



OXFORD

INSIGHT
MATHEMATICS
STANDARD

11

JOHN LEY
MICHAEL FULLER

ADDITIONAL RESOURCE
CONTRIBUTORS

BARBARA MARINAKIS
ANDREW HOLLAND

OXFORD

OXFORD

INSIGHT
MATHEMATICS
STANDARD

11

JOHN LEY
MICHAEL FULLER

ADDITIONAL RESOURCE
CONTRIBUTORS

BARBARA MARINAKIS
ANDREW HOLLAND

OXFORD
UNIVERSITY PRESS
AUSTRALIA & NEW ZEALAND

Oxford University Press is a department of the University of Oxford.

It furthers the University's objective of excellence in research, scholarship, and education by publishing worldwide. Oxford is a registered trademark of Oxford University Press in the UK and in certain other countries.

Published in Australia by
Oxford University Press
253 Normanby Road, South Melbourne, Victoria 3205, Australia

© John Ley, Michael Fuller 2018

The moral rights of the author have been asserted

First published 2018
Third Edition

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, without the prior permission in writing of Oxford University Press, or as expressly permitted by law, by licence, or under terms agreed with the appropriate reprographics rights organisation. Enquiries concerning reproduction outside the scope of the above should be sent to the Rights Department, Oxford University Press, at the address above.

You must not circulate this work in any other form and you must impose this same condition on any acquirer.

National Library of Australia Cataloguing-in-Publication data

Ley, John (author)

Oxford Insight Mathematics Standard (Year 11) Student book + obook assess / John Ley, Michael Fuller.

ISBN 978 0 19 031051 6 (paperback)

For secondary school age.

Mathematics—Study and teaching (Secondary).

Mathematics—New South Wales—Textbooks.

Other Creators/Contributors:

Fuller, Michael (author)

Reproduction and communication for educational purposes

The Australian *Copyright Act 1968* (the Act) allows a maximum of one chapter or 10% of the pages of this work, whichever is the greater, to be reproduced and/or communicated by any educational institution for its educational purposes provided that the educational institution (or the body that administers it) has given a remuneration notice to Copyright Agency Limited (CAL) under the Act.



For details of the CAL licence for educational institutions contact:

Copyright Agency Limited
Level 15, 233 Castlereagh Street
Sydney NSW 2000
Telephone: (02) 9394 7600
Facsimile: (02) 9394 7601
Email: info@copyright.com.au

Edited by Scharlaine Cairns
Typeset by Newgen KnowledgeWorks Pvt. Ltd., Chennai, India
Indexed by Max McMaster
Printed by Sheck Wah Tong Printing Press Ltd

Disclaimer

Indigenous Australians and Torres Strait Islanders are advised that this publication may include images or names of people now deceased.

*Links to third party websites are provided by Oxford in good faith and for information only.
Oxford disclaims any responsibility for the materials contained in any third party website referenced in this work.*

CONTENTS

Using Oxford Insight Mathematics Standard (Year 11)	vi
Top tips for study success.....	vii

1 Earning and managing money 2

FINANCIAL MATHEMATICS (MS-F1 Money Matters F1.2)

Are you ready?	3
1A Earning an income	4
1B Allowances, bonuses and loadings.....	10
1C Commission, piecework and royalties.....	16
1D Government allowances and pensions.....	22
1E Deductions and net income	28
1F Taxable income	32
1G Calculating tax	36
Chapter review	40

2 Practicalities of measurement 46

MEASUREMENT (MS-M1 Applications of Measurement M1.1)

Are you ready?	47
2A Significant figures	48
2B Scientific notation	50
2C Metric units of measurement	54
2D Prefixes for units of measurement.....	58
2E Error and accuracy in measurement.....	60
2F Absolute error and limits of accuracy	64
Chapter review	70

3 Formulas and equations 74

ALGEBRA

(MS-A1 Formulae and Equations)

Are you ready?	75
3A Substitution into expressions and formulas.....	76
3B Solving linear equations	80
3C Solving further linear equations.....	86
3D Finding the value of a variable in a formula.....	92
3E Travel calculations.....	96
3F Blood alcohol content	100
3G Medication calculations	104
Chapter review	108

Chapters 1–3 Cumulative review 114

4 Classifying data and sampling 116

STATISTICAL ANALYSIS

(MS-S1 Data Analysis S1.1)

Are you ready?	117
4A Collecting and classifying data.....	118
4B Census or sample and self-selected sampling.....	122
4C Random sampling	126
4D Stratified random sampling.....	132
4E Systematic sampling.....	134
4F Suitability of sample types.....	136
Chapter review	140

5 Perimeter, area and volume 146

MEASUREMENT

(MS-M1 Applications of Measurement M1.2)

Are you ready?	147
5A Perimeter	148
5B Area of simple shapes	154
5C Area of composite figures	160
5D Perimeter and area of irregular shapes	166
5E Surface area of prisms	174
5F Surface area of cylinders and spheres	180
5G Volume	186
5H Capacity	192
Chapter review	196

6 Interest and depreciation 202

FINANCIAL MATHEMATICS

(MS-F1 Money Matters F1.1)

Are you ready?	203
6A Percentage applications	204
6B Simple interest calculations	210
6C Simple interest graphs	214
6D Straight-line depreciation	218
6E Compound interest	222
Chapter review	226

Chapters 4–6 Cumulative review 230

7 Representing data 234

STATISTICAL ANALYSIS

(MS-S1 Data Analysis S1.1)

Are you ready?	235
7A Frequency distribution tables	236
7B Cumulative and relative frequency	240
7C Dot plots and stem-and-leaf plots	246
7D Column and bar graphs	252
7E Line graphs	258
7F Frequency histograms and polygons	262
7G Cumulative frequency graphs	266
Chapter review	272

8 Relative frequency and probability 278

STATISTICAL ANALYSIS

(MS-S2 Relative Frequency and Probability)

Are you ready?	279
8A Sample space and number of outcomes	280
8B Relative frequency	286
8C Probability	290
8D Complementary events	294
8E Probability for multistage events	298
8F Probability tree diagrams	302
8G Expected frequency and simulations	306
Chapter review	314

9 Working with time 320

MEASUREMENT

(MS-M2 Working with Time)

Are you ready?	321
9A Latitude and longitude	322
9B Position on Earth's surface	326
9C Time units and intervals	330
9D World time zones	334
9E Australian time zones	340
9F Interpreting timetables	342
Chapter review	346

Chapters 7–9 Cumulative review 352

10 Budgeting and household expenses 354

FINANCIAL MATHEMATICS

(MS-F1 Money Matters F1.3)

Are you ready?	355
10A Household bills	356
10B Purchasing a car	362
10C Financing a purchase	368
10D Fuel consumption	372
10E Running costs of a car	376
10F Budgeting	380
Chapter review	384

11 Exploring and describing data 390

STATISTICAL ANALYSIS

(MS-S1 Data Analysis S1.2)

Are you ready? 391

11A Mean and mode.....392

11B Median.....398

11C Relative merits of mean,
mode and median 404

11D Range and interquartile range.....408

11E Deciles and percentiles.....414

11F Box-plots 418

11G Standard deviation 422

11H Identifying outliers 428

11I Graphical data displays.....432

Chapter review.....440

12 Linear relationships 446

ALGEBRA

(MS-A2 Linear Relationships)

Are you ready? 447

12A Straight-line graphs.....448

12B Gradient and vertical intercept 452

12C The equation $y = mx + c$ 458

12D Direct variation relationships 462

12E Linear modelling.....468

Chapter review.....474

13 Units of energy and mass 480

MEASUREMENT

(MS-M1 Applications of Measurement M1.3)

Are you ready? 481

13A Units of mass 482

13B Units of energy 486

13C Food and nutrition: energy intake.....490

13D Energy expenditure 494

13E Energy consumption 502

Chapter review.....506

Chapters 10–13 Cumulative review 512

Answers 516

Glossary 579

Index 582

Acknowledgements 586

Using Oxford Insight Mathematics Standard (Year 11)

New South Wales' most trusted Mathematics series has been updated for the new Mathematics Standard Stage 6 syllabus. The new edition includes comprehensive exercise sets, carefully graded exercises and worked examples embedded where students need them. Enhanced opportunities for support and extension, as well as consolidation and practice are offered in reviews, cumulative reviews, exam-style questions and integrated technology.

helpful resources are outlined at the beginning of each unit

visual mathematics glossary boosts understanding of key concepts

The collage includes several pages from the textbook:

- Page 116: Standard deviation** - This page introduces the concept of standard deviation, explaining it as a measure of dispersion. It includes a definition, a formula for standard deviation ($s = \sqrt{\frac{1}{n} \sum (x_i - \bar{x})^2}$), and a worked example (Example 116-1) showing how to calculate the standard deviation for a set of scores.
- Page 117: Standard deviation** - This page continues the discussion on standard deviation, providing more examples and exercises. It includes a table for calculating the standard deviation of a set of scores.
- Page 118: Standard deviation** - This page shows a table for calculating the standard deviation of a set of scores, with columns for 'Height (cm)', 'Class centre (x)', 'Frequency (f)', and 'fx'.
- Page 119: Standard deviation** - This page shows a table for calculating the standard deviation of a set of scores, with columns for 'Height (cm)', 'Class centre (x)', 'Frequency (f)', and 'fx'.

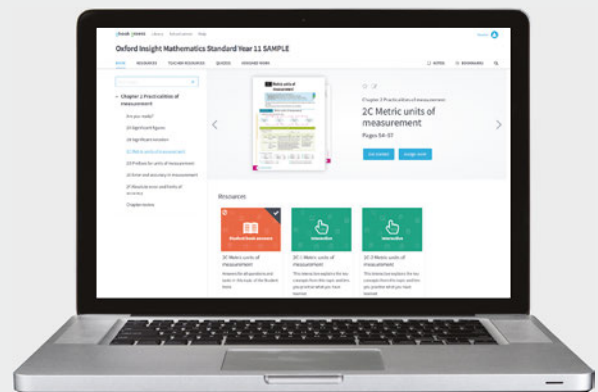
worked examples and visuals located next to the relevant exercise

'Working Mathematically' syllabus components clearly signposted

Student obook assess

Oxford Insight Mathematics Standard (Year 11) is supported by a range of engaging and relevant digital resources via obook assess.

- > a complete digital version of the Student book with notetaking and bookmarking functionality
- > targeted instructional videos by a team of Australia's most experienced Mathematics Standard teachers designed to help students prepare for assessment tasks and exams
- > interactive auto-correcting multiple-choice quizzes.
- > access to teacher-assigned work including readings, homework, tests and assignments



Teacher obook assess

In addition to the student resources, teachers also receive:

- > detailed planning resources
- > printable (and editable) class tests with exam-style questions and answers
- > the ability to set up classes, set assignments, monitor progress and graph results, and to view all available content and resources in one place.

Top tips for study success

Tip 1 – read key documents

The first step to success is to gather all key documents and read them carefully.

- > Your most important tool is the syllabus. It sets out all of the information about the course, including what you are expected to learn and how you will be assessed. You can download a copy from the NESA website.
- > Keep all documents from your teacher relating to assessment tasks and copies of any assessment advice (e.g. marking criteria or assessment rubrics). Understanding exactly what is required in an assessment task is crucial.

Tip 2 – study regularly

If you're going to perform at your best, you need to allocate time for regular periods of study and revision. Studying regularly will help you to reinforce concepts and avoid the stress of last-minute cramming. During your study you might:

- > summarise key points in your own words
- > focus on topics you find difficult and work through the relevant examples and questions
- > test your understanding with revision questions, practice papers and past exams.

Tip 3 – manage your study time

When studying, it helps to put some practical strategies in place to stay on track. Try the following time management strategies.

- > **Create a study timetable** to set up periods of regular study and revision around your school and personal schedule.
- > **Use a diary, wall planner or calendar** to record the dates of upcoming assessment tasks, tests or exams and allow you to adequately prepare.
- > **Make lists** of daily, weekly or monthly goals. It helps to keep the bigger picture in mind and breaks down big tasks into smaller, more manageable tasks, so that you gain a sense of achievement.

Tip 4 – take care of yourself

Looking after yourself during your HSC is important:

- > eat a balanced diet and stay hydrated – try to avoid too much caffeine and junk food
- > get enough sleep and regular exercise
- > have breaks – a walk to get some fresh air will help you reset before the next study session.

Tip 5 – know the structure of exams

It's important for you to become familiar with the format of the exam and the types of questions that typically appear. In an exam you should also:

- > show your working when answering a question – even if a question is incorrect or left unfinished, you may still get some marks for your working
- > keep an eye on the clock to make sure you have enough time to answer every question
- > re-read questions so you know that you have provided a complete and accurate answer.

Tip 6 – understand key terms

Assessment tasks will likely include key terms. These range in level of difficulty. Some, such as *solve* or *find*, are simple to understand and master. Others, such as *justify*, are more challenging and will take practice to master. Below is a list of common key terms and an explanation of what they mean.

TERM	DEFINITION
analyse	examine something complex by breaking it down into smaller parts and show how they relate to one another
calculate	work out an answer mathematically
classify	categorise into groups
convert	change to a different form without changing the value
describe	give a detailed account of the features
evaluate	determine the value
explain	make something clear by describing the relationships between different aspects and giving reasons
express	represent an answer as a number, figure, formula or symbol
find	determine the value or answer to a problem
identify	determine and state clearly
justify	present an argument and provide evidence
solve	work out the solution to a question

1

Earning and managing money

The main mathematical ideas in this chapter are:

- ▶ using different methods for calculating income
- ▶ calculating allowances, bonuses, royalty payments, commission and holiday loading
- ▶ calculating piecework payments
- ▶ calculating government allowances, pensions and health payments
- ▶ evaluating superannuation
- ▶ calculating net pay and gross pay
- ▶ calculating allowable deductions
- ▶ calculating taxable income
- ▶ calculating the Medicare levy
- ▶ calculating tax refunds or payments.

ARE YOU READY?

- 1A ▶ 1** What is the result of calculating $28.67 + 139.48$?
A 168.15 **B** 42.618
C 110.81 **D** 157.05
- 1A ▶ 2** What is the result of calculating $243.81 - 185.25$?
A 142.64 **B** 69.66
C 429.06 **D** 58.56
- 1A ▶ 3** What is the result of calculating 47.86×2.7 ?
A 45.16 **B** 17.726
C 129.222 **D** 50.56
- 1A ▶ 4** What is the result of calculating $2.9796 \div 0.04$?
A 0.7449 **B** 7.449
C 74.49 **D** 744.9
- 1A ▶ 5** How many minutes in 3 hours?
A 24 **B** 60
C 120 **D** 180
- 1A ▶ 6** How many days in 168 hours?
A 7 **B** 6
C 5 **D** 4
- 1A ▶ 7** How many weeks in 2 years?
A 8 **B** 24
C 52 **D** 104
- 1A ▶ 8** How many months in 3 years?
A 12 **B** 24
C 36 **D** 52
- 1A ▶ 9** How many hours have elapsed from 6:00 am to 8:00 pm?
A 2 **B** 10
C 12 **D** 14
- 1A ▶ 10** How many hours have elapsed from 9:30 am to 4:30 pm?
A 5 **B** 7
C 7.5 **D** 13
- 1B ▶ 11** What is 65% of \$240?
A \$156 **B** \$1560
C \$27 **D** \$2708
- 1C ▶ 12** What is 42 as a percentage of 70?
A 60% **B** 42%
C 167% **D** 29.4%
- 1C ▶ 13** What is \$27 as a percentage of \$60?
A 22% **B** 27%
C 45% **D** 60%
- 1C ▶ 14** Georgia scored 65 marks out of 80 in a mathematics test. What is her percentage mark?
A 81.25% **B** 80%
C 65.25% **D** 8.125%

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook assess](#).

- Q1–2** Support sheet 1A.1 Adding and subtracting decimal numbers
Q3–4 Support sheet 1A.2 Multiplying and dividing decimal numbers
Q5–8 Support sheet 1A.3 Converting units of time
Q9–10 Support sheet 1A.4 Calculating elapsed time
Q11 Support sheet 1B.1 Calculating the percentage of a quantity
Q12–14 Support sheet 1C.1 Calculating an amount as a percentage of another

1A Earning an income

These resources are available on your obook assess:

- **Spreadsheet 1A:** Convert between annual, monthly, weekly and hourly income
- **assess quiz 1A:** Test your skills with an auto-correcting multiple-choice quiz

wage

a fixed regular payment earned for work or services, usually based on an hourly rate and typically paid daily, weekly, or fortnightly

salary

a fixed regular payment, typically paid monthly but often expressed as an annual sum, made by an employer to an employee

per annum (p.a.)

for each year (used in financial contexts)

Money earned by working is the basis of most people's income. Without an income people are unable to satisfy their basic needs. There are many ways people receive money for work done. Most people work for an employer who pays them for their work. An amount earned based on hours worked is called a **wage**. A fixed amount earned regardless of the number of hours spent working is called a **salary**. This section deals with salary and wage payments.

A salary is an income that is usually described as an amount **per annum (p.a.)** or per year, although the salary may be paid weekly, fortnightly or monthly. People who are paid salaries include nurses, government employees, teachers and other professionals. A wage is based on an hourly rate for a certain number of hours worked. Wages are usually paid on a weekly or fortnightly basis. People who are paid wages include factory workers, shop assistants, building workers and mechanics.

When converting from yearly salaries to monthly, fortnightly and weekly amounts, the following are used.

1 year = 365 days
= 12 months
= 52 weeks
= 26 fortnights

Employees may be permanent or casual, and may work full-time or part-time. Permanent employees receive benefits such as sick leave and holiday pay.

EXAMPLE 1A-1 Calculating salary payments

Jenny works as a pastry chef. Her salary is \$51 064 per annum. Express this salary as an amount:

a per month

b per week

c per fortnight.

	Solve	Think	Apply
a	Monthly = $\$51\,064 \div 12$ = \$4255.33	There are 12 months in a year. Divide the annual income by 12.	When converting from an annual income, divide by the required number:
b	Weekly = $\$51\,064 \div 52$ = \$982	There are 52 weeks in a year. Divide the annual income by 52.	12 for monthly 52 for weekly
c	Fortnightly = $\$51\,064 \div 26$ = \$1964	There are 26 fortnights in a year. Divide the annual income by 26.	26 for fortnightly

EXERCISE 1A Earning an income

- 1 Amy earns \$19 884.80 per annum for part-time work. Complete the following to find how much she earns:
 - a Monthly = $\frac{\quad}{12}$
= $\frac{\quad}{\quad}$
 - b Weekly = $\frac{\$19\,884.80}{\quad}$
= $\frac{\quad}{\quad}$
 - c Fortnightly = $\frac{\quad}{\quad}$
= $\frac{\quad}{\quad}$
- 2 Caterina earns \$86 924 per annum. Calculate her weekly pay.
- 3 Express each of the following annual salaries as an amount:

i per month	ii per week	iii per fortnight.
a \$38 903	b \$27 106	c \$30 098
d \$109 870	e \$75 076	f \$50 990



EXAMPLE 1A-2 Calculating annual and monthly salary payments

Michiko works part-time and earns \$283 per week.

- a How much does Michiko earn per year?
- b How much does Michiko earn per month?

	Solve	Think	Apply
a	Yearly = $\$283 \times 52$ = \$14 716	There are 52 weeks in a year. Multiply the weekly wage by 52.	When converting weekly amounts to annual amounts, multiply by 52.
b	Monthly = $\frac{\$14\,716}{12}$ = \$1226.33	There are 12 months in a year. Divide the annual amount by 12. One month is <i>not</i> 4 weeks, because $12 \times 4 = 48$, not 52 weeks.	Always use the annual income divided by 12 to find the monthly income. If the annual income is not provided, convert to it first.

- 4 Jason earns \$421 per week for weekend work. Complete the following to find:

a Jason's annual income Annual income = $\$421 \times \frac{\quad}{\quad}$ = $\frac{\quad}{\quad}$	b Jason's monthly income Monthly income = $\frac{\$21\,892}{\quad}$ = $\frac{\quad}{\quad}$
- 5 Ashley earns \$379 per week for part-time work. How much does Ashley earn:

a per year?	b per month?
- 6 Calculate the annual salary amount for each of these workers.

Name	Weekly wage	Annual salary
C. Cooke	\$847.94	
D. Moussa	\$1021.06	
B. Boje	\$630.54	
T. Lambert	\$570.38	

- 7**
- a** David works part-time and receives an annual salary of \$41 398.45. How much does he earn each week?
 - b** Debra works part-time and receives a monthly salary of \$2933. How much does she earn each week?
 - c** Ali works part-time and receives an annual salary of \$46 837.25. How much does he earn each fortnight?
 - d** Kristy works full-time and receives an annual salary of \$78 500. How much does she earn each fortnight?
 - e** Iain works part-time and receives a monthly salary of \$3599.60. How much does he earn each fortnight?
 - f** Ngoc works full-time and receives an annual salary of \$74 292. How much does she earn each month?
- 8** Convert each of the following to an annual salary, if necessary, and calculate the difference between the annual salaries.
- a** Kylie: \$1136.60 a week and Stacey: \$54 642 a year
 - b** Roland: \$1803 a fortnight and Paul: \$55 271 a year
 - c** Binh: \$2610 a month and Aaron: \$693.20 a week

EXAMPLE 1A-3 Calculating wage payments

Monica is paid \$21.65 per hour for casual work. How much does she earn if she works:

- a** 6 hours in one day **b** 6 hours per day for five days?

	Solve	Think	Apply
a	$\text{Pay} = \$21.65 \times 6$ $= \$129.90$	The find the amount earned, multiply the hourly rate, (\$21.65) by the number of hours (6).	Incomes stated as hourly rates need to be multiplied by the number of hours to find the total income.
b	$\text{Pay} = \$21.65 \times 6 \times 5$ $= \$649.50$	Multiply the daily rate (\$21.65 $\times$ 6) by the number of days (5).	

- 9** Clare works in a jewellery store earning \$28.45 per hour. Complete the following to find the amount Clare earns:
- a** in an 8-hour day
Pay = $\_\_\_ \times 8 = \_\_\_$
 - b** for a 38-hour week
Pay = $\_\_\_ \times \_\_\_ = \_\_\_$
- 10** Aaron works as an assistant in a clothing store and is paid \$21.75 an hour for a 35-hour week. Calculate his weekly wage.
- 11** Madonna works as a receptionist. If she is paid \$20.90 an hour, how much does she earn in a 38-hour week?



- ### EXAMPLE 1A-4 Calculating annual income

Solve	Think	Apply
Pay per week = $\$22.80 \times 35$ = \$798 Annual income = 798×52 = \$41 496 Angus earns \$ 41 496 per year.	Find the pay per week by multiplying \$22.80 by 35, then multiply by 52 to find the annual income.	Hourly income is the basic building block that, by multiplication, will allow us to calculate income over any time period.

- ### EXAMPLE 1A-5 Converting annual salaries to shorter time periods

	Solve	Think	Apply
a	Weekly = $\$47\,424 \div 52$ = \$912	Divide the annual salary of \$47 424 by 52 to convert to weekly salary. Next, divide the weekly salary (\$912) by 35 to convert to the hourly rate.	Converting annual salaries to shorter time periods requires division. Divide by 52 for weekly or by 12 for monthly incomes.
b	Hourly = $\$912 \div 35$ = \$26.06		

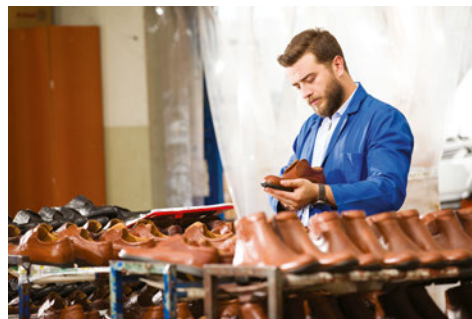
- 7

- 20** Rodney and Anastasia both work at a shoe factory for an hourly rate of pay of \$26.80, including the time taken for their meal breaks.

- i Calculate the total number of hours worked in one week by each employee.
- ii Calculate each employee's weekly wage.

a Name: Rodney

Day	In	Out	Hours
Monday	7:00 am	4:00 pm	
Tuesday	8:00 am	4:00 pm	
Wednesday	7:00 am	5:00 pm	
Thursday	11:00 am	4:00 pm	
Friday	9:30 am	2:30 pm	
		Total	



b Name: Anastasia

Day	In	Out	Hours
Monday	7:40 am	5:40 pm	
Tuesday	10:20 am	4:20 pm	
Wednesday	9:00 am	3:00 pm	
Thursday	8:30 am	3:30 pm	
Friday	8:35 am	3:35 pm	
		Total	



- 21** Complete the following wage table.

Name	Day					Hours worked	Hourly rate	Weekly rate
	Mon	Tues	Wed	Thurs	Friday			
S. Smith	7	5	–	$4\frac{1}{2}$	9		\$29.40	
D. De Souza	11	4	$3\frac{1}{2}$	$6\frac{1}{2}$	7		\$26.30	
A. Bentley	$6\frac{1}{2}$	–	–	3	$9\frac{1}{2}$			\$611.80
C. Johnson	4	$2\frac{1}{2}$	7	2	$1\frac{1}{2}$		\$30.34	
K. Kernell	3	$7\frac{1}{2}$	5	$4\frac{1}{2}$	–			\$374.80
P. Patrick	$5\frac{1}{2}$	9	$2\frac{1}{2}$	–	$6\frac{1}{2}$		\$19.83	

- 22** Jim is a junior and works 26 days per month. He averages 7 hours per day. Find his annual income if his hourly rate is \$18.17.

- 23** Nato is paid \$35.20 per hour. He works 4 hours per day seven days a week. Find his pay for 1 year.

- 24** Isla is paid \$1710 per fortnight.

a Find:

- i her weekly pay
- ii her monthly pay

b If Isla works 6 hours per five days a week, what is her hourly rate?

- 25** James is a junior and works as a casual employee at the local supermarket. He is paid \$16.05 an hour from Monday to Friday and \$20.35 an hour on weekends. During a week of his school holidays he worked from 3:00 pm until 6:00 pm Monday to Friday and 8:30 am until 1:30 pm on Saturday and Sunday.
- How many hours did he work on weekdays?
 - How many hours did he work on weekends?
 - Calculate his income for the week.



- 26** Steve is paid a salary of \$78 904 per annum.
- Calculate Steve's weekly pay rate.
 - What is his hourly rate if he works 45 hours per week?
- 27** Which pay option offers the higher weekly wage? By how much?
- | | | |
|---|----|------------------------------|
| a Option A: \$903.20 a week | or | Option B: \$45 103.20 a year |
| b Option A: \$953.29 a fortnight | or | Option B: \$2405 a month |
| c Option A: \$3276 a month | or | Option B: \$47 024 a year |

- 28** A spreadsheet is provided on your obook assess to enable you to easily convert between annual, monthly, fortnightly, weekly and hourly income. Use it to check some of your answers for questions in this exercise.

- 29** Armin and Richard were comparing their pays. Richard said that he earns more per week than Armin. Richard earns \$33 800 per annum, which he said is \$704.17 per week. Armin earns \$680 per week. The yearly pay for Richard is correct, but he has made an error in calculating his weekly pay.
- Find Armin's annual income.
 - Find Richard's correct weekly pay.
 - Who really earns more?
 - Richard said that he divided the annual income by 12 to find his monthly pay and then by 4 to find his weekly pay. Find the mistake that Richard made. Explain your answer.



1B Allowances, bonuses and loadings

overtime

extra time worked by employees in excess of their normal working hours

time-and-a-half

overtime rate where employees are paid $1\frac{1}{2}$ times their normal hourly rate

double-time

overtime rate where employees are paid 2 times their normal hourly rate

bonus

extra payment paid as an incentive to employees who work hard

annual leave loading

extra payment added to the holiday pay of employees taking their annual holidays

These resources are available on your obook assess:

- **Video tutorial 1B:** Watch and listen to an explanation of Example 1B-5
- **Worksheet 1B:** Practise your skills with extra problems on earning money
- **Spreadsheet 1B:** Calculate wages including overtime
- **assess quiz 1B:** Test your skills with an auto-correcting multiple-choice quiz

Most employees work under an award that states the maximum number of normal work hours they should work per day or per week, and also their normal working days. Hours worked in excess of these (including weekend, late night and public holiday work) are **overtime** and are paid at a higher hourly rate than normal time. The two main overtime rates are **time-and-a-half** and **double-time**.

Employees who are required to work under difficult, unpleasant or dangerous conditions are often paid an allowance that is above the normal rate of pay. Some examples include an allowance for cleaning toilets, working in confined spaces, working in hot or very cold conditions, or dealing with toxic substances.

Allowances may also be paid for uniforms, dry-cleaning of uniforms, travelling, meals, tools and so on.

Employees sometimes are paid extra payments in addition to their normal wages as described below.

A **bonus** is an extra one-off payment paid as an incentive for employees who work hard over a period of time. Employers use bonuses to encourage the employees to work even harder!

Employees receive four weeks of annual leave per year. **Annual leave loading** is an increase in an employee's pay while on holidays. The holiday loading is a fixed percentage of the employee's normal pay. It is usually paid at a rate of $17\frac{1}{2}\%$.

EXAMPLE 1B-1 Calculating overtime rates

Convert an hourly rate of \$26.20 to an overtime rate of:

a time-and-a-half

b double-time.

	Solve	Think	Apply
a	Time-and-a-half rate = $\$26.20 \times 1.5$ = \$39.30	Multiply the hourly rate of \$26.20 by the overtime loading of 1.5. This is the overtime hourly rate.	To calculate the overtime rate multiply the normal hourly rate by the overtime factor.
b	Double-time = $\$26.20 \times 2$ = \$52.40	Multiply the hourly rate of \$26.20 by the overtime loading of 2. This is the overtime hourly rate.	For time-and-a-half the factor is 1.5, and for double-time the factor is 2.

EXERCISE 1B Allowances, bonuses and loadings

- 1 Jasmine earns \$22.24 per hour. Copy and complete the following to find her overtime rate at:
 - a time-and-a-half
Overtime rate = $\frac{\text{normal rate}}{2} \times 1.5$
= _____
 - b double-time.
Overtime rate = $\frac{\text{normal rate}}{2} \times 2$
= _____
- 2 Convert these normal hourly pay rates to overtime rates at:
 - i time-and-a-half
a \$11.83 b \$14.35 c \$21.59 d \$18.70
 - ii double-time.

EXAMPLE 1B-2 Calculating wages using overtime rates

Janice is a junior and works 10 hours on a Sunday. The Sunday rate of pay is \$12.80 per hour for the first 4 hours, then 'time-and-a-half' for additional hours worked as overtime. Calculate Janice's wage for Sunday.

Solve	Think	Apply
Number of hours worked is 10. This is 4 hours at normal rate and 6 hours at time-and-a-half. Wage at normal rate = $4 \times \$12.80$ = \$51.20 Wage at time-and-a-half = $6 \times 1.5 \times \$12.80$ = \$115.20 Janice's wage = \$51.20 + \$115.20 = \$166.40	The first 4 hours are at the normal rate, so \$12.80 is multiplied by 4. The next 6 hours are at time-and-a-half. Multiply 6 by 1.5 and \$12.80 to calculate the overtime pay. Add \$51.20 and \$115.20 to find the total earned.	The normal hours are multiplied by the hourly rate. The overtime hours are multiplied by the hourly rate and the overtime factor. The normal and overtime totals are added.

- 3 Brian is a junior and is paid \$18.42 per hour. Calculate his weekly wage if he works 38 hours at the normal rate and 6 hours overtime at time-and-a-half.
- 4 Karen earns \$27.70 an hour. Calculate her weekly wage if she worked 33 hours at the normal rate and 6 hours at time-and-a-half.
- 5 Vikash is a junior and is paid \$13.24 per hour for a 35-hour week. He is paid overtime at double-time for the hours he works in excess of 35 hours. Calculate his pay in a week in which he works 44 hours.
- 6 Anna is a junior and earns \$14.35 per hour for the first 30 hours in a week. She receives time-and-a-half for the next 5 hours and double-time after that. Find her pay for a 42-hour week.
- 7 Alexia works 16 hours at \$20.20, 4 hours at time-and-a-half, and 3 hours at double-time. Calculate her weekly wage.



EXAMPLE 1B-3 Calculating weekly pay with an allowance

Vicki earns \$25.45 per hour as a cleaner. If she cleans the toilets, she receives an allowance of \$24.00 per day. Find her weekly pay if she works for 4 hours each weekday and cleans the toilets on Wednesday and Friday.

Solve	Think	Apply
$\text{Normal pay} = \$25.45 \times 4 \times 5$ $= \$509$ $\text{Toilet allowance} = \$24 \times 2$ $= \$48$ $\text{Total pay} = \$509 + \$48$ $= \$557$	Normal weekly pay is \$25.45 multiplied by 4 hours multiplied by 5 days. She cleans toilets on 2 days. The allowance is then multiplied by 2, giving \$48. Add the normal pay and allowance to get the total pay.	Allowances are added to normal income. First, calculate the normal pay. Next, calculate the amounts of each allowance. Add the allowance pay to the normal pay to find the total income.

- 8** Jovan earns \$34.20 per hour for an eight-hour day. He is paid a supervisor allowance of \$30 per day. Complete the following to find his pay for a week in which he works 5 days and is supervisor for 3 of the days.

a Normal pay	b Supervisor allowance	c Total pay
$= \$34.20 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$	$= \$30 \times \underline{\hspace{1cm}}$	$= \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$
$= \underline{\hspace{1cm}}$	$= \underline{\hspace{1cm}}$	$= \underline{\hspace{1cm}}$

- 9** Rose is paid \$20.70 per hour welding bins. If the temperature is over 30°C, she is paid an extra \$13 per day. Find her pay for a 35-hour week in which there are two days over 30°C.
- 10** Harry works in a timber yard and is paid \$21.40 per hour. He receives a dust allowance of \$12.50 per day. Find his pay for a 5-day, 40-hour week.
- 11** Georgina is a junior who works in a restaurant and receives a uniform allowance of \$9.50 per night. Her hourly rate of pay is \$18.85. Find her pay if she works 5 nights for 6 hours per night.

EXAMPLE 1B-4 Calculating the value of a bonus

Renee worked extremely hard throughout the year. Her reward was a bonus of 6% of her yearly income. If her normal yearly income was \$58 000, calculate the value of her bonus.

Solve	Think	Apply
$\text{Bonus} = 6\% \text{ of } \$58\,000$ $= \frac{6}{100} \times \$58\,000$ $= \$3480$	$6\% \text{ is } \frac{6}{100}$ Multiply this by \$58 000 to get \$3480.	If the bonus is a percentage, then that percentage amount is multiplied by the income.

- 12** Jeremy receives a bonus of 3% of his annual salary of \$43 600. Complete to find the value of his bonus:
- Bonus = $\underline{\hspace{1cm}}$ % of \$43 600
- $= \frac{3}{100} \times \underline{\hspace{1cm}}$
- $= \underline{\hspace{1cm}}$
- 13** Determine the bonus for each of these annual salaries.
- | | | |
|---------------------------|---------------------------|---------------------------|
| a 5% of \$53 000 | b 8% of \$92 500 | c 9% of \$123 468 |
| d 10% of \$196 320 | e 10% of \$253 452 | f 15% of \$271 096 |
- 14** Zofia earns \$590 per week. What is her total weekly wage in weeks with these bonuses added?
- | | | | |
|-------------|--------------|--------------|--------------|
| a 5% | b 12% | c 20% | d 14% |
|-------------|--------------|--------------|--------------|

EXAMPLE 1B-5 Calculating holiday pay with a leave loading

Lee is a permanent part-time employee and is entitled to 4 weeks annual leave with an annual leave loading of $17\frac{1}{2}\%$. If his weekly pay is \$368, find his holiday pay.

Solve	Think	Apply
$4 \text{ weeks pay} = \$368 \times 4$ $= \$1472$ $\text{Loading} = 17\frac{1}{2}\% \text{ of } \$1472$ $= 17\frac{1}{2} \div 100 \times \$1472$ $= \$257.60$ $\text{Holiday pay} = \$1472 + \$257.60$ $= \$1729.60$	Calculate 4 weeks pay by multiplying \$368 by 4. Find $17\frac{1}{2}\%$ of \$1472. Add the 4 weeks pay and the holiday loading to get the holiday pay.	First find 4 weeks pay; the holiday loading is $17\frac{1}{2}\%$ of that amount. Total holiday pay is the normal pay for 4 weeks plus the holiday loading. Total holiday pay can also be calculated by multiplying the normal 4 weeks pay by $117\frac{1}{2}\%$.

- 15** Caleb earns \$673 per week and is paid a holiday loading of $17\frac{1}{2}\%$ on 4 weeks pay. Copy and complete the following to find his holiday pay.

$$4 \text{ weeks pay} = \$673 \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$\text{Loading} = \underline{\hspace{1cm}}\% \text{ of } \$2692$$

$$= \frac{17.5}{100} \times \underline{\hspace{1cm}}$$

$$= \underline{\hspace{1cm}}$$

$$\text{Holiday pay} = \$2692 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

- 16** Allie has permanent part-time work as a personal assistant and is paid \$538 per week. If her 4 weeks of holiday pay attracts a loading of $17\frac{1}{2}\%$, find her holiday pay.

- 17** The holiday loading is $17\frac{1}{2}\%$. Find the 4 weeks holiday pay for employees who earn the following:

- a** a weekly wage of \$611
- b** a fortnightly wage of \$1054
- c** an hourly rate of \$28.45 for a 38-hour week

- 18** If $17\frac{1}{2}\%$ holiday loading is paid on 4 weeks normal pay, find the holiday pay for:

- a** John, who earns \$1700 in 4 weeks
- b** Divit, who earns \$720 in a fortnight
- c** Sid, who earns \$430 in a week
- d** Riya, who earns \$22.50 an hour for a 30-hour week
- e** Angie, who earns \$25.65 an hour for a 35-hour week.

- 19** Ximena was paid 4 weeks holiday pay plus $17\frac{1}{2}\%$ holiday loading. If her normal weekly pay is \$637.52, calculate:

- a** her normal pay for 4 weeks **b** her holiday loading for 4 weeks **c** her total holiday pay.

- 20** Paul earns \$28.60 an hour and works a 38-hour week. Calculate his holiday loading if he is paid $17\frac{1}{2}\%$ of 4 weeks wages.



21 Shane is paid \$21.30 per hour as a labourer. His normal working week is 35 hours. He is paid time-and-a-half for the first 5 hours worked beyond 35 hours and double-time for any hours after that. Calculate shane's weekly wage if he worked 49 hours.

22 a Jontey works from 8:00 am to 4:00 pm on Sunday. He is paid time-and-a-half for the morning work and double-time for the afternoon. If his normal hourly rate is \$26.90, calculate Jontey's earnings for Sunday.

b Jontey's employer is removing overtime rates and changing the normal rate to \$42 per hour. Is Jontey better off with the change? Explain.

c What would be the equivalent hourly rate so Jontey still receives the same Sunday pay?

23 The following wage table shows the hours worked by the junior employees at Active Sports. Calculate the weekly wage for each employee if:

- the normal hourly rate is \$19.40
- overtime of time-and-a-half is paid for hours worked beyond 4 hours from Monday to Friday
- time-and-a-half is the hourly rate on Saturdays.

	Employee	Day						Normal time (h)	Overtime (h) time-and-a-half	Weekly wage
		Mon	Tues	Wed	Thur	Frid	Sat			
a	M. Goodwyn	8	$8\frac{1}{2}$	8	9	4	6			
b	J. Osbourne	9	9	9	7	8	$3\frac{1}{2}$			
c	G. Dale	10	$9\frac{1}{2}$	9	$9\frac{1}{2}$	10	2			
d	E. Evans	$4\frac{1}{2}$	–	–	10	9	9			
e	M. Harkin	7	6	$4\frac{1}{2}$	5	2	$7\frac{1}{2}$			
f	A. Fenech	10	9	8	9	10	5			

24 Raymond earns \$96 325 per year. At the end of the year he is given a bonus of 13% of his yearly wage. Calculate his bonus.

25 Louie works as a plumber's apprentice and is paid \$13.80 per hour. He is paid a digging allowance of \$13.50 per day if he has to dig trenches and \$15.80 per day if he works on open sewers. Find his pay in a week in which he works 50 hours, digs on three days and works on an open sewer for two days.

26 Benny is a security guard and is paid \$24.25 per hour. If he works on the door he is paid \$25 per night danger money. Find Benny's wage for a week in which he is on the door for 3 nights and works 35 hours.

27 Nabil works in a factory and is paid \$25.54 per hour. If he works the furnace he is paid a heat allowance of \$5.27 per hour in addition to his normal rate. Find his weekly pay if he works a total of 32 hours of which 8 hours are spent working the furnace.

28 Jenny works in a bar and is paid \$23.90 per hour. If she makes cocktails she receives an extra \$3.20 per hour. Find her pay for a week in which she works 20 normal hours and a further 10 hours making cocktails.



- 29** Slavo earns \$35 600 per year. At the end of the year his company made a profit of \$129 561. As a reward, Slavo was given a bonus of 1% of the profits.
- What was Slavo's bonus?
 - What was his total income for the year?
- 30**
- Casey works 6 hours on Saturday at time-and-a-half and is paid \$234. Calculate her normal hourly pay rate.
 - Corey works 8 hours on Saturday at time-and-a-half and 5 hours on Sunday at double time. He earns \$539 for the week. Calculate his normal hourly pay rate.
- 31** Samantha received a $17\frac{1}{2}\%$ holiday loading on 4 weeks normal wages. She normally works a 35-hour week. Her 4 weeks holiday pay and loading is \$2533.30.
- Find her normal weekly pay.
 - Find her normal hourly pay rate.
- 32** On your obook assess, a spreadsheet for calculating an employee's wage including overtime at time-and-a-half is provided. The following view shows the formulas used.

	A	B	C	D	E	F
1	Wage calculator					
2	Hours	Mon	Tue	Wed	Thur	Fri
3	Normal	8	8	8	5	8
4	Overtime	2	–	3	–	1
5	Normal hours	=SUM(B3:F3)	Normal pay	=B5*B7		
6	Overtime hours	=SUM(B4:F4)	Overtime pay	=B6*1.5*B7		
7	Hourly rate	13.6	Total pay	=D5+D6		

- Use this spreadsheet or create your own to check your answers for questions 3 and 4.
- How would you change cell D6 in the spreadsheet if the overtime rate was double-time instead of time-and-a-half?
- Change the spreadsheet so you can use it to check your answer for question 5.
- Create a new spreadsheet to check your answers for questions 6 and 7.

- 33** Research and investigate the rates paid to various workers as special allowances.
- 34** Investigate the history of annual leave loading. Many employers are phasing out or trying to phase out leave loading. On the basis of your research, state your opinion about why this is the case.



1C Commission, piecework and royalties

commission

payment based on the percentage of the total price of the goods sold

piecework

payment based on an amount per item produced

royalty

payment based on a percentage of the continuing sales of a product that someone has made or created

These resources are available on your [obook assess](#):

- **assess quiz 1C:** Test your skills with an auto-correcting multiple-choice quiz

Real-estate agents and car salespersons are examples of people who are paid a **commission**. Their commission is a percentage of the total cost of the goods they sell. The more they sell, the greater their income. A person earning a salary is paid a fixed amount regardless of sales, and a wage earner is paid a fixed rate per hour. A person who works on commission may also be paid a retainer. A retainer is a fixed amount of money, independent of the total cost of the goods sold. People employed to make items may be paid for each item produced. This is called **piecework**. Authors will receive a percentage of the selling price of books they have written. This is called a **royalty**.

EXAMPLE 1C-1 Calculating commission

Larry sold a new car valued at \$22 800. His commission for selling the car was 6% of the selling price. Calculate Larry's commission.

Solve	Think	Apply
$\text{Commission} = 6\% \text{ of } \$22\,800$ $= \frac{6}{100} \text{ of } \$22\,800$ $= \$1368$ Larry earned \$1368 from his sale.	Larry sold goods worth \$22 800. His commission is 6%, so find 6% of \$22 800.	Commission is usually a percentage of the value of sales. Calculating the percentage of the amount sold finds the commission.

EXERCISE 1C Commission, piecework and royalties

- Holly holds a make-up party and sells \$1350 of make-up to her friends. Her commission is 7% of sales. Complete the following to find the amount earned.
 Commission amount
 $= \underline{\hspace{1cm}}\% \text{ of } \$1350$
 $= \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$
 $= \underline{\hspace{1cm}}$



- 2 Find the amount of commission earned for each of these sales amounts. Answer to the nearest dollar.

	Commission rate	Sales amount
a	25%	\$42 000
b	18%	\$6 782
c	7%	\$840
d	$15\frac{1}{2}\%$	\$132
e	2.7%	\$80 000
f	$17\frac{1}{3}\%$	\$80 000

- 3 Darren sold a house valued at \$650 000. Calculate Darren's commission from the sale if his rate of commission is 4.8%.
- 4 Josephine sold a car valued at \$48 000. Calculate her commission if the rate of commission is 7%.
- 5 An investment advisor receives 6% commission on money invested. How much does he earn for advising on a \$753 000 investment?

EXAMPLE 1C-2 Calculating commission and total earnings

A salesperson earns a retainer of \$330 a week plus 13% commission on sales. For a week in which her sales were \$1780, find:

a her commission

b her total earnings.

	Solve	Think	Apply
a	$\text{Commission} = 13\% \text{ of } \$1780$ $= \frac{13}{100} \times \$1780$ $= \$231.40$	The commission is 13% of \$1780. The salesperson is paid \$330 as well. Add \$330 and the commission (\$231.40) to find the total income.	Some salespeople are paid a retainer plus commission. The retainer is an amount paid even if there are no sales. It is independent of any sales. The amount of commission earned from sales is added to the retainer to calculate the total income.
b	Total earnings $= \text{retainer} + \text{commission}$ $= \$330 + \$231.40$ $= \$561.40$		

- 6 Luke earns \$280 per week plus 9% commission on sales. He makes \$2800 in sales in one week. Complete the following to find his commission amount and his weekly pay.
- a Commission amount = ____% of ____
 $= \frac{9}{100} \times \text{____}$
 $= \text{____}$
- b Weekly pay = ____ + \$280
 $= \text{____}$
- 7 Jenny earns \$265 per week plus $10\frac{1}{2}\%$ commission on sales. Calculate her total earnings for the weeks when she sold the following amounts.
- a \$124 b \$881 c \$2763 d \$3882 e \$4009 f \$5720
- 8 Samir earns \$310 per week plus 15% commission on sales. Calculate his total earnings for a week in which his sales are \$2050.
- 9 In one week, Mark sold goods to the value of \$143 000. If his retainer is \$145, calculate his wage that week if his rate of commission on sales is 13.7%.

EXAMPLE 1C-3 Calculating percentage rate of commission

An auctioneer sells 40 chairs at \$18.75 each.

a What is the total value of the sale?

b Determine the auctioneer's percentage rate of commission if she earns \$75 commission.

	Solve	Think	Apply
a	Sales = $40 \times \$18.75$ = \$750	Total sales is 40 chairs multiplied by \$18.75 each.	When finding the percentage rate of commission, two amounts are needed.
b	Commission = $\frac{75}{750} \times 100\%$ = 10%	Total commission = $\frac{\text{commission earned}}{\text{total sale value}} \times 100\%$ Divide 75 by 750 and multiply by 100.	First, the total of all sales. Second, the dollar amount of commission paid. The formula $\frac{\text{commission earned}}{\text{total sale value}} \times 100\%$ is used to calculate the rate of commission as a percentage.

- 10** Emily earns \$350 commission on sales of \$3723. Copy and complete the following to find her percentage rate of commission to one decimal place.

$$\begin{aligned} \text{Rate of commission} &= \frac{\text{commission earned}}{\text{total sale value}} \times 100\% \\ &= \frac{\square}{\$3723} \times 100\% \\ &\approx \underline{\quad\quad} \% \end{aligned}$$

- 11** A car salesperson earned \$7853.20 commission for 1 week. If his total sales were \$54 160, calculate the rate of commission as a percentage.

- 12** Determine the rate of commission as a percentage if Sue earned \$9348 on sales valued at \$76 000.

- 13** Copy and complete the following table.

	Employee	Rate of commission	Total sales	Commission earned
a	E. Swift	4.4%	\$72 000	
b	M. Keane		\$115 872	\$4634.88
c	S. Wise		\$64 500	\$9481.50
d	G. Walsh	8.2%	\$183 600	
e	G. Potts		\$286 472	\$16 472.14
f	L. Monroe	$12\frac{1}{3}\%$	\$350 700	

- 14** In one week a used car salesman sold three Ford Focus cars each priced at \$25 360 and earned $4\frac{1}{2}\%$ commission on his total sales. Find his commission for the week.

- 15** A real-estate agent sold a house valued at \$576 000. If she received $2\frac{1}{2}\%$ commission on the first \$150 000 and $\frac{1}{2}\%$ commission on the remainder, find her total commission.

- 16** Rhonda sells paintings. She is paid 3% of the first \$5000 of sales, 2% on the next \$5000 and 1% thereafter. Find her commission on sales of:

- a** \$4500 **b** \$6725
c \$9422 **d** \$14 680



- 17** Reginaldo is paid commission of 5% on the first \$1000 of sales, 4% on the next \$3000 and 1% thereafter. Find his commission on sales of:
- a** \$5467 **b** \$4321 **c** \$560 **d** \$9800

EXAMPLE 1C-4 Calculating pay for piecework

Enrique paints wooden fire engines. He is paid \$4.38 per fire engine painted. In 1 week he paints 143 fire engines. How much is he paid?

Solve	Think	Apply
$\text{Pay} = \$4.38 \times 143$ $= \$626.34$	Multiply \$4.38 by 143.	To find the total earned multiply the amount per item by the number of items.

- 18** Angelina paints Christmas decorations and is paid \$1.80 per decoration. Complete the following to find her pay for 300 decorations.
- Pay = $\underline{\hspace{1cm}}$ $\times$ 300 = $\underline{\hspace{1cm}}$



- 19** Austin spray paints wooden toys and is paid \$6.90 per toy. Find his pay if he paints 168 toys.
- 20** Shania assembles skateboards. She is paid \$15.00 per assembled board. How much does she earn for a week in which she assembles 43 skateboards?
- 21** Ricky waxes surfboards for the surf hire shop. He is paid \$6.70 per waxed board. Find his pay for a day in which he waxes 15 boards.
- 22** Ellen works in a belt factory. Each belt completed earns her \$4.20. Find her wage for a week in which she makes 157 belts.

EXAMPLE 1C-5 Calculating hourly rate

Antonia makes dolls which she sells at a craft market. She sells each doll for \$36.

- a** How much does Antonia earn if she sells 18 dolls?
- b** If it takes Antonia 5 h to make a doll, what is her hourly rate?

	Solve	Think	Apply
a	$\text{Income} = 18 \times \$36$ $= \$648$	Multiply \$36 by 18 dolls.	To find the total earned, multiply the amount per item by the number of items.
b	$\text{Hourly rate} = \frac{\$36}{5}$ $= \$7.20$	Divide \$36 by 5 to get the hourly rate.	Hourly rate is found by dividing the amount paid per item by the time taken in hours.

- 23** Roberto makes cricket bats and is paid \$130 per bat. In a 35-hour week he makes 8 bats. Complete the following to find his weekly pay and hourly rate:
- a** Weekly pay = $\underline{\hspace{1cm}} \times 8$
 $= \underline{\hspace{1cm}}$
- b** Hourly rate = weekly pay $\div$ hours worked
 $= \underline{\hspace{1cm}} \div 35$
 $= \underline{\hspace{1cm}}$

- 24** Ivan refelts billiard tables and is paid \$290 per completed table. If he completes 8 tables in a 35-hour week, find his:
a weekly pay **b** hourly rate.
- 25** Allan strings new tennis racquets and is paid \$7.50 per racquet. In a 40-hour week Allan strings 120 racquets. Find his:
a weekly pay **b** hourly rate.

EXAMPLE 1C-6 Calculating income for piecework paid at different rates

Employees at the Fancy Hanky factory are paid on a piecework basis at the rate of 39 cents per article produced, up to 200 items per day, and 53 cents per article for each piece produced in excess of 200 items per day. Calculate Romina's income on Tuesday if she produces 224 hankies that day.

Solve	Think	Apply
$\text{Tuesday} = 200 \times \$0.39 + 24 \times \$0.53$ $= \$90.72$ $\text{Income} = \$90.72$	Romina makes more than 200 hankies. The number over 200 is multiplied by 53 cents and the first 200 are multiplied by 39 cents.	Where more than one rate is paid, find the times where this occurs. Split the number into the parts required and multiply by the correct rate.

- 26** Stephanie makes savouries and is paid 80 cents per savoury for the first 50 per day and 95 cents for each savoury in excess of 50. On Monday she makes 45 savouries and on Tuesday she makes 110. Complete the following to find her total pay for the two days.

Monday = $\_\_\_ \times \$0.80 = \_\_\_$

Tuesday = $50 \times \_\_\_ + \_\_\_ \times \$0.95 = \_\_\_$

Total pay = $\_\_\_ + \_\_\_ = \_\_\_$

- 27** Calculate the weekly wage of the following employees who are paid on a piecework basis at the rate of 57 cents for each piece up to 200 pieces per day and then 65 cents for each piece produced in excess of 200 pieces.



	Name	Monday	Tuesday	Wednesday	Thursday	Friday
a	N. Solo	230	250	186	223	192
b	I. Kuryakin	290	210	175	238	177

- 28** Paula is paid a royalty of 12.3% on all sales. Complete the following to find her royalty on sales of \$68 000.

Royalty = $\_\_\_ \%$ of \$68 000

$= \frac{12.3}{100} \times \_\_\_$

$= \_\_\_$

- 29** Calculate the royalties for these authors.

	Author	Royalty rate	Sales
a	M. Golovchenko	8.7%	\$43 590
b	N. Smythe	7.3%	\$367 098
c	R. Ireland	12.9%	\$23 098
d	P. Goodacre	10.5%	\$358 900

- 30** Karen and Tony both work for Up Market Realty. Karen sold a home for \$535 000 and Tony sold two units, each valued at \$285 000. The commission rate was 4%.

- Calculate Karen's commission.
- Calculate Tony's commission.
- Who made more from the sales? By how much?

- 31** Bart earned \$446.20 on sales valued at \$4600. Karla earned \$213.18 on sales valued at \$2090.

- Who has the better rate of commission?
- What is that better commission rate?

- 32** Madison works in sales. She is given the choice of being paid 8% commission on sales only, or a base salary of \$200 per week plus $2\frac{1}{2}\%$ commission on sales. If Madison sells \$3050 worth of goods in a week, which method of payment would pay her the most? By how much would she be better off?

- 33** Jenny was offered a retainer of \$640 per month, \$140 per month car allowance, plus $1\frac{1}{2}\%$ commission on any houses she sells. If during her first month of work for her new company she sold two units valued at \$249 000 and \$317 000 respectively, what was Jenny's total income for the month?

- 34** A doctor charges each person \$35.80 per consultation. If the doctor sees, on average, 4 people an hour, calculate her earnings for an 8-hour day.

- 35** Nadia is a hairdresser who works at home. She charges \$17 for a haircut, \$42 for a cut and colour and \$65.20 for a perm. In one week she completes 7 haircuts, 14 cuts and colours, and 2 perms.

- Find her income for the week.
- Calculate her profit for the week, if her expenses were \$287.30.
- If Nadia worked a total of 32 h during this week, calculate her hourly rate of pay after expenses.

- 36** A dressmaker charges \$19.00 for major alterations and \$11.20 for minor alterations.

- Calculate her wage if, in one week, she completed 15 major alterations and 23 minor alterations.
- Determine her hourly rate if she spent a total of 16 hours working on alterations.

- 37** Ayansh sells books and is paid a retainer of \$875 per month and a percentage commission on total sales made. Calculate the percentage commission rate for a month where he earns a total of \$1562.50 on sales of \$27 500.

- 38** George sells confectionary. He is paid \$2440 per month plus commission on all sales. His commission is calculated using the scale shown in the table below.

Sales value	Commission rate
Up to \$15 000	No commission
\$15 001 to \$25 000	4% of sales over \$15 000
\$25 001 to \$40 000	\$600 plus 3% of sales over \$25 000
\$40 001 and over	\$1050 plus 1.5% of sales over \$40 000

Calculate George's earnings for the following months.

- March sales totalled \$24 890
- April sales totalled \$36 002
- May sales totalled \$14 050
- June sales totalled \$53 821



1D Government allowances and pensions

These resources are available on your obook assess:

- **Spreadsheet 1D:** Calculate part pensions and part allowances
- **assess quiz 1D:** Test your skills with an auto-correcting multiple-choice quiz

Some people may receive pensions or allowances from the government. These include pensions for old age, disabilities and sole parenthood. Some allowances include Youth Allowance and Job Start. Youth Allowance is paid to young people who are studying full-time, undertaking a full-time apprenticeship, training, looking for work or are sick.

Youth Allowance rates (as at July 2017)

Circumstance	Fortnightly payment
Single, no children	
Under 18, at home	\$239.50
Under 18, away from home	\$437.50
18 and over, at home	\$288.10
18 and over, away from home	\$437.50
Single with children	\$573.30
Partnered, no children	\$437.50
Partnered with children	\$480.50

Students and Australian apprentices personal income test

Students and Australian apprentices	If you earn between \$437 and \$524 your fortnightly payment reduces by	If you earn more than \$524 your fortnightly payment reduces by	Your payment reduces to \$0 once your income reaches the maximum of
Single, under 18 years, at home	50 cents for each dollar you earn over \$437	\$43.50 plus 60 cents for each dollar you earn over \$524	\$857.17
Single, 18 years and over, at home	50 cents for each dollar you earn over \$437	\$43.50 plus 60 cents for each dollar you earn over \$524	\$939.34
Single or couple, no dependants, away from home	50 cents for each dollar you earn over \$437	\$43.50 plus 60 cents for each dollar you earn over \$524	\$1192.34
Couple with dependants	50 cents for each dollar you earn over \$437	\$43.50 plus 60 cents for each dollar you earn over \$524	\$1265.17
Single with dependants	50 cents for each dollar you earn over \$437	\$43.50 plus 60 cents for each dollar you earn over \$524	\$1422.34

If you earn less than \$437 per fortnight, your Youth Allowance payment is not affected.

Job seekers personal income test

Job seekers	If you earn between \$143 and \$250 your fortnightly payment reduces by	If you earn more than \$250 your fortnightly payment reduces by	Your payment reduces to \$0 once your income reaches the maximum of
Single, under 18 years, at home	50 cents for each dollar you earn over \$143	\$53.50 plus 60 cents for each dollar you earn over \$250	\$566.50
Single, 18 years and over, at home	50 cents for each dollar you earn over \$143	\$53.50 plus 60 cents for each dollar you earn over \$250	\$648.67
Single or couple, no dependants, away from home	50 cents for each dollar you earn over \$143	\$53.50 plus 60 cents for each dollar you earn over \$250	\$901.67
Couple with dependants	50 cents for each dollar you earn over \$143	\$53.50 plus 60 cents for each dollar you earn over \$250	\$974.50
Single with dependants	50 cents for each dollar you earn over \$143	\$53.50 plus 60 cents for each dollar you earn over \$250	\$1131.67
Single principal carer of dependent children, granted an exemption for foster caring, home schooling, distance education or large family	50 cents for each dollar you earn over \$143	\$53.50 plus 60 cents for each dollar you earn over \$250	\$1427.67

If you earn less than \$143 per fortnight, your Youth Allowance payment is not affected.

EXAMPLE 1D-1 Calculating Youth Allowance for a student living at home

Jenny is a student living at home. She is 17 and entitled to Youth Allowance.

- What is her Youth Allowance per fortnight if she has no income?
- How much could she earn and still keep full Youth Allowance?
- How much could she earn before she loses her allowance completely?

	Solve	Think	Apply
a	From the Youth Allowance rates table, Jenny is entitled to \$239.50.	Look in the Youth Allowance rates table for 'single, no children, under 18, at home'. The rate is \$239.50.	It is important to look in the correct part of the table. Find the category you require, then check for the correct amounts.
b	From the personal income test table Jenny could earn up to \$437.	Look in the Students and Australian apprentices personal income table for 'single, under 18, at home'. Consider the meaning of the heading for the second column.	
c	From the personal income test table, Jenny must earn less than \$857.17.	Look in the last column of the 'Students and Australian apprentices personal income test' table.	

EXERCISE 1D Government allowances and pensions

- 1 Andrea is a student living away from home. She is 18 and entitled to Youth Allowance.
 - a What is her Youth Allowance if she has no income?
(Hint: Look in the 'Youth Allowance rates' table.)
 - b How much could she earn and still keep the full Youth Allowance?
(Hint: Look in the 'Students and Australian apprentices personal income test' table and find the value at the end of the table.)
 - c How much could she earn before she loses her allowance completely?
(Hint: As for part b, but look in the 'Your payment reduces to \$0 once your income reaches the maximum of' column.)
- 2 Goran is a student living at home. He is 19 and entitled to Youth Allowance.
 - a What is his Youth Allowance if he has no income?
 - b How much could he earn and still keep the full Youth Allowance?
 - c How much could he earn before he loses his allowance completely?
- 3 Andres is a job seeker and living at home. He is 19 and entitled to Youth Allowance.
 - a What is his Youth Allowance if he has no income?
 - b How much could he earn and still keep the full Youth Allowance?
 - c How much could he earn before he loses his allowance completely?
- 4 Sylvana is a job seeker and living away from home. She is 17 and entitled to Youth Allowance.
 - a What is her Youth Allowance if she has no income?
 - b How much could she earn and still keep the full Youth Allowance?
 - c How much could she earn before she loses her allowance completely?
- 5 Tito is a student living away from home. He is 16 and entitled to Youth Allowance.
 - a What is his Youth Allowance if he has no income?
 - b If he starts a part-time job earning \$350 per fortnight, would his Youth Allowance be affected? Explain.
 - c If he started a part-time job earning \$225 per week, would his Youth Allowance be affected? Explain.
 - d How much could he earn before he loses his allowance completely?
- 6 Leia is a job seeker living at home. She is 20 and entitled to Youth Allowance.
 - a What is her Youth Allowance if she has no income?
 - b If she starts a part-time job earning \$160 per fortnight, would her Youth Allowance be affected? Explain.
 - c If she started a part-time job earning \$70 per week, would her Youth Allowance be affected? Explain.
 - d How much could she earn before she loses her allowance completely?



EXAMPLE 1D-2 Calculating Youth Allowance when earning a small income

Marcella is 19, living at home and a job seeker. She earns \$187 per fortnight, and is entitled to Youth Allowance. Calculate the amount of Youth Allowance that Marcella will receive.

Solve	Think	Apply
Income in excess of \$143 $= \$187 - \$143$ $= \$44$ Reduction $= 44 \times 0.50$ $= \$22$ Marcella's allowance $= \$288.10 - \$22$ $= \$266.10$	Full Youth Allowance for Marcella would be \$288.10. She loses some because her income exceeds \$143. Reduction for income between \$143 and \$250 is 50 cents in the dollar.	Find the normal full Youth Allowance. Compare the income with the reduction conditions in the 'Job seekers personal income test' table. Calculate the cents in the dollar reduction first. If applicable, add the fixed reduction to the dollar reduction. Subtract the total reduction from the full allowance. Be careful to read all the tables accurately.

- 7** Amy is 18, living at home and a student entitled to Youth Allowance. She earns \$501 per fortnight. Complete the following to calculate the amount of allowance Amy receives.

- From the 'Youth Allowance rates' table, full allowance for Amy is ____.
- Income in excess of \$437 = ____ - \$437 = ____
- Reduction = $64 \times$ ____ = ____
- Amy's allowance = ____ - \$32 = ____

- 8** Ingrid is 17, living at home and a job seeker. She earns \$205 per fortnight, and is entitled to Youth Allowance. Calculate the allowance Ingrid will receive each fortnight.

- 9** George is 19, living away from home and a job seeker. He earns \$211 per fortnight, and is entitled to Youth Allowance. Calculate the amount of Youth Allowance that George will receive each fortnight.

- 10** Theresa is 18, living at home and a student. She earns \$414 per fortnight, and is entitled to Youth Allowance. Calculate the amount of Youth Allowance that Theresa will receive each fortnight.



- 11** Christos is 20, living at home and a job seeker. He earns \$85 per week and is entitled to Youth Allowance. Calculate the amount of Youth Allowance that Christos will receive each fortnight.
- 12** Piera is 16, living at home and a student. She earns \$250 per week and is entitled to Youth Allowance. Calculate the amount of Youth Allowance that Piera will receive each fortnight.

EXAMPLE 1D-3 Calculating Youth Allowance when working part-time

Dukan is 18 and has a partner and one child. He is a full-time student and is entitled to Youth Allowance. Dukan works part-time and earns \$614 per fortnight. Calculate his fortnightly payment.

Solve	Think	Apply
Dukan earns more than \$524 but less than \$1265.17 per fortnight, so he is entitled to a part allowance. From the 'Youth Allowance rates' table, full allowance in these circumstances is \$480.50. Income in excess of \$524 $= \$614 - \$524$ $= \$90$ Reduction = 90×0.60 $= \$54.00$ Total reduction = $\\$43.50 + \\$54.00$ $= \$97.50$ Dukan's allowance $= \$480.50 - \$97.50$ $= \$383.00 \text{ per fortnight}$	Look up the full allowance in the 'partnered with children' section of the 'Youth Allowance rates' table. Dukan's income exceeds \$524 and so the fixed reduction applies, along with the 60 cents in the dollar for the amount in excess of \$524. Subtract \$524 from income of \$614, then multiply the result by 0.60 (being \$0.60 or 60 cents). Then add the fixed amount of \$43.50 to that amount. Subtract the total from \$480.50.	Find the normal full Youth Allowance. Compare the income with the amounts requiring reduction to determine whether there are multiple reduction amounts needed. Calculate the cents in the dollar reduction first. If applicable, add the fixed reduction to the dollar reduction. Subtract the total reduction from the full allowance. Be careful to read all the tables accurately.

- 13** Sami is 19, single and has one child. She is a full-time student entitled to Youth Allowance. Sami's part-time income is \$564 per fortnight. Complete the following to calculate her Youth Allowance.

- Youth Allowance rate from table is ____.
- Income in excess of \$524 = $\$564 - \underline{\hspace{1cm}}$
= \$40
- Reduction = $\underline{\hspace{1cm}} \times 0.60 = \underline{\hspace{1cm}}$
- Fixed reduction for income over \$524 = ____
- Total reduction = $\underline{\hspace{1cm}} + \$43.50 = \underline{\hspace{1cm}}$
- Sami's allowance = $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

- 14** Venus is 16, living at home and a student. She earns \$580 per fortnight and is entitled to Youth Allowance. Calculate the amount of Youth Allowance that Venus will receive.

- 15** Xavier is 17 and lives at home. He is a full-time student and entitled to Youth Allowance. Xavier works part-time and earns \$544 per fortnight. Calculate his Youth Allowance.

- 16** Yoshii is 20, lives away from home and has a partner. He is a full-time student and is entitled to Youth Allowance. He earns \$626.00 per fortnight. Calculate his Youth Allowance.

- 17** Nerita is 20, living at home and a job seeker. She earns \$175 per week and is entitled to Youth Allowance. Calculate the amount of Youth Allowance that Nerita will receive each fortnight.



The table below shows the personal income test for pensions. Use the table to answer questions 18–21.

Family situation	Income test for pensions	
	For full pension (per fortnight)*	For part pension (per fortnight)
Single	up to \$164	less than \$1940.60
Couple (combined)	up to \$292	less than \$2970.40
Couple living apart for health reasons	up to \$292	less than \$3845.20

*Income over these amounts reduces the rate of pension payable by 50 cents in the dollar.

- 18** Joan and Don are entitled to the age pension. They are a couple with a combined income of \$453.00 per fortnight. The full pension for each person is \$609.30 per fortnight. Calculate the total pension paid to Joan and Don.

- 19** Maurice is single with an income of \$620 per fortnight. He is entitled to the age pension. If the full pension is \$808.30 per fortnight, calculate the pension Maurice will receive.

- 20** George and Cheryl are a couple entitled to the age pension but living apart due to health reasons. George earns \$320 per fortnight. The full pension is \$808.30 each. Calculate the total pension amount they would receive each fortnight.

- 21** Jessica is single and has been receiving the full age pension of \$808.30 per fortnight. Her financial situation is about to change and she anticipates receiving an income of \$1000 per week. How will this income affect the age pension Jessica has been receiving?

- 22** Use the internet to find the latest values for Youth Allowance and pensions. Recalculate the answers to some of the questions using these. The website address for the Australian Department of Human Services is: <http://www.humanservices.gov.au>

- 23** Design a spreadsheet that will allow you to calculate part pensions and part allowances. A prepared spreadsheet is provided on your obook assess. Modify this spreadsheet with the latest data from the internet or the Australian Tax Office.

- 24** Research the following questions to investigate further examples of pensions or allowances.

- Find at least three examples of pensions or allowances.
- List the criteria for obtaining these pensions or allowances.
- Find the amount paid to people on these pensions or allowances.
- Find the amounts that people may earn and still retain the full pension or allowance.
- Find the reasons for having these pensions or allowances.
- Where does the money come from to pay for these pension or allowances?



1E Deductions and net income

gross income

total amount earned by an employee

net income

reduced amount earned by an employee after subtracting deductions from gross income

These resources are available on your obook assess:

- **assess quiz 1E:** Test your skills with an auto-correcting multiple-choice quiz

The total amount earned by an employee is called **gross income**. The **net income** is calculated by subtracting deductions from the gross income.

Net income = gross income – all deductions

The most common deductions include income tax, health insurance, employee superannuation contributions, social club contributions, voluntary savings and union fees.

EXAMPLE 1E-1 Calculating net income

Won Tae receives a gross weekly income of \$865.43. His weekly deductions are income tax of \$139.00, union fees of \$6.20, health insurance of \$9.40 and employee superannuation contribution of \$28.20. Calculate his net income.

Solve	Think	Apply
$\begin{aligned}\text{Total deductions} &= \$139.00 + \$6.20 + \$9.40 + \$28.20 \\ &= \$182.80 \\ \text{Net income} &= \text{gross income} - \text{deductions} \\ &= \$865.43 - \$182.80 \\ &= \$682.63\end{aligned}$	Add together all the deductions (tax, union fees, health insurance and superannuation). Subtract this total of \$182.80 from the gross income of \$865.43.	To calculate net income, subtract all deductions from the gross income.

EXERCISE 1E Deductions and net income

- Pooja has a gross weekly wage of \$642.90 and tax of \$56 and union fees of \$6.80 are deducted from her weekly wage. Complete the following to calculate her total deductions and her net income.
 - Total deductions = ____ + ____ = ____
 - Net income = ____ – \$62.80 = ____
- Tony's gross weekly wage is \$763.90. His weekly wage deductions are tax of \$67, union fees of \$4.30, health insurance of \$10.40 and employee superannuation contribution of \$18.45. Calculate his net income.
- Calculate net income if gross income is \$980.25 per week, tax is \$159, employee superannuation contribution is \$13.68 and union fees are \$7.30 per week.
 - Calculate net income if gross pay is \$611.92 per week, tax is \$51, employee superannuation contribution is \$12.65, union fees are \$3.00, and health insurance is \$15.95 per week.
- Calculate gross income if net income is \$564.70, tax is \$80.00, health insurance is \$14.80, and employee superannuation contribution is \$10.50 per week.
 - Calculate gross income if net income is \$629.90, tax is \$116.00, union fees are \$3.75 and employee superannuation contribution is \$50.35 per week.

The table below shows health insurance costs (the 30% rebate has been deducted from these premiums).

NSW health insurance premiums – single

Hospital cover	Direct debit/Payroll deduction			Advance Pay		
	Weekly	Fortnightly	Monthly	Quarterly	Half-yearly	Yearly
Top hospital no excess	\$25.64	\$51.27	\$111.09	\$333.26	\$666.51	\$1310.00
Intermediate hospital no excess	\$19.32	\$38.63	\$83.70	\$251.10	\$502.19	\$996.00
<i>Intermediate hospital excess</i>						
Level 1: \$250	\$18.59	\$37.18	\$80.56	\$241.67	\$483.34	\$950.00
Level 2: \$500	\$16.48	\$32.96	\$71.41	\$214.24	\$428.48	\$845.00
Level 3: \$1000	\$13.21	\$26.42	\$57.24	\$171.73	\$343.46	\$680.00
<i>Basic hospital excess</i>						
Level 1: \$250	\$16.21	\$32.42	\$70.24	\$210.73	\$421.46	\$835.00
Level 2: \$500	\$14.76	\$29.52	\$63.96	\$191.88	\$383.76	\$762.00
Level 3: \$1000	\$11.13	\$22.25	\$48.21	\$144.63	\$289.25	\$575.00

EXAMPLE 1E-2 Calculating the annual premium for hospital cover

Jenny is single. She takes out top hospital cover with no excess and pays her premium in advance. Find her annual cost.

Solve	Think	Apply
From the table, Jenny pays \$1310.	The table shows premiums for singles. Read across the 'Top hospital no excess' row to the 'Yearly' column.	First check the table headings. Match the row and column headings to find the value.

- 5 Harry is single and takes out intermediate hospital cover with no excess. Find his premium:
 - a per week, if paid by payroll deduction
 - b per month, if paid by payroll deduction
 - c per quarter, if paid in advance
 - d per year, if paid in advance.
- 6 Lasallian takes out basic hospital cover for singles. She chooses the \$1000 excess.
 - a Find her weekly premium.
 - b Find the amount she pays for a year by making weekly payments.
 - c How much would she save by paying the annual premium in advance?
 - d How much does she save per week by having a \$1000 excess rather than a \$250 excess?
- 7 For each of these rates, calculate the saving by paying yearly in advance instead of paying weekly.
 - a Top cover, no excess
 - b Intermediate cover, no excess
- 8 Paula pays \$57.24 per month for health insurance. Which cover does she have?
- 9 Find the cover the following premiums pay for.
 - a \$63.96 per month
 - b \$32.96 per fortnight
 - c \$80.56 per month
 - d \$680 annually

Superannuation (super) is a way of saving for retirement. Employers contribute a base rate of 9.5% of an employee's normal income directly into the employee's specified super fund as an extra payment above the employee's income. Super funds invest the money in assets such as shares, property and managed funds.

EXAMPLE 1E-3 Calculating superannuation contributions

Paul earns \$56 092 per annum. His employer pays superannuation of 9.5% on his behalf and Paul contributes 4% himself.

- How much superannuation is paid on Paul's behalf by his employer?
- How much does Paul contribute himself?
- What is Paul's income less his superannuation contribution?
- What is the total amount paid into Paul's superannuation fund per year?

	Solve	Think	Apply
a	Employer's contribution $= \frac{9.5}{100} \times \$56\,092$ $= \$5328.74$	Employer contributions are 9.5%, so find 9.5% of \$56 092.	The employer contribution is a percentage of the total income, paid over and above that income. <i>Note:</i> The personal superannuation contribution is subtracted from the total income, but a tax of 15% is deducted by the superannuation fund, if applicable.
b	Paul's contribution $= \frac{4}{100} \times \$56\,092$ $= \$2243.68$	Paul contributes 4%, so find 4% of \$56 092.	
c	Paul's income (less super) $= \$56\,092 - \$2243.68$ $= \$53\,848.32$	The employer contribution of \$5328.74 is additional to Paul's income and so is not subtracted.	
d	Total super contribution $= \$5328.74 + \$2243.68$ $= \$7572.42$	Total superannuation put into the fund is \$5328.74 plus \$2243.68.	

- 10** Arif earns \$41 922 per annum. His employer pays superannuation of 9.5% on his behalf and Arif contributes 4%.

- How much superannuation is paid on Arif's behalf by his employer?
Employer superannuation contribution $= \frac{9.5}{100} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
- How much does Arif contribute himself?
Employee superannuation contribution $= \frac{\square}{100} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
- What is Arif's income less his superannuation contribution?
Income $= \underline{\hspace{1cm}} - \$1676.88 = \underline{\hspace{1cm}}$
- What is the total amount paid into Arif's superannuation fund?
Total superannuation paid into the fund $= \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

- 11** For the following people, calculate:

- employer superannuation contributions
- employee superannuation contributions
- net income
- total superannuation paid into the fund.

	Name	Annual income	Employer contribution	Employee contribution
a	Fielding	\$37 764	9.5%	1%
b	Louise	\$48 193	9.5%	7%
c	Bianca	\$12 943	9.5%	0%
d	Peter	\$41 098	9.5%	5%
e	Nato	\$22 940	9.5%	0%
f	Ito	\$63 901	9.5%	6%

- 12** Wai earns \$949 per week. His employer contributes 9.5% of that amount for superannuation and Wai contributes 3%. Calculate Wai's net weekly income and the total weekly contributions to his superannuation.

EXAMPLE 1E-4 Calculating annual net income after deductions are made

Calculate the annual net income for an employee whose gross yearly income is \$73 926. He has union fees of \$276, tax of \$14 521.89, employee superannuation contributions of 4% of gross income and basic private (single) health insurance with \$500 excess, which is paid yearly in advance.

Solve	Think	Apply
Deductions: $\text{Superannuation} = \frac{4}{100} \times \$73\,926$ $= \$2957.04$ Health insurance = \$762 Net income $= \$73\,926 - \$14\,521.89 - \$2957.04$ $- \$762 - \$276$ $= \$55\,409.07$	Calculate the amount of superannuation, using 4% of \$73 926. Find the health contribution from the table on page 29. Net income is \$73 926 minus the amounts calculated and the other deductions of \$276 and \$14 521.89.	Calculate the value of any deductions and subtract these from the gross income to find the net income. Note that the compulsory employer super contribution is not included in the calculations.

- 13** Complete the following to calculate the annual net income for Anita whose gross income is \$42 965 p.a. She has union fees of \$365 and personal super contributions of 5% of gross income.

$$\text{Superannuation} = \frac{5}{100} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$\text{Net income} = \underline{\hspace{2cm}} - \$365 - \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

- 14** Calculate the annual net income for Jahmalia whose gross income is \$47 955 p.a. if, annually, she has tax of \$6339.78, union fees of \$421, employee superannuation contributions of 5% of gross income and intermediate private (single) health insurance with no excess, paid in advance.

- 15** Theresa earns \$38 902 per annum. She has 11% tax taken out, 6% employee superannuation contributions and health insurance of \$672.70 per annum.

- Calculate the tax Theresa has taken out of her gross annual income.
- How much superannuation does Theresa contribute?
- What is Theresa's net annual income?
- Calculate Theresa's net weekly pay.

- 16** Carol is retired and is paid a yearly superannuation payment of 68% of her final year's salary, which was \$138 000. Calculate her:

- yearly superannuation payment
- monthly income.

- 17** A particular superannuation scheme pays its members a retirement benefit of 20% of their average salary over the last 5 years multiplied by the number of years they have been in the scheme.

Retirement payment = 20% of average salary $\times$ number of years

Find the retirement benefit for a person with:

- an average salary of \$46 981 who has contributed for 15 years
- an average salary of \$39 243 who has contributed for 20 years
- salaries of \$38 942, \$39 210, \$29 881, \$33 125 and \$40 010 for the last 5 years who has contributed for 8 years.

1F Taxable income

These resources are available on your obook assess:

- **Video tutorial 1F:** Watch and listen to an explanation of Example 1F-1
- **Spreadsheet 1F:** Calculate the Medicare levy payable on taxable income
- **assess quiz 1F:** Test your skills with an auto-correcting multiple-choice quiz

taxable income

income for one financial year calculated by subtracting allowable deductions from total income

total income

all income received by a person during one financial year

Medicare levy

extra tax to fund the health care system, calculated as a percentage of taxable income

The **taxable income** is calculated using **total income** and allowable deductions. Total income includes income from all sources throughout the year. It could include wages and salaries, bonuses, interest earned, commissions and allowances. The total income may be subject to deductions that reduce the amount used to calculate the tax payable. Tax deductions may include the cost of tools used for work, the cost of safety equipment, self-education expenses, union fees, charitable donations over \$2, car travel expenses, the cost of uniforms and their cleaning, and tax agent fees. The deductions allowable will depend on the profession.

$$\text{Taxable income} = \text{total income} - \text{allowable deductions}$$

The **Medicare levy** is an extra tax that may be payable. Medicare is the public hospital medical system available without charge for Australians. The Medicare levy is currently calculated at 2% of taxable income. In 2017, the full Medicare levy was payable if a person's taxable income exceeded \$26 668. For incomes between \$21 335 and \$26 668, a reduced levy was payable, and for incomes of less than \$21 335 no levy was payable. These thresholds were higher in some circumstances. A Medicare levy surcharge may also apply if a person does not have private hospital health insurance.

For taxation purposes, income is usually calculated in whole dollars, so always round down to the nearest whole dollar. (It does not follow the usual rules for rounding.)



EXAMPLE 1F-1 Calculating taxable income

Amita uses the following information to calculate her taxable income: wages \$35 980, interest \$569 (joint account), bonus \$200, cost of uniforms \$140, tax agent fee \$80, work-related car costs \$298. Calculate her taxable income.

Solve	Think	Apply
$\begin{aligned} \text{Income} &= \$35\,980 + (\$569 \div 2) + \$200 \\ &= \$36\,464.50 \\ &= \$36\,464 \text{ (rounded down to the nearest dollar)} \end{aligned}$	Interest on a joint account is shared between both account holders.	Determine all income amounts and all deductions.
$\begin{aligned} \text{Deductions} &= \$140 + \$80 + \$298 \\ &= \$518 \end{aligned}$	Add all income items and round down to the nearest dollar, to give \$36 464.	Subtract to find the taxable income.
$\begin{aligned} \text{Taxable income} &= \$36\,464 - \$518 \\ &= \$35\,946 \end{aligned}$	Add all deductions, to get \$518. Subtract to find the taxable income of \$35 946.	

EXERCISE 1F Taxable income

- Annie uses the following information to calculate her taxable income: wages \$45 320, interest \$665 (joint account), bonus \$800, cost of uniforms \$340, tax agent fee \$60 and work-related car costs \$45. Complete the following to calculate Annie's taxable income.

$$\text{Income} = \underline{\hspace{1cm}} + (\underline{\hspace{1cm}} \div 2) + \$800 = \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \text{ (rounded down to the nearest dollar)}$$

$$\text{Deductions} = \underline{\hspace{1cm}} + \$60 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$

$$\text{Taxable income} = \underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$$
- Use the following information to calculate Gary's taxable income: wages \$42 330, interest \$2355 (joint account), cost of uniforms \$300, cleaning of uniforms \$156, tax agent fee \$100 and work-related car costs \$3220.
- Asha uses the following information to calculate her taxable income: wages \$29 555, interest \$243, tax agent fee \$70, work-related car costs \$452, donations to charity \$120. Calculate Asha's taxable income.
- Paul uses the following information to calculate his taxable income: wages \$53 022, interest \$3659, bonus \$1000, tax agent fee \$120, work-related car costs \$1256, donations to charity \$500. Calculate Paul's taxable income.
- Calculate the taxable income for the following people.

Income/deductions	Tasman	Polding	Flockhart	Gen	Old
Wage	\$45 996	\$36 899	\$72 665	\$12 990	\$26 773
Allowances	\$590	\$100	–	\$390	\$225
Interest	\$32	\$12	\$366	\$5	\$15
Bonus/tips	\$100	–	\$2000	–	\$25
Uniforms	\$50	–	–	–	\$150
Cleaning	\$35	–	–	–	\$55
Tax agent fee	\$80	\$60	\$200	\$20	\$45
Union fee	\$300	–	–	\$60	\$190
Donations	–	–	\$100	\$2	\$20
Car expenses	\$423	\$19	\$2333	–	–

EXAMPLE 1F-2 Calculating the Medicare levy on taxable income

Calculate the Medicare levy payable on a taxable income of \$35 908.

Solve	Think	Apply
$\begin{aligned} \text{Medicare levy} &= 2\% \text{ of taxable income} \\ &= \frac{2}{100} \times \$35\,908 \\ &= \$718.16 \end{aligned}$	The Medicare levy is 2% of taxable income. 2% of \$35 908 is \$718.16.	Taxable incomes that exceed the minimum threshold pay a 2% Medicare levy.

- Complete the following to calculate the Medicare levy on a taxable income of \$42 756.

$$\text{Medicare levy} = \frac{\square}{100} \times \underline{\hspace{1cm}}$$

$$= \underline{\hspace{1cm}}$$
- Calculate the Medicare levy on these taxable incomes.

a \$39 021	b \$44 906	c \$43 879
d \$46 110	e \$60 958	f \$10 365

EXAMPLE 1F-3 Calculating the Medicare levy on taxable income between \$21 335 and \$26 668

The Medicare levy on taxable incomes between \$21 335 and \$26 668 is 10 cents in the dollar for each dollar in excess of \$21 335. Find the Medicare levy on \$21 536.

Solve	Think	Apply
$\text{Excess} = \$21\,536 - \$21\,335$ $= \$201$ $\text{Medicare levy} = \$0.10 \times 201$ $= \$20.10$	Calculate the excess by subtracting \$21 335. Multiply 201 by 10 cents (or \$0.10) to get \$20.10.	Taxable incomes between \$21 335 and \$26 668 have the Medicare levy calculated on the excess income over the \$21 335 amount.

- 8 Complete the following to calculate the Medicare levy on a taxable income of \$24 369.

$$\text{Excess} = \underline{\hspace{2cm}} - \$21\,335 = \underline{\hspace{2cm}}$$

$$\text{Medicare levy} = \$0.10 \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

- 9 Calculate the Medicare levy on these taxable incomes.

a \$22 000

b \$22 530

c \$12 450

d \$24 041

Government loans

For people with loans that assisted them when studying, or other forms of financial support, those loans are repayable when taxable income exceeds a threshold. The loan repayment is paid as additional tax. The repayment thresholds and rates for the compulsory repayment of debts to the following are updated annually: Higher Education Loan Program (HELP), Student Start-up Loan (SSL), ABSTUDY Student Start-up Loan (ABSTUDY SSL), Trade Support Loan (TSL) and Student Financial Supplement Scheme (SFSS). The repayment amount is a percentage of RI (Repayment Income), where $\text{RI} = \text{taxable income plus any total net investment loss (which includes net rental losses), total reportable fringe benefits amounts, reportable superannuation contributions and exempt foreign employment income}$. For this course, we will assume RI is equal to taxable income.

2016–17 repayment income thresholds and rates for HELP, SSL, ABSTUDY SSL and TSL

Repayment income (RI*)	Repayment rate
Below \$54 869	Nil
\$54 869–\$61 119	4.0%
\$61 120–\$67 368	4.5%
\$67 369–\$70 909	5.0%
\$70 910–\$76 222	5.5%
\$76 223–\$82 550	6.0%
\$82 551–\$86 894	6.5%
\$86 895–\$95 626	7.0%
\$95 627–\$101 899	7.5%
\$101 900 and above	8.0%

EXAMPLE 1F-4 Calculating study loan repayments

Christopher has a student loan and a taxable income of \$68 537. Calculate the amount of additional tax he will pay due to the HELP loan.

Solve	Think	Apply
Amount to pay = 5% of \$68 537 = $0.05 \times 68\,537$ = \$3426.85	\$68 537 is between \$67 369 and \$70 909, so the repayment rate is 5%. $5\% = \frac{5}{100} = 0.05$	Determine the repayment income and use the corresponding repayment rate to calculate the amount of extra tax allocated for the repayment.

- 10 Use the '2016–17 repayment income thresholds and rates' table to calculate the extra tax allocated to repay the HELP debts of people with taxable incomes of:
- a \$65 124 b \$88 223 c \$50 111 d \$57 899

- 11 Use the PAYG payment summary shown on the right to find and calculate:

- a gross income
b taxable income if there are deductions of \$2854
c tax paid

- 12 Choose three different professions and find the deductions to which they are entitled.

- 13 Examine the Tax Pack to determine allowable deductions.

- 14 Visit the Australian Taxation Office internet site at www.ato.gov.au Write about the latest tax information.

- 15 What changes have been made to Medicare throughout its history?

Australian Government Australian Taxation Office PAYG payment summary – individual non-business

All sections of this form must be completed. For help, refer to *How to complete the PAYG payment summary – individual non-business* (NAT 3388).

Payment summary for year ending 30 June 2017

If you are amending a payment summary you have already sent, place X in this box ☐

Section A: Payee details

Payee's surname or family name: MURRAY

Payee's given name: FRANK JOHN

Payee's residential address: 25 THE WAY CRS

Suburb/town/locality: HOMETOWN State/territory: NSW Postcode: 2013

Date of birth (if known): / /

Section B: Payment details

Period during which payments were made: 01 / 07 / 2016 to 30 / 06 / 2017

Payee's tax file number: 456 789 123

TOTAL TAX WITHHELD \$ 9,832

Total tax withheld – whole dollars only in words: NINE EIGHT THREE TWO

If the information on this payment summary relates to payments of a pension or annuity, place X in this box ☐

GROSS PAYMENTS (do not include amounts shown under 'Allowances', 'Lump sum payments', 'CDEP payments' and 'Lump sum superannuation contributions')

Community Development Employment Projects (CDEP) payments: \$ 54,277

Reportable fringe benefits amount: \$

Reportable employer superannuation contributions (do not include voluntary super guarantee amounts. For more information see the back page of this form): \$

Deductible amount of the undeducted purchase price of an annuity: \$

Exempt foreign employment income: \$

Allowances (provide details): \$

Section C: Payer details

1 You must also complete this section

Payer's Australian business number (ABN) or withholding payer number (WPN): 12 378 945 123 Branch number: 123

Payer's name (use the same name that appears on your activity statement): ACCOUNT-WHIZ CO

DECLARATION – I declare that the information given on this form is complete and correct.

Signature of authorised person: Date: / /

Section C: Payer details

1 You must also complete this section

Payer's Australian business number (ABN) or withholding payer number (WPN): 277 Branch number: 123

Payer's name (use the same name that appears on your activity statement):

DECLARATION – I declare that the information given on this form is complete and correct.

Signature of authorised person: Date: / /

1G Calculating tax

These resources are available on your obook assess:

- **Video tutorial 1G:** Watch and listen to an explanation of Example 1G-2
- **Spreadsheet 1G:** Calculate tax payable on taxable income
- **assess quiz 1G:** Test your skills with an auto-correcting multiple-choice quiz

Income tax is calculated using a tax table. The following table shows the tax rates for 2017–18.

Taxable income	Tax on this income
0–\$18 200	Nil
\$18 201–\$37 000	19c for each \$1 over \$18 200
\$37 001–\$87 000	\$3572 plus 32.5c for each \$1 over \$37 000
\$87 001–\$180 000	\$19 822 plus 37c for each \$1 over \$87 000
\$180 001 and over	\$54 232 plus 45c for each \$1 over \$180 000

The above rates do not include the 2% Medicare levy.

EXAMPLE 1G-1 Calculating tax payable using a tax table

Use the tax table to calculate the income tax payable on these incomes.

a \$53 251

b \$26 784

c \$105 631

	Solve	Think	Apply
a	\$53 251 is in the range \$37 001–\$87 000, in the third row of the table. $\text{Excess} = \$53\,251 - \$37\,000 = \$16\,251$ $\text{Tax} = \$3572 + 0.325 \times \$16\,251 = \$8853.58$	Subtract \$37 000 to find the excess. Tax is \$3572 plus $0.325 \times$ excess.	Locate the taxable income range in the table. Use that row of the tax table to calculate the income tax payable by subtracting the ‘over’ amount from the taxable income and multiplying by the rate, in dollars. Add the first amount given in that cell of the tax column if appropriate.
b	\$26 784 is in the range \$18 201–\$37 000, in the second row of the table. $\text{Excess} = \$26\,784 - \$18\,200 = \$8584$ $\text{Tax} = 0.19 \times \$8584 = \$1630.96$	Subtract \$18 200 to find the excess. Tax is $0.19 \times$ excess.	
c	\$105 631 is in the range \$87 001–\$180 000, in the fourth row of the table. $\text{Excess} = \$105\,631 - \$87\,000 = \$18\,631$ $\text{Tax} = \$19\,822 + 0.37 \times \$18\,631 = \$26\,715.47$	Subtract \$87 000 to find the excess. Tax is \$19 822 plus $0.37 \times$ excess.	

EXERCISE 1G Calculating tax

- Complete to find the income tax payable on \$47 953.
\$47 953 is in the range _____ in the _____ row of the tax table.
Excess = _____ - \$37 000 = _____
Tax = _____ + $0.325 \times$ _____ = _____
- What is the income tax payable on each of these taxable incomes?
a \$96 564 **b** \$48 911 **c** \$8100 **d** \$31 000
- The table below shows the pay rates for state MPs, not including electoral allowances. Calculate the amount of income tax that would be payable on these annual salaries. Round each amount to the nearest dollar.

Office	Pay
Premier	\$377 780
Deputy premier	\$318 937
Minister	\$287 515
Parliamentary secretary	\$235 668
Government whip	\$185 392

- The table shows the estimated 2017 starting salaries for new graduates. Calculate the amount of income tax that would be payable on these annual salaries.

Profession	Starting salary
Dentistry	\$85 000
Medicine	\$61 000
Optometry	\$80 000
Engineering	\$66 000
Computer science	\$59 000
Veterinary science	\$50 000
Education	\$65 600
Accounting	\$55 000
Psychology	\$63 000



- Julie is paid an annual salary of \$33 700 and receives an income of \$896 from other sources.
a Calculate Julie's total annual income.
b Use the tax table on the opposite page to help you determine the amount of income tax Julie will need to pay on her total income.
- Joshua earns \$47 500 per year.
a Calculate the amount of income tax he must pay.
b Calculate the Medicare levy Joshua must pay. (*Hint: the full Medicare levy is 2% of taxable income.*)
c What is the total amount of tax Joshua must pay?
- Sam has a few odd jobs. In total, he earns \$22 500 per year. He receives a tax deduction of \$2600. Calculate:
a his taxable income **b** the income tax payable on his taxable income
c the Medicare levy payable **d** the total tax payable.

EXAMPLE 1G-2 Calculating tax refund or balance payable

Sarah works as a secretary and receives a yearly salary of \$34 479 plus an income of \$1950 per year from babysitting. Her total tax deductions are \$1570. During the year she paid tax instalments of \$3950.

- a** Find her taxable income. **b** Calculate the Medicare levy she needs to pay.
c Find the income tax payable on her taxable income. **d** Find her tax refund or tax balance payable.

	Solve	Think	Apply
a	$\begin{aligned}\text{Total income} &= \$34\,479 + \$1\,950 \\ &= \$36\,429 \\ \text{Taxable income} &= \$36\,429 - \$1\,570 \\ &= \$34\,859\end{aligned}$	Add Sarah's salary and babysitting income. Taxable income $= \text{total income} - \text{tax deductions}$	Calculate the taxable income by subtracting all tax deductions from total income. Use the tax table to calculate the income tax payable. Compare the tax paid with the total tax liability to determine if a refund is payable.
b	$\begin{aligned}\text{Medicare levy} &= 2\% \text{ of } \$34\,859 \\ &= \$697.18\end{aligned}$	The full Medicare levy applies for taxable incomes in excess of \$26 668.	
c	$\begin{aligned}\text{Tax} &= 19 \text{ cents per } \$ \text{ in excess of } \$18\,200 \\ &= 0.19 \times (\$34\,859 - \$18\,200) \\ &= \$3165.21\end{aligned}$	Income tax payable on \$34 859 is in the range \$18 201–\$37 000 in row 2 of the tax table.	
d	$\begin{aligned}\text{Total tax due} &= \text{Medicare levy} + \text{income tax} \\ &= \$697.18 + \$3165.21 \\ &= \$3862.39 \\ \text{Tax refund} &= \$3950 - \$3862.39 \\ &= \$87.61\end{aligned}$	Sarah has already paid \$3950. She only needed to pay \$3862.39. She has paid too much and will receive a refund.	

- 8** Brittany earns \$89 452 as a company manager and \$5203 as a part-time singer. She has tax deductions of \$2612 and, throughout the year, pays a total of \$21 867.60 in tax instalments. Complete the following to find:

- a** her total income = $\$89\,452 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
b her taxable income = $\underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
c the tax payable = $\underline{\hspace{1cm}} + 0.37 \times (\underline{\hspace{1cm}} - \$87\,000)$
 $\hspace{10em} = \underline{\hspace{1cm}}$
d her tax refund = $\$21\,867.60 - \underline{\hspace{1cm}}$
 $\hspace{10em} = \underline{\hspace{1cm}}$

- 9** Celine earns \$32 056 per annum as an environmentalist and \$8159 as a part-time analyst. She has tax deductions of \$4903 and has paid \$5123.90 in tax instalments.

- a** Find her taxable income.
b Calculate the Medicare levy she needs to pay.
c Find the income tax payable on her taxable income.
d Find her tax refund or tax balance payable.

- 10** Mr Benton works part time and has a yearly salary of \$29 760. He receives a salary of \$2500 from his hobby of wood carving. His tax deductions amount to \$1090 and throughout the year he has paid PAYG instalments of \$3215.90.

- a** Calculate his taxable income.
b Calculate the Medicare levy he needs to pay.
c Calculate income tax payable on his taxable income.
d Calculate his tax refund or tax balance payable.



- 11** Ricky earns \$84 365 as a cabaret artist and \$4620 from his share portfolio. He has tax deductions of \$13 884 and, throughout the year, pays a total of \$15 988 in tax instalments.
- Find his taxable income.
 - Calculate the Medicare levy Ricky needs to pay.
 - Find the income tax payable on his taxable income.
 - Find the tax refund or tax balance payable.
- 12** Use the PAYG payment summary on the right to find and calculate:
- gross income
 - taxable income if there are deductions of \$1350
 - the Medicare levy payable on the taxable income
 - income tax payable on the taxable income
 - tax paid
 - tax refund or tax balance payable. (Note: ignore the allowances shown on the PAYG payment summary)

- 13** Use the PAYG payment summary shown on page 35 and your answers to question 11 in Exercise 1F to find and calculate:
- income tax payable on the taxable income
 - the Medicare levy payable on the taxable income
 - tax refund or tax balance payable.

- 14** Find the starting salaries for three apprenticeships of your choice. Work out the tax payable and the take-home pay. Websites for organisations such as Fair Work Australia could be useful.

- 15** The spreadsheet shown on the right is provided in your obook assess. It allows you to calculate the amount of income tax payable on a given taxable income.

The formula typed into cell B5 is:
 $\text{=IF}(B3>180000,(B3-180000)*0.45+54232,$
 $\text{IF}(B3>87000,(B3-87000)*0.37+19822,$
 $\text{IF}(B3>37000,(B3-37000)*0.325+3572,$
 $\text{IF}(B3>18200,(B3-18200)*0.19,$
 $\text{IF}(B3>0,"NO TAX"))))$

Explain why the formula works. Try using it to calculate tax.

Australian Government Australian Taxation Office PAYG payment summary individual non-business

All sections of this form must be completed. For help, refer to How to complete the PAYG payment summary individual non-business (NAT 3388).

Payment summary for year ending 30 June 2017

If you are amending a payment summary you have already sent, place X in this box. ☐

Section A: Payee details

Payee's surname or family name: MURPHY

Payee's given name: JANICE SONIA

Payee's residential address: 15 THE WAY CRS

Suburb/town/locality: HOMETOWN State/territory: NSW Postcode: 2013

Date of birth (if known): / /

Section B: Payment details

Period during which payments were made: 01/07/2016 to 30/06/2017

Payee's tax file number: 456 123 789

TOTAL TAX WITHHELD \$ 35,803

Total tax withheld - whole dollars only in words: THREE FIVE EIGHT ZERO THREE

GROSS PAYMENTS (do not include amounts shown under 'Allowances', 'Lump sum payments', 'CEEP payments' and 'Exempt foreign employment income') \$ 129,340

Community Development Employment Projects (CEEP) payments \$

Reportable fringe benefits amount \$

Reportable employer superannuation contributions (do not include voluntary contributions; for more information see the back page of this form) \$

Deductible amount of the undeducted purchase price of an annuity \$

Exempt foreign employment income \$

Allowances (provide details): MEALS \$ 1,236 TRAVEL \$ 2,110

Total allowances \$ 3,346

Section C: Payer details

You must also complete this section

Payer's Australian business number (ABN) or withholding payer number (WPN): 12 378 945 123 Branch number

Payer's name (use the same name that appears on your activity statement): BELLA RESTAURANT

DECLARATION - I declare that the information given on this form is complete and correct.

Signature of authorised person: Date: / /

DECLARATION - I declare that the information given on this form is complete and correct.

Signature of authorised person: Date: / /

	A	B
1	Tax calculator	
2		
3	Taxable income =	\$35 641
4		
5	Tax payable =	\$3313.79
6		

CHAPTER 1 REVIEW EARNING AND MANAGING MONEY

You should be able to:

- ✓ convert between annual, weekly, fortnightly and monthly incomes
- ✓ calculate incomes that include special allowances or loadings, such as holiday loading
- ✓ calculate pay when overtime is paid
- ✓ calculate incomes based on commissions, piecework and royalties
- ✓ understand and calculate bonus payments
- ✓ use charts and tables to calculate pensions, government allowances and health insurance payments
- ✓ calculate the Medicare levy and student loan repayments.
- ✓ calculate superannuation contributions and payments
- ✓ calculate net income and understand the difference between gross income and net income
- ✓ calculate tax payable on taxable income.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

REVIEW MULTIPLE-CHOICE QUESTIONS

- 1A** ➤ **1** Katrina is a part-time employee and receives an annual salary of \$19 370. What is her weekly salary?
A \$1616.17 B \$403.54 C \$372.50 D \$745
- 1A** ➤ **2** Zoltan is paid \$17.90 per hour. What are his earnings for a 26-hour week?
A \$465.40 B \$17.90 C \$930.80 D \$698.10
- 1A** ➤ **3** Andre is an apprentice and works 9 hours per day, five days per week. He is paid \$8.43 per hour. What is his annual income?
A \$75.84 B \$379.35 C \$18 208.80 D \$19 726.20
- 1A** ➤ **4** Mal receives a salary of \$63 428 per annum. He works six days per week, 8 hours per day. What is his hourly rate?
A \$25.41 B \$27.52 C \$27.53 D \$50.82
- 1B** ➤ **5** Paula is a junior and works during the school holidays. She is paid \$15.70 per hour normal rate. She works 30 normal hours, 6 hours at time-and-a half and 3 hours at double-time. What is her pay?
A \$471 B \$612.30 C \$642.96 D \$706.50
- 1B** ➤ **6** Zanthia receives a bonus at Christmas of 6.2% of her \$27 695 salary. What is the amount of her bonus?
A \$1717.09 B \$276.95 C \$33.02 D \$6.20
- 1B** ➤ **7** Paulette is a permanent part-time employee and is paid \$456 per week. She receives a holiday loading of $17\frac{1}{2}\%$ on her 4 weeks holiday pay. What is her total holiday pay?
A \$79.80 B \$319.20 C \$775.20 D \$2143.20
- 1C** ➤ **8** Marek is paid a weekly retainer of \$400 and 13% commission on sales in excess of \$5000. What is his pay in a week when he has sales of \$8350?
A \$1085.50 B \$435.50 C \$835.50 D \$1685.50

- 1C ➤ 9** Alexia is paid \$362 commission on sales of \$4525. What is her percentage rate of commission?
A 0.08% **B** 8% **C** 12.5% **D** 3.62%
- 1C ➤ 10** Sue picks mushrooms and is paid \$2.15 per box. What is her pay for picking 53 boxes?
A \$24.65 **B** \$53 **C** \$4.05 **D** \$113.95
- 1C ➤ 11** Charles is paid a royalty of 8% on total sales. What is his royalty for sales of \$356 990?
A \$28 559.20 **B** \$3569.90 **C** \$44 623.75 **D** \$35 699
- 1D ➤ 12** The fortnightly living-at-home student rate for a single person under 18 years is \$239.50. It reduces by 50 cents for every dollar in excess of \$437 earned per fortnight. Sharon is 17, living at home and earning \$451 per fortnight. What is her allowance?
A \$7 **B** \$239.50 **C** \$232.50 **D** \$690.50
- 1E ➤ 13** Bernadette receives a gross income of \$35 866. She has the following deductions: \$3159 PAYG tax; health insurance of \$425; and superannuation contribution of \$1240. What is her net income?
A \$31 467 **B** \$31 042 **C** \$40 690 **D** \$200 134
- 1E ➤ 14** Trina has an annual salary of \$41 908. when she retires, her superannuation will pay her 63% of her salary each year. What will be her annual income from her superannuation?
A \$68 310.04 **B** \$2 640 204 **C** \$15 505.96 **D** \$26 402.04
- 1F ➤ 15** Ossie works part-time for a wage of \$23 879 per annum. He receives a bonus of \$800, pays tax agent fees of \$60, makes donations to charity of \$75 and is entitled to a deduction for work-related car expenses of \$356. What is Ossie's taxable income?
A \$24 415 **B** \$25 397 **C** \$24 188 **D** \$23 961
- 1F ➤ 16** The Medicare levy is 2% of taxable income provided the taxable income exceeds \$26 668. What is the Medicare levy on a taxable income of \$34 677?
A \$120.14 **B** \$693.54 **C** \$1 201.40 **D** \$8009
- 1G ➤ 17** The income tax on \$100 666 is \$19 822 plus 37 cents per dollar over \$87 000. What is the tax payable on \$100 666?
A \$24 878.42 **B** \$19 822 **C** \$14 765.58 **D** \$5056.42

REVIEW SET 1

- 1 An electrician works a 42-hour week. Find his yearly pay if he earns \$36.50 per hour.
- 2 Indera receives an annual salary of \$57 801. Convert this to a weekly salary and find Indera's average hourly rate if she works 48 hours per week.
- 3 Brian is paid \$17.63 per hour. Calculate his weekly wage if he works 35 hours at normal time and 7 hours overtime at time-and-a-half.
- 4 Jack is a security guard and is paid \$26.50 per hour. If he works on the door, he is paid an extra \$30 danger money. Find Jack's wage for a week in which he is on the door for 3 nights and works 30 hours.
- 5 Georgina works as a personal assistant and is paid \$696 per week. If her 4 weeks of holiday pay attracts a loading of $17\frac{1}{2}\%$, find her pay for her holidays.
- 6 Tony is paid 1.2% commission on any sales made. Find his commission on a house that sells for \$676 500.

- 7** Mark waxes surfboards for the surf hire shop. He is paid \$7.40 per waxed board. Find his pay for a day in which he waxes 11 boards.
- 8** Martin is paid a royalty of 10.8% on sales of his book. His sales for a year are \$63 580. Calculate his royalty payment.
- 9** Lorena is a job seeker and living away from home. She is 16 and entitled to Youth Allowance. Use the charts in Topic 1D to answer these questions.
 - a** What is her Youth Allowance if she has no income?
 - b** How much could she earn before she loses her allowance completely?
- 10** Mary and Ian are entitled to the age pension. They are a couple with a combined income of \$459.00 per fortnight. The full pension for a couple is \$1073.40 per fortnight. Use the table in Topic 1D to calculate the combined pension paid to them.
- 11** Calculate net income if gross income is \$790.25 per week, tax is \$93.75, superannuation contribution is \$39.51 and union fees are \$6.30.
- 12** Calculate the gross income of a part-time employee if her net income is \$373.21, tax is \$82.10, health insurance is \$27.60 and superannuation contribution is \$55.40.
- 13** Harry is single and takes out basic hospital cover with a \$500 excess. Use the table in Topic 1E to find his weekly premium.
- 14** Martha uses the following information to calculate her taxable income: wages \$36 522, interest \$64, tax agent fee \$65, work-related car expenses \$1274 and donations to charity \$225. Calculate Martha's taxable income.
- 15** The Medicare levy is 2% of taxable income over \$26 668. Calculate the Medicare levy payable on a taxable income of \$39 359.
- 16** Use the tax table in Topic 1G to calculate the income tax payable on each of these taxable incomes.

a \$57 368	b \$31 259	c \$6422	d \$82 330
- 17** Gloria earns \$47 366 per annum as a company manager and \$4865 as a part-time tax agent. She has tax deductions of \$3698 and, throughout the year, pays a total of \$8978.40 in PAYG tax instalments. Calculate:

a her taxable income	b the Medicare levy she needs to pay
c the income tax payable on her taxable income	d her tax refund or tax balance payable.



REVIEW SET 2

- 1** Jan earns \$62 475 per annum. Calculate her weekly pay and her hourly rate if she works 35 hours per week.
- 2** Akbar works a 7-hour day, six days per week. His hourly rate is \$22.56. Find his pay for 1 year.
- 3** Visnoo is paid \$11.95 per hour for a 30-hour week. He is paid overtime at double-time for all other hours. Calculate his pay for a week in which he works 38 hours.
- 4** Hewey works as an apprentice plumber and is paid \$11.30 per hour. He is paid a digging allowance of \$11.50 per day if he has to dig trenches and \$15.80 per day if he works on open sewers. Find his pay for a week in which he works 42 hours, digs on 2 days and works on an open sewer for 3 days.

- 5** Tabitha received a $17\frac{1}{2}\%$ holiday loading on 4 weeks normal wages. She normally works a 32-hour week. Her 4 weeks holiday pay and loading is \$2587.90.
- a** Find her normal weekly pay. **b** What is her normal hourly pay rate?
- 6** Robert sells new cars valued at \$35 675. Find his pay if his commission is 4.5%.
- 7** Mitchell is a student living at home. He is 17 and entitled to Youth Allowance. Use the charts in Topic 1D to calculate the allowance Mitchell receives each fortnight if he earns \$270 per week.
- 8** Calculate net weekly income if gross income is \$635.28 per week, tax is \$128.54, superannuation contribution is \$27.60, and health insurance is \$26.20.
- 9** Calculate gross weekly income if net income is \$372.55, tax is \$89.10, health insurance is \$34.80 and superannuation is \$15.50.
- 10** Brent is single and takes out top hospital cover with no excess. Use the table in Topic 1G to find his weekly premium.
- 11** Kang earns \$1728 per week. His employer contributes 9% of that amount to Kang's superannuation fund and Kang contributes 3%. Calculate Kang's net weekly income and the total weekly contribution to his superannuation.
- 12** Giselle uses the following information to calculate her taxable income: wages \$67 220, interest \$985, meal allowance \$612, bonus \$5000, tax agent fee \$180, work-related car expenses \$1255 and donations to charity \$600. Calculate Giselle's taxable income.
- 13** The Medicare levy on taxable incomes between \$21 335 and \$26 668 is 10 cents in the dollar for each dollar in excess of \$21 335. Calculate the Medicare levy on \$22 317.
- 14** Use the tax table in Topic 1G to calculate the income tax payable on each of these taxable incomes.
- a** \$66 889 **b** \$33 699 **c** \$19 369 **d** \$5522
- 15** Shania earns \$38 996 per annum as a cocktail attendant and \$7560 as a part-time singer. She has tax deductions of \$3561 and, throughout the year, pays a total of \$6963.30 in tax instalments. Find:
- a** her taxable income **b** the Medicare levy she needs to pay
- c** income tax payable on her taxable income **d** her tax refund or tax balance payable.

REVIEW SET 3

- 1** Erica works part-time and earns \$29 881 per annum. Calculate her weekly pay.
- 2** Ronaldo works 5 hours a day, five days per week. Find his hourly rate if his annual income is \$26 031.
- 3** Mona earns \$14.28 for the first 35 hours in a week. She receives time-and-a-half for the next 8 hours and double-time after that. Find her pay for a 52-hour week.
- 4** Regina works in a cafe and receives a uniform allowance of \$8.50 per day. Her hourly rate of pay is \$15.23. Find her pay if she works 5 days for 4 hours per day.
- 5** Ling works as a personal assistant and is paid \$753 per week. If her 4 weeks of holiday pay attracts a loading of $17\frac{1}{2}\%$, what is her pay for her holidays?
- 6** Robyn is paid a commission of 5.8% of the price of any item of jewellery she sells. If Robyn sells a ring for \$5250, how much is her commission?

- 7** Steven is paid a royalty of 9.8% on sales of his book. If his sales for the year are \$265 800, what is his royalty?
- 8** Fabrice is a job seeker and living at home. He is 17 and entitled to Youth Allowance. Use the charts in Topic 1D to calculate the allowance Fabrice receives each fortnight if he earns \$280 per fortnight.
- 9** Maurice is single with an income of \$625 per fortnight. He is entitled to the age pension. If the full pension is \$808.30 per fortnight, calculate the pension Maurice will receive. Use the chart in Topic 1D.
- 10** Calculate net weekly income if gross income is \$956.31 per week, tax is \$148.24, superannuation contribution is \$38.25, union fees are \$7.00 and health insurance is \$23.50.
- 11** Larry is single and takes out basic hospital cover with the maximum excess. Use the table in Topic 1E to find his weekly premium.
- 12** Vanja uses the following information to calculate his taxable income: wages \$53 005, interest \$134 (joint account), cost of uniforms \$600, cleaning of uniforms \$260, tax agent fee \$100 and work-related car expenses \$2551. Calculate Vanja's taxable income.
- 13** The Medicare levy is 2% of taxable income over \$26 668. Calculate the Medicare levy payable on a taxable income of \$30 080.
- 14** Use the tax table in Topic 1G to calculate the income tax payable on each of these taxable incomes.

a \$72 556	b \$38 694	c \$21 211	d \$5006
- 15** Tania earns \$48 259 per annum as a musician and \$9634 in royalties from her recordings. She has tax deductions of \$4980 and throughout the year pays a total of \$8413.44 in tax instalments. Find:

a her taxable income	b the Medicare levy she needs to pay
c income tax payable on her taxable income	d her tax refund or tax balance payable.

REVIEW SET 4

- 1** Pauline earns \$62 408 per annum. Calculate her hourly rate if she works 38 hours per week.
- 2** Fielding is paid \$23.37 per hour. He works 5 hours per day, six days a week. Find his pay for one year.
- 3** James is an apprentice and is paid \$11.22 per hour. Calculate his weekly wage if he works 35 hours at normal time and 5 hours overtime at time-and-a-half.
- 4** Ross works in a timber yard and is paid \$25.40 per hour. He receives a dust allowance of \$12.50 per day. Find his pay for a five-day, 38-hour week.
- 5** Endora received a $17\frac{1}{2}\%$ holiday loading on 4 weeks normal wages. She normally works a 30-hour week. Her 4 weeks holiday pay and loading is \$2185.70.

a Find her normal weekly pay.	b Find her normal hourly pay rate.
- 6** A factory worker is paid \$4.35 for each completed garment. Calculate his wage if he completes 429 garments.
- 7** Willow is paid a royalty of 8.3% on sales of her book. Her sales for a year are \$149 000. Find her royalty.
- 8** Agneska is a student living away from home. She is 19 and entitled to Youth Allowance. Use the charts in Topic 1D to calculate the allowance Agneska receives each fortnight if she earns \$300 per week.

- 9** Carol and John are entitled to the age pension. They are a couple with a combined income of \$428.00 per fortnight. The full pension for each person is \$609.30 per fortnight. Calculate the total pension paid to Carol and John. Use the chart in Topic 1D.
- 10** Sam receives a gross weekly wage of \$911.03. His weekly deductions are tax of \$133.23, union fees of \$5.20, health insurance of \$11.80, and superannuation contribution of \$19.50. Calculate his net income.
- 11** Calculate gross income if net income is \$540.61, tax is \$58.12, health insurance is \$15.50, and superannuation contribution is \$38.00.
- 12** Perry is single and takes out intermediate hospital cover with the minimum payable excess. Use the table in Topic 1E to help you find his weekly premium.
- 13** Ito uses the following information to calculate her taxable income: wages \$28 369, interest \$32 (joint account), bonus \$300, cost of uniforms \$250, tax agent fee \$50 and work-related car expenses \$25. Calculate Ito's taxable income.
- 14** Use the tax table in Topic 1G to calculate the income tax payable on each of these taxable incomes.
a \$55 233 **b** \$28 775 **c** \$7998 **d** \$22 225
- 15** Natalie earns \$38 555 per annum as a recording artist and \$23 997 in royalties. She has tax deductions of \$8550 and, throughout the year, pays a total of \$10 044.44 in tax instalments. Find:
a her taxable income **b** the Medicare levy she needs to pay
c income tax payable on her taxable income **d** her tax refund or tax balance payable.

REVIEW PRACTICE EXAMINATION QUESTION

- 1 a** Bimo is paid \$652 for a 38-hour week.
i Convert this to an annual salary. (1 mark)
ii What is Bimo's hourly rate? (1 mark)
iii Bimo's mother says his monthly pay is \$2608. Is she correct? Explain using calculations. (2 marks)
- b** Leanne received $17\frac{1}{2}\%$ holiday loading for 4 weeks holiday calculated on her normal weekly wage of \$946. Calculate the holiday pay for her 4 weeks holiday. (2 marks)
- c** Turay is paid a commission of 8% on all sales and a retainer of \$200 per week. He is offered a new pay deal of 10% on all sales. By calculating Turay's weekly pay based on average weekly sales of \$9000, decide if he should accept the new pay deal. Give reasons for your answer. (3 marks)
- d** Jenny is single and receives the age pension of \$808.60 per fortnight. She takes a part-time job earning \$300 per fortnight. She loses 50 cents of her pension for each dollar she earns in excess of \$164. Find her adjusted pension payment. (3 marks)
- e** Maria's gross income is \$46 832 p.a. She pays 4% superannuation, health insurance of \$758 and \$425 union fees.
i Calculate her superannuation payment. (1 mark)
ii Find Maria's net income. (2 marks)

TOTAL: 15 marks

2

Practicalities of measurement

The main mathematical ideas in this chapter are:

- ▶ understanding approximations and significant figures
- ▶ expressing numbers in scientific notation
- ▶ converting between metric units of measurement
- ▶ using prefixes for units of measurement
- ▶ calculating error in measurement
- ▶ understanding accuracy of measurement.

ARE YOU READY?

- 2A ▶ 1** What does the first non-zero digit in the number 5083 represent?
A ones **B** tens
C hundreds **D** thousands
- 2A ▶ 2** What is the result when 482 096 is rounded to the nearest 1000?
A 480 000 **B** 482 000
C 483 000 **D** 48 000
- 2A ▶ 3** How many decimal places does 56.08 have?
A 1 **B** 2
C 3 **D** 4
- 2A ▶ 4** What is the result when 0.639 74 is rounded to three decimal places?
A 0.63 **B** 0.64
C 0.639 **D** 0.640
- 2A ▶ 5** Which decimal value is equivalent to the fraction $\frac{7}{8}$?
A 0.0875 **B** 0.78
C 0.87 **D** 0.875
- 2B ▶ 6** What is the result of 3.74×1000 ?
A 374 **B** 3740
C 37 400 **D** 374 000
- 2B ▶ 7** What is the result of $5.031 \div 100$?
A 0.050 31 **B** 0.0531
C 0.5031 **D** 0.005 031
- 2C ▶ 8** Which length measurement is equivalent to 2354 mm?
A 2.354 m **B** 23.54 m
C 235.4 m **D** 0.2354 m
- 2C ▶ 9** Which area measurement is equivalent to 3 m²?
A 300 cm² **B** 3000 cm²
C 30 000 cm² **D** 300 000 cm²
- 2C ▶ 10** Which volume measurement is equivalent to 5 cm³?
A 50 mm³ **B** 500 mm³
C 5000 mm³ **D** 50 000 mm³
- 2E ▶ 11** The ruler below is marked in centimetres. To the nearest centimetre, what length does the line segment show?
- 
- A** 3 cm **B** 4 cm
C $4\frac{1}{2}$ cm **D** 5 cm
- 2F ▶ 12** What is the area of a rectangle that is 12 cm long and 7 cm wide?
A 19 cm² **B** 38 cm²
C 84 cm² **D** 168 cm²
- 2F ▶ 13** What is the perimeter of the rectangle in question 12?
A 19 cm **B** 38 cm
C 84 cm **D** 168 cm

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook assess](#).

- Q1–2** Support sheet 2A.1 Understanding place value in decimals
Q3–4 Support sheet 2A.2 Decimal places and rounding
Q5 Support sheet 2A.3 Converting between fractions and decimals
Q6–7 Support sheet 2B.1 Multiplying and dividing by 10, 100, 1000, etc.
Q8 Support sheet 2C.1 Length conversions
Q9 Support sheet 2C.2 Area conversions
Q10 Support sheet 2C.3 Volume conversions
Q11 Support sheet 2E.1 Accuracy in measurement
Q12 Support sheet 2F.1 Area of a rectangle
Q13 Support sheet 2F.2 Finding perimeter

2A Significant figures

These resources are available on your obook assess:

- **Video tutorial 2A:** Watch and listen to an explanation of Example 2A-1
- **assess quiz 2A:** Test your skills with an auto-correcting multiple-choice quiz

1010

significant figures
the number of
digits in a number
that indicate its
accuracy

The **significant figures** in a number are the important or meaningful figures. A crowd of 61 348 is approximately 61 000, indicating that only the first two figures (digits) are important. It is impossible to cut a piece of timber to a length of 1.333 333... m. The digits after the fourth figure, are completely meaningless in this case.

EXAMPLE 2A-1 Rounding to a given number of significant figures

The first significant figure in a number is the first non-zero digit, reading from left to right. Round each of the following to:

- i** one significant figure **ii** two significant figures **iii** three significant figures
- a** 293 568 **b** 0.07604

	Solve	Think
a i	300 000	Locate the relevant significant figure and then round appropriately using the next digit. The first non-zero digit is 2. This is the first significant figure. The next digit (9) is bigger than 5; so, rounded to one significant figure, $293\,568 \approx 300\,000$. (This is the same as rounding 293 568 to the nearest 100 000 because the first significant figure is in the 100 000s column.)
ii	290 000	The second significant figure is 9. The next digit (3) is smaller than 5; so, rounded to two significant figures, $293\,568 \approx 290\,000$. (This is the same as rounding 293 568 to the nearest 10 000 because the second significant figure is in the 10 000s column.)
iii	294 000	The third significant figure is 3. The next digit is 5; so, rounded to three significant figures, $293\,568 \approx 294\,000$. (This is the same as rounding 293 568 to the nearest 1000 because the third significant figure is in the 1000s column.)
b i	0.08	The first non-zero digit is 7. This is the first significant figure. The next digit (6) is bigger than 5; so, rounded to one significant figure, $0.07604 \approx 0.08$. (This is the same as rounding 0.07604 to two decimal places because the first significant figure is in the second place after the decimal point; or rounding to the nearest hundredth because the first significant figure is in the hundredths column.)
ii	0.076	The second significant figure is 6. The next digit (0) is smaller than 5; so, rounded to two significant figures, $0.07604 \approx 0.076$. (This is the same as rounding 0.07604 to three decimal places because the second significant figure is in the third place after the decimal point; or rounding to the nearest thousandth because the second significant figure is in the thousandths column.)
iii	0.0760	The third significant figure is 0. The next digit (4) is smaller than 5; so, rounded to three significant figures, $0.07604 \approx 0.0760$. (This is the same as rounding 0.07604 to four decimal places because the third significant figure is in the fourth place after the decimal point; or rounding to the nearest ten-thousandth, because the third significant figure is in the ten-thousandths column.)

EXERCISE 2A Significant figures

- 1 Complete the following to round:
 - a 5368 to three significant figures.
The first non-zero digit is _____. This is the first significant figure. The third significant figure is _____.
The digit after this is smaller than/bigger than/equal to 5, indicating that, when rounded, the number is closer to _____ than to _____. So, rounded to three significant figures, $5368 \approx$ _____.
 - b 0.062 53 to three significant figures.
The first non-zero digit is _____. This is the first significant figure. The third significant figure is _____.
The digit after this is smaller than/bigger than/equal to 5, indicating that, when rounded, the number is closer to _____ than to _____. So, rounded to three significant figures, $0.062\,53 \approx$ _____.
- 2 Round each of the following to one significant figure.

a 42 600	b 59	c 4.6	d 108
e 0.6529	f 0.0082	g 0.025	h 990
- 3 Round each of the following to two significant figures.

a 290 365	b 3960	c 24.9	d 2653
e 8.63	f 0.0487	g 0.000 162 8	h 0.003 97
- 4 Round each of the following to three significant figures.

a 3688	b 20 657	c 154 299	d 813.4
e 14.294	f 0.003 508 1	g 0.039 14	h 1.999
- 5 Round each of the following to:

i one significant figure	ii two significant figures	iii three significant figures.
a 17.256	b 0.450 72	c 521 500
		d 0.002 095
- 6 Express $3\frac{2}{7}$ as a decimal correct to four significant figures.
- 7 Express each of the following numbers as a decimal correct to three significant figures and arrange the numbers in ascending order (from smallest to largest).
 $\sqrt{2}$, 1.4, $1\frac{2}{5}$, $\frac{10}{7}$
- 8 A town's average rainfall in summer over six successive years is 246.5 mm, 237.6 mm, 366.9 mm, 287.4 mm, 412.8 mm and 348.2 mm. Calculate the average rainfall for the town over this six-year period, correct to four significant figures.
- 9 A car travels for $3\frac{1}{2}$ hours at 71 km/h and then for $2\frac{1}{4}$ hours at 75 km/h.
 - a Find the total distance travelled by the car, correct to four significant figures.
 - b Calculate the average speed for the whole trip, correct to two significant figures.

- 10 A satellite orbits the Earth at a height of 32 km above Earth's surface. The diameter of the Earth is 12 740 km.
 - a Find the radius, to five significant figures, of the satellite's orbit, assuming that orbit is circular.
 - b Calculate the length (l) of the satellite's orbit, using $l = 2\pi r$ where r is the radius. Write your answer to five significant figures.
 - c If the satellite travels at a speed of 26 000 km/h, calculate how long, in minutes, correct to three significant figures it will take to complete one orbit.



2B Scientific notation

scientific notation

a value written as a number from 1 up to, but not including, 10 (with any number of decimal places) multiplied by a power of 10

These resources are available on your [obook assess](#):

- **Interactive 2B:** Explore key ideas for scientific notation
- **Investigation 2B:** Investigate measurements in our solar system
- **assess quiz 2B:** Test your skills with an auto-correcting multiple-choice quiz

Scientific notation (or standard notation) is a convenient way of writing very large and very small numbers. A number written in scientific notation is written as the product of a number between 1 and 10 and a power of 10; that is, it is put in the form $A \times 10^n$ where A lies between 1 and 10, and n is an integer (whole number).

EXAMPLE 2B-1 Identifying numbers written in scientific notation

State whether the following numbers are expressed in scientific notation.

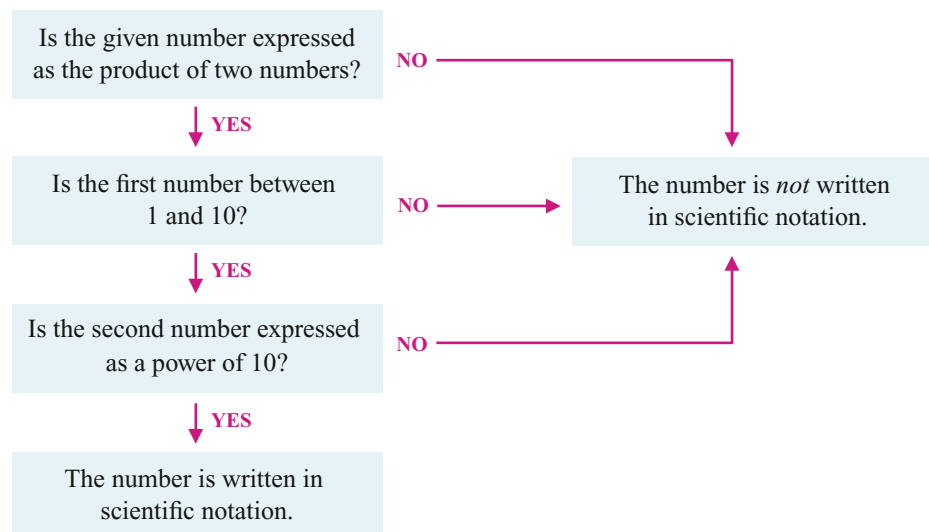
- a** 5.3×10^7 **b** 78×10^5 **c** $4.9 \times 10\,000$ **d** 3×10^{-4} **e** 294 000

	Solve	Think	Apply
a	Yes	The first number in the product (5.3) is between 1 and 10, the second number (10^7) is a power of 10.	A number is written in scientific notation if it is written as the product of a number from 1 up to 10 and a power of 10.
b	No	The first number (78) is not between 1 and 10.	
c	No	The second number (10 000) is not written as a power of 10.	
d	Yes	The first number in the product (3) is between 1 and 10, the second number (10^{-4}) is a power of 10.	
e	No	294 000 is not written as a product of two numbers.	

EXERCISE 2B Scientific notation

- 1 Use the flow diagram below to determine whether the given numbers are expressed in scientific notation.

- a** 2.91×10^{-17} **b** 53×10^6 **c** $3.8 \times 100\,000$ **d** 3^{26}



2 State whether the following numbers are written in scientific notation.

a 3.6×10^5

b 5.2×10000

c 21×10^5

d 2.87×10^{-6}

e $6.07 \times \frac{1}{1000000}$

f 594×10^{-5}

g 70×10^8

h 3.06×10^{-9}

EXAMPLE 2B-2 Writing numbers in scientific notation

Write these numbers in scientific notation.

a 138 000

b 0.000 486

	Solve	Think	Apply
a	1.38×10^5	Move the decimal point so that it is positioned between the first and second digits. This always produces a number between 1 and 10, in this case 1.380 00. Count the number of places back to the original position of the decimal point: 1.38000 Number of places = five to the right = +5 This becomes the power of 10: $138\,000 = 1.38000 \times 10^5$ $= 1.38 \times 10^5$ (leave off the zeros)	Move the decimal point so that it is positioned between the first two digits. This produces a number between 1 and 10. Count the number of places back to the original position of the decimal point. This becomes the power of 10.
b	4.86×10^{-4}	Move the decimal point so that it is positioned between the first and second digits, in this case 4.86. Count the number of places back to the original position of the decimal point: 00004.86 Number of places = four to the left = -4 This becomes the power of 10: $0.000486 = 4.86 \times 10^{-4}$	<i>Note:</i> When counting back to the original position of the decimal point, counting to the right produces a positive power of 10 and counting to the left produces a negative power of 10.

3 Complete the following to write each number in scientific notation.

a 243 000

Position the decimal point between the first two digits → _____.

The number of places to the original position of the decimal point = _____ to the _____
= +____.

So, $243\,000 = \_\_\_\_ \times 10^\square$

b 0.000 586

Position the decimal point between the first two digits → _____.

The number of places to the original position of the decimal point = _____ to the _____
= -____.

So, $0.000\,586 = \_\_\_\_ \times 10^\square$

4 Use the method in Example 2B-2 to write each of these numbers in scientific notation.

a 526 000

b 28 000

c 7 000 000

d 49 800

e 28 000 000

f 603 000 000

g 910 000

h 13 200 000 000

5 Use the method in Example 2B-2 to write each of these numbers in scientific notation.

a 0.000 43

b 0.008 21

c 0.000 007

d 0.000 029

e 0.065

f 0.000 387

g 0.000 008 2

h 0.000 06

EXAMPLE 2B-3 Changing from scientific notation to an ordinary number

Write the following as ordinary (or basic) numbers.

a 4.83×10^7

b 9.2×10^{-6}

	Solve	Think	Apply
a	48 300 000	Because the power of 10 is +7, the decimal point is moved 7 places to the right. $\overbrace{48\,300\,000}^{7 \text{ places}}$ so $4.83 \times 10^7 = 48\,300\,000$	The magnitude of the power of 10 tells us how many places to move the decimal point. If the power of 10 is positive, move the decimal point to the right. If the power of 10 is negative, move it to the left.
b	0.000 009 2	Because the power of 10 is -6, the decimal point is moved 6 places to the left. $\overbrace{0.000\,009\,2}^{6 \text{ places}}$ so $9.2 \times 10^{-6} = 0.000\,009\,2$	

6 Complete the statements to write each of these numbers as an ordinary number.

a 5.48×10^6 Move the decimal point ____ places to the _____. $5.48 \times 10^6 =$ _____.

b 3.09×10^{-5} Move the decimal point ____ places to the _____. $3.09 \times 10^{-5} =$ _____.

7 Express these as ordinary numbers.

a 3.4×10^6

b 8.3×10^8

c 2.94×10^7

d 2.58×10^5

e 5.26×10^5

f 3.02×10^{12}

g 2.9×10^7

h 8.75×10^8

8 Write the basic number for:

a 5.9×10^{-4}

b 3.2×10^{-6}

c 7.1×10^{-8}

d 2×10^{-3}

e 8×10^{-7}

f 2.64×10^{-5}

g 8.67×10^{-9}

h 2.97×10^{-6}

EXAMPLE 2B-4 Calculating numbers in scientific notation

Use your calculator to find:

a $(3.5 \times 10^7) \times (2.4 \times 10^9)$

b $(6.4 \times 10^8) \div (2.5 \times 10^{-6})$

c $\sqrt{2.4 \times 10^{10}}$

d $(1.5 \times 10^7)^3$

	Solve	Think	Apply
a	8.4×10^{16}	Possible steps using a Casio calculator are: Press 3.5 $\times 10^x$ 7 $\times$ 2.4 $\times 10^x$ 9 $=$ Answer: 8.4×10^{16}	If the answer is not displayed in scientific notation, you could use the SCI function on the calculator to express the answer in this form.
b	2.56×10^{14}	Press 6.4 $\times 10^x$ 8 $\div$ 2.5 $\times 10^x$ -6 $=$ Answer: 2.56×10^{14}	
c	1.55×10^5	Press $\sqrt{}$ 2.4 $\times 10^x$ 10 $=$ Answer: 1.55×10^5 to three significant figures	
d	3.375×10^{21}	Press 1.5 $\times 10^x$ 7 x^y 3 $=$ Answer: 3.375×10^{21}	

9 Calculate the following, correct to three significant figures. Write the answers in scientific notation.

a $(2.6 \times 10^8) \times (4.1 \times 10^7)$	b $(5.8 \times 10^9) \times (8.2 \times 10^{12})$
c $(9.2 \times 10^{24}) \times (3.5 \times 10^{-8})$	d $(5.8 \times 10^{-6}) \times (2.4 \times 10^{-9})$
e $(8.4 \times 10^{18}) \div (2.5 \times 10^7)$	f $(5.25 \times 10^{12}) \div (4.2 \times 10^{-8})$
g $(1.82 \times 10^{-6}) \times (2.9 \times 10^{-10})$	h $(2.8 \times 10^8)^4 \times (1.6 \times 10^6) \div (2.1 \times 10^{15})$
i $\sqrt{5.76 \times 10^{16}}$	j $\sqrt{6.8 \times 10^{17}}$
k $\sqrt{(3.1 \times 10^8)^5}$	l $(8 \times 10^{-10})^6$

10 Express the following numbers in scientific notation.

- The distance of Mars from the Sun is approximately 229 000 000 km.
- The diameter of the hydrogen atom is 0.000 000 000 025 4 m.
- The Sun produces the same amount of light as 3 000 000 000 000 000 000 000 000 000 000 000 000 candles.
- There are approximately 130 000 hairs on a person's head.
- There are approximately 10 000 000 000 000 cells in the human body.



11 Change the numbers in the following to ordinary numbers.

- There are approximately 3.16×10^7 s in a year.
- A molecule's diameter is 8.9×10^{-7} mm.
- The number of different possible hands in the card game Poker is approximately 2.6×10^6 .
- Swarms of locusts have been known to contain as many as 3×10^{10} locusts.
- The size of the influenza virus is approximately 2.6×10^{-4} mm.

12 a Light travels at a velocity of 300 000 km/s. Express 300 000 in scientific notation.

b How far does light travel in:

i 1 minute?	ii 1 hour?	iii 1 day?	iv 1 year?

c If light takes 4.1 min to reach Earth from Mars, what is the distance from Earth to Mars?

13 The radius of the Earth is approximately 6400 km.

- Calculate the area of the Earth's surface, to two significant figures. (Use $A = 4\pi r^2$.)
- Calculate the volume of the Earth, to two significant figures. (Use $V = \frac{4}{3}\pi r^3$.)

14 The radius of the Earth's orbit around the Sun is approximately 1.49×10^8 km. Assuming that the orbit is circular, calculate to two significant figures the distance travelled by the Earth in one orbit. (Use $C = 2\pi r$.)

15 a The human brain contains about 10^{10} cells. Write this as an ordinary number.

b Each human brain cell is about 2.8×10^{-5} m long. If all the brain cells could be placed next to each other, in a straight line, how long would this line be?

16 Measure your pulse to determine the number of times your heart beats in a minute. If you live to be 75 years of age, how many times will your heart have beaten in this time? (Assume your pulse remains constant.)

17 Light travels at approximately 3×10^5 km/s and sound travels at 330 m/s.

- After the starter fires the starting gun, how long does it take:
 - the sight of the smoke from the gun to reach a timekeeper standing at the end of a 100 m running track. Give your answer in scientific notation to four significant figures.
 - the sound of the gun to reach the timekeeper, in scientific notation to four significant figures.
- What is the difference between the times from your answers to part **a**?
- If the timekeeper's stopwatch measures time to the nearest hundredth of a second, does it matter if she uses the sight of the smoke or the sound of the gun to start the stopwatch?

2C Metric units of measurement

These resources are available on your [obook assess](#):

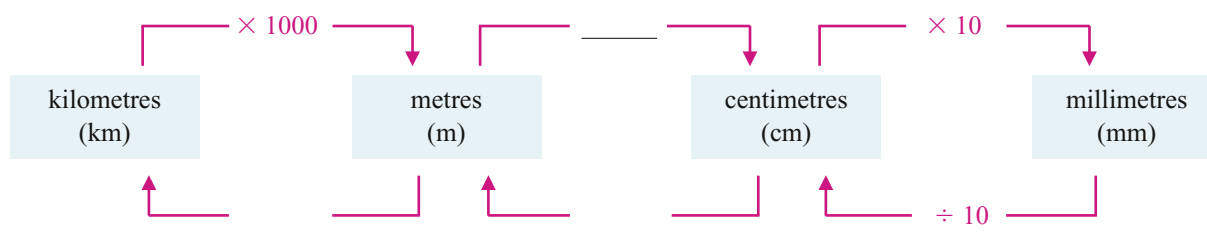
- **Interactive 2C.1:** Explore key ideas for converting units of length
- **Interactive 2C.2:** Explore key ideas for converting units of area and volume
- **assess quiz 2C:** Test your skills with an auto-correcting multiple-choice quiz

The metric system of measurement uses base units for quantities such as length, mass, capacity, area and volume.

The prefix of each unit name indicates the factor of 10 by which the base unit is multiplied.

EXERCISE 2C Metric units of measurement

- 1 Complete this conversion diagram for length.



EXAMPLE 2C-1 Converting units of length

Convert these lengths.

a $2.5 \text{ km} = \underline{\hspace{1cm}} \text{ m}$

b $430 \text{ mm} = \underline{\hspace{1cm}} \text{ m}$

	Solve	Think	Apply
a	$2.5 \text{ km} = (2.5 \times 1000) \text{ m}$ $= 2500 \text{ m}$	$1 \text{ km} = 1000 \text{ m}$, so conversion factor is 1000. Multiply by the conversion factor since we are converting from km to the smaller unit of m.	To convert to a smaller unit, multiply by the conversion factor.
b	$430 \text{ mm} = (430 \div 10) \text{ cm}$ $= 43 \text{ cm}$ $= (43 \div 100) \text{ m}$ $= 0.43 \text{ m}$	First convert from mm to cm (conversion factor of 10) and then convert from cm to m (conversion factor of 100). In each case, divide by the conversion factor because we are converting to a larger unit.	To convert to a larger unit, divide by the conversion factor.

- 2 Convert these lengths.

a $3.6 \text{ km} = \underline{\hspace{1cm}} \text{ m}$

b $8.4 \text{ m} = \underline{\hspace{1cm}} \text{ mm}$

c $34.82 \text{ m} = \underline{\hspace{1cm}} \text{ cm}$

d $0.56 \text{ km} = \underline{\hspace{1cm}} \text{ m}$

e $2.9 \text{ m} = \underline{\hspace{1cm}} \text{ cm}$

f $0.964 \text{ m} = \underline{\hspace{1cm}} \text{ mm}$

g $0.658 \text{ m} = \underline{\hspace{1cm}} \text{ cm}$

h $45.2 \text{ cm} = \underline{\hspace{1cm}} \text{ mm}$

i $15.68 \text{ km} = \underline{\hspace{1cm}} \text{ m}$

j $3.69 \text{ cm} = \underline{\hspace{1cm}} \text{ mm}$

k $16.37 \text{ m} = \underline{\hspace{1cm}} \text{ mm}$

l $4.265 \text{ km} = \underline{\hspace{1cm}} \text{ m}$

- 3 **a i** Complete: $1 \text{ km} = \underline{\hspace{1cm}} \text{ cm}$.

- ii** Express the answer in scientific notation.

- b i** Complete: $1 \text{ km} = \underline{\hspace{1cm}} \text{ mm}$.

- ii** Express the answer in scientific notation.

4 Convert these lengths.

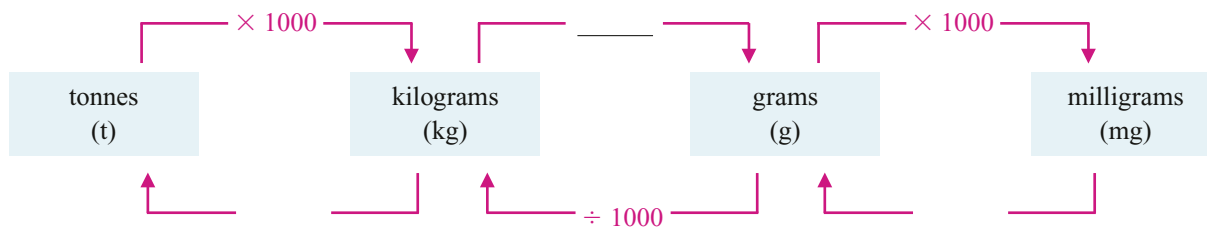
a 7000 m = ___ km	b 594 cm = ___ m	c 8930 m = ___ km	d 6000 mm = ___ m
e 40 mm = ___ cm	f 85 m = ___ km	g 800 cm = ___ m	h 328 mm = ___ cm
i 620 mm = ___ m	j 14 300 m = ___ km	k 86 cm = ___ m	l 630 m = ___ km
m 94 mm = ___ cm	n 70 mm = ___ m	o 24 895 m = ___ km	p 23 000 mm = ___ m
q 14 960 mm = ___ m	r 16 270 cm = ___ m	s 3600 cm = ___ m	t 72 945 mm = ___ m

5 What would be a convenient unit (millimetres, centimetres, metres, kilometres) to use for measuring the following?

- width of the classroom
- length of a textbook
- height of a student
- length of a baby
- length of your foot
- length of a matchstick
- distance from Sydney to Brisbane
- length of material for a dress
- length of a driveway
- distance between railway stations



6 Complete this conversion diagram for mass.



7 Convert these masses.

a 3.6 t = ___ kg	b 7.1 g = ___ mg	c 17.84 kg = ___ g	d 0.63 t = ___ kg
e 4.8 kg = ___ g	f 2.465 t = ___ kg	g 21.59 t = ___ kg	h 0.6 g = ___ mg
i 6.06 kg = ___ g	j 0.489 kg = ___ g	k 1.07 t = ___ kg	l 0.03 g = ___ mg

- Complete: 1 t = ___ g.
- Complete: 1 t = ___ mg.
- Complete: 1 kg = ___ g.
- Write the answer in scientific notation.
- Write the answer in scientific notation.
- Write the answer in scientific notation.

9 Convert the following.

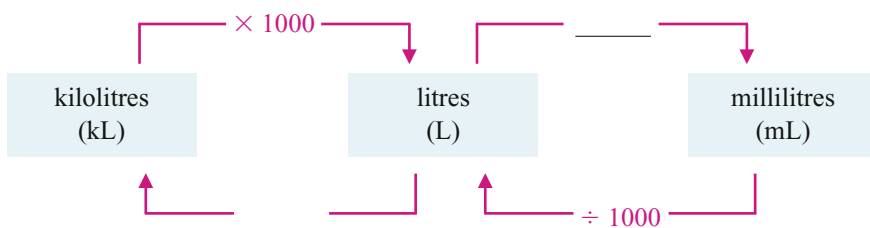
a 8000 kg = ___ t	b 4300 g = ___ kg	c 2740 mg = ___ g	d 690 g = ___ kg
e 65 mg = ___ g	f 2320 g = ___ kg	g 700 kg = ___ t	h 460 mg = ___ g
i 80 g = ___ kg	j 7 mg = ___ g	k 9 kg = ___ t	l 300 g = ___ kg

10 State a convenient unit to use (milligrams, grams, kilograms, tonnes) for measuring the mass of the following.

a a woman	b a jar of honey
c a packet of sugar	d a bag of cement
e a large SUV	f a rhinoceros
g a headache tablet	h a maths textbook
i a sewing needle	j a tennis ball



11 Complete this conversion diagram for capacity.



12 Convert the following.

a 35 kL = ___ L	b 15.9 L = ___ mL
c 1.65 L = ___ mL	d 0.85 kL = ___ L
e 0.06 L = ___ mL	f 1.08 kL = ___ L
g 0.015 L = ___ mL	h 0.005 kL = ___ L

13 How many millilitres are there in 1 kL? Express your answer in scientific notation.

14 Convert the following.

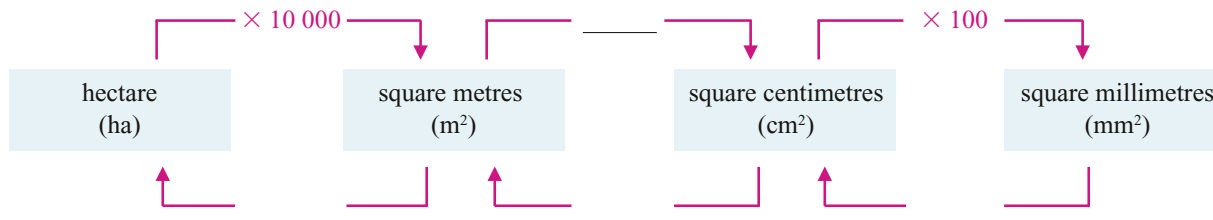
a 15 000 mL = ___ L	b 8000 L = ___ kL
c 7600 mL = ___ L	d 800 mL = ___ L
e 9280 L = ___ kL	f 725 L = ___ kL
g 95 mL = ___ L	h 40 L = ___ kL



15 State an appropriate unit to use (millimetres, litres, kilolitres) for measuring the capacity of a:

a teaspoon	b swimming pool
c bucket	d fish tank
e laundry tub	f farm dam
g car's petrol tank	h kettle

16 Complete this conversion diagram for area.



17 Convert these areas.

a 2.6 ha = ___ m ²	b 4.9 m ² = ___ cm ²	c 14 cm ² = ___ mm ²	d 0.752 m ² = ___ cm ²
e 1.65 ha = ___ m ²	f 24.8 cm ² = ___ mm ²	g 8.294 km ² = ___ m ²	h 5.671 km ² = ___ cm ²

18 Convert the following.

a $63\,000\text{ m}^2 = \underline{\hspace{2cm}}\text{ ha}$

d $45\,680\text{ cm}^2 = \underline{\hspace{2cm}}\text{ m}^2$

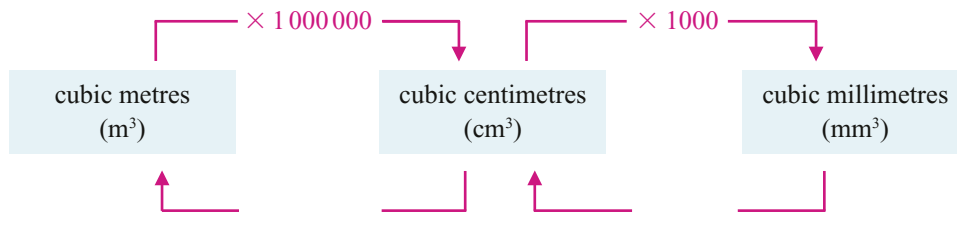
b $127\,000\text{ cm}^2 = \underline{\hspace{2cm}}\text{ m}^2$

e $298\,000\text{ m}^2 = \underline{\hspace{2cm}}\text{ km}^2$

c $810\text{ mm}^2 = \underline{\hspace{2cm}}\text{ cm}^2$

f $2.4\text{ km}^2 = \underline{\hspace{2cm}}\text{ ha}$

19 Complete this conversion diagram for volume.



20 Convert these volumes.

a $5\text{ cm}^3 = \underline{\hspace{2cm}}\text{ mm}^3$

b $3.9\text{ m}^3 = \underline{\hspace{2cm}}\text{ cm}^3$

c $25.6\text{ cm}^3 = \underline{\hspace{2cm}}\text{ mm}^3$

d $0.64\text{ m}^3 = \underline{\hspace{2cm}}\text{ cm}^3$

e $0.415\text{ cm}^3 = \underline{\hspace{2cm}}\text{ mm}^3$

f $7.39\text{ m}^3 = \underline{\hspace{2cm}}\text{ mm}^3$

21 Convert the following.

a $7\,400\,000\text{ cm}^3 = \underline{\hspace{2cm}}\text{ m}^3$

b $56\,700\text{ mm}^3 = \underline{\hspace{2cm}}\text{ cm}^3$

c $690\,000\text{ cm}^3 = \underline{\hspace{2cm}}\text{ m}^3$

d $4258.5\text{ mm}^3 = \underline{\hspace{2cm}}\text{ cm}^3$

- 22 a** A bottle of wine has a mass of 1140 g. What would be the mass of a case of 1 dozen bottles if the cardboard packaging has a mass of 320 g. Write the answer in kilograms.
- b** If each bottle contains 750 mL of wine, what is the total capacity of one case. Answer in litres.



- 23** The catchment area of a dam is 25.6 km^2 . If 25 mm of rain falls over the catchment area, what will be the increase in capacity of the dam, given that 1 m^3 holds 1000 kL of water?
- 24** The average mass of an adult hippopotamus is 1.5 t. Baby hippopotami, which are born underwater, have an average mass of 37 kg. Express the mass of a baby hippo as a percentage of its mass as an adult.



25 A house brick has dimensions of $76\text{ mm} \times 230\text{ mm} \times 110\text{ mm}$.

- a** When travelling to a work site, trucks typically carry 6 pallets each containing 500 bricks. Calculate the total space in cubic metres occupied by these 6 pallets.
- b i** Calculate the area of the largest face of a brick.
- ii** Assume that, when laid, each brick has a 10 mm thickness of mortar along one 230 mm edge and one 110 mm edge. What is the area covered by a brick plus the mortar?
- iii** How many bricks, surrounded by mortar, are needed to build a wall of area 60 m^2 ?

2D Prefixes for units of measurement

These resources are available on your [obook assess](#):

- **Video tutorial 2D:** Watch and listen to an explanation of Example 2D-1
- **Investigation 2D:** Investigate prefixes used for file sizes of digital data
- **assess quiz 2D:** Test your skills with an auto-correcting multiple-choice quiz

Prefixes are used to indicate the factor of 10 by which the base metric unit of measurement is multiplied. For example, the unit 'kilogram' uses the name of the base unit for mass, the gram, with the prefix *kilo*, which indicates a multiplying factor of 1000: $1 \text{ kg} = 1000 \text{ g}$.

This table summarises the most common prefixes used for very large and very small measurements.



Prefix	Multiplying factor
tera (T)	$10^{12} = 1\,000\,000\,000\,000$
giga (G)	$10^9 = 1\,000\,000\,000$
mega (M)	$10^6 = 1\,000\,000$
kilo (k)	$10^3 = 1000$
centi (c)	$10^{-2} = 0.01$
milli (m)	$10^{-3} = 0.001$
micro (μ)	$10^{-6} = 0.000\,001$
nano (n)	$10^{-9} = 0.000\,000\,001$

EXAMPLE 2D-1 Converting length measurements

a Convert the following to metres.

i 3.6 Gm

ii $7 \mu\text{m}$

b Convert 5.6 m to:

i kilometres

ii micrometres

	Solve	Think	Apply
a i	$3.6 \text{ Gm} = 3.6 \times 10^9 \text{ m}$ or $3\,600\,000\,000 \text{ m}$	$1 \text{ Gm} = 1 \times 10^9 \text{ m}$	Apply the multiplying factor for the prefix.
ii	$7 \mu\text{m} = 7 \times 10^{-6} \text{ m}$ or $0.000\,007 \text{ m}$	$1 \mu\text{m} = 1 \times 10^{-6} \text{ m}$	
b i	$5.6 \text{ m} = \frac{5.6}{10^3} \text{ km}$ $= 5.6 \times 10^{-3}$ or 0.0056 km	Divide 5.6 by the number of metres in a kilometre. $\frac{1}{10^3} = 10^{-3}$	Divide by the number of metres in the required unit.
ii	$5.6 \text{ m} = \frac{5.6}{10^{-6}} \mu\text{m}$ $= 5.6 \times 10^6$ or $5\,600\,000 \mu\text{m}$	Divide 5.6 by the number of metres in a micrometre. $\frac{1}{10^{-6}} = 10^6$	

EXERCISE 2D Prefixes for units of measurement

- 1
 - a Convert these to metres.
 - i 5.7 Mm
 - ii 9 cm
 - iii 8 nm
 - b Convert these to grams.
 - i 8 Gg
 - ii 4.2 mg
 - iii 5 μ g
 - c Convert these to litres.
 - i 3 TL
 - ii 2.8 ML
 - iii 7 mL
- 2
 - a Convert 4.9 m to:
 - i kilometres
 - ii micrometres
 - iii nanometres
 - b Convert 2.4 g to:
 - i megagrams
 - ii teragrams
 - iii micrograms
 - c Convert 6.5 L to:
 - i megalitres
 - ii millilitres
 - iii giganlitres

EXAMPLE 2D-2 Converting mass measurements

Convert the following.

a $3.56 \text{ Tg} = \text{___ Mg}$

b $9.4 \text{ mg} = \text{___ } \mu\text{g}$

	Solve	Think	Apply
a	$3.56 \text{ Tg} = \frac{3.56 \times 10^{12}}{10^6} \text{ Mg}$ $= 3.56 \times 10^6 \text{ or } 3\,560\,000 \text{ Mg}$	Convert 3.56 Tg into grams and divide by the number of grams in a megagram.	Convert to grams and divide by the number of grams in the required unit.
b	$9.4 \text{ mg} = \frac{9.4 \times 10^{-3}}{10^{-6}} \mu\text{g}$ $= 9.4 \times 10^3 \text{ or } 9400 \mu\text{g}$	Convert 9.4 mg into grams and divide by the number of grams in a microgram.	

- 3 Convert the following.
 - a $7.2 \text{ Gg} = \text{___ Mg}$
 - b $2.9 \text{ cg} = \text{___ ng}$
 - c $8 \text{ Tm} = \text{___ km}$
 - d $4.3 \text{ cm} = \text{___ } \mu\text{m}$
 - e $8.8 \text{ TL} = \text{___ ML}$
 - f $9 \text{ kL} = \text{___ mL}$
 - g $5.3 \text{ ms} = \text{___ ns}$
 - h $1.2 \mu\text{s} = \text{___ ns}$
- 4 Light travels approximately $9.46 \times 10^{15} \text{ m}$ in a year. Express this in terametres.
- 5 Warragamba dam holds approximately 2 580 000 ML of water at full capacity. How many giganlitres is this?
- 6 The distance from Mars to the Sun is 0.228 Tm. Convert this distance to kilometres.
- 7 The mass of a hydrogen atom is $1.67 \times 10^{-24} \text{ g}$. What is the mass in nanograms of 1 million hydrogen atoms?
- 8 A computer can access its memory in 24 ns. Convert this to microseconds.
- 9 Prefixes are also used for file sizes of digital data. You can access an extension task related to these file sizes (Investigation 2D) from your [obook assess](#).



2E Error and accuracy in measurement

These resources are available on your obook assess:

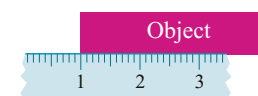
- **Investigation 2E:** Investigate different types of error
- **assess quiz 2E:** Test your skills with an auto-correcting multiple-choice quiz

o
b
o
o
k

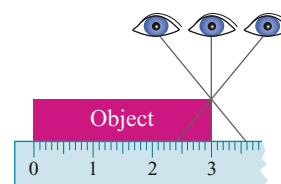


When physically measuring using a measuring instrument, there are several sources of possible error and uncertainty.

- Errors occur if the zero on the scale of the measuring instrument does not coincide with the end of the object or with the pointer on the measuring instrument.
- An error occurs if the end of the measuring instrument has been damaged. In this case start measuring from the 1, for example, instead of 0.



- Calibration error can occur if the scale is not accurately marked on the measuring instrument
- Parallax error occurs if your eye is not directly above the scale on the measuring instrument.
- There is always an error due to the limit of reading the measuring instrument.



Repeating a measurement a number of times and averaging the values can reduce the effect of any errors.

As a result of the accumulating effect of errors when calculations are performed with measured values, the following conventions are usually applied.

- When adding or subtracting measured quantities, the degree of accuracy of the answer is limited by the measurement that is accurate to the least number of decimal places.
- When multiplying or dividing with measured quantities, the degree of accuracy of the answer is limited by the measurement with the least number of significant figures.

!

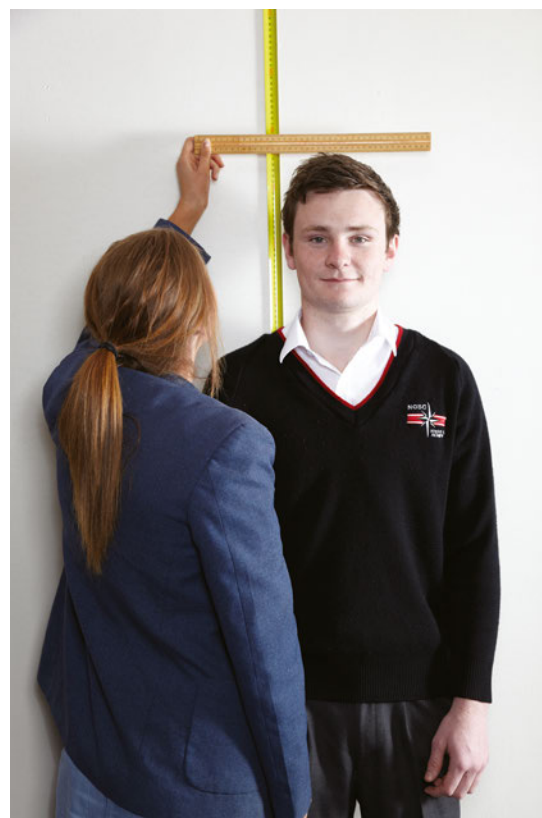
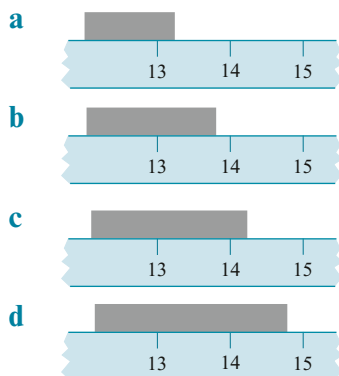
EXAMPLE 2E-1 Averaging measurements to find an approximation

John measured the width of his maths textbook five times using a ruler marked in millimetres. The results were 190 mm, 189 mm, 190 mm, 192 mm and 190 mm. Average these measurements to give an approximation for the width of the book, to the nearest millimetre.

Solve	Think	Apply
Average $= \frac{190 + 189 + 190 + 192 + 190}{5}$ $= 190.2 \text{ mm}$ $= 190 \text{ mm (to the nearest millimetre)}$	Determine the average by finding the sum of the measurements and then dividing by the number of measurements. The answer is 190 mm, to the nearest millimetre, because 190.2 is closer to 190 than to 191.	Averaging measurements reduces the effects of any errors. The answer should be given to the same degree of accuracy as the given measurements (in this case, to the nearest millimetre). The differences in the measurements could have been caused by any of the errors discussed on the previous page.

EXERCISE 2E Error and accuracy in measurement

- A student measured the length of his textbook using a ruler marked in millimetres. The results were 256 mm, 255 mm, 255 mm, 254 mm and 254 mm. Average these measurements to give an approximation of the length of the book, to the nearest millimetre.
- Average the following to give an approximation of the true measurement. In each case, the measurements were taken to the same degree of accuracy.
 - 83 mm, 85 mm, 84 mm, 85 mm, 85 mm, 84 mm
 - 4.9 kg, 4.8 kg, 4.9 kg, 5.0 kg, 4.9 kg
 - 162 mL, 162 mL, 160 mL, 161 mL, 161 mL, 162 mL
 - 22.49 s, 22.61 s, 22.54 s, 22.56 s, 22.52 s
- Ask five students to measure your height to the nearest centimetre. Average these measurements to give an approximation of your true height.
- The diagrams show several steel rods being measured with a ruler divided into centimetres. Write the length of each rod, using the scale given on the ruler.



- 5 The length of a rod is measured using the ruler in question 4, and the measurement is recorded as 14 cm.
- Would this be the exact length of the rod?
 - Between what values would the actual length lie?
 - What is the absolute possible error in stating that the length is 14 cm?
 - How could we find a more accurate value for the length of the rod?

EXAMPLE 2E-2 Finding a sensible approximation for the result of an addition (or subtraction) calculation

Write a sensible approximation for the result of this calculation: $15.642 \text{ m} + 8 \text{ m} + 19.21 \text{ m}$.

Solve	Think	Apply
$15.642 \text{ m} + 8 \text{ m} + 19.21 \text{ m}$ $= 42.852 \text{ m}$ $= 43 \text{ m}$ (to the nearest metre)	15.642 m is accurate to three decimal places. 8 m is accurate to the nearest whole number. 19.21 m is accurate to two decimal places. The least precise measurement is 8 m (the nearest whole number), so the answer should be rounded to the nearest whole number.	When adding or subtracting measured quantities, the degree of accuracy of the answer is limited by the measurement with the least decimal place accuracy.

- 6 Complete the following to calculate $13.65 \text{ L} + 10.9 \text{ L} + 12.624 \text{ L}$.
- 13.65 is accurate to _____ decimal place(s).
 10.9 is accurate to _____ decimal place(s).
 12.624 is accurate to _____ decimal place(s).
 The least precise measurement is _____ L to _____ decimal place(s).
 Hence, $13.65 \text{ L} + 10.9 \text{ L} + 12.624 \text{ L} = \text{_____ L} = \text{_____ L}$ to _____ decimal place(s).
- 7 Write a sensible approximation for the results of the following calculations.
- $9.87 \text{ m} + 15.219 \text{ m} + 11 \text{ m}$
 - $27.3 \text{ L} + 21.475 \text{ L} + 16.54 \text{ L}$
 - $6.132 \text{ km} - 3.46 \text{ km}$
 - $10.528 \text{ kg} + 11.607 \text{ kg} - 9.2 \text{ kg}$

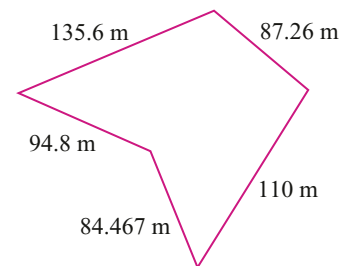
EXAMPLE 2E-3 Finding a sensible approximation for the result of a multiplication (or division) calculation

Write a sensible approximation for the result of this calculation: $15.2 \text{ m} \times 9.8 \text{ m}$.

Solve	Think	Apply
$15.2 \text{ m} \times 9.8 \text{ m}$ $= 148.96 \text{ m}^2$ $= 150 \text{ m}^2$ (to two significant figures)	15.2 is accurate to three significant figures. 9.8 is accurate to two significant figures. The measurement with the least number of significant figures is 9.8 m (two significant figures), so the answer should be rounded to two significant figures.	When multiplying or dividing with measured quantities, the degree of accuracy of the answer is limited by the measurement with the least number of significant figures.

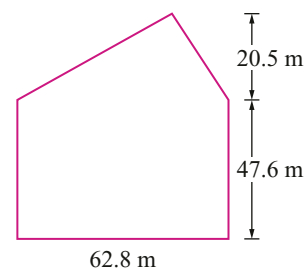
- 8** Complete the following to calculate $7.5 \text{ m} \times 12.3 \text{ m}$.
 7.5 m is accurate to ____ significant figures.
 12.3 m is accurate to ____ significant figures.
The least precise measurement is ____ m (to ____ significant figures).
So, $7.5 \text{ m} \times 12.3 \text{ m} = \text{____ m}^2 = \text{____ m}^2$ (to ____ significant figures)
- 9** Write a sensible approximation for the result of each of the following calculations.
- | | |
|---|--|
| a $23.6 \text{ m} \times 5.7 \text{ m}$ | b $405.2 \text{ cm} \times 58.6 \text{ cm}$ |
| c $88 \text{ cm}^3 \div 65 \text{ cm}^3$ | d $37.7 \text{ mm} \div 12 \text{ mm}$ |

- 10** Five students are given the task of each measuring one of the sides of an irregular 5-sided block of land. Each student measured their own side to a different degree of accuracy, as shown on the figure on the right. Write a sensible approximation for the perimeter of the land.
- 11** The diameter of a 20-cent coin is measured to be 2.3 cm . Write a sensible approximation for the area of the face of the coin.



- 12** The base and perpendicular height of a triangle are measured to be 5.6 m and 3.89 m respectively. Write a sensible approximation for the area of the triangle.
- 13** The mass and height of a woman are measured to be 63.8 kg and 175 cm respectively. Write a sensible approximation for the body mass index (BMI) of the woman using the formula $\text{BMI} = \frac{\text{mass}}{(\text{height})^2}$ where mass is in kilograms and height is in metres.

- 14** Consider the diagram on the right, showing a composite figure made up of a rectangle and a triangle.
- a** Calculate a sensible approximation for the area of the rectangle.
- b**
- How many significant figures are there in the answer to part **a**?
 - Has the answer to part **a** been rounded to the nearest whole square metre, 10 m^2 or 100 m^2 ?
- c** Calculate a sensible approximation for the area of the triangle.
- d**
- How many significant figures are there in the answer to part **c**?
 - Has the answer to part **c** been rounded to the nearest whole square metre, 10 m^2 or 100 m^2 ?
- e** Calculate a sensible approximation for the area of the composite figure.



2F Absolute error and limits of accuracy

These resources are available on your obook assess:

- **Video tutorial 2F:** Watch and listen to an explanation of Example 2F-2
- **Investigation 2F:** Compare the relative size of errors in calculations
- **assess quiz 2F:** Test your skills with an auto-correcting multiple-choice quiz

precision

smallest unit on a measurement instrument

absolute error

equals half the precision

lower bound of true measurement

result of subtracting the absolute error from the given measurement

upper bound of true measurement

result of adding the absolute error to the given measurement

In question 5 in the previous section, a steel rod was measured to the nearest centimetre because this was the smallest unit on the ruler: the length was closer to 14 cm than to 13 cm or 15 cm. The greatest possible error for this measurement is 0.5 cm, or half of the smallest scale unit (centimetre) on the ruler.

The actual length will lie between 13.5 cm and 14.5 cm; that is, between $14 - 0.5$ cm and $14 + 0.5$ cm. To obtain a more accurate measurement, we would need to use a more accurate ruler, one that has a scale marked in smaller units.

Because there is always some degree of error in a numerical value found by measurement, it follows that the results of any calculations involving this value will also contain a degree of error.

The smallest unit on a measuring instrument is called the **precision** of the instrument.

The **absolute error** when measuring a quantity (sometimes called the greatest possible error) is equal to half the precision.

The smallest and largest values between which the actual measurement lies are called the **lower and upper bounds of the true measurement**. These are the limits of accuracy of the measurement.

EXAMPLE 2F-1 Finding precision and absolute error

For each of the measurements below, find:

i the smallest unit of measurement (the precision)

ii the absolute error.

a 18 cm

b 2.4 kg

	Solve	Think	Apply
a i	The smallest unit of measurement is 1 cm; that is, the measurement has been made to the nearest centimetre. Precision = 1 cm	The last significant figure of the number is in the units column. So, the smallest scale on the measuring instrument is 1 cm.	The position of the last digit in the number determines the smallest scale on the measuring instrument used. This is the precision of the instrument. The absolute error is half the precision.
ii	Absolute error = $\frac{1}{2} \times 1 \text{ cm} = 0.5 \text{ cm}$	Absolute error = $\frac{1}{2} \times \text{precision}$	
b i	The smallest unit of measurement is 0.1 kg; that is, the measurement has been made to the nearest 0.1 of a kilogram. Precision = 0.1 kg	The last significant figure of the number is in the tenths column. So, the smallest scale on the measuring instrument is 0.1 kg.	The position of the last digit in the number determines the smallest scale on the measuring instrument used. This is the precision of the instrument. The absolute error is half the precision.
ii	Absolute error = $\frac{1}{2} \times 0.1 \text{ kg} = 0.05 \text{ kg}$	Absolute error = $\frac{1}{2} \times \text{precision}$	

EXERCISE 2F Absolute error and limits of accuracy

- 1 Complete the following.
 - a For a measurement given as 138 cm, the last significant figure is in the ____ column.
 So, the smallest scale on the measuring instrument is ____.
 The measurement has been made to the nearest ____.
 So, Precision = ____.
 Absolute error = $\frac{1}{2} \times$ ____ = ____
 - b For a measurement given as 11.7 s, the last significant figure is in the ____ column.
 So, the smallest scale on the measuring instrument is ____.
 The measurement has been made to the nearest ____.
 So, Precision = ____.
 Absolute error = $\frac{1}{2} \times$ ____ = ____
- 2 For each of the following measurements, find:
 - i the smallest unit of measurement (the precision)
 - ii the absolute error.

a 16 cm	b 286 g	c 38 m	d 16 L
e 3.6 kg	f 15.3 s	g 2.8 L	h 3.76 m

EXAMPLE 2F-2 Finding absolute error and limits of accuracy

For each of the following measurements, find:

- i the smallest unit of measurement (the precision)
 - ii the absolute error
 - iii the lower and upper bounds of the true measurement.
- a 16 s b 9.38 m

	Solve	Think	Apply
a i	The smallest unit of measurement is 1 s; that is, this measurement of time has been made to the nearest second. Precision = 1 s	Find the precision (1 s) and the absolute error (0.5 s) as in Example 2F-1.	Find the precision and the absolute error. Lower bound = measurement – absolute error
ii	Absolute error = $\frac{1}{2} \times 1 = 0.5$ s	Lower bound = 16 s – 0.5 s	Upper bound = measurement + absolute error
iii	Lower bound = 16 – 0.5 = 15.5 s Upper bound = 16 + 0.5 = 16.5 s True measurement is between 15.5 s and 16.5 s.	Upper bound = 16 s + 0.5 s	<i>Note:</i> The true measurement is greater than or equal to the lower bound, but is less than the upper bound; that is, lower bound $\leq$ true measurement < upper bound.
b i	Smallest unit of measurement = 0.01 m; that is, this measurement of length has been made to the nearest 0.01 of a metre. Precision = 0.01 m	Find the precision (0.01 m) and absolute error (0.005 m) as in Example 2F-1.	
ii	Absolute error = $\frac{1}{2} \times 0.01$ m = 0.005 m	Lower bound = 9.38 m – 0.005 m	
iii	Lower bound = 9.38 – 0.005 = 9.375 m Upper bound = 9.38 + 0.005 = 9.385 m True measurement is between 9.375 m and 9.385 m.	Upper bound = 9.38 m + 0.005 m	

- 3** Complete the following for a measurement of 2.6 kg.
- a** Precision = ____ kg **b** Absolute error = ____ kg
- c** Lower bound of measurement = $2.6 - \text{____}$ kg. Upper bound of measurement = $2.6 + \text{____}$ kg
The true measurement lies between ____ and ____.
- 4** For each of the measurements below, find:
- i** the precision **ii** the absolute error in the measurement
iii the lower and upper bounds of the true measurement.
- a** 12 mm **b** 348 g **c** 375 mL **d** 8.2 km
e 18.4 s **f** 4.9 kg **g** 2.37 m **h** 5.81 L
- 5** The capacity of a container is given as 750 mL, to the nearest 50 mL. Complete the following.
- a** The measurement has been given to the nearest ____ mL. Precision = ____ mL.
b Absolute error = $\frac{1}{2} \times \text{____}$ mL = ____ mL.
c Lower bound of measurement = $750 - \text{____}$ mL.
Upper bound of measurement = $750 + \text{____}$ mL.
The true measurement lies between ____ and ____.
- 6** For each of the measurements below, find:
- i** the smallest unit of measurement
ii the absolute error in the measurement
iii the lower and upper bounds of the true measurement.
- a** The mass of a can of soup is 420 g, to the nearest 30 g.
b The capacity of a drink bottle is 380 mL, to the nearest 20 mL.
c The crowd at a cricket match was 38 000, to the nearest 1000.
d The time taken for a plane flight was $6\frac{1}{2}$ hours, to the nearest $\frac{1}{2}$ hour.



EXAMPLE 2F-3 Finding absolute error as a percentage of the measurement

- a** Find the absolute error for the measurement 18 kg.
b Express the absolute error as a percentage of the measurement.

	Solve	Think	Apply
a	Smallest unit of measurement = 1 kg Absolute error = 0.5 kg	Find the precision (1 kg) and absolute error (0.5 kg) as in Example 2F-1.	The percentage error is the absolute error expressed as a percentage of the given measurement.
b	Percentage error = $\frac{0.5}{18} \times 100\%$ = 2.8% (to one decimal place)	Percentage error = $\frac{0.5}{18} \times 100\%$	Percentage error = $\frac{\text{absolute error}}{\text{measurement}} \times 100\%$

- 7** For each of the measurements below, find:
- i** the absolute error
ii the percentage error.
- a** 10 cm **b** 32 s **c** 250 g **d** 14 min
e 6 L **f** 2.4 kg **g** 13.5 s **h** 12.56 m
- 8** Explain how to determine the percentage error for a measurement.

EXAMPLE 2F-4 Finding limits of true perimeter and maximum error

The length and breadth of a rectangle were measured to be 8 cm and 6 cm respectively.

- Calculate the perimeter of the rectangle using these measurements.
- Find the lower and upper bounds of the rectangle's true perimeter.
- Hence find the maximum error in the answer to part a.

	Solve	Think	Apply
a	Perimeter = $2 \times 8 + 2 \times 6$ = 28 cm	Perimeter (using measurements) = $2 \times l + 2 \times b = 28$ cm	Calculate the perimeter using the measured length and breadth.
b	Now $7.5 \text{ cm} \leq \text{length} < 8.5 \text{ cm}$ and $5.5 \text{ cm} \leq \text{breadth} < 6.5 \text{ cm}$. So, $2 \times 7.5 + 2 \times 5.5 \text{ cm}$ $\leq \text{perimeter} < 2 \times 8.5 + 2 \times 6.5 \text{ cm}$. Thus $26 \text{ cm} \leq \text{perimeter} < 30 \text{ cm}$.	The absolute error of each measurement is 0.5 cm: length lies between 7.5 cm and 8.5 cm and breadth between 5.5 cm and 6.5 cm. Lower bound of perimeter = $2 \times 7.5 + 2 \times 5.5 = 26$ cm Upper bound of perimeter = $2 \times 8.5 + 2 \times 6.5 = 30$ cm	Determine the lower and upper bounds of each given measurement. Calculate the perimeter using the lower and upper bounds of length and breadth. Find the difference between the perimeter, calculated using the given measurements, and the perimeter using the lower (or upper) bound of each measurement.
c	Maximum error = $28 \text{ cm} - 26 \text{ cm}$ (or $28 \text{ cm} - 30 \text{ cm}$) = 2 cm	Maximum error = perimeter (using given measurements) – lower bound of perimeter (or upper bound of perimeter).	

- 9 The length and breadth of a rectangular recreation room are measured to be 7 m and 4 m, respectively. Complete the following.

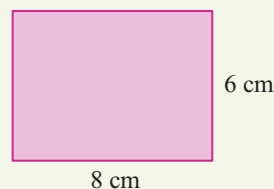
- Using the given measurements,
Perimeter = $2 \times \underline{\hspace{1cm}} + 2 \times \underline{\hspace{1cm}}$
= $\underline{\hspace{1cm}}$ m
- Now $6.5 \text{ m} \leq \text{length} < \underline{\hspace{1cm}} \text{ m}$
and $\underline{\hspace{1cm}} \text{ m} \leq \text{breadth} < 4.5 \text{ m}$
Lower bound of perimeter
= $2 \times 6.5 + 2 \times 3.5$
= $\underline{\hspace{1cm}}$ m
Upper bound of perimeter
= $2 \times \underline{\hspace{1cm}} + 2 \times \underline{\hspace{1cm}}$
= $\underline{\hspace{1cm}}$ m
So, $\underline{\hspace{1cm}} \text{ m} \leq \text{perimeter} < \underline{\hspace{1cm}} \text{ m}$
- Maximum error = $\underline{\hspace{1cm}} \text{ m} - \underline{\hspace{1cm}} \text{ m}$
= $\pm \underline{\hspace{1cm}} \text{ m}$



- 10 The length and breadth of a rectangle were measured to be 9 cm and 5 cm respectively.
- Calculate the perimeter of the rectangle using these measurements.
 - Find the lower and upper bounds of the rectangle's true length and breadth.
 - Hence, find the lower and upper bounds of the rectangle's true perimeter.
 - Find the maximum error in the answer to part a.

EXAMPLE 2F-5 Finding limits of true area and maximum error

The length and breadth of a rectangle were measured to be 8 cm and 6 cm respectively.



- a** Calculate the area, using the measurements given.
- b** Find the lower and upper bounds of the true area.
- c** Hence, find the maximum error in the answer to part **a**.

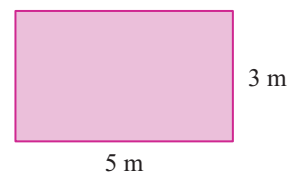
	Solve	Think	Apply
a	Area = $8 \times 6 = 48 \text{ cm}^2$	Using the given measurements: Area = $l \times b$ $= 48 \text{ cm}^2$	Calculate the area using the measured length and breadth.
b	Now $7.5 \text{ cm} \leq \text{length} < 8.5 \text{ cm}$ and $5.5 \text{ cm} \leq \text{breadth} < 6.5 \text{ cm}$. $(7.5 \times 5.5) \leq \text{area} < (8.5 \times 6.5)$. So $41.25 \text{ cm}^2 \leq \text{area} < 55.25 \text{ cm}^2$.	The absolute error of each measurement is 0.5 cm. So, the length lies between 7.5 cm and 8.5 cm and the breadth lies between 5.5 cm and 6.5 cm. Lower bound of area $= 7.5 \times 5.5$ $= 41.25 \text{ cm}^2$ Upper bound of area $= 8.5 \times 6.5$ $= 55.25 \text{ cm}^2$	Determine the lower and upper bounds of each given measurement. Calculate the area using the lower bounds of the length and breadth, and calculate the area using the upper bounds of the length and breadth. Find the difference between the area calculated using the given measurements and the area using the lower (or upper) bound of each measurement.
c	$48 - 41.25 = 6.75 \text{ cm}^2$ $48 - 55.25 = -7.25 \text{ cm}^2$ Maximum error = 7.25 cm^2	Maximum error = area (using given measurements) – lower bound of area (or upper bound of area).	

- 11** The length and breadth of a rectangle are measured to be 7 cm and 4 cm respectively. Complete the following.

- a** Using the given measurements: Area = (____ $\times$ ____) $\text{cm}^2 =$ ____ cm^2
- b** Now $6.5 \text{ cm} \leq \text{length} < \text{____ cm}$ and ____ $\text{cm} \leq \text{breadth} < 4.5 \text{ cm}$
Lower bound of area = $(6.5 \times 3.5) \text{ cm}^2 =$ ____ cm^2
Upper bound of area = (____ $\times$ ____) $\text{cm}^2 =$ ____ cm^2
So ____ $\text{cm}^2 \leq \text{area} < \text{____ cm}^2$
- c** ____ – lower bound of area = ____ cm^2
____ – upper bound of area = ____ cm^2
Maximum error = ____ cm^2

- 12** A rectangle was measured to be 5 m long by 3 m wide.

- a** Calculate the area of the rectangle using these measurements.
- b** Find the lower and upper bounds of the true length and width.
- c** What are the lower and upper bounds of the true area?
- d** Find the maximum error in the answer to part **a**.



- 13** Two pieces of timber were measured to be 164 cm and 128 cm respectively.
- If the two pieces were placed end to end, what would be their total length, using the measurements given?
 - Find the lower and upper bounds of the true length of each piece.
 - Hence, calculate the lower and upper bounds of the true total length of these two pieces of timber.
 - Find the maximum error in the answer to part **a**.
- 14** The masses of two bags of sand were measured and found to be 47 kg and 52 kg.
- What is the total mass of the two bags?
 - Find the lower and upper bounds of the true mass of each bag.
 - Calculate the lower and upper bounds of the true total mass.
 - What is the maximum error in the answer to part **a**?
- 15** Repeat question **14** given that the masses of the sand bags were 47.4 kg and 51.9 kg.
- 16** A rectangular room was measured to be 5.4 m long by 3.2 m wide.
- Calculate the area of the room using these measurements.
 - Find the lower and upper bounds of the room's true length and width.
 - What are the lower and upper bounds of the room's true area?
 - Find the maximum error in the answer to part **a**.
 - Compare your answers to those obtained in question **12**.
- 17** The diameter of a circular pizza tray is measured to be 28.6 cm.
- Calculate the area of the tray using the measurement given. (Use $A = \frac{\pi d^2}{4}$.)
 - What are the lower and upper bounds of the true length of the tray's diameter?
 - Find the lower and upper bounds of the true area of the tray.
 - What is the maximum error in the answer to part **a**?
- 18** A (rectangular) billiards table is measured to be 2.84 m by 1.42 m.
- Write the absolute error of each measurement.
 - Find the percentage error of each measurement, to two significant figures.
 - Calculate the area of the table using the given measurements.
 - Calculate the lower and upper bounds of the table's true area.
 - What is the maximum error in the answer to part **c**?
 - Express the error stated in part **e** as a percentage of the calculated area in part **c**, to two significant figures.
 - Is the percentage error in part **f** the sum of the percentage errors in part **b**?



CHAPTER 2 REVIEW PRACTICALITIES OF MEASUREMENT

You should be able to:

- ✓ round numbers using significant figures
- ✓ express decimal numbers in scientific notation, and vice versa
- ✓ perform calculations with numbers expressed in scientific notation
- ✓ convert between the commonly used metric units for length, mass, capacity, area and volume
- ✓ understand the possible sources of error in measuring and how to reduce their effect
- ✓ determine the precision, the absolute error, the upper and lower bounds and the percentage error for a measurement
- ✓ find the maximum possible error when measurements are used in calculations
- ✓ make sensible approximations for the results of calculations using measurements.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

REVIEW MULTIPLE-CHOICE QUESTIONS

- 2A** ➤ **1** When rounded to two significant figures, 3950.628 becomes:
A 3900 B 4000 C 39 D 3950.63
- 2B** ➤ **2** Which of the following numbers is written in scientific notation?
A $5 \times 10\,000$ B 50000 C 5×10^4 D 50×10^3
- 2B** ➤ **3** 7.06×10^{-6} is equivalent to
A 0.00000706 B 0.0000706 C 706000 D 7060000
- 2B** ➤ **4** The result of the calculation $(4 \times 10^5) \div (8 \times 10^{-3})$ is:
A 5×10^8 B 5×10^7 C 5000 D 5×10^2
- 2C** ➤ **5** Which one of the following is equivalent to 5.06 kg?
A 0.00506 g B 5060 g C 0.0506 g D 506 g
- 2C** ➤ **6** Which one of the following is *not* equivalent to 5.3 m?
A 530 cm B 5300 mm C 0.0053 km D 0.053 km
- 2C** ➤ **7** The capacity of a drinking glass would be closest to:
A 2 mL B 20 mL C 200 mL D 2 L
- 2D** ➤ **8** 6 ML is equivalent to:
A 60000 kL B 6000 kL C 600 kL D 60 kL
- 2F** ➤ **9** The absolute error in the measurement 3.6 L is:
A 0.1 L B 0.05 L C 0.5 L D 3.55 L
- 2F** ➤ **10** The mass of a can of soup was 250 g, to the nearest 10 g. The percentage error in this measurement is:
A 4% B 2% C 0.4% D 0.2%
- 2F** ➤ **11** The side length of a square was measured to be 8 cm. The maximum error in stating that the perimeter is 32 cm is:
A 0.5 cm B 1 cm C 2 cm D 4 cm

REVIEW SET 1

- Round 3659.063 to:
 - the nearest 100
 - the nearest whole number
 - two decimal places
 - three significant figures
 - five significant figures
 - one significant figure
- State whether each of these numbers is written in scientific notation.
 - 6×1000
 - 15×10^7
 - 2.04×10^{-6}
- Express these numbers in scientific notation.
 - 105 000 000
 - 0.000 062
 - 3179
- Calculate the following, writing the answers in scientific notation.
 - $(4.1 \times 10^8) \times (6 \times 10^5)$
 - $(1.96 \times 10^{-3}) \div (1.4 \times 10^7)$
 - $(8 \times 10^5)^4$
 - $\sqrt{8.41 \times 10^{-12}}$
- Convert the following.
 - $5.6 \text{ cm}^2 = \text{___ mm}^2$
 - $43\,000 \text{ m}^2 = \text{___ ha}$
 - $2.9 \text{ m}^3 = \text{___ cm}^3$
 - $5600 \text{ mm}^3 = \text{___ cm}^3$
- Convert the following.
 - $2.3 \text{ Gm} = \text{___ m}$
 - $52 \text{ ML} = \text{___ kL}$
 - $3 \text{ ms} = \text{___ } \mu\text{s}$
 - $7.2 \text{ Tg} = \text{___ mg}$
- For each of the following measurements, find the:
 - precision
 - absolute error
 - lower and upper bounds of the true measurement
 - percentage error (to one decimal place).
 - 7.5 m
 - 280 g
- The length and breadth of a rectangle were measured to be 6 cm and 4 cm.
 - Calculate the rectangle's perimeter using these measurements.
 - Write the lower and upper bounds of the true length and breadth of the rectangle.
 - Find the lower and upper bounds of the rectangle's true perimeter.
 - What is the maximum error in the answer in part a?
 - Calculate the rectangle's area using the measurements given.
 - Find the lower and upper bounds of the rectangle's true area.
 - Find the maximum error in the answer to part e.
- Write sensible approximations for the results of the following calculations.
 - $17.3 \text{ m} + 15.89 \text{ m}$
 - $17.3 \text{ m} \times 15.89 \text{ m}$

REVIEW SET 2

- Round 1472.634 to:
 - the nearest 10
 - two significant figures
 - two decimal places.
- Express each of the following in scientific notation.
 - 749 000
 - 0.000 003
 - 0.0105
- Calculate $(1.4 \times 10^7) \times (4.5 \times 10^8)$, expressing the answer in scientific notation.
- Convert the following.
 - $2.1 \text{ ha} = \text{___ m}^2$
 - $780 \text{ mm}^2 = \text{___ cm}^2$
 - $9\,500\,000 \text{ cm}^3 = \text{___ m}^3$
 - $72 \text{ cm}^3 = \text{___ mm}^3$

- 5** Convert the following.
a $4.3 \text{ cm} = \text{ } \mu\text{m}$ **b** $2 \text{ Tg} = \text{ } \text{Mg}$ **c** $52\,000 \text{ kL} = \text{ } \text{ML}$ **d** $9.1 \text{ mm} = \text{ } \text{nm}$
- 6** Write sensible approximations for the results of the following calculations.
a $15.36 \text{ m} + 9.7 \text{ m} + 11.62 \text{ m}$ **b** $16.5 \text{ cm} \times 4.7 \text{ cm}$
- 7** For each of the following measurements, find the:
i precision **ii** absolute error
iii lower and upper bounds of the true measurement **iv** percentage error (to one decimal place).
a 12.8 kg **b** 12.56 m
- 8** The length and breadth of a table were measured to be 154 cm and 80 cm , to the nearest centimetre.
a Calculate the table's area using the measurements given.
b Find the lower and upper bounds of the table's true area.
c Find the maximum error in the answer to part **a**.
d Express the maximum error from part **c** as a percentage of the area (from part **a**).

REVIEW SET 3

- 1** Round $0.005\,06$ to:
a two significant figures **b** two decimal places **c** one significant figure
- 2** State whether or not each of the following numbers is expressed in scientific notation.
a $4.9 \times 100\,000$ **b** 7.0×10^8 **c** 70×10^7
- 3** Convert each of the following numbers to scientific notation.
a $67\,000$ **b** $0.000\,0809$ **c** 1230
- 4** Write the basic numeral for:
a 3.4×10^6 **b** 8.7×10^{-5} **c** 2.053×10^4
- 5** Convert the following.
a $11.2 \text{ cm}^2 = \text{ } \text{mm}^2$ **b** $129\,000 \text{ m}^2 = \text{ } \text{ha}$
c $3.4 \text{ m}^3 = \text{ } \text{cm}^3$ **d** $73\,000 \text{ mm}^3 = \text{ } \text{cm}^3$
- 6** Convert the following.
a $4.5 \text{ Mm} = \text{ } \text{m}$ **b** $2 \text{ Mg} = \text{ } \text{t}$ **c** $7 \mu\text{s} = \text{ } \text{ns}$ **d** $3.5 \text{ kL} = \text{ } \text{GL}$
- 7** The masses of two bags of potatoes were measured, to be 49 kg and 51 kg , to the nearest kilogram.
a What is the total mass of the two bags using these measurements?
b Write the lower and upper bounds of the true mass of each bag.
c Calculate the lower and upper bounds of the total mass of the two bags.
d Determine the absolute error in the answer to part **a**.
e Express the absolute error as a percentage of the total mass of the bags.
- 8** Write sensible approximations for the results of the following calculations.
a $43.2 \text{ kg} - 8 \text{ kg}$ **b** $125.345 \text{ L} \div 0.85 \text{ L}$

REVIEW SET 4

- 1** Round 2.0695 to these numbers of significant figures:
a one significant figure **b** two significant figures **c** three significant figures **d** four significant figures.

- 2** Convert the following.
- a** 13.65 m = ___ cm **b** 3460 kg = ___ t **c** 276 mL = ___ L **d** 8.3 m = ___ mm
- 3** Convert the following.
- a** 13.65 ha = ___ m² **b** 1960 mm² = ___ cm² **c** 3700000 cm³ = ___ m³ **d** 6.8 cm³ = ___ mm³
- 4** Convert the following.
- a** 6 mL = ___ μ L **b** 4.2 Gg = ___ kg **c** 8.1 μ s = ___ ns **d** 560 ML = ___ TL
- 5** Express in scientific notation, correct to two significant figures.
- a** 643 700 000 **b** 0.000 000 304
- 6** Calculate the following, expressing the answer in scientific notation.
- a** $(1.08 \times 10^{-6}) \div (7.2 \times 10^{-5})$ **b** $\sqrt{1.96 \times 10^{20}}$
- 7** The base and perpendicular height of a triangle were measured to be 15.4 cm and 12.5 cm respectively.
- a** Find the area of the triangle using these measurements.
b Calculate the range within which the triangle's true area lies.
c What is the maximum error in using part **a** as the area?
d Express the maximum error from part **c** as a percentage of the area (from part **a**).
- 8** The length and breadth of a bed are measured to be 1.9 m by 0.84 m respectively. Write a sensible approximation for:
- a** the perimeter of the bed **b** the area of the bed.

REVIEW PRACTICE EXAMINATION QUESTION

- 1 a** Write 0.001 306 in scientific notation, rounded to three significant figures. (2 marks)
- b** How many significant figures are there in the measurement 5.0×10^6 metres? (1 mark)
- c** Evaluate $\sqrt{6.724 \times 10^{11}}$. Write your answer in scientific notation. (1 mark)
- d** Convert each measurement:
- i** 94 630 mm = ___ m **ii** 2.5 m³ = ___ cm³. (2 marks)
- e** The scale on a thermometer measures temperature to the nearest 0.5°C.
- i** What is the absolute error in stating that the temperature is 19.5°C? (1 mark)
- ii** Calculate the percentage error for this measurement. (1 mark)
- f** Bottle A contains 2.64 L of saline solution and bottle B contains 2.88 L of the same solution. They are both poured into a large container.
- i** Calculate the volume of saline solution in the large container, using the measurements given. (1 mark)
- ii** Find the lower and upper bounds of the true volume of saline solution. (2 marks)
- iii** Calculate the maximum error in the answer to part **i**. (1 mark)
- iv** Express the error from part **iii** as a percentage of the answer in part **i**. (1 mark)
- g** The base and height of a triangle are measured to be 28.4 cm and 9.6 cm respectively. Write a sensible approximation for the area of the triangle, based on the accuracy of each measurement. (2 marks)

TOTAL: 15 marks

3

Formulas and equations

The main mathematical ideas investigated are:

- ▶ substituting into algebraic expressions
- ▶ substituting into formulas and finding values
- ▶ solving linear equations
- ▶ using formulas to calculate distance, speed and time
- ▶ using formulas to calculate an estimate for blood alcohol content
- ▶ using formulas to calculate required medication dosages.

ARE YOU READY?

- 3A ▶ 1** What is the result of calculating $10 - 3 \times 5$?
A 35 **B** 25
C -2 **D** -5
- 3A ▶ 2** What is the value of 6^2 ?
A 6 **B** 8
C 12 **D** 36
- 3A ▶ 3** What is the value of $\sqrt{64}$?
A 4 **B** 8
C 16 **D** 32
- 3A ▶ 4** Which expression is the result of simplifying $4a + 7 - 9a - 4$?
A $-5a + 3$ **B** $-5a - 3$
C $5a + 3$ **D** $5a - 3$
- 3A ▶ 5** Which expression is the result of simplifying $5x - 3y + 8y - 7x$?
A $2x - 5y$ **B** $5y - 2x$
C $2x + y$ **D** $12x + 11y$
- 3A ▶ 6** Which expression is the result of simplifying $8d \times -6g$?
A $2dg$ **B** $48dg$
C $-48dg$ **D** $-2dg$
- 3A ▶ 7** Which expression is the result of simplifying $5a \times 3b \times 4ab$?
A $19ab$ **B** $534aab$
C $60ab$ **D** $60a^2b^2$
- 3A ▶ 8** Which expression is the result of simplifying $\frac{14k}{21}$?
A $\frac{k}{3}$ **B** $\frac{2k}{3}$
C $2k$ **D** $3k$
- 3A ▶ 9** Which expression is the result of simplifying $\frac{6xy^3}{4xyz}$?
A $\frac{3y}{z}$ **B** $\frac{3y}{2z}$
C $\frac{3y^2}{z}$ **D** $\frac{3y^2}{2z}$
- 3A ▶ 10** What is the expansion of $-7x(2x - 5)$?
A $-14x^2 + 35x$ **B** $-14x + 35$
C $14x^2 - 35x$ **D** $14x - 35$
- 3A ▶ 11** What is the complete factorisation of $15pq - 12q$?
A $3(5pq - 4q)$ **B** $q(15p - 12)$
C $3q(5p - 4)$ **D** $3pq(5 - 12p)$
- 3A ▶ 12** What is the value of $6a - 4b$ if $a = 3$ and $b = -5$?
A 38 **B** -2
C 18 **D** 2
- 3B ▶ 13** What is the value of a in the equation $a + 4 = 24$?
A 2 **B** 6
C 20 **D** 28
- 3C ▶ 14** What is the lowest common multiple of 4 and 6?
A 24 **B** 12
C 6 **D** 4

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook assess](#).

- Q1** Support sheet 3A.1 Understanding order of operations
Q2-3 Support sheet 3A.2 Finding powers and square roots
Q4-5 Support sheet 3A.3 Simplifying like terms
Q6-7 Support sheet 3A.4 Multiplying algebraic terms
Q8-9 Support sheet 3A.5 Dividing algebraic terms
Q10 Support sheet 3A.6 Expanding expressions to remove brackets
Q11 Support sheet 3A.7 Factorising simple expressions
Q12 Support sheet 3A.8 Evaluating expressions
Q13 Support sheet 3B.1 Solving simple linear equations
Q14 Support sheet 3C.1 Finding the lowest common multiple

3A Substitution into expressions and formulas

These resources are available on your obook assess:

- **Worksheet 3A:** Practise your skills with extra problems for substitution
- **Investigation 3A:** Write and evaluate expressions relating to costs at a cinema
- **assess quiz 3A:** Test your skills with an auto-correcting multiple-choice quiz

pronumeral

letter or symbol that takes the place of a number

formula

relationship or rule between two or more variables that contains an equals sign; e.g. $v = u + at$

variable (algebra)

numerical quantity that can have varying or different value and can be represented by a pronumeral; e.g. time or t

subject of the formula

single variable usually on the left-hand side of a formula; e.g. v is the subject of the formula $v = u + at$

When the **pronumeral** x is replaced by a number, for example 2, in an expression such as $3x - 5$, we say that 2 was substituted for x , and the value of the expression is 1 when $x = 2$ because $3(2) - 5 = 6 - 5 = 1$. When substituting a number into an expression, we usually place the substituted value within brackets.

A mathematical **formula** can be used in many applications. Usually it starts with a single **variable** on the left-hand side followed by the equals sign and the rest of the formula on the right-hand side. The single variable on the left-hand side is called the **subject of the formula** and its value is found by substituting other values into the right-hand side of the formula and evaluating.

(Note: the words 'formulas' and 'formulae' both mean more than one formula.)

EXAMPLE 3A-1 Evaluating expressions by substituting given values for pronumerals

For $a = 5$, $b = -2$ and $c = 3$, find the value of:

a $2a - c$

b $\sqrt{a - 2b}$

c $c - 4b^2$

	Solve	Think	Apply
a	$2a - c = 2(5) - (3)$ $= 10 - 3$ $= 7$	Substitute 5 for a and 3 for c . Use order of operations to evaluate.	When asked to evaluate (or find the value of) an expression, substitute the given values into the expression.
b	$\sqrt{a - 2b} = \sqrt{(5) - 2(-2)}$ $= \sqrt{5 + 4}$ $= \sqrt{9}$ $= 3$	Substitute 5 for a and -2 for b . Evaluate under the square root sign first, then find the square root.	When substituting, put the number in brackets. Use the correct order of operations to find the value.
c	$c - 4b^2 = (3) - 4(-2)^2$ $= 3 - 4 \times 4$ $= 3 - 16$ $= -13$	Substitute 3 for c and -2 for b . When -2 is squared the answer is $+4$. Use order of operations to evaluate.	When using a calculator, enter the information from the expression from left to right.

EXERCISE 3A

Substitution into expressions and formulas

- 1 For $x = 7$ and $y = -3$, complete the following to find the value of:
 - a $3x + 2y = 3(7) + 2(\underline{\quad})$
 $= \underline{\quad}$
 - b $x - 2y = (\underline{\quad}) - 2(-3)$
 $= \underline{\quad}$
- 2 For $a = 2$ and $b = 5$, find the value of:
 - a $a + b$
 - b $b - a$
 - c $2a + 3b$
 - d $a - 2b$
 - e b^2
 - f $2b^2$
 - g $(2b)^2$
 - h b^3
 - i $4a + b$
 - j $4(a + b)$
 - k $(a + b)^2$
 - l $a^2 + b^2$
 - m $\frac{ab}{20}$
 - n $5(2a + 3b)$
 - o $\frac{ab}{4}$
 - p $\frac{a^2b}{10}$
- 3 For $p = 2$ and $q = -5$, find the value of:
 - a $p + q$
 - b $p - q$
 - c $2p + 5q$
 - d $3p - 2q$
 - e $\frac{1}{3}(p + q)$
 - f $\frac{p + q}{2}$
 - g $(p - q)^2$
 - h $p^2 - q^2$
 - i $-5(3p - 2q)$
 - j $4 - (p - q)$
 - k $8 - (3p - 4q)$
 - l $(p + q)(p - q)$
- 4 For $m = 3$ and $n = 7$, find the value of:
 - a $m + n$
 - b $m - n$
 - c $n - m$
 - d $n + m$
 - e $m^2 + n^2$
 - f $n^2 - m^2$
 - g $3(m + n)$
 - h $-2(m - n)$
 - i $\sqrt{6m + n}$
 - j $\sqrt{10(m + n)}$
 - k $\sqrt{4(n - m)}$
 - l $\sqrt{7m + 4n}$

EXAMPLE 3A-2 Substituting into a formula to find a value

Given that $v = u + at$, find v when $u = 50$, $a = 10$ and $t = 6$.

Solve	Think	Apply
$v = u + at$ $= 50 + 10 \times 6$ $= 50 + 60$ $= 110$	Substitute 50 for u , 10 for a and 6 for t in the equation. Evaluate using order of operations or your calculator.	When finding the value of a variable in an equation or a formula, substitute the given values into the formula and evaluate.

- 5 Complete the following to find t using the formula $t = a + (n - 1)d$ when $a = -3$, $n = 20$ and $d = 2$.

$$t = -3 + (20 - 1) \times \underline{\quad}$$

$$= \underline{\quad}$$
- 6
 - a Given that $V = IR$, find V when $I = 10$ and $R = 250$.
 - b Given that $V = IR$, find V when $I = 50$ and $R = 20$.
 - c Given that $W = Fs$, find W when $F = 100$ and $s = 12$.
 - d Given that $s = \frac{d}{t}$, find s when $d = 50$ and $t = 10$.
- 7
 - a Given that $v = u + at$, find v when $u = 5$, $a = 10$ and $t = 7$.
 - b Given that $s = ut + \frac{1}{2}at^2$, find s when $u = -3$, $a = 10$ and $t = 5$.
 - c Given that $d = 180 - \frac{360}{n}$, find d when $n = 12$.
 - d Given that $a = \frac{2Rn}{n + 1}$, find a when $R = 8$ and $n = 24$.

- 8** The volume, V , of a sphere is found using the formula $V = \frac{4}{3}\pi r^3$, where r is the radius. Find, to one decimal place, the volume of a sphere with radius:

a 6 cm **b** 8.5 cm **c** 1.3 m

- 9** The amount of money in an investment that pays compound interest is given by $A = P(1 + r)^n$. Find A when:

a $P = 1000$, $r = 0.1$ and $n = 5$
b $P = 600$, $r = 0.05$ and $n = 20$
c $P = 2340$, $r = 0.035$ and $n = 10$
d $P = 625$, $r = 0.112$ and $n = 8$

- 10** Pythagoras' theorem is $c = \sqrt{a^2 + b^2}$. Find c when:

a $a = 3$, $b = 4$ **b** $a = 5$, $b = 12$
c $a = 15$, $b = 20$ **d** $a = 16$, $b = 12$.

- 11** Using $a = 2$ and $b = -5$, and showing all working, check whether these pairs of expressions are the same. For those pairs that are not the same, rewrite the second expression to make it the same as the first.

a $4(a + b)$ and $4a + 4b$
b $4(a + b)$ and $4a + b$
c $2(3a + b)$ and $6a + 3b$
d $3a - 5(a + b)$ and $2a - 5b$
e $3a - 5(a + b)$ and $-2a - b$
f $3a - 5(a + b)$ and $-2a - 5b$
g $(3a - 2b)(4a + 7b)$ and $12a^2 - 14ab$
h $(3a - 2b)(4a + 7b)$ and $12a^2 - 8ab - 14b^2$
i $(a + b)^3$ and $a^3 + b^3$
j $(3a - 2b)(4a + 7b)$ and $12a^2 + 13ab - 14b^2$

- 12 a** Using $a = 3$ and $b = -4$, evaluate the following.

i $4(a + b)$ **ii** $a^2 + b - 9$ **iii** $4a + 4b$ **iv** $-10 + 2a$

b What do you notice about the value of all these expressions?
c Does this necessarily mean that all these expressions are always equal?
d Test using other values of a and b to find which, if any, of the expressions are always equal.

- 13** The body mass index, b , is a measure used to determine if a person's mass is within a recommended range. The range is $21 < b < 25$.

The body mass index formula is given by $b = \frac{m}{h^2}$ where m = mass in kilograms and h = height in metres.

a Calculate the value of b for a person of mass 65 kg and height 1.65 m. Is the value within the recommended range?
b Peter has a mass of 100 kg and is 1.95 m tall. He states that his mass is within the recommended range. By calculating the value of b for Peter, determine if he is correct.
c Kristy has a mass of 50 kg, is 1.5 m tall and feels that she is below the recommended body mass. Is she correct? Explain using calculations.



- 14** To hire beach chairs a \$50 deposit must be paid as well as \$10 per chair hired.
- Write an algebraic expression for the total hire cost, using n to represent the number of chairs hired.
 - How much would it cost to hire 6 chairs?
- 15** In basketball, players score points by throwing the ball through the basket, in an act known as 'shooting'. Different points are awarded to players based on where they are standing when they shoot.

3 points	Awarded to players successfully goaling from behind the three-point line
2 points	Awarded to players successfully goaling from anywhere inside the three-point line
1 point	Awarded to players successfully goaling when playing a foul shot

- Using appropriate variables, write an algebraic expression to represent the total number of points scored during a basketball game. Remember to carefully define your variables.
- Calculate the total number of points scored by a team during part of a practice game in which the players successfully scored 5 goals from behind the three-point line, 12 goals from inside the three-point line and 4 goals when playing foul shots.
- Find three different ways of achieving a score of 68 points.



- 16** Emma walks along a beach. She starts 5 kilometres from the lighthouse and travels at 6 kilometres per hour.
- Write an algebraic expression to represent Emma's distance from the lighthouse, using t to represent the number of hours she has been walking.
 - How far will Emma have walked if she has been walking for half an hour?
 - For how long would Emma have walked if she is 14 km from the lighthouse?

- 17** A formula for calculating the bend allowance, B , of

a sheet of metal is $B = \frac{2\pi\left(R + \frac{T}{2}\right) \times A}{360}$, where

B = bend allowance

T = thickness of the sheet, in millimetres

A = number of degrees in the angle of bend

R = radius of curvature in millimetres

By calculating the value of B determine which sheet metal below has the greatest bend allowance.

Sheet 1: $T = 3, A = 5, R = 12$

Sheet 2: $T = 15, A = 50, R = 60$



3B Solving linear equations

These resources are available on your obook assess:

- **Interactive 3B:** Explore key ideas for solving linear equations
- **Video tutorial 3B:** Watch and listen to an explanation of Example 3B-1
- **Investigation 3B.1:** Solve linear equations relating to household bills
- **Investigation 3B.2:** Write formulas relating to body measurements
- **assess quiz 3B:** Test your skills with an auto-correcting multiple-choice quiz

equation

collection of two or more algebraic terms separated by mathematical operation symbols and containing an equals sign; e.g. $2x + 1 = 7$ or $3x^2 = 5x - 2$

solve

to find the value of the pronumeral that makes the equation a true statement

linear equation

equation in which the highest positive integer power of the pronumeral is 1; e.g. $2x + 1 = 7$ is a linear equation but $3x^2 = 5x - 2$ is not because its highest power of x is 2

solution

the value of the pronumeral that makes an equation a true statement; e.g. the solution of $2x + 1 = 7$ is $x = 3$

An algebraic **equation** contains two algebraic expressions that are equal to each other. The aim of solving an equation is to find the value of the pronumeral that makes the equation a true statement.

To **solve** a **linear equation**:

- 1 Keep the equation balanced by performing the same operation to both sides of the equation; that is:
 - add the same number or term to both sides of the equation
 - subtract the same number or term from both sides of the equation
 - multiply both sides of the equation by the same number or term
 - divide both sides of the equation by the same number or term.
- 2 Continue until you obtain the simplest equivalent equation with the pronumeral on one side of the equation and a number on the other. For example, to solve $x + 4 = 10$, we subtract 4 from both sides of the equation to obtain the **solution** $x = 6$.

EXAMPLE 3B-1 Solving two-step linear equations

Solve these equations.

a $5x + 7 = 37$

b $8x - 9 = 79$

	Solve	Think	Apply
a	$5x + 7 = 37$ $5x + 7 - 7 = 37 - 7$ $5x = 30$ $\frac{5x}{5} = \frac{30}{5}$ $x = 6$ Check: $(5 \times 6) + 7 = 30 + 7 = 37$ $x = 6$ is correct.	<i>Step 1:</i> Subtract 7 from both sides, then divide both sides by 5. <i>Step 2:</i> Substituting $x = 6$ gives 37, which is correct.	Perform the same operation on both sides of the equation to keep equivalent equations in each step. Always substitute the answer back into the equation to check it is correct. Answers are not always whole numbers.
b	$8x - 9 = 79$ $8x - 9 + 9 = 79 + 9$ $8x = 88$ $\frac{8x}{8} = \frac{88}{8}$ $x = 11$ Check: $(8 \times 11) - 9 = 88 - 9 = 79$ $x = 11$ is correct.	Add 9 to both sides, giving $8x = 88$, then divide by 8 to get $x = 11$. Substitute $x = 11$ to get 79, which shows that $x = 11$ is correct.	

EXERCISE 3B Solving linear equations

1 Complete to solve these equations.

a $7x - 3 = 32$
 $7x - 3 + \underline{\quad} = 32 + \underline{3}$
 $7x = \underline{\quad}$
 $\frac{7x}{7} = \frac{35}{7}$
 $x = \underline{\quad}$

b $4x + 7 = 15$
 $4x + 7 - \underline{7} = 15 - \underline{\quad}$
 $\underline{\quad} = 8$
 $\frac{\square}{4} = \frac{8}{\square}$
 $x = 2$

2 Solve the following equations.

a $3x - 9 = 3$

d $6x + 7 = 61$

g $2x - 32 = 16$

j $4q - 21 = 115$

m $7T - 8 = 27$

b $4p + 8 = 24$

e $4p + 12 = 52$

h $11p - 10 = 34$

k $8s - 12 = 252$

n $11s + 16 = 115$

c $6x - 2 = 82$

f $5y - 6 = 44$

i $7m + 8 = 50$

l $10r - 30 = 100$

o $4m + 9 = 73$

EXAMPLE 3B-2 Checking whether a given value is a solution to an equation

Substitute the given value into the equation to check if it is a solution to the equation.

a $3x - 9 = 36$ $[x = 11]$

b $49 - 5x = 29$ $[x = 4]$

	Solve	Think	Apply
a	$3x - 9 = (3 \times 11) - 9$ $= 33 - 9$ $= 24$ $\neq 36$ $x = 11$ is not the solution.	Substitute $x = 11$ into $3x - 9$; the answer is 24. But $24 \neq 36$, so $x = 11$ is not the solution.	Substitute the given value into the left-hand side of the equation. If both sides of the equation have the same number, then that value is a solution.
b	$49 - 5x = 49 - (5 \times 4)$ $= 49 - 20$ $= 29$ $x = 4$ is the solution.	Substitute $x = 4$ into $49 - 5x$ to get 29. This means $x = 4$ is the correct solution.	If the two sides are not the same, then it is not a solution.

3 Complete the following to check whether:

a $x = 5$ is a solution to $7x - 3 = 28$
 $7x - 3 = (7 \times \underline{\quad}) - 3$
 $= 35 - 3$
 $= \underline{\quad}$
 $x = 5$ is $\underline{\quad}$.

b $a = 8$ is a solution to $7 + 3a = 31$.
 $7 + 3a = 7 + (3 \times \underline{\quad})$
 $= 7 + \underline{\quad}$
 $= 31$
 $a = 8$ is $\underline{\quad}$.

4 Substitute the given value for the pronumeral into the equation to determine if it is a solution to the equation.

a $4x + 8 = 60$ $[x = 13]$

d $9 - 2p = -15$ $[p = 12]$

g $6d - 5 = 31$ $[d = 9]$

b $5x - 9 = 11$ $[x = 3]$

e $10 + 5m = -20$ $[m = -6]$

h $11y - 72 = -193$ $[y = -11]$

c $3x + 4 = 13$ $[x = 3]$

f $22 + 3c = 46$ $[c = 7]$

i $14q + 18 = 116$ $[q = 7]$

EXAMPLE 3B-3 Solving linear equations containing a negative pronumeral term

Solve the following.

a $9 - 7x = 5$

b $17 = 8 - 4x$

	Solve	Think	Apply
a	$ \begin{aligned} 9 - 7x &= 5 \\ 9 - 7x - 9 &= 5 - 9 \\ -7x &= -4 \\ \frac{-7x}{-7} &= \frac{-4}{-7} \\ x &= \frac{4}{7} \end{aligned} $	First, subtract 9 from both sides, then divide both sides by -7 to get $x = \frac{4}{7}$.	The solving process is the same whether the equation appears as $9 - 7x = 5$ or as $-7x + 9 = 5$, so we start by subtracting 9 from each side. Similarly, $17 = 8 - 4x$ is the same as $8 - 4x = 17$ or $-4x + 8 = 17$, so we subtract 8 from each side.
b	$ \begin{aligned} 17 &= 8 - 4x \\ 17 - 8 &= 8 - 4x - 8 \\ 9 &= -4x \\ \frac{9}{-4} &= \frac{-4x}{-4} \\ -\frac{9}{4} &= x \\ x &= -\frac{9}{4} \\ \text{or } x &= -2\frac{1}{4} \end{aligned} $	First, subtract 8 from both sides, then divide by -4 . It doesn't matter which side of the equation the $-4x$ and 9 are on. The answer is $x = -2\frac{1}{4}$.	The strategy is to have the pronumeral on one side of the equation and the numbers on the other side.

5 Complete to solve these equations.

a $6 - 5x = 14$
 $6 - 5x - 6 = 14 - \underline{\hspace{1cm}}$
 $\frac{-5x}{-5} = \frac{\square}{-5}$
 $\underline{\hspace{1cm}} = \frac{8}{-5}$
 $x = \underline{\hspace{1cm}}$

b $-11 = 7 + 5x$
 $-11 - \underline{\hspace{1cm}} = 7 + 5x - \underline{\hspace{1cm}}$
 $-18 = \underline{\hspace{1cm}}$
 $\frac{-18}{\square} = \frac{5x}{\square}$
 $x = \underline{\hspace{1cm}}$

6 Solve the following for x .

a $5 = 3x + 7$
d $6 - x = -5$
g $5 - 4x = -7$
j $11 = 3 - 2x$
m $6 = -1 - 7x$

b $6x - 7 = -1$
e $3 - 4x = 15$
h $3 - 7x = -2$
k $15 - 2x = -1$
n $-15 = 3 - 6x$

c $-1 = 2x + 6$
f $3 - 2x = 7$
i $17 - 2x = -1$
l $8 = 3 - 2x$
o $11 = -4 - 3x$

EXAMPLE 3B-4 Solving a linear equation

Solve $\frac{m}{3} - 5 = -2$.

Solve	Think	Apply
$ \begin{aligned} \frac{m}{3} - 5 &= -2 \\ \frac{m}{3} - 5 + 5 &= -2 + 5 \\ \frac{m}{3} &= 3 \\ \frac{m}{3} \times 3 &= 3 \times 3 \\ m &= 9 \end{aligned} $	Add 5 to both sides, then multiply both sides by 3 to get $m = 9$.	Continue to use the same technique of adding or subtracting first before multiplying or dividing.

- 7 Complete to solve the equation.

$$\begin{aligned}\frac{x}{5} + 3 &= 7 \\ \frac{x}{5} + 3 - \underline{\quad} &= 7 - \underline{\quad} \\ \frac{x}{5} &= \underline{\quad} \\ \frac{x}{5} \times \underline{\quad} &= 4 \times 5 \\ x &= \underline{\quad}\end{aligned}$$

- 8 Solve each of these equations for x .

- a $\frac{x}{2} + 3 = 8$
b $\frac{x}{3} - 1 = 4$
c $\frac{x}{5} + 2 = -3$
d $\frac{x}{6} + 3 = -4$
e $\frac{x}{7} - 2 = 4$
f $\frac{x}{10} - 6 = -1$

- 9 Check the given solution by substitution and state whether or not it is correct.

- a $2x + 8 = 15$ $[x = 7]$
b $7 - 5x = 9$ $[x = -\frac{2}{5}]$
c $-15 = 6 - 7x$ $[x = 3]$
d $\frac{x}{5} - 3 = 6$ $[x = \frac{9}{5}]$



EXAMPLE 3B-5 Solving a linear equation resulting from substitution

Given $y = 5x - 3$, find x when $y = -18$.

Solve	Think	Apply
$y = 5x - 3$ $-18 = 5x - 3$ $-18 + 3 = 5x - 3 + 3$ $-15 = 5x$ $\frac{-15}{5} = \frac{5x}{5}$ $x = -3$	First, substitute the value for y, giving an equation that can be solved. Add 3 to both sides, simplify, then divide both sides by 5.	A single equation with two different pronumerals can't be solved unless one value is known. Substitute that value and solve the resulting equation in the usual way.

- 10 Given $y = 7x + 11$, complete the following to find x when $y = -3$.

$$\begin{aligned}\underline{\quad} &= 7x + 11 \\ -3 - \underline{11} &= 7x + 11 - \underline{\quad} \\ \underline{\quad} &= 7x \\ \frac{-14}{7} &= \frac{7x}{\square} \\ x &= \underline{\quad}\end{aligned}$$

- 11 a Given $y = 3x - 5$, find x when $y = -14$.
c Given $y = 7 - 5x$, find x when $y = 8$.
e Given $y = 5 - 7x$, find x when $y = -5$.

- b Given $y = 4x + 2$, find x when $y = 11$.
d Given $y = 4 - 3x$, find x when $y = -3$.
f Given $y = 3x - 5$, find x when $y = 0$.

EXAMPLE 3B-6 Developing and solving an equation from a worded description

When five is subtracted from twice a number the answer is three. Write an equation and solve it to find the original number

Solve	Think	Apply
Let the original number be x. The equation becomes: $2x - 5 = 3$ $2x - 5 + 5 = 3 + 5$ $2x = 8$ $\frac{2x}{2} = \frac{8}{2}$ $x = 4$ The original number is 4.	Twice the number is $2x$. Subtracting 5 gives $2x - 5$. Form the equation using $2x - 5$ and 3. Now solve the equation. Add 5 to both sides. Divide both sides by 2. Checking: $2 \times 4 - 5 = 3$, therefore it is correct.	Begin by defining the variable so that you can ensure the answer makes sense. Form the equation using the information in the question. After solving, make sure the answer makes sense in the context of the question.

12 By first forming an equation, solve each of the following worded questions.

- When five is subtracted from three times a number, the answer is thirteen. Find the original number.
- When seven is subtracted from four times a number, the answer is thirteen. Find the original number.
- When nine is added to five times a number, the answer is thirty-four. Find the original number.
- When eight is added to seven times a number, the answer is twenty-nine. Find the original number.

13 The sum of two **consecutive integers** is 275.

- If the first number is x , write an algebraic expression for the second number.
- Write an equation in terms of x to represent this problem.
- Solve the equation and find the two numbers.

consecutive integers

whole numbers that follow one another in order, with a difference of 1 between them (e.g. 7 and 8; -4, -3 and -2)

14 For each of these problems, form an equation to solve.

- Two consecutive integers add to 127. Find the numbers.
- Three consecutive integers have a sum of 27. Find the smallest of the three numbers.
- Four consecutive integers have a sum of -6. Find the largest of them.
- Three consecutive even integers have a sum of 132. Find the smallest of the three numbers.

15 Tyler is saving to buy a cricket bat that costs \$235. He is able to save \$28 per week and currently has \$67. Follow the steps below to work out when he will be able to buy the bat.

- Choose a pronumeral to represent the unknown quantity in the problem.
- Use the pronumeral to write an equation to represent the problem.
- Solve the equation and write your answer.

16 For each of these problems, form an equation to solve. Remember to clearly define the pronumeral you use to represent the unknown variable.

- Rachel bought three equally-priced dresses online for a total cost of \$196, which included the delivery charge of \$19. What is the cost of each dress?



- b** The cost of hiring a tennis court is \$9 plus \$14 per hour. For how many hours can you hire the tennis court if you and your friends have a total of \$85?
 - c** Andrew and Todd scored a total of 37 goals between them in a basketball match. Todd scored five more goals than Andrew. How many goals did Andrew score?
 - d** Tina has a budget of \$1200 for her birthday party. The cost of hiring a local hall is \$325 and catering is \$28 per person. What is the maximum number of people that can attend?
- 17** The length of a rectangular playing field is 14 m longer than its width.
- a** If w represents the width of the field, write an expression for the length of the field in terms of w .
 - b** Form an equation for the perimeter of the field in terms of w .
 - c** If the perimeter of the field is 156 m, solve an equation to find the dimensions of the playing field.
- 18** In an AFL match, players can score points for their team in two ways. They can kick a goal, which is worth six points, and they can score a behind, which is worth one point.
- a** Using appropriate variables, write an algebraic equation to calculate the total number of points scored during an AFL game. Remember to carefully define your variables.
 - b** Calculate the number of points scored if a team kicks 12 goals and 3 behinds.
 - c** Calculate the number of points scored by both teams in each of the following games and identify the winner. Show all working.
 - i** *Kangaroos*: 6 goals and 12 behinds, *Swans*: 7 goals and 8 behinds
 - ii** *Eagles*: 23 goals and 3 behinds, *Suns*: 17 goals and 25 behinds
 - d** The scoring in the following games is incomplete. Calculate the number of behinds scored in each of the games. Show all working.
 - i** *Cats*: total points was 97, number of goals was 15
 - ii** *Power*: total points was 128, number of goals was 19
 - e** The scoring for the games below is also incomplete. Calculate the number of goals scored in each of the games. Show all working.
 - i** *Saints*: total points was 87, number of behinds was 9
 - ii** *Hawks*: total points was 131, number of behinds was 17
 - f** There has been some discussion about the possibility of changing the scoring system so that players can receive three points if they hit the post instead of kicking a goal. How would this change your equation?
 - g** If the change described in part **f** was implemented and a team scored 15 goals and 14 behinds, and hit the post a number of times for a total score of 119 points, how many times did the ball hit the goalpost?

- 19** For each of these problems, form an equation to solve.
- a** When four is subtracted from one-third of a number, the answer is eight. What was the original number?
 - b** When six is subtracted from one-fifth of a number, the answer is seven. Find the original number.
 - c** A box of chocolates contains 24 chocolates that are mint, caramel or strawberry. There are four more mint than caramel and twice as many strawberry as caramel. How many of the chocolates are caramel?
- 20** Hayley, Davina and Marcel go fishing. Davina catches four more fish than Marcel, and Hayley catches twice as many fish as Marcel.
- a** Form an equation to find the number of fish each person catches. Explain why this problem cannot be solved.
 - b** Hayley believes they caught a total of 65 fish. Does this seem reasonable? Explain.

3C Solving further linear equations

These resources are available on your obook assess:

- **Interactive 3C.1:** Explore key ideas for solving more linear equations
- **Interactive 3C.2:** Explore equations with an unknown on both sides
- **Video tutorial 3C:** Watch and listen to an explanation of Example 3C-3
- **Spreadsheet 3C:** Solve linear equations
- **Worksheet 3C:** Practise your skills with extra problems for solving equations
- **Investigation 3C:** Solve linear equations relating to costs at a cinema
- **assess quiz 3C:** Test your skills with an auto-correcting multiple-choice quiz

To solve equations involving algebraic fractions, multiply both sides by the lowest common multiple (LCM) of the denominators.

EXAMPLE 3C-1 Solving linear equations involving fractions

Solve the following for x .

a $\frac{x}{3} = \frac{2}{5}$

b $\frac{4}{7} = \frac{x}{3}$

	Solve	Think	Apply
a	$\frac{x}{3} = \frac{2}{5}$ $\frac{x}{3} \times 15 = \frac{2}{5} \times 15$ $5x = 6$ $\frac{5x}{5} = \frac{6}{5}$ $x = \frac{6}{5}$ $\text{or } x = 1\frac{1}{5}$	The LCM of 3 and 5 is 15 so multiply both sides of the equation by 15, to remove fractions. Divide both sides by 5 to solve.	To solve equations containing more than one fraction, multiply both sides by the LCM of the denominators to remove the fractions. Alternatively, multiply both sides by the product of the denominators. Solve the resulting equation in the normal manner.
b	$\frac{4}{7} = \frac{x}{3}$ $\frac{4}{7} \times 21 = \frac{x}{3} \times 21$ $12 = 7x$ $\frac{12}{7} = \frac{7x}{7}$ $x = \frac{12}{7}$ $\text{or } x = 1\frac{5}{7}$	The LCM of 7 and 3 is 21 so multiply both sides of the equation by 21. Divide both sides by 7 to solve.	

EXERCISE 3C Solving further linear equations

- 1 Complete the following to solve for x .

$$\begin{aligned}\frac{x}{5} &= \frac{7}{8} \\ \frac{x}{5} \times 40 &= \frac{7}{8} \times \underline{\hspace{2cm}} \\ 8x &= \underline{\hspace{2cm}} \\ \frac{8x}{8} &= \frac{35}{8} \\ \underline{\hspace{1cm}} &= \underline{\hspace{1cm}} \\ x &= \underline{\hspace{2cm}}\end{aligned}$$

- 2 Solve the following for x .

a $\frac{x}{3} = \frac{5}{2}$

b $\frac{x}{7} = \frac{1}{3}$

c $\frac{x}{5} = \frac{1}{2}$

d $\frac{x}{9} = \frac{2}{5}$

e $\frac{x}{7} = \frac{3}{4}$

f $\frac{x}{5} = \frac{2}{3}$

EXAMPLE 3C-2 Solving more linear equations involving fractions

Solve for x .

a $\frac{4x+3}{5} = -2$

b $\frac{1}{3}(2x-1) = -4$

	Solve	Think	Apply
a	$\frac{4x+3}{5} = \frac{-2}{1}$ $\frac{4x+3}{5} \times 5 = \frac{-2}{1} \times 5$ $4x+3 = -10$ $4x+3 - 3 = -10 - 3$ $4x = -13$ $x = -\frac{13}{4}$ $\text{or } x = -3\frac{1}{4}$	Multiply both sides of the equation by 5 to eliminate fractions. Subtract 3 from both sides of the equation, and then divide both sides by 4.	When solving equations involving fractions, multiply both sides by the same number, then expand any brackets and simplify. Solve the resulting equation which should not contain any fractions.
b	$\frac{1}{3}(2x-1) = -4$ $\frac{1}{3}(2x-1) \times 3 = -4 \times 3$ $2x-1 = -12$ $2x-1 + 1 = -12 + 1$ $2x = -11$ $x = -\frac{11}{2}$ $\text{or } x = -5\frac{1}{2}$	Multiply both sides of the equation by 3, then expand the brackets. Add 1 to both sides of the equation, and then divide both sides by 2.	

- 3 Complete the following to solve for x .

a $\frac{5x-2}{3} = 4$

$$\frac{5x-2}{3} \times \underline{\hspace{2cm}} = 4 \times \underline{\hspace{2cm}}$$

$$5x-2 = \underline{\hspace{2cm}}$$

$$5x-2 + \underline{\hspace{2cm}} = 12 + \underline{\hspace{2cm}}$$

$$5x = \underline{\hspace{2cm}}$$

$$x = \underline{\hspace{2cm}}$$

b $\frac{1}{4}(3x+1) = 5$

$$\frac{1}{4}(3x+1) \times \underline{\hspace{2cm}} = 5 \times \underline{\hspace{2cm}}$$

$$3x + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$3x + 1 - \underline{\hspace{2cm}} = 20 - \underline{\hspace{2cm}}$$

$$3x = \underline{\hspace{2cm}}$$

$$x = \underline{\hspace{2cm}}$$

- 4 Solve the following for x .

a $\frac{x-2}{4} = -3$

d $\frac{2x+7}{3} = -1$

g $\frac{1-2x}{2} = 0$

b $\frac{x-1}{2} = 8$

e $\frac{1}{2}(3x+1) = -3$

h $\frac{1}{5}(4-3x) = -2$

c $\frac{x+5}{3} = -5$

f $\frac{1+2x}{7} = 7$

i $\frac{1}{4}(5-2x) = -1$

EXAMPLE 3C-3 Solving linear equations with a pronumeral term on both sides

Solve these equations.

a $5x + 2 = 3x - 5$

b $15 - 2x = 11 + x$

	Solve	Think	Apply
a	$5x + 2 = 3x - 5$ $5x + 2 - 3x = 3x - 5 - 3x$ $2x + 2 = -5$ $2x + 2 - 2 = -5 - 2$ $2x = -7$ $\frac{2x}{2} = \frac{-7}{2}$ $x = -\frac{7}{2}$ $\text{or } x = -3\frac{1}{2}$	First, subtract $3x$ from both sides of the equation. Then, subtract 2 from both sides. Finally, divide both sides by 2.	When solving equations with pronumerals on both sides, it may be necessary to not only add and subtract numbers from both sides, but also to add and subtract pronumerals from both sides. The first step is to add or subtract the pronumerals to move them to one side, it does not matter which side. Next add or subtract to move the numbers to the other side of the equation from the pronumerals. Then solve as normal by multiplying or dividing.
b	$15 - 2x = 11 + x$ $15 - 2x + 2x = 11 + x + 2x$ $15 = 11 + 3x$ $15 - 11 = 11 + 3x - 11$ $4 = 3x$ $\frac{4}{3} = \frac{3x}{3}$ $x = \frac{4}{3}$ $\text{or } x = 1\frac{1}{3}$	First, add $2x$ to both sides of the equation. Then, subtract 11 from both sides. Finally, divide both sides by 3.	

5 Complete to solve the following.

a

$$4x - 2 = 4 + 2x$$

$$4x - 2 - \underline{\hspace{1cm}} = 4 + 2x - 2x$$

$$2x - 2 = 4$$

$$2x - 2 + \underline{\hspace{1cm}} = 4 + 2$$

$$2x = \underline{\hspace{1cm}}$$

$$\frac{2x}{\square} = \frac{6}{2}$$

$$x = \underline{\hspace{1cm}}$$

b

$$7 - 3x = 5x - 9$$

$$7 - 3x - 5x = 5x - 9 - \underline{\hspace{1cm}}$$

$$7 - \underline{\hspace{1cm}} = -9$$

$$7 - 8x - \underline{\hspace{1cm}} = -9 - \underline{\hspace{1cm}}$$

$$\underline{\hspace{1cm}} = -16$$

$$\frac{-8x}{-8} = \frac{-16}{\square}$$

$$x = \underline{\hspace{1cm}}$$

6 Solve the following equations with integer solutions.

a $6x + 2 = 2x + 18$

b $3x + 7 = 12 - 2x$

c $5 + x = 9 - 3x$

d $4x - 4 = 5x - 1$

e $3 - x = 2x + 9$

f $9 - 2x = 3 - 5x$

g $3x - 12 = x + 6$

h $5x - 9 = 11 + 7x$

i $3x - 5 = 10 - 2x$

7 Solve these equations.

a $6x + 7 = 4x - 2$

b $5x + 3 = 2x + 7$

c $5 + 2x = 11 - 3x$

d $2x - 3 = 5x + 7$

e $3 + x = 17 + 5x$

f $15 - 3x = 2 - 5x$

g $2x + 5 = 9 - 3x$

h $4x - 5 = 5x + 9$

i $5 - 7x = 3x + 7$

j $5a + 3 = a - 3$

k $4 - 3s = 2s + 15$

l $9x - 4 = 3 + 5x$

m $11a - 7 = 5a + 15$

n $3y - 5 = -14 - 5y$

o $7p = 15 - 4p$

EXAMPLE 3C-4 Checking solutions to linear equations

Substitute the value given in square brackets to check whether it is the solution to each equation.

a $2x - 5 = 10 - 3x$ $[x = 3]$

b $5x + 2 = 2x - 7$ $[x = 2]$

	Solve	Think	Apply
a	$2x - 5 = 2(3) - 5$ $= 6 - 5$ $= 1$ $10 - 3x = 10 - 3(3)$ $= 10 - 9$ $= 1$ $x = 3$ is the solution.	Substitute $x = 3$ into both sides of the equation. Both sides of the equation are equal to 1, so $x = 3$ is the solution.	When checking a solution, substitute the value into the equation and check that the two sides of the equation are equal. If they are not equal, the given value is not the solution.
b	$5x + 2 = 5(2) + 2$ $= 10 + 2$ $= 12$ $2x - 7 = 2(2) - 7$ $= 4 - 7$ $= -3$ $x = 2$ is <i>not</i> the solution.	Substitute $x = 2$ into both sides of the equation. One side is 12 and the other side is -3 , so $x = 2$ is not the solution.	

- 8** Complete to show that $x = 3$ is the solution of $7 - 3x = x - 5$.

$$7 - 3x = 7 - 3(\underline{\quad}) = 7 - \underline{\quad} = \underline{\quad}$$

$$x - 5 = \underline{\quad} - 5 = -2$$

$$x = 3 \underline{\quad} \text{ the solution.}$$

- 9** Substitute the value given in square brackets to check whether it is the solution to the equation each time.

a $3x + 9 = 4 - 2x$ $[x = -1]$

b $9a + 2 = 7a - 4$ $[a = -3]$

c $7a - 5 = 3 - a$ $[a = 2]$

d $15 - 2x = 6 + x$ $[x = 3]$

e $2x - 3 = 7 - 4x$ $[x = 1\frac{2}{3}]$

f $5x - 7 = 3 + x$ $[x = 3\frac{1}{2}]$

EXAMPLE 3C-5 Solving a problem by first forming an equation to solve

If one more than twice a number is five more than the number, what is the number?

Solve	Think	Apply
Let the number be x . The equation becomes: $2x + 1 = x + 5$ $2x + 1 - x = x + 5 - x$ $x + 1 = 5$ $x + 1 - 1 = 5 - 1$ $x = 4$ The number is 4.	$2x + 1$ is one more than twice the number. $x + 5$ is five more than the number. Form the equation using $2x + 1$ and $x + 5$.	Carefully define the pronumeral. Form an equation and solve it.

- 10** Form an equation and then solve it to find the number in each of these problems.
- Three more than twice a number is seven more than the number.
 - Eight more than three times a number is four more than the number.
 - Six more than twice a number is four more than four times the number.
 - Nine less than five times a number is three less than twice the number.
- 11** A formula for converting temperature is $C = \frac{5}{9}(F - 32)$ where C represents the temperature in degrees Celsius ($^{\circ}\text{C}$) and F represents the temperature in degrees Fahrenheit ($^{\circ}\text{F}$).
- For $C = 30$, write the formula as a linear equation involving the variable F .
 - Solve your linear equation to find F .
 - What is the equivalent temperature in degrees Fahrenheit for a temperature of 30°C ?
 - Solve a linear equation to find the equivalent temperature in degrees Fahrenheit for a temperature of 18°C .
 - Which temperature is higher: 25°C or 76°F ? Show steps of working to justify your answer.
- 12** The number of biscuits in a packet is unknown. However, Lisa can fill the family biscuit barrel with six more biscuits than the contents of two packets, or five less biscuits than the contents of three packets.
- If n represents the number of biscuits in a packet, which equation (A, B, C or D) best matches this situation?
 - $2n - 6 = 3n - 5$
 - $2n - 6 = 3n + 5$
 - $2n + 6 = 3n - 5$
 - $2n + 6 = 3n + 5$
 - Solve the equation to find the number of biscuits in a packet.
- 13** Kristen and Daniel have the same amount of money. Kristen buys 5 kg of cherries and has \$2 left over. Daniel buys 2 kg of cherries and has \$20 left over.
- Write an equation to represent this situation.
 - Solve the equation to find the cost of 1 kg of cherries.
- 14** James and Sue have the same amount of money. James buys seven sushi rolls and has \$1.50 left over. Sue buys four sushi rolls and has \$12 left over. Form an equation and then solve it to find the cost of a sushi roll.
- 15** The average of Hayden's four test results for the semester is 78%. He remembers that three of his test results were 91%, 69% and 88%. Form an equation and then solve it to work out the result he received in the fourth test.
- 16** For three consecutive even integers, the sum of the two smaller numbers is equal to six more than the largest number. Form an equation and then solve it to find the integers.



- 17** Use a spreadsheet program to solve the equation $3x - 7 = 9 - x$, given that the solution is an integer. The spreadsheet needs to have three columns labelled x , $3x - 7$ and $9 - x$. In cell A1 enter x , in cell B1 enter $3x - 7$, in cell C1 enter $9 - x$, in cell A2 enter 0, in cell B2 enter $=A2*3-7$ and in cell C2 enter $=9-A2$.

	A	B	C
1	x	$3x - 7$	$9 - x$
2	0	-7	9
3	1		
4	2		
5	3		
6	4		
7	5		
8	6		

Use the **fill down** command to find the values for each side of the equation. The answer is the value of x that gives the same value in columns B and C.

- 18** Change the spreadsheet in question 17 to solve each of these equations with positive integer solutions.

a $5x - 3 = 53 - 2x$ **b** $3x + 5 = 35 - 2x$ **c** $19 - 2x = 7x - 44$
d $6x + 11 = 41 - 4x$ **e** $3x - 17 = 33 - 7x$ **f** $9x + 15 = 79 - 7x$

- 19** Use a spreadsheet to solve these equations with negative integer solutions.

a $4x - 3 = 13 + 6x$ **b** $7x - 3 = -25 + 5x$ **c** $8 - 7x = 2 - 8x$

- 20** Explain how to modify the spreadsheet used in questions 18 and 19 to solve equations that do not have integer solutions.

- 21** Solve these equations using a spreadsheet.

a $3x - 7 = 6 - 2x$ **b** $5x + 23 = 2x - 8$ **c** $8 - 7x = 8x + 59$

- 22** Solve the following problems by first forming an equation.

- a** A man is currently three times as old as his son. In 11 years from now he will be twice as old as his son will be then. How old is his son now?
b At present Jen is 8 years older than Wesley. In 3 years from now, Jen's age will be double Wesley's age. How old is Wesley?
c The sum of Kelly and Sam's ages is 45 years. If Kelly's age was doubled, it would be 5 years more than three times Sam's age. How old is Sam?

- 23** Solve the following linear equations for x .

a $\frac{4x+1}{3} = \frac{1}{2}$ **b** $\frac{3x+1}{5} = \frac{2}{3}$ **c** $\frac{7x-2}{4} = -\frac{2}{5}$
d $\frac{2x+1}{5} = \frac{3}{4}$ **e** $\frac{3x+1}{4} = \frac{3}{5}$ **f** $\frac{3x+2}{5} = x - \frac{1}{3}$
g $\frac{1-x}{5} = x + \frac{1}{3}$ **h** $\frac{x+1}{2} = \frac{x}{7}$ **i** $\frac{2x-1}{4} = \frac{3x}{5}$

3D Finding the value of a variable in a formula

These resources are available on your obook assess:

- **Worksheet 3D:** Practise your skills with extra problems for solving equations
- **Investigation 3D.1:** Write and solve equations relating to a circus scenario
- **Investigation 3D.2:** Investigate the concept of a balanced linear equation
- **assess quiz 3D:** Test your skills with an auto-correcting multiple-choice quiz

We know that a formula links two or more variables according to a rule. Each variable can be represented by a pronumeral. The pronumeral that is expressed in terms of the others is called the subject of the formula. If the pronumeral to be found is the subject of the formula, its value is found directly by substitution as in Topic 3A.

If the pronumeral to be found is not the subject of the formula, there are two methods for finding its value:

- 1 substitute the known values into the formula and solve the resulting equation
- 2 rearrange the formula, so that the subject of the formula is the pronumeral to be found and then substitute the known values. To rearrange the formula to change its subject, we use the same rules as those used for solving equations.

EXAMPLE 3D-1 Substituting known values into a formula and solving the resulting equation

a Find the value of t when $v = 117$, $u = 5$, $a = 8$ and $v = u + at$.

b Find the value of N when $R = 23$, $I = 4$ and $R = \frac{I}{N} + 1$.

	Solve	Think	Apply
a	$v = u + at$ $117 = 5 + 8t$ $112 = 8t$ $\frac{112}{8} = t$ $t = 14$	In the formula, replace v with 117, u with 5 and a with 8. Solve the resulting equation by adding 5 to both sides and then dividing both sides by 8.	Substitute the given values into the formula and solve the resulting equation.
b	$R = \frac{I}{N} + 1$ $23 = \frac{4}{N} + 1$ $22 = \frac{4}{N}$ $22N = 4$ $N = \frac{4}{22}$ $N = \frac{2}{11}$	Replace R with 23 and I with 4. Solve the resulting equation by subtracting 1 from both sides, multiplying both sides by N and then dividing both sides by 22.	

EXERCISE 3D

Finding the value of a variable in a formula

UNDERSTANDING, FLUENCY AND COMMUNICATING

- 1 If $v = u + at$ find:
 - a t when $v = 102$, $u = 18$ and $a = 7$
 - b a when $v = 54$, $u = 12$ and $t = 14$
- 2 Use the formula $d = \frac{1}{2}ct$ to find the value of:
 - a c when $d = 100$ and $t = 8$
 - b t when $d = 320$ and $c = 16$
- 3 If $M = \frac{5k}{18}$, find k when $M = 15$.
- 4 If $V^2 = gR$, find R when $V = 12$ and $g = 10$.
- 5 If $b = \frac{m}{h^2}$, find m when $b = 23$ and $h = 1.63$.
- 6 Use the formula $I = Prn$ to find the value of:
 - a P when $I = 19\,500$, $r = 0.03$ and $n = 25$
 - b n when $I = 2100$, $P = 5000$ and $r = \frac{6}{100}$
- 7 Given $s = ut + \frac{1}{2}at^2$, find u when $s = 360$, $t = 8$ and $a = 10$.
- 8 If $S = \frac{n}{2}(a + l)$, find a when $S = 560$, $n = 20$ and $l = 53$.
- 9 If $S = \frac{n}{2}[2a + (n - 1)d]$, find the value of a when $S = 610$, $d = 3$ and $n = 20$.
- 10 Use the formula $A = 180 - \frac{360}{n}$ to find n when $A = 120$.
- 11 Use the formula $a = \frac{2Rn}{n + 1}$ to find the value of:
 - a R when $a = 12$ and $n = 24$
 - b R when $a = 19.5$ and $n = 39$
- 12 Use the formula $I = \frac{E}{R + r}$ to find the value of:
 - a E when $I = 8$, $R = 15$ and $r = 3$
 - b R when $I = 2$, $E = 24$ and $r = 3$

EXAMPLE 3D-2 Changing the subject of a formula (one step with two pronumerals)

Make x the subject of each formula below.

a $x + 3 = y$

b $3x = p$

c $\frac{x}{2} = t$

d $x - 5 = r$

Solve				
a		b	c	d
$x + 3 = y$ $x + 3 - 3 = y - 3$ $x = y - 3$		$3x = p$ $\frac{3x}{3} = \frac{p}{3}$ $x = \frac{p}{3}$	$\frac{x}{2} = t$ $\frac{x}{2} \times 2 = t \times 2$ $x = 2t$	$x - 5 = r$ $x - 5 + 5 = r + 5$ $x = r + 5$
Apply				
Apply the rules for solving equations to rearrange the formula so that x is the subject.				

13 Make x the subject of each of these formulas.

a $x + 2 = y$

b $x + 5 = y$

c $y = x + 8$

d $5x = p$

e $7x = q$

f $r = 2x$

g $\frac{x}{3} = t$

h $\frac{x}{5} = r$

i $m = \frac{x}{8}$

j $x - 3 = p$

k $x - 7 = m$

l $z = x - 1$

EXAMPLE 3D-3 Changing the subject of a formula (one step with three pronumerals)

Make x the subject of each formula below.

a $x + a = b$

b $x - k = t$

c $mx = z$

d $\frac{x}{w} = p$

Solve		
$x + a = b$ $x + a - a = b - a$ $x = b - a$	b	$x - k = t$ $x - k + k = t + k$ $x = t + k$
$mx = z$ $\frac{mx}{m} = \frac{z}{m}$ $x = \frac{z}{m}$	d	$\frac{x}{w} = p$ $\frac{x}{w} \times w = p \times w$ $x = pw$
Apply		
Apply the rules for solving equations to rearrange the formula so that x is the subject.		

14 Make x the subject of each formula below.

a $x + y = z$

b $x + n = m$

c $x - p = q$

d $x - k = n$

e $ax = c$

f $dx = e$

g $\frac{x}{v} = w$

h $\frac{x}{k} = t$

i $p = x + q$

j $m = x - k$

k $a = bx$

l $t = \frac{x}{b}$

EXAMPLE 3D-4 Changing the subject of a formula (one step with four pronumerals)

Make x the subject of each formula.

a $\frac{x}{a} = \frac{b}{c}$

b $\frac{z}{x} = \frac{w}{y}$

	Solve	Think	Apply
a	$\frac{x}{a} = \frac{b}{c}$ $\frac{x}{a} \times a = \frac{b}{c} \times a$ $x = \frac{ba}{c}$ or $\frac{ab}{c}$	Multiply both sides of the equation by a and simplify.	Apply the rules for solving equations to rearrange the formula so that x is the subject.
b	$zx = \frac{w}{y}$ $\frac{zx}{1} \times \frac{1}{z} = \frac{w}{y} \times \frac{1}{z}$ $x = \frac{w}{yz}$	Divide both sides of the equation by z and simplify. To divide by z , multiply by its reciprocal $\frac{1}{z}$.	

15 Make x the subject of each formula below.

a $\frac{x}{m} = \frac{p}{q}$

b $\frac{x}{t} = \frac{k}{m}$

c $\frac{x}{y} = \frac{a}{b}$

d $ax = \frac{k}{t}$

e $zx = \frac{d}{e}$

f $kx = \frac{m}{n}$

EXAMPLE 3D-5 Changing the subject of a formula (two steps)

Make x the subject of each formula below.

a $y = 2x + 3$

b $m = 4 - 3x$

c $y = mx + b$

	Solve	Think	Apply
a	$2x + 3 = y$ $2x + 3 - 3 = y - 3$ $2x = y - 3$ $\frac{2x}{2} = \frac{y - 3}{2}$ $x = \frac{y - 3}{2}$	Subtract 3 from both sides of the equation, then divide both sides by 2.	Apply the rules for solving equations to rearrange the formula so that x is the subject.
b	$4 - 3x = m$ $4 - 3x - 4 = m - 4$ $-3x = m - 4$ $\frac{-3x}{-3} = \frac{m - 4}{-3}$ $x = \frac{-(m - 4)}{3}$ $x = \frac{-m + 4}{3}$	Subtract 4 from both sides of the equation, then divide both sides by -3 . The expression $-(m - 4)$ is the same as $-1(m - 4)$ or $-1 \times m - 1 \times -4$ or $-m + 4$.	
c	$mx + b = y$ $mx + b - b = y - b$ $mx = y - b$ $\frac{mx}{m} = \frac{y - b}{m}$ $x = \frac{y - b}{m}$	Subtract b from both sides of the equation, then divide both sides by m .	

16 Make x the subject of each formula below.

a $y = 3x + 2$

b $y = 5x + 1$

c $y = 2x + 5$

d $y = 7x - 4$

e $y = 6x - 5$

f $y = 3x - 5$

g $m = 3 + 2x$

h $m = 5 + 3x$

i $m = 7 + 5x$

j $m = 6 - 5x$

k $m = 4 - 3x$

l $m = -8 - 6x$

m $y = mx + 4$

n $y = mx - 3$

o $y = mx + c$

p $y = mzx$

17 Make y the subject of each of these equations.

a $a + by = c$

b $p + qy = r$

c $xy - p = r$

d $a - b = cy$

e $x - ky = t$

f $c = 4 - ry$

g $a + by + r = 0$

h $k = r - py$

i $r + px = qy$

18 Given $R = \frac{I}{N} + 1$, find the value of N when:

a $R = 3$ and $I = 7$

b $R = 11$ and $I = 5$

19 Use $t = \sqrt{\frac{l}{g}}$ to find l when $t = 1.3$ and $g = 10$.

20 Use $r = \sqrt[3]{\frac{V}{k}}$ to find V when $r = 3$ and $k = 4.2$.

21 Use the formula $E = \frac{1}{2}mv^2$ to find the value of v when $E = 135$ and $m = 2.8$. Give your answer correct to one decimal place.

3E Travel calculations

These resources are available on your obook assess:

- **Video tutorial 3E:** Watch and listen to an explanation of Example 3E-1
- **Investigation 3E:** Investigate travel calculations for constant speed
- **assess quiz 3E:** Test your skills with an auto-correcting multiple-choice quiz

oi
ci

average speed

calculated by dividing the distance travelled during a time interval by the length of the time interval; e.g. average speed of a car that travels 120 km in 2 hours is 60km/h

total stopping distance

total distance travelled in the time it takes for a driver to react, apply the brakes and for the car to come to a stop

reaction-time distance

distance travelled in the time it takes a driver to react and move his or her foot to apply the brakes

braking distance

distance a motor vehicle travels between the time the brakes are first applied and when the vehicle comes to a complete stop

The formula for the **average speed** of an object is given below. This formula is used to calculate the average speed, distance and time travelled.

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

$$\text{This is usually written as } S = \frac{D}{T}$$

where S is the average speed, D is the distance travelled and T is the time taken.

$$\text{So it follows that } D = S \times T, \text{ and } T = \frac{D}{S}.$$

When drivers react to situations in which they need to stop a motor vehicle, the **total stopping distance** of the vehicle depends on the distance travelled while the driver reacts and the distance travelled after applying the brakes.

The distance a car travels in the time it takes to stop is:

$$\text{Total stopping distance} = \text{reaction-time distance} + \text{braking distance}$$

The reaction-time distance is the distance travelled in the time it takes a driver to react to a situation; that is, to realise there is a problem and move his or her foot to the brake. The usual reaction time, for drivers unaffected by alcohol, drugs or fatigue has been found to be about 2.5 s.

The braking distance is the distance a car travels after the brakes have been applied. This distance depends on (the square of) the speed of the car.

Note: Factors such as the condition of the road (wet or dry) and the condition of the car's brakes and tyres, as well as the state of the driver, all have an effect on the total stopping distance.



The braking distance is a function of the square of the speed of the car. For a car with good brakes and tyres, travelling in dry conditions on a good road, the relationship can be approximated by the formula $d = 0.01v^2$, where d is the braking distance in metres and v is the speed of the car in km/h.

For the same car travelling on a slippery road, the formula for braking distance becomes $d = 0.014v^2$.

UNDERSTANDING, FLUENCY AND COMMUNICATING

	Solve	Think	Apply
a	$S = \frac{232 \text{ km}}{4.283 \text{ 33... h}}$ $= 54.16... \text{ km/h}$ $= 54 \text{ km/h (to the nearest whole number)}$	Change 17 minutes to hours. $17 \text{ min} = (17 \div 60) \text{ h} = 0.283 \text{ 33... h}$ $4 \text{ h } 17 \text{ min} = 4.283 \text{ 33... hours}$ Or use the degrees, minutes, seconds key on your calculator.	$S = \frac{D}{T}$
b	$D = 83 \text{ km/h} \times 2.4 \text{ h}$ $= 199.2 \text{ km}$	Change 24 minutes to hours. $24 \text{ min} = (24 \div 60) \text{ h} = 0.4 \text{ h}$	$D = S \times T$
c	$T = \frac{34 \text{ km}}{52 \text{ km/h}}$ $= 0.6538... \text{ hours}$ $= 39 \text{ min (to the nearest minute)}$	$0.6538... \text{ hours} = (0.653 \text{ 8...} \times 60) \text{ min}$ $= 39.23... \text{ min}$	$T = \frac{D}{S}$

- 1** Calculate the average speed for these distances and times.
 - a** 185 km is travelled in 4 h
 - b** 720 km is travelled in 9 h and 50 min
 - c** 154 km is travelled in 3 h and 15 min
 - d** 272 km is travelled in 4 h and 35 min
- 2**
 - a** Calculate the distance travelled in 3 h and 30 min at an average speed of 64 km/h.
 - b** Calculate the distance travelled in 3 h and 20 min at an average speed of 56 km/h.
 - c** Calculate the distance travelled in 5 h and 47 min at an average speed of 82 km/h.
 - d** Calculate the distance travelled in 2 h and 13 min at an average speed of 75 km/h.
- 3** How long will it take to travel:
 - a** 486 km at 60 km/h?
 - b** 298 km at 74 km/h?
 - c** 365 km at 82 km/h?
 - d** 88 km at 95 km/h?

Convert:

	Solve	Think	Apply
a	$65 \text{ km/h} = \frac{65 \times 1000}{60 \times 60} \text{ m/s}$ $= 18.1 \text{ m/s}$ (to one decimal place)	$65 \text{ km} = (65 \times 1000) \text{ m}$ and $1 \text{ h} = (60 \times 60) \text{ s}$	Change kilometres to metres (by multiplying by 1000), change hours to seconds (by multiplying by 60×60) and divide.
b	$9.8 \text{ m/s} = \frac{9.8 \times 60 \times 60}{1000} \text{ km/h}$ $= 35.28 \text{ km/h}$	$9.8 \text{ m in } 1 \text{ s}$ $= (9.8 \times 60) \text{ m in } 1 \text{ min}$ $= (9.8 \times 60 \times 60) \text{ m in } 1 \text{ h}$ Divide by 1000 to change metres into kilometres.	Change m/s to m/h by multiplying by 60×60 . Change metres to kilometres by dividing by 1000.

- 4 Complete the following to convert the units of speed.

a $70 \text{ km/h} = \frac{70 \times \square}{\square \times \square}$

b $8 \text{ m/s} = \frac{8 \times \square \times \square}{\square}$

= ____ m/s (to one decimal place)

= ____ km/h

- 5 Convert each of the following to m/s, correct to one decimal place.

a 45 km/h

b 76 km/h

c 110 km/h

- 6 Convert each of the following to km/h.

a 15 m/s

b 12.5 m/s

c 25 m/s

EXAMPLE 3E-3 Calculating reaction-time distance

Calculate the reaction-time distance for a car travelling at 60 km/h. Assume a reaction time of 2.5 s.

Solve	Think	Apply
Reaction-time distance $= \frac{60 \times 1000}{60 \times 60} \times 2.5$ $\approx 42 \text{ m}$	$60 \text{ km/h} = \frac{60 \times 1000}{60 \times 60} \text{ m/s}$ Multiply the speed in m/s by the reaction time of 2.5 s.	Convert the speed to m/s and use $D = S \times T$ where $T = 2.5 \text{ s}$. T is the time it takes the driver to react.

- 7 Calculate the reaction-time distance for a car travelling at 80 km/h. Assume a reaction time of 2.5 s.
- 8 Calculate the reaction-time distance for a car travelling at 100 km/h. Assume a reaction time of 2.5 s.
- 9 a Use the results of questions 7 and 8 to draw a (straight-line) graph of the relationship between reaction-time distance (m) and speed (km/h), given a reaction time of 2.5 s.
- b Use the graph to estimate the reaction-time distances for cars travelling at these speeds.
- i 120 km/h ii 45 km/h
- c If the speed of a car increases by 10 km/h, what is the increase in the stopping distance?

EXAMPLE 3E-4 Calculating braking distance

Calculate the braking distance for a car travelling in dry conditions at 60 km/h.

Solve	Think	Apply
Braking distance $= 0.01 \times 60^2$ $= 36 \text{ m}$	Substitute $v = 60$ into the formula $d = 0.01v^2$.	Substitute the value of v into the dry conditions formula.

- 10 Calculate the braking distance for a car travelling at 80 km/h in good conditions.
- 11 a Calculate the braking distance for a car travelling at 100 km/h in good conditions.
- b What is the braking distance for a car travelling at 100 km/h in wet conditions?
- c What is the difference between the braking distances at 100 km/h in good conditions and in wet conditions?

EXAMPLE 3E-5 Calculating total stopping distance

Find the total stopping distance, to the nearest metre, for a car travelling at 70 km/h in good conditions, assuming a reaction time of 2.5 s.

Solve	Think	Apply
Total stopping distance $= \frac{70 \times 1000}{60 \times 60} \times 2.5 + 0.01 \times 70^2$ $= 97.611 \dots \text{ m}$ $= 98 \text{ m (to the nearest metre)}$	Reaction-time distance $= \frac{70 \times 1000}{60 \times 60} \times 2.5 = 48.611 \dots \text{ m}$ Braking distance $= 0.01 \times 70^2 = 49 \text{ m}$	Add the reaction-time distance to the braking distance to find the stopping distance.

- 12** Complete the following to find the total stopping distance for a car travelling at 90 km/h in good conditions, assuming a reaction time of 2.5 s.

$$\text{Stopping distance} = \frac{\square}{\square \times \square} \times 2.5 + 0.01 \times \square^2$$

$$= \square \text{ m (to the nearest metre)}$$

- 13** Find the total stopping distance for a car travelling at 110 km/h in good conditions, assuming a reaction time of 2.5 s.

- 14 a** If a driver affected by fatigue has a reaction time of 3.5 s, what will be the reaction-time distance of a car being driven by this driver at 60 km/h?
- b** For this driver, what is the difference between the stopping distance with a reaction time of 3.5 s and the stopping distance with a reaction time of 2.5 s?
- 15** If a driver is affected by alcohol and has a reaction time of 4.5 s, what difference will this make to the reaction-time distance of a car travelling at 100 km/h compared with the usual reaction time of 2.5 s?
- 16** The reaction-time distance, d m, for a car travelling at v km/h, assuming a reaction time of t s, can be approximated using the formula $d = 0.28vt$. Use this formula to check your answers for questions **14** and **15**.
- 17** If the speed of a car increases from 50 km/h to 60 km/h, what is the increase in total stopping distance?
- 18** Find the stopping distance for a car travelling at 70 km/h in good conditions. Assume a reaction time of 2.5 s, and use the formula $d = 0.7v + 0.01v^2$, where d is the stopping distance (m) and v is the speed (km/h).
- 19 a** Use the formula $d = 0.7v + 0.01v^2$, where d is the stopping distance in metres and v is the speed in km/h, to complete the following table.

Speed (km/h)	0	20	40	60	80	100
Stopping distance (m)		$0.7 \times 20 + 0.01 \times 20^2 = 18$				

- b** Use the information in the completed table from part **a** to graph the relationship between speed and stopping distance. (Join the points with a smooth curve.)
- c** Using your graph from part **b**, estimate the stopping distance for a car travelling at these speeds.
- i** 50 km/h **ii** 120 km/h
- 20** For a driver under the influence of alcohol and driving in poor road conditions, the formula for stopping distance becomes $d = 1.2v + 0.018v^2$.
- a** Prepare a table similar to the one in question **19**. Use the values from your completed table to draw, on the same set of axes as the graph from question **19**, a graph of the relationship between speed and stopping distance for a driver under the influence of alcohol.
- b** Compare your graphs to find the difference in stopping distances at:
- i** 50 km/h **ii** 60 km/h **iii** 110 km/h.

3F Blood alcohol content

These resources are available on your obook assess:

- **Spreadsheet 3F:** Calculate blood alcohol content
- **assess quiz 3F:** Test your skills with an auto-correcting multiple-choice quiz

blood alcohol content (BAC)

percentage concentration of alcohol in a person's blood; e.g. a BAC of 0.05 (%) is 0.05 g of alcohol in every 100 mL of blood

standard drink

any drink that contains 10 g of alcohol

Blood alcohol content (BAC) is a measure of the concentration of alcohol in a person's blood. It is expressed as a percentage mass per unit of volume.

For example, a person with a BAC of 0.02 (%) has $\frac{0.02}{100}$ g of alcohol in every millilitre of their blood. This is equivalent to 0.02 g/100 mL or 20 mg/100 mL.

A BAC measure can be estimated using tables, formulas and on-line calculators; but it is very important to remember that these are only approximations because they are based on average values and do not apply equally to everyone.

A person's BAC reading is affected by factors such as whether the person is male or female, how much the person drank, the length of time that the person spent drinking and the length of time since his or her last drink, the person's weight, whether the person is physically fit, the state of the person's liver, whether the person is a regular drinker, or the person's mood at the time.

The only way to measure a person's BAC accurately is with an approved breath analysing unit known as a 'breathalyser'.

An estimate of a person's BAC can be determined by counting the number of standard drinks consumed. A **standard drink** is any drink that contains 10 g of alcohol. A standard drink always contains the same amount of alcohol irrespective of the container size or the type of drink (beer, wine or spirits).

The number of standard drinks in a container can be calculated using the formula:

$$N = 0.789 \times V \times A$$

where N = number of standard drinks

V = the volume of the container in litres

A = percentage of alcohol (% alc/vol) in the drink. (This is stated on the container.)

Note: 0.789 is the specific gravity of ethyl alcohol, the type of alcohol used in beverages.

EXAMPLE 3F-1 Calculating the number of standard drinks

Calculate the number of standard drinks in a 150 mL glass of red wine, given that the alcohol content of the wine is 14.5% alc/vol.

Solve	Think	Apply
$N = 0.789 \times V \times A$ $= 0.789 \times 0.15 \times 14.5$ ≈ 1.7	The volume of the glass is 150 mL or 0.15 L. So $V = 0.15$. The alcohol content is 14.5% alc/vol. So $A = 14.5$.	Convert the volume of the container to litres and substitute the values of V and A into the formula $N = 0.789 \times V \times A$.

EXERCISE 3F Blood alcohol content

- 1 Complete the following to calculate the number of standard drinks in these beers:
 - a a 375 mL stubby of full-strength beer with an alcohol content of 4.8% alc/vol

$$N = 0.789 \times V \times A$$

$$= 0.789 \times \underline{\quad} \times 4.8$$

$$= \underline{\quad}$$
 - b a 375 mL stubby of light beer with an alcohol content of 2.7% alc/vol

$$N = 0.789 \times V \times A$$

$$= 0.789 \times 0.375 \times \underline{\quad}$$

$$= \underline{\quad}$$
- 2 Calculate (to one decimal place) the number of standard drinks in:
 - a a 120 mL glass of wine with an alcohol content of 12% alc/vol
 - b a 90 mL glass of fortified wine with an alcohol content of 16.5% alc/vol
 - c a 375 mL can of bourbon and coke with an alcohol content of 6% alc/vol
 - d a 750 mL bottle of white wine with an alcohol content of 11.5% alc/vol
 - e a 275 mL bottle of vodka and orange with an alcohol content of 5% alc/vol
 - f a six-pack (6×330 mL) of full strength beer with an alcohol content of 4.9% alc/vol.

An estimate of a person's BAC can be found using these formulas:

$$\text{BAC}_{\text{male}} = \frac{10N - 7.5H}{6.8M} \quad \text{and} \quad \text{BAC}_{\text{female}} = \frac{10N - 7.5H}{5.5M}$$

where N = number of standard drinks consumed

H = number of hours spent drinking

M = person's mass in kg

Note: One standard drink per hour will raise BAC by between 0.01% and 0.03%. A person's BAC will increase at a greater rate if the person:

- is female
- is drinking highly carbonated drinks
- has a low body weight
- is unfit
- has not eaten recently
- has an unhealthy liver.

EXAMPLE 3F-2 Calculating blood alcohol content (BAC)

Calculate, correct to two decimal places, the BAC of:

- a a 78 kg male who has consumed five standard drinks in 3 hours
- b a 46 kg female who has consumed four standard drinks in 4 hours.

	Solve	Think	Apply
a	$\text{BAC}_{\text{male}} = \frac{10N - 7.5H}{6.8M}$ $= \frac{10 \times 5 - 7.5 \times 3}{6.8 \times 78}$ ≈ 0.05	Substitute $N = 5$, $H = 3$ and $M = 78$ into the formula $\text{BAC}_{\text{male}} = \frac{10N - 7.5H}{6.8M}$	Substitute the number of standard drinks consumed, the number of hours spent drinking and the mass of the person into the relevant formula.
b	$\text{BAC}_{\text{female}} = \frac{10N - 7.5H}{5.5M}$ $= \frac{10 \times 4 - 7.5 \times 4}{5.5 \times 46}$ ≈ 0.04	Substitute $N = 4$, $H = 4$ and $M = 46$ into the formula $\text{BAC}_{\text{female}} = \frac{10N - 7.5H}{5.5M}$	

3 Complete the following, using the formulas given in Example 3F-2, to calculate the BAC of:

- a** an 83 kg male who has consumed six standard drinks in 3 hours

$$\begin{aligned} \text{BAC}_{\text{male}} &= \frac{10N - 7.5H}{6.8M} \\ &= \frac{10 \times \square - 7.5 \times \square}{6.8 \times \square} \\ &= \end{aligned}$$

- b** a 58 kg female who has consumed five standard drinks in 4 hours

$$\begin{aligned} \text{BAC}_{\text{female}} &= \frac{10N - 7.5H}{5.5M} \\ &= \frac{10 \times \square - 7.5 \times \square}{5.5 \times \square} \\ &= \end{aligned}$$

4 Use the formulas for calculating BAC to complete the following table.

	Gender	Mass (kg)	Number of standard drinks consumed	Number of hours spent drinking	BAC (%)
a	Male	70	3	2	
b	Female	50	4	2	
c	Male	95	6	3	
d	Female	57	2	2	

5 An 80 kg adult male wants to keep his $\text{BAC} \leq 0.05$. Complete the following to find how many drinks he can consume in 5 hours to achieve this.

Let $\text{BAC} = 0.05$.

$$\begin{aligned} \text{So: } 0.05 &= \frac{10N - 7.5 \times 5}{6.8 \times 80} \\ &= \frac{10N - \square}{\square} \\ \square &= 10N - \square \\ \square &= 10N \\ N &= \end{aligned}$$



If he has $\square$ standard drinks, his $\text{BAC} = 0.05$.

For his $\text{BAC} < 0.05$, he can have up to $\square$ standard drinks.

6 A 56 kg woman wants a BAC that is less than 0.05. How many drinks can she consume in 3 hours?

7 A rule of thumb can be used by the holder of a full drivers' licence to stay under the 0.05 legal limit in NSW:

For males: no more than two standard drinks in the first hour and one standard drink per hour after that.

For females: no more than one standard drink per hour.

Use this rule of thumb to calculate the maximum number of standard drinks that can be consumed by a person who wants to stay under 0.05 BAC if the person is:

- a** a male and drinking for 4 hours **b** a male and drinking for 6 hours
c a female and drinking for 4 hours **d** a female and drinking for 6 hours

8 Use the rule of thumb from question 7 to calculate the answers to questions 5 and 6.

Note: After drinking ceases, the only thing that will reduce BAC is the passing of time. Drinking coffee, exercising, taking a cold shower or inducing vomiting will not reduce BAC. Alcohol is eliminated from the body by the liver at a rate between 4 g/h and 12 g/h, or at an average of 7.5 g/h or 0.75 standard drinks per hour (since a standard drink contains 10 g of alcohol), and can vary considerably depending on the person's health. This means that it can take the liver more than an hour to eliminate one standard drink.



EXAMPLE 3F-3 Calculating time needed to eliminate one standard drink

If a person's liver can break down alcohol at the rate of 6 g/h, how long will it take for this person's body to eliminate one standard drink?

Solve	Think	Apply
Time needed $= \frac{10}{6} \text{ h}$ $= 1.666... \text{ h}$ $= 1 \text{ h } 40 \text{ min}$	One standard drink contains 10 g of alcohol. If the liver can eliminate 6 g every hour, then the time needed is $(10 \div 6)$ hours.	If the rate at which the liver eliminates alcohol is given in g/h, then time in hours needed to eliminate one standard drink is $\frac{10}{\text{rate of elimination}}$.

- 9 Calculate the time it takes different people's bodies to eliminate one standard drink if their livers break down alcohol at these rates.

a 5 g/h **b** 10 g/h **c** 4 g/h **d** 12 g/h **e** 7.5 g/h

The liver breaks down alcohol at an average rate of 0.75 standard drinks per hour.

A formula that can be used to calculate the time it takes for BAC to fall to zero is: $T = \frac{\text{BAC}}{0.015}$

where T = the number of hours the drinker must wait.

- 10 Complete the following to calculate how long a person must wait for BAC to drop to zero from 0.05%.

$$T = \frac{\text{BAC}}{0.015} = \frac{\square}{0.015} = \text{___ h} = \text{___ h ___ min}$$

- 11 Calculate how long a person must wait for BAC to drop to zero from these levels.

a 0.04% **b** 0.02% **c** 0.06% **d** 0.035%

- 12 **a** Calculate the BAC for an 80 kg male and a 52 kg female, both with provisional licences, who consume four standard drinks in 3 hours.

b A zero BAC is a requirement of NSW law for all learner and provisional drivers. How long would the two people described in part **a** need to wait before they could legally drive a motor vehicle?

- 13 Damien and Nicole go to a party and start drinking at 8:00 pm. Damien drinks eight schooners of full strength beer (12 standard drinks) over the next 5 hours. Nicole has six mixed drinks (9 standard drinks) in the same time. Damien has a mass of 86.6 kg and Nicole's mass is 56.1 kg.

a Calculate the BAC of both Damien and Nicole at 1:00 am.

b At what time will they be able to legally drive if they both have provisional licences?

- 14 Ben goes to a party and consumes two stubbies (375 mL) of full strength beer (4.9% alc/vol) in the first hour and one stubby per hour for the next 3 hours.

a Calculate the number of standard drinks he has consumed.

b Use the formula to calculate his BAC if his mass is 72 kg.

c How long will it be before Ben's BAC drops to zero?

- 15 Create a spreadsheet to show the blood alcohol content for a 75 kg male and a 55 kg female, where N is the number of standard drinks consumed compared with the number of hours spent drinking. Investigate what the BAC would be for a given number of standard drinks in a given time. Remember it is only an approximation! (You may like to refer to the spreadsheet supplied in your obook assess.)

3G Medication calculations

These resources are available on your obook assess:

- **Spreadsheet 3G:** Calculate medication dosage for children
- **assess quiz 3G:** Test your skills with an auto-correcting multiple-choice quiz

oi
gi

This section examines dosages of various medications. Some terms are defined here.

- The dose is the amount of drug taken at any one time.
- The dosage regimen is the frequency at which the drug doses are given.
- The total daily dose is calculated from the dose and the number of times the dose is taken.
- The dosage form is the physical form of a dose of the drug. Common dosage forms include tablets, capsules, creams, ointments, aerosols and patches.
- The optimal dosage is the dosage that gives the desired effect with minimal side effects.

EXERCISE 3G Medication calculations

- 1 The dosage for a particular painkiller is as follows.

Age: 7–12 years: $\frac{1}{2}$ – 1 tablet every 4–6 hours (maximum 4 tablets in 24 hours)

Age: 12 years–adult: 1–2 tablets every 4–6 hours (maximum 8 tablets in 24 hours)

- How many tablets can an adult take in one dose?
- An adult plans to take two tablets every 4 hours for 24 hours.
 - How many tablets would that person take over 24 hours? Why shouldn't they do this?
 - How many doses of two tablets can be taken over 24 hours?
- A child takes $\frac{1}{2}$ a tablet every 4 hours for 24 hours. Has the maximum dosage been exceeded? Explain.

- 2 For a very strong painkiller, dosage for adults and children from 12 years is: two caplets, then 1–2 caplets every 4–6 hours as necessary. (Maximum 6 caplets in 24 hours.)

- An adult takes two caplets now and then two more after 4 hours. How many more caplets can that person take in that 24-hour period?
- Is two caplets initially, two more after 4 hours and two more after 6 hours, then no more, an acceptable dosage? Explain your answer.

There are three different formulas that can be used to calculate dosage for children, depending on the age of the child.

Fried's formula for children 1 to 2 years old is: $\text{Dosage} = \frac{\text{age (in months)} \times \text{adult dosage}}{150}$

or $D = \frac{mA}{150}$ where D = infant dosage, m = age of infant in months and A = adult dosage.

Young's formula for children 1 to 12 years old is: $\text{Dosage} = \frac{\text{age of child (in years)} \times \text{adult dosage}}{\text{age of child (in years)} + 12}$

or $D = \frac{yA}{y + 12}$ where y = age of child in years and A = adult dosage.

Clark's formula for children of any age is: $\text{Dosage} = \frac{\text{child's weight (in kilograms)} \times \text{adult dose}}{70}$

or $D = \frac{kA}{70}$ where k = mass of child in kilograms and A = adult dosage.

!

EXAMPLE 3G-1 Using Fried's formula to calculate medication dosage

The adult dose of a medication is 40 mL. Use Fried's formula for children 1 to 2 years old to calculate the dosage for a 20-month-old child.

Solve	Think	Apply
Dose = $\frac{20 \times 40}{150} = 5.3 \text{ mL}$	Use the formula $D = \frac{mA}{150}$ and substitute $m = 20$ months and $A = 40 \text{ mL}$.	Ensure that the units are correct for the formula; that is, age is in months.

- 3 Use Fried's formula to calculate each child's dosage below.
- a adult dose of 50 mL, child's age 15 months
 - b adult dose of 40 mL, child's age 21 months
 - c adult dose of 30 mL, child's age 18 months
 - d adult dose of 50 mL, child's age 13 months
 - e adult dose of 100 mL, child's age 17 months
 - f adult dose of 80 mL, child's age 23 months
- 4 a A child aged 17 months is given a dosage of 6 mL. Use Fried's formula to calculate the adult dosage.
b A child aged 11 months is given a dosage of 7 mL. Use Fried's formula to calculate the adult dosage.

EXAMPLE 3G-2 Using Young's formula to calculate medication dosage

Use Young's formula to calculate the dosage for a $5\frac{1}{2}$ -year-old child if the adult dose is 60 mL.

Solve	Think	Apply
Dose = $\frac{5.5 \times 60}{5.5 + 12} \approx 19 \text{ mL}$	Use the formula $D = \frac{yA}{y + 12}$ where $y = 5.5$ and $A = 60 \text{ mL}$.	Check the units before substituting. Some formulas use age in years, others use age in months.

- 5 Use Young's formula to calculate each child's dosage, to the nearest millilitre.
- a adult dose of 50 mL, child's age 6 years
 - d adult dose of 20 mL, child's age 7.5 years
 - b adult dose of 40 mL, child's age 8 years
 - e adult dose of 100 mL, child's age 6.2 years
 - c adult dose of 80 mL, child's age 4.5 years
 - f adult dose of 75 mL, child's age 8.4 years
- 6 a Use Young's formula to calculate the adult dose if the dosage for an 8-year-old child was 10 mL.
b Use Young's formula to calculate the adult dose if the dosage for a 6-year-old child was 5 mL.

EXAMPLE 3G-3 Using Clark's formula to calculate medication dosage

Use Clark's formula to calculate the dosage for a child weighing 19 kg. The adult dose is 50 mL

Solve	Think	Apply
Dose = $\frac{19 \times 50}{70} \approx 14 \text{ mL}$	Use the formula $D = \frac{kA}{70}$ where $k = 19$ and $A = 50$.	Check units before substituting. The formula uses 70 kg as the average adult weight.

- 7 Use Clark's formula to calculate each child's dosage, to the nearest millilitre.
- a adult dosage of 18 mL, child's weight 40 kg
 - b adult dosage of 27 mL, child's weight 60 kg
 - c adult dosage of 51 mL, child's weight 80 kg
 - d adult dosage of 39 mL, child's weight 75 kg
 - e adult dosage of 32 mL, child's weight 100 kg
 - f adult dosage of 40 mL, child's weight 35 kg

- 8 a** Use Clark's formula to calculate the adult dose if a 35 kg child has a dose of 18 mL.
- b** Use Clark's formula to calculate the adult dose if a 25 kg child has a dose of 17 mL.
- c** Use Clark's formula to calculate the weight of a child receiving a dose of 40 mL, given that the adult dose is 140 mL.

EXAMPLE 3G-4 Calculating flow rate in millilitres per hour

A patient is to receive 1.6 L of fluid over 10 h. What is the flow rate in mL/h?

Solve	Think	Apply
$\text{Flow rate} = \frac{\text{volume (mL)}}{\text{time (h)}}$ $= \frac{1600 \text{ mL}}{10 \text{ h}}$ $= 160 \text{ mL/h}$	Convert 1.6 L to mL by multiplying by 1000.	Ensure that units are converted before dividing to find the rate.

- 9** Calculate the flow rate in mL/h for these volumes of fluid and these times.
- | | |
|-------------------------------------|-------------------------------------|
| a volume of 1.4 L over 8 h | b volume of 1.7 L over 5 h |
| c volume of 0.8 L over 6 h | d volume of 0.6 L over 5 h |
| e volume of 0.085 L over 3 h | f volume of 4.26 L over 12 h |
- 10**
- The flow rate is 150 mL/h for 6 h. How much fluid is delivered? Give your answer in litres.
 - The flow rate is 200 mL/h for 7 h. How much fluid is delivered? Give your answer in litres.
 - The flow rate is 180 mL/h and 600 mL is delivered. For how long was the fluid delivered?

EXAMPLE 3G-5 Calculating flow rate in drops per minute

A patient is to receive 1.2 L of fluid over 4 h through an intravenous drip. There are 15 drops/mL. How many drops per minute are required?

Solve	Think	Apply
$\text{Flow rate} = \frac{1200\text{mL}}{240\text{min}} = 5\text{ mL/min}$ $\text{Drops} = 5 \times 15\text{ drops/min}$ $= 75\text{ drops/min}$	Convert 1.2 L to mL by multiplying by 1000. Convert 4 h to minutes by multiplying by 60.	The unit for drops/min is actually gtt/min. Large drops per mL are called ‘macrodrops’ and are used if the rate is greater than 100 mL/h.

- 11** Calculate the number of drops per minute required at a rate of 15 drops/mL in these situations.
- a** volume of 1.4 L over 7 h
 - b** volume of 1.5 L over 6 h
 - c** volume of 800 mL over 4 h
 - d** volume of 600 mL over 2 h
 - e** volume of 750 mL over 5 h
 - f** volume of 900 mL over 6 h
- 12** What is the drip rate per minute for:
- a** 1.3 L of fluid over 6 h with a drip size giving 12 drops/mL?
 - b** 850 mL of fluid over 5 h with a drip size giving 8 drops/mL?



EXAMPLE 3G-6 Calculating delivery time for an intravenous drip

An intravenous drip is delivering 30 drops/min. There are 20 drops/mL and 900 mL of liquid to be delivered. How long will the drip take to deliver the required dose?

Solve	Think	Apply
$\begin{aligned}\text{Number of drops} &= 900 \times 20 \\ &= 18\,000 \\ \text{Time} &= \frac{18\,000}{30} \text{ min} \\ &= 600 \text{ min} \\ &= 10 \text{ h}\end{aligned}$	First calculate the number of drops needed. Calculate the time using number of drops/min. Divide minutes by 60 to convert to hours.	Be aware of the units and convert where necessary. The drop size can be varied as well as the drop rate.

- 13** Calculate the time it will take to deliver intravenous liquid at these rates.
- 800 mL delivered at 20 drops/min and there are 15 drops/mL
 - 600 mL delivered at 15 drops/min and there are 10 drops/mL
 - 500 mL delivered at 10 drops/min and there are 12 drops/mL
 - 1.2 L delivered at 25 drops/min and there are 20 drops/mL
 - 1.5 L delivered at 15 drops/min and there are 12 drops/mL
 - 1.8 L delivered at 20 drops/min and there are 15 drops/mL
- 14** A sick child requires medication. The adult dose of the medication is 20 mL. The child is 16 months old and has a mass of 7.9 kg.
- Find the dosage using Fried's rule.
 - Find the dosage using Young's rule.
 - Find the dosage using Clark's rule.
 - How much would you give the child? Explain.
- 15** Repeat question 14 for a 20-month-old child weighing 12 kg, given that the adult dose of the medication is 30 mL.
- 16** A child is 10 years of age and requires a dose of a medicine for which the adult dosage would be 50 mL. The dosage is calculated using Fried's rule, without realising that rule is used for children under 2 years only. A dose could be an overdose if it is double the correct dose.
- Calculate the dosage according to Fried's rule.
 - Calculate the correct dosage using Young's rule.
 - If the entire amount calculated using Fried's rule is taken at the one time, has the child received an overdose? Explain.
 - The dosage is given through a drip at 20 drops per minute with 15 drops per millilitre. At what time should the drip be stopped to result in the correct dosage being given?

- 17** Create a spreadsheet to calculate the dosage for children under 12 years of age. Fried's rule is applicable for children under 2 years, Young's formula is for children 1 to 12 years and Clark's formula is for children of any age. Set your spreadsheet up so, when you enter the three variables for the child, the spreadsheet calculates the dosage. (You may like to refer to the spreadsheet supplied in your *obook assess*.)

	A	B	
1	Calculation of medicine dosage for child 12 years or less.		
2	Child's information		
3	Age in months	11	
4	Weight in kilograms	6.2	
5	Adult dosage in millilitres	40	
6	Child dosage in millilitres		
7	Fried's rule	Young's rule	Clark's rule
8	2.9	Not suitable	3.5

CHAPTER 3 REVIEW FORMULAS AND EQUATIONS

You should be able to:

- ✓ substitute for pronumerals and evaluate expressions
- ✓ find the value of the subject of a formula by substituting for variables
- ✓ solve linear equations
- ✓ solve equations by substitution into formulas
- ✓ change the subject of a formula
- ✓ solve problems related to speed, distance and time
- ✓ calculate the distance a motor vehicle travels in the time it takes to bring it to a stop
- ✓ calculate the number of standard drinks in a container, the BAC for males and females, and the time it takes for someone's BAC to fall to zero
- ✓ calculate medical dosages.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

REVIEW MULTIPLE-CHOICE QUESTIONS

- 3A** 1 Let $a = 7$ and $b = -3$. What is the value of $\frac{a^2 - 5b}{8}$?
A 64 B 3 C $50\frac{7}{8}$ D 8
- 3A** 2 What is the value of $s = ut + \frac{1}{2}at^2$ if $u = 0$, $t = 5$ and $a = 10$?
A 125 B 130 C 2500 D 2505
- 3A** 3 The volume of a cone with $h = 10$ and $r = 3$ is calculated using $V = \frac{1}{3}\pi r^2 h$. Which value is closest to the volume?
A 31 B 94 C 314 D 30
- 3B** 4 What is the solution to $p - 16 = 42$?
A $p = 26$ B $p = 36$ C $p = 48$ D $p = 58$
- 3B** 5 What is the solution to $11x - 10 = 25$?
A $x = 25$ B $x = 3\frac{2}{11}$ C $x = \frac{11}{35}$ D $x = \frac{15}{11}$
- 3B** 6 Which equation has solution $x = -3$?
A $3x - 7 = 2$ B $4x - 3 = 15$ C $x + 5 = 8$ D $5x + 2 = -13$
- 3B** 7 Which equation does *not* have $x = 4$ as a solution?
A $x - 4 = 0$ B $\frac{x}{4} = 16$ C $4x = 16$ D $x + 5 = 9$
- 3C** 8 What is the solution to $\frac{3a + 4}{5} = 2$?
A $a = 2$ B $a = 3$ C $a = 4$ D $a = 5$
- 3C** 9 What is the solution to $7x - 3 = 4x + 9$?
A $x = 2$ B $x = 3$ C $x = 4$ D $x = 5$
- 3D** 10 Using the formula $V = IR - E$, what is the value of I when $V = 13$, $R = 4$ and $E = 7$?
A $1\frac{1}{2}$ B $\frac{1}{4}$ C 5 D $\frac{2}{3}$

- 3D ▶ 11** The formula $V = IR - E$ is rearranged to make R the subject. What is the new formula?
A $R = \frac{V + E}{I}$ **B** $R = \frac{V - I}{E}$ **C** $R = \frac{E}{VI}$ **D** $R = \frac{E - V}{I}$
- 3E ▶ 12** A car travels 280 km in 3 h 25 min. What is its average speed?
A 86 km/h **B** 82 km/h **C** 93 km/h **D** 42 km/h
- 3E ▶ 13** What is 70 km/h expressed in m/s?
A 1.2 m/s **B** 0.02 m/s **C** 1167 m/s **D** 19.4 m/s
- 3E ▶ 14** What is 15 m/s expressed in km/h?
A 250 km/h **B** 4.2 km/h **C** 54 km/h **D** 41.7 km/h
- 3E ▶ 15** What is the distance a car travels in 2.8 s at 80 km/h?
A 6.2 m **B** 7.9 m **C** 62.2 m **D** 373.3 m
- 3F ▶ 16** Paul goes to a party and consumes three stubbies (375 mL) of full strength beer (4.9% alc/vol) in the first hour and one stubby per hour for the next 4 hours. Approximately, how many standard drinks has he consumed?
A 1.4 **B** 4.3 **C** 5.8 **D** 10.1
- 3F ▶ 17** A 75 kg woman consumes seven standard drinks in 4 hours. What is her BAC?
A 0.078 **B** 0.097 **C** 0.010 **D** 0.012
- 3G ▶ 18** The adult dose of a medication is 50 mL. Using Fried's formula below, what is the dosage for a 1-year-old child?
 Child dose = $\frac{\text{age (in months)} \times \text{adult dose}}{150}$
A 4 mL **B** 0.3 mL **C** 3 mL **D** 0.4 mL
- 3G ▶ 19** A patient is to receive 1.2 L of fluid over 8 h. What is the flow rate in mL/h?
A 9.6 mL/h **B** 15 mL/h **C** 150 mL/h **D** 6.6 mL/h
- 3G ▶ 20** A patient is to receive 800 mL of liquid through an intravenous drip delivering 25 drops/min. If there are 16 drops/mL, how long will it take to deliver the required dose?
A 0.52 h **B** 8 h 32 min **C** 12 h **D** 20 h 50 min

REVIEW SET 1

- 1** If $x = 4$ and $y = 6$, find the value of $3x - 5y$.
- 2** The volume, V , of a sphere is found using the formula $V = \frac{4}{3}\pi r^3$, where r is the radius. Find the volume of a sphere with radius:
a 7 cm **b** 3.5 cm **c** 0.6 m
- 3** The formula for converting temperature measurements from degrees Celsius, C , to degrees Fahrenheit, F , is $F = \frac{9}{5}C + 32$. Find F when:
a $C = 200^\circ$ **b** $C = 20^\circ$ **c** $C = 55^\circ$
- 4** Solve the following equations.
a $d - 8 = 36$ **b** $x + 9 = 20$ **c** $\frac{x}{8} = 72$
d $11x = 55$ **e** $4x - 7 = 39$ **f** $3x - 12 = 60$
g $10 + 8n = 72$ **h** $12 - 6c = 85$ **i** $3(2x + 1) = 4$

- 5** Use the formula $E = \frac{1}{2}mv^2$ to find the value of E when $m = 50$ and $v = 20$.
- 6** Make x the subject of each formula below.
a $3x = t$ **b** $y = mx - 7$ **c** $a = \frac{x+d}{c}$
- 7** For a car travelling on a slippery road, the formula for braking distance becomes $d = 0.015v^2$. What is the braking distance of a car travelling at 100 km/h on this road?
- 8** A train averages 84 km/h for 2 h 36 min. How far does it travel?
- 9** Calculate how long an average person must wait for BAC to drop to zero from 0.07%.
- 10** Use Fried's formula to calculate the dosage of medicine for a 9-month-old child if the adult dose is 60 mL.
- 11** Use Young's formula to calculate the dosage of medicine for a $6\frac{1}{2}$ -year-old child if the adult dose is 45 mL.
- 12** A patient is to receive 1.5 L of fluid over 6 h through an intravenous drip. If there are 12 drops/mL, how many drops per minute are required to deliver the dose?

REVIEW SET 2

- 1** If $x = -2$ and $y = 3$ find the value of $5x - 7y$.
- 2** The surface area, A , of a sphere is found using the formula $A = 4\pi r^2$, where r is the radius. Find the surface area of a sphere with radius:
a 4 cm **b** 7.5 cm **c** 0.8 m
- 3** The formula for converting temperature measurements from degrees Fahrenheit, F , to degrees Celsius, C , is $C = \frac{5}{9}(F - 32)$. Find C when:
a $F = 248^\circ$ **b** $F = 50^\circ$ **c** $F = 32^\circ$
- 4** Solve the following equations.
a $d - 9 = 23$ **b** $x + 4 = 65$ **c** $\frac{x}{3} = 5$
d $11x = 121$ **e** $3x - 7 = 30$ **f** $4x - 12 = 15$
g $12 - 9c = 43$ **h** $7(2x + 5) = 4$ **i** $4x - 2 = x + 9$
- 5** Use the formula $A = \frac{1}{2}bh$ to find b , given that $A = 72$ and $h = 8$.
- 6** Make x the subject of each formula below.
a $x + y = m$ **b** $n = k - 3x$ **c** $m = \frac{x}{k} - 4$
- 7** A car travels 252 km in 4 h 19 min. Calculate its average speed.
- 8** Calculate the BAC of a 98 kg male who has consumed six standard drinks in 4 hours.
- 9** Use Young's formula to calculate the dosage of medicine for a $7\frac{1}{2}$ -year-old child if the adult dose is 25 mL.
- 10** Use Clark's formula to calculate the dosage of medicine for a child weighing 15 kg. The adult dose is 30 mL.
- 11** A patient is to receive 600 mL of saline solution. An intravenous drip delivers 30 drops/min and there are 12 drops/mL. How long will it take to deliver the required dose?

REVIEW SET 3

- 1 Given $x = -5$ and $y = 8$, find the value of $4x^2 - 6y$.
- 2 The surface area, A , of a cylinder is found using the formula $A = 2\pi r(r + h)$, where r is the radius and h is the height. Find the surface area of a cylinder with radius 15 cm and height 10 cm.
- 3 The time T s for a pendulum of length L m to swing back and forth once is give by the formula $T = 2\pi\sqrt{\frac{L}{g}}$ where $g \approx 10 \text{ m/s}^2$. Find how long it takes for a pendulum of length 2.5 m to swing back and forth once.
- 4 Solve the following equations.

a $d - 9 = 11$	b $x + 4 = 14$	c $\frac{x}{3} = 2$
d $11x = 77$	e $3x - 8 = 12$	f $7x - 12 = 19$
g $14 + 3n = 23$	h $23 - 4c = 11$	i $2(4x + 5) = 12$
- 5 Use the formula $T = a + (n - 1)d$ to find n , given that $T = 41$, $a = 3$ and $d = 2$.
- 6 Make x the subject of each formula below.

a $\frac{x}{y} = t$	b $y = k - mx$	c $\frac{x - t}{w} = y$
- 7 Find the total stopping distance for a car travelling at 90 km/h in good conditions. Assume a reaction time of 2.5 s and a braking distance of $d = 0.01v^2$.
- 8 Calculate the number of standard drinks in a 120 mL glass of red wine, given that the alcohol content of the wine is 14.6% alc/vol.
- 9
 - a Calculate the BAC of a 76 kg male who has consumed five standard drinks in 3 hours.
 - b Calculate the BAC of a 52 kg female who has consumed four standard drinks in 4 hours.
- 10 Calculate how long an average person must wait for BAC to drop to 0 from 0.06%.
- 11
 - a Calculate the BAC for a 75 kg male and a 53 kg female, both with provisional licences, who consume four standard drinks in 3 hours.
 - b A zero BAC is a requirement of NSW law for all learner and provisional drivers. How long would the two people have to wait before they could legally drive a motor vehicle?
- 12 What is the drip rate per minute for 1.25 L of fluid over 8 h, with a drip size giving 14 drops/mL?
- 13 Use Clark's formula to calculate the dosage of medicine for a child weighing 38 kg, given the adult dose is 35 mL.



REVIEW SET 4

- 1 Given $x = 12$ and $y = -4$, find the value of $6x^2 - 11y$.
- 2 The volume, V , of a cylinder is found using the formula $V = \pi r^2 h$, where r is the radius and h is the height. Find the volume of a cylinder with radius 12 cm and height 25 cm.
- 3 A formula for calculating the bend allowance, in millimetres, of sheet metal is $B = 2\pi\left(R + \frac{T}{2}\right) \times \frac{A}{360}$ where B is the bend allowance, T is the thickness in millimetres, A is the number of degrees in the angle of bend, and R is the radius of curvature in millimetres. Find B when $T = 1.5$, $R = 4$ and $A = 113$.
- 4 Solve the following equations.

a $d - 5 = 11$	b $x + 7 = 14$	c $\frac{x}{6} = 2$
d $7x = 77$	e $3x - 4 = 92$	f $8x - 12 = 11$
g $11 + 7n = 53$	h $27 - 9c = 31$	i $4(3x + 5) = 13$
- 5 Use the formula $b = \frac{m}{h^2}$ to find m if $b = 31.25$ and $h = 1.6$.
- 6 Use the formula $a = \frac{2Rn}{n+1}$ to find the value of R when $a = 95$ and $n = 19$.
- 7 If a cyclist can average 15 km/h, how long will it take her to travel 28 km?
- 8 Convert the following.

a 55 km/h to m/s	b 21.4 m/s to km/h
- 9 Calculate the reaction-time distance for a car travelling at 60 km/h. Assume a reaction time of 2.5 s.
- 10 Use the formula $d = 0.01v^2$ to calculate the braking distance for a car travelling at 80 km/h in good conditions.
- 11 Find the total stopping distance for a car travelling at 80 km/h in good conditions.
Use the formula $d = 0.7v + 0.01v^2$, where d is the stopping distance in metres and v is the speed in km/h.
- 12 For a driver under the influence of alcohol and driving in poor road conditions, the formula for the stopping distance of a car becomes $d = 1.1v + 0.018v^2$.
 - a Calculate the stopping distance to the nearest metre to complete the table for this driver.

Speed (km/h)	0	20	40	60	80	100
Stopping distance (m)						

 - b Use the table from part a to draw a graph of the relationship between speed and stopping distance.
 - c From the graph drawn for part b estimate the stopping distance if the car is travelling:

i 50 km/h	ii 70 km/h	iii 110 km/h
- 13 Use Young's formula to calculate the dosage of medicine for a 10-year-old child. The adult dose is 20 mL.
- 14 A patient is to receive 1.1 L of fluid over 9 h.
 - a What is the flow rate in mL/h?
 - b Find the drip rate in drops/min if there are 18 drops/mL.

REVIEW PRACTICE EXAMINATION QUESTION

- 1 a i** Solve $\frac{x}{5} = -3$. (1 mark)
- ii** Find the value of $\sqrt{10a + 2b}$ when $a = 5$ and $b = 7$. (2 marks)
- b** The body mass index, b , is used to determine whether a person's mass is within the acceptable range of $21 < b < 25$. To calculate her body mass index, Stella used the formula $b = \frac{m}{h^2}$, where m = mass in kilograms and h = height in metres. Here are her calculations.
- $$b = \frac{m}{h^2}$$
- When $m = 55$ kg and $h = 168$ cm,
- $$b = \frac{55}{(168)^2}$$
- $$= 0.00195$$
- i** Explain the error in her calculations. (1 mark)
- ii** Calculate the correct value of b . (1 mark)
- iii** Is Stella in the healthy mass range? Explain. (1 mark)
- c** George's solution to the equation $6 - 5x = 8$ is shown below.
- $$6 - 5x = 8$$
- $$6 - 5x + 6 = 8 + 6$$
- $$-5x = 14$$
- $$x = 8$$
- He has made some errors. Identify the errors and find the correct solution. (2 marks)
- d** The total stopping distance, d metres, of a car travelling at v km/h under good conditions is given by the formula $d = 0.7v + 0.01v^2$.
- i** Calculate the stopping distance of a car travelling at 60 km/h. (1 mark)
- ii** If the speed of the car is increased by 10 km/h, what is the increase in the stopping distance? (2 marks)
- e** Harry goes to a party and drinks seven standard drinks over 4 hours.
- i** Use the formula $BAC_{\text{male}} = \frac{10N - 7.5H}{6.8M}$ to calculate Harry's blood alcohol content after 4 hours if his mass is 78 kg. (2 marks)
- ii** Use the formula $T = \frac{BAC}{0.015}$ to find how long it will take for his BAC to drop to zero. (1 mark)
- f** Use Young's formula below to calculate the dosage of medicine for a $4\frac{1}{2}$ -year-old child, given the adult dose is 100 mL.
- $$\text{Dosage for children 1 to 12 years} = \frac{\text{age of child (in years)} \times \text{adult dosage}}{\text{age of child (in years)} + 12}$$
- (1 mark)

TOTAL: 15 marks

- 1
 - a Pauline earns \$56 028 per annum. Calculate her weekly pay.
 - b Fielding is a junior and is paid \$16.58 per hour. He works 5 hours per day, six days a week. Find his pay for 1 year.
 - c Louise is paid \$24.40 per hour. Calculate her weekly wage if she works 40 hours at normal time and 8 hours overtime at time-and-a-half.
 - d Hunter works in a timber yard and is paid \$28.40 per hour. He receives a dust allowance of \$12.50 per day. Find his pay for a 5-day, 37-hour week.
 - e An art salesperson is paid 6.2% of the selling price of any works he sells. Find his commission on a painting that sells for \$6250.
 - f Endora received a $17\frac{1}{2}\%$ holiday loading on 4 weeks normal wages. She normally works a 32-hour week. Her 4 weeks holiday pay with loading is \$2185.70.
 - i Find her normal weekly pay.
 - ii Find her normal hourly pay rate.
 - g A factory worker is paid \$5.35 for each garment completed. Calculate his wage if he completes 324 garments.
 - h Micky earns \$968 per week. His employer pays superannuation of 9.5% of that amount on his behalf and Micky contributes 3% of his salary to his superannuation fund.
 - i What is Micky's weekly income less his superannuation contribution?
 - ii What is the total weekly amount paid into Micky's superannuation fund?
 - i Harry and Sally are a couple entitled to the age pension. They have a combined income of \$522.00 per fortnight. The full pension would be \$609.30 each per fortnight. Calculate the total pension paid to Harry and Sally. Use the tables in Topic 1D to help you.
- 2
 - a Giselle uses the following information to calculate her taxable income: wages \$47 220, interest \$385, bonus \$3000, tax agent fee \$180, work-related car expenses \$1255 and donations to charity \$600. Calculate Giselle's taxable income.
 - b The Medicare levy is 2% of taxable income for taxable incomes over \$26 668. Calculate the Medicare levy payable on a taxable income of \$41 894.
 - c Use the following table, showing the tax rates for 2017–18, to calculate the tax payable on these incomes.

- i \$62 889
- ii \$37 699
- iii \$19 369
- iv \$3522

Taxable income	Tax on this income
0–\$18 200	Nil
\$18 201–\$37 000	19c for each \$1 over \$18 200
\$37 001–\$87 000	\$3572 plus 32.5c for each \$1 over \$37 000
\$87 001–\$180 000	\$19 822 plus 37c for each \$1 over \$87 000
\$180 001 and over	\$54 232 plus 45c for each \$1 over \$180 000

- d Simon earns \$29 996 p.a. as a cocktail attendant and \$17 560 as a part-time singer. He has tax deductions of \$3561 and, throughout the year, pays a total of \$8725 in tax instalments. Find:
 - i his total income
 - ii his taxable income
 - iii the Medicare levy he must pay
 - iv tax payable on his taxable income
 - v his tax refund or balance payable.

- 3 a** Round 4263.091 to:
- i** the nearest 100
 - ii** the nearest whole number
 - iii** two decimal places
 - iv** three significant figures
 - v** five significant figures
 - vi** one significant figure
- b** Are the following numbers in scientific notation?
- i** 6×10000
 - ii** 15×10^8
 - iii** 2.04×10^{-6}
- c** Express the following in scientific notation.
- i** 105 000
 - ii** 0.0000627
 - iii** 45.6
- d** Calculate the following, writing the answers in scientific notation.
- i** $(4.1 \times 10^8) \times (7 \times 10^5)$
 - ii** $(8 \times 10^5)^3$
 - iii** $(6.24 \times 10^{-3}) \div (1.43 \times 10^{-7})$
- e** Convert the following.
- i** 6.73 km = ____ m
 - ii** 1750 cm = ____ m
 - iii** 4.3 g = ____ mg
 - iv** 47 000 kg = ____ t
 - v** 6.35 L = ____ mL
 - vi** 90 mL = ____ L
 - vii** 3.2 Gm = ____ m
 - viii** 52 ML = ____ kL
 - ix** $16\,500 \text{ cm}^2 = \text{____ m}^2$
- f** For the measurement 6.8 m, find:
- i** the precision
 - ii** the absolute error
 - iii** the lower and upper bounds of the true measurement
 - iv** the percentage error (to one decimal place).
- 4 a** Given $x = -5$ and $y = 8$, find the value of $4x^2 - 6y$.
- b** The surface area, A , of a cylinder is found using the formula $A = 2\pi r(r + h)$, where r is the radius and h is the height. Find the surface area of a cylinder with radius 12 cm and height 8 cm.
- 5** Solve each of the following equations.
- a** $d - 9 = 12$
 - b** $x + 4 = 12$
 - c** $\frac{x}{3} = 6$
 - d** $11x = 55$
 - e** $3x - 8 = 19$
 - f** $7x - 12 = 16$
 - g** $14 + 3n = 29$
 - h** $23 - 4c = 14$
 - i** $\frac{4x + 1}{5} = \frac{1}{7}$
- 6 a** If $m = 10$, $n = 15$ and $p = -6$, find the value of $\frac{(pm)^2}{n}$.
- b** If $A = 750$ and $h = 12.6$, find r correct to one decimal place given that $r = \sqrt{\frac{A}{\pi h}}$.
- 7** Solve each of the following equations.
- a** $6x = 96$
 - b** $\frac{x}{4} = -2$
 - c** $x - 5 = 8$
 - d** $x + 6 = 9$
 - e** $3x - 5 = 7$
 - f** $4 - 2x = -8$
 - g** $7 - 3x = 4 - 5x$
 - h** $-\frac{4}{x} = 7$
 - i** $\frac{x-5}{2} = 11$
- 8** Make y the subject of each of these equations.
- a** $\frac{y}{3} = p$
 - b** $x = 4 - 3y$
 - c** $\frac{y-x}{w} = 2$
- 9** Find the total stopping distance, to the nearest metre, for a car travelling at 80 km/h in slippery conditions. Assume a reaction time of 2.5 s and use the formula $d = 0.014v^2$, where d is the braking distance in metres and v is the speed of the car in km/h. Give your answer correct to the nearest metre.
- 10** Lisa goes to a party and drinks four standard drinks over 2 hours. Use the formulas in Topic 3F to help you answer the following.
- a** Calculate Lisa's estimated blood alcohol content (BAC) after 2 hours if her mass is 60 kg.
 - b** Find how long it will be until her BAC drops to zero?

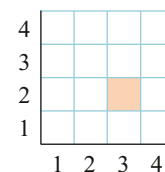
4

Classifying data and sampling

The main mathematical ideas investigated are:

- ▶ classifying data as categorical or numerical
- ▶ identifying the target population
- ▶ investigating bias in sampling, including self-selected sampling
- ▶ choosing a random sample
- ▶ choosing a stratified random sample
- ▶ choosing a systematic sample
- ▶ determining the suitability of sample types for a given situation
- ▶ evaluating the effect of a sample size in estimating the nature of a population.

ARE YOU READY?



- 4A ▶ 1** What type of data is 'the eye colour of students in a class'?
- A** numerical **B** systematic
C central **D** categorical
- 4A ▶ 2** A survey is conducted to find the number of children in a family. What type of data is being collected?
- A** categorical ordinal
B categorical nominal
C numerical discrete
D numerical continuous
- 4B ▶ 3** What is the term given to data collected from every member of a population?
- A** sample **B** census
C number **D** category
- 4B ▶ 4** Which of the following survey questions indicate bias in the collection process?
- A** What type of music do you like listening to?
B Do you like classical music?
C Pop is your favourite style of music, isn't it?
D I like to listen to hip hop. Circle your opinion of this statement.
- 1 strongly agree
2 agree
3 disagree
4 strongly disagree
- 4C ▶ 5** What are the grid references for the shaded region?
- A** (3, 1) **B** (1, 2)
C (2, 3) **D** (3, 2)
- 4D ▶ 6** What is 16 as a fraction of 40, in its simplest form?
- A** $\frac{16}{40}$ **B** $\frac{8}{20}$
C $\frac{4}{10}$ **D** $\frac{2}{5}$
- 4D ▶ 7** What is $\frac{5}{8} \times 30$?
- A** $3\frac{3}{4}$ **B** 6
C $18\frac{3}{4}$ **D** 48
- 4D ▶ 8** What is 17 as a percentage of 25?
- A** 17% **B** 25%
C 68% **D** 147%
- 4D ▶ 9** What is 35% of 160?
- A** 125 **B** 56
C 22 **D** 5
- 4E ▶ 10** What is the next number in the sequence 17, 38, 59, 80, ...?
- A** 21 **B** 81
C 101 **D** 111
- 4E ▶ 11** What is the hundredth term in the sequence 5, 8, 11, 14, ...?
- A** 305 **B** 302
C 300 **D** 100

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook assess](#).

- Q1–2** Support sheet 4A.1 Types of data
Q3 Support sheet 4B.1 Populations and samples
Q4 Support sheet 4B.2 Survey questions
Q5 Support sheet 4C.1 Grid references
Q6 Support sheet 4D.1 Writing and simplifying fractions
Q7 Support sheet 4D.2 Multiplying fractions
Q8–9 Support sheet 4D.3 Calculating percentages
Q10–11 Support sheet 4E.1 Number patterns

4A Collecting and classifying data

These resources are available on your obook assess:

- **Video tutorial 4A:** Watch and listen to an explanation of Example 4A-1
- **Interactive 4A:** Explore key ideas for classifying data
- **assess quiz 4A:** Test your skills with an auto-correcting multiple-choice quiz

statistics

the study of collecting, organising, analysing and interpreting data

data

pieces of collected information

Statistics is the science of collecting facts and information. The information is organised in a form that can be analysed easily and conclusions and predictions can accurately be based on. The facts or pieces of information collected are called **data**. Data may be collected by counting, measuring and asking questions.

The process of statistical inquiry includes the following steps:

- identifying a problem and posing a statistical question
- collecting or obtaining data
- representing and analysing that data
- communicating and interpreting the findings.

In this chapter we will investigate the first two of these steps.

Questioning can be by interview, either in person or by phone, or by questionnaire. The usefulness of the responses to the questions asked will depend on the relevance, clarity and structure of the questions. Questions should use simple language, be unambiguous so that there can be no doubt about their meaning, and they should be free from bias; that is, free from unfair influence.

The main types of questions used in questionnaires are:

- Free-response or open-ended questions
- Yes or No questions
- True or False questions
- Tick-a-box type questions
- Response scale questions
- Continuum scale questions

Here are some examples of these question types that could be used to start an investigation into the school uniform, with a discussion of some of the advantages and disadvantages of each type.

Free-response: ‘What do you like/dislike about the present school uniform?’

This question would possibly collect a lot of information, but, because there would be many different responses, it might be difficult to organise and interpret the responses.

Yes or No: ‘Would you like to change the present school uniform?’

The responses to this question will indicate whether there is a desire to change the uniform, but will not indicate what changes, if any, are wanted. (This, of course, could be done by asking further questions.)

True or False: ‘The present school uniform is satisfactory.’

As for the Yes and No questions, the responses will not indicate any changes wanted.

EXERCISE 4A Classification of data

- List some advantages and disadvantages of the following methods for collecting information from people. You could consider factors such as time, cost, the response rate (the number of people prepared to respond to the questions) and the truthfulness of answers. Discuss your answers with the class.
 - personal interviews
 - phone interviews
 - questionnaires
- For each of the following investigations, write a short questionnaire using each of the different types of questions discussed on the previous pages.
 - an investigation of what change, if any, students would like to make to your school uniform (considering how many students want a change, boys/girls, juniors/seniors, which items, etc.)
 - an investigation of which are the most popular radio stations and why (considering types of programs, types of music, boys/girls, when they listen, how often, etc.)
 - an investigation of whether students are satisfied with the food available at the school canteen (considering who uses the canteen when, how often, how much is spent, types of food, etc.)
- Trial your questionnaires from question 2 on a small group and, if necessary, modify the questions.



EXAMPLE 4A-1 Classifying data

Classify the following variables as categorical nominal, categorical ordinal, numerical discrete or numerical continuous.

- the number of heads when three coins are tossed
- the brands of toothpaste used by the students in a class
- the heights of a group of 15-year-old children
- the energy star ratings of refrigerators

	Solve	Think	Apply
a	numerical discrete	The values of the variables are obtained by counting the number of heads. The result can only be one of the exact values 0, 1, 2 or 3. It is numerical discrete data.	A numerical discrete variable is one that only takes exact or distinct values.
b	categorical nominal	The variable describes the brands (names) of the toothpaste used.	A categorical nominal variable is one that names the categories (such as brands of toothpaste). It cannot be ordered.
c	numerical continuous	This is numerical data obtained by measuring. The results can take any value between certain limits determined by the degree of accuracy of the measuring device. It is numerical continuous data.	A numerical continuous variable is one that can take any value in a given range.
d	categorical ordinal	This variable ranks (orders) the relative efficiency of refrigerators.	An ordinal variable is a qualitative variable with an intrinsic order.

- 4 For each of the following investigations, classify the variable as categorical nominal, categorical ordinal, numerical discrete or numerical continuous.
 - a the number of goals scored weekly by a netball team
 - b the heights of the members of a football team
 - c the most popular radio stations
 - d the numbers of children in Australian families
 - e the number of loaves of bread bought each week by a family
 - f the types of pets owned by students in a class
 - g the energy ratings of different washing machines
 - h the number of leaves on the stems of different plants
 - i the amount of sunshine each day
 - j the number of people who die from cancer each year
 - k students' opinions of their school uniform (like, no opinion, dislike)
 - l the amount of rainfall in each month of the year
 - m the countries of origin of immigrants
 - n the sizes of apples picked (small, medium, large)
 - o the numbers of pets owned by the students in a class
 - p the genders of school principals
 - q the health of patients after being given medication (worse, no different, better)



- 5 Give three of your own examples of:
 - a categorical nominal data
 - b categorical ordinal data
 - c numerical discrete data
 - d numerical continuous data.Discuss your examples with the class. Be prepared to justify your answers.
 - 6 Design a short questionnaire to investigate the use of calculators by students in your class. Include at least one question for collecting information about each of categorical nominal, categorical ordinal, numerical discrete and numerical continuous variables.
 - 7 Write a short questionnaire to investigate use of the school canteen by members of your school population and others. Include at least one question for collecting information about each of categorical nominal, categorical ordinal, numerical discrete and numerical continuous variables.
- CHALLENGE**
- 8 Have you ever taken part in market research? Explain what market research is and how market research companies use statistics. Write a short report.
 - 9 What statistics does the World Health Organization (WHO) collect and how are they used? The internet address is www.who.int
 - 10 The United Nations collects statistics. Research what data is collected and how they are used. The internet address is www.un.org

4B Census or sample and self-selected sampling

These resources are available on your obook assess:

- **Investigation 4B:** Develop an understanding of the National Census
- **assess quiz 4B:** Test your skills with an auto-correcting multiple-choice quiz

population

entire group about whom, or from which, information is sought

census

involves collecting information about every member of the entire population

sample

involves collecting information about a small selection from a population

biased sample

when the method of collecting data produces a sample that does not accurately reflect the characteristics of the population

One of the first decisions to be made when collecting data is deciding from whom, or what, the information is to be collected. The target **population** is the entire group about whom, or from which, information is sought. For instance, if we were interested in collecting information about the cleanliness of trains, the target population would be all commuters who use rail transport. If we wanted to determine the most popular colour for new cars purchased in one particular year, the target population would be all new cars sold in that year.

There are two ways of collecting the information: a **census** or a **sample**.

A census involves collecting data about every individual in the whole population. The individuals may be people or objects. A census is detailed and accurate but is expensive, time-consuming and often impractical.

A sample involves collecting data about a portion of the population. It is cheaper and quicker than a census but is not as detailed or as accurate. Conclusions drawn from samples always involve some degree of error.

The most common way of collecting information is by using a sample. The purpose of a sample is to provide an estimate of a particular characteristic of the whole population. Therefore, the challenge in selecting a sample is to make it as free from prejudice (bias) as possible and large enough to be representative of the whole population.

A **biased sample** is one in which the data has been unfairly influenced by the collection process and is not truly representative of the whole population.

EXERCISE 4B Target population and census or sample

- 1 What would be the target population if we wanted to:
 - a investigate the suitability of bus timetables on a certain route?
 - b collect information about the effects of road tax on company profits?
 - c investigate the success of a new treatment for ticks and fleas on dogs?
 - d investigate bank charges?

EXAMPLE 4B-1 Deciding whether a census or sample would be used

Would a census or a sample be used to investigate:

- a** the length of time an electric light globe will last?
- b** the causes of car accidents in New South Wales?
- c** the number of people who use Britex toothpaste?

	Solve	Think	Apply
a	Sample	It is obviously impractical to test every light globe produced – there would be none left for sale!	A sample collects data about a subset of the population.
b	Census	An accurate analysis of all accidents would be required.	A census would collect information about every accident.
c	Sample	It would be very time consuming to interview the whole population to find out who uses, or does not use, Britex toothpaste.	A sample collects data about a subset of the population.

2 State whether a census or a sample would be used for each of the following investigations. Discuss any problems in collecting such data with the class.

- a** the number of goals scored weekly by a netball team
- b** the heights of the members of a football team
- c** the most popular radio stations
- d** the number of children in Australian families
- e** the number of loaves of bread bought each week by a family
- f** the types of pets owned by students in a class
- g** the energy ratings of different washing machines
- h** the number of leaves on the stems of different plants
- i** the amount of sunshine each day
- j** the number of people who die from cancer each year
- k** students' opinion of their school uniform (like, no opinion, dislike)
- l** the amount of rainfall in each month of the year
- m** the countries of origin of immigrants
- n** the sizes of apples picked (small, medium, large)
- o** the numbers of pets owned by the students in a class
- p** the genders of school principals
- q** the health of patients after being given medication (worse, no different, better)

3 Give three examples of data that would be collected using:

- a** a census
- b** a sample.



EXAMPLE 4B-2 Identifying possible bias in a survey

Suggest the possible bias in each of the following surveys.

- a** a phone survey during a week day
- b** a survey of people on a train station
- c** a survey of a football crowd
- d** a survey of 10 people who have been tested with a new drug developed to cure the common cold

	Solve	Think	Apply
a	The sample would be biased towards people who are at home during the day.	The sample does not include people who go out to work.	The sample is not representative of the whole population or is not large enough to be representative of the population.
b	The sample would be biased towards people who catch the train.	The sample does not include people who use other forms of transport or work at home.	
c	The sample would be biased towards people who attend football matches.	Most probably more males than females attend football matches.	
d	The sample is not large enough.	As only 10 people were tested, a valid conclusion cannot be made.	

4 Explain and discuss any possible bias in the following samples.

- a** a phone survey on a Saturday night
- b** a survey of the people at a bus stop
- c** a survey of people at the beach
- d** people selected from the electoral roll
- e** a survey of businesses selected from the *Yellow Pages* phone directory

5 Discuss and comment on any possible bias in the following situations.

- a** Year 7 students are interviewed about changes to the school uniform.
- b** Motorists stopped in peak hour are interviewed about traffic problems.
- c** Real-estate agents are interviewed about the prices of houses.
- d** Politicians are interviewed about the state of the country's economy.
- e** A survey of 20 people indicated that 80% of people watch the Channel 9 News.
- f** A company marketing team claims that four out of five dentists recommend its brand of toothbrush.
- g** An advertisement claims that 'Dog breeders recommend Buddy dog food'.

EXAMPLE 4B-3 Selecting a sample to support a point of view

People may use biased samples to enhance their product claims or to support a point of view. For example, if you wanted the local council to upgrade its swimming pools, what sample of people would you choose to survey?

Solve	Think	Apply
A sample of people at the swimming pool.	You would expect the people who use the pool to be biased favourably towards this proposal.	The sample is not representative of the whole population.

6 Discuss and describe the sample you would choose if you wanted to gather support to:

- a** stop smoking in public places
- b** improve the local bus service
- c** increase the number of books in the library
- d** improve the school's senior common room.

A **self-selected sample** (or a voluntary response sample) is a sample that the participants choose to be part of. They are not directly approached by the researcher to participate in the survey/research. A radio phone-in survey is a common example of a self-selected sample.



self-selected sample

sample in which participants choose to take part by responding to a request for volunteers

This type of sampling is useful when we want people (or organisations) to volunteer to take part in research. There may be a wide range of reasons why people do this. They may have strong opinions about the research topic, or a specific interest in the research and its findings, or may simply want to assist a researcher.

Participants usually select themselves to be part of a study by responding to an advertisement in a newspaper, magazine, journal or an online noticeboard. The volunteers are then checked to make sure that their participation would be relevant to the study. For example, for a study of type 1 diabetes in 20-to-30-year-old males the researcher would check that the volunteers do satisfy all of the criteria before accepting them.

Advantages: Since the volunteers contact the researcher, this can reduce the time and expense of finding suitable subjects who meet the selection criteria. Volunteers are likely to be committed to taking part in the study, which can help ensure attendance; for example, at focus group meetings. They may also show a greater willingness to provide more insight into the subject of the research by giving full detailed responses to open-ended questions.

Disadvantages: Because the people chosen are volunteers, there is likely to be a degree of self-selection bias. For example, people volunteering to answer a questionnaire on a proposal to reduce company tax, may have extreme political views on this and may wish to push their point of view. This can either lead to the sample not being representative of the population being studied, or could exaggerate some particular finding from the study.

Despite the potential disadvantages of self-selection sampling, it is a popular and effective sampling strategy in experimental research settings.

- 7 Read the text above and answer the questions below.
 - a Write a definition of what is meant by a self-selected sample.
 - b What other term is sometimes used to describe this type of sample?
 - c Describe the process of creating a self-selected sample.
 - d List three advantages of a self-selected sample.
 - e List three disadvantages of a self-selected sample.
- 8 The careers adviser at your school wants to gain an insight into the career goals of the students in their final year at school. She puts a notice in the school newsletter asking for volunteers.
 - a What type of sample is this?
 - b Describe the process the careers adviser could use to find this information.
 - c Does this process introduce any bias into the sample? For example, consider the number of boys and girls selected. Discuss with your class.
- 9 The Australian Bureau of Statistics (ABS) is responsible for the Australian census. Research the role of the ABS and the information collected by the ABS. (The ABS produces year books and many other free publications. The ABS can be found on the internet at www.abs.gov.au)
 - a How often is the Australian census taken? What information is collected in the census?
 - b What information is openly made available?
 - c How is the information used by governments and businesses?
- 10 Investigate the aims and purpose of *Choice* magazine. How does this publication use statistics? Give some examples. How do the researchers avoid bias in their results? (*Choice* magazine is probably available in your school library or can be found on the internet at www.choice.com.au)

4C Random sampling

These resources are available on your obook assess:

- **Video tutorial 4C:** Watch and listen to an explanation of Example 4C-1
- **Spreadsheet 4C:** Generate random numbers
- **Investigation 4C:** Investigate the effect of sample size
- **assess quiz 4C:** Test your skills with an auto-correcting multiple-choice quiz

random sample
sample in which each member of the population has an equal chance of being selected

In a **random sample**, each member of the population has an equal chance of being selected. There are a number of ways in which a random sample may be taken. For example, suppose we wanted to choose a random sample of 10 students from a class of 30. We could:

- write the students' names on pieces of paper, place the names in a hat, mix them and draw 10 names
- write the students' names on cards, shuffle the cards and deal 10
- assign the numbers 1 to 30 to each member of the class. Select 10 random numbers from 1 to 30 using a list of random numbers (see below). The students whose numbers are selected form the sample.

(Note: since we are selecting a sample of students, we reject any repeated number.)

If the population is large, we can use random numbers generated by tables, calculators or computers.

The following list is an excerpt from a table of random numbers.

48047	45381	33232	35178	46971	85879	31458	22016
08695	58112	96070	91910	18868	52251	99827	32581
90070	98873	89846	50953	92529	68249	54949	83829
10132	27359	13017	41045	13817	65603	87615	55691
58547	01331	62538	79181	33071	63766	73613	24470
85566	81574	71965	20977	48005	83418	58738	98771
40942	42373	38710	39916	08187	00133	16288	64277
80553	58331	62724	74004	09344	91315	25791	40296
27328	85758	45342	98884	36034	79836	94902	80442
64584	20776	86792	42340	83522	62139	14038	88433



EXAMPLE 4C-1 Using a list of random numbers to select a random sample

Use the list of random numbers on the opposite page to select a random sample of five different:

a one-digit numbers

b two-digit numbers

c three-digit numbers.

	Solve/Think	Apply
a	Start at any digit in the table, say the 7 in row 5 and 16 digits across, as shown. Select the numbers to the right, left, down or up until you have five different numbers. Moving to the right generates the random numbers 7, 9, 1, 8 and 1. Since 1 is a repeated number, continue to the next number (3). Therefore, the random sample of five different numbers is 7, 9, 1, 8, 3. Moving to the left generates the random numbers 7, 8, 3, 5, 2. Moving down the column generates the random numbers 7, 2, 3, 7, 9. Since 7 is a repeated number, continue to the next number (4). Therefore, the random sample of five different numbers is 7, 2, 3, 9, 4. Moving up the column generates the random numbers 7, 4, 5, 9, 3. Any of these sets of numbers would be satisfactory.	Start at any digit and select groups of one, two or three digits by moving right, left, up or down. <i>Note:</i> if you run out of digits in a row (or column), continue by moving down (or up) to the next row, or across (left or right) to the next column. If, for example, a three-digit number such as 005 is generated, it is read as the number 5. If a number has already been selected, reject it and continue to the next number.
b	Start at any digit, say the 6 in row 5 and 11 digits across. Select pairs of digits by moving to the right, left, down or up. Moving to the right generates 62, 53, 87, 91, 81. Moving to the left generates 62, 31, 13, 70, 54. Moving down generates 62, 71, 38, 62, 45. Rejecting the repeated 62, the random sample of five different numbers is 62, 71, 38, 45, 86. Moving up generates 62, 13, 89, 96, 33.	
c	Start at any digit, say the 8 in row 6 and 22 digits across. Select groups of three digits by moving to the right, left, down or up. Moving to the right generates 800, 583, 418, 587, 389. Moving to the left generates 800, 774, 209, 965, 471. Moving down generates 800, 818, 934, 603, 352. Moving up generates 800, 307, 381, 252, 886.	

EXERCISE 4C Random sampling

For questions 1–6 in this exercise, use the list of random numbers on the opposite page.

- Start at the 2 in row 6 and 16 digits across, and select five random one-digit numbers by moving:
a to the right **b** to the left **c** down the column **d** up the column.
- Use the method in question 1 to select five two-digit random numbers.
- Use the method in question 1 to select five three-digit random numbers.
- Select 20 one-digit random numbers.
 - Check the randomness of these numbers by counting the number of odd numbers. Are about half of all the numbers odd numbers?
 - How many of the numbers are:
 - less than 5?
 - greater than 5?
 - equal to 5?

EXAMPLE 4C-2 Selecting a random sample of five students from a group of 60 students

Use the list of random numbers on page 126 to select five students at random from a group of 60 students.

Solve/Think	Apply
We assign each student a number from 1 to 60 and then select five random two-digit numbers from the list of random numbers. Start at any digit, say the 2 in row 3 and 24 digits across. Move across the row to the right. The first two-digit number is 29. The next is 68, but this is larger than 60, so we reject it. The next number is 24, etc. We reject the repeated appearance of 49 and any numbers larger than 60. Our final list will be 29, 24, 49, 01, 32.	Choose two-digit numbers, as shown in Example 4C-1, rejecting any that are larger than 60 or any number that is repeated. If selecting a random sample from a group with less than 10 items, use one-digit random numbers. If selecting a random sample from a group with less than 100 items, use two-digit random numbers. If selecting a random sample from a group with less than 1000 items, use three-digit random numbers.

- 5** Start at the 2 in row 4 and 6 digits across, then move across the row to the right to select 10 students at random from a group of:

a 80 students **b** 35 students.

- 6** Start at the 1 in row 6 and 12 digits across, then move down the column, to select 10 students at random from a group of:

a 700 students **b** 280 students

- 7** A factory supervisor wishes to interview a sample of workers about the use of safety equipment.

- a** Explain two methods that she could use to choose a random sample of five from the 25 factory workers.
- b** Which workers would be selected if she used this list of random numbers and started at the first number?

49246 36704 13499 48926 61279 36535
 32337 15455 26345 95085 05701 30998
 95907 47062 44630 49743 60358 26022

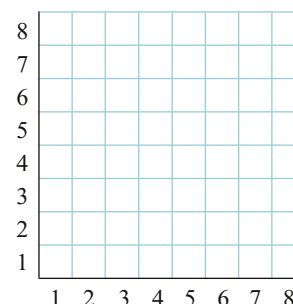
- 8** A supermarket manager wants a sample of his shop assistants to complete a questionnaire about rosters.

- a** Explain how 10 shop assistants could be selected at random from the 70 employed by the supermarket.
- b** Which shop assistants would be selected if the manager used the list of random numbers from question 7 and also started at the first number?

- 9** Explain how a random sample of 100 students could be selected from a school of 760 students.



- 10** A medical researcher wants to collect data from a random sample of hospitals in NSW that perform heart bypass surgery.
- Explain how he could select a random sample of six hospitals from the 12 that perform this type of surgery.
 - Which hospitals would be selected if the researcher used the following list of random numbers to do this, starting with the first digit?
 48047 45381 33232 35178
 46971 85879 31458 22016
 08695 58112 96070 91910
 18868 52251 99827 32581
- 11** **a** Use the list of random numbers from question 7 to select 10 squares at random from the grid shown. For example, starting at the first pair of digits and moving right, the first suitable ordered pair is (2, 4), the next is (6, 3), etc.
- b** Copy the grid and shade the squares selected.



Generating random numbers by calculator

Most modern scientific calculators have a random number generator. On Casio calculators this function key is labelled RAN#. By pressing **SHIFT** **.** (RAN#) **=** the calculator produces a random number from 0 to 0.999.

To obtain another random number it is only necessary to press **=** again.

To obtain a one-digit random number press **SHIFT** **.** **×** 10 **=** and truncate the digits after the decimal point, to produce an integer.

(Truncate means to shorten by cutting off a part.)

To obtain a two-digit random number press **SHIFT** **.** **×** 100 **=** and truncate the digits after the decimal point, to produce an integer.

To obtain a three-digit random numbers press **SHIFT** **.** **×** 1000 **=** and truncate the digits after the decimal point, to produce an integer.

To obtain a random number from a to b press **SHIFT** **.** **×** $(b - a) + a$ **=**

- 12** Use your calculator for the following questions.

- Produce a list of 20 one-digit numbers.
- Produce a list of 20 two-digit numbers.
- Produce a list of 20 three-digit numbers.
 - Are about half the numbers odd?
 - Are about half the numbers less than 500?
 - Should the answers for parts **ii** and **iii** always be 'yes'?

- 13** Produce a list of 10 random numbers:

- from 0 to 50
- from 10 to 50
- from 1 to 100
- from 50 to 100
- so $50 \leq \text{number} \leq 60$
- so $15 \leq \text{number} \leq 45$

Generating random numbers by computer

Random numbers can be generated using a spreadsheet. The following instructions apply to Microsoft Excel. Random numbers from 0 to 0.999... are generated using the RAND function.

To produce a list of one-digit random numbers:

- format cells to 0 decimal places
- type into the formula bar: =TRUNC(RAND()*10)
- press enter; highlight the first cell and drag down (or across) as many cells as needed and select **Fill Down** (or **Fill Right**).

The number of cells needed = the number of random numbers needed.

The TRUNC function truncates a number to an integer by removing the decimal part of the number.

To produce a list of two-digit random numbers:

- format cells to 0 decimal places
- type into the formula bar: =TRUNC(RAND()*100)
- press enter; highlight the first cell and drag down (or across) as many cells as needed and select **Fill Down** (or **Fill Right**).

To produce a list of three-digit random numbers:

- format cells to 0 decimal places
- type into the formula bar: =TRUNC(RAND()*1000)
- continue as above for two-digit random numbers.

To produce a list of random numbers from a to b :

- format cells to 0 decimal places
- replace a and b with your chosen values and type into the formula bar: =TRUNC(RAND()*($b-a$)+ a)
- continue as above for two-digit random numbers.

14 Create a spreadsheet using the instructions above. An example of this is shown below.

	A	B	C
1	Two-digit numbers	Three-digit numbers	Numbers from 30 to 45
2			
3	92	333	32
4	99	675	31
5	61	955	44
6	81	967	43
7	8	297	30
8	70	682	44
9	47	510	40
10	51	989	35
11	47	826	44
12	73	948	31
13			

The formula in cell A3 is =TRUNC(RAND()*100)

The formula in cell B3 is =TRUNC(RAND()*1000)

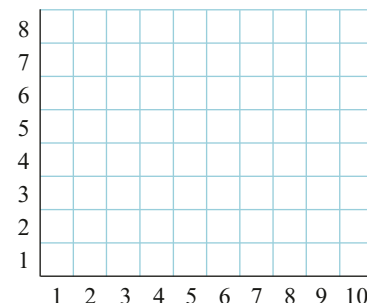
The formula in cell C3 is =TRUNC(RAND()*(45-30)+30)

- 15** Use a spreadsheet to produce a list of 20 random numbers that are:
- a** one-digit numbers
 - b** two-digit numbers
 - c** three-digit numbers
- 16** Use a spreadsheet and **Fill Down** and **Fill Right** to make a table of single-digit random numbers.
- 17** Use a spreadsheet to produce a list of 10 random numbers:
- a** from 0 to 50
 - b** from 10 to 50
 - c** from 1 to 100
 - d** from 50 to 100
 - e** so $50 \leq \text{number} \leq 60$
 - f** so $15 \leq \text{number} \leq 45$

- 18** A gardener spreads flower seeds over a large plot of ground. To estimate the number of seeds that germinate, he divides the plot of land into eighty squares, measuring 1 m by 1 m, as shown in this grid.

He selects 10 squares at random and counts the number of seedlings in each square. By using the numbers he counted to find an average number of seedlings per square, he can estimate the total number of seedlings in the garden.

- a** Explain how the gardener could select a random sample of 10 squares.
- b** Use a list of random numbers to select 10 squares at random.
- c** If the numbers of seedlings in the squares selected are 21, 17, 18, 13, 27, 9, 22, 10, 18, 25:
 - i** calculate the average number of seedlings per square
 - ii** estimate the total number of seedlings in the plot.



- 19** A school principal wishes to select a random sample of teachers to find out the amount of homework that is being set each day for Year 11 students.
- a** Explain two methods that the principal could use to choose a random sample of 10 teachers from the 62 teachers on the staff.
 - b** Which teachers would be selected if the principal used the following list of random numbers to do this, starting with the first digit?
 49246 36701 13499 48926 61279 36535 32337 15455 26345
 95085 05701 30998 95907 47062 44630 49743 60358 26022
 - c** If the number of minutes of homework set by the teachers selected is 20, 45, 30, 35, 40, 20, 25, 15, 50, 25, calculate the average amount of homework set by these teachers.
 - d** If each Year 11 student has six teachers, estimate the total amount of homework each student is set each day.
- 20** Complete this class activity.
- a** Assign a number to each student in your class.
 - b** Write your age, in years and months on the board, next to the number assigned to you.
 - c** Choose a random sample of five students from your class.
 - d** Calculate the average age of the five students in your sample.
 - e** Write your sample average on the board and compare the average of your sample with the averages of the other students' samples.
 - f** Calculate the average age of the class using the information on the board.
 - g** Compare the sample averages with the actual class average. Discuss and comment on any findings.
 - h** Find the average of the sample averages. How does this compare with the actual class average (from part f)? Discuss with the class.

- 21** Find out how geographers use grid sampling to determine percentages of land types.

4D Stratified random sampling

These resources are available on your obook assess:

- **Video tutorial 4D:** Watch and listen to an explanation of Example 4D-1
- **assess quiz 4D:** Test your skills with an auto-correcting multiple-choice quiz



stratified random sample

sample made up of random samples taken from subgroups (strata) of the population, proportional to the relative size of each subgroup

When we select a random sample of 10 students from a mixed class of 30 students, it is possible to get, for example, 10 boys. Similarly, when we select a random sample of 80 students from a school of 950, it is possible that we could get 80 students from the same year group. If it is important for our purpose to have a proportional mix, then simple random sampling may not produce a satisfactory sample. In these cases we can use a **stratified random sample**.

For a stratified random sample, the population is divided up into subgroups (strata), based on different characteristics such as age, gender, income, and so on. The stratified random sample is made up of random samples from each subgroup. The size of each random sample is proportional to the relative size of the subgroup in the population.

EXERCISE 4D Stratified random sampling

- 1 Complete the following to calculate the proportion of boys and girls that should be chosen in a stratified random sample of students selected on the basis of gender, if there are 75 boys and 65 girls.
Proportion of boys = $\frac{75}{140} = \frac{\square}{28}$ or ____% Proportion of girls = $\frac{\square}{140} = \frac{\square}{28}$ or ____%
- 2 Calculate the proportion of boys and girls that should be chosen in a stratified random sample of students selected on the basis of gender, if there are 180 students, of which 76 are boys and 104 are girls.

EXAMPLE 4D-1 Selecting a stratified random sample based on year groups

A school of 950 students is made up of the following numbers of students at each year level.

Year	7	8	9	10	11	12
Number of students	130	165	180	190	150	135

- a If a stratified random sample is to be taken on the basis of these subgroups (school years), how many students should be chosen from each year if we want a stratified random sample of 80 students?
- b Explain how to obtain this sample.

	Solve	Think	Apply
a	Number of Year 7 students = $\frac{130}{950} \times 80 \approx 11$ Number of Year 8 students = $\frac{165}{950} \times 80 \approx 14$ Number of Year 9 students = $\frac{180}{950} \times 80 \approx 15$ Number of Year 10 students = $\frac{190}{950} \times 80 = 16$ Number of Year 11 students = $\frac{150}{950} \times 80 \approx 13$ Number of Year 12 students = $\frac{135}{950} \times 80 \approx 11$	The proportion of students in Year 7 is 130 out of 950 or $\frac{130}{950}$. So, the number of Year 7 students to be selected is $\frac{130}{950} \times 80$. Repeat the calculation for each year group.	Determine the proportion and, therefore, the number of students to be selected from each year group. The stratified sample is then made up of random samples from each year group.
b	We select a random sample of 11 Year 7 students, 14 Year 8 students, 15 Year 9 students, 16 Year 10 students, 13 Year 11 students and 11 Year 12 students.		

- 3 A school of 859 students is made up of the following numbers of students at each year level.

Year	7	8	9	10	11	12
Number of students	125	142	175	160	134	123

- If a stratified random sample is to be taken on the basis of these subgroups (school years), what proportion of students should be chosen from each year?
- How many students should be chosen from each year if we want a stratified random sample of 60 students?
- Explain how to obtain this sample.

- 4 A school of 439 students is made up of the following numbers of students at each year level.

Year	7	8	9	10	11	12
Number of students	96	84	78	65	56	60

- If a stratified random sample is to be taken on the basis of these subgroups (school years), what proportion of students should be chosen from each year?
- How many students should be chosen from each year if we want a stratified random sample of 40 students?
- Explain how to obtain this sample.

- 5 In Year 11 at Greengate High School there are 69 girls and 51 boys.

- In a stratified random sample that is selected on the basis of gender, what proportion of girls and boys should be in the sample?
- How many girls and boys should we select for a stratified random sample of 40 students?
- Explain how to obtain this sample.

- 6 The ages of people who live in a small country town are shown in the following table. It is proposed to do a dental check by choosing a stratified random sample of the population on the basis of these age groups.

Age	0–9	10–19	20–29	30–39	40–49	50–59	60–69	70+
Number of people	180	835	260	350	280	250	200	120

- What proportion of each age group should be included in the sample?
- What number of people in each age group should be selected for a sample of 40?
- Explain how to obtain this sample.

- 7 Discuss what subgroups could be used in a stratified random sample to investigate:

- the number of people with private healthcare cover
- the political party that people intend to vote for at the next election.

- 8 Of the 150 students in Year 11, 87 travel to school by bus, 23 ride bicycles, 30 travel by private car and 10 walk. In order to find their average time of travel to school, it is decided to take a stratified random sample on the basis of these subgroups (methods of travel).

- What proportion of each subgroup should be in the sample?
- How many of each subgroup should be in a sample of 30 students?
- Explain how to obtain this sample.



4E Systematic sampling

These resources are available on your obook assess:

- **assess quiz 4E:** Test your skills with an auto-correcting multiple-choice quiz

systematic sample

sample chosen in a methodical way when members of the population are put in order, the first number is chosen at random and then every n th number is chosen for the sample

In systematic sampling, a sample is chosen in a systematic way. To obtain a **systematic sample**, each member of the population is assigned a number, the first number is selected at random, and then we select every n th number; that is, we select numbers at regular intervals. The value of n , the size of the regular interval, depends on the size of the sample wanted and the size of the population.

For example, to find the interval size when selecting a systematic sample of 20 items from a population of 240 items, we calculate $240 \div 20 = 12$. That is, we should select every 12th item.

EXERCISE 4E Systematic sampling

- 1 A machine produces 300 items per day. At what interval should we select items to obtain a systematic sample of:
a 25 items? **b** 50 items? **c** 10 items? **d** 15 items?
- 2 An electoral roll has 1200 names on it. At what interval should we select names to obtain a systematic sample of:
a 100 names? **b** 200 names? **c** 150 names? **d** 40 names?

EXAMPLE 4E-1 Explaining how to select a systematic sample

Explain how to obtain a systematic sample of 10 students from a year level of 150 students.

Solve	Think	Apply
$150 \div 10 = 15$, so start at any number then select every 15th number.	Assign the numbers 1 to 150 to each student. Divide the size of the population by the size of the sample wanted: $150 \div 10 = 15$. Start at any number then select every 15th number. For example: 1 If we started at the student who was assigned number 7, then we would select the numbers 7, 22, 37, 52, 67, 82, 97, 112, 127, 142 to form our sample. 2 If we started at the student who was assigned number 24 we would select the numbers 24, 39, 54, 69, 84, 99, 114, 129, 144, 9 (because 159 would have been the next number, but there are only 150 numbers, so we start at the beginning again and $159 - 150 = 9$). 3 If we started at the student with the number 56, then we would select the numbers 56, 71, 86, 101, 116, 131, 146, 11 (which is $161 - 150 = 11$), 26, 41. <i>Note:</i> in the 'Think' examples 2 and 3 above we need to go back to the start of the list.	Assign the numbers 1 to 150 to the students. Divide the size of the population by the sample size to get the interval size. Start at any random number and select students at the interval size.

- 3** **a** Explain how to obtain a systematic sample of 10 students from a year level of 200 students.
b Which students would be selected if we started at the student numbered:
i 10? **ii** 15? **iii** 30? **iv** 45? **v** 113?
- 4** **a** Explain how to obtain a systematic sample of 15 names from a roll of 180 names.
b Which names would be selected if we started at the name numbered:
i 5? **ii** 10? **iii** 25? **iv** 50? **v** 120?

EXAMPLE 4E-2 Selecting a systematic sample from different populations

Explain how to select a systematic sample of 20 names from a list of:

a 146 names

b 156 names.

	Solve	Think	Apply
a	$146 \div 20 = 7.3 \approx 7$ Start at any number and select every 7th number until we have a sample of 20 names.	Assign the numbers 1 to 146 to the names. Population size is 146, sample size is 20. $146 \div 20 = 7.3$. Round 7.3 to the nearest whole number to get the interval size ($n = 7$).	Assign numbers to each name. Divide the population size by the sample size. Round to the nearest whole number. Start at any random number and select names at intervals according to the interval size.
b	$156 \div 20 = 7.8 \approx 8$ Start at any number and select every 8th number until we have a sample of 20 names.	Assign the numbers 1 to 156 to the names. Population size is 156, sample size is 20. $156 \div 20 = 7.8$. Round 7.8 to the nearest whole number to get the interval size ($n = 8$).	

- 5** Explain how to obtain a systematic sample of 10 names from a list of:
a 62 names **b** 87 names **c** 124 names **d** 239 names **e** 185 names.
- 6** A machine produces 200 numbered engine parts in a day.
a At what intervals should a part be selected for a quality check if a systematic sample is required each day of:
i 5 parts? **ii** 12 parts?
b Choose a starting number at random and write the numbers of the engine parts you would select for each sample in part **a**.
- 7** A physical education teacher has a class of 30 students. At the start of the day's lesson he wants to use a systematic sample to select the captains of six teams with equal numbers of students in each team.
a Explain how he would do this.
b The teacher tosses two dice and uses the total, 9, as the number for the first captain selected. Which other students will be selected as captains?
c For the second half of the lesson the teacher requires five teams of equal size. If he again starts with student number 9 as the first captain selected, which other students will be selected as captains?
- 8** An army battalion consists of 600 men and women.
a A systematic sample of 10 soldiers is to be selected for a health check. Explain how this could be done.
b If the first soldier, selected at random, is number 23, list the numbers of the other soldiers selected.
c **i** Which soldiers should be selected for a health check if a systematic sample of 50 soldiers is required?
ii If the first soldier selected is number 17, what will be the number of the last soldier selected?

4F Suitability of sample types

These resources are available on your obook assess:

- **Interactive 4F:** Explore key ideas for methods of sampling
- **Worksheet 4F:** Practise your skills with extra problems for sample types
- **Investigation 4F:** Investigate differences between three sample types
- **assess quiz 4F:** Test your skills with an auto-correcting multiple-choice quiz

A good sample is one that has almost exactly the same characteristics as the total population. We say that a good sample is ‘representative’ of the population. In this section we will decide which sampling methods are most suitable in given situations and investigate the effect the size of a sample has on how well it represents the whole population.

Sampling methods

Random sampling: each member of the population has an equal chance of being selected in the sample.

Stratified random sampling: the population is divided into subgroups and then a random sample, proportional to the relative size of each subgroup, is taken from each subgroup.

Self-selected sampling: the participants choose to be part of the sample by responding to a request for volunteers.

Systematic sampling: the sample is chosen in a systematic way, with the first number being selected at random from the population and then further numbers being selected at regular intervals from the first number.

EXERCISE 4F Suitability of sample types

- 1 Discuss which sampling methods would be suitable for the following situations.
 - a A machine produces 5000 bolts in a day. A sample of 50 bolts per day is required for a quality-control check.
 - b There are 600 people on a town’s electoral roll. A sample of 40 people is required to complete a questionnaire.
 - c A sample of people is wanted to investigate average weekly expenditure on food.
 - d A sample is wanted of the people attending a football game.
 - e The school principal wants a sample of teachers to estimate how much homework is set each night.
 - f A shoe company employs five machinists. A weekly quality-control check of their work is required.
 - g A medical journal advertises for volunteers from the profession to participate in a research project.
 - h A sample of Year 11 students is required to estimate the number of left-handed people in the group.
 - i A grid sample is required by a geographer to estimate the proportions of various types of land use (for example, grazing, crops, forest, buildings).
 - j A sample is wanted in order to estimate the average weekly income of senior students.

nnnny	nnnyn	ynnnn	ynyny	ynyny	ynnyn	nyynn	ynyny	ynnyn	ynyny
nnyyy	yyyyy	nnnyy	nnnnn	nnyny	ynnny	nynnn	ynyny	nnyny	ynyyy
ynnnn	yyyyn	ynnnn	nynyn	nynnn	ynnny	nyynn	ynyny	ynynn	nyyyy
ynnyy	nyyny	ynynn	nyynn	nnnyy	ynyny	yyyny	ynnyy	nnyny	ynnnn

- When arranged within the categories male and female, the replies are:

<i>Male</i>	nnyny	nnnny	ynyyy	nyynn	yynyn	ynnyn	yynyy	ynyyy	yyynn
	nnynn	nnyny	nyynn	nnyny	nnynn	nyynn	yyny	nnyny	yynn
	yynyn	yynnn	nyyy	yyyn	ynn				
<i>Female</i>	nnyyy	yynyy	nnnyy	nynnn	nnyny	yynny	nynnn	yyny	nnny
	ynnyy	nyyny	ynynn	nyynn	nnnyy	yyny	ynnyy	nnny	nn

-

EXAMPLE 4F-1 Estimating the responses of a population from a representative sample

A sample of 50 people is chosen from a population of 70 000 people. The sample was carefully chosen to be representative of the entire population. The people in the sample were asked the following questions. Their responses are shown in the table.

	Yes responses	No responses	Unsure responses
Should a new cinema be built in the centre of town?	21	8	21
Is the public transport system adequate?	18	27	5

- Calculate the percentage of the sample who answered 'Yes' to a new cinema.
- Estimate the number of people from the entire population who would want a new cinema.
- Estimate the number of people from the entire population who would think the public transport system is inadequate.
- Estimate the number of people from the entire population who would be unsure about whether the public transport system is adequate.

	Solve	Think	Apply
a	$\frac{21}{50} \times 100\% = 42\%$	21 of the 50 people surveyed said 'Yes' to the new cinema.	A sample does not have to be large to be representative of the entire population. If the sample is chosen so that it is representative, then the results of the sample can be applied to the entire population.
b	$\frac{42}{100} \times 70\,000 = 29\,400$	The sample is representative of the entire population. Find 42% of the entire population.	
c	$\frac{27}{50} \times 70\,000 = 37\,800$	27 people surveyed thought the public transport system was inadequate. Find $\frac{27}{50}$ of the population of 70 000.	
d	$\frac{5}{50} \times 70\,000 = 7\,000$	Of the 50 surveyed, 5 people were unsure. Find $\frac{5}{50}$ of the population of 70 000.	

- A sample of 20 people is chosen from a population of 4000 people. The sample was carefully chosen to be representative of the entire population. The people in the sample were asked the following questions. The results are shown in the table.

	Yes responses	No responses	Unsure responses
Should a new fast-food restaurant be built in the centre of town?	12	6	2
Is the access to childcare adequate?	5	7	8

- Calculate the percentage of the sample who said 'Yes' to a new fast-food restaurant.
Percentage = $\frac{12}{\square} \times 100 = \underline{\hspace{2cm}}$
- Estimate the number of people from the entire population who would want a new fast-food restaurant.
Number = $\frac{60}{\square} \times \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

- c Estimate the number of people from the entire population who would think access to childcare is adequate.
 Adequate childcare = $\frac{5}{\square} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
- d Estimate the number of people from the entire population who would be unsure about the access to childcare.
 Unsure = $\frac{\square}{20} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

- 4 A sample of 40 people is chosen from a population of 50 000 people. The sample was carefully chosen to be representative of the entire population. The people in the sample were surveyed on the following questions. The results are shown in the table.

	Yes responses	No responses	Unsure responses
Should a new skate park be built in the town centre?	8	25	7
Are there enough parks?	28	3	9

- a Calculate the percentage of the sample who said 'No' to a new skate park.
- b Estimate the number of people from the entire population who would not want a new skate park.
- c Estimate the number of people from the entire population who would think there are enough parks.
- d Estimate the number of people from the entire population who would answer 'No' to the question 'Are there enough parks?'
- 5 A sample is selected from a school population of 800. The 40 students chosen are representative of the entire school population. Of the students surveyed, 28 said they were happy with the school uniform and the rest were not.
- a Calculate the percentage of the sample who are happy with the uniform.
- b How many students from the entire school population would be happy with the uniform?
- c How many students from the entire school population would be unhappy with the uniform?
- 6 A sample is selected from a school population of 600. The 20 students chosen are representative of the entire school population. Of the students surveyed, 6 said they were happy with the school canteen and the rest were not.
- a What percentage of the students in the sample are happy with the school canteen?
- b How many students from the entire school population would be happy with the school canteen?
- c How many students from the entire school population would be unhappy with the school canteen?
- 7 State whether you think the following samples would be satisfactory in estimating the nature of the whole population.
- a Using the proportion of boys and girls in your class to estimate the proportion of boys and girls in Year 11 at your school.
- b Using the proportion of boys and girls in your class to estimate the proportion of boys and girls in Year 11 across the state.
- 8 Write a short analysis of the following statistical statements. Is there anything misleading about them?
- a Four out of five dentists recommend Britex toothpaste.
- b A newspaper reported that 10% of the Citico Company workforce had been dismissed.
- c Of the two major political parties, one claimed that unemployment was rising and the other claimed that employment was rising. Could both be correct?
- 9 Investigate the contribution made to statistics by the following people: John Graunt, Carl Friedrich Gauss, Florence Nightingale, Sir Ronald Fisher, W. Edwards Deming and John Tukey. Justify why their contribution to statistics is so important.

CHAPTER 4 REVIEW CLASSIFYING DATA AND SAMPLING

You should be able to:

- ✓ classify data as nominal, ordinal, discrete or continuous
- ✓ identify whether a census or a sample is appropriate for data collection
- ✓ recognise that the purpose of a sample is to estimate a particular characteristic of the whole population
- ✓ understand the concept of bias in a sample
- ✓ describe and use random, stratified, self-selected and systematic samples to collect data
- ✓ generate random numbers using a table, calculator or spreadsheet to establish random samples
- ✓ determine the appropriate sample type for a given situation.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

REVIEW MULTIPLE-CHOICE QUESTIONS

- 4A** 1 Which of the following is *not* an example of a numerical continuous variable?
A heights of students B wages of workers C daily temperatures D time to get to work
- 4B** 2 Which of the following is an example of data that could not be collected by a census?
A the life of a light globe B the number of vehicle accidents caused by speed
C the number of tries scored by a rugby team D the age of school teachers in NSW
- 4B** 3 Which of the following samples are *not* biased?
A a phone survey on a Friday night
B a survey of the audience at the opera
C 10 students are selected from a class by writing all names on slips of paper, putting them in a hat, shaking them and then selecting 10 slips
D a factory tests a sample of its products every Friday afternoon
- 4B** 4 Which of the following is an example of a self-selected sample?
A a researcher stands at the entrance to a bank and asks the first five people who arrive to participate in her survey
B from an alphabetical list of names, one is chosen at random and then every tenth name is selected
C a deck of cards is shuffled and then one card is selected at random
D five people respond to a phone-in poll on radio
- 4C** 5 Use the line of random numbers below to answer this question.
40942 42373 38710 39916 08187 00133 16288 64277
Starting at the first digit (4) and moving to the right, the first two-digit number less than 20 is:
A 3 B 9 C 10 D 16
- 4D** 6 Of the 150 students in Year 11, 70 are boys and 80 are girls. In a stratified random sample based on gender, the proportion of boys who should be chosen is:
A $\frac{7}{8}$ B $\frac{7}{15}$ C $\frac{8}{15}$ D $\frac{8}{7}$
- 4D** 7 For the information in question 6, the number of girls who should be chosen for a sample of 60 students is:
A 53 B 28 C 32 D 69

- 4E** ▶ **8** A machine produces 6000 items in a week. To obtain a systematic sample of 200, we could start with the 50th item and then select the items numbered:
- A 80, 110, 140, 170, ... B 200, 400, 600, 800, ...
 C 110, 170, 230, 300, ... D 250, 450, 650, 850, ...
- 4F** ▶ **9** To investigate the average time it takes for Year 11 students to get to school, the most suitable sample type would be:
- A a random sample B a stratified random sample based on mode of travel
 C a systematic sample D a self-selected sample.
- 4F** ▶ **10** To estimate the number of boys and girls in Year 11 in NSW, a suitable sample would be the number of boys and girls in:
- A your class B the mathematics classes at your school
 C Year 11 at your school D Year 11 in a selection of schools in different regions.
- 4F** ▶ **11** A sample is selected from a school population of 900. The 30 students chosen are representative of the entire school population. Of the students surveyed, 25 said they were happy with the school canteen and the rest were not. How many students from the entire school population would not be happy with the school canteen?
- A 5 B 25 C 150 D 750

REVIEW SET 1

- Classify this data as categorical nominal, categorical ordinal, numerical discrete or numerical continuous.
 - number of cloudy days each month
 - heights of the girls in a netball team
 - dress sizes (small, medium, large, etc.) in a store
 - types of fish caught by anglers
- State whether a census or a sample would be used for each of the investigations in question 1.
- Comment on any possible bias in the following situations:
 - a phone survey on Saturday night
 - a questionnaire is sent to a sample of people chosen from the electoral roll.
- Explain how a random sample of 12 students could be chosen from a class of 30.
- Use the list of random numbers given below to find a random selection of five squares from the grid shown.

08695	58112	96070	91910	18868	52251	99827	32581
90070	98873	89846	50953	92529	68249	54949	83829
10132	27359	13017	41045	13817	65603	87615	55691
 - Copy the grid and shade the squares selected.



5					
4					
3					
2					
1					
	1	2	3	4	5

- 6** A school population is made up of the following numbers of students at each year level. A stratified random sample on the basis of year groups is to be taken.

Year	7	8	9	10	11	12
Number of students	85	82	89	96	75	73

- a** What proportion of each year should be selected?
b How many from each year should be selected for a sample of 100 students?
- 7** Explain how to obtain a systematic sample of 20 students from a school roll of 300.
- 8** Give two examples of a self-selected sample.
- 9** A sample of 20 students is selected from a school population of 400 students. The sample chosen is representative of the entire school population. Of the students surveyed, six said they were happy with the school uniform and the rest were not.
- a** What percentage of the students from the sample are happy with the uniform?
b How many students from the entire school population would be happy with the uniform?
c How many students from the entire school population would be unhappy with the uniform?
- 10** The manager of a bus depot wants a sample of drivers to complete a questionnaire about duty rosters.
- a** Explain how eight drivers could be selected at random from the 74 who work at the depot.
b Which drivers would be selected if the manager used the list of random numbers given in question 5, by starting at the first digit in the second row and moving to the right?
- 11** State whether or not you think the size of the following samples would be satisfactory for estimating the characteristics of the whole population.
- a** Using the number of students in your class who approve of the school uniform to estimate approval of the school uniform by all Year 11 students (consider three classes, six classes, etc.)
b Using the cars passing the front of the school to estimate the most popular colours of cars



REVIEW SET 2

- 1** Write two examples of categorical data (one categorical nominal data and one categorical ordinal date).
- 2** Would a census or a sample be used to investigate:
- a** the number of people who use Freshspray deodorant?
b the consumption of alcohol by senior students in your school?
- 3** Describe the sample you would choose if you wanted to gather support for:
- a** improved netball facilities at the local park
b more lines for serving senior students at the school canteen.
- 4** Describe samples with the opposite bias to those in question 3.
- 5** Choose five random three-digit numbers less than 200 from the table below. Start at the first digit and move right.

08695	58112	96070	91910	18868	52251	99827	32581
90070	98873	89846	50953	92529	68249	54949	83829
10132	27359	13017	41045	13817	65603	87615	55691

- 6** This table shows the number of pairs of shoes each of four machinists produces in a week. The factory manager wants to carry out a quality check using a stratified random sample of the shoes produced by each machinist.

Machinist	Vince	Sophie	Talia	Mohammed
Pairs of shoes	56	43	51	50

- a** What proportion of shoes from each machinist should be checked?
b How many shoes produced by each machinist should be selected for a sample of 20 shoes?
- 7** A university administrator places an advertisement on an online notice board asking final year students to volunteer for a series of focus studies on access to tutors. What type of sampling is this?
- 8** List the steps you would use to produce one-digit numbers on your calculator.
- 9** Explain two methods that could be used to choose a random sample of six students from a class of 24.

REVIEW SET 3

- 1** Classify the following as categorical nominal, categorical ordinal, numerical discrete or numerical continuous.
- a** the eye colours of a group of students **b** the amount of sunshine each day
c the numbers of children in families **d** the quality of dishwashers (poor, satisfactory, good)
- 2** Give an example of data that could be collected using a:
- a** census **b** sample.
- 3** Describe a suitable sample to survey if you wanted to collect data about the:
- a** punctuality of the local bus service
b cleanliness of the council swimming pool.
- 4** **a** A machine produces 300 numbered engine parts in a day. At what intervals should a part be selected for a quality check if a systematic sample is required each day of:
i 20 parts? **ii** 12 parts?
b Choose a starting number at random and write the numbers of the engine parts you would select for each sample in part **a**.
- 5** Write two advantages and two disadvantages of a self-selected sample.
- 6** A sample of 15 students is selected from a school population of 300 students. The students chosen represent the entire school population. Of the students surveyed, 12 said they were happy with the school canteen and the rest were not.
- a** What percentage of the students in the sample are happy with the school canteen?
b How many students from the entire school population would be happy with the school canteen?
c How many students from the entire school population would be unhappy with the school canteen?
- 7** Outline the steps you would use to produce a list of random numbers from 1 to 100 using an Excel spreadsheet.
- 8** Use your calculator to obtain 10 random numbers between, and including, 60 and 80.
- 9** State whether or not you think the size of the following samples would be satisfactory in estimating the characteristic of the whole population.
- a** Surveying the people at a bus stop to estimate the suitability of timetables for all users of the bus route (consider the number of bus stops on the route)
b Surveying the people in a train carriage to estimate the cleanliness of all trains



REVIEW SET 4

- 1** A television station invites viewers to phone in their response to a question. What type of sample is this?
- 2** Write two examples of discrete data.
- 3** State the advantages and disadvantages of using a census to collect information.
- 4** Comment on any possible bias in the following samples.
 - a** Car tyre distributors are interviewed about the best-value tyre to buy.
 - b** A systematic sample is taken of the people listed on the first five pages of the telephone directory.
- 5** A factory has 120 employees. A sample of 25 employees is to be chosen to complete a questionnaire.
 - a** Explain how a random sample using a table of random numbers could be used to select the sample.
 - b** Explain how to obtain a random sample without using random numbers.
 - c** Explain how to obtain a systematic sample of employees.
- 6** Explain the meaning of the term 'bias'.
- 7** What is the purpose and characteristics of a good sample?
- 8** The list of random numbers below is to be used to select five random numbers less than 200. List the first five numbers selected if you start at the first digit (0) and move to the right.
08695 58112 96070 91910 18868 52251 99827 32581
90070 98873 89846 50953 92529 68249 54949 83829
10132 27359 13017 41045 13817 65603 87615 55691
- 9** A factory supervisor wants to interview a sample of workers about the use of the new equipment in the factory.
 - a** Explain how five workers could be selected at random from the 24 who work at the factory.
 - b** Which workers would be selected if the supervisor used the list of random numbers given in question 8, by starting at the first digit in the first row and moving to the right?



- 1 a** Classify the following data as categorical nominal, categorical ordinal, numerical discrete or numerical continuous.
 - i** marks scored in the yearly examination (1 mark)
 - ii** favourite ice-cream flavour (1 mark)
- b** State whether a census or a sample would be more appropriate to use to collect the following data.
 - i** the number of left-handed children in Sydney primary schools (1 mark)
 - ii** the number of motorcyclists killed on NSW roads in a year (1 mark)
- c** The list of random numbers below is to be used to select five random numbers less than 80. List the first five numbers selected if you start at the first digit in the second row (9) and move to the right.

08695	58112	96070	91910	18868	52251	99827	32581
90070	98873	89846	50953	92529	68249	54949	83829
10132	27359	13017	41045	13817	65603	87615	55691

 (1 mark)
- d** The manager of a bus depot wants a sample of drivers to complete a questionnaire on duty rosters.
 - i** Explain two methods that could be used to choose six drivers at random from the 48 drivers who work at the depot. (2 marks)
 - ii** Explain the likely bias if a self-selected sample was used to choose the six drivers. (1 mark)
- e** There are 67 boys and 53 girls in Year 11 at a high school. A stratified random sample is to be selected based on gender.
 - i** What proportion of boys and girls should be selected? (1 mark)
 - ii** How many boys and how many girls should be selected for a stratified random sample of 30 students? (2 marks)
- f** Explain how to obtain the sample in part e. (1 mark)
- g** A machine produces 600 numbered electrical parts each day. A systematic sample of 20 parts each day is required for a quality check.
 - i** At what intervals should parts be selected? (1 mark)
 - ii** If the first part selected at random is the 37th, what is the number of the last part selected for the sample? (1 mark)
- h** A sample is selected from a school population of 700. The 28 students chosen are representative of the entire school population. Of the students surveyed, 11 said they were happy with the school canteen and the rest were not.
How many of the students from the entire school population would be unhappy with the school canteen? (1 mark)

TOTAL: 15 marks

5

Perimeter, area and volume

The main mathematical ideas investigated are:

- ▶ calculating the perimeter of simple and complex shapes
- ▶ calculating the area of triangles, quadrilaterals, circles, sectors and composite shapes
- ▶ calculating perimeter and area of irregularly shaped blocks of land
- ▶ solving practical area problems
- ▶ calculating the surface area of prisms, cylinders, spheres and composite solids
- ▶ calculating the volume of right prisms, cylinders, spheres and composite solids
- ▶ converting between units of volume and capacity.

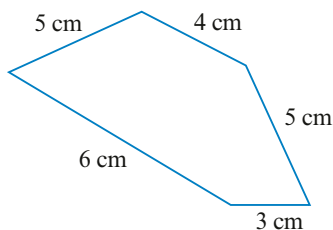
ARE YOU READY?

5A ▶ 1 What is 3.7 m converted to centimetres?

- A** 0.0037 cm **B** 0.037 cm
C 370 cm **D** 3700 cm

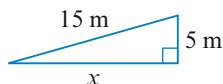
5A ▶ 2 What is the perimeter of this shape?

- A** 18 cm
B 20 cm
C 23 cm
D 1800 cm



5A ▶ 3 What is the length of the side labelled x in this triangle?

- A** 10 m **B** 14.1 m
C 15.8 m **D** 75 m



5A ▶ 4 A circle is divided into three equal sectors. What is the size of each sector angle?

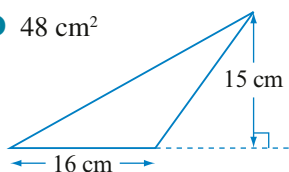
- A** 360° **B** 180°
C 120° **D** 60°

5B ▶ 5 What is the area of a rectangle with length 8 cm and breadth 3 cm?

- A** 11 cm^2 **B** 22 cm^2
C 24 cm^2 **D** 48 cm^2

5B ▶ 6 What is the perpendicular height of this triangle?

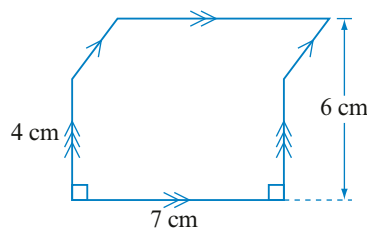
- A** 15 cm **B** 16 cm
C 18 cm **D** 30 cm



5B ▶ 7 What is the area of the triangle shown in question 6?

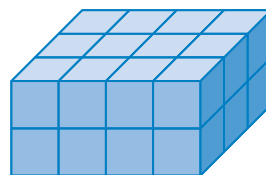
- A** 64 cm^2 **B** 120 cm^2
C 240 cm^2 **D** 288 cm^2

5C ▶ 8 This composite shape can be divided into two simpler shapes. What are the names of the two simpler shapes?



- A** rectangle and parallelogram
B rectangle and trapezium
C rectangle and triangle
D square and trapezium

5G ▶ 9 This rectangular prism is made up of blocks. Each block is 1 cm long, 1 cm wide and 1 cm high. What is the volume of this prism?



- A** 7 cm^3 **B** 8 cm^3
C 12 cm^3 **D** 24 cm^3

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your obook assess.

- Q1** Support sheet 5A.1 Converting units of length
Q2 Support sheet 5A.2 Finding perimeter
Q3 Support sheet 5A.3 Pythagoras' theorem
Q4 Support sheet 5A.4 Circles and sectors
Q5 Support sheet 5B.1 Area of a rectangle
Q6–7 Support sheet 5B.2 Area of a triangle
Q8 Support sheet 5C.1 Identifying simple shapes within composite shapes
Q9 Support sheet 5G.1 Finding volume

5A Perimeter

These resources are available on your obook assess:

- **Interactive 5A.1:** Explore key ideas for perimeter
- **Interactive 5A.2:** Explore key ideas for circumference of a circle
- **Video tutorial 5A:** Watch and listen to an explanation of Example 5A-5
- **Worksheet 5A:** Practise your skills with extra problems for perimeter
- **assess quiz 5A:** Test your skills with an auto-correcting multiple-choice quiz

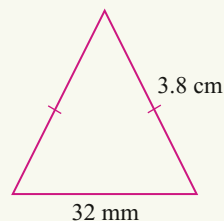
perimeter

the distance around the boundary of a closed figure

The **perimeter** of a closed figure is the distance around its boundary. The boundary may consist of straight or curved sides. The perimeter of a figure is found by adding the lengths of all its sides. All measurements must be expressed in the same unit before the side lengths are added.

EXAMPLE 5A-1 Calculating the perimeter after converting units

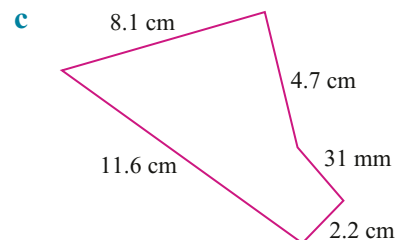
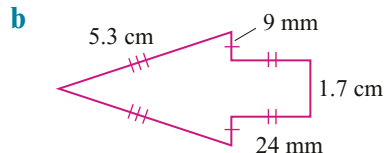
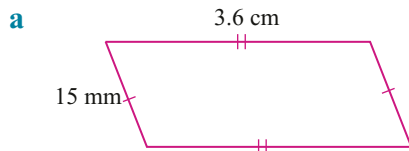
Calculate the perimeter of this figure, in millimetres



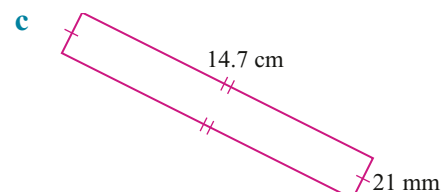
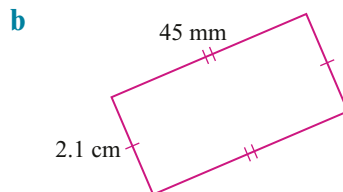
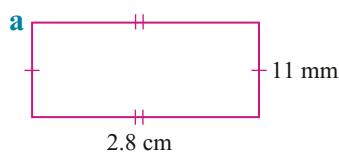
Solve	Think	Apply
Convert 3.8 cm to 38 mm. $P = 32 + 2 \times 38$ $= 108 \text{ mm}$	$1 \text{ cm} = 10 \text{ mm}$ $3.8 \times 10 = 38 \text{ mm}$ Multiply 38 by 2 because there are two sides marked as equal.	Convert units so all the sides are expressed in the same unit. Sides marked with identical markings are of equal length.

EXERCISE 5A Perimeter

1 Calculate the perimeter of each of these shapes, in millimetres.

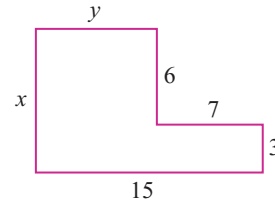


2 Calculate the perimeter of each of these rectangles. Give the answers in centimetres.

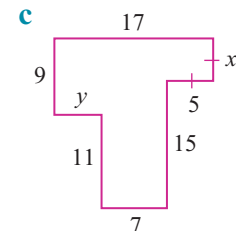
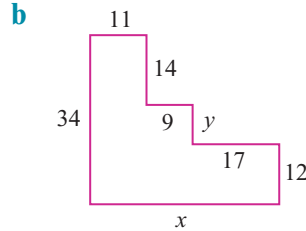
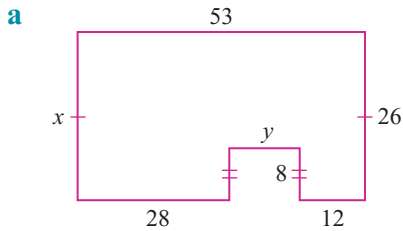


- 3 Complete the following to find the perimeter of this figure. All measurements are in centimetres.

$$\begin{aligned}x &= 6 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \\y &= \underline{\hspace{1cm}} - 7 = \underline{\hspace{1cm}} \\P &= 15 + 3 + 7 + 6 + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} \\&= \underline{\hspace{1cm}} \text{ cm}\end{aligned}$$

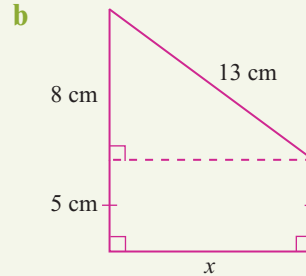
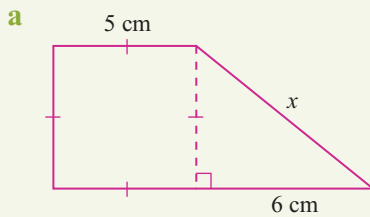


- 4 Find the unknown side lengths and calculate the perimeter of each of these figures. All measurements are in centimetres.



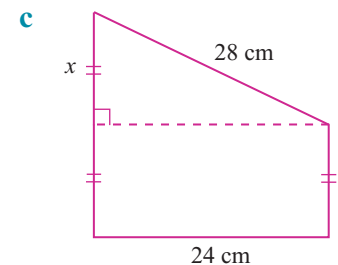
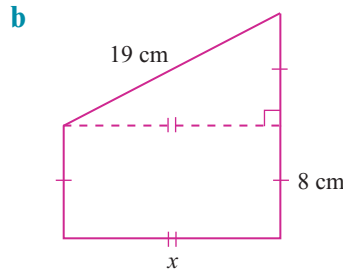
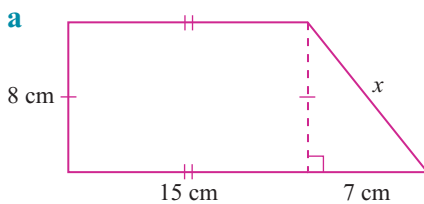
EXAMPLE 5A-2 Using Pythagoras' theorem to find perimeter

Find the perimeter of each of the following figures.



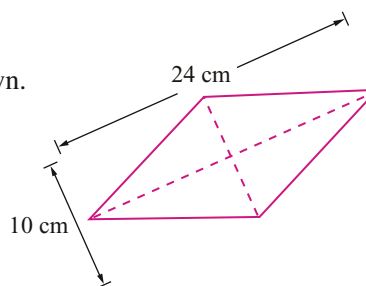
	Solve	Think	Apply
a	$x^2 = 5^2 + 6^2 = 61$ $x = \sqrt{61} \approx 7.8 \text{ cm}$ $P = 7.8 + 6 + 3 \times 5$ $= 28.8 \text{ cm}$	Use Pythagoras' theorem to find the hypotenuse. Then add all sides.	If any side length of a right-angled triangle is unknown, it can be calculated using Pythagoras' theorem. For finding the hypotenuse use $c^2 = a^2 + b^2$, where c is the hypotenuse and a and b are the two shorter sides. For finding a shorter side use $a^2 = c^2 - b^2$.
b	$x^2 = 13^2 - 8^2 = 105$ $x = \sqrt{105} \approx 10.2 \text{ cm}$ $P = 10.2 + 8 + 13 + 2 \times 5$ $= 41.2 \text{ cm}$	Use Pythagoras' theorem to find the third side of the triangle. Then add all sides.	

- 5 Find x and calculate the perimeter of each of these figures, correct to one decimal place.



6 A rhombus has diagonals measuring 24 cm and 10 cm, as shown.

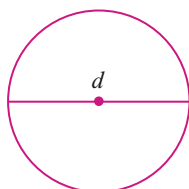
- a** Calculate the side length of the rhombus.
b Calculate the perimeter of the rhombus.



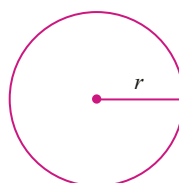
Circumference (or perimeter) of a circle:

Circumference = $\pi \times \text{diameter}$ or Circumference = $2 \times \pi \times \text{radius}$

$$C = \pi d$$



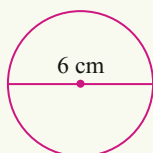
$$C = 2\pi r$$



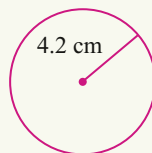
EXAMPLE 5A-3 Calculating the circumference of circles

Find the circumference (perimeter) of each of these circles, correct to one decimal place.

a



b



c

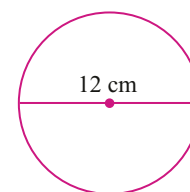


	Solve	Think	Apply
a	$C = \pi d$ $= \pi \times 6$ $\approx 18.8 \text{ cm}$	Use the formula $C = \pi d$ because the diameter is given.	Because the diameter and radius are related by $d = 2r$, either formula can be used at any time. Simply convert the diameter or radius to suit the formula used. This means only one formula needs to be learnt.
b	$C = 2\pi r$ $= 2 \times \pi \times 4.2$ $\approx 26.4 \text{ cm}$	Use the formula $C = 2\pi r$ because the radius is given.	
c	$C = 2\pi r$ $= 2 \times \pi \times 42$ $\approx 263.9 \text{ mm}$	The formula $C = 2\pi r$ can be used even when the diameter is given.	

7 Complete the following to find the circumference of this circle in two ways.

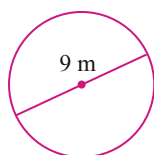
a $C = \pi d$
 $= \pi \times \underline{\hspace{1cm}}$
 $\approx \underline{\hspace{1cm}} \text{ cm}$

b $C = 2\pi r$
 $r = \underline{\hspace{1cm}} \div 2 = \underline{\hspace{1cm}}$
 $C = 2 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$
 $\approx \underline{\hspace{1cm}} \text{ cm}$



8 Calculate the circumference of each of these circles, correct to one decimal place.

a



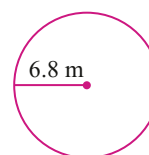
b



c



d



e diameter = 12.6 m

f radius = 7.9 m

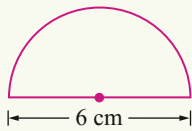
g radius = 2.5 m

h diameter = 8.64 m

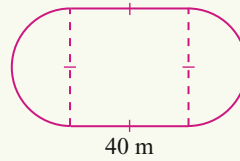
EXAMPLE 5A-4 Finding the perimeter of figures containing part of a circle

Find the perimeter of each of these figures, correct to one decimal place.

a



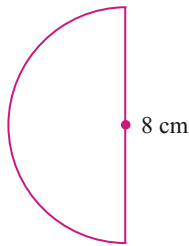
b



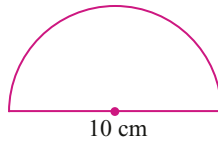
	Solve	Think	Apply
a	$\begin{aligned}\text{Curved part} &= \pi d \div 2 \\ &= \pi \times 6 \div 2 \\ &\approx 9.4 \text{ cm} \\ P &\approx 9.4 + 6 \\ &= 15.4 \text{ cm}\end{aligned}$	A semicircle is half a circle, so divide the circumference of a whole circle by 2 to find its arc length. Add the diameter to this length.	Divide the circumference of a whole circle depending on what part of the circumference is required. Sometimes there are two equal semicircles making one circle. Remember to add all lengths.
b	$\begin{aligned}\text{Curved part} &= \pi \times 40 \\ &\approx 125.7 \text{ m} \\ P &\approx 125.7 + 2 \times 40 \\ &= 205.7 \text{ m}\end{aligned}$	The two semicircles make one whole circle. Markings show all sides of the square are equal, so the diameter of the circle is 40 m. Add all lengths.	

9 Find the perimeter of each of these figures, correct to one decimal place.

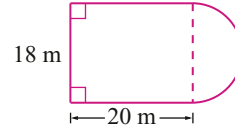
a



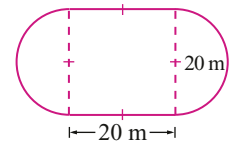
b



c

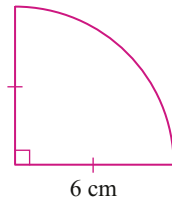


d

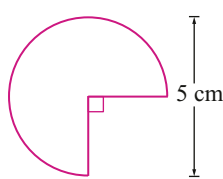


10 Find the perimeter of each of these figures, correct to one decimal place.

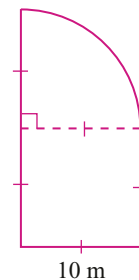
a



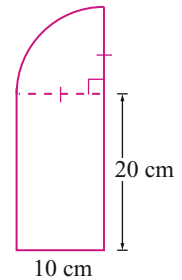
b



c

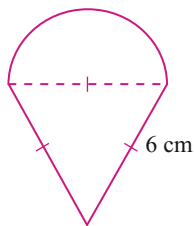


d

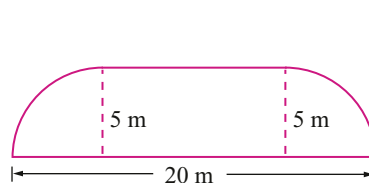


11 Find the perimeter of each of these figures, correct to four significant figures.

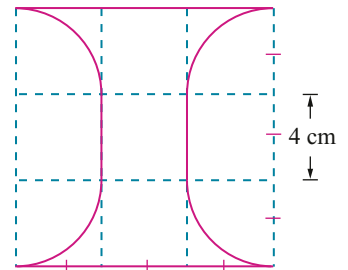
a



b



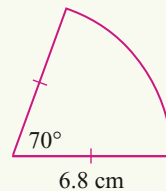
c



EXAMPLE 5A-5 Finding the perimeter of a sector

a Determine what fraction of a circle is drawn.

b Determine the perimeter of the sector, correct to one decimal place.



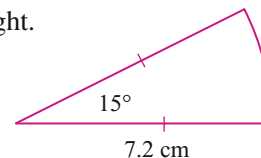
	Solve	Think	Apply
a	Fraction = $\frac{70^\circ}{360^\circ} = \frac{7}{36}$	There are 360° in a circle. The fraction is the sector angle 70° divided by 360° .	Calculate the fraction of the circle by dividing the sector angle by 360° , then multiply by the normal circumference formula to obtain the length of the curved part. Remember to add all lengths.
b	Curved part $= \frac{7}{36}$ of circumference of circle $= \frac{7}{36} \times 2\pi r$ $= \frac{7}{36} \times 2 \times \pi \times 6.8$ $= 8.307\dots$ ≈ 8.3 cm $P \approx 8.3 + 6.8 + 6.8$ $= 21.9$ cm		

12 a Complete the following to determine what fraction of a circle is drawn on the right.

$$\text{Fraction} = \frac{\square^\circ}{360^\circ} = \underline{\hspace{1cm}}$$

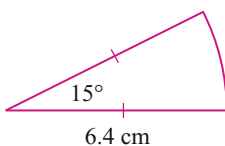
b Determine the perimeter of the sector shown, correct to one decimal place.

$$P = \frac{1}{\square} \times 2 \times \pi \times \underline{\hspace{1cm}} + 7.2 + \underline{\hspace{1cm}} \approx \underline{\hspace{1cm}} \text{ cm}$$

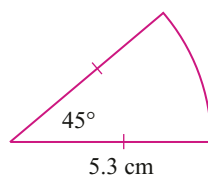


13 Determine what fraction of a circle is drawn each time, then calculate the perimeter of each sector, correct to one decimal place.

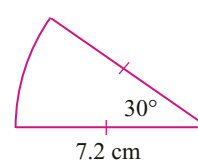
a



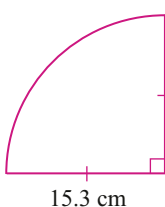
b



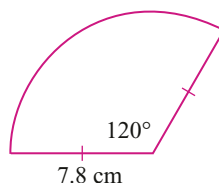
c



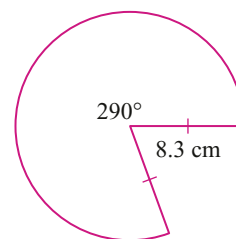
d



e

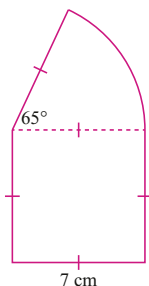


f

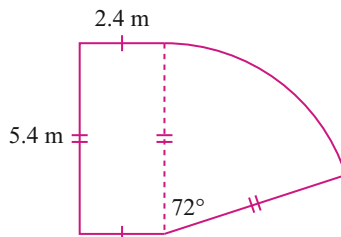


14 Find the perimeter of each of these figures, correct to one decimal place.

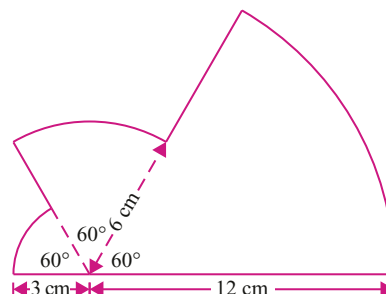
a



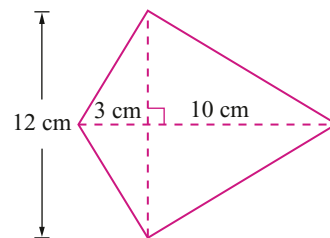
b



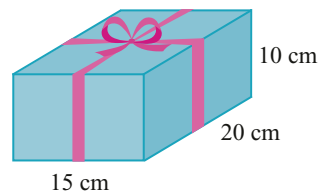
c



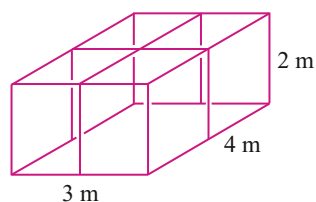
- 15** A kite has diagonals as shown.
- Calculate the length of one short side of the kite.
 - Calculate the length of one long side of the kite.
 - Calculate the perimeter of the kite.



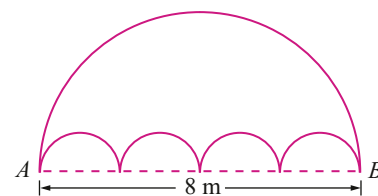
- 16** Find the total length of ribbon used to gift wrap a box as shown. An extra 15 cm is required for the knot and bow.



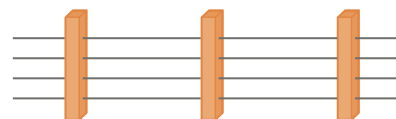
- 17** The frame of a tool shed consists of square galvanised tubing that costs \$4.65 a metre. Find the total cost of the tubing needed to make the frame of this shed.



- 18 a** Find the lengths of the paths from A to B along the four small semicircles of equal diameter, and along the larger semicircle, correct to two decimal places.
- Which distance is shorter?
 - What is the difference between the lengths of the two paths?



- 19** A farmer decides to fence a 400 m by 350 m paddock with a four-strand wire fence. Find the total cost of the wire required, given that single strand wire costs 12.4 cents per metre.



- 20** A circular plate has a circumference of 50 cm. Find its radius, correct to one decimal place.
- 21** Georgette runs around a circular track with a circumference of 200 m. Find the radius of the track, to one decimal place.
- 22** A satellite has a circular orbit 800 km above the Earth's surface.
- If the radius of the Earth is 6400 km, find the radius of the orbit of the satellite.
 - Find the circumference of the satellite's orbit, to the nearest kilometre.
 - If the satellite makes one orbit in a day, find the speed of the satellite.

- 23** The production director of a newspaper company decides that a plastic wrapper needs to be placed around its newspapers. Each wrapper is 50% longer than the circumference of the rolled-up paper, and the average diameter of a rolled-up paper is 5 cm. Find the number of kilometres of wrapper required to wrap the 275 000 newspapers printed daily. Give your answer correct to two decimal places.



5B Area of simple shapes

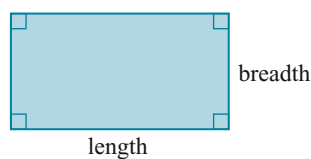
These resources are available on your [obook assess](#):

- **Interactive 5B.1:** Explore key ideas for area of a parallelogram
- **Interactive 5B.2:** Explore key ideas for area of a triangle
- **Interactive 5B.3:** Explore key ideas for area of a circle
- **Worksheet 5B:** Practise your skills with extra problems for area
- **Investigation 5B:** Investigate finding maximum area given the perimeter
- **assess quiz 5B:** Test your skills with an auto-correcting multiple-choice quiz

Rectangle

Area = length $\times$ breadth

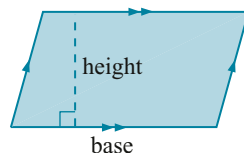
$$A = lb$$



Parallelogram

Area = base $\times$ height

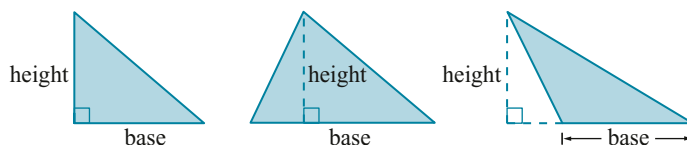
$$A = bh$$



Triangle

Area = $\frac{1}{2} \times$ base $\times$ height

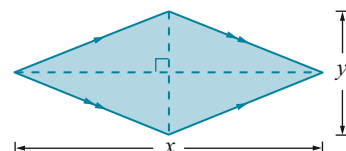
$$A = \frac{1}{2}bh$$



Rhombus

Area = $\frac{1}{2}$ product of the lengths of the diagonals

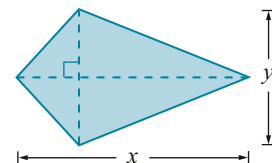
$$A = \frac{1}{2}xy, \text{ where } x \text{ and } y \text{ are the lengths of the diagonals}$$



Kite

Area = $\frac{1}{2}$ product of the lengths of the diagonals

$$A = \frac{1}{2}xy, \text{ where } x \text{ and } y \text{ are the lengths of the diagonals}$$

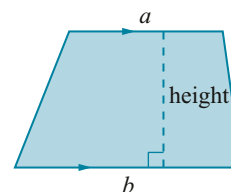


Trapezium

Area = $\frac{1}{2} \times$ height $\times$ sum of the parallel sides

$$A = \frac{1}{2}h(a + b)$$

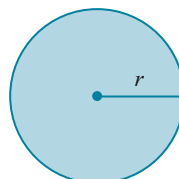
$$\text{or } A = \left(\frac{a+b}{2}\right)h \text{ or } A = \frac{h}{2}(a + b)$$



Circle

Area = $\pi \times$ radius squared

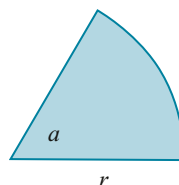
$$A = \pi r^2$$



Sector

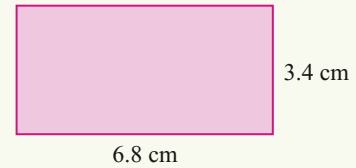
Area = fraction of circle $\times$ area of circle

$$A = \frac{a}{360^\circ} \times \pi r^2$$



EXAMPLE 5B-1 Finding the area of a rectangle

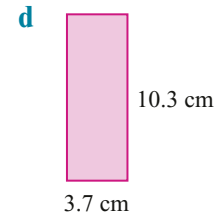
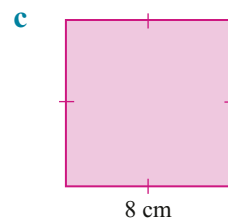
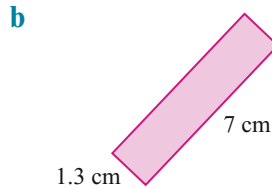
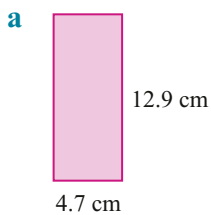
Find the area of the rectangle shown.



Solve	Think	Apply
$A = 6.8 \times 3.4 \text{ cm}^2$ $= 23.12 \text{ cm}^2$	Area = length $\times$ breadth $= lb$	Multiply side lengths to find the area.

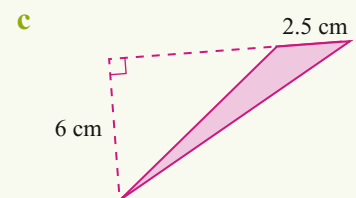
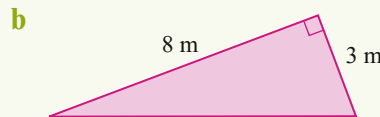
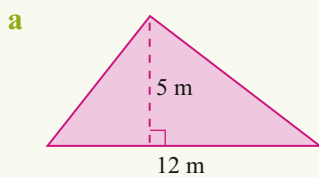
EXERCISE 5B Area of simple shapes

1 Find the area of each of these rectangles.



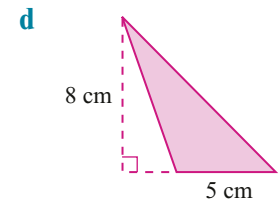
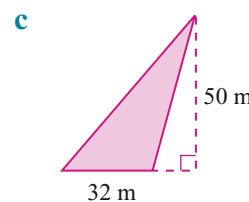
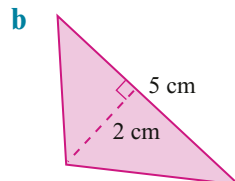
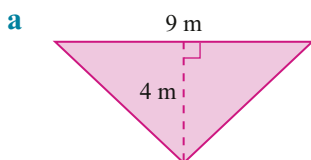
EXAMPLE 5B-2 Finding the area of a triangle

Find the area of each of these triangles.



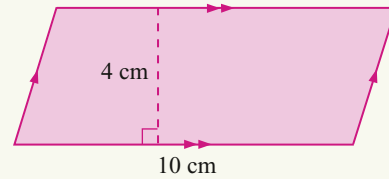
	Solve	Think	Apply
a	$A = \frac{1}{2} \times 12 \times 5 = 30 \text{ m}^2$	Area = $\frac{1}{2} \times \text{base} \times \text{height}$ $= \frac{1}{2}bh$	The area of a triangle is half the area of the rectangle with the same base and height.
b	$A = \frac{1}{2} \times 8 \times 3 = 12 \text{ m}^2$		
c	$A = \frac{1}{2} \times 2.5 \times 6 = 7.5 \text{ cm}^2$		

2 Find the area of each of these triangles.



EXAMPLE 5B-3 Finding the area of a parallelogram

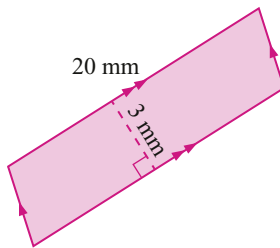
Find the area of this parallelogram.



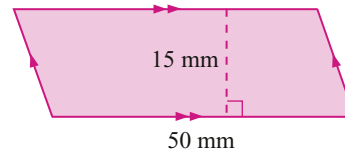
Solve	Think	Apply
$A = 10 \times 4$ $= 40 \text{ cm}^2$	Area = base $\times$ height $= bh$	The height is the perpendicular distance between the two parallel sides.

3 Find the area of each of these parallelograms.

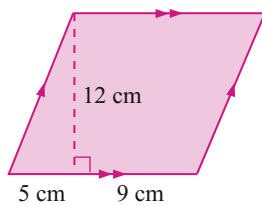
a



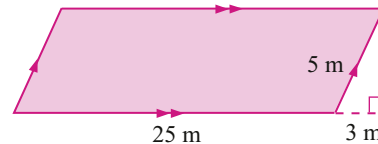
b



c

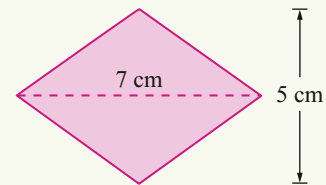


d



EXAMPLE 5B-4 Finding the area of a rhombus

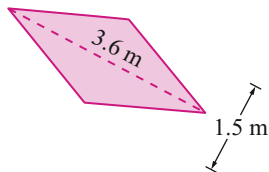
Find the area of a rhombus with diagonals of length 5 cm and 7 cm.



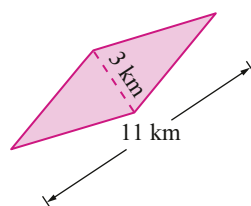
Solve	Think	Apply
$A = \frac{1}{2} \times 5 \times 7$ $= 17.5 \text{ cm}^2$	Area = $\frac{1}{2}$ product of the lengths of the diagonals $= \frac{1}{2} xy$	A rhombus has two congruent triangles, so the area formula is similar to that for a triangle.

4 Find the area of each rhombus below.

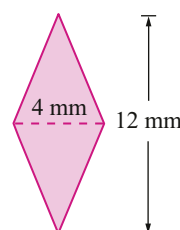
a



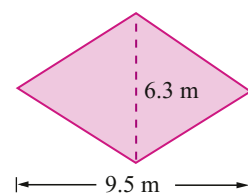
b



c

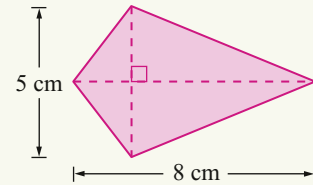


d



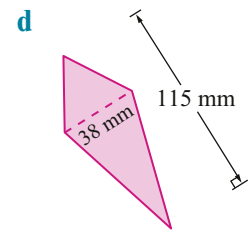
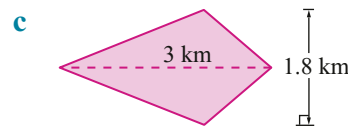
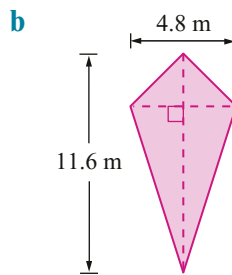
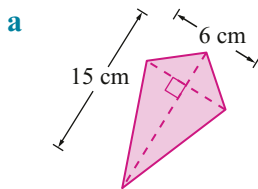
EXAMPLE 5B-5 Finding the area of a kite

Find the area of this kite.



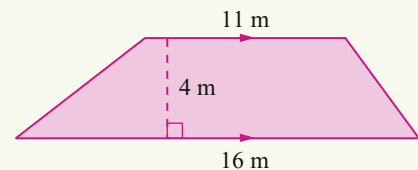
Solve	Think	Apply
$A = \frac{1}{2} \times 5 \times 8$ $= 20 \text{ cm}^2$	Area = $\frac{1}{2}$ product of the lengths of the diagonals $= \frac{1}{2}xy$	The diagonals are perpendicular. The formula is the same as for a rhombus.

5 Find the area of each kite below.



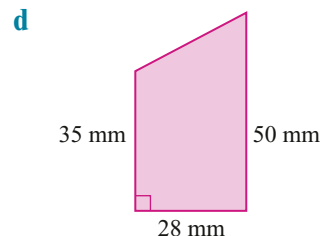
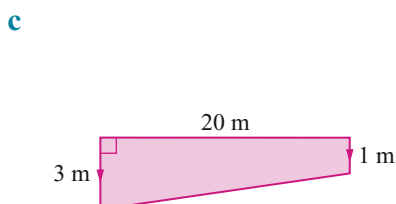
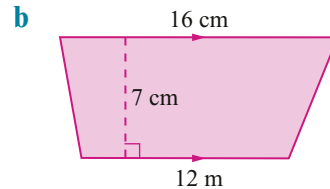
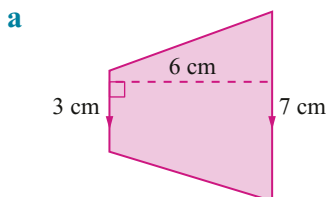
EXAMPLE 5B-6 Finding the area of a trapezium

Find the area of this trapezium. First identify the height, then use the formula.



Solve	Think	Apply
$A = \frac{1}{2} \times 4(11 + 16)$ $= 54 \text{ m}^2$	Area = $\frac{1}{2} \times \text{height} \times \text{sum of the parallel sides}$ $A = \frac{1}{2}h(a + b)$	The height of the trapezium is perpendicular to the parallel sides.

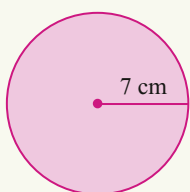
6 Find the area of each of these trapeziums. Identify the height first.



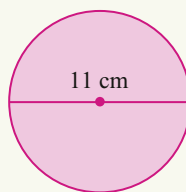
EXAMPLE 5B-7 Calculating the area of a circle

Calculate the area of each of these circles, correct to one decimal place.

a



b



a

$$A = \pi \times 7^2 \approx 153.9 \text{ cm}^2$$

Calculator: $\pi \times 7^2 =$

Think

$$A = \pi r^2$$

Substitute $r = 7$ into πr^2 .

Apply

The formula for the area of a circle requires the radius. If the diameter is given, halve it to find the radius.

b

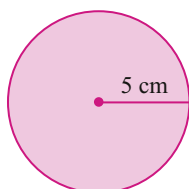
$$A = \pi \times 5.5^2 \approx 95.0 \text{ cm}^2$$

Calculator: $\pi \times 5.5^2 =$

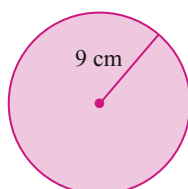
Halve the given diameter of 11 to get $r = 5.5$.
Substitute into πr^2 .

7 Calculate the area of each of these circles, correct to one decimal place.

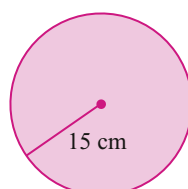
a



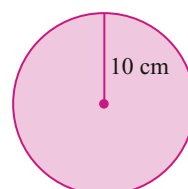
b



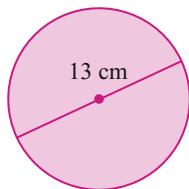
c



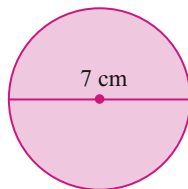
d



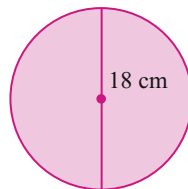
e



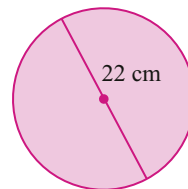
f



g



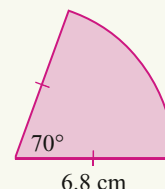
h



EXAMPLE 5B-8 Finding the area of a sector

a Determine what fraction of a circle is drawn.

b Determine the area to the nearest square centimetre (cm^2).



a

$$\text{Fraction} = \frac{70^\circ}{360^\circ} = \frac{7}{36}$$

b

$$\begin{aligned} \text{Area} &= \frac{7}{36} \text{ of area of circle} \\ &= \frac{7}{36} \times \pi r^2 \\ &= \frac{7}{36} \times \pi \times (6.8)^2 \\ &= 28.246... \\ &\approx 28 \text{ cm}^2 \end{aligned}$$

There are 360° in a circle. The fraction is the sector angle 70° divided by 360° .

Calculate the fraction of the circle by dividing the sector angle by 360° , then multiply by the area of the circle.

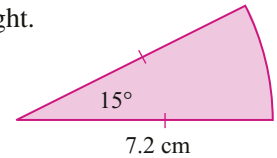
- 8 a** Complete the following to determine what fraction of a circle is drawn on the right.

$$\text{Fraction} = \frac{\square^\circ}{360^\circ} = \frac{\square}{\square}$$

- b** Determine the area of the sector to the nearest square centimetre (cm²).

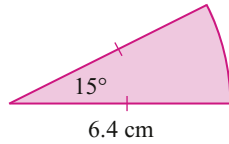
$$A = \frac{1}{\square} \times \pi \times (\square)^2$$

$$\approx \square$$

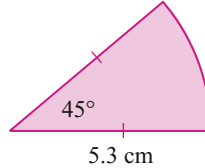


- 9** Determine the fraction of a circle that is drawn each time, then calculate each area to the nearest cm².

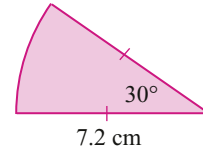
a



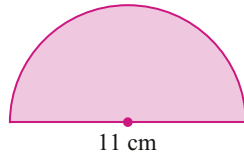
b



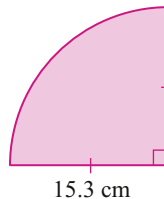
c



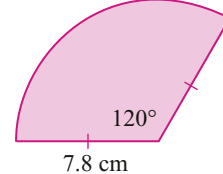
d



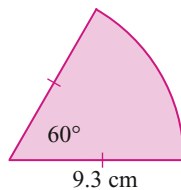
e



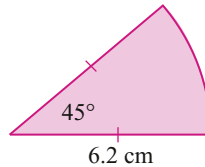
f



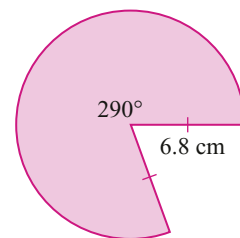
g



h



i



- 10 a** A square has an area of 81 cm². Find the side length.
b A rectangle has an area of 56 cm². Find the rectangle's length if its breadth is 7 cm.
c A rhombus has an area of 72 cm². If one diagonal is 18 cm long, how long is the other diagonal?
d Find the radius of a circle with an area of 100 m².
- 11** A farm covers land in the shape of a trapezium. The length of the northern boundary of the farm is 265 m and the length of the southern boundary is 180 m. The perpendicular distance from the northern end to the southern end is 90 m.
a Find the area of the farm to the nearest square metre.
b Write your answer for part **a** to the nearest hectare. (*Hint*: 1 ha = 10 000 m²).
c The farmer decides to purchase her neighbour's farm, which is in the shape of a rectangle but has the same area as her farm. What are possible dimensions (in metres) of her neighbour's farm?
- 12** A crop duster can carry 480 kg of fertiliser in its tanks. The pilot is required to fertilise a rectangular field of dimensions 2.8 km by 1.4 km. How many flights will be necessary if the fertiliser is to be spread at the rate of 40 kg/ha?

- 13** A circular drink coaster has a circumference of 28 cm. Calculate its area to the nearest square centimetre (cm²).
- 14** A cafe has square tables of side-length 1.3 m. They are covered by round tablecloths of diameter 1.9 m. What percentage of the tablecloth hangs over the edges of the table?



5C Area of composite figures

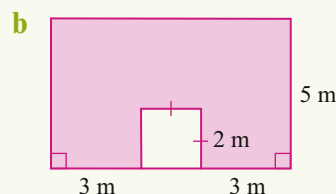
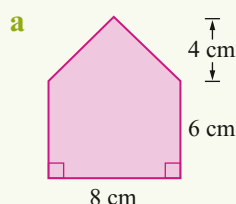
These resources are available on your obook assess:

- **Interactive 5C:** Explore key ideas for area of composite figures
- **Worksheet 5C:** Practise your skills with extra problems for area of shapes
- **Investigation 5C:** Identify using area or perimeter to solve problems
- **assess quiz 5C:** Test your skills with an auto-correcting multiple-choice quiz

Composite figures are made up of more than one plane shape. To find the area of a composite figure, first divide the figure into its component shapes and then add or subtract the areas.

EXAMPLE 5C-1 Finding the area of a composite figure

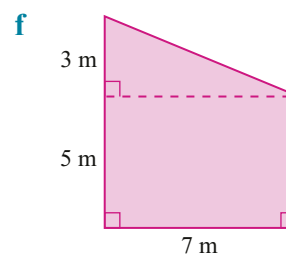
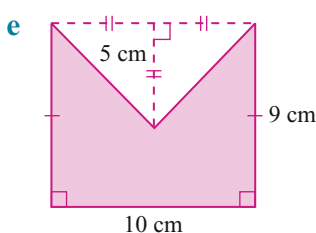
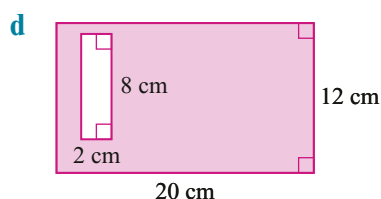
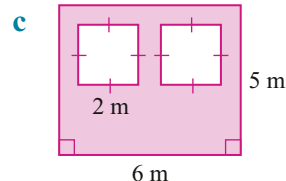
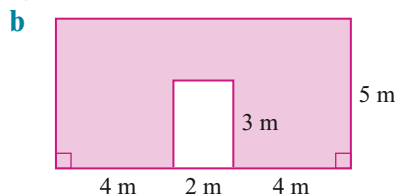
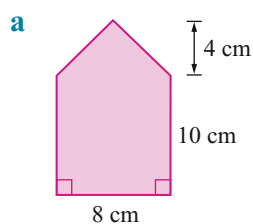
Find the shaded area of each of these figures.



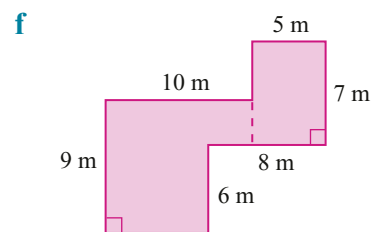
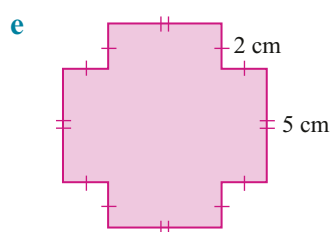
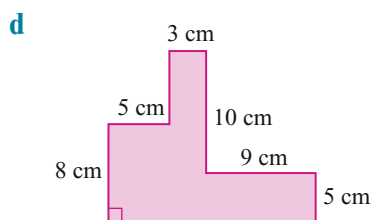
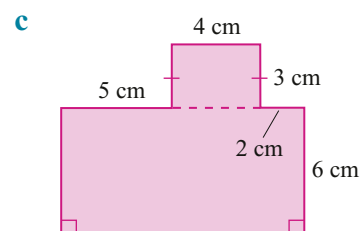
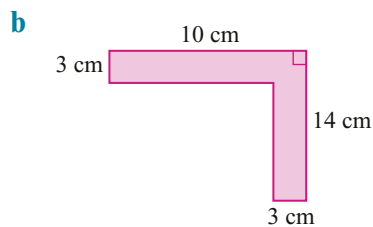
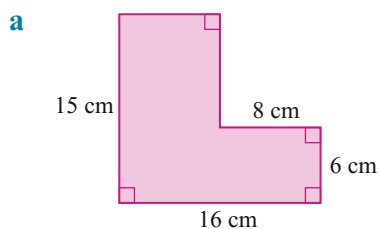
	Solve	Think	Apply
a	$\begin{aligned} \text{Area} &= A_{\text{rectangle}} + A_{\text{triangle}} \\ &= 8 \times 6 + \frac{1}{2} \times 8 \times 4 \\ &= 64 \text{ cm}^2 \end{aligned}$	The figure is made up of a rectangle and a triangle. The rectangle has length 8 cm and breadth 6 cm. The triangle has base 8 cm and height 4 cm.	Divide the composite figure into its component shapes. Find all the areas, then add or subtract.
b	$\begin{aligned} \text{Area} &= A_{\text{large rectangle}} - A_{\text{square}} \\ &= 5 \times 8 - 2 \times 2 \\ &= 36 \text{ m}^2 \end{aligned}$	Subtract the area of the small rectangle (square) from the area of the large rectangle.	

EXERCISE 5C Area of composite figures

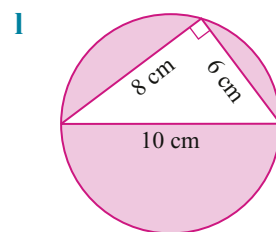
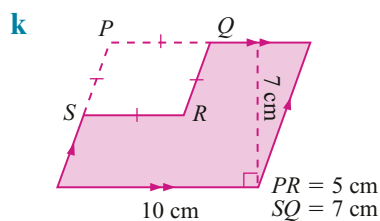
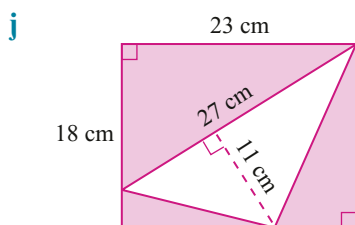
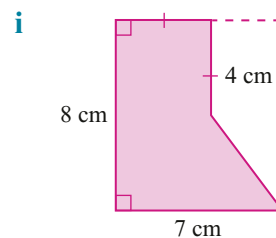
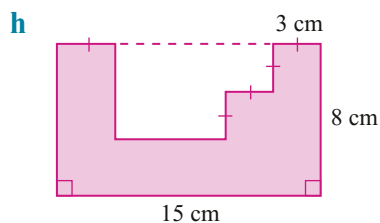
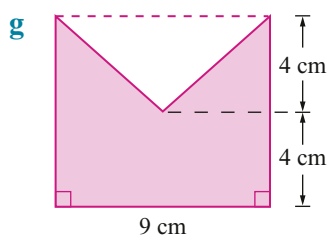
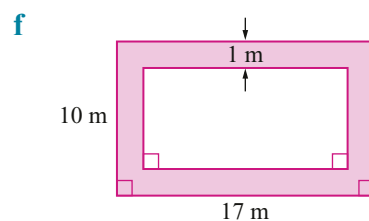
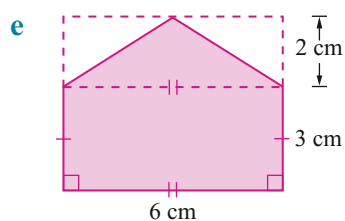
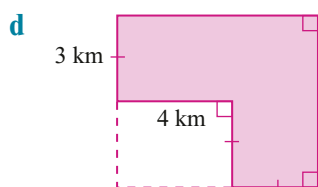
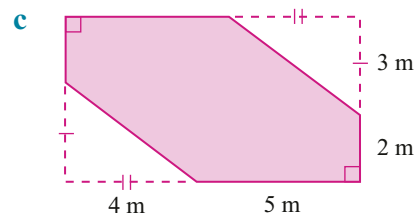
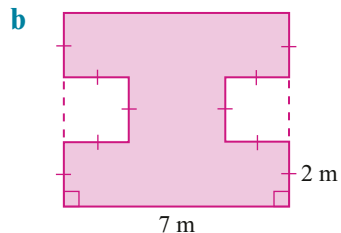
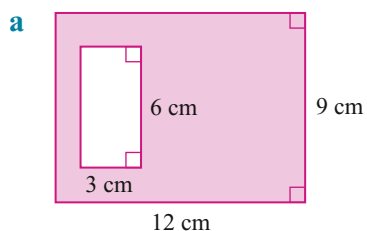
1 Find the shaded area of each of these figures.



2 Find the area of each of these composite figures made up of rectangles.

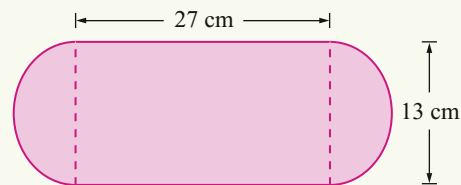


3 Find the shaded area of each of these figures, correct to two decimal places where necessary.



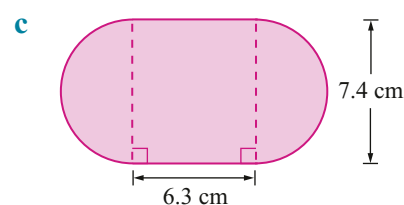
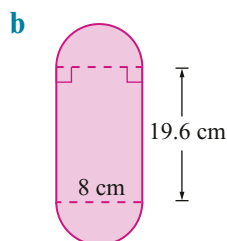
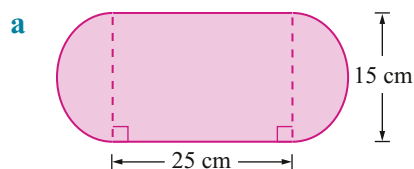
EXAMPLE 5C-2 Finding the area of a composite figure containing part of a circle

Calculate the area enclosed by this composite figure. Give your answer to the nearest square centimetre (cm²).

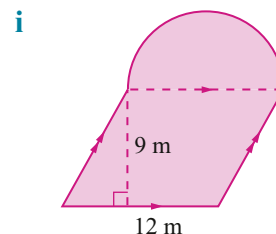
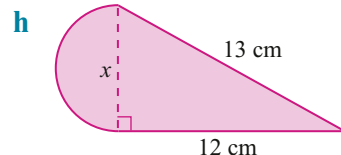
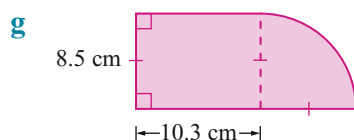
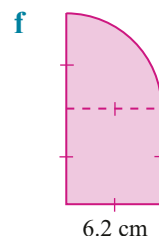
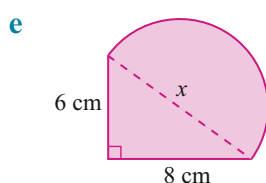
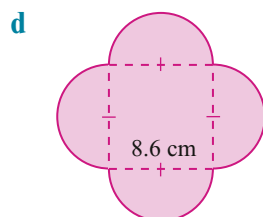
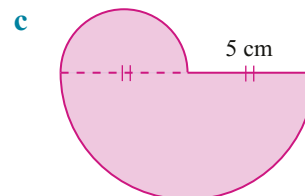
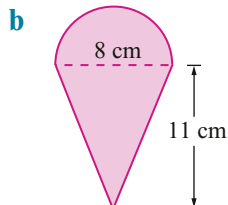
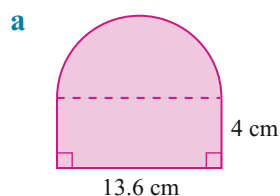


Solve	Think	Apply
The ends combine to make a circle with radius 6.5 cm. $\begin{aligned} \text{Area 1} &= \pi r^2 \\ &= \pi \times (6.5)^2 \\ &= 132.732\dots \text{ cm}^2 \\ &\approx 132.7 \text{ cm}^2 \end{aligned}$ Area 2 = length $\times$ breadth $\begin{aligned} &= 13 \text{ cm} \times 27 \text{ cm} \\ &= 351 \text{ cm}^2 \end{aligned}$ Total area = Area 1 + Area 2 $\begin{aligned} &\approx 132.7 + 351 \\ &= 483.7 \\ &\approx 484 \text{ cm}^2 \end{aligned}$	There are two semicircles and one rectangle. Add the areas of the semicircles and the rectangle together.	Divide the composite shape into simpler plane shapes. Calculate the areas and add them together.

4 Calculate the area enclosed by each of these composite figures to the nearest square centimetre (cm²).



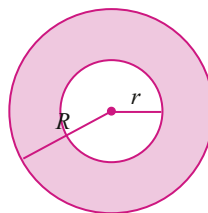
5 Find the area of each of these figures, correct to one decimal place if necessary.



The shaded area in this diagram is called an **annulus**. An annulus is the area between two circles with the same centre but different radii. Circles with the same centre are concentric circles. The formula for the area of an annulus is

$$A = \pi R^2 - \pi r^2 \quad \text{or} \quad A = \pi(R^2 - r^2)$$

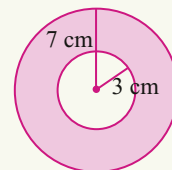
where R is the radius of the large circle, and r is the radius of the small circle.



annulus
the area between two circles with the same centre but different radii

EXAMPLE 5C-3 Finding the area of an annulus

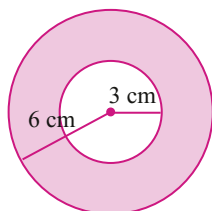
Calculate the area of this annulus to the nearest square centimetre (cm^2).



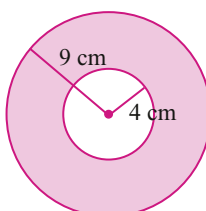
Solve	Think	Apply
$A = \pi \times 7^2 - \pi \times 3^2$ $= 125.663\dots$ $\approx 126 \text{ cm}^2$	The radius of the large circle is 7 cm. The radius of the small circle is 3 cm. Area of the large circle is $\pi \times 7^2$ and area of the small circle is $\pi \times 3^2$. Subtract the area of the small circle from the area of the large circle.	The formula for the area of an annulus is $A = \pi R^2 - \pi r^2$, or $A = \pi(R^2 - r^2)$, where R is the radius of the large circle, and r is the radius of the small circle. Substitute the value of the large and small radii into the formula.

6 Calculate the area of each annulus below, to the nearest square centimetre (cm^2).

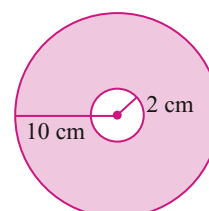
a



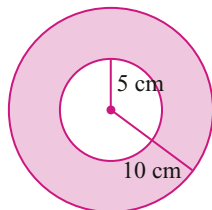
b



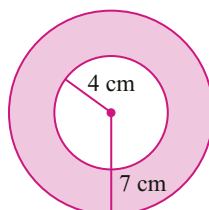
c



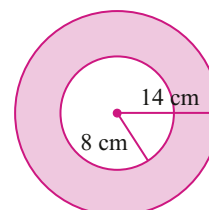
d



e

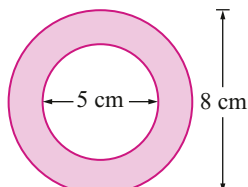


f

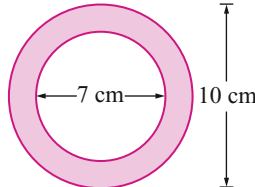


7 Calculate the area of each annulus below, to the nearest square centimetre (cm^2).

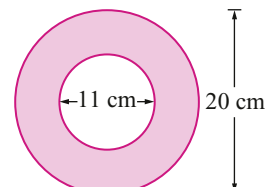
a



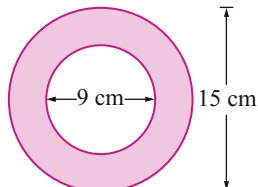
b



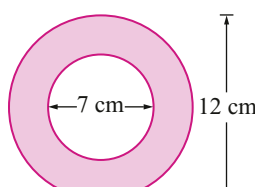
c



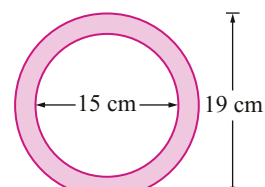
d



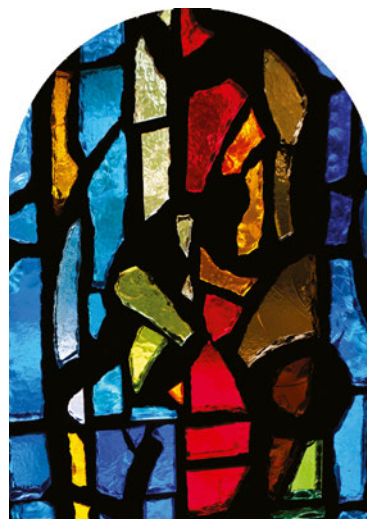
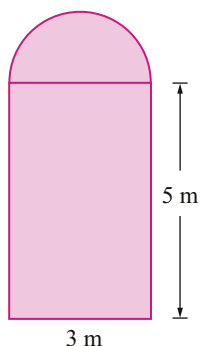
e



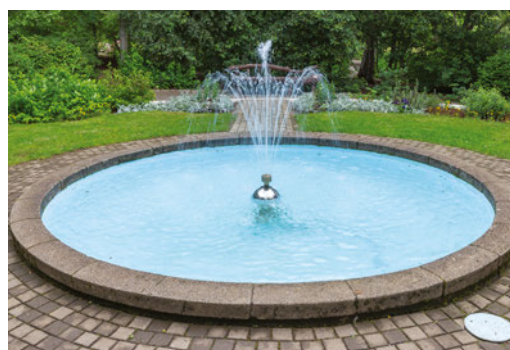
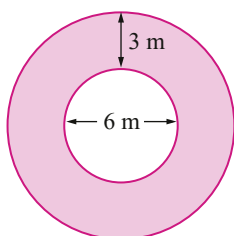
f



- 8** The diagram shows a stained glass window in a church.
- a** Calculate the area of the window, correct to two decimal places.
- b** The cost of replacing the window is \$540/m². Calculate the cost of replacing this window to the nearest dollar.

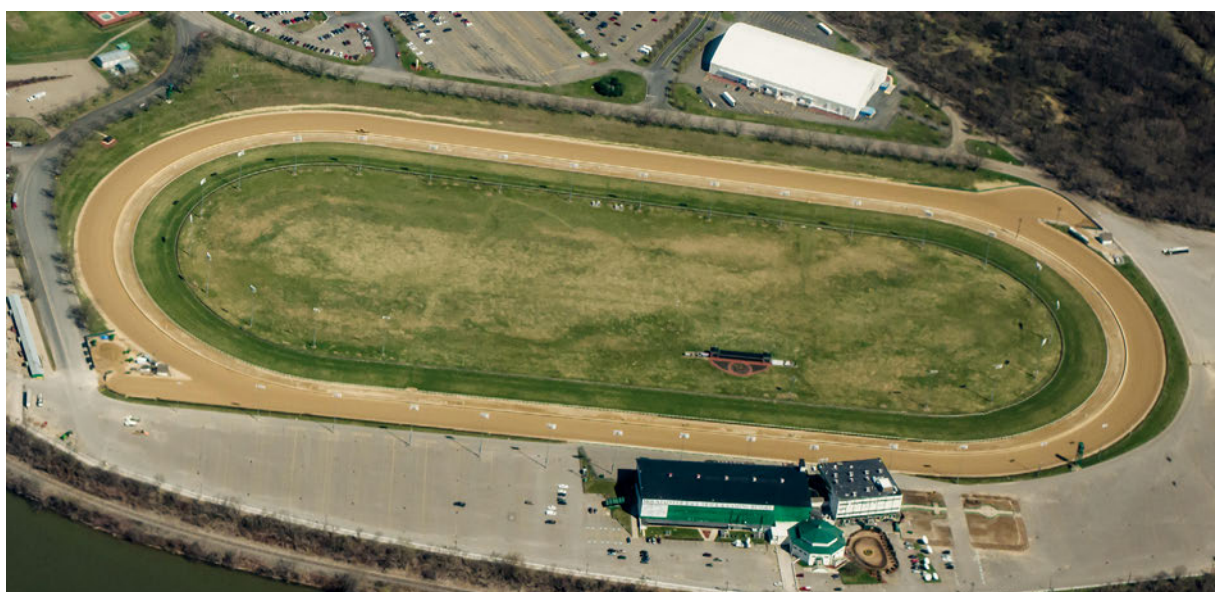
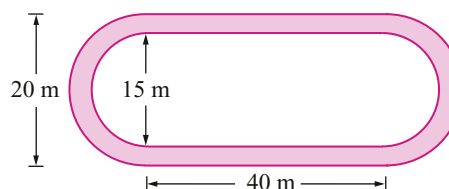


- 9** A 3 m wide path is placed around a circular pond that has a diameter of 6 m.
- Find the area of the path correct to one decimal place.

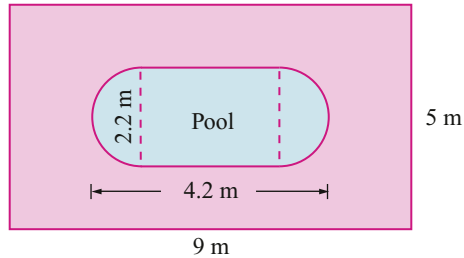


- 10** The race track shown shaded in the diagram on the right is to be resurfaced.

- a** Calculate the area to be resurfaced, to the nearest square metre.
- b** Calculate the cost of the resurfacing, if the cost is \$9.90/m². Give your answer to the nearest \$10.



- 11** The diagram shows a pool surrounded by a rectangular paved area.
- Calculate the paved area (correct to the nearest square metre).
 - If paving costs \$34.90/m², or part thereof, calculate the cost of paving the area surrounding the pool.

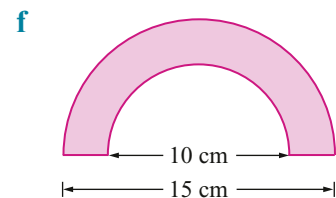
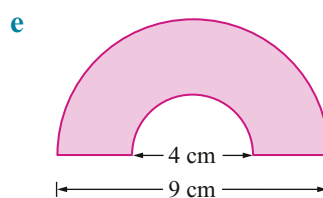
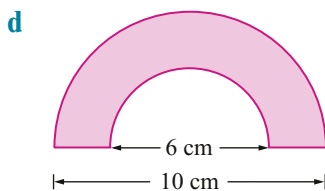
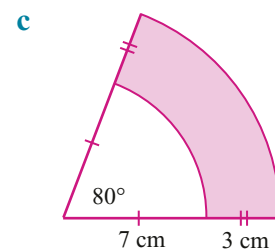
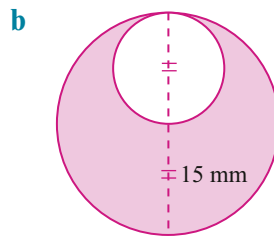
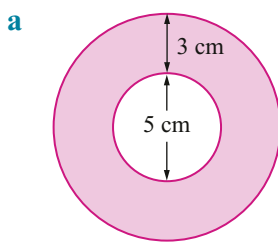


CHALLENGE

- 12** A clock shows the time is 8 o'clock. The clock's minute-hand is 9 cm long and its hour-hand is 6 cm long. As each hand moves, it sweeps over an area of the clock face.
- What is the area covered by the hour-hand as it sweeps through a full revolution?
 - What is the area covered by the minute-hand as it sweeps through a full revolution?
 - What is the difference in the area covered by each hand in a full revolution?
 - If the tip of the minute-hand moves from 12 to 2 on the clock face, what is the area covered by the minute-hand?
 - If the tip of the hour-hand moves from 8 to 11 on the clock face, what is the area covered by the hour-hand?



- 13** Determine the area of each shaded region below, correct to one decimal place.



- 14** An arrow shape can be drawn using a rectangle and a triangle. Draw three different arrows that could have an area of 50 cm².

5D Perimeter and area of irregular shapes

These resources are available on your obook assess:

- **Video tutorial 5D:** Watch and listen to an explanation of Example 5D-6
- **Investigation 5D:** Calculate the area of different parcels of land
- **assess quiz 5D:** Test your skills with an auto-correcting multiple-choice quiz

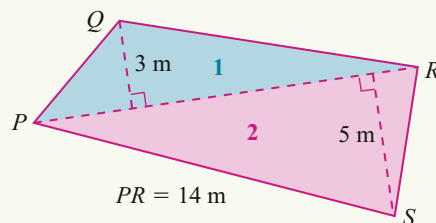
offset

a short distance measured perpendicularly from a main survey line

Although the shapes of paddocks or fields might be plane shapes, they may not be shapes whose perimeter and area can be calculated using simple formulas. But even irregularly shaped blocks can be dissected into familiar shapes for which formulas can be used. This section demonstrates the calculations for perimeter and area of irregular shapes, including estimates for those with curved boundaries.

EXAMPLE 5D-1 Finding the area of a quadrilateral

Find the area of quadrilateral $PQRS$.

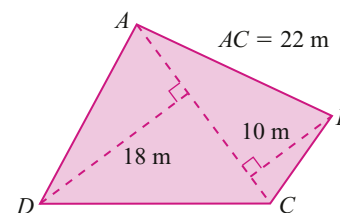


Solve	Think	Apply
$\begin{aligned}\text{Area} &= \text{area of triangle 1} + \text{area of triangle 2} \\ &= \frac{1}{2} \times 14 \times 3 + \frac{1}{2} \times 14 \times 5 \\ &= 56 \text{ m}^2\end{aligned}$	Find the area of each triangle. The diagonal, PR, of the quadrilateral is the base of both triangles and the offset measurement is the height each time.	Using the entire diagonal of the quadrilateral as the base measurement means the areas of only two triangles need to be calculated, instead of four.

EXERCISE 5D Perimeter and area of irregular shapes

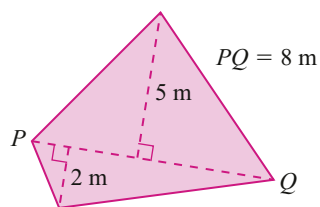
- 1 Complete the following to find the area of $ABCD$.

$$\begin{aligned}\text{Area} &= \text{area of triangle } ABC + \text{area of triangle } ACD \\ &= \frac{1}{2} \times 22 \times \underline{\hspace{1cm}} + \frac{1}{2} \times \underline{\hspace{1cm}} \times 18 \\ &= \underline{\hspace{1cm}} \text{ m}^2\end{aligned}$$

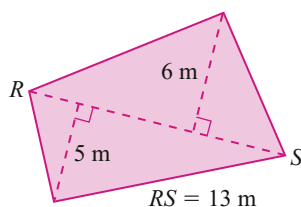


- 2 Find the area of each of these quadrilaterals.

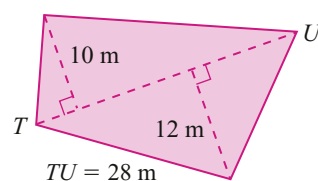
a



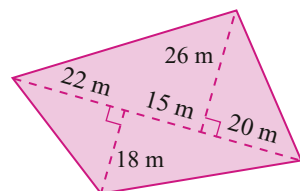
b



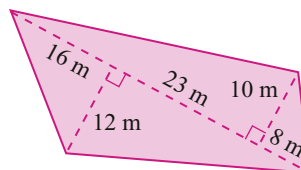
c



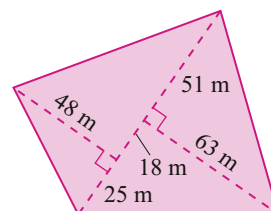
d



e

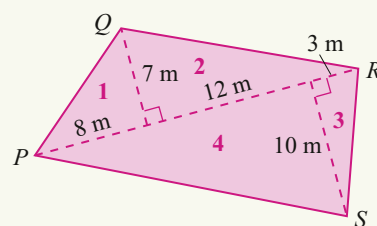


f



EXAMPLE 5D-2 Finding the perimeter of a quadrilateral

Use Pythagoras' theorem to help you find the perimeter of quadrilateral PQRS, correct to one decimal place.



Solve	Think	Apply
Triangle 1: $(PQ)^2 = 8^2 + 7^2 = 113$ $PQ = \sqrt{113} \approx 10.63$ m Triangle 2: $(QR)^2 = 7^2 + (12 + 3)^2 = 274$ $QR = \sqrt{274} \approx 16.55$ m Triangle 3: $(RS)^2 = 10^2 + 3^2 = 109$ $RS = \sqrt{109} \approx 10.44$ m Triangle 4: $(PS)^2 = 10^2 + (8 + 12)^2 = 500$ $PS = \sqrt{500} \approx 22.36$ m Perimeter $\approx 10.63 + 16.55 + 10.44 + 22.36$ $= 59.98$ ≈ 60.0 m (to one decimal place)	Divide the quadrilateral into four right-angled triangles. To use Pythagoras' theorem to find QR, we need to add 12 and 3 to find the length of the third side. To use Pythagoras' theorem to find PS, we need to add 8 and 12 to find the length of the third side. It is best not to round answers until all calculations are complete.	Divide the field into right-angled triangles. It might be necessary to add the measurements of parts of sides to find the whole side length for some triangles.

- 3 Complete the following to find the perimeter of quadrilateral ABCD, correct to one decimal place.

$$(BC)^2 = 13^2 + (5 + \underline{\hspace{1cm}})^2 = \underline{\hspace{1cm}}$$

$$BC = \sqrt{569} \approx \underline{\hspace{1cm}}$$

$$(AD)^2 = \underline{\hspace{1cm}} + (15 + \underline{\hspace{1cm}})^2 = \underline{\hspace{1cm}}$$

$$AD = \sqrt{\hspace{1cm}} \approx \underline{\hspace{1cm}}$$

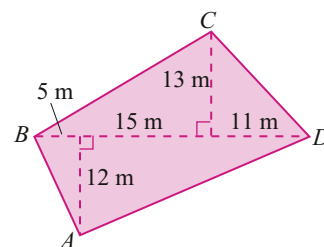
$$\text{Perimeter} \approx 23.85 + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \text{ m}$$

$$(CD)^2 = 13^2 + \underline{\hspace{1cm}}^2 = 290$$

$$CD = \sqrt{\hspace{1cm}} \approx \underline{\hspace{1cm}}$$

$$(AB)^2 = \underline{\hspace{1cm}}^2 + \underline{\hspace{1cm}}^2 = \underline{\hspace{1cm}}$$

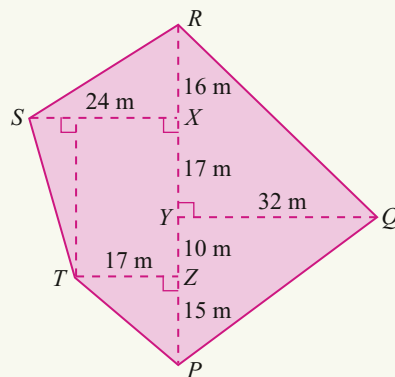
$$AB = \sqrt{\hspace{1cm}} \approx \underline{\hspace{1cm}}$$



- 4 Calculate the perimeter of the quadrilaterals in questions 2d, 2e and 2f.

EXAMPLE 5D-3 Finding the area of a field involving triangles and trapeziums

Find the area of the field $PQRST$, to the nearest square metre.



Solve	Think	Apply
$\begin{aligned}\text{Area of triangle } RQP &= \frac{1}{2} \times RP \times YQ \\ &= \frac{1}{2} \times 58 \times 32 \\ &= 928 \text{ m}^2\end{aligned}$	$RP = 16 + 17 + 10 + 15 = 58 \text{ m}$	Divide the whole field into suitable shapes and find the area of each shape. It might be necessary to add the measurements of parts of sides to find the whole side length for some shapes.
$\begin{aligned}\text{Area of triangle } PTZ &= \frac{1}{2} \times PZ \times TZ \\ &= \frac{1}{2} \times 15 \times 17 \\ &= 127.5 \text{ m}^2\end{aligned}$	For trapezium $TSXZ$: $\text{Area} = \frac{1}{2} \times \text{height} \times \text{sum of the parallel sides.}$	
$\begin{aligned}\text{Area of triangle } SRX &= \frac{1}{2} \times RX \times SX \\ &= \frac{1}{2} \times 16 \times 24 \\ &= 192 \text{ m}^2\end{aligned}$	The height is $17 + 10 = 27 \text{ m}$ and the parallel sides are 24 m and 17 m . Add all the areas.	
$\begin{aligned}\text{Area of trapezium } TSXZ &= \frac{1}{2} \times 27 \times (24 + 17) \\ &= 553.5 \text{ m}^2\end{aligned}$		
$\begin{aligned}\text{Total area} &= 928 + 127.5 + 553.5 + 192 \\ &= 1801 \text{ m}^2\end{aligned}$		

5 Complete the following to find the area of the field $MNOPQ$.

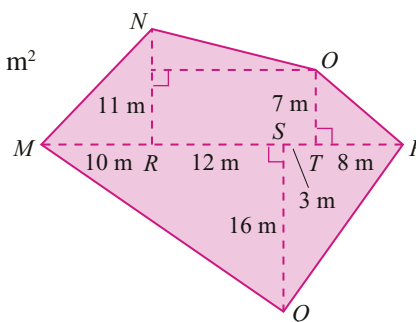
$$\text{Area of triangle } MQP = \frac{1}{2} \times (10 + 12 + \underline{\hspace{1cm}} + \underline{\hspace{1cm}}) \times 16 = \underline{\hspace{1cm}} \text{ m}^2$$

$$\text{Area of triangle } MRN = \frac{1}{2} \times 10 \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \text{ m}^2$$

$$\text{Area of triangle } TOP = \frac{1}{2} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = 28 \text{ m}^2$$

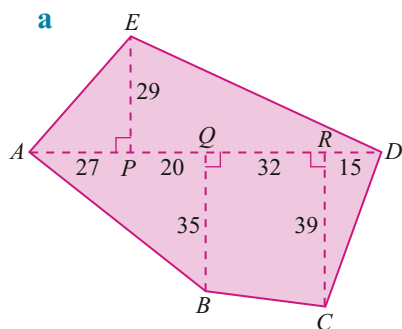
$$\begin{aligned}\text{Area of } NOTR &= \frac{1}{2} \times (12 + 3) \times (\underline{\hspace{1cm}} + \underline{\hspace{1cm}}) \\ &= \frac{1}{2} \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \text{ m}^2\end{aligned}$$

$$\text{Total area} = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + 28 + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \text{ m}^2$$

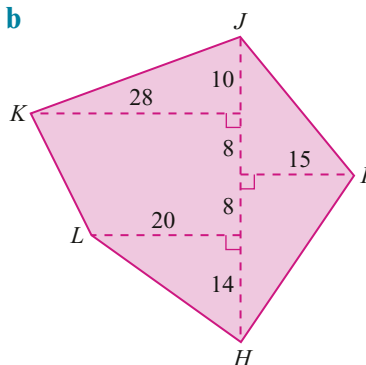


6 Find the area of each of the following fields, to the nearest square metre. All measurements are in metres.

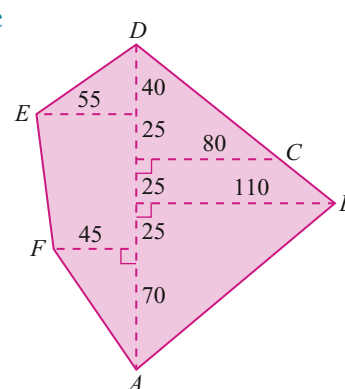
a



b



c



EXAMPLE 5D-4 Calculating the perimeter of a field involving triangles and trapeziums

Calculate the perimeter of the field $PQRST$ (shown in Example 5D-3), to the nearest metre.

Solve	Think	Apply
$(SR)^2 = 24^2 + 16^2 = 832$ $SR = \sqrt{832} \approx 28.84 \text{ m}$ $(RQ)^2 = 32^2 + (16 + 17)^2 = 2113$ $RQ = \sqrt{2113} \approx 45.97 \text{ m}$ $(QP)^2 = 32^2 + (10 + 15)^2 = 1649$ $QP = \sqrt{1649} \approx 40.61 \text{ m}$ $(PT)^2 = 17^2 + 15^2 = 514$ $PT = \sqrt{514} \approx 22.67 \text{ m}$ $(ST)^2 = (10 + 17)^2 + (24 - 17)^2 = 778$ $ST = \sqrt{778} \approx 27.89 \text{ m}$ Perimeter $\approx 28.84 + 45.97 + 40.61 + 22.67 + 27.89$ $= 165.98$ $\approx 166 \text{ m}$	Find the side lengths of each triangle (as we did in Example 5D-3). Divide the trapezium $TSXZ$ into a rectangle and a right-angled triangle. The length, ZX , of the triangle and the rectangle is: $17 + 10 = 27$. The height of the triangle is: $24 - 17 = 7$. Add all the side lengths.	Any trapeziums need to be divided into a rectangle and right-angled triangle. Add or subtract measurements to find side lengths.

- 7** Complete the following calculation to find the perimeter of field $MNOPQ$ from question 5.

$$(MN)^2 = 10^2 + \underline{\hspace{1cm}}^2 = \underline{\hspace{1cm}}$$

$$MN = \sqrt{221} \approx \underline{\hspace{1cm}} \text{ m}$$

$$(OP)^2 = \underline{\hspace{1cm}}^2 + 8^2 = \underline{\hspace{1cm}}$$

$$OP = \sqrt{\hspace{1cm}} \approx \underline{\hspace{1cm}} \text{ m}$$

$$(QM)^2 = \underline{\hspace{1cm}}^2 + (10 + \underline{\hspace{1cm}})^2 = 740$$

$$QM = \sqrt{740} \approx \underline{\hspace{1cm}} \text{ m}$$

$$\text{Perimeter} \approx \underline{\hspace{1cm}} + 15.52 + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$$

$$= \underline{\hspace{1cm}}$$

$$\approx \underline{\hspace{1cm}} \text{ m}$$

$$(NO)^2 = (11 - \underline{\hspace{1cm}})^2 + (12 + \underline{\hspace{1cm}})^2 = \underline{\hspace{1cm}}$$

$$NO = \sqrt{\hspace{1cm}} \approx 15.52 \text{ m}$$

$$(PQ)^2 = (3 + \underline{\hspace{1cm}})^2 + 16^2 = \underline{\hspace{1cm}}$$

$$PQ = \sqrt{377} \approx \underline{\hspace{1cm}} \text{ m}$$

- 8** Find the perimeter of the field represented by each field diagram in question 6, to the nearest metre.

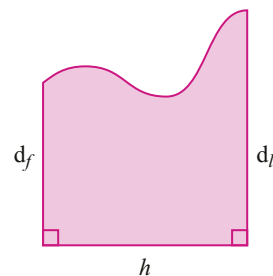


Trapezoidal rule

If a field has a boundary that is not straight (for example a river) then its area can be approximated by replacing the curved boundary with a straight line, creating a trapezium. The area of the trapezium is then an estimate of the area of the field. This is known as the Trapezoidal rule and is usually written as:

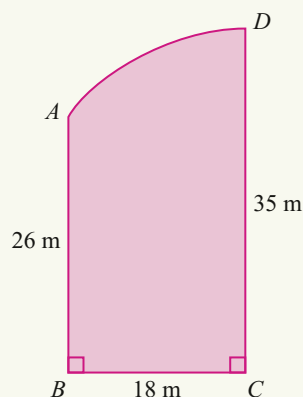
$$A \approx \frac{h}{2} (d_f + d_l)$$

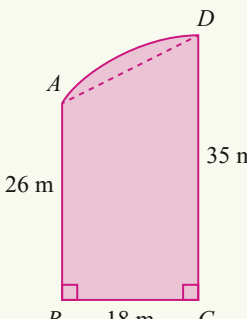
where d_f and d_l are the lengths of the parallel sides of the trapezium and h is the perpendicular distance between them. (d_f is the first length and d_l is the last length.)



EXAMPLE 5D-5 Estimating areas using the Trapezoidal rule

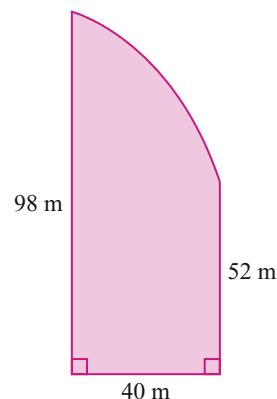
Use the Trapezoidal rule to estimate the area of the field shown.



Solve	Think	Apply
$\begin{aligned} \text{Area} &\approx \frac{18}{2}(26 + 35) \\ &= 9 \times 61 \\ &= 549 \text{ m}^2 \end{aligned}$	 Draw a straight line from A to D. The required area is approximately equal to the area of trapezium ABCD. Area of trapezium = $\frac{1}{2}(a + b) \times h$ or $A = \frac{h}{2}(a + b)$	The area of the field can be estimated by finding the area of a trapezium. The Trapezoidal rule can be used, with $d_f = 26$ m, $d_l = 35$ m and $h = 18$ m.

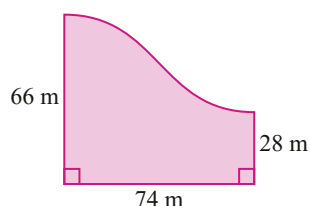
- 9 Complete the following to estimate the area of this field using the Trapezoidal rule.

$$\begin{aligned}\text{Area} &\approx \frac{\square}{2} (\square + 52) \\ &= \square \times \square \\ &= \square \text{ m}^2\end{aligned}$$

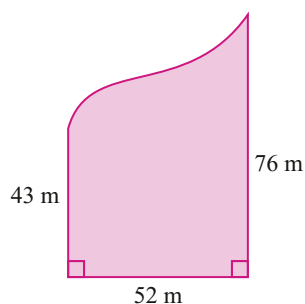


- 10 Use the Trapezoidal rule to estimate the area of each of the following fields.

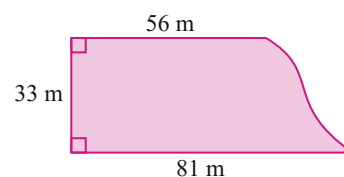
a



b

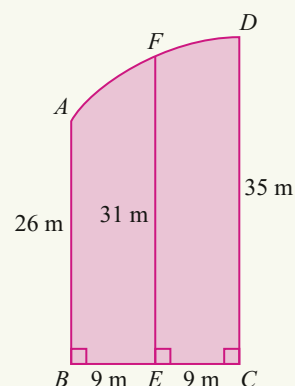


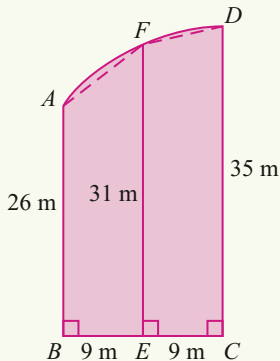
c



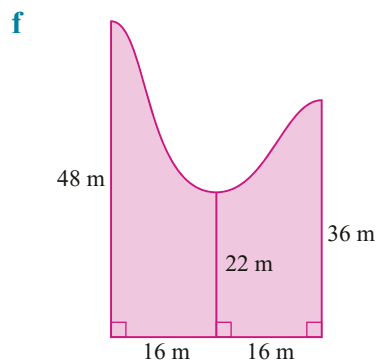
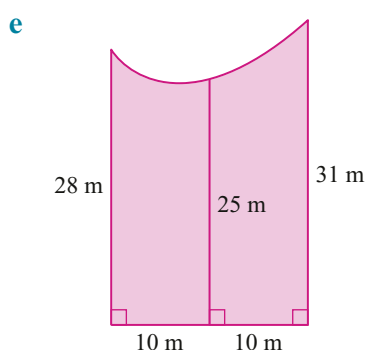
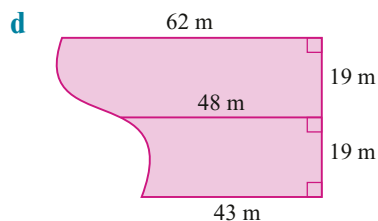
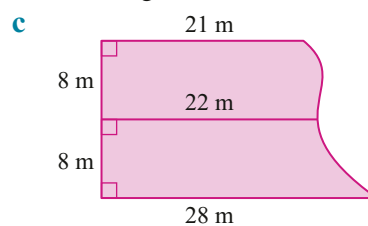
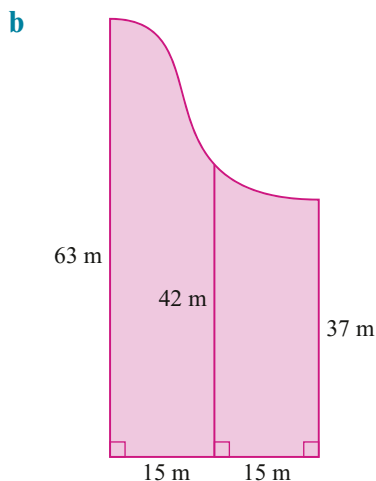
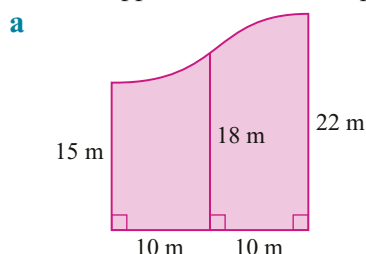
EXAMPLE 5D-6 Estimating area using two applications of the Trapezoidal rule

The field in Example 5D-5 has been dissected as shown here. The baseline has been bisected at E and the length of the perpendicular offset EF is 31 m. Use two applications of the Trapezoidal rule to estimate the area of the field.



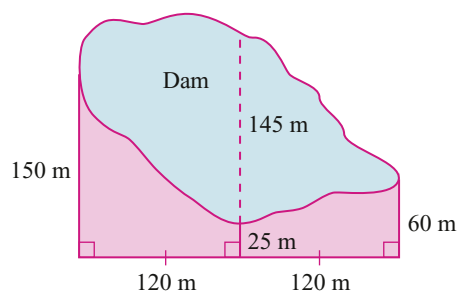
Solve	Think	Apply
$\begin{aligned}\text{Area} &\approx \frac{9}{2}(26+31) + \frac{9}{2}(31+35) \\ &= 4.5 \times 57 + 4.5 \times 66 \\ &= 256.5 + 297 \\ &= 553.5 \\ &\approx 534 \text{ m}^2 \text{ (to the nearest m}^2\text{)}\end{aligned}$	 Area of field $ABCD$ $\approx$ area of trapezium $ABEF$ $+ \text{ area of trapezium } FECD$	Divide the total area into two strips and apply the Trapezoidal rule to each strip. Add the results. To divide the area into two or more strips, the baseline BC is divided into two or more equal intervals and the lengths of the perpendicular offsets are measured. <i>Note:</i> the more strips the field is divided into, the closer the estimate will be to the actual area.

11 Use two applications of the Trapezoidal rule to estimate the area of each of the following fields.

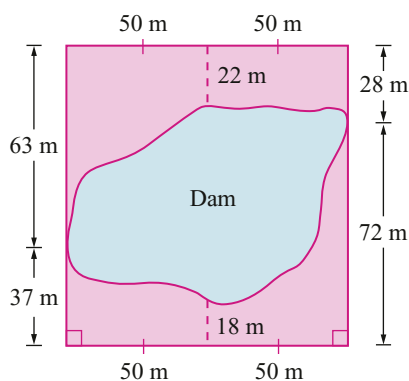


12 This diagram shows an aerial view of a dam.

- Use the measurements and two applications of the Trapezoidal rule to find the combined area of the dam and land.
- Use two applications of the Trapezoidal rule to find the land area.
- Find the area of the dam.
- The average depth of water is 10 m. Give an estimate for the volume of water in the dam.

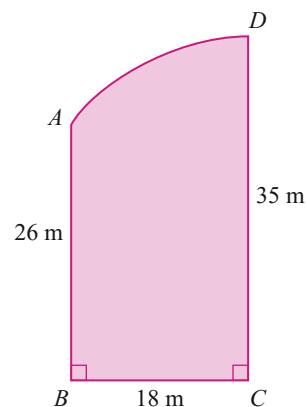
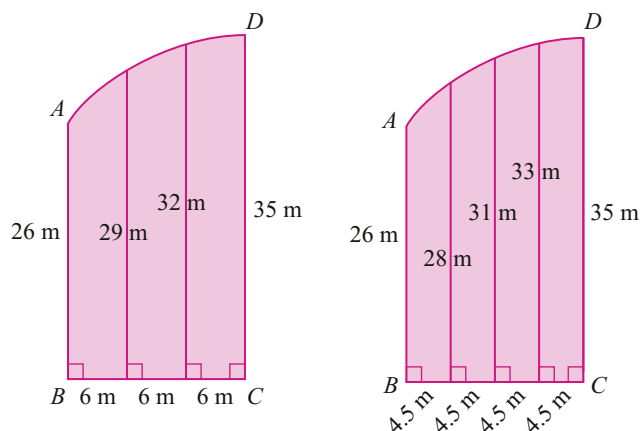


- Use two applications of the Trapezoidal rule to find the area of the dam shown below.
- If the average depth is 18 m, find an estimate for the volume of water in the dam.



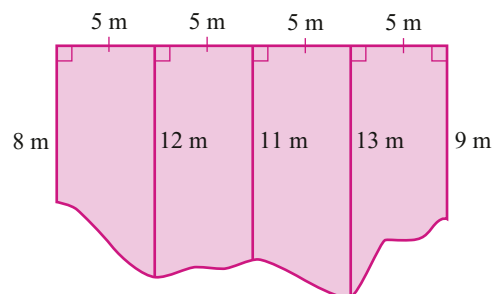
- 14** For the field shown on the right (used in Examples 5D-5 and 5D-6), the more strips the field is divided into the closer the estimate will be to the actual area. To divide the area into two or more strips, the baseline BC is divided into two or more equal intervals and the lengths of the perpendicular offsets are measured.

- a** If the field is divided into three strips as shown below left, use three applications of the Trapezoidal rule to estimate the area of the field.
- b** If the field is divided into four strips as shown below right, use four applications of the Trapezoidal rule to estimate the area of the field.



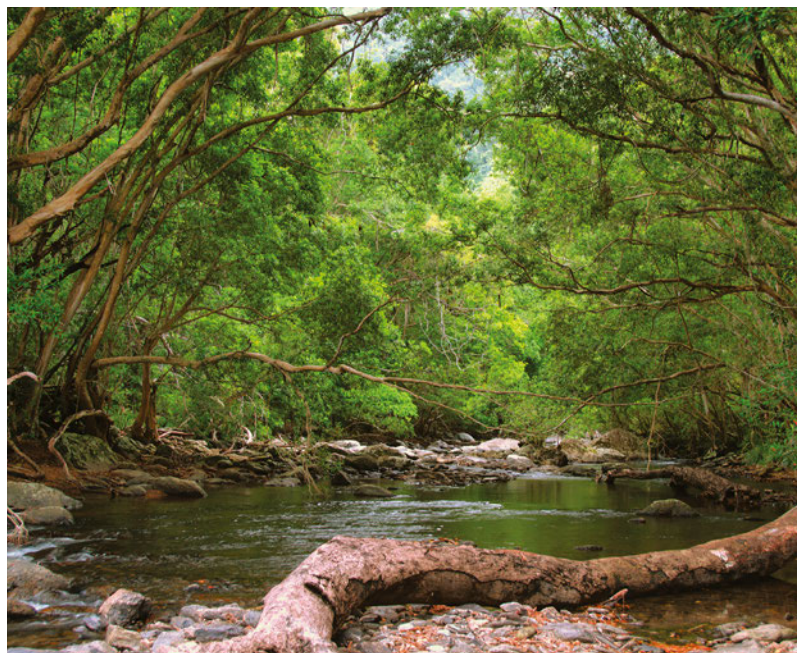
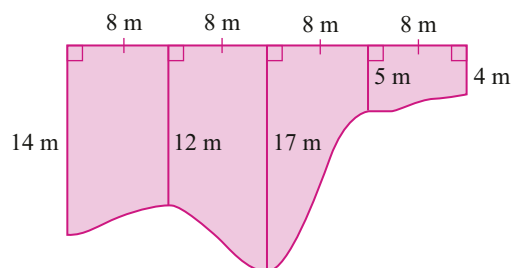
- 15** The diagram on the right shows the cross-section of a river.

- a** Use four applications of the Trapezoidal rule to find an estimate of the area of the cross-section.
- b** If the water flows at 10 metres per minute, find the volume of water passing a particular point on the river in 1 minute. *Hint:* this is the same as finding the volume with a 'height' of 10 m.
- c** How much water flows past in 1 hour?



- 16** The diagram below shows the cross-section of a river.

- a** Use four applications of the Trapezoidal rule to find an estimate of the area of the cross-section.
- b** If water flows past a particular point at 2 metres per minute, find the volume of water passing that point in 10 minutes.
- c** How much water flows past in an hour?



5E Surface area of prisms

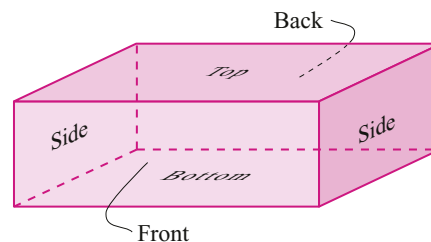
These resources are available on your [obook assess](#):

- **Interactive 5E.1:** Explore key ideas for surface area of a rectangular prism
- **Interactive 5E.2:** Explore key ideas for surface area of a triangular prism
- **Interactive 5E.3:** Explore key ideas for surface area of a pyramid
- **Video tutorial 5E:** Watch and listen to an explanation of Example 5E-3
- **Worksheet 5E.1:** Practise your skills with extra problems for prisms
- **Worksheet 5E.2:** Calculate the surface area of pyramids
- **Investigation 5E.1:** Calculate quotes for painting different rooms
- **Investigation 5E.2:** Calculate the surface area of a tiered wedding cake
- **assess quiz 5E:** Test your skills with an auto-correcting multiple-choice quiz

surface area

the sum of the areas of all the three faces of a three-dimensional solid

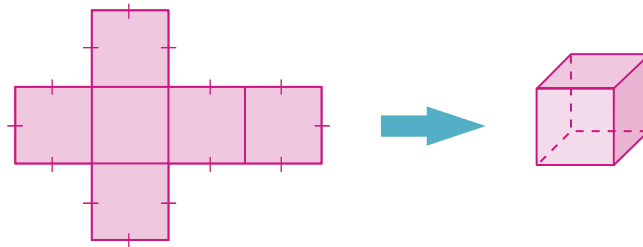
The **surface area** of any shape refers to the sum of the areas of all the faces. This box has six faces: two sides, a front and a back, and a top and a bottom.



net

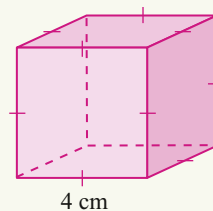
a plane shape that can be folded to form a solid

The **net** of a solid is a plane shape that can be folded to form the solid. The diagram below shows a net of a cube.



EXAMPLE 5E-1 Calculating the surface area of a cube

Calculate the surface area of this cube.

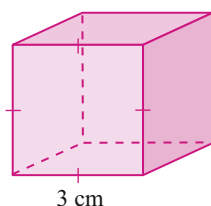


Solve	Think	Apply
$\begin{aligned}\text{Surface area} &= 6 \times \text{area of one face} \\ &= 6 \times 4 \times 4 \\ &= 96 \text{ cm}^2\end{aligned}$	All faces are identical. Multiply the area of one face by 6.	Count all faces. A cube always has six identical faces.

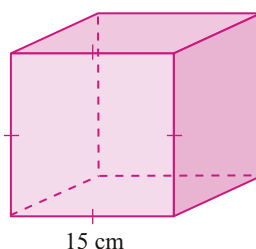
EXERCISE 5E Surface area of prisms

- 1 Calculate the surface area of each of these cubes.

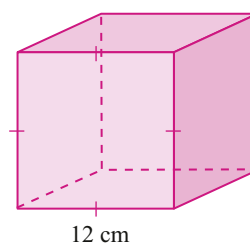
a



b



c



- 2 Calculate the surface area of cubes with sides of the following lengths.

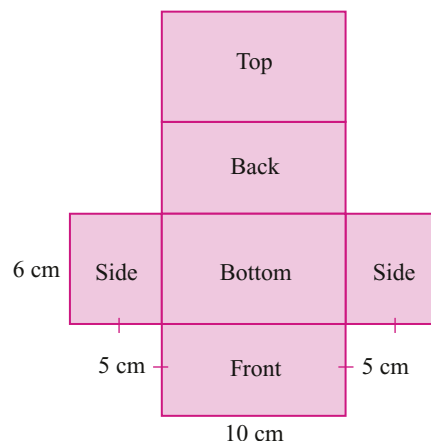
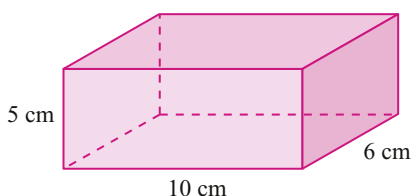
a 4.8 cm

b 17 mm

c 0.31 m

- 3 Complete the following to calculate the surface area of this rectangular prism, given its net.

Hint: rectangular prisms have three pairs of equal rectangles: top and bottom, front and back, and the two sides.



Top and ____: $A = 2 \times 10 \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \text{ cm}^2$

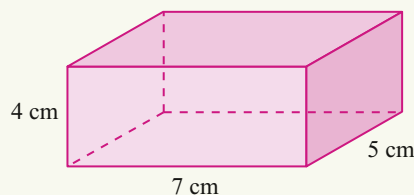
____ and back: $A = \underline{\hspace{1cm}} \times 5 \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \text{ cm}^2$

Two sides: $A = 2 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \text{ cm}^2$

Surface area = $\underline{\hspace{1cm}} + \underline{\hspace{1cm}} + 60 \text{ cm}^2 = \underline{\hspace{1cm}} \text{ cm}^2$

EXAMPLE 5E-2 Calculating the surface area of a rectangular prism

Calculate the surface area of this rectangular prism.



Solve	Think	Apply
Area of front and back = $2 \times \text{area of front}$ $= 2 \times 7 \times 4$ $= 56 \text{ cm}^2$ Area of sides = $2 \times \text{area of one side}$ $= 2 \times 5 \times 4$ $= 40 \text{ cm}^2$ Area of top and bottom = $2 \times \text{area of top}$ $= 2 \times 7 \times 5$ $= 70 \text{ cm}^2$ Surface area = $56 + 40 + 70$ $= 166 \text{ cm}^2$	Rectangles Rectangles Rectangles	Count and compare all faces. If some faces have the same dimensions, then the number of calculations is reduced. A rectangular prism has three pairs of identical faces.

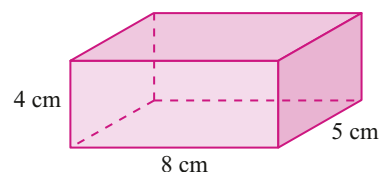
- 4 Complete the following to calculate the surface area of this rectangular prism.

Area of front and _____ = $2 \times 8 \times \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \text{ cm}^2$

Area of both sides = $2 \times \underline{\hspace{1cm}} \times 5 = \underline{\hspace{1cm}} \text{ cm}^2$

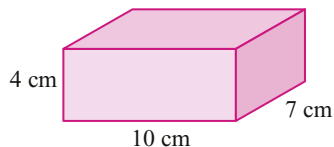
Area of _____ and bottom = $2 \times \underline{\hspace{1cm}} \times 5 = \underline{\hspace{1cm}} \text{ cm}^2$

Surface area = $64 + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \text{ cm}^2$

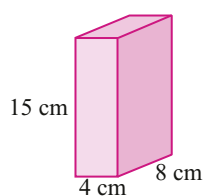


- 5 Calculate the surface area of each of these rectangular prisms.

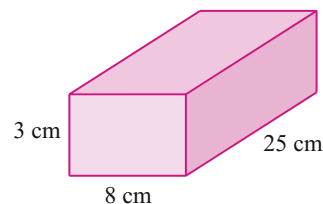
a



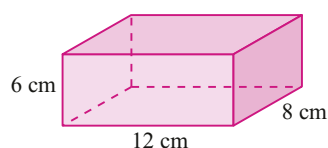
b



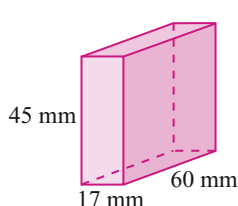
c



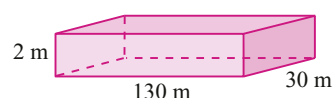
d



e



f



- 6 Calculate the surface area of the rectangular prisms with the dimensions given. Drawing a diagram can help.

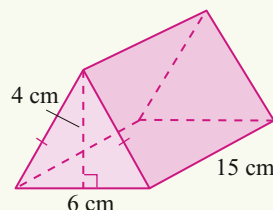
a Length 8 m, breadth 9 m and height 4 m

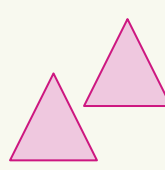
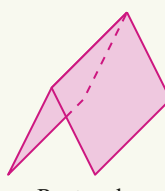
b Length 12 mm, breadth 15 mm and height 6 mm

c Length 15.3 m, breadth 6.4 m and height 4.3 m

EXAMPLE 5E-3 Calculating the surface area of a triangular prism

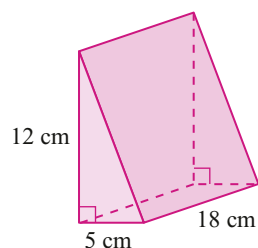
Calculate the surface area of this triangular prism.



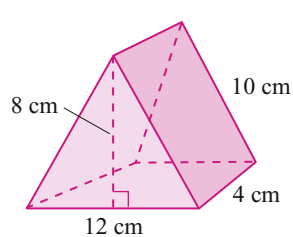
Solve	Think	Apply
$A_1 = 2 \times \text{area of one triangle}$ $= 2 \times \frac{1}{2} \times 6 \times 4$ $= 24 \text{ cm}^2$ $A_2 = 2 \times \text{area of one side rectangle}$ $= 2 \times \text{length} \times \text{breadth}$ Breadth = $\sqrt{4^2 + 3^2} = \sqrt{25} = 5 \text{ cm}$ so: $A_2 = 2 \times 15 \times 5$ $= 150 \text{ cm}^2$ $A_3 = \text{area of rectangular base}$ $= 15 \times 6$ $= 90 \text{ cm}^2$ Total surface area $= 24 + 150 + 90$ $= 264 \text{ cm}^2$	<i>Step 1:</i> Surface area of two triangles (front and back)  Triangles <i>Step 2:</i> Surface area of two rectangles (identical). Use Pythagoras' rule to find the breadth of the rectangle.  Rectangles <i>Step 3:</i> Surface area of one rectangle (base)  Rectangle	All prisms are solid objects that have one pair of identical ends. All other sides are rectangles. The cross-section is the same all along the prism's length. The shape of the ends give the prism its name (for example, triangular prism).

7 Calculate the surface area of each of these triangular prisms.

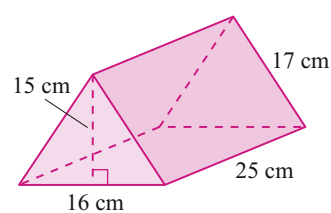
a



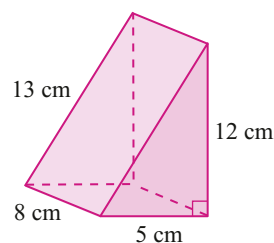
b



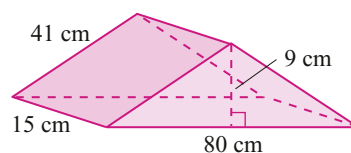
c



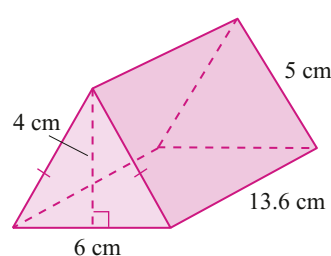
d



e

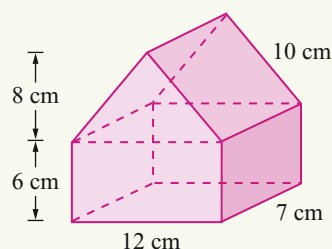


f



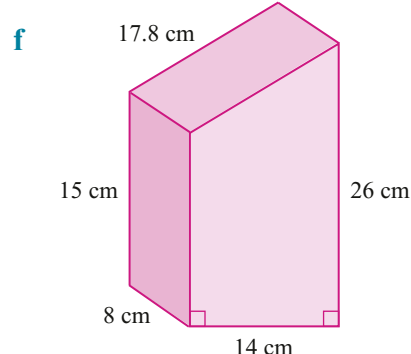
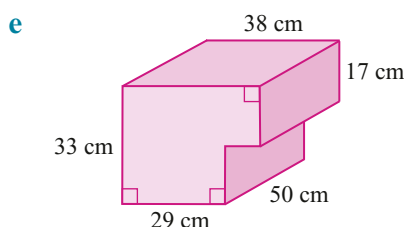
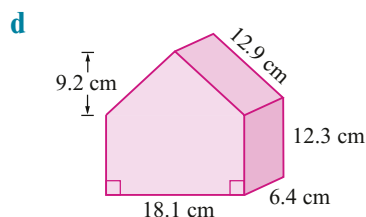
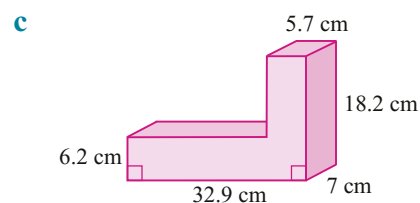
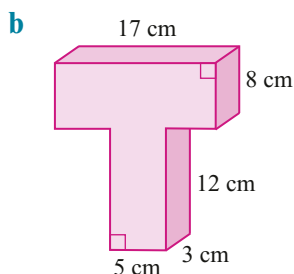
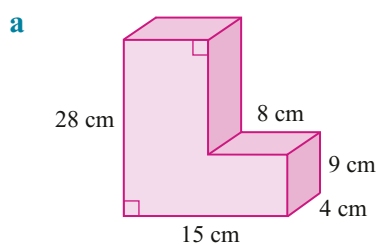
EXAMPLE 5E-4 Calculating the surface area of a composite prism

Calculate the surface area of this composite prism.



Solve	Think	Apply
$A_1 = \frac{1}{2} \times 12 \times 8$ $= 48 \text{ cm}^2$ $A_2 = 12 \times 6$ $= 72 \text{ cm}^2$ Composite area = $A_1 + A_2$ $= 48 + 72$ $= 120 \text{ cm}^2$ Area front and back = 2×120 $= 240 \text{ cm}^2$ Area of sides = $2 \times 7 \times 6$ $= 84 \text{ cm}^2$ Area of roof = $2 \times 10 \times 7$ $= 140 \text{ cm}^2$ Area of base = 12×7 $= 84 \text{ cm}^2$ Total surface area $= \text{area of front and back} + \text{area of sides}$ $+ \text{area of roof} + \text{area of base}$ $= 240 + 84 + 140 + 84$ $= 548 \text{ cm}^2$	<i>Step 1:</i> For the front and back, find the composite area of the triangle and the rectangle and multiply by 2. <i>Step 2:</i> The sides are rectangles. Area of sides is two times $l \times b$. <i>Step 3:</i> The roof is rectangles. Area of roof is two times $l \times b$. <i>Step 4:</i> The base is a rectangle. Area is $l \times b$.	There are several pairs of faces of equal size. Count the faces to ensure that all areas are calculated.

- 8 Calculate the total surface area of each of these composite prisms.

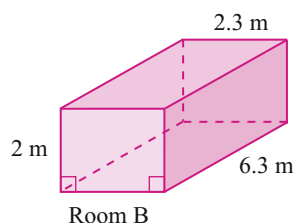
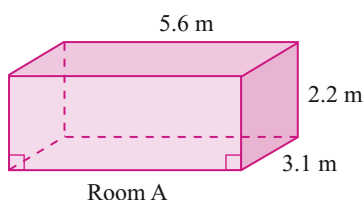


- 9 A gift box measures 10 cm by 12 cm by 20 cm.

- a** Calculate its total surface area.
b Gift wrapping paper costs \$3.50 a sheet. If each sheet covers 290 cm^2 , calculate the total cost of wrapping the gift box. (*Remember: you must purchase complete sheets.*)

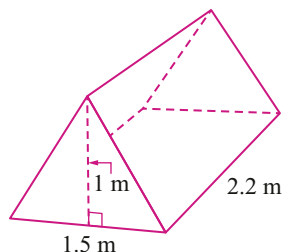


- 10 **a** Two rooms with the following dimensions are to be painted. Which room has the greater wall area?



- b** A 4 L tin of paint covers approximately 10 square metres and costs \$54.95. Calculate the number of 4 L tins of paint needed to paint room B.
c Calculate the cost of purchasing paint for room B.

- 11 A floorless tent is to be made with dimensions as shown. How much material is needed to make the tent? (Ignore overlaps for seams, etc.)

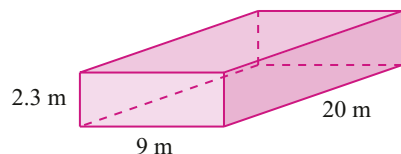


- 12 Which of these cereal boxes has the greater surface area?



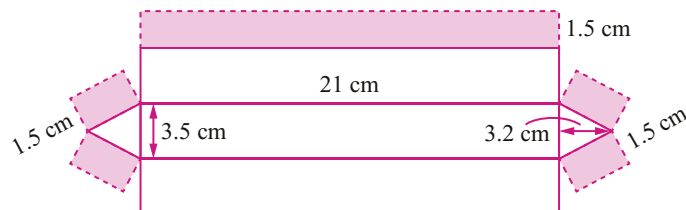
- 13 The interior walls and floor of an in-ground swimming pool are to be repainted.

- Calculate the total surface area of the four walls and floor of the pool.
- How many cans of paint are needed if one can covers 70 m^2 ?
- What is the cost of repainting, if each can costs \$82.50?

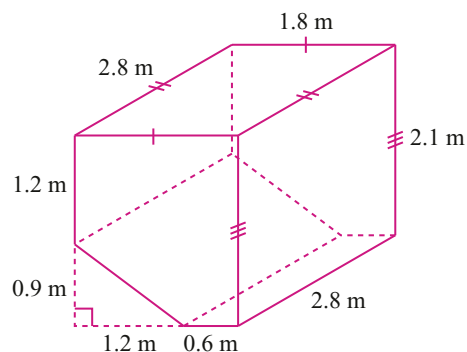


- 14 The packaging of a 100 g Toblerone bar has the following dimensions: base = 3.5 cm, height = 3.2 cm and length = 21.0 cm.

- Calculate the surface area of the packet.
- To make each packet, flaps 1.5 cm wide must be added, as shown below on the net of the solid, so that it can be glued together. How much cardboard is needed to make this packet?



- 15 A grain storage chute is designed as shown below. Calculate the area of metal needed to make the chute. (Ignore the thickness of the metal.)



- 16 As an extension task, access worksheet 5E.2 from your [gbook assess](#) to calculate the surface area of pyramids.

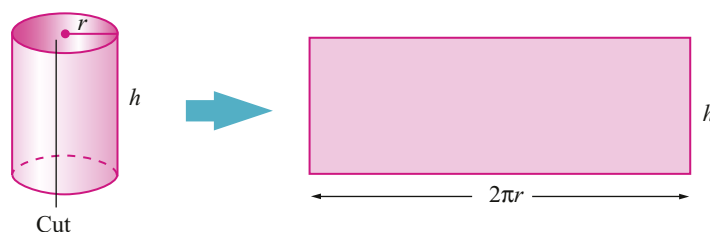
5F Surface area of cylinders and spheres

These resources are available on your obook assess:

- **Interactive 5F:** Explore key ideas for surface area of a cylinder
- **Worksheet 5F:** Practise your skills with extra problems for cylinders and spheres
- **assess quiz 5F:** Test your skills with an auto-correcting multiple-choice quiz

In order to find formulas for the surface area of an open cylinder (open ends) and for a closed cylinder (closed ends), we can examine the following diagrams.

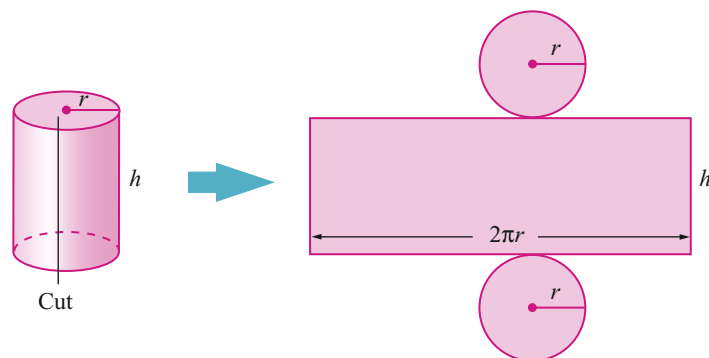
An open cylinder that has been cut perpendicular to the base, and folded out, forms a rectangle. The width of the rectangle is equal to the height of the cylinder. The length of the rectangle is equal to the circumference of the circle that formed the top or bottom.



The circumference of a circle is given by $C = 2\pi r$. Therefore the area of the curved surface of the cylinder is given by $A = 2\pi rh$.

A closed cylinder has a circle on both ends. The diagram below shows a closed cylinder that has been cut perpendicular to the base and unfolded.

$$\begin{aligned} A &= 2 \times \text{area of circle} + \text{area of rectangle} \\ &= 2 \times \pi r^2 + 2\pi r \times h \\ &= 2\pi r^2 + 2\pi rh \end{aligned}$$

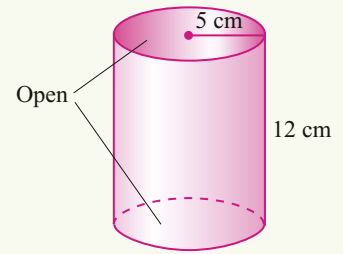


The surface area of a cylinder of radius r and height h , open at both ends, is given by $A = 2\pi rh$.

The surface area of a cylinder, of radius r and height h , closed at both ends, is given by $A = 2\pi r^2 + 2\pi rh$.

EXAMPLE 5F-1 Finding the surface area of an open cylinder

Find the outer surface area of this open cylinder, correct to one decimal place.

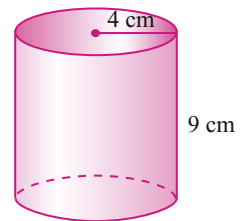


Solve	Think	Apply
$A = 2\pi rh$ $= 2 \times \pi \times 5 \times 12$ $= 376.991\dots$ $\approx 377.0 \text{ cm}^2$	Use $A = 2\pi rh$, where $r = 5$ and $h = 12$.	The cylinder is open at both ends, so only the curved surface area needs to be calculated. Use the formula $A = 2\pi rh$.

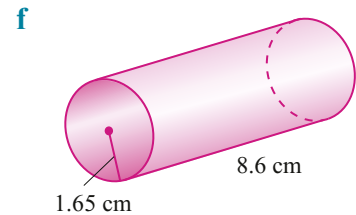
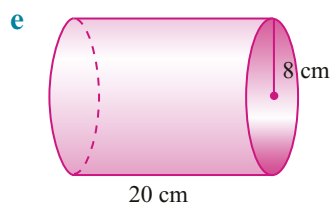
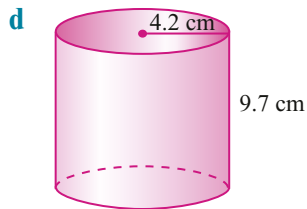
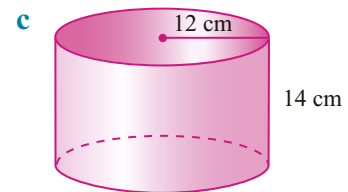
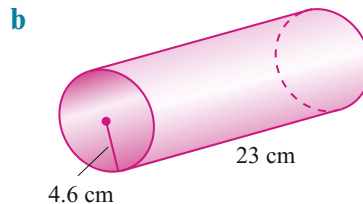
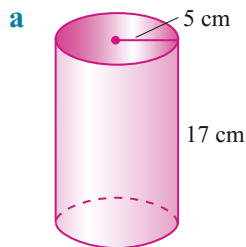
EXERCISE 5F Surface area of cylinders and spheres

- 1 Complete the following to find the outer surface area of this open cylinder, correct to one decimal place.

$$\begin{aligned}
 A &= 2\pi rh \\
 &= 2 \times \pi \times \underline{\quad} \times \underline{\quad} \\
 &= \underline{\quad} \dots \\
 &\approx \underline{\quad} \text{ cm}^2
 \end{aligned}$$

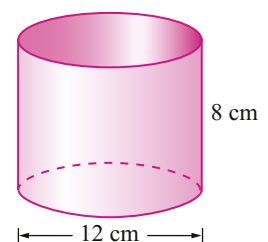


- 2 Find the outer surface area of each of these open cylinders, correct to one decimal place.

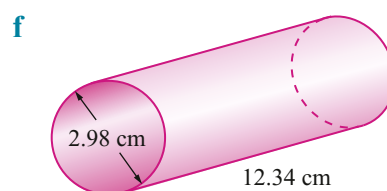
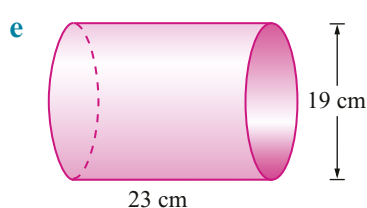
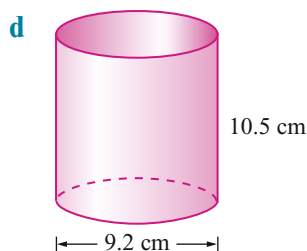
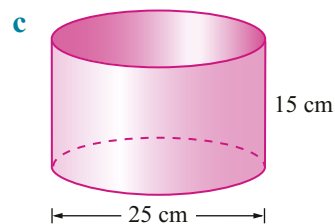
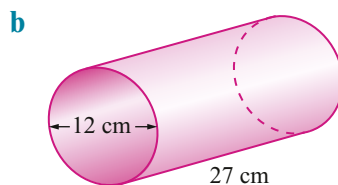
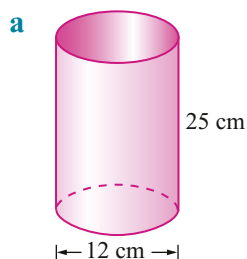


- 3 Complete the following to calculate the outer surface area of this open cylinder, correct to one decimal place.

$$\begin{aligned}
 \text{Diameter} &= \underline{\quad} & \text{so radius} &= \underline{\quad} \\
 A &= 2\pi rh \\
 &= 2 \times \pi \times \underline{\quad} \times \underline{\quad} \\
 &= \underline{\quad} \dots \\
 &\approx \underline{\quad} \text{ cm}^2
 \end{aligned}$$



- 4 Find the outer surface area of each of these open cylinders, correct to one decimal place.



- 5 Find the outer surface area of the open cylinders with these dimensions, correct to one decimal place.

a Radius = 4 cm, Height = 12 cm

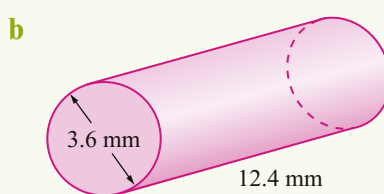
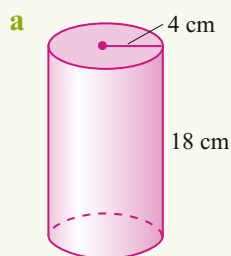
b Diameter = 18 cm, Height = 13 cm

c Radius = 3 cm, Height = 5 cm

d Diameter = 11.2 cm, Height = 23.6 cm

EXAMPLE 5F-2 Finding the surface area of a closed cylinder

Calculate the surface areas of these closed cylinders, correct to one decimal place.



	Solve	Think	Apply
a	$A = 2\pi rh + 2\pi r^2$ $= 2 \times \pi \times 4 \times 18 + 2 \times \pi \times 4^2$ $= 552.920\dots$ $\approx 552.9 \text{ cm}^2$	Use $A = 2\pi rh + 2\pi r^2$ with $r = 4$ and $h = 18$.	The cylinders are closed, so the area of the top and bottom circles is added to the curved surface area.
b	$A = 2\pi rh + 2\pi r^2$ $= 2 \times \pi \times 1.8 \times 12.4 + 2 \times \pi \times 1.8^2$ $= 160.598\dots$ $\approx 160.6 \text{ mm}^2$	Halve the diameter of 3.6 mm to find the radius. $r = 1.8 \text{ mm}$ $h = 12.4 \text{ mm}$	

- 6 Complete the following to calculate the surface area of this closed cylinder, correct to one decimal place.

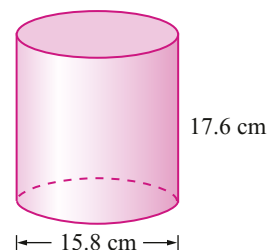
Radius = $\_\_\_ \div 2 = \_\_\_$

$A = 2\pi rh + 2\pi r^2$

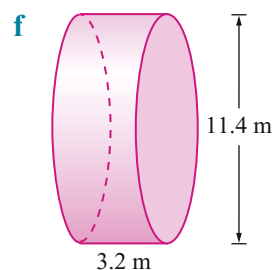
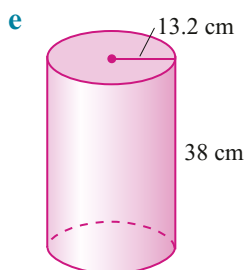
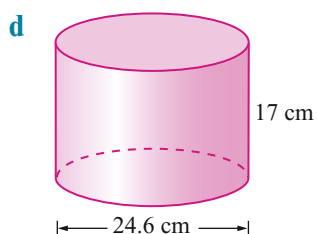
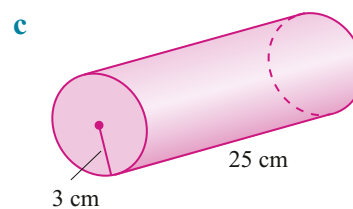
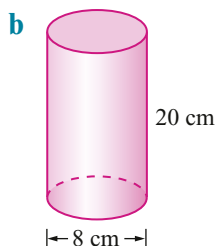
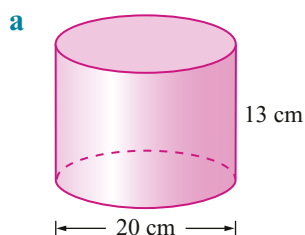
$= 2 \times \pi \times \_\_\_ \times 17.6 + 2 \times \pi \times (\_\_\_)^2$

$= \_\_\_$

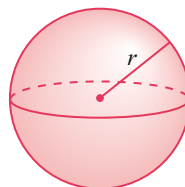
$\approx \_\_\_ \text{ cm}^2$



7 Find the surface area of each of these closed cylinders, correct to one decimal place.

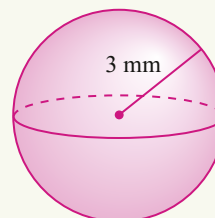


The surface area of a sphere of radius r is given by $A = 4\pi r^2$.



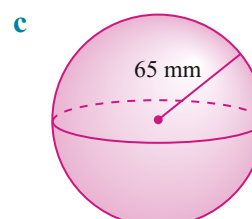
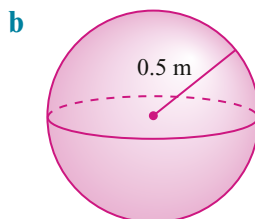
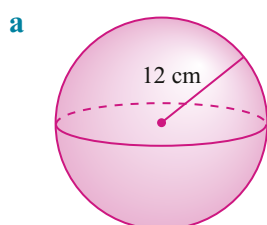
EXAMPLE 5F-3 Finding the surface area of a sphere with given radius

Find the surface area of this sphere, correct to one decimal place.



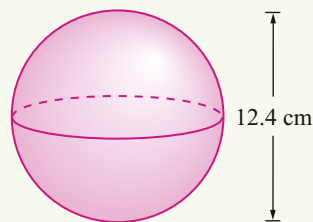
Solve	Think	Apply
$ \begin{aligned} A &= 4\pi r^2 \\ &= 4 \times \pi \times 3^2 \\ &= 113.097\dots \\ &\approx 113.1 \text{ mm}^2 \end{aligned} $	Use $A = 4\pi r^2$ with $r = 3 \text{ mm}$.	The only variable in the formula is the radius.

8 Find the surface area of each of these spheres, correct to one decimal place.



EXAMPLE 5F-4 Finding the surface area of a sphere with given diameter

Find the surface area of this sphere, correct to one decimal place.



Solve	Think	Apply
$A = 4\pi r^2$ $= 4 \times \pi \times (6.2)^2$ $= 483.051\dots$ $\approx 483.1 \text{ cm}^2$	Diameter is 12.4 cm so $r = 6.2$ cm.	The only variable in the formula is the radius. If the diameter is given, halve it to find the radius.

- 9 Complete the following to find the surface area of this sphere, correct to one decimal place.

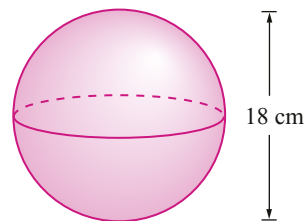
$$r = \underline{\hspace{1cm}} \div 2 = \underline{\hspace{1cm}} \text{ cm}$$

$$A = 4\pi r^2$$

$$= 4 \times \pi \times (\underline{\hspace{1cm}})^2$$

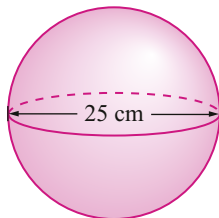
$$= \underline{\hspace{1cm}}$$

$$\approx \underline{\hspace{1cm}} \text{ cm}^2$$

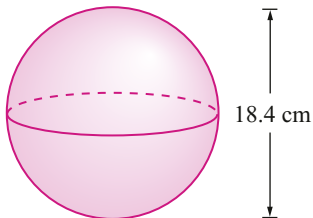


- 10 Find the surface area of each of these spheres, correct to one decimal place.

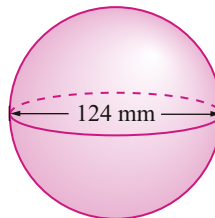
a



b

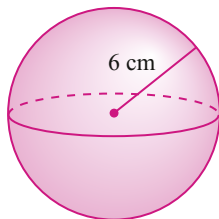


c

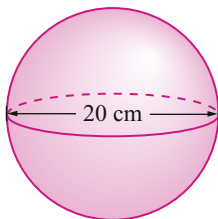


- 11 Find the surface area of each of these spheres, correct to one decimal place.

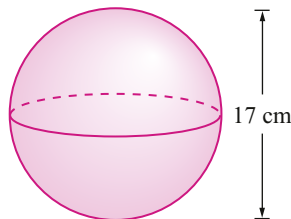
a



b

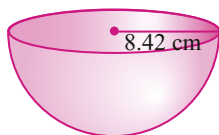


c

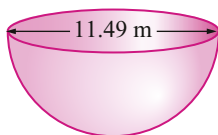


- 12 Find the outer surface area of each of these open hemispheres, correct to two decimal places.

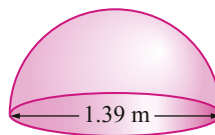
a



b



c



- 13 a** A closed cylindrical tank, 40 m high and 20 m in diameter, sits on a concrete slab. Find the outer surface area of the tank.
- b** If paint covers $8 \text{ m}^2/\text{L}$, find the number of 10 L cans of paint needed to paint the outside of the tank with one coat.
- c** Paint costs \$115.20 per 10 L can. Find the cost of paint for the tank.



- 14** How many spheres of 15 cm diameter can be painted with 1 L of glitter paint if the paint covers 10 m^2 ?
- 15** Determine the total surface area of leather used to cover 20 dozen cricket balls each with diameter 7 cm.

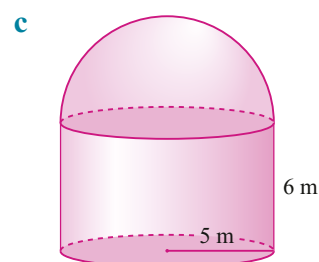
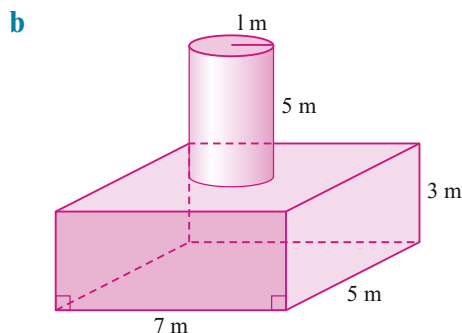
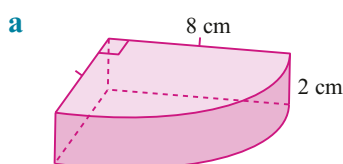
- 16** Fabian has the choice of two paint rollers. One roller is 25 cm long and has a diameter of 8 cm. The other roller is 30 cm long and has a radius of 3 cm. Assuming the rollers have the same absorbency, which roller would need to be redipped in paint the least often?



- 17** A can of baked beans has a cylindrical shape with a base diameter of 7 cm and a height of 10 cm. A label is placed around the curved surface of the can, covering it completely. What percentage of the total surface area of the can is covered by the label?

- 18** If you double the height of a cylinder, do you double its surface area? Explain, using an example.

- 19** Calculate the total surface area of each of the following closed composite figures to one decimal place.



- 20** Consider the composite figure in part **b** of question **19**. Explain why you could calculate the total surface area of this figure without knowing the radius of the cylinder.

- 21** Tennis balls are packed in aluminium cylindrical cans which hold four balls. The diameter of the can is 74 mm and its height is 272 mm. The curved surface is manufactured from sheet aluminium which costs \$4.80/m². The top and bottom are made from stronger aluminium sheeting which costs \$5.60/m². At the Australian Open Tennis Championships about 48 000 balls are used. Find the cost of producing the cans to hold this number of balls.



5G Volume

These resources are available on your [obook assess](#):

- **Interactive 5G.1:** Explore key ideas for volume of a rectangular prism
- **Interactive 5G.2:** Explore key ideas for volume of a triangular prism
- **Interactive 5G.3:** Explore key ideas for volume of a cylinder
- **Worksheet 5G.1:** Practise your skills with extra problems for prisms
- **Worksheet 5G.2:** Calculate the volume of pyramids, cones and annular cylinders
- **Investigation 5G:** Compare volume of different sized rubbish ships
- **assess quiz 5G:** Test your skills with an auto-correcting multiple-choice quiz

volume

the amount of space a solid occupies

cross-section

the shape of the face formed when a solid is sliced by a plane

The **volume** of a solid is the amount of space it occupies.

Prisms have two identical and parallel ends (or faces) joined by rectangles. Any slice through the prism parallel to these ends will result in the same **cross-section**.

When a solid has a uniform cross-section (face), the volume of the solid can be determined using:

$$\begin{aligned} \text{Volume} &= \text{area of face} \times \text{height} && (\text{or Volume} = \text{cross-sectional area} \times \text{length}) \\ &= \text{the area of the face of the shape multiplied by the height.} \\ V &= Ah \end{aligned}$$

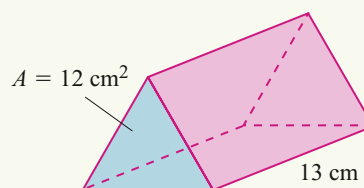
Even though a cylinder is not classed as a prism, its volume is found using the same method.

The volume of a prism of face area A and height h is given by $V = Ah$.

The volume of a cylinder of radius r and height h is given by $V = \pi r^2 h$.

EXAMPLE 5G-1 Finding the volume of a prism using face area and height

Find the volume of this triangular prism with a front face area of 12 cm^2 and height (depth) of 13 cm .

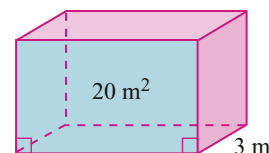


Solve	Think	Apply
$V = 12 \text{ cm}^2 \times 13 \text{ cm}$ $= 156 \text{ cm}^3$	Volume = area of face $\times$ height The area of the front face (or base) is 12 cm^2 and the height is 13 cm .	The volume is the area of the base multiplied by the perpendicular height. The height is perpendicular to the front face or base.

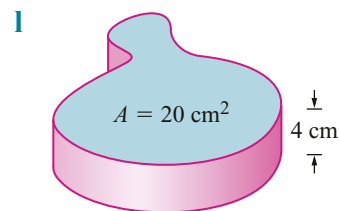
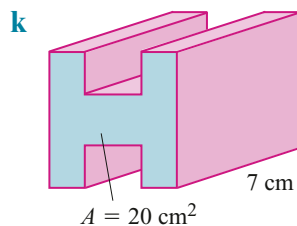
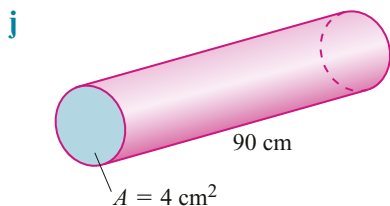
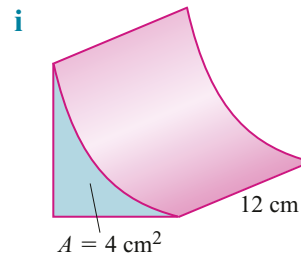
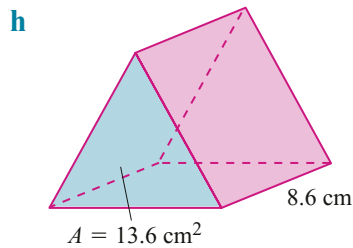
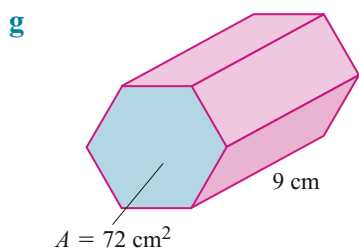
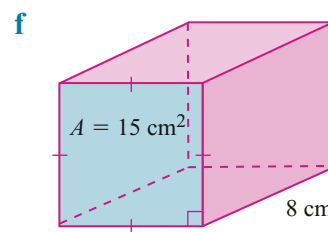
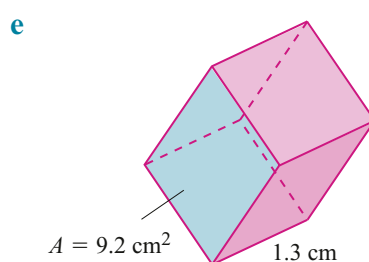
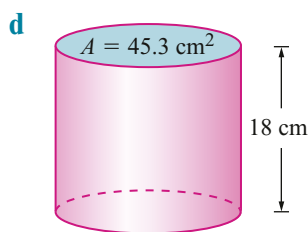
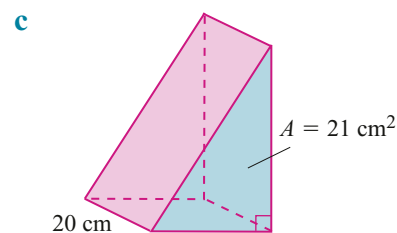
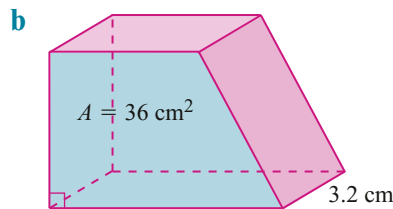
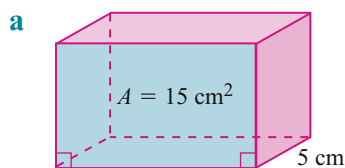
EXERCISE 5G Volume

- 1 Complete the following to find the volume of this solid.

$$\begin{aligned} V &= A \times \underline{\hspace{1cm}} \\ &= \underline{\hspace{1cm}} \times 3 \\ &= \underline{\hspace{1cm}} \text{ m}^3 \end{aligned}$$

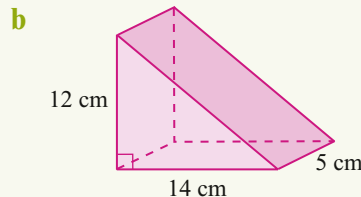
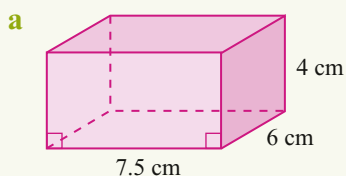


2 Calculate the volume of each of the following solids, given the area of the face of the shape and its height.



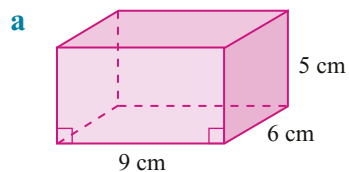
EXAMPLE 5G-2 Finding the volume of different solids

Find the volume of each of these solids.

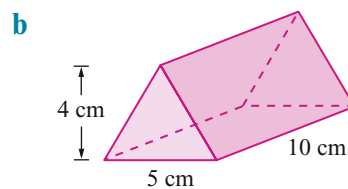


	Solve	Think	Apply
a	$V = 7.5 \times 6 \times 4$ $= 180 \text{ cm}^3$	Volume = length $\times$ breadth $\times$ height The base is a rectangle 7.5 cm by 6 cm and the perpendicular height is 4 cm.	These solids are prisms. The base area (or cross-sectional area) is calculated first, then multiplied by the perpendicular height (or depth). The solid need not be 'standing' on its base.
b	$V = \frac{1}{2} \times 12 \times 14 \times 5$ $= 420 \text{ cm}^3$	Volume = area of end $\times$ height The base is a right-angled triangle so Area = $\frac{1}{2}bh$. The height is 5 cm.	

3 Complete the following to find the volume of each of these solids.

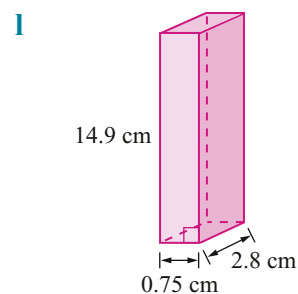
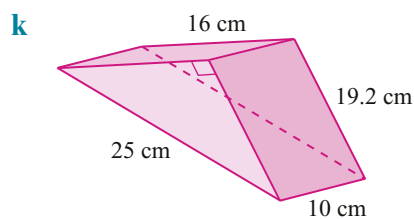
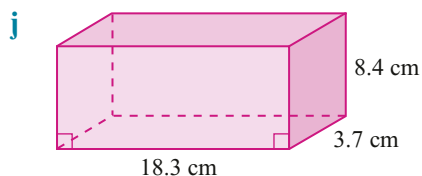
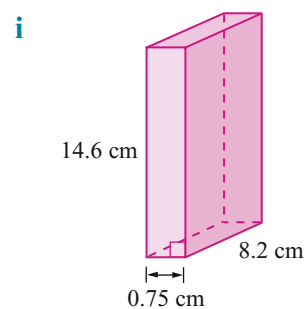
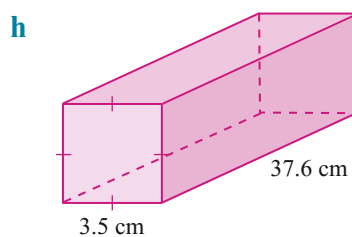
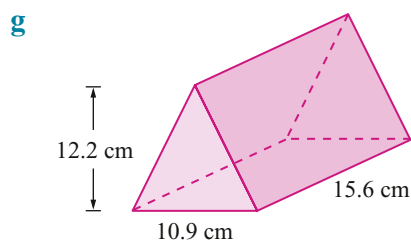
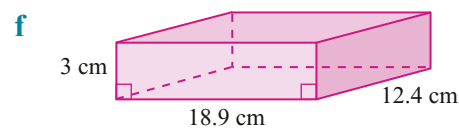
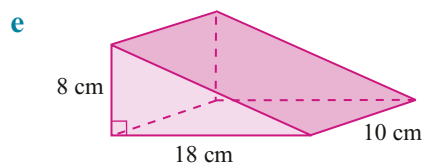
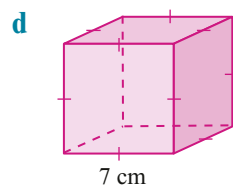
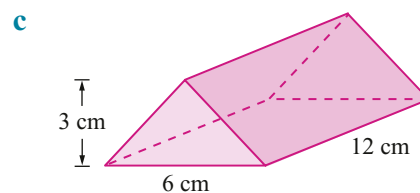
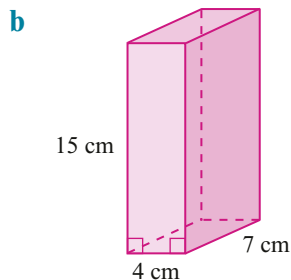
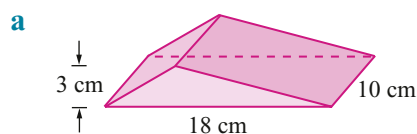


$$\begin{aligned} V &= A \times \text{---} \\ &= (9 \times \text{---}) \times 5 \\ &= \text{---} \text{ cm}^3 \end{aligned}$$



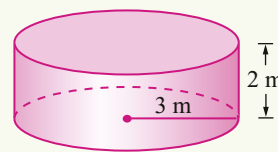
$$\begin{aligned} V &= \text{---} \times h \\ &= \left(\frac{1}{2} \times 5 \times \text{---} \right) \times \text{---} \\ &= \text{---} \text{ cm}^3 \end{aligned}$$

4 Calculate the volume of each solid.



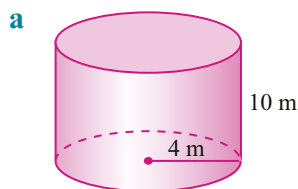
EXAMPLE 5G-3 Calculating the volume of a cylinder

Calculate the volume of this cylinder, correct to two decimal places.

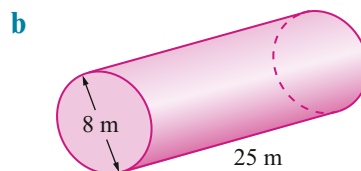


Solve	Think	Apply
$V = \pi r^2 \times h$ $= \pi(3)^2 \times 2$ $\approx 56.55 \text{ m}^3$	Volume = area of base $\times$ height The base is a circle, so $A = \pi r^2$. In this case $A = \pi \times 3^2$.	Using $A = \pi r^2$ for the area of the base, the formula becomes $V = \pi r^2 h$ and can be used directly.

- 5 Complete the following to find the volume of each of these cylinders, correct to two decimal places.

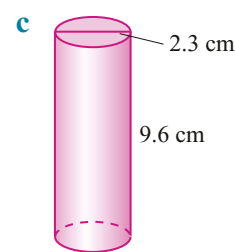
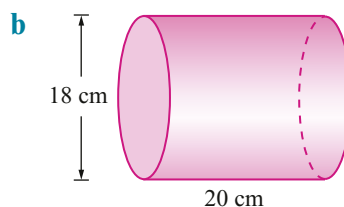
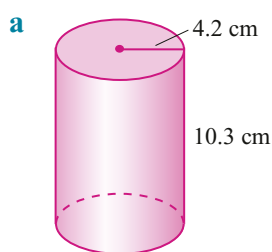


$$\begin{aligned}
 V &= A \times \underline{\hspace{1cm}} \\
 &= \pi \underline{\hspace{1cm}} \times h \\
 &= \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}^2 \times 10 \\
 &\approx \underline{\hspace{1cm}} \text{ m}^3
 \end{aligned}$$



$$\begin{aligned}
 V &= \underline{\hspace{1cm}} \times h \\
 &= \underline{\hspace{1cm}} \times 4^2 \times 25 \\
 &\approx \underline{\hspace{1cm}} \text{ m}^3
 \end{aligned}$$

- 6 Determine the volume of each of these cylinders, correct to the nearest cubic centimetre (cm³).

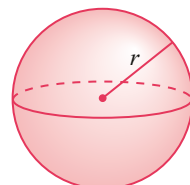


- 7 Use the dimensions below to calculate the volume of each cylinder, correct to one decimal place.

- a** Radius = 5 cm, Height = 12 cm
c Radius = 6.4 cm, Height = 16 cm
e Diameter = 37.4 cm, Height = 100 cm

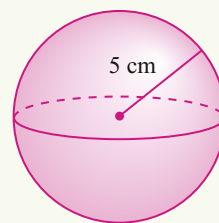
- b** Diameter = 13 cm, Height = 25 cm
d Diameter = 9 cm, Height = 40 cm
f Radius = 14.6 cm, Height = 70 cm

The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.



EXAMPLE 5G-4 Finding the volume of a sphere

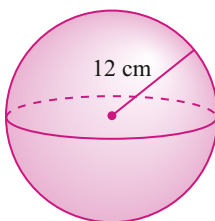
Find the volume of this sphere, correct to one decimal place.



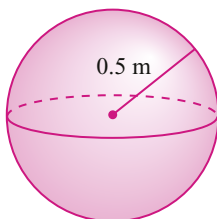
Solve	Think	Apply
$V = \frac{4}{3}\pi r^3$ $= \frac{4}{3} \times \pi \times 5^3$ $= 523.598\dots$ $\approx 523.6 \text{ cm}^3$	Use $V = \frac{4}{3}\pi r^3$ with $r = 5 \text{ mm}$.	The only variable in the formula is the radius.

8 Find the volume of each of these spheres, correct to one decimal place.

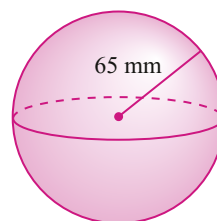
a



b

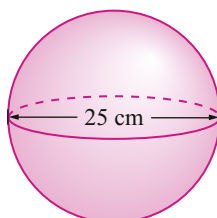


c

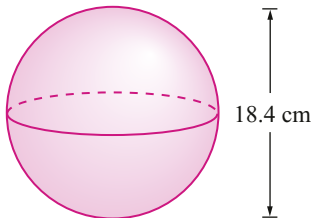


9 Find the volume of each of these spheres, correct to one decimal place.

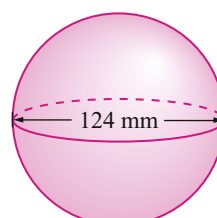
a



b

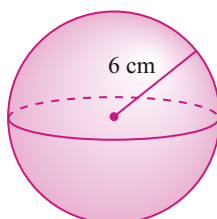


c

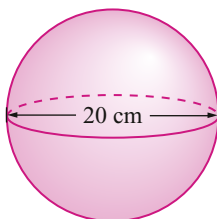


10 Find the volume of each of these spheres, correct to one decimal place.

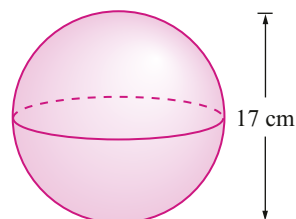
a



b

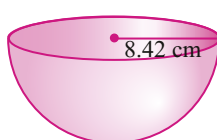


c

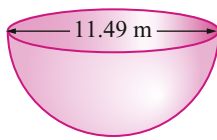


11 Find the volume of each of these hemispheres, correct to two decimal places.

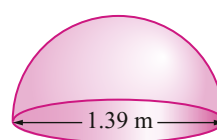
a



b

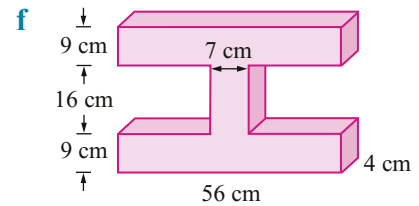
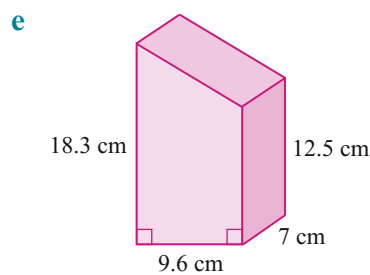
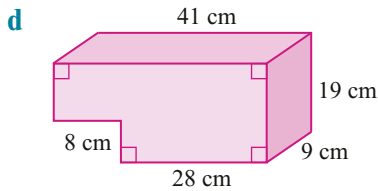
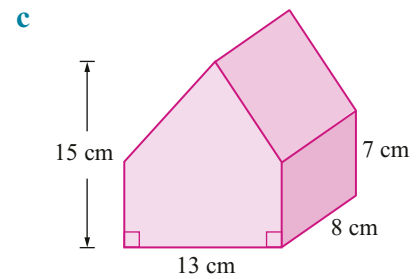
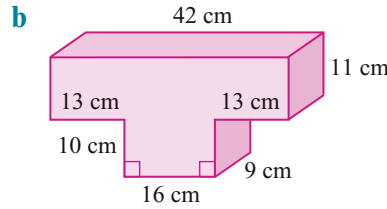
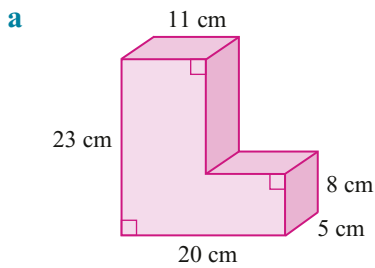


c



12 Calculate:

- the cross-sectional area of each of these composite prisms
- the volume of the composite prisms.



13 Which has the greatest volume, a cylinder of radius 3 cm and height 4 cm or a cylinder of radius 4 cm and height 3 cm?

14 a Find the base area of a prism with height 10 cm and volume 280 cm^3 .

b Find the height of a rectangular prism with length 10 cm and width 8 cm if its volume is 560 cm^3 .

c Find the height of a cylinder with radius 8 cm and volume 120 cm^3 .

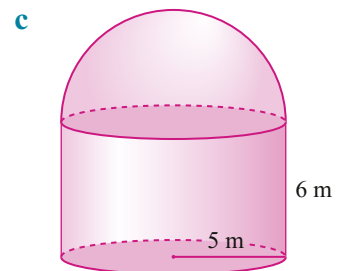
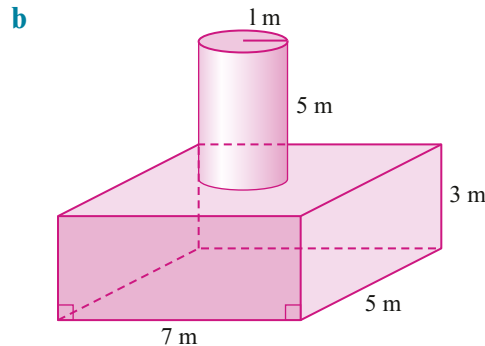
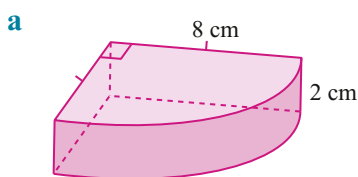
15 A sheet of cardboard 1000 mm by 800 mm has squares of side length 200 mm cut from each corner.

a Draw a diagram of the cardboard sheet with the corners cut out.

b The sides are folded up to form an open rectangular box. Calculate the volume of the box.



16 Calculate the volume of each of the following composite solids to one decimal place.



17 As an extension task, access worksheet 5G.2 from your obook assess to calculate the volume of pyramids, cones and annular cylinders.

5H Capacity

These resources are available on your obook assess:

- **Video tutorial 5H:** Watch and listen to an explanation of Example 5H-2
- **Worksheet 5H:** Practise your skills with extra problems for capacity
- **Investigation 5H.1:** Investigate volume and capacity of a vegetable garden
- **Investigation 5H.2:** Explore capacity measurements in cooking and medicine
- **assess quiz 5H:** Test your skills with an auto-correcting multiple-choice quiz

capacity

amount of fluid
that a container
can hold

The **capacity** of a container is the amount of fluid that it may contain. The common units of capacity are:

- millilitre (mL)
- litre (L)
- kilolitre (kL)
- megalitre (ML)

Note: 1 ML = 1000 kL = 1 000 000 L



Capacity and volume are related by the following conversions.

$$1 \text{ mL} = 1 \text{ cm}^3$$

$$1 \text{ L} = 1000 \text{ mL} = 1000 \text{ cm}^3$$

$$1 \text{ kL} = 1000 \text{ L} = 1 \text{ m}^3$$

EXAMPLE 5H-1 Converting between units of volume and capacity

- a** Calculate the volume of 5.2 L of liquid.
b Calculate the capacity of a container with volume 1800 cm^3 . Answer in litres.

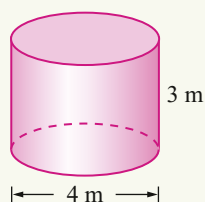
	Solve	Think	Apply
a	$V = (5.2 \times 1000) \text{ cm}^3$ $= 5200 \text{ cm}^3$	Use $1 \text{ L} = 1000 \text{ cm}^3$ as the conversion.	Choose the correct conversion, then multiply.
b	Capacity $= (1800 \times 1) \text{ mL}$ $= 1800 \text{ mL}$ $= (1800 \div 1000) \text{ L}$ $= 1.8 \text{ L}$	Use $1 \text{ cm}^3 = 1 \text{ mL}$ to convert to millilitres then use $1000 \text{ mL} = 1 \text{ L}$.	Be careful not to mix up the various conversions.

EXERCISE 5H Capacity

- 1 Complete the following to find:
 - a the volume of 6.5 L of liquid
 $\text{Volume} = (\text{ } \times 1000) \text{ cm}^3$
 $= \text{ } \text{ cm}^3$
 - b the capacity, in litres, with volume 25 600 cm^3
 $\text{Capacity} = (25\,600 \times 1) \text{ mL}$
 $= 25\,600 \text{ } \text{ }$
 $= (25\,600 \text{ mL} \div \text{ }) \text{ L}$
 $= \text{ } \text{ L}$
- 2 Find, in cubic centimetres, the space occupied by the following amounts of liquid.
 a 2 L b 12 L c 2.6 L d 450 mL e 975 mL
- 3 When completely submerged, various solids displaced the following amounts of water. Calculate the volume of each solid.
 a 650 mL b 7.5 L c 1.25 L d 3.78 L e 0.845 L
- 4 Calculate the capacity, in millilitres, of containers with the following volumes.
 a 1495 cm^3 b 3000 cm^3 c 500 cm^3 d 24 000 cm^3 e 6740 cm^3
- 5 Write each of your answers for question 4 in litres.
- 6 Use the conversion $1 \text{ m}^3 = 1000 \text{ L}$ to calculate the space occupied by the following amounts of liquid.
 a 2000 L b 5 kL c 6.7 kL d 0.9 kL e 83 200 L
- 7 Convert each of the following to kilolitres, using $1 \text{ m}^3 = 1 \text{ kL}$.
 a 0.02 m^3 b 0.7 m^3 c 28.4 m^3 d 302 m^3 e 5660 m^3

EXAMPLE 5H-2 Finding the capacity of a cylinder

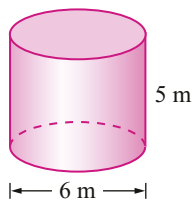
Find the capacity, in kilolitres, of a cylindrical rainwater tank with height 3 m and diameter 4 m.



Solve	Think	Apply
$\text{Volume} = \text{area of base} \times \text{height}$ $= \pi \times 2^2 \times 3$ $= 37.6991 \text{ m}^3$ $\text{Capacity} = 37.6991 \text{ kL}$ $\approx 37.70 \text{ kL}$	Halve the diameter to find the radius. Find the volume of the tank in cubic metres first, then use $1 \text{ kL} = 1 \text{ m}^3$ to find the capacity.	Calculate volume first, using $V = Ah$, then convert to capacity using an appropriate conversion.

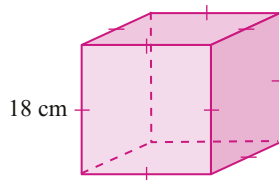
- 8** Complete the following to find the capacity, in kilolitres, of a cylindrical tank with diameter 6 m and height 5 m.

$$\begin{aligned}\text{Volume} &= \text{area of } \underline{\hspace{1cm}} \times \underline{\hspace{1cm}} \\ &= (\pi \times 3 \times \underline{\hspace{1cm}}) \times \underline{\hspace{1cm}} \text{ m}^3 \\ &\approx \underline{\hspace{1cm}} \text{ m}^3 \\ \text{Capacity} &\approx \underline{\hspace{1cm}} \text{ kL}\end{aligned}$$

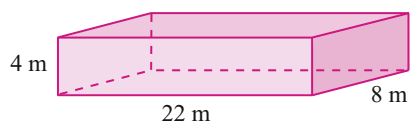


- 9** For the cube shown, find:

- a** the volume in cm^3
- b** the capacity in millilitres
- c** the capacity in litres.



- 10** A rectangular prism measures 22 m by 4 m by 8 m. Find:
- a** its volume in cm^3
 - b** its capacity in millilitres
 - c** its capacity in litres
 - d** its capacity in kilolitres.

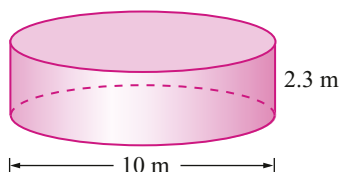


- 11** How much water can a cylindrical bottle cap hold if it has a diameter of 2.8 cm and a height of 1.1 cm? Write your answer in millilitres.
- 12** A hemispherical bowl has a diameter of 24 cm. What is its capacity in litres?
- 13** Find the capacity, in megalitres, of a dam that has a cross-sectional area of 5000 m^2 and average depth of 8 m.
- 14** Aileen has two cylindrical vases. The first vase has a radius of 5 cm and a height of 10 cm. The second vase has a diameter of 6.5 cm and a height of 22 cm. Which vase holds more, and by how much? Write your answer in millilitres.

- 15** A rectangular fish tank is 50 cm long, 20 cm wide and 60 cm tall.
- a** Calculate the volume of the fish tank.
 - b** What is the capacity of the fish tank if it is filled to the brim? Give your answer in:
 - i** millilitres
 - ii** litres.
 - c** The fish tank can only be filled to a line 6 cm from the top. How many litres of water will be required to fill the tank to this height?



- 16** A water trough in the shape of half a cylinder has radius 40 cm and length 6 m. What is the capacity of the trough, in litres, when it is full?
- 17** The internal dimensions of a refrigerator are height 1.8 m by width 84 cm and depth 60 cm. Find:
- the internal volume in cubic centimetres (cm^3)
 - the capacity in litres.
- 18** A circular swimming pool has a diameter of 10 m and a depth of 2.3 m.
- Calculate its volume to the nearest cubic metre (m^3).
 - Calculate the number of kilolitres of water required to fill the pool.



- 19** Ten oranges, assumed to be spherical, have a uniform diameter of 6 cm. If 65% of each orange is made up of juice, how much juice could be squeezed from these 10 oranges? Give your answer to the nearest millilitre.
- 20** When a balloon is blown up it forms a sphere.
- What would be the capacity of a balloon of diameter 26 cm? Give your answer in litres.
 - By taking a large breath, a teenage boy can expel 1.2 L of air from his lungs. How many of these breaths would be needed to fill the balloon to a diameter of 26 cm?
- 21** The capacity of a cylindrical water tank with radius 10 m is 4000 kL. Calculate the height of the tank to the nearest metre.
- 22** The fuel tank of a car has a capacity of 60 L. How big is the tank? Suggest some possible dimensions if the tank is in the shape of a rectangular prism.
- 23** Water drops falling from a tap are spheres of diameter 0.4 cm. If 15 drops per minute leak from the tap, how much water is wasted in one day?



CHAPTER 5 REVIEW PERIMETER, AREA AND VOLUME

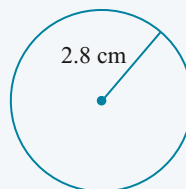
You should be able to:

- ✓ calculate the perimeter and area of triangles, quadrilaterals, circles, sectors and composite shapes
- ✓ calculate the perimeter and area of irregularly shaped blocks of land, including the use of the Trapezoidal rule
- ✓ calculate the surface area of right prisms, cylinders, spheres and composite solids
- ✓ calculate the volume of right prisms, cylinders, spheres and composite solids
- ✓ convert between units of volume and capacity
- ✓ solve problems involving perimeter, area, surface area, volume and capacity.

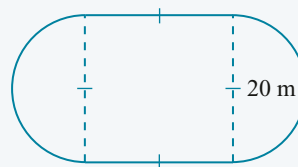
Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

REVIEW MULTIPLE-CHOICE QUESTIONS

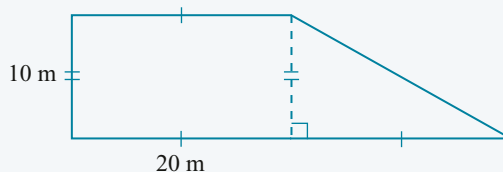
- 5A** 1 What is the circumference of this circle?
- A 5.6 cm B 8.8 cm
C 14.4 cm D 17.6 cm



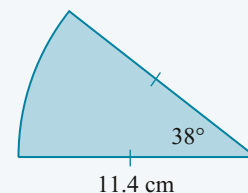
- 5A** 2 What is the perimeter of this shape?
- A 71 m B 80 m
C 103 m D 143 m



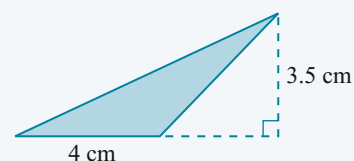
- 5A** 3 What is the perimeter of this shape?
- A 92 m B 70 m
C 50 m D 30 m



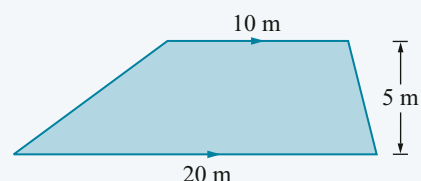
- 5A** 4 The perimeter of this sector, to the nearest square centimetre is:
- A 94 cm² B 66 cm²
C 53 cm² D 30 cm²



- 5B** 5 The area of this triangle is:
- A 7 cm² B 14 cm²
C 1.75 cm² D 2 cm²



- 5B** 6 The area of this quadrilateral is:
- A 35 m² B 75 m²
C 150 m² D 300 m²

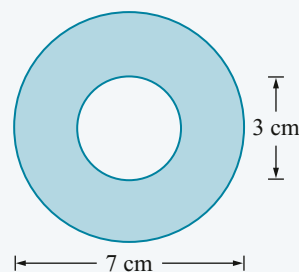


5B ▶ 7 The area of the sector shown in question 4 is closest to:

- A 1.2 cm^2 B 408.3 cm^2
C 43.1 cm^2 D 433.2 cm^2

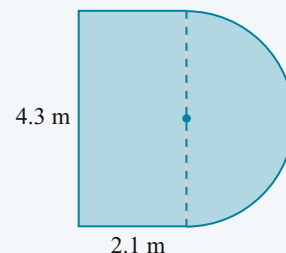
5C ▶ 8 The area of the shaded annulus is closest to:

- A 4 cm^2 B 31.4 cm^2
C 125.6 cm^2 D 125.7 cm^2



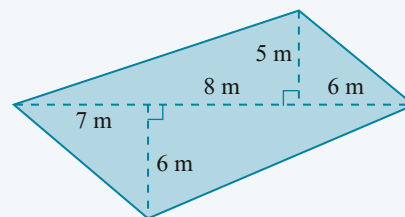
5C ▶ 9 The shaded area of the composite shape on the right is closest to:

- A 16.3 m^2 B 67.1 m^2
C 9.03 m^3 D 11.8 m^2



5D ▶ 10 What is the area of this quadrilateral?

- A 231 m^2 B 168 m^2
C 115.5 m^2 D 32 m^2

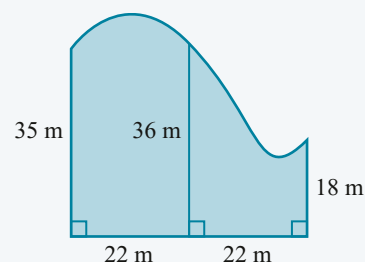


5D ▶ 11 What is the perimeter of the quadrilateral in question 10?

- A 32 m B 39 m
C 48 m D 1521 m

5D ▶ 12 Using two applications of the Trapezoidal rule, an estimate of the area of the field on the right, to the nearest square metre, is:

- A 688 m^2 B 1066 m^2
C 1375 m^2 D 2750 m^2

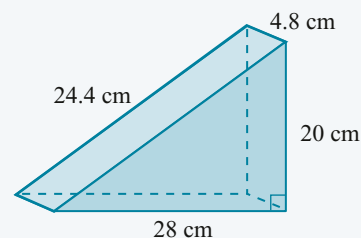


5E ▶ 13 The surface area of a rectangular prism with length 12 cm, breadth 8 cm and height 5 cm is closest to:

- A 480 cm^2 B 392 cm^2
C 296 cm^2 D 196 cm^2

5E ▶ 14 what is the surface area of the triangular prism on the right?

- A 627 cm^2 B 772 cm^2
C 811 cm^2 D 907 cm^2



5F ▶ 15 The total surface area of a closed cylinder of radius 5 cm and height 12 cm is closest to:

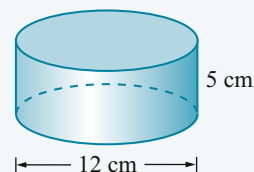
- A 377 cm^2 B 456 cm^2
C 534 cm^2 D 603 cm^2

5F ▶ 16 The surface area of a sphere with diameter 17.8 cm, correct to two decimal places, is:

- A 995.38 cm^2 B 3981.52 cm^2 C 3981.53 cm^2 D 2952.97 cm^2

5G ▶ 17 The volume of the cylinder on the right is closest to:

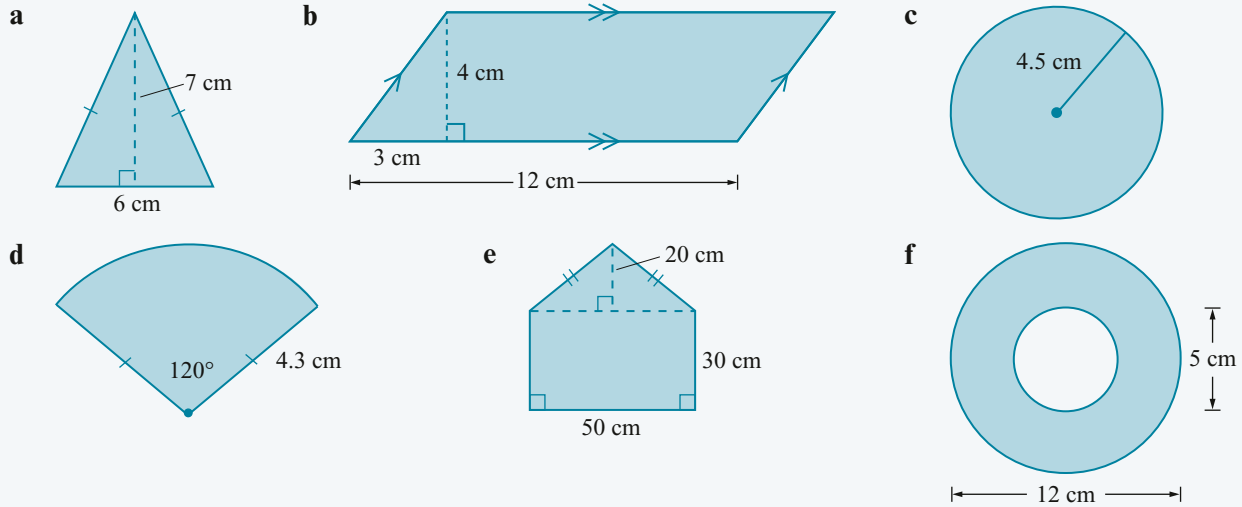
- A 565 cm^3 B 266 cm^3
C 188 cm^3 D 259 cm^3



- 5G** **18** The volume of a sphere with diameter 8 m is closest to:
A 67.0 m^3 **B** 804.2 m^3 **C** 268.1 m^3 **D** 2144.7 m^3
- 5H** **19** The capacity, in kilolitres, of a cubic tank with side length 8 m is closest to:
A 64 kL **B** 512 kL **C** 640 kL **D** 512000 kL

REVIEW SET 1

1 Find the perimeter and area of each of the following figures.



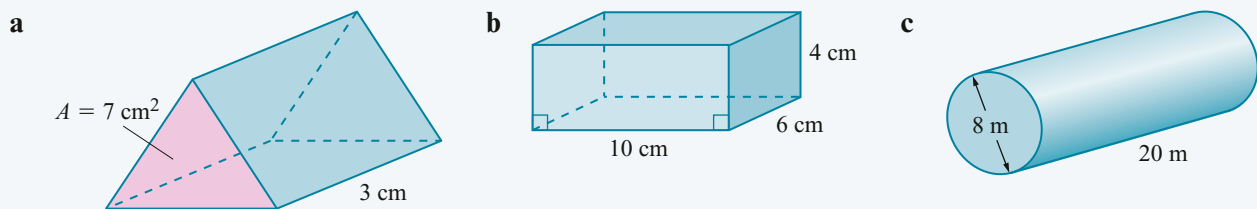
2 a Calculate the area of the field represented by the diagram below. **b** Use the Trapezoidal rule to estimate the area of this field.



3 Calculate the surface area of each of these solids.

- a** a rectangular prism with length 8 m, breadth 4 m and height 3 m
b a closed cylinder with radius 8 cm and height 12 cm
c a sphere of radius 7.5 cm

4 Calculate the volume of each of these solids.

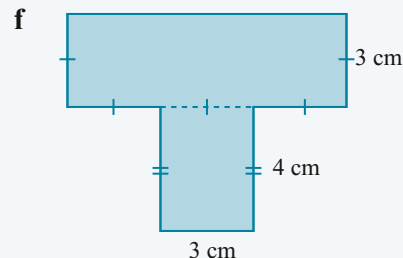
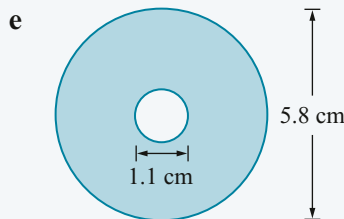
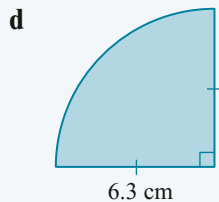
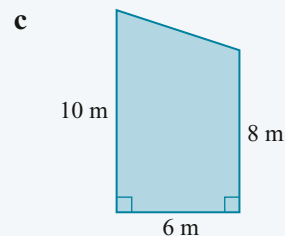
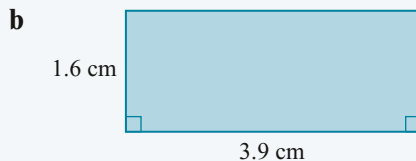
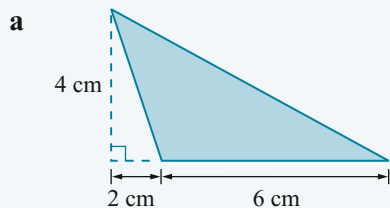


5 Calculate the volume of a sphere with radius 26 mm, to the nearest cubic millimetre (mm^3).

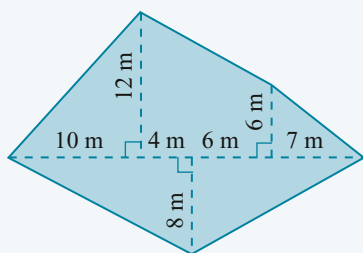
6 Find the capacity in millilitres of a cylinder with radius 7 cm and perpendicular height 14 cm.

REVIEW SET 2

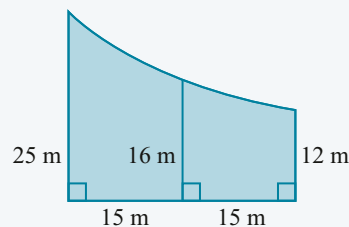
1 Find the perimeter and area of each of these figures.



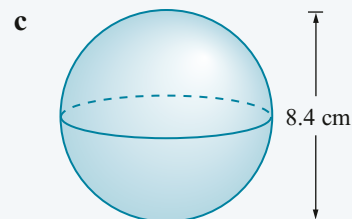
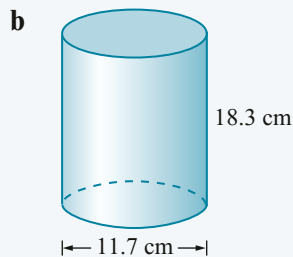
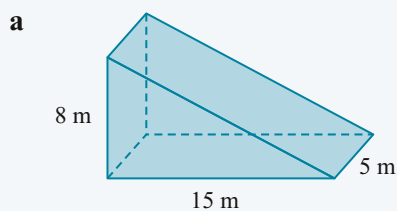
2 **a** Calculate the perimeter and area of this field.



b Use two applications of the Trapezoidal rule to estimate the area of this field.



3 Find the total surface area of each of these solids.

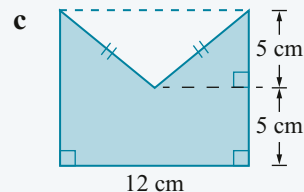
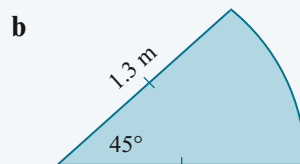
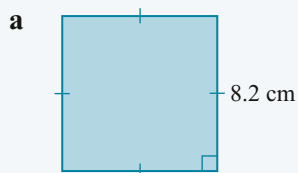


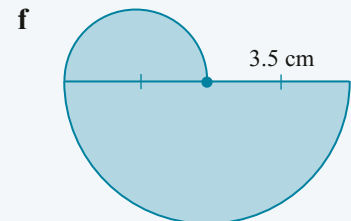
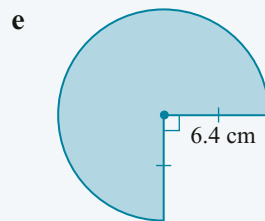
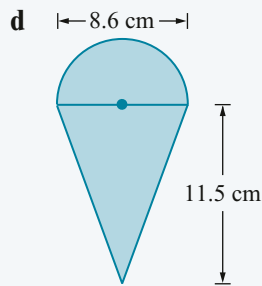
4 Calculate the volume of each solid in question 3.

5 Find the capacity, in megalitres, of a dam with cross-sectional area of 6000 m^2 and an average depth of 10 m.

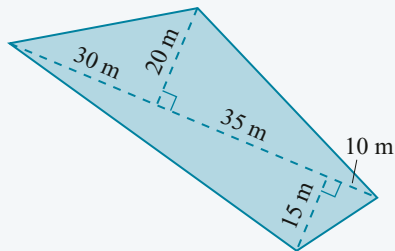
REVIEW SET 3

1 Find the perimeter and area of each of these figures.

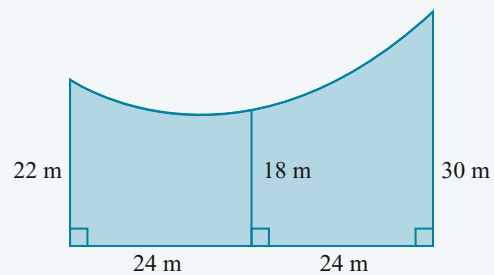




- 2 a** Calculate the perimeter and area of the field from the diagram below.



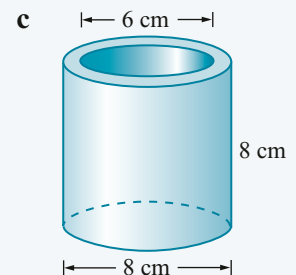
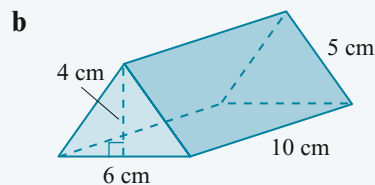
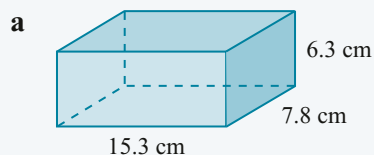
- b** Use two applications of the Trapezoidal rule to estimate the area of the field below.



- 3** Calculate the surface area, correct to one decimal place, of:

- a** a cube of length 7.2 mm
b a closed cylinder with diameter 17 cm and height 8 cm
c a sphere of radius 13 mm.

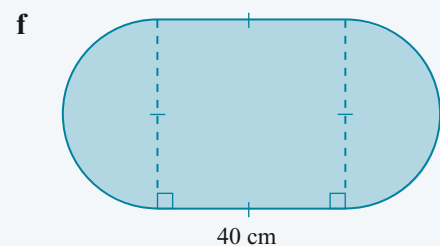
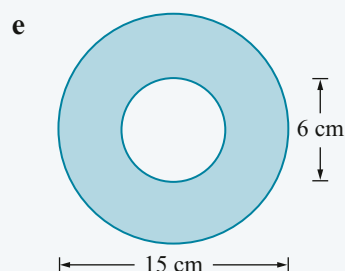
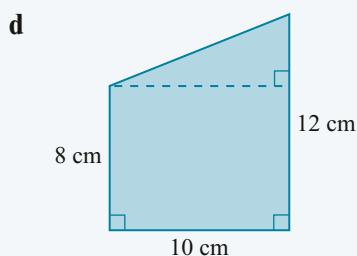
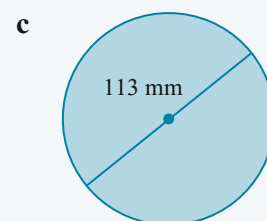
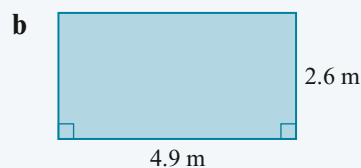
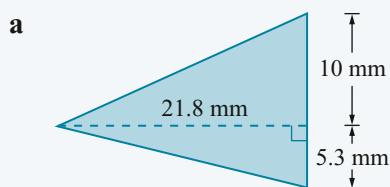
- 4** Calculate the volume of each of these solids, correct to one decimal place.



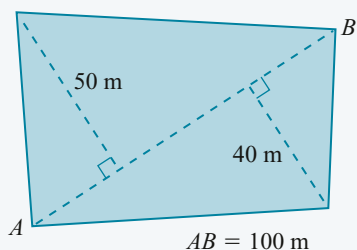
- 5** Find the capacity, in kilolitres, of a spherical tank with diameter 8 m.

REVIEW SET 4

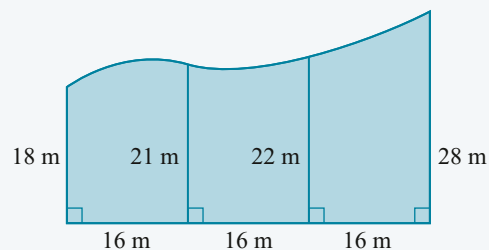
- 1** Find the perimeter and area of each of these figures. Give your answers correct to one decimal place.



- 2 a** Calculate the area of the field from the diagram below.

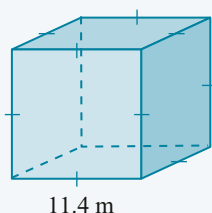


- b** Use three applications of the Trapezoidal rule to estimate the area of the field below.

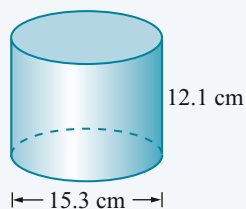


- 3** Find the total surface area of each of these solids, to one decimal place.

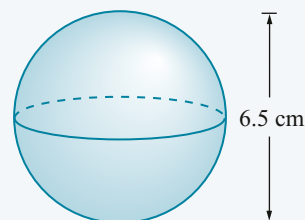
a



b



c

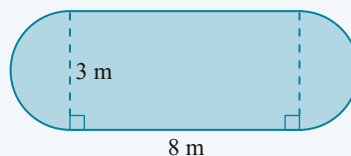


- 4** Calculate the volume of each solid in question 3, to one decimal place.

- 5** Find the capacity, in kilolitres, of a rectangular tank with length 8 m, breadth 6 m and height 4 m.

REVIEW PRACTICE EXAMINATION QUESTION

- 1 a** Find the perimeter of this figure.

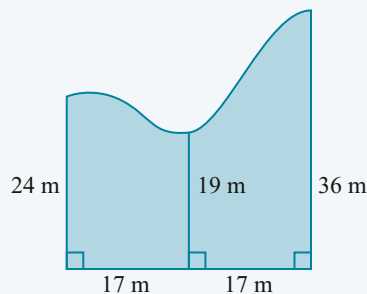


(2 marks)

- b** Find the perimeter and area of a sector which has radius 8 cm and a sector angle of 66° .

(2 marks)

- c** Use two applications of the Trapezoidal rule to estimate the area of the field represented by this diagram.



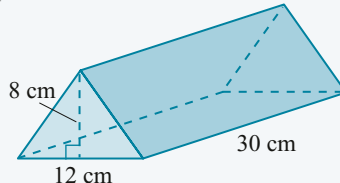
(2 marks)

- d** Calculate the surface area and volume of a sphere of diameter 11 cm.

(4 marks)

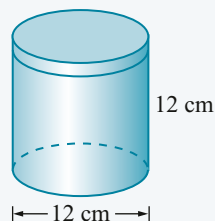
- e** Find the volume of this triangular prism.

(2 marks)



- f** Find the volume of this cylindrical ice-cream container.

(2 marks)



- g** Find the capacity of the container in part f.

(1 mark)

TOTAL: 15 MARKS

6

Interest and depreciation

The main mathematical ideas investigated are:

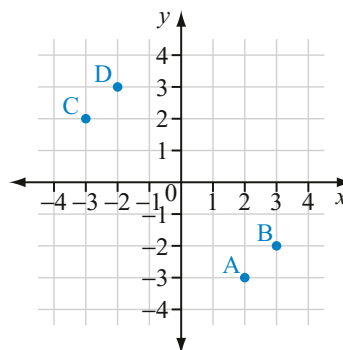
- ▶ applying percentage increase or decrease
- ▶ calculating profit or loss
- ▶ calculating percentage profit or percentage loss
- ▶ exploring calculations involving GST
- ▶ calculating simple interest over various time periods
- ▶ graphing simple interest
- ▶ calculating depreciation of an asset using the straight-line method
- ▶ drawing graphs of compound interest.

ARE YOU READY?

- 6A ▶ 1** What percentage is equivalent to $\frac{2}{5}$?
A 40% **B** 25%
C 2% **D** 50%
- 6A ▶ 2** What is 52% as a fraction in simplest form?
A $\frac{52}{100}$ **B** $\frac{1}{2}$
C $\frac{26}{50}$ **D** $\frac{13}{25}$
- 6A ▶ 3** What is 0.12 as a percentage?
A 0.12% **B** 1.2%
C 12% **D** 120%
- 6A ▶ 4** What is the equivalent decimal to 0.3%?
A 0.3 **B** 0.03
C 0.003 **D** 0.0003
- 6A ▶ 5** Which value is equivalent to 16 written as a percentage of 25?
A 64% **B** 32%
C 16% **D** 25%
- 6A ▶ 6** What is 125% of 80?
A 20 **B** 40
C 60 **D** 100
- 6A ▶ 7** What is 35% of \$1200?
A \$42 **B** \$420
C \$4200 **D** \$42 000
- 6A ▶ 8** A 30% discount is given on an item originally priced at \$120. How much is the discount?
A \$156 **B** \$120
C \$84 **D** \$36

- 6A ▶ 9** An item originally priced at \$160 is marked up by 25%. What is the resulting price?
A \$200 **B** \$160
C \$120 **D** \$40

Use the following diagram to answer questions 10 and 11.



- 6C ▶ 10** What are the coordinates of point B?
A $(-2, 3)$ **B** $(2, 3)$
C $(3, 2)$ **D** $(3, -2)$
- 6C ▶ 11** Which point has the coordinates $(-3, 2)$?
A A **B** B
C C **D** D
- 6C ▶ 12** Which set of coordinates matches the values in this table?
- | | | | | | | |
|----------|---|---|---|----|----|----|
| x | 0 | 1 | 2 | 3 | 4 | 5 |
| y | 0 | 4 | 8 | 12 | 16 | 20 |
- A** $\{(0, 1), (2, 3), (4, 5), (0, 4), (8, 12), (16, 20)\}$
B $\{(0, 0), (1, 4), (2, 8), (3, 12), (4, 16), (5, 20)\}$
C $\{(0, 0), (4, 1), (8, 2), (12, 3), (16, 4), (20, 5)\}$
D $\{(0, 0), (1, 4), (2, 8), (3, 10), (4, 12), (5, 14)\}$

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook assess](#).

- Q1** Support sheet 6A.1 Writing percentages
Q2–4 Support sheet 6A.2 Converting percentages, fractions and decimals
Q5 Support sheet 6A.3 Writing one amount as a percentage of another
Q6–7 Support sheet 6A.4 Percentage of a quantity
Q8–9 Support sheet 6A.5 Financial percentage calculations
Q10–12 Support sheet 6C.1 Plotting points

6A Percentage applications

These resources are available on your [obook assess](#):

- **Interactive 6A.1:** Explore key ideas for finding the percentage of an amount
- **Interactive 6A.2:** Explore key ideas for percentage profit or loss
- **Video tutorial 6A:** Watch and listen to an explanation of Example 6A-1
- **Worksheet 6A:** Practise your skills with extra problems for percentages
- **Investigation 6A:** Explore applications of percentages
- **assess quiz 6A:** Test your skills with an auto-correcting multiple-choice quiz

1010

EXAMPLE 6A-1 Increasing or decreasing an amount by a percentage

a Increase \$75 by 20%.

b Decrease 3 m by 12%.

	Solve	Think	Apply
a	$120\% \text{ of } \$75 = \frac{120}{100} \times \$75$ $= \$90$	$\begin{aligned} &\$75 + 20\% \text{ of } \$75 \\ &= 100\% \text{ of } \$75 + 20\% \text{ of } \$75 \\ &= 120\% \text{ of } \$75 \\ &= \frac{120}{100} \times \$75 \\ &= \$90 \end{aligned}$	To increase a quantity by $x\%$, find $(100 + x)\%$ of the quantity.
b	$88\% \text{ of } 3 \text{ m} = \frac{88}{100} \times 3 \text{ m}$ $= 2.64 \text{ m}$	$\begin{aligned} &3 \text{ m} - 12\% \text{ of } 3 \text{ m} \\ &= 100\% \text{ of } 3 \text{ m} - 12\% \text{ of } 3 \text{ m} \\ &= 88\% \text{ of } 3 \text{ m} \\ &= \frac{88}{100} \times 3 \text{ m} \\ &= 2.64 \text{ m} \end{aligned}$	To decrease a quantity by $x\%$, find $(100 - x)\%$ of the quantity.

EXERCISE 6A Percentage applications

1 Increase the following by the given percentages.

a \$450 by 28%

b 15 m by 75%

c 850 kg by 8%

d 160 cm by 12%

e 240 min by 35%

f \$6 by 200%

g 2 t by 1.5%

h 40 s by $62\frac{1}{2}\%$

i 300 L by $6\frac{2}{3}\%$

2 Decrease the following by the given percentages.

a 115 mm by 30%

b \$480 by 24%

c 350 mL by 40%

d 4.8 km by 19%

e 86 g by 25%

f \$456 by 8.5%

g \$3000 by $2\frac{1}{4}\%$

h 57 s by $33\frac{1}{3}\%$

i 4.2 ha by 15.6%

$$\text{Percentage change} = \frac{\text{change in value}}{\text{original value}} \times 100\%$$

!

EXAMPLE 6A-2 Calculating percentage increase or decrease

- a** John's weight increased from 64 kg to 68 kg. Find the percentage increase in his weight.
b The value of a car decreased from \$18 500 to \$14 900 in 1 year. Calculate the percentage decrease in value.

	Solve	Think	Apply
a	<i>Method 1:</i> Percentage increase $= \frac{68 - 64}{64} \times 100\%$ $= 6.25\%$ <i>Method 2:</i> $\frac{68}{64} = \frac{68}{64} \times 100\%$ $= 106.25\%$ Percentage increase $= 106.25\% - 100\%$ $= 6.25\%$	<i>Method 1:</i> Increase = 68 - 64 kg $= 4$ kg Percentage increase $= \frac{4}{64} \times 100\%$ $= 6.25\%$ <i>Method 2:</i> $\frac{68}{64} \times 100\% = 106.25\%$ The increased weight is 106.25% of the original weight. Percentage increase $= 106.25\% - 100\%$ $= 6.25\%$	<i>Method 1:</i> Find the increase in weight and express this as a percentage of the original weight. Percentage increase $= \frac{\text{increase in quantity}}{\text{original quantity}} \times 100\%$ <i>Method 2:</i> Express the second weight as a percentage of the first and then subtract 100%. Percentage increase $= \frac{\text{second quantity}}{\text{first quantity}} \times 100\% - 100\%$
b	<i>Method 1:</i> Percentage decrease $= \frac{18\,500 - 14\,900}{18\,500} \times 100\%$ $= 19.5\%$ (to one decimal place) <i>Method 2:</i> $\frac{14\,900}{18\,500} = \frac{14\,900}{18\,500} \times 100\%$ $= 80.5\%$ (to one decimal place) Percentage decrease $= 100\% - 80.5\%$ $= 19.5\%$ (to one decimal place)	<i>Method 1:</i> Decrease = \$18 500 - \$14 900 $= \\$3600$ Percentage decrease $= \frac{3600}{18\,500} \times 100\%$ $\approx 19.5\%$ <i>Method 2:</i> $\frac{14\,900}{18\,500} \times 100\% \approx 80.5\%$ The decreased value is 80.5% of the original value. Percentage decrease $= 100\% - 80.5\%$ $= 19.5\%$	<i>Method 1:</i> Find the decrease in value and express this as a percentage of the original value. Percentage decrease $= \frac{\text{decrease in value}}{\text{original value}} \times 100\%$ <i>Method 2:</i> Express the second value as a percentage of the first and then subtract this result from 100%. Percentage decrease $= 100\% - \frac{\text{second quantity}}{\text{first quantity}} \times 100\%$

- 3 a** Complete the following to find the percentage increase in weight from 52 kg to 55 kg.
 Increase in weight = 55 - ____ kg = ____ kg
 Percentage increase = $\frac{\square}{\square} \times 100\% = \text{____}\% = \text{____}\%$ (to one decimal place)
- b** Complete the following to find the percentage decrease in cost from \$186 to \$154.
 Decrease in cost = \$____ - \$____ = \$____
 Percentage decrease = ____ $\times 100\% = \text{____}\% = \text{____}\%$ (to one decimal place)

- 4** Find the percentage increase (to one decimal place) from:
- a** \$350 to \$425 **b** 7.2 m to 7.8 m **c** 63 kg to 68 kg
d \$80 to \$215 **e** 4.2 kg to 8.4 kg **f** 480 mL to 530 mL
- 5** Find the percentage decrease (to one decimal place) from:
- a** \$256 to \$190 **b** 55 kg to 51 kg **c** 15.8 s to 15.5 s
d 430 ha to 385 ha **e** \$5400 to \$1800 **f** 13 s to 12.2 s

EXAMPLE 6A-3 Calculating profit or loss and percentage profit or loss

- a** A signed 2005 West Tigers jersey was purchased for \$800. It was later sold for \$1040. Calculate the percentage profit.
- b** A drone was purchased for \$480 and later sold for \$360. Calculate the percentage loss.

	Solve	Think	Apply
a	Profit = selling price – cost price $= 1040 - 800$ $= 240$ Profit is \$240. Percentage profit $= \frac{240}{800} \times 100\%$ $= 30\%$	To find the profit, subtract the cost price from the selling price. The percentage profit is the profit divided by the cost price, multiplied by 100%.	If the selling price exceeds the cost price, a profit is made.
b	Loss = selling price – cost price $= 360 - 480$ $= -120$ Loss is \$120. Percentage loss $= \frac{120}{480} \times 100\%$ $= 25\%$	To find the loss, subtract the cost price from the selling price. The percentage loss is the loss divided by the cost price, multiplied by 100%.	If the selling price is less than the cost price, a loss is made.

- 6** Calculate the following for the sales listed below:
- i** the profit or loss
- ii** the percentage profit or loss.
- a** A car purchased for \$22 000 is sold for \$16 500.
- b** A Star Wars collectable purchased for \$100 is sold for \$170.
- c** A piano purchased for \$4200 is sold for \$2520.
- d** A \$50 000 share portfolio is sold for \$57 500.
- e** A surfboard purchased for \$600 is sold for \$280.
- f** A set of golf clubs purchased for \$2100 is sold for \$800.



EXAMPLE 6A-4 Calculating multiple percentage applications

A stamp collection was bought for \$3600. In the first year its value increased by 5%. In the second year it increased in value by a further 6%.

- Calculate the stamp collection's value at the end of the first year.
- Calculate its value at the end of the second year.
- What is the overall change in its value after 2 years?
- Find the percentage profit if it is sold for the value at the end of 2 years.

	Solve	Think	Apply
a	Value after 1 year = 105% of \$3600 $= \frac{105}{100} \times 3600$ $= \$3780$	Calculate the value at the end of the first year. Use this result to calculate the value at the end of the second year. Find the overall change in value over these 2 years. Calculate the percentage change in value over these 2 years.	Percentage change $= \frac{\text{change in value}}{\text{original value}} \times 100\%$ If the percentage change is positive, it is a profit. If the percentage change is negative, it is a loss.
b	Value after 2 years = 106% of \$3780 $= \frac{106}{100} \times 3780$ $= \$4006.80$		
c	Overall change in value = \$4006.80 - \$3600 $= \$406.80$		
d	Percentage profit over 2 years $= \frac{\text{change in value}}{\text{original value}} \times 100\%$ $= \frac{406.8}{3600} \times 100\% = 11.3\%$		

- 7** A painting was bought for \$3600. In the first year its value increased by 6%. In the second year it increased by a further 8%. Complete the following.

- Value after 1 year = 106% of \$____

$$= \frac{\square}{100} \times \$\_\_\_\_\_\_$$

$$= \$\_\_\_\_\_\_$$
- Value after 2 years = ____% of \$____

$$= \frac{\square}{100} \times \$\_\_\_\_\_\_$$

$$= \$\_\_\_\_\_\_$$
- Overall change in value after 2 years = \$____ - \$3600

$$= \$\_\_\_\_\_\_$$
- Percentage profit after 2 years = $\frac{\square}{\square} \times 100\%$

$$= \_\_\_\_\_\% \text{ (to one decimal place)}$$

- 8** A piece of antique jewellery was bought for \$2400. In the first year of ownership its value increased by 15%. In the second year, it increased in value by a further 8%.

- Calculate the value of the jewellery at the end of the first year.
- Calculate its value at the end of the second year.
- What is the overall change in the value of the jewellery after 2 years?
- Find the percentage profit after 2 years.

- 9** A \$50 000 share portfolio increased in value by 12% in the first year and decreased by 3% in the second year. Find the percentage profit or loss if it is sold after 2 years.

- 10** An investor bought \$80 000 worth of gold. In the first year the gold increased in value by 10% but, in the second year, it decreased in value by 10%. Find the overall percentage profit or loss of the investment after these 2 years.

GST

a tax of 10% that is added to the price of goods and services

GST stands for Goods and Services Tax, which we have in Australia. Other countries have a tax similar to Australia's GST, called VAT, which stands for Value Added Tax. The GST is a tax added to the price of goods and services. GST is payable on most goods (such as all new goods) and services (such as hospitality, electrical work, plumbing, building). A GST of 10% was introduced in Australia in July 2000 and has remained at 10%. Most unprepared food items, including meat and vegetables, are GST free.

EXAMPLE 6A-5 Calculating GST

Calculate the GST payable on:

- a** a restaurant meal with a pre-GST cost of \$84.00 **b** a pre-GST bill for \$368.00 from an electrician.

	Solve	Think	Apply
a	$\begin{aligned}\text{GST} &= 10\% \text{ of } \$84 \\ &= \frac{10}{100} \times \$84 \\ &= \$8.40\end{aligned}$	Find 10% of the amount.	GST on an amount is 10% of that amount.
b	$\begin{aligned}\text{GST} &= 10\% \text{ of } \$368 \\ &= \frac{10}{100} \times \$368 \\ &= \$36.80\end{aligned}$		

- 11** Complete the following to calculate the GST on a plasma television with a pre-GST price of \$750.
 GST = 10% of ____

$$= \frac{10}{\square} \times \text{____}$$

$$= \text{____}$$
- 12** Calculate the 10% GST payable on goods with a pre-GST price of:
- | | | | |
|-------------------|-------------------|-----------------|-------------------|
| a \$46.00 | b \$150.00 | c \$8.40 | d \$459.00 |
| e \$975.50 | f \$10.99 | g \$1980 | h \$15.42 |
- 13** Michael purchased a mobile phone listed at a pre-GST price of \$654.
- a** Calculate the GST payable on the phone.
b Find the total amount Michael paid for the phone.
- 14** Aaron had a new bathroom installed in his house. The bill was \$8742.80 and was subject to GST of 10%.
- a** Calculate the GST payable on the bathroom.
b Find the actual amount Aaron paid for the bathroom.
- 15** Angelica bought a watch listed at a pre-GST price of \$389.00.
- a** Calculate the GST payable on the watch.
b Find the total amount Angelica paid for the watch.

The easiest way to work out how much GST has been paid on a particular item is to divide the total price, including tax, by 11. This is a GST rule of thumb.

Note: The rule of thumb of dividing by 11 only works if the GST is 10%.

EXAMPLE 6A-6 Calculating GST when it is included in the total price

Guido buys a sofa for \$720 which includes GST.

- Find the GST paid on the sofa.
- Find the original cost of the sofa before the GST was added.

	Solve	Think	Apply
a	$\text{GST paid} = \$720 \div 11$ $= \$65.45$	Because GST is 10%, divide by 11 to find the GST.	To find the GST on a price that already includes GST, divide by 11. For GST amounts other than 10%, the unitary method should be used.
b	$\text{Original cost} = \$720 - \$65.45$ $= \$654.55$ <i>Unitary method:</i> $\text{Price} = \text{cost price plus } 10\%$ $110\% = \$720$ $1\% = \frac{720}{110}$ $= 6.54545\dots$ $100\% = \$654.55$	Original cost is the price paid minus the GST. <i>Unitary method:</i> Price is the cost price plus 10%, or 110% of the cost price.	

- Complete the following to find the GST on a laptop computer that is priced at \$1200 including GST.

$$\text{GST paid} = \underline{\hspace{2cm}} \div 11$$

$$= \underline{\hspace{2cm}}$$
- Use the given rule of thumb to find the GST included in the price of goods and services costing a total of:

a \$458.00	b \$264	c \$34.56	d \$219
e \$19.80	f \$1904	g \$2980	h \$14.90
- Find the cost of each item in question 17 before the GST was added.
- Keong bought a Blu-ray 3D disc player listed at a pre-GST price of \$634. Find the total amount Keong paid for the disc player.
- An antique watch bought for \$15 000 increased in value by 5% in the first year of ownership and increased by 8% in the second year. It is then sold for this value after the second year. Calculate the percentage profit.
- Shannon is travelling overseas and is being refunded the GST on the purchase of a camera that was \$475 including GST. She calculates the GST refund to be 10% of \$475 which is \$47.50. Is she correct? Explain using calculations.
- Aimee has two quotes for repairs on her car. The first is a pre-GST price of \$2450. The second is \$2680 including GST. Which quote is cheaper and by how much?
- A car that cost \$28 900 new depreciated in value by 22% in the first year, by 20% in the second year and by 18% in the third year. What was its value at the end of 3 years?
- The GST in New Zealand is 15%. Jarrod has purchased a kiwi carving for NZ\$95.00 including GST. He is entitled to a refund of the GST when returning to Australia. The rule of thumb of dividing by 11 does not work because the GST is not 10%. Calculate Jarrod's GST refund.

6B Simple interest calculations

These resources are available on your obook assess:

- **Interactive 6B.1:** Explore key ideas for understanding simple interest
- **Interactive 6B.2:** Explore key ideas for simple interest investments
- **Video tutorial 6B:** Watch and listen to an explanation of Example 6B-4
- **Worksheet 6B:** Practise your skills with extra problems for simple interest
- **Investigation 6B:** Explore interest rates on a loan for a car
- **assess quiz 6B:** Test your skills with an auto-correcting multiple-choice quiz

simple interest

an amount calculated on the full amount of a loan or investment for the full term

principal

sum of money invested or amount borrowed as a loan

rate

an amount of interest usually expressed as a percentage

flat rate

simple interest rate as a percentage

term

period of time for an investment or loan

Simple interest is the name given to the type of interest that is calculated on the full amount borrowed over the full term of the loan; or invested for the full term of the investment.

The amount borrowed (or invested) is called the **principal**. The simple interest **rate** as a percentage is often called the **flat rate** of interest. The **term** of the loan (or investment) is usually expressed in years, but may be given as any time period.

Simple interest is calculated using the following formula.

Simple interest formula

$I = Prn$ where:

I is the amount of interest in dollars

P is the principal, the amount borrowed (or invested)

r is the percentage interest rate per time period, expressed as a decimal

n is the number of time periods.

Note: Sometimes t instead of n is used in the simple interest formula.

EXAMPLE 6B-1 Calculating simple interest

Calculate the simple interest on \$5000 at a flat rate of 8% p.a. over 4 years.

Solve	Think	Apply
$I = Prn$ $I = \$5000 \times 0.08 \times 4$ $= \$1600$ The interest is \$1600.	Convert 8% to a decimal and substitute the given values: $r = 8 \div 100 = 0.08$ $P = \$5000$ $n = 4$	Identify all the values for each pronumeral. Convert the percentage rate to a decimal by dividing by 100.

EXERCISE 6B Simple interest calculations

- 1 Complete the following to calculate the simple interest on \$800 at a flat rate of 5% p.a. over 6 years.

$$P = \underline{\hspace{2cm}}$$

$$r = \underline{\hspace{2cm}} \div 100 = \underline{\hspace{2cm}}$$

$$n = \underline{\hspace{2cm}}$$

$$I = Prn$$

$$= \underline{\hspace{2cm}} \times 0.05 \times \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

- 2** Calculate the simple interest on:
- a** \$300 at a flat rate of 6% p.a. over 3 years **b** \$750 at a flat rate of 9% p.a. over 2 years
c \$1500 at a flat rate of 11% p.a. over 5 years **d** \$3650 at a flat rate of 3% p.a. over 6 years.
- 3** Complete the following to calculate the simple interest on \$4650 at a flat rate of 6.47% p.a. for 5 years.
- $$P = \underline{\hspace{2cm}} \qquad I = Prn$$
- $$n = \underline{\hspace{2cm}} \qquad = \underline{\hspace{2cm}} \times \underline{\hspace{2cm}} \times 5$$
- $$r = 6.47 \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}} \qquad = \underline{\hspace{2cm}}$$
- 4** Calculate the simple interest on:
- a** \$1268 at a flat rate of 9.62% p.a. over 3 years **b** \$5360 at a flat rate of 5.36% p.a. over 6 years
c \$33 000 at a flat rate of 6.35% p.a. over 4 years **d** \$11 000 at a flat rate of 7.28% p.a. over 7 years.
- 5** Calculate the amount of simple interest earned on the following investments.

	Principal	Interest rate p.a.	Years
a	\$2680	8%	5
b	\$5990	7%	8
c	\$21 000	12%	6
d	\$55 000	3%	2
e	\$12 500	9%	4
f	\$68 500	14.5%	8
g	\$4225	11.96%	2

EXAMPLE 6B-2 Finding interest rates for different time intervals

An investment pays 5% interest p.a.

- a** Express this percentage as a decimal.
b Find the six-monthly interest rate as a percentage and as a decimal.
c Find the quarterly interest rate as a percentage and as a decimal.
d Find the monthly interest rate as a percentage and as a decimal.

	Solve	Think	Apply
a	Rate = 5% = 0.05	Divide 5% by 100 to express as a decimal: $5 \div 100 = 0.05$	Find the number of times the time period divides into a year and divide the rate by that number.
b	Rate = $5\% \div 2$ = 2.5% = 0.025	Six months is half a year, so divide by 2.	
c	Rate = $5\% \div 4$ = 1.25% = 0.0125	For a quarter of a year, divide by 4.	
d	Rate = $5\% \div 12$ = 0.466 666...% $\approx 0.467\%$ = 0.004 67	There are 12 months in a year, so divide by 12.	

- 6** An investment pays 8% interest p.a.
a Express this percentage as a decimal.
b Find the six-monthly interest rate as a percentage and as a decimal.
c Find the quarterly interest rate as a percentage and as a decimal.
d Find the monthly interest rate as a percentage and as a decimal.
- 7** An investment pays 3% interest p.a.
a Express this percentage as a decimal.
b Find the six-monthly interest rate as a percentage and as a decimal.
c Find the quarterly interest rate as a percentage and as a decimal.
d Find the monthly interest rate as a percentage and as a decimal.
- 8** Complete this table of percentage interest rates.

	Annual rate	Six-monthly rate	Quarterly rate	Monthly rate
a	9%			
b	10%			
c	15%			
d	4.5%			
e		3.8%		
f			2.6%	
g				1.3%
h				0.065%
i		1.95%		

EXAMPLE 6B-3 Calculating simple interest using monthly interest rate

Calculate the simple interest on \$3000 at a flat rate of 8.56% p.a. over 16 months.

Solve	Think	Apply
$P = \$3000$ $r = 8.56 \div 12 \div 100$ $= 0.00713333$ per month $n = 16$ months $I = Prn$ $= 3000 \times 0.00713333 \times 16$ $= \$342.40$ The interest is \$342.40.	Convert 8.56% p.a. to a monthly rate as a decimal and substitute the values for P , r and n .	The units for r and n must be the same; both in years or both in months. It is easier to convert the annual rate to the same time period as n .

- 9** Complete the following to calculate the simple interest of \$5300 at a flat rate of 6.47% p.a. over 17 months.
- | | |
|---|--|
| $P = \underline{\hspace{2cm}}$ | $I = Prn$ |
| $n = \underline{\hspace{2cm}}$ | $= 5300 \times \underline{\hspace{1cm}} \times \underline{\hspace{1cm}}$ |
| $r = 6.47 \div \underline{\hspace{1cm}} \div 100$ | $= \underline{\hspace{2cm}}$ |

- 10** Calculate the amount of simple interest earned on the following investments.

	Principal	Interest rate	Months
a	\$3000	4%	16
b	\$6000	7%	18
c	\$45 500	9%	20
d	\$7500	8.2%	28
e	\$2360	4.6%	30
f	\$5630	8.5%	14

EXAMPLE 6B-4 Calculating annual simple interest rate

If \$3500 is invested for 3 years the amount of interest earned is \$735. Calculate the annual simple interest rate as a percentage.

Solve	Think	Apply
$I = Prn$ $735 = 3500 \times r \times 3$ $= 10\,500r$ $\frac{735}{10\,500} = r$ $r = 0.07$ $\text{Annual rate} = 0.07 \times 100\%$ $= 7\%$	Substitute the values $P = 3500$, $n = 3$ and $I = 735$ into $I = Prn$. Multiply 3500 by 3 and then divide the interest by 10 500. The rate is a decimal. Multiply by 100 to make a percentage.	Substitute the values into the formula and solve for the required value.

- 11** If \$4000 is invested for 4 years, the amount of interest earned is \$1280. Calculate the annual simple interest rate as a percentage.
- 12** If \$6200 is invested for 5 years, the amount of interest earned is \$1240. Calculate the annual simple interest rate as a percentage.
- 13** If \$190 is invested for 10 years, the amount of interest earned is \$123.50. Calculate the annual simple interest rate as a percentage.
- 14** If \$3200 is invested at 6.35% p.a., the amount of interest earned is \$952. Find the number of years of the investment, to one decimal place.
- 15** Ayush invests an amount of money for 2 years at 3.2% p.a. simple interest and receives \$154 in interest. What amount of money did he invest?
- 16** Eden invests \$1300 for two years at a simple interest rate of 2.8% p.a. After the first year, the interest rate changes to 3.1%. Calculate the amount of extra interest Eden receives due to the rate change.
- 17** Hanna has \$1000 to invest and wants to earn \$50 in interest. She wants a rate between 2.1% and 3.5% and a time period between 1 and 2 years.
- a** An investment at 5% for 1 year would earn \$50 in interest. Why would that not be suitable?
- b** Determine four different investment scenarios with rates and time periods suitable for Hannah that will result in interest of \$50.

6C Simple interest graphs

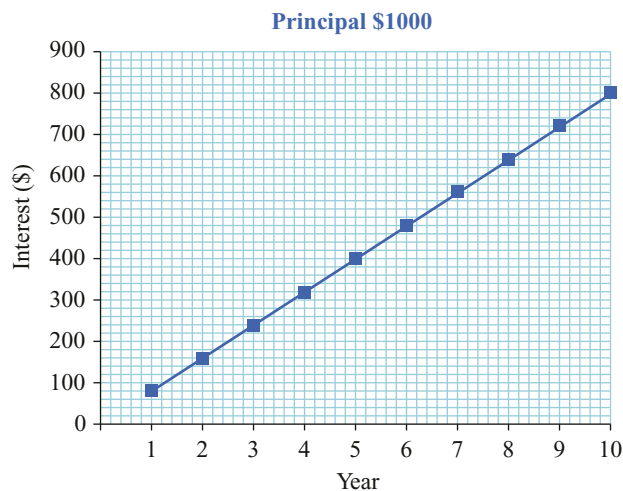
These resources are available on your obook assess:

- **Spreadsheet 6C:** Generate simple interest graphs
- **assess quiz 6C:** Test your skills with an auto-correcting multiple-choice quiz

01
GI

A mathematical model for simple interest may be developed using a graph that compares the interest earned and the rate for a fixed amount of money over a number of years.

This section is best attempted using a graphics calculator or, if one is not available, a spreadsheet. This enables the variables to be changed and the new graph to be seen without having to calculate and plot points.



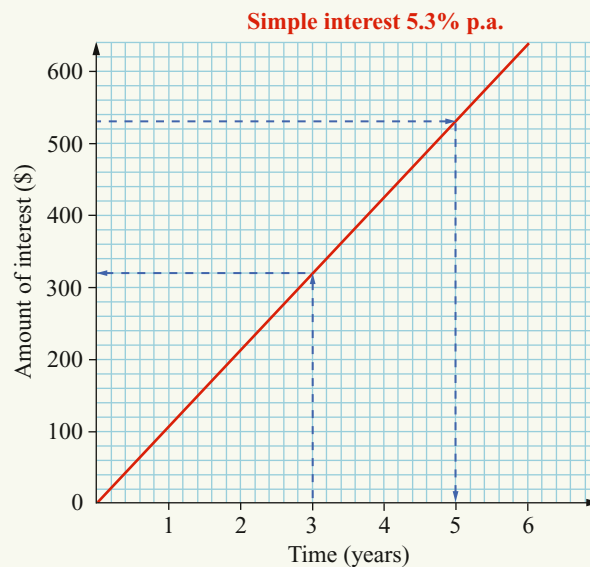
EXAMPLE 6C-1 Interpreting a simple interest graph

The graph shows the simple interest earned on an investment of \$2000 at 5.3% p.a. interest.

Use the graph to find:

- the amount of interest earned after 3 years
- the time taken to earn \$530 interest.

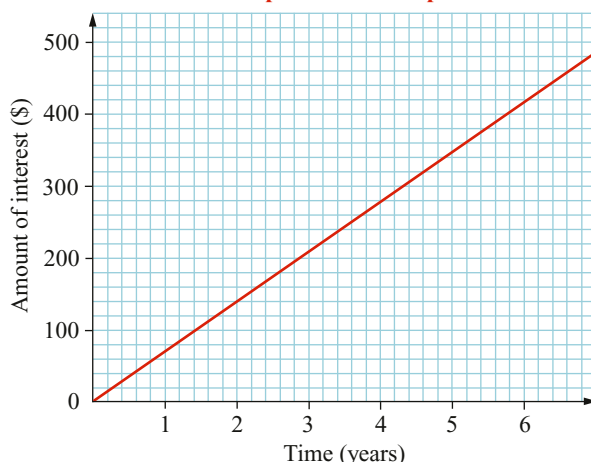
	Solve	Think
a	About \$320	Go up from 3 on the 'Time' axis to the line, then across to the 'Amount of interest' axis. The value is \$320.
b	About 5 years	Go across from \$530 on the 'Amount of interest' axis to the line, then down to the 'Time' axis. The time is 5 years.
Apply		
To find values, move from one axis to the line, then to the other axis.		



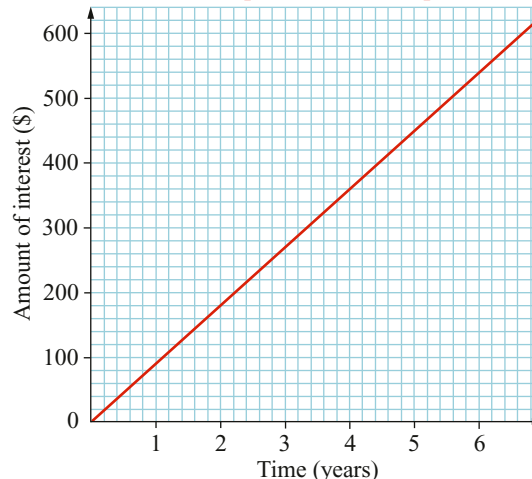
EXERCISE 6C Simple interest graphs

Use the following graphs to answer questions 1–4.

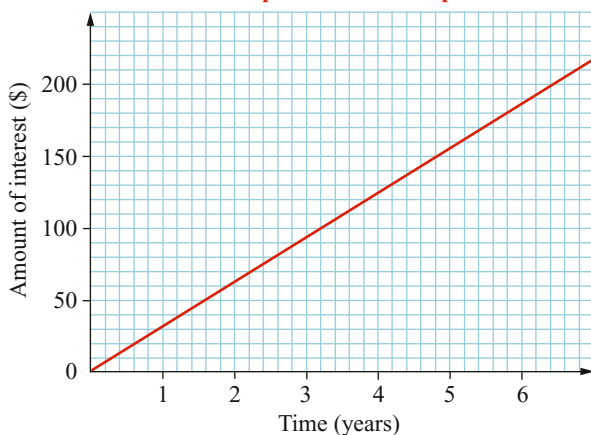
Graph A
Simple interest 7% p.a.



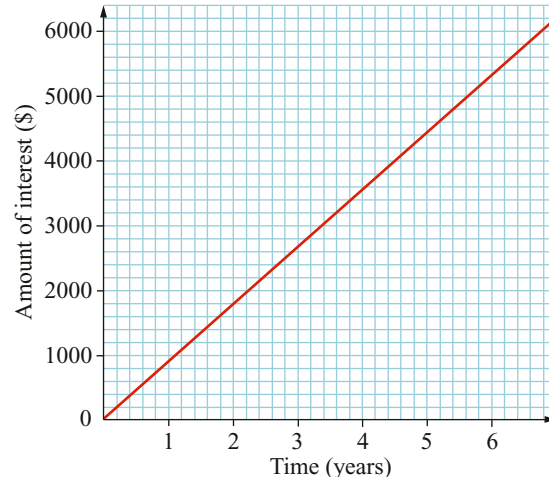
Graph B
Simple interest 4.5% p.a.



Graph C
Simple interest 6.3% p.a.



Graph D
Simple interest 8.9% p.a.



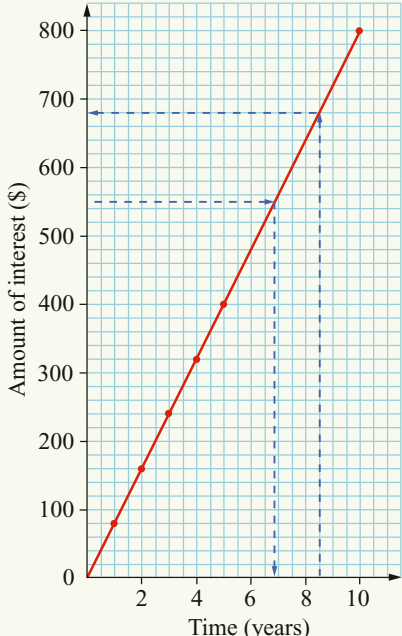
- 1 Graph A above shows the interest earned when \$1000 is invested at 7% p.a.
Use Graph A to help you complete the following to find:
 - a the amount of interest earned after 4 years.
Go up from ____ on the ____ axis to the line, then ____ to the 'Amount of interest' axis, to get ____.
 - b the time taken to earn \$400 interest.
Go ____ from ____ on the 'Amount of interest' axis to the line, then ____ to the 'Time' axis, to get ____.
- 2 Graph B above shows the interest earned when \$2000 is invested at 4.5% p.a. simple interest.
Use this graph to find:
 - a the amount of interest earned after 6 years
 - b the time taken to earn \$350 interest.
- 3 Graph C above shows the interest earned when \$500 is invested at 6.3% p.a. simple interest. Use this graph to find:
 - a the amount of interest earned after 2 years
 - b the time taken to earn \$100 interest.
- 4 Graph D above shows the interest earned when \$10000 is invested at 8.9% p.a. simple interest. Use this graph to find:
 - a the amount of interest earned after 4.5 years
 - b the time taken to earn \$5000 interest.

EXAMPLE 6C-2 Drawing and interpreting a simple interest graph

Time (years)	1	2	3	4	5	10
Interest (\$)	80	160	240	320	400	800

From the table of values, draw a graph showing the amount of interest earned over a period of 10 years when \$1000 is invested at 8% p.a. Use the graph to find:

- a** the interest earned after $8\frac{1}{2}$ years **b** the time taken to earn interest of \$550.

Solve	Think
Simple interest 8% p.a. 	Plot the points from the table and draw the straight-line graph. Show 'Time (years)' on the horizontal axis and 'Amount of interest (\$)' on the vertical axis.
a From the graph, the interest after $8\frac{1}{2}$ years is about \$680.	Go up from $8\frac{1}{2}$ on the horizontal axis (Time) to the graph, then across to the vertical axis (Amount of interest): about \$680.
b The time taken to earn interest of \$550 is a little less than 7 years.	Go across from \$550 on the vertical axis (Amount of interest) to the graph, then down to the horizontal axis (Time): less than 7 years.
Apply Plot the points from the table and draw the graph. To find values, move from one axis to the line, then to the other axis. If using a graphics calculator, use the TRACE function to find the values.	

- 5** An amount of \$1000 is invested for 10 years at 5% p.a. simple interest. The table shows some interest values.

Time (years)	1	2	4	5	10
Interest (\$)	50	100	200	250	500

- a** Use the values in the table to draw a graph showing the amount of interest earned over a period of 10 years. (If a graphics calculator is not available, refer to question 9.)
- b** Find the interest after $7\frac{1}{2}$ years.
- c** Find the time taken to earn interest of \$180.

- 6** An amount of \$1000 is invested for 10 years at 6% p.a. simple interest.

a Use the simple interest formula to complete this table of values.

Time (years)	1	2	4	5	10
Interest (\$)					

- b** Use the values from your completed table to draw a graph showing the amount of interest earned over a period of 10 years.
c Find the interest after $9\frac{1}{2}$ years.
d Find the time taken to earn interest of \$250.

- 7** On the same set of axes, draw graphs to show \$1000 invested at:

i 9% p.a. over 10 years

ii 11% p.a. over 10 years

iii 8.5% p.a. over 10 years.

- a** How much has each investment earned over 10 years?
b How much has each investment earned after $4\frac{1}{2}$ years?
c How much more than the others has the best investment earned after 10 years?
d How long does each investment take to earn \$500 of interest?

- 8** On the same set of axes, draw graphs to show \$2000 invested at:

i 8% p.a. over 10 years

ii 12% p.a. over 10 years

iii 10.5% p.a. over 10 years.

- a** How much has each investment earned over 10 years?
b How much has each investment earned after $8\frac{1}{2}$ years?
c How much more than the others has the best investment earned after 10 years?
d How long does each investment take to earn \$1000 of interest?

- 9 a** Create a spreadsheet to draw simple interest graphs like the example shown on the right in which \$1000 is invested at 8% p.a. The formula in cell B4 is $=B\$1*B\$2/100*A4$. Use the **fill down** function to complete the spreadsheet.

- b** Highlight the column values for 'Year' and 'Interest' and then select **Charts** from the **Insert** menu (choose a line graph style) to produce the matching graph.
c Modify your spreadsheet to check your results for questions 7 and 8.

	A	B
1	Principal (\$)	1000
2	Percentage rate (%)	8
3	Year	Interest (\$)
4	1	80
5	2	160
6	3	240
7	4	320
8	5	400
9	6	480
10	7	560
11	8	640
12	9	720
13	10	800

- 10 a** Research a number of bank accounts to compare fees and interest rates. Some accounts have low fees, but low interest rates, and others vary.

- b** Compare fees and interest rates per month for the following deposits.

i \$500

ii \$1000

iii \$2000

- 11 a** Find some examples of the current annual interest rates for investments. List them.

- b** Draw graphs to show the investment of \$100 over 10 years at varying simple interest rates.

- 12 a** Find some examples of the current annual interest rates for loans. List them.

- b** Draw graphs to show the simple interest paid on a loan of \$100 over 10 years at these rates.

6D Straight-line depreciation

These resources are available on your obook assess:

- **assess quiz 6D:** Test your skills with an auto-correcting multiple-choice quiz

o
i
c
i

depreciation

the loss in the value of an asset due to its age and usage

salvage value (book value, scrap value, written-down value)

the value of an asset after depreciation

straight-line method of depreciation

a method for which an item loses value by a constant amount each year

The **depreciation** of an item is its loss in value due to age and usage. The value of an item after depreciation is called its **salvage value**, **book value**, **scrap value** or **written-down value**.

In this section, we will look at the **straight-line method of depreciation**, which assumes that the item depreciates by a constant amount each year.



EXAMPLE 6D-1 Calculating the value of an asset after depreciation

Construct a table to calculate the value of a \$20 000 car after 3 years if it depreciates \$3400 each year.

Solve				Think	Apply
Year	Value (\$)	Depreciation (\$)	Depreciated value (\$)	Value at end of year 1 = \$20 000 – \$3400 = \$16 600 Value at end of year 2 = \$16 600 – \$3400 = \$13 200 etc.	Subtract the amount of depreciation from the value at the beginning of the year. <i>Note:</i> Value at beginning of year = value at end of previous year.
1	20 000	3400	16 600		
2	16 600	3400	13 200		
3	13 200	3400	9800		

EXERCISE 6D

Straight-line depreciation

- 1 Complete this table to calculate the depreciated value of a \$35 000 car after 3 years. The rate of depreciation is \$3800 per year.

Year	Value (\$)	Depreciation (\$)	Depreciated value (\$)
1	35 000	3800	
2			
3			

The formula for the straight-line method of depreciation is $S = V_0 - Dn$

where S = salvage (current) value of the asset

V_0 = purchase price of the asset

D = amount of depreciation per time period

n = total number of time periods.



EXAMPLE 6D-2 Calculating the value of an asset using the formula for the straight-line method of depreciation

A car purchased for \$14 900 depreciates \$1660 per year. Calculate

- a its book value after 5 years
b the scrap value if the useful life is 8 years.

	Solve	Think	Apply
a	$S = V_0 - Dn$ $= 14\,900 - 1660 \times 5$ $= \$6600$	Substitute $V_0 = 14\,900$, $D = 1660$, $n = 5$ into the formula.	Substitute the values of D , n and V_0 into the formula $S = V_0 - Dn$.
b	$S = V_0 - Dn$ $= 14\,900 - 1660 \times 8$ $= \$1620$	Substitute $V_0 = 14\,900$, $D = 1660$, $n = 8$ into the formula.	The scrap value is the depreciated value at the end of an item's usable life. However the market value may be greater than the depreciated value.

- 2 A car purchased for \$26 990 depreciates \$3300 per year. Complete the following to calculate its book value:
- a after 2 years
 $S = V_0 - Dn$
 $= \$26\,990 - \$\_\_\_ \times 2$
 $= \$\_\_\_$
- b after 4 years.
 $S = V_0 - Dn$
 $= \$\_\_\_ - \$\_\_\_ \times \_\_\_$
 $= \$\_\_\_$
- 3 A car purchased for \$18 700 depreciates \$1980 per year. Calculate its book value after:
- a 2 years
b 5 years.

- 4 A new sewing machine costs \$3800 and depreciates by \$400 per year. Calculate the book value after:

a 3 years b 8 years.

- 5 A car purchased for \$38 999 depreciates \$4200 per year. Calculate its book value after:

a 2 years b 5 years.

- 6 A donut-making machine is purchased new for \$5900. It depreciates at \$700 per year.

a Calculate its book value after 4 years.
b The machine has a usable life of 7 years. Find its scrap value.



- 7 A cash register machine is purchased for \$2770 and depreciates by \$360 per year. It has a usable life of 7 years. Find:

a its value after 4 years b its scrap value.

EXAMPLE 6D-3 Calculating the annual amount of depreciation using the formula for the straight-line method

A car purchased for \$21 990 was worth \$11 990 after 4 years, using the straight-line method of depreciation. Calculate the annual amount of depreciation.

Solve	Think	Apply
$S = V_0 - Dn$ $\$11\,990 = \$21\,990 - D \times 4$ $4D + \$11\,990 = \$21\,990$ $4D = \$21\,990 - \$11\,990$ $= \$10\,000$ $D = \$2500$	Substitute $S = \\$11\,990$, $V_0 = \\$21\,990$, $n = 4$ into the formula. Add $4D$ to both sides. Subtract \$11 990 from both sides. Divide both sides by 4.	Substitute the given values into the formula $S = V_0 - Dn$ and solve the resulting equation.

- 8 A car purchased for \$26 900 was worth \$14 260 after 4 years, using the straight-line method of depreciation. Complete the following to calculate the annual amount of depreciation.

$$\begin{aligned}
 S &= V_0 - Dn \\
 \underline{\hspace{1cm}} &= \underline{\hspace{1cm}} - D \times 4 \\
 4D + \underline{\hspace{1cm}} &= \underline{\hspace{1cm}} \\
 4D &= \underline{\hspace{1cm}} - \underline{\hspace{1cm}} \\
 &= \underline{\hspace{1cm}} \\
 D &= \underline{\hspace{1cm}}
 \end{aligned}$$

- 9 A car purchased for \$45 900 was worth \$35 150 after five years, using the straight-line method of depreciation. Calculate the annual amount of depreciation.

- 10 A boat depreciates in value from \$23 000 to \$15 000 in two years. Calculate the annual amount of depreciation.



- 11** A car that was purchased for \$36 760 was worth \$14 460 after five years, using the straight-line method of depreciation. Calculate the annual amount of depreciation.
- 12** A pastry making machine is purchased new for \$8600. After 6 years, its book value was \$3500. Calculate the annual amount of depreciation.
- 13** A laminator is purchased new for \$1450. After 3 years, its book value was \$610. Calculate the annual amount of depreciation.
- 14** An engine analyser is purchased new for \$8550. After 7 years, its book value was \$2250. Calculate the annual amount of depreciation.



- 15** A car purchased for \$15 570 depreciates \$3120 each year. According to the straight-line method of depreciation, after how many years is the car worthless?
- 16** A commercial dishwasher was purchased for \$6200. It depreciates at \$950 per year. Its scrap value is \$500. Calculate the dishwasher's useable life.
- 17** A motorcycle purchased for \$22 880 depreciates \$3200 each year. After how many years is the motorcycle worthless?
- 18** Calculate the usable life of a washing machine purchased for \$4800, depreciating at \$575 per year with a scrap value of \$1350.

- 19** A Porsche motor car is purchased for \$125 000. The annual rate of depreciation is \$15 000.
 - a** Calculate the depreciated value of the car after 7 years.
 - b** After 7 years, the car has low kilometres and is in excellent condition. It is sold for \$65 000. The cost price can be considered to be the depreciated value after 7 years. Calculate the profit on the sale over the depreciated value.
 - c** Calculate the percentage profit over the depreciated price.
 - d** Calculate the total loss as a percentage of the original purchase price.



6E Compound interest

These resources are available on your obook assess:

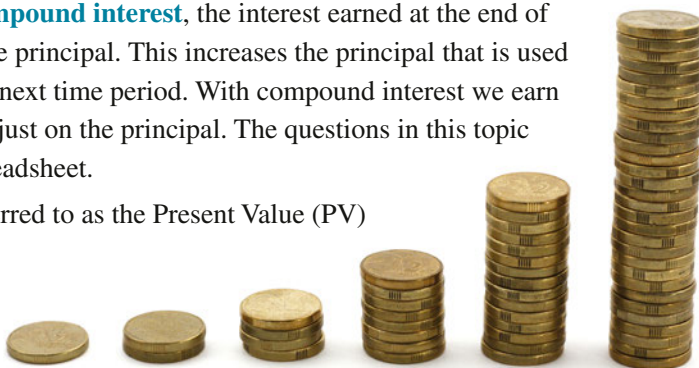
- **Video tutorial 6E:** Watch and listen to an explanation of Example 6E-1
- **Worksheet 6E:** Practise your skills with extra problems for compound interest
- **Spreadsheet 6E:** Generate compound interest graphs
- **assess quiz 6E:** Test your skills with an auto-correcting multiple-choice quiz

compound interest

interest that is added to the principal at regular time intervals during an investment or loan period so future interest is calculated including that amount

When money is invested at **compound interest**, the interest earned at the end of each time period is added to the principal. This increases the principal that is used to calculate the interest for the next time period. With compound interest we earn interest on the interest and not just on the principal. The questions in this topic are best completed using a spreadsheet.

Note: The principal can be referred to as the Present Value (PV) while the final amount can be referred to as the Future Value (FV).



EXAMPLE 6E-1 Calculating total value of an investment and compound interest earned

\$2000 is invested for 3 years at 7% p.a. interest compounded annually.

- Find the amount the \$2000 will grow to after 3 years.
- Find the amount of interest earned.

Solve

a

Time	Working	Total balance	Interest
Start of first year		\$2000	$\frac{7}{100} \times 2000 = \$140$
Start of second year	$\$2000 + \$140$	\$2140	$\frac{7}{100} \times \$2140 = \$149.80$
Start of third year	$\$2140 + \$149.80$	\$2289.80	$\frac{7}{100} \times \$2289.80 = \$160.29$
Start of fourth year	$\$2289.80 + \$160.29$	\$2450.09	

The amount the \$2000 will grow to after 3 years is \$2450.09.

b

The amount of interest earned = $\$2450.09 - \$2000 = \$450.09$.

Think

a

Use $I = Prn$ with $P = 2000$, $r = 0.07$ and $n = 1$ to find the interest for the first year (\$140). Add \$140 to \$2000 to get a new principal of \$2140, then calculate the interest on \$2140. Repeat this last step for the required number of years.

b

Subtract \$2000 from the total.

Apply

Use $I = Prn$ with $n = 1$ to calculate interest each year. The principal each year is the previous principal plus the interest for that year. The interest earned is the total balance less the original principal.

EXERCISE 6E Compound interest

- 1 A spreadsheet can be used for the compound interest calculations in Example 6E-1. Open a new spreadsheet and type in three of the column headings from the example into cells A1 to C1 as shown below. Then follow these instructions.

Type 2000 into cell B2.

Type the formula $=7/100 * B2$ into cell C2 and **fill down** to C5.

Type the formula $=B2 + C2$ into cell B3 and **fill down** to B5.

	A	B	C
1	Time	Total balance	Interest
2	Start	\$ 2,000.00	\$ 140.00
3	Year 1	\$ 2,140.00	\$ 149.80
4	Year 2	\$ 2,289.80	\$ 160.29
5	Year 3	\$ 2,450.09	\$ 171.51

- a Compare your results to those in Example 6E-1. Do you obtain the same values for the total balance after 3 years and the total amount of interest earned?
- b Highlight the values in the columns for 'Time' and 'Total balance' and then select **Charts** from the **Insert** menu to produce a graph which shows the value of the investment over the three years.

You can modify the spreadsheet from question 1 to answer the following questions.

- 2 a Complete the table below to determine the final value of \$2400 invested at 6% p.a. compound interest for 3 years.

Time	Balance + interest	Total balance	Interest
Start of first year		\$2400	\$144
Start of second year	$\$2400 + \$144$	\$2544	\$152.64
Start of third year	$\$2544 + \$152.64$		
Start of fourth year			

- b Calculate the total interest earned.
Interest = _____ - \$2400
= _____
- c Use a spreadsheet to produce a graph of the value of the investment over 3 years.
- 3 a Complete the table below to determine the final value of \$3600 invested at 5% p.a. compound interest for 3 years.

Time	Balance + interest	Total balance	Interest
Start of first year		\$3600	\$180
Start of second year	$\$3600 + \$180$	\$3780	\$189
Start of third year	$\$3780 + \underline{\hspace{2cm}}$		
Start of fourth year			

- b Calculate the total interest earned.
- c Use a spreadsheet to produce a graph of the value of the investment over 3 years.

- 4 a Complete the table to determine the final value of \$980 invested at 3% p.a. compound interest for 4 years.

Time	Balance + interest	Total balance	Interest
Start of first year			
Start of second year			
Start of third year			
Start of fourth year			

- b Calculate the total interest earned.
c Use a spreadsheet to produce a graph of the value of the investment over 4 years.

- 5 Adele decided to invest her savings of \$7890 for 5 years at 8.9% p.a. compound interest.

- a Complete the table below for Adele's investment.

Time	Balance + interest	Total balance	Interest
Start of first year			
Start of second year			
Start of third year			
Start of fourth year			
Start of fifth year			
Start of sixth year			

- b If Adele intends to buy a car that is expected to be valued at \$12 990 when her investment matures, will she have enough to buy the car? Explain.
c By how much will her matured investment be over or under the value of the car?



- 6**
- a** \$1000 is invested at 7% p.a. interest compounding annually. Use a spreadsheet to help you complete this table to show the value of the investment.
 - b** Produce a graph showing the value of the investment over a period of 10 years.
 - c** Find the value of the investment after $6\frac{1}{2}$ years.
 - d** Find the time taken for the investment to be worth \$1500.
 - e** On the same set of axes for part **b**, draw a simple interest graph for the same time period but with a rate of 9% p.a.
 - f** Use the graph drawn in part **e** to determine the number of years it takes to have the same total amount.

Time (years)	Investment (\$)
0	1000.00
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

- 7**
- a** \$2000 is invested at 9.3% p.a. interest compounding annually. Use a spreadsheet to help you complete this table to show the value of the investment.
 - b** Produce a graph showing the value of the investment over a period of 10 years.
 - c** Find the value of the investment after $4\frac{1}{2}$ years.
 - d** Find the interest earned after $4\frac{1}{2}$ years.
 - e** Find the time taken for the investment to be worth \$4000.
 - f** On the same set of axes for part **b**, draw a straight line joining the point representing the initial value of the investment and the point representing the investment value after 10 years.
 - g** Calculate the gradient of the straight line drawn for part **f** to help you determine the equivalent simple interest rate.

Time (years)	Investment (\$)
0	2000.00
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

- 8** An amount of \$100 000 is to be invested for 10 years.
- a** Use a spreadsheet to determine the final value of the investment if the initial amount is invested at:
 - i** 8.5% p.a. simple interest
 - ii** 8.5% p.a. compound interest.
 - b** Explain why the final value is different for the two investment options.
 - c** On the same set of axes, draw graphs to show how the value of the investment changes for each option over the 10 years.



CHAPTER 6 REVIEW INTEREST AND DEPRECIATION

You should be able to:

- ✓ apply percentage increase or decrease in various contexts
- ✓ calculate profit or loss, and percentage profit or percentage loss
- ✓ calculate GST
- ✓ calculate simple interest over different time periods, altering the rate where necessary
- ✓ draw simple interest graphs using pen and paper and technology
- ✓ calculate the depreciation of an asset using the straight-line method
- ✓ draw compound interest graphs as a result of repeated applications of the simple interest formula.

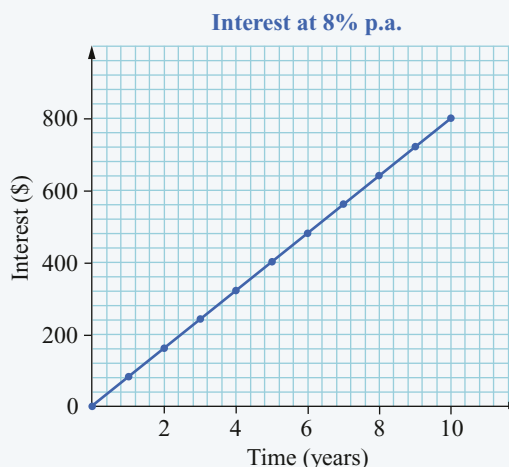
Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

REVIEW MULTIPLE-CHOICE QUESTIONS

- 6A** 1 What is 60 kg increased by 5%?
 A 65 kg B 5 kg C 63 kg D 3 kg
- 6A** 2 What is the percentage decrease from 80 kg to 74 kg?
 A 92.5% B 7.5% C 94% D 6%
- 6A** 3 Melissa receives a 20% discount on a pair of jeans. She pays \$68 for the jeans. How much does she save?
 A \$13.60 B \$17 C \$85 D \$51
- 6A** 4 Sharee buys a dining table for \$576 including GST. What is the GST on the table?
 A \$52.36 B \$633.60 C \$523.64 D \$57.60
- 6B** 5 What is the simple interest on \$650 at 8% p.a. over 4 years?
 A \$52 B \$208 C \$758 D \$20 800
- 6B** 6 What is the simple interest on \$1580 at 7.2% p.a. over 15 months?
 A \$1706.40 B \$3286.40 C \$113.76 D \$142.20

Use this graph to answer questions 7 and 8.

- 6C** 7 From the graph, what is the amount of interest earned after 7 years?
 A \$560 B \$860 C \$500 D \$600
- 6C** 8 From the graph, what is the number of years required to earn \$700 interest?
 A 8.8 B 8 C 9 D 8.4
- 6D** 9 Using the straight-line method, what is the value after 4 years of a \$25 000 car that depreciates \$2650 per year?
 A \$22 350 B \$17 050 C \$19 700 D \$14 400
- 6D** 10 A car that was purchased for \$29 900 was worth \$14 300 after 5 years, using the straight-line method of depreciation. What was the annual amount of depreciation?
 A \$14 300 B \$15 600 C \$3120 D \$11 180



REVIEW SET 1

- 1**

a Increase 80 kg by 3%.

b Decrease \$290 by 15%.
- 2**

a What would be the value of a \$25 000 vintage car after 2 years if it increases in value by 8% in the first year and then decreases in value by 5% in the second year?

b If the car is sold for its value at the end of the second year calculate the overall percentage profit made.
- 3** Penny received a discount of \$25.70 on a mobile phone. This was 5% of the original price. Find the original price of the phone.
- 4** Mark has a new kitchen installed in his house. The bill is \$12 546.30, and is subject to GST of 10%.

a Calculate the GST payable on the kitchen.

b Find the total amount Mark pays for the kitchen.
- 5** Use the rule of thumb in Topic 6A to find the GST paid on goods and services costing the following amounts.

a \$1235 **b** \$359 **c** \$42.18 **d** \$266
- 6** Calculate the simple interest on \$850 at a flat rate of 7% p.a. over 3 years.
- 7**

a An investment pays 10% interest p.a. Express this percentage as a decimal.

b Use your answer to part **a** to find each of these rates as a percentage and as a decimal.

i the six-monthly interest rate **ii** the quarterly interest rate **iii** the monthly interest rate
- 8** Calculate the simple interest on \$4000 at a flat rate of 7.92% p.a. over 17 months.
- 9** Draw a graph showing the amount of interest earned over a period of 10 years if \$1000 is invested at 7% p.a. simple interest.

a Find the interest after $6\frac{1}{2}$ years.

b Find the time to earn interest of \$250.

REVIEW SET 2

- 1** Kieren invests \$8000 in a share fund. In the first year the fund increases in value by 12% and in the second year by 15%. Calculate the overall percentage change in the value of the fund.
- 2** Julie's weekly wage increased from \$450 to \$494.40. Calculate the percentage increase.
- 3** Harry wants to buy a camera that is listed at the pre-GST price of \$849.
 - a** Calculate the GST payable on the camera.
 - b** Find the total amount Harry pays for the camera.
- 4** Calculate the simple interest on \$3550 at a flat rate of 11% p.a. over 3 years.
- 5** Calculate the simple interest on \$6000 at a flat rate of 9.74% p.a. over 13 months.
- 6** If \$1590 is invested for 8 years, the amount of interest earned is \$699.60. Calculate the annual simple interest rate as a percentage.
- 7** A car purchased for \$15 800 depreciates \$1760 per year. Calculate its book value after 5 years.
- 8** A car that was purchased for \$22 990 had a book value of \$15 190 after 4 years, using the straight-line method of depreciation. Calculate the annual amount of depreciation.

REVIEW SET 3

- 1 Sean weighed 75 kg before he went on a diet and, in the first week, he lost 1.5 kg. Calculate the percentage decrease in Sean's weight.
- 2 Cedric has a new roof put on his house. The bill is \$22 586 and is subject to GST of 10%.
 - a Calculate the GST payable on the roof.
 - b Find the total amount Cedric pays for the roof.
- 3 Lisa buys a bracelet listed at the pre-GST price of \$425.
 - a Calculate the GST payable on the bracelet.
 - b Find the total amount Lisa pays for the bracelet.
- 4 Use the rule of thumb discussed in Topic 6A to find the GST already paid on goods and services costing the following amounts.
 - a \$7500
 - b \$1268
 - c \$25.99
 - d \$401
- 5 Calculate the simple interest on \$1600 at a flat rate of 8.5% p.a. over 6 years.
- 6 An investment pays 6% interest p.a.
 - a Express this percentage as a decimal.
 - b Find the six-monthly interest rate as a percentage and as a decimal.
 - c Find the quarterly interest rate as a percentage and as a decimal.
 - d Find the monthly interest rate as a percentage and as a decimal.
- 7 Calculate the simple interest on \$9000 at a flat rate of 3.97% p.a. over 19 months.
- 8 If \$11 000 is invested for 7 years, the amount of interest earned is \$4312. Calculate the annual simple interest rate as a percentage.
- 9 Draw a graph showing the amount of interest earned over a period of 10 years if \$1000 is invested at 9% p.a. simple interest.
 - a Find the interest after $5\frac{1}{2}$ years.
 - b Find the time taken to earn interest of \$160.

REVIEW SET 4

- 1 Bill the builder purchased \$2800 worth of materials from the hardware store. He receives a trade discount of 16% and then a further 5% discount if he pays within 3 days of receiving the account. What is the overall percentage discount if he pays within 3 days?
- 2 An LCD television costs \$578. Paul receives a 15% discount. How much does he pay for the television?
- 3 Lui-Min has his house painted. The bill is \$3258.50, and is subject to GST of 10%.
 - a Calculate the GST payable on the cost of the painting.
 - b Find the total amount Lui-Min pays for the painting of his house.



- 4 Gunter wants to buy a television that is listed at the pre-GST price of \$520.
 - a Calculate the GST payable on the television.
 - b Find the total amount Gunter pays for the television.
- 5 Use the rule of thumb in Topic 6A to find the GST paid on goods and services costing the following amounts.
 - a \$9953
 - b \$855
 - c \$39.51
 - d \$399
- 6 Calculate the simple interest on \$700 at a flat rate of 7% p.a. over 7 years.
- 7 An investment pays 5% interest p.a.
 - a Express this percentage as a decimal.
 - b Find the six-monthly interest rate as a percentage and as a decimal.
 - c Find the quarterly interest rate as a percentage and as a decimal.
 - d Find the monthly interest rate as a percentage and as a decimal.
- 8 If \$9200 is invested for 3 years, the amount of interest earned is \$2263.20. Calculate the annual simple interest rate as a percentage.
- 9 Calculate the simple interest on \$7000 at a flat rate of 5.53% p.a. over 11 months.
- 10 Draw a graph showing the amount of interest earned over a period of 10 years if \$1000 is invested at 15% p.a. simple interest. Find:
 - a the interest after $9\frac{1}{2}$ years.
 - b the time taken to earn interest of \$850.
- 11 A photocopier is bought for \$15 000. If it depreciates by \$4200 each year, how long will it take for the photocopier's value to fall below its scrap value of \$2400?

REVIEW PRACTICE EXAMINATION QUESTION

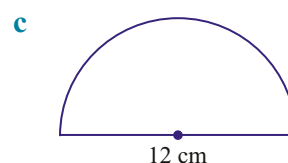
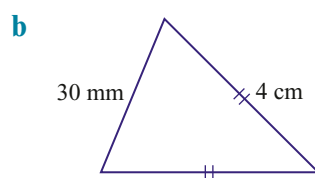
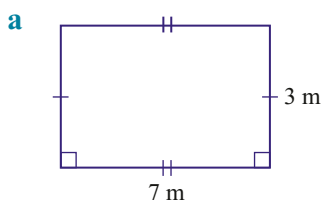
- 1 a An investment of \$40 000 increases in value by 12% in the first year and decreases in value by 12% in the second year of investment.
 - i Calculate the value of the investment at the end of the first year. (1 mark)
 - ii Calculate the value of the investment at the end of the second year. (1 mark)
 - iii Find the overall percentage change in the value of the investment over these 2 years. (1 mark)
- b Elni has a new kitchen installed for the pre-GST price of \$15 685.
 - i Calculate the GST. (1 mark)
 - ii How much does Elni pay? (1 mark)
- c Onska purchases an LCD television for \$2138 including GST. How much is the television excluding GST? (2 marks)
- d A car is bought for \$29 800. If it depreciates by \$1850 per year, what will its book value be after 4 years? (1 mark)
- e
 - i Convert 8% p.a. to a rate per quarter as a decimal. (1 mark)
 - ii How many quarters in 5 years? (1 mark)
 - iii Calculate the amount \$6250 grows to when invested at 8% p.a. compounding quarterly for 5 years. (2 marks)
 - iv How much interest is earned in part iii? (1 mark)
- f Calculate the simple interest on \$1475 at a flat rate of 9.31% p.a. interest for 7 years. (2 marks)

TOTAL: 15 Marks

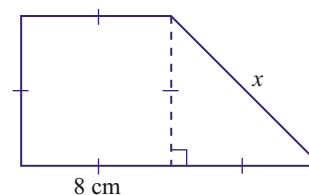
Chapters 4–6 CUMULATIVE REVIEW

- 1 a** Classify the following data as nominal, ordinal, discrete or continuous.
- i** number of rainy days each month
 - ii** the hair colours of students in your class
 - iii** the lengths of fish in a pond
- b** Explain how a random sample of 10 students may be chosen from a class of 120 students.
- c** Explain the statistical term ‘bias’.
- d** Starting at the first digit on the left and moving to the right in this list of random numbers, write the first five numbers less than 200. (*Hint*: choose three-digit numbers from the list.)
- | | | | | |
|-------|-------|-------|-------|-------|
| 90070 | 98873 | 08695 | 58112 | 96070 |
| 18523 | 72899 | 52251 | 86118 | 91910 |
| 83829 | 54949 | 68249 | 92592 | 35905 |
- e** An office supervisor wants to interview a sample of workers about a new computer system.
- i** Explain how four workers could be selected from the 27 people who work in the office.
 - ii** Which workers would be selected if the supervisor used the list of random numbers in part **d**, starting with the first digit in the second row and moving right.
- f** The numbers of students in each year at a school are shown in this table. In a stratified random sample selected on the basis of year groups, what proportion of students should be chosen from each year level?
- | Year | 7 | 8 | 9 | 10 | 11 | 12 | Total |
|--------------------|-----|-----|-----|-----|----|----|-------|
| Number of students | 135 | 144 | 170 | 120 | 94 | 68 | 731 |
- g** A sample is selected from a school population of 700. The 35 students chosen are representative of the entire school population. Of the students surveyed, 16 said they were happy with the school uniform and the rest were not.
- i** What percentage of students in the sample said they were happy with the uniform?
 - ii** How many students from the entire school population would be happy with the uniform?
- h** An electoral roll has 1800 names on it. How often should we select names to obtain a systematic sample of:
- i** 100 names?
 - ii** 200 names?
 - iii** 150 names?
 - iv** 40 names?

- 2** Find the perimeter of each of these figures, correct to one decimal place, if necessary.

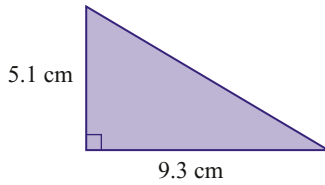


- 3** Consider the figure on the right. Calculate:
- a** the value of x
 - b** the perimeter of this figure, correct to one decimal place.

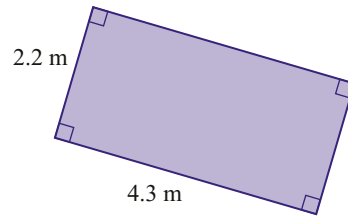


4 Find the area of each of these figures.

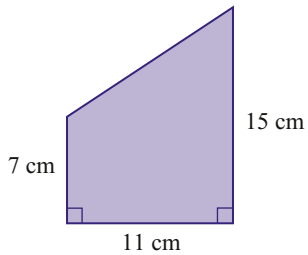
a



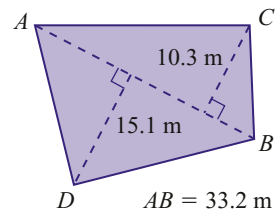
b



c

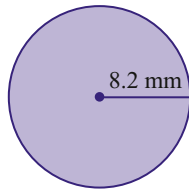


d

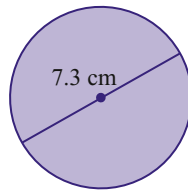


5 Find the area of each of these figures, correct to one decimal place.

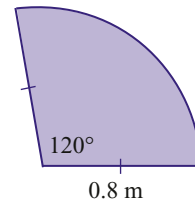
a



b

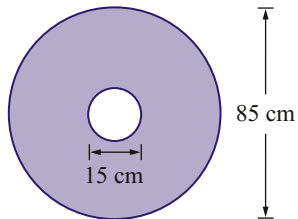


c

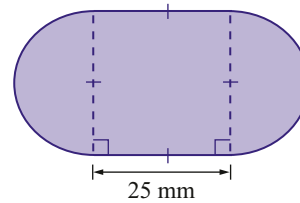


6 Find the shaded area of each of these figures, correct to one decimal place.

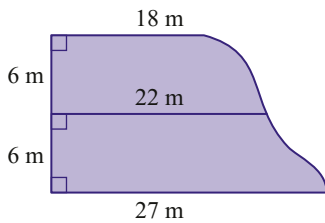
a



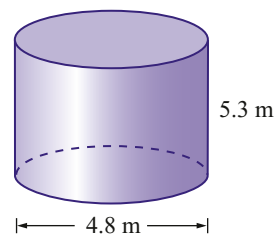
b



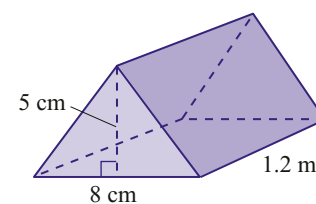
7 Use two applications of the Trapezoidal rule to estimate the area of this field to the nearest m^2 .



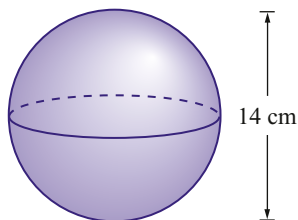
8 a If this cylinder is closed at both ends, find its surface area, correct to one decimal place.



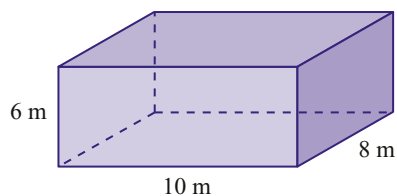
b Find the surface area of this triangular prism, correct to one decimal place.



- 9 a** Find the surface area of this sphere, to the nearest whole square centimetre.

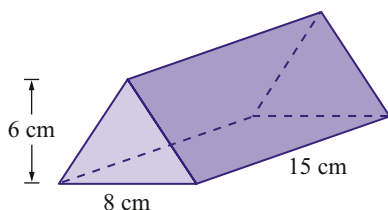


- b** Find the surface area of this prism.

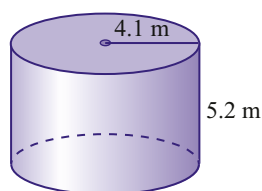


- 10** Calculate the volume of each of these solids, correct to one decimal place, if necessary.

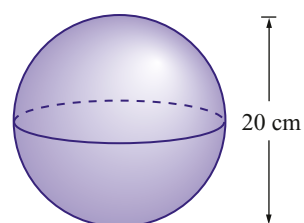
a



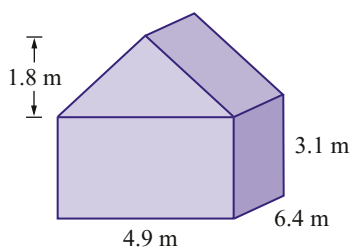
b



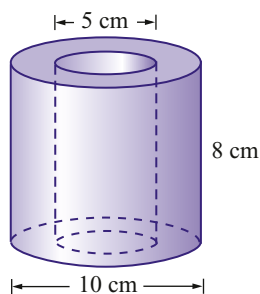
c



- 11 a** Find the volume of this barn.

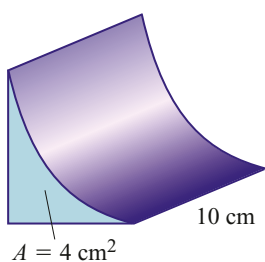


- b** Find the volume of this hollow cylinder, correct to one decimal place.

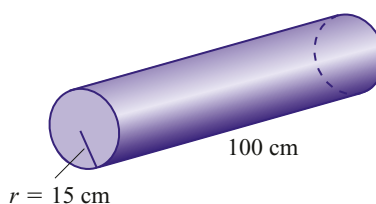


- 12** Calculate the volume of these solids, rounded to the nearest whole number, if necessary.

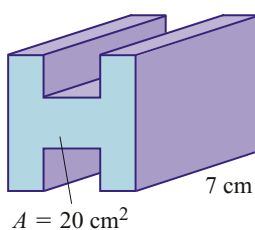
a



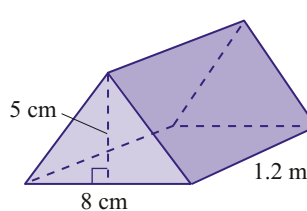
b



c



d



- 13** Find the capacity, in kilolitres, of a rectangular tank with length 10 m, breadth 4 m and height 3 m.

- 14** Find the capacity, to the nearest millilitre, of a cylindrical drinking glass with diameter 7 cm and height 11 cm.

- 15 a** Increase 80 kg by 5%.
- b** Find the percentage decrease from 70 kg to 65.8 kg.
- c** A Melbourne Storm signed rugby jersey was purchased for \$1200. It was sold for \$425. Calculate the percentage loss.
- d** Ollie had a new billiard room built as an extension to his house. The bill for labour and materials was \$14 360 and was subject to GST of 10%.
- Calculate the GST payable on the bill.
 - Find the amount Ollie paid for the billiard room.
- e** Harry wants to buy a camera that is listed at the pre-GST price of \$725.
- Calculate the GST payable on the camera.
 - Find the total amount Harry will pay for the camera.
- f** Use the rule of thumb discussed in Topic 6A to find the GST already paid on goods and services costing \$3254.
- 16 a** Calculate the simple interest on \$4250 at a flat rate of 5% p.a. over 3 years.
- b** An investment pays 4% interest p.a.
- Express this percentage as a decimal.
 - Find the six-monthly interest rate as a percentage and as a decimal.
 - Find the quarterly interest rate as a percentage and as a decimal.
 - Find the monthly interest rate as a percentage and as a decimal.
- c** Calculate the simple interest on \$4000 at a flat rate of 6.74% p.a. over 17 months.
- d** If \$1750 is invested for 6 years, the amount of interest earned is \$840. Calculate the annual simple interest percentage rate.
- e**
- Draw a graph showing the amount of interest earned over a period of 10 years if \$1000 is invested at 7.2% p.a. simple interest.
 - Find the interest after $3\frac{1}{2}$ years.
 - Find the time taken to earn interest of \$300.
- f** A printer is purchased new for \$6900. It depreciates at \$900 per year.
- Calculate the printer's book value after 4 years.
 - The printer has a usable life of 7 years. Find its scrap value.
- g** A boat depreciates in value from \$22 000 to \$17 000 in two years. Find the annual amount of depreciation.
- h** Complete the table below to determine the final value of \$4200 invested at 4% p.a. compound interest for 3 years.



Time	Balance + interest	Total balance	Interest
Start of first year		\$4200	\$168
Start of second year	\$4200 + \$168		
Start of third year			
Start of fourth year			

7

Representing data

The main mathematical ideas investigated are:

- ▶ constructing frequency distribution tables
- ▶ using cumulative and relative frequency
- ▶ using grouped data, class and class centre
- ▶ constructing stem-and-leaf plots
- ▶ creating and interpreting dot plots
- ▶ creating and interpreting column and bar graphs
- ▶ creating and interpreting line graphs
- ▶ recognising misleading graphs
- ▶ constructing frequency histograms and polygons
- ▶ constructing cumulative frequency graphs
- ▶ construct and interpret Pareto charts.

ARE YOU READY?

This frequency table shows the favourite pets of a group of animal lovers. Use the table to answer questions 1 and 2.

Pet type	Tally
Dog	
Cat	
Bird	
Rat	

- 7A** 1 How many people stated that a dog was their favourite pet?

A 21 **B** 11 **C** 9 **D** 6

- 7A** 2 How many people were surveyed?

A 10 **B** 19 **C** 22 **D** 37

- 7A** 3 Which list of class intervals would be the most appropriate way of grouping this data?

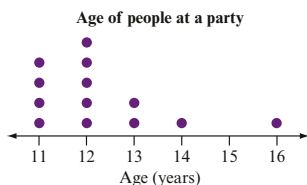
2 12 20 13 20 15 1
1 4 16 10 12 3 10
3 18 11 4 17 13 16

A 0–5, 5–10, 10–15, 15–20
B 0–4, 5–10, 11–19, 20–24
C 0–4, 5–9, 10–14, 15–20
D 0–4, 10–14, 15–19, 20–24

- 7B** 4 What is 35 as a percentage of 40?

A 87.5% **B** 80% **C** 70% **D** 35%

- 7C** 5 For the dot plot shown, which statement is *not* correct?



- A** Five children were 12 years old.
B The difference between the ages of the youngest and oldest children was 3 years.
C No-one at the party was 15 years old.
D Thirteen children attended the party.

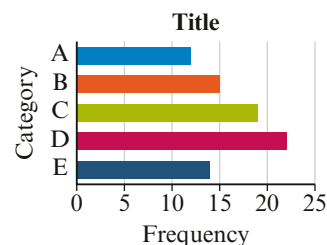
- 7C** 6 For the stem-and-leaf plot shown, which statement is correct?

Stem	Leaf
1	3 5 7
2	1 4 6 8
3	0 2 9

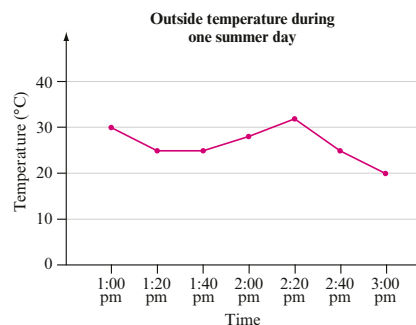
- A** The score of 1 occurs three times.
B The score of 3 occurs once.
C The lowest score is 13.
D The highest score is 9.

- 7D** 7 What type of graph is shown below?

- A** vertical column graph
B dot plot
C horizontal bar graph
D line graph



- 7E** 8 For this graph, which statement is incorrect?



- A** The lowest and highest temperatures were 20°C and 30°C respectively.
B The temperature was recorded between 1:00 pm and 3:00 pm.
C The temperature went up between 2:00 pm and 2:20 pm.
D The temperature at 1:40 pm was 25°C.

- 7F** 9 Which number is midway between 1 and 6?

A 2.5 **B** 3 **C** 3.5 **D** 4

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching SupportSheets available on your [obook assess](#).

- Q1–2** Support sheet 7A.1 Presenting data in a frequency table
Q3 Support sheet 7A.2 Class intervals
Q4 Support sheet 7B.1 Calculating percentages
Q5 Support sheet 7C.1 Dot plots
Q6 Support sheet 7C.2 Stem-and-leaf plots
Q7 Support sheet 7D.1 Types of data displays
Q8 Support sheet 7E.1 Interpreting line graphs
Q9 Support sheet 7F.1 Finding the average of two numbers

7A Frequency distribution tables

frequency table

table that displays the number of times each category of data occurs

tally marks

lines or marks that help record data in a table with every fifth tally mark drawn through the preceding four marks

These resources are available on your [obook](#) [assess](#):

- **Interactive 7A:** Explore key ideas for frequency distribution tables
- **Worksheet 7A:** Practise your skills with extra problems for grouping data in classes
- **assess quiz 7A:** Test your skills with an auto-correcting multiple-choice quiz

In Chapter 4 we discussed the planning and management of data collection. We will now look at recording and displaying the data that has been collected. Raw data collected by observation or experiment is usually in random order. It is difficult to gain information from the data when it is in this form.

One of the aims of statistics is to rearrange and condense raw information into a form that is more easily read, so that patterns and characteristics may be identified, conclusions can be drawn and inferences may be made.

A **frequency table** displays the frequency (number of occurrences) for each of the categories of data. **Tally marks** are often used to help record the data in the table. Every fifth tally mark is drawn through the preceding four marks to make bundles of five which are easy to count.

EXAMPLE 7A-1 Organising data into a frequency distribution table

The method by which students from a particular class travelled to school on a particular day is recorded below, using the code walk (W), cycle (C), bus (B), train (T) and motor car (M)

W C B T C B B B W B B B C B T
C M C B T M M T M M M T C C B

Rearrange this information into a frequency distribution table using a tally column.



Solve			Think	Apply
Method of travel	Tally	Frequency	Work through the data from left to right using tally marks to record each time a category occurs. Add the tally marks and record the total in the frequency column.	The total of the tally marks for each category gives the frequency for that category; that is, the number of times a particular mode of travel is used. Check that the total of the frequency column is the same as the number of observations recorded.
Walk	II	2		
Cycle	IIII II	7		
Bus	IIII II	10		
Motor car	IIII I	6		
Train	IIII	5		
	Total	30		

EXERCISE 7A Frequency distribution tables

- 1** The colours of cars passing the front of a school in a 30-minute period are recorded below using the code white (W), blue (B), grey (G), red (R), other (O).

B W R O W B R G W O
 B G R W W G B R W O
 G B R W G B R R G W
 B R G O W B W G R B
 W W B R G W B R W B

- a** Rearrange this information into a frequency distribution table using a tally column.
b How many cars passed the front of the school in the 30-minute time period?
c What was the most popular colour of car in this survey?
d Calculate the percentage of cars of each colour.



- 2** The minimum daily temperatures (in °C) recorded for a month were as follows.

22 21 20 22 23 22 21 23 19 21 21 23 21 20 22
 21 20 20 22 20 23 19 20 19 20 18 19 17 18 22

- a** Organise this information into a frequency distribution table using a tally column.
b On how many days was the temperature recorded?
c On how many days did the lowest minimum temperature occur?
d On how many days was the minimum temperature 20°C?
e What month of the year could this be? Discuss.

- 3** The eye colours of a group of students are recorded below using the code blue (B), brown (b), green (G), grey (g).

B b g B b b G b B g b B g b G
 b b B g B B g b B G B b g b B

- a** Organise this data into a frequency distribution table using a tally column.
b How many students were observed?
c What is the most common eye colour in this sample?

- 4** Consider the following marks (out of a possible 100) gained by 50 students in a test.

70 31 49 54 55 74 50 34 60 68 68 57 45 56 54 85 44
 66 56 60 22 45 53 29 61 20 13 52 41 50 74 51 48 69
 75 61 40 41 33 62 65 55 51 42 94 64 38 3 51 79

- a** Complete this frequency distribution table for the data.

Score	3	13	20	22	29	31	33	34	38	40	41	42	44
Frequency													

Score	45	48	49	50	51	52	53	54	55	56	57	60	61
Frequency													

Score	62	64	65	66	68	69	70	74	75	79	85	94
Frequency												

- b** How useful is the frequency distribution table you completed? Explain.

- c** In cases such as these, where the rearrangement of the raw data gives us very little information, we can condense the data by grouping it into **classes**, or **class intervals**, and finding the frequency of each class, again using a tally column.
- Consider the following incomplete frequency distribution table in which the data is grouped into classes of 10 scores.

classes (class intervals)
equal-sized groupings of data

- i** Explain what a class of 0–9 means.
 - ii** Explain what a class of 60–69 means.
 - iii** Are the classes the same size? What is the size of each class interval?
 - iv** Complete the frequency distribution table for the given data.
 - v** How many of the test scores lie in the interval 50 to 59?
 - vi** What percentage of students obtained a test score in the range 80 to 89?
- d** Consider this incomplete frequency distribution table which includes less classes.
- i** What is the size of each class?
 - ii** Complete this frequency distribution table for the given data.
 - iii** How many of the test scores lie in the interval 20 to 39?
 - iv** How many students obtained a test score below 60?
- e** List at least one advantage and one disadvantage of grouping data in classes.

Class	Tally	Frequency
0–9	I	1
10–19	I	1
20–29	III	3
30–39		
40–49		
50–59		
60–69		
70–79		
80–89		
90–99		
Total		50

Class	Tally	Frequency
0–19	II	2
20–39		
40–59		
60–79		
80–99		
Total		50

When grouping data we ensure that:

- the classes do not overlap. For example, if we form classes 0–10, 10–20, 20–30, ... then we would not know in which class to put score 10 or score 20
- there are no gaps between class intervals. For example, if we organise the data in question 4 into classes 0–4, 5–9, 10–14, etc. then, even though there are no scores in the class interval 5–9, this class should be included
- each class interval is the same size. (When there are extremely low or high scores it may be convenient to make the bottom and top class intervals open; for example, less than 300, 300–319, 320–329, ..., 390–399, 400 or more. However, any further statistical analysis then becomes very difficult.)



- 5** The marks scored by 30 students in a test are shown below.

28 35 8 61 45 68 47 23 52 44 22 36 20 44 54
51 32 57 24 61 44 34 42 60 39 47 32 76 53 62

- a** Organise this data into frequency distribution tables, using these classes:

i 0–4, 5–9, 10–14, 15–19, ...

ii 0–9, 10–19, 20–29, 30–39, ...

iii 0–<20, 20–<40, 40–<60, 60–<80, ...

iv 0–49, 50–99

- b** Which of the groupings from part **a** gives the clearest ‘picture’ of the results of the tests? Discuss your choice.

- c** Explain why the use of the following groupings of scores would be unsatisfactory.

i 0–5, 5–10, 10–15, ...

ii 0–9, 20–29, 30–39, 40–49, ...

iii 0–4, 5–14, 15–29, 30–34, ...

- 6** A random sample of light globes from a production line was tested and the time ($\times 100$ hours) each globe lasted, to the nearest 100 hours, is recorded below.

10	9	13	5	2	11	8	12
4	12	7	13	8	14	10	12
9	7	11	15	10	11	10	4
8	8	11	12	14	13	10	10
9	11	13	12	5	11	8	16

- a** Organise the given data into frequency distribution tables using these class intervals:

i 0–4, 5–9, 10–14, 15–19

ii 1–<6, 6–<11, 11–<16, 16–<21

- b** Would the following class intervals be suitable to use?

i 0–3, 3–6, 6–9, ...

ii 0–3, 4–7, 8–11, ...

iii 0–9, 10–19, 20–29, ...

- c** How many light globes were tested?

- d** Using both tables from part **a** where necessary, determine the number of light globes that had a recorded life of:

i less than 1000 hours

ii more than 1000 hours

iii 1000 hours.



- 7** Investigate the modes of travel of the students in your class. Present the data in a frequency distribution table and write a short report on your findings. Would the results for your class be typical of the whole school?



7B Cumulative and relative frequency

These resources are available on your obook assess:

- **Video tutorial 7B:** Watch and listen to an explanation of Example 7B-1
- **Spreadsheet 7B:** Find cumulative and relative frequencies
- **assess quiz 7B:** Test your skills with an auto-correcting multiple-choice quiz

cumulative frequency

sum of the frequencies of all the scores up to and including a particular score

relative frequency

the number of times a score occurs as a fraction of the total number of scores

percentage relative frequency

relative frequency expressed as a percentage

A convenient way of answering many of the questions in Exercise 7A is to add a **cumulative frequency** column and a **relative frequency** column to the frequency distribution tables. The cumulative frequency of each score is found by adding the frequencies of all the scores up to and including that particular score. The relative frequency of a score is its frequency as a fraction of the total number of scores. For the purpose of comparison, it is often convenient to express this fraction as a percentage (sometimes called the **percentage relative frequency**).

EXAMPLE 7B-1 Adding a cumulative frequency column and a relative frequency column to a frequency distribution table

- a** Copy this frequency distribution table showing the number of Candy Clicks in a packet.

Number of Candy Clicks	Frequency
36	1
37	5
38	8
39	13
40	7
41	4
42	2

- i** Add a cumulative frequency column to your table.
ii Add a relative frequency column.
- b** How many packets contained:
- i** 40 or fewer Candy Clicks?
ii fewer than 38 Candy Clicks?
- c** What fraction of packets contained:
- i** 37 Candy Clicks?
ii 40 Candy Clicks?
- d** What percentage of packets contained:
- i** 38 Candy Clicks?
ii 39 Candy Clicks?



Solve/Think				Apply	
a	Number of Candy Clicks	Frequency	Cumulative frequency	Relative frequency	The cumulative frequency of a score is found by ‘accumulating’ the frequencies of all the scores up to and including that score. The relative frequency of a score is found by expressing its frequency as a fraction of the total number of scores. This fraction is often expressed as a percentage.
	36	1	1	$\frac{1}{40} = 2.5\%$	
	37	5	(1 + 5 =) 6	$\frac{5}{40} = 12.5\%$	
	38	8	(6 + 8 =) 14	$\frac{8}{40} = 20\%$	
	39	13	(14 + 13 =) 27	$\frac{13}{40} = 32.5\%$	
	40	7	(27 + 7 =) 34	$\frac{7}{40} = 17.5\%$	
	41	4	(34 + 4 =) 38	$\frac{4}{40} = 10\%$	
	42	2	(38 + 2 =) 40	$\frac{2}{40} = 5\%$	
	Total	40		1 = 100%	
b	i Number of packets containing 40 or fewer Candy Clicks = cumulative frequency of the score 40 = 34 ii Number of packets containing fewer than 38 Candy Clicks = number of packets containing 36 or 37 Candy Clicks = cumulative frequency of 37 = 6				
c	From the table: i $\frac{5}{40}$ or $\frac{1}{8}$ ii $\frac{7}{40}$				
d	From the table: i 20% ii 32.5%				

EXERCISE 7B Cumulative and relative frequency

- 1** The numbers of children in the families of a class of students are summarised in the given frequency distribution table.

- a** Copy the frequency distribution table and add:

- i** a cumulative frequency column
ii a relative frequency column.

- b** How many families had:

- i** 3 or fewer children?
ii fewer than 5 children?

- c** What fraction of families had:

- i** 3 children?
ii 4 children?

- d** What percentage of families had:

- i** 2 children?
ii 5 children?

Number of children	Frequency
1	3
2	9
3	7
4	4
5	2

- 2** The number of goals scored in a season by the goal shooter in a netball team is shown in this frequency distribution table.

- a** Copy the frequency distribution table and add:
i a cumulative frequency column
ii a relative frequency column.
- b** In how many games did the goal shooter score:
i 7 or fewer goals? **ii** fewer than 10 goals?
- c** In what fraction of games did she score:
i 8 goals? **ii** 5 goals?
- d** In what percentage of games did she score:
i 6 goals? **ii** 11 goals?

Goals per match	Frequency
5	4
6	3
7	7
8	5
9	0
10	3
11	2
12	1

EXAMPLE 7B-2 Determining frequency from a cumulative frequency table

Given the cumulative frequency table on the right, determine the frequency of each score.

Score	Cumulative frequency
7	2
8	5
9	7
10	12
11	16

Solve/Think			Apply
Score	Cumulative frequency	Frequency	Use the fact that the cumulative frequency of a score is the sum of the frequencies of all the scores up to and including that score.
7	2	2	
8	5	$5 - 2 = 3$	
9	7	$7 - 5 = 2$	
10	12	$12 - 7 = 5$	
11	16	$16 - 12 = 4$	
The cumulative frequency of the first score is the same as its frequency. So, the frequency of the score 7 is 2. The cumulative frequency of the score 8 is 5, which is the sum of the frequencies of 7 and 8. So, the frequency of the score 8 is $(5 - 2 =) 3$. The cumulative frequency of the score 9 is 7, which is the sum of the frequencies of 7, 8 and 9. So, the frequency of 9 is $(7 - 5 =) 2$. The cumulative frequency of the score 10 is 12, which is the sum of the frequencies of 7, 8, 9 and 10. So, the frequency of 10 is $(12 - 7 =) 5$. The cumulative frequency of the score 11 is 16, which is the sum of the frequencies of 7, 8, 9, 10 and 11. So, the frequency of 11 is $(16 - 12 =) 4$.			

- 3 Given the cumulative frequency tables below, determine the frequency of each score.

a

Score	Cumulative frequency
3	4
4	6
5	10
6	15
7	18

b

Score	Cumulative frequency
12	5
13	11
14	19
15	29
16	34
17	36

c

Score	Cumulative frequency
8	11
9	17
10	27
11	27
12	32
13	40

d

Score	Cumulative frequency
87	23
88	48
89	78
90	106
91	127
92	138

- 4 The numbers of matches in 60 boxes of matches were counted and the results are shown below.

49 51 50 50 48 49 50 49 53 50 48 51 49 50 51
 52 50 48 49 51 50 53 48 49 51 50 52 49 50 52
 51 48 50 49 50 51 52 50 49 48 52 50 51 49 50
 50 48 53 52 49 50 51 53 48 50 51 49 50 53 48

- a** Organise this data into a frequency distribution table and add a cumulative frequency column and a relative frequency column.
- b** How many matchboxes contained:
- i** exactly 50 matches?
 - ii** fewer than 50 matches?
 - iii** 50 or fewer matches?
 - iv** more than 50 matches?
- c** Find the percentage of matchboxes that contained:
- i** 49 matches
 - ii** 50 matches
 - iii** 51 matches
 - iv** 50 or fewer matches
 - v** 50 or more matches.

- 5 A marriage counsellor asked his clients to keep a record of the number of arguments they had in a one-week period. The results were collected and are shown in this table.

Number of arguments	Frequency
0–2	8
3–5	18
6–8	10
9–11	3
12–14	1

- a** How many different responses are included in each class interval?
- b** How many clients responded to the counsellor's request?
- c** What percentage of these clients had fewer than three arguments in the week?
- d** What percentage had more than eight arguments?
- e** Can you determine how many clients had six arguments? Explain.
- f** Can you determine the lowest number and highest number of arguments? Explain.

- 6** A group of Year 11 golfers had a competition to find out who could hit a golf ball the furthest. Each student's best shot, to the nearest metre, is recorded below.

244 220 232 195 242 200 229 205 213 224
 216 250 227 186 207 236 225 212 227 230
 209 235 234 251 195 206 221 190 198 210
 222 235 230 209 254 193 203 228 191 242
 204 218 212 218 239 223 252 215 220 198
 207 205 226 235 202 196 188 224 245 230

- a** Copy and complete the frequency distribution table for this data.
b How many golfers entered the competition?
c Add a cumulative frequency column and a relative frequency column to your table from part **a**.
d For how many golfers was the length of their best shot recorded as less than:
i 200 m? **ii** 230 m?
e What percentage of golfers had the length of their best shot recorded as:
i 250 m or more?
ii 220 m or more, but less than 230 m?
iii less than 200 m?



Distance (m)	Tally	Frequency
180–189		
190–199		
200–209		
210–219		
220–229		
230–239		
240–249		
250–259		
Total		

- 7** A nurseryman takes a random sample of seedlings and measures their heights to the nearest millimetre. The results are recorded in the frequency distribution table below.

Height (mm)	Frequency
300–324	3
325–349	18
350–374	47
375–399	32
400–424	14
425–449	6

- a** How many different heights, to the nearest millimetre, are possible in each class interval?
b How many seedlings are there in this sample?
c Can you determine how many plants were 350 mm high? Explain.
d Can you determine the shortest and tallest plants in this sample? Explain.
e Copy the table and add a cumulative frequency column and a relative frequency column.
f How many seedlings had a recorded height of less than:
i 350 mm? **ii** 400 mm?
g What percentage of seedlings were 425 mm or more?
h Any seedlings that are less than 325 mm tall are discarded. What percentage of seedlings will be discarded?



- 8 The maximum daily temperature (in °C) was recorded over the month of April and the results are in the frequency distribution table on the right.

- Copy the table and add a cumulative frequency column and a relative frequency column.
- How many days in April had a maximum daily temperature of less than 20°C?
- What percentage of days in April had a maximum daily temperature of 19°C or less?
- What proportion of days in April had a maximum daily temperature of 21°C or more?

Maximum daily temperature (°C)	Frequency
17	3
18	4
19	8
20	6
21	5
22	4

- 9 A spreadsheet can be used to add a cumulative frequency column and a relative frequency column to a frequency distribution table. The following spreadsheet uses the data from Example 7B-1. The relative frequencies are expressed as decimals, set to three decimal places.

	A	B	C	D
1	Frequency distribution table			
2	Score	Frequency	Cumulative frequency	Relative frequency
3	36	1	1	0.025
4	37	5	6	0.125
5	38	8	14	0.200
6	39	13	27	0.325
7	40	7	34	0.175
8	41	4	38	0.100
9	42	2	40	0.050
10		40		1.000

Here is the formula view. Use **fill down** to complete the columns. **Fill down** for additional rows and use **AutoSum** to calculate the totals.

Note: the calculation of the relative frequency will need to be changed if additional rows are added.

	A	B	C	D
1	Frequency distribution table			
2	Score	Frequency	Cumulative frequency	Relative frequency
3	36	1	=B3	=B3/\$B\$10
4	37	5	=C3+B4	=B4/\$B\$10
5	38	8	=C4+B5	=B5/\$B\$10
6	39	13	=C5+B6	=B6/\$B\$10
7	40	7	=C6+B7	=B7/\$B\$10
8	41	4	=C7+B8	=B8/\$B\$10
9	42	2	=C8+B9	=B8/\$B\$10
10		=SUM(B3:B9)		=SUM(D3:D9)

Create a spreadsheet to complete questions in this exercise again. You may like to use the prepared spreadsheet provided on your obook assess.

7C Dot plots and stem-and-leaf plots

These resources are available on your obook assess:

- **Interactive 7C:** Explore key ideas for stem-and-leaf plots
- **Worksheet 7C.1:** Practise your skills with extra problems for dot plots
- **Worksheet 7C.2:** Practise your skills with extra problems for stem-and-leaf plots
- **assess quiz 7C:** Test your skills with an auto-correcting multiple-choice quiz

dot plot

a display of data where each piece of data is represented by a dot above a matching number or category on a horizontal scale

A **dot plot** displays data as columns or rows of dots. The number of dots is the frequency of the variable.

A dot plot is a convenient and efficient method for illustrating a small data set consisting of a small number of categories or possible outcomes. It also has the advantage that it can be used directly for unsorted data.

For large data sets, a dot plot is very time-consuming to prepare and, conversely, a completed dot plot is difficult to read. A dot plot can only be used when the outcomes can be counted.

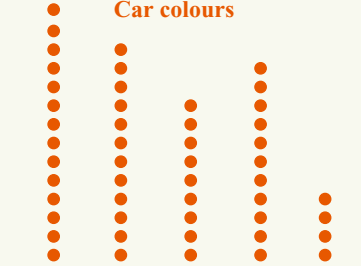
Dot plots are useful for drawing attention to clusters, gaps and outliers.

EXAMPLE 7C-1 Drawing a dot plot

The colours of cars passing the front of a school in a 30-minute period are recorded below using the code white (W), blue (B), grey (G), red (R), other (O).

B W R O W B R G W O B G R W W G B R W O G B R W G
B R R G W B R G O W B W G R B W W B R G W B R W B

- Draw a dot plot to display the information given.
- What is the frequency of each colour?

Solve	Think	Apply												
Car colours  White Blue Grey Red Other	Work through the data from left to right, placing a dot above each colour on the scale as it occurs.	The frequency of each category can be found by counting the dots.												
<table><tr><th>Colour</th><th>Frequency</th></tr><tr><td>White</td><td>14</td></tr><tr><td>Blue</td><td>12</td></tr><tr><td>Grey</td><td>9</td></tr><tr><td>Red</td><td>11</td></tr><tr><td>Other</td><td>4</td></tr></table>	Colour	Frequency	White	14	Blue	12	Grey	9	Red	11	Other	4	Count the dots in each category.	
Colour	Frequency													
White	14													
Blue	12													
Grey	9													
Red	11													
Other	4													

EXERCISE 7C

Dot plots and stem-and-leaf plots

- 1 The eye colours of students is a group are recorded using the code blue (B), brown (b), green (G), grey (g).

B b g B b b G b B g b B g b g
G b b B g B B g b B G B b B b

- a Draw a dot plot for this data.
b Determine the frequency of each eye colour.

- 2 The number of children in 25 families is recorded below.

2 1 3 5 2 3 1 4 2 2 3 4 1
3 2 5 2 3 2 4 3 2 4 2 3

- a Draw a dot plot to display this information.
b Write the frequency of each number of children.

Stem-and-leaf plots are another way of displaying information. They are used to group and rank data to show the range and distribution. The **leaf** is the final digit of a number, the preceding digits form the **stem**.

A stem may have any number of digits but a leaf has exactly one. The stem is written once in the left column and the leaves are listed in numerical order beside the appropriate stem. In a stem-and-leaf plot:

- all of the data is used and displayed
- the largest and smallest measurements can be found
- the clustering of data can be seen more easily
- the length of the leaf row indicates the number of scores belonging to that stem.

stem-and-leaf plot

A display of data where each piece of data is split into two parts: the last digit becoming the leaf and the other digits becoming the stem

leaf

last digit of a piece of data; e.g. for the data score 256, the leaf is 6

stem

digit or digits preceding the last digit of a piece of data; e.g. for the data score 256, the stem is 25

EXAMPLE 7C-2 Drawing a stem-and-leaf plot

The results in a mathematics class test are given below.

43 45 46 22 65 65 23 53 45 26 46 61 51 57 55
55 66 57 42 41 63 70 57 65 48 23 67 62 70 46

- a Draw a stem-and-leaf plot to represent this data.
b What are the lowest and highest scores?
c How many students scored:
i 46 marks? ii 50 marks? iii 70 marks? iv a mark in the sixties?

Solve/Think		Apply														
In this stem-and-leaf plot, the tens digit forms the stem and the units digit forms the leaf. This means that, for the mark 45, the stem is the 4 and the leaf is the 5. <table><thead><tr><th>Stem</th><th>Leaf</th></tr></thead><tbody><tr><td>2</td><td>2 3 6 3</td></tr><tr><td>3</td><td></td></tr><tr><td>4</td><td>3 5 6 5 6 2 1 8 6</td></tr><tr><td>5</td><td>3 1 7 5 5 7 7</td></tr><tr><td>6</td><td>5 5 1 6 3 5 7 2</td></tr><tr><td>7</td><td>0 0</td></tr></tbody></table> ← This row represents 22, 23, 26 and 23. ← There were no scores in the thirties.		Stem	Leaf	2	2 3 6 3	3		4	3 5 6 5 6 2 1 8 6	5	3 1 7 5 5 7 7	6	5 5 1 6 3 5 7 2	7	0 0	The data is displayed in numerical order using the first digit (or digits) as the stem and the last digit as the leaf.
Stem	Leaf															
2	2 3 6 3															
3																
4	3 5 6 5 6 2 1 8 6															
5	3 1 7 5 5 7 7															
6	5 5 1 6 3 5 7 2															
7	0 0															

Solve/Think		Apply														
The leaves are now put into ascending numerical order.																
<table><thead><tr><th>Stem</th><th>Leaf</th></tr></thead><tbody><tr><td>2</td><td>2 3 3 6</td></tr><tr><td>3</td><td></td></tr><tr><td>4</td><td>1 2 3 5 5 6 6 6 8</td></tr><tr><td>5</td><td>1 3 5 5 7 7 7</td></tr><tr><td>6</td><td>1 2 3 5 5 5 6 7</td></tr><tr><td>7</td><td>0 0</td></tr></tbody></table>	Stem	Leaf	2	2 3 3 6	3		4	1 2 3 5 5 6 6 6 8	5	1 3 5 5 7 7 7	6	1 2 3 5 5 5 6 7	7	0 0		
Stem	Leaf															
2	2 3 3 6															
3																
4	1 2 3 5 5 6 6 6 8															
5	1 3 5 5 7 7 7															
6	1 2 3 5 5 5 6 7															
7	0 0															
Lowest score = 22, highest score = 70		List the first and last entry in the stem-and-leaf plot.														
- i Number of students who scored $46 = 3$ - ii Number of students who scored $50 = 0$ - iii Number of students who scored $70 = 2$ - v Number of students who scored a mark in the sixties = 8		Count how many entries are in the table for each score.														

- 3 a** Draw a stem-and-leaf plot, using the stems 3, 4, 5 and 6, for these scores:
32 45 37 65 48 57 33 42 59 62 47 51 50 47 36 68 60 50 63 47
- b** What are the lowest and highest scores?
- c** How many times do the following scores occur?
- i** 50 **ii** 40
- d** Which score occurs the most often?
- e** How many scores are in the sixties?
- f** How many scores are less than 50?
- 4 a** Draw a stem-and-leaf plot, using the stems 12, 13, 14 and 15, for these scores:
132 154 124 156 125 145 130 141 126 151 140 139 128 145 126
146 128 130 140 158 154 142 149 145 150 126 130 148 158 145
- b** What are the lowest and highest scores?
- c** How many times do the following scores occur?
- i** 150 **ii** 158 **iii** 120
- d** Which score occurs the most often?
- e** How many scores are:
- i** less than 130? **ii** 150 or more?
- 5** The results of a test are shown in this stem-and-leaf plot.
- | Stem | Leaf |
|------|---------------------------|
| 5 | 0 0 0 1 4 4 4 4 5 6 7 8 8 |
| 6 | 0 0 1 2 3 3 5 7 8 8 9 9 |
| 7 | 4 4 5 7 8 8 |
| 8 | 1 1 5 5 7 7 |
| 9 | 0 0 |
- a** What are the lowest and highest scores?
- b** How many times do the following scores occur?
- i** 50 **ii** 60
- iii** 70 **iv** 80
- c** Which score occurs the most often?
- d** How many scores are in the 50s?
- e** How many scores are:
- i** less than 60? **ii** 80 or more?

Stem	Leaf
5	0 0 0 1 4 4 4 4 4 5 6 7 8 8
6	0 0 1 2 3 3 5 7 8 8 9 9
7	4 4 5 7 8 8
8	1 1 5 5 7 7
9	0 0

- 6 The stem-and-leaf plot on the right shows the time (hours) a group of students spent watching television in one particular week.
- How many students were surveyed?
 - What was the least number and the greatest number of hours spent watching television that week?
 - How many students watched fewer than 10 hours of television that week?
 - How many students watched more than 30 hours of television that week?
 - Draw a grouped frequency table to represent this data, using these class intervals:
 - 1–5, 6–10, 11–15, etc.
 - 0–9, 10–19, 20–29, etc.
 - Comment on the advantages and disadvantages of a stem-and-leaf plot compared with a grouped frequency table.

Stem	Leaf
0	3 5 6 8 9
1	0 2 2 3 5 5 5 9
2	2 4 5 5 5 7 8
3	0 1 1 4 6

EXAMPLE 7C-3 Drawing a stem-and-leaf plot with split stems

The number of people who live in each house in a particular street is recorded below.

4 2 3 4 5 2 4 3 2 5
1 4 5 5 3 7 2 3 4 4

Draw a stem-and-leaf plot for this information.



Solve/Think		Apply
Stem	Leaf	If the data has one stem only, it is often useful to split the leaves into two groups, one containing the leaves 0, 1, 2, 3, 4 and one containing the leaves 5, 6, 7, 8, 9.
0	1 2 2 2 2 3 3 3 3 4 4 4 4 4 4 5 5 5 5 7	
This plot gives no more information than an ordered list of numbers.		
To show the information in a more meaningful way, the stem is split.		
The stem 0 will contain the leaves 0, 1, 2, 3, 4.		
The stem 0* will contain the leaves 5, 6, 7, 8, 9.		
The stem-and-leaf plot then becomes:		
Stem	Leaf	
0	1 2 2 2 2 3 3 3 3 4 4 4 4 4 4	
0*	5 5 5 5 7	
It can be seen from the second plot that most households have 4 or fewer people.		

- 7 The number of hours per week a group of Year 11 students spent doing homework is shown below.
- 27 22 38 30 26 34 18 27 32 31
28 33 33 29 24 27 30 31 33 27
28 22 26 32 34 26 26 37 29 28
31 33 23 34 29 26 33 29 32 27
- How many students were surveyed?
 - Draw a stem-and-leaf plot for this data using stems 1, 2 and 3.
 - Draw another stem-and-leaf plot for the data using stems 1, 1*, 2, 2*, 3, 3*.
 - What information is shown in the second stem-and-leaf plot that is not evident in the first?

EXAMPLE 7C-4 Drawing a back-to-back stem-and-leaf plot

The heights of girls and boys in Year 12 were measured in centimetres. The results are listed below.

Girls: 152, 162, 167, 170, 180, 193, 174, 157, 163, 172, 169, 174, 167, 160, 170, 171, 167, 174

Boys: 169, 180, 185, 179, 174, 185, 195, 181, 163, 176, 174, 174, 179, 183, 188, 188, 192, 192

- Represent this data in a back-to-back stem-and-leaf plot.
- From the data, what comparisons can be made between the heights of boys and girls?

Solve			Think	Apply
Boys	Stem	Girls	Use stems beginning with 15 up to a stem of 19. The values are in order as usual, but for the left-hand side of the plot, the smallest numbers are closest to the stem.	The stem is the centre of the stem-and-leaf plot. The numbers in the leaves are always smallest at the stem and larger further from the stem. The scores can be compared by looking at the two distributions. In this case, clearly the values for heights of boys are larger than the values for heights of girls.
	15	2 7		
9 3	16	0 2 3 7 7 7 9		
9 9 6 4 4 4	17	0 0 1 2 4 4 4		
8 8 5 5 3 1 0	18	0		
5 2 2	19	3		
The boys are generally taller than the girls, even though some girls are taller than some boys.			The values for the boys are higher than the values for the girls.	

- The average rainfall, in millimetres, for Adelaide and Melbourne is shown in the table.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Adelaide	17	19	22	38	59	52	65	50	43	38	24	23
Melbourne	48	48	52	58	58	50	49	51	59	68	60	59

- Copy and complete this back-to-back stem-and-leaf plot for the rainfall data.
- What comparisons can be made between the rainfall of the two cities?

Melbourne Leaf	Stem	Adelaide Leaf
	1	— —
	2	2 3 4
	3	— —
— 8 8	4	3
— — — — —	5	0 2 —
8 0	6	—

- The average rainfall, in millimetres, for Canberra and Hobart is shown in the table.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Canberra	58	57	56	53	49	37	40	47	50	67	63	53
Hobart	48	40	47	52	49	56	54	52	52	64	55	52

- Represent this data in a back-to-back stem-and-leaf plot.
- What comparisons can be made between the rainfall of the two cities?

- The weights, in kilograms, of students in a group are shown.

Girls: 62, 45, 53, 58, 50, 48, 56, 65, 78, 54

Boys: 68, 75, 84, 77, 86, 70, 55, 68, 67, 69

- Represent the data in a back-to-back stem-and-leaf plot.
- What comparisons can be made about the weights of the girls and boys in this sample?

- 11** Two primary school classes are surveyed and asked to nominate their favourite vegetable from the given list. The results are shown below.

Class 4 Blue	Favourite vegetable	Class 4 Red
• • • • • • • • • •	carrots	• • • • •
•	peas	• • • • • • •
• • • • • • • • • •	potato	• • • • • • • • • • • • • •
• • • • •	broccoli	• • • • •
• • • •	snow peas	• •
• •	pumpkin	•

- a** How many students are there in Class 4 Blue?
b How many students in Class 4 Red nominated peas as their favourite?
c Which is the most popular vegetable among students in:
i Class 4 Blue? **ii** Class 4 Red?
d Which is the least popular vegetable among students in:
i Class 4 Blue? **ii** Class 4 Red?
e If the two classes are combined, which vegetable is the:
i most popular? **ii** least popular?
- 12** The back-to-back stem-and-leaf plot shows the marks of two Year 11 classes for a mathematics test.

Class 11A Leaf	Stem	Class 11B Leaf
	0	2
9 9 9 8 8 5 5 5 4 4 2	1	9
9 9 8 7 6 5 5 5 1 0 0 0 0	2	0 0 2 2 3 3 5 6 6 7 9
9	3	0 0 1 2 2 4 4 4 5 8 8 9

- Answer the following questions for:
- i** Class 11A **ii** Class 11B.
- a** What was the lowest mark? **b** What was the highest mark?
c What was the most common mark? **d** In what range are the marks clustered?
e In this test a mark of 18 or more was considered satisfactory. How many marks were rated unsatisfactory?
f Based on your analysis of parts **a–e**, which class performed better on this test?

- 13** The stem-and-leaf plot shows the weights (in kilograms) of the boys and girls in a Year 11 class.

Boys Leaf	Stem	Girls Leaf
	4	7 9
7 0	5	3 7 7 7 9
9 9 4 3	6	3 6 7
6 5 3 1 0	7	1 5
9 2 2	8	0

- a** How many students were in the class?
b How many girls weighed less than 60 kg?
c How many boys weighed more than 70 kg?
d How many students weighed less than 66 kg?
e If Range = highest score – the lowest score, what is the range for the boys' weights?

7D Column and bar graphs

These resources are available on your obook assess:

- **Interactive 7D:** Explore key ideas for column graphs
- **Video tutorial 7D:** Watch and listen to an explanation of Example 7D-3
- **Spreadsheet 7D.1:** Create a column graph for given data
- **Spreadsheet 7D.2:** Create a bar graph for given data
- **Investigation 7D:** Construct dot plots and column graphs to interpret data
- **assess quiz 7D:** Test your skills with an auto-correcting multiple-choice quiz

Instead of a column or row of dots, it is often easier to draw a plain column, or row.

EXAMPLE 7D-1 Drawing a vertical column graph

The number of each brand of dishwasher sold by an electrical store in one week is shown in the table.

Draw a vertical column graph to illustrate this information.

Brand	Frequency
Dishlex	10
Hoover	8
LG	4
Bosch	7
Asko	2

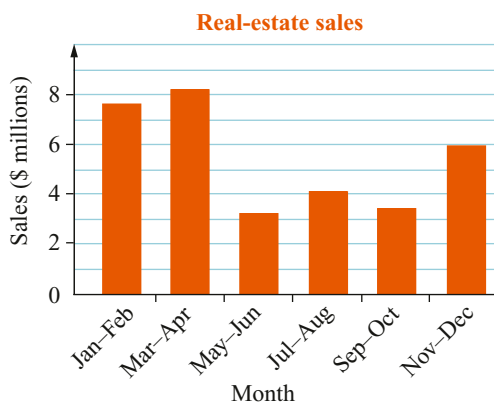
Solve	Think	Apply
Dishwasher sales 	The brands are placed on the horizontal axis and the frequency on the vertical axis. The highest frequency is 10, so a suitable scale is chosen for this axis. A rectangular column 10 units high is drawn to represent Dishlex dishwashers, a column 8 units high is drawn to represent Hoover dishwashers, etc.	Place the categories on the horizontal axis and the frequency, with a suitable scale, on the vertical axis. The rectangular columns are the same width and evenly spaced. The height of each column is determined by the frequency of each category.

EXERCISE 7D Column and bar graphs

- 1 This table shows the average percentage of protein in certain foods. Draw a vertical column graph to illustrate this information.

Food	Protein (%)
Beef	35
Rice	5
Eggs	13
Fish	30
Milk	3.5

- 2 The column graph below shows the sales for a real-estate company, for the months given. Complete the table below for the data.



Month	Sales (\$million)
Jan/Feb	
Mar/Apr	
May/Jun	
Jul/Aug	
Sept/Oct	
Nov/Dec	

- 3 The ages of the vehicle occupants killed on NSW roads in one year are shown in the table below. Represent this information using a column graph.

Age group	Number killed
0–9	42
10–19	110
20–29	175
30–39	65
40–49	80
50–59	55
60–69	40
70+	60

EXAMPLE 7D–2 Drawing a horizontal bar graph

The average house price for each of the capital cities of Australia, in April 2016, is shown in this table. Illustrate this information on a horizontal bar graph.

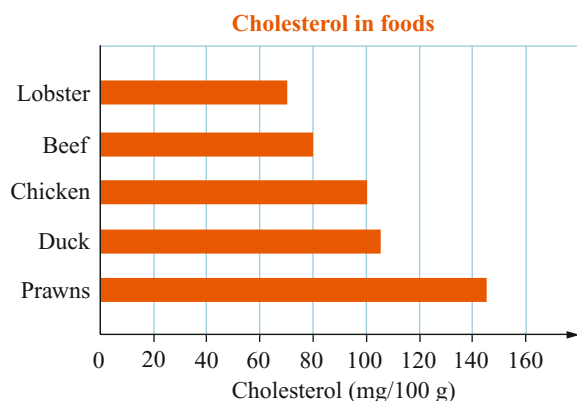
City	Average price (\$'000)
Sydney	1200
Melbourne	860
Canberra	705
Brisbane	530
Perth	560
Adelaide	510
Hobart	390

Solve	Think	Apply
Prices of houses in Australian capital cities	The cities are placed on the vertical axis and the price is on the horizontal axis. The highest price is \$1200, so a suitable scale is chosen for this axis. A bar 1200 units long is drawn to represent Sydney, a bar 860 units long is drawn to represent Melbourne, etc.	When rectangles are drawn horizontally, the graph is usually called a horizontal bar graph. Place the categories on the vertical axis. The rectangular bars are the same width and evenly spaced. The length of each bar represents the data.

- 4 The table on the right shows the number of injuries in eight of the main sports played in Australia. Illustrate this information using a horizontal bar graph.

Sport	Injuries (number per year)
AFL	270
Soccer	160
Cricket	99
Rugby League	88
Rugby Union	58
Netball	40
Hockey	32
Indoor Cricket	30
Basketball	23

- 5 The number of milligrams of cholesterol per 100 g of various foods is shown in the horizontal bar graph below. Complete the given table for the data.

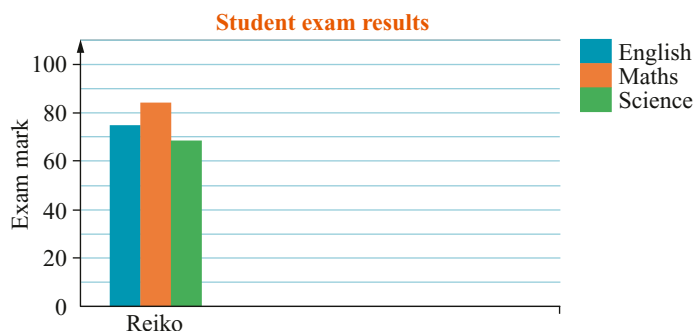


Food	Cholesterol (mg/100 g)
Lobster	
Beef	
Chicken	
Duck	
Prawns	



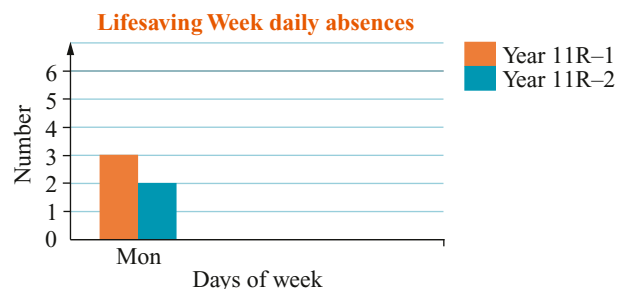
- 6 The English, mathematics and science exam marks for five students are shown in the table below. Complete the column graph to show these results by grouping the subjects together for each student as shown.

Student	English	Maths	Science
Reiko	75	84	68
James	65	76	84
Wendy	89	65	60
Hassan	54	78	75
Vincent	62	90	88



- 7 The number of absences each day of Lifesaving Week for two roll classes is shown in this table. Complete the column graph, grouping the classes together each day, for comparison purposes.

Day	Number of absences	
	Year 11R-1	Year 11R-2
Mon	3	2
Tue	3	3
Wed	5	6
Thu	6	4
Fri	2	3



Misleading graphs

Misleading graphs give a false impression of the data, either by mistake or deliberately. The main causes of graphs being misleading are:

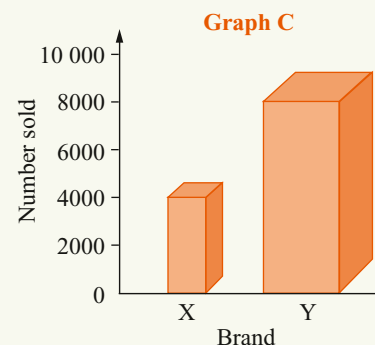
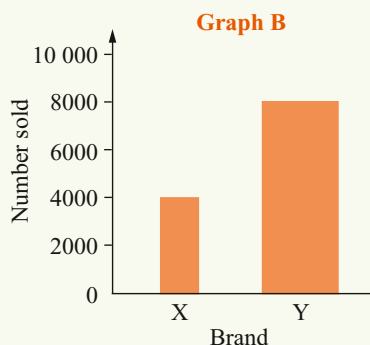
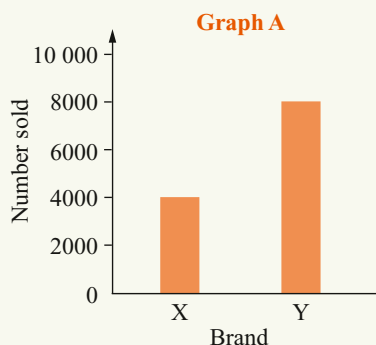
- the scale on the vertical axis does not start at zero
- the scale on the vertical or horizontal axis is irregular
- the scale on the vertical or horizontal axis is missing
- the use of area or volume creates a false impression.

EXAMPLE 7D-3 Identifying misleading features of column graphs

This table shows the number of televisions of each of two brands sold.

Below are shown three ways of presenting the information in the table graphically. Describe each graph, indicating whether or not it is misleading.

Brand	Number of televisions sold
X	4000
Y	8000



Solve

Graph A: This column graph correctly shows (as the table does) that the sales of brand X are double those of brand Y. The width of both columns are the same and the height of the second column is twice that of the first.

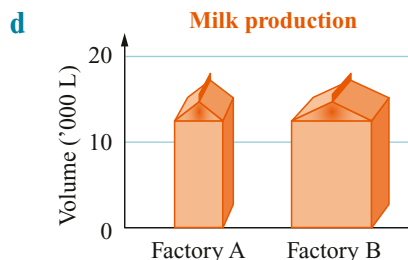
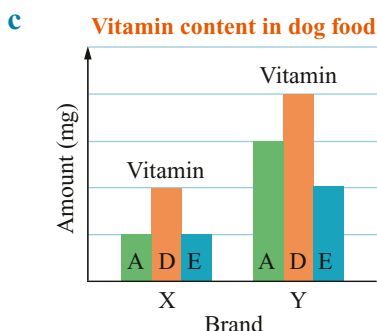
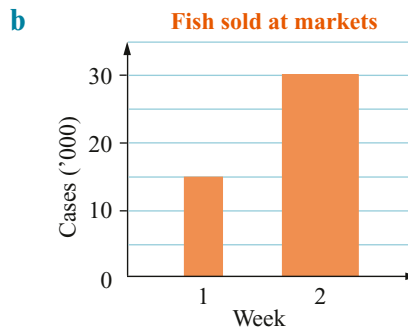
Graph B: In this graph, both the height and the width of the column for brand Y have been doubled, creating the impression that the sales of brand Y are 4 times (2×2 times) those of brand X.

Graph C: The three dimensions in this graph have been doubled for brand Y, creating the impression that the sales of brand Y are 8 times ($2 \times 2 \times 2$ times) those of brand X.

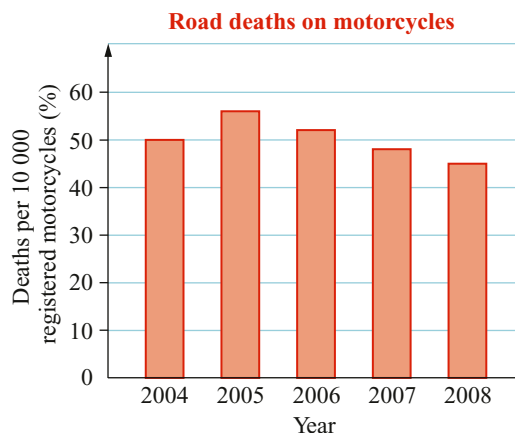
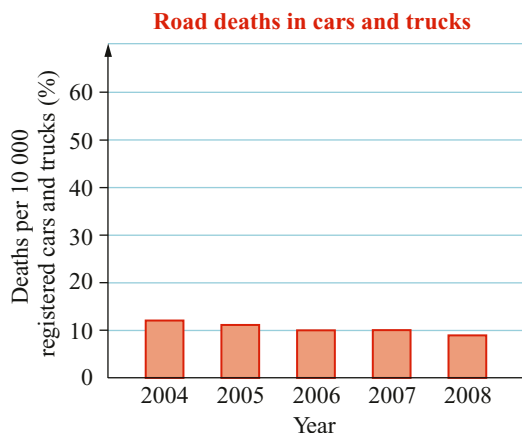
Think/Apply

For column graphs, the area of a 2D column and the volume of a 3D column indicate the values. Check that the use of area or volume does not create a false impression of the data.

- 8 Describe the misleading or poor features of the following graphs.



- 9 The percentage of deaths, per 10 000 vehicles, for all registered cars and trucks and motorcycles between 2004 and 2008 are shown on the graphs below. Approximately how many times more likely is a motorcyclist to be killed than a driver of any other type of vehicle?



- 10 The top 10 most downloaded artists for July 2012 are listed in this table.

- Add a column to the table and write each number of downloads as a number in scientific notation, correct to two significant figures.
- Draw a column graph displaying this information. Start at zero on the vertical axis and use a scale of millions of downloads.
- Redraw the column graph starting at 25 million sales instead of zero on the vertical axis. How do the two graphs compare?

Artist	Number of downloads
Rihanna	47 571 000
Black Eyed Peas	42 405 000
Eminem	42 290 000
Lady Gaga	42 078 000
Taylor Swift	41 821 000
Katy Perry	36 788 000
Lil Wayne	36 518 000
Beyonce	30 439 000
Kanye West	30 242 000
Britney Spears	28 665 000

- 11** The data for fatal crashes in NSW in 2010, grouped by the licence status of the car drivers, are shown in Table A.

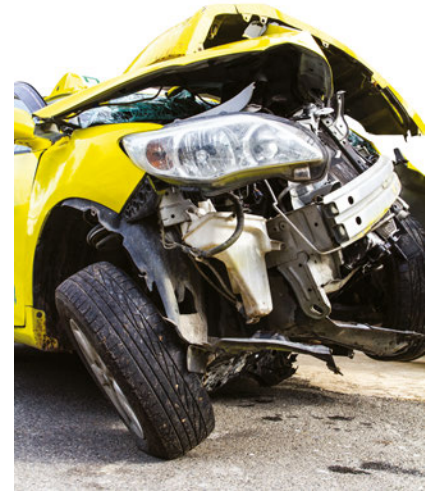
- Draw a column graph to illustrate this data.
- What is the relative frequency of unlicensed drivers involved in fatal accidents?
- What percentage of provisional licence holders were involved in fatal accidents?
- What percentage of standard licence holders were involved in fatal accidents?
- Which licence type do you think should be targeted by safety authorities?
- Table B shows the number and type of licences held by car drivers in 2010. Use both tables to answer the following.
 - How many provisional (P1 and P2) licence drivers per 100 000 were involved in fatal accidents?
 - How many standard licence drivers per 100 000 were involved in fatal accidents?
 - Reassess your answer to part e and change it if necessary.

Table A

Licence status	Fatal crashes
Learner	6
Provisional	60
Standard	221
Unlicensed	27
Unknown status	5
Total	319

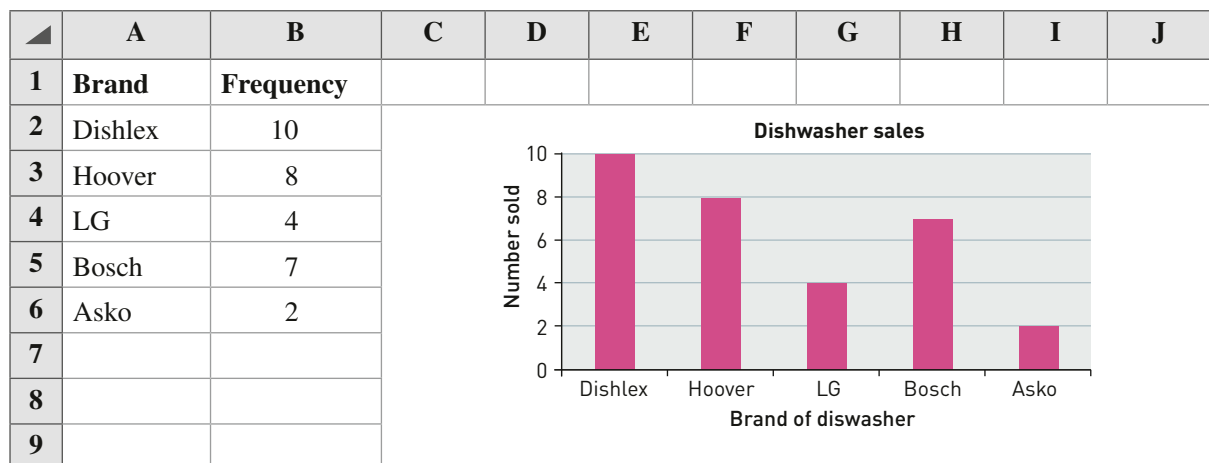
Table B

Licence type	Number
L	263 552
P1	145 295
P2	251 327
Standard	3 647 936
Total	4 308 110



- 12** Many different types of graphs can be drawn using the **Chart** option (or equivalent) of spreadsheet software.

The vertical column graph below was drawn after entering the information from Example 7D-1 into the first two columns of a spreadsheet. Highlight the table, select **Chart** and choose **Column graph**. Investigate the options available such as the labelling, including the table, etc.



Similarly, you can create a bar graph for given data. *Note:* to change from one graph type to another, right click on the graph and select **Change Series Chart Type**.

- Produce a spreadsheet to create a column graph for the data in Example 7D-1.
- Produce a spreadsheet to create a bar graph for the data in Example 7D-2.
- Now use a spreadsheet to draw the graphs in this exercise.

You may like to use the prepared spreadsheet files provided on your obook assess to help you.

7E Line graphs

These resources are available on your obook assess:

- **Worksheet 7E:** Practise your skills with extra problems for misleading graphs
- **Spreadsheet 7E:** Create a line graph for given data
- **assess quiz 7E:** Test your skills with an auto-correcting multiple-choice quiz

Line graphs are useful for showing upward and downward trends in data. They are also useful for finding 'in between' values when the quantity on the horizontal axis is continuous.

EXAMPLE 7E-1 Drawing a line graph

This table shows the variation in price of a share at end of trading for each day over one week. Draw a line graph to show this information.

Day	Mon	Tue	Wed	Thu	Fri
Price (cents)	45	48	55	52	46

Solve	Think	Apply
Share performance	Days of the week are placed on the horizontal axis and price on the vertical axis. The points are plotted and joined by straight lines.	Plot the points corresponding to the information given in the table. Join the points with straight lines.

EXERCISE 7E Line graphs

- 1** This table shows the temperature variation ($^{\circ}\text{C}$) on a summer day in Sydney, from 7:00 am to 7:00 pm. Draw a line graph to illustrate this data.

Time (hour)	7 am	8 am	9 am	10 am	11 am	12 pm	1 pm	2 pm	3 pm	4 pm	5 pm	6 pm	7 pm
Temperature ($^{\circ}\text{C}$)	13	14	16	17	19	22	25	28	27	27	24	22	18

- 2** The monthly sales figures for a computer firm are shown below. Draw a line graph for this information.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sales (\$'000)	54	36	30	30	28	34	25	26	31	38	44	56

- 3 The percentage of each age group of the male population that is overweight is shown below. Illustrate this information on a line graph.

Age group (years)	10–19	20–29	30–39	40–49	50–59	60–69
Overweight (%)	25	28	39	42	55	50

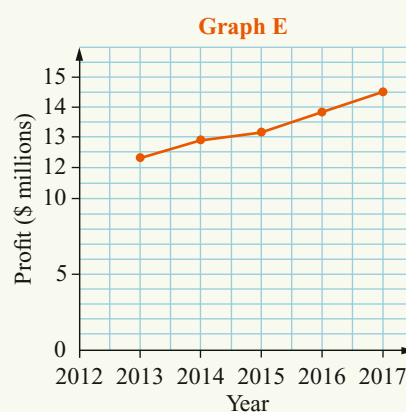
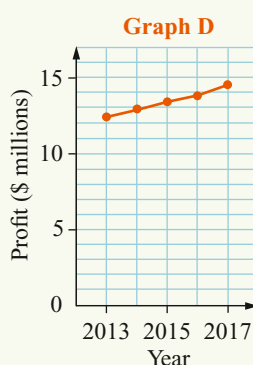
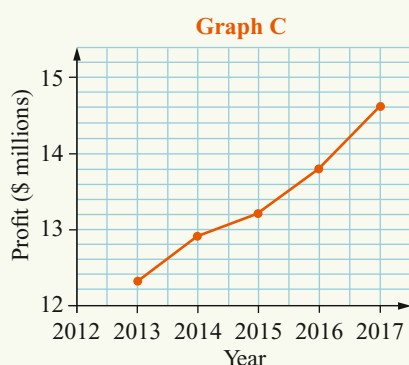
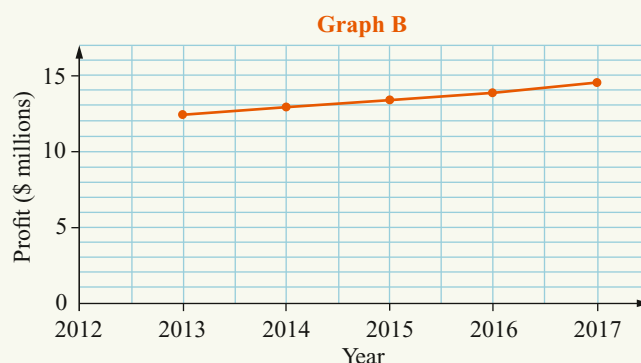
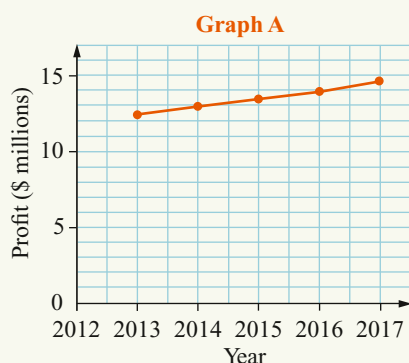
- 4 The weight of a baby at various ages is shown below. Illustrate this information on a line graph.

Age (months)	0	3	6	9	12	15	18	21	24
Weight (kg)	3.2	5.1	7.0	8.8	10.0	10.7	11.2	11.8	12.5

EXAMPLE 7E-2 Identifying misleading features of line graphs

This table shows the profits of a company over the 5-year period from 2013 to 2017. Graphs A to E below are five ways of presenting this information graphically. Which features of the graphs, if any, are misleading?

Year	2013	2014	2015	2016	2017
Profit (\$ millions)	12.3	12.9	13.2	13.8	14.6



Solve

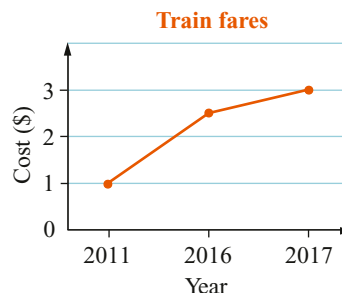
Graphs A, B and D are fair, although each gives a different impression: Graph B makes the increase in profit look smaller because the horizontal scale is enlarged. Graph D makes the increase in profit look larger because the horizontal scale is compressed.

Graphs C and E are misleading: Graph C has exaggerated the increase in profit by not starting the vertical scale at 0, thus enlarging this scale. Graph E has an irregular vertical scale, so the graph is incorrect.

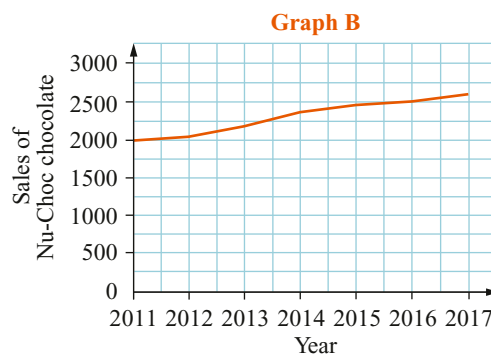
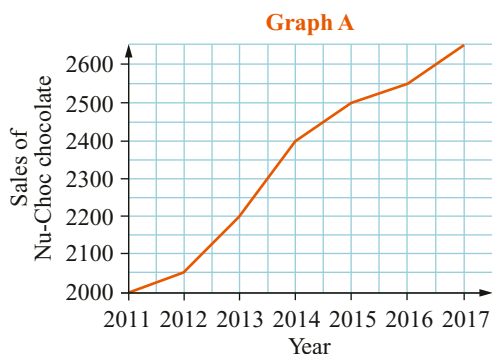
Think/Apply

Check that a scale is present on the vertical axis and that it starts at zero and is regular.

- 5 Describe the misleading or poor features of this graph.



- 6 a Which graph gives the impression of rapidly increasing sales?



- b Have sales in fact increased rapidly over this 7-year period?
 c According to graph A, the sales for 2014 appear to be double those of 2013. Is this true?

- 7 a A large 4WD has a fuel consumption of 15 L/100 km when running on unleaded petrol (ULP). The cost of ULP is \$1.50/L. Complete the table below to show the fuel cost of driving this car.

Distance travelled (km)	0	10 000	20 000	30 000	40 000	50 000
Fuel cost (\$)	0	2250				

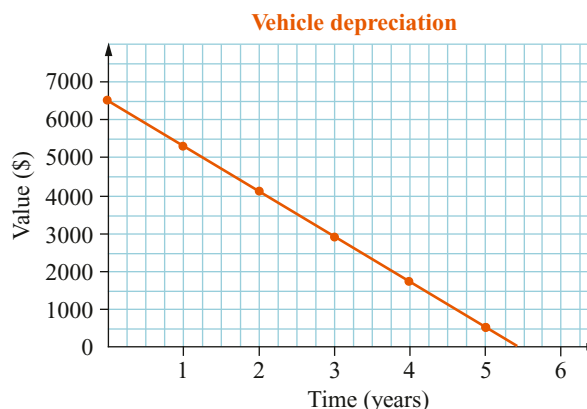
- b Use the information from the table to draw a graph of fuel cost versus distance travelled.
 c When converted to LPG the fuel consumption of this vehicle is 19.5 L/100 km. The cost of converting this vehicle to LPG is \$3500 and the cost of LPG is 70c/L. Complete the following table to show the cost of driving this vehicle on LPG.

Distance travelled (km)	0	10 000	20 000	30 000	40 000	50 000
Fuel cost (\$)	3500	4865				

- d On the same axes used in part b, draw a graph of fuel cost versus distance travelled for the LPG vehicle.
 e From the graph, estimate the distance travelled to reach the break-even point.

- 8 The graph show the depreciation of a motorcycle over time.

- a What was the purchase price?
 b What is the value of the motorcycle after:
 i 1 year? ii $2\frac{1}{2}$ years?
 c When is it worth:
 i \$4000? ii \$1700?
 d The motorcycle is scrapped after 5 years. What is its final written-down value after this time?
 e What is the annual amount of depreciation?



- 9 A car bought for \$26 990 depreciates \$3300 per year.

- a Complete the table to show the car's value after 0, 1, 2, 3 years.

Number of years	0	1	2	3
Value (\$)	26 990			

- b Draw a line graph to illustrate the data in the table.
- c Extend the graph to find the value of the car after 6 years.
- d From the graph, when will the car be worth half its purchase price?

- 10 Consider the following data about road fatalities in NSW from 1950 to 2015.

Year	1950	1955	1960	1965	1970	1975	1980
Number killed	634	820	978	1151	1309	1288	1303

Year	1985	1990	1995	2000	2005	2010	2015
Number killed	1067	797	620	603	508	405	350

- a Draw a line graph for the data in the table and comment on any trends in these figures.
- b In which 5-year period did:
- i the largest increase in fatalities occur? ii the largest decrease in fatalities occur?
- c Discuss why the number of fatalities has decreased since 1980 even though the number of registered vehicles has increased.

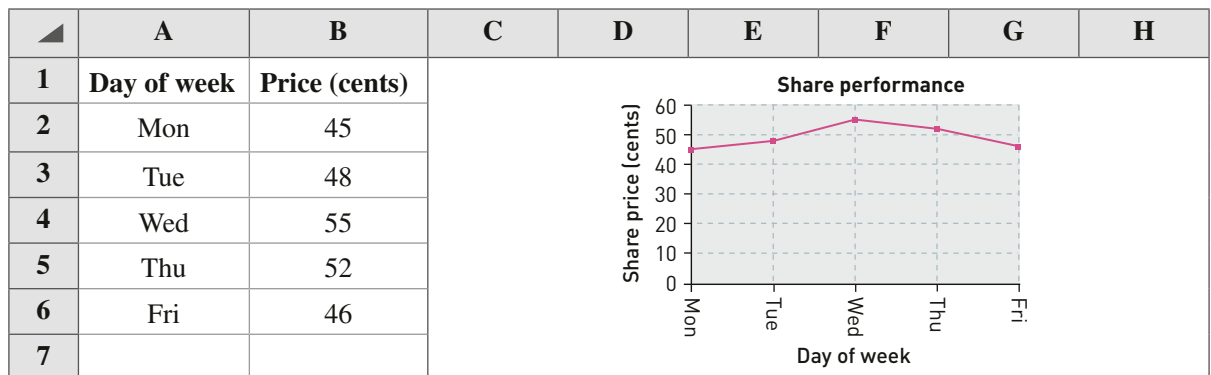
- 11 A 75-kg male drinks for 3 hours. His blood alcohol content (BAC) after 3 hours can be approximated by the formula $BAC = \frac{10N - 22.5}{510}$ where N is the number of standard drinks consumed in this time.

- a Complete the table using the given formula.

N	1	2	3	4	5	6
BAC						

- b Draw a line graph to illustrate the information in the table.
- c Extend the graph to find the man's BAC after: i 7 drinks ii 8 drinks.
- d If he wants his BAC to be under 0.05 after 3 hours, so that he can drive home, what is the maximum number of standard drinks he can consume?
- e A zero BAC is a requirement of NSW law for all learner and provisional licence holders. If he wants his BAC to be zero after 3 hours, what is the maximum number of drinks the man can consume?

- 12 The line graph below, representing the data in Example 7E-1, was drawn using a spreadsheet. The data is entered in the first two columns. Highlight the table, select **Chart** and choose **Line graph**.



Use a spreadsheet to draw some of the line graphs in this exercise. You may like to use the prepared spreadsheet file provided on your [obook assess](#) to help you.

7F Frequency histograms and polygons

histogram

column graph with no gaps between the columns that shows data scores along the horizontal axis and frequency on the vertical axis

frequency polygon

line graph made by joining the midpoints of the tops of the columns of a frequency histogram, with the first and last points joined to the horizontal axis

These resources are available on your obook assess:

- **Worksheet 7F:** Practise your skills with extra problems for representing data
- **Investigation 7F:** Determine data sets from frequency histograms
- **assess quiz 7F:** Test your skills with an auto-correcting multiple-choice quiz

A convenient method for representing a frequency distribution graphically is by means of a frequency histogram.

A **histogram** is a column (or bar) graph in which the values of the variable are placed on the horizontal axis and the frequency of the variable is on the vertical axis. There are no gaps between the columns.

A **frequency polygon** is a line graph made by joining the midpoints of the tops of the columns of a frequency histogram, with the first and last points joined to the horizontal axis to form a polygon. The first and last point of the polygon are located on the horizontal axis at half the column width before the first column and at half the column width after the last column in the histogram.

EXAMPLE 7F-1 Drawing a frequency histogram and frequency polygon

The maximum temperature (in $^{\circ}\text{C}$) on each day in September was recorded, and the results were summarised in a frequency distribution table, as shown.

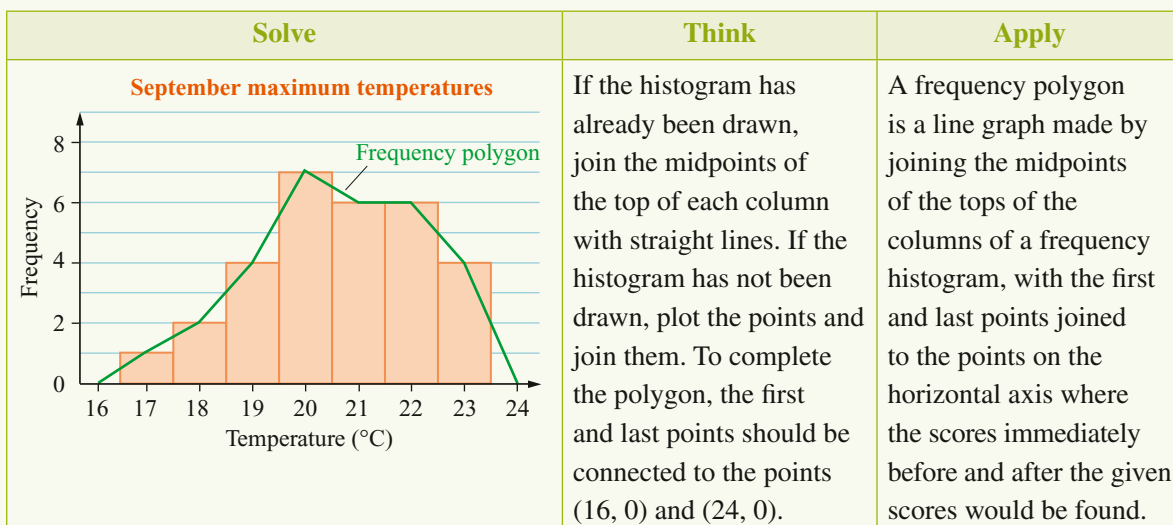
- Draw a frequency histogram of the temperature data.
- Draw a frequency polygon of the distribution.

Temperature ($^{\circ}\text{C}$)	17	18	19	20	21	22	23
Frequency	1	2	4	7	6	6	4

a

Solve	Think	Apply
September maximum temperatures	Choose suitable scales. Put temperature on the horizontal axis and frequency on the vertical axis. Draw the columns the same width, joined together, and centred above the temperatures they represent. The width of each column is the distance between successive scores on the horizontal axis. The height of each column is the frequency of each score (temperature).	A frequency histogram is a column graph with frequency on the vertical axis. The height of each column represents the frequency of each score.

b



EXERCISE 7F Frequency histograms and polygons

- 1 Draw a frequency histogram for each of the following distributions.

a	Number of snacks	0	1	2	3	4	5	6
	Frequency	5	7	9	7	6	4	2

b	Score	10	11	12	13	14
	Frequency	7	11	17	0	5

- 2 Draw a frequency polygon for each of the following distributions.

a	Number of goals	0	1	2	3	4
	Frequency	8	9	6	3	1

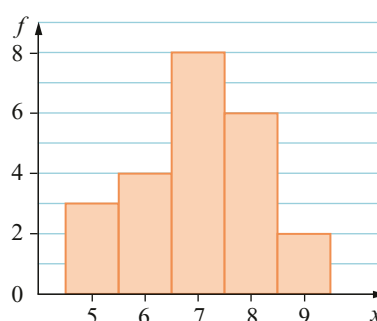
b	Salary (\$)	30 000	40 000	50 000	60 000	70 000	80 000
	Frequency	8	6	7	3	0	1

- 3 On the same diagram, draw a frequency histogram and a frequency polygon for the data given below.

Mark	14	15	16	17	18	19	20
Frequency	4	6	9	0	2	3	1

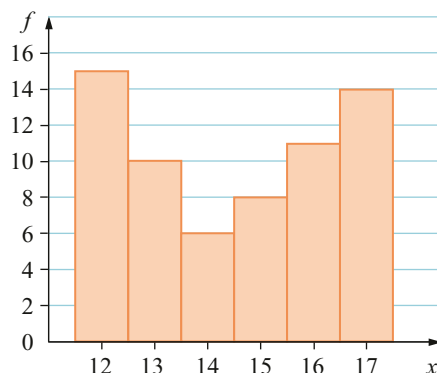
- 4 Complete the frequency distribution tables given the following histograms.

a	Score (x)	Frequency (f)
	5	
	6	
	7	
	8	
	9	



b

Score (x)	Frequency (f)
12	
13	
14	
15	
16	
17	



EXAMPLE 7F-2 Drawing a grouped frequency histogram and frequency polygon

Draw a grouped frequency histogram and polygon for this data about the heights of Year 11 students.

Height (cm)	155–159	160–164	165–169	170–174	175–179
Frequency	4	7	10	5	2

Solve	Think	Apply																		
Height of students	For grouped data, we need to find the midpoint of each class interval, called the class centre. <table> <tr> <th>Height (cm)</th><th>Class centre</th><th>Frequency</th></tr> <tr> <td>155–159</td><td>$\frac{155 + 159}{2} = 157$</td><td>4</td></tr> <tr> <td>160–164</td><td>$\frac{160 + 164}{2} = 162$</td><td>7</td></tr> <tr> <td>165–169</td><td>$\frac{165 + 169}{2} = 167$</td><td>10</td></tr> <tr> <td>170–174</td><td>$\frac{170 + 174}{2} = 172$</td><td>5</td></tr> <tr> <td>175–179</td><td>$\frac{175 + 179}{2} = 177$</td><td>2</td></tr> </table> The class centres are placed on the horizontal axis and the histogram and polygon completed.	Height (cm)	Class centre	Frequency	155–159	$\frac{155 + 159}{2} = 157$	4	160–164	$\frac{160 + 164}{2} = 162$	7	165–169	$\frac{165 + 169}{2} = 167$	10	170–174	$\frac{170 + 174}{2} = 172$	5	175–179	$\frac{175 + 179}{2} = 177$	2	For grouped data, the class centres are used for the scores on the horizontal axis, and then the histogram and polygon are completed as before.
Height (cm)	Class centre	Frequency																		
155–159	$\frac{155 + 159}{2} = 157$	4																		
160–164	$\frac{160 + 164}{2} = 162$	7																		
165–169	$\frac{165 + 169}{2} = 167$	10																		
170–174	$\frac{170 + 174}{2} = 172$	5																		
175–179	$\frac{175 + 179}{2} = 177$	2																		

- 5** For the data in the following tables, find the class centre of each class interval. Draw a grouped frequency histogram and polygon, for the data in each table.

a

Mass (kg)	Frequency
60–62	2
63–65	5
66–68	7
69–71	4
72–74	2

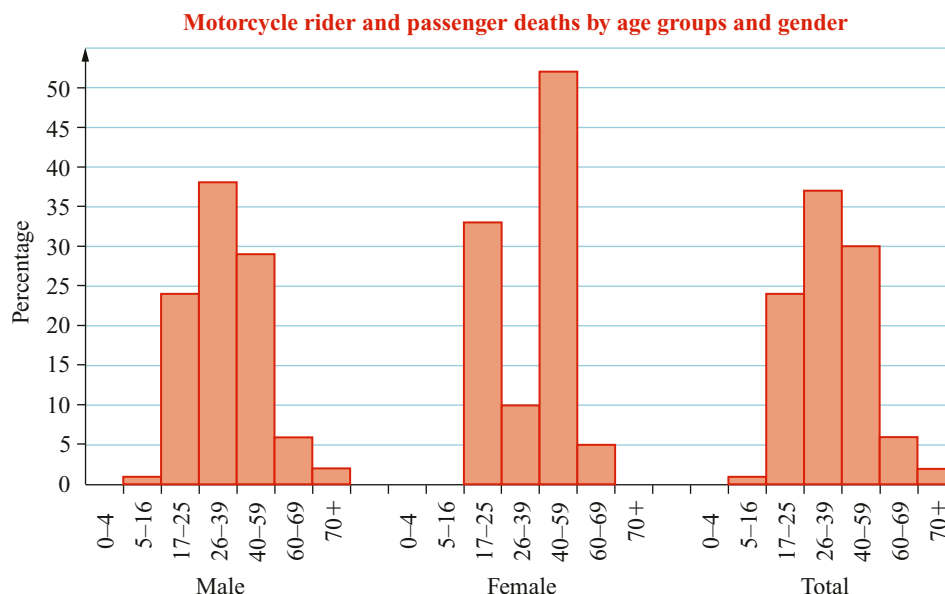
b

Score	Frequency
41–50	3
51–60	0
61–70	10
71–80	8
81–90	7
91–100	2

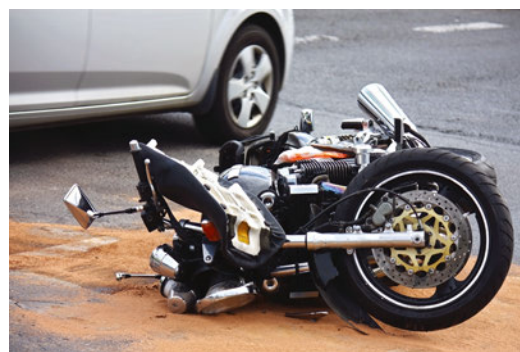
c

Patients per day	Frequency
10–19	13
20–29	15
30–39	16
40–49	10
50–59	4
60–69	2

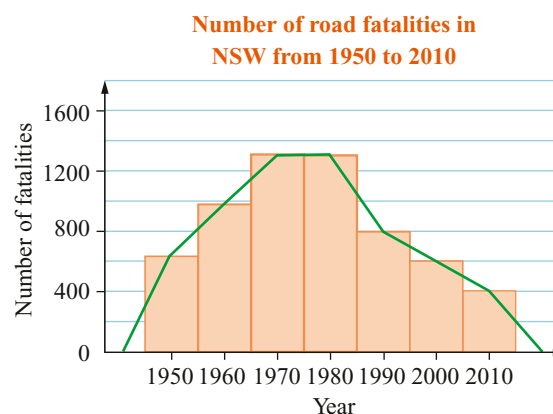
- 6 The graph below shows the percentage of riders and passengers killed in motorcycle accidents in 2008, categorised by gender and age. Use the graph to answer the following questions.



- In 2008, approximately what percentage of all male riders or passengers killed in motorcycle accidents were:
 - 17–25 years old?
 - 60–69 years old?
- Approximately what percentage of all female riders or passengers killed in motorcycle accidents were:
 - 17–25 years old?
 - 60–69 years old?
- Which age group of male motorcycle riders and passengers had the:
 - highest number of fatalities?
 - lowest number of fatalities?
- Which age group of female motorcycle riders and passengers had the:
 - highest number of fatalities?
 - lowest number of fatalities?
- Which age group of all motorcycle riders and passengers had the:
 - highest number of fatalities?
 - lowest number of fatalities?



- 7 The graph on the right shows the number of road fatalities in NSW from 1950 to 2010.
- In which year was the number of fatalities:
 - the highest?
 - the lowest?
 - In which 10-year period did:
 - the largest increase in fatalities occur?
 - the largest decrease in fatalities occur?
 - Explain any trends shown by the graph.
 - Discuss why the number of fatalities has decreased since 1980 even though the number of registered vehicles on the road has increased.



7G Cumulative frequency graphs

These resources are available on your obook assess:

- **Video tutorial 7G:** Watch and listen to an explanation of Example 7G-4
- **assess quiz 7G:** Test your skills with an auto-correcting multiple-choice quiz

cumulative frequency histogram

column graph (histogram) with data scores along the horizontal axis and cumulative frequency on the vertical axis

cumulative frequency polygon (ogive)

line graph formed by joining the upper right-hand corners of each column of the cumulative frequency histogram

A **cumulative frequency histogram** is a histogram with cumulative frequency on the vertical axis. A **cumulative frequency polygon** is a line graph formed by joining the upper right-hand corners of each column of the cumulative frequency histogram. A cumulative frequency polygon is also called an **ogive**.

EXAMPLE 7G-1 Drawing a cumulative frequency histogram and cumulative frequency polygon

The maximum temperature, in $^{\circ}\text{C}$, on each day in September, (as given in Example 7F-1) is shown in the table below. Draw a cumulative frequency histogram and a cumulative frequency polygon for this data.

Temperature ($^{\circ}\text{C}$)	17	18	19	20	21	22	23
Frequency	1	2	4	7	6	6	4

Solve	Think																								
September maximum temperatures Cumulative frequency Temperature ($^{\circ}\text{C}$)	Add a cumulative frequency column to the table. <table><tr><th>Temperature ($^{\circ}\text{C}$)</th><th>Frequency</th><th>Cumulative frequency</th></tr><tr><td>17</td><td>1</td><td>1</td></tr><tr><td>18</td><td>2</td><td>3</td></tr><tr><td>19</td><td>4</td><td>7</td></tr><tr><td>20</td><td>7</td><td>14</td></tr><tr><td>21</td><td>6</td><td>20</td></tr><tr><td>22</td><td>6</td><td>26</td></tr><tr><td>23</td><td>4</td><td>30</td></tr></table> Draw a cumulative frequency histogram, using the cumulative frequency of each score. Next, use straight lines to join the top right-hand corners of the columns to form the cumulative frequency polygon.	Temperature ($^{\circ}\text{C}$)	Frequency	Cumulative frequency	17	1	1	18	2	3	19	4	7	20	7	14	21	6	20	22	6	26	23	4	30
Temperature ($^{\circ}\text{C}$)	Frequency	Cumulative frequency																							
17	1	1																							
18	2	3																							
19	4	7																							
20	7	14																							
21	6	20																							
22	6	26																							
23	4	30																							

Apply

Note: when drawing a cumulative frequency polygon, we join the upper-right hand vertices, not the midpoints, of the columns. We do not join the last corner to the horizontal axis, so the graph is not strictly a polygon.

To draw a cumulative frequency polygon without first drawing the histogram requires careful consideration of the upper and lower boundaries of each class interval.

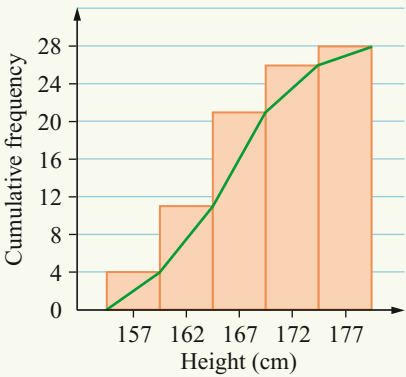
EXERCISE 7G Cumulative frequency graphs

- 1 For each of the frequency distribution tables given in question 1 of Exercise 7F:
 - a add a cumulative frequency column
 - b draw a cumulative frequency histogram and a cumulative frequency polygon.
- 2 For each of the frequency distribution tables given in question 2 of Exercise 7F:
 - a add a cumulative frequency column
 - b draw a cumulative frequency histogram and a cumulative frequency polygon.

EXAMPLE 7G-2 Drawing a cumulative frequency histogram and cumulative frequency polygon for grouped data

Given the information about the heights of students in the table below, draw a cumulative frequency histogram and a cumulative frequency polygon for the data.

Height (cm)	155–159	160–164	165–169	170–174	175–179
Frequency	4	7	10	5	2

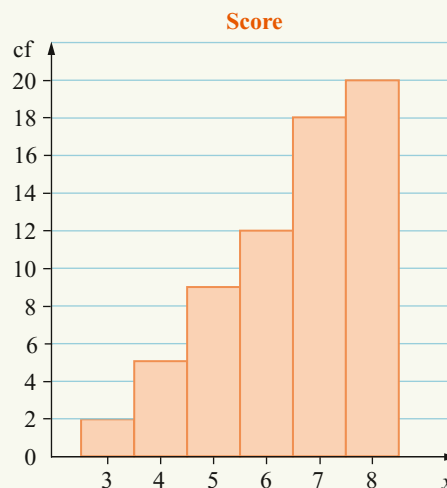
Solve	Think																								
Height of students 	Find the class centre and add a cumulative frequency column. <table border="1"> <thead> <tr> <th>Height (cm)</th> <th>Class centre</th> <th>Frequency</th> <th>Cumulative frequency</th> </tr> </thead> <tbody> <tr> <td>155–159</td> <td>$\frac{155 + 159}{2} = 157$</td> <td>4</td> <td>4</td> </tr> <tr> <td>160–164</td> <td>$\frac{160 + 164}{2} = 162$</td> <td>7</td> <td>11</td> </tr> <tr> <td>165–169</td> <td>$\frac{165 + 169}{2} = 167$</td> <td>10</td> <td>21</td> </tr> <tr> <td>170–174</td> <td>$\frac{170 + 174}{2} = 172$</td> <td>5</td> <td>26</td> </tr> <tr> <td>175–179</td> <td>$\frac{175 + 179}{2} = 177$</td> <td>2</td> <td>28</td> </tr> </tbody> </table> Draw the cumulative frequency histogram using the class centres. Join the <i>top right-hand corners</i> of the columns of the histogram to draw the cumulative frequency polygon (the ogive).	Height (cm)	Class centre	Frequency	Cumulative frequency	155–159	$\frac{155 + 159}{2} = 157$	4	4	160–164	$\frac{160 + 164}{2} = 162$	7	11	165–169	$\frac{165 + 169}{2} = 167$	10	21	170–174	$\frac{170 + 174}{2} = 172$	5	26	175–179	$\frac{175 + 179}{2} = 177$	2	28
Height (cm)	Class centre	Frequency	Cumulative frequency																						
155–159	$\frac{155 + 159}{2} = 157$	4	4																						
160–164	$\frac{160 + 164}{2} = 162$	7	11																						
165–169	$\frac{165 + 169}{2} = 167$	10	21																						
170–174	$\frac{170 + 174}{2} = 172$	5	26																						
175–179	$\frac{175 + 179}{2} = 177$	2	28																						
Apply Use the class centres as the values on the horizontal axis and use the cumulative frequency on the vertical axis. Join the top right-hand corners of the columns of the histogram to form the ogive.																									

- 3
 - a For each part of question 5 in Exercise 7F, add a cumulative frequency column to the table.
 - b Use the tables from part a to help you draw a cumulative frequency histogram and ogive for each set of data, on the same diagram.

EXAMPLE 7G-3 Constructing a frequency distribution table from a cumulative frequency histogram

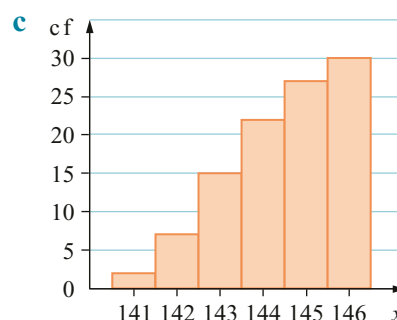
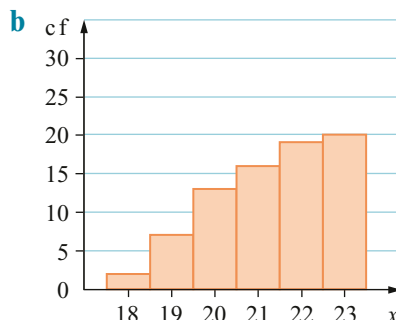
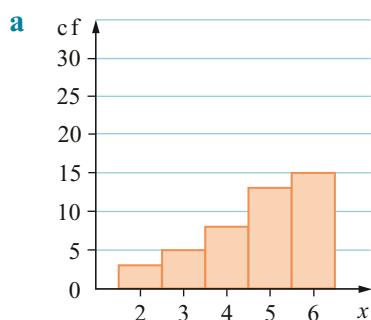
Complete the frequency distribution table given the cumulative frequency histogram.

Score (x)	Cumulative frequency (cf)	Frequency
3		
4		
5		
6		
7		
8		



Solve			Think	Apply
Score (x)	Cumulative frequency (cf)	Frequency	From the graph, write the cumulative frequency of each score. The cumulative frequency of the first score is the same as its frequency: the frequency of the score 3 is 2. The cumulative frequency of the score 4 is 5; this is the sum of the frequencies of the scores 3 and 4. Hence, the frequency of the score 4 is $(5 - 2)$ or 3. The cumulative frequency of the score 5 is 9; this is the sum of the frequencies of 3, 4 and 5. Hence, the frequency of 5 is $(9 - 5)$ or 4, and so on.	From the graph, determine the cumulative frequency for each score. Use the cumulative frequency column to find the frequency of each score.
3	2	2		
4	5	$5 - 2 = 3$		
5	9	$9 - 5 = 4$		
6	12	$12 - 9 = 3$		
7	18	$18 - 12 = 6$		
8	20	$20 - 18 = 2$		

4 Complete frequency distribution tables given these cumulative frequency histograms.



Pareto charts

A **Pareto chart** is used as a tool in quality control when analysing data about the frequency of problems in a process or their causes (in particular, when we want to focus on the most significant problems or causes).

A Pareto chart combines a vertical column graph and a cumulative relative frequency graph. The statistical variable (problems/causes) is placed on the horizontal axis and frequency is placed on the vertical axis. The heights of the vertical bars represent the frequencies of the problems/causes and are arranged with the longest bars on the left and the shortest on the right (in descending order of frequency). This means that the problems/causes that have the most impact are clearly shown on the left of the chart. The cumulative relative frequency graph helps us to judge the added contribution of each problem/cause. If the cumulative graph rises steeply then levels out, this indicates that the first two or three causes are these having the most impact on the problem. If the columns are roughly the same height, the cumulative graph will rise at a steady rate, indicating that all causes are having a roughly equal impact on the problem.



Pareto chart
chart that combines a column graph and a cumulative frequency graph

EXAMPLE 7G-4 Drawing a Pareto chart

A bank wants to identify the most significant problems it experiences in the delay of processing credit card applications. A random survey was conducted and the results are shown in the table.

- Draw a Pareto chart to illustrate this information.
- What area(s) should the manager concentrate on improving?
- What percentage of her problem does this solve?

Cause of delay	Frequency (f)
Incorrect address	36
Can't read	17
No signature	73
Wrong form	11
Other	3

a

Solve

Think

Cause of delay	Frequency(f)
No signature	73
Incorrect address	36
Can't read	17
Wrong form	11
Other	3
Total (Σf)	140

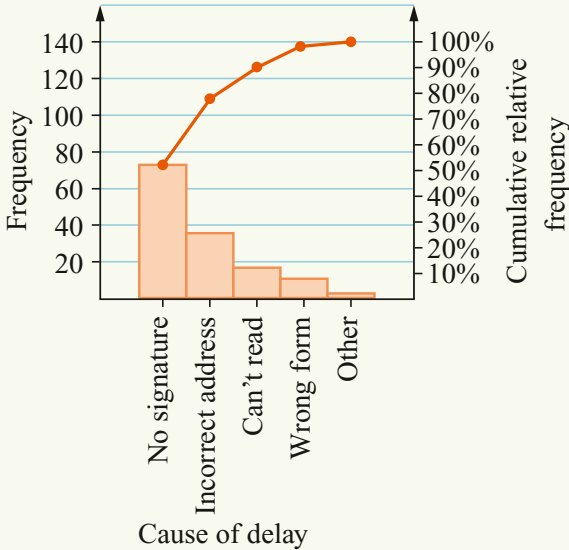
Rearrange the categories in descending order of frequency.

Find the total of the frequencies.

Note: Σf means 'sum of f '.

Cause of delay	Frequency
No signature	73
Incorrect address	36
Can't read	17
Wrong form	11
Other	3

Draw a vertical column graph using the data from the table. The frequency axis should be at least 140 units high (140 is the total of the frequency column). Make the widths of the columns the same. The columns are usually connected.

Solve				Think
				Add a relative frequency and cumulative relative frequency column to the table. Relative frequency $= \frac{f}{\Sigma f} \times 100\%$. The relative frequencies have been rounded to the nearest whole number percentage
Cause of delay	Frequency (f)	Relative frequency (%)	Cumulative relative frequency (%)	
No signature	73	52	52	
Incorrect address	36	26	78	
Can't read	17	12	90	
Wrong form	11	8	98	
Other	3	2	100	
Total	140	100		
				Draw another vertical axis on the right-hand side of the graph. On this axis, label the point directly opposite 140 on the frequency axis as 100%. Divide the scale of the second vertical axis into 100 equal divisions, starting with zero at the level of the horizontal axis to create a cumulative relative frequency axis. In this case $140 \div 100 = 1.4$. That is, 1 unit on the percentage axis (1%) represents 1.4 units on the frequency axis, or 10 units on the percentage axis (10%) represents 14 units on the frequency axis, and so on. Mark the points corresponding to the cumulative relative frequencies and join using straight lines. <i>Note:</i> if the left and right vertical scales match (i.e. $\Sigma f = 140$ on the frequency axis corresponds to 100% on the cumulative relative frequency axis), the cumulative relative frequency graph will always start at the top of the first column. If the scales do not match, then the cumulative relative frequency graph will not start at the top of the first column.
The two main areas that need to be fixed to improve the process are ‘No signature’ and ‘Incorrect address’.				
Fixing the two main causes will solve 78% of the problem.				
Apply				
Place the categories (problems/causes), in descending order of relative frequency, on the horizontal axis. Place frequency on the vertical axis. Draw a vertical column graph. Add a cumulative relative frequency graph.				
<i>Note:</i> since the causes are placed in order of relative frequency, the graph clearly reveals the factors having the greatest impact and where attention is likely to be the most beneficial. In our example, if the manager concentrates on fixing the two causes ‘No signature’ and ‘Incorrect address’, she will have solved 78% of her problem.				

- 5** A bank manager wants to identify the most significant problems the bank experiences in the delay of processing loan applications. A random survey was conducted and the results are shown in the table on the right.

- Draw a Pareto chart to illustrate this information.
- What area(s) should the manager concentrate on improving?
- What percentage of his problems would this solve?

Cause of delay	Frequency
No address	20
Can't read	8
No signature	84
Other	3
Wrong form	5

- 6** A random survey of customer complaints yielded the data shown on the right.

- Draw a Pareto chart to illustrate this information.
- What area(s) should this company concentrate on improving?
- What percentage of the problems would this solve?

Type of complaint	Frequency
Packaging	11
Delivery	7
Invoice wrong	36
Product quality	22
Other	4

- 7** The owner of a shoe store takes a random sample of customer complaints. The results are shown in the table on the right.

- Draw a Pareto chart to illustrate this information.
- Before the survey, the manager thought that it was the limited range of styles being offered that was the main reason for the decline in her business and she blamed the supplier. What percentage of the problem was caused by limited styles?
- If you were the shoe store owner, what area(s) would you concentrate on improving?
- What percentage of the problems would your improvements from part **c** solve?

Type of complaint	Frequency
Difficult parking	77
Salesperson rude	9
Poor lighting	5
Layout confusing	8
Limited sizes	37
Limited styles	11
Other	3

- 8** Pareto charts can be drawn using a spreadsheet. After entering the data as you would for a table, go to:
Insert > Chart > Histogram > Pareto.

- Use your computer spreadsheet to draw Pareto charts for questions **5–7**.
- What is different about the charts drawn by Excel compared to your hand-drawn charts?

- 9** A restaurant manager takes a random survey of customer complaints in order to increase the patronage of his restaurant. The results are shown in the table on the right.

- Draw a Pareto chart to illustrate this information.
- If you were the manager, what area(s) would you concentrate on improving?
- What percentage of the problems would your improvements from part **b** solve?

Type of complaint	Frequency
Rude staff	6
No atmosphere	8
Small portions	17
Too noisy	19
Too expensive	93
Limited menu	5
Dirty washrooms	8
Long delays in serving	35
Cramped seating	9

CHAPTER 7 REVIEW REPRESENTING DATA

You should be able to:

- ✓ construct frequency distribution tables
- ✓ understand and use cumulative and relative frequency graphs and tables
- ✓ group data into classes
- ✓ create and interpret dot plots
- ✓ construct and interpret stem-and-leaf plots
- ✓ create column and bar graphs
- ✓ create and interpret line graphs
- ✓ understand how graphs may be misleading
- ✓ construct and interpret frequency histograms and polygons
- ✓ construct and interpret cumulative frequency histograms and ogives
- ✓ construct and interpret Pareto charts
- ✓ link types of data with appropriate displays
- ✓ describe the strengths and weaknesses of various forms of display.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

REVIEW MULTIPLE-CHOICE QUESTIONS

- 7A** 1 The marks out of 10 for a spelling test were:

5 3 9 8 6 4 5 6 6 5
7 8 10 6 7 5 6 8 6 7

By organising this information into a frequency distribution table, the number of students who scored less than 6 is:

- A 6 B 5 C 4 D 3

- 7A** 2 The distance, in kilometres, the members of a group of students travel to school are shown below.

0.6 3.4 0.8 2.8 0.9 1.1 1.9 2.4
3.5 3.6 0.8 1.4 2.7 0.6 1.3 3.9
2.1 2.3 3.0 1.6 1.5 3.2 2.1 1.8
0.7 2.3 3.6 3.5 1.6 1.4 0.4 2.8
3.8 2.9 0.8 1.7 3.7 2.5 1.9 2.0

By organising this data into a frequency table using classes 0–0.4, 0.5–0.9, 1.0–1.4, 1.5–1.9, ... the number of students who travel more than 2.4 km is:

- A 25 B 21 C 15 D 12

- 7B** 3 For the data in question 1, the cumulative frequency and the percentage relative frequency of the score 7 are respectively:

- A 3 and 15% B 15 and 3% C 15 and 15% D 12 and 15%

- 7C ▶ 4** This stem-and-leaf plot shows the time, in hours, a group of Year 11 students spent studying during one particular week. The lowest and highest scores are respectively:

A 7 and 9 **C** 2 and 9
B 0 and 9 **D** 2 and 39

Stem	Leaf
0	2 3 3 6 7 7
1	0 2 2 5 8 8 8 9 9
2	1 1 2 3 4 4 5 6 6 7 8
3	0 2 2 9

- 7C ▶ 5** From the data in question 4, how many students spent less than 20 hours studying?

A 9 students **B** 19 students **C** 15 students **D** 21 students

- 7C ▶ 6** Which of the following statements is *not* true?

A A dot plot is convenient for illustrating small sets of data.
B A dot plot can be used directly with unsorted data.
C A dot plot is very time-consuming for large sets of data.
D A dot plot can be used to show the percentage of each category in the data.

- 7D ▶ 7** Consider the sales data shown in the table on the right. If a horizontal bar graph was used to display the information in this table, with a scale of 10 mm representing \$1 million, the length of the bar representing sales for March/April would be:

A 0.31 mm **B** 3.1 mm
C 31 mm **D** 310 mm

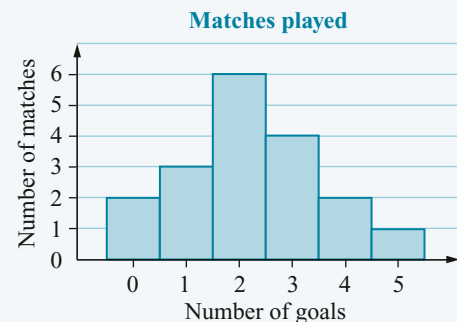
Months	Sales (\$ million)
Jan/Feb	2.4
Mar/Apr	3.1
May/Jun	1.8
Jul/Aug	1.4
Sep/Oct	2.8
Nov/Dec	3.6

- 7E ▶ 8** Which of the following is *not* a cause of graphs being misleading?

A The scale on the vertical axis does not start at zero.
B The scale on the horizontal axis does not start at zero.
C The scale on the vertical axis is irregular.
D The incorrect use of area or volume creates a false impression.

- 7F ▶ 9** From this frequency histogram, the total number of matches played was:

A 17 **B** 5
C 6 **D** 18

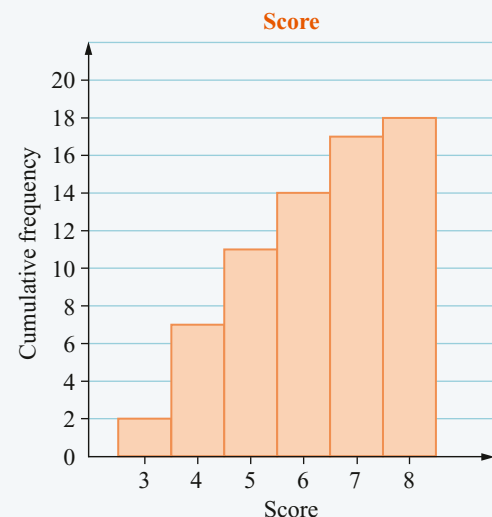


- 7F ▶ 10** To draw a frequency polygon using the histogram in question 9, the first and last column midpoints should be joined to:

A any two points on the horizontal axis
B the two points on the horizontal axis where the previous and the next score would be
C any two points on the vertical axis
D the bottom left and right corners of the histogram.

- 7G ▶ 11** From the cumulative frequency histogram on the right, the frequency of the score 5 is:

A 3 **B** 4
C 5 **D** 11



REVIEW SET 1

-
- A group of approximately 15 people of various ages and ethnicities are standing together, smiling. They are dressed in casual clothing like t-shirts, jeans, and sweaters. The group is diverse in age, including young adults, middle-aged, and older individuals. They are standing on a light-colored floor against a plain white background.

Day	Mon	Tue	Wed	Thu	Fri
Price (\$)	2.80	3.50	3.60	2.60	2.55

- Draw a cumulative frequency histogram and ogive for the data shown in the table.

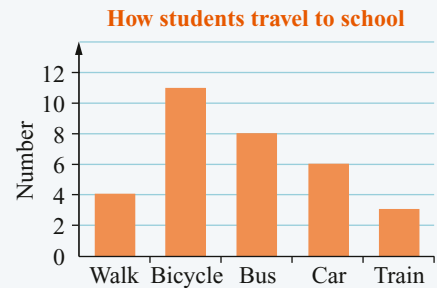
REVIEW SET 2

11	12	11	14	18	15	13	14	19	18
17	15	15	12	18	17	13	10	9	14
16	16	13	14	11	14	18	10	14	15

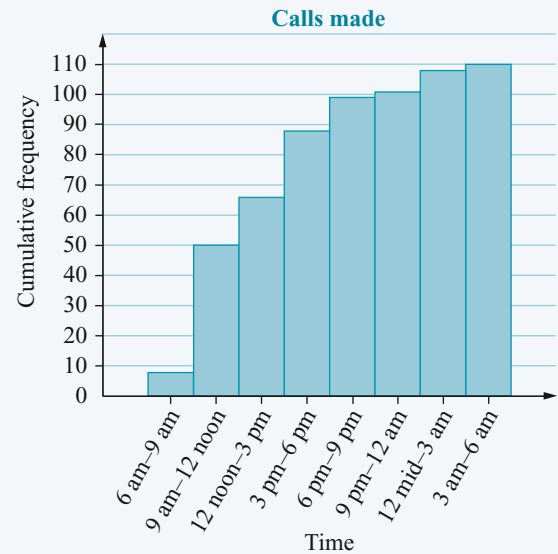
- 274

7 From the column graph, how many students:

- a** walk to school?
- b** don't catch a train?



8 Using the cumulative frequency graph on the right, how many calls were made between 12:00 noon and 3:00 pm?



REVIEW SET 3

1 The numbers of each brand of sound system sold by an electrical store in one week are shown in the table.

- a** How many sound systems were sold during the week?
- b** Draw a horizontal bar graph to display this information.
- c** Draw a vertical column graph to display this information.

Brand	Number of sales
Sony	15
National	8
Pioneer	12
Aiwa	10
Phillips	5

2 a Draw a stem-and-leaf plot, using the stems 3, 4, 5 and 6, for these scores:

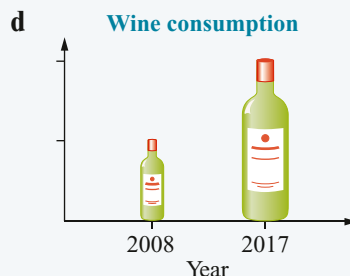
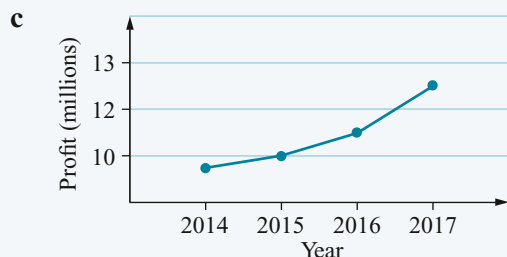
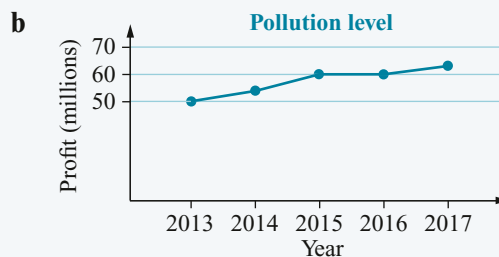
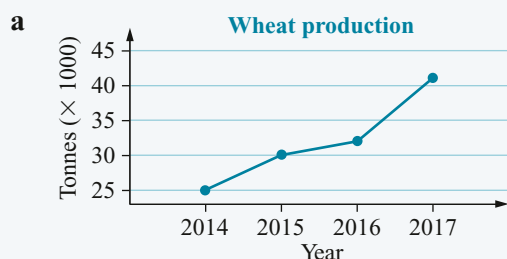
33 46 38 66 49 58 34 43 60 63
48 52 51 48 37 69 61 51 64 48

- b** What are the lowest and highest scores?
- c** How many times do the following scores occur?
 - i** 51
 - ii** 41
- d** Which score occurs the most often?
- e** How many scores are in the sixties?
- f** How many scores are less than 50?

3 A bank manager wants to identify the most significant problems the bank experiences in the delay of processing credit card applications. A random survey was conducted and the results are shown in the table. Draw a Pareto chart to illustrate this information

Cause of delay	Frequency
Incorrect address	47
Can't read	22
No signature	92
Wrong form	14
Other	5

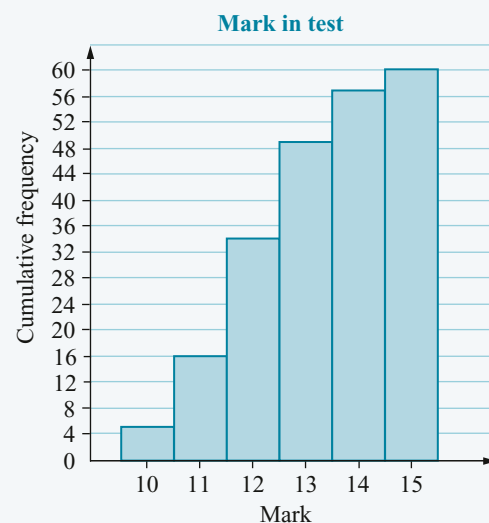
4 Describe the misleading or poor features of the following graphs.



REVIEW SET 4

Use the cumulative frequency histogram on the right to answer questions 1–4.

- 1 **a** Copy the cumulative frequency histogram and draw the ogive.
- b** How many students scored:
 - i less than 12?
 - ii 12 or less?
 - iii more than 12?
- 2 Construct a frequency distribution table for the data.
- 3 What percentage of students scored:
 - a 10?
 - b 15?
- 4 Draw a frequency histogram and polygon for the data.



REVIEW PRACTICE EXAMINATION QUESTION

1 **a**

Student score	Class centre	Frequency	Cumulative frequency
0–4		7	
5–9		11	
10–14		15	
15–19		14	
20–24		3	

- i Complete this frequency distribution table. (2 marks)
- ii How many students scored less than 15? (1 mark)
- iii What is the relative frequency of students scoring 5, 6, 7, 8 or 9? (1 mark)
- iv What percentage of students scored more than 15? (1 mark)
- v Draw a cumulative frequency histogram and polygon for this data. (2 marks)

- b Find the number missing from the table below.

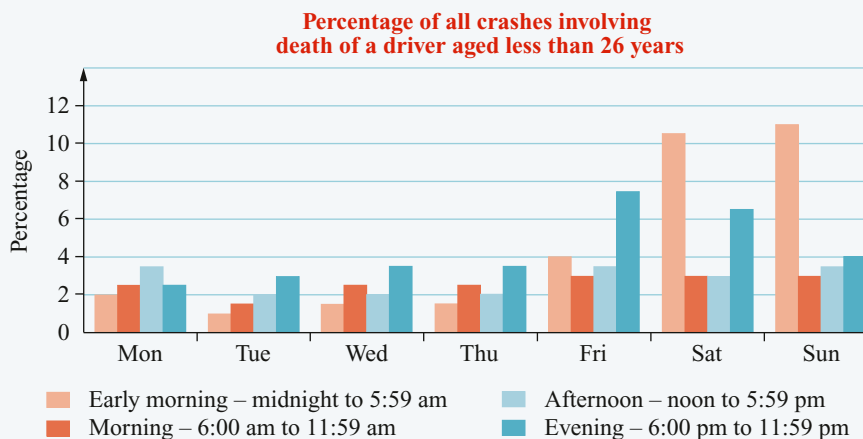
(1 mark)

Score	Cumulative frequency	Frequency
9	7	7
10	13	6
11	16	3
12	21	

- c This stem-and-leaf plot shows the time, in hours, the members of a group of university students spent studying per week.

Stem	Leaf
3	1 6
4	0 1 2 2 5 8 9
5	0 1 1 3 4 7 8 9 9
6	2 3 5 8
7	0 1 1

- i What is the lowest score? (1 mark)
- ii How many scores are in the forties? (1 mark)
- iii How many students studied for more than 50 hours per week? (1 mark)
- iv What percentage of students studied for fewer than 44 hours per week? (1 mark)
- d The graph below shows the percentage of crashes resulting in the death of a driver aged less than 26 years, by time of the day and day of the week.



- i What percentage of fatal crashes involving a driver under 26 occurred between noon and 5:59 pm on a Tuesday? (1 mark)
- ii At what time of the week do most fatal crashes occur? (1 mark)
- iii On what day of the week do the most fatal crashes occur? (1 mark)

TOTAL: 15 MARKS

8

Relative frequency and probability

The main mathematical ideas investigated are:

- ▶ listing the sample space for simple experiments and counting the number of possible outcomes
- ▶ investigating equally likely outcomes
- ▶ listing the sample space for multistage experiments
- ▶ determining relative frequency or experimental probability
- ▶ determining theoretical probability
- ▶ comparing calculated probabilities with experimental results
- ▶ finding the probability of complementary events
- ▶ calculating probability for multistage events
- ▶ using probability tree diagrams to calculate probabilities
- ▶ finding the expected frequency of an event and comparing it with experimental results.
- ▶ solving problems involving simulations or trials of experiments.

ARE YOU READY?

- 8A ▶ 1** In a standard deck of playing cards, how many cards are in each suit?

A 2 **B** 4 **C** 13 **D** 14

- 8A ▶ 2** A card is selected at random from a deck of playing cards and its suit noted. What is the sample space of this experiment?

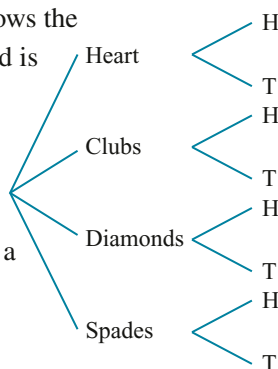
A {red, black}
B {jack, queen, king}
C {hearts, diamonds, clubs, spades}
D {4}

- 8A ▶ 3** What is the sample space when this spinner is spun?



A {red, blue, green, yellow}
B {red, red, blue, blue, green, yellow}
C {4} **D** {6}

- 8A ▶ 4** This tree diagram shows the outcomes when a card is selected from a deck of playing cards and the suit (hearts, diamonds, clubs or spades) is noted, and a coin is tossed. How many outcomes are possible?



A 2 **B** 4 **C** 6 **D** 8

- 8B ▶ 5** What is $\frac{9}{40}$ as a decimal?

A 0.09 **B** 0.225 **C** 0.444 **D** 0.940

- 8B ▶ 6** What is $\frac{57}{80}$ as a percentage?

A 0.007 125% **B** 0.7125%
C 7.125% **D** 71.25%

- 8B ▶ 7** The numbers 1 to 10 were written on cards and placed in a box.

Value	Frequency
≤ 3	21
> 3	59
Total	80

One card was chosen at random, its value noted

and then it was replaced in the box. This was repeated to obtain 80 results. These results are shown in the table.

Using the data in the table, what is the relative frequency of selecting a number that is greater than 3 when one card is chosen randomly from the ten cards?

A 59 **B** 59% **C** $\frac{21}{59}$ **D** $\frac{59}{80}$

- 8C ▶ 8** A marble is chosen at random from a bag containing 6 red, 1 green and 2 blue marbles. What is the best description of the chance of choosing a green marble?

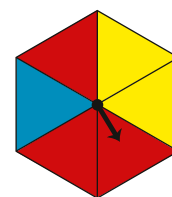
A impossible **B** unlikely
C likely **D** certain

- 8C ▶ 9** For the bag of marbles in question 8, what is the best description of the chance of choosing a yellow marble?

A impossible **B** unlikely
C likely **D** certain

- 8C ▶ 10** This spinner is divided into six equal sections. What is the chance of spinning yellow?

A $\frac{1}{6}$ **B** $\frac{1}{3}$
C $\frac{1}{2}$ **D** 2



- 8G ▶ 11** What is $\frac{5}{8}$ of 120?

A 0.0052 **B** 62.5 **C** 75 **D** 192

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook assess](#).

- Q1** Support sheet 8A.1 Understanding a standard deck of playing cards
Q2–3 Support sheet 8A.2 Sample space and outcomes
Q4 Support sheet 8A.3 Introducing tree diagrams
Q5–6 Support sheet 8B.1 Converting fractions, decimals and percentages
Q7 Support sheet 8B.2 Simple experimental probability
Q8–9 Support sheet 8C.1 Using key words to describe probability
Q10 Support sheet 8C.2 Simple theoretical probability
Q11 Support sheet 8G.1 Multiplying fractions

8A Sample space and number of outcomes

These resources are available on your obook assess:

- **Worksheet 8A:** Practise your skills with extra problems for sample space
- **Investigation 8A:** Investigate the rolling of two Sicherman dice
- **assess quiz 8A:** Test your skills with an auto-correcting multiple-choice quiz

sample space

set of all possible outcomes of an experiment

experiment

a trial (or trials) performed to obtain data to predict the chances of an event occurring

event

one outcome or collection of outcomes from performing an experiment

multistage experiment

experiment made up of more than one simpler experiment

multistage event

one outcome or collection of outcomes from performing a multistage experiment

equally likely outcomes

outcomes that have an equal chance of occurring

In probability, the **sample space** of an **experiment** is the set of all possible outcomes of the experiment. For example, rolling a six-sided die is an experiment with possible outcomes of 1, 2, 3, 4, 5 or 6.

An **event** is either one outcome or a collection of outcomes. For example, when a die is rolled, 'getting a 2' is one of the six equally likely outcomes. The event is 'getting a 2'. Another event could be 'getting an even number' which includes the outcomes 2, 4 or 6. This event is a collection of outcomes.

A **multistage experiment** is one that is made up of simpler experiments. For example, a two-stage experiment could be flipping a coin and then rolling a die. When finding the sample space of multistage experiments, it is often useful to use a systematic method such as a list, table or tree diagram.

A **multistage event** is either one outcome or a collection of outcomes of a multistage experiment. For example, when flipping a coin and rolling a die, a two-stage event could be 'getting a tail and a 2' (one outcome) or 'getting a tail and an even number' (collection of outcomes).



Equally likely outcomes are outcomes that have exactly the same (equal) chance of occurring. For example, for a fair die, we would expect rolling a 2 to have the same chance of occurring as rolling a 5 (or any number from 1 to 6).

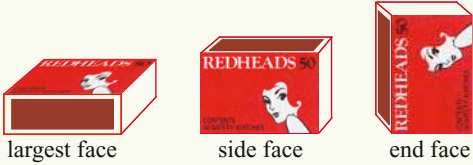
EXAMPLE 8A-1 Listing the sample space

List the sample space, S , when:

a a coin is tossed

b a six-sided die is rolled

c a matchbox is tossed.

	Solve	Think
a	$S = \{H, T\}$	When a coin is tossed, the possible outcomes are a head (H) or a tail (T).
b	$S = \{1, 2, 3, 4, 5, 6\}$	When a six-sided die is rolled, the possible outcomes are a 1, 2, 3, 4, 5 or 6.
c	If each of the six faces can be identified: $S = \{\text{top, bottom, front face, back face, right end, left end}\}$. If each of the six faces cannot be identified, and the matchbox can land on one of the largest faces, one of the side faces or one of the end faces: $S = \{\text{largest face, side face, end face}\}$.	The matchbox has six faces and they are either dealt with as three pairs or as six separate faces. 

Apply

The sample space is the list of all possible outcomes.

EXERCISE 8A Sample space and number of outcomes

1 For each of the following experiments:

i list the sample space

ii state the number of possible outcomes.

a A 12-sided die is tossed.

b A card is chosen from cards numbered 1 to 7.

c A marble is chosen from 3 black and 5 red marbles.

d A letter is chosen from the word MATHS.

e An eight-sided die is thrown.

f A card is selected from the four aces in a standard deck of 52 cards.

2 List the sample space for each of the following experiments.

a A boy or girl is born.

b A drawing pin is tossed.

c A cylindrical can is tossed.

d A marble is chosen from a bag containing 5 red, 3 blue and 2 white marbles.

e The colour of a set of traffic lights is noted as a car approaches.

f A vowel is chosen from the English alphabet.

g A letter is chosen from the word INSIGHT.

h The day of the week on which a child is born is recorded.

i The result (not the score) of a soccer match.

j A card is chosen from a standard deck of playing cards (there are several answers depending on what is being investigated).



EXAMPLE 8A-2 Deciding whether outcomes are equally likely to occur

State whether the outcomes of each of the sample spaces in Example 8A-1 are equally likely to occur.

From Example 8A-1, the sample spaces for the experiments are:

- a** a coin is tossed: $S = \{H, T\}$
- b** a six-sided die is rolled: $S = \{1, 2, 3, 4, 5, 6\}$
- c** a matchbox is tossed: $S = \{\text{top, bottom, front face, back face, right end, left end}\}$ or $S = \{\text{largest face, side face, end face}\}$

	Solve	Think	Apply
a	Equally likely	If the coin is 'fair' (not biased in some way), then we would expect each outcome to occur an equal number of times.	Each outcome must have the same chance of occurring.
b	Equally likely	Again, if the die is 'fair' we would expect the outcomes to be equally likely.	
c	Not equally likely	The matchbox is more likely to fall onto one of its larger faces; so the outcomes are not equally likely.	

- 3** State whether you think the outcomes of each of the sample spaces for the following experiments (from question 2) are equally likely to happen (we will test some of these experimentally, later). Discuss any problems as a class.
- a** A boy or girl is born.
 - b** A drawing pin is tossed.
 - c** A cylindrical can is tossed.
 - d** A marble is chosen from a bag containing 5 red, 3 blue and 2 white marbles.
 - e** The colour of a set of traffic lights is noted as a car approaches.
 - f** A vowel is chosen from the English alphabet.
 - g** A letter is chosen from the word INSIGHT.
 - h** The day of the week on which a child is born is recorded.
 - i** The result (not the score) of a soccer match.
 - j** A card is chosen from a standard deck of playing cards (there are several answers depending on what is being investigated).
- 4** Comment on the following statements.
- a** As it either rains or is fine, there is an even chance of it raining tomorrow.
 - b** There are six swimmers in a final, and each has an equal chance of winning.
 - c** The chance that a person's family name starts with Z is the same as the chance that it starts with any other letter of the alphabet.
 - d** As there are only two candidates for the election, each has a 50–50 chance of winning.
 - e** Each student in the class has an equal chance of getting his or her driving licence this year.
- 5** Which of the events in question 4 are equally likely?



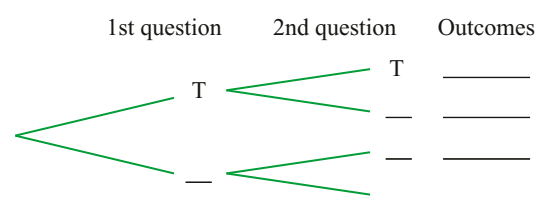
EXAMPLE 8A-3 Finding the sample space for a two-stage experiment

Tim has three pairs of trousers (blue, brown and grey) and two shirts (one long-sleeved and one short-sleeved) in his wardrobe. Find all the possible combinations of trousers and shirts he can wear by:

- a** using a table **b** using a tree diagram.

Solve					Think	Apply
Shirt	Trousers				List all possibilities in a table and list the outcomes.	A table assists in finding all possibilities. This is called a two-stage experiment because it is made up of two simpler experiments: selecting trouser colour (three possible outcomes) and selecting shirt type (two possible outcomes).
		Blue	Brown	Grey		
	Long	Bl, L	Br, L	Gr, L		
	Short	Bl, S	Br, S	Gr, S		
There are six equally likely outcomes: • blue trousers with long-sleeved shirt • brown trousers with long-sleeved shirt • grey trousers with long-sleeved shirt • blue trousers with short-sleeved shirt • brown trousers with short-sleeved shirt • grey trousers with short-sleeved shirt.						
Using a tree diagram yields the same results.					Draw three tree branches, one for each trouser type, then two branches extending from each trouser type showing long-sleeved or short-sleeved shirt.	Tables and tree diagrams are convenient ways of identifying the sample space for two-stage experiments.
Trousers Shirt Outcomes blue long Bl and L blue short Bl and S brown long Br and L brown short Br and S grey long Gr and L grey short Gr and S						

- 6** Complete this tree diagram to find all the possible ways you can answer the first two questions on a True or False test. (Use T for true and F for false.)



- 7** Find the sample space when two coins are tossed. It does not matter if the coins are tossed simultaneously or one after the other. (Use H for heads and T for tails.) Are all the outcomes equally likely?
- 8** List all the outcomes possible when a coin is tossed and a die is rolled.

- 9 a** Complete this table to see the sample space when two six-sided dice are rolled.
- b** How many outcomes are in the sample space?
- c** Are the outcomes equally likely to occur?
- d** How many outcomes show that a 'double' has been rolled?

		Second die					
		1	2	3	4	5	6
First die	1	(1, 1)	(1, 2)	(1, 3)			
	2	(2, 1)					
	3	(3, 1)					
	4						
	5						
	6						

- 10 a** Use the table in question 9 to find all the possible sums of the two numbers uppermost, when two dice are rolled. This table, recording those sums, has been started for you.
- b** How many different outcomes are shown in the table?
- c** Is it equally likely to obtain a sum of 4 as it is to obtain a sum of 6? Explain.
- d** Which outcome is the most likely to occur?
- e** Which outcomes have the same chance of occurring?
- f** How many of the 36 outcomes show a sum greater than 8?

		Second die					
		1	2	3	4	5	6
First die	1	2	3	4			
	2	3					
	3	4					
	4						
	5						
	6						

- 11 a** Use the table in question 9 to find all the possible differences between the two numbers uppermost when two dice are rolled. This table has been started for you.
- b** How many different outcomes are shown in the table?
- c** Is it equally likely to obtain a difference of 0 as it is to obtain a difference of 3? Explain.
- d** Which outcome is the most likely to occur?
- e** How many of the 36 outcomes show a difference less than 3?

		Second die					
		1	2	3	4	5	6
First die	1	0	1	2			
	2	1					
	3	2					
	4						
	5						
	6						

EXAMPLE 8A-4 Finding the sample space for a three-stage experiment

Find all the possible ways in which you can answer the first three questions on a True or False test. How many outcomes are there in total?

Solve					Think	Apply
This is a three-stage experiment because it is the combination of the outcomes of three simpler experiments (the possible answers to three questions). It is not possible to use a table for three-stage experiments. Using a tree diagram, we can find the eight outcomes listed below. 1st question 2nd question 3rd question Outcomes					Each branching of the tree has two possibilities: T or F. There needs to be three branchings of the tree, one for each question.	Each branching of a tree diagram must include all possibilities for a particular event.

- 12** What is the possible make-up (including the order of birth) of the three children in a family. (Use B for boy and G for girl.) Are the outcomes equally likely?
- 13** Find all the 'words' that can be made using the letters shown.
First letter: b, c, d Second letter: a, e Third letter: x, y
- 14** List all the ways in which John, Paul and George can be arranged in a line.
- 15** How many different responses are possible for a True or False test with four questions?
- 16** Ann has three pairs of jeans and four shirts. How many 'mix and match' outfits can she make?
- 17** Sarah travels through two sets of traffic lights on her way to the station each morning. What are all the possible sequences of coloured traffic lights she could encounter?
Do you think that all these outcomes are equally likely?
- 18** A special 'Sports' model of a small car is produced. It comes with 1.8 L or 2.2 L engines, in automatic (A) or manual (M) and is available in metallic orange (O), blue (B) or red (R) paint. List all the different combinations of features available in this model.
- 19** Cathy, Melinda and Rebecca are the only three runners in a race. Find all the possible orders in which they could finish.
- 20** A bag contains one red, one blue and one green marble. Two marbles are drawn from the bag in succession. List the sample space if the first marble is:
a replaced before the second marble is drawn
b not replaced before the second marble is drawn.
- 21** The numbers 5, 7 and 9 are written on three cards and the cards are placed in a hat. Two cards are selected, in succession, from the hat and the numbers are written in the order they were drawn. List all the two-digit numbers it is possible to make if the first card is:
a replaced before the second is drawn
b not replaced before the second is drawn.
- 22** A spinner has the numbers 1 to 4 on it, each with an equally likely chance of occurring. A coin is tossed and the spinner is spun. Find the sample space for the results of this two stage experiment.
- 23** Part of a mathematics test contains five multiple-choice questions, each having four possible responses. How many different ways are there of answering these five questions?
- 24** Use a table to list all the possible outcomes if Samantha plays Venus in a *best of* three sets tennis match.
- 25** Design a two-stage experiment that has 12 possible outcomes.
- 26** Design a three-stage experiment that has 36 possible outcomes.



8B Relative frequency

These resources are available on your obook assess:

- **assess quiz 8B:** Test your skills with an auto-correcting multiple-choice quiz

probability

chance of an event occurring, expressed as a number in the range 0 to 1 or as the equivalent percentage

You often hear people talk about the ‘chance’ of something happening, or how likely something is. Words and numbers can both be used to describe the **probability** of an event occurring. A probability, P , is expressed as a value in the range 0 (impossible) to 1 (certain), or as the equivalent percentage.

One way to predict the likelihood of an event happening is to investigate the occurrence of the event in the past. This assumes that patterns in the immediate past will not change greatly in the future.

We can perform an experiment a number of times to observe the frequency of the outcomes for an event.

EXAMPLE 8B-1 Calculating the relative frequency of an event

The students in a class combined to toss two coins 2000 times. The results are recorded in the table. Use this information to predict the chance of getting ‘2 heads’ when you toss two coins.

Result	Frequency
2 heads	563
1 head and 1 tail	987
2 tails	450

Solve	Think
$P(2 \text{ heads}) = \frac{563}{563+987+450}$ $= \frac{563}{2000}$ $= 0.2815$ $\approx 28\%$	If we assume that the frequency of occurrence will be the same for future tosses, we could predict that, when two coins are tossed, the chance of getting 2 heads is approximately 28%. Similarly, we could predict from the data that the chance of getting 1 head and 1 tail is $\frac{987}{2000}$, or approximately 49%, and the chance of getting 2 tails is $\frac{450}{2000}$, or approximately 23%.

relative frequency (experimental probability)

an estimate of the probability of an event occurring, determined by performing an experiment a number of times and finding the event’s relative frequency

In Example 8B-1, we assigned a number (or a percentage) to the chance of an event happening.

The number we have used above is called the **relative frequency** of the event.

Relative frequency gives an estimate of the probability of an event occurring.

Relative frequency is often referred to as **experimental probability**.

$$\begin{aligned} \text{Experimental probability} &= \text{relative frequency} \\ &= \frac{f}{\Sigma f} \text{ or } \frac{f}{\Sigma f} \times 100\% \end{aligned}$$

where f is the frequency and Σf is the total number of possibilities.

EXERCISE 8B Relative frequency

- 1 A cylindrical can was tossed 200 times. The number of times it landed on its side and on an end is shown in the table.
 - a Find the relative frequency of each outcome.
 - b Estimate the probability that in a future toss the can will land:
 - i on an end
 - ii on its side.
- 2 A card was selected at random from a standard deck of playing cards and its value noted. The card was replaced and this was repeated 120 times. The results were recorded as shown in the table on the right. (An ace was counted as a one and the court cards [jack, queen, king] were counted as 10.) Using this data, estimate the probability that a card selected at random from a standard deck is:
 - a less than 5
 - b greater than or equal to 5.
- 3 Five hundred drivers were randomly selected and asked the question 'In what country was your car manufactured?' The results are shown in the table on the right.
 - a Find the relative frequency for each country of manufacture.
 - b Using this data, estimate the probability that another driver chosen at random will own a car manufactured in Australia.
 - c Estimate the probability that the driver's car (from part b) has been manufactured in Korea.
- 4 A restaurant sold 5284 bottles of wine last year. Of these, 107 were returned because the wine was faulty. Based on this information, estimate the probability that a bottle of wine will be faulty when it is opened.
- 5
 - a In a Federal Government health survey of 10 000 people, 1150 people admitted to skipping breakfast. Based on this data, what is the probability that a person chosen at random has skipped breakfast?
 - b Of the 1080 girls aged 14–18 in the survey, 172 said that they had skipped breakfast. What is the probability that a girl aged 14–18 chosen at random has skipped breakfast?
- 6 The table on the right summarises the answers to a question concerning religion in an Australian census, when the population was 21.5 million. Estimate the probability that the religion of a person chosen at random will be:
 - a Catholic
 - b Anglican
 - c non-Christian.

Can tosses

Outcome	Frequency
End	76
Side	124

Card selection

Value	Frequency
<5	36
≥5	84

Car manufacture

Country	Frequency
Australia	169
Japan	153
Germany	77
Korea	62
Other	39



Religions in Australia

Religion	Number (millions)
Catholic	5.44
Anglican	3.68
Other Christian	4.03
Non-Christian	1.55
No stated religion	6.80

- 7** Data for the past 50 years, showing the number of days it rained in the month of September, is shown in the table. Estimate the probability that next September it will rain on:

- a** 12 to 14 days
- b** more than 8 days
- c** fewer than 3 days
- d** 5 or fewer days.

Rain in September

Number of days	Frequency
0–2	21
3–5	18
6–8	8
9–11	2
12–14	0
15–17	1

- 8** One hundred batteries were selected at random from a production line and tested. The results are shown in the table on the right. Determine the approximate probability that a battery selected at random from those produced on this line will have a life of:

- a** less than 10 hours
- b** 40 or more hours
- c** less than 20 hours
- d** 20 or more hours.

Battery test results

Battery life (h)	Frequency
<10	2
10–19	22
20–29	59
30–39	16
40–49	1

- 9** The table of life expectancy shown on the right is similar to those used by insurance companies. It is the result of the collection of statistics on 100 000 males and 100 000 females.

- a**
 - i** In this survey, how many of the original 100 000 males were still alive at age 15?
 - ii** What is the relative frequency of ‘males surviving to 15 years of age’?
 - iii** On the basis of this data, estimate the probability of a male baby born today being alive in 15 years time.
- b** Using the above procedure, estimate the probability that a female baby will survive to 15 years of age.
- c** Estimate the probabilities of a male and a female living to age:
 - i** 40
 - ii** 60
 - iii** 80
- d**
 - i** How many males were still alive at 50 years of age?
 - ii** How many males were still alive at 75 years of age?
 - iii** What percentage of males who survived to 50 were still alive at 75?
 - iv** Estimate the probability of a 50-year-old male living to 75.
- e** Repeat part **d** for females.

Life expectancy

Age (years)	Number surviving	
	Male	Female
0	100 000	100 000
5	99 378	99 482
10	99 322	99 438
15	99 260	99 393
20	99 008	99 272
25	98 626	99 130
30	98 194	98 965
35	97 697	98 751
40	97 096	98 443
45	96 292	97 971
50	95 141	97 251
55	93 447	96 191
60	90 925	94 618
65	86 931	92 152
70	80 738	88 398
75	71 589	82 305
80	57 770	72 308
85	39 651	56 373
90	20 529	34 635
95	7 113	14 319
100	1 649	3 634

- 10** In a survey of 4000 marriages it was found that 1356 ended in divorce. Phil and Jenny are getting married next weekend. On the basis of the evidence from the survey, what is the approximate probability that their marriage will end in divorce?



Questions **11–13** involve performing a number of trials of different experiments to calculate the relative frequency or experimental probability of an event.

- 11**
- a** Drop a drawing pin 100 times and complete the table to record whether it lands point up or point down.
 - b** How many times did the drawing pin land point up? How many times did it land point down?
 - c** Calculate the experimental probability of each outcome.
 - d** Based on your results, are the outcomes equally likely events? Explain.
 - e** Combine the results for the whole class. Compare the results.
 - f** Based on the class results, are the outcomes equally likely? Discuss this with the class.

Outcome	Tally	Frequency
Point up		
Point down		



- 12**
- a** Toss an empty matchbox 100 times and record the number of times it lands on its largest faces, a side face or an end face.
 - b** Calculate the experimental probability of each outcome.
 - c** Based on your results, are the outcomes equally likely events? Explain.
 - d** Enter the results into a spreadsheet and use the **Chart** option to make a graph of the results.
 - e** Combine the results for the whole class.
 - f** Enter the combined results into your spreadsheet and use the **Chart** option to compare the results.
 - g** Based on the class results, are the outcomes equally likely? Discuss your answer with the class.

Outcome	Tally	Frequency
Largest face		
Side face		
End face		



- 13** Count and record the number of boys and girls born on a particular day by examining the birth notices in a daily newspaper. On the basis of this information, are the births of a boy and a girl equally likely?

8C Probability

These resources are available on your obook assess:

- **Interactive 8C:** Compare theoretical and experimental probability
- **Worksheet 8C:** Practise your skills with extra problems for probability
- **assess quiz 8C:** Test your skills with an auto-correcting multiple-choice quiz

10
10

If all the possible outcomes are equally likely, then the theoretical probability of an event, E , happening is given by:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

The probability of an event may be expressed as a fraction, a decimal or a percentage.

!

EXAMPLE 8C-1 Calculating theoretical probability

When a six-sided die is rolled, what is the probability of getting:

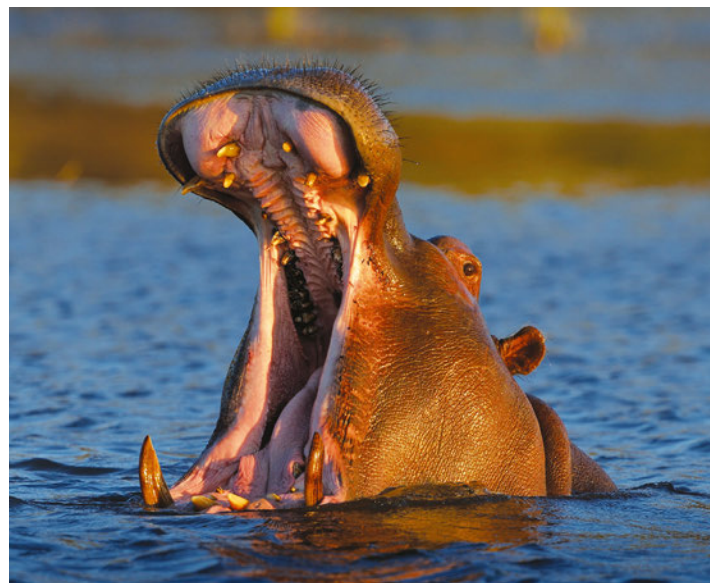
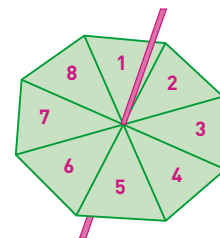
- a** a 2? **b** an even number?

	Solve	Think	Apply
a	There is only one favourable outcome for $P(2)$. $P(2) = \frac{1}{6}$	The sample space is $S = \{1, 2, 3, 4, 5, 6\}$ and all these outcomes are equally likely. So the total number of outcomes is 6.	Find the number of favourable outcomes and divide by the number in the sample space.
b	$P(\text{even number}) = \frac{3}{6} = \frac{1}{2}$ (or 0.5 or 50%).	There are three favourable outcomes for getting an even number: 2, 4 and 6.	$P(2)$ means the probability of getting a 2.

EXERCISE 8C Probability

- A six-sided die is rolled. What is the probability of getting:
 - a 5?
 - a 5 or a 6?
 - an odd number?
 - a 9?
 - a number greater than 4?
 - a number less than 7?
- In Erin's maths class there are 11 girls and 12 boys. If the teacher selects one student at random from the class, what is the probability that it is:
 - Erin?
 - a girl?
 - a boy?
 - a girl or a boy?
- A six-sided die has three faces painted black, two faces painted red and one face painted yellow. If the die is rolled, what is the probability that the uppermost face is:
 - black?
 - black or red?
 - red?
 - green?
 - yellow?
 - black or red or yellow

- 4** The spinner shown in the diagram is spun. What is the probability that it stops on:
- | | | |
|--------------------------------|-------------------------|-----------------------------------|
| a 6? | b 7? | c 6 or 7? |
| d an even number? | e an odd number? | f a number greater than 5? |
| g a number less than 3? | h 9? | i a number less than 9? |
- 5** A card is selected at random from a standard deck of playing cards. Determine:
- | | | |
|----------------------------------|-------------------------------|--|
| a $P(7 \text{ of clubs})$ | b $P(7)$ | c $P(\text{club})$ |
| d $P(\text{black } 7)$ | e $P(\text{red } 7)$ | f $P(\text{black})$ |
| g $P(\text{red})$ | h $P(\text{green } 7)$ | i $P(\text{either red or black})$ |
- 6** If there are 200 tickets in a raffle, what is the probability of winning first prize if you buy:
- | | | |
|----------------------|----------------------|-----------------------|
| a 1 ticket? | b 2 tickets? | c 5 tickets? |
| d 10 tickets? | e 20 tickets? | f 200 tickets? |
- 7** A letter is chosen at random from the word HIPPOPOTAMUS. Find:
- | | |
|----------------------------------|-------------------------------|
| a $P(P)$ | b $P(O)$ |
| c $P(T)$ | d $P(\text{a vowel})$ |
| e $P(\text{a consonant})$ | f $P(\text{not an A})$ |
| g $P(U \text{ or } S)$ | h $P(Q)$ |
- 8** A marble is selected at random from a bag containing four red, three blue and two green marbles. Find the probability that the marble selected is:
- | | |
|---------------------------|------------------------------|
| a red | b blue |
| c green | d red or blue |
| e not red | f not blue |
| g not red or green | h not red or blue |
| i yellow | j red, blue or green. |
- 9** Discs numbered 1 to 20 are placed in a hat and one is chosen at random. Find the probability that the number on the disc chosen is:
- | | |
|-----------------------------|------------------------------|
| a 7 | b not 7 |
| c even | d not even |
| e less than 10 | f not less than 10 |
| g divisible by 3 | h not divisible by 3 |
| i greater than 20 | j less than 20 |
| k a one-digit number | l a two-digit number. |



You will have discovered from the previous questions that:

If an event E is impossible, then: $P(E) = 0$

If an event E is certain, then: $P(E) = 1$

So, $0 \leq P(E) \leq 1$

- 10** For the experiments below, write events which have the following probabilities:
- i** $P(E) = 0$
 - ii** $P(E) = 1$
- a** A die is rolled.
 - b** A ball is chosen from a bag containing three red balls and two white balls.
 - c** A card is selected from a standard playing deck.
- 11** In a class discussion, Matthew gave the following answers to three questions on probability. What would you tell Matthew about his answers?
- a** 1.2
 - b** -0.6
 - c** 99%

Remember that:

Relative frequency or experimental probability gives an estimate of the probability that is based on performing trials of an experiment.

Theoretical probability is the expected probability when all the possible outcomes are equally likely and it can be calculated for an event, E , using the formula:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}.$$



- 12 a** Toss a coin 100 times and record the results in a table similar to the one shown on the right.
- | | | |
|------------------|---|---|
| Outcome | H | T |
| Frequency | | |
- b i** From your results, are the outcomes equally likely?
 - ii** Calculate the experimental probability of each outcome.
 - c i** Combine your results with those of the rest of the class by writing them on the board.
 - ii** Calculate the experimental probability of each outcome using the combined results.
 - d** Compare the experimental probability with the expected (theoretical) probability.
 - e** What do you think would happen if you tossed a coin 1 000 000 times?
 - f** Does the number of trials have an effect on the results? (Number of trials is the number of times the experiment is repeated.) Comment on your findings.
- 13 a** When a die is rolled, what is the theoretical probability of rolling a number less than 3?
- b** Roll a die 100 times and find the experimental probability of rolling a number less than 3.
 - c** Compare the theoretical and experimental results from parts **a** and **b**. Comment on your findings.
 - d** Combine your results from part **b** with those of the rest of the class by writing them on the board. Calculate the experimental probability using the combined results.
 - e** Compare the results from parts **a** and **d**. Comment on your findings.
- 14** A cup contains five red, three green and two blue cubes. The cup is shaken and one cube is selected.
- a** What is the theoretical probability that the cube is:
 - i** red?
 - ii** green?
 - iii** blue?
 - b** Perform this experiment 100 times and find the experimental probability of selecting a cube that is:
 - i** red
 - ii** green
 - iii** blue.
 - c** Compare the experimental results from part **b** with the theoretical results from part **a**. Comment on your findings.
 - d** Combine your results from part **b** with those of the rest of the class by writing them on the board. Calculate the experimental probability of each outcome using the results of the whole class.
 - e** Compare the results from part **d** with the results from part **a**. Comment on your findings.

- 15 a** If a card is selected at random from a standard deck, what is the theoretical probability that the card is:
- a heart?
 - a diamond?
 - a spade?
 - a club?
- b**
- Select a card from a standard deck and record its suit.
 - Replace the card and shuffle the pack.
 - Repeat this 50 times.
- c** Use your results from part **b** to determine the experimental probability that a card chosen at random is:
- a heart
 - a diamond
 - a spade
 - a club.
- d**
- Combine your results with those of the rest of the class by writing them on the board.
 - Repeat part **c** using the class results.
- e** Compare the experimental results with the theoretical results. Comment on your findings.
- 16** Consider your answers to questions **12–15**. Describe what happens to the relative frequency (experimental probability) when the number of trials of an experiment is increased. How does it compare to the expected (theoretical) probability?



- 17** The digits 0 to 9 are written on cards that are placed in a hat. One card is drawn at random from the hat. Write an event that has a probability of:
- $\frac{1}{10}$
 - $\frac{1}{2}$
 - 0
 - 1
 - $\frac{3}{10}$
 - $\frac{9}{10}$
 - $\frac{11}{10}$
 - $-\frac{7}{10}$
- 18** The following is a list of colloquial terms and expressions used to describe chance. Research and discuss the meaning of these terms.
- pigs might fly
 - once in a blue moon
 - as scarce as hens' teeth
 - a long shot
 - a sure thing
 - in the box seat
 - Buckley's chance
 - bank on it
 - put your house on it
 - a one-in-300-year flood
 - a snowflake's chance in hell



8D Complementary events

These resources are available on your obook assess:

- **Interactive 8D:** Explore key ideas for complementary events
- **Video tutorial 8D:** Watch and listen to an explanation of Example 8D-1
- **Worksheet 8D:** Practise your skills with extra problems for complementary events
- **assess quiz 8D:** Test your skills with an auto-correcting multiple-choice quiz

complement of E ($\tilde{E}$ or E')

remaining group of outcomes in the sample space that do not belong to the event E

complementary events

events are complementary if the sum of the number of outcomes in each event equals the total number of possible outcomes in the sample space

An event, E , is an outcome or collection of outcomes. It is a subgroup of the sample space. The remaining group of outcomes of the sample space that do not belong to E is called the **complement of E** . The complement of E is written $\tilde{E}$ or E' .

E and $\tilde{E}$ are called **complementary events**. From the definition it follows that:

$$\begin{aligned} \text{Number of outcomes in } E + \text{number of outcomes in } \tilde{E} \\ = \text{total number of possible outcomes.} \end{aligned}$$

EXAMPLE 8D-1 Identifying the complement of an event

- a** A six-sided die is rolled. List all the possible outcomes.
b What is the complement of the event:
- i** rolling a 6? **ii** rolling an even number?
 - iii** rolling a number less than 3? **iv** not rolling a 4?



	Solve	Think
a	$S = \{1, 2, 3, 4, 5, 6\}$	List each possibility.
b i	If $E = \{6\}$ then $\tilde{E} = \{1, 2, 3, 4, 5\}$.	The complement of 'rolling a 6' is 'rolling a 1, 2, 3, 4 or 5' or 'not rolling a 6'.
ii	If $E = \{2, 4, 6\}$ then $\tilde{E} = \{1, 3, 5\}$. The outcomes that do not result in an even number are 1, 3, 5.	The complement of 'rolling an even number' is 'rolling a 1, 3 or 5' or 'not rolling an even number' or 'rolling an odd number'.
iii	If $E = \{1, 2\}$ then $\tilde{E} = \{3, 4, 5, 6\}$. The outcomes that do not result in a number less than 3 are 3, 4, 5, 6.	The complement of 'rolling a number less than 3' is 'rolling a 3, 4, 5 or 6' or 'not rolling a number less than 3' or 'rolling a number greater than 2'.
iv	In this case $E = \{1, 2, 3, 5, 6\}$, so $\tilde{E} = \{4\}$.	The complement of 'not rolling a 4' is 'rolling a 4'.
Apply		
List all possible outcomes. Cross out the outcomes in the sample space that are in the event. The remaining outcomes are the complement of the event.		

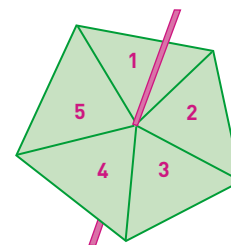
EXERCISE 8D Complementary events

- 1 **a** A coin is tossed. List the possible outcomes.
b Determine the complement of the event:
 - i** tossing a head
 - ii** tossing a tail.
- 2 **a** The five-sided spinner shown is spun. List all the possible outcomes.
b List the outcomes and write in words the complement of the event:
 - i** spinning a 3
 - ii** spinning an odd number
 - iii** spinning a number greater than 3
 - iv** not spinning a 5.
- 3 A box contains one red (R), one blue (B) and one green (G) counter. One counter is selected at random. Match the following events with their complement.

<i>Event:</i>	a R	b G	c R or B	d B or G	e not B	f R, B or G
<i>Complement:</i>	A G	B G or B	C not R, B or G	D R or B	E R	F B
- 4 A card is selected from a standard playing deck. Write the complement of each event below.

a selecting a diamond	b selecting a diamond or a heart
c selecting a diamond, a heart or a club	d selecting a red card
e selecting an ace	f not selecting a king
- 5 **a** List the sample space when two coins are tossed.
b Write the complement of each of these events.

i two heads	ii head, tail in that order	iii exactly one head
iv no heads	v at least one head	vi at least one tail



EXAMPLE 8D-2 Calculating probability of complementary events

A fair six-sided die is rolled. Find the probability of:

- a i** rolling a 6
- ii** not rolling a 6
- b i** rolling an even number
- ii** not rolling an even number
- c i** rolling a number less than 3
- ii** not rolling a number less than 3.

	Solve	Think
a i	$P(6) = \frac{1}{6}$	$S = \{1, 2, 3, 4, 5, 6\}$ If $E = \{6\}$ then $P(6) = \frac{1}{6}$ $\tilde{E} = \{1, 2, 3, 4, 5\}$ so $P(\text{not } 6) = \frac{5}{6}$
ii	$P(\text{not } 6) = \frac{5}{6}$	Note: $P(6) + P(\text{not } 6) = 1$ or: $P(\text{not } 6) = 1 - P(6)$ or: $P(6) = 1 - P(\text{not } 6)$
b i	$P(\text{even number}) = \frac{3}{6}$ $= \frac{1}{2}$	If $E = \{2, 4, 6\}$ then $P(E) = \frac{3}{6}$ $\tilde{E} = \{1, 3, 5\}$ so $P(\text{not even}) = \frac{3}{6}$
ii	$P(\text{not even}) = \frac{3}{6}$ $= \frac{1}{2}$	Note: $P(\text{even}) + P(\text{not even}) = 1$ or: $P(\text{not even}) = 1 - P(\text{even})$ or: $P(\text{even}) = 1 - P(\text{not even})$

	Solve	Think
c i	$P(\text{number less than 3}) = \frac{2}{6}$ $= \frac{1}{3}$	If $E = \{1, 2\}$ then $P(E) = \frac{2}{6}$
ii	$P(\text{number not less than 3}) = \frac{4}{6}$ $= \frac{2}{3}$	$\tilde{E} = \{3, 4, 5, 6\}$ so $P(\text{number not less than 3}) = \frac{4}{6}$ Note: $P(\text{number less than 3}) + P(\text{number not less than 3}) = 1$ or: $P(\text{number not less than 3}) = 1 - P(\text{number less than 3})$ or: $P(\text{number less than 3}) = 1 - P(\text{number not less than 3})$
Apply		
Determine the outcomes of the complementary event and find its probability.		

In general, for complementary events: $P(E) + P(\tilde{E}) = 1$

So: $P(E) = 1 - P(\tilde{E})$

and: $P(\tilde{E}) = 1 - P(E)$



The results described above are often used when it is inconvenient or time-consuming to count all the favourable outcomes for an event.

EXAMPLE 8D-3 Using the complementary events formula to calculate probability

a Calculate the probability of event E if $P(\tilde{E}) = 0.24$.

b Calculate the probability of the complement of event E if $P(E) = \frac{5}{6}$.

	Solve	Think	Apply
a	$P(E) = 1 - P(\tilde{E})$ $= 1 - 0.24$ $= 0.76$	Use $P(E) = 1 - P(\tilde{E})$.	The probability of an event plus the probability of the complement of the event must add to 1.
b	$P(\tilde{E}) = 1 - P(E)$ $= 1 - \frac{5}{6}$ $= \frac{1}{6}$	Use $P(\tilde{E}) = 1 - P(E)$.	

6 Calculate the probability of event E if:

a $P(\tilde{E}) = \frac{3}{4}$

b $P(\tilde{E}) = 0.3$

c $P(\tilde{E}) = 15\%$

7 Calculate the probability of the complement of event E if:

a $P(E) = \frac{7}{8}$

b $P(E) = 0.67$

c $P(E) = 48\%$

EXAMPLE 8D-4 Deciding whether to consider an event or its complement to calculate probability

The numbers 1 to 200 are written on cards. The cards are shuffled and one card is selected at random. Determine the probability that the number on the card is:

- a** 93 **b** not 93 **c** divisible by 10 **d** not divisible by 10.

	Solve	Think	Apply
a	$P(93) = \frac{1}{200}$	1 outcome out of 200.	Read the question and decide whether the event or its complement would be easier to count. If the event is easier, use $P(E)$. If the complement is easier, use $P(E) = 1 - P(\tilde{E})$.
b	$P(\text{not } 93) = 1 - P(93)$ $= 1 - \frac{1}{200}$ $= \frac{199}{200}$	Use $P(\tilde{E}) = 1 - P(E)$	
c	$P(\text{divisible by } 10) = \frac{20}{200}$ $= \frac{1}{10}$	20 numbers are divisible by 10.	
d	$P(\text{not divisible by } 10)$ $= 1 - P(\text{divisible by } 10)$ $= 1 - \frac{1}{10}$ $= \frac{9}{10}$	Use $P(E) = 1 - P(\tilde{E})$.	

- 8** The numbers 1 to 100 are written on cards. The deck of cards is shuffled and one card is selected at random. Determine the probability that the card is:

- a** 37 **b** not 37 **c** divisible by 10
d not divisible by 10 **e** a one-digit number **f** not a one-digit number
g a multiple of 12 **h** not a multiple of 12 **i** a number with last digit 9
j a number with last digit not 9 **k** less than 5 **l** not less than 5
m greater than 93 **n** not greater than 93.

- 9** A card is selected from a standard playing deck. Find the probability that it is:

- a** not the 6 of clubs **b** not a black 6 **c** not the ace or king of diamonds
d not a 6 **e** not an ace or king **f** not a heart.

- 10** If three coins are flipped and H represents flipping a head, $P(3H) = \frac{1}{8}$, $P(\text{exactly } 2H) = \frac{3}{8}$ and $P(\text{exactly } 1H) = \frac{3}{8}$.

- a** Write the probability of:

- i** no heads **ii** no tails **iii** exactly one tail
iv exactly two tails **v** three tails.

- b i** What is the complement of the event 'at least one tail'?

- ii** Find the probability of at least one tail.

- c i** What is the complement of the event 'at least one head'?

- ii** Find the probability of at least one head.

- 11** The numbers from 1 to 100 are written on cards and one card is selected at random.

What is the probability that the card selected is:

- a** divisible by 12? **b** not divisible by 12? **c** divisible by 18?
d not divisible by 18? **e** divisible by both 12 and 18? **f** not divisible by both 12 and 18?
g divisible by 12 or 18? **h** not divisible by either 12 or 18?

8E Probability for multistage events

These resources are available on your obook assess:

- **Interactive 8E:** Explore key ideas for tree diagrams
- **Worksheet 8E:** Practise your skills with extra problems using tree diagrams
- **Investigation 8E:** Display multistage experiments using tree diagrams
- **assess quiz 8E:** Test your skills with an auto-correcting multiple-choice quiz

Multistage experiments are those that are comprised of two or more simple experiments. For example, tossing a coin with rolling a die. Finding probabilities of multistage events is made easier if we use a table, tree diagram or lattice diagram to show all the possible outcomes.

EXAMPLE 8E-1 Calculating the probability of two-stage events

- a** A shop sells jeans in three colours (blue, grey and white) with either front or side pockets. There is an equal number of each style on the shelves. Use a table to find the sample space.
- b** Find the probability that a pair of jeans chosen at random from the shelves will:
- be blue with front pockets
 - be grey with side pockets
 - be white
 - have side pockets.



Solve				Think
Colour	Pockets			There are six possible and equally likely outcomes.
		Front	Side	
	B	BF	BS	
	G	GF	GS	
	W	WF	WS	
$P(\text{BF}) = \frac{1}{6}$				There is only one favourable outcome: BF.
$P(\text{GS}) = \frac{1}{6}$				There is only one favourable outcome: GS.
$P(\text{white}) = \frac{2}{6} = \frac{1}{3}$				There are two favourable outcomes: WF, WS.
$P(\text{side pockets}) = \frac{3}{6} = \frac{1}{2}$				There are three favourable outcomes: BS, GS, WS.
Apply				
A table ensures that all possible outcomes can be listed.				

EXERCISE 8E Probability for multistage events

- 1
 - a Find all the possible ways of answering the first two questions on a True or False test.
 - b Determine the probability that:
 - i the first two answers are True
 - ii the first answer is True and the second is False
 - iii exactly one answer is True
 - iv both answers are False.

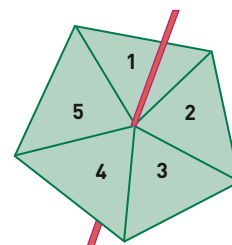
- 2
 - a A coin is tossed and a six-sided die is rolled. Find the sample space.
 - b Find the probability of getting:
 - i a head and 6
 - ii a tail and 3
 - iii a head and an even number
 - iv a tail and a number less than 3.

- 3
 - a The numbers 1 to 5 appear on this five-sided spinner, each with equal probability of occurring. The spinner is spun and a coin is tossed. List the sample space.
 - b Find the probability of getting:
 - i a head and a 5
 - ii a head and an even number
 - iii a tail and an odd number.

- 4
 - a Two four-sided dice (regular tetrahedra) with the numbers 1 to 4 on their faces are tossed. Use a table to find all the possible outcomes.
 - b Find the probability of tossing:
 - i double 4
 - ii any double
 - iii a 1 and a 4, in any order
 - iv two even numbers
 - v two odd numbers
 - vi a sum of 5.
 - c What is the probability that:
 - i exactly one of the numbers is a 4?
 - ii at least one of the numbers is a 4?

- 5
 - a Two fair six-sided dice are tossed. Use a table to find all the possible outcomes.
 - b Find the probability of tossing:
 - i double 5
 - ii any double
 - iii a 2 and a 3, in any order
 - iv a 1 and a 6, in any order
 - v two even numbers
 - vi two odd numbers
 - vii a sum of 5
 - viii a sum of 11.
 - c Which sum is the most likely to occur?
 - d What is the probability that:
 - i exactly one of the numbers is a 2?
 - ii at least one of the numbers is a 2?

- 6
 - a List the sample space for a family with two children, using G for a girl and B for a boy.
 - b Assuming that the chances of having a boy or a girl are the same, determine the probability of having:
 - i two boys
 - ii a boy first and then a girl
 - iii a girl first and then a boy
 - iv a boy and a girl in any order
 - v two girls.



EXAMPLE 8E-2 Calculating the probability of three-stage events

- a** Find all the possible ways in which you can answer the first three questions on a True or False test.
- b** Find the probability that:
- i** the first three answers are True
 - ii** the first two answers are True and the third is False
 - iii** two answers are True and one is False, in any order
 - iv** only one answer is True
 - v** all answers are False.

	Solve	Think
a	Possible outcomes: TTT, TTF, TFT, TFF, FTT, FTF, FFT, FFF.	A tree diagram shows there are eight possible and equally likely outcomes. <pre> graph LR Q1((1st question)) -- T --> Q2T((2nd question)) Q1 -- F --> Q2F((2nd question)) Q2T -- T --> Q3TT((3rd question)) Q2T -- F --> Q3TF((3rd question)) Q2F -- T --> Q3FT((3rd question)) Q2F -- F --> Q3FF((3rd question)) Q3TT --> O1[TTT] Q3TF --> O2[TTF] Q3FT --> O3[TFT] Q3FT --> O4[TFF] Q3FT --> O5[FTT] Q3FT --> O6[FTF] Q3FT --> O7[FFT] Q3FT --> O8[FFF] </pre>
b i	$P(\text{TTT}) = \frac{1}{8}$.	There is only one favourable outcome: TTT.
ii	$P(\text{TTF}) = \frac{1}{8}$	There is only one favourable outcome: TTF.
iii	$P(\text{two True, one False}) = \frac{3}{8}$	There are three favourable outcomes: TTF, TFT, FTT.
iv	$P(1 \text{ True}) = \frac{3}{8}$	There are three favourable outcomes: TFF, FTF, FFT.
v	$P(\text{FFF}) = \frac{1}{8}$	There is only one favourable outcome: FFF.
Apply		
Each branching of the tree must contain all possible outcomes each time. The number of branches depends on the number of times the event is repeated.		

- 7 a** List all the possible outcomes for the genders of three children in a family.
- b** Calculate the probability of having:
- i** three girls
 - ii** two girls and one boy, in that order
 - iii** two girls and one boy, in any order
 - iv** exactly one girl
 - v** no girls
 - vi** at least one boy
 - vii** at least two boys
 - viii** three boys.



- 8 a** The menu at Joe's Cafe is shown below. List all the possible three-course meals.
b Mike decides to choose a three-course meal at random.

What is the probability that he chooses:

- i** pasta, steak and fruit salad?
- ii** BBQ prawns, fish and apple cake?
- iii** a meal including pasta and chicken?
- iv** a meal including steak?
- v** a meal not including steak?
- vi** a meal not including BBQ prawns?
- vii** a meal including pasta and apple cake?

<i>Entree</i>
Pasta or BBQ Prawns
<i>Main Course</i>
Steak or Fish or Chicken
<i>Sweets</i>
Apple Cake or Fruit Salad



- 9 a** A bag contains one red, one blue and one white marble. One marble is chosen at random from the bag, and then replaced into the bag. A second marble is chosen. Find the sample space.
b Find the probability that:
- i** both marbles are red
 - ii** both marbles are the same colour
 - iii** both marbles are different colours
 - iv** the first marble is red
 - v** there is one blue and one white marble
 - vi** exactly one marble is red
 - vii** at least one marble is red
 - viii** no marble are blue.
- 10** Repeat question 9 but, this time, the first marble is not replaced in the bag before the second marble is chosen.
- 11** A standard deck of playing cards is shuffled and one card is drawn from the deck. The suit of the card is noted and the card is replaced in the deck. This procedure is repeated once more.
- a** Find all the possible outcomes.
b Find the probability that the two cards will be:
- i** both diamonds
 - ii** a diamond followed by a heart
 - iii** a diamond and a heart, in any order
 - iv** the same suit
 - v** different suits
 - vii** both black
 - vi** both red
 - viii** a red card followed by a black card
 - ix** a red and black card, in any order
 - x** a club followed by a red card.
- c** Find the probability that:
- i** exactly one of the cards is a spade
 - ii** at least one of the cards is a spade
 - iii** at least one of the cards is black
 - iv** neither of the cards is a heart.
- 12 a** List all the ways that Kylie, Marie and Helen can form a queue in the canteen line.
b Find the probability that Kylie will be:
- i** first in the queue
 - ii** second in the queue
 - iii** third in the queue
 - iv** in front of Helen
 - v** behind Marie
 - vi** in front of Helen and behind Marie
 - vii** not last in the queue.

8F Probability tree diagrams

These resources are available on your obook assess:

- **Video tutorial 8F:** Watch and listen to an explanation of Example 8F-3
- **Interactive 8F.1:** Explore key ideas for experiments with replacement
- **Interactive 8F.2:** Explore key ideas for experiments without replacement
- **Worksheet 8F:** Practise your skills with extra problems for probability tree diagrams
- **Investigation 8F:** Investigate selecting marbles without replacement
- **assess quiz 8F:** Test your skills with an auto-correcting multiple-choice quiz

Constructing a tree diagram can help us determine the outcomes for a simple multistage experiment and this can help us determine the probability of a multistage event.

EXAMPLE 8F-1 Constructing a tree diagram to determine the number of equally likely outcomes

A bag contains one red and one blue marble, and another bag contains one red, one blue and one green marble. One marble is drawn from each bag. Find the probability of getting two red marbles.



Solve			Think
1st bag	2nd bag	Outcomes	Draw a tree diagram using R for red marble, B for blue marble and G for green marble. From the tree diagram, there are six equally likely outcomes and one outcome favourable to this event.
		RR RB RG BR BB BG	
$P(RR) = \frac{1}{6}$			Apply $P(E) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$

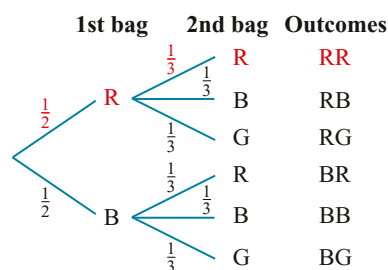
probability tree diagram

tree diagram on which the probabilities of simpler events are shown along its branches

Note: if we write the probabilities of each simpler event along the branches of the tree diagram, as shown, the diagram is called a **probability tree diagram**.

The probability of drawing a red marble from the first bag is $\frac{1}{2}$. The probability of drawing a red marble from the second bag is $\frac{1}{3}$.

So, the probability of drawing two red marbles is equal to the product of the probabilities along the branches leading to that two-stage event: $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$.



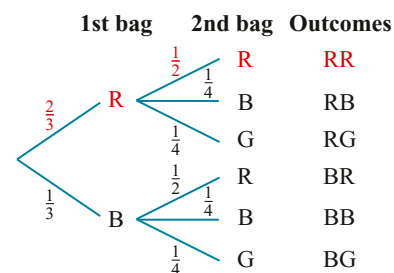
EXAMPLE 8F-2 Constructing a tree diagram to determine the number of equally-likely outcomes and the number of favourable outcomes

A bag contains two red and one blue marble, and another bag contains two red, one blue and one green marble. One marble is drawn from each bag. What is the probability of drawing two red marbles?

Solve	Think
1st bag 2nd bag Outcomes <pre> R / \ / \ / \ / \ R R / \ / \ / \ / \ / \ / \ B G RR RB \ / \ \ / \ \ / \ \ / \ R RG / \ / \ / \ / \ / \ / \ / \ R RR / \ / \ / \ / \ / \ / \ B G RB RG \ / \ \ / \ \ / \ \ / \ R BR / \ / \ / \ / \ / \ / \ / \ R BR / \ / \ / \ / \ / \ / \ B G BB BG \ / \ \ / \ \ / \ \ / \ R BG / \ / \ / \ / \ / \ / \ / \ R BG </pre> $P(RR) = \frac{4}{12} = \frac{1}{3}$	Draw a tree diagram using R for red marble, B for blue marble and G for green marble. From the tree diagram, the number of (equally likely) possible outcomes is 12 and the number of favourable outcomes is 4.
	Apply $P(E) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$

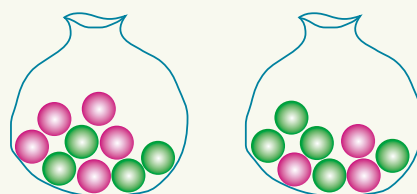
Note: draw a probability tree diagram showing only the simpler events at each stage, with the probability of each along its branches.

The probability of a red marble from the first bag is $\frac{2}{3}$ and the probability of a red marble from the second bag is $\frac{2}{4} = \frac{1}{2}$. So the probability of getting two red marbles is equal to the product of the probabilities along the branches leading to that event: $\frac{2}{3} \times \frac{1}{2} = \frac{2}{6}$ or $\frac{1}{3}$.



EXAMPLE 8F-3 Constructing a probability tree diagram to show outcomes that are not equally likely to occur

A bag contains five purple and four green marbles, and a second bag contains three purple and five green marbles. If a marble is selected at random from each bag, what is the probability of selecting two purple marbles?



Solve	Think	Apply
1st bag 2nd bag Event <pre> 5/9 / \ / \ / \ / \ P G / \ / \ / \ / \ / \ / \ G G PG GP \ / \ \ / \ \ / \ \ / \ P GG / \ / \ / \ / \ / \ / \ / \ P GG / \ / \ / \ / \ / \ / \ G G GG GG \ / \ \ / \ \ / \ \ / \ P GG / \ / \ / \ / \ / \ / \ / \ P GG </pre> $P(PP) = \frac{5}{9} \times \frac{3}{8} = \frac{5}{24}$	Draw a probability tree diagram showing each of the simpler events at each stage. Use P for purple marbles and G for green marbles. Find the product of the probabilities along the branches leading to PP (two purple marbles).	Draw a probability tree diagram and find the product of the probabilities along the branches leading to the required event.

EXERCISE 8F Probability tree diagrams

Remember that: To find the probability of a two-stage event, draw a probability tree diagram and find the product of the probabilities along the branches leading to that event.



- 1 A bag contains three red and four blue marbles, and a second bag contains 5 red and 4 blue marbles. A marble is selected from each bag.
 - a Complete this probability tree diagram for this two-stage event.
 - b What is the probability of getting two blue marbles?
- | | 1st bag | 2nd bag | Event |
|-------------------|---------|---------|-------|
| $\frac{3}{7}$
 | R | R | RR |
| | | B | RB |
| | B | R | BR |
| | | B | BB |
- 2 A hand of cards consists of two aces and two kings and a second hand consists of three aces and a queen. One card is selected at random from each hand.
 - a Draw a probability tree diagram for the selection of the card.
 - b What is the probability of selecting two aces?
 - 3 Four green discs and five yellow discs are placed in a box. A disc is selected and its colour is noted. The disc is replaced and a second disc is chosen.
 - a Draw a probability tree diagram for this two-stage event.
 - b Find the probability that:

i both discs are green	ii both discs are yellow
iii a green disc is followed by a yellow disc	iv a yellow disc is followed by a green disc.
 - 4 Class 11A is made up of 8 boys and 12 girls. Class 11B has 11 boys and 11 girls.
 - a If one student is chosen at random from class 11A, what is the probability of choosing:

i a boy?	ii a girl?
 - b If one student is chosen at random from class 11B, what is the probability of choosing:

i a boy?	ii a girl?
 - c One student is chosen at random from each class. Draw a probability tree diagram for the possible outcomes.
 - d What is the probability that the students chosen in part c are:

i both boys?	ii both girls?	iii a boy from class 11A and a girl from class 11B?
 - 5 When a child is born, the actual probability that it is a boy is 0.51 and the probability that it is a girl is 0.49.
 - a Draw a probability tree diagram for a family's two children.
 - b Calculate the probability of:

i two boys	ii a boy and a girl, in that order
iii a girl and a boy, in that order	iv two girls.
 - 6 a A coin is biased so that it comes up tails 60% of the time. When this coin is tossed, what is the probability of getting:

i a tail?	ii a head?

 - b Draw a probability tree diagram for two tosses of this coin.
 - c For two tosses of the coin, find the probability of getting:

i two heads	ii a head and a tail, in that order
iii a tail and a head, in that order	iv two tails.

EXAMPLE 8F-4 Calculating probabilities using a probability tree diagram

The probability that a chicken will survive the first 6 weeks after hatching is 0.7.
Find the probability that, of two chickens hatched:

- a both will survive
- b the first will survive but the second will die
- c both will die.



	Solve			Think	Apply
a	1st chicken	2nd chicken	Event	Draw a probability tree diagram showing each event. Use S for chicken survives and D for chicken dies. Multiply along the appropriate branches.	Draw a probability tree diagram and find the product of the probabilities along the branches leading to the required event.
	0.7 S	0.7 S	SS		
		0.3 D	SD		
	0.3 D	0.7 S	DS		
		0.3 D	DD		
	$P(SS) = 0.7 \times 0.7 = 0.49$				
b	$P(SD) = 0.7 \times 0.3 = 0.21$				
c	$P(DD) = 0.3 \times 0.3 = 0.09$				

- 7 The probability that a 60-year-old man will live to be 70 is 0.8 and the probability that a 60-year-old woman will live to be 70 is 0.9. For a 60-year-old married couple, find the probability that, in 10 years time:
- a both will be alive
 - b only the husband will be alive
 - c only the wife will be alive
 - d neither will be alive.
- 8 A student enters the 100 m and 400 m races at the school athletics carnival. His chance of winning the 100 m is 0.8 and his chance of winning the 400 m is 0.6. Find the probability that:
- a he wins both races
 - b he wins the 100 m but not the 400 m race
 - c he wins the 400 m but not the 100 m race
 - d he does not win either race.
- 9 Melanie and Tina are archers. The probability that Melanie hits the bullseye of the target is $\frac{3}{5}$ and the probability that Tina hits the bullseye is $\frac{3}{4}$. If both shoot at the target, what is the probability that:
- a both hit the bullseye?
 - b Melanie hits the bullseye but Tina misses?
 - c only Tina hits the bullseye?
 - d both archers miss the bullseye?
- 10 The probability that a particular ear infection will be cured when treated with an antibiotic is 0.9. If two patients with this ear infection are treated with this antibiotic, find the probability that:
- a both will be cured
 - b neither will be cured
 - c one will be cured but not the other
 - d at least one will be cured.
- 11 Ken plays Lew in a best of three set tennis match; that is, the first player to win two sets wins the match.
- a If each player has an equal chance of winning each set, what is the probability that:
 - i Lew wins in two sets
 - ii Lew wins in three sets
 - iii Lew loses in two sets
 - iv Lew loses in three sets?
 - b On past performances Lew has won 6 out of every 10 sets they have played against each other. On the basis of this information, calculate the probability that:
 - i Lew wins in two sets
 - ii Lew wins in three sets
 - iii Lew loses in two sets
 - iv Lew loses in three sets.

8G Expected frequency and simulations

These resources are available on your obook assess:

- **Interactive 8G.1:** Explore key ideas for expected frequency
- **Worksheet 8G.1:** Practise your skills with extra problems for expected frequency
- **Interactive 8G.2:** Explore key ideas for simulating an experiment
- **Video tutorial 8G:** Watch and listen to an explanation of Example 8G-3
- **Spreadsheet 8G.1:** Simulate the toss of a coin
- **Spreadsheet 8G.2:** Simulate the roll of a die
- **Worksheet 8G.2:** Practise your skills with extra problems for simulations
- **Investigation 8G:** Perform a simulation using a spreadsheet
- **assess quiz 8G:** Test your skills with an auto-correcting multiple-choice quiz

expected frequency (expected number)

expected number of times an event will occur, calculated by multiplying the probability of the event occurring by the number of trials

The probability of tossing a head with a single toss of a coin is $\frac{1}{2}$. So, if a coin is tossed 100 times, we would expect half of the tosses (50) to result in a head.

When a six-sided die is rolled, the probability of rolling a 4 is $\frac{1}{6}$. So, if a die is rolled 600 times, we would expect $\frac{1}{6}$ of the tosses (100) to result in a 4.

These are simple examples of what is called the **expected frequency** or **expected number** of an event. It is the *expected number of times* the event would occur. This idea is formalised as follows.

Expected frequency = probability of event occurring $\times$ number of trials

EXAMPLE 8G-1 Calculating expected frequency of an event using a given probability

The probability that a basketball player scores from a free throw is 0.75. If the player has 60 free throws, what is the expected frequency of scoring? (How many times would you expect the player to score?)



Solve/Think	Apply
Expected frequency = $0.75 \times 60 = 45$ The basketball player would be expected to score 45 times.	Expected frequency = probability of scoring $\times$ number of throws

EXERCISE 8G Expected frequency and simulations

- 1 Complete the following to find the number of times you would expect a tail to occur when a fair coin is tossed 400 times.
Expected number of tails
= probability of a tail $\times$ number of tosses
= $\underline{\hspace{1cm}}$ $\times$ $\underline{\hspace{1cm}}$
= $\underline{\hspace{1cm}}$
- 2 The probability that a male child is born is 0.51 and the probability of a female child being born is 0.49. If 300 babies are born in a hospital in one year, how many would you expect to be:
a male? **b** female?
- 3 The probability that a 50-year-old male lives to be 70 years of age is 0.73. For a group of six hundred 50-year-old males, what is the expected frequency of living to 70 years? (How many males would be expected to live to 70 years?)
- 4 The probability that a 50-year-old female is diagnosed as having cancer is 0.07. Of a group of five hundred 50-year-old females, how many would be expected to be diagnosed as having cancer?
- 5 The probability that a school-age child has defective eyesight is 0.08. In a school of 850 students, how many would you expect to have defective eyesight?
- 6 The probability that a worker in an abattoir has an accident in any year is 0.14. If an abattoir employs 150 workers, how many accidents could be expected in the next year?
- 7 In the game of poker, a hand of five cards is dealt to each player. The approximate probability of various hands is shown in the table below. If one million hands are dealt, what is the expected frequency of each of the hands in the table?



Hand	Probability
Royal flush	0.000 002
Four of a kind	0.000 240
Full house	0.001 441
Straight	0.003 925
Three of a kind	0.021 128
Two pairs	0.047 539
One pair	0.422 569



- 8 Fifty people are given the drug septamycin to cure an infection. This drug has a 90% success rate. How many of the 50 people can be expected to be treated successfully?

EXAMPLE 8G-2 Finding expected frequency after first calculating the probability of the event

- a** In a single roll of a six-sided die, what is the probability of getting:
- i** a 2?
 - ii** an even number?
 - iii** a number less than 5?
- b** If the die is rolled 300 times, what is the expected frequency of:
- i** a 2?
 - ii** an even number?
 - iii** a number less than 5?

	Solve	Apply
a i	The probability of a 2 in one roll of the die = $\frac{1}{6}$.	$P(E) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$
ii	The probability of an even number = $\frac{1}{2}$.	
iii	The probability of a number less than 5 = $\frac{4}{6}$ or $\frac{2}{3}$.	
b i	Expected frequency of a 2 = $\frac{1}{6} \times 300$ = 50 We would expect a 2 to occur 50 times.	Expected frequency = probability of event $\times$ number of trials
ii	Expected frequency of even numbers = $\frac{1}{2} \times 300$ = 150	
iii	Expected frequency of numbers less than 5 = $\frac{2}{3} \times 300$ = 200	

- 9 a** A six-sided die is rolled once. Find the probability of getting:

- i** a 5
- ii** an odd number
- iii** a number less than 3
- iv** a number greater than 2

- b** If the die is rolled 600 times, calculate the expected frequency of:

- i** a 5
- ii** an odd number
- iii** a number less than 3
- iv** a number greater than 2.

- 10 a** A bag contains three red, four blue and one green marble. If one marble is drawn at random from the bag, what is the probability that it is:

- i** red?
- ii** blue?
- iii** green?

- b** This process is performed 400 times, and the marble replaced after each draw. What is the expected frequency of:

- i** red marbles drawn?
- ii** blue marbles drawn?
- iii** green marbles drawn?



- 11 a** A card is selected at random from a standard playing deck. Find the probability of selecting:
- i** a black card
 - ii** a heart
 - iii** a king
 - iv** a red ace.
- b** If this process is performed 260 times and the card is replaced each time, find the expected number of:
- i** black cards selected
 - ii** hearts selected
 - iii** kings selected
 - iv** red aces selected.



- 12 a** If two coins are tossed simultaneously, what is the probability of tossing:
- i** 0 heads?
 - ii** exactly one head?
 - iii** exactly two heads?
- b** If two coins are tossed 100 times, what is the expected frequency of:
- i** 0 heads?
 - ii** exactly one head?
 - iii** exactly two heads?
- 13 a** Use a tree diagram to find the sample space for the genders of three children in a family.
- b** Assuming the chances of a boy or a girl are equally likely, find the probability of having:
- i** no girls
 - ii** exactly one girl
 - iii** exactly two girls
 - iv** three girls.
- c** In a survey of 200 families with three children, how many would be expected to have:
- i** no girls?
 - ii** exactly one girl?
 - iii** exactly two girls?
 - iv** three girls?

- 14 a** Write the theoretical probability of throwing a head when a coin is tossed.
- b**
- i** Calculate the expected frequency of heads when a coin is tossed 100 times.
 - ii** In pairs, toss a coin 100 times and record the results in a table.
 - iii** What is the difference between the expected number and the actual number of heads tossed?
 - iv** Use your results to calculate the experimental probability of throwing a head. (Out of 100 tosses, how many times did you toss a head?)
- c**
- i** Calculate the expected frequency of heads when a coin is tossed 1000 times.
 - ii** Combine the results of 10 pairs of your classmates and find the actual number of heads that occurred in 1000 tosses.
 - iii** What is the difference between the expected number and the actual number of heads tossed?
 - iv** Use the combined results to calculate the experimental probability of throwing a head.
- d** Use your answers to parts **b** and **c** to help you answer the following.
- i** As the number of trials increases, what happens to the difference between the expected number and the actual number of heads tossed?
 - ii** As the number of trials increases, what happens to the experimental probability compared with the theoretical probability of tossing a head?

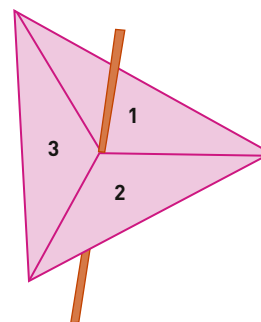
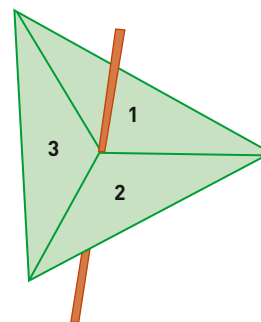


- 15 a** What is the theoretical probability of selecting a diamond when a card is selected from a standard 52-card deck?
- b**
- i** Calculate the expected frequency of diamonds when a card is selected from a deck 104 times.
 - ii** In pairs, select a card from a deck 104 times and record whether or not it is a diamond. List the results in a table.
 - iii** What is the difference between the expected number and the actual number of diamonds selected?
 - iv** Use your results to calculate the experimental probability of selecting a diamond.
- c**
- i** Calculate the expected frequency of diamonds when a card is selected from a deck 1040 times.
 - ii** Combine the results of 10 pairs of your classmates and find the actual number of diamonds that occurred in 1040 selections.
 - iii** What is the difference between the expected number and the actual number of diamonds selected?
 - iv** Use your results to calculate the experimental probability of selecting a diamond.
- d** Use your answers to parts **b** and **c** to help you answer the following.
- i** As the number of trials increases, what happens to the difference between the expected number and actual number of diamonds selected?
 - ii** As the number of trials increases, what happens to the experimental probability compared with the theoretical probability of selecting a diamond?

- 16 a** Two three-sided spinners, with the numbers 1, 2 and 3, each with an equal probability of occurring, are spun. Use a table similar to the one shown below to see the sample space.

		Second spinner		
		1	2	3
First spinner	1			
	2			
	3			

- b** Find the probability of spinning:
- i** double three
 - ii** any double
 - iii** a total of five.
- c** If the two spinners are spun simultaneously 450 times, what would be the expected number of times the following events would occur?
- i** double three
 - ii** any double
 - iii** a total of five



Simulations

A simulation is a simple mathematical model used to investigate a more complicated, time consuming or costly situation. Situations are modelled using the outcomes of tossing coins, rolling dice, selecting cards from a pack and using random numbers generated from tables, calculators and computers. Events are modelled so that simulated outcomes closely match real-world outcomes.

EXAMPLE 8G-3 Simulating an experiment using a table of random numbers

Based on past performances, a rugby player has a 70% chance of kicking a goal from 40 metres out from the posts.

- Simulate five kicks, using the table of random numbers below, and estimate the number of goals he is likely to score (out of the five attempts).
- Repeat the simulation to estimate the number of goals the player would kick from 50 attempts.
- Compare the results of the simulation in part **b** with the theoretical expected frequency of goals.

48047	45381	33232	35178	46971	85879	31458	22016
08695	58112	96070	91910	18868	52251	99827	32581
90070	98873	89846	50953	92529	68249	54949	83829
10132	27359	13017	41045	13817	65603	87615	55691
58547	01331	62538	79181	33071	63766	73613	24470
85566	81574	71965	20977	48005	83418	58738	98771
40942	42373	38710	39916	08187	00133	16288	64277
80553	58331	62724	74004	09344	91315	25791	40296
27328	85758	45342	98884	36034	79836	94902	80442
64584	20776	86792	42340	83522	62139	14038	88433

	Solve/Think	Apply
a	Let each digit in the table represent a kick at goal. Let the digits 0, 1, 2, 3, 4, 5, 6 signify 'a goal' and let the digits 7, 8, 9 signify 'a miss'; so seven of the ten possible digits in the table (70%) represent 'a goal' and three of the ten possible digits (30%) represent 'a miss'. Start at any digit in the table and read off five digits. Suppose we start at the 5 in row 2 and column 6. Reading to the right the digits selected are 5, 8, 1, 1, 2. This represents 4 goals and 1 miss in five attempts; meaning, in five attempts at kicking a goal, he would be successful four times.	Describe the possible outcomes ('a goal', or 'a miss'). Link each outcome to one or more random numbers (0, 1, 2, ..., 6 for 'a goal' and 7, 8, 9 for 'a miss'). Choose a suitable source of random numbers. (From a table or generated by a computer or calculator.) Choose a random number and note the simulated outcome. (5 is 'a goal', 8 is 'a miss', 1 is 'a goal', etc.) <i>Note:</i> we could have chosen any seven of the ten digits to represent a goal.
b	Starting at the same place, and selecting 50 digits, the digits selected are 5811296070 9191018868 5225199827 3258190070 9887389846 representing 28 goals and 22 misses.	
c	The expected number of goals scored out of 50 attempts is $0.7 \times 50 = 35$. This is higher than the results of the simulation.	

- 17** The chance that the goal shooter in a netball team will score a goal is 90%.
- Assign digits to represent 'a goal' and 'a miss'.
 - Using simulation, select 50 digits from the table of random numbers in Example 8G-3 and estimate how many goals she will score out of 50 attempts.
 - Compare the results of your simulation with the theoretical expected frequency.

- 18** Repeat question 17 by assigning a different set of digits to represent 'a goal' and 'a miss'.

- 19** A plant seedling has an 80% chance of surviving the first 6 weeks after planting. A nurseryman plants 100 seedlings.
- Using simulation, select 100 digits from the table of random numbers in Example 8G-3 and estimate the number of seedlings that will survive.
 - Compare the results of your simulation with the theoretical expected frequency.

- 20** Assume that the chance of a baby being male and the chance of the baby being female are both 0.5.
- Assigning the odd digits 1, 3, 5, 7, 9 to represent boys and the digits 0, 2, 4, 6, 8 to represent girls, use the table of random numbers in Example 8G-3 to estimate the number of boys and girls in a family with four children.
 - Repeat part i to produce results for 10 trials and record the results in a table similar to the one below.

Trial	1	2	3	4	5	6	7	8	9	10	
Number of boys											Average =
Number of girls											Average =

- Calculate the average number of boys and girls in these 10 simulated families with four children.
 - From your results, in a family of four children, what are the probabilities that there are 0, 1, 2, 3, 4 boys?
 - Put your average on the classroom board and calculate the average of all the class averages.
 - Combine the results in your table (number of boys) with the class. From the class data, in a family of four children, what are the probabilities that there are 0, 1, 2, 3, 4 boys?
 - What is the expected number of boys in a family with four children? Compare this with the results of parts iii and v. Discuss with the class.
 - The theoretical probabilities of 0, 1, 2, 3, 4 boys in a family of four are $\frac{1}{16}, \frac{1}{4}, \frac{3}{8}, \frac{1}{4}, \frac{1}{16}$ respectively. Compare the results of your simulations (parts iv and vi) with the theoretical probabilities. Discuss with the class.
- Assigning the outcome 'heads' to represent boys and 'tails' to represent girls, toss a coin four times (or toss four coins simultaneously) to estimate the number of boys and girls in a family with four children. Repeat parts ii to vi from part a.
 - Assigning the outcomes 1, 2, 3 to represent boys and 4, 5, 6 to represent girls, roll a die four times to estimate the number of boys and girls in a family with four children. Repeat parts ii to vi from part a.
 - Assigning the suits spades and clubs to represent boys, and hearts and diamonds to represent girls, shuffle a standard deck of 52 playing cards and select four cards from the deck to estimate the number of boys and girls in a family with four children. Repeat parts ii to vi from part a.



- 21** Using random numbers, a coin or a die to simulate the birth of a boy or a girl, investigate what would happen if couples stopped having children after the first girl was born, by:
- a** simulating the birth of boys and girls until a girl is born
 - b** recording the number of girls (obviously one) and number of boys in this family
 - c** repeating the simulation until you have the results for 20 families.
- Record the results in a table similar to the one below

Family	1	2	3	4	5	6	...	20	Total
Number of boys									
Number of girls	1	1	1	1	1	1	1	1	20
Number of children									

- d** Calculate the average number of children a family will have if they stop after having a girl.
- 22** As a class, research and discuss the factors that could complicate the simulation of real-world events.

- 23** For a large number of trials, an experiment can be simulated using random numbers generated by a spreadsheet function. The rolling of a die can be simulated by generating a list of random numbers from 1 to 6. The tossing of a coin can be simulated by generating a list of random numbers from 1 to 2 and assigning the number 1 to represent 'a head' and the number 2 to represent 'a tail' (or vice versa).

In Microsoft Excel, to produce a list of random numbers from a to b , type: $\text{=RAND()}*(b-a)+a$ into the formula bar.

For example, suppose we wish to simulate the tossing of a coin 100 times. We can generate a list of 100 random numbers from 1 to 2 by the following steps.

Step 1: Format the cells to 0 decimal places.

Step 2: Type $\text{=RAND()}*(2-1)+1$ into the formula bar.

Step 3: Highlight 100 cells and **fill down**.

Step 4: Count the number of heads (1s) and tails (2s).

Use the method shown above to simulate the experiments in questions **24** and **25** below.

- 24**
- a** Roll a die 100 times. Use tally marks to record the results.
 - b** Calculate the relative frequency of each outcome.
 - c** Do your results show that each number occurs the same number of times?
 - d** Enter the results into a spreadsheet and use the **Chart** option to make a graph of the results.
 - e** Combine the results for the whole class.
 - f** Enter the combined class results into your spreadsheet and use the **Chart** option to compare the results.
 - g** Are the outcomes equally likely? Discuss your answer with the class.
- 25**
- a** Make a spinner with four colours, each occupying $\frac{1}{4}$ of the area of the spinner. Spin the spinner 100 times to perform 100 trials of the experiment.
 - b** Record the results in a frequency table.
 - c** Enter the results into a spreadsheet and use the **Chart** option to make a graph of the results.
 - d** Calculate the relative frequency of obtaining each colour.
 - e** Is each colour equally likely? Explain your answer.

CHAPTER 8 REVIEW

RELATIVE FREQUENCY AND PROBABILITY

You should be able to:

- ✓ list the sample space of single and multistage experiments
- ✓ use tables and tree diagrams to list the sample space
- ✓ determine relative frequency or experimental probability
- ✓ calculate theoretical probability
- ✓ understand the relationship between experimental and theoretical probability
- ✓ find the probability of complementary events
- ✓ calculate the probability for multistage events
- ✓ use probability tree diagrams to calculate probability
- ✓ calculate the expected number of times an event will occur
- ✓ compare expected frequency of an event with experimental results
- ✓ solve problems involving simulations or trials of experiments.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

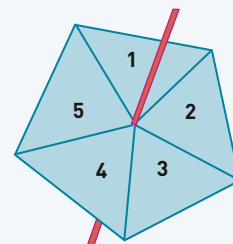
REVIEW MULTIPLE-CHOICE QUESTIONS

- 8A** 1 The number of possible ways in which you can answer the first three questions on a True or False test is:
A 3 **B** 6 **C** 8 **D** 16

- 8B** 2 The relative frequency of the score 3 shown in the table is:
A $\frac{3}{10}$ **B** 6
C 14 **D** $\frac{7}{10}$

Score	1	2	3	4	5
Frequency	3	5	6	2	4

- 8C** 3 The spinner shown on the right is spun once. The probability of getting an odd number is:
A $\frac{2}{5}$ **B** $\frac{3}{5}$
C $\frac{1}{5}$ **D** $\frac{4}{5}$



- 8C** 4 If an event, E , is certain to happen, then:
A $P(E) = 0$ **B** $P(E) = \frac{1}{2}$
C $P(E) = 1$ **D** $0 < P(E) < 1$

- 8D** 5 If E is the event 'a number less than 3' when a six-sided die is rolled, then the complement of the event, $\tilde{E}$, is:
A {1, 2, 3} **B** {3, 4, 5, 6} **C** {5, 6} **D** {1, 2, 3, 4}

- 8E** 6 Two three-sided spinners with the numbers 1, 2, and 3 on them are spun. If the probability of each number occurring is the same, then the probability of two 3s is:
A $\frac{1}{3}$ **B** $\frac{2}{9}$ **C** $\frac{2}{3}$ **D** $\frac{1}{9}$

- 8E** ▶ **7** A bag contains one red, one black and one white marble. A marble is selected from the bag and not replaced before a second marble is selected. The probability of selecting a red and a white marble in any order is:
- A $\frac{2}{9}$ B $\frac{1}{9}$ C $\frac{1}{3}$ D $\frac{1}{6}$
- 8E** ▶ **8** What is the probability of correctly guessing the first three answers to a multiple-choice test that has four alternative answers for each question?
- A $\frac{1}{3}$ B $\frac{3}{4}$ C $\frac{3}{16}$ D $\frac{1}{64}$
- 8F** ▶ **9** A bag contains three red and four blue marbles. A second bag contains two red and three blue marbles. If a marble is chosen from each bag, what is the probability that both are red?
- A $\frac{5}{12}$ B $\frac{3}{7}$ C $\frac{1}{2}$ D $\frac{6}{35}$
- 8G** ▶ **10** A card is selected from a standard deck 500 times and is replaced each time. The expected number of times an ace would be selected is:
- A 4 B 125 C 13 D 38 or 39

REVIEW SET 1

- 1** List the sample space for:
- a rolling a six-sided die b boys and girls in a two-child family.
- 2** A coffee tin was tossed 100 times and the way it landed was recorded. What is the experimental probability that this tin will land on one of its ends, when tossed?
- | Landed on | Frequency |
|-------------|-----------|
| End | 18 |
| Curved face | 82 |
- 3** A card is chosen from a standard playing deck. Determine the probability that the card is:
- a the jack of hearts b a jack c a heart
- d a picture card (not an ace) e black f a red queen.
- 4** Write an example of an event E for which:
- a $P(E) = 0$ b $P(E) = \frac{1}{2}$ c $P(E) = 1$
- 5** An eight-sided die has three faces painted black, two painted white, two painted yellow and one painted green. Find the probability that, when the die is rolled, the uppermost face is:
- a black b white c yellow d not green
- e black or white f not black, white or yellow g pink.
- 6** A coin is tossed and a six-sided die is rolled. Determine the probability of getting:
- a a head and a 3 b a head and an odd number
- c a tail and a 5 d a tail and a number less than 3.
- 7** A coin is tossed and a card is selected from a standard playing deck. What is the probability of getting:
- a a head and a diamond? b a head and a red card?
- c a tail and a club? d a tail and a black card?
- 8** A six-sided die is rolled 300 times. How many times would you expect to roll:
- a a 6? b an even number? c a number less than 5?
- 9** A football player has a 90% chance of kicking a goal from in front of the posts. If this footballer has 50 shots at goal, how many times would he be expected to score a goal?

REVIEW SET 2

- Percy, Ming and Yuki form a queue. List the sample space for determining their order in the queue.
- One hundred batteries were selected at random from a production line and tested. The results are shown in the table. Determine the approximate probability that a battery selected at random from those produced on this line will have a life of:

Battery life (h)	Frequency
<10	3
10–19	25
20–29	61
30–39	10
40–49	1

 - less than 10 hours
 - 40 or more hours
 - less than 20 hours
 - 20 or more hours.
- Of the 24 students in a class, six watch the evening news. Find the probability that a student chosen at random watches the news.
- A spinner is shaped like a regular octagon with the numbers 1 to 8 on it. Each number has an equal chance of being spun. If the spinner is spun, find the probability of getting:
 - a 5
 - a 7
 - an odd number
 - a 9
 - a number less than 9
 - a number divisible by 3.
- If $P(E) = 35\%$, find the probability of the complementary event occurring.
- One card is selected at random from a standard playing deck. What is the probability that the card is:
 - a diamond
 - not a diamond
 - a black jack
 - not a black jack
 - not a black card
 - black or red
- Two fair six-sided dice are tossed.
 - List the sample space in a table.
 - Find the probability of getting:
 - double 6
 - any double
 - a total of 7
 - a total of 13.
- A manufacturer randomly tested 150 light globes from the week's production and found that six were faulty.
 - Estimate the probability that a light globe produced that week is faulty.
 - If 2700 globes were produced that week, how many would be expected to be faulty?
- A plant seedling has an 85% chance of surviving the first 6 weeks after planting. If 400 seedlings are planted, how many would be expected to survive the first 6 weeks?

REVIEW SET 3

- A jar contains one red, one blue and one green disc.
 - A disc is selected, its colour noted and the disc replaced. A second disc is selected. List the sample space.
 - List the sample space if the first disc is not replaced before the second disc is selected.
- The results of a survey of the number of children in each of 100 families in a certain town is shown in the table. What is the probability that a family chosen at random from this town will have:
 - no children?
 - two children?
 - more than six children?

Number of children	0	1	2	3	4	5	6
Number of families	11	24	34	19	8	3	1

- 8** A sharemarket investor predicts that, in any week, the probability of a particular share price rising is 0.3, the share price remaining steady is 0.5, and the share price falling is 0.2.
- a** Complete this probability tree diagram to show all the possible outcomes for a two-week period.
- b** What is the probability that, in the two-week period, these shares will
- i** rise both weeks?
 - ii** remain steady both weeks?
 - iii** fall both weeks?
 - iv** rise for at least one of the weeks?

	1st week	2nd week	Outcomes
	R	R	—
		S	—
		F	—
	S	—	—
		—	—
		—	—
	F	—	—
		—	—
		—	—

REVIEW SET 4

- 1** A letter is chosen at random from the word WOOLLOOMOOLOO. Find the probability that the letter is:
a W **b** O **c** L **d** M **e** P
- 2** Write the value of $P(E) + P(\bar{E})$.
- 3** A bag contains three red, four blue and five green marbles. If a marble is chosen at random, find the probability that it is:
a red **b** blue **c** green **d** red or blue **e** not blue.
- 4** A deck of cards consists of 200 cards numbered from 1 to 200. The deck is shuffled and a card is selected at random. Find the probability that it is:
a 53 **b** not 53
c divisible by 100 **d** not divisible by 100
e a one-digit number **f** not a one-digit number
g less than 56 **h** not less than 56
i not less than 5 **j** not greater than 150.
- 5** A multiple-choice test has two questions, each with four alternative answers A, B, C and D.
a Use a tree diagram to find all the possible ways of answering these two questions.
b Find the probability that:
i the answers are A then B **ii** the answers are B then A
iii the answers are both A **iv** neither answer is A
v at least one answer is A.
- 6** A card is selected from a standard deck, its suit is noted and the card is replaced in the deck. A second card is then selected and its suit is also noted.
a List the sample space.
b Find the probability that:
i both cards are hearts **ii** one card is a heart and one is a diamond
iii both cards are black **iv** one card is black and one is red.
- 7** Two fair six-sided dice are rolled.
a How many different outcomes are possible?
b Use a table to list the sample space.
c Find the probability of:
i a double 3 **ii** any double
iii at least one 6 **v** two numbers less than 4
iv one number that is double the other **vi** two even numbers.
d Consider the total of the numbers on the two dice. Find the probability that the total is:
i 3 **ii** 7 **iii** 12 **iv** 9 or 10 **v** 1
e Consider the difference between the two numbers on the dice. Find the probability that the difference is:
i 0 **ii** 1 **iii** 5 **iv** 3 or 4 **v** 6
- 8** The probability that a sales representative will have a car accident in a year is 0.08. How many car accidents would a company employing 25 sales representatives expect its sales team to have in a year?

- 1 a** A bag contains a red, a blue and a green marble. A marble is chosen at random from the bag and its colour is noted. A second marble is then chosen and its colour is also noted. Find the sample space if the first marble is:
- i** replaced before the second marble is chosen (1 mark)
 - ii** not replaced before the second marble is chosen. (1 mark)

- b** Data for the past 50 years, giving the number of days in the month of October on which it rained, is shown in the table. Estimate the probability that next October it will rain on:
- i** more than 8 days (1 mark)
 - ii** 5 or fewer days. (1 mark)

Number of days	Frequency
0–2	16
3–5	17
6–8	9
9–11	7
12–14	1

- c** If $P(E) = 41\%$, what is the probability that the complementary event occurs? (1 mark)
- d** A deck of cards contains 200 cards numbered 1 to 200. One card is selected at random.
- i** What is the probability it is 67? (1 mark)
 - ii** What is the probability it is not 67? (1 mark)
 - iii** What is the probability it is not divisible by 5? (1 mark)
- e** Two four-sided spinners, with the numbers 1, 2, 3 and 4, each with an equal probability of occurring, are spun.
- i** Find the probability of spinning any double (2 marks)
 - ii** If the two spinners are spun simultaneously 450 times, what would be the expected number of times any double would occur? (1 mark)
- f** For a family with two children the probability of a boy is 0.55 and the probability of a girl is 0.45.
- i** Draw a probability tree diagram to represent this information. (1 mark)
 - ii** What is the probability of having a boy and a girl in any order? (2 marks)
- g** The chance that the colour of a car passing the front of the school during the middle of the day is silver is 30%. In a simulation, the digits 3, 5 and 7 are assigned to represent a silver coloured car and 0, 1, 2, 4, 6, 8 and 9 to represent a colour other than silver. Using the set of random digits below (produced by a computer), determine the number of silver cars in this sample of 20 cars. (1 mark)

10132 27359 91315 25791

TOTAL: 15 MARKS

9

Working with time

The main mathematical ideas investigated are:

- ▶ measuring latitude and longitude
- ▶ locating positions on Earth's surface
- ▶ using world time zones to determine standard time in different cities (including daylight saving time)
- ▶ using Coordinated Universal Time and the International Date Line
- ▶ calculating time differences around the world
- ▶ interpreting timetables.

ARE YOU READY?

- 9A ▶ 1** Which direction is opposite to east?
A north **B** south
C east **D** west
- 9A ▶ 2** Which direction is opposite to north?
A north **B** south
C east **D** west
- 9A ▶ 3** Which direction is 90° clockwise from west?
A north **B** south
C east **D** west
- 9C ▶ 4** How many hours in a day?
A 12 hours **B** 24 hours
C 60 hours **D** 365 hours
- 9C ▶ 5** How many minutes in an hour?
A 10 minutes **B** 24 minutes
C 60 minutes **D** 365 minutes
- 9C ▶ 6** How many months in 5 years?
A 12 months **B** 24 months
C 60 months **D** 365 months
- 9C ▶ 7** How many seconds in 4 minutes 25 seconds?
A 29 seconds **B** 65 seconds
C 240 seconds **D** 265 seconds
- 9C ▶ 8** What is 7:30 pm written in 24-hour time?
A 0730 **B** 1730
C 1930 **D** 7300
- 9C ▶ 9** What is the 24-hour time 1520 written in 12-hour time?
A 3:20 am **B** 3:20 pm
C 5:20 pm **D** 15:20 pm
- 9C ▶ 10** What is the sum of 54 seconds and 48 seconds written in minutes and seconds?
A 102 seconds
B 6 seconds
C 1 minute 2 seconds
D 1 minute 42 seconds
- 9C ▶ 11** What is the sum of 3 hours 47 minutes and 87 minutes written in hours and minutes?
A 3 hours 134 minutes
B 3 hours 34 minutes
C 5 hours 14 minutes
D 5 hours 4 minutes
- 9C ▶ 12** What is the difference between 5 hours 15 minutes and 2 hours 38 minutes?
A 3 hours 37 minutes
B 2 hours 37 minutes
C 3 hours 23 minutes
D 2 hours 23 minutes
- 9C ▶ 13** What is the time difference between 10:38 am and 12:00 pm?
A 1 hour 22 minutes
B 2 hours 22 minutes
C 1 hour 38 minutes
D 2 hours 38 minutes
- 9C ▶ 14** What is the time difference between 9:47 am and 3:12 pm?
A 6 hours 25 minutes
B 5 hours 25 minutes
C 6 hour 59 minutes
D 5 hours 59 minutes

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook assess](#).

Q1–3 Support sheet 9A.1 Understanding a directional compass

Q4–7 Support sheet 9C.1 Converting units of time

Q8–9 Support sheet 9C.2 Understanding 24-hour time

Q10–12 Support sheet 9C.3 Time calculations

Q13–14 Support sheet 9C.4 Differences in time

9A Latitude and longitude

great circle

circle formed when a slice is taken through the centre of Earth

small circle

circle formed when a slice is taken that does not pass through the centre of Earth

axis

the diameter about which Earth rotates with endpoints at the North Pole and the South Pole

parallels of latitude (circles of latitude)

circles formed when slices perpendicular to Earth's axis are taken

equator

great circle that is a parallel of latitude and whose centre is the centre of Earth

meridians of longitude

semicircles that are part of great circles with endpoints at the North Pole and South Pole

prime meridian (Greenwich Meridian)

reference line for longitude (0°) that passes through Greenwich in London, England

These resources are available on your obook assess:

- **Interactive 9A:** Explore key ideas for latitude and longitude
- **assess quiz 9A:** Test your skills with an auto-correcting multiple-choice quiz

o i o i

Earth is not exactly a sphere but, because the difference in shape is so small, it is considered to be for many navigational purposes. Earth's radius is approximately 6400 kilometres.

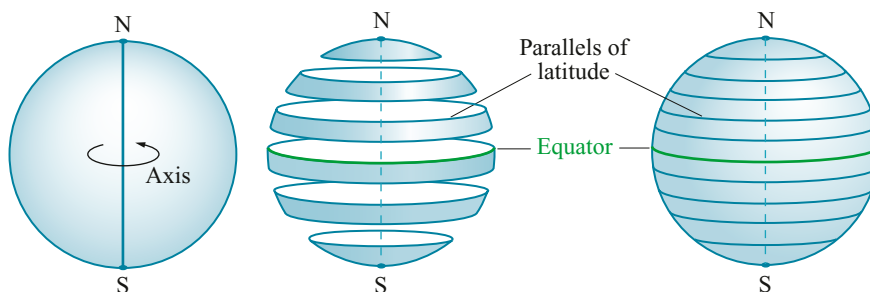
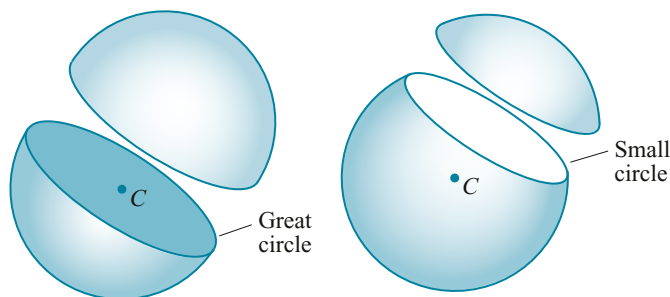
A **great circle** on Earth's surface is the circle formed when a slice is taken through the centre of Earth. The radius of a great circle is equal to the radius of Earth.

A **small circle** on Earth's surface is the circle formed when a slice is taken that does not pass through the centre of Earth. The radius of a small circle is less than the radius of Earth.

The diameter about which Earth rotates is called its **axis**. The endpoints of Earth's axis are the North Pole and the South Pole.

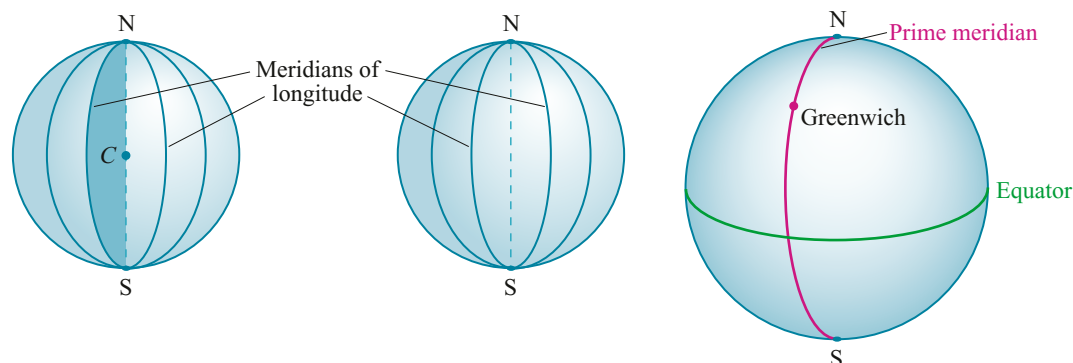
Slices perpendicular to Earth's axis cut the surface in small circles called **parallels of latitude** or **circles of latitude**.

The parallel of latitude whose centre is the centre of Earth is a great circle called the **equator**.



Great circles that pass through the North Pole and South Pole are cut into semicircles by the poles. These semicircles are called **meridians of longitude**.

The reference line for longitude is called the **prime meridian**. By international agreement, it was decided in 1884 that the prime meridian passes through Greenwich, now a suburb of London, England. Hence, it is often referred to as the **Greenwich Meridian**.



EXERCISE 9A Latitude and longitude

- 1 Use the diagram on the right to state whether the following are on parallels of latitude or meridians of longitude:

a CDI b NDS c AFG
 d DEF e SAB f BEH

- 2 For the diagram in question 1, state the direction from:

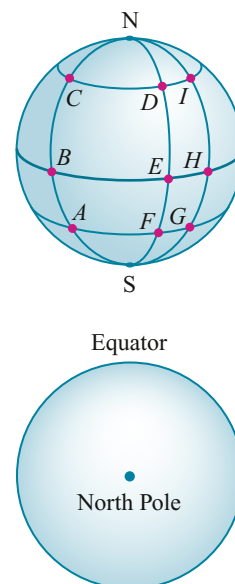
a D to E b A to F c H to B d B to C .

- 3 a Mark the North and South poles on a sphere and draw some parallels of latitude on it.

b Copy the diagram on the right and draw the view looking down on Earth from above the North Pole. Show some parallels of latitude.

- 4 a Mark the North and South poles on a sphere and draw some meridians of longitude on it.

b Copy the diagram on the right again and draw the view looking down on Earth from above the North Pole. Show some meridians of longitude.



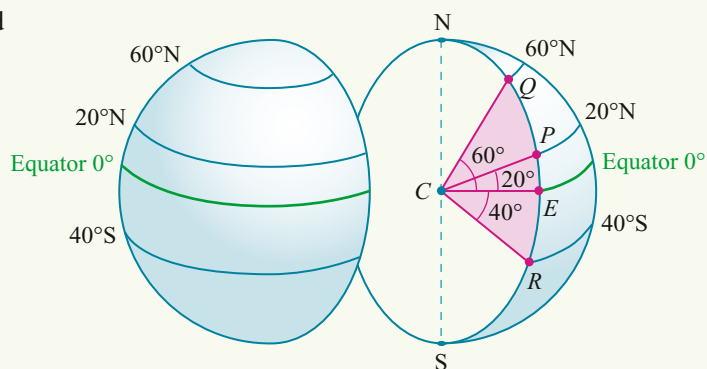
Parallels of latitude are labelled according to whether the circle is north or south of the equator and according to the size of the angle made at the centre of Earth. The reference line for measuring latitude is the equator. The latitude of the equator is 0° .

EXAMPLE 9A-1 Identifying points on parallels of latitude

This diagram shows parallels of latitude and angles from the equator.

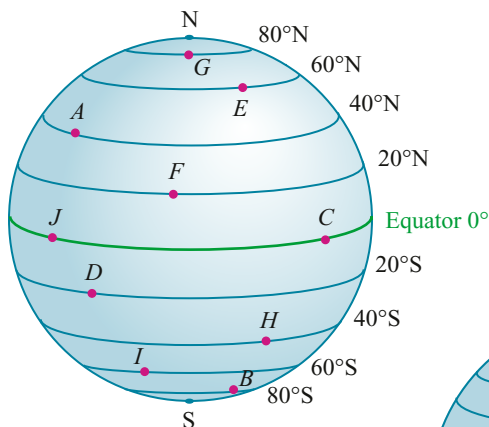
Write the latitude of each of the following points.

- a Q
 b R
 c P



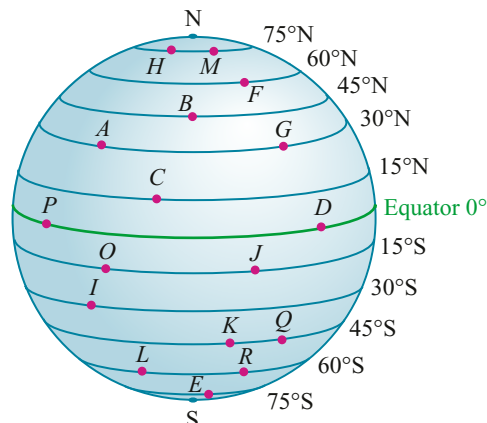
	Solve	Think	Apply
a	The point Q is on the 60°N parallel of latitude.	Angle $ECQ = 60^\circ$ and Q is north of the equator.	To find the parallel of latitude, first determine if the parallel of latitude is north or south of the equator. The angle made with the equator combined with 'North' or 'South' names the parallel of latitude.
b	The point R is on the 40°S parallel of latitude.	Angle $ECR = 40^\circ$ and R is south of the equator.	
c	The point P is on the 20°N parallel of latitude.	Angle $ECP = 20^\circ$ and P is north of the equator.	

- 5** Write the latitude of points A to J.



- 6** Use the diagram on the right to name the points that lie on the following parallels of latitude.

a 15°N	b 30°S	c 75°N
d 0°	e 45°N	f 60°S
g 30°N	h 15°S	i 45°S
j 60°N	k 75°S	



- 7** What is the latitude of:

- a** the North Pole?
b the South Pole?

- 8 a** Write the latitude of the circles through these points on the diagram.

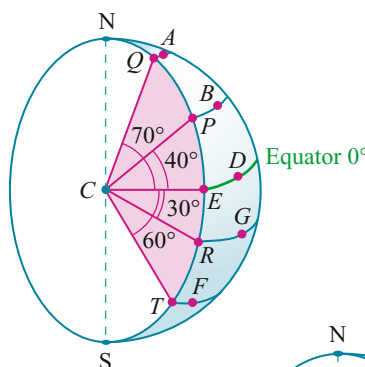
- i** P **ii** Q **iii** E

- iv** R **v** T

- b** Write the latitude of these points.

- i** A **ii** B **iii** D

- iv** G **v** F



- 9 a** Write the latitude of these points from the diagram on the right.

- i** H **ii** I **iii** J

- iv** E **v** K **vi** L

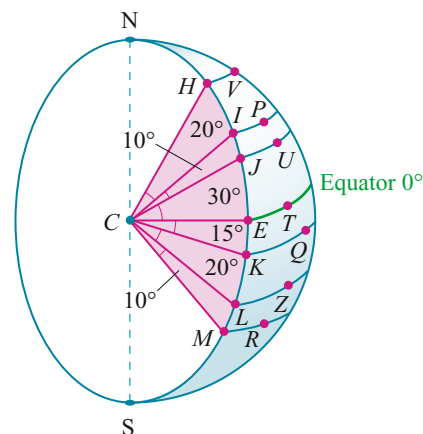
- vii** M

- b** Write the latitude of these points.

- i** P **ii** Q **iii** R

- iv** T **v** U **vi** V

- vii** Z



- 10 a** Use a globe or map of the world to find the latitude of:

- i** Sydney **ii** London, UK **iii** New York, USA

- iv** the Tropic of Capricorn **v** the Tropic of Cancer.

- b** Use digital technology to check your answers to part **a**.

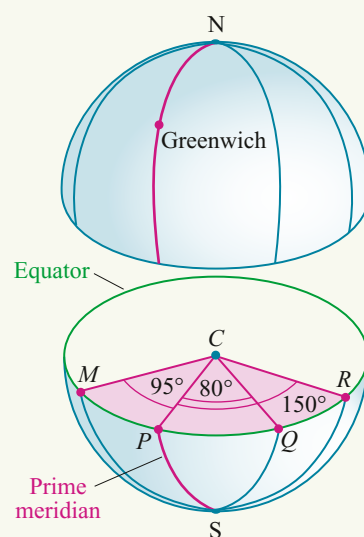
Meridians of longitude are labelled according to whether they are east or west of the prime meridian and according to the angle made at the centre of Earth. The longitude of the prime meridian is 0°.

EXAMPLE 9A-2 Identifying points on meridians of longitude

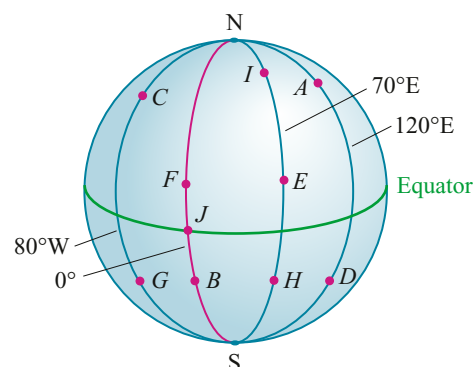
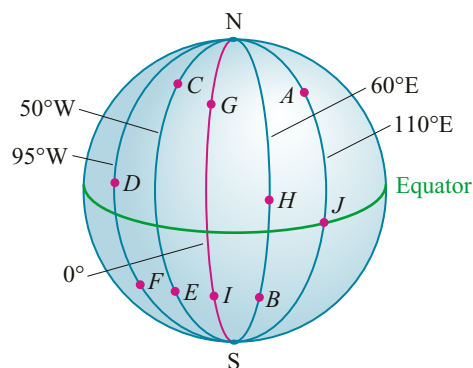
The diagram shows meridians of longitudes and angles from the prime meridian.

- a** On which meridian of longitude is the point Q ?
- b** On which meridian of longitude is the point R ?
- c** Which point lies on the 95°W meridian of longitude?

Solve	Think
a Q is 80°E .	Angle $PCQ = 80^\circ$ and Q is east of the prime meridian.
b R is 150°E .	Angle $PCR = 150^\circ$ and R is east of the prime meridian.
c The point M	Angle $PCM = 95^\circ$ and M is west of the prime meridian.
Apply	
To find the meridian of longitude, first determine if the meridian of longitude is east or west of the prime meridian. The angle made with the prime meridian combined with 'E' (for east) or 'W' (for west) names the meridian of longitude.	



- 11 a** Use the diagram below to help you write the longitude of the points A to J .
- b** Name the points that lie on the following meridians in the diagram below.
 - i** 80°W
 - ii** 70°E
 - iii** 120°E
 - iv** 0°



- 12 a** Write the longitude of the meridian passing through each of these points in Figure 1 below.

i T **ii** V **iii** W

- b** Write the longitude of these points.

i K **ii** L **iii** M **iv** N

- 13 a** Write the longitude of the meridian through these points in Figure 2 on the right.

i K **ii** L **iii** M

iv R **v** Q

- b** Write the longitude of these points.

i T **ii** U **iii** V

iv X **v** Y **vi** Z

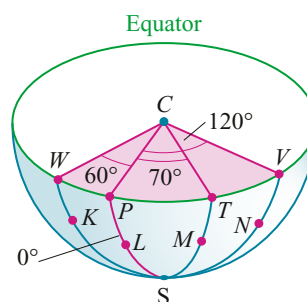


Figure 1

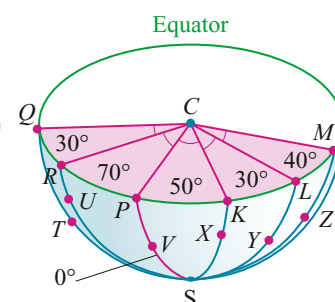


Figure 2

- 14 a** Use a globe or map of the world to find the longitude of:

i Sydney

ii London, UK

iii New York, USA

- b** Use digital technology to check your answers to part **a**.

9B Position on Earth's surface

These resources are available on your obook assess:

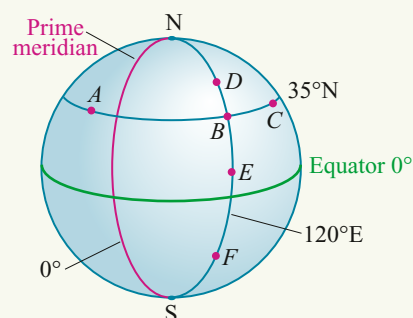
- **Video tutorial 9B:** Watch and listen to an explanation of Example 9B-2
- **Worksheet 9B:** Practise your skills with extra problems on coordinates
- **Investigation 9B.1:** Explore position coordinates to specify different locations
- **Investigation 9B.2:** Compare location coordinates using an atlas and Google Earth
- **assess quiz 9B:** Test your skills with an auto-correcting multiple-choice quiz

o i o i

The parallels of latitude and meridians of longitude form a grid on Earth's surface. Lines of latitude run east to west and meridians of longitude run north to south. The position of a point can be given by the ordered pair of coordinates (latitude, longitude).

EXAMPLE 9B-1 Writing position coordinates of a point using latitude and longitude

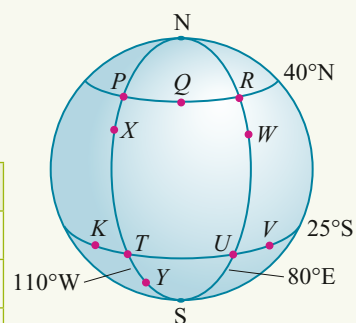
- a** List all the points that lie on:
- the 35°N parallel of latitude
 - the 120°E meridian of longitude
 - both the 35°N parallel of latitude and the 120°E meridian of longitude.
- b** Write the position coordinates of point B .



	Solve	Think	Apply
a i	A, B, C	The 35°N is a horizontal slice.	Latitudes are horizontal circles and longitudes are vertical circles. The intersection of the two circles gives the location.
ii	D, B, E, F	The 120°E meridian is a vertical slice.	
iii	B	This is the point where they intersect.	
b	$(35^\circ\text{N}, 120^\circ\text{E})$	Write latitude first, then longitude.	

EXAMPLE 9B-2 Finding position coordinates of points and naming points from position coordinates

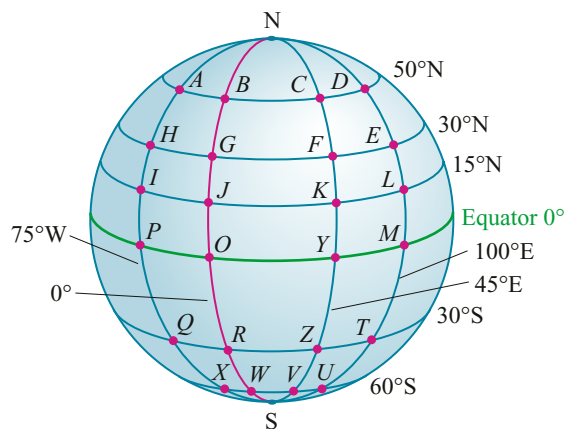
- a** What are the position coordinates of the following points?
- P
 - U
- b** Name the points with the following position coordinates.
- $(25^\circ\text{S}, 110^\circ\text{W})$
 - $(40^\circ\text{N}, 80^\circ\text{E})$



	Solve	Think
a i	$(40^\circ\text{N}, 110^\circ\text{W})$	Latitude 40°N , longitude 110°W
ii	$(25^\circ\text{S}, 80^\circ\text{E})$	Latitude 25°S , longitude 80°E
b i	T	Intersection of 25°S and 110°W
ii	R	Intersection of 40°N and 80°E
Apply		
Position is shown by latitude (N or S) first, then longitude (E or W).		

EXERCISE 9B Position on Earth's surface

Use this diagram to answer questions 1 to 3.



- 1 a Name the points that are on:
 - i the 50°N parallel of latitude
 - ii the 45°E meridian of longitude
 - iii both the 50°N parallel of latitude and the 45°E meridian of longitude.
- b Name the points that are on:
 - i the 30°S parallel of latitude
 - ii the 75°W meridian of longitude
 - iii both the 30°S parallel of latitude and the 75°W meridian of longitude.
- 2 Write the position coordinates of points A, E, G, I, K, M, O, Z, U, W.
- 3 Name the points with the following position coordinates.

a (50°N, 100°E)	b (15°N, 75°W)	c (60°S, 45°E)
d (30°S, 0° longitude)	e (15°N, 0° longitude)	f (30°N, 75°W)
g (30°S, 100°E)	h (60°S, 100°E)	i (0° latitude, 75°W)
j (30°N, 45°E)	k (60°S, 75°W)	l (0° latitude, 0° longitude)
m (15°N, 100°E)	n (50°N, 0° longitude)	o (50°N, 45°E)
- 4 On a globe or map of the world, find the country in which these lines of latitude and longitude meet.

a 30°S and 135°E	b 30°N and 105°E	c 75°S and 75°W
d 20°N and 100°W	e 0° latitude and 38°E	f 45°N and 0° longitude
- 5 a Point A has position (5°S, 130°W). State whether the following points are due north or due south of A.

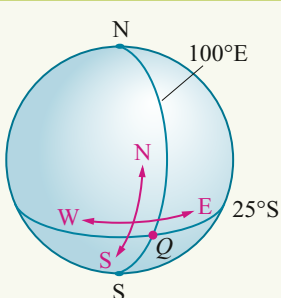
i (20°S, 130°W)	ii (0° latitude, 130°W)	iii (15°N, 130°W)
- b State whether the following points are due east or due west of A.

i (5°S, 150°W)	ii (5°S, 120°W)	iii (5°S, 0° longitude)

EXAMPLE 9B-3 Determining position coordinates of points relative to a given point

If Q is the point (25°S, 100°E), determine the position coordinates of the point that is:

- a 20° due north of Q b 20° due south of Q c 20° due east of Q d 20° due west of Q.

Solve	Think	Apply
a (5°S, 100°E)	Draw a diagram showing the parallel of latitude and meridian of longitude through the point Q. Move north 20° along the meridian to 5°S, 100°E. 	Movements north or south affect the latitude. Movements east or west affect the longitude.
b (45°S, 100°E)	Move south 20° along the meridian to 45°S, 100°E.	
c (25°S, 120°E)	Move east 20° along the parallel of latitude to 25°S, 120°E.	
d (25°S, 80°E)	Move west 20° along the parallel of latitude to 25°S, 80°E.	

- 6 a** If Q is the point (35°S , 95°E), determine the coordinates of the point that is:
- i** 10° due north of Q
 - ii** 10° due south of Q
 - iii** 10° due east of Q
 - iv** 10° due west of Q .

Repeat part **a** given these coordinates of point Q .

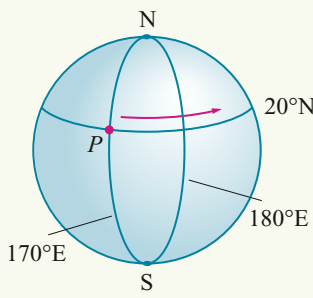
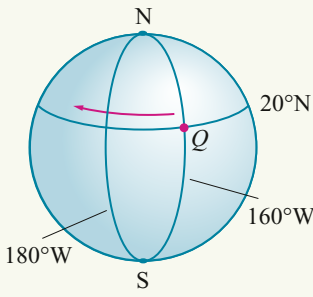
- b** (25°N , 150°E) **c** (20°S , 90°W) **d** (50°S , 40°W)

- 7** If P is the point (10°N , 75°W), find the coordinates of the point that is:
- a** 15° due north of P
 - b** 5° due south of P
 - c** 25° due south of P
 - d** 60° due south of P .

EXAMPLE 9B-4 Determining position coordinates of a point relative to a point close to the 180°E or 180°W meridian

Find the coordinates of the point that is:

- a** 30° due east of P (20°N , 170°E) **b** 30° due west of Q (20°N , 160°W).

Solve	Think
a (20°N , 160°W)	Draw a diagram showing the parallel of latitude and meridian of longitude through P. Indicate the 180°E meridian (the 180°E meridian and the 180°W meridian coincide). So the point that is 30° east of P is (20°N, 160°W). 
b (20°N , 170°E)	Draw a diagram showing the parallel of latitude and meridian of longitude through Q. Indicate the 180°W meridian. Note that the 180°W meridian and 180°E meridian coincide. So the point that is 30° west of Q is (20°N, 170°E). 

Apply

The 180°E and 180°W meridians are the same. When this meridian is crossed, the longitude counts backwards from 180° .

Note: $190^{\circ}\text{W} = 170^{\circ}\text{E}$

- 8** Find the coordinates of the point that is:
- a** 40° due east of (15°N , 155°E)
 - b** 20° due east of (32°S , 170°E)
 - c** 30° due west of (18°S , 170°W)
 - d** 50° due west of (34°N , 160°W)

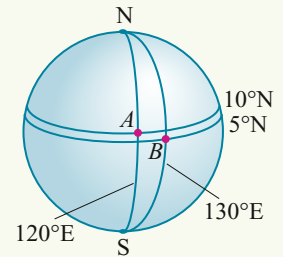
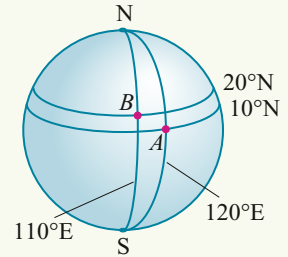
EXAMPLE 9B-5 Describing the position of a point relative to another point

Give the position of point B relative to point A .

a $A(10^\circ\text{N}, 120^\circ\text{E}), B(20^\circ\text{N}, 110^\circ\text{E})$

b $A(10^\circ\text{N}, 120^\circ\text{E}), B(5^\circ\text{N}, 130^\circ\text{E})$

Solve	Think
a B is 10° north of A and 10° west of A .	Using a sketch and comparing latitudes, B is 10° to the north of A . Comparing longitudes, B is 10° to the west of A .
b B is 5° south of A and 10° east of A .	Using a sketch and comparing latitudes, B is 5° to the south of A . Comparing longitudes, B is 10° to the east of A .
Apply Drawing a sketch will make finding the solution much easier. Be careful if crossing the equator or the $180^\circ\text{E}/180^\circ\text{W}$ meridian.	



9 Give the position of point B relative to point A .

a $A(25^\circ\text{N}, 110^\circ\text{E}), B(10^\circ\text{N}, 130^\circ\text{E})$

b $A(15^\circ\text{S}, 80^\circ\text{E}), B(20^\circ\text{S}, 70^\circ\text{E})$

c $A(30^\circ\text{S}, 75^\circ\text{W}), B(15^\circ\text{S}, 65^\circ\text{W})$

d $A(45^\circ\text{N}, 80^\circ\text{W}), B(30^\circ\text{N}, 120^\circ\text{W})$

e $A(10^\circ\text{S}, 120^\circ\text{E}), B(20^\circ\text{N}, 10^\circ\text{W})$

f $A(15^\circ\text{N}, 150^\circ\text{E}), B(25^\circ\text{S}, 150^\circ\text{W})$

10 On a globe or map of the world, find:

a two cities with the same latitude as Paris (France)

b two cities with the same longitude as Berlin (Germany)

c the city closest to:

i $(34^\circ\text{S}, 151^\circ\text{E})$

ii $(40^\circ\text{N}, 116^\circ\text{E})$

iii $(42^\circ\text{N}, 12^\circ\text{E})$

iv $(26^\circ\text{S}, 28^\circ\text{E})$

v $(55^\circ\text{N}, 37^\circ\text{E})$

vi $(34^\circ\text{N}, 118^\circ\text{W})$

d the position coordinates (to the nearest degree) of the following cities.

i Tokyo (Japan)

ii London (UK)

iii New York (USA)

iv Tehran (Iran)

v Rio de Janeiro (Brazil)

vi Vancouver (Canada)

11 Use a map of Australia with lines of latitude and longitude marked to answer the following questions.

a Name the city or town that is closest to:

i $(35^\circ\text{S}, 149^\circ\text{E})$

ii $(12^\circ\text{S}, 131^\circ\text{E})$

iii $(43^\circ\text{S}, 147^\circ\text{E})$

iv $(17^\circ\text{S}, 146^\circ\text{E})$

b Which is further south of the equator?

i Sydney or Perth

ii Rockhampton or Alice Springs

c Which is further east of the prime meridian?

i Sydney or Brisbane

ii Cairns or Hobart

12 Angelina is in the desert and heading to Cairo $(30^\circ\text{N}, 31^\circ\text{E})$. Her GPS coordinates place her at $(28^\circ\text{N}, 35^\circ\text{E})$. What is her current position relative to Cairo?

13 Klaus is sailing to Honolulu $(21^\circ\text{N}, 58^\circ\text{W})$. According to GPS navigation, his position is $(18^\circ\text{N}, 64^\circ\text{W})$. What is his position relative to Honolulu?

9C Time units and intervals

These resources are available on your [obook assess](#):

- **Interactive 9C.1:** Explore key ideas for 12-hour and 24-hour time
- **Interactive 9C.2:** Explore key ideas for elapsed time
- **Investigation 9C:** Investigate prefixes used with the time unit of seconds
- **assess quiz 9C:** Test your skills with an auto-correcting multiple-choice quiz

The measurement of time is important in our lives. The units of time and their conversions are listed below.

- 60 seconds = 1 minute
- 60 minutes = 1 hour
- 24 hours = 1 day
- 7 days = 1 week
- 2 weeks = 1 fortnight
- 52 weeks = 1 year (actually 52 weeks and 1 day)
- 12 months = 1 year
- 365 days = 1 year (366 for a leap year)
- 10 years = 1 decade
- 100 years = 1 century
- 1000 years = 1 millennium

EXAMPLE 9C-1 Converting units of time

Convert:

a 150 minutes to hours

c 110 hours to days and hours

b 143 minutes to hours and minutes

d 3 days 15 hours to hours.

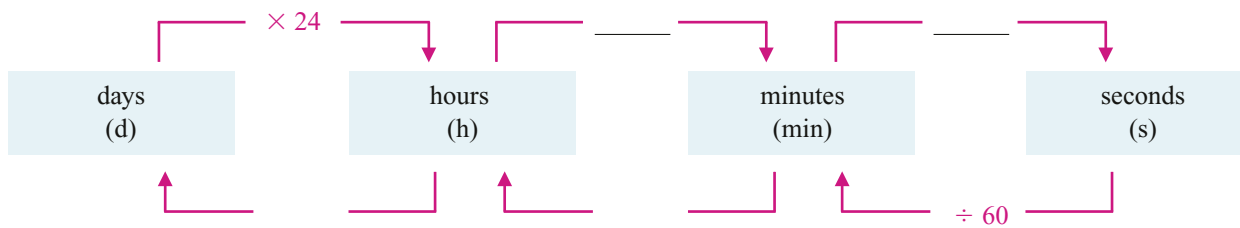
	Solve	Think
a	Number of hours = $150 \div 60$ $= 2.5$ Therefore 150 minutes is $2\frac{1}{2}$ hours.	There are 60 minutes in an hour. Divide by 60. When expressed in hours, the answer is a decimal.
b	Number of hours = $143 \div 60$ $= 2.3833\dots$ hours There are two whole hours in 143 minutes. Number of minutes remainder = $143 - 2 \times 60$ $= 23$ Therefore 143 minutes is 2 hours and 23 minutes.	Divide by 60. Then multiply the whole number of hours by 60, and subtract that from 143 to calculate the remainder. Or use the degrees, minutes and seconds button on your calculator.
c	Number of days = $110 \div 24$ $= 4.5833\dots$ (4 whole days) Number of hours = $110 - 4 \times 24$ $= 14$ Therefore 110 hours is 4 days 14 hours.	There are 24 hours in a day. Divide by 24. Calculate the number of hours remaining.
d	Number of hours = $3 \times 24 + 15$ $= 87$	Multiply 3 days by 24 hours to convert to hours and then add the extra 15 hours.

Apply

First, determine the conversion factor. Second, decide whether to multiply or divide. Third, find the remainder.

EXERCISE 9C Time units and intervals

- 1 Complete this time conversion diagram.



- 2 Convert:

a 150 minutes to hours	b 200 minutes to hours
c 300 seconds to minutes	d 193 minutes to hours and minutes
e 485 seconds to minutes	f 635 minutes to hours and minutes
g 4 hours 37 minutes to minutes	h 47 minutes 12 seconds to seconds
i 5 days 17 hours to hours	j 100 hours to days and hours
k 1 week to seconds	l 10000 seconds to hours, minutes and seconds

EXAMPLE 9C-2 Converting between 12-hour (analogue) and 24-hour time

- a** Write the 24-hour time 1020 as an analogue time.
b Write the 24-hour time 1324 as an analogue time.
c Convert 8:47 am to 24-hour time.
d Convert 6:58 pm to 24-hour time.

	Solve	Think	Apply
a	1020 is 10:20 am.	Because the first two digits are less than 12 this is 'am' time.	24-hour time always has four digits. If the time is before 10:00 am, a zero is put at the start so it does have four digits. If the 24-hour time is later than 1200, we subtract 12 from the hours and write the time as 'pm' time.
b	1324 is 1:24 pm.	The first two digits are greater than 12 so we subtract 12 hours from the 24-hour time and the resulting time is 'pm' time.	
c	8:47 am is 0847.	Time that is 'am' time has the same digits when expressed in 24-hour time and if the time has less than 10 hours then a zero is put at the start.	
d	6:58 pm is 1858.	Time that is 'pm' time has 12 added to the hours when converting to 24-hour time.	

- 3 Write these 24-hour times as analogue times (am or pm).

a 1406	b 0051	c 2005	d 0122
e 0925	f 2121	g 1915	h 1212
i 1254	j 1749	k 0622	l 2254

- 4 Write these analogue times in 24-hour notation.

a 7:43 am	b 5:20 pm	c 1:35 am	d 12:43 pm
e 4:20 pm	f 1:46 am	g 6:08 am	h 8:27 pm
i 9:20 am	j 11:08 pm	k 12:00 noon	l 12:00 midnight

EXAMPLE 9C-3 Performing calculations involving 24-hour time

- a** Calculate the time 2 hours 10 minutes after 0920. **b** Calculate the time 1 hour 12 minutes before 1528.
c Calculate the time 5 hours 26 minutes after 0850. **d** Calculate the time 6 hours 38 minutes before 1524.

	Solve	Think	Apply
a	0920 + 2 hours 10 minutes = 1130	Add the hours: $9 + 2 = 11$ Add the minutes: $20 + 10 = 30$	Either add or subtract the hours and minutes.
b	1528 – 1 hour 12 minutes = 1416	Subtract the hours: $15 - 1 = 14$ Subtract the minutes: $28 - 12 = 16$	If, when adding, the total minutes exceed 60, convert 60 minutes to 1 hour and add. If subtracting the minutes will not work because there are not enough minutes to subtract from, convert 1 hour to 60 minutes and subtract.
c	0850 + 5 hours 26 minutes = 1350 + 26 minutes = 1416	Add the hours: $8 + 5 = 13$ Add the minutes: $50 + 26 = 76$ The minutes exceed 60, so convert to hours and minutes. 76 minutes = 1 hour and 16 minutes	
d	1524 – 6 hour 38 minutes = 1424 + 60 min – 6 h 38 min = 1424 – 6 h + 60 min – 38 min = 0824 + 22 min = 0846	We can't subtract 38 from 24. Convert 1 hour to 60 minutes to make the time 1424 + 60 min. Then subtract the hours and minutes.	

5 Complete each of the following time calculations, giving the answer in 24-hour time.

- a** Add 2 hours 30 minutes to the 24-hour time 0600.
b Add 1 hour 15 minutes to the 24-hour time 1020.
c Subtract 1 hour 40 minutes from the 24-hour time 0916.
d Add 1 hour 48 minutes to the 24-hour time 1422.
e Add 2 hours 46 minutes to the 24-hour time 1509.
f Subtract 3 hours 51 minutes from the 24-hour time 1223.
g Subtract 3 hours 43 minutes from the 24-hour time 1605.
h Add 3 hours 46 minutes to the 24-hour time 0854.
i Subtract 4 hours 38 minutes from the 24-hour time 1127.

EXAMPLE 9C-4 Performing calculations involving time intervals

- a** Calculate 2 days 4 hours 55 minutes + 1 day 3 hours 31 minutes.
b Calculate 3 days 7 hours 25 minutes – 1 day 15 hours 12 minutes.

	Solve	Think	Apply
a	2 days 4 hours 55 minutes + 1 day 3 hours 31 minutes = 3 days 7 hours 86 minutes = 3 days 8 hours 26 minutes	Add each of the time units together. Since the number of minutes exceeds 60, convert 60 minutes to 1 hour. $86 \text{ min} = 1 \text{ h } 26 \text{ min}$	When adding the time units it might be necessary to convert the answer to a larger unit. When subtracting time units, a larger unit might need to be converted to a smaller unit for subtraction to be possible.
b	3 days 7 hours 25 minutes – 1 day 15 hours 12 minutes = 2 days 31 hours 25 minutes – 1 day 15 hours 12 minutes = 1 day 16 hours 13 minutes	Convert 1 day to 24 hours so the hours can be subtracted. 3 days 7 hours becomes 2 days + 24 hours + 7 hours or 2 days 31 hours.	

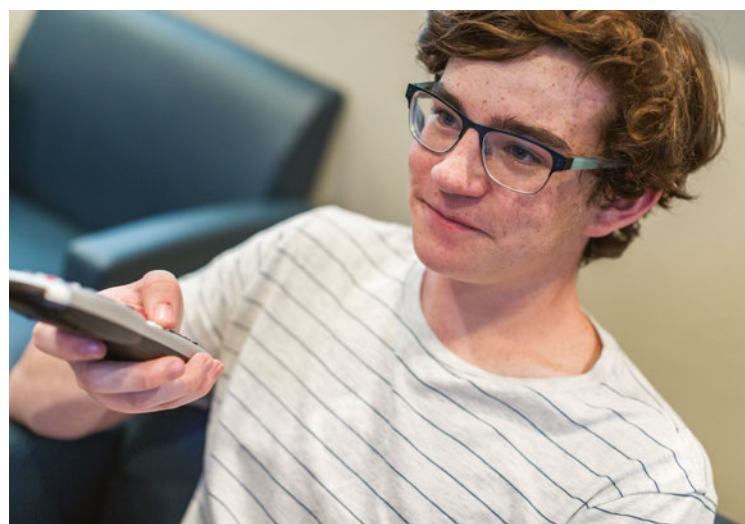
- 6** Perform each of the following calculations.
- a** 3 days 16 hours + 4 days 18 hours
 - b** 2 days 21 hours + 3 days 23 hours
 - c** 5 days 17 hours – 2 days 10 hours
 - d** 4 days 2 hours 20 minutes + 1 day 17 hours 11 minutes
 - e** 6 days 15 hours 38 minutes – 2 days 7 hours 15 minutes
 - f** 5 days 18 hours 51 minutes + 1 day 13 hours 28 minutes
- 7** Calculate the time (in days/hours/minutes) that elapses between:
- a** 4:38 pm Tuesday and 6:30 am on the following Thursday
 - b** 7:09 am Saturday to 12:16 pm on the following Wednesday
 - c** 6:52 am Monday to 8:15 pm on the following Friday
 - d** 1346 on Sunday to 1925 on the following Monday
 - e** 1047 on Wednesday to 1638 on the following Saturday
 - f** 0024 Friday to 1915 on the following Wednesday.

- 8** Pooja leaves for school camp on Friday at 0900. The bus trip takes 3 hours 42 minutes.
- a** At what time does Pooja arrive at camp?
 - b** If dinner is at 1800, how long after she arrived at camp does Pooja wait until dinner?
 - c** Everyone is being woken at 5:45 am to watch the sun rise. Pooja needs 9 hours 25 minutes sleep. She needs 15 minutes to fall asleep. At what time will she need to go to bed?
 - d** Pooja arrives back at school at 1545 on Monday. How long is it since she left for camp?



- 9** George is allowed 1 hour and 10 minutes screen time per weekday and double that time on each day of the weekend. He intends to watch the entire first season of *Small Pop Hypothesis* comprising 35 episodes, each of 24 minutes duration. He starts watching at 7:00 pm on Monday 7 May and starts his screen time at 7:00 pm every day.

- a** At what time and on what day and date does George complete watching Season 1?
- b** Due to a poor half-yearly report, George has the screen time he is allowed reduced by 15 minutes per weekday but it is not reduced on weekends. Season 2 of *Small Pop Hypothesis* has 38 episodes of 23 minutes duration and he starts watching it on Wednesday 23 May at 7:00 pm. At what time and on what day and date does he finish watching Season 2?



- c** After coming first in his class mathematics assessment, George is permitted to watch Season 3 of *Small Pop Hypothesis* over one week, starting on a Friday at 7:00 pm. There are 33 episodes of 24 minutes duration. He is allowed double his weekday screen time on each day of the weekend. How long must his weekday screen time be in order for him to finish watching the whole season after exactly one week?

time zone

region of the world that observes the same time throughout

standard time

time adopted for a particular region within a time zone

World Time Zones

time zones around the world that are expressed as positive or negative offsets from UTC

Coordinated Universal Time (UTC)

primary time standard for clocks and time, replacing Greenwich Mean Time (GMT)

9D World time zones

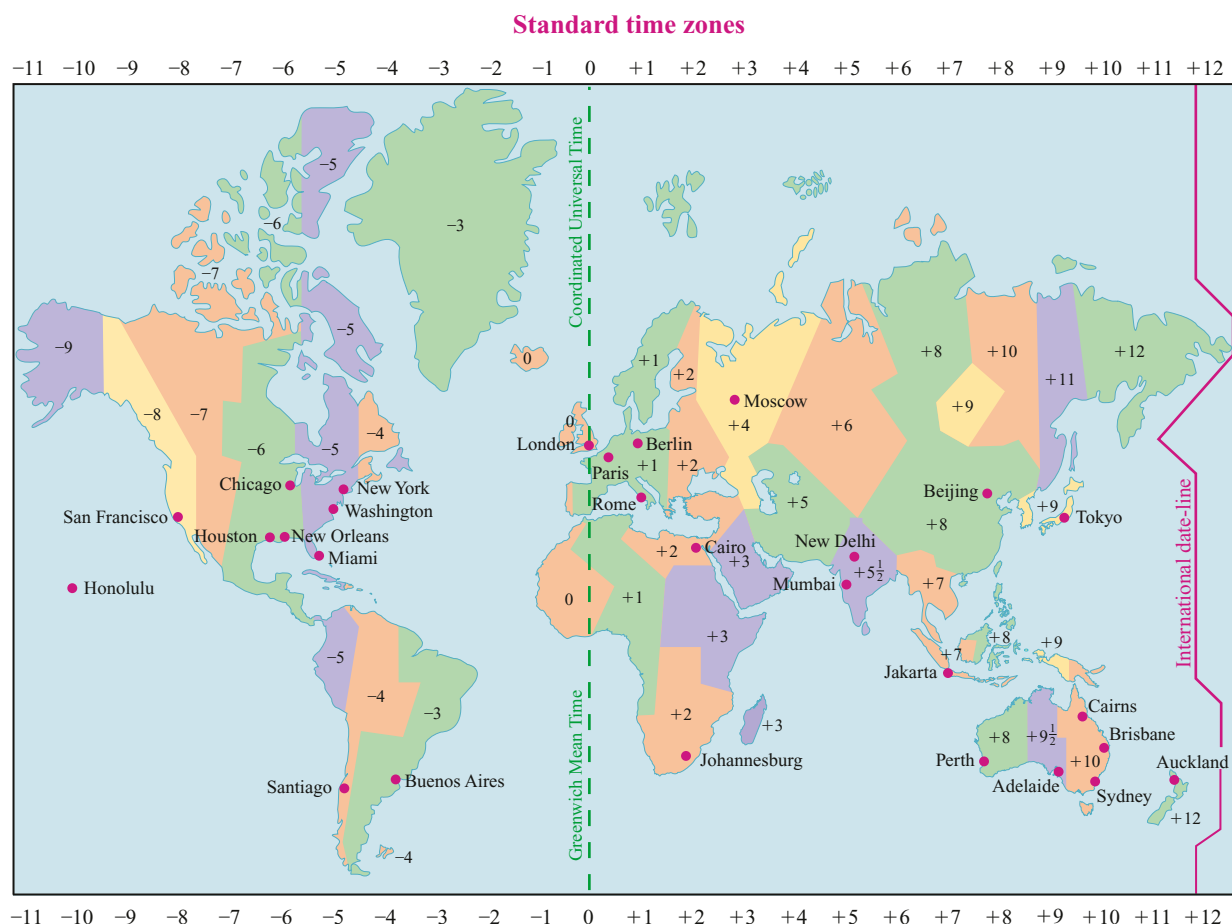
These resources are available on your obook assess:

- **Interactive 9D:** Explore key ideas for world time zones
- **Video tutorial 9D.1:** Watch and listen to an explanation of Examples 9D-3 and 9D-4
- **Video tutorial 9D.2:** Watch and listen to an explanation of Example 9D-7
- **Worksheet 9D:** Practise your skills with extra problems for time zones
- **Investigation 9D.1:** Explore the time zones and time differences around the world
- **Investigation 9D.2:** Investigate how the International Date Line affects time
- **assess quiz 9D:** Test your skills with an auto-correcting multiple-choice quiz

Lines of longitude are used as the basis of **time zones**. There are 12 time zones to the east of the prime meridian and 12 time zones to its west. Each zone covers about 15° of longitude. Every place within a zone uses the same time. The time within the zone is known as **standard time**.

The borders of time zones do not necessarily follow meridians of longitude. For convenience, they often follow country or state boundaries. Australia, for example, has three time zones.

World Time Zones are shown on the simplified map below. The numbers in each zone indicate the number of hours to be added to or subtracted from **Coordinated Universal Time (UTC)** to get the standard time for that zone. The term UTC has replaced Greenwich Mean Time (GMT).



EXAMPLE 9D-1 Finding standard times

Use the world time zone map to find the standard time in the following cities when UTC is 12:00 noon.

a Sydney

b Houston (USA)

	Solve	Think	Apply
a	Standard time in Sydney would be 10:00 pm.	Sydney is in the time zone marked +10, so standard time in Sydney is 10 hours ahead of (later than) UTC.	Find the time zone for the city and add or subtract that number of hours from Coordinated Universal Time (UTC).
b	Standard time in Houston would be 6:00 am.	Houston is in the time zone marked -6, so standard time in Houston is 6 hours behind (earlier than) UTC.	

EXERCISE 9D World time zones

- 1** Use the world time zone map to find the standard time in each of these cities when UTC is 12:00 noon.

- a** Perth
- b** San Francisco
- c** Cairo
- d** Santiago
- e** Johannesburg
- f** Moscow



EXAMPLE 9D-2 Finding standard times involving am and pm

Use the world time zone map to find the standard time in the following cities when UTC is 10:00 am.

a Sydney

b Houston

	Solve	Think	Apply
a	Standard time in Sydney is 8:00 pm.	Sydney is in the time zone marked +10, so standard time in Sydney is 10 hours ahead of (later than) 10:00 am.	Find the time zone for the city and add or subtract that number of hours from the stated UTC time.
b	Standard time in Houston would be 4:00 am.	Houston is in the time zone marked -6, so standard time in Houston is 6 hours behind (earlier than) 10:00 am.	

- 2** Use the world time zone map to find the standard time in the following cities when UTC is 2:00 pm.

- a** Tokyo
- b** Mumbai
- c** New York
- d** Houston
- e** Rome
- f** Adelaide

EXAMPLE 9D-3 Finding standard times involving adding days

Use the world time zone map to find the standard time in Sydney when UTC is 8:00 pm Wednesday.

Solve	Think	Apply
Standard time in Sydney is 6:00 am Thursday.	Sydney is in the time zone marked +10, so standard time in Sydney is 10 hours ahead of (later than) 8:00 pm Wednesday. This means it is 6:00 am Thursday.	Find the time zone for the city and add or subtract that number of hours from the stated UTC time. This may mean that it is the following day or the previous day.

3 Use the world time zone map to find the standard time in these cities when UTC is 9:00 pm Tuesday.

a Sydney	b New Delhi	c Moscow
d Beijing	e Brisbane	f Jakarta

EXAMPLE 9D-4 Finding standard times involving subtracting days

Use the world time zone map to find the standard time in Houston when UTC is 2:00 am Monday.

Solve	Think	Apply
Standard time in Houston would be 8:00 pm Sunday.	Houston is in the time zone marked -6, so standard time in Houston is 6 hours behind (earlier than) 2:00 am Monday.	Find the time zone for the city and add or subtract that number of hours from the stated UTC time. This may mean that it is the following day or the previous day.

4 Use the world time zone map to find the standard time in these cities when UTC is 4:00 am Monday.

a Houston	c San Francisco	e New York
d Santiago	e Miami	f Buenos Aires

EXAMPLE 9D-5 Finding the difference in standard time between two cities

What is the difference in standard times between:

- a** New York and San Francisco? **b** New York and Paris?

	Solve	Think	Apply
a	The difference in standard times is 3 hours.	New York is in the -5 time zone and San Francisco is in the -8 time zone. Difference in standard times $= -5 - -8$, or $-8 - -5 = \pm 3$ h.	Find the time zone for each city. Subtract the time zones to find the difference.
b	The difference in standard times is 6 hours.	New York is in the -5 time zone and Paris is in the +1 time zone. Difference in standard times $= 1 - -5$, or $-5 - 1 = \pm 6$ h.	

5 What is the difference in standard times between:

a New York and Houston?	b San Francisco and Houston?
c Cairo and Mumbai?	d Beijing and Sydney?
e New York and Moscow?	f Johannesburg and San Francisco?

EXAMPLE 9D-6 Finding the standard time in another city

When it is 9:00 am in Cairo, what is the standard time in:

- a Mumbai?
- b New York?



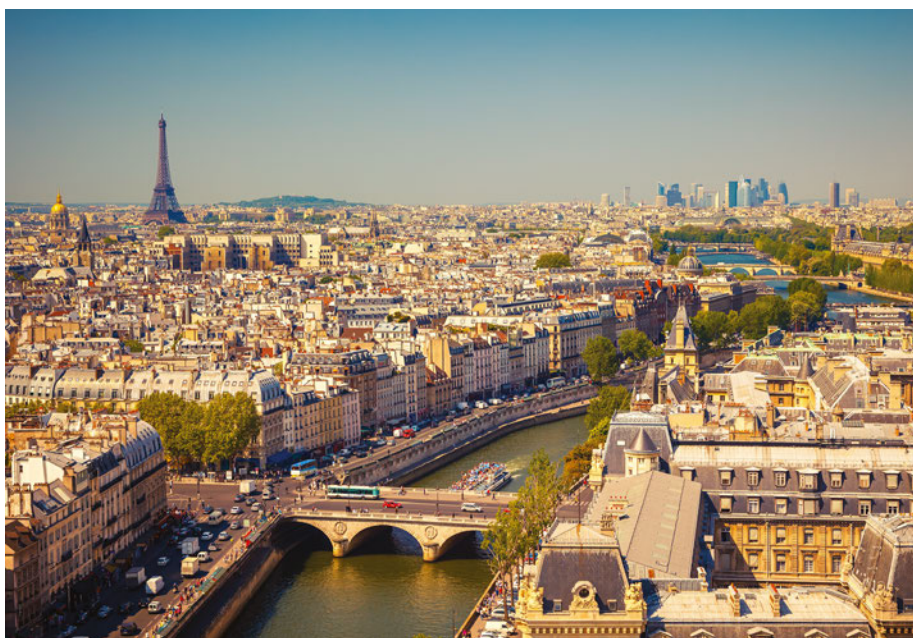
	Solve	Think	Apply
a	The standard time in Mumbai is 12:30 pm.	Difference in standard times = $5\frac{1}{2} - 2$ $= 3\frac{1}{2}$ h Mumbai is east of Cairo. It has already had its 9:00 am local time. The time in Mumbai will be $3\frac{1}{2}$ h later than 9:00 am.	Find the time zone for each city. Subtract the time zones to find the difference.
b	The standard time in New York is 2:00 am.	Difference in standard times = $2 - (-5)$ $= 7$ h New York is west of Cairo. It has not yet had its 9:00 am local time. The time in New York is 7 h earlier than 9:00 am.	Add or subtract that number of hours from the stated time.

- 6 When it is 11:00 am in Cairo, what is the standard time in:

- a Perth?
- b Brisbane?
- c Johannesburg?
- d New York?
- e San Francisco?
- f Buenos Aires

- 7 When it is 3:00 pm in Paris, what is the standard time in:

- a Moscow?
- b Tokyo?
- c Houston?
- d Mumbai?
- e Santiago?
- f Jakarta



- 8 When it is 8:00 pm Saturday in New York, what is the standard time in:

- a Berlin?
- b Johannesburg?
- c Beijing?

- 9 When it is 5:00 am Tuesday in Sydney, what is the standard time in:

- a Cairo?
- b London?
- c New York?

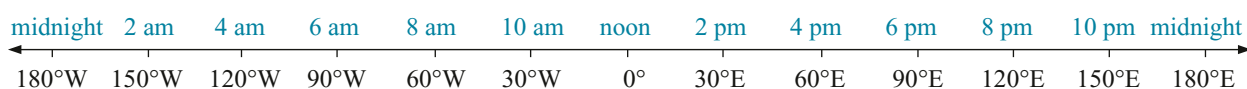
10 An international company has offices in San Francisco, London and Sydney.

- Use the world standard time zone map to find the local time in San Francisco and Sydney when it is 12:00 noon in London.
- Copy and complete the table on the right, to compare standard times in the three cities.
- The company directors in the three cities wish to organise a three-way teleconference. What would be a convenient time to hold the conference?

San Francisco	London	Sydney
	12:00 noon (Monday)	
	2:00 pm	
	4:00 pm	
	.	
	.	
	.	
	12:00 noon (Tuesday)	

11 Use the following number line to complete the table below to show the local time for places east and west of the prime meridian (0°) when it is 12:00 noon Wednesday UTC.

This number line shows times and longitudes relative to noon in Greenwich, which is on the prime meridian.



The 180°W and 180°E meridians are the same meridian but time differs by 1 day. This is the approximate location of the International Date Line.

Longitude west	Local time	Longitude east	Local time
0°	12:00 noon Wednesday	0°	12:00 noon Wednesday
45°W		45°E	
90°W		90°E	
135°W		135°E	
180°W		180°E	

12 Use your completed table from question **11** to help you answer the following questions.

- On a globe, find the 180°E and 180°W meridians of longitude. What do you notice?
- Compare local times for the 180°E and 180°W meridians from the table.
- Discuss the need for the International Date Line.
- Find the International Date Line on a map or globe. Does it follow the 180° meridian exactly? Discuss why this happens.
- What happens if you cross the International Date Line from:
 - east to west?
 - west to east?

13 A ship travelling from San Francisco (USA) to Brisbane crosses the International Date Line at 8:30 pm on Saturday 25 March. What will be the time and date just after it has crossed?



EXAMPLE 9D-7 Finding standard times involving the International Date Line

Georgia leaves Sydney (34°S, 151°E), at 11:00 am Friday, flying to Honolulu (21°N, 158°W).

- What is the standard time and day in Honolulu when Georgia leaves Sydney?
- The flight, time is $11\frac{1}{2}$ hours. What is the day and time in Honolulu when she arrives?

	Solve	Think	Apply
a	The standard time in Honolulu is 3:00 pm on Thursday. This is due to the International Date Line.	Sydney standard time is +10. Honolulu standard time is -10. Difference in standard time is 20 hours. Sydney is east of Honolulu. It has already had its 11:00 am local time. The time in Honolulu will be 20 hours earlier than 11:00 am.	Find the time zone for each city. Subtract the time zones to find the difference. Add or subtract that number of hours from the stated time.
b	The time is 2:30 am Friday.	Add $11\frac{1}{2}$ hours to 3:00 pm on Thursday.	Crossing the International Date Line from east to west takes the time back to the previous day.

- Reynold leaves Beijing (31°N, 121°E) at 7:00 am Tuesday, flying to San Francisco (38°N, 122°W).

- What is the standard time and day in San Francisco when Reynold leaves Beijing?
- The flight time is 12 hours. What is the day and time in San Francisco when he arrives?

- Elliot leaves San Francisco (38°N, 122°W) at 1:00 pm Sunday, flying to Sydney (34°S, 151°E).

- What is the standard time and day in Sydney when Elliot leaves San Francisco?
- The flight time is 16 hours. What is the day and time in Sydney when he arrives?

- Kaya leaves Houston (30°N, 95°W) at 6:00 pm Saturday, flying to Sydney (34°S, 151°E).

- What is the standard time and day in Sydney when Kaya leaves Houston?
- The flight time is $15\frac{1}{2}$ hours. What is the day and time in Sydney when she arrives?

- Kelly and Erin travel on a container ship from Perth (32°S, 116°E) to Buenos Aires (34°S, 58°W).

- They leave Perth at 0700 Saturday, 1 December 2018, and travel 12 500 km at an average speed of 20 km/h.
 - What is the local time, day and date in Buenos Aires when they leave Perth?
 - How long is the journey?
 - What is the local time, day and date in Buenos Aires when they arrive?
- They holiday until the return journey which leaves 25 days and 5 hours after they arrived. The return journey covers 13 000 km at an average speed of 18 km/h.
 - What is the local time, day and date in Perth when they leave Buenos Aires to go back?
 - How long is the journey?
 - What is the local time, day and date in Perth when they arrive?
- How long was the entire journey?



9E

Australian time zones

These resources are available on your obook assess:

- **Interactive 9E:** Explore key ideas for Australian time zones
- **Worksheet 9E:** Practise your skills with extra problems
- **Investigation 9E.1:** Investigate daylight saving time around the world
- **Investigation 9E.2:** Explore Australian time zones and daylight saving time
- **assess quiz 9E:** Test your skills with an auto-correcting multiple-choice quiz

We observed in the previous section that the borders of time zones do not necessarily follow meridians of longitude. For convenience, they often follow country or state boundaries. Australia, for example, has three time zones.

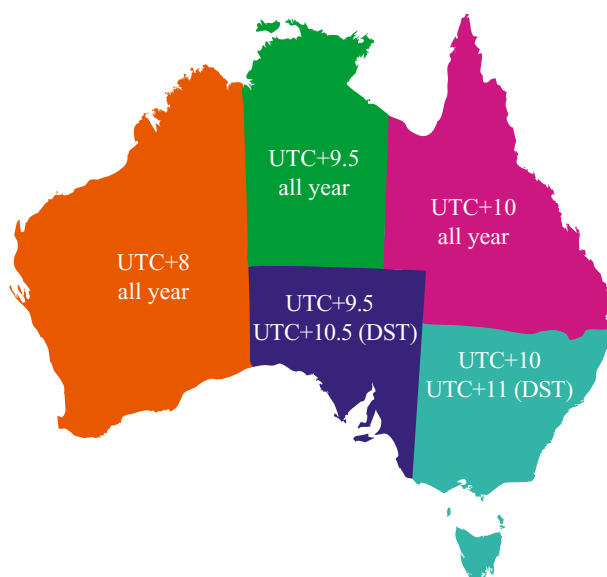
- **Eastern Standard Time (EST)** is based on the 150°E meridian of longitude and is used by the eastern states: Queensland, NSW (except for a region around Broken Hill), Victoria and Tasmania.
- **Central Standard Time (CST)** is 30 minutes behind Eastern Standard Time and is used by South Australia, Northern Territory and Broken Hill.
- **Western Standard Time (WST)** is based on the 120°E meridian and is 2 hours behind Eastern Standard Time. It is used by all of Western Australia.

daylight saving time (DST)

the practice of setting clocks forward one hour from standard time during the summer months (from October to April in Australia)

Some states have **daylight saving time (DST)** during summer and change the time by one hour. During those times the word 'Standard' is replaced with 'Summer'. In Australia, daylight saving time usually starts from the first Sunday in October, when clocks are put forward by one hour, and ends on the first Sunday in April, when clocks are put back one hour. New South Wales, the Australian Capital Territory, Victoria, Tasmania and South Australia observe daylight saving time each year.

The diagram below shows the times zones within Australia. *Note:* the diagram indicates how many hours have been added to the UTC time standard.



EXERCISE 9E Australian time zones

- 1 Use the map showing the Australian time zones to help you find the time in:
 - a Darwin when it is 8:30 am in Cairns
 - b Hobart when it is 10:00 am in Sydney
 - c Perth when it is 7:00 pm in Brisbane
 - d Adelaide when it is 5:00 pm in Perth
 - e Melbourne when it is 5:20 pm in Perth
 - f Sydney when it is 2:30 pm in Alice Springs.
- 2 a The flight from Sydney to Perth takes 4 hours flying time. If a plane leaves Sydney at 5:00 pm, at what time will it arrive in Perth (Perth time)?
 b A plane leaves Perth at 6:00 am for the return trip to Sydney. Due to head winds, the trip takes $4\frac{1}{2}$ hours flying time. At what time will the plane arrive in Sydney (Sydney time)?
- 3 A plane leaves Sydney at 7:00 am Saturday, EST, and flies directly to London. If the trip takes 16 hours, at what time will it be in London when the plane arrives?
- 4 Some states of Australia use daylight saving time.
 - a Explain the meaning of daylight saving.
 - b Which states adopt daylight saving?
 - c When does daylight saving start and finish in the states you listed in part b?

- 5 Complete the table on the right to compare times in the given cities during the months when daylight saving operates if the time in Sydney is 1:00 pm.
- 6 Aryun is in Adelaide and rings his mother to say Happy New Year at midnight Adelaide time. If his mother is in Brisbane, at what local time does she receive the call?
- 7 Paula lives in Sydney and rings her friend in Perth during summer when daylight saving time applies. If it is 8:00 pm in Sydney, what is the time in Perth?
- 8 The Socceroos are playing in Jakarta in summer and the game is at 8:00 pm local time. Jakarta is 1 hour behind standard Perth time. The match is broadcast live. At what time in Sydney will the game start, using local daylight saving time?
- 9 A summer concert is held in Perth starting at 7:00 pm and is live streamed around Australia. Calculate the starting time of the concert as local time in Adelaide, Sydney and Brisbane.

City	Time
Brisbane	
Sydney	1:00 pm
Canberra	
Melbourne	
Hobart	
Adelaide	
Darwin	
Perth	



- 10 A plane leaves Adelaide at 8:00 am, Adelaide daylight saving time, and flies to Hobart. The flight time is $1\frac{1}{2}$ hours. What time will it be in Hobart when the plane arrives?

9F Interpreting timetables

These resources are available on your obook assess:

- **Interactive 9F:** Explore key ideas for timetables
- **Worksheet 9F:** Practise your skills with extra problems for timetables
- **Investigation 9F:** Use a timetable to read information about tides
- **assess quiz 9F:** Test your skills with an auto-correcting multiple-choice quiz

timetables
information
displayed in
tables about
the times of
scheduled events

Tables are a useful way of representing a lot of information easily and quickly. **Timetables** are tables containing information about the times of scheduled events.

When reading timetables check these points.

- 1 Are you looking at the right timetable or section? (Check the heading.)
- 2 Should you read the timetable in rows or in columns?
- 3 Is there a key to any symbols, abbreviations or shading?

EXERCISE 9F Interpreting timetables

Here is part of the Carlingford to City train timetable. Use it to answer question 1.

Note: although 24-hour time is usually written without a colon, the train and bus timetables in NSW are written with a colon included.



Tuesday 18 April 2017

T6 Carlingford Line: Carlingford to Clyde

Stations										
Carlingford	---	16:05	---	16:45	---	17:20	---	18:02	---	18:47
Teloepa	---	16:07	---	16:47	---	17:22	---	18:04	---	18:49
Dundas	---	16:10	---	16:50	---	17:25	---	18:07	---	18:52
Rydalmere	---	16:12	---	16:52	---	17:27	---	18:09	---	18:54
Camellia	---	16:13	---	16:53	---	17:28	---	18:10	---	18:55
Rosehill	---	16:15	---	16:55	---	17:30	---	18:12	---	18:57
Clyde (arr)	---	16:18	---	16:58	---	17:33	---	18:15	---	19:00
Clyde (dep)	15:50	---	16:35	---	17:05	---	17:50	---	18:20	19:05
Auburn	15:52	---	16:37	---	17:07	---	17:52	---	18:22	19:07
Lidcombe	15:55	---	16:40	---	17:10	---	17:55	---	18:25	19:10
Strathfield	16:02	---	16:47	---	17:17	---	18:02	---	18:32	19:17
Burwood	16:04	---	16:49	---	17:19	---	18:04	---	18:34	19:19
Redfern	16:13	---	16:58	---	17:28	---	18:13	---	18:43	19:28
Central	16:17	---	17:02	---	17:32	---	18:17	---	18:47	19:32
Town Hall	16:20	---	17:05	---	17:35	---	18:20	---	18:50	19:35
Wynyard	16:23	---	17:08	---	17:38	---	18:23	---	18:53	19:38
Milsons Point	16:26	---	17:11	---	17:41	---	18:26	---	18:56	19:41
North Sydney	16:29	---	17:14	---	17:44	---	18:29	---	18:59	19:44

Source: http://www.sydneytrains.info/timetables/timetables_by_line

- 1 a i I am meeting a friend at Town Hall station. What is the latest train from Carlingford that I can catch to arrive before 7:00 pm?
 - ii At what time does this train pass through Lidcombe station?
- b If I miss the 17:20 train at Carlingford, what would be the earliest time I could arrive at Wynyard?
- c i How long does the 4:47 pm train from Strathfield take to get to Wynyard?
 - ii How long does the 4:47 pm train from Telopea take to get to Central?
 - iii How long does the 6:07 pm train from Dundas take to get to Town Hall?
 - iv How long does the 4:15 pm train from Rosehill take to get to Strathfield?
- d i Calculate the time taken by the 16:05, 16:45, 17:20, 18:02 and 18:47 Carlingford trains to travel to Wynyard.
 - ii Why are the times taken different?

Below is the Chatswood to Richmond/Emu Plains train timetable. Use it to answer question 2 on the next page.



T1 Western Line: City to Emu Plains, City to Richmond

Stations											
Chatswood	08:46	---	08:52	08:58	---	09:13	09:16	09:28	---	09:43	---
Artarmon	08:48	---	08:54	09:00	---	09:15	09:18	09:30	---	09:45	---
St Leonards	08:50	---	08:56	09:02	---	09:17	09:20	09:32	---	09:47	---
Wollstonecraft	08:53	---	08:59	09:05	---	09:20	09:23	09:35	---	09:50	---
Waverton	08:55	---	09:01	09:07	---	09:22	09:25	09:37	---	09:52	---
North Sydney	08:59	---	09:05	09:11	---	09:26	09:29	09:41	---	09:56	09:59
Milsons Point	09:01	---	09:07	09:13	---	09:28	09:31	09:43	---	09:58	10:01
Wynyard	09:05	---	09:11	09:17	---	09:32	09:35	09:47	---	10:02	10:05
Town Hall	09:09	---	09:15	09:21	---	09:36	09:39	09:51	---	10:06	10:09
Central	09:13	09:18i	09:19	09:25	---	09:40	09:43	09:55	---	10:10	10:13
Redfern	09:15	---	09:21	09:27	---	09:42	09:45	09:57	---	10:12	10:15
Burwood	09:24	---	09:30	---	---	---	09:54	---	---	---	10:24
Strathfield	09:27	09:31	09:33	09:38	---	09:53	09:57	10:08	---	10:23	10:27
Lidcombe	09:32	---	---	09:43	---	09:58	10:02	10:13	---	10:28	10:32
Auburn	09:35	---	---	---	---	---	10:05	---	---	---	10:35
Clyde	09:38	---	---	---	---	---	10:08	---	---	---	10:38
Granville	09:40	---	---	---	---	---	10:10	---	---	---	10:40
Harris Park	09:43	---	---	---	09:54	---	10:13	---	10:24	---	10:43
Parramatta	09:45	09:43	09:46	09:52	09:57	10:07	10:15	10:22	10:27	10:37	10:45
Westmead	09:48	---	09:49	09:55	10:00	10:10	10:18	10:25	10:30	10:40	10:48
Wentworthville	09:50	---	---	---	10:02	---	10:20	---	10:32	---	10:50
Pendle Hill	09:52	---	---	---	10:04	---	10:22	---	10:34	---	10:52
Toongabbie	09:55	---	---	---	10:07	---	10:25	---	10:37	---	10:55
Seven Hills	09:58	---	09:55	10:01	10:10	10:16	10:28	10:31	10:40	10:46	10:58
Blacktown	10:02	09:52	10:00	10:06	10:20	10:20	10:32	10:36	10:50	10:50	11:02
Marayong	---	---	---	10:09	10:23	---	---	10:39	10:53	---	---
Quakers Hill	---	---	---	10:12	10:26	---	---	10:42	10:56	---	---
Schofields	---	---	---	10:16	10:29	---	---	10:46	10:59	---	---
Riverstone	---	---	---	10:21	---	---	---	10:51	---	---	---
Vineyard	---	---	---	10:25	---	---	---	10:55	---	---	---
Mulgrave	---	---	---	10:29	---	---	---	10:59	---	---	---
Windsor	---	---	---	10:32	---	---	---	11:02	---	---	---
Clarendon	---	---	---	10:37	---	---	---	11:07	---	---	---
East Richmond	---	---	---	10:40	---	---	---	11:10	---	---	---
Richmond	---	---	---	10:42	---	---	---	11:12	---	---	---
Doonside	10:06	---	---	---	---	10:24	10:36	---	---	10:54	11:06
Rooty Hill	10:09	---	---	---	---	10:27	10:39	---	---	10:57	11:09
Mount Druitt	10:12	---	---	---	---	10:30	10:42	---	---	11:00	11:12
St Marys	10:16	---	---	---	---	10:34	10:46	---	---	11:04	11:16
Werrington	10:19	---	---	---	---	10:37	10:49	---	---	11:07	11:19
Kingswood	10:23	---	---	---	---	10:41	10:53	---	---	11:11	11:23
Penrith	10:27	10:06	---	---	---	10:44	10:57	---	---	11:14	11:27
Emu Plains	10:30	10:09	---	---	---	---	11:00	---	---	---	11:30
Proceeds to	---	MVR	---	---	---	---	---	---	---	---	---

Source: http://www.sydneytrains.info/timetables/timetables_by_line

- 2 a**
- i** How long does the 9:29 am train from North Sydney take to get to Penrith?
 - ii** How long does the 10:06 am train from Town Hall take to get to Seven Hills?
 - iii** How long does the 9:57 am train from Strathfield take to get to Werrington?
 - iv** How long does the 10:20 am train from Wentworthville take to get to Riverstone?
- b**
- i** If I arrive at Blacktown at 10:36 am, what is the latest train I could have caught from Milsons Point?
 - ii** If I arrive at Auburn at 10:05 am, what is the latest train I could have caught from Redfern?
 - iii** If I arrive at Werrington at 10:49 am, what is the latest train I could have caught from Clyde?
 - iv** If I arrive at Emu Plains at 11:00 am, what is the latest train I could have caught from Westmead?
- c** I am travelling from Burwood to Clarendon to meet a friend for morning tea at 11:00 am.
- i** Can I catch a train directly from Burwood to Clarendon?
 - ii** What is the closest station to Burwood from which I can get a connecting train to Clarendon?
 - iii** If I am to be on time for morning tea, at what time should I depart Burwood?
 - iv** At what station and at what time would I catch the connecting train to Clarendon?
 - v** At what time would I arrive at Clarendon?
 - vi** How long would the entire trip from Burwood to Clarendon take?
 - vii** If I am being picked up by my friend from the station at 10:50 am, how long must I wait at the station?

Use this bus timetable to answer question 3.

Saturday (continued...)													
	Route Number	325	324	325	324	325	324	325	324	325	324	325	324
A	Watsons Bay Military Road	08:51	09:13	09:20	09:43	09:50	10:13	10:20	10:43	10:50	11:13	11:20	11:43
B	Vaucluse Heights Old South Head Rd	...	09:18	...	09:48	...	10:18	...	10:48	...	11:18	...	11:48
C	Vaucluse Hopetoun Avenue	08:56	...	09:25	...	09:55	...	10:25	...	10:55	...	11:25	...
D	Dover Heights Military Road	...	...	...	...	...	...	...	...	...	...	...	...
E	Rose Bay Dover Road	09:07	09:22	09:37	09:52	10:07	10:22	10:37	10:52	11:07	11:22	11:37	11:52
F	Double Bay Manning Road	09:18	09:33	09:48	10:03	10:18	10:33	10:48	11:03	11:18	11:33	11:48	12:03
G	Edgecliff Station New South Head Rd	09:20	09:35	09:50	10:05	10:20	10:35	10:50	11:05	11:20	11:35	11:50	12:05
H	Kings Cross Bayswater Road	09:25	09:40	09:55	10:10	10:25	10:40	10:55	11:10	11:25	11:40	11:55	12:10
I	City - Town Hall Park Street	09:33	09:48	10:03	10:18	10:33	10:48	11:03	11:18	11:33	11:48	12:03	12:18
J	City - Walsh Bay Hickson Road	09:45	10:00	10:15	10:30	10:45	11:00	11:16	11:31	11:46	12:01	12:16	12:31

Source: <http://www.sydneybuses.info>

- 3 a** On Friday, Harper decided to catch a bus from Watsons Bay Military Road to meet her friend at City–Town Hall Park Street.
- i** If Harper arrived at Watsons Bay Military Road at 10:05 am, how long did she need to wait for her bus to arrive?
 - ii** At what time would Harper have arrived at City–Town Hall Park Street?
 - iii** Harper’s friend arrived at City–Town Hall Park Street at 11:00 am. How long did Harper wait for her friend?
- b** Aaron caught the 9:18 am bus from Vaucluse Heights Old South Head Road to Edgecliff Station New South Head Road.
- i** How long did Aaron’s journey take?
 - ii** Aaron waited at the station for 35 minutes until his friend arrived by train. They then caught a bus together to City–Walsh Bay Hickson Road. At what time did they arrive?
 - iii** If Aaron arrived home at 2:42 pm, how long had it been from the time he caught the first bus?



Use this extract from a tide table to answer question 4.

May / June 2017									
SYDNEY (FORT DENISON)									
LAT 33° 51' S LONG 151° 14' E									
TIMES AND HEIGHTS OF HIGH AND LOW WATERS									
Times are in local standard time (UTC +10:00) or daylight savings time (UTC +11:00) when in effect.									
MAY 2017					JUNE 2017				
Time	m	Time	m	Time	m	Time	m	Time	m
1 MO 0627 0.37 1232 1.41 1811 0.62		12 FR 0313 0.48 0909 1.44 1450 0.58 2118 1.77		23 TU 0543 1.64 1155 0.38 1819 1.80		1 TH 0118 1.71 0810 0.47 1424 1.41 2006 0.74		12 MO 0408 0.47 1004 1.36 1535 0.63 2206 1.77	
2 TU 0038 1.80 0732 0.43 1340 1.36 1916 0.70		13 SA 0350 0.49 0947 1.41 1524 0.61 2153 1.76		24 WE 0036 0.42 0637 1.66 1242 0.35 1907 1.93		2 FR 0221 1.60 0904 0.52 1525 1.44 2116 0.75		13 TU 0447 0.49 1046 1.35 1616 0.66 2245 1.74	
3 WE 0143 1.71 0839 0.48 1451 1.36 2030 0.73		14 SU 0430 0.51 1027 1.37 1600 0.65 2230 1.73		25 TH 0130 0.32 0731 1.66 1330 0.34 1956 2.02		3 SA 0323 1.52 0954 0.55 1620 1.49 2224 0.73		14 WE 0530 0.51 1131 1.33 1700 0.70 2327 1.69	
4 TH 0252 1.64 0940 0.50 1558 1.40 2143 0.73		15 MO 0511 0.54 1110 1.33 1640 0.70 2309 1.68		26 FR 0225 0.24 0828 1.63 1418 0.36 2045 2.08		4 SU 0422 1.46 1040 0.56 1710 1.55 2324 0.69		15 TH 0615 0.52 1220 1.33 1750 0.73	
5 FR 0359 1.60 1035 0.51 1655 1.46 2249 0.69		16 TU 0556 0.57 1156 1.30 1724 0.75 2352 1.63		27 SA 0320 0.21 0924 1.59 1510 0.41 2137 2.08		5 MO 0515 1.43 1121 0.56 1753 1.62		16 FR 0013 1.63 0702 0.53 1313 1.35 1846 0.75	
								23 FR 0119 0.29 0716 1.55 1306 0.37 1937 2.05	
								24 SA 0215 0.22 0814 1.54 1359 0.37 2030 2.09	
								25 SU 0309 0.18 0910 1.53 1545 0.40 2121 2.09	
								26 MO 0402 0.19 1005 1.51 1545 0.44 2213 2.03	
								27 TU 0455 0.24 1100 1.48 1640 0.50 2303 1.93	

Indicates high tide of 1.7 m or more Indicates low tide of 0.3 m or less

Source: <http://www.rms.nsw.gov.au/documents/maritime/usingwaterways/tides-weather/tide-tables-2016-2017.pdf>

- 4 a Consider Wednesday 3 May. There are four entries indicating high or low tides.
- How many high tides occurred on that day?
 - What is the time difference between each high tide and the following low tide?
 - What is the height difference between each high tide and the following low tide?
- b Consider the five weekdays from 12 June to 16 June.
- What is the time and date of the highest tide?
 - What is the time and date of the lowest tide?
 - Calculate the time between high tides on Wednesday 14 June.
 - Why does Thursday 15 June have only three tides?
- c Sally surfs every day between 7:00 am and 9:00 am. She says the waves are at their best three hours after low tide. On which days does she surf when the waves are at their best?



- 5 Charles catches the 08:51 bus from Watsons Bay Military Road to City–Walsh Bay Hickson Road. Unfortunately Charles becomes travel sick if he stays on the bus for more than 13 minutes and he needs a break of at least 5 minutes before he can travel again. Refer to the bus timetable above question 3 on the opposite page to describe Charles' journey, including when and where he changes buses and the time that he finally arrives at City–Walsh Bay Hickson Road.

CHAPTER 9 REVIEW WORKING WITH TIME

You should be able to:

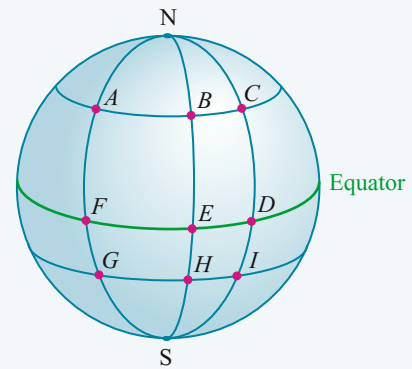
- ✓ explain the terms great circle, small circle, axis, poles, parallel (or circle) of latitude, meridian
- ✓ understand how latitude and longitude are measured
- ✓ use the equator and prime meridian as lines of reference for locating position on Earth's surface
- ✓ determine local time in different cities (including daylight saving time)
- ✓ use a world map of time zones to determine the standard time in different cities
- ✓ use and understand the need for the International Date Line in travel questions
- ✓ interpret different timetables such as bus, train and ferry timetables.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

REVIEW MULTIPLE-CHOICE QUESTIONS

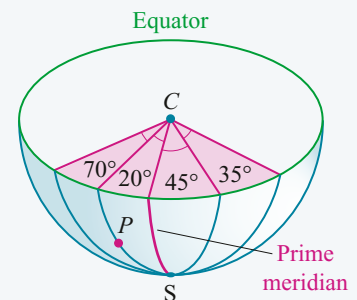
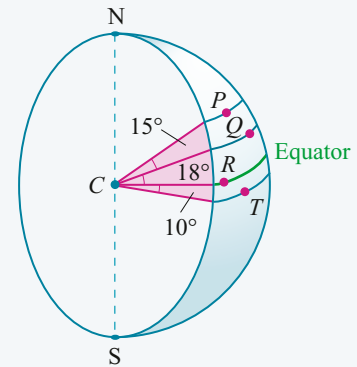
Use the diagram on the right for questions 1 and 2.

- 9A 1 Which of the following sets of points lie on the same parallel of latitude?
- A A, F, G B A, E, I
C G, H, I D C, E, H
- 9A 2 Which of the following points lie on the same meridian of longitude?
- A A, F, G B A, E, I
C G, H, I D C, E, H



Use the diagram on the right for questions 3 and 4.

- 9A 3 What is the latitude of point P?
- A 15°N B 18°N
C 33°N D 43°N
- 9A 4 What is the difference in latitude between points P and T?
- A 5° B 25°
C 23° D 43°
- 9A 5 What is the reference line for longitude on Earth's surface?
- A the axis NS
B the equator
C the prime meridian
D the parallel of latitude through Greenwich
- 9A 6 In the diagram on the right, what is the longitude of point P?
- A 20°W C 65°W
B 30°E D 100°W



- 9B ▶ 7** In the diagram on the right, what is the point with position coordinates (30°S, 85°E)?

A P B Q C U D V

- 9B ▶ 8** What is the point that is 15° north of (10°S, 20°W)?

A (25°N, 20°W) B (10°S, 35°W)
C (10°S, 5°W) D (5°N, 20°W)

- 9B ▶ 9** If point P has position (12°N, 25°W) and point Q has position (10°S, 30°W), what is the position of Q relative to P?

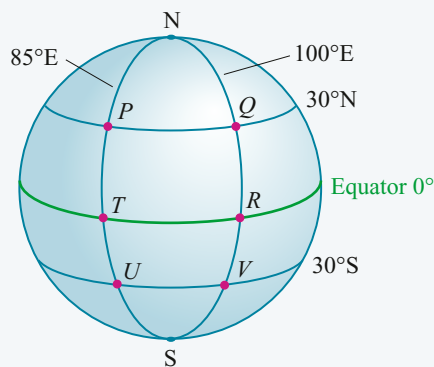
A Q is 22° south and 5° west of P B Q is 22° south and 5° east of P
C Q is 22° north and 5° west of P D Q is 22° north and 5° east of P

- 9C ▶ 10** What is the 24-hour time 1535 written as an analogue time?

A 5:35 pm B 3:35 pm C 5:35 am D 3:35 am

- 9C ▶ 11** What is 1625 – 6 hours 38 minutes?

A 0947 B 1013 C 2303 D 1047



Use the world standard time zones map from Topic 9D to answer questions 12 to 14.

- 9D ▶ 12** What is the time difference between Tokyo and Houston?

A 3:00 hours B 6:00 hours C 9:00 hours D 15:00 hours

- 9D ▶ 13** If it is 2:00 pm in Berlin, what is the standard time in Houston?

A 7:00 am B 11:00 pm C 8:00 am D 3:00 pm

- 9D ▶ 14** When it is 9:00 pm Friday in New York, what is the standard time in Beijing?

A 6:00 pm Friday B 8:00 am Friday C midnight Friday D 10:00 am Saturday

- 9D ▶ 15** Eden leaves Beijing (31°N, 121°E) at 10:00 am Tuesday flying to Honolulu (21°N, 158°W). The flight takes 13 hours. What day and time is it in Honolulu when Eden arrives?

A Tuesday 11:00 pm B Wednesday 1:00 am C Tuesday 6:00 am D Tuesday 5:00 pm

- 9D ▶ 16** When you cross the International Date Line from east to west, what should you do?

A put the clock forward one day B put the clock back one day
C put the clock forward 12:00 hours D put the clock back 12:00 hours

Use the Australian time zones map from Topic 9E to answer questions 17 and 18.

- 9E ▶ 17** What is the time in Sydney when it is 3:00 pm in Perth?

A 1:00 pm B 3:00 pm C 5:00 pm D 6:00 pm

- 9E ▶ 18** It is 7:30 am daylight saving time in Adelaide. What is the standard time in Brisbane, which does not have daylight saving time?

A 6:00 am B 6:30 am C 7:00 am D 8:00 am

- 9F ▶ 19** Use the Carlingford–City train timetable from Topic 9F. Odette catches the 17:22 train from Telopea. At what time will she arrive at Central?

A 18:17 B 17:32 C 17:33 D 18:47

- 9F ▶ 20** Use the bus timetable from Topic 9F. Charlotte leaves home at 0920 and arrives at the Vacluse Heights Old South Head Road bus stop at 0930. She catches the next bus to City–Walsh Bay Hickson Road then walks for 18 minutes to meet her friend. How long is her entire journey?

A 48 minutes B 58 minutes C 68 minutes D 88 minutes

REVIEW SET 1

- 1** Complete the following.
- a** On Earth's surface, a small circle is _____. **b** On Earth's surface, a great circle is _____.

- 2** Refer to Figure 1 on the right and write the latitude of the following points.

- a** J
- b** K
- c** L
- d** M

- 3** Refer to Figure 2 on the right and write the longitude of the following points.

- a** D
- b** E
- c** F
- d** G

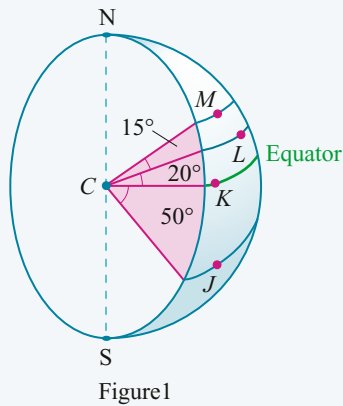


Figure 1

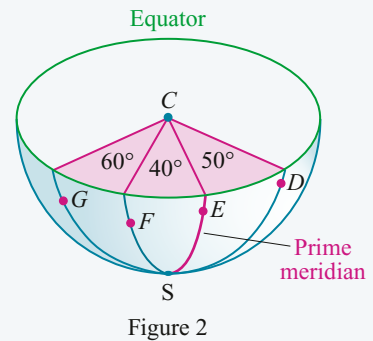
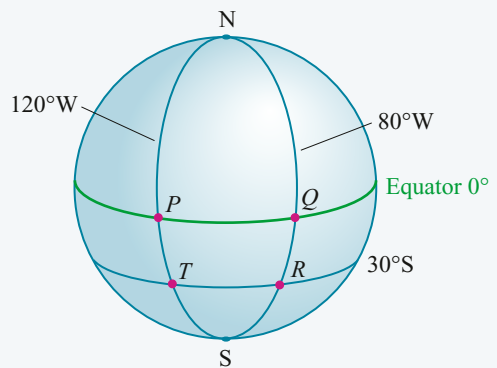


Figure 2

- 4** Use the diagram on the right to name the points with these position coordinates.

- a** (30°S, 120°W)
b (0°, 80°W)
c (30°S, 80°W)
d (0°, 120°W)



- 5** Perform each of the following calculations.

- Convert 4 hours 15 minutes to minutes.
- Write the 24-hour time 1046 as analogue time.
- Calculate $0957 + 5$ hours 49 minutes.
- Calculate 3 days 9 hours 25 minutes $-$ 1 day 15 hours 37 minutes.

- 6 a** When it is noon in Greenwich, what is the local time in Canberra?
- b** If it is 2:00 pm in Sydney, what is the standard time in Greenwich?
- c** When it is 11:30 am local time in Darwin, what is the local time in Hobart?

- 7** Jess leaves Beijing (31°N, 121°E) at 8:00 am Wednesday, flying to Honolulu (21°N, 158°W).

- a** What is the standard time and day in Honolulu when she leaves Beijing?
- b** The flight takes 11 hours. What is the day and time in Honolulu when she arrives?

- 8** A plane leaves Perth at 11:00 am for Sydney. Due to head winds, the trip takes $4\frac{1}{2}$ hours flying time. At what time will the plane arrive in Sydney (local Sydney time)?

- 9** Use the tide table from Topic 9F to answer these questions.

- Calculate the height difference between the high tide and low tide on Sunday 4 June?
- What is the time difference between high tide and low tide on Sunday 25 June?



REVIEW SET 2

1 Explain the meaning of these terms.

a parallel of latitude

b meridian of longitude

2 Use the diagram on the right to write the coordinates of these points.

a C

b D

c E

d F

e G

f H

3 Given that point A has position (26°N, 105°E), write the position coordinates of the point that is:

a 15° north of A

b 15° south of A

c 15° east of A

d 15° west of A.

4 Point P has position (16°S, 140°W) and point Q has position (5°N, 120°W). State whether P is:

a to the north or south of Q

b to the east or west of Q.

5 Perform each of the following calculations.

a Convert 4 days 17 hours to hours.

b Convert 9:42 pm to 24-hour time.

c Calculate 3 days 17 hours 55 minutes + 1 day 14 hours 31 minutes.

6 When it is 11:00 pm Thursday standard time in Sydney, what is the standard time in Auckland?

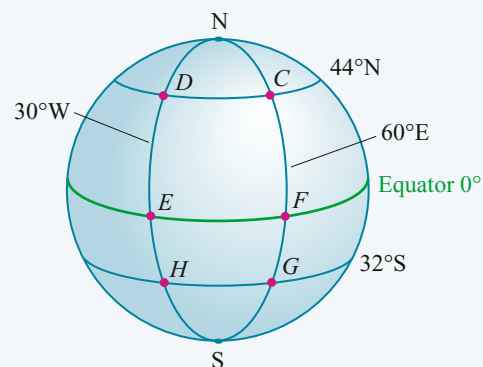
7 Use the world time zone map from Topic 9D to find the standard time in Houston when UTC is 8:00 am.

8 A plane leaves Sydney at 5:00 pm daylight saving time to travel to Perth. The trip takes 3 hours 50 minutes. At what time does the plane arrive in Perth?

9 Use the Chatswood–Richmond/Emu Plains train timetable from Topic 9F to answer these questions.

a How long does the 9:26 am train from North Sydney take to get to Penrith?

b If I arrive at Auburn at 10:35 am, which train did I catch from Redfern?



REVIEW SET 3

Use the diagram on the right to answer questions 1 to 4.

1 Write the coordinates of these points.

a E

b H

c P

d F

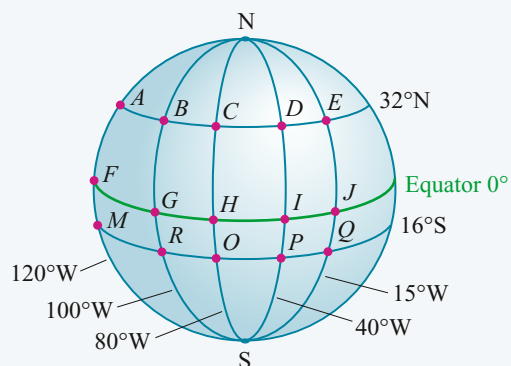
2 Name a point that is:

a due north of I

b due south of F

c due east of C

d due west of R.



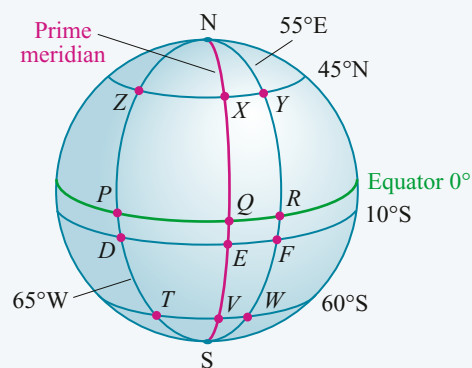
- 3 a** State whether point E is:
i north or south of I **ii** east or west of I .
b State whether point Q is:
i north or south of H **ii** east or west of H .
c State whether point G is:
i north or south of O **ii** east or west of O .
d State whether point R is:
i north or south of C **ii** east or west of C .
- 4** Write the coordinates of the point that is:
a 5° west of D **b** 30° north of Q **c** 12° south of B **d** 15° east of R .
- 5** Perform each of the following calculations.
a Convert 157 hours to days and hours.
b Write the 24-hour time 1534 as an analogue time.
c Convert 3:28 pm to 24-hour time.
d Calculate the time (in days/hours/minutes) that elapses between 7:38 pm Monday and 11:30 am Thursday.
- 6** When the time in Chicago is noon Thursday, standard time, what is the standard time in Tokyo?
- 7** A plane leaves Sydney at 11:00 am Friday, EST, and flies directly to Paris. If the trip takes 15 hours, at what time does the plane arrive in Paris (Paris standard time)?
- 8** Use the Australian time zones to find the time in Darwin when it is 11:28 am in Cairns.
- 9** A summer concert is held in Perth starting at 6:30 pm and it is live streamed around Australia. Calculate the local time in Adelaide, Sydney and Brisbane when the concert starts.
- 10** Use the Carlingford–City train timetable from Topic 9F to answer these questions.
a I am meeting a friend at Milsons Point station. Which is the latest train from Dundas that I can catch to arrive before 6:00 pm?
b At what time does the train from part **a** pass through Lidcombe station?

REVIEW SET 4

Use the diagram on the right for questions **1** to **3**.

- 1** Write the position coordinates of these points.
a Y **b** P
c Q **d** D
- 2** Name a point that is:
a due east of T **b** due south of E
c due north of P **d** due west of F .
- 3** Write the position coordinates of the point that is:
a 20° due north of P **b** 30° due west of D
c 15° due south of R **d** 50° due east of W .
- 4** Perform each of the following calculations.
a Convert 227 minutes to hours and minutes.
c Convert 11:23 am to 24-hour time.

- b** Write the 24-hour time 1127 as an analogue time.
d Calculate $0950 + 11$ hours 33 minutes.



- 5** When it is noon in Hobart, what is UTC in Greenwich?
- 6** When the time in San Francisco is 5:00 pm Monday, standard time, what is the standard time in Moscow?
- 7** A plane leaves Perth at 11:00 am and flies to Johannesburg. If the flight takes $12\frac{1}{2}$ hours, at what time will the plane arrive in Johannesburg (Johannesburg standard time)?
- 8** Saayng is in Adelaide and rings her mother to say Happy New Year at midnight Adelaide time. If her mother is in Brisbane, at what local time does she receive the call?
- 9** Use the bus timetable from Topic 9F to answer the following questions.
Angus caught the 9:48 am bus from Vacluse Heights Old South Head Road to Edgecliff Station New South Head Road.
 - a** How long did the journey take?
 - b** Angus waited at the station for 25 minutes until his friend, Stuart, arrived by train. They then caught a bus to City–Walsh Bay Hickson Road. At what time did they arrive?
 - c** If Angus arrived home at 1:23 pm, how long was he out from the time he caught the first bus?

REVIEW PRACTICE EXAMINATION QUESTION

- 1 a** What is the reference line for longitude on Earth's surface. (1 mark)
- b**
 - i** Write the 24-hour time 1827 as an analogue time. (1 mark)
 - ii** Convert 223 minutes to hours and minutes. (1 mark)
- c** Find the position coordinates of point *B*, which is 20°N and 15°E of point *A* (40°N, 45°W). (2 marks)
- d** It is 9:00 am in Adelaide during daylight saving time. What is the time in Brisbane which does not have daylight saving? (2 marks)
- e** Perry flew from Rome to Sydney via Singapore where he stopped for 6 hours. He left Rome at 2:40 pm Sunday (Rome time) and arrived at 1:15 am Tuesday (Sydney time).
 - i** Use the world map in Topic 9D to calculate the time difference between Rome and Sydney. (1 mark)
 - ii** Calculate the time in Sydney when the plane left Rome. (1 mark)
 - iii** Calculate the total flight time. (2 marks)
- f** Use the Chatswood–Richmond/Emu Plains train timetable from Topic 9F to answer the following questions.
Margo leaves home at 9:00 am and walks for 17 minutes to Milsons Point station. She then catches the next train to Westmead to visit her friend Robbie in hospital.
 - i** At what time does Margo catch the train at Milsons Point station? (1 mark)
 - ii** At what time does she arrive at Westmead Station? (1 mark)
 - iii** She buys a gift and arrives at the hospital 20 minutes after she arrived at Westmead station. She stays with Robbie for 1 hour and 20 minutes. At what time does she leave the hospital? (2 marks)

TOTAL: 15 MARKS

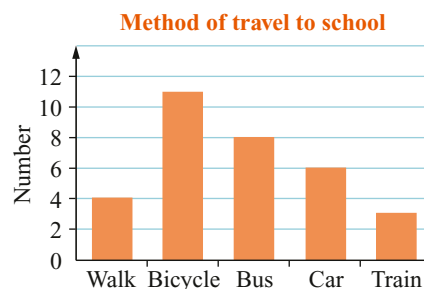
- 1** The results of a survey of the hair colours of a group of people are shown below using the code black (B), brown (b), blond (w), red (r), grey (g).

B B w w b b B B r w
 w b g B w B B b b w
 w w w b B g g g B B

- a** Organise the information into a frequency distribution table.
b How many people were surveyed?
c What was the most common hair colour in this group of people?
d Add a relative frequency column to the table above and find the relative frequency of people with:
i black hair **ii** blond hair.
e Display the data above using:
i a dot plot **ii** a horizontal bar graph.

- 2** Use this column graph to answer the following questions.

- a** How many students catch a bus to school?
b How many students don't travel by car?



- 3** The results in a test given to two classes are shown below.

Class A: 29, 39, 21, 23, 35, 31, 41, 45, 27, 31, 37, 25, 13, 43, 19, 33, 37, 35, 29

Class B: 11, 21, 33, 29, 29, 39, 43, 31, 27, 25, 33, 23, 29, 17, 41, 41, 27, 15, 7

- a** Construct back-to-back stem-and-leaf plots for this data.
b Which class performed better? Explain your answer.
- 4** **a** Complete the table shown on the right.
b Draw a cumulative frequency histogram and polygon of the data.

Score	Class centre	Frequency	Cumulative frequency
1–4	2.5	1	
5–8		4	
9–12		7	
13–16		12	
17–20		9	
21–24		8	
25–28		4	
29–32		5	

- 5** **a** List the sample space for the children in a three-child family.

- b** A card is chosen from a standard deck of playing cards. Determine the probability that the card is:

- i** the jack of hearts **ii** a jack
iii a heart **iv** black.

- c** A coffee tin was tossed 100 times and the way it landed is recorded in the table on the right. What is the experimental probability that this tin will land on one of its ends when tossed?

Lands on	Frequency
End	23
Curved face	77

- d** A coin is tossed and a die is rolled. Find the probability of getting:

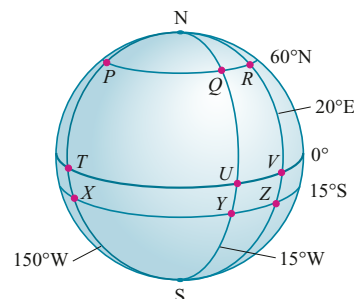
- i** a head and a 3 **ii** a head and an odd number
iii a tail and a 5 **iv** a tail and a number less than 3.

- 6** A six-sided die is rolled. List the outcomes of these events and write in words the complement of each event:

- a** rolling a 3 **b** rolling an odd number **c** rolling a number greater than 3.

- 7 a** A bag contains one green, one blue and one white marble. One marble is chosen at random from the bag, and then replaced into the bag. A second marble is chosen. Find the sample space.
- b** Find the probability that:
- i** both marbles are green
 - ii** the first marble is green
 - iii** exactly one marble is green
 - iv** both marbles are the same colour
 - v** the marbles are different colours
 - vi** the first marble is blue and the second marble is white
 - vii** there is one blue and one white marble
 - viii** at least one marble is green.
- 8** Repeat question 7 but, this time, the first marble is not replaced in the bag before the second marble is chosen.
- 9** A six-sided die is rolled 600 times. How many times would you expect to roll:
- a** a 3?
 - b** a number less than 3?
 - c** an odd number?

- 10** Use the diagram on the right to answer these questions.



- a** List the points on the equator.
 - b** Write the latitude and longitude of the points *P*, *U* and *Y*.
 - c** Determine whether the following are true or false.
 - i** *P* is west of *Q*.
 - ii** *R* is north of *Z*.
 - iii** *X* is east of *Y*.
 - iv** *Z* is south of *X*.
 - d** Determine the position coordinates of the point that is:
 - i** 30° west of *Z*
 - ii** 8° south of *Q*.
- 11 a** Convert 4 days and 19 hours to hours. **b** Convert 289 minutes to hours and minutes.
- c** Calculate the time (in days/hours/minutes) that elapses between 7:38 pm Tuesday and 9:30 am Friday.
- 12 a** Moscow is in the +4 time zone and Miami is in the −5 time zone. What time is it in:
- i** Moscow when it is 4:00 am in Miami
 - ii** Miami when it is 2:00 pm in Moscow.
- b** Reynold leaves Beijing (31°N, 121°E), at 11:00 am Friday, flying to San Francisco (38°N, 122°W). Beijing is in the +8 time zone and San Francisco is in the −8 time zone.
- i** What is the standard time and day in San Francisco when Reynold leaves Beijing?
 - ii** The flight time is 12½ hours. What is the day and time in San Francisco when he arrives?
- c** Use the Australian time zones to find the summer time in:
- i** Darwin when it is 8:30 am in Cairns
 - ii** Hobart when it is 10:00 am in Sydney
 - iii** Perth when it is 7:00 pm in Brisbane
 - iv** Adelaide when it is 5:00 pm in Perth.
- 13** Use this bus timetable to answer the following questions.

Saturday									
map ref	Route Number	333	333	333	333	333	333	333	333
C	North Bondi Military Rd	06:28	06:57	07:28	07:56	08:28	08:42	08:54	09:09
D	Bondi Beach Campbell Pde	06:30	06:59	07:30	07:58	08:30	08:44	08:56	09:11
G	Bondi Jn Interchange Stand N	06:41	07:10	07:41	08:09	08:41	08:55	09:10	09:25
H	Darlinghurst Taylor Square	06:51	07:20	07:51	08:19	08:51	09:06	09:21	09:36
J	City - Circular Quay Young St	07:00	07:29	08:00	08:28	09:00	09:15	09:30	09:45

- a** Mick arrives at North Bondi Military Rd at 7:00 am. How long must he wait for the next bus to Circular Quay?
- b** At what time does Mick arrive at Circular Quay?
- c** How long was the bus trip?
- d** Mick's friend catches the 7:58 am bus from Bondi beach. At what time does she arrive at Circular Quay?
- e** How long did Mick wait at Circular Quay for his friend to arrive?

10

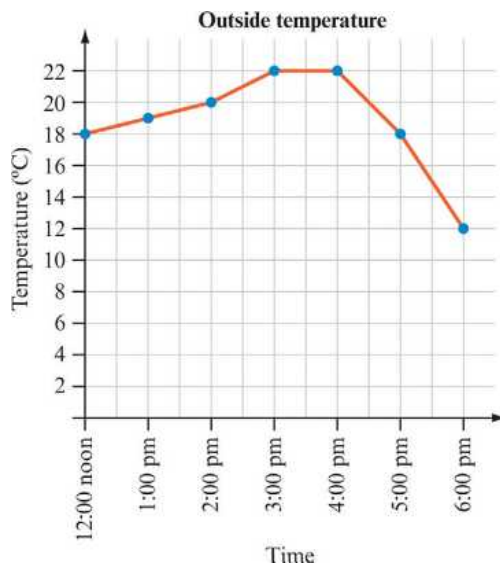
Budgeting and household expenses

The main mathematical ideas in this chapter are:

- ▶ calculating and comparing household running, maintenance and repair costs
- ▶ interpreting tables and graphs to calculate motor vehicle purchase price and insurance costs
- ▶ calculating the cost of finance
- ▶ calculating fuel consumption
- ▶ calculating depreciation
- ▶ calculating the running costs of motor vehicles
- ▶ preparing personal budgets.

ARE YOU READY?

Use this graph to answer questions 1 to 4.



- 10A ▶ 1** What is the title of the vertical axis?
A Outside temperature
B 6:00 pm
C Temperature
D Time
- 10A ▶ 2** Between which times is the temperature constant?
A 12:00 pm to 1:00 pm
B 2:00 pm to 3:00 pm
C 3:00 pm to 4:00 pm
D 5:00 pm to 6:00 pm
- 10A ▶ 3** Between which times was the greatest decrease in temperature?
A 12:00 pm to 1:00 pm
B 2:00 pm to 3:00 pm
C 3:00 pm to 4:00 pm
D 5:00 pm to 6:00 pm
- 10A ▶ 4** When was the temperature 18°C?
A 12:00 pm and 5:00 pm
B 2:00 pm and 3:00 pm
C 3:00 pm and 4:00 pm
D 1:00 pm and 6:00 pm
- 10A ▶ 5** What is the result for $15.687 - 7.884$?
A 8.203 **B** 8.207
C 8.803 **D** 7.803
- 10A ▶ 6** What is the result for $35.209 + 142.36$?
A 494.45 **B** 49.445
C 177.245 **D** 177.569
- 10A ▶ 7** What is the result for 23.08×8.5 ?
A 196.18 **B** 31.58
C 14.58 **D** 2.715
- 10A ▶ 8** What is the result for $6 + 4 \times 5 - 8$?
A 42 **B** 18
C 6 **D** -6
- 10A ▶ 9** A car travels 140 km in 4 hours. What is its average speed?
A 560 km/h **B** 144 km/h
C 140 km/h **D** 35 km/h
- 10B ▶ 10** What is the value of the expression $3a - 5b + 5$ when $a = 2$ and $b = -4$?
A 31 **B** 21
C -12 **D** -17
- 10C ▶ 11** A car uses 55 L of petrol to travel 506 km. What is the fuel consumption in litres per 100 km?
A 9.2 L/100 km **B** 10.2 L/100 km
C 10.5 L/100 km **D** 10.9 L/100 km

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook assess](#).

- Q1-4** Support sheet 10A.1 Interpreting data in graphical form
Q5-7 Support sheet 10A.2 Operations with decimals
Q8 Support sheet 10A.3 Understanding order of operations
Q9 Support sheet 10A.4 Writing rates
Q10 Support sheet 10B.1 Substituting for pronumerals
Q11 Support sheet 10C.1 Calculating rates

10A

Household bills

These resources are available on your obook assess:

- **assess quiz 10A:** Test your skills with an auto-correcting multiple-choice quiz

10
10

Organisations that provide services such as electricity, gas and water are called utilities. In this section you will read, interpret and perform calculations relating to various utility bills.

EXERCISE 10A Household bills

UNDERSTANDING, FLUENCY AND COMMUNICATING

- Use the sample water bill on the opposite page to answer questions 1 and 2.
 - What is the supply period for this bill?
 - How many days are there in the supply period?
 - The fixed charges are those payable for having water and sewerage available to your property. What is the total of the fixed charges for this account?
 - How much water did this household use for the period shown?
 - Using the answer for part a ii, what is the average daily usage?
 - The usage charge is what we pay for the amount of water that we use.
 - What is the cost per kilolitre of water?
 - What is the total cost (fixed and usage) for this period?
 - Using the answer for part a ii, what is the average daily cost?
 - Assuming the same fixed and usage charges, was this bill smaller or larger than the last bill?
 - Was this the bill smaller or larger than for the same period last year?
 - Assume the same fixed and usage charges.
 - Calculate the water usage charge for the last bill.
 - Calculate the total charges for the last bill.
 - The water usage charge last year was \$2.13 per kilolitre. What is the difference between the water usage charge for this bill and that for the bill for the same period last year?
 - How did the average daily water consumption for this bill compare with the local area average?
 - The table 'Targets for water-efficient households' in the sample water bill allows us to compare a household's water usage with the supplier's targets for ideal water-efficient households. This bill is for a medium-sized property with five occupants.
 - Is this household's usage smaller or larger than the ideal efficient usage shown in the table?
 - By how much would this household have to change its daily usage to reach the ideal?
 - If a bucket can hold 12 L of water, to how many buckets is your answer for part ii equivalent?
- Refer to the 'Targets for water-efficient households' table on the water bill to answer these questions. What is the ideal daily water consumption for the following households?

a a large property with four people	b a small property with two people
c a 600 m ² property with three people	d an 18 m by 40 m property with six people
- In 2018, Hunter Water charged residential properties \$2.25 per kilolitre for water usage. The water availability charge was \$25.69 p.a. Calculate the total annual cost for a property that used 190 kL of water.

MONT BAY WATER

Mr S Ample
12 Water Street
NORTH WATERVILLE

Customer Number	22988701
Due Date	23 April 2018
Amount	\$348.99

ACCOUNT DETAILS

Account for residential property

Usage Charges (GST does not apply)
For period 01/01/2018 to 31/03/2018 (90 days)

Water Usage	
68 kL @ \$2.17 per kL =	\$147.56
Total Usage Charges	\$147.56

Service Charge Details (GST does not apply)

Water Service Charge	\$47.68
Sewerage Service Charge	\$153.75
Total Service Charges	\$201.43

Total Current Charges **\$348.99**

YOUR CHARGES EXPLAINED

Water Usage

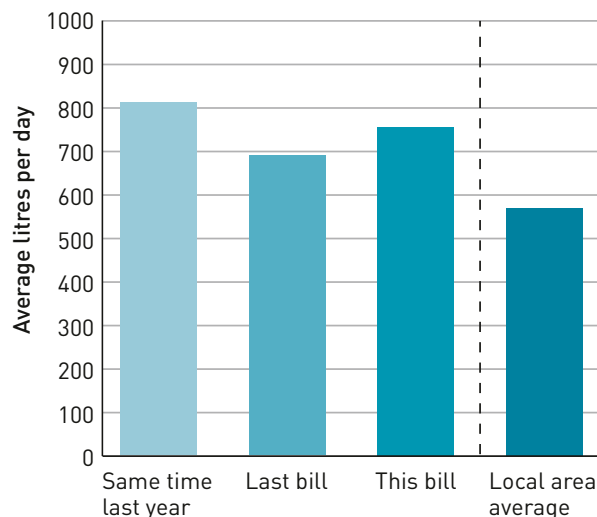
Recorded by your water meter, this charge covers the amount of water used at your property. This charge is billed per kilolitre (kL) of water you use.

Service Charges

Your water and sewerage service charges are fixed charges for access to our water supply and sewerage systems. They also help us maintain, renew and expand these systems so we can continue to provide you with high quality drinking water and safe sewerage removal now and into the future.

NOTICEBOARD

Your average daily water usage comparison



Targets for water-efficient households

People per household	Property size		
	Small	Medium	Large
1 person	247 L/day	249 L/day	267 L/day
2 people	367 L/day	375 L/day	396 L/day
3 people	463 L/day	477 L/day	498 L/day
4 people	546 L/day	565 L/day	587 L/day
5 people	621 L/day	645 L/day	666 L/day
6 people	689 L/day	718 L/day	739 L/day

Note:

A small property has an area less than 500 m².

A medium property has an area between 501 m² and 700 m².

A large property has an area between 701 m² and 900 m².

- 4 a** In 2018, Sandhaven Water charged residential properties \$1.55 per kilolitre for water usage up to 450 kL and \$1.75 per kilolitre for each kilolitre more than 450 kL. The water availability charge was \$92. What was the annual cost of water for a household that used:

- i** 270 kL? **ii** 480 kL?

- b** For non-residential properties, the water usage charge was \$1.55 per kilolitre. The water availability charge is based on the size (diameter) of the water meter service connection(s), as shown in the table on the right. If there is more than one meter connection to a property, each is charged separately. What would be the water availability charge for a non-residential property that is supplied by:

- i** one 32 mm connection?
ii one 25 mm and one 40 mm connection?
iii one 50 mm connection?
iv two 25 mm connections?

Meter connection	Charge (\$)
20 mm	81
25 mm	127
32 mm	207
40 mm	324
50 mm	506
100 mm	2025

- c** A non-residential property has one 20 mm, one 32 mm and one 40 mm connection. If the amount of water used was 560 kL, what was the total annual cost?

- 5** Use the sample electricity account on the opposite page to help you answer the following questions.

- a i** What is the supply period for this bill? **ii** How many days are in the supply period?
- b** The Electricity Service Availability charge is the charge payable for having electricity available to a property (the cost of supplying and maintaining the poles and wires).
i What is the daily charge on the bill? **ii** What is the total charge for this account?
- c** The usage charge is a payment for the amount of electricity used.
i What was the total amount of electricity used by this household for the period shown?
ii What is the average daily usage?
- d** The cost of usage is broken down into peak energy rate, shoulder energy charge and off-peak energy rate. What percentage of usage occurred in:
i the shoulder period? **ii** the off-peak period?
- e** What is the peak energy rate per kilowatt-hour?
- f** What is the difference between the peak and off-peak rates per kilowatt-hour?
- g i** What is the total energy usage charge for this quarter?
ii What is the total cost (availability and usage) for this quarter, after GST?
iii What is the average daily cost?
- h** Assuming the same fixed and usage charges, was this bill smaller or larger than:
i the previous bill? **ii** the bill for the same period last year?
- i** Consider the bill for the same period last year.
i What was the average daily usage?
ii Calculate the total amount of electricity used for that period.
- j** If the total electricity usage on last year's bill (calculated in part **i ii**) is broken down in the same proportion as the current bill, what would have been the amount of electricity used last year in the:
i peak period? **ii** shoulder period? **iii** off-peak period?
- k** Using your answers for part **j**, and assuming the same rates as the current bill, calculate the:
i energy usage charge for the same period last year **ii** service availability charge
iii total of the usage and availability charges for this period
iv total charges after the discount is applied and the GST is added.
- l** Consider the electricity usage comparison. In which month was the:
i most energy consumed? **ii** least shoulder rate energy used?
iii most off-peak rate energy used?

FIRST COMBINATION *energy*

Mr P Smith
71 Winter Grove
SUMMERTON

Customer Number 0123456

Due Date 18 March 2018

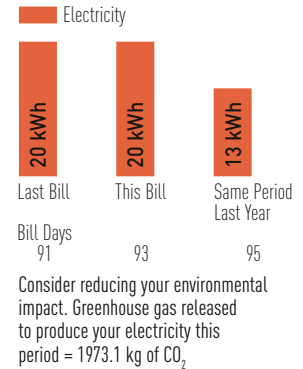
Amount Payable \$621.41

Quarterly Electricity Account

LOCATION: 71 Winter Grove, SUMMERTON

Previous Amount Payable	529.44
Payment Received — Thank You	–529.44 Cr
Electricity (22/11/2017 to 22/02/2018)	594.65
Electricity Discount 5% (22/11/2017 to 22/02/2018)	–29.73 Cr
Subtotal of Charges before GST	564.92
Total GST Payable 10%	56.49
Total Charges including GST	621.41
Total Amount Payable	\$621.41

AVERAGE DAILY USAGE



Energy Used & Costs

METER ID	THIS READING	–	LAST READING	=	ENERGY USED	x	RATE	=	COST
Peak Energy Rate — Contract (22/11/17–22/02/18)									
EDX009745/001	649.9		0.0		649.9 kWh		47.7700c		\$310.44
Shoulder Energy Charge — Contract (22/11/17–22/02/18)									
EDX009745/002	940.8		0.0		940.8 kWh		19.4000c		\$182.52
Off-Peak (Night Rate) Energy Rate — Contract (22/11/17–22/02/18)									
EDX009745/003	270.7		0.0		270.7 kWh		11.9000c		\$32.22
Electricity Service Availability Charge			93 days				74.7000c/day		\$69.47
Total Electricity Before GST					1861.4 kWh				\$594.65

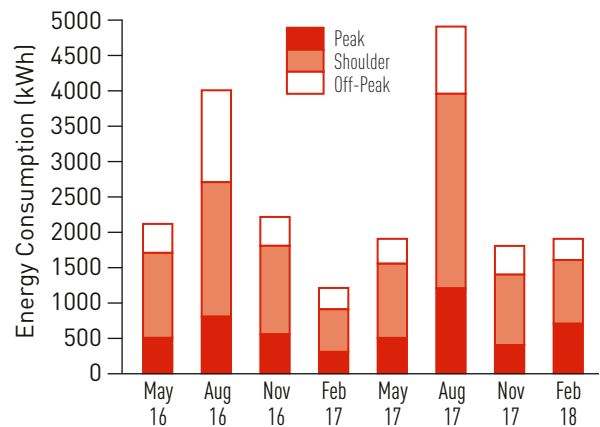
PowerSmart Home Electricity Usage Summary

Supply Period:
22 November 2017 to 22 February 2018 — 93 days

USAGE BREAKDOWN

Peak	650 kWh	34.91%
Shoulder	941 kWh	50.54%
Off-Peak	271 kWh	14.54%
TOTAL ENERGY	1861 kWh	

ELECTRICITY USAGE COMPARISON



Mr A Winters
2 Spring Street
GREENHILL

Your account number 0123456

Due date 27 May 2018

Total amount due \$187.87

Your account summary

Supply period 2 Feb 2018 to 3 May 2018

Previous balance \$183.47
Payment received \$183.47 cr

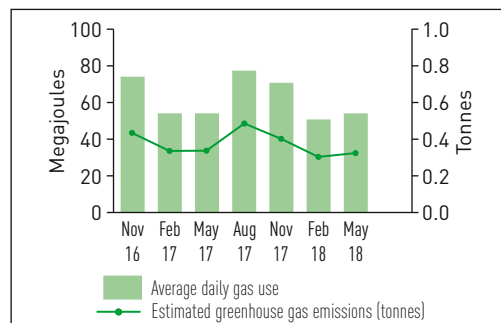
Balance brought forward \$0.00

New charges and credits (see details below)

Usage and supply charges \$170.79
Total GST for new charges \$17.08

Total amount due \$187.87

Your overall picture.



Usage

Average usage per day in this billing period **53.30 MJ**

Same time last year **53.33 MJ**

Average cost per day **\$2.06**

ESTIMATED GREENHOUSE GAS EMISSIONS FOR THIS BILL **0.3 tonnes**

Your account in detail

DPI 51408525661

Reading type Actual read on 3 May 2018 for 91 days

Tariff description Residential Standard

Meter number EA186971

Previous reading	Current reading	Units	Multiplier value	Heating factor	Conversion	Usage MJ
7486	7611.01	125.01	1	38.27165	1.013730	4850

Balance brought forward **Charge \$0.00**

New charges and credits

Usage and supply charges

Gas Consumption 3740 MJ @ \$0.02967 \$110.97
Next 1110 MJ @ \$0.01758 \$19.51
Supply Charge \$40.31
Total usage and supply charges \$170.79
Total GST for new charges \$17.08

Total amount due \$187.87

- 6** Use the sample gas account on the opposite page to answer the following questions.
- a** **i** What is the supply period for this account? **ii** If the dates are inclusive, how many days is this?
 - b** **i** How much GST is included in the total amount due?
ii What percentage is this of the usage and supply charges?
 - c** What was the total number of megajoules used in this period?
 - d** What is the cost/MJ for: **i** the first 3740 MJ used? **ii** the next 1110 MJ used?
 - e** **i** What is the supply charge?
ii Assuming that the usage and supply charges were the same for the previous bill, calculate the total amount due if this household used 4510 MJ.
 - f** For this billing period, what was: **i** the average daily usage? **ii** the average cost per day?
 - g** What were the estimated greenhouse gas emissions for this billing period?
 - h** Calculate the total usage for the same time last year if the billing period was 90 days.

- 7** The table below shows part of the rates and charges notice from a Sydney council.

Details	Rate (\$)	Rateable value (\$)	Amount (\$)
Residential: on land value	0.001 454 74 per \$ rateable value	599 000	871.39
Residential: base amount	465		465
Domestic waste availability charge	70		70
Domestic waste-management charge	291		291
Total of this notice			

- a** **i** What is the residential rate charge, in dollars, for properties in this council area?
ii Multiply the rate from part **i** by the rateable value of the land. Is the answer the amount in the last column for 'Residential: on land value'?
 - b** Calculate the total charge for domestic waste services.
 - c** Calculate the total rates payable.
 - d** Calculate the total rates payable for another residential property in this municipality if this property has a rateable land value of \$726 000.
- 8** **a** The average rate of flow of a bathroom shower with a normal shower rose is 18 L/min. Jenny has two 8-minute showers each day. Calculate the annual cost of her showers if the water usage charge is \$2.45/kL. (Use 1 year = 365 days.)
- b** Jenny's brother Sam has one 20-minute shower each day. What is the annual cost of Sam's showers?
- c** How much could Jenny and Sam save each year by installing a water-efficient shower rose that has a flow rate of 8.5 L/min?

- 9** **a** There are four people in the Lee family. Use the table to help you find the number of litres per year they would save by replacing their full-flush toilet with a 3-star rated dual-flush toilet.

Toilet	Litres of water used
Full-flush	54 L per person per day
3-star rated dual-flush	18 L per person per day

- b** What would be the cost savings if the water usage charge is \$2.75/kL?
- 10** A 7.0 kg top-loading washing machine that costs \$556 uses 170 L of water per wash. A 7.0 kg front-loading machine of the same brand costs \$660 and uses 80 L per wash. The Lee family who use their washing machine four times per week are considering whether to buy the top-loading or front-loading machine.
- a** Calculate the annual water usage charges for each machine if water costs \$2.25/kL. (Use 1 year = 52 weeks.)
 - b** What is the annual saving in water costs if they buy the front-loading machine?
 - c** **i** What is the difference in the purchase price of the machines?
ii If they purchase the front-loading machine, how long will it take to break-even on total costs (purchase price and water usage)?

10B Purchasing a car

These resources are available on your obook assess:

- **Video tutorial 10B:** Watch and listen to an explanation of Example 10B-3
- **Spreadsheet 10B:** Compare the purchase costs for different vehicles
- **assess quiz 10B:** Test your skills with an auto-correcting multiple-choice quiz

There are a number of extra costs added to the retail price when purchasing a new car. Registration, stamp duty and compulsory third-party insurance are compulsory costs. There may also be a dealer delivery charge, other insurance costs and, if borrowing money to purchase the car, interest charges on the loan.

Registration

The fee to transfer the registration of a vehicle from one owner to another was \$32 in 2017. The cost of registration of a new vehicle depends on the weight of the vehicle without a load (this is also referred to as the 'kerb weight' or tare weight) and whether the vehicle is being used mostly for private use or mostly for business use. The table below shows some of these costs.

Table 10-1: Registration of motor vehicles in NSW

Size of vehicle	Tare weight	Private use (\$)	Business use (\$)
Cars, station wagons, trucks	up to 975 kg	207	336
	976 kg to 1154 kg	240	382
	1155 kg to 1504 kg	293	463
	1505 kg to 2504 kg	448	697
Motorcycles		62	62

Source: www.rta.nsw.gov.au

Stamp duty

Stamp duty is a state tax that is based on the market value of the vehicle or the price paid for it, whichever is the greater. It is charged at the following rate:

- 3% of the market value up to \$45 000 plus
- 5% of the value over \$45 000.

(Go to the RTA website and click on the link to stamp duty to see the stamp duty costs for NSW.)

Insurance

Compulsory third-party (CTP) insurance, often referred to as a 'green slip', is necessary to register a vehicle in NSW. This CTP insurance is paid each time a vehicle is registered. It provides compensation to other people injured or killed when the insured driver's vehicle is involved in an accident. (It does not cover damage to other vehicles, property or your vehicle or theft of your vehicle. Other types of insurance are available for these.)

The cost of a green slip varies and depends on a number of factors, including the type and age of the vehicle, where it is garaged, and the age and driving record of the drivers. You can find examples of the cost of CTP (premiums) on the State Insurance Regulatory Authority website at www.sira.nsw.gov.au.

There are types of insurance, other than CTP, available for motor vehicles, such as comprehensive insurance and third-party property insurance. These are not compulsory, but are advisable.

- Comprehensive insurance covers damage to, or theft of, your vehicle as well as damage to other vehicles and property in the case of an accident.
- Third-party property damage vehicle insurance covers you for damage caused by your vehicle to someone else's vehicle or property. (It does not cover damage to, or theft of, your vehicle.)

Comprehensive insurance is more expensive than third-party property insurance because it covers more types of incidents. Both types of insurance vary in price according to factors such as the driver's age, driving experience and driving record, the type and age of the vehicle and where it is garaged. Many insurance companies will give an online quote for car insurance.

Dealer delivery

When purchasing a new car, the dealer will often charge a fee for 'dealer delivery'. This fee represents the cost to the dealer of preparing the car for delivery to the buyer. The fee varies from dealer to dealer and is sometimes waived or reduced in order to induce people to buy from one dealer rather than another.

Trade-in

As part of the agreement when you purchase a new car, the dealer may buy your current car, if you have one, at an agreed price. When you sell your current car to the dealer in this way, it is called a trade-in.

EXAMPLE 10B-1 Calculating the cost of car registration

Use Table 10-1 to determine how much extra it costs to register a new Toyota Corolla, which weighs 1250 kg, for business use compared with private use.

Solve	Think	Apply
$\text{Extra cost} = \$463 - \$293$ $= \$170$	In Table 10-1, 1250 kg is in the range 1155 kg to 1504 kg. Cost for business use = \$463 Cost for private use = \$293	In the table, find the weight range in which the vehicle lies and read the costs for business and private registration. Find the difference between these two costs.

EXERCISE 10B Purchasing a car

- Complete the following to calculate the cost of registration, for private use, of a vehicle that weighs 1060 kg. In Table 10-1, 1060 kg is in the range ____ kg to ____ kg.
Cost to register for private use = ____.
- Use Table 10-1 to help you calculate the cost of registration, for private use, of a vehicle that weighs:
 - 1820 kg
 - 1150 kg
 - 1450 kg
- How much more expensive is it to register a vehicle for business use than for private use, if the vehicle weighs:
 - 2130 kg?
 - 975 kg?
 - 1200 kg?

EXAMPLE 10B-2 Calculating the stamp duty on a new car

Calculate the stamp duty to be paid on the purchase of a new Mazda that has a recommended retail price of \$57 000.

Solve	Think	Apply
Stamp duty $= 0.03 \times 45\,000 + 0.05 \times (57\,000 - 45\,000)$ $= \$1950$	$3\% = 0.03$ $3\% \text{ of } \$45\,000 = 0.03 \times \$45\,000$ The value over \$45 000 $= \$57\,000 - \$45\,000$ $5\% = 0.05$ $5\% \text{ of the value over } \$45\,000$ $= 0.05 \times (\$57\,000 - \$45\,000)$	If the price paid for the car is not more than \$45 000, the stamp duty is 3% of its price. If the price is more than \$45 000, the stamp duty is 3% of \$45 000 (\$1350) plus 5% of \$(price - 45 000).

4 Complete the following to calculate the stamp duty.

a Market value = \$17 900

Stamp duty

= 3% of \$_____

= $0.03 \times \$$ _____

= \$_____

b Market value = \$52 380

Stamp duty

= _____% of \$45 000 + _____% of \$(_____ - \$45 000)

= $0.\text{_____} \times \$45\,000 + 0.\text{_____} \times \$$ _____

= \$_____

5 Calculate the stamp duty that would be charged on a vehicle that has a market value of:

a \$21 990

b \$35 699

c \$49 000

d \$93 600

EXAMPLE 10B-3 Calculating the total cost of buying a new car

- a** Calculate the total cost of purchasing a new Ford with a recommended retail price (RRP) of \$37 000 and weight of 1704 kg. The CTP insurance for the car is \$487 and the dealer delivery charge is \$528. The car is for private use and the owner decides to take out comprehensive insurance that costs \$960 for the first year.
- b** If the buyer receives a trade-in of \$15 000 on her old vehicle, what is the total cost for changing to the new car?

	Solve	Think	Apply
a	RRP = \$37 000 Registration = \$448 $\text{Stamp duty} = 0.03 \times \$37\,000$ $= \$1110$ CTP insurance = \$487 Dealer delivery = \$528 Comprehensive insurance = \$960 Total = \$40 533	The weight of the car (1704 kg) is in the range 1505 kg to 2504 kg. From Table 10-1, the registration cost for private use is \$448. The RRP of the car is less than \$45 000; so stamp duty is 3% of its price (\$37 000). Add all costs to the RRP of the car.	Use the weight of the vehicle to determine the registration cost from Table 10-1. Calculate the stamp duty. Add all the extra costs to the RRP of the car.
b	Changeover price $= \$40\,533 - \$15\,000$ $= \$25\,533$	Total cost = \$40 533 Trade-in price = \$15 000 Changeover price is the difference.	The changeover price is the total price of the vehicle less the trade-in.

- 6 Complete the following table to calculate the total cost to purchase each of the following new vehicles.

Costs	Holden	Mazda	Toyota sedan	Toyota SUV	Yamaha motorcycle
RRP (\$)	37 990	31 450	18 000	52 870	14 999
Weight (kg)	1637	1440	1040	1970	
Private (P) or business (B)	B	P	P	B	P
Registration (\$)					
Stamp duty (\$)					
CTP insurance (\$)	620	487	528	660	280
Dealer delivery (\$)	790	585	499	887	389
Insurance (\$)	1089	790	560	1185	299
Total					

EXAMPLE 10B-4 Calculating the total cost of buying a used car

Find the total cost of purchasing and insuring a three-year-old Honda hatchback that has an advertised price of \$10 500. Third-party property damage insurance is \$479.

Solve	Think	Apply
Dealer price = \$10 500 Transfer of registration = \$32 Stamp duty = $0.03 \times \$10\,500$ = \$315 Insurance = \$479 Total = \$11 326	Transfer of registration fee is \$32 for used vehicles. The price of the car is less than \$45 000, therefore stamp duty is 3% of its \$10 500 price.	The transfer of registration fee must be paid. Stamp duty is charged when there is a change of ownership of a vehicle. The CTP insurance was paid when the car was last registered, so is not due until the next time registration is due.

- 7 Complete the following to find the total cost of purchasing and insuring a four-year-old Nissan sports car that is advertised for \$43 900. Comprehensive insurance is \$1560.

Dealer price = \$43 900

Transfer of registration = \$_____

Stamp duty = $0.03 \times \$\_\_\_\_\_\_ = \$\_\_\_\_\_\_$

Insurance = \$_____

Total = \$_____

- 8 Find the total cost of purchasing and insuring a one-year-old Toyota sedan that has an advertised price of \$23 900. Third-party property insurance is \$463.
- 9 Find the total cost of purchasing and insuring a two-year-old Kawasaki 1400cc motorcycle that has an advertised price of \$14 890. Comprehensive insurance is \$678.



- 10 a** Stamp duty is calculated as 3% of the price of the vehicle up to \$45 000, plus 5% of the price over \$45 000. Complete the following table.

Stamp duty on vehicle

Price (\$'000)	3% of the price up to \$45 000	5% of the price over \$45 000	Total
10	$0.03 \times \$10\,000 = \$300$		
20			
30			
40			
45	$0.03 \times \$45\,000 = \$1350$		
50	$0.03 \times \$45\,000 = \$1350$	$0.05 \times (\$50\,000 - \$45\,000) = \$250$	\$1600
60	$0.03 \times \$45\,000 = \$1350$	$0.05 \times (\$60\,000 - \$45\,000) = \$750$	
70			
80			

- b** Use the information in the table to draw a piecewise graph, with price (the independent variable) on the horizontal axis and stamp duty (the dependent variable) on the vertical axis.
- c** Use the graph to estimate the stamp duty on a vehicle purchased for:
- i** \$35 000 **ii** \$65 000 **iii** \$78 000
- 11 a** Go to the State Insurance Regulatory Authority website at www.sira.nsw.gov.au and use the following information to get a quote on a green slip for Richard.
- Richard is buying a new car. There is no entitlement to GST input credit. The start date for the insurance will be the 1st of the month/next month/this year. The vehicle is a new Mazda 6 sedan that is normally garaged at Parramatta, postcode 2150. The car will be privately owned and registered for private use. It does not have a current CTP insurance policy because it is a new car. It will also be covered by comprehensive insurance with NRMA Insurance. Richard has continuously held comprehensive insurance with NRMA for 2 years. The policy does not have a no-claim discount. Richard, the sole owner/driver, is 20 years old. He has not had any accidents in the 3 years he has had his licence and has not lost any demerit points. Richard is not a member of NRMA roadside service.
- b** What is the cheapest quote for Richard's green slip?
- c** What would the cheapest quote be if Richard had had one at-fault accident and lost 4 demerit points.
- 12 a** Search the internet for an insurance company (such as NRMA insurance at www.nrma.com.au) and use the following information to get a quote for comprehensive insurance for this new car.
- Vehicle details:* Toyota Corolla hatchback 1.8 that is usually garaged at Manly, postcode 2095. Insurance cover required is \$21 990. The car is for private use and the owner has no finance owing on the car. The driver is a 20-year-old male (enter a date of birth that makes the driver 20-years old) with 2 years driving experience and has had no accidents. He wants an excess of \$600 on the policy and has no other relevant policies or memberships. As this is his first car there is no previous insurer.
- b** Vary the age of the driver (say 20, 25, 30, 35 years, etc.) and record and compare costs.
- c** Vary the gender of the driver for the ages used in part **b** and compare costs.
- d** Using a map and a list of postcodes, vary the locality where the vehicle is garaged and compare costs.
- e** Investigate the change in costs for a driver who has had an at-fault accident.
- f** Vary the type of vehicle and compare costs. For example, compare the costs for small versus large passenger vehicles, 4WDs, people movers and light commercial vehicles.
- g** Compare the costs from other insurers. (Try www.comparethemarket.com.au/car-insurance/australia/new-south-wales)
- h** Is there an age excess to be paid on top of the basic excess?

- 13** Investigate and compare the advantages and disadvantages of comprehensive and third-party property insurance for cars. In what circumstances might one form of insurance be more suitable than the other?
- 14** Investigate and compare the costs of third-party property insurance at www.nrma.com.au/car-insurance. Use the example and investigations suggested in question **12**.
- 15** From the information gathered in the previous questions, make a list of the factors that affect insurance premiums (such as type of vehicle, driver experience, etc.)
- 16** A spreadsheet can be used to calculate the total purchase cost of a vehicle and to compare costs. The spreadsheet below has two vehicles for comparison but using **fill across** will give more information. Enter the RRP, weight and type of use, and then the registration and stamp duty will be calculated automatically. Then enter CTP, dealer delivery and comprehensive insurance.

	A	B	C
1	Comparing the costs of purchasing vehicles		
2	Cost items	Mazda sedan	Holden sedan
3	RRP	\$28,500.00	\$57,888.00
4	Weight (kg)	1440	1720
5	Private (P) or business (B)	P	B
6	Registration	\$385.00	\$789.00
7	Stamp Duty	\$855.00	\$1,994.40
8	CTP Insurance		
9	Dealer Delivery		
10	Comprehensive Insurance		
11	Total	\$29,740.00	\$60,671.40
12			
13			
14			
15	Registration Costs		
16	Enter weight in kilograms	1440	1720
17	Private Use	\$385.00	\$540.00
18	Business Use	\$555.00	\$789.00

In cell B6, the formula to calculate the registration of the Mazda sedan is =IF(B5 = "P",B17,B18)

In cell B7, the formula to calculate the stamp duty is

=IF(B3>45000,(B3-45000)*0.05+1350,B3*0.03)

In cell B11, the formula to calculate the total cost is =SUM(B6:B10)+B3

In cell B17, the formula to calculate the private use registration is

=IF(B16>1504,448,IF(B16>1154,293,IF(B16>975,240,207)))+92

In cell B18, the formula to calculate the business registration is

=IF(B16>1504,697,IF(B16>1154,463,IF(B16>975,382,336)))+92.

Use **fill across** to find the costs for the second vehicle (the Holden sedan).

Enter data for other vehicles.

To help you, you might like to use the prepared spreadsheet file (Spreadsheet 10B) provided on your [obook assess](#).

10C Financing a purchase

These resources are available on your obook assess:

- **Spreadsheet 10C:** Calculate monthly repayments on a reducing-balance loan
- **assess quiz 10C:** Test your skills with an auto-correcting multiple-choice quiz

Often people need to borrow money to buy a car. There are many financial institutions that will provide a car loan or personal loan for this purpose.

The following table shows the monthly repayments (\$) for every \$1000 borrowed on a reducing-balance car loan.

Table 10-2: Reducing-balance loan

Interest rate (%p.a.)	Term of loan (months)				
	12	24	36	48	60
8	86.99	45.23	31.34	24.41	20.28
9	87.45	45.68	31.80	24.89	20.76
10	87.92	46.14	32.27	25.36	21.25
11	88.38	46.61	32.74	25.85	21.74
12	88.85	47.07	33.21	26.33	22.24
13	89.32	47.54	33.69	26.83	22.75
14	89.79	48.01	34.18	27.33	23.27

EXAMPLE 10C-1 Performing loan calculations

Use Table 10-2 to answer the following questions.

- Calculate the monthly repayments on a loan of \$23 600 at 9% p.a. reducible over 5 years.
- What is the total amount of interest paid on this loan?
- How much would be saved by repaying the loan over 4 years instead of 5 years?

	Solve	Think	Apply
a	Monthly repayment $= \$20.76 \times 23.6$ $\approx \$489.94$	5 years = 60 months From Table 10-2, monthly repayment on \$1000 over 60 months at 9% p.a. is \$20.76. The number of \$1000s being borrowed $= \frac{23\,600}{1000} = 23.6$. Monthly repayment $= \$20.76 \times 23.6$	Convert the term of the loan to months. From Table 10-2, find the monthly repayment on \$1000 for the interest and term. Divide the amount of the loan by 1000 to determine the number of thousands (\$) borrowed. Multiply the monthly repayment for \$1000 by the number of 1000s borrowed.

	Solve	Think	Apply
b	Total amount repaid $= \$489.94 \times 60$ $= \$29\,396.40$ Interest paid $= \$29\,396.40 - \$23\,600$ $= \$5796.40$	Total amount repaid $= \$489.94 \times 60$. Amount borrowed was \$23 600. Interest paid over 5 years $= \$489.94 \times 60 - \$23\,600$.	Total amount repaid is the monthly repayment by the number of months. The difference between this and the amount borrowed is the interest paid on the loan.
c	Monthly repayment over 4 years $= \$24.89 \times 23.6$ $\approx \$587.40$ Total amount repaid $= \$587.40 \times 48$ $= \$28\,195.20$ Amount saved $= \$29\,396.40 - \$28\,195.20$ $= \$1201.20$	Amount saved $= \text{amount repaid over 5 years} - \text{amount repaid over 4 years}$ Amount saved is the difference between \$29 396.40 and \$28 195.20.	The amount saved is the difference between the total amount repaid over the longer term and the total amount repaid over the shorter term.

Note: most financial institutions have an online calculator that can be used to calculate the monthly repayments on a loan. Visit www.aussie.com.au/home-loan-calculator/repayment.html

EXERCISE 10C Financing a purchase

Use Table 10-2, or an online calculator, to answer the following questions.

- 1** Complete the following to calculate the monthly repayments on a car loan of \$22 700 at 12% p.a. reducible over 4 years.
 4 years = ____ months
 From Table 10-2, the monthly repayment on \$1000 over ____ months at 12% p.a. = \$____.
 Number of \$1000s being borrowed = $\frac{22\,700}{1000} = \underline{\hspace{2cm}}$
 Monthly repayment on \$22 700 = \$____ $\times$ ____ = \$____
- 2** Calculate the monthly repayments on these car loans:
 - a** \$25 000 at 11% p.a. reducible over 4 years
 - b** \$13 600 at 9% p.a. reducible over 3 years
 - c** \$38 900 at 14% p.a. reducible over 5 years
- 3** For the following loans, calculate:
 - i** the monthly repayment
 - ii** the total amount of interest paid.
 - a** \$18 200 at 10% p.a. over 4 years
 - b** \$8700 at 8% p.a. over 2 years
 - c** \$34 800 at 12% p.a. over 5 years



Depreciation

The depreciation of an item is its decrease in value due to age and usage. The value of an item after depreciation is called its salvage value, book value, scrap value or written-down value.

As seen in Chapter 6, the formula for the straight-line method of depreciation is:

$$S = V_0 - Dn$$

where S = salvage (current) value of the asset

V_0 = purchase price of the asset

D = amount of depreciation per time period

n = the number of time periods

EXAMPLE 10C-2 Calculating the book value of an asset using the straight-line method of depreciation

A new car is purchased for \$32 000. It depreciates in value at a rate of 16% of the original price per year. Calculate the book value of the car after 3 years.

Solve	Think	Apply
$V_0 = \$32\,000$ $n = 3$ $D = 16\% \text{ of } \$32\,000$ $= 0.16 \times 32\,000$ $= \$5120$ $S = V_0 - Dn$ $= 32\,000 - 5120 \times 3$ $= \$16\,640$	Identify the values for V_0 and n . Calculate the amount of the annual depreciation, D , by finding 16% of \$32 000.	Substitute the values of D , n and V_0 into the formula $S = V_0 - Dn$.

- 4 A new car is purchased for \$35 000. It depreciates in value at a rate of 15% of the initial cost per year. Complete the following to find the book value of the car after 3 years.

$$15\% \text{ of } \$35\,000 = \underline{\hspace{2cm}}$$

$$V_0 = \underline{\hspace{2cm}}, D = \underline{\hspace{2cm}}, n = \underline{\hspace{2cm}}$$

$$S = V_0 - Dn$$

$$= \underline{\hspace{2cm}} - \underline{\hspace{2cm}} \times 3$$

$$= \$\underline{\hspace{2cm}}$$

- 5 A new car is purchased for \$19 990. It depreciates in value at a rate of 18% of the initial cost per year. Calculate the book value of the car after 5 years.

- 6 A new car is purchased for \$56 000. It depreciates in value at a rate of 21% of the initial cost per year. Calculate the book value of the car after 4 years.



7 Heidi wants to buy a motor bike. She is offered a loan of \$14 100 at 9% p.a. over either 3 years or 4 years. How much would she save if she chose the shorter term?

8 a Jack needs to borrow \$19 600 to buy a car and can pay a maximum of \$450 per month. He is offered a loan at 12% p.a. Can he afford the loan? Give details.

b Could he afford the loan if the interest rate was 14%? What advice would you give Jack?

9 a Jo's monthly repayment on a loan at 13% p.a. over 3 years is \$808.56. How much did Jo borrow?

b Ben's monthly repayment on a loan at 10% p.a. over 4 years is \$798.84. How much did Ben borrow?

10 a A car depreciates in value from \$36 000 to \$19 000 in 2 years. Use the straight-line method to calculate the annual amount of depreciation.

b Calculate the annual depreciation as a percentage of the purchase price.

11 A car depreciates in value from \$44 900 to \$23 440 in 3 years.

a Use the straight line method to calculate the annual amount of depreciation.

b Calculate the annual depreciation as a percentage of the purchase price.

12 Create a spreadsheet to calculate the monthly repayments for every \$1000 borrowed on a reducing-balance car loan. Start with the information in Table 10-2 and adapt the spreadsheet so that you can easily change the interest rate and the term of the loan. You may like to use the prepared spreadsheet file (Spreadsheet 10C) provided on your obook assess to help you.

13 Investigate and write a report on the purchase of a motor vehicle. Include selecting the vehicle, making calculations for any funding needed, the type of lending institution and the lending rate, and the amount payable in stamp duty, registration and insurance. New and used vehicle prices can be found in motoring magazines and on internet websites such as www.redbook.com.au



10D Fuel consumption

These resources are available on your obook assess:

- **Video tutorial 10D:** Watch and listen to an explanation of Example 10D-1
- **assess quiz 10D:** Test your skills with an auto-correcting multiple-choice quiz

Fuel consumption is a comparison between two types of quantities: the distance travelled and the amount of fuel used. It is a rate.

The rate of fuel consumption can be determined by dividing the distance travelled by the amount of fuel used. This gives the number of kilometres travelled on 1 L of petrol. The rate can also be determined by dividing the amount of fuel used by the distance travelled, which gives the amount of fuel consumed in travelling 1 km. For ease of comparison, this second rate is usually expressed as L/100 km.

To calculate fuel consumption, we can use one of these formulas.

$$\text{Fuel consumption (km/L)} = \frac{\text{distance travelled (km)}}{\text{amount of fuel used (L)}}$$

$$\text{Fuel consumption (L/km)} = \frac{\text{amount of fuel used (L)}}{\text{distance travelled (km)}}$$

$$\text{Fuel consumption (L/100 km)} = \frac{\text{amount of fuel used (L)}}{\text{distance travelled (km)}} \times 100$$

These formulas can be rearranged to help you find the distance travelled or the amount of fuel used when the other two quantities are known.

The most common way of expressing fuel consumption is in L/100 km.

EXAMPLE 10D-1 Calculating the fuel consumption of a car

A car uses 50 L of petrol to travel 416 km. Calculate the fuel consumption of the car on this trip in:

- a** km/L **b** L/km **c** L/100 km.

	Solve	Think	Apply
a	Fuel consumption $= \frac{416 \text{ km}}{50 \text{ L}}$ $= 8.32 \text{ km/L}$ Car travels 8.32 km on 1 L of petrol.	To calculate the fuel consumption in km/L, divide the distance travelled (416 km) by the amount of petrol used (50 L).	Use the appropriate formula to obtain the fuel consumption in km/L, L/km or L/100 km. Fuel consumption (km/L) $= \frac{\text{distance travelled (km)}}{\text{amount of fuel used (L)}}$
b	Fuel consumption $= \frac{50 \text{ L}}{416 \text{ km}}$ $= 0.12 \text{ L/km}$ Car uses 0.12 L of petrol for every 1 km travelled.	To calculate the fuel consumption in L/km, divide the amount of petrol used (50 L) by the distance travelled (416 km).	Fuel consumption (L/km) $= \frac{\text{amount of fuel used (L)}}{\text{distance travelled (km)}}$

	Solve	Think	Apply
c	Fuel consumption $= 0.12 \text{ L/km} \times 100$ $= 12 \text{ L/km}$ So the car consumes 12 L of petrol for every 100 km travelled.	To calculate the fuel consumption in km/100 L, first calculate the fuel consumption in km/L and then multiply this value by 100. This is equivalent to: $0.12 \text{ L/km} = \frac{0.12 \text{ L}}{1 \text{ km}}$ $= \frac{0.12 \text{ L}}{1 \text{ km}} \times \frac{100}{100}$ $= \frac{12 \text{ L}}{100 \text{ km}}$ $= 12 \text{ L/100 km}$	Fuel consumption (L/100 km) $= \frac{\text{amount of fuel used (L)}}{\text{distance travelled (km)}} \times 100$

EXERCISE 10D Fuel consumption

- 1** If a car uses 35 L of petrol on a trip of 400 km, complete the following to calculate the rate of fuel consumption to two decimal places.
- a Fuel consumption in km/L = $\frac{400 \text{ km}}{\square \text{ L}} = \underline{\hspace{2cm}} \text{ km/L}$
- b Fuel consumption in L/km = $\frac{35 \text{ L}}{\square \text{ km}} = \underline{\hspace{2cm}} \text{ L/km}$
- c Fuel consumption in L/100 km = $\underline{\hspace{1cm}} \text{ L/km} \times \underline{\hspace{1cm}} \text{ km} = \underline{\hspace{2cm}} \text{ L/100 km}$
- 2** Calculate the fuel consumption for each of the following trips in:
- i km/L ii L/km iii L/100 km
- a A car travels 260 km on 28 L of petrol.
- b A car travels 220 km on 19 L of petrol.
- c A car travels 420 km on 48 L of petrol.

When the rate of fuel consumption is expressed in L/100 km, we can use the following formula to find the distance travelled in kilometres.

$$\text{Distance travelled (km)} = \frac{\text{amount of fuel (L)}}{\text{fuel consumption (L/100 km)}} \times 100$$

EXAMPLE 10D-2 Calculating the distance a car can travel on a given amount of fuel

How far can a Toyota hatchback travel on 48 L of petrol if its petrol consumption is 7.4 L/100 km?

Solve	Think	Apply
Distance travelled $= \frac{48}{7.4} \times 100$ $= 649 \text{ km}$	The number of 'lots of 7.4 L' used is $\frac{48}{7.4}$. The car travels 100 km for each 'lot of 7.4 L' used.	Distance travelled (km) $= \frac{\text{amount of fuel}}{\text{fuel consumption}} \times 100$ Fuel consumption is in L/100 km.

- 3** Complete the following to calculate how far a vehicle can travel on 45 L of fuel if the fuel consumption is 6.4 L/100 km.
- Distance travelled = $\frac{45}{\square} \times 100 = \underline{\hspace{2cm}} \text{ km}$

- 4 How far can a vehicle travel on:
- 35 L of fuel if the fuel consumption is 8.4 L/100 km?
 - 66 L of fuel if the fuel consumption is 9.6 L/100 km?
 - 94 L of fuel if the fuel consumption is 12.2 L/100 km?

When the rate of fuel consumption is expressed in L/100 km, we can use the following formula to find the amount of fuel used in litres.

$$\text{Amount of fuel used (L)} = \frac{\text{distance travelled (km)}}{100} \times \text{fuel consumption (L/100 km)}$$

EXAMPLE 10D-3 Calculating the amount of petrol needed to travel a given distance

Calculate the amount of petrol used by a Holden sedan on a trip of 640 km if the car's petrol consumption is 11 L/100 km.

Solve	Think	Apply
Amount of petrol used $= \frac{640}{100} \times 11$ $= 70.4 \text{ L}$	The number of 'lots of 100 km' travelled is $\frac{640}{100}$ (or 6.4). Each 'lot of 100 km' uses 11 L of petrol.	Amount of fuel used (L) $= \frac{\text{distance travelled}}{100} \times \text{fuel consumption}$ Fuel consumption is in L/100 km and distance travelled is in kilometres.

- 5 Complete the following to calculate the amount of fuel used by a vehicle on a trip of 1160 km, if the fuel consumption is 10.6 L/100 km.

$$\text{Fuel used} = \frac{\square}{100} \times \square = \square \approx \square \text{ L}$$

- 6 Calculate the amount of fuel used by a vehicle on a trip of:
- 325 km, if the fuel consumption is 8.4 L/100 km
 - 540 km, if the fuel consumption is 12.2 L/100 km
 - 270 km, if the fuel consumption is 6.7 L/100 km.

- 7 A sales representative averages 3400 km of city driving each month in a Ford sedan that has a fuel consumption of 11.4 L/100 km. Calculate the cost of petrol used in a month in which the average price of unleaded petrol (ULP) is 149.9c/L.

- 8 In 2006 the average fuel consumption of Australian vehicles was 13.8 L/100 km and the average yearly distance travelled was 17 600 km. If the average price of fuel was 135.9c/L, what was the average yearly fuel cost?

- 9 A Citroën hatchback uses 7.6 L/100 km of ULP and the diesel model of the same car uses 6 L/100 km of diesel fuel.
- Calculate the cost of driving the petrol model 780 km if ULP is 152.9c/L.
 - Calculate the cost of driving the diesel model 780 km if diesel fuel is 162.2c/L.
 - How much cheaper is the diesel option over this distance?



- 10** Harry owns a Holden sedan that runs on ULP and has a fuel consumption of 10.6 L/100 km. When converted to run on liquid petroleum gas (LPG), the car's fuel consumption is 13.5 L/100 km. Harry averages 19 000 km per year.
- Calculate the annual cost for each type of fuel (assuming that the car only runs on one type of fuel for the year) if the average price of ULP is 149.9c/L and LPG is 67.8c/L.
 - How much would Harry save in fuel costs for the year if he converted to the LPG model?
 - What would be the savings per month?
 - The cost of converting the car to LPG is \$2600. How many months would Harry take to break even, if he converted the car to LPG?
 - What distance would Harry travel before reaching the break-even point? (The break-even point is the point at which the cost of running on each type of fuel is the same.)
- 11** Jenny averages 13 000 km per year and wants to buy a new car. The car she likes is available with a petrol motor or diesel motor. The petrol model uses 12.8 L/100 km and the diesel model uses 7.8 L/100 km.
- If Jenny bought the car with the petrol engine, what would be her annual fuel cost if ULP is 152.9c/L?
 - If Jenny bought the car with the diesel engine, what would be her annual fuel cost if diesel fuel is 169.9c/L?
 - How much per year would she save by buying the car with the diesel engine?
 - What is the average monthly saving with the diesel engine?
 - The diesel car costs \$1200 more to buy than the petrol car. How many months would it take for Jenny to break even if she buys the diesel car?
 - What distance would Jenny travel before reaching her break-even point?



- 12** Search the internet for fuel-watch websites to investigate trends in fuel prices for:
- different types of fuel
 - different locations (for example, compare city and country prices).
- Collect and present the data in tables and graphs.

- 13** Investigate cycles in the price of ULP. Describe a strategy that could be used to save money on fuel costs. Some useful websites include:
- www.mynrma.com.au
 - www.motormouth.com.au
 - www.fueltrac.com.au

10E

Running costs of a car

These resources are available on your obook assess:

- **Spreadsheet 10E:** Compare the running costs for different vehicles
- **assess quiz 10E:** Test your skills with an auto-correcting multiple-choice quiz

standing costs

fixed costs; e.g. depreciation, loan interest, on-road costs such as registration, insurance and motoring organisation membership

operating costs

variable running costs that depend on how a vehicle is driven; e.g. cost of fuel, tyres, servicing and repairs

The total running cost of a vehicle is made up of **standing costs** (fixed) and **operating costs**.

Standing costs include the depreciation in value of the vehicle (the loss in value due to age and use), the interest charged on the loan used to purchase it, and on-road costs such as registration, CTP insurance and membership of a motoring organisation that provides roadside assistance (for example NRMA).

Operating costs are running costs that depend on how the vehicle is driven, such as the cost of fuel, tyres, servicing and repairs.



EXERCISE 10E Running costs of a car

- 1 Calculate the values missing from the table below, showing average annual running costs for some vehicles. The calculations are based on buying a new vehicle for private use and operating it for 5 years. The interest charges are based on the total cost of the new vehicle being financed by a loan. It is assumed that the vehicle travels 15 000 km each year.

Table 10-3: Average annual running costs

Average annual running costs	Ford hatchback (small)	Toyota hatchback (small)	Ford sedan (large)	Holden sedan (large)	Honda SUV	Toyota compact SUV
<i>Standing costs:</i>						
Depreciation (\$)	3120	3136	5980	5928	4628	4784
Interest (\$)	1524	1560	2444	2444	2340	2288
On-road costs (\$)	1248	957	1092	962	988	1061
<i>Operating costs:</i>						
Fuel (\$)	1898	1872	2947	2626	2444	
Tyres (\$)	146	146	132	146	177	208
Service and repairs (\$)	671	848	614	634		952
Total (\$)	8607		13 209	12 740	11 445	11 633
<i>Average costs:</i>						
Total cost/week (\$/week)	165.52	163.83		245	220.10	223.71
Total cost/km (c/km)	57.4	56.8	88.1		76.3	77.6

Use Table 10-3 from question 1 to help you answer questions 2 to 20.

- 2 For which vehicle is the fuel consumption rate:
 - a best?
 - b worst?
- 3 For which vehicle are the on-road costs:
 - a highest?
 - b lowest?
- 4 For which vehicle is the service cost:
 - a highest?
 - b lowest?
- 5 What percentage of the total running cost are the fuel costs for the:
 - a Toyota hatchback?
 - b Toyota SUV?
- 6 Calculate the fuel cost per kilometre to run the:
 - a Ford hatchback
 - b Toyota SUV.
- 7 What percentage of the total running cost are the service and repairs for the:
 - a Holden sedan?
 - b Toyota SUV?
- 8 What would be the fuel cost to drive a Ford sedan from Sydney to Melbourne, a distance of 885 km?
- 9 What would be the difference in fuel cost when driving a Toyota hatchback and a Holden sedan from Sydney to Brisbane, a distance of 928 km?
- 10 If the cost of fuel increased by 10%, what would be the new running costs (per week and per kilometre) for the Honda SUV?
- 11 If the cost of fuel decreased by 5%, what would be the new running costs (per week and per kilometre) for the Toyota SUV?
- 12 The interest charges are based on 100% of the cost of the new vehicle being financed by a loan.
 - a What would be the savings over 5 years if cash had been paid for the Ford sedan instead of the purchase price being borrowed?
 - b What would be the annual running costs for a new Honda SUV if cash had been paid instead of the purchase price being borrowed?
- 13 What is the weekly cost of tyres for the:
 - a Ford sedan?
 - b Honda SUV?
- 14 Assuming 1 year = 365 days, what is the daily cost of tyres for the:
 - a Ford hatchback?
 - b Toyota SUV?
- 15 If the price of the Ford hatchback is \$22 500, what is the depreciated value after 5 years?
- 16 The cost of the Honda SUV is \$34 000. What will be its depreciated value at the end of 5 years?
- 17 If the Holden sedan cost \$37 000 to buy, what would be the average annual rate of interest charged on the loan?



- 18** Add another column to Table 10-3 in question 1 and use the information given below to calculate the total cost/week and total cost/kilometre to run a Nissan 4WD (dealer price \$61 440). Round your answers to the nearest dollar.

- Depreciation each year is 13.7% of the original dealer price.
- Average annual rate of loan interest is 6.6% (assume the total cost of the vehicle is financed by the loan).
- On-road costs are \$24 per week.
- Fuel consumption is 14 L/100 km and fuel costs 165.9c/L. Assume the vehicle travels 15 000 km each year.
- Tyres cost \$276 per year.
- Services and repairs are \$19.80 per week.



- 19 a** Using the information from Table 10-3, calculate the average operating costs per day for the Ford sedan, the Holden sedan, the Honda SUV and the Toyota SUV (use 1 year = 365 days).
- b** Four people, who live in the same general area and work in the same location, drive to work 5 days per week. Richard drives a Ford sedan, Stephanie drives a Holden sedan, Lilly drives a Honda SUV and Paul drives a Toyota SUV.
- Over a 4-week period, how much does it cost each person to drive to work, assuming the average daily operating cost is completely work related?
 - The four drivers decide to form a car pool, so that each person drives everyone to work 1 week out of every 4 weeks. How much does each person save every 4 weeks by taking part in the car pool?

- 20 a** Using the information from Table 10-3, calculate the average operating costs per kilometre for the Ford hatchback, the Toyota hatchback and the Toyota SUV.
- b** Three people drive to work over an average distance of 64 km return, from each of their homes. They work a normal 5-day week. Helen owns a Ford hatchback, Greg owns a Toyota hatchback and Peta owns a Toyota SUV.
- Over a 3-week period, how much does it cost each person to drive to work?
 - The three drivers form a car pool and take turns driving each other to work on a weekly basis. How much does each person save every 3 weeks by sharing the driving?



- 21** Use an online motoring costs calculator to estimate the running costs for several different types of vehicles, including motorcycles. Visit www.mynrma.com.au

- 22** A spreadsheet can be used to compare the costs of different motor vehicles. Extra columns may be added to compare more vehicles. There are tables that calculate fuel cost and registration costs after the variables are entered. This spreadsheet compares the running costs for a Ford hatchback with the costs for a large Toyota SUV.

	A	B	C
1	Comparing the running costs of vehicles driven 15 000 km per year		
2	Average running costs	Ford hatchback	Toyota SUV (large)
3	Fuel	\$1,653.90	\$3,465.00
4	Tyres	\$184.00	\$455.00
5	Service and repairs	\$670.00	\$1,150.00
6	Green Slip	\$525.00	\$598.00
7	Registration	\$332.00	\$540.00
8	Comprehensive Insurance	\$592.00	\$1,035.00
9	Other		
10			
11	Total	\$3,956.90	\$7,243.00
12	Difference	–\$3,286.10	\$3,286.10
13			
14			
15	Fuel cost		
16	Economy in L/100km	7.4	14
17	Kilometres travelled	15 000	15 000
18	Fuel cost per litre	\$1.49	\$1.65
19	Total fuel cost	\$1,653.90	\$3,465.00
20			
21	Registration		
22	Enter weight in kilograms	1100	2055
23	Cost of registration	\$332.00	\$540.00

In cell B11, the formula to calculate the total cost for the Ford hatchback is =SUM(B3:B10)

In cell B12, the formula to calculate the difference is =B11–C11

In cell B19, the formula to calculate the fuel cost is =B16*B17/100*B18.

In cell B23, the formula for registration cost is

=IF(B22>1504,448,IF(B22>1154,293,IF(B22>975,240,207)))+92

Use **fill across** to find the costs for the second vehicle (the Toyota SUV).

Enter the data for other vehicles.

To help you, you might like to use the prepared spreadsheet file (Spreadsheet 10E) provided on your [ebook assess](#).

10F Budgeting

budget

a financial plan in which expenses need to be less than income

fixed spending

includes expenses like rent or board that cannot be varied

discretionary spending

includes expenses such as entertainment, clothes and gifts that can be varied based on choice

These resources are available on your obook assess:

- **Video tutorial 10F:** Watch and listen to an explanation of Example 10F-1
- **Spreadsheet 10F:** Prepare a budget
- **assess quiz 10F:** Test your skills with an auto-correcting multiple-choice quiz

A **budget** is a financial plan for the future. It is a means by which we can save for future purchases and avoid overspending. To prepare a budget we need to determine our expected income and estimate our expected expenses. Expenses include **fixed spending**, such as rent or board, and **discretionary spending**, such as entertainment, clothes and gifts. Our income needs to be greater than our expenses if we are to live within our means.

To prepare a budget for a given time period, such as a week, month or year:

- 1 calculate your total income
- 2 estimate your total expenses
- 3 calculate income minus expenses
- 4 adjust income or expenses if necessary.

EXAMPLE 10F-1 Preparing an annual budget

Karen has just started work and lives at home with her parents. Her weekly take-home pay is \$480. Each week she pays \$110 for board, \$49 for fares and \$35 for lunches. She spends \$120 per week on entertainment, \$95 per fortnight on personal items and \$330 per month on clothes.

- Prepare an annual budget for Karen.
- Karen is saving to go on an overseas holiday in 3 years. She needs to save \$7899. Determine whether she will be able to afford her holiday.
- If Karen will not have sufficient money to take her holiday, how could she adjust the discretionary expenses in her budget so that she will be able to afford the holiday?

	Solve	Think
a	<i>Income:</i> $(\\$480 \times 52) = \\$24\,960$ <i>Expenses:</i> Board $(\\$110 \times 52) = \\$5\,720$ Fares $(\\$49 \times 52) = \\$2\,548$ Lunches $(\\$35 \times 52) = \\$1\,820$ Entertainment $(\\$120 \times 52) = \\$6\,240$ Personal items $(\\$95 \times 26) = \\$2\,470$ Clothes $(\\$330 \times 12) = \\$3\,960$ Total = \$22 758 Income – expenses = \$2202	Karen's net income is greater than her expenses, so she is able to live within her budget and save some money towards her holiday.
b	If Karen saves all her money, in 3 years she will have $\\$2202 \times 3 = \\$6606$. She is short of her target by $\\$7899 - \\$6606 = \\$1293$.	The amount she can save over the 3 years is the difference between income and expenses multiplied by 3.

Solve		Think
c	Karen must either increase her income or decrease her expenses by at least $(\$1293 \div 3 =) \$431$ per year. She could increase her income by finding employment with a higher income or getting a second job. She could decrease her discretionary expenses by, for example, reducing her spending on clothes to \$290 per month. She would then save $(\$40 \times 12 =) \$480$ per year. Or, if she reduced her spending on entertainment to \$110 per week, she would save $(\$10 \times 52 =) \$520$ per year. She would then be able to afford to take the holiday.	
Apply		
Find the net income and the total of the expenses for the year. Subtract the expenses from the income to determine how much can be saved.		

EXERCISE 10F Budgeting

- 1 Alison is in Year 11 and lives at home with her parents. She has a part-time job and earns \$160 per week. She pays \$20 per week for board, spends \$140 per month on clothes, \$45 per fortnight on entertainment, and \$80 per fortnight on driving lessons. She saves the rest.
 - a Prepare a budget for Alison.
 - b Alison wants to buy a car at the end of the year. How much will she expect to have available to spend?
- 2 Moses is in Year 11 and lives at home with his parents. He has a part-time job and earns \$270 per fortnight. He pays \$25 per week board, spends \$150 per month on personal items, and \$450 per year for his junior golf club membership. He saves the remainder.
 - a Prepare a budget for Moses.
 - b He wants to buy new golf clubs at the end of the year. The clubs cost \$1200. Will he have enough money saved to buy them?
- 3 Naomi lives at home with her parents. Her weekly take-home pay is \$590. Each week she pays \$100 for board, \$53 for fares and \$42 for lunches. She spends \$150 per week on entertainment, \$84 per fortnight on personal items and \$380 per month on clothes. She saves the rest of her money.
 - a Prepare an annual budget for Naomi.
 - b Naomi wants to buy a car in 3 years. The cost of the car is \$12550. Determine whether Naomi will be able to buy the car.
- 4 Matthew's net earnings are \$640 per week. He shares a house for which he pays \$120 per week rent. Each week he spends \$110 on food, \$145 on entertainment and \$75 on personal items. The loan repayments on his car are \$380 per month. He spends \$95 per week on petrol and the 6-monthly car service is \$380. Annual car registration and insurance add to \$1148. His mobile phone costs him \$64 per month.
 - a Prepare an annual budget for Matthew.
 - b How could Matthew adjust his budget so that he can live within his means?

- 5** George is a full-time TAFE student. He receives an allowance of \$407.70 per fortnight from the government and averages net earnings of \$120 per week from his part-time job. His expenses are \$320 per month for rent, \$90 per week for food, \$110 per quarter for the phone, \$70 per week for entertainment and \$350 per year for books.
- Prepare an annual budget for George.
 - George has saved the money to buy a car. He estimates that a car will cost him \$40 per week for petrol, \$40 per month for maintenance and \$840 per year for registration and insurance. Can George afford to own a car?

- 6** Julie and Robert are students working part-time. They want to prepare a budget for next year and have gathered the following information.

Income:

Julie's take-home pay is \$490 per week.

Robert takes home \$380 per week.

Interest of \$230 from investments is expected in February and August each year.

Expenses:

Home loan repayments, \$980 per month

Food, \$160 per week

Electricity, \$480 each quarter

Telephone, \$110 per month

Council rates, \$340 each quarter

Water rates, \$186 per quarter

Car registration and insurance, \$780 per year

Comprehensive car insurance, \$810 per year

Car loan repayments, \$108 per week

Car running expenses, average \$190 per month

Clothing, average \$350 per month

Personal items, \$45 per week

- Prepare an annual budget for Julie and Robert.
- In order to reduce the cost of their loans, Julie and Robert wish to increase their loan repayments. Can they afford to do this? What advice would you give them?



- 7** Tiarne is in Year 11 and lives at home with her parents. She has a part-time job with a beautician and earns \$180 per week. She assists with the make-up at six weddings per year and is paid \$100 for each. She does not pay board, but spends \$200 per month on clothes and entertainment, \$45 per week on make-up and \$30 per fortnight on bus fares. She saves the rest of her money.
- Prepare a budget for Tiarne.
 - If Tiarne completes a specific make-up course she will then be paid \$250 per wedding and her beautician earnings will increase to \$270 per week. The course costs \$1200.
 - If Tiarne intends to do the course at the end of the year, will she have enough money saved to pay for it?
 - Her parents offer to lend her the money for the course at the beginning of the year and Tiarne will repay the loan at a rate of \$50 per fortnight for the year. Modify Tiarne's budget using this extra information.
 - Tiarne repays \$50 per fortnight for the entire year. How much interest does she pay on the \$1200 loan?
 - How much more money does Tiarne have at the end of the year by taking the loan?

- 8 The spreadsheet below shows part of a monthly budget for a year. Set up a similar spreadsheet of your own. Enter the formulas as shown and **fill down** and **fill across** to show the entire year. There is a conversion table to convert weekly, fortnightly and monthly payments into annual payments in cells A19 to C23.

	A	B	C	D	E
1	Monthly Budget				
2	Item	Annual Amount	Jan	Feb	Mar
3	Income				
4			=\$B4/12	=\$B4/12	=\$B4/12
5			=\$B5/12	=\$B5/12	=\$B5/12
6	Total income		=SUM(C4:C5)	=SUM(D4:D5)	=SUM(E4:E5)
7	Expenditure				
8			=\$B8/12	=\$B8/12	=\$B8/12
9			=\$B9/12	=\$B9/12	=\$B9/12
10			=\$B10/12	=\$B10/12	=\$B10/12
11			=\$B11/12	=\$B11/12	=\$B11/12
12			=\$B12/12	=\$B12/12	=\$B12/12
13			=\$B13/12	=\$B13/12	=\$B13/12
14	Total expenditure		=SUM(C8:C13)	=SUM(D8:D13)	=SUM(E8:E13)
15	Monthly difference		=C6–C14	=D6–D14	=E6–E14
16	Cumulative monthly difference		=C15	=C16+D15	=D16+E15
17					
18					
19	Convert to annual amount				
20	Period	Amount	Annual Amount		
21	weekly		=52*B21		
22	fortnightly		=26*B22		
23	monthly		=12*B23		

- a Enter the data from question 1 to create the budget.
 b Enter your own data into the spreadsheet to make your annual budget.

To help you, you might like to use the prepared spreadsheet file (Spreadsheet 10F) provided on your obook assess.

CHAPTER 10 REVIEW

BUDGETING AND HOUSEHOLD EXPENSES

You should be able to:

- ✓ read, interpret and perform calculations based on common household bills
- ✓ calculate the costs of common repairs
- ✓ calculate the registration, stamp duty and insurance costs for new and used motor vehicles
- ✓ calculate the cost to finance the purchase of a motor vehicle
- ✓ solve problems related to the fuel consumption of a motor vehicle
- ✓ calculate the total running cost, including the standing costs and operating costs, of a motor vehicle
- ✓ prepare a budget.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

REVIEW MULTIPLE-CHOICE QUESTIONS

- 10A** 1 In Bindora Shire the water usage charge for residential properties is \$1.76/kL for the first 250 kL and \$2.02/kL for any consumption over 250 kL. If the amount of water consumed is 490 kL, what is the total annual cost?
A \$422.40 B \$862.40 C \$924.80 D \$989.80
- 10A** 2 A front-loading washing machine uses 90 L of water per wash. A family uses the washing machine five times a week. Water costs \$2.26/kL. What is the annual water usage charge for this machine (use 1 year = 52 weeks)?
A \$1.02 B \$10.58 C \$52.88 D \$528.84
- 10A** 3 The usage charges set by a gas supplier are:
- \$0.02998/MJ for usage less than or equal to 3560 MJ
 - \$0.01578/MJ for usage greater than 3560 MJ
- What is the cost of using 4236 MJ for the supply period?
A \$127.00 B \$117.40 C \$76.44 D \$66.84
- 10B** 4 The stamp duty charged when buying a car is 3% of the market value up to \$45 000, plus 5% of the value over \$45 000. What is the stamp duty to be paid on the purchase of a new car worth \$56 000?
A \$2800 B \$1900 C \$1680 D \$550
- 10B** 5 What is the cost to register a new car that weighs 1504 kg and is used mostly for business? Use Table 10-1 in Topic 10B to help you.
A \$382 B \$463 C \$293 D \$448
- 10C** 6 The monthly repayment on a loan of \$12 000 over 5 years is \$256.80. What is the total amount of interest paid on this loan?
A \$15 408 B \$3408 C \$1284 D \$13 284
- 10D** 7 A car travels 480 km on 60 L of petrol. What is the fuel consumption?
A 12.5 L/100 km B 0.125 L/100 km C 8 L/100 km D 28.8 L/100 km

- 10D ▶ 8** How far can a motor vehicle travel on 45 L of petrol if its fuel consumption is 8.4 L/100 km?
A 5.36 km **B** 536 km **C** 18.7 km **D** 187 km
- 10E ▶ 9** A car is bought for \$18 500. It depreciates in value by \$4070 per year. What is the book value of the car after 4 years?
A \$2220 **B** \$4070 **C** \$6848 **D** \$16 280
- 10E ▶ 10** A car depreciates in value from \$36 800 to \$23 550 in 3 years. Using the straight line method, what is the annual amount of depreciation?
A \$4417 **B** \$7850 **C** \$8833 **D** \$13 250
- 10E ▶ 11** The total annual running costs of a small car were \$8960. The car travelled 14 800 km in the year. What is the average cost/kilometre?
A \$1.65/km **B** \$16.50/km **C** \$0.61/km **D** \$6.10/km

REVIEW SET 1

- 1** In 2018 Hunter Water charged residential properties \$2.25 per kilolitre for water usage. The water availability charge was \$25.69 p.a. Calculate the total annual cost of water for a property that used 208 kL.
- 2** The average rate of flow of a bathroom shower with a normal shower rose is 18 L/min.
 - a** Lorraine has two 6-minute showers each day. Calculate the annual cost of Lorraine's showers if the water usage charge is \$2.17/kL. (Use 1 year = 365 days.)
 - b** How much could Lorraine save each year by using a water-efficient shower rose that has a flow rate of 8.5 L/min?
- 3** The usage charges set by a gas supplier are:
 - \$0.029 98/MJ for usage less than or equal to 3560 MJ
 - \$0.015 78/MJ for usage greater than 3560 MJ
 What would be the cost of using 3889 MJ in one billing period?
- 4** How much more expensive is it to register a new Toyota sedan, which weighs 1460 kg, for business use rather than for private use?
- 5** Calculate the stamp duty to be paid on the purchase of a new BMW car that has a recommended retail price of \$76 000.
- 6**
 - a** Calculate the total cost of purchasing and registering a new Holden sedan that has a recommended retail price of \$36 000, weighs 1637 kg, with CTP insurance costing \$477 and a dealer delivery charge of \$630. The car is for private use and the owner decides to take out comprehensive insurance that costs \$1150 for the first year.
 - b** If the buyer of the Holden receives \$16 500 for the trade-in of her current vehicle, what is the changeover price to purchase the new sedan?
- 7** Find the total cost of purchasing a three-year-old Mazda that has an advertised price of \$11 699, and for which third-party property damage insurance is \$479. Include the cost of registration transfer and stamp duty.
- 8** Use the loan repayment table in Topic 10C (Table 10-2) to answer the following questions.
 - a** Calculate the monthly repayments on a loan of \$25 900 at 10% p.a. reducible over 5 years.
 - b** What is the total amount of interest paid on this loan?
 - c** How much would be saved by repaying the loan over 4 years instead of 5 years?

- 9 a** Karen needs to borrow \$17 000 to buy a car and can pay a maximum of \$390 per month. She is offered a loan at 12% p.a. Use Table 10-2 in Topic 10C to calculate if she can afford to take out the loan? Give details.
- b** Could she afford the loan if the interest rate was 14% p.a.? What advice would you give Karen?
- 10** Jo's monthly repayment on a loan at 9% p.a. over 3 years is \$381.60. How much did Jo borrow?

REVIEW SET 2

- 1** Refer to the sample water bill in Topic 10A.
- a** What is the cost per kilolitre of water?
- b** What are the fixed charges for water availability and sewerage availability?
- c** Calculate the bill for a period in which 72.3 kL of water is consumed.
- 2** Refer to the Sydney council rates and charges notice in question 7 of Exercise 10A. From that information, calculate the total rates payable for a residential property that has a rateable land value of \$683 000.
- 3** Calculate the stamp duty that would be charged on a vehicle that has a market value of \$55 600.
- 4** Find the total cost of purchasing a two-year-old Honda 1100cc motorcycle that has an advertised price of \$16 890. Comprehensive insurance is \$685. Include the cost of registration transfer and stamp duty.
- 5** For a loan of \$10 450 at 9% p.a. reducible over 3 years, use Table 10-2 in Topic 10C to calculate:
- a** the monthly repayment **b** the total amount of interest paid.
- 6** If a car uses 55 L of petrol on a trip of 560 km, calculate its fuel consumption in:
- a** km/L **b** L/km **c** L/100 km
- 7** How far can a vehicle travel on 38 L of fuel if its fuel consumption is 9.4 L/100 km?
- 8** Calculate the amount of fuel used by a vehicle on a trip of 315 km, if the fuel consumption is 10.2 L/100 km.
- 9** A salesman averages 5400 km of city driving each month in a Ford sedan that uses 11.4 L/100 km. Calculate the monthly cost of the petrol used by the salesman if the average price of ULP is 139.9c/L.
- 10** A Citroën hatchback uses 7.6 L/100 km of ULP. The diesel model of this car uses 6 L/100 km of diesel fuel. Which car would be cheaper to drive a distance of 680 km, if the price of ULP is 142.9c/L and the price of diesel is 162.2c/L? How much cheaper is it?
- 11** Barry owns a Holden sedan that runs on ULP and has a fuel consumption of 10.6 L/100 km. When converted to run on liquid petroleum gas (LPG), the car's fuel consumption will be 13.5 L/100 km. Barry drives an average of 18 000 km per year.
- a** Calculate the annual fuel cost of running the car on each type of fuel if the average price of ULP is 149.9 c/L and of LPG is 67.8c/L.
- b** How much per year would Barry save in fuel costs if he converted the car to LPG?
- c** What would be Barry's saving per month?
- d** The cost of converting the car to LPG is \$2500. How many months would it take to break even, if Barry converts the car to LPG?
- e** What distance would Barry need to travel before he broke even?
- 12** A car depreciates in value from \$33 000 to \$12 000 in 4 years. Use the straight-line formula to calculate the annual amount of depreciation.

REVIEW SET 3

- 1** Refer to the sample electricity bill in Topic 10A. Using the 'Energy Used & Costs' section, calculate the total electricity cost (before GST) if the energy used for a property was Peak 674.6 kW, Shoulder 961.3 kW and Off-peak 242.8 kW.
- 2** Refer to the Sydney council rates and charges notice in question 7 of Exercise 10A. Calculate the total rates payable for a residential property which has a rateable land value of \$759 000.
- 3**
 - a** Calculate the total cost of purchasing a new Ford with a recommended retail price (RRP) of \$54 000 and weight of 1804 kg. The CTP insurance for the car is \$655 and the dealer delivery charge is \$600. The car is for private use and the owner decides to take out comprehensive insurance that costs \$1160 for the first year.
 - b** If the buyer receives a trade-in of \$18 000 on her old vehicle, what is the price to change to the new car?
 - c** To make the purchase, a loan is taken out on the balance of the car's price.
 - i** Use Table 10-2 in Topic 10C to calculate the monthly loan repayment at 8% p.a. over 4 years.
 - ii** What is the total amount of interest paid on the loan?
 - d** The new Ford travels 580 km on 72 litres of petrol. Calculate its fuel consumption in:
 - i** km/L
 - ii** L/km
 - iii** L/100km.
- 4** Three people, who live in the same general area and work at the same location, drive to work each weekday. Elizabeth owns a Ford hatchback, Monique a Holden sedan and Tanya a Toyota SUV.
 - a** Using the information in Table 10-3 in Topic 10E, calculate the average operating costs per day for each car (use 1 year = 365 days).
 - b** Over a 3-week period, how much does it cost each person to drive to work, assuming the average daily operating cost is completely work related?
 - c** The three drivers decide to form a car pool, so that each person drives everyone to work 1 week out of every 3 weeks. How much does each person save every 3 weeks by taking part in the car pool?
- 5** Allira is in Year 11 and lives at home with her parents. She has a part-time job and earns \$170 per week. She pays \$30 per week for board and, spends \$170 per month on clothes, \$85 per fortnight on entertainment and \$80 per fortnight on driving lessons. She saves what she has left.
 - a** Prepare a budget for Allira.
 - b** Allira wants to buy a car at the end of the year. How much will she expect to have available to spend?



REVIEW SET 4

- 1 In 2018 Sandhaven Water charged residential properties \$1.55/kL for water usage up to 450 kL and \$1.75/kL for each kilolitre more than 450 kL. The water availability charge was \$81. What is the annual cost of water for a household that uses 510 kL?

- 2 a Use the table on the right to find the number of litres of water per year that a household of five people would save if they replaced their full-flush toilet with a 3-star rated dual-flush toilet.

Toilet	Litres used
Full-flush	52 L per person per day
3-star rated dual-flush	18 L per person per day

- b What would be the cost savings if the water usage charge is \$2.85/kL?

- 3 The usage charges set by a gas supplier are:
- \$0.029 98/MJ for usage less than or equal to 3560 MJ
 - \$0.015 78/MJ for usage greater than 3560 MJ

What would be the cost of using 3674 MJ in the billing period?

- 4 The table below shows the average annual running costs for the vehicles listed. The calculations are based on the cost of buying a new vehicle for private use and operating it for 5 years. The interest charges are based on the total cost of the new vehicle being financed by a loan. It is assumed that the vehicle travels 15 000 km each year.

Average annual running costs	Ford hatchback (small)	Holden sedan (large)	Toyota SUV
<i>Standing costs:</i>			
Depreciation (\$)	3120	5928	
Interest (\$)		2444	2288
On-road costs, road service membership (\$)	1248	962	1061
<i>Operating costs:</i>			
Fuel (\$)	1898	2626	2340
Tyres (\$)	146	146	208
Service and repairs (\$)	671	634	952
Total (\$)	8607		
<i>Average costs:</i>			
Total cost/week (\$/week)	165.52	245	223.71
Total cost/kilometre (c/km)	57.4		

- a Calculate the values missing from the table.
- b Add another column to the table and use the information below to calculate the cost per week and cost per kilometre to run a Nissan 4WD (dealer price \$52 000):
- Depreciation each year is \$7250.
 - Average annual rate of loan interest is 6.5% (assume the total cost of the vehicle is financed by the loan).
 - On-road costs are \$23 per week.
 - Fuel consumption is 14 L/100 km and fuel price is 145.9 c/L. Assume that the vehicle travels 15 000 km each year.
 - Tyres cost \$294 per year.
 - Services and repairs are \$18.60 per week.

- 1 a** The average rate of flow of a bathroom shower with a normal shower rose is 18 L/min.
- i** Aria has two 7-minute showers each day. Calculate the annual cost of her showers if the water usage charge is \$2.65/kL. (Use 1 year = 365 days.) (1 mark)
 - ii** Aria's brother Ezra has one 25-minute shower each day. What is the annual cost of Ezra's showers? (1 mark)
 - iii** How much could Aria and Ezra save each year by installing a water-efficient shower rose that has a flow rate of 8.5 L/min? (1 mark)
- b** This table gives the monthly repayments (\$) for every \$1000 borrowed on a reducing-balance loan.

Interest rate (%p.a.)	Term of loan (months)				
	12	24	36	48	60
8	86.99	45.23	31.34	24.41	20.28
9	87.45	45.68	31.80	24.89	20.76
10	87.92	46.14	32.27	25.36	21.25

- i** Jenny borrows \$21 500 to buy a new car. Use the table to calculate the monthly repayment on this loan at 9% p.a. over 3 years. (1 mark)
- ii** What is the total amount of interest Jenny pays on her loan? (1 mark)
- iii** Stamp duty is 3% of the purchase price. Calculate the stamp duty payable on Jenny's car purchase. (1 mark)
- iv** The cost of registration is \$92 plus the cost depending on the car's weight. Use the table below to calculate the cost of registering Jenny's car for private use if it weighs 1255kg. (1 mark)

Registration of motor vehicles in NSW

Size of vehicle	Tare weight	Private use (\$)	Business use (\$)
Cars, station wagons, trucks	up to 975 kg	207	336
	976 kg to 1154 kg	240	382
	1155 kg to 1504 kg	293	463
	1505 kg to 2504 kg	448	697

- v** The value of Jenny's car depreciates to \$7600 after 4 years. Calculate the annual amount of depreciation. (1 mark)
- vi** Jack's monthly repayment on a loan at 8% over 5 years is \$334.62. Use the table at the start of part **b** above to calculate how much Jack borrowed. (1 mark)
- c** How far can a car travel on 54 L of fuel if its fuel consumption is 7.8 L/100 km? (1 mark)
- d** To calculate annual residential rates, a council charges a base rate of \$495 plus \$0.001 562 74 for every dollar of rateable value of the property. The waste service charges are \$387. Calculate the total charges for a residential property with a rateable value of \$716 000. (2 marks)
- e** Karen is in Year 12, lives at home with her parents and has a part-time job. Her weekly take-home pay is \$272. Each week she pays \$70 for board, \$12 for fares and \$15 for lunches. She spends \$50 per week on entertainment, \$25 per fortnight on personal items and \$100 per month on clothes. She saves the remainder of her money.
 - i** Prepare an annual budget for Karen. (2 marks)
 - ii** Karen is saving to go to schoolies at the end of the year. She needs to save \$4400. Determine whether she will be able to afford to go to schoolies. (1 mark)

TOTAL: 15 marks



Exploring and describing data

The main mathematical ideas investigated are:

- ▶ calculating the mean, mode and median for grouped data
- ▶ selecting and using the appropriate measure to describe features of a data set
- ▶ comparing the summary statistics of samples from the same population
- ▶ dividing data into quartiles, deciles and percentiles
- ▶ calculating range and interquartile range
- ▶ drawing and interpreting box-plots and parallel box plots
- ▶ calculating population and sample standard deviation.
- ▶ describing, comparing and interpreting the distributions of graphical displays.

ARE YOU READY?

- 11A ▶ 1** What is the mean of this set of scores?
5, 4, 6, 5, 8, 5
A 5.3 **B** 5.5
C 5 **D** 33

- 11A ▶ 2** For the set of scores 8, 7, 5, 8, 6, 7, 8, what is the mode?
A 8 **B** 7
C 6 **D** 5

- 11A ▶ 3** What is the range of the scores given in question 2?
A 0 **B** 1
C 2 **D** 3

- 11B ▶ 4** What is the median of the scores 8, 7, 5, 8, 6, 7, 8?
A 5 **B** 6
C 7 **D** 8

- 11A ▶ 5** The table below shows the maximum temperatures in Sydney for one month. Which of the following statements is *not* true?

Maximum temperature (°C)	17	18	19	20	21
Number of days	1	12	9	7	2

- A** The number of days in this month is 31.
B The temperature was 12°C on 18 days.
C The temperature was 20°C on 7 days.
D The temperature was less than 19°C on 13 days.

- 11A ▶ 6** A set of data was organised into a table as shown. What is the total of all the scores of 9 in the data set?

Score	9	10	11	12	13
Frequency	8	6	9	10	5

- A** 72 **B** 99
C 19 **D** 17

- 11A ▶ 7** Using the frequency table in question 6, what is the range and mode of the scores, in that order?
A 4, 12 **B** 4, 10
C 5, 12 **D** 5, 10

Use the following table to answer questions 8 and 9.

Class interval	Frequency
0–9	3
10–19	5
20–29	6
30–39	4
40–49	2

- 11A ▶ 8** What is the midpoint of the class interval 10–19?
A 14 **B** 14.5
C 15 **D** 15.5
- 11A ▶ 9** What is the size of each class interval?
A 9 **B** 10
C 20 **D** 49

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook assess](#).

Q1–4 Support sheet 11A.1 Mean, median, mode and range

Q5–7 Support sheet 11A.2 Interpreting tables

Q8 Support sheet 11A.3 Finding the average of two numbers

Q9 Support sheet 11A.4 Class intervals

11A Mean and mode

measure of central tendency (measure of location)

a single number that indicates the location of the centre or most typical score in a set of scores; e.g. the mean, the mode and the median

mean

the sum of all the scores divided by the number of scores, also known as the average

These resources are available on your obook assess:

- **assess quiz 11A:** Test your skills with an auto-correcting multiple-choice quiz

In Chapter 7 we collected and organised raw data into tables and graphs that are easily read, so that patterns and characteristics may be identified and conclusions can be drawn. Often it is convenient and useful to try to find a single number that is typical or representative of all the data. This number is sometimes called an ‘average’, but statisticians prefer to call it a **measure of central tendency** or a **measure of location**. The three commonly used measures of central tendency are the mean, the mode and the median.

The **mean** of a set of scores is the sum of all the scores divided by the number of scores.

$\bar{x}$ is the symbol we use to represent the mean.

EXAMPLE 11A-1 Finding the mean from a frequency distribution table

Find the mean of the scores given in this frequency distribution table.

Score	4	5	6	7	8
Frequency	3	2	4	8	6

Solve/Think				Apply
We add an $f \times x$ column.				First the sub-totals of all the 4s, 5s, 6s, 7s and 8s are found by using an $f \times x$ column. Then the sum of these sub-totals is determined, to find the sum of all the scores for each set of data. To find the mean we divide the sum of the sub-totals by the sum of the scores.
Score (x)	Frequency (f)	$f \times x$		
4	3	$3 \times 4 = 12$	This is the sum of all the 4s.	
5	2	$2 \times 5 = 10$	This is the sum of all the 5s.	
6	4	$4 \times 6 = 24$	This is the sum of all the 6s.	
7	8	$8 \times 7 = 56$	This is the sum of all the 7s.	
8	6	$6 \times 8 = 48$	This is the sum of all the 8s.	
Total	$\Sigma f = 23$	$\Sigma fx = 150$	The sum of all the 4s, 5s, 6s, 7s and 8s.	
Σf = the sum of the frequencies; that is, the total number of scores.				
Σfx = the sum of the sub-totals 12, 10, 24, 56 and 48				
= the sum of all the scores.				
Mean ($\bar{x}$) = $\frac{\Sigma fx}{\Sigma f} = \frac{150}{23} = 6.5$ (to one decimal place).				

EXERCISE 11A Mean and mode

- 1 Consider these frequency distribution tables.
 - i Copy each table and add an fx column.
 - ii Find the mean.

a

Score (x)	3	4	5	6	7
Frequency (f)	2	3	6	4	1

b

Score (x)	12	13	14	15	16
Frequency (f)	4	6	5	3	2

c

Score (x)	50	51	52	53	54	55
Frequency (f)	3	5	8	6	2	4

d

Score (x)	8	9	10	11	12
Frequency (f)	12	28	25	26	9

- 2 Use your calculator to find the mean of the data given in each part of question 1. Use your calculator's instruction booklet to determine the appropriate steps for your calculator, or ask your teacher.

EXAMPLE 11A-2 Finding the mean for grouped data

This grouped frequency table gives the heights of a sample of plants in a nursery. Find the mean height.

Height (mm)	300–324	325–349	350–374	375–399	400–424	425–449
Frequency	3	18	47	32	14	6

Solve				Think	Apply
Height (mm)	Class centre (x)	Frequency (f)	$f \times x$	Determine the class centre for each class interval. Class centre for 300–324 is $\frac{300+324}{2} = 312$. Class centre for 325–349 is $\frac{325+349}{2} = 337$, etc. Add an $f \times x$ column using the class centres for x. Calculate the mean as before: Mean = $\frac{\Sigma fx}{\Sigma f}$.	When the data is grouped into classes, we consider all the scores to be evenly distributed throughout the class and use the class centre as the ‘score’, or ‘x’, when we calculate the mean.
300–324	312	3	936		
325–349	337	18	6066		
350–374	362	47	17 014		
375–399	387	32	12 384		
400–424	412	14	5768		
425–449	437	6	2622		
		$\Sigma f = 120$	$\Sigma fx = 44\,790$		
$\begin{aligned}\text{Mean} &\approx \frac{\Sigma fx}{\Sigma f} \\ &= \frac{44\,790}{120} \\ &= 373.25 \text{ mm}\end{aligned}$					

Notes:

- You can also use your calculator to find these means, as for the ungrouped data.
- Because we do not know the exact scores and have assumed that the scores are evenly distributed throughout each class, the answer is an approximation for the real mean.

3 Complete the following tables to find an approximation for the mean of the distribution.

a

Mass (kg)	Class centre (x)	Frequency (f)	$f \times x$
60–62	61	2	122
63–65	64	5	
66–68		7	
69–71		4	
72–74		2	
		$\Sigma f = 20$	$\Sigma fx =$

b

Patients per day	Class centre (x)	Frequency (f)	$f \times x$
10–19	14.5	13	188.5
20–29	24.5	15	
30–39		16	
40–49		10	
50–59		4	
60–69		2	
		$\Sigma f =$	$\Sigma fx =$

4 Consider these frequency distribution tables.

- Copy each table and add a column for the class centres.
- Use your calculator to find an approximation for the mean.

a

Height	Frequency (f)
155–159	4
160–164	7
165–169	10
170–174	5
175–179	2

b

Number of arguments	Frequency (f)
0–2	8
3–5	18
6–8	10
9–11	3
12–14	1

c

Score	41–50	51–60	61–70	71–80	81–90	91–100
Frequency	3	0	10	8	7	2

The **mode** is the score that occurs most often; that is, it is the score with the highest frequency. A set of scores may be bimodal. For example, 2, 3, 3, 3, 4, 4, 4, 5, 6, 8 is bimodal because it has two modes, namely 3 and 4.



mode
the score that occurs the most often in a set of scores

EXAMPLE 11A-3 Finding the mode from a frequency distribution table

Find the mode of the scores shown.

a

Score	Frequency
4	3
5	2
6	4
7	8
8	6

b

Score	Frequency
0–4	1
5–9	3
10–14	6
15–19	4
20–24	2

	Solve	Think	Apply
a	Mode = 7	From the table, the score with the highest frequency is 7.	The score that occurs most often is the score with the highest frequency.
b	For grouped data, we can find the class with the highest frequency. It is called the modal class. For this data: Modal class = 10–14.	For grouped data the actual score with the highest frequency cannot be found unless we also have the raw data. From the table, the class with the highest frequency is 10–14.	The modal class is the class that occurs most often.

5 Find the mode, or modal class, for the following distributions.

a

<i>x</i>	3	4	5	6	7
<i>f</i>	2	3	6	4	1

b

<i>x</i>	12	13	14	15	16
<i>f</i>	4	6	5	3	2

c

<i>x</i>	50	51	52	53	54	55
<i>f</i>	3	5	8	6	2	4

d

<i>x</i>	8	9	10	11	12
<i>f</i>	12	28	23	28	9

e

<i>x</i>	<i>f</i>
24	9
25	11
26	18
27	0
28	12

f

Mass (kg)	Frequency (<i>f</i>)
60–62	2
63–65	5
66–68	7
69–71	4
72–74	2

g

Score (x)	Frequency (f)
41–50	3
51–60	0
61–70	9
71–80	9
81–90	7
91–100	2

h

Patients per day (x)	Frequency (f)
10–19	13
20–29	15
30–39	16
40–49	10
50–59	4
60–69	2

EXAMPLE 11A-4 Finding the mean and the mode from a stem-and-leaf plot

For the data in this stem-and-leaf plot, find the:

a mean

b mode.

Stem	Leaf
2	2 3 3 6
3	
4	1 2 3 5 5 6 6 6 8
5	1 5 5 7 7 7 7
6	1 2 3 5 5 5 6 7
7	0 0

	Solve	Think	Apply
a	$\bar{x} = 51.3$	Using the statistics function on your calculator, enter the scores and determine the mean.	Find the sum of the scores and divide that by the number of scores, or use the statistics function on a calculator.
b	Mode = 57	The leaf that occurs most often is the 7 in the 50s row. So: Mode = 57.	Find the leaf that occurs most often: this is part of the score that occurs most often.

6 For the data given in the following stem-and-leaf plots, find the:

i mean

ii mode.

a

Stem	Leaf
5	1 2
6	0 2 5
7	1 4 5 5
8	6 7
9	3

b

Stem	Leaf
2	4 6 6
3	0 1 1 5
4	5 8 8 9
5	1 6 8
6	7

c

Stem	Leaf
10	5 6 8 8 8
11	2 3 3 6
12	0 1
13	8
14	3 5 5 6
15	1 1 3 7

d

Stem	Leaf
1	2 3 3 4
1*	5 6
2	0 1 1 3
2*	6 6 6 8 9

Note: the stems have been split into 1 and 1*, and 2 and 2*. The stems of 1 and 2 can have leaves of 0, 1, 2, 3 and 4. The stems of 1* and 2* can have leaves of 5, 6, 7, 8 and 9.

- 7** The marks scored by 30 students in a test are shown below.
- 25 35 8 61 45 70 47 21 52 72 56 36 20 44 54
51 32 60 16 61 44 34 42 60 41 47 61 76 53 62
- a** Calculate the mean of this data.
- b** Organise the data into frequency distribution tables, using these classes:
- i** 0–4, 5–9, 10–14, 15–19, ... **ii** 0–9, 10–19, 20–29, 30–39, ...
iii 0–19, 20–39, 40–59, ... **iv** 0–39, 40–79, ...
- c** Calculate an approximation for the mean of this data using each of the grouped frequency tables from part **b**.
- d** Compare the approximations for the mean in part **c** with the actual mean found in part **a**. What is the effect of the size of the class interval on the approximations for the mean?
- 8** **a** The mean mark in a test for the 24 students in Class 11M1 was 70 out of 100. What was the total of the marks scored by this class?
- b** The mean mark for the same test done by the 17 students in Class 11M2 was 56. What was the total of the marks scored by Class 11M2?
- c** What was the combined mean mark of both classes?
- 9** The distance from Broken Hill to Tibooburra is 360 km. A motorist travels from Broken Hill to Tibooburra at an average (mean) speed of 90 km/h and returns at an average speed of 60 km/h. What is the average speed of the motorist for the whole journey?



- 10** **a** Find the mean of these x scores: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
This is the theoretical mean of the one-digit numbers in a table of random numbers, because, theoretically, each occurs, with the same frequency.
- b** From the table of random numbers given below, take samples of one-digit numbers and find the mean and mode of each sample. To make this task simpler, work in pairs and take samples of the following sizes.
- i** 5 **ii** 10 **iii** 20 **iv** 50
- | | | | | | | | | | | | |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 48047 | 45381 | 33232 | 35178 | 46971 | 85879 | 31458 | 22016 | 08695 | 58112 | 96070 | 91910 |
| 18868 | 52251 | 99827 | 32581 | 90070 | 98873 | 89846 | 50953 | 92529 | 68249 | 54949 | 83829 |
| 10132 | 27359 | 13017 | 41045 | 13817 | 65603 | 87615 | 55691 | 58547 | 01331 | 62538 | 79181 |
| 33071 | 63766 | 73613 | 24470 | 85566 | 81574 | 71965 | 20977 | 48005 | 83418 | 58738 | 98771 |
| 40942 | 42373 | 38710 | 39916 | 08187 | 00133 | 16288 | 64277 | 80553 | 58331 | 62724 | 74004 |
| 09344 | 91315 | 25791 | 40296 | 27328 | 85758 | 45342 | 98884 | 36034 | 79836 | 94902 | 80442 |
- c** Compare the means you found in part **b** with the theoretical mean found in part **a**. Is it true that, as the sample gets larger, the sample mean gets closer to the theoretical mean? Discuss this with your class.
- d** **i** On the blackboard, write the mean of your sample of size 10, from part **bii**. (All the other students in the class should do likewise.)
ii Find the mean of all the sample means on the blackboard.
iii Compare your result from part **ii** with the theoretical mean from part **a**. Is it true that the mean of a large number of sample means approaches the value of the theoretical mean? Discuss this with your class.
- e** Comment on the modes for your samples, found in part **b**.

11B Median

These resources are available on your obook assess:

- **Interactive 11B.1:** Explore mean, median, mode and range
- **Interactive 11B.2:** Explore summary statistics of stem-and-leaf plots
- **assess quiz 11B:** Test your skills with an auto-correcting multiple-choice quiz

oi
gi

The median divides the data into two parts, with equal numbers of scores below it and above it.

median

the middle score after the set of scores has been arranged in ascending order

The **median** is the middle score after the scores have been arranged in ascending order; that is, from the smallest to the largest. For an odd number of scores, there is one middle score. For an even number of scores, the median is the number midway between the two middle scores.

To find the location of the median in a set of n scores arranged in numerical or ascending order:

- 1 cross off equal numbers of scores from each end until there is one or two middle scores, or
- 2 calculate the value of $\frac{n+1}{2}$ to determine where the middle score is placed within the set.

!

EXAMPLE 11B-1 Finding the median of an odd number of scores

Find the median of the following scores.

7 9 3 9 8

Solve	Think	Apply
Median = 8	First arrange the five scores in ascending order: 3, 7, 8, 9, 9. For an odd number of scores there is one middle score. By observation, the middle score is 8 because there are two scores below it and two scores above it: Median = 8.	Sort the scores into ascending order and locate the middle score.

Note:

- The median can be found by crossing off equal numbers of scores from each end of the ordered arrangement:

~~3~~ ~~7~~ 8 ~~9~~ ~~9~~
 ↑
 Median = 8

- If the number of scores, n , is odd, the median is the value of the score in the $\frac{n+1}{2}$ th position.
In this example, $n = 5$, so $\frac{n+1}{2} = 3$; meaning the median is the 3rd score (which is 8).

EXAMPLE 11B-2 Finding the median of an even number of scores

Find the median of the scores 3, 4, 8, 9, 8, 9, 3, 5.

Solve	Think	Apply
$\text{Median} = \frac{5+8}{2}$ $= 6.5$	Arrange the scores in numerical order: 3 3 4 5 8 8 9 9 Cross off equal numbers of scores from each end: ~~3~~ ~~3~~ ~~4~~ 5 8 ~~8~~ ~~9~~ ~~9~~ In this case there are two middle scores, 5 and 8. Median = the number midway between 5 and 8 = the mean of 5 and 8 $= \frac{5+8}{2}$ $= 6.5$ 3 3 4 5 8 8 9 9 ↑ Median = 6.5	Sort the scores into ascending order and, because there is an even number of scores locate the <i>two</i> middle scores. The median is the number midway between the two middle scores. This is found by averaging these scores.

Note:

- There are four scores less than 6.5 and four scores greater than 6.5.
- The median is not always one of the scores.
- For this example, $n = 8$ and $\frac{n+1}{2} = 4.5$.
This indicates that the median is the number midway between the 4th and 5th scores.

EXERCISE 11B Median

- Find the median of the following sets of data by:
 - rearranging the scores into numerical order, where necessary
 - crossing off equal numbers of scores from each end.
 - 2, 4, 5, 6, 9, 9, 10
 - 11, 13, 13, 16, 17
 - 0, 2, 4, 5, 7, 6, 4, 5, 4, 0, 1
 - 51, 52, 54, 55, 57, 57, 58, 59
 - 240, 243, 245, 246, 244, 243
 - 2, 3, 3, 4, 5, 6, 7, 8, 9
 - 27, 28, 29, 27, 30, 31, 27, 31, 30
 - 20, 20, 20, 23, 25, 27
 - 1, 1, 2, 4, 4, 4, 4, 7, 7, 8, 9, 10
 - 104, 101, 104, 102, 104, 105, 106, 101
- For the following sets of data:
 - rearrange the scores, into numerical order, where necessary.
 - calculate the value of $\frac{n+1}{2}$, where n = number of scores.
 - find the median.
 - 6, 6, 7, 7, 8, 9, 10, 10, 11
 - 15, 15, 16, 16, 17, 17, 18, 19
 - 48, 49, 50, 51, 53, 49, 50, 50, 51, 55, 53
 - 34, 35, 38, 40, 37, 36, 33, 38, 34, 39, 36, 37, 38, 37
 - 20, 21, 21, 22, 22, 23, 23
 - 48, 49, 49, 49, 50, 50, 50, 51, 52, 52
 - 2, 1, 0, 3, 4, 5, 4, 4, 3, 1, 0, 2

EXAMPLE 11B-3 Finding the median from a frequency distribution table

Find the median of the set of scores given below.

x	20	21	22	23	24
f	5	7	11	9	4

Solve			Think			
			Add a cumulative frequency column to the table.			
x	f	Cumulative frequency	x	f	Cumulative frequency	Scores
20	5	5	20	5	5	1st, 2nd, 3rd, 4th, 5th
21	7	12	21	7	12	6th, 7th, 8th,..., 11th, 12th
22	11	23	22	11	23	13th, 14th, 15th,..., 22th, 23rd
23	9	32	23	9	32	24th, 25th, 26th,..., 31st, 32nd
24	4	36	24	4	36	33rd, 34th, 35th, 36th
Because $n = 36$, the median is the average of the 18th and 19th scores. From the table, the 18th and 19th scores are both 22. So: Median = 22			From the table, $n = \Sigma f = 36$, so, $\frac{n+1}{2} = 18.5$. This indicates that the median is the number midway between the 18th and 19th scores; that is: Median = the mean of the 18th and 19th scores From the cumulative frequency column, we can see that the 18th and 19th scores are both 22, so: Median = $\frac{22+22}{2} = 22$.			
Apply						
Add a cumulative frequency column to the table and use it to find the middle score(s).						

3 Complete the following tables to find the median of each set of scores.

a

x	f	Cumulative frequency	Scores
3	2	2	1st, 2nd
4	3	5	3rd, 4th, 5th
5	7	12	6th, 7th, ..., 12th
6	4		13th, ...
7	1		

b

x	f	Cumulative frequency	Scores
12	4	4	1st, 2nd, 3rd, 4th
13	7	11	5th, 6th, ..., 11th
14	5	16	12th, ...
15	3		
16	2		

4 Find the median of each set of scores below.

a

x	50	51	52	53	54	55
f	3	5	8	6	2	4

b

x	33	34	35	36	37
f	2	4	9	10	5

c

x	8	9	10	11	12
f	12	28	25	26	9

d

x	24	25	26	27	28
f	9	11	18	0	12

EXAMPLE 11B-4 Finding the median for grouped data

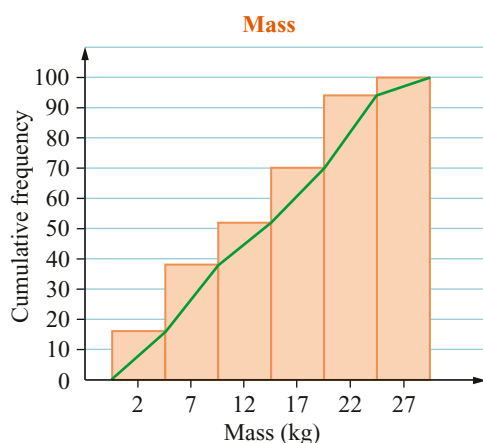
Find the median of the scores given in this frequency distribution table.

Height (mm)	Frequency
300–324	3
325–349	18
350–374	47
375–399	32
400–424	14
425–449	6

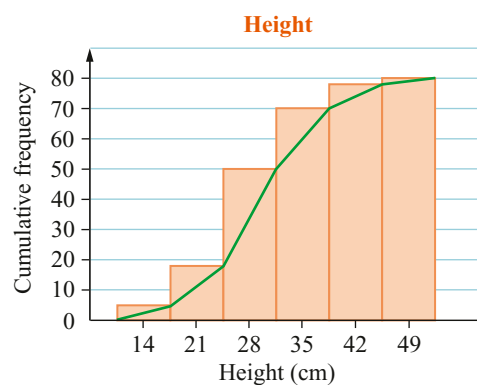
Solve	Think	Apply																					
Add a cumulative frequency column to the table and use the data to draw a cumulative frequency polygon. <table> <tr> <th>Height (mm)</th><th>Frequency</th><th>Cumulative frequency</th></tr> <tr> <td>300–324</td><td>3</td><td>3</td></tr> <tr> <td>325–349</td><td>18</td><td>21</td></tr> <tr> <td>350–374</td><td>47</td><td>68</td></tr> <tr> <td>375–399</td><td>32</td><td>100</td></tr> <tr> <td>400–424</td><td>14</td><td>114</td></tr> <tr> <td>425–449</td><td>6</td><td>120</td></tr> </table> $\frac{n+1}{2} = \frac{120+1}{2} = 60.5$ So the median is the mean of the 60th and 61st scores. The 60th and 61st scores are in the 350–374 class so an exact value for the median cannot be found. The best estimate is found by reading the value from the polygon when the cumulative frequency is 60. So: Median $\approx$ 370.	Height (mm)	Frequency	Cumulative frequency	300–324	3	3	325–349	18	21	350–374	47	68	375–399	32	100	400–424	14	114	425–449	6	120	The 60th and 61st scores are in the 350–374 class, but it is not possible to state exact values from the table. The class containing the median is sometimes called the median class. We could estimate the median to be at the class centre (362), but a more accurate estimate can be found by using the class centres and cumulative frequency to help you draw the cumulative frequency histogram and polygon (ogive). To find the median, locate the point half way up the cumulative frequency axis, $\frac{1}{2} \times 120 = 60$, and draw a horizontal line across to the polygon. Draw a vertical line to the score axis (Height) and use it to read the median, on the horizontal scale. In this example: Median $\approx$ 370	Draw a cumulative frequency polygon. Estimate the median by locating the point halfway (vertically) up the polygon.
Height (mm)	Frequency	Cumulative frequency																					
300–324	3	3																					
325–349	18	21																					
350–374	47	68																					
375–399	32	100																					
400–424	14	114																					
425–449	6	120																					

5 Estimate the median from the ogives drawn below.

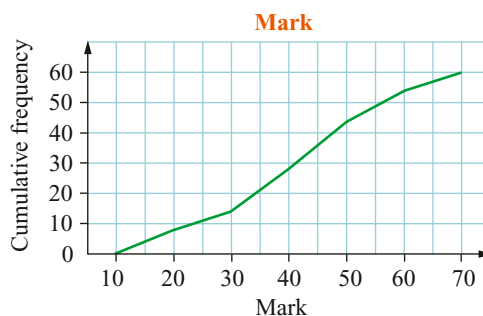
a



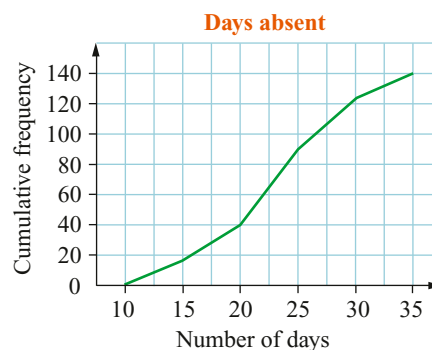
b



c



d



6 Consider each of the grouped frequency distributions below.

- i Copy each table and add columns for class centre and cumulative frequency.
- ii Draw the ogive for each distribution.
- iii Use your ogives from part ii to help you estimate the median score for each set of data.

a

Score	Frequency
41–50	5
51–60	0
61–70	12
71–80	10
81–90	9
91–100	4

b

Mass (kg)	Frequency
60–62	4
63–65	7
66–68	9
69–71	6
72–74	4

c

Height (cm)	Frequency
155–159	19
160–164	21
165–169	24
170–174	19
175–179	17

d

Patients per day	Frequency
10–19	13
20–29	15
30–39	16
40–49	10
50–59	4
60–69	2

EXAMPLE 11B-5 Finding the median from a stem-and-leaf plot

Find the median for the data in this stem-and-leaf plot.

Stem	Leaf
2	2 3 3 6
3	
4	1 2 3 5 5 6 6 6 8
5	1 5 5 7 7 7 7
6	1 2 3 5 5 5 6 7
7	0 0

Solve		Think	Apply
Stem	Leaf	Cross off numbers in pairs, starting with the lowest and highest numbers, 22 and 70. The two middle scores remaining are 55 and 55. So: Median = $\frac{55+55}{2} = 55$ Or, because there are 30 scores, the median is the average of the 15th and 16th scores. The scores are already in order, and the 15th and 16th scores are both 55. So: Median = 55.	The median can be found by crossing off pairs of numbers from the top and bottom of the stem-and-leaf plot or by identifying the middle score(s).
2	~~2 3 3 6~~		
3			
4	~~1 2 3 5 5 6 6 6 8~~		
5	~~1 5 5 7 7 7 7~~		
6	~~1 2 3 5 5 5 6 7~~		
7	~~0 0~~		
Median = 55			

7 Find the median of the data in each of these stem-and-leaf plots.

a	Stem	Leaf	b	Stem	Leaf	c	Stem	Leaf	d	Stem	Leaf
	5	1 2		2	4 6 6		10	5 6 8 8 8		1	2 3 3 4
	6	0 2 5		3	0 1 1 5		11	2 3 3 6		1*	5 6
	7	1 4 5 5		4	5 8 8 9		12	0 1		2	0 1 1 3
	8	6 7		5	1 6 8		13	8		2*	6 6 6 8 9
	9	3		6	7		14	3 5 5 6			
							15	1 1 3 7			

8 Use the data from the stem-and-leaf plot on the right to answer the following questions.

Stem	Leaf
3	0 2 2 3 8
4	1 3 6
5	2 4 4 4 8 8 9
6	1 5 5 6 7

- Find the mean, mode and median of the data.
- Complete a frequency distribution table using the classes 30–39, 40–49, etc.
- Calculate the mean from the grouped data. Compare this with the mean found in part a.
- Add a cumulative frequency column to your table from part b.
- Draw the ogive and estimate the median. Compare this with the exact median value found in part a.
- Why are some later answers different from those in part a?

9 Repeat question 8 using classes of 30–34, 35–39, etc. Explain any differences from the question 8 answers in your results this time.

- Show that the median of the scores 2, 5, 5, 5, 8, 11, 14, 17 and 44 lies between the mode and the mean.
 - Show that the mean of the scores 1, 3, 3, 3, 5, 5, 6, 6, 7 and 7 lies between the mode and the median.
 - Find a set of scores for which the mode lies between the median and the mean.
 - Find a set of scores for which the mean lies between the median and the mode.
 - Find a set of scores that has the same mean, mode and median.

11C Relative merits of mean, mode and median

These resources are available on your obook assess:

- **Video tutorial 11C:** Watch and listen to an explanation of Example 11C-2
- **assess quiz 11C:** Test your skills with an auto-correcting multiple-choice quiz

The mean, the mode and the median were introduced in this chapter as measures of central tendency; that is, single numbers that are central to or typical of all the numbers under consideration. Which of these measures is the most appropriate in a given situation? There is no general rule; rather it depends on the nature of the data and the relative merits of the measures. We will investigate some of the properties of the mean, mode and median.

EXAMPLE 11C-1 Finding the mode and comparing it to the mean and median

a Find the mode of each of these sets of scores.

- i** 14, 15, 15, 16, 16, 16, 17, 18 **ii** 14, 15, 16, 17, 19, 21 **iii** 14, 15, 16, 17, 26, 26, 26

b Consider the relative merits of the mode, the mean and the median for each of the cases above.

	Solve/Think	Apply
a i	Mode = 16, because the number 16 occurs the most times.	The mode is the number that occurs with the highest frequency.
ii	There is no mode because all the numbers have the same frequency.	
iii	Mode = 26, because the number 26 occurs the most times.	
b i	The mode (16) is central and typical of the scores. The mean (15.9) and the median (16) are also good measures of central tendency.	There are cases in which the mode is important even if it is not a central value. For example, if the numbers are the sizes of hats sold by a shop, then the size that is sold most often would be important to the shop owner and hat manufacturer.
ii	There is no mode, so the mode is not a satisfactory measure of central tendency. The mean (17) and the median (16.5) are both good measures of central tendency.	
iii	The mode (26) occurs at one end of the set of scores; it is not a central value. The mean (20) and the median (17) are both good measures of central tendency.	

EXERCISE 11C Relative merits of mean, mode and median

1 a Find the mode of each of the following sets of scores.

- i** 5, 6, 6, 7, 7, 8, 9 **ii** 2, 3, 4, 5, 7, 9 **iii** 23, 24, 25, 26, 35, 35, 35

b Is the mode a satisfactory measure of central tendency for each set of data in part **a**?

c Consider the relative merits of the mean and the median for each of the sets of data from part **a**.

EXAMPLE 11C-2 Finding the mean and comparing it to the mode and median

- a** Find the mean of each of these sets of scores.
i 5, 5, 6, 7, 8 **ii** 5, 5, 6, 7, 80
b Consider the relative merits of the mean, the mode and the median for each of the sets of data above.

	Solve/Think	Apply
a i	Mean = $\frac{5+5+6+7+8}{5} = 6.2$	The mean is the sum of the scores divided by the number of scores.
ii	Mean = $\frac{5+5+6+7+80}{5} = 20.6$	
b i	The mean (6.2) is central and typical of the data, so it is a good measure of central tendency. The mode (5) is not a central value, but the median (6) is both central and typical.	The mean has the disadvantage of being affected by extremely large (or small) values. Very small or very large atypical scores are called outliers . In some cases it may be useful to ignore outliers when calculating the mean. <i>Note:</i> If we ignore the outlier 80 in part b ii : Mean = 5.75
ii	Four out of five scores are less than the mean (20.6), so the mean is not a central value and is not typical of the size of any of the scores. The mode (5) is not a central value. The median (6) is unaffected by the extreme value and would be the best measure of central tendency.	

- 2 a** Find the mean of each of the following sets of scores.
i 8, 8, 9, 10, 11 **ii** 8, 8, 9, 10, 110
b Is the mean a satisfactory measure of central tendency for each set of data in part **a**?
c Find the median and the mode, and consider the relative merits of these measures for each set of data in part **a**.

outlier

an extreme piece of data or a score that is much higher or much lower than the rest of the data in the data set

EXAMPLE 11C-3 Finding the median and comparing it to the mean and mode

- a** Find the median of this data about the number of goals scored by a footballer over 10 games:
1 2 2 3 6 6 6 6 6 6
b Consider the relative merits of the median, mode and mean for this data.

	Solve/Think	Apply
a	Median = average of 5th and 6th scores = 6	The median is the middle score, after the scores have been arranged in ascending order.
b	For this pattern of scores the median happens to be the end score (which is also the mode). In this case, the mean (4.4) would be a better measure of central tendency.	The most appropriate measure is the one that is a central value.

- 3 a** Find the median of these scores: 2, 3, 3, 3, 5, 5, 5, 5, 5.
b Is the median a satisfactory measure of central tendency for these scores?
c Consider the relative merits of the mode and the mean for this data.
4 Which measure of central tendency depends on the value of every score in a data set?
5 Which is the only measure suitable for categorical data?

Summary – choosing the appropriate measure

Mean

- This is the most commonly used, is easy to understand and is easy to calculate.
- Its value depends on the value of every score in the data set.
- Its main disadvantage is that its value is easily distorted by very large or very small atypical scores (outliers).
- It has the best sampling stability: if we choose random samples of the same size from a large population, the mean is the measure that varies the least from sample to sample.
- It is not suitable for categorical data.

Mode

- This is the most frequent value in a set of data.
- Its value is easy to determine and it is not affected by outliers.
- There may be no mode, or more than one mode, in a data set.
- It may not be a central value.
- When comparing random samples of the same size taken from a large population, the mode is the measure that varies the most.
- It is the only measure suitable for categorical data.

Median

- This is easy to understand: there are equal numbers of scores above and below it.
- It is not affected by outliers, since its value depends on the number of scores above and below it, not on the values of these scores.
- For some unusual patterns of scores it may not be a central value; e.g. 0, 1, 1, 5, 5, 5, 5
- When comparing samples, it varies more than the mean but much less than the mode.
- It is not suitable for categorical data.



- 6** A shoe store had a discount sale of women's running shoes and sold shoes of the following sizes in one day.

6 10 4 7 8 7 6 5 7 8 7 5 6 4 3 7

- a** For the sizes of shoes sold, find the:

i mean

ii mode

iii median.

- b** Which of these measures would be of most use to the shop owner?

- 7** The Beacon Lamp Company has a total of 30 employees whose annual salaries are listed on the right.

- a** What is the total annual wage bill for this company?

- b** Calculate the mean wage for the employees.

- c** How many employees earn:

i less than the mean wage from part **b**?

ii more than the mean wage from part **b**?

- d** What is the median wage?

- e** What is the modal wage?

- f** In a wage determination case for the employees of this company, which measure of central tendency would you use to support your argument if you were the representative for:

i the general manager?

ii the production workers?

- g** Which measure of central tendency is the most appropriate to represent the wages of the employees of this company? Give reasons for your answer.

1 general manager	\$260 000
1 marketing manager	\$100 000
1 accountant	\$120 000
1 engineer	\$100 000
1 warehouse manager	\$100 000
15 production workers	\$50 000 each
10 tradespeople	\$60 000 each

- 8** The times to the nearest minute that 10 patients, chosen at random, spent in the waiting room of a hospital were:
18 19 19 20 20 27 28 29 29 29
a Find the mean, mode and median of these waiting times.
b Consider the relative merits of these three measures for this data.
- 9** Geoff scored these marks in a series of topic tests: 6 3 6 3 6 2 6 1
a Find the mean, mode and median for Geoff's marks.
b Which of these 'averages' would Geoff prefer to tell his parents?
- 10** The prices of land sold by a real-estate agent in a particular suburb in one month were:
\$510 000 \$630 000 \$960 000 \$540 000 \$560 000
a The real-estate agent claimed that, for this month, the 'average' price of houses in the suburb was \$640 000. To which 'average' is the estate agent referring?
b Is the 'average' from part **a** an appropriate 'average' to use? Give reasons for your answer.
- 11** The eye colours of a group of 10 students are recorded below.
brown blue brown green blue grey brown brown blue green
Which, if any, of the mean, mode and median can be used with this categorical (qualitative) data?

- 12** The number of mistakes made by 100 students in a spelling test is shown (no-one made more than 9 mistakes).

4 8 0 4 7 4 5 3 8 1 3 3 2 3 2 3 5 1 7 8
4 6 9 7 1 8 5 8 7 9 3 1 4 5 8 2 2 0 1 6
0 8 6 9 4 5 8 1 1 2 9 6 0 7 0 9 1 9 1 0
1 8 8 6 8 5 2 2 5 1 9 9 8 2 7 3 2 6 8 1
9 0 0 7 0 9 8 8 7 3 8 9 8 4 6 5 0 9 5 3

- a i** Complete the frequency distribution table for the first 20 students. (Start at 4 and work to the right.)
ii Use your calculator to find the mean, mode and median of the number of mistakes made by the 20 students from part **i**.

Number of mistakes	Tally	Frequency
0		
1		
2		
3		
...		
9		

- b** Repeat part **a** for the:
i second group of 20 students **ii** third group of 20 students
iii fourth group of 20 students **iv** fifth group of 20 students.

- c** Complete a table similar to the one on the right to summarise the results for the samples of 20 students in parts **a** and **b**.

	1st 20	2nd 20	3rd 20	4th 20	5th 20
Mean					
Mode					
Median					

- d** Which of these measures (mean, mode or median) varies the most between samples?
e Which of the measures varies the least?

- 13** The weights, in kilograms, of the players in a women's hockey team are:
55, 56, 63, 48, 58, 56, 48, 54, 55, 62, 64, 157
a Are there any outliers in this data set? Discuss possible reasons for this score.
b Calculate the mean weight of the team, with and without the outlier.
c Find the median weight of the team. What is the modal weight?
d Which measure of central tendency is the most appropriate to represent the weights of the players?

- 14** The number of injuries sustained in workplace accidents in a factory over a period of 12 months are:
2, 0, 1, 0, 5, 0, 0, 3, 0, 1, 2, 2
a For the number of injuries, find the: **i** mean **ii** mode **iii** median.
b Which of these measures of central tendency is the most appropriate for this data? Explain your answer.

11D Range and interquartile range

measure of dispersion

a single number that indicates the spread of scores, also known as a measure of spread; e.g. the range, the interquartile range and the standard deviation

range

the difference between the highest score and the lowest score in a data set

These resources are available on your obook assess:

- **Worksheet 11D:** Practise your skills with extra problems on summary statistics
- **Investigation 11D:** Explore the effect on the IQR when data values are changed
- **assess quiz 11D:** Test your skills with an auto-correcting multiple-choice quiz

Consider these two sets of data:

3, 4, 5, 6, 7 and 0, 1, 5, 9, 10.

For both sets, the mean is 5. However, it is obvious that these sets of scores are quite different in nature. The first set of scores is closely grouped around the mean, but the second set is quite scattered about the mean. So, as well as having a central value for a set of scores, it is useful to have some measure of the scatter, or spread, of the scores. Such measures of spread are called **measures of dispersion**. The first two we will investigate in this topic are the **range** and the interquartile range.

The range is the difference between the highest score and the lowest score in a data set.

Range = highest score – lowest score

EXAMPLE 11D-1 Finding the range

Find the range of each of these sets of scores.

a 3, 4, 4, 5, 5, 5, 7

b 1, 9, 0, 5, 10

	Solve	Think	Apply
a	Range = $7 - 3$ = 4	Highest score = 7, Lowest score = 3	Range = highest score – lowest score
b	Range = $10 - 0$ = 10	Highest score = 10, Lowest score = 0 Range = highest score – lowest score = $10 - 0$ = 10	

EXERCISE 11D Range and interquartile range

1 Find the range of each of these sets of scores.

a 2, 4, 5, 6, 9, 9, 10

c 11, 13, 13, 16, 170

e 0, 2, 4, 5, 7, 6, 4, 5, 4, 0, 1

g 51, 52, 54, 55, 57, 57, 58, 59

b 2, 3, 3, 4, 5, 6, 7, 8, 9

d 27, 28, 29, 27, 30, 31, 27, 31, 30

f 20, 20, 20, 23, 25, 27

h 104, 101, 104, 102, 104, 105, 106, 101

2 Find the range of the scores in each of these sets of data.

a

x	f
3	2
4	3
5	6
6	4
7	1

b

x	f
50	3
51	5
52	8
53	6
54	2
55	4

c

x	f
12	4
13	3
14	5
15	6
16	2

Some situations in which the range is useful are the measurement of temperature, height, length and blood pressure, but a major disadvantage of the use of the range is that it depends only on the extreme values of the data. The extreme values can be atypical scores (outliers), such as the 170 in question 1c. The outliers are often caused by an error of measurement and, in these cases, the range is of little use.

To avoid this problem, we can find the difference between two numbers that are not at the extremes of the data. One such measure is called the **interquartile range**. We are already familiar with the concept of the median, which divides the data into two equal parts. In the same way, we can find measures that divide the data into four parts, with equal numbers of scores in each part. These measures are called **quartiles**. Each set of data has three quartiles: the lower quartile, the middle quartile (this is the median) and the upper quartile, usually denoted by Q_1 , Q_2 and Q_3 respectively.

The interquartile range (IQR) is the difference between the upper and lower quartiles:

$$\text{Interquartile range} = Q_3 - Q_1$$

interquartile range

measure of dispersion found by calculating the difference between the upper quartile and lower quartile; this, $\text{IQR} = Q_3 - Q_1$

quartiles

three measures that divide a set of scores (in ascending order) into four parts with an equal number of scores in each part; the measures are the lower quartile (Q_1), middle quartile or median (Q_2), and upper quartile (Q_3)

EXAMPLE 11D-2 Finding the interquartile range (quartiles are scores in the set)

Find the lower, middle and upper quartiles and the interquartile range for these scores:

2 2 3 5 6 6 7

Solve	Think	Apply
2 2 3 5 6 6 7 ↑ ↑ ↑ Q_1 Q_2 Q_3 Interquartile range $= Q_3 - Q_1$ $= 6 - 2$ $= 4$	Write the scores in numerical order, if necessary. Find the median (2nd quartile): $\frac{n+1}{2}$ The median is the 4th score: $Q_2 = 5$ <i>Note:</i> the median divides the data into two parts with three scores in each. The lower part contains the scores 2, 2, 3. The lower quartile is the score that divides this set into two equal parts; it is the middle score of this set. So, $Q_1 = 2$. The upper part contains the scores 6, 6, 7. The upper quartile is the score that divides this set into two equal parts; it is the middle score of this set. So, $Q_3 = 6$. We can see that the quartiles have divided the data into four parts with one score in each. Interquartile range $= Q_3 - Q_1 = 6 - 2 = 4$	Find the median (Q_2). This divides the data into two groups with equal numbers in each. The middle score of the lower group is the lower quartile (Q_1). The middle score of the upper group is the upper quartile (Q_3). The interquartile range (IQR) $= Q_3 - Q_1$.

- 3** Complete the following to find the interquartile range of these scores.

$$\begin{array}{ccccccc} 7 & 8 & 10 & 12 & 13 & 13 & 14 \\ & & & \uparrow & & \uparrow & \\ & & & Q_2 & & Q_3 & \end{array}$$

$$Q_1 = \underline{\hspace{2cm}}$$
$$IQR = \underline{\hspace{2cm}} - \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

- 4** Find the interquartile range for each of these sets of scores.

a 15, 16, 17, 20, 22, 23, 25

b 10, 11, 11, 13, 13, 14, 14, 15, 18, 20, 20

EXAMPLE 11D-3 Finding the interquartile range (quartiles are not scores in the set)

Find the lower, middle and upper quartiles and the interquartile range for these scores:

a 1 2 2 3 4 5 5 6 6

b 23 27 25 23 24 25

c 1 2 2 3 4 5 6 6

	Solve	Think	Apply
a	1 2 2 3 4 5 5 6 6 ↑ ↑ ↑ Q_1 Q_2 Q_3 $Q_2 = 4$ $Q_1 = \frac{2+2}{2} = 2, Q_3 = \frac{5+6}{2} = 5.5$ $IQR = Q_3 - Q_1$ $= 5.5 - 2$ $= 3.5$	The scores are already in numerical order. In this case $n = 9$, so $\frac{n+1}{2} = 5$. So, the median is the 5th score = 4. Q_1 is the middle score of the set 1, 2, 2, 3. Q_3 is the middle score of the set 5, 5, 6, 6. The quartiles have divided the data into four parts with two scores in each.	First find the median (Q_2). This divides the data into two groups with equal numbers in each. The middle score of the lower group is the lower quartile (Q_1). The middle score of the upper group is the upper quartile (Q_3). $IQR = Q_3 - Q_1$
b	23 23 24 25 25 27 ↑ ↑ ↑ Q_1 Q_2 Q_3 $Q_2 = \frac{24+25}{2} = 24.5$ $Q_1 = 23, Q_3 = 25$ $IQR = 25 - 23 = 2$	Arrange the scores in numerical order. In this case $n = 6$, so $\frac{n+1}{2} = 3.5$. The median is the number midway between the 3rd and 4th scores. Q_1 is the middle score of the set 23, 23, 24. Q_3 is the middle score of the set 25, 25, 27.	
c	1 2 2 3 4 5 6 6 ↑ ↑ ↑ Q_1 Q_2 Q_3 $Q_2 = \frac{3+4}{2} = 3.5$ $Q_1 = \frac{2+2}{2} = 2, Q_3 = \frac{5+6}{2} = 5.5$ $IQR = 5.5 - 2 = 3.5$	The scores are already in numerical order. In this case $n = 8$, so $\frac{n+1}{2} = 4.5$. The median is the mean of the 4th and 5th scores. Q_1 is the middle score of the set 1, 2, 2, 3. Q_3 is the middle score of the set 4, 5, 6, 6.	

- 5** Complete the following to find the interquartile range of the following scores.

$$\begin{array}{ccccccccccc} 42 & 44 & 45 & 48 & 53 & 61 & 64 & 68 & 71 \\ & \uparrow & & & \uparrow & & & & \\ & Q_1 & & & Q_2 & & & & \end{array}$$

$$\begin{aligned} Q_1 &= \underline{\hspace{1cm}} \\ Q_3 &= \underline{\hspace{1cm}} \\ \text{IQR} &= \underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}} \end{aligned}$$

- 6** Find the interquartile range of each of these sets of scores.

a 170, 170, 170, 185, 188, 189, 194, 196, 203

b 2, 3, 5, 8, 8, 8, 9, 9, 9, 11, 15, 17, 18

12 By first estimating the values of the lower quartile and upper quartile, find the interquartile range for each ogive drawn in question 5 of Exercise 11B.

13 By first drawing an ogive, find the interquartile range of the scores (discrete) in each of these frequency distribution tables. (For help with drawing an ogive, refer to Topic 7G on page 266.)

a

x	f
33	2
34	4
35	9
36	10
37	5

b

x	f
8	12
9	28
10	25
11	26
12	9

c

x	f
24	9
25	11
26	18
27	0
28	12

14 Consider the following grouped data.

i Draw a cumulative frequency histogram and polygon for the data in each table.

ii Estimate the interquartile range from an ogive for each set of data.

a

Score	Frequency
41–50	5
51–60	0
61–70	12
71–80	10
81–90	9
91–100	4

b

Patients per day	Frequency
10–19	13
20–29	15
30–39	16
40–49	10
50–59	4
60–69	2

c

Mass (kg)	Frequency
60–62	4
63–65	7
66–68	9
69–71	6
72–74	4

d

Height (cm)	Frequency
155–159	3
160–164	8
165–169	12
170–174	7
175–179	6

EXAMPLE 11D–5 Interpreting what the interquartile range indicates

In the term test, the median result for Kelly’s class was 65 and the interquartile range was 20. For Sean’s class, the median was 65 and the interquartile range was 12. What conclusions could be made about these results?

Solve	Think	Apply
The marks for Sean’s class are more closely clustered about the median than those for Kelly’s class.	The median mark for both classes is 65: half of the students scored less than 65 and half scored more than 65. However, the interquartile range for Sean’s class is smaller, indicating that the marks for Sean’s class are more closely clustered around the median than the marks for Kelly’s class (or the marks for Kelly’s class are more spread out from the median).	The smaller the value of the interquartile range, the closer the scores are clustered about the median.

Note:

- For a relatively large number of scores:
25% of the scores are less than Q_1 and 75% are greater than Q_1 .
50% of the scores are less than Q_2 and 50% are greater than Q_2 .
75% of the scores are less than Q_3 and 25% are greater than Q_3 .
So the interquartile range contains the middle 50% of the scores.
- The interquartile range gives an indication of the spread of the scores from the median. The greater its value, the greater the spread. A smaller value indicates that the scores are more closely clustered around the median.
- The advantage of using the interquartile range instead of the range is that it is not affected by atypical extreme scores (outliers).

- 15** In a test, the median result for Linda's class was 58 and the interquartile range was 21. In Lara's class, the median was also 58 but the interquartile range was 10. From this information, what conclusions could be made about the results?

- 16** At the end of the cricket season, Tom's median batting score was 31 with an interquartile range of 8. Matthew's median score was 31 and the interquartile range was 24. On the basis of the information given:

- which batsman was more consistent?
- who is more likely to make a high score?
- who is more likely to make a low score?

- 17** Can you find the range of the scores, given the information in the table? Explain your answer.

Height (mm)	Frequency
300–324	3
325–349	18
350–374	47
375–399	32
400–424	14
425–449	6



- 18** The summary statistics of two sets of data are given.
- By comparing the range and interquartile range, determine which set of data, A or B, has an outlier.
 - Which measures are most affected by the outlier?

	A	B
Mean	20.6	21.9
Median	22	18
Range	26	50
IQR	7	9
Upper quartile	24	25
Lower quartile	17	16
Lowest score	4	15
Highest score	30	65

11E Deciles and percentiles

These resources are available on your [obook assess](#):

- **assess quiz 11E:** Test your skills with an auto-correcting multiple-choice quiz

deciles

nine numbers that separate the set of ordered scores into ten parts with an equal number of scores in each part

percentiles

99 numbers that separate the set of ordered scores in 100 parts with an equal number of scores in each part.

For large sets of data, a natural and useful extension of the concept of quartiles is to divide the data into 10 parts with equal numbers of scores in each. The scores that separate these 10 parts are called **deciles**.

Decile 1 is the score that has 10% of the scores below it and 90% above it.

Decile 2 is the score that has 20% of the scores below it and 80% above it.

Decile 3 is the score that has 30% of the scores below it and 70% above it, etc.

Decile 5 is the median.

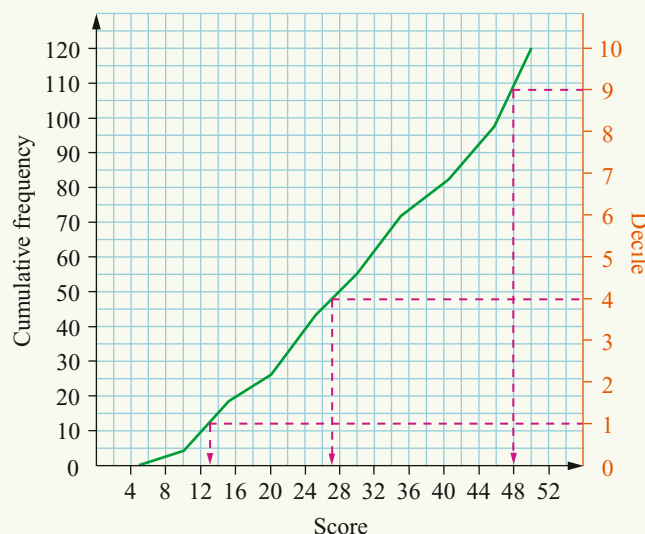
If a large set of data is divided into 100 equal parts with equal numbers of scores in each, the scores that separate these 100 parts are called **percentiles**.

The 35th percentile (P_{35}) is the score that has 35% of the scores below it (and 65% of the scores above it).

The 90th percentile (P_{90}) is the score that has 90% of the scores below it (and 10% of the scores above it). This is equivalent to decile 9.

EXAMPLE 11E-1 Finding deciles from an ogive

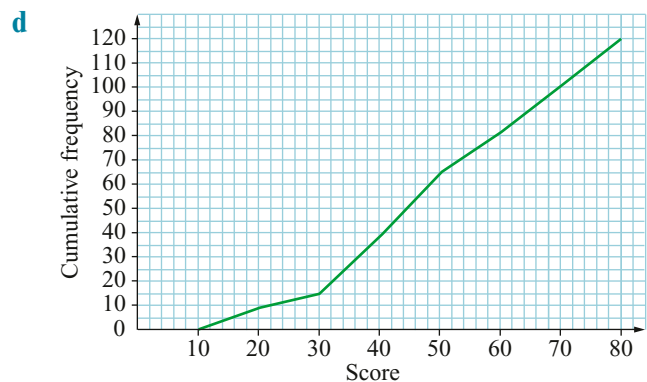
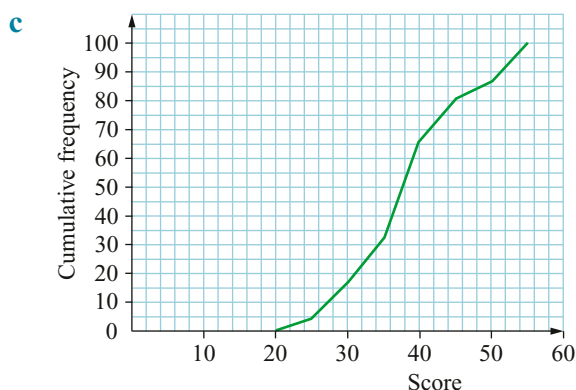
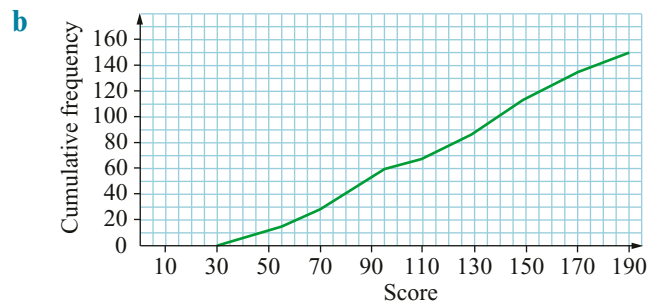
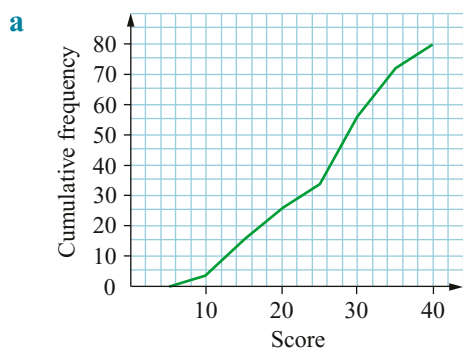
Use the ogive shown to find the 1st, 4th and 9th deciles for the given data.



Solve	Think	Apply
Decile 1 = 13 Decile 4 = 27 Decile 9 = 48	Add another vertical scale to the graph and divide this scale into 10 equal parts. The deciles can be found as shown on the graph above. From the ogive, Decile 1 = 13, Decile 4 = 27 and Decile 9 = 48. So 10% of the scores are less than 13 (and 90% are greater than 13); 40% of the scores are less than 27 (and 60% are greater than 27); and 90% of the scores are less than 48 (and 10% are greater than 48).	Divide the vertical scale into 10 equal parts. The scores that separate these parts are deciles.

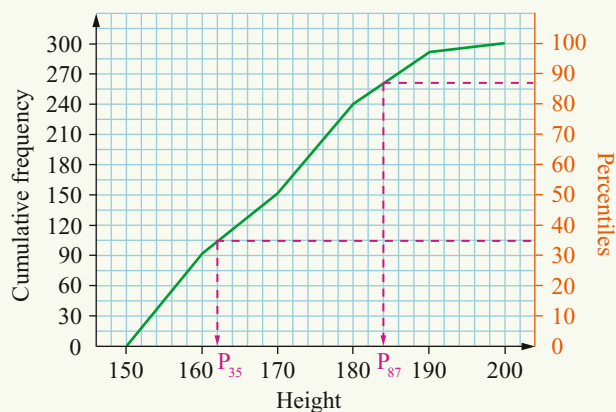
EXERCISE 11E Deciles and percentiles

1 Use the ogives drawn below to find the 1st, 4th and 9th deciles for the given data.



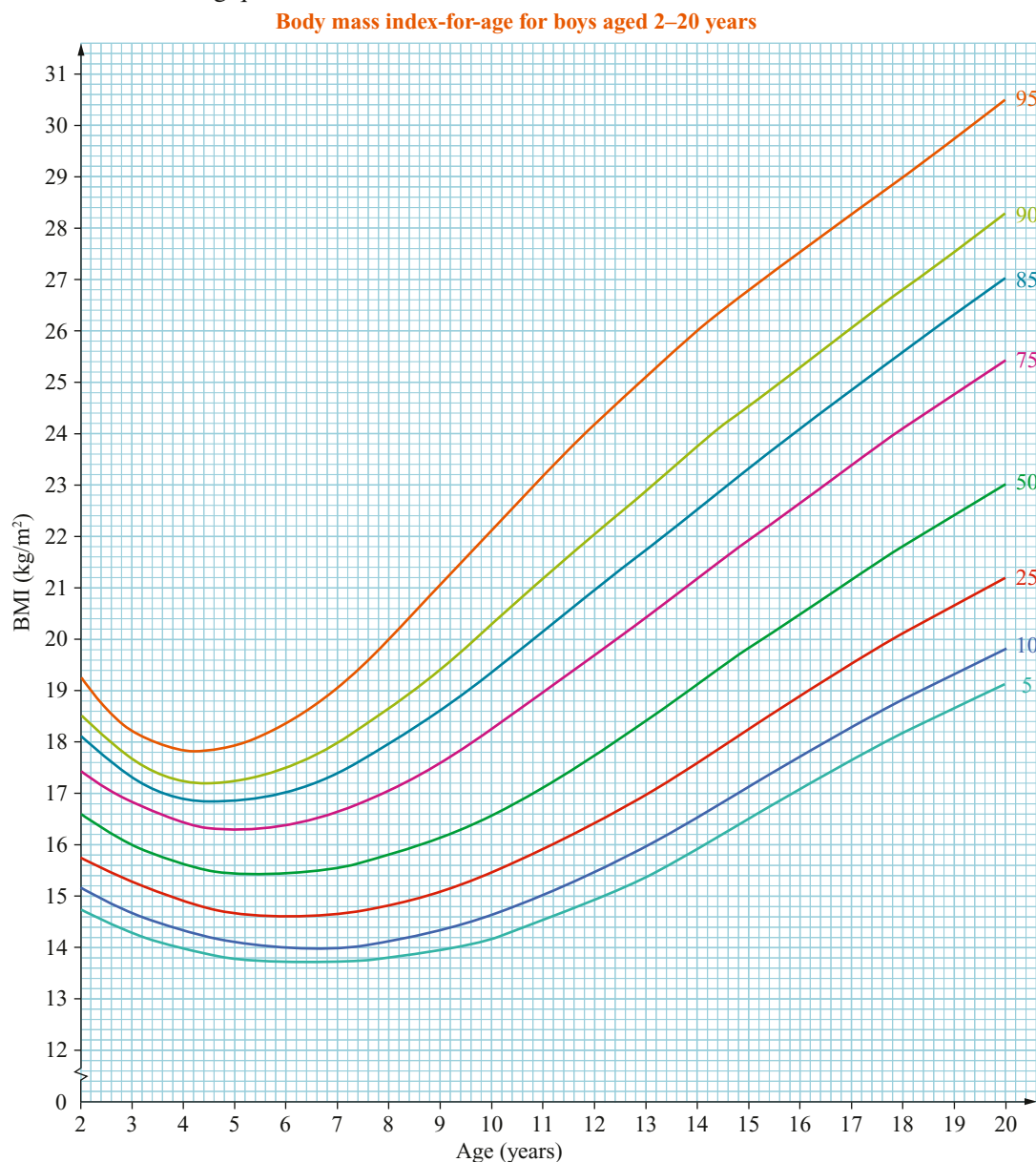
EXAMPLE 11E-2 Finding percentiles from an ogive

For the ogive on the right, estimate P_{35} and P_{87} .



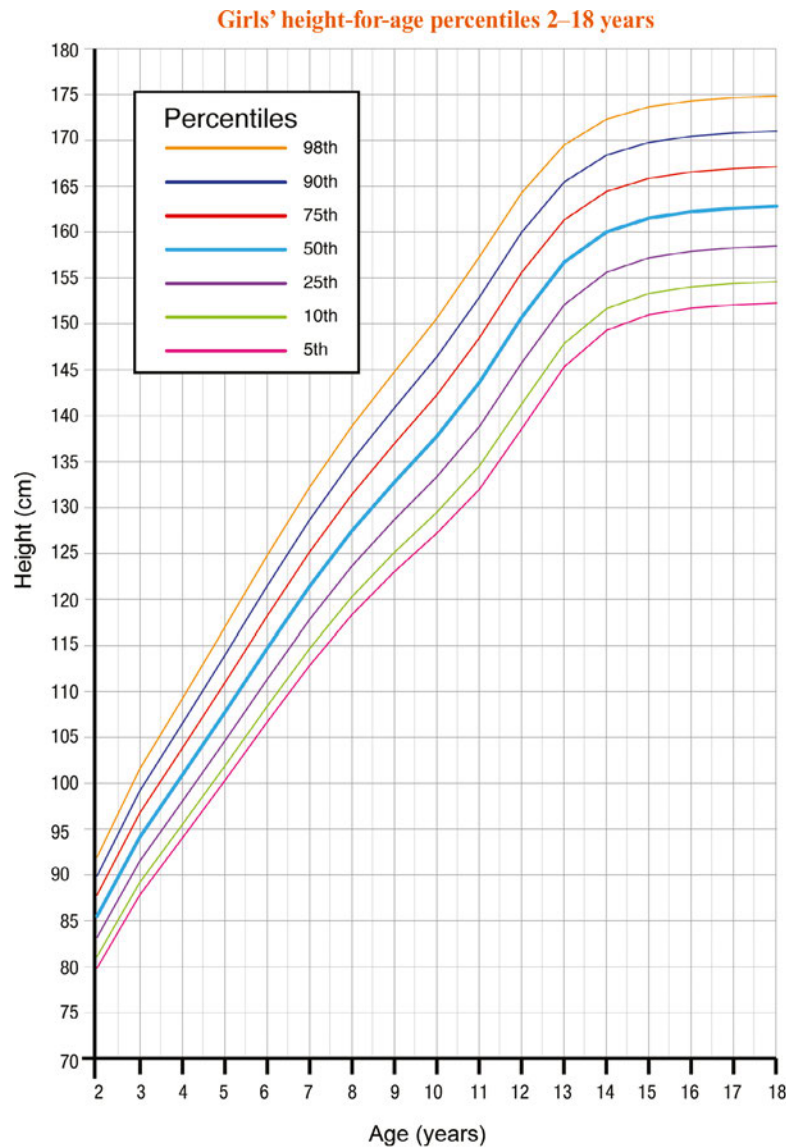
Solve	Think	Apply
$P_{35} \approx 162$ $P_{87} \approx 184$	Add another vertical scale to the graph and divide this into 100 equal parts. The percentiles can be estimated from the ogive. $P_{35} \approx 162$ $P_{87} \approx 184$ So 35% of the scores are less than 162 and 87% of the scores are less than 184.	Divide the vertical scale into 100 equal parts. The scores that separate these parts are percentiles.

- 2 Use the ogives from question 1 parts **a** and **b** to estimate the 15th, 45th and 78th percentiles of each data set.
- 3 In a frequency distribution, what percentage of scores are:
 - a** less than P_{47} ?
 - b** more than P_{95} ?
- 4 In a frequency distribution, determine the percentile that is equivalent to:
 - a** decile 4
 - b** decile 8
 - c** the lower quartile
 - d** the upper quartile
 - e** the median.
- 5 The chart below shows the body mass index-for-age (BMI) for boys aged 2 to 20 years. Use the chart to answer the following questions.



- a** What is the BMI for 12-year-old boys in the 85th percentile?
- b** What percentage of 15-year-old boys have a body mass index that is less than 17.1?
- c** For what age group is the median BMI equal to 15.4?
- d** Boys are considered to be obese if their BMI is greater than the 95th percentile for their age.
 - i** What is the BMI above which 19-year-old boys are considered obese?
 - ii** John is a 9-year-old boy whose BMI is 21.5. Is he considered to be obese, according to this chart?

- e Boys are considered to be underweight if their BMI is less than the 5th percentile for their age.
- i What is the BMI below which 4-year-old boys are considered to be underweight?
- ii A 13-year-old boy has a BMI of 15.8. According to the chart, is he considered to be underweight?
- f The BMI of boys is considered to be normal (not underweight, and not overweight) if it lies between the 5th and 85th percentiles for their age. What range of BMIs is considered to be normal for 17-year-old boys?
- g Calculate the interquartile range for the BMI of 8-year-old boys.
- 6 Use this chart to answer the following questions.



Source: Developed by the National Center for Health Statistics [USA] in collaboration with the National Center for Chronic Disease Prevention and Health Promotion [USA] (2000). <https://www.cdc.gov/growthcharts>

- a What is the height of 16-year-old girls in the 5th percentile?
- b In what percentile band is a 15-year-old girl who is 157 cm tall?
- c What percentage of 12-year-old girls are shorter than 156 cm?
- d What percentage of 18-year-old girls are taller than 175 cm?
- e A 17-year-old girl is 173 cm tall. In which decile band is her height?

box plot (box-and-whisker plot)

a diagram made up of a box and two 'whiskers' displaying five key values for a data set; these values are known as the five-number summary

five-number summary

five key values for a data set; lowest score, highest score, median, lower quartile and upper quartile

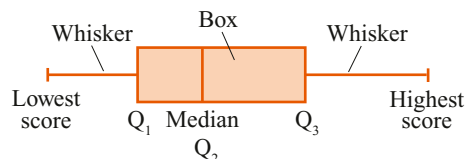
11F Box plots

These resources are available on your obook assess:

- **Worksheet 11F:** Practise your skills with extra problems for box plots
- **Investigation 11F:** Analyse data for the men's 100 m sprint in the Olympic Games
- **assess quiz 11F:** Test your skills with an auto-correcting multiple-choice quiz

A useful way of displaying and comparing sets of data is by means of a **box plot** (or a **box-and-whisker plot**). A box plot is a diagram that displays the extremes (the highest and lowest scores), the median and the upper and lower quartiles of the data.

These five values are also known as the **five-number summary**.



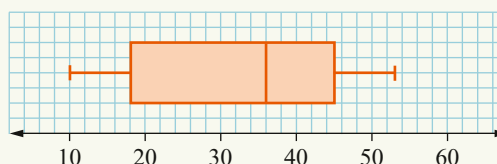
EXAMPLE 11F-1 Finding the range, median and interquartile range from a box plot

For the box plot shown, find the:

- highest score
 - lowest score
 - range of the scores.

b For the data displayed, find the:

- median
- upper quartile
- lower quartile
- interquartile range.



	Solve	Think	Apply
a i	Highest score = 53	The highest score is at the right-hand end of the right-hand whisker. Using the scale, this is 53.	The ends of the 'whiskers' indicate the lowest score and highest score. The ends of the 'box' indicate the lower quartile and upper quartile. The vertical line drawn inside the box indicates the median. The five numbers used (lowest, Q_1 , Q_2 , Q_3 and highest) are referred to as a five-number summary of the data set.
ii	Lowest score = 10	The lowest score is the first point on the left-hand whisker. Using the scale, this is 10.	
iii	Range = $53 - 10 = 43$	The range is the highest score minus the lowest score.	
b i	Median = 36	The median is the score indicated by the vertical line drawn inside the box. This is 36.	
ii	$Q_3 = 45$	The upper quartile is the score indicated by the right-hand end of the box. This is 45.	
iii	$Q_1 = 18$	The lower quartile is the score indicated by the left-hand end of the box. This is 18.	
iv	$IQR = 45 - 18 = 27$	The interquartile range is the upper quartile minus the lower quartile.	

Note: From these results we can say:

- the bottom 25% of the scores take values from 10 up to, but less than, 18
- the top 25% of the scores take values from, but not including, 45 up to 53
- the middle 50% of the scores lie between and including 18 and 45.

Because the median is closer to the upper quartile than to the lower quartile, we can say that the top half of the scores are clustered closer to the median than the bottom half of the scores.

EXERCISE 11F Box plots

1 For each of the box plots shown, find these values.

i highest score

ii lowest score

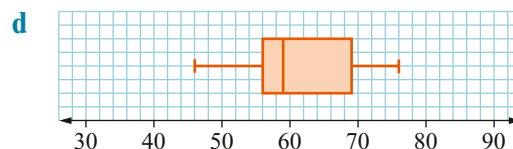
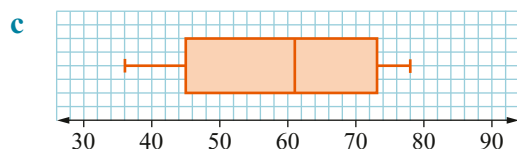
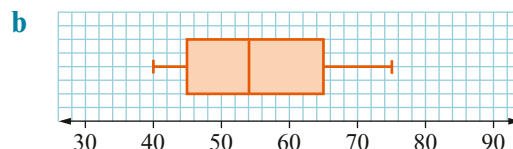
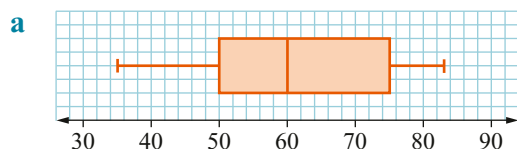
iii range of the scores

iv median

v upper quartile

vi lower quartile

vii interquartile range



EXAMPLE 11F-2 Drawing a box plot from a given five-number summary

Draw a box plot using the following five-number summary of a data set.

Highest score = 82 Lowest score = 25 Median = 64 Lower quartile = 49 Upper quartile = 71

Solve	Think	Apply
Draw a number line to include scores from 25 to 82. Mark the position of each of the five scores given and draw the box plot.	Draw a number line to include scores from 25 to 82. On the number line, mark the position of each of the five scores given and draw the box plot.	Mark the positions of the five scores on a number line. Draw the box using the lower quartile and upper quartile as endpoints and draw the median line inside the box. Draw the whiskers by joining the lowest score and highest score to the box using straight lines.

2 Draw a box plot using the five-number summary given for each data set below.

	Lowest score	Highest score	Lower quartile	Median	Upper quartile
a	15	40	23	28	32
b	130	153	139	141	148
c	6	28	7	10	18
d	71	83	73	78	80
e	1	9	3	5	7

EXAMPLE 11F-3 Drawing a box plot

Draw a box plot for the following set of data.

35 35 36 36 37 38 39 39 39 40

Solve	Think/Apply
	Find the five scores required and draw the box plot as before. Lowest score = 35, Highest score = 40. Use a diagram: 35 35 36 36 37 38 39 39 39 40 ↑ ↑ ↑ Q_1 Q_2 Q_3 $Q_1 = 36$, $Q_2 = 37.5$, $Q_3 = 39$

3 Draw a box plot for each of the following sets of data.

a 4, 5, 8, 8, 10, 12, 12, 14, 15, 19

b 21, 21, 23, 24, 24, 24, 26, 28, 30

c 89, 90, 92, 95, 95, 98, 102, 103

d 18, 20, 22, 23, 25, 29, 30, 30, 30, 31

EXAMPLE 11F-4 Drawing a box plot from a stem-and-leaf plot

Consider the data given in this stem-and-leaf plot.

a Determine the five-number summary.

b Draw a box plot.

Stem	Leaf
2	1 4
3	0 2 2 5 8
4	1 3 4 6 9
5	4 7 8

Solve/Think

a Lowest score = 21, Highest score = 58.

There are 15 scores, So Q_2 = 8th score = 41

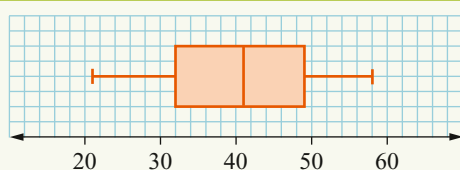
Q_1 = middle of bottom seven scores (21, 24, 30, 32, 32, 35, 38)
= 32

Q_3 = middle of top seven scores (43, 44, 46, 49, 54, 57, 58)
= 49

Apply

Find the five scores required and draw the box plot.

b



4 Consider the data given in these stem-and-leaf plots.

i Determine the five-number summary for each set of data.

ii Draw a box plot for each set of data.

a

Stem	Leaf
5	3
6	1 2 5 8 9
7	0 2 3 6 7 9
8	4 4 6

b

Stem	Leaf
1	7 7 9
2	0 1 1 4
3	2 5 6 6 7
4	1 3 3 8
5	4 5

c

Stem	Leaf
4	2
5	4 6
6	3 3 7
7	1 8 8 9
8	5 6

5 This parallel box plot shows the times (in seconds) male and female competitors take to complete a triathlon.

a What was the fastest time for the event?

b What was the difference in time between the slowest male and slowest female competitor?

c How long did it take the first half of the male competitors to finish?

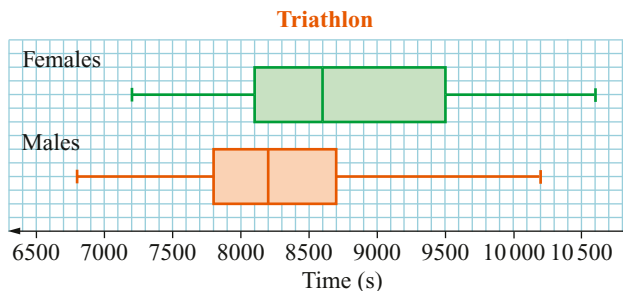
d How long did it take the top 75% of the males to finish?

e How long did it take the first 25% of the females to finish?

f Which group had the greatest range of times?

g Which group had the smallest interquartile range of times?

h Which group had the fastest overall times?



- 6 The average monthly rainfall, in millimetres, for Adelaide and Melbourne is shown in this table.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Adelaide	17	19	22	38	59	52	65	50	43	38	24	23
Melbourne	48	48	52	58	58	50	49	51	59	68	60	59

Use the rainfall data to help you complete the following and draw two box plots on the same scale.

First write the data in ascending order.

Adelaide: 17, 19, 22, 23, 24, 38, 38, 43, 50, 52, 59, 65

Melbourne: 48, 48, 49, 50, 51, 52, 58, 58, 59, 59, 60, 68

Adelaide median = ____

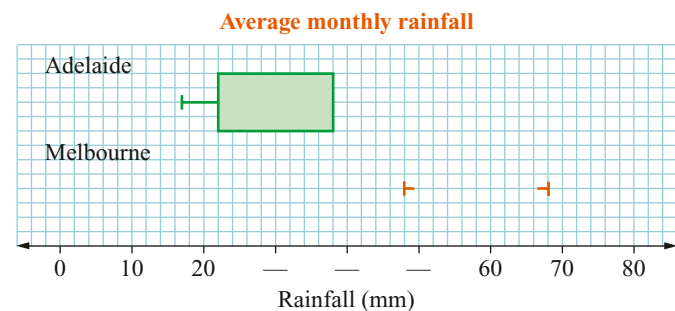
Q_1 = ____

Q_3 = ____

Melbourne median = ____

Q_1 = ____

Q_2 = ____



- 7 The average monthly rainfall, in millimetres, for Canberra and Hobart is shown in this table.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Canberra	58	57	56	53	49	37	40	47	50	67	63	53
Hobart	48	40	47	52	49	56	54	52	52	64	55	52

a Use the given rainfall data to draw two box plots on the same scale.

b What comparisons can be made using the box plots?

- 8 The weights in kilograms of a group of students are shown.

Girls: 62, 45, 53, 58, 50, 48, 56, 65, 78, 54

Boys: 68, 75, 84, 77, 86, 70, 55, 68, 67, 69

a Display the data for weights in kilograms using two box plots on the same scale.

b Make comparisons between the weights of boys and girls using the fact that box plots divide the data into four groups of 25%.

- 9 The towns of Karuah and Buladelah both have a speed limit of 60 km/h. The speeds of the first 100 cars travelling through these towns, from 9:00 am to 10:00 am on Saturday, were measured and the results are shown in the table.

a Draw box plots to display this data.

b Use this data to give a reason why the police might target Karuah for speeding. Also give a reason why police might target Buladelah.

	Karuah	Buladelah
Mean	59	60
Median	58	58
Lower quartile	52	50
Interquartile range	18	9
Highest speed	85	105
Lowest speed	35	40

11G Standard deviation

These resources are available on your obook assess:

- **Worksheet 11G:** Practise your skills with extra problems on centre and spread
- **Investigation 11G.1:** Compare the mean and standard deviation of different samples
- **Investigation 11G.2:** Analyse the data from a sporting team
- **assess quiz 11G:** Test your skills with an auto-correcting multiple-choice quiz

The interquartile range is a better measure of the spread of a set of scores than the range, but its weakness is that its value depends on only two scores, the upper quartile and lower quartile. A more commonly used measure, whose value depends on all the scores, is the standard deviation.

standard deviation

a measure of dispersion or spread whose value depends on all the scores; determined by calculating an 'average' of the distance of each score from the mean

Standard deviation, σ_n , is determined by finding the distances of all the scores from the mean and then calculating an 'average' of these distances.

In the same way as the mean, mode and median are 'typical' scores of the set of data, the standard deviation is a 'typical' distance of the scores from the mean. The smaller the standard deviation, the smaller the 'typical' distance of the scores from the mean, therefore the smaller the spread of the scores.

Standard deviation can be calculated using an algebraic formula, but it is simpler and quicker to use the statistics mode on a scientific calculator.

Note: the interquartile range is a measure of the spread of the scores about the median. The standard deviation is a measure of the spread of the scores about the mean.

EXAMPLE 11G-1 Finding the standard deviation

Find the standard deviation of the following scores.

2 3 5 1 4 6 5 2 1 4

Solve	Think	Apply
$\sigma_n = 1.7$ (to one decimal place)	Use your calculator's instruction manual or ask your teacher to help you determine the appropriate steps for your calculator.	Put the calculator into statistics mode and enter the data. To obtain the standard deviation press the keys for σ_n .

Note: there is another key on the calculator labelled σ_{n-1} . This is another kind of standard deviation which we will investigate later in this section.

EXERCISE 11G Standard deviation

1 Use your calculator to find the mean and standard deviation of each of the following sets of scores.

a 4, 5, 6, 7, 8

b 2, 4, 6, 8, 10

c 0, 3, 6, 9, 12

d 25, 26, 27, 28, 29, 33

e 88, 89, 89, 91, 91, 92

f 239, 245, 248, 250, 253, 253, 254, 258

g 7, 5, 9, 9, 3, 3

h 5, 5, 5, 5, 5

2 Determine the mean and standard deviation of each of the following sets of data.

a

Score	2	3	4	5	6
Frequency	5	7	4	8	1

b

Score	15	16	17	18	19
Frequency	3	8	11	5	3

c

Score	0	1	2	3	4
Frequency	12	15	24	14	5

d

Score	121	122	123	124	125
Frequency	4	6	6	3	5

EXAMPLE 11G-2 Finding the mean and standard deviation for grouped data

Find the mean and standard deviation for the following grouped data.

Height (mm)	300–324	325–349	350–374	375–399	400–424	425–449
Frequency	3	18	47	32	14	6

Solve			Think	Apply
Height (mm)	Class centre (x)	Frequency (f)	For grouped data we again assume that the scores are evenly distributed throughout each class and we use the class centres. Apply the appropriate steps to enter the data into your calculator.	Put the calculator into statistics mode and enter the data using the class centres as the ‘scores’. Press the keys for $\bar{x}$ (mean) and σ_n (standard deviation).
300–324	312	3		
325–349	337	18		
350–374	362	47		
375–399	387	32		
400–424	412	14		
425–449	437	6		
$\bar{x} = 373.3$ (to one decimal place)				
$\sigma_n = 27.7$ (to one decimal place)				

3 Find the mean and standard deviation for each of these data sets.

a

Mass (kg)	Frequency
60–62	2
63–65	5
66–68	7
69–71	4
72–74	2

b

Score	Frequency
41–50	3
51–60	0
61–70	10
71–80	8
81–90	7
91–100	2

c

Patients per day	Frequency
10–19	13
20–29	15
30–39	16
40–49	10
50–59	4
60–69	2

d

Height	Frequency
155–159	4
160–164	7
165–169	10
170–174	5
175–179	2

- 4 **a** Find the mean and standard deviation of these scores: 3, 3, 4, 5, 6, 6.
- b** **i** Add 5 to each of the scores from part **a** and recalculate the mean and standard deviation.
- ii** What do you think would happen to the mean and standard deviation if we added 10 to each score from part **a**? Check by calculation.
- c** **i** Multiply each score from part **a** by 5 and recalculate the mean and standard deviation.
- ii** What do you think would happen to the mean and standard deviation if we multiplied each score from part **a** by 10? Check by calculation.
- 5 For a set of scores, the mean is 17 and the standard deviation is 4. Determine the new mean and new standard deviation if:
- a** 3 is added to each score in the set **b** each score is multiplied by 5 **c** 5 is subtracted from each score
- 6 If the standard deviation of a set of scores is zero, describe the data.

In the same way that there are different averages, there are two types of standard deviation. The standard deviation we have been calculating above is called the **population standard deviation**. However, it can be shown that when σ_n is calculated for a sample from a large population, its value tends to underestimate the spread of scores in the whole population. For this reason, the algebraic formula for standard deviation is altered slightly and this value, denoted by σ_{n-1} , called the **sample standard deviation**, is used when determining the standard deviation for a sample. The value of σ_{n-1} may be determined on a calculator using a similar method to that for σ_n .



population standard deviation

standard deviation for a population, denoted by σ_n

sample standard deviation

standard deviation for a sample, denoted by σ_{n-1}

EXAMPLE 11G-3 Finding the sample standard deviation

A random sample of 16-year-old boys was selected and their weights measured. The results are shown in the table. Find the mean and standard deviation for this data.

Weight (kg)	35–39	40–44	45–49	50–54	55–59	60–64	65–69	70–74	75–79	80–84
Frequency	1	8	11	12	7	5	4	0	1	1

Solve			Think	Apply
Weight	Class centre	Frequency	Find the class centres. Enter the data into a calculator using class centres for scores. Press the $\bar{x}$ key for mean. Because we are dealing with a sample from a larger population, press the σ_{n-1} key for sample standard deviation.	Enter the scores into a calculator and press appropriate keys for mean ($\bar{x}$) and sample standard deviation (σ_{n-1}).
35–39	37	1		
40–44	42	8		
45–49	47	11		
50–54	52	12		
55–59	57	7		
60–64	62	5		
65–69	67	4		
70–74	72	0		
75–79	77	1		
80–84	82	1		
Using the class centres for scores and the frequency, the calculator values are: $\bar{x} = 53$, $\sigma_{n-1} = 9.4$ (to one decimal place).				

Note:

- For the distribution in Example 11G-3, $\sigma_n = 9.3$.
- For large numbers of scores there is very little difference between σ_{n-1} and σ_n .
- For situations in which both population values (parameters) and sample-based estimates (statistics) are being considered, different notations are often used to distinguish between them. The following table is a guide.

Sample statistic	Population value	Description
n	N	number of members of sample or population
$\bar{x}$ (x bar)	μ (mu)	mean
s, σ_{n-1}	σ, σ_n	standard deviation

- 7 The shoe sizes of a random sample of Year 11 girls are recorded below. Find the sample standard deviation of this data.

5, 4, 7, 5, 6, 7, 4, 8, 5, 6, 4, 6, 7, 5, 9

- 8 A sample of 20 families is chosen at random and the number of children in each family is recorded in this table. Calculate the mean and sample standard deviation.

Number of children	Frequency
0	2
1	3
2	10
3	4
4	0
5	1

- 9 A woman measures the time it takes her to get to work each day. The times, in minutes, for a two week period are summarised in the table below. Determine the mean and sample standard deviation of these times.

Time	Frequency
24–26	2
27–29	3
30–32	3
33–35	1
36–38	1

- 10 The weights (in kilograms) of 17-year-old soccer players were: 69, 64, 58, 77, 85, 90, 72, 61, 73, 59.

- Find the population standard deviation (σ_n) and the sample standard deviation (σ_{n-1}) of these weights.
- Compare the values obtained in part a. Which would be more appropriate to use?

- 11 Two machines produce chocolates with an advertised weight of 60 g. Samples of the output from these machines were selected and weighed. The weights of the sample chocolates, in grams, are listed.

Machine 1: 50, 60, 60, 60, 60, 60, 60

Machine 2: 50, 50, 55, 55, 60, 60, 60

- For machine 1 and machine 2 find the:
 - range
 - interquartile range
 - standard deviation.
- Where are the scores clustered for machine 1?
- Which machine produces chocolates of a more consistent weight? Explain your answer.



- 12** In a game of golf, the player with the lowest score wins the hole. Usually the player with the lowest total score wins the match. Carrie and Laura play nine holes of golf. Their scores are listed in the table below.

- a** For each player, find the:
- i** mean score
 - ii** median score.
- b** How many holes did each player win?
- c** For each set of scores, compare the:
- i** range
 - ii** interquartile range
 - iii** standard deviation.
- d** Use the data to present an argument that Laura is the better player.
- e** Use the data to present an argument that Carrie is the better player.

Hole	Carrie	Laura
1	5	4
2	8	6
3	6	5
4	3	3
5	5	4
6	8	8
7	4	4
8	7	6
9	8	15
Total	54	55



- 13** The marks scored by Julie and Alison in the eight topic tests they did in mathematics over the year are shown.

Test	1	2	3	4	5	6	7	8
Julie	72	71	81	74	72	65	78	44
Alison	71	70	76	73	78	62	76	69

- a** Find the total marks scored by each student.
- b** For the marks scored by each student, find the:
- i** mean mark
 - ii** median mark.
- c** In how many tests did Julie score higher marks than Alison?
- d** For each student's marks, compare and comment on the:
- i** range
 - ii** interquartile range.
- e** Calculate the standard deviation of each student's marks.
- f** Which student's performances were more consistent?
- g** The Mathematics Prize is to be given to the better of these two students. Use the data to present a case for each student to receive the award.

- 14 a** Find the standard deviation (σ_n) for each of these data sets.

- i** 5, 6, 6, 7, 8, 9
- ii** 5, 6, 6, 7, 8, 90

- b** Discuss the effect of outliers on the standard deviation.
- c** Find the range and interquartile range for each data set.
- d** Does the outlier affect the range or the interquartile range?

- 15 a** From the table of random numbers below, choose an arbitrary starting point and select a random sample of 20 one-digit numbers. Find the standard deviations for your sample.

48047	45381	33232	35178	46971	85879	31458	22016
08695	58112	96070	91910	18868	52251	99827	32581
90070	98873	89846	50953	92529	68249	54949	83829
10132	27359	13017	41045	13817	65603	87615	55691
58547	01331	62538	79181	33071	63766	73613	24470
85566	81574	71965	20977	48005	83418	58738	98771
40942	42373	38710	39916	08187	00133	16288	6427
80553	58331	62724	74004	09344	91315	25791	40296
27328	85758	45342	98884	36034	79836	94902	80442
64584	20776	86792	42340	83522	62139	14038	88433

- b** Use a summary table on the board to combine your results with those of the rest of the class.
- c** For a table of random numbers the theoretical value of the population standard deviation is 2.87, correct to two decimal places. (This is based on the assumption that each digit occurs the same number of times.)
- Compare your results for part **b** with the theoretical standard deviation of the population.
 - Which standard deviation (σ_n or σ_{n-1}) is usually closer to the theoretical value?
 - Discuss the results from parts **i** and **ii** with the class.
- 16** Investigate examples from the media to illustrate appropriate and inappropriate use or misuse of measures of central tendency and spread.

- 17** Spreadsheets have inbuilt functions to find various statistics for a set of data. The spreadsheet below left contains the marks out of 50 obtained on some tests and the formulas for finding statistics. The spreadsheet below right contains the statistics generated by the formulas in the first spreadsheet shown.

The marks in cells B2 down to B6 can be added using the sum function in the formula =SUM(B2:B6).

The mean mark can be found using the formula =AVERAGE(B2:B6) or =AVG(B2:B6).

The standard deviation can be found using =STDEV(B2:B6).

	A	B
1	Test	Mark
2	1	33
3	=A2+1	23
4	=A3+1	45
5	=A4+1	18
6	=A5+1	44
7	Total	=SUM(B2:B6)
8	Mean	=AVERAGE(B2:B6)
9	Standard deviation	=STDEV(B2:B6)

	A	B
1	Test	Mark
2	1	33
3	2	23
4	3	45
5	4	18
6	5	44
7	Total	163
8	Mean	32.6
9	Standard deviation	12.14

- a** Enter some marks of your own and find the total, mean and standard deviation.
- b** Try these formulas:
- =COUNT(B2:B6) =MAX(B2:B6) =MIN(B2:B6)
- Explain what each of these formulas does.
- c** Investigate the FORMULA functions of the spreadsheet.

11H Identifying outliers

These resources are available on your obook assess:

- **Video tutorial 11H:** Watch and listen to an explanation of Example 11H-1
- **Investigation 11H:** Explain the effect of outliers on summary statistics
- **assess quiz 11H:** Test your skills with an auto-correcting multiple-choice quiz

As previously discussed, outliers are scores which appear to be abnormally higher or lower than the rest of the scores. It is up to statisticians to decide what is abnormal. Here are two general methods that statisticians use to identify outliers.

1 Using the median and quartiles:

Outliers are scores which lie outside the range: $Q_1 - 1.5 \times IQR$ to $Q_3 + 1.5 \times IQR$
This is the preferred method for identifying outliers because the median and quartiles are usually not affected by outliers. The method can be used for skewed data.

2 Using the mean and standard deviation:

Outliers are scores which lie outside the range:
mean $- 3 \times$ standard deviation to mean $+ 3 \times$ standard deviation
This method is less favoured because the calculation of the mean and standard deviation have been affected by the presence of the outlier(s). The method is not suitable for skewed data.

EXAMPLE 11H-1 Identifying outliers from a set of scores

Consider the sample of scores 30, 50, 50, 52, 55, 55, 57, 57, 58, 60, 60.

Identify any outliers using:

- a** the median and quartiles **b** the mean and sample standard deviation.

	Solve/Think	Apply
a	30 appears to be much lower than the other scores which are clustered from 50 to 60. 30, 50, 50, 52, 55, 55, 57, 57, 58, 60, 60 $\uparrow$ $\uparrow$ $\uparrow$ Q_1 Q_2 Q_3 Median = $Q_2 = 55$, $Q_1 = 50$, $Q_3 = 58$ $IQR = Q_3 - Q_1 = 58 - 50 = 8$ Lower limit = $Q_1 - 1.5 \times IQR$ $= 50 - 1.5 \times 8$ $= 38$ Since $30 < 38$ that means 30 is an outlier.	For this method, outliers are scores which lie outside the range $Q_1 - 1.5 \times IQR$ to $Q_3 + 1.5 \times IQR$.
b	Sample mean = $\bar{x} = 53.1$ Sample standard deviation = $\sigma_{n-1} = 8.4$ Lower limit = $\bar{x} - 3 \times \sigma_{n-1}$ $= 53.1 - 3 \times 8.4$ $= 27.9$ Since $30 > 27.9$, it is not regarded as an outlier using this method.	Outliers are scores which lie outside the range: mean $- 3 \times$ standard deviation to mean $+ 3 \times$ standard deviation.

EXERCISE 11H Identifying outliers

Consider each of the data sets in this exercise to be samples from a larger population.

1 Identify any outliers in each data set using:

i the median and quartiles

ii the mean and sample standard deviation.

a 3, 24, 25, 26, 26, 26, 28, 29, 30, 32

b 5, 5, 6, 7, 7, 7, 8, 10, 12, 13, 21

c 17, 31, 32, 32, 34, 35, 35, 36, 38, 41, 44, 45

d 49, 50, 51, 51, 51, 53, 54, 56, 60, 63, 64, 75

EXAMPLE 11H-2 Identifying outliers from a frequency table

For the given data set, identify any outliers using:

a the median and quartiles

b the mean and sample standard deviation.

x	f
3	1
4	1
5	1
6	3
7	5
8	1

	Solve/Think	Apply																						
a	<table><tr><th>x</th><th>f</th><th>cf</th></tr><tr><td>3</td><td>1</td><td>1</td></tr><tr><td>4</td><td>1</td><td>2</td></tr><tr><td>5</td><td>1</td><td>3</td></tr><tr><td>6</td><td>3</td><td>6</td></tr><tr><td>7</td><td>5</td><td>11</td></tr><tr><td>8</td><td>1</td><td>12</td></tr></table> Median = $Q_2 = 6.5$ $Q_1 = 5.5$ $Q_3 = 7$ $IQR = Q_3 - Q_1 = 7 - 5.5 = 1.5$ Lower limit = $Q_1 - 1.5 \times IQR$ $= 5.5 - 1.5 \times 1.5$ $= 3.25$ Any score below 3.25 is an outlier, so 3 is an outlier. Upper limit = $Q_3 + 1.5 \times IQR$ $= 7 + 1.5 \times 1.5$ $= 9.25$ So 8 is not an outlier.	x	f	cf	3	1	1	4	1	2	5	1	3	6	3	6	7	5	11	8	1	12	Add a cumulative frequency column. There are 12 scores so the median is the average of the 6th score (which is 6) and the 7th score (which is 7); i.e. $Q_2 = 6.5$. Q_1 is the middle of the first 6 scores = 5.5. Q_3 is the middle of the last 6 scores = 7.	Find Q_1, Q_2, Q_3 and the IQR and use $Q_1 - 1.5 \times IQR$ for the lower limit and $Q_3 + 1.5 \times IQR$ for the upper limits.
x	f	cf																						
3	1	1																						
4	1	2																						
5	1	3																						
6	3	6																						
7	5	11																						
8	1	12																						
b	Sample mean = $\bar{x} = 6.1$ Sample standard deviation = $\sigma_{n-1} = 1.4$ Lower limit = $6.1 - 3 \times 1.4 = 1.9$ Upper limit = $6.1 + 3 \times 1.4 = 10.3$ Using this method, none of the scores are outliers.	Use your calculator to find the mean and sample standard deviation and substitute into: mean $- 3 \times$ standard deviation and mean $+ 3 \times$ standard deviation.																						

2 Identify any outliers in each of these data sets using:

- i the median and quartiles
- ii the mean and sample standard deviation.

a

x	f
3	1
6	1
7	4
8	8
9	5
10	3

b

x	f
12	1
13	2
14	12
15	3
18	1

3 For each data set in question 2, which method of identifying outliers is the most suitable?

EXAMPLE 11H-3 Identifying outliers from a stem-and-leaf plot

Consider the data in the stem-and-leaf plot shown.

- a** Calculate the median, Q_1 , Q_3 and IQR for the data shown.
- b** Calculate the limits for outliers in this data.
- c** Is 50 an outlier? Explain your answer.

Stem	Leaf
5	0
6	
7	9 9
8	1 2 8
9	1 1 4 6 6 6 7 8
10	0 0 1 3 4 5 8
11	3 6

	Solve	Think	Apply
a	Median = 96 $Q_1 = 88$ $Q_3 = 103$ IQR = $103 - 88$ = 15	There are 23 scores, so the median is the 12th score. There are 11 scores on either side of the median. Q_1 is the 6th score and Q_3 is the 18th score.	The interquartile range is calculated as usual. The scores can be crossed off from either end to find the median, Q_1 and Q_3 .
b	Lower limit = $Q_1 - 1.5 \times \text{IQR}$ = $88 - 1.5 \times 15$ = 65.5 Upper limit = $Q_3 + 1.5 \times \text{IQR}$ = $103 + 1.5 \times 15$ = 125.5	Find the lower limit and the upper limit using the appropriate formula.	The formulas used for the limits are $Q_1 - 1.5 \times \text{IQR}$ and $Q_3 + 1.5 \times \text{IQR}$.
c	Since $50 < 65.5$, it is an outlier.	The score of 50 is below the lower limit of 65.5. There are no very high outliers.	

4 Consider the data in the stem-and-leaf plots shown below.

- Calculate the median, Q_1 , Q_3 and IQR.
- Calculate the limits for outliers in this data.
- Are any of the scores outliers?

a

Stem	Leaf
1	2 3 6 8 8
2	2 5 6 7 7 8 9
3	4 4
4	
5	9

b

Stem	Leaf
0	5
1	3 7
2	2 2 4 6
3	0 1 1 5 8 8 9
4	2 3 3 6
5	1 5
6	4

c

Stem	Leaf
3	9
4	2 5
5	2 2 3
6	0 1 3 4 4 7 8
7	0 0 2 4 5 5 7 9 9
8	1 3

d

Stem	Leaf
8	3
9	0 1 5
10	6 6 7 7 8 9 9
11	0 1 3 5 7 8 9
12	0 0 3 1
13	4

e

Stem	Leaf
6	5 5 6 7 8 9
7	0 1 3 3 4 6 9 9
8	2 2 5 8
9	1 3
10	0 2
11	8

f

Stem	Leaf
1	9 9
2	0 2 4
3	8 8 8 9
4	0 5 5 9 9
5	2 2 2 6 7 7
6	3 4 5 8 9

5 The weights, in kilograms, of the players in a women's hockey team are recorded as:

55, 56, 63, 48, 58, 56, 48, 54, 55, 62, 64, 157

- Show that 157 is an outlier, using both methods discussed earlier.
- Discuss possible reasons for the score of 157.
- Calculate the following measures, with and without the outlier:
 - mean and median weight of the team
 - the interquartile range
 - the population standard deviation.
- Which measures are affected the most by the outlier?
- Describe the weights of the team members using the most appropriate measures of location and spread.

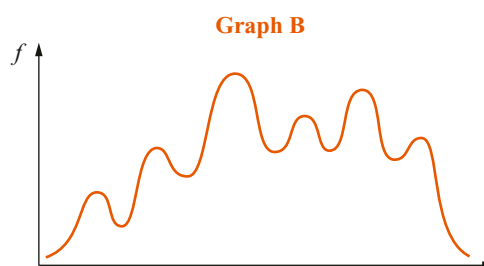
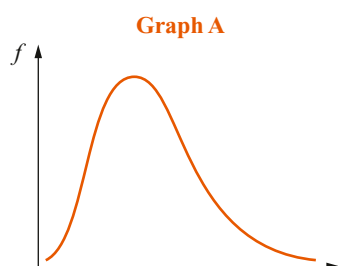


11 | Graphical data displays

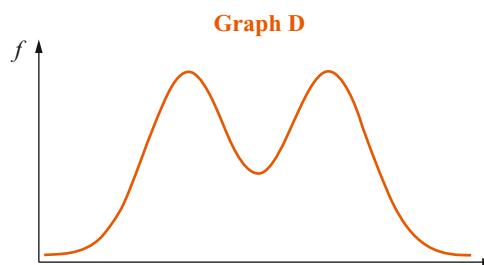
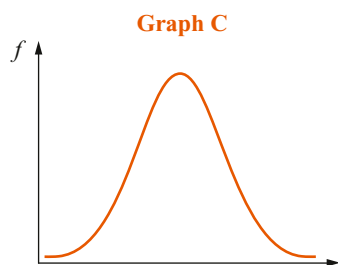
These resources are available on your [obook assess](#):

- **Video tutorial 11I:** Watch and listen to an explanation of Examples 11I-1 and 11I-2
- **Investigation 11I:** Examine ways of displaying the distribution of data
- **assess quiz 11I:** Test your skills with an auto-correcting multiple-choice quiz

The shape of frequency curves may be described in terms of smoothness, symmetry and number of modes. The mode is the score with the highest frequency, so has the highest value on the graph. Graph A is a smooth curve, Graph B is not smooth.

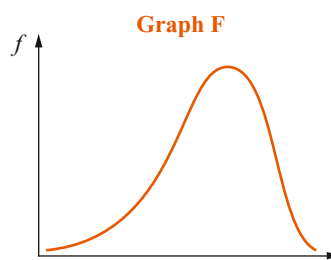
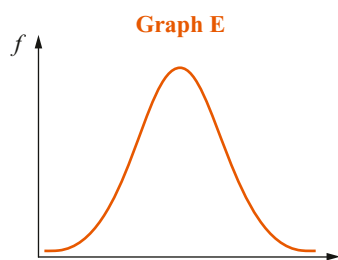


Graph C is **unimodal** (has one mode) and graph D is **bimodal** (has two modes).

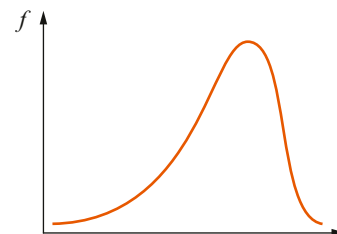


Note: A curve with two distinct humps, even if the frequency at each hump is not the same, is called bimodal. If a curve has three or more modes it is sometimes described as being **multimodal**. (*Note:* bimodal can be considered as a special case of multimodal.)

Graph E is symmetrical, Graph F is asymmetrical. Graphs that are not symmetrical are said to be **skewed**.



If the longer tail of the graph is to the left, then the distribution is **negatively skewed**. This would occur, for example, if we graphed the distribution of the results of a very easy test. Most of the students would score high marks and only a few would score low marks.



unimodal

description of a distribution graph for a data set with one mode

bimodal

description of a distribution graph for a data set with two modes

multimodal

description of a distribution graph for a data set with many modes

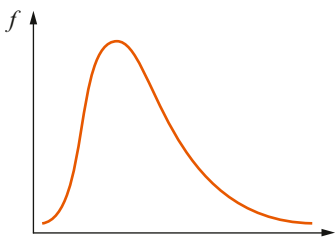
skewed

description of a distribution graph of a data set for which the shape is not symmetrical

negatively skewed

description of a skewed graph which has a longer tail to the left

If the longer tail is to the right, then the distribution is **positively skewed**. This would occur for the distribution of the results of a very hard test.



positively skewed
description of a skewed graph which has a longer tail to the right

The curve on the right is described as **bell-shaped**. It is often referred to as a normal distribution. It occurs for many naturally occurring characteristics, such as the number of tomatoes on a plant, the number of peas in a pod, the heights of a particular age group of females. More will be learnt about this curve in Year 12.

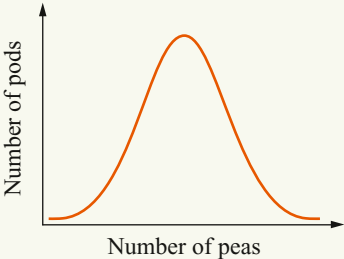


bell-shaped
description of a distribution graph of a set for which the shape is symmetrical

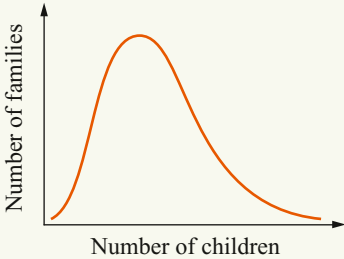
EXAMPLE 11I-1 Describing graphs

Describe the shape, smoothness, symmetry and modality of the following curves.

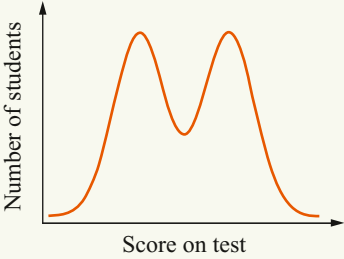
a



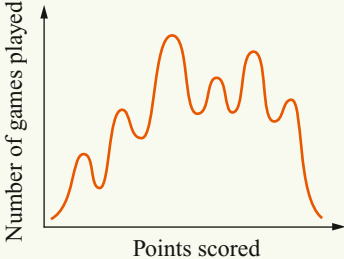
b



c



d

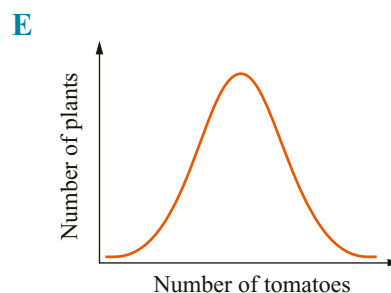
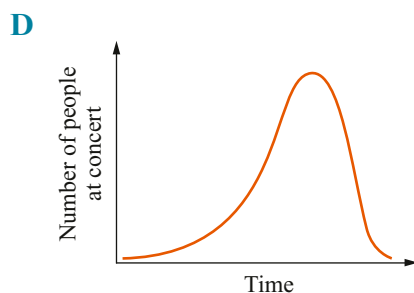
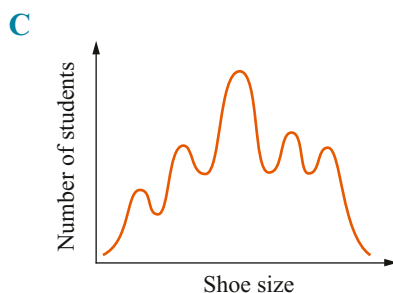
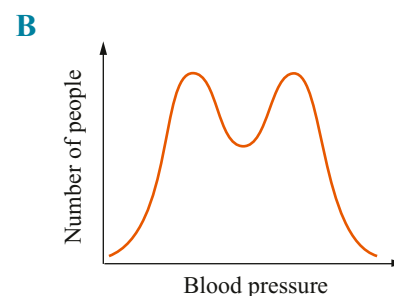
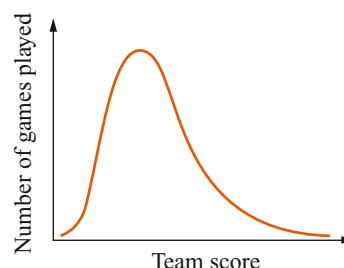


	Solve	Think/Apply
a	The curve is bell-shaped, smooth, symmetrical and has one mode.	Symmetry is the first feature examined, followed by the number of 'bumps' or modes.
b	The curve is positively skewed, smooth, not symmetrical and has one mode.	
c	The curve is smooth, symmetrical and bimodal.	
d	The curve is not smooth or symmetrical and is multimodal.	

EXERCISE 111 Graphical data displays

1 Match each description with one of the curves **A** shown.

- a** smooth, unimodal, positively skewed
- b** smooth, unimodal, negatively skewed
- c** smooth, symmetrical, bell-shaped, unimodal
- d** smooth, symmetrical, bimodal
- e** not smooth, asymmetrical, multimodal



EXAMPLE 11I-2 Describing the nature of a graph

Comment on the nature of the data graphed in Example 11I-1.

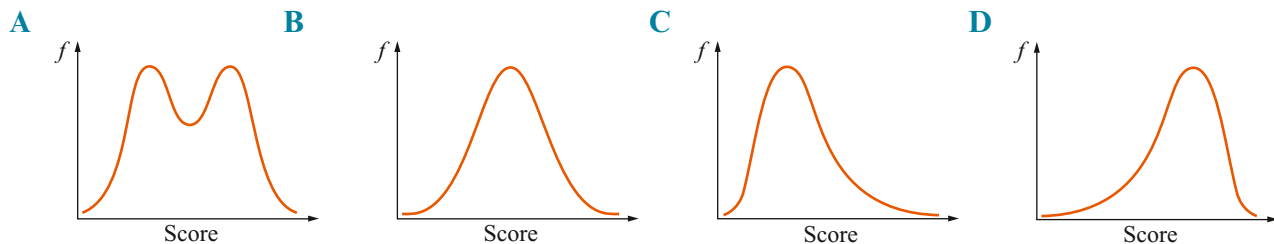
	Solve	Think	Apply
a	The number of peas in a pod is evenly spread on both sides of the mode.	This graph is a symmetrical distribution.	Smooth symmetrical graphs give detail about populations.
b	More families have small numbers of children and fewer have large families.	This graph's positive skew indicates that there are more families with fewer children at the left side of the curve.	
c	A large number of students scored high marks on a test and a large number of students did poorly.	This graph is symmetrical with two modes. The graph could show, for example, that a large number of engineering students scored high marks on a test and a large number of commerce students did poorly.	
d	There is no pattern of points scored in the games recorded.	No real pattern.	

2 Comment on the nature of the data graphed in question 1.

3 For graphs **A–E** in question 1, place a cross on the horizontal axis to indicate the mode(s).

4 Match each of the following descriptions to one of the curves, **A** to **D**, shown on the next page.

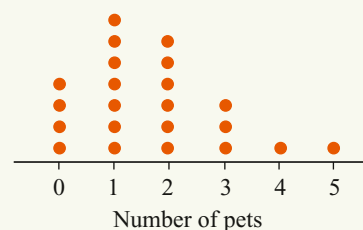
- a** the heights of 1000 randomly selected adult women
- b** the results in a difficult examination
- c** the weights of jockeys and wrestlers
- d** the results in an easy examination



EXAMPLE 11I-3 Describing the shape of a distribution from a dot plot

The number of pets that each student in a class owns are graphed in this dot plot.

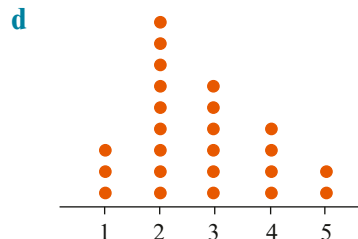
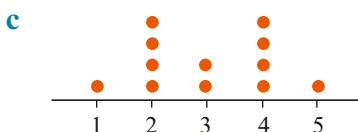
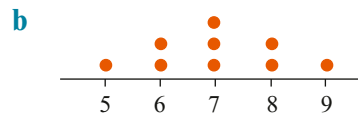
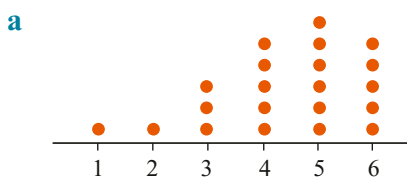
- Draw a curve showing the shape of the distribution.
- Describe the distribution.
- Mark the location of the mode on the horizontal axis.



	Solve	Think	Apply
a		Draw the curve showing the shape of the distribution.	The smooth curve shows the shape of the distribution. The mode is the score where the graph peaks.
b	Positively skewed distribution. More students have a small number of pets.	The tail points right, so the skew is positive. This means there are more small numbers than large numbers.	
c	The mode is marked with a cross on the scale of the graph in part a.	The highest point on the graph shows the location of the mode.	

5 For each dot plot below:

- Draw a curve showing the shape of the distribution.
- Plot the position of the mode, if one exists, by placing a cross on the horizontal axis.
- Describe the shape of the distribution.

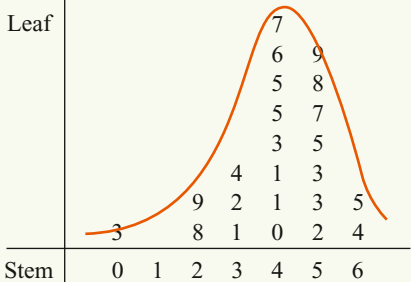


EXAMPLE 11I-4 Describing the shape of the distribution from a stem-and-leaf plot

For the data given in the stem-and-leaf plot:

- a** Determine whether the distribution of scores is symmetrical, positively skewed or negatively skewed.
- b** Are there any clusters? If so, where are they located?

Stem	Leaf
0	3
1	
2	8 9
3	1 2 4
4	0 1 1 3 5 5 6 7
5	2 3 3 5 7 8 9
6	4 5

Solve	Think	Apply
a The frequency curve is negatively skewed.	Turn the stem-and-leaf plot on its side and draw a curve to fit the height of each column. 	Outliers and clusters show properties of distributions.
b This set of scores is clustered in the 40s and 50s.	The rows with the most numbers can form a cluster of scores.	

6 Consider the data given in the stem-and-leaf plots below.

- i** Determine whether the distribution of scores is symmetrical, positively skewed or negatively skewed.
- ii** Are there any clusters? If so, where are they located?

a

Stem	Leaf
1	2 3 6 8 8
2	2 5 6 7 7 8 9
3	4 4
4	
5	9

b

Stem	Leaf
0	5
1	3 7
2	2 2 4 6
3	0 1 1 5 8 8 9
4	2 3 3 6
5	1 5
6	4

c

Stem	Leaf
3	9
4	2 5
5	2 2 3
6	0 1 3 4 4 7 8
7	0 0 2 4 5 5 7 9 9
8	1 3

d

Stem	Leaf
8	3
9	0 1 5
10	6 6 7 7 8 9 9
11	0 1 3 5 7 8 9
12	0 0 3 1
13	4

e

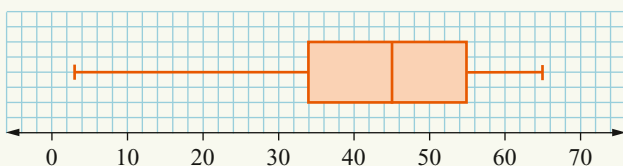
Stem	Leaf
6	5 5 6 7 8 9
7	0 1 3 3 4 6 9 9
8	2 2 5 8
9	1 3
10	0 2
11	5

f

Stem	Leaf
1	9 9
2	0 2 4
3	8 8 8 9
4	0 5 5 9 9
5	2 2 2 6 7 7
6	3 4 5 8 9

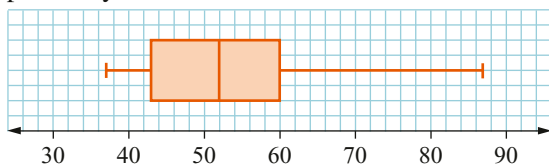
EXAMPLE 11I-5 Describing the shape of a distribution from a box plot

The data from the stem-and-leaf plot in Example 11I-4 has been drawn as a box plot. Describe the shape of the distribution.



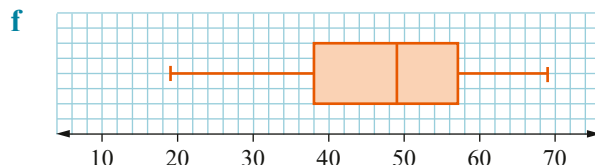
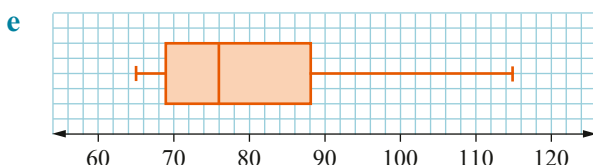
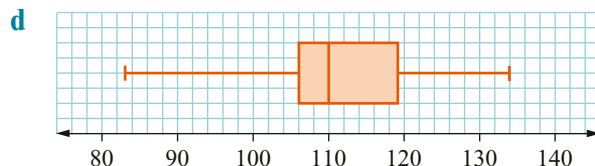
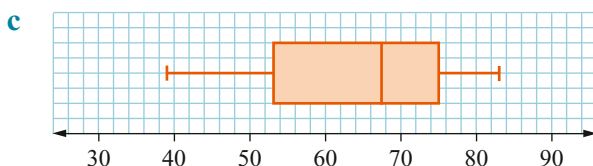
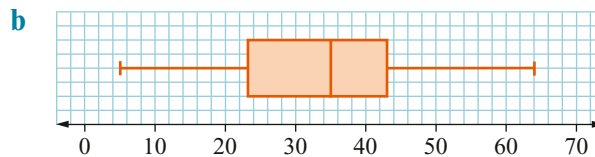
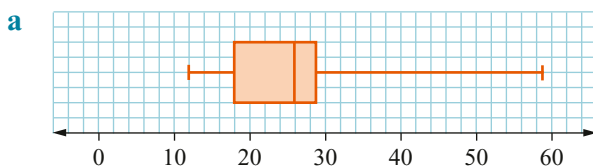
Solve	Think	Apply
The distribution is negatively skewed.	Each of the four parts of the box plot contain 25% of the scores. The first 25% of scores are spread from 3 to 34. The first 50% are spread from 3 to 45. The last 25% of scores are spread from 55 to 65. The second 50% are spread from 45 to 65. Therefore the first 25% and 50% of scores are spread out much further than the last 25% and 50% of scores. This gives a tail pointing left; that is, it is negatively skewed.	Check the first and last 25% and 50% of scores to see how spread out they are. This will indicate skewness. If all four parts of the box plot are evenly spread, then the distribution is symmetrical. If not, then it is skewed.

- 7 The box plot below is drawn from the data in the stem-and-leaf plot shown. Explain why the distribution is positively skewed.



Stem	Leaf
3	7 8 9
4	0 3 5 7 8
5	2 2 2 6 9 9
6	0 1 8
7	4
8	7

- 8 These box plots are drawn from the stem-and-leaf plots in question 6. What is the shape of each distribution? What aspect of the box plot shows this?

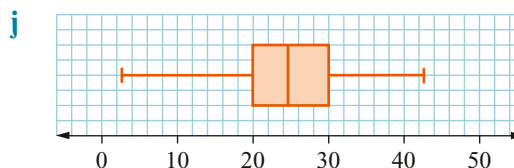
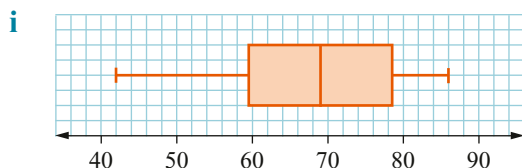
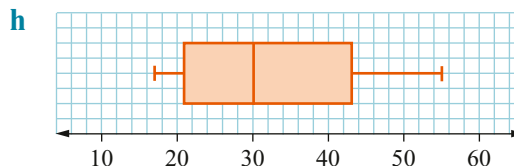
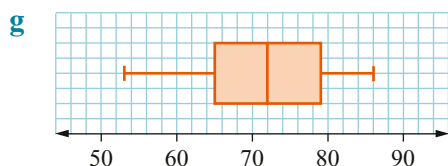
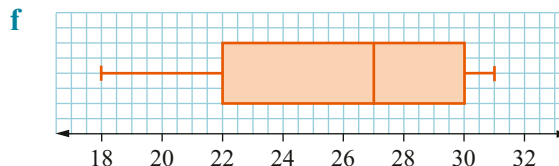
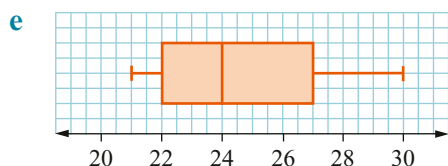
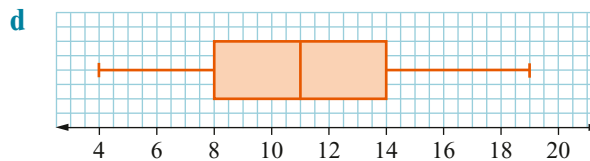
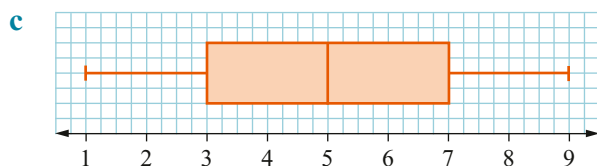
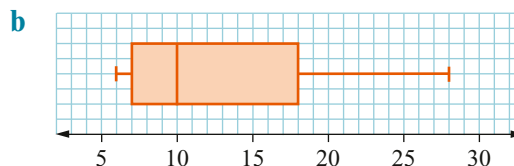
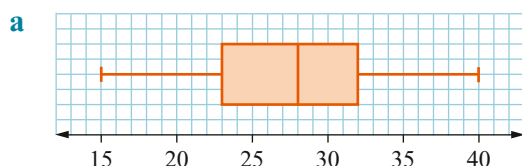


9 Answer the following questions for each of the box plots below.

i Describe the shape of the data distribution in each box plot.

ii Determine Q_1 , Q_2 , Q_3 and IQR.

iii Are there any outliers?

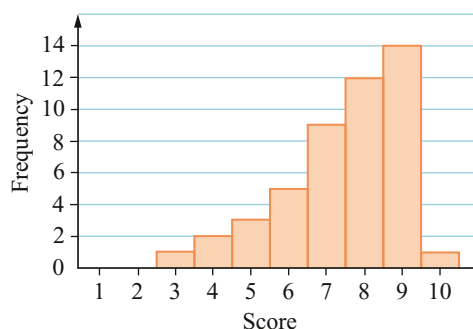


10 The histogram on the right shows the results of 47 students for a test marked out of 10.

a Draw a smooth curve on the histogram.

b Describe the shape of the curve.

c What comments can be made about the test based on the shape of the curve?

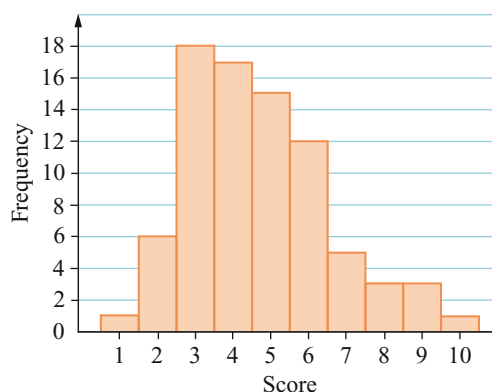


11 The histogram on the right shows the results of 81 students for a test marked out of 10.

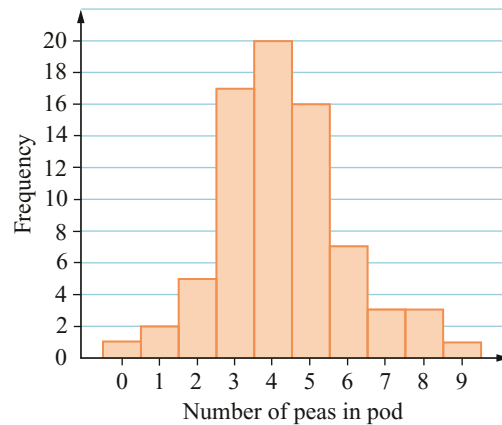
a Draw a smooth curve on the histogram.

b Describe the shape of the curve.

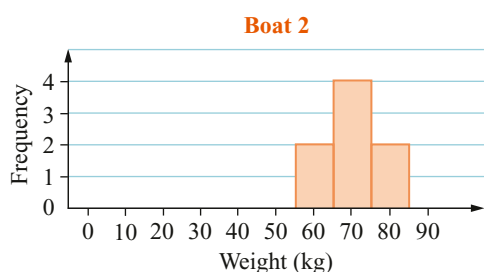
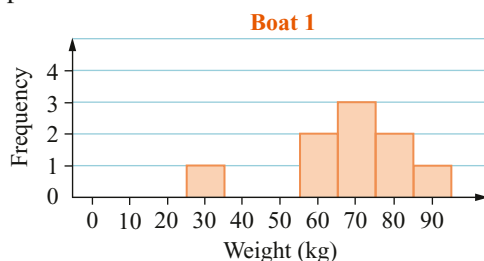
c What comments can be made about the test based on the shape of the curve?



- 12** The histogram on the right shows the number of peas in 75 pods.
- Draw a smooth curve on the histogram.
 - Describe the shape of the curve.
 - What comments can be made about the number of peas in a pod based on the shape of the curve?



- 13** The histograms below show the weights, to the nearest 10 kg, of the people in two row boats. Row boats can also carry a non-rower called a cox. Generally the cox will be as lightweight as possible.



- Draw a smooth curve on each histogram.
- Describe the shape of the curve on each histogram.
- Which boat has a cox? Explain in terms of the shape of the distribution and any outliers.



- 14** Use summary statistics and statistical displays to compare and comment on related pairs of data. Examples are provided below.
- Collect data. Determine the means, modes, medians and spreads of the sets of scores you collect.
 - Display the data using multiple displays, including comparative box plots.
 - Comment on similarities and differences between the following data sets.
 - Home scores and away scores for national sporting competitions
 - The ages of Oscar-winning male actors and female actors
 - The blood pressure of males versus females
 - The heights (or weights) of males versus females
 - The waiting times of customers at a fast-food outlet on two different days of the week



CHAPTER 11 REVIEW EXPLORING AND DESCRIBING DATA

You should be able to:

- ✓ calculate mean, mode and median
- ✓ select the appropriate measure to use by recognising its merits and limitations
- ✓ determine the range and interquartile range for a set of data
- ✓ find deciles and percentiles for a data set
- ✓ interpret and draw box plots and parallel box plots
- ✓ calculate and use population and sample standard deviation
- ✓ identify outliers and describe their effect on summary statistics
- ✓ analyse data using measures of central tendency (mean, mode, median) and measures of dispersion (range, interquartile range, standard deviation)
- ✓ describe, compare and interpret the distributions of graphical displays.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

REVIEW MULTIPLE-CHOICE QUESTIONS

11A 1 Which are the correct values for the data set in the frequency table on the right?

- A Mean = 36.9, Mode = 11 B Mean = 36.9, Mode = 24
C Mean = 24.6, Mode = 11 D Mean = 24.6, Mode = 24

Score	Frequency
23	4
24	11
25	9
26	6

11A 2 The mean of the scores in the table on the right is:

- A 2.5 B 9.3 C 9.5 D 35

Class	Frequency
0–4	3
5–9	4
10–14	6
15–19	2

11A 3 The modal class of the data in the table in question 2 is:

- A 0–4 B 5–9 C 10–14 D 15–19

11B 4 The median of the data in the table in question 1 is:

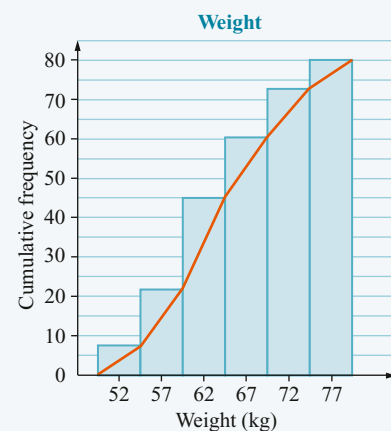
- A 24 B 25 C 24.5 D 10

11B 5 For the scores 7, 8, 8, 8, 8, 9, 10, 11, 12 and 12, which of the following values are correct?

- A Mean = 9.3, Mode = 8, Median = 8.5
B Mean = 9, Mode = 8.5, Median = 8
C Mean = 9.3, Mode = 8, Median = 9
D Mean = 9, Mode = 9, Median = 8.5

11B 6 The median of the data shown in the ogive on the right is:

- A 64.5 B 62 C 67 D 63.5



11B 7 The data in this stem-and-leaf plot has:

- A Mode = 6, Median = 6 B Mode = 6, Median = 4
C Mode = 46, Median = 46 D Mode = 46, Median = 4

Stem	Leaf
3	0 2 4
4	1 3 6 6 6
5	0 4 8 8

11C ▶ 8 The most appropriate measure of central tendency for the scores 5, 5, 5, 9, 10 and 44 is the:

- A mode B mean C median D range

11C ▶ 9 If we choose random samples from a large population, which of the following is the measure that varies the most?

- A mean B mode C median D all vary the same amount

11D ▶ 10 For the ogive shown in question 6, the interquartile range is approximately:

- A 15 B 40 C 69 D 10

11D ▶ 11 Which of the following statements is *not* true for the data in this back-to-back stem-and-leaf plot?

- A The median for class B is greater than the median for class A.
 B The mean for class B is greater than the mean for class A.
 C The interquartile range for class B is greater than the interquartile range for class A.
 D The range for class B is greater than the range for class A.

Class A Leaf	Stem	Class B Leaf
8 5 2 2 0	4	2 5
9 7 6 3 1	5	3 3 6 9 9
8 7 4 3 2	6	0 1 4 8 8 8
9 8 6 5	7	1 1 3 7
9	8	0 1 1

11E ▶ 12 For the data graphed in question 6, the 7th decile is approximately;

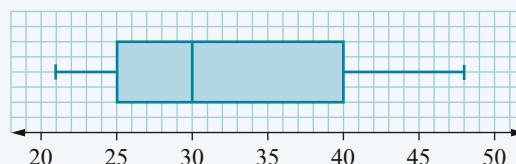
- A 53 B 66 C 68 D 74

11E ▶ 13 For the data graphed in question 6, the 65th percentile is approximately;

- A 45 B 66 C 72 D 77

11F ▶ 14 For the box plot shown, the:

- A Range = 20, Interquartile range = 23
 B Range = 27, Interquartile range = 15
 C Range = 10, Interquartile range = 5
 D Range = 15, Interquartile range = 27



11G ▶ 15 What is the population standard deviation of the data in the following table?

- A 1.21 B 1.41 C 2.28 D 2.55

Mark	5	6	7	8	9
Number of students	5	5	8	7	2

11G ▶ 16 The sample standard deviation of the scores 23, 23, 24, 25, 25, 25 and 26 is:

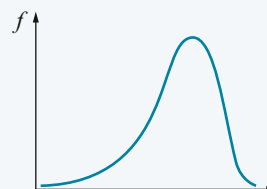
- A 1.13 B 1.05 C 171 D 7

11H ▶ 17 Which of the following can be used to identify outliers in a sample of data?

- A $\text{mean} - 1.5 \times \text{IQR}$ B $Q_3 + 1.5 \times \text{IQR}$
 C $\text{median} + 1.5 \times \text{IQR}$ D $Q_1 - 5 \times \sigma_{n-1}$

11I ▶ 18 The graph shown on the right is:

- A smooth, unimodal, positively skewed
 B smooth, unimodal, negatively skewed
 C smooth, symmetrical, bimodal
 D smooth, symmetrical, U-shaped.



11I ▶ 19 The graph in question 18 could represent:

- A the class marks for an easy test B the class marks for a hard test
 C a uniform distribution of class marks D no recognisable pattern of marks.

REVIEW SET 1

1 Find the mean, mode, median and range of these scores: 6, 6, 6, 7, 8, 8, 9, 9, 10.

2 Consider the grouped data in the table on the right.

- a** Copy the table and add a cumulative frequency column.
- b** Draw a cumulative frequency histogram and ogive.
- c** From the ogive, you drew in part **b** determine the:
 - i** median
 - ii** fourth decile
 - iii** 65th percentile.

Class	Frequency
1–5	12
6–10	8
11–15	18
16–20	31
21–25	11

3 For each of the following data sets, find the:

- i** 1st, 2nd and 3rd quartiles
- ii** interquartile range.
 - a** 9, 9, 10, 11, 12, 12, 13
 - b** 8, 9, 9, 10, 11, 12, 12, 13, 14
 - c** 8, 9, 9, 10, 11, 12, 12, 12, 14, 14
 - d** 8, 8, 9, 10, 10, 11, 12, 12, 13, 14, 14, 15

4 Draw a box plot for this five-number summary:

Highest score = 42, Lowest score = 25, Upper quartile = 36, Lower quartile = 30, Median = 32.

5 Find the mean and sample standard deviation of these scores: 17, 25, 19, 20, 23, 19, 21.

6 For the scores 5, 5, 6, 7, 7, 8, 8, 8 and 10 find the:

- a** mean
- b** median
- c** population standard deviation.

7 The prices of apartments sold in one month by a real-estate agent are listed below.

\$250 000 \$280 000 \$240 000 \$300 000 \$290 000 \$550 000

- a** Calculate the:
 - i** mean price
 - ii** median price.
- b** Identify any outliers.
- c** Which of these measures (mean or median) is the more appropriate average to use for these prices? Give reasons for your answer.

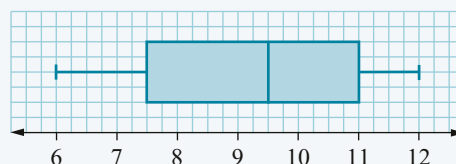
8 The maximum temperature recorded each month for two towns is shown in the table below.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Town A	31	30	28	26	23	18	17	19	24	27	28	30
Town B	36	34	30	26	22	17	13	20	25	27	32	35

- a** For each town, find the:
 - i** mean temperature
 - ii** median temperature.
- b** For each town's temperatures, find the:
 - i** range
 - ii** interquartile range
 - iii** population standard deviation.
- c** Write a short description comparing the temperatures for the two towns. In your description include the shape of each distribution and any clustering or outliers.

9 a Describe the shape of the distribution represented by this box plot.

b Identify any outliers.



REVIEW SET 2

- 1 Find the mean, mode, median and range for the data in this table.

Score	85	86	87	88
Frequency	9	15	24	12

- 2 Consider each of the following sets of scores.

i Find the mean, mode and median.

ii Discuss the relative merits of each measure of central tendency for representing the data.

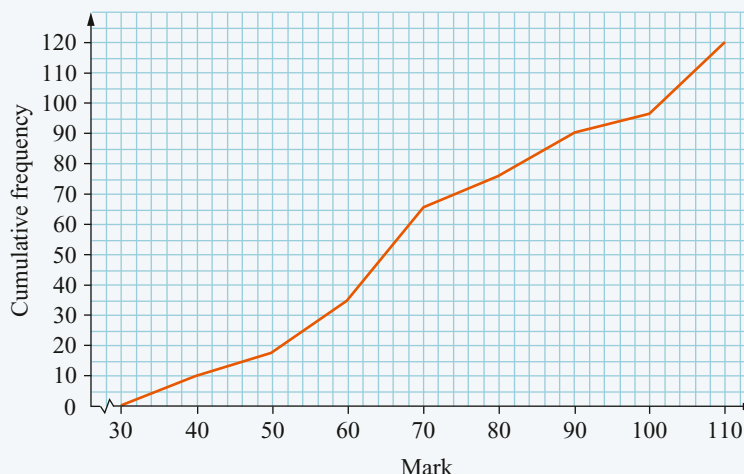
a 4, 5, 5, 6, 100

b 4, 4, 4, 5, 6, 6, 7

c 1, 1, 1, 1, 5, 6

- 3 Use the ogive shown on the right to find the:

- a median
b upper quartile
c lower quartile
d interquartile range
e 2nd decile
f 45th percentile.



- 4 Which of the following measures is suitable for analysing categorical data?

- A range B mean
C mode D median

- 5 Calculate the mean and population standard deviation of the data given in this table.

Weight	Frequency
4.3	6
4.4	9
4.5	4
4.6	5
4.7	6

- 6 For the data 22, 23, 23, 25, 26, 26, 27, 29, 31, 31, find the:

- a mean
b median
c range
d interquartile range
e standard deviation.

- 7 a Draw a smooth curve representing the data in the histogram on the right.
b Describe the curve in terms of skewness.

- 8 Two groups of students are given pairs of shoes to wear to school. The students in group X are given rubber-soled shoes and those in group Y are given shoes with soles made of a new synthetic material. The thickness (in millimetres) of the soles of the shoes after 6 months is shown below.

Group X: 5, 3, 4, 5, 4, 3, 5, 6, 2, 6, 4, 4, 5, 3, 5, 6, 7, 6, 6, 3, 4, 5, 4, 5

Group Y: 3, 4, 5, 6, 5, 6, 4, 8, 5, 3, 4, 7, 6, 5, 2, 8, 5, 6, 5, 6, 8, 7, 3, 4

- a For each group of results, find the:

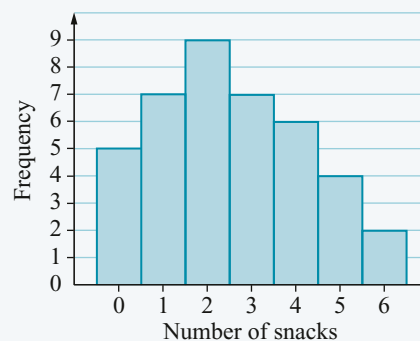
i mean ii median.

- b For each group of results determine the:

i range ii sample standard deviation.

- c For the given data draw two box plots on the same scale.

- d Is the new synthetic sole an improvement on the rubber soles? Justify your answer.



REVIEW SET 3

- 1** Consider the data in the table on the right.
- Determine the modal class.
 - Complete the table by adding a column for the class centres.
 - Find the mean.

Class	Frequency
30–39	5
40–49	10
50–59	14
60–69	13
70–79	0
80–89	8

- 2** Draw a box-plot for these scores:
6 6 7 8 8 9 10 10 11 11 11 12

- 3** Use the data given in the table in question 1.

- Draw the ogive.
- Using your ogive from part **a** estimate the:
 - lower quartile
 - upper quartile
 - interquartile range
 - median
 - 9th decile.

- 4** The number of goals scored by two basketballers in 20 games are shown below:

Adrian: 8, 9, 8, 6, 7, 7, 9, 4, 8, 8, 6, 7, 5, 7, 8, 8, 8, 9, 9, 10

Zoltan: 8, 7, 5, 7, 8, 9, 9, 9, 8, 5, 7, 10, 6, 6, 9, 5, 7, 8, 8, 10

- For each player, calculate the:
 - mean
 - median
 - range
 - interquartile range
 - sample standard deviation.
- Draw two box plots of the basketballers data on the same scale.
 - Compare the box plots.
- Who is the better player, Adrian or Zoltan?
Given reasons for your answer.

- 5** **a** Draw a smooth curve representing the data in this histogram.

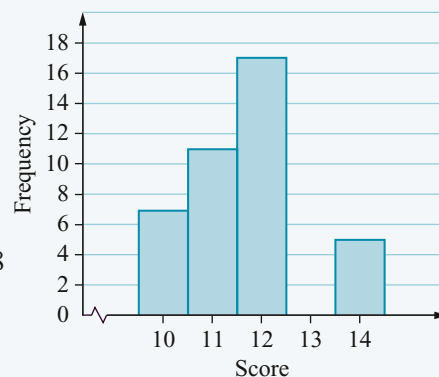
b Describe the curve in terms of skewness.

c Are there any outliers?

- 6** The following are the scores in a language test:

60 61 59 50 65 77 63 58 36 88 66 74 61 43 58

- Draw a stem-and-leaf plot for this data.
- Describe the shape of the distribution.
- Use your stem-and-leaf plot to determine:
 - the range
 - the mode



iii the median

REVIEW SET 4

- 1** Find the mean, mode, median, range and standard deviation (σ_n) of these scores:

2 2 3 3 4 5 5 5 6

- 2** The results in a test given to two classes are shown below.

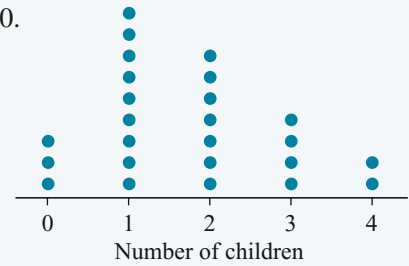
Class A: 24, 34, 16, 18, 30, 26, 36, 40, 22, 26, 32, 20, 8, 38, 14, 28, 32, 30, 24

Class B: 6, 16, 28, 24, 24, 34, 38, 26, 22, 20, 28, 18, 24, 12, 36, 36, 22, 10, 2

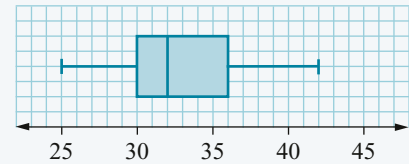
- Construct back-to-back stem-and-leaf plots for this data.
- For each class find the:
 - mean
 - median
 - range
 - population standard deviation.
- Describe the shape of the distribution for each class.
- Use the median and interquartile range to identify any outliers.
- Which class performed better? Explain.

- 3** Find the range and interquartile range of these scores: 5, 5, 6, 7, 7, 8, 8, 8, 10.

- 4 a** Draw a smooth curve representing the data in this dot plot.
b Describe the curve in terms of skewness.
c What does the shape of the curve tell you about the data?



- 5** Consider the box plot on the right.
a From the box plot, determine the value of the:
i lowest score **ii** lower quartile
iii median **iv** upper quartile
v highest score.
b Describe the shape of the distribution.
c Identify any outliers.

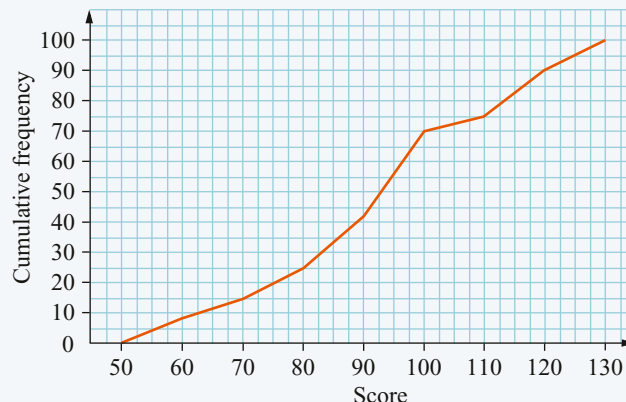


REVIEW PRACTICE EXAMINATION QUESTION

- 1 a** The following information is given about a set of 12 scores:
 Lowest score = 3, Highest score = 17, Mean = 9.5, Median = 10, Mode = 11.
 The highest score (17) is replaced by 71. For the new data set, find the:
i median **ii** mode **iii** mean. (3 marks)
b Calculate the population standard deviation for the scores given in this stem-and-leaf plot. (1 mark)

Stem	Leaf
1	0 6
2	1 1 2 5 8 8
3	3 5 6
4	1 1

- c** The annual incomes (\$) for a sample of 12 people are given below.
 40 000 43 000 32 000 29 000 37 000 75 000
 60 000 51 000 52 000 45 000 38 000 25 000
i Determine the lower quartile, middle quartile and upper quartile for this data. (3 marks)
ii Construct a box plot for the data. (2 marks)
iii Use the median and interquartile range to identify any outliers. (3 marks)
iv Calculate the sample standard deviation. (1 mark)
d The ogive for a set of data is shown below.
 From the graph determine the:
i 9th decile (1 mark)
ii 15th percentile (1 mark)



TOTAL: 15 marks

12

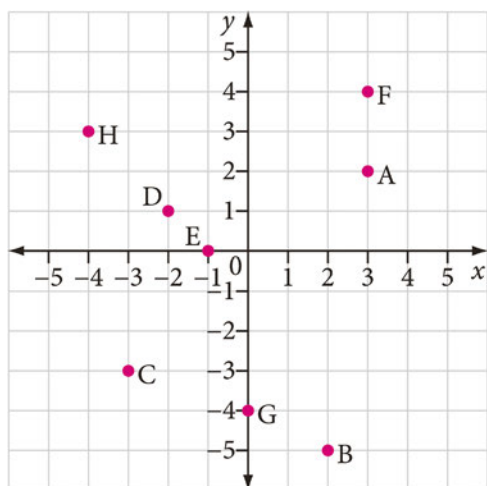
Linear relationships

The main mathematical ideas investigated are:

- ▶ graphing linear relationships
- ▶ calculating and interpreting gradients
- ▶ finding and interpreting the vertical intercept
- ▶ using and interpreting graphs of the form $y = mx + c$
- ▶ developing equations from descriptions of how one quantity directly varies with another
- ▶ constructing and analysing linear models
- ▶ solving direct variation problems
- ▶ recognising the limitations of models.

ARE YOU READY?

Use this diagram to answer questions 1 to 3.



- 12A ▶ 1** What are the coordinates of point F?
A (4, 3) **B** (3, 4)
C (2, 3) **D** (3, 2)
- 12A ▶ 2** What are the coordinates of point B?
A (2, -5) **B** (-5, 2)
C (-2, 5) **D** (5, -2)
- 12A ▶ 3** For which two points is the x -coordinate the same?
A G and E **B** D and H
C B and C **D** A and F
- 12A ▶ 4** What is the value of the expression $5p - 4q$ when $p = 3$ and $q = -5$?
A 35 **B** 8
C -5 **D** -37
- 12A ▶ 5** What is the value of the expression $3a - 5b + 5$ when $a = -2$ and $b = 4$?
A 31 **B** -21
C -12 **D** -17

- 12A ▶ 6** Which table shows x and y values for the rule $y = 3x - 5$?

A

x	4	6	7
y	7	9	10

B

x	2	6	8
y	1	5	7

C

x	1	4	5
y	-2	7	10

D

x	2	3	7
y	-1	4	14

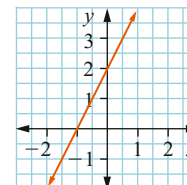
- 12A ▶ 7** Which rule links the values of x and y in this table?

x	2	6	8
y	6	22	30

- A** $y = 2x + 2$ **B** $y = 2x - 5$
C $y = 4x - 2$ **D** $y = 3x$

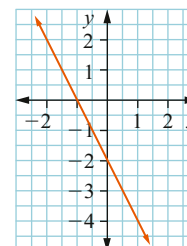
- 12B ▶ 8** What is the gradient of this straight-line graph?

- A** 2 **B** -2
C 1 **D** -1



- 12B ▶ 9** What is the gradient of this straight-line graph?

- A** 2 **B** -2
C 1 **D** -1



If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook assess](#).

- Q1** Support sheet 12A.1 The Cartesian plane (positive numbers)
Q2-3 Support sheet 12A.2 The Cartesian plane (four quadrants)
Q4-5 Support sheet 12A.3 Substituting for pronumerals
Q6-7 Support sheet 12A.4 Tables of values and rules
Q8-9 Support sheet 12B.1 Finding the gradient of a linear graph

12A Straight-line graphs

These resources are available on your obook assess:

- **Investigation 12A:** Plot and interpret a linear relationship for height of an aircraft
- **assess quiz 12A:** Test your skills with an auto-correcting multiple-choice quiz

independent variable

variable whose outcomes are not due to those of another variable; usually represented on the horizontal axis of a graph; e.g. for the linear relationship $y = 2x + 1$, the independent variable is x

dependent variable

variable whose value depends upon that of another; usually represented on the vertical axis of a graph; e.g. for the relationship $y = 2x + 1$, the dependent variable is y (as it depends on the value of x)

Many situations are modelled using straight-line graphs. Sometimes, points are plotted on a number plane and a line is drawn through them. At other times, a rule or relation is given and ordered pairs are calculated. The graph of a linear relationship between two variables produces a straight line. The values of the **dependent variable** depend on the values of the **independent variable**.

EXAMPLE 12A-1 Drawing a straight-line graph from a table of values

Bulk washing powder is sold for \$1.40 per kilogram. The table shows weight versus cost for washing powder.

Weight (kg)	1	2	5	10	15	20
Cost (\$)	1.40	2.80	7.00	14.00	21.00	28.00

- Draw the graph of weight versus cost.
- Use the graph to find the cost of 12.5 kg of washing powder.

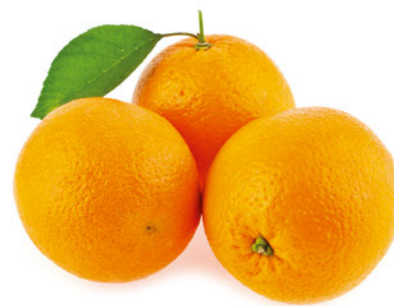
	Solve	Think
a		Because cost depends on the number of kilograms, it is the dependent variable, and is placed on the vertical axis. The number of kilograms, the independent variable, is placed on the horizontal axis. Choose a suitable scale for each axis. Plot the points from the table, (1, 1.40), (2, 2.80), etc. and draw a straight line through them.
b	The cost of 12.5 kg of washing powder (read from the graph) is \$17.50.	Find the point 12.5 kg on the 'Weight' axis. Draw a vertical line to intersect the graph. From this point, draw a horizontal line to the 'Cost' axis. Read the value at this point: \$17.50.
Apply Place the dependent variable on the vertical axis and the independent variable on the horizontal axis. Remember to label the axes and use equal divisions on the scale. Plot the points and draw a straight line through them. Find the required value on the horizontal axis and draw a line to intersect the graph. From this point, draw a line to meet the vertical axis. Read the value.		

EXERCISE 12A Straight-line graphs

- 1** Oranges are sold for \$3.50 per kilogram. The table shows weight versus cost for the oranges.

Weight (kg)	1	2	3	4	5	6
Cost (\$)	3.50	7.00	10.50	14.00	17.50	21.00

- a** Draw the graph of weight versus cost. Which is the dependent variable?
b Use your graph from part **a** to find the cost of 2.5 kg of oranges.



- 2** Mobile telephone calls cost 65 cents per minute. This table shows the cost of calls of different duration.

Time (min)	1	2	3	4	5	6
Cost (\$)	0.65	1.30	1.95	2.60	3.25	3.90

- a** Draw the graph of time versus cost. Which is the independent variable?
b Use your graph from part **a** to find the cost of a call of 2.5 minutes duration.
c A call cost \$3.00. How long was that call?

- 3** Rema earns \$18 per hour. The table shows her income versus hours worked.

Time (h)	2	4	6	8	10
Income (\$)	36	72	108	144	180

- a** Draw the graph of income versus hours worked.
b Use your graph from part **a** to find Rema's income for 7 hours.
c How many hours must she work to earn \$90?

EXAMPLE 12A-2 Completing a table of values to draw a graph

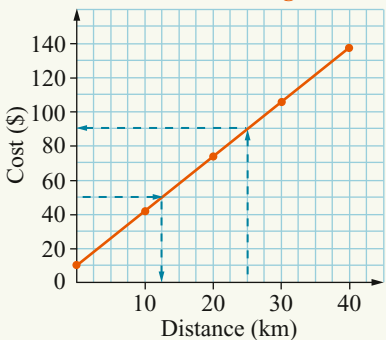
The cost of travelling in a hire car is \$10 booking fee plus \$3.20 per kilometre.

- a** Complete the table of values below for the hire car.
b Plot the points and sketch the graph of hire car costs.
c Find the cost of a journey of 25 km.
d How far can the hire car be driven for \$50?

Distance (km)	0	10	20	30	40
Cost (\$)					



Solve/Think							Apply
a	Distance (km)	0	10	20	30	40	Calculate the cost by multiplying the cost per kilometre by the number of kilometres and adding this result to the booking fee.
	Cost (\$)	10	42	74	106	138	
	For a journey of 10 km: Cost = \$10 + \$3.20 × 10 = \$42						
	For a journey of 20 km: Cost = \$10 + \$3.20 × 20 = \$74						
	For a journey of 30 km: Cost = \$10 + \$3.20 × 30 = \$106						
For a journey of 40 km: Cost = \$10 + \$3.20 × 40 = \$138							

	Solve/Think	Apply
b	Hire car charges  Put cost on the vertical axis. Plot the points ((0, 10), (10, 42), etc.) and draw a straight line through them.	Place the dependent variable on the vertical axis. Plot the points from the table and draw a straight line through them.
c	The cost of a journey of 25 km is \$90. Find the point on the line with first coordinate 25 km and read the second coordinate, \$90, on the vertical axis.	Find the point on the line with the given coordinate and read the unknown coordinate on the appropriate axis.
d	The hire car can be driven 12.5 km for \$50. Find the point on the line with second coordinate \$50 and read off the first coordinate, 12.5 km.	

- 4** The cost of hiring a taxi is \$3.00 flagfall and \$2.40 per kilometre travelled.

a Copy and complete this table of costs for taxi hire.

Distance (km)	0	10	20	30	40
Cost (\$)	3	27			

- b** Draw the graph showing the cost of hiring the taxi.
c How much does it cost to travel 35 km?
d How far could you travel for \$15?

- 5** The cost of hiring a taxi at night is \$5.00 flagfall and \$3.20 per kilometre travelled.

a Copy and complete this table of costs for taxi hire.

Distance (km)	0	10	20	30	40
Cost (\$)					

- b** Draw a graph showing the cost of hiring the taxi.
c How much does it cost to travel 15 km?
d How far could you travel for \$75?



- 6** A person drives non-stop from Taree to Sydney, a distance of 300 km. The rule giving the distance from Sydney is $d = 300 - 100t$ where d is distance in kilometres and t is time in hours.

a Copy and complete this table of values for the given rule.

b Draw a graph of distance from Sydney versus time.

c How far is the driver from Sydney after $1\frac{1}{2}$ h?

d How far has the driver travelled after 1 h?

Time (h)	0	1	2	3
Distance (km)	300		100	

- 7** On a particular day, \$100 Australian buys \$60 US. Use this and the fact that A\$0 = US\$0 to draw a conversion graph for A\$ and US\$. Put A\$ on the horizontal axis (values up to \$100) and US\$ on the vertical axis. How many A\$ can be bought for US\$40?

- 8 Sumeyra is travelling to Europe. At the time Sumeyra is travelling, \$100 Australian will buy 70 euros. Draw a conversion graph with A\$ on the horizontal axis (values up to \$100) and euros on the vertical axis. From the graph, how many A\$ could be bought for 60 euros?
- 9 Shailesh is filling his car with petrol. He puts in 80 litres of fuel and the cost is \$110.
- Draw a straight-line conversion graph for this information.
 - How many litres of fuel would \$70 buy?
 - What would be the cost for 55 litres of fuel?
- 10 Karolin is filling her car with petrol. She puts in 50 litres of fuel and the cost is \$65.
- Draw a straight-line conversion graph for this information.
 - How many litres of fuel would \$25 buy?
 - What would be the cost for 35 litres of fuel?

- 11 This table shows the rates for taxi travel in the day and at the special night rate.
- Plot the points and draw the graphs for both rates on the same set of axes.
 - Use your graph to determine how much further you can travel for \$75 during the day compared with the special night rate.
 - Use your graph to find the difference in cost between a 42 km journey at the day rate and at the special night rate.

Distance (km)	Cost for day rate (\$)	Cost for special night rate (\$)
0	3.60	6.10
5	14.55	19.25
10	25.50	32.40
15	36.45	45.55
20	47.40	58.70
25	58.35	71.85
30	69.30	85.00
35	80.25	98.15
40	91.20	111.30
45	102.15	124.45
50	113.10	137.60
55	124.05	150.75
60	135.00	163.90

- 12 Here is part of a spreadsheet for the data in question 11 showing the formulas for calculating the costs.
- Create a spreadsheet or use the one provided in your [obook assess](#). **Fill down** to complete the spreadsheet for the data provided in the table in question 11.
 - Use your spreadsheet to answer question 11.
 - Edit the kilometre rate and flagfall to see how that affects the graph.

	A	B	C
1	Taxi rates		
2	km rate (\$/km)	2.19	2.63
3	Flagfall (\$)	3.6	6.1
4			
5	Distance (km)	Day rate	Special night rate
6	0	=B3	=C3
7	5	=A7*\$B\$2+\$B\$3	=A7*\$C\$2+\$C\$3
8	10	=A8*\$B\$2+\$B\$3	=A8*\$C\$2+\$C\$3
9	15	=A9*\$B\$2+\$B\$3	=A9*\$C\$2+\$C\$3

12B Gradient and vertical intercept

These resources are available on your obook assess:

- **Interactive 12B:** Explore gradient and intercepts of a linear graph
- **Worksheet 12B:** Practise your skills with extra problems for gradient
- **assess quiz 12B:** Test your skills with an auto-correcting multiple-choice quiz

gradient (slope)

steepness of a line (defined as m) that can be measured using any two points on the line or interval

The words **slope** and **gradient** are used when talking about the steepness of a line graph. Gradient or slope is associated with house roofs, escalators, hills, etc. The steeper the slope, the larger the gradient.

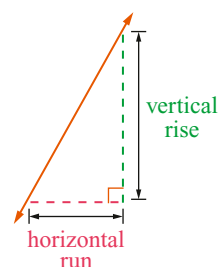
$$\text{Slope} = \frac{\text{vertical rise}}{\text{horizontal run}}$$

The slope is also referred to as the gradient.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}}$$

Note: for a horizontal line the vertical rise is 0, therefore the slope is 0.

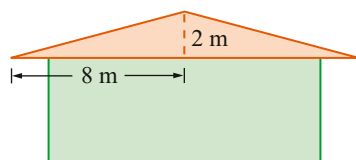
For a vertical line the horizontal run is 0, so the slope is infinite or undefined.



The following illustrations indicate slopes of varying amounts.

a

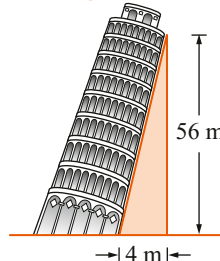
House roof



$$\text{Slope} = \frac{2}{8} = \frac{1}{4}$$

b

Leaning Tower of Pisa

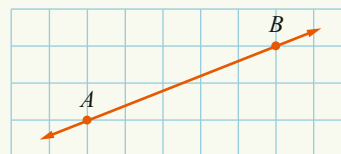


$$\text{Slope} = \frac{56}{4} = 14$$

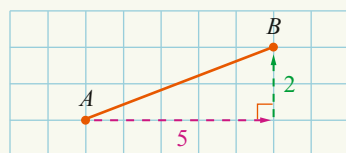
When line segments are drawn on graph paper we can easily determine the slope of the line segments by drawing horizontal and vertical lines to complete a right-angled triangle.

EXAMPLE 12B-1 Finding the slope of a line segment (positive)

Find the slope of AB .



Solve



$$\text{Slope of } AB = \frac{\text{vertical rise}}{\text{horizontal run}} = \frac{2}{5}$$

Think

Complete a right-angled triangle using AB as the hypotenuse.

Vertical rise = 2

Horizontal run = 5

$$\text{Slope} = \frac{2}{5}$$

Apply

Complete a right-angled triangle using the line segment between two points on the line as the hypotenuse.

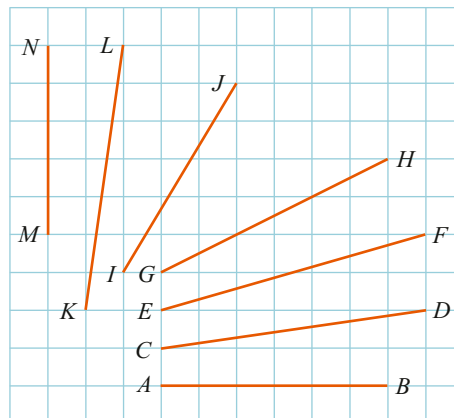
Determine the vertical rise and horizontal run between the two points

and calculate: Gradient = $\frac{\text{rise}}{\text{run}}$

EXERCISE 12B Gradient and vertical intercept

- 1 a Complete the table using the line segments shown.

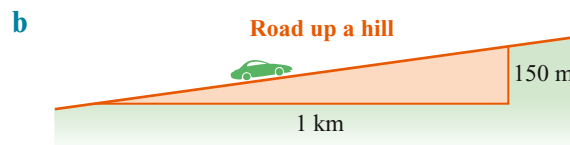
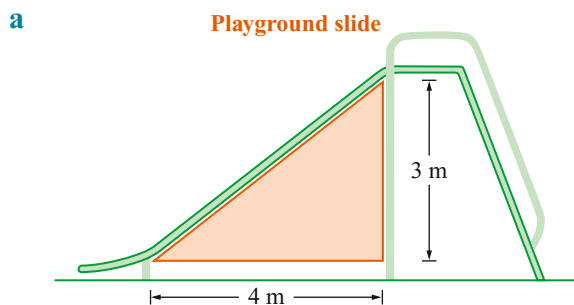
Line segment	Horizontal run	Vertical rise	Slope
<i>AB</i>			
<i>CD</i>			
<i>EF</i>			
<i>GH</i>			
<i>IJ</i>			
<i>KL</i>			
<i>MN</i>			



- b Complete the following.

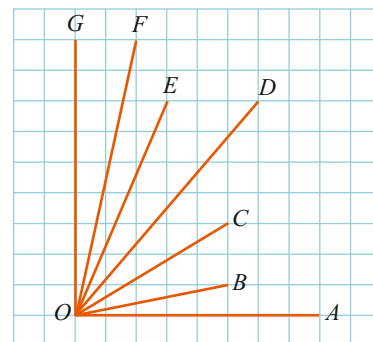
The slope of a horizontal line is _____. The slope of a vertical line is _____ or _____.
As the line segments become steeper, their slopes _____.

- 2 Find the slope in each of these situations.



- 3 Determine the slope of each of the line segments on the right.

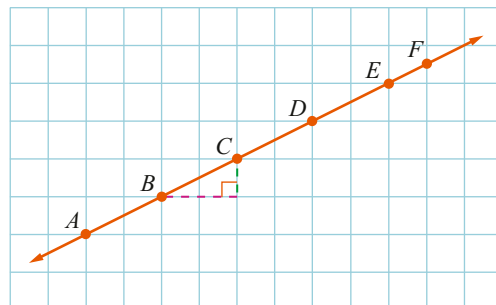
- a *OA* b *OB* c *OC* d *OD*
e *OE* f *OF* g *OG*



- 4 a Complete the table using the line shown below right.

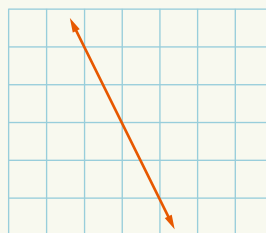
- b State, in sentence form, any conclusions you can draw from the graph and table.

Line segment	run	rise	$\frac{\text{rise}}{\text{run}}$
<i>BC</i>	2	1	$\frac{1}{2}$
<i>DE</i>			
<i>AC</i>			
<i>BE</i>			
<i>AE</i>			
<i>AF</i>			

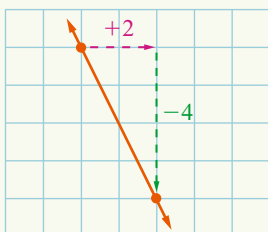


EXAMPLE 12B-2 Finding the gradient of a line (negative)

Find the gradient (slope) of this line.



Solve



$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{-4}{+2} = -2$$

Think

Choose two convenient points on the line. To get from the left-hand point to the right-hand point, we move 2 units to the right horizontally and 4 units down vertically.

(By convention, moving horizontally to the right is taken as the positive direction, and to the left as the negative direction. Moving vertically, up is the positive direction, and down is the negative direction.)

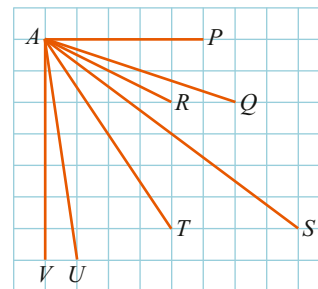
Apply

Calculate the slope using: $\text{Gradient} = \frac{\text{rise}}{\text{run}}$

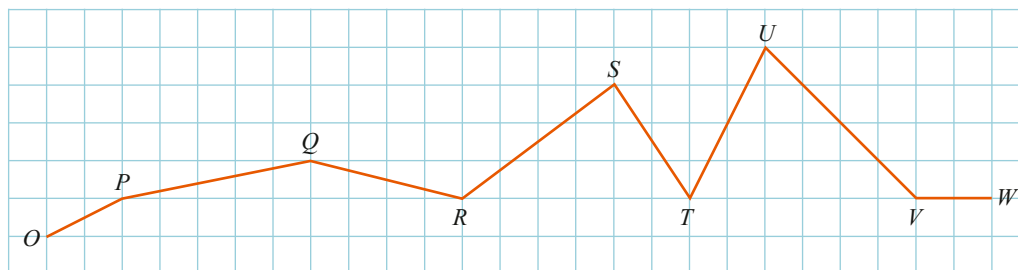
Note: Moving from left to right, 'uphill' lines have positive slopes and 'downhill' lines have negative slopes.

- 5** Determine the slope of each of the line segments on the right.

- a** AP **b** AQ **c** AR **d** AS
e AT **f** AU **g** AV



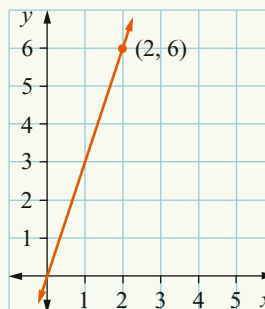
- 6** Imagine you are walking across the countryside from O to W (from left to right). This graph represents the profile of your walk.



- a** When are you going uphill? **b** When are you going downhill?
c Where is the steepest positive slope? **d** Where is the steepest negative slope?
e Where is the slope zero? **f** Where is the slope not zero but least?

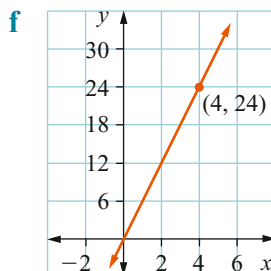
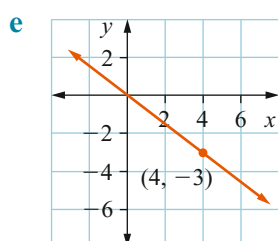
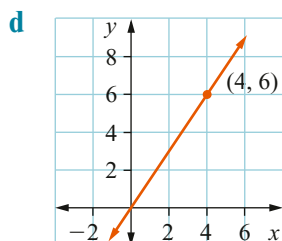
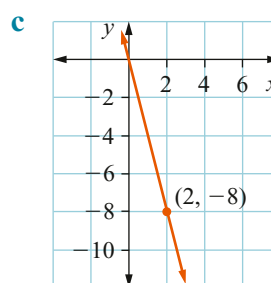
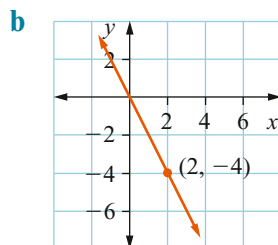
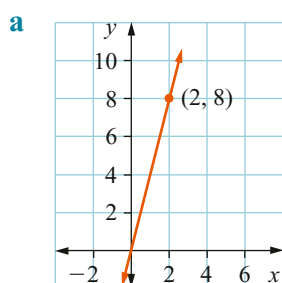
EXAMPLE 12B-3 Finding the gradient of a line that passes through the origin

Find the gradient of this line.



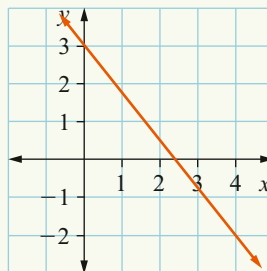
Solve	Think	Apply
$\begin{aligned}\text{Gradient} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{+6}{+2} \\ &= +3\end{aligned}$	Draw a right-angled triangle with the points (0, 0) and (2, 6) as the endpoints of the hypotenuse. Moving from left to right, the horizontal run is +2 and the vertical rise is +6.	Draw a right-angled triangle with (0, 0) and the given point as the endpoints of the hypotenuse. Determine the vertical rise and the horizontal run and calculate the gradient. $\text{Gradient} = \frac{\text{rise}}{\text{run}}$

7 Find the gradient of each of these lines.



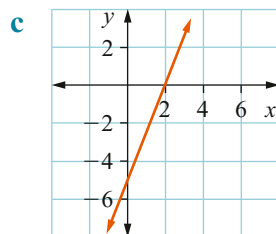
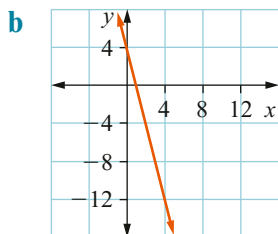
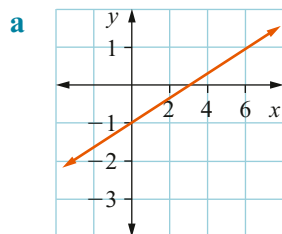
EXAMPLE 12B-4 Finding the gradient and y-intercept of a line

- a** Find the gradient of this line.
- b** Find where the graph cuts the vertical axis (**y-intercept**).



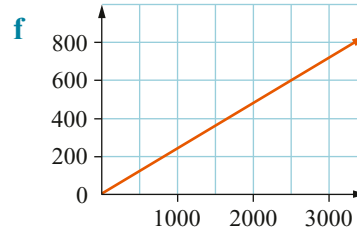
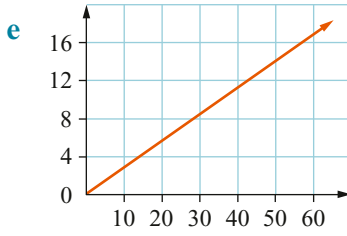
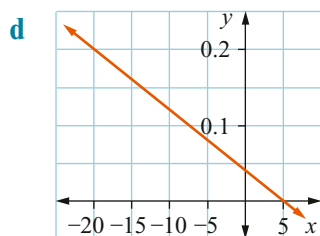
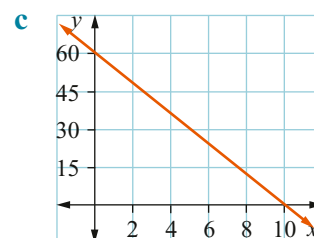
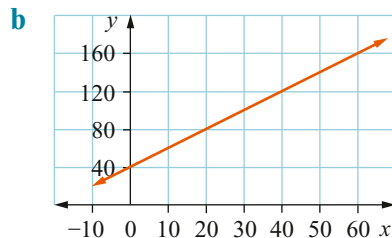
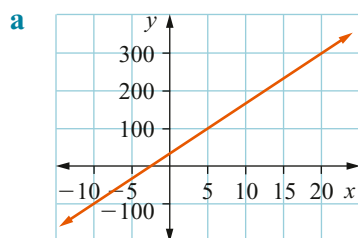
Solve	Think	Apply
a Gradient = $\frac{\text{rise}}{\text{run}}$ $= \frac{-5}{+4}$ $= -\frac{5}{4}$	Choose any two points on the line; for example, (0, 3) and (4, -2). The rise is -5 and the run is +4.	Choose two convenient points on the line and complete a right-angled triangle. Determine the vertical rise and horizontal run and calculate the gradient. Gradient = $\frac{\text{rise}}{\text{run}}$
b The line cuts the vertical axis at the point where $y = 3$. (3 is called the y-intercept of the line.)		

- 8** Find the gradient and y-intercept of each line below.



y-intercept
point where a line crosses the y-axis of a number plane

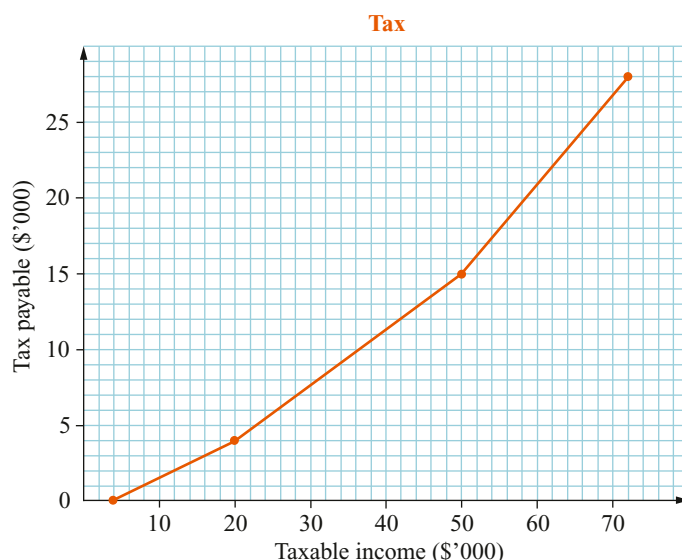
- 9** Find the slope of each of these lines. Be careful, because the scales are not the same.



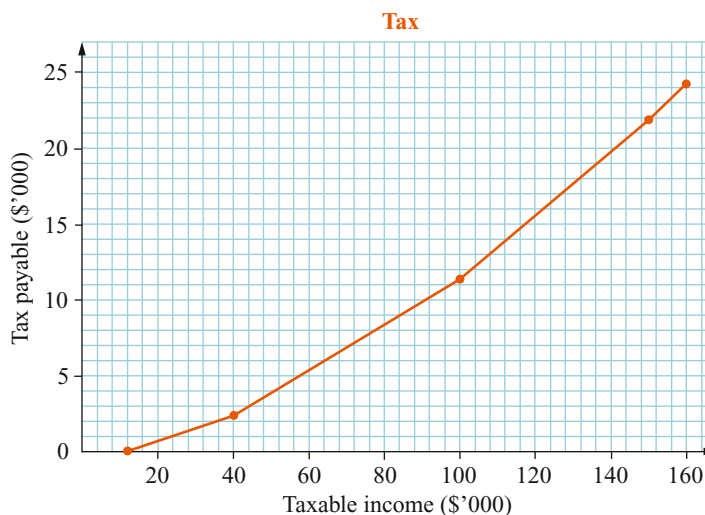
- 10** Find where each line in question 9 cuts the vertical axis.

- 11** Consider the tax graph shown below. It is a **piecewise graph**. The points where the tax rate changes are marked.
- Find the tax payable on \$40 000.
 - Find the income on which tax of \$20 000 is payable.
 - Find the gradient of the section between taxable incomes of \$30 000 and \$40 000. What does the gradient represent?
 - Find the gradient of the section between taxable incomes of \$60 000 and \$70 000. What does the gradient represent?

piecewise graph
graph made up of several joined straight lines with different gradients, not just one single straight line



- 12** This is another piecewise graph. The points where the tax rate changes are marked.
- Find the tax payable on \$90 000.
 - Find the income on which tax of \$20 000 is payable.
 - Find the gradient of the section between taxable incomes of \$60 000 and \$80 000. What does the gradient represent?
 - What is the tax rate for taxable incomes between \$100 000 and \$150 000?
 - Find the gradient of the section of the graph between taxable incomes of \$20 000 and \$40 000.
 - Find the tax rate for incomes over \$150 000.



- 13** One application of gradient is to give an indication to drivers of the steepness of the road going up or down a steep incline. Examples of steep inclines include Mount Victoria Pass, Mount Ousley and Bulli Pass. Some examples of signs are shown.

- Find examples of road signs that express gradient and copy them.
- Find the steepest gradient that can be driven up and driven down by:
 - trucks
 - cars
 - four wheel drives
 - trains.



12C The equation $y = mx + c$

These resources are available on your [obook assess](#):

- **Interactive 12C.1:** Explore key ideas for plotting linear graphs
- **Interactive 12C.2:** Explore key ideas for sketching linear graphs
- **Video tutorial 12C:** Watch and listen to an explanation of Example 12C-3
- **Worksheet 12C:** Practise your skills with extra problems for linear graphs
- **assess quiz 12C:** Test your skills with an auto-correcting multiple-choice quiz

12C

The rules or equations for straight-line or linear graphs may all be written in a similar form.

The convention is to write the rule for straight-line graphs in the form $y = mx + c$

where x is the independent variable, because any value may be used

y is the dependent variable, because it depends on the value of x

m is the gradient of the line

c is the y -intercept.

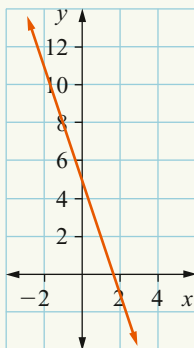
This relationship connects every point on a straight-line graph.

!

EXAMPLE 12C-1 Drawing a linear graph by first completing a table of values

Draw the graph of $y = -3x + 5$ by first completing a table of values.

Solve						Think	Apply
x	-2	-1	0	1	2	First set up a table. Substitute values of x into the equation $y = -3x + 5$. $x = -2$ gives $y = -3 \times -2 + 5 = 11$ $x = -1$ gives $y = -3 \times -1 + 5 = 8$ $x = 0$ gives $y = -3 \times 0 + 5 = 5$, etc. Plot the points $(-2, 11)$, $(-1, 8)$, $(0, 5)$, etc. Draw a straight line through these points.	Choose at least three convenient values of x for the table. Substitute these values into the equation to find the corresponding value of y for each x . Plot the points from the table and draw a straight line through them.
y	11	8	5	2	-1		



EXERCISE 12C The equation $y = mx + c$

- 1 Draw the graphs of the following linear relationships by first completing tables of values similar to the one shown on the right.

a $y = 3x + 1$

b $y = 2x + 2$

c $y = -2x - 1$

x	-2	-1	0	1	2
y					

- 2** Draw the graphs of the linear relationships with the following equations.
- a** $y = 2x + 7$ **b** $y = 4x - 6$ **c** $y = x - 1$ **d** $y = -x + 3$
e $y = -3x + 2$ **f** $y = -2x - 2$ **g** $y = \frac{1}{2}x - 1$ **h** $y = -\frac{3}{4}x + 1$
- 3** From the graphs you drew, find the gradient and y-intercept of each of the lines in question 2.

EXAMPLE 12C-2 Writing the gradient and y-intercept of a line from its equation

Write the gradient and y-intercept of each of the following lines.

- a** $y = 7x + 8$ **b** $y = -3x - 1$ **c** $y = \frac{2x}{3}$

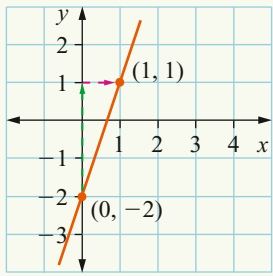
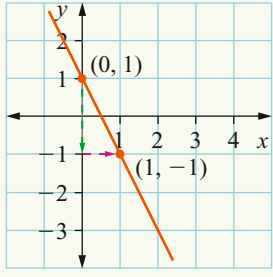
	Solve	Think	Apply
a	Gradient = 7, y-intercept = 8	From question 3 above, the number by which x is being multiplied is the gradient, and the constant term is the y-intercept.	When the equation of a line is written in the form $y = mx + c$, m is the gradient of the line and c is the y-intercept.
b	Gradient = -3, y-intercept = -1		
c	Gradient = $\frac{2}{3}$, y-intercept = 0		

- 4** Write the gradient and y-intercept for each of the following lines.
- a** $y = 5x - 1$ **b** $y = -2x + 11$ **c** $y = \frac{3x}{4}$ **d** $y = -x - 2$
e $y = \frac{x}{2} + 3$ **f** $y = x - 15$ **g** $y = -\frac{7x}{5} - 1$ **h** $y = \frac{x}{6} - \frac{2}{5}$

EXAMPLE 12C-3 Sketching a linear graph

Sketch the following straight lines without using a table of values.

- a** $y = 3x - 2$ **b** $y = -2x + 1$

	Solve	Think	Apply
a		Gradient = 3 and y-intercept = -2. Start at the y-intercept (0, -2). Because the gradient is $\frac{3}{1}$, move up 3 units then right 1 unit. This gives a second point (1, 1) on the line. Rule a line through these two points. <i>Note:</i> we could have moved up 6 units and right 2 units. Discuss why.	Find the gradient and y-intercept of the line. Use (0, c) as the first point on the line. Use Gradient = $\frac{\text{rise}}{\text{run}}$ to find a second point on the line.
b		Gradient = -2 and y-intercept = 1. Start at the y-intercept (0, 1). Because the gradient is $-\frac{2}{1}$, move down 2 units then right 1 unit. This gives a second point (1, -1) on the line. Rule a line through these two points.	

- 5** Sketch each of the lines in question 4 without using a table of values.

EXAMPLE 12C-4 Finding the equation that describes a simple linear relationship

Find a linear equation that describes the pattern shown in each of the following tables.

a

x	2	3	4	5	6
y	5	6	7	8	9

b

x	1	2	3	4	5
y	-3	-2	-1	0	1

c

x	0	1	2	3	4
y	0	3	6	9	12

d

x	0	1	2	3	4
y	3	6	9	12	15

	Solve	Think	Apply
a	$y = x + 3$	By inspection y is 3 more than x each time.	For simple patterns, find the equation by inspection.
b	$y = x - 4$	By inspection y is 4 less than x each time.	
c	$y = 3x$	By inspection y is 3 times x each time.	
d	$y = 3(x + 1)$	By inspection y is 3 times one more than x each time.	

6 Find a linear equation that describes the pattern shown in each of the following tables.

a

x	1	2	3	4	5
y	6	7	8	9	10

b

x	2	3	4	5	6
y	-1	0	1	2	3

c

x	0	1	2	3	4
y	0	5	10	15	20

d

x	0	1	2	3	4
y	-2	0	2	4	6

7 The KNOT ME Tie Company has fixed costs of \$800 per day and a variable production cost of \$2 per tie. The daily cost can be modelled using the straight-line equation $C = 2n + 800$.

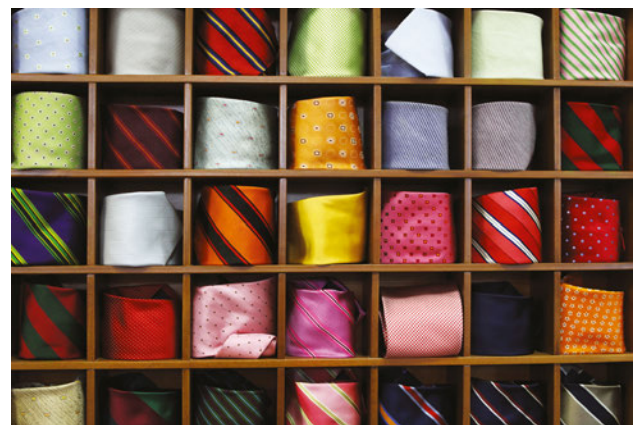
a Explain each term in the formula $C = 2n + 800$.

b Complete this table of values.

n	0	50	100	150	200	250
C						

c Draw the straight-line graph $C = 2n + 800$.

d Use your graph from part **c** to find the number of ties produced when the daily cost is \$940.



8 The Chilly Frozen Yoghurt Shop has a monthly fixed expense of \$1500 and the cost of making each serve of yoghurt is \$1.40.

a Write an equation modelling this information. Use C for monthly cost in dollars and n for the number of serves of yoghurt made.

b Complete this table of values.

n	0	500	1000	1500	2000	2500
C						

c Draw the straight-line graph.

d What is the value of the gradient?

e Use the graph to find the number of serves of yoghurt made if the cost is \$3250.

- 9** The Pulp-Free Fruit Juice company has weekly fixed costs of \$750. The cost to produce a cup of fruit juice is \$1.80.
- Write an equation modelling this information. Use C for the weekly cost in dollars and n for the number of cups of fruit juice.
 - What value is used for the gradient? Why?
 - Where does the graph cut the vertical axis?
 - Complete this table of values.
 - Draw the straight-line graph.
 - Use the graph to find the number of cups of juice made if the cost is \$1560.

n	0	200	400	600	800	1000
C						

Questions **10–13** may be completed manually, or by using a graphics calculator, online graphing software or other digital technology. There are many websites that offer free access to an online graphing calculator (for example, <https://www.desmos.com>).

The following instructions are for drawing the graph of $y = 2x + 3$ with a CASIO graphics calculator.

Step 1: Press **MENU** to obtain the **MAIN MENU** screen and select **GRAPH**.

Step 2: At **Y1:**, enter the right-hand side of the equation $y = 2x + 3$ by pressing 2 **X, θ , T** $+$ 3 and then **EXE**.
(You can enter more equations at **Y2:**, **Y3:**, etc. to draw more than one relationship on the same number plane.)

Step 3: Select **DRAW** to draw the graph.

Step 4: The graph should appear on the screen. If the scale of the axes needs adjustment, access **V-Window** and adjust as needed. Then **EXIT** to return to the graph.

- Draw graphs of $y = 3x + 1$, $y = 3x - 1$, $y = 3x$, and $y = 3x + 2$ on the same number plane.
 - What do you notice about all four graphs? Explain.
 - Without plotting any points, add the graph of $y = 3x + 3$ to the number plane. Explain how you knew what to draw.
- Draw the graphs of $y = 2x + 1$, $y = 2x - 1$ and $y = 2x + 3$ on the same number plane. What do you notice?
 - Draw the graphs of $y = -3x$, $y = -3x - 1$ and $y = -3x + 2$ on the same number plane. What do you notice?
 - Draw the graphs of $y = x + 2$, $y = x$ and $y = x - 5$ on the same number plane. What do you notice?
 - What conclusion can be drawn about straight lines with the same gradient?
- Draw the graphs of $y = 2x + 1$, $y = 3x + 1$ and $y = x + 1$ on the same number plane.
 - What do you notice about all three graphs?
 - Without plotting points, add the graph of $y = 4x + 1$ to the number plane. Explain how you knew what to draw.
 - Draw the graphs of $y = 3x - 1$, $y = 2x - 1$ and $y = -4x - 1$ on the same number plane. What do you notice?
 - What conclusion can be drawn about straight lines with the same constant term?
- On the same number plane, draw the graphs of $y = x + 1$ and $y = -x + 1$.
 - What do you notice about the two graphs from part **a**? Explain.
 - On a second number plane, draw the graphs of $y = x$ and $y = -x$.
 - What do you notice about the two graphs from part **c**? Explain.
 - On a third number plane, draw graphs of $y = 2x + 1$ and $y = -2x + 1$.
 - What do you notice about the two graphs from part **e**? Explain.

12D Direct variation relationships

These resources are available on your [obook assess](#):

- **Video tutorial 12D:** Watch and listen to an explanation of Example 12D-3
- **assess quiz 12D:** Test your skills with an auto-correcting multiple-choice quiz

direct variation relationship

relationship between two quantities or variables in which one of the quantities can be expressed as a constant multiplied by the other quantity

directly

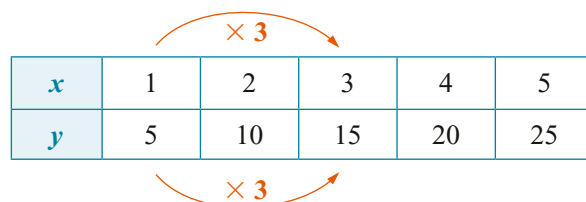
proportional

two quantities or variables are directly proportional when an increase (or a decrease) in one causes a proportional increase (or decrease) in the other; e.g. y is directly proportional to x if $\frac{y}{x}$ is a constant value

Quantities such as time, distance, cost and length change depending on the situation. These are called variables.

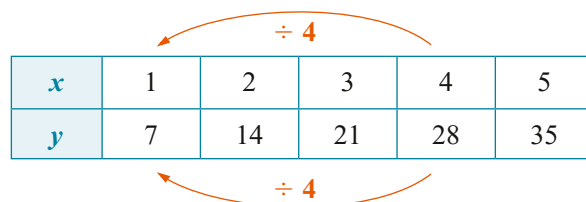
Mathematics uses the relationship between variables in many circumstances. When there is a **direct variation relationship** between two quantities or variables, one of the quantities can be expressed as a constant multiplied by the other quantity.

In the table below, when x is multiplied by 3, y is also multiplied by 3. As one quantity increases, the second quantity increases.



x	1	2	3	4	5
y	5	10	15	20	25

In the table below, when x is divided by 4 (or multiplied by $\frac{1}{4}$) so is y . As one quantity decreases, the second quantity also decreases.



x	1	2	3	4	5
y	7	14	21	28	35

Many applications of this exist in the real world, such as:

- a person's wage and the number of hours worked
- the cost of fuel and the amount of fuel bought
- the amount of paint used and the area painted
- the distance travelled at constant speed and the time taken
- the total cost of drinks and the number of drinks purchased.

If two quantities are **directly proportional**, then an increase (or a decrease) in one causes a proportional increase (or decrease) in the other.

EXAMPLE 12D-1 Graphing a direct variation relationship

A train is travelling at a constant speed of 75 km/h.
The distance, d km, the train travels in t hours is shown in the table.

t	0	1	2	3
d	0	75		

- Complete the table.
- Graph this information on a number plane.
- Write an equation to describe the relationship between time and distance.



	Solve	Think	Apply										
a	<table><tr><td>t</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>d</td><td>0</td><td>75</td><td>150</td><td>225</td></tr></table>	t	0	1	2	3	d	0	75	150	225	Multiply t by 75 to obtain d .	The constant speed is the gradient of the graph. Because $t = 0$ and $d = 0$,
t	0	1	2	3									
d	0	75	150	225									
b	Train travel Distance in kilometres (d) Time in hours (t)	Plot points and draw the line. Notice that this graph passes through the origin.	the graph starts at the origin $(0, 0)$. Any straight line through the origin indicates direct variation. Because the line passes through the origin, $c = 0$ and, therefore, the equation is of the form $y = mx$.										
c	The equation is $d = 75t$.	The gradient of the line is 75.											

The graph in Example 12D-1 shows a direct correlation between the two quantities time and distance. To find distance, time is multiplied by 75; that is, the ratio of distance to time is always 75:1.

If $\frac{y}{x}$ is always a constant, then we say that y *varies directly* as x .

The equation connecting two quantities is $\frac{y}{x} = k$ or $y = kx$ where k is called the **constant of variation** or **constant of proportionality**.

The notation for variation is $y \propto x$ and is read as:

‘ y is proportional to x ’ or ‘ y varies directly as x ’.

If $y \propto x$, the graph of y plotted against x is a straight line through the origin, with gradient k .

constant of variation (or proportionality)

constant value (k) for $\frac{y}{x}$ when there is a direct variation relationship between y and x . The equation connecting y and x can be written as $y = kx$ where k is the constant of variation.

EXERCISE 12D Direct variation relationships

- 1 a Complete this table, which shows the price of packets of pencils.

Number of packets (N)	1	2	3	4	5
Price (\$ P)	1.50		4.50		7.50

- b Plot the values in the table on a number plane and join the points, extending the line through the origin.
- c Find the gradient which is the constant of variation k where $k = \frac{P}{N}$.
- d Write the linear equation for P in terms of N .
- e Find the cost of 9 packs of pencils.
- f If the price for N packs of pencils is \$27, find N .



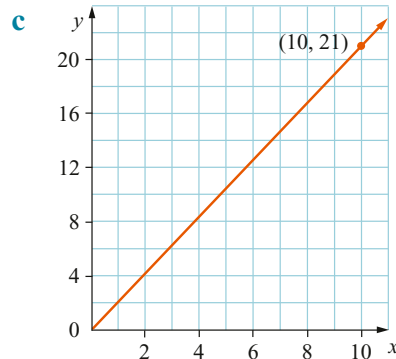
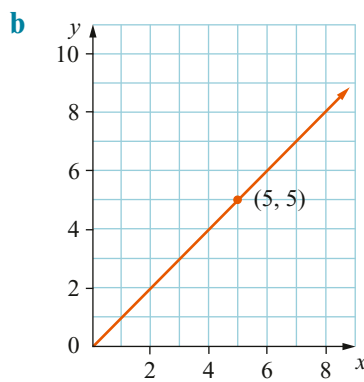
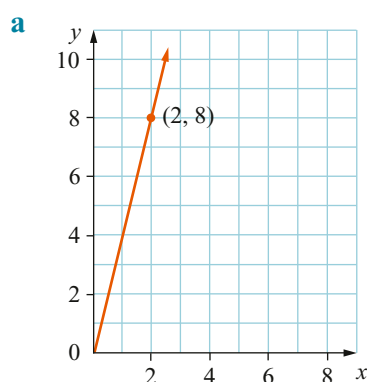
- 2 a Complete this table for the equation $C = \pi D$.

Diameter (D)	1	2	3	4
Circumference (C)				

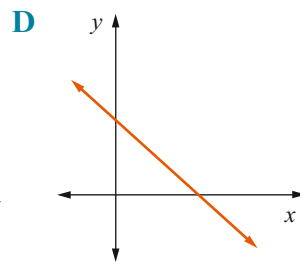
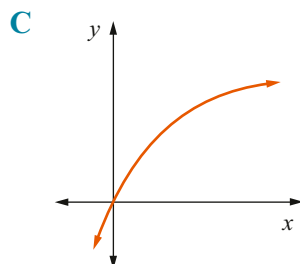
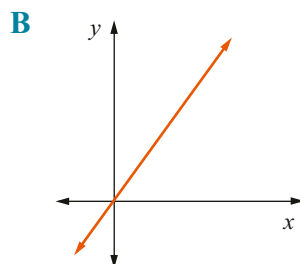
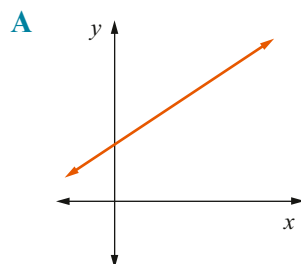
- b Graph the information from your completed table on a number plane.
- c Does the circumference of a circle vary directly with the length of its diameter? What feature of the graph supports your answer?
- d What is the constant of variation, which is the gradient of the line?
- e From the graph, find the approximate value for:
- the circumference of a circle with diameter 2.5 cm
 - the diameter of a circle with circumference 16 cm.

- 3 Consider each of the direct variation graphs below.

- i By drawing a right-angled triangle using the given point and the origin as vertices each time, calculate the gradient of each line.
- ii State the relationship between x and y as a linear equation.



- 4 Which of the following graphs indicates that y is directly proportional to x ?



EXAMPLE 12D-2 Determining whether a direct variation relationship exists

By calculating the values for $\frac{y}{x}$ for each of these tables, determine if $y \propto x$.

a

x	1	3	5
y	8	24	40

b

x	3	7	9
y	12	21	36

	Solve	Think	Apply												
a	<table><tr><td>x</td><td>1</td><td>3</td><td>5</td></tr><tr><td>y</td><td>8</td><td>24</td><td>40</td></tr><tr><td>$\frac{y}{x}$</td><td>8</td><td>8</td><td>8</td></tr></table> $y \propto x$ and the constant of proportionality is 8.	x	1	3	5	y	8	24	40	$\frac{y}{x}$	8	8	8	Calculate $\frac{y}{x}$ for each entry in the table. Because $\frac{y}{x}$ is the same number for every entry, y varies directly as x. In this case, $y = 8x$.	Direct proportion requires $\frac{y}{x}$ to equal the same value for all ordered pairs (x, y). The origin $(0, 0)$ satisfies any direct proportion equation.
x	1	3	5												
y	8	24	40												
$\frac{y}{x}$	8	8	8												
b	<table><tr><td>x</td><td>3</td><td>7</td><td>9</td></tr><tr><td>y</td><td>12</td><td>21</td><td>36</td></tr><tr><td>$\frac{y}{x}$</td><td>4</td><td>3</td><td>4</td></tr></table> Direct variation does not exist.	x	3	7	9	y	12	21	36	$\frac{y}{x}$	4	3	4	$\frac{y}{x}$ is not the same number for every entry, so y does not vary directly as x.	
x	3	7	9												
y	12	21	36												
$\frac{y}{x}$	4	3	4												

5 Calculate values for $\frac{y}{x}$ for each of the following tables.

a

x	1	2	5
y	4	8	21

b

x	2	3	6
y	14	21	42

c

x	-4	-1	2
y	-8	-2	4

d

x	-7.5	-8.4	11.1
y	2.5	2.8	-3.7

- In each case, state whether or not direct proportionality exists between x and y .
- When direct proportionality exists, find the constant of variation and express y in terms of x .

EXAMPLE 12D-3 Finding the equation of a direct variation relationship

Given that $y \propto x$ and $y = 4.5$ when $x = 13.5$, find:

- the constant of variation
- the equation for y in terms of x
- y , when $x = 42$
- x , when $y = 7$.

	Solve	Think	Apply
a	$y \propto x$ $y = kx$ $4.5 = k \times 13.5$ $\frac{4.5}{13.5} = k$ $k = \frac{1}{3}$	If $y \propto x$ then $y = kx$ and $\frac{y}{x} = k$. Substitute the values for y and x into $y = kx$. Alternatively, find $\frac{4.5}{13.5}$ directly.	If $y \propto x$ then $y = kx$ and dividing related x and y values will give the value of k. The value k can be a whole number, a fraction or a decimal. The line with equation $y = kx$ is the same as the line with equation $y = mx$. The gradient and the constant of variation are equal.
b	$y = kx$ $= \frac{1}{3}x$ $= \frac{x}{3}$	The equation is always $y = kx$ with $k = \frac{y}{x}$.	

	Solve	Think	Apply
c	$y = \frac{x}{3}$ $= \frac{42}{3}$ $= 14$	Substitute $x = 42$ into $y = \frac{x}{3}$.	Substitute the known value for x or y into the equation and solve it to find the unknown quantity.
d	$y = \frac{x}{3}$ $7 = \frac{x}{3}$ $x = 7 \times 3$ $= 21$	Substitute $y = 7$ into $y = \frac{x}{3}$.	

- 6 a** Given that $y \propto x$ and $y = 42$ when $x = 5$, find:
- i** the constant of variation **ii** the equation connecting x and y
 - iii** y when $x = 7$ **iv** x when $y = 51$.
- b** Given that $a \propto b$ and $a = 9.6$ when $b = 1.6$, find:
- i** the constant of variation **ii** the equation connecting a and b
 - iii** a when $b = 21$ **iv** b when $a = 11$.

- 7 i** Find the values of a , b and c in the tables below given that $y \propto x$.
- ii** For each table write the relationship between x and y as a linear equation.

a

x	2	3	5	b
y	36	54	a	20

b

x	4	12	1	b
y	2	6	a	24

c

x	11.1	-12	34.2	c
y	3.7	a	b	17.1

d

x	a	5.1	-7.8	c
y	14.4	22.95	b	31.05

- 8** The amount of petrol used by a car travelling at a constant speed is directly proportional to the distance travelled. A car uses 6 L of petrol to travel 50 km.
- a** Complete the working below to find an equation linking the amount of petrol used, P , with the distance travelled, d .
- $P \propto \underline{\hspace{2cm}}$
- $P = kd$
- $\underline{\hspace{2cm}} = k \times \underline{\hspace{2cm}}$
- $k = \underline{\hspace{2cm}}$
- So $P = \underline{\hspace{2cm}}$
- b** Use your equation to find the amount of petrol used to travel 180 km at the same speed?
- 9** The amount of petrol used by a car travelling at a constant speed is directly proportional to the distance travelled. A car uses 6 L of petrol to travel 56 km. How much petrol will be used to travel 140 km at the same speed?
- 10** The amount of petrol used by a car travelling at a constant speed is directly proportional to the distance travelled. A car uses 8 L of petrol to travel 75 km. What distance will the car cover if it travels at the same speed and uses 24 L of petrol?

- 11** The amount of paint needed to paint a wall is directly proportional to the area covered. A 4-litre can of paint will cover 12 m^2 .

- Find an equation linking the number of litres of paint, P , with the area covered, A .
- Find the number of litres of paint needed to paint a wall 63 m^2 in area.

- 12** The amount of varnish needed to paint a timber deck is directly proportional to the area covered. A 4-litre can of varnish will cover 14 m^2 . Find the number of litres of varnish needed to varnish a deck 46 m^2 in area.

- 13** A 10-litre can of paint will cover 35 m^2 .

- Find the number of litres of paint needed to paint a wall 175 m^2 in area.
- How many cans of paint must be purchased?

- 14** Three packets of chips cost \$6.45.

- Find an equation linking the cost, C , with the number of packets, P .
- Use the equation from part **a** to find the cost of 17 packets.

- 15** The amount of water dripping into a bucket from a tap is directly proportional to the length of time the tap drips. Water dripping from a tap fills a 5-litre bucket in 8 hours. How long will it take to fill a 12-litre bucket?

- 16** The cost of a 2.8 m length of hose is \$4.20.

- Find an equation linking the cost, C , with the number of metres of hose, H .
- Find the cost of 6.7 m of hose.
- How much hose can be purchased for \$30?

- 17** The resistance, R ohms, to the flow of electricity in a wire varies directly with the length, l cm, of the wire and, when $l = 20 \text{ cm}$, $R = 0.08 \text{ ohms}$.

- Find R when $l = 85 \text{ cm}$.
- Find l when $R = 6 \text{ ohms}$.

- 18** The currency exchange rate on a particular day was $\text{A\$1} = 0.651 \text{ British pounds}$.

- Write the rule connecting British pounds, P , and Australian dollars, D , on that day.
- How many Australian dollars could be bought with 850 pounds on that day?

- 19** The current required to operate a car's starter motor varies directly as the voltage. The current is 150 amps when the battery voltage is 12 volts.

- Write the rule connecting current, I amps, and voltage, V volts.
- What current is drawn if the battery voltage drops to 10 volts?
- The car cannot start if the current falls below 95 amps. What is the lowest battery voltage that will allow the car to start?



12E Linear modelling

These resources are available on your obook assess:

- **Video tutorial 12E:** Watch and listen to an explanation of Example 12E-2
- **assess quiz 12E:** Test your skills with an auto-correcting multiple-choice quiz

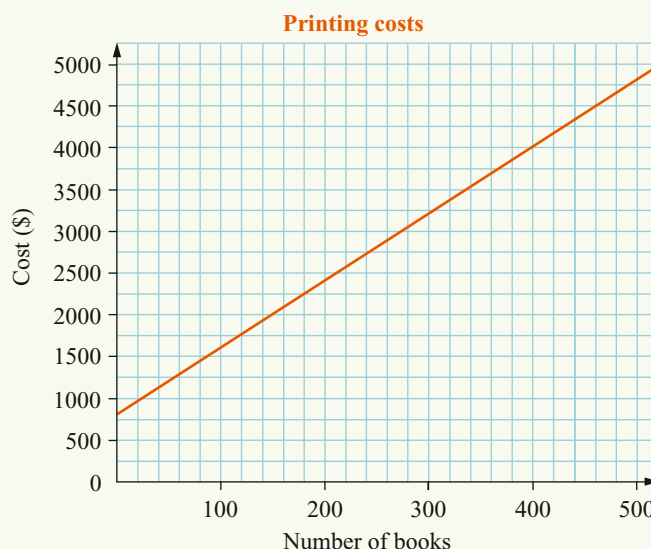
o
i
a
s
s
e
s
s

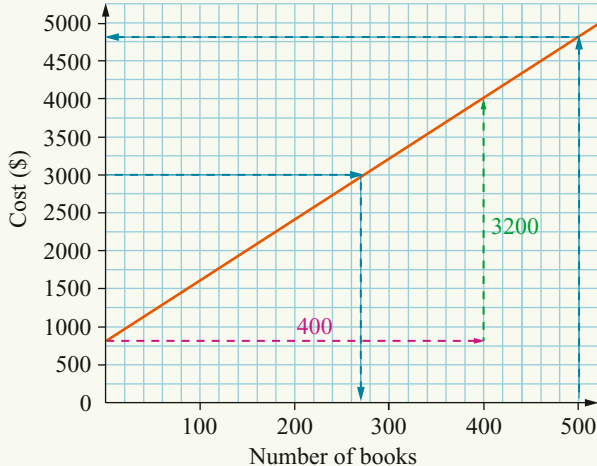
Graphs and equations can be used to model real situations. A graph allows a comparison between two variables. In this topic, we will look at straight-line graphs to model physical situations. The gradient and the intercept on the vertical axis of linear graphs have meaning in practical situations. This section examines these applications. A straight-line graph of the form $y = mx$ (or $y = kx$) shows a direct variation relationship between the two variables. A straight-line graph of the form $y = mx + c$ shows a relationship between x and y , the gradient m , and a constant term c .

EXAMPLE 12E-1 Interpreting the graph of a linear model

The graph on the right models printing costs.

- Find the cost of printing 500 books.
- Find the number of books that can be printed for \$3000.
- Find the gradient of the line. What is its meaning in this situation?
- Find the vertical intercept. What is its meaning in this situation?



	Solve	Think	Apply
a	\$4800	Find the point on the line whose horizontal coordinate (number of books) is 500. The vertical coordinate (cost) is \$4800. Printing costs 	Find the point on the line with the given coordinate and read off the unknown coordinate on the appropriate axis.
b	270 books	Find the point on the line whose vertical coordinate (cost) is 3000. The horizontal coordinate (number of books) is 270.	
c	Gradient = $\frac{3200}{400} = 8$ This is the cost per book after the initial set up costs. Cost = \$8/book	Using points (0, 800) and (400, 4000): Gradient = $\frac{\text{rise}}{\text{run}} = \frac{3200}{400}$. Because the units of the vertical rise are dollars and the units of the horizontal run are books, the unit for the gradient is \$/book.	The gradient is expressed as the unit of the rise per unit of the run.
d	Vertical intercept is \$800. This is the fixed cost for setting up.	The vertical intercept is the cost of printing zero books.	The vertical intercept is the point where the line cuts the vertical axis.

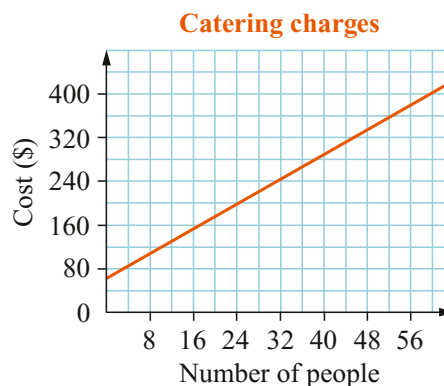
EXERCISE 12E Linear modelling

1 This graph models taxi charges.

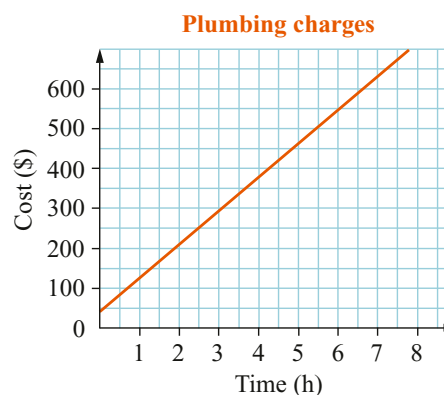
- From the graph, find the cost of travelling 22 km by taxi.
- How far can a passenger travel for \$30?
- Find the gradient. What is its meaning in this situation?
- Find the intercept on the vertical axis. What is its meaning in this situation?



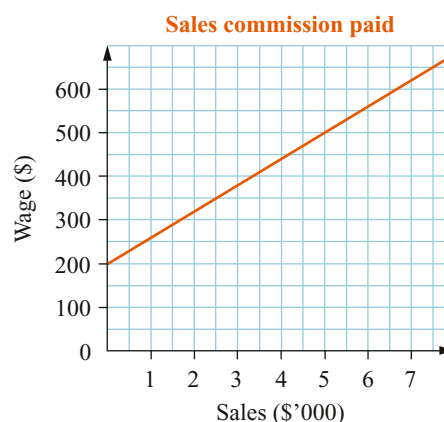
- 2** This graph models catering charges.
- How much would it cost to cater for 24 people?
 - How many people could be catered for \$360?
 - Find the gradient. What is its meaning in this situation?
 - Find the intercept on the vertical axis. What is its meaning in this situation?



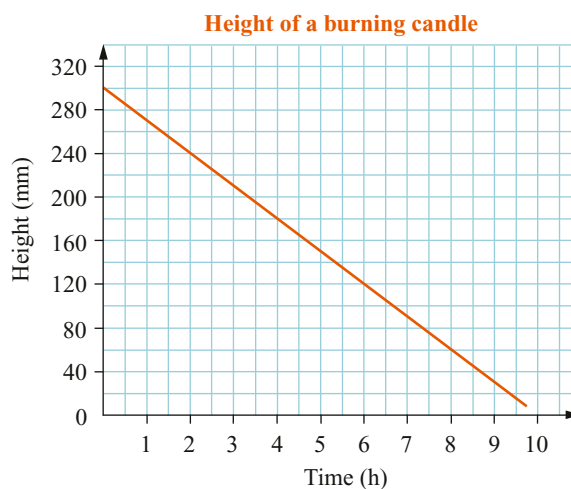
- 3** This graph models plumbing charges.
- Find the cost of the plumber for $4\frac{1}{2}$ hours.
 - Find the number of hours worked if the charge is \$250.
 - Find the gradient. What is its meaning in this context?
 - Find the intercept on the vertical axis. What is its meaning in this context?



- 4** The graph models the income of a salesperson.
- What was the value of sales if the salesperson was paid \$320 for the week?
 - What wage would be paid if the salesperson sold \$3500 worth of goods?
 - Find the gradient. What is its meaning in this context?
 - Find the intercept on the vertical axis. What is its meaning in this context?



- 5** This graph models the height of a burning candle.
- What is the height of the candle:
 - initially?
 - after 5 hours?
 - How long does it take for the height to decrease to 210 mm?
 - How long does it take for 50 mm of the candle to be used?
 - Find the gradient. What is its meaning in this context?
 - Find the intercept on the vertical axis. What is its meaning in this context?



EXAMPLE 12E-2 Exploring direct variation applications

The interest, $\$I$, earned on a principal of $\$P$ for 1 year is given by the formula $I = kP$, where k is a constant.

- a** If an investment of $\$6000$ earns interest of $\$300$, find the value of the constant k .

P	2000	4000	6000	8000	10 000
I			300		

- b** Use the value of k from part **a** to complete the table.
c Draw a graph of this linear relationship.
d Calculate the gradient of the graph line. What is the meaning of the gradient in this context?

Solve	Think	Apply												
$300 = k \times 6000$ $\frac{300}{6000} = k$ $k = 0.05$	$I = \$300$ when $P = \$6000$. Substituting into $I = k \times P$ gives $300 = k \times 6000$. Solve the equation by dividing both sides by 6000.	The value of the constant is found by substituting the given values into the formula. Use the value of the constant to complete the table. Plot points and draw the straight line.												
<table><tr><td>P</td><td>2000</td><td>4000</td><td>6000</td><td>8000</td><td>10 000</td></tr><tr><td>I</td><td>100</td><td>200</td><td>300</td><td>400</td><td>500</td></tr></table>	P		2000	4000	6000	8000	10 000	I	100	200	300	400	500	The equation connecting I and P is $I = 0.05P$. Substituting $P = 2000$ gives $I = 0.05 \times 2000 = 100$. Substituting $P = 4000$ gives $I = 0.05 \times 4000 = 200$, etc.
P	2000		4000	6000	8000	10 000								
I	100	200	300	400	500									
Interest earned in 1 year	Plot the points (2000, 100), (4000, 200), ... and draw a straight line through them.													
$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{200}{4000} = 0.05$ The gradient is the rate of interest being earned on the investment.	$\text{Gradient} = \frac{\text{amount of interest}}{\text{amount invested}}$ expressed as a decimal. This is dollars earned per dollar invested; that is, the rate of interest (equivalent to 5%).	$\text{Gradient} = \frac{\text{rise}}{\text{run}}$ The meaning is indicated by the units of the gradient.												

- 6** The interest, $\$I$, earned on a principal of $\$P$ for 1 year is given by the formula $I = kP$, where k is a constant.
- a** If an investment of $\$8000$ earns interest of $\$240$, find the value of the constant k .
b Use the value of k from part **a** to complete the table below.

P	2000	4000	6000	8000	10 000
I				240	

- c** Draw a graph of this linear relationship.
d What is the meaning of the gradient of this line in this context?

- 7** The distance, d km, travelled by a train in a time of t hours is given by the relationship $d = mt$, where m is a constant.

t	1	3	5	7	9
d			400		

- Use the information in the table to find the value of the constant m .
- Use the value of m from part **a** to complete the table.
- Draw a graph of this linear relationship.
- What is the meaning of the gradient of this line in this context?

- 8** The volume, V cm³, of a solid with a fixed cross-section and height h cm is given by $V = kh$, where k is a constant.

h	4	8	12	16	20
V		34			

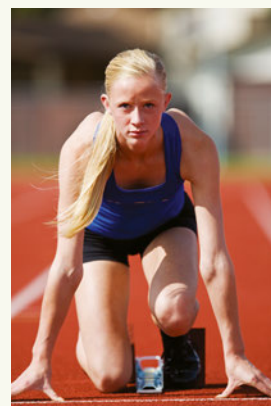
- Use the information in the table on the right to find the value of the constant k .
- Use the value of k from part **a** to complete the table.
- Draw a graph of this linear relationship.
- Calculate the gradient of this line. What is its meaning in this context?

EXAMPLE 12E-3 Modelling direct variation

At an athletics carnival, Celine runs 100 m in 16 s.

It is assumed that she runs at a constant rate.

- Draw a straight-line graph to model Celine's run.
- Find the gradient of the graph from part **a**.
- Write an equation to model the relationship between distance and time for Celine's run.



	Solve	Think	Apply
a	Celine's 100-m run	Put time on the horizontal axis, and distance on the vertical axis. Draw a straight line through the points (0, 0) and (16, 100), because Celine runs 0 m in 0 s and 100 m in 16 s.	Draw the straight line between (0, 0) and the point whose coordinates are given by the information supplied in the question.
b	Gradient = $\frac{\text{rise}}{\text{run}} = \frac{100}{16} = 6.25$ The gradient is Celine's speed in m/s. She runs at an average speed of 6.25 m/s.	The vertical rise between these two points is 100 m and the horizontal run is 16 s. The units for the gradient are m/s. This is speed.	Then calculate the gradient using $\text{Gradient} = \frac{\text{rise}}{\text{run}}$. The meaning of the gradient is indicated by its units.
c	If d is distance (in m) and t is time (in s) then $d = 6.25t$.	Use pronumerals to define the two variables and write an equation in the form $y = mx$.	

- 9** Christopher runs 100 m in 25 s.
- Draw a straight-line graph to model Christopher's run, assuming he runs at a constant rate.
 - How long would it take him to run 30 m?
 - Find the gradient, and hence, Christopher's speed in metres per second.
 - Write an equation to model the relationship between distance and time.
- 10** Craig drives 200 km in 4 hours.
- Draw a straight-line graph to model Craig's drive, assuming he drives at a constant rate.
 - How far does Craig drive in $1\frac{1}{2}$ hours?
 - Find the gradient of the graph you drew in part a.
 - What are the units of the gradient in this case?
 - Write an equation to model the relationship between distance and time.

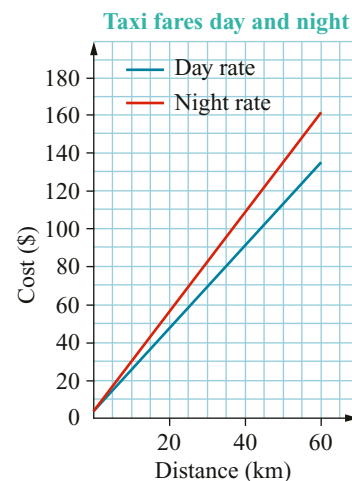


- 11** On a particular day A\$100 buys 78 euros.
- Draw a conversion graph with A\$ on the x -axis.
 - How many A\$ are needed to purchase 50 euros?
 - Find the gradient of the graph you drew in part a.
 - What is the meaning of the gradient in this context?
 - Write an equation to model the relationship between A\$ and euros.
 - Extend your graph to A\$500. Is it still accurate as an exchange model?



- 12** Use the information in question 9 and extend the graph to cover a distance of 500 m.
- Is the gradient still the same? Why or why not?
 - The graph is no longer accurate as a model for Christopher's run. Explain, including some reference to the gradient.
- 13** Use the information in question 10 and extend the graph to cover a distance of 1000 km.
- Is the gradient still the same? Why or why not?
 - Is the graph still accurate as a mathematical model for Craig's drive? (*Hint: consider petrol.*)

- 14** The rate for taxi fares varies depending on the time of day. The graph on the right shows the cost in dollars compared to the distance travelled in kilometres for travel during the day and at night.
- What is the meaning of the gradient in this context?
 - Find the gradient of each line. What does the steeper slope of one of the lines indicate?
 - For each line, find the intercept on the vertical axis. What is the meaning of the intercept for each situation?
 - For each line, write an equation which models the linear relationship.



- 15** Write a report based on one of the algebraic models presented in this chapter. Explain how the model works and its limitations. Give examples of ways of improving the model. Explain the value of having the model.

CHAPTER 12 REVIEW LINEAR RELATIONSHIPS

You should be able to:

- ✓ generate a table of values from a linear equation
- ✓ produce straight-line graphs with pencil and paper, and using technology
- ✓ calculate and interpret gradient
- ✓ find and interpret the vertical intercept
- ✓ complete tables, plot points, and draw graphs of the form $y = mx + c$
- ✓ generalise linear number patterns using algebraic symbols
- ✓ graph and interpret quantities that vary directly
- ✓ construct and analyse linear models
- ✓ solve practical direct variation problems
- ✓ recognise the limitations of models.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

REVIEW MULTIPLE-CHOICE QUESTIONS

Use the graph on the right, showing the distance a car travels over a period of time, to answer questions 1 to 3.

12A 1 What is the time taken to travel 200 km?

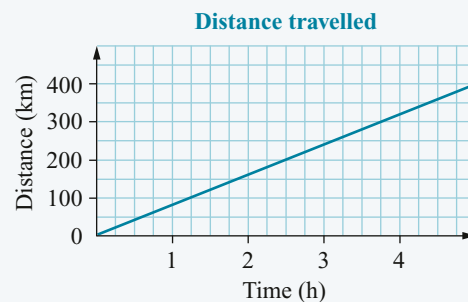
- A 2 h B $2\frac{1}{2}$ h
C 3 h D $3\frac{1}{2}$ h

12A 2 What is the distance travelled after $3\frac{3}{4}$ h?

- A 5 km B 250 km
C 300 km D 6 km

12B 3 What is the slope of the graph?

- A $\frac{4}{5}$ B $\frac{5}{4}$ C 100 D 80



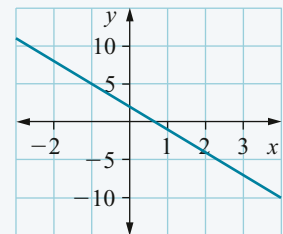
Use the graph shown on the right to answer questions 4 and 5.

12B 4 What is the gradient of the line?

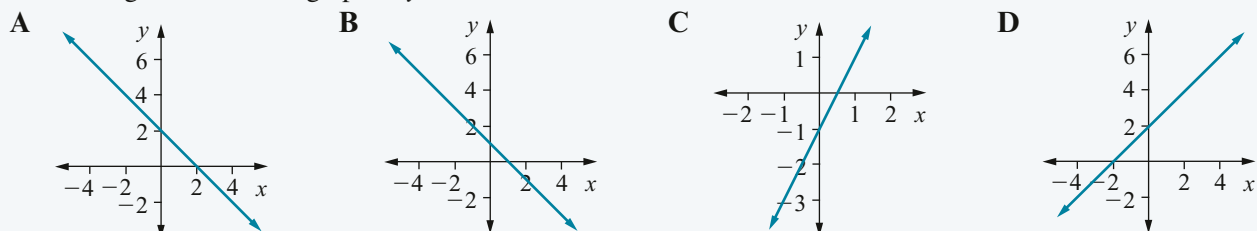
- A 3 B $\frac{1}{3}$ C $-\frac{1}{3}$ D -3

12B 5 What is the y-intercept?

- A 2 B 34 C -3 D -2



12C 6 Which diagram shows the graph of $y = 2x - 1$?



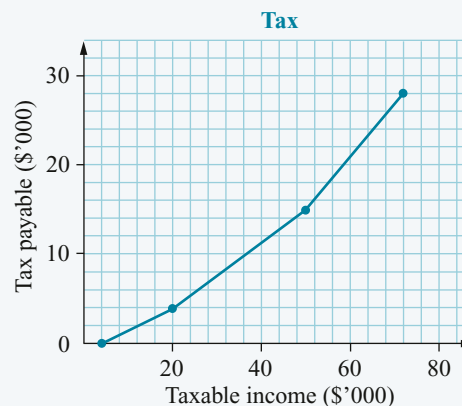
- 12C ▶ 7** What are the gradient and y-intercept of the line $y = \frac{3x}{2} - 1$?
A 3 and -1 **B** $\frac{3}{2}$ and -1 **C** $\frac{3}{2}$ and 1 **D** $\frac{3}{2}$ and $-\frac{1}{2}$

- 12C ▶ 8** What is the form of the linear equation describing the pattern in this table?
A $y = 2x + c$ **B** $y = 3x + c$
C $y = 4x + c$ **D** $y = 5x + c$

x	1	2	3	4	5
y	3	7	11	15	19

Use the tax graph shown to answer questions 9 and 10.

- 12C ▶ 9** The tax payable on \$65 000 is closest to:
A \$20 000 **B** \$21 000
C \$23 000 **D** \$25 000
- 12C ▶ 10** The income on which tax of \$10 000 is payable is closest to:
A \$2000 **B** \$36 000
C \$30 000 **D** \$2500



- 12C ▶ 11** The Clear View Water Company purifies water. The weekly fixed costs are \$250 and the cost per glass is \$3. Which of the following equations represents this information?
A $C = 3n + 250$ **B** $C = 250n + 3$ **C** $C = 250n - 3$ **D** $C + 250 = 3n$
- 12C ▶ 12** The equation $C = 4n + 520$ models an ice-cream shop's costs. What could the 520 represent?
A number of ice creams sold **B** the cost per ice cream
C fixed daily cost **D** number of ice creams made

- 12D ▶ 13** Which table of values below represents a direct variation relationship?

A

x	1	3	5
y	10	26	42

B

x	1	3	5
y	8	24	40

C

x	1	3	5
y	4	6	8

D

x	1	3	5
y	8	10	12

- 12D ▶ 14** The table on the right shows values of x and y which are in direct proportion. What are the values of a and b ?
A $a = 8, b = 4$ **B** $a = 8, b = 36$
C $a = 72, b = 4$ **D** $a = 72, b = 36$

x	6	12	24	b
y	2	4	a	12

- 12D ▶ 15** The amount of petrol used by a car travelling at a constant speed is directly proportional to the distance travelled. A car used 7 L of petrol to travel 52.5 km. The amount of petrol used to travel 126 km at the same rate is:
A 2.4 L **B** 7.5 L **C** 16.8 L **D** 945 L
- 12E ▶ 16** What does the slope of the distance–time graph for question 1 represent?
A speed **B** time taken **C** distance travelled **D** all of these

REVIEW SET 1

- 1** Dog shampoo is sold in bulk amounts for \$4.50 per litre. This table shows capacity, in litres, versus cost for the liquid.

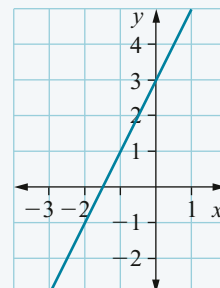
Capacity (L)	1	2	3	4	5	6
Cost (\$)	4.50	9.00	13.50	18.00	22.50	27.00

- a** Draw the graph of capacity versus cost. Which is the dependent variable?
b Find the cost of 3.5 L of shampoo.

- 2 A person drives non-stop from Condobolin to Sydney, a distance of 400 km. The linear relationship for finding the distance the driver is from Sydney is $d = 400 - 100t$, where d is distance in kilometres and t is time in hours.
- Complete this table of values.
 - Draw the graph of distance to Sydney.
 - How far is it to Sydney after $1\frac{1}{2}$ h of driving?
 - How far has the car travelled after $2\frac{1}{2}$ h?

Time (h)	0	1	2	3	4
Distance (km)	400		200		

- 3 Find the gradient and y-intercept of the straight-line graph shown on the right.



- 4 Sketch the graph of $y = 3x + 5$.

- 5 On a particular day, A\$100 buys 68 euros.
- Draw a conversion graph with A\$ on the horizontal axis.
 - How many A\$ are needed to purchase 50 euros?
 - Find the gradient.
 - What is the meaning of the gradient in this context?
 - Write an equation to model the relationship between A\$ and euros.
 - Extend your graph to A\$500. Is it still accurate as an exchange model?

x	0	1	2	3	4
y	-5	-2	1	4	7

- 6 Find a linear equation that describes the pattern shown in this table.

- 7 The distance, d kilometres, travelled by a train over time t hours is $d = 70t$.

- Complete this table of values for t and d .
- Draw the graph showing the distance travelled by the train versus time.
- How far away is the train after $3\frac{1}{2}$ hours?
- When is the train 200 km away?
- Is this an example of direct variation? Explain.

t	1	2	3	4	5
d					

REVIEW SET 2

- 1 Chocolate frogs are sold by weight. This table shows weight, in kilograms, versus cost for the chocolate frogs.

Weight (kg)	1	2	3	4	5	6
Cost (\$)	10	20	30	40	50	60

- Draw the graph of weight versus cost. Which is the dependent variable?
- Use the graph to find the cost of 4.5 kg of chocolate frogs.

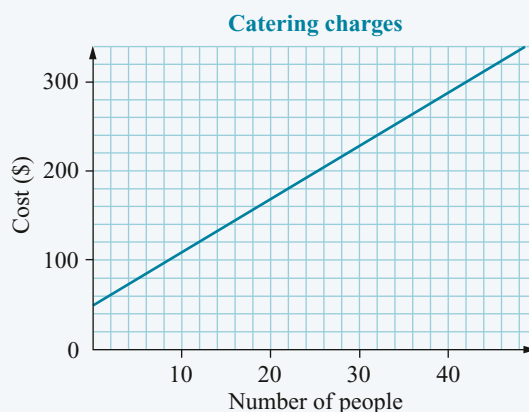
- 2 The distance, d km, travelled by a bus in time t hours is given by $d = 60t$.

Time (h)	0	1	2	3	4
Distance (km)	0		120		

- Complete this table of values.
- Draw the graph of distance travelled versus time.
- How far away is the bus after $1\frac{1}{2}$ h?
- When is the bus 100 km away?

- 3 Sketch the straight-line graph of $y = 3 - 2x$.

- 4 The graph on the right models catering charges.
- How much would it cost to cater for 35 people?
 - For how many people would catering cost \$300?
 - Find the gradient. What is its meaning in this context?
 - Find the intercept on the vertical axis. What is its meaning in this concept?



- 5 The Hats Are Tops company has fixed costs of \$900 per day and a variable cost of \$5 per hat. The daily cost can be modelled using the straight-line equation $C = 5n + 900$.

n	0	50	150	200	250
C					

- Explain each term in the formula $C = 5n + 900$.
- Complete the table of values.
- Draw the straight-line graph $C = 5n + 900$, for $0 \leq n \leq 250$.
- Use the graph to find the number of hats produced when the cost is \$1500.

- 6 By calculating the values for $\frac{y}{x}$ from the table on the right, determine whether $y \propto x$.

x	1	2	5
y	6	14	30

REVIEW SET 3

- 1 Cashew nuts are sold by weight. This table shows weight, in kilograms, versus cost for the nuts.

Weight (kg)	1	2	3	4	5	6
Cost (\$)	15	30	45	60	75	90

- Draw the graph of weight versus cost. Which is the dependent variable?
- Use the graph to find the cost of 3.8 kg of nuts.
- How many kilograms of nuts can be purchased for \$50?

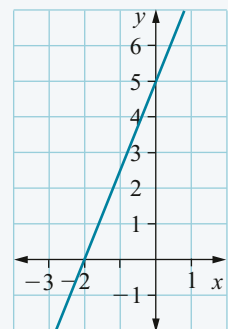
- 2 The cost of hiring a night-time taxi is \$7.00 flagfall and \$3.80 per kilometre travelled.

- Complete this table of costs for taxi hire.
- Draw the graph showing the cost of hiring the taxi.
- How much does it cost to travel 15 km?
- How far can a passenger travel for \$75?

Distance (km)	0	10	20	30	40
Cost (\$)					

- 3 Find the gradient and y -intercept of the straight-line graph on the right.

- 4 Sketch the straight-line graph of $y = 3x - 4$.



- 5 On a particular day, A\$100 buys 87 euros.

- Draw a conversion graph with A\$ on the horizontal axis.
- How many A\$ are needed to purchase 50 euros?
- Find the gradient. What is its meaning in this context?
- Extend your graph to \$500 Australian. Is it still accurate as an exchange model?

- 6 Find a linear equation that describes the pattern shown in this table.

x	1	2	3	4	5
y	-1	-4	-7	-10	-13

- 7 The Socks Are Us company sells socks for \$16 a pair. Sales may be modelled using the straight-line equation $I = 16n$.

- Explain each term in the formula $I = 16n$.
- Complete the table of values.
- Draw the straight-line graph $I = 16n$.
- Use the graph to find the number of pairs of socks sold when the income is \$1840.
- Is this a direct variation relationship? Explain.

n	0	50	150	200	250
I					

REVIEW SET 4

- 1 Mobile telephone calls cost 85 cents per minute. This table shows time versus cost for the calls.

Time (min)	1	2	3	4	5	6
Cost (\$)	0.85	1.70	2.55	3.40	4.25	5.10

- Draw the graph of time versus cost. Which is the independent variable?
- Use the graph to find the cost of a 2.5 min call.
- What is the maximum length of a call that costs \$3.00?

- 2 a The cost of hiring a car is \$10 flagfall and \$3.40/km for each kilometre travelled. Complete this table of values for car hire.

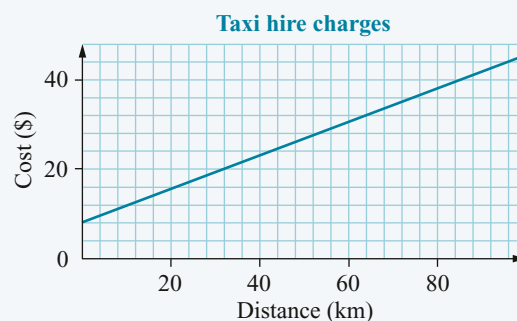
Distance (km)	0	10	20	30	40
Cost (\$)	10.00	44.00			

- Draw the graph showing the cost of hiring the car.
- How much does it cost to travel 35 km?
- How far can a driver travel for \$15?
- Is this an example of direct variation? Explain

- 3 Sketch the straight-line graph of $y = -2 - 4x$.

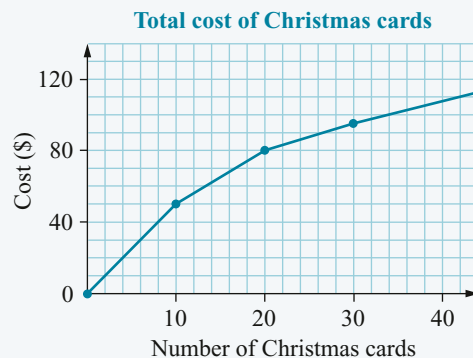
- 4 The graph on the right models taxi charges.

- Find the cost of travelling 65 km.
- How far can a passenger travel for \$40?
- Find the gradient. What is its meaning in this context?
- Find the intercept on the vertical axis. What is its meaning in this context?



- 5 This graph shows the total cost of hand-made Christmas cards.

- What is the cost of buying:
 - 6 cards?
 - 18 cards?
 - 30 cards?
- How many cards can be bought for:
 - \$20?
 - \$65?
 - \$100?
- If you buy 10 cards what is the cost per card?
- What is the gradient of the line segment for the cost of buying more than 30 cards?
 - What does the gradient represent in this situation?



- 6 The Tight Squeeze belt company has weekly fixed costs of \$400. The cost per belt is \$7.

- Write an equation to represent this information.
Use A for cost and n for the number of belts.

n	0	200	400	600	800	1000
A						

- Complete the table of values.
- Draw a straight-line graph.
- Use the graph to find the number of belts made when the cost is \$2000.

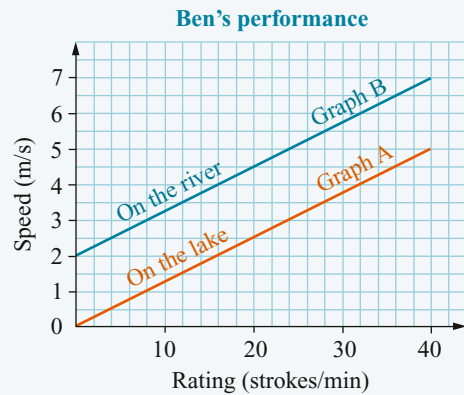
- 7 a Find the values of a , and b in the table on the right, given that $y \propto x$.

- From the table, write the relationship between x and y as a linear equation.

x	2	3	5	b
y	32	48	a	8

- 1 a** The speed, s m/s, at which Ben can row his boat depends on the number of strokes, r , he takes per minute. This is called his rating. Graph A shows his performance when he is training on a lake.

- Use the graph to find the rating necessary for a speed of 2 m/s.
- Find Ben's speed if his stroke rating is 32 strokes/min.
- Calculate the time, in minutes and seconds, that it takes to row one kilometre at the speed found in part ii.



(1 mark)

(1 mark)

(1 mark)

- b** Graph B represents Ben's performance on a river when he rows downstream, with the same rating as his rating on the lake.

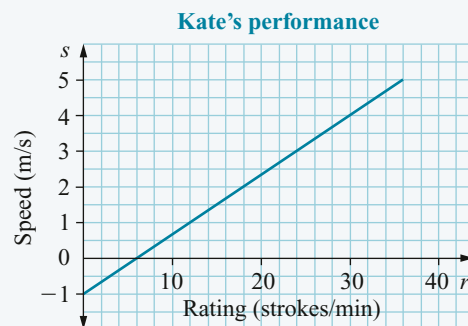
- Find his speed on the river if his stroke rating is 20 strokes/min.
- How fast is the river flowing?

(1 mark)

(1 mark)

- c** Ben's coach records the speed and rating of another rower, Kate. The graph on the right shows Kate's performance when she is rowing on the same river as Ben, but it is flowing at a different speed.

- Is Kate rowing upstream against the flow or downstream? Give reasons.
- How fast is this river flowing?



(1 mark)

(1 mark)

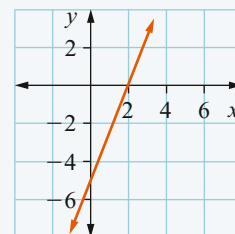
- d** Find the linear equation that describes the pattern shown in the table below.

(2 marks)

x	1	2	3	4	5
y	7	3	-1	-5	-9

- e** The diagram on the right shows a straight-line graph.

- Calculate the gradient of the line.
- What is the y -intercept?



(1 mark)

(1 mark)

- f** The cost of pies varies directly with the number purchased. Three pies cost \$13.50.

- Form an equation linking cost, $\$C$, and the number of pies purchased, n .
- What is the constant of variation k ?
- Draw the graph of C versus n for $0 \leq n \leq 10$.
- How much would 8 pies cost?

(1 mark)

(1 mark)

(1 mark)

(1 mark)

TOTAL: 15 marks

13

Units of energy and mass

The main mathematical ideas in this chapter are:

- ▶ converting between metric units of mass
- ▶ solving problems using metric units of mass
- ▶ converting between metric units of energy
- ▶ solving problems using metric units of energy
- ▶ using units of energy and mass to solve problems related to food and nutrition
- ▶ using units of energy to solve problems involving the amount of energy expended in activities
- ▶ using units of energy to solve problems involving the consumption of electricity.

ARE YOU READY?

- 13A ▶ 1** What is 35.41×1000 ?
A 3541 **B** 35 410
C 0.3541 **D** 0.035 41
- 13A ▶ 2** What is $35.41 \div 1000$?
A 3541 **B** 35 410
C 0.3541 **D** 0.035 41
- 13A ▶ 3** What is $1452.63 + 324 - 96.97$?
A 1873.6 **B** 1031.66
C 1679.66 **D** 1262.87
- 13A ▶ 4** What is $12.18 \div 2.1 \times 6.7$?
A 38.86 **B** 0.87
C 171.37 **D** 1.16
- 13A ▶ 5** What is 17.096 rounded to two decimal places?
A 17.09 **B** 17.1
C 17.10 **D** 17.16
- 13A ▶ 6** What is 234.6 rounded to the nearest whole number?
A 230 **B** 234
C 235 **D** 236
- 13A ▶ 7** How many kilograms are there in 454 g?
A 454 000 kg **B** 45.4 kg
C 4.54 kg **D** 0.454 kg
- 13C ▶ 8** Which of the following rates is equivalent to 20 errors in 16 games of tennis?
A 0.2 errors/game
B 0.25 errors/game
C 0.75 errors/game
D 1.25 errors/game
- 13C ▶ 9** What is $\frac{5}{6}$ of 78?
A 93.6 **B** 65
C 0.83 **D** 1.2
- 13D ▶ 10** What is the value of $3x + 4y - 2z$ if $x = 7$, $y = 5$ and $z = 3$?
A 59 **B** 47
C 35 **D** 18
- 13D ▶ 11** How many centimetres are there in 2.01 km?
A 201 cm **B** 2010 cm
C 20 100 cm **D** 201 000 cm
- 13D ▶ 12** What is the solution of the equation $84 = a \times 4 \times 3$?
A $a = 21$ **B** $a = 7$
C $a = 28$ **D** $a = 112$

If you had difficulty with any of these questions or would like further practice, complete one or more of the matching Support sheets available on your [obook assess](#).

- Q1–2** Support sheet 13A.1 Multiplying and dividing by 10, 100, 1000, etc.
Q3–4 Support sheet 13A.2 Operations with decimal numbers
Q5–6 Support sheet 13A.3 Decimal places and rounding
Q7 Support sheet 13A.4 Converting between kilograms and grams
Q8 Support sheet 13C.1 Writing rates
Q9 Support sheet 13C.2 Finding a fraction of a whole number
Q10 Support sheet 13D.1 Substituting for pronumerals
Q11 Support sheet 13D.2 Converting units of length
Q12 Support sheet 13D.3 Solving simple linear equations

13A Units of mass

These resources are available on your obook assess:

- **Interactive 13A:** Explore key ideas for units of mass
- **Worksheet 13A:** Practise your skills with extra problems for units of mass
- **assess quiz 13A:** Test your skills with an auto-correcting multiple-choice quiz

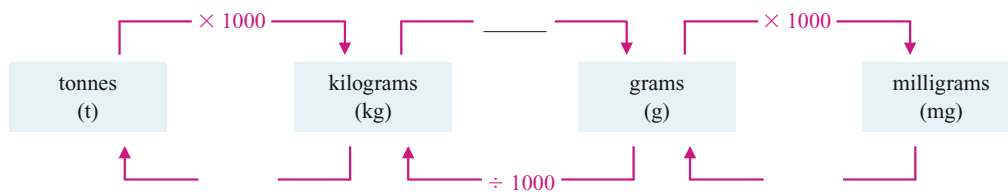
The metric system of measurement uses base units to measure quantities such as length, mass, capacity, area and volume.

The international standard (SI) unit for mass is kilograms. Common units of mass are tonnes (t), kilograms (kg), grams (g) and milligrams (mg).

Prefixes such as ‘kilo-’ and ‘milli-’ are used to indicate the factor of 10 by which a unit is multiplied.

EXERCISE 13A Units of mass

- 1 Complete this conversion diagram for mass.



EXAMPLE 13A-1 Converting grams to milligrams

Convert each of the following to milligrams.

a 2 g

b 0.6 g

c 0.35 g

	Solve	Think	Apply
a	$2 \text{ g} = (2 \times 1000) \text{ mg}$ $= 2000 \text{ mg}$	Multiply each measurement by 1000, because $1 \text{ g} = 1000 \text{ mg}$.	Grams are larger than milligrams. So the measurement in grams must be multiplied by the conversion factor of 1000.
b	$0.6 \text{ g} = (0.6 \times 1000) \text{ mg}$ $= 600 \text{ mg}$		
c	$0.35 \text{ g} = (0.35 \times 1000) \text{ mg}$ $= 350 \text{ mg}$		

- 2 Convert each of the following to milligrams.

a 3 g

b 5 g

c 7 g

d 9 g

e 2.5 g

f 2.2 g

g 1.3 g

h 3.4 g

i 0.4 g

j 0.3 g

k 0.15 g

l 0.22 g

m 0.05 g

n 0.037 g

o 0.002 g

p 0.003 g

EXAMPLE 13A-2 Converting milligrams to grams

Convert each of the following to grams.

a 3000 mg

b 200 mg

c 43 mg

	Solve	Think	Apply
a	$3000 \text{ mg} = (3000 \div 1000) \text{ g}$ $= 3 \text{ g}$	Divide each measurement in milligrams by 1000, because $1 \text{ g} = 1000 \text{ mg}$.	Milligrams are smaller than grams. So the measurement in milligrams must be divided by the conversion factor of 1000.
b	$200 \text{ mg} = (200 \div 1000) \text{ g}$ $= 0.2 \text{ g}$		
c	$43 \text{ mg} = (43 \div 1000) \text{ g}$ $= 0.043 \text{ g}$		

3 Convert the following to grams.

a 3000 mg

b 7000 mg

c 4000 mg

d 8000 mg

e 2500 mg

f 4200 mg

g 7500 mg

h 6200 mg

i 400 mg

j 350 mg

k 270 mg

l 120 mg

m 60 mg

n 38 mg

o 4 mg

p 2.5 mg

EXAMPLE 13A-3 Converting units of mass involving decimal values

Convert:

a 3.6 t to kilograms

b 780 g to kilograms

	Solve	Think	Apply
a	$3.6 \text{ t} = (3.6 \times 1000) \text{ kg}$ $= 3600 \text{ kg}$	$1 \text{ t} = 1000 \text{ kg}$ Tonnes are bigger than kilograms, so we multiply by the conversion factor.	Multiply tonnes by 1000 to get kilograms.
b	$780 \text{ g} = (780 \div 1000) \text{ kg}$ $= 0.78 \text{ kg}$	$1 \text{ kg} = 1000 \text{ g}$ Grams are smaller than kilograms, so we divide by the conversion factor.	Divide grams by 1000 to get kilograms.

4 Convert these masses.

a 2.7 t to kilograms

b 4.5 g to milligrams

c 23.92 kg to grams

d 0.34 t to kilograms

e 5.6 kg to grams

f 1.758 t to kilograms

g 23.49 t to kilograms

h 0.8 g to milligrams

i 4.05 kg to grams

j 0.875 kg to grams

k 2.05 t to kilograms

l 0.05 g to milligrams

5 Convert the following.

a 4000 kg to tonnes

b 8500 g to kilograms

c 1650 mg to grams

d 750 g to kilograms

e 45 mg to grams

f 1480 g to kilograms

g 400 kg to tonnes

h 950 mg to grams

i 90 g to kilograms

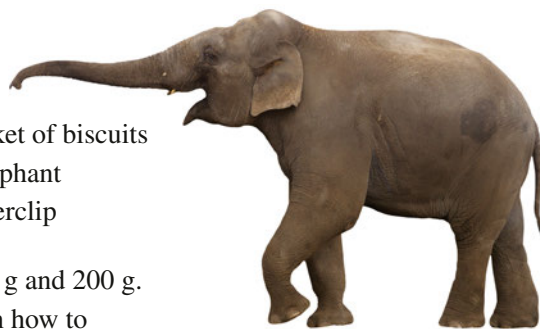
j 9 mg to grams

k 5 kg to tonnes

l 200 g to kilograms

- 6** Choose a convenient unit (tonnes, kilograms, grams, milligrams) to use for measuring the mass of each of these items.

a a man	b a jar of jam	c a packet of biscuits
d a bag of sand	e a truck	f an elephant
g a vitamin pill	h a knitting needle	i a paperclip



- 7** Jay adds up the following masses: 2.4 kg, 421 g, 0.7 kg, 35 g and 200 g. He obtains an answer of 659.1 kg. Give Jay some advice on how to answer the question correctly, and state the correct answer.

- 8** The average mass of an egg is 70 g.

- a** What is the mass of each of the following quantities of eggs?

- i** half a dozen eggs
- ii** a dozen eggs
- iii** four cartons of a dozen eggs

- b** Are there other factors to consider when determining the mass of four cartons of eggs? Explain.

- 9** A can of soup has a mass of 265 g. If the soup is delivered to the supermarket in cartons of 12 cans, what is the total mass of five cartons? The mass of each empty cardboard carton is 210 g. Give your answer in kilograms.

- 10** The total mass of a transport vehicle when fully laden with six cars is 9.43 t. If the unloaded car transport vehicle has a mass of 2700 kg, what is the average mass of each car, to the nearest kilogram?

- 11** Kyna needs to find the mass of one sheet of paper, but does not have a scale sensitive enough to measure something that light.

- a** Suggest a possible way Kyna could find this mass.
- b** If the mass of a ream (500 sheets) of paper is 2.25 kg, calculate the mass of one sheet of paper.

- 12** Look at the tin of tomatoes shown on the right.

- a** Net mass refers to the mass of the contents of the container. What is the net mass of this tin?
- b** Gross mass is the mass of the container plus contents. If you measured the gross mass as 450 g, what is the mass of the tin itself?
- c** If you know that the mass of the empty tin is actually 75 g and the gross mass is 450 g, what does this mean that the net mass is?
- d** Is the net mass, calculated in part **c**, more or less the advertised net mass?



EXAMPLE 13A-4 Calculating the amount of medication

A patient is prescribed 400 mg of a painkiller. The medication the chemist provides contains 80 mg of the painkiller in 10 mL. How much of this medication should be given to the patient?

Solve	Think
$400 \div 80 = 5$ Amount = $5 \times 10 \text{ mL}$ = 50 mL or: Volume required = $\frac{\text{strength required}}{\text{stock strength}} \times \text{volume of stock}$ = $\frac{400}{80} \times 10$ = 50 mL	Calculate the number of doses of 10 mL required, by dividing the amount prescribed by the amount contained in 10 mL of medication. Multiply this answer by 10 mL to obtain the amount required. In this formula: Strength required = 400 mg Stock strength = 80 mg Volume of stock = 10 mL
Apply	
Deciding whether more or less than 10 mL is to be given is the key to answering this question. Because the amount of painkiller required is greater than the amount in 10 mL of the medication, more than 10 mL of the medication needs to be given.	

- 13** A patient is prescribed 600 mg of a painkiller. Calculate how much of each of the following medications he must be given if the medication is available in these concentrations.

a 20 mg in 5 mL	b 30 mg in 10 mL	c 50 mg in 1 mL
d 120 mg in 5 mL	e 100 mg in 20 mL	f 60 mg in 5 mL
g 5 mg in 1 mL	h 50 mg in 5 mL	i 75 mg in 5 mL

- 14** A patient is prescribed 800 mg of an anti-nausea drug. Calculate how much of each of the following medications she must be given if the medication is available in these concentrations.

a 100 mg in 5 mL	b 10 mg in 1 mL	c 50 mg in 5 mL
d 160 mg in 10 mL	e 200 mg in 20 mL	f 80 mg in 5 mL
g 20 mg in 5 mL	h 40 mg in 5 mL	i 80 mg in 10 mL

- 15** Lana needs to give her dog some antibiotics. The instructions say that for every 5 kg of body mass, the dog needs half a tablet.

- a** How many antibiotic tablets would Lana's dog need if it has a mass of 30 kg?
- b** How many antibiotic tablets would be given to dogs of the following masses?
- | | |
|------------------|-------------------|
| i 10 kg | ii 15 kg |
| iii 20 kg | iv 12.5 kg |
- c** Rhys gives his dog 2.5 tablets. What is the mass of his dog?

- 16** A pharmaceutical company produces paracetamol tablets using a machine which holds a maximum of 47 kg of paracetamol powder. What is the maximum number of 60 tablet packets of 480 mg tablets that can be produced from the 47 kg?



13B Units of energy

These resources are available on your obook assess:

- **assess quiz 13B:** Test your skills with an auto-correcting multiple-choice quiz

Energy is measured in kilojoules (kJ) or calories (cal).

The international standard (SI) unit of energy is the joule (J). One joule is roughly equal to the energy required to lift a mass of 1 kg through 10 cm on the surface of Earth. The joule is a very small unit, so the kilojoule is often used instead.

$$1 \text{ kJ} = 1000 \text{ J}$$

The calorie was originally defined as the amount of energy needed to heat 1 gram of water by 1°C . It is now defined in terms of the joule as:

$$1 \text{ cal} = 4.184 \text{ J}$$

While the calorie is the unit of energy mainly used in physics and chemistry, 1 calorie is too small to be used for describing the energy content of food. Another energy unit called the Calorie or Cal (using a capital C) is used for representing the energy contained in the food we eat. The Calorie is equivalent to 1000 calories (using a lower case c) or 1 kilocalorie (kcal).

$$\text{So: } 1 \text{ Cal} = 1 \text{ kcal} = 1000 \text{ cal} = 4184 \text{ J} = 4.184 \text{ kJ}$$

Confusingly, in labels describing the energy content of food, we sometimes see references to both calories and Calories interchangeably, even though they do describe different amounts of energy. To avoid this confusion, in nearly every country, including Australia, kilojoules instead of Calories are used as the official unit for the energy value of food or drink. Some nutrition labels may show kilocalories or Calories as well as kilojoules.

EXAMPLE 13B-1 Converting between kilojoules and joules

If 1 kilojoule (kJ) is 1000 joules, convert:

a 15.36 kJ to joules

b 7584 J to kilojoules

	Solve	Think	Apply
a	Number of joules = 15.36×1000 = 15 360	$1 \text{ kJ} = 1000 \text{ J}$ The kilojoule unit is bigger than the joule unit, so we multiply by the conversion factor.	To convert kilojoules to joules, multiply the number of kilojoules by 1000.
b	Number of kilojoules = $7584 \div 1000$ = 7.584	$1000 \text{ J} = 1 \text{ kJ}$ The joule unit is smaller than the kilojoule unit, so we divide by the conversion factor.	To convert joules to kilojoules, divide the number of joules by 1000.

EXERCISE 13B Units of energy

- Convert each of the following to joules.
a 7.2 kJ **b** 12.08 kJ **c** 1.209 kJ **d** 0.64 kJ **e** 0.054 kJ
- Convert each of the following to kilojoules.
a 8200 J **b** 2503 J **c** 895 J **d** 88 J **e** 15 406 J

EXAMPLE 13B-2 Converting between kilocalories and calories

If 1 kilocalorie (kcal) is 1000 calories (cal), convert:

- a** 9.103 kcal to calories **b** 740 cal to kilocalories

	Solve	Think	Apply
a	Number of calories = 9.103×1000 = 9103	1 kcal = 1000 cal The kilocalorie unit is bigger than the calorie unit, so we multiply by the conversion factor.	To convert kilocalories to calories, multiply the number of kilocalories by 1000.
b	Number of kilocalories = $740 \div 1000$ = 0.74	1000 cal = 1 kcal The calorie unit is smaller than the kilocalorie unit, so we divide by the conversion factor.	To convert calories to kilocalories, divide the number of calories by 1000.

- Convert each of the following to calories.
a 7.254 kcal **b** 16.169 kcal **c** 0.78 kcal **d** 26.3 kcal **e** 1.008 kcal
- Convert each of the following to kilocalories.
a 22 560 cal **b** 7604 cal **c** 687 cal **d** 48 300 cal **e** 98 cal

EXAMPLE 13B-3 Converting calories to joules

Convert each of the following to joules.

- a** 38 cal **b** 4.23 kcal

	Solve	Think	Apply
a	Number of joules = 38×4.184 = 158.992 = 159 (to the nearest whole number)	1 cal = 4.184 J The calorie unit is bigger than the joule unit, so we multiply by the conversion factor.	To convert calories to joules, multiply the number of calories by 4.184.
b	Number of joules = $4.23 \times 1000 \times 4.184$ = 17 698.32 = 17 698 (to the nearest whole number)	4.23 kcal = 4.23×1000 cal Then convert to joules.	To convert kilocalories to joules, first convert kilocalories to calories and then multiply the number of calories by 4.184.

- 5** Convert each of the following to joules (to the nearest whole number).
a 256 cal **b** 5840 cal **c** 3.6 kcal **d** 20.3 kcal **e** 0.47 kcal

EXAMPLE 13B-4 Converting joules to calories

Convert each of the following to calories.

- a** 260 J **b** 18 kJ

	Solve	Think	Apply
a	Number of calories $= 260 \div 4.184$ $= 62.141 \dots$ $= 62$ (to the nearest whole number)	$4.184 \text{ J} = 1 \text{ cal}$ The joule unit is smaller than the calorie unit, so we divide by the conversion factor.	To convert joules to calories, divide the number of joules by 4.184.
b	Number of calories $= 18 \times 1000 \div 4.184$ $= 4302.103 \dots$ $= 4302$ (to the nearest whole number)	$18 \text{ kJ} = 18 \times 1000 \text{ J}$ Then convert to calories.	To convert kilojoules to calories, first convert kilojoules to joules and then divide by 4.184.

- 6** Convert each of the following to calories (to the nearest whole number).
a 580 J **b** 6923 J **c** 15.9 kJ **d** 0.65 kJ **e** 252 J

Example 13B-5 Converting between kilojoules and Calories or kilocalories

Convert each of the following energy values for food.

- a** 120 Cal to kilojoule **b** 850 kJ to Calories **c** 350 kcal to kilojoules

	Solve	Think	Apply
a	Number of kilojoules $= 120 \times 4.184$ $= 502.08$ $= 502 \text{ kJ}$ (to the nearest whole number)	For food energy values, $1 \text{ Cal} = 4.184 \text{ kJ}$. The Calorie unit is bigger than the kilojoule unit, so we multiply by the conversion factor.	To convert Calories into kilojoules, we multiply the number of Calories by 4.184. Note the difference between Calories and calories (1 Cal is equivalent to 1000 cal or 1 kcal).
b	Number of Calories $= 850 \div 4.184$ $= 203.15 \dots$ $= 203 \text{ Cal}$ (to the nearest whole number)	The kilojoule unit is smaller than the Calorie unit, so we divide by the conversion factor.	To convert kilojoules into Calories, divide the number of kilojoules by 4.184.
c	Number of kilojoules $= 350 \times 4.184$ $= 1464.4$ $= 1464 \text{ kJ}$ (to the nearest whole number)	$1 \text{ kcal} = 4.184 \text{ kJ}$. The kilocalorie unit is bigger than the kilojoule unit, so we multiply by the conversion factor.	To convert kilocalories into kilojoules, multiply the number of kilocalories by 4.184.

- 7** Convert each of the following energy values in Calories to kilojoules (to the nearest whole number).
a 30 Cal **b** 150 Cal **c** 248 Cal **d** 305 Cal **e** 650 Cal
- 8** Convert each of the following energy values in kilojoules to Calories (to the nearest whole number).
a 1000 kJ **b** 926 kJ **c** 1240 kJ **d** 20000 kJ **e** 7590 kJ
- 9** Convert each of the following energy values in kilocalories to kilojoules (to the nearest whole number).
a 40 kcal **b** 690 kcal **c** 137 kcal **d** 1050 kcal **e** 2520 kcal
- 10** Convert each of the following energy values into kilocalories (to the nearest whole number).
a 730 kJ **b** 2690 kJ **c** 9500 kJ **d** 1950 Cal **e** 3425 Cal
- 11** The estimated daily energy requirement from food for an average adult is stated as 8700 kJ. How many Calories is this?
- 12** Individual dietary requirements may be higher or lower than the average, depending on age, gender, height, weight and physical activity levels. The questions below relate to the estimated daily energy requirements for people with a low level of physical activity (for example office workers or students).
 - a** The estimated daily energy requirement for a 17-year-old male who weighs 70 kg is 12 700 kJ. Convert this energy requirement to kilocalories.
 - b** The estimated daily energy requirement for a 17-year-old female who weighs 60 kg is 2400 Calories. Convert this energy requirement to kilojoules.
 - c** The estimated daily energy requirement for a 32-year-old female who weighs 65 kg is 9200 kJ. Convert this energy requirement to Calories.
 - d** The estimated daily energy requirement for a 55-year-old male who weighs 80 kg is 2865 kilocalories. Convert this energy requirement to kilojoules.
- 13** Use this nutrition label to answer the following questions.



- a** How many Calories of energy are in each serving?
- b** How many Calories of energy are in the contents of the whole container?
- c** How many kilocalories of energy are in each serving?
- d** How many kilojoules of energy are in the contents of the whole container?

13C Food and nutrition: energy intake

These resources are available on your obook assess:

- **Investigation 13C.1:** Compare the sugar and fibre content of common fruits
- **Investigation 13C.2:** Analyse information from nutrition labels
- **assess quiz 13C:** Test your skills with an auto-correcting multiple-choice quiz

The food we eat provides our bodies with the energy they need to function. Most of that energy comes from the carbohydrates, protein, fat and dietary fibre in the food we consume. In Australia, the energy content of food and drinks is measured in kilojoules.

The average amount of energy provided by these nutrients is:

- protein 17 kJ/g
- fat 38 kJ/g
- carbohydrate 17 kJ/g
- dietary fibre 8 kJ/g

To calculate our total energy intake from a serving of food, we take the amount of each of these nutrients, multiply each by the energy it provides and add the subtotals.



EXAMPLE 13C-1 Calculating the energy intake from foods

One 265 g serve of chicken soup contains 7.7 g of protein, 13.0 g of fat, 18.8 g of carbohydrate and 4.0 g of dietary fibre. Calculate the total energy intake from eating one serve of this soup.

Solve	Think	Apply
Energy intake from protein = 7.7×17 = 130.9 kJ	Calculate the number of grams of protein $\times 17$ + the number of grams of fat $\times 38$ + the number of grams of carbohydrate $\times 17$ + the number of grams of fibre $\times 8$.	To calculate the total energy intake, multiply the amount of each nutrient, in grams, by the amount of energy it produces per gram and then add the results.
Energy intake from fat = 13.0×38 = 494.0 kJ		
Energy intake from carbohydrate = 18.8×17 = 319.6 kJ		
Energy intake from fibre = 4.0×8 = 32.0 kJ		
Total energy intake from one serving = $130.9 + 494.0 + 319.6 + 32.0$ = 976.5 kJ = 977 kJ (to the nearest kilojoule)		

EXERCISE 13C Food and nutrition: energy intake

- 1 One serve of baked beans contains 6.2 g of protein, 0.7 g of fat, 16.8 g of carbohydrate and 7.0 g of dietary fibre. Calculate the total energy intake from eating one serve of baked beans.
- 2 There is 7.6 g of protein, 4.6 g of fat, 16.5 g of carbohydrate and 2.0 g of dietary fibre per 100 g of savoury biscuits. Calculate the total energy intake from consuming 100 g of biscuits.
- 3 A 100 g serving of ice cream contains 3.3 g of protein, 4.9 g of fat, 30.6 g of carbohydrates and no fibre. Calculate the total energy in one 100 g serving.
- 4 Sea salt potato chips contain 2.2 g of protein, 6.3 g of fat, 17.8 g of carbohydrates and 1.1 g of dietary fibre per serve.
 - a Calculate the total energy in one serve of these potato chips.
 - b Convert your answer for part a to:
 - i joules
 - ii calories
 - iii kilocalories.
 - c If the serving size of the potato chips is 28 g, calculate the amount of energy provided by eating 60 g of chips, in:
 - i kilojoules
 - ii joules
 - iii calories
 - iv kilocalories.
 - d Calculate the energy intake per 100 g of chips in:
 - i kilojoules
 - ii joules
 - iii calories
 - iv kilocalories
- 5 Rice contains 7.0 g of protein, 0.4 g of fat, 79.8 g of carbohydrates and 1.4 g of dietary fibre per 100 g.
 - a What is the total energy intake from the consumption of 100 g of rice?
 - b The average size of a serving of rice with a meal is 60 g. How much energy is provided by one serving?
 - c Convert your answer for part b to kilocalories.
 - d Show that 1 kcal = 4.184 kJ.
 - e Complete the following table using the conversion shown in part d.

Nutrient	Energy kJ/g	Energy kcal/g
Protein	17	
Fat	38	
Carbohydrate	17	
Dietary fibre	8	

- f Using the table from part e, calculate the total energy provided by 100 g of rice, in kilocalories.
- g What is the energy intake, in kilocalories, from eating a 60 g serve of rice? Compare your answer with your answer to part c.
- 6 Milk contains 3.4 g of protein, 3.4 g of fat, 4.8 g of carbohydrate and no fibre per 100 mL.
 - a Calculate the total energy intake per 100 mL of milk.
 - b Health experts recommend that women in the 19–50 age group should have a minimum of $2\frac{1}{2}$ servings of milk (or other approved dairy food) per day. If one serving is 250 mL, what would be the energy provided by $2\frac{1}{2}$ servings of milk?
 - c Convert your answer for part b to kilocalories.



- 7** Every 100 g of chocolate biscuits contains about 5.2 g of protein, 30.1 g of fat, and 59.9 g of carbohydrate. Chocolate biscuits contain almost no fibre.
- What is the total energy intake from the consumption of 100 g of chocolate biscuits?
 - If one chocolate biscuit has a mass of 13.3 g, what would be the energy intake from eating two biscuits?
 - Convert your answer for part **b** to kilocalories.
 - Using the table you completed for question **5e**, what is the total energy intake, in kilocalories, from eating 100 g of chocolate biscuits?
 - If one chocolate biscuit has a mass of 13.3 g, what is the energy intake, in kilocalories, from eating two biscuits? Compare your answer with your answer for part **c**.

Table 13-1 below shows a list of common foods, their serving sizes and the average number of kilojoules of energy in each serving. *Note:* tablespoon is written as tbs, and teaspoon is written as tsp.

Table 13-1: Energy intake per serve of common foods

Food	Serving	Energy (kJ)
Carbohydrates		
Grain bread	2 slices	830
Cornflakes	1 cup	455
Plain boiled pasta	1 cup	670
Instant porridge	1 sachet	595
Natural muesli	$\frac{1}{3}$ cup	510
White bread	2 slices	753
Dairy foods		
Skimmed milk	1 cup	370
Reduced fat milk	1 cup	530
Regular milk	1 cup	730
Low-fat yoghurt	1 tub (200 g)	650
Cheddar cheese	1 slice	345
Meat, poultry and fish		
Roast lamb	1 slice	350
Sausage (grilled)	1	850
Steak (grilled)	140 g	1040
Beef stir-fry	$\frac{1}{2}$ cup	965
Skinless baked chicken	1 thigh	620
Egg (boiled)	1 medium	310
Takeaway food		
Crumbed fish (fried)	1 fillet	1380
Hot chips	140 g	1520
Meat pie	1	1750
Sausage roll	1	1720
Sushi	3 long rolls	1760
Pizza	3 slices	2750
Burger	1	2290

Food	Serving	Energy (kJ)
Fruit and vegetables		
Leafy salad (no dressing)	1 cup	65
Broccoli	$\frac{1}{2}$ cup	65
Carrot	$\frac{1}{2}$ cup	110
Peas	$\frac{1}{2}$ cup	220
Corn	1 cobette	385
Potato (boiled)	1 small	295
Banana	1	420
Orange	1	230
Apple	1	330
Beverages		
Flavoured milk	600 mL	1980
Water	600 mL	0
Regular soft drink	600 mL	895
Fruit juice	1 cup	420
Chocolate powder	1 tbs	110
Beer (full strength)	375 mL	575
White wine	200 mL	550
Red wine	200 mL	390
Spreads and sauces		
Butter	1 tbs	580
Peanut butter	1 tbs	630
Vegemite	1 tsp	45
Jam	2 tsp	150
Margarine	1 tbs	500
Honey	2 tsp	190
Tomato pasta sauce	250 g	570
Tomato sauce	1 tbs	90

EXAMPLE 13C-2 Calculating the total energy intake from meals

Jack consumes the following meals on one day.

Breakfast: 1 cup of orange juice, 1 cup of cornflakes, 2 cups of regular milk, 3 slices of white bread with 3 tbs of butter and 3 tbs of jam

Lunch: 2 sausage rolls with 2 tbs of tomato sauce, 1 flavoured milk (600 mL)

Dinner: 2 slices of roast lamb with 1 small boiled potato, $\frac{1}{2}$ cup of peas and $\frac{1}{2}$ cup of carrots

Snacks: 2 bananas

Use the information in Table 13-1 to help you calculate Jack's total energy intake for the day. Express the energy intake for each food item to the nearest kilojoule.

Solve	Think	Apply
1 cup of orange juice = 420 kJ 1 cup of cornflakes = 455 kJ 2 cups of regular milk = $2 \times 730 = 1460$ kJ 3 slices of white bread = $\frac{753}{2} \times 3 = 1130$ kJ 3 tbs of butter = $3 \times 580 = 1740$ kJ 3 tsp of jam = $\frac{150}{2} \times 3 = 225$ kJ 2 sausage rolls = $2 \times 1720 = 3440$ kJ 2 tbs of tomato sauce = $2 \times 90 = 180$ kJ 1 flavoured milk (600 mL) = 1980 kJ 2 slices of roast lamb = $2 \times 350 = 700$ kJ 1 small potato = 295 kJ $\frac{1}{2}$ cup of peas = 220 kJ $\frac{1}{2}$ cup of carrots = 110 kJ 2 bananas = $2 \times 420 = 840$ kJ Total energy intake $\approx 13\,195$ kJ	Find the required information for each food in the table and adjust for the quantity consumed, where necessary.	Use the table to find the energy intake from each food and add to find the total energy intake for the day.

- 8 Annabel consumes the following meals during one day. Use the information from Table 13-1 to help you calculate her total energy intake for the day.

Breakfast: 1 sachet of instant porridge, 1 cup of reduced fat milk, 1 slice of grain bread with $\frac{1}{2}$ tbs of margarine, 1 medium boiled egg, 1 banana

Lunch: 1 baked chicken thigh (skin removed), 1 cup of leafy salad (no dressing), 1 apple

Dinner: 1 cup of beef stir-fry, $\frac{1}{2}$ cup of broccoli, 1 corn cobette, 1 glass of red wine (200 mL)

- 9 Josh consumes the following meals on one day. Use the information from Table 13-1 to help you calculate his total energy intake for the day.

Breakfast: 2 grilled sausages, 2 tbs of tomato sauce, 2 slices of white bread, 1 tbs of margarine, 1 tbs of peanut butter, 1 tsp of vegemite, 2 tbs of chocolate powder in 1 cup of regular milk

Lunch: 1 burger, 1 regular soft drink (600 mL)

Dinner: 2 fried fillets of crumbed fish, 1 serve of hot chips, 2 glasses (each 375 mL) of full strength beer



13D Energy expenditure

These resources are available on your obook assess:

- **Video tutorial 13D.1:** Watch and listen to an explanation of Example 13D-1
- **Video tutorial 13D.2:** Watch and listen to an explanation of Example 13D-5
- **Video tutorial 13D.3:** Watch and listen to an explanation of Example 13D-6
- **assess quiz 13D:** Test your skills with an auto-correcting multiple-choice quiz

energy out

amount of energy the body uses to sustain basic functions and perform normal daily physical activities

Energy out refers to the amount of energy the body uses to sustain its basic physiological functions (breathing, digestion, maintaining body temperature, muscle activity, etc.) and to perform normal daily physical activities. The amount of energy the body needs to perform these activities is influenced by a number of factors including gender, age, weight, height and level of fitness.

Table 13-2 below shows the estimated number of kilojoules burned per kilogram of body weight for every 30 minutes of common activities.

Table 13-2: Energy burned per kilogram of weight in a 30-minute time period

Activity	kJ/kg/30 min
Aerobics	10.8
Basketball	20.3
Cycling (15 km/h)	11.7
Dancing	9.1
Golf	9.3
Hiking (with a 10 kg load)	18.2
Jogging	17.8
Running	27.9
Tennis (singles)	14.7
Tennis (doubles)	10.0
Walking	8.1
Weight training	17.5



Please note that the figures above are estimates only, for individuals of average weight, average height and average fitness.

EXAMPLE 13D-1 Estimating the energy burned to perform activities

Use Table 13-2 above to estimate the energy burned by the following people's bodies during the given activities.

- a** A 60 kg girl walks for 30 minutes.
- b** A 65 kg woman does aerobics for 50 minutes.
- c** A 70 kg boy plays basketball for 20 minutes.



	Solve	Think	Apply
a	Energy out = $8.1 \times 60 \times 1$ = 486 kJ	From Table 13-2, find the rate of energy use, substitute weight (60 kg) and the number of 30-minute time intervals of activity (1).	Choose the appropriate rate from the table, then use the formula: $E = r \times W \times t$
b	Energy out = $10.8 \times 65 \times \frac{50}{30}$ = 1170 kJ	From Table 13-2, find the rate of energy use, substitute weight (65 kg) and the number of 30-minute time intervals of activity ($\frac{50}{30}$).	where: E = energy burned r = rate of energy used (in kJ/kg/30 min) W = weight (in kilograms)
c	Energy out = $20.3 \times 70 \times \frac{20}{30}$ = 947 kJ (to the nearest kilojoule)	From Table 13-2, find the rate of energy use, substitute weight (70 kg) and the number of 30-minute time intervals of activity ($\frac{20}{30}$).	t = number of 30-minute time intervals of the activity

EXERCISE 13D Energy expenditure

- 1 Use Table 13-2 to estimate the energy that is burned by the following people's bodies during the given activities.

- a A 76 kg boy runs for 30 minutes.
- b A 55 kg girl takes part in a dance class for 45 minutes.
- c A 62 kg woman cycles for 25 minutes.
- d A 84 kg man plays golf for 4 hours.
- e A 96 kg man walks for 40 minutes.
- f A 58 kg girl jogs for 20 minutes.
- g A 92 kg boy does weight training for 40 minutes.



Another method for calculating the amount of energy used in an activity involves using the concepts of basal metabolic rate and a physical activity factor.

The **basal metabolic rate (BMR)** of a person refers to the minimum amount of energy needed to sustain basic bodily functions (that is, to keep the person alive) for a 24-hour period. An estimate can be calculated using either the Schofield formula or the Harris–Benedict formula, discussed later (pages 496 and 497).

The **physical activity level (PAL)** is the result of the comparison:

$$\frac{\text{total energy needed for the activity}}{\text{BMR}}$$

For example, a physical activity factor of 1.7 indicates that the activity requires 1.7 times more energy than the BMR.

The energy output needed is then given by:

$$\text{Energy out} = \text{basal metabolic rate} \times \text{physical activity level}$$

or:

$$\text{Energy out} = \text{BMR} \times \text{PAL}$$



basal metabolic rate (BMR)

minimum amount of energy needed to sustain basic bodily functions for a 24-hour period, measured in kJ/day

physical activity level (PAL)

the total energy needed for an activity compared to the BMR

Table 13-3 below shows how to calculate the estimated BMR for females and males of a particular age using their weight. The formula used is known as the Schofield formula.

Table 13-3: The Schofield formula for using weight (W kg) to estimate BMR, in kJ/day

Age	BMR (kJ/day)	
	Females	Males
<3	$244 \times W - 130$	$249 \times W - 127$
3–10	$85 \times W + 2033$	$95 \times W + 2110$
11–18	$56 \times W + 2898$	$74 \times W + 2754$
19–30	$62 \times W + 2036$	$63 \times W + 2896$
31–60	$34 \times W + 3538$	$48 \times W + 3653$
>60	$38 \times W + 2755$	$49 \times W + 2459$

EXAMPLE 13D-2 Calculating BMR using the Schofield formula

Use the Schofield formula, shown in Table 13-3, to calculate the basal metabolic rate (BMR) for:

- a** a 17-year-old female who weighs 58 kg
- b** a 35-year-old man who weighs 92 kg.

	Solve	Think	Apply
a	$\text{BMR} = 56 \times 58 + 2898$ $= 6146 \text{ kJ/day}$	The age of 17 is in the 11–18 age group. From the information in Table 13-3, for females, the appropriate formula to use is: $\text{BMR} = 56 \times W + 2898$	Select the formula for the appropriate gender and age group, and substitute the weight.
b	$\text{BMR} = 48 \times 92 + 3653$ $= 8069 \text{ kJ/day}$	The age of 35 is in the 31–60 age group. Using the information from Table 13-3, for males, the appropriate formula to use is: $\text{BMR} = 48 \times W + 3653$	

2 Use the Schofield formula to calculate the basal metabolic rate (BMR) for:

- a** a 24-year-old woman who weighs 55 kg
- b** a 16-year-old boy who weighs 73 kg
- c** a two-year-old girl who weighs 12.1 kg
- d** a 70-year-old man who weighs 72 kg.

The **Harris–Benedict formula** for estimating BMR, in kJ/day, from weight in kilograms, height in centimetres and age in years is shown below.

For females:

$$\text{BMR (kJ/day)} = 2783 + (40.0 \times \text{weight in kg}) + (7.7 \times \text{height in cm}) - (19.6 \times \text{age in years})$$

For males:

$$\text{BMR (kJ/day)} = 278 + (57.5 \times \text{weight in kg}) + (20.9 \times \text{height in cm}) - (28.2 \times \text{age in years})$$

EXAMPLE 13D-3 Calculating BMR using the Harris-Benedict formula

Use the Harris-Benedict formula to calculate the basal metabolic rate (BMR) for:

- a a 17-year-old boy who weighs 82 kg and whose height is 178 cm
- b a 16-year-old girl who weighs 59 kg and is 164 cm tall.

	Solve	Think
a	$\text{BMR} = 278 + (57.5 \times 82) + (20.9 \times 178) - (28.2 \times 17)$ $= 8234 \text{ kJ/day (to the nearest kJ/day)}$	Into the formula, substitute: Weight = 82 kg, Height = 178 cm, Age = 17
b	$\text{BMR} = 2783 + (40.0 \times 59) + (7.7 \times 164) - (19.6 \times 16)$ $= 6092 \text{ kJ/day (to the nearest kJ/day)}$	Into the formula, substitute: Weight = 59 kg, Height = 164 cm, Age = 16
Apply		
Substitute the values for weight, height and age into the appropriate formula.		

- 3 Use the Harris-Benedict formula to calculate the basal metabolic rate (BMR) for:
 - a an 18-year-old boy who weighs 88 kg and whose height is 183 cm
 - b a 15-year-old girl who weighs 55 kg and is 160 cm tall
 - c a 37-year-old woman who weighs 56 kg and whose height is 1.64 m
 - d a 70-year-old man who weighs 82 kg and is 1.73 m tall.
- 4 Use Table 13-2 on page 494 to help you answer the following questions.
 - a Estimate the energy used by a 52 kg girl who plays tennis doubles for 45 minutes.
 - b How much more energy would the girl have used if she had played singles, instead of doubles, for the same length of time?
- 5 Ben (86 kg) and his brother Noah (93 kg) go on a 75-minute run together. Who burns the most energy during the run and by how much?

Table 13-4 below shows the physical activity level (PAL) for different lifestyles.

Table 13-4: Energy expenditure for different lifestyles

	Lifestyle	Examples	PAL
1	Little to no exercise, sedentary, lying (on bed)	Hospital patient, or infirm person with limited ability to move	1.2
2	Little physical activity at work or leisure	Office worker	1.45
3	Seated work with some occasional walking and standing, no strenuous leisure activity	Laboratory assistants, drivers, students, assembly line workers	1.65
4	Predominantly standing or walking	Homemakers, salespeople, waiters, mechanics	1.85
5	Heavy occupational work or highly active leisure	Construction workers, fitness trainers	2.2
6	Significant amounts of exercise in addition to 2, 3, or 4 above		Add extra PAL units



EXAMPLE 13D-4 Estimating the energy output required to maintain lifestyle

A 17-year-old male student who weighs 82 kg does little exercise.

- a** Use the Schofield formula to estimate the student's BMR.
- b** Use Table 13-4 (page 497) and the following formula to estimate the energy required by the student to maintain this lifestyle:

$$\text{Energy out} = \text{BMR} \times \text{PAL}$$

	Solve	Think	Apply
a	$\text{BMR} = 74 \times 82 + 2754$ $= 8822 \text{ kJ/day}$	Since 17 is in the 11–18 age group, the appropriate Schofield formula for males is: $\text{BMR} = 74 \times W + 2754.$	Select the formula for the appropriate gender and age and substitute the weight.
b	Energy out $= 8822 \times 1.65$ $\approx 14\,556 \text{ kJ/day (to the nearest kJ/day)}$	Multiply the BMR by the physical activity level for the lifestyle. From Table 13-4, this person's lifestyle matches lifestyle 2. So $\text{PAL} = 1.65$.	$\text{Energy out} = \text{BMR} \times \text{PAL}$

Note: it is important to remember that the calculations above are broad general guidelines only. Energy output depends on a number of variables including age, gender, body size and composition, and the amount and intensity of physical activity or exercise. The lifestyles and corresponding PALs in Table 13-4 are generalisations only. Each calculation needs to be adjusted to suit the characteristics of each individual. The energy output estimated above corresponds to the amount of energy that must be consumed, by eating food, to maintain the current weight of the person (that is, the amount of food that must be consumed to maintain the balance between energy in and energy out for the nominated lifestyle).

- 6** A 68-year-old woman who weighs 48 kg has a totally sedentary lifestyle.
 - a** Use the Schofield formula to estimate the woman's BMR.
 - b** Using Table 13-4, estimate the energy output this woman requires to maintain her lifestyle.
- 7** A 36-year-old male construction labourer weighs 94 kg and is 184 cm tall.
 - a** Use the Harris–Benedict formula to estimate the labourer's BMR.
 - b** Estimate the energy the labourer requires to maintain his lifestyle.
- 8** A 28-year-old woman, who weighs 57 kg and is 160 cm tall, works as a salesperson and stands for most of the day.
 - a** Use the Harris–Benedict formula to estimate the woman's BMR.
 - b** Estimate the energy the woman requires to maintain her lifestyle.
- 9** A 24-year-old male truck driver weighs 83 kg.
 - a** Use the Schofield formula to estimate the truck driver's BMR.
 - b** Estimate the energy required by the truck driver to maintain this lifestyle.



- 10** A 17-year-old female office worker weighs 55 kg and does little exercise.
- Use the Schofield formula to estimate the office worker's BMR.
 - Estimate the energy the office worker requires to maintain her lifestyle.

The table below shows the physical activity level (PAL) for different activities and sports.

Table 13-5 Energy expenditure for various activities

Activity/Sport	PAL
Walking 4 km/h	2.9
Walking 7 km/h	5.2
Running 9 km/h	8.6
Running 14 km/h	12.8
Cycling 16 km/h	4.0
Swimming 1.2 km/h	3.3
Swimming 3 km/h	10.0
Soccer	10.0
Cross-country skiing	7.5
Table tennis	5.4
Volleyball	6.5



EXAMPLE 13D-5 Estimating the energy required to perform individual activities

A 35-year-old woman who weighs 52 kg plays table tennis for 2 hours.

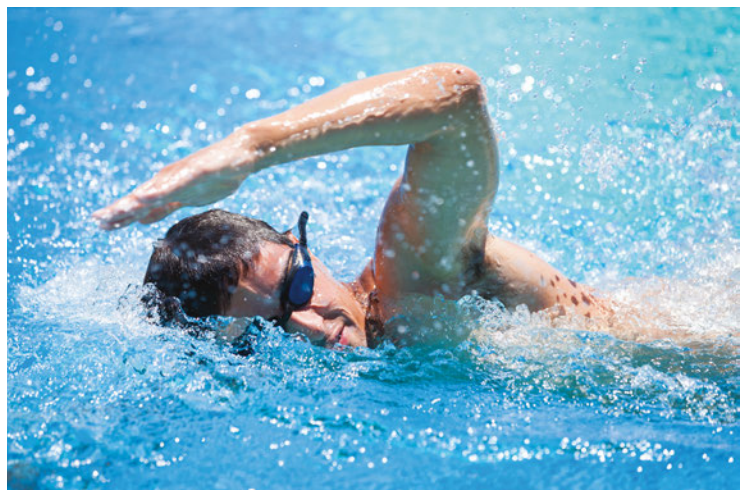
- Use the Schofield formula to estimate the woman's BMR.
- Use the information from Table 13-5 and the following formula to calculate the amount of energy required by the woman to perform the exercise described:

$$\text{Energy out} = (\text{duration of activity in hours}) \times (\text{BMR in kJ/hour}) \times \text{PAL}$$

	Solve	Think	Apply
a	$\text{BMR} = 34 \times 52 + 3538$ $= 5306 \text{ kJ/day}$	Use the information for females from Table 13-3 on page 496. The age of 35 is in the 31–60 age group. So the appropriate formula is: $\text{BMR} = 34 \times W + 3538$ where $W = 52$.	Select the Schofield formula for the appropriate gender and age group and substitute the weight.
b	Energy out $= 2 \times \frac{5306}{24} \times 5.4$ $\approx 2388 \text{ kJ (to the nearest kilojoule)}$	Activity duration = 2 h $\text{BMR} = \frac{5306}{24} \text{ kJ/h}$ $\text{PAL} = 5.4 \text{ (from Table 13-5)}$	Substitute the appropriate values into the formula: $\text{Energy out} = (\text{activity duration in hours}) \times (\text{BMR in kJ/hour}) \times \text{PAL}.$ Note that each formula for BMR gives the result in kJ/day. The formula for this calculation requires the BMR in kJ/hour.

- 11** A 38-year-old man who weighs 88 kg plays volleyball for $1\frac{1}{2}$ hours.
- Use the Schofield formula to estimate the man's BMR.
 - Use the information from Table 13-5 and the following formula to calculate the amount of energy required by the man to perform the exercise described:

$$\text{Energy out} = (\text{duration of activity in hours}) \times (\text{BMR in kJ/hour}) \times \text{PAL}$$
- 12** A 27-year-old man, who weighs 76 kg and is 174 cm tall, swims at 3 km/h for 45 minutes.
- Use the Harris–Benedict formula to estimate the man's BMR.
 - Calculate the amount of energy required for the man to perform the exercise described.
- 13** A 48-year-old man, who weighs 86 kg, runs at 9 km/h for 80 minutes.
- Use the Schofield formula to estimate the man's BMR.
 - Calculate the amount of energy required for the man to perform the exercise described.
- 14** A 16-year-old girl, who weighs 47 kg and is 157 cm tall, goes cross-country skiing for 3 hours.
- Use the Harris–Benedict formula to estimate the girl's BMR.
 - Calculate the amount of energy required for the girl to perform this exercise.
- 15** A 52-year-old woman, who weighs 58 kg, walks at 4 km/h for $2\frac{1}{2}$ hours.
- Use the Schofield formula to estimate the woman's BMR.
 - Calculate the amount of energy required for the woman to perform this exercise.



EXAMPLE 13D-6 Estimating the time needed to burn kilojoules

A 25-year-old woman weighs 62 kg.

- Use the Schofield formula to estimate the woman's BMR.
- How long would it take the woman to burn 1500 kJ of energy by cycling at 16 km/h?

	Solve	Think	Apply
a	$\text{BMR} = 62 \times 62 + 2036$ $= 5880 \text{ kJ/day}$	Use the information for females from Table 13-3 on page 496. The age of 25 is in the 19–30 age group. So the appropriate formula is: $\text{BMR} = 62 \times W + 2036$ where $W = 62$.	Select the Schofield formula for the appropriate gender and age group, and substitute the weight.
b	$\text{Energy out} = (\text{activity duration in hours}) \times (\text{BMR in kJ/hour}) \times \text{PAL}$ Let T = activity duration, in hours. $1500 = T \times \frac{5880}{24} \times 4.0$ $1500 = T \times 980$ $T = \frac{1500}{980}$ $= 1.5306\dots \text{ h}$ $= 1 \text{ h } 32 \text{ min (to the nearest minute)}$	$\text{Energy out} = 1500 \text{ kJ}$ $\text{BMR} = 5880 \text{ kJ/day} = \frac{5880}{24} \text{ kJ/h}$ $\text{PAL} = 4.0 \text{ (from Table 13-5)}$ Multiply the number of hours by 60 to convert to minutes.	Substitute the relevant values into the formula: $\text{Energy out} = (\text{activity duration in hours}) \times (\text{BMR in kJ/hour}) \times \text{PAL}$ Solve the resulting equation.

- 16** A 17-year-old girl weighs 51 kg.
- Use the Schofield formula to estimate the girl's BMR.
 - How long would it take the girl to burn 1500 kJ of energy playing soccer?
- 17** A 42-year-old man weighs 86 kg.
- Use the Schofield formula to estimate the man's BMR.
 - How long would it take the man to burn 1800 kJ of energy by walking at 7 km/h?
- 18** A 35-year-old woman weighs 67 kg.
- Use the Schofield formula to estimate the woman's BMR.
 - How long would it take the woman to burn 1000 kJ of energy playing table tennis?
- 19** A 65-year-old man weighs 76 kg.
- Use the Schofield formula to estimate the man's BMR.
 - How long would it take the man to burn 1200 kJ of energy by swimming at 1.2 km/h?
- 20** A 23-year-old woman weighs 60 kg.
- Use the Schofield formula to estimate the woman's BMR.
 - How long would it take the woman to burn 1000 kJ of energy by running at 9 km/h?



- 21** Ben consumes the following meals in one day.
- Breakfast:* $\frac{1}{3}$ cup of natural muesli, 1 cup of regular milk, 3 slices of grain bread, 3 tbs of margarine, 1 grilled sausage, 1 boiled egg, 1 orange
- Lunch:* 4 slices of pizza, 1 flavoured milk (600 mL), 1 banana
- Dinner:* a 140 g piece of grilled steak, 1 serve of hot chips, $\frac{1}{2}$ cup of carrots, $\frac{1}{2}$ cup of peas, glass of water
- Snacks:* 1 orange, 1 apple, 1 tub of low-fat yoghurt, 2 slices of cheddar cheese
- Using the information in Table 13-1 on page 492, calculate Ben's total energy intake on this day.
 - In order to stay the same weight we need to consume about the same amount of energy as we burn. The average energy intake recommended to maintain a normal lifestyle for a healthy adult is 8700 kJ/day. What percentage of the average daily intake for an adult is Ben's energy intake?
 - What could Ben do to maintain his body weight?
 - Ben is 28 years old and weighs 98 kg.
 - Use the Schofield formula to estimate Ben's BMR.
 - For how long would Ben need to run at 9 km/h to burn the energy he consumed on the day described?



13E Energy consumption

These resources are available on your obook assess:

- **assess quiz 13E:** Test your skills with an auto-correcting multiple-choice quiz

1010

power rating

rate at which an appliance uses electricity, measured in watts (W) or kilowatts (kW)

energy

consumption

energy used by an appliance, measured in kilowatt-hours (kWh)

Electricity is a form of energy used in homes and businesses for room heating and cooling, lighting, cooking, water heating and running machinery. Domestic users are usually charged for their household electricity by way of a quarterly bill. Electricity bills are based on the cost of getting the electricity to the user – that is, the cost of generating the electricity and the cost of supplying and maintaining the poles and wires. This is often referred to as the ‘electricity service availability charge’ or the ‘supply charge’. The users are also charged for the amount of electricity they use.

All electrical appliances have a **power rating**. This is the rate at which the appliance uses electricity. Power is measured in units called watts (W). One kilowatt (kW) = 1000 W.

A satisfactory unit for measuring the **energy consumption** of an appliance is the kilowatt-hour (kWh or kW-h). This is the energy used by a one kilowatt appliance in one hour. The energy consumption of an appliance can be calculated using:

$$\text{Energy (kWh)} = \text{power (kW)} \times \text{time (h)}$$

EXERCISE 13E Energy consumption

- 1 Use the sample electricity account on the opposite page to answer these questions.
 - a
 - i What is the supply period for this bill?
 - ii How many days is the supply period?
 - b The electricity service availability charge is an amount electricity users pay for having electricity available to their properties (that is, the cost of generating the electricity and supplying and maintaining the poles and wires).
 - i What is the daily electricity service availability charge?
 - ii Write the total electricity service availability charge for this account.
 - c The usage charge is what we pay for the amount of electricity we use.
 - i What was the total amount of electricity used by the Smith household for the billing period shown on the bill?
 - ii What is their average daily usage?
 - d The cost of usage is broken down into peak energy rate, shoulder energy charge and off-peak energy rate. What percentage of usage occurred in the:
 - i shoulder period?
 - ii off-peak period?
 - e
 - i What is the peak energy rate per kilowatt-hour?
 - ii What is the difference between the peak and off-peak rates per kilowatt-hour?
 - f
 - i What is the total energy usage charge for this quarter?
 - ii What is the total cost (availability and usage charges) for this quarter, including the 5% discount and GST?
 - iii What is the average daily cost?

FIRST COMBINATION *energy*

Mr P Smith
71 Winter Grove
SUMMERTON

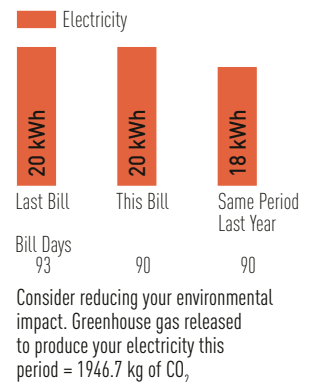
Customer Number	0123456
Due Date	17 June 2018
Amount Payable	\$688.22

Quarterly Electricity Account

LOCATION: 71 Winter Grove, SUMMERTON

Previous Amount Payable	711.45
Payment Received — Thank You	-711.45 Cr
Electricity (23/02/2018 to 23/05/2018)	\$658.58
Electricity Discount 5% (23/02/2018 to 23/05/2018)	-32.93 Cr
Subtotal of Charges before GST	625.65
Total GST Payable 10%	62.57
Total Charges including GST	688.22
Total Amount Payable	\$688.22

AVERAGE DAILY USAGE



METER ID	THIS READING	-	LAST READING	=	ENERGY USED	x	RATE	=	COST
Peak Energy Rate — Contract (23/02/2018 to 23/05/2018)									
EDX009745/001	595.6		0.0		595.6 kWh		52.50c		\$312.69
Shoulder Energy Charge — Contract (23/02/2018 to 23/05/2018)									
EDX009745/002	950.2		0.0		950.2 kWh		24.40c		\$231.85
Off-Peak (Night Rate) Energy Rate — Contract (23/02/2018 to 23/05/2018)									
EDX009745/003	290.7		0.0		290.7 kWh		12.76c		\$37.09
Electricity Service Availability Charge			90 days				85.5c/day		\$76.95
Total Electricity Before GST					1836.5 kWh				\$658.58

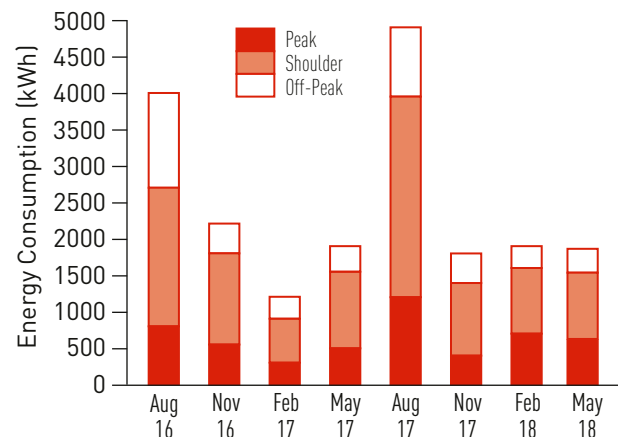
PowerSmart Home Electricity Usage Summary

Supply Period:
22 February 2018 to 23 May 2018 — 90 days

USAGE BREAKDOWN

Peak	596 kWh	32.44%
Shoulder	950 kWh	51.71%
Off-Peak	291 kWh	15.84%
TOTAL ENERGY	1,837 kWh	

ELECTRICITY USAGE COMPARISON



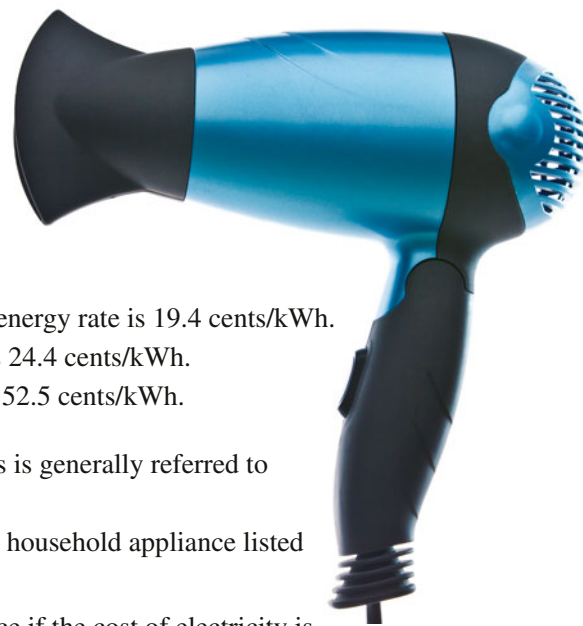
- g** Assuming the same fixed and usage charges, was this bill smaller or larger than:
- i** the previous bill? **ii** the bill for the same period last year?
- h** Consider the bill for the same period last year.
- i** What was the average daily usage?
 - ii** Calculate the total amount of electricity used for that period.
- i** Consider the electricity usage comparison. In which quarter was the:
- i** most energy consumed? **ii** least shoulder rate energy used?
 - iii** most off-peak rate energy used?

EXAMPLE 13E-1 Calculating the cost of running an appliance

Calculate the cost of running a 300 W television for 6 hours if the domestic rate is 47.77 cents/kWh.

Solve	Think	Apply
$\text{Energy used} = 0.3 \times 6$ $= 1.8 \text{ kWh}$ $\text{Cost} = 1.8 \times 0.4777$ $= \$0.86$	$300 \text{ W} = 0.3 \text{ kW}$ $47.77 \text{ cents} = \$0.4777$	$\text{Energy (kWh)} = \text{power (kW)} \times \text{time (h)}$ and: $\text{Cost} = \text{energy used (kWh)} \times \text{rate (\$/kWh)}$

- 2** Complete the following to find the cost of running a 2400 W heater for 5 hours per day for 60 days if the energy rate is 39.95 cents/kWh.
- Energy used = ____ (kW) $\times$ ____ (number of hours) = ____ kWh
 Cost = ____ (kWh) $\times$ \$0.3995 = \$____
- 3** Calculate the cost of running each of the following appliances.
- a** An 8000 W electric oven is used for 3 hours. The domestic energy rate is 47.77 cents/kWh.
 - b** A 1500 W hair dryer is used for 10 minutes every day for 30 days. The energy rate is 19.4 cents/kWh.
 - c** A 1200 W iron is used for 2 hours per day for 5 days. The energy rate is 24.4 cents/kWh.
 - d** A 100 W light globe is left on for 5 hours per day for a year. The rate is 52.5 cents/kWh.
- 4** Many appliances consume electricity even when they are not operating. This is generally referred to as stand-by power consumption.
- i** Calculate the annual energy consumption in kilowatt-hours for each household appliance listed below if it is left on stand-by for 24 hours a day for a year.
 - ii** Determine the annual cost of stand-by energy used for each appliance if the cost of electricity is 26 cents/kWh.
- a** Cordless phone with an average stand-by power consumption of 3 W
 - b** Television with an average stand-by power consumption of 10 W
 - c** Computer monitor with an average stand-by power consumption of 5 W
 - d** Clock radio with an average stand-by power consumption of 4 W
 - e** Personal computer with an average stand-by power consumption of 2 W
- 5**
- a** Calculate the cost of running a 1400 W pool filter for 2 hours per day for a year if the peak rate is 47.77 cents/kWh.
 - b** How much could be saved by programming the filter to operate in the off-peak period of the day when the cost is 19.4 cents/kWh?



- 6** What is the difference in the annual running costs for the two models of dishwasher in each of the following pairs of washers, given that the energy cost is 39.65 cents/kWh?
- a** *Gold DL10*: energy consumption of 230 kWh/year
and: *Gold DL20*: energy consumption of 275 kWh/year
- b** *Speedy G123*: energy consumption of 231 kWh/year
and: *Speedy G456*: energy consumption of 324 kWh/year
- 7** What is the difference in the annual running costs for the two models of television in each of the following pairs of televisions, given that the energy cost is 42.84 cents/kWh?
- a** *Sonic LED 138.8 cm screen*: energy consumption of 261 kWh/year
and: *Sonic LED 98 cm screen*: energy consumption of 141 kWh/year
- b** *Yanso LCD 66 cm screen*: energy consumption of 241 kWh/year
and: *Yanso LCD 100 cm screen*: energy consumption of 561 kWh/year
- 8** The energy consumption of four 450 L refrigerators is shown in this table.
- a** Calculate the annual cost of running each of the refrigerators if the cost of electricity is \$0.47/kWh.
- b** How much can be saved per year using a refrigerator with a rating of $3\frac{1}{2}$ stars compared with a refrigerator with each of the following ratings?
- i** 3 stars
- ii** $2\frac{1}{2}$ stars
- iii** $1\frac{1}{2}$ stars
- 9 a** A house uses 12 standard 100 W light globes. Assuming that, on average, each globe is used for 3 h/day, calculate the annual cost of the lighting for the house if electricity costs 39 cents/kWh.
- b** How much could be saved each year by replacing the standard globes with energy-efficient 20 W globes?

Brand	Energy rating	Energy consumption (kWh/year)
A	$3\frac{1}{2}$ stars	352
B	3 stars	396
C	$2\frac{1}{2}$ stars	484
D	$1\frac{1}{2}$ stars	586

- 10** Power is the rate at which energy is used; that is, $\text{Power} = \frac{\text{energy}}{\text{time}}$.

The international standard (SI) unit for power is the watt (W), which is defined as:

1 watt = 1 joule/second, where the amount of energy is measured in joules (J)

So: 1 kilowatt (kW) = 1000 W = 1000 J/s

Investigate the rate of energy consumption (the power rating) of these common appliances and physical activities:

a an electric kettle	b fast running
c playing tennis	d an electric room heater
e doing aerobics	f a car engine
g swimming or cycling	h a laptop computer

Hint: the power ratings of electrical appliances can be found directly from labels on the appliances or from power rating tables. For the physical activities, convert the rate of energy use to joules/second (watts).



CHAPTER 13 REVIEW UNITS OF ENERGY AND MASS

You should be able to:

- ✓ convert between the commonly used metric units for mass and energy
- ✓ calculate the total energy intake from food based on the amount of protein, fat, carbohydrate and dietary fibre it contains
- ✓ calculate the total energy intake consumed in meals
- ✓ given the length of time a physical activity is performed, estimate the number of kilojoules burned per kilogram of body weight
- ✓ use the Schofield formula and the Harris–Benedict formula to calculate a person’s basal metabolic rate (BMR)
- ✓ use basal metabolic rate and physical activity level to estimate the energy output required to maintain a lifestyle
- ✓ use basal metabolic rate and physical activity level to estimate the energy required to perform individual activities
- ✓ estimate the time required to burn kilojoules
- ✓ interpret information about a household electricity bill
- ✓ interpret the energy ratings of household appliances and calculate the cost of running these appliances.

Create a summary overview of this chapter. Include your own descriptions of key terms and strategies.

REVIEW MULTIPLE-CHOICE QUESTIONS

- 13A** 1 Which of these mass measurements is equivalent to 5.06 g?
A 0.005 06 mg B 5060 mg C 0.0506 mg D 506 mg
- 13A** 2 A patient is prescribed 480 mg of a painkiller. The painkiller is supplied in a syrup with a concentration of 16 mg/3 mL. The amount of this syrup that should be given to the patient is:
A 9 mL B 90 mL C 2560 mL D 10 mL
- 13B** 3 Which of the following is equivalent to 570 joules?
A 0.057 kJ B 0.57 kJ C 57 000 kJ D 570 000 kJ
- 13B** 4 To the nearest joule, which of the following is equivalent to 1.4 kcal?
A 1.4 kJ B 0.586 kJ C 5.858 kJ D 0.335 kJ
- 13C** 5 Two slices of wholegrain bread contain 8.6 g of protein, 4.0 g of fat, 24.8 g of carbohydrate and 6.2 g of dietary fibre. The average amount of energy provided by these nutrients is:
- protein 17 kJ/g
 - fat 38 kJ/g
 - carbohydrate 17 kJ/g
 - dietary fibre 8 kJ/g
- What is the energy supplied by one slice of this bread?
A 192.35 kJ B 384.7 kJ C 769.4 kJ D 1538.5 kJ
- 13D** 6 A person who performs aerobics burns 10.8 kJ/kg/30 minutes. The energy burned by a 58 kg woman who does 50 minutes of aerobics is:
A 376 kJ B 626 kJ C 1044 kJ D 1253 kJ

- 13D ▶ 7** The Harris–Benedict formula for estimating basal metabolic rate (BMR) in kJ/day from weight in kilograms, height in centimetres and age in years is:
Females:

$$\text{BMR (kJ/day)} = 2783 + (40.0 \times \text{weight in kilograms}) + (7.7 \times \text{height in centimetres}) - (19.6 \times \text{age in years})$$
Males:

$$\text{BMR (kJ/day)} = 278 + (57.5 \times \text{weight in kilograms}) + (20.9 \times \text{height in centimetres}) - (28.2 \times \text{age in years})$$
 Using the appropriate formula, an estimate of the BMR of a 45-year-old man, who weighs 87 kg and is 1.78 m tall, is:
A 5395 kJ **B** 6752 kJ **C** 4049 kJ **D** 7732 kJ
- 13D ▶ 8** A woman who weighs 52 kg has a BMR of 5306 kJ/day. The physical activity level (PAL) for cross-country skiing is 7.5. The energy burned by the woman if she goes cross-country skiing for 2 hours is:
A 1531 kJ **B** 3316 kJ **C** 79 590 kJ **D** 172 432 kJ
- 13D ▶ 9** A man has a BMR of 6854 kJ/day. How long, to the nearest minute, would it take him to burn 2000 kJ by running if the PAL for this exercise is 10.1?
A 5768 min **B** 177 min **C** 84 min **D** 42 min
- 13E ▶ 10** Given that the cost of electricity is 52.68 cents/kWh, which of the following is the cost of running a 2200 W electric heater for 5 hours a day for 70 days?
A \$81.13 **B** \$579.49 **C** \$162.26 **D** \$405.64

REVIEW SET 1

- 1** Convert:
 - a** 2.4 g to kilograms
 - b** 5630 g to kilograms
 - c** 4.846 t to kilograms
- 2** A patient is prescribed 400 mg of a painkiller. Calculate how much medication must be given if it is available in these concentrations.
 - a** 5 mg in 1 mL
 - b** 50 mg in 5 mL
 - c** 100 mg in 5 mL.
- 3** Convert:
 - a** 19.28 kJ to joules
 - b** 2634 J to kilojoules
 - c** 820 kJ to Calories.
- 4** Convert each of the following to joules.
 - a** 47 cal
 - b** 1.096 kcal
 - c** 2 Cal
- 5** Every 100 g of chicken and sweetcorn soup contains 2.0 g of protein, 2.3 g of fat, 7.0 g of carbohydrate and 1.0 g of dietary fibre. If one serving of the soup is 265 g, calculate the total energy content in one serving.
- 6** Use Table 13-2 on page 494 to estimate the energy burned by the body when performing each of the following activities.
 - a** A 55 kg girl walks for 45 minutes.
 - b** A 62 kg woman does aerobics for 40 minutes.
 - c** A 78 kg boy plays basketball for 1 hour.
- 7** Use the Harris–Benedict formula to calculate the basal metabolic rate for:
 - a** a 27-year-old man who weighs 86 kg and whose height is 176 cm
 - b** a 36-year-old woman who weighs 62 kg and is 167 cm tall.

- 8** A 25-year-old woman, who weighs 58 kg, plays table tennis for $1\frac{1}{2}$ hours.
- Use the Schofield formula to estimate her BMR.
 - Use Table 13-5 on page 499 to help you calculate the amount of energy required to perform the exercise described.
- 9** Calculate the cost of running a 350 W television for 8 hours when the cost of electricity is 47.77 cents/kWh.

REVIEW SET 2

- 1** Convert:
- 2900 mg to grams
 - 7.06 kg to grams
 - 940 kg to tonnes.
- 2** A patient is prescribed 600 mg of a painkiller. Calculate how much medication must be given to the patient if the medication is available in these concentrations.
- 30 mg in 4 mL
 - 60 mg in 3 mL
 - 100 mg in 10 mL
- 3** Convert:
- 1.608 kcal to calories
 - 5205 cal to kilocalories
 - 10000 cal to Calories
- 4** Convert each of the following to calories.
- 345 J
 - 2.84 kJ
 - 7.65 Cal
- 5** Use Table 13-1 on page 492 to help you calculate Eliza's total energy intake for the day if she consumes the following meals.
- Breakfast:* $\frac{1}{3}$ cup of natural muesli, 1 cup of reduced fat milk, 1 boiled egg, 1 slice of grain bread,
 $\frac{1}{2}$ tbs of margarine
- Lunch:* 2 long sushi rolls, 1 tub of low-fat yoghurt (200 mL), 1 apple, 1 bottle of water (600 mL)
- Dinner:* 2 baked chicken thighs (skin removed), 1 boiled potato, $\frac{1}{2}$ cup of broccoli, $\frac{1}{2}$ cup of carrots,
 1 glass of white wine (200 mL)
- 6** Use the Schofield formula to calculate the basal metabolic rate for:
- a 31-year-old female who weighs 59 kg
 - a 65-year-old man who weighs 88 kg.
- 7** A 42-year-old male bus driver weighs 83 kg and does no strenuous leisure activities.
- Use the Schofield formula to estimate his BMR.
 - Using the information in Table 13-4 on page 497, estimate the energy output required to maintain the bus driver's lifestyle.
- 8** A 28-year-old woman weighs 66 kg.
- Use the Schofield formula to estimate her BMR.
 - Using the information in Table 13-5 on page 499, how long would it take for the woman to burn 1200 kJ by swimming at 3 km/h?
- 9** Calculate the cost of running a 175 watt ceiling fan for 4 hours a day for 60 days when the cost of electricity is 54.6 cents/kWh.



REVIEW SET 3

- 1 Convert:
 - a 17.61 g to milligrams
 - b 62 805 g to kilograms
 - c 3.604 t to kilograms.
- 2 A man has been prescribed 750 mg of penicillin for an infection. The penicillin is available in a syrup which contains 125 mg of penicillin in every 8 mL. How much syrup should be given?
- 3 Convert:
 - a 23.05 kJ to joules
 - b 2900 J to kilojoules
 - c 415 kJ to Calories.
- 4 Convert each of the following to joules.
 - a 124 cal
 - b 1.08 kcal
 - c 7.5 Cal
- 5 Every 100 g of canned spaghetti and tomato sauce contains 1.7 g of protein, 0.2 g of fat, 11.0 g of carbohydrate and 0.8 g of dietary fibre. Calculate the total energy intake if you ate one 220 g can of this spaghetti and tomato sauce.
- 6 Use Table 13-2 on page 494 to estimate the energy that is burned by the following people's bodies during the given activities.
 - a A 70 kg girl walks for $1\frac{1}{2}$ h.
 - b A 95 kg man plays golf for $3\frac{1}{2}$ h.
 - c A 65 kg boy plays tennis for 80 minutes.
- 7 Use the Harris–Benedict formula to calculate the basal metabolic rate for:
 - a a 48-year-old woman who weighs 72 kg and whose height is 170 cm
 - b a 14-year-old girl who weighs 48 kg and is 1.55 m tall.
- 8 A 52-year-old man, who weighs 79 kg, plays volleyball for 50 minutes.
 - a Use the Schofield formula to estimate his BMR.
 - b Use the information from Table 13-5 on page 499 to help you calculate the amount of energy required for the man to perform the exercise described.
- 9 a Complete this table to find the total usage charges for the electricity bill to which it refers.

	Energy used (kWh)	Rate (\$/kWh)	Cost (\$)
Peak	583.6	0.5144	
Flexi	892.8	0.2136	
Off-peak	291.7	0.139	
		Total	

- b The service availability charge is 78.67 cents/day. If the billing period was 91 days, calculate the total availability charge for the bill.
 - c Find the total amount payable for the bill if a 5.5% discount is applied and then GST is added.
 - d Would it make any difference to the amount paid if the discount was applied after the GST had been added?
- 10 a A computer monitor has a stand-by power usage of 4 watts. Calculate the annual energy consumption in kilowatt-hours if the monitor is left on stand-by for 20 hours every day for a year.
 - b Determine the cost of the stand-by energy used when the cost of electricity is 26.9 cents/kWh.



REVIEW SET 4

- 1 Convert:
 - a 780 mg to grams
 - b 31.206 kg to grams
 - c 289.6 kg to tonnes.
- 2 A woman is prescribed 60 mg of an anti-nausea drug which is available in a suspension containing 15 mg/8 mL. How much of the suspension should she be given?
- 3 Convert:
 - a 3.14 kcal to calories
 - b 980 cal to kilocalories
 - c 60 Cal to kilojoules.
- 4 Convert each of the following to calories.
 - a 178 J
 - b 3.05 kJ
 - c 4.93 Cal
- 5 Use Table 13-1 on page 492 to calculate Jamie's total energy intake for the day if he consumes the following meals.

Breakfast: 1 cup of cornflakes, 1 cup of regular milk, 3 slices of white bread (toasted) with 2 tbs of butter and 3 tsp of honey, 1 banana

Lunch: 1 meat pie, 1 serve of hot chips, 1 flavoured milk (600 mL)

Dinner: 1 cup plain boiled pasta, 250 g of tomato pasta sauce, 2 glasses of red wine (each 200 mL)
- 6 Use the Schofield formula to calculate the basal metabolic rate for:
 - a a 10-year-old boy who weighs 32 kg
 - b a 7-year-old girl who weighs 23 kg.
- 7 A 27-year-old woman, who weighs 54 kg, works as a fitness trainer.
 - a Use the Schofield formula to estimate her BMR.
 - b Use the information in Table 13-4 on page 497 to help you estimate the energy output required for this fitness trainer to maintain her lifestyle.
- 8 A 50-year-old man weighs 72 kg.
 - a Use the Schofield formula to estimate his BMR.
 - b Use the information in Table 13-5 on page 499 to help you calculate how long it would take for the man to burn 1500 kJ by walking at 7 km/h?
- 9 Calculate the annual cost of running the average Sydney home, which uses 21 kWh of energy each day, given that the cost of electricity is 47.77 cents/kWh.



REVIEW PRACTICE EXAMINATION QUESTION

- 1 a** A large park of area 1.2 ha is to be fertilised at the rate of 14 g/m².
- Calculate, in kilograms, the amount of fertiliser needed. (2 marks)
 - If the fertiliser is only available in 25 kg bags, how many bags are needed? (1 mark)
- b** Convert 3.6 kcal to kilojoules. (Answer to the nearest kilojoule.) (2 marks)
- c** Every 100 g of savoury biscuits contains 7.8 g of protein, 23.1 g of fat, 62.2 g of carbohydrates and 3.7 g of dietary fibre. The average amount of energy provided by these nutrients is:
- protein 17 kJ/g
 - fat 38 kJ/g
 - carbohydrate 17 kJ/g
 - dietary fibre 8 kJ/g
- Calculate the energy intake from eating 30 g of these biscuits. (2 marks)
- d i** In order to stay the same weight, we need to consume about the same amount of energy as we burn. For an 18-year-old girl, who does an average level of activity, to maintain a normal lifestyle, the recommended average energy intake is 9100 kJ/day. Jennifer is 18 years old and her energy intake from meals in one day was 10 200 kJ. What percentage of the average daily intake for an 18-year-old girl is Jennifer's energy intake? (1 mark)
- ii** Jennifer weighs 64 kg. Calculate her BMR using the Schofield formula, shown in the table below, where W is weight in kilograms. (1 mark)

Age	Formula (kJ/day)
<3	$244 \times W - 130$
3–10	$85 \times W + 2033$
11–18	$56 \times W + 2898$
19–30	$62 \times W + 2036$
31–60	$34 \times W + 3538$
>60	$38 \times W + 2755$

- iii** For how long would Jennifer need to play soccer to burn the extra energy she consumed on this day, given that the PAL for this exercise is 9.8? (3 marks)
- e i** Calculate the amount of energy needed to run a 2200 watt room heater for 4 hours a day for 90 days. (2 marks)
- ii** If the price of electricity is 33.6 cents/kWh, how much could be saved in a year by only putting the heater on for 3 hours a day instead of 4 hours a day? (1 mark)

TOTAL: 15 MARKS

- 1** The table below shows part of the rates and charges notice for a Sydney council.

Details	Rate (\$)	Rateable value (\$)	Amount (\$)
Residential – on land value	0.001 334 74 per \$ rateable value	499 000	666.04
Residential – base amount	525		525
Domestic waste availability charge	180		180
Domestic waste-management charge	388		388
Total of this notice			

- What is the rateable value of the land on which this property is situated?
 - Consider the charges for residential land.
 - What is the residential rate, in dollars, for properties in this council area?
 - Multiply the residential rate from part **i** by the rateable value of the land. Is this the amount in the last column for ‘Residential – on land value’?
 - Consider the charges for domestic waste services.
 - What is the availability charge?
 - What is the management charge?
 - Calculate the total charge for waste services.
 - Calculate the total rates payable.
 - Calculate the total rates payable for another residential property in this municipality if that property has a rateable land value of \$635 000.
- 2** The stamp duty on the purchase of a motor car is 3% of the value up to \$45 000 and 5% for amounts in excess of \$45 000.
- Calculate the stamp duty on a car valued at \$16 950.
 - Calculate the stamp duty on a car valued at \$105 000.

- 3** The following table gives the monthly repayments (\$) for every \$1000 borrowed on a reducing-balance car loan.

Interest rate (% p.a.)	Term of loan (months)				
	12	24	36	48	60
8	86.99	45.23	31.34	24.41	20.28
9	87.45	45.68	31.80	24.89	20.76
10	87.92	46.14	32.27	25.36	21.25

- Calculate the monthly repayments on a loan of \$28 500 at 9% p.a. reducible over 5 years.
 - What is the total amount of interest paid on this loan?
 - How much would be saved by repaying the loan over 4 years instead of 5 years?
- 4** A car depreciates in value from \$40 900 to \$25 440 in 3 years.
- Use the straight-line method to calculate the annual amount of depreciation.
 - Calculate the annual depreciation as a percentage of the purchase price.

- 5 a** A car travels 520 km on 48 L of petrol. Calculate the fuel consumption for the trip in:
- i** km/L **ii** L/km **iii** L/100 km.
- b** How far can a vehicle travel on 45 L of fuel if fuel consumption is 8.4 L/100 km?

- 6** Tugre is in Year 11 and lives at home with her parents. She has a part-time job and earns \$170 per week. She pays \$40 per week for board, spends \$100 per month on clothes, \$65 per fortnight on entertainment, and \$85 per fortnight on driving lessons. She saves whatever is left.



- Prepare a budget for Tugre.
- Tugre wants to buy a car at the end of the year. How much will she expect to have saved to spend?
- If Tugre wants to spend more on a car, what aspects of her budget can she modify to help her save more?

- 7 a** Find the mean, mode, median and range of these scores:

6 6 6 7 8 8 8 8 10

- b** Consider the grouped data shown in the table on the right.
 - i** Copy the table and add a cumulative frequency column.
 - ii** Draw a cumulative frequency histogram and ogive.
 - iii** From the ogive, determine the median and fourth decile.
 - iv** Estimate the interquartile range.

Class	Frequency
1–5	16
6–10	8
11–15	18
16–20	31
21–25	9

- c** For the scores 8, 9, 10, 10, 10, 11, 11, 12, 12, 12, 12 and 13, find:
- i** the 1st, 2nd and 3rd quartiles **ii** the interquartile range.

- d** Organise the following data into a stem-and-leaf plot:

56, 58, 74, 62, 54, 50, 51, 73, 60, 71, 59, 65, 49, 78, 65, 56, 48, 48, 64, 57

- e** Use your stem-and-leaf plot from part **d** to determine the:

- i** range **ii** mode **iii** median **iv** interquartile range.

- f** Draw a box-plot for these scores: 6, 6, 7, 8, 8, 9, 10, 10, 10, 10, 11, 12.

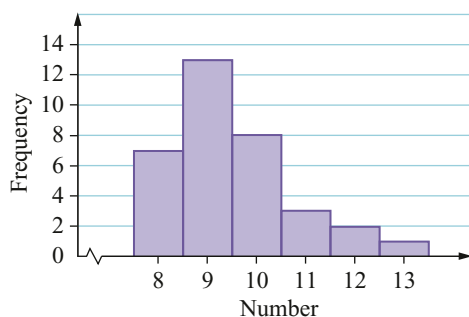
- 8** The mean of a set of data is 11.3 and the standard deviation is 2.9. What is the new mean and standard deviation if:

- i** 5 is added to each score? **ii** each score is multiplied by 5?

- h** Find the mean, mode, median, range and population standard deviation (σ_n) of these scores: 2, 2, 3, 3, 3, 5, 5, 5, 6.

- 8 a** Draw a smooth curve representing the data in this histogram.

- b** Describe this curve in terms of skewness.



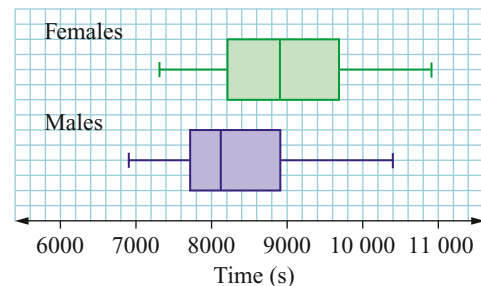
- 9** The following are scores in a mathematics test:
40, 41, 39, 30, 45, 57, 43, 38, 16, 68, 36, 54, 41, 23, 38.
- Draw a stem-and-leaf plot of this data.
 - Determine whether the distribution of scores is symmetrical, positively skewed or negatively skewed.

- 10** For the scores 3, 7, 7, 8, 9, 9, 9, 10 and 10, determine the:

- mean
- median
- range
- interquartile range
- population standard deviation.

- 11** The comparative box-plots on the right show the times (in seconds) for male and female competitors to complete a triathlon.

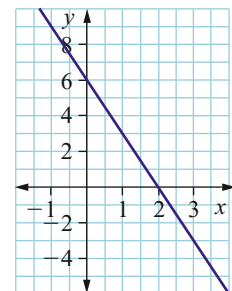
- What was the fastest time for this event?
- How long did it take the first half of the male competitors to finish?
- How long did it take the first quarter of the female competitors to finish?
- Which group had the greatest range of times?
- Which group had the smallest interquartile range of times?



- 12 a** Mobile telephone calls cost 75 cents per minute. This table shows time versus cost for the calls.

Time (min)	1	2	3	4	5	6
Cost (\$)	0.75	1.50	2.25	3.00	3.75	4.50

- Draw the graph of time versus cost. Which is the dependent variable?
- Use the graph to find the cost of a 2.5 min call.
- What is the maximum length of a call that costs \$2.50?

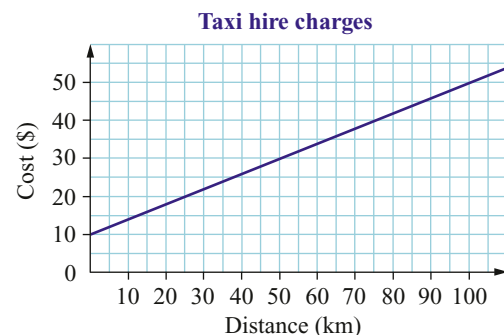


- b** Find the gradient and y-intercept of the straight-line graph on the right.

- c** Sketch the straight-line graph $y = -3 - 4x$.

- d** A graph modelling taxi charges is shown on the right.

- Find the cost of travelling 75 km.
- How far can you travel for \$50?
- Find the gradient. What is the meaning of the gradient in this context?
- Find the intercept on the vertical axis. What is its meaning in this context?
- Is this an example of direct variation? Explain.



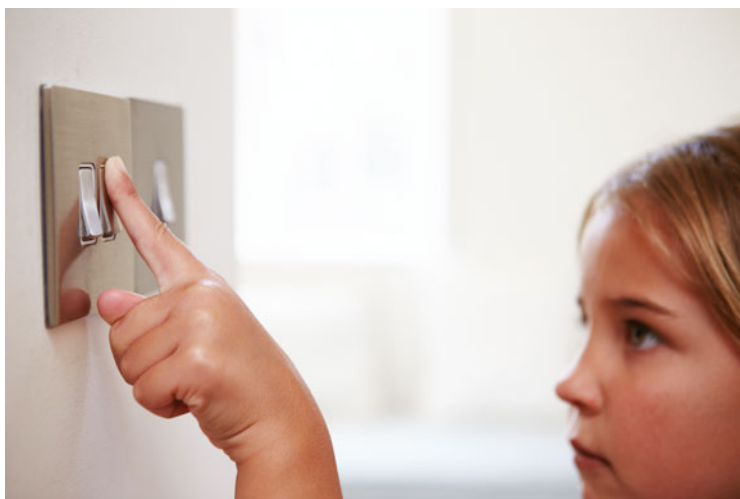
- e** Given that $y \propto x$, and $y = 4.5$ when $x = 18$, find:

- the constant of variation
- the equation for y in terms of x
- y , when $x = 48$
- x , when $y = 8$.

- 13** Complete this table of values for $y = 3x - 2$ and draw the straight-line graph on a number plane.

x	-3	-2	-1	0	1	2	3
y							

- 14** For its bow ties, the Polka Dot Tie Company has fixed costs of \$900 per day and a variable cost of \$8 per tie. The daily cost may be modelled using the straight-line equation $C = 8n + 900$.
- Explain each term in the formula.
 - Complete this table of values.
- | | | | | | | |
|-----|---|----|-----|-----|-----|-----|
| n | 0 | 50 | 100 | 150 | 200 | 250 |
| C | | | | | | |
- Draw the straight-line graph of $C = 8n + 900$.
 - Use your graph from part **c** to find the number of ties produced if the cost is \$1940.
- 15 a** Convert each of the following masses to kilograms.
- 3.4 g
 - 14 630 g
 - 4.113 t
- b** A patient is prescribed 500 mg of a painkiller. Calculate how much must be given if the medication is available in these concentrations.
- 5 mg in 1 mL
 - 50 mg in 5 mL
 - 10 mg in 5 mL
- c** Convert:
- 1.428 kcal to calories
 - 6605 cal to kilocalories.
 - 1200 kJ to Calories
- d** Convert each energy measurement to calories.
- 245 J
 - 1.84 kJ
 - 6.5 Cal
- e** Every 100 g of canned ravioli contains 1.8 g of protein, 0.3 g of fat, 12.0 g of carbohydrate and 0.7 g of dietary fibre. Calculate the total energy intake if one 220 g can is eaten.
- 16 a** Use the Schofield formula to calculate the basal metabolic rate for:
- a 34-year-old female who weighs 58 kg
 - a 62-year-old man who weighs 92 kg.
- b** Use the Harris–Benedict formula to calculate the basal metabolic rate for:
- a 39-year-old woman who weighs 75 kg and whose height is 170 cm
 - a 14-year-old girl who weighs 55 kg and is 1.45 m tall.
- 17** Calculate the annual cost of running the average Sydney home, which uses 27 kWh of energy each day, given that the cost of electricity is 47.77 cents/kWh.



ANSWERS

CHAPTER 1 EARNING AND MANAGING MONEY

ARE YOU READY?

- 1 A 2 D 3 C 4 C
5 D 6 A 7 D 8 C
9 D 10 B 11 A 12 A
13 C 14 A

EXERCISE 1A EARNING AN INCOME

- 1 a \$19 884.80, \$1657.07 b 52, \$382.40
c \$19 884.80, 26, \$764.80
2 \$1671.62
3 a i \$3241.92 ii \$748.13 iii \$1496.27
b i \$2258.83 ii \$521.27 iii \$1042.54
c i \$2508.17 ii \$578.81 iii \$1157.62
d i \$9155.83 ii \$2112.88 iii \$4225.77
e i \$6256.33 ii \$1443.77 iii \$2887.54
f i \$4249.17 ii \$980.58 iii \$1961.15
4 a 52, \$21 892.00 b 12, \$1824.33
5 a \$19 708 b \$1642.33
6 Cooke: \$44 092.88, Moussa: \$53 095.12,
Boje: \$32 788.08, Lambert: \$29 659.76
7 a \$796.12 b \$676.85 c \$1801.43
d \$3019.23 e \$1661.35 f \$6191
8 a Kylie earns \$4461.20 more than Stacey.
b Paul earns \$8393 more than Roland.
c Aaron earns \$4726.40 more than Joshua.
9 a \$28.45, \$227.60 b \$28.45, 38, \$1081.10
10 \$761.25 11 \$794.20
12 a \$355.20 b \$532.80 c \$777.00 d \$910.20
13 a \$259.74 b \$505.05 c \$577.20 d \$692.64
14 a 38, \$820.80 b 52, \$42 681.60
15 \$115 398.40 16 \$44 005.52
17 a \$54 527.20, \$1048.60 b \$1048.6, 35, \$29.96
18 \$23.41 19 \$21.37
20 a Rodney: i 37 ii \$991.60
b Anastasia: i 36 ii \$964.80

21

Name	Hours worked	Hourly rate	Weekly wage
S. Smith	25 $\frac{1}{2}$	\$29.40	\$749.70
D. De Souza	32	\$26.30	\$841.60
A. Bentley	19	\$32.20	\$611.80
C. Johnson	17	\$30.34	\$515.78
K. Kernell	20	\$18.74	\$374.80
P. Patrick	23.5	\$19.83	\$466.01

- 22 \$39 683.28 23 \$51 251.20
24 a i \$855 ii \$3705
b \$28.50
25 a 15 b 10 c \$444.25
26 a \$1517.38 per week b \$33.72 per hour
27 a Option A by \$35.83 b Option B by \$78.36
c Option B by \$148.31
28 a \$35 360 b \$650
c Armin by \$30 per week
d This assumes 48 weeks in a year but there are 52.

EXERCISE 1B ALLOWANCES, BONUSES AND LOADINGS

- 1 a \$22.24, 5, \$33.36 b \$22.24, 2, \$44.48
2 a i \$17.75 ii \$23.66

- b i \$21.53 ii \$28.70
c i \$32.39 ii \$43.18
d i \$28.05 ii \$37.40
3 \$865.74 4 \$1163.40
5 \$701.72 6 \$739.03
7 \$565.60
8 a 8, 5, \$1368 b 3, \$90
c \$1368, \$90, \$1458
9 \$750.50 10 \$918.50
11 \$613 12 3, \$43 600, \$1308
13 a \$2650 b \$7400 c \$11 112.12
d \$19 632 e \$25 345.20 f \$40 664.40
14 a \$619.50 b \$660.80 c \$708 d \$672.60
15 4, \$2692; 17 $\frac{1}{2}$, \$2692, \$471.10; \$471.10, \$3163.10
16 \$2528.60
17 a \$2871.70 b \$2476.90 c \$5081.17
18 a \$1997.50 b \$1692 c \$2021
d \$3172.50 e \$4219.43
19 a \$2550.08 b \$446.26 c \$2996.34
20 \$760.76 21 \$1288.65
22 a \$376.60 b No, \$336 c \$47.08 per hour
23
- | | Normal hours | Overtime hours | Weekly wage |
|---|--------------|------------------|-------------|
| a | 20 | 23 $\frac{1}{2}$ | \$1071.85 |
| b | 20 | 25 $\frac{1}{2}$ | \$1130.05 |
| c | 20 | 30 | \$1261.00 |
| d | 12 | 20 $\frac{1}{2}$ | \$829.35 |
| e | 18 | 14 | \$756.60 |
| f | 20 | 31 | \$1290.10 |
- 24 \$12 522.25
25 \$762.10 26 \$923.75
27 \$859.44 28 \$749
29 a \$1295.61 b \$36 895.61
30 a \$26 b \$24.50
31 a \$539 b \$15.40
32 b =B6*2*B7

EXERCISE 1C COMMISSION, PIECEWORK AND ROYALTIES

- 1 7, $\frac{7}{100}$, \$1350, \$94.50
2 a \$10 500 b \$1221 c \$59
d \$20 e \$2160 f \$13 867
3 \$31 200 4 \$3360
5 \$45 180
6 a 9, \$2800, \$2800, \$252 b \$252, \$532
7 a \$278.02 b \$357.51 c \$555.12
d \$672.61 e \$685.95 f \$865.60
8 \$617.50 9 \$19 736
10 \$350, 9.4 11 14.5%
12 12.3%
13 a \$3168 b 4% c 14.7%
d \$15 055.20 e 5.75% f \$43 253
14 \$3423.60 15 \$5880
16 a \$135 b \$184.50 c \$238.44 d \$296.80
17 a \$184.67 b \$173.21 c \$28 d \$228
18 \$1.80, \$540 19 \$1159.20
20 \$645 21 \$100.50
22 \$659.40
23 a \$130, \$1040 b \$1040, \$29.71
24 a \$2320 b \$66.29
25 a \$900 b \$22.50

- 26 45, \$36; \$0.80, 60, \$97; \$36, \$97, \$133
 27 a \$624.41 b \$632.34
 28 12.3, \$68 000, \$8364
 29 a \$3792.33 b \$26 798.15
 c \$2979.64 d \$37 684.50
 30 a \$21 400 b \$22 800 c Tony by \$1400
 31 a Karla b 10.2%
 32 The second method is better by \$32.25.
 33 \$9270 34 \$1145.60
 35 a \$837.40 b \$550.10 c \$17.19
 36 a \$542.60 b \$33.91
 37 2.5%
 38 a \$2835.60 b \$3370.06 c \$2440 d \$3697.32

EXERCISE 1D GOVERNMENT ALLOWANCES AND PENSIONS

- 1 a \$437.50 per fortnight b \$437.00 per fortnight
 c \$1192.34 per fortnight
 2 a \$288.10 per fortnight b \$437.00 per fortnight
 c \$939.34 per fortnight
 3 a \$288.10 per fortnight b \$143.00 per fortnight
 c \$648.67 per fortnight
 4 a \$437.50 per fortnight b \$143.00 per fortnight
 c \$901.67 per fortnight
 5 a \$437.50 per fortnight b No
 c Yes, fortnightly payment reduces by \$6.50.
 d \$1192.34 per fortnight
 6 a \$288.10 per fortnight
 b Yes, fortnightly payment reduces by \$8.50
 c No d \$648.67 per fortnight
 7 a \$288.10 b \$501, \$64
 c 0.50, \$32 d \$288.10, \$256.10
 8 \$208.50 9 \$403.50
 10 \$288.10 11 \$274.60 per fortnight
 12 \$208 per fortnight
 13 a \$573.30 b \$524 c \$40, \$24
 d \$43.50 e \$24, \$67.50
 f \$573.30, \$67.50, \$505.80
 14 \$162.40 per fortnight 15 \$184 per fortnight
 16 \$332.80 per fortnight 17 \$174.60 per fortnight
 18 \$1138.10 per fortnight 19 \$580.30 per fortnight
 20 \$1602.60
 21 Jessica will no longer receive an age pension.

EXERCISE 1E DEDUCTIONS AND NET INCOME

- 1 a \$56, \$6.80, \$62.80 b \$642.90, \$580.10
 2 \$663.75
 3 a \$800.27 b \$529.32
 4 a \$670 b \$800
 5 a \$19.32 b \$83.70 c \$251.10 d \$996
 6 a \$11.13 b \$578.76 c \$3.76 d \$5.08
 7 a \$23.28 b \$8.64
 8 Intermediate hospital Level 3 (\$1000 excess)
 9 a Basic hospital Level 2 (\$500 excess)
 b Intermediate hospital Level 2 (\$500 excess)
 c Intermediate hospital Level 1 (\$250 excess)
 d Intermediate hospital Level 3 (\$1000 excess)
 10 a \$41 922, \$3982.59
 b 4, \$41 922, \$1676.88
 c \$41 922, \$40 245.12
 d \$1676.88, \$3982.59, \$5659.47
 11 a i \$3587.58 ii \$377.64
 iii \$37386.36 iv \$3965.22
 b i \$4578.34 ii \$3373.51
 iii \$44 819.49 iv \$7951.85

- c i \$1229.59 ii \$0
 iii \$12943 iv \$1229.59
 d i \$3904.31 ii \$2054.90
 iii \$39043.10 iv \$5959.21
 e i \$2179.30 ii \$0
 iii \$22 940 iv \$2179.30
 f i \$6070.60 ii \$3834.06
 iii \$60 066.94 iv \$9904.66
 12 \$920.53, \$118.63
 13 \$42 965, \$2148.25; \$42 965, \$2148.25, \$40 451.75
 14 \$37 800.47
 15 a \$4279.22 b \$2334.12
 c \$31 615.96 d \$608.00
 16 a \$93 840 b \$7820
 17 a \$140 943 b \$156 972 c \$57 973.76

EXERCISE 1F TAXABLE INCOME

- 1 \$45 320, \$665, \$46 452.50, \$46 452; \$340, \$45, \$445;
 \$46 452, \$445, \$46 007
 2 \$39 731 3 \$29 156 4 \$55 805
 5 Tasman: \$45 830, Polding: \$36 932, Flockhart: \$72 398,
 Gen: \$13 303, Old: \$26 578
 6 2, \$42 756, \$855.12
 7 a \$780.42 b \$898.12 c \$877.58
 d \$922.20 e \$1219.16 f \$0
 8 \$24 369, \$3034, 3034, \$303.40
 9 a \$66.50 b \$119.50 c \$0 d \$0
 10 a \$2930.58 b \$6175.61 c \$0 d \$2315.96
 11 a \$54 227 b \$51 373 c \$9832

EXERCISE 1G CALCULATING TAX

- 1 \$37 001–\$87 000, third; \$47 953, \$10 953; \$3572, \$10 953,
 \$7131.73
 2 a \$23 360.68 b \$7443.08 c \$0 d \$2432
 3 Premier: \$143 233, Deputy: \$116 754, Minister: \$102 614,
 Secretary: \$79 283, Whip: \$56 658
 4 Dentistry: \$19 172, Medicine: \$11 372,
 Optometry: \$17 547, Engineering: \$12 997,
 Computer science: \$10 722, Veterinary science: \$7797,
 Education: \$12 867, Accounting: \$9422,
 Psychology: \$12 022
 5 a \$34 596 b \$3115.24
 6 a \$6984.50 b \$950 c \$7934.50
 7 a \$19 900 b \$323 c \$0 d \$323
 8 a \$5203, \$94 655
 b \$94 655, \$2612, \$92 043
 c \$19 822, \$92 043, \$21 687.91
 d \$21 687.91, \$179.69
 9 a \$35 312 b \$706.24
 c \$3251.28 d \$1166.38 refund
 10 a \$31 170 b \$623.40
 c \$2464.30 d \$128.20 refund
 11 a \$75 101 b \$1502.02
 c \$15 954.83 d \$33.17 refund
 12 a \$129 340 b \$127 990 c \$2559.80
 d \$34 988.30 e \$35 803 f \$1745.10 refund
 13 a \$8243.23 b \$1027.46 c \$561.32 refund

REVIEW MULTIPLE-CHOICE QUESTIONS

- 1 C 2 A 3 D 4 A 5 D
 6 A 7 D 8 C 9 B 10 D
 11 A 12 C 13 B 14 D 15 C
 16 B 17 A

REVIEW SET 1

- 1 \$79 716 2 \$23.16 3 \$802.17
 4 \$885 5 \$3271.20 6 \$8118
 7 \$81.40 8 \$6866.64
 9 a \$437.50 per fortnight
 b \$1192.34 per fortnight
 10 \$989.90 per fortnight
 11 \$650.69 12 \$538.31 13 \$14.76
 14 \$35 022 15 \$253.82
 16 a \$10 191.60 b \$2481.21 c \$0
 d \$18 304.25
 17 a \$48 533 b \$970.66
 c \$7320.23 d \$687.51 refund

REVIEW SET 2

- 1 \$1201.44, \$34.33 2 \$49 271.04
 3 \$549.70 4 \$545
 5 a \$550.62 b \$17.21
 6 \$1605.38 7 \$186.40 per fortnight
 8 \$452.94 9 \$511.95
 10 \$25.64 11 \$1676.16, \$207.36
 12 \$71 782 13 \$98.20
 14 a \$13 285.93 b \$2994.81
 c \$222.11 d \$0
 15 a \$42 995 b \$859.90
 c \$5520.38 d \$583.02 refund

REVIEW SET 3

- 1 \$574.63 2 \$20.02 3 \$928.20
 4 \$347.10 5 \$3539.10 6 \$304.50
 7 \$26 048.40 8 \$168.00 per fortnight
 9 \$577.80 10 \$739.32 11 \$11.13
 12 \$49 561 13 \$601.60
 14 a \$15 127.70 b \$4122.55
 c \$572.09 d \$0
 15 a \$52 913 b \$1058.26
 c \$8743.73 d \$1388.55 payable

REVIEW SET 4

- 1 \$31.58 2 \$36457.20
 3 \$476.85 4 \$597.70
 5 a \$465.04 b \$15.50
 6 \$1866.15 7 \$12 367
 8 \$348.40 per fortnight
 9 \$1150.60 per fortnight
 10 \$741.30 11 \$652.23
 12 \$18.59 13 \$28 360
 14 a \$9497.73 b \$2009.25
 c \$0 d \$764.75
 15 a \$54 002 b \$1080.04
 c \$9097.65 d \$133.25 payable

REVIEW PRACTICE EXAMINATION QUESTION

- 1 a i \$33 904 ii \$17.16
 iii No. $\$33\ 904 \div 12 = \$2825.33$
 b \$4446.20
 c No; he would receive \$20 less per week.
 d \$740.68 per fortnight
 e i \$1873.28 ii \$43 755.72

CHAPTER 2 PRACTICALITIES OF MEASUREMENT

ARE YOU READY?

- 1 D 2 B 3 B 4 D
 5 D 6 B 7 A 8 A
 9 C 10 C 11 B 12 C
 13 B

EXERCISE 2A SIGNIFICANT FIGURES

- 1 a 5; 6; bigger than, 5370, 5360; 5370
 b 6; 5; smaller than, 0.0625, 0.0626; 0.0625
 2 a 40 000 b 60 c 5 d 100
 e 0.7 f 0.008 g 0.03 h 1000
 3 a 290 000 b 4000 c 25 d 2700
 e 8.6 f 0.049 g 0.000 16 h 0.0040
 4 a 3690 b 20 700 c 154 000 d 813
 e 14.3 f 0.003 51 g 0.0391 h 2.00
 5 a i 20 ii 17 iii 17.3
 b i 0.5 ii 0.45 iii 0.451
 c i 500 000 ii 520 000 iii 522 000
 d i 0.002 ii 0.0021 iii 0.002 10
 6 3.286
 7 $1\frac{2}{5}$ (= 1.4), $\sqrt{2}$ (= 1.41...), $\frac{10}{7}$ (= 1.42...), $1.\dot{4}$ (= 1.44...)
 8 316.6 mm
 9 a 417.3 km b 73 km/h
 10 a 6402.0 km b 40 225 km c 92.8 min

EXERCISE 2B SCIENTIFIC NOTATION

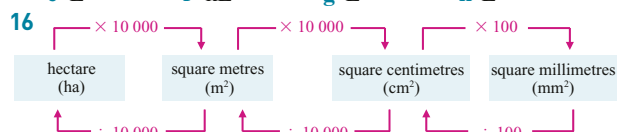
- 1 a Yes b No c No d No
 2 a Yes b No c No d Yes
 e No f No g No h Yes
 3 a 2.43000; 5, right, 5; 2.43×10^5
 b 5.86; 4, left, 4; 5.86×10^{-4}
 4 a 5.26×10^5 b 2.8×10^4 c 7×10^6
 d 4.98×10^4 e 2.8×10^7 f 6.03×10^8
 g 9.1×10^5 h 1.32×10^{10}
 5 a 4.3×10^{-4} b 8.21×10^{-3} c 7×10^{-6}
 d 2.9×10^{-5} e 6.5×10^{-2} f 3.87×10^{-4}
 g 8.2×10^{-6} h 6×10^{-5}
 6 a 6, right, 5 480 000 b 5, left, 0.000 030 9
 7 a 3 400 000 b 830 000 000
 c 29 400 000 d 258 000
 e 526 000 f 3 020 000 000 000
 g 29 000 000 h 875 000 000
 8 a 0.000 59 b 0.000 003 2 c 0.000 000 071
 d 0.002 e 0.000 000 8 f 0.000 026 4
 g 0.000 000 008 67 h 0.000 002 97
 9 a 1.07×10^{16} b 4.76×10^{22} c 3.22×10^{17}
 d 1.39×10^{-14} e 3.36×10^{11} f 1.25×10^{20}
 g 5.28×10^{-16} h 4.68×10^{24} i 2.40×10^8
 j 8.25×10^8 k 1.69×10^{21} l 2.62×10^{-55}
 10 a 2.29×10^8 b 2.54×10^{-11} c 3×10^{27}
 d 1.3×10^5 e 1×10^{13}
 11 a 31 600 000 b 0.000 000 89 c 2 600 000
 d 30 000 000 000 e 0.000 26
 12 a 3×10^5
 b i 1.8×10^7 km ii 1.08×10^9 km
 iii 2.592×10^{10} km iv 9.4608×10^{12} km
 c 7.38×10^7 km

- 13 a $5.1 \times 10^8 \text{ km}^2$ b $1.1 \times 10^{12} \text{ km}^3$
 14 $9.4 \times 10^8 \text{ km}$
 15 a 10 000 000 000 b 280 km
 17 a i 3.333×10^{-7} ii 3.030×10^{-1}
 b 3.030×10^{-1}
 c If the timekeeper starts the stopwatch when he or she sees the smoke, he or she will actually start the stopwatch 0.000 000 333 3 seconds late (because this is the time it takes for the light to travel this distance). Since the stopwatch only measures to the nearest 0.01 of a second, this will make no difference to the timing of the race. If the timekeeper starts the stopwatch when he or she hears the gun, he or she will actually start the stopwatch 0.0303 seconds late (because this is the time it takes for the sound to travel this distance). This will make the timing of the race 0.03 seconds less than the real time.

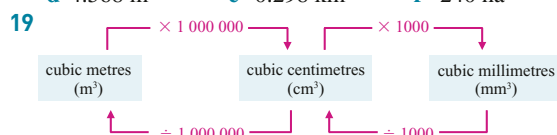
EXERCISE 2C METRIC UNITS OF MEASUREMENT

- 1
-
- 2 a 3600 m b 8400 mm c 3482 m
 d 560 m e 290 cm f 964 mm
 g 65.8 cm h 452 mm i 15 680 m
 j 36.9 mm k 16 370 mm l 4265 m
- 3 a i 100 000 ii 1×10^5
 b i 1 000 000 ii 1×10^6
- 4 a 7 km b 5.94 m c 8.93 km
 d 6 m e 4 cm f 0.085 km
 g 8 m h 32.8 cm i 0.62 m
 j 14.3 km k 0.86 m l 0.63 km
 m 9.4 cm n 0.07 m o 24.895 km
 p 23 m q 14.96 m r 162.7 m
 s 36 m t 72.945 m
- 5 a m b cm or mm c cm d cm
 e cm or mm f mm g km h cm
 i m j km
- 6
-
- 7 a 3600 kg b 7100 mg c 17840 g d 630 kg
 e 4800 g f 2465 kg g 21590 kg h 600 mg
 i 6060 kg j 489 g k 1070 kg l 30 mg
- 8 a i 1 000 000 ii 1×10^6
 b i 1 000 000 000 ii 1×10^9
 c i 1000 ii 1×10^3
- 9 a 8 t b 4.3 kg c 2.74 g d 0.69 kg
 e 0.065 g f 2.32 kg g 0.7 t h 0.46 g
 i 0.08 kg j 0.007 g k 0.009 t l 0.3 kg
- 10 a kg b g c g d kg e t or kg
 f t or kg g mg h g i mg j g
- 11
-

- 12 a 35 000 L b 15 900 mL c 1650 mL d 850 L
 e 60 mL f 1080 L g 15 mL h 5 L
 13 1 000 000 = 10^6 mL
 14 a 15 L b 8 kL c 7.6 L d 0.8 L
 e 9.28 kL f 0.725 kL g 0.095 L h 0.04 kL
 15 a mL b kL c L d L
 e L f kL g L h L



- 17 a 26 000 m² b 49 000 cm² c 1400 mm²
 d 7520 cm² e 16 500 m² f 2480 mm²
 g 8 294 000 m² h $5.671 \times 10^{10} \text{ cm}^2$
 18 a 6.3 ha b 12.7 m² c 8.1 cm²
 d 4.568 m² e 0.298 km² f 240 ha



- 20 a 5000 mm³ b 3 900 000 cm³ c 25 600 mm³
 d 640 000 cm³ e 415 mm³ f 7 390 000 000 mm³
 21 a 7.4 m³ b 56.7 cm³ c 0.69 m³ d 4.258 5 cm³
 22 a 14 kg b 9 L
 23 $6.4 \times 10^8 \text{ kL}$ 24 2.5%
 25 a 5.8 m³ (to one decimal place)
 b i 0.0253 m² ii 0.0288 m² iii 2084

EXERCISE 2D PREFIXES FOR UNITS OF MEASUREMENT

- 1 a i 5.7×10^6 (5 700 000) m
 ii 9×10^{-2} (0.09) m
 iii 8×10^{-9} (0.000 000 008) m
 b i 8×10^9 (8 000 000 000) g
 ii 4.2×10^{-3} (0.0042) g
 iii 5×10^{-6} (0.000 005) g
 c i 3×10^{12} (3 000 000 000 000) L
 ii 2.8×10^6 (2 800 000) L
 iii 7×10^{-3} (0.007) L
- 2 a i 4.9×10^{-3} (0.0049) km
 ii 4.9×10^6 (4 900 000) μm
 iii 4.9×10^9 (4 900 000 000) nm
 b i 2.4×10^{-6} (0.000 002 4) Mg
 ii 2.4×10^{-12} (0.000 000 000 002 4) Tg
 iii 2.4×10^6 (2 400 000) μg
 c i 6.5×10^{-6} (0.000 006 5) ML
 ii 6.5×10^3 (6500) mL
 iii 6.5×10^{-9} (0.000 000 006 5) GL
- 3 a 7.2×10^3 (7200) Mg
 b 2.9×10^7 (29 000 000) ng
 c 8×10^9 (8 000 000 000) km
 d 4.3×10^4 (43 000) μm
 e 8.8×10^6 (8 800 000) ML
 f 9×10^6 (9 000 000) mL
 g 5.3×10^6 (5 300 000) ns
 h 1.2×10^3 (1200) ns
- 4 9.46×10^3 (9460) Tm
 5 2580 GL
 6 228 000 000 km
 7 1.67×10^{-9} ng
 8 0.024 μs

EXERCISE 2E ERROR AND ACCURACY IN MEASUREMENT

- 1 255 mm
 2 a 84 mm b 4.9 kg c 161 mL d 22.54 s
 4 a 13 cm b 14 cm c 14 cm d 15 cm
 5 a No b 13.5 and 14.5 cm c 0.5 cm
 d Use a ruler with smaller units.
 6 two; one; three; 10.9 L, one; 37.174 L, 37.2 L, one
 7 a 36 m b 65.3 L c 2.67 km d 12.9 kg
 8 two; three; 7.5 m, two; 92.25 m², 92 m², two
 9 a 130 m² b 23 700 cm² c 1.4 cm³ d 3.1 mm
 10 512 m 11 4.2 cm² 12 11 m² 13 20.8
 14 a 2990 m²
 b i three ii 10 m²
 c 644 m²
 d i three ii whole square metre
 e 3630 m²

EXERCISE 2F ABSOLUTE ERROR AND LIMITS OF ACCURACY

- 1 a units; 1 cm; cm; 1 cm; 1 cm, 0.5 cm
 b tenths; 0.1 s; tenth of 1 s; 0.1 s; 0.1 s, 0.05 s
 2 a i 1 cm ii 0.5 cm
 b i 1 g ii 0.5 g
 c i 1 m ii 0.5 m
 d i 1 L ii 0.5 L
 e i 0.1 kg ii 0.05 kg
 f i 0.1 s ii 0.05 s
 g i 0.1 L ii 0.05 L
 h i 0.01 m ii 0.005 m
 3 a 0.1 b 0.05
 c 0.05, 0.05, 2.55 kg, 2.65 kg
 4 a i 1 mm ii 0.5 mm iii 11.5 mm, 12.5 mm
 b i 1 g ii 0.5 g iii 347.5 g, 348.5 g
 c i 1 mL ii 0.5 mL iii 374.5 mL, 375.5 mL
 d i 0.1 km ii 0.05 km iii 8.15 km, 8.25 km
 e i 0.1 s ii 0.05 s iii 18.35 s, 18.45 s
 f i 0.1 kg ii 0.05 kg iii 4.85 kg, 4.95 kg
 g i 0.01 m ii 0.005 m iii 2.365 m, 2.375 m
 h i 0.01 L ii 0.005 L iii 5.805 L, 5.815 L
 5 a 50, 50 b 50, 25
 c 25, 25, 725 mL, 775 mL
 6 a i 30 g ii 15 g iii 405 g, 435 g
 b i 20 mL ii 10 mL iii 370 mL, 390 mL
 c i 1000 ii 500 iii 37 500, 38 500
 d i $\frac{1}{2}$ h ii $\frac{1}{4}$ h iii $6\frac{1}{4}$ h, $6\frac{3}{4}$ h
 7 a i 0.5 cm ii 5%
 b i 0.5 s ii 1.6%
 c i 0.5 g ii 0.2%
 d i 0.5 min ii 3.6%
 e i 0.5 L ii 8.3%
 f i 0.05 kg ii 2.1%
 g i 0.05 s ii 0.4%
 h i 0.005 m ii 0.04%
 8 Percentage error = $\frac{\text{absolute error}}{\text{measurement}} \times 100\%$
 9 a 7, 4, 22 b 7.5, 3.5, 20, 7.5, 4.5, 24, 20, 24
 c 24, 22, 2
 10 a 28 cm b 8.5 cm, 9.5 cm and 4.5 cm, 5.5 cm
 c 26 cm, 30 cm d 2 cm
 11 a 7, 4, 28
 b 7.5, 3.5; 22.75, 7.5, 4.5, 33.75; 22.75, 33.75
 c 28, 5.25; 28, -5.75; 5.75
 12 a 15 m² b 4.5 m, 5.5 m and 2.5 m, 3.5 m
 c 11.25 m², 19.25 m²
 d 4.25 m²

- 13 a 292 cm
 b 163.5 cm, 164.5 cm and 127.5 cm, 128.5 cm
 c 291 cm, 293 cm d 1 cm
 14 a 99 kg
 b 46.5 kg, 47.5 kg and 51.5 kg, 52.5 kg
 c 98 kg, 100 kg d 1 kg
 15 a 99.3 kg
 b 47.35 kg, 47.45 kg and 51.85 kg, 51.95 kg
 c 99.2 kg, 99.4 kg d 0.1 kg
 16 a 17.28 m²
 b 5.35 m, 5.45 m and 3.15 m, 3.25 m
 c 16.8525 m², 17.7125 m² d 0.4325 m²
 17 a 642.4 cm² b 28.55 cm, 28.65 cm
 c 640.2 cm², 644.7 cm² d 2.3 cm²
 18 a 0.005 m b 0.18% and 0.35%
 c 4.0328 m² d 4.011 525 m², 4.054 125 m²
 e 0.021 325 m² f 0.53%
 g Yes

REVIEW MULTIPLE-CHOICE QUESTIONS

- 1 B 2 C 3 A 4 B 5 B
 6 D 7 C 8 B 9 B 10 B
 11 C

REVIEW SET 1

- 1 a 3700 b 3659 c 3659.06
 d 3660 e 3659.1 f 4000
 2 a No b No c Yes
 3 a 1.05×10^8 b 6.2×10^{-5} c 3.179×10^3
 4 a 2.46×10^{14} b 1.4×10^{-10}
 c 4.096×10^{23} d 2.9×10^{-6}
 5 a 560 mm² b 4.3 ha
 c 2 900 000 cm³ d 5.6 cm³
 6 a 2.3×10^9 m b 52 000 kL
 c 3000 μ s d 7 200 000 000 000 000 mg
 7 a i 0.1 m ii 0.05 m iii 7.45, 7.55 m
 iv 0.7%
 b i 10 g ii 5 g iii 275, 285 g iv 1.8%
 8 a 20 cm b 5.5 cm, 6.5 cm and 3.5 cm, 4.5 cm
 c 18 cm, 22 cm d 2 cm e 24 cm²
 f 19.25 cm², 29.25 cm² g 5.25 cm²
 9 a 33.2 m b 275 m²

REVIEW SET 2

- 1 a 1470 b 1500 c 1472.63
 2 a 7.49×10^5 b 3×10^{-6} c 1.05×10^{-2}
 3 6.3×10^{15}
 4 a 21 000 m² b 7.8 cm²
 c 9.5 m³ d 72 000 mm³
 5 a 43 000 μ m b 2 000 000 Mg
 c 52 ML d 9 100 000 nm
 6 a 36.7 m b 78 cm²
 7 a i 0.1 kg ii 0.05 kg
 iii 12.75 kg, 12.85 kg iv 0.4%
 b i 0.01 m ii 0.005 m
 iii 12.555 m, 12.565 m iv 0.04%
 8 a 12 320 cm² b 12 203.25 cm², 12 437.25 cm²
 c 117.25 cm² d 0.95%

REVIEW SET 3

- 1 a 0.0051 b 0.01 c 0.005
 2 a No b Yes c No
 3 a 6.7×10^4 b 8.09×10^{-5} c 1.23×10^3

- 4 a 3 400 000 b 0.000 087 c 20 530
 5 a 1120 mm² b 12.9 ha
 c 3 400 000 cm³ d 73 cm³
 6 a 4 500 000 m b 2 t
 c 7000 ns d 0.000 003 5 GL
 7 a 100 kg b 48.5, 49.5 kg, 50.5, 51.5 kg
 c 99 kg, 101 kg d 1 kg e 1%
 8 a 35 kg b 1.5 g/mL

REVIEW SET 4

- 1 a 2 b 2.1 c 2.07 d 2.070
 2 a 1365 cm b 3.46 t
 c 0.276 L d 8300 mm
 3 a 136 500 m² b 19.6 cm²
 c 3.7 m³ d 6800 m³
 4 a 6000 µL b 4 200 000 kg
 c 8100 ns d 0.000 56 TL
 5 a 6.4×10^8 b 3.0×10^{-7}
 6 a 1.5×10^{-2} b 1.4×10^{10}
 7 a 96.25 cm²
 b $95.55\ 375\text{ cm}^2 \leq \text{area} < 96.94\ 875\text{ cm}^2$
 c $0.698\ 75\text{ cm}^2 \approx 0.7\text{ cm}^2$
 d 0.7%
 8 a 5.5 m b 1.6 m²

REVIEW PRACTICE EXAMINATION QUESTION

- 1 a 1.31×10^{-3} b two
 c 8.2×10^5
 d i 94.63 m ii 2 500 000 cm³
 e i 0.25°C ii 1.3%
 f i 5.52 L ii 5.51 L, 5.53 L
 iii 0.01 L iv 0.18%
 g 140 cm²

CHAPTER 3 FORMULAS AND EQUATIONS**ARE YOU READY?**

- 1 D 2 D 3 B 4 A
 5 B 6 C 7 D 8 B
 9 D 10 A 11 C 12 A
 13 C 14 B

EXERCISE 3A SUBSTITUTION INTO EXPRESSIONS AND FORMULAS

- 1 a -3, 15 b 7, 13
 2 a 7 b 3 c 19 d -8
 e 25 f 50 g 100 h 125
 i 13 j 28 k 49 l 29
 m $\frac{1}{2}$ n 95 o $2\frac{1}{2}$ p 2
 3 a -3 b 7 c -21 d 16
 e -1 f $-\frac{3}{2}$ g 49 h -21
 i -80 j -3 k -18 l -21
 4 a 10 b -4 c 4 d 10
 e 58 f 40 g 30 h 8
 i 5 j 10 k 4 l 7
 5 2, 35
 6 a 2500 b 1000 c 1200 d 5
 7 a 75 b 110 c 150 d 15.36
 8 a 904.8 cm³ b 2572.4 cm³ c 9.2 m³
 9 a \$1610.51 b \$1591.98 c \$3300.80 d \$1461.23

- 10 a 5 b 13 c 25 d 20
 11 a Yes b No, $4a + 4b$
 c No, $6a + 2b$ d No, $-2a - 5b$
 e No, $-2a - 5b$ f Yes
 g No, $12a^2 + 13ab - 14b^2$
 h No, $12a^2 + 13ab - 14b^2$
 i No, $a^3 + 3a^2b + 3ab^2 + b^3$
 j Yes

- 12 a i -4 ii -4 iii -4 iv -4
 b All equal -4. c No
 d Only expressions a i and iii are always equal.

- 13 a 23.8, yes b 26.3, no c 22.2, yes

- 14 a $10n + 50$ b \$110

- 15 a $3a + 2b + c$ where a represents the number of goals scored from behind the three-point line, b represents the number of goals scored inside the three-point line and c represents the number of goals scored on a foul shot

b 43

c One possible answer is $a = 10$, $b = 17$ and $c = 4$.

- 16 a $6t + 5$ where t represents time in hours

b 8 km c 1.5 h

- 17 Sheet 1: 1.18, Sheet 2: 58.9; Sheet 2 has greater bend allowance

EXERCISE 3B SOLVING LINEAR EQUATIONS

- 1 a 3, 35, 7, 5
 b 7, 4x, 4x, 4
 2 a $x = 4$ b $p = 4$ c $x = 14$ d $x = 9$
 e $p = 10$ f $y = 10$ g $x = 24$ h $p = 4$
 i $m = 6$ j $q = 34$ k $s = 33$ l $r = 13$
 m $T = 5$ n $s = 9$ o $m = 16$
 3 a 5, 32, not a solution b 8, 24, a solution
 4 a Yes b No c Yes
 d Yes e Yes f No
 g No h Yes i Yes
 5 a 6, 8, x , $-1\frac{3}{5}$ b 7, 7, $5x$, 5, 5, $-3\frac{3}{5}$
 6 a $x = -\frac{2}{3}$ b $x = 1$ c $x = -3\frac{1}{2}$ d $x = 11$
 e $x = -3$ f $x = -2$ g $x = 3$ h $x = \frac{5}{7}$
 i $x = 9$ j $x = -4$ k $x = 8$ l $x = -2\frac{1}{2}$
 m $x = -1$ n $x = 3$ o $x = -5$
 7 3, 3, 4, 5, 20
 8 a $x = 10$ b $x = 15$ c $x = -25$
 d $x = -42$ e $x = 42$ f $x = 50$
 9 a No b Yes c Yes d No
 -3, 11, -14, 7, -2
 11 a $x = -3$ b $x = \frac{9}{4} = 2\frac{1}{4}$ c $x = -\frac{1}{5}$
 d $x = \frac{7}{3} = 2\frac{1}{3}$ e $x = \frac{10}{7} = 1\frac{3}{7}$ f $x = \frac{5}{3} = 1\frac{2}{3}$
 12 a $3x - 5 = 13$, $x = 6$ b $4x - 7 = 13$, $x = 5$
 c $9 + 5x = 34$, $x = 5$ d $7x + 8 = 29$, $x = 3$
 13 a $x + 1$
 b $x + x + 1 = 275$ or $2x + 1 = 275$
 c $x = 137$; two consecutive numbers are 137 and 138
 14 a Let x be the first integer, hence the second integer is $x + 1$; $x + x + 1 = 127$ or $2x + 1 = 127$, $x = 63$; two consecutive integers are 63 and 64
 b Let x be the first integer, hence the second integer is $x + 1$ and the third integer is $x + 2$; $x + x + 1 + x + 2 = 27$ or $3x + 3 = 27$, $x = 8$; smallest number is 8
 c Let x be the first integer, hence the second integer is $x + 1$, the third integer is $x + 2$ and the fourth integer is $x + 3$; $x + x + 1 + x + 2 + x + 3 = -6$ or $4x + 6 = -6$, $x = -3$; largest number is 0

- d** Let x be the first even integer, hence the second even integer is $x + 2$ and the third even integer is $x + 4$; $x + x + 2 + x + 4 = 132$ or $3x + 6 = 132$, $x = 42$; smallest number is 42
- 15 a** Let x be the number of weeks.
b $67 + 28x = 235$
c $x = 6$; Tyler can buy the bat in 6 weeks
- 16 a** Let d be the cost of one dress in dollars; $3d + 19 = 196$; $d = 59$; cost of each dress is \$59
b Let t be the time in hours; $14t + 9 = 85$; $t = 5.4$; tennis court can be hired for 5 hours (for a total cost of \$79)
c Let n be the number of goals scored by Andrew; $n + n + 5 = 37$ or $2n + 5 = 37$; $n = 16$; Andrew scored 16 goals
d Let n be the number of people; $28n + 325 = 1200$; $n = 31.25$; 31 people can attend the party
- 17 a** $w + 14$ **b** $P = 4w + 28$
c $w = 32$; length is 46 m and width is 32 m
- 18 a** Let g be the number of goals scored, b the number of behinds scored and n be the total number of points scored; $n = 6g + b$
b 75 points
c i Kangaroos: 48 points, Swans: 50 points; winner is Swans
ii Eagles: 141 points, Suns: 127 points; winner is Eagles
- d i** 7 **ii** 14
e i 13 **ii** 19
f $n = 6g + b + 3p$ where p is the number of times a player hits the goal post instead of kicking a goal
- g** 5
- 19 a** Let x be the unknown number; $\frac{x}{3} - 4 = 8$; $x = 36$
b Let x be the unknown number; $\frac{x}{5} - 6 = 7$; $x = 65$
c Let n be the number of caramel chocolates; hence there are $(n + 4)$ mint chocolates and $2n$ strawberry chocolates; $n + 4 + 2n + n = 30$ or $4n + 4 = 30$; $n = 5$; so there are 5 caramel chocolates
- 20** Let n be the number of fish caught by Marcel; Davina catches $(n + 4)$ fish, Hayley catches $2n$ fish.
a The total number of fish caught is unknown so we cannot form an equation to solve.
b $n + 4 + 2n + n = 65$ or $4n + 4 = 65$; $n = 15.25$; this would indicate that Marcel caught 15.25 fish which is not possible
- d** $x = -3\frac{1}{3}$ **e** $x = -3\frac{1}{2}$ **f** $x = -6\frac{1}{2}$
g $x = \frac{4}{5}$ **h** $x = -14$ **i** $x = -\frac{1}{5}$
j $a = -1\frac{1}{2}$ **k** $s = -2\frac{1}{5}$ **l** $x = 1\frac{3}{4}$
m $a = 3\frac{2}{3}$ **n** $y = -1\frac{1}{8}$ **o** $p = 1\frac{4}{11}$
- 8** 3, 9, -2, 3, is
9 a Yes **b** Yes **c** No
d Yes **e** Yes **f** No
- 10** Let x be the unknown number in each case.
a $2x + 3 = x + 7$; $x = 4$ **b** $3x + 8 = x + 4$; $x = -2$
c $2x + 6 = 4x + 4$; $x = 1$ **d** $5x - 9 = 2x - 3$; $x = 2$
- 11 a** $30 = \frac{5}{9}(F - 32)$ **b** $F = 86$ **c** 86°F
d 64.4°F **e** 25°C because $25^\circ\text{C} = 77^\circ\text{F}$
- 12 a** C
b $n = 11$, so there are 11 biscuits in a box
- 13 a** Let x be the cost (\$) for 1 kg of cherries;
 $5x + 2 = 2x + 20$
b $x = 6$; 1 kg of cherries costs \$6
- 14** Let x be the cost (\$) for a sushi roll; $7x + 1.5 = 4x + 12$; $x = 3.5$; sushi roll costs \$3.50
- 15** Let x be the result (%) Hayden received in the fourth test; $\frac{91 + 69 + 88 + x}{4} = 78$; $x = 64$; fourth test result is 64%
- 16** Let x be the first even integer; $x + x + 2 = x + 4 + 6$ or $2x + 2 = x + 10$; $x = 8$; integers are 8, 10 and 12
- 17** $x = 4$
18 a $x = 8$ **b** $x = 6$ **c** $x = 7$
d $x = 3$ **e** $x = 5$ **f** $x = 4$
19 a $x = -8$ **b** $x = -11$ **c** $x = -6$
- 20** Find the integers between which the answer lies. Then use smaller increments between these values. Otherwise guess, check and refine.
- 21 a** $x = 2.6$ **b** $x = -7.75$ **c** $x = -3.4$
- 22 a** Let x be the current age (in years) of the son; $3x + 11 = 2(x + 11)$; $x = 11$; son is currently 11 years old
b Let x be the current age (in years) of Wesley; $x + 8 + 3 = 2(x + 3)$; $x = 5$; Wesley is currently 5 years old
c Let x be the current age (in years) of Sam; $2(45 - x) = 3x + 5$; $x = 17$; Sam is currently 17 years old

- 23 a** $x = \frac{1}{8}$ **b** $x = \frac{7}{9}$ **c** $x = \frac{2}{35}$
d $x = 1\frac{3}{8}$ **e** $x = \frac{7}{15}$ **f** $x = 1\frac{5}{6}$
g $x = -\frac{1}{9}$ **h** $x = -1\frac{2}{5}$ **i** $x = -2\frac{1}{2}$

EXERCISE 3C SOLVING FURTHER LINEAR EQUATIONS

- 1** 40, 35, 8, 8, $4\frac{3}{8}$
2 a $x = 7\frac{1}{2}$ **b** $x = 2\frac{1}{3}$ **c** $x = 2\frac{1}{2}$
d $x = 3\frac{3}{5}$ **e** $x = 5\frac{1}{4}$ **f** $x = 3\frac{1}{3}$
- 3 a** 3, 3, 12, 2, 2, 14, $2\frac{4}{5}$
b 4, 4, 1, 20, 1, 1, 19, $6\frac{1}{3}$
- 4 a** $x = -10$ **b** $x = 17$ **c** $x = -20$
d $x = -5$ **e** $x = -2\frac{1}{3}$ **f** $x = 24$
g $x = \frac{1}{2}$ **h** $x = 4\frac{2}{3}$ **i** $x = 4\frac{1}{2}$
- 5 a** 2x, 2, 6, 2, 3
b 5x, 8x, 7, 7, -8x, -8, 2
- 6 a** $x = 4$ **b** $x = 1$ **c** $x = 1$
d $x = -3$ **e** $x = -2$ **f** $x = -2$
g $x = 9$ **h** $x = -10$ **i** $x = 3$
- 7 a** $x = -4\frac{1}{2}$ **b** $x = 1\frac{1}{3}$ **c** $x = 1\frac{1}{5}$

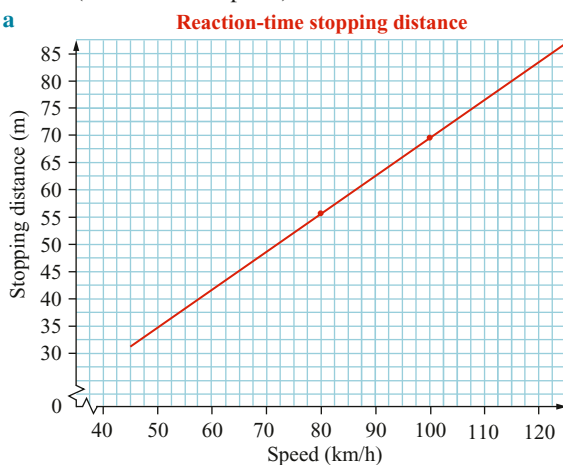
EXERCISE 3D FINDING THE VALUE OF A VARIABLE IN A FORMULA

- 1 a** $t = 12$ **b** $a = 3$
2 a $c = 25$ **b** $t = 40$
3 $k = 54$ **4** $R = 14.4$
5 $m = 61.1$ (to one decimal place)
6 a $P = 26000$ **b** $n = 7$ **c** $r = 4$
7 $u = 5$ **8** $a = 3$ **9** $a = 2$ **10** $n = 6$
- 11 a** $R = 6\frac{1}{4}$ **b** $R = 10$
12 a $E = 144$ **b** $R = 9$
13 a $x = y - 2$ **b** $x = y - 5$ **c** $x = y - 8$
d $x = \frac{p}{5}$ **e** $x = \frac{q}{7}$ **f** $x = \frac{r}{2}$
g $x = 3t$ **h** $x = 5r$ **i** $x = 8m$
j $x = p + 3$ **k** $x = m + 7$ **l** $x = z + 1$

- 14 a $x = z - y$ b $x = m - n$ c $x = p + q$
 d $x = n + k$ e $x = \frac{c}{a}$ f $x = \frac{e}{a}$
 g $x = vw$ h $x = kt$ i $x = p - q$
 j $x = k + m$ k $x = \frac{a}{b}$ l $x = bt$
- 15 a $x = \frac{mp}{q}$ b $x = \frac{kt}{m}$ c $x = \frac{ay}{b}$
 d $x = \frac{k}{at}$ e $x = \frac{d}{ez}$ f $x = \frac{m}{kn}$
 16 a $x = y - \frac{2}{3}$ b $x = \frac{y-1}{5}$ c $x = \frac{y-5}{2}$
 d $x = \frac{y+4}{7}$ e $x = \frac{y+5}{6}$ f $x = \frac{y+5}{3}$
 g $x = \frac{m-3}{2}$ h $x = \frac{m-5}{3}$ i $x = \frac{m-7}{5}$
 j $x = \frac{6-m}{5}$ k $x = \frac{4-m}{3}$ l $x = \frac{-(m+8)}{6}$
 m $x = \frac{y-4}{m}$ n $x = \frac{y+3}{m}$ o $x = \frac{y-c}{m}$
 p $x = \frac{y}{mz}$
- 17 a $y = \frac{c-a}{b}$ b $y = \frac{r-p}{q}$ c $y = \frac{r+p}{x}$
 d $y = \frac{a-b}{c}$ e $y = \frac{x-t}{k}$ f $y = \frac{4-c}{r}$
 g $y = \frac{-r-a}{b}$ h $y = \frac{r-k}{p}$ i $y = \frac{r+px}{q}$
- 18 a $N = 3.5$ b $N = 0.5$
 19 $l = 16.9$ 20 $V = 113.4$ 21 $v = \pm 9.8$

EXERCISE 3E TRAVEL CALCULATIONS

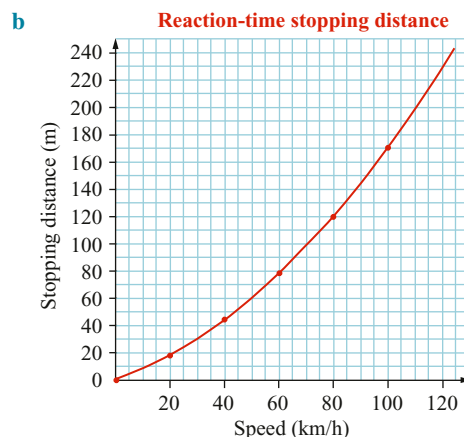
- 1 a 46 km/h b 73 km/h c 47 km/h d 59 km/h
 2 a 224 km b 187 km c 474 km d 166 km
 3 a 8 h 6 min b 4 h 2 min c 4 h 27 min d 56 min
 4 a $70 \text{ km/h} = \frac{70 \times 1000}{60 \times 60} \text{ m/s} = 19.4 \text{ m/s}$ (to one decimal place)
 b $8 \text{ m/s} = \frac{8 \times 60 \times 60}{1000} \text{ km/h} = 28.8 \text{ km/h}$
 5 a 12.5 m/s b 21.1 m/s c 30.6 m/s
 6 a 54 km/h b 45 km/h c 90 km/h
 7 55.6 m (to one decimal place)
 8 69.4 m (to one decimal place)
 9 a



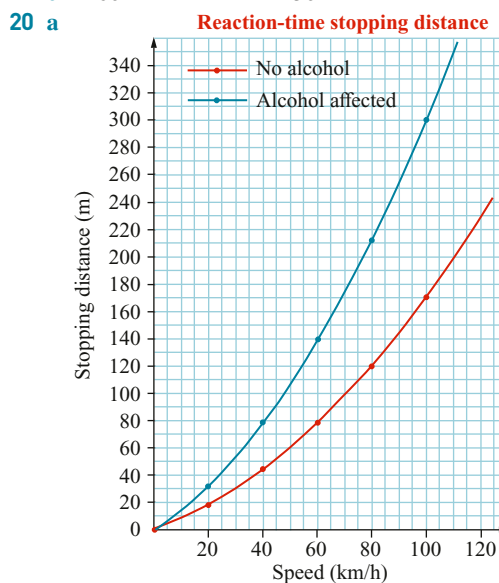
- b i 83 m ii 31 m c About 7 m
 10 64 m
 11 a 100 m b 140 m c 40 m
 12 $\frac{90000}{60 \times 60} \times 2.5 + 0.01 \times 90^2 = 144 \text{ m}$ (to the nearest metre)
 13 197.4 m
 14 a 58.3 m b 16.7 m
 15 55.6 m 16 58.8 m, 16.8 m, 56 m
 17 18 m 18 98 m

19 a

Speed (km/h)	0	20	40	60	80	100
Stopping distance (m)	0	18	44	78	120	170



- c i 60 m ii $\approx 230 \text{ m}$



- b i 45 m ii 60 m iii 150 m

EXERCISE 3F BLOOD ALCOHOL CONTENT

- 1 a 0.375, 1.4 b 2.7, 0.8
 2 a 1.1 b 1.2 c 1.8
 d 6.8 e 1.1 f 7.7
 3 a $\text{BAC}_{\text{male}} = \frac{10 \times 6 - 7.5 \times 3}{6.8 \times 83} \approx 0.07$
 b $\text{BAC}_{\text{female}} = \frac{10 \times 5 - 7.5 \times 4}{5.5 \times 58} \approx 0.06$
 4 a $\text{BAC} \approx 0.03$ b $\text{BAC} \approx 0.09$
 c $\text{BAC} \approx 0.06$ d $\text{BAC} \approx 0.02$
 5 $0.05 = \frac{10N - 7.5 \times 5}{6.8 \times 80} = \frac{10N - 37.5}{544}$
 $27.2 = 10N - 37.5$
 $64.7 = 10N$
 $N = 6.47$
 If he has 6.47 standard drinks his $\text{BAC} = 0.05$.
 For his $\text{BAC} < 0.05$ he can have up to 6 standard drinks.
 6 Up to 3 standard drinks
 7 a 5 b 7 c 4 d 6
 8 6 standard drinks, 3 standard drinks
 9 a 2 h b 1 h c 2 h 30 min
 d 50 min e 1 h 20 min

- 10 $T = \frac{0.05}{0.015} = 3.33... \text{ h} = 3 \text{ h } 20 \text{ min}$
 11 a 2 h 40 min b 1 h 20 min c 4 h d 2 h 20 min
 12 a Male: 0.0321... $\approx$ 0.03, Female: 0.0611... $\approx$ 0.06
 b Male: 2 h 9 min, Female: 4 h 5 min
 13 a Damien: 0.14, Nicole: 0.17
 b Damien: 10:20 am, Nicole: 12:20 pm
 14 a 7.2 b 0.0858 ... $\approx$ 0.09 c 5 h 43 min

EXERCISE 3G MEDICATION CALCULATIONS

- 1 a 2
 b i 12, maximum is 8. ii 4
 c No; took 3 tablets in 24 hours, allowed 4.
 2 a 2 b Yes, maximum is 6.
 3 a 5 mL b 5.6 mL c 3.6 mL
 d 4.3 mL e 11.3 mL f 12.3 mL
 4 a 53 mL b 95 mL
 5 a 17 mL b 16 mL c 22 mL
 d 8 mL e 34 mL f 31 mL
 6 a 25 mL b 15 mL
 7 a 10 mL b 23 mL c 58 mL
 d 42 mL e 46 mL f 20 mL
 8 a 36 mL b 48 mL c 20 kg
 9 a 175 mL/h b 340 mL/h c 133 mL/h
 d 120 mL/h e 28 mL/h f 355 mL/h
 10 a 0.9 L b 1.4 L c $3\frac{1}{3}$ hours
 11 a 50 drops/min b 63 drops/min c 50 drops/min
 d 75 drops/min e 38 drops/min f 38 drops/min
 12 a 43 drops/min b 23 drops/min
 13 a 10 h b 6 h 40 min c 10 h
 d 16 h e 20 h f 22 h 30 min
 14 a 2.1 mL b 1.9 mL c 2.3 mL
 d 2 mL; all about 2 mL
 15 a 4 mL b 3.7 mL c 5.1 mL
 d 4 mL; closest to the mean of all three
 16 a 40 mL b 23 mL
 c No, but close to overdose
 d After 17 minutes

REVIEW MULTIPLE-CHOICE QUESTIONS

- 1 D 2 A 3 B 4 D 5 B 6 D
 7 B 8 A 9 C 10 C 11 A 12 B
 13 D 14 C 15 C 16 D 17 B 18 A
 19 C 20 B

REVIEW SET 1

- 1 -18
 2 a 1437 cm³ b 180 cm³ c 0.9 m³
 3 a $F = 392^\circ$ b $F = 68^\circ$ c $F = 131^\circ$
 4 a $d = 44$ b $x = 11$ c $x = 576$
 d $x = 5$ e $x = 11\frac{1}{2}$ f $x = 24$
 g $n = 7.75$ h $c = -12\frac{1}{6}$ i $x = \frac{1}{6}$
 5 $E = 10000$
 6 a $x = \frac{t}{3}$ b $x = \frac{y+7}{m}$ c $x = ac - d$
 7 150 m 8 218.4 km 9 4 h 40 min
 10 3.6 mL 11 15.8 mL 12 50 drops/min

REVIEW SET 2

- 1 -31
 2 a 201 cm² b 707 cm² c 8.0 m²
 3 a $C = 120^\circ$ b $C = 10^\circ$ c $C = 0^\circ$

- 4 a $d = 32$ b $x = 61$ c $x = 15$
 d $x = 11$ e $x = 12\frac{1}{3}$ f $x = 6\frac{3}{4}$
 g $c = -3\frac{4}{9}$ h $x = -2\frac{13}{14}$ i $x = 3\frac{2}{3}$
 5 $b = 18$
 6 a $x = m - y$ b $x = \frac{k-n}{3}$ c $x = k(m+4)$
 7 58.4 km/h 8 0.045 9 2.7 mL
 10 6.4 mL 11 4 h

REVIEW SET 3

- 1 52 2 2356 cm² 3 3.1 s
 4 a $d = 20$ b $x = 10$ c $x = 6$
 d $x = 7$ e $x = 6\frac{2}{3}$ f $x = 4\frac{3}{7}$
 g $n = 3$ h $c = 3$ i $x = \frac{1}{4}$
 5 $n = 20$
 6 a $x = ty$ b $x = \frac{k-y}{m}$ c $x = wy + t$
 7 143.5 m 8 1.4
 9 a 0.05 b 0.035
 10 4 h
 11 a Male: 0.0343, female: 0.06
 b Male: 2 h 17 min, female: 4 h
 12 36 drops/min 13 19 mL

REVIEW SET 4

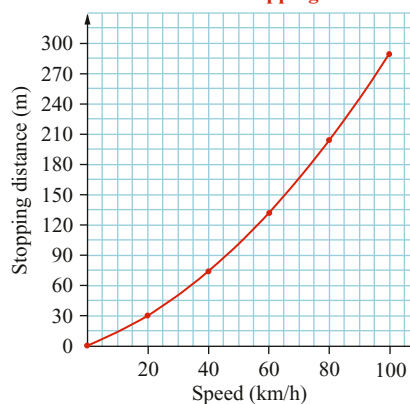
- 1 908 2 11 310 cm³ 3 $B = 9.4 \text{ mm}$
 4 a $d = 16$ b $x = 7$ c $x = 12$
 d $x = 11$ e $x = 32$ f $x = 2\frac{7}{8}$
 g $n = 6$ h $c = -\frac{4}{9}$ i $x = -\frac{7}{12}$
 5 $m = 80$ 6 $R = 50$ 7 1 h 52 min
 8 a 15.3 m/s b 77 km/h
 9 42 m 10 64 m 11 120 m

12 a

Speed (km/h)	0	20	40	60	80	100
Stopping distance (m)	0	29	73	131	203	290

b

Reaction-time stopping distance



- c i 100 m ii 165 m iii 340 m

13 9 mL

- 14 a 122 mL/h b 37 drops/min

REVIEW PRACTICE EXAMINATION QUESTION

- 1 a i $x = -15$ ii 8
 b i The height must be in metres; that is, 1.68 m not 168 cm.
 ii 19.5 iii No, her BMI is too low.
 c He added 6 instead of subtracting 6, then divided by 14 not -5. Solution is $x = -\frac{2}{5}$.
 d i 78 m ii 20 m

- e i 0.075... $\approx$ 0.08 ii 5 h
f 27 mL

CHAPTERS 1–3 CUMULATIVE REVIEW

- 1 a \$1077.46 b \$25 864.80 c \$1268.80
d \$1113.30 e \$387.50
f i \$465.04 ii \$14.53
g \$1733.40
h i \$938.96 ii \$121
i \$1103.60
- 2 a \$48 570 b \$837.88
c i \$11 985.93 ii \$3799.18
iii \$222.11 iv \$0
d i \$47 556 ii \$43 995 iii \$879.90
iv \$5845.38 v \$1999.72 refund
- 3 a i 4300 ii 4263 iii 4263.09
iv 4260 v 4263.1 vi 4000
b i No ii No iii Yes
c i 1.05×10^5 ii 6.27×10^{-5} iii 4.56×10^1
d i 2.87×10^{14} ii 5.12×10^{17} iii 4.36×10^4
e i 6730 m ii 17.5 m iii 4300 mg
iv 47 t v 6350 mL vi 0.09 L
vii 3.2×10^9 m
viii 52 000 kL
ix 1.65 m^2
f i 0.1 m ii 0.05 m
iii 6.75 m to 6.85 m iv 0.7%
- 4 a 52 b 1508 cm^2
- 5 a $d = 21$ b $x = 8$ c $x = 18$
d $x = 5$ e $x = 9$ f $x = 4$
g $n = 5$ h $c = \frac{9}{4} = 2\frac{1}{4}$ i $x = -\frac{1}{14}$
- 6 a 240 b 4.4
- 7 a $x = 16$ b $x = -8$ c $x = 13$
d $x = 3$ e $x = 4$ f $x = 6$
g $x = -\frac{3}{2}$ h $x = -\frac{4}{7}$ i $x = 27$
- 8 a $y = 3p$ b $y = \frac{4-x}{3}$ c $y = 2w + x$
- 9 145 m
- 10 a 0.08 b 5 h 20 min

CHAPTER 4 CLASSIFYING DATA AND SAMPLING

ARE YOU READY?

- 1 D 2 C 3 B 4 C 5 D 6 D
7 C 8 C 9 B 10 C 11 B

EXERCISE 4A COLLECTING AND CLASSIFYING DATA

- 4 a Numerical discrete b Numerical continuous
c Categorical nominal d Numerical discrete
e Numerical discrete f Categorical nominal
g Categorical ordinal h Numerical discrete
i Numerical continuous j Numerical discrete
k Categorical ordinal l Numerical continuous
m Categorical nominal n Categorical ordinal
o Numerical discrete p Categorical nominal
q Categorical ordinal

EXERCISE 4B CENSUS OR SAMPLE AND SELF-SELECTED SAMPLING

- 1 a Commuters who use buses on the bus route
b Companies that pay road tax

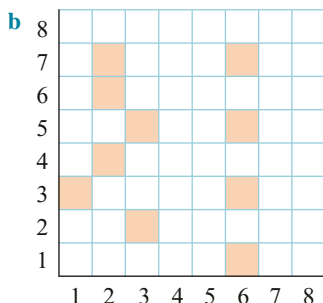
- c Dog owners who have used the new treatment
d Bank customers

- 2 a Census b Census c Sample
d Sample e Sample f Census
g Census or sample h Sample
i Census j Census k Census or sample
l Census m Sample n Census or sample
o Census p Sample q Sample
- 7 a A self-selected sample is a sample that the participants choose to be part of.
b A voluntary response sample
c Participants usually select themselves to be part of a study by responding to an advertisement in a newspaper, magazine, journal or an online noticeboard. The volunteers are then checked to make sure that they satisfy the criteria of the study.
d Can reduce the time and expense of finding suitable subjects that meet the selection criteria. Volunteers are likely to be committed to take part in the study. They may also show a greater willingness to provide more insight into the subject of the research by giving full detailed responses to open ended questions.
e Because the people chosen are volunteers, there is likely to be a degree of self-selection bias. This can either lead to the sample not being representative of the population being studied, or exaggerating some particular finding from the study.
- 8 a Self-selected sample

EXERCISE 4C RANDOM SAMPLING

- 1 a 2, 0, 9, 7, 4 b 2, 5, 6, 9, 1
c 2, 3, 7, 9, 4 d 2, 7, 4, 5, 9
- 2 a 20, 97, 74, 80, 5 b 25, 69, 17, 47, 51
c 20, 39, 74, 98, 42 d 20, 79, 41, 50, 91
- 3 a 209, 774, 800, 583, 418 b 256, 917, 475, 186, 655
c 209, 399, 740, 988, 423 d 209, 791, 410, 509, 919
- 5 a Assign each student a number from 1 to 80; random sample contains students with numbers 27, 35, 30, 17, 41, 4, 51, 38, 65, 60
b Assign each student a number from 1 to 35; random sample contains students with numbers 27, 30, 17, 4, 15, 1, 33, 16, 25, 18
- 6 a Assign each student a number from 1 to 700; random sample contains students with numbers 182, 562, 80, 597, 374, 136, 124, 678, 504, 223
b Assign each student a number from 1 to 280; random sample contains students with numbers 182, 80, 136, 124, 223, 190, 219, 154, 84, 41
- 7 a Write the workers' names on pieces of paper, place the names in a hat, mix and draw 5 names. Write the workers' names on cards, shuffle the cards and deal 5.
b 24, 4, 13, 5, 9
- 8 a Assign the numbers 1 to 70 to each shop assistant. Select 10 random numbers from 1 to 70 using a table of random numbers. The shop assistants whose numbers are selected form the sample.
b 49, 24, 63, 67, 4, 13, 49, 26, 61, 27
- 9 Assign the numbers 1 to 760 to each student. Select 100 random numbers from 1 to 760 using a table of random numbers. The students whose numbers are selected form the sample.

- 10 a Number the hospitals 1 to 12. Use a table of random numbers to select 6 random numbers less than or equal to 12. The hospitals whose numbers are selected form the sample.
b 4, 8, 12, 7, 9, 10
11 a (2, 4), (6, 3), (6, 7), (1, 3), (2, 6), (6, 1), (2, 7), (6, 5), (3, 5), (3, 2)



- 18 a Start at first pair of digits and, moving right, select pairs whose first and second members are each a number from 1 to 8.
c i 18 seedlings ii 1440 seedlings
19 a Method 1: Put all the teachers' names in a hat, mix and select 10 names.
Method 2: Assign the numbers 1 to 62 to the teachers. Use a table of random numbers to select 10 random numbers less than or equal to 62. The teachers whose numbers are selected form the sample.
b 49, 24, 1, 13, 49, 26, 61, 27, 35, 32
c i 30.5 minutes ii 3 hours and 3 minutes

EXERCISE 4D STRATIFIED RANDOM SAMPLING

- 1 Boys: $\frac{75}{140} = \frac{15}{28}$ or 53.6%, girls: $\frac{65}{140} = \frac{13}{28}$ or 46.4%
2 Boys: $\frac{19}{45}$ or 42.2%, girls: $\frac{26}{45}$ or 57.8%
3 a Year 7: $\frac{125}{859}$, Year 8: $\frac{142}{859}$, Year 9: $\frac{175}{859}$, Year 10: $\frac{160}{859}$, Year 11: $\frac{134}{859}$, Year 12: $\frac{123}{859}$
b 9, 10, 12, 11, 9, 9
c Choose at random 9 students from Year 7, 10 from Year 8, 12 from Year 9, 11 from Year 10, 9 from Year 11, 9 from Year 12.
4 a $\frac{96}{439}, \frac{84}{439}, \frac{78}{439}, \frac{65}{439}, \frac{56}{439}, \frac{60}{439}$
b 9, 8, 7, 6, 5, 5
c Choose at random the required number of students from each year.
5 a Girls: $\frac{69}{120}$, boys: $\frac{51}{120}$ b 23 girls, 17 boys
c Choose at random 23 girls and 17 boys.
6 a 0–9: $\frac{180}{2475} = \frac{4}{55}$, 10–19: $\frac{835}{2475} = \frac{167}{495}$, 20–29: $\frac{260}{2475} = \frac{52}{495}$, 30–39: $\frac{350}{2475} = \frac{14}{99}$, 40–49: $\frac{280}{2475} = \frac{56}{495}$, 50–59: $\frac{250}{2475} = \frac{10}{99}$, 60–69: $\frac{200}{2475} = \frac{8}{99}$, 70+: $\frac{120}{2475} = \frac{8}{165}$
b 0–9: 3, 10–19: 13, 20–29: 4, 30–39: 6, 40–49: 5, 50–59: 4, 60–69: 3, 70+: 2
c Choose at random the required number of people from each age group.
8 a Bus: $\frac{87}{150} = \frac{29}{50}$, ride: $\frac{23}{150}$, car: $\frac{30}{150} = \frac{1}{5}$, walk: $\frac{10}{150} = \frac{1}{15}$
b Bus: 17, ride: 5, car: 6, walk: 2
c Choose at random 17 students who travel by bus, 5 students who ride bicycles, 6 students who travel by car and 2 students who walk.

EXERCISE 4E SYSTEMATIC SAMPLING

- 1 a Every 12th item b Every 6th item
c Every 30th item d Every 20th item
2 a Every 12th name b Every 6th name
c Every 8th name d Every 30th name
3 a Start at any random number and select every 20th student.
b i 10, 30, 50, 70, 90, 110, 130, 150, 170, 190
ii 15, 35, 55, 75, 95, 115, 135, 155, 175, 195
iii 30, 50, 70, 90, 110, 130, 150, 170, 190, 10
iv 45, 65, 85, 105, 125, 145, 165, 185, 5, 25
v 113, 133, 153, 173, 193, 13, 33, 53, 73, 93
4 a Start at any random number and select every 12th name.
b i 5, 17, 29, 41, 53, 65, 77, 89, 101, 113, 125, 137, 149, 161, 173
ii 10, 22, 34, 46, 58, 70, 82, 94, 106, 118, 130, 142, 154, 166, 178
iii 25, 37, 49, 61, 73, 85, 97, 109, 121, 133, 145, 157, 169, 1, 13
iv 50, 62, 74, 86, 98, 110, 122, 134, 146, 158, 170, 2, 14, 26, 38
v 120, 132, 144, 156, 168, 180, 12, 24, 36, 48, 60, 72, 84, 96, 108
5 Start at any number and select every:
a 6th name b 9th name c 12th name
d 24th name e 19th name
6 a i Select every 40th part.
ii Select every 17th part.
7 a Assign the numbers 1 to 30 to the students, choose the first number at random and then select every 5th number until he has 6 students.
b (9), 14, 19, 24, 29, 4
c (9), 15, 21, 27, 3
8 a Assign the numbers 1 to 600 to the soldiers, choose the 1st number at random and then select every 60th number until a sample of 10 has been chosen.
b (23), 83, 143, 203, 263, 323, 383, 443, 503, 563
c i Every 12th soldier. ii 5

EXERCISE 4F SUITABILITY OF SAMPLE TYPES

- 1 Possible answers are:
a Systematic sampling b Random sampling
c Stratified random sampling d Random sampling
e Stratified random sampling f Systematic sampling
g Self-selected sampling h Random sampling
i Random sampling
j Stratified random sampling
2 a Census b Categorical c $\frac{95}{200} = \frac{19}{40}$
d i $\frac{1}{5}$, no ii $\frac{1}{5}$, no
f i every 40th student, answers will vary
ii every 20th student, answers will vary
iii every 10th student, answers will vary
iv every 5th student, answers will vary
g i ≈ 3 male; ≈ 2 female, answers will vary
ii ≈ 6 male; ≈ 4 female, answers will vary
iii ≈ 11 male; ≈ 9 female, answers will vary
iv ≈ 23 male; ≈ 17 female, answers will vary
h The larger the sample, the closer it approximates to the population response.
3 a Percentage = $\frac{12}{20} \times 100 = 60\%$
b Number = $\frac{60}{100} \times 4000 = 2400$

c Adequate childcare = $\frac{5}{20} \times 4000 = 1000$

d Unsure = $\frac{8}{20} \times 4000 = 1600$

4 a 62.5% b 31 250 c 35 000 d 3750

5 a 70% b 560 c 240

6 a 30% b 180 c 420

- 7 a It may or may not be representative; it depends on how students are allocated to classes in your school.
 b The sample is too small to be representative of the whole population of Year 11 students in your state. It may also be biased.
 8 a Dentists may be paid by Britex. The sample is not representative of the whole population.
 b Is the 10% across all positions from senior management down or is it 10% of the lowest paid workers? This could be misleading as, for example, if it was 10% of the 10 managers then it would be 1 person. If it was 10% of the 500 factory workers then it would be 50 people.
 c Yes, depending on how the sample was grouped.

REVIEW MULTIPLE-CHOICE QUESTIONS

- 1 B 2 A 3 C 4 D 5 A 6 B
 7 C 8 A 9 B 10 D 11 C

REVIEW SET 1

- 1 a Quantitative discrete b Quantitative continuous
 c Categorical ordinal d Categorical nominal
 2 a Census b Census c Census d Sample
 3 a Only people at home on Saturday night are surveyed.
 b Only people on electoral roll are surveyed.
 4 Write the names of the students on cards, shuffle the cards and select 12.
 5 a Moving left across the list gives (5, 5), (1, 2), (2, 2), (5, 1), (2, 5).
 b

5					
4					
3					
2					
1					
	1	2	3	4	5

6 a Year 7: $\frac{85}{500} = \frac{17}{100}$, Year 8: $\frac{82}{500} = \frac{41}{250}$, Year 9: $\frac{89}{500}$,
 Year 10: $\frac{96}{500} = \frac{24}{125}$, Year 11: $\frac{75}{500} = \frac{3}{20}$, Year 12: $\frac{73}{500}$

b 17, 16, 18, 19, 15, 15

- 7 Assign the numbers 1 to 300 to the students, choose the first number at random and then select every 15th number.
 9 a 30% b 120 c 280
 10 a Assign the numbers 1 to 74 to the drivers. Use a table or calculator to select 8 random numbers less than or equal to 74. The drivers whose numbers are selected form the sample.
 b 7, 9, 73, 65, 53, 52, 49, 54
 11 a Yes, if the number of classes is small (2 or 3).
 b No

REVIEW SET 2

- 2 a Sample b Census
 3 a Members of the local netball club
 b Senior students
 4 a Members of another sporting club who may want their own facilities improved, or non-sporting people.
 b Junior students

5 86, 70, 101, 190, 13

6 a Vince: $\frac{56}{200} = \frac{7}{25}$, Sophie: $\frac{43}{200}$, Talia: $\frac{51}{200}$,
 Mohammed: $\frac{50}{200} = \frac{1}{4}$

b Vince: 6, Sophie: 4, Talia: 5, Mohammed: 5

7 Self-selected sampling

- 9 *Method 1:* Write names of students on pieces of paper and put in a box. Select 6 pieces of paper at random from the box.
Method 2: Assign the numbers 1 to 24 to the students. Use a table of random numbers to select 6 numbers which are less than or equal to 24.

REVIEW SET 3

- 1 a Categorical nominal b Quantitative continuous
 c Quantitative discrete d Categorical ordinal
 3 a People who use the bus service
 b People who use the pool.
 4 a i Every 15th part ii Every 25th part
 5 Advantages: Can reduce the time and expense of finding suitable subjects that meet the selection criteria. Volunteers are likely to be committed to take part in the study. They may also show a greater willingness to provide more insight into the subject of the research by giving full detailed responses to open ended questions.
 Disadvantages: Because the people chosen are volunteers, there is likely to be a degree of self-selection bias. This can either lead to the sample not being representative of the population being studied, or exaggerating some particular finding from the study.
 6 a 80% b 240 c 60
 9 a If there are only a few bus stops on the route then the number of people at one bus stop would be suitable. If there are many bus stops then one bus stop is not satisfactory.
 b No. One carriage on one train out of many trains in service is not suitable.

REVIEW SET 4

- 1 Self-selected sample
 3 Advantage: collects data about every member of the population so is detailed and accurate.
 Disadvantage: often expensive, time-consuming and impractical.
 4 a The tyre distributor would only recommend tyres that they sell.
 b The sample is biased towards people whose names start with one (or two) letters of the alphabet.
 5 a Number every employee from 1 to 120. Use a table of random numbers to choose 25 different numbers from 1 to 120.
 b Put names in a hat and draw names at random. Other answers are possible.
 c Start at any number and select every 5th employee.
 6 'Biased' means 'not free from prejudice'.
 7 A good sample is one whose characteristics are representative of the whole population.
 8 86, 70, 101, 190, 13
 9 a Assign the numbers 1 to 24 to the workers.
 Use a table or calculator to select five random numbers less than or equal to 24. The workers whose numbers are selected form the sample.
 b 8, 12, 7, 9, 19

REVIEW PRACTICE EXAMINATION QUESTION

- 1 a i Numerical discrete ii Categorical nominal
b i Sample ii Census
c 7, 9, 73, 65, 53
d i Write names of drivers on pieces of paper and put them in a hat. Shake and select 6 names. Assign the numbers 1 to 48 to the drivers and select 6 numbers less than or equal to 48 from a table of random numbers (or use a calculator to generate the random numbers).
e i Boys: $\frac{67}{120}$, girls: $\frac{53}{120}$ ii Boys: 17, girls: 13
f Take a random sample of 17 boys and a random sample of 13 girls.
g i intervals of 30 ii 7
h 425

CHAPTER 5 PERIMETER, AREA AND VOLUME

ARE YOU READY?

- 1 C 2 C 3 B 4 C 5 C
6 A 7 B 8 A 9 D

EXERCISE 5A PERIMETER

- 1 a 102 mm b 189 mm c 297 mm
2 a 7.8 cm b 13.2 cm c 33.6 cm
3 $x = 6 + 3 = 9$, $y = 15 - 7 = 8$,
 $P = 15 + 3 + 7 + 6 + 8 + 9 = 48$ cm
4 a $x = 26$, $y = 13$, $P = 174$ cm
b $x = 37$, $y = 8$, $P = 142$ cm
c $x = 5$, $y = 5$, $P = 74$ cm
5 a $x \approx 11.3$ cm, $P \approx 43.3$ cm
b $x \approx 17.2$ cm, $P \approx 60.2$ cm
c $x \approx 14.4$ cm, $P \approx 95.3$ cm
6 a 13 cm b 52 cm
7 a $C = \pi d = \pi \times 12 \approx 37.7$ cm
b $C = 2\pi r$, $r = 12 \div 2 = 6$, $C = 2 \times \pi \times 6 \approx 37.7$ cm
8 a 28.3 cm b 20.1 m c 56.9 m d 42.7 m
e 39.6 m f 49.6 m g 15.7 m h 27.1 m
9 a 20.6 cm b 25.7 cm c 86.3 m d 102.8 m
10 a 21.4 cm b 16.8 cm c 55.7 m d 75.7 cm
11 a 21.42 cm b 45.71 m c 57.13 cm
12 a Fraction = $\frac{15^\circ}{360^\circ} = \frac{1}{24}$
b $P = \frac{1}{24} \times 2 \times \pi \times 7.2 + 7.2 + 7.2 \approx 16.3$ cm
13 a $\frac{1}{24}$, $P \approx 14.5$ cm b $\frac{1}{8}$, $P \approx 14.8$ cm
c $\frac{1}{12}$, $P \approx 18.2$ cm d $\frac{1}{4}$, $P \approx 54.6$ cm
e $\frac{1}{3}$, $P \approx 31.9$ cm f $\frac{29}{36}$, $P \approx 58.6$ cm
14 a 35.9 cm b 22.4 cm c 46.0 cm
15 a 6.7 cm b 11.7 cm c $P \approx 36.8$ cm
16 125 cm
17 \$237.15
18 a 12.57 m, 12.57 m b Same distance c No difference
19 \$744 20 8.0 km 21 31.8 m
22 a 7200 km b 45 239 km c 1885 km/h
23 64.80 km

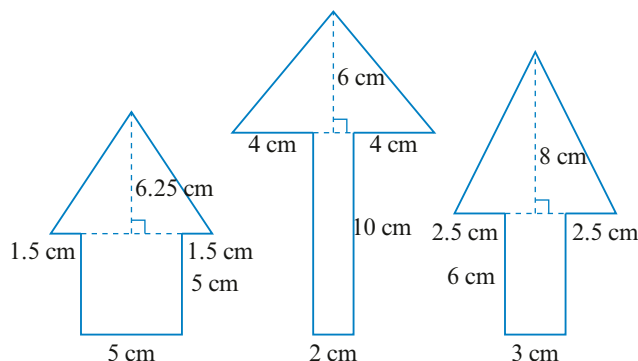
EXERCISE 5B AREA OF SIMPLE SHAPES

- 1 a 60.63 cm² b 9.1 cm² c 64 cm² d 38.11 cm²
2 a 18 m² b 5 cm² c 800 m² d 20 cm²

- 3 a 60 mm² b 750 mm² c 168 cm² d 100 m²
4 a 2.7 m² b 16.5 km² c 24 mm² d 29.9 m²
5 a 45 cm² b 27.84 m² c 2.7 km² d 2185 mm²
6 a 30 cm² b 98 m² c 40 m² d 1190 mm²
7 a 78.5 cm² b 254.5 cm² c 706.9 cm² d 314.2 cm²
e 132.7 cm² f 38.5 cm² g 254.5 cm² h 380.1 cm²
8 a Fraction = $\frac{15^\circ}{360^\circ} = \frac{1}{24}$
b $A = \frac{1}{24} \times \pi \times 7.2^2 \approx 7$ cm²
9 a 5 cm² b 11 cm² c 14 cm²
d 48 cm² e 184 cm² f 64 cm²
g 45 cm² h 15 cm² i 117 cm²
10 a 9 cm b 8 cm
c 8 cm d 5.6 m
11 a 20025 m² b 2 ha
c One possible set of dimensions is 225 m by 89 m.
12 33 13 62 cm² 14 40%

EXERCISE 5C AREA OF COMPOSITE FIGURES

- 1 a 96 cm² b 44 m² c 22 m²
d 224 cm² e 65 cm² f 45.5 m²
2 a 168 cm² b 63 cm² c 78 cm²
d 130 cm² e 65 cm² f 107 m²
3 a 90 cm² b 34 m² c 33 m²
d 30 km² e 24 cm² f 50 m²
g 54 cm² h 75 cm² i 38 cm²
j 265.5 cm² k 52.5 cm² l 54.54 cm²
4 a 552 cm² b 207 cm² c 90 cm²
5 a 127.0 cm² b 69.1 cm² c 49.1 cm²
d 190.1 cm² e 63.3 cm² f 68.6 cm²
g 144.3 cm² h 39.8 cm² i 164.5 cm²
6 a 85 cm² b 204 cm² c 302 cm²
d 236 cm² e 104 cm² f 415 cm²
7 a 31 cm² b 40 cm² c 219 cm²
d 113 cm² e 75 cm² f 107 cm²
8 a 18.53 m² b \$10 006
9 84.8 m²
10 a 337 m² b \$3340
11 a 37 m² b \$1291.30
12 a 113.1 cm² b 254.5 cm² c 141.4 cm²
d 42.4 cm² e 28.3 cm²
13 a 75.4 cm² b 530.1 mm² c 35.6 cm²
d 25.1 cm² e 25.5 cm² f 49.1 cm²
14 Some possible answers are:



EXERCISE 5D PERIMETER AND AREA OF IRREGULAR SHAPES

- 1 $A = \frac{1}{2} \times 22 \times 10 + \frac{1}{2} \times 22 \times 18 = 308$ m²
2 a 28 m² b 71.5 m² c 308 m²
d 1254 m² e 517 m² f 5217 m²

- 3 $(BC)^2 = 13^2 + (5 + 15)^2 = 569$
 $BC = \sqrt{569} \approx 23.9$
 $(CD)^2 = 13^2 + 11^2 = 290$
 $CD = \sqrt{290} \approx 17.0$
 $(AD)^2 = 12^2 + (15 + 11)^2 = 820$
 $AD = \sqrt{820} \approx 28.6$
 $(AB)^2 = 12^2 + 5^2 = 169$
 $AB = \sqrt{169} = 13$
Perimeter $\approx 23.9 + 17.0 + 28.6 + 13 = 82.5$ m
- 4 d 145.8 m e 91.1 cm f 295.6 m
- 5 Area $\triangle MQP = \frac{1}{2} \times (10 + 12 + 3 + 8) \times 16 = 264$ m²
Area $\triangle MRN = \frac{1}{2} \times 10 \times 11 = 55$ m²
Area $\triangle TOP = \frac{1}{2} \times 7 \times 8 = 28$ m²
Area $\triangle NOTR = \frac{1}{2} \times (12 + 3) \times (7 + 11)$
 $= \frac{1}{2} \times 15 \times 18 = 135$ m²
Area $MNOPQ = 264 + 55 + 28 + 135 = 482$ m²
- 6 a 3662 m² b 964 m² c 16 625 m²
- 7 $(MN)^2 = 10^2 + 11^2 = 221$
 $MN = \sqrt{221} \approx 14.87$ m
 $(NO)^2 = (11 - 7)^2 + (12 + 3)^2 = 241$
 $NO = \sqrt{241} \approx 15.52$ m
 $(OP)^2 = 7^2 + 8^2 = 113$
 $OP = \sqrt{113} \approx 10.63$ m
 $(PQ)^2 = (3 + 8)^2 + 16^2 = 377$
 $PQ = \sqrt{377} \approx 19.42$ m
 $(QM)^2 = 16^2 + (10 + 12)^2 = 740$
 $QM = \sqrt{740} \approx 27.20$ m
Perimeter $\approx 14.87 + 15.52 + 10.63 + 19.42 + 27.20$
 $= 87.64$ m
 ≈ 88 m
- 8 a 245.27 m $\approx$ 245 m b 122.09 m $\approx$ 122 m
c 514.36 m $\approx$ 514 m
- 9 Area $\approx \frac{40}{2}(98 + 52) = 20 \times 150 = 3000$ m²
- 10 a 3478 m² b 3094 m² c 2260.5 m²
- 11 a 365 m² b 1380 m² c 372 m²
d 1909.5 m² e 545 m² f 1024 m²
- 12 a 33 000 m² b 15 600 m² c 17 400 m²
d 174 000 m³
- 13 a 3000 m² b 54 000 m³
- 14 a 549 m² b 551.25 m²
- 15 a 225.5 m² b 2255 m³ c 133 500 m³
- 16 a 344 m² b 6880 m³ c 41 280 m³

EXERCISE 5E SURFACE AREA OF PRISMS

- 1 a 54 cm² b 1350 cm² c 864 cm²
- 2 a 138.24 cm² b 1734 mm² c 0.5766 m²
- 3 Top and bottom: $A = 2 \times 10 \times 6 = 120$ cm²
Front and back: $A = 2 \times 5 \times 10 = 100$ cm²
Two sides: $A = 2 \times 5 \times 6 = 60$ cm²
Surface area = $120 + 100 + 60 = 280$ cm²
- 4 Area of front and back: $A = 2 \times 8 \times 4 = 64$ cm²
Area of both sides: $A = 2 \times 5 \times 4 = 40$ cm²
Area of top and bottom: $A = 2 \times 8 \times 5 = 80$ cm²
Surface area = $64 + 80 + 40 = 184$ cm²
- 5 a 276 cm² b 424 cm² c 598 cm²
d 432 cm² e 8970 mm² f 8440 m²
- 6 a 280 m² b 684 mm² c 382.46 m²
- 7 a 600 cm² b 224 cm² c 1490 cm²
d 300 cm² e 3150 cm² f 241.6 cm²

- 8 a 880 cm² b 614 cm² c 1260.16 cm²
d 1050.18 cm² e 9320 cm² f 1156.40 cm²
- 9 a 1120 cm² b \$14
- 10 a Room A = 38.28 m², Room B = 34.4 m². Room A has the greater area.
b 4 tins c \$219.80
- 11 7 m²
- 12 Weet-Bix: 1161 cm², Sultana Bran: 1262.5 cm². Sultana Bran box has the greater surface area.
- 13 a 313.4 m² b 5 cans c \$412.50
- 14 a 231.7 cm² b 285.1 cm²
- 15 25.56 m²

EXERCISE 5F SURFACE AREA OF CYLINDERS AND SPHERES

- 1 $A = 2\pi rh = 2 \times \pi \times 4 \times 9 = 226.19... \approx 226.2$ cm²
- 2 a 534.1 cm² b 664.8 cm² c 1055.6 cm²
d 256.0 cm² e 1005.3 cm² f 89.2 cm²
- 3 Diameter = 12 cm, so radius = 6 cm
 $A = 2 \times \pi \times 6 \times 8 = 301.593... \approx 301.6$ cm²
- 4 a 942.5 cm² b 1017.9 cm² c 1178.1 cm²
d 303.5 cm² e 1372.9 cm² f 115.5 cm²
- 5 a 301.6 cm² b 735.1 cm² c 94.2 cm²
d 830.4 cm²
- 6 Radius = $15.8 \div 2 = 7.9$ cm
 $A = 2 \times \pi \times 7.9 \times 17.6 + 2 \times \pi \times (7.9)^2$
 $= 1265.747...$
 ≈ 1265.7 cm²
- 7 a 1445.1 cm² b 603.2 cm² c 527.8 cm²
d 2264.4 cm² e 4246.4 cm² f 318.7 m²
- 8 a 1809.6 cm² b 3.1 m² c 53 092.9 mm²
- 9 $r = 18 \div 2 = 9$ cm
 $A = 4\pi r^2 = 4 \times \pi \times 9^2 = 1017.876... \approx 1017.9$ cm²
- 10 a 1963.5 cm² b 1063.6 cm² c 48 305.1 mm²
- 11 a 452.4 cm² b 1256.6 cm² c 907.9 cm²
- 12 a 445.46 cm² b 207.38 m² c 3.03 m²
- 13 a 2827 m² b 36 cans c \$4147.20
- 14 141 spheres
- 15 36 945 cm²
- 16 The roller with a diameter of 8 cm and length 25 cm has a larger surface area and so would not need to be re-dipped as often.
- 17 74%
- 18 No
- 19 a 157.7 cm² b 173.4 m²
c 424.1 m²
- 20 The area of the top surface of the cylinder is the same as the area not counted when calculating the surface area of the top of the rectangular prism.
- 21 \$4220.31

EXERCISE 5G VOLUME

- 1 $V = A \times h = 20 \times 3 = 60$ m³
- 2 a 75 cm³ b 115.2 cm³ c 420 cm³
d 815.4 cm³ e 11.96 cm³ f 120 cm³
g 648 cm³ h 116.96 cm³ i 48 cm³
j 360 cm³ k 140 cm³ l 80 cm³
- 3 a $V = A \times h = (9 \times 6) \times 5 = 270$ cm³
b $V = A \times h = (\frac{1}{2} \times 5 \times 4) \times 10 = 100$ cm³
- 4 a 270 cm³ b 420 cm³ c 108 cm³
d 343 cm³ e 720 cm³ f 703.08 cm³
g 1037.244 cm³ h 460.6 cm³ i 89.79 cm³
j 568.764 cm³ k 1536 cm³ l 31.29 cm³

- 5 a $V = A \times h = \pi \times r^2 \times h = \pi \times 4^2 \times 10 \approx 502.65 \text{ m}^3$
b $V = A \times h = \pi \times 4^2 \times 25 \approx 1256.64 \text{ m}^3$
- 6 a 571 cm^3 b 5089 cm^3 c 40 cm^3
- 7 a 942.5 cm^3 b 3318.3 cm^3 c 2058.9 cm^3
d 2544.7 cm^3 e $109\,858.4 \text{ cm}^3$ f $46\,876.3 \text{ cm}^3$
- 8 a 7238.2 cm^3 b 0.5 m^3
c $1\,150\,346.5 \text{ mm}^3$
- 9 a 8181.2 cm^3 b 3261.8 cm^3
c $998\,306.0 \text{ mm}^3$
- 10 a 904.8 cm^3 b 4188.8 cm^3 c 2572.4 cm^3
- 11 a 1250.24 cm^3 b 397.13 m^3 c 0.70 m^3
- 12 a i 325 cm^2 ii 1625 cm^3
b i 622 cm^2 ii 5598 cm^3
c i 143 cm^2 ii 1144 cm^3
d i 675 cm^2 ii 6075 cm^3
e i 147.84 cm^2 ii 1034.88 cm^3
f i 1120 cm^2 ii 4480 cm^3
- 13 If $r = 3 \text{ cm}$, $h = 4 \text{ cm}$, then $V = 113.1 \text{ cm}^3$.
If $r = 4 \text{ cm}$, $h = 3 \text{ cm}$, then $V = 150.8 \text{ cm}^3$.
Cylinder of radius 4 cm and height 3 cm has the greater volume.
- 14 a 28 cm^2 b 7 cm c 0.6 cm
- 15 b $48\,000\,000 \text{ mm}^3$ or $48\,000 \text{ cm}^3$
- 16 a 100.5 cm^3 b 120.7 m^3 c 733.0 m^3

EXERCISE 5H CAPACITY

- 1 a Volume $= (6.5 \times 1000) \text{ cm}^3 = 6500 \text{ cm}^3$
b Capacity $= (25\,600 \times 1) \text{ mL} = 25\,600 \text{ mL}$
 $= (25\,600 \div 1000) \text{ L} = 25.6 \text{ L}$
- 2 a 2000 cm^3 b $12\,000 \text{ cm}^3$ c 2600 cm^3
d 450 cm^3 e 975 cm^3
- 3 a 650 cm^3 b 7500 cm^3 c 1250 cm^3
d 3780 cm^3 e 845 cm^3
- 4 a 1495 mL b 3000 mL c 500 mL
d $24\,000 \text{ mL}$ e 6740 mL
- 5 a 1.495 L b 3 L c 0.5 L
d 24 L e 6.74 L
- 6 a 2 m^3 b 5 m^3 c 6.7 m^3
d 0.9 m^3 e 83.2 m^3
- 7 a 0.02 kL b 0.7 kL c 28.4 kL
d 302 kL e 5660 kL
- 8 Volume $= \text{area of base} \times \text{height} = (\pi \times 3 \times 3) \times 5 \text{ m}^3$
 $\approx 141.4 \text{ m}^3$
Capacity $\approx 141.4 \text{ kL}$
- 9 a 5832 cm^3 b 5832 mL c 5.832 L
- 10 a $704\,000\,000 \text{ cm}^3$ b $704\,000\,000 \text{ mL}$
c $704\,000 \text{ L}$ d 704 kL
- 11 6.8 mL 12 3.6 L 13 40 ML
- 14 Vase 1: 785.4 mL ; Vase 2: 730.0 mL ; Vase 1 holds
 55.4 mL more
- 15 a $60\,000 \text{ cm}^3$ b i $60\,000 \text{ mL}$ ii 60 L c 54 L
- 16 1508 L
- 17 a $907\,200 \text{ cm}^3$ b 907.2 L
- 18 a 181 m^3 b 181 kL
- 19 735 mL
- 20 a 9.2 L b 8 breaths
- 21 13 m
- 22 Inside volume of tank $= 60\,000 \text{ cm}^3$.
Suggest dimensions in cm for a rectangular prism where
length $\times$ width $\times$ height $= 60\,000 \text{ cm}^3$. One possible
answer is 100 cm by 60 cm by 10 cm .
- 23 724 mL

REVIEW MULTIPLE-CHOICE QUESTIONS

- 1 D 2 C 3 A 4 D 5 A
6 B 7 C 8 B 9 A 10 C
11 C 12 C 13 B 14 D 15 C
16 A 17 A 18 C 19 B

REVIEW SET 1

- 1 a $P = 21.2 \text{ cm}$, $A = 21 \text{ cm}^2$
b $P = 34 \text{ cm}$, $A = 48 \text{ cm}^2$
c $P = 28.3 \text{ cm}$, $A = 63.6 \text{ cm}^2$
d $P = 17.6 \text{ cm}$, $A = 19.4 \text{ cm}^2$
e $P = 174.0 \text{ cm}$, $A = 2000 \text{ cm}^2$
f $P = 53.4 \text{ cm}$, $A = 93.5 \text{ cm}^2$
- 2 a 405 m^2 b 514.5 m^2
- 3 a 136 m^2 b 1005.3 cm^2 c 706.9 cm^2
- 4 a 21 cm^3 b 240 cm^3 c 1005.3 m^3
- 5 $73\,622 \text{ mm}^3$ 6 2155 mL

REVIEW SET 2

- 1 a $P = 19.4 \text{ cm}$, $A = 12 \text{ cm}^2$
b $P = 11 \text{ cm}$, $A = 6.24 \text{ cm}^2$
c $P = 30.3 \text{ m}$, $A = 54 \text{ m}^2$
d $P = 22.5 \text{ cm}$, $A = 31.2 \text{ cm}^2$
e $P = 21.7 \text{ cm}$, $A = 25.5 \text{ cm}^2$
f $P = 32 \text{ cm}$, $A = 39 \text{ cm}^2$
- 2 a $P = 67.9 \text{ m}$, $A = 279 \text{ m}^2$ b 517.5 m^2
- 3 a 320 m^2 b 887.7 cm^2 c 221.7 cm^2
- 4 a 300 m^3 b 1967.5 cm^3 c 310.3 cm^3
- 5 0.06 ML

REVIEW SET 3

- 1 a $P = 32.8 \text{ cm}$, $A = 67.24 \text{ cm}^2$
b $P = 3.6 \text{ m}$, $A = 0.7 \text{ m}^2$
c $P = 47.6 \text{ cm}$, $A = 90 \text{ cm}^2$
d $P = 38.1 \text{ cm}$, $A = 78.5 \text{ cm}^2$
e $P = 43.0 \text{ cm}$, $A = 96.5 \text{ cm}^2$
f $P = 20.0 \text{ cm}$, $A = 24.1 \text{ cm}^2$
- 2 a $P = 170.0 \text{ m}$, $A = 1312.5 \text{ m}^2$
b 1056 m^2
- 3 a 311.0 mm^2 b 881.2 cm^2 c 2123.7 mm^2
- 4 a 751.8 cm^3 b 120 cm^3 c 175.9 cm^3
- 5 268.1 kL

REVIEW SET 4

- 1 a $P = 61.7 \text{ mm}$, $A = 166.8 \text{ mm}^2$
b $P = 15 \text{ m}$, $A = 12.7 \text{ m}^2$
c $P = 355.0 \text{ mm}$, $A = 10\,028.7 \text{ mm}^2$
d $P = 40.8 \text{ cm}$, $A = 100 \text{ cm}^2$
e $P = 66.0 \text{ cm}$, $A = 148.4 \text{ cm}^2$
f $P = 205.7 \text{ cm}$, $A = 2856.6 \text{ cm}^2$
- 2 a 4500 m^2 b 1056 m^2
- 3 a 779.8 m^2 b 949.3 cm^2 c 132.7 cm^2
- 4 a 1481.5 m^3 b 2224.6 cm^3 c 143.8 cm^3
- 5 192 kL

REVIEW PRACTICE EXAMINATION QUESTION

- 1 a 25.4 m
b $P = 25.2 \text{ cm}$, $A = 36.9 \text{ cm}^2$
c 833 m^2
d $SA = 380.1 \text{ cm}^2$, $V = 696.9 \text{ cm}^3$
e 1440 cm^3 f 1357.2 cm^3 g 1357 mL

CHAPTER 6 INTEREST AND DEPRECIATION

ARE YOU READY?

- 1 A 2 D 3 C 4 C 5 A 6 D
7 B 8 D 9 A 10 D 11 C 12 B

EXERCISE 6A PERCENTAGE APPLICATIONS

- 1 a \$576 b 26.25 m c 918 kg
d 179.2 cm e 324 min f \$18
g 2.03 t h 65 s i 320 L
2 a 80.5 mm b \$364.80 c 210 mL
d 3.888 km e 64.5 g f \$417.24
g \$2932.50 h 38 s i 3.5448 ha
3 a Increase = $55 - 52 = 3$ kg
Percentage increase = $\frac{3}{52} \times 100\% = 5.76\% \approx 5.8\%$
b Decrease = $\$186 - \$154 = \$32$
Percentage decrease = $\frac{32}{186} \times 100\% = 17.20\% \approx 17.2\%$
4 a 21.4% b 8.3% c 7.9%
d 168.8% e 100% f 10.4%
5 a 25.8% b 7.3% c 1.9%
d 10.5% e 66.7% f 6.2%
6 a i loss \$5500 ii 25% b i profit \$70 ii 70%
c i loss \$1680 ii 40% d i profit \$7500 ii 15%
e i loss \$320 ii $53\frac{1}{3}\%$ f i loss \$1300 ii 61.9%
7 a Value after 1 year = 106% of \$3600
 $= \frac{106}{100} \times \$3600 = \$3816$
b Value after two years = 108% of \$3816
 $= \frac{108}{100} \times \$3816 = \$4121.28$
c Change = $\$4121.28 - \$3600 = \$521.28$
d Percentage profit = $\frac{521.28}{3600} \times 100\% \approx 14.5\%$
8 a \$2760 b \$2980.80 c \$580.80 d 24.2%
9 Value after 1 year = \$56 000
Value after 2 years = $\frac{97}{100} \times \$56 000 = \$54 320$
Change = $\$54 320 - \$50 000 = \$4320$
Percentage change = $\frac{4320}{50000} \times 100\% = 8.64\%$; that is
8.64% profit
10 Value after 1 year = $\frac{110}{100} \times 80 000 = \$88 000$
Value after 2 years = $\frac{90}{100} \times 88 000 = \$79 200$
Change = $\$79 200 - \$80 000 = -\$800$
Percentage change = $\frac{-800}{80000} \times 100\% = -1\%$; that is,
1% loss
11 GST = 10% of \$750 = $\frac{10}{100} \times \$750 = \$75$
12 a \$4.60 b \$15 c \$0.84 d \$45.90
e \$97.55 f \$1.10 g \$198 h \$1.54
13 a \$65.40 b \$719.40
14 a \$874.28 b \$9617.08
15 a \$38.90 b \$427.90
16 GST paid = $1200 \div 11 = \$109.09$
17 a \$41.64 b \$24 c \$3.14
d \$19.91 e \$1.80 f \$173.09
g \$270.91 h \$1.35
18 a \$416.36 b \$240 c \$31.42
d \$199.09 e \$18 f \$1730.91
g \$2709.09 h \$13.55
19 \$697.40 20 13.4%

- 21 No, GST = $\$475 \div 11 = \$43.18$
22 The second quote (\$2680 including GST) is cheaper
by \$15.
23 \$14 787.55 24 \$12.39

EXERCISE 6B SIMPLE INTEREST CALCULATIONS

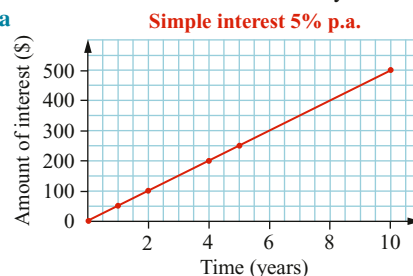
- 1 $P = \$800, r = 5 \div 100 = 0.05, n = 6$
 $I = \$800 \times 0.05 \times 6 = \$240$
2 a \$54 b \$135
c \$825 d \$657
3 $P = \$4650, n = 5$
 $r = 6.47 \div 100 = 0.0647$
 $I = \$4650 \times 0.0647 \times 5 = \$1504.28$
4 a \$365.94 b \$1723.78
c \$8382 d \$5605.60
5 a \$1072 b \$3354.40
c \$15 120 d \$3300
e \$4500 f \$79 460 g \$1010.62
6 a 0.08 b 4%, 0.04
c 2%, 0.02 d 0.666...%, 0.0066...
7 a 0.03 b 1.5%, 0.015
c 0.75%, 0.0075 d 0.25%, 0.0025

	Annual rate (%)	6-monthly rate (%)	Quarterly rate (%)	Monthly rate (%)
a	9	4.5	2.25	0.75
b	10	5	2.5	0.8333...
c	15	7.5	3.75	1.25
d	4.5	2.25	1.125	0.375
e	7.6	3.8	1.9	0.6333...
f	10.4	5.2	2.6	0.8666...
g	15.6	7.8	3.9	1.3
h	0.78	0.39	0.195	0.065
i	3.9	1.95	0.975	0.325

- 9 $P = \$5300, n = 17$
 $r = 6.47 \div 12 \div 100 = 0.005391666...$
 $I = \$5300 \times 0.005391666... \times 17 = \$485.79$
10 a \$160 b \$630 c \$6825
d \$1435 e \$271.40 f \$558.31
11 8% 12 4% 13 6.5%
14 4.7 years 15 \$2406.25 16 \$3.90
17 a interest rate too high b various

EXERCISE 6C SIMPLE INTEREST GRAPHS

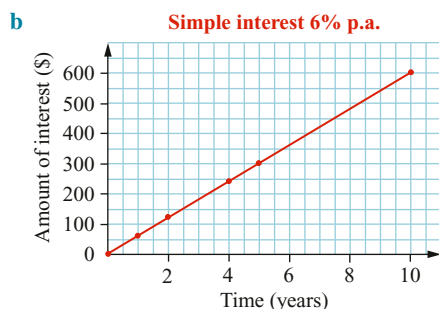
- 1 a 4, Time, across, \$280
b across, \$400, down, 5.8 years
2 a \$540 b 3.9 years
3 a \$65 b 3.2 years
4 a \$4000 b 5.6 years
5 a



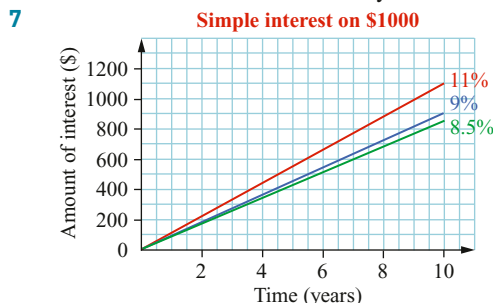
- b \$380 c $3\frac{1}{2}$ years

6 a

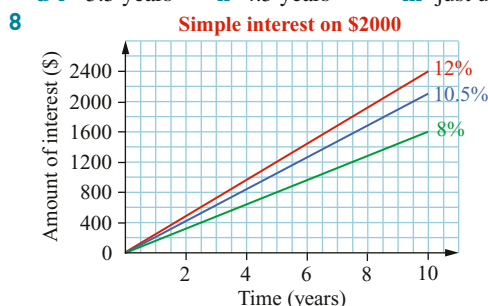
Time (years)	0	1	2	4	5	10
Interest (\$)	0	60	120	240	300	600



- c \$570
- d a little over 4 years



- a i \$900 ii \$1100 iii \$850
- b i \$400 ii \$500 iii \$380
- c \$250, \$200
- d i 5.5 years ii 4.5 years iii just under 6 years



- a i \$1600 ii \$2400 iii \$2100
- b i just under \$1400 ii just over \$2000
- iii just under \$1800
- c \$800, \$300
- d i $6\frac{1}{4}$ years ii a little over 4 years
- iii $4\frac{3}{4}$ years

EXERCISE 6D STRAIGHT-LINE DEPRECIATION

1

Year	Value (\$)	Depreciation (\$)	Depreciated value (\$)
1	35 000	3800	31 200
2	31 200	3800	27 400
3	27 400	3800	23 600

- 2 a $S = \$26\,990 - \$3300 \times 2 = \$20\,390$
- b $S = \$26\,990 - \$3300 \times 4 = \$13\,790$
- 3 a \$14 740 b \$8800
- 4 a \$2600 b \$600
- 5 a \$30 599 b \$17 999
- 6 a \$3100 b \$1000
- 7 a \$1330 b \$250

- 8 $14\,260 = 26\,900 - D \times 4$
 $4D + 14\,260 = 26\,900$
 $4D = 26\,900 - 14\,260$
 $= 12\,640$
 $D = \$3160$
- 9 \$2150 10 \$4000 11 \$4460
- 12 \$850 13 \$280 14 \$900
- 15 5 years 16 6 years 17 8 years
- 18 6 years
- 19 a \$20 000 b \$45 000 c 225% d 48%

EXERCISE 6E COMPOUND INTEREST

2 a

Time	Balance + interest	Total balance	Interest
Start of 1st year		\$2400	\$144
Start of 2nd year	\$2400 + \$144	\$2544	\$152.64
Start of 3rd year	\$2544 + \$152.64	\$2696.64	\$161.80
Start of 4th year	\$2696.64 + \$161.80	\$2858.44	

- b \$2858.44, \$458.44

3 a

Time	Balance + interest	Total balance	Interest
Start of 1st year		\$3600	\$180
Start of 2nd year	\$3600 + \$180	\$3780	\$189
Start of 3rd year	\$3780 + \$189	\$3969	\$198.45
Start of 4th year	\$3969 + \$198.45	\$4167.45	

- b \$567.45

4 a

Time	Balance + interest	Total balance	Interest
Start of 1st year		\$980	\$29.40
Start of 2nd year	\$980 + \$29.40	\$1009.40	\$30.28
Start of 3rd year	\$1009.40 + \$30.28	\$1039.68	\$31.19
Start of 4th year	\$1039.68 + \$31.19	\$1070.87	\$32.13
Start of 5th year	\$1070.87 + \$32.13	\$1103.00	

- b \$123.00

5 a

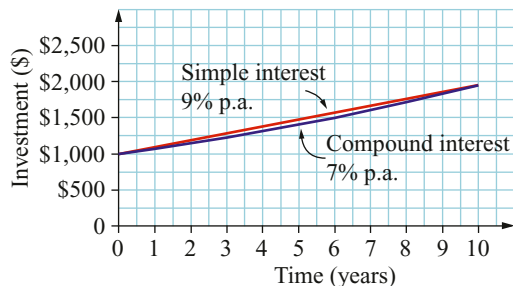
Time	Balance + interest	Total balance	Interest
Start of 1st year		\$7890	\$702.21
Start of 2nd year	\$7890 + \$702.21	\$8592.21	\$764.71
Start of 3rd year	\$8592.21 + \$764.71	\$9356.92	\$832.77
Start of 4th year	\$9356.92 + \$832.77	\$10 189.69	\$906.88
Start of 5th year	\$10 189.69 + \$906.88	\$11 096.57	\$987.59
Start of 6th year	\$11 096.57 + \$987.59	\$12 084.16	

- b No, not enough money, only \$12 084.16
- c Needs another \$905.84

6 a

Time (years)	Investment (\$)
0	1000.00
1	1070.00
2	1144.90
3	1225.04
4	1310.80
5	1402.55
6	1500.73
7	1605.78
8	1718.19
9	1838.46
10	1967.15

b, e



c approx. \$1550

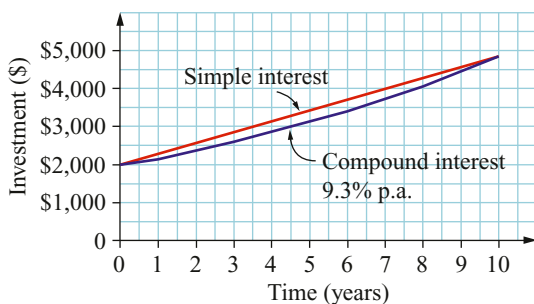
d about 6 years

f between 9 and 10 years

7 a

Time (years)	Investment (\$)
0	2000.00
1	2186.00
2	2389.30
3	2611.50
4	2854.37
5	3119.83
6	3409.97
7	3727.10
8	4073.72
9	4452.58
10	4866.67

b, f



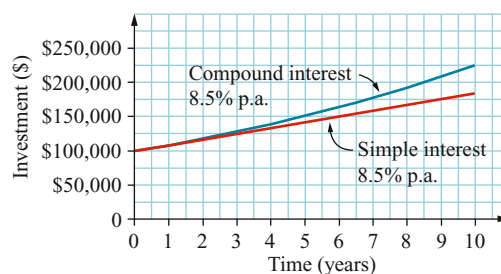
c about \$3000 d about \$1000 e Just under 8 years

g 14.3%

8 a i \$185 000 ii \$226 098.34

b Compound interest gives interest on the interest earned each year. Simple interest is the same amount every year.

c

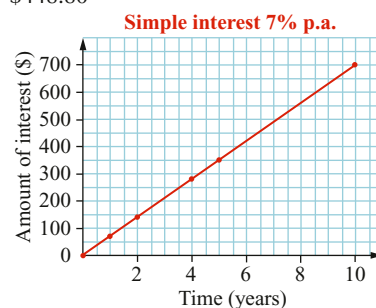


REVIEW MULTIPLE-CHOICE QUESTIONS

- 1 C 2 B 3 B 4 A 5 B
6 D 7 A 8 A 9 D 10 C

REVIEW SET 1

- 1 a 82.4 kg b \$246.50
2 a \$25 650 b 2.6%
3 \$514
4 a \$1254.63 b \$13 800.93
5 a \$112.27 b \$32.64 c \$3.83 d \$24.18
6 \$178.50
7 a 0.1
b i 5%, 0.05 ii 2.5%, 0.025 iii 0.83%, 0.0083
8 \$448.80
9



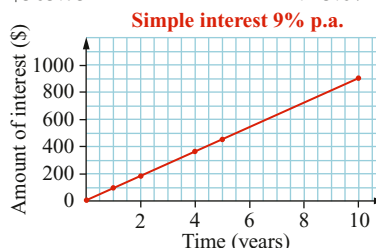
- a \$450 or \$460 b $3\frac{1}{2}$ years

REVIEW SET 2

- 1 28.8% 2 9.87%
3 a \$84.90 b \$933.90
4 \$1171.50 5 \$633.10 6 5.5%
7 \$7000 8 \$1950

REVIEW SET 3

- 1 2%
2 a \$2258.60 b \$24 844.60
3 a \$42.50 b \$467.50
4 a \$681.82 b \$115.27 c \$2.36 d \$36.45
5 \$816
6 a 0.06 b 3%, 0.03
c 1.5%, 0.015 d 0.5%, 0.005
7 \$565.73 8 5.6%
9

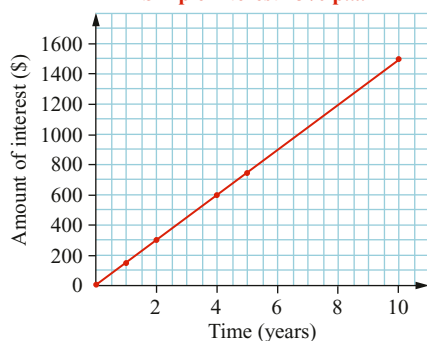


- a \$500 b $1\frac{3}{4}$ years

REVIEW SET 4

- 1 20.2% 2 \$491.30
 3 a \$325.85 b \$3584.35
 4 a \$52 b \$572
 5 a \$904.82 b \$77.73 c \$3.59 d \$36.27
 6 \$343
 7 a 0.05 b 2.5%, 0.025
 c 1.25%, 0.0125 d 0.417%, 0.004 17
 8 8.2% 9 \$354.84

10 **Simple interest 15% p.a.**



- a Just over \$1450 b $5\frac{3}{4}$ years
 11 3 years

REVIEW PRACTICE EXAMINATION QUESTION

- 1 a i \$44 800 ii \$39 424 iii Decrease 1.44%
 b i \$1568.50 ii \$17 253.50
 c \$1943.64
 d \$22 400
 e i 0.02 ii 20 iii \$9287.17
 iv \$3037.17
 f \$961.26

CHAPTERS 4-6 CUMULATIVE REVIEW

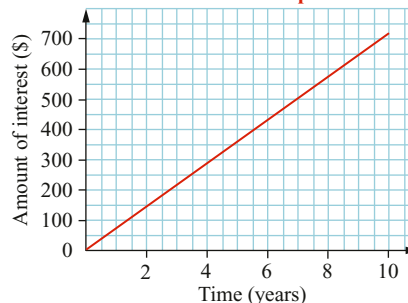
- 1 a i discrete ii nominal iii continuous
 b Select 10 numbers randomly from 120.
 c A sample that is biased is one in which data has been unfairly influenced by the collection process and is not truly representative of the whole population.
 d 129, 18, 186, 118, 108
 e i Assign numbers 1-27 to the workers. Use a table or calculator to select 4 random numbers ≤ 27 .
 ii 18, 25, 8, 23
 f Proportions are provided as percentages, correct to one decimal place.

Year	7	8	9	10	11	12
Number of students	18.5%	19.7%	23.3%	16.4%	12.9%	9.3%

- g i 46% ii 320
 h i 18 ii 9 iii 12 iv 45
 2 a 20 m b 11 cm or 110 mm c 30.8 cm
 3 a $x = 11.3$ b 43.3 cm
 4 a 23.715 cm² b 9.46 m²
 c 121 cm² d 421.64 m²
 5 a 211.2 mm² b 41.9 cm² c 0.7 m²
 6 a 5497.8 cm² b 1115.9 mm²
 7 267 m²
 8 a 116.1 cm² b 2536.7 cm²
 9 a 616 cm² b 376 m²

- 10 a 360 cm³ b 274.6 m³ c 4188.8 cm³
 11 a 125.44 m³ b 471.2 cm³
 12 a 40 cm³ b 70 686 cm³
 c 140 cm³ d 2400 cm³
 13 120 kL 14 423 mL
 15 a 84 kg b 6% c 64.6%
 d i \$1436 ii \$15 796
 e i \$72.50 ii \$797.50
 f \$295.82
 16 a \$637.50
 b i 0.04 ii 2%, 0.02
 iii 1%, 0.01 iv $\frac{1}{3}\%$, 0.003
 c \$381.93 d 8%
 e i

Interest 7.2% p.a.



- ii about \$250 iii about 4.2 years
 f i \$3300 ii \$600
 g \$2500

Time	Balance + interest	Total balance	Interest
Start of first year		\$4200	\$168
Start of second year	\$4200 + \$168	\$4368	\$174.72
Start of third year	\$4368 + \$174.72	\$4542.72	\$181.71
Start of fourth year	\$4542.72 + \$181.71	\$4724.43	

Total interest = \$524.43

CHAPTER 7 REPRESENTING DATA

ARE YOU READY?

- 1 B 2 C 3 C 4 A
 5 C 6 B 7 C 8 A

EXERCISE 7A FREQUENCY DISTRIBUTION TABLES

1 a

Colour	Number of cars
White	14
Blue	12
Green	9
Red	11
Other	4

- b 50 c White
 d W 28%, B 24%, G 18%, R 22%, O 8%

2 a

Temperature	Number of days
17	1
18	2
19	4
20	7
21	6
22	6
23	4

b 30 **c** 1 **d** 7

3 a

Colour	Number of students
Blue	10
Brown	11
Green	3
Grey	6

b 30 **c** Brown

4 a

Score	3	13	20	22	29	31	33	34	38	40	41	42	44
Frequency	1	1	1	1	1	1	1	1	1	1	2	1	1

Score	45	48	49	50	51	52	53	54	55	56	57	60	61
Frequency	2	1	1	2	3	1	1	2	2	2	1	2	2

Score	62	64	65	66	68	69	70	74	75	79	85	94
Frequency	1	1	1	1	2	1	1	2	1	1	1	1

b We are unable to obtain very much information from this table apart from getting a rank order of the scores. It is also very time consuming to prepare.

c i The scores from 0 to 9

ii The scores from 60 to 69

iii Yes. Ten possible scores.

iv

Class	Tally	Frequency
0–9		1
10–19		1
20–29		3
30–39		4
40–49		9
50–59		14
60–69		11
70–79		5
80–89		1
90–99		1
Total		50

v 14

vi 2%

d i Twenty possible scores

ii

Class	Tally	Frequency
0–19		2
20–39		7
40–59		23
60–79		16
80–99		2
Total		50

iii 7

iv 32

5 a i

Mark	Frequency
0–4	0
5–9	1
10–14	0
15–19	0
20–24	4
25–29	1
30–34	3
35–39	3
40–44	4
45–49	3
50–54	4
55–59	1
60–64	4
65–69	1
70–74	0
75–79	1

ii

Mark	Frequency
0–9	1
10–19	0
20–29	5
30–39	6
40–49	7
50–59	5
60–69	5
70–79	1

iii

Mark	Frequency
0–< 20	1
20–< 40	11
40–< 60	12
60–< 80	6

iv

Mark	Frequency
0–49	19
50–99	11

c i Classes overlap

ii Class 10–19 missing

iii Classes are different sizes.

6 a i

Time ($\times 100$ h)	Number of globes
0–4	3
5–9	12
10–14	23
15–19	2

ii

Time ($\times 100$ h)	Number of globes
1–<6	5
6–<11	16
11–<16	18
16–<21	1

b i No, classes overlap

ii Yes

iii Yes, but class size too big

c 40

d i 15

ii 19

iii 6

EXERCISE 7B CUMULATIVE AND RELATIVE FREQUENCY

1 a

Number of children	Frequency	Cumulative frequency	Relative frequency
1	3	3	12%
2	9	12	36%
3	7	19	28%
4	4	23	16%
5	2	25	8%

b i 19 ii 23

c i $\frac{7}{25}$ ii $\frac{4}{25}$

d i 36% ii 8%

2 a

Goals/match	Frequency	Cumulative frequency	Relative frequency
5	4	4	16%
6	3	7	12%
7	7	14	28%
8	5	19	20%
9	0	19	0%
10	3	22	12%
11	2	24	8%
12	1	25	4%

b i 14 ii 19

c i $\frac{1}{5}$ ii $\frac{4}{25}$

d i 12% ii 8%

3 a

Score	Frequency
3	4
4	2
5	4
6	5
7	3

b

Score	Frequency
12	5
13	6
14	8
15	10
16	5
17	2

c

Score	Frequency
8	11
9	6
10	10
11	0
12	5
13	8

d

Score	Frequency
87	23
88	25
89	30
90	28
91	21
92	11

4 a

Number of matches	Frequency	Cumulative frequency	Relative frequency
48	9	9	15%
49	12	21	20%
50	18	39	30%
51	10	49	$16\frac{2}{3}\%$
52	6	55	10%
53	5	60	$8\frac{1}{3}\%$

b i 18 ii 21 iii 39 iv 21

c i 20% ii 30% iii $16\frac{2}{3}\%$ iv 65%

v 65%

5 a 3 b 40 c 20% d 10%

e No, the grouped data only shows how many clients had 6, 7 or 8 arguments.

f No, the grouped data shows three responses in each class interval so lowest could be 0, 1 or 2 and highest could be 12, 13 or 14.

6 a c

Distance	Tally	Frequency	Cumulative frequency	Relative frequency
180–189		2	2	$3\frac{1}{3}\%$
190–199		8	10	$13\frac{1}{3}\%$
200–209		11	21	$18\frac{1}{3}\%$
210–219		8	29	$13\frac{1}{3}\%$
220–229		13	42	$21\frac{2}{3}\%$
230–239		10	52	$16\frac{2}{3}\%$
240–249		4	56	$6\frac{2}{3}\%$
250–259		4	60	$6\frac{2}{3}\%$

b 60

d i 10 ii 42

e i $6\frac{2}{3}\%$ ii $21\frac{2}{3}\%$ iii $16\frac{2}{3}\%$

7 a 25 b 120

c No, the grouped data only shows how many plants were 350–374 mm high.

d No, the grouped data only shows that the shortest plant is in the class interval 300–324 and the tallest plant is in the class interval 425–449.

e

Height	Frequency	Cumulative frequency	Relative frequency
300–324	3	3	2.5%
325–349	18	21	15%
350–374	47	68	39.2%
375–399	32	100	26.7%
400–424	14	114	11.7%
425–449	6	120	5%

f i 21 ii 100

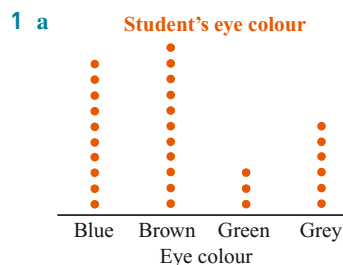
g 5% h 2.5%

8 a

Maximum daily temperature ($^{\circ}\text{C}$)	Frequency	Cumulative frequency	Relative frequency
17	3	3	10%
18	4	7	13.3%
19	8	15	26.7%
20	6	21	20%
21	5	26	16.7%
22	4	30	13.3%

b 15 days c 50% d 0.3

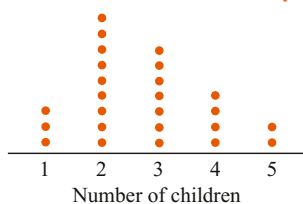
EXERCISE 7C DOT PLOTS AND STEM-AND-LEAF PLOTS



b

Eye colour	Frequency
Blue	10
Brown	11
Green	3
Grey	6

2 a Number of children in family



b

Number of children	Frequency
1	3
2	9
3	7
4	4
5	2

3 a

Stem	Leaf
3	2 3 6 7
4	2 5 7 7 7 8
5	0 0 1 7 9
6	0 2 3 5 8

b Lowest = 32, highest = 68

c i 2 **ii** 0

d 47 **e** 5 **f** 10

4 a

Stem	Leaf
12	4 5 6 6 6 8 8
13	0 0 0 2 9
14	0 0 1 2 5 5 5 6 8 9
15	0 1 4 4 6 8 8

b Lowest = 124, highest = 158

c i 1 **ii** 2 **iii** 0

d 145

e i 7 **ii** 7

5 a 50, 90

b i 3 **ii** 2 **iii** 0 **iv** 0

c 54 **d** 14

e i 14 **ii** 8

6 a 25 **b** Least = 3, greatest = 36

c 5 **d** 4

e i

Time	Frequency
1–5	2
6–10	4
11–15	6
16–20	1
21–25	5
26–30	3
31–35	3
36–40	1

ii

Time	Frequency
0–9	5
10–19	8
20–29	7
30–39	5

7 a 40

b

Stem	Leaf
1	8
2	2 2 3 4 6 6 6 6 6 7 7 7 7 7 8 8 8 9 9 9 9
3	0 0 1 1 1 2 2 2 3 3 3 3 3 4 4 4 7 8

c

Stem	Leaf
1	
1*	8
2	2 2 3 4
2*	6 6 6 6 6 7 7 7 7 7 8 8 8 9 9 9 9
3	0 0 1 1 1 2 2 2 3 3 3 3 3 4 4 4 4
3*	7 8

d Most students spend between 25 and 35 hours per week doing homework.

8 a

Melbourne	Stem	Adelaide
	1	7 9
	2	2 3 4
	3	8 8
9 8 8	4	3
9 9 8 8 2 1 0	5	0 2 9
8 0	6	5

b Melbourne has more rain than Adelaide.

Melbourne's rainfall is clustered more than Adelaide's.

9 a

Hobart	Stem	Canberra
	3	7
9 8 7 0	4	0 7 9
6 5 4 2 2 2 2	5	0 3 3 6 7 8
4	6	3 7

b Hobart has a more constant rainfall than Canberra. Canberra has a greater range.

10 a

Boys	Stem	Girls
	4	5 8
5	5	0 3 4 6 8
9 8 8 7	6	2 5
7 5 0	7	8
6 4	8	

b The boys in this sample are heavier than the girls; most boys are heavier than most girls.

11 a 30 **b** 6

c i Carrots **ii** Potato
d i Peas **ii** Pumpkin
e i Potato **ii** Pumpkin

12 a i 12 **ii** 2
b i 39 **ii** 39
c i 20 **ii** 34

d i 10s and 20s ii 20s and 30s

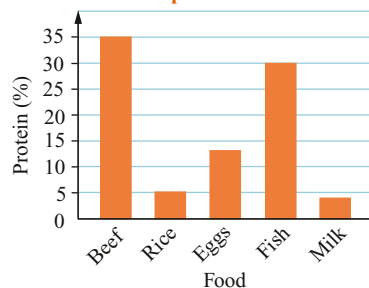
e i 6 ii 1

f Class 11B

13 a 27 b 7 c 8 d 12 e 39

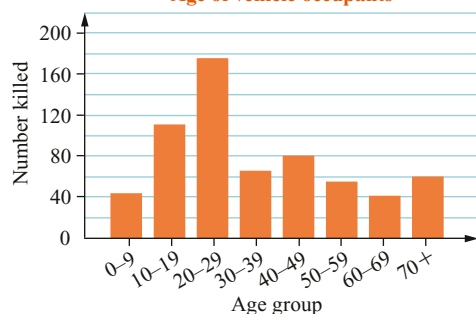
EXERCISE 7D COLUMN AND BAR GRAPHS

1 % protein in foods

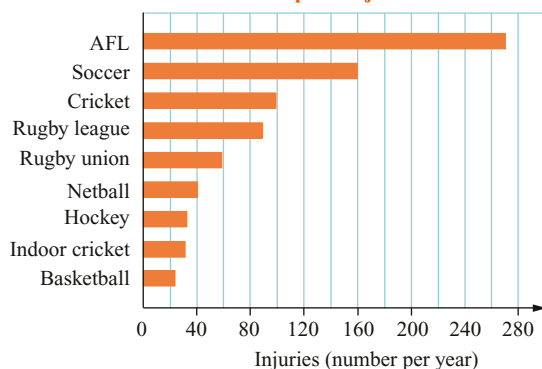


Month	Sales (\$ million)
Jan/Feb	7.7
Mar/Apr	8.2
May/Jun	3.2
Jul/Aug	4.1
Sep/Oct	3.4
Nov/Dec	6.0

3 Age of vehicle occupants

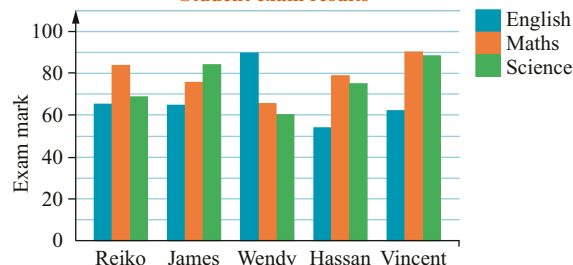


4 Sports injuries

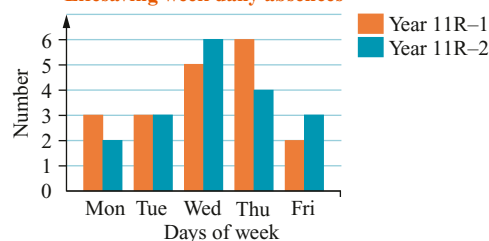


Food	Cholesterol (mg/100 g)
Lobster	70
Beef	80
Chicken	100
Duck	105
Prawns	145

6 Student exam results



7 Lifesaving week daily absences



8 a Scale on vertical axis is irregular.

b Use of area.

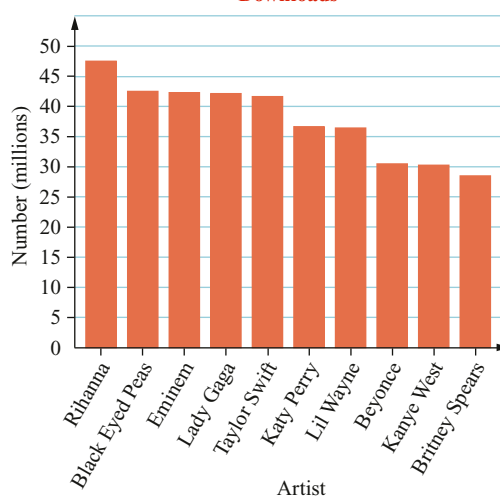
c No scale on vertical axis.

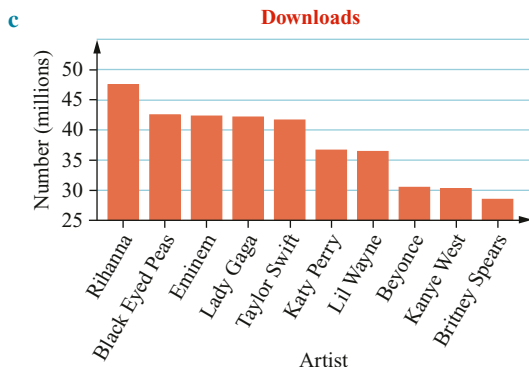
d Use of volume.

9 5 times

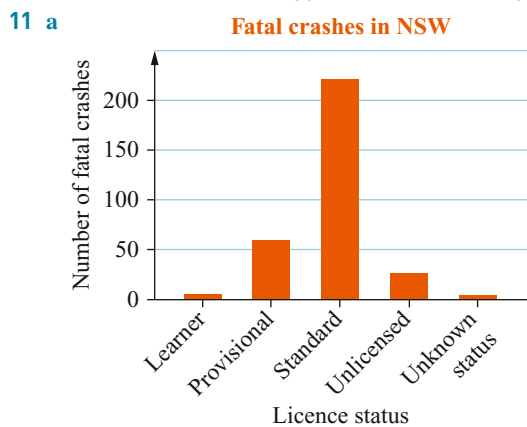
Artist	Number of downloads	Scientific notation
Rihanna	47 571 000	4.8×10^7
Black Eyed Peas	42 405 000	4.2×10^7
Eminem	42 290 000	4.2×10^7
Lady Gaga	42 078 000	4.2×10^7
Taylor Swift	41 821 000	4.2×10^7
Katy Perry	36 788 000	3.7×10^7
Lil Wayne	36 518 000	3.7×10^7
Beyonce	30 439 000	3.0×10^7
Kanye West	30 242 000	3.0×10^7
Britney Spears	28 665 000	2.9×10^7

b Downloads



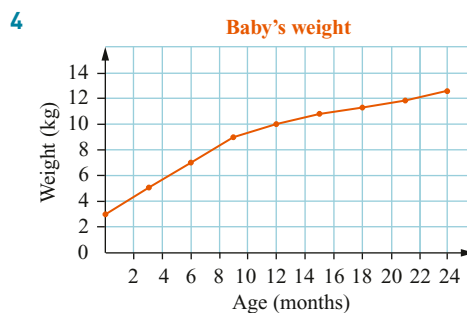
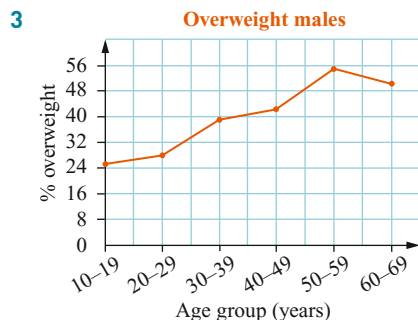
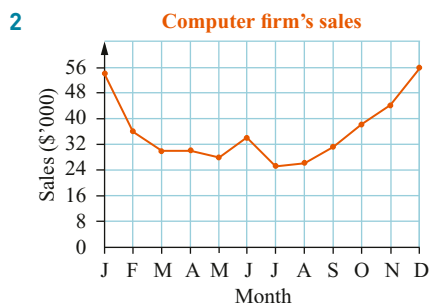
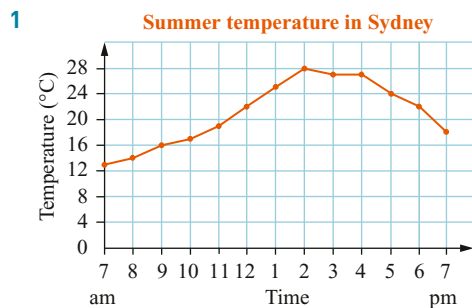


The differences are exaggerated in the second graph.



- b** $\frac{27}{312} \approx 0.085$ **c** 18.8% **d** 69.3%
- e** Class discussion
- f** **i** 15.1 **ii** 6.1 **iii** Class discussion

EXERCISE 7E LINE GRAPHS

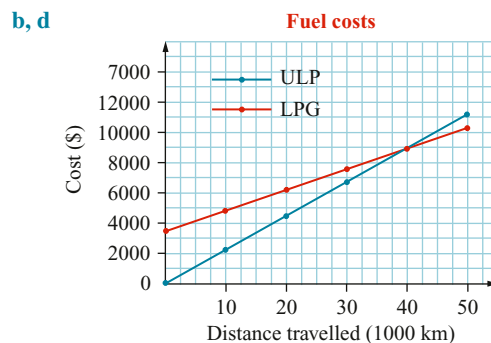


5 Scale on horizontal axis is irregular.

6 a Graph A **b** No **c** No

7 a

Distance (km)	0	10 000	20 000	30 000	40 000	50 000
Fuel cost (\$)	0	2250	4500	6750	9000	11 250



c

Distance (km)	0	10 000	20 000	30 000	40 000	50 000
Fuel cost (\$)	3500	4865	6230	7595	8960	10 325

e $\approx 40\,000$ km

8 a \$6500

b **i** \$5300 **ii** \$3500

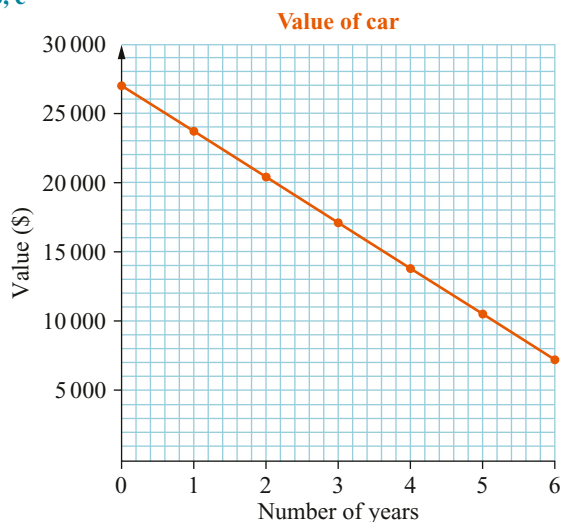
c **i** After 2 years **ii** After 4 years

d \$500 **e** \$1200

9 a

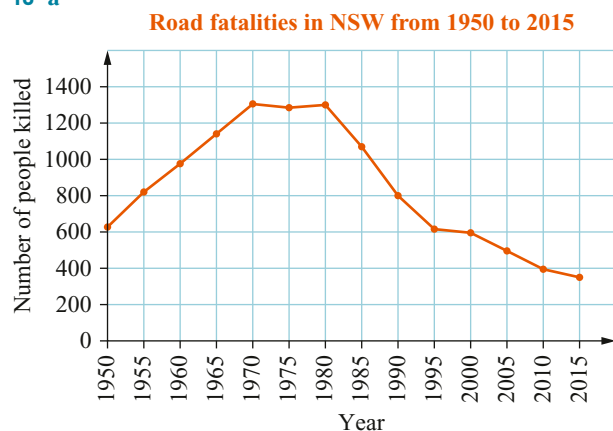
Number of years	0	1	2	3
Value (\$)	26 990	23 690	20 390	17 090

b, c



d After about 4 years.

10 a



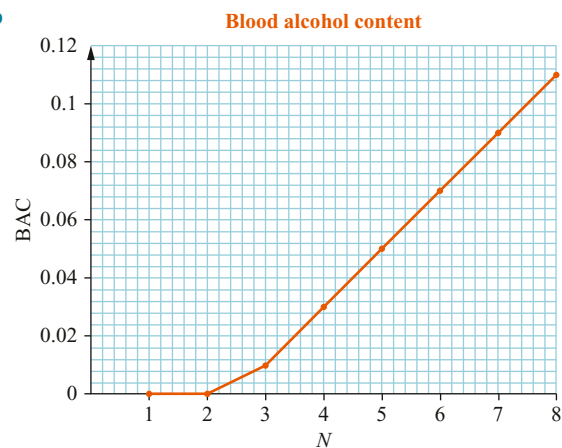
The number of fatalities rose sharply from about 600 in 1950 to 1300 in 1970 where it remained steady for the next decade. It then decreased from 1980 to 2015, quickly at first and then at a relatively slower rate.

b i 1950–55 ii 1985–90

11 a

N	1	2	3	4	5	6
BAC	0	0	0.01	0.03	0.05	0.07

b

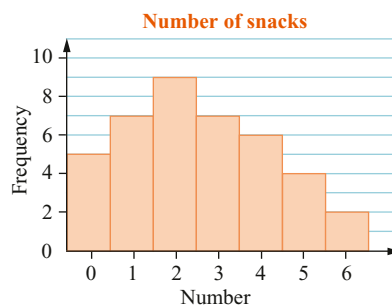


c i 0.09 ii 0.11

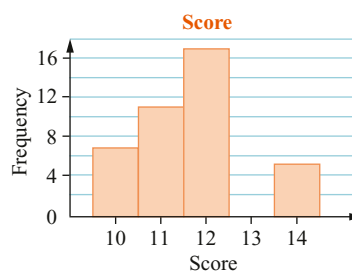
d 4 e 2

EXERCISE 7F FREQUENCY HISTOGRAMS AND POLYGONS

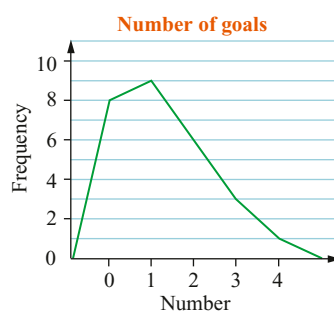
1 a



b



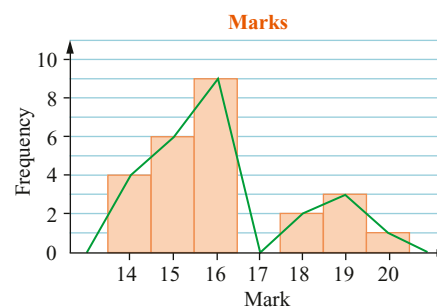
2 a



b



3



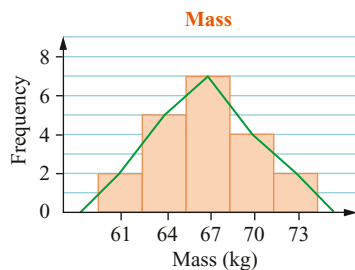
4 a

Score (x)	Frequency (f)
5	3
6	4
7	8
8	6
9	2

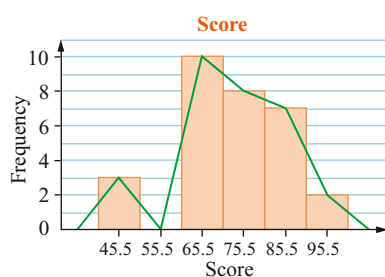
b

Score (x)	Frequency (f)
12	15
13	10
14	6
15	8
16	11
17	14

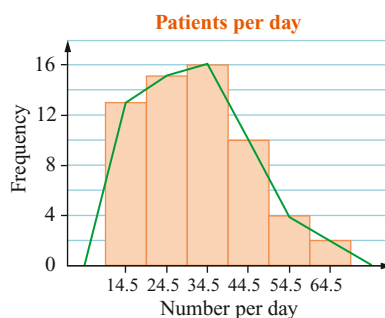
5 a



b



c



- 6 a i** 24% **ii** 6%
b i 33% **ii** 5%
c i 26–39 **ii** 0–4
d i 40–59 **ii** 0–4, 5–16
e i 26–39 **ii** 0–4

- 7 a i** 1970, 1980 **ii** 2010
b i 1950–60 **ii** 1980–90

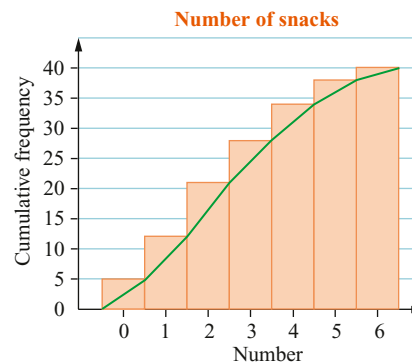
c The number of fatalities rose sharply from about 600 in 1950 to 1300 in 1970 where it remained steady for the next decade. It then decreased from 1980 to 2010, quickly at first and then at a relatively slower rate

EXERCISE 7G CUMULATIVE FREQUENCY GRAPHS

1 a i

Number of snacks	Frequency	Cumulative frequency
0	5	5
1	7	12
2	9	21
3	7	28
4	6	34
5	4	38
6	2	40

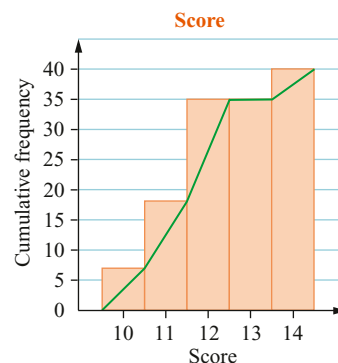
ii



b i

Score	Frequency	Cumulative frequency
10	7	7
11	11	18
12	17	35
13	0	35
14	5	40

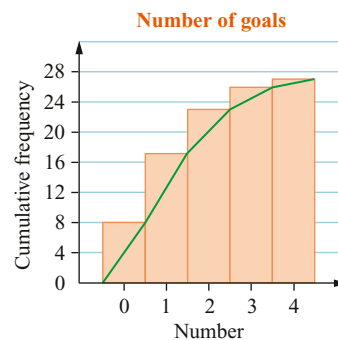
ii



2 a i

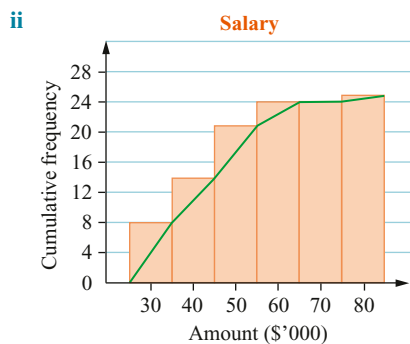
Number of goals	Frequency	Cumulative frequency
0	8	8
1	9	17
2	6	23
3	3	26
4	1	27

ii



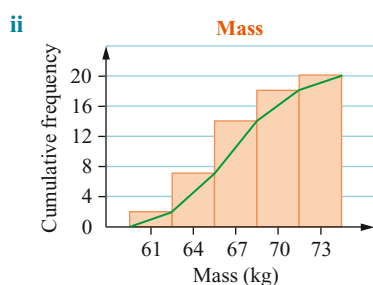
b i

Salary (\$'000)	Frequency	Cumulative frequency
30	8	8
40	6	14
50	7	21
60	3	24
70	0	24
80	1	25



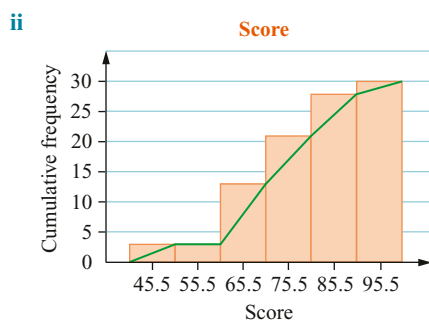
3 a i

Mass (kg)	Frequency	Cumulative frequency
60–62	2	2
63–65	5	7
66–68	7	14
69–71	4	18
72–74	2	20



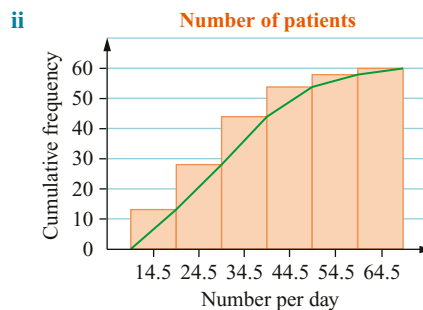
b i

Score	Frequency	Cumulative frequency
41–50	3	3
51–60	0	3
61–70	10	13
71–80	8	21
81–90	7	28
91–100	2	30



c i

Patients/day	Frequency	Cumulative frequency
10–19	13	13
20–29	15	28
30–39	16	44
40–49	10	54
50–59	4	58
60–69	2	60



4 a

Score	Cumulative frequency	Frequency
2	3	3
3	5	2
4	8	3
5	13	5
6	15	2

b

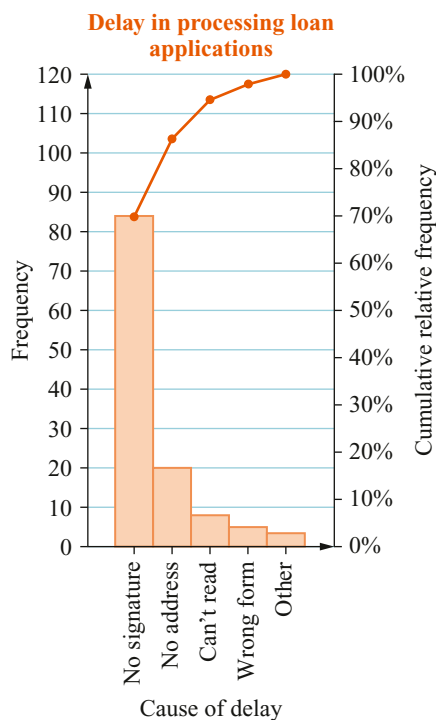
Score	Cumulative frequency	Frequency
18	2	2
19	7	5
20	13	6
21	16	3
22	19	3
23	20	1

c

Score	Cumulative frequency	Frequency
141	2	2
142	7	5
143	15	8
144	22	7
145	27	5
146	30	3

5 a

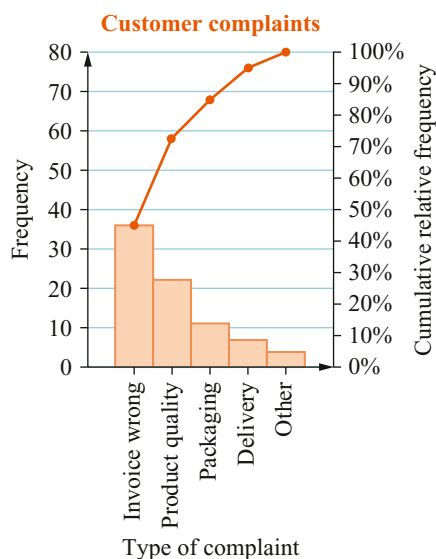
Cause of delay	Frequency	Relative frequency (%)	Cumulative relative frequency (%)
No signature	84	70	70
No address	20	17	87
Can't read	8	7	94
Wrong form	5	4	98
Other	3	3	100
Total	120		



b Check signature, check address **c** 87%

6 a

Type of complaint	Frequency	Relative frequency (%)	Cumulative relative frequency (%)
Invoice wrong	36	45	45
Product quality	22	27.5	72.5
Packaging	11	13.8	86.3
Delivery	7	8.7	95
Other	4	5	100
Total	80		

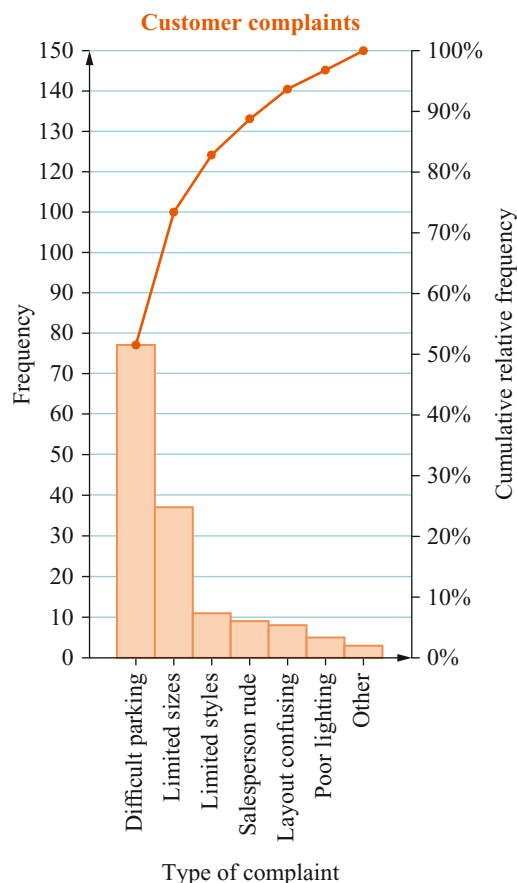


b Check invoice, check product quality, (ensure correct packaging)

c 72.5% (86.3% if packaging included)

7 a

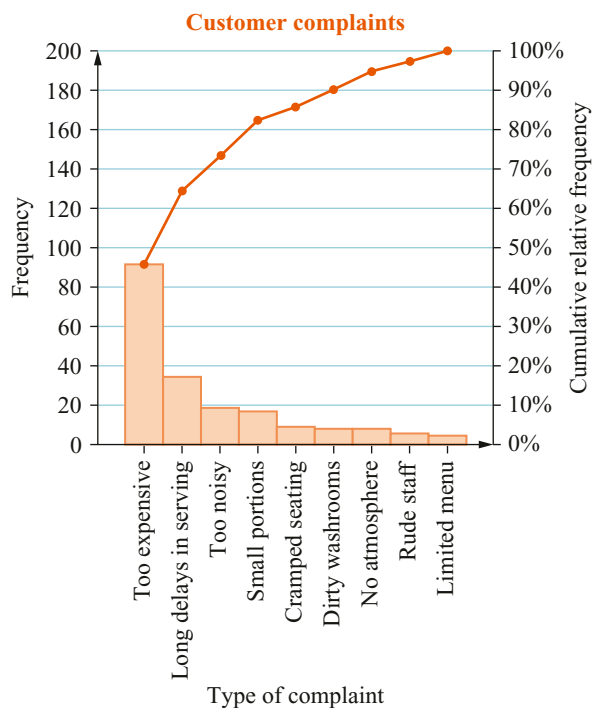
Type of complaint	Frequency	Relative frequency (%)	Cumulative relative frequency (%)
Difficult parking	77	51	51
Limited sizes	37	25	76
Limited styles	11	7	83
Salesperson rude	9	6	89
Layout confusing	8	5	94
Poor lighting	5	3	97
Other	3	2	100
Total	150		



b 7% **c** Parking, range of sizes **d** 76%

9 a

Type of complaint	Frequency	Relative frequency (%)	Cumulative relative frequency (%)
Too expensive	93	46.5	46.5
Long delays in serving	35	17.5	64
Too noisy	19	9.5	73.5
Small portions	17	8.5	82
Cramped seating	9	4.5	86.5
Dirty washrooms	8	4	90.5
No atmosphere	8	4	94.5
Rude staff	6	3	97.5
Limited menu	5	2.5	100
Total	200		



- b** Cost of meals, delays in serving, noise, (portion size)
c 74% (82% if portion size included)

REVIEW MULTIPLE-CHOICE QUESTIONS

- 1** A **2** C **3** C **4** D **5** C **6** D
7 C **8** B **9** D **10** B **11** B

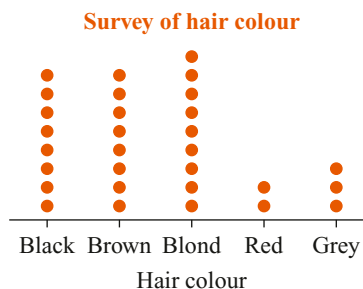
REVIEW SET 1

1 a

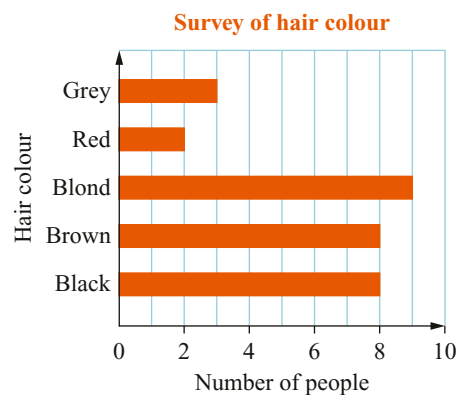
Hair colour	Frequency	Relative frequency
Black	8	$26\frac{2}{3}\%$
Brown	8	$26\frac{2}{3}\%$
Blond	9	30%
Red	2	$6\frac{2}{3}\%$
Grey	3	10%

- b** 30 **c** Blond
d See table above **i** $26\frac{2}{3}\%$ **ii** 30%

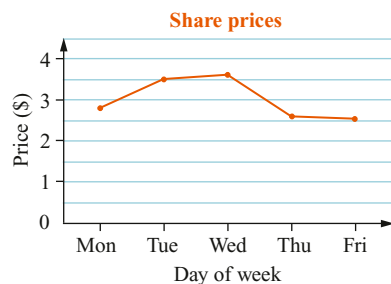
e i



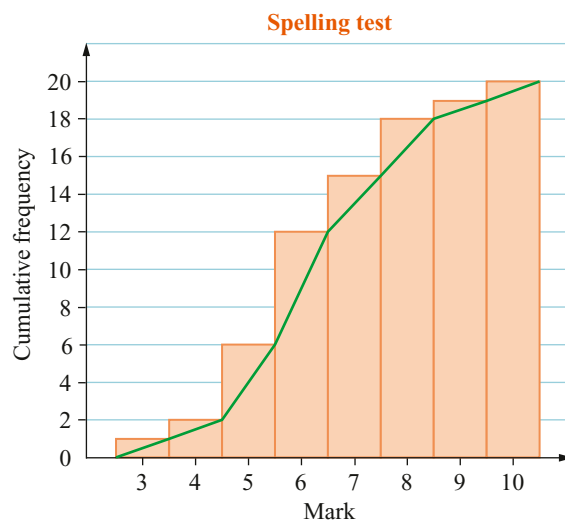
ii



2



3



REVIEW SET 2

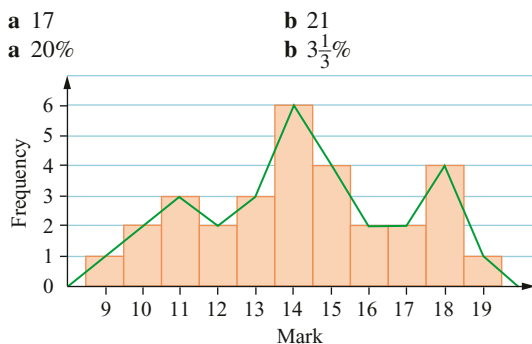
1, 2

Mark	Frequency	Cumulative frequency	Relative frequency
9	1	1	$3\frac{1}{3}\%$
10	2	3	$6\frac{2}{3}\%$
11	3	6	10%
12	2	8	$6\frac{2}{3}\%$
13	3	11	10%
14	6	17	20%
15	4	21	$13\frac{1}{3}\%$
16	2	23	$6\frac{2}{3}\%$
17	2	25	$6\frac{2}{3}\%$
18	4	29	$13\frac{1}{3}\%$
19	1	30	$3\frac{1}{3}\%$

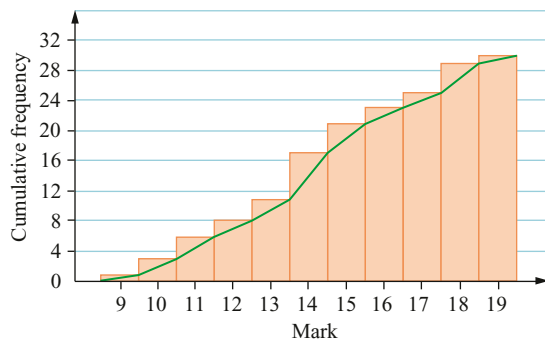
3 a 17

4 a 20%

5



6



7 a 4

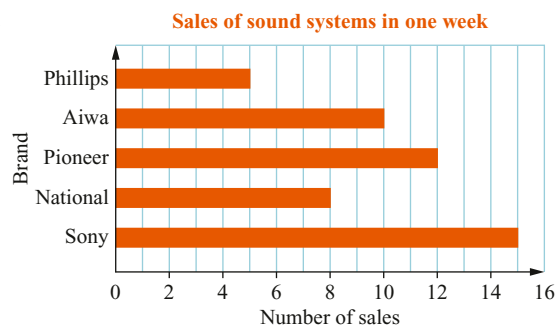
8 15

b 29

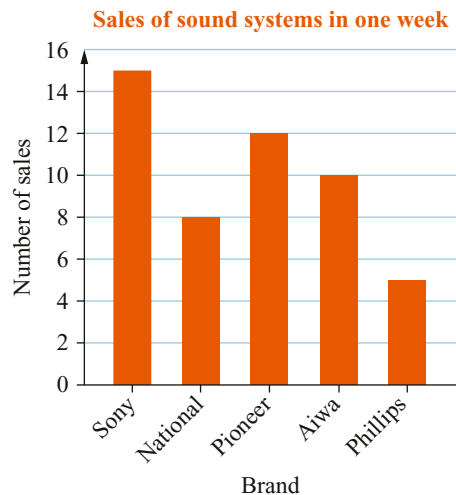
REVIEW SET 3

1 a 50

b



c



2 a

Stem	Leaf
3	3 4 7 8
4	3 6 8 8 8 9
5	1 1 2 8
6	0 1 3 4 6 9

b 33, 69

c i 2 times

ii 0 times

d 48

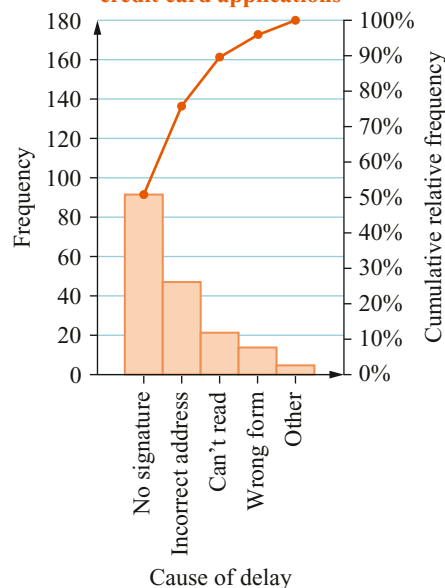
e 6

f 10

3

Cause of delay	Frequency	Relative frequency (%)	Cumulative relative frequency (%)
No signature	92	51	51
Incorrect address	47	26	77
Can't read	22	12	89
Wrong form	14	8	97
Other	5	3	100
Total	180		

Delays in processing credit card applications



4 a Vertical axis does not start at zero.

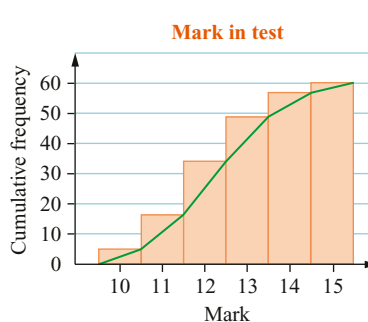
b Vertical axis does not start at zero.

c Irregular scale on vertical axis.

d Use of area.

REVIEW SET 4

1 a



b i 16

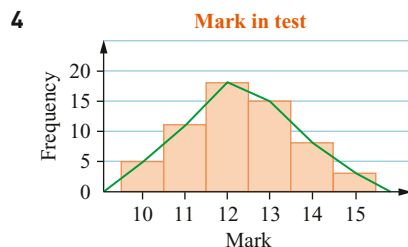
ii 34

iii 26

2

Mark	Frequency
10	5
11	11
12	18
13	15
14	8
15	3

3 a $8\frac{1}{3}\%$ b 5%

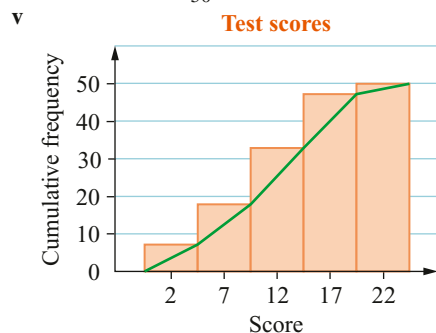


REVIEW PRACTICE EXAMINATION QUESTION

1 a i

Class centre	Cumulative frequency
2	7
7	18
12	33
17	47
22	50

ii 33 iii $\frac{11}{50}$ or 22% iv 34%



b 5
c i 31 ii 7 iii 15 iv 24%
d i $\approx 2\%$ ii Midnight to 5:59 am Sunday
iii Saturday

CHAPTER 8 RELATIVE FREQUENCY AND PROBABILITY

ARE YOU READY?

1 C 2 C 3 B 4 D 5 B 6 D
7 D 8 B 9 A 10 B 11 C

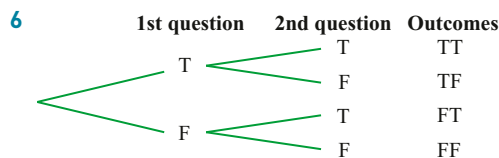
EXERCISE 8A SAMPLE SPACE AND NUMBER OF OUTCOMES

1 a i {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
ii 12
b i {1, 2, 3, 4, 5, 6, 7} ii 7
c i {R, R, R, R, R, B, B, B} ii 8

d i {M, A, T, H, S} ii 5
e i {1, 2, 3, 4, 5, 6, 7, 8} ii 8
f i {A♥, A♦, A♣, A♠} ii 4
2 a {boy, girl} b {point up, point down}
c {lands on an end, lands on curved face}
d {R, R, R, R, R, B, B, B, W, W}
e {red, green, amber}
f {a, e, i, o, u}
g {I, I, N, S, G, H, T}
h {Mon, Tue, Wed, Thu, Fri, Sat, Sun}
i {win, lose, draw}
j Discuss with class

3 a Yes b Don't know c No
d No e No
f Yes g No h Yes
i Don't know j Discuss with class
4 a False. The prevailing weather pattern determines the chance of rain.
b False. Not all swimmers are of equal ability.
c False. The letter Z is more common (or less common) in different cultures.
d False. The candidates do not necessarily have the same qualifications, financial backing, etc.
e False. Not all students may be old enough, have access to a car, etc.

5 None of them



7 {HH, HT, TH, TT}, yes
8 {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}

9 a

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

b 36 c yes d 6

10 a

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b 11
c No, 4 appears three times and 6 appears five times in the table
d 7
e 2 and 12, 3 and 11, 4 and 10, 5 and 9, 6 and 8
f 10

11 a

		Second die					
		1	2	3	4	5	6
First die	1	0	1	2	3	4	5
	2	1	0	1	2	3	4
	3	2	1	0	1	2	3
	4	3	2	1	0	1	2
	5	4	3	2	1	0	1
	6	5	4	3	2	1	0

b 6

c Yes, 3 and 6 each appear six times in the table

d 1

e 24

12 {BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG}, yes

13 {bax, bay, bex, bey, cax, cay, cex, cey, dax, day, dex, dey}

14 {John Paul George, John George Paul, Paul John George, Paul George John, George John Paul, George Paul John}

15 16

16 12

17 {RR, RG, RA, GR, GG, GA, AR, AG, AA}, no

18 {1.8AO, 1.8AB, 1.8AR, 1.8MO, 1.8MB, 1.8MR, 2.2AO, 2.2AB, 2.2AR, 2.2MO, 2.2MB, 2.2MR}

19 {CMR, CRM, MCR, MRC, RCM, RMC}

20 a {RR, RB, RG, BR, BB, BG, GR, GB, GG}

b {RB, RG, BR, BG, GR, GB}

21 a {55, 57, 59, 75, 77, 79, 95, 97, 99}

b {57, 59, 75, 79, 95, 97}

22 $S = \{H1, H2, H3, H4, T1, T2, T3, T4\}$

23 1024

24 {SS, SVS, SVV, VSS, VSV, VV}

EXERCISE 8B RELATIVE FREQUENCY1 a On its end: $\frac{76}{200} = \frac{19}{50}$ or 38%On its side: $\frac{124}{200} = \frac{31}{50}$ or 62%b i $\frac{19}{50}$ or 38% ii $\frac{31}{50}$ or 62%2 a $\frac{3}{10}$ or 30% b $\frac{7}{10}$ or 70%3 a Australia $\frac{169}{500}$, Japan $\frac{153}{500}$, Germany $\frac{77}{500}$,Korea $\frac{62}{500} = \frac{31}{250}$, Other $\frac{39}{500}$ b $\frac{169}{500}$ or 33.8% c $\frac{31}{250}$ or 12.4%4 $\frac{107}{5284}$ or 2%5 a $\frac{23}{200}$ or 11.5%b $\frac{43}{270}$ or 15.9%

6 a 25.3% b 17.1% c 7.2%

7 a 0 b $\frac{3}{50}$ or 6% c $\frac{21}{50}$ or 42% d $\frac{39}{50}$ or 78%8 a $\frac{1}{50}$ or 2% b $\frac{1}{100}$ or 1% c $\frac{6}{25}$ or 48% d $\frac{19}{25}$ or 76%9 a i 99.260 ii $\frac{99260}{100000}$ iii 99.26%

b 99.393%

c i Male 97.1%, Female 98.4%

ii Male 90.9%, Female 94.6%

iii Male 57.8%, Female 72.3%

d i 95 141 ii 71 589 iii 75.2% iv 75.2%

e i 97 251 ii 82 305 iii 84.6% iv 84.6%

10 $\frac{339}{1000}$ or 33.9%**EXERCISE 8C** PROBABILITY1 a $\frac{1}{6}$ b $\frac{1}{2}$ c $\frac{1}{3}$ d $\frac{1}{3}$ e 0 f 12 a $\frac{1}{23}$ b $\frac{11}{23}$ c $\frac{12}{23}$ d 13 a $\frac{1}{2}$ b $\frac{1}{3}$ c $\frac{1}{6}$ d $\frac{5}{6}$ e 0 f 14 a $\frac{1}{8}$ b $\frac{1}{8}$ c $\frac{1}{4}$ d $\frac{1}{2}$ e $\frac{1}{2}$ f $\frac{3}{8}$ g $\frac{1}{4}$ h 0 i 15 a $\frac{1}{52}$ b $\frac{1}{13}$ c $\frac{1}{4}$ d $\frac{1}{26}$ e $\frac{1}{26}$ f $\frac{1}{2}$ g $\frac{1}{2}$ h 0 i 16 a $\frac{1}{200}$ b $\frac{1}{100}$ c $\frac{1}{40}$ d $\frac{1}{20}$ e $\frac{1}{10}$ f 17 a $\frac{1}{4}$ b $\frac{1}{6}$ c $\frac{1}{12}$ d $\frac{5}{12}$ e $\frac{7}{12}$ f $\frac{11}{12}$ g $\frac{1}{6}$ h 08 a $\frac{4}{9}$ b $\frac{1}{3}$ c $\frac{2}{9}$ d $\frac{7}{9}$ e $\frac{5}{9}$ f $\frac{2}{3}$ g $\frac{1}{3}$ h $\frac{2}{9}$ i 0 j 19 a $\frac{1}{20}$ b $\frac{19}{20}$ c $\frac{1}{2}$ d $\frac{1}{2}$ e $\frac{9}{20}$ f $\frac{11}{20}$ g $\frac{3}{10}$ h $\frac{7}{10}$ i 0 j $\frac{19}{20}$ k $\frac{9}{20}$ l $\frac{11}{20}$

10 Other answers possible.

a i $P(7) = 0$ ii $P(\text{odd or even}) = 1$ b i $P(\text{yellow}) = 0$ ii $P(\text{red or white}) = 1$ c i $P(17) = 0$ ii $P(\text{red or black}) = 1$ 11 a Wrong, $0 \leq P(E) \leq 1$ b Wrong, $0 \leq P(E) \leq 1$

c Possible

13 a $\frac{1}{3}$ b-e Discuss with class14 a i $\frac{1}{2}$ ii $\frac{3}{10}$ iii $\frac{1}{5}$ 15 a i $\frac{1}{4}$ ii $\frac{1}{4}$ iii $\frac{1}{4}$ iv $\frac{1}{4}$

17 Other answers possible.

a $P(3) = \frac{1}{10}$ b $P(\text{odd}) = \frac{1}{2}$ c $P(10) = 0$ d $P(\text{number less than } 10) = 1$ e $P(\text{number divisible by } 3) = \frac{3}{10}$ f $P(\text{number less than } 9) = \frac{9}{10}$

g Impossible h Impossible

EXERCISE 8D COMPLEMENTARY EVENTS

1 a {H, T}

b i {T} ii {H}

2 a {1, 2, 3, 4, 5}

b i {not spinning a 3} = {1, 2, 4, 5}

ii {not spinning an odd number} = {spinning an even number} = {2, 4}

iii {not spinning a number greater than 3} = {spinning a number less than or equal to 3} = {1, 2, 3}

iv {spinning a 5} = {5}

3 a B b D c A d E e F f C

4 a {not selecting a $\spadesuit$ } = {selecting a $\heartsuit$, $\clubsuit$ or $\diamondsuit$ }b {not selecting a $\diamondsuit$ or $\heartsuit$ } = {selecting a $\clubsuit$ or $\spadesuit$ }c {not selecting a $\diamondsuit$, $\heartsuit$ or $\clubsuit$ } = {selecting a $\spadesuit$ }

d {not selecting a red card} = {selecting a black card}

e {not selecting an ace} f {selecting a king}

5 a {HH, HT, TH, TT}

b i {HT, TH, TT} ii {HH, TH, TT} iii {HH, TT}

iv {HT, TH, HH} v {TT} vi {HH}

- 6 a $\frac{1}{4}$ b 0.7 c 85%
- 7 a $\frac{1}{8}$ b 0.33 c 52%
- 8 a $\frac{1}{100}$ b $\frac{99}{100}$ c $\frac{1}{10}$ d $\frac{9}{10}$ e $\frac{9}{100}$
 f $\frac{91}{100}$ g $\frac{2}{25}$ h $\frac{23}{25}$ i $\frac{1}{10}$ j $\frac{9}{10}$
 k $\frac{1}{25}$ l $\frac{24}{25}$ m $\frac{7}{100}$ n $\frac{93}{100}$
- 9 a $\frac{51}{52}$ b $\frac{25}{26}$ c $\frac{25}{26}$ d $\frac{12}{13}$ e $\frac{11}{13}$ f $\frac{3}{4}$
- 10 a i $\frac{1}{8}$ ii $\frac{1}{8}$ iii $\frac{3}{8}$ iv $\frac{3}{8}$ v $\frac{1}{8}$
 b i No tails ii $\frac{7}{8}$
 c i No heads ii $\frac{7}{8}$
- 11 a $\frac{2}{25}$ b $\frac{23}{25}$ c $\frac{1}{20}$ d $\frac{19}{20}$
 e $\frac{1}{50}$ f $\frac{49}{50}$ g $\frac{11}{100}$ h $\frac{89}{100}$

EXERCISE 8E PROBABILITY FOR MULTISTAGE EVENTS

- 1 a {TT, TF, FT, FF}
- b i $\frac{1}{4}$ ii $\frac{1}{4}$ iii $\frac{1}{2}$ iv $\frac{1}{4}$
- 2 a {H1, H2, H3, H4, H5, H6, T1, T2, T3, T4, T5, T6}
- b i $\frac{1}{12}$ ii $\frac{1}{12}$ iii $\frac{1}{4}$ iv $\frac{1}{6}$
- 3 a {H1, H2, H3, H4, H5, T1, T2, T3, T4, T5}
- b i $\frac{1}{10}$ ii $\frac{1}{5}$ iii $\frac{3}{10}$

4 a

		Second die			
		1	2	3	4
First die	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)

- b i $\frac{1}{16}$ ii $\frac{1}{4}$ iii $\frac{1}{8}$
 iv $\frac{1}{4}$ v $\frac{1}{4}$ vi $\frac{1}{4}$
 c i $\frac{3}{8}$ ii $\frac{7}{16}$

5 a

		Second die					
		1	2	3	4	5	6
First die	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

- b i $\frac{1}{36}$ ii $\frac{1}{6}$ iii $\frac{1}{18}$ iv $\frac{1}{18}$
 v $\frac{1}{4}$ vi $\frac{1}{4}$ vii $\frac{1}{9}$ viii $\frac{1}{18}$
 c 7
 d i $\frac{5}{18}$ ii $\frac{11}{36}$

6 a {BB, BG, GB, GG}

- b i $\frac{1}{4}$ ii $\frac{1}{4}$ iii $\frac{1}{4}$ iv $\frac{1}{2}$ v $\frac{1}{4}$

7 a {BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG}

- b i $\frac{1}{8}$ ii $\frac{1}{8}$ iii $\frac{3}{8}$ iv $\frac{3}{8}$
 v $\frac{1}{8}$ vi $\frac{7}{8}$ vii $\frac{1}{2}$ viii $\frac{1}{8}$

8 a {PSA, PSF, PFA, PFF, PCA, PCF, BSA, BSF, BFA, BFF, BCA, BCF}

- b i $\frac{1}{12}$ ii $\frac{1}{12}$ iii $\frac{1}{6}$ iv $\frac{1}{3}$
 v $\frac{2}{3}$ vi $\frac{1}{2}$ vii $\frac{1}{4}$

9 a {RR, RB, RW, BR, BB, BW, WR, WB, WW}

- b i $\frac{1}{9}$ ii $\frac{1}{3}$ iii $\frac{2}{3}$ iv $\frac{1}{3}$
 v $\frac{2}{9}$ vi $\frac{4}{9}$ vii $\frac{5}{9}$ viii $\frac{4}{9}$

10 a {RB, RW, BR, BW, WR, WB}

- b i 0 ii 0 iii 1 iv $\frac{1}{3}$
 v $\frac{1}{3}$ vi $\frac{2}{3}$ vii $\frac{2}{3}$ viii $\frac{1}{3}$

11 a {HH, HD, HS, HC, DH, DD, DS, DC, SH, SD, SS, SC, CH, CD, CS, CC}

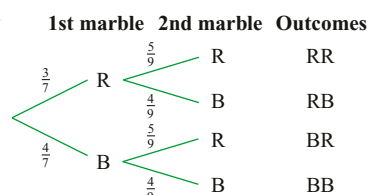
- b i $\frac{1}{16}$ ii $\frac{1}{16}$ iii $\frac{1}{8}$ iv $\frac{1}{4}$ v $\frac{3}{4}$
 vi $\frac{1}{4}$ vii $\frac{1}{4}$ viii $\frac{1}{4}$ ix $\frac{1}{2}$ x $\frac{1}{8}$
 c i $\frac{3}{8}$ ii $\frac{7}{16}$ iii $\frac{3}{4}$ iv $\frac{9}{16}$

12 a {KMH, KHM, MKH, MHK, HKM, HMK}

- b i $\frac{1}{3}$ ii $\frac{1}{3}$ iii $\frac{1}{3}$ iv $\frac{1}{2}$
 v $\frac{1}{2}$ vi $\frac{1}{6}$ vii $\frac{2}{3}$

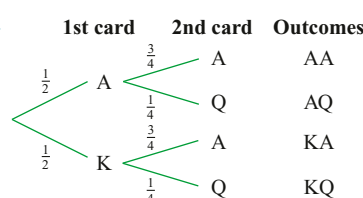
EXERCISE 8F PROBABILITY TREE DIAGRAMS

1 a



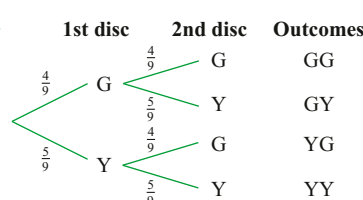
- b $\frac{16}{63}$

2 a



- b $\frac{3}{8}$

3 a

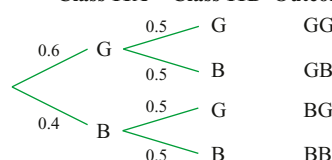


- b i $\frac{16}{81}$ ii $\frac{25}{81}$ iii $\frac{20}{81}$ iv $\frac{20}{81}$

4 a i 0.4 ii 0.6

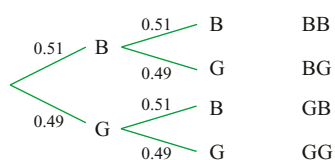
- b i 0.5 ii 0.5

c Class 11A Class 11B Outcomes



- d i 0.2 ii 0.3 iii 0.2

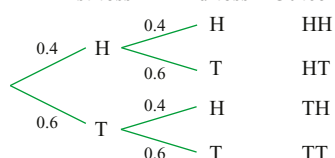
5 a 1st child 2nd child Outcomes



b i 0.2601 ii 0.2499
iii 0.2499 iv 0.2401

6 a i 0.6 ii 0.4

b 1st toss 2nd toss Outcomes



c i 0.16 ii 0.24 iii 0.24 iv 0.36

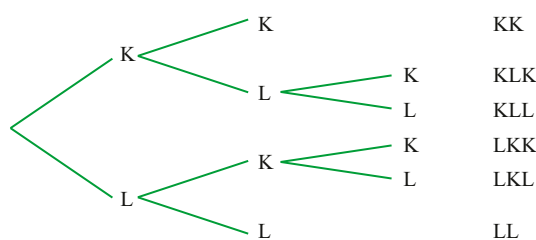
7 a 0.72 b 0.08 c 0.18 d 0.02

8 a 0.48 b 0.32 c 0.12 d 0.08

9 a $\frac{9}{20}$ b $\frac{3}{20}$ c $\frac{3}{10}$ d $\frac{1}{10}$

10 a 0.81 b 0.01 c 0.18 d 0.99

11 a 1st set 2nd set 3rd set Outcome



i 0.25 ii 0.25 iii 0.25 iv 0.25

b i 0.36 ii 0.288 iii 0.16 iv 0.192

EXERCISE 8G EXPECTED FREQUENCY AND SIMULATIONS

1 Expected number of tails = $\frac{1}{2} \times 400 = 200$

2 a 153 b 147

3 438 4 35

5 68 6 21

7 Royal flush 2, Four of a kind 240, Full house 1441, Straight 3925, Three of a kind 21 128, Two pairs 47 539, One pair 422 569

8 45

9 a i $\frac{1}{6}$ ii $\frac{1}{2}$ iii $\frac{1}{3}$ iv $\frac{2}{3}$

b i $\frac{1}{6} \times 600 = 100$

ii $\frac{1}{2} \times 600 = 300$

iii $\frac{1}{3} \times 600 = 200$

iv $\frac{2}{3} \times 600 = 400$

10 a i $\frac{3}{8}$ ii $\frac{1}{2}$ iii $\frac{1}{8}$

b i 150 ii 200 iii 50

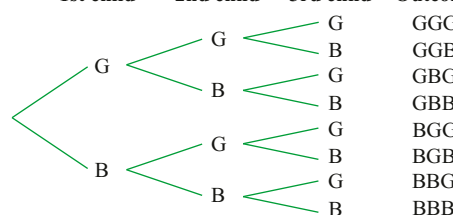
11 a i $\frac{1}{2}$ ii $\frac{1}{4}$ iii $\frac{1}{13}$ iv $\frac{1}{26}$

b i 130 ii 65 iii 20 iv 10

12 a i $\frac{1}{4}$ ii $\frac{1}{2}$ iii $\frac{1}{4}$

b i 25 ii 50 iii 25

13 a 1st child 2nd child 3rd child Outcome



b i $\frac{1}{8}$ ii $\frac{3}{8}$ iii $\frac{3}{8}$ iv $\frac{1}{8}$

c i 25 ii 75 iii 75 iv 25

14 a $\frac{1}{2}$ 15 a $\frac{1}{4}$

b i 50 b i 26

c i 500 c i 260

16 a Second spinner

First spinner	Second spinner		
	1	2	3
1	(1, 1)	(1, 2)	(1, 3)
2	(2, 1)	(2, 2)	(2, 3)
3	(3, 1)	(3, 2)	(3, 3)

b i $\frac{1}{9}$ ii $\frac{1}{3}$ iii $\frac{2}{9}$

c i 50 ii 150 iii 100

REVIEW MULTIPLE-CHOICE QUESTIONS

1 C 2 A 3 B 4 C 5 B 6 D
7 C 8 D 9 D 10 D

REVIEW SET 1

1 a {1, 2, 3, 4, 5, 6} b {BB, BG, GB, GG}

2 0.18

3 a $\frac{1}{52}$ b $\frac{1}{13}$ c $\frac{1}{4}$
d $\frac{3}{13}$ e $\frac{1}{2}$ f $\frac{1}{26}$

4 a Any impossible event

b e.g. Getting heads when a coin is tossed.

c Any certain event

5 a $\frac{3}{8}$ b $\frac{1}{4}$ c $\frac{1}{4}$ d $\frac{7}{8}$

e $\frac{5}{8}$ f $\frac{1}{8}$ g 0

6 a $\frac{1}{12}$ b $\frac{1}{4}$ c $\frac{1}{12}$ d $\frac{1}{6}$

7 a $\frac{1}{8}$ b $\frac{1}{4}$ c $\frac{1}{8}$ d $\frac{1}{4}$

8 a 50 b 150 c 200

9 45

REVIEW SET 2

1 {PMY, PYM, MPY, MYP, YPM, YMP}

2 a 0.03 b 0.01 c 0.28 d 0.72

3 $\frac{1}{4}$

4 a $\frac{1}{8}$ b $\frac{1}{8}$ c $\frac{1}{2}$ d 0

e 1 f $\frac{1}{4}$

5 65%

6 a $\frac{1}{4}$
e $\frac{1}{2}$

b $\frac{3}{4}$
f 1

c $\frac{1}{26}$

d $\frac{25}{26}$

7 a

		Second die					
		1	2	3	4	5	6
First die	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

b i $\frac{1}{36}$

ii $\frac{1}{6}$

iii $\frac{1}{6}$

iv 0

8 a $\frac{1}{25}$

b 108

9 340

REVIEW SET 3

1 a {RR, RB, RG, BR, BB, BG, GR, GB, GG}

b {RB, RG, BR, BG, GR, GB}

2 a 0.11

b 0.34

c 0

3 a 0.01

b 0.09

c 0.1

d 0.9

e 0.01

f 0.09

g 0.99

h 0

i 0.8

j 0.5

4 a

		Second set of cards				
		1	2	3	4	5
First set of cards	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)

b i $\frac{1}{25}$

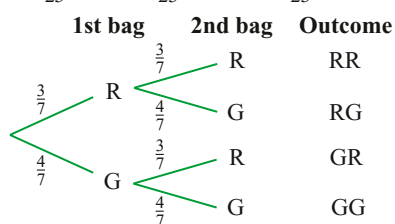
ii $\frac{2}{25}$

iii $\frac{1}{25}$

iv $\frac{4}{25}$

v 0

5 a



b i $\frac{15}{56}$

ii $\frac{29}{56}$

iii $\frac{11}{14}$

6 a 1300

b 650

c 200

d 100

7 a

		Second die					
		1	2	3	4	5	6
First die	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b i $\frac{1}{6}$

ii $\frac{1}{36}$

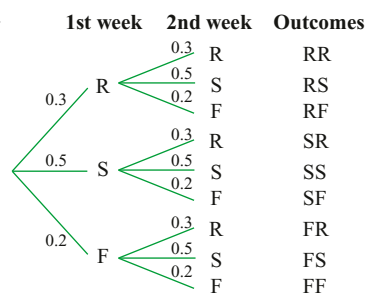
iii $\frac{5}{36}$

c i 10

ii 1 or 2

iii 8 or 9

8 a



b i 0.09

ii 0.25

iii 0.04

iv 0.51

REVIEW SET 4

1 a $\frac{1}{13}$

b $\frac{8}{13}$

c $\frac{3}{13}$

d $\frac{1}{13}$

e 0

2 1

3 a $\frac{1}{4}$

b $\frac{1}{3}$

c $\frac{5}{12}$

d $\frac{7}{12}$

e $\frac{2}{3}$

4 a $\frac{1}{200}$

b $\frac{199}{200}$

c $\frac{1}{100}$

d $\frac{99}{100}$

e $\frac{9}{200}$

f $\frac{191}{200}$

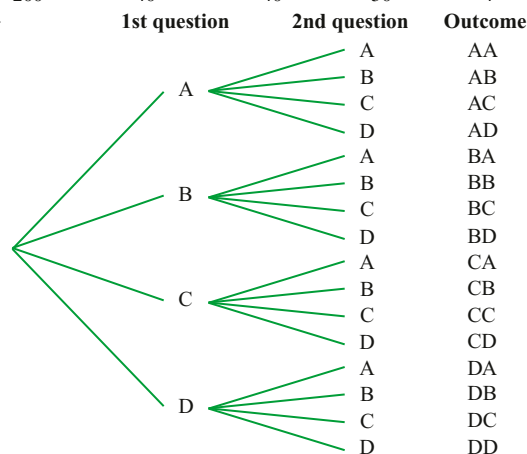
g $\frac{11}{40}$

h $\frac{29}{40}$

i $\frac{49}{50}$

j $\frac{3}{4}$

5 a



b i $\frac{1}{16}$

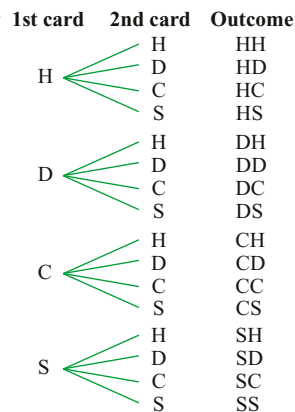
ii $\frac{1}{16}$

iii $\frac{1}{16}$

iv $\frac{9}{16}$

v $\frac{7}{16}$

6 a



b i $\frac{1}{16}$

ii $\frac{1}{8}$

iii $\frac{1}{4}$

iv $\frac{1}{2}$

7 a 36

b

		Second die					
		1	2	3	4	5	6
First die	1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
	2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
	3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
	4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
	5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
	6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

- c i $\frac{1}{36}$ ii $\frac{1}{6}$ iii $\frac{11}{36}$ iv $\frac{1}{6}$
 v $\frac{1}{4}$ vi $\frac{1}{4}$
 d i $\frac{1}{18}$ ii $\frac{1}{6}$ iii $\frac{1}{36}$ iv $\frac{7}{36}$ v 0
 e i $\frac{1}{6}$ ii $\frac{5}{18}$ iii $\frac{1}{18}$ iv $\frac{5}{18}$ v 0

8 2

REVIEW PRACTICE EXAMINATION QUESTION

- 1 a i {RR, RG, RB, GR, GG, GB, BR, BG, BB}
 ii {RG, RB, GR, GB, BR, BG}

b i 0.16 ii 0.66

c 59%

d i $\frac{1}{200}$ ii $\frac{199}{200}$ iii $\frac{4}{5}$ e i $\frac{1}{4}$ ii 112 or 113

		1st child	2nd child	Outcome
0.55	B	0.55	B	BB
		0.45	G	BG
0.45	G	0.55	B	GB
		0.45	G	GG

ii 0.495

g 8

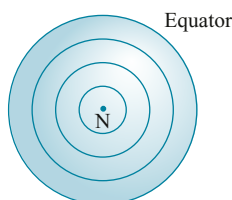
CHAPTER 9 WORKING WITH TIME**ARE YOU READY?**

- 1 D 2 B 3 A 4 B 5 C 6 C
 7 D 8 C 9 B 10 D 11 C 12 B
 13 A 14 B

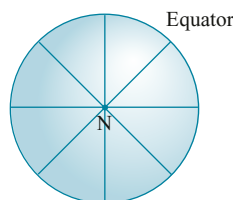
EXERCISE 9A LATITUDE AND LONGITUDE

- 1 a Parallel of latitude b Meridian of longitude
 c Parallel of longitude d Meridian of longitude
 e Meridian of longitude f Parallel of latitude
 2 a S b E
 c W d N

3 b



4 b



- 5 a A 40°N, B 80°S, C 0°, D 20°S, E 60°N, F 20°N, G 80°N,
 H 40°S, I 60°S, J 0°

- 6 a C b I c H, M
 d D, P e B f L, R
 g A, G h J, O i K, Q
 j F k E

- 7 a 90°N b 90°S
 8 a i 40°N ii 70°N iii 0° iv 30°S
 v 60°S

- b i 70°N ii 40°N iii 0° iv 30°S
 v 60°S

- 9 a i 60°N ii 40°N iii 30°N iv 0°
 v 15°S vi 35°S vii 45°S
 b i 40°N ii 15°S iii 45°S iv 0°
 v 30°N vi 60°N vii 35°S

- 10 i 34°S ii 51.5°N iii 41°N iv 23.5°S v 23.5°N

- 11 a A 110°E, B 60°E, C 50°W, D 95°W, E 50°W, F 95°W, G
 0°, H 60°E, I 0°, J 110°E

- b C, G b E, H, I c A, D d B, F, J

- 12 a i 70°E ii 120°E iii 60°W
 b i 60°W ii 0° iii 70°E iv 120°E

- 13 a i 50°E ii 80°E iii 120°E
 iv 70°W v 100°W
 b i 100°W ii 70°W iii 0°
 iv 50°E v 80°E vi 120°E

- 14 i 151°E ii 0° iii 74°W

EXERCISE 9B POSITION ON EARTH'S SURFACE

- 1 a i A, B, C, D ii C, F, K, Y, Z, V iii C
 b i Q, R, Z, T ii A, H, I, P, Q, X iii Q

- 2 A(50°N, 75°W), E(30°N, 100°E), G(30°N, 0°), I(15°N,
 75°W), K(15°N, 45°E), M(0°, 100°E), O(0°, 0°), Z(30°S,
 45°E), U(60°S, 100°E), W(60°S, 0°)

- 3 a D b I c V d R e J
 f H g T h U i P j F
 k X l O m L n B o C

- 4 a Australia b China
 c Antarctica d Mexico
 e Kenya f France

- 5 a i S ii N iii N
 b i W ii E iii E

- 6 a i (25°S, 95°E) ii (45°S, 95°E)
 iii (35°S, 105°E) iv (35°S, 85°E)
 b i (35°N, 150°E) ii (15°N, 150°E)
 iii (25°N, 160°E) iv (25°N, 140°E)
 c i (10°S, 90°W) ii (30°S, 90°W)
 iii (20°S, 80°W) iv (20°S, 100°W)
 d i (40°S, 40°W) ii (60°S, 40°W)
 iii (50°S, 30°W) iv (50°S, 50°W)

- 7 a (25°N, 75°W) b (5°N, 75°W)
 c (15°S, 75°W) d (50°S, 75°W)
 8 a (15°N, 165°W) b (32°S, 170°W)
 c (18°S, 160°E) d (34°N, 150°E)

- 9 a B is 15°S and 20°E of A
 b B is 5°S and 10°W of A
 c B is 15°N and 10°E of A

- d** B is 15°S and 40°W of A
e B is 30°N and 130°W of A
f B is 40°N and 60°E of A
- 10 c** **i** Sydney **ii** Beijing
iii Rome **iv** Johannesburg
v Moscow **vi** Los Angeles
- d i** (36°N, 140°E) **ii** (52°N, 0°)
iii (41°N, 74°W) **iv** (36°N, 51°E)
v (23°S, 43°W) **vi** (49°N, 123°W)
- 11 a i** Canberra **ii** Darwin
iii Hobart **iv** Cairns
b i Sydney **ii** Alice Springs
c i Brisbane **ii** Hobart
- 12** 2°S, 4°E **13** 3°S, 6°W

EXERCISE 9C TIME UNITS AND INTERVALS

- 1**
-
- 2 a** 2.5 hours **b** $3\frac{1}{3}$ hours
c 5 minutes **d** 3 hours 13 minutes
e 8 minutes 5 seconds **f** 10 hours 35 minutes
g 277 minutes **h** 2832 seconds
i 137 hours **j** 4 days 4 hours
k 604 800 seconds **l** 2 hours 46 minutes 40 seconds
- 3 a** 2:06 pm **b** 12:51 am **c** 8:05 pm
d 1:22 am **e** 9:25 am **f** 9:21 pm
g 7:15 pm **h** 12:12 pm **i** 12:54 pm
j 5:49 pm **k** 6:22 am **l** 10:54 pm
- 4 a** 0743 **b** 1720 **c** 0135
d 1243 **e** 1620 **f** 0146
g 0608 **h** 2027 **i** 0920
j 2308 **k** 1200 **l** 0000
- 5 a** 0830 **b** 1135 **c** 0736
d 1610 **e** 1755 **f** 0832
g 1222 **h** 1240 **i** 0649
- 6 a** 8 days 10 hours **b** 6 days 20 hours
c 3 days 7 hours **d** 5 days 19 hours 31 minutes
e 4 days 8 hours 23 minutes **f** 7 days 8 hours 19 minutes
- 7 a** 1 day 13 hours 52 minutes **b** 4 days 5 hours 7 minutes
c 4 days 13 hours 23 minutes **d** 1 day 5 hours 39 minutes
e 3 days 5 hours 51 minutes **f** 5 days 18 hours 51 minutes
- 8 a** 1242 **b** 5 hours 18 minutes
c 2005 **d** 3 days 6 hours 45 minutes
- 9 a** 8:10 pm Wednesday 16 May
b 7:14 pm Sunday 3 June
c 1 hour 28 minutes

EXERCISE 9D WORLD TIME ZONES

- 1 a** 8:00 pm **b** 4:00 am **c** 3:00 pm
d 8:00 am **e** 2:00 pm **f** 4:00 pm
- 2 a** 11:00 pm **b** 7:30 pm **c** 9:00 am
d 8:00 am **e** 3:00 pm **f** 11:30 pm
- 3 a** 7:00 am Wednesday **b** 2:30 am Wednesday
c 1:00 am Wednesday **d** 5:00 am Wednesday
e 7:00 am Wednesday **f** 4:00 am Wednesday
- 4 a** 10:00 pm Sunday **b** 8:00 pm Sunday
c 11:00 pm Sunday
d 12:00 midnight end of Sunday
e 11:00 pm Sunday **f** 1:00 am Monday

- 5 a** 1 hour **b** 2 hours **c** $3\frac{1}{2}$ hours
d 2 hours **e** 9 hours **f** 10 hours
- 6 a** 5:00 pm **b** 7:00 pm **c** 11:00 am
d 4:00 am **e** 1:00 am **f** 6:00 am
- 7 a** 6:00 pm **b** 11:00 pm **c** 8:00 am
d 7:30 pm **e** 10:00 am **f** 9:00 pm
- 8 a** 2:00 am Sunday **b** 3:00 am Sunday **c** 9:00 am Sunday
- 9 a** 9:00 pm Monday **b** 7:00 pm Monday
c 2:00 pm Monday
- 10 a** San Francisco 4:00 am, Sydney 10:00 pm

b

San Francisco	London	Sydney
4:00 am	12:00 noon (Monday)	10:00 pm
6:00 am	2:00 pm	12:00 am
8:00 am	4:00 pm	2:00 am
10:00 am	6:00 pm	4:00 am
12:00 pm	8:00 pm	6:00 am
2:00 pm	10:00 pm	8:00 am
4:00 pm	12:00 midnight	10:00 am
6:00 pm	2:00 am	12:00 pm
8:00 pm	4:00 am	2:00 pm
10:00 pm	6:00 am	4:00 pm
12:00 am	8:00 am	6:00 pm
2:00 am	10:00 am	8:00 pm
4:00 am	12:00 noon (Tuesday)	10:00 pm

- c** 10:00 pm London time

11

Longitude west	Local time	Longitude east	Local time
0°	12:00 noon Wed	0°	12:00 noon Wed
45°W	9:00 am Wed	45°E	3:00 pm Wed
90°W	6:00 am Wed	90°E	6:00 pm Wed
135°W	3:00 am Wed	135°E	9:00 pm Wed
180°W	12:00 midnight Tue	180°E	12:00 midnight Wed

- 12 a** Same meridian **b** Differ by one day
c Change of one day
d No, due to countries needing to be on the same day.
- e i** Add one day **ii** Subtract one day
- 13** Sunday 26 March 8:30 pm
- 14 a** 3:00 pm Monday **b** 3:00 am Tuesday
- 15 a** 7:00 am Monday **b** 11:00 pm Monday
- 16 a** 10:00 am Sunday **b** 1:30 am Monday
- 17 a i** 8:00 pm Friday 30 November 2018
ii 26 days 2 hours 30 minutes
iii 10:30 pm Wednesday 26 December 2018
- b i** 2:30 pm Monday 21 January 2019
ii 30 days 2 hours 13 minutes
iii 4:43 pm Wednesday 20 February 2019
- c** 81 days 9 hours 43 minutes

EXERCISE 9E AUSTRALIAN TIME ZONES

- 1 a** 8:00 am **b** 10:00 am **c** 5:00 pm
d 6:30 pm **e** 7:20 pm **f** 3:00 pm
- 2 a** 7:00 pm **b** 12:30 pm
- 3** 1:00 pm Saturday

- 4 a To allow greater use of daylight hours in summer
 b ACT, NSW, SA, Tas, Vic
 c Start first Sunday in October and finish first Sunday in April

5

City	Time
Brisbane	12:00 pm
Sydney	1:00 pm
Canberra	1:00 pm
Melbourne	1:00 pm
Hobart	1:00 pm
Adelaide	12:30 pm
Darwin	11:30 am
Perth	10:00 am

- 6 11:30 pm 7 5:00 pm 8 12:00 midnight
 9 Adelaide 9:30 pm, Sydney 10:00 pm, Brisbane 9:00 pm
 10 10:00 am

EXERCISE 9F INTERPRETING TIMETABLES

- 1 a i 1802 ii 1825
 b 1853
 c i 21 minutes ii 45 minutes
 iii 43 minutes iv 32 minutes
 d i 1 hour 3 minutes, 53 minutes, 1 hour 3 minutes, 51 minutes, 51 minutes
 ii delay at Clyde
 2 a i 88 minutes ii 40 minutes
 iii 52 minutes iv 31 minutes, change at Blacktown
 b i 0943 ii 0945 iii 1008 iv 1018
 c i No ii Strathfield iii 0930
 iv Strathfield 0938 v 1037
 vi 1 hour 7 minutes vii 53 minutes
 3 a i 8 minutes ii 1048 iii 12 minutes
 b i 17 minutes ii 1045 iii 5 hours 24 minutes
 4 a i 2
 ii 6 hours 56 minutes, 5 hours 39 minutes
 iii 1.23 m and 0.63 m
 b i 12 June 2206, 1.77 m
 ii 12 June 0408, 0.47 m
 iii 11 hours 56 minutes
 iv Times of the tides
 c 14 May, 15 May, 16 May, 23 May, 4 June, 5 June, 12 June, 13 June, 14 June, 26 June and 27 June
 5 Off at 0856 Vacluse, on at 0925 Vacluse, off Rose Bay 0937, on 0952 Rose Bay, off Double Bay 1003, on Double Bay 1018, off Kings Cross 1025, on Kings Cross 1040, off City - Town Hall 1048, on City - Town Hall 1103, arrives City - Walsh Bay 1116.

REVIEW MULTIPLE-CHOICE QUESTIONS

- 1 C 2 A 3 C 4 D 5 C 6 A
 7 C 8 D 9 A 10 B 11 A 12 D
 13 A 14 D 15 C 16 B 17 C 18 C
 19 A 20 D

REVIEW SET 1

- 1 a one whose centre is not the centre of Earth
 b one whose centre is the centre of Earth
 2 a 50°S b 0° c 20°N d 35°N

- 3 a 50°E b 0° c 40°W d 100°W
 4 a T b Q c R d P
 5 a 255 minutes b 10:46 am
 c 1546 d 1 days 17 hours 48 minutes
 6 a 10:00 pm b 4:00 am c 12:00 pm
 7 a 3:00 pm Tuesday b 2:00 am Wednesday
 8 5:30 pm
 9 a 0.9 m and 0.86 m
 b 7 hours 1 minute and 6 hours 29 minutes

REVIEW SET 2

- 1 a Circle parallel to the equator
 b Circle passing through both poles
 2 a (44°N, 60°E) b (44°N, 30°W)
 c (0°, 30°W) d (0°, 60°E)
 e (32°S, 60°E) f (32°S, 30°W)
 3 a (41°N, 105°E) b (11°N, 105°E)
 c (26°N, 120°E) d (26°N, 90°E)
 4 a South b West
 5 a 113 hours b 2142
 c 5 days 8 hours 26 minutes
 6 1:00 am Friday 7 2:00 am 8 5:50 pm
 9 a 1 hour 18 minutes b 10:15 am

REVIEW SET 3

- 1 a E(32°N, 15°W) b H(0°, 80°W)
 c P(16°S, 40°W) d F(0°, 120°W)
 2 a D b M c D or E d M
 3 a i North ii East
 b i South ii East
 c i North ii West
 d i South ii West
 4 a (32°N, 45°W) b (14°N, 15°W)
 c (20°N, 100°W) d (16°S, 85°W)
 5 a 6 days 13 hours b 3:34 pm
 c 1528 d 2 days 15 hours 52 minutes
 6 3:00 am Friday 7 5:00 pm Friday 8 11:58 am
 9 Adelaide 9:00 pm, Sydney 9:30 pm, Brisbane 8:30 pm
 10 a 4:50 pm b 5:10 pm

REVIEW SET 4

- 1 a (45°N, 55°E) b (0°, 65°W)
 c (0°, 0°) d (10°S, 65°W)
 2 a V, W b V c Z d E, D
 3 a (20°N, 65°W) b (10°S, 95°W)
 c (15°S, 55°E) d (60°S, 105°E)
 4 a 3 hours 47 minutes b 11:27 am
 c 1123 d 2123
 5 2:00 am 6 5:00 am Tuesday 7 5:30 pm
 8 11:30 pm
 9 a 17 minutes b 11:00 am c 3 hours 35 minutes

REVIEW PRACTICE EXAMINATION QUESTION

- 1 a UTC 0°
 b i 6:27 pm ii 3 hours 43 minutes
 c B(60°N, 30°W)
 d 8:30 am
 e i 9 hours ii 11:40 pm Sunday
 iii 19 hours 35 minutes
 f i 9:28 am ii 10:10 am iii 11:50 am

CHAPTERS 7–9 CUMULATIVE REVIEW

1 a

Colour	Number	Relative frequency
Black	10	$\frac{10}{30} = \frac{1}{3}$
Brown	6	$\frac{6}{30} = \frac{1}{5}$
Blond	9	$\frac{9}{30} = \frac{3}{10}$
Red	1	$\frac{1}{30}$
Grey	4	$\frac{4}{30} = \frac{2}{15}$

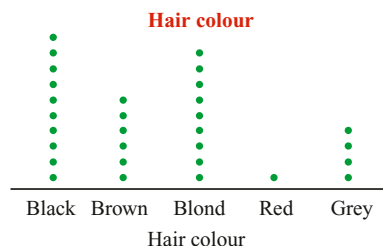
b 30

c black

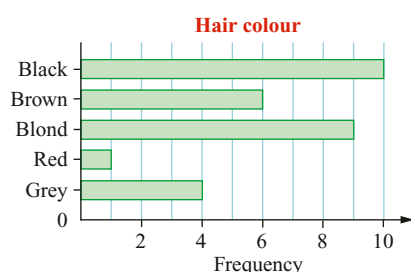
d i $\frac{1}{3}$

ii $\frac{3}{10}$

e i



ii



2 a 8

b 26

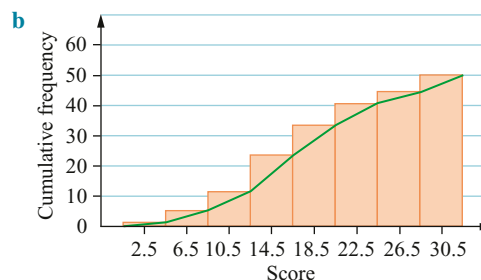
3 a

Class B	Stem	Class A
7	0	
7 5 1	1	3 9
9 9 9 7 7 5 3 1	2	1 3 5 7 9 9
9 3 3 1	3	1 1 3 5 5 7 7 9
3 1 1	4	1 3 5

b Class A because it had more higher scores than class B.

4 a

Score	Class centre	Frequency	Cumulative frequency
1–4	2.5	1	1
5–8	6.5	4	5
9–12	10.5	7	12
13–16	14.5	12	24
17–20	18.5	9	33
21–24	22.5	8	41
25–28	26.5	4	45
29–32	30.5	5	50



5 a {GGG, GGB, GBG, GBB, BGG, BGB, BBG, BBB}

b i $\frac{1}{52}$ ii $\frac{1}{13}$ iii $\frac{1}{4}$ iv $\frac{1}{2}$

c $\frac{23}{100}$

d i $\frac{1}{12}$ ii $\frac{1}{4}$ iii $\frac{1}{12}$ iv $\frac{1}{6}$

6 a 3; not rolling a 3

b 1, 3, 5; rolling an even number

c 4, 5, 6; rolling a number less than 4

7 a GG, GB, GW, BF, BB, BW, WG, WB, WW

b i $\frac{1}{9}$ ii $\frac{1}{3}$ iii $\frac{1}{3}$ iv $\frac{1}{3}$
v $\frac{2}{3}$ vi $\frac{1}{9}$ vii $\frac{2}{9}$ viii $\frac{4}{9}$

8 a GB, GW, BG, BW, WG, WB

b i 0 ii 0 iii 1

iv $\frac{1}{3}$ v $\frac{1}{3}$ vi $\frac{1}{2}$

vii $\frac{1}{3}$ viii $\frac{2}{3}$ ix $\frac{2}{3}$

9 a 100 b 200

c 300

10 a i PQR, TUV, XYZ

ii TUV

iii PTX, QUY, RVZ

b P(60°N, 150°W), U(0°, 15°W), Y(15°S, 15°W)

c i True ii True iii False iv False

d i (15°S, 10°W) ii (52°N, 15°W)

11 a 115 hours b 4 hours 49 minutes

c 2 days 13 hours 52 minutes

12 a i 1:00 pm ii 5:00 am

b i 7:00 pm Thursday ii 7:30 am Friday

c i 8:00 am ii 10:00 am

iii 5:00 pm iv 7:30 pm

13 a 28 minutes b 8:00 am c 32 minutes

d 8:28 am e 28 minutes

CHAPTER 10 BUDGETING AND HOUSEHOLD EXPENSES

ARE YOU READY?

1 C 2 C 3 D 4 A 5 D 6 D
7 A 8 B 9 D 10 A 11 D

EXERCISE 10A HOUSEHOLD BILLS

1 a i 1/01/18 to 31/03/18 ii 90 days

b \$201.43

c i 68 kL ii 756 L

d i \$2.17 ii \$348.99 iii \$3.88

e i Larger ii Smaller

f i \$134.76 ii \$336.19

g \$7.72

h This bill was 185 L greater than the local average.

i i Larger ii 111 L iii 9.25 buckets

2 a 587 L/day b 367 L/day

c 477 L/day d 739 L/day

- 3 \$453.19
- 4 a i \$510.50 ii \$842
b i \$207 ii \$451 iii \$506 iv \$254
c \$1480
- 5 a i 22/11/17 to 22/02/18 (inclusive)
ii 93 days
b i 74.7 cents/day ii \$69.47
c i 1861.4 kWh ii 20 kWh
d i 50.54% ii 14.54%
e 47.77 cents/kWh f 35.87 cents/kWh
g i \$525.18 ii \$621.41 iii \$6.68/day
h i Larger ii Larger
i i 13 kWh ii 1235 kWh
j i 431.1 kWh ii 624.2 kWh iii 179.6 kWh
k i \$348.40 ii \$70.97
iii \$419.37 iv \$438.24
l i August 2017 ii February 2017 iii August 2016
- 6 a i 2/02/18 to 3/05/18 ii 91
b i \$17.08 ii 10%
c 4850 MJ
d i \$0.029 67 ii \$0.017 58
e i \$40.31 ii \$181.30
f i 53.30 MJ ii \$2.06
g 0.3 t h 4799.7 MJ
- 7 a i \$0.001 454 74 ii \$871.39, yes
b \$361 c \$1697.39 d \$1882.14
- 8 a \$257.54 b \$321.93 c \$305.83
- 9 a 52 560 L b \$144.54
- 10 a Top-loader \$79.56, front-loader \$37.44
b \$42.12
c i \$104 ii about $2\frac{1}{2}$ years

EXERCISE 10B PURCHASING A CAR

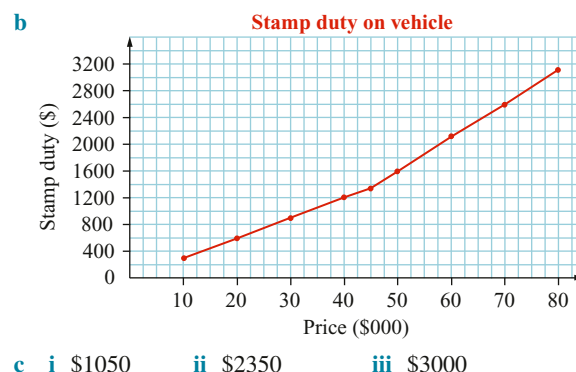
- 1 976 to 1154, \$240
- 2 a \$448 b \$240 c \$293
- 3 a \$249 b \$129 c \$170
- 4 a 3% of \$17 900 = $0.03 \times \$17\,900 = \$537$
b 3% of \$45 000 + 5% of (\$52 380 - \$45 000)
= $0.03 \times \$45\,000 + 0.05 \times \$7380$
= \$1719
- 5 a \$659.70 b \$1070.97
c \$1550 d \$3780
- 6

Costs	Holden	Mazda	Toyota sedan	Toyota SUV	Yamaha motorcycle
RRP (\$)	37 990	31 450	18 000	52 870	14 999
Weight (kg)	1 637	1 440	1 040	1 970	
Private (P) or business (B)	B	P	P	B	P
Registration (\$)	697	293	240	697	62
Stamp duty (\$)	1139.70	943.50	540	1743.50	449.97
CTP insurance (\$)	620	487	528	660	280
Dealer delivery (\$)	790	585	499	887	389
Insurance (\$)	1089	790	560	1185	299
Total (\$)	42 325.70	34 548.50	20 367.00	58 042.50	16 498.97

- 7 \$32, $0.03 \times \$43\,900 = \$1317$, \$1560, \$46 809
- 8 \$25 112 9 \$16 046.70

10 a

Price (\$000)	3% of the price up to \$45 000	5% of the price over \$45 000	Total
10	\$300	–	\$300
20	\$600	–	\$600
30	\$900	–	\$900
40	\$1200	–	\$1200
45	\$1350	–	\$1350
50	\$1350	\$250	\$1600
60	\$1350	\$750	\$2100
70	\$1350	\$1250	\$2600
80	\$1350	\$1750	\$3100



EXERCISE 10C FINANCING A PURCHASE

- 1 48; 48, \$26.33; 22.70; \$26.33, 22.70, \$597.69
- 2 a \$646.25 b \$432.48 c \$905.20
- 3 a i \$461.55 ii \$3954.40
b i \$393.50 ii \$744
c i \$773.95 ii \$11 637
- 4 \$5250, \$35 000, \$5250, 3, 35 000, 5250, 19 250
- 5 \$1999 6 \$8960 7 \$703.87
- 8 a At 12% over 60 months, the monthly repayment is \$435.90, so yes.
b At 14% over 60 months, the monthly repayment is \$456.09, so no.
- 9 a \$24 000 b \$31 500
- 10 a \$8500 b 23.6%
- 11 a \$7153.33 b 15.93%

EXERCISE 10D FUEL CONSUMPTION

- 1 a Fuel consumption = $\frac{400 \text{ km}}{35 \text{ L}} = 11.43 \text{ km/L}$
b Fuel consumption = $\frac{35 \text{ L}}{400 \text{ km}} = 0.0875 \text{ L/km}$
c Fuel consumption = $0.0875 \text{ L/km} \times 100 = 8.75 \text{ L/100 km}$
- 2 a i 9.3 km/L ii 0.108 L/km
iii 10.8 L/100 km
b i 11.6 km/L ii 0.086 L/km
iii 8.6 L/100 km
c i 8.8 km/L ii 0.114 L/km
iii 11.4 L/100 km
- 3 Distance travelled = $\frac{45}{6.4} \times 100 = 703 \text{ km}$
- 4 a 417 km b 688 km c 770 km
- 5 Fuel used = $\frac{1160}{100} \times 10.6 = 122.96 \approx 123 \text{ L}$
- 6 a 27.3 L b 65.88 L c 18.09 L
- 7 \$581.01

- 8 \$3300.74
 9 a \$90.64 b \$75.91 c \$14.73
 10 a ULP \$3018.99, LPG \$1739.07
 b \$1279.92 c \$106.66
 d ≈ 24.4 months e $\approx 38\,600$ km
 11 a i \$2544.26 ii \$1722.79 iii \$821.47
 b \$68.46 c ≈ 17.5 months d $\approx 19\,000$ km

EXERCISE 10E RUNNING COSTS OF A CAR

- 1 Toyota hatchback total = \$8519
 Honda SUV service and repairs = \$868
 Toyota SUV fuel = \$2340
 Ford sedan cost/week = \$254.02
 Holden sedan cost/km = 84.9c
 2 a Toyota hatchback b Ford sedan
 3 a Ford hatchback b Toyota hatchback
 4 a Toyota SUV b Ford sedan
 5 a 22% b 20%
 6 a 12.7c/km b 15.6c/km
 7 a 5% b 8%
 8 \$173.87
 9 \$46.65 more for Holden sedan
 10 \$224.80, 77.93c/km 11 \$221.46, 76.77c/km
 12 a \$12 220 b \$9105
 13 a \$2.54 b \$3.40
 14 a 40 c/day b 57c/day
 15 \$6900 16 a \$10 860 17 6.6%

Nissan 4WD	
Depreciation	\$8 417
Interest	\$4 055
On-road costs	\$1 248
Fuel	\$3 484
Tyres	\$276
Service/repairs	\$1 030
Total	\$18 510

Cost/week = \$356.96

Cost/km = 123.4c/km

	Richard	Stephanie	Lilly	Paul
a	\$10.12	\$9.33	\$9.56	\$9.59
b i	\$202.40	\$186.60	\$191.20	\$191.80
ii	\$151.80	\$139.95	\$143.40	\$143.85

	Helen	Greg	Peta
a	18.1c	19.1c	23.3c
b i	\$173.76	\$183.36	\$223.68
ii	\$115.84	\$122.24	\$149.12

EXERCISE 10F BUDGETING

- 1 a *Income* : $(\$160 \times 52) = \$8320$
Expenses:
 Board $(\$20 \times 52) = \$1040$
 Lessons $(\$80 \times 26) = \$2080$
 Entertainment $(\$45 \times 26) = \$1170$
 Clothes $(\$140 \times 12) = \$1680$
 Total = \$5970
 Income – expenses = \$2350

- b \$2350
 2 a *Income*: $(\$270 \times 26) = \$7020$
Expenses:
 Board $(\$25 \times 52) = \$1300$
 Golf = \$450
 Personal items $(\$150 \times 12) = \$1800$
 Total = \$3550
 Income – expenses = \$3470
 b Yes
 3 a *Income*: $(\$590 \times 52) = \$30\,680$
Expenses:
 Board $(\$100 \times 52) = \$5200$
 Fares $(\$53 \times 52) = \$2756$
 Lunches $(\$42 \times 52) = \$2184$
 Entertainment $(\$150 \times 52) = \$7800$
 Personal items $(\$84 \times 26) = \$2184$
 Clothes $(\$380 \times 12) = \$4560$
 Total = \$24 684
 Income – expenses = \$5996
 b No, she will only have \$17 988.
 4 a *Income*: $(\$640 \times 52) = \$33\,280$
Expenses:
 Rent $(\$120 \times 52) = \$6240$
 Food $(\$110 \times 52) = \$5720$
 Entertainment $(\$145 \times 52) = \$7540$
 Personal items $(\$75 \times 52) = \$3900$
 Car loan repayments $(\$380 \times 12) = \$4560$
 Petrol $(\$95 \times 52) = \$4940$
 Car servicing $(\$380 \times 2) = \$760$
 Car registration and insurance = \$1148
 Mobile phone $(\$64 \times 12) = \$768$
 Total = \$35 576
 Income – expenses = \$2296
 b He needs to reduce his expenses by approximately \$45 per week; for example, cut entertainment to \$100 per week.
 5 a *Income*:
 Allowance $(\$407.70 \times 26) = \$10\,600.20$
 Job $(\$120 \times 52) = \$6240$
 Total = \$16 840.20
Expenses:
 Rent $(\$320 \times 12) = \$3840$
 Food $(\$90 \times 52) = \$4680$
 Phone $(\$110 \times 4) = \$440$
 Entertainment $(\$70 \times 52) = \$3640$
 Books = \$350
 Total = \$12 950
 Income – expenses = \$3890.20
 b Car costs = \$3400, yes
 6 a *Income*:
 Julie $(\$490 \times 52) = \$25\,480$
 Robert $(\$380 \times 52) = \$19\,760$
 Interest $(\$230 \times 2) = \$460$
 Total = \$45 700
Expenses:
 Home loan repayments $(\$980 \times 12) = \$11\,760$
 Food $(\$160 \times 52) = \$8320$
 Electricity $(\$480 \times 4) = \$1920$
 Telephone $(\$110 \times 12) = \$1320$
 Council rates $(\$340 \times 4) = \$1360$
 Water rates $(\$186 \times 4) = \$744$
 Car registration and insurance = \$780 per year
 Comprehensive car insurance = \$810 per year

Car loan repayments ($\$108 \times 52$) = \$5616
 Car running expenses ($\$190 \times 12$) = \$2280
 Clothes ($\$350 \times 12$) = \$4200
 Personal items ($\$45 \times 52$) = \$2340
 Total = \$41 450

Income – expenses = \$4250

- b** Yes, they could increase them by at most \$354 per year. Should leave some savings for emergencies. Look at reducing expenses.

7 a Income:

Beautician ($\$180 \times 52$) = \$9360

Wedding ($\$100 \times 6$) = \$600

Total = \$9960

Expenses:

Bus fares ($\$30 \times 26$) = \$780

Make up ($\$45 \times 52$) = \$2340

Clothes and entertainment ($\$200 \times 12$) = \$2400

Total = \$5520

Income – expenses = \$4440

- b i** Yes

ii Income:

Beautician ($\$270 \times 52$) = \$14 040

Wedding ($\$250 \times 6$) = \$1500

Total = \$15 540

Expenses:

Bus fares ($\$30 \times 26$) = \$780

Make up ($\$45 \times 52$) = \$2340

Clothes and entertainment ($\$200 \times 12$) = \$2400

Loan repayment ($\$50 \times 26$) = \$1300

Total = \$6820

Income – expenses = \$8720

iii \$100

iv \$4280

REVIEW MULTIPLE-CHOICE QUESTIONS

- 1** C **2** C **3** B **4** B **5** B **6** B
7 A **8** B **9** A **10** A **11** C

REVIEW SET 1

- 1** \$493.69
2 a \$171.08 **b** \$90.29
3 \$111.92 **4** \$170 **5** \$2900
6 a \$39 785 **b** \$23 285
7 \$12560.97
8 a \$550.38 **b** \$7122.80 **c** \$1495.25
9 a Monthly payment over 5 years is \$378.08, so yes.
b Minimum payment at 14% is \$395.59, so no.
10 \$12 000

REVIEW SET 2

- 1 a** \$2.17 **b** \$201.43 **c** \$358.32
2 \$1819.59 **3** \$1880 **4** \$18 113.70
5 a \$332.31 **b** \$1513.16
6 a 10.2 km/L **b** 0.098 L/km **c** 9.82 L/100km
7 404 km **8** 32.13 L **9** \$861.22
10 Diesel by \$7.67
11 a ULP cost = \$2860.09, LPG = \$1647.54
b \$1212.55 **c** \$101.05
d 25 months **e** 37 500 km
12 \$5250

REVIEW SET 3

- 1** \$537.64 **2** \$1930.15
3 a \$58 663 **b** \$40 663
c i \$992.58 **ii** \$6980.84
d i 8.06 km/h **ii** 0.12 L/km **iii** 12 L/100 km
4 a Elizabeth \$7.44, Monique \$9.33, Tanya \$9.59
b Elizabeth \$111.60, Monique \$139.95, Tanya \$143.85
c Elizabeth \$74.40, Monique \$93.30, Tanya \$95.90
5 a Income: ($\$170 \times 52$) = \$8840
Expenses:
 Board ($\$30 \times 52$) = \$1560
 Lessons ($\$80 \times 26$) = \$2080
 Entertainment ($\$85 \times 26$) = \$2210
 Clothes ($\$170 \times 12$) = \$2040
 Total = \$7890
 Income – expenses = \$950
b \$950

REVIEW SET 4

- 1** \$883.50
2 a 62.05 kL **b** \$176.84
3 \$108.53

4 a

Average annual running costs	Ford hatchback (small)	Holden sedan (large)	Toyota SUV
Depreciation (\$)	3120	5928	4784
Interest (\$)	1524	2444	2288
Road service (\$)	1248	962	1061
Fuel (\$)	1898	2626	2340
Tyres (\$)	146	146	208
On-road costs (\$)	671	634	952
Total (\$)	8607	12 740	11 633
Cost/week (\$)	165.52	245	223.71
Cost/km (c/km)	57.4	84.9	77.6

b

Running cost	Nissan 4WD
Depreciation (\$)	7250
Interest (\$)	3380
On-road costs (\$)	1196
Fuel (\$)	3064
Tyres (\$)	294
Service and repairs (\$)	967
Total (\$)	16 151
Cost/week (\$)	310.60
Cost/km (c/km)	107.7

REVIEW PRACTICE EXAMINATION QUESTION

- 1 a i** \$243.75 **ii** \$435.26
iii Aria \$128.65, Ezra \$229.72
b i \$683.70 **ii** \$3113.20
iii \$645 **iv** \$385
v \$3475 **vi** \$16 500
c 692 km **d** \$2000.92

- e i *Income:* $(\$272 \times 52) = \$14\,144$
Expenses:
Board $(\$70 \times 52) = \$3640$
Fares $(\$12 \times 52) = \$624$
Lunch $(\$15 \times 52) = \$780$
Entertainment $(\$50 \times 52) = \$2600$
Personal items $(\$25 \times 26) = \$650$
Clothes $(\$100 \times 12) = \$1200$
Total = \$9494
Income – expenses = \$4650
ii Yes, with \$250 to spare.

CHAPTER 11 EXPLORING AND DESCRIBING DATA

ARE YOU READY?

- 1 B 2 A 3 D 4 C 5 B 6 A
7 A 8 B 9 B

EXERCISE 11A MEAN AND MODE

1 a i

Score (x)	Frequency (f)	$f \times x$
3	2	6
4	3	12
5	6	30
6	4	24
7	1	7
	$\Sigma f = 16$	$\Sigma fx = 79$

ii 4.9

b i

Score (x)	Frequency (f)	$f \times x$
12	4	48
13	6	78
14	5	70
15	3	45
16	2	32
	$\Sigma f = 20$	$\Sigma fx = 273$

ii 13.65

c i

Score (x)	Frequency (f)	$f \times x$
50	3	150
51	5	255
52	8	416
53	6	318
54	2	108
55	4	220
	$\Sigma f = 28$	$\Sigma fx = 1467$

ii 52.4

d i

Score (x)	Frequency (f)	$f \times x$
8	12	96
9	28	252
10	25	250
11	26	286
12	9	108
	$\Sigma f = 100$	$\Sigma fx = 992$

ii 9.92

3 a

Mass (kg)	Class centre (x)	Frequency (f)	$f \times x$
60–62	61	2	122
63–65	64	5	320
66–68	67	7	469
69–71	70	4	280
72–74	73	2	146
		$\Sigma f = 20$	$\Sigma fx = 1337$

$$\text{Mean} \approx \frac{\Sigma fx}{\Sigma f} = \frac{1337}{20} = 66.85$$

b

Patients per day	Class centre (x)	Frequency (f)	$f \times x$
10–19	14.5	13	188.5
20–29	24.5	15	367.5
30–39	34.5	16	552.0
40–49	44.5	10	445.0
50–59	54.5	4	218.0
60–69	64.5	2	129.0
		$\Sigma f = 60$	$\Sigma fx = 1900$

$$\text{Mean} \approx \frac{\Sigma fx}{\Sigma f} = \frac{1900}{60} \approx 31.7$$

4 a i

Height (mm)	Class centre (x)	Frequency (f)
155–159	157	4
160–164	162	7
165–169	167	10
170–174	172	5
175–179	177	2

ii 165.9 mm

b i

Number of arguments	Class centre (x)	Frequency (f)
0–2	1	8
3–5	4	18
6–8	7	10
9–11	10	3
12–14	13	1

ii 4.8

c i

Score	Class centre (x)	Frequency (f)
41–50	45.5	3
51–60	55.5	0
61–70	65.5	10
71–80	75.5	8
81–90	85.5	7
91–100	95.5	2
		$\Sigma f = 30$

ii 72.83

- 5 a 5 b 13 c 52 d 9, 11 e 26
f 66–68 g 61–70, 71–80 h 30–39
6 a i 70.9 ii 75
b i 41.7 ii 26, 31, 48
c i 127.95 ii 108
d i 20.2 ii 26

7 a 46.2

b i

Mark	Frequency
0–4	0
5–9	1
10–14	0
15–19	1
20–24	2
25–29	1
30–34	2
35–39	2
40–44	4
45–49	3
50–54	4
55–59	1
60–64	6
65–69	0
70–74	2
75–79	1

ii

Mark	Frequency
0–9	1
10–19	1
20–29	3
30–39	4
40–49	7
50–59	5
60–69	6
70–79	3

iii

Mark	Frequency
0–19	2
20–39	7
40–59	12
60–79	9

iv

Mark	Frequency
0–39	9
40–79	21

c i 46.7 ii 47.5 iii 48.2 iv 47.5

d The smaller the class interval, the closer the approximation to the real mean.

8 a 1680 b 952 c 64.2

9 72 km/h 10 a $\frac{45}{10} = 4.5$

EXERCISE 11B MEDIAN

- 1 a 6 b 5 c 13 d 29 e 4
f 21.5 g 56 h 4 i 243.5 j 104
- 2 a ii 5 iii 8
b ii 4 iii 22
c ii 4.5 iii 16.5
d ii 5.5 iii 50

e ii 6 iii 50

f ii 6.5 iii 2.5

g ii 7.5 iii 37

3 a

x	f	Cumulative frequency	Scores
3	2	2	1st, 2nd
4	3	5	3rd, 4th, 5th
5	7	12	6th, 7th, ..., 12th
6	4	16	13th, ..., 16th
7	1	17	17th

Median = 5 (9th score)

b

x	f	Cumulative frequency	Scores
12	4	4	1st, 2nd, 3rd, 4th
13	7	11	5th, 6th, ..., 11th
14	5	16	12th, ..., 16th
15	3	19	17th, 18th, 19th
16	2	21	20th, 21st

Median = 13 (11th score)

4 a 52

b 35.5

c 10

d 26

5 a 14

b 29

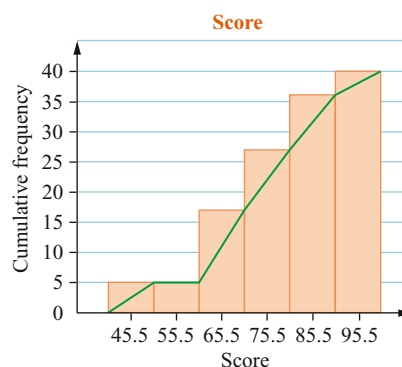
c 41

d 23

6 a i

Score	Frequency	Class centre	Cumulative frequency
41–50	5	45.5	5
51–60	0	55.5	5
61–70	12	65.5	17
71–80	10	75.5	27
81–90	9	85.5	36
91–100	4	95.5	40

ii

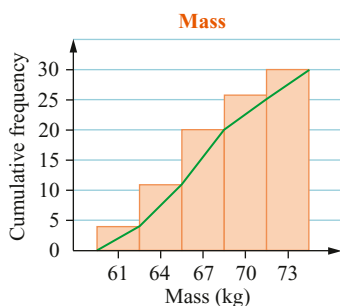


iii 74

b i

Mass (kg)	Frequency	Class centre	Cumulative frequency
60–62	4	61	4
63–65	7	64	11
66–68	9	67	20
69–71	6	70	26
72–74	4	73	30

ii

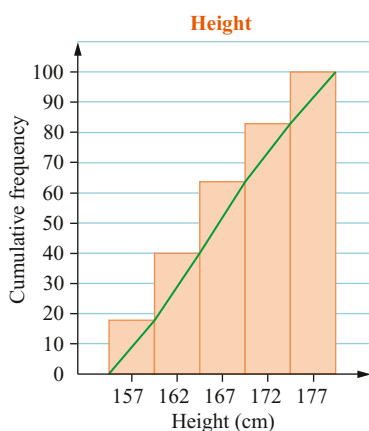


iii 67

c i

Height (cm)	Frequency	Class centre	Cumulative frequency
155–159	19	157	19
160–164	21	162	40
165–169	24	167	64
170–174	19	172	83
175–179	17	177	100

ii

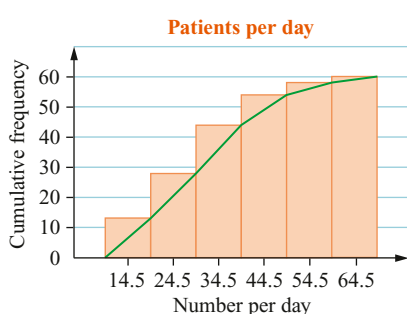


iii 167

d i

Patients per day	Frequency	Class centre	Cumulative frequency
10–19	13	14.5	13
20–29	15	24.5	28
30–39	16	34.5	44
40–49	10	44.5	54
50–59	4	54.5	58
60–69	2	64.5	60

ii



iii 32

7 a 72.5 b 45 c 120.5 d 21

8 a mean = 50.4, mode = 54, median = 54

b

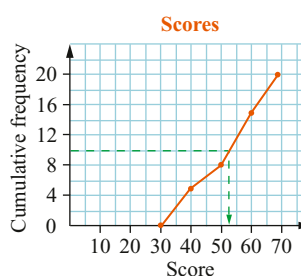
Scores	Frequency
30–39	5
40–49	3
50–59	7
60–69	5

c 50.5; reasonably close

d

Scores	Frequency	Cumulative frequency
30–39	5	5
40–49	3	8
50–59	7	15
60–69	5	20

e



Median is approximately 52.

This is smaller than the exact median of 54.

f Median of a stem-and-leaf plot is exact, but median of grouped data is an estimate.

9 b

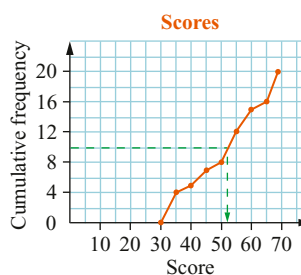
Scores	Frequency
30–34	4
35–39	1
40–44	2
45–49	1
50–54	4
55–59	3
60–64	1
65–69	4

c Mean is 50.25, which is close to 50.4.

d

Scores	Frequency	Cumulative frequency
30–34	4	4
35–39	1	5
40–44	2	7
45–49	1	8
50–54	4	12
55–59	3	15
60–64	1	16
65–69	4	20

e



Median = 52

- e Smaller than the exact median
 f Median of a stem-and-leaf plot is exact, but median of grouped data is an estimate.
 10 a Median = 8, mode = 5, mean = 12.3
 b Mode = 3, mean = 4.6, median = 5

EXERCISE 11C RELATIVE MERITS OF MEAN, MODE AND MEDIAN

- 1 a i 7 ii No mode iii 35
 b i Yes ii No iii No, not a central value
 c i Mean = 6.9 and median = 7 are both good measures.
 ii Mean = 5 and median = 4.5 are both good measures.
 iii Mean = 29 and median = 26 are both good measures.
 2 a i 9.2 ii 29
 b i Yes ii No; it is affected by outlier 110.
 c i Median = 9 is good measure. Mode = 8 is not a central value.
 ii Median = 9 is satisfactory.
 Mode = 8 is not a central value.
 3 a 5 b No, it is not a central value.
 c Mode = 5 is not central; mean = 4 is satisfactory.
 4 Mean 5 Mode
 6 a i 6.25 ii 7 iii 6.5
 b Mode, as it is the shoe size with highest sales.
 7 a \$2 030 000 b \$67 667
 c i 25 ii 5
 d \$55 000 e \$50 000
 f i Mean ii Mode
 g Median: half the employees earn more and half earn less.
 8 a Mean = 23.8, mode = 29, median = 23.5
 b Mean and median are central and typical. Mode is not central but is important because more people had to wait this length of time than any other.
 9 a Mean = 4.1, mode = 6, median = 4.5
 b Mode
 10 a Mean = \$640 000
 b The mean has been affected by the high price \$960 000.
 No mode. Median = \$560 000 and is the most appropriate.
 11 Mode
 12 a, b Results are summarised in part c.

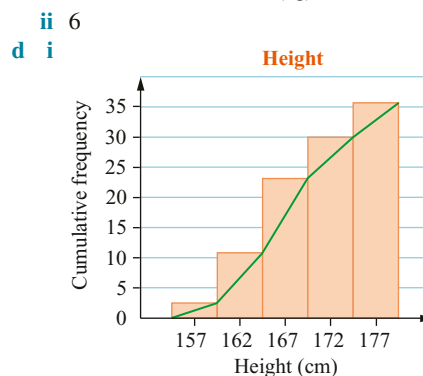
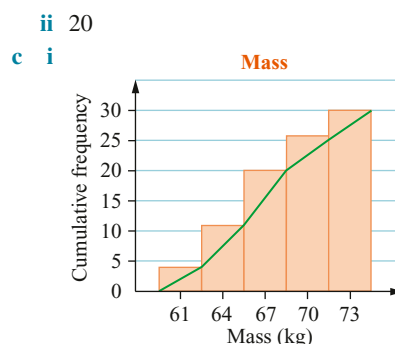
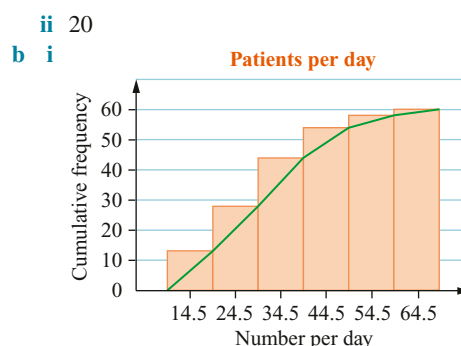
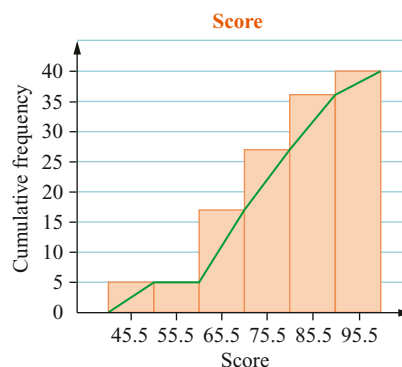
	1st 20	2nd 20	3rd 20	4th 20	5th 20
Mean	4.05	4.8	4.3	5.05	5.4
Mode	3	1, 8	0, 1, 9	8	0, 8, 9
Median	3.5	5	4.5	5.5	6.5

- d Mode e Mean
 13 a 157 is an outlier possibly caused by an error in measurement or recording of measurement.
 b 56.3 kg (without), 64.7 kg (with)
 c i 56 kg ii 48, 55, 56 kg
 d median or mean without outlier
 14 a i 1.3 ii 0 iii 1
 b Mode; number of injuries of main concern and need to be minimised

EXERCISE 11D RANGE AND INTERQUARTILE RANGE

- 1 a 8 b 7 c 159 d 4
 e 7 f 7 g 8 h 5
 2 a 4 b 5 c 4
 3 $Q_1 = 8, Q_3 = 13, \text{IQR} = 13 - 8 = 5$
 4 a 7 b 7
 5 $Q_1 = 44.5, Q_3 = 66, \text{IQR} = 66 - 44.5 = 21.5$
 6 a 25 b 6.5

- 7 $Q_2 = 21, Q_1 = 19, Q_3 = 24, \text{IQR} = 5$
 8 a 6 b 19
 9 $Q_1 = 4.5, Q_3 = 9, \text{IQR} = 4.5$
 10 a 6.5 b 3
 11 a 3 b 2.5 c 2 d 2
 12 a ≈ 14 b ≈ 10 c ≈ 15 d ≈ 9
 13 a 1 b 2 c 1
 14 a i

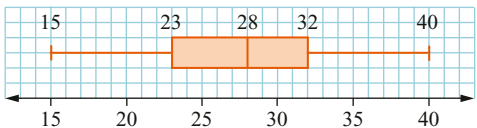
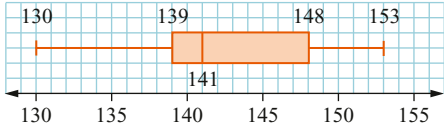
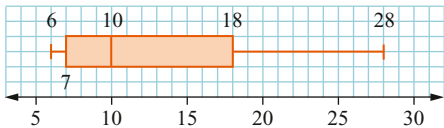
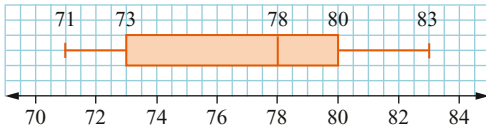
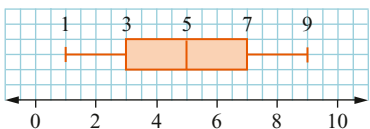
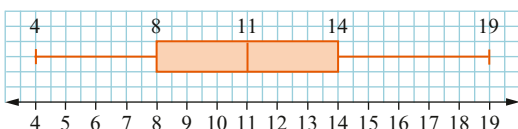
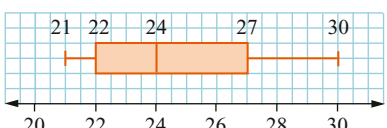
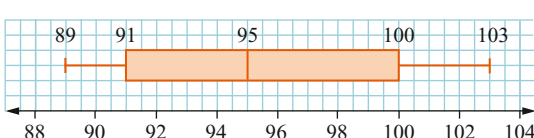


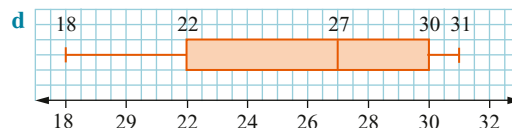
- ii 9
 15 The IQR is smaller for the results for Lara's class and so there is less spread than for Linda's class.
 16 a Tom b Matthew c Matthew
 17 No; as the individual heights are not given.
 18 a Set B b Range, mean

EXERCISE 11E DECILES AND PERCENTILES

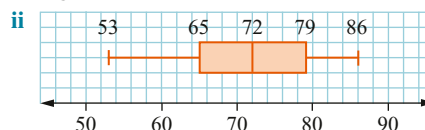
- 1 a Decile 1 = 12, decile 4 = 24, decile 9 = 35
 b Decile 1 = 55, decile 4 = 96, decile 9 = 170
 c Decile 1 = 27, decile 4 = 36, decile 9 = 51
 d Decile 1 = 25, decile 4 = 44, decile 9 = 74
- 2 a 14, 26, 32 b 65, 110, 151
- 3 a 47% b 5%
- 4 a P_{40} b P_{80} c P_{25} d P_{75} e P_{50}
- 5 a 21 b 10 c 5 year olds
- d i 29.8 ii Yes
 e i 14 ii No
 f 17.6 to 24.9 g 2.2
- 6 a 152 cm b 25th c 75% d 2% e 90th

EXERCISE 11F BOX-PLOTS

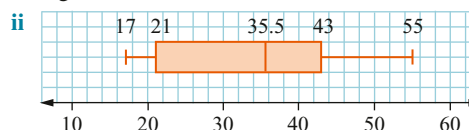
- 1 a i 83 ii 35 iii 48 iv 60
 v 75 vi 50 vii 25
- b i 75 ii 40 iii 35 iv 54
 v 65 vi 45 vii 20
- c i 78 ii 36 iii 42 iv 61
 v 73 vi 45 vii 28
- d i 76 ii 46 iii 30 iv 59
 v 69 vi 56 vii 13
- 2 a 
- b 
- c 
- d 
- e 
- 3 a 
- b 
- c 



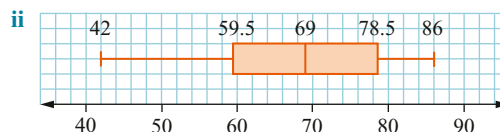
- 4 a i Lowest = 53, $Q_1 = 65$, $Q_2 = 72$, $Q_3 = 79$, highest = 86



- b i Lowest = 17, $Q_1 = 21$, $Q_2 = 35.5$, $Q_3 = 43$, highest = 55

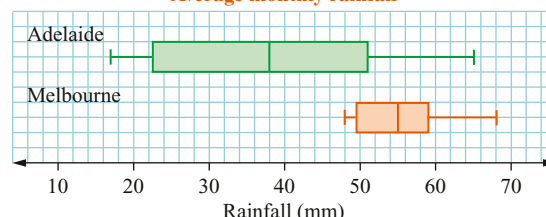


- c i Lowest = 42, $Q_1 = 59.5$, $Q_2 = 69$, $Q_3 = 78.5$, highest = 86



- 5 a 6800 s b 400 s
 c 8200 s d 8700 s
 e 8100 s f Ranges were the same.
 g Males h Males

6 Average monthly rainfall



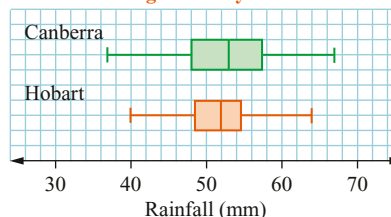
$$\text{Adelaide median} = \frac{38 + 38}{2} = 38,$$

$$Q_1 = \frac{22 + 23}{2} = 22.5, Q_3 = \frac{50 + 52}{2} = 51$$

$$\text{Melbourne median} = \frac{52 + 58}{2} = 55,$$

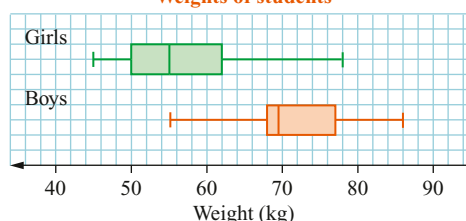
$$Q_1 = \frac{49 + 50}{2} = 49.5, Q_3 = \frac{59 + 59}{2} = 59$$

7 a Average monthly rainfall

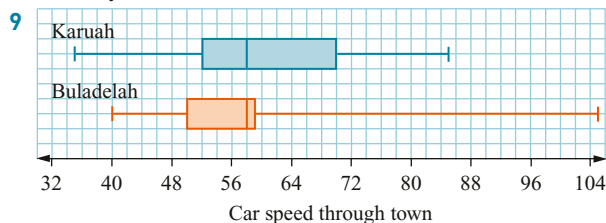


- b Canberra has a greater spread of rainfall than Hobart. The two cities have similar rainfall in the middle 50% of values.

8 a Weights of students



- b More than 75% of boys weigh more than 75% of girls.
75% of girls weigh less than the lower quartile of 25% of boys.



Karuah has more drivers exceeding the speed limit and so more drivers would be fined for speeding. Buladelah has fewer speeding drivers but the highest speed is very dangerous, so it would be targeted for safety.

EXERCISE 11G STANDARD DEVIATION

- 1 a 6, 1.4 b 6, 2.8 c 6, 4.2
d 28, 2.6 e 90, 1.4 f 250, 5.6
g 6, 2.5 h 5, 0
- 2 a 3.72, 1.22 b 16.9, 1.11
c 1.79, 1.16 d 122.96, 1.37
- 3 a 66.85, 3.35 b 72.83, 12.89
c 31.67, 13.30 d 165.93, 5.57
- 4 a 4.5, 1.26
b i 9.5, 1.26 ii $(4.5 + 10)$, 1.26
c i 22.5, 6.29 ii (4.5×10) , (1.26×10)
- 5 a 20, 4 b 85, 20 c 12, 4
- 6 The scores are all the same.
- 7 1.5
- 8 mean = 2, sample s.d. = 1.1
- 9 mean = 29.8, sample s.d. = 3.8
- 10 a population s.d. = 10.3, sample s.d. = 10.9
b sample s.d.
- 11 a i Range: machine 1 is 10, machine 2 is 10.
ii Interquartile range: machine 1 is 0, machine 2 is 10.
iii Standard deviation: machine 1 is 3.5, machine 2 is 4.2.
b Around 60
c Machine 1 as it has a lower standard deviation, with scores clustered around 60.
- 12 a Carrie: i 6 ii 6
Laura: i $6\frac{1}{9}$ ii 5
b Carrie 1, Laura 5
c Carrie: i 5 ii 3.5 iii 1.76
Laura: i 12 ii 3 iii 3.45
d Laura won more holes and had a lower median.
e Carrie player fewer strokes and her mean was lower.
- 13 a Julie 557, Alison 575
b Julie: i 69.6 ii 72
Alison: i 71.9 ii 72
c 6
d Julie: i 37 ii 8
Alison: i 16 ii 6.5
e Julie 10.7, Alison 4.8 f Alison
g Julie had higher marks than Alison in 6 of the 8 tests.
Alison had a higher mean and lower standard deviation.
- 14 a i 1.3 ii 31.2
b Outliers have a large effect on the standard deviation.
c i Range = 4, IQR = 2
ii Range = 85, IQR = 2
d Range not IQR

EXERCISE 11H IDENTIFYING OUTLIERS

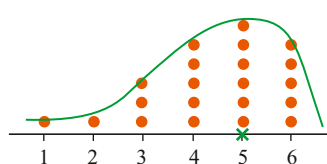
- 1 a i 3 ii No outlier
b i 21 ii No outlier
c i 17 ii No outlier
d i No outlier ii No outlier
- 2 a i 3 ii No outlier
b i No outlier ii 18
- 3 a Median and IQR b Mean and standard deviation
- 4 a i $Q_1 = 18$, $Q_2 = 26$, $Q_3 = 29$, IQR = 11
ii 1.5, 45.5 iii 59
b i $Q_1 = 23$, $Q_2 = 35$, $Q_3 = 43$, IQR = 20
ii -7, 73 iii No outlier
c i $Q_1 = 56.5$, $Q_2 = 67.5$, $Q_3 = 75$, IQR = 18.5
ii 28.75, 102.75 iii No outlier
d i $Q_1 = 106$, $Q_2 = 110$, $Q_3 = 119$, IQR = 13
ii 86.5, 138.5 iii 83
e i $Q_1 = 69$, $Q_2 = 76$, $Q_3 = 88$, IQR = 19
ii 40.5, 116.5 iii 118
f i $Q_1 = 38$, $Q_2 = 49$, $Q_3 = 57$, IQR = 19
ii 9.5, 85.5 iii No outlier
- 5 b No girl will weigh 157 kg so this is probably an error in measuring, or recording the correct weight.
c With outlier: i mean = 64.7 kg, median = 56 kg
ii IQR = 8 iii 28.3
Without outlier: i mean = 56.3 kg, median = 56 kg
ii IQR = 8 iii 5.1
d mean, standard deviation
e Ignore outlier because it is an obvious error. Mean = 56.3, median = 56, IQR = 8 and population s.d. = 5.1

EXERCISE 11I GRAPHICAL DATA DISPLAYS

- 1 a A b D c E d B e C
- 2 A More matches result in low scores than high scores.
B The two humps might represent the clustering of blood pressures for males and females.
C There is no pattern of shoe sizes except that there are fewer pairs of small and large sizes.
D More people arrive closer to the concert starting time than early.
E The number of tomatoes/plant is evenly distributed about the mode.
- 3 A
-
- Number of matches
- Team score
- B
-
- Number of people
- Blood pressure
- C
-
- Number of students
- Shoe size
- D
-
- Number of plants
- Number of tomatoes
- E
-
- Number of people at concert
- Time

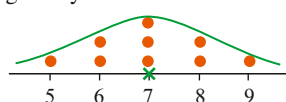
4 a B b C c A d D

5 a i, ii



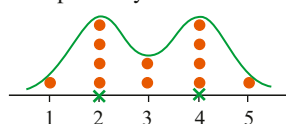
iii negatively skewed

b i, ii



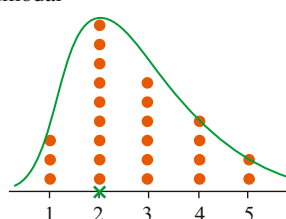
iii bell shaped or symmetrical

c i, ii



iii bimodal

d i, ii

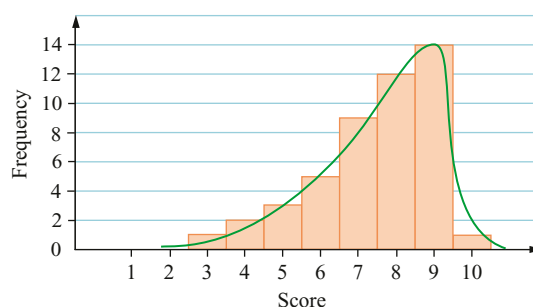


- iii positively skewed
- 6 a i positively skewed ii 10s and 20s
b i symmetrical about mode ii 30s
c i negatively skewed ii 60s and 70s
d i symmetrical ii 100s and 110s
e i positively skewed ii 60s and 70s
f i negatively skewed ii no clusters
- 7 The last 25% of scores are much more spread out than the first 25% of scores.
- 8 a Positively skewed. The last 25% of scores are more spread out than first 25% of scores.
b Symmetrical. Even spread of scores.
c Negatively skewed. The first 25% of scores are more spread out than last 25% of scores.
d Symmetrical. Even spread of scores.
e Positively skewed. The last 25% of scores are more spread out than first 25% of scores.
f Negatively skewed. The first 25% of scores are more spread out than last 25% of scores.

9 i, ii, iii

	Shape	Q_1	Q_2	Q_3	IQR	Outliers
a	Symmetrical	23	28	32	9	No
b	Positively skewed	7	10	18	11	No
c	Symmetrical	3	5	7	4	No
d	Symmetrical	8	11	14	6	No
e	Positively skewed	22	24	27	5	No
f	Negatively skewed	22	27	30	8	No
g	Negatively skewed	65	72	79	14	No
h	Positively skewed	21	30	43	22	No
i	Negatively skewed	60	69	79	19	No
j	Symmetrical	20	25	30	10	Yes

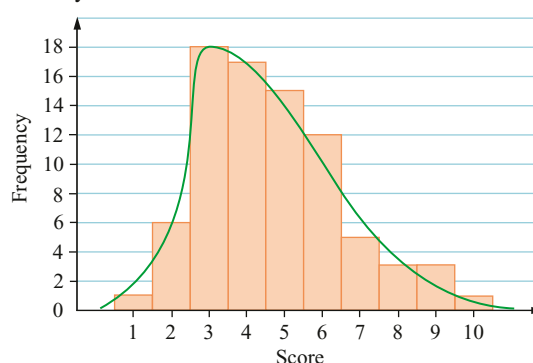
10 a



b Negatively skewed

c An easy test

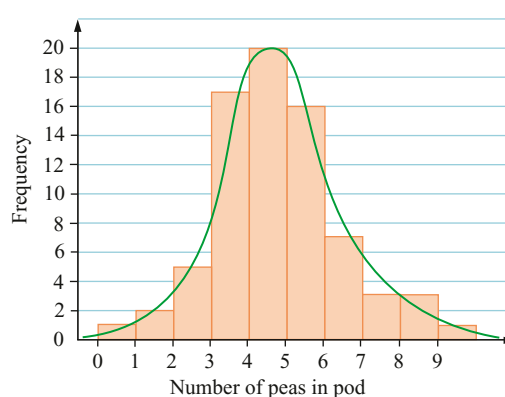
11 a



b Positively skewed

c A hard test

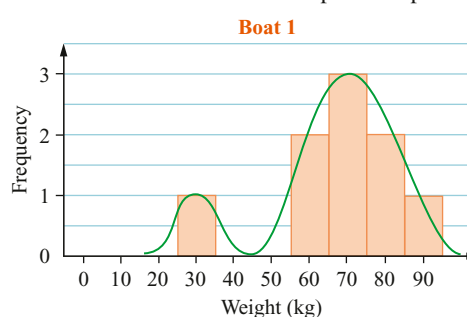
12 a

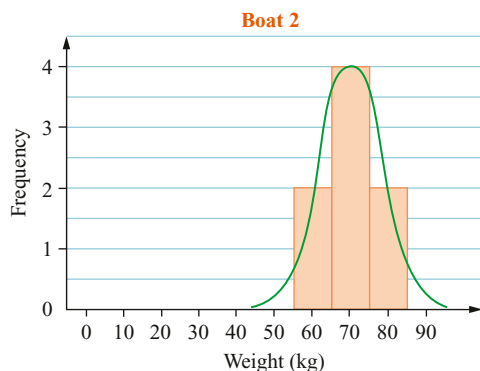


b Symmetrical

c The number is centred around 4 peas in a pod.

13 a





- b** Weights for boat 1 show a bimodal distribution and weights for boat 2 are symmetrically distributed around 70 kg.
- c** Boat 1, as there is an outlier, 30 kg, that represents the weight of the cox.

CHAPTER 11 REVIEW EXPLORING AND DESCRIBING DATA

REVIEW MULTIPLE-CHOICE QUESTIONS

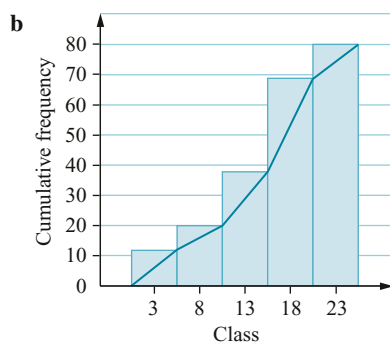
- 1 D 2 B 3 C 4 C 5 A 6 D
 7 C 8 C 9 B 10 D 11 D 12 C
 13 B 14 B 15 A 16 A 17 B 18 B
 19 B

REVIEW SET 1

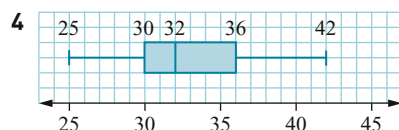
- 1 Mean = 7.7, mode = 6, median = 8, range = 4

2 a

Class centre	Cumulative frequency
3	12
8	20
13	38
18	69
23	80



- c** i 16 ii 14 iii 18
3 a i 9, 11, 12 ii 3
b i 9, 11, 12.5 ii 3.5
c i 9, 11.5, 12 ii 3
d i 9.5, 11.5, 13.5 ii 4

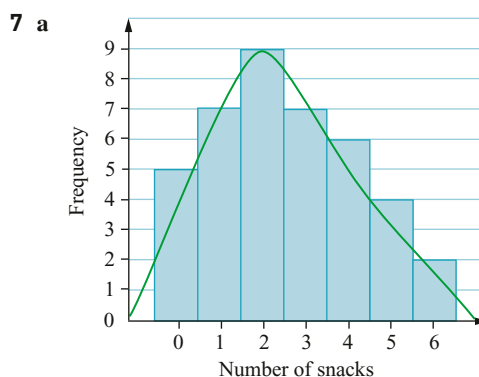


- 5 20.6, 2.7

- 6 a 7.1 b 7 c 1.5
 7 a i \$318 333 ii \$285 000
b \$550 000.
c Median, because it is not affected by the outlier of \$550 000.
 8 a Town A: i 25 ii 26.5
 Town B: i 26 ii 26.5
b Town A: i 14 ii 8 iii 4.7
 Town B: i 23 ii 12 iii 7.1
c Town A negatively skewed, cluster above median.
 Town B symmetrical, no clustering.
 9 a Negatively skewed b No outliers

REVIEW SET 2

- 1 Mean = 86.65, mode = 87, median = 87, range = 3
 2 a i Mean = 24, mode = 5, median = 5
 ii Mean has been affected by the outlier 100. Mode and median are appropriate measures.
b i Mean = 5.1, mode = 4, median = 5
 ii Mode is not a central score. Mean and median are appropriate measures.
c i Mean = 2.5, mode = 1, median = 1
 ii Mode and median are not central. Mean is an appropriate measure.
 3 a 68 b 90 c 57
 d 33 e 53 f 66
 4 C 5 4.49, 0.14
 6 a 26.3 b 26 c 9
 d 6 e 3.07



- b** Slightly positively skewed with mode of 2 snacks
 8 a Group X: i 4.6 ii 5
 Group Y: i 5.2 ii 5
b Group X: i 5 ii 1.2
 Group Y: i 6 ii 1.7



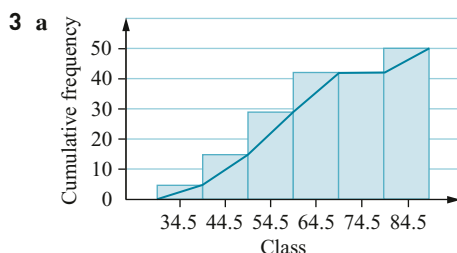
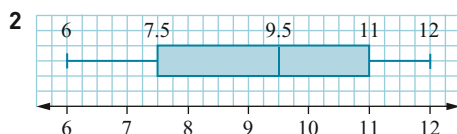
- d** Much the same for lower 50% but group Y students have more shoes that last longer.

REVIEW SET 3

1 a 50–59

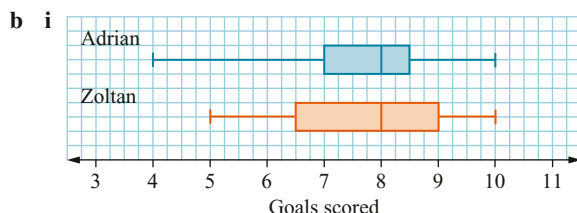
Class	Class centre	Frequency
30–39	34.5	5
40–49	44.5	10
50–59	54.5	14
60–69	64.5	13
70–79	74.5	0
80–89	84.5	8

c 57.9



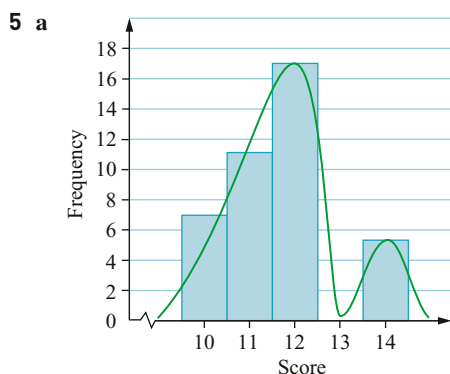
b i 47 or 48 ii 66 or 67 iii 19
iv 57 v 83

4 a Adrian: i 7.6 ii 8
iii 6 iv 1.5 v 1.5
Zoltan: i 7.6 ii 8
iii 5 iv 2.5 v 1.6



ii The scores for Adrian are negatively skewed and clustered between 7 and 8.5. The scores for Zoltan are almost symmetrical.

c Zoltan has more games with a higher number of goals than Adrian. Zoltan's lowest number of goals per game was 5 whereas Adrian's was 4.



b Distribution is bimodal.

c Yes, 14.

6 a

Stem	Leaf
3	6
4	3
5	0 8 8 9
6	0 1 1 3 5 6
7	4 7
8	8

b Distribution is symmetrical.

REVIEW SET 4

1 Mean = 3.9, Mode = 5, Median = 4, Range = 4,
Standard deviation (σ_n) = 1.37

2 a

Class B	Stem	Class A
6 2	0	8
8 6 2 0	1	4 6 8
8 8 6 4 4 4 2 2 0	2	0 2 4 4 6 6 8
8 6 6 4	3	0 0 2 2 4 6 8
	4	0

b Class A: i 26.2 ii 26 iii 32 iv 8.3

Class B: i 22.4 ii 24 iii 36 iv 9.9

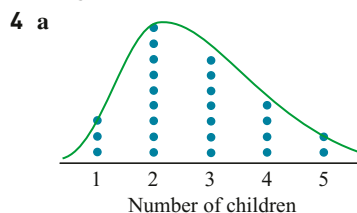
c Class A slightly negatively skewed

Class B almost symmetrical, slight negative skew

d No outliers in class A. In class B, 2 is an outlier.

e Class A had more high scores than class B and had the highest score.

3 Range = 5, IQR = 2.5



b Positively skewed

c There are more families with a smaller number of children.

5 a i 25 ii 30 iii 32 iv 36 v 42

b Positively skewed

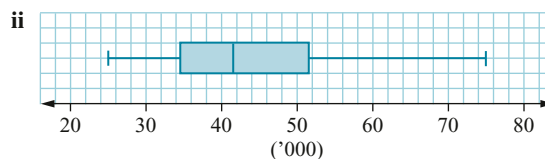
c No outliers

REVIEW PRACTICE EXAMINATION QUESTION

1 a i 10 ii 11 iii 14

b 9.1

c i \$34 500, \$41 500, \$51 500



iii No outliers; all incomes sit within the interval \$9000 and \$77 000.

iv Approx. \$14 022

d i 120 ii 70

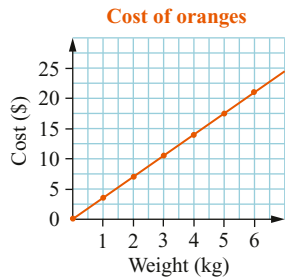
CHAPTER 12 LINEAR RELATIONSHIPS

ARE YOU READY?

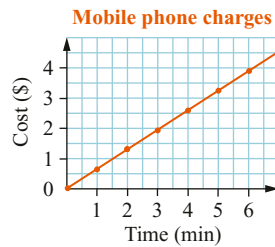
- 1 B 2 A 3 D 4 A 5 B 6 C
7 C 8 A 9 B

EXERCISE 12A STRAIGHT-LINE GRAPHS

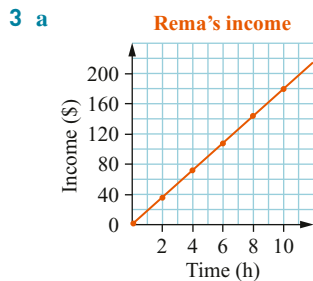
- 1 a Dependent variable is cost. b About \$9.00



- 2 a Independent variable is time.



- b \$1.60 c 4.7 min



- b \$130 c 5 h

4 a

Distance (km)	0	10	20	30	40
Cost (\$)	\$3.00	\$27.00	\$51.00	\$75.00	\$99.00

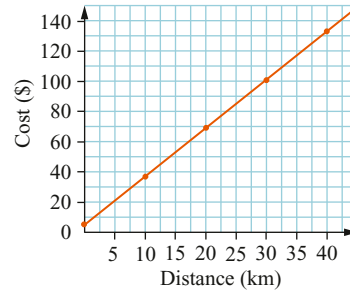


- c \$87 d 5 km

5 a

Distance (km)	0	10	20	30	40
Cost (\$)	\$5.00	\$37.00	\$69.00	\$101.00	\$133.00

b Cost of night-time taxi hire

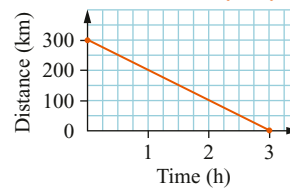


- c \$53 d 22 km

6 a

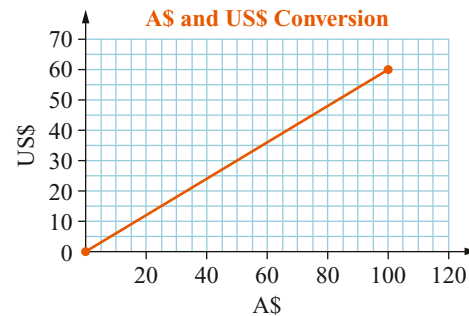
Time (h)	0	1	2	3
Distance (km)	300	200	100	0

b Distance to Sydney

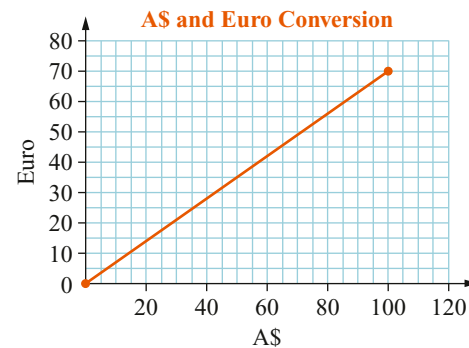


- c 150 km d 100 km

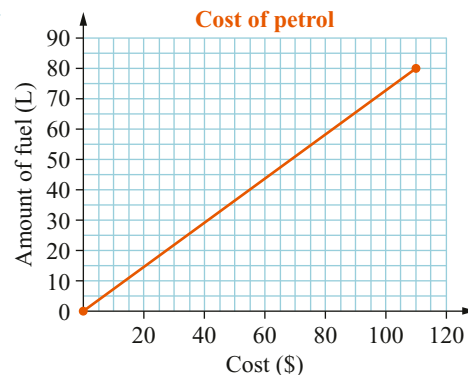
- 7 US\$40 is about A\$67.



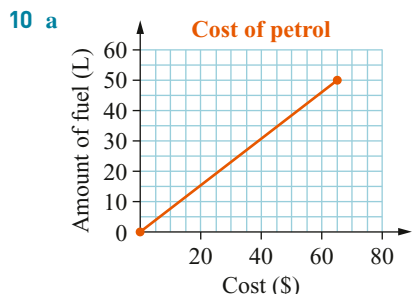
- 8 60 Euros is about A\$85.



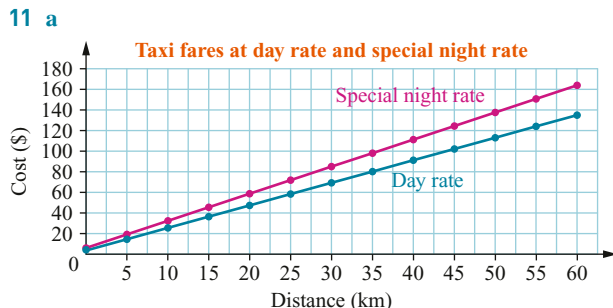
- 9 a



- b about 50 L c about \$75



- b about 20 L c about \$45



- b about 8 km c about \$20

EXERCISE 12B GRADIENT AND VERTICAL INTERCEPT

1 a

Line segment	Horizontal run	Vertical rise	Slope
AB	6	0	0
CD	7	1	$\frac{1}{7}$
EF	7	2	$\frac{2}{7}$
GH	6	3	$\frac{1}{2}$
IJ	3	5	$\frac{5}{3}$
KL	1	7	7
MN	0	5	∞

- b 0; ∞ or undefined; increase

2 a $\frac{3}{4}$ b $\frac{150}{1000} = \frac{3}{20}$

3 a 0 b $\frac{1}{5}$ c $\frac{3}{5}$ d $\frac{7}{6}$
 e $\frac{7}{3}$ f $\frac{9}{2}$ g ∞ or undefined

4 a

Line segment	run	rise	rise/run
BC	2	1	$\frac{1}{2}$
DE	2	1	$\frac{1}{2}$
AC	4	2	$\frac{1}{2}$
BE	6	3	$\frac{1}{2}$
AE	8	4	$\frac{1}{2}$
AF	9	$4\frac{1}{2}$	$\frac{1}{2}$

- b No matter which pair of points on a straight line are used, the gradient is the same.

5 a 0 b $-\frac{1}{3}$ c $-\frac{1}{2}$ d $-\frac{3}{4}$
 e $-\frac{3}{2}$ f -7 g $-\infty$ or ∞ or undefined

6 a OP, PQ, RS, TU b QR, ST, UV
 c TU d ST e VW f PQ

7 a 4 b -2 c -4 d $\frac{3}{2}$ e $-\frac{3}{4}$ f 6

8 a Gradient = $\frac{1}{3}$, y-intercept = -1
 b Gradient = -4, y-intercept = 4
 c Gradient = $\frac{5}{2}$, y-intercept = -5

9 a $13\frac{1}{3}$ b 2 c -6

d $-\frac{2}{250}$ e $\frac{14}{50} = \frac{7}{25}$ f $\frac{6}{25}$

10 a 30 b 40 c 60 d 0.04 e 0 f 0

11 a about \$11 300

b just over \$58 000

c Gradient = $\frac{11}{30} \approx 0.37$; that is, 37 cents per dollar in this tax bracket.

d Gradient = $\frac{13}{22} \approx 0.59$; that is, 59 cents per dollar in this tax bracket

12 a \$10 000

b Just over \$140 000

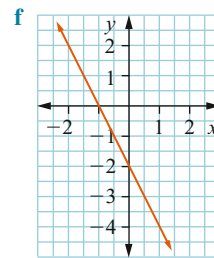
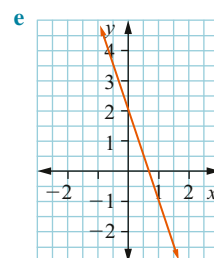
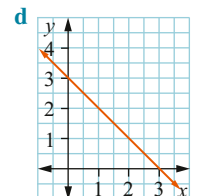
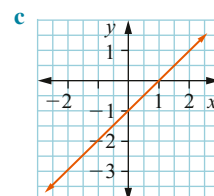
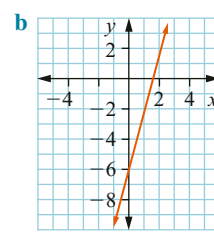
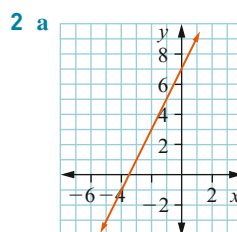
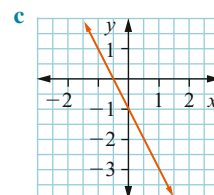
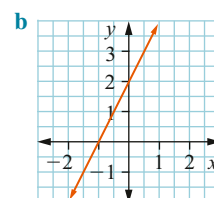
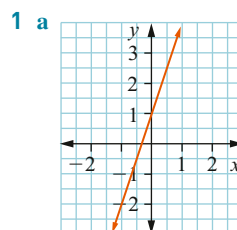
c Gradient = 0.15; that is, 15 cents per dollar in this tax bracket

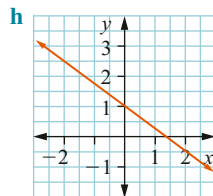
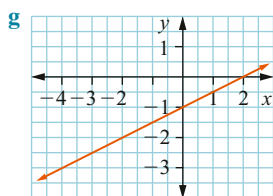
d Gradient ≈ 0.21 ; that is, 21 cents per dollar in this tax bracket

e Gradient ≈ 0.08 ; that is, 8 cents per dollar in this tax bracket

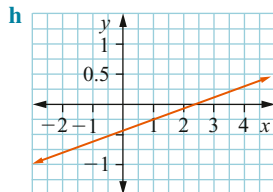
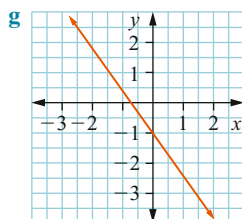
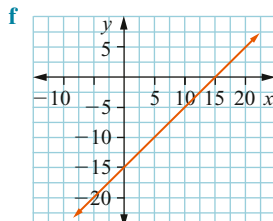
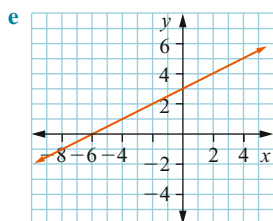
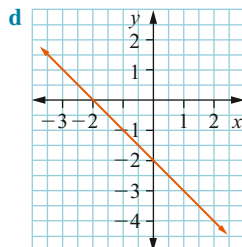
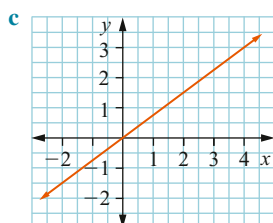
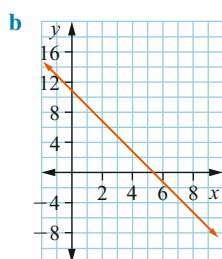
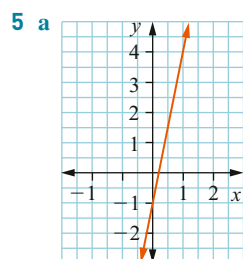
f Approx. 23.5 cents in the dollar

EXERCISE 12C THE EQUATION $y = mx + c$





- 3** **a** Gradient = 2, y-intercept = 7
b Gradient = 4, y-intercept = -6
c Gradient = 1, y-intercept = -1
d Gradient = -1, y-intercept = 3
e Gradient = -3, y-intercept = 2
f Gradient = -2, y-intercept = -2
g Gradient = $\frac{1}{2}$, y-intercept = -1
h Gradient = $-\frac{3}{4}$, y-intercept = 1
- 4** **a** Gradient = 5, y-intercept = -1
b Gradient = -2, y-intercept = 11
c Gradient = $\frac{3}{4}$, y-intercept = 0
d Gradient = -1, y-intercept = -2
e Gradient = $\frac{1}{2}$, y-intercept = 3
f Gradient = 1, y-intercept = -15
g Gradient = $-\frac{7}{5}$, y-intercept = -1
h Gradient = $\frac{1}{6}$, y-intercept = $-\frac{2}{5}$

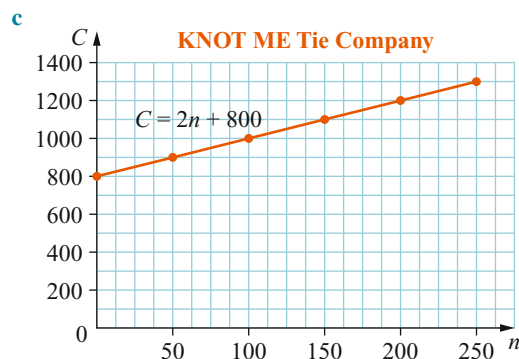


- 6** **a** $y = x + 5$ **b** $y = x - 3$
c $y = 5x$ **d** $y = 2x - 2$ or $y = 2(x - 1)$

- 7** **a** C is the daily cost in dollars, n is the number of ties sold each day, 800 is the fixed cost per day in dollars

b

n	0	50	100	150	200	250
C	800	900	1000	1100	1200	1300

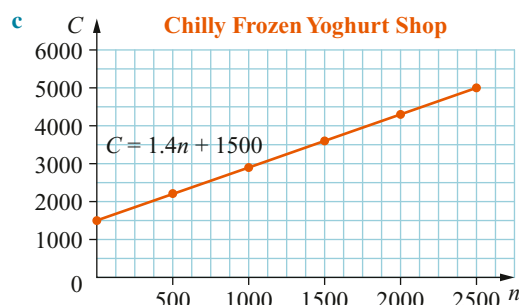


- d** 70 ties

- 8** **a** $C = 1.4n + 1500$

b

n	0	500	1000	1500	2000	2500
C	1500	2200	2900	3600	4300	5000



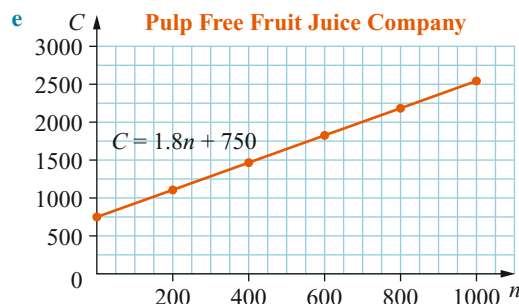
- d** 1.4 **e** 1250 serves of yoghurt

- 9** **a** $C = 1.8n + 750$

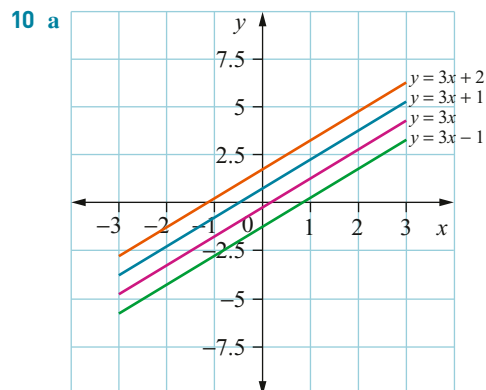
- b** 1.8 is cost per cup **c** 750

d

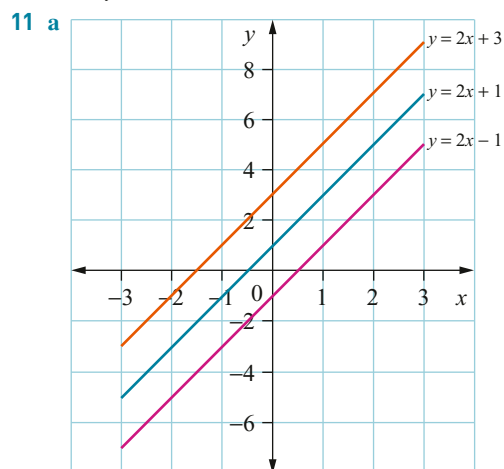
n	0	200	400	600	800	1000
C	750	1110	1470	1830	2190	2550



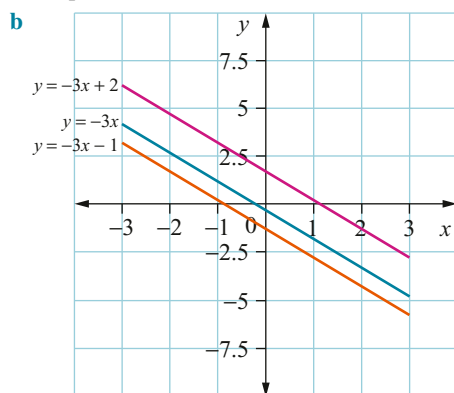
- f** 450 cups of juice



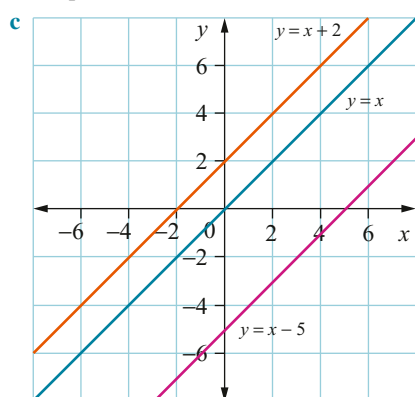
- b** All parallel
c Parallel to $y = 3x + 2$ and one unit further up, cutting the y -axis at 3



All parallel

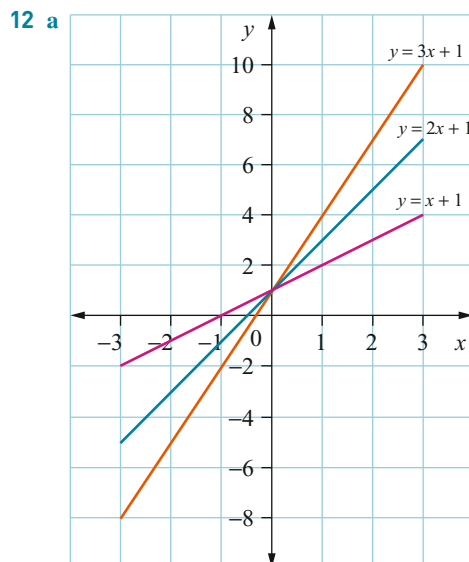


All parallel

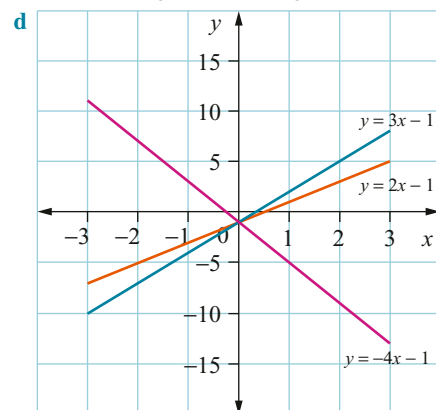


All parallel

- d** Lines with the same gradient are parallel.

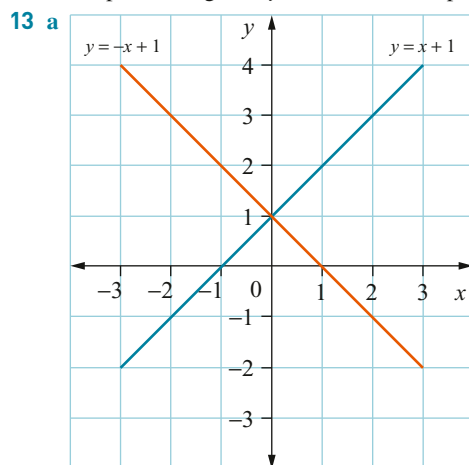


- b** Not parallel but all have the same y -intercept of 1
c Passes through $(0, 1)$ with gradient 4

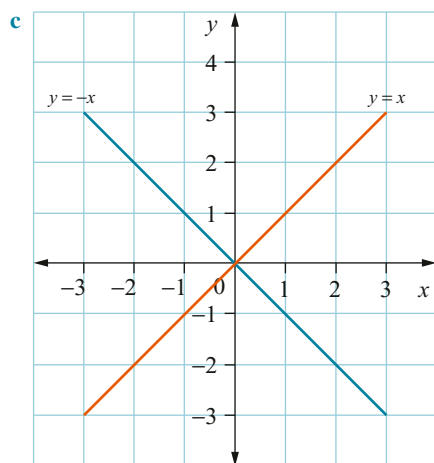


Not parallel but all pass through $(0, -1)$; that is, all have a y -intercept of -1

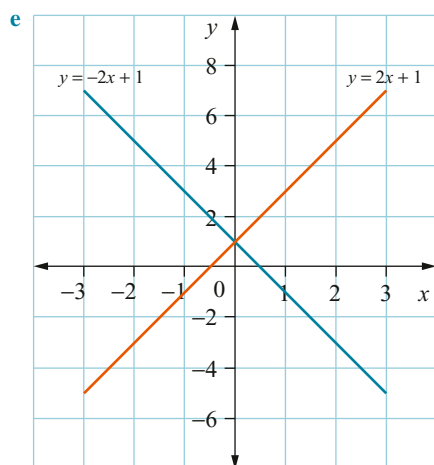
- e** All pass through the y -axis at the same point.



- b** Not parallel but have the same y -intercept of 1



d Not parallel but have the same y-intercept of 0

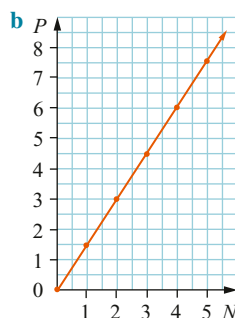


f Not parallel but have the same y-intercept of 1

EXERCISE 12D DIRECT VARIATION RELATIONSHIPS

1 a

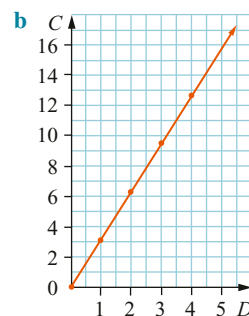
Number of packets (N)	1	2	3	4	5
Price (\$ P)	1.50	3.00	4.50	6.00	7.50



c 1.5 **d** $P = 1.5N$
e \$13.50 **f** 18

2 a

D	1	2	3	4
C	3.1	6.3	9.4	12.6



c Yes; it is a straight line through the origin.

d $\pi \approx 3.1$

e **i** 7.9 cm **ii** 5 cm

3 a **i** 4 **ii** $y = 4x$

b **i** 1 **ii** $y = x$

c **i** 2.1 **ii** $y = 2.1x$

4 Graph B

5 a **i** not proportional

b **i** $\frac{y}{x} = 7$, proportional **ii** $y = 7x$

c **i** $\frac{y}{x} = 2$, proportional **ii** $y = 2x$

d **i** $\frac{y}{x} = -\frac{1}{3}$, proportional **ii** $y = -\frac{x}{3}$

6 a **i** 8.4 **ii** $y = 8.4x$ **iii** 58.8 **iv** $6\frac{1}{14} \approx 6.07$

b **i** 6 **ii** $a = 6b$ **iii** 126 **iv** $1\frac{5}{6} \approx 1.83$

7 a **i** $a = 90, b = 1\frac{1}{9}$ **ii** $y = 18x$

b **i** $a = \frac{1}{2}, b = 48$ **ii** $y = \frac{1}{2}x$

c **i** $a = -4, b = 11.4, c = 51.3$ **ii** $y = \frac{1}{3}x$

d **i** $a = 3.2, b = -35.1, c = 6.9$ **ii** $y = \frac{9x}{2}$ or $y = 4.5x$

8 a $P \propto d, P = kd, 6 = k \times 50, k = \frac{3}{25}, P = \frac{3}{25}d$

b 21.6 L

9 15 L

10 225 km

11 a $P = \frac{1}{3}A$

b 21 L

12 13.1 L

13 a 50 L

b 5 cans

14 a $C = 2.15P$

b \$36.55

15 19.2 hours

16 a $C = 1.5H$

b \$10.05

c 20 m

17 a 0.34 ohms

b 1500 cm

18 a $P = 0.651D$

b \$1305.68

19 a $I = 12.5V$

b 125 amps

c 7.6 volts

EXERCISE 12E LINEAR MODELLING

1 a \$60 **b** 10 km

c Gradient = 2.5; cost per km after flagfall = \$2.50

d Vertical intercept = 5; flag fall = \$5

2 a \$200 **b** 52 people

c Gradient $\approx \frac{160}{28} = \frac{40}{7}$, $\$ \frac{40}{7} \approx \$5.70$ per person (using the points in parts **a** and **b**); \$6 per person

d Vertical intercept = 60; set-up cost = \$60

3 a \$420 **b** $2\frac{1}{2}$ hours

c Gradient ≈ 85 ; cost/hour $\approx \$85$

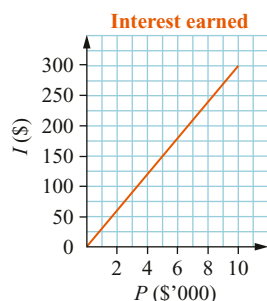
d Vertical intercept $\approx \$40$; call-out charge = \$40

- 4 a \$2000 b \$410
 c Gradient is 0.06; it is the commission as a decimal (6%).
 d Vertical intercept is 200; it is a retainer of \$200.
- 5 a i 300 mm ii 150 mm
 b 3 h c $\approx 1\frac{2}{3}$ h or $1\frac{1}{2}$ h
 d Gradient is -30 ; height decreases 30 mm/h.
 e Vertical intercept is 300; it is the initial height of the candle.

- 6 a $k = 0.03$

P	2000	4000	6000	8000	10000
I	60	120	180	240	300

c

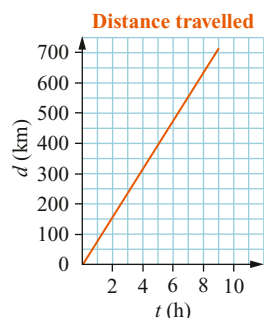


- d Gradient is 0.03; it is the rate of interest as a decimal.

- 7 a 80

t	1	3	5	7	9
d	80	240	400	560	720

c

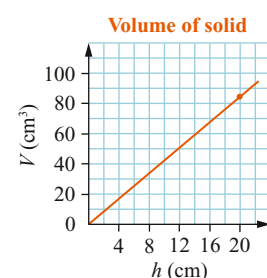


- d Gradient = $m = 80$; the speed of the train is 80 km/h

- 8 a 4.25

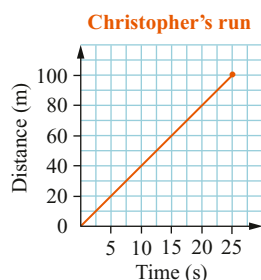
h	4	8	12	16	20
V	17	34	51	68	85

c



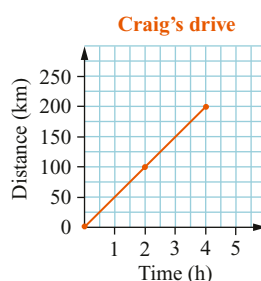
- d Gradient is 4.25; it is the cross-sectional area of the solid.

- 9 a



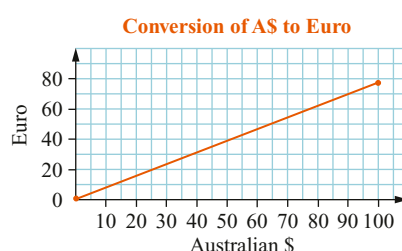
- b 7.5 s c Gradient = 4, speed = 4 m/s
 d If d is distance (in m) and t is time (in s) then $d = 4t$.

- 10 a



- b 75 km c Gradient = 50 d km/h
 d If d is distance (in km) and t is time (in h) then $d = 50t$.

- 11 a



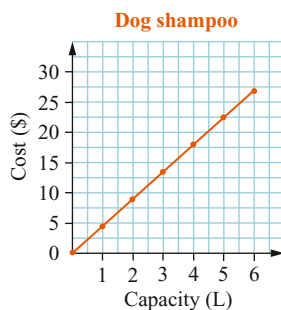
- b A\$65 c Gradient = 0.78
 d It is the conversion rate.
 e If e is the amount of euros and d is the amount of Australian dollars then $e = 0.78d$.
 f Yes
- 12 a Yes, the line has the same gradient as it is straight.
 b The runner would get tired and so their speed would decrease. A longer race would have to be run at a slower speed to cover the distance.
- 13 a Yes, the line is straight; the speed has not changed.
 b Not necessarily, as Craig may need petrol stops, there may be traffic, he should also take a rest, etc.
- 14 a cost (in dollars) per kilometre
 b Day rate: gradient ≈ 2.22 , Night rate: gradient ≈ 2.73 ; cost of a taxi trip at night is higher than during the day.
 c \$5; fixed cost of \$5 for each trip
 d If C is the total cost in dollars to travel d km, then:
 Day rate: $C = 2.22d + 5$; Night rate: $C = 2.73d + 5$

REVIEW MULTIPLE-CHOICE QUESTIONS

- 1 B 2 C 3 D 4 D 5 A
 6 C 7 B 8 C 9 C 10 B
 11 A 12 C 13 B 14 B 15 C 16 A

REVIEW SET 1

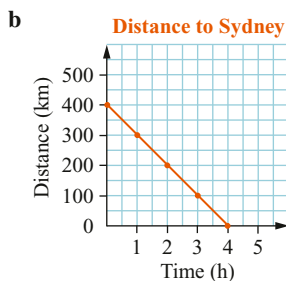
- 1 a Dependent variable is cost.



- b About \$16

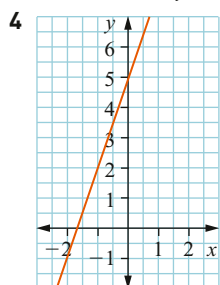
2 a

Time (h)	0	1	2	3	4
Distance (km)	400	300	200	100	0

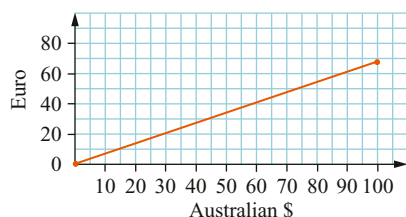


- c 250 km d 250 km

- 3 Gradient = 2, y-intercept = 3



- 5 a **Conversion of A\$ to euro**



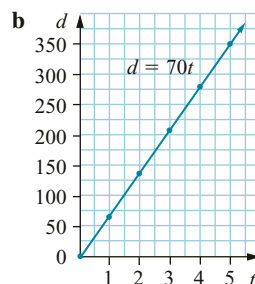
- b About A\$73 c Gradient = 0.68
 d Conversion rate of A\$1 to euro
 e If e is the amount of euros and d is the amount of Australian dollars then $e = 0.68d$.

- f Yes

- 6 $y = 3x - 5$

7 a

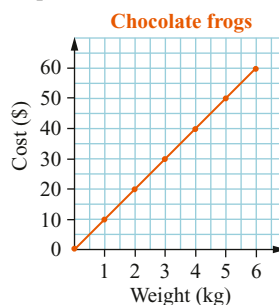
t	1	2	3	4	5
d	70	140	210	280	350



- c 245 km d ≈ 2 h 50 min
 e Yes, as it is a straight-line graph passing through the origin.

REVIEW SET 2

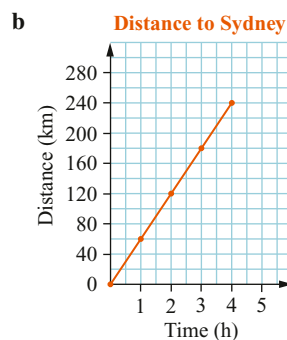
- 1 a Dependent variable is cost.



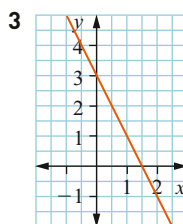
- b \$45

2 a

Time (h)	0	1	2	3	4
Distance (km)	0	60	120	180	240



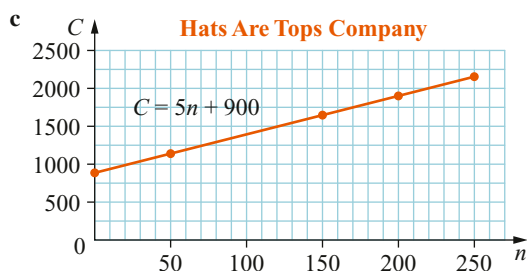
- c About 90 km d Just over $1\frac{1}{2}$ h



- 4 a Just under \$260 b 42 people
 c Gradient = 6; this is the cost per person
 d \$50; this is the set up cost of \$50
 5 a C is the daily cost in dollars, n is the number of hats produced each day, 5 is the cost per item in dollars and 900 is the fixed cost in dollars

b

n	0	50	150	200	250
C	900	1150	1650	1900	2150

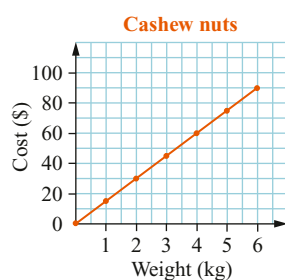


d 120 hats

6 No, as $\frac{y}{x}$ does not produce a constant value

REVIEW SET 3

1 a Dependent variable is cost.

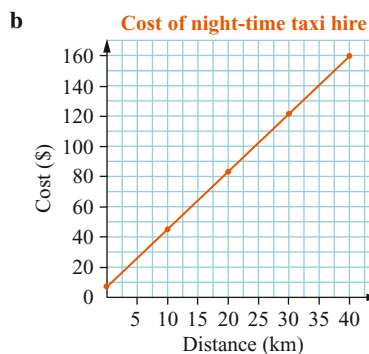


b About \$57

c About 3.3 kg

2 a

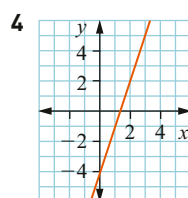
Distance (km)	0	10	20	30	40
Cost (\$)	7.00	45.00	83.00	121.00	159.00



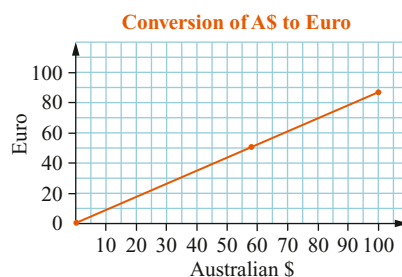
c About \$65

d About 18 km

3 Gradient = $\frac{5}{2}$, y-intercept = 5



5 a



b About A\$57 to A\$58

c Gradient is 0.87; conversion rate for number of euros you get for A\$1.

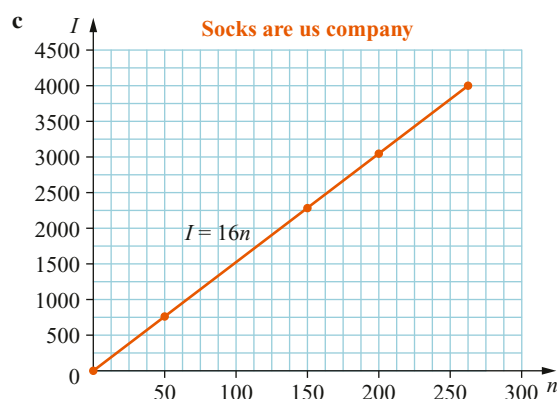
d Yes

6 $y = -3x + 2$

7 a I is the income in dollars, n is the number of pairs of socks produced, 16 is the cost (in dollars) per pair of socks

b

n	0	50	150	200	250
I	0	800	2400	3200	4000

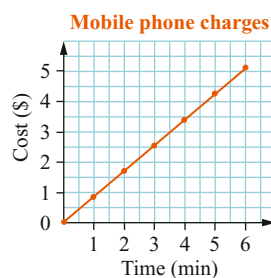


d 115 pairs of socks

e Yes, as it is a straight line passing through the origin.

REVIEW SET 4

1 a Independent variable is time.

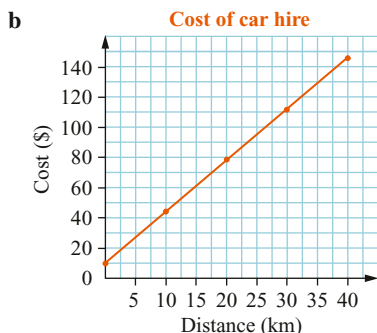


b about \$2.10

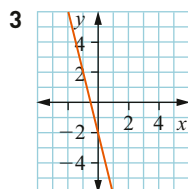
c about 3.5 min

2 a

Distance (km)	0	10	20	30	40
Cost (\$)	10.00	44.00	78.00	112.00	146.00



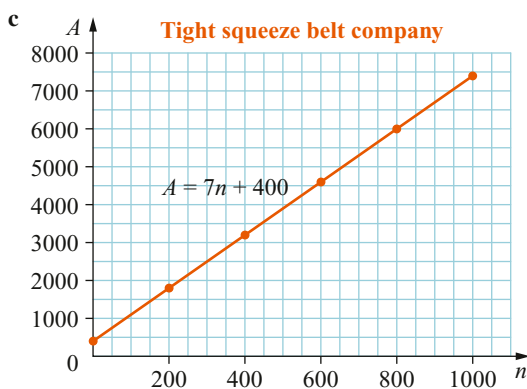
- c** About \$130 **d** About 1.5 km
e No, as the straight line does not pass through the origin.



- 4 a** About \$32 **b** About 84 km
c About $\frac{2}{5}$; the charge in \$ per km
d \$8; the flagfall is \$8
5 a i \$30 **ii** \$75 **iii** \$95
b i 4 **ii** 15 **iii** 34
c \$5
d i 1.25
ii The cost for each extra card, having already bought 30 cards

- 6 a** $A = 7n + 400$

n	0	200	400	600	800	1000
A	400	1800	3200	4600	6000	7400

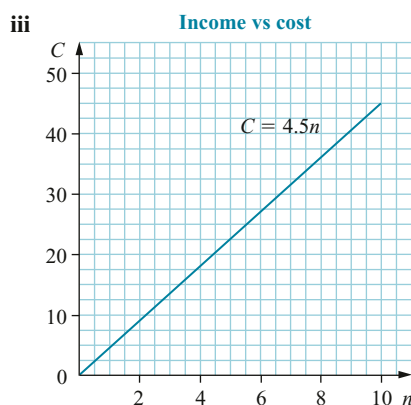


- d** about 230 belts
7 a $a = 80, b = 0.5$ **b** $y = 16x$

REVIEW PRACTICE EXAMINATION QUESTION

- 1 a i** 16 strokes/min **ii** 4 m/s **iii** 4 min 10 s
b i 4.5 m/s **ii** 2 m/s
c i Upstream. When $r = 0$, s is negative: she is going backwards.
ii 1 m/s
d $y = -4x + 11$
e i $\frac{5}{2}$ or $2\frac{1}{2}$ **ii** -5

- f i** $C = 4.5n$ **ii** 4.5



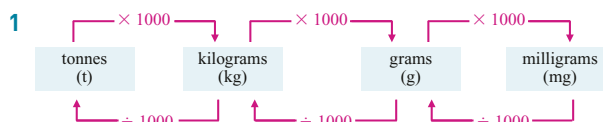
- iv** \$36

CHAPTER 13 ENERGY AND MASS

ARE YOU READY?

- 1** B **2** D **3** C **4** A **5** C **6** C
7 D **8** D **9** B **10** C **11** D **12** B

EXERCISE 13A UNITS OF MASS



- 2 a** 3000 mg **b** 5000 mg **c** 7000 mg **d** 9000 mg
e 2500 mg **f** 2200 mg **g** 1300 mg **h** 3400 mg
i 400 mg **j** 300 mg **k** 150 mg **l** 220 mg
m 50 mg **n** 37 mg **o** 2 mg **p** 3 mg
3 a 3 g **b** 7 g **c** 4 g **d** 8 g
e 2.5 g **f** 4.2 g **g** 7.5 g **h** 6.2 g
i 0.4 g **j** 0.35 g **k** 0.27 g **l** 0.12 g
m 0.06 g **n** 0.038 g **o** 0.004 g **p** 0.0025 g
4 a 2700 kg **b** 4500 mg **c** 23 920 g **d** 340 kg
e 5600 g **f** 1758 kg **g** 23 490 kg **h** 800 mg
i 4050 g **j** 875 g **k** 2050 kg **l** 50 mg
5 a 4 t **b** 8.5 kg **c** 1.65 g **d** 0.75 kg
e 0.045 g **f** 1.48 kg **g** 0.4 t **h** 0.95 g
i 0.09 kg **j** 0.009 g **k** 0.005 t **l** 0.2 kg
6 a kg **b** g **c** g **d** kg
e t or kg **f** t or kg **g** mg **h** g
i mg

- 7** All masses need to be converted to the same unit before adding them; 3.756 kg

- 8 a i** 420 g **ii** 840 g **iii** 3.36 kg
b the mass of the cartons

- 9** 16.95 kg **10** 1122 kg

- 11 a** Use the scale to find the mass of a larger number of pages and then divide by the number of pages.

- b** 4.5 g

- 12 a** 400 g **b** 50 g **c** 375 g **d** under
13 a 150 mL **b** 200 mL **c** 120 mL **d** 25 mL
e 120 mL **f** 50 mL **g** 120 mL **h** 60 mL
i 40 mL

- 14 a 40 mL b 80 mL c 80 mL d 50 mL
e 80 mL f 50 mL g 200 mL h 100 mL
i 100 mL
15 a 3 tablets
b i 1 tablet ii $1\frac{1}{2}$ tablets
iii 2 tablets iv $1\frac{1}{4}$ tablets
c 25 kg
16 1631 packets

EXERCISE 13B UNITS OF ENERGY

- 1 a 7200 J b 12080 J c 1209 J d 640 J e 54 J
2 a 8.2 kJ b 2.503 kJ c 0.895 kJ d 0.088 kJ
e 15.406 kJ
3 a 7254 cal b 16 169 cal c 780 cal
d 26 300 cal e 1008 cal
4 a 22.56 kcal b 7.604 kcal c 0.687 kcal
d 48.3 kcal e 0.098 kcal
5 a 1071 J b 24 435 J c 15 062 J
d 84 935 J e 1966 J
6 a 139 cal b 1655 cal c 3800 cal
d 155 cal e 60 229 cal
7 a 126 kJ b 628 kJ c 1038 kJ
d 1276 kJ e 2720 kJ
8 a 239 Cal b 221 Cal c 296 Cal
d 4780 Cal e 1814 Cal
9 a 167 kJ b 2887 kJ c 573 kJ
d 4393 kJ e 10 544 kJ
10 a 174 kcal b 643 kcal c 2271 kcal
d 1950 kcal e 3425 kcal
11 2079 Cal
12 a 3035 kcal b 10042 kJ c 2199 Cal d 11 987 kJ
13 a 50 Cal b 125 Cal c 50 kcal d 523 kJ

EXERCISE 13C FOOD AND NUTRITION: ENERGY INTAKE

- 1 474 kJ 2 601 kJ 3 763 kJ
4 a 588.2 kJ
b i 588 200 J ii 140 583 cal iii 140.583 kcal
c i 1260.429 kJ ii 1 260 429 J iii 301 250 cal
iv 301.25 kcal
d i 2100.714 kJ ii 2 100 714 J iii 502 083 cal
iv 502.083 kcal
5 a 1502 kJ b 901.2 kJ c 215.39 kcal

e

Nutrient	kJ/g	kcal/g
Protein	17	4.06
Fat	38	9.08
Carbohydrates	17	4.06
Fibre	8	1.91

- f 358.7 kcal g 215 kcal
6 a 268.6 kJ b 1678.8 kJ c 401.2 kcal
7 a 2250.5 kJ b 598.6 kJ c 143.1 kcal
d 537.6 kcal e 143 kcal
8 6305 kJ 9 13 373 kJ

EXERCISE 13D ENERGY EXPENDITURE

- 1 a 2120 kJ b 751 kJ c 605 kJ
d 6250 kJ e 1037 kJ f 688 kJ g 2147 kJ

- 2 a 5446 kJ/day b 8156 kJ/day c 2822 kJ/day
d 5987 kJ/day
3 a 8655 kJ/day b 5921 kJ/day c 5561 kJ/day
d 6635 kJ/day
4 a 780 kJ b 367 kJ
5 Noah by 488 kJ
6 a 4579 kJ/day b 5495 kJ/day
7 a 8513 kJ/day b 18 729 kJ/day
8 a 5746 kJ/day b 10 630 kJ/day
9 a 8125 kJ/day b 13 406 kJ/day
10 a 5978 kJ/day b 8668 kJ/day
11 a 7877 kJ/day b 3200 kJ
12 a 7523 kJ/day b 2351 kJ
13 a 7781 kJ/day b 3718 kJ
14 a 5558 kJ/day b 5211 kJ
15 a 5510 kJ/day b 1664 kJ
16 a 5754 kJ/day b 38 min
17 a 7781 kJ/day b 64 min
18 a 5816 kJ/day b 46 min
19 a 6183 kJ/day b 1 h 25 min
20 a 5756 kJ/day b 29 min
21 a 16 232 kJ b 187% c Eat less and/or exercise more
d i 9070 kJ/day ii 5 h

EXERCISE 13E ENERGY CONSUMPTION

- 1 a i 23/02/18 to 23/05/18 (inclusive) ii 90 days
b i 85.5 cents/day ii \$76.95
c i 1836.5 kWh ii 20.4 kWh
d i 51.71% ii 15.84%
e i 52.5 cents/kWh ii 39.74 cents/kWh
f i \$581.63 ii \$688.22 iii \$7.65/day
g i Smaller ii Larger
h i 18 kWh ii 1620 kWh
i i August 2017 ii February 2017
iii August 2016
2 Energy used = $2.4 \text{ (kW)} \times 300 \text{ (number of hours)}$
= 720 kWh
Cost = $720 \text{ (kWh)} \times \$0.3995 = \$287.64$
3 a \$11.46 b \$1.46
c \$2.93 d \$95.81
4 a i 26.28 kWh ii \$6.83
b i 87.6 kWh ii \$22.78
c i 43.8 kWh ii \$11.39
d i 35.04 kWh ii \$9.11
e i 17.52 kWh ii \$4.56
5 a \$488.21 b \$289.94
6 a \$17.84 b \$36.87
7 a \$51.41 b \$137.09
8 a Brand A \$165.44, Brand B \$186.12, Brand C \$227.48,
Brand D \$275.42
b i \$20.68 ii \$62.04 iii \$109.98
9 a \$512.46 b \$409.97

REVIEW MULTIPLE-CHOICE QUESTIONS

- 1 B 2 B 3 B 4 C 5 B 6 C
7 D 8 B 9 D 10 D

REVIEW SET 1

- 1 a 0.0024 kg b 5.63 kg c 4846 kg
2 a 80 mL b 40 mL c 20 mL
3 a 19 280 J b 2.634 kJ c 196 Cal

- 4 a 197 J b 4586 J c 8368 J
 5 658 kJ
 6 a 668 kJ b 893 kJ c 3167 kJ
 7 a 8140 kJ b 5843 J
 8 a 5632 kJ/day b 1901 kJ
 9 \$1.34

REVIEW SET 2

- 1 a 2.9 g b 7060 g c 0.94 t
 2 a 80 mL b 30 mL c 60 mL
 3 a 1608 cal b 5.205 kcal c 10 Cal
 4 a 82 cal b 679 cal c 7650 cal
 5 6428 kJ
 6 a 5544 kJ/day b 6771 kJ/day
 7 a 7637 kJ/day b 12 601 kJ/day
 8 a 6128 kJ/day b 28 min
 9 \$22.93

REVIEW SET 3

- 1 a 17 610 mg b 62.805 kg c 3604 kg
 2 48 mL
 3 a 23 050 J b 2.9 kJ c 99 Cal
 4 a 519 J b 4519 J c 31 380 J
 5 506 kJ
 6 a 1701 kJ b 6185 kJ c 1274 kJ
 7 a 6031 kJ/day b 5622 kJ/day
 8 a 7445 kJ/day b 1680 kJ
 9 a

	Energy used (kWh)	Rate (\$/kWh)	Cost (\$)
Peak	583.6	0.5144	300.20
Flexi	892.8	0.2136	190.70
Off-peak	291.7	0.139	40.55
		Total	531.45

- b \$71.59 c \$626.86 d No
 10 a 29.2 kWh b \$7.85

REVIEW SET 4

- 1 a 0.78 g b 31 206 g c 0.2896 t
 2 32 mL
 3 a 3140 cal b 0.98 kcal c 251 kJ
 4 a 43 cal b 729 cal c 4930 cal
 5 11 450 kJ
 6 a 5150 kJ/day b 3988 kJ/day
 7 a 5384 kJ/day b 11 845 kJ/day
 8 a 7109 kJ/day b 58 min
 9 \$3661.57

REVIEW PRACTICE EXAMINATION QUESTION

- 1 a i 168 kg ii 7 bags
 b 15 kJ c 629 kJ
 d i 112% ii 6482 kJ/day iii 25 min
 e i 792 kWh ii \$66.53

CHAPTERS 10–13 CUMULATIVE REVIEW

- 1 a \$499 000
 b i \$0.00133474 ii Yes
 c i \$180 ii \$388 iii \$568
 d \$1759.04 e \$1940.56
 2 a \$508.50 b \$4350

- 3 a \$591.66 b \$6999.60 c \$1450.08
 4 a \$5153.33 b 12.6%
 5 a i 10.8 km/L ii 0.092 km/L iii 9.23 L/100 km
 b 535.7 km
 6 a Income: $(\$170 \times 52) = \$8840$

Expenses:Board $(\$40 \times 52) = \$2080$ Clothes $(\$100 \times 12) = \$1200$ Entertainment $(\$65 \times 26) = \$1690$ Lessons $(\$85 \times 26) = \$2210$

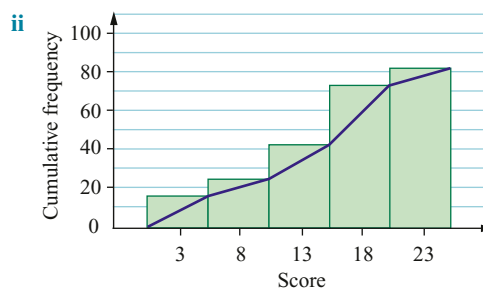
Total expenses = \$7180

Excess = \$1660

- b \$1660 c Reduce expenses or increase income
 7 a Mean = 7.4, mode = 8, median = 8

b i

Class	Frequency	Class centre	Cumulative frequency
1–5	16	3	16
6–10	8	8	24
11–15	18	13	42
16–20	31	18	73
21–25	9	23	82

iii Median ≈ 15 , fourth decile ≈ 13

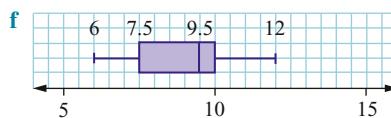
iv 10

c i 10, 11, 12 ii 2

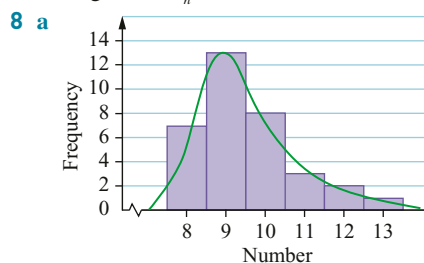
d

Stem	Leaf
4	8 8 9
5	0 1 4 6 6 7 8 9
6	0 2 4 5 5
7	1 3 4 8

e i Range = 30 ii 48, 56, 65
 iii 58.5 iv 12.5

g i Mean = 16.3, $\sigma_n = 2.9$ ii Mean = 56.5, $\sigma_n = 14.5$

h Mean = $3\frac{7}{9}$, mode = 3 and 5, median = 3,
 range = 4, $\sigma_n \approx 1.397$



b Positively skewed

9 a

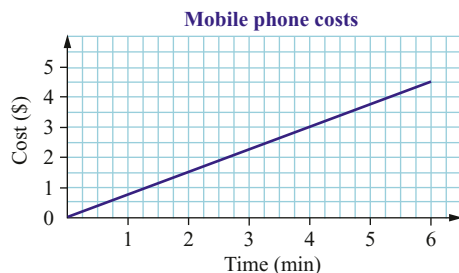
Stem	Leaf
1	6
2	3
3	0 6 8 8 9
4	0 1 1 3 5
5	4 7
6	8

b Symmetrical

10 a 8 b 9 c 7 d 2.5 e 2.05

11 a 6900 s b 8100 s c 8200 s d Females e Males

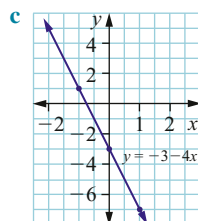
12 a i Dependent variable is cost.



ii About \$1.90

iii About $3\frac{1}{3}$ min

b Gradient = -3, y-intercept = 6



d i \$40 ii 100 km

iii Gradient is 0.4; cost per km

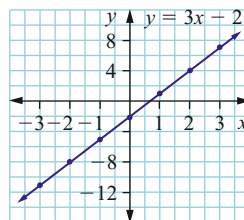
iv Intercept is \$10; flag fall

v No, as the y-intercept is not at the origin.

e i $\frac{1}{4}$ ii $y = \frac{x}{4}$ iii 12 iv 32

13

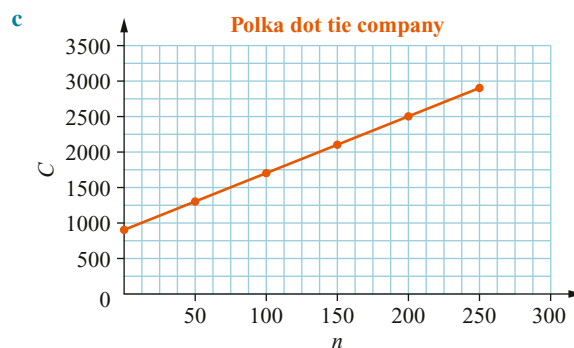
x	-3	-2	-1	0	1	2	3
y	-11	-8	-5	-2	1	4	7



14 a C = total cost, \$8 is the cost per tie, n is the number of ties, \$900 is the fixed cost

b

n	0	50	100	150	200	250
C	900	1300	1700	2100	2500	2900



d About 130 ties

15 a i 3400 kg ii 14.63 kg iii 4113 kg

b i 100 mL ii 50 mL iii 250 mL

c i 1428 cal ii 6.605 kcal iii 287 Cal

d i 58.56 cal ii 439.77 cal iii 6500 cal

e 553.52 kJ

16 a i 5510 kJ/day ii 6967 kJ/day

b i 6327.6 kJ/day ii 5825.1 kJ/day

17 \$4707.73

GLOSSARY

absolute error equals plus or minus half the precision

annual leave loading extra payment added to the holiday pay of employees taking their annual holidays

annulus the area between two circles with the same centre but different radii

average speed calculated by dividing the distance travelled during a time interval by the length of the time interval; e.g. average speed of a car that travels 120 km in 2 hours is 60 km/h

axis the diameter about which Earth rotates with endpoints at the North Pole and the South Pole

basal metabolic rate (BMR) minimum amount of energy needed to sustain basic bodily functions for a 24 hour period, measured in kJ/day

bell-shaped description of a distribution graph of a set for which the shape is symmetrical

biased sample when the method of collecting data produces a sample that does not accurately reflect the characteristics of the population

bimodal description of a distribution graph for a data set with two modes

blood alcohol content (BAC) percentage concentration of alcohol in a person's blood; e.g. a BAC of 0.05 (%) is 0.05 g of alcohol in every 100mL of blood

bonus extra payment paid as an incentive to employees who work hard

box plot (box-and-whisker plot) a diagram made up of a box and two 'whiskers' displaying five key values for a data set; these values are known as the five-number summary

braking distance distance a motor vehicle travels between the time the brakes are first applied and when the vehicle comes to a complete stop

budget a financial plan in which expenses need to be less than income

capacity amount of fluid that a container can hold

categorical variable a qualitative characteristic of data that can be divided into groups (or categories)

census involves collecting information about every member of the entire population

commission payment based on the percentage of the total price of the goods sold

complement of E ($\bar{E}$ or E') remaining group of outcomes in the sample space that do not belong to the event E

complementary events events are complementary if the sum of the number of outcomes in each event equals the total number of possible outcomes in the sample space

compound interest interest that is added to the principal at regular time intervals during an investment or loan period so future interest is calculated including that amount

consecutive integers whole numbers that follow one another in order, with a difference of 1 between them (e.g. 7 and 8; -4 , -3 and -2)

constant of variation (or proportionality) constant value (k) for $\frac{y}{x}$ when there is a direct variation relationship between y and x . The equation connecting y and x can be written as $y = kx$ where k is the constant of variation

continuous variable type of numerical variable within a range of values, usually resulting from measuring; e.g. heights of students

Coordinated Universal Time (UTC) primary time standard for clocks and time, replacing Greenwich Mean Time (GMT)

cross-section the shape of the face formed when a solid is sliced by a plane

cumulative frequency sum of the frequencies of all the scores up to and including a particular score

cumulative frequency histogram column graph (histogram) with data scores along the horizontal axis and cumulative frequency on the vertical axis

cumulative frequency polygon (ogive) line graph formed by joining the upper right-hand corners of each column of the cumulative frequency histogram

data pieces of collected information

daylight saving time (DST) the practice of setting clocks forward one hour from standard time during the summer months (from October to April in Australia)

deciles nine numbers that separate the set of ordered scores into ten parts with an equal number of scores in each part

dependent variable variable whose value depends upon that of another; usually represented on the vertical axis of a graph; e.g. for the relationship $y = 2x + 1$, the dependent variable is y (as it depends on the value of x)

depreciation the loss in the value of an asset due to its age and usage

direct variation relationship relationship between two quantities or variables in which one of the quantities can be expressed as a constant multiplied by the other quantity

directly proportional two quantities or variables are directly proportional when an increase (or a decrease) in one causes a proportional increase (or decrease) in the other; e.g. y is directly proportional to x if $\frac{y}{x}$ is a constant value

discrete variable type of numerical variable that is exact, usually resulting from counting; e.g. number of pets in a family

discretionary spending includes expenses such as entertainment, clothes and gifts that can be varied based on choice

dot plot a display of data where each piece of data is represented by a dot above a matching number or category on a horizontal scale

double-time overtime rate where employees are paid 2 times their normal hourly rate

energy consumption energy used by an appliance, measured in kilowatt-hours (kWh)

energy out amount of energy the body uses to sustain basic functions and perform normal daily physical activities

equally likely outcomes outcomes that have an equal chance of occurring

equation collection of two or more algebraic terms separated by mathematical operation symbols and containing an equals sign; e.g. $2x + 1 = 7$ or $3x^2 = 5x - 2$

equator great circle that is a parallel of latitude and whose centre is the centre of Earth

event one outcome or collection of outcomes from performing an experiment

expected frequency (expected number) expected number of times an event will occur, calculated by multiplying the probability of the event occurring by the number of trials

experiment a trial (or trials) performed to obtain data to predict the chances of an event occurring

five-number summary five key values for a data set; lowest score, highest score, median, lower quartile and upper quartile

fixed spending includes expenses like rent or board that cannot be varied

flat rate simple interest rate as a percentage

formula relationship or rule between two or more variables that contains an equals sign; e.g. $v = u + at$

frequency polygon line graph made by joining the mid points of the tops of the columns of a frequency histogram, with the first and last points joined to the horizontal axis

frequency table table that displays the number of times each category of data occurs

gradient (slope) steepness of a line (defined as m) that can be measured using any two points on the line or interval

great circle circle formed when a slice is taken through the centre of Earth

gross income total amount earned by an employee

GST a tax of 10% that is added to the price of goods and services

histogram column graph with no gaps between the columns that shows data scores along the horizontal axis and frequency on the vertical axis

independent variable variable whose outcomes are not due to those of another variable; usually represented on the horizontal axis of a graph; e.g. for the linear relationship $y = 2x + 1$, the independent variable is x

interquartile range measure of dispersion found by calculating the difference between the upper quartile and lower quartile; this is, $IQR = Q_3 - Q_1$

leaf last digit of a piece of data; e.g. for the data score 256, the leaf is 6

linear equation equation in which the highest positive integer power of the pronumeral is 1; e.g. $2x + 1 = 7$ is a linear equation but $3x^2 = 5x - 2$ is not because its highest power of x is 2

lower limit of true measurement result of subtracting the absolute error from the given measurement

mean the sum of all the scores divided by the number of scores, also known as the average

measure of central tendency (measure of location) a single number that indicates the location of the centre or most typical score in a set of scores; e.g. the mean, the mode and the median

measure of dispersion a single number that indicates the spread of scores, also known as a measure of spread; e.g. the range, the interquartile range and the standard deviation

median the middle score after the set of scores has been arranged in ascending order

Medicare levy extra tax to fund the health care system, calculated as a percentage of taxable income

meridians of longitude semicircles that are part of great circles with endpoints at the North Pole and South Pole

mode the score that occurs the most often in a set of scores

multimodal description of a distribution graph for a data set with many modes

multistage event one outcome or collection of outcomes from performing a multistage experiment

multistage experiment experiment made up of more than one simpler experiment

negatively skewed description of a skewed graph which has a longer tail to the left

net a plane shape that can be folded to form a solid

net income reduced amount earned by an employee after subtracting deductions from gross income

nominal variable type of categorical variable for which the categories are named but not ordered; e.g. hair colour

numerical variable a quantitative characteristic of data that can be counted or measured

offset a short distance measured perpendicularly from a main survey line

operating costs variable running costs that depend on how a vehicle is driven; e.g. cost of fuel, tyres, servicing and repairs

ordinal variable type of categorical variable for which the categories are ordered on a scale or as a ranking; e.g. rating system from 1–5

outlier an extreme piece of data or a score that is much higher or much lower than the rest of the data in the data set

overtime extra time worked by employees in excess of their normal working hours

parallels of latitude (circles of latitude) circles formed when slices perpendicular to Earth's axis are taken

pareto chart chart that combines a column graph and a cumulative frequency graph

per annum (p.a.) for each year (used in financial contexts)

percentage relative frequency relative frequency expressed as a percentage

percentiles 99 numbers that separate the set of ordered scores in 100 parts with an equal number of scores in each part

perimeter the distance around the boundary of a closed figure

Physical activity level (PAL) the total energy needed for an activity compared to the BMR

piecewise graph graph made up of several joined straight lines with different gradients, not just one single straight line

piecework payment based on an amount per item produced

population entire group about whom, or from which, information is sought

population standard deviation standard deviation for a population, denoted by σ_n

positively skewed description of a skewed graph which has a longer tail to the right

power rating rate at which an appliance uses electricity, measured in watts (W) or kilowatts (kW)

precision smallest unit on a measurement instrument

prime meridian (Greenwich Meridian) reference line for longitude (0°) that passes through Greenwich in London, England

principal sum of money invested or amount borrowed as a loan

probability chance of an event occurring, expressed as a number in the range 0 to 1 or as the equivalent percentage

probability tree diagram tree diagram on which the probabilities of simpler events are shown along its branches

pronumeral letter or symbol that takes the place of a number

quartiles three measures that divide a set of scores (in ascending order) into four parts with an equal number of scores in each part; the measures are the lower quartile (Q_1), middle quartile or median (Q_2), and upper quartile (Q_3)

random sample sample in which each member of the population has an equal chance of being selected

range the difference between the highest score and the lowest score in a data set

rate an amount of interest usually expressed as a percentage

reaction-time distance distance travelled in the time it takes a driver to react and move his or her foot to apply the brakes

relative frequency the number of times a score occurs as a fraction of the total number of scores

relative frequency (experimental probability) an estimate of the probability of an event occurring, determined by performing an experiment a number of times and finding the event's relative frequency

royalty payment based on a percentage of the continuing sales of a product that someone has made or created

salary a fixed regular payment, typically paid monthly but often expressed as an annual sum, made by an employer to an employee

salvage value (book value, scrap value, written-down value) the value of an asset after depreciation

sample involves collecting information about a small selection from a population

sample space set of all possible outcomes of an experiment

sample standard deviation standard deviation for a sample, denoted by σ_{n-1}

scientific notation a value written as a number from 1 up to, but not including, 10 (with any number of decimal places) multiplied by a power of 10

self-selected sample sample in which participants choose to take part by responding to a request for volunteers

significant figures the number of digits in a number that indicate its accuracy

simple interest an amount calculated on the full amount of a loan or investment for the full term

skewed description of a distribution graph of a data set for which the shape is not symmetrical

small circle circle formed when a slice is taken that does not pass through the centre of the Earth

solution the value of the pronumeral that makes an equation a true statement; e.g. the solution of $2x + 1 = 7$ is $x = 3$

solve to find the value of the pronumeral that makes the equation a true statement

standard deviation a measure of dispersion or spread whose value depends on all the scores; determined by calculating an 'average' of the distance of each score from the mean

standard drink any drink that contains 10 g of alcohol

standard time time adopted for a particular region within a time zone

standing costs fixed costs; e.g. depreciation, loan interest, on-road costs such as registration, insurance and motoring organisation membership

statistics the study of collecting, organising, analysing and interpreting data

stem digit or digits preceding the last digit of a piece of data; e.g. for the data score 256, the stem is 25

stem-and-leaf plot A display of data where each piece of data is split into two parts: the last digit becoming the leaf and the other digits becoming the stem

straight-line method of depreciation a method for which an item loses value by a constant amount each year

stratified random sample sample made up of random samples taken from subgroups (strata) of the population, proportional to the relative size of each subgroup

subject of the formula single variable usually on the left-hand side of a formula; e.g. v is the subject of the formula $v = u + at$

surface area the sum of the areas of all the three faces of a three-dimensional solid

systematic sample sample chosen in a methodical way where members of the population are put in order, the first number is chosen at random and then every n th number is chosen for the sample

tally marks lines or marks that help record data in a table with every fifth tally mark drawn through the preceding four marks

taxable income income for one financial year calculated by subtracting allowable deductions from total income

term period of time for an investment or loan

time zone region of the world that observes the same time throughout

time-and-a-half overtime rate where employees are paid $1\frac{1}{2}$ times their normal hourly rate

timetables information displayed in tables about the times of scheduled events

total income all income received by a person during one financial year

total stopping distance total distance travelled in the time it takes for a driver to react, apply the brakes and for the car to come to a stop

unimodal description of a distribution graph for a data set with one mode

upper limit of true measurement result of adding the absolute error to the given measurement

variable (algebra) numerical quantity that can have varying or different value and can be represented by a pronumeral; e.g. time or t

variable (statistics) a particular characteristic that we are interested in when collecting data

volume the amount of space a solid occupies

wage a fixed regular payment earned for work or services, usually based on an hourly rate and typically paid daily, weekly or fortnightly

World Time Zones time zones around the world that are expressed as positive or negative offsets from UTC

y-intercept point where a line crosses the y-axis of a number plane

INDEX

A

absolute error 65–69
 accuracy of measurement 61–62
 alcohol, blood alcohol content (BAC) 100–03
 algebraic equations *see* linear equations
 algebraic expressions, substitution into 76–77, 80
 allowances 10–15
 annual income 7
 annual leave loading 10, 15
 approximations 61, 62–63
 area
 composite figures 160–65
 estimating using Trapezoidal rule 170–73
 of a field involving triangles and trapeziums 169
 formulas 154
 irregular shapes 166–73
 limits of true 68
 metric units 54
 quadrilateral 166–67
 simple shapes 154–69
 Australian time zones 340–41
 average speed 96–97
 axis 322

B

back-to-back stem-and-leaf plot 246–51
 bar graphs 252–57
 basal metabolic rate (BMR) 495–97
 using Harris–Benedict formula 497
 using Schofield formula 496
 bell-shaped curve 433–34
 bias in sampling 124
 biased sample 122, 124
 bimodal distribution 432
 bimodal scores 394
 blood alcohol content (BAC) 100–03
 bonus 10–15
 book value 218–21, 370
 box plots (box-and-whisker plots) 418–21
 describing the shape of a distribution from 434
 drawing 419–20
 drawing from five-number summary 419
 drawing from stem-and-leaf plot 420
 range, median and interquartile range from 418
 braking distance 96–99
 breathalyser 100
 budget 380
 budgeting 380–83

C

calibration error 60
 calories 486–89
 capacity 192–95
 converting to/from volume 192
 cylinder 193–94
 metric units 54
 car loans 368–71
 carbohydrates 490–93
 cars
 average speed, distance and time 96–99
 braking distance 96–99
 cost of purchase 362–67
 dealer delivery 363
 depreciation 370–71
 financing the purchase 368–71
 fuel consumption 372–75
 insurance 363
 reaction-time difference 96, 98–99
 registration 362–65
 running costs 376–79
 stamp duty 362, 364–67
 total stopping distance 96, 99
 trade-in 363
 categorical variables 119–21
 census 122–25
 Central Standard Time (CST) 340
 changing the subject of the formula 93–95
 circle
 area 154, 158, 159
 circumference 150–53
 circles of latitude 322
 Clark's formula 104–07
 classes (class intervals) 238–39
 classification of data 118–21
 column graphs 252–57
 commission 16–21
 complement of E ($\bar{E}$ or E') 294
 complementary events 294–97
 composite figures, area 160–62
 composite prisms
 surface area 177–79
 volume 191
 compound interest 222–25
 comprehensive insurance 363–67
 compulsory third-party (CTP) insurance 362–67
 consecutive integers 84
 constant of proportionality 463
 constant of variation 463
 continuous variable 119
 Coordinated Universal Time (UTC) 334
 cross-section 186
 cube, surface area 174–76
 cumulative frequency 240–45

cumulative frequency
 histogram 266–71
 cumulative frequency
 polygon 266–71
 see also ogive
 cumulative reviews 114–15, 230–33, 352–53, 512–15
 cylinders
 capacity 193–5
 surface area 180–85
 volume 186–91

D

data 118
 classification of 118–21
 daylight saving time (DST) 340
 dealer delivery (cars) 363
 deciles 414–17
 from an ogive 414
 decimal values, units of mass 482–85
 deductions 28–31
 dependent variable 448–51
 depreciation 218–21
 straight-line method 218–21, 370–71
 dietary fibre 490–91
 direct variation relationships 462–67
 finding the equation of 465–67
 graphing 463–64
 modelling 472–73
 directly proportional 462
 discrete variable 119–21
 discretionary spending 380
 distance, speed and time 96–97
 distribution (curves) 432–36
 describing 433–39
 describing the nature of a graph 434–35
 describing the shape from a box plot 437–38
 describing the shape from a dot plot 435
 describing the shape from a stem-and-leaf plot 436
 dot plots 246–51
 describing shape of distribution from 435
 double-time 10
 drivers, blood alcohol content 100–03

E

Earth's surface
 latitude and longitude 322–25
 position on 326–29
 Eastern Standard Time (EST) 340

electrical appliances
 cost of running 504–05
 power rating 502
 electricity accounts 359, 503
 energy, units of 486–89
 energy consumption 502–05
 energy expenditure 494–501
 time needed to burn
 kilojoules 500–01
 to maintain lifestyle 498–99
 to perform individual
 activities 499–500
 energy intake from foods 490–93
 energy out 494–95
 equally likely outcomes 280, 283
 equations
 with fractions 86–87
 linear 80–91, 458
 equator 322
 errors in measurement 60–63
 absolute error 64–69
 maximum error 68–69
 event 280
 expected frequency (expected
 number) 306–13
 experiment 280–85
 experimental probability 286–89

F

fats 490
 five-number summary 418–21
 drawing box plots from 419
 fixed costs 376
 fixed spending 380
 flow rates (intravenous drip) 106
 food, energy intake from 490–93
 formulas 7
 area 154–59
 blood alcohol content
 (BAC) 100–03
 changing the subject of the 93–95
 complementary events 294
 finding the value of a
 variable in 92–95
 simple interest 210
 straight-line method of
 depreciation 218
 subject of the 76
 substitution into 76–79, 92
 volume 186
 frequency curves *see* distribution (curves)
 frequency distribution tables 236–39
 adding cumulative and relative
 frequency to 240–45
 constructing from cumulative
 frequency histogram 268
 grouped data 264–65, 395–401,
 412
 identifying outliers from 428–31
 mean from 392–93
 median from 400
 mode from 395

frequency histograms 262–65
 frequency polygons 262–65
 Fried's formula 105
 fuel consumption (cars) 372–75

G

gas accounts 360–61
 government allowances and
 pensions 22–27
 gradient (slope) 452–57
 grams 482–84
 graphical data displays 432–37
 graphing, direct variation
 relationship 462–67
 graphs
 bar 252–57
 column 252–57, 262
 cumulative frequency 266–69
 line 258–61
 misleading 259–60, 261–62
 piecewise 366, 457
 simple interest 210–17
 sketching 459
 straight-line 458–73
 great circle 322
 greatest possible error (GPE) 64
 green slip 362
 Greenwich Mean Time (GMT) 334
 Greenwich meridian 322
 gross income 28
 grouped data 242–43
 mean 392–95, 423–24
 median 401–02
 standard deviation 422–27
 GST (Goods and Services Tax)
 208–09

H

Harris–Benedict formula 495–501
 health insurance 28–31
 histograms 262–64
 horizontal bar graphs 253–54
 household bills 356–58, 502–05
 hypotenuse 149

I

income, earning an 4–9
 income tax calculation 36–39
 independent variable 448–49
 insurance (cars) 363
 interest rates 211–13
 International Date Line 338, 399
 interquartile range 408–13
 from box plot 418–19
 from ogive 414–15
 interpreting 412
 irregular shapes, perimeter and
 area 166–73

J

joules 486–89

K

kilocalories 488–89
 kilograms 482–84
 kilojoules 486–89
 energy expenditure 494–501
 kite, area 154, 157

L

latitude 322–24, 326–29
 and position coordinates 326–29
 length, metric units 54–55
 limit of reading 60
 line graphs 258–61
 see also straight-line graphs
 linear equations 80–91
 checking solutions to 89
 containing negative
 pronumerals 82
 involving fractions 86–87
 with pronumerals on both sides 88
 resulting from substitution 83
 worded descriptions 84–85
 writing the gradient and y-intercept
 from 459
 linear graphs *see* straight-line graphs
 linear modelling 468–73
 direct variation 472–73
 linear models, interpreting graphs
 of 468–69
 linear relationships, finding equations
 that describe 460–61
 loadings 10–15
 see also annual leave loading
 loan calculations 368–71
 longitude 322–29
 and position coordinates 326–29
 lower limit of true measurement
 64–66
 lower quartile 409

M

mass
 converting units of mass 482–84
 metric units 54, 482–85
 mathematical formulas *see* formulas
 maximum error 67–69
 mean 392–97
 comparison with mode and
 median 405–07
 from frequency distribution
 table 392–93
 from stem-and-leaf plot 396
 for grouped data 393–94, 423–24
 relative merits 404–07
 measurement
 accuracy of 60–63
 errors in 64–69
 prefixes for units of 58–61
 units of 54–57
 upper and lower limits of
 true 64–69

measures of central tendency 392, 404–07
see also mean; median; mode
 measures of dispersion (spread) 408
 measures of location 392
 median 398–403
 of an even number of scores 399
 of an odd number of scores 398–99
 comparison with mean and mode 405–06
 from box plot 418–19
 from frequency distribution table 400
 from stem-and-leaf plot 403
 for grouped data 401–02
 relative merits 404–07
 Medicare levy 32–35
 medication dosages 104–07
 meridians of longitude 322–27
 and position coordinates 326–29
 metric units of measurement 54–57
 area 56
 capacity 56
 energy 496–99
 length 54–55
 mass 55, 482–85
 volume 57
 middle quartile 409
 milligrams 482–84
 misleading graphs 255–57, 259–60
 mode 394–97
 comparison with mean and median 404–07
 from frequency distribution table 395–96
 from stem-and-leaf plot 396–97
 relative merits 404–07
 multimodal distribution 432
 multistage events 298–301
 probability 290–93
 multistage experiments 298
 sample space for 298–99

N

negatively skewed distribution 434
 net 174
 net income 28–31
 nominal variable 119
 normal distribution 433
 numerical variables 119
 nutrients, energy intake from 490–95

O

ogive 266–67
 deciles from 414–15
 interquartile range from 408–13
 percentiles from 414–15
 operating costs 376, 378
 ordinal variable 119–20

outcomes and sample space 280–85
 outliers 405, 406, 428
 identifying from a frequency table 429–30
 identifying from a set of scores 428
 identifying from a stem-and-leaf plot 430–31
 overtime 14–15

P

parallax error 60
 parallelogram, area 154, 156
 parallels of latitude 322–24
 and position coordinates 326
 Pareto chart 269–71
 pensions 22, 24, 27
 per annum (p.a.) 4
 percentage applications 204, 207
 percentage change 207
 percentage error 66
 percentage increase or decrease 205–06
 percentage profit or loss 206–09
 percentage relative frequency 240
 percentiles 414–17
 from an ogive 415
 perimeter 148–53
 circle 150
 of a field involving triangles and trapeziums 169
 irregular shapes 166–73
 limits of true 67
 Pythagoras' theorem to find 149–50
 quadrilateral 166
 sector 152
 physical activity level (PAL) 495, 497–500
 piecewise graphs 457
 piecework 16, 19–20
 population 122–25
 population standard deviation 424, 425, 427
 position on Earth's surface 326–29
 positively skewed distribution 433
 power rating (electrical appliances) 502
 precision 64–66
 prefixes for units of measurement 58–59
 prime meridian 322, 324, 325
 principal 210
 prisms
 surface area 174–78
 volume 186–87, 191
 probability 286–89, 290
 complementary events 294–97
 expected frequency 306–12
 experimental 286–89
 for multistage events 298–301
 relative frequency 286–89
 and simulations 307–13
 theoretical 290–93
 tree diagrams 302–05
 profit or loss 206

pronumeral 76
 protein 490–93
 purchasing a car 362–67
 financing the purchase 368–71
 Pythagoras' theorem 149–50

Q

quadrilateral
 area 166–67
 perimeter 167
 quartiles 409–11
 questions in questionnaires 118–19

R

random number generator 129–31
 random sampling 126–27, 132–33
 range 408–13
 from box plot 418–19
 rates (interest) 210, 211–13
 reaction-time distance 96, 98–99
 rectangle, area 154, 155
 rectangular prism, surface area 175–76
 reducing-balance loan 368–69
 registration (cars) 363–65, 367
 relative frequency (data) 240–45
 relative frequency (experimental probability) 286–89
 rhombus, area 154, 156
 right-angled triangle 149
 royalties 16, 21
 running costs (cars) 382–85

S

salary 4–9
 salvage value 218, 370
 sample 122–25
 biased 122, 124
 random 126–31
 self-selected 125, 136
 stratified random 132–33, 136–37
 systematic 134–35, 137
 sample size, effect of 134–35
 sample space 280–85
 and equally likely outcomes 282
 for multistage experiments 283–85
 sample standard deviation 424–25
 sample types, suitability 136–37
 sampling methods 136
 Schofield formula 469, 498–99, 500–01
 scientific notation 50–53
 scrap value 218–21, 370
 sector
 area 156, 158–59, 160–61
 perimeter 154
 self-selected sampling 125, 136
 shorter side (right-angled triangle) 149
 significant figures 48–49

- simple interest 210–13
 - formula 210
 - graphs 214–17
 - simulations 307, 310
 - sketching graphs 459
 - skewed distribution 432, 435
 - slope (gradient) 452–57
 - small circle 322
 - speed, distance and time 96–97
 - sphere
 - surface area 183–85
 - volume 190–91
 - stamp duty 362, 364–67
 - standard deviation 422–27
 - grouped data 423–24
 - standard drinks 100–03
 - standard times 335–39
 - standing costs 376
 - statistics 118
 - stem-and-leaf plots 246–51
 - back-to-back 250–51, 254–55
 - describing the shape of a distribution from 436
 - drawing a box plot from 420
 - identifying outliers from 430–31
 - mean and mode from 396–97
 - median from 403
 - with split stems 249
 - straight-line graphs 448–51
 - equation $y = mx + c$ 458–61
 - gradient (slope) 452–57
 - sketching 459
 - y-intercept 456, 459
 - straight-line method of
 - depreciation 219–22, 370
 - formula 219
 - stratified random
 - sampling 132–33, 136
 - study loan repayment 35
 - subject of the formula 76
 - changing the 92, 93–95
 - substitution
 - into algebraic expressions 76–77, 78
 - into mathematical formulas 77–79
 - superannuation 29–30
 - surface area 174–79
 - cylinders 180–85
 - prisms 174–79
 - sphere 180–85
 - systematic sampling 134–35, 136
- T**
- tally marks 236
 - target population 122–25
 - tax payable 36–39
 - tax rates 36
 - tax table 36
 - taxable income 32–35
 - term (loan) 210
 - theoretical probability 290–93
 - third-party property damage vehicle
 - insurance 363
 - time
 - calculations 330–33
 - converting between 12-hour and 24-hour time 331
 - converting units of 330–31
 - distance and speed 96–97
 - time zones
 - Australian 340–41
 - world 334–39
 - time-and-a-half 10
 - timetables, interpreting 342–43
 - total income 32
 - total stopping distance 96, 99
 - trade-in 363
 - trapezium, area 154, 157, 168
 - Trapezoidal rule 170–73
 - travel calculations 96–99
 - tree diagrams 302–05
 - triangle, area 154, 155, 168
 - triangular prism, surface area 176–78
- U**
- unimodal distribution 432
 - units of energy 486–89
 - units of mass 55, 482–85
 - units of measurement
 - metric units 54–57
 - prefixes 58–61
 - upper limit of true
 - measurement 64, 65–67
 - upper quartile 409–11
 - utility bills 356–61, 503–05
- V**
- variables 76, 119–21
 - VAT (Value Added Tax) 208
 - vertical column graphs 252–53
 - vertical intercept (straight-line graph) 452–53
 - volume 186–91
 - converting to/from capacity 192–93
 - cylinder 189, 191
 - formulas 186
 - metric units 54
 - prism 186–88, 191
 - sphere 190–91
- W**
- wages 4, 6–7
 - water bills 262–64, 356–58
 - Western Standard Time (WST) 340
 - World Time Zones 334–39
 - written-down value 218, 370
- Y**
- $y = mx + c$ 458–61
 - y-intercept (straight-line graph) 456, 459
 - Young's formula 104, 105
 - Youth Allowance 22–27

ACKNOWLEDGEMENTS

The author and the publisher wish to thank the following copyright holders for reproduction of their material.

Cover: Getty Images/Alexander Kesselaar. **Chapter 1:** Alamy/ableimages, p. 16; Getty Images/Gillianne Tedder, p. 14; Getty Images/Tom Grill, p. 18; Getty Images/istock/Jacob Wackerhusen, p. 6; imagefolk/Image Source, p. 42; Shutterstock, pp. 4, 5, 8, 9, 11, 13, 15, 19, 20, 21, 24, 25, 26, 27, 32, 37, 38. **Chapter 2:** Alamy, 57 top; Brent Parker Jones, 60; Getty Images/Dorling Kindersley, 56 bottom; Getty Images/istockphoto, 56 top; Getty Images/iStockphoto/craftvision, 67; Ian Trower/AWL Images Ltd/imagefolk, 66; imagefolk/Mark Garlick, 53; Shutterstock, 49, 55, 57 bottom, 58, 59, 63, 69 bottom. **Chapter 3:** iStockphoto, 106; iStockphoto/Imgorhand, 111; Shutterstock, 78, 84, 79 bottom, 79 top, 90 bottom, 90 top, 96, 102. **Chapter 4:** Alamy/Rob Francis, 121; Shutterstock, 120, 123, 126, 128, 137, 141, 142, 143, 144. **Chapter 5:** Alamy/John Trax, 164 bottom; Lindsay Edwards, 179 top left and right; istockphoto/GregC, 194 top; Shutterstock, 153, 159, 164 centre and top, 165, 169, 172, 173, 178, 179 centre and bottom, 185, 191, 192, 193, 194 bottom, 195. **Chapter 6:** All photographs Shutterstock. **Chapter 7:** Alamy/Sindre Ellingsen, 236; iStockphoto/kristian sekulic, 274; iStockphoto/pamspix, 249; iStockphoto/nycshooter, p. 244 top; Shutterstock, 239, 244 bottom, 254, 257, 265. **Chapter 8:** Alamy/Blend Images, 307 top; iStockphoto/Carolyn Woodcock, 301 bottom right; iStockphoto/Lauri Patterson, 301 left; Shutterstock, 289 bottom, 289 centre, 289 top, 293 bottom, 301 top, 307 bottom. **Chapter 9:** istockphoto/nathanphoto, p. 338; all other photographs Shutterstock. **Chapter 10:** 123rf/Luca Bertolli, p. 376; istockphoto/Henrick5000, p. 387; all other photographs Shutterstock. **Chapter 11:** Alamy/David Foster, p. 397; Centers for Disease Control and Prevention, National Center for Health Statistics, p. 416; Shutterstock, pp. 413, 426, 431. **Chapter 12:** Alamy/Martin Berry, p. 450; Alamy/PhotoAlto, p. 468; all other photographs Shutterstock. **Chapter 13:** Alamy/ScotStock, p. 484 bottom; all other photographs Shutterstock.

Every effort has been made to trace the original source of copyright material contained in this book. The publisher will be pleased to hear from copyright holders to rectify any errors or omissions.

OXFORD
UNIVERSITY PRESS
AUSTRALIA & NEW ZEALAND

ISBN 978-0-19-031051-6



9 780190 310516

visit us at: oup.com.au or
contact customer service: cs.au@oup.com