of outcomes.	(iii) $\Pr(\text{two different pieces of fruit})$ $= \frac{12}{20} = \frac{6}{10} = \frac{3}{5}$

Note that for sampling without replacement, each of the outcomes on the bottom/left of the table has a matching outcome, in the opposite order, on the top/right of the array, so the probabilities could have been worked out using one part of the array or the other. This is not true for when the repeated outcomes are present in the sample space.

8.7 Representing probability

Navigator

Answers p. 685

1, 2, 3, 4, 5, 6 (a, b), 9, 11	1, 2, 3, 4, 5, 6, 8, 9, 12	1, 2, 3, 4, 5, 7, 8, 9, 10, 12

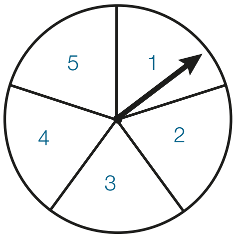
Fluency

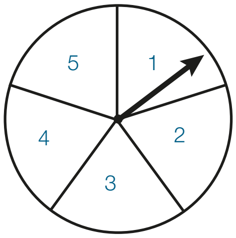
For the following questions, it may be a good idea to first write the complete sample space.

W.E. 12

1 Con has two spinners as shown.

(a) Complete the table to show the sample space.





		Second spinner				
		1	2	3	4	5
First spinner	1	(1, 1)	(1, 2)			
	2					
	3					
	4					
	5					

(b) If Con spins both spinners, find the probability of getting:

(i) a total of 6

(ii) a total of 10

(iii) a total of less than 7

(iv) a total not equal to 5.

2 A coin is tossed and then a six-sided die is rolled.

W.E. 13

(a) Complete the tree diagram to show the sample space.

(b) Find the probability of:

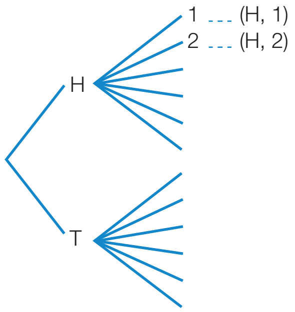
(i) heads followed by a 6

(ii) heads followed by an even number

(iii) heads followed by a number less than 5

(iv) tails followed by a multiple of 3

(v) not getting heads or a 5.



3 A bowl contains four apples and two bananas. Joshua wants two pieces of fruit and doesn't care what he has so he just randomly chooses two pieces of fruit from the bowl.

W.E. 14

(a) Complete the table to show the sample space.

	A_1	A_2	A_3	A_4	B_1	B_2
A_1		(A_1, A_2)	(A_1, A_3)			

(b) Find the probability that Joshua chooses:

(i) two apples **(ii)** two bananas **(iii)** two different pieces of fruit.

4 The menu at Shirley's local restaurant has four starters (pulled-pork sliders, oysters, kimchi and edamame) and five main courses (lamb, fish, beef, pork and poultry).

(a) List all the possible different meals.

(b) Find, assuming random selection:

(i) Pr(kimchi and pork)

(ii) Pr(oysters or fish)

(c) Shirley does not like edamame or lamb. If she chooses a dish at random, what are the chances she has a meal she will enjoy?

5 Heath has four suits (black, blue, grey and brown) and five shirts (white, blue, yellow, pink and beige). Heath randomly chooses his clothes each day.

(a) The probability that Heath wears the grey suit is:

A $\frac{1}{4}$ **B** $\frac{1}{9}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$

(b) The probability that Heath wears the blue suit and the blue shirt is:

A $\frac{1}{20}$ **B** $\frac{7}{24}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$

(c) The probability that Heath wears the black suit or the yellow shirt is:

A $\frac{1}{60}$ **B** $\frac{1}{9}$ **C** $\frac{2}{9}$ **D** $\frac{2}{5}$

Understanding

6 Aldo is making his choice for elective classes for Year 10. He must choose one subject from each of these subject blocks.

Block 1	Block 2
French	German
Art	Metalwork
Literature	Bonus Maths
	Drama

(a) List all the possible subject combinations that Aldo can choose.

(b) Find the following, assuming the subjects are chosen randomly:

(i) Pr(Art and Drama)

(ii) Pr(chooses exactly one language)

(iii) Pr(not Literature)

(iv) Pr(at least one language)

(v) Pr(neither Art nor German)

(c) If Aldo *must* do Bonus Maths, what is the probability that he also does French?

7 Angela is on holiday at the Solaris Beach Resort. She needs to choose her activities for the next day from the choices shown in the following table.

Session 1	Session 2
Wind surfing	Water skiing
Scuba diving	Wave boarding
Body boarding	Wave pool
	Surfing

(a) How many different combinations of two activities can Angela select?

(b) Find the following probabilities, assuming Angela selects at random:

(i) Pr(wind surfing and wave boarding)

(ii) Pr(scuba diving and any of the Session 2 activities)

(iii) Pr(body boarding and not surfing)

(c) Angela decides to finish the day in the wave pool. How many different combinations can she select now?

8 A bowl contains two blue marbles, two green marbles and two yellow marbles. Two marbles are drawn from the bowl.

(a) If the marbles are not replaced after being drawn, find the probability that:

(i) each marble is blue

(ii) one marble is blue and the other yellow

(iii) each marble is a different colour

(iv) neither marble is green.

(b) Recalculate the same probabilities if the marbles are now replaced after being drawn.

Reasoning

9 **(a)** A coin is tossed. List the elements in the sample space.

(b) How many outcomes are possible?

(c) A 10c coin and a 20c coin are tossed. List the elements in the sample space.

(d) How many outcomes are possible?

(e) A 10c coin, 20c coin and a 50c coin are tossed. Write out the sample space.

(f) How many events are possible?

(g) Is there a connection between the number of outcomes possible for each coin and the total number of outcomes possible for the combined events? Explain your answer.

10 A particular cube has two green faces, two blue faces and two red faces. It is rolled eight times and the face on top is recorded.

(a) How many elements are there in the sample space? (Hint: Don’t try to write them all—apply the rule from Question **9**.)

(b) What is the probability that eight green faces in a row will occur?

(c) Why is part **(b)** one of the easiest questions that could be asked about this event?

(d) What are the other equivalent questions with the same answer?

(e) Consider the question ‘*What is the probability that none of the faces will be green?*’ This seems like an application of complementary probabilities to the event in **(b)**, but it is not quite that easy. Why not?

Open-ended

11 Construct a situation that has a sample space for a multiple event with at least 15 elements. Make up at least four questions to go with your sample space.

12 Make up some questions to do with choosing different drinks and snacks from a vending machine. Make sure your questions include some with ‘and‘, ‘or’ and ‘not’.

Problem solving

Birthday cards

Two of your friends happen to have their birthdays close together. You decide to post them each a birthday card. You purchase the cards and write a nice message inside each card. You then address the two envelopes but get distracted when you are putting the cards in the envelopes. What is the probability that you put the cards in the correct envelopes?

Now consider the same question if there were three friends, so three cards and three envelopes. What is the probability that you get all the cards correct? What about getting just one of them correct? What about getting none of them correct? Can you get exactly two of them correct? Explain your answer.

Strategy options

- Make a table.
- Test all possible combinations.

Challenge 8

1 This is a set of scores: 18, 20, 20, 25, 37.

A new whole number score between 18 and 37 is added to the set of scores.

For the new set of scores, indicate whether each of the following statements is correct or incorrect.

(a) The mean of the scores could stay the same.

(b) The mean of the scores could decrease.

(c) The median of the scores could stay the same.

(d) The median of the scores could decrease.

2 **(a)** A set of five numbers has a range of 5 and a mean of 5. None of the numbers is 5. Write a set of possible numbers.

(b) Add a sixth number to your set so as to increase the mean to 6. (The range will change.)

3 A factor of the number 48 is chosen at random. What is the probability that the chosen factor does not itself have a factor of 3?

4 A six-sided prime number die has the numbers 2, 3, 5, 7, 11 and 13 on it. Two of these dice are rolled. What is the probability that:

(a) the sum of the two dice is even

(b) the number on one die is larger than the number on the other die?

5 A bowl contains 18 grapes that are coloured white, black or green. If you draw one grape at random, then you are most likely to draw a black grape, and you are twice as likely to draw a white grape than a green one. What could the contents of the bowl be? (There are three possible answers.)

6 It is estimated that 85% of students in Australia have internet access at home.

(a) Two students are selected at random. What is the probability that neither of them has internet access at home?

(b) Based on a recent survey, 60% of students who have internet access at home also have a Facebook account. A student is selected at random. What is the probability that the student has internet access at home but not a Facebook account?

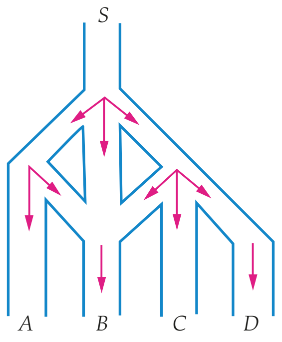
7 Rémy the mouse is put in a maze. He starts at Point *S* and can only move in the direction of the arrows; he cannot double back. At any junction he is equally likely to choose any of the forward paths.

(a) Show that the probability that Rémy ends up at Point *A* is $\frac{1}{6}$.

(b) Find the probability that Rémy ends up at Point *B*.

(c) Find the probability that Rémy ends up at Point *C*.

(d) Find the probability that Rémy ends up at Point *D*.



Chapter review 8

Maths literacy

back-to-back histogram
back-to-back stem-and-leaf plot
categorical data
certain
chance
complementary events
continuous data
discrete data
impossible
measure of centre
measure of location
median class interval
mutually exclusive
negative skew
numerical data
observational data
ordinal data
positive skew
primary data
probability
qualitative data
random sample
relative frequency
sample space
sampling with replacement
sampling without replacement
secondary data
skew
skewed graph
stem-and-leaf plot
symmetrical
tree diagram
two-way table

Copy and complete the following using the words and phrases from the list, where appropriate. A word or phrase may be used more than once.

1 The __________ or __________ of an event is a value between 0 and 1 inclusive.

2 If you collect data directly, such as recording the types of vehicles that pass the school, you are dealing with __________.

3 Data collected by another person or agency that you have accessed are called __________.

4 Non-numerical data that are simply divided into categories are called __________.

5 Exact data values that are countable are called __________.

6 The mean and the median are the two most commonly used __________.

7 If a data set has its values bunched to the right of the histogram, it is said to display __________.

8 An event with a probability of 1 is said to be __________.

9 You can use a __________ or a __________ to assist in finding the sample space.

10 If the mean, median and mode are close together, then a histogram of the data is likely to be __________.

11 Any data value that is from measurement is going to be __________.

12 Events that cannot occur at the same time are said to be __________.

Fluency

1 State whether the following data sets are discrete or continuous. **8.1**

(a) the times taken by competitors to complete the Sydney–Hobart yacht race in a particular year

(b) the number of occupants in cars that go through a toll booth

2 A survey was conducted at the local football club concerning a possible change to the club jumper. The following table summarises the percentage support for each of the three new designs as well as the current design, among different groups of people at the club. **8.1**

	Current design	New design #1	New design #2	New design #3
Junior players	36	25	19	20
Junior parents	78	12	8	2
Senior players	46	31	18	5
Supporters	42	25	12	21
Sponsors	19	34	21	26

As the member of the club committee in charge of the survey, what recommendation would you make to the club? Give as much detail as you think appropriate.

3 Which one of these numbers could be obtained using a survey?

A the percentage of people in the crowd at the SCG who travelled by public transport

B the number of historic sites along the Hume Highway between Melbourne and Sydney

C the number of paintings in the MONA art museum

D the number of red cars in the long-term car park at Tullamarine Airport

4 For each of the following stem-and-leaf plots, calculate the **(i)** mean and **(ii)** median, and **(iii)** comment on the skewness of the data. Where necessary, give your answers correct to 2 decimal places. **8.3**

(a)

Stem	Leaf
4	0 1
5	4 5
6	2 4 5
7	4 5 6 7 8
8	1 3 5 5 6 6
9	0 0 2 4 5

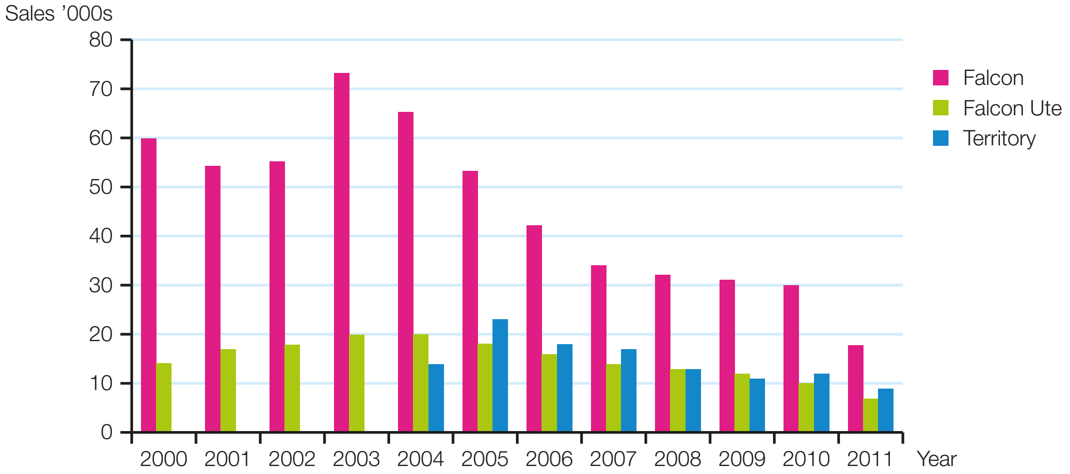
Key: 4 | 0 = 40

(b)

Stem	Leaf
23	0 3 7 8 8
24	2 5 6 9
25	3 4 7 8 9
26	3
27	5 6
28	0

Key: 23 | 0 = 230

5 Consider the following graph of vehicle sales data.



(a) What has happened to the annual sales of the Falcon since the introduction of the Territory?

(b) Compare the trend in the sales of the Falcon with the sales of the Falcon Ute across the years shown in the graph.

(c) For the year 2004, estimate the sales of the three vehicle types shown in the graph.

(d) What is the number of Falcon Utes sold in 2004 as a percentage of the number of Falcons sold in 2004?

(e) What is the number of Territorys sold in 2004 as a percentage of the number of Falcons sold in 2004?

(f) Repeat parts **(c)–(e)** for the year 2010.

(g) Make a comment about the percentages you have just calculated.

(h) What could you say about the total sales of these three Ford vehicles?

6 Last year I bought a packet of a particular brand of biscuits every week. Of these, 40 packets contained broken biscuits. Based on these previous results, what is the estimated probability that the next packet I buy will contain broken biscuits?

8.5

7 A jar contains five coloured blocks, one of each of the colours red, white, blue, green and gold. In an experiment in which 20 blocks were drawn from the jar, one at a time with replacement, the following results were obtained.

8.5

Colour	Red	White	Blue	Green	Gold
Frequency	4	3	1	7	5

(a) What is the relative frequency of drawing out a white block?

(b) What is the relative frequency of drawing out a gold block?

8 A spinner, coloured as shown on the right, is spun once. Find:

8.6

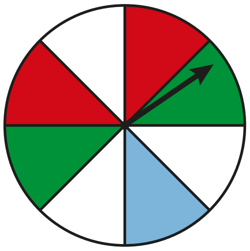
(a) Pr(white)

(b) Pr(green or red)

(c) Pr(not white)

(d) Pr(neither green nor red)

(e) Pr(white or green or red)



9 Two standard six-sided dice are rolled at the same time. **8.7**

(a) How many elements are there in the sample space?

(b) Find the probability of the following events:

(i) Pr(total of 5) **(ii)** Pr(total less than 11)

(iii) Pr(at least one even number in the pair) **(iv)** Pr(no prime numbers in the pair)

Understanding

10 An area 20 m × 20 m is marked out in a nature reserve and the number of various species within the area is counted. The count includes three Coast Wattle, 12 Bidgee Widgee, seven Karkalla, four White Correa, six Running Postman and two Moonah. The nature reserve has a total area of 7.5 hectares. (1 hectare = 10 000 m^2) **8.2**

(a) If the control area is representative of the entire reserve, state the number of each species you would expect to find in the reserve.

(b) Do you think it is reasonable to make this assumption about the relationship between the control area and the entire reserve? What would you do to improve the reliability of your estimate?

11 Do Year 9 girls have longer or shorter arm spans than Year 9 boys? **8.2, 8.4**

The following data from a sample of 50 Year 9 Victorian students were obtained from the Census at Schools website.

Male arm span (cm):	166	174	163	178	198	145	180	175	176	171
	133	178	165	152	163	191	169	173	178	182
Female arm span (cm):	158	165	166	170	153	157	151	180	168	160
	170	165	155	153	171	164	159	170	169	164
	173	163	156	149	171	129	154	150	166	172

(a) Draw a back-to-back stem-and-leaf plot of the data. Use a class interval of 10.

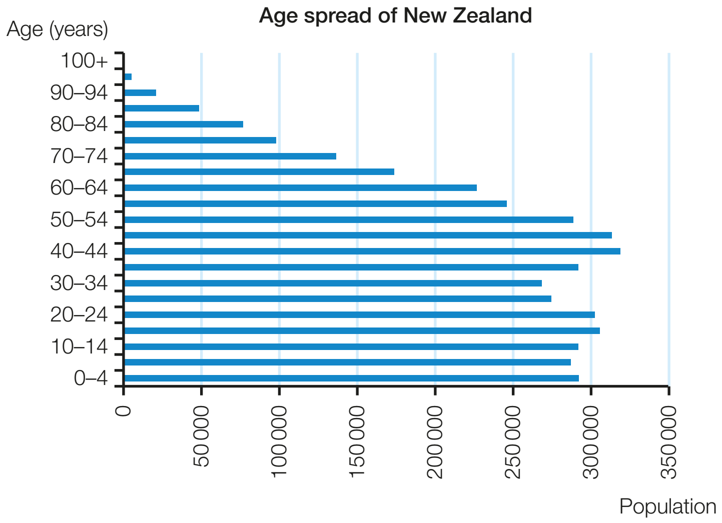
(b) What conclusion would you take from the stem-and-leaf plot drawn?

(c) Is it a concern that there are more girls than boys in the sample?

(d) Does this mean there are more Year 9 girls than boys in Victorian schools? Explain your reasoning.

(e) How different would the results be if they were for Year 7 students or Year 11 students? Give some detail in your answer.

12 The following graph shows the age spread of the population of New Zealand 10 years ago. **8.4, 8.5**



Describe this age spread of the population of New Zealand using as much detail as you can.

13 The coach of the local junior soccer team has recorded the number of goals the team scores each week in the competition over a number of years.

Number of goals	Number of games
0	5
1	9
2	7
3	4
4	3
5	1

8.5, 8.6

Assume that the figures in the table are an indication of what can be expected from his team in the future.

(a) What is the estimated probability that in the next game the number of goals scored will be:

(i) Five goals

(ii) Less than two goals

(iii) No more than four goals

(iv) Either two or four goals?

(Give your answers in fractional form.)

(b) State each of the probabilities from part **(a)** as a decimal, correct to 2 decimal places, and as a whole number percentage.

14 A standard six-sided die is rolled twice. What is the probability of getting:

8.5, 8.6

(a) a pair of sixes

(b) any pair of identical numbers

(c) two even numbers?

Reasoning

15 Two five-sided dice, one green and one blue, both with faces numbered 1–5, are rolled at the same time.

8.5, 8.6

(a) Make a table to show the sample space.

(b) Find the following probabilities:

(i) Pr(total of 9)

(ii) Pr(total less than 8)

(iii) Pr(total no more than 5)

(iv) Pr(at least one prime number)

(v) Pr(difference between numbers less than 3)

(vi) Pr(at least one odd number)

(vii) Pr(both numbers even)

(viii) Pr(not a double)

(c) Are the events ‘getting a 5 on the green die’ and ‘getting a 3 on the blue die’ mutually exclusive? Explain your answer.

16 Every year *The Economist* publishes what it calls the Big Mac Index. This compares the cost of a Big Mac in various countries around the world using US dollars as the comparative currency. This index can give the reader a simple view of the relative cost of (some) goods in these various countries. Consider the following figures.

Country	Big Mac price (US$)	Country	Big Mac price (US$)	Country	Big Mac price (US$)
United States	3.73	Hong Kong	1.90	Russia	2.33
Argentina	3.56	Hungary	3.33	Saudi Arabia	2.67
Australia	3.84	Indonesia	2.51	Singapore	3.08
Brazil	4.91	Israel	3.86	South Africa	2.45
Britain	3.48	Japan	3.67	Sri Lanka	2.82
Canada	4.00	Latvia	2.80	Sweden	1.86
Chile	3.34	Lithuania	2.71	Switzerland	6.56
China	1.95	Malaysia	2.19	South Korea	6.19
Colombia	4.39	Mexico	2.50	Taiwan	2.34
Costa Rica	3.83	New Zealand	3.59	Thailand	2.17
Czech Republic	3.43	Norway	7.20	Turkey	3.89
Denmark	4.90	Pakistan	2.46	UAE	2.99
Egypt	2.28	Peru	3.54	Ukraine	1.84
Estonia	2.62	Philippines	2.19	Uruguay	1.74
Eurozone	4.33	Poland	2.60		

(a) Which country has the lowest price and what is that price?

(b) Which country has the highest price and what is it?

(c) What is the range of prices?

(d) To draw a histogram showing this data, why would a price range of 50 cents be appropriate?

(e) Draw a histogram of the data, using 1.50–<2.00 as the first interval.

The mean of the data is 3.336 and the median is 3.205.

8.1, 8.2, 8.4

(f) Take a random sample of five prices and find the mean and median of those five prices.

(g) Take another random sample of five prices and find the mean and median of those five prices.

(h) Take two different random samples of 10 prices and find the mean and median for each group of 10.

(i) Which size sample would you expect to do a better job of representing the whole list? Did your sampling process confirm this?

Numeracy practice 8

Non-calculator

1 The probability of rolling a 3 on a standard six-sided die is:

A $\frac{1}{2}$ **B** $\frac{1}{3}$ **C** $\frac{1}{6}$ **D** $\frac{3}{4}$

2 For each of the following data sets, state whether it is discrete or continuous.

(a) the number of potatoes in each 5 kg bag at the supermarket

(b) the wrist circumference of each player in a football team

(c) the time taken by each competitor to complete a particular stage of the Tour de France

(d) the number of brothers and sisters for each of the students in your class

Calculator allowed

Questions **3** and **4** refer to the following frequency table.

Value	2	3	4	5	6	7	8	9
Frequency	4	5	2	1	8	5	7	2

3 The median of the data is:

A 4 **B** 5.5 **C** 6 **D** 8

4 The interquartile range of the data is:

A 3 **B** 5 **C** 6 **D** 8

5 The spellchecker on Bob's computer rearranges the letters in a word in a random order. If the letters in the word FRIEND are rearranged randomly, what is the probability that the new arrangement ends with either an I or an E?

A $\frac{1}{3}$ **B** $\frac{2}{5}$ **C** $\frac{2}{4}$ **D** $\frac{2}{3}$

6 Ros has a 70% chance of passing her English exam and an 80% chance of passing her Mathematics exam. The probability that Ros will pass both exams is:

A 1.5 **B** 0.56 **C** 0.44 **D** 0.06

7 This list shows the number of films that nine members of a film club watched in April.

Number of films watched	0, 1, 2, 2, 3, 4, 5, 5, 5

Which of the following is true for this data?

A Mean > median = mode **B** Mean < median < mode

C Mean = median = mode **D** Mean = median < mode

9

Non-linear relationships and proportion

9

The mathematics of fear. Roller-coasters are designed to give thrills with no spills, so it is very important to get the maths right!

The very first roller-coasters were probably slides built in the 16th century in Russia. By 1817 there were two roller-coasters in France that used cars locked to the track, similar to roller-coasters in operation today.

The SideWinder, a high-speed gravity roller-coaster, opened at Dreamworld in December 2001, and it is still one of the tallest high-speed gravity roller-coasters in the Southern Hemisphere. It reaches speeds of up to 85 km/h.

Roller-coasters are mathematically marvellous machines. One of the curves frequently used in the design of roller-coasters is the parabola. A parabola is the graph of a type of equation known as a quadratic!

Forum

How is the speed of the SideWinder generated? What effect do you think there would be on the speed or fear factor if the drop was made steeper? Is there a limit to how steep the drop could be? Why? Would more people in each roller-coaster car increase or decrease the speed?

Why learn this?

Linear equations in the form of $y = mx + b$ are only one way that variables can be related. Non-linear equations have variables related in other ways that do not graph as straight lines. If you can recognise the shapes of these different graphs, know how to transform them and how to solve their non-linear equations, then you can use mathematics to solve a greater range of real-life problems. For instance, the shape of a quadratic curve is the same as the path of a thrown ball or other object (projectile) such as a bullet or arrow. Circles appear everywhere in everyday life: in cylinders, in kitchenware and in architecture. Growth and decay curves are modelled with exponential relationships. Hyperbolas are used to graph inverse relationships between variables such as gas pressure and volume. Non-linear relationships such as these are used extensively in science and engineering to explore and interpret the world and to understand how things work.

After completing this chapter you will be able to:

- plot, sketch and transform graphs of basic non-linear relationships between two variables
- recognise the equations for parabolas, circles, exponentials and hyperbolas
- use algebra to solve some types of quadratic equations algebraically
- understand how solutions to a non-linear equation are related to the graph of the relationship
- solve real-life applications of non-linear relationships, taking constraints into account
- solve problems involving direct proportion and inverse proportion.

Recall 9

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, you can download a Recall Worksheet from the eBook or the Pearson Places website.

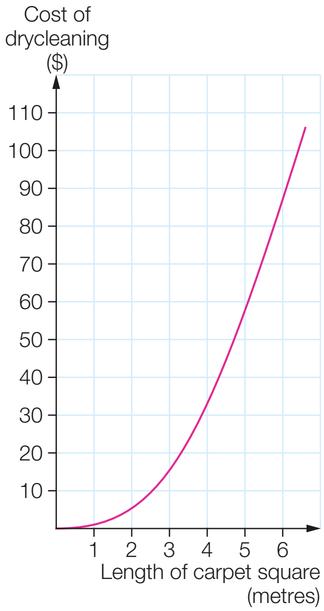
1 **(a)** If $p = 2m^2 - 3m + 5$, find the value of p when:

(i) $m = 1$ **(ii)** $m = 0$ **(iii)** $m = -1$

(b) Find the value of $x(x - 3)$ when:

(i) $x = 5$ **(ii)** $x = -3$ **(iii)** $x = 0$ **(iv)** $x = 3$

2 Consider this graph showing the cost charged to dryclean carpet squares of different lengths.



(a) Find the approximate cost to dryclean a carpet square with a length of:

(i) 3 m **(ii)** 5.5 m

(b) Find the approximate length of the carpet square if the drycleaning cost is:

(i) \$48.00 **(ii)** \$18.75

3 Expand the following.

(a) $3x(x + 4)$ **(b)** $(x + 2)(x + 5)$

(c) $(x - 7)(x + 3)$ **(d)** $(x - 5)(x + 5)$

4 Factorise the following.

(a) $x^2 + 2x$ **(b)** $x^2 - 16$ **(c)** $x^2 + 12x + 36$

5 Factorise the following.

(a) $x^2 + 2x + 5x + 10$ **(b)** $x^2 - 2x - 3x + 6$

(c) $x^2 - 14x + x - 14$ **(d)** $x^2 - 7x - 4x + 28$

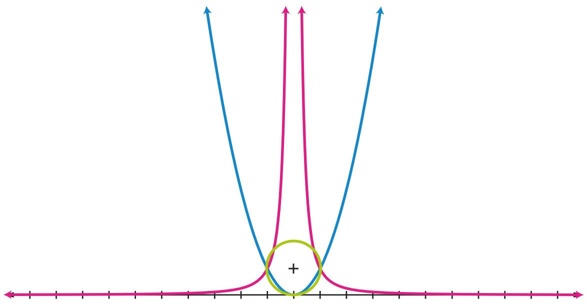
6 Solve for x.

(a) $3x + 5 = 7$ **(b)** $5x - 2 = 11$ **(c)** $6x - 5 = -11$

Exploration Task

You can download this activity from the eBook or the Pearson Places website.

The range of non-linear relationships



In this activity, you will explore why certain non-linear relations can only be positive.

Quadratic relationships

9.1

You should be familiar with linear relationships between two variables. Linear relationships graph as straight lines and a linear equation is written in the form $y = mx + b$, where m is the gradient and b is the y-intercept of the graph.

$y = 3x - 4$ and $y = -2x + 7$ are examples of linear equations.

However, not all relationships between two variables are linear.

A **quadratic relationship** is a very common type of non-linear relationship. The graph of a quadratic relationship is a special shape called a **parabola**.

A **quadratic** equation can be written in the form $y = ax^2 + bx + c$.

$y = x^2 + 3x - 4$ and $y = -2x^2 + 9$ are examples of quadratic equations. Here x is the independent variable and y is the dependent variable.

All quadratic equations have a term where the highest power of the independent variable is 2, the dependent variable has a highest power of 1, and all powers are integers.

> The general equation of a quadratic relation is given by:
> $y = ax^2 + bx + c$, where $a \neq 0$

Worked example 1

W.E. 1

State which of the following are quadratic relationships.

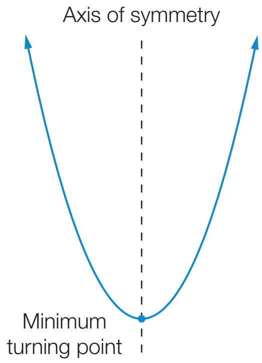
(a) $g = 8f + 6f^2$ **(b)** $p^2 = r - 3r^2$ **(c)** $d = 4b^3 - b^2 + 5$

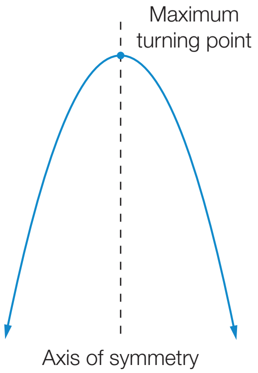
Thinking	Working
(a) Identify the highest power of the independent variable. If it is 2, check that the highest power of the dependent variable is 1 before deciding that it is a quadratic.	(a) The highest power of f is 2 and the highest power of g is 1. This is a quadratic relationship.
(b) Identify the highest power of the independent variable. If it is 2, check that the highest power of the dependent variable is 1 before deciding that it is a quadratic.	(b) The highest power of r is 2 and the highest power of p is 2. This is not a quadratic relationship.
(c) Identify the highest power of the independent variable. If it is 2, check that the highest power of the dependent variable is 1 before deciding that it is a quadratic.	(c) The highest power of b is 3 and the highest power of d is 1. This is not a quadratic relationship.

Key features of a parabola

Axis of symmetry and turning point

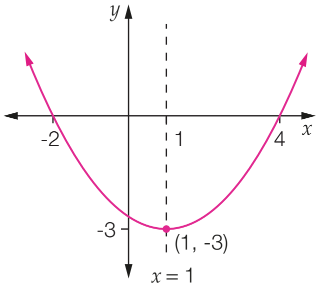
A parabola can be drawn as  or . Each parabola has an axis of symmetry and a maximum or minimum turning point.





An **axis of symmetry** is the imaginary line that divides the parabola into two symmetrical (mirror-image) halves. The **turning point** or **vertex** is where the graph changes direction.

A minimum turning point is where the graph changes direction at the lowest point (the lowest y-value) of the graph. A maximum turning point is where the graph changes direction at the highest point (the highest y-value) of the graph.



The parabola shown at right has a minimum turning point at (1, -3) and an axis of symmetry at $x = 1$.

To plot the graph of a quadratic:

1. Complete a table of values.
2. Draw a set of appropriately sized and scaled axes.
3. Plot the points from the table.
4. Join the points with a smooth curve.
5. Label the graph, the turning point and the axis of symmetry.

Although the graph of a quadratic is always a parabola, graphs that have the same axis of symmetry and turning point as $y = x^2$ can be thinner or wider than $y = x^2$ and can have a maximum turning point instead of the minimum turning point of $y = x^2$.

Worked example 2

W.E. 2

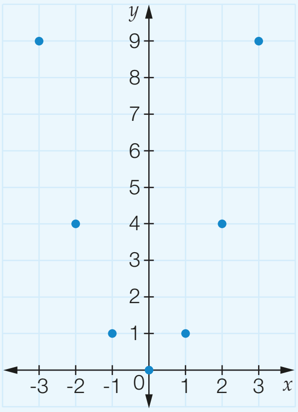
Plot the graph of the following quadratic equations for integer values of x between -3 and 3 inclusive.

(a) $y = x^2$ **(b)** $y = 2x^2$ **(c)** $y = -x^2$

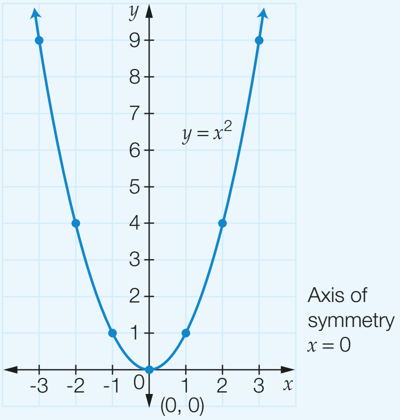
Thinking	Working
(a) 1 Draw a table of values using the x-values in the question (here -3, -2, -1, 0, 1, 2, 3) and substitute the values of x into the equation to find the corresponding values of y.	(a) $y = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

2 Draw a set of appropriately scaled and labelled axes, taking careful note of the smallest and largest x- and y-values. Plot the points from the table.



3 Join the points with a smooth curve and label the graph, the turning point and the axis of symmetry.

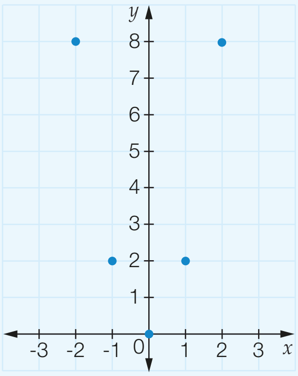


(b) 1 Draw a table of values using the x-values in the question (here -2, -1, 0, 1, 2) and substitute the values of x into the equation to find the corresponding values of y.

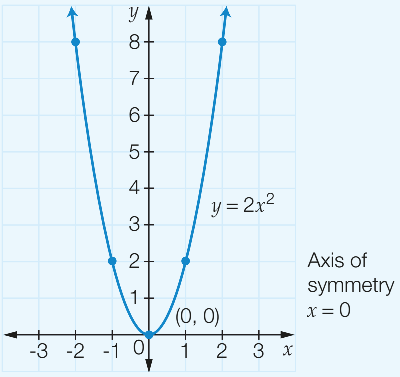
(b) $y = 2x^2$

x	-2	-1	0	1	2
y	8	2	0	2	8

2 Draw a set of appropriately scaled and labelled axes, taking careful note of the smallest and largest x- and y-values. Plot the points from the table.



3 Join the points with a smooth curve and label the graph, the turning point and the axis of symmetry.

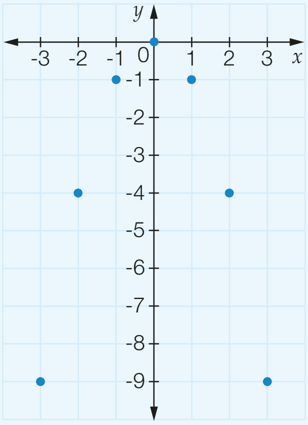


(c) **1** Draw a table of values using the x-values in the question (here -3, -2, -1, 0, 1, 2, 3) and substitute the values of x into the equation to find the corresponding values of y.

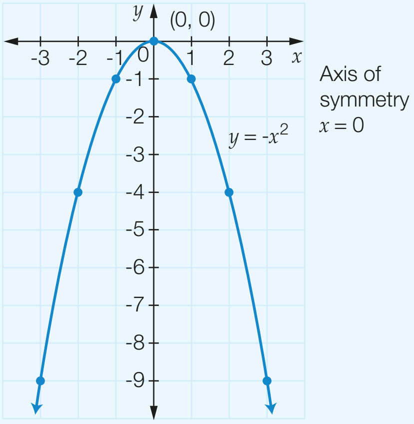
(c) $y = -x^2$

x	-3	-2	-1	0	1	2	3
y	-9	-4	-1	0	-1	-4	-9

2 Draw a set of appropriately scaled and labelled axes, taking careful note of the smallest and largest x- and y-values. Plot the points from the table.



3 Join the points with a smooth curve and label the graph, the turning point and the axis of symmetry.



Note: Not all parabolas have a turning point at the origin, and not all parabolas are symmetrical at the y-axis.

Worked example 3

W.E. 3

Plot the graph of the following quadratic equations for integer values of x between -3 and 3 inclusive.

(a) $y = 2x^2 - 3$ **(b)** $y = 2 - x^2$ **(c)** $y = x^2 - 2x - 3$

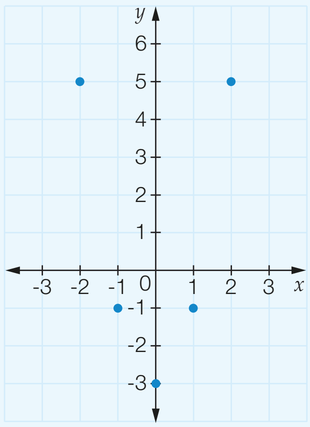
Thinking / Working

(a) **1** Draw a table of values using the x-values in the question (here -2, -1, 0, 1, 2) and substitute the values of x into the equation to find the corresponding values of y.

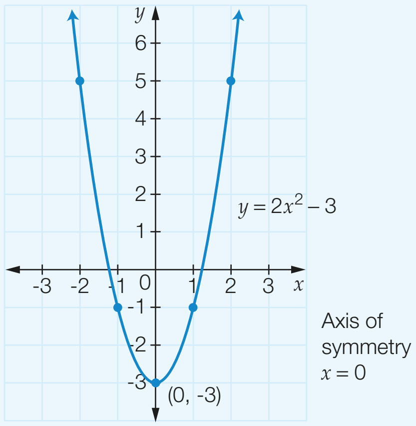
(a) $y = 2x^2 - 3$

x	-2	-1	0	1	2
y	5	-1	-3	-1	5

2 Draw a set of appropriately scaled and labelled axes, taking careful note of the smallest and largest *x*- and *y*-values. Plot the points from the table.



3 Join the points with a smooth curve and label the graph, the turning point and the axis of symmetry.

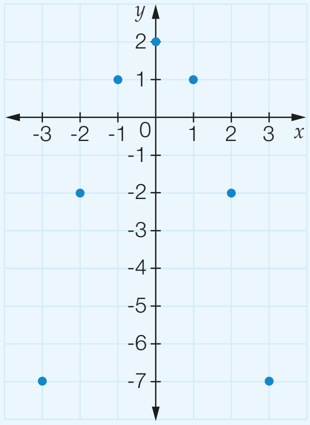


(b) 1 Draw a table of values using the *x*-values in the question, (here -3, -2, -1, 0, 1, 2, 3) and substitute the values of *x* into the equation to find the corresponding values of *y*.

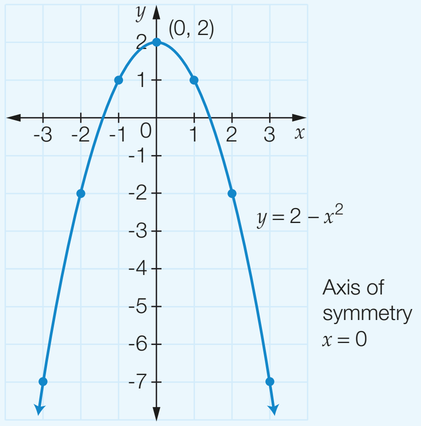
(b) $y = 2 - x^2$

x	-3	-2	-1	0	1	2	3
y	-7	-2	1	2	1	-2	-7

2 Draw a set of appropriately scaled and labelled axes, taking careful note of the smallest and largest *x*- and *y*-values. Plot the points from the table.



3 Join the points with a smooth curve and label the graph, the turning point and the axis of symmetry.

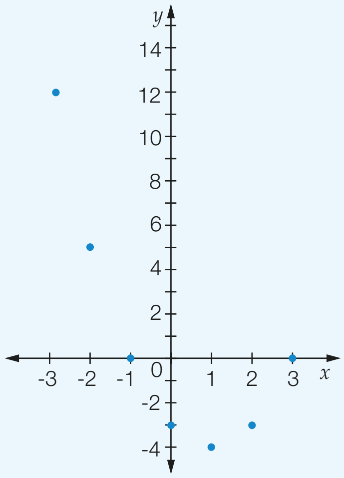


(c) **1** Draw a table of values using the x-values in the question (here -3, -2, -1, 0, 1, 2, 3) and substitute the values of x into the equation to find the corresponding values of y.

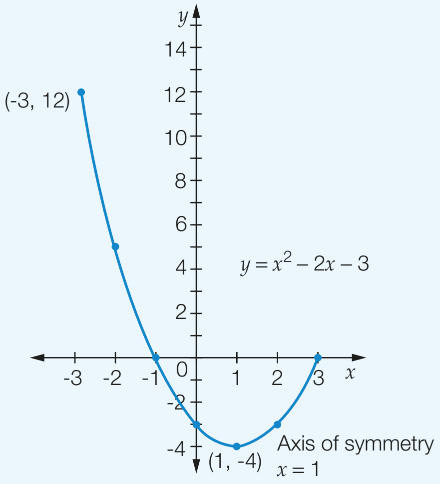
(c) $y = x^2 - 2x - 3$

x	-3	-2	-1	0	1	2	3
y	12	5	0	-3	-4	-3	0

2 Draw a set of appropriately scaled and labelled axes, taking careful note of the smallest and largest x- and y-values. Plot the points from the table.



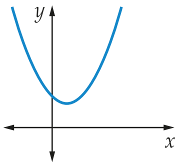
3 Join the points with a smooth curve and label the graph, the turning point and the axis of symmetry.



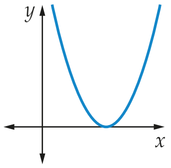
x-intercepts and *y*-intercepts of parabolas

All quadratic graphs will intersect the *y*-axis. This intersection point is called the *y*-intercept.

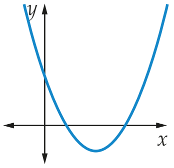
Many quadratic graphs never intersect the *x*-axis. Some quadratic graphs only touch the *x*-axis at one point. All other quadratics intersect the *x*-axis at two points.



No *x*-intercepts



One *x*-intercept



Two *x*-intercepts

All quadratics have a *y*-intercept.

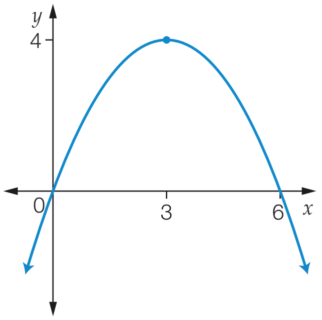
A quadratic graph may have one, two or no *x*-intercepts.

Worked example 4

W.E. 4

For the graph shown, write:

- **(a)** the coordinates of the turning point
- **(b)** the type of turning point
- **(c)** the equation of the axis of symmetry
- **(d)** the *y*-intercept
- **(e)** the *x*-intercepts.



Thinking	Working
(a) Identify the point where the graph changes direction. This is the turning point.	(a) There is a turning point at (3, 4).
(b) Determine whether the turning point is at the maximum (or greatest) value of *y* or the minimum (or least) value of *y*.	(b) Because $y = 4$ is the maximum value of y, then this is a maximum turning point.
(c) Find the line that divides the graphs into symmetrical halves. Notice that the axis of symmetry goes through the turning point.	(c) The axis of symmetry is the line $x = 3$.
(d) Find the point where the graph cuts the *y*-axis (*y*-intercept).	(d) The y-intercept is (0, 0).
(e) Find the points where the graph cuts the *x*-axis (*x*-intercepts).	(e) The x-intercepts are (0, 0) and (6, 0).

Worked example 5

W.E. 5

For each of the following equations:

(a) $y = x^2 - 4x + 5$ **(b)** $y = x^2 - 4x + 4$ **(c)** $y = x^2 - 4x + 3$

(i) Complete the table of values.

x	-2	-1	0	1	2	3	4	5
y								
(x, y)								

(ii) Plot the points on a Cartesian plane for the values in the table and join with a smooth curve.

(iii) Write the coordinates of the turning point.

(iv) Does the parabola have a maximum or a minimum turning point?

(v) Write the equation of the axis of symmetry of the parabola.

(vi) Write the coordinates of the y-intercept.

(vii) Write the coordinates of the x-intercepts.

(viii) From the graph, find the values of x when $y = 8$. Hence, state the values of x when the given equation is equal to 8. (Answer correct to 1 decimal place.)

Thinking

Working

(a) **(i)** Complete the table of values by substituting values. For example, when $x = -2$:

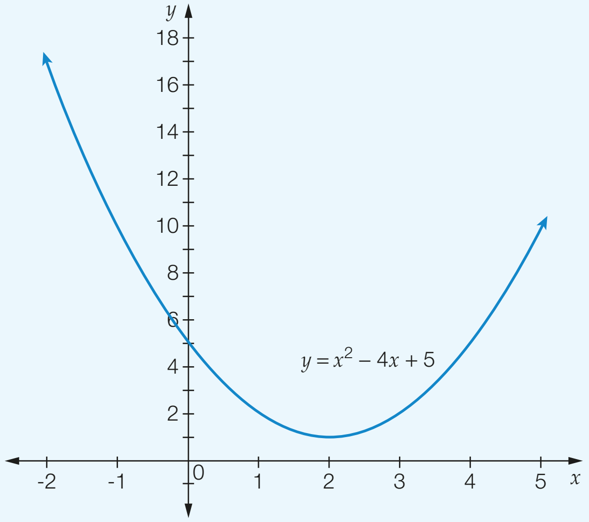
$$\begin{aligned} y &= (-2)^2 - 4 \times (-2) + 5 \\ &= 4 + 8 + 5 \\ &= 17 \end{aligned}$$

(a) (i) $y = x^2 - 4x + 5$

x	-2	-1	0	1	2	3	4	5
y	17	10	5	2	1	2	5	10
(x, y)	(-2, 17)	(-1, 10)	(0, 5)	(1, 2)	(2, 1)	(3, 2)	(4, 5)	(5, 10)

(ii) Draw a set of axes, label them, plot the points and join them with a smooth curve.

(ii)



(iii) Identify the point where the graph changes direction.

(iii) The turning point is (2, 1).

(iv) Determine whether the turning point is where the maximum or the minimum value of y occurs.

(iv) $y = 1$ is the minimum value of y, so this is a minimum turning point.

(v) The axis of symmetry is a vertical line passing through the turning point.

(v) The axis of symmetry is $x = 2$.

(vi) Identify the point where the graph cuts the y-axis.

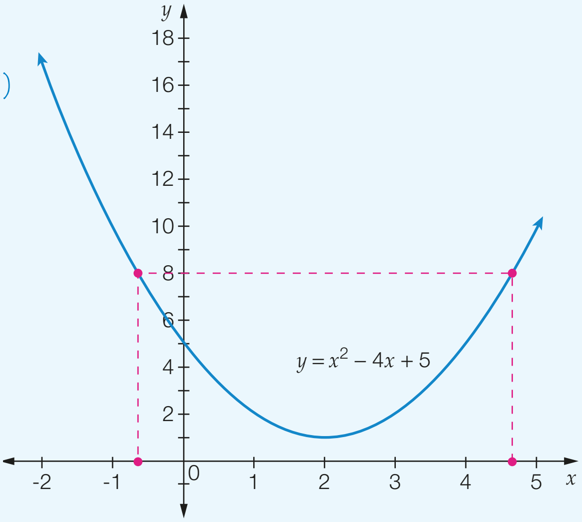
(vi) The y-intercept is (0, 5).

(vii) Identify the points, if any, where the graph cuts the x-axis.

(vii) There are no x-intercepts.

(viii) Find x at the specified value of y and move horizontally to find where it cuts the graph. See the corresponding values of x.

(viii)



The graph shows $x \approx 4.6$ and -0.6 when $y = 8$.

So, $x^2 - 4x + 5 = 8$ when $x \approx 4.6$ and -0.6.

(b) **(i)** Complete the table of values by substituting values. For example, where $x = -2$:

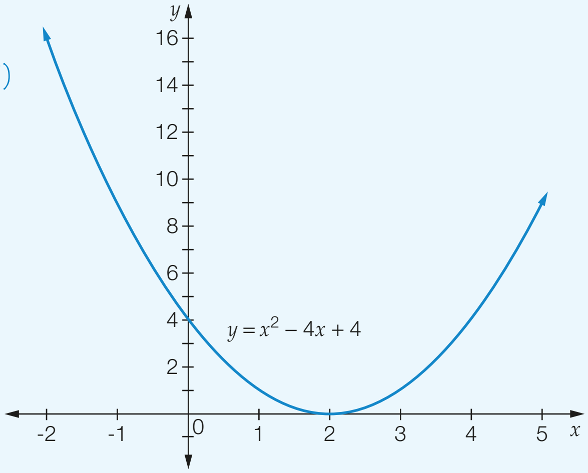
$$y = (-2)^2 - 4 \times (-2) + 4$$
$$= 4 + 8 + 4$$
$$= 16$$

(b) (i) $y = x^2 - 4x + 4$

x	-2	-1	0	1	2	3	4	5
y	16	9	4	2	01	1	4	9
(x, y)	(-2, 16)	(-1, 9)	(0, 4)	(1, 1)	(2, 0)	(3, 1)	(4, 4)	(5, 9)

(ii) Draw a set of axes, label them, plot the points and join them with a smooth curve.

(ii)



(iii) Identify the point where the graph changes direction.

(iii) The turning point is (2, 0).

(iv) Determine whether the turning point is where the maximum or minimum value y occurs.

(iv) $y = 0$ is the minimum value of y, so this is a minimum turning point.

(v) The axis of symmetry is a vertical line passing through the turning point.

(v) The axis of symmetry is $x = 2$.

(vi) Identify the point where the graph cuts the y-axis.

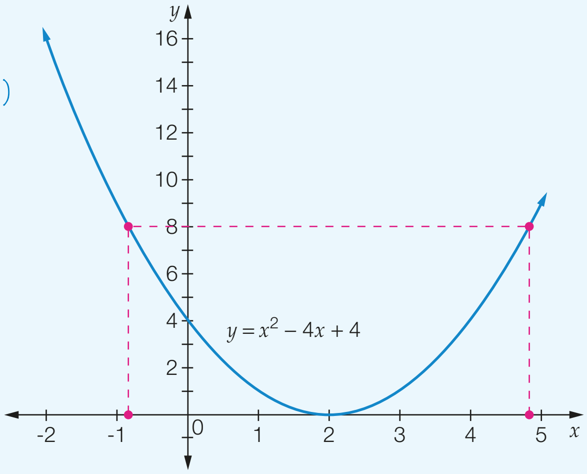
(vi) The y-intercept is (0, 4).

(vii) Identify the point, if any, where the graph cuts the x-axis.

(vii) There is one x-intercept at (2, 0).

(viii) Find x at the specified value of y and move horizontally to find where it cuts the graph. Read off the corresponding values of x.

(viii)



The graph shows $x \approx 4.8$ and -0.8 when $y = 8$.

So, $x^2 - 4x + 4 = 8$ when $x \approx 4.8$ and -0.8.

(c) **(i)** Complete the table of values by substituting values. For example, when $x = -2$:

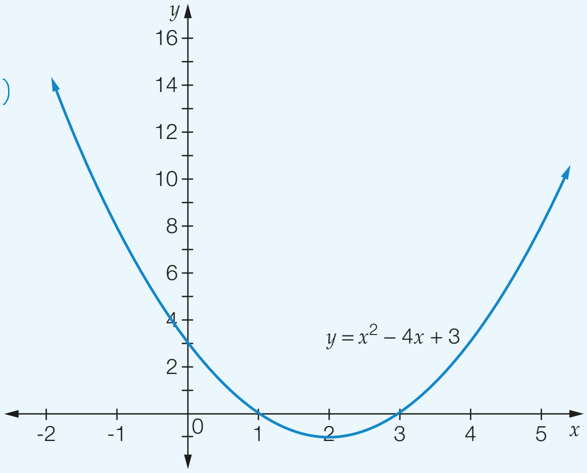
$$\begin{aligned} y &= (-2)^2 - 4 \times (-2) + 3 \\ &= 4 + 8 + 3 \\ &= 15 \end{aligned}$$

(c) (i) $y = x^2 - 4x + 3$

x	-2	-1	0	1	2	3	4	5
y	15	8	3	0	-1	0	3	8
(x, y)	(-2, 15)	(-1, 8)	(0, 3)	(1, 0)	(2, -1)	(3, 0)	(4, 3)	(5, 8)

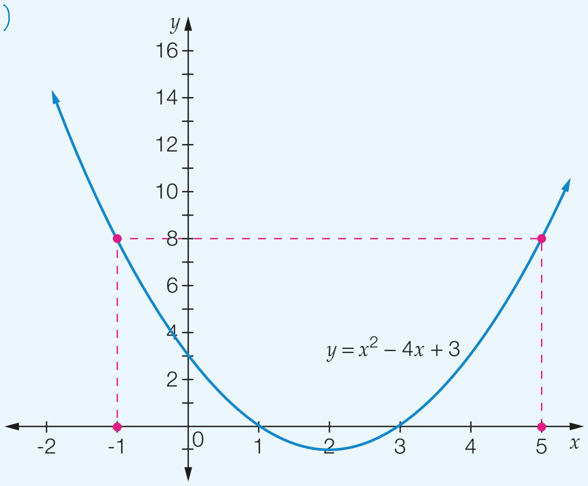
(ii) Draw a set of axes, label them, plot the points and join them with a smooth curve.

(ii)



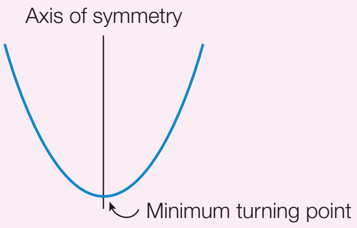
(iii) Identify the point where the graph changes direction.

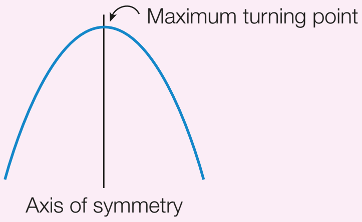
(iii) The turning point is (2, -1).

(iv) Determine whether the turning point is where the maximum value of y occurs, or where its minimum value occurs.	(iv) $y = -1$ is the minimum value of y, so this is a minimum turning point.
(v) The axis of symmetry is a vertical line passing through the turning point.	(v) The axis of symmetry is $x = 2$.
(vi) Identify the point where the graph cuts the y-axis.	(vi) The y-intercept is $(0, 3)$.
(vii) Identify the points where the graph cuts the x-axis.	(vii) There are two x-intercepts, one at $(1, 0)$ and one at $(3, 0)$.
(viii) Find x at the specified value of y and move horizontally to find where it cuts the graph. Read off the corresponding values of x.	(viii)  The graph shows $x = -1$ and 5 when $y = 8$. So, $x^2 - 4x + 3 = 8$ when $x = -1$ and 5.

Features of a parabola:

- Every parabola is symmetrical about the axis of symmetry, which is a vertical line with an equation of the form $x = a$ where a is a constant.
- Every parabola has a vertex or turning point, which is where the graph changes direction.
- A parabola can have a minimum turning point or a maximum turning point.
- All parabolas have a y-intercept.
- A parabola can have zero, one or two x-intercepts.





9.1 Quadratic relationships

Navigator

Answers p. 687

1, 2, 3 (a–c), 4, 5 (a), 6, 7, 8, 9, 11, 12, 14, 17, 22, 23, 25	1, 2, 3 (a–c), 4, 5 (b), 6, 7, 8, 9, 11, 12, 14, 15, 16, 17, 18, 21, 22, 24, 25	1, 2, 3 (d–f), 4, 5 (a), 6, 7, 8, 10, 12, 13, 15, 16, 18, 19, 20, 22, 23, 24, 25

Equipment required: graph paper for most questions

Fluency

W.E. 1

1 State which of the following are quadratic relationships.

A $m = 4k^2 - 6$ B $a = -b^2$ C $s = 4t^4 - 5t^2 + 7$

D $r = 4w - 3$ E $l = 2 - 3n^2$ F $a^2 = 9 - x^2$

W.E. 2

2 Plot the graph of the following quadratic equations for integer values of x between -3 and 3 inclusive.

(a) $y = 3x^2$ **(b)** $y = -2x^2$ **(c)** $y = \frac{x^2}{2}$

W.E. 3

3 Plot the graph of the following quadratic equations for integer values of x between -4 and 4 inclusive.

(a) $y = 3x^2 - 3$ **(b)** $y = -2x^2 + 3$ **(c)** $y = \frac{x^2}{2} - 2$

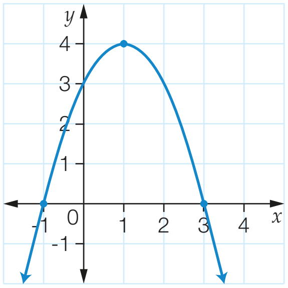
(d) $y = 3x^2 - 6x$ **(e)** $y = -2x^2 + 5x + 3$ **(f)** $y = \frac{x^2}{2} - \frac{3x}{2} - 2$

W.E. 4

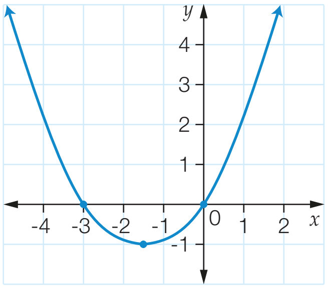
4 For each of the following graphs, write:

- **(i)** the coordinates of the turning point
- **(ii)** the type of turning point
- **(iii)** the equation of the axis of symmetry
- **(iv)** the y-intercept
- **(v)** the x-intercepts.

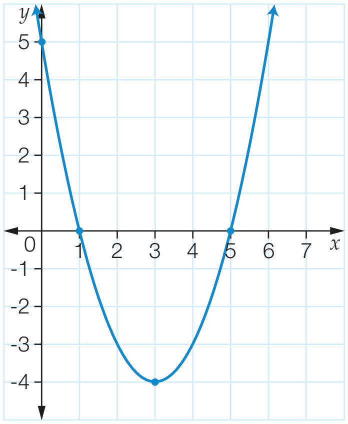
(a)



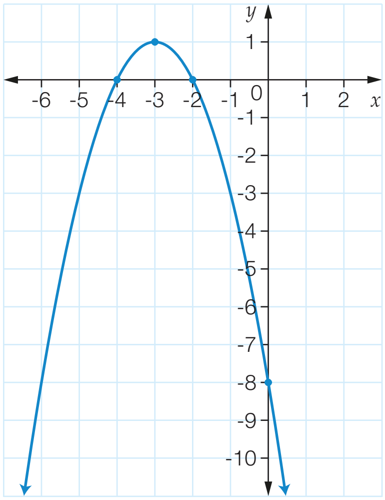
(b)



(c)



(d)



5 For each of the following equations:

W.E. 5

(a) $y = x^2 - 2x + 2$ **(b)** $y = x^2 - 2x - 3$ **(c)** $y = x^2 - 2x + 1$

x	-2	-1	0	1	2	3	4	5
y								

(i) Copy and complete the table of values.

(ii) Plot the points on a Cartesian plane for the values in the table and join with a smooth curve.

(iii) Write the coordinates of the turning point.

(iv) Does the parabola have a maximum or a minimum turning point?

(v) Write the equation of the axis of symmetry of the parabola.

(vi) Write the coordinates of the y-intercept.

(vii) Write the coordinates of the x-intercepts.

(viii) From the graphs, find the values of x when $y = 5$. Hence, state the values of x when the given equation is equal to 5.

6 **(a)** Which of the following does not show a quadratic relationship?

A $v = w - 4w^2$ **B** $f^2 = 3d^2 - 1$ **C** $p = 6r^2 - 7$ **D** $k^2 = t$

(b) If $p = 3m^2 + 4$, then if m equals 5, p will be equal to:

A 19 **B** 34 **C** 75 **D** 79

(c) A parabola has a turning point at (-4, 3) and a y-intercept at (0, 13). The axis of symmetry is located at:

A $y = 13$ **B** $x = -4$ **C** $y = 3$ **D** $x = 13$

(d) A parabola has x-intercepts at (-3, 0) and (-1, 0) and a y-intercept at $y = 3$. The x-value of the turning point is:

A $x = -3$ **B** $x = -1$ **C** $x = -2$ **D** $x = 3$

(e) A parabola with a y-intercept at $y = -6$ and turning point at (-1, -8) cuts the x-axis at (1, 0). At what other point will the parabola cut the x-axis?

A $x = -1$ **B** $y = -6$ **C** $x = -3$ **D** $x = -2$

7 Which statement is not true about the key features of a parabola?

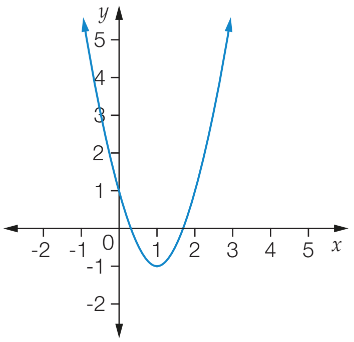
A All parabolas have a y-intercept.

B Every parabola has a turning point, which can be a minimum turning point or a maximum turning point.

C Every parabola is symmetrical about the axis of symmetry.

D A parabola can have only one or two x-intercepts.

8 The graph of $y = 2x^2 - 4x + 1$ is shown.



(a) For this graph the turning point would be:

A (1, 0) B (1, 1)

C (1, -1) D (-1, 1)

(b) The value(s) of x for $2x^2 - 4x + 1 = 0$ would be approximately:

A -0.8 and 1.8 B 1 and 0

C 0.3 and 1.7 D 0 and 2

(c) The value(s) of x for $2x^2 - 4x + 1 = 1$ would be approximately:

A 0 and 2 B 0.3 and 1.7 C 0 and 1 D -0.8 and 1.8

Understanding

9 Compare your graphs in Question **2 (a)–(c)** to your graphs in Questions **3 (a)–(c)**.

Comment on any similarities and differences.

10 Compare your graphs in Question **2 (a)–(c)** to your graphs in Questions **3 (d)–(f)**.

Comment on any similarities and differences.

11 **(a)** Complete the following table of values for $y = x^2 - 4$. (The first column has been completed for you.)

x	-4	-3	-2	-1	0	1	2	3	4
y	12								
(x, y)	(-4, 12)								

(b) Plot the points onto a number plane. Join them with a smooth curve.

(c) What are the coordinates of the turning point?

(d) Does the parabola have a maximum or a minimum turning point?

(e) Write the equation of the axis of symmetry of the parabola.

(f) Write the coordinates of the y-intercept.

(g) Write the coordinates of the x-intercepts.

(h) From the graph, find the value of y where $x = 3.5$ (correct to 1 decimal place).

(i) From the graph, find the values of x where $y = -2$ (correct to 1 decimal place).

(j) State the values of x where $x^2 - 4 = 5$. Hence, find the values of x where $x^2 - 9 = 0$.

12 **(a)** Complete the following table of values for $y = -2x^2 - 6x$.

x	-5	-4	-3	-2	-1	0	1	2
x^2								

(b) Plot the graph of $y = -2x^2 - 6x$ using graph paper.

(c) Write the coordinates of the turning point. State whether the parabola has a maximum or a minimum turning point.

(d) Write the coordinates of any x- and y-intercepts.

(e) What are the x-values where $y = 4$?

(f) If $y = 4$, what does $-2x^2 - 6x$ equal? Hence, state the x-values where $-2x^2 - 6x = 4$. These x-values are the solution to the equation $-2x^2 - 6x = 4$.

(g) Use the graph to find the x-values that solve the equation $-2x^2 - 6x = -8$.

(h) Find the x-values that solve $-2x^2 - 6x = 0$.

13 **(a)** Complete a table of values for $y = x^2 - x - 2$ for integer values of x between -3 and 5.

(b) Plot the graph of $y = x^2 - x - 2$ for the points in your table.

(c) Write the coordinates of the turning point. Does the parabola have a maximum or a minimum turning point?

(d) Write the coordinates of any x- and y-intercepts.

(e) Use the parabola to solve x in each of the following equations.

(i) $x^2 - x - 2 = 0$ **(ii)** $x^2 - x - 2 = 4$ **(iii)** $x^2 - x - 2 = 2$

(iv) $x^2 - x - 2 = -2.25$ **(v)** $x^2 - x - 2 = -4$ **(vi)** $x^2 - x - 2 = 1\frac{1}{2}$

14 **(a)** Plot the graph of $y = x^2 + 6x + 10$. Use integer values of x between -7 and 1.

(b) What are the coordinates of the turning point?

(c) Write the coordinates of any x- and y-intercepts.

(d) Use the graph to solve each of the following equations.

(i) $x^2 + 6x + 10 = 1$ **(ii)** $x^2 + 6x + 10 = 10$

(iii) $x^2 + 6x + 10 = 0$ **(iv)** $x^2 + 6x + 10 = 15$

15 **(a)** Plot the graphs of the following parabolas on separate axes on graph paper. (Remember to complete a table of values first.)

(i) $y = -x^2 + 4x$ for $-1 \le x \le 5$

(ii) $y = x^2 + 3x - 10$ for $-6 \le x \le 3$

(iii) $y = -x^2 + 2x + 3$ for $-2 \le x \le 4$

(b) Use your graphs to solve the following equations to find x.

(i) $-x^2 + 4x = 0$ **(ii)** $x^2 + 3x - 10 = 0$ **(iii)** $-x^2 + 2x + 3 = 4$

(iv) $-x^2 + 4x = 4$ **(v)** $x^2 + 3x - 10 = 8$ **(vi)** $-x^2 + 2x + 3 = 0$

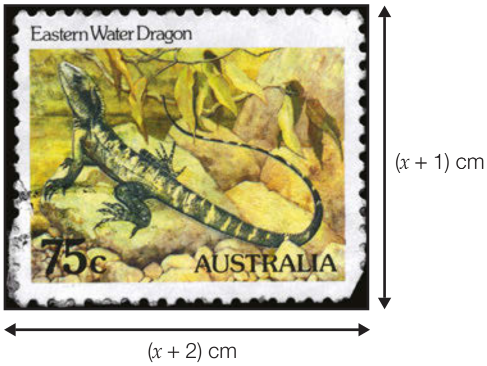
(vii) $-x^2 + 4x = 2$ **(viii)** $x^2 + 3x - 10 = 4$ **(ix)** $-x^2 + 2x + 3 = -2$

Reasoning

Hint

Use only realistic values of x for your table of values.

16 A postage stamp has a length of $(x + 2)$ cm and a width of $(x + 1)$ cm.



(a) Write an equation for the area A of the postage stamp in expanded form.

(b) Complete a table of values for x and A.

(c) Plot a graph of your equation showing x on the horizontal axis (use 1 cm = 1 cm) and A on the vertical axis (use 1 cm = 1 cm^2).

(d) Use the graph to find the value of x if the area of the stamp is 8 cm^2. Are there one or two values for x in this problem? Explain.

(e) What is the length and width of the stamp if its area is 8 cm^2? (Give your answers correct to 1 decimal place.)

17 Karrie, a keen golfer, hits a golf ball along the fairway. The golf ball's height h (in metres) above the ground at any time t (in seconds) during its flight is given by the equation $h = -5t^2 + 20t$.

(a) Complete a table of values for t and h.

(b) Plot the graph of $h = -5t^2 + 20t$. Show the values of t on the horizontal axis (use 1 cm = 0.25 s) and values for h on the vertical axis (use 1 cm = 2 m).

Use the graph to answer the following questions.

(c) How high does the ball get above the ground?

(d) How much time does it take to reach this maximum height?

(e) When does the ball first reach a height of 12 m?

(f) How much time does it take before the ball hits the ground?

(g) For how many seconds is the ball at least 8 m above the ground?

18 A worker's drink bottle is accidentally dropped from the top of a multi-storey building. The relationship between the bottle's height h (in metres) above the ground and the time t (in seconds) for it to fall to that height is $h = 125 - 5t^2$.

(a) Complete a table of values for t and h.

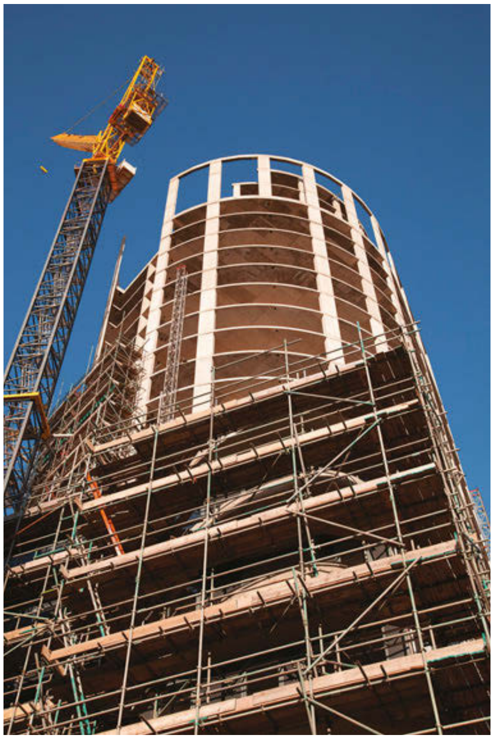
(b) Plot the graph of $h = 125 - 5t^2$. Show values of t on the horizontal axis (use 1 cm = 1 s) and values for the height h on the vertical axis (use 1 cm = 20 m).

(c) What is the height of the building?

(d) How much time does it take for the bottle to hit the ground?

(e) How high above the ground is the bottle after 2.5 s? (Answer correct to the nearest metre.)

(f) What distance, to the nearest metre, has the bottle fallen after 2.5 s?



(g) How much time does it take for the bottle to fall 50 m? (Answer correct to 1 decimal place.)

(h) How much time does it take for the bottle to fall 100 m? (Answer correct to 1 decimal place.)

(i) What distance does the bottle fall during the fourth second?

(j) During which second does the bottle fall the greatest distance?

Hint

The first second occurs between $t = 0$ and $t = 1$.

19 The speed of a car is tracked by radar in a police helicopter for 10 minutes. The police find that the car's speed follows the equation $v = 3t^2 - 30t + 135$, where v is the speed of the car in km/h after t minutes have passed.

(a) Plot a graph representing this information, showing values of t on the horizontal axis (use 1 cm = 2 minutes) and values of v on the vertical axis (use 1 cm = 20 km/h).

Use the graph to answer the following questions.

(b) What is the maximum speed recorded for the car?

(c) What is the minimum speed recorded for the car?

(d) What is the speed of the car after 3 minutes?

(e) What is the speed of the car after 7 minutes?

(f) When is the car travelling at 80 km/h?

(g) When is the car travelling at 120 km/h?

(h) The speed limit along this section of road is 100 km/h. In which time intervals is the car driven above the speed limit?

20 The entrance to a one-way railway tunnel forms an arch. At any particular distance d across the base of the tunnel, starting from the left, the height h of the arch can be found using the equation $h = -d^2 + 6d$. Both h and d are in metres.

(a) Plot the graph of $h = -d^2 + 6d$. (Show d on the horizontal axis.)

Use the graph to answer the following questions.

(b) How wide is the base of the tunnel?

(c) What is the maximum height of the tunnel?

(d) What is the height of the tunnel 1 m along the base from each end of the entrance?

For a train to pass safely through the tunnel there must be a minimum vertical clearance of 1 m between the train and the arch. Assume the train has a rectangular cross-section. Use this information to answer the following questions.

(e) If a particular train is 4 m wide, what is its maximum allowable height?

(f) What is the maximum allowable height of a train that is 3 m wide?

(g) Find the maximum width of a train that just fits through the tunnel and is 4.8 m high.

(h) Decide whether trains of the following dimensions would fit through the tunnel.

(i) width 2.6 m, height 6 m **(ii)** width 3.8 m, height 4.5 m.

21 The volume of liquid in a particular container can be found using the equation $V = 3t^2 + 4t$, where V is in cm^3 and t is the number of seconds after pouring commences.

(a) Plot the graph of $V = 3t^2 + 4t$. (Use values of t from 0 to 7 and show t on the horizontal axis.)

Use your graph to answer the following questions.

(b) Find the amount of liquid in the container at the start.

(c) Find the amount of liquid in the container after 4 seconds.

(d) The container can hold only 132 cm^3. How long, in seconds, does it take to fill the container?

(e) How much liquid is added in the first 2 seconds?

(f) What are the possible and sensible values for time and for volume?

Open-ended

22 (a) Write at least three expressions that are quadratic expressions.

(b) Write at least three expressions that are not quadratic expressions.

23 Make up at least two quadratic equations. For each equation:

(a) Draw up a table of values.

(b) Plot the graph.

(c) Find the turning point.

(d) Find the value of any x- and y-intercepts.

24 (a) If the length of a rectangle is 3 cm longer than its width, write the area of the rectangle.

(b) Draw a graph of this relationship.

(c) Choose a value for the area of the rectangle and use your graph to find the dimensions of your rectangle correct to 2 decimal places.

25 Josh is having trouble filling in the tables of values for the following quadratic equations. Explain what he is doing wrong.

(a) $y = -x^2 + 2x$

x	-3	-2	-1	0
y	3	0	-1	0

(b) $y = 2x^2 + 1$

x	-2	-1	0	1	2
y	17	5	1	5	17

Puzzle

Hitori

Copy and complete the following grid by colouring in cells according to the rules given below.

5	2	2	3	4
5	6	1	2	2
5	5	4	5	1
6	3	1	1	5
5	1	3	4	2

Rules:

1 Each number 1–9 must not appear more than once in each row and column. Where there is more than one number (e.g. where there are two 4s), all of the 4s except one must be coloured in.

2 No cell that is coloured in can have another coloured-in cell next to it, except diagonally (i.e. coloured cells cannot touch horizontally or vertically, but diagonally is OK).

3 The white cells must all be joined or continuous.

Correct

Incorrect (adjacent cells are coloured)

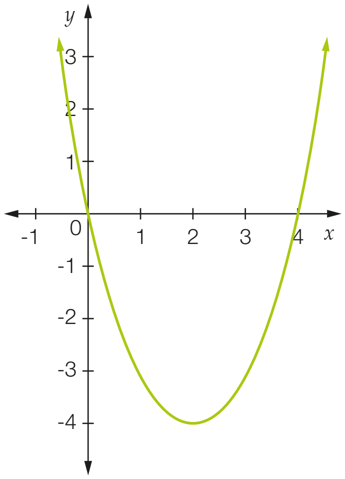
Incorrect (uncoloured cells don't form a single area)

It may be helpful to circle numbers to be not coloured (e.g. numbers directly next to coloured cells).

Solving quadratic equations

9.2

To solve an equation means to find the values that make the equation true. You can find solutions to quadratic equations such as $x^2 - 4x = 0$ by locating the points where the curve of $y = x^2 - 4x$ intersects the line of $y = 0$ (the x-axis).



The x-intercepts of $y = x^2 - 4x$ are 0 and 4.
The solutions to $x^2 - 4x = 0$ are $x = 0$ and $x = 4$.

Graphical methods of solution are time-consuming and can be inaccurate if the solutions involve fractions or surds. You need to be able to solve quadratic equations algebraically.

You can use the null factor law to solve these equations.

Null factor law

If the product of two or more numbers is zero, then at least one of the numbers must be zero.

If $ab = 0$, then $a = 0$, $b = 0$, or both $a = 0$ and $b = 0$.

To use the null factor law to solve quadratic equations, there are two requirements:

1. One side of the equation must equal zero.
2. The other side must be the product of factors. (Factorisation may be necessary to achieve this.)

When quadratic equations were plotted in the previous section, all of the parabolas had a y-intercept. Some parabolas had no x-intercepts, some had one x-intercept, and the rest had two x-intercepts. The x-intercepts occur where the graph crosses the x-axis, where $y = 0$. You can read off these values from a graph, but you need to be able to find these values algebraically. You can do this by solving the quadratic equation when it equals zero.

Use the null factor law to obtain solutions without having to draw graphs.

For linear equations there is only one x-intercept, so there is only one solution where $y = 0$.

$$\begin{aligned} 2x &= 0 \\ x &= 0 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 2(x-4) &= 0 \\ x - 4 &= 0 \\ x &= 4 \end{aligned}$$

Because there can be zero, one or two x-intercepts, there can be zero, one or two solutions where $y = 0$.

$x^2 + 4 = 0$ no solutions	$x^2 = 0$ $x = 0$ one solution	$x^2 - 4x = 0$ $x(x - 4) = 0$ $x = 0$ or $(x - 4) = 0$ $x = 0$ or $x = 4$ two solutions

Worked example 6

W.E. 6

Solve the following equations using the null factor law.

(a) $x(x + 3) = 0$ **(b)** $(x + 3)(x - 2) = 0$ **(c)** $(x - 1)^2 = 0$

Thinking		Working
(a)	**1** Write the equation.	(a) $x(x + 3) = 0$
	2 Use the null factor law.	$x = 0$ or $x + 3 = 0$
	3 Solve each individual equation.	$x = 0$ or $x = -3$
	4 State your solution.	The solution to $x(x + 3) = 0$ is $x = 0$ or $x = -3$.
(b)	**1** Write the equation.	(b) $(x + 3)(x - 2) = 0$
	2 Use the null factor law.	$x + 3 = 0$ or $x - 2 = 0$
	3 Solve each individual equation.	$x = -3$ or $x = 2$
	4 State your solution.	The solution to $(x + 3)(x - 2) = 0$ is $x = -3$ or $x = 2$.
(c)	**1** Write the equation.	(c) $(x - 1)^2 = 0$
	2 Rewrite as a product.	$(x - 1)(x - 1) = 0$
	3 Use the null factor law. Because both factors are identical (it is a repeated factor), there is only one equation to solve.	$x - 1 = 0$
	4 Solve each individual equation.	$x = 1$
	5 State your solution.	The solution to $(x - 1)^2 = 0$ is $x = 1$.

If the quadratic expression is not in a factorised form, you must try to factorise it first. Not all quadratics will factorise to give real number solutions.

Solving non-factorised quadratics

For equations in the form of $ax^2 + bx = 0$:

- make sure the right-hand side (RHS) is equal to zero
- identify the highest common factor (HCF) and factorise
- use the null factor law to find solutions.

Worked example 7

W.E. 7

Solve the following equations using the null factor law.

(a) $2x^2 + 4x = 0$ **(b)** $7x^2 = x$

Thinking	Working
(a) 1 Write the equation.	(a) $2x^2 + 4x = 0$
2 Factorise the left-hand side (LHS) by finding the HCF.	$2x(x + 2) = 0$
3 Apply the null factor law.	$2x = 0$ or $x + 2 = 0$
4 Solve each equation.	$x = 0 \qquad x = -2$
5 State your answer.	$x = 0$ or -2
(b) 1 Write the equation.	(b) $7x^2 = x$
2 Rearrange to make one side equal to zero.	$7x^2 - x = 0$
3 Factorise the LHS of the equation by finding the HCF.	$x(7x - 1) = 0$
4 Apply the null factor law.	$x = 0$ or $7x - 1 = 0$
5 Solve each equation.	$x = 0$ or $\quad 7x = 1$ $x = \frac{1}{7}$
6 State your answer.	$x = 0$ or $\frac{1}{7}$

For equations in the form of $x^2 - a^2 = 0$:

- Make sure the RHS is equal to zero.
- Recognise the difference of two squares.
- Factorise using $a^2 - b^2 = (a + b)(a - b)$.
- Use the null factor law to find solutions.

Worked example 8

W.E. 8

Solve the following equations using the null factor law.

(a) $x^2 - 49 = 0$ **(b)** $4x^2 = 81$ **(c)** $x^2 - 7 = 0$

Thinking	Working
(a) 1 Write the equation.	(a) $x^2 - 49 = 0$
2 Factorise the LHS using $a^2 - b^2 = (a + b)(a - b)$.	$(x - 7)(x + 7) = 0$
3 Apply the null factor law.	$x - 7 = 0$ or $x + 7 = 0$
4 Solve each equation and state your answer.	$x = 7$ or $x = -7$

(b) 1 Write the equation.	(b) $4x^2 = 81$
2 Rearrange the equation so that the RHS = 0.	$4x^2 - 81 = 0$
3 Write the LHS of the equations as the difference of two squares.	$(2x)^2 - (9)^2 = 0$
4 Factorise the LHS using $a^2 - b^2 = (a + b)(a - b)$.	$(2x - 9)(2x + 9) = 0$
5 Apply the null factor law.	$2x - 9 = 0$ or $2x + 9 = 0$
6 Solve each equation and state your answer.	$x = \frac{9}{2}$ or $x = -\frac{9}{2}$ $x = 4\frac{1}{2}$ or $x = -4\frac{1}{2}$
(c) 1 Write the equation.	(c) $x^2 - 7 = 0$
2 Write the LHS of equation as the difference of two squares.	$x^2 - (\sqrt{7})^2 = 0$
3 Factorise the LHS using $a^2 - b^2 = (a + b)(a - b)$.	$(x - \sqrt{7})(x + \sqrt{7}) = 0$
4 Apply the null factor law.	$(x - \sqrt{7}) = 0$ or $(x + \sqrt{7}) = 0$
5 Solve each equation and state your answer.	$x = \sqrt{7}$ or $x = -\sqrt{7}$

For equations in the form of $x^2 + 2ax + a^2 = 0$ or $x^2 - 2ax + a^2 = 0$

- Make sure the RHS is equal to zero.
- Recognise a perfect square.
- Factorise using $a^2 + 2ab + b^2 = (a + b)^2$ and $a^2 - 2ab + b^2 = (a - b)^2$.
- Use the null factor law to find solutions.

Worked example 9

W.E. 9

Solve the following equations using the null factor law.

(a) $x^2 - 6x + 9 = 0$

(b) $2x^2 + 48x = -288$

Thinking	**Working**
(a) 1 Use the perfect square rule to factorise the LHS.	(a) $x^2 - 6x + 9 = 0$ $(x - 3)^2 = 0$
2 Write as the product of factors.	$(x - 3)(x - 3) = 0$
3 Solve the equation and state your answer. Because there is a repeated factor, there is only one equation to solve.	$x - 3 = 0$ $x = 3$

(b) **1** Rearrange to make one side equal to zero.

(b) $2x^2 + 48x = -288$

2 Factorise the LHS by:

- taking out the HCF
- using the perfect square rule to factorise the LHS.

$$2x^2 + 48x + 288 = 0$$
$$2(x^2 + 24x + 144) = 0$$
$$2(x + 12)^2 = 0$$

3 Divide both sides by 2.

$$(x + 12)^2 = 0$$

4 Write as the product of factors.

$$(x + 12)(x + 12) = 0$$

5 Solve the equation and state your answer. Because there is a repeated factor, there is only one equation to solve.

$$x + 12 = 0$$
$$x = -12$$

Notice that there is only one solution for these quadratics. This is because some parabolas only touch the x-axis at one point. The equation to find the x-intercept of these parabolas will have only one solution.

9.2 Solving quadratic equations

Navigator

1 (column 1), 2 (column 1), 3 (column 1), 4 (column 1), 5, 6, 7, 8, 10, 12, 13, 15	1 (column 2), 2 (column 2), 3 (column 2), 4 (column 2), 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16	1 (column 3), 2 (column 3), 3 (column 3), 4 (column 3), 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16

Answers p. 691

Fluency

W.E. 6

1 Solve the following equations using the null factor law.

(a) $x(x - 1) = 0$ **(b)** $x(x + 8) = 0$ **(c)** $x(x + 5) = 0$
(d) $(x - 4)(x - 1) = 0$ **(e)** $(x - 2)(x - 6) = 0$ **(f)** $(x - 3)(x - 9) = 0$
(g) $(x - 4)(x + 1) = 0$ **(h)** $(x - 3)(x + 7) = 0$ **(i)** $(x - 5)(x + 2) = 0$
(j) $(x + 4)(x + 6) = 0$ **(k)** $(x + 7)(x + 2) = 0$ **(l)** $(x + 3)(x + 10) = 0$
(m) $(x - 4)^2 = 0$ **(n)** $(x + 7)^2 = 0$ **(o)** $(x - 9)^2 = 0$

W.E. 7

2 Solve the following equations using the null factor law.

(a) $x^2 - 6x = 0$ **(b)** $x^2 - x = 0$ **(c)** $x^2 - 8x = 0$
(d) $x^2 + 7x = 0$ **(e)** $x^2 + 5x = 0$ **(f)** $x^2 + x = 0$
(g) $2x^2 - 8x = 0$ **(h)** $3x^2 - 9x = 0$ **(i)** $7x^2 - 28x = 0$
(j) $3x^2 + 27x = 0$ **(k)** $5x^2 + 45x = 0$ **(l)** $12x^2 + 4x = 0$
(m) $9x^2 = -x$ **(n)** $8x^2 = 2x$ **(o)** $x^2 = 11x$

W.E. 8

3 Solve the following equations using the null factor law.

(a) $x^2 - 9 = 0$
(b) $x^2 - 1 = 0$
(c) $x^2 - 16 = 0$
(d) $9x^2 - 25 = 0$
(e) $25x^2 - 36 = 0$
(f) $49x^2 - 81 = 0$
(g) $2x^2 - 98 = 0$
(h) $3x^2 - 27 = 0$
(i) $7x^2 - 28 = 0$
(j) $5x^2 = 125$
(k) $4x^2 = 49$
(l) $9x^2 = 100$
(m) $16 - x^2 = 0$
(n) $36 - x^2 = 0$
(o) $9 - x^2 = 0$
(p) $x^2 - 11 = 0$
(q) $x^2 - 29 = 0$
(r) $x^2 - 47 = 0$

W.E. 9

4 Solve the following equations using the null factor law.

(a) $x^2 - 6x + 9 = 0$
(b) $x^2 - 2x + 1 = 0$
(c) $x^2 - 8x + 16 = 0$
(d) $x^2 + 10x + 25 = 0$
(e) $x^2 + 12x + 36 = 0$
(f) $x^2 + 18x + 81 = 0$
(g) $2x^2 - 28x + 98 = 0$
(h) $3x^2 + 18x + 27 = 0$
(i) $7x^2 - 28x + 28 = 0$
(j) $x^2 - 4x = -4$
(k) $x^2 - 12x = -36$
(l) $x^2 - 10x = -25$
(m) $x^2 + 9 = 6x$
(n) $x^2 + 1 = 2x$
(o) $x^2 + 49 = 14x$

5 (a) What is/are the solution/s to $x(x + 1) = 0$?

A $x = -1$ B $x = 1$ C $x = 0, x = -1$ D $x = 0, x = 1$

(b) What is/are the solution/s to $(x + 2)(x - 3) = 0$?

A $x = 2, x = 3$ B $x = -2, x = 3$ C $x = 2, x = -3$ D $x = -2, x = -3$

(c) What is/are the solution/s to $x^2 + 10x + 25 = 0$?

A $x = 2, x = 5$ B $x = -5$ C $x = 10, x = 25$ D $x = 5$

(d) What is/are the solution/s to $x^2 - 121 = 0$?

A $x = 11, x = -11$ B $x = 11$ C $x = 0, x = -11$ D $x = 0, x = 11$

6 How many solutions are there for the quadratic equation $(x - 5)^2 = 0$?

A no solutions B 1 solution C 2 solutions D 3 solutions

Understanding

7 The null factor law is used to find solutions to quadratic equations without needing to draw the graph of the parabola. What do the solutions represent?

A the turning point of the parabola
B the axis of symmetry and the y-intercept
C the x-intercepts of the parabola
D the values where the parabola does not exist

8 When three times a number is added to the square of that number, the result is seven times the number. Use an equation to find the number if it is greater than zero.

9 Find the values of x where the product of $(2x + 1)$ and $(2x - 3)$ is equal to the product of $(x + 2)$ and $(5x - 6)$.

10 The height above the ground of a golf ball can be represented by the equation $h = 4t - t^2$, where t represents the time in seconds. When will the golf ball be 4 m above the ground?

A 0 seconds B 4 seconds C 2 seconds D Never

Reasoning

11 A square vegetable garden has one side extended by 4 m and the other side reduced by 4 m. The area is now 65 m^2. What was the side length of the original vegetable garden?

Hint

Write an equation and solve it to find the original length of the vegetable garden.

12 A graph has equation $y = (x + 1)(x - 4)$. Find the x-intercepts of this graph.

13 Determine the x-intercepts of the following quadratic relationships.

(a) $y = x^2 - 144$ **(b)** $y = 6x^2 + 24x$ **(c)** $y = x^2 - 12x + 36$

14 **(a)** Expand $(x - 6)(x + 2)$.

(b) Use your answer in part **(a)** to find the solution to $x^2 - 4x - 12 = 0$.

(c) How are your solutions in part **(b)** related to the constant term in $y = x^2 - 4x -12$?

(d) Expand $(x - 5)(x + 1)$.

(e) Use your answer in part **(d)** to find the solution to $x^2 - 4x - 5 = 0$.

(f) How are your solutions for part **(e)** related to the constant term in $y = x^2 - 4x - 5$?

(g) Solve the quadratic equation $x^2 - 4x = 0$.

(h) What is the value of the constant term in $x^2 - 4x$?

(i) How are your solutions for part **(g)** related to the constant term in $y = x^2 - 4x$?

(j) Expand $(x - 1)(x - 3)$.

(k) Use your answer in part **(j)** to find the solution to $x^2 - 4x + 3 = 0$.

(l) How are your solutions for part **(k)** related to the constant term in $y = x^2 - 4x + 3$?

(m) Solve the quadratic equation $x^2 - 4x + 4 = 0$.

(n) How are your solutions in part **(m)** related to the constant term in $y = x^2 - 4x + 4$?

(o) What does the constant term in the equations in parts **(c)**, **(f)**, **(i)** and **(n)** represent graphically?

(p) What do the equations in parts **(b)**, **(e)**, **(g)**, **(k)** and **(m)** all have in common?

(q) What information does the solution to these five equations give you about the graphs of each of these equations?

(r) What happens to the x-intercepts of the graphs of these equations as the constant term increases?

(s) Write a statement about the solutions to $x^2 - 4x + c = 0$ where $c > 4$. Hence, make a statement about the x-intercepts of graphs in the form of $y = x^2 - 4x + c$ where $c > 4$.

Open-ended

15 Decide on two values of x that will be the x-intercepts of a quadratic graph. Write at least two possible equations for the graph.

16 Louise is solving the equation $8x - x^2 = 16$. She writes

Explain to Louise what she has done incorrectly and write the correct solution.

$8x - x^2 = 16$
$x(8 - x) = 16$
$x = 16$ or $8 - x = 16$
$x = 16$ or $x = -8$

Problem solving

Stopping distances

The distance a car needs to stop could be described by the equation $y = 0.005x^2 + 0.12x$, where y is the distance (in m) that the car takes to stop when x is the speed (in km/h) of the car.

(a) If the car is travelling at 60 km/h instead of 40 km/h through a school zone, what is the extra distance it must travel before it stops?

(b) What is the extra distance a car needs to stop if it is travelling 10 km/h faster than the allowed 100 km/h?

(c) Show that the speed of the car must be less than 12 km/h to ensure it does not hit a stationary car 2 m in front of it.

(d) What is the effect on the stopping distance if the car drives 95 km/h instead of 100 km/h?

Strategy options

- Have I seen a similar problem?
- Break problem into manageable parts.

Sketching parabolas using transformations

9.3

You have seen how to graph parabolas by plotting points from a table of values. You can also sketch a parabola by using its key features instead of plotting points. This section will explore how to sketch parabolas using key features and transformations.

A **transformation** is any change that can be made to a graph or shape using a **dilation**, **translation**, reflection or rotation.

Dilation of parabolas

The quadratic equation in the form of $y = x^2$ forms a parabola when graphed.

Compared to the graph of $y = x^2$, the graph of $y = ax^2$ is dilated by a factor of a, where a is a constant. The size of this coefficient of x^2, a, is called the **dilation factor**.

Worked example 10

W.E. 10

(a) Plot the graph of $y = x^2$ for $-2 \leq x \leq 2$.

(b) Plot the graphs of **(i)** $y = -x^2$, **(ii)** $y = 2x^2$ and **(iii)** $y = \frac{1}{2}x^2$ on the same set of axes as your graph in part **(a)**, using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$ using words like narrower, wider or inverted (upside down).

(d) State the dilation factor for each of the graphs in part **(b)**.

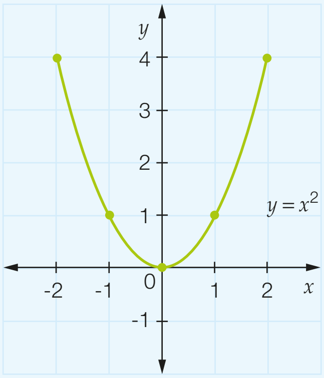
Thinking

Working

(a) Plot the graph of $y = x^2$.

(a)

x	-2	-1	0	1	2
y	4	-1	0	1	4



(b) Plot each of the graphs on the same set of axes using an appropriate table of values.

(b) (i) $y = -x^2$

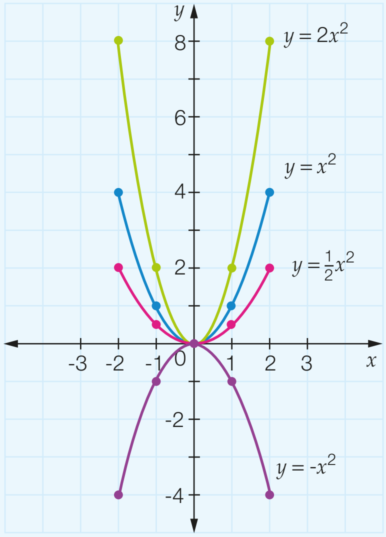
x	-2	-1	0	1	2
y	-4	-1	0	-1	-4

(ii) $y = 2x^2$

x	-2	-1	0	1	2
y	8	2	0	2	8

(iii) $y = \frac{1}{2}x^2$

x	-2	-1	0	1	2
y	2	0.5	0	0.5	2



(c) Use your graph to decide whether each graph is narrower, wider or inverted.

(c) (i) $y = -x^2$ is inverted but the same width.

(ii) $y = 2x^2$ is narrower than $y = x^2$.

(iii) $y = \frac{1}{2}x^2$ is wider than $y = x^2$.

(d) Identify the coefficient of x^2 in each equation. This is the value of a, the dilation factor.

(d) (i) The dilation factor, a, is -1.

(ii) The dilation factor, a, is 2.

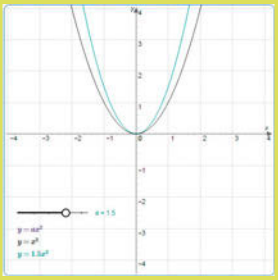
(iii) The dilation factor, a, is $\frac{1}{2}$.

Interactive

Shape of a quadratic function

Explore how the coefficient of x^2 affects the dilation of a quadratic graph.

Go to the eBook or the Pearson Places website to access this interactive.



For equations of the form $y = ax^2$:

- When the coefficient of x^2 is negative ($a < 0$) the parabola is inverted. These graphs are reflected in the x-axis.
- When the coefficient of x^2 is greater than 1 ($a > 1$), the parabola is narrower than $y = x^2$.
- When the coefficient of x^2 is greater than zero but less than 1 ($0 < a < 1$), the parabola is wider than $y = x^2$.
- When the coefficient of x^2 is greater than -1 but less than zero ($-1 < a < 0$) the parabola is also wider than $y = x^2$.

Horizontal translations of parabolas

Equations that shift only the value of x in $y = x^2$, by adding or subtracting a constant to x, will translate the parabola left or right (horizontally).

Worked example 11

W.E. 11

(a) Plot the graph of $y = x^2$ for $-4 \le x \le 4$.

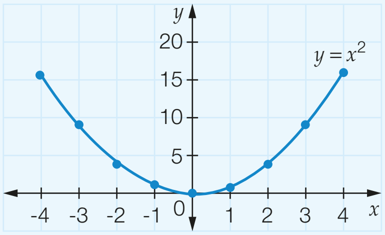
(b) Plot the graphs of **(i)** $y = (x - 1)^2$ and **(ii)** $y = (x + 2)^2$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$.

(d) State the x-intercept for each graph in part **(b)** and identify the x-value of the intercept from the equation.

Thinking	Working
(a) Plot the graph of $y = x^2$.	(a) $y = x^2$

x	-4	-3	-2	-1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16



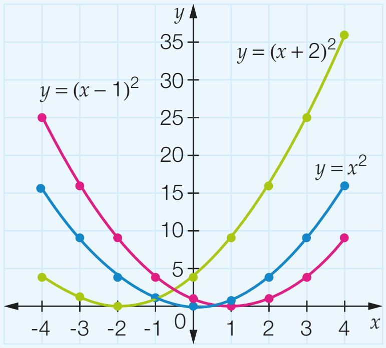
(b) Plot each of the graphs on the same set of axes using an appropriate table of values.

(b) (i) $y = (x - 1)^2$

x	-4	-3	-2	-1	0	1	2	3	4
y	25	16	9	4	1	0	1	4	9

(ii) $y = (x + 2)^2$

x	-4	-3	-2	-1	0	1	2	3	4
y	4	1	0	1	4	9	16	25	36



(c) Use your graph to decide what translation has happened to the graph of $y = x^2$.

(c) (i) The graph of $y = (x - 1)^2$ has been translated 1 unit to the right.

(ii) The graph of $y = (x + 2)^2$ has been translated 2 units to the left.

(d) State the x-intercept from the graph and identify the x-value of the intercept from the equation.

(d) (i) The x-intercept of $y = (x - 1)^2$ is the point (1, 0). The x-value of the intercept is $x = 1$. This is the number subtracted from x before squaring in the equation.

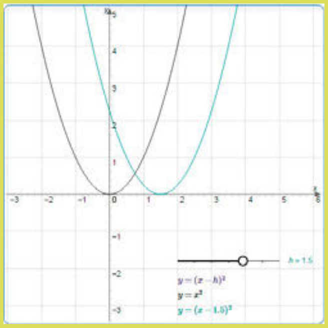
(ii) The x-intercept of $y = (x + 2)^2$ is the point (-2, 0). The x-value of the intercept is $x = -2$. This is the number subtracted from x before squaring in the equation.

Interactive

Horizontal translation of a quadratic

Explore how adding or subtracting a constant to x affects the translation of a quadratic graph.

Go to the eBook or the Pearson Places website to access this interactive.



The graph of $y = (x - h)^2$ is the graph of $y = x^2$ translated h units to the right.
If the value of h is negative, this will mean a translation to the left.

Vertical translations of parabolas

Equations that shift only the value of y in $y = x^2$, by adding or subtracting a constant to y, will translate the parabola up or down (vertically).

Worked example 12

W.E. 12

(a) Plot the graph of $y = x^2$ for $-4 \leq x \leq 4$.

(b) Plot the graphs of **(i)** $y = x^2 + 3$ and **(ii)** $y = x^2 - 4$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$.

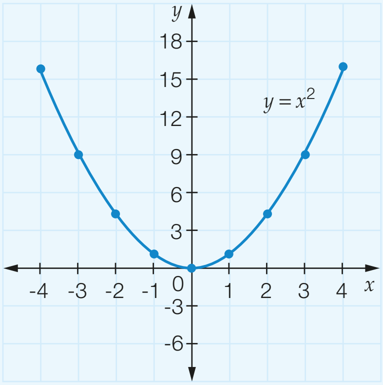
(d) State the y-intercept for each graph in part **(b)** and identify the y-intercept from the equation.

Thinking	Working

(a) Plot the graph of $y = x^2$.

(a)

x	-4	-3	-2	-1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16



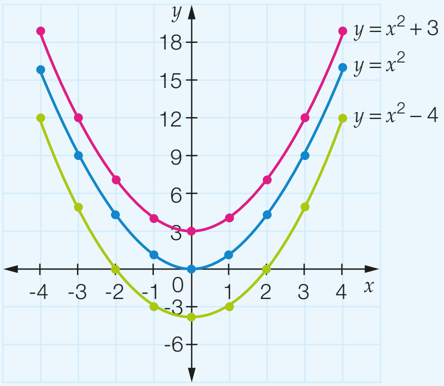
(b) Plot each of the graphs on the same set of axes using an appropriate table of values.

(b) (i) $y = x^2 + 3$

x	-4	-3	-2	-1	0	1	2	3	4
y	19	12	7	4	3	4	7	12	19

(ii) $y = x^2 - 4$

x	-4	-3	-2	-1	0	1	2	3	4
y	12	5	0	-3	-4	-3	0	5	12



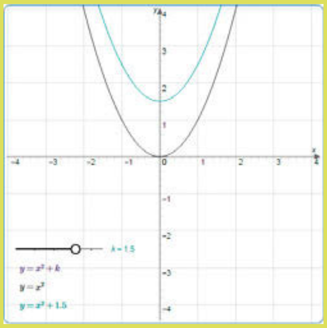
(c) Use your graph to decide what translation has happened to the graph of $y = x^2$.	(c) (i) The graph of $y = x^2$ has been translated 3 units upwards. (ii) The graph of $y = x^2$ has been translated 4 units downwards.
(d) State the y-intercept from the graph and identify the y-value of the intercept from the equation.	(d) (i) The y-intercept of $y = x^2 + 3$ is the point (0, 3). The y-value of the intercept is $y = 3$. This is the constant in the equation. (ii) The y-intercept of $y = x^2 - 4$ is the point (0, 4). The y-value of the intercept is $y = -4$. This is the constant in the equation.

Interactive

Vertical translation of a quadratic

Explore how adding or subtracting a constant to y affects the translation of a quadratic graph.

Go to the eBook or the Pearson Places website to access this interactive.



The graph of $y = x^2 + k$ is the graph of $y = x^2$ translated k units upwards. If the value of k is negative, this will mean a translation downwards.

Combined horizontal and vertical translations

You can combine horizontal and vertical translations with equations in the form of $y = (x - h)^2 + k$, where h is the translation to the right and k is the translation upwards.

Worked example 13

W.E. 13

(a) Plot the graph of $y = x^2$ for $-5 \le x \le 5$.

(b) Plot the graphs of **(i)** $y = (x - 3)^2 + 1$ and **(ii)** $y = (x + 1)^2 - 5$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$.

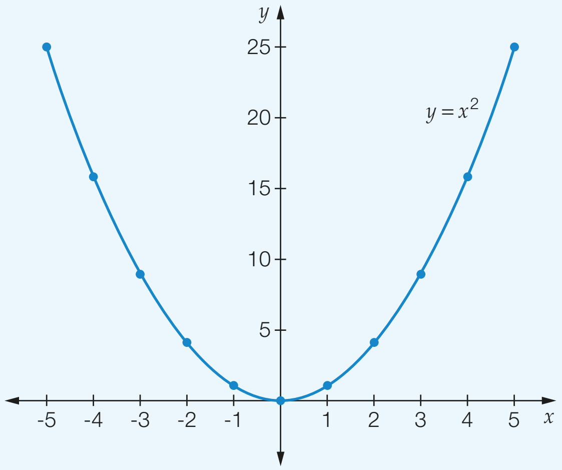
(d) Find the y-intercept for each graph in part **(b)**.

(e) For each graph, identify whether there are zero, one or two x-intercepts.

Thinking	**Working**
(a) 1 Construct a table of values for the given values of x.	(a) $y = x^2$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	25	16	9	4	1	0	1	4	9	16	25

2 Plot the graph of $y = x^2$.



(b) **1** Construct a table of values for each of the equations for the given values of x.

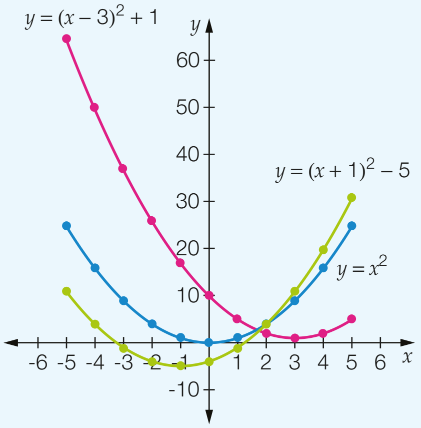
(b) (i) $y = (x - 3)^2 + 1$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	65	50	37	26	17	10	5	2	1	2	5

(ii) $y = (x + 1)^2 - 5$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	11	4	-1	-4	-5	-4	-1	4	11	20	31

2 Plot each of the graphs on the same set of axes.



(c) Use your graph to decide what translation has happened to the graph of $y = x^2$.

(c) (i) The graph of $y = x^2$ has been translated 3 units to the right and 1 unit upwards.

(ii) The graph of $y = x^2$ has been translated 1 unit to the left and 5 units down.

(d) Identify the y-value of the intercept from the table of values for the equation (when $x = 0$).

(d) (i) The y-intercept is (0, 10).

(ii) The y-intercept is (0, -4).

(e) Use your graph to determine how many x-intercepts there are.

(e) (i) The graph has no x-intercepts.

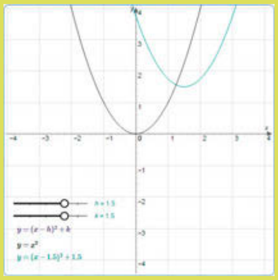
(ii) The graph has two x-intercepts.

Interactive

Combined transformations of a quadratic

Explore how the shape and position of a quadratic graph is affected by changing its values.

Go to the eBook or the Pearson Places website to access this interactive.



The graph of $y = (x - h)^2 + k$ is the graph of $y = x^2$ translated h units to the right and k units upwards. If the value of h is negative, this will mean a translation to the left. If the value of k is negative, this will mean a translation downwards.

Locating *x*-intercepts and *y*-intercepts

You have seen that the y-intercept is where the parabola intersects the y-axis (that is, where $x = 0$). The y-value of the y-intercept can therefore be found by substituting $x = 0$ into the quadratic equation:

$$\begin{aligned} y &= x^2 - 5x + 6 \\ &= 0 - 5 \times 0 + 6 \quad \text{(substituting } x = 0\text{)} \\ &= 6 \end{aligned}$$

So the coordinates of the y-intercept would be (0, 6).

You have also seen that the x-intercept is where the parabola intersects the x-axis where $y = 0$. The x-value of the x-intercept can therefore be found by substituting $y = 0$ into the quadratic equation:

$$\begin{aligned} &\text{If } y = (x - 2)(x - 3) && \text{(this quadratic, when expanded, gives } y = x^2 - 5x + 6\text{)} \\ &0 = (x - 2)(x - 3) && \text{(substituting } y = 0\text{)} \\ &(x - 2) = 0 \text{ or } (x - 3) = 0 && \text{(using the null factor law)} \\ &x = 2 \text{ or } \quad x = 3 && \end{aligned}$$

So the coordinates of the x-intercepts would be (2, 0) or (3, 0).

The turning point

All quadratic graphs have a vertex or turning point. This can be a minimum value or a maximum value.

When a graph has two x-intercepts, the x-coordinate of the turning point must be half-way between the two x-intercepts.

For the quadratic equation $y = (x - 3)(x + 5)$, the x-intercepts are (3, 0) and (-5, 0), so $x = -1$ is the x-coordinate of the turning point.

The corresponding y-coordinate can be found by substituting the x-value into the quadratic equation:

$$\begin{aligned} y &= (-1 - 3)(-1 + 5) \\ &= -4 \times 4 \\ &= -16 \end{aligned}$$

When a graph has only one x-intercept, the turning point is the x-intercept.

For the quadratic equation $y = (x - 4)^2$, the x-intercept is (4, 0).

When there are no x-intercepts, the turning point is more difficult to find, as in the example below.

Worked example 14

W.E. 14

(a) Plot the graph of $y = (x - 1)^2 - 9$ for $-5 \leq x \leq 5$.

(b) From your graph, identify the turning point.

(c) By comparing the equation in part **(a)** to $y = (x - h)^2 + k$, write the values of h and k. Is the point (h, k) the same point as the turning point found in part **(b)**?

(d) Expand the given equation and expand $y = (x - 4)(x + 2)$ to show that they are equivalent expressions. Use this information to find the x-coordinates of the x-intercepts. Find the average of these two values. Is this value the same as the x-coordinate of the turning point found in part **(b)**?

(e) Substitute this value into the equation in part **(a)** to find the corresponding y-value. Is this value the same as the y-coordinate of the turning point found in part **(b)**?

(f) By comparing the expanded equation found in part **(d)** to $y = ax^2 + bx + c$, write the values of a, b and c. Find the value of $-\frac{b}{2a}$. Is this value the same as the x-coordinate of the turning point found in part **(b)**?

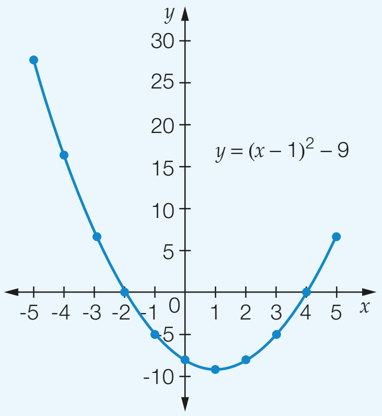
Thinking

Working

(a) Draw up a table of values and plot the points on a Cartesian plane. Label the axes and the graph.

(a) $y = (x - 1)^2 - 9$

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	27	16	7	0	-5	-8	-9	-8	-5	0	7



(b) Look at the graph to see where it changes direction.

(b) The turning point is (1, -9).

(c) Write the values of h and k and then an ordered pair (h, k). Compare this point to the point found in part **(b)**.

(c) $h = 1$ and $k = -9$

$(h, k) = (1, -9)$

Yes, it is the same point.

(d) **1** Expand the brackets using the perfect square rule and simplify.

(d) $y = (x - 1)^2 - 9$

$= x^2 - 2x + 1 - 9$

$= x^2 - 2x - 8$

2 Expand the second expression and simplify.

$(x - 4)(x + 2)$

$= x^2 - 4x + 2x - 8$

$= x^2 - 2x - 8$

	3	Compare the two expressions.	$(x-1)^2 - 9 = (x-4)(x+2)$
	4	Equate the factorised expression to 0.	$(x-4)(x+2) = 0$
	5	Use the null factor law to find the x-intercepts.	$x = 4$ and $x = -2$
	6	Find the average of the two x-values.	Average of 4 and -2 = $\frac{4 + -2}{2} = 1$
	7	Compare this value with the value of x found in part **(b)**.	Yes, this is the same value.
(e)		Substitute the value of x found in part **(d)** into the equation.	(e) $y = (x-1)^2 - 9 = (1-1)^2 - 9 = -9$ Yes, this is the same value.
(f)	1	Write the values of a, b and c.	(f) $a = 1, b = -2, c = -8$
	2	Find $-\frac{b}{2a}$.	$-\frac{b}{2a} = -\frac{-2}{2} = 1$
	3	Compare this value with the value of x found in part **(b)**.	Yes, this is the same value.

When the quadratic equation is written as $y = a(x-h)^2 + k$, you can always identify the turning point as the point with coordinates (h, k).

For the quadratic equation $y = (x-2)^2 + 7$ the coordinates of the turning point are (2, 7).

In a quadratic equation of the form $y = ax^2 + bx + c$, the x-value of the turning point can be found using $x = -\frac{b}{2a}$.

9.3 Sketching parabolas using transformations

Navigator

Answers p. 692

1, 2, 3, 4, 5, 6, 7 (a–c), 8 (a–c), 9 (a–c), 10, 11, 12, 13, 14 (a–f), 15, 18, 20, 21	1, 2, 3, 4, 5, 6, 7 (a–c), 8 (a–c), 9 (a–c), 10, 11, 12, 13, 14 (a–h), 15, 17, 18, 19, 20, 21	1, 2, 3, 4, 5, 6, 7 (e–h), 8 (e–h), 9 (d–f), 10, 11, 12, 13, 14 (i–p), 15, 16, 17, 18, 19, 21

Fluency

W.E. 10

1 **(a)** Plot the graph of $y = x^2$ for $-3 \le x \le 3$.

(b) Plot the graphs of **(i)** $y = -2x^2$, **(ii)** $y = 3x^2$ and **(iii)** $y = \frac{x^2}{4}$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$ using the words narrower, wider or inverted (upside down).

(d) State the dilation factor for each of the graphs in part **(b)**.

2 **(a)** Plot the graph of $y = x^2$ for $-3 \leq x \leq 3$.

W.E. 11

(b) Plot the graphs of **(i)** $y = (x - 2)^2$ and **(ii)** $y = (x + 1)^2$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$.

(d) State the x-intercept for each graph in part **(b)** and identify the x-value of the intercept from the equation.

3 **(a)** Plot the graph of $y = x^2$ for $-3 \leq x \leq 3$.

W.E. 12

(b) Plot the graphs of **(i)** $y = x^2 + 4$ and **(ii)** $y = x^2 - 2$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$.

(d) State the y-intercept for each graph in part **(b)** and identify the value of the y-intercept from the equation.

4 **(a)** Plot the graph of $y = x^2$ for $-3 \leq x \leq 3$.

W.E. 13

(b) Plot the graphs of **(i)** $y = (x - 1)^2 - 2$ and **(ii)** $y = (x + 2)^2 + 1$ on the same set of axes as your graph in part **(a)** using the same values for x.

(c) Compare each of the graphs in part **(b)** to $y = x^2$.

(d) Find the y-intercept for each graph in part **(b)**.

(e) For each graph, identify whether there are zero, one or two x-intercepts.

5 **(a)** Plot the graph of $y = (x - 2)^2 - 1$ for $-5 \leq x \leq 5$.

W.E. 14

(b) From your graph, identify the turning point.

(c) By comparing the equation in part **(a)** to $y = (x - h)^2 + k$, write the values of h and k. Is the point (h, k) the same point as the turning point found in part **(b)**?

(d) Expand the given equation and expand $y = (x - 1)(x - 3)$ to show that they are equivalent expressions. Use this information to find the x-coordinates of the x-intercepts. Find the average of these two values. Is this value the same as the x-coordinate of the turning point found in part **(b)**?

(e) Substitute this value into the equation in part **(a)** to find the corresponding y-value. Is this value the same as the y-coordinate of the turning point found in part **(b)**?

(f) By comparing the expanded equation found in part **(d)** to $y = ax^2 + bx + c$, write the values of a, b and c. Find the value of $-\frac{b}{2a}$. Is this value the same as the x-coordinate of the turning point found in part **(b)**?

6 **(a)** State whether the graph of each of these equations will be narrower or wider than the parabola $y = x^2$.

(i) $y = 5x^2$ **(ii)** $y = \frac{7}{8}x^2$ **(iii)** $y = -\frac{1}{6}x^2$ **(iv)** $y = -14x^2$

(b) Which of the graphs of the equations in part **(a)** would be upside down (or inverted) compared with $y = x^2$?

(c) State the dilation factor of each of the following.

(i) $y = 4x^2$ **(ii)** $y = \frac{1}{5}x^2$ **(iii)** $y = 7x^2 + 2x$ **(iv)** $y = \frac{2}{3}x^2 - 3x + 1$

(d) Will a parabola be narrower or wider than $y = x^2$ if it has a dilation factor of:

(i) 3 **(ii)** $\frac{4}{5}$ **(iii)** 16 **(iv)** $\frac{1}{7}$?

7 For each of the following:

(i) find the translation of the graph of $y = x^2$

(ii) state the y-intercept.

(a) $y = x^2 + 4$
(b) $y = x^2 - 7$
(c) $y = x^2 - 1$
(d) $y = x^2 + 2$
(e) $y = x^2 - 8$
(f) $y = x^2 + 5$
(g) $y = x^2 + \frac{1}{2}$
(h) $y = x^2 - \frac{3}{4}$

8 For each of the following:

(i) find the translation of the graph of $y = x^2$

(ii) identify the x-value of the x-intercept and hence the coordinates of the turning point.

(a) $y = (x - 5)^2$
(b) $y = (x + 4)^2$
(c) $y = (x + 1)^2$
(d) $y = (x - 7)^2$
(e) $y = (x + 6)^2$
(f) $y = (x - 3)^2$
(g) $y = (x - \frac{2}{3})^2$
(h) $y = (x + \frac{3}{2})^2$

9 For each of the following quadratic relationships:

(i) find the transformations of the graph $y = x^2$

(ii) find the turning point.

(a) $y = 2(x - 1)^2 + 3$
(b) $y = -3(x + 2)^2 - 1$
(c) $y = 4(x + 3)^2 - 2$
(d) $y = \dfrac{1}{3}(x + 5)^2 + 3$
(e) $y = (x + 4)^2 - 3$
(f) $y = 2(x + 3)^2 + \dfrac{1}{2}$

10 **(a)** To obtain the graph of $y = x^2 + 6$, you need to move the graph of $y = x^2$:

A 6 units to the right
B 6 units to the left
C 6 units up
D 6 units down

(b) To obtain the graph of $y = (x + 6)^2$, you need to move the graph of $y = x^2$:

A 6 units to the right
B 6 units to the left
C 6 units up
D 6 units down

(c) To obtain the graph of $y = (x - 2)^2$, you need to move the graph of $y = x^2$:

A 2 units to the right
B 2 units to the left
C 2 units up
D 2 units down

(d) To obtain the graph of $y = x^2 - 4$, you need to move the graph of $y = x^2$:

A 4 units to the right
B 4 units to the left
C 4 units up
D 4 units down

11 **(a)** To obtain the graph of $y = (x - 1)^2 - 3$, you need to move the graph of $y = x^2$:

A 1 unit to the right and 3 units down
B 3 units to the right and 1 unit down
C 1 unit to the left and 3 units up
D 3 units to the right and 1 unit down

(b) To obtain the graph of $y = (x + 4)^2 - 2$, you need to move the graph of $y = x^2$:

A 2 units to the right and 4 units up
B 2 units to the left and 4 units down
C 4 units to the right and 2 units down
D 4 units to the left and 2 units down

(c) To obtain the graph of $y = -(x - 3)^2 + 1$, you need to change the graph of $y = x^2$ by:

A inverting it, moving it 1 unit to the right and 3 units up
B inverting it, moving it 1 unit to the left and 3 units down
C inverting it, moving it 3 units to the left and 1 unit down
D inverting it, moving it 3 units to the right and 1 unit up

(d) To obtain the graph of $y = 2(x - 1)^2 - 3$, you need to change the graph of $y = x^2$ by:

A dilating by a factor of 2 (makes it narrower) and moving it 1 unit to the right and 3 units down

B dilating by a factor of 3 (makes it narrower) and moving it 1 unit to the left and 3 units down

C dilating by a factor of $\frac{1}{2}$ (makes it wider) and moving it 1 unit to the right and 3 units down

D dilating by a factor of 2 (makes it narrower) and moving it 1 unit to the right and 3 units up

(e) To obtain the graph of $y = -\frac{1}{2}(x - 4)^2 + 1$, you need to change the graph of $y = x^2$ by:

A dilating by a factor of $\frac{1}{2}$ (makes it wider), moving it 4 units to the right and 1 unit up

B inverting it, dilating by a factor of $\frac{1}{2}$ (makes it wider), moving it 4 units to the left and 1 unit up

C inverting it, dilating by a factor of $\frac{1}{2}$ (makes it wider), moving it 4 units to the left and 1 unit down

D inverting it, dilating by a factor of $\frac{1}{2}$ (makes it wider), moving it 4 units to the right and 1 unit up

Understanding

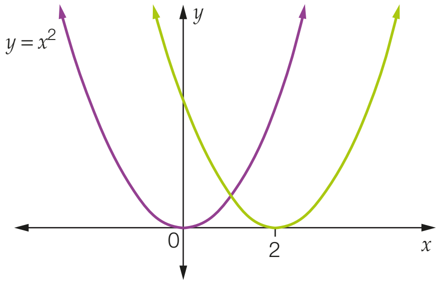
12 **(a)** The equation of the unlabelled parabola could be:

A $y = x^2 + 2$

B $y = x^2 - 2$

C $y = (x - 2)^2$

D $y = (x + 2)^2$



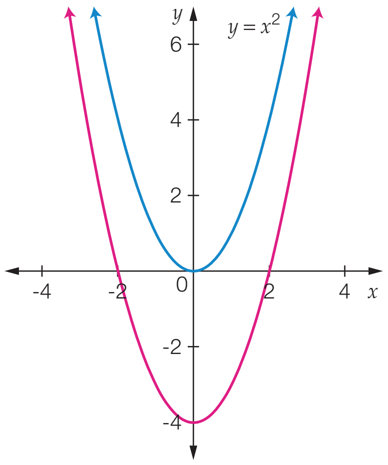
(b) The equation of the unlabelled parabola could be:

A $y = (x + 4)^2$

B $y = (x - 4)^2$

C $y = x^2 + 4$

D $y = x^2 - 4$



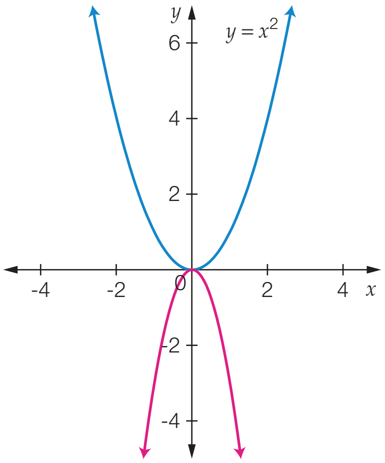
(c) The equation of the unlabelled parabola could be:

A $y = 3x^2$

B $y = \frac{1}{3}x^2$

C $y = -3x^2$

D $y = -\frac{1}{3}x^2$



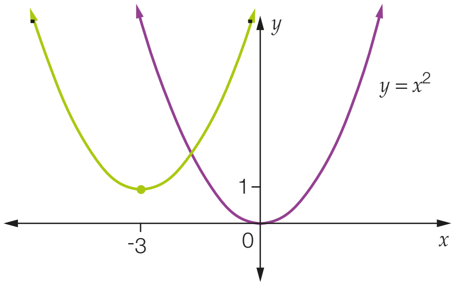
(d) The equation of the unlabelled parabola could be:

A $y = x^2 + 1$

B $y = (x + 3)^2 + 1$

C $y = 3x^2 + 1$

D $y = (x - 3)^2 + 1$



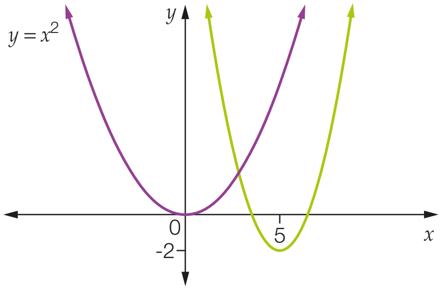
(e) The equation of the unlabelled parabola could be:

A $y = 2(x - 5)^2 - 2$

B $y = \frac{1}{2}(x + 5)^2 - 2$

C $y = 2(x - 5)^2 + 2$

D $y = \frac{1}{2}(x - 5)^2 - 2$



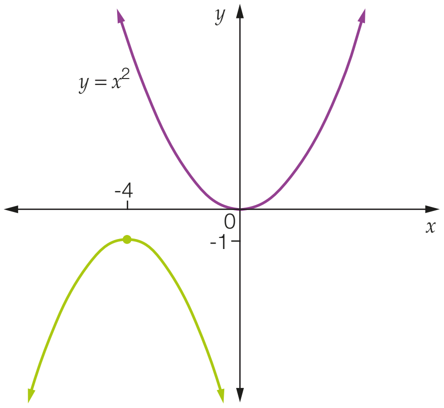
(f) The equation of the unlabelled parabola could be:

A $y = (x + 4)^2 - 1$

B $y = -(x + 4)^2 - 1$

C $y = (x - 4)^2 - 1$

D $y = 4(x - 4)^2 - 1$



13 For each of the following equations, state the translation made to $y = x^2$. Use this to find the turning point.

(a) $y = 4x^2$

(b) $y = (x - 6)^2$

(c) $y = x^2 + 3$

(d) $y = (x + 2)^2 - 6$

(e) $y = -2(x - 1)^2 + 4$

14 State the changes needed to transform a graph of $y = x^2$ into each of the following.

		Dilation	Reflection in *x*-axis	Translation in *x*-direction	Translation in *y*-direction
(a)	$y = (x + 1)^2 + 2$				
(b)	$y = (x - 3)^2 + 1$				
(c)	$y = 5x^2 + 4$				
(d)	$y = -x^2 - 6$				
(e)	$y = 2(x + 5)^2 - 3$				
(f)	$y = -(x - 2)^2 + 3$				
(g)	$y = -3(x + 6)^2 - 4$				
(h)	$y = \frac{1}{2}(x - 3)^2 + 1$				
(i)	$y = (x - 4)^2 - 5$				
(j)	$y = (x + 2)^2 - 7$				
(k)	$y = 3x^2 - 5$				
(l)	$y = -x^2 - 2$				
(m)	$y = 5(x - 6)^2 - 4$				
(n)	$y = -(x - 5)^2 + 2$				
(o)	$y = -4(x + 1)^2 - 3$				
(p)	$y = \frac{3}{4}(x + 4)^2 + 6$				

15 When graphed, how many of the following equations would have a shape like the parabola shown? Identify them.

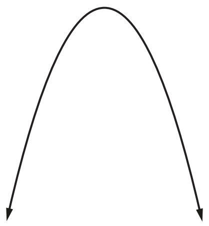
A $y = x^2 + 2x - 5$

B $y = -x^2 - 2x + 4$

C $y = 1 - x^2$

D $y = 3 - 4x + x^2$

E $y = 6 - 2x - x^2$



16 If the x-intercepts of a parabola are -1 and 5, the turning point could be at:

A (0, 3) **B** (-2, 2) **C** (2, 4) **D** (1, -6)

Reasoning

17 **(a)** A quadratic graph has a turning point at (-2, 5) and a dilation factor of 1. Find the equation of the graph.

(b) Find the y-intercept (where $x = 0$).

(c) Draw the graph, clearly labelling the turning point and the y-intercept.

18 Find the new equation when the following transformations are completed on the graph $y = x^2$.

(a) The graph is dilated by a factor of 5, translated 3 units to the right and 2 units up.

(b) The graph is dilated by a factor of $\frac{1}{3}$, translated down 2 units and to the right 4 units.

19 The sum of two numbers, x and y, is 20.

(a) Express y in terms of x.

(b) Write the numbers' product P in terms of x and sketch a graph of P.

(c) Use the graph to find the values of x and y that will make the product as large as possible.

Open-ended

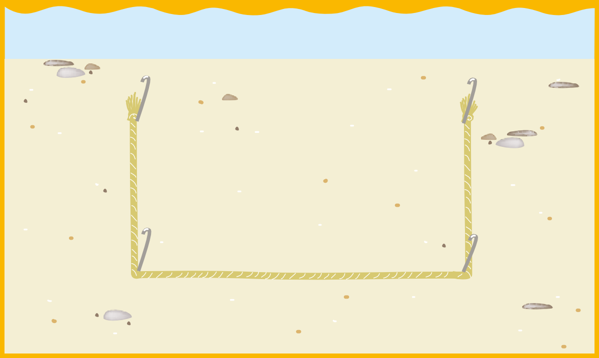
20 Write at least two equations that show a quadratic relationship in the form $y = a(x - b)^2 + c$. For each one, write what transformations need to be applied to the graph of $y = x^2$ to produce it.

21 Write equations for three parabolas that have no x-intercepts.

Problem solving

Let's go camping

You decide to go camping at a camping ground near a lake. The manager of the camping ground gives you a 60 m long piece of rope and four pegs, which you must use to mark your campsite. The only condition is that your campsite must be rectangular. You decide to pitch your tent by the lake so that you only need to use the rope on three sides. Find the largest campsite that you could make.



Strategy options

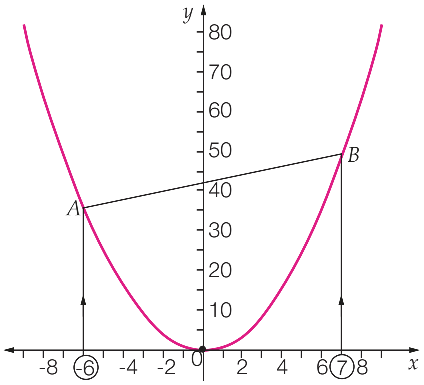
- Test all possible combinations.
- Make a table.
- Make a model.
- Draw a diagram.

Investigation

A parabolic nomogram

Equipment required: graph paper (1 cm grid with smaller divisions)

A *nomogram* is a group of scaled lines that are usually parallel and are used to perform a calculation. The nomogram in this investigation uses a parabolic graph to calculate multiplications.



The Big Question

What makes a parabolic nomogram work?

Engage

1 **(a)** Construct an accurate parabola $y = x^2$ on graph paper. Circle the values of -6 and 7 on the x-axis, then draw two vertical lines from these points, going upwards to meet the parabola. Label the points where these lines meet the parabola A and B. Draw a line joining the points A and B. Your graph should look like the graph shown above.

(b) Write the y-value of the y-intercept.

2 **(a)** Write the y-value as the y-intercept of the line segment AB.

(b) What do you notice about the numbers circled on the graph shown above?

3 Use your parabolic nomogram to find the approximate answers to the following:

(a) 3.5×8.5 **(b)** 6.2×7.4

(c) $60 \div 4.6$ **(d)** $59 \div 5.4$

Explore

4 The equation of a line can be found using $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$. This equation connects the gradient of the line joining the point (x, y) to the point (x_1, y_1) with the gradient of the line joining the point (x_2, y_2) to the point (x_1, y_1). For a straight line, these gradients should be equal.

Use this formula to find the equation of the line joining the points A and B on the given parabola.

Compare the y-value of the y-intercept in your equation to your answer in **2**.

5 Find the equation of the line joining the points on the parabola with the following x-values.

(a) $x = -3.5$ and $x = 8.5$. Compare the y-value of the y-intercept in your equation to your answer in **3 (a)**.

(b) $x = -6.2$ and $x = 7.4$. Compare the y-value of the y-intercept in your equation to your answer in **3 (b)**.

Strategy options

- Solve a simpler problem.
- Test all possible combinations.
- Look for a pattern.

Explain

6 Can you see a relationship between the values that are multiplied and the following?

(a) the gradient of your equation

(b) the y-value of the y-intercept of your equation

Write the possible relationships.

7 What was different about the x-intercept used in the graph from the values multiplied in **3**?

Elaborate

8 **(a)** Find the general equation of the line joining the point (a, a^2) and the point (b, b^2). Remember $a^2 - b^2 = (a + b)(a - b)$.

(b) Write the gradient and the y-value of the y-intercept.

(c) Does this confirm what you found in **6**?

(d) Use this information to answer the Big Question.

9 What happens when you multiply a number by itself? Why does the nomogram still work?

Evaluate

10 Did you draw your graph accurately enough for your nomogram to work properly?

11 Did this activity give you a better understanding of gradients and y-intercepts? If so, explain how.

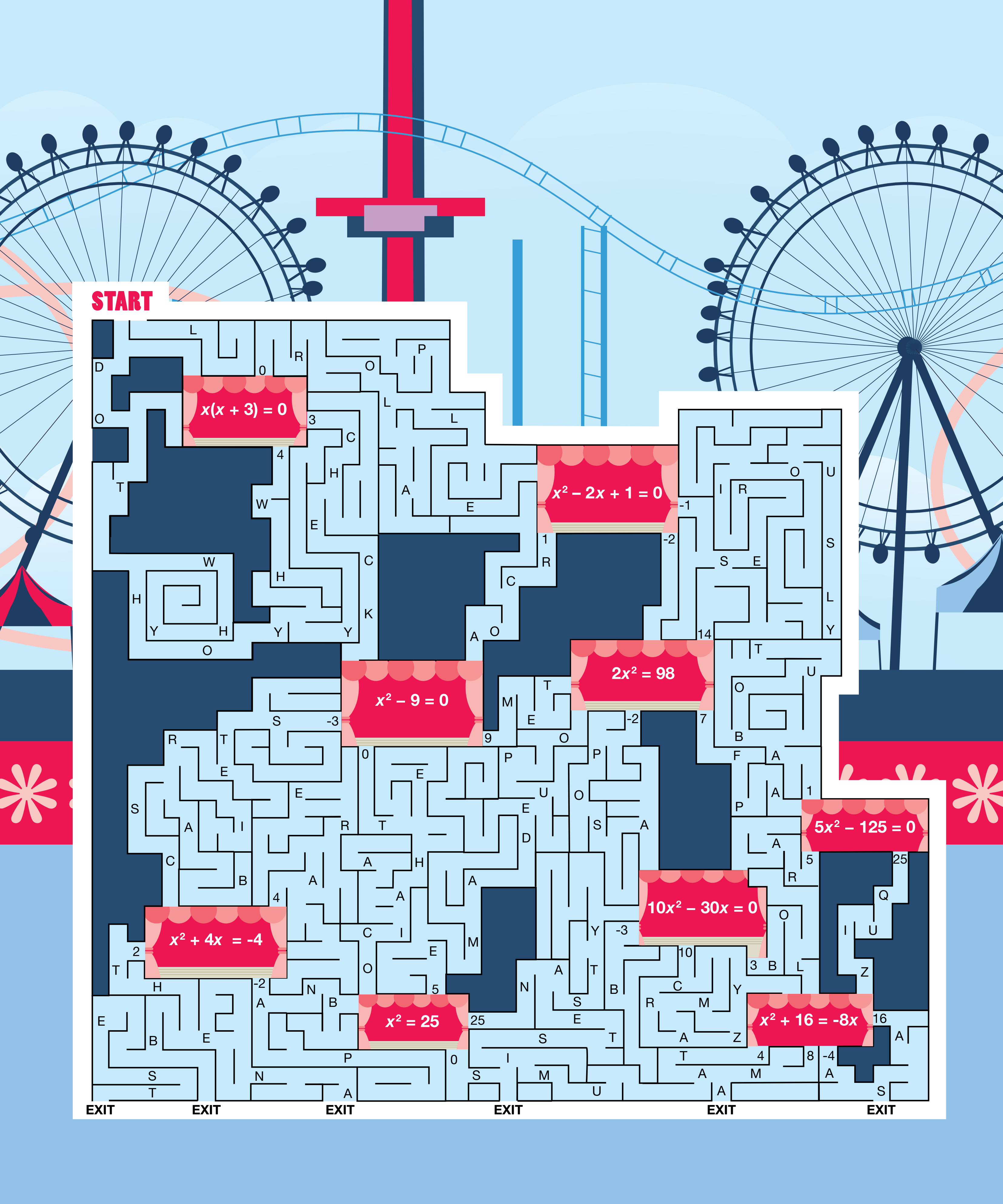
Extend

12 Describe how this nomogram could be used to find the square root of a number.

Gamespace

Quadratic maze

Welcome to 'Parabolas', a local amusement park. You can win free entry if you can find your way correctly through this quadratic maze. As you move through the maze you will need to collect the letters that lie in your path. There are nine sideshows in the maze. You must pass through all of them. When you enter a sideshow, you need to solve the equation shown, then move out through the exit that shows the correct solution. You must enter each sideshow through an opening that does not have a number across it. Once you have exited the maze, the collected letters will spell a secret code. Crack the code and entry is free! Good luck!


START
$x(x + 3) = 0$
$x^2 - 2x + 1 = 0$
$2x^2 = 98$
$x^2 - 9 = 0$
$5x^2 - 125 = 0$
$10x^2 - 30x = 0$
$x^2 + 4x = -4$
$x^2 = 25$
$x^2 + 16 = -8x$
EXIT
EXIT
EXIT
EXIT
EXIT
EXIT

Half-time 9

9.3 1 Consider the equation $y = -x^2 + 1$.

(a) Set up a table of values using integer values of x between -3 and 3 inclusive.

(b) Plot the points on a number plane and join them with a smooth curve.

(c) Find the coordinates of the turning point.

(d) Find the coordinates of the y-intercept.

(e) Find the coordinates of the x-intercepts.

9.1 2 Which of the following is not a quadratic expression?

A $2x^2 + 3x + 5$ **B** $-x + 3x^2$ **C** $3x + 2$ **D** $4x^2 - 3x - 1$

9.2 3 Solve each of the following.

(a) $x(x + 4) = 0$ (b) $2x(x - 3) = 0$ (c) $(x + 2)(x - 3) = 0$

9.1 4 Li throws a ball straight up. The equation of the flight of the ball is $h = -5t^2 + 8t + 1.3$, where h is the height of the ball in metres and t is the time in seconds.

(a) Draw a graph showing this relationship.

(b) From the graph, find when the ball first reaches a height of 3 m.

(c) How high is the ball after 1.2 seconds?

9.2 5 Solve each of the following.

(a) $x^2 + 4x + 4 = 0$ (b) $x^2 + 12x + 36 = 0$ (c) $x^2 - 18x + 81 = 0$

9.3 6 (a) Complete a table of values for $y = 2x^2 + 4x$ from $x = -3$ to $x = 3$ inclusive.

(b) Plot the points on a number plane and join them using a smooth curve.

(c) Write the coordinates of the turning point.

(d) State whether the parabola has a maximum or a minimum turning point.

(e) Write the coordinates of any x-intercepts and y-intercepts.

9.2 7 Solve each of the following.

(a) $x^2 - 6x = 0$ (b) $2x^2 + 20x + 50 = 0$ (c) $3x^2 = 27x$

9.2 8 Solve each of the following.

(a) $x^2 - 64 = 0$ (b) $(x + 1)^2 = 0$ (c) $(x - 2)^2 = 4$

9.1 9 (a) Plot the graph of $y = x^2 - 2x + 3$. Use integer values of x between -3 and 5 inclusive.

(b) What are the coordinates of the turning point?

(c) Write the coordinates of any x-intercepts and y-intercepts.

(d) Use the graph to solve the following equations.

(i) $x^2 - 2x + 3 = 3$ (ii) $x^2 - 2x + 3 = 2$

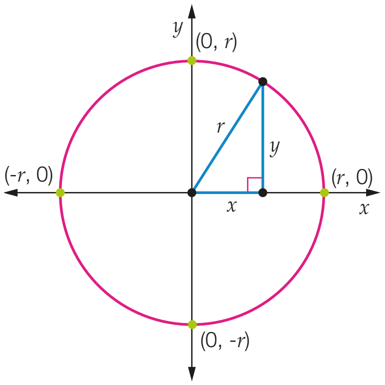
(iii) $x^2 - 2x + 3 = 6$ (iv) $x^2 - 2x + 3 = 0$

Circle relationships

9.4

Circles are important non-linear relationships. The important features of a **circle** are:

- centre with coordinates (*h*, *k*)
- radius *r*



To find the equation of a circle with the centre at the origin, use Pythagoras' theorem. Using the graph of a circle, you can write an equation $x^2 + y^2 = r^2$.

In circle equations, both the *x* and *y* variables are squared.

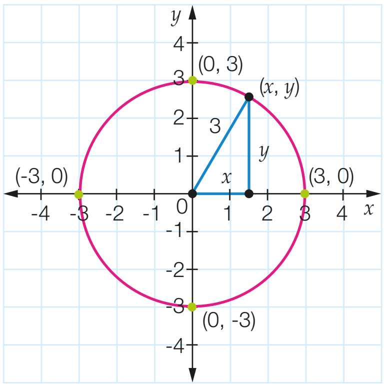
You can find four points on the circle circumference to give maximum and minimum values by adding and subtracting the radius from the centre coordinates.

For a circle with centre (0, 0) and radius *r*, the four points would be (*r*, 0), (0, *r*) (-*r*, 0) and (0, -*r*).

In the diagram shown, the radius is 3 units, so:

$x^2 + y^2 = z^2$
$x^2 + y^2 = 9$

In this case, the four points on the circumference would be (3, 0), (0, 3) (-3, 0) and (0, -3).



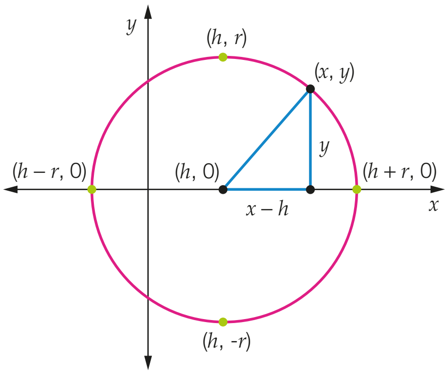
Translated circles

A circle equation can be translated horizontally.

If the circle centre is moved *h* units to the right, it will have the coordinates (*h*, 0), and all points on the circle will move *h* units to the right:

$(x - h)^2 + y^2 = r^2$

The four points on the circumference would be (*h* + *r*, 0), (*h*, *r*), (*h* − *r*, 0) and (*h*, -*r*).

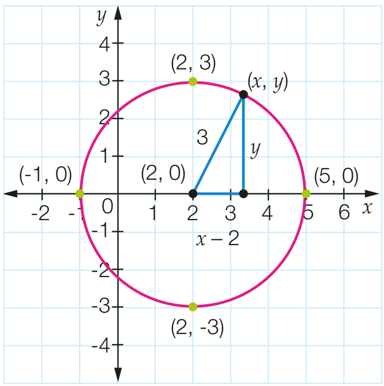


For a radius of 3 units and a centre (2, 0) the equation of the circle is:

$(x - 2)^2 + y^2 = 3^2$

$(x - 2)^2 + y^2 = 9$

The four points on the circumference would be (5, 0), (2, 3), (-1, 0) and (2, -3).



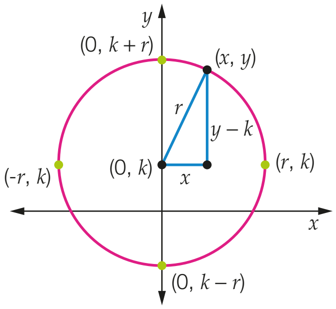
A circle equation can be translated vertically.

If the circle centre is moved k units up, it will have the coordinates (0, k) and all points on the circle will move k units up.

The equation becomes:

$x^2 + (y - k)^2 = r^2$

The four points on the circumference would be (r, k), (0, $k + r$), ($-r$, k) and (0, $k - r$).

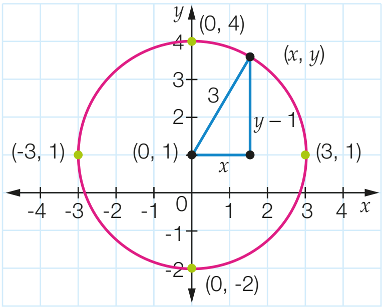


For a radius of 3 units and a centre of (0, 1), the equation of the circle is:

$x^2 + (y - 1)^2 = 3^2$

$x^2 + (y - 1)^2 = 9$

The four points on the circumference would be (3, 1), (0, 4), (-3, 1) and (0, -2).



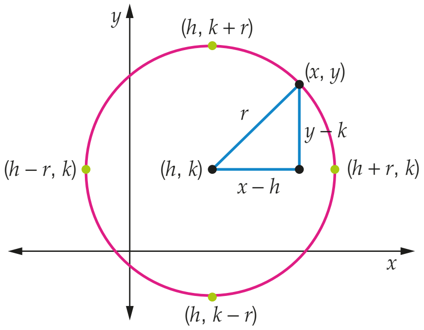
A circle can be translated both horizontally and vertically.

If the circle centre is moved h units to the right and k units up, it will have the coordinates (h, k) and all points on the circle will move h units to the right and k units up.

The equation becomes:

$(x - h)^2 + (y - k)^2 = r^2$

The four points on the circumference would be ($h + r$, k), (h, $k + r$), ($h - r$, k) and (h, k - r).

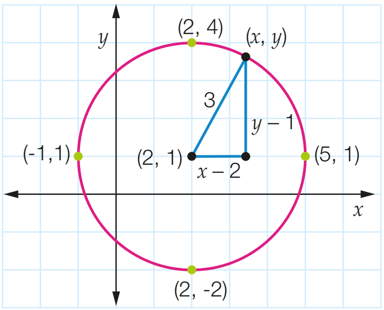


For a radius of 3 units and a centre of (2, 1), the equation will be:

$(x - 2)^2 + (y - 1)^2 = z^2$

$(x - 2)^2 + (y - 1)^2 = 9$

The four points on the circumference would be (5, 1), (2, 4), (-1, 1) and (2, -2).



The equation of a circle with centre (0, 0) and radius *r* is:

$x^2 + y^2 = r^2$

The general equation of a circle with centre (*h*, *k*) and radius *r* is:

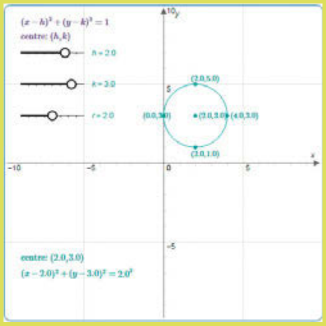
$(x - h)^2 + (y - k)^2 = r^2$

Interactive

Equation of a circle

Explore how the graph of a circle changes as the values change in its equation.

Go to the eBook or the Pearson Places website to access this interactive.

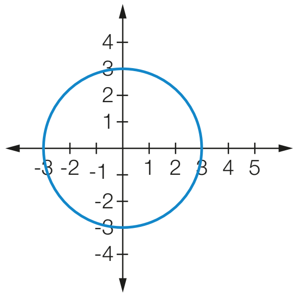


Worked example 15

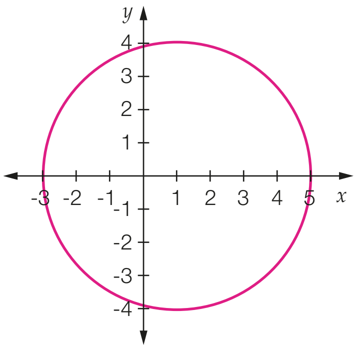
W.E. 15

Determine the centre and radius of each of the following to find the equation of the circle.

(a)



(b)



Thinking	Working
(a) 1 Determine the centre by looking at the *x*- and *y*-values that are the maximum and minimum; add them together and divide by 2.	(a) $x = \frac{3 + (-3)}{2} = 0$ $y = \frac{3 + (-3)}{2} = 0$ Centre: (0, 0)
2 Determine the radius by subtracting the minimum from the maximum for the *x*- or *y*-values and then dividing by 2.	Radius: $\frac{3 - (-3)}{2} = 3$
3 State the equation by substituting for *h*, *k* and *r* in the general form of the equation for a circle, $(x - h)^2 + (y - k)^2 = r^2$.	$(x - 0)^2 + (y - 0)^2 = 3^2$ $x^2 + y^2 = 9$

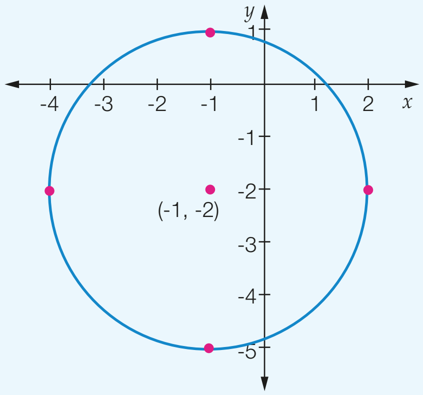
(b) 1 Determine the centre by looking at the x- and y-values that are the maximum and minimum; add them together and divide by 2.	(b) $x = \frac{5 + (-3)}{2} = 1$ $y = \frac{4 + (-4)}{2} = 0$ Centre: (1, 0)
2 Determine the radius by subtracting the minimum from the maximum for the x- and y-values and then dividing by 2.	Radius: $\frac{5 - (-3)}{2} = 4$
3 State the equation by substituting for h, k and r in the general form of the equation for a circle, $(x - h)^2 + (y - k)^2 = r^2$.	$(x - 1)^2 + (y - 0)^2 = 4^2$ $(x - 1)^2 + y^2 = 16$

Worked example 16

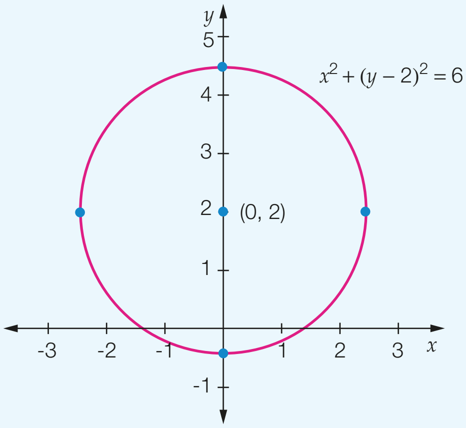
W.E. 16

Sketch each of the following graphs of circles.

(a) $(x + 1)^2 + (y + 2)^2 = 9$ **(b)** $x^2 + (y - 2)^2 = 6$

Thinking	Working
(a) 1 Determine the centre of the circle and the radius from the equation.	(a) $(x + 1)^2 + (y + 2)^2 = 9$ Centre: (-1, -2) Radius: $\sqrt{9} = 3$ Points: (2, -2) (-1, 1) (-4, -2) (-1, -5)
2 Set up appropriate axes to fit the graph.	
3 Using the radius and the centre, find four points of the circle and then sketch between them.	
(b) 1 Determine the centre of the circle and the radius from the equation.	(b) $x^2 + (y - 2)^2 = 6$ Centre: (0, 2) Radius: $\sqrt{6}$ Points: $(\sqrt{6}, 2)$ $(0, \sqrt{6} + 2)$ $(-\sqrt{6}, 2)$, $(0, \sqrt{6} - 2)$
2 Set up appropriate axes to fit the graph.	≈ (2.45, 2), (0, 4.45), (-2.45, 2), (0, -0.45) (correct to 2 decimal places)

3 Using the radius and the centre, find four points of the circle and then sketch between them. If the radius is not rational, use an approximation.



A circle can be sketched by plotting the four points where the radius is added vertically or horizontally to the centre point (directly above, below, left and right). The other points of a circle are less easy to plot by hand.

Worked example 17

W.E. 17

A path around an area on a coordinate map is described by the equation $(x - 3)^2 + (y - 2)^2 = 9$.

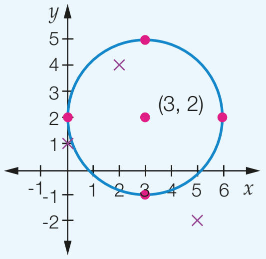
(a) Determine the centre and radius of the circle.

(b) Find the four points on the circle in line with the centre horizontally and vertically.

(c) Use these points to plot the graph.

(d) Determine whether the following points are inside or outside the circle.

(i) (0, 1) **(ii)** (2, 4) **(iii)** (5, -2)

Thinking	Working
(a) Compare the given equation to the general equation of a circle $(x - h)^2 + (y - k)^2 = r^2$ to determine h, k and r.	(a) Centre (3, 2) and radius = 3
(b) Add and subtract the radius onto the h value to find the two points on the circle in line with the centre horizontally. Add and subtract the radius to the k value to find the two points on the circle in line with the centre vertically.	(b) (0, 2), (6, 2) (3, -1) (3, 5)
(c) Plot the points on the circumference on Cartesian coordinates.	(c) 
(d) Use your graph to decide whether the given points are inside or outside the circle.	(d) (i) (0, 1) is outside the circle. (ii) (2, 4) is inside the circle. (iii) (5, -2) is outside the circle.

9.4 Circle relationships

Navigator

Answers p. 695

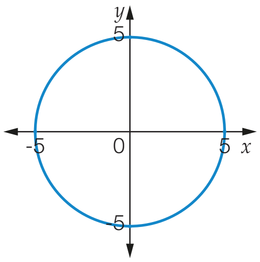
1, 2 (column 1), 3, 4, 5, 6 (a–d), 7, 11, 12	1, 2 (column 1), 3, 4, 5, 6, 7, 8, 11, 12	1, 2 (column 2), 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

Fluency

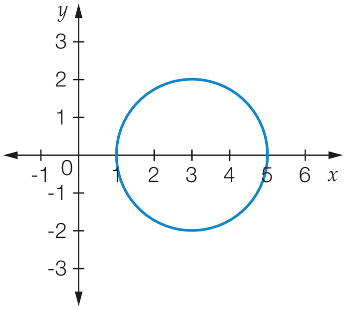
W.E. 15

1 Determine the centre and radius of each of the following, and hence the equation of the circle.

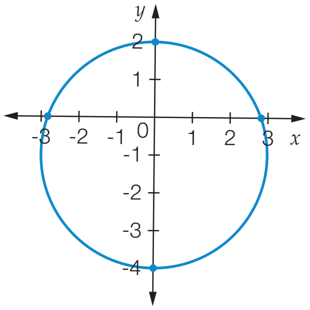
(a)



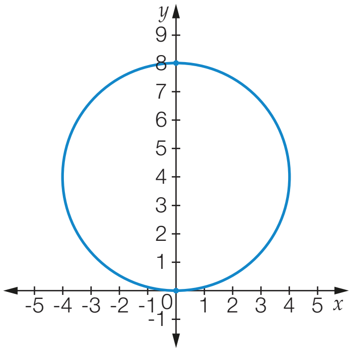
(b)



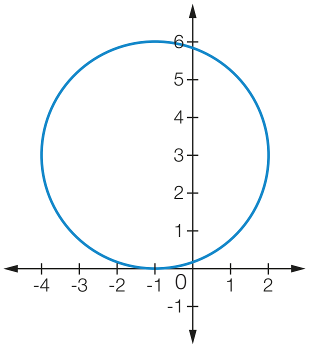
(c)



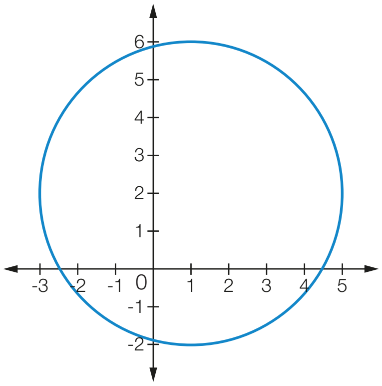
(d)



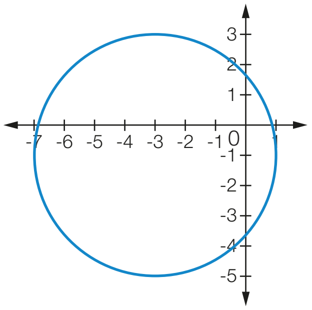
(e)



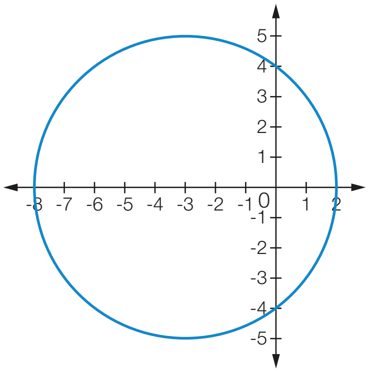
(f)



(g)



(h)



2 Sketch each of the following graphs of circles. **W.E. 16**

(a) $x^2 + y^2 = 36$ **(b)** $(x - 4)^2 + y^2 = 49$

(c) $(x + 2)^2 + (y + 3)^2 = 25$ **(d)** $(x - 2)^2 + (y - 1)^2 = 16$

(e) $(x + 1)^2 + (y - 1)^2 = 4$ **(f)** $x^2 + (y - 1)^2 = 9$

(g) $(x + 2)^2 + y^2 = 7$ **(h)** $x^2 + y^2 = 18$

3 A path around an area on a coordinate map is described by the equation $(x + 1)^2 + (y - 3)^2 = 16$. **W.E. 17**

(a) Determine the centre and radius of the circle.

(b) Find the four points on the circle in line with the centre horizontally and vertically.

(c) Use these points to plot the graph.

(d) Determine whether the following points are inside or outside the circle.

(i) (1, 2) **(ii)** (3, 6) **(iii)** (-2, 0)

Understanding

4 Which of the following equations would graph as a circle?

A $x^2 + y = 4$ **B** $x^2 + (y - 3)^2 = 5$ **C** $y^2 = x^2 + 9$ **D** $y^2 = x - 3$

5 Write the equation of a circle with the following features.

(a) a centre located at (0, 0) and a radius of 7 units

(b) a centre located at (0, 3) and a radius of 10 units

(c) a centre located at (-8, 0) and a radius of 9 units

(d) a centre located at (5, -2) and a radius of 11 units

6 Write the equation for the circle with equation $x^2 + y^2 = 36$ after the following transformations.

(a) a move of 4 units to the right and 3 units up

(b) a move of 6 units to the left and 4 units up

(c) a move of 7 units to the right

(d) a move of 5 units down

(e) a dilation to a radius of 7

7 What are the radius and centre coordinates of the circle with the following equation?

(a) $(x - 1)^2 + y^2 = 36$

A (-1, 0), 6 **B** (-1, 0), 36 **C** (1, 0), 36 **D** (1, 0), 6

(b) $x^2 + (y + 8)^2 = 144$

A (0, 8), 144 **B** (0, -8), 12 **C** (0, 8), 12 **D** (0, -8), 144

(c) $(x - 5)^2 + (y + 3)^2 = 81$

A (5, 3), 9 **B** (-5, 3), 9 **C** (5, -3), 81 **D** (5, -3), 9

Reasoning

8 A circle has an equation $(x + 4)^2 + (y + 2)^2 = 25$. What is the equation of this circle after it is translated 2 units left and 6 units up and the radius is decreased by 1 unit?

9 **(a)** Show, by substitution, that the equations $(x-8)^2+(y+8)^2=169$, $(x+9)^2+(y-9)^2=169$ and $x^2+y^2=25$ are three circles that intersect at (3, 4) and (-4, -3).

(b) Draw a graph of each of these three circles on the same set of axes.

10 A circular path with a radius of 5 metres has its centre 2 metres east and 3 metres north of a sculpture. It has been decided to move the path 9 metres further east and 1 metre further north and that its radius is to be increased by 8 metres.

(a) Write an equation to represent the original circle if the sculpture is positioned at the origin.

(b) Write an equation to represent the circle after it has been moved.

(c) Show that a point that is 1 metre west and 1 metre south of the sculpture lies on both circles.

Open-ended

11 **(a)** Write two equations that would graph as circles with a centre at the origin.

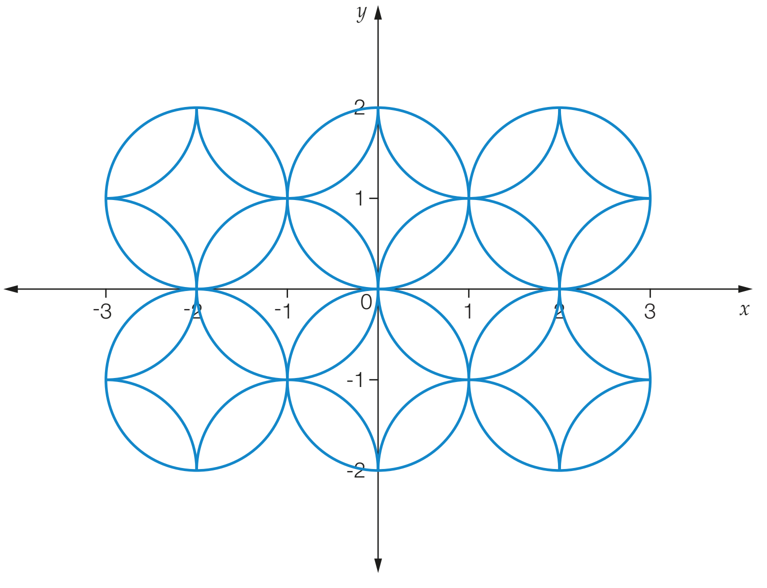
(b) Write two equations that would graph as circles with the same radius but with centres in different quadrants.

12 Make a design with three overlapping circles and write the equations for the circles you used in your design.

Problem solving

Circles within circles

Find the equations for the eight full circles in the pattern shown.



Strategy options

- Draw a diagram.
- Solve a simpler problem.
- Look for a pattern.
- Guess and check.

Exploration CAS

Equipment required: TI-Nspire CAS or ClassPad CAS

Investigating non-linear equations

Investigating parabolas

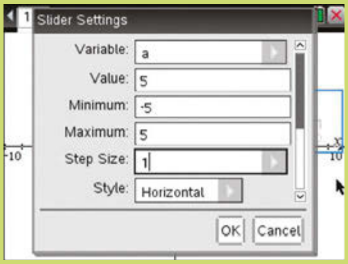
All parabolas can be represented by an equation $y = ax^2 + bx + c$, where a represents the dilation and c represents the y-coordinate of the y-intercept (where the line cuts the y-axis).

You will now create parabolas using different values of a, b and c and investigate how the graphs change when you change these values.

Using TI-Nspire CAS

Add **Graphs** to your document and enter the equation to be graphed as ***f1*(*x*)=*x*²**. A parabola will appear.

Then press **tab** and enter another equation to be graphed as ***f2*(*x*)=*a*×*x*²**. To create a slider to change the value of a, select **menu** > **Actions** > **Insert Slider** and set the **Slider Settings** as shown, with the **Variable** as **a**.



Slider Settings

Variable: a

Value: 5

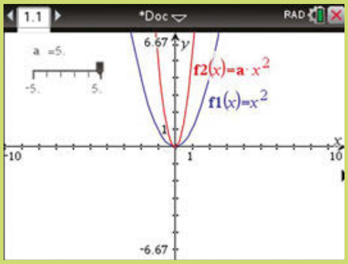
Minimum: -5

Maximum: 5

Step Size: 1

Style: Horizontal

The slider will now appear. Use the cursor to move the slider so that it does not cover the graphs.

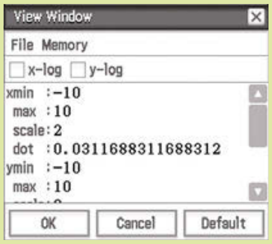


You can now select the slider and adjust the values to watch what happens to the parabola as the value of a changes.

Using Casio ClassPad CAS

From the menu select **Graph & Table**. Enter the equation to be graphed as **y1=x^2** and tick the box next to it.

You should adjust the graph window settings by selecting the view icon and then changing the values as shown. Set the **scale** for both axes to 2. Then select the draw graph icon and the parabola will appear in the lower screen.



View Window

File Memory

☐x-log ☐y-log

xmin :−10

max :10

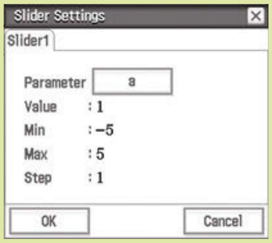
scale:2

dot :0.0311688311688312

ymin :−10

max :10

Now enter another equation to be graphed as **y2=*a*x^2**, using the **Var** tab from the **Keyboard** to enter the ***a***, and tick the box next to it. To create a slider to change the value of a, select ◆ > **Dynamic Graph** and the graph will appear with a slider for the value of a. Select ☰ and set the slider values from -5 to 5 as shown.



Slider Settings

Slider1

Parameter a

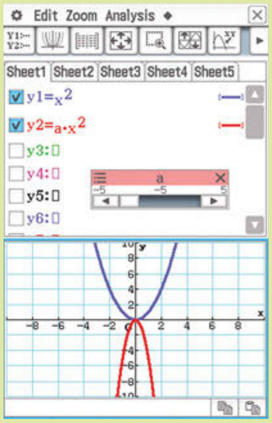
Value :1

Min :−5

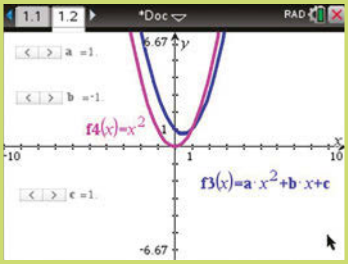
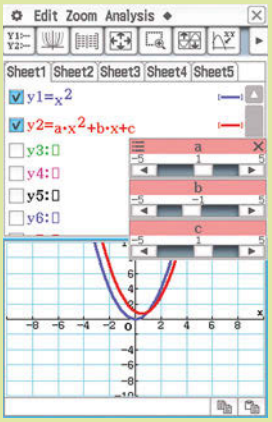
Max :5

Step :1

You can now select the slider and adjust the values to watch what happens to the parabola as the value of a changes.



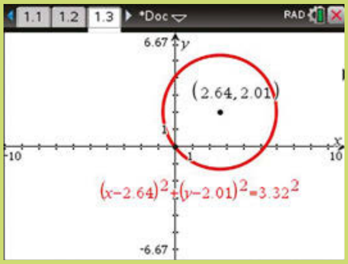
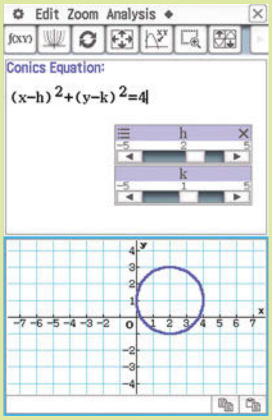
What happens when $a = 1$? What happens when $a = 0$? What happens when $a = -1$? Write a sentence to describe what you see happening.

Using TI-Nspire CAS	Using Casio ClassPad CAS
Press **tab** to return to the equation entry and use the **History Menu** to return to ***f2(x)***. Change this to **$f2(x)=x^2+c$** and then change the slider variable from **a** to **c**. Use the slider to change the value of c. Write any connection you find between the graphs and the c value.	Change the equation being graphed to **y2=x^2+c**, using the **Var** tab from the **Keyboard** to enter the **c**, and tick the box next to it. To create a slider to change the value of c, select ◆ > **Dynamic Graph** and the graph will appear with a slider for the value of c. Select ☰ and set the slider values from -5 to 5 as before. Use the slider to change the value of c. Write any connection you find between the graphs and the c value.
Now again press **tab** to return to the equation entry and use the **History Menu** to return to ***f2(x)***. Change this to **$f2(x)=x^2+b\times x$** and then change the slider variable from **c** to **b**. Use the slider to change the value of b. Write any connection you find between the graphs and the b value.	Now again change the equation being graphed to **y2=x^2+bx**, using the **Var** tab from the **Keyboard** to enter the **b**, and tick the box next to it. To create a slider to change the value of b, select ◆ > **Dynamic Graph** and the graph will appear with a slider for the value of b. Select ☰ and set the slider values from -5 to 5 as before. Change the value of b. Write any connection you find between the graphs and the b value.
To investigate the general equation $y = ax^2 + bx + c$, again press **tab** to return to the equation entry and use the **History Menu** to return to ***f2(x)***. Change this to **$f2(x)=a\times x^2+b\times x+c$**. To create sliders for each of a, b and c, select **menu** > **Actions** > **Insert Slider** and set the **Slider Settings** as before, with the **Variable** first as **a**, then again as **b** and as **c**, as needed. You may need to minimise and move the sliders so that everything is clearly visible. To minimise a slider, select it and then **ctrl** > **menu** > **Minimize**. You may also need to adjust the graph window settings, by selecting **menu** > **Window/Zoom** > **Window Settings...** and changing the values.  	To investigate the general equation $y = ax^2 + bx + c$, change the equation being graphed to **y2=ax^2+bx+c**, using the **Var** tab from the **Keyboard** to enter the **a**, **b** and **c**, and then create sliders to change the values by selecting ◆ > **Dynamic Graph**. The graph will appear with sliders for the values of a, b and c. Select ☰ and set each of the slider values from -5 to 5 as before. 

1 With $a = 1$, find a value for each of b and c so that the parabola moves to the *right* compared to the graph of $y = x^2$. Explain why you chose those values. Check that these values of b and c have the same effect even if $a \neq 1$.

2 With $a = 1$, find a value for each of b and c so that the parabola moves to the *left* compared to the graph of $y = x^2$. Explain why you chose these values. Check that these values of b and c have the same effect even if $a \neq 1$.

Investigating circles

Using TI-Nspire CAS	Using Casio ClassPad CAS
Add **Graphs** to your document. Select **menu** > **Geometry** > **Shapes** > **Circle** to select the centre point and then the size of a circle. Then select **menu** > **Actions** > **Coordinates and Equations** and select the centre point of the circle, to place a caption showing the centre coordinates. Then select the circumference of the circle, to place a caption showing the equation of the circle.  $(x-2.64)^2+(y-2.01)^2=3.32^2$ Now select the centre point and move it around. What happens to the circle equation and the centre point coordinates, as the centre point moves around?	From the menu select **Conics**. Enter the equation to be graphed as **(x–*h*)^2+(y–*k*)^2=4**, using the **Var** tab from the **Keyboard** to enter the ***h*** and ***k***. Then create sliders to change the values by selecting ◆ > **Dynamic Graph**. The graph will appear with sliders for the values of h and k. Select ☰ and set each of the slider values from -5 to 5 as before. Set the value of h to 2 and set the value k to 1.  Conics Equation: $(x-h)^2+(y-k)^2=4$ Now use the sliders to change the values of h and k. What happens to the circle and its centre point, as the values change?

3 What do you think is the connection between the circle equation and the centre point?

Ext

Investigating exponentials

Using TI-Nspire CAS	Using Casio ClassPad CAS
Add **Graphs** to your document and clear any previous equations by selecting **menu** > **Actions** > **Delete all**. Enter ***f1*(*x*)=*a*×*x*^*h*+*k*** and create sliders for each of a, h and k as for other sliders before. Set the **Slider Settings** for **a** with a starting **Value** of 2, **Minimum** of 0 and **Maximum** of 5. For **h** and **k**, use a starting **Value** of 0, **Minimum** of -5 and **Maximum** of 5. Describe the graph that appears. You may need to minimise and move the sliders so that everything is clearly visible. To minimise a slider, select it and then **ctrl** > **menu** > **Minimize**. You may also need to adjust the graph window settings, by selecting **menu** > **Window/Zoom** > **Window Settings…** and changing the values.	From the menu select **Graph & Table** and delete any previous equations. Enter a new equation to be graphed as **y1=*ax*^*h*+*k***, using the **Var** tab from the **Keyboard** to enter the ***a***, ***h*** and ***k***, and tick the box next to it. Then create sliders to change the values by selecting ◆ > **Dynamic Graph**. The graph will appear with sliders for the values of a, h and k. Select ☰ and set each of the slider values from -5 to 5 as before. Set the value of a to 2 and set the values of h and k both to 0. Describe the graph that appears.

4 With $k = 0$ and $a = 2$, change the slider value of h from -5 to 5. What happens to the graph?

5 With $h = 0$, change the slider value of k from -5 to 5. What happens to the graph?

6 What happens when neither h nor k is zero? Choose integer values for h and k. Write these values and write the equation of the asymptote for the graphs that are displayed. What do you find?

7 With $h = 0$ and $k = 0$, change the value of a from 0 to 5. What happens to the graph?

Ext

Investigating hyperbolas

Using TI-Nspire CAS	Using Casio ClassPad CAS
Add **Graphs** to your document and clear any previous equations by selecting **menu** > **Actions** > **Delete all**. Enter ***f1*(*x*)=a/(x–h)+k** and create sliders for each of a, h and k as for other sliders before. Set the **Slider Settings** for **a** with a starting **Value** of 1, **Minimum** of -5 and **Maximum** of 5. For **h** and **k**, use a starting **Value** of 0, **Minimum** of -5 and **Maximum** of 5. Describe the graph that appears. You may need to minimise and move the sliders so that everything is clearly visible. To minimise a slider, select it and then **ctrl** > **menu** > **Minimize**. You may also need to adjust the graph window settings, by selecting **menu** > **Window/Zoom** > **Window Settings…** and changing the values.	From the menu select **Graph & Table** and delete any previous equations. Enter a new equation to be graphed as **y1=*a*/(x–*h*)+*k***, using the **Var** tab from the **Keyboard** to enter the ***a***, ***h*** and ***k***, and tick the box next to it. Then create sliders to change the values by selecting ◆ > **Dynamic Graph**. The graph will appear with sliders for the values of a, h and k. Select ☰ and set each of the slider values from -5 to 5 as before. Set the value of a to 1 and set the values of h and k both to 0. Describe the graph that appears.
Now set the value of a to -1. Describe the graph and compare it with the previous graph.	Now set the value of a to -1. Describe the graph and compare it with the previous graph.

8 With $k = 0$ and $a = 1$, change the slider value of h from -5 to 5. What happens to the graph?

9 What happens when neither h nor k is zero? Choose integer values for h and k. Write these values and write the equations of the asymptotes for the graphs that are displayed. What do you find?

10 With $h = 0$ and $k = 0$, change the value of a from 1 to 5. What happens to the graph?

Exponentials and hyperbolas

9.5

Extension

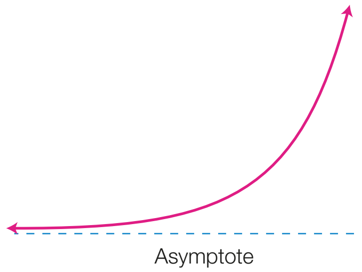
Exponential relationships

Consider cell division: one cell divides into two, and each of these cells then divides into two. Now there are four cells. If this process continues, four cells become eight, eight become 16 and so on. The way these numbers increase at a constantly increasing rate is called **exponential** growth. In the case of cell division, this can be represented by an equation $y = 2^x$.

An exponential relationship is a relationship between two quantities involving an exponent (or index), where the exponent is a variable. Many natural processes in the real world involve some form of exponential growth or decay.

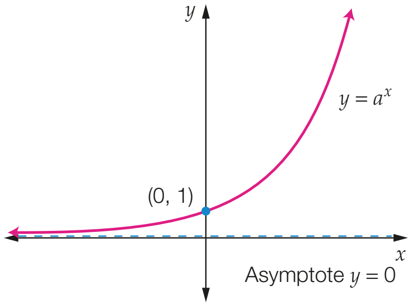
Exponential graphs always go in the same direction, either continually getting bigger or continually getting smaller. Exponential graphs also have a value that they get closer and closer to but never reach, called the **asymptote** of the graph.

A standard exponential curve looks like this:



The simplest exponential equations are in the form $y = a^x$ where a is the base (any non-zero number) and x is the index or exponent.

Because $a^0 = 1$, all these exponential graphs will have a y-intercept of (0, 1), given $a \neq 0$. As x gets more negative, the value of y gets closer and closer to 0, but never reaches it. The line with the equation $y = 0$ (the x-axis) is the asymptote.

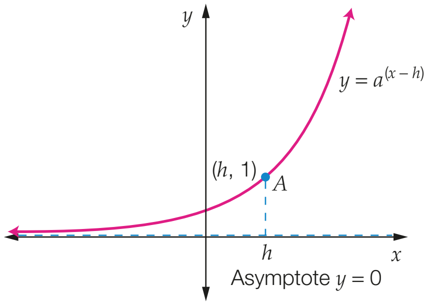


Translations of exponential graphs

An exponential graph with equation $y = a^x$ can be translated h units to the right to give the equation $y = a^{(x-h)}$.

If the value of h is negative, this will mean a translation to the left.

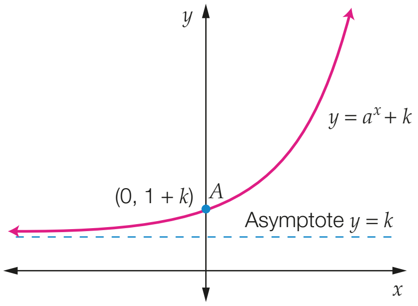
The asymptote remains $y = 0$.



An exponential graph with an equation $y = a^x$ can be translated k units up to give the equation $y = a^x + k$.

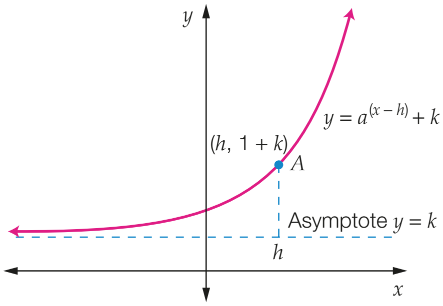
If the value of k is negative, this will mean a translation downwards.

The asymptote becomes $y = k$.



An exponential graph with the equation $y = a^x$ can be translated h units to the right and k units up to give the equation $y = a^{(x-h)} + k$.

If the value of h is negative, this will mean a translation to the left. If the value of k is negative, this will mean a translation downwards.



The general equation of an exponential relationship is $y = a^{(x-h)} + k$, where

- a is the base
- x is the exponent
- $y = k$ is the asymptote
- h indicates the translation across the x-axis
- k indicates the translation across the y-axis

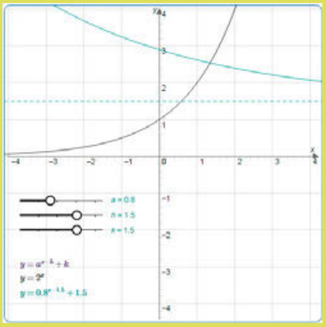
Extension

Interactive

Exponential growth and decay

Explore the exponential growth or decay relationship of a graph by changing the values of the exponential equation.

Go to the eBook or the Pearson Places website to access this interactive.



Worked example 18

W.E. 18

Plot the following graphs by recording tables of values from -3 to 3 inclusive.

(a) $y = 2^x$ **(b)** $y = 2^{(x+1)}$ **(c)** $y = 2^x + 3$

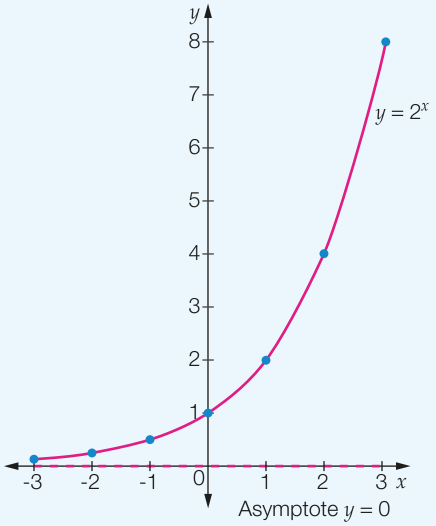
Thinking **Working**

(a) 1 Set up the table and replace x with each value in the equation, taking care with negative indices.

(a)

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

2 Plot the points and join them with a smooth curve.

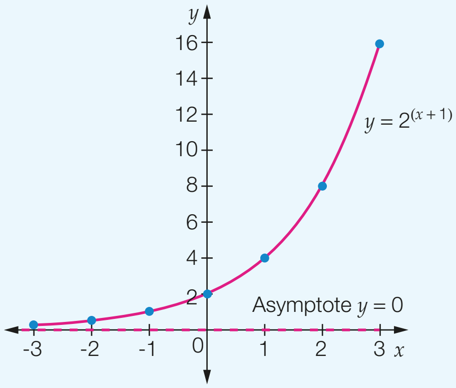


(b) **1** Set up the table and replace x with each value in the equation, taking care with negative indices.

(b)

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16

2 Plot the points and join them with a smooth curve.

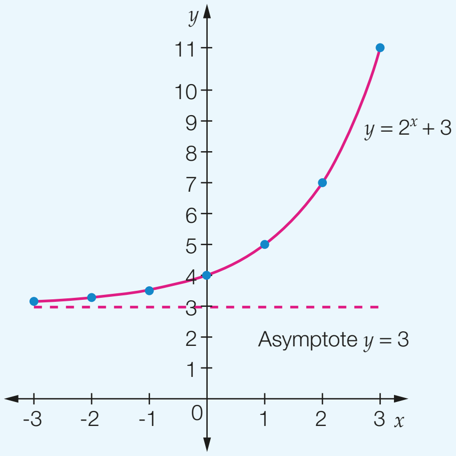


(c) **1** Set up the table and replace x with each value in the equation, taking care with negative indices.

(c)

x	-3	-2	-1	0	1	2	3
y	$3\frac{1}{8}$	$3\frac{1}{4}$	$3\frac{1}{2}$	4	5	7	11

2 Plot the points and join them with a smooth curve.



Worked example 19

W.E. 19

Bacteria in a culture are multiplying rapidly. Each bacterial cell divides into two, then each of these cells divides into two, and so on. This population growth can be expressed as $N = 2^t$ where N is the number of bacteria and t is the time in hours.

(a) For the relationship $N = 2^t$, copy and complete the table of values for t and N.

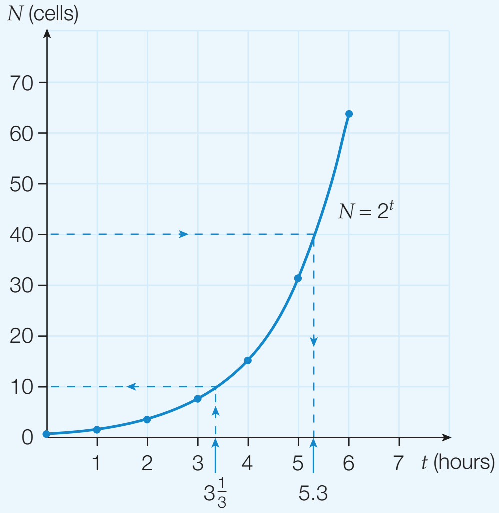
t	0	1	2	3	4	5	6
N							

(b) Use this table of values to plot a graph of $N = 2^t$. Join the points with a smooth curve.

(c) Use your graph to find approximately how much time it takes for the number of bacteria to exceed 40.

(d) Evaluate N for $t = 5.3$ (give your answer correct to the nearest integer) and explain why this value is not useful in this situation.

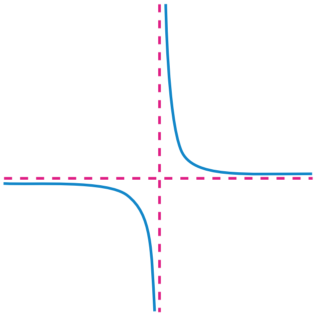
(e) Determine how many bacteria there are after 3 hours 20 minutes.

<table>
<tr><th>Thinking</th><th>Working</th></tr>
<tr><td>(a) Substitute for t in $N = 2^t$ to fill in the table.</td><td>(a)
<table>
<tr><td>t</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>N</td><td>1</td><td>2</td><td>4</td><td>8</td><td>16</td><td>32</td><td>64</td></tr>
</table></td></tr>
<tr><td>(b) Plot points, using t for the horizontal axis and N for the vertical axis.</td><td>(b)

</td></tr>
<tr><td>(c) Using the graph, draw a horizontal line from $N = 40$ to the curve, then drop a vertical line to the x-axis.</td><td>(c) From the graph, to exceed the number of 40 bacteria, t must be greater than 6 hours.</td></tr>
<tr><td>(d) Calculate $2^{5.3}$ using your calculator.</td><td>(d) The answer using a calculator is 39 (to the nearest integer), but if cells are only completely divided at the end of each hour, then this is not a realistic value for the actual number of bacteria.</td></tr>
<tr><td>(e) Change the time into fractional units, then move up to the curve and across to the vertical axis.</td><td>(e) At 3 hours 20 minutes there will still only be 8 cells, as the next complete cell division does not occur until $t = 4$.</td></tr>
</table>

Rectangular hyperbolas

A **rectangular hyperbola** is the graph created by a reciprocal relationship between variables. The simplest rectangular hyperbola is represented by the equation $y = \frac{1}{x}$.

A hyperbola has the shape shown at right, with both horizontal and vertical asymptotes.

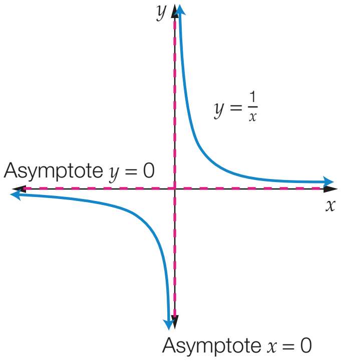


A table of values for the equation $y = \frac{1}{x}$ is shown below.

x	-100	-10	-5	-1	-0.1	-0.01	0	0.01	0.1	1	5	10	100
y	-0.01	-0.1	-0.2	-1	-10	-100	undefined	100	10	1	0.2	0.1	0.001

Plotting points, you can see there are two separate parts to the graph. Where $x = 0$, the y-value is undefined. When x is just greater than 0, y is extremely large. Where x is just a little less than 0, y is an extremely large negative number. The graph is discontinuous at $x = 0$ and there is an asymptote at $x = 0$, the y-axis.

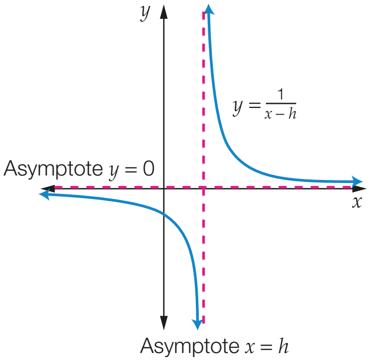
Where x is extremely large, y becomes extremely small but never reaches 0. Where x is an extremely large negative number, y is an extremely small negative number (close to 0). Once again, y can never reach 0, because there is no solution to the equation $0 = \frac{1}{x}$. So the line $y = 0$ (that is, the x-axis) is also an asymptote.



Translations of rectangular hyperbolas

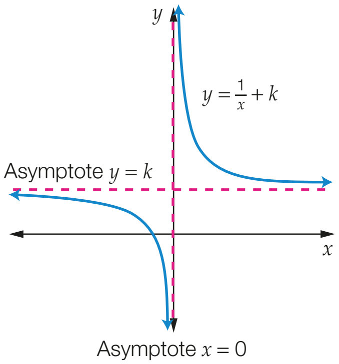
$y = \frac{1}{x-h}$ is the graph of $y = \frac{1}{x}$ translated h units to the right.

If the value of h is negative, this will mean a translation to the left.

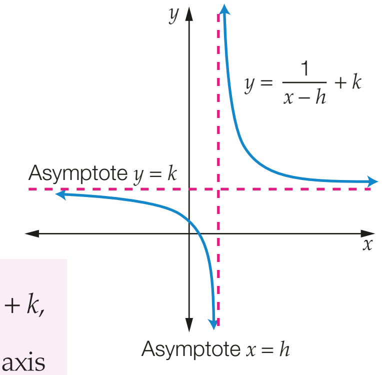


$y = \frac{1}{x} + k$ is the graph of $y = \frac{1}{x}$ translated k units upwards.

If the value of k is negative, this will mean a translation downwards.



$y = \frac{1}{x-h} + k$ is the graph of $y = \frac{1}{x}$ translated h units to the right and k units upwards. If the value of h is negative, this will mean a translation to the left. If the value of k is negative, this will mean a translation downwards.



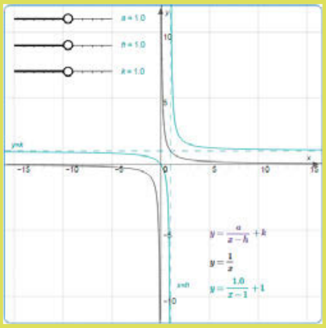
The general equation of a rectangular hyperbola is $y = \frac{a}{x-h} + k$, where a is the dilation factor, h is the translation across the x-axis and k is the translation across the y-axis.

Interactive

Hyperbola

Explore how the shape and position of a rectangular hyperbola is affected by the relationship between the values.

Go to the eBook or the Pearson Places website to access this interactive.

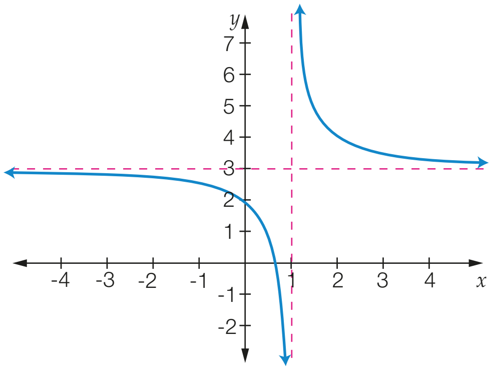


Worked example 20

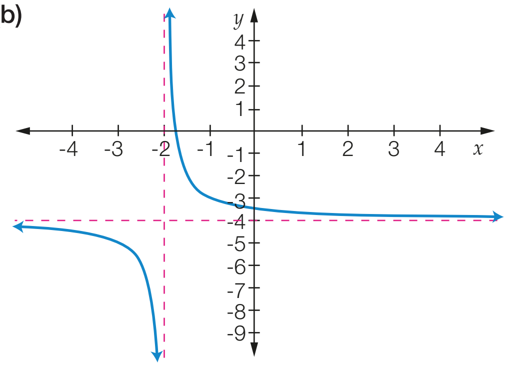
W.E. 20

Determine the equation of each of the following hyperbolas from their graphs. Use $a = 1$ in the general equation $y = \frac{a}{x-h} + k$.

(a)



(b)



Thinking	Working
(a) 1 Determine the equations of the asymptotes.	(a) $a = 1$ Asymptotes: $x = 1$, $y = 3$
2 Use the equations of the asymptote to identify h and k. (Here $h = 1$ and $k = 3$.)	$h = 1$ and $k = 3$

3 Use the values of h and k in the general equation $y = \dfrac{a}{x-h} + k$.

Equation: $y = \dfrac{1}{x-1} + 3$

(b) 1 Determine the equations of the asymptotes.

(b) $a = 1$
Asymptotes: $x = -2$, $y = -4$

2 Use the equations of the asymptote to identify h and k. (Here $h = -2$ and $k = -4$.)

$h = -2$ and $k = -4$

3 Use the values of h and k in the general equation $y = \dfrac{a}{x-h} + k$.

Equation: $y = \dfrac{1}{x-(-2)} + -4$

$y = \dfrac{1}{x+2} - 4$

9.5 Exponentials and hyperbolas

Navigator

1, 2 (a–c), 3, 4, 6, 7, 9	1, 2, 3, 4, 5, 6, 7, 9	1, 2 (a–c), 3, 4, 5, 6, 7, 8, 9

Answers p. 697

Fluency

1 Plot the following graphs by setting up tables of values from -3 to 3 inclusive. **W.E. 18** **Extension**

(a) $y = 3^x$ **(b)** $y = 3^{(x+1)}$ **(c)** $y = 3^x - 2$

2 A gardener decides to propagate an orchid (make new plants). This is done by dividing the bulb on the plant into three parts after it finishes flowering. The gardener then plants these smaller bulbs giving three orchids in total. This is done at the same time every year with every orchid that has grown from the original orchid. This situation can be modelled by the equation $P = 3^t$ where P is the number of orchids the gardener has and t is the time in years. **W.E. 19**

(Assume that a healthy orchid grows from every divided bulb and all orchids survive for at least 6 years.)

(a) For the relationship of orchids to the time in years where $P = 3^t$, copy and complete the table of values below.

***t* (years)**	0	1	2	3	4	5	6
***P* (orchids)**							

(b) Use this table of values to plot a graph of $P = 3^t$. Join the points with a smooth curve.

(c) Use your graph to determine approximately how many years it takes for the number of orchids to exceed 125.

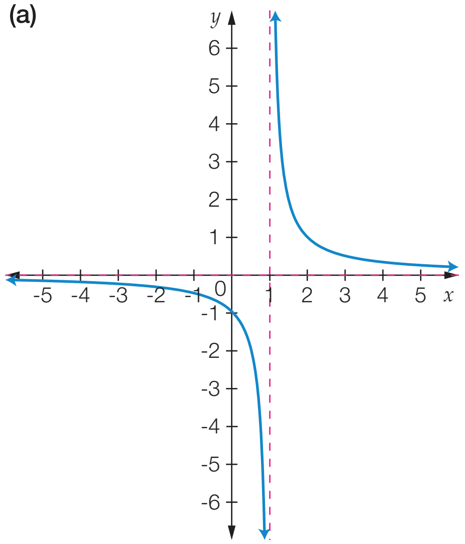
(d) Evaluate P for $t = 3.8$ (give your answer correct to the nearest integer) and explain why this value is not useful in this situation.

(e) Determine how many orchids there are after $5\frac{1}{2}$ years.

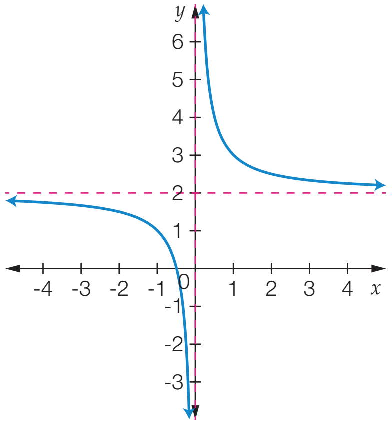
W.E. 20

3 Determine the equation of each of the following hyperbolas from their graphs. Use $a = 1$ in the general equation $y = \frac{a}{x-h} + k$.

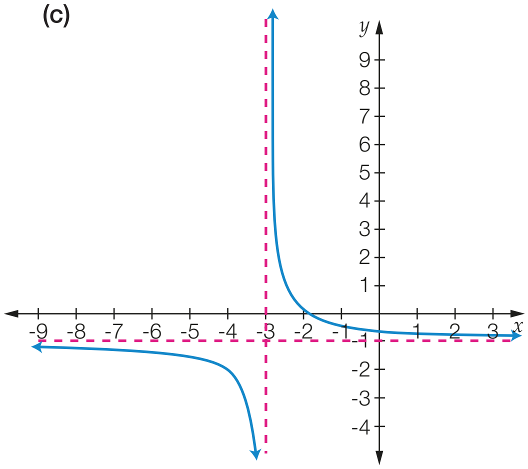
(a)



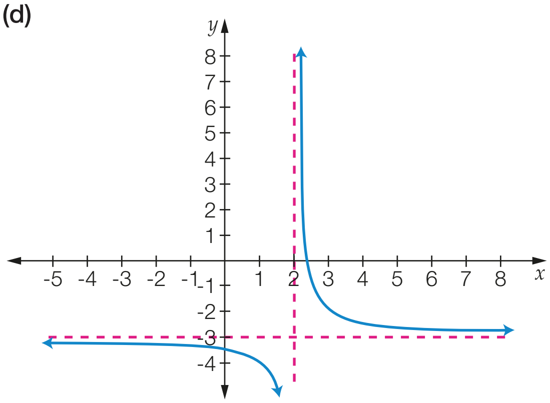
(b)



(c)



(d)



Extension

Understanding

4 (a) Plot the graphs of $y = 2^x$ and $y = 3^x$ on the same set of axes.

(b) By comparing asymptotes, y-intercepts and shape, describe the similarities and differences between the two graphs.

5 Which of the following equations will not graph as a hyperbola?

A $y = \frac{2}{x+5} - 6$ **B** $y = 3 + \frac{2}{x}$ **C** $x(y-3) = 1$ **D** $y = \frac{2}{x^2} - 1$

6 An exponential graph of $y = 5^x$ is shifted 4 units left and 3 units down. Which of the following is the equation of the new graph?

A $y = 5^{(x-4)} + 3$ **B** $y = 5^{(x-4)} - 3$ **C** $y = 5^{(x+4)} + 3$ **D** $y = 5^{(x+4)} - 3$

Reasoning

7 The table below gives values for the temperature of the metal surface of an electric iron as it cools. The temperature T is in degrees Celsius and the time m is in minutes.

m	0	2	4	6	8	10
T	160	108	76	55	42	34

(a) Use graph paper to plot a graph of T versus m. (Put m along the horizontal axis.) Join the points with a smooth curve.

(b) What is the initial temperature of the metal surface of the iron?

(c) What is the temperature of the iron after 3 minutes?

(d) When is the temperature of the iron 50 °C?

(e) Extend the curve. Predict the temperature of the iron after 12 minutes.

8 The population of a town has been increasing at a rate of 10% each year since the beginning of 2005. This can be modelled by the equation $P = 2000 \times 1.1^t$ where P is the number of people in the town t years after the beginning of 2005. Initially, when $t = 0$, the population numbered 2000.

(a) Set up a table to show the population of the town for the first 10 years after 2005.

(b) Draw a graph showing this information.

(c) From the graph, approximately how long did it take for the town to double in size?

(d) Jason said it would take 10 years for the population to double. Explain why this isn't the case.

Extension

Open-ended

9 **(a)** Substitute your own values for h and k into the equation of an exponential in the form of $y = 2^{(x-h)} + k$ and draw a graph of this relationship.

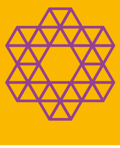
(b) Substitute your own values for h and k into the equation of a hyperbola in the form of $y = \frac{1}{x-h} + k$ and draw a graph of this relationship.

Problem solving

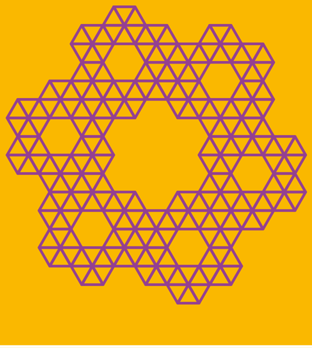
How many triangles?

Six equilateral triangles are joined to make a hexagon.

Six hexagons are joined to make the hexagon pattern as shown.



Six of these hexagon patterns are joined to make a larger pattern.



Six of these larger hexagon patterns are joined in the same way to make an even larger pattern. How many triangles would there be in this pattern?

Strategy options

- Look for a pattern.
- Make a table.

Maths 4 Real

The space race

The ‘Space Race’ was the name given to the competition between the United States of America and the USSR (Soviet Russia) during their Cold War from the 1950s until 1989, as each raced to explore space faster and further than the other. The USSR launched Sputnik 1 on 4 October 1957, the first artificial object launched into orbit. The USSR was also first to put a live animal into space (the dog Laika) and first to launch humans into space. However, the USA was first to put humans on the Moon.

Rocket science

When an object is projected vertically straight up with an initial velocity of v metres per second, its approximate height, h metres above the ground after t seconds is given by the equation $h = vt - 5t^2$.

1 A toy rocket is fired into the air with the initial velocities given in the table below. Calculate the height above the ground after the times given.

	Initial velocity (m/s)	Time (s)	Height (m)
(a)	10	1	
(b)	50	7	
(c)	25	4	
(d)	40	2	
(e)	1200	120	
(f)	300	60	

2 A toy rocket is fired into the air with an initial velocity of 20 m/s.

(a) Copy and complete this table of values of h for the values of t shown.

t	0	1	2	3	4	5
h						

(b) Use your completed table of values to plot a graph of h and t.

(c) Use your graph to determine:

(i) the maximum height of the rocket

(ii) the time the rocket spent in the air.

3 Given that h is the height above ground, is it sensible to calculate h for values of t greater than 4?

(a) Substitute the values $h = 0$ and $v = 20$ m/s and factorise the right-hand side of the equation.

(i) Find the solutions to the equation.

(ii) Which two stages of the rocket's journey do the solutions correspond to?

4 Find the time t when the rocket hits the ground, if it has been fired with an initial velocity of:

(a) 10 m/s **(b)** 100 m/s **(c)** 500 m/s

(d) 1300 m/s **(e)** 1800 m/s **(f)** 2100 m/s

Research

Research one of these landmark events in space exploration. How was mathematics involved in these developments?

Timeline of some major events in space exploration	
4 October 1957	USSR Sputnik 1: First satellite
17 March 1958	USA Vanguard 1: First solar-powered satellite
2 January 1959	USSR Luna 1: First rocket reaching Earth escape velocity
7 August 1959	USA Explorer 6: First photograph of Earth from space
12 April 1961	USSR Vostok 1: First human in space
3 February 1966	USSR Luna 9: First soft landing on the Moon
21 July 1969	USA Apollo 11: First humans on the Moon
23 April 1971	USSR Salyut 1: First space station
15 July 1975	USSR and USA combine in the Apollo–Soyuz Test Project

9.6

Direct proportion

Many types of relationships between quantities occur frequently in real life. The simplest type of relationship between two quantities is one in which any change in one quantity results in an identical change to the other quantity. For example, if you double one quantity, the other will double also; halve one quantity and the other will halve also.

The relationship between two quantities a and b is called **direct proportion** when a increases as b increases. The graph of the relationship will be a straight line that passes through the origin.

A simple example of direct proportion is the relationship between distance and speed. For example, if you double the speed of travel, the distance covered in a time interval will double also; if you halve the speed, the distance travelled will halve also. In this case the graph showing direct variation is linear. It also passes through the origin, because an object that has no speed is stationary and covers no distance.

$a \propto b$ means 'a is directly proportional to b'.

$a \propto b$ is not the same as $a = b$.

Consider this example. Emi and Michael sail their boat from Queenscliff at a speed of 30 km/h. After 1 hour they have travelled 30 km, after 2 hours they have travelled 60 km, and after 3 hours they have travelled 90 km.

Time t (hours)	0	1	2	3
Distance d (km)	0	30	60	90

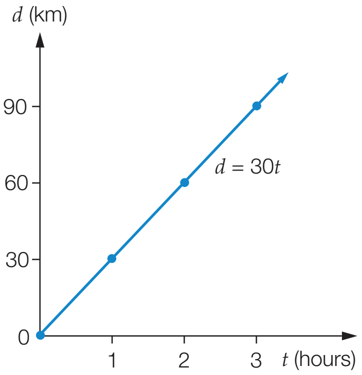
By considering the ratio $\frac{d}{t}$ for each pair of values, a constant value is found.

This value k is called the **constant of proportionality**.

In this case, $\frac{d}{t} = k = 30$, so $d = 30t$.

A graph of distance d versus time t can be drawn.

Notice that the graph of d versus t is a straight line that passes through the origin, (0, 0). As t increases, d increases at a proportional rate. The distance is in direct proportion to time, so $d \propto t$. In other words, the relationship between d and t is a relationship of direct proportion.



In a directly proportional relationship with a positive k, one variable increases as the other increases. (If k is negative, then as one variable increases, the other decreases. For this chapter, assume that k is always positive.) Not all increasing trends are necessarily direct proportion, so it is important to check if there is a constant of proportionality before deciding whether two increasing variables are in direct proportion.

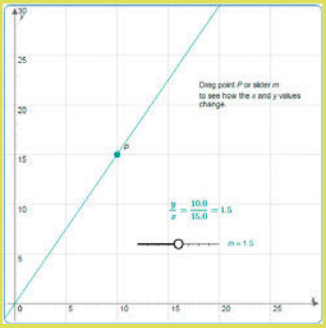
- Two variables, x and y, have a directly proportional linear relationship when $y = kx$.
- k is the constant of proportionality.
- The graph of a directly proportional relationship is a straight line that passes through the origin (0, 0).
- For $k > 0$, y increases as x increases.

Interactive

Direct proportion

Explore the relationship between two quantities when these are directly proportional to each other.

Go to the eBook or the Pearson Places website to access this interactive.



Drag point P or slider m to see how the x and y values change.

Worked example 21

W.E. 21

Which of the following could be examples of direct proportion?

(a)

x	1	3	6	8
y	2	6	12	18

(b)

s	-1	0	2	4
t	1	2	4	6

(c)

v	1	2	3	4
w	4	5	1	0.5

Thinking	Working
(a) y is increasing as x is increasing.	(a) Could be direct proportion
(b) t is increasing as s is increasing but the graph does not pass through the origin.	(b) Not direct proportion
(c) w is fluctuating as v is increasing.	(c) Not direct proportion

It may be that the direct proportion occurs between one quantity and another quantity raised to a positive power. The relationship is then of the form $y \propto x^n$ or, as an equation, $y = kx^n$.

Consider this table of values:

x	0	1	2	3
y	0	3	12	27

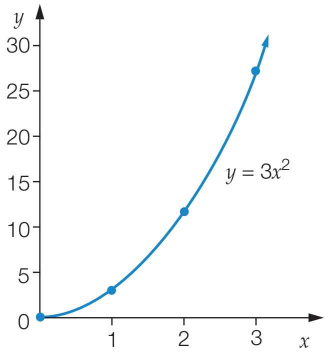
By calculating the ratio of each pair of values you can see k is not constant. However, the relationship may still be directly proportional, because y increases as x increases and it passes through the origin. This relationship may be of the type $y \propto x^n$.

Set up a new table using x^2 instead of x.

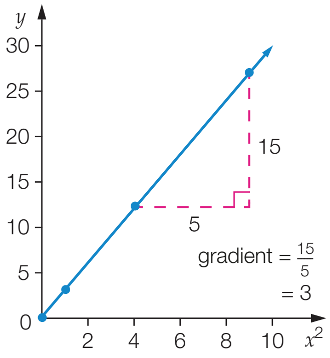
x^2	0	1	4	9
y	0	3	12	27

By calculating k for each pair of values you can see that $\frac{y}{x^2} = k = 3$, so $y = 3x^2$.

By plotting the graph y against x you get a curve, indicating that y is not directly proportional to x.



However, by plotting y against x^2 you get a straight line passing through the origin with a gradient of 3. That is, $y \propto x^2$, so y is directly proportional to x^2.



- y is directly proportional to x^n, $y \propto x^n$, where n is a positive number, so $y = kx^n$.
- k is the constant of proportionality.
- The graph of y plotted against x^n is a straight line passing through the origin, (0, 0).
- As x increases, y also increases (for $k > 0$).

A *direct proportional relationship* exists if a graph of one variable against another results in a straight line through the origin.

A *linear proportional relationship* exists if the graph is a straight line (not necessarily through the origin).

You can use this knowledge to find the equation that represents the proportional relationship between two variables.

Worked example 22

W.E. 22

Find the equation for each of the following tables of values.

(a)

x	1	3	6	8
y	2	6	12	16

(b)

s	0	1	2	3	5
t	0	2	8	18	50

(c)

a	0	1	4	9
b	0	3	6	9

Thinking	Working
(a) **1** Determine the ratio $\frac{y}{x}$ for each pair of values.	(a) $\frac{y}{x} = \frac{2}{1} = \frac{6}{3} = \frac{12}{6} = \frac{16}{8} = 2$ $\therefore k = 2$
2 The ratio is constant, so y is directly proportional to x.	$y \propto x$
3 Write the equation.	$y = kx$
4 Substitute $k = 2$ into the equation.	$y = 2x$
(b) **1** Determine the ratio $\frac{t}{s}$ for each pair of values.	(b) $\frac{t}{s} = \frac{2}{1} = 2$, $\frac{t}{s} = \frac{8}{2} = 4$, $\frac{t}{s} = \frac{18}{3} = 6$, $\frac{t}{s} = \frac{50}{5} = 10$
2 The ratio is not constant, so t is not linearly proportional to s. Since there is a point at the origin, the relationship may be of the form $t \propto s^n$. Notice that the ratio is increasing, so $n > 1$. Set up a table of values of t and s^2.	$\begin{array}{\|c\|c\|c\|c\|c\|c\|} \hline s^2 & 0 & 1 & 4 & 9 & 25 \\ \hline t & 0 & 2 & 8 & 18 & 50 \\ \hline \end{array}$
3 Determine the ratio $\frac{t}{s^2}$ for each pair of values.	$\frac{t}{s^2} = \frac{2}{1} = \frac{8}{4} = \frac{18}{9} = \frac{50}{25} = 2$ $\therefore k = 2$
4 The ratio is constant, so t is directly proportional to s^2.	$t \propto s^2$
5 Write the equation.	$t = ks^2$
6 Substitute $k = 2$ into the equation.	$t = 2s^2$
(c) **1** Determine the ratio $\frac{b}{a}$ for each pair of values.	(c) $\frac{b}{a} = \frac{3}{1} = 3$, $\frac{b}{a} = \frac{6}{4} = \frac{3}{2}$, $\frac{b}{a} = \frac{9}{9} = 1$
2 The ratio is not constant, so b is not linearly proportional to a. Since there is a point at the origin, the relationship may be of the form $b \propto a^n$. Notice that the ratio is decreasing, so $0 < n < 1$. Set up a table of values of b and $\sqrt{a}$.	$\begin{array}{\|c\|c\|c\|c\|c\|} \hline \sqrt{a} & 0 & 1 & 2 & 3 \\ \hline b & 0 & 3 & 6 & 9 \\ \hline \end{array}$
3 Determine the ratio $\frac{b}{\sqrt{a}}$ for each pair of values.	$\frac{b}{\sqrt{a}} = \frac{3}{1} = \frac{6}{2} = \frac{9}{3} = 3$ $\therefore k = 3$
4 The ratio is constant, so b is directly proportional to $\sqrt{a}$.	$b \propto \sqrt{a}$
5 Write the equation.	$b = k\sqrt{a}$
6 Substitute $k = 3$ into the equation.	$b = 3\sqrt{a}$

9.6 Direct proportion

Navigator

Answers p. 698

1, 2 (a–f), 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 16

1, 2 (column 1), 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16

1, 2 (column 2), 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16

Fluency

W.E. 21

1 Which of the following could be examples of direct proportion? Give a reason for your answer.

(a)

v	0	2	4	5
w	0	3	3	1

(b)

p	1	2	3	4
q	3	4	5	0

(c)

x	1	2	3	4
w	0	1	2	4

(d)

s	1	2	6	5
t	5	10	30	25

(e)

p	1	3	4	10
q	30	90	120	300

(f)

d	2	4	6	8
f	0	2	1	3

W.E. 22

2 Find the equation for each of the following tables of values.

(a)

a	0	1	2	3
b	0	4	8	12

(b)

w	0	1	2	3
v	0	5	10	15

(c)

c	0	1	4	6
d	0	0.5	2	3

(d)

g	0	4	6	8
h	0	1	1.5	2

(e)

e	0	1	2	3
h	0	3	6	9

(f)

p	0	2	4	6
q	0	3	6	9

(g)

i	1	2	3	0
j	2	8	18	0

(h)

a	0	1	2	3
d	0	3	12	27

(i)

m	0	1	2	3
n	0	4	32	108

(j)

g	1	2	3	0
h	2	16	54	0

(k)

p	0	1	2	3
q	0	0.5	2	4.5

(l)

b	0	1	2	3
f	0	0.2	0.8	1.8

(m)

r	0	1	2	3
s	0	3	24	81

(n)

p	0	1	2	3
m	0	0.25	2	6.75

(o)

t	0	1	2	3
u	0	0.5	4	13.5

(p)

v	0	1	4	9
w	0	2	4	6

(q)

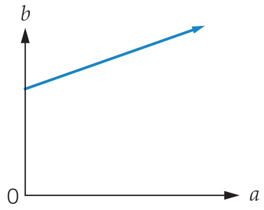
u	0	1	4	9
t	6	0.1	0.2	0.3

(r)

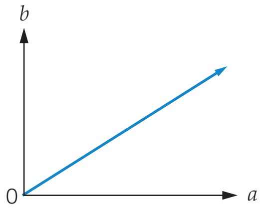
d	0	1	4	9
l	0	$\frac{1}{3}$	$\frac{2}{3}$	1

3 State whether or not each graph may indicate direct proportion.

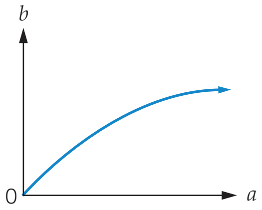
(a)



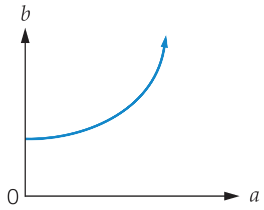
(b)



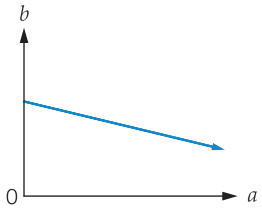
(c)



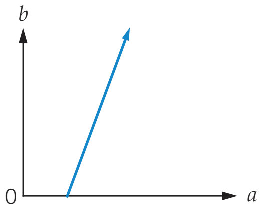
(d)



(e)



(f)



4 Express each of these relationships involving direct proportion as an equation, using k as the constant.

(a) $m \propto n$

(b) $a \propto b^2$

(c) $g \propto h^3$

(d) t is proportional to w.

(e) N is proportional to p^2.

(f) y is proportional to the cube of x.

(g) V is proportional to the square of r.

(h) L is proportional to the square root of d.

5 **(a)** The volume of a sphere (V) is directly proportional to the cube of its radius (r). This may be written as:

A $V \propto r^2$ **B** $V \propto r^3$ **C** $V \propto r$ **D** $V \propto \sqrt{r}$

(b) The radius of a cylinder (r) is directly proportional to the square root of the volume (V). This may be written as:

A $r \propto \sqrt{V}$ **B** $r \propto V^2$ **C** $r \propto V^3$ **D** $r \propto V$

6 State the value of the constant of proportionality in each of the following.

(a) $p = 3q$ **(b)** $a = 0.2b^2$ **(c)** $M = \frac{1}{4}n$ **(d)** $y = 200\sqrt{x}$

(e) $c = n^5$ **(f)** $F = \frac{a}{6}$ **(g)** $x = \frac{w^3}{20}$ **(h)** $P = \frac{r^4}{35}$

7 **(a)** n is directly proportional to m. If $n = 18$ where $m = 6$, then what is the value of the constant of proportionality k?

A 12 **B** 108 **C** $\frac{1}{3}$ **D** 3

(b) b is directly proportional to a and the constant of proportionality is $k = 5$. What is the value of b where $a = 20$?

A 4 **B** 100 **C** 80 **D** 5

(c) t is directly proportional to s. If $t = 45$ where $s = 3$, what is the value of s where $t = 150$?

A 15 **B** 135 **C** 2250 **D** 10

8 a is directly proportional to b^2.

(a) If $a = 12$ where $b = 2$, what is the value of the constant of proportionality k?

(b) What is the value of a where $b = 4$?

(c) What is the value of b where $a = 75$?

9 In each of the following table of values find the constant of proportionality and hence find the missing values in the table.

(a) $p \propto q$

q	5	7	9	
p	15	21		33

(b) $m \propto n^2$

n	1	3	5	
m	7		175	252

(c) $d \propto c^3$

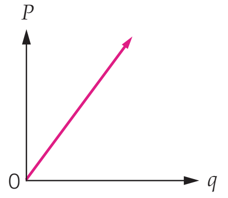
c	1		4	5
d	1	8		125

(d) $w \propto \sqrt{x}$

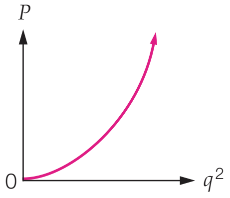
x	1		16	49
w	3	6	12	

10 Which graph shows direct proportion of the form $p \propto q^2$?

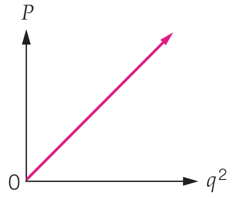
A



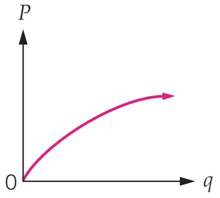
B



C



D



11 Consider the table of values:

f	2	5	9	12
g	16	40	72	96

(a) What is the value of the constant of proportionality?

(b) Write the relationship between the two quantities f and g as an equation.

(c) Graph the relationship between f and g, for $f = 0$ to $f = 12$. Plot f on the horizontal axis and g on the vertical axis.

Understanding

12 A student measures the current and voltage of a simple electrical circuit to determine the value of a resistor within the circuit. The results obtained are shown in the table.

Current *I* (amps)	0.001	0.002	0.003	0.004
Voltage *V* (volts)	3	6	9	12

(a) Graph the relationship between *I* amperes (amps) and *V* volts. Show *I* on the horizontal axis and *V* on the vertical axis.

(b) Calculate the constant of proportionality, in this case called the resistance *R* (ohms).

(c) Calculate the voltage if 0.01 amps of current is flowing through the circuit.

(d) What is the current flowing through the circuit when a voltage of 45 volts is measured in the circuit?

13 The following table shows the relationship between the surface area of a sphere and its radius.

Radius *r* (cm)	2	7	15	20	50
Surface area *A* (cm^2)	50.24	615.44	2 826	5 024	31 400

(a) Determine the constant of variation for this relationship.

(b) Express this relationship as an equation using $\pi = 3.14$.

Reasoning

14 The distance to the horizon is proportional to the square root of your height above sea level. A person standing on a cliff 32 m above sea level can see a distance of 18 km to the horizon.

(a) How far could a person see if standing on the viewing platform of a lighthouse 18 m above the cliff top?

(b) What height above sea level would be needed to see a distance of 100 km?

15 The following table shows the circumference, surface area and volume of spheres of various radii, in the same units.

Radius r	Circumference C	Surface area A	Volume V
2	12.56	50.24	33.49
4	25.12	200.96	267.95
6	37.68	452.16	904.32
8	50.24	803.84	2143.57
10	62.80	1256.00	4186.67
12	75.36	1808.64	7234.56

(a) Find the ratios $\frac{C}{r}$, $\frac{A}{r}$ and $\frac{V}{r}$ for each pair of values.

(b) Which of the ratios is constant? Describe your results.

(c) Plot graphs of C against r, A against r and V against r.

(d) Plot a graph of A against r^2. Describe the graph.

(e) Plot a graph of V against r^3. Describe the graph.

(f) Which of the graphs show direct proportion between the quantities? Write a relationship for each of these.

(g) Calculate the constant of proportionality, k, for each of these relationships. Express each as an equation in terms of π (where $\pi = 3.14$).

Open-ended

16 **(a)** Write two different relationships of the form $y = kx^n$, where $n > 0$ and $0 < k < 1$. Use the same value of n for both of your relationships.

(b) Sketch the graph of $y = x^n$ for your chosen value of n and, on the same set of axes, sketch the two relationships you have written. Describe what happens as k decreases from 1 to 0.

Problem solving

Relationships between variables

Can you determine the relationship between x and y for each table of values?

Strategy options

- Look for a pattern.
- Guess and check.

1

x	0	1	2	3	4
y	0	3	12	27	48

2

x	0	1	2	3	4
y	1	5	25	125	625

Inverse proportion

9.7

Extension

It is possible that an increase in one quantity will result in the *decrease* of the other quantity, or vice versa.

When a gas is confined to a certain volume, it exerts a pressure on the sides of the container. If the volume of the container increases, the pressure exerted by the gas decreases. If the volume of the container decreases, the pressure exerted by the gas increases. The product of the pressure and volume stays constant, so $PV = k$ where P is the pressure, V is the volume and k is a constant. This can written as $P = \frac{k}{V}$.

Consider the relationship between average speed and time of travel. If you double the average speed of travel, you will halve the time taken to travel a distance. Similarly, if the average speed is divided by a factor of 4, then it will take four times as long to travel a distance. This type of proportion is called **inverse proportion**. The graph of P plotted against V will be a **hyperbola**, which is a non-linear graph.

For example: Emi and Michael decide to take a trip in their cruiser from Queenscliff to Sorrento, following the coast of Port Phillip Bay. The trip is 200 km long. The speed they will sail at depends on how much time they want the trip to take.

For example, if the trip is to take 5 hours, the average speed of the cruiser must be $\frac{200}{5}$ or 40 km/h.

For 10 hours, the average speed must be $\frac{200}{10}$ or 20 km/h.

For 20 hours, the average speed must be $\frac{200}{20}$ or 10 km/h.

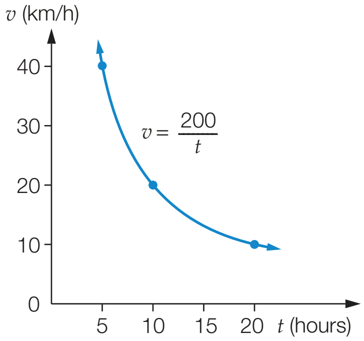
This can be shown in a table:

Time (hours)	5	10	20
Average speed (km/h)	40	20	10

For a trip of t hours, the average speed of the cruiser must be $\frac{200}{t}$ km/h.

For average speed v you can write the formula $v = \frac{200}{t}$.

A graph of v plotted against t can be drawn.



The shape of the graph is called a hyperbola.

As t increases, v decreases, so the average speed *is inversely* proportional to the time, i.e. $v \propto \frac{1}{t}$.

In other words, the relationship between v and t is a relationship of inverse proportion.

Note that only the first quadrant of a rectangular hyperbola is graphed.

The formula $v = \frac{200}{t}$ or $v = 200 \times \frac{1}{t}$ has a constant of proportionality of 200.

The constant of inverse proportionality, k, can be found using the product of the two variables, e.g. $vt = k = 200$.

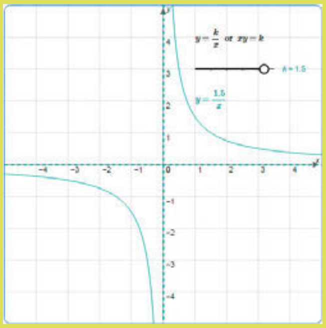
In an inverse proportional relationship with a positive k, one variable increases as the other decreases. (If k is negative, then as one variable increases, the other also increases. For this chapter, assume that k is always positive.) Not all decreasing trends are necessarily inverse proportion, so it is important to check if there is a constant of proportionality before deciding whether a particular relationship is inversely proportional.

Interactive

Inverse proportion

Explore the graphical relationship between two variables where, as one variable increases, the other decreases.

Go to the eBook or the Pearson Places website to access this interactive.



Extension

Worked example 23

W.E. 23

Which of the following could be examples of inverse proportion (with $k > 0$)?

(a)

a	1	3	6	8
b	2	6	12	16

(b)

c	1	2	3	5
d	10	5	3	1

(c)

e	1	2	3	4
f	4	2	1	0.5

Thinking	Working
(a) b is increasing as a is increasing.	(a) Not inverse proportion
(b) d is decreasing as c is increasing.	(b) Could be inverse proportion
(c) f is decreasing as e is increasing.	(c) Could be inverse proportion

If y is inversely proportional to x, then:

$$y \propto \frac{1}{x} \text{ where '}\propto\text{' means 'is proportional to',}$$

or, as an equation:

$$y = \frac{k}{x} \quad \text{or} \quad xy = k, \text{ where } k \text{ is the constant of proportionality}$$

This means that when x is doubled, y is halved. If x is tripled, y is divided by 3.

It may also be that inverse proportion occurs between one quantity and another quantity raised to a positive power. The simplest case is when this power is 1.

In general:

- y is inversely proportional to x^n, $y \propto \frac{1}{x^n}$, where n is a positive number, so $y = \frac{k}{x^n}$.
- k is the constant of proportionality.
- As x increases, y decreases (for $k > 0$).
- The graph of an inversely proportional relationship is a hyperbola.

Worked example 24

W.E. 24

Find the equation of the relationship between the variables in each of the following tables of values.

(a)

x	5	10	15	20	30
y	60	30	20	15	10

(b)

F	1	2	5	10
d	200	50	8	2

Thinking	Working
(a) 1 As x increases, y decreases, so the relationship could be inverse proportion.	(a) $y \propto \frac{1}{x^n}$
2 Calculate xy to see whether there is a constant of proportionality.	$5 \times 60 = 300$ $10 \times 30 = 300$ $15 \times 20 = 300$ $20 \times 15 = 300$ $30 \times 10 = 300$
3 If the product gives a constant, the variables are inversely proportional and the constant value calculated is the constant of proportionality.	$y \propto \frac{1}{x}$ and $k = 300$
4 Express as an equation.	$y = \frac{k}{x}$ or $xy = k$
5 Substitute the value of k into the equation. Here $k = 300$.	$y = \frac{300}{x}$ or $xy = 300$
(b) 1 As F increases, d decreases, so the relationship could be inverse proportion.	(b) $d \propto \frac{1}{F^n}$
2 Calculate the product of each pair of values to see whether there is a constant of proportionality.	$1 \times 200 = 200$ $2 \times 50 = 100$ $5 \times 8 = 40$ $10 \times 2 = 20$
3 If the product does not give a constant, set up a new table using F^2 and d.	F^2: 1, 4, 25, 100 d: 200, 50, 8, 2

4	Repeat step 2 using F^2 and d.	$1 \times 200 = 200$ $4 \times 50 = 200$ $25 \times 8 = 200$ $100 \times 2 = 200$
5	If the product gives a constant, the variables are inversely proportional and the value calculated is the constant of proportionality.	$d \propto \frac{1}{F^2}$ and $k = 200$
6	Express as an equation.	$d = \frac{k}{F^2}$ or $dF^2 = k$
7	Substitute the value of k into the equation. Here $k = 200$.	$d = \frac{200}{F^2}$ or $dF^2 = 200$

9.7 Inverse proportion

Navigator

Answers p. 699

1, 2, 3, 4, 5 (column 1), 6, 7, 8, 12	1, 2, 3, 4, 5 (column 2), 6, 7, 8, 9, 11 (a–c), 12	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

Fluency

Extension

W.E. 23

1 Considering general trends, which of the following could be examples of inverse proportion?

(a)

v	1	2	4	5
w	7	3	2	1

(b)

p	1	2	3	4
q	3	4	5	0

(c)

x	1	2	3	4
y	24	12	8	6

(d)

s	1	2	3
t	9	2.25	1

W.E. 24

2 Find the equation of the relationship between the variables in each of the following tables of values.

(a)

a	1	2	4	5
b	40	20	10	8

(b)

c	3	4	5	6
d	20	15	12	10

(c)

e	3	6	9	12
f	72	36	24	18

(d)

g	4	8	16	24
h	36	18	9	6

(e)

k	1	2	4	10
l	240	60	15	2.4

(f)

m	1	2	4	6
n	144	36	9	4

3 Express each of these relationships involving inverse proportion as an equation, using k as the constant.

(a) $p \propto \frac{1}{q}$ **(b)** $y \propto \frac{1}{x^2}$ **(c)** $a \propto \frac{1}{\sqrt{b}}$

(d) t is inversely proportional to w.

(e) V is inversely proportional to h^2.

(f) f is inversely proportional to the cube of g.

(g) L is inversely proportional to the square root of m.

(h) a is inversely proportional to the square root of d.

4 **(a)** The cost per passenger on a bus tour (c) is inversely proportional to the number of passengers on the tour (n). This may be written as:

A $c \propto n$ **B** $c \propto n^2$ **C** $c \propto \frac{1}{n}$ **D** $c \propto \frac{1}{n^2}$

(b) The thickness (t) of a pizza is inversely proportional to the square of its radius (r). This may be written as:

A $t \propto r$ **B** $t \propto r^2$ **C** $t \propto \frac{1}{r}$ **D** $t \propto \frac{1}{r^2}$

(c) The radius (r) of a cylinder is inversely proportional to the square root of its height (h). This may be written as:

A $r \propto h$ **B** $r \propto \frac{1}{h}$ **C** $r \propto \sqrt{h}$ **D** $r \propto \frac{1}{\sqrt{h}}$

Extension

5 State the value of the constant of proportionality in each of the following.

(a) $y = \frac{7}{x}$ **(b)** $a = \frac{6.3}{b^2}$ **(c)** $d = \frac{12}{g}$

(d) $p = \frac{0.97}{q^2}$ **(e)** $w = 5 \times \frac{1}{\sqrt{n}}$ **(f)** $B = 3 \times \frac{1}{\sqrt{c}}$

(g) $M = \frac{1}{2p^3}$ **(h)** $L = \frac{1}{5a^3}$ **(i)** $g = \frac{1}{0.4h}$

(j) $w = \frac{1}{1.6x}$ **(k)** $V = \frac{1}{0.08c^2}$ **(l)** $n = \frac{1}{1.25q^3}$

6 s is inversely proportional to r. If $s = \frac{1}{2}$ where $r = 10$, what is the value of the constant of proportionality?

A 20 **B** 10 **C** 5 **D** $\frac{1}{2}$

Understanding

7 y is inversely proportional to the cube of x. If $y = 16$ where $x = 2$, find:

(a) y where $x = 5$ **(b)** x where $y = 2$.

8 Average speed is inversely proportional to the time taken to travel a given distance. At an average speed of 100 km/h, a certain journey takes 2 hours to complete. However, due to a roadworks speed limit the average speed was reduced to 40 km/h. How long does it now take to travel this distance?

9 The frequency of waves of electromagnetic radiation is inversely proportional to their wavelength. If gamma rays have a frequency of 3×10^{22} Hz with a wavelength of 1×10^{-14} m, what is the wavelength of ultraviolet waves with a frequency of 1.5×10^{16} Hz?

10 The gravitational force of attraction between two bodies is inversely proportional to the square of their distance apart. The force between two objects 40 km apart is 85 newtons. What is the distance between the two objects when the force is 340 newtons?

Reasoning

11 The following results were obtained during an experiment measuring light intensity, I, from a light source on an object a distance d metres away from the light.

Distance, *d*	1	2	3	4
Intensity, *I*	720	180	80	45

Extension

(a) Plot the graph of light intensity (I) against distance (d).

(b) Plot the graph of light intensity (I) against $\frac{1}{d}$.

(c) Is I inversely proportional to d? Justify your answer.

(d) Copy and complete the following table.

d	1	2	3	4	6	8	12
d^2	1						
I	720	180	80	45			

(e) Calculate the value of d^2I for each pair of values. Is this value constant?

(f) Write an equation showing the relationship between I and d.

(g) Plot the graph of I against $\frac{1}{d^2}$. Comment on the result.

(h) What is the effect on the value for I when d is doubled?

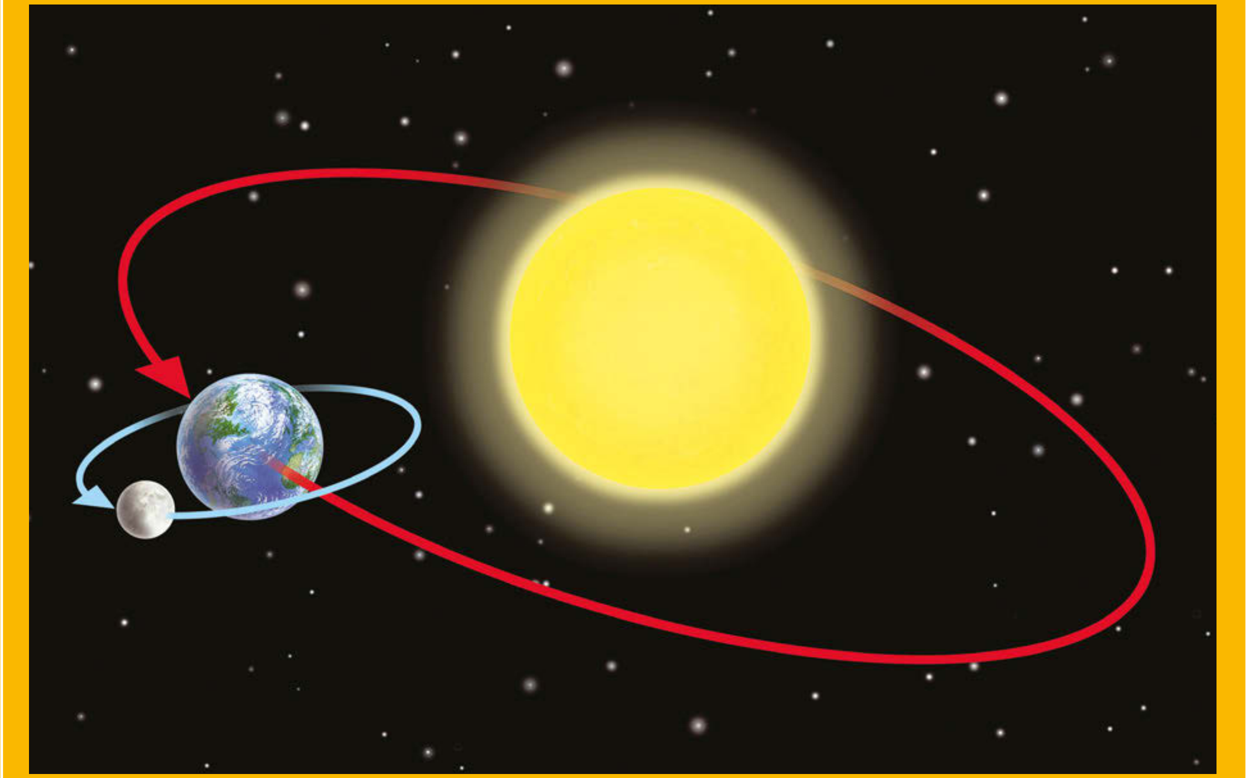
(i) What is the effect on the value for I when d is tripled?

(j) What is the effect of halving the value of d?

Open-ended

12 Write three pairs of coordinate points for three different equations of the type $y = \frac{k}{x^n}$. One pair of coordinate points must be common to all three equations.

Problem solving



The mass of Earth

In 1611, Johannes Kepler (1571–1630) proposed a law relating the radius (R) of orbit and time (T) in years for one orbit of any planet or satellite orbiting the same central body (e.g. the Sun). R can be measured in gigametres, 1×10^9 m.

- The Earth orbits the Sun every year and its radius of orbit is 228 gigametres.
- The Moon orbits the Earth every 27.3 days and its radius of orbit is 380 000 km.

This law, Kepler's third law of planetary motion, states that for any planets or satellites orbiting the same central body the ratio $\frac{R^3}{T^2}$ is a constant. This constant is $\frac{GM}{4\pi^2}$ where approximately $G = 6.67 \times 10^{-11}$ $\left(\text{in } N\frac{\text{m}^2}{\text{kg}^2}\right)$ and M is the mass of the central body (in kg). Use Kepler's law to calculate the mass of the Earth.

Strategy options

- Break problem into manageable parts.
- Guess and check.

Challenge 9

1 The solutions of the equation $x^2 - 13x - 48 = 0$ are:

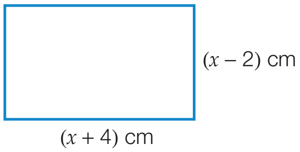
A -3, 16 **B** 3, -16 **C** 4, -12 **D** -6, 8

2 How many positive integers less than or equal to 3000 can be written down without using the digits 4, 5 or 6?

3 If $(x - 2)(2x + 3) = 0$, then possible values for $2x + 3$ are:

A 0 only **B** 0 and 3 **C** 0 and 7 **D** $-\frac{3}{2}$ and $-\frac{7}{2}$

4 The perimeter of the rectangle in the diagram is 112 cm. Its total area is:



A 44 cm^2 **B** 187 cm^2

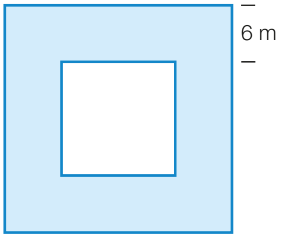
C 775 cm^2 **D** 3127 cm^2

5 If $u \# v$ means $2u - v$, what is the value of $6 \# (2 \# 5)$?

6 The mean of the solutions of the quadratic equation $x^2 + 10x - 11 = 0$ is:

A -10 **B** -5 **C** 5 **D** 10

7 A 6 m wide strip is mown around the inside edges of a square lawn. If eight-ninths of the lawn has been mowed (shaded area), then calculate the area (in square metres) that still needs to be mown (unshaded area).



8 Two three-digit numbers, *abc* and *def* (where the letters represent the digits in each number), have the following property:

$$\begin{array}{cccc} & a & b & c \\ + & d & e & f \\ \hline 1 & 0 & 0 & 0 \end{array}$$

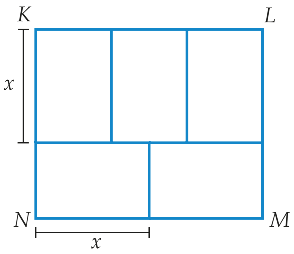
None of a, b, c, d, e and f is zero. Therefore, the value of $a + b + c + d + e + f$ is:

A 10 **B** 19 **C** 21 **D** 28

9 If $x + \frac{1}{x} = 5$, then $x^2 + \frac{1}{x^2}$ equals:

A 25 **B** 23 **C** 27 **D** 3

10 Five identical rectangles are arranged to form a larger rectangle $KLMN$, as shown. The area of $KLMN$ is 4000 cm^2. The length, x cm, of each of the identical rectangles is closest to:



A 33 cm **B** 35 cm

C 37 cm **D** 39 cm

Chapter review 9

Maths literacy

- asymptote
- axis of symmetry
- circle
- coefficient
- constant of proportionality
- dilation
- dilation factor
- direct proportion
- exponential
- hyperbola
- inverse proportion
- null factor law
- parabola
- quadratic equation
- quadratic relationship
- radius
- rectangular hyperbola
- reflection
- transformation
- translation
- turning point
- vertex

Copy and complete the following using the words and phrases from the list, where appropriate. A word or phrase may be used more than once.

1 A ____________ is the type of graph produced by a quadratic relationship.

2 A ____________ is an expression where the highest power of x is 2 and the highest power of y is 1.

3 The ____________ passes through the ____________ of a quadratic graph.

4 Another name for the turning point is the ____________.

5 The graph $y = x^2$ can be changed into $y = 2(x - 1)^2 + 3$ by undergoing a number of ____________s.

6 The graph $y = x^2$ is a ____________ of the graph $y = -x^2$.

7 The equation $(x - 3)^2 + (y + 4)^2 = 9$ graphs as a ____________ with a ____________ of 3.

8 The main features that a sketch curve of a quadratic relationship has are: y-intercept, x-intercepts and the ____________.

9 Using the ____________ it is possible to find the solutions for x when an equation equals 0.

Ext **10** The graph of $y = \frac{1}{x}$ is called a ____________.

Fluency

9.1

1 Plot the graph of $y = x^2 - 2x - 8$.

(a) Write the coordinates of the x- and y-intercepts.

(b) Write the coordinates of the turning point. Does the parabola have a maximum or a minimum turning point?

(c) Write the equation of the axis of symmetry.

(d) From the graph, find the value of y when $x = 3$.

(e) What are the values of x when $y = 7$?

(f) Hence state the values of x when $x^2 - 2x - 8 = 7$.

(g) Write the values of x when $x^2 - 2x - 8 = 0$.

9.2

2 Solve the following using the null factor law.

(a) $2x(x + 4) = 0$ **(b)** $(x - 7)(x + 1) = 0$ **(c)** $x^2 - 16 = 0$

(d) $x^2 + 6x + 9 = 0$ **(e)** $x^2 = 8x$ **(f)** $x^2 + 36 = 12x$

3 Solve these quadratic equations.

9.2

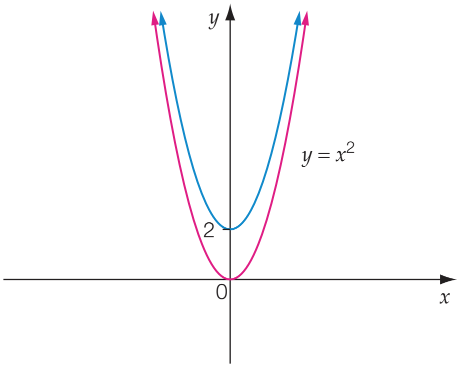
(a) $x^2 - 4x = 0$ **(b)** $x^2 - 6 = 0$ **(c)** $x^2 + 12 = 0$

(d) $3x^2 = 27$ **(e)** $(x - 3)(x + 7) = 11$ **(f)** $(x - 3)(x + 2) = 6$

4 **(a)** The equation of the unlabelled parabola could be:

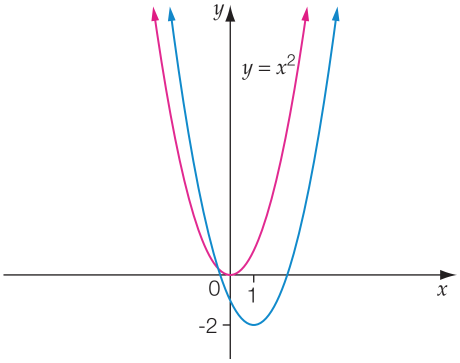
9.3

- **A** $y = x^2 + 2$
- **B** $y = x^2 - 2$
- **C** $y = (x - 2)^2$
- **D** $y = (x + 2)^2$



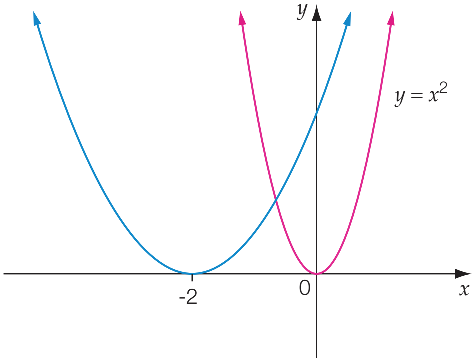
(b) The equation of the unlabelled parabola could be:

- **A** $y = (x - 1)^2 - 2$
- **B** $y = (x - 1)^2 + 2$
- **C** $y = (x + 1)^2 + 2$
- **D** $y = (x + 1)^2 - 2$



(c) The equation of the unlabelled parabola could be:

- **A** $y = 2(x - 2)^2$
- **B** $y = \frac{1}{2}(x + 2)^2$
- **C** $y = \frac{1}{2}(x - 2)^2$
- **D** $y = 2(x + 2)^2$



5 State the changes that should be made to the graph of $y = x^2$ to obtain the graph of each of the following.

9.3

		Dilation	Reflection in *x*-axis	Translation in *x*-direction	Translation in *y*-direction
(a)	$y = (x - 1)^2 + 3$				
(b)	$y = -x^2 + 4$				
(c)	$y = -(x + 2)^2 - 1$				
(d)	$y = 3(x - 4)^2 + 2$				
(e)	$y = \frac{1}{2}(x - 1)^2 + 1$				
(f)	$y = -2(x + 2)^2 - 2$				

6 Sketch the graphs of:

9.3

(a) $y = x^2 + 8x + 16$ **(b)** $y = -x^2 + 2x - 1$

(c) $y = 3x^2 + 9x$ **(d)** $y = x^2 - 4x + 4$

7 **(a)** The graph of the quadratic relationship $y = x^2 - 64$ crosses the x-axis at: **9.3**

A $x = -8$ and $x = 8$ **B** $x = 8$ **C** $x = -8$ **D** $x = 64$

(b) The graph of the quadratic relationship $y = x^2 - 8x + 16$ crosses the x-axis at:

A $x = 2$ and $x = 8$ **B** $x = -4$ and $x = 4$ **C** $x = 4$ **D** $x = -4$

8 Sketch graphs for each of the following circle equations. **9.4**

(a) $x^2 + (y - 2)^2 = 25$ **(b)** $(x + 3)^2 + (y - 1)^2 = 16$

Ext **9** Sketch graphs for each of the following equations, showing the asymptotes. **9.5**

(a) $y = \frac{1}{x+2} - 1$ **(b)** $y = 2^{(x-1)} + 3$

10 Find the equation for each of the following tables. **9.6**

(a)

a	1	3	4
b	3	9	12

(b)

c	0	4	6
d	0	2	3

(c)

g	0	4	9
h	0	6	9

11 **(a)** The volume of a cube is directly proportional to the cube of its height. This may be written as: **9.6**

A $V \propto H^3$ **B** $V = H^3 + 1$ **C** $V \propto H^{\frac{1}{3}}$ **D** $V = H^{\frac{1}{3}}$

(b) The surface area of a cylinder is directly proportional to the square of its radius. This may be written as:

A $A = r^2 + 3r$ **B** $A \propto r^2$ **C** $A \propto \sqrt{r}$ **D** $A \propto \frac{1}{r^2}$

(c) The profit of a company is directly proportional to the square root of its number of employees. This may be written as:

A $p \propto n^2$ **B** $p = n^2$ **C** $p = \sqrt{n^2}$ **D** $p \propto \sqrt{n}$

Ext **12** Find the equation for each table of values. **9.7**

(a)

j	1	2	4
k	12	6	3

(b)

p	1	4	9
q	6	3	2

Ext **13** **(a)** The flexibility of a ball is inversely proportional to the cube root of its volume. This may be written as: **9.7**

A $f \propto V^3$ **B** $f \propto \frac{1}{V^3}$ **C** $f \propto \sqrt{V}$ **D** $f \propto \frac{1}{\sqrt[3]{V}}$

(b) The intensity of candlelight is inversely proportional to the square of the distance from the candle. This may be written as:

A $I \propto d^2$ **B** $I = d^2$ **C** $I \propto \frac{1}{d^2}$ **D** $I = \frac{1}{\sqrt{d}}$

Understanding

14 A softball thrown by an outfielder to the pitcher followed a parabolic path where its height h metres at any time t seconds is given by the relationship: **9.1, 9.2**

$$h = -5t^2 + 10t + 1$$

(a) Complete a table of values for this relationship.

(b) Plot the graph of this relationship on graph paper.

(c) What was the height of the softball after 0.5 seconds?

(d) When was the ball first at a height of 5 m above the ground?

(e) During which time interval was the ball above a height of 2 m?

(f) What was the maximum height of the softball during its flight?

(g) How long was the ball in flight if it was caught at a height of 1.6 m above the ground on its downward path?

15 A jet of water from an outlet at the surface of a pond follows a parabolic path. The height of the water at any point can be described by the equation $h = -d^2 + 6d$. **9.1, 9.2**

Here h is the height of the water a horizontal distance d from the water outlet. Both h and d are in metres.

(a) Sketch the graph of $h = -d^2 + 6d$.

(b) From your sketch, state the maximum height of the water jet above the surface of the pond.

(c) What horizontal distance does the jet of water cover from its outlet back to the surface of the pond?

16 Describe the transformation that has been performed when $(x + 2)^2 + (y - 1)^2 = 25$ has been changed to $(x - 4)^2 + (y - 3)^2 = 16$. **9.4**

Ext **17** **(a)** Show that the graphs of $y = 2^{(x+1)} + 5$ and $y = 3^{(x+2)} - 2$ have the same y-intercept. **9.5**

(b) Write the equation of the asymptotes for each graph.

Reasoning

Ext **18** Nova and Sam found the following table of values, but some values were missing. Nova could not see a pattern, but Sam thought she could see one. **9.5**

x	1	2	3	4	5	
y	0.1		2.7		12.5	21.6

(a) Is there a relationship between the two values? If so, write the relationship.

(b) Use the relationship to fill in the missing values.

19 Sergio and Emina want to develop a vegetable garden in their backyard. They have 20 m of chicken wire and enough posts to erect a fence around the four sides of the rectangular garden. **9.1, 9.2**

(a) If the length of the rectangular garden is x m, find an expression (in terms of x) for the width of the garden.

(b) Show that the area of the garden, A m^2, is given by the quadratic expression $A = -x^2 + 10x$.

(c) Sketch the graph of the area of the vegetable garden, A m^2, against the length of the garden, x m. (Show length on the horizontal axis.)

(d) What should the dimensions of the garden be, if the area is to be as large as possible?

(e) What is the area of the largest vegetable garden that Sergio and Emina can make?

20 A book cover has an area of 64 cm^2. If the length is x cm and the length and width add to 16 cm, find the dimensions of the book cover. **9.2**

21 The sum of two numbers is 50. What are the numbers if their product is to be as large as possible? **9.2**

22 A parabola has a minimum turning point at (3, 5) and a dilation factor of 1. What is the equation of the curve? **9.3**

23 A circle has the equation $(x + 1)^2 + (y - 3)^2 = 121$. What would be the equation of this circle if it is translated 4 units right and 3 units down and the radius is decreased by 2 units? **9.4**

24 $v^2 = u^2 + 2as$ is a formula used in motion. If $u = 0$, the formula becomes $v^2 = 2as$. If a is a constant: **9.2**

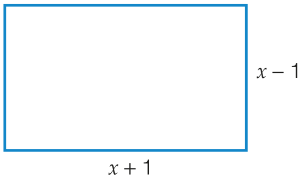
(a) what happens to v when s is doubled?

(b) what happens to s when v is reduced to $\frac{3}{4}$ of its original value?

Numeracy practice 9

Non-calculator

1 Find the value of x in the following rectangle given that the area is 8 m^2.



2 Here is a table of values for x and y.

x	-3	-2	-1	0	1
y	18	8	2	0	2

Which of the following is a correct rule for y in terms of x?

A $y = 6x$ **B** $y = 4x$ **C** $y = x^2 + 3$ **D** $y = 2x^2$

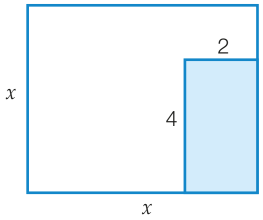
3 The solution/s to $x^2 - 4 = 0$ is/are:

A $x = -2$ **B** $x = 0$ **C** $x = 2$ and -2 **D** $x = 2$

Calculator allowed

4 Jane drew this plan for her square block of land. (All measurements are in metres.)

The shaded area shows the garage. Apart from the garage, what is the area of the land?



A $x^2 + 4x$ **B** $(x + 4)(x - 2)$

C $(x - 4)(x - 2)$ **D** $x^2 - 8$

5 Complete the table of values below that would be used to plot points for the relationship $y = x^2 - 3x + 2$.

x	-3	-2	-1	0	1	2	3
y							

6 If $x = 2$, then which of these expressions will be equal to 6?

A $x^2 + 5x - 3$ **B** $(x + 1)(x - 4)$ **C** $4x^2 - 2x + 3$ **D** $-x^2 + 4x + 2$

Mixed review D

Fluency

1.5

1 An investment of \$5000 is made at 3.5% p.a. for 6 years. Find the difference in interest earned if interest is compounded monthly compared with annually.

3.8

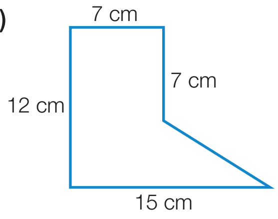
2 Factorise the following quadratic expressions.

(a) $6a^2 - 24$ **(b)** $3a^2 + 6a + 3$ **(c)** $x^2 + 5x - 7x - 35$

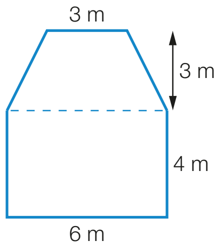
4.2

3 Find the area of each of the following shapes.

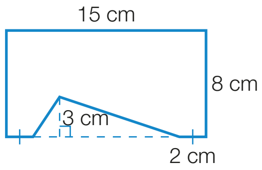
(a)



(b)



(c)



5.1

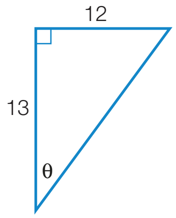
4 Solve the following equations.

(a) $\dfrac{3a+4}{7} = -2$ **(b)** $\dfrac{4a+3}{2} = \dfrac{2a}{5}$ **(c)** $\dfrac{3a-2}{3} = \dfrac{4a-5}{2}$

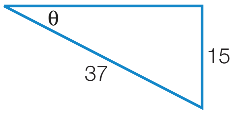
7.4

5 Find the value of the pronumeral in each of the following, correct to 1 decimal place.

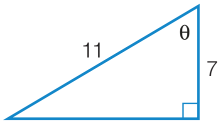
(a)



(b)



(c)



8.3

6 Find the mean for each of the following grouped data frequency tables. Give your answers correct to 2 decimal places.

(a)

Class interval	Frequency
20–<25	8
25–<30	6
30–<35	3
35–<40	2
40–<45	5
45–<50	5

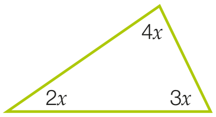
(b)

Class interval	Frequency
111–<115	12
116–<120	15
121–<125	17
126–<130	19
131–<135	13
136–<140	11

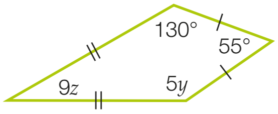
5.2

7 Find the value of the pronumeral in each figure.

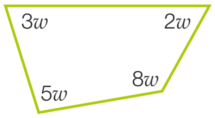
(a)



(b)



(c)



9.2

8 Solve each of the following for x.

(a) $2x^2 - 50 = 0$ **(b)** $4x^2 + 2x = 0$ **(c)** $x^2 - 6x + 9 = 0$

9 Sketch the graphs of each of the following, clearly showing the turning point. 9.1

(a) $y = (x + 3)(x - 2)$ **(b)** $y = x^2 + 2x + 5$ **(c)** $y = 2x^2 - 4x$

10 Sketch the graph of each of the following quadratics, showing all important features. 9.1

(a) $y = x^2 + 6x - 4$ **(b)** $y = x^2 - 4x + 4$ **(c)** $y = -2x^2 + 3x$

11 Consider the equation $y = 2x^2 + x$. 9.1

(a) Draw up and fill in a table of values with x-values from -3 to 3.

(b) Use the table of values to draw the graph of the equation.

(c) Find the turning point of the graph.

(d) Determine the x- and y-intercepts for the graph.

Understanding

12 Jodie is standing on the top of a sand dune and sees a dolphin in the water. The distance from Jodie to the dolphin along her line of sight is 100 m. Find the angle of depression from Jodie to the dolphin if Jodie is 4.3 m above sea level, correct to 1 decimal place. 7.7

13 The 26 letters of the alphabet are each written on 26 identical cards and placed in a bag. A card is chosen at random. What is the probability that the card chosen is: 8.6

(a) a vowel

(b) one of the letters in the word 'card'

(c) not a letter in the word 'letter'?

Reasoning

14 A parabola has x-intercepts at $x = 4$ and $x = 6$. 9.3

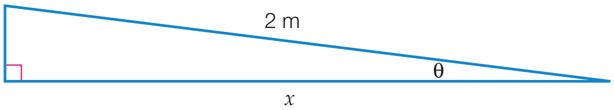
(a) Determine the x-coordinate of the turning point.

(b) If the coefficient of x^2 is 1, determine the equation of the graph.

15 A book cover has an area of 60 cm^2. If the width is 7 cm less than the length, find the dimensions of the book cover. 9.2

16 To meet Australian standards, ramps for wheelchair access that are longer than 152 cm must only rise 1 cm vertically for every 14 cm horizontal distance. 7.5

(a) What is the largest angle θ between the ramp and the horizontal that can be used to build a wheelchair ramp 2 m long? Answer to the nearest degree.



(b) Use your answer to **(a)** to find (to the nearest mm):

(i) the horizontal length x

(ii) the height of the top of the ramp.

Answers

Worked solutions and answers to *all* activities appear in the Teacher Companion.

Chapter 1

Recall 1

1 (a) 0.17 (b) 0.057 (c) 0.00321

2 (a) 0.64 (b) 1.125 (c) 0.575

(d) $0.1\dot{6}$ (e) $0.\overline{90}$ (f) $0.\overline{714285}$

3 (a) $\frac{79}{100}$ (b) $\frac{3}{1000}$ (c) $\frac{7}{500}$

(d) $\frac{213}{200}$ (e) $\frac{507}{50}$ (f) $\frac{451}{5000}$

4 (a) 8.7 (b) 9.1 (c) 7.8 (d) $206\frac{2}{3}$

5 (a) 70% (b) 91% (c) 3.2% (d) 82.14%

6 (a) $\frac{17}{20}$ (b) $\frac{11}{50}$ (c) $\frac{67}{1000}$ (d) $\frac{1}{200}$

7 (a) 0.49 (b) 0.3 (c) 0.025 (d) 3.7

8 (a) 75 m (b) 78.75 kg (c) $28 160

9 (a) 1 m (b) 6.35 km (c) 3.982 cm

Exercise 1.1

1 (a) $100 (b) $100 (c) $7.50

(d) $96 (e) $55 (f) $3.80

(g) $10 (h) $22.50 (i) $45

(j) 6.6L (k) 264 m (l) 26.95 kg

(m) $17.50 (n) 187.5 m (o) 1.08 L

(p) $977.50 (q) $5.58 (r) $1333.33

2 (a) (i) 55 (ii) 75 (iii) 62.5

(b) (i) 360 (ii) 324 (iii) 259.2 (iv) 576

(c) (i) 4560 (ii) 1425 (iii) 3078 (iv) 114

(d) (i) $70.57 (ii) $230.94 (iii) $68.96 (iv) $68.48

(e) (i) 4.9 s (ii) 8.53 s (iii) 9.18 s (iv) 9.47 s

3 (a) 85% (b) 32% (c) 45% (d) 34%

(e) 38%

4 (a) 69.2% (b) 16.8% (c) 63.6% (d) 18.9%

(e) 66.7% (f) 21.5% (g) 75.9% (h) 26.0%

5 Pineapple: 1.95 L
Orange: 0.388 L
Passionfruit: 0.113 L
Guava: 0.05 L

6 37.7 kg

7 (a) (i) 60% (ii) 8.5% (iii) 16%

(b) (i) 17% (ii) 28.3%

8 (a) 8.56 s (b) 3.22%

9 (a) $\frac{7}{9}$ (b) 22.22% (c) $\frac{8}{10}$ or $\frac{4}{5}$, 20%

10 (a) 0.25, 25% (b) 0.5, 50% (c) 1.00, 100%

(d) 0.2, 20% (e) 0.75, 75% (f) 0.4, 40%

11 (a) 0.3, 30% (b) $\frac{9}{20}$, 45% (c) $\frac{18}{25}$, 0.72

(d) 1.25, 125% (e) $\frac{14}{25}$, 56% (f) $\frac{1}{8}$, 0.125

12 (a) 131 250 t (b) 87.5%

13 (a) $4.00 (b) $3.64 (c) 7.06%

14 16 884

15 (a) (i) 120 (ii) 96

(b) (i) 390 (ii) 273

(c) When you are increasing and decreasing by the same percentage, you are not multiplying and dividing by the same number.

16 79.6%

Open-ended—Sample answers

17 45, 59 and 80

18 (a) No, 25% of $299 is $74.75, a greater saving than $31.

(b) 25% is equivalent to $\frac{1}{4}$, which Trent could have found by dividing $300 (rounding up slightly) by 4 to get $75. Because this is rounded up the actual answer will be slightly less, but still greater than $31.

Exercise 1.2

1 (a) 3 × 10% (b) 3 × 25% or 50% + 25%

(c) 25% + 10% or 3 × 10% + 5%

(d) 10% + 5% or 25% – 10%

(e) 8 × 10% (f) 100% – 5%

2 (a) $20.40 (b) $71.25 (c) $38.39 (d) $262.47

3 (a) 70% (b) 85% (c) 62.5% (d) 75%

(e) 40% (f) 25% (g) 88% (h) 60%

4 (a) (i) 40% (ii) 28.6%

(b) (i) 3.6% (ii) 3.7%

5 (a) $75.45 (b) $16.80

6 (a) $2 (b) $27.57

7 (a) $175 (b) $2699

8 A **9** D **10** C

11 (a) $69.60 (b) $96

12 $735 **13** $88.80

14 (a) $1434 (b) 149.4%

15 (a) $19.99 (b) $1.82

16 (a) $27.18 (b) $317.27

17 54.4%

18 (a) $227.29 (b) $11 591.79 (c) $756.21 (d) 6.52%

19 She would save $4.99 by going to her favourite shoe shop.

20 (a) $1088 (b) $598

(c) 45% is applied to two different prices. First, 45% of the cost price of $750 is found and added on. Second, 45% of the selling price of $1088 is found and subtracted. This is a larger amount than the 45% mark up.

(d) 31%

21 $7857.14

Open-ended—Sample answers

22 (a) 24.2%: $999 × (1 – 0.242) = $999 × 0.758 = $757.24
23%: $999 × (1 – 0.23) = $999 × 0.77 = $769.23
20.8%: $999 × (1 – 0.208) = $999 × 0.792 = $791.21

(b) Example:
Discount 20%, marked price $3062.50;
$3062.50 × 0.8 = $2450
Discount 21.5%, marked price $3121.02;
$3121.02 × 0.785 = $2450
Discount 23%, marked price $3181.82;
$3181.82 × 0.77 = $2450

23 Example: A $100 pair of shoes.

A 30% discount gives a sale price of $70. An additional 20% discount means that 20% of $70 (not the original $100) is subtracted. After a 20% discount, sale price = $56.

One 50% discount off the original $100 gives a sale price of $50.

Exercise 1.3

1 (a) $779.00 (b) $1094.40 (c) $1362.30 (d) $748.60

2 (a) $84.15 (b) $74.44 (c) $145.95 (d) $50.37

3 (a) time-and-a-half: 3 hours
double time: 3 hours

(b) time-and-a-half: 3 hours
double time: 7.5 hours

(c) time-and-a-half: 3 hours
double time: 1.5 hours

4 $690.90 **5** $795 **6** $1530

7 A

8 B **9** C

10 Adam—$623.20 Britta—$639.60
Con—$623.20 Deng—$442.80

11 (a) $656 ÷ 38 × 0.8 = $13.81 per hour (b) $339.73

12 $661.30

13 (a) $4800 (b) $5400 (c) $6950 (d) $8000

14 (a) $675 (b) 31.2%

15 $150 000

16 (a) 41 hours (b) 45 hours (c) 50 hours

(d) 20.5 hours overtime is exactly the same

17 (a) (i) $78.75 (ii) $1057.50

(b) $2937.50

18 (a) Option A: $40 000; Option B: $35 714

(b) $25 000

(c) If Nabil expects sales less than $25 000 per week, he should choose Option A.

19 (a) $1873.08 (b) $23.41 (c) $18.73

Open-ended—Sample answers

20 (a) Currently Malik is earning $16.78 × 0.4 × 32 + 17
= $231.78 a week.

Working the same number of hours in a fast food outlet, he would earn $600 ÷ 38 × 0.7 × 32 = $353.68.

Next year (assuming the same number of hours), his wages would be:

Apprentice: $16.78 × 0.52 × 32 + 17 = $296.22 a week

Fast food: $600 ÷ 38 × 0.8 × 32 = $404.21 a week

In the third year, his wages would be:

Apprentice: $16.78 × 0.7 × 32 + 17 = $392.87 a week

Fast food: $600 ÷ 38 × 0.9 × 32 = $454.74 a week

In the fourth year, his wages would be:

Apprentice: $16.78 × 0.82 × 32 + 17 = $457.31 a week

Fast food: $600 ÷ 38 × 32 = $505.26

(b) Malik does not earn more than a casual fast food employee during his apprenticeship. However, he is training to become a fully qualified tradesperson capable of earning far more than a casual fast food employee. Also, this calculation assumes 32 hours a week as a casual worker, but for casuals there is no guaranteed number of hours, and no leave. Malik's current position is more stable and secure.

21 16 hours during the week: $15.80 × 1.25 × 16 = $316

14 hours on a weekend = ($15.80 × 1.25 × 1.1 × 14) = $304.15

10 weekday hours + 5 weekend hours:
(10 × $15.80 × 1.25) + (5 × $15.80 × 1.25 × 1.1) = $306.13

Exercise 1.4

1 (a) $46 253 (b) $55 410.73

(c) $56 686.48 (d) $48 796

2 (a) (i) $6391.05 (ii) $39 282.95

(b) (i) $16 690.63 (ii) $60 674.38

(c) (i) $34 197 (ii) $90 803

(d) (i) $2161.44 (ii) $27 414.56

(e) (i) $0 (ii) $14 033

(f) (i) $63 547 (ii) $136 453

(g) (i) $11 034 (ii) $48 926

(h) (i) $0 (ii) $5976

3 (a) (i) $25 (ii) $700

(b) (i) $800.33 (ii) $3829.67

(c) (i) $106.85 (ii) $1155.54

(d) (i) $0 (ii) $176

(e) (i) $2190.62 (ii) $6444.62

(f) (i) $35.54 (ii) $501.50

4 (a) $57 765 (b) $10 320.63

5 (a) $220 (b) $146 510 (c) $42 155.70

6 (a) $1963 (b) $41 149 (c) $4920.43

7 B

8 (a) $48 256 (b) $278.09 (c) $1577.92 (d) $176.32

9 (a) $532 (b) $7344 (c) $1115.58 (d) $4404.42

10 (a) $24 518.52 (b) $24 296.52 (c) $1158.34

11 (a) $9259.50 (b) $1048.08 (c) $178.07

(d) $3312.65 (e) $2741.35

12 (a) $17 547 (b) 22% (c) $9094

(d) 11% (e) $8453

(f) If the income is split then each individual gets the benefit of the tax-free threshold and a lower percentage of income tax.

13 (a) $0.33 (b) $0.37 (c) $0.45

(d) These figures are the amount of tax paid up to the beginning of that tax bracket.

Open-ended—Sample answers

14 The tax table is designed so that those who earn more, pay more tax. This could be argued as being unfair because those who work hard and earn more are penalised, or fair because those who can afford more, can contribute more.

15 Compulsory superannuation means that Australian workers should all have an income after they retire, and are therefore less likely to fall into poverty after retirement.

Half-time 1

1 (a) 73.3% (b) 14.5%

2 $65.45

3 (a) $79.20 (b) $275.80 (c) $55.28 (d) $199.33

4 Lucas should buy the $165 skateboard with a 25% discount in shop B (it costs $123.75 compared with the other one costing $130).

5 $1851.29

6 (a) 28.4% (b) 22.1%

7 (a) $6994.25 (b) $1559.07

8 Jeans—$72.22
Jacket—$72.31
Scarf—$13.75
In total Naomi saved $35.28.

9 $758

Exercise 1.5

1 (a) $16 200 (b) $980 (c) $4694.50 (d) $233.75

2 (a) 8 quarters (b) 6 half-years

(c) 24 quarters (d) 24 half-years

3 (a) $176 (b) $378 (c) $11.07 (d) $34.98

4 (a) $937.50 (b) $21 562.50

5 (a) $33.75 (b) $5303.75

6 (a) $2.03 (b) $1.37

7 8.9%

8 B 9 $13 427.13

10 (a) $R = 0.090$, 9.0% (b) $R = 0.140$, 14.0%

11 (a) 7.9 years (b) 3.8 years

12 (a) $200 (b) $1350

13 4.6 years 14 $1455.23 15 $120 16 $8036

17 $3.29

18 207 days 19 $24 000 20 $7000

21

Date	Details	Amount	Balance
			$0
Jun 20	Deposit	$4000.00	$4000.00
Jul 1	Interest	$3.62	$4003.62
Jul 30	Withdrawal	$500.00	$3503.62
Aug 1	Interest	$5.06	$3508.68

22 (a) $2250 (b) $17 250 (c) 479.17

(d) 13% (e) $1500 (f) $1000

(g) Part (e) assumes the $15 000 is owed for the whole 3 years, whereas part (f) doesn't.

Open-ended—Sample answers

23 4% for 2 years, 8% for 1 year, 16% for 6 months

24 Michael has overcalculated the amount of days the balance was in his account. The opening balance was in his account for 8 days instead of 9, and the second balance was in his account for 12 days instead of 13.

25 As a borrower you are in debt and must repay the debt out of your own income, which gives you less money to spend. If you pay interest you will end up paying back more than you borrowed. As a lender, you are relying on people to pay back the money you loaned. If they don't or can't repay, you have lost money.

Exercise 1.6

1 (a) $11.60 (b) $14.10 (c) $10 (d) $12

2 (a) 0.0148% (b) 0.0266% (c) 0.0195% (d) 0.0041%

3 (a) 91 (b) 91 (c) 184 (d) 62

4 $44.76

5 (a) $41 (b) $989.70 (c) $45.55

6 (a) 7 weeks (b) $8.95

7 (a) 31 days (b) $251.33 (c) $3.70

8 (a) $95.80 (b) $106.03 (c) $179.17/month

9 (a) $2.10

(b) Electronic transfer into their landlord's account, because this saves $3.90 each month

10 (a) $48 600 (b) $8605 (c) 4.3%

11 (a) $124.48 (b) $8825.05 (c) $12 623.05

(d) $179.32 (e) $321.80

(f) People often use a credit card because they want something but do not have the cash to pay for it at the time. If they cannot pay the bill at the end of the month, interest and late fees are added to the balance. If several payments are missed, interest is charged on the interest, and paying off the debt becomes more and more difficult.

Open-ended—Sample answers

12 (a) four ATM withdrawals, two other bank ATM withdrawals, three EFTPOS transactions and five online internet transfers = $10.75

(b) Kezia could try to stop using other banks' ATMs altogether, and withdraw more money from her own bank's ATM each time, so she is only being charged an ATM fee a couple of times each month.

Exercise 1.7

1 (a) (i) 1.35 kWh (ii) $0.25

(b) (i) 0.76 kWh (ii) $0.14

(c) (i) 0.422 5 kWh (ii) $0.08

(d) (i) 0.5 kWh (ii) $0.09

(e) (i) 19.2 kWh (ii) $3.55

(f) (i) 38.4 kWh (ii) $7.10

(g) (i) 1.275 kWh (ii) $0.24

(h) (i) 20.25 kWh (ii) $3.75

2 (a) 60 days (b) 86 m^3 (c) 3429 MJ

(d) $41.31 (e) $76.46

3 (a) The computer uses more energy (0.06 kWh compared with 0.053 kWh).

(b) The dishwasher uses more energy (0.615 kWh compared with 0.55 kWh)

4 (a) Sewage Disposal, Service Charges, Waterways and Drainage Charge.

(b) $91.19 (c) 92 days

(d) 68 kL (e) 185 L/day

5 (a) 18.75 kWh (b) $3.71

6 (a) 1.08 kWh (b) $74.90

7 (a) 49 L (b) 18 kL

8 (a) 0.354 kWh (b) $0.07 (c) $24.55

9 (a) $0.217 per kL

(b) A higher charge for extra water could discourage people from using it, and encourage them to save water.

10 (a) 9.5 kWh (b) 1.2 tonnes

(c) 13.3 kg

(d) Consumption is higher during the winter months.

(e) Higher heating costs during winter (house heating, hot water). Alternatively, a greater number of people in the house between April and December.

Open-ended—Sample answers

11 Games console—1.5 hours—saving of $1.30
Television (LCD)—2 hours—saving of $18.72

12 For a 4-minute shower:

(a) 60 L (b) 21.9 kL (c) $27.59

(d) 10.95 kL of water and $13.80 saved

13 (a) heating, hot water, cooking

(b) The house has gas heating, which is in use during winter.

Exercise 1.8

1 (a) $588 (b) $1920 (c) $2352

(d) $456 (e) $480 (f) $4752

2 (a) $668 (b) $611 (c) $228 (d) $114

3 (a) $3.90 (b) $26.10

4 (a) $2.99 (b) $6.36

5 $18.40

6 (a) $1896 (b) $1440

(c) '$1206 upfront' costs $456 more than the '$60 per month' plan

(d) $420

7 (a) $5.40 (b) $151.20 (c) Unlimited 1.5 GB

8 (a) $8.49 (b) $0.79

9 (a) $4.29 (b) $21.36 (c) $640.80

(d) (i) $561.80 (ii) $6741.60

Open-ended—Sample answers

10 three calls of 2 minutes, two calls of 5 minutes, 17 SMSs, ($19.21)
two calls of 4 minutes, one call of 6 minutes, one call of 7 minutes, four SMS, one MMS ($20.00)
five calls of 90 seconds, two calls of 3 minutes, 16 SMS, five MMS ($19.99)

11 Students' own answers.

12 Students' own answers.

Challenge 1

1 Numbers are x, $x + 1$, $x + 2$, $x + 3$, $x + 4$, $x + x + 4 = 1708$, $2x = 1704$, $x = 852$.

Numbers are 852, 853, 854, 855, 856.

2 $7^1 = 7, 7^2 = 49, 7^3 = 343, 7^4 = 2401, 7^5 = 16\,807, 7^6 = 117\,649, 7^7 = 823\,543, 7^8 = 5\,764\,801, \ldots$

The powers of 7 end in 07, 49, 43 or 01 in that order and then the pattern is repeated.

2048 is divisible by 4, so the last two digits of 7^{2048} will be the same as the last two digits of 7^4, that is 01.

Hence, $7^{2048} \div 100$ has a remainder of 1.

3 The numbers will end in 5 because numbers ending in 0 have 2 as their smallest prime factor. Will be ≥ 25.

$5 = 1 \times 5$ (1 is not prime), $25 = 5 \times 5$, $35 = 5 \times 7$, $55 = 5 \times 11$, $65 = 5 \times 13$, $85 = 5 \times 17$, $95 = 5 \times 19$.

Numbers are 5, 25, 35, 55, 65, 85, 95.

4 If $m = n = 1$ then $m^3 = n^4$. But m and n are different.
$(2^4)^3 = (2^3)^4$, $m = 16$, $n = 8$.

$(3^4)^3 = (3^3)^4$, $m = 81$, $n = 27$ etc. Smallest value is $m = 16$.

5 (a) 21/02/2012, 22/02/2022, 23/02/2032, 11/02/2011, 10/02/2001 etc.

(b) 10/02/2001, 2 October, 2001; 11/02/2011, 2 November, 2011; 09/02/2090, 2 September, 2090; 01/02/2010, 2 January, 2010

6 The pattern is that the numbers increase by the next multiple of 3. The missing terms are 45, 63 and 84.

7 This is dividing the interval between

$\frac{1}{12}$ and $\frac{1}{7}$ into three equal parts.

$$\left(\frac{1}{7} - \frac{1}{12}\right) \div 3 = \frac{5}{84} \times \frac{1}{3} = \frac{5}{252}\text{; the numbers are}$$

$$\frac{1}{12} + \frac{5}{252} = \frac{26}{252} = \frac{13}{126} \text{ and } \frac{26}{252} + \frac{5}{252} = \frac{31}{252}$$

8 12 students, 30 min, 100 dishes; four students, 90 min, 100 dishes; eight students, 45 min, 100 dishes; eight students, 4.5 min, 10 dishes; 8 students, 40.5 min, 90 dishes. It will take 40.5 min.

9 D

10 D
1000^1 has four digits $= 3 \times 1 + 1$; $1000^2 = 1\,000\,000$ has seven digits $= 3 \times 2 + 1$; so, 1000^{2010} has $3 \times 2010 + 1 = 6031$ digits

11 C

12 A

13 A, C

Chapter review 1

1 (a) $62.21 (b) $260

2 (a) 81.25% (b) 25%

3 (a) 52.50 (b) 26.24

4 C

5 (a) (i) $6.16 (ii) $41.09

(b) (i) $117.27 (ii) $35.41

6 $415.80

7 $831.25

8 (a) $0 (b) $7111.25 (c) $11950.50

9 (a) $2380 (b) $9380 (c) $195.42/month

10 $64.13

11 (a) (i) 1.935 kWh (ii) $0.38

(b) (i) 42 kWh (ii) $8.32

(c) (i) 0.225 kWh (ii) $0.04

12 $24.95

13 $989.20

14 8.4%

15 (a) $138.75 (b) $124.88 (c) 66.5%

16 $1004.25

17 (a) $44 720 (b) $43 280 (c) $5613

18 (a) $16 (b) 5 weeks

19 $1.11

20 (a) (i) $1440 (ii) $5560 (iii) $109

(b) $291.67/month (c) $132.53

21 (a) Tanya earned more money ($516.91 compared with Freya's $493.76).

(b) Casual workers receive more because they don't receive work benefits, including sick pay and holiday leave, and their job isn't guaranteed to be ongoing.

22 $19.76 profit 23 $136.89

24 (a) $473.70 (b) $9952.80

Numeracy practice 1

1 B 2 A 3 B 4 C 5 B

6 C 7 B 8 $776.80 9 $10 740 10 $43.22

Chapter 2

Recall 2

1 (a) 45.79 (b) 12.23 (c) 4.55

2 (a) 144 (b) 3025 (c) 1406.25

(d) 25 (e) 250 (f) 56.66

3 (a) 9 (b) 13 (c) 16

4 (a) 8.06 (b) 25.65 (c) 17.93

5 (a) $x = 6$ (b) $x = 81$ (c) $x = 33$

6 (a) $x = 6$ (b) $x = 6.24$ (c) $x = 18.03$

(d) $x = 8$ (e) $x = 11$ (f) $x = 36.06$

Exercise 2.1

1 (a) E (b) A (c) B (d) D (e) F (f) C

2 (a) right-angled, $\angle ACB$ (b) not right-angled

(c) not right-angled (d) not right-angled

(e) right-angled, $\angle DFE$ (f) right-angled, $\angle DFE$

(g) right-angled, $\angle PRQ$ (h) right-angled, $\angle PRQ$

(i) not right-angled

3 **(a)** A **(b)** D **(c)** D

4 **(a)** $AB, \angle ACB$ **(b)** $AB, \angle ACB$

(c) $AC, \angle ABC$ **(d)** $BC, \angle BAC$

5 C

6 **(a)** $a^2 + b^2 \neq c^2$ so this triangle is not right-angled.

(b) $a^2 + b^2 = c^2$ so this triangle is right-angled.

7 **(a)** yes **(b)** no **(c)** no **(d)** yes **(e)** yes **(f)** yes

8 **(a)** Not square. $a^2 + b^2 = 34$ m; $c^2 = 36$ m.

(b) Decrease the length of the diagonal to $\sqrt{34}$ m (5.83 m).

9 **(a)**

a	b	c	a^2	b^2	$a^2 + b^2$	c^2
3 cm	4 cm	5 cm	9	16	25	25
12 cm	16 cm	20 cm	144	256	400	400
60 mm	80 mm	100 mm	3600	6400	10 000	10 000
5 cm	12 cm	13 cm	25	144	169	169

(b) the last two columns

(c) The values in these columns should be the same.

10 **(a)** Yes. $15^2 + 36^2 = 39^2$.

(b) Yes. $2.5^2 + 6^2 = 6.5^2$.

(c) Pythagoras' theorem holds when all side lengths are multiplied or divided by the same value.

11 A and C are both right-angled triangles. Their side lengths are double the side lengths of the given right-angled triangles.

Open-ended—Sample answers

12 An example is 3 cm, 3 cm, 4.2 cm: $3^2 + 3^2 = 4.2^2$ (approximately).

13 Examples are triangles with side lengths:
3 m, 4 m, 5 m
5 m, 12 m, 13 m
8 m, 15 m, 17 m.

Multiples of these side lengths (e.g. 6 m, 8 m, 10 m) also give right-angled triangles.

14 Students' own answers.

Exercise 2.2

1 **(a)** 13 m **(b)** 10 cm **(c)** 15 mm

(d) 39 m **(e)** 26 cm **(f)** 65 mm

2 **(a)** $\sqrt{185}$ m **(b)** $\sqrt{180}$ m **(c)** $\sqrt{29}$ m

(d) $\sqrt{10}$ cm **(e)** $\sqrt{73}$ cm **(f)** $\sqrt{452}$ mm

3 **(a)** 7.28 m **(b)** 2.83 m **(c)** 14.14 cm **(d)** 14.76 km

4 **(a)** $AD = 12$ m; $DE = 5$ m **(b)** 13 m

5 $a = 3$ cm; $b = 4$ cm; $c = 5$ cm

6 **(a)** 17 cm **(b)** 34 m **(c)** 10 cm

(d) 5 m **(e)** 35 m **(f)** 10 cm

(g) 9.90 cm **(h)** 27.46 mm **(i)** 19.21 cm

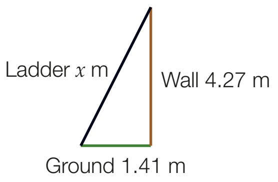
(j) 10.20 m **(k)** 5.52 km **(l)** 13.89 m

7 **(a)** $x = 17$ m, $y = \sqrt{338}$ m

(b) $x = 13$ cm, $y = \sqrt{194}$ cm

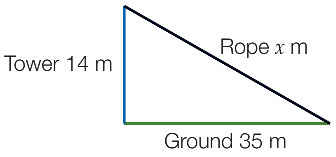
8 B **9** B **10** A

11 **(a)**



(b) 4.50 m

12 **(a)**



(b) 38 m

13 **(a)** $a = \sqrt{89} = 9.43$, $b = \sqrt{80} = 8.94$

(b) $d = \sqrt{337} = 18.36$, $e = \sqrt{562} = 23.71$, $f = \sqrt{4162} = 64.51$

(c) $a = \sqrt{34} = 5.83$, $b = \sqrt{68} = 8.25$

(d) $a = \sqrt{65} = 8.06$, $b = 9$, $c = \sqrt{97} = 9.85$

14 C

15 **(a)** Hypotenuse: $\sqrt{200} = 14.14$ cm
$14.14 \div 10 = 1.414$
$\sqrt{2} \approx 1.414$
The values appear to be equal.

(b) $\sqrt{2}$; it is always $\sqrt{2}$

16 **(a)** $A = \sqrt{2} = 1.41$ m, $B = \sqrt{3} = 1.73$ m, $C = 2$ m,
$D = \sqrt{5} = 2.24$ m, $E = \sqrt{6} = 2.45$ m, $F = \sqrt{7} = 2.65$ m

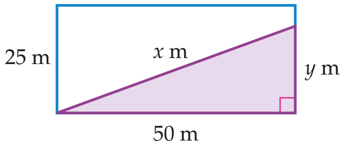
(b) Of the 26 side lengths, 22 cannot be written as an exact decimal value. The exact values are 2 (side C), 3 (side H), 4 (side O) and 5 (side X).

17 **(a)** A to E to H to K **(b)** $3 + 3\sqrt{2}$

18 106 cm

Open-ended—Sample answers

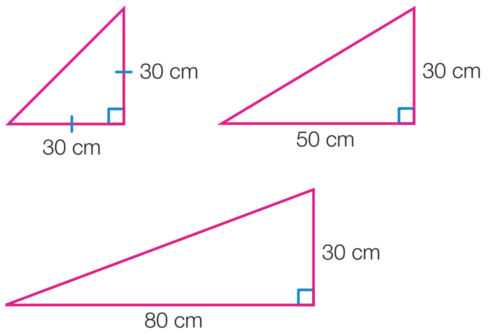
19



Distance from corner, y m	Diagonal distance, x m
1	50.01 m
2	50.04 m
5	50.25 m
10	50.99 m
15	52.20 m
20	53.85 m
25	55.90 m

20 **(a)** horizontal length of 30 cm, ramp length of 42 cm; horizontal length of 50 cm, ramp length of 58 cm; horizontal length of 80 cm, ramp length of 85 cm.

(b)



(c) The ramp with the longest length will be the easiest for getting wheelchairs up the ramp.

Exercise 2.3

1 (a) $a = 35$ (b) $b = 24$ (c) $c = 9$

(d) $w = \sqrt{21}$ (e) $x = \sqrt{3}$ (f) $y = \sqrt{133}$

(g) $g = \sqrt{55}$ (h) $t = \sqrt{14\,104}$ (i) $q = \sqrt{112.5}$ m

(j) $s = \sqrt{290.72}$ m

2 (a) 5 m; 13 m; 169; $\sqrt{144}$ (b) $a = 21$ m; $h = 20$ m

(c) 25^2; 312.5; 17.68 cm (d) 14

3 (a) $a = 25.40$ (b) $b = 19.74$ (c) $c = 31.43$ (d) $d = 17.05$

(e) $e = 9$ (f) $f = 9.90$ (g) $g = 3$ (h) $h = 8$

(i) $x = 7.48$, $y = 8.31$ (j) $x = 6.63$, $y = 17.32$

4 (a) B (b) C

5 (a) B (b) D

6 2.42 m 7 2.46 m

8 (a) 14.14 cm (b) 28.28 cm

9 (a) Sides are each 4 m long. (b) 3.46 m

Open-ended—Sample answers

10 Students' own answers; should range between 2.77 m and 3.1 m.

11 (a) 12 cm and 16 cm; 10 cm and 17.32 cm

(b) 11.25 cm and 33.14 cm; 21.58 cm and 27.56 cm

Half-time 2

1 (a) 106.15 m (b) 45 m (c) 98.49 m

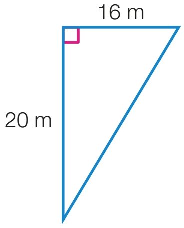
2 (a) 39.69 m (b) 12 m (c) 4.33 m

3 $a^2 + b^2 = 10^2 + 24^2$ $c^2 = 26^2$

$= 676$ $= 676$

$a^2 + b^2 = c^2$, so triangle is right-angled.

4 (a)

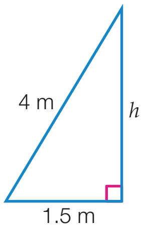


(b) 25.61 m

5 (a) yes (b) yes (c) no

6 $a = 65$; $b = 72$ 7 2.29 cm

8 (a)



(b) 3.71 m

Exercise 2.4

1 (a) 5.83 m (b) 15.80 m

2 9.95

3 (a) 1.70 m (b) 2.33 m

4 D 5 3.47 m

6 (a) 44.82 m (b) 18.18 m

7 0.51 m 8 1.88 km 9 D 10 D

11 (a) 58 m; 8 m longer (b) 86 m (c) 30 m

12 11.31 cm 13 3.14 m 14 $1005.33

15 Through mountains $157 429; around mountains $132 500; it is cheaper to build around the mountains.

16 $1840 (for 8 girders)

17 Two sides are 85 cm long and one is 120 cm long.

18 (a) 9 m (b) 3.31 m (c) 15.62 m

19 (a) 1.73 m (b) 3.87 m (c) 29.21 m

Open-ended—Sample answers

20 After 40 minutes one candle is of length 25 cm and the other candle is of length 26 cm.

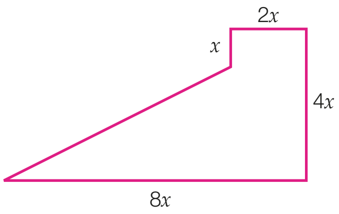
Distance $= \sqrt{1^2 + 20^2} = 20.02$ cm

After 3 hours one candle is of length 7.5 cm and the other candle is of length 12 cm.

Distance $= \sqrt{4.5^2 + 20^2} = 20.5$ cm

21 (a) $x = 10$

(b) 217.08 m



Exercise 2.5

1 (a) 3; 7; 20

(b) (70, 240, 250); (3.5, 12, 12.5); (0.7, 2.4, 2.5)

(c) 26; 2.5; 72

2 (a) 15 m (b) 50 mm (c) 13 cm (d) 25 cm

(e) 2.5 m (f) 8.5 mm (g) 12 mm (h) 24 m

(i) 24 cm (j) 16 cm (k) 12 m (l) 6 cm

3 (a) 26 (b) 39 (c) 15

(d) 45 (e) 82 (f) 130

(g) 65 (h) 1300 (i) 116

4 B 5 C

6 **(a)** yes **(b)** yes **(c)** yes **(d)** no

(e) no **(f)** yes **(g)** no **(h)** no

7 192 cm

8 **(a)**

7	24	25
9	40	41
11	60	61

(b) The three numbers are a Pythagorean triple.

(c) Take an odd number and square it. Subtract 1 from this number and divide by 2. These are the first two numbers in the triple. The third is now more than the second.

9 **(a)**

12	35	37
16	63	65
20	99	101

(b) The three numbers form a Pythagorean triple.

(c) Take any two consecutive odd numbers and add them together. This is the first number in the triple. Now multiply the two odd numbers together to give the second number in the triple. The third number is two more than the second number.

10 **(a)** (48, 64, 80) or (36, 48, 60) **(b)** (20, 48, 52)

(c) (48, 90, 102) **(d)** no **(e)** no

(f) (48, 140, 148) **(g)** no **(h)** (48, 286, 290)

11 (5, 12, 13) × 32 = (160, 384, 416)

12 **(a)** All of the triples except for (16, 63, 65) and (36, 77, 85) satisfy Derek's rule.

(b) Neither (16, 63, 65) nor (36, 77, 85) includes a prime number.

(c) You don't need to mention the condition of having at least one prime number.

(d) In column 1 the pattern is 'Take the next odd number as the first number, square it and then halve it and use the two whole numbers either side of this.' The next triple is (15, 112, 113).

In column 2 the pattern is 'Get the next multiple of 4 as the first number, times by which multiple of 4 it is and subtract 1. Use this and the number 2 more than the second number.' The next triple is (32, 255, 257).

In column 3 the pattern is 'From the previous triple the first number is 8 more than the previous first number.'

The third number is found by adding the square of an odd number to the first number (the odd number is the next consecutive odd in the sequence 3, 5, 7, 9, …). The second number is 8 less than the third number. The next triple is (60, 221, 229).

Open-ended—Sample answers

13 Students' own answers. For example, (11, 60, 61) gives (22, 120, 122), (33, 180, 183), (44, 240, 244).

14 **(a)** example: (8, 15, 17) **(b)** (16, 30, 34), (24, 45, 51)

Challenge 2

1 **(a)** $b = 3 - a,\ a + b = 3$

(b) $KL = a\sqrt{2}$

(c) $LM = b\sqrt{2}$

(d) $KL + LM = (a + b)\sqrt{2} = 3\sqrt{2}$

2 **(a)** $AC = 5,\ AD = 13,\ DE^2 = 85^2 - 13^2 = 84$

(b) Area = 0.5 × (4 × 3 + 5 × 12 + 13 × 84) = 582 cm^2.

3 $7^2 = 49,\ 8^2 = 64$. Any whole number between 49 and 64.

4 2016 = 4 × 504 = 4 × 4 × 126 = 4 × 4 × 9 × 14. The smallest number is 14.

5 **(a)** $(m^2 - n^2)^2 + (2mn)^2 = m^4 - 2m^2n^2 + n^4 + 4m^2n^2$
$= m^4 + 2m^2n^2 + n^4 = (m^2 + n^2)^2$

(b) **(i)** $m = 2,\ n = 1$: 3, 4, 5

(ii) $m = 3,\ n = 1$: 8, 6, 10 → 3, 4, 5

$m = 3,\ n = 2$: 5, 12, 13

(iii) $m = 4,\ n = 2$: 12, 16, 20 → 3, 4, 5

$m = 4,\ n = 3$: 7, 24, 25

(iv) $m = 4,\ n = 1$: 15, 8, 17 → 8, 15, 17

(v) $m = 5,\ n = 1$: 24, 10, 26 → 5, 12, 13

$m = 5,\ n = 2$: 21, 20, 29 → 20, 21, 29

$m = 5,\ n = 3$; 16, 30, 34 → 8, 15, 17

$m = 5,\ n = 4$: 9, 40, 41

Note that when m and n are both even or odd then the triple formed has a common factor. Thus, to generate triples with no common factor, it appears that one value must be even and the other odd. This hypothesis could be investigated further.

6 D

$AB = 3\sqrt{2},\ BC = 3\sqrt{5},$

Perimeter $= 3\sqrt{2} + 3\sqrt{5} + 6 + 9 = 3(5 + \sqrt{2} + \sqrt{5})$ cm

Chapter review 2

1 **(a)** no **(b)** yes **(c)** no

2 **(a)** 75 cm **(b)** $\sqrt{605}$ mm **(c)** $\sqrt{4050}$ m

3 **(a)** $x = 27.33$ **(b)** $x = 78.33$ **(c)** $x = 15.23$

4 **(a)** $a = 40.82$ **(b)** $x = 9.49$ **(c)** $x = 9.64$ **(d)** $x = 13$

5 A

6 **(a)** 20 **(b)** 205 **(c)** 407

7 **(a)** It is not square. **(b)** 72 cm or 73 cm

8 **(a)** $x = \sqrt{115};\ y = \sqrt{556}$

(b) $x = \sqrt{40}\ ;\ y = \sqrt{15}$

9 Ming wins by 1.58 s. **10** 7.03 m

11 **(a)** 2 m

(b) Possibly—many races are won by less than 2 m.

12 **(a)** yes **(b)** no **(c)** yes

13 **(a)** 19.71 m, 10 sleepers **(b)** $350

14 (a) 4 cm (b) 5.66 cm (c) 21.66 cm

Numeracy practice 2

1 D **2** B **3** A **4** A

5 D **6** A **7** C **8** D

Mixed review A

1 9 months

2 (a) 56.25 m (b) 79.04 kg (c) \$192.50

3 not right-angled

4 (a) 72.3% (b) 17.4% (c) 6.4%

5 (a) \$1401.65 (b) \$777

6 (a) \$0 (b) \$3192 (c) \$9097

7 \$853.62

8 (a) (i) 3.15 kWh (ii) 60.48 cents

(b) (i) 12.6 kWh (ii) 241.92 cents (\$2.4192)

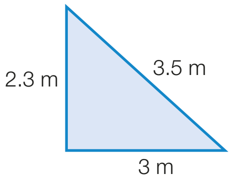
9 \$11 025 **10** $b = 48$, $c = 50$

11 (a) $x = 52$ (b) $x = 80$

12 Mustafa: 87.5%; Adam: 81.8%

Mustafa is the more accurate shooter.

13 (a)



(b) No; $3^2 + 2.3^2 \neq 3.52$

14 65 cm

15 10.41% p.a.

16 685 km

17 (a) \$16.45 (b) \$12.67 (c) 170%

18 No, there is a 4.69 cm drop from the ramp to the platform.

19 28.93 cm

Chapter 3

Recall 3

1 (a) $14x$ (b) $8y$ (c) k^2

2 (a) $4h + 2y$ (b) $8y^3 + 6x^2 - 2x$

3 (a) $12xy$ (b) $20k^2$ (c) $-60g^3$

4 (a) 2^{13} (b) 6^6 (c) 5^4 (d) 1

5 (a) $\frac{4^6}{5^9}$ (b) $2^{16} \times 5^8$ (c) $\frac{7^4 \times 3^8}{4^{10}}$

6 (a) (i) 20 cm (ii) 25 cm^2

(b) (i) 22 m (ii) 18 m^2

(c) (i) 16 mm (ii) 12 mm^2

7 (a) $4x + 8$ (b) $6x - 18$ (c) $-2x - 8$

8 (a) $6(x - 2)$ (b) $5(2x - 5)$ (c) $-3(x - 5)$

Exercise 3.1

1 (a) 2^9 (b) 3^{11} (c) 5^{18} (d) a^7 (e) b^{11}

(f) $5c^5$ (g) $6d^5$ (h) $40b^8$ (i) $28b^4$ (j) $12a^6$

(k) $14a^4$ (l) $200a^7$ (m) $24a^9b^2$ (n) $27a^4b^5$ (o) $30b^7c^7$

(p) $-35a^4b^8$ (q) $36e^6f^6$ (r) $-24a^6b^{10}$

2 (a) 3^3 (b) 4^5 (c) 9^2 (d) m^3 (e) $4b^2$

(f) $5g^4$ (g) n^2 (h) $2b^2$ (i) $2c^4$ (j) $2a$

(k) $\frac{3}{2}a^4$ (l) $\frac{7}{5}e^3f^3$ (m) $2a^3b^4$ (n) $-\frac{8}{7}a^3b^2$ (o) $-12ab$

3 (a) 2^{12} (b) 3^{10} (c) 5^8 (d) 3^{18} (e) 4^{21}

(f) 7^{18} (g) 5^{10} (h) 8^{34} (i) 11^5 (j) b^{21}

(k) k^{32} (l) a^{12} (m) y^{17} (n) k^{27} (o) m^{20}

(p) g^{18} (q) y^{23} (r) t^{21} (s) p^2 (t) n^6

(u) b^3

4 (a) C (b) A

5 (a)

$$\frac{x^7 \times x^{12}}{x^{12}} = \frac{x^{16}}{x^{12}} = x^4$$

(b)

$$\frac{6y^4 \times y^6}{y^2 \times y^5} = \frac{6y^{10}}{y^7} = 6y^3$$

6 (a) x^9 (b) $\frac{t}{3}$ (c) c^7 (d) j^5 (e) $12a^8$ (f) $2q^3$

7 (a) A (b) C (c) C (d) C

8 (a) $5a^2$ (b) $3a^2 \times 5a^2$

9 (a) C (b) B

10 (a) m^3 (b) p^{14} (c) k^3 (d) n^{10}

11 (a) A (b) C

12 (a) $12a^3b^5$ (b) $4m^2p^4$ (c) $4d^2f^2$

13 (a) 3 (b) 0 (c) -2 (d) 1

14 (a) 10 000, 10^4 (b) 1111 (c) 6

15 (a), (b)

Week	Amount saved each week (c)	Amount saved each week in index form (c)
1	5	5×2^0
2	10	5×2^1
3	20	5×2^2
4	40	5×2^3
5	80	5×2^4

(c) $a = 5 \times 2^{(n-1)}$ (d) \$51.20

(e) \$1 677 721.60

Open-ended—Sample answers

16 $-3a$ and ab^3, a^2 and $-3b^3$, $3b$ and $-a^2b^2$

17 $10a^5b^4 \div 2a^3b$, $5a^2b^5 \div b^2$, $15a^4b^3 \div 3a^2$

18 $24a^2b^3 \div 22a^2b^2 = \frac{12b}{11}$

$2 \times 3^2a \div 3ab = \frac{6}{b}$

$48a^5b^6 \div 24a^2b^3 = 2a^3b^3$

19 $m = 3, n = 8; m = 4, n = 6; m = 12, n = 2$

20 **(a)** The indices have been multiplied instead of added. The correct answer is x^6.

(b) The indices have been divided instead of subtracted. The correct answer is x^6.

(c) The index inside the brackets has been raised to the power of the index outside the brackets, when instead they should have been multiplied together. The correct answer is x^8.

Exercise 3.2

1 **(a)** $6^2 \times 5^2$ **(b)** $11^4 \times 3^4$ **(c)** $12^5 \times 9^5$ **(d)** $16x^2$
(e) p^6q^2 **(f)** $a^{12}f^3$ **(g)** $32p^5$ **(h)** $p^{21}q^{35}$
(i) c^8d^{28} **(j)** b^3c^3 **(k)** $216a^{12}$ **(l)** $81p^{10}q^8$
(m) $49m^6n^{12}$ **(n)** $8a^3b^6$ **(o)** $81e^4f^{16}$

2 **(a)** $\frac{12^2}{5^2}$ **(b)** $\frac{7^4}{3^4}$ **(c)** $\frac{9^8}{2^8}$ **(d)** $\frac{y^3}{x^3}$
(e) $\frac{d^5}{e^5}$ **(f)** $\frac{m^{12}}{n^4}$ **(g)** $\frac{w^8}{z^2}$ **(h)** $\frac{r^{20}}{s^4}$
(i) $\frac{d^6}{g^2}$ **(j)** $\frac{3^4}{w^{24}}$ **(k)** $\frac{8^3}{m^6}$ **(l)** $\frac{2^3}{r^{12}}$
(m) $\frac{h^{15}}{g^{10}}$ **(n)** $\frac{a^{12}}{c^{20}}$ **(o)** $\frac{t^{12}}{v^9}$ **(p)** $\frac{5^3a^3}{b^3}$
(q) $\frac{c^{24}}{64x^6}$ **(r)** $\frac{f^{18}}{216g^3}$

3 **(a)** 12^2 **(b)** 20^6 **(c)** 56^4 **(d)** $(4h)^8$ **(e)** $(3p)^4$
(f) $(7r)^9$ **(g)** $(am)^5$ **(h)** $(kt)^7$ **(i)** $(pn)^8$

4 **(a)** 5^4 **(b)** 4^3 **(c)** 12^5 **(d)** $\left(\frac{r}{3}\right)^5$ **(e)** $\left(\frac{n}{6}\right)^7$
(f) $\left(\frac{g}{4}\right)^9$ **(g)** $\left(\frac{x}{y}\right)^6$ **(h)** $\left(\frac{k}{n}\right)^{11}$ **(i)** $\left(\frac{g}{h}\right)^{10}$

5 **(a)** 1 **(b)** 7 **(c)** 8 **(d)** b^2
(e) 3 **(f)** c^3 **(g)** $-3a^4$ **(h)** $-8g^{11}$
(i) -1 **(j)** 1 **(k)** 3 **(l)** 3
(m) $3x^4$ **(n)** k^8 **(o)** $\frac{n^4}{2}$ **(p)** a^3
(q) 3 **(r)** $\frac{3}{2}$ **(s)** $\frac{3f^2}{4}$ **(t)** $\frac{5s^9}{9}$
(u) $\frac{5p^2}{3}$

6 **(a)** $\frac{1}{16}$ **(b)** $\frac{1}{8}$ **(c)** $\frac{1}{625}$ **(d)** $\frac{1}{27}$
(e) $\frac{1}{25}$ **(f)** $\frac{1}{64}$ **(g)** $\frac{1}{x^5}$ **(h)** $\frac{1}{y^3}$
(i) $\frac{1}{b^7}$ **(j)** $\frac{3}{e^3}$ **(k)** $\frac{7}{x^5}$ **(l)** $\frac{3}{g^4}$
(m) $\frac{8}{p^5q^2}$ **(n)** $\frac{2}{x^3y^5}$ **(o)** $\frac{12}{a^3b^9}$ **(p)** $\frac{3p^3}{q}$
(q) $\frac{5x^2}{y^7}$ **(r)** $\frac{7b^2}{a^5}$

7 **(a)** C **(b)** A **(c)** B **(d)** B

8 **(a)** x^6y^6 **(b)** g^9h^9 **(c)** $432x^5$ **(d)** $400x^2y^4$
(e) $c^{12}d^9$ **(f)** x^6y^8

9 **(a)** $\frac{f^5}{g^5}$ **(b)** $\frac{p^{10}}{q^{10}}$ **(c)** $\frac{d^{27}}{q^{14}}$ **(d)** a^2b^2
(e) $\frac{4a^5}{b^5}$ **(f)** $\frac{108x^8}{y^5}$

10 **(a)** w^2z^5 **(b)** c^2e^4 **(c)** p^6q^3 **(d)** v^3w^5

11 **(a)** 1 **(b)** 1 **(c)** 1 **(d)** 1
(e) m^{12} **(f)** f^{20} **(g)** t^4 **(h)** 1
(i) 1 **(j)** $\frac{3}{5}$ **(k)** $\frac{4}{7}$ **(l)** 1
(m) $\frac{5b^6}{2}$ **(n)** $\frac{d^6}{6}$ **(o)** $\frac{d}{10}$ **(p)** $\frac{q^5}{15}$

12 **(a)** $\frac{1}{16}$ **(b)** $\frac{1}{27}$ **(c)** $\frac{1}{625}$ **(d)** 2 **(e)** 64 **(f)** 9
(g) $\frac{4}{9}$ **(h)** $\frac{3}{64}$ **(i)** $\frac{2}{25}$ **(j)** $-\frac{7}{27}$ **(k)** -3 **(l)** -27

13 **(a)** d^2 **(b)** k^5 **(c)** $3a$
(d) x^3y^6 **(e)** p^7q^2 **(f)** $\frac{n^4}{m^4}$
(g) $\frac{b^{12}}{a^8}$ **(h)** $\frac{n^{10}}{m^8}$ **(i)** $x^{15}y^6$

14 **(a)** C **(b)** A **(c)** B **(d)** B

15 Eva's method is faster because simplifying the expression in the brackets first before raising it to the index makes the mathematics simpler.

16 **(a)** 3^{2n} **(b)** 2^{3n+2} **(c)** 5^{n+1}

17 **(a)** $5^3, 5^2, 5^1, 5^0$ **(b)** $5^{-1}, 5^{-2}, 5^{-3}, 5^{-4}, 5^{-5}$
(c) $125, 25, 5, 1, \frac{1}{5}, \frac{1}{25}, \frac{1}{125}, \frac{1}{625}, \frac{1}{3125}$

18 **(a)** 1 divided by the original number
(b) $\frac{7}{a}$ m

19 **(a)** $\frac{n^4T^6p^2}{b^2}$

(b) (i) 10^{12}

(ii) The answer will be positive, because the number inside the brackets will be squared and a negative or a positive number squared gives a positive answer.

Open-ended—Sample answers

20 (a) $(6m^2)^2 \times (n^2)^4$, $(mn^2)^4 \times 6^2$

(b) $(2m^2)^2 \div 5(n^3)^2$, $\dfrac{(4m^2n)^2}{20n^8}$

21 $(2x)^3$, x^3y^2, $(3xy)^3$

22 Students' own answers.

23 $3^2 = 9$ not 6. $(y^4)^2 = y^8$ not y^6. $6 \div 2 = 3$ not 4. $x^4 \div x^{-2} = x^6$ not x^2.

$$\frac{(3x^2y^4)^2}{2x^{-2}y^5} = \frac{9x^4y^8}{2x^{-2}y^5} = \frac{9x^6y^3}{2}$$

Exercise 3.3

1 (a) 9800 (b) 65 000 000 (c) 3 250 000
(d) 420 710 (e) 0.002 (f) 0.000 05
(g) 0.000 000 07 (h) 0.050 32 (i) 0.008 26
(j) 0.000 041 8 (k) 0.000 714 (l) 0.000 003 55

2 (a) 2.56×10^3 (b) 9.002×10^4 (c) 6.14×10^5
(d) 1.9×10^5 (e) 8.05×10^6 (f) 5.6×10^7
(g) 3.05×10^{-1} (h) 3×10^{-2} (i) 4×10^{-4}
(j) 2.01×10^{-3} (k) 7.2466×10^{-5} (l) 5.3218×10^{-7}

3 (a) 390 000 (b) 0.054 0 (c) 670
(d) 0.003 00 (e) 1700 000 (f) 0.900

4 (a) 3 (b) 4 (c) 3 (d) 2 (e) 4 (f) 2

5 (a) 3×10^{-3} seconds (b) 1.34×10^{-1} seconds
(c) $3–4 \times 10^{-1}$ seconds

6 (a) 4.0075×10^4 km (b) 1×10^{-7} m
(c) 1.5×10^{-3} m (d) 5.9×10^9 km
(e) 3×10^{-7} m (f) 1.063×10^3 °C

7 (a) 300 000 000 m/s
(b) 0.000 000 000 000 001 5 m
(c) 0.000 12 m
(d) 4 000 000 000 000 000 000 000 000 atoms
(e) 778 000 000 km
(f) 15 000 000 °C

8 A

9 (a) 1.416×10^{11} (b) 2.1286×10^{-2}
(c) 3.0×10^{11} (d) 2.0×10^{12}
(e) 6.066×10^{-4} (f) 4.299×10^{-10}
(g) $-1.379\,914 \times 10^8$ (h) $-1.349\,545 \times 10^{10}$
(i) $2.689\,98 \times 10$ (j) $1.580\,64 \times 10^{12}$
(k) $2.376\,923\,077 \times 10^{11}$ (l) $3.395\,358\,605 \times 10^{14}$

10 (a) 1×10^{-14} m (b) 4×10^3 m (c) 2×10^{-5} m
(d) 2.73×10^3 °C (e) 1.0651×10^4 athletes
(f) 1×10^{14} atoms (g) 1.5×10^8 km, 6×10^3 °C
(h) 1.4×10^5 cases

11 (a) 6×10^{-3} seconds (b) 2.5×10^{-3} seconds
(c) 2×10^{-3} seconds (d) 2×10^{-3} seconds

12 (a) $9.99–9.997 \times 10^{-7}$ seconds (b) $1.9–1.95 \times 10^{-7}$ seconds

13 (a) 4.57×10^9 years (b) 6.4×10^8 years
(c) 2.4×10^9 years ago (d) 2.2×10^8 years
(e) 5×10^8 years (f) 5.3×10^8 years
(g) 1.2×10^8 years (h) 1.65×10^8 years
(i) 1.44×10^8 years
(j) 6.5×10^7 years ago and 2.51×10^8 years ago
(k) 4.5698×10^9 years

14 (a) (i) 3.6×10^{10} m (ii) $1.088\,64 \times 10^{15}$ m
(iii) $9.467\,28 \times 10^{15}$ m (b) 8.3 minutes

15 1.28 seconds

16 (a) paper (b) 1.8×10^{-3} m

17 (a) 9.0×10^3 revolutions per hour
(b) 9000 revolutions per hour
(c) 150 revolutions per minute

18 (a) 6.235×10^{11} cells (b) 3.9904×10^{13} cells
(c) 39 904 000 000 000 cells

19 (a) 1.08×10^8 km (b) 1.08×10^{11} m
(c) 6.79×10^8 km or 6.79×10^{11} m
(d) 4.2×10^7 km or 4.2×10^{10} m

Open-ended—Sample answers

20 5×10^{-3} cm 21 2×10^{13}

Exercise 3.4

1 (a) (i) $k = \dfrac{P}{m}$ (ii) -8
(b) (i) $b = y - mx$ (ii) -1
(c) (i) $h = \dfrac{C + g}{a}$ (ii) $-\dfrac{1}{2}$
(d) (i) $x = \dfrac{a - c}{m}$ (ii) -6
(e) (i) $p = \dfrac{kn + 2}{m}$ (ii) -53
(f) (i) $w = v(u + xy)$ (ii) 30

2 (a) $u = v - at$ (b) $h = \dfrac{2A}{b}$ (c) $W = Pt$
(d) $b = 2a - P$ (e) $a = \dfrac{P + b}{2}$ (f) $h = \dfrac{3V}{\pi r^2}$

3 (a) $r = \sqrt{\frac{V}{\pi h}}$ (b) $v = \sqrt{\frac{2E}{m}}$ (c) $h = \sqrt{\frac{Fk}{d}}$

(d) $m = \frac{k^2 n}{49}$ (e) $b = \frac{d^2 c}{4}$ (f) $y = \frac{m^2 z}{x^2}$

4 (a) $x = z - y$ (b) $x = 3a + 2b$ (c) $x = \frac{m + n}{k}$

(d) $x = \frac{f - c}{d}$ (e) $x = rt - p$ (f) $x = vy + w$

(g) $x = \frac{kp - y}{2}$ (h) $x = \frac{mw - t}{r}$ (i) $x = \frac{mn - mp}{p}$

(j) $x = \frac{gh - e}{cd}$ (k) $x = \frac{mn + k}{ap}$ (l) $x = \frac{s - vw}{tu}$

(m) $x = \frac{y(f - b)}{d}$ (n) $x = \frac{n(t - kp)}{m}$ (o) $x = \frac{c(ab - f)}{d}$

5 B

6 (a) $C = \frac{5F - 160}{9}$ or $C = \frac{5(F - 32)}{9}$

(b) $C = 160$ °C

7 (a) $t = \frac{v - u}{a}$

(b) (i) $t = 4$ s (ii) $t = 3$ s

8 (a) $r = \frac{C}{2\pi}$

(b) (i) $r = 99.95$ (ii) $r = 2.61$

9 $h = \frac{V}{\pi r^2}$; $h = 2.00$

10 $w = \frac{P}{2} - l$; $w = 19$ m

11 (a) 1.44 (b) 4 (c) ±700

(d) When calculating a square root of a number there is a negative and positive solution. The negative solution means that the object is travelling in the opposite direction to the positive solution.

12 (a) 21 g (b) Yes (the overall density is 1.02 g/cm^3)

13 (a) $\frac{2A}{h} - a$ (b) $b = 13$

(c) If you had to recalculate b for different values of A, a and h, it would be easier to have b as the subject.

14 $s = an + \frac{dn^2}{2} - \frac{dn}{2}$

It's difficult to make n the subject as there are terms with n and n^2 in this expression.

Open-ended—Sample answers

15 She has made a mistake in the final line. She should have taken b away from $\frac{2A}{h}$. $a = \frac{2A}{h} - b$; $b = \frac{2A}{h} - a$

Half-time 3

1 (a) $12x^7y^2$ (b) $4x^2y^3$ (c) $\frac{3m^6n}{4}$

2 $x = \frac{z + 3y}{2y}$, $x = 2.2$

3 (a) $9x^8y^{10}$ (b) $\frac{a^6}{36b^2}$ (c) $\frac{125}{3}$

4 (a) 3.84×10^5 km (b) 5×10^2 mL

5 (a) 4 (b) 3 (c) 3

6 (a) $2^7 \times 5^4$ (b) 3×5^6 (c) 2^5

7 (a) 110 000 000 000 000 bacteria

(b) 0.000 004 4 m

8 (a) $\frac{1}{q^7}$ (b) $\frac{1}{c^9b^5}$ (c) 2 (d) $\frac{8n^4}{m^4}$

9 (a) $\frac{1}{x^5}$ (b) $\frac{3}{p^4}$ (c) $\frac{5n}{m^3}$ (d) $\frac{6r^2}{p^4q}$

10 (a) 0.300 (b) 80.0 (c) 62 100

Exercise 3.5

1 (a) $2x + 10$ (b) $4a + 36$ (c) $8b + 24$

(d) $8x - 16$ (e) $9x - 18$ (f) $3k - 12$

(g) $-7x - 7$ (h) $-4d - 32$ (i) $-6x - 18$

(j) $-9c + 54$ (k) $-2x + 10$ (l) $-5x + 30$

(m) $x^2 + 3x$ (n) $m^2 + 5m$ (o) $x^2 + 9x$

(p) $-x^2 + 8x$ (q) $-x^2 + 4x$ (r) $-n^2 + 9n$

(s) $3x^2 + 6x$ (t) $2u^2 - 12u$ (u) $5x^2 - 5x$

(v) $-2x^2 - 14x$ (w) $-7q^2 - 7q$ (x) $-9b^2 + 54b$

2 (a) $5x + 2$ (b) $5y - 4$ (c) $2x - 11$

(d) $5a + 13$ (e) $2a^2 - 3a + 10$ (f) $3x^2 + 2x + 4$

(g) $7y^2 - 26y - 10$ (h) $9k^2 + 27k$

3 (a) $xy + 5x + 6y + 30$ (b) $xy + 7x + 4y + 28$

(c) $ce + 3c + de + 3d$ (d) $rt - 8r + 5t - 40$

(e) $ab - a + 2b - 2$ (f) $pq - 2p + 3q - 6$

(g) $ay - 4a - 3y + 12$ (h) $mn - 6m - 2n + 12$

(i) $km - 4k - 5m + 20$ (j) $4mp + 4np + mq + nq$

(k) $7ax + 7kx + ay + ky$ (l) $12ps + 3pt + 8rs + 2rt$

(m) $2ab - 6a + b - 3$ (n) $3x^2 + 5x - 2$

(o) $3mn - 3m + 2n - 2$ (p) $10am - 6an - 15bm + 9bn$

(q) $15cx - 20cy - 6dx + 8dy$ (r) $14pr - 21pt - 4qr + 6qt$

4 (a) $a^2 + 5a + 6$ (b) $c^2 + 8c + 15$

(c) $y^2 + 13y + 42$ (d) $x^2 - 2x - 3$

(e) $y^2 + 7y - 18$ (f) $a^2 + 5a - 84$

(g) $b^2 - 3b + 2$ (h) $x^2 - 7x + 12$

(i) $p^2 - 15p + 56$ (j) $6m^2 + 11m + 3$

(k) $8d^2 + 22d + 15$ (l) $21k^2 + 47k + 20$

(m) $8y^2 - 2y - 3$ (n) $10k^2 - 29k - 21$
(o) $8a^2 + 14a - 15$ (p) $12a^2 - 16a + 5$
(q) $12p^2 - 29p + 15$ (r) $24r^2 - 37r + 14$

5 (a) $x^2 + 2xy + y^2$ (b) $m^2 + 2mn + n^2$
(c) $a^2 + 2ab + b^2$ (d) $p^2 - 2pr + r^2$
(e) $c^2 - 2cd + d^2$ (f) $g^2 - 2gh + h^2$
(g) $4c^2 + 12cd + 9d^2$ (h) $9m^2 + 12mn + 4n^2$
(i) $25a^2 + 30ab + 9b^2$ (j) $25x^2 - 40xy + 16y^2$
(k) $4j^2 - 12jk + 9k^2$ (l) $49c^2 - 70cd + 25d^2$

6 (a) $m^2 - n^2$ (b) $p^2 - q^2$
(c) $a^2 - b^2$ (d) $9a^2 - 4b^2$
(e) $16x^2 - 9y^2$ (f) $49m^2 - 9n^2$

7 A **8** D **9** C

10 (a) $(x + 7)(x + 2)\,\text{cm}^2$ (b) $204\ \text{cm}^2$
(c) 6 (d) $x^2 + 9x + 14$
(e) $10^2 + 9 \times 10 + 14 = 204$

11 (a) (i) $(15 + x)$ m (ii) $(3 + x)$ m
(iii) $(15 + x)(3 + x)\,\text{m}^2$ (iv) $(45 + 18x + x^2)\,\text{m}^2$
(b) New garden bed has $(18x + x^2)\ \text{m}^2$ more area.
(c) $40\ \text{m}^2$ extra area

12 (a) $x(x + 3)\ \text{cm}^2$ (b) $(10x - 12)$ cm (c) 3 cm, 4 cm

13 Two possible methods are to substitute values for x and to use the areas of rectangles.

14 (a) $x + 4$ (b) $A = x(x + 4)$
$A = x^2 + 4x$ (c) $320\ \text{m}^2$
(d) width = 24 metres, length = 28 metres

15 (a) (i) $x^2\ \text{m}^2$ (ii) $(x - 4)$ m
(iii) $(x - 2)$ m (iv) $(x - 2)(x - 4)\ \text{m}^2$
(b) $(6x - 8)\ \text{m}^2$

Open-ended—Sample answers

16 length = l, width = w; increase length by 5 units, increase width by 7 units; area of new rectangle = $(l + 5)(w + 7)$

17 Correct answer is $-15x + 6$

18 Correct answer is $x^2 + x - 12$

Exercise 3.6

1 (a) $d^2 - 2d + 1$ (b) $m^2 + 10m + 25$
(c) $k^2 + 4k + 4$ (d) $p^2 - 6p + 9$
(e) $x^2 - 16x + 64$ (f) $y^2 - 24y + 144$
(g) $m^2 + 2mn + n^2$ (h) $w^2 - 2wk + k^2$
(i) $c^2 - 12c + 36$ (j) $y^2 + 12y + 36$
(k) $x^2 + 18x + 81$ (l) $k^2 - 14k + 49$
(m) $16y^2 + 24y + 9$ (n) $4w^2 + 36w + 81$
(o) $9x^2 + 30x + 25$ (p) $36k^2 - 12k + 1$
(q) $49 - 28a + 4a^2$ (r) $9 - 48d + 64d^2$
(s) $1 - 8c + 16c^2$ (t) $36 + 84y + 49y^2$
(u) $16g^2 - 8gh + h^2$ (v) $9a^2 + 6ad + d^2$
(w) $4a^2 - 12ab + 9b^2$ (x) $25m^2 + 80mn + 64n^2$

2 (a) $c^2 - 25$ (b) $a^2 - 16$ (c) $k^2 - 64$
(d) $81 - x^2$ (e) $9 - y^2$ (f) $1 - k^2$
(g) $m^2 - n^2$ (h) $x^2 - y^2$ (i) $c^2 - d^2$
(j) $v^2 - w^2$ (k) $p^2 - q^2$ (l) $h^2 - g^2$
(m) $4m^2 - 1$ (n) $9x^2 - 16$ (o) $36b^2 - 49$
(p) $1 - 25y^2$ (q) $64 - 25a^2$ (r) $9 - 4d^2$
(s) $16a^2 - 25c^2$ (t) $9m^2 - 4n^2$ (u) $4x^2 - 49y^2$
(v) $16 - a^2b^2$ (w) $9 - c^2d^2$ (x) $25 - j^2k^2$

3 B **4** D **5** A **6** C

7 (a) (i) $x^2 + 14x + 49$ (ii) $4x^2 - 12x + 9$
(b) (i) $121\ \text{cm}^2$ (ii) $25\ \text{cm}^2$

8 (a) $x^2\ \text{m}^2$ (b) $(x^2 - 4)\,\text{m}^2$
(c) Square playing field is larger by $4\ \text{m}^2$.

9 $21^2 = 441$

10 (a) 529 (b) 2401 (c) 12 544
(d) 14.44 (e) 36.120 1 (f) 143.041 6
(g) 9996 (h) 3.91 (i) 35.998 4

11 (a) π (b) The area remains constant (π).

12 (a) $\frac{1}{2}(L + l)(L - l)$ (b) $\frac{1}{2}L^2 - \frac{1}{2}l^2$
(c) $(L + l)(L - l) = L^2 - l^2$
(d) (i) $12l^2\ \text{cm}^2$ (ii) $12l^2\ \text{cm}^2$
(e) Area of large triangle = $125l^2$

$$\text{\% of area shaded} = \frac{12l^2}{12.5l^2} \times 100\% = 96\%$$

Open-ended—Sample answers

13 $a = 5, b = 4; a = 13, b = 5; a = 10, b = 6$; any Pythagorean triple where a = length of hypotenuse and b = either of the other side lengths.

Exercise 3.7

1 (a) $3(x + 3)$ (b) $12(y + 2)$ (c) $6(1 + 7a)$
(d) $2(k - 3)$ (e) $4(m - 5)$ (f) $4(2b - 1)$
(g) $x(7 - y)$ (h) $s(4 - 9t)$ (i) $a(3 - b)$
(j) $5(3y + w + 2x)$ (k) $2(a + 2b - 4c)$ (l) $3(x - 5y + 3)$
(m) $ac(3 + 2b)$ (n) $xy(5y + 2x)$ (o) $3fg(3fh - 4g)$
(p) $6gh(h + 3g)$ (q) $8xy(2x - 5y)$ (r) $6ab(3 - 7a)$
(s) $-4(m + 5)$ (t) $-7(d + 7)$ (u) $-12(r + 5)$
(v) $(a + 3)(x + 5)$ (w) $(n - 2)(m + 9)$ (x) $(x + 5)(3 + y)$

2 (a) $5(2x + 5)$ (b) $4(4y + 9)$ (c) $7(2a - 5)$
(d) $6(2d - 3)$ (e) $9(2 - 3m)$ (f) $3(12 - 5k)$
(g) $b(24ac - 10 + 7c)$ (h) $k(5m - 3n + 2gh)$ (i) $c(1 + 4b + 2a)$
(j) $x(x + 3)$ (k) $y(y + 6)$ (l) $k(5 + k)$

(m) $m(8-3m)$ **(n)** $p(4p-7)$ **(o)** $a(9a-5)$

(p) $3d(2-3d)$ **(q)** $2g(11-7g)$ **(r)** $8a(3-4a)$

3 **(a)** $-2(m+6)$ **(b)** $-4(k+6)$ **(c)** $-5(h+5)$

(d) $-x(x+1)$ **(e)** $-y(y+3)$ **(f)** $-p(p+4)$

(g) $-4(4a-1)$ **(h)** $-7(3b-1)$ **(i)** $-6(5r-1)$

(j) $-8(3p-5)$ **(k)** $-2(9w-8)$ **(l)** $-9(5k+7)$

(m) $-3x(5x+12)$ **(n)** $-7y(2y+7)$ **(o)** $-4r(5r+11)$

(p) $-2(4d+2f+3g)$ **(q)** $-5(3a+2b+c)$ **(r)** $-7(2c+4d+f)$

(s) $-4x(3x+4-5y-z)$

(t) $-2k(m-2m^2+3+4k)$

(u) $-2b(5b-2c+4c^2-8a)$

4 **(a)** $(y-1)(4+w)$ **(b)** $(m-3)(5+n)$

(c) $(p-2)(5+r)$ **(d)** $(q+5)(p-2)$

(e) $(2x+3)(y-1)$ **(f)** $(3p+5)(a-1)$

(g) $(f-1)(d-6)$ **(h)** $(d-2)(5d-4)$

(i) $(m-6)(3m-7)$

5 C **6** A **7** C

8 **(a)** $8ab+12a$ **(b)** $4a(2b+3)$

9 **(a)** $(c+2)(d+5)$ **(b)** $c=d+3$

10 **(a)** $(5-x)$ m **(b)** $(5-x)(y+3)$ m^2

11 **(a)** $4x^2+xy$ cm^2 **(b)** $x(4x+y)$ cm^2

(c) $\pi(x-1)^2$ cm^2 **(d)** $\pi x^2-2\pi x+\pi$ cm^2

(e) $4x^2+xy-(\pi x^2-2\pi x+\pi)$
$=(4-\pi)x^2+(y+2\pi)x-\pi$ cm^2

(f) $\pi x^2+xy-2\pi x+\pi$ cm^2

12 **(a)** $(3p+2)(2p-1)$ cm^2 **(b)** $(2p-1)(r+2)$ cm^2

(c) $(2p-1)(3p-r)$

(d) If you calculate the length of the lasagne after removing the slice, it's easier to then calculate the area after the removing the slice.

New length $=3p+2-(r+2)=3p-r$

New area $=(2p-1)(3p-r)$

Open-ended—Sample answers

13 $3x+12xy$; $6x+9xb$; $12abx+15cx$

14 C

Answer A has the wrong sign in front of the constant in the brackets, and answer B the wrong sign and value of the constant in the brackets.

Exercise 3.8

1 **(a)** $(x+4)(y+3)$ **(b)** $(p+1)(q+5)$

(c) $(m+2)(n+9)$ **(d)** $(a+f)(b+d)$

(e) $(p+t)(q+r)$ **(f)** $(k+6)(m+3n)$

(g) $(p-7)(n+1)$ **(h)** $(a-5)(b+1)$

(i) $(m-2)(n+7)$ **(j)** $(a-6)(b-4)$

(k) $(x+2)(x-8)$ **(l)** $(y+4)(y-6)$

2 **(a)** $(k+2)(p+3)$ **(b)** $(m+p)(n+q)$

(c) $(d+6)(c+2)$ **(d)** $(a-k)(d+h)$

(e) $(x-4)(y+2)$ **(f)** $(e-1)(g+3)$

(g) $(b+1)(c-1)$ **(h)** $(y+3)(x-1)$

(i) $(k+3)(m-n)$

3 **(a)** **(i)** $(x+2)(x+7)$
  (ii) $x^2+9x+14$
  (iii) $x^2+9x+14=(x+2)(x+7)$
  (iv) $2\times7=14$

(b) **(i)** $(j+5)(j+3)$
  (ii) $j^2+8j+15$
  (iii) $j^2+8j+15=(j+5)(j+3)$
  (iv) $5\times3=15$

(c) **(i)** $(z+8)(z+1)$
  (ii) z^2+9z+8
  (iii) $z^2+9z+8=(z+8)(z+1)$
  (iv) $8\times1=8$

(d) **(i)** $(x-8)(x+2)$
  (ii) $x^2-6x-16$
  (iii) $x^2-6x-16=(x-8)(x+2)$
  (iv) $-8\times2=-16$

(e) **(i)** $(y-6)(y+4)$
  (ii) $y^2-2x-24$
  (iii) $y^2-2x-24=(y-6)(y+4)$
  (iv) $-6\times4=-24$

(f) **(i)** $(x-4)(x+3)$
  (ii) x^2-x-12
  (iii) $x^2-x-12=(x-4)(x+3)$
  (iv) $-4\times3=-12$

(g) **(i)** $(a+5)(a-4)$
  (ii) a^2+a-20
  (iii) $a^2+a-20=(a+5)(a-4)$
  (iv) $5\times-4=-20$

(h) **(i)** $(k+5)(k-2)$
  (ii) $k^2+3k-10$
  (iii) $k^2+3k-10=(k+5)(k-2)$
  (iv) $5\times-2=-10$

(i) **(i)** $(m+4)(m-7)$
  (ii) $m^2-3m-28$
  (iii) $m^2-3m-28=(m+4)(m-7)$
  (iv) $4\times-7=-28$

(j) **(i)** $(y-7)(y-3)$
  (ii) $y^2-10y+21$
  (iii) $y^2-10y+21=(y-7)(y-3)$
  (iv) $-7\times-3=21$

(k) **(i)** $(c-5)(c-9)$

(ii) $c^2 - 14c + 45$

(iii) $c^2 - 14c + 45 = (c-5)(c-9)$

(iv) $-5 \times -9 = 45$

(l) **(i)** $(d-12)(d-8)$

(ii) $d^2 - 20d + 96$

(iii) $d^2 - 20d + 96 = (d-12)(d-8)$

(iv) $-12 \times -8 = 96$

(m) **(i)** $(m+3)^2$

(ii) $m^2 + 6m + 9$

(iii) $m^2 + 6m + 9 = (m+3)^2$

(iv) $3 \times 3 = 9$

(n) **(i)** $(r+6)^2$

(ii) $r^2 + 12r + 36$

(iii) $r^2 + 12r + 36 = (r+6)^2$

(iv) $6 \times 6 = 36$

(o) **(i)** $(k+11)^2$

(ii) $k^2 + 22k + 121$

(iii) $k^2 + 22k + 121 = (k+11)^2$

(iv) $11 \times 11 = 121$

(p) **(i)** $(p-5)^2$

(ii) $p^2 - 10p + 25$

(iii) $p^2 - 10p + 25 = (p-5)^2$

(iv) $-5 \times -5 = 25$

(q) **(i)** $(d-7)^2$

(ii) $d^2 - 14d + 49$

(iii) $d^2 - 14d + 49 = (d-7)^2$

(iv) $-7 \times -7 = 49$

(r) **(i)** $(n-8)^2$

(ii) $n^2 - 16n + 64$

(iii) $n^2 - 16n + 64 = (n-8)^2$

(iv) $-8 \times -8 = 64$

(s) **(i)** $(c-3)(c+3)$

(ii) $c^2 - 9$

(iii) $c^2 - 9 = (c-3)(c+3)$

(iv) $-3 \times 3 = -9$

(t) **(i)** $(t-2)(t+2)$

(ii) $t^2 - 4$

(iii) $t^2 - 4 = (t-2)(t+2)$

(iv) $-2 \times 2 = -4$

(u) **(i)** $(b-9)(b+9)$

(ii) $b^2 - 81$

(iii) $b^2 - 81 = (b-9)(b+9)$

(iv) $-9 \times 9 = -81$

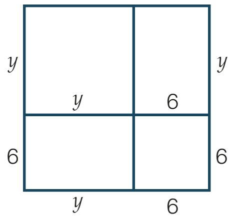
4 A

5 **(a)** $x^2 - 4x + 12 - 3x$ **(b)** $x^2 - 7x + 12$

(c) $(x-4)(x-3)$

6 **(a)** $y^2 + 12y + 36$ **(b)** $(y+6)^2$

(c)



7 **(a)** $(a+7)(b+c+d)$

(b) The expression would be equal to $(a+7)(a+7)$, which is a perfect square: $(a+7)^2$.

8 **(a)** $2x$, $5x$ **(b)** $(x+2)(x+5)$

9 **(a)** $7y(5y-3)$ **(b)** $7y$ cm by $(5y-3)$ cm

(c) $(7y+3)(5y-3)$ **(d)** 3 cm by $(5y-3)$ cm

10 **(a)** $\frac{1}{2}(3x+2y)(x-2)$ **(b)** $3x + 2y$

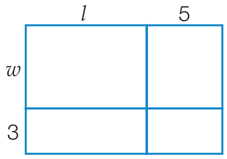
11 **(a)** Tess: $A = (x+2)(x+20)$
Ying: $A = 22(x+2)$

(b) $(x+2)(x+20) - 22(x+2)$ **(c)** $(x+2)(x-2)$

Open-ended—Sample answers

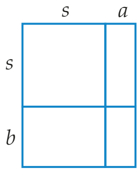
12 **(a)** xy and ay **(b)** $(x+a)(x+y)$

13 **(a)**



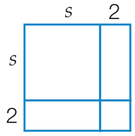
Area $= lw + 3l + 5w + 15$
or
Area $= (l+5)(w+3)$

(b)



Area $= s^2 + as + bs + ab$
or
Area $= (s+a)(s+b)$

(c)



Area $= s^2 + 2s + 2s + 4$
or
Area $= (s+2)^2$

Challenge 3

1 $72 = 6 \times 12 = 8 \times 9 = 4 \times 18 = 3 \times 24$. Numbers are 3 and 24.

2 Let today be x. The day before yesterday is $x-2$. These two days need to be in different years for the age difference to be possible. If today is 1 January, year y, then on 30 December, year $(y-1)$, she is 15. Her birthday is on 31 December. So on 31 December, year $(y-1)$, she turns 16. On 31 December, year y, she is 17; on 31 December, year $(y+1)$, next year, she is 18. She said this on 1 January in the year of her 17th birthday.

3 $12^3 = 1728$, $13^3 = 2197$, $20^3 = 8000$, $21^3 = 9261$, $22^3 = 10\,648$.
There are nine perfect cubes between 2000 and 10 000.

4 D
Shaded area $= (2x + 1)(x - 2) - 10 = 2x^2 - 3x - 12$.

5 $(3 \times 174)^2 = 272\,484$, so ♦ represents 4.

6 If a, b and c are positive whole numbers then at least two of them are either both odd or both even. The difference between two odd numbers will be even and the difference between two even numbers will be even; hence, at least one of $(a - b)$, $(b - c)$ or $(c - a)$ will be even and so the product will have a factor of 2.

7 Powers of 3 end in 3, 9, 7 or 1, and the pattern is repeated. $404 \div 4 = 101$, so 3^{404} ends in 1.

Powers of 4 end in 4 or 6, and the pattern is repeated. Hence, odd powers of 4 end in 4.

$3^{404} + 4^{495}$ ends in $(1 + 4 = 5)$, so it is divisible by 5.

8 $x + y = 6$; $1 + 5, 2 + 4$
$x + z = 7$; $1 + 6, 2 + 5, 3 + 4$
$x + w = 9$; $1 + 8, 2 + 7, 3 + 6, 4 + 5$
$y + z = 9$; $1 + 8, 2 + 7, 3 + 6, 4 + 5$
$y + w = 11$; $1 + 10, 2 + 9, 3 + 8, 4 + 7, 5 + 6$
$z + w = 12$; $1 + 11, 2 + 10, 3 + 9, 4 + 8, 5 + 7$
By trial and error: if $x = 1$, $y = 5$, $z = 6$, $w = 8$, $z = 4$, so not the answer. Same result if $x = 5$. If $s = 2$, $y = 4$, $z = 5$, $w = 7$, and these results satisfy the other equations.
Numbers are 2, 4, 5 and 7.

Or, using pairs of equations you can get: $z - y = 1$, $w - y = 3$, $y - x = 2$, $w - x = 5$, so $y + x = 9$ and $z - y = 1$, so $2z = 10$, $z = 5$
$y + w = 11$ and $w - y = 3$, so $2w = 14$, $w = 7$
$x + y = 6$ and $y - x = 2$, so $2y = 8$, $y = 4$
$x + w = 9$ and $w - x = 5$, so $2x = 4$, $x = 2$

9 (a) $f = \frac{1}{u} + \frac{1}{v} = \frac{1}{f}, \frac{v + u}{uv} = \frac{1}{f}, f = \frac{uv}{u + v}$

(b) $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}, = \frac{1}{v} = \frac{u - f}{fu}, v = \frac{uf}{u - f}$

10 (a) Let the two-digit number be $10a + b$. The number formed by reversing the digits is $10b + a$. The difference $= 10a + b - (10b + a) = 9a - 9b = 9(a - b)$, which is divisible by 9.

(b) The three digit number is $100a + 10b + c$. The number formed by reversing the digits is $100c + 10b + a$. The difference $= 100a + 10b + c - (100c + 10b + a) = 99a - 99c = 99(a - c)$, which is divisible by 99 (and hence by the factors of 99).

(c) The four digit number is $1000a + 100b + 10c + d$. The number formed by reversing the digits is $1000d + 100c + 10b + a$. The difference $= 1000a + 100b + 10c + d - (1000d + 100c + 10b + a) = 999a + 99b - 99c - 999d = 9(111a + 11b - 11c - 111d)$, which is divisible by 9 (and hence by 3).

Chapter review 3

1 (a) a^5b^7 (b) m^7n^8 (c) $\frac{P}{q^3}$
(d) x^2 (e) y^5 (f) c^2d

2 D

3 (a) $\frac{-5}{m}$ (b) $\frac{a^2}{b^6}$ (c) $\frac{1}{x^2y^3}$ (d) $\frac{1}{a^4b^9}$

4 (a) 2×10^{-15} mm (b) 1.5×10^{-4} kg
(c) 3.2×10^8 L (d) 4.8×10^{14}

5 (a) 54 000 000 (b) 0.000 827 3
(c) 0.000 014 (d) 7162

6 (a) $x = \frac{3 - 2z}{y}$ (b) $x = \frac{2a}{3}$
(c) $x = \frac{s - 8t^2}{t}$ (d) $x = \frac{1 + 2z}{15}$

7 A

8 (a) $4x + 28$ (b) $x^2 - 3x$ (c) $-10x^2 + 5x$

9 (a) (i) $13x - 7$ (ii) $18x^2 + 19x - 6$
(b) (i) 45 (ii) 358

10 (a) $x^2 + 10x + 16$ (b) $6a^2 + 7a - 20$

11 (a) $x^2 + 24x + 144$ (b) $25c^2 - 20cd + 4d^2$
(c) $16x^2 - 9$

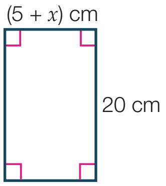
12 (a) $4(2a + 3)$ (b) $-5k(3 + 4k)$
(c) $pq(5p - 3 + 2q - r)$ (d) $-6(4a + ab + 12)$

13 (a) $(a + 4)(b + 3)$ (b) $(b + 1)(a - 2)$
(c) $(b - 6)(a - 5)$ (d) $(a + 7)(b - 1)$

14 (a) 1.5×10^{-3} (b) 5.9×10^9 km

15 (a) 778 000 000 km (b) 15 000 000 °C

16 (a)



(b) $(50 + 2x)$ cm (c) $2(25 + x)$
(d) $20(5 + x)\text{cm}^2$ (e) $100 + 20x$
(f) sample answer: $x = 10$; $20 \times 15 = 100 + 20 \times 10$

17 $2x + 6$

18 $(m - 2)(k + 8)$; $m = 5$, $k = 8$

19 $5y - 3$

20 (a) 7×10^3 L
(b) (i) 4.9×10^4 L (i) 0.1 L (iii) 7.665×10^6 L
(c) No, the amount of blood each person has will vary between people, as well as at different times for the same person. However, this provides a suitable approximation.

21 (a) $10 + 2x$ (b) $x^2 + 5x$
(c) (i) $x^2 + 5x - 14$
(ii) $x^2 + 7x - 2x - 14$, they are the same expression
(iii) There are no sheep.

Numeracy practice 3

1 A   2 42   3 $7(2x+3)$

4 B   5 $x = \frac{12-y}{3}$ 6 D   7 A

Chapter 4

Recall 4

1 (a) 72.1   (b) 18 210   (c) 15.7123   (d) 0.000074

2 (a) 42 mm   (b) 390 cm   (c) 7100 m
(d) 0.45 m   (e) 0.186 m   (f) 5.06 km
(g) 0.011 km   (h) 0.003 72 km

3 (a) 5 m   (b) 1.6 mm   (c) 15.1 cm

4 (a) D   (b) A   (c) C   (d) B

5 (a) 44 cm, 112 cm^2 (b) 48 cm, 96 cm^2
(c) 44 cm, 154 cm^2

Exercise 4.1

1 (a) 34 cm   (b) 24 cm   (c) 60 cm
(d) 45 cm   (e) 130 mm   (f) 140 cm

2 (a) (i) 4.8 m   (ii) 10.2 m   (iii) 45.0 mm   (iv) 60.0 mm
(b) (i) 14.8 km   (ii) 11.3 km   (iii) 128.0 cm   (iv) 0.2 cm

3 (a) (i) 15 cm   (ii) 43 cm
(b) (i) 7 m   (ii) 13 m
(c) (i) 1470 mm   (ii) 2212 mm
(d) (i) 139 m   (ii) 191 m
(e) (i) 68 cm   (ii) 121 cm
(f) (i) 152 cm   (ii) 480 cm

4 B

5 (a) 9.1 km   (b) 111.4 cm   (c) 42.8 cm
(d) 112.8 cm   (e) 314.2 cm   (f) 127.1 cm
(g) 94.2 cm   (h) 268.5 cm   (i) 88.8 cm

6 (a) 17.83 cm
(b) (i) 30°   (ii) $\frac{28}{3}$ cm   (iii) 44.98 cm

7 40 200 km

8 (a) 36 cm   (b) 226 cm   (c) 11.3 s
(d) (i) 350 cm   (ii) 55.7 cm   (iii) 19.7 cm

9 2.342 km

10 35.5 km

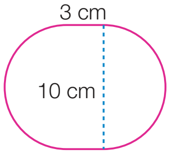
11 almost 16 corrugations (15.9 to 1 d.p.)

Open-ended—Sample answers

12 Students' own answers.

13 Any whole number value from 16 to 28 cm (e.g. a diameter of 20 cm gives a circumference of ≈ 62.83 cm; a diameter of 25 cm gives a circumference of ≈ 78.54 cm; a diameter of 28 cm gives a circumference of ≈ 87.96 cm).

14 possible shape with $P \approx 37.4$ cm:



Exercise 4.2

1 (a) 30 000 cm^2 (b) 1720 cm^2 (c) 7250 mm^2
(d) 60 mm^2 (e) 90 000 m^2 (f) 40 700 m^2
(g) 0.0035 ha   (h) 6.95 cm^2 (i) 0.0127 m^2
(j) 0.0835 m^2 (k) 438 000 m^2 (l) 0.685 ha
(m) 0.029 m^2 (n) 3960 mm^2 (o) 15 km^2
(p) 100.8 km^2

2 (a) 8196.9 cm^2 (b) 16.6 m^2 (c) 506.3 cm^2
(d) 5.3 m^2 (e) 4.3 m^2 (f) 1.3 km^2

3 (a) 1853 cm^2 (b) 143 cm^2 (c) 1750 mm^2
(d) 240 m^2 (e) 293 cm^2 (f) 506 cm^2
(g) 6514 cm^2 (h) 628 cm^2 (i) 184 cm^2
(j) 468 cm^2 (k) 1087 cm^2 (l) 1973 cm^2

4 C

5 (a) 64π m^2 (b) 1408 m^2 (c) 176 kg

6 (a) (i) 112 mm^2 (ii) 1.12 cm^2
(b) (i) 225 mm^2 (ii) 2.25 cm^2
(c) (i) 127.23 mm^2 (ii) 1.27 cm^2
(d) (i) 4500 mm^2 (ii) 45 cm^2

7 (a) trapezium, rectangle   (b) 0.75 m^2, 8.2 m^2
(c) 8.95 m^2

8 (a) $30\,625\pi$ cm^2 (b) 265 cm   (c) 124 407 cm^2
(d) 200 cm^2 (e) 622 bricks

9 167.6 cm^2

10 (a) 46 cm   (b) 451 cm   (c) 800π cm^2
(d) 1546 cm^2 (e) 62%

11 (a) 311.1 m^2 (b) 69.1 m^2 (c) 82%

12 5.53 m^2 13 289.6 m^2 14 807 m^2 15 375 blocks

16 (a) 200 cm^2 (b) 14.14 cm

17 127°   18 11.5 m^2

19 (a) (i) 3 cm   (ii) 2 cm
(b) They are the same.

20 C

Open-ended—Sample answers

21 The area of the top of the pizza is 616 cm^2, so any rectangle that gives this area with integer values: approximately 25 × 25; 22 × 28; 31 × 20 etc.

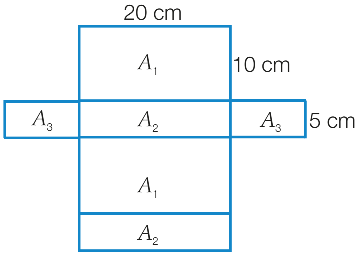
22 Area = 200 cm^2 (length 20 cm, width 10 cm);
Area = 216 cm^2 (length 18 cm, width 12 cm).

Half-time 4

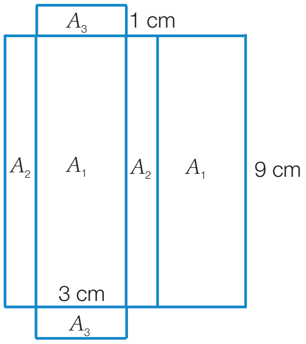
1 **(a)** 540 cm **(b)** 29 mm **(c)** 30 m
(d) 111 mm **(e)** 57 cm **(f)** 107 m
2 **(a)** 1344 cm^2 **(b)** 273 cm^2 **(c)** 57 cm^2
3 10 100 km
4 30 cm
5 **(a)** 400 mm^2 **(b)** 75 000 cm^2 **(c)** 260 000 m^2
(d) 198 000 m^2 **(e)** 305 ha **(f)** 0.0274 m^2
6 270 cm^2

Exercise 4.3

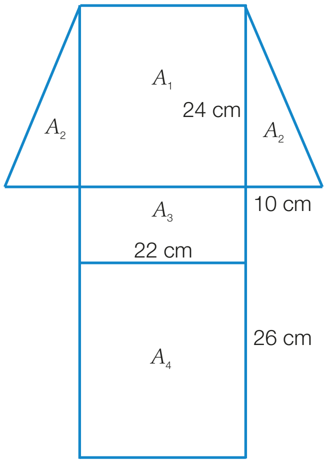
1 **(a)** 700 cm^2



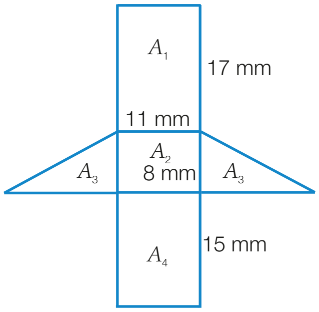
(b) 78 cm^2



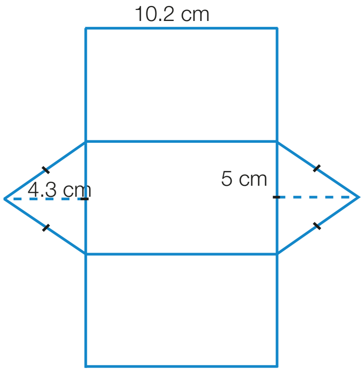
(c) 1560 cm^2



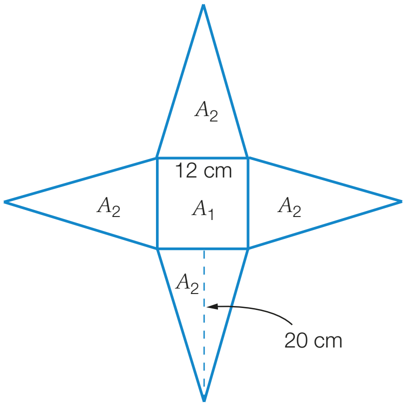
(d) 560 mm^2



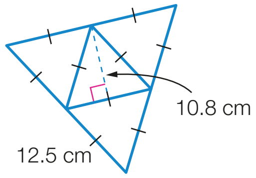
(e) 174.5 cm^2



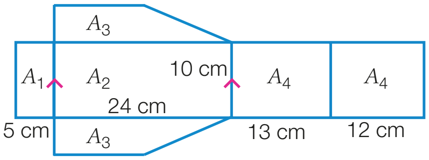
(f) 624 cm^2



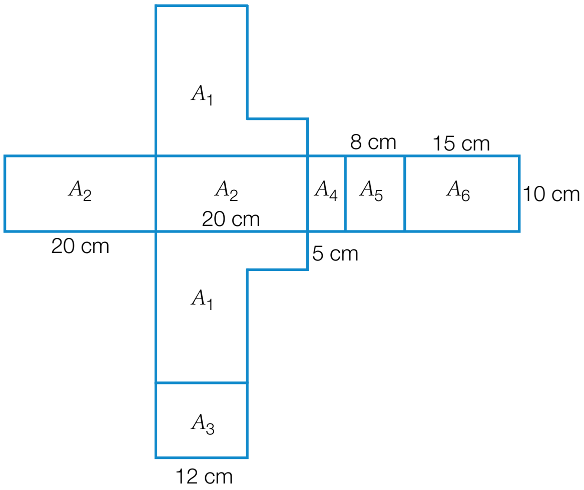
(g) 270 cm^2



(h) 720 cm^2



(i) 1360 cm^2



2 **(a)** 31.7 m^2 **(b)** 6622.5 cm^2

(c) 18.8 m^2 **(d)** 6432.4 mm^2

3 **(a)** C **(b)** D

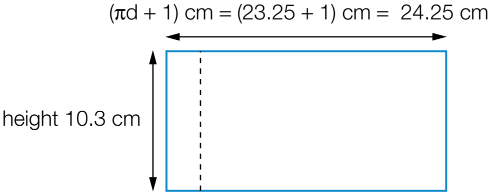
4 **(a)** Type 1: 352 cm^2 and Type 2: 132 cm^2

(b) The surface area of Type 2 is smaller, so it uses less cardboard.

5 **(a)** 519.21 cm^2 **(b)** 2.55 m^2 **(c)** 10 796.31 mm^2

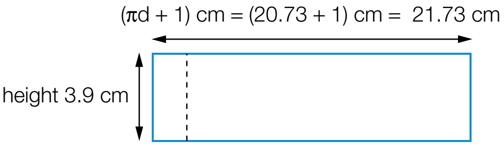
6 109.93 m^2

7 **(a)** **(i)**



(ii) 250 cm^2

(b) **(i)**

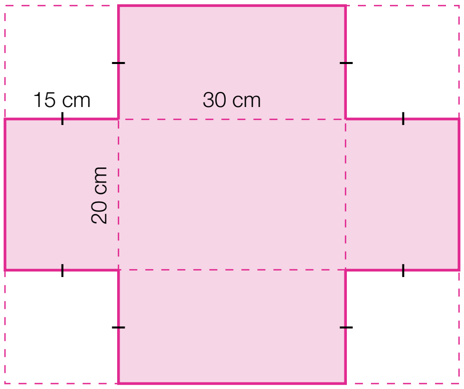


(ii) 85 cm^2

8 34 m^2

9 **(a)** 2100 cm^2

(b)

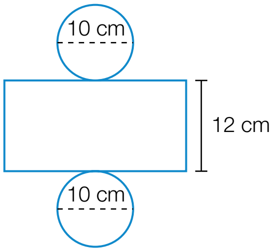


(c) 60 cm long, 50 cm wide

(d) 3000 cm^2 **(e)** 900 cm^2 **(f)**30%

10 **(a)** 54 cm^2 **(b)** 162 cm^2 **(c)** 3 times

11 **(a)**



(b) B **(c)** C **(d)** C

Open-ended—Sample answers

12 four times larger; possible choice, $l = 10$ cm, 20 cm, SA = 600 cm^2, 2400 cm^2

13 any of these combinations (length, width, height) in centimetres: (2, 2, 119), (4, 4, 58), (6, 6, 37), (8, 8, 26), (10, 10, 19), (12, 12, 14), (16, 16, 7), (20, 20, 2)

Exercise 4.4

1 **(a)** 382 cm^3 **(b)** 105.78 mm^3

(c) 2.25 m^3 **(d)** 45 cm^3

2 **(a)** 325 mL **(b)** 2.34 L **(c)** 580 L **(d)** 63.5 kL

(e) 0.145 kL **(f)** 8.5 ML **(g)** 1.25 mL **(h)** 0.75 L

3 **(a)** 600 cm^3 **(b)** 512 m^3 **(c)** 1260 mm^3

(d) 210 cm^3 **(e)** 63 m^3 **(f)** 28 274 cm^3

4 **(a)** 827 mL **(b)** 3.2 L **(c)** 195 kL **(d)** 18.2 ML

5 3040 m^3

6 **(a)** 20 cm **(b)** 12.4 m

(c) 12 cm **(d)** 30 mm

7 **(a)** D **(b)** C **(c)** B **(d)** A

8 **(a)** 20 cm **(b)** 25 132.7 mL **(c)** 100 drinks

9 **(a)** 8 m^3 **(b)** 4 m^2 **(c)** 113 cm **(d)** 160 days

10 6048 cm^3

11 **(a)** 15 944 cm^3 **(b)** 10 cm

12 **(a)** 76 027 mL **(b)** 14 451 cm^3 **(c)** 273.7 kL

Open-ended—Sample answers

13 **(a)** medicine, cans of drink, bottles of juice

(b) the amount of petrol in the tank of a car, daily water usage in a household

(c) capacity of a swimming pool, water tank capacity

14 **(a)** 2 cm, 5 mm, 1 cm **(b)** 20 cm, 5 cm, 10 cm

(c) 2 m, 50 cm, 1 m **(d)** 20 m, 5 m, 10 m

15 **(a)** **(i)** 56 700 cm^3 = 0.0567 m^3. Need to move 2 decimal places for each length conversion, hence 6 places overall.

(ii) 144 765 mm^3 ≈ 145 cm^3. Need to move 1 decimal place for each length conversion, hence 3 places overall.

(b) Work with converted units from the start

(i) 0.45 m, 0.18 m, 0.7 m **(ii)** 4.8 cm, 2 cm

Challenge 4

1 $A = 6s^2$. $v = s^3 = 19\,683$, $s = 27$. $A = 6 \times 27^2 = 4374$ cm^2.

2 3 years of 366 days and 12 years of 365 days = 5478 days. Number of seconds = 5478 × 24 × 60 × 60 = 473 299 200.

3 time off field = $\frac{2}{13} \times 40$ min = 6.15 min = 6 min 9 s

4 Time: $1 + 2 + 4 + 8 + \ldots = 24 \times 60 = 1440$ min.

Number of clicks = 1 more than number of terms in above time sum.

$1 + 2 = 3$, $1 + 2 + 4 = 7$, $1 + 2 + 4 + 8 = 15 = 2^4 - 1$

Find which power of 2 is ≤ 1440. $2^{10} = 1024$. He clicks his fingers 10 + 1 = 11 times.

5 Require 360 mL in each of the two containers.

Fill the 330 mL jug and use it to fill the 150 mL jug. Pour the remaining 180 mL into one container. Pour the 150 mL back into the 720 mL jug.

Repeat, giving 360 mL in one container. Pour the remaining 360 mL from the 720 mL jug into the second container.

6 Volume increases by a factor of 8, the surface area by a factor of 4.

7 Perimeter of first figure = $12\pi + 12\pi = 24\pi$. Perimeter of second figure = $3 \times 4\pi + 12\pi = 24\pi$.

Ratio = 1:1.

8 Each small square has an area of 4 cm^2. Length of side = 2 cm

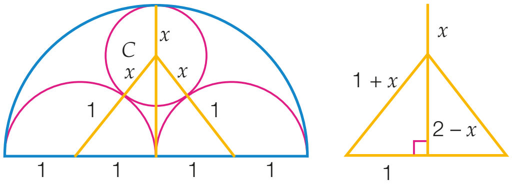
Perimeter = 14 × 2 + 8 × 2 = 44 cm

9 Let $OB = x$ cm, $OA = 2x$ cm, $AD = 4x$ cm.

Area of square = $AD^2 = 16x^2$ cm^2. Area of annulus = $\pi(2x + x)(2x - x) = 3\pi x^2$ cm^2.

Fraction covered $\frac{3\pi x^2}{16x^2} = \frac{3\pi}{16}$.

10



Let the radius of the circle, centre C, be x cm.

In the right-angled triangle:

$$\begin{aligned}(1 + x)^2 &= 1^2 + (2 - x)^2\\ 1 + 2x + x^2 &= 1 + 4 - 4x + x^2\\ 6x &= 4\\ x &= \frac{2}{3} \text{ cm}\end{aligned}$$

Chapter review 4

1 **(a)** 27 cm **(b)** 60 cm **(c)** 57 cm

2 **(a)** **(i)** 134 mm **(ii)** 304 mm

(b) **(i)** 66 cm **(ii)** 108 cm

(c) **(i)** 1.3 cm **(ii)** 3.7 cm

(d) **(i)** 204 mm **(ii)** 356 mm

(e) **(i)** 98.2 cm **(ii)** 151.8 cm

(f) **(i)** 6.95 m **(ii)** 9.65 m

3 **(a)** 420 mm^2 **(b)** 2 ha **(c)** 68.25 m^2

(d) 5.28 km^2 **(e)** 807 000 m^2 **(f)** 90 cm^2

4 **(a)** 5675 mm^2 **(b)** 693 cm^2 **(c)** 0.75 cm^2

(d) 7762 mm^2 **(e)** 1316.2 cm^2 **(f)** 4.69 m^2

5 **(a)** 164 cm^2 **(b)** 3792 cm^2 **(c)** 356 cm^2

(d) 603 cm^2 **(e)** 7.51 m^2

6 **(a)** 1550 cm^2 **(b)** 7.49 m^2 **(c)** 190.5 cm^2

(d) 179 cm^2

7 **(a)** 360 cm^3 **(b)** 2011 cm^3 **(c)** 1380 cm^3

(d) 3000 cm^3 **(e)** 110 mm^3 **(f)** 450 m^3

8 **(a)** 0.017 6 ha **(b)** 1.96 cm^2

(c) 1800 mm^2 **(d)** 0.002 4 m^2

9 **(a)** 61.34 m^2 **(b)** 12.30 m^2

10 147 575 m^2 **11** 308.86 mL **12** 384 cm^2

13 **(a)** 7527 m^2 **(b)** 361 m

14 148 cm^2 **15** 598 cm^2

16 **(a)** 143 m^2 **(b)** 14 900 **(c)** 145 200 L

17 **(a)** 12 160 cm^3 **(b)** 32 cm **(c)** 16.75 cm

18 22 cm

19 **(a)** 117 810 **(b)** 16 h 21 min 45 s

20 **(a)** 8.48 m

(b) hour hand: 0.04 cm/min,
minute hand: 0.63 cm/min

Numeracy practice 4

1 C **2** 62.4 cm^2 **3** B **4** 7 cm

5 D **6** A **7** 62 cm **8** C

Mixed review B

1 **(a)** 212.5 **(b)** $0.08 **(c)** 72

2 **(a)** $80g^2h^3$ **(b)** $-36k^4$ **(c)** $108e^3fg$

3 **(a)** $-8x - 16$ **(b)** $30y - 55y^2$ **(c)** $12p + 58$

4 **(a)** $4(m - 7)$ **(b)** $7d(ed + 5)$ **(c)** $2y(4xy - 1)$

5 **(a)** 14.29% **(b)** 75% **(c)** 12.5% **(d)** 250%

6 132 cm^2

7 **(a)** 20 mm^2 = 0.2 cm^2

(b) 23.67 km^2 = 23 670 000 m^2

(c) 0.007 m^2 = 70 cm^2 **(d)** 14.5 ha = 145 000 m^2

8 (a) x^8 (b) x^4 (c) x^4

9 (a) $48.50 (b) $77.60 (c) $92.15 (d) $145.50

10 (a) 1 (b) 5 (c) 5 (d) 5

11 (a) 178.5 cm^2 (b) 92 m^2 (c) 198 cm^2
(d) 2045.2 cm^2 (e) 17.4 cm^2 (f) 47.1 cm^2

12 (a) $0 (b) $76 (c) $7033.25 (d) $18 546

13 (a) $x^2 + 4x - 21$ (b) $x^2 + 12x + 36$
(c) $4x^2 - 20x + 25$ (d) $x^2 - 64$

14 (a) 4.5×10^7 (b) 5.27×10^9
(c) 3×10^{-4} (d) 2.98×10^{-7}

15 (a) (i) 153.68 m^2 (ii) 110.98 m^3
(b) (i) 20.36 m^2 (ii) 6.87 m^3

16 (a) 7238.2 cm^3 (b) 7238.2 mL

17 (a) $12g^4h^4$ (b) $120a^7b^{11}$ (c) $\frac{xy^5}{3}$

18 216 days

19 (a) $560 (b) $481

20 (a) 2 (b) $\frac{3p^5q^7}{2}$ (c) $5x^5y^7$

21 5.1 m^2 22 (a) $m = \frac{2E}{v^2}$ (b) 5.3 kg

23 (a) $364 (b) $487.50 (c) $606.25 (d) $649.95

24 (a) 0.8 m (approx.) (b) 12 days

25 Surface area is multiplied by four.

26 (a) Option A: at least $16 000; Option B: at least $12 667.
(b) $6000
(c) Option A is better if Bettina expects to sell less than $6000 worth of goods per week.

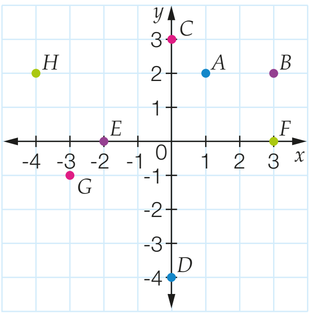
27 (a) 21.46%
(b) 21.46%
(c) They are equally efficient.

Chapter 5

Recall 5

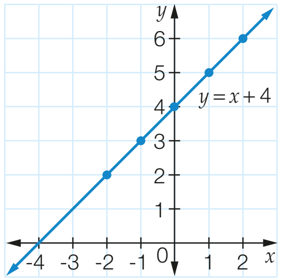
1 (a) $P = 5$ (b) $P = 11$ (c) $P = 3$ (d) $P = -13$ (e) $C = 5$
(f) $C = 17$ (g) $C = 9$ (h) $C = -7$

2



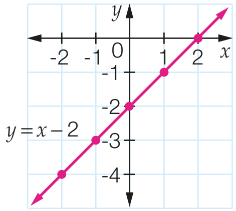
3 (a)

x	-2	-1	0	1	2
y	2	3	4	5	6



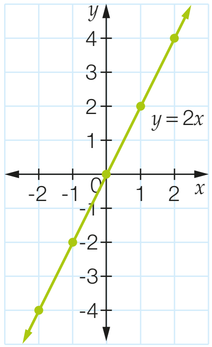
(b)

x	-2	-1	0	1	2
y	-4	-3	-2	-1	0



(c)

x	-2	-1	0	1	2
y	-4	-2	0	2	4



4 (a) $x = 6$ (b) $y = 15$ (c) $a = -10$
(d) $b = 21$ (e) $c = 12$ (f) $d = -14$

5 (a) $g = 19$ (b) $x = 42$ (c) $y = 6$
(d) $x = 35$ (e) $b = -12$ (f) $k = 36$

6 (a) $-15x - 6$ (b) $20 - 6x$ (c) $15x + 43$

7 $y = 6 - 2x$

8 (a) $h = 13$ (b) $x = 7$

Exercise 5.1

1 (a) $x = 4$ (b) $b = 5$ (c) $x = -1$ (d) $h = 7$ (e) $f = -4$
(f) $k = \frac{2}{5}$ (g) $a = 5$ (h) $x = -9$ (i) $y = 0$ (j) $b = 6$
(k) $a = -2$ (l) $b = -3$ (m) $p = -15$ (n) $e = 4$ (o) $g = 8$
(p) $y = 2\frac{3}{5}$ (q) $p = 2\frac{3}{4}$ (r) $y = 2$

2 (a) $x = 20$ (b) $x = 7$ (c) $x = 9$ (d) $x = -15$ (e) $x = 12$
(f) $x = 6$ (g) $x = 7$ (h) $x = -2$ (i) $x = 2$ (j) $x = 5$
(k) $x = -1$ (l) $x = 6$ (m) $x = 19$ (n) $x = -8$ (o) $x = -5$

3 (a) $c = 2$ (b) $b = -8$ (c) $f = -2$ (d) $d = 3$ (e) $e = 4$
(f) $a = 6$ (g) $g = -3$ (h) $h = 20$ (i) $x = 3$ (j) $k = \frac{3}{2}$
(k) $j = 2$ (l) $y = 3$ (m) $m = 4$ (n) $n = 2$ (o) $x = 1$
(p) $y = 1$ (q) $p = -3$ (r) $x = 3$ (s) $a = 2$ (t) $x = 9$
(u) $p = 4$

4 (a) $x = 2$ (b) $x = -10$ (c) $x = -5$ (d) $x = 12$
(e) $x = -3\frac{1}{4}$ (f) $x = 5\frac{5}{6}$ (g) $x = 1$ (h) $x = -\frac{4}{27}$
(i) $x = 7$

5 (a) $b = -2$ (b) $a = 21$ (c) $c = -5$ (d) $d = -8$
(e) $x = 23$ (f) $y = 10$ (g) $g = 18$ (h) $k = 1$
(i) $m = 14$ (j) $j = \frac{3}{2}$ (k) $x = 2\frac{1}{2}$ (l) $p = -19$
(m) $x = 6$ (n) $y = \frac{26}{41}$

6 (a) $x = 6$ (b) $x = 8$ (c) $x = 0$ (d) $x = -36$ (e) $x = -42$
(f) $x = 12$ (g) $x = 3\frac{3}{4}$ (h) $x = 0$ (i) $x = -14$ (j) $x = 15$
(k) $x = 8$ (l) $x = 22$ (m) $x = 22$ (n) $x = 57$ (o) $x = 55$

7 (a) B (b) B (c) C (d) C

8 (a) B (b) C (c) D (d) A (e) D (f) C

9 (a) D (b) B

10 (a) B (b) D (c) D (d) A

11 (a) $x + 7 = 19, x = 12$ (b) $y - 5 = 3, y = 8$
(c) $3x = 14, x = 4\frac{2}{3}$ (d) $\frac{x}{4} = -5, x = -20$
(e) $2x + 3 = 13, x = 5$ (f) $4x - 7 = -12, x = -1\frac{1}{4}$

12 (a) $x = 5$ (b) $x = -2$ (c) $x = 2$ (d) $x = -9$ (e) $x = -6$
(f) $x = 4$ (g) $x = 5$ (h) $x = 1$ (i) $x = -2$ (j) $x = 0$
(k) $x = 6$ (l) $x = -2$

13 (a) $x = -\frac{5}{8}$ (b) $x = -\frac{23}{12}$ (c) $x = -\frac{8}{15}$ (d) $x = 8\frac{1}{2}$
(e) $x = \frac{1}{4}$ (f) $x = -\frac{11}{14}$ (g) $x = -\frac{1}{6}$ (h) $x = \frac{2}{3}$
(i) $x = 2\frac{5}{6}$

14 $2x + 4 = x - 2, x = -6$

15 $x + 36 = 2(x + 6), x = 24$

16 $\frac{3x + 1}{2} = \frac{x - 4}{5}, x = -1$

17 (a) $n + n + 2 + n + 4 = 195, n = 63$. The three numbers are 63, 65 and 67.
(b) $n + n + 2 + n + 4 + n + 6 = 204, n = 48$. The four numbers are 48, 50, 52 and 54.
(c) $n + n + 7 + n + 14 + n + 21 + n + 28 + n + 35 = 609, n = 84$. The six numbers are 84, 91, 98, 105, 112 and 119.

18 (a) $x + \frac{x}{6} = 28, x = 24$ (b) $x + \frac{x}{4} = 45, x = 36$
(c) $x + \frac{x}{8} = 99, x = 88$

Open-ended—Sample answers

19 If $x = -1, 2x + 7 = 5$. If $x = -2, 2x + 7 = 3$. If $x = -10, 2x + 7 = -13$.

20 $\frac{x}{2} + 1 = x - 1, \frac{3x + 4}{2} = 8, 5(x - 1) = 3(x + 1)$

21 $x + 2 = 2x + 1, 2x - 7 = x - 3, 20 - x = x + 18$

Exercise 5.2

1 $2j + 25 = 34$. Jen is 4.5 years old.
2 Kate is 15 years old and her father is 45 years old.
3 Andrew has 11 coins and Lori has 6 coins.
4 Liu is 16 years old. Huan is 32 years old.
5 B
6 length = 13 m, width = 10 m
7 Annette: 300 m; Mark: 600 m; Sue: 1.2 km
8 \$1.65 9 $\frac{7b}{2} = 63$. Bianca has 18 marbles.
10 13 cm, 22 cm, 32 cm
11 21 years old 12 49°, 55°, 76° 13 11 cm
14 A mineral water costs \$4.
An iced coffee costs \$5.
15 $\frac{3(3a - 8)}{4} = \frac{7(a + 3)}{5}, a = \$12$

Open-ended—Sample answers

16 23, 26, 30

Exercise 5.3

1 (a) 5 (b) 13 (c) 17 (d) 10
(e) 10 (f) 25 (g) $\sqrt{10}$ (h) $\sqrt{37}$
(i) $\sqrt{61}$ (j) $\sqrt{53}$ (k) $2\sqrt{13}$ (l) $4\sqrt{5}$

2 (a) (2, 4) (b) (-1, 4) (c) (-1, -7) (d) (7.5, 9.5)
(e) (-1, 7.5) (f) (1.5, -0.5) (g) (3.5, 4.5) (h) (2.5, -2.5)
(i) (4.5, 5.5)

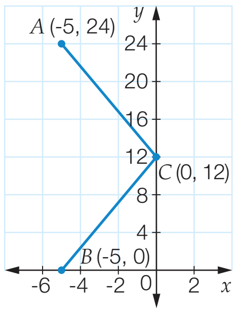
3 C 4 17 units 5 (1, 5.5)

6 (a) (9, 2) (b) (5, -10) (c) (9, 0)

7 $AB = \sqrt{(1 - -3)^2 + (1 - -2)^2} = 5$
$BC = \sqrt{(-2 - 1)^2 + (5 - 1)^2} = 5$
$AC = \sqrt{(-2 - -3)^2 + (5 - -2)^2} = \sqrt{50}$
Two sides are of equal length, therefore ΔABC is isosceles.

$\sqrt{AB^2 + BC^2} = \sqrt{5^2 + 5^2} = \sqrt{50}$. Pythagoras' theorem holds so ΔABC is a right-angled triangle.

8 (a) Side lengths are 3.61, 3.61 and 5.10 (2 d.p.).
(b) Isosceles

9 

$d(AC) = 13$

$d(BC) = 13$

10 **(a)** 5 km **(b)** (3, 2) **(c)** (7, 1)

(d) from *B* to *C*, 11.18 km (2 d.p.)

(e) parallelogram

11 **(a)** midpoint of $AC = (-3, -5)$, midpoint of $BD = (-3, -5)$

(b) All sides are 4.47 (2 d.p.). **(c)** $AC = 8.49$, $BD = 2.83$

(d) rhombus

12 $AB = \sqrt{20}$, $BC = \sqrt{45}$, $CD = \sqrt{20}$, $DA = \sqrt{45}$. Therefore opposite sides are of equal length. $AC = \sqrt{65}$, $BD = \sqrt{65}$. Therefore diagonals are also of equal length, so *ABCD* is a rectangle.

13 (4, 1)

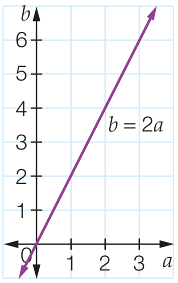
14 Coordinates of *PQRS* are $P(-2, 4)$, $Q(1, 2)$, $R(-1, 0)$, $S(-4, 2)$. *PQ* has length $\sqrt{13}$ and gradient $-\frac{2}{3}$, *QR* has length $\sqrt{8}$ and gradient 1, *RS* has length $\sqrt{13}$ and gradient $-\frac{2}{3}$, *SP* has length $\sqrt{8}$ and gradient 1. Therefore opposite sides have equal length and gradient, so *PQRS* is a parallelogram.

Open-ended—Sample answers

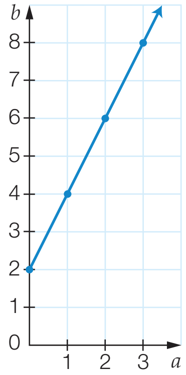
15 $A(-1, 0)$, $B(1, 0)$ and $C(0, \sqrt{3})$. $AB = 2$, $BC = 2$ and $CA = 2$. Therefore ΔABC is an equilateral triangle.

16 $A(0, 8)$, $B(3, 12)$, $C(6, 8)$ and $D(3, 0)$. $AB = 5$, $BC = 5$, $CD = \sqrt{73}$ and $DA = \sqrt{73}$. *ABCD* has two pairs of adjacent, equal sides and is therefore a kite.

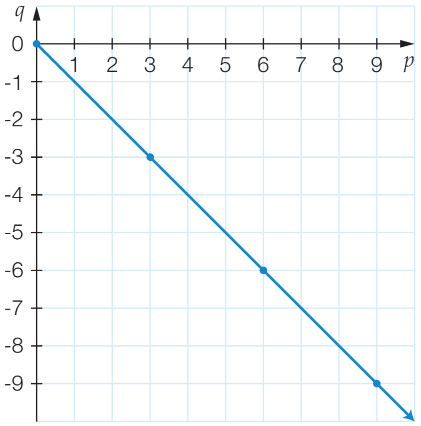
Exercise 5.4

1 **(a)** **(i)** 

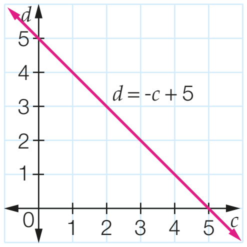
(ii) $b = 2a$

(b) **(i)** 

(ii) $b = 2a + 2$

(c) **(i)** 

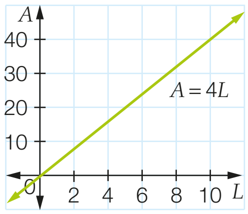
(ii) $q = -p$

(d) **(i)** 

(ii) $d = -c + 5$

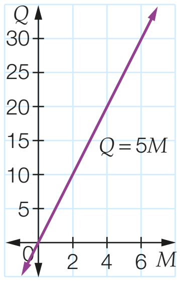
2 **(a)** **(i)**

L	0	10	20	30
A	0	40	80	120

(ii) 

(b) **(i)**

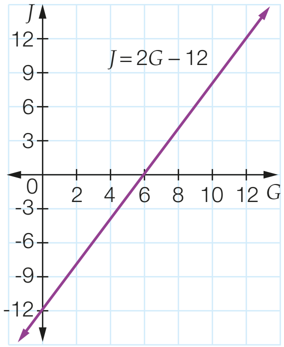
M	0	2	4	6
Q	0	10	20	30

(ii) 

(c) **(i)**

G	0	4	8	12
J	-12	-4	4	12

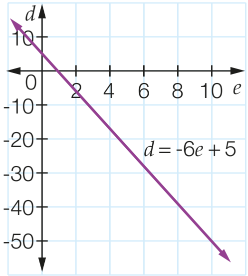
(ii)



(d) **(i)**

e	0	3	6	9
d	5	-13	-31	-49

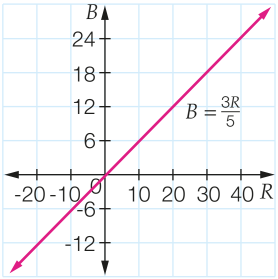
(ii)



(e) **(i)**

R	-20	0	20	40
B	-12	0	12	24

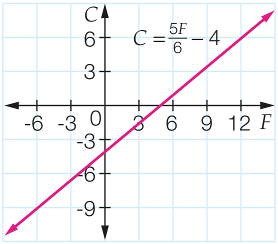
(ii)



(f) **(i)**

F	-6	0	6	12
C	-9	-4	1	6

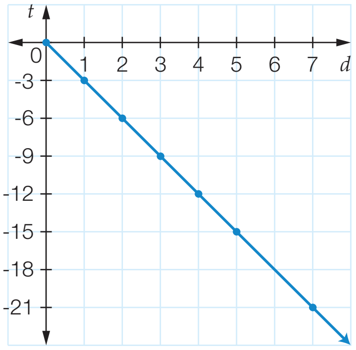
(ii)



3 **(a)**

Day (d)	0	1	2	3	4	5
Total time lost (t)	0	-3	-6	-9	-12	-15

(b)

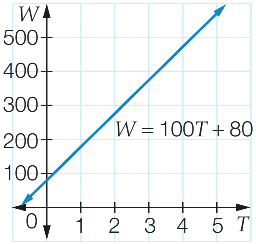


(c) yes **(d)** $t = -3d$ **(e)** 21 seconds

4 **(a)**

Number of televisions sold	0	1	2	3	4	5
Daily income ($)	80	180	280	380	480	580

(b)



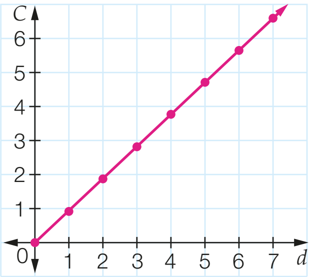
(c) yes **(d)** $W = 100T + 80$ **(e)** $880

5 **(a)** D **(b)** B **(c)** C

6 **(a)**

Number of litres	0	1	2	3	4	5	6	7
Cost ($)	0	0.95	1.90	2.85	3.80	4.75	5.70	6.65

(b)



(c) Yes, the graph is in a straight line.

(d) **(i)** $2.85 **(ii)** $2.28 **(iii)** $5.42

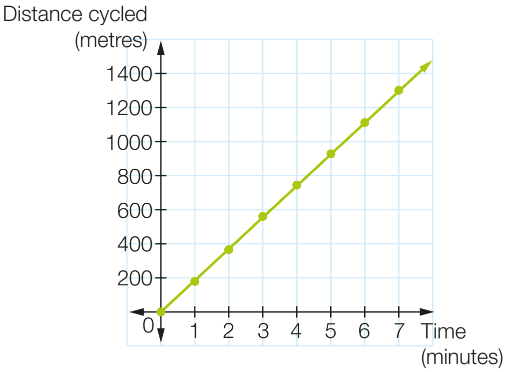
(e) **(i)** 4 L **(ii)** 2.6 L **(iii)** 5.5 L

(f) $C = 0.95d$

7 **(a)**

Time (min)	0	1	2	3	4	5	6	7
Distance cycled (m)	0	185	370	555	740	925	1110	1295

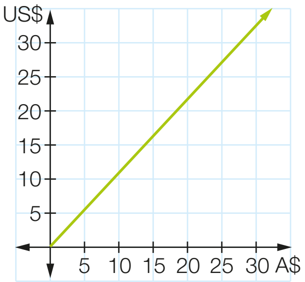
(b)



(c) 499.5 m **(d)** 3.8 min **(e)** 10.8 min

(f) $D = 185t$ **(g)** 7400 m (7.4 km)

8 **(a)**

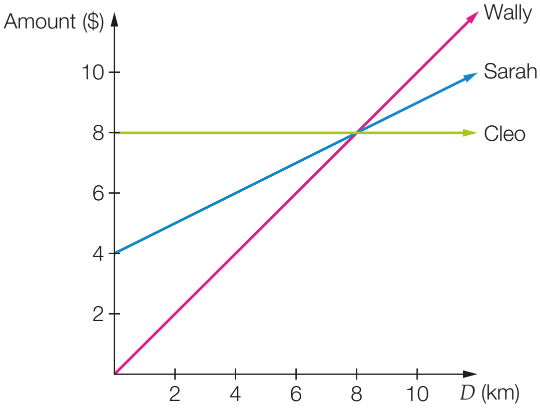


(b) US$7.35 **(c)** US$18.50 **(d)** A$5.20 **(e)** A$9.10

9 **(a)**

D	Wally	Sarah	Cleo
0	0	4	8
1	1	4.5	8
2	2	5	8
3	3	5.5	8
4	4	6	8
5	5	6.5	8
6	6	7	8
7	7	7.5	8
8	8	8	8
9	9	8.5	8
10	10	9	8

(b)



(c) 8 km

(d) **(i)** Wally collects the most money above 8 km.

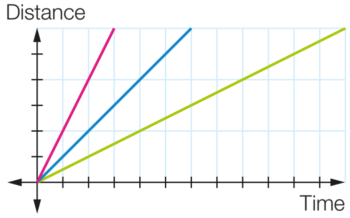
(ii) Sarah never collects the highest amount of money for any distance.

(iii) Cleo collects the most money between 0 and 8 km.

10 In the first 2 years Frank earns more than Bruno; after that, Bruno earns more.

Open-ended—Sample answers

11



As the time taken to travel the same distance increases, the line becomes less steep (has a smaller gradient).

Half-time 5

1 **(a)** yes **(b)** no **(c)** no **(d)** yes

2 (4.5, 5)

3 **(a)** $x = 9$ **(b)** $y = 5$ **(c)** $x = 16$ **(d)** $x = -9$ **(e)** $x = 3$

(f) $x = 3$ **(g)** $p = \frac{5}{2}$ **(h)** $a = -5$ **(i)** $m = 13$

4 13

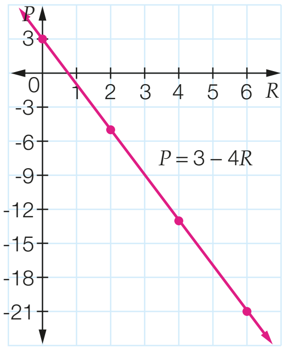
5 Kiara weighs 54 kg; Rob weighs 60 kg.

6 C **7** $x = 2$ m

8 **(a)**

R	0	2	4	6
P	3	-5	-13	-21

(b)



Exercise 5.5

1 *AB*: 3; *AC*: 1; *AD*: -2; *AE*: $-\frac{2}{5}$

2 *a*: 1; *b*: $\frac{2}{5}$; *c*: $-\frac{1}{3}$; *d*: -2

3 *a*: 0; *b*: 0; *c*: undefined; *d*: undefined

4 **(a)** 2 **(b)** 3 **(c)** -4 **(d)** 6 **(e)** 2

(f) 1 **(g)** 1 **(h)** -1 **(i)** $-\frac{1}{3}$ **(j)** 5

(k) 0 **(l)** 0 **(m)** 7

(n) undefined **(o)** undefined

5 **(a)** positive **(b)** negative

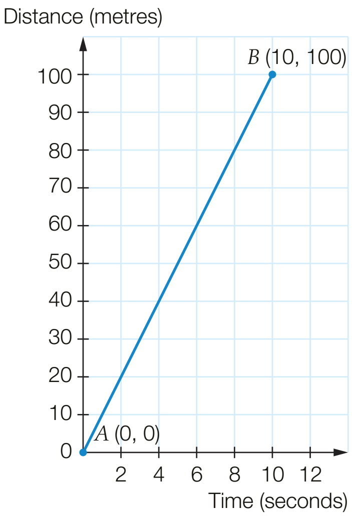
(c) zero **(d)** undefined

6 *a*: B; *b*: A; *c*: A; *d*: C; *e*: B

7 **(a)** a: -1; b: 3; c: $\frac{1}{3}$; d: 0; e: -$\frac{1}{3}$ **(b)** b **(c)** a

8 **(a)** C **(b)** A **(c)** A **(d)** C

9 **(a)**



(b) 10 metres per second

(c) The gradient is an indication of how fast Sia is travelling.

(d) $d = 10t$

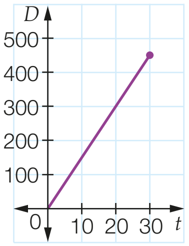
10 **(a)** 1200 m **(b)** 20 min **(c)** 60 m **(d)** 60

(e) 60 m/min

(f) **(i)** 360 m **(ii)** 720 m **(iii)** 1080 m **(iv)** $60t$ m

(g) $D = 60t$

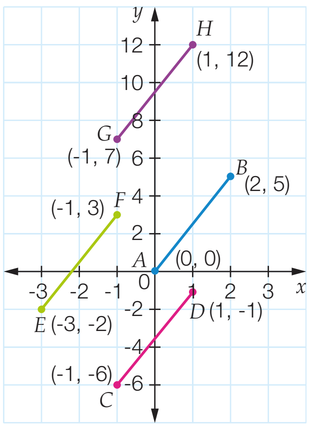
11 **(a)**



(b) 15 m/s or 54 km/h **(c)** 15

(d) $D = 15t$ **(e)** 795 m

12 **(a)**



(b) gradient of line segments: $AB = \frac{5}{2}$; $CD = \frac{5}{2}$; $EF = \frac{5}{2}$; $GH = \frac{5}{2}$

(c) Part **(b)** shows that the four line segments have the same gradient.

13 555 cm

14 24 gradients; there are 22 (11 positive and 11 negative due to a mirror image) that are either whole numbers or fractions. The 23rd is $m = 0$ and the 24th is undefined.

15 **(a)** 320 L **(b)** 8 min **(c)** -40

(d) 40 L/min **(e)** The volume of water is decreasing.

(f) **(i)** 160 L **(ii)** 80 L **(iii)** 240 L **(iv)** $(320 - 40t)$ L

(g) $V = 320 - 40t$

(h) Neither time nor the volume of water can be a negative value.

16 **(a)** 48 km/h **(b)** 96 km/h **(c)** 7 h

Open-ended—Sample answers

17 (-2, 8), (4, 4), (7, 2)

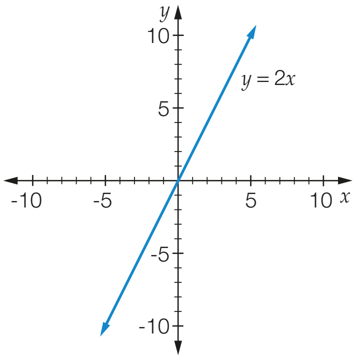
18 The mistake is adding the negative values of x and y to the positive values instead of subtracting them from the positive values.

Exercise 5.6

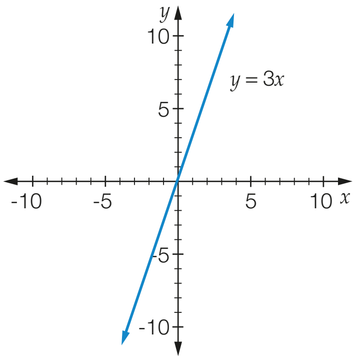
1

	Gradient	y-intercept
(a)	1	(0, 3)
(b)	2	(0, 1)
(c)	7	(0, 3)
(d)	3	(0, -2)
(e)	1	(0, -4)
(f)	-4	(0, 3)
(g)	-1	(0, 4)
(h)	-2	(0, 6)
(i)	2	(0, -5)

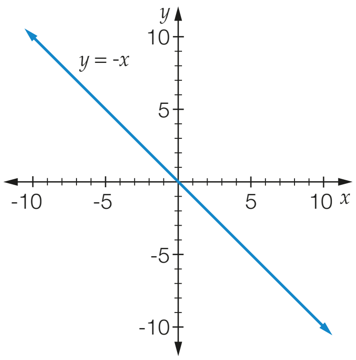
2 **(a)**



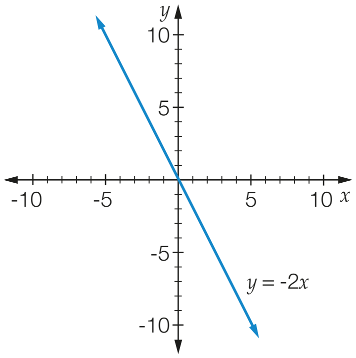
(b)



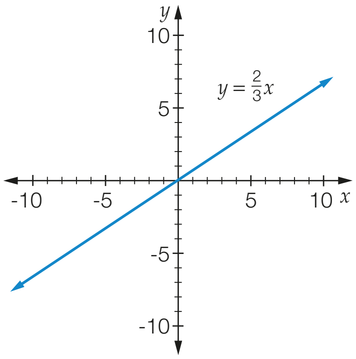
(c)



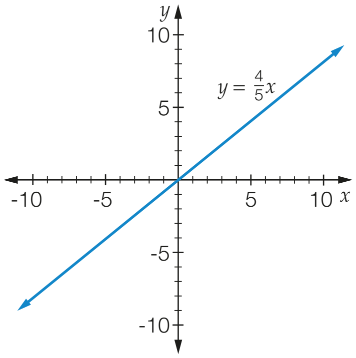
(d)



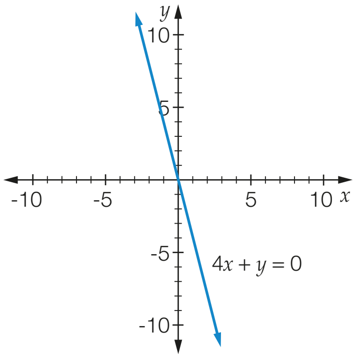
(e)



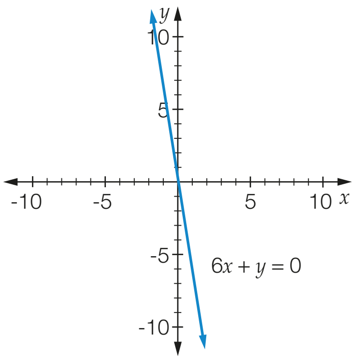
(f)



(g)



(h)



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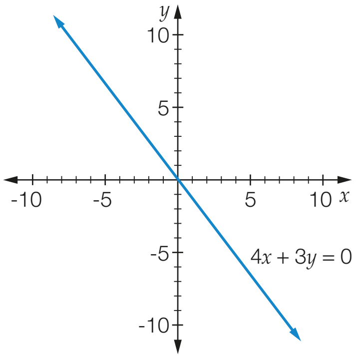
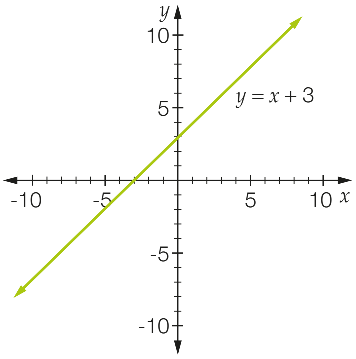
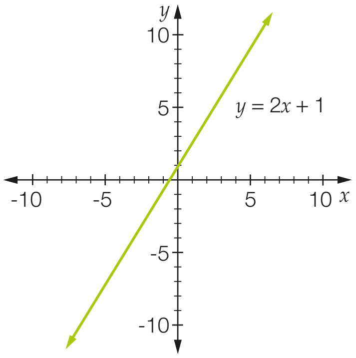


3 (a)



(b)



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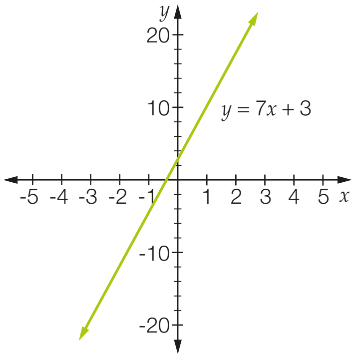

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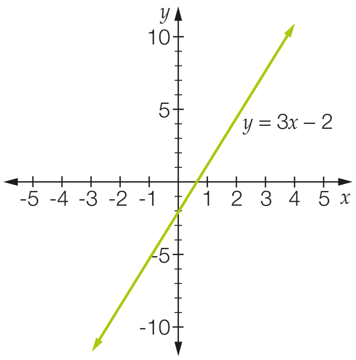

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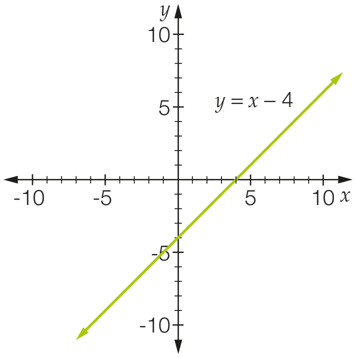

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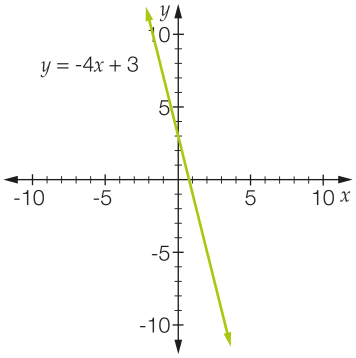

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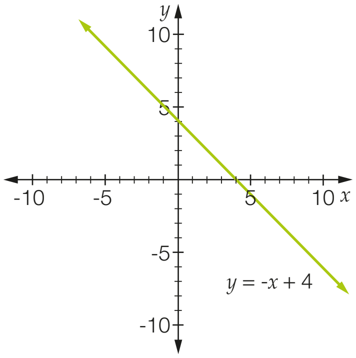

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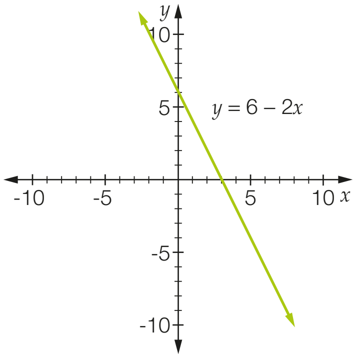
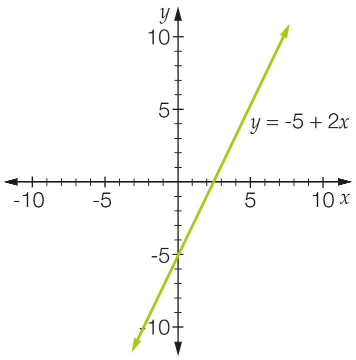
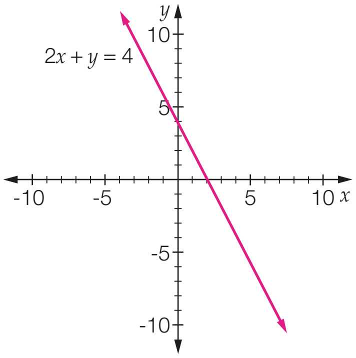


(i)



4 (a)



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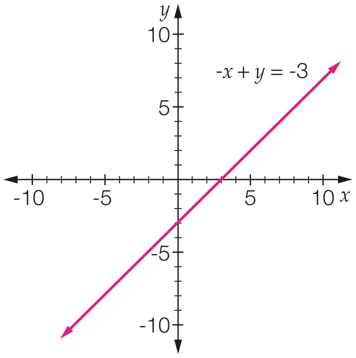

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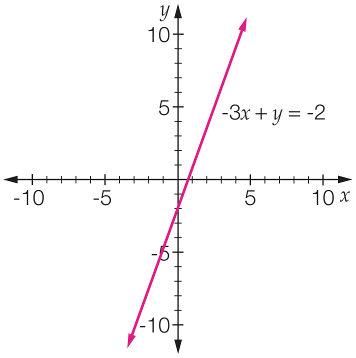

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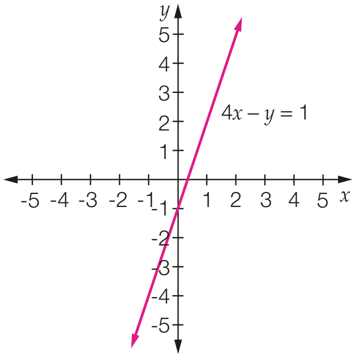

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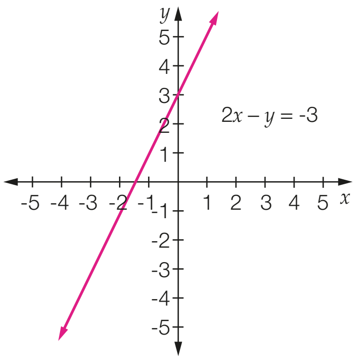

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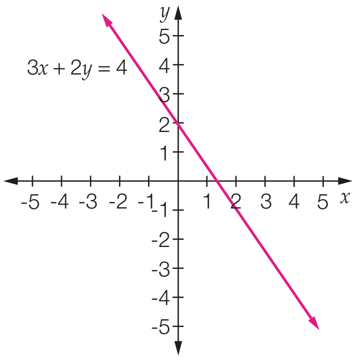
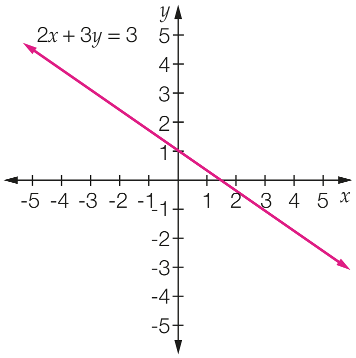


(g)



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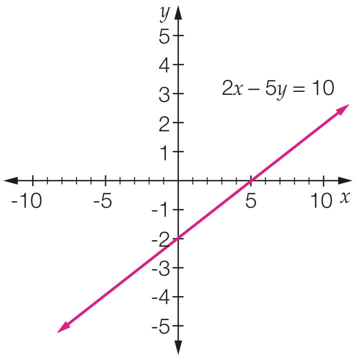
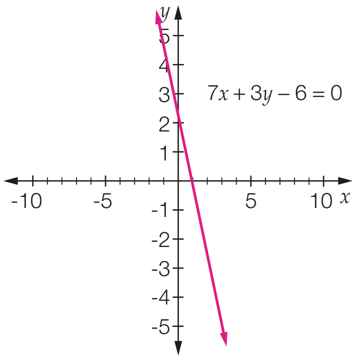


(i)



5 **(a)** $m = 3$ **(b)** $m = -4$ **(c)** $m = 3$

(d) $m = 3$ **(e)** $m = -\frac{2}{5}$ **(f)** $m = \frac{4}{3}$

6 **(a)** $-\frac{1}{3}$ **(b)** $\frac{1}{4}$ **(c)** $-\frac{1}{3}$

(d) $-\frac{1}{3}$ **(e)** $\frac{5}{2}$ **(f)** $-\frac{3}{4}$

7 **(a)** $y = 3x + 4$ **(b)** $y = x + 5$ **(c)** $y = 2x$

(d) $y = 4x$ **(e)** $y = -5x + 1$ **(f)** $y = 3x - 4$

8 **(a)** D **(b)** C

9 **(a)** B **(b)** B **(c)** A **(d)** B

10 **(a)** D **(b)** A **(c)** C

11 **(a)** $-\frac{1}{2}$ **(b)** undefined **(c)** $-\frac{1}{7}$ **(d)** $-\frac{5}{2}$

12 **(a)** B **(b)** D **(c)** A **(d)** F

(e) G **(f)** C **(g)** E

13 **(a)** The lines are horizontal. **(b)** The lines are vertical.

14 **(a)** (0, 50) **(b)** 25 **(c)** $d = 25t + 50$

(d) 25 m/min **(e)** 550 m

15 **(a)** 30 **(b)** 30 km/h **(c)** 20 km

(d) $d = 100t + 90$

16 Comparisons:

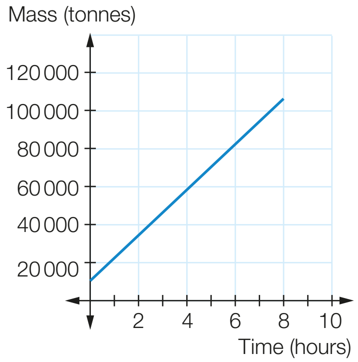
Both Jack and Jill take the same time to go to the shops: they arrive at the supermarket and the milk bar at the same time.

Differences:
The supermarket is 100 metres away from their home. The milk bar is 40 metres away. Therefore Jack goes 100 metres, while Jill goes 40 metres, so Jack goes 60 metres further than Jill.

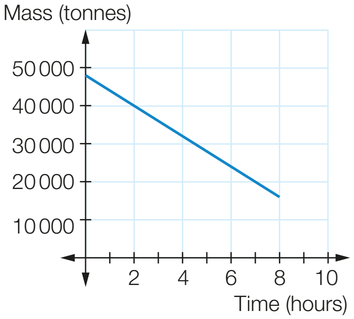
Jack moves at 5 metres per second while Jill moves at 2 metres per second, so Jack is faster than Jill by 3 metres per second.

17 $m_a = -1$, $m_b = \frac{3}{2}$; for m_a and m_b to be perpendicular,
$m_a \times m_b = -1$.
$-\frac{3}{2} \neq -1$, so lines a and b are not perpendicular.

18 **(a)**



(b) 12 000 **(c)** $m = 12\,000t + 10\,000$

(d) 

(e) $m = 4000t$ **(f)** 16 000 tonnes **(g)** 12 hours

19 Ajay is 110 km away from town A, Safiya is 40 km away from town A.

Open-ended—Sample answers

20 **(a)** (0, -4), (10, 6), (15, 11) **(b)** (0, 2), (-5, 0), (10, 6)

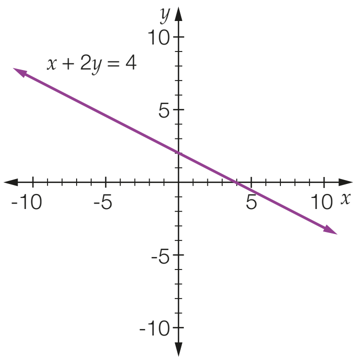
(c) (0, -0.5), (4, 0.5), (2, 0)

21 $y = x$, $y = x + 1$, $y = -x + 2$, $y = -x + 3$

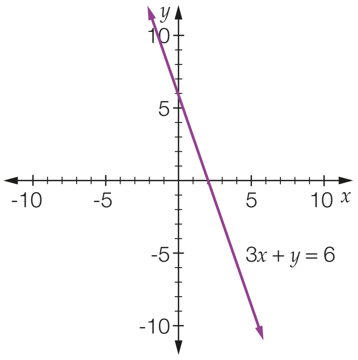
22 **(a)** $y = 4x - 9$ **(b)** $y = -\frac{1}{4}x + 2$

Exercise 5.7

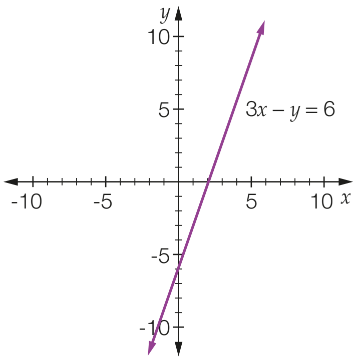
1 **(a)**



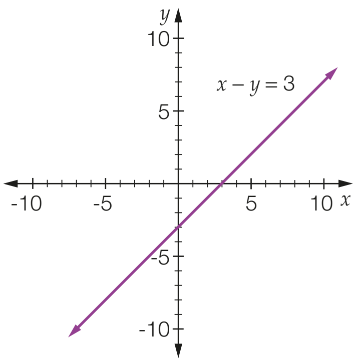
(b)



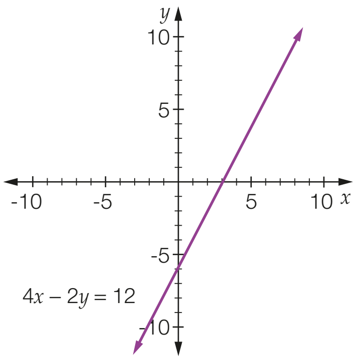
(c)



(d)



(e)



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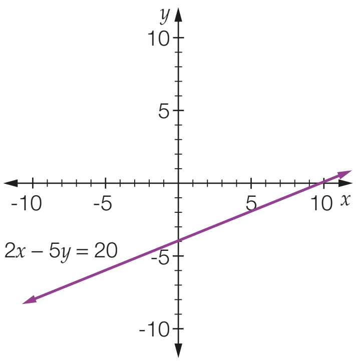

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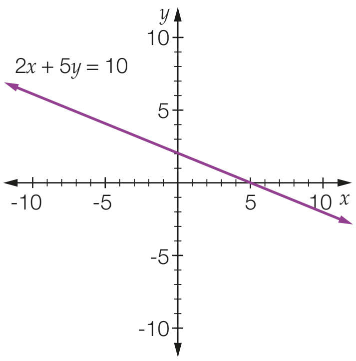

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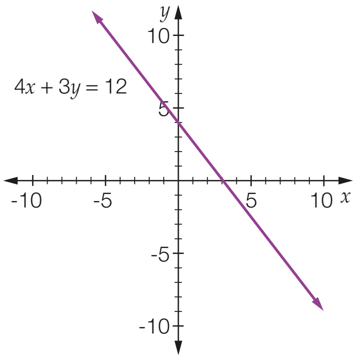

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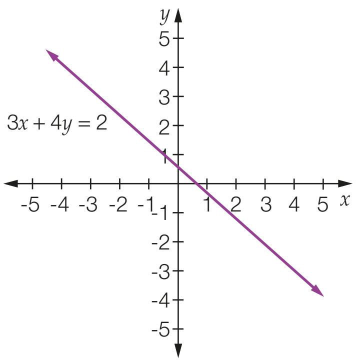
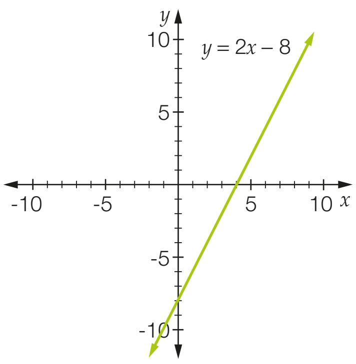
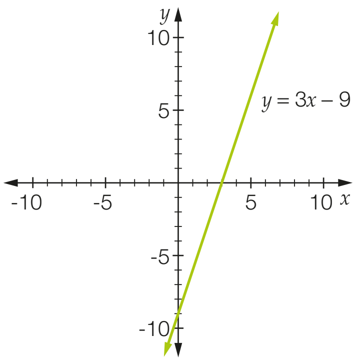


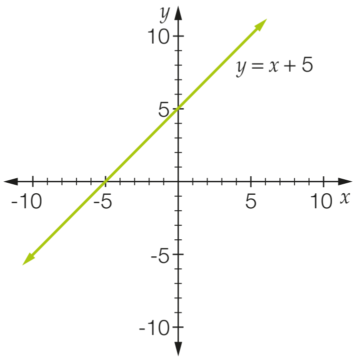
2 (a)



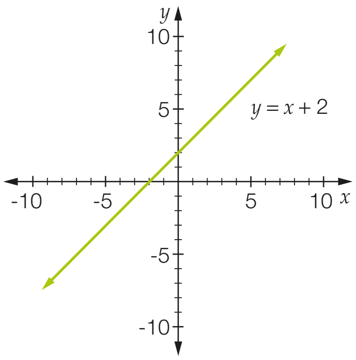
(b)



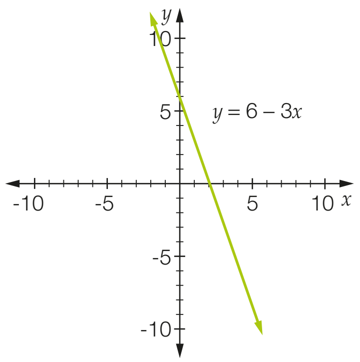
(c)



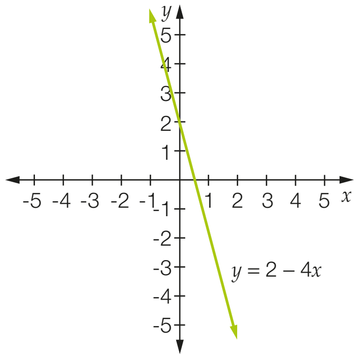
(d)



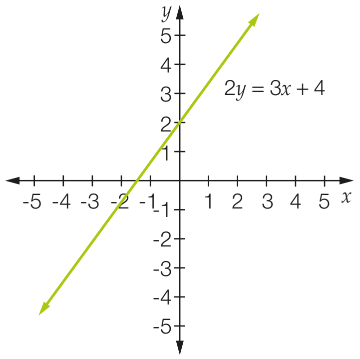
(e)



(f)



(g)



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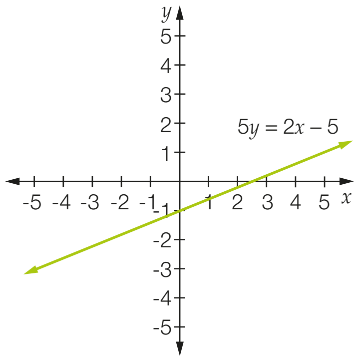

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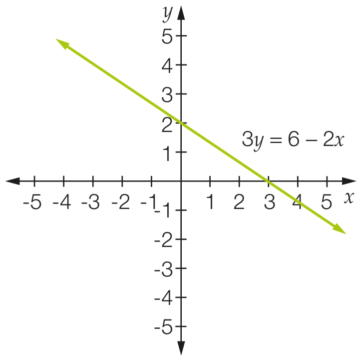
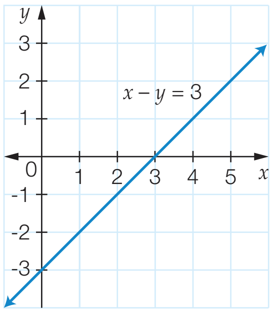


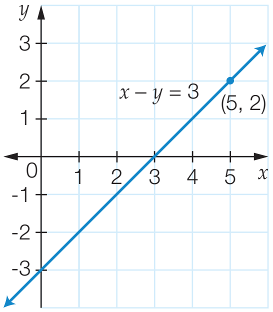
3 **(a)** C **(b)** B

4 **(a)** $x - y = 3$

(b)

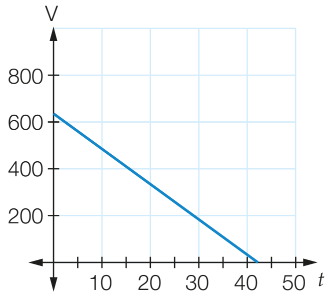


(c)



5 **(a)** 555 L **(b)** 630 L **(c)** 42 min

(d)



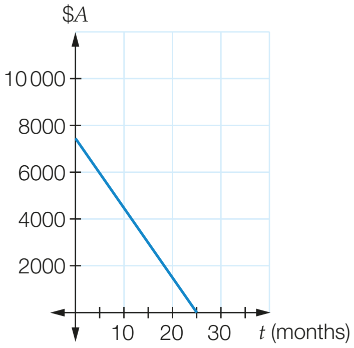
6 **(a)** $b = 32$, $m = \frac{9}{5}$, $y = F$ and $x = C$, so $F = \frac{9}{5}C + 32$

(b) colder, $37\frac{7}{9}$ °C ≈ 37.8 °C

(c) $-17\frac{7}{9}$ °C

7 **(a)** $7500 **(b)** 2 years and 1 month

(c)



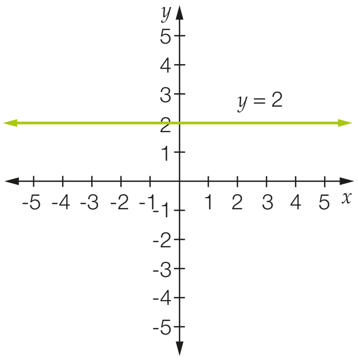
(d) $\$A = 7500 - 300t$ **(e)** $5700 **(f)** $4200

Open-ended—Sample answers

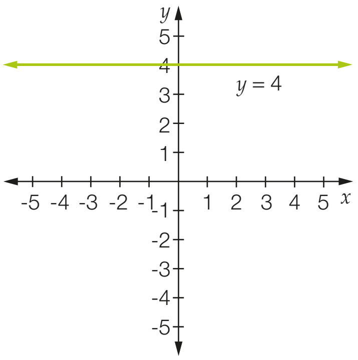
8 (-1, 0) and (0, 3); (1, 0) and (0, -3); (2, 0) and (0, -6)

Exercise 5.8

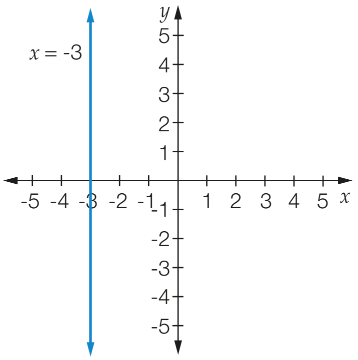
1 **(a)**



(b)



(c)



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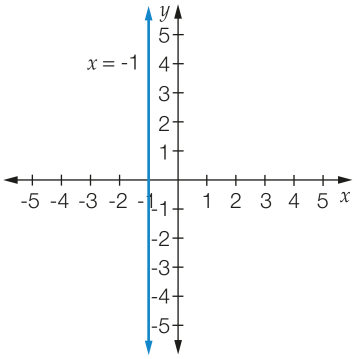
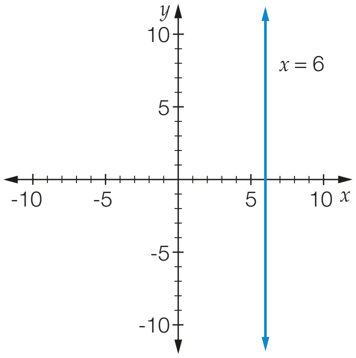
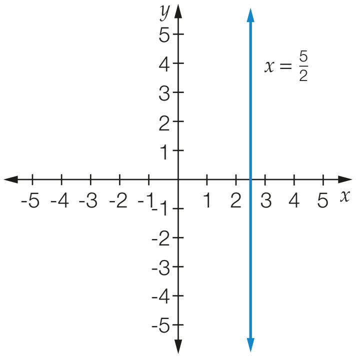


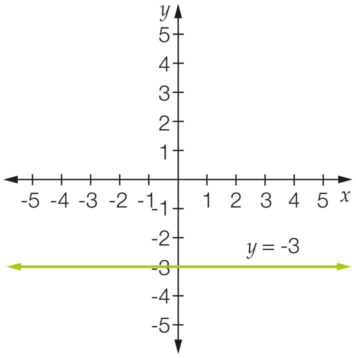
(e)



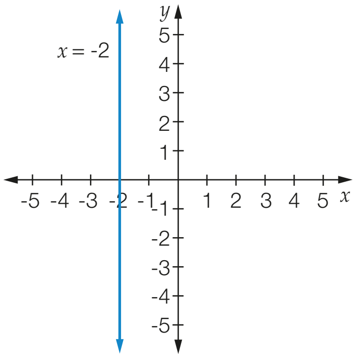
(f)



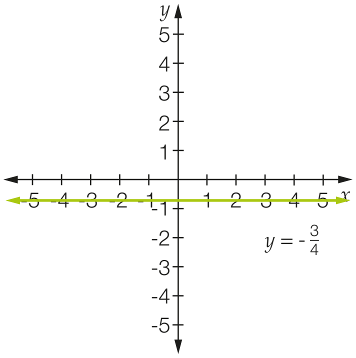
(g)



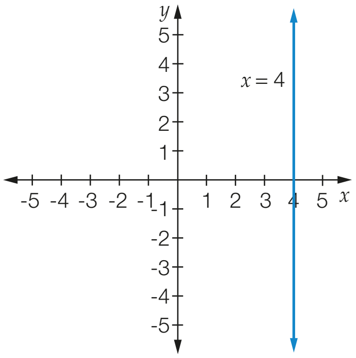
(h)



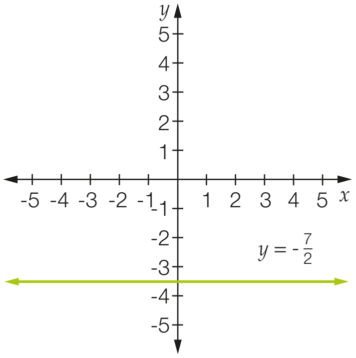
(i)



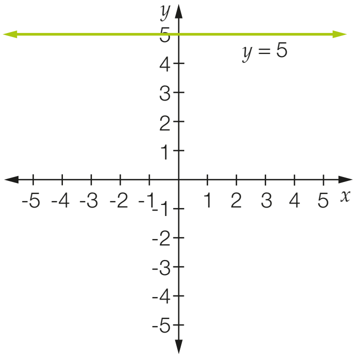
(j)



(k)



(l)



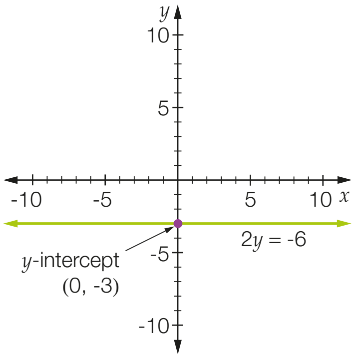
2 Line A: $y = 3$ Line B: $x = 5$

Line C: $x = -2$ Line D: $y = -9$

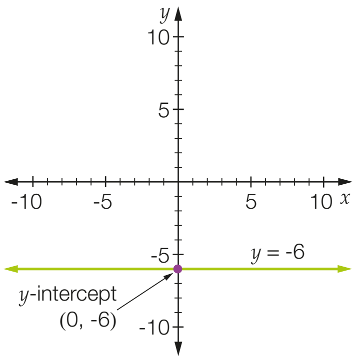
3 **(a)** $y = -6$ **(b)** $y = 2$ **(c)** $y = \frac{2}{3}$ **(d)** $y = \frac{3}{4}$

4 **(a)** B **(b)** B **(c)** C **(d)** D

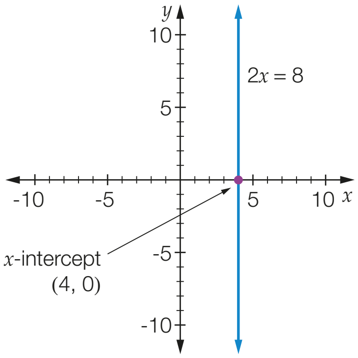
5 **(a)**



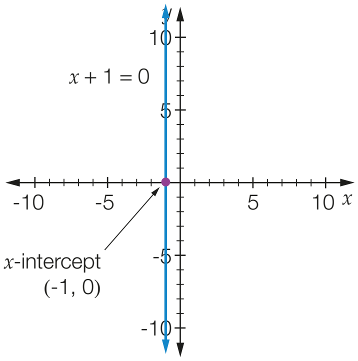
(b)


y
10
5
-10
-5
5
10 x
-5
-10
y = -6
y-intercept
(0, -6)

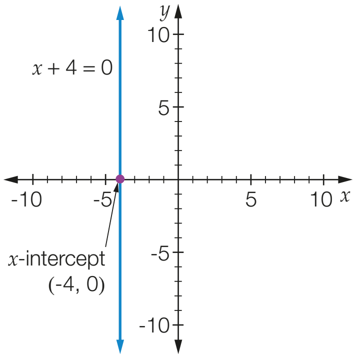
(c)


y
10
5
-10
-5
5
10 x
-5
-10
2x = 8
x-intercept
(4, 0)

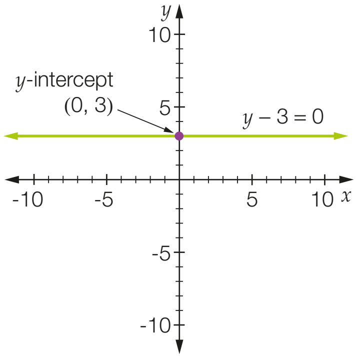
(d)


y
10
5
-10
-5
5
10 x
-5
-10
x + 1 = 0
x-intercept
(-1, 0)

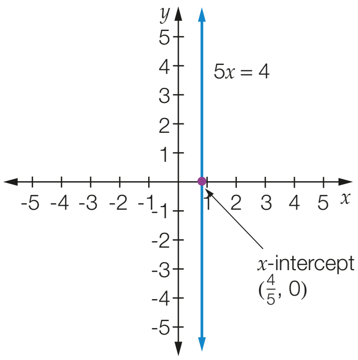
(e)


y
10
5
-10
-5
5
10 x
-5
-10
x + 4 = 0
x-intercept
(-4, 0)

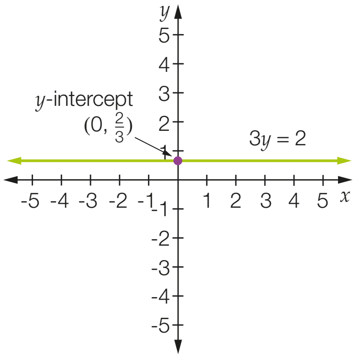
(f)


y
10
5
-10
-5
5
10 x
-5
-10
y – 3 = 0
y-intercept
(0, 3)

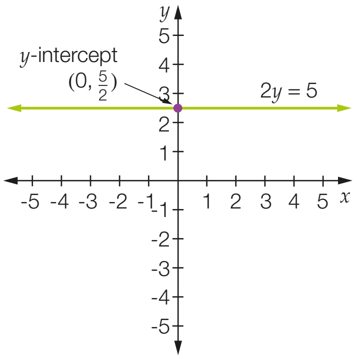
(g)


y
5
4
3
2
1
-1
-2
-3
-4
-5
-5 -4 -3 -2 -1
1 2 3 4 5 x
5x = 4
x-intercept
($\frac{4}{5}$, 0)

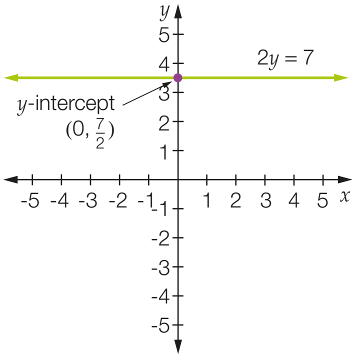
(h)


y
5
4
3
2
1
-1
-2
-3
-4
-5
-5 -4 -3 -2 -1
1 2 3 4 5 x
3y = 2
y-intercept
(0, $\frac{2}{3}$)

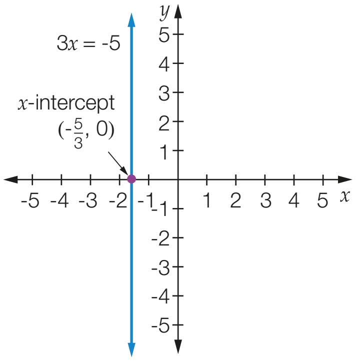
(i)


y
5
4
3
2
1
-1
-2
-3
-4
-5
-5 -4 -3 -2 -1
1 2 3 4 5 x
2y = 5
y-intercept
(0, $\frac{5}{2}$)

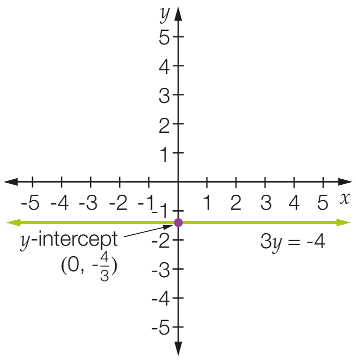
(j)


y
5
4
3
2
1
-1
-2
-3
-4
-5
-5 -4 -3 -2 -1
1 2 3 4 5 x
2y = 7
y-intercept
(0, $\frac{7}{2}$)

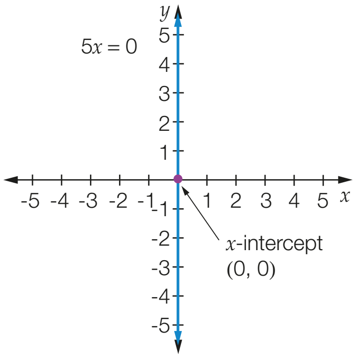
(k)


y
5
4
3
2
1
-1
-2
-3
-4
-5
-5 -4 -3 -2 -1
1 2 3 4 5 x
3x = -5
x-intercept
(-$\frac{5}{3}$, 0)

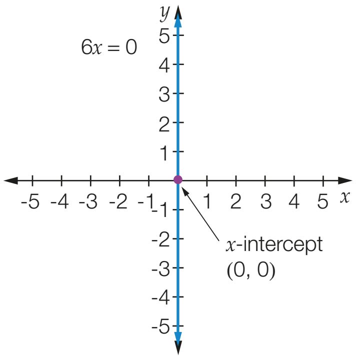
(l)


y
x
-5 -4 -3 -2 -1 1 2 3 4 5
y-intercept
(0, $-\frac{4}{3}$)
3y = -4

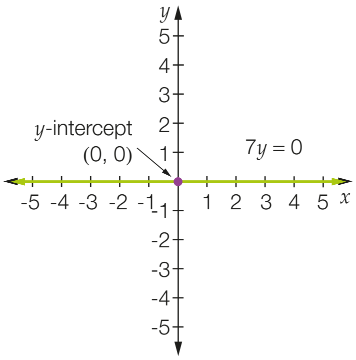
(m)


y
x
5x = 0
x-intercept
(0, 0)

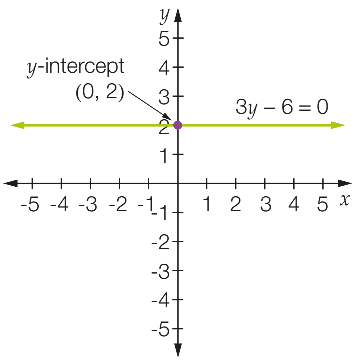
(n)


y
x
6x = 0
x-intercept
(0, 0)

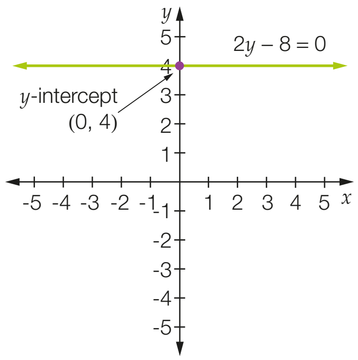
(o)


y
x
y-intercept
(0, 0)
7y = 0

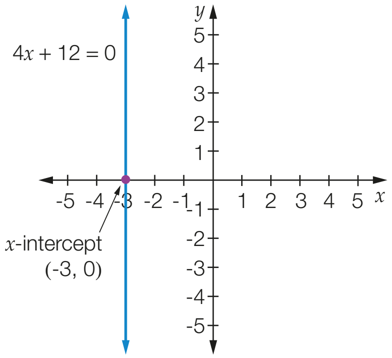
(p)


y
x
y-intercept
(0, 2)
3y – 6 = 0

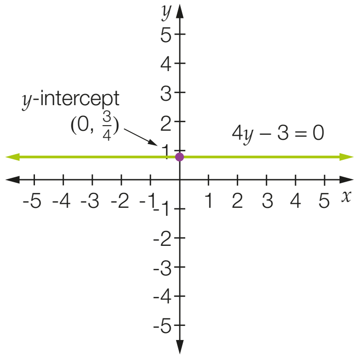
(q)


y
x
2y – 8 = 0
y-intercept
(0, 4)

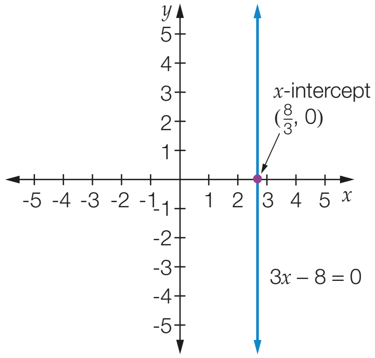
(r)


y
x
4x + 12 = 0
x-intercept
(-3, 0)

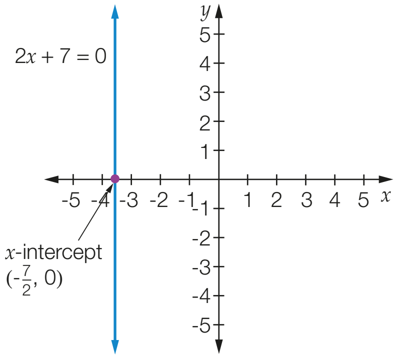
(s)


y
x
y-intercept
(0, $\frac{3}{4}$)
4y – 3 = 0

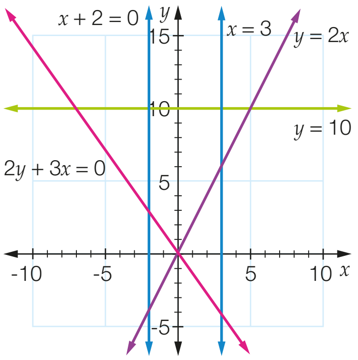
(t)


y
x
x-intercept
($\frac{8}{3}$, 0)
3x – 8 = 0

(u)

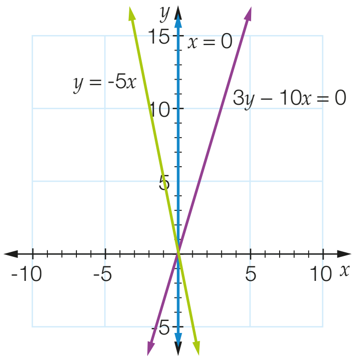

y
x
2x + 7 = 0
x-intercept
($-\frac{7}{2}$, 0)

6 (a)



(b) (0, 0), (3, 6), (3, 10), (-2, 3), (-2, 10), $(-6\frac{2}{3}, 10)$, (5, 10)

(c)



(d) Yes. It is located in the path of the road.

Open ended–Sample answers

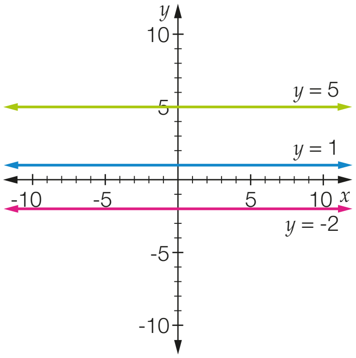
7 (a) Sophie's answer is $16 \times 11 = 176$ square units, which is correct. Henry's rectangle is $16 \times 12 = 192$ square units, which is incorrect.

(b) Explanations may vary, but Sophie could say:

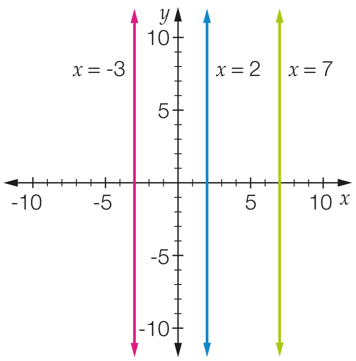
'Move the current line segment with equation $x = -5$, length of line segment $y = -7$ to $y = 9$, to the new equation $x = -4$, length of line segment $y = -7$ to $y = 9$,' or:

'Move the current line segment with equation $x = 7$, length of line segment $y = -7$ to $y = 9$, to the new equation $x = 6$, length of line segment $y = -7$ to $y = 9$.'

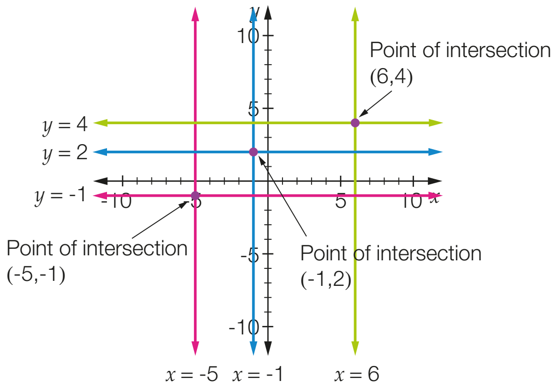
8 $y = -2$, $y = 1$, $y = 5$



9 $x = -3$, $x = 2$, $x = 7$



10



Challenge 5

1 Jesse travels twice as fast as Vinny. He leaves 13 minutes after Vinny, so in the next 13 minutes he will travel twice as far as Vinny has travelled in the 26 minutes since he started. Thus, Jesse will catch up with Vinny at this time: 5:48 pm + 13 min = 6:01 pm

2 Let the bicycle cost $\$x$ per day and the car cost $\$y$ per day.

$2x + 3y = 150$ and $4x + y = 90$
$12x + 3y = 270$
$10x = 120$
$x = 12$ and $y = 42$

The bike costs \$12 per day and the car costs \$42 per day.

3 The equation of the line is $y = 2x + 3$; $w = 17$.

4 D

5 (a) From A to B is across 2 and up 1. From B to A it is back 2 and down 1. Hence, from C to D is back 2 and down 1, so D is (0, 3).

(b) The gradient of $AB = \frac{2-1}{3-1} = \frac{1}{2}$; the gradient of $BC = \frac{4-2}{2-3} = \frac{2}{-1} = -2$. The product of the gradients is -1, i.e. one gradient is the negative reciprocal of the other.

(c) The gradient of $AD = -2$, the gradient of $DC = \frac{1}{2}$. The gradients have a product of -1 and are the same as the gradients of their opposite sides.

(d) The parallel sides of the square have the same gradient.

(e) The product of the gradients is -1, i.e. one gradient is the negative reciprocal of the other.

6 M (-3, 6) and N (-6, 2); or M (5, 0) and N (2, -4).

Either plot the points, or by analysis see that from K to L is 3 across and 4 up. From L, go back 4 and up 3 to give M (-3, 6). From K, go back 4 and up 3 to give N (-6, 2). For the second set of values, from L go across 4 and down 3 to give M (5, 0). From K, go across 4 and down 3 to give N (2, -4). This process is based on the fact that the gradient of the perpendicular sides is the negative reciprocal of the gradient of KL. The hypotenuse of the triangle is still 5, the length of the side.

7 (1, -1)

8 **(a)** $\frac{x-a}{x-b} = 1 - \frac{x}{x-b}, \frac{x-a}{x-b} = \frac{x-b-x}{x-b}, x - a = -b, x = a - b$

(b) $\frac{a}{bx} - \frac{b}{ax} = a^2 - b^2, \frac{a^2-b^2}{abx} = a^2 - b^2, abx = 1, x = \frac{1}{ab}$

(c) $\frac{a-b}{x-c} = \frac{a+b}{x+c}, (a-b)(x+c) = (a+b)(x-c),$
$ax + ac - bx - bc = ax - ac + bx - bc, 2ac = 2bx, x = \frac{ac}{b}$

Chapter review 5

1 **(a)** $x = -5$ **(b)** $x = -1$ **(c)** $x = -45$
(d) $x = 72$ **(e)** $x = 0$ **(f)** $x = \frac{5}{2}$

2 **(a)** $x = 9$ **(b)** $x = 19$ **(c)** $x = \frac{11}{9}$ **(d)** $x = 3$

3 **(a)** $x = \frac{13}{6}$ **(b)** $x = -19$ **(c)** $x = -9$ **(d)** $x = 1$
(e) $x = \frac{3}{2}$ **(f)** $x = \frac{41}{5}$

4 **(a)** B **(b)** C

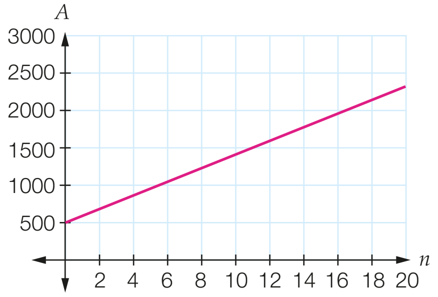
5 **(a)** $p + 4$ **(b)** $p - 11$
(c) $10 = p - 11, p = 21$. Pete had 21 mice originally.

6 **(a)** 10.82 (2 d.p.) **(b)** (1.5, 8)

7 **(a)**

Number of weeks	0	1	2	3	4	5	6
Amount paid ($)	500	590	680	770	860	950	1040

(b)



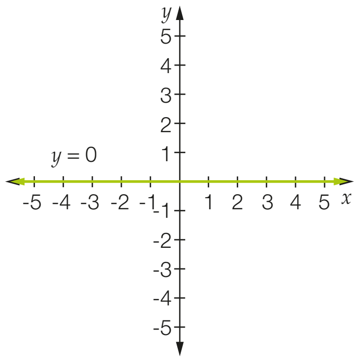
(c) \$1490 **(d)** 15 weeks
(e) $A = 500 + 90n$ **(f)** \$1310

8 5

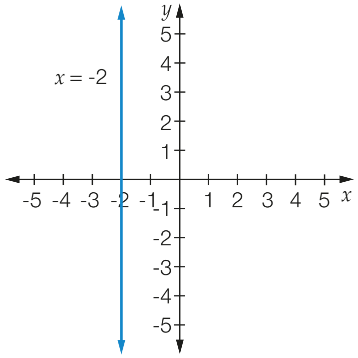
9 **(a)** A **(b)** A

10 gradient = 2, x-intercept = (-2, 0), y-intercept = (0, 4)

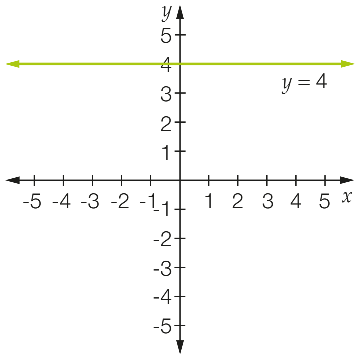
11 **(a)** (-2, 0), (0, 0), (1, 0), (3, 0) or any points that have a y-value of 0



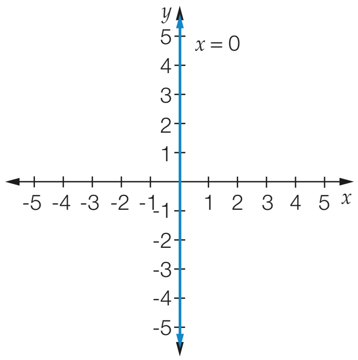
(b) (-2, -5), (-2, -1), (-2, 1), (-2, 4) or any points that have an x-value of -2



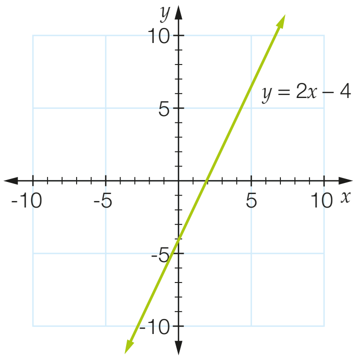
(c) (-2, 4), (0, 4), (1, 4), (3, 4) or any points that have a y-value of 4



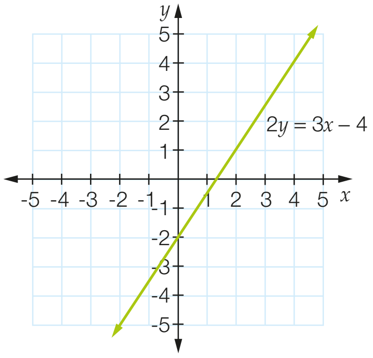
(d) (0, -5), (0, -1), (0, 1), (0, 4) or any points that have an x-value of 0



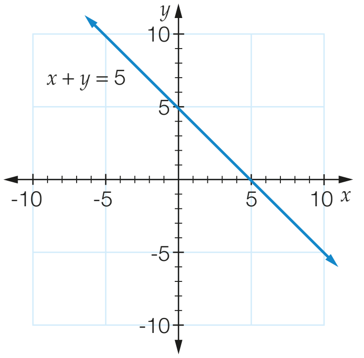
12 **(a)** y-intercept: (0, -4), gradient = 2



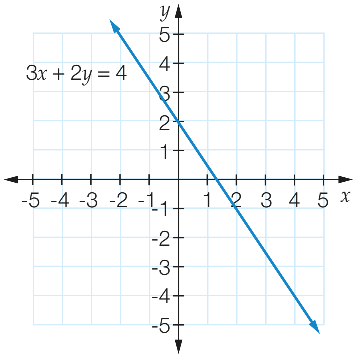
(b) y-intercept: (0, -2), gradient = $\frac{3}{2}$



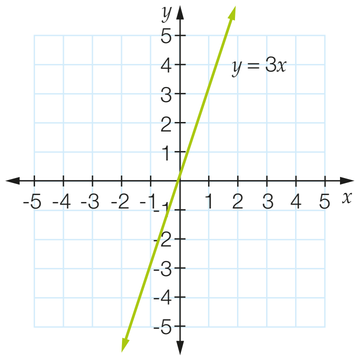
(c) y-intercept: (0, 5), gradient = -1



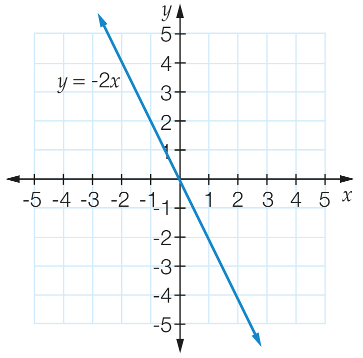
(d) y-intercept: (0, 2), gradient = $-\frac{3}{2}$



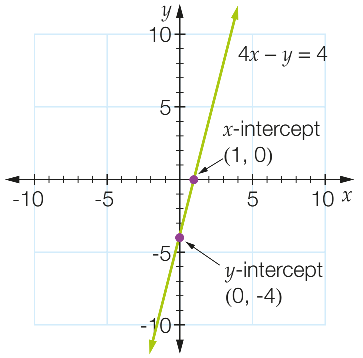
(e) y-intercept: (0, 0), gradient = 3



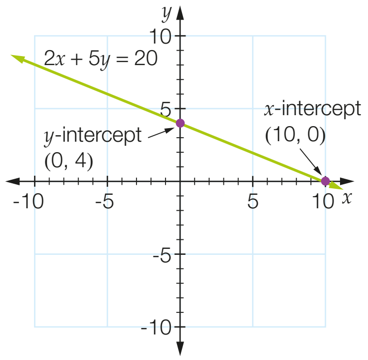
(f) y-intercept: (0, 0), gradient = -2



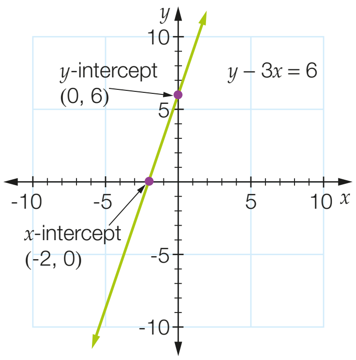
13 **(a)**



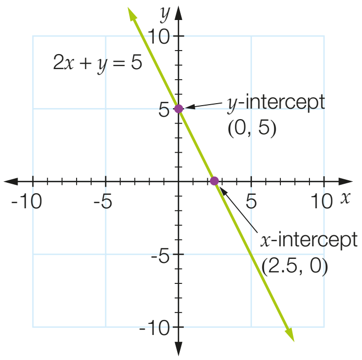
(b)



(c)

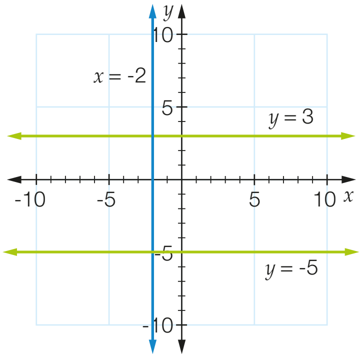


(d)



14 **(a)** D **(b)** C **(c)** D

15



16 **(a)** (0, 0) and (5, 350) **(b)** 70

(c) 70 m/min **(d)** $d = 70t$ **(e)** 560 m

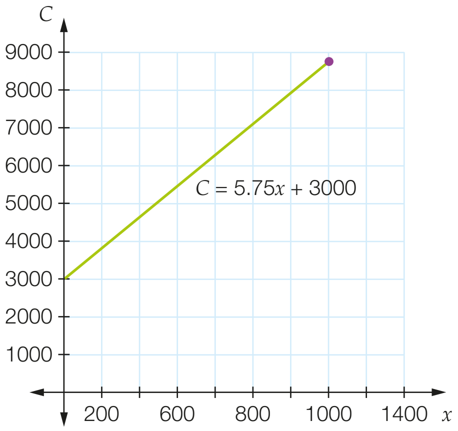
17 **(a)** 21, 22, 23 **(b)** 22, 24, 26, 28

18 $58 = 2(2x - 7) + 2x$, $x = 12$. Therefore the length is 17 m and the width is 12 m.

19 $k = 9$

20 **(a)** $C = 5.75x + 3000$ **(b)** $5875 **(c)** 1043

(d)



The cost of 1000 components is $8750.

21 **(a)** $C = 70 + 105t$ **(b)** $910

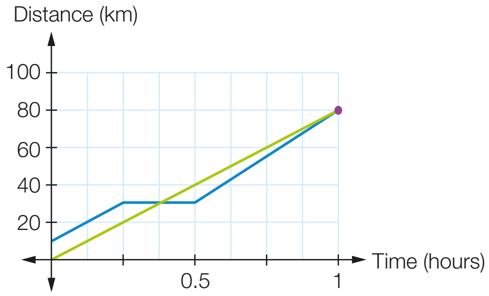
(c) 2 hours 15 minutes

22 **(a)** Students should check graphs carefully.

(b) 80 km **(c)** 80 km/h **(d)** 15 min

(e) 100 km/h **(f)** 70 km

(g)



(h) 80 km/h **(i)** $d = 80t$

(j) 22.5 min after she left **(k)** 50 km from home

Numeracy practice 5

1 C **2** D **3** $b = 7$ **4** B

5 C **6** 125 **7** A **8** 5

9 C **10** 38 cm

Chapter 6

Recall 6

1 Students' own angles.

2 **(a)** acute **(b)** obtuse **(c)** reflex **(d)** reflex

3 **(a)** 35° **(b)** 245°

4 D

5 $a = 46°$, $b = 46°$, $c = 46°$

6 **(a)** 173 **(b)** 64 **(c)** 70

7 **(a)** 4.8 m **(b)** 1.5 m

8 **(a)** 6 : 1 **(b)** 3.5 : 1 **(c)** 21.88 : 1 **(d)** 686.17 : 1

9 4 cups

Exercise 6.1

1 (a) vertically opposite (b) co-interior

(c) alternate (d) supplementary

(e) corresponding

2 (a) $y = 60^\circ$ (co-interior angle with 120° between parallel lines)

(b) $w = 97^\circ$ (vertically opposite angle to 97°)

(c) $p = 53^\circ$ (alternate angle to 53° on parallel lines)

(d) $m = 148^\circ$ (corresponding angle to 148° on parallel lines)

$n = 32^\circ$ (supplementary angle to m)

3 (a) $c = 43^\circ$ (supplementary angle to 137°); $d = 43^\circ$ (corresponding angle to c on parallel lines)

(b) $h = 80^\circ$ (co-interior angle between parallel lines with 100°)

(c) $a = 38^\circ$ (alternate angles on parallel lines); $b = 82^\circ$ (corresponding angles on parallel lines)

(d) $x = 56^\circ$ (co-interior angle between parallel lines with 124°); $y = 154^\circ$ (co-interior angle between parallel lines with 26°)

(e) $m = 33^\circ$ (vertically opposite angle to 33°); $n = 147^\circ$ (co-interior angle between parallel lines with m)

(f) $f = 125^\circ$ (straight angle); $g = 125^\circ$ (corresponding angles on parallel lines); $h = 70^\circ$ (alternate angles on parallel lines)

(g) $a = 322^\circ$ (makes an angle of revolution with b and c); $b = 18^\circ$ (alternate angle to 18° on parallel lines); $c = 20^\circ$ (co-interior angle between parallel lines with 160°)

(h) $d = 41^\circ$ (supplementary angle to 41°); $f = 128^\circ$ (exterior angle)

(i) $r = 94^\circ$ (supplementary angle to 41°); $s = 25^\circ$ (alternate angle to 25° between parallel lines)

(j) $w = 25^\circ$ (straight angle); $x = 85^\circ$ (vertically opposite angle); $y = 25^\circ$ (alternate angles on parallel lines); $z = 110^\circ$ (exterior angle equals sum of two interior opposite angles)

4 (a) $x = 20^\circ$ ($6x$ and $3x$ are co-interior angles between parallel lines.)

(b) $w = 76^\circ$ (corresponding angles)

(c) $x = 50^\circ$ (supplementary angles)

(d) $m = 30^\circ$ (angles in a revolution)

(e) $a = 47^\circ$ (co-interior angles between parallel lines); $b = 94^\circ$ (alternate angles on parallel lines)

(f) $a = 90^\circ$ (corresponding angles on parallel lines); $b = 117^\circ$ (corresponding angles on parallel lines); $c = 63^\circ$ (straight angle); $d = 153^\circ$ (straight angle)

5 B

6 (a) $a = 30^\circ$ (angles on a straight line)
$b = 30^\circ$ (alternate angles on parallel lines)

(b) $b = 150^\circ$ (alternate angles on parallel lines)

(c) $x = 100^\circ$ (co-interior angles between parallel lines)
$y = 30^\circ$ (alternate angles on parallel lines)

(d) $a = 50^\circ$ (co-interior angles on parallel lines); $2b + 40 = 130$, $b = 45^\circ$ (two opposite interior angles equal to exterior angle of triangle)

(e) $m = 40$ (two opposite interior angles equal to exterior angle of triangle); $n = 40^\circ$ (alternate angles on parallel lines); $r = 140^\circ$ (straight angle)

(f) $x = 36^\circ$ (straight angle); $y = 144^\circ$ (corresponding angles on parallel lines)

7 (a) false (b) false (c) true (d) false

8 $x = 102^\circ$

9 $p = 75^\circ$, equal angles of an isosceles triangle; $r = 75^\circ$, corresponding angles on parallel lines

10 (a) $x = 30^\circ$; $y = 50^\circ$ (b) $a = 75^\circ$; $b = 50^\circ$; $c = 55^\circ$

(c) $a = 15^\circ$; $b = 20^\circ$

11 $y = 5x + 45$, alternate angles on parallel lines, hence $9x + 45 = 180^\circ$, $x = 15^\circ$, sum of angles in a triangle; $y = 5 \times 15 + 45 = 120^\circ$; $z = 60^\circ$, sum of angles in a triangle

12 (a) $3x = 40 + (x + 30)$, vertically opposite angles; $2x = 70$, $x = 35^\circ$

(b) $(2y - 40) + 40 = 180$, co-interior angles supplementary on parallel lines; $2y = 180$, $y = 90^\circ$

(c) $65 + (90 + z - 35) = 180$, co-interior angles supplementary on parallel lines; $120 + z = 180$; $z = 60^\circ$

Open-ended—Sample answers

13 y creates a straight angle with 2 angles. These both have the same value as x; one because it is an equal angle to x in an isosceles triangle, the other because it is an alternate angle on parallel lines to x. Therefore, $2x + y = 180$. If $x = 40^\circ$, $y = 100^\circ$. If $x = 50^\circ$, $y = 80^\circ$. If $x = 70^\circ$, $y = 40^\circ$.

14 (a) x and y must sum to 180°. For example, $x = 100^\circ$ and $y = 80^\circ$.

(b) m and n must be equal. For example, $m = n = 120^\circ$.

(c) $t = 180 - y - (180 - n)$, $t = -y + n$

(d) $t = -(180 - x) + m$ (e) $t = -80 + 120 = 40^\circ$

(f) $t = -(180 - 100) + 120 = 40^\circ$ (g) yes

Exercise 6.2

1 (a) SSS (b) SSS (c) RHS

(d) SAS (e) SAS (f) SSS

(g) RHS (h) ASA (i) ASA

2 (a) $\angle STO = \angle PRO$ (alternate angles, $ST \parallel RP$)
$\angle SOT = \angle POR$ (vertically opposite angles)
$ST = PR = 10$ cm (given)
$\Delta STO \equiv \Delta PRO$ (ASA)

(b) $TO = RO$ (matching sides in congruent triangles)
$x = 6$ cm

3 (a) In ΔEFH and ΔEGH:

$FH = GH$ (given)

$\angle FHE = \angle GHE$ (both 90°)

EH is a common side.

$\therefore \Delta EFH \equiv \Delta EGH$ (SAS)

$\therefore \angle EFH = \angle EGH$ (matching angles in congruent triangles)

(b) ΔEFG is isosceles because it has two equal angles, $\angle EFH = \angle EGH$.

(c) If the perpendicular bisector of a side of a triangle passes through the opposite vertex, then the triangle is isosceles with the other two sides equal in length.

(d) $EF = EG$ because they are matching sides in the congruent triangles EFH and EGH.

4 $\Delta MNO \equiv \Delta DFE$ (SSS), $\Delta ABC \equiv \Delta JKL$ (SAS), $\Delta GHI \equiv \Delta RQP$ (ASA)

5 (a) In ΔXYZ and ΔSRT, $XY = SR$ (both 17 m)

$\angle XYZ = \angle SRT$ (both 90°)

$YZ = RT$ (both 12 m)

$\therefore \Delta XYZ \equiv \Delta SRT$ (SAS)

(b) In ΔABC and ΔRQS, $\angle ABC = \angle RQS$ (both 90°)

$AB = RQ$ (both 11 cm)

$AC = RS$ (both 15 cm)

$\therefore \Delta ABC \equiv \Delta RQS$ (RHS)

(c) In ΔKLM and ΔQSR,

$\angle LKM = \angle SQR$ (both 50°)

$\angle LMK = \angle SRQ$ (both 30°)

$KM = QR$ (both 32 cm)

$\therefore \Delta KLM \equiv \Delta QSR$ (ASA)

(d) In ΔFGH and ΔPQR,

$\angle FGH = \angle PQR$ (both 80°)

$\angle FHG = \angle PRQ$ (both 70°)

$GF = QP$ (both 5 cm)

$\therefore \Delta FGH \equiv \Delta PQR$ (ASA)

(e) In ΔDEF and ΔXVW, $\angle DFE = \angle XWV$ (both 90°)

$DE = \mathrm{XV}$ (both 16 cm)

$FE = WV$ (both 7 cm)

$\therefore \Delta DEF \equiv \Delta XVW$ (RHS)

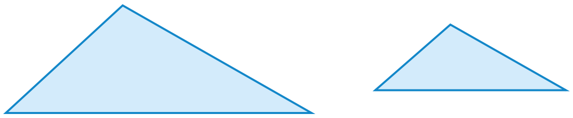
(f) In ΔDEF and ΔXZY, $DE = XZ$ (both 6 cm)

$EF = ZY$ (both 6 cm)

$DF = XY$ (both 8 cm)

$\therefore \Delta DEF \equiv \Delta XZY$ (SSS)

6 Two triangles with three equal angles are equiangular and so are similar. To be congruent they must have at least one pair of matching sides equal. The following triangles are equiangular but not congruent.



7 Proof: In ΔABE and ΔDCE,

$\angle ABE = \angle DCE$ (alternate angles, $AB \parallel CD$)

$BE = CE$ (given)

$\angle AEB = \angle DEC$ (vertically opposite angles)

$\therefore \Delta ABE \equiv \Delta DCE$ (ASA congruency)

$\therefore AB = CD$ (matching sides of congruent triangles)

8 (a) (i) SAS (ii) SAS (iii) ASA

(iv) SAS (v) ASA (vi) RHS

(b) (i) $c = 35°$ (ii) $p = 38°$ (iii) $x = 15$ cm

(iv) $g = 5$ cm (v) $m = 75°$ (vi) $x = 50$ mm

9 In ΔTUR and ΔTSR, $\angle TUR = \angle TSR$ (both 90°)

TR is a common side.

$TU = TS$ (given)

$\therefore \Delta TUR \equiv \Delta TSR$ (RHS)

10 In ΔSQP and ΔRPQ,

$SP = RQ$ (given)

$\angle SPQ = \angle RQP$ (given)

PQ is a common side.

$\therefore \Delta SQP \equiv \Delta RPQ$ (SAS)

11 (a) In ΔABC and ΔCDA, $AB = CD$ (given)

$BC = DA$ (given)

AC is a common side.

$\therefore \Delta ABC \equiv \Delta CDA$ (SSS)

$x = 32°$ (matching angles in congruent triangles)

(b) In ΔADC and ΔABC, $\angle ADC = \angle ABC = 35°$ (given)

$\angle ACD = \angle ACB = 120°$ (given)

AC is a common side.

$\therefore \Delta ADC \equiv \Delta ABC$ (ASA)

$b = 23$ cm (matching sides in congruent triangles)

(c) In ΔABE and ΔCDE, $\angle ABE = \angle CDE = 56°$ (given)

$\angle AEB = \angle CED$ (vertically opposite angles)

$AB = CD$ (given)

$\therefore \Delta ABE \equiv \Delta CDE$ (ASA)

$x = 23$ m (matching sides in congruent triangles)

(d) In ΔOAB and ΔOCB, $\angle OBA = \angle OBC = 90°$

$OA = OC$ (radii)

OB is a common side.

$\therefore \Delta OAB \equiv \Delta OCB$ (RHS)

$y = 26°$ (matching angles in congruent triangles)

(e) In ΔABO and ΔCDO, $AB = CD$ (given)

$AO = CO$ (radii)

$BO = DO$ (radii)

$\therefore \Delta ABO \equiv \Delta CDO$ (SSS)

$y = 9$ m (matching sides in congruent triangles)

(f) In ΔABE and ΔDCE,

$\angle BAE = \angle CDE = 75°$ (given)

$AB = DC$ (given)

$\angle AEB = \angle DEC$ (vertically opposite angles)

$\therefore \Delta ABE \equiv \Delta DCE$ (ASA)

$k = 8$ cm (matching sides in congruent triangles)

12 (a) In ΔDEG and ΔDFG,

$DE = DF$ (given)

$EG = FG$ (G is the midpoint of EF)

GD is a common side.

$\therefore \Delta DEG \equiv \Delta DFG$ (SSS)

$\therefore \angle DGE = \angle DGF$ (matching angles of congruent triangles)

But $\angle DGE + \angle DGF = 180°$, therefore $\angle DGE = 90°$ and hence $DG \perp EF$.

(b) If ΔDEF is equilateral, then all interior angles are 60°.

Because $\Delta DEG \equiv \Delta DFG$ (SSS)

then $\angle EDG = \angle FDG = 60 \div 2 = 30°$

13 In ΔKLN and ΔKMN,

$KL = KM$ (given)

$\angle LKN = \angle MKN$ (given)

KN is a common side.

$\therefore \Delta KLN \equiv \Delta KMN$ (SAS)

$\therefore LN = NM$ (matching sides of congruent triangles)

$\angle KNL = \angle KNM$ (matching angles of congruent triangles)

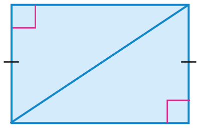
But $\angle KNL + \angle KNM = 180°$, hence $KN \perp LM$.

Therefore KN is the perpendicular bisector of LM.

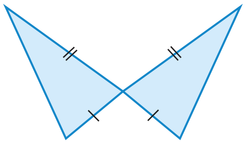
14 In ΔQPS and ΔRPS there are only two pairs of equal sides. To prove congruence, either another pair of sides needs to be equal, the angles between the equal sides need to be equal, or PS needs to be perpendicular to QR. The equal angles that are given are not included by the sides.

Open-ended—Sample answers

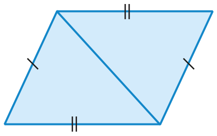
15



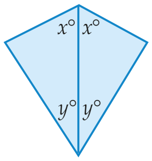
RHS



SAS

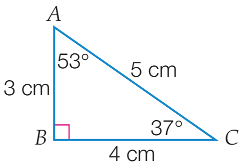


SSS

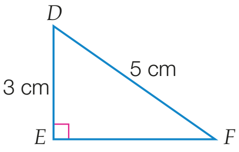


ASA

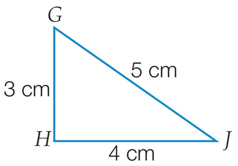
16



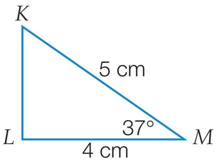
Original ΔABC



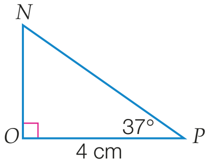
RHS



SSS



SAS



ASA

Exercise 6.3

1 (a) $x = 36°$ **(b)** $y = 15°, z = 110°$ **(c)** $w = 27°$

2 (a) Join DB. In ΔDAB and ΔBCD,

$\angle ABD = \angle CDB$ (alternate angles, $AB \parallel DC$)

DB is a common side.

$\angle ADB = \angle CBD$ (alternate angles, $AD \parallel BC$)

$\Delta DAB \equiv \Delta DCB$ (ASA)

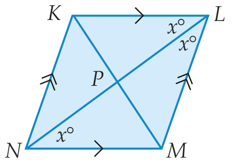
$\angle DAB = \angle BCD$ (matching angles in congruent triangles)

(b) $AB = DC$ (matching sides in congruent triangles); $AD = BC$ (matching sides in congruent triangles)

(c) If the diagonal AC is drawn, then the same steps as in **(a)** can be used to prove that $\Delta ADC \equiv \Delta CBD$. $\angle ADC = \angle CBA$ (matching angles in congruent triangles).

(d) The opposite angles of a parallelogram are equal. The opposite sides of a parallelogram are equal.

3 **(a)** Let $\angle KLN = x^\circ$ and mark other angles that are equal to x° on the diagram.



$\angle LNM = \angle KLN = x^\circ$ (alternate angles, $KL \parallel NM$)

$LM = MN$ (adjacent sides of a rhombus)

ΔLMN is isosceles

$\angle MLN = \angle LNM = x^\circ$ (equal angles of an isosceles triangle)

$\angle KLN = \angle MLN$ (both equal x°)

(b) In ΔKLP and ΔMLP,

$KL = LM$ (adjacent sides of a rhombus)

$\angle KLP = \angle MLP$ (proved in **(a)**)

LP is a common side.

$\Delta KLP \equiv \Delta MLP$ (SAS)

(c) $\angle KPL = \angle MPL$ (matching angles in congruent triangles)

$\angle KPL + \angle MPL = 180^\circ$ ($\angle KPM = 180^\circ$)

$\angle KPL = \angle MPL = 90^\circ$

(d) $KP = MP$ (matching sides in congruent triangles)

(e) The diagonals of a rhombus bisect each other at right angles and bisect the angles of the rhombus.

4 $PT = TR$, $QT = TS$, $\angle PTQ = \angle QTR = \angle RTS = \angle STP = 90^\circ$

In ΔPTQ and ΔQTR, $PT = TR$ (given)

$\angle PTQ = \angle QTR = 90^\circ$ (given)

QT is a common side.

$\Delta PTQ \equiv \Delta QTR$ (SAS)

$PQ = QR$ (matching sides in congruent triangles)

In ΔPTQ and ΔRTS, $PT = TR$ (given)

$\angle PTQ = \angle RTS = 90^\circ$ (vertically opposite angles)

$QT = TS$ (given)

$\Delta PTQ \equiv \Delta RTS$ (SAS)

$\angle PQT = \angle RST$ (matching angles in congruent triangles)

$\angle PQS$ and $\angle RSQ$ are a pair of equal alternate angles contained between PQ and SR.

$PQ \parallel SR$

Repeating these steps in ΔPTS and ΔRTQ allows you to prove that $PS \parallel QR$.

Hence, $PQRS$ is a parallelogram with a pair of adjacent sides equal, so $PQRS$ is a rhombus.

5 C

6 **(a)** **(i)** parallelogram **(ii)** $y = 122^\circ$; $x = 58^\circ$; $z = 58^\circ$

(b) **(i)** quadrilateral **(ii)** $a = 50^\circ$

(c) **(i)** rhombus **(ii)** $m = 150^\circ$; $n = 75^\circ$

7 **(a)** $x = 21^\circ$ **(b)** $w = 124^\circ$, $y = 33^\circ$, $z = 40^\circ$ **(c)** $u = 115^\circ$

8 **(a)** false **(b)** true **(c)** false **(d)** true

9 hexagon **10** A **11** B

12 **(a)** 1080° **(b)** 135°

13 **(a)** Proof: In ΔADC and ΔCBA,
$\angle ACD = \angle CAB$ (alternate angles, $AB \parallel DC$)
$\angle DAC = \angle BCA$ (alternate angles, $AD \parallel BC$)
AC is a common side.

$\therefore\ \Delta ADC \equiv \Delta CBA$ (ASA)

$\therefore\ \angle ADC = \angle CBA$ (matching angles in congruent triangles)

(b) Repeat **(a)** in ΔABD and ΔCDB to prove that they are congruent. Hence, $\angle DAB = \angle BCD$ (matching angles in congruent triangles).

14 **(a)** $\angle ADC = \angle CBA$ and $\angle DAB = \angle BCD$, hence the opposite angles of a parallelogram are equal.

(b) Because $\Delta ADC \equiv \Delta CBA$, then $AB = DC$ (matching sides) and $AD = BC$ (matching sides). Hence the opposite sides of a parallelogram are equal.

(c) In ΔAEB and ΔCED,

$\angle ABE = \angle CDE$ (alternate angles, $AB \parallel DC$)

$\angle AEB = \angle CED$ (vertically opposite angles)

$AB = DC$ (opposite sides of a parallelogram are equal, proved above)

$\Delta AEB \equiv \Delta CED$ (ASA)

(d) $AE =$ EC (matching sides of congruent triangles);
$BE = ED$ (matching sides of congruent triangles).
Hence, AC and BD bisect each other at E.

15 **(a)** Proof: In ΔPRS and ΔQSR,
$\angle PSR = \angle QRS = 90^\circ$ (given)

$PQ = SR$ (given)

$PS = QR$ (given)

$\therefore\ \Delta PRS \equiv \Delta QSR$ (SAS)

(b) $PR = QS$ (matching sides of congruent triangles)

(c) The diagonals of a rectangle are equal.

16 **(a)** In ΔKLM and ΔKNM, $KL = KN$ (given)

$LM = NM$ (given)

KM is a common side.

$\Delta KLM \equiv \Delta KNM$ (SSS)

Hence, $\angle KLM = \angle KNM$ (matching angles of congruent triangles)

(b) In ΔLKX and ΔNKX, $KL = KN$ (given)

$\angle LKX = \angle NKX$ (matching angles of congruent triangles KLM and KNM)

KX is a common side.

$\Delta LKX \equiv \Delta NKX$ (SAS)

(c) $\angle LXK = \angle NXK$ (matching angles of congruent triangles), but, $\angle LXK + \angle NXK = 180°$ (LXN is a straight angle). Hence, $\angle LXK = \angle NXK = 90°$.

(d) One pair of opposite angles of a kite are equal; these are the angles between the unequal sides. The diagonals of a kite intersect at right angles.

17 (a) In ΔWYX and ΔWYZ, $WX = WZ$ (given)

$XY = ZY$ (given)

WY is a common side.

$\Delta WYX \equiv \Delta WYZ$ (SSS)

$\angle WYX = \angle WYZ$ (matching angles of congruent triangles)

(b) Repeat **(a)** for ΔWZX and ΔYXZ, then $\angle WZX = \angle YZX$ (matching angles of congruent triangles).

(c) The diagonals of a rhombus bisect the angles of the rhombus.

18 (a) In ΔACD and ΔACB, $AD = BC$ (given)

$DC = AB$ (given)

AC is a common side.

$\Delta ACD \equiv \Delta ACB$ (SSS)

$\angle ACD = \angle ACB$ (matching angles of congruent triangles)

(b) Because $\angle DCB = 90°$, $\angle ACD = \angle ACB = 45°$.

(c) The diagonals of a square bisect the angles of the square.

19 (a) In ΔKLM and ΔLKN, $LM = KN$ (given)

$\angle KLM = \angle LKN = 90°$ (given)

KL is a common side.

$\Delta KLM \equiv \Delta LKN$ (SAS)

$KM = NL$ (matching sides of congruent triangles)

(b) ΔLKN is a right-angled isosceles triangle so $\angle KLN = \angle KNL = 45°$

ΔLMN is a right-angled isosceles triangle so $\angle MLN = \angle MNL = 45°$

Hence $\angle KLO = \angle LKO = 45°$

ΔKLO is isosceles so $KO = OL$.

Similarly you can show that ΔMLO is isosceles and $OL = OM$.

Hence $KO = OM$ (both equal OL)

As ΔKNO is isosceles, $KO = NO$, so $LO = ON$ (both equal KO)

(c) In ΔKLO since $\angle KLO = \angle LKO = 45°$ then $\angle KOL = 90°$ (angle sum of ΔKLO is 180°)

(d) The diagonals of a square bisect each other at right angles and bisect the angles of the square.

20 In ΔUXW and ΔWVU,

$UX = WV$ (given)

$\angle XUW = \angle VWU$ (alternate angles, $UX \parallel VW$)

UW is a common side.

$\Delta UXW \equiv \Delta WVU$ (SAS)

Hence $\angle XWU = \angle VUW$ (matching angles of congruent triangles)

But these are a pair of equal alternate angles between XW and UV, so $XW \parallel UV$.

Hence $UX \parallel VW$ and $XW \parallel UV$, so $UVWX$ is a parallelogram.

21 In ΔABC and ΔADC, $AB = AD$ (given)

$BC = DC$ (given)

AC is a common side.

$\Delta ABC \equiv \Delta ADC$ (SSS)

Hence, $\angle ADC = \angle ABC = 90°$ (matching angles of congruent triangles). Because ΔABC is right-angled and isosceles, then $\angle BAC = \angle BCA = 45°$. Similarly, $\angle DAC = \angle DCA = 45°$. Hence, $\angle DAB = \angle DCB = 90°$. Now $\angle DAB$ and $\angle ADC$ are a pair of supplementary co-interior angles between AB and DC so $AB \parallel DC$, and $\angle DAB$ and $\angle ABC$ are a pair of supplementary co-interior angles between AD and BC. Thus, $AD \parallel BC$. Hence, $ABCD$ is a parallelogram with a pair of adjacent sides equal, so $ABCD$ is a rhombus. But $ABCD$ is a rhombus with one angle a right angle, so $ABCD$ is a square.

22 In ΔEFH and ΔGFH, $EF = HG$ (given)

$EH = GF$ (given)

FH is a common side.

$\Delta EFH \equiv \Delta GFH$ (SSS)

Hence $\angle EHF = \angle GFH$ (matching angles of congruent triangles). But these are a pair of equal alternate angles between EH and FG, so $EH \parallel FG$. Similarly, $\angle DFH = \angle GHF$ (matching angles of congruent triangles). But these are a pair of equal alternate angles between EF and HG, so $EF \parallel HG$. Hence, $EFGH$ is a parallelogram. But $EF = FG$, so EFGH is a parallelogram with a pair of adjacent sides equal. Thus, $EFGH$ is a rhombus.

23 (a) $2x + 2y = 360°$ (angle sum of a quadrilateral); $x + y = 180°$. Hence, $\angle MJK + \angle JML = 180°$.

(b) $\angle MJK$ and $\angle JML$ are a pair of supplementary co-interior angles between JK and ML. Hence $JK \parallel ML$. But $\angle MJK + \angle JKL = 180°$, so $\angle MJK$ and $\angle JKL$ are a pair of supplementary co-interior angles between JM and KL. Hence, $JM \parallel KL$. $JKLM$ has both pairs of opposite sides parallel, so it is a parallelogram.

24 In ΔSPT and ΔQRT, $ST = QT$ (given)

$\angle STP = \angle QTR$ (vertically opposite angles)

$TP = TR$ (given)

$\Delta SPT \equiv \Delta QRT$ (SAS)

Hence $\angle PST = \angle TQR$ (matching angles of congruent triangles). But these are a pair of equal alternate angles between SP and RQ, so $SP \parallel RQ$. Similarly, you can prove that $\Delta SRT \equiv \Delta QPT$ (SAS) so that $\angle RST = \angle TQP$ (matching angles of congruent triangles). But these are a pair of equal alternate angles between SR and PQ, so $SR \parallel PQ$. Hence, $SP \parallel RQ$ and $SR \parallel PQ$, so $SPQR$ is a parallelogram.

Open-ended—Sample answers

25 **(a)** Not true. A rhombus has four equal sides but no right angle, so it is not a square.

(b) All rectangles have their opposite sides parallel. The converse is: 'All quadrilaterals with their opposite sides parallel are rectangles'. This is false, as parallelograms have opposite sides parallel but are not always rectangles.

26 A square is a rectangle because it has two pairs of parallel sides all meeting at 90° angles, the defining features of a rectangle. However, for a rectangle to be a square its four sides must also be of equal length, which is not necessarily the case.

Half-time 6

1 **(a)** $d = 130°$, $e = 125°$ **(b)** $n = 60°$, $m = 140°$, $p = 100°$

(c) $n = 47°$, $p = 133°$

2 **(a)** $a = 54°$, $b = 36°$ (RHS) **(b)** $x = 11$ cm (ASA)

(c) $y = 5$ cm (SAS) **(d)** $y = 108°$ (SSS)

3 **(a)** $a = 77°$, $b = 138°$ **(b)** $u = 110°$

(c) $w = 114°$, $y = 37°$, $z = 35°$

4 Proof: In ΔABE and ΔCDE,
$\angle ABE = \angle CDE = 40°$ (given)
$\angle AEB = \angle DEC$ (vertically opposite angles)
$BE = DE$ (given)
$\therefore \Delta ABE \equiv \Delta CDE$ (SAS)
$\therefore AB = CD$ (matching sides of congruent triangles)

5 *D*

6 **(a)** False: a rectangle does not have to have four equal sides.

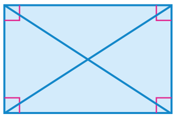
(b) True: all parallelograms have four sides.

(c) True: the diagonals of a square cut the square into four congruent isosceles triangles.

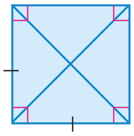
(d) False: the diagonals of a parallelogram only bisect each other at right angles when it is a rhombus.

7 Proof: In ΔKLM and ΔKNM, $KL = KN$ (given)
$LM = NM$ (given)
MK is a common side.
$\therefore \Delta KLM \equiv \Delta KNM$ (SSS)
$\angle KLM = \angle KNM$, $\angle LMK = \angle NMK$, $\angle MKL = \angle MKN$.

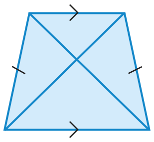
8 Rectangle



Square



Isosceles trapezium



9 $(2y - 20) + 140 = 180$ (allied angles on parallel lines), $y = 30°$
$3x = x + 10 + 40$ (vertically opposite angles), $x = 25°$
$30 + z - 25 + 35 = 180$ (allied angles on parallel lines), $z = 140°$

10 Only two sides are given equal. For congruence you need the third sides equal or the angles between the equal sides equal (i.e. need $PQ = PR$ or $\angle PSQ = \angle PSR$).

11 **(a)** In ΔABC and ΔADC,
$AB = AD$ (given)
$BC = DC$ (given)
AC is a common side.
$\therefore \Delta ABC \equiv \Delta ADC$ (SSS)
$\therefore \angle ABC = \angle ADC$ (matching angles in congruent triangles)

(b) A kite has one pair of opposite angles that are equal.

Exercise 6.4

1 **(a)** 1.5 **(b)** 0.5 **(c)** 2 **(d)** 1

2 Students' own answers—check by measuring that pairs of sides are in the same ratio as the scale factor.

3 Students' own answers—check by measuring that pairs of sides are in the same ratio as the scale factor.

4 **(a)** **(i)** 1 cm : 2 cm **(ii)** 1.5 cm : 3 cm

(iii) 1.75 cm : 3.5 cm

(b) 0.5

5 **(a)** **(i)** 3 cm : 2 cm **(ii)** 1.5 cm : 1 cm

(iii) 3 cm : 2 cm **(iv)** 1.5 cm : 1 cm

(b) 1.5

6 C

7 Students' own answers—the dimensions of $A'B'C'D'$ should be 6 cm × 9 cm.

8 Students' own answers—side lengths of $J'K'L'M'N$, should be 10 cm.

9 **(a)** 3.5 **(b)** $\frac{3}{7}$

(c) Enlarged by a scale factor of 1.5

(d) It is not a true enlargement because only the length has been stretched, altering the actual shape of the original image.

10 4 cm^2 **11** 1.75

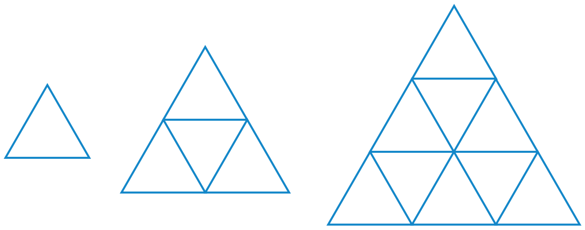
Open-ended—Sample answers

12 Students' own answers—check by measuring that pairs of sides are in the same ratio as the scale factors.

13 The area of an enlarged rectangle is the area of the original rectangle multiplied by the square of the scale factor. Area of original rectangle $= 72 \div 3^2 = 8$. The dimensions of the original rectangle are either 1×8 or 2×4.

14 Students' own answers—check by measuring that pairs of sides are in the same ratio as the scale factors.

15 **(a)**



(b) Scale factor of 4 has a side length of 4 cm (or area of 16 small triangles); scale factor of 9 has a side length of 9 cm (or an area of 81 small triangles).

(c) Because the side length of the medium triangle has length of two small triangles, the scale factor is 2; the small triangle has been enlarged by a scale factor of 3 to obtain the large triangle because it has a side length of three triangles.

Exercise 6.5

1 **(a)** SAS **(b)** AAA **(c)** RHS **(d)** SSS

2 **(a)** $x = 14$ cm **(b)** $x = 8$ m **(c)** $x = 7.5$ cm **(d)** $x = 20$ mm

3 **(a)** B **(b)** C

4 **(a)** C **(b)** C

5 **(a)** 2.5 **(b)** 0.5 **(c)** 0.8 **(d)** 1.5

6 **(a)** $\Delta ABC \sim \Delta EDF$ (SSS) **(b)** $\Delta GHK \sim \Delta GIJ$ (AAA)

(c) $\Delta LMN \sim \Delta OPN$ (AAA)

7 **(a)** $\frac{AC}{LN} = \frac{AB}{LM} \neq \frac{BC}{MN}$

As not all the ratios are equal, ΔABC is not similar to ΔLMN.

(b) $BC = QR = 6$ cm
$CA = RP = 14$ cm
$\angle BCA = 150° \neq \angle QRP = 65°$

There are two pairs of matching side lengths but the included angles are not equal. Therefore ΔBCA is not similar to ΔQRP.

(c) $\frac{TU}{XU} \neq \frac{VU}{ZY}$

Therefore ΔXYZ is not similar to ΔTUV.

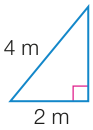
8 **(a)** **(i)** $\frac{2}{3}$ **(ii)** 7.5 m

(b) **(i)** 2.5 **(ii)** 8.75 cm

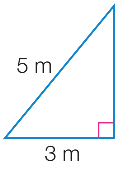
(c) **(i)** $\frac{3}{4}$ **(ii)** 5.625 m

(d) **(i)** 1.25 **(ii)** 1.28 km

9 **(a)**



(b)



(c) Not similar; matching side lengths do not have the same ratio.

10 **(a)** 1.5

(b) Students' own answers—check carefully that $QR = 5.25$ cm and $QPR = 35°$.

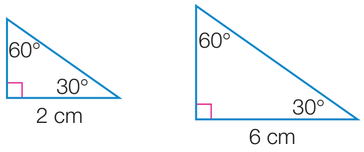
11 All angles remain the same (AAA).

12 **(a)** Matching angles are the same and matching side lengths are of the same ratio.

(b) $b = 20$ cm, $d = 100°$, $r = 55°$, $s = 12$ cm

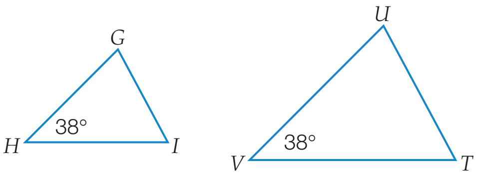
Open-ended—Sample answers

13



Scale factor = 3

14 **(a)**



(b) Two side lengths (which enclose the given angle) or one more angle must be stated to justify similarity.

Exercise 6.6

1 **(a)** AAA; $x = 10$ **(b)** AAA; $x = 6$

(c) AAA; $x = 5.56$ **(d)** AAA; $x = 26.4$

2 **(a)** AAA means that the triangles are similar.

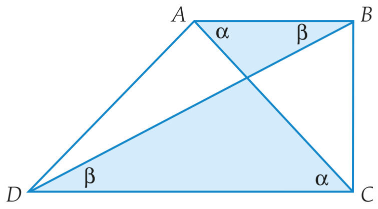
(b) 12 m

3 **(a)** C **(b)** C

4 6 m **5** 316.67 cm **6** 7.2 m

7 **(a)** Equally opposite angles at X mean that SAS shows the shaded triangles are similar.

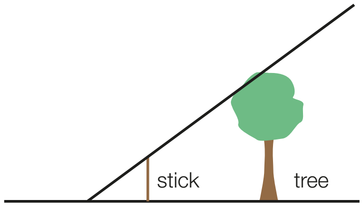
(b)



As the triangles are similar and the alternate angles are equal, AB and CD must be parallel. Hence $ABCD$ is a trapezium.

(c) No, the remaining two triangles only have one pair of vertically opposite angles, which is not enough to justify similarity.

8 **(a)**



Measure these three distances: from the stick to the tree; from the stick to the place on the ground you look from; and the length of the stick. Use similar triangles to find the height of the tree.

(b) Students' own answers.

Open-ended—Sample answers

9 Ratio of corresponding sides must be equal $\frac{a}{e}=\frac{b}{d}=\frac{c}{f}$.

One possible solution is: $a = 3$, $b = 2$, $c = 4$, $e = 6$, $d = 4$, $f = 8$

$\therefore \frac{3}{6}=\frac{2}{4}=\frac{4}{8}$

10 (a) The triangles might not be similar, so they cannot be calculated in this way.

(b) Pythagoras' theorem

Exercise 6.7

1 D

2 several possibilities for each—teacher to check

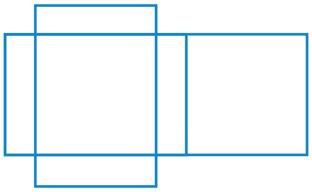
3 (a) tetrahedron (b) cylinder

(c) cone (d) hexagonal prism

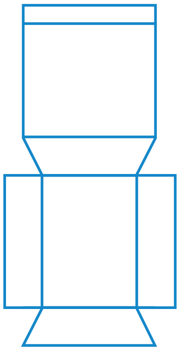
4 D 5 B

6 sample answers—teacher to check

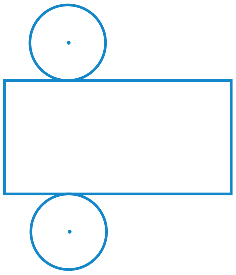
(a)



(b)



(c)



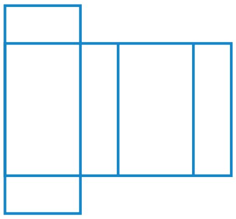
7 Students' own answers—check construction measurements carefully.

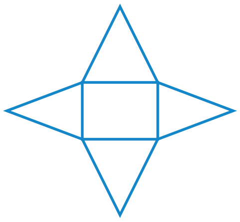
8 Students' own constructions.

9 Top part has five faces, five vertices and eight edges. Bottom part has six faces, eight vertices and twelve edges.

10 (a) Rectangular prism: edges that are meant to meet are of different lengths, no top to match the base. Rectangular pyramid: the triangles all need to have the same sloping side lengths to meet at the apex of the pyramid.

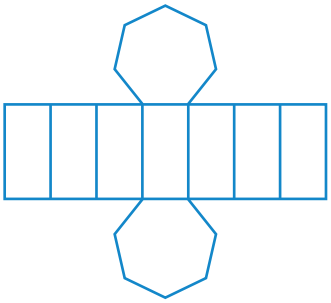
(b)





Open-ended—Sample answers

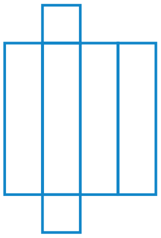
11

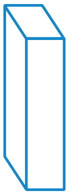


12 tetrahedron, octahedron, dodecahedron

13 No; a face is a flat surface, so a cylinder is not suitable. A polyhedron always has more than three faces. The simplest polygon used to construct a polyhedron is a triangle and it requires a minimum of four faces to create a three-dimensional shape.

14 rectangular prism





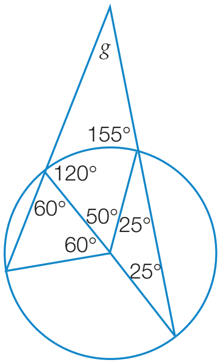
Challenge 6

1 D

2 Arrange the matches as a three-dimensional tetrahedron. The four faces are equilateral triangles.

3 B

4 $g = 360 - (120 + 50 + 155) = 35°$



5 B

6 B, C, D

7 B

Chapter review 6

1 **(a)** $x = 163°$ **(b)** $x = 41°$ **(c)** $x = 46°$
(d) $x = 85°$ **(e)** $x = 32°$ **(f)** $x = 110°$

2 **(a)** $y = 53°, z = 127°$ **(b)** $a = 95°, b = 110°, c = 95°, d = 85°$
(c) $m = 55°, n = 65°, p = 115°$

3 **(a)** $x = 72°, y = 141°$ **(b)** $c = 118°, d = 62°, e = 118°$
(c) $f = 90°$

4 **(a)** B **(b)** D **(c)** A **(d)** C

5 **(a)** ASA **(b)** SAS **(c)** ASA

6 **(a)** rectangle **(b)** rhombus **(c)** parallelogram

7 **(a)** 0.4 **(b)** 2.5

8 Students' own diagrams.

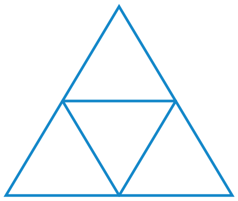
9 The missing angles in the triangles are 40° and 115°, so the triangles are similar (AAA).

10 **(a)** 5 **(b)** 9

11 A vertically opposite angle and an alternate angle on parallel lines means the triangles are similar by AAA.

$v = 4$ m, $x = 7.5$ m

12



13 **(a)** $x = 77°$ **(b)** $x = 60°, y = 120°$
(c) $w = 72°$ $y = 108°, z = 72°$ **(d)** $x = 60°$
(e) $x = 40°$ **(f)** $x = 40°$

14 **(a)** $x = 56°$ **(b)** $x = 24°, y = 128°$ **(c)** $x = 43.6°$
(d) $x = 30°$ **(e)** $x = 34°, y = 124°$

15 **(a)** $BD \perp AC$

In ΔABD and ΔCBD, $\angle BDA = \angle BDC = 90°$ ($BD \perp AC$)

$AB = CB$ (given)

BD is a common side.

$\therefore \Delta ABD \equiv \Delta CBD$ (RHS)

$\therefore \angle BAD = \angle BCD$ (matching angles of congruent triangles)

$\therefore \angle BAC = \angle BCA$

(b) The angles opposite equal sides in an isosceles triangle are equal.

16 **(a)** $AE = CE$

$DE = BE$

$\Delta ABE \equiv \Delta BEC \equiv \Delta CED \equiv \Delta DEA$ (SSS)

$\therefore \angle AEB = \angle BEC = \angle CED = \angle DEA = 90°$

$\therefore AC \perp BD$

(b) The diagonals of a rhombus bisect each other at right angles.

17 **(a)** In ΔABC and ΔADC, $AB = CD$ (given)

$BC = DA$ (given)

AC is a common side.

$\therefore \Delta ABC \equiv \Delta ADC$ (SSS)

$\therefore \angle BAC = \angle DCA$ (matching angles of congruent triangles)

These are a pair of equal alternate angles between AB and DC, so $AB \parallel DC$.

Similarly, $\angle BCA = \angle DAC$ and hence $BC \parallel AD$.

Hence $ABCD$ is a parallelogram because it is a quadrilateral with both pairs of opposite sides parallel.

(b) In ΔPQT and ΔRQT, $PT = TR$ (given)

$\angle PTQ = \angle RTQ$ (both 90°, given)

QT is a common side.

$\therefore \Delta PQT \equiv RQT$ (SAS)

$\therefore PQ = QR$ (matching sides of congruent triangles)

Similarly, you can prove that $\Delta PQT \equiv RST$ (SAS) so that $PQ = SR$.

$\angle QPT = \angle SRT$ (matching angles of congruent triangles)

These are a pair of equal alternate angles between PQ and SR, so $PQ \parallel SR$.

Similarly, you can prove that $\Delta PST \cong \Delta RQT$ (SAS).

$\angle SPT = \angle QRT$ (matching angles of congruent triangles)

These are a pair of equal alternate angles between PS and QR, so $PS \parallel QR$.

Hence, $PQRS$ is a parallelogram with a pair of adjacent sides equal, so it is a rhombus.

But ΔPQT and ΔRQT are right-angled isosceles triangles, so $\angle PQT = \angle RQT = 45°$.

Hence $\angle PQR = 90°$.

$\therefore PQRS$ is a square.

18 12.6 m **19** 2.43 m

20 Let $\angle TSU = x°$.

$\angle TSR = (180 - x)°$ ($RSUV$ a straight line)

$\angle TUS = (90 - x)°$ (angle sum of ΔTSU)

$\angle TUV = (90 + x)°$ ($RSUV$ a straight line)

$\therefore \angle TSR + \angle TUV = (180 - x)° + (90 + x)° = 270°$

21 $a + b = 180°$ (AD a straight line)

Similarly $c + d = 180°$

Now $\angle ZWX = a$ (vertically opposite angles)

and $\angle YXW = d$ (vertically opposite angles)

$\angle YZW = b$ (corresponding angles, $AD \parallel HE$)

$\angle ZYX = c$ (corresponding angles, $AD \parallel HE$)

$\angle ZWX + \angle YXW + \angle ZYX + \angle YZW = a + d + c + b$
$= 180° + 180° = 360°$

22 **(a)** 1.2 **(b)** 13.2 cm by 10.2 cm

(c) **(i)** 93.5 cm^2 **(ii)** 134.64 cm^2

(d) 1.44—this ratio is different from the scale factor as each side has been increased by the scale factor, so the area has increased by the square of the scale factor.

(e) 144%

Numeracy practice 6

1 60° **2** C **3** 50° **4** 40°

5 95° **6** 40° **7** C **8** B

9 12.15 m^2

Mixed review C

1 61.70m

2 **(a)** 86° **(b)** 77° **(c)** 87°

3 16.25 cm^2

4 **(a)** 7.5 **(b)** $k = 46°$

5 351.86 cm^2

6 **(a)** \$705 **(b)** \$90.05

7 **(a)** Scale factor = 1.4, $x = 28$ cm

(b) Scale factor = $\frac{5}{12}$, $x = 3.75$ cm

8 **(a)** $A = \frac{xy}{2}$ **(b)** $A = \frac{xy}{2}$ **(c)** $A = \frac{1}{2}h(a+b)$

9 **(a)** $(y+7)(y+3)$ **(b)** $(d+3)(c+4)$ **(c)** $(t-4)(t-5)$

10 Students' own answers.

11 **(a)** 2 **(b)** -1 **(c)** $\frac{1}{2}$

12 **(a)** $x = \frac{55}{3}$ **(b)** $x = \frac{13}{4}$ **(c)** $x = -\frac{13}{3}$

13 **(a)** AAA **(b)** SAS, scale factor 2

14 **(a)** 2.4 **(b)** $\frac{1}{3}$

15 $a = 30°, b = 80°, c = 70°, d = 150°, e = 100°$

16 9 m

17 **(a)** $x^2 + 4x - 21$ **(b)** $(x+7)(x-3)$ **(c)** $x = 103$

(d) Students to show the equality of the expressions.

18 Any of ΔAEH, ΔAFI, ΔFEG, ΔACE, ΔABF, ΔFDE.
All triangles have one right angle and two 45° angles
∴ they are similar triangles (AAA).

19 5.4

20 **(a)** 3

(b) **(i)** 150 cm^2 **(ii)** 1350 cm^2

(c) 9

(d) $9 = 3^2$, the scale factor of the area enlargement is the square of the scale factor of the enlarged lengths.

Chapter 7

Recall 7

1 **(a)** 12.5 **(b)** 13 **(c)** ±8 **(d)** ±7

(e) 5.34 **(f)** 62.7221

2 **(a)** equilateral and acute-angled

(b) scalene and right-angled

(c) scalene and acute-angled

(d) isosceles and right-angled

(e) equilateral and acute-angled

(f) scalene and obtuse-angled

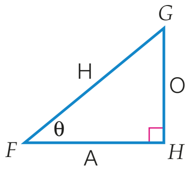
3 **(a)** 42 **(b)** 40 **(c)** 54 **(d)** 4

4 **(a)** 64° **(b)** 25° **(c)** 53°

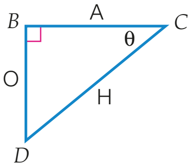
5 **(a)** $\frac{4}{5}$ **(b)** $\frac{3}{2}$ **(c)** $\frac{5}{4}$ **(d)** $\frac{4}{3}$ **(e)** $\frac{3}{4}$ **(f)** $\frac{4}{5}$

Exercise 7.1

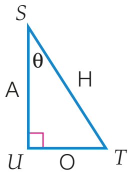
1 **(a)**



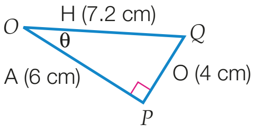
(b)



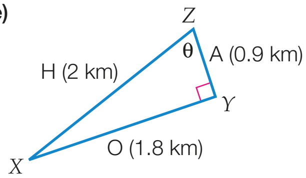
(c)



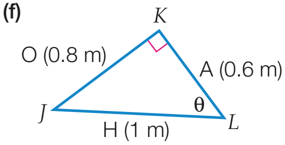
(d)



(e)



(f)



2 **(a)** **(i)** 10.6 cm **(ii)** 7 cm **(iii)** 8 cm

(b) **(i)** 12 m **(ii)** 3.8 m **(iii)** 11.4 m

(c) **(i)** 3.4 km **(ii)** 2.5 km **(iii)** 2.3 km

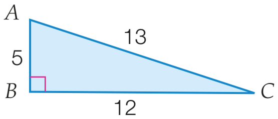
3 A **4** C

5 **(a)** D **(b)** B **(c)** C

6 **(a)** 5.3 m **(b)** 1.8 m **(c)** 70°

7 **(a)** B **(b)** A **(c)** C

8 **(a)** 13 **(b)** Sample answer:



9 **(a)** Triangle 1: H: 2.25 cm; O: 0.55 cm; A: 2.2 cm

Triangle 2: H: 4.25 cm; O: 1 cm; A: 4.1 cm

Triangle 3: H: 6.25 cm; O: 1.55 cm; A: 6.15 cm

(b) $\frac{0.55}{2.25}; \frac{1}{4.25}; \frac{1.55}{6.25}$ and $\frac{2.2}{2.25}; \frac{4.1}{4.25}; \frac{6.15}{6.25}$

10 **(a)** Students' own diagram.

(b) $\frac{0.9}{1} = 0.9;\ \frac{1.8}{2} = 0.9;\ \frac{2.7}{3} = 0.9;\ \frac{3.6}{4} = 0.9$

(c) Corresponding sides have an equal ratio in similar triangles.

11 **(a)** Students' own diagrams.

(b) Length measurements will depend on students' diagrams, but measured ratios should be approximately as shown.

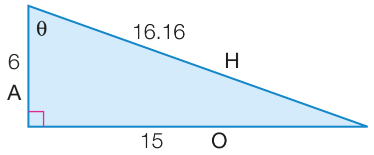
Triangle	Reference angle	Hypotenuse length	Opposite side length	Adjacent side length	$\frac{\text{Opposite}}{\text{Adjacent}}$	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$
1	30°				≈0.577	≈0.5	≈0.866
2	30°				≈0.577	≈0.5	≈0.866
3	30°				≈0.577	≈0.5	≈0.866
1	60°				≈1.732	≈0.866	≈0.5
2	60°				≈1.732	≈0.866	≈0.5
3	60°				≈1.732	≈0.866	≈0.5

(c) The ratios of corresponding sides are the same for each triangle.

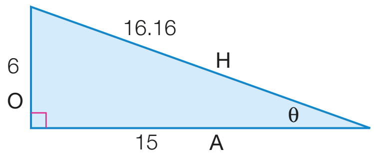
(d) Students' own responses.

12 **(a)** The teacher has not marked in the angle.

(b) Nash



Aisha



Open-ended—Sample answers

13 The adjacent side is between the right angle and the reference angle. It is not always on the bottom of a triangle.

Exercise 7.2

1 **(a)** **(i)** $\frac{7}{24}$ **(ii)** $\frac{4}{3}$ or $1\frac{1}{3}$ **(iii)** $\frac{21}{20}$ or $1\frac{1}{20}$ or 1.05

(b) **(i)** $\frac{7}{25}$ **(ii)** $\frac{3}{5}$ **(iii)** $\frac{21}{29}$

(c) **(i)** $\frac{3}{5}$ or 0.6 **(ii)** $\frac{8}{17}$ **(iii)** $\frac{9}{41}$

2 **(a)** **(i)** $x = 6.\dot{6}$ **(ii)** $p = 39$ **(iii)** $n = 0.7$

(b) **(i)** $x = 13$ **(ii)** $y = 6$ **(iii)** $r = 20.8$

(c) **(i)** $n = 48$ **(ii)** $s = 12$ **(iii)** $r = 22.4$

3 0.50 **4** 0.50 **5** 1

6 **(a)** 0.59 **(b)** 0.81 **(c)** 0.73

7 **(a)** **(i)** 0.58 **(ii)** 0.87 **(iii)** 0.50

(iv) 0.87 **(v)** 1.73 **(vi)** 0.50

(b) same **(c)** same

8 **(a)** **(i)** 0.54 **(ii)** 1.54 **(iii)** 0.84

(iv) 0.54 **(v)** 0.84 **(vi)** 0.65

(b) same **(c)** same

9 **(a)** B **(b)** A **(c)** B

10 B **11** 100 m **12** 2.8 m

13 27 m **14** 0.82 cm

15 **(a)** Students to draw a scale diagram

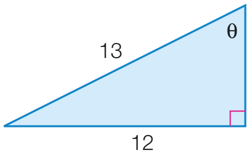
(b) right-angled isosceles triangle

(c) 5.3 m

(d) The two shorter sides of a right-angled isosceles triangle are equal.

16 **(a)** 3 m **(b)** Angle from the ground could be lowered to 30° or less.

17 **(a)**



(b) 5 **(c)** $\frac{5}{13}$

18 **(a)** sin (50°) = 0.766; cos (50°) = 0.643; tan (50°) = 1.192

(b) tan (50°) has a value greater than 1. The opposite side is longer than the adjacent side.

(c) **(i)** Any right-angled trianlge with a reference angle less than 45°.

(ii) Any right-angled triangle with a reference angle greater than 45°.

(d) sin (θ) and cos (θ) can not be greater than 1 because the hypotenuse (the bottom number on the ratio) will always be longer than the other two sides.

Open-ended—Sample answers

19 Any, as long as the ratio between b and a is 7 : 10. Sample answers: $a = 10$, $b = 7$; $a = 20$, $b = 14$; $a = 100$, $b = 70$.

20 The opposite and adjacent side change position with the two reference angles, so sin (θ) is the same as cos (β).

Exercise 7.3

1 **(a)** 26.63 **(b)** 7.51 **(c)** 18.87 **(d)** 33.27

(e) 4.04 **(f)** 0.92

2 **(a)** 313.49 cm **(b)** 137.73 mm **(c)** 13.62 m

(d) 229.40 km **(e)** 89.71 cm **(f)** 150.13 m

(g) 5.93 cm **(h)** 5.72 cm **(i)** 3.25 cm

3 **(a)** C **(b)** D

4 **(a)** C **(b)** D

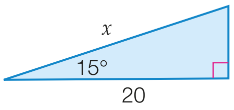
5 2.40 m **6** 5.79 m **7** 1.39 m **8** 7.60 m

9 54.64 m **10** 10.72 m **11** 47.57 cm **12** 5.52 mm

Open-ended—Sample answers

13 The adjacent side divided by the hypotenuse should be approximately equal to 0.45.

14 **(a)** A question might have been to find the hypotenuse, as shown in the triangle below.



(b) 5.36

15 (a) (i) Use of the wrong reference angle in the equation. Student has calculated the length of j.

(ii) Incorrect method of rearranging the equation. The unknown is the denominator in the equation, so the known side should be divided by the trigonometric ratio.

(b) (i)

$$\cos(\theta) = \frac{A}{H}$$
$$\cos(50°) = \frac{q}{8}$$
$$\cos(50°) \times 8 = q$$
$$q = 5.14 \text{ m}$$

(ii)

$$\sin(\theta) = \frac{O}{H}$$
$$\sin(55°) = \frac{13}{x}$$
$$x = \frac{13}{\sin(55°)}$$
$$x = 15.87 \text{ km}$$

16 To find a side length with Pythagoras' theorem, the side lengths of two sides must be known. Trigonometry can calculate side lengths with one side length and one angle known.

Half-time 7

1 (a) O **(b)** A

2 (a) 7.26 **(b)** 16.68 **(c)** 9.14

3 (a) 0.59 **(b)** 0.81 **(c)** 0.73

4 (a) $d = 19.04$ **(b)** $e = 81.94$ **(c)** $f = 28.59$

5 (a) 4 m = opposite, 7 m = adjacent, unknown = hypotenuse

(b) 9 km = hypotenuse, 6.5 km = opposite, unknown = adjacent

(c) 10.3 km = hypotenuse, 6.9 km = adjacent, unknown = opposite

6 $f = 6$ cm

Exercise 7.4

1 (a) 15° **(b)** 39° **(c)** 11° **(d)** 78°

2 (a) 62° **(b)** 35° **(c)** 28° **(d)** 28°

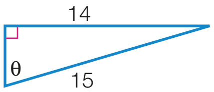
3 54°

4 (a) B **(b)** C **(c)** D

5 15° **6** 24° **7** 37°

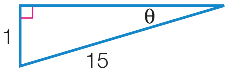
8 35° **9** 8°

10 (a)



$\sin(\theta) = 0.933$

(b)



$\sin(\theta) = 0.067$

(c) (i) close to 1 **(ii)** close to 0

11 (a) A ratio of 1 means that the two sides are equal in length.

(b) Tangent ratio—the other two ratios involve the hypotenuse, which is always longer than the other two sides.

(c) isosceles triangle

(d) 45°

Open-ended—Sample answers

12 There are four possible triangles with reference angles:

$\sin^{-1}\left(\frac{9}{15}\right) = 36.87°$, $\cos^{-1}\left(\frac{9}{15}\right) = 53.13°$, $\tan^{-1}\left(\frac{9}{15}\right) = 30.96°$,

$\tan^{-1}\left(\frac{15}{9}\right) = 59.04°$.

13 ramp length = 450 cm, $\theta = 8.3°$

ramp length = 500 cm, $\theta = 7.5°$

ramp length = 550 cm, $\theta = 6.8°$

Exercise 7.5

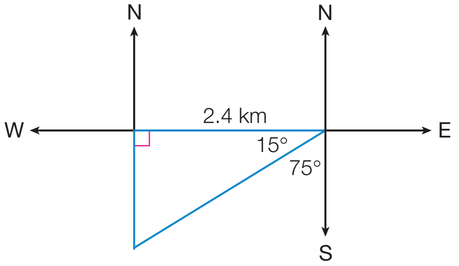
1 122.06 m **2** 16° **3** 34.64 km **4** 130.76 m

5 63° **6** A **7** B **8** C

9 9.58 m **10** 43.56 m **11** 9.71 m **12** 24°

13 (a) 33.44 km **(b)** 30.11 km

14 (a)



(b) 0.64 km **(c)** 2.48 km

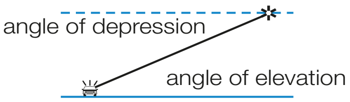
15 (a) N54°W or 306°T **(b)** S54°E or 126°T

16 76 m **17** 26.17 m

Open-ended—Sample answers

18 height of tree ÷ distance from car ≈ 0.51

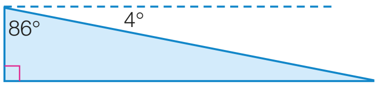
19



The two horizontal lines are parallel. The angle of elevation and the angle of depression are alternate angles, which we know are equal.

20 (a) Germaine has based her calculations on the reference angle of 4° between the rope and the boat; however, 4° is the angle of depression. The correct answer is 14.96 m.

(b) The angle of depression must go down from the horizontal, which means in this case it is not inside the triangle. The reference angle inside the triangle (to use the sine ratio) is 86°.



Challenge 7

1 (a)

$$\begin{aligned}(m^2 - n^2)^2 + (2mn)^2 &= m^4 - 2m^2n^2 + n^4 + 4m^2n^2 \\ &= m^4 + 2m^2n^2 + n^4 \\ &= (m^2 + n^2)^2\end{aligned}$$

(b) (i) $m = 2, n = 1{:}3, 4, 5$

(ii) $m = 3, n = 1:8, 6, 10 \rightarrow 3, 4, 5$
$m = 3, n = 2:5, 12, 13$

(iii) $m = 4, n = 2:12, 16, 20 \rightarrow 3, 4, 5$
$m = 4, n = 3:7, 24, 25$

(iv) $m = 4, n = 1:15, 8, 17 \rightarrow 8, 15, 17$

(v) $m = 5, n = 1:24, 10, 26 \rightarrow 5, 12, 13$
$m = 5, n = 2:21, 20, 29 \rightarrow 20, 21, 29$
$m = 5, n = 3:16, 30, 34 \rightarrow 8, 15, 17$
$m = 5, n = 4:9, 40, 41$

2 D
$AB = 3\sqrt{2}, BC = 3\sqrt{5}$,
Perimeter $= 3\sqrt{2} + 3\sqrt{5} + 6 + 9 = 3(5 + 2\sqrt{5})$ cm

3 **(a)** isosceles **(b)** $PS^2 = 100 - 36; PS = 8$ cm **(c)** A

4 D

5 C

6 **(a)** $EC = DE = x$

(b) $\frac{CF}{CB} = \cos(60°), CF = 2x \times 0.5 = x$

(c) $BF^2 = CB^2 - CF^2 = 4x^2 - x^2, BF = \sqrt{3}x$

(d) $\tan(\angle EBF) = \frac{EF}{BF} = \frac{2x}{\sqrt{3}x}, \angle EBF = 49°6'$

(e) $\angle CBF = 30°, \angle EBC = 49°6' - 30° = 19°6'$

7 **(a)** $\sin(\theta) = \frac{30}{132}, \theta = 13.13° \approx 13°$

(b) $BD = \frac{44}{\sin(27°)} = 96.9 \approx 97$ m

(c) total length of track $= 132 + 97 = 229$ m

average speed $= \frac{229}{90} = 2.54 \approx 2.5$ m/s

Chapter review 7

1 **(a)** 25 m = opposite, x = hypotenuse, sin

(b) 8.3 cm = hypotenuse, x = adjacent, cos

(c) 3.2 km = adjacent, x = opposite, tan

2 **(a)** 1.3333 **(b)** 0.225 **(c)** 5.4545

3 **(a)** $x = 4.00$ cm **(b)** $t = 3.70$ cm **(c)** $p = 7.90$ m

4 **(a)** $\frac{3}{5}$ **(b)** $\frac{15}{17}$ **(c)** $\frac{20}{29}$

5 **(a)** 2.19 cm **(b)** 15.00 cm **(c)** 55°

(d) 111.87 m **(e)** 1766.89 m **(f)** 68°

6 A **7** 7.57 km south and 10.81 km east

8 33.58 m **9** 48°

10 26.11 m

11 76.9 km

12 89.6 m

13 50.75 km

14 151°T or S29°E

15 46.1°

16 **(a)** 2.23 cm **(b)** 15.61 cm^2

17 63.4°

Numeracy practice 7

1 B **2** B **3** A **4** D

5 D **6** A **7** B **8** C

Chapter 8

Recall 8

1 mean = 20.9, median = 20.5, range = 18

2 **(a)** $\frac{1}{6}$ **(b)** $\frac{2}{3}$ **(c)** $\frac{1}{3}$

3 mean 2.36; median 2

4 spade, club, diamond, heart

5 **(a)** **(i)** mean = 66.47 **(ii)** median = 65.00

(iii) positively skewed

(b) **(i)** mean = 15.11 **(ii)** median = 15.75

(iii) negatively skewed

Exercise 8.1

1 **(a)** discrete **(b)** continuous **(c)** continuous

(d) discrete **(e)** continuous **(f)** continuous

(g) discrete **(h)** continuous

2 **(a)** categorical **(b)** categorical **(c)** numerical

(d) numerical **(e)** numerical **(f)** numerical

(g) categorical **(h)** categorical

3 **(i)** **(c)** discrete **(d)** discrete

(e) continuous **(f)** continuous

(ii) **(a)** nominal **(b)** nominal

(g) ordinal **(h)** nominal

4 **(a)** D **(b)** C

5 The safest state, with the lowest registered deaths per 10 000 registered motorcyclists, is Victoria. Although the most motorcyclists are killed in Queensland, the rate is much worse in the Northern Territory. The deaths per 10 000 motorcyclists gives a more accurate picture than the raw figures.

6 **(a)** A, D **(b)** D **(c)** B, C **(d)** C **(e)** B

7 **(a)** B **(b)** A **(c)** D

8 Sample answers:
Question 1: Every day; once a week; at least once a month etc.
Question 2: Range of products: prices; quality of food; service etc.
Question 3: Students to provide own answer based on their own school canteen.

9 The results are discrete because there is a fixed, countable number of values possible.

10 Joe's answer is incorrect because measured data is always continuous; it doesn't matter how the numbers are rounded. Ngarla's answer is correct, although she provides more information than necessary. It would be sufficient to say that height is a measured variable and so continuous.

Open-ended—Sample answers

11 **(a)** number of pets per household, number of students in each year

(b) time taken to swim 50 m, height of seedlings in a science experiment

(c) favourite football team, type of pet owned

(d) genre of movie: drama, comedy, romance, animation; gender: male, female

12 **(a)** Students' own answers.

(b) It is unlikely that every student would produce exactly the same list.

(c) If it is intended to collate class results together at the end, then it would be a problem if the lists were not the same.

Exercise 8.2

1 **(a)** **(i)** 3.8 **(ii)** 3.5

(b) **(i)** 25.4 **(ii)** 25

(c) **(i)** 61.08 **(ii)** 61.2

(d) **(i)** 0.438 **(ii)** 0.465

2 Student' own answers.

3 A

4 **(a)** 5

(b) No, every sample will have its own unique composition.

(c) Students will produce their own answers and comment on the similarities and differences in their samples.

5 **(a)** **A** 26.25 **B** 27.25

(b) **A** 26.5 **B** 26.5

(c) **A** 27 **B** 27

(d) the mean

6 mean = 26 mm, median = 19.5 mm

7 **(a)** 10

(b) test 5: 8, test 6: 10; test 5: 9, test 6: 9 or 10; test 5: 10, test 6: 8, 9 or 10

8 52.6

9 **(a)** 160

(b) No; this would favour executives and managers.

(c) Conduct a census instead.

Open-ended—Sample answers

10 **(a)** mean = 4.67, median = 4.65

(b) Answers may vary.

(c) Answers may vary.

(d) Possible answer: Sample means and medians were sometimes higher and sometimes lower than the population mean and median. Usually when the sample mean was higher than the population mean then the sample median was higher than the population median, so both sample averages either overestimated or underestimated the population averages. The sample averages seem to 'average out' at similar values to the population averages.

11 **(a)** Lexus

(b) 140

(c) mean

(d) possible answers: mean = 145.4 p/h; median = 121 p/h

(e) possible answers: mean = 159.55 p/h; median = 156.5 p/h

(f) Total class means and medians could be either higher or lower than the population values. The samples of size 10 overall had values closer to the population, but this was not always the case.

Exercise 8.3

1 **(a)** 1.3 siblings **(b)** 1 sibling **(c)** 1 sibling

2 **(a)** negatively skewed

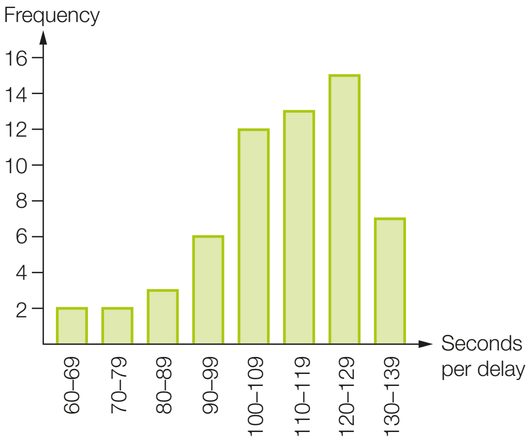
(b) positively skewed

(c) symmetrical

3 **(a)** **(i)** 110.83

(ii) 110–119

(iii)

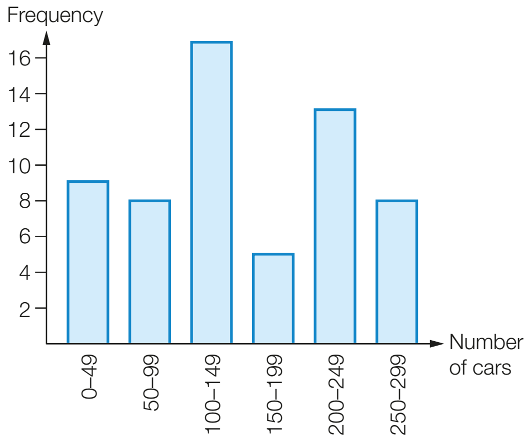


(iv) The graph is not symmetrical. It is negatively skewed and there are no outliers.

(b) **(i)** 148.67

(ii) 100–149

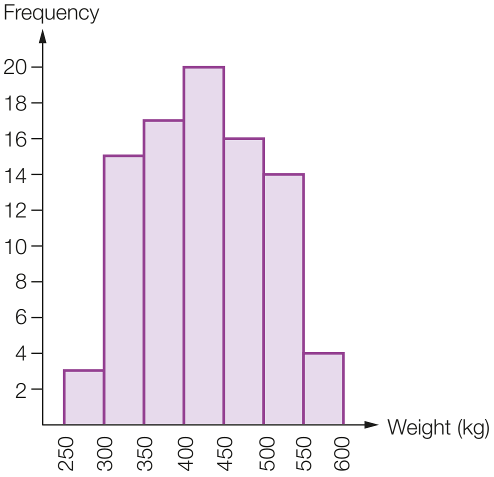
(iii)



(iv) The graph is relatively symmetrical, although the 150–199 class interval is a little low for true symmetry.

4 (a) 427 kg (b) 400–<450 kg

(c)



(d) The data looks symmetrical.

5 (a) B (b) B

6 (a) A: mean = 7, median = 7; B: mean = 7.4, median = 7

(b) For negatively skewed data, with a tail to the left, mean < median.

7 (a) The data set is positively skewed.

(b) Students will provide their own answer, but a value about 2 would seem reasonable.

(c) 1.72; students will make their own comment about the reasonableness of their estimate.

(d) Students will make their own comment, but may suggest conducting a survey within their own school.

8 (a) 1–10; 11–20; 21–30; 31–40; 41–50

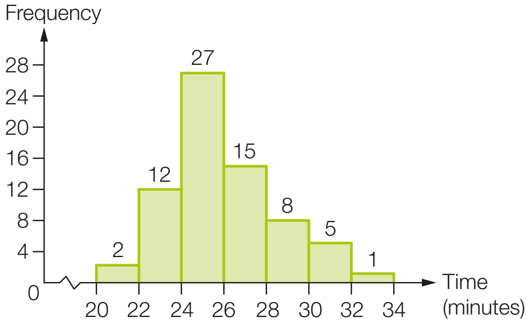
(b) 0–4; 5–9; 10–14; 15–19; 20–24

(c) 0–49; 50–99; 100–149; 150–199; 200–249

(d) 1–50; 51–100; 101–150; 151–200; 201–250

9 (a) 25.97 minutes

(b)



The data set is moderately positively skewed.

10 (a) The data set is slightly positively skewed.

(b) Somewhere around 3 km would seem reasonable.

11 (a) 45.1 (b) 20–39 (c) median

(d) mean = 39.1
median class = 20–39

(e) The median was unchanged and is still more representative as the data remains positively skewed. The removal of the outlier has made the mean and median values quite similar.

12 (a) (i) positively skewed (ii) symmetrical

(b) (i) 5.7 (ii) 174.6

(c) (i) 5.61 (ii) 174.0

(d) (i) Estimated mean is 0.09 lower than the actual mean.

(ii) Estimated mean is 0.06 lower than the actual mean.

(e) bi-modal, not symmetric

Open-ended—Sample answers

13 (a) Students' own answers.

(b) Students' own answers.

14 (a) 20.6

(b) Possible raw data: 4.6, 8.0, 9.5, 10.7, 11.4, 13.9, 15.6, 15.7, 18.3, 22.8, 24.5, 25.1, 27.0, 29.2, 29.8, 34.4, 37.1, 42.6

(c) for the possible data in **(b)** mean = 21.1

(d) For the possible data in **(b)**, the estimated mean is 0.5 less than the actual mean.

Exercise 8.4

1 (a) 37 for Fremantle and 27 for West Coast Eagles

(b)

Fremantle	Stem	West Coast Eagles
9 8 7 6 4	17	6 9 9
9 9 8 8 7 7 6 6 6 5 5 2 2 2 2 1 1 0	18	1 1 3 3 3 3 3 4 5 5 5 5 5 6 6 7 7 7 8
9 7 7 7 6 5 3 2 2 2 0	19	0 0 0 1 1 3 4 5 5 6 6 6 7
3 1 1	20	1 1 3
1	21	

Key: 17 | 6 = 176 cm

We have already seen that Fremantle has a much greater range of heights, with one player taller and one shorter than any West Coast Eagles player, but there is no obvious difference in location of height. This is confirmed by the median heights of 187.5 cm for Fremantle and 187 cm for the West Coast Eagles.

2 (a) All year round Kuala Lumpur is wetter than Port Moresby, with the exception of February. Kuala Lumpur has very high rainfall in March to May and October to December, whereas Port Moresby has a single wet season from December to April.

(b) Kuala Lumpur: mean = 218.3 mm, median = 219.5 mm, range = 140 mm

Port Moresby: mean = 95.8 mm, median = 75 mm, range = 182 mm

The monthly rainfall for Kuala Lumpur is, on average, more than double that of Port Moresby for both mean and median, and yet Port Moresby experiences greater variation. You could say that Port Moresby has a more distinct wet and dry season.

(c) Canberra has a very consistent (and quite low) monthly average all year round whereas Lahore has a distinct wet season from July to September. It is only during this wet season that Lahore's monthly rainfall is greater than that of Canberra.

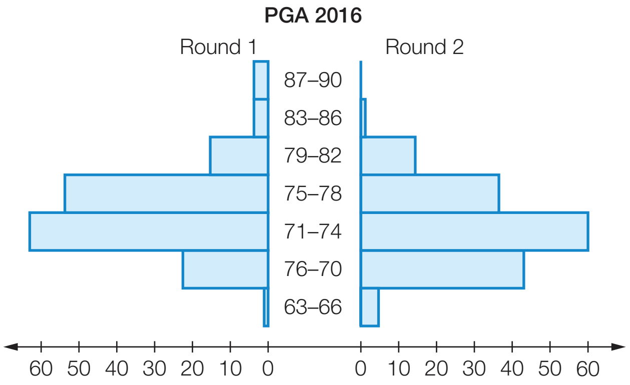
(d) Lahore: mean = 52.3 mm, median = 26 mm, range = 198 mm

Canberra: mean = 52.8 mm, median = 51.5 mm, range = 27 mm

The mean monthly rainfalls are nearly the same. Lahore has a much lower median rainfall due to the positive skew in the data. Monthly variation for Canberra is very small compared with Lahore.

3 (a)

Score range	Frequency Day 1	Frequency Day 2
63–66	1	5
67–70	21	40
71–74	62	60
75–78	53	35
79–82	12	12
83–86	2	1
87–90	2	0



(b) Grouping this way gave six or seven columns for each part of the graph. This fits well into the preferred number of 5 to 10.

(c) Day 2 appears to have had lower scores.

(d) Round 1: mean = 74.1, median = 74

Round 2: mean = 72.9, median = 72

This confirms that it was easier to score well in round 2.

4 (a) 100%

(b) 80–84 is a 5-year span, whereas 85+ includes women from 85–100 and beyond.

(c) The graph does not show the size of the population, but it shows that the proportion of young people is smaller and the proportion of older people is larger. There are still more young boys than young girls, and still more elderly women than elderly men.

5 (a)

Male	Stem	Female
7	14	4 5
9 7 4	15	3 3 4 5
6 4	16	0 0 0 1 2 2 2 3 4 4 5 5 7 7
7 7 6 5 5 4 3	17	7 8 8 9
8 5 3 1 0	18	0 1 2 4 4 8
1 0	19	

Key: 14 | 4 = 144

(b) In Victoria, Year 9 boys are clearly taller on average than girls.

6 (a) The following table shows the results.

	Mean	Median	Range
A	95.03	46.35	296.7
B	31.88	24.5	49.8
C	21.33	21.2	7.1
D	161.31	164.15	148.3

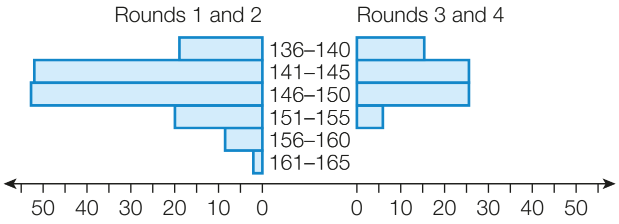
(b) A has a wet season (basically summer) and a dry season (winter and spring). This is best reflected in its very high range. It is likely to be in the tropics.
B has low annual rainfall with most rain occurring in the winter.
C is quite a dry place with a consistent rainfall pattern. This is reflected in the closeness of the mean and median values and the very low range.
D is the wettest of the four places with most of its rain coming in the winter months. However, its high range value indicates it has some relatively dry months. These occur in the summer.

(c) Tropical places have high rainfall in the summer months, so A must be Townsville. Broken Hill and Port Lincoln are both dry places, but as Port Lincoln is on the coast it will likely have more rain. So B is Port Lincoln and C is Broken Hill. This leaves D to be Weeaproinah, which is the wettest place in Victoria.

7 (a) This grouping would give six groups for rounds 1 and 2, so it fits neatly into the preferred range of 5 to 10.

(b) There are only four groups for rounds 3 and 4 because no one scored over 153.

(c)



(d) There are only half as many results on the right-hand side and none at all in the highest two categories. However, it appears that the scores for the second pair of rounds are lower because there are almost as many scores in the lowest score category as on the first pair of rounds. As approximately half the players are eliminated after the first two rounds, it is safe to say that the better performing players will proceed to Rounds 3 and 4.

(e) For rounds 1 and 2: mean = 146.4 and median = 146;
For rounds 3 and 4: mean = 144.6 and median = 145.
This confirms that scoring was lower for rounds 3 and 4.

8 The number axis for Tonga exaggerates the size 125 times compared to the graphs for Australia and Cambodia, so there is a much smaller population for Tonga. In terms of proportion at each age group, Tonga and Cambodia are quite similar. Australia seems to have a comparatively small proportion of children compared to Cambodia and Tonga. A greater proportion of Australians has a very long life span compared to Cambodia and Tonga.

Open-ended—Sample answers

9 Possible answer:

Basketball	Stem	Tennis
	14	9
6 4 3 2 1 1 0 0	15	0 1 1 2 3 3 4 5 5 6 7 8 9
9 8 7 7 6 6 6 4 2 1 0	16	0 4 4 5 6 7 9 9
3 2 1 0 0	17	1 2 8
1	18	

Key: 15 | 0 = 150 cm

The data above might be the heights (in cm) of a school's basketball team and tennis team.

Basketball: mean = 162 cm, median = 166 cm
Tennis: mean = 159.9 cm, median = 158 cm

10 Students' own answers.

Half-time 8

1 (a) (i) 54.32 (ii) 53 (iii) positive skew

(b) (i) 14.946 (ii) 14.95 (iii) symmetrical

2 (a) B (b) B (c) A

3 Students will provide their own answer, but it should contain at least some of the following elements:

- Construct a survey instrument that shows the three new logos as well as the current one.
- Decide on important stakeholders to be surveyed—employees, customers, general public etc.
- How many of each will be surveyed?
- Do their opinions count equally?
- What will represent success for your survey?

4

Group 1	Stem	Group 2
9	14_H	
4 3	15_L	1 3 4
6 5 5	15_H	5 5 5 8
4 0	16_L	0 0 1
7 7 6	16_H	6 6 6 7
3 0	17_L	1 1 2 2 3
7 7 5	17_H	9
3 1 0	18_L	
9	18_H	

Key: 15_L | 1 = 151 cm

It is likely that Group 1 is the boys.

5 (a) Gold Coast Suns appear to be slightly taller. They have the tallest player and more players in the second tallest band. The Brisbane Lions have a lot of players in the 180–<185 band, which will bring down their average height.

(b) The mean and median would be the least you would require. Having the actual heights would be most useful.

(c) This extra information confirms that the Gold Coast Suns are slightly taller than the Brisbane Lions. In fact the statistics are very close.

Exercise 8.5

1 (a) $\frac{1}{8}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{1}{2}$ (e) $\frac{3}{8}$

2 $\frac{13}{27}$ 3 42

4 (a) 0.16 (b) 0.3 (c) 0.81

5 Elliot: 0.3; Kris: 0.25; Austin: 0.45

6 (a) B (b) D (c) B

7 B

8 (a) B (b) B (c) B

9 (a) $\frac{1}{12}$ (b) $\frac{5}{12}$ (c) $\frac{2}{3}$

10 $\frac{5}{9}$

11 black: 4; blue: 5; white: 2

Even though this gives 11 marbles, it is still the 'best estimate' for each colour.

12 (a) (i) 72.1 years (ii) 72.8 years (iii) 73.8 years

(iv) 76.7 years (v) 80 years (vi) 84.6 years

(b) As you get older your expected age at death increases.

(c) $\frac{1}{4}$ (half of a half)

(d) Insurance companies; it helps them set the premiums (prices) for policies.

13 (a) $\frac{1}{365}$ (b) $\frac{6}{73}$ (c) $\frac{12}{365}$ (d) $\frac{24}{73}$ (e) $\frac{337}{365}$

14 (a) $\frac{1}{6}$ (b) 0 (c) $\frac{1}{3}$ (d) $\frac{1}{2}$ (e) $\frac{2}{3}$ (f) 1

15 Students' own answers.

16 (a) 200 (b) 250 (c) 500

17 The coach is wrong. Pr(success) = $\frac{2}{3}$ for every free throw

18 (a) $\Pr(1) = \Pr(2) = \Pr(6) = \frac{1}{12}, \Pr(3) = \frac{1}{3}, \Pr(4) = \frac{1}{4}, \Pr(5) = \frac{1}{6}$

(b) 4 ones, 4 twos, 17 threes, 12 or 13 fours, 8 fives, 4 sixes

Open-ended—Sample answers

19 Examples of solutions:

(a) A = getting a multiple of 4

(b) B = getting an odd number

(c) C = not getting a composite number

20 (a) No, the probability of rolling any double number, whether it is a double 6, double 3 or double 1, is the same, $\frac{1}{36}$.

(b) Students' own answers.

Exercise 8.6

1 (a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $\frac{1}{2}$ (d) $\frac{5}{8}$

(e) $\frac{1}{2}$ (f) $\frac{5}{8}$ (g) $\frac{1}{2}$ (h) $\frac{5}{8}$

2 (a) Derek, Ahn and Ngaio

(b) Ahn, Ngaio, Samantha and Zarko

(c) Derek

(d) Maha, Erica, Phillipa, Derek, Ahn and Ngaio

(e)

	Sport	No sport
Sister	1	2
No sister	3	2

3 (a) $\frac{7}{9}$ (b) $\frac{5}{9}$ (c) $\frac{2}{3}$

4 (a) $\frac{1}{15}$ (b) $\frac{8}{15}$ (c) $\frac{3}{5}$ (d) $\frac{7}{30}$ (e) $\frac{1}{3}$ (f) $\frac{7}{30}$

5 B

6 (a) (i) no (ii) yes (iii) no

(iv) no (v) no (vi) no

(b) $\Pr(A) = \frac{1}{4}$ $\Pr(B) = \frac{3}{10}$ $\Pr(C) = \frac{1}{4}$ $\Pr(D) = \frac{3}{10}$

7 (a) A (b) D (c) B

8 (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{7}{26}$ (e) $\frac{3}{13}$ (f) $\frac{3}{52}$

(g) $\frac{15}{52}$ (h) $\frac{12}{13}$ (i) $\frac{3}{4}$ (j) $\frac{11}{13}$ (k) $\frac{4}{13}$ (l) $\frac{15}{26}$

9 (a) $\frac{2}{13}$ (b) $\frac{5}{26}$ (c) $\frac{4}{13}$

(d) You can write the sample space or use Pr(either) = Pr(left) + Pr(right) – Pr(both)

(e) $\frac{1}{26}$ (f) $\frac{11}{13}$ (g) $\frac{21}{26}$ (h) $\frac{9}{13}$

10 (a) Pr(not C and not D) = 1 – Pr(C or D)

(b) Pr(D) = 1 – Pr(C) – Pr(not C and not D) + Pr(C and D)

11 (a)

		Red die					
		1	2	3	4	5	6
Blue die	1	1, 1	2, 1	3, 1	4, 1	5, 1	6, 1
	2	1, 2	2, 2	3, 2	4, 2	5, 2	6, 2
	3	1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
	4	1, 4	2, 4	3, 4	4, 4	5, 4	6, 4
	5	1, 5	2, 5	3, 5	4, 5	5, 5	6, 5
	6	1, 6	2, 6	3, 6	4, 6	5, 6	6, 6

(b) (i) $\frac{1}{6}$ (ii) $\frac{1}{6}$ (iii) $\frac{5}{36}$ (iv) $\frac{5}{18}$

(c) (i) $\frac{11}{36}$ (ii) $\frac{5}{18}$ (iii) $\frac{7}{18}$ (iv) $\frac{5}{18}$

(v) $\frac{7}{18}$ (vi) $\frac{5}{12}$

(d) yes, but C and D are the only mutually exclusive pair

Open-ended—Sample answers

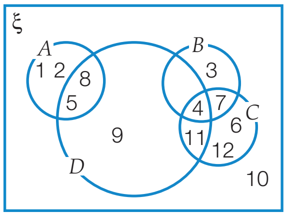
12 (a) possible sets:

$B = \{3, 4, 7\}$

$C = \{4, 6, 7, 11, 12\}$

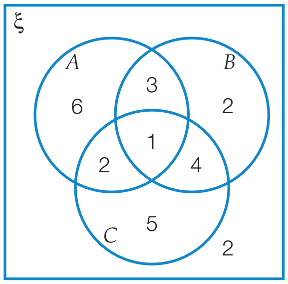
(b) A possible set to go with the answer given for **(a)** is $D = \{4, 5, 8, 9, 11\}$.

(c)



(d) (i) $\frac{7}{12}$ (ii) $\frac{5}{12}$ (iii) $\frac{1}{2}$ (iv) $\frac{1}{2}$

13 (a)



(b) possible answers:

(i) $\frac{12}{25}$ (ii) $\frac{2}{5}$ (iii) $\frac{12}{25}$

(c) possible answers:

(i) $\frac{13}{25}$ (ii) $\frac{3}{5}$ (iii) $\frac{13}{25}$

(d) to match possible answers given above:

(i) $\frac{4}{25}$ (ii) $\frac{17}{25}$ (iii) $\frac{4}{25}$

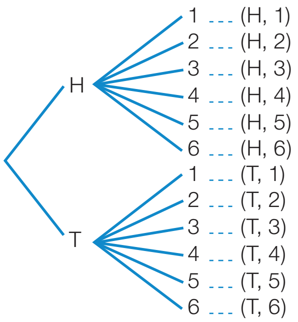
Exercise 8.7

1 (a)

		Second spinner				
		1	2	3	4	5
First spinner	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)

(b) (i) $\frac{1}{5}$ (ii) $\frac{1}{25}$ (iii) $\frac{3}{5}$ (iv) $\frac{21}{25}$

2 (a)



(b) (i) $\frac{1}{12}$ (ii) $\frac{1}{4}$ (iii) $\frac{1}{3}$ (iv) $\frac{1}{6}$ (v) $\frac{5}{12}$

3 (a)

	A_1	A_2	A_3	A_4	B_1	B_2
A_1	—	(A_1, A_2)	(A_1, A_3)	(A_1, A_4)	(A_1, B_1)	(A_1, B_2)
A_2	(A_2, A_1)	—	(A_2, A_3)	(A_2, A_4)	(A_2, B_1)	(A_2, B_2)
A_3	(A_3, A_1)	(A_3, A_2)	—	(A_3, A_4)	(A_3, B_1)	(A_3, B_2)
A_4	(A_4, A_1)	(A_4, A_2)	(A_4, A_3)	—	(A_4, B_1)	(A_4, B_2)
B_1	(B_1, A_1)	(B_1, A_2)	(B_1, A_3)	(B_1, A_4)	—	(B_1, B_2)
B_2	(B_2, A_1)	(B_2, A_2)	(B_2, A_3)	(B_2, A_4)	(B_1, B_2)	—

(b) (i) $\frac{2}{5}$ (ii) $\frac{1}{15}$ (iii) $\frac{8}{15}$

4 (a) sliders, lamb; sliders, fish; sliders, beef; sliders, pork; sliders, poultry; oysters, lamb; oysters, fish; oysters, beef; oysters, pork; oysters, poultry; kimchi, lamb; kimchi, fish; kimchi, beef; kimchi, pork, kimchi, poultry; edamame, lamb; edamame, fish; edamame, beef; edamame, pork; edamame, poultry

(b) (i) $\frac{1}{20}$ (ii) $\frac{2}{5}$ (c) $\frac{3}{5}$

5 (a) A (b) A (c) D

6 (a) French and German, French and Metalwork, French and Bonus Maths, French and Drama, Art and German, Art and Metalwork, Art and Bonus Maths, Art and drama, Literature and German, Literature and Metalwork, Literature and Bonus Maths, Literature and Drama

(b) (i) $\frac{1}{12}$ (ii) $\frac{5}{12}$ (iii) $\frac{2}{3}$ (iv) $\frac{1}{2}$ (v) $\frac{1}{2}$

(c) $\frac{1}{3}$

7 (a) 12

(b) (i) $\frac{1}{12}$ (ii) $\frac{1}{3}$ (iii) $\frac{1}{4}$

(c) 3

8 (a) (i) $\frac{1}{15}$ (ii) $\frac{4}{15}$ (iii) $\frac{4}{5}$ (iv) $\frac{2}{5}$

(b) (i) $\frac{1}{9}$ (ii) $\frac{2}{9}$ (iii) $\frac{2}{3}$ (iv) $\frac{4}{9}$

9 (a) H, T (b) 2 (c) HH, HT, TH, TT (d) 4

(e) HHH, HHT, HTH, HTT, THH, THT, TTH, TTT

(f) 8

(g) Yes. To find the number of events for a combination you multiply together the number possible for each event within the combination.

10 (a) $3^8 = 6561$ (b) $\frac{1}{6561}$

(c) There is only one way it can happen: GGGGGGGG.

(d) 'What is the probability that all the faces will be blue?' and 'What is the probability that all the faces will be red?'

(e) The complementary question to the statement in (b) would actually be: 'What is the probability that not all of the faces are green?'

Open-ended—Sample answers

11 Students' own answers.

12 Students' own answers.

Challenge 8

1 (a) correct (b) correct (c) correct (d) incorrect

2 (a) 2, 4, 6, 6, 7 (other answers may be possible)

(b) 11

3 The factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48.

3, 6, 12, 24, 48 all have factors of 3; the other five factors do not have a factor of 3. Therefore, the probability of not having a factor of 3 is $\frac{1}{2}$.

4

+	2	3	5	7	11	13
2	4	5	7	9	13	15
3	5	6	8	10	14	16
5	7	8	10	12	16	18
7	9	10	12	14	18	20
11	13	14	16	18	22	24
13	15	16	18	20	24	26

(a) $\frac{13}{18}$

(b) $\frac{5}{6}$, except when the numbers are the same, one number is always larger.

5 9 black, 6 white, 3 green; or 12 black, 4 white, 2 green; or 15 black, 2 white, 1 green

6 (a) $\frac{3}{20} \times \frac{3}{20} = \frac{9}{400} = 0.022\,5$

(b) $\frac{17}{20} \times \frac{2}{5} = \frac{17}{50} = 0.34$

7 (a) At first junction he has three choices, one of which leads to A. At the second junction he has two choices, one of which leads to A. The probability that he ends up at $A = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$.

(b) Probability that he ends up at $B = \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} = \frac{11}{18}$.

(c) Probability that he ends up at $C = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

(d) Probability that he ends up at $D = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$.

Chapter review 8

1 (a) continuous (b) discrete

2 For every group except sponsors the existing jumper has the greatest level of support. If the sponsors are more important than the other groups, then a decision to go with new design #1 may be justified, but the sponsors are not really overwhelming in their support for this. The current design should probably be retained.

3 A

4 (a) (i) 75.13 (ii) 78 (iii) negative skew

(b) (i) 251.83 (ii) 251 (iii) positive skew

5 (a) The sales of the Falcon have fallen considerably. In 2004 there were approximately 64 000 sold, and by 2010 this had fallen to approximately 30 000. So, the number sold had more than halved. In this same time the sales of the Ford Territory have fluctuated but have usually been in the 10 000–15 000 range.

(b) The sales of the Falcon fell initially before seeing a resurgence in 2003, but have then fallen steadily, although up to the end of 2010 the number seeemed to have become relatively stable. The value for 2011 sees another fall. In the same time the sales of the Falcon Ute rose steadily to a peak in 2004 and then fell at about the same slight rate as they rose.

(c) Falcon: 65 000; Falcon Ute: 20 000; Territory: 13 000

(d) 31% (will depend on estimates)

(e) 20% (will depend on estimates)

(f) Falcon: 30 000; Falcon Ute: 10 000; Territory: 11 000; 33%; 37%

(g) The percentage comparisions for both the Falcon Ute and the Territory have increased. The Territory has almost doubled its comparative value.

(h) The total number has fallen enormously. The peak year was 2004 with approximately 98 000 vehicles and the worst year is 2011. The total number of vehicles sold in these categories has more than halved.

6 $\frac{10}{13}$

7 (a) $\frac{3}{20}$ (b) $\frac{1}{4}$

8 (a) $\frac{3}{8}$ (b) $\frac{1}{2}$ (c) $\frac{5}{8}$ (d) $\frac{1}{2}$ (e) $\frac{7}{8}$

9 (a) 36

(b) (i) $\frac{1}{9}$ (ii) $\frac{11}{12}$ (iii) $\frac{3}{4}$ (iv) $\frac{1}{4}$

10 (a) Coast Wattle: 562; Bidgee Widgee: 2250; Karkalla: 1312; White Correa: 750; Running Postman: 1125; Moonah: 375

(b) It is probably not reasonable to apply this across the entire reserve because different areas could support different vegetation types. To improve reliability you should take a few more samples from widely spaced areas of the reserve.

11 (a)

Male	Stem	Female
	12	9
3	13	
5	14	9
2	15	0 1 3 3 4 5 6 7 8 9
9 6 5 3 3	16	0 3 4 4 5 5 6 6 8 9
8 8 8 6 5 4 3 1	17	0 0 0 1 1 2 3
2 0	18	0
8 1	19	

Key: 12 | 9 = 129

(b) The arm span for boys is longer than for girls.

(c) Not in this case, because the longest three arm spans are all boys.

(d) No, this does not mean there are more girls than boys; it is just the way this sample was constructed.

(e) You would expect the values to be lower for Year 7 students and higher for Year 11 students. For Year 11 you would probably expect that the boys would have larger arm spans than the girls. Things are not so clear for year 7 students because the younger male students are less likely to be larger than the younger female students.

12 The spread of the population of New Zealand shows a bump around the ages 35–54. This is where the so-called 'baby boomers' can be found in the graph. The rest of the graph shows that the population is relatively stable until about the age of 65, when things start to decline fairly rapidly. This is the age when the number of deaths starts to increase. By the late 70s or 80s there is only a small number of people left.

13 (a) (i) $\frac{1}{29}$ (ii) $\frac{14}{29}$ (iii) $\frac{28}{29}$ (iv) $\frac{10}{29}$

(b) (i) 0.03, 3% (ii) 0.48, 48%

(iii) 0.97, 97% (iv) 0.34, 34%

14 (a) $\frac{1}{36}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$

15 (a)

		Green die				
		1	2	3	4	5
Blue die	1	1, 1	2, 1	3, 1	4, 1	5, 1
	2	1, 2	2, 2	3, 2	4, 2	5, 2
	3	1, 3	2, 3	3, 3	4, 3	5, 3
	4	1, 4	2, 4	3, 4	4, 4	5, 4
	5	1, 5	2, 5	3, 5	4, 5	5, 5

(b) (i) $\frac{2}{25}$ (ii) $\frac{19}{25}$ (iii) $\frac{2}{5}$ (iv) $\frac{21}{25}$
(v) $\frac{19}{25}$ (vi) $\frac{21}{25}$ (vii) $\frac{4}{25}$ (viii) $\frac{4}{5}$

(c) no; Pr(5 on green) + Pr(3 on blue) ≠ Pr(5 on green or 3 on blue)

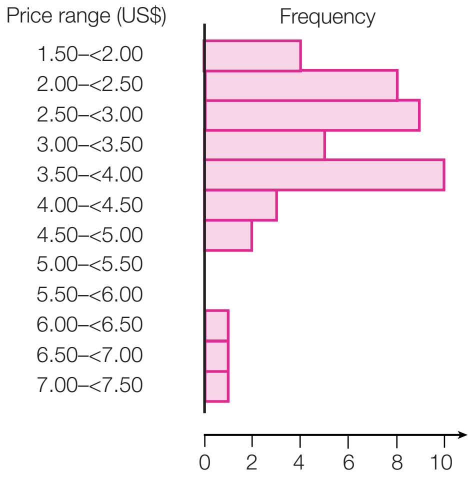
16 (a) Uruguay, US\$1.74

(b) Norway, US\$7.20

(c) US\$5.46

(d) Only 50 cents or \$1 would be sensible intervals, and 50 cents gives 12 intervals, which is just a bit larger than usual. However, there are lots of prices in the \$2–\$4 range, so this helps separate those prices.

(e)



(f) Students' own answers.

(g) Students' own answers.

(h) Students' own answers.

(i) A sample of 10 would be expected to do a better job, but this may not always be the case.

Numeracy practice 8

1 C

2 (a) discrete (b) continuous

(c) continuous (d) discrete

3 C 4 B 5 A 6 B

7 D

Chapter 9

Recall 9

1 (a) (i) 4 (ii) 5 (iii) 10

(b) (i) 10 (ii) 18 (iii) 0 (iv) 0

2 (a) (i) \$16 (ii) \$75

(b) (i) 4.6 m (ii) 3.2 m

3 (a) $3x^2 + 12x$ (b) $x^2 + 7x + 10$

(c) $x^2 - 4x - 21$ (d) $x^2 - 25$

4 (a) $x(x + 2)$ (b) $(x - 4)(x + 4)$

(c) $(x + 6)^2$

5 (a) $(x + 2)(x + 5)$ (b) $(x - 2)(x - 3)$

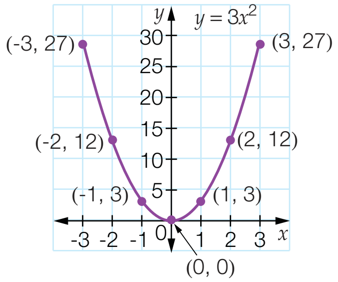
(c) $(x - 14)(x + 1)$ (d) $(x - 7)(x - 4)$

6 (a) $x = \frac{2}{3}$ (b) $x = \frac{13}{5}$ (c) $x = -1$

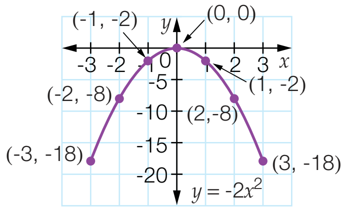
Exercise 9.1

1 A, B, E

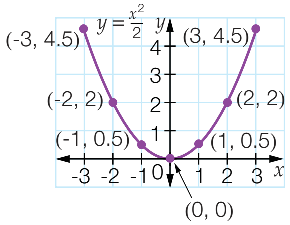
2 (a)



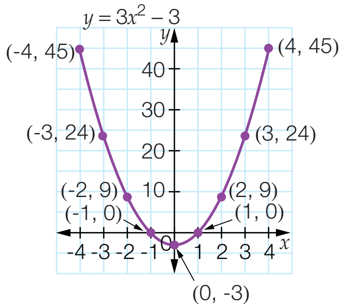
(b)



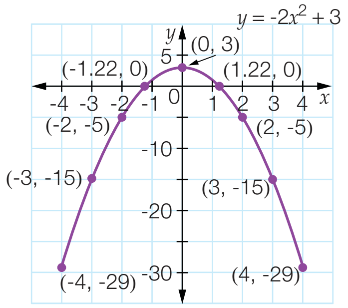
(c)



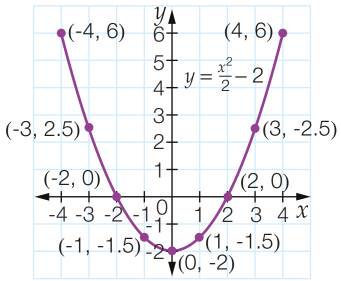
3 (a)



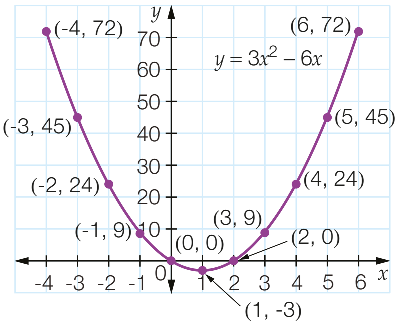
(b)



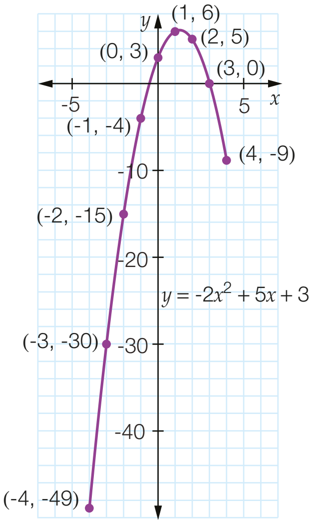
(c)



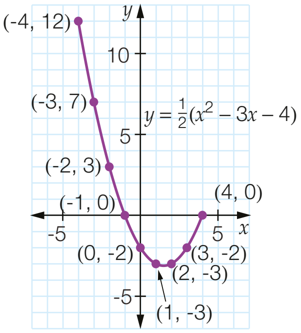
(d)



(e)



(f)



4 **(a)** **(i)** (1, 4) **(ii)** maximum **(iii)** $x = 1$

(iv) (0, 3) **(v)** (-1, 0) and (3, 0)

(b) **(i)** $(-\frac{3}{2}, -1)$ **(ii)** minimum **(iii)** $x = -\frac{3}{2}$

(iv) (0, 0) **(v)** (-3, 0) and (0, 0)

(c) **(i)** (3, -4)

(ii) minimum turning point

(iii) $x = 3$

(iv) (0, 5)

(v) (1, 0) and (5, 0)

(d) **(i)** (-3, 1)

(ii) maximum turning point

(iii) $x = -3$

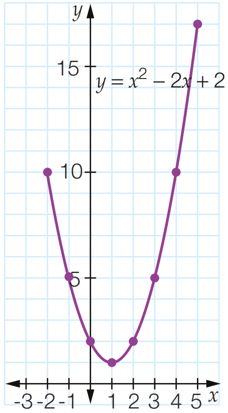
(iv) (0, -8)

(v) (-4, 0) and (-2, 0)

5 **(a)** **(i)**

x	-2	-1	0	1	2	3	4	5
y	10	5	2	1	2	5	10	17

(ii)



(iii) (1, 1) **(iv)** minimum turning point

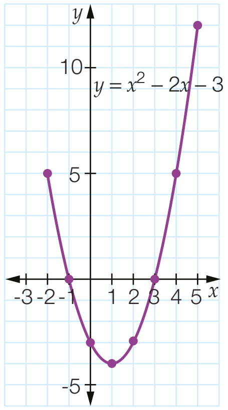
(v) $x = 1$ **(vi)** (0, 2)

(vii) no x-intercepts **(viii)** $x = -1$ or $x = 3$

(b) **(i)**

x	-2	-1	0	1	2	3	4	5
y	5	0	-3	-4	-3	0	5	12

(ii)



(iii) (1,-4) **(iv)** minimum turning point

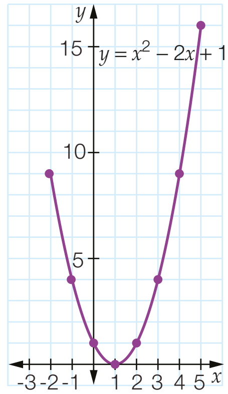
(v) $x = 1$ **(vi)** (0, -3)

(vii) (-1, 0) and (3, 0) **(viii)** $x = -2$ or $x = 4$

(c) **(i)**

x	-2	-1	0	1	2	3	4	5
y	9	4	1	0	1	4	9	16

(ii)



(iii) (1, 0) **(iv)** minimum turning point

(v) $x = 1$ **(vi)** (0, 1)

(vii) (1, 0) **(viii)** $x = -1.2$ or $x = 3.2$

6 **(a)** B **(b)** D **(c)** B **(d)** C **(e)** C

7 D

8 **(a)** C **(b)** C **(c)** A

9 **(a)** Both graphs concave up. If you move the graph of $y = 3x^2$ down 3 units, you get the graph of $y = 3x^2 - 3$.

(b) Both graphs concave down. If you move the graph of $y = -2x^2$ up 3 units, you get the graph of $y = -2x^2 + 3$.

(c) Both graphs concave up. If you move the graph of $y = \frac{x^2}{2}$ down 2 units you get the graph of $y = \frac{x^2}{2} - 2$.

10 **(a)** Both graphs concave up. The graph of $y = 3x^2 - 6x$ has the same dilation as $y = 3x^2$, but is moved to the right and down compared to $y = 3x^2$.

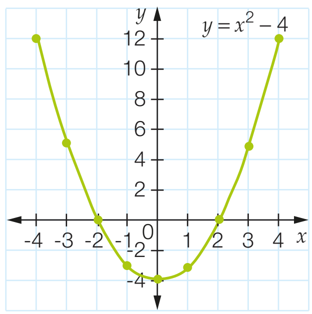
(b) Both graphs concave down. The graph of $y = -2x^2 + 5x + 3$ has the same dilation as $y = -2x^2$, but is moved to the right and upwards compared to $y = -2x^2$.

(c) Both graphs concave up. The graph of $y = \frac{x^2}{2} - \frac{3x}{2} - 2$ has the same dilation as $y = \frac{x^2}{2}$, but is moved to the right and down compared to $y = \frac{x^2}{2}$.

11 **(a)**

x	-4	-3	-2	-1	0	1	2	3	4
y	12	5	0	-3	-4	-3	0	5	12
(x, y)	(-4, 12)	(-3, 5)	(2, 0)	(-1, -3)	(0, -4)	(1, -3)	(2, 0)	(3, 5)	(4, 12)

(b)



(c) (0, -4) **(d)** minimum **(e)** $x = 0$

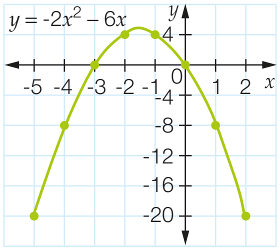
(f) (0, -4) **(g)** (-2, 0) and (2, 0) **(h)** 8.3

(i) -1.4; 1.4 **(j)** $x = -3$ or 3

12 **(a)**

x	-5	-4	-3	-2	-1	0	1	2
y	-20	-8	0	4	4	0	-8	-20

(b)



(c) $\left(-1\frac{1}{2}, 4\frac{1}{2}\right)$ maximum turning point

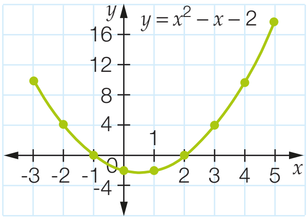
(d) (-3, 0) (0, 0); (0, 0) **(e)** -2; -1

(f) 4; $x = -2, -1$ **(g)** $x = -4, 1$ **(h)** $x = -3, 0$

13 **(a)**

x	-3	-2	-1	0	1	2	3	4	5
y	10	4	0	-2	-2	0	4	10	18

(b)



(c) $\left(\frac{1}{2}, -2\frac{1}{4}\right)$; minimum turning point

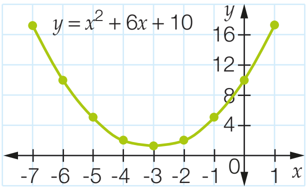
(d) (-1, 0); (2, 0); (0, -2)

(e) **(i)** $x = -1, 2$ **(ii)** $x = -2, 3$ **(iii)** $x \approx -1.6, 2.6$

(iv) $x = 0.5$ **(v)** no real x-values

(vi) $x \approx -1.4, 2.4$

14 **(a)**

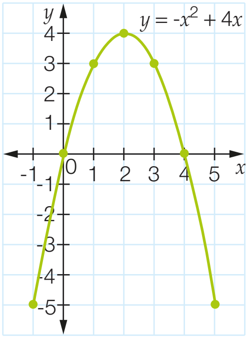


(b) (-3, 1) **(c)** no x-intercepts, (0, 10)

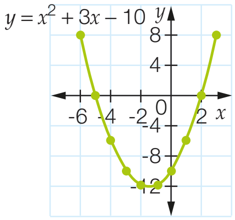
(d) **(i)** $x = -3$ **(ii)** $x = -6, 0$

(iii) no real x-values **(iv)** $x \approx -6.7, 0.7$

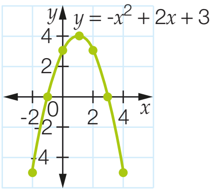
15 **(a)** **(i)**



(ii)



(iii)



(b) **(i)** $x = 0, 4$ **(ii)** $x = -5, 2$ **(iii)** $x = 1$

(iv) $x = 2$ **(v)** $x = -6, 3$ **(vi)** $x = -1, 3$

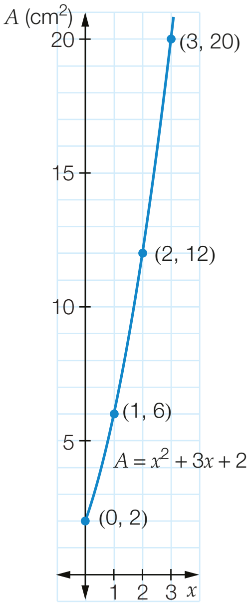
(vii) $x \approx 0.6, 3.4$ **(viii)** $x \approx -5.5, 2.5$ **(ix)** $x \approx -1.4, 3.4$

16 **(a)** $A = x^2 + 3x + 2$

(b)

x	0	1	2	3
A	2	6	12	20

(c)



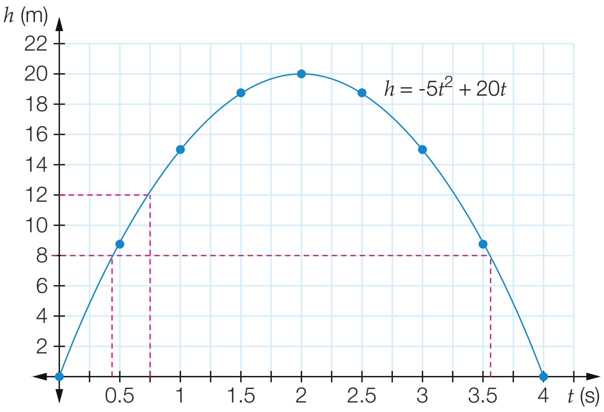
(d) $x = 1.4$. This should be the only value, because it is unrealistic for x to be negative.

(e) length = 3.4 cm, width = 2.4 cm

17 **(a)**

t	0	0.5	1	1.5	2	2.5	3	3.5	4
h	0	8.75	15	18.75	20	18.75	15	8.75	0

(b)

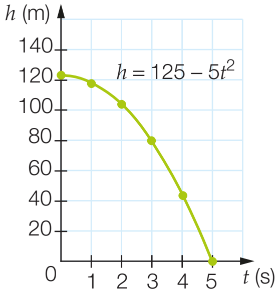


(c) 20 m **(d)** 2 s **(e)** ≈ 0.75 s **(f)** 4 s **(g)** ≈ 3.2 s

18 **(a)**

t	0	1	2	3	4	5
h	125	120	105	80	45	0

(b)

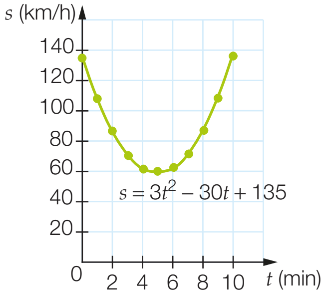


(c) 125 m **(d)** 5 s **(e)** 94 m **(f)** 31 m

(g) 3.2 s **(h)** 4.5 s **(i)** 35 m

(j) fifth second i.e. between $t = 4$ and $t = 5$ s

19 **(a)**

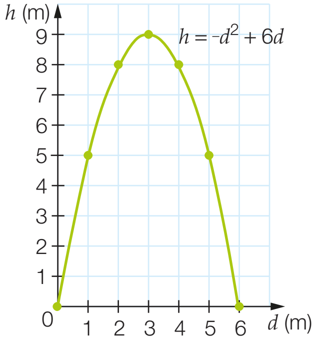


(b) 135 km/h **(c)** 60 km/h **(d)** 72 km/h

(e) 72 km/h **(f)** 2.4 min; 7.6 min

(g) 0.5 min; 9.5 min **(h)** $0 \leq t < 1.35$ and $8.65 < t \leq 10$

20 **(a)**

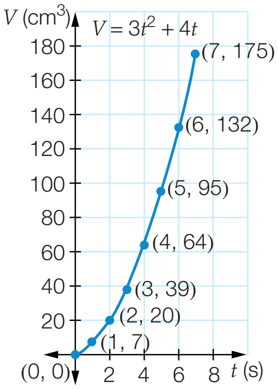


(b) 6 m (c) 9 m (d) 5 m

(e) 4 m (f) 5.75 m (g) 3.6 m

(h) (i) yes (ii) no

21 (a)



(b) 0 (c) 64 cm^3 (d) 6 s (e) 20 cm^3

(f) $0 \le t \le 6, 0 \le V \le 132$

Open-ended—Sample answers

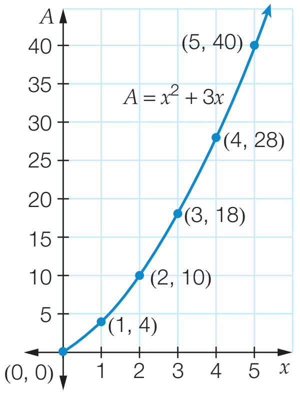
22 (a) Students' own answers; any expressions such as $x^2 + 2x$, $3x^2 - 6x$, $\frac{1}{2}(x^2 - 3x - 4)$.

(b) Students' own answers; any expressions such as $y + 4$, $x^2 + y^2$, $4x^3 + 3x^2 + 2x$.

23 Students' own answers.

24 (a) $A = x^2 + 3x$, where x is the width of the rectangle.

(b)



(c) If $A = 10$, width = 2 and length = 5.

25 (a) Josh thought that $-x^2 = (-x)^2$.

(b) Josh thought that $2x^2 = (2x)^2$.

Exercise 9.2

1 (a) $x = 0$ or $x = 1$ (b) $x = 0$ or $x = -8$ (c) $x = 0$ or $x = -5$

(d) $x = 1$ or $x = 4$ (e) $x = 2$ or $x = 6$ (f) $x = 3$ or $x = 9$

(g) $x = -1$ or $x = 4$ (h) $x = -7$ or $x = 3$ (i) $x = -2$ or $x = 5$

(j) $x = -6$ or $x = -4$ (k) $x = -7$ or $x = -2$

(l) $x = -10$ or $x = -3$ (m) $x = 4$

(n) $x = -7$ (o) $x = 9$

2 (a) $x = 0$ or $x = 6$ (b) $x = 0$ or $x = 1$ (c) $x = 0$ or $x = 8$

(d) $x = -7$ or $x = 0$ (e) $x = -5$ or $x = 0$ (f) $x = -1$ or $x = 0$

(g) $x = 0$ or $x = 4$ (h) $x = 0$ or $x = 3$ (i) $x = 0$ or $x = 4$

(j) $x = -9$ or $x = 0$ (k) $x = -9$ or $x = 0$ (l) $x = -\frac{1}{3}$ or $x = 0$

(m) $x = 0$ or $x = -\frac{1}{9}$ (n) $x = 0$ or $x = \frac{1}{4}$ (o) $x = 0$ or $x = 11$

3 (a) $x = -3$ or $x = 3$ (b) $x = -1$ or $x = 1$ (c) $x = -4$ or $x = 4$

(d) $x = -\frac{5}{3}$ or $x = \frac{5}{3}$ (e) $x = -\frac{6}{5}$ or $x = \frac{6}{5}$

(f) $x = -\frac{9}{7}$ or $x = \frac{9}{7}$ (g) $x = -7$ or $x = 7$ (h) $x = -3$ or $x = 3$

(i) $x = -2$ or $x = 2$ (j) $x = -5$ or $x = 5$ (k) $= -\frac{7}{2}$ or $x = \frac{7}{2}$

(l) $x = -\frac{10}{3}$ or $x = \frac{10}{3}$ (m) $x = -4$ or $= 4$

(n) $x = -6$ or $x = 6$ (o) $x = -3$ or $x = 3$

(p) $x = -\sqrt{11}$ or $x = \sqrt{11}$ (q) $x = -\sqrt{29}$ or $x = \sqrt{29}$

(r) $x = -\sqrt{47}$ or $x = \sqrt{47}$

4 (a) $x = 3$ (b) $x = 1$ (c) $x = 4$

(d) $x = - 5$ (e) $x = -6$ (f) $x = -9$

(g) $x = 7$ (h) $x = -3$ (i) $x = 2$

(j) $x = 2$ (k) $x = 6$ (l) $x = 5$

(m) $x = 3$ (n) $x = 1$ (o) $x = 7$

5 (a) C (b) B (c) B (d) A

6 B 7 C

8 Let n be the number, $n^2 + 3n = 7n$; $n^2 - 4n = 0$; $n = 4$

9 $x = -9$ or $x = 1$

10 C 11 9 m 12 (-1, 0), (4, 0)

13 (a) (12, 0), (-12, 0) (b) (0, 0), (-4, 0) (c) (6, 0)

14 (a) $x^2 - 4x - 12$ (b) $x = -2$ or $x = 6$

(c) The product of the solutions equals the constant term.

(d) $x^2 - 4x - 5$ (e) $x = -1$ or $x = 5$

(f) The product of the solutions equals the constant term.

(g) $x = 0$ or $x = 4$ (h) 0

(i) The product of the solutions equals the constant term.

(j) $x^2 - 4x + 3$ (k) $x = 1$ or $x = 3$

(l) The product of the solutions equals the constant term.

(m) $x = 2$

(n) The product of the repeated solutions equals the constant term.

(o) The constant term represents the y-intercept.

(p) All equations have the first two terms, x^2 and $-4x$, in common.

(q) The solution tells you where the graphs intersect the x-axis.

(r) As the constant term increases, the distance between the x-intercepts decreases until the case where there is only one x-intercept.

(s) When $c > 4$, the equation $x^2 - 4x + c = 0$ has no solution. Hence, a graph of $y = x^2 - 4x + c$, $c > 4$ has no x-intercepts.

Open-ended—Sample answers

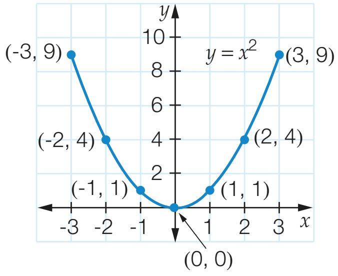
15 Let $x = 0$ or $x = 2$; two possible equations are $y = x^2 - 2x$ and $y = 2x^2 - 4x$.

16 Louise needed to use the null factor law (i.e. she had to first set the equation to equal 0).

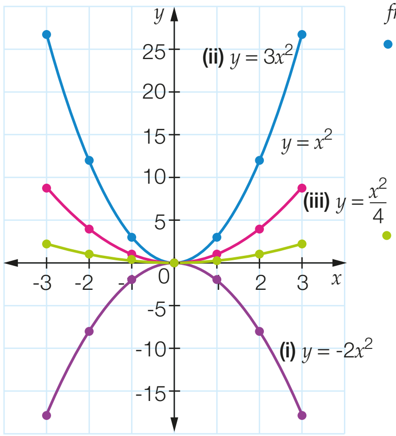
$$0 = x^2 - 8x + 16$$
$$0 = (x - 4)^2$$
$$x - 4 = 0$$
$$x = 4$$

Exercise 9.3

1 **(a)**



(b)



from left to right

● (-3, 27)	● (-3, 9)
(-2, 12)	(-2, 4)
(-1, 3)	(-1, 1)
(0, 0)	(0, 0)
(1, 3)	(1, 1)
(2, 12)	(2, 4)
(3, 27)	(3, 9)

● (-3, 2.25)	● (-3, -18)
(-2, 1)	(-2, -8)
(-1, 0.25)	(-1, -2)
(0, 0)	(0, 0)
(1, 0.25)	(1, -2)
(2, 1)	(2, -8)
(3, 2.25)	(3, -18)

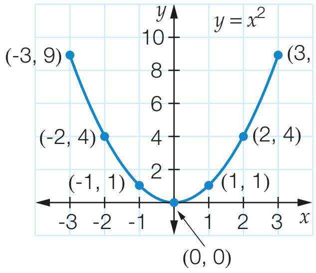
(c) **(i)** $y = -2x^2$ is reflected in the x-axis and narrower than $y = x^2$

(ii) $y = 3x^2$ is narrower than $y = x^2$

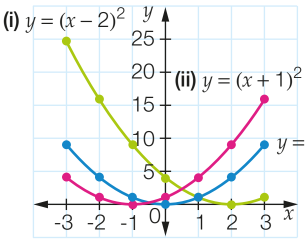
(iii) $y = \frac{1}{4}x^2$ is wider than $y = x^2$

(d) **(i)** -2 **(ii)** 3 **(iii)** $\frac{1}{4}$

2 **(a)**



(b)



from left to right

● (-3, 4)	● (-3, 9)	● (-3, 25)
(-2, 1)	(-2, 4)	(-2, 16)
(-1, 0)	(-1, 1)	(-1, 9)
(0, 1)	(0, 0)	(0, 4)
(1, 4)	(1, 1)	(1, 1)
(2, 9)	(2, 4)	(2, 0)
(3, 16)	(3, 9)	(3, 1)

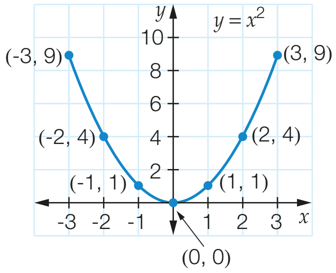
(c) **(i)** The graph of $y = x^2$ has been translated 2 units to the right to give the graph of $y = (x - 2)^2$.

(ii) The graph of $y = x^2$ has been translated 1 unit to the left to give the graph of $y = (x + 1)^2$.

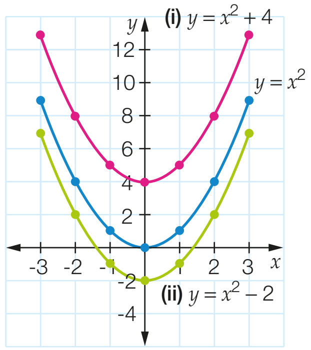
(d) **(i)** (2, 0). The x-value of the intercept, 2, is the number subtracted from x before squaring in the equation.

(ii) (-1, 0). The x-value of the intercept, -1, is the number subtracted from x before squaring in the equation.

3 **(a)**



(b)



from left to right

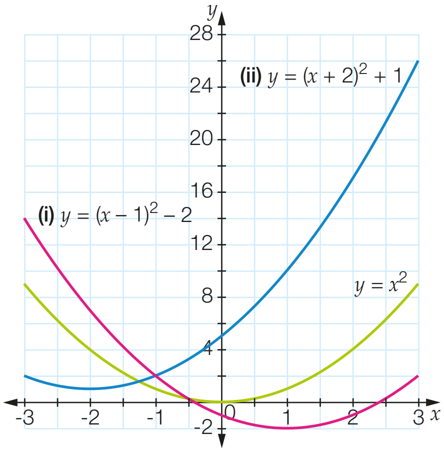
● (-3, 13)	● (-3, 9)	● (-3, 7)
(-2, 8)	(-2, 4)	(-2, 2)
(-1, 5)	(-1, 1)	(-1, -1)
(0, 4)	(0, 0)	(0, -2)
(1, 5)	(1, 1)	(1, -1)
(2, 8)	(2, 4)	(2, 2)
(3, 13)	(3, 9)	(3, 7)

(c) **(i)** The graph of $y = x^2$ has been translated 4 units upwards to give the graph of $y = x^2 + 4$.

(ii) The graph of $y = x^2$ has been translated 2 units downwards to give the graph of $y = x^2 - 2$.

(d) **(i)** (0, 4) **(ii)** (0, -2)

4 **(a)–(b)**



(c) **(i)** The graph of $y = (x - 1)^2 - 2$ is the graph of $y = x^2$ translated 1 unit right and 2 units down.

(ii) The graph of $y = (x + 2)^2 + 1$ is the graph of $y = x^2$ translated 2 units left and 1 unit up.

(d) **(i)** $y = (x - 1)^2 - 2$ has a y-intercept of -1.

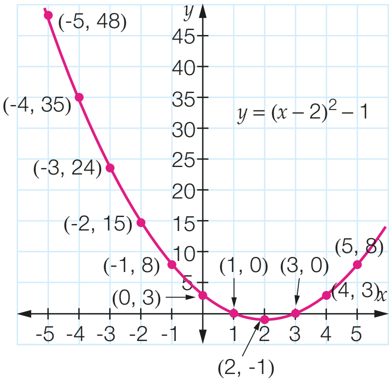
(ii) $y = (x + 2)^2 + 1$ has a y-intercept of 5.

(e) $y = x^2$ has one x-intercept.

(i) $y = (x - 1)^2 - 2$ has two x-intercepts.

(ii) $y = (x + 2)^2 + 1$ has zero x-intercepts.

5 **(a)**



(b) (2, -1)

(c) $h = 2, k = -1$. Yes

(d) Both equations expand to $x^2 - 4x + 3$. The x-coordinates of the x-intercepts are 1 and 3, so the value of the x-coordinate of the turning point is the same as the average of these two values.

(e) $y = -1$, which is the same as the y-coordinate of the turning point.

(f) $a = 1, b = -4, c = 3$. The value of $-\dfrac{b}{2a}$ is 2, which is the same as the value of the x-coordinate of the turning point.

6 **(a)** **(i)** narrower **(ii)** wider **(iii)** wider **(iv)** narrower

(b) **(iii)** and **(iv)**

(c) **(i)** 4 **(ii)** $\frac{1}{5}$ **(iii)** 7 **(iv)** $\frac{2}{3}$

(d) **(i)** narrower **(ii)** wider **(iii)** narrower **(iv)** wider

7 **(a)** **(i)** up 4 units **(ii)** (0, 4)

(b) **(i)** down 7 units **(ii)** (0, -7)

(c) **(i)** down one unit **(ii)** (0, -1)

(d) **(i)** up 2 units **(ii)** (0, 2)

(e) **(i)** down 8 units **(ii)** (0, -8)

(f) **(i)** up 5 units **(ii)** (0, 5)

(g) **(i)** up $\frac{1}{2}$ unit **(ii)** $(0, \frac{1}{2})$

(h) **(i)** down $\frac{3}{4}$ unit **(ii)** $(0, -\frac{3}{4})$

8 **(a)** **(i)** 5 units to the right

(ii) x-value of the x-intercept: $x = 5$; turning point: (5, 0)

(b) **(i)** 4 units to the left

(ii) x-value of the x-intercept: $x = -4$; turning point: (-4, 0)

(c) **(i)** 1 unit to the left

(ii) x-value of the x-intercept: $x = -1$; turning point: (-1, 0)

(d) **(i)** 7 units to the right

(ii) x-value of the x-intercept: $x = 7$; turning point: (7, 0)

(e) **(i)** 6 units to the left

(ii) x-value of the x-intercept: $x = -6$; turning point: (-6, 0)

(f) **(i)** 3 units to the right

(ii) x-value of the x-intercept: $x = 3$; turning point: (3, 0)

(g) **(i)** $\frac{2}{3}$ unit to the right

(ii) x-value of the x-intercept: $x = \frac{2}{3}$; turning point: $(\frac{2}{3}, 0)$

(h) **(i)** $\frac{3}{2}$ units to the left

(ii) x-value of the x-intercept: $x = -\frac{3}{2}$; turning point: $(-\frac{3}{2}, 0)$

9 **(a)** **(i)** dilation factor 2, moved right 1 up 3 **(ii)** (1, 3)

(b) **(i)** reflected in x-axis, dilation factor 3, moved left 2 down 1 **(ii)** (-2, -1)

(c) **(i)** dilation factor 4, moved left 3 down 2

(ii) (-3, -2)

(d) **(i)** dilation factor $\frac{1}{3}$, moved left 5 up 3 **(ii)** (-5, 3)

(e) **(i)** moved left 4, down 3 **(ii)** (-4, -3)

(f) **(i)** dilation factor 2, moved left 3 up $\frac{1}{2}$ **(ii)** $(-3, \frac{1}{2})$

10 **(a)** C **(b)** B **(c)** A **(d)** D

11 **(a)** A **(b)** D **(c)** D **(d)** A **(e)** D

12 **(a)** C **(b)** D **(c)** C **(d)** B **(e)** A **(f)** B

13 **(a)** no translation; (0, 0)

(b) translated 6 units to the right; (6, 0)

(c) translated 3 units up; (0, 3)

(d) translated 2 units to the left and 6 units down; (-2, -6)

(e) translated 1 unit to the right and 4 units up; (1, 4)

14

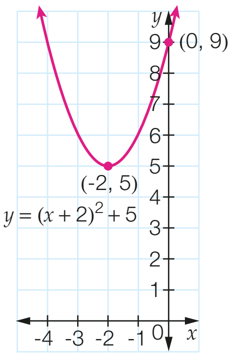
		Dilation	Reflection in x-axis	Translation in x-direction	Translation in y-direction
(a)	$y = (x + 1)^2 + 2$	–	–	1 unit left	2 units up
(b)	$y = (x - 3)^2 + 1$	–	–	3 units right	1 unit up
(c)	$y = 5x^2 + 4$	by factor of 5 (narrower)	–	–	4 units up
(d)	$y = -x^2 - 6$	–	invert	–	6 units down
(e)	$y = 2(x + 5)^2 - 3$	by factor of 2 (narrower)	–	5 units left	3 units down
(f)	$y = -(x - 2)^2 + 3$	–	invert	2 units right	3 units up
(g)	$y = -3(x + 6)^2 - 4$	by factor of 3 (narrower)	invert	6 units left	4 units down
(h)	$y = \frac{1}{2}(x - 3)^2 + 1$	by factor of $\frac{1}{2}$ (wider)	–	3 units right	1 unit up
(i)	$y = (x - 4)^2 - 5$	–	–	4 units right	5 units down
(j)	$y = (x + 2)^2 - 7$	–	–	2 units left	7 units down
(k)	$y = 3x^2 - 5$	by factor of 3 (narrower)	–	–	5 units down
(l)	$y = -x^2 - 2$	–	invert	–	2 units down
(m)	$y = 5(x - 6)^2 - 4$	by factor of 5 (narrower)	–	6 units right	4 units down
(n)	$y = -(x - 5)^2 + 2$	–	invert	5 units right	2 units up
(o)	$y = -4(x + 1)^2 - 3$	by factor of 4 (narrower)	invert	1 unit left	3 units down
(p)	$y = \frac{3}{4}(x + 4)^2 + 6$	by factor of $\frac{3}{4}$ (wider)	–	4 units left	6 units up

15 3: B, C, E

16 C

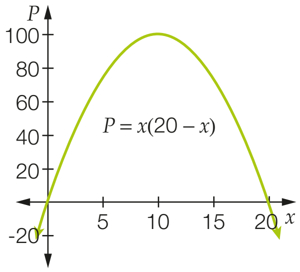
17 **(a)** $y = (x + 2)^2 + 5$ **(b)** $y = 9$

(c)



18 **(a)** $y = 5(x - 3)^2 + 2$ **(b)** $y = \frac{1}{3}(x - 4)^2 - 2$

19 **(a)** $y = 20 - x$ **(b)** $P = x(20 - x)$



(c) $x = 10$ and $y = 10$, $P = 100$

Open-ended—Sample answers

20 $y = 4(x - 2)^2 + 3$. Dilation factor of 4, translation 2 units to the right and 3 units up.

$y = -\frac{1}{2}(x + 3)^2 - 4$. Reflected in the x-axis and a dilation factor of $\frac{1}{2}$, translated 3 units to the left and 4 units down.

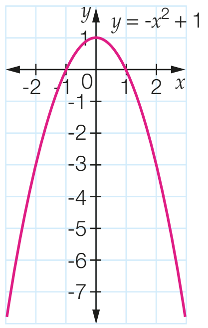
21 $y = (x - 5)^2 + 2$; $y = (x + 4)^2 + 7$; $y = -2(x - 5)^2 - 3$

Half-time 9

1 **(a)** $y = -x^2 + 1$

x	-3	-2	-1	0	1	2	3
y	-8	-3	0	1	0	-3	-8

(b)

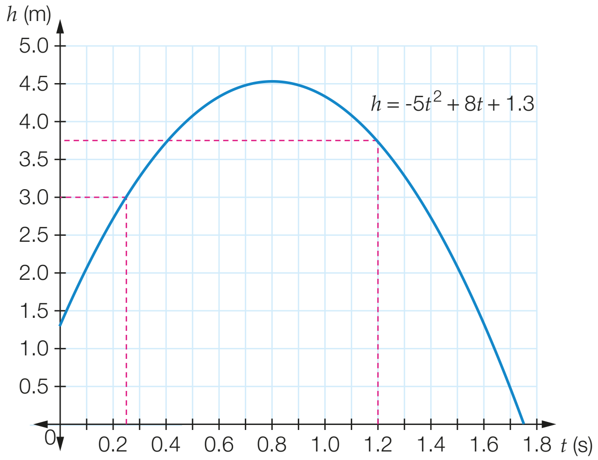


(c) (0, 1) **(d)** (0, 1) **(e)** (-1, 0) and (1, 0)

2 C

3 **(a)** $x = -4$ or $x = 0$ **(b)** $x = 0$ or $x = 3$ **(c)** $x = -2$ or $x = 3$

4 **(a)**



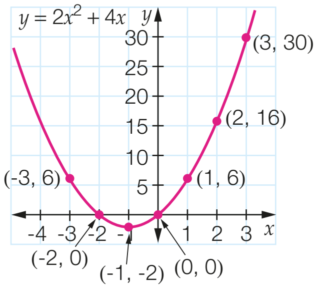
(b) 0.25 s **(c)** 3.7 m

5 **(a)** $x = -2$ **(b)** $x = -6$ **(c)** $x = 9$

6 **(a)** $y = 2x^2 + 4x$

x	-3	-2	-1	0	1	2	3
y	6	0	-2	0	6	16	30

(b)



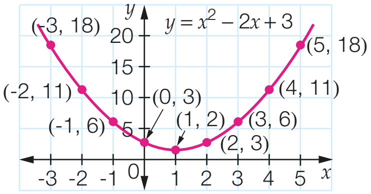
(c) (-1, -2) **(d)** minimum turning point

(e) x-intercepts (-2, 0) and (0, 0); y-intercept (0, 0)

7 **(a)** $x = 0$ or $x = 6$ **(b)** $x = -5$ **(c)** $x = 0$ or $x = 9$

8 **(a)** $x = -8$ or $x = 8$ **(b)** $x = -1$ **(c)** $x = 0$ or $x = 4$

9 **(a)**



(b) (1, 2)

(c) y-intercept is (0, 3); there are no x-intercepts

(d) **(i)** $x = 0$ or $x = 2$ **(ii)** $x = 1$

(iii) $x = -1$ or $x = 3$ **(iv)** No solutions

Exercise 9.4

1 **(a)** centre: (0, 0); radius = 5; $x^2 + y^2 = 25$

(b) centre: (3, 0); radius = 2; $(x - 3)^2 + y^2 = 4$

(c) centre: (0, -1); radius = 3; $x^2 + (y + 1)^2 = 9$

(d) centre: (0, 4); radius = 4; $x^2 + (y - 4)^2 = 16$

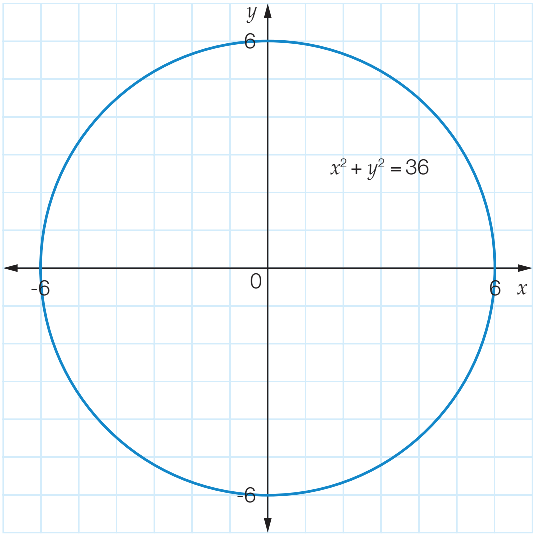
(e) centre: (-1, 3); radius = 3; $(x + 1)^2 + (y - 3)^2 = 9$

(f) centre: (1, 2); radius = 4; $(x - 1)^2 + (y - 2)^2 = 16$

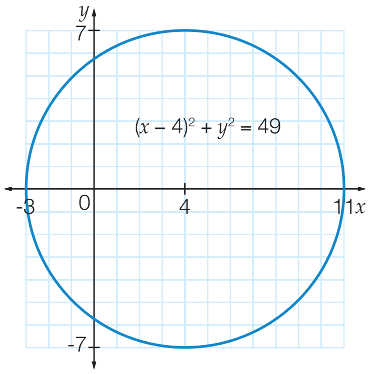
(g) centre (-3, -1); radius = 4; $(x + 3)^2 + (y + 1)^2 = 16$

(h) centre (-3, 0); radius = 5; $(x + 3)^2 + y^2 = 25$

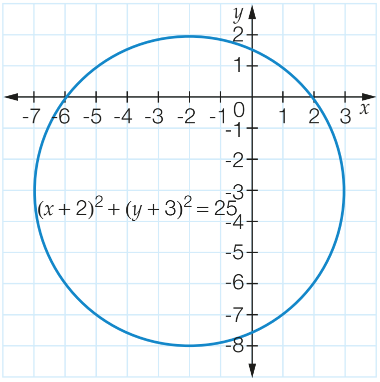
2 **(a)**



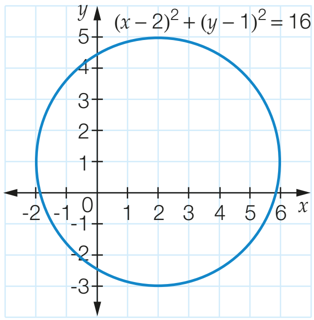
(b)



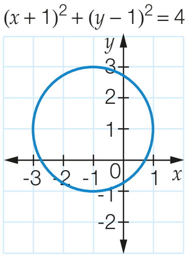
(c)



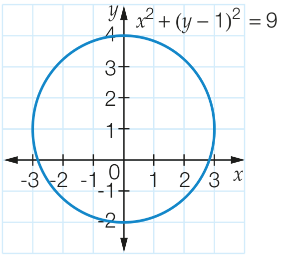
(d)



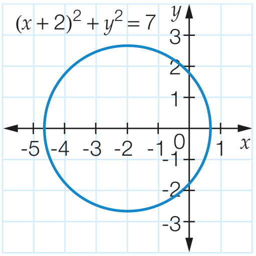
(e)



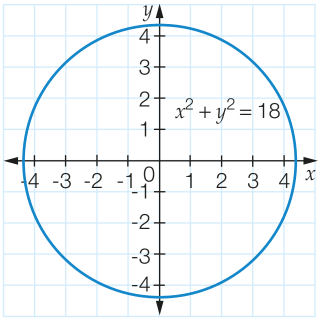
(f)



(g)



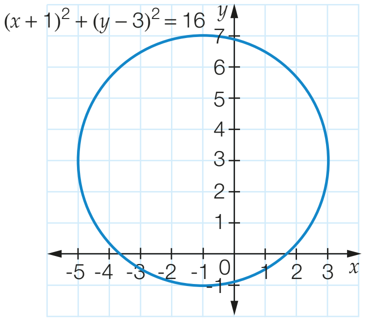
(h)



3 **(a)** centre = (-1, 3), radius = 4

(b) Points on circle are (3, 3), (-1, 7), (-5, 3), (-1, -1).

(c)



(d) **(i)** inside **(ii)** outside **(iii)** inside

4 B

5 **(a)** $x^2 + y^2 = 49$ **(b)** $x^2 + (y - 3)^2 = 100$

(c) $(x + 8)^2 + y^2 = 81$ **(d)** $(x - 5)^2 + (y + 2)^2 = 121$

6 **(a)** $(x - 4)^2 + (y - 3)^2 = 36$ **(b)** $(x + 6)^2 + (y - 4)^2 = 36$

(c) $(x - 7)^2 + y^2 = 36$ **(d)** $x^2 + (y + 5)^2 = 36$

(e) $x^2 + y^2 = 49$

7 **(a)** D **(b)** B **(c)** D

8 $(x + 6)^2 + (y - 4)^2 = 16$

9 **(a)** Point (3, 4)

$(x - 8)^2 + (y + 8)^2 = 169$
$(3 - 8)^2 + (4 + 8)^2 = 169$
$(-5)^2 + 12^2 = 169$
$25 + 144 = 169$—true

$(x + 9)^2 + (y - 9)^2 = 169$
$(3 + 9)^2 + (4 - 9)^2 = 169$
$12^2 + (-5)^2 = 169$
$25 + 144 = 169$—true

$x^2 + y^2 = 25$
$3^2 + 4^2 = 25$
$9 + 16 = 25$—true

Point (-4, -3)

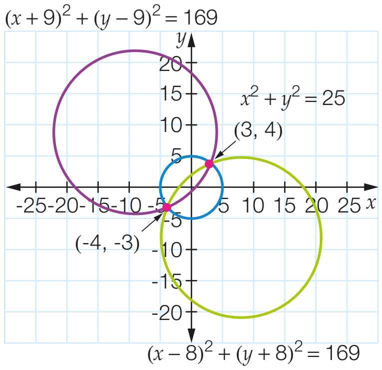
$(x - 8)^2 + (y + 8)^2 = 169$
$(-4 - 8)^2 + (-3 + 8)^2 = 169$
$(-12)^2 + 5^2 = 169$
$144 + 25 = 169$—true

$(x + 9)^2 + (y - 9)^2 = 169$
$(-4 + 9)^2 + (-3 - 9)^2 = 169$
$5^2 + (-12)^2 = 169$
$25 + 144 = 169$—true

$x^2 + y^2 = 25$
$(-4)^2 + (-3)^2 = 25$
$16 + 9 = 25$—true

The three circles intersect at the points (3, 4) and (-4, -3).

(b)



10 **(a)** $(x - 2)^2 + (y - 3)^2 = 25$ **(b)** $(x - 11)^2 + (y - 4)^2 = 169$

(c) Testing point (-1, -1) in part **(a)** equation:

$(-1 - 2)^2 + (-1 - 3)^2 = 25$—true

Testing point (-1, -1) in part **(b)** equation:

$(-1 - 11)^2 + (-1 - 4)^2 = 169$—true

Open-ended—Sample answers

11 **(a)** $x^2 + y^2 = 9$, $x^2 + y^2 = 100$

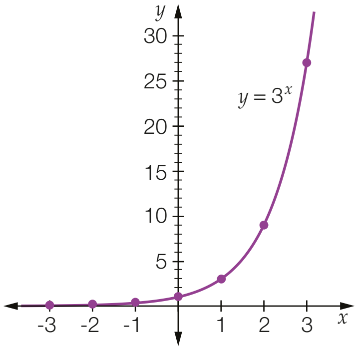
(b) $(x - 5)^2 + (y - 6)^2 = 9$, $(x + 14)^2 + (y + 8)^2 = 9$

12 Students' own design.

Exercise 9.5

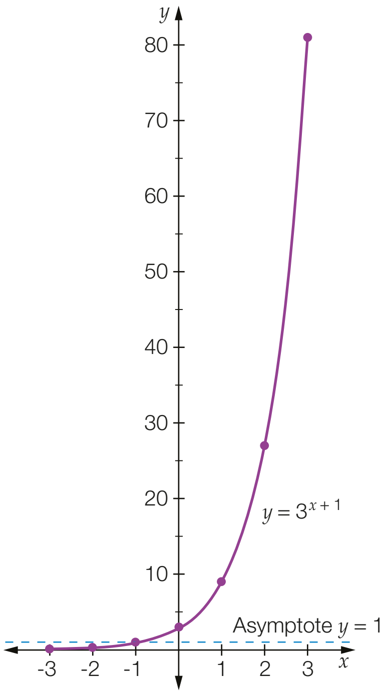
1 **(a)**

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27



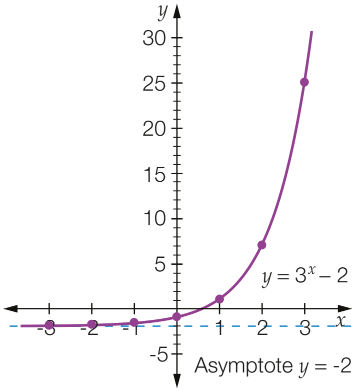
(b)

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27	81



(c)

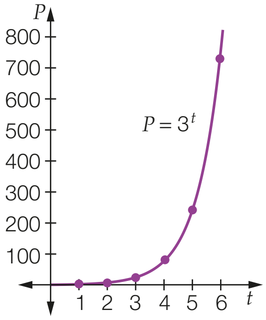
x	-3	-2	-1	0	1	2	3
y	$-\frac{53}{27}$	$-\frac{17}{9}$	$-\frac{5}{3}$	-1	1	7	25



2 **(a)**

t	0	1	2	3	4	5	6
P	1	3	9	27	81	243	729

(b)



(c) 5 years

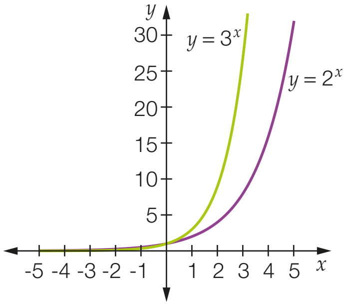
(d) 65. This value is not useful as the gardener has 27 orchids until the bulbs are ready at the end of the year. The values are not continous in this situation.

(e) 243 orchids

3 **(a)** $y = \frac{1}{x-1}$ **(b)** $y = \frac{1}{x} + 2$

(c) $y = \frac{1}{x+3} - 1$ **(d)** $y = \frac{1}{x-2} - 3$

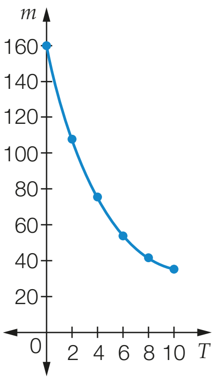
4 **(a)**



(b) Both graphs have a y-intercept of 1 and $y = 0$ is the equation of the asymptote for both graphs. $y = 3^x$ is steeper than $y = 2^x$.

5 D **6** D

7 (a)



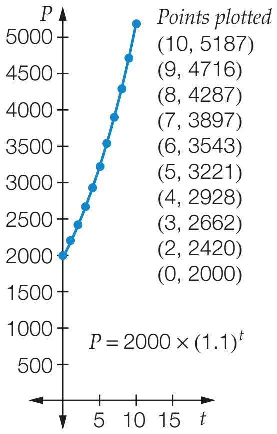
(b) 160° (c) ≈ 91° (d) ≈ 6 min 45 s (e) ≈ 28°

8 (a)

	Beginning of year					
t	**2005**	**2006**	**2007**	**2008**	**2009**	**2010**
Population, *P*	2000	2200	2420	2662	2928	3221

	Beginning of year					
t	**2011**	**2012**	**2013**	**2014**	**2015**	
Population, *P*	3543	3897	4287	4716	5187	

(b)



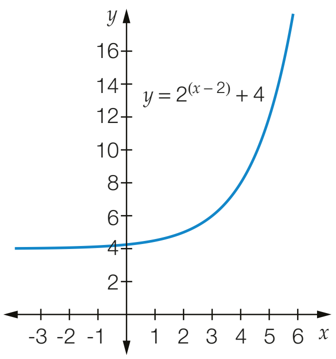
(c) 7.3 years

(d) The increase is happening on the previous year's population; the 10% is compounded annually and it would therefore take less than 10 years for the population to double.

Open-ended—Sample answers

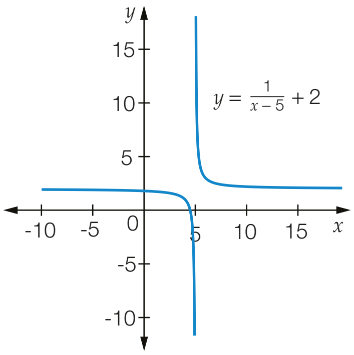
9 (a) $h = 2, k = 4$

$y = 2^{(x-2)} + 4$



(b) $h = 5, k = 2$

$y = \frac{1}{x-5} + 2$



Exercise 9.6

1 (a) not direct proportion (w fluctuates as v increases)

(b) not direct proportion (q fluctuates as p increases)

(c) not direct proportion (not through origin)

(d) could be direct proportion (t increases as s increases)

(e) could be direct proportion (q increases as p increases)

(f) not direct proportion (f fluctuates as d increases)

2 (a) $b = 4a$ (b) $v = 5w$ (c) $d = \frac{1}{2}c$

(d) $h = \frac{1}{4}g$ (e) $h = 3e$ (f) $q = 1.5p$

(g) $j = 2i^2$ (h) $d = 3a^2$ (i) $n = 4m^3$

(j) $h = 2g^3$ (k) $q = \frac{1}{2}p^2$ (l) $f = \frac{1}{5}b^2$

(m) $s = 3r^3$ (n) $m = \frac{1}{4}p^3$ (o) $u = \frac{1}{2}t^3$

(p) $w = 2\sqrt{v}$ (q) $t = \frac{1}{10}\sqrt{u}$ (r) $l = \frac{1}{3}\sqrt{d}$

3 (b) is the only graph that could be an example of direct proportion

4 (a) $m = kn$ (b) $a = kb^2$ (c) $g = kh^3$

(d) $t = kw$ (e) $N = kp^2$ (f) $y = kx^3$

(g) $V = kr^2$ (h) $L = k\sqrt{d}$

5 (a) B (b) A

6 (a) 3 (b) 0.2 (c) $\frac{1}{4}$ (d) 200

(e) 1 (f) $\frac{1}{6}$ (g) $\frac{1}{20}$ (h) $\frac{1}{35}$

7 (a) D (b) B (c) D

8 (a) 3 (b) 48 (c) 5

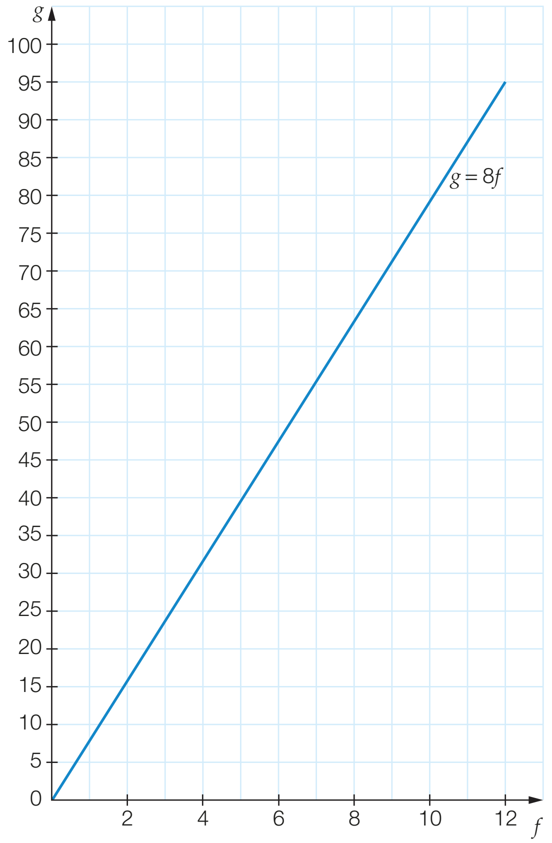
9 (a) $k = 3, q = 11, p = 27$ (b) $k = 7, m = 63, n = 6$

(c) $k = 1, c = 2, d = 64$ (d) $k = 3, x = 4, w = 21$

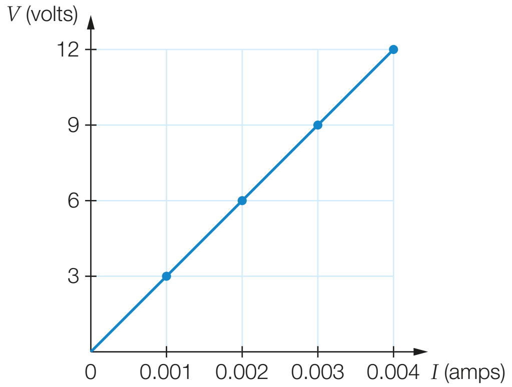
10 C

11 (a) 8 (b) $g = 8f$

(c)



12 (a)



(b) 3000 ohms (c) 30 volts (d) 0.015 amps

13 (a) $k = 12.56$ (b) $A = 4\pi r^2$

14 (a) 22.5 km (b) 988 m

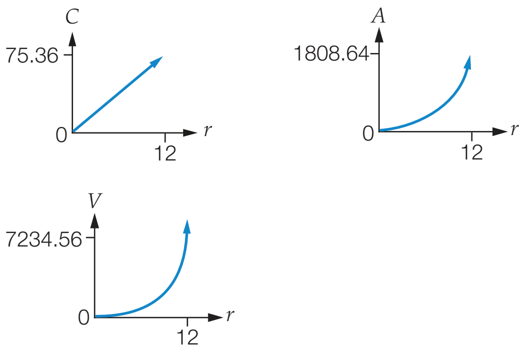
15 (a) $\frac{C}{r} = 6.28$

$\frac{A}{r} = 25.12, 50.24, 75.36, 100.48, 125.60, 150.72$

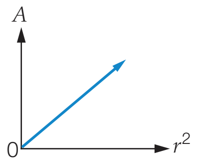
$\frac{V}{r} = 16.745, 66.9875, 150.72, 267.95, 418.67, 602.88$

(b) $\frac{C}{r} = 6.28$; $\frac{A}{r}$ is increasing by a constant value with each pair of values; $\frac{V}{r}$ seems to be no relationship

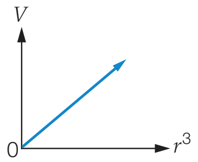
(c)



(d) linear relationship



(e) linear relationship



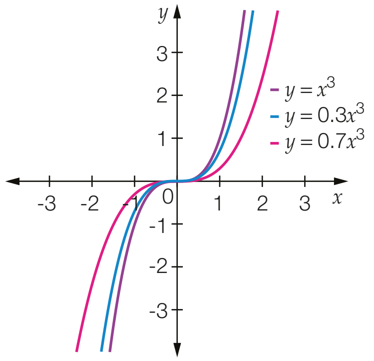
(f) $C \propto r$; $A \propto r^2$; $V \propto r^3$

(g) $k = 6.28$, $C = 2\pi r$; $k = 12.56$, $A = 4\pi r^2$; $k = 4.19$, $V = \frac{4}{3}\pi r^3$

Open-ended—Sample answers

16 (a) $y = 0.3x^3$, $y = 0.7x^3$

(b)



As k decreases from 1 to 0, the graph is dilated horizontally.

Exercise 9.7

1 (a) could be inverse proportion

(b) not inverse proportion (fluctuates)

(c) could be inverse proportion

(d) could be inverse proportion

2 (a) $b = \frac{40}{a}$ (b) $d = \frac{60}{c}$ (c) $f = \frac{216}{e}$

(d) $h = \frac{144}{g}$ (e) $l = \frac{240}{k^2}$ (f) $n = \frac{144}{m^2}$

3 **(a)** $p = \frac{k}{q}$ **(b)** $y = \frac{k}{x^2}$ **(c)** $a = \frac{k}{\sqrt{b}}$ **(d)** $t = \frac{k}{w}$

(e) $V = \frac{k}{h^2}$ **(f)** $f = \frac{k}{g^3}$ **(g)** $L = \frac{k}{\sqrt{m}}$ **(h)** $a = \frac{k}{\sqrt{d}}$

4 **(a)** C **(b)** D **(c)** D

5 **(a)** 7 **(b)** 6.3 **(c)** 12 **(d)** 0.97

(e) 5 **(f)** 3 **(g)** $\frac{1}{2}$ **(h)** $\frac{1}{5}$

(i) $\frac{1}{0.4} = 2.5$ **(j)** $\frac{1}{1.6} = \frac{5}{8}$

(k) $\frac{1}{0.08} = 12.5$ **(l)** $\frac{1}{1.25} = 0.8$

6 C

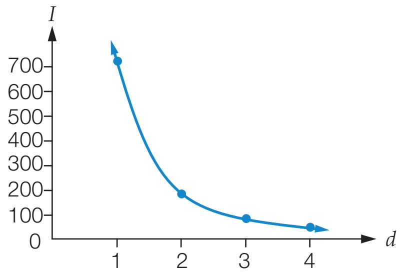
7 **(a)** $\frac{128}{125}$ **(b)** 4

8 5 hours

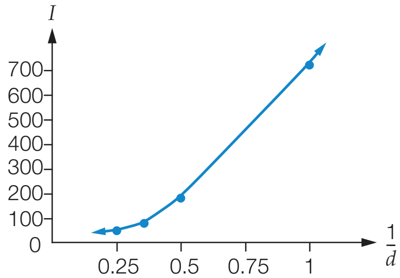
9 2×10^{-8} m

10 20 km

11 **(a)**



(b)



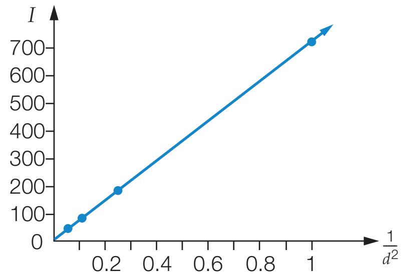
(c) No, because it is not a linear relationship.

(d)

d	1	2	3	4	6	8	12
d^2	1	4	9	16	36	64	144
I	720	180	80	45	20	11.25	5

(e) 720; yes **(f)** $I = \frac{720}{d^2}$

(g)



It is a linear relationship: $I \propto \frac{1}{d^2}$

(h) I is reduced by 4 times.

(i) I is reduced by 9 times.

(j) I is increased by 4 times.

Open-ended—Sample answers

12 possible solution $y = \frac{16}{x}$: (1, 16), (2, 8), (4, 4);

$y = \frac{16}{x^2}$: (1, 16), (2, 4), (4, 1); $y = \frac{16}{\sqrt{x}}$: (1, 16), (4, 8), (16, 4)

Challenge 9

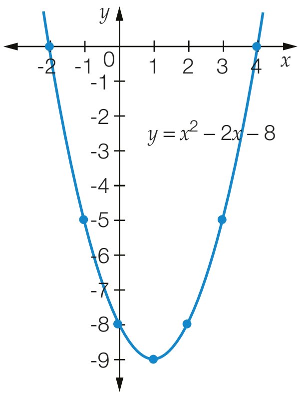
1 A **2** 1028 **3** C **4** C

5 13 **6** B **7** 36 m^2 **8** D

9 B **10** B

Chapter review 9

1



(a) x-intercepts (-2, 0) and (4, 0); y-intercept (0, -8)

(b) (1, -9) minimum turning point

(c) $x = 1$ **(d)** $y = -5$ **(e)** -3 and 5

(f) $x = -3$ or $x = 5$ **(g)** $x = -2$ or $x = 4$

2 **(a)** $x = -4$ or $x = 0$ **(b)** $x = -1$ or $x = 7$ **(c)** $x = -4$ or $x = 4$

(d) $x = -3$ **(e)** $x = 0$ or $x = 8$ **(f)** $x = 6$

3 **(a)** $x = 0$ or $x = 4$ **(b)** $x = -\sqrt{6}$ or $x = \sqrt{6}$

(c) no solution **(d)** $x = -3$ or $x = 3$ **(e)** $x = -8$ or $x = 4$

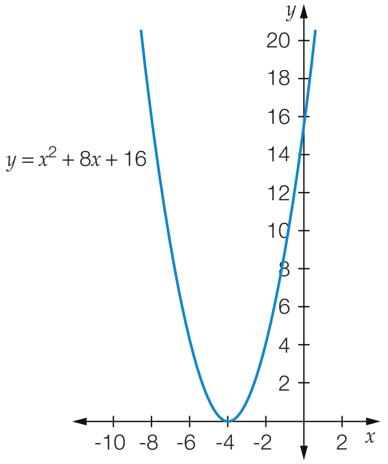
(f) $x = -3$ or $x = 4$

4 **(a)** A **(b)** A **(c)** B

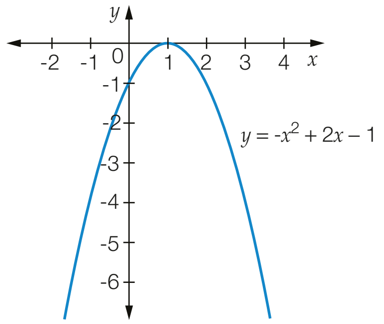
5

		Dilation	Reflection in x-axis	Translation in x-direction	Translation in y-direction
(a)	$y = (x - 1)^2 + 3$	No dilation	No reflection	1 unit right	3 units up
(b)	$y = -x^2 + 4$	No dilation	Invert	No translation	4 units up
(c)	$y = -(x + 2)^2 - 1$	No dilation	Invert	2 units left	1 unit down
(d)	$y = 3(x - 4)^2 + 2$	By factor 3 (narrower)	No reflection	4 units right	2 units up
(e)	$y = \frac{1}{2}(x - 1)^2 + 1$	By factor $\frac{1}{2}$ (wider)	No reflection	1 unit right	1 unit up
(f)	$y = -2(x + 2)^2 - 2$	By factor 2 (narrower)	Invert	2 units left	2 units down

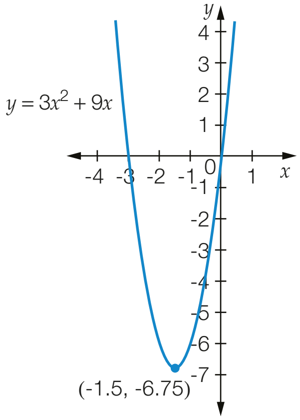
6 (a)



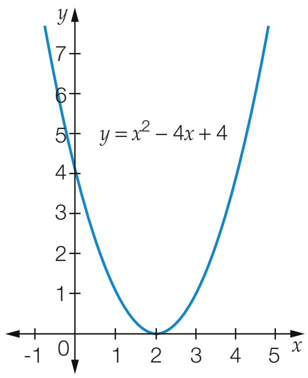
(b)



(c)

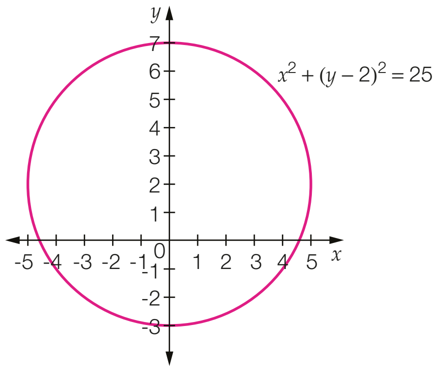


(d)

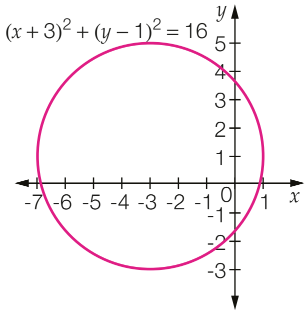


7 (a) A (b) C

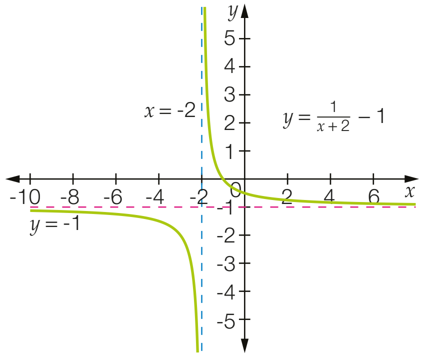
8 (a)



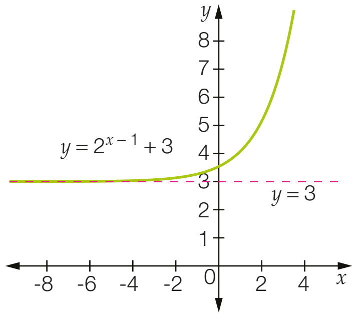
(b)



9 (a)



(b)



10 **(a)** $b = 3a$ **(b)** $d = \frac{c}{2}$ **(c)** $h = 3\sqrt{g}$

11 **(a)** A **(b)** B **(c)** D

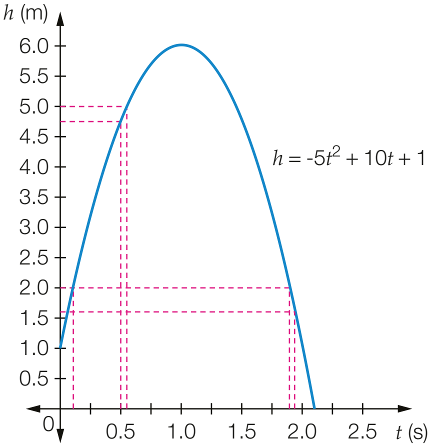
12 **(a)** $k = \frac{12}{j}$ **(b)** $q = \frac{6}{\sqrt{p}}$

13 **(a)** D **(b)** C

14 **(a)**

t	0	0.5	1	1.5	2
h	1	4.75	6	4.75	1

(b)

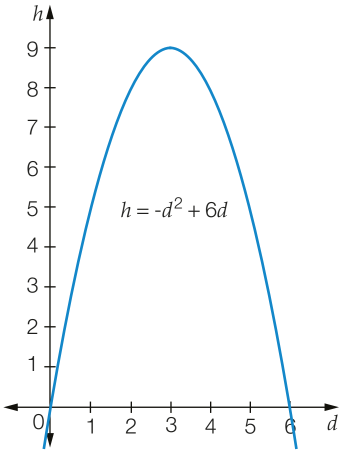


(c) 4.75 m **(d)** ≈ 0.55 s

(e) between 0.1 and 1.9 s

(f) 6 m **(g)** ≈ 1.95 s

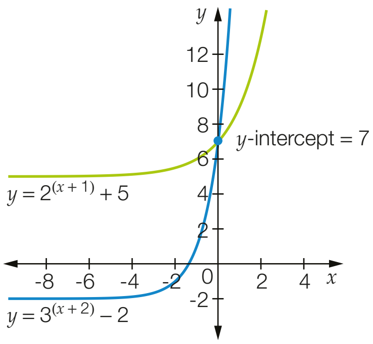
15 **(a)**



(b) 9 m **(c)** 6 m

16 The circle $(x - 4)^2 + (y - 3)^2 = 16$ has been translated 6 units to the right, 2 units upwards and the radius decreased by 1 unit compared with the circle $(x + 2)^2 + (y - 1)^2 = 25$.

17 **(a)**



(b) $y = 2^{(x+1)} + 5$ horizontal asymptote $y = 5$; $y = 3^{(x+2)} - 2$ horizontal asymptote $y = -2$

18 **(a)** yes, $y = \frac{1}{10}x^3$

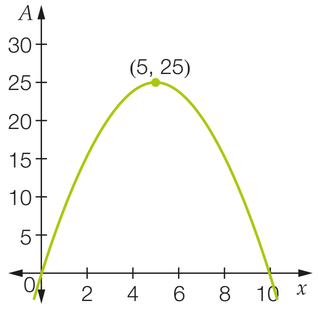
(b)

x	1	2	3	4	5	6
y	0.1	0.8	2.7	6.4	12.5	21.6

19 **(a)** width = $10 - x$

(b) $A = \text{length} \times \text{width} = x(10 - x) = 10x - x^2 = -x^2 + 10x$

(c)



(d) 5 m × 5 m **(e)** 25 m^2

20 length = 8 cm, width = 8 cm

21 25 and 25

22 $y = (x - 3)^2 + 5$

23 $(x - 3)^2 + y^2 = 81$

24 **(a)** When s is doubled, v increases by $\sqrt{2}$.

(b) When v is reduced by $\frac{1}{4}$, s is reduced by $\frac{7}{16}$.

Numeracy practice 9

1 $x = 3$ **2** D **3** C **4** D

5 $y = x^2 - 3x + 2$

x	-3	-2	-1	0	1	2	3
y	20	12	6	2	0	0	2

6 D

Mixed review D

1 6166.51 – 6146.28 = $20.23

2 **(a)** $6(a + 2)(a - 2)$ **(b)** $3(a + 1)^2$ **(c)** $(x + 5)(x - 7)$

3 **(a)** 104 cm^2 **(b)** 37.5 m^2 **(c)** 103.5cm^2

4 **(a)** $a = -6$ **(b)** $a = -\frac{15}{16}$ **(c)** $a = \frac{11}{6}$

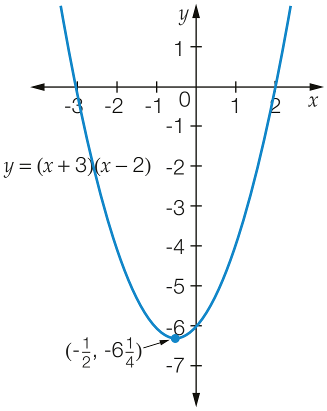
5 **(a)** 42.7° **(b)** 23.9° **(c)** 50.5°

6 **(a)** 33.36 **(b)** 125.24

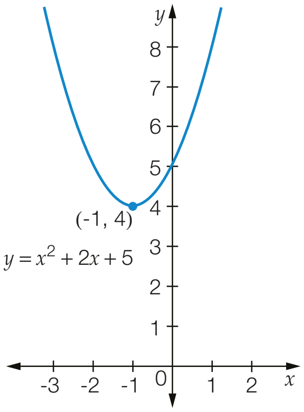
7 **(a)** $x = 20°$ **(b)** $y = 26°, z = 5°$ **(c)** $w = 20°$

8 **(a)** $x = -5$ or $x = 5$ **(b)** $x = -\frac{1}{2}$ or $x = 0$ **(c)** $x = 3$

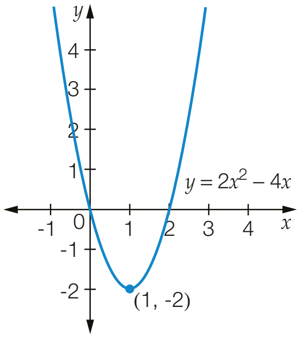
9 **(a)**



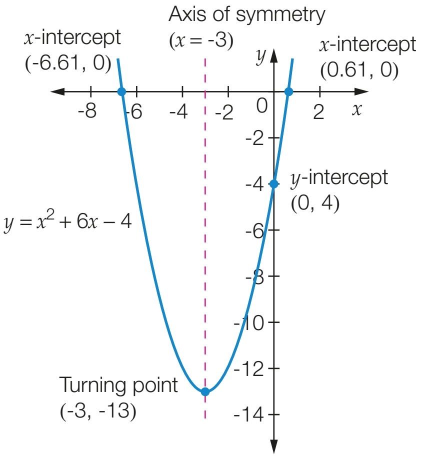
(b)



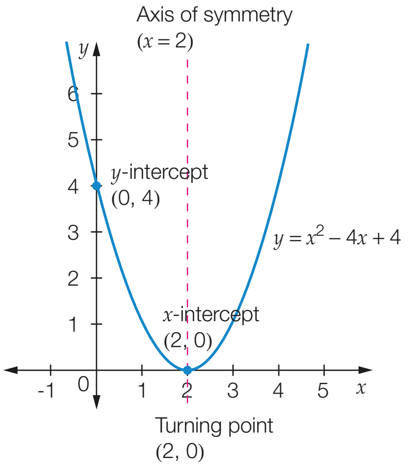
(c)



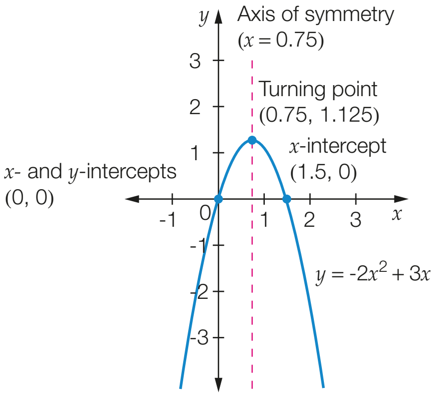
10 **(a)**



(b)



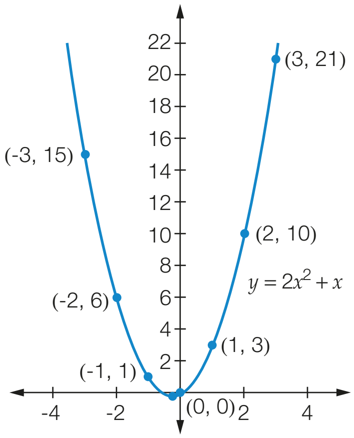
(c)



11 **(a)** $y = 2x^2 + x$

x	-3	-2	-1	0	1	2	3
y	15	6	1	0	3	10	21

(b)



(c) $(-\frac{1}{4}, -\frac{1}{8})$

(d) x-intercepts $(-\frac{1}{2}, 0)$ and $(0, 0)$; y-intercept $(0, 0)$

12 2.5°

13 **(a)** $\frac{5}{26}$ **(b)** $\frac{2}{13}$ **(c)** $\frac{11}{13}$

14 **(a)** $x = 5$ **(b)** $y = (x - 4)(x - 6)$ or $y = x^2 - 10x + 24$

15 length = 12 cm, width = 5 cm

16 **(a)** 4°

(b) **(i)** 1.995 m **(ii)** 140 mm

Glossary and index

adjacent side the side of a right-angled triangle next to the reference angle that is not the hypotenuse (p. 413)

alternate angles angles found between two lines but on opposite sides of a transversal. Alternate angles between parallel lines are equal (p. 334)

angle of depression the angle between the horizontal and the line of sight to a lower object (p. 442)

angle of elevation the angle between the horizontal and the line of sight to a higher (elevated) object (p. 442)

annulus the area between two circles that have the same centre but different radii (p. 221)

arc a section of the circumference of a circle (p. 209)

area the amount of surface within a boundary (p. 215)

asymptote a line that a graph approaches but does not cross or touch (p. 593)

average *see* mean (p. 469)

award a document that sets out the minimum working conditions of employment for employees in a certain occupation or industry (p. 24)

axis of symmetry a line that divides a figure into two identical but flipped (mirror-image) parts (p. 536)

back-to-back histogram two horizontal histograms of similar data, with the same y-axis but plotted in opposite directions on the x-axis (p. 488)

back-to-back stem-and-leaf plot two stem-and-leaf plots, one with leaves going to the left from the stem and the other with leaves going to the right, used to compare two sets of (related) data (p. 488)

base the number that is raised to an index or power (p. 133)

bearing indicates the direction of one object from another object or point (p. 444)

bi-modal a data distribution that has two modes (p. 474)

binomial an expression with two terms (p. 176)

buying on terms a method of purchase where goods can be taken home before payment, under certain conditions; also known as hire purchase (p. 55)

capacity the volume of gas or liquid that a container can hold, usually measured in litres (p. 236)

categorical data non-numerical data that consists of categories or descriptive terms (p. 462)

census when an entire population is surveyed, not just a sample from the population (p. 463)

centre of enlargement a point of reference (called O) from which the distance to each of the points on the original shape is measured and multiplied by the scale factor, to determine the distance from O to the matching corners of the enlarged or reduced shape (p. 365)

certain associated with a probability of 1 (p. 499)

chance the likelihood of an event (p. 499)

cheque a form on which an order is written to a bank to pay the stated sum from the drawer's account (p. 52)

circle a plane curve with the equation $x^2 + y^2 = r^2$ (p. 581)

circumference the perimeter of a circle (p. 208)

coefficient the number factor in a term

co-interior angles angles found between two lines and on the same side of a transversal. Co-interior angles between parallel lines are supplementary. (p. 334)

commission payment for work done based on a percentage of sales made or business gained (p. 25)

compass bearing a direction measured as an angle either side of north or south (p. 444)

complementary events two outcomes of an event (often called success and failure) whose probabilities add together to 1 (p. 499)

congruent exactly the same in size and shape (p. 342)

congruent triangles triangles that are exactly the same shape and size. There are four tests that can be used to determine if two triangles are congruent: AAA (Angle, Angle, Angle), SAS (Side, Angle, Side), ASA (Angle, Side, Angle) and RHS (Right angle, Hypotenuse, Side) (p. 342)

constant of proportionality the constant value in a relationship that is a direct proportion (p. 604)

continuous data numerical data for which the possible values are all of the real numbers within a particular range; hence, there are an infinite number of possible outcomes that cannot be counted (p. 461)

converse a statement that reverses a statement that has been made (p. 85)

coordinates numbers used to describe a position in space, for example on a Cartesian plane (p. 273)

corresponding angles angles found either both above or both below two lines and on the same side of a transversal. Corresponding angles on parallel lines are equal (p. 334)

cosine (**cos**) the ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ for a reference angle in a right-angled triangle (p. 418)

credit card a card issued by a bank or financial institution that gives the user credit to spend money immediately but pay later (p. 52)

cross-section a slice through a solid that makes a shape parallel to the base. In a prism, the cross-section is uniform (identical to the base) (p. 227)

debit card similar to a credit card, but the money is deducted from the account within a few days and no interest is charged (p. 52)

deferred payment a payment option for education fees, where payment is only made if the borrower is earning over a certain amount (p. 52)

definition the minimum amount of information necessary to describe a figure exactly (p. 351)

degree a unit used to measure the size of angles; a full turn is equal to 360° (p. 413)

dependent variable a variable whose value is dependent on the value of another variable (the independent variable) in a rule (p. 259)

difference of two squares a technique for factorising quadratics: $a^2 - b^2 = (a - b)(a + b)$ (p. 183)

dilation a change in the size of a graph or shape (p. 561)

dilation factor the number that compares an original figure to its dilated size (p. 561)

direct proportion a relationship between two variables such that one variable increases as the other increases, and when one variable is 0, the other variable is also 0 (p. 604)

discount a reduction in the marked price of goods for sale (p. 14)

discrete data exact values that are also countable (e.g. number of students present) (p. 461)

distributive law the way to 'multiply out the brackets', e.g. $a(b + c) = ab + ac$ (p. 174)

earnings income received for performing work (p. 33)

edge a line that joins two faces of a solid (p. 392)

EFTPOS Electronic Funds Transfer at Point Of Sale, where a card is used at the time of purchase and the funds are withdrawn directly from the purchaser's account (p. 52)

enlargement when a figure has corresponding sides in the same ratio as another figure, with a scale factor greater than 1. The figures are said to be similar (p. 364)

equation two expressions, on either side of an equals symbol (p. 168)

equivalence the left-hand side of an equation is equal to the right-hand side of the equation (p. 259)

expanded form an expression with all factors shown, written as a multiplication (p. 133)

expected frequency the number of times that an event is expected to occur over a number of trials, based on its relative frequency or probability (p. 501)

exponent *see* index

exponential a relationship between two quantities that involves an exponent (or index), where the exponent is the variable (p. 593)

expression a collection of terms added or subtracted together (p. 146)

exterior angle an angle formed when the side of a polygon is extended outside the shape; the angle is between the extended side and the adjacent side (p. 334)

face a flat surface of a solid (p. 392)

factorise to write an expression as a product of its factors (p. 186)

formula (*plural* **formulas** or **formulae**) a rule that is expressed using pronumerals and an equals symbol (p. 168)

frequency column graph a column graph that shows the frequency of different values in a data set (p. 474)

frequency table a table that shows the frequency of different values in a data set (p. 473)

general form an equation written with pronumerals to represent all values; the general form of a linear equation is $ax + by + \mathrm{c} = 0$ (p. 300)

gradient the measure of a line's steepness, also called slope (p. 287)

gradient–intercept form a linear equation in the form $y = mx + b$, where the gradient is m and the y-value of the intercept is b (p. 300)

gross income earnings before deductions have been made (p. 33)

grouping in pairs a technique for factorising a four-term expression so that terms are grouped in pairs with a common factor (p. 192)

highest common factor (HCF) the largest number that occurs in each of the sets of factors of a given group of numbers (p. 186)

hire-purchase *see* buying on terms

histogram a frequency column graph for continuous numerical data (p. 474)

hyperbola the shape of the graph of an inverse proportionality relationship (p. 613)

hypotenuse the longest side of a right-angled triangle, opposite the right angle (pp. 83, 413)

impossible associated with a probability of 0 (p. 499)

income money received for work or from business activities or investments, such as shares, property or savings (p. 24)

income tax a tax paid on earnings (income) (p. 33)

independent variable a variable in a rule that determines the value of another variable (the dependent variable) in the rule (p. 259)

index (*plural* **indices**) a number, written smaller, to the right and above another number (the base), which shows the number of times the base is multiplied by itself; also called a power or exponent (p. 133)

index form a number multiplied by itself, written as a base raised to an index (p. 133)

intercept the value at which a line cuts (meets) another line or axis (p. 300)

interest additional money that is paid or earned when money is borrowed or invested, calculated at regular times as a proportion of the total amount (p. 43)

interest rate the percentage per year (per annum) applied to an amount borrowed or invested to calculate interest (p. 43)

interior angle an angle inside a shape (p. 334)

interval line segment formed when two points are joined (p. 273)

inverse operation an 'opposite' operation; for example, + and – are inverse operations of each other (p. 259)

inverse proportion a relationship between two variables such that one variable increases as the other decreases (p. 613)

irrational number a number that cannot be expressed in the form $\frac{a}{b}$, a and b are whole numbers and where $b \neq 0$ (p. 90)

lay-by a system of payment where an item is paid for at intervals before being taken home after payment in full (p. 52)

linear equation an equation in which the highest power of the pronumeral(s) is 1 (p. 259)

linear graph a straight-line graph (p. 259)

linear relationship a relationship between two variables that gives a straight line when graphed (p. 259)

loss when a product's selling price is less than its cost price, the difference between these prices (p. 15)

mark-up the amount added to the cost price of an item to make its selling price (p. 15)

mean the average value of a set of a data (that is, the sum of the data values divided by the number of data values) (p. 469)

measure of centre or **measure of location** a single value that represents key information about a data set (p. 469)

median the middle data value when a set of data values are put in order (p. 469)

median class interval a class interval in grouped data that contains the median (middle) value (p. 475)

midpoint half-way between two points on an interval (p. 274)

mode the most frequently occurring data value in a data set (p. 469)

mutually exclusive not able to occur at the same time; if one event occurs, then the other cannot (pp. 499, 511)

negative skew when a set of data is not symmetrical, with data values (including the mean) appearing as a 'long tail' on the left (negative) side of the median (p. 474)

net a diagram consisting of each face of a solid, used to help construct these shapes (pp. 227, 392)

net income earnings after deductions have been made, also known as take-home pay (p. 33)

nominal data non-numerical data that names categories but has no order associated with it (p. 462)

null factor law if the product of factors is zero, then one or more of the factors must equal zero (p. 553)

numerical data data that consists of discrete or continuous numerical values (p. 461)

observational data primary data obtained by direct observation (p. 464)

opposite side the side of a right-angled triangle opposite the reference angle (p. 413)

ordinal data non-numerical data that has some sort of order associated with it (p. 462)

origin the point of intersection of the vertical and horizontal axes (p. 273)

overtime hours worked in excess of standard hours set for a time period (p. 24)

parabola the shape of the graph of a quadratic relationship (p. 535)

parallel when lines in the same plane are always the same distance apart and will never intersect (p. 305)

PAYG Pay As You Go, the system by which income tax is paid by employers directly to the government (p. 33)

perfect square a quadratic trinomial expression that can be factorised to give $(a \pm b)^2$ (p. 182)

perimeter the distance around the outside of a shape (p. 207)

perpendicular lines lines that meet or intersect at right angles to each other (p. 305)

piece work work for a payment per item produced (p. 26)

polygon a many-sided (or many-angled) plane shape (p. 350)

polyhedron (*plural* **polyhedra**) a solid where all faces are polygons (pp. 227, 392)

positive skew when a set of data is not symmetrical, with data values (including the mean) appearing as a 'long tail' on the positive (right) side of the median (p. 474)

power *see* index

primary data data collected directly by an observer (p. 462)

principal the initial amount of money borrowed or invested (p. 43)

prism a three-dimensional shape with a uniform cross-section, where the cross-section is a polygon and the edges are straight (pp. 227, 392)

probability the likelihood of a specific event expressed as a number from 0 to 1 (p. 499)

profit when a product's selling price is more than its cost price, the difference between these prices (p. 15)

proof a step-by-step logical connection of statements known to be true, which shows that a particular statement is always true (p. 83)

property any characteristic of a particular geometrical figure (p. 351)

Pythagoras' theorem in any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares for the other two sides; often expressed as $c^2 = a^2 + b^2$ (p. 83)

Pythagorean triple or **Pythagorean triad** any three whole numbers that can be the exact side lengths of a right-angled triangle. The simplest is (3, 4, 5) (p. 115)

quadratic an expression or equation where the highest power of a pronumeral is 2 (p. 535)

quadratic relationship a relationship between two variables that gives a parabola when graphed, that can be expressed as a polynomial where the highest power of the pronumeral is 2 (p. 535)

quadratic trinomial a polynomial consisting of three terms where the highest power of the pronumeral is two (p. 178)

qualitative data *see* categorical data

quantitative data *see* numerical data

random sample a sample for which every member of a group has the same chance of being selected as any other member of that group (p. 468)

rational approximation an approximate value for an irrational number (p. 91)

reciprocal the result of dividing an expression, term or number into 1. The product of an expression, term or number and its reciprocal is 1 (p. 148)

rectangular hyperbola a non-linear graph of equation

$$y = \frac{a}{x-h} + k \quad \text{(p. 597)}$$

reduction when a figure has corresponding sides in the same ratio as another figure, with a scale factor less than 1. The figures are said to be similar (p. 364)

reference angle the angle in a right-angled triangle that determines the opposite and adjacent sides used in trigonometric ratios (p. 413)

regular polygon a polygon with all sides of equal length and all angles of equal size (p. 350)

regular polyhedron a polyhedron where all faces are identical regular polygons (p. 392)

relative frequency an estimate of an event's probability, found by dividing the number of successful trials of an event by the total number of trials (p. 500)

retainer a fixed amount paid to an employee as a base payment, to which any commission earned is added (p. 25)

right-angled triangle a triangle that contains a right angle (p. 83)

rise the vertical distance between two points on the Cartesian plane, used in the equation

$$\text{gradient} = \frac{\text{rise}}{\text{run}} \quad \text{(p. 287)}$$

run the horizontal distance between two points on the Cartesian plane, used in the equation $\text{gradient} = \frac{\text{rise}}{\text{run}}$ (p. 287)

salary a fixed annual income paid at regular intervals (p. 25)

sample space a listing of all the possible outcomes for a particular situation (pp. 510, 517)

sampling with replacement random sampling where the sample taken from any population is returned to the population before another sample is taken, so the size of the population remains constant (p. 519)

sampling without replacement random sampling where the sample taken from any population is not returned to the population before another sample is taken, so the size of the population decreases with each sample taken (p. 519)

scale factor the number that relates the size of an enlarged or reduced figure to its original (p. 364)

scientific notation (standard form) a number written as $a \times 10^b$ where $1 \leq a < 10$ and b is an integer (p. 157)

secondary data data not collected directly, instead taken from another source (p. 462)

sector the area within two radii of a circle and the arc that joins them (p. 209)

significant figures the number of digits used in a measurement value to indicate the measurement's accuracy (p. 158)

similar having the same shape but not necessarily the same size (p. 374)

similar triangles triangles where corresponding sides are in the same proportion and where corresponding angles are equal (p. 375)

simple interest interest calculated on the initial principal for the entire term of a loan/investment (p. 43)

sine (sin) the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$ for a reference angle in a right-angled triangle (p. 418)

skewed when a set of data is not symmetrical; *see* negative skew, *see* positive skew (p. 474)

slope *see* gradient

solid a three-dimensional object that has length, width and depth (p. 392)

stem-and-leaf plot a statistical graph in which the leaf value represents a units digit for each value and the stem represents the other digit(s); leaf values are ordered from smallest to largest, usually on the right-hand side of the corresponding stem values

subtended when an angle is formed by lines that come directly from the endpoints of an arc (or other curve or line) (p. 209)

superannuation regular payments made to a fund to be used after retirement (p. 33)

surd an irrational number that can only be expressed exactly using the radical ($\sqrt{\ }$) symbol (p. 90)

surface area the total area of all the surfaces of a solid (p. 227)

symmetrical when there is at least one axis of symmetry that divides the figure or graph into identical mirror-image halves (p. 474)

tangent (tan) the ratio $\frac{\text{opposite}}{\text{adjacent}}$ for a reference angle in a right-angled triangle (p. 418)

tax money collected by a government to pay for public services (p. 33)

taxable income gross income minus allowable tax deductions (p. 35)

tax deduction a work-related expense that may be deducted from earnings to reduce taxable income (p. 35)

tax return a document completed at the end of the financial year to determine whether a refund is due or more tax needs to be paid (p. 35)

theorem a mathematical statement that is proved to be always true (p. 83)

transformation any change that can be made to a graph or figure using rotation, dilation, reflection or translation (p. 561)

translation a transformation of a graph or figure that keeps the shape, size and orientation exactly the same, but changes the location (p. 561)

tree diagram a diagram, consisting of lines branching out from other lines, used to represent a sample space; probabilities can be assigned to each branch and then used to calculate the probability of a particular outcome (p. 518)

trigonometric ratio a ratio of sides of a right-angled triangle, for a given reference angle; *see* cosine, *see* sine, *see* tangent (pp. 413, 418)

trigonometry the mathematics of the relationships between sides and angles in triangles (p. 413)

true bearing a direction measured as an angle clockwise from north (p. 444)

turning point a point where the gradient of a curve changes from positive to negative or from negative to positive (p. 536)

two-way table a table that categorises information in rows and columns according to two different criteria (p. 510)

undefined a value that cannot be calculated (p. 290)

variable in algebra, an unknown amount that may vary (p. 134)

vertex (*plural* **vertices**)

1. a point where lines or edges meet
2. *see* turning pointthe corner of a shape (pp. 392, 536); *see also* turning point

volume the amount of space occupied by a three-dimensional object (p. 235)

wages income based on the amount of time worked, usually at an hourly rate; the rate may be higher for extra hours outside of standard set hours (p. 24); *see also* overtime

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