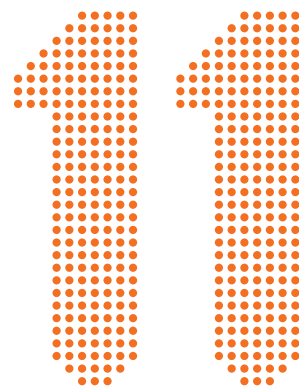


PEARSON

GENERAL MATHEMATICS

QUEENSLAND

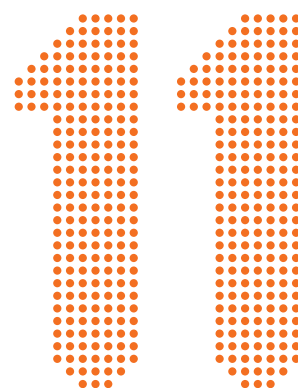
STUDENT BOOK



UNITS 1 & 2



PEARSON
GENERAL
MATHEMATICS
QUEENSLAND
STUDENT BOOK



UNITS 1 & 2

Pearson Australia

(a division of Pearson Australia Group Pty Ltd)
707 Collins Street, Melbourne, Victoria 3008
PO Box 23360, Melbourne, Victoria 8012
www.pearson.com.au

Copyright © Pearson Australia 2018
(a division of Pearson Australia Group Pty Ltd)
First published 2018 by Pearson Australia
2021 2020 2019 2018
10 9 8 7 6 5 4 3 2 1

Reproduction and communication for educational purposes

The Australian *Copyright Act 1968* (the Act) allows a maximum of one chapter or 10% of the pages of this work, whichever is the greater, to be reproduced and/or communicated by any educational institution for its educational purposes provided that that educational institution (or the body that administers it) has given a remuneration notice to the Copyright Agency under the Act. For details of the copyright licence for educational institutions contact the Copyright Agency (www.copyright.com.au).

Reproduction and communication for other purposes

Except as permitted under the Act (for example any fair dealing for the purposes of study, research, criticism or review), no part of this book may be reproduced, stored in a retrieval system, communicated or transmitted in any form or by any means without prior written permission. All enquiries should be made to the publisher at the address above.

This book is not to be treated as a blackline master; that is, any photocopying beyond fair dealing requires prior written permission.



Publisher: Daniel Hernandez
Project Manager: Sarah Daglish
Production Editor: Natalie Lincoln
Development Editor: Lindy Bayles
Editor: Kay Waters
Series Designer/Cover Designer: Nicole Melbourne
Illustrators: diacriTech
Typesetters: Aptara
Proofreader: Jane Fitzpatrick
Rights & Permissions: Amirah Fatin
Senior Publishing Services Analyst: Rob Curulli
Production Controller: Dominic Harman
Printed in Malaysia by Vivar



A catalogue record for this book is available from the National Library of Australia

ISBN: 978 1 4886 2139 0

Pearson Australia Group Pty Ltd ABN 40 004 245 943

Attributions

Cover: Ingo Oeland/Alamy Stock Photo

Alamy Stock Photo: William Caram, p. 55; VPC Animals Photo, p. 449.

Fotolia: p. 200 (mulch); Anatolii, p. 167; ikonoklast_hh, p. 16; jonasginter, p. 5; Kesu, p. 192 (small pizza); Kzenon, p. 15; Pius Lee, p. 178; leekris, p. 191; Derek Miller, p. 67; Picture Partners, p. 199; Marc Xavier, p. 70.

Getty Images: FLYFILM.TV, pp. 208–9; Posnov, pp. 278–9.

Shutterstock: AnemStyle, pp. 398–9; aquariagirl1970, p. 137; artoflogic, p. 113; biDaala_studio, pp. 80–1; BlankaB, p. 406; Bokeh Blur Background, p. 408; cloki, p. 139; dny3d, p. 245; FastMotion, pp. 338–9; Hitdelight, pp. 150–1; isak55, pp. 2–3; Markus Mainka, p. 192 (large pizza); Natan86, p. 200 (ball); Luis Padilla-Fotografia, p. 148 (mosquito); Sompol, p. 125; urfin, p. 108; Peter Waters, p. 148 (spider).

Disclaimer/s

The selection of internet addresses (URLs) provided for this book was valid at the time of publication and was chosen as being appropriate for use as a secondary education research tool. However, due to the dynamic nature of the internet, some addresses may have changed, may have ceased to exist since publication, or may inadvertently link to sites with content that could be considered offensive or inappropriate. While the authors and publisher regret any inconvenience this may cause readers, no responsibility for any such changes or unforeseeable errors can be accepted by either the authors or the publisher.

Some of the images used in *Pearson General Mathematics 11 (Units 1 & 2) Queensland* might have associations with deceased Indigenous Australians. Please be aware that these images might cause sadness or distress in Aboriginal or Torres Strait Islander communities.

PEARSON
**GENERAL
MATHEMATICS**
QUEENSLAND

11
UNITS 1 & 2

Writing and development team

We are grateful to the following people for their time and expertise in contributing to **Pearson General Mathematics 11**.

Nicola Silva

Mathematics teacher, QLD
Author

Geoff Phillips

Mathematics author, VIC
Contributing author

Daniel Hernandez Navas

Lead publisher
Content and learning specialist
Pearson Australia

Lindy Bayles

Mathematics teacher, VIC
Development editor

Kylie Agnew

Senior Mathematics teacher, QLD
Reviewer

Rodney Anderson

Mathematics teacher, QLD
Teacher support material author

Scott Brown

Leading teacher – STEAM Programs, VIC
Reviewer and answer checker

Linda Brook

Mathematics and STEM teacher, QLD
Answer checker

Phillip Campbell

Mathematics education consultant, QLD
Reviewer and answer checker

Greg Carroll

Principal, Exam assessor, author, VIC
Contributor

David Coffey

Mathematics author, VIC
Teacher support material author

Elizabeth Constable

Mathematics teacher, QLD
Answer checker

Shelley Cross

Mathematics teacher, QLD
Answer checker

Peter Flynn

Lecturer, IB assessor, trainer, VIC
Contributor

Emily Frazer

Mathematics teacher, QLD
Answer checker

Emily Hobbs

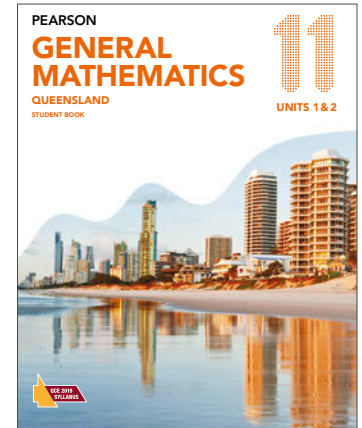
Mathematics teacher, HoD, VIC
Teacher support material author
Reviewer and answer checker

Robyn Johnson

Mathematics teacher, QLD
Teacher support material author

Karleigh Nicholls

Senior Mathematics teacher, QLD
Reviewer



Contents

Pearson General Mathematics 11

Writing and development team

How to use this book

iii

vi

UNIT 1

CHAPTER 1 Consumer arithmetic

Recall

4

1.1	Percentages, decimals and rates	5
1.2	Making money	15
1.3	Taxation and other government charges	22
1.4	Investing money	31
1.5	Shares	42
1.6	Profit, loss and inflation	49
1.7	Making the most of your money	56
1.8	Working with finance	66

Summary

73

Chapter review

75

CHAPTER 2 Measurement

Recall

82

2.1	Pythagoras' theorem	83
2.2	Perimeter and area	92
2.3	Volume of prisms and cylinders	104
2.4	Volume of tapered solids and spheres	116
2.5	Surface area of prisms and cylinders	126
2.6	Surface area of tapered solids and spheres	133

Summary

141

Chapter review

144

CHAPTER 3 Similar figures

Recall

152

3.1	Similar shapes and length	153
3.2	Scale drawing	172
3.3	Similar shapes and area	182
3.4	Similar solids and volume	193

Summary

201

Chapter review

202

CHAPTER 4 Linear equations and their graphs

Recall

210

4.1	Linear equations	211
4.2	Linear graphs	220
4.3	Sketching linear graphs	231
4.4	Simultaneous linear equations	246

4.5	Piece-wise linear graphs and step graphs	259

Summary

268

Chapter review

270

Exam review Unit 1 (Chapters 1–4)

274

UNIT 2

CHAPTER 5 Applications of trigonometry

Recall

280

5.1	Trigonometric ratios	281
5.2	Angles of elevation and depression	289
5.3	Directions and bearings	296
5.4	The sine rule	305
5.5	The cosine rule	313
5.6	Alternative formulas for triangle area	322

Summary

330

Chapter review

332

CHAPTER 6 Algebra and matrices

Recall

340

6.1	Linear and non-linear expressions	341
6.2	Matrix properties	350
6.3	Matrix addition, subtraction and scalar multiplication	357
6.4	Matrix multiplication	363
6.5	Networks and power matrices	376

Summary

390

Chapter review

392

CHAPTER 7 Univariate data analysis

Recall

400

7.1	Categorising and organising data	401
7.2	Categorical data	409
7.3	Numerical data	417
7.4	Median, range and interquartile range	429
7.5	Cumulative frequency	443
7.6	Mean and standard deviation	454
7.7	Comparing data	466

Summary

476

Chapter review

479

Exam review Unit 2 (Chapters 5–7)

484

Exam review Units 1 & 2 (Chapters 1–7)

487

Answers

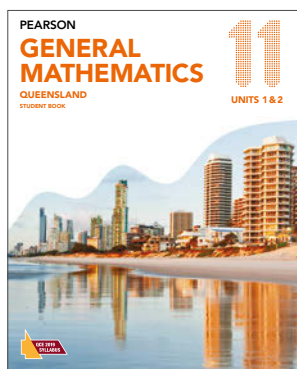
493

PEARSON GENERAL MATHEMATICS

QUEENSLAND

11

UNITS 1 & 2



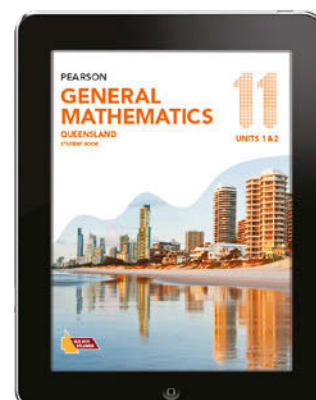
General Mathematics 11
Student book

Student book

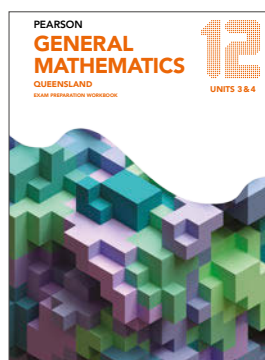
The student book has been authored by local authors, ensuring quality content and complete curriculum coverage for Queensland, enabling students to prepare with ease and confidence. We have covered the breadth of the content within our exercise questions, from simpler skills-focused questions to those using unfamiliar contexts and application of the theory learnt. The theory, worked examples and question sets are written in line with the assessment objectives, with the aim of familiarising students with QCE cognitive verbs in the process of dependent and guided instruction. Additional interactives that help explain the theory and consolidate concepts have been included throughout all chapters.

Pearson Reader+

Pearson Reader+ is our next-generation eBook. This is an electronic textbook that students can access on any device, online or offline, and is linked to features, interactives and visual media that will help consolidate their understanding of concepts and ideas, as well as other useful content developed specifically for senior mathematics. It supports students with appropriate online resources and tools for every section of the student book, providing access to exemplar worked solutions that demonstrate high levels of mathematical and everyday communication. Students have the opportunity to learn independently through the Explore further tasks and Making connections interactive widgets, designed to engage and support conceptual understanding. Additionally, teachers have access to syllabus maps, a teaching program, sample exams, problem-solving and modelling tasks, and additional banks of questions for extra revision.



General Mathematics 11
eBook



General Mathematics 12
Exam preparation workbook

Exam preparation workbook

Additional component for Year 12 only

The exam preparation workbook provides additional support in preparing students for the external exam. It has been constructed to guide students through a sequence of preparatory steps and build confidence leading up to the external exam.

How to use this book

Pearson General Mathematics 11 Queensland Units 1 & 2

This Queensland senior mathematics series has been written by a team of experienced Queensland teachers for the QCE 2019 syllabus. It offers complete curriculum coverage, rich content and comprehensive teacher support.

Additional information

These interactives appear in the eBook in two forms, as videos explaining specific concepts or as interactive questions to check students' understanding.

Making connections

This eBook feature provides teachers and students with a visual interactive of specific mathematics concepts or ideas to aid students in their understanding.

Explore further

This eBook feature provides an opportunity for students to consolidate their understanding of concepts and ideas with the aid of technology, and answer a small number of questions to deepen their understanding and broaden their skills base. These activities should take approximately 5–15 minutes to complete.

2.2

Additional information

Perimeter

Consolidate your understanding of perimeter.

Additional information

Arc length

Review the process of finding arc lengths.

Area

Area A is the size of the surface within the boundary of a two-dimensional shape. Some of the most common area formulas are shown below.

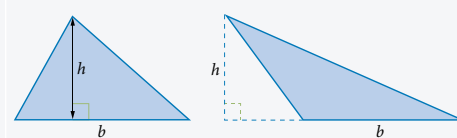
Rectangle

$$A = lw$$



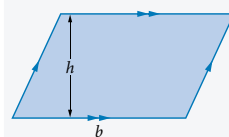
Triangle

$$A = \frac{1}{2}bh$$



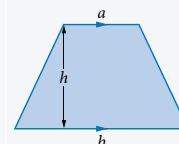
Parallelogram

$$A = bh$$



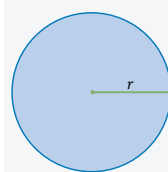
Trapezium

$$A = \frac{1}{2}(a + b)h$$



Circle

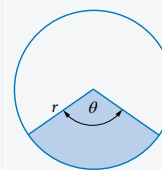
$$A = \pi r^2$$



Sector

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

(Note: $\frac{\theta}{360^\circ}$ is the fraction of the circle occupied by the sector.)



Making connections

Sector area

Determine the effect on the area of the sector when:

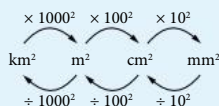
- the radius increases
- the angle subtended increases.

Explore further

Area formulas

Create formulas to solve area problems using a spreadsheet.

To convert between square factors, use the square of the linear conversion factor.



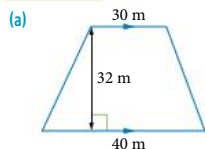
Additional information

Converting units of length

Revise converting metric units of area.

7 Area

Determine the area of the shape, to 2 decimal places where necessary.



THINKING

1 **Recall** the formula for the area of the shape.

2 Substitute the known measurements into the formula to calculate A .

3 **Interpret** the answer.

WORKING

The shape is a trapezium.

$$A = \frac{1}{2}(a + b)h$$

$$A = \frac{1}{2}(30 + 40) \times 32$$

$$= \frac{1}{2} \times 70 \times 32$$

$$= 1120 \text{ m}^2$$

The area of the trapezium is 1120 m^2 .

Key information

Key information and rules are highlighted throughout the chapter.

Every worked example and question is graded

Every example and question is graded using three levels of difficulty, as specified in the QCE syllabus:

- simple familiar (1 bar)
- complex familiar (2 bars)
- complex unfamiliar (3 bars)

The visibility of this grading helps ensure all levels of difficulty are well covered.

Meeting the needs of the QCE syllabus

The authors have integrated both the **cognitive verbs** and the language of the **syllabus objectives** throughout the worked examples and questions.

11 An Aussie cap has been discounted by 15% to a sale price of \$21.25.

(a) Which alternative shows a correct calculation for finding the original price?

- A $\$21.25 \times 1.15$ B $\frac{\$21.25}{1.15}$ C $\$21.25 \times 0.85$ D $\frac{\$21.25}{0.85}$

(b) **Explain the common error** in the first of the incorrect alternatives.

(c) Which alternatives give a value that is less than the sale price?

Highlighting common errors

Throughout the exercises authors have integrated questions designed to **highlight common errors** frequently made by students. Explanations are given in the worked solutions.

Worked solutions

Fully worked solutions are provided for every question in the student textbook and can be accessed from the accompanying eBook.

WARNING

A common error when determining the original price is to increase the discounted price by the given percentage.

From Worked example 5: $\frac{49.80}{0.83} \neq 49.80 \times 1.17$.

Warning boxes

Warning boxes are located throughout the chapter to alert students to common errors and misconceptions.

Recall

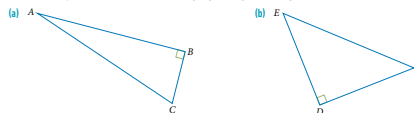
Each chapter begins with a review of assumed knowledge for the chapter.

2

Recall

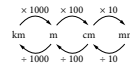
Locate and label the hypotenuse in right-angled triangles

1 Name the hypotenuse in the following right-angled triangles.



Convert metric units of length

2 Convert the following, using the conversion chart as a reference.



- (a) 8560 m to kilometres (b) 3400 cm to metres (c) 0.14 m to millimetres

Round to a given number of decimal places

3 Round to the number of decimal places given in the brackets.

- (a) 30.546 (2) (b) 5.999522 (3) (c) 3.1415926 (2)

Substitute values into a formula

4 Calculate the value of the subject in each formula.

- (a) $A = \frac{1}{2}bh$ given $b = 20$, $h = 7$ (b) $V = \frac{1}{3}\pi r^2 h$ given $r = 3$, $h = 8$
(c) $A = \frac{1}{2}(a+b)h + \frac{1}{2}\pi r^2$ given $a = 5$, $b = 9$, $h = 12$, $r = 3$

Solve equations involving squares

5 Determine the value of the pronumeral, given that it has a positive value. Round your answer to 2 decimal places, where necessary.

- (a) $a^2 = 81$ (b) $x^2 = 47$ (c) $c^2 = 64 + 121$

6 Calculate the unknown variable in the formula.

- (a) $A = lw$ where $A = 45$, $w = 5$ (b) $P = 2(l + w)$ where $P = 30$, $l = 12$

Calculate what percentage one value is of the total

7 A water tank has a capacity of 5000 L, but contains only 1750 L of water. Calculate the amount of water as a percentage of the capacity, rounded to the nearest whole per cent.

Calculate percentage increase or decrease

8 Determine the percentage of change, rounded to the nearest whole per cent.

- (a) With a renovation, the kitchen bench is increased in length from 1700 mm to 2000 mm.
(b) Due to the drought, the surface area of a reservoir decreases from 6200 m² to 5700 m².

Summary

At the end of each chapter is a summary of the key facts and rules used in the chapter.

4

Summary

Linear equations

A linear equation contains an equals symbol (=), and one or more variables that are all raised to the power of 1.

To solve an equation, visualise the steps necessary to create the equation and then reverse the operations to find the solution. The steps used to create an equation must be reversed in the opposite order in which they were applied. These reverse operations are more formally known as inverse operations.

Operation	Inverse operation
+	−
−	+
×	÷
÷	×

Linear graphs

The horizontal x -axis and the vertical y -axis divide the grid into four quadrants. The intersection of the two axes is called the origin.

A point is located by a coordinate pair (x, y) .

The point where the graph crosses the x -axis is called the x -intercept. To find the x -intercept, substitute $y = 0$ into the equation and solve for x .

The point where the graph crosses the y -axis is called the y -intercept. To find the y -intercept, substitute $x = 0$ into the equation and solve for y .

A linear graph is represented by the general form of $y = a + bx$ where a is the y -coordinate of the y -intercept and b is gradient.

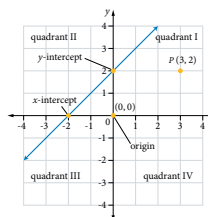
A straight-line graph has a constant slope or gradient. The gradient b is a measure of the steepness of a graph and is found using $b = \frac{y}{x}$. From two points (x_1, y_1) and (x_2, y_2) the formula becomes

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$

Simultaneous equations

The simultaneous solution to two equations with two variables can be found algebraically by the method of substitution. This involves isolating a variable in one equation and substituting into the other.

The intersection of the graphs of the two equations displays the simultaneous solution – that is, the value for each variable that satisfies both equations.



Chapter review 7

- Classify each of the following data types.
 - The size of shirts worn by the pupils in a class
 - The height jumped by each of the competitors in the school pole vault competition
- State the data type for each of the following examples.
 - The favourite colour of each of the students in your class
 - The highest level of education achieved by each of the respondents to a survey
- Consider the data set shown in the table.

x	f
82	7
83	16
84	12
85	1
86	2

- Determine the mode.
 - Calculate the relative frequency of each data value as a percentage, to the nearest per cent.
- Construct a stem-and-leaf plot for each of the following data sets.
 - 25, 38, 65, 32, 77, 21, 79, 81, 66, 50, 47, 53, 25, 30, 42, 60, 70, 29, 51, 63, 68, 82, 40, 33, 22, 45, 37, 65, 74, 70, 35, 61, 81, 77, 65, 66
 - 5.8, 2.1, 7.6, 3.0, 4.1, 2.1, 6.5, 6.4, 3.4, 2.8, 6.3, 3.2, 6.0, 5.9, 3.1, 5.7, 3.8, 4.6, 7.2, 7.1, 4.4, 4.5, 3.4, 7.5, 2.9, 4.6, 3.8
 - Construct a dot plot for the data set shown.
Number of pets: 2, 0, 1, 3, 3, 2, 5, 2, 0, 3, 6, 0, 5, 1, 3, 3, 1, 2, 2, 4, 3
 - Determine the median of the data set.

x	f
16	3
17	5
18	2
19	4
20	2

Chapter 7 Univariate data analysis 479

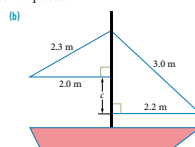
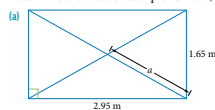
Chapter review

Every chapter review follows the QCAA examination proportions for levels of difficulty, which is 60% simple familiar, 20% complex familiar and 20% complex unfamiliar.

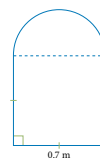
Exam review UNIT 1

- Calculate the hourly rate of pay for Amy, who earns \$287.50 for 25 h of work.
- Logan invested \$24 000 at a simple interest rate of 5.25% p.a. for 3 years.
 - Determine the total interest earned.
 - Calculate the balance of his investment.
- Sara needs to make some quick sales, so she decides to reduce prices on a variety of goods. For each of the following, calculate the price reduction to be applied.
 - 7% off \$450
 - 37.5% off \$880

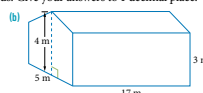
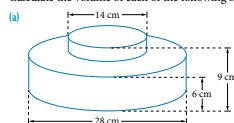
- Determine the value of each pronumeral, to 2 decimal places.



- Determine the perimeter and area of the shape. Give your answers to 2 decimal places.



- Calculate the volume of each of the following solids. Give your answers to 1 decimal place.



Exam review

Exam reviews give cumulative practice at content already covered, to prepare students for the end-of-year exam. They have been placed at the end of each Unit.



1

Consumer arithmetic

• • • • •

Recall	4
1.1 Percentages, decimals and rates	5
1.2 Making money	15
1.3 Taxation and other government charges	22
1.4 Investing money	31
1.5 Shares	42
1.6 Profit, loss and inflation	49
1.7 Making the most of your money	56
1.8 Working with finance	66
Summary	73
Chapter review	75

Recall



Convert a mixed number to an improper fraction

1 Convert the following mixed numbers to improper fractions.

(a) $2\frac{2}{5}$

(b) $1\frac{3}{8}$

Round to a given number of decimal places

2 Round the following to the number of decimal places shown in brackets.

(a) 0.6842 (2)

(b) 0.84559 (3)

(c) 9.897 (2)

Substitute values into a formula

3 Substitute the given values into the following formulas.

(a) Given $a = 2$, $b = 5$ and $c = 4$, determine the value of y if $y = 3a + bc$.

(b) Given $j = 5$, $k = 3$ and $l = 1$, determine the value of a if $a = j^2 - 2(k + l)$.

(c) Given $I = 500$, $P = 5000$ and $n = 2$, determine the value of i if $i = \frac{100I}{Pn}$

Calculate the mean of a group of numbers

4 Calculate the mean of each group of numbers.

(a) 4, 5, 6, 8, 12

(b) 15, 25, 30, 30, 45, 65

(c) 8.4, 0.5, 3.2, 1.7

Convert percentages to decimals

5 Convert the following percentages to decimals.

(a) 15%

(b) 3.5%

(c) 152%

(d) 0.75%

Convert decimals to percentages

6 Convert the following decimals to percentages.

(a) 0.73

(b) 0.026

(c) 1.09

Percentages, decimals and rates

Percentages

You will use percentages throughout your life, such as in calculating:

- a pay increase
- a discount at a shop
- GST added to a purchase
- tax on your earnings.

You will recall that 'per cent' means 'out of 100' and the symbol for percentage is %.



1 Converting to a percentage

Express the following as a percentage, to 1 decimal place where necessary.

(a) $\frac{4}{5}$

THINKING

Multiply the fraction by 100%.

WORKING

$$\frac{4}{5} \times 100\% = 80\%$$

(b) 53 out of 78

1 Express the figures as a fraction.

$$\frac{53}{78}$$

2 Multiply the fraction by 100%. Include 1 more decimal place than is required in the answer.

$$\frac{53}{78} \times 100\% = 67.94\ldots\%$$

3 Round as directed.

$$= 67.9\% \text{ (1 decimal place)}$$

(c) 0.9465

1 Multiply the number by 100%.

$$0.9465 \times 100\% = 94.65\%$$

2 Round as directed.

$$= 94.7\% \text{ (1 d.p.)}$$

i Additional information

Fractions, decimals and percentages

Practise converting between fractions, decimals and percentages.

2 Percentages greater than 100

Express the following as a percentage, to the nearest whole percentage.

(a) $2\frac{3}{8}$

THINKING

1 Separate the parts of the mixed number.

2 Multiply each part by 100%.

3 Round as directed.

WORKING

$$2\frac{3}{8} = 2 + \frac{3}{8}$$

$$\begin{aligned} 2 \times 100\% + \frac{3}{8} \times 100\% \\ = 200\% + 37.5\% \\ = 237.5\% \end{aligned}$$

$$= 238\% \text{ (nearest \%)}$$

(b) 123 out of 65

1 Express the figures as a fraction.

2 Multiply the fraction by 100%. Include 1 more decimal place than is required in the answer.

3 Round as directed.

$$\frac{123}{65}$$

$$\begin{aligned} \frac{123}{65} \times 100\% \\ = 189.2\ldots\% \end{aligned}$$

$$= 189\% \text{ (nearest \%)}$$

(c) 34.067

1 Multiply the number by 100%.

2 Round as directed.

$$\begin{aligned} 34.067 \times 100\% \\ = 3406.7\% \end{aligned}$$

$$= 3407\% \text{ (nearest \%)}$$

3 Calculating a percentage of a quantity

Determine the specified percentage of each quantity.

(a) 6.1% of \$280

THINKING

1 Express the percentage as a decimal.

2 Multiply the decimal by the quantity.

3 Calculate the answer.

WORKING

$$6.1\% \text{ of } \$280$$

$$= 0.061 \times \$280$$

$$= \$17.08$$

(b) 102% of 64 kg

1 Express the percentage as a decimal.	102% of 64 kg
2 Multiply the decimal by the quantity.	$= 1.02 \times 64 \text{ kg}$
3 Calculate the answer.	$= 65.28 \text{ kg}$

(c) 0.3% of 40 km

1 Express the percentage as a decimal.	0.3% of 40 km
2 Multiply the decimal by the quantity.	$= 0.003 \times 40 \text{ km}$
3 Calculate the answer and change to a more appropriate unit, if necessary.	$= 0.12 \text{ km}$ $= 120 \text{ m}$

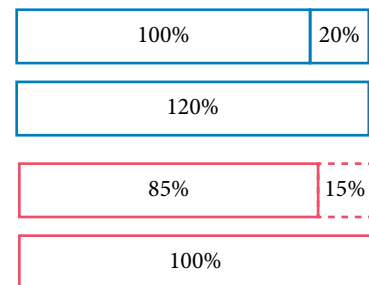
Increasing or decreasing a quantity by a set percentage

There are two basic methods for increasing or decreasing a quantity by a set percentage.

Method 1: Percentage as a scale factor

If you consider the initial number as being 100%, then an increase of 20% will mean you now have 120% of the original number (a decimal ratio of 1.2).

Similarly, if a number is reduced by 15%, then 100% less 15% means you have 85% remaining (a decimal ratio of 0.85).



Method 2: Adding or subtracting a percentage of the original quantity

The second method is to find the stated percentage of the original quantity, then add or subtract this amount to calculate the new increased or reduced quantity.

See how both methods are used in the following worked examples.

4 Percentage increase and decrease

Change each amount by the specified percentage.

(a) Increase 125 by 20%.

METHOD 1

THINKING	WORKING
1 Create a mathematical expression that represents the problem.	$(100 + 20)\%$ of 125 $= 120\%$ of 125
2 Convert the percentage to a decimal by dividing by 100.	$= 1.2 \times 125$
3 Multiply the original quantity by the decimal.	$= 150$

METHOD 2

1 Calculate the stated percentage of the original quantity.	$20\% \text{ of } 125 = \frac{20}{100} \times 125$ $= 0.2 \times 125$ $= 25$
2 Add the amount to the original quantity.	Increased amount: $125 + 25 = 150$

(b) Decrease 420 by 14%.

METHOD 1

1 Write an expression that represents the problem.	$(100 - 14)\% \text{ of } 420$ $= 86\% \text{ of } 420$
2 Convert the percentage to a decimal by dividing by 100.	$= 0.86 \times 420$
3 Multiply the original quantity by the decimal.	$= 361.2$

METHOD 2

1 Calculate the stated percentage of the original quantity.	$14\% \text{ of } 420 = \frac{14}{100} \times 420$ $= 0.14 \times 420$ $= 58.8$
2 Subtract the amount from the original quantity.	Decreased amount: $420 - 58.8 = 361.2$

When given the increased or decreased value, you can use the percentage change to determine the original value.

5 Calculating the original value

After a 17% discount, a shirt is advertised for \$49.80. Calculate the original price before the discount.

THINKING

- 1 Construct an equation showing the discounted price as a percentage of the original price.
- 2 Convert the percentage discount to a scale factor, expressed as a decimal.
- 3 Solve the equation to determine the value of the original price.
- 4 Interpret the result.

WORKING

$$(100 - 17)\% \text{ of original price} = \$49.80$$

$$83\% \times \text{original price} = 49.80$$

$$0.83 \times \text{original price} = 49.80$$

$$\text{Original price} = \frac{49.80}{0.83}$$

$$= \$60$$

The original price of the item (before the discount) was \$60.

WARNING

A common error when determining the original price is to increase the discounted price by the given percentage.

From Worked example 5: $\frac{49.80}{0.83} \neq 49.80 \times 1.17$.

i Additional information**Percentage change**

Watch the video and practise calculating percentage change.

The unitary method is another way of solving problems where backtracking is involved.

	Advertised price (83%)	1%	Original (100%)
Amount	\$49.80	$\$49.80 \div 83 = \$0.60$	$\$0.60 \times 100 = \$60$

Rates

Whether you realise it or not, you deal with the mathematical concept of *rates* every day. Rates are when one quantity is expressed in terms of another. For example:

- call costs expressed as a cost per minute (cents/min)
- data charges expressed as a cost per megabyte (\$/MB)
- hourly rates of pay expressed as dollars per hour (\$/h)
- speed expressed in kilometres per hour (km/h).

i Additional information**Rate, distance and speed**

Watch the videos and practise using rates to solve problems.

6 Calculating a rate

Bernard takes his car on a road trip and drives for 5 h at an average speed of 85 km/h. Over the trip his car's fuel economy is 7 L per 100 km. Fuel costs \$1.45/L.

(a) Determine the distance travelled.

THINKING

- 1 Interpret the rate.
- 2 Multiply by the number of hours to give the distance in kilometres.

WORKING

85 km/h means 85 km travelled each 1 h
 Distance travelled in 5 h:
 $85 \text{ km} \times 5 = 425 \text{ km}$

(b) Calculate the amount of fuel used over the trip. Use the distance travelled calculated in the previous part of the question.

- 1 Interpret the rate.

7 L per 100 km means:
 7 L of petrol used for every 100 km travelled

- 2 Divide the distance by 100 km then multiply the number of lots of 100 km by 7 L.

$$\frac{425 \text{ km}}{100 \text{ km}} \times 7 \text{ L} = 29.75 \text{ L}$$

(c) Calculate the cost of fuel to the nearest dollar.

Multiply the number of litres used by the cost per litre.

$$\begin{aligned} \$1.45 \times 29.75 &= \$43.1375 \\ &= \$43 \text{ (nearest \$)} \end{aligned}$$

EXERCISE

1.1

Percentages, decimals and rates

Worked
Example

1

1 Convert the following to percentages. Round your answer to 2 decimal places where necessary.

(a) $\frac{2}{5}$

(b) $\frac{4}{9}$

(c) $\frac{3}{8}$

(d) $\frac{7}{20}$

(e) 45 out of 50

(f) 68 out of 90

(g) 34 out of 55

(h) 3 out of 500

(i) 0.28

(j) 0.876

(k) 0.66666

(l) 0.00123

2

2 Convert the following to percentages. Round your answer to the nearest whole number where necessary.

(a) $\frac{12}{5}$

(b) $1\frac{2}{9}$

(c) $\frac{709}{20}$

(d) 3.43

(e) 24.567

(f) 7.8009

(g) 145 compared to 30

(h) 56 out of 25

(i) 75 out of 40

3

3 Determine the specified percentage of each quantity. Give your answer in the most appropriate units.

(a) 18% of 420 g

(b) 68% of \$45

(c) 2.4% of 30 m

(d) 115% of \$35

(e) 105.6% of 400 m

(f) 205% of 44 kg

(g) 0.5% of \$3400

(h) 0.01% of 20 s

(i) 0.11% of \$500

4

4 Make the percentage change shown for each of the following. Round your answer to 2 decimal places where necessary.

(a) Increase 300 by 12%.

(b) Decrease 25 by 20%.

(c) Increase 42.5 by 15%.

(d) Decrease 83.5 by 16%.

(e) Increase 0.08 by 50%.

(f) Decrease 0.05 by 20%.

(g) Increase 23.6 by 4.5%.

(h) Decrease 56.75 by 3.8%.

6

5 Trinh takes her car on a road trip and drives for 6 h at an average speed of 90 km/h. Over the trip the car's fuel economy is 7.2 L per 100 km.

(a) Determine the distance travelled.

(b) Calculate the amount of fuel used, to the nearest litre.

6 Calculate the hourly rate of pay for the following workers.

(a) Amy earns \$287.50 for 25 h of work.

(b) Ben earns \$740 for his 40 h week.

(c) Cathy is paid monthly and earns \$6400 for 170 h of work.

(d) Danesh has a yearly income of \$72 000 and works a 38 h week, assuming the pay is across all 52 weeks of the year.

7 A small dam on Brian's farm has a capacity of 2 million litres of water. Flooding rain comes and the dam is overflowing. The next morning Brian notices the rain has damaged the retaining wall of the dam at the bottom, and water is slowly leaking out. In 24 h the dam has lost 1000 L of water.

(a) Determine the rate at which the water is leaking, to 1 decimal place.

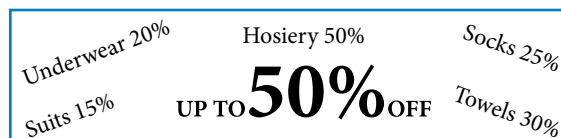
(b) From the moment when the dam began to leak, how long will it take for the dam to be completely empty?

- 8 Sara needs to make some quick sales, so she decides to reduce prices on a variety of goods. For each of the following, calculate the price reduction to be applied.

(a) 10% off \$23.40 (b) 15% off \$650 (c) 12.5% off \$64

- 9 Gold has a current value of \$825 per ounce. Determine the value of the price reduction if 5 ounces of gold is reduced by 17%.

- 10 Based on the information in this advertisement, what is the discounted price of a \$450 suit?



- 11 An Aussie cap has been discounted by 15% to a sale price of \$21.25.

(a) Which alternative shows a correct calculation for finding the original price?

A $\$21.25 \times 1.15$ B $\frac{\$21.25}{1.15}$ C $\$21.25 \times 0.85$ D $\frac{\$21.25}{0.85}$

(b) Explain the common error in the first of the incorrect alternatives.

(c) Which alternatives give a value that is less than the sale price?

Worked
Example

- 12 Maria takes over a discount store and mistakenly throws out the original price tags, so the customer can only see what the percentage discount is. For each of the following items, determine the original price before the discount was given.

(a) discounted price \$42.50, 15% discount	(b) discounted price \$52, 20% discount
(c) discounted price \$24.23, 8% discount	(d) discounted price \$76, 5% discount
(e) discounted price \$41.40, 8% discount	(f) discounted price \$67.68, 6% discount
(g) discounted price \$3.15, 10% discount	(h) discounted price \$8.64, 20% discount
(i) discounted price \$131.37, 14% discount	

- 13 A DVD is advertised as 15% off the original price and you are charged \$29.75.

(a) Calculate the original price of the DVD.

A \$44.75 B \$25.30 C \$35 D \$34.20

(b) Explain the common error made by a student who selected the last of the incorrect options.

- 14 Amena takes her American friends Cathy and Richard on a road trip to Central Australia. They travel a distance of 6400 km and use 450 L of fuel at a cost of \$810.

- (a) Calculate the rate of fuel consumption in litres per kilometre, to 2 decimal places.
- (b) It is more usual to state the fuel consumption in L per 100 km to compare fuel efficiency. Calculate the rate of fuel consumption in L per 100 km, to 1 decimal place.
- (c) Determine the average cost of fuel in dollars per litre.

5

- 15 Mosi and Liam are comparing their results during the year. This table shows their results for three mathematics tests.

	Mosi	Liam
Test 1	15 out of 20	17 out of 20
Test 2	43 out of 50	32 out of 50
Test 3	24 out of 30	28 out of 30

Answer the following questions, expressing all answers to 1 decimal place.

- Convert the result for each test to a percentage to complete the table at bottom right.
- Calculate Mosi's mean percentage.
- Calculate Liam's mean percentage.
- Based on the mean percentages, which student has better results overall?
- The final mark is calculated by adding the individual test scores to obtain a mark out of 100. Calculate Mosi and Liam's final mark to determine who has the better result.
- Explain the discrepancy between the two comparisons of overall results for Mosi and Liam.

	Percentages	
	Mosi	Liam
Test 1		
Test 2		
Test 3		

- 16 Tim's boss offers him a choice: either a lump sum bonus of 15% of his current salary of \$45 000, or a \$100 per week pay increase for the whole of the year. Ignoring tax implications, determine which choice is better for Tim and by how much.

- 17 Answer the following questions based on the values shown in the table. Round your answers to the nearest whole percentage.

State or territory	Population (1000s)
Australian Capital Territory	397.4
New South Wales	7 480.2
Northern Territory	228.8
Queensland	4 703.2
South Australia	1 676.7
Tasmania	510.0
Victoria	5 926.6
Western Australia	2 474.4
Total	23 397.3

- What percentage of the entire Australian population live in the Australian Capital Territory?
- What percentage of the Australian population live in Queensland, New South Wales or Victoria?
- What percentage of the Australian population live outside Queensland, New South Wales and Victoria?

- 18 FIT Fun and Fitness is having a grand opening sale.

Alina and Jonas are going to trial the centre and want to take advantage of the special introductory offers on a set of 5 or 10 discounted sessions. (Discounts are given on the price of casual passes.)

Classes with 10 discounted sessions:

Class	Original price (per session)	Discount
Personal training	\$42	5%
Spin	\$18	20%
Boxing	\$35	10%

Classes with 5 discounted sessions:

Class	Original price (per session)	Discount
Gym and swim	\$15	30%
Hiit Fit	\$20	15%
Intervals and circuits	\$16	25%

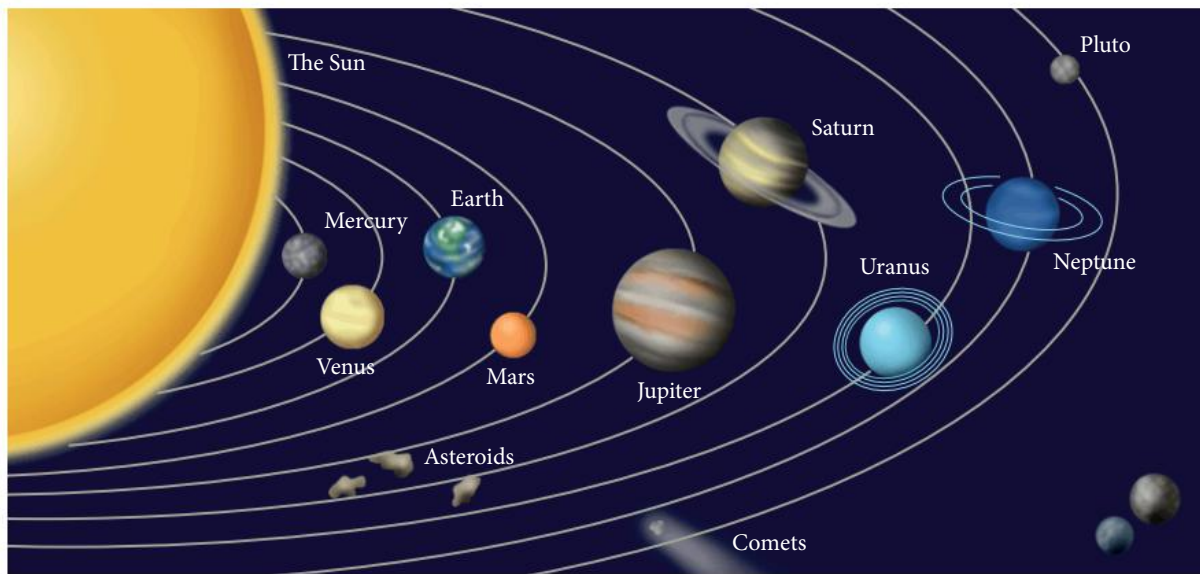
- (a) Alina would like to try a class that runs for only 5 sessions. Compare the savings to determine which class she should try, in order to get the most value from the introductory discount.
- (b) Alina enjoys the class and decides to sign up for a yearly pass for \$1040. If on average she attends 5 sessions per week, calculate the cost per session for her yearly membership.
- (c) Jonas wants to sign up for two different 10 discounted sessions to kick start his fitness program. Which sessions should he purchase to make the most of the discounts offered?
- (d) After his trial, Jonas decides to purchase an annual membership to work with a personal trainer. The yearly subscription costs \$1820. Jonas attends on average 5 sessions per week over the course of the year. Calculate the total cost per session for the year.

- 19 The table gives the area of Australian states and territories with their estimated populations, taken from the 2016 Census data. Use a spreadsheet or calculator to complete the following tasks. Give your answers in a table.

Region	Area (km ²)	Population (1000s)
Australian Capital Territory	2 280	397.4
New South Wales	800 642	7 480.2
Northern Territory	1 349 129	228.8
Queensland	1 730 648	4 703.2
South Australia	983 482	1 676.7
Tasmania	68 401	510.0
Victoria	227 146	5 926.6
Western Australia	2 529 875	2 474.4
Total	7 691 603	23 397.3

- (a) (i) Calculate the percentage area of each state and territory relative to the whole of Australia. Calculate answers to the nearest whole percentage.
- (ii) Determine the largest and smallest of the mainland states.
- (b) (i) Calculate the population density (the rate of persons per km²) for each state and territory and for the whole of Australia. Calculate answers to 1 decimal place.
- (ii) Which state or territory has a population density closest to the average?

- 20 Use the information in the table of planets to answer the questions, to the nearest whole percentage.



Planet	Diameter (km)	Distance from the Sun (millions of km)	Length of year (Earth days or years)
Mercury	4 880	57.9	59 days
Venus	12 100	108.2	224.7 days
Earth	12 756	149.6	365.3 days
Mars	6 794	227.4	687 days
Jupiter	143 200	778.3	11.9 years
Saturn	120 000	1 427	295 years
Uranus	51 800	2 870	84 years
Neptune	49 500	4 497	165 years

- What percentage of the diameter of Mars is Earth's diameter?
- What percentage of the diameter of the largest planet is the smallest planet's diameter?
- Earth's distance from the Sun is what percentage of the distance from the Sun to Neptune?
- A year on Earth is what percentage of a year on Mercury?
- The Sun has a diameter of 1 391 000 km. What percentage is this of the largest planet's diameter?
- The Sun's diameter is what percentage of the distance from Neptune to the Sun? Give your answer to 2 decimal places.
- Do you think the diagram of the solar system shown is drawn to scale? Explain.

Wages and salaries

Incomes are earned in different ways. Workers most often earn wages or salaries. A salary is usually an agreed annual amount, whereas a wage may depend on the hours worked and may include extra income for overtime or work outside normal hours.



7 Calculate a weekly wage

Laura works as a music teacher and is paid a standard \$55 an hour up until 6 pm and then overtime at time-and-a-half. ('Time-and-a-half' means that she is paid 1.5 times her hourly rate for every additional hour after 6 pm.)

(a) Calculate Laura's weekly wage if she works 30 standard hours.

THINKING

Multiply the number of hours by the hourly rate.

WORKING

Laura's weekly wage:
 $55 \times 30 = \$1650$

(b) Calculate Laura's weekly wage if she works 28 h at the standard rate and 4 h at the penalty rate.

1 Calculate the wage earned from the standard hours.

Amount for standard hours:
 $28 \times 55 = \$1540$

2 Calculate the hourly rate at time-and-a-half.

Rate for time-and-a-half:
 $1.5 \times 55 = \$82.50$ per hour

3 Calculate the wage earned at time-and-a-half.

Amount at time-and-a-half:
 $4 \times 82.50 = \$330$

4 Calculate the total wage.

$1540 + 330 = \$1870$

5 Interpret the answer.

Laura's total wage is \$1870

Commission and piecework

Some employees get paid for each piece of work completed and others are paid on commission, i.e. they receive a percentage of the value of the work they produce once it is sold.



8 Other ways of calculating a wage

Olivia, Lachie and Cam all get summer jobs while they are waiting for Year 12 to start. Olivia picks fruit and is paid \$40 for every 20 kg of fruit she picks, Lachie produces a short puppet play for the local shopping centre and is paid \$50 per performance, while Cam sells ukuleles door to door and is paid 15% commission of his total sales.

- (a) Calculate Olivia's weekly wage if she picks 340 kg of fruit for the week.

THINKING

- 1 Determine the number of units she is paid for, by dividing the weight of fruit picked by the unit weight.
- 2 Multiply the number of units by the price per unit.
- 3 Interpret the answer.

WORKING

$$\begin{aligned}\text{Number of units} &= 340 \div 20 \\ &= 17\end{aligned}$$

$$\begin{aligned}\text{Wage} &= 17 \times 40 \\ &= \$680\end{aligned}$$

Olivia's weekly wage is \$680.

- (b) Calculate Lachie's wage if the play is performed three times each weekday and once each day over the weekend.

- 1 Calculate the total number of performances.
- 2 Multiply the number of performances by the fee per performance.
- 3 Interpret the answer.

$$\begin{aligned}\text{Number of performances} &= 5 \times 3 + 2 \\ &= 17\end{aligned}$$

$$\begin{aligned}\text{Wage} &= 17 \times 50 \\ &= \$850\end{aligned}$$

Lachie's wage is \$850.

- (c) Calculate Cam's earnings if the total revenue from his ukulele sales is \$3700.

- 1 Calculate the commissioned percentage of the total sales.
- 2 Interpret the answer.

$$\begin{aligned}\text{Wage} &= 15\% \text{ of sales} \\ &= 0.15 \times 3700 \\ &= \$555\end{aligned}$$

Cam's earnings amount to \$555.

9 Commission with a retainer

Jasmin sells cars. She earns a weekly retainer of \$400 plus a commission of 5% of her sales for the week. Calculate her earnings if she sells only one car in the week, valued at \$32 000.

THINKING

1 Determine the commission earned.

2 Add the commission (the percentage of sales) to the weekly retainer.

3 Interpret the answer.

WORKING

$$5\% \text{ of } 32\,000 = 0.05 \times \$32\,000 \\ = \$1600$$

$$400 + \$1600 = \$2000$$

Jasmin earns \$2000 for the week.

10 Graduated commission

Elmar works in real estate and is paid monthly. He is paid a commission of 2% on the first \$50 000 of the value of each house sold, 1% on the next \$200 000 and 0.5% on the remainder of each house's value. Calculate Elmar's commission on a property he sold for \$580 000.

THINKING

1 Break down the total into the three graduated amounts.

2 Calculate the commissions for each percentage bracket.

3 Interpret the answer.

WORKING

$$\$580\,000 = \$50\,000 + \$200\,000 + \$330\,000$$

$$\begin{aligned} \text{Commission} &= 2\% \text{ of } 50\,000 + 1\% \text{ of } 200\,000 + 0.5\% \text{ of } 330\,000 \\ &= 0.02 \times 50\,000 + 0.01 \times 200\,000 + 0.005 \times 330\,000 \\ &= 1000 + 2000 + 1650 \\ &= \$4650 \end{aligned}$$

Elmar's commission is \$4650.

Explore further

Graduated commission

Complete the spreadsheet to determine the total amount of commission earned. Add your own formulas to compare commission rates.

Government allowances and pensions

However money is earned, the total amount for a given time period is called the *gross* amount. From this gross amount, tax is paid. The amount remaining after tax is the *net* amount. Taxation and net income are explored in the next section.

The Australian government allocates money from taxes to pay allowances and pensions to people who qualify for short-term or long-term assistance. Such benefits include youth allowances, unemployment benefits, disability pensions and study allowances.

11 Government allowances

Rauna is a 17-year-old single apprentice, living away from home. Use the table to answer the following questions, assuming that Rauna is eligible for the government allowance of \$455.20.

Students and Australian apprentices	If you earn \$437 – \$524, your fortnightly payment reduces by ...	If you earn more than \$524, your fortnightly payment reduces by ...	Your payment reduces to \$0 once your income reaches the maximum of ...
Single, under 18 years, at home	50 cents for each dollar you earn over \$437	\$43.50 plus 60 cents for each dollar you earn over \$524	\$857.17

- (a) Calculate the fortnightly income that Rauna can claim from the government if his employer pays him \$250 per week.

THINKING	WORKING
1 Convert salary from the employer to a fortnightly amount.	Salary per fortnight = $\$250 \times 2$ = \$500
2 Find the column on the table that includes the fortnightly amount.	If you earn \$437 – \$524, your fortnightly payment reduces by 50 cents for each dollar you earn over \$437
3 Calculate the reduction.	Reduction = $(500 - 437) \times 50 \text{ cents}$ = $63 \times \$0.50$ = \$31.50
4 Subtract the reduction from the maximum income.	Government allowance = $\$455.20 - \$31.50$ = \$423.70
5 Interpret the answer.	Rauna can claim \$423.70 from the government each fortnight.

- (b) What would be Rauna’s annual gross earnings if this situation continued for a full year?

1 Convert salary from the employer and allowance from the government to annual amounts and calculate the total.	Annual gross earnings = $\$423.70 \times 26 + \$250 \times 52$ = $\$11\,016.20 + \$13\,000$ = \$24 016.20
2 Interpret the answer.	Rauna’s annual gross earnings amount is \$24 016.20.

EXERCISE

1.2

Making money

Worked
Example

- 1 Anora works in a music store and earns an hourly rate of \$14. If she works on weekends or after 6 pm during the week, she is paid a penalty rate of time-and-a-half.
- Calculate her weekly earnings for a standard 35 h week.
 - Calculate her weekly earnings for a standard 35 h week plus 8 h at time-and-a-half.
 - Another employee leaves, so the shifts are rearranged. Anora now works from 3 pm to 8 pm on Monday, Tuesday and Wednesday, and from 9 am to 2 pm on Friday and Saturday. Calculate her wage for this week (assuming no unpaid breaks).
 - Do Anora's new shifts earn her more or less than if she works a standard 35 hour week, and by how much per week?

7

- 2 Ben is offered three new holiday jobs. He can pack school books and be paid \$6 per box, or he can collect trolleys at the local shopping centre for 50c per trolley, or he can sell ice-cream and earn 25% of total sales.
- Calculate his weekly pay if he packs an average of 94 boxes per week.
 - Calculate his weekly pay if he collects an average of 1250 trolleys per week.
 - Calculate his weekly pay if the average ice-cream sales are \$2460 per week.
 - Which job will earn the most money if the average numbers are achieved?

8

- 3 Quon is paid a base amount of \$500 per week plus a commission of 6% on all sales he makes for the week. In one week, he sells \$300 worth of goods. What is his wage for the week?

9

- 4 Lindy works as a sales representative and is paid a weekly amount of \$600 plus a commission of 12% of the sales she makes. Over 2 weeks, her sales total \$31 450. Calculate her gross wage for the fortnight.

- 5 Christy has a summer job picking cherries. She is paid a daily rate of \$40 plus an additional \$0.60 for each punnet she fills on the day. The table shows the number of punnets she fills each day over a 2 week period. Tuesday of Week 1 was Christmas day, and the picking restarted on Boxing day with double pay for the day, including the rate per punnet.

	Week 1 (punnets)	Week 2 (punnets)
Monday	360	300
Tuesday	0	240
Wednesday	340	320
Thursday	320	280
Friday	350	290

- How many days did Christy work over the 2 weeks?
 - Calculate her wage in week 1.
 - Calculate her wage in week 2.
 - Calculate Christy's average daily wage, to the nearest dollar.
- 6 Andrea is offered two salary packages. She could take a set wage of \$1250 a week, or a base salary of \$500 a week plus commission of 4% on her weekly sales. If her average sales are \$27 000 per week, which package is better, and by how much?

- 7 Kate works as a laboratory assistant and is paid an annual salary of \$65 000 plus any overtime she works. She is paid \$40 per hour for overtime. If Kate works an extra 4 hours overtime each fortnight, which option is closest to her fortnightly wage? Assume there are 26 fortnights in a year.

A \$1250 B \$2500 C \$2660 D \$1410

- 8 A cafe employs Djaali, Bridie and Namish at a standard rate of \$18 per hour. The workers have an unpaid break of half an hour after every 4 hour period worked. Time-and-a-half applies after 6 pm on weekdays and all day Saturday. The employer pays double-time on Sundays, even though this is no longer compulsory. The cafe is closed on Mondays.

Answer the questions below, based on the times worked in a particular week.

	Tue	Wed	Thu	Fri	Sat	Sun
Djaali	8 am–4 pm	8 am–4 pm	8 am–4 pm	8 am–4 pm		
Bridie		1 pm–9 pm	1 pm–9 pm	1 pm–9 pm	1 pm–9 pm	1 pm–9 pm
Namish				10 am–2 pm	10 am–2 pm	

- (a) Determine the weekly wage for each employee.
- (b) If Djaali were to swap shifts with Bridie on Wednesday to Friday, how much extra will he earn in the week?
- (c) Namish actually worked until 9 pm on Friday but did not note the change on his time sheet. How much back pay will he receive when the issue is sorted out?

Worked
Example

10

- 9 Tuan sells high-performance motor vehicles on commission. Each week she earns 5% on the first \$40 000 of her sales for the week and 2% on her sales above that. In one week, she sells a sports car for \$117 000. Calculate her commission for the week.
- 10 Sally is paid a base of \$500 a week but earns an additional commission of 12% on all sales she makes for the week. Sally sets herself a goal of earning at least \$1000 per week. Calculate the minimum sales (in whole dollars) she needs to achieve this goal.
- 11 A sales assistant earned \$270 on the sale of a TV, which was 5% of the selling price.
- (a) Which alternative shows a correct calculation for finding the selling price of the TV?
- A 270×0.05 B $\frac{270}{0.05}$ C 270×95 D 270×19.5
- (b) Which of the incorrect alternatives gives a reasonable answer? Demonstrate, by a check step, that this alternative is not correct.
- 12 A salesperson receives a commission of 6% on his weekly sales. His commission for the week is \$1740. Calculate his total sales for the week, to the nearest dollar.
- 13 A commission of \$15 500 is paid on total sales of \$199 166. If 5% is paid on the first \$120 000, calculate the rate paid on the remainder.

- 14 In a certain year, the Newstart Allowance for an unemployed single person over 22 with no children and very few assets is \$492.60 per fortnight. An unemployed person can earn up to \$62 a fortnight without affecting this payment. After that, the allowance is reduced according to the table below.

Extra fortnightly income	Allowance reduction
\$0–62	No reduction
\$62–250	50 c for every \$1 over \$62
Greater than \$250	\$94 plus 60 c for every \$1 over \$250

- (a) Trevor is a single man with no children who was on the Newstart Allowance for 12 weeks while seeking full-time employment. If he did no other work, calculate to the nearest cent his income over the 12 weeks.

- (b) Ellie is a single woman with no children who was on the Newstart Allowance for 8 weeks. During that time she found some part-time work and earned the amounts shown in the table.

Weeks	Earnings (part-time work)
1–4	\$0
5 and 6	\$150
7 and 8	\$350

- (i) Calculate her Newstart payment for weeks 1 to 4.
 (ii) Calculate her Newstart payment for weeks 5 and 6.
 (iii) Calculate her Newstart payment for weeks 7 and 8.
 (iv) Calculate Ellie's total income for weeks 7 and 8.

- 15 In a certain year, seniors on the Age Pension are entitled to \$712 per fortnight for a single person or \$1317.40 per fortnight for a couple if they have no other income. All pensioners can also earn up to \$250 a week, to a maximum of \$6500 a year, without affecting the pension they receive.

- (a) Calculate the total annual income of a single pensioner who relies solely on the pension.
 (b) Calculate the annual income for a couple who both earn the maximum amount of extra income and maintain their full pension.

- 16 Garth produces sporting memorabilia in his workshop. His framed photographs sell for \$230 each. He pays \$480 each week in fixed expenses (rent and utilities) for the workshop, and each framed picture costs him \$110 in materials.

- (a) How many photos must Garth sell in a week to make at least \$1200 profit?
 (b) Each photograph takes Garth 1 h 15 min to produce. Calculate how much time it will take to produce the photographs required for the \$1200 profit in the previous question.
 (c) Calculate Garth's hourly rate of pay to the nearest 10c.

- 17 Alan and Nancy live on the Age Pension, which is \$1317.40 per fortnight for a couple with no other income. Their pension is reduced by 50c for every \$1 more than \$268 that they earn. Alan earns 5% royalties on sales of his country and western album.
 Calculate their pension in a fortnight when Alan sells \$600 worth of albums and Nancy receives \$300 for her part-time work.

- 18 Keaton sells digital devices in a pop-up store. He has two options to earn a weekly wage: 12% of total sales or 7% of total sales plus a retainer of \$200 per week. What average amount of sales per week is needed for 'commission only' to be the better option?

1.3

Taxation and other government charges

All kinds of tax

Local, state and federal governments need money to provide services and infrastructure for the community such as roads, libraries and schools. The government collects money in the form of taxes, fines, stamp duty, and excises on alcohol, cigarettes and fuel. The system is quite complex, so in this section you will just look at the taxes most relevant to young people: the Goods and Services Tax (GST) and PAYG ('pay as you go') income tax. You will also look at the HELP scheme of repayments for tertiary education and training.

The GST

The GST is a 10% charge added to the cost of most goods and services. The provider of the goods or services forwards the GST amount to the Australian Taxation Office (ATO).

The GST-inclusive price adds 10% to the pre-tax price. The formulas shown at right are useful when solving problems.

$$\text{GST} = 10\% \text{ of pre-GST price}$$

$$\begin{aligned}\text{GST-inclusive price} &= (100 + 10)\% \text{ of pre-tax price} \\ &= 110\% \text{ of pre-tax price} \\ &= 1.1 \times \text{pre-tax price}\end{aligned}$$

$$\text{Pre-GST price} = \frac{\text{GST-inclusive price}}{1.1}$$

12 Adding GST

Calculate:

- (a) the GST added to the cost of a massage with a pre-tax price of \$35.

THINKING

Calculate 10% of the pre-tax price.
GST = 10% of pre-GST price.

WORKING

$$\begin{aligned}\text{GST amount} &= 10\% \text{ of } \$35 \\ &= 0.1 \times 35 \\ &= \$3.50\end{aligned}$$

- (b) the GST-inclusive price of a bicycle with a pre-tax price of \$345.

The GST-inclusive price is 110% of the pre-GST price.

$$\begin{aligned}\text{GST-inclusive price} &= 110\% \text{ of pre-tax price} \\ &= 1.1 \times 345 \\ &= \$379.50\end{aligned}$$

- (c) the pre-GST price of a car advertised at \$10670 including GST.

Divide the GST-inclusive price by 110% to calculate the pre-GST price.

$$\begin{aligned}\text{Pre-GST price} &= \frac{\text{GST-inclusive price}}{1.1} \\ &= \frac{10670}{1.1} \\ &= \$9700\end{aligned}$$

Additional information

GST

Calculate the pre-GST price of several items.

Pay as you go (PAYG) income tax

Employees can earn up to \$18 200 a year before they begin paying tax. If your wage indicates that you are likely to earn more than \$18 200 in a year, your employer withholds some of your salary as tax and sends it straight to the ATO.

Tax rates

The rate of income tax increases as you earn more money, so lower-income employees pay a smaller proportion of their salary in tax than people earning a higher income. This table shows how the rate increases for each income bracket. (An 'income bracket' is a range of incomes that have been grouped together.)

Tax brackets and tax rates for the financial year 1 July 2017 to 30 June 2018

Taxable income	Tax on this income
\$0–\$18 200	Nil
\$18 201–\$37 000	19c for each \$1 over \$18 200
\$37 001–\$87 000	\$3572 plus 32.5c for each \$1 over \$37 000
\$87 001–\$180 000	\$19 822 plus 37c for each \$1 over \$87 000
\$180 001 and over	\$54 232 plus 45c for each \$1 over \$180 000

To keep things simple, the examples here don't include complications such as the Medicare levy or tax rebates. The following examples use the tax table shown here. If you want to work out your own approximate tax situation, you will need to find the tax table applicable to the year the tax was collected.

13 Calculating tax

Use the *Tax brackets and tax rates* table to calculate the annual tax, if any, that should be paid on each of these incomes.

- (a) an annual taxable income of \$77 000

THINKING

- 1 Identify the correct tax bracket.
- 2 Recall the formula shown in the table.
Convert cents to dollars, e.g. 37c = \$0.37
- 3 Interpret your answer.

WORKING

\$77 000 is in the bracket \$37 001–\$87 000.
 The tax is \$3572 + 32.5c for each \$1 over \$37 000.
 $77\,000 - 37\,000 = \$40\,000$
 $40\,000 \times 0.325 = \$13\,000$
 Total tax = $3572 + 13\,000$
 $= \$16\,572$
 The tax payable for an annual taxable income of \$77 000 is \$16 572.

(b) a weekly income of \$200.

1 Use the weekly wage to calculate the expected yearly salary.	Yearly salary = 200×52 = \$10 400
2 Identify the correct tax bracket.	\$10 400 is in the bracket \$0–\$18 200.
3 Interpret your answer.	There is no tax payable.

(c) a weekly income of \$540.

1 Use the weekly wage to calculate the expected yearly salary.	Yearly salary = 540×52 = \$28 080
2 Identify the correct tax bracket.	\$28 080 is in the bracket \$18 201–\$37 000.
3 Recall the formula shown in the table.	The tax is 19c for each \$1 over \$18 200.
4 Convert cents to dollars and calculate the tax payable.	$(28\,080 - 18\,200) \times 0.19 = \$1877.20$
5 Interpret your answer.	The tax payable for a weekly income of \$540 is \$1877.20 per year.

Tax calculators are available online to help employers work out the amount of PAYG tax to remove from workers' wages before giving them their *take-home* pay.

Explore further

PAYG

Complete the tax table in order to create your own tax calculator.

Tax returns

At the end of the financial year, taxpayers complete a *tax return* to allow an accurate tax assessment. If the assessment shows that too much tax has been paid already, then the ATO will give a refund. If not enough tax was paid, then the outstanding amount must be paid to the ATO.

Deductions and taxable income

The ATO recognises that some of the money people earn is spent on work-related expenses and should not be taxed. When completing a tax return, these legitimate work-related expenses are deducted from the *gross income* to calculate the *taxable income*, and the final tax is then calculated based on this (reduced) taxable income. Donations to recognised charities may also be deducted. In this way, the tax system rewards those who spend money on appropriate work expenses and those who donate to charities.

The classification of which expenses are tax-deductible is complex, and it varies depending on the type of work being done. For simplicity, consider all the claims made in the following examples and questions to be legitimate deductions unless otherwise stated.

14 Calculating the taxable income

Anna works part-time as a builder and had a gross income of \$47 000 for the year. During the year she spent \$250 on tools and \$50 on safety gear, and she sponsored a child through a registered charity at a cost of \$60 per month.

Calculate Anna's taxable income.

THINKING	WORKING
1 Calculate the total work-related expenses (that are tax-deductible).	Total work-related expenses = $250 + 50$ = \$300
2 Determine the annual total of charity donations.	Total charity donations = 60×12 = \$720
3 Add the deductions for expenses and for charity donations.	Total deductions = $300 + 720$ = \$1020
4 Taxable income = gross income – deductions	Taxable income = $47\,000 - 1020$ = \$45 980

Tax refunds

For example, Ash normally earns \$100 a week working Saturdays in a pet store. This is \$5200 a year and below the threshold of \$18 200, so no tax is withheld by the taxation office.

If Ash works extra shifts and earns \$500 in one week, then she pays tax. At \$500 per week her annual salary would be \$26 000, above the \$18 200 threshold.

At the end of the financial year, when Ash submits her annual tax return, the ATO will return any excess tax paid.

15 Calculating a tax refund or debt

Mayra works as a research biologist and had a gross income of \$75 400 for the year. During the year she spent \$250 on scientific journals, \$80 on work-related phone calls, \$45 on work-related internet data charges, \$45 on the purchase of a lab coat and \$2 each of 10 times the coat was cleaned. She also donated \$200 to a fundraiser for a registered charity.

(a) Calculate Mayra's taxable income.

THINKING	WORKING
1 Calculate the total of work-related expenses and gifts to charities.	Total deductions = $250 + 80 + 45 + 45 + 2 \times 10 + 200$ = \$640
2 Taxable income = gross income – deductions	Taxable income = $75\,400 - 640$ = \$74 760

(b) Calculate her tax to be paid for the year.

1	Refer to the <i>Tax brackets and tax rates</i> table to identify the correct tax bracket.	\$74 760 is in the tax bracket: \$37 001 – 87 000
2	Apply the formula shown in the <i>Tax brackets and tax rates</i> table.	The tax is \$3572 + 32.5 c for each \$1 over \$37 000. $74\,760 - 37\,000 = \$37\,760$ $37\,760 \times \$0.325 = \$12\,272$ Total tax = 3572 + 12 272 = \$15 844

(c) If Mayra's tax summary shows that she has already paid \$16 430, calculate her tax refund or debt.

Subtract the tax to be paid from the tax already paid, as shown on the group certificate, and interpret your answer.	$16\,430 - 15\,844 = \$586$ Mayra paid \$586 too much tax and will get this refunded to her.

The HELP scheme

To allow more people access to tertiary studies, a loans scheme called HELP (Higher Education Loan Program) was introduced to pay for courses and allow students to repay the debt later, when they are working. Depending on the length of the course, the cost of delivering the subjects and the total number of units studied, a graduate may leave university with a HELP debt between \$10 000 and \$50 000. This debt will be automatically paid back via the tax system, although extra repayments can be made at any time.

The table shows that the rate of repayment is zero until an annual income of \$55 874 is reached, and then the rate gradually increases to a maximum of 8.0% for an income over \$103 766.

HELP repayment income thresholds for 2017–2018

Repayment income	Repayment rate (percentage of repayment income)
Below \$55 874	Nil
\$55 874–\$62 238	4.0%
\$62 239–\$68 602	4.5%
\$68 603–\$72 207	5.0%
\$72 208–\$77 618	5.5%
\$77 619–\$84 062	6.0%
\$84 063–\$88 486	6.5%
\$88 487–\$97 377	7.0%
\$97 378–\$103 765	7.5%
\$103 766 and above	8.0%

Source: © Australian Taxation Office for the Commonwealth of Australia

HELP repayment = repayment rate % of taxable income

16 Calculating HELP repayments

Penny studied for 5 years to become a primary school music and drama teacher. In that time she accumulated a HELP debt of \$36 000. In her first full financial year of work, her taxable income was \$57 560.

- (a) Calculate the compulsory HELP repayment for that year.

THINKING

- 1 Use the *HELP repayments* table to identify which repayment income bracket applies.
- 2 Identify the associated percentage repayment.
- 3 Apply the rate to the repayment income to calculate the repayment amount.

WORKING

\$57 560 is in the following income bracket:
\$55 874–\$62 238

The percentage repayment is 4.0% of the taxable income.

$$\begin{aligned}\text{HELP repayment} &= 4\% \text{ of } 57\,560 \\ &= 0.04 \times 57\,560 \\ &= \$2302.40\end{aligned}$$

- (b) If the HELP repayments were spread over the whole year, how much would be deducted
- (i) weekly?

Divide the annual total to be repaid by the number of weeks in a year. Round to the nearest cent.

$$\begin{aligned}\text{Weekly repayment} &= \frac{\text{annual amount}}{52} \\ &= \frac{\$2302.40}{52} \\ &= \$44.28\end{aligned}$$

- (ii) monthly?

Divide the annual total to be repaid by the number of months in a year. Round to the nearest cent.

$$\begin{aligned}\text{Monthly repayment} &= \frac{\text{annual amount}}{12} \\ &= \frac{\$2302.40}{12} \\ &= \$191.87\end{aligned}$$

WARNING

A common error made when calculating a monthly amount is to multiply the weekly amount by 4. To calculate the monthly amount correctly, multiply the weekly amount by 52 to determine the annual amount and then divide the annual amount by 12.

EXERCISE

1.3

Taxation and other government charges

Worked
Example

- 1 Calculate the following to the nearest cent.
 - (a) the GST to be added to the cost of a chair with a pre-tax price of \$85
 - (b) the GST-inclusive price of a sound system with a pre-GST price of \$564
 - (c) the pre-GST price of a motorbike advertised for \$5780

12

- 2 In the year 2000, Sydney hosted the Olympic Games and Australia introduced the GST. The GST replaced some existing taxes and introduced a flat rate of 10% on most goods and services, many being taxed for the first time.

Calculate the cost of the following items after 10% GST is added.

- (a) a camera originally priced at \$450
- (b) a theatre ticket originally priced at \$120
- (c) a haircut originally priced at \$28

Worked
Example

13

- 3 Mette owns a roller door business and charges customers \$650 for each door, including installation. The 10% GST is added to the bill. How much GST will Mette have to pay in a year if she averages 15 installations per week and takes 4 weeks annual leave?

- 4 Use the *Tax brackets and tax rates* table to calculate the annual tax, if any, that should be paid on each of the following.

- (a) an annual income of \$107 000
- (b) an annual income of \$46 000
- (c) a weekly income of \$300
- (d) a weekly income of \$1400
- (e) a monthly income of \$1120
- (f) a fortnightly income of \$3850
- (g) a monthly income of \$6500
- (h) a fortnightly income of \$585
- (i) a weekly income of \$1273

14

- 5 Bik works as a mechanic and has a gross income of \$46 000 for the year. During the year his tax-deductible expenses were \$400 on tools, \$100 on work-related phone calls and a donation of \$300 to the Children's Hospital. Calculate his taxable income.

- 6 Use the *HELP repayments* table to calculate the compulsory HELP repayment for the following taxable incomes.

- (a) \$85 200
- (b) \$52 100
- (c) \$68 900
- (d) \$101 600
- (e) \$22 064
- (f) \$56 713

16

- 7 Scott studied for 4 years to become a nurse. In that time he accumulated a HELP debt of \$29 000. In his first full financial year of work his taxable income was \$56 820.

- (a) Use the *HELP repayments* table to calculate Scott's compulsory HELP repayment for that year.
- (b) Calculate his weekly HELP repayment amount for that year.

- 8 Doug wants the final GST-inclusive price of his fishing lures to be no more than \$20. Select the option that is closest to the maximum pre-tax price he can charge.

- A \$2
- B \$18.15
- C \$18.20
- D \$19.99

- 9 Select the option that is closest to the taxable income of a driver with a fortnightly income of \$1860 and deductions of \$865 annually.

- A \$47 495
- B \$48 360
- C \$95 855
- D \$96 720

- 10 An employee earns a gross weekly wage of \$1285 and has annual deductions of \$310.

- (a) Determine the monthly taxable income to the nearest dollar.

- A \$4445.60
- B \$5258.33
- C \$5114.17
- D \$5542.50

- (b) Explain the common misconception made by a student who selected the last of the incorrect options.

- 11 Use the *Tax brackets and tax rates* table to calculate the annual tax payable by a florist who has a weekly income of \$1292 and annual deductions of \$1022.

- 12 Explain the mistake that has been made in each of the following calculations, which are based on a taxable income of \$70 000. Write the correct calculation each time.

(a) HELP debt rate: $\$68\,603 - \$72\,207$: 5.0%

HELP repayment: $(\$70\,000 - \$67\,369) \times 0.05 = \$131.55$

(b) Tax rate: $\$37\,001 - \$87\,000$: \$3572 plus 32.5 c for each \$1 over \$37 000

Tax on taxable income: $\$0.325 \times 70\,000 = \$22\,750$

Worked
Example

15

- 13 Than works as a school laboratory technician and had a gross income of \$32 320 for the year. During the year she spent \$70 on the purchase and cleaning of a lab coat, \$20 on work-related phone calls, and donated \$100 to registered charities.

(a) Calculate Than's taxable income.

(b) Calculate the tax to be paid for the year.

(c) According to Than's group certificate (official statement of income), she already paid \$3345 in tax during the year. Calculate her tax refund or debt.

- 14 Before the GST was introduced, the price of a particular bicycle was \$218.50, which included a 15% sales tax. The sales tax was abolished when the GST was introduced. Calculate the new GST-inclusive cost.

- 15 Gary owns a discount clothing store and imports his stock from overseas. He adds 100% mark-up to the cost of the item and then adds GST before putting the items out for sale.

(a) Calculate the selling price of a dress (cost \$23).

(b) Calculate the selling price of a T-shirt (cost \$6).

(c) Calculate the selling price of a jacket (cost \$15).

- 16 Milosh is an electrician who earned \$96 543 for the financial year. During the year his allowable deductions were the cost of a new trailer and tools (\$2750), the purchase and cleaning of his work uniform (\$320) and his union fees (\$530). According to his group certificate, he has already paid \$23 600 in tax.

(a) Calculate the taxable income.

(b) Calculate the amount of tax payable.

(c) Calculate the tax refund or debt.

- 17 Elise earned \$68 754 for the year as an engineer. Her tax-deductible expenses are shown in the table. According to Elise's group certificate, she has already paid \$14 960 in tax. How much is her tax refund or debt?

Tax-deductible expense	Amount (\$)
Purchase of a new hard hat	75
Professional association membership	430
Travel between work sites	380
Purchase of a CAS calculator	210
Charity donations	500

- 18 After completing his final year of secondary school, Tamlyn starts work as a children's drama coach in February. He has not had any other employment before this. He earns a set rate of \$18 an hour during the week and \$25 an hour on weekends. His average working week is 25 hours during the week and 10 hours at the weekend.

(a) Calculate Tamlyn's average gross weekly wage (before tax).

(b) Calculate his yearly salary if he works a full 52 weeks.

- (c) Calculate the tax to be paid if Tamlyn works a full 52 weeks.
 - (d) Calculate the amount of tax per week.
 - (e) Calculate his average net weekly wage (after tax).
 - (f) Calculate his earnings if he works only 21 weeks in the financial year.
 - (g) Calculate the tax payable if Tamlyn's total taxable income is that generated in 21 weeks of work.
 - (h) For those 21 weeks, Tamlyn's employer has taken tax out of each weekly wage at the rate calculated in part (d) for year-round earnings. Calculate the tax refund due.
- 19 Tony works as a gardener and has worked for three different companies throughout the year. The information from his group certificates is shown in the table.

Employer	Income (\$)	Tax paid (\$)
Greg's Garden Centre	24 876	4 870
Jan's Craft Cottage	31 009	5 740
Heathmead Primary School	6 256	1 245

His work-related tax-deductible expenses are shown in the table.

Tax-deductible expense	Amount (\$)
Gardening gloves (10 pairs)	58
Union subscription	340
Tools maintenance and purchase	790
Sunscreen	45
Cleaning of work clothes	210
- (a) Calculate the total (gross) income.
 - (b) Calculate the taxable income.
 - (c) Calculate the tax to be paid for the year.
 - (d) Calculate the tax refund or debt.
 - (e) Tony also has a HELP debt of \$4300. Use the *HELP repayments* table to calculate his annual compulsory HELP repayment based on his taxable income, and use this to recalculate his refund or debt.
- 20 Fy has an annual taxable income of \$85 400 and a HELP debt of \$110 178. Determine Fy's net weekly income after her tax and HELP repayments.
- 21 The following questions should help you understand the *Tax brackets and tax rates* table more clearly.
- (a) Calculate 19c in the dollar (for each dollar) for \$18 800.
 - (b) Calculate 32.5c in the dollar for \$43 000.
 - (c) Calculate 37c in the dollar for \$100 000.
 - (d) Compare your answer to part (a) with the figures in the *Tax brackets and tax rates* table.
 - (e) Add your answers to parts (a) and (b) and compare this with the figures in the *Tax brackets and tax rates* table.
 - (f) Add your answers to parts (a), (b) and (c) and compare this with the figures in the *Tax brackets and tax rates* table.
 - (g) Explain how the *Tax brackets and tax rates* table is organised.

Investments

Imagine you have been working hard at your new job and want to begin saving for something like a car, a house or a trip overseas. How would you invest this money? Factors such as the length of time you want to invest, whether you need easy access to those savings and how big a risk you are willing to take will determine what type of investment is right for you.

Simple interest

You may remember simple interest from earlier years, but you should revise the simple interest formula before moving on to more complex examples.

$$I = Pin$$

- I is the amount of interest paid (\$) (or charged if you are borrowing money).
- n is the number of periods for which the interest is paid.
- P is the principal (\$) (the amount of money invested or borrowed).
- i is the rate of interest for the same time period as n , expressed as a decimal.
- $A = P + I$ where A is the account balance (\$).

WARNING

It is essential that n and i are in the same time units. If they are different in the question, you must convert either i or n so that the units match.

17 Investing with simple interest

Robin invests \$2000 for 3 years at a simple interest rate of 5% p.a.

(a) Calculate the interest earned.

THINKING

- 1 Identify the values of I , n , P , i .
- 2 Check that n and i have the same units of time. If they do not, then convert one to the same units as the other.
- 3 Recall the simple interest formula $I = Pin$. Substitute the known values and calculate I .
- 4 Interpret the answer.

WORKING

$$I = ?, n = 3 \text{ years}, P = \$2000, i = 5\% \text{ p.a.} = 0.05$$

n and i are both in years, so no change is needed.

$$\begin{aligned} I &= Pin \\ &= 2000 \times 0.05 \times 3 \\ &= \$300 \end{aligned}$$

The interest earned is \$300.

- (b) Calculate the balance in the account after 3 years.

1 The balance is the sum of the principal and the interest.	$A = P + I$ $= 2000 + 300$ $= \$2300$
2 Interpret the answer.	The balance after 3 years is \$2300.

- (c) After 3 years, Robin decides to reinvest the money for a further 18 months at a simple interest rate of 6%. Determine the balance after the further 18 months.

1 Identify the values of I , n , P , i .	$I = ?$ $n = 18 \text{ months}$ $= 1.5 \text{ years}$ $P = \$2300$ $i = 6\% \text{ p.a.} = 0.06$
2 Recall the simple interest formula $I = Pin$. Substitute the known values and calculate the interest earned, I .	$I = Pin$ $= 2300 \times 0.06 \times 1.5$ $= \$207.00$
3 The balance is the sum of the principal and the interest.	$A = P + I$ $= 2300 + 207$ $= \$2507$
4 Interpret the answer.	The balance after the invested amount is reinvested for 18 months is \$2507.

Calculating n , i or p using the simple interest formula

You may know the interest you want to achieve and need to calculate one of n , P or i . For this you can either rearrange the simple interest formula using your algebra skills to find the unknown, or use the rearranged formulas shown here.

$$n = \frac{I}{iP} \quad i = \frac{I}{nP} \quad P = \frac{I}{in}$$

18 Using the simple interest formula

Determine the rate of simple interest that would result in an initial investment of \$8000 growing to \$9480 in 5 years. Calculate the interest earned.

THINKING

- 1 Identify the values of I , n , P , i , A .
- 2 There is not enough information to use the simple interest formula, so use $A = P + I$ first to calculate I .
- 3 Recall the simple interest formula $I = Pin$.
Substitute the known values and calculate i .
- 4 Interpret the answer.

WORKING

$$\begin{aligned}
 I &= ?, n = 5 \text{ years}, P = \$8000, i = ?, A = \$9480 \\
 A &= P + I \\
 9480 &= 8000 + I \\
 I &= 9480 - 8000 \\
 &= 1480 \\
 I &= Pin \\
 1480 &= 8000 \times i \times 5 \\
 i &= \frac{1480}{(8000 \times 5)} \\
 &= 0.037 \\
 &= 3.7\% \\
 \text{The interest rate is } 3.7\% \text{ p.a.}
 \end{aligned}$$

Understanding compound interest

Simple interest assumes that the principal remains the same throughout and the interest is added at the end of the investment. With compound interest, the interest is added to the principal during the investment, so the principal changes and the next calculation earns interest on the previously added interest as well as on the principal. In real life, interest is usually calculated as compound interest.

19 Understanding compound interest

Ryan invests \$4000 at an interest rate of 4% p.a. for 3 years.

- (a) Calculate the final balance if the interest is calculated as simple interest.

THINKING

- 1 Identify the values of I , n , P , i .
- 2 Check that n and i are in the same units of time. If they are not, then convert i to the same units as n .
- 3 Recall the simple interest formula $I = Pin$.
Substitute the known values and calculate I .
- 4 The balance is the sum of the principal and the interest.
- 5 Interpret the answer.

WORKING

$$\begin{aligned}
 I &= ?, n = 3 \text{ years}, P = \$4000, i = 4\% \text{ p.a.} = 0.04 \\
 n \text{ and } i &\text{ are both in years, so no change is needed.} \\
 I &= Pin \\
 &= 4000 \times 0.04 \times 3 \\
 &= \$480 \\
 \text{Balance} &= P + I \\
 &= 4000 + 480 \\
 &= \$4480 \\
 \text{The final balance with simple interest} &\text{ is } \$4480.
 \end{aligned}$$

- (b) Calculate the final balance if the simple interest is calculated and added to the principal at the end of each year.

1 Identify the values of I , n , P , i for the first yearly interest calculation.	$I = ?$, $n = 1$ year, $P = \$4000$, $i = 4\%$ p.a. $= 0.04$
2 Recall the simple interest formula $I = Pin$. Substitute the known values and calculate I .	$I = Pin$ $= 4000 \times 0.04 \times 1$ $= \$160$
3 The balance is the sum of the principal and the interest.	$\text{Balance} = P + I$ $= 4000 + 160$ $= \$4160$
4 Repeat the previous steps, using the new principal to calculate the balance at the end of the second year.	$I = ?$, $n = 1$ year, $P = \$4160$, $i = 4\%$ p.a. $= 0.04$ $I = Pin$ $= 4160 \times 0.04 \times 1$ $= \$166.40$ $\text{Balance} = P + I$ $= 4160 + 166.40$ $= \$4326.40$
5 Repeat the previous steps, using the new principal to calculate the balance at the end of the final year.	$I = ?$, $n = 1$ year, $P = \$4326.40$, $i = 4\%$ p.a. $= 0.04$ $I = Pin$ $= 4326.40 \times 0.04 \times 1$ $= \$173.06$ $\text{Balance} = P + I$ $= 4326.40 + 173.06$ $= \$4499.46$
6 Interpret the answer.	The final balance with simple interest added to the principal each year is \$4499.46

- (c) Explain why the two results are different.

What is different about the two cases? Remember that for compound interest, the principal changes each year.	As the number of years increase, the interest increases because it is calculated on a higher principal each time.

The calculations in the previous worked example require three repetitive calculations. Repeating calculations like this can become very tedious, especially when interest is normally calculated daily by financial institutions. If the interest had been calculated daily in the previous example, it would have needed 1095 calculations (3×365). To more easily track interest over time, you can use a spreadsheet.

20 Compound interest using a spreadsheet

Alex invests \$20 000 at an interest rate of 5% p.a. for 3 years, with interest added quarterly.

- (a) Use a spreadsheet to determine the principal after 9 months.

THINKING

- 1 Interest is added quarterly (where 1 quarter is 3 months).
- 2 Convert the rate to the correct time units.
- 3 Construct a spreadsheet using the values of i , P and n to cover all parts of the question.

- 4 Read the appropriate answer from your spreadsheet.

- 5 Interpret the answer.

WORKING

9 months is 3 quarters.

$$i = 5 \div 4 \\ = 1.25\% \text{ per quarter}$$

	A	B	C	D
1	Quarter	Principal	Interest	Amount
2	1	20000	250	20250
3	2	20250.00	253.13	20503.13
4	3	20503.13	256.29	20759.41
5	4	20759.41	259.49	21018.91

	A	B	C	D
1	Quarter	Principal	Interest	Amount
2	1	20000	250	20250
3	2	20250.00	253.13	20503.13
4	3	20503.13	256.29	20759.41
5	4	20759.41	259.49	21018.91

The principal amount to be reinvested after 9 months is \$20759.41

- (b) Use a spreadsheet to determine the principal after 18 months.

- 1 Convert the time values to the correct time units.

18 months is 6 quarters.

- 2 Read the appropriate answer from the spreadsheet you created (see cell A7).

The principal amount to be reinvested after 18 months is \$21547.66

- (c) Use a spreadsheet to determine the principal after 3 years.

- 1 Convert the time values to the correct time units.

3 years is 12 quarters.

- 2 Read the appropriate answer from the spreadsheet you created earlier (see cell D13).

The final amount of the investment after 3 years is \$23215.09

Compound interest formula

Spreadsheets are very useful for calculating the amount owed or earned at any point in time and in situations where only one variable changes, but if a single answer is required you can use the compound interest formula:

$$A = P(1 + i)^n$$

- A is the total amount (\$), which includes the principal and interest.
- P is the original principal (\$).
- i is the rate of interest with the same time units as n , expressed as a decimal.
- n is the number of time periods (when interest is calculated).
- $I = A - P$ where I is the amount of interest (\$).

C Making connections

Simple vs compound interest

Compare the simple interest and compound interest earned under the same investment conditions.

WARNING

Never round your results until the final answer.

21 Compound interest using the formula

Kylie invests \$12 000 for 3 years at an interest rate of 8% p.a.

- (a) Calculate the value of the investment after 3 years if the interest is compounded annually.

THINKING

- 1 Identify the values of A , n , P , i .
- 2 Check that n and i have the same units of time as the compounding time period.
- 3 Recall the compound interest formula
 $A = P(1 + i)^n$, substitute the known values and calculate A .
- 4 Interpret the answer.

WORKING

$A = ?$, $n = 3$ years, $P = \$12\,000$,
 $i = 8\% \text{ p.a.} = 0.08$

All time units are in years, so no change is needed.

$$\begin{aligned} A &= P(1 + i)^n \\ &= 12\,000 \times (1 + 0.08)^3 \\ &= \$15\,116.54 \end{aligned}$$

The total value of the investment after 3 years with the interest compounded annually is \$15 116.54.

- (b) Calculate the value of the investment after 3 years if the interest is compounded quarterly.

- 1 Identify the values of A , n , P , i .
- 2 Check that n and i have the same units of time as the compounding time period.

$A = ?$, $n = 3$ years, $P = \$12\,000$, $i = 8\% \text{ p.a.}$

$$\begin{aligned} n &= 3 \times 4 \\ &= 12 \text{ quarters} \\ i &= 8 \div 4 \\ &= 2\% \text{ per quarter} \\ &= 0.02 \end{aligned}$$

- 3 Recall the compound interest formula
 $A = P(1 + i)^n$, substitute the known values
 and calculate A .

$$\begin{aligned} A &= P(1 + i)^n \\ &= 12\,000 \times (1 + 0.02)^{12} \\ &= \$15\,218.90 \end{aligned}$$

- 4 Interpret the answer.

The total value of the investment after 3 years with the interest compounded quarterly is \$15 218.90.

- (c) Calculate the value of the investment after 3 years if the interest is compounded daily.

- 1 Identify the values of A , n , P , i .
 2 Check that n and i have the same units of time as the compounding time period.

$$\begin{aligned} A &= ?, n = 3 \text{ years}, P = \$12\,000, i = 8\% \text{ p.a.} \\ n &= 3 \times 365 \\ &= 1095 \text{ days} \\ i &= (8 \div 365)\% \text{ per day} \\ &= \frac{0.08}{365} \end{aligned}$$

- 3 Recall the compound interest formula
 $A = P(1 + i)^n$, substitute the known values
 and calculate A .

$$\begin{aligned} A &= P(1 + i)^n \\ &= 12\,000 \times \left(1 + \frac{0.08}{365}\right)^{1095} \\ &= \$15\,254.59 \end{aligned}$$

- 4 Interpret the answer.

The total value of the investment after 3 years with the interest compounded daily is \$15 254.59.

- (d) How much interest is earned when the interest is compounded daily?

- 1 Interest earned is the total amount minus the original principal.

$$\begin{aligned} \text{Interest earned} &= A - P \\ &= 15\,254.59 - 12\,000 \\ &= \$3\,254.59 \end{aligned}$$

- 2 Interpret the answer.

The interest earned when interest is compounded daily is \$3254.59.

- (e) Comment on these results.

- 1 Identify a pattern.

The amount increased as the compounding period decreased.

- 2 Determine an explanation of that pattern.

Interest was being added more frequently, so there was more interest being earned on the interest previously added.

WARNING

In Worked example 21, if the fraction $\frac{8}{365}$ was expressed as the decimal 0.0219178... and rounded to 2 decimal places, 0.02, the final answer would have been \$14 937.65 instead of \$15 254.59.

i Additional information**Earning interest**

Watch the video and complete the activity about earning interest.

22 Using a spreadsheet to see the effect of varying the compounding period

Use the compound interest formula to create a spreadsheet that compares the growth in the balance of an account over 10 years where the initial principal was \$5000. Let the interest rate be 6% p.a. in each case and use separate columns to show annual, monthly and daily compounding periods. Complete your analysis with a graph and comments on the trends observed.

THINKING

- 1 Set up headings with values of time in whole numbers for the required number of years. Enter the initial principal for each Amount column.
- 2 Insert formulas in the first row of increased balance according to the formula $A = P(1 + i)^n$.
- 3 Select the row containing the formulas and use it to fill down the rows, completing the values for all years.

WORKING

	A	B	C	D
1	Time (years)	A (annual)	A (monthly)	A (daily)
2	0	5000	5000	5000
3	1			
⋮	⋮			
12	10			

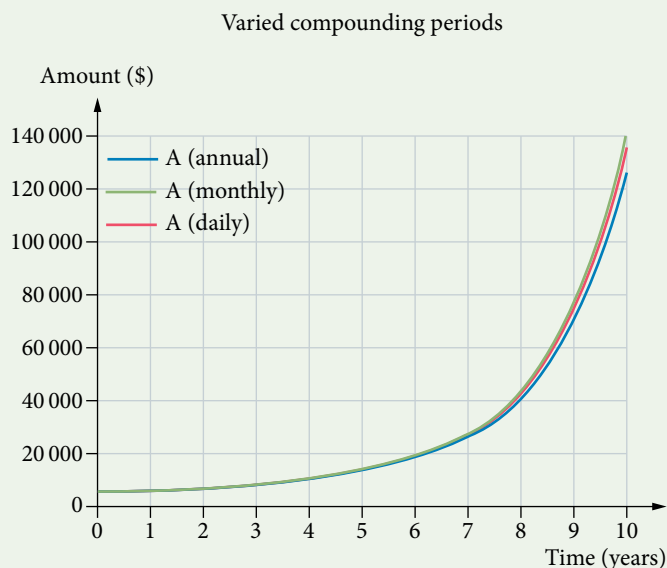
For annual compounding,
cell B3: = B2 * (1 + 0.06)^A3

For monthly compounding,
cell C3: = C2 * (1 + 0.06/12)^(A3 * 12)

For daily compounding,
cell D3: = D2 * (1 + 0.06/365)^(A3 * 365)

	A	B	C	D
1	Time (Years)	A (annual)	A (monthly)	A (daily)
2	0	5000	5000	5000
3	1	5300.00	5308.39	5309.16
4	2	5955.08	5983.40	5986.00
5	3	7092.60	7160.22	7166.44
6	4	8954.24	9096.98	9110.14
7	5	11982.79	12270.47	12297.11
8	6	16997.82	17571.85	17625.28
9	7	25558.43	26715.71	26824.08
10	8	40736.26	43122.97	43347.99
11	9	68823.05	73899.82	74382.15
12	10	123251.61	134453.08	135526.43

- 4 Insert a scatterplot for the entire set of values.



- 5 Comment on the trends.

Specify the way in which the graphs are similar and the ways in which they differ.

For all three compounding periods the growth was slow at first, then much more rapid. From the table you can see that the balance for the annual compounding situation is always less than for monthly compounding, and that the balance for monthly compounding is less than for daily compounding. The difference increases as time goes on.

EXERCISE

1.4

Investing money

Worked
Example

- 1 Use the simple interest formula to calculate the earnings (interest paid) on the following investments.

(a) \$1500 invested for 4 years at 4.5% p.a.	(b) \$18000 for 10 years at 6% p.a.
(c) \$12000 for 6 months at 7% p.a.	(d) \$100000 for 20 months at 4% p.a.
- 2 Use the simple interest formula to calculate the earnings (interest paid) on the following investments.

(a) \$8450 invested for 200 days at 2.9% p.a.
(b) \$20 000 invested for 90 days at 4.25% p.a.
(c) \$1.2 million invested for 40 weeks at 3.2% p.a.
(d) \$5000 invested for 2 years 3 weeks at 6.7% p.a.
(e) \$63 700 invested for 6 years 2 weeks 4 days at 5.8% p.a.
- 3 An amount of \$480 is invested for 6 years at 7% p.a. simple interest.

(a) The amount of annual interest is:	A \$33.60	B \$201.60	C \$513.60	D \$681.60

(b) Use a spreadsheet to draw a graph of the value of the investment over the 6 years. Describe the trend in your graph.

17

Worked
Example

- 4 Calculate the final balance on the following simple interest investments.
- (a) \$4000 invested for 12 years at 5.4% p.a. (b) \$957 invested for 20 months at 2.94% p.a.
(c) \$83 200 invested for 3 weeks at 8.8% p.a. (d) \$200 000 invested for 291 days at 7.15% p.a.
- 19 5 Shona invested \$5000 for 2 years at an interest rate of 6% p.a. Use the simple interest formula for the following questions.
- (a) How much interest would be earned if it were calculated at the end of the period?
(b) How much interest would be earned altogether if the interest was calculated and added to the principal at the end of the first year?
(c) Explain the difference between the two figures.
- 21 6 Cal invests \$8000 at 6.1% p.a. over 4 years.
- (a) Calculate the value of the investment after 4 years if the interest is compounded:
(i) annually (ii) quarterly (iii) monthly (iv) daily.
(b) Explain the trend in your answers to the previous parts of this question.
- 7 Indira invests \$8000 at 6.1% p.a. over 4 years. Interest is compounded daily.
- (a) Calculate the balance of the investment at the end of the 4 years.
(b) Calculate the interest earned on the investment.
- 8 Calculate the interest earned on the following investments.
- (a) \$10000 for 4 years at 5.9% p.a. compounded annually.
(b) \$8000 for 5 years at 6.3% p.a. compounded weekly.
(c) \$85000 for 3 years at 6.87% p.a. compounded monthly.
(d) \$106000 for 10 months at 5.7% p.a. compounded monthly.
(e) \$68000 for 6 years 3 months at 3.87% p.a. compounded weekly.
(f) \$1.5 million for 3 years 6 weeks at 4.59% p.a. compounded weekly.
- 9 Use a spreadsheet to answer this question. Ji-Koo invests \$40000 for 5 years at a rate of 4.5% compounding half-yearly. Create a table that shows the amount of money in the account after each half-year period.

..

..

- 18 10 Determine the annual rate of simple interest for each of the following investments.
- (a) \$400 invested for 2 years amounts to \$468
(b) \$6000 invested for 15 months amounts to \$6315
(c) \$1560 invested for 10 weeks amounts to \$1568.70
- 11 Determine the initial amount invested for each of the following.
- (a) An investment earns \$450 interest after 3 years of simple interest at 6.3%.
(b) An investment earns \$806.25 interest after 5 months of simple interest at 4.3%.
(c) An investment earns \$81 interest after 40 weeks of simple interest at 2.7%.
- 12 Determine the time taken for each of the following.
- (a) An investment of \$10000 grows to \$11610 at 4.6% simple interest.
(b) An investment of \$4200 grows to \$4541.25 at 6.5% simple interest.
(c) An investment of \$960 grows to \$1300.20 at 5.25% simple interest.

20

- 13 Mia invests \$3000 in an account offering 5.2% p.a. over 4 years. Interest is added quarterly. Set up a spreadsheet using simple interest formulas to calculate her account balance (to 2 decimal places) after:
- (a) 2 years
 - (b) 30 months.

- 14 \$8000 is invested at 6% p.a. compounding monthly for 10 months.
- (a) Which alternative shows a correct calculation to determine the compound interest earned?

A $8000 \times (1 + 0.06)^{10} - 8000$

B $8000 \times (1 + 0.005)^{10} - 8000$

C $8000 \times (1 + 0.06)^{\frac{10}{12}} - 8000$

D $8000 \times (1 + 0.005)^{\frac{10}{12}} - 8000$

- (b) Explain the common error in the first incorrect response.
 - (c) Explain why the other two incorrect alternatives need not be considered.
- 15 Tulay sells some artwork and has \$7000 to invest for a period of 2 years. After investigating, she narrows her choice to the following three options: 5% p.a. compounded monthly, 4.5% p.a. compounded daily or 5.5% p.a. simple interest.
- Which investment will earn the most interest?

- 16 Use a spreadsheet to answer this question. An inheritance of \$12000 is placed in an investment account offering 6% p.a., compounding monthly. The money is invested for 1 year.
- (a) Calculate the account balance and interest earned for each month. Record your results in a table.
 - (b) Describe how the interest is changing each month. Explain the changes that are occurring.

- 17 Use the compound interest formula to create a spreadsheet that compares the growth in the balance in an account over 5 years where the initial principal was \$8200. Let the interest rate be 4.9% p.a. in each case and use separate columns to show annual, quarterly and weekly compounding periods. Complete your analysis with a graph and comment on the trends observed.

22

- 18 Use a spreadsheet or other method to answer this question.

Trinh and Lewis open a short-term investment account and deposit \$25000. The interest is compounded daily for a total of 40 days. Answer the following to the nearest cent.

- (a) If the interest rate is 6.7% p.a., what is the investment worth after: 10 days, 20 days, 30 days and 40 days?
 - (b) If the interest rate is 7.8% p.a., what is the investment worth after: 10 days, 20 days, 30 days and 40 days?
- 19 A bank account has an initial balance of \$490 and no deposits or withdrawals are made by the account holder over 3 years. Determine the balance over the 3 years, given that interest rates varied from 3.1% p.a. to 2.8% p.a. and finally to 4.3% p.a. and the bank charges a \$5 annual fee for balances under \$500. Make any necessary assumptions and present your answer as a table.

1.5 Shares

Shares and dividends

Companies list on the share market when they need funds from the public to set up or expand without having to borrow money.

When you buy shares in a company you are buying a portion of the company, and you are entitled to share in any profits the company makes.

Some shareholders choose a company that pays a dividend, or return (some of the company's profits), each year. The dividend is given as an amount of money, dollars per share (DPS).

$$\text{Dividend} = \frac{\text{returned profit}}{\text{total number of shares}}$$

23 Dividends

Emmy buys 2000 shares in a phone company that has issued a total of 12 billion shares. Some time later the company declares a profit of \$3.4 billion for distribution to the shareholders.

(a) Calculate the dividend per share, rounding down to the nearest cent.

THINKING

Divide the profit being distributed by the total number of shares.

WORKING

$$\begin{aligned}\frac{3.4 \text{ billion}}{12 \text{ billion}} &= \$0.28333\dots \text{ per share} \\ &= 28 \text{ c per share}\end{aligned}$$

(b) Using the rounded value of the dividend, calculate Emmy's total dividend from this investment.

Multiply the dividend per share by the number of shares held.

$$2000 \times \$0.28 = \$560$$

Some companies pay dividends more frequently, or may pay a special dividend at the time of a significant event.

Capital gain

It is not a requirement for a company to pay dividends, as they may choose to reinvest some or all of their profits.

As a shareholder, you will benefit if your shares increase in value. If you sell your shares, the profit you make at the time of sale is called a *capital gain*.

24 Capital gain

Emmy bought 2000 shares in a phone company at \$3.80 each and sold them for \$4.00. What is her capital gain from the sale?

THINKING

- 1 The capital gain per share is the selling price minus the cost price.
- 2 To determine the total capital gain, multiply the capital gain per share by the number of shares.

WORKING

$$\begin{aligned}\text{Capital gain per share} &= \$4.00 - \$3.80 \\ &= \$0.20\end{aligned}$$

$$\begin{aligned}\text{Capital gain} &= 2000 \times \$0.20 \\ &= \$400\end{aligned}$$

Market price of shares

While the stock market is operating, the market price (current selling price) fluctuates, based on buyer perception of the value of the shares. At the time of the payment of a dividend, a percentage dividend can be calculated, based on the market price at that time.

$$\text{Percentage dividend (yield)} = \frac{\text{dividend value per share}}{\text{market value per share}} \times 100\%$$

25 Percentage dividend

Emmy's shares had a market value of \$3.95 at the time the 28 c per share dividends were paid. Calculate the percentage dividend (yield), to 1 decimal place.

THINKING

- 1 Calculate the dividend as a percentage of current market value.

WORKING

$$\begin{aligned}\text{Percentage dividend} &= \frac{\text{dividend value per share}}{\text{market value per share}} \times 100\% \\ &= \frac{0.28}{3.95} \times 100\% \\ &= 7.1\% (1 \text{ d.p.})\end{aligned}$$

- 2 Interpret the answer.

The dividend represents a yield of 7.1%.

Brokerage

There are likely to be hidden costs whenever large purchases are made. Buying and selling shares does not attract a government charge (as it does in many other countries), as stamp duty was abolished in Australia when the GST was introduced.

Shares cannot be sold in bundles of less than \$500 in value, unless the trader is topping up the number of existing shares in a particular company.

A stockbroker, associated with a brokerage firm, charges a fee to buy and sell shares on behalf of individual traders. Since online buying and selling of shares became popular, brokerage fees have diminished significantly, currently averaging just under \$20 for up to \$10 000 worth of shares.

26 Brokerage and break-even point

Toren bought a parcel of 500 shares online at \$14.20 each and paid a \$20 brokerage fee. He expects to have to pay the same fee when he sells the shares. What is the minimum value per share that Toren must receive in order to break even (no profit or loss) on this venture?

THINKING

- 1 Calculate the total outlay, including both brokerage fees.
- 2 For break-even point:
total selling price = total outlay.
Divide the total selling price by the number of shares.
- 3 Interpret the answer.

WORKING

$$\begin{aligned}\text{Total outlay} &= \text{total share value} + \text{brokerage} \times 2 \\ &= 500 \times \$14.20 + \$20 \times 2 \\ &= \$7140\end{aligned}$$

For break-even point:

$$\text{total selling price} = \$7140$$

$$\begin{aligned}\text{Selling price per share} &= \frac{\text{total selling price}}{\text{number of shares}} \\ &= \frac{\$7140}{500} \\ &= \$14.28\end{aligned}$$

Toren must receive \$14.28 per share in order to break even.

Price-to-earnings ratio

To compare the returns from different companies, one measure used is the price-to-earnings ratio.

The price to earnings ratio of a share (P/E ratio) is defined as:

$$\text{P/E ratio} = \frac{\text{market price per share}}{\text{annual earnings per share}}$$

The price-to-earnings ratio is an indicator of how many years it will take for the dividends, or other earnings such as capital gains, to pay for the purchase of shares. It is one way to compare companies prior to purchasing shares.

WARNING

Because price-to-earnings calculations are based on current market price and past earning capacity, the ratio is unlikely to be reliable in the long term.

27 Price-to-earnings ratio

Shares in Oily, an oil and gas company, have a current market value of \$42.30. The company published its annual net profit at \$1.48 per share. Shares in another oil and gas company, Gassy, have a current market value of \$112.86. Gassy's annual net profit was \$4.62 per share. Calculate the price-to-earnings ratio for these companies and explain what the figures mean for prospective investors.

THINKING

- 1 Identify the market price and annual earnings.

- 2 Recall the formula for the price-to-earnings ratio: $\text{P/E ratio} = \frac{\text{market price per share}}{\text{annual earnings per share}}$ for each company.

- 3 Explain, in terms of the price-to-earnings ratio, how many years it will take for the earnings to pay for the purchase of shares.

WORKING

Oily:

Market price per share: \$42.30

Annual earnings per share: \$1.48

Gassy:

Market price per share: \$112.86

Annual earnings per share: \$4.62

Oily:

$$\begin{aligned}\text{P/E ratio} &= \frac{\text{market price per share}}{\text{annual earnings per share}} \\ &= \frac{42.30}{1.48} \\ &= 28.6 \text{ (1 d.p.)}\end{aligned}$$

Gassy:

$$\begin{aligned}\text{P/E ratio} &= \frac{\text{market price per share}}{\text{annual earnings per share}} \\ &= \frac{112.86}{4.62} \\ &= 24.4 \text{ (1 d.p.)}\end{aligned}$$

On the basis of the P/E ratio alone, a purchaser would choose Gassy, as Gassy shares would pay back their purchase price in 24.4 years, whereas Oily shares would take 28.6 years.

In 2017 the average P/E ratio for the oil and gas sector was about 25.4, which was typical across all sectors of the share market. For comparison, the banking sector had an average P/E ratio of around 17.8. If the inflation rate increases then the average ratios generally decrease.

EXERCISE

1.5

Shares

Worked
Example

- 1 William buys 5000 shares in a company that has issued a total of 10 million shares. The company declares a profit of \$2.5 million to be returned in dividends.
 - (a) Calculate the dividend per share.
 - (b) Calculate the total share dividend from William's investment for the year.

23

- 2 Calculate the dividend per share for each of the following companies. Assume the net profits are returned entirely to the shareholders. Round answers to 2 decimal places where necessary.
- A company with 5 million shares made a net profit of \$860 000.
 - A company with 50 million shares made a net profit of \$60 million.
 - A company with 5.5 million shares made a net profit of \$65 million.
 - A company with 500 million shares made a net profit of \$40 million.
 - Which of the companies in the previous parts of this question would be the best to have shares in?
 - The one with the largest profit.
 - The one with the most shares.
 - The one that paid the highest dividend per share.
 - You cannot make a sensible choice unless you know the market value of the shares.
- 3 Calculate the total dividend for the shareholder, for each of the following portfolios of shares.
- Heidi has 2000 shares in Bello paying dividend per share (DPS) of \$0.29, and 500 in Pepsico paying DPS of \$4.36.
 - Mikail has 5000 shares in RWW paying DPS of \$0.14, and 100 in ZtZ paying DPS of \$0.55.
 - Soltan has 500 shares in Lam paying DPS of \$0.10, 10 000 shares in CCt paying DPS of \$0.08, and 1000 in SoK paying DPS of \$5.06.
 - Peyton has 8000 shares in XbJ paying DPS of \$1.04, 1500 shares in BCL paying DPS of \$0.33, and 2000 in KTY paying DPS of \$0.58.
 - Kai has 500 shares in AAB paying DPS of \$0.91, 500 shares in XML paying DPS of \$1.57, and 2000 in QLZ paying DPS of \$3.09.
 - Hunter has 1000 shares in PoP paying DPS of \$1.87, 10 000 shares in WMW paying DPS of \$0.85, and 5500 in BaaY paying DPS of \$2.20.
- 24 4 William paid \$9.70 each for his 5000 shares and finds they are now worth \$10.00 each. Ignoring dividends and brokerage fees, calculate the capital gain William would make by selling his shares.
- 25 5 Calculate the percentage dividend (yield), to 1 decimal place, for each of the following.
- market value of \$13.48, dividend 77 c per share
 - market value of \$104.20, dividend \$4.13 per share
 - market value of \$12.06, dividend 18 c per share
 - market value of \$85.90, dividend 62 c per share
 - market value of \$7.24, dividend 45 c per share
 - market value of \$64.50, dividend 90 c per share
- 6 Ronin bought 1000 shares at \$15.22 per share and paid brokerage of \$22. The break-even point (where Ronin makes no profit or loss) for the market value of the shares at time of sale (assuming the same brokerage at time of sale) is:
- | | | | |
|---|---|---|--|
| A | \$15.22 | B | $\frac{\$15.22 \times 1000 + \$22 \times 2}{1000}$ |
| C | $\frac{\$15.22 \times 1000 + \$22}{1000}$ | D | $\frac{\$15.22 \times 1000 - \$22}{1000}$ |

Worked
Example

26]

- •

• • • • • • • • • • • •

27

- 12 Asif borrows \$10 000 to buy shares in a 'sure thing' that his neighbour's sister's friend told him about. He makes no repayments on his \$10 000 loan during the year because he intends to collect a dividend and sell his shares after 12 months to repay the loan plus interest. The interest on the loan is charged at 8% p.a., compounding monthly. The shares cost \$1.25 each. After 12 months the shares yield a dividend of 6c each and can be sold for \$1.30 each.
- How many shares did Asif buy?
 - How much does Asif earn from the share dividend on his investment for the year?
 - Calculate the total value of his shares after 1 year.
 - Calculate the amount to be repaid for the loan (to the nearest dollar).
 - Did Asif make a profit or a loss? How much is this profit or loss?

- 13 Eden used an inheritance to purchase 10 000 CMN shares at \$2.75, 10 years ago. She recently sold the shares at \$5.82 in order to invest in her first home. At the time of purchase of the shares, Eden was offered a term deposit account at her bank that paid 4.3% p.a. compounding annually for 10 years. Given that brokerage and solicitor's fees for the purchase and sale of the shares totalled \$280, determine whether Eden made a wise decision in purchasing the shares.
- 14 Georgia takes out a 5 year personal loan to buy 20 000 LoL shares when they are at a low value of \$5.60. Georgia pays the bank interest of 6.8% at the end of each year. What will the market value of the shares need to be at the end of 5 years in order for Georgia to pay back the bank and make an overall profit of \$30 000 on the venture? Assume brokerage of \$100 at time of purchase and sale of the shares, and bank fees of \$150 added to the amount borrowed. State any other assumptions you need to make to answer the question.
- 15 Alistair owns shares in four companies as shown.

Company	Initial share price (\$)	Number of shares held	Market price (\$)	Dividend (\$)
Y Mining	54.70	1000	58.30	0.34
Indilantic Holdings	21.42	2000	12.40	0.10
GCA Leasing	2.10	1000	2.40	0.25
Brushwell Pty Ltd	0.85	10000	1.80	0.18

- Calculate Alistair's total share dividend earnings from these investments for the year.
- Which investment was the best value? Explain your answer.
- Calculate Alistair's overall loss if he sells all his shares at this time. (Ignore the income from dividends and any taxation implications or other charges.)

Profit, loss and inflation

1.6

Profit and loss

The profit (or loss) on an item is the difference between the cost price and the selling price.

$$\begin{aligned}\text{Profit} &= \text{selling price (SP)} - \text{cost price (CP)} \\ \text{Percentage profit or loss} &= \frac{\text{profit (or loss)}}{\text{cost price}} \times 100\%\end{aligned}$$

28 Calculating percentage profit or loss

Calculate the percentage profit on an item that cost \$450 and was sold for \$620.

THINKING

- 1 Determine whether the difference between CP and SP represents a profit or a loss.
- 2 Calculate the profit by finding the difference between the cost and selling prices.
- 3 Express the difference as a fraction over the cost price and multiply by 100%.

WORKING

Given that $SP > CP$ a profit was made.

$$\begin{aligned}\text{Profit} &= \text{selling price (SP)} - \text{cost price (CP)} \\ &= 620 - 450 \\ &= \$170\end{aligned}$$

$$\begin{aligned}\text{Percentage profit} &= \frac{\text{profit}}{\text{cost price}} \times 100\% \\ &= \frac{170}{450} \times 100\% \\ &= 37.8\%\end{aligned}$$

To establish a selling price for goods, many businesses will increase the cost price by a set percentage. This is referred to as the *profit margin* or *mark-up*.

$$SP = CP + \text{mark-up \% of CP} \text{ or } SP = (100 + \text{mark-up})\% \text{ of CP}$$

Additional information

Cost price and selling price

Complete the activity to practise calculating buying and selling price.

29 Calculating mark-ups

An import company has a fixed mark-up of 60% on every item.

Calculate the selling price of a desk with a cost price of \$380.

THINKING

Use the formula to increase the cost price by the required percentage.

WORKING

$$\begin{aligned}SP &= (100 + \text{mark-up})\% \text{ of CP} \\ &= (100 + 60)\% \text{ of } \$380 \\ &= 160\% \text{ of } \$380 \\ &= 1.6 \times \$380 \\ &= \$608\end{aligned}$$

Percentage mark-up can be found if both the cost price (CP) and mark-up are known or can be found.

$$\text{Mark-up} = \text{selling price} - \text{cost price} \quad \text{Percentage mark-up} = \frac{\text{mark-up}}{\text{cost price}} \times 100\%$$

Inflation

Many products you purchase will lose value over time. This is known as *depreciation*. On the other hand, the value of some items, such as gold or an art collection, may increase over time. This is called *appreciation*.

When prices rise over time, this is known as *inflation*. One method of measuring the health of the economy is the *Consumer Price Index* (CPI), which is a way of measuring inflation across the economy. The CPI is calculated by looking at the price variation of a set 'basket' of household expenses every 3 months. This 'basket' includes things such as housing, transport, communication, education, recreation, insurance, energy, food and clothing. The annual CPI (also simply called the 'inflation rate') is found by averaging the quarterly results.

Inflation can be affected by many factors. For example, when people are feeling confident about the future they tend to spend more money; this leads to increased demand and prices go up, increasing inflation.

The Reserve Bank of Australia aims to keep the economy running efficiently and to keep inflation below 3%.

To slow down the economy and control inflation, the Reserve Bank may raise interest rates. Similarly, if the economy is slowing down, the Reserve Bank may reduce interest rates to stimulate spending.

30 Calculating the CPI

The Australian Bureau of Statistics quarterly CPI figures for two consecutive years are shown in the table. All CPI figures are given as percentage change from the previous year.

Year	Jan-Mar	Apr-Jun	Jul-Sep	Oct-Dec
2008	4.2	4.5	5.0	3.7
2009	2.5	1.5	1.3	2.1

(a) Calculate the annual CPI for both years.

THINKING

Calculate the annual mean by adding the quarterly CPI figures and dividing by 4.

The data are given to 1 decimal place, so give the answer to 1 decimal place.

WORKING

2008:

$$\text{CPI} = \frac{4.2\% + 4.5\% + 5.0\% + 3.7\%}{4} \\ = 4.4\%$$

2009:

$$\text{CPI} = \frac{2.5\% + 1.5\% + 1.3\% + 2.1\%}{4} \\ = 1.9\%$$

- (b) Given the annual CPI for each year, how might the Reserve Bank alter interest rates for the next 2 years?

The inflation target is 3%. When inflation is higher, interest rates are raised, and when the economy slows down, the rates are lowered.

In the first year, inflation is above 3%, so interest rates are likely to be increased.

In the second year, inflation is below 3%, so interest rates are likely to be lowered.

- (c) A basket of goods cost \$278 at the start of the first year. How much did the same goods cost at the end of the first year?

1 Recall the annual CPI for the first year.

2008 CPI: 4.4%

2 Determine the inflation factor by adding 100% to the inflation rate and then convert the result to a decimal.

$100\% + 4.4\% = 104.4\%$
 $= 1.044$

3 Multiply the cost of the goods by this inflation factor.

Cost at end of a year:
 $\$278 \times 1.044 = \$290.23$

4 Interpret the result.

After a single year, due to inflation only, the basket of goods is now worth \$290.23.

If you are using an inflation rate over several years, you may need to multiply by the same inflation figure for each year. For example, if annual inflation had averaged 2% for 4 years, the inflated cost of a \$20 item would be $20 \times 1.02 \times 1.02 \times 1.02 \times 1.02$ or, more simply, 20×1.02^4 . This is exactly the same kind of multiplication as is found in compound interest, so you could also use the compound interest formula $A = P(1 + i)^n$ to give $A = 20(1 + 0.02)^4$.

Additional information

Inflation

Calculate the cost of items after inflation.

31 The effect of inflation

Assuming an average annual inflation rate of 3.4% over the last 10 years, calculate the value of a house that 10 years ago was priced at \$240 000.

THINKING

1 Identify the original price, the number of years and the average inflation rate.

2 Determine the inflation factor by adding 100% to the inflation rate and then converting the answer to a decimal.

3 Calculate the current value by multiplying the original value by the inflation factor for every year that has passed.

WORKING

Original price = \$240 000

Time = 10 years

Inflation rate = 3.4%
 $= 0.034$

$103.4\% = 1.034$

Current value = $\$240\,000 \times 1.034^{10}$
 $= \$335\,287$

If there is a time delay between buying and selling an item, then inflation can have an impact on the *effective profit*. For example, if you make an investment that returns 2% but inflation is at 3%, then you would make a monetary profit, but your money would be able to purchase less than it could a year ago.

$$\text{Effective profit} = \text{sale price} - \text{inflated value of purchase price}$$

$$\text{Effective percentage profit} = \frac{\text{effective profit}}{\text{current value}} \times 100\%$$

32 Calculating effective profit

A tile importer buys a pallet of tiles for \$6000 and sells them after 2 years for \$7500. Annual inflation averages 3.2% over that time period.

(a) Calculate the importer's percentage profit.

THINKING

- 1 Calculate the profit by finding the difference between the cost price and the selling price.

WORKING

$$\begin{aligned}\text{Profit} &= \text{SP} - \text{CP} \\ &= 7500 - 6000 \\ &= \$1500\end{aligned}$$

- 2 To calculate percentage profit, express the profit as a fraction of the cost price and multiply by 100%.

$$\begin{aligned}\text{Percentage profit} &= \frac{\text{profit}}{\text{cost price}} \times 100\% \\ &= \frac{1500}{6000} \times 100\% \\ &= 25\%\end{aligned}$$

(b) Calculate the importer's effective profit.

- 1 Determine the value of the tiles after repeated annual inflation on the cost price, using the compound interest formula.

$$\begin{aligned}P &= \$6000, i = 3.2\% = 0.032, n = 2 \text{ years} \\ \text{Inflated value} &= P(1 + i)^n \\ &= 6000 \times (1 + 0.032)^2 \\ &= \$6390.14\end{aligned}$$

- 2 To calculate effective profit subtract the inflated value from the selling price.

$$\begin{aligned}\text{Effective profit} &= \text{SP} - \text{inflated value} \\ &= 7500 - 6390.14 \\ &= \$1109.86\end{aligned}$$

(c) Calculate the importer's effective percentage profit.

To calculate effective percentage profit, express the effective profit as a fraction of the inflation-adjusted value and multiply by 100%.

$$\begin{aligned}\text{Effective percentage profit} &= \frac{\text{effective profit}}{\text{current value}} \times 100\% \\ &= \frac{1109.86}{6390.14} \times 100\% \\ &= 17.4\%\end{aligned}$$

For calculations involving effective percentage profit, the profit is adjusted to allow for inflation, so the cost price must also be adjusted for inflation.

EXERCISE

1.6

Profit, loss and inflation

Worked
Example

- 1 Answer the questions for each item in the table.

- (a) Calculate the profit.
(b) Calculate the percentage profit. Round your answer to 1 decimal place, where necessary.

Item	Cost price	Selling price
T-shirt	\$10.50	\$15.40
Jacket	\$34.80	\$40
Pen	\$2.30	\$4.80
Hamburger	\$1.20	\$3.50

- 2 Answer the questions for each item in the table.

- (a) Calculate the loss.
(b) Calculate the percentage loss. Round your answer to 1 decimal place, where necessary.

Item	Cost price	Selling price
TV	\$800	\$780
Chocolate bar	\$1.20	\$0.72
Boat	\$65 000	\$42 500
Book	\$32.50	\$5.00

- 3 A fashion label import company has a fixed mark-up of 70% on every item. Calculate the selling price of the items in the table.

Item	Jeans	Shoes	Jacket	Scarf
Cost price	\$47	\$80	\$124	\$5.70

- 4 Assuming that housing prices have kept pace with the average inflation rate of 2.9%, what is the value of a house that 5 years ago was worth \$450 000?

- 5 The mark-up in a music shop is 80%. Select the option that is closest to the selling price of a book of sheet music with a cost price of \$18.

A \$14.40 B \$18.80 C \$21.60 D \$32.40

- 6 Calculate the percentage mark-up on each of the following items to 1 decimal place.

- (a) A sun hat: cost price = \$12.50, selling price = \$15.00
(b) A split-cycle air conditioner: cost price = \$500, selling price = \$980
(c) A speedboat: cost price = \$10 786, selling price = \$15 000
(d) An icy pole: cost price = \$0.72, selling price = \$1.80

- 7 The mark-up in an online kitchenware store is 15%. Calculate the selling prices of the following items.

- (a) A toaster: cost price = \$17
(b) A bottle opener: cost price = \$2.80
(c) A kettle: cost price = \$10.20
(d) A blender: cost price = \$23

Worked
Example

- 8 If inflation continues at the 2015–16 rate of 1% p.a. predict the value of each of the following in 5 years, to the nearest dollar.

- (a) A night out for a family, now \$250
- (b) The weekly earnings of a barista, now \$670
- (c) The cost of a week at the Gold Coast (accommodation only), now \$1500

30

- 9 The quarterly CPI figures for two consecutive years are shown in the table. All CPI figures are given as percentage change from the previous year.

	Jan–Mar	Apr–Jun	Jul–Sep	Oct–Dec
2012	2.9	3.1	2.8	2.7
2013	3.3	3.6	3.5	3.1

- (a) Calculate the annual CPI for the first year to 1 decimal place.
- (b) Calculate the annual CPI for the second year to 1 decimal place.
- (c) The Reserve Bank's target for inflation is 3%. How is the Reserve Bank likely to have adjusted the interest rates?
- (d) Use the annual inflation rate to determine the cost of a basket of goods at the end of the second year if they cost \$450 at the end of the first year.

..

..

32

- 10 A gold speculator buys gold bars for \$840 per ounce and sells them after 3 years for \$920. Inflation averages 2.7% over that time.

- (a) Calculate the percentage profit to 1 decimal place.
- (b) Calculate the effective profit.
- (c) Calculate the effective percentage profit to 1 decimal place.

- 11 Ben buys an antique clock for \$250 and sells it 3 years later at a profit of \$50. Inflation averages 2.2% over the 3 years. Ben calculates his effective percentage profit as follows:

$$\frac{300 - 250 \times 1.022^3}{250} \times 100\% = 13.25\%$$

Correct Ben's calculation.

- 12 A collection of rare stamps was purchased for \$2450 and sold 8 years later for \$3300. Over that period, inflation averaged 2.1% p.a.

- (a) Calculate the profit.
- (b) Calculate the percentage profit (to 1 decimal place).
- (c) Calculate the effective profit.
- (d) Calculate the effective percentage profit (to 1 decimal place).

- 13 These questions use constant or average annual CPI rates over a number of years.

- (a) Assume that the price of a hamburger follows an annual CPI that is a constant 2.3% p.a. over time. If the price of a hamburger in 2017 is \$4.20, calculate the price of a hamburger in 2050.
- (b) Assuming the CPI averages 2.6% over the next 10 years, calculate the price of a \$4 loaf of bread in 18 months if the price follows the CPI.

-
- A photograph of a classic cream-colored Volkswagen Beetle parked on a green lawn. The car is positioned in the lower half of the frame, angled slightly towards the right. To its right stands a small, vibrant yellow tree. The background features a dry, golden-brown field under a clear blue sky, with distant hills and a few scattered buildings visible on the horizon.

1.7

Making the most of your money

Unit cost

In the supermarket the same product may come in different sizes, and different brands will often offer a similar product in slightly different sizes, making it difficult to compare the value. One method of comparing prices is by comparing the cost per unit. By law, supermarkets must display the cost per unit measure on the ticketed price on the shelf, but this is often lost in the other information.

$$\text{Unit cost} = \frac{\text{total cost}}{\text{number of units}}$$

33 Calculating the best value

The same brand of coffee is available in three different sizes and prices: 50 g for \$4.60, 125 g for \$9.40 and 225 g for \$15.40.

Based on cost alone, which size is the best buy?

THINKING

- 1 To calculate the cost per unit, convert each price into cents and divide by the number of grams.

WORKING

$$\begin{aligned} \text{50 g jar:} \\ \frac{\$4.60}{50 \text{ g}} &= \frac{460 \text{ cents}}{50 \text{ g}} \\ &= 9.2 \text{ cents/g} \end{aligned}$$

$$\begin{aligned} \text{125 g jar:} \\ \frac{\$9.40}{125 \text{ g}} &= \frac{940 \text{ cents}}{125 \text{ g}} \\ &= 7.5 \text{ cents/g} \end{aligned}$$

$$\begin{aligned} \text{225 g jar:} \\ \frac{\$15.40}{225 \text{ g}} &= \frac{1540 \text{ cents}}{225 \text{ g}} \\ &= 6.8 \text{ cents/g} \end{aligned}$$

- 2 Compare the costs per gram to determine which is the best value.

The 225 g jar has the lowest cost per gram and represents the best value.

Cost is not the only thing to take into account when deciding on the best purchase. For example, purchasing a large jar of coffee might not be a good idea if you don't drink coffee often or you haven't tried that brand before; or maybe you have only enough money to purchase the small jar.

Mobile phone plans

The cost of a pre-paid phone is usually easy to understand, but the many different features available make it hard to compare different mobile phone plans. One way of investigating a new plan is to keep track of your current usage for a period of time and then apply that information to a potential new plan.

34 Calculating the cost of a mobile phone plan

Vinh keeps a record of his phone usage for a month. He makes 70 calls averaging 2 min each and 20 calls averaging 5 min. He also sends 180 texts. This usage was covered by his current plan, which costs \$55 per month.

He is offered a new plan for \$45 per month that allows \$100 worth of calls and 200 free texts. Calls are charged at 15 c per connection ('flagfall') plus 17 c for each 30 s of call time.

Is it worth changing plans?

THINKING	WORKING
1 Calculate the cost of a 2 min call. Include flagfall and cost for time taken.	$\begin{aligned}\text{Cost} &= \text{flagfall} + \text{cost per } 30\text{ s} \times \text{time (min)} \times 2 \\ &= 15 + 17 \times 2 \times 2 \\ &= 83\text{c}\end{aligned}$
2 Calculate the cost of a 5 min call. Include flagfall and cost for time taken.	$\begin{aligned}\text{Cost} &= \text{flagfall} + \text{cost per } 30\text{ s} \times \text{time (min)} \times 2 \\ &= 15 + 17 \times 5 \times 2 = 185\text{c}\end{aligned}$
3 Determine the total cost of calls. Give the cost in dollars.	$\begin{aligned}\text{Total cost} &= 70 \times 83 + 20 \times 185 \\ &= 9510\text{c} = \$95.10\end{aligned}$
4 Compare the costs of the two plans and choose the cheaper plan.	The new plan offers 200 free texts, which is more than his current usage of 180 texts. The new plan offers \$100 worth of calls, which is more than his current usage of \$95.10 worth of calls. Saving under new plan: $55 - 45 = \\$10$ per month The new plan covers his existing usage and is also cheaper, so he should change plans, assuming there are no hidden costs such as a fee for terminating his current plan early.

Creating a budget

Time passes quickly and it won't be long before you will want to purchase a car or move out of home, or maybe go on an overseas trip. Creating a budget will help you achieve your goals. To create a budget you could follow steps like these:

STEP 1: Have a goal in mind. Research the costs associated with achieving your goal. Decide on a realistic time frame and work out how much you will need to save each week to achieve your goal.

STEP 2: Keep track of your expenses for a month so you have an idea of where your money goes. Some of your costs are unavoidable: board, food or transport to work. Other costs are 'discretionary', meaning they are non-essential: things you can live without. Holidays, dining out, clothes (beyond what is absolutely needed) and entertainment can all be considered discretionary costs.

STEP 3: Think about avoiding or reducing your discretionary expenses. Start to spend amounts that you have budgeted for, rather than impulse spending.

A spreadsheet is ideal for creating and monitoring a budget.

Approximating months and weeks

A monthly cost is not simply a weekly cost multiplied by 4, because a month is not exactly 4 weeks. There is a better approximation:

$$\text{Monthly cost} = \text{weekly cost} \times \frac{52}{12}$$

This is still an approximation, because 52 weeks is actually only 364 days compared with the calendar year of 365 days (or 366 days in a leap year). However, it is still more accurate than $4 \times$ weekly costs, which would give only 48 weeks in a year: $4 \text{ weeks per month} \times 12 \text{ months} = 48 \text{ weeks}$.

35 Creating a budget

Lily is a 17-year-old girl living at home and paying \$20 per week for board. In addition to school, she works 6 h per week at the local supermarket, earning \$75 per week. She wants to buy a 128 GB tablet that costs \$789. She currently spends all her discretionary money on music, food (eating out and snack food), clothes and social activities.

(a) What is Lily's monthly discretionary income to the nearest dollar?

THINKING

- 1 Deduct the essential weekly expenses from the weekly income.
- 2 Multiply by 52 weeks to convert the weekly discretionary spending to an annual amount, then divide by 12 to determine the monthly amount.
- 3 Interpret the answer.

WORKING

$$\begin{aligned} \text{Discretionary income} &= 75 - 20 \\ &= \$55 \text{ per week} \\ &= 55 \times \frac{52}{12} \text{ per month} \\ &\approx \$238 \text{ per month} \end{aligned}$$

Lily has an approximate discretionary income of \$238.

(b) By budgeting to spend, each month, \$30 on music, \$40 on food, \$50 on her social life and buy no new clothes for the next 3 months, how much can Lily save?

- 1 Calculate the expenses under this budget.

$$\begin{aligned} \text{Expenses:} \\ 30 + 40 + 50 &= \$120 \end{aligned}$$

- 2 Determine the savings per month (the difference between the current expenses and the future budgeted expenses).

$$\begin{aligned} \text{Monthly savings:} \\ 238 - 120 &= \$118 \end{aligned}$$

- 3 Calculate the savings over 3 months.

$$\begin{aligned} \text{Savings over 3 months:} \\ 3 \times 118 &= \$354 \end{aligned}$$

- 4 Interpret the answer.

If Lily can stick to her budget, she has the potential to earn \$354 over the 3 month period.

(c) How much time will it take Lily to save the necessary money for a new tablet?

- 1 Determine the saving time frame by dividing the savings target by the amount saved each month.

$$\begin{aligned}\text{Months required} &= \frac{\text{savings target}}{\text{savings per month}} \\ &= \frac{789}{118} \\ &= 6.7 \text{ months}\end{aligned}$$

- 2 Interpret the answer, rounding up to the nearest whole month.

It will take Lily 7 months.

(d) Suggest how Lily might achieve her goal earlier.

THINKING

There are two ways to save faster: earn more income or spend less.

WORKING

Suggestions may vary. For example:

Lily could try to work extra shifts to increase her income.

Lily could spend even less of her discretionary money on music, eating out and social activities.

e Explore further

Budgeting

Complete the spreadsheet budget to determine your cost of living.

Spreadsheet formulas for different time frames

Spreadsheets are useful for converting weekly amounts to monthly, quarterly or annual amounts, and for finding totals.

36 Spreadsheet formulas for a budget

Use spreadsheets formulas to complete the table for car expenses. State the monthly total, rounded up to the nearest \$10, that must be allocated to car expenses.

Item	Weekly	Monthly	Annually
Petrol	\$80		
Cleaning		\$20	
Tyres			\$200
Registration			\$560
Insurance			\$350
Service			\$150
Repairs			\$400
Total			

THINKING

- 1 Input the headings and given values. Do not use units or spacing of any kind with the values.

- 2 Fill in the amounts, beginning with annual.

For D 2: = B 2 * 52 For D 3: = B 2 * 12

For weekly amounts, use the annual amounts.
Enter one formula and fill the rest.

For B 3: = D 3 / 52

For monthly amounts, use the annual amounts.
Enter one formula and fill the rest.

For C 4: = D 4 / 12

For any of the total values, use the automatic summation formula, then drag across to fill the other totals.

For D 9: = SUM(D 2 : D 8).

- 3 Interpret the answer.

WORKING

	A	B	C	D
1		Weekly	Monthly	Annually
2	Petrol	80		
3	Cleaning		20	
4	Tyres			200
5	Registration			560
6	Insurance			350
7	Service			150
8	Repairs			400
9	Total			

	A	B	C	D
1		Weekly	Monthly	Annually
2	Petrol	80	347	4160
3	Cleaning	5	20	240
4	Tyres	4	17	200
5	Registration	11	47	560
6	Insurance	7	29	350
7	Service	3	13	150
8	Repairs	8	33	400
9	Total	117	505	6060

Enter whole numbers only, as most values will be estimates.

\$505 each month must be allocated to car expenses.

Using a budget to plan

Budgets are useful for estimating how much can be put aside after the essentials have been paid, to plan future ventures.

37 Saving using a budget

Eli has an after-tax annual salary of \$43 000. He moves into shared rental accommodation and will contribute \$350 per week towards rent, bills, food and cleaning. His car costs him \$2800 each year for registration, insurance and maintenance, and he spends \$45 each week on petrol. He has a \$39 monthly mobile phone plan, which he never exceeds, and he spends the rest of his money on entertainment, clothes, holidays, sporting events, music, takeaway food and snacks.

- (a) Calculate the non-discretionary expenses for the year.

THINKING

Add all the essential expenses.

WORKING

Non-discretionary expenses annually:

$$\begin{aligned}\text{Accommodation} &= 350 \times 52 \\ &= \$18\,200\end{aligned}$$

$$\begin{aligned}\text{Car} &= 2800 + 45 \times 52 \\ &= \$5140\end{aligned}$$

$$\begin{aligned}\text{Mobile plan} &= 39 \times 12 \\ &= \$468\end{aligned}$$

Total non-discretionary expenses for the year:

$$18\,200 + 5140 + 468 = \$23\,808$$

- (b) Calculate the weekly discretionary income.

Subtract the non-discretionary costs from annual income. Divide by 52 to find the weekly amount.

Discretionary income:

$$\begin{aligned}43\,000 - 23\,808 &= \$19\,192 \text{ per year} \\ &= 19\,192 \div 52 \text{ per week} \\ &= \$369 \text{ per week (nearest \$)}\end{aligned}$$

- (c) Create a budget that will allow Eli to save \$100 per week and maintain his current expenses.

Allocate the weekly discretionary money to cover expenses, including the savings target.

Allocations may vary. Sample answer:

Saving: \$100

Clothes: \$40

Entertainment: \$100

Holidays: \$0

Music: \$35

Sporting events: \$40

Takeaway food and snacks: \$50

Total: \$365

This gives Eli \$4 leeway.

- (d) Suggest how Eli might increase his savings to buy a new car in a year's time.

To increase savings, there must be more income and/or less discretionary spending.

Eli has a salaried job, so it is not easy to increase income; it might be possible to work overtime, be promoted or change jobs.

Any of the discretionary expenses might be reduced.

EXERCISE

1.7

Making the most of your money

Worked
Example

33

- 1 Corny Brits breakfast cereal is available in three sizes, and prices are given in the table. Determine the unit cost in cents per gram for each. Based on unit cost alone, explain which size is the best buy.

Cost	\$3.60	\$7.00	\$10.50
Size	400 g	750 g	1.25 kg

- 2 Select the option that is closest to the cost per gram of a 750 g pack of cereal priced at \$4.24.

A 0.48 cents

B 0.57 cents

C 0.80 cents

D 1.77 cents

- 3 The costs of various sizes of breakfast cereals are shown in the table.

- (a) Based purely on price, which size is the best-value buy for Crispex?
- (b) In a choice between Brantastic and Rice is Nice, which brand represents the better value?
- (c) Assuming that price is the only consideration, which is the best-value brand and packet size?
- (d) If you only had \$4 to spend, which option would be the best choice?

Brand	Size (g)	Cost (\$)
Crispex	275	2.83
	550	4.65
	800	5.40
Sprinkles	425	4.25
	675	5.95
Brantastic	525	5.10
Popbix	300	3.40
Rice is Nice	425	3.40

- 4 A supermarket sells canned tomatoes in the sizes shown in the table.

Brand A	220 g at \$0.54	425 g at \$1.05	810 g at \$1.69
Brand B		410 g at \$0.85	800 g at \$1.35

- (a) If price is the only issue, which tin represents the best value?
- (b) If you only need 400 g of tomatoes, which can is the best value?
- (c) You need 750 g of tomatoes, but you don't like the taste of Brand B. Which cans of Brand A should you buy to get the best value for money?

34

- 5 Phone plans consist of many charges. Calculate the charge for each of the following situations.

- (a) Mateo's monthly phone plan includes 200 MB of data and charges 25 c per MB over 200 MB. Calculate the excess data charge if he uses 500 MB per month.
- (b) Simone's \$29 monthly smartphone plan is advertised to give \$250 of value, which includes \$25 of texts, \$175 of calls and \$50 of data. She is charged for any extra data, calls or texts beyond this. The charges are 50 c per MB of data, 30 c 'flagfall' plus 15 c for every 20 s of call time for calls, and 25 c per SMS text. Her actual monthly usage is 200 calls at an average of 2 min each, 120 texts and 240 MB of data.
- (i) Based on her usage, calculate the combined cost of her calls, texts and data.
- (ii) Calculate her excess charge.
- (iii) Calculate her total charge for the month.
- (c) Benito likes the inclusions in a particular phone plan. The summary states that total costs over 24 months amount to \$2807.90. Calculate the amount Benito needs to allocate to phone use each week. Round up to the nearest dollar.

- 6 A two-week holiday in New Zealand in six months' time sounds like a good idea to Kate, so she researches all the costs she can think of and creates a list.

Item	Estimated cost
Return airfares	\$600
Accommodation	\$120 per night
Food	\$100 a day
Rental car	\$600 for 2 weeks
Entry fees	\$50 a day
Gifts/souvenirs	\$400 total

- (a) Calculate the estimated cost of the trip.
- (b) Calculate the cost of her airfares, accommodation and car hire.
- (c) Kate currently earns \$1500 per week and spends it all. How much does she need to save per week if she wants to pay for her airfare, car hire and accommodation 2 months before she leaves? Round appropriately.
- (d) After paying for her airfare, car hire and accommodation, how much will Kate need to budget per day to meet her expected expenses for the rest of the trip? Round appropriately.

Worked
Example

- 7 Determine Calvin's total annual expenses from the following estimates: electricity: \$105/quarter; rent: \$250/fortnight; groceries: \$65/week; public transport: \$42/week for 48 weeks each year.

35

- 8 Create a spreadsheet to display Calvin's expenses from the previous question. Use the spreadsheet to specifically determine the amount Calvin will have left over from his fortnightly take-home pay of \$650 after putting aside money for the essential expenses on his list. Round up the fortnightly expenses to the nearest \$10.

36

- 9 Rashid is considering a new phone plan, so he keeps a record of his phone usage for a month. In one month he makes 90 calls averaging 3 min each and 10 calls averaging 10 min each. He also sends 150 texts. His current phone plan costs \$35 per month and covers his existing needs. He can switch to a new plan that costs \$29 per month and will allow \$100 of calls and 200 free texts. Calls are charged at 40c per connection ('flagfall') plus 25c for each 30s of call time.

- (a) Calculate the cost of a 3 min call.
- (b) Calculate the cost of a 10 min call.
- (c) Calculate the total cost of calls on the new plan.
- (d) Is it worth changing plans?

- 10 Jackson signs up to a smartphone plan offering \$550 of value for \$17.99 a month. This includes 1GB of data, call charges of 40c flagfall plus 99c per minute, and 50c per SMS text. Excess data is charged at 50c per MB and calls at the standard rate.

- (a) Calculate his monthly charge for 800 MB of data, 105 calls lasting a total of 380 min and 120 texts.
- (b) Calculate his monthly charge for 1200 MB of data, 85 calls lasting a total of 210 min and 70 texts.
- (c) Calculate his monthly charge for 2000 MB of data, 120 calls lasting a total of 410 min and 30 texts.

- 11 Alina has a take-home pay of \$3200/month. She estimates her regular expenses as follows: electricity \$85/quarter; rent \$350/fortnight; groceries \$95/week; car (petrol, registration and insurance) \$4000/year.

- (a) Calculate Alina's weekly leisure allowance.
- (b) If Alina wants to save \$10000 each year, what will be her reduced weekly leisure allowance? Round down to the nearest whole dollar.

37

- 12 Jesse is backpacking in Spain. He has one final night in a hostel and requires 100 g of rice, 150 g of tuna and 5 apples to see him through. The table shows what he can buy at the local supermarket.
- Nothing will be wasted, as the other hungry backpackers will consume the leftovers.
- What should Jesse purchase, and what is the total cost?

Item	Quantity	Price (converted to A \$)
Rice and tuna combo	100 g each	\$4.20
Tuna	250 g	\$3.10
Rice	250 g	\$0.95
Apples	10	\$3
Apple	individual	\$0.50

- 13 Bluey is considering the full costs of moving out of home, before deciding whether to move out. Many of the expenses depend on personal priorities, so he draws up a list of possible expenses and estimated costs. Use Bluey’s research to draw up a list of costs as if you are considering moving out of home yourself. (This is an ideal situation for a spreadsheet, because the total cost can be automatically updated every time you make a change.)

Before you start, see if you can think of any extra expenses that Bluey hasn’t considered.

Expense	Expense details/notes	Costs (one-off costs unless noted)
Bond	Moving into a rental property	\$1000–2000
Rent	Total rent is divided by the number of people sharing the space.	\$100–400 per week per person
Food		\$100–200 per week per person
Utilities	Electricity, gas, internet, phone	\$50–100 per week per person plus connection fees (unless moving into existing sharehouse): \$0–500
Furniture	Can be hired, borrowed, bought second-hand. Not everything is needed at once. Some furniture costs may be divided by the number of people sharing the furniture.	Bed: \$50–2000 Table and chairs: \$50–1000 TV: \$200–2000
Whitegoods, other appliances	Can be hired, borrowed, bought second-hand. Not everything is needed at once. Costs may be divided by the number of people sharing the appliances.	Fridge: \$100–1500 Washing machine: \$50–1000
Clothes		\$10–50 per week
Entertainment/sport		\$10–50 per week
Transport		Running a car: \$80–200 per week Public transport: \$50–100 per week
Unpredictable/fun spending		\$0–200 per week
Insurance		Health: \$0–120 per week Car: \$1000 per year House contents: \$0–50 per week

- 
- 14 Martin works 10 h each week helping at a childcare centre and is paid, on average, \$17.50 per hour after tax. He wants to save for a weekend holiday in New Zealand to watch a Wallabies and All-Blacks match. He finds that the cost of his flights, accommodation, food and match ticket would total approximately \$800. He decides to budget \$50 each month for music, \$100 for food, \$80 on his social life and nothing on clothes, as well as paying \$300 for board. To the nearest month, how long will it take Martin to save the necessary money for his trip?
- 15 Zaria has an after-tax annual income of \$31 000. She moves into shared rental accommodation and will contribute \$300 per week towards rent, bills, food and cleaning. She has a car, which costs \$2200 per year for registration, insurance and maintenance, and she spends \$55 per week on petrol. She has a \$29 monthly smartphone plan, which she considers essential and never exceeds. She spends her discretionary money on entertainment, clothes, holidays, sporting events, music, dining out and snacks. Create a weekly budget that will allow Zaria to save \$100 per week and maintain her current expenses.
- 16 You want to market a new 400 g jar of coffee, so you research the current prices and find that for this type of coffee you can already buy 50 g for \$2.47, 100 g for \$4.65 and 250 g at \$9.51. At what price should you market your product, if price is the only consideration? Justify your answer.

1.8

Working with finance

Marketers, retailers and those who work in the finance industry generally have a better understanding of how the system works than their customers. Remember, the job of business is to maximise the profit for themselves or for their clients, so be aware of the 'fine print' of common transactions.

Buying on credit

Credit cards often offer an interest-free period of up to 60 days from the time of purchase. While this sounds attractive, if you miss paying in full by the final day then you may be charged interest on the full amount back to the original purchase date.

Credit card interest is charged on the average daily amount overdue, using the simple interest formula, then added to the amount owing at the next due date.

38 Credit card calculation

Rowan uses his credit card to purchase a new sound system for \$1450. Unfortunately, he can only pay \$1000 at the end of the 60 day interest-free period.

- (a) Calculate the amount that Rowan still owes, before interest is charged.

Up to 60 days
interest free

19% p.a.
interest*

HB
HappyBank

*Calculated daily from the date the debt is incurred if not paid by the due date.

THINKING

- 1 Subtract the amount paid from the original amount owed.
- 2 Write your answer in a sentence.

WORKING

$$P = 1450 - 1000$$

$$= \$450$$

Rowan owes \$450.

- (b) Calculate the interest Rowan will see on his next credit card statement if he pays the balance of the amount owed a week after the due date.

- 1 Identify the variables needed to use the simple interest formula.
Note: The period of time is the interest-free period plus the number of days after the due date.

$$P = \$450, i = 19\% \text{ p.a.} = 0.19$$

$$n = 60 + 7$$

$$= 67 \text{ days}$$

$$= \frac{67}{365} \text{ years}$$

- 2 Recall the simple interest formula $I = Pin$, substitute the given information and calculate the interest using the simple interest formula.

$$\begin{aligned} I &= Pin \\ &= 450 \times 0.19 \times \frac{67}{365} \\ &= \$15.69 \end{aligned}$$

- 3 Interpret the answer.

Rowan will see \$15.69 interest on his next statement.

Cars, appliances and furniture are sometimes available to be purchased under schemes similar to credit card debt. These schemes allow you to purchase an item 'on credit' or 'on finance' (without paying the total cost up front), and then to pay back the total cost slowly, with 12, 24 or even up to 48 months interest free. In these schemes, 'interest free' means that no interest is charged on the amount owed as long as regular payments are made, so you simply pay back the amount owed slowly over time. Cash buyers are usually given a discount, so in this way the purchase over time will cost more.

Car dealers often advertise finance in this way. The asterisk (*) after the price shows that you must read the fine print to see the conditions and understand the full costs. In this case, as well as the advertised cost per week a deposit is required (an extra cash amount paid upfront), as well as a balloon payment (an extra cash amount paid at the end of the finance period). A balloon payment is also called a residual payment.



39 Buying a car on finance

Use the image above to answer the following questions.

Edgar is intrigued by 'hot rod' cars and wants to buy one on finance.

- (a) Calculate the deposit required.

THINKING

Express the deposit percentage as a decimal and multiply by the cash price.

WORKING

$$\text{Deposit: } 0.2 \times 23\,899 = \$4779.80$$

- (b) Calculate the total repayments.

Multiply the repayment amount by the number of repayments to be made.

$$\text{Total repaid: } 403 \times 60 = \$24\,180$$

- (c) Calculate the total cost that Edgar must pay if paying for the car over 60 months.

Add the deposit, repayments and balloon payment.

$$\begin{aligned} \text{Total cost:} \\ 4779.80 + 24\,180 + 5000 = \$33\,959.80 \end{aligned}$$

- (d) How much extra will Edgar pay if he buys on finance rather than with cash?

Calculate the difference between the cash price and the total cost on finance.	Extra due to finance (I): $33\,959.80 - 23\,899 = \$10\,060.80$
This is equivalent to an amount of interest I charged on the amount owed P .	

- (e) What is the equivalent flat (simple interest) rate charged on the amount owed?

1 Subtract the deposit from the purchase price to determine the principal of the money owed.	Amount borrowed (P): $23\,899 - 4779.80 = \$19\,119.20$
2 Calculate the interest rate i using the interest formula.	$i = \frac{I}{nP}$ $= \frac{10\,060.8}{5 \times 19\,119.20}$ $= 0.1052\dots$ $= 10.52\% \text{ p.a. (2 d.p.)}$

Transactions that are 'on credit' or 'on finance', as in the previous worked example, are sometimes also called *hire-purchase* arrangements.

Foreign exchange

Other countries usually do not accept Australian currency (money), so when you travel overseas you must use a foreign currency. The exchange rates between different currencies change daily, so the example rates given in this section are unlikely to be accurate today. Travellers usually remember an approximate exchange rate, so they can quickly calculate in their head whether a product or service is good value when it is expressed in a foreign currency.

Note that in real life there are different exchange rates depending on whether you are buying or selling currency, to allow whoever is exchanging the currency to make a profit from the exchange. For the following problems, for simplicity assume there is only a single exchange rate, whether buying or selling.

Number of Australian dollars = value in foreign currency $\div$ exchange rate

Value in foreign currency = number of Australian dollars $\times$ exchange rate

40 Exchanging money

Abigail has just returned from a holiday in Asia. She has 142 000 Vietnamese dong, 1500 Malaysian ringgit and 4000 Thai baht that she wants to exchange for Australian dollars.

- (a) How much will she get back using the following exchange rates?

A \$1 = 21 940 dong; A \$1 = 3.19 ringgit; A \$1 = 31.14 baht

THINKING

- 1 Recall the formula for currency conversion:

$$\text{Number of Australian dollars} = \text{value in foreign currency} \div \text{exchange rate}$$
- 2 Add the amounts (in the same currency) to determine the total amount.
- 3 Interpret the answer.

WORKING

From 142000 dong:
 $142000 \div 21940 = \text{A } \$6.47$

From 1500 ringgit: $1500 \div 3.19 = \text{A } \$470.22$

From 4000 baht: $4000 \div 31.14 = \text{A } \$128.45$

Total = $6.47 + 470.22 + 128.45$
 $= \text{A } \$605.14$

Abigail's currency will be exchanged for A\$605.14.

- (b) Abigail takes a trip later in the year and exchanges A \$200 for US dollars. How much will she get if A \$1 = US \$0.75?

- 1 Recall the formula for currency conversion:

$$\text{Value in foreign currency} = \text{number of Australian dollars} \times \text{exchange rate}$$
- 2 Interpret the answer.

From A \$200:
 $200 \times 0.75 = \text{US } \$150$

Abigail will receive US \$150.

WARNING

Always check the reasonableness of your answer. For example, if the exchange rate is A \$1 = US \$0.75, this indicates that for every Australian dollar you exchange, you will receive only 75 cents US. Therefore, the numerical value of the AU currency will always be greater than the US.

EXERCISE

1.8

Working with finance

- 1 Athalia uses her D-mart credit card to purchase a new computer system for \$1250. Unfortunately she can only pay \$800 at the end of the 40 day interest-free period.
 - (a) Calculate the amount Athalia will still owe before any interest is charged.
 - (b) Calculate the interest that will appear on her next statement if she pays the balance owed 2 weeks after the due date.

D-mart Credit Card

UP TO 40 DAYS INTEREST FREE

14% p.a. interest*

*Calculated daily from the date the debt is incurred if not paid by the due date.



Worked Example

38

- 2 Determine the equivalent of A \$600 in each of the following foreign currencies.
- New Zealand dollars when A \$1 = NZ \$1.09
 - Japanese yen when A \$1 = JPY 83.27
 - US dollars when A \$1 = US \$0.75
- 3 Determine the equivalent in Australian dollars for each of the following foreign currency amounts.
- 600 Chinese yuan when A \$1 = CNY 5.16
 - 6000 Egyptian pounds when A \$1 = EGP 13.56
 - 50 euros when A \$1 = EUR 0.69
- 4 Determine the value of P , the amount borrowed, in each of the following situations.
- A car is sold for \$80 233 and a deposit of \$50 000 is required.
 - A house is sold for \$690 500 and a deposit of 15% is required.
 - A washing machine is sold for \$1425 and a deposit of 30% is required.
- 5 Virginia has just returned from a European holiday and has some leftover euros and British pounds. She has €430 and £54. She wants to exchange this money for Australian currency.
- If the exchange rates are A \$1 = €0.789 and A \$1 = £0.631, calculate how much Virginia will receive for her foreign money, and justify the reasonableness of your answer.
 - Later in the year, Virginia takes a trip to New Zealand and exchanges A \$300 for New Zealand dollars. Given an exchange rate of A \$1 = NZ \$1.26, calculate how much Virginia will receive in New Zealand dollars.
- 6 Select the option that is closest to the deposit required for the car in the advertisement.
- \$425
 - \$4160
 - \$6800
 - \$19 200

Worked
Example

40



- 7 Barry takes a \$250 cash advance on his credit card, thinking he has 45 days to pay it back before interest is charged. Unfortunately, he has not read the fine print and he is charged interest at 12.4% p.a. simple interest for 45 days. How much interest will he owe the bank for this cash advance?

- 8 Dan wants to compare the savings of paying cash over paying on credit for a used car. Where rounding is required, round your answers to 2 decimal places.
- Calculate the deposit required.
 - Calculate the total of the monthly repayments if Dan buys the car on the finance terms offered.
 - Calculate the total cost of the car on finance.
 - How much less will Dan pay if he pays cash rather than buying on credit?
 - Calculate the equivalent flat (simple interest) rate charged if the car is bought on finance.



- 9 When buying an item on finance, a residual or 'balloon' payment is often required at the end of the repayment period, as well as a deposit at the start. It would seem easier to just ask for a bigger deposit at the start and no balloon payment, so why is this done? See if you can understand the advantage (to the lender) of a balloon payment by working through the following.
- Ruby buys a \$2000 TV for 20% deposit and 24 equal monthly repayments calculated at a flat rate of 15% p.a. interest. Calculate:
 - the deposit
 - the interest on the outstanding balance
 - the amount of each monthly repayment
 - the total cost of the TV.
 - Kathryn also buys a \$2000 TV, but her terms require a 10% deposit and a 10% residual ('balloon') payment, with 24 equal monthly repayments calculated at a flat rate of 15% p.a. interest. Calculate:
 - the deposit
 - the cost of the balloon payment
 - the interest on the outstanding balance (after paying the deposit)
 - the monthly repayments. You can use the following formula:

$$\text{Repayment} = ((\text{amount owing} - \text{balloon payment}) + \text{interest}) \div \text{number of repayments}$$
 - the total cost of the TV.
 - How do the two systems of repayment compare?
- 10 Ray goes to the UK with A \$1000 worth of British pounds on his travel card. When he changed the money originally, the exchange rate was A \$1 = £0.622. He doesn't use his travel card on the trip and changes the money back to Australian dollars on his return. In the meantime the exchange rate has changed to A \$1 = £0.689. How much money does Ray make or lose?

- 11 Gina begins working for Lenny the Lender's Finance Company and finds that she doesn't understand the reasons behind quite a few of the company policies. Explain the mathematical thinking behind each policy that gives an advantage to Lenny.
- For savings accounts, the interest is calculated monthly on the lowest balance for the month.
 - A special savings account offers double the normal interest rate each month, provided no withdrawals are made in the month. If there are any withdrawals then no interest is paid.
- 12 Megan has a travel card with her bank and pays no fees for transferring money from one currency to another. She has JPY 2200 (Japanese yen), \$US 132 and €51 (euros) left on the card from previous trips. Now she wants to travel to New Zealand. If she exchanges all her currency for NZD, how much in AUD must be transferred to ensure she has NZ\$1000 in the account for her trip?
- Exchange rates at the time of transfer are A\$1 buys NZ\$1.08, US\$0.75, €0.71 or JPY 83.33.
- 13 Marcos uses a hire-purchase deal to buy a \$3200 bed. The deal requires a 10% deposit and 36 equal monthly repayments, where the repayments include a flat rate of 8% p.a. interest on the amount that is 'financed' (that is, the amount still owed after paying the deposit). Calculate:
- the monthly repayment
 - the total cost of the bed under this hire-purchase deal.
-
- 14 Romney wants to buy an investment property, which she will rent out. She knows of a house for sale in Mackay for \$450 000, for which she will be able to charge rent of \$1650 per month. Having just returned from an overseas holiday in Malaysia, she also knows she can buy a block of six units in Kuala Lumpur for 1.4 million Malaysian ringgit and each unit can be rented out for 800 ringgit per month. Assume an exchange rate of A \$1 = 3.18 ringgit.
- Calculate the cost of the Malaysian block of units in AUD.
 - Calculate the monthly rent from the six Malaysian units in AUD.
 - Which property represents the better buy? Explain your answer.
- 15 A new home theatre system has a cash price of \$1150. It can be bought on interest-free terms of \$50 per month over 24 months. The fine print states that a one-off account-keeping fee of \$50 is also payable and that if any payment is late then interest is charged at a flat rate of 8% p.a. on the original debt (including the account-keeping fee) for the entire period.
- Assuming all payments are made on time, calculate the cost on credit.
 - Calculate the interest charged if the final payment is late but all other payments are on time.
 - Do you think this is fair? Explain your reasons.

Summary

Percentages

To convert a fraction or a decimal to a percentage: multiply by 100 and attach the percentage symbol.

To increase an amount by a percentage: add 100% to the percentage, convert to a decimal and then multiply the amount by the decimal formed.

To decrease an amount by a percentage: subtract the percentage from 100%, convert to a decimal and then multiply the amount by the decimal formed.

Taxation

The Goods and Services Tax (GST) adds 10% to the price of most products and services.

Personal income tax is paid from your wage as you earn it. This is why it is also known as PAYG ('pay as you go') tax. The rate of taxation increases as you earn more money.

At the end of the financial year a tax return is required, to assess whether an individual has paid enough or too much tax during the year. Deductions are allowed for particular costs incurred in earning the income. A *Tax brackets and tax rates* table showing tax to be paid according to income is used to calculate what is due.

Taxable income = gross income – deductions

GST price = pre-tax price $\times 1.1$ or Pre-tax price = $\frac{\text{GST}}{1.1}$

Simple interest

$$I = Pin$$

where:

I is the interest paid (\$)

n is the number of times interest is added

P is the principal (\$)

i is the interest rate for the same time period as n expressed as a decimal.

Compound interest

$$A = P(1 + i)^n$$

where:

A is the amount or balance at a given time

n is the number of times interest is added

P is the principal (\$)

i is the interest rate for the same time period as n expressed as a decimal.

Profit and loss

$$\text{Profit} = \text{sale price} - \text{cost price}$$

The percentage profit or loss is calculated using the formula:

$$\% \text{ Profit or loss} = \frac{\text{profit (or loss)}}{\text{cost price}} = 100\%$$

Comparing prices

To compare prices, calculate the cost per unit for each product and identify the one costing the least per unit.

$$\text{Unit cost} = \frac{\text{total cost}}{\text{number of units}}$$

Share market

$$\text{Dividend} = \frac{\text{returned profit}}{\text{number of shares}}$$

$$\text{Price-to-earnings ratio} = \frac{\text{market price per share}}{\text{annual earnings per share}}$$

Foreign exchange

$$\text{Number of Australian dollars} = \text{value of foreign currency} \div \text{exchange rate}$$

$$\text{Value of foreign currency} = \text{number of Australian dollars} \times \text{exchange rate}$$

where exchange rate is given as the amount of foreign currency per A \$1

Chapter review

1

- 1 Which option is closest to 8.5% of 659?

A 7.8 B 54.0 C 56.0 D 77.5

Exercise 1.1

- 2 Complete the following percentage increases and decreases.

Exercise 1.1

- (a) Increase \$840 by 30%.
 (b) Decrease \$840 by 18%.
 (c) If \$840 is the result of a 5% increase, what was the original amount?

- 3 Consider a vehicle travelling at 65 km/h. Write your answers in appropriate units, to 1 decimal place.

Exercise 1.1

- (a) Calculate the distance travelled in:
 (i) 3h (ii) 10 min (iii) 5 s
 (b) Calculate the time it takes to go:
 (i) 300 km (ii) 10 km (iii) 2 m

- 4 Henri earns a base salary of \$400 a week plus a commission of 5% on his sales. If sales were \$2850 for the week, determine his weekly salary.

Exercise 1.2

- 5 Jake is paid a commission of \$623.25 on sales of \$13850. What was the percentage commission?

Exercise 1.2

A 4% B 4.5% C 4.6% D 4.8%

- 6 From a commission of 11% Linley earns \$975. Calculate the value of her sales to the nearest dollar.

Exercise 1.2

- 7 Sasha, Sanjeev and Chen all work at the same electronics shop. Use the timesheet shown to calculate the pay of Sasha, Sanjeev and Chen.

Exercise 1.2

Employee	Standard hours worked	Overtime hours (paid at time-and-a-half)	Base rate per hour
Chen	35	0	\$17.50
Sanjeev	6	10	\$15.00
Sasha	40	2	\$16.80

- 8 Use the *Tax brackets and tax rates* table for these questions.

Brandon worked three jobs. His gross income and tax paid for the financial year are shown.

- (a) Calculate Brandon's total gross income.
 (b) How much tax did Brandon pay?
 (c) He has deductions of \$1025. What amount is his tax payable?
 (d) Calculate the amount of his refund or debt.

Workplace	Gross income	Tax paid
Secondary school	\$51634	\$8122
Education Union	\$4800	\$1560
Theatre Company	\$17540	\$5865

Exercise 1.3

- Exercise 1.3 9 For the income bracket \$37 001–\$87 000, the tax payable is \$3572 plus 32.5c in every \$1 above \$37 000. What is the tax payable on an income of \$64 553?
- Exercise 1.3 10 For the income bracket \$87 001–\$180 000, the tax payable is \$19 822 plus 37c in every \$1 above \$87 000 (as shown in the *Tax brackets and tax rates* table). Pahkip earns \$95 650 and has \$6300 of work-related deductions. Calculate the tax payable on his taxable income.
- Exercise 1.3 11 People in the income bracket \$55 874–\$62 238 who have a HELP debt for tertiary education are required to pay 4.0% of their income towards their debt. Sungku had a taxable income of \$59 000 and has a HELP debt of \$22 340. Calculate Sungku’s remaining HELP debt after making the required repayment.
- Exercise 1.4 12 Calculate the simple interest paid over 3 years on a \$7000 investment at a rate of 4.8% p.a.
A \$336 B \$1000 C \$1008 D \$5992
- Exercise 1.4 13 Determine the amount of interest earned on \$780 invested at 6% p.a., compounded monthly over 2 years.
- Exercise 1.4 14 Ngaio invests \$50 000 for 2 years at 5.1% p.a. Calculate the value of the investment to the nearest cent if the interest is compounded at the following time intervals.
(a) annually (b) quarterly (c) monthly (d) daily
- Exercise 1.6 15 An importer gets her T-shirts for a cost price of \$18.50 and adds a profit margin of 60%. What is the selling price?
A \$7.40 B \$11.20 C \$19.10 D \$29.60
- Exercise 1.6 16 Lawrence buys model trains and resells them to make a profit when he can. He purchases a replica of *Stephenson’s Rocket*, an early steam locomotive, for \$1200 and sells it 3 years later for a profit of \$800.
(a) Calculate his percentage profit to 1 decimal place.
(b) If inflation averaged 2.6% p.a. over those 3 years, calculate his effective profit to the nearest cent.
- Exercise 1.6 17 A chocolate bar in 2015 costs \$4.30. Assuming the price rises with a steady rate of inflation of 3.1% p.a., what is the price of a chocolate bar in 2017?
- Exercise 1.7 18 Which option is closest to the cost per gram of a 1.25 kg bag of flour priced at \$2.80?
A 0.022c B 0.22c C 22c D 2.24c
- Exercise 1.7 19 Sports drinks are available in three sizes: 600 mL for \$6.40, 325 mL for \$3.50 and 500 mL for \$5.15. Based on price alone, which size represents the best buy?
- Exercise 1.8 20 On returning to Australia from an overseas holiday, Sedera wants to exchange his USD for AUD. The rate offered is A \$1 to US \$0.78. There is also a A \$5 fee to make the exchange. How many AUD will Sedera get for his US \$250?
- Exercise 1.3 21 A store takes delivery of 1000 packets of soap, which cost the store \$2750. The store policy is to add a 70% mark-up and then add GST.
(a) Calculate the marked-up price of a single packet of soap before adding GST.
(b) Calculate the selling price of a single packet of soap including GST.
(c) Increase \$2.75 by 80%. Explain why the sale price is higher than this amount.

- 22 Khloe takes out an \$18000 loan to buy a car at a flat rate of 10.95% p.a. She agrees to pay the loan in equal monthly instalments over 4 years. Calculate her monthly repayments rounded up to the next dollar. Exercise 1.4
- 23 Kamil purchased his share portfolio one year ago and now wants to revise his investment. The portfolio is shown in the table. Exercise 1.5

Company	Number of shares	Purchase price	Current price	Annual dividend per share
Mike's Corporation	3000	\$2.30	\$2.80	\$0.80
Sunstart Communications	5000	\$1.25	\$1.30	\$0.20
CIO Mining	4000	\$23.80	\$22.50	\$1.10
YHSC Bank	800	\$1.65	\$2.10	\$0.10

- (a) Calculate the total price paid for the shares.
- (b) Calculate Kamil's total dividend from his investments.
- (c) To 1 decimal place, calculate Kamil's percentage return on his initial investment.
- (d) Calculate the price-to-earnings ratio for each company to determine which company represents the best investment, assuming all profits are paid in dividends.
- (e) If Kamil sells his shares in CIO Mining, calculate his capital gain or capital loss from the sale, not including the dividend and ignoring any taxation implications.
- 24 Mary buys some sporting memorabilia as an investment. She pays \$3775 and sells it 5 years later for \$4500. During that time inflation averaged 1.8% p.a. What is her effective profit or loss? Exercise 1.6
- 25 Nick purchases a new car for \$24500 and 2 years later the value has dropped to \$18200. Determine the annual percentage decrease in the value of the car. Give your answer to the nearest whole percentage. State any assumptions you made in choosing your calculation method. Exercise 1.6

- 26 Mackenzie falls in love with a kitten at the pet shop and finds that the price is \$90 for the wormed and vaccinated kitten. Her father says that there are more costs to be considered and makes a list of the ones he can think of.

Item	Cost
Bedding	\$20 a year
Kitty litter	\$2 a week
De-sexing	\$130
Food	\$9 a week
Toys	\$20 a year
Vet bills	\$200 a year

- (a) Calculate the cost of having a cat for the first 12 months, assuming it is de-sexed at 6 months of age. Exercise 1.7
- (b) Assuming that the costs rise with the CPI and this averages 2.3% p.a. over the next 5 years, calculate the total cost of the cat for the first 5 years. (A spreadsheet could be used here.) Make any necessary assumptions, and round your answer up to the nearest \$100.
- (c) The cat lives for a total of 12 years. Calculate the cost in the twelfth year (to the nearest dollar), assuming inflation has remained steady at 2.3% p.a. State another assumption relating to this estimate.

Exercise 1.8

- 27 A car is offered for a cash-sale price of \$23 000 or on credit with a deposit of 20% and 60 monthly repayments of \$425. What is the flat rate of simple interest charged? Give your answer to 1 decimal place.

Exercise 1.4

- 28 The value of a car starts to decrease the moment it is driven from the dealers. A new 'K-Star Cruiser' costs \$38 950 drive away, no more to pay. A spreadsheet would be ideal for this task.

- (a) The manufacturer suggests the depreciation rate will be a steady 8% p.a.

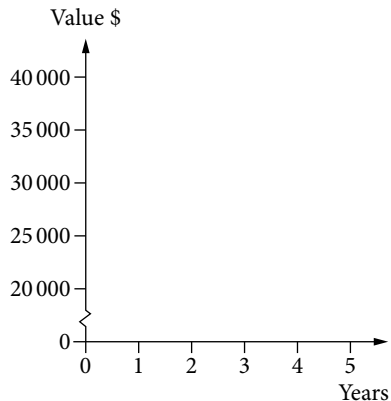
Copy and complete the table of book values (theoretical values based on regular amounts or percentages of depreciation). Round each amount to the nearest dollar.

Age (years)	0	1	2	3	4	5
Value (beginning of year)	\$38 950					

- (b) In reality the depreciation did average 8% over the first 5 years, but was not a steady 8%. The depreciation was 15% in year 1, 10% in year 2 and 5% in each of the following years. Complete the table of values in your workbook, rounding to whole dollars as you go.

Age (years)	0	1	2	3	4	5
Value	\$38 950					

- (c) Draw a set of axes as shown and use your results from the first two parts of the question to draw a graph of both methods of depreciation.



Exercise 1.4

- 29 To expand her small business, Phuoc borrows \$80 000 at a fixed interest rate of 6% p.a. compounded quarterly.

- (a) How much will Phuoc owe after 2 years if she makes no repayments?
(b) Hoping to repay the loan in full after 2 years, Phuoc puts \$1000 each month from her extra profits into a different account. Over the 2 years this money earns \$2300 in interest. Calculate Phuoc's account balance after 2 years of saving.
(c) How much will she have left or still owe?

Exercise 1.8

- 30 Kumar is an Australian citizen for tax purposes, but last financial year his taxable income of 4 650 000 Indian rupees came from India. The exchange rate was A \$1 = 57.24 Indian rupees.
- Calculate Kumar's taxable income in AUD. Round down to the nearest dollar.
 - Calculate Kumar's tax payable. Refer to the *Tax brackets and tax rates* table to find the rate.
 - Kumar also has a HELP debt. Given that his income is in the \$77 619–\$84 062 bracket, he must contribute 6.0% of his income towards repaying his debt. Calculate the amount of HELP debt he must repay.
 - Kumar paid \$23 850 in PAYG tax. Calculate the amount of his refund or debt for the year.

Exercise 1.8

- 31 Anna and Juan have saved \$25 000 for a deposit for the purchase of a unit. They find a unit for \$280 000, but could also rent the same unit for \$1400 a month. A loan of \$255 000 has repayments of \$1705.88 a month for 25 years, calculated on 6.4% p.a. Assume the unit will have a resale value of \$780 000 after 25 years.
- Calculate the total that needs to be repaid on the loan over 25 years if they choose to buy.
 - If Anna and Juan invest their deposit at a fixed interest rate of 4.5% p.a. compounded monthly for 25 years, how much will their investment be worth?
 - Assuming the rent stays constant over the 25 years, calculate the total rent paid.
 - Given that:
Cost of renting = rent paid in 25 years – value of the investment after 25 years
determine the cost of renting over 25 years.
 - Given that:
Cost of buying = deposit + repayments made in 25 years – value of selling the unit after 25 years
determine how much they would make or lose after 25 years.
 - Which method would Anna and Juan choose on these calculations alone? State some variations to the assumptions that were made and the consequences of altering these assumptions.





2

Measurement

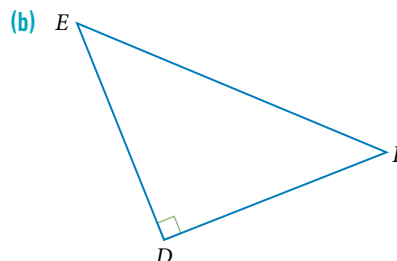
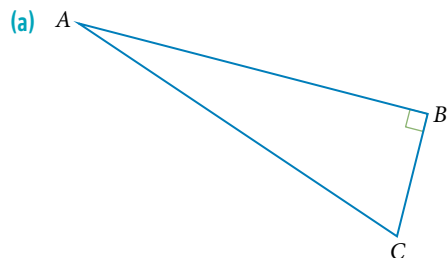
• • • • •

Recall	82
2.1 Pythagoras' theorem	83
2.2 Perimeter and area	92
2.3 Volume of prisms and cylinders	104
2.4 Volume of tapered solids and spheres	116
2.5 Surface area of prisms and cylinders	126
2.6 Surface area of tapered solids and spheres	133
Summary	141
Chapter review	144

Recall

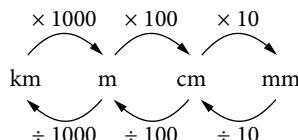
Locate and label the hypotenuse in right-angled triangles

1 Name the hypotenuse in the following right-angled triangles.



Convert metric units of length

2 Convert the following, using the conversion chart as a reference.



(a) 8560 m to kilometres

(b) 3400 cm to metres

(c) 0.14 m to millimetres

Round to a given number of decimal places

3 Round to the number of decimal places given in the brackets.

(a) 30.546 (2)

(b) 5.999522 (3)

(c) 3.1415926 (2)

Substitute values into a formula

4 Calculate the value of the subject in each formula.

(a) $A = \frac{1}{2}bh$ given $b = 20$, $h = 7$

(b) $V = \frac{1}{3}\pi r^3 h$ given $r = 3$, $h = 8$

(c) $A = \frac{1}{2}(a + b)h + \frac{1}{2}\pi r^2$ given $a = 5$, $b = 9$, $h = 12$, $r = 3$

Solve equations involving squares

5 Determine the value of the pronumeral, given that it has a positive value. Round your answer to 2 decimal places, where necessary.

(a) $a^2 = 81$

(b) $x^2 = 47$

(c) $c^2 = 64 + 121$

6 Calculate the unknown variable in the formula.

(a) $A = lw$ where $A = 45$, $w = 5$

(b) $P = 2(l + w)$ where $P = 30$, $l = 12$

Calculate what percentage one value is of the total

7 A water tank has a capacity of 5000 L, but contains only 1750 L of water. Calculate the amount of water as a percentage of the capacity, rounded to the nearest whole per cent.

Calculate percentage increase or decrease

8 Determine the percentage of change, rounded to the nearest whole per cent.

(a) With a renovation, the kitchen bench is increased in length from 1700 mm to 2000 mm.

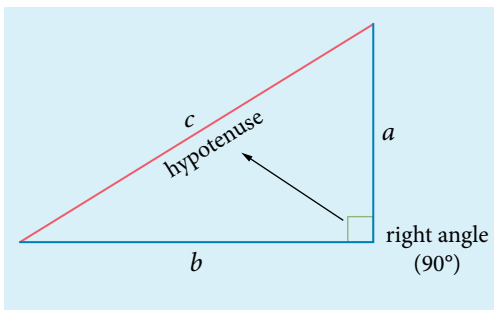
(b) Due to the drought, the surface area of a reservoir decreases from 6200 m^2 to 5700 m^2 .

Pythagoras' theorem

Introduction

Pythagoras' theorem is perhaps the most famous maths equation of all. It can be used to find the side length of any side in a *right-angled* triangle, provided the two other side lengths are known (and also to find two equal unknown sides, when given the hypotenuse).

Pythagoras' theorem states that, in any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the two shorter sides, or $c^2 = a^2 + b^2$ where c is the hypotenuse (the side opposite the right angle), which will always be the longest side, and a and b are the two shorter sides.



$$c^2 = a^2 + b^2$$

$$c = \pm \sqrt{a^2 + b^2}$$

Given that c is a measurement of length (of the hypotenuse), for the triangle to exist, $c > 0$. Therefore $c = \sqrt{a^2 + b^2}$

Also $a^2 = c^2 - b^2$ and $b^2 = c^2 - a^2$

$$a = \sqrt{c^2 - b^2} \quad \text{and} \quad b = \sqrt{c^2 - a^2}$$

i Additional information

Pythagoras' theorem

Review your knowledge of Pythagoras' theorem.

Pythagorean triads

In most cases, using Pythagoras' theorem to find a side in a right-angled triangle results in an irrational number (a number with no repeating pattern of digits). You have to round this number as required.

However, there are some right-angled triangles with three sides that are all whole numbers. These are called Pythagorean triads, a set of three whole numbers that satisfies Pythagoras' theorem $c^2 = a^2 + b^2$. They are also referred to as Pythagorean triples. The best-known triad is the triangle formed by side lengths 3, 4 and 5.

$$c^2 = a^2 + b^2$$

$$5^2 = 3^2 + 4^2$$

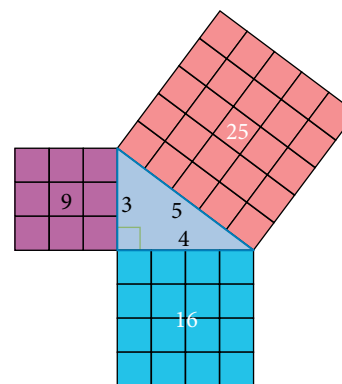
$$25 = 9 + 16$$

This triad was so well known that farmers used a 12m chain marked at 3m and 7m ((3m + 4m) with 5m for the remainder) to ensure that right angles were created when erecting fences.

New triads can be obtained by multiplying all three values by the same number. These triads are said to belong to the same family, as they all originate from the 3, 4, 5 triad. For example, multiply by 2 to get a new triad 6, 8, 10.

$$6, 8, 10 : \quad 6^2 + 8^2 = 10^2$$

$$36 + 64 = 100$$



Here are some more Pythagorean triads.

$$\begin{array}{lll}
 5, 12, 13 : & 5^2 + 12^2 = 13^2 & 7, 24, 25 : \quad 7^2 + 24^2 = 25^2 \\
 & 25 + 144 = 169 & 49 + 576 = 625
 \end{array}
 \quad
 \begin{array}{l}
 8, 15, 17 : \quad 8^2 + 15^2 = 17^2 \\
 64 + 225 = 289
 \end{array}$$

Knowing a few common Pythagorean triads may speed up the solving of problems involving Pythagoras' theorem.

1 Right angled or not?

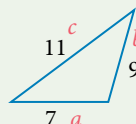
Determine whether the triangle with the following side lengths is right angled or not.

(a) 7, 9, 11

THINKING

- 1 Draw a labelled diagram.
- 2 Square each of the two shorter sides and add.
- 3 Square the longest side.
- 4 Compare the sum of the squares of the shorter sides to the square of the longest side.
- 5 Interpret the answer.

WORKING



$$\begin{aligned}
 a^2 + b^2 &= 7^2 + 9^2 \\
 &= 49 + 81 \\
 &= 130
 \end{aligned}$$

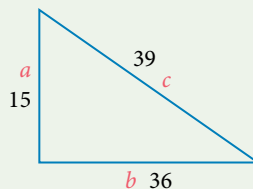
$$\begin{aligned}
 c^2 &= 11^2 \\
 &= 121
 \end{aligned}$$

$$\begin{aligned}
 121 &\neq 130 \\
 c^2 &\neq a^2 + b^2
 \end{aligned}$$

Because the values 7, 9 and 11 do not form a Pythagorean triad, the triangle is not right angled.

(b) 15, 36, 39

- 1 Draw a labelled diagram.



- 2 Square each of the two shorter sides and add.
- 3 Square the longest side.
- 4 Compare the sum of the squares of the shorter sides to the square of the longer side.
- 5 Interpret the result.

$$\begin{aligned}
 a^2 + b^2 &= 15^2 + 36^2 \\
 &= 225 + 1296 \\
 &= 1521
 \end{aligned}$$

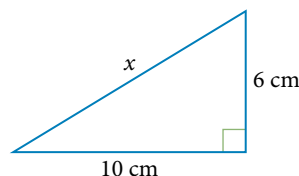
$$\begin{aligned}
 c^2 &= 39^2 \\
 &= 1521
 \end{aligned}$$

$$a^2 + b^2 = c^2$$

The triangle is right angled because the values 15, 36 and 39 form a Pythagorean triad.

2 Calculating the length of the hypotenuse

Determine the length of the hypotenuse x , to 2 decimal places.



THINKING

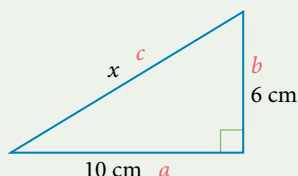
1 Label the hypotenuse c and the two shorter sides a and b .

2 Recall Pythagoras' theorem and substitute the values into the formula.

3 Simplify and then solve for the unknown, rounding your answer to 2 decimal places.

4 Check that the answer is reasonable.

WORKING



$$c^2 = a^2 + b^2$$

$$x^2 = 6^2 + 10^2$$

$$x^2 = 36 + 100$$

$$x^2 = 136$$

$$\therefore x = \sqrt{136}$$

$$x = 11.661\dots$$

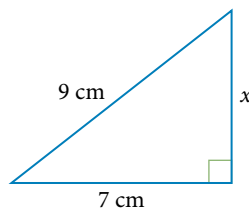
$$= 11.66 \text{ cm (2 d.p.)}$$

The value for x is reasonable because it is larger than the other sides but not larger than the total of the other sides.

When solving equations with square roots, such as $x^2 = 136$, there are two solutions: the positive square root and the negative square root, so $x = \pm\sqrt{136}$. When you are dealing with lengths, which are always positive in a 'practical' context, you can reject the negative value.

3 Calculating the length of a shorter side

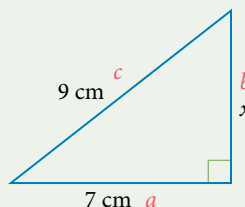
Determine the length of the shorter side x , to 2 decimal places.



THINKING

1 Label the hypotenuse c and the two shorter sides a and b .

WORKING



- 2 Recall Pythagoras' theorem and substitute the values into the formula.

$$c^2 = a^2 + b^2$$

$$9^2 = 7^2 + x^2$$

- 3 Transpose the equation and then solve for the unknown, rounding your answer to 2 decimal places.

$$x^2 = 9^2 - 7^2$$

$$x^2 = 81 - 49$$

$$x^2 = 32$$

$$\therefore x = \sqrt{32}$$

$$= 5.656\dots$$

$$= 5.66 \text{ cm (2 d.p.)}$$

- 4 Check that your answer is reasonable.

The value for x is reasonable because it is smaller than the hypotenuse.

e Explore further

Creating Pythagorean triads

Create a spreadsheet to calculate Pythagorean triads and missing side lengths in right-angled triangles.

i Additional information

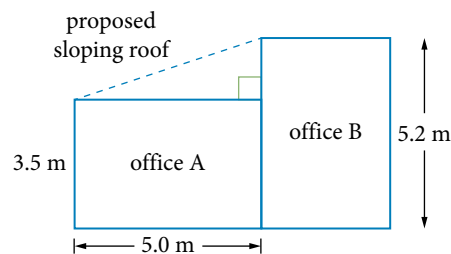
Calculating the length of a shorter side length

Practise calculating the length of an unknown shorter side length in a right-angled triangle.

4 Application of Pythagoras' theorem

As part of a school building upgrade, a sloping roof will be built between two existing offices as shown.

Determine the length of the proposed new roof.



THINKING

- 1 Identify a right-angled triangle that includes the unknown length. Label the hypotenuse c and the two shorter sides a and b .

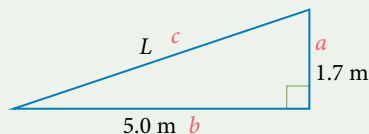
Determine the known lengths from the given dimensions.

- 2 Recall Pythagoras' theorem and substitute the values into the formula.
- 3 Simplify and then solve for the unknown, rounding your answer to 1 decimal place.

WORKING

Subtract building heights to find the unknown side length:

$$5.2 - 3.5 = 1.7$$



$$c^2 = a^2 + b^2$$

$$L^2 = 1.7^2 + 5.0^2$$

$$L^2 = 25 + 2.89$$

$$L^2 = 27.89$$

$$\therefore L = \sqrt{27.89}$$

$$= 5.281\dots$$

$$= 5.3 \text{ m (1 d.p.)}$$

- 4 Interpret the answer, checking the reasonableness of the answer.

The length of the proposed roof is 5.3 m.

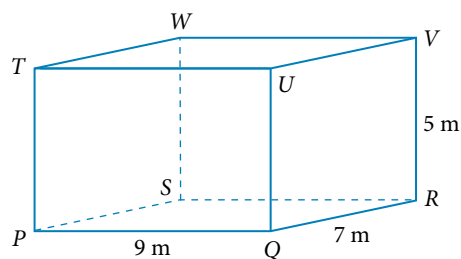
The length of the roof is the hypotenuse and therefore must be longer than 5 m.

- 5 State any assumptions you made.

The assumption is that the roof does not overhang the wall. If it did, the roof would be longer.

5 Pythagoras in 3D

A rectangular prism has length 9 m, width 7 m and height 5 m.

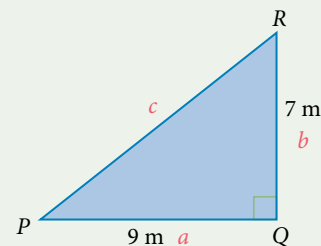
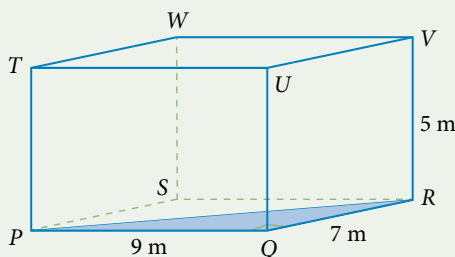


- (a) Calculate the length of PR to 2 decimal places.

THINKING

- 1 Identify the right-angled triangle with the unknown length as its hypotenuse (the side opposite the right angle).

WORKING



- 2 Substitute the values into Pythagoras' theorem.

$$c^2 = a^2 + b^2$$

$$(PR)^2 = 9^2 + 7^2$$

- 3 Solve to find the unknown length.

$$(PR)^2 = 81 + 49$$

$$(PR)^2 = 130$$

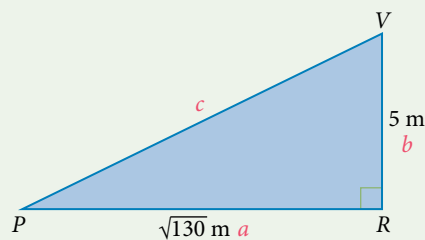
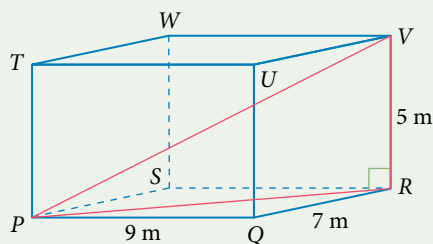
$$\therefore PR = \sqrt{130}$$

$$= 11.4017\dots$$

$$= 11.40 \text{ m (2 d.p.)}$$

- (b) Calculate the length of PV to 2 decimal places.

- 1 Identify the right-angled triangle with the unknown length as its hypotenuse (the side opposite the right angle).



- 2 Substitute the exact values into Pythagoras' theorem.

$$c^2 = a^2 + b^2$$

$$(PV)^2 = 5^2 + (\sqrt{130})^2$$

- 3 Solve to find the unknown length.

$$(PV)^2 = 25 + 130$$

$$= 155$$

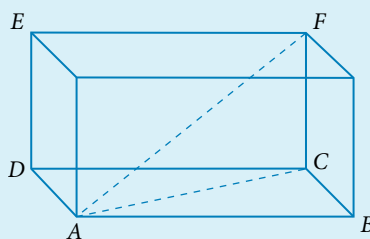
$$\therefore PV = \sqrt{155}$$

$$= 12.4498\dots$$

$$= 12.45 \text{ m (2 d.p.)}$$

A version of Pythagoras' theorem can be used to find the internal diagonal, provided all other dimensions are known:

$$(AF)^2 = (AC)^2 + (CF)^2$$



i Additional information

Pythagoras' theorem in three dimensions

View the video to review the application of Pythagoras' theorem in three dimensions.

EXERCISE

2.1

Pythagoras' theorem

Worked Example

1

- 1 For each of the following triangles, determine whether the side lengths given will form a right-angled triangle or not.

(a) (3, 4, 5)

(b) (6, 8, 12)

(c) (9, 12, 15)

(d) (7, 9, 13)

(e) (10, 24, 26)

(f) (30, 72, 80)

(g) (70, 240, 250)

(h) (35, 120, 125)

- 2 A particular right-angled triangle has side lengths 12 cm, 35 cm and 37 cm. Select the set of side lengths that belongs to a similar right-angled triangle.

A 13 cm, 37 cm, 40 cm

B 22 cm, 45 cm, 47 cm

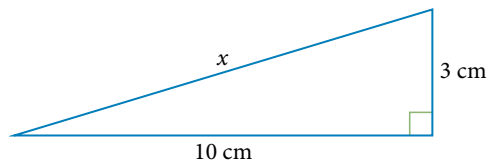
C 24 cm, 70 cm, 77 cm

D 60 cm, 175 cm, 185 cm

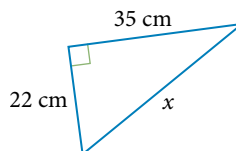
2

- 3 Calculate the length of the hypotenuse x in each of the triangles shown, to 2 decimal places where necessary.

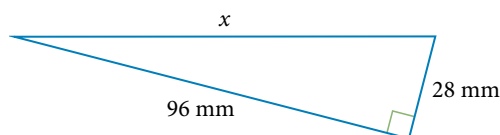
(a)



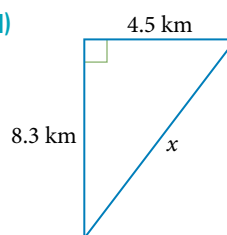
(b)

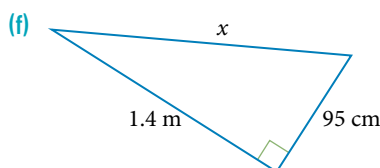
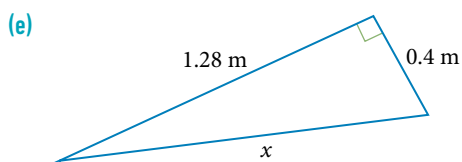


(c)

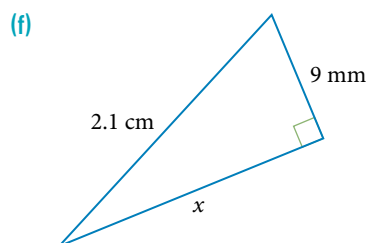
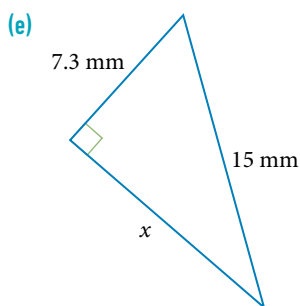
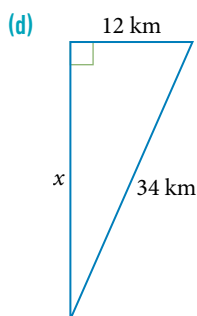
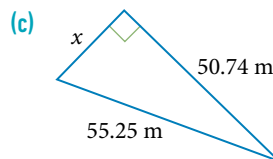
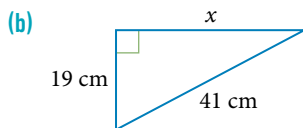
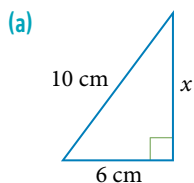


(d)

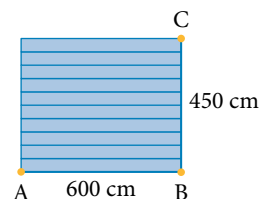




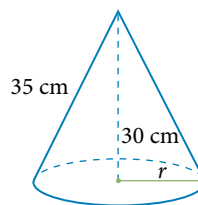
- 4 Calculate the shorter side length labelled x in each of the triangles shown, to 2 decimal places where necessary.



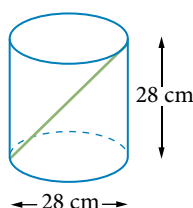
- 5 Determine the length of the hypotenuse of a right-angled triangle, given the shorter sides measure 12.5 m and 14.7 m. Give your answer to 1 decimal place.
- 6 The hypotenuse of a right-angled triangle is 28 m. If one of the sides is 12 m, determine the length of the remaining side. Give your answer to 1 decimal place.
- 7 Mohan is setting out a string line joining points A , B and C before constructing a rectangular deck, making measurements as shown. Calculate the length of string required to connect corner A to corner C . Answer in metres.



- 8 A 5 m long rope joins the top of a flagpole to an anchor point on the ground 220 cm from the base of the pole.
- (a) How tall is the flagpole? (Answer in metres, to 2 decimal places.)
- (b) If you made the common error of calculating the height as if it was the hypotenuse, how would you recognise your error from your answer?
- 9 A ship travels from its port 18 km due east, then 10 km due south. Determine how far it is from its port. State your answer to 2 decimal places.
- 10 Determine, to 1 decimal place, the radius r of a cone with a perpendicular height of 30 cm and a slant height of 35 cm.



- 11 A restaurant stores chopsticks in a cylindrical tin with a lid. Calculate the longest chopstick that can be stored in this tin with the flat lid closed. Express your answer to 1 decimal place. State any assumptions you made.

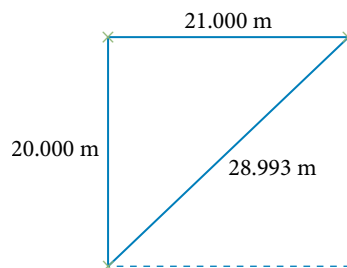


Worked
Example

4

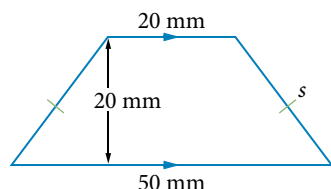
- 12 Building inspector Craig measures the length, width and diagonal of a concrete slab for a house, as shown. The slab will be declared 'square' if the hypotenuse conforms to Pythagoras' theorem to within 10 mm.

Assess Craig's construction to determine whether the slab will be declared 'square'.

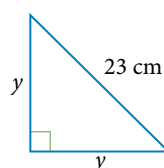


- 13 Determine the length of the side marked with a pronumeral in each case.

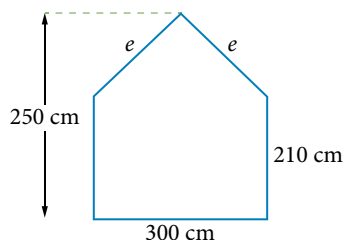
(a)



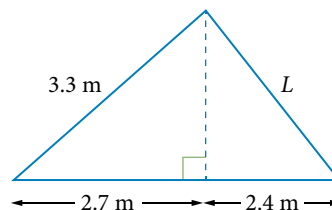
(b)



- 14 An adventure playground contains a building as shown. Determine the length e of a sloping edge of the roof, rounded to the nearest centimetre.

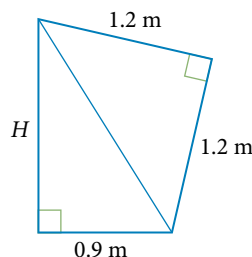


- 15 A builder studies some plans for a roof and wonders what length of beam is needed for the section labelled L . Calculate the value of L for the builder, giving your answer to 1 decimal place.



- 16 A contestant on a home renovation program, *The Blot*, designs a window that consists of two right-angled triangles joined at the hypotenuse, as shown.

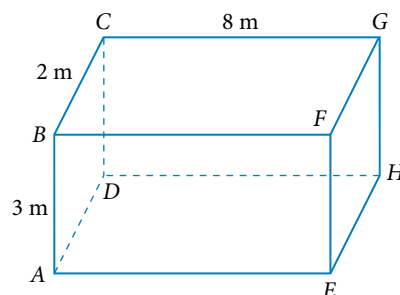
What is the height H of the window, to 2 decimal places?



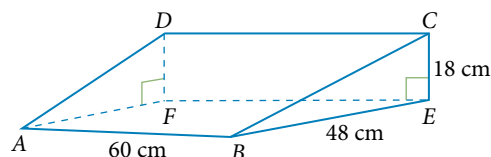
5

- 17 The rectangular prism shown has length 3 m, width 2 m and height 8 m.

Calculate the following lengths, to 3 decimal places.

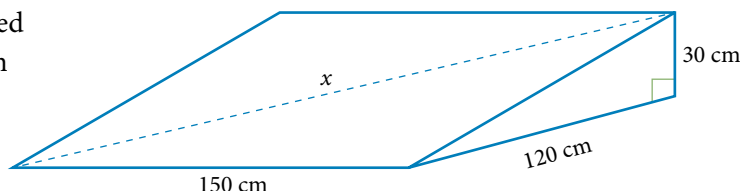
(a) AC (b) DG (c) AH (d) AG 

- 18 P is a point anywhere along AB , and Q is a point anywhere on DC . Determine the longest possible length of PQ , to the nearest centimetre.



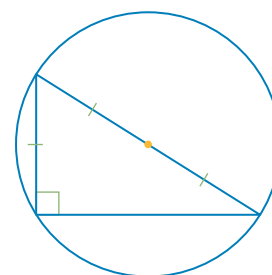
- 19 Farah is trying to fit a variety of ski poles into the boot of her car. If the boot is approximated as a rectangular prism with length 1.3 m, depth 0.9 m and height 0.5 m, determine the length of the longest pole that will fit inside. Express your answer to 2 decimal places. Discuss any assumptions you made.
- 20 A square-based pyramid has a base of length of 5 m and a perpendicular height of 6 m. Calculate the length s , of the pyramid's sloping edge. Answer to 1 decimal place. Discuss any assumptions you made.

- 21 A rectangular access ramp is constructed as shown. Calculate the distance x from the bottom left corner to the top right corner of the ramp. Give your answer to the nearest centimetre.

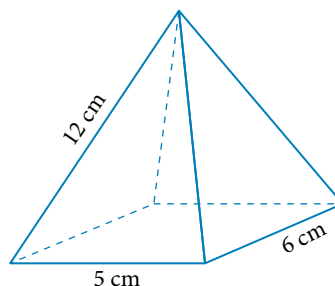


- 22 A major company's new logo is based on a right-angled triangle in a circle, as shown. The hypotenuse of the triangle passes through the centre of the circle, and the shortest side of the triangle is equal in length to the radius of the circle.

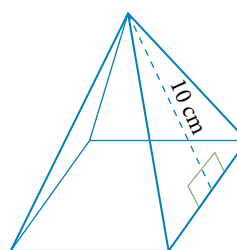
A large version of the logo is painted on the wall of the company headquarters within a circle of radius 2.75 m. Determine the length of the second-largest side of the triangle, to 2 decimal places.



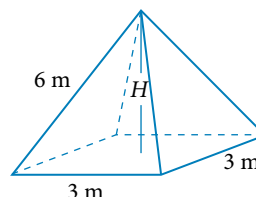
- 23 Determine the height of the pyramid shown. Identify any assumptions you needed to make to carry out your calculations. Give your answer to 2 decimal places.



- 24 In the square-based pyramid shown, its base side length is twice its height. Calculate the height H of the pyramid, to 2 decimal places, if the slant face of the pyramid has a height of 10 cm.



- 25 A museum curator is designing a scaled-down pyramid, and submits the plan shown to the museum board for approval. The board worries that the pyramid will not fit in the display area, which has a ceiling height of 5.7 m. Demonstrate, using calculations, that the model pyramid will fit and determine how much clearance (spare vertical distance) there will be, to the nearest centimetre.



2.2

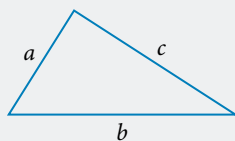
Perimeter and area

Perimeter

The perimeter P of a shape is the length of its boundary. Circumference C is a special name given to the perimeter of a circle. Some of the most common perimeter formulas are shown here.

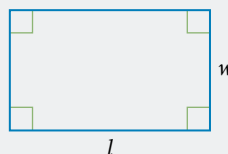
Triangle

$$P = a + b + c$$



Rectangle

$$P = 2(l + w)$$



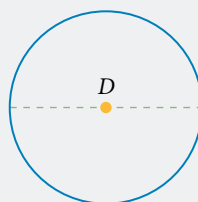
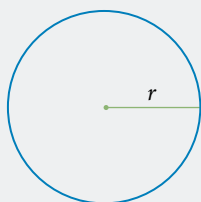
Circle

r = radius

$$C = 2\pi r \text{ or } C = \pi D$$

where:

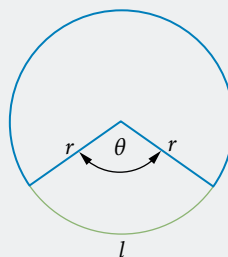
D = diameter ($D = 2r$)



Arc length (l) and perimeter (P) of a sector

$$l = \frac{\theta}{360^\circ} \times 2\pi r$$

$$P = l + 2r$$



Making connections

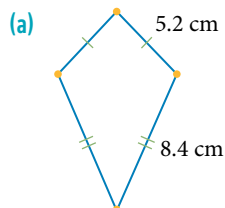
Arc length

Determine the effect on the arc length when the subtended angle increases or the radius increases.

Units for perimeter include kilometres (km), metres (m), centimetres (cm) and millimetres (mm).

6 Perimeter

Use the measurements given in the diagram to determine the perimeter of each shape, to 2 decimal places where necessary.



THINKING

- 1 Construct a formula for the perimeter of the shape.
- 2 Substitute the given values into the formula and calculate P .
- 3 Interpret the answer.

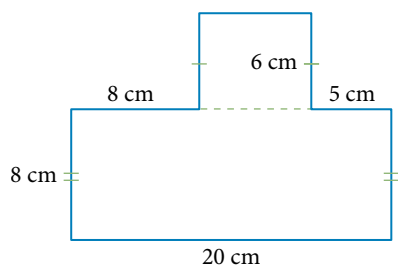
WORKING

$$P = 2a + 2b$$

$$P = 2 \times 5.2 + 2 \times 8.4 \\ = 27.2 \text{ cm}$$

The perimeter of the kite is 27.2 cm.

(b)

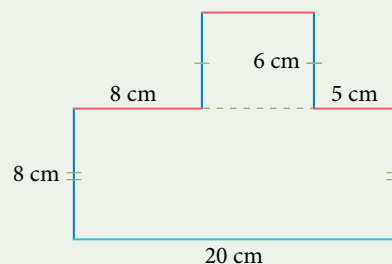


1 Account for the unknown side lengths.

2 Identify a suitable approach to determine the formula for the perimeter of the shape.

3 Substitute the given values into the formula and calculate P .

4 Interpret the answer.



Total of upper horizontal lengths: 20 cm

$P = \text{sum of all side lengths}$

$P = \text{horizontal lengths} + \text{vertical lengths}$

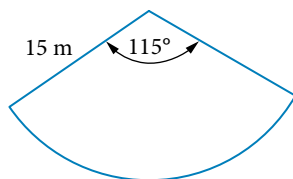
$$P = 20 \times 2 + 8 \times 2 + 6 \times 2$$

$$= 40 + 16 + 12$$

$$= 68 \text{ cm}$$

The perimeter of the shape is 68 cm.

(c)



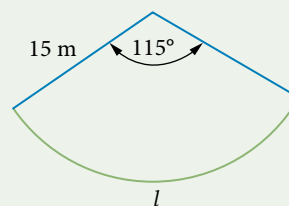
THINKING

1 Recall the formula for the perimeter of the shape.

2 Substitute the given values into the formula and calculate P .

3 Interpret the result.

WORKING



Sector:

$$P = \ell + 2r$$

$$= \frac{\theta}{360^\circ} \times 2\pi r + 2r$$

$$P = \frac{115^\circ}{360^\circ} \times 2\pi \times 15 + 2 \times 15$$

$$P = 30.11 + 30$$

$$= 60.11 \text{ m (2 d.p.)}$$

The perimeter of the sector is 60.11 m.

i Additional information

Perimeter

Consolidate your understanding of perimeter.

i Additional information

Arc length

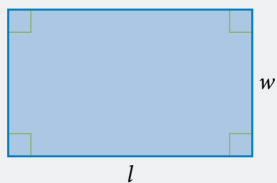
Review the process of finding arc lengths.

Area

Area A is the size of the surface within the boundary of a two-dimensional shape. Some of the most common area formulas are shown below.

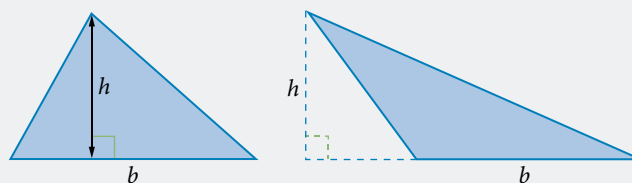
Rectangle

$$A = lw$$



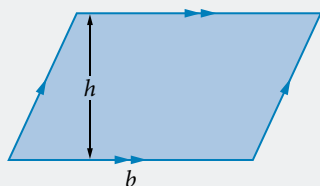
Triangle

$$A = \frac{1}{2}bh$$



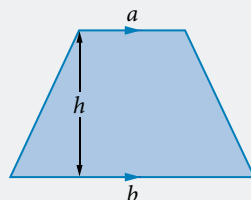
Parallelogram

$$A = bh$$



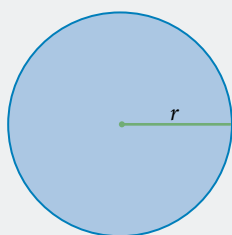
Trapezium

$$A = \frac{1}{2}(a+b)h$$



Circle

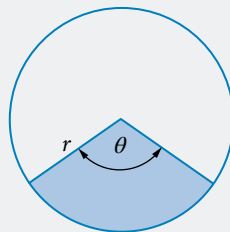
$$A = \pi r^2$$



Sector

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

(Note: $\frac{\theta}{360^\circ}$ is the fraction of the circle occupied by the sector.)



C Making connections

Sector area

Determine the effect on the area of the sector when:

- the radius increases
- the angle subtended increases.

e Explore further

Area formulas

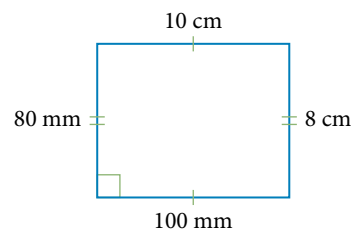
Create formulas to solve area problems using a spreadsheet.

Area units

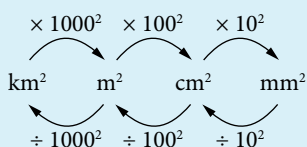
Standard units for area include square kilometres (km^2), square metres (m^2), square centimetres (cm^2) and square millimetres (mm^2).

If a question involves mixed units, you should convert measurements before finding the area. If you need to convert between area units, consider the rectangle shown at right.

The area of the rectangle can be calculated as $10\text{ cm} \times 8\text{ cm} = 80\text{ cm}^2$ or $100\text{ mm} \times 80\text{ mm} = 8000\text{ mm}^2$. To change centimetres to millimetres you need to multiply by 10, but to change square centimetres to square millimetres you need to multiply by 10^2 or 100.



To convert between square factors, use the square of the linear conversion factor.



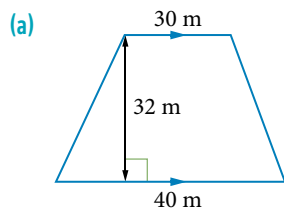
i Additional information

Converting units of area

Revise converting metric units of area.

7 Area

Determine the area of the shape, to 2 decimal places where necessary.



THINKING

- 1 Recall the formula for the area of the shape.
- 2 Substitute the known measurements into the formula to calculate A .
- 3 Interpret the answer.

WORKING

The shape is a trapezium.

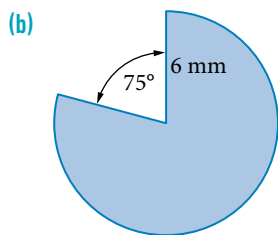
$$A = \frac{1}{2}(a + b)h$$

$$A = \frac{1}{2}(30 + 40) \times 32$$

$$= \frac{1}{2} \times 70 \times 32$$

$$= 1120\text{ m}^2$$

The area of the trapezium is 1120 m^2 .



1 Identify the shape.

The shape is a sector.

2 Recall the formula for the area of the shape.

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

3 Substitute the given values into the formula and calculate A. (First determine the angle within the sector.)

The angle of the sector is:

$$\begin{aligned}\theta &= 360 - 75 \\ &= 285^\circ\end{aligned}$$

$$\begin{aligned}A &= \frac{(285^\circ)}{360^\circ} \times \pi \times (6)^2 \\ &= 89.54 \text{ mm}^2 \quad (2 \text{ d.p.})\end{aligned}$$

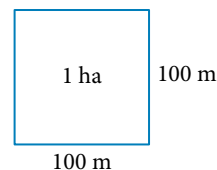
4 Interpret the answer.

The area of the sector to 2 decimal places is 89.54 mm^2 .

Area in hectares

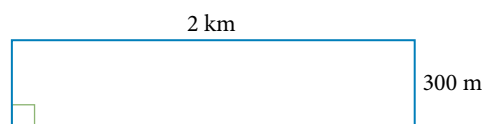
Fields and sporting ovals have areas suitable for measuring in hectares. A square of side length 100 m has an area of one hectare, or 1 ha, so:

$$1 \text{ ha} = 10000 \text{ m}^2$$



8 Area in hectares

Determine the area of a rectangular section of beachfront, in hectares.



THINKING

- Write all measurements in the same units, converting units where necessary.
- Recall the formula for area, with suitable units.
- Convert to hectares.
- Interpret the answer, checking the reasonableness of the value.

WORKING

$$2 \text{ km} = 2000 \text{ m}$$

$$\begin{aligned}A &= lw \\ &= 2000 \times 300 \\ &= 600000 \text{ m}^2\end{aligned}$$

$$10000 \text{ m}^2 = 1 \text{ ha}$$

$$600000 \text{ m}^2 = 60 \text{ ha}$$

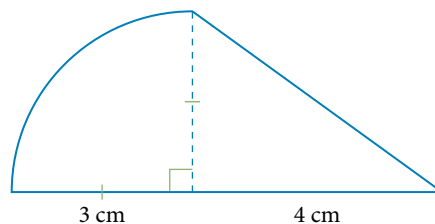
The area of beachfront is 60 ha. This is the same as 60 squares, each $100 \text{ m} \times 100 \text{ m}$.

Compound areas

To calculate the areas of compound shapes, break the shape into smaller shapes for which the formulas are known.

9 Compound areas

Calculate the area of the shape to 2 decimal places.



THINKING

- 1 Recognise common or known shapes within the compound shape.

Calculate the lengths of any unknown sides that are required.

- 2 Recall the formulas of the known shapes.

Create a strategy to calculate the area of the shape.

- 3 Substitute the given values and calculate the area.

- 4 Interpret the answer, checking that the value is reasonable.

WORKING

The compound shape can be divided into a quarter circle and a right-angled triangle with a height of 3 cm.

$$\text{Quarter circle: } A = \frac{1}{4}\pi r^2$$

$$\text{Triangle: } A = \frac{1}{2}bh$$

$$A_{\text{total}} = A_{\text{quarter circle}} + A_{\text{triangle}}$$

$$\begin{aligned} A &= \frac{1}{4}\pi r^2 + \frac{1}{2}bh \\ &= \frac{1}{4}\pi \times 3^2 + \frac{1}{2} \times 4 \times 3 \\ &= 13.068\dots \\ &= 13.07 \text{ cm}^2 \text{ (2 d.p.)} \end{aligned}$$

The area of the shape is 13.07 cm^2 .

This is slightly more than the size of a $2 \text{ cm} \times 6 \text{ cm}$ rectangle.

EXERCISE

2.2

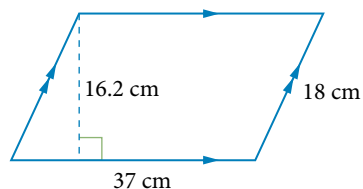
Perimeter and area

Worked
Example

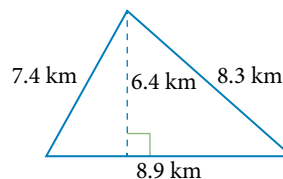
6

- 1 Determine the perimeter of each shape. Round your answers to 2 decimal places where appropriate.

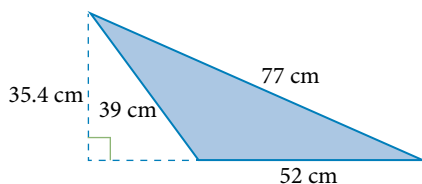
(a)



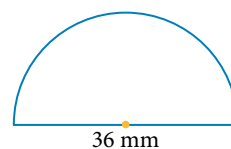
(b)



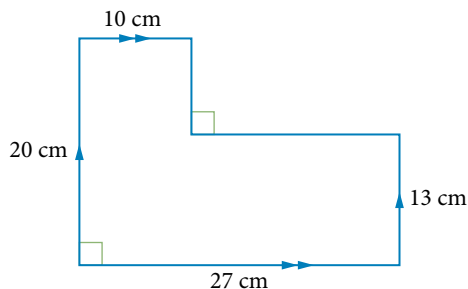
(c)



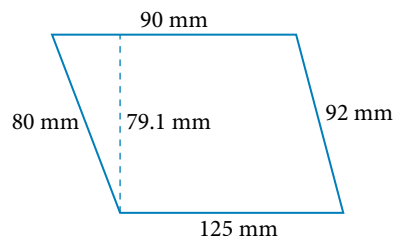
(d)



(e)



(f)



- 2 Select the option that shows how many times a 30 m length of cotton string will wrap around a post of diameter 20 cm. Ignore the width of the string.

A 24

B 38

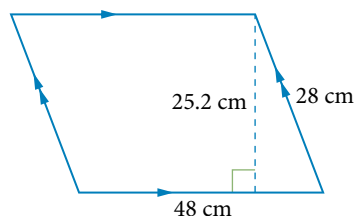
C 47

D 150

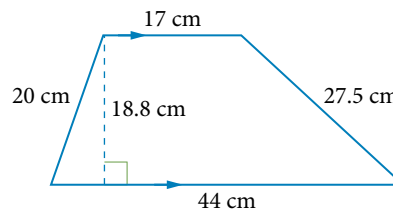
7

- 3 Determine the area of each shape. Round your answers to 2 decimal places where appropriate.

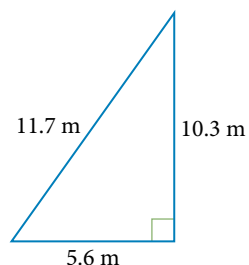
(a)



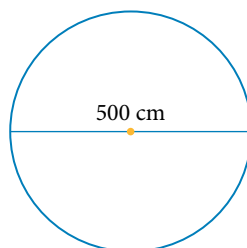
(b)



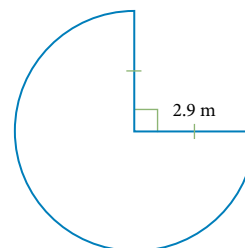
(c)



(d)



(e)

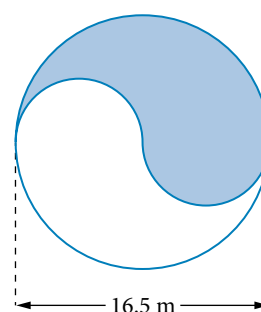
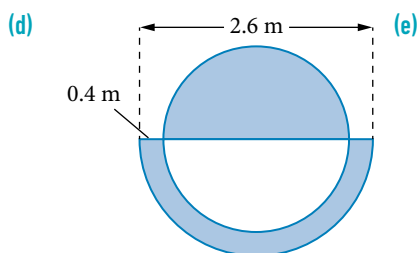
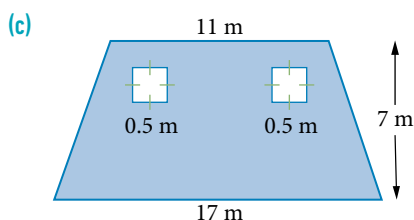
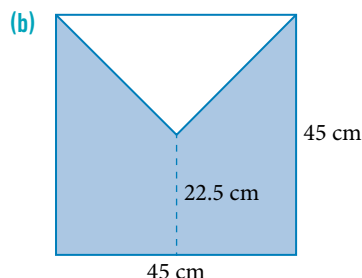
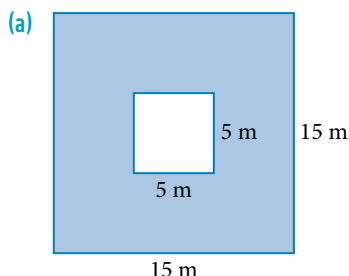


8

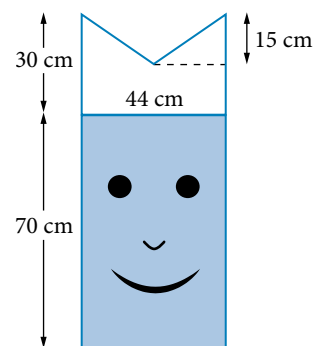
- 4 A park is in the shape of a rectangle 150 m by 400 m.
- Calculate the area of the park in hectares.
 - Justify the reasonableness of your answer to part (a) by drawing the shape of the park broken into areas of 1 hectare each.

9

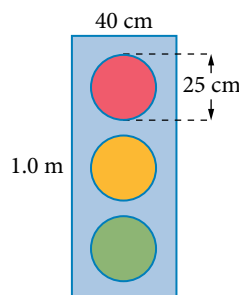
- 5 Determine the area of each of the following regions, stating any formulas used. Where shading is present, determine the shaded area. Round your answers for area to 2 decimal places, where necessary.



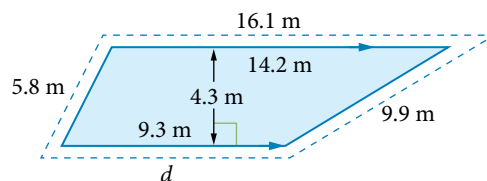
- 6 The Happy Cook restaurant is having a new sign installed. The sign is to be made of sheet metal, then painted to appear as shown. Determine the total area of sheet metal in the finished sign.



- 7 Trafficorp manufactures traffic lights of dimensions as shown. Each set of three coloured sections is embedded in a plastic panel. Calculate the area of the plastic panel, to 2 decimal places.

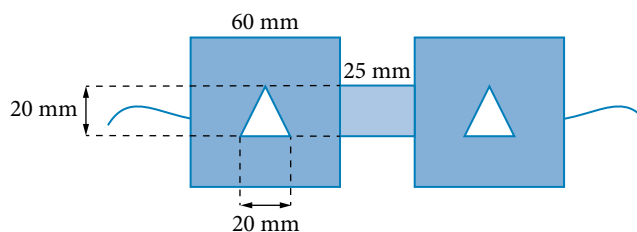


- 8 Kieran draws a plan for a backyard swimming pool. The dashed lines show the perimeter safety fence. Give answers to 1 decimal place.

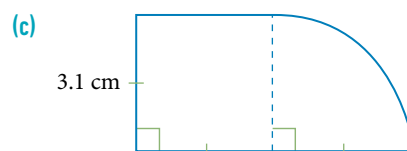
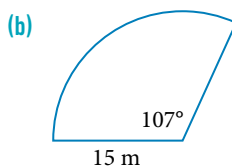
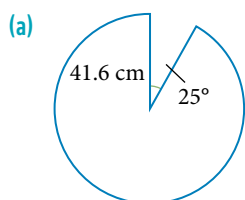


- Kieran remembers that the perimeter of the fence is 42.0 m, but has noticed that one of the sides of the fence does not have a measurement marked. Determine the missing measurement d .
- Determine the area of the swimming pool (blue area).
- The fence is 0.5 m from the edges of the pool. Determine the area enclosed by the fence.
- Kieran decides to pave the area between the pool and the fence with pebble mix. What area is to be covered with the pebble mix?

- 9 Yuki is invited to a fancy dress party but hates the extra effort of having to find a costume. She decides to simply make a mask and go as a superhero, 'The Geometer'. The design for the mask includes squares with triangular eye-holes and a rectangular nose band.



- Before the eye-holes are cut out, what area of material is required?
 - What area does the mask cover when it is complete?
 - Yuki's friend thinks the mask is a little drab, and suggests attaching gold thread around the outer edges of the mask (excluding the edges of the eye-holes). What length of thread is required?
- 10 Determine the perimeter and the area of each shape. Round your answers to 2 decimal places.



..

..

..

..

..

..

..

..

..

..

..

..

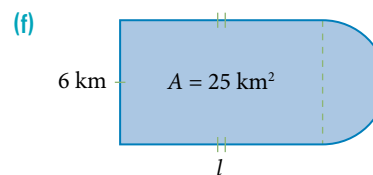
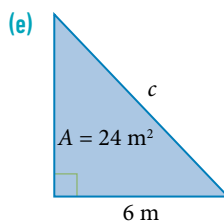
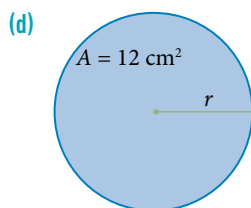
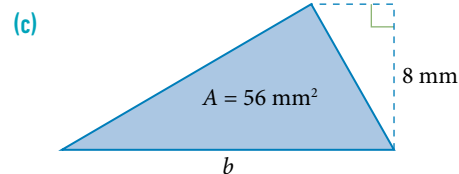
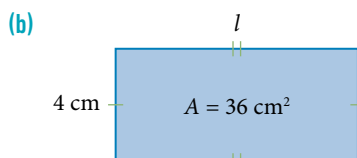
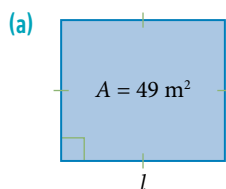
..

..

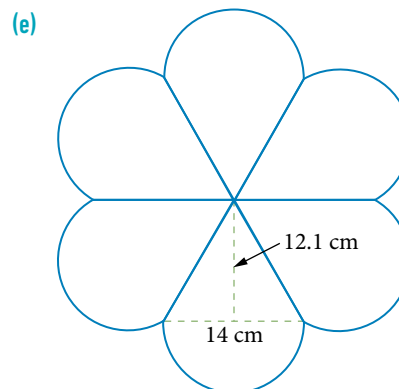
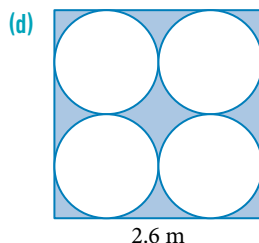
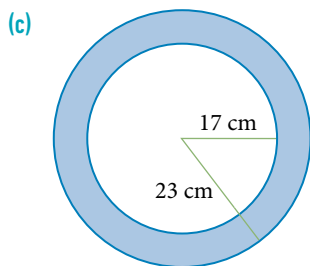
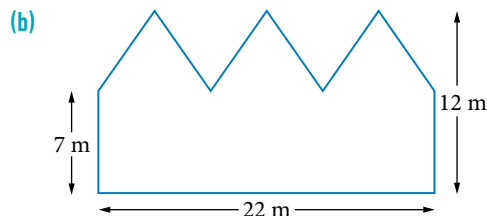
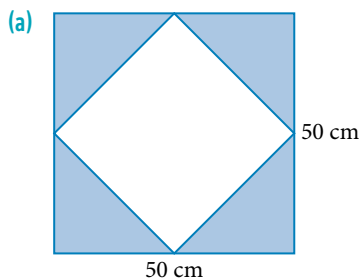
..



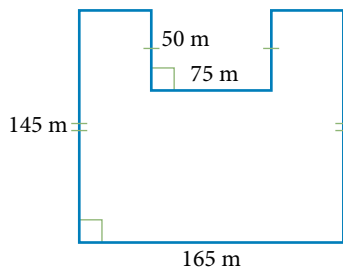
- 11 Determine the unknown length for the following shapes. Round your answers to 2 decimal places where necessary.



- 12 Rachel runs 20 times around a rectangular basketball court that is 28 m long and 15 m wide. Her friend Mauz runs laps of a circular track of radius 60 m. What is the minimum number of laps Mauz must run to cover at least the same distance as Rachel?
- 13 Determine the area of each of the following regions. Where shading is present, determine the shaded area. Round your answers for area to 2 decimal places, where necessary.



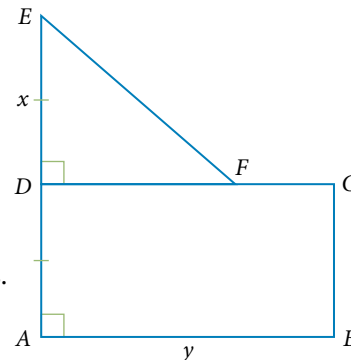
- 14 Determine the area of the shape in hectares.



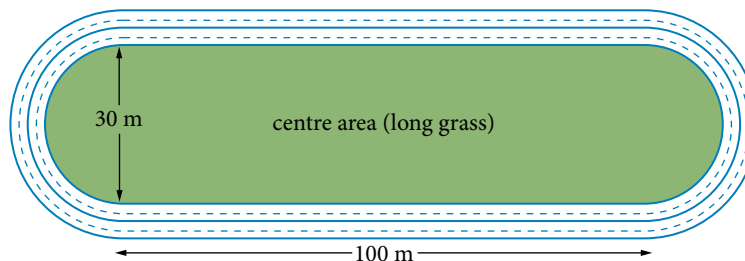
- 15 A giant television screen at a major sports ground is made up of several smaller screens joined together with negligible space between them. The individual small screens measure $120\text{ cm} \times 55\text{ cm}$ and the giant screen is $24\text{ m} \times 11\text{ m}$. Select the number of small screens that make up the giant screen.
- A 66 B 254 C 350 D 400

- 16 Consider the diagram with rectangle $ABCD$ and a right-angled triangle DEF , where F is a moveable point along the line DC .

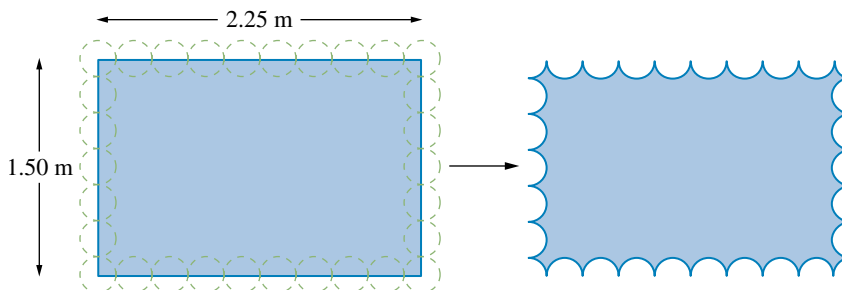
- (a) Determine the maximum possible area of the shape in terms of x and y .
- (b) Determine the difference between the maximum area and the area formed if $DF = FC$.
- (c) A straight line EF that passes through point F has a length of 5 units.
- (i) Use Pythagoras' theorem to show that $4x^2 + y^2 = 25$.
- (ii) The triangle formed from ABE has an area of 6 square units. Form an equation in terms of x and y .
- (d) If $x = \frac{2}{3}y$, determine the dimensions and maximum perimeter of the shape, to 2 decimal places.



- Calculate the following to 1 decimal place.

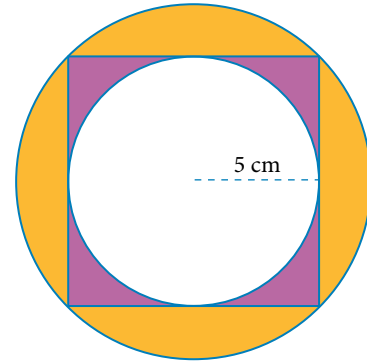


- (a) What is the distance run by a teacher completing one lap in the inside lane?
 - (b) Compare the distance run by a teacher and a student, to determine how much further a student must run in the outside lane.
 - (c) The centre area of the track will stay as it is, with long grass. What is the area of the centre of the track?
 - (d) Gordon only mows the actual running lanes. What area of the track must he mow?
- 18 Annika has prepared a paved area of width 4 m and length 7.5 m, and wants to add a decorative border using different coloured pavers. How many square metres of area will the addition of a 0.25 m wide border using coloured pavers add to the paved area?
-
- 19 Shabazz is planning an upgrade to his kitchen island bench. He has selected a polished marble finish that costs \$235 per square metre (which includes the cutting and fitting of any fixtures). The length of the bench is fixed at 2.8 m. He has a budget of \$840. Determine the maximum possible width of Shabazz's new bench, to the nearest centimetre.
- 20 Terrence is building a giant stamp to promote a stamp collectors' convention. The stamp is to be made from a rectangular sheet of plywood, and will have semicircles and quarter circles (all of the same radius) cut from the perimeter using a large circular drill. The original and finished shapes are shown.

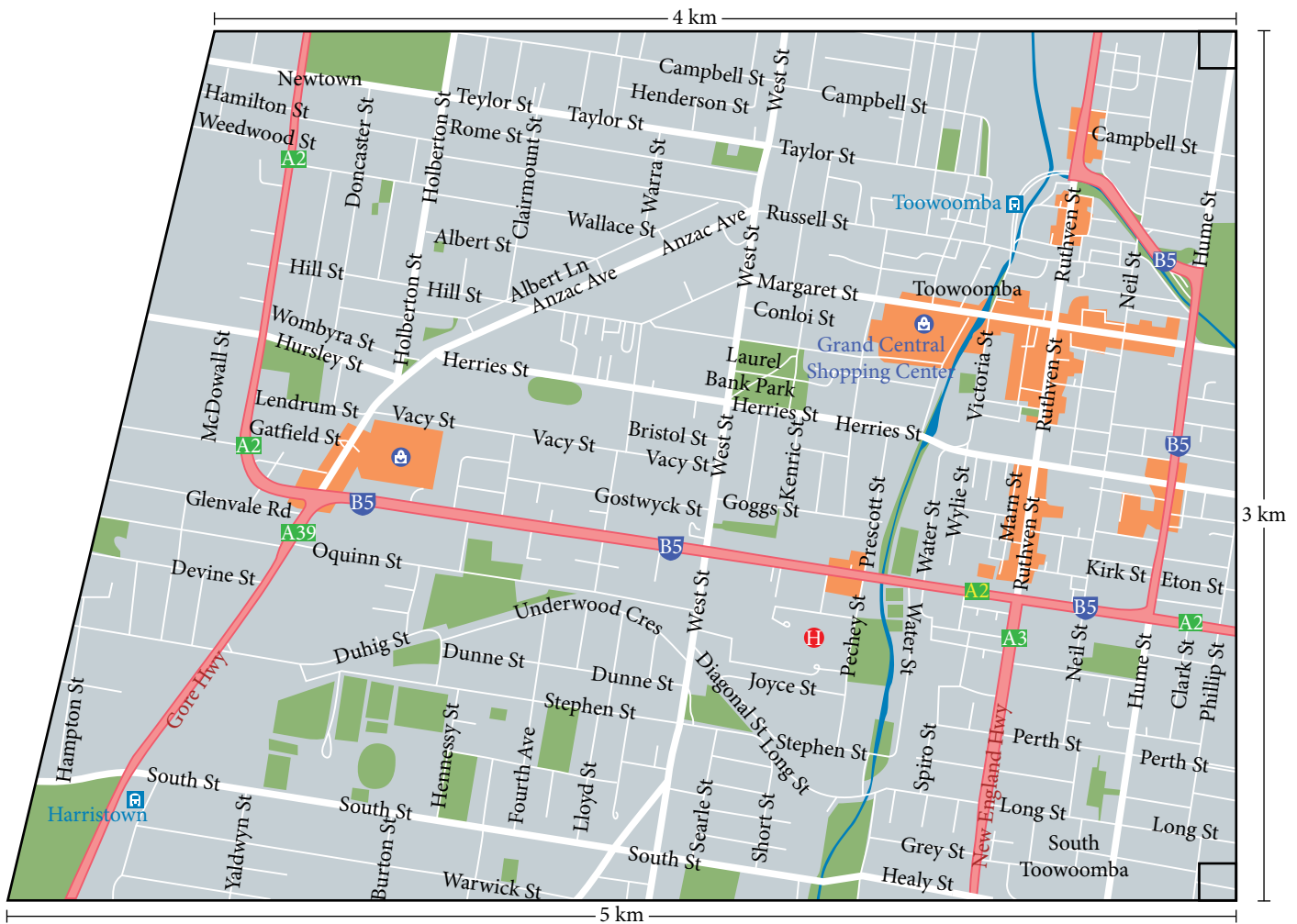


21 An equilateral triangle has a side length of 5 cm. A square has the same area as the triangle. Determine the side length of the square. Write your answer to 1 decimal place.

- 22 Drago, a graphic artist, is designing a company logo for a client. The logo must be based on a circle inside a square, which is in turn inside another circle, as shown. Determine the individual areas of pink and yellow in Drago's design. Round your answers to 2 decimal places.



- 23 A trapezium-shaped suburb contains 15 ha of parkland. After allocation to road reserves, house blocks take up 80% of the remaining land. If the house blocks have an average area of 600 m^2 , how many are there in this suburb?

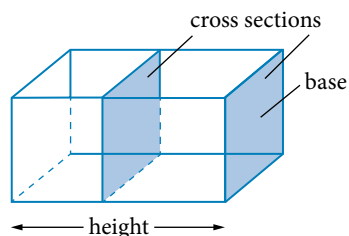


2.3

Volume of prisms and cylinders

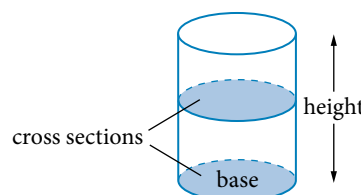
Prisms

A prism is an object that has identical parallel polygon faces at each end (sometimes called 'base' and 'top') and a uniform (i.e. the same) cross-section throughout.



Cylinders

A cylinder is technically not a prism, because the end faces are not polygons. However, the cross-section of a cylinder is constant and the ends are parallel.



Volume

Volume is the amount of three-dimensional space within the surface of an object.

There are specific formulas for determining the volume of particular solids, but the following generalised formula always applies to any three-dimensional solid with a uniform cross-section and parallel ends, like the solids shown below.

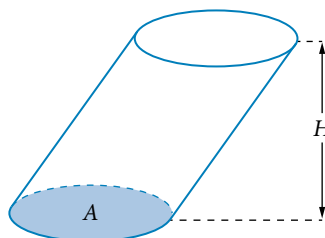
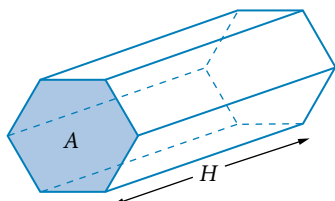
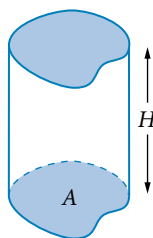
Making connections

Volume of a rectangular prism

View the changing volume of a rectangular prism.

$$\text{Volume} = \text{area of cross-section} \times \text{height}$$

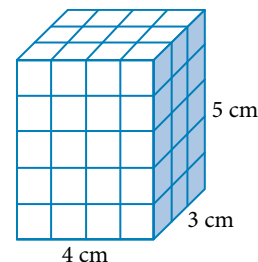
$$\text{or } V = AH$$



The base is the cross-section of the three-dimensional solid of area A , and the height H is the perpendicular distance between the parallel cross-sections at each end (even if the object is not sitting on its base).

Volume is measured in cubic kilometres (km^3), cubic metres (m^3), cubic centimetres (cm^3), cubic millimetres (mm^3) and so on.

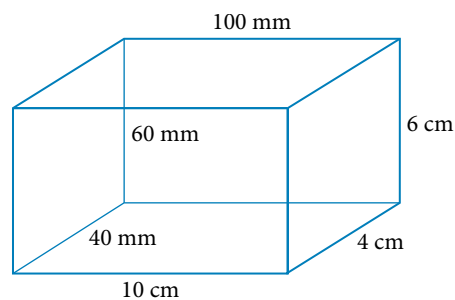
The volume of each of the cubes in the solid shown at right is 1 cm^3 .



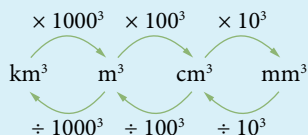
Care needs to be taken when converting between volume units.

Consider the prism shown at right.

The volume can be calculated as $10\text{ cm} \times 4\text{ cm} \times 6\text{ cm} = 240\text{ cm}^3$ or $100\text{ mm} \times 40\text{ mm} \times 60\text{ mm} = 240\,000\text{ mm}^3$. The conversion factor to change cm to mm is to multiply by 10, but the conversion factor to change cm^3 to mm^3 is to multiply by 10^3 or 1000.



To convert between cubic units, use the cube of the linear conversion factor.

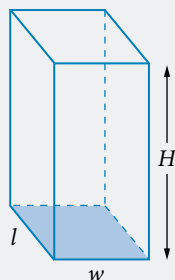


If a question involves mixed units, you must convert measurements before finding the volume.

Using the general volume formula $V = AH$, you can derive specific formulas for various objects with consistent cross-sections.

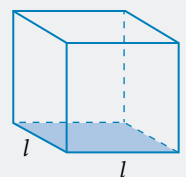
Rectangular prism

$$\begin{aligned} V &= AH \\ &= (lw)H \\ &= lwh \end{aligned}$$



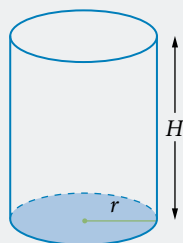
Cube

$$\begin{aligned} V &= AH \\ &= l^2 \times l \\ &= l^3 \end{aligned}$$



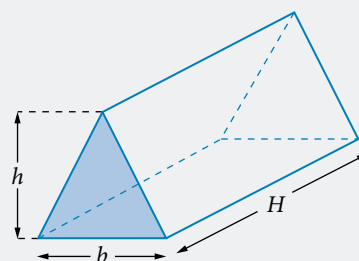
Cylinder

$$\begin{aligned} V &= AH \\ &= (\pi r^2)H \end{aligned}$$



Triangular prism

$$\begin{aligned} V &= AH \\ &= \left(\frac{1}{2}bh\right)H \end{aligned}$$



i Additional information

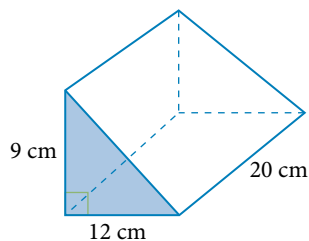
Volume of a cylinder

Practise calculating the volume of a cylinder.

10 Volume of a prism

Calculate the volume of the prism, to 2 decimal places.

(a)



THINKING

- 1 Recall the formula for the volume.
- 2 Identify the shape of the cross-sectional area.
- 3 Calculate $V = AH$ where A represents the area of the uniform cross-section and interpret the answer.

WORKING

Prism volume:

$$V = AH$$

The cross-section is a triangle, so:

$$\begin{aligned} A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 12 \times 9 \\ &= 54 \text{ cm}^2 \end{aligned}$$

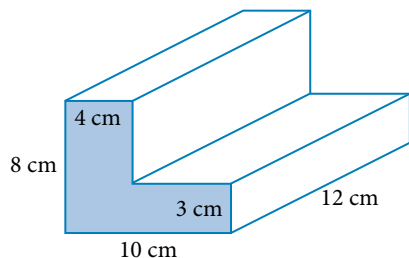
$$V = AH$$

$$= 54 \times 20$$

$$= 1080 \text{ cm}^3$$

The volume of the triangular prism is 1080 cm^3 .

(b)

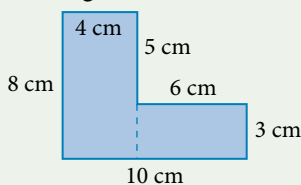


- 1 Recall the formula for the volume.
- 2 Identify common shapes that combine to form the cross-sectional area.
- 3 Determine the area of the cross-section (base).

Prism volume:

$$V = AH$$

The base is a compound shape, comprising two rectangles.



$$\begin{aligned} A &= LW + lw \\ &= (8 \times 4) + (6 \times 3) \\ &= 32 + 18 \\ &= 50 \text{ cm}^2 \end{aligned}$$

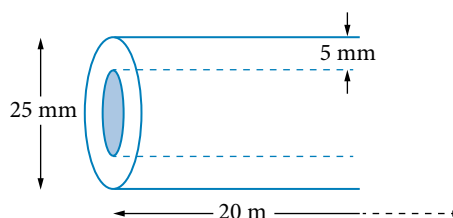
- 4 Calculate $V = AH$ where A represents the area of the uniform cross-section and interpret the answer.

$$\begin{aligned} V &= AH \\ &= 50 \times 12 \\ &= 600 \text{ cm}^3 \end{aligned}$$

The volume of the prism is 600 cm^3 .

11 Volumes of hollow solids

A 20 m long hollow plastic water pipe has thickness 5 mm. Determine the volume of plastic required to make the pipe. Answer in cubic centimetres, rounded to the nearest whole number.



THINKING

- 1 Explain the steps for finding the required volume.
- 2 Calculate the entire volume. Calculate any dimensions not given. Ensure matching units.
- 3 Determine the volume to be removed. Calculate any dimensions not given. Ensure matching units.
- 4 Calculate the required volume, rounding as directed.

WORKING

Volume required:

$$V_{\text{outer cylinder}} - V_{\text{inner cylinder}}$$

Radius of outer cylinder:

$$\begin{aligned} r_o &= 25 \div 2 \\ &= 12.5 \text{ mm} \\ &= 1.25 \text{ cm} \end{aligned}$$

$$20 \text{ m} = 2000 \text{ cm}$$

Volume of outer cylinder:

$$\begin{aligned} V_o &= \pi r^2 H \\ &= \pi \times 1.25^2 \times 2000 \\ &= 9817.5 \text{ cm}^3 \text{ (1 d.p.)} \end{aligned}$$

Volume of inner cylinder:

$$\begin{aligned} r_i &= (25 - 10) \div 2 \\ &= 7.5 \text{ mm} \\ &= 0.75 \text{ cm} \end{aligned}$$

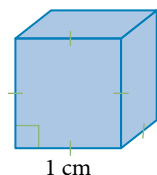
$$\begin{aligned} V_i &= \pi r^2 H \\ &= \pi \times 0.75^2 \times 2000 \\ &= 3534.3 \text{ cm}^3 \text{ (1 d.p.)} \end{aligned}$$

$$\begin{aligned} V &= V_o - V_i \\ &= 9817.5 - 3534.3 \\ &= 6283.2 \text{ cm}^3 \text{ (1 d.p.)} \end{aligned}$$

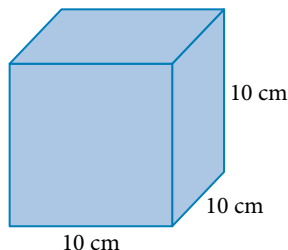
To the nearest cubic centimetre, the volume of the plastic in the water pipe is 6283 cm^3 .

Capacity

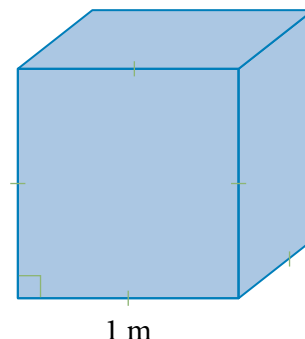
Capacity is used to describe how much a three-dimensional object can hold (for example, how much water can be stored in a hollow container).



$V = 1 \times 1 \times 1 = 1 \text{ cm}^3$, which holds 1 mL of water, which has a mass of 1 g.



$V = 10 \times 10 \times 10 = 1000 \text{ cm}^3$, which holds 1000 mL = 1 L of water, which has a mass of 1000 g = 1 kg.



$V = 100 \times 100 \times 100 = 1\,000\,000 \text{ cm}^3$, which holds 1 000 000 mL = 1000 L = 1 kL of water, which has a mass of 1 000 000 g = 1000 kg = 1 tonne.

Litres (L) and millilitres (mL) are measurement units used for capacity of household containers. Kilolitres (kL) and megalitres (ML) are used for swimming pools and reservoirs.

Decimal prefixes

Common conversions between volume and capacity:

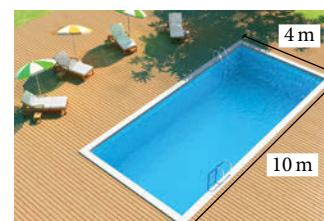
$1 \text{ cm}^3 = 1 \text{ mL}$
$1000 \text{ cm}^3 = 1 \text{ L}$
$1 \text{ m}^3 = 1 \text{ kL}$

mega	kilo	centi	milli
1 000 000	1000	$\frac{1}{100}$	$\frac{1}{1000}$
M	k	c	m

12 Capacity

A rectangular swimming pool 10 m by 4 m is filled with water to a depth of 2 m at the deep end and 1.5 m at the shallow end, with the floor a steady slope from shallow to deep end.

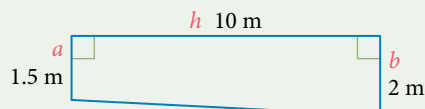
- (a) Determine the number of kilolitres (kL) of water in the pool. State the limitations of the solution.



THINKING

- 1 Recognise the shape of the cross-section of the pool and identify required lengths.
- 2 Recall the formula for the cross-sectional area. Substitute values and calculate the area.

WORKING



Trapezium:

$$\begin{aligned}
 A &= \frac{1}{2}(a+b)h \\
 &= \frac{1}{2}(1.5+2) \times 10 \\
 &= 17.5 \text{ m}^2
 \end{aligned}$$

- 3 Calculate the volume, using appropriate units.

$$\begin{aligned} V &= AH \\ &= 17.5 \times 4 \\ &= 70 \text{ m}^3 \end{aligned}$$

- 4 Convert to units of capacity and interpret the answer.

$$1 \text{ m}^3 = 1 \text{ kL}, \text{ so the pool holds } 70 \text{ kL of water.}$$

- 5 State the limitations of the solution.

The solution is correct only when the depth of the pool increases at a constant rate and the cross-section from side to side is constant. The total capacity will decrease if steps, ladders and rails are present.

- (b) How many standard cups of 250 mL each would it take to fill the pool?

- 1 Convert one or both of the amounts so that the units are the same.

$$\begin{aligned} 1000 \text{ mL} &= 1 \text{ L} \\ \Rightarrow 250 \text{ mL} &= 0.25 \text{ L} \end{aligned}$$

- 2 Use division to find how many lots of the smaller quantity are in the total quantity.

$$\begin{aligned} 1 \text{ kL} &= 1000 \text{ L} \\ \Rightarrow 70 \text{ kL} &= 70\,000 \text{ L} \end{aligned}$$

Number of cups needed:

$$\frac{70\,000}{0.25} = 280\,000$$

- 3 Interpret the answer, checking that it is reasonable.

280 000 cups of water would be needed, i.e. 4 cups for each litre.

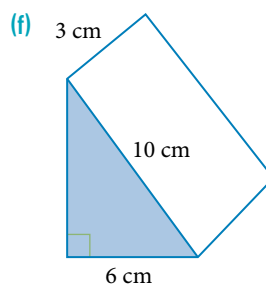
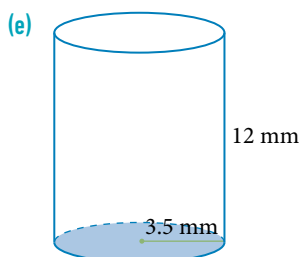
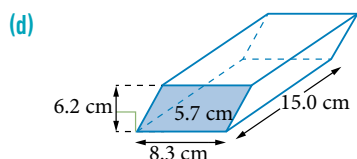
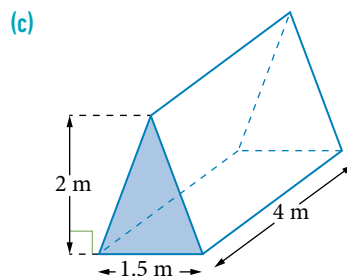
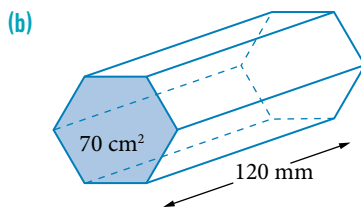
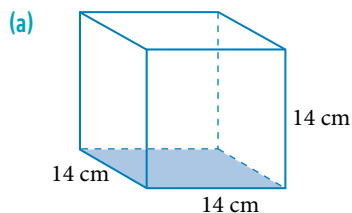
EXERCISE

2.3

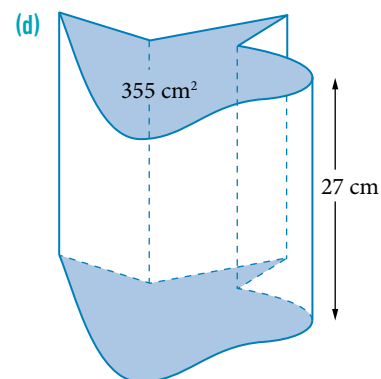
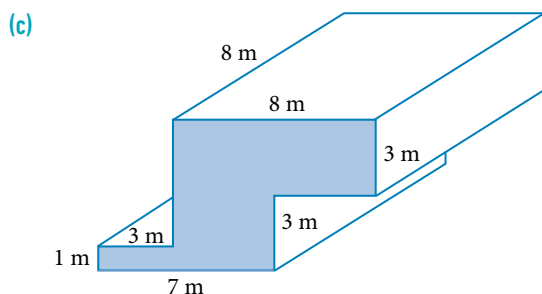
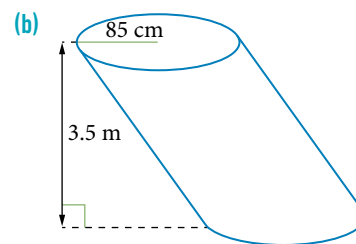
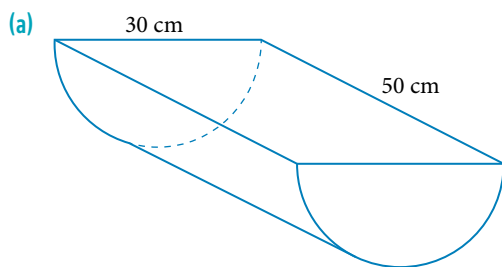
Volume of prisms and cylinders

Worked
Example

- 1 Determine the volume of each solid.



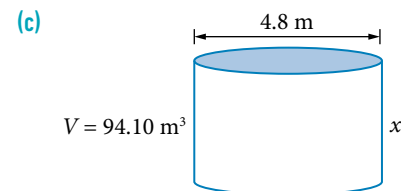
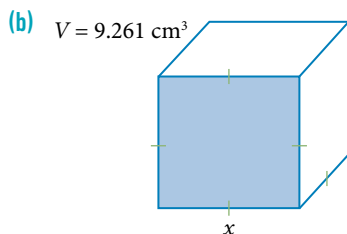
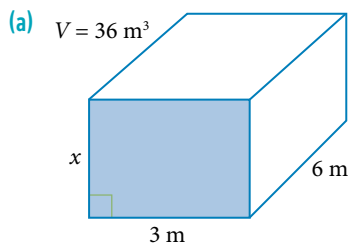
2 Calculate the capacity of each container in litres, to the nearest whole number.



3 A rectangular prism has length 20 cm and width 10 cm. If the volume of the prism is 1100 cm^3 , calculate the height of the prism.

- A 5 cm B 5.5 cm C 6 cm D 6.5 cm

4 Calculate the side length x , to 2 decimal places where necessary.



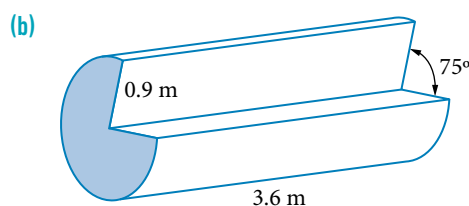
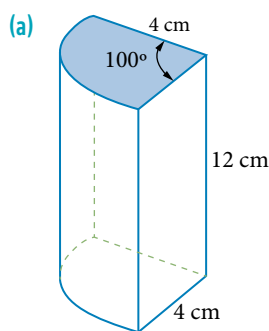
5 An open plastic cube has a side length of 50 cm.

(a) What is the capacity (in L) of the cup?

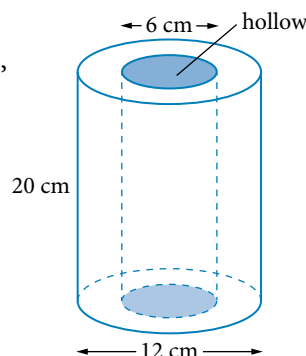
- A 125 B 150 C 250 D 125 000

(b) What common error is made by a student who chooses the last of the incorrect options?

- 6 Determine the capacity of each of the following containers. Give your answer in the most suitable unit, to the nearest whole number.

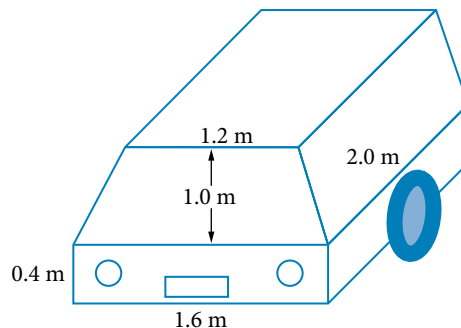


- 7 A metal toilet paper dispenser with an inner diameter of 6 cm is shown. Determine the volume, to the nearest cubic centimetre, occupied by the dispenser.

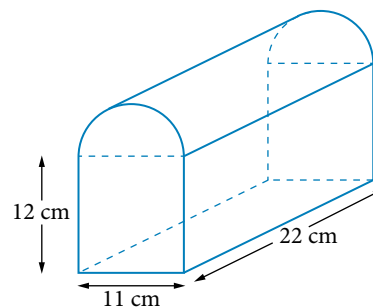


- 8 Anya hires a storage compartment for 9 weeks from a business that has various-sized storage options. If the weekly hire rate is \$3 per cubic metre, how much does it cost her to hire a rectangular prism-shaped space of length 4 m, width 3 m and height 2.5 m for the required time?
- 9 Bavneet wants to buy a tent. He has narrowed his choices to two tents. One tent is 2.6 m long and has a semicircular cross-section of diameter 1.3 m, while the other tent is 3 m long and has a triangular cross-section of width 1.5 m and height 0.75 m. Compare the sizes so Bavneet buys the tent with the greater volume.

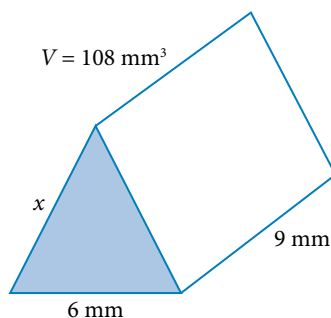
- 10 Layal wants to add a canopy to her trailer. The dimensions of the trailer are $2\text{ m} \times 1.6\text{ m} \times 0.4\text{ m}$. The proposed canopy has a trapezium-shaped cross-section of height 1.0 m that sits perfectly on top of the trailer, and tapers to a width of 1.2 m at the top. Determine the total capacity (or volume) of the trailer-canopy combination.



- 11 Booros the baker produces identical loaves of bread in the shape shown. What volume does each loaf occupy? Give your answer to the nearest cubic centimetre.



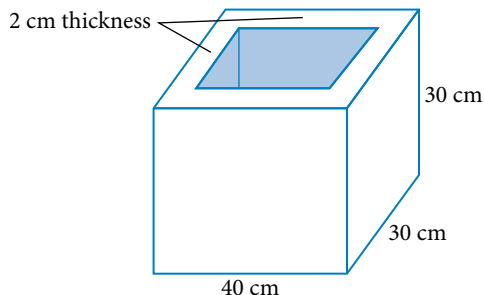
- 12 Calculate the missing side length x .



Worked
Example

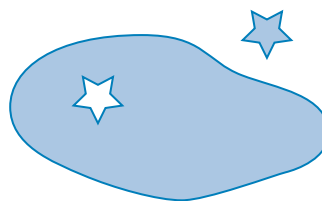
11

- 13 Determine the volume of the hollow solid.



- 14 Walid cuts biscuits from a 12 mm thick sheet of rolled dough using a star-shaped biscuit cutter that encloses an area of 5000 mm^2 per biscuit.

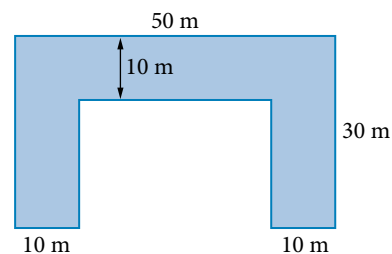
If Walid makes 24 biscuits and the biscuits expand to three times their volume when baked, what volume of baked biscuits does Walid produce? Give your answer in cubic centimetres.



12

- 15 The swimming pool at a large resort has the dimensions shown (viewed from above).

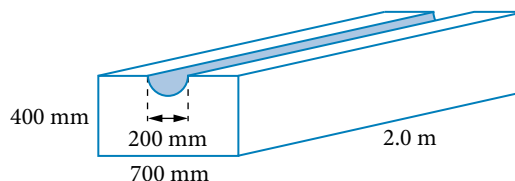
- (a) If the depth of water in the pool is on average 1.6 m, how much water is there? Give your answer in litres.
- (b) How long does it take to fill the pool if a hose delivers 30 L/min? Give your answer to the nearest day.



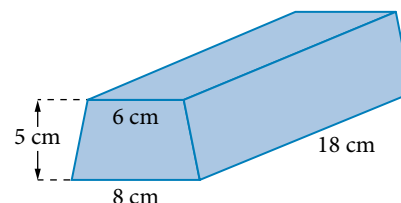
- 16 Mei installs a cylindrical water tank of radius 2.7 m and height 2.4 m to collect rainwater from her house's roof, which covers an area of 300 m^2 . Just after the empty tank is installed, a storm deposits 50 mm of rain on Mei's roof.

- (a) How much rain (in cubic metres) falls on the roof?
- (b) If all the rain is collected in the tank, to what depth is it filled? Give your answer to the nearest centimetre.

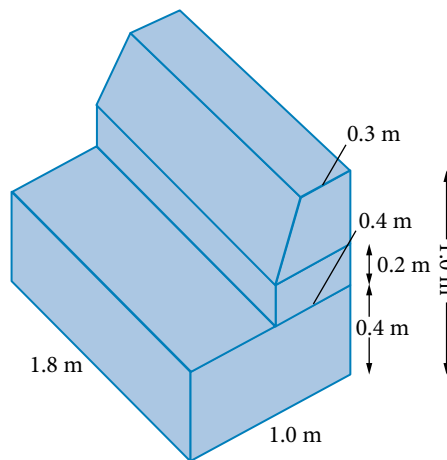
- 17 Connie is preparing a 2 m long section of a concrete open drain, as shown, and must calculate the amount of concrete to be delivered to the site. If she allows 10% extra for wastage, how many cubic metres of concrete should she order? Write your answer to 2 decimal places.



- 18 Aurum has five gold bars with the size and shape shown. She melts them down and forms them into a single gold cube. What is the edge length of Aurum's cube? Give your answer to 2 decimal places.



- 19 The ComfortKing sofa is made of 100% high-density foam, covered with a thin layer of fabric. Determine the volume of foam used to make the sofa.



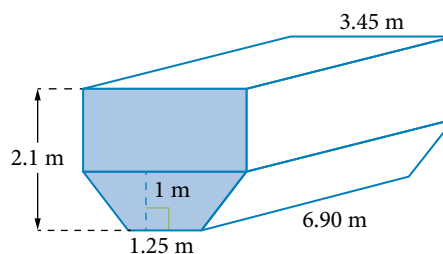
- 20 Farmer Marshall drives a combine harvester to harvest grain at a rate of 1 paddock per day. The dimensions of the rectangular paddocks are $1000\text{ m} \times 800\text{ m}$.

- (a) Determine the area of the paddock to be harvested.
- (b) The draper front of the harvester (the part that cuts and catches the grain) is 12 m wide.

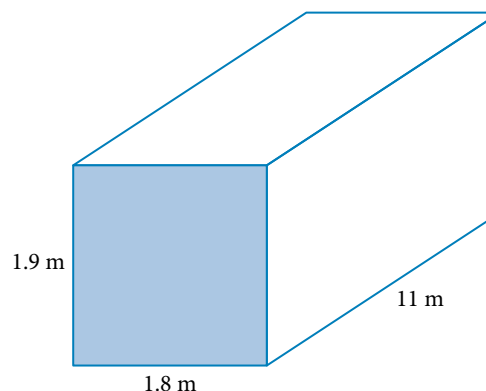


- If the harvester collected the grain in neat straight rows end to end, determine the distance (to the nearest kilometre) that Marshall would drive. State the limitations of the calculated distance and any assumptions you have made.
- (c) Marshall knows that to harvest a paddock with all the stops and starts required, his average speed will be 5 km/h.
- Determine to the nearest minute how long he anticipates it will take him to harvest the paddock.
- (d) From previous experience Marshall has estimated that the paddock will yield 3 tonnes per hectare. Calculate the mass of grain he is expecting to harvest from this paddock.
- (e) The harvester has a storage capacity of 12 tonnes. Calculate the rate of collection in kilograms per minute to 2 decimal places, and determine how long Marshall can drive the header before the bin is full.

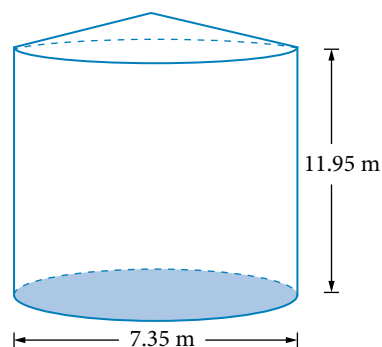
- (f) To empty the grain efficiently, the farm hand Summer will drive the chaser bin to the harvester to collect the grain. The approximate mass of grain is $1 \text{ m}^3 = 750 \text{ kg}$. Calculate the capacity of the chaser bin, to the nearest 10 tonnes.



- (g) Summer empties the chaser bin into a prime mover (truck) with the bin dimensions shown. Determine the maximum number of tonnes the prime mover can hold.

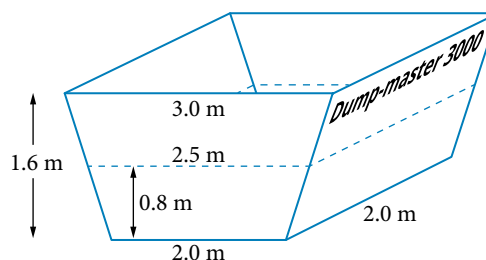


- (h) The net mass of the prime mover is 15.5 tonnes. The maximum allowable gross mass of a prime mover in its category is 42.5 tonnes. Calculate the maximum mass that can be transported to the grain storage facility per trip.
- (i) While Marshall is harvesting the paddock, his team is able to deliver 7 maximum prime mover loads to the storage facility. Calculate the mass of grain (in tonnes) from this paddock that Marshall will need to store on his farm.
- (j) Marshall stores excess grain in cylindrical silos like the one shown. If Marshall and his team continue to work at this rate during harvest, how many days will it take to fill a silo with the excess grain?



- 21 Rotundo is a water runner for his soccer club. For Saturday's game, he needs enough water for 60 cylindrical cups of radius 3.5 cm filled to a depth of 10 cm. He uses a large cylindrical container of radius 20 cm and height 70 cm to transport the water to the game. To what height must Rotundo fill the large container so he has enough water for the game? Answer in whole centimetres.

- 22 This is the Dump-master 3000 waste collection bin. When it is filled to half its depth, what percentage of its capacity is used?



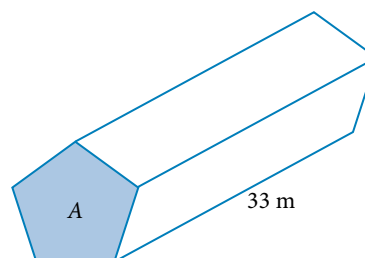
- 23 A nuclear waste storage facility consists of a rectangular prism-shaped room of length 22 m, width 16 m and height 9 m. Cylinders containing radioactive waste with diameter 1.5 m and height 2.5 m are stored upright in the room, which is surrounded by a thick layer of concrete shielding.

- Calculate the volume of storage space available inside the room.
- Calculate the maximum number of cylinders that could be stored in the room. Explain any assumptions you made.

- 24 Storage modules for a proposed space station are to be pentagonal prisms of length 33 m. Designers are considering pentagonal ends of different areas, which will in turn affect the volume of the storage modules.

The area of a regular pentagon based on its side length L , involving a rounded value, is: $A = 1.72 \times L^2$.

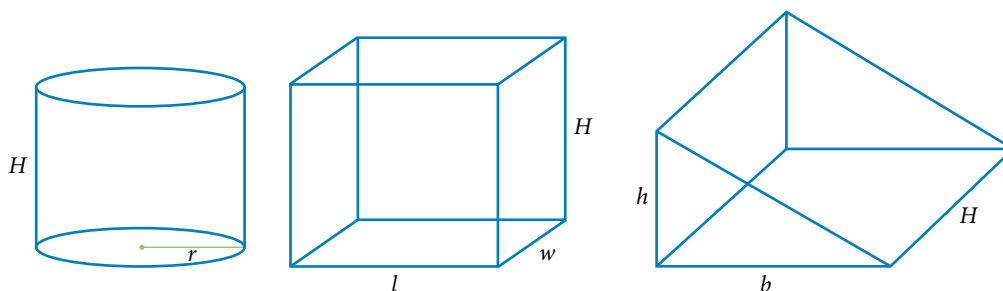
What side length would produce volumes of 100 m^3 and 5000 m^3 respectively? Answer to 2 decimal places.



- 25 A teacher is showing a Maths class the relationship between cubic centimetres and millilitres. The equipment for each group is a measuring cylinder, some water and a standard plastic 'stick' 1 cm by 1 cm by 10 cm. In the demonstration, the teacher begins with 31.2 mL of water in the measuring cylinder, then drops in the plastic stick, pushing it just below the surface with a pencil.

- What will be the reading in the measuring cylinder with the submerged stick?
- Some groups have a measuring cylinder of diameter 2 cm and others have a cylinder of diameter 5 cm. For each cylinder size, calculate how many millimetres the water level will rise by, when the stick is submerged.

- 26 Consider the following solids.



- Determine the relationship between the volume of the shape V_1 and the volume of the shape if all dimensions doubled in length, V_2 .
- Determine the relationship between the volume of the shape V_1 and the volume of the shape if all dimensions tripled in length, V_3 .
- Determine the relationship between the volume of the solids V_1 and the volume of the solids if each dimension increased to 5 times the length, V_5 .
- Create a formula to show the relationship between V_1 and V_k , where V_k represents the volume of a solid if all dimensions in V_1 are increased by a factor of k .

2.4

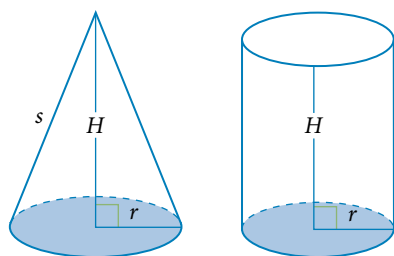
Volume of tapered solids and spheres

Tapered solids

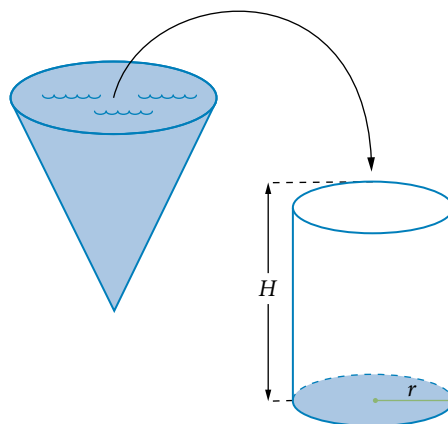
A tapered solid is a three-dimensional object with a decreasing cross-sectional area moving from the base of the solid to the top.

Cones and pyramids are examples of tapered solids. How do you calculate their volume?

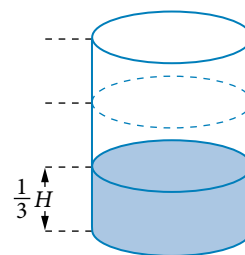
Consider two solids that have identical bases and heights: one a cylinder and the other a cone. The cone has less volume than the cylinder.



Imagine the cone turned upside down and filled with water, then picture the cone filling the cylinder.



Exactly three cones worth of water will fill the cylinder.



The volume of the cone is:

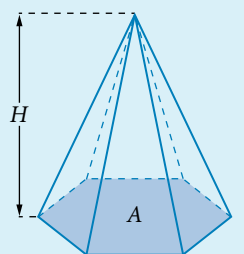
$$V = \frac{1}{3}AH \text{ where } A \text{ represents the area of the base, or } V = \frac{1}{3}\pi r^2 H$$

Pyramids are also tapered solids. A pyramid is a polyhedron with a polygonal base and triangular sides that meet at a point. The volume of a pyramid be calculated using $V = \frac{1}{3}AH$ where A represents the area of the base and H is the perpendicular height.

For all pyramids and tapered solids:

$$V = \frac{1}{3}AH$$

where A is the area of the base and H represents the perpendicular height of the solid.



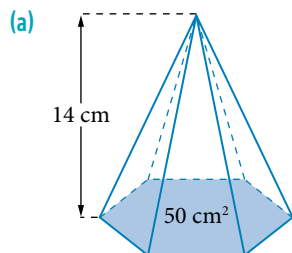
i Additional information

Volume of a pyramid

Revise the process of calculating the volume of a pyramid.

13 Volume of tapered solids

Determine the volume of the tapered solid to the nearest whole number.



THINKING

- 1 Recall the formula for the volume.
- 2 Identify the required lengths and area, then substitute into the formula to calculate V .
- 3 Interpret the answer.

WORKING

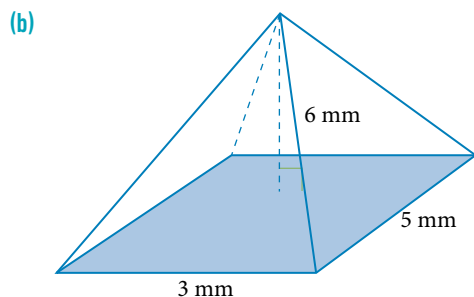
Volume of a tapered solid:

$V = \frac{1}{3}AH$ where A is the area of the base and H is the perpendicular height between the base and the highest point.

$$V = \frac{1}{3} \times 50 \times 14$$

$$V = 233.33\dots$$

To the nearest cubic centimetre, the volume of the pyramid is 233 cm^3 .



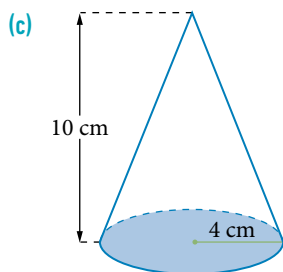
- 1 Recall the formula for the volume.
- 2 Identify appropriate side lengths and substitute the values into the formula to calculate V .
- 3 Interpret the answer.

Volume of a tapered solid:

$V = \frac{1}{3}AH$ where the base is a rectangle, so $A = lw$.

$$\begin{aligned} V &= \frac{1}{3}lwH \\ &= \frac{1}{3} \times 5 \times 3 \times 6 \\ &= 30\text{ mm}^3 \end{aligned}$$

The volume of the pyramid is 30 mm^3 .



1 Recall the formula for the volume.

Volume of a cone:

$V = AH$ where the base is a circle, so $A = \pi r^2$

$$V = \frac{1}{3} \pi r^2 H$$

2 Identify the required side lengths and substitute into the formula to calculate V .

$$V = \frac{1}{3} \pi \times 4^2 \times 10$$

$$= 167.55 \dots$$

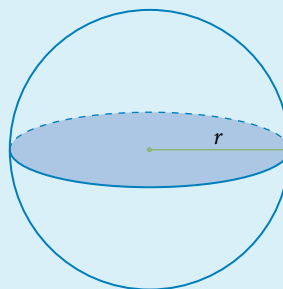
3 Interpret the answer.

The volume of the cone, to the nearest cubic centimeter, is 168 cm^3 .

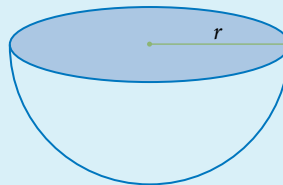
Spheres

A sphere is a solid (like a ball) with all points on the surface the same distance from the centre.

The volume of a sphere (ball) is given by the formula: $V = \frac{4}{3} \pi r^3$ where r is the radius of the sphere.



The volume of a half-sphere (or 'hemisphere' or 'semi-sphere') is given by: $V = \frac{2}{3} \pi r^3$



i Additional information

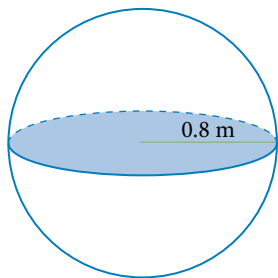
Volume of a sphere

Determine the volume of a sphere, by watching the videos and then practising on your own with the follow-up questions.

14 Volume of a sphere and a hemisphere

Calculate the volume of each solid, to 2 decimal places.

(a)



THINKING

- 1 Recall the formula for the volume.
- 2 Identify the length of the radius, then substitute into the formula to calculate V .
- 3 Interpret the answer.

WORKING

Volume of a sphere:

$$V = \frac{4}{3}\pi r^3$$

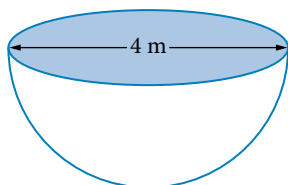
$$V = \frac{4}{3}\pi \times 0.8^3$$

$$V = 2.144\dots$$

$$= 2.14\text{ m}^3 \text{ (2 d.p.)}$$

The volume of the sphere to 2 decimal places is 2.14 m^3 .

(b)



- 1 Recall the formula for the volume.
- 2 Identify the length of the radius, then substitute into the formula to calculate V .
- 3 Interpret the answer.

Volume of a hemisphere:

$$V = \frac{2}{3}\pi r^3$$

$$r = 4 \div 2$$

$$= 2\text{ m.}$$

The radius of the hemisphere is 2 m.

$$V = \frac{2}{3}\pi \times 2^3$$

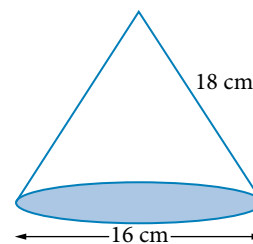
$$V = 16.755\dots$$

$$= 16.76\text{ m}^3 \text{ (2 d.p.)}$$

The volume of the hemisphere to 2 decimal places is 16.76 m^3 .

15 Volume involving Pythagoras' theorem

Determine the volume of the cone, to the nearest whole number.



THINKING

- 1 Redraw the diagram to identify the dimensions that are needed.

Use a suitable variable for dimensions that are to be calculated.

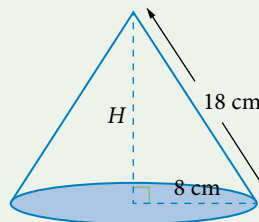
- 2 Use Pythagoras' theorem to calculate the unknown. Do not round your answer.

- 3 Recall the formula for the volume.

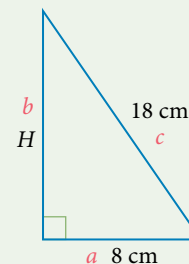
- 4 Substitute the given values into the formula to calculate V .

- 5 Interpret the answer.

WORKING



$$\begin{aligned} c^2 &= a^2 + b^2 \\ 18^2 &= 8^2 + H^2 \\ H^2 &= 18^2 - 8^2 \\ &= 260 \\ \therefore H &= \sqrt{260} \\ &= 16.124\dots \text{cm} \end{aligned}$$



The exact height of the cone is $\sqrt{260}$ cm.

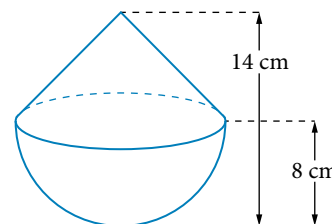
Volume of a cone:

$$\begin{aligned} V &= \frac{1}{3}\pi r^2 H \\ V &= \frac{1}{3}\pi \times 8^2 \times \sqrt{260} \\ &= 1080.675\dots \text{cm}^3 \end{aligned}$$

To the nearest cubic centimetre the volume of the cone is 1081 cm^3 .

16 Volume of a composite solid

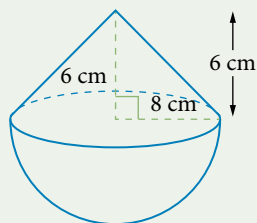
Determine the volume of the solid, to the nearest whole number.



THINKING

- 1 Redraw the diagram, identifying the dimensions that are needed.

WORKING



- 2 Recall the formulas for the volume.

Volume of a cone with a hemisphere:

$$V = \frac{1}{3}\pi r^2 H + \frac{2}{3}\pi r^3$$

- 3 Substitute the given values into the formula to calculate V .

$$\begin{aligned} V &= \frac{1}{3}\pi \times 8^2 \times 6 + \frac{2}{3}\pi \times 8^3 \\ &= 1474.454\dots \text{cm}^3 \end{aligned}$$

- 4 Interpret the answer.

To the nearest cubic centimetre the volume is 1474 cm^3 .

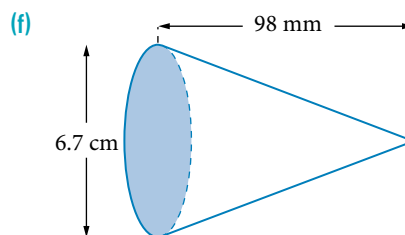
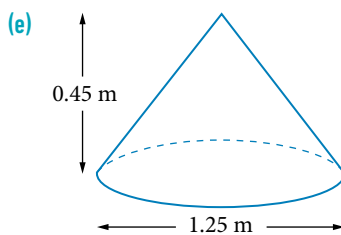
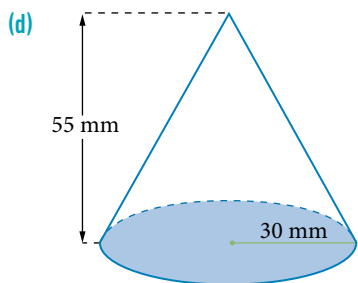
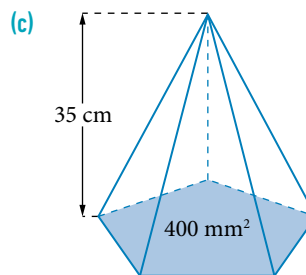
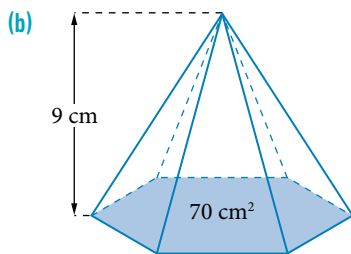
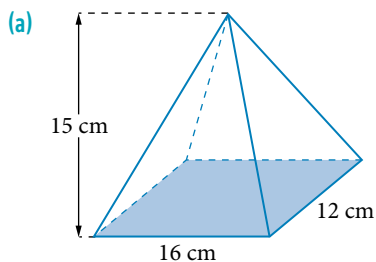
EXERCISE

2.4

Volume of tapered solids and spheres

Worked
Example

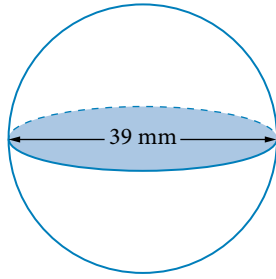
- 1 Determine the volume of each tapered solid. Give your answers to 2 decimal places where necessary.



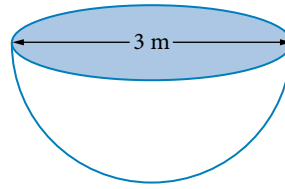
14

- 2 Determine the volume of each sphere or hemisphere to 2 decimal places.

(a)

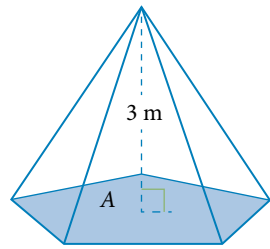


(b)

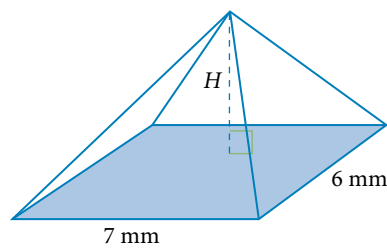


- 3 Calculate the value of the unknown, to 2 decimal places if necessary.

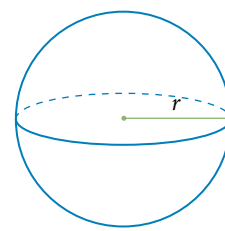
(a) $V = 84 \text{ m}^3$



(b) $V = 210 \text{ mm}^3$



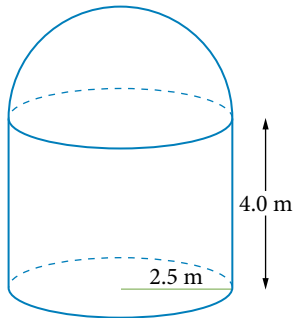
(c) $V = 310 \text{ cm}^3$



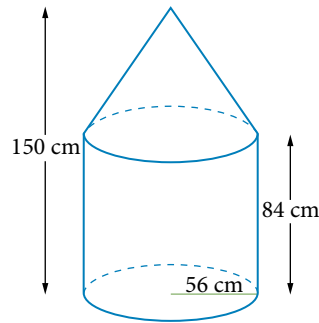
16

- 4 Determine the volume of each of the following composite solids. Round your answers to 2 decimal places where necessary.

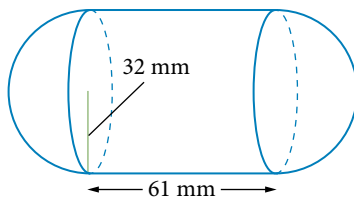
(a)



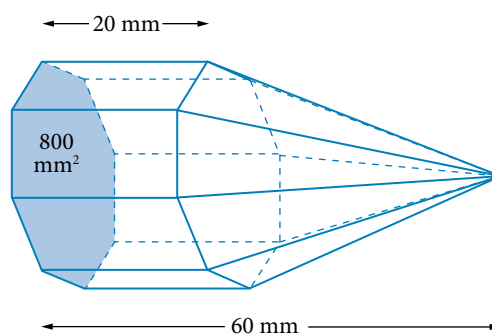
(b)



(c)

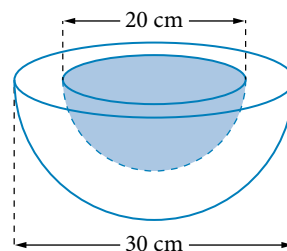


(d)

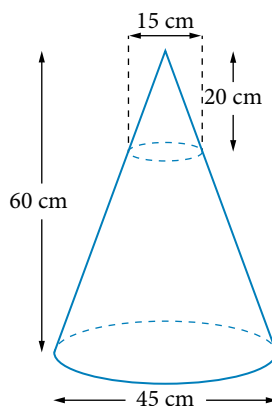


- 5 Chen works for a homewares company that sells hemispherical ceramic bowls like the one shown.

- (a) Determine the volume of ceramic material used to make such a bowl, to the nearest cubic centimetre.
- (b) What is the capacity of the bowl, to the nearest millilitre?



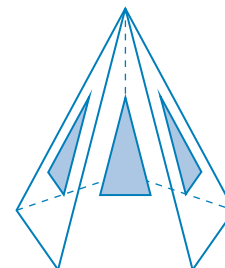
- 6 A cone of height 60 cm has its top 20 cm removed. Determine the volume of the remaining solid, to 2 decimal places.



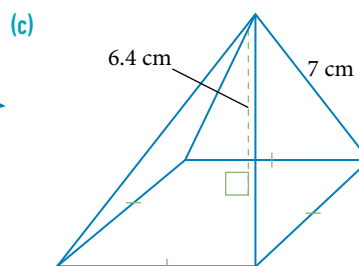
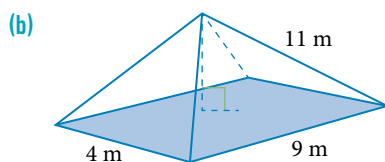
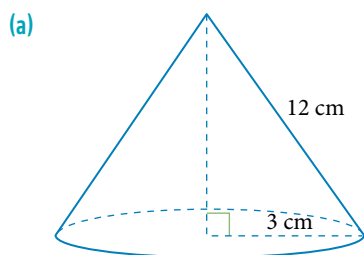
- 7 In an effort to appear on a well-known television program featuring unusual homes, Feda plans to build a home in the shape of a pentagonal pyramid.

The home will be 13 m tall and occupy a base area of 220 m^2 .

- (a) What volume will the home occupy? Answer to the nearest cubic metre.
- (b) To satisfy council requirements, Feda is forced to reduce the height of the home to 9.5 m. By what volume does this reduce the size of the home? Use your rounded answer from part (a) and answer to the nearest cubic metre.
- (c) Calculate the percentage decrease in volume, to the nearest per cent.

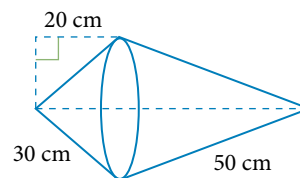


- 8 Calculate the volume of these solids, to 2 decimal places.



Worked
Example

- 9 Determine the volume of the solid, to the nearest cubic centimetre.



- 10 A sphere has volume 1000 cm^3 .

- (a) Which option is closest to the diameter of the sphere?

A 6.20 cm

B 7.52 cm

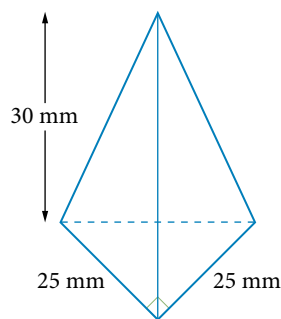
C 12.41 cm

D 15.03 cm

- (b) Explain the common error that would lead to the first incorrect option.

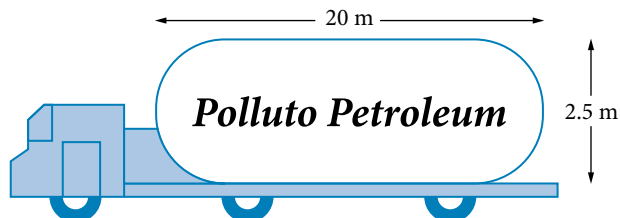
- 11 A novelty squash ball signed by athletes at a tournament has the same volume as 3 squash balls of radius 2 cm. Determine the radius of the novelty squash ball.
- 12 On a hot day in the summer holidays, Gurkan is using a 2 L water bottle to fill balloons in readiness for a water fight with his friends. He wants each balloon to have a diameter of 7 cm.
- How many balloons of diameter 7 cm can Gurkan prepare?
 - After losing the initial water fight, Gurkan decides he needs to make 20 identically filled water balloons from his next 2 L of water. What diameter must each of these balloons now have? Round the answer to 1 decimal place.

- 13 Carrina the chocolate maker wants to make individual chocolates in the shape of right-angled triangular pyramids, as shown. Each box will contain 12 identical chocolates. Determine the volume of chocolate in each box, in millilitres.

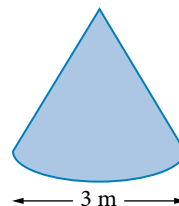


- 14 A cone with a height of 8 cm has the same volume as a sphere of radius 5 cm. Calculate the base diameter of the cone to 1 decimal place.
- 15 The Earth has a diameter of 12 742 km, which is 9268 km wider than the Moon. How many times the Moon's volume is the volume of the Earth? Write your answer to the nearest whole number. State any assumptions you had to make.

- 16 A petrol company's delivery vehicle carries a 20 m long, 2.5 m tall tank in the shape of a cylinder with hemispherical ends.
- The delivery vehicle has a full tank. What volume of fuel is it carrying? Express your answer in cubic metres, to 1 decimal place.



- The vehicle delivers fuel to an empty underground tank at a petrol station. The tank is a rectangular prism with length 15 m, width 7 m and height 4 m. What is the volume of the tank?
 - The delivery vehicle empties its load of fuel into the station's underground tank. What is the depth of the delivered fuel in the underground tank? Answer to the nearest centimetre.
 - Determine the minimum number of deliveries required to fill the underground tank.
- 17 Grottie's Garden Supplies delivers a 20 m^3 truckload of sand to a building site. The tipped sand forms a 3 m wide conical pile on the site. How high is the delivered pile of sand? Answer in metres, to 1 decimal place.



- 18 A confectionery company manufactures a new spherical jelly lolly. The diameter of the lolly is 15 mm. The manufacturing process involves producing hemispheres and then joining them to form a sphere. The quality-control department discovers that many lollies are falling apart during transport to retailers. The company decides to avoid this problem by making hemispherical lollies instead, but the lollies must have the same volume as the original spherical version. Determine the required diameter of the hemispherical version of the lolly, to the nearest millimetre.

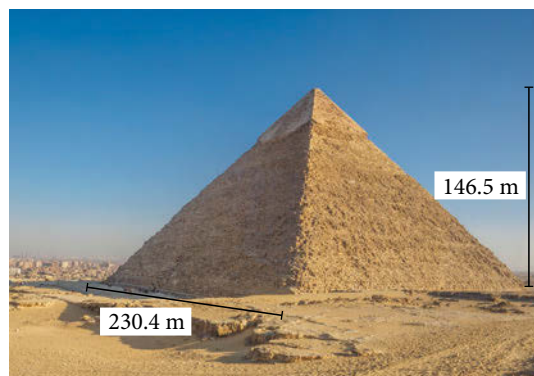
- 19 The Great Pyramid of Giza is estimated to be constructed from 2.3 million limestone blocks. It is 146.5 m tall with a square base of length 230.4 m.

(a) Calculate the volume of the pyramid to the nearest cubic metre. State any assumptions you made.

(b) The density of limestone is approximately 2.71 g/cm^3 .

Calculate the average mass of each block to 2 decimal places.

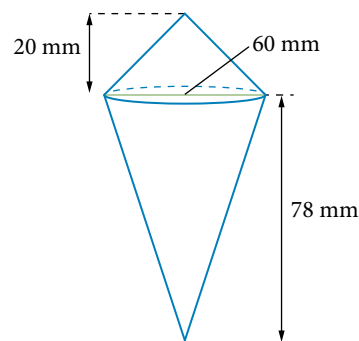
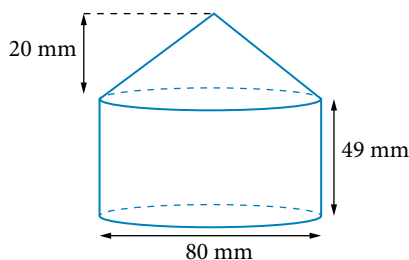
(c) One of the tallest stationary cranes in the world is Kockums Crane (Tears of Malmo), built in 1934. It can lift 1500 tonnes at a time up to approximately 130 m. What percentage of the pyramid would this crane be able to build? Write your answer to 2 decimal places. State any assumptions you made.



- 20 Calculate the percentage increase in volume produced by adding a 1 mm layer of chocolate to a 2 cm solid spherical core of caramel. Give your answer to the nearest whole percentage.

- 21 An ice-cream store sells ice-cream in a waffle cone and in a cup. Both have an additional 20 cm of ice-cream swirled in the shape of a cone.

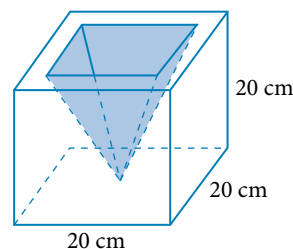
The waffle cone adds 30 cents to the cost. Calculate the volume of ice-cream in each cone, to determine the price of the cup if the price of an ice-cream served in a cone is \$3.15. State any assumptions you made.



- 22 Neil has designed a glass vase in the shape of a cube with an inverted hollow square-based pyramid section into which water can be poured. The top opening of the vase is $18 \text{ cm} \times 18 \text{ cm}$ and the hollow section is 18 cm deep.

(a) Calculate the volume of glass required to make the vase.

(b) How many millilitres of water are needed to fill the vase to a depth of 15 cm?

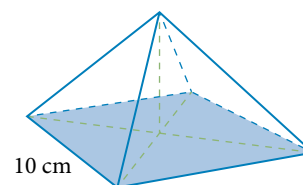


- 23 Bakir produces a sphere of dough of diameter 12 cm. He rolls this out to form a flat sheet of thickness 8 mm. First he cuts a circle of diameter 20 cm. He then re-rolls the remaining pastry to the same thickness to make a second circle. Determine the diameter of the second circle, to the nearest centimetre.

- 24 A square-based symmetrical pyramid (tetrahedron) has all edges 10 cm.

(a) Determine the diameter of a sphere with the same volume as the tetrahedron.

(b) Determine the radius of a cone with the same height and volume as the tetrahedron.



2.5

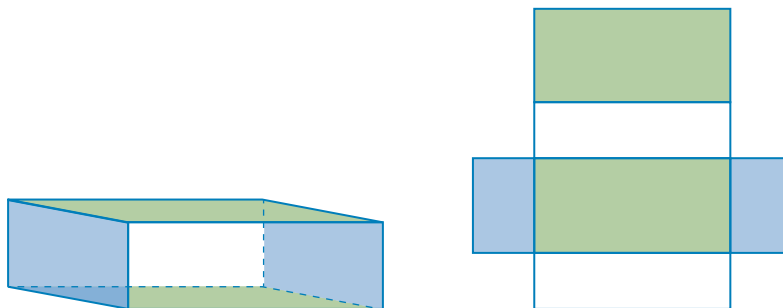
Surface area of prisms and cylinders

Closed objects

Surface area, or SA, is the sum of the areas of all the faces of a solid.

There are formulas for calculating the SA of various objects, but it is often easier to just calculate the area of each individual face and add these areas up. Be careful to include all the required surfaces.

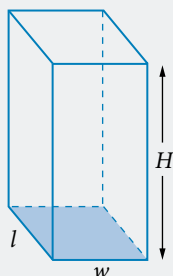
It may help to visualise a net of the object being considered. For example, a rectangular prism has six faces (three pairs: left and right, top and bottom, front and back). The prism can be 'unfolded', and the faces then form the flat net shown.



SA formulas

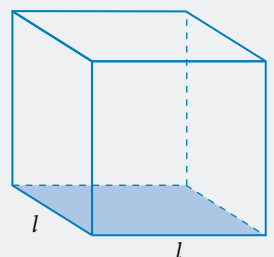
Rectangular prism

$$SA = 2(lw + lH + wH)$$



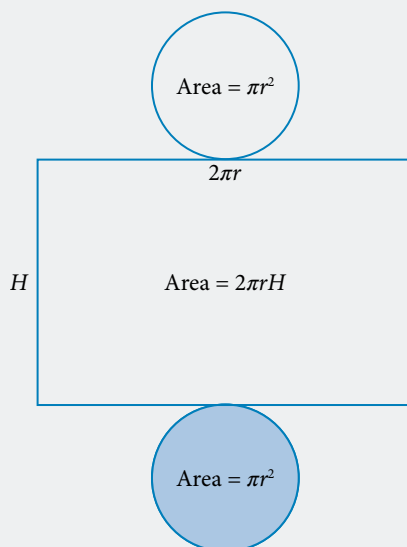
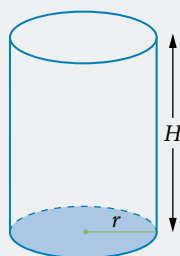
Cube

$$SA = 6l^2$$



Cylinder

$$SA = 2\pi r^2 + 2\pi rH$$



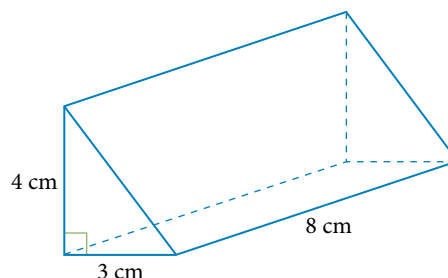
i Additional information

Surface area of a cylinder

Revise your understanding of cylinder surface area.

17 Surface area of a prism

Determine the surface area of the triangular prism.



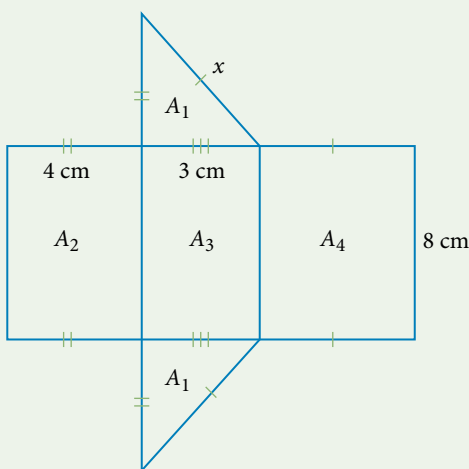
THINKING

- 1 Identify the shapes that make up the surface area.
- 2 Draw a labelled diagram and determine the lengths of any required sides.

WORKING

There are 2 identical triangular ends.

There are 3 rectangular faces of equal length.



Applying Pythagoras' theorem:

$$\begin{aligned}x^2 &= 3^2 + 4^2 \\&= 9 + 16 \\&= 25 \\\therefore x &= \sqrt{25} \\&= 5 \text{ cm}\end{aligned}$$

- 3 Recall suitable area formulas to calculate the area of each face by substituting known side lengths.

Triangular end (each):

$$\begin{aligned}A_1 &= \frac{1}{2}bh \\&= \frac{1}{2} \times 3 \times 4 \\&= 6 \text{ cm}^2\end{aligned}$$

Rectangular base:

$$\begin{aligned}A_3 &= lw \\&= 3 \times 8 \\&= 24 \text{ cm}^2\end{aligned}$$

Rectangular back:

$$\begin{aligned}A_2 &= lw \\&= 4 \times 8 \\&= 32 \text{ cm}^2\end{aligned}$$

Sloping face:

$$\begin{aligned}A_4 &= lw \\&= 5 \times 8 \\&= 40 \text{ cm}^2\end{aligned}$$

- 4 Add the areas of each surface to determine the total surface area (SA).

$$\begin{aligned} SA &= 2 \times A_1 + A_2 + A_3 + A_4 \\ &= 6 \times 2 + 32 + 24 + 40 \\ &= 108 \text{ cm}^2 \end{aligned}$$

- 5 Interpret the answer.

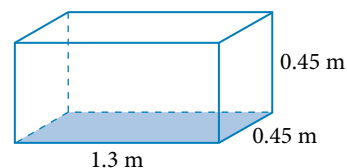
The total surface area of the prism is 108 cm^2 .

Open objects

The surface area of an open object generally refers to the total area of the external surfaces. Care must be taken to ensure you include the correct faces, as one of the 'usual' faces will be missing.

18 Surface area of an open object

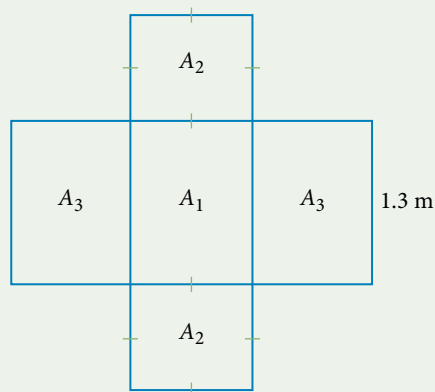
Determine the total area of the glass of this open-topped fish tank.



THINKING

- 1 Recognise the shape of each face.
Recall the formulas to determine the area and substitute the known values.

WORKING



$$A = lw$$

Rectangular bottom:

$$\begin{aligned} A_1 &= 0.45 \times 1.3 \\ &= 0.585 \text{ m}^2 \end{aligned}$$

Rectangular end (each):

$$\begin{aligned} A_2 &= 0.45 \times 0.45 \\ &= 0.2025 \text{ m}^2 \end{aligned}$$

Rectangular side (each):

$$\begin{aligned} A_3 &= 0.45 \times 1.3 \\ &= 0.585 \text{ m}^2 \end{aligned}$$

- 2 Add the areas of each surface to determine the total surface area (SA).

$$\begin{aligned} SA &= A_1 + 2A_2 + 2A_3 \\ &= 0.585 + 2 \times 0.2025 + 2 \times 0.585 \\ &= 2.16 \text{ m}^2 \text{ (2 d.p.)} \end{aligned}$$

- 3 Interpret the answer.

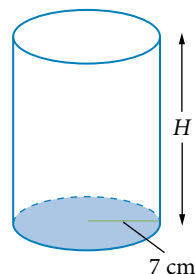
The total area of the glass is 2.16 m^2 .

Calculating a dimension from a known surface area

In practical situations you may need to calculate the dimensions required to produce a given surface area.

19 Backtracking from SA to determine the length of a dimension

The surface area of a cylindrical drinking glass of radius 7 cm is 682 cm^2 . Calculate the height of the glass, to the nearest centimetre.



THINKING

1 Identify the shapes that make up the surface area.

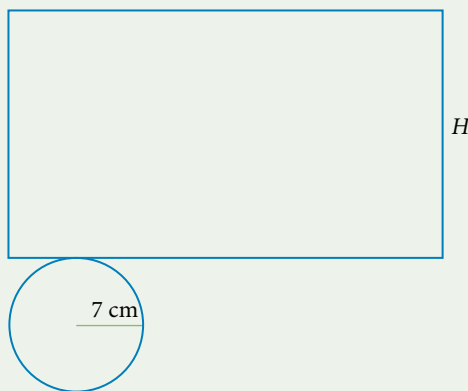
2 Recall the formula for the surface area (SA).

3 Substitute the known values into the formula.

4 Rearrange the equation and solve for the unknown.

5 Interpret the answer.

WORKING



$$SA = \pi r^2 + 2\pi rH$$

$$682 = \pi \times 7^2 + 2\pi \times 7 \times H$$

$$2\pi \times 7 \times H = 682 - \pi \times 7^2$$

$$H = \frac{682 - \pi \times 7^2}{2\pi \times 7}$$

$$H = 12.006\dots$$

$$= 12 \text{ cm (nearest cm)}$$

The height of the cylinder is 12 cm.

EXERCISE

2.5

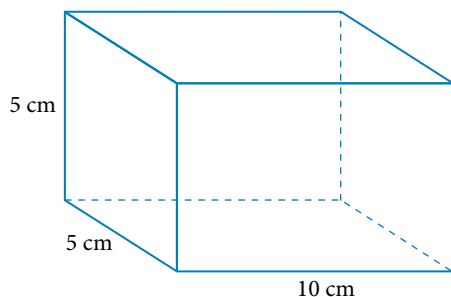
Surface area of prisms and cylinders

Worked
Example

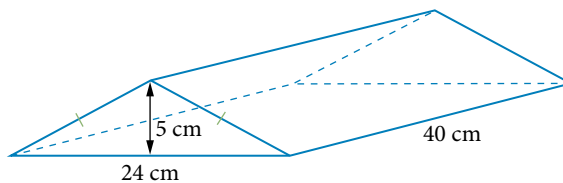
17

- 1 Determine the surface area of each object.

(a)

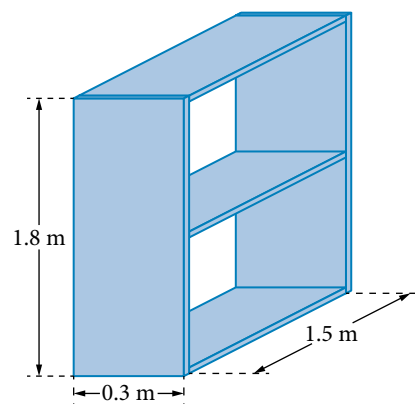


(b)



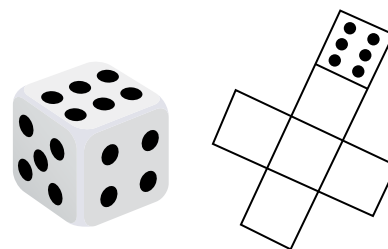
18

- 2 Mamta builds an open bookshelf using rectangular pieces of particle board, as shown. The pieces were cut from a single large flat sheet of particle board. What was the area of the flat sheet used?



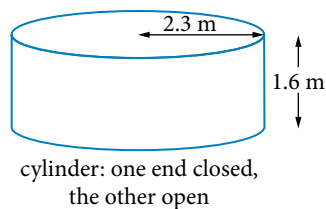
- 3 A standard die is a cube with the numbers 1 to 6 or dots arranged so that the total of opposite faces adds to 7, so if 6 is on the upper face then 1 is hidden underneath.

Copy and complete the net of the die by filling in dots to match the description.

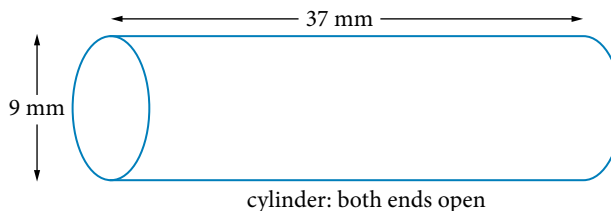


- 4 Determine the total surface area of each of the following objects. Answer to 1 decimal place.

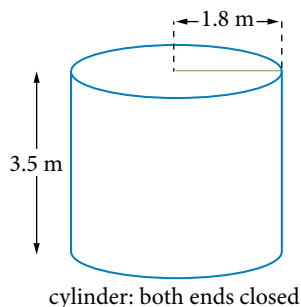
(a)



(b)

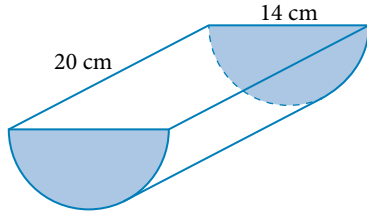


(c)

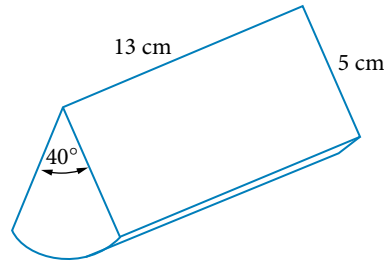


- 5 Determine the surface area of the following solids, to the nearest square centimetre.

(a)



(b)



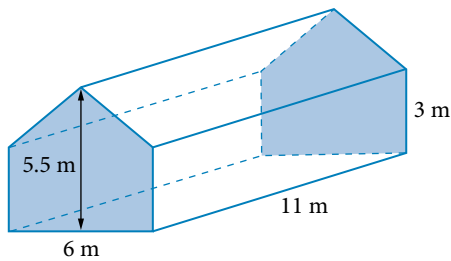
- 6 Jo is a pastry chef and wants to make 12 pies, each with an outer surface made of her special pastry. Each pie is shaped as a cylinder with radius 8.5 cm and height 5.2 cm. Assuming no wasted pastry, determine the total area of pastry needed. Answer to the nearest square centimetre.

..

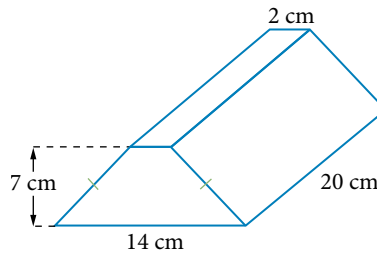
.....

- 7 Determine the surface area of each object, to the nearest whole number.

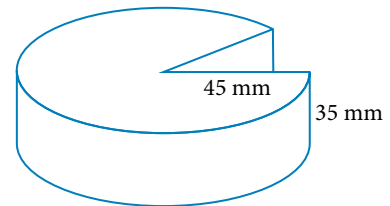
(a)



(b)

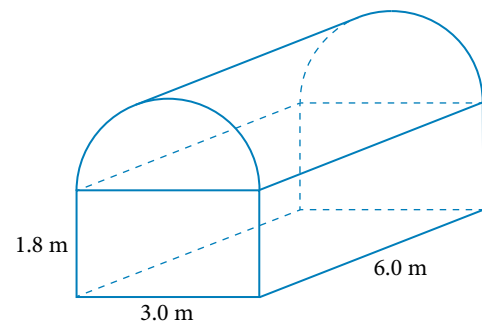


- 8 Mr Gorgonzola sells cylindrical cheeses with one 30° slice missing (he insists on tasting each block). Determine the total surface area of the remaining piece of cheese (as shown). Answer to the nearest square millimetre.



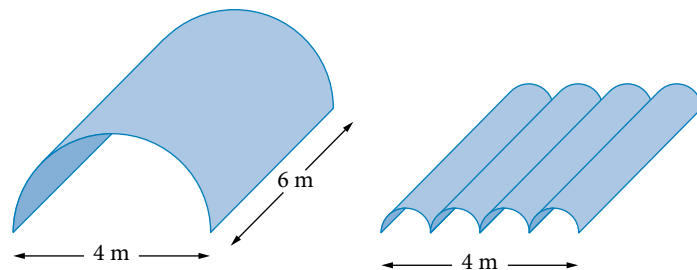
- 9 Robert manufactures polythene-covered greenhouses using steel frames and polythene sheeting. Each greenhouse has a semicircular roof and no base.

Calculate the area of polythene sheeting in one of Robert's greenhouses, assuming no gaps in the sides or roof. Round your answer to the nearest square metre. Explain any assumptions you made.

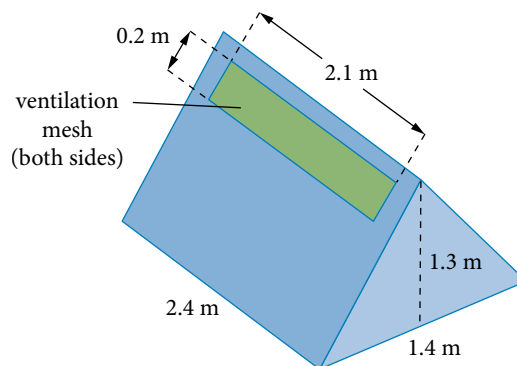


- 10 Ginevra wants to construct a hothouse for growing seedlings, and is considering either one large or four narrow semi-cylindrical tunnels made of polythene sheet.

Which design (left or right) uses the most polythene sheeting? Discuss the limitations of each of the designs.



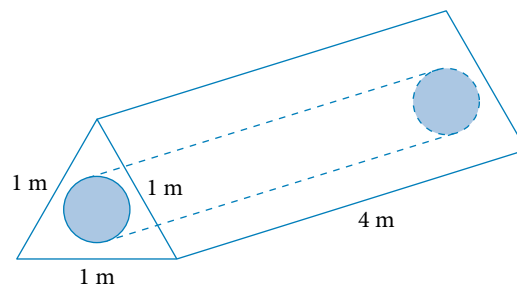
- 11 The Freezemaster tent is made of blue canvas, except for two fly-wire ventilation mesh panels (one on each side), and it has the dimensions shown. A rectangular canvas floor is attached to the two sides and two ends.
- (a) Calculate the area of canvas used in the construction of the tent, to 2 decimal places, assuming no overlap.
- (b) To be certified as a 'student ready' tent, each ventilation panel must be at least 10% of the area of the base. Can the tent be certified 'student ready'?



- 12 A 4 m long plastic section of play equipment is shaped as a triangular prism with a cylindrical tunnel of diameter 0.7 m from one end to the other.

- (a) Select the option that shows the surface area of the plastic section of equipment, including the tunnel, to the nearest square metre.

A 17 m^2 B 20 m^2
C 21 m^2 D 25 m^2



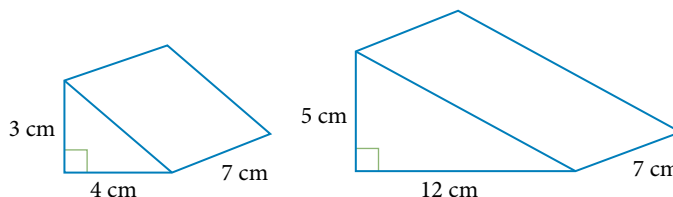
- (b) Which incorrect option would be chosen by a student who assumed the base was not included?

Worked
Example

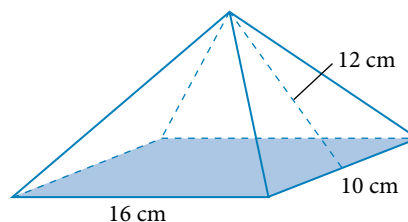
19

- 13 Determine the height of an open square-based vase of width 80 mm, given that its surface area is 800 cm^2 . State any assumptions you made.

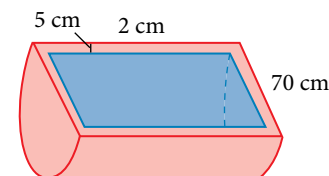
- 14 Compare the surface areas of the two right-angled triangular prisms, by stating how much more surface area the larger prism has, as a quantity and as a percentage of the smaller prism.



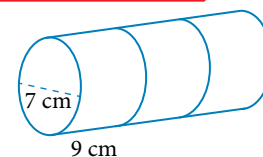
- 15 Determine the surface area of the rectangular-based pyramid to the nearest square centimetre.



- 16 The hollow half cylinder of thickness 5 cm has outer dimensions of 2 m and 70 cm. The inside is painted blue and the outside red. What percentage of the surface is red? Write your answer to the nearest whole per cent.



- 17 Three cylinders of height 9 cm and diameter 7 cm are snapped together at the circular ends.
- (a) What percentage of the surface area is lost by joining the cylinders?
- (b) Would the percentage change if there were four cylinders? Confirm your answer with calculations.



Surface area of tapered solids and spheres

2.6

Pyramids

The faces of pyramids are triangular, with the possible exception of the base, for which the solid is named.

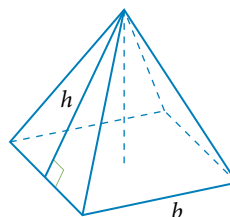
Square-based pyramid

A square-based pyramid has one square face and four triangular faces.

Area of base = b^2

Area of each triangular face = $\frac{1}{2}bh$

SA = $b^2 + 4 \times \frac{1}{2}bh$



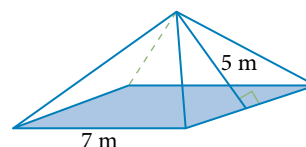
WARNING

It is best to leave formulas for SA in separate pieces so you can use parts of the formula for open objects or similar shapes. When finding surface area, always check that you have accounted for every face.

20 Surface area involving tapered solids

Determine the surface area of each solid.

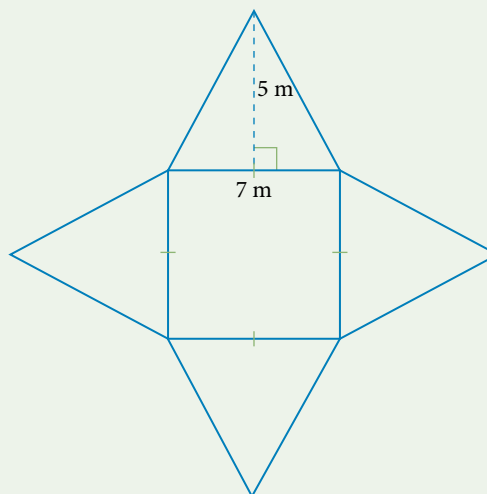
- (a) a square-based pyramid with each sloping face the same



THINKING

- 1 Recognise the shapes that comprise the overall solid. Identify required lengths.

WORKING



- 2 Recall appropriate formulas to determine the surface area (SA).

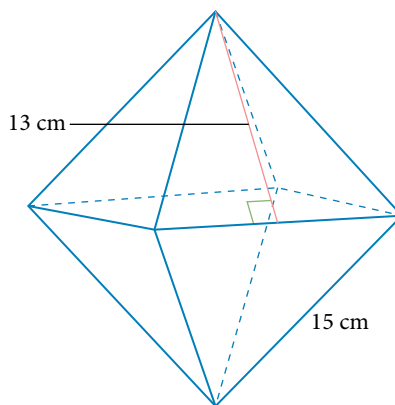
$$SA = l^2 + 4 \times \frac{1}{2}bh$$

- 3 Substitute the appropriate values to calculate and interpret SA.

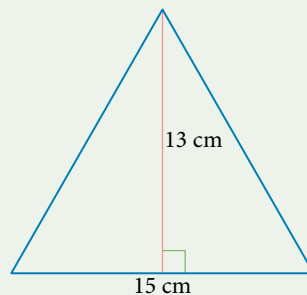
$$\begin{aligned} SA &= 7^2 + 4 \times \frac{1}{2} \times 7 \times 5 \\ &= 49 + 70 \\ &= 119\text{m}^2 \end{aligned}$$

The surface area of the pyramid is 119m^2 .

- (b) an octahedron, given that each edge is 15 cm long and the height of each triangular face is 13 cm



- 1 Redraw required faces, identifying and labelling the required dimensions.



- 2 State the shape, dimensions and number of each different face.
- 3 Recall appropriate formulas to determine the surface area (SA).
- 4 Substitute the known values and calculate SA.
- 5 Interpret the answer.

There are 8 triangular faces:

$$b = 15\text{ cm}, h = 13\text{ cm}$$

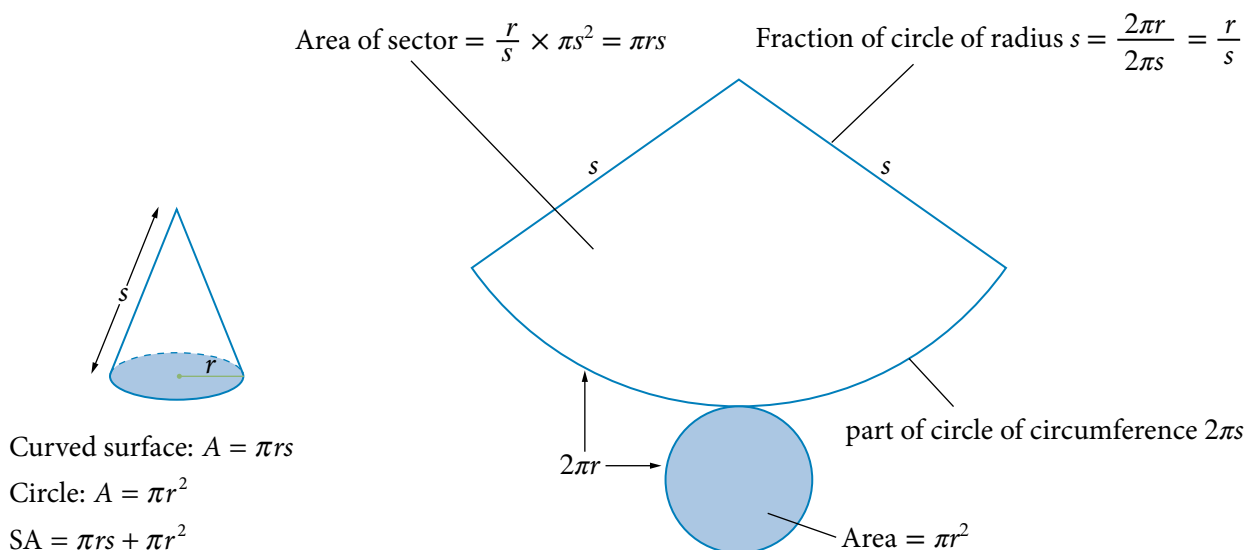
$$SA = 8 \times \frac{1}{2}bh$$

$$\begin{aligned} SA &= 8 \times \frac{1}{2} \times 15 \times 13 \\ &= 780\text{cm}^2 \end{aligned}$$

The surface area of the shape is 780cm^2 .

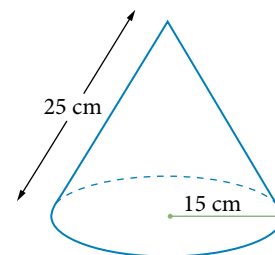
Cones

A cone has a curved surface that can be imagined flattened out to form a sector. Its base is a circle. The rim of the curved surface matches the rim of the circular base exactly, and there are formulas you can use to find the area of the curved surface.



21 Surface area of a cone

Determine the surface area of the party hat, to the nearest square centimetre.
State any assumptions you made in your calculations.



THINKING

- 1 Identify the shapes that make up the surface area.
- 2 Recall the formula for the surface area (SA).
- 3 Substitute the known values into the formula.
- 4 Calculate and interpret the answer.
- 5 State any assumptions. (You may think of others.)

WORKING

A party hat has no base, so its SA is just the area of the curved face.

$$\begin{aligned} \text{SA} &= \pi rs \\ &= \pi \times 15 \times 25 \\ &= 1178.097 \dots \text{ cm}^2 \end{aligned}$$

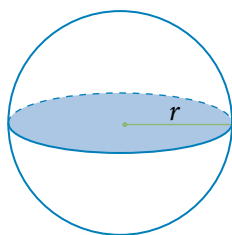
To the nearest square centimetre $\text{SA} = 1178 \text{ cm}^2$.

It was assumed that the solid was a perfectly shaped cone with no overlap of cardboard to be accounted for. The usual SA definition was assumed – that is, a single outside surface rather than outside and inside.

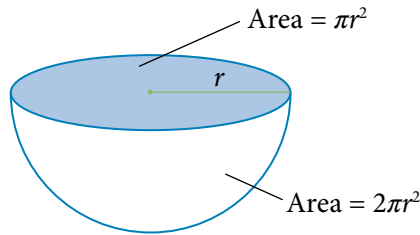
Spheres and hemispheres

A sphere has a single surface that cannot be flattened. The formula for calculating the surface area of a sphere is known.

A hemisphere has the surface area of half of a sphere added to the area of the flat circular face.



$$SA = 4\pi r^2$$



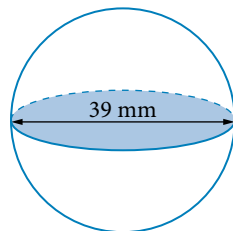
$$\text{Curved surface: } A = 2\pi r^2$$

$$\text{Circle: } A = \pi r^2$$

$$SA = 3\pi r^2$$

22 Surface area of a sphere

Calculate the surface area of the world globe, to the nearest square millimetre.



THINKING

- 1 Recall the formula for the surface area (SA).
- 2 Calculate the length of any required values.
- 3 Substitute the known values into the formula.
- 4 Calculate the answer, rounding as directed.

WORKING

$$\text{Sphere: } SA = 4\pi r^2$$

$$r = \frac{39}{2}$$

$$= 19.5 \text{ mm}$$

The radius of the sphere is 19.5 mm.

$$SA = 4\pi \times 19.5^2$$

$$= 4778.362... \text{ mm}^2$$

To the nearest square millimetre the surface area of the sphere is 4778 mm^2 .

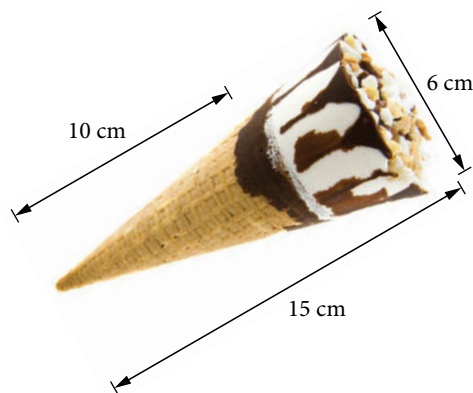
i Additional information

Surface area of a sphere

Revise your knowledge of the surface area of a sphere.

23 Application of surface area

A cone-shaped treat of diameter 6 cm and waffle-cone diameter 4 cm has ice cream protruding 5 cm from the waffle cone. What area of ice-cream is visible? Write your answer as a whole number of square centimetres.



THINKING

- 1 Identify an appropriate strategy.
- 2 Recall the formula to calculate the surface area of the individual parts.
- 3 Substitute the required values into the strategy identified initially and then calculate, rounding as required.
- 4 Interpret the answer.

WORKING

$$SA_{\text{ice cream}} = SA_{\text{treat}} - SA_{\text{waffle cone}}$$

Entire treat:

$$r = 3 \text{ cm}, S = 15 \text{ cm}$$

$$\begin{aligned} SA_t &= \pi rs + \pi r^2 \\ &= \pi \times 3 \times 15 + \pi \times 3^2 \\ &= 169.64... \text{ cm}^2 \end{aligned}$$

Waffle cone:

$$r = 2 \text{ cm}, S = 10 \text{ cm}$$

$$\begin{aligned} SA_c &= \pi rS \\ &= \pi \times 2 \times 10 \\ &= 62.83... \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} SA_{\text{ice cream}} &= SA_t - SA_c \\ &= 169.64... - 62.83... \\ &= 106.81 \text{ cm}^2 \text{ (2 d.p.)} \end{aligned}$$

The visible area of ice-cream to the nearest square centimetre is 107 cm^2 .

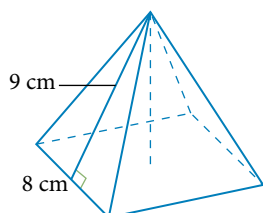
EXERCISE

2.6

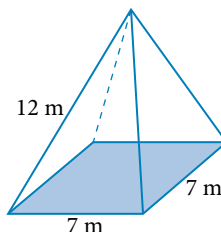
Surface area of tapered solids and spheres

- 1 Determine the surface area of the following square-based pyramids, to the nearest square centimetre.

(a)



(b)

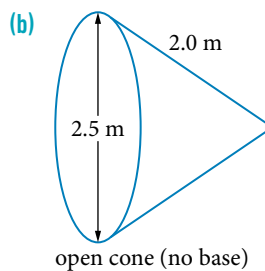
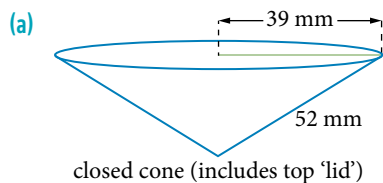


Worked
Example

20

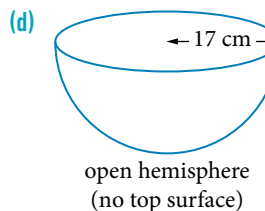
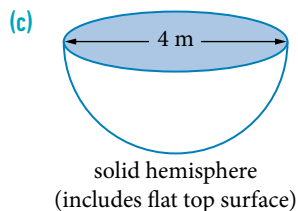
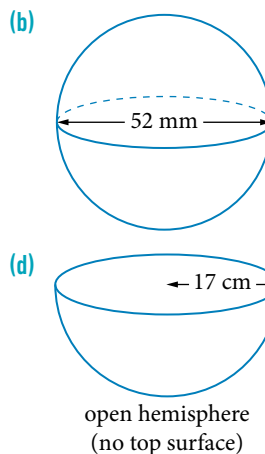
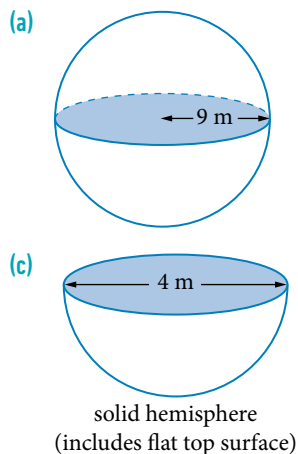
21

- 2 Determine the surface area of the cones, to 1 decimal place:

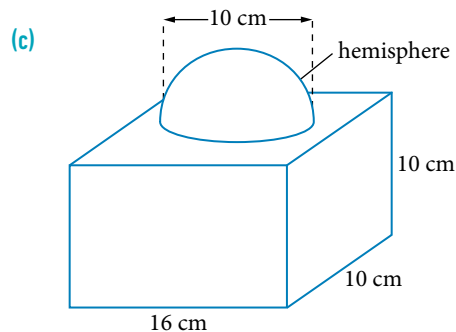
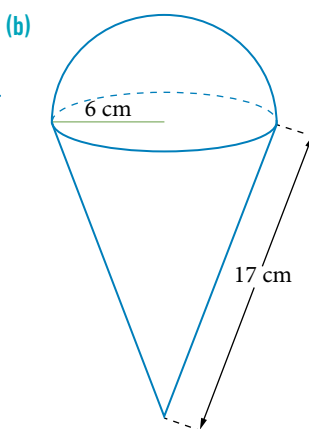
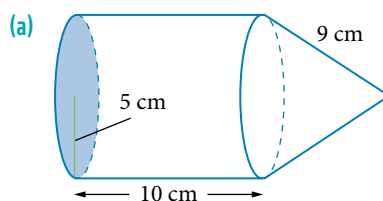


22

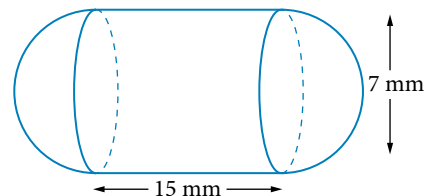
- 3 Determine the surface area of each of the following solids. Answer to the nearest whole number.



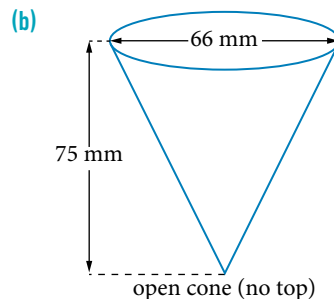
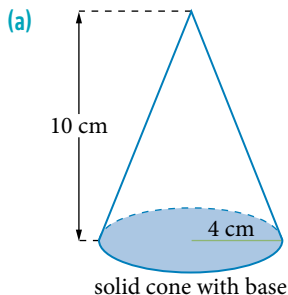
- 4 Determine the surface area of each of these solid objects. Round your answer to the nearest whole number.



- 5 A pharmaceutical company is making a new capsule with a special slippery coating, with dimensions as shown.
What area of coating is required to produce 1 million capsules?
Answer to the nearest square metre.



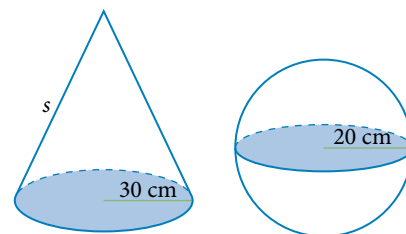
- 6 For each of the following, determine the surface area to the nearest whole number.



- 7 A cone of base radius 30 cm has the same curved surface area as a sphere of radius 20 cm.

- (a) Select the option that is closest to the length of the sloping edge length s of the cone.

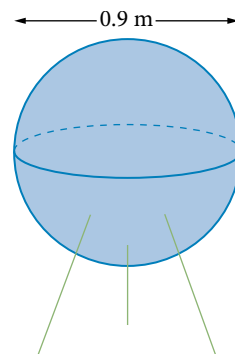
A 23 cm B 45 cm
C 50 cm D 53 cm



- (b) Which incorrect option would be chosen by a student who included the base of the cone?

- 8 The Wizzer kettle barbecue is spherical and is coated on the outside with gloss black paint.

The factory that manufactures Wizzers buys the black paint in 5 L containers. Each 5 L container has enough paint for 12 Wizzers. Determine how many square metres of barbecue each litre of paint covers. Answer to 1 decimal place.

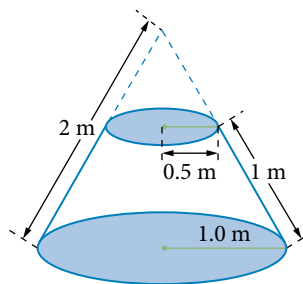


- 9 The radius of the Earth is 6371 km.

- (a) Calculate the area of habitable land on Earth, to the nearest 100 000 km², given that 71% of the surface is oceans, 33% of the land area is desert, and 24% of the land area is mountainous.
(b) Forests occupy 30% of the land. Calculate the area of forests on Earth, to the nearest 100 000 km².
(c) The area of Australia is 7.692 million km². Determine the percentage of the land area of Earth occupied by Australia. Answer to the nearest whole percentage.



- 10 Minh is building a display stand for a museum. The stand, excluding the base, will be covered in black velvet. What area of velvet will be required? Answer in square metres, to 2 decimal places.



11 A chrome ‘plumb bob’ (used by builders to check that a structure is vertical) is shown.

The diagram shows a composite figure consisting of a large inverted cone with a smaller upright cone on top. The large cone has a base radius of 6 cm and a height of 9 cm. The small cone has a base radius of 2 cm and a height of 3 cm. Dashed lines indicate the dimensions.

- 1

-
- The diagram illustrates three stages of surface evolution. The first stage is a flat surface with dimensions 10 mm by 10 mm. The second stage shows a surface with small, regular bumps, with a dimension of 2 mm indicated. The third stage shows a surface with large, irregular spikes, with a dimension of 3 mm indicated.

Compare the final surface area with the surface area of the flat sheet.

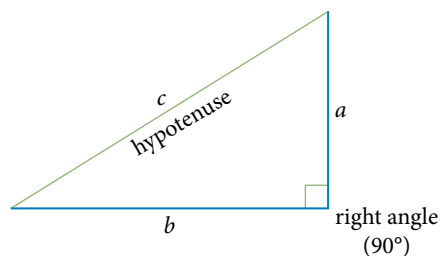
-
- A 3D geometric figure composed of several triangular faces. The faces are colored green, blue, and red. The figure is a complex polyhedron with multiple vertices and edges, showing a non-symmetrical arrangement of the colored faces.

Summary

Pythagoras' theorem

Pythagoras' theorem can be used to find the side length of any side in a right-angled triangle, provided the other side lengths are known.

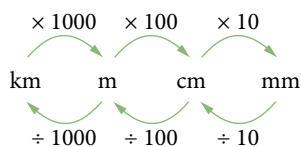
The formula is $c^2 = a^2 + b^2$ in any right-angled triangle where c is the hypotenuse (the longest side, opposite the right angle).



Perimeter

Perimeter is the distance along the boundary of a shape.

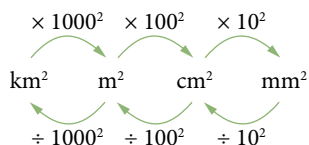
Unit conversion:



Area

Area is the amount of surface within the boundary of a shape.

Unit conversion:

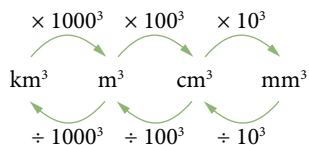


$$1 \text{ ha} = 10000 \text{ m}^2$$

Volume

Volume is the amount of three-dimensional space inside an object.

Unit conversion:



Capacity equivalence:

$$1 \text{ cm}^3 = 1 \text{ mL}$$

$$1000 \text{ cm}^3 = 1 \text{ L}$$

$$1 \text{ m}^3 = 1 \text{ kL}$$

Surface area (SA)

Surface area, or SA, is the area of all the outside surfaces of a solid added together.

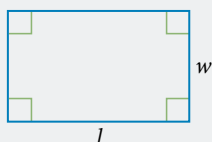
It is often easier to add the required surface areas individually rather than use a formula.

Perimeter and area formulas

Rectangle

$$P = 2(l + w)$$

$$A = lw$$

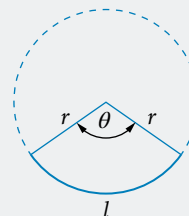


Sector

$$P = l + 2r$$

$$\text{where } l = \frac{\theta}{360} \times 2\pi r$$

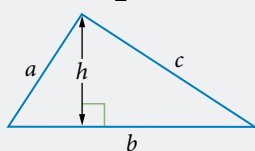
$$A = \frac{\theta}{360} \times \pi r^2$$



Triangle

$$P = a + b + c$$

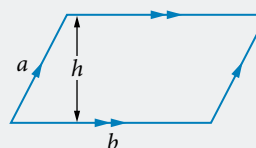
$$A = \frac{1}{2}bh$$



Parallelogram

$$P = 2(a + b)$$

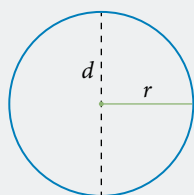
$$A = bh$$



Circle

$$C = 2\pi r \text{ or } C = \pi d$$

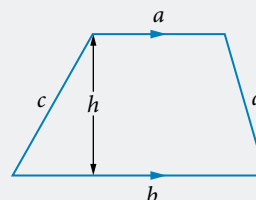
$$A = \pi r^2$$



Trapezium

$$P = a + b + c + d$$

$$A = \frac{1}{2}(a + b)h$$

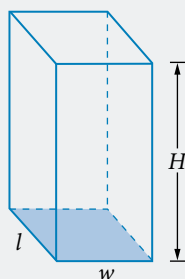


Volume and surface area formulas

Rectangular prism

$$V = lwh$$

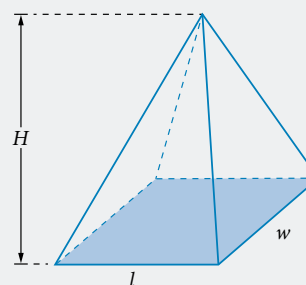
$$SA = 2lw + 2lH + 2wH$$



Pyramid

$$V = \frac{1}{3}lwh$$

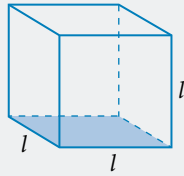
Calculate required SA using given dimensions



Cube

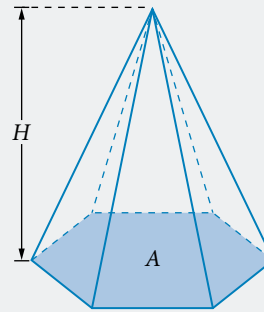
$$V = l^3$$

$$SA = 6l^2$$

**Tapered solid**

$$V = \frac{1}{3}AH$$

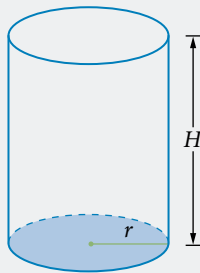
Calculate required SA using given dimensions

**Cylinder**

$$V = \pi r^2 H$$

$$\text{Curved SA} = 2\pi rH$$

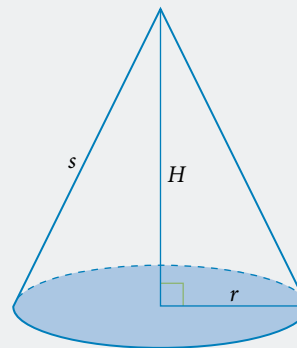
$$SA = 2\pi r + 2\pi rH$$

**Cone**

$$V = \frac{1}{3}\pi r^2 H$$

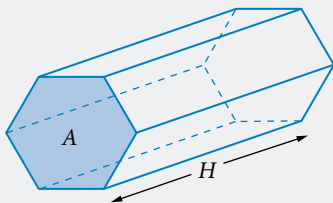
$$\text{Curved SA} = \pi rs$$

$$SA = \pi r^2 + \pi rs$$

**Any prism**

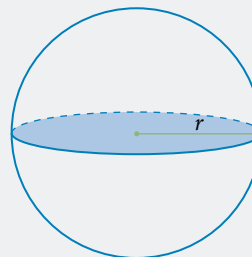
$$V = AH$$

Calculate required SA using given dimensions

**Sphere**

$$V = \frac{4}{3}\pi r^3$$

$$SA = 4\pi r^2$$

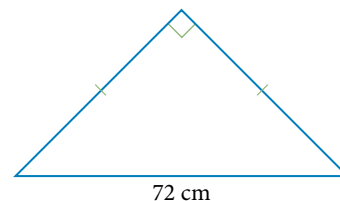


Chapter review

2

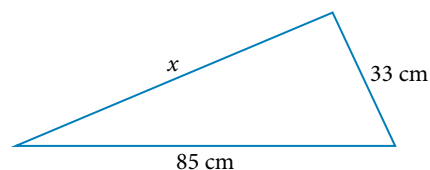
Exercise 2.1

- 1 Determine the length of the short sides in the triangle, to 1 decimal place.



Exercise 2.1

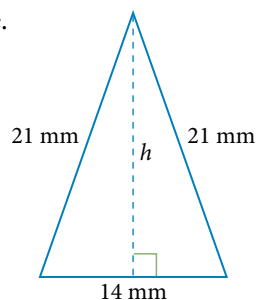
- 2 Determine the values of x , to 1 decimal place, that would make this triangle right angled.
Note: The right angle need not be at the top.



Exercise 2.1

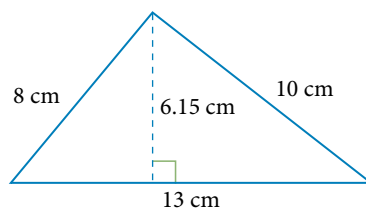
- 3 Choose the option that shows the height h of the triangle.

- A 5.9 mm
- B 19.8 mm
- C 15.7 mm
- D 22.1 mm



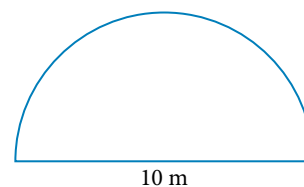
Exercise 2.2

- 4 Calculate the perimeter of the triangle.



Exercise 2.2

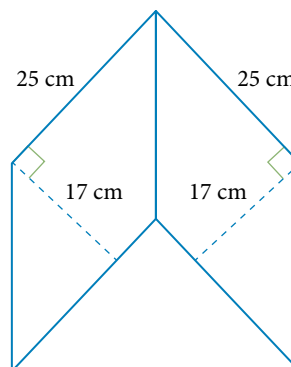
- 5 Calculate the perimeter of the semicircle, to 1 decimal place.



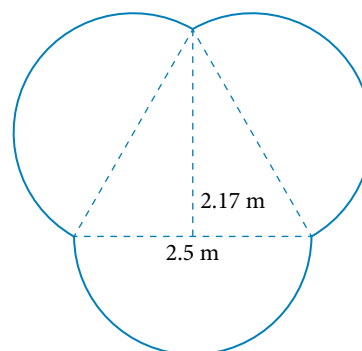
Exercise 2.2

- 6 A design for the wings of a paper dart is shown. Select the option that shows the total area of the wings.

- A 84 cm^2
- B 425 cm^2
- C 850 cm^2
- D 212.5 cm^2



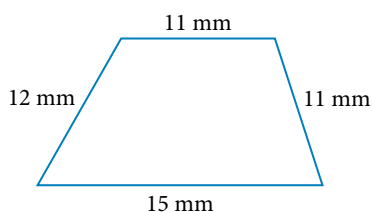
- 7 A plan for a pond consists of an equilateral triangle with each side bounded by a semi-circle, as shown. Determine the area of the pond to the nearest square metre.



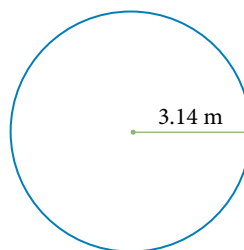
Exercise 2.2

- 8 Calculate the perimeter of each of the following shapes. Round to 2 decimal places where necessary.

(a)



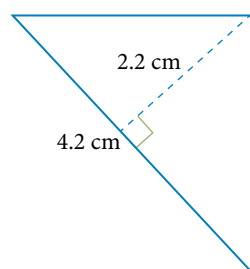
(b)



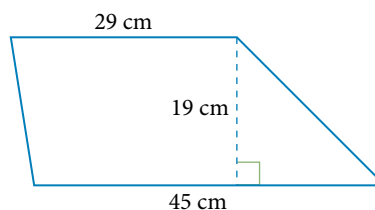
Exercise 2.2

- 9 Calculate the area of each of the following.

(a)

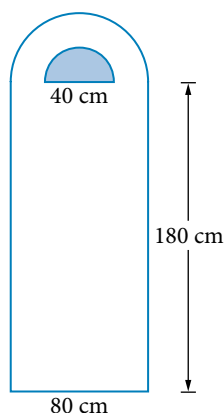


(b)



Exercise 2.2

- 10 A new unpainted wooden door contains a semicircular glass viewing panel. Determine the area of wood, on the outside only, that requires painting. Give your answer to the nearest square centimetre.



Exercise 2.2

- 11 An ice cube tray has 20 rectangular prism-shaped compartments of length 5 cm, width 3 cm and depth 3 cm. Calculate the capacity of the ice cube tray.
- 12 A cylindrical chemical storage tank has a diameter of 0.85 m and stands 1.5 m tall. Choose the option that shows the volume of the tank, accurate to 2 decimal places.

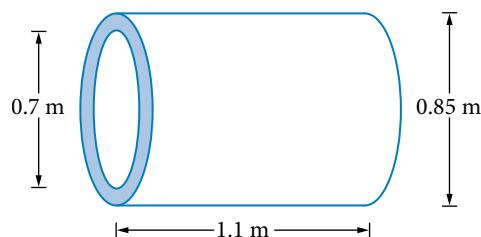
Exercise 2.3

Exercise 2.3

A 0.85 m^3 B 3.40 m^3 C 4.01 m^3 D 5.14 m^3

- 13 A garden roller used to compress soil consists of two cylinders. The space between the cylinders can be filled with water to give the roller extra weight if required. What is the maximum volume of water that the roller could hold? Write your answer as a whole number of litres.

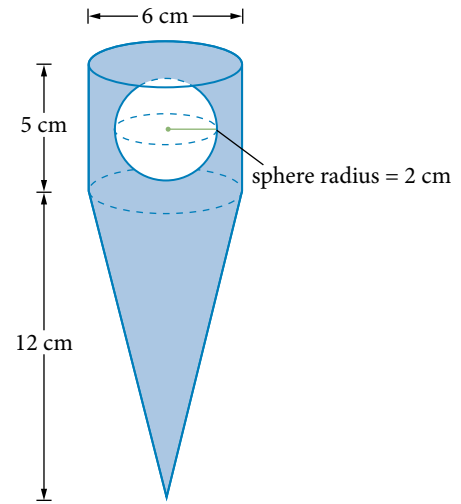
Exercise 2.3



Exercise 2.3, 2.4

- 14 A new ice-cream dessert consists of a joined waffle cone and cylinder filled with chocolate ice-cream, except for a sphere of white chocolate of radius 2 cm embedded inside.

Determine the volume of ice-cream in this dessert, to the nearest millilitre.



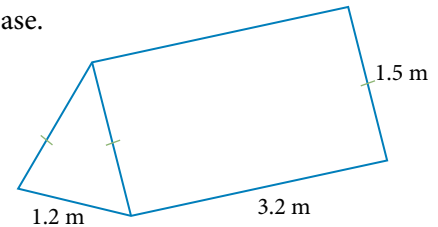
Exercise 2.4

- 15 Popcorn is sold in a paper cone of radius 8 cm and height 24 cm. Select the option that shows the volume of the cone, to the nearest cubic centimetre.

A 402 cm^3 B 603 cm^3 C 1206 cm^3 D 1608 cm^3

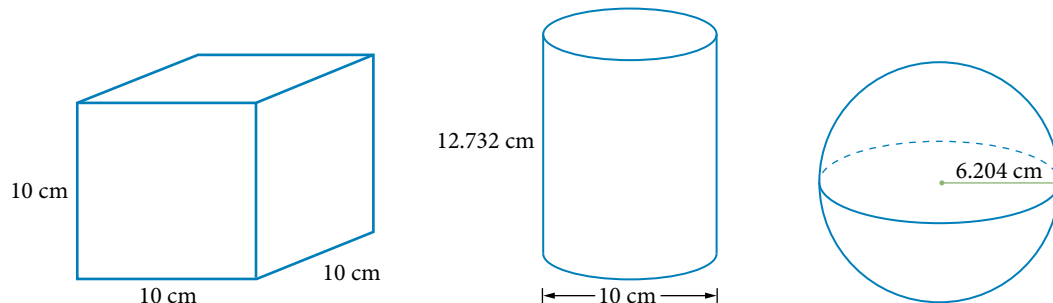
Exercise 2.5

- 16 Determine the surface area of the tent shown, including its base. Give your answer to 2 decimal places.



Exercise 2.3, 2.4, 2.5, 2.6

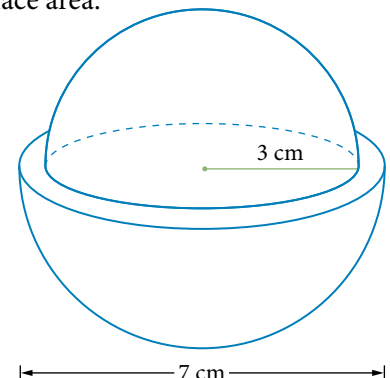
- 17 Compare the following solids, then select the correct statement. In your calculations, round your results to the nearest square centimetre or cubic centimetre.



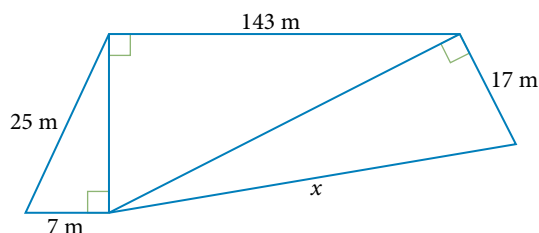
- A They all have the same volume and the same surface area.
 B They all have the same volume but each has a different surface area.
 C They each have a different volume but all have the same surface area.
 D They all have different volumes and different surface areas.

Exercise 2.4, 2.6

- 18 The solid shown here is made from two hemispheres.
 (a) Determine the volume of the solid to the nearest cubic centimetre.
 (b) Determine the surface area of the solid to the nearest square centimetre.

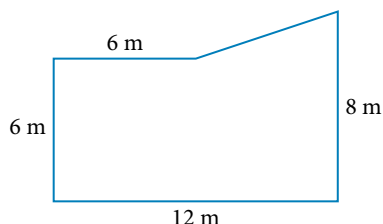


- 19 Determine the value of x in the diagram, to the nearest whole number.



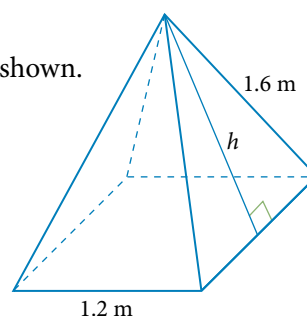
Exercise 2.1

- 20 The outline of a proposed swimming pool is shown. Calculate the perimeter of the pool, to 1 decimal place, making assumptions if necessary.



Exercise 2.2

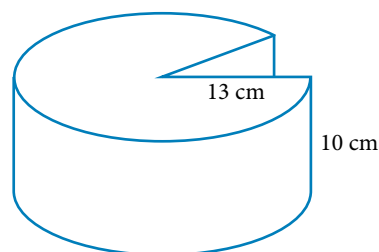
- 21 An ice sculptor uses a chainsaw to craft the square-based ice pyramid shown. Determine the surface area of the visible part of the pyramid, to 2 decimal places.



Exercise 2.4

- 22 Layla bakes a birthday cake, but before she can ice the cake, her brother steals a 40° slice.

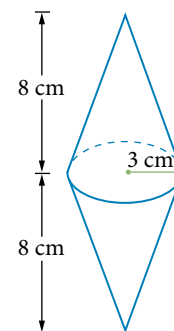
- (a) Layla decides to go ahead and cover the top and curved surface of the remaining cake with icing. Determine the area of cake that needs to be covered, to the nearest square centimetre.
- (b) What is the volume of the remaining cake, including icing that is 3 mm thick? Write your answer to the nearest cubic centimetre.
- (c) What percentage of the remaining cake is icing? Write your answer to the nearest whole number.



Exercise 2.3, 2.5

- 23 A solid is made of two identical cones.

- (a) Determine the volume of the solid to the nearest cubic centimetre.
- (b) Determine the surface area of the solid to the nearest square centimetre.

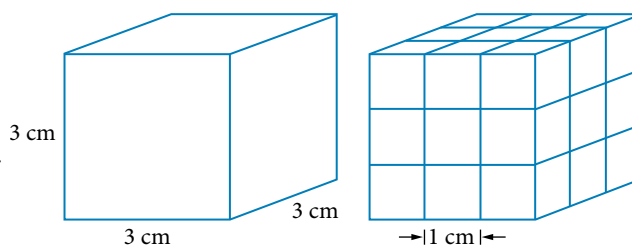


Exercise 2.4, 2.6

- 24 A chef shapes a potato into a cube of side length 3 cm.

The chef then cuts the cube into smaller 1 cm cubes as shown, and separates the small cubes.

Investigate the effect of dividing and separating a cube into 27 identical smaller cubes on the resulting total volume and total surface area.

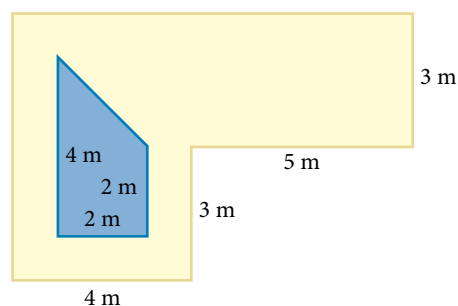


Exercise 2.3, 2.5

Exercise 2.2,
2.3, 2.5

- 25 Xiao Ming's plan for a spa surrounded by a paved area is shown. The outer paved area is to be bordered by a regulation pool fence.

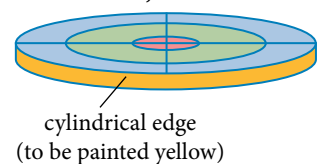
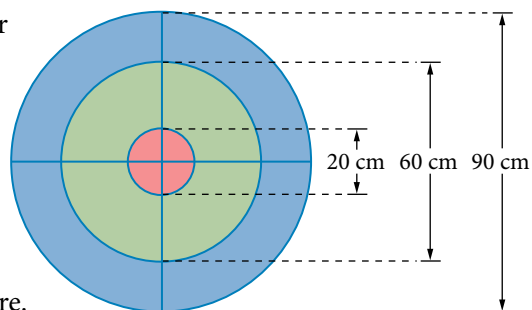
- Calculate the length of fencing needed.
- Determine the total area to be paved.
- How many litres of water are required to fill the spa to a depth of 1.5 m?



Exercise 2.2,
2.3, 2.5

- 26 Daria is constructing a large dart board that is easier to hit than a conventional one. She has a circular piece of dense corkboard and uses wire to form three circular rings and two straight segments (diameters), dividing the board into 12 sections as shown. (The wire is shown as blue lines.)

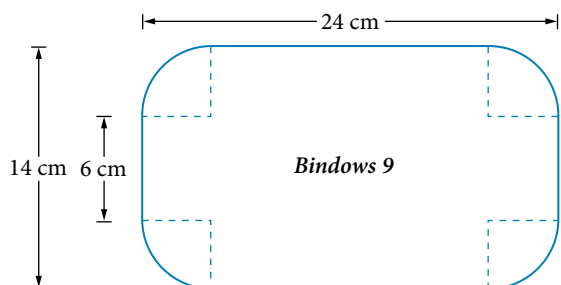
- Determine the length of wire, in metres, that Daria will need. Answer to the nearest centimetre.
- Determine the area, to the nearest square centimetre, of each of the three coloured sections: the inner circle, the middle ring and the outer ring.
- If the dart board is 2 cm thick, determine the volume of cork used in the board, to the nearest cubic centimetre.
- Daria paints the front as shown, and also the outer cylindrical edge. What is the total surface area she will paint? Express your answer to the nearest square centimetre.



Exercise 2.2,
2.3, 2.5

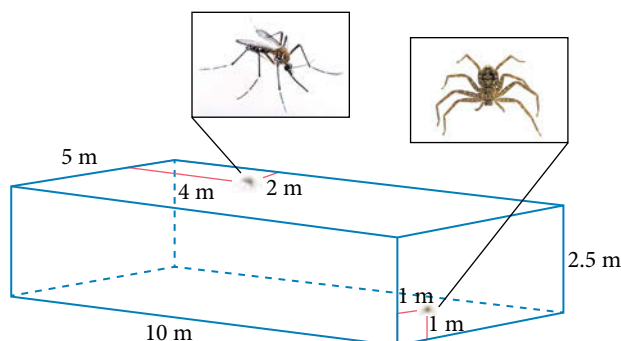
- 27 A tablet computer has the dimensions shown (where the screen spans the entire face). The corners of the tablet are quarter circles.

- Determine the perimeter of the tablet, to the nearest millimetre.
- Calculate the screen area, to the nearest square centimetre.
- The tablet is 9 mm thick. Calculate the total volume of the tablet, to the nearest cubic centimetre.
- The tablet is sold in a rectangular box of length 30 cm, width 20 cm and height 15 cm, with specially shaped foam packing material that completely fills the gaps between the tablet and the box. What volume of packing material is required for each box? Answer to the nearest cubic centimetre.



Exercise 2.1

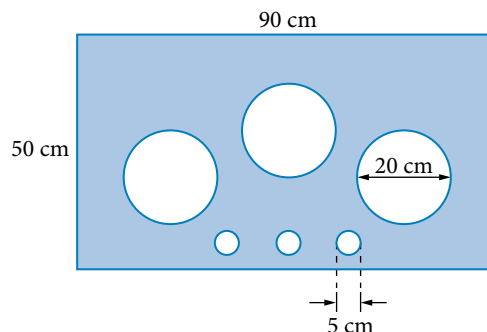
- 28 A huntsman spider crouched on a wall plans to sneak up on a mosquito that is sitting upside down on the ceiling. The room is 10 m by 5 m with a ceiling height of 2.5 m. The mosquito is 2 m and 4 m from the nearest walls as shown, and the spider starts off at 1 m from each of a wall and the floor.



What is the shortest distance from the spider to the mosquito? (Hint: Use a net of the room.)
Write your answer to the nearest centimetre.

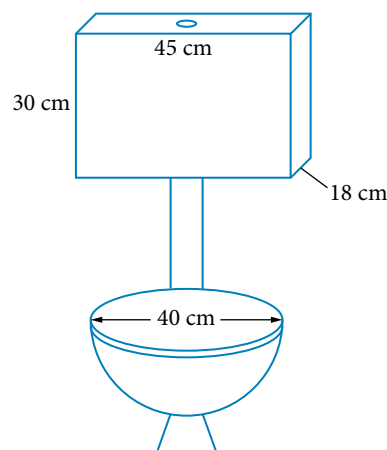
State an assumption you made and test the validity of that assumption.

- 29 Peter has ordered a new cooktop for his renovated kitchen. The cooktop contains three gas burners and three control knobs, which all protrude through circular sections cut out of a glass surface 6 mm thick. Each cubic centimetre of the glass weighs 10 g. Calculate the weight (mass) of the glass part of the cooktop, to the nearest kilogram.



Exercise 2.2, 2.3

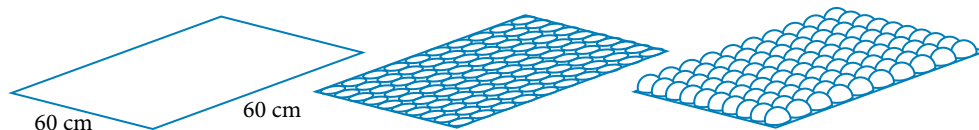
- 30 Kenny has constructed a toilet for his renovated bathroom using discarded parts of other systems. The top tank is a rectangular prism and the bowl is a perfect hemisphere. When the toilet flushes, all the water flows into the bowl before flushing away. Determine the maximum water level in the tank that will prevent any overflow in the bowl. Write your answer in terms of distance below the rim, to the nearest millimetre.



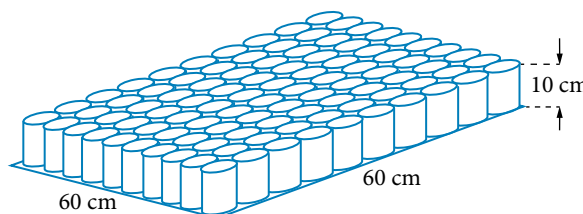
Exercise 2.4

- 31 A new type of sound-absorbing panel is made by attaching 100 hemispheres of diameter 6 cm to a flat, square piece of rubber of side length 60 cm, as shown. The idea behind this design is that sound waves get trapped between the gaps in the hemispheres and are eventually absorbed.

Exercise 2.6



- Determine the total surface area (flat and curved) on the top of a finished panel, to the nearest square centimetre.
- By what percentage, to the nearest whole percentage, has the surface area been increased by the addition of the hemispheres?
- An alternative design involves attaching cylinders of base diameter 6 cm and height 10 cm instead of hemispheres. Calculate the surface area of the top of this new sheet, to the nearest square centimetre.
- Calculate the height that the cylinders must be, to provide a panel with the same surface area as a panel containing the hemispheres.







3

Similar figures

• • • • •	
Recall	152
3.1 Similar shapes and length	153
3.2 Scale drawing	172
3.3 Similar shapes and area	182
3.4 Similar solids and volume	193
Summary	201
Chapter review	202

Recall

Convert metric units of length

1 Convert the following.

(a) 20.7 m to centimetres

(b) 350 mm to metres

(c) 5.2 km to metres

Round to a given number of decimal places

2 Round to the number of decimal places given in the brackets.

(a) 9.0909 (2)

(b) 5.656656 (3)

Express a ratio between two given quantities as a decimal

3 Express the following as decimals. For answers that involve more than 3 decimal places, round to 3 decimal places.

(a) $\frac{10}{4}$

(b) $\frac{15}{70}$

Solve equations involving squares and cubes

4 Determine the value of x . For answers that are not whole numbers, round to 2 decimal places if necessary.

(a) $x^2 = 289$

(b) $x^3 = 174.68$

Solve equations involving ratios

5 Calculate the value of x . For answers that are not whole numbers, round to 2 decimal places if necessary.

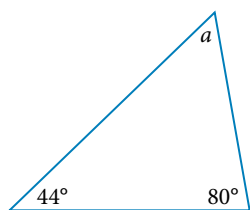
(a) $\frac{x}{4} = \frac{25}{2}$

(b) $\frac{19}{3} = \frac{16}{x}$

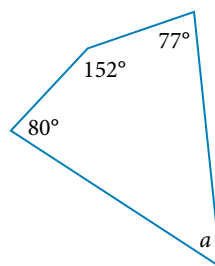
Calculate a missing angle in a triangle or quadrilateral

6 Calculate the size of angle a in each shape.

(a)



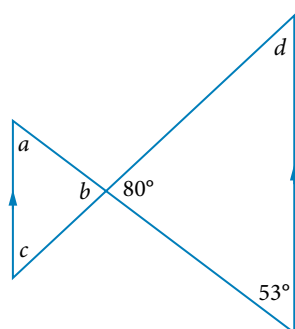
(b)



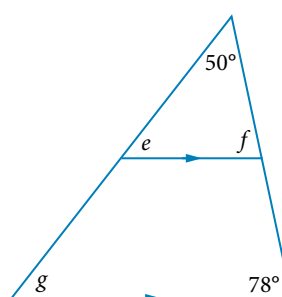
Determine angles in triangles involving parallel lines

7 Determine the value of each pronumeral.

(a)



(b)



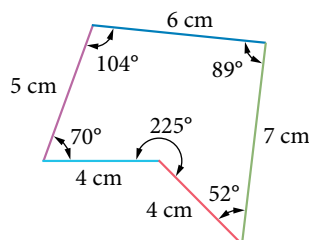
Similar shapes and length

What are similar shapes?

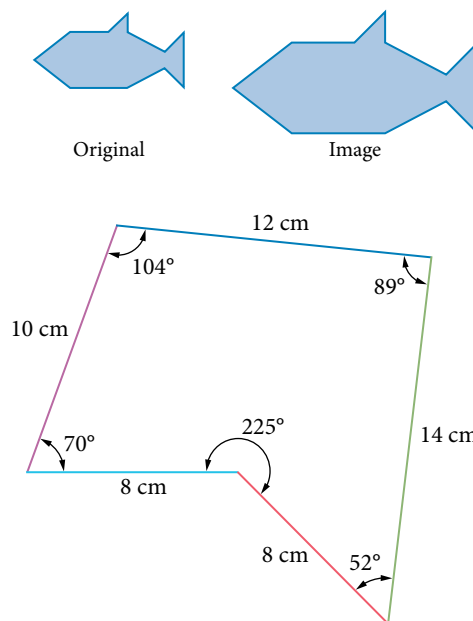
Two shapes (or figures) are *similar* if they are exactly the same shape, but not usually the same size, so one shape is an enlargement or reduction of the other. Each angle in a pair of similar shapes is the same size as the corresponding angle in the other shape.

Specifically, polygons are similar if two things are true:

- 1 The corresponding sides of each are in the same proportion.
- 2 The corresponding interior angles are equal.



lengths $\times k$
($k = 2$)



Side length ratio in similar shapes

If two shapes are similar then each pair of corresponding sides is in the same ratio. Sides are said to be corresponding if they are in the same position, between the same angles.

This ratio is the *scale factor* k .

$$k = \frac{\text{side length in image } (L)}{\text{corresponding side length in original } (l)}$$

$$k = \frac{L}{l}$$

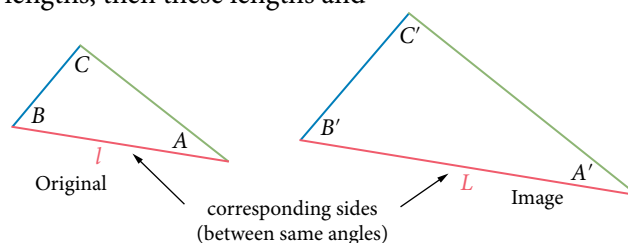
Note: For $k < 1$ the image is reduced. For $k > 1$ the image is enlarged. If $k = 1$ the images are said to be congruent (exactly the same size and shape).

For any similar figures, if L and l are corresponding lengths, then these lengths and k are related in three ways.

$k = \frac{L}{l}$ is the scale factor formula, where L and l have the same units.

$L = kl$ means that the length of a side in the image may be found by multiplying a corresponding side in the original by k .

$l = \frac{L}{k}$ means that the length of a side in the original may be found by dividing a corresponding side in the image by k .



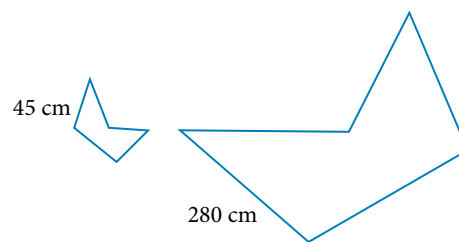
Additional information

Calculating scale factor

Practise identifying similar shapes and finding scale factors.

1 Determining the scale factor using lengths in similar shapes

Determine the scale factor k for a pair of similar shapes, given the corresponding side lengths 45 cm for the original and 280 cm for the image. If necessary, round your answer to 2 decimal places.



THINKING

- 1 Recall the required formula for scale factor.
- 2 Identify and substitute the given values for L (image length) and l (corresponding length in original) into the formula.
- 3 Interpret k , rounding the answer to 2 decimal places where necessary.

WORKING

$$k = \frac{\text{side length in image } (L)}{\text{corresponding side length in original } (l)}$$

Let $L = 280$ and $l = 45$

Therefore:

$$k = \frac{280}{45}$$

$$k = 6.22 \text{ (2 d.p.)}$$

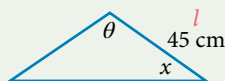
The image is 6.22 times larger than the original.

2 Calculating the image side length, using the original side length and the scale factor

Calculate the image length corresponding to the original side length 45 cm, if the scale factor k between a pair of similar figures is 3.7.

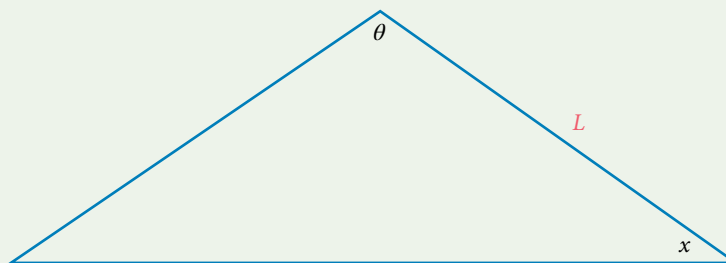
THINKING

- 1 Construct a diagram from the information given.



WORKING

The image is 3.7 times larger than the original.



- 2 Recall the formula for calculating image length.
- 3 Calculate the image length L by multiplying the original length l by k .

$$L = kl$$

Let the original side length $l = 45$

$$L = 3.7 \times 45$$

$$= 166.5 \text{ cm}$$

4 Interpret the result.

The length of the corresponding image side length L is 166.5 cm.

5 Check the reasonableness of your answer.

The image is larger than the original length and the ratio of the two sides will be 3.7.

$$\begin{aligned} k &= \frac{L}{l} \\ &= \frac{166.5}{45} \\ &= 3.7 \end{aligned}$$

3 Calculating the original side length, using a similar shape and the scale factor

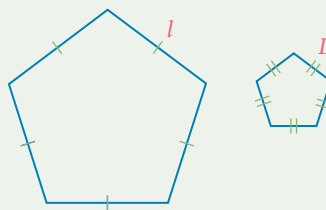
Calculate the original length corresponding to the image side length of 7 mm if the scale factor k between a pair of similar figures is 0.06. Round your answer to 1 decimal place.

THINKING

1 Construct a diagram from the information given.

WORKING

Given that the scale factor is less than 1, the image is a reduction of the original.



2 Recall the formula for calculating the original length.

$$l = \frac{L}{k}$$

3 Calculate the original length l by dividing the image length L by k .

Let $L = 7$ and $k = 0.06$

$$\begin{aligned} l &= \frac{L}{k} \\ &= \frac{7}{0.06} \\ &= 116.7 \text{ mm (1 d.p.)} \end{aligned}$$

4 Interpret the result.

The original side length of the pentagon is 116.7 mm.

- 5 Check the reasonableness of your answer.

The original side length is larger than the image side length and the ratio of the two sides will be 0.06.

$$\begin{aligned} k &= \frac{L}{l} \\ &= \frac{7}{116.7} \\ &= 0.059... \\ &= 0.06 \text{ (2 d.p.)} \end{aligned}$$

The slight variation occurs because of the rounded value.

Similar triangles

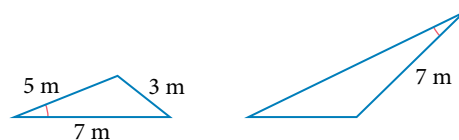
Similar triangles have three corresponding angles of the same size and three corresponding sides in the same proportion—that is, connected by the same scale factor.

Even after rotation, you may need to flip (reflect) one shape so it is in the same orientation as the other shape, to make it easier to identify corresponding sides and angles.

Scale factor for similar triangles

4 Calculating the scale factor for similar triangles

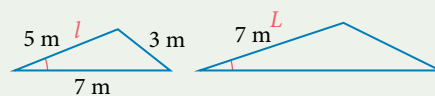
The pairs of triangles in the diagram are similar. Determine the scale factor k needed to enlarge the image on the left to produce the image on the right.



THINKING

- 1 By recognising equal angles, draw the triangles in the same orientation.
- 2 Recall the formula for determining scale factor.
- 3 Identify a matching pair of sides where l is a side length on the original triangle and L is the corresponding side length on the similar shape. Substitute the values into the formula.
- 4 Interpret the value of k , checking the reasonableness of your answer.

WORKING



$$k = \frac{L}{l}$$

Original length: $l = 5 \text{ m}$

Image length: $L = 7 \text{ m}$

$$k = \frac{7}{5}$$

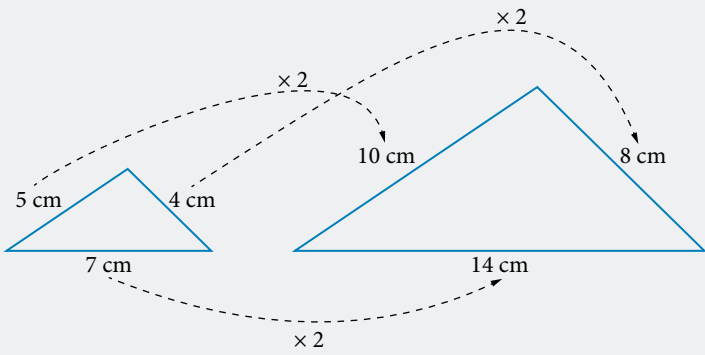
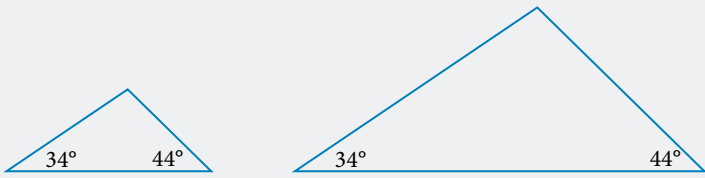
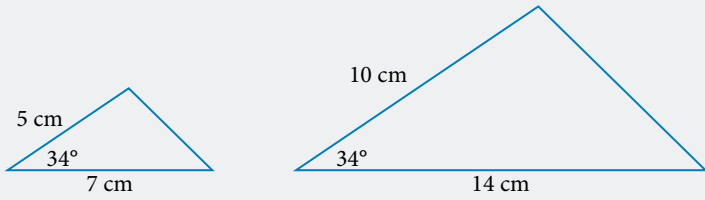
$$k = 1.4$$

The image is 1.4 times larger than the original.

Given that the scale factor represents an enlargement, you would expect $k > 1$.

Tests for similarity

To determine whether or not two shapes are similar, you need to know whether all matching pairs of angles are the same and whether matching pairs of sides are in the same ratio. For triangles it is not necessary to have information on all six measurements. In fact, two triangles are similar if they satisfy any one of the following tests. (An example is shown in each case.)

Test	Example
Each pair of corresponding sides is in the same ratio ('side-side-side' or SSS).	 $k = \frac{L}{l} = \frac{10}{5} = \frac{8}{4} = \frac{14}{7} = 2$
At least two pairs of angles are the same ('angle-angle' or AA).	 All the pairs of angles are the same, because the angle sum is 180° for all triangles.
The lengths of two pairs of corresponding sides are in the same ratio <i>and</i> the angles between these sides are equal ('side-angle-side' or SAS).	 $k = \frac{10}{5} = \frac{14}{7} = 2, \text{ and the angles between these sides are congruent (equal).}$

WARNING

The tests for similarity apply to triangles only. For other polygons, more information may be needed.

Additional information

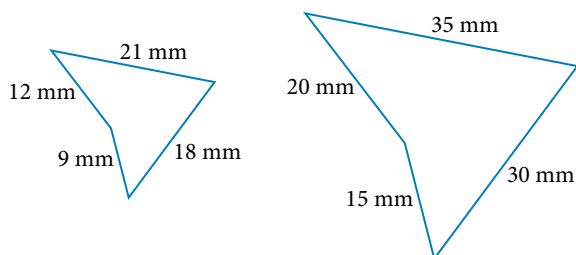
Similar triangles

Try the activities, to revise the properties of similar triangles.

5 Similar or not?

Determine whether the pairs of shapes in the diagram are similar shapes, not similar or possibly similar. Explain your answer using mathematical reasoning.

(a)



THINKING

- 1 Calculate the exact ratios of all corresponding sides using $\frac{L}{l}$.

- 2 Interpret the result.

If all ratios (or scale factors) are the same for corresponding sides, then the shapes may be similar. If the shapes were triangles, this would be sufficient to say they are similar (SSS).

WORKING

Start with the smallest pair of sides and move clockwise:

$$\frac{L_1}{l_1} = \frac{15}{9} = \frac{5}{3}$$

$$\frac{L_2}{l_2} = \frac{20}{12} = \frac{5}{3}$$

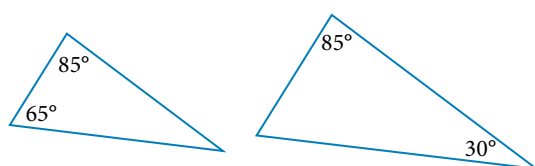
$$\frac{L_3}{l_3} = \frac{35}{21} = \frac{5}{3}$$

$$\frac{L_4}{l_4} = \frac{30}{18} = \frac{5}{3}$$

Given that $\frac{L_1}{l_1} = \frac{L_2}{l_2} = \frac{L_3}{l_3} = \frac{L_4}{l_4}$, all side ratios are identical, so the shapes may be similar.

Because the shapes are not triangles, you would need to know more information about the angles to be sure.

(b)



- 1 Calculate any missing angles.

Missing angle in the first triangle:

$$180^\circ - (85^\circ + 65^\circ) = 30^\circ$$

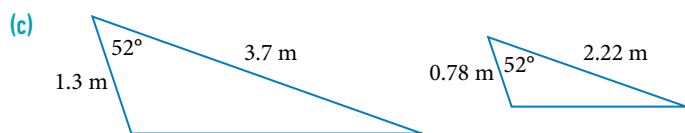
- 2 Interpret the result.

If at least two angles are the same, then the triangles are similar (AA similarity test).

Both triangles have angles of 30° and 85° .

At least two angles are the same, so the third is automatically the same because the angle sum of a triangle is constant.

The triangles are similar by AA.



1 Two sides and an identical included angle are given for each triangle.

2 Calculate the ratio of each pair of corresponding sides on either side of the given angle using $\frac{L}{l}$.

3 Interpret the result.

$$\frac{0.78}{1.3} = 0.6$$

$$\frac{2.22}{3.7} = 0.6$$

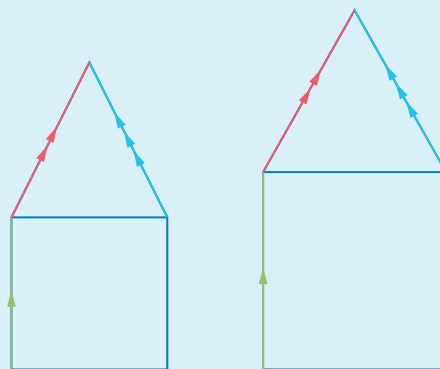
If the ratios of the corresponding sides on either side of the identical angle are the same, then the triangles are similar by SAS.

i Additional information

Calculating unknown lengths in similar triangles

Try the activity to practise your skills in finding unknown sides and angles in similar triangles.

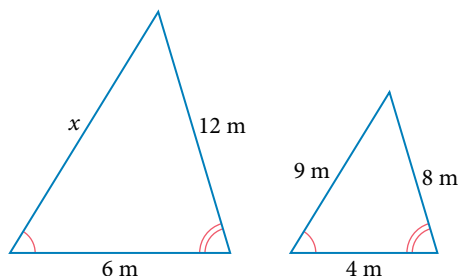
If the orientation of an image is the same as the original, then the sides of the image will be parallel to the corresponding sides in the original.



6 Calculating a length using similar triangles

Calculate the value of the pronumeral by first proving similarity for each pair of shapes.

(a)



THINKING

Prove similarity.

WORKING

At least two angles are the same, so the third is automatically the same because the angle sum of a triangle is constant. The triangles are similar by AA.

METHOD 1

THINKING

- 1 Calculate the scale factor k using the given lengths.

The triangles are oriented the same way, so this can be done by matching the corresponding sides.

- 2 Calculate the original length x by dividing the image length L by k .

WORKING

The original triangle (left): $l_1 = 6$, $l_2 = 12$

The image (right): $L_1 = 4$, $L_2 = 8$

$$\text{Scale factor: } k = \frac{L_1}{l_1} = \frac{L_2}{l_2}$$

$$k = \frac{4}{6} = \frac{8}{12}$$

$$k = 0.666\dots$$

When rounded to 1 decimal place, $k = 0.7$, meaning the original triangle has been reduced in size.

$$\begin{aligned} l &= \frac{L}{k} \\ &= \frac{9}{0.666\dots} \\ &= 13.5 \text{ m} \end{aligned}$$

The missing side length x is 13.5 m.

METHOD 2

- 1 Equate the corresponding ratios.

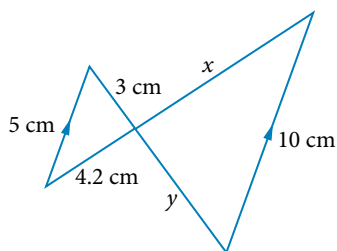
$$\frac{x}{9} = \frac{12}{8}$$

- 2 Solve for the unknown.

$$\begin{aligned} x &= \frac{12}{8} \times 9 \\ &= 13.5 \text{ m} \end{aligned}$$

The missing side length x is 13.5 m.

(b)



- 1 Recognise alternate and vertically opposite angles.

Prove similarity using angle relationships by determining pairs of equal angles.

- 2 Recall the formula for scale factor.

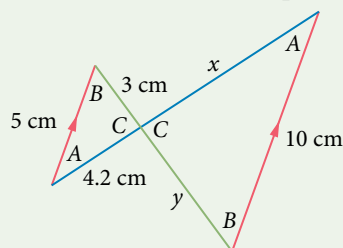
- 3 Determine the scale factor k by substituting the given lengths and interpret the result.

- 4 Equate ratios of corresponding sides and then solve for x .

- 5 Calculate the length y , multiplying by k to find the image length.

- 6 Interpret the result.

Colours indicate corresponding sides.



Angles marked A and B form alternate angles and angles marked C are vertically opposite.

The triangles are similar by AA.

$$k = \frac{L}{l}$$

With the triangle on the right as the image:

Let $l = 5$ and $L = 10$

$$\begin{aligned} k &= \frac{L}{l} \\ &= \frac{10}{5} \\ &= 2 \end{aligned}$$

Therefore, the image is twice as long as the original (if the image is the triangle on the right).

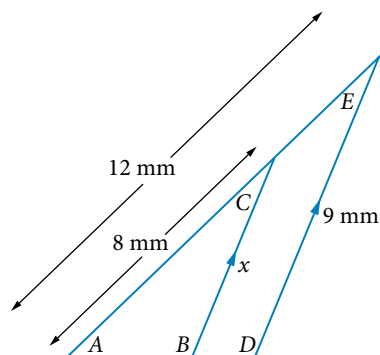
$$\begin{aligned} \frac{x}{4.2} &= \frac{10}{5} \\ x &= \frac{10}{5} \times 4.2 \\ &= 8.4 \text{ cm} \end{aligned}$$

$$\begin{aligned} y &= 2 \times 3 \\ &= 6 \text{ cm} \end{aligned}$$

The image is twice as long as the original.

$$x = 2 \times 4.2 \text{ and } y = 2 \times 3$$

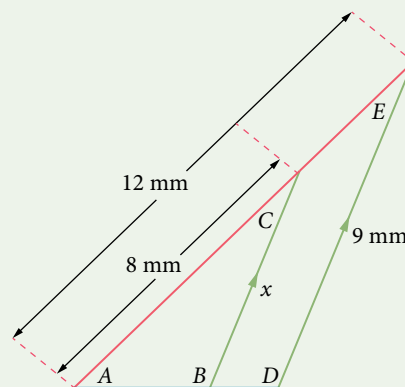
(c)



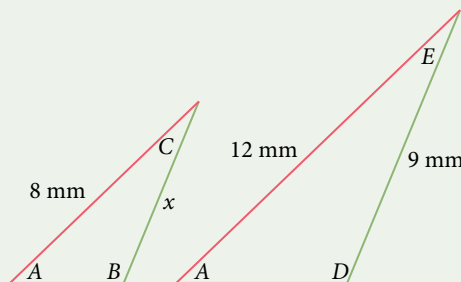
1 Prove similarity.

There is a common angle (A) and corresponding angles (pairs B, D and C, E). The triangles are similar by AA.

The colours indicate corresponding sides.



2 Construct individual triangles to ensure correct dimensions are being matched.



3 Recall the formula for scale factor.

$$k = \frac{L}{l}$$

4 Determine the scale factor k using the given lengths. Interpret the answer.

With the triangle on the right as the image:

Let $L = 12$ and $l = 8$

$$k = \frac{12}{8} \\ = 1.5$$

The image is 1.5 times as long as the original triangle.

5 Calculate the length x by equating equal ratios.

$$\frac{x}{9} = \frac{8}{12} \\ x = \frac{8}{12} \times 9 \\ = 6 \text{ mm}$$

6 Check the reasonableness of the calculation.

The image length 9 mm should be 1.5 times the original length.

$$6 \times 1.5 = 9$$

EXERCISE

3.1

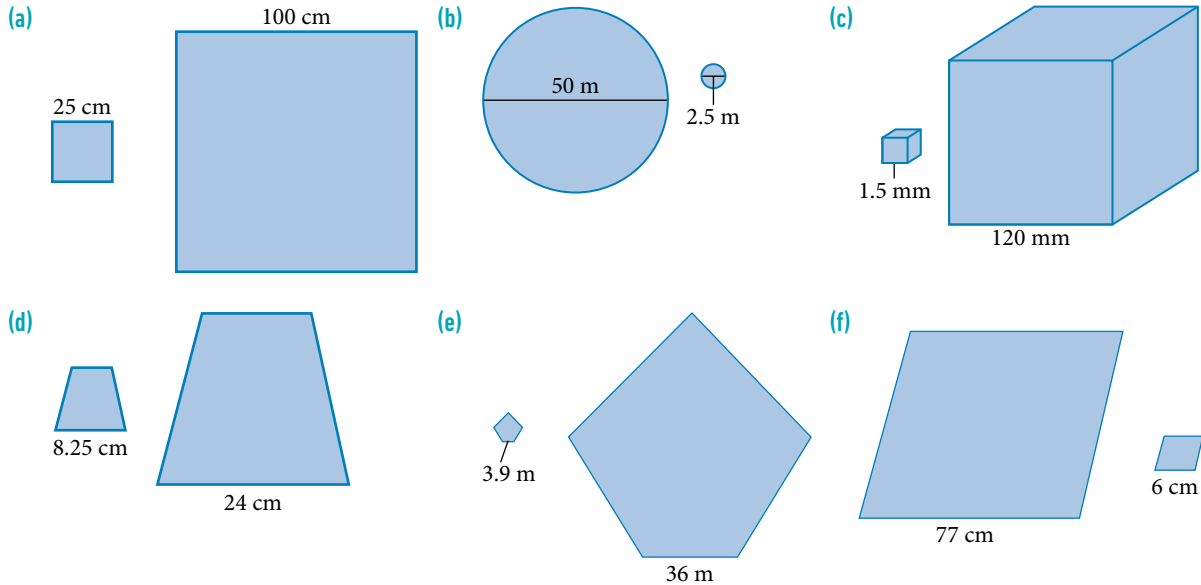
Similar shapes and length

Worked
Example

In the following questions give your answers to 2 decimal places where necessary, unless otherwise specified.

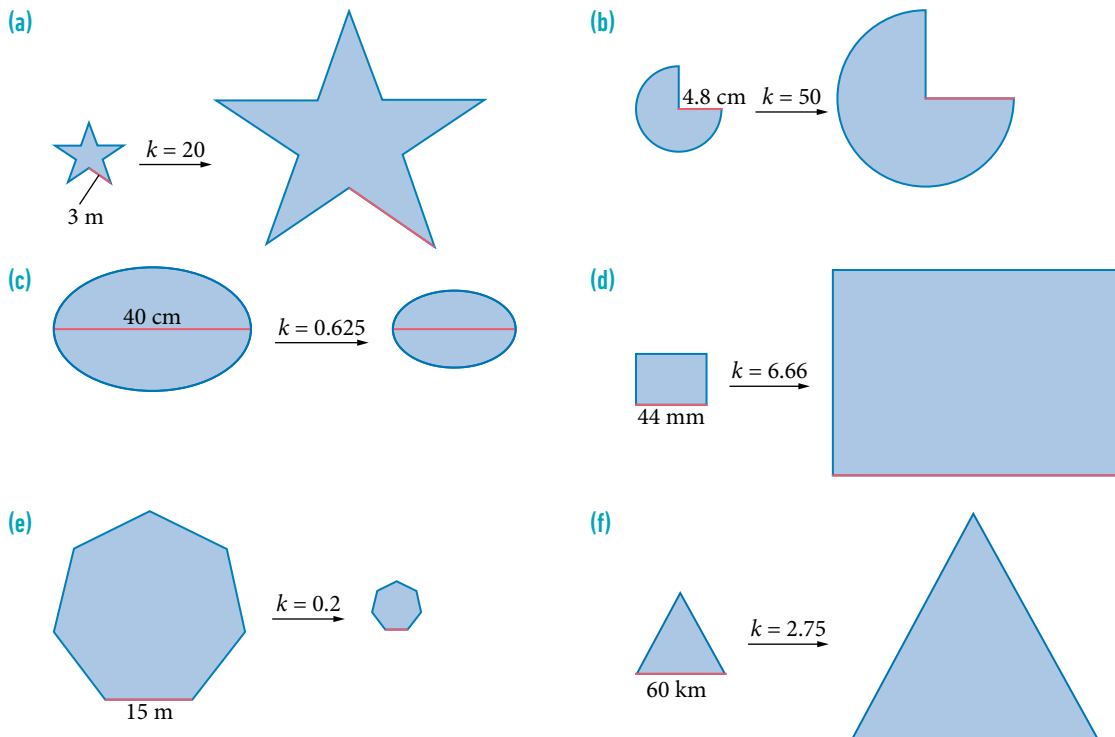
- 1 Calculate the scale factor k for each pair of similar shapes, given that the image is on the right in each case.

1



- 2 Determine the image length corresponding to the known original length, given the scale factor k in each case.

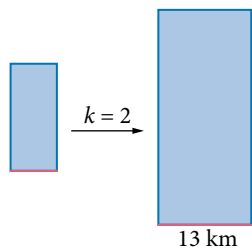
2



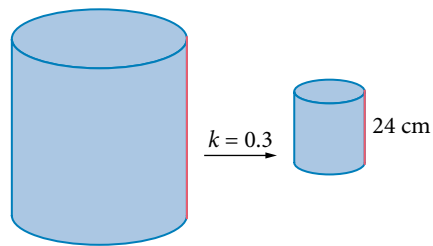
3

- 3 Determine the original length corresponding to the known image length, given the scale factor k in each case.

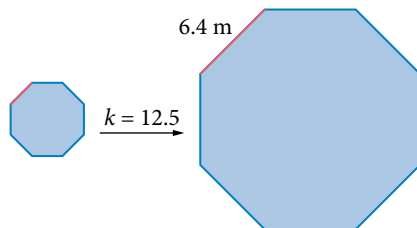
(a)



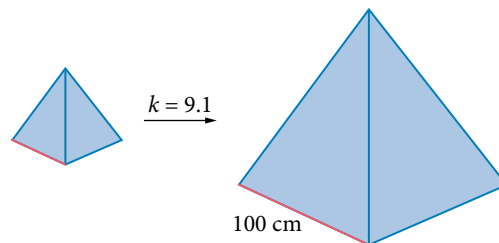
(b)



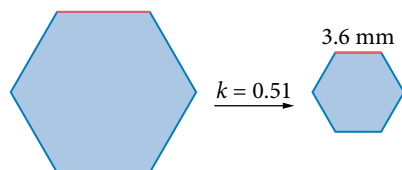
(c)



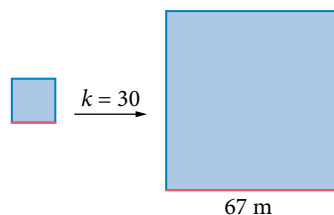
(d)



(e)



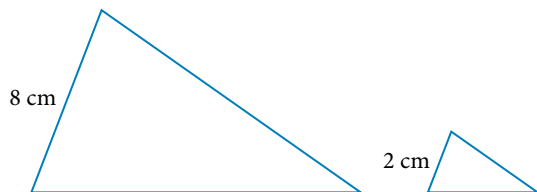
(f)



4

- 4 Calculate the scale factor connecting the following pairs of similar shapes, given that the image is on the right in each case.

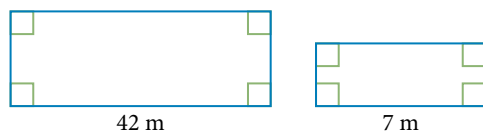
(a)



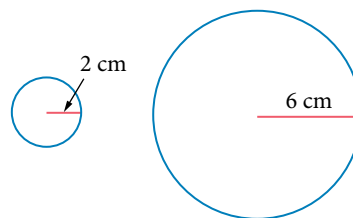
(b)



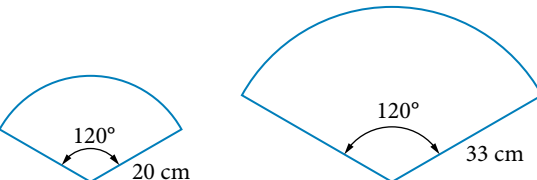
(c)



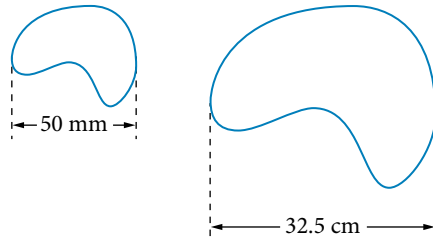
(d)



(e)



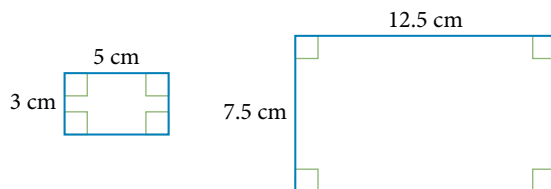
(f)



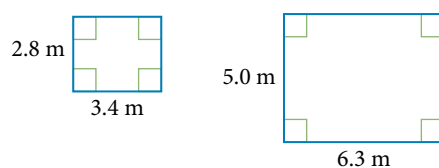
5

5 Describe each pair of shapes as similar, not similar or not necessarily similar.

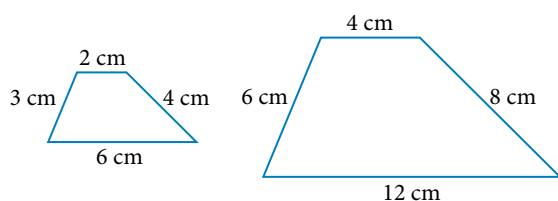
(a)



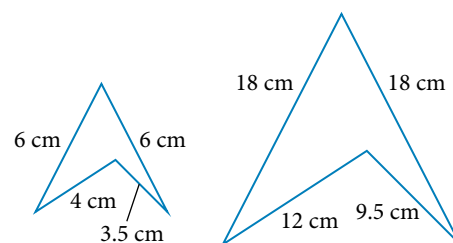
(b)



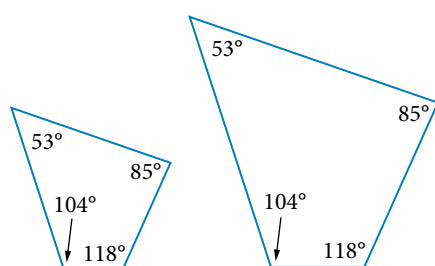
(c)



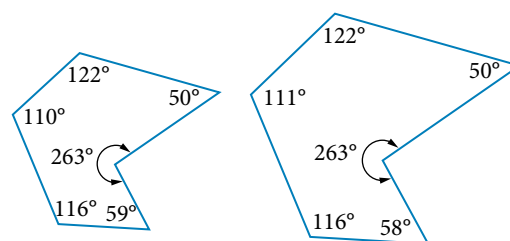
(d)



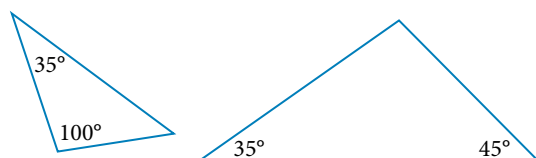
(e)



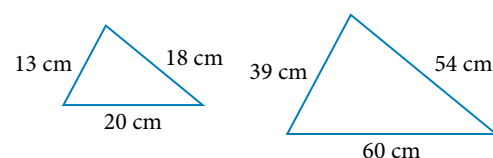
(f)



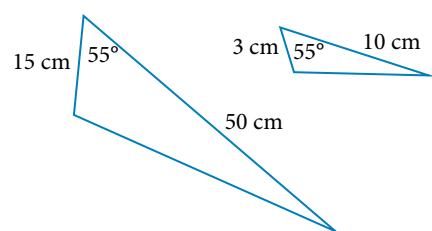
(g)



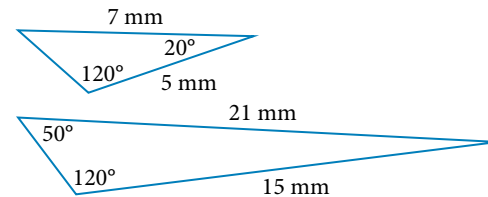
(h)



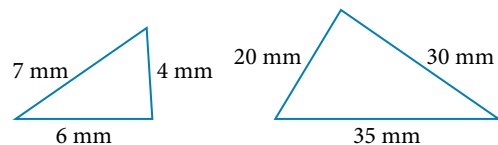
(i)



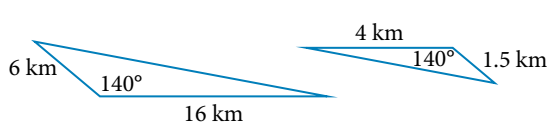
(j)



(k)

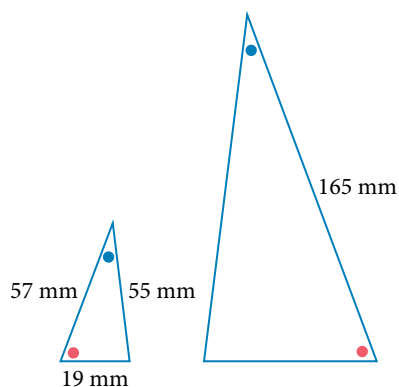


(l)

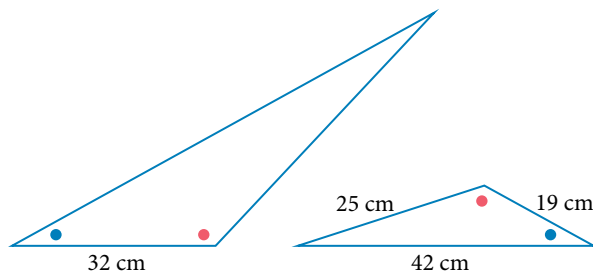


- 6 Each of the following pairs of shapes are similar. Determine the scale factor in each case, assuming the image is on the right.

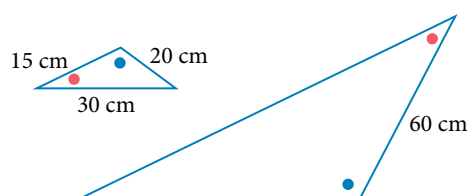
(a)



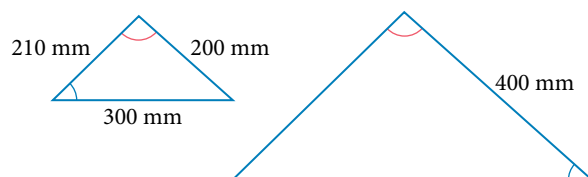
(b)



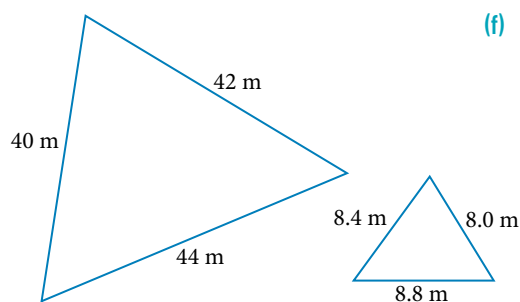
(c)



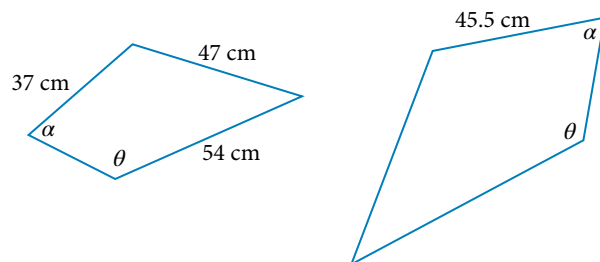
(d)



(e)

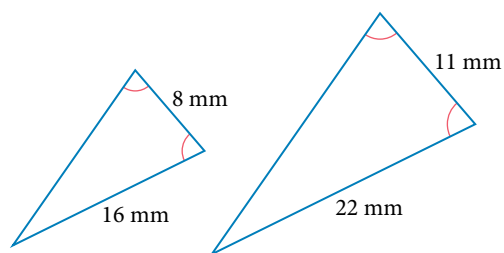


(f)



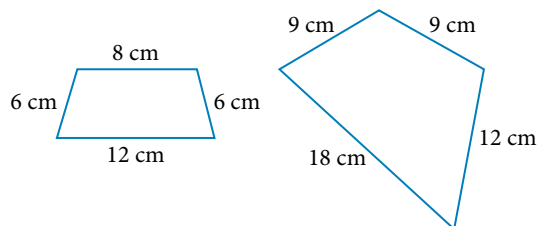
- 7 For the pair of similar triangles shown, with the triangle on the right as the image, which alternative shows the scale factor k ?

A $\frac{1}{2}$ B $\frac{11}{16}$ C $\frac{11}{8}$ D $\frac{16}{11}$



- 8 For the two quadrilaterals, choose the correct statement concerning possible similarity.

- A The quadrilaterals could be similar with scale factor 1.5 but you cannot be sure because there are no known angles.
 B The quadrilaterals are not similar because the sides are not in corresponding positions.
 C The quadrilaterals are similar because of the 'all sides proportional' test.
 D The two sets of lengths given cannot form a pair of similar shapes.



- 9 Anton is drawing illustrations for a computer game, and is basing a drawing of an ant on the specimen shown.

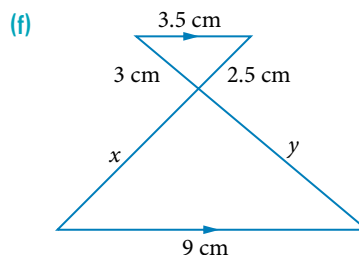
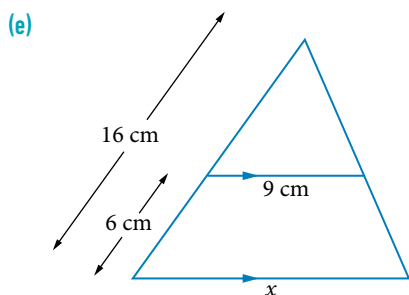
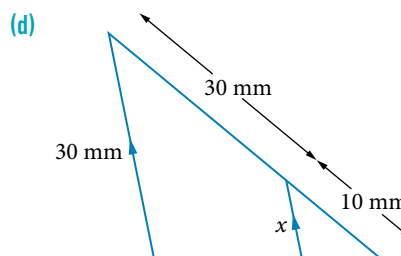
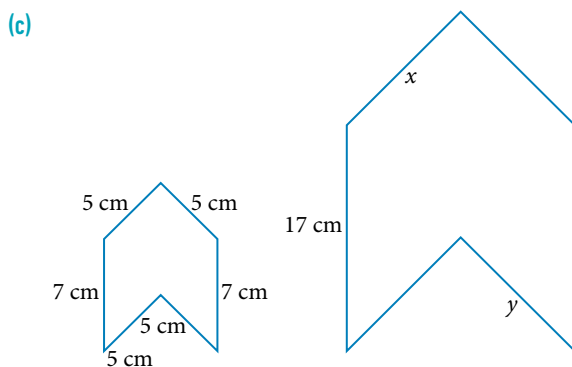
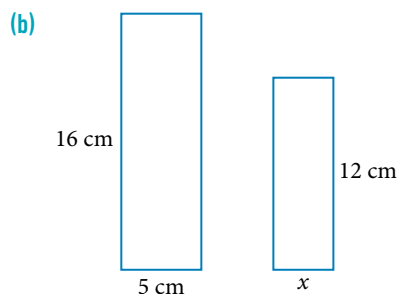
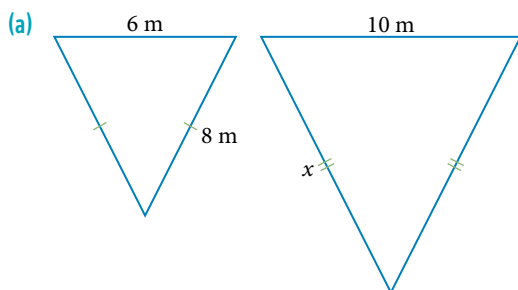


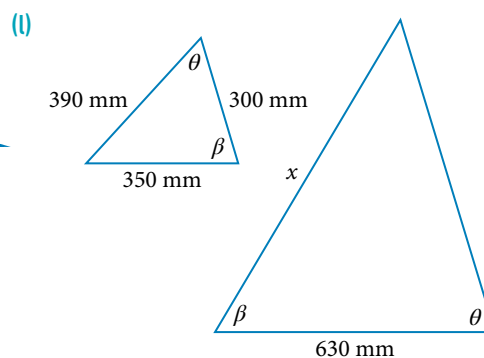
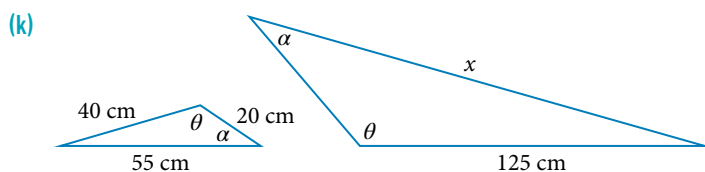
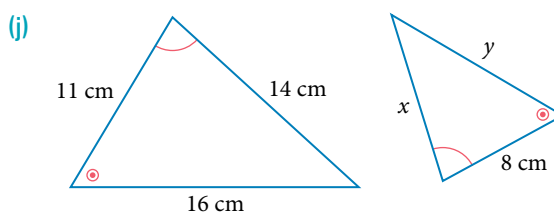
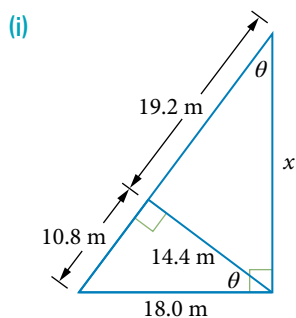
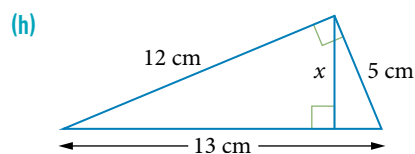
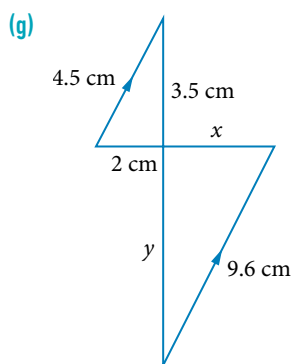
The length of the ant is 9 mm. Anton wants to draw it so it is 12 cm long in his illustration.

- (a) Determine the scale factor for this enlargement.
- (b) If the actual length of one of the ant's legs is 7.2 mm, how long, in centimetres, should Anton draw it? Answer to 1 decimal place.
- 10 Patricia studies dinosaur fossils and concludes that most adults of a particular species and gender are similar in shape. By studying a complete skeleton, she measures the length of a thigh bone to be 62 cm, and the total length of the dinosaur to be 8.6 m. Patricia later finds an incomplete skeleton of another dinosaur of the same gender and species, including a thigh bone of length 45 cm. Estimate the total length, in metres, of the second dinosaur.

Worked
Example

- 11 Determine the value of each pronumeral, given that each pair of shapes is similar.

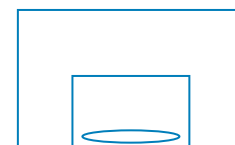
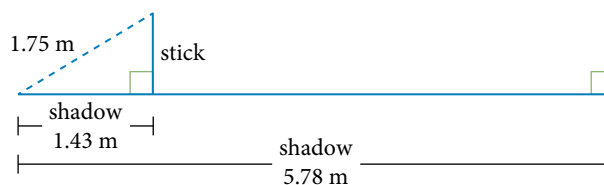




- 12 LeMarcus and Carmelo are trying to determine the height of the top of an outdoor basketball backboard. LeMarcus takes a tape measure and clinometer outside. He finds a stick, stands the stick perpendicular to the ground and measures the direct length from the top of the stick to the clinometer, and then measures the direct distance from the clinometer to the top of the stick as shown.

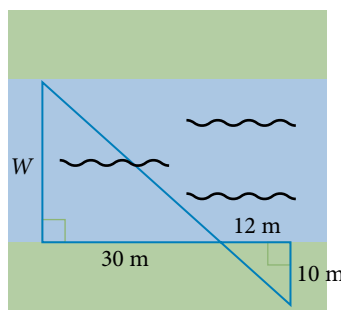
Meanwhile, Carmelo measures the distance from the clinometer to the base pole of the basketball ring.

Determine the height of the basketball ring from the ground to the top of the backboard. Explain any assumptions you made, and explain a simpler method the boys could have used.

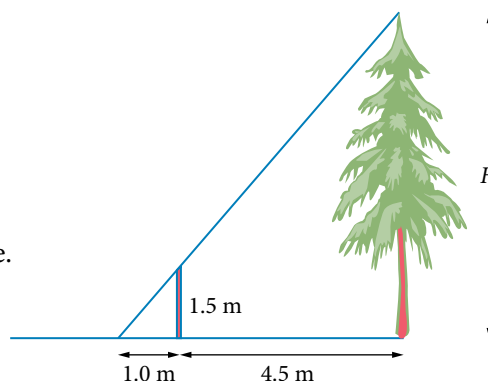


- 13 Trent surveys the information shown in the diagram in order to calculate the width W of a river using similar triangles.

Demonstrate that the triangles are similar and then calculate the width W of the river.

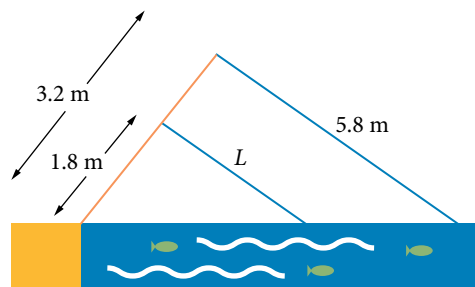


- 14 Meena must cut down a tree in her backyard, and wants to avoid the tree falling on her house. She decides to calculate the height H of the tree using a 1.5 m measuring stick and shadows of the stick and the tree. The shadow of the stick and the shadow of the tree end at the same point. She records the measurements shown in the diagram. Use similar triangles to determine the height of Meena's tree.

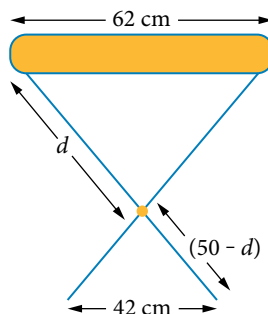


- 15 Eshaan stands up straight, casting a shadow of 284 cm. His little brother, Djaali, comes and stands within Eshaan's shadow and 90 cm from Eshaan, with the sunlight just missing the top of his head. Construct a diagram of the situation and use similar triangles to calculate Djaali's height, to the nearest centimetre, given that Eshaan is 175 cm tall.

- 16 Rui attaches two fishing lines to the one rod and casts into the ocean so that the lines are parallel. Determine the visible length of the shorter fishing line. Assume both lines and the rod are straight-line segments.

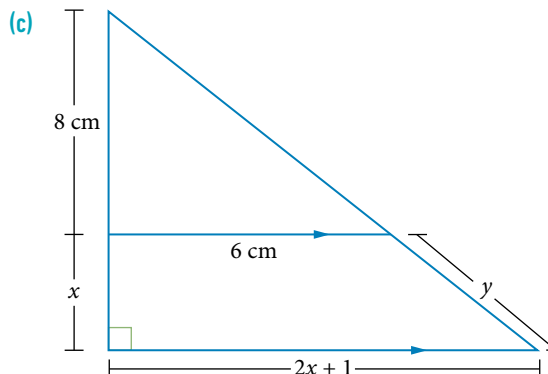
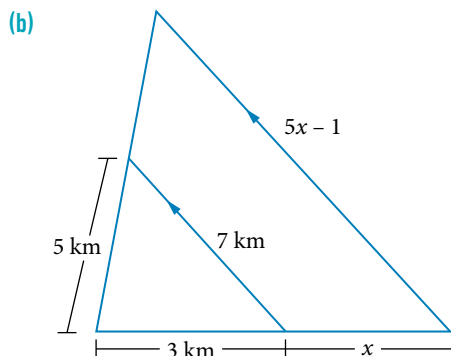
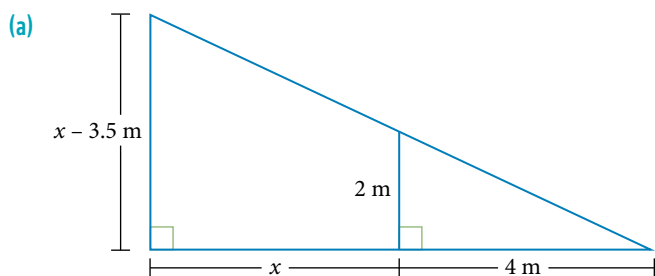


- 17 Cameron's folding director's chair sits on two diagonal struts, each of total length 50 cm, as shown. Calculate the distance d to the nearest centimetre.

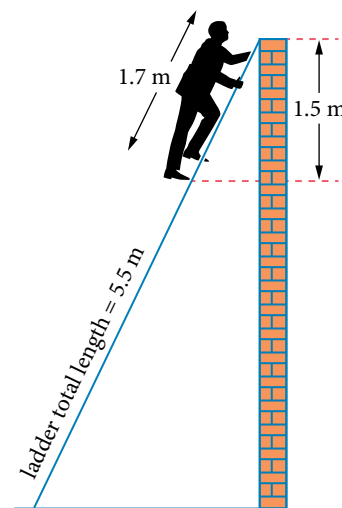


- 18 Hans secures his 4 m ladder against a much taller wall, so that the bottom of his ladder is 1.5 m from the base of the wall. His daughter Footsie has a 1.0 m long toy ladder that she leans against the same wall, copying her father. If Footsie's ladder leans against the wall at the same angle as Hans' ladder, how far from the base of the wall is the bottom of the toy ladder? Answer in centimetres.

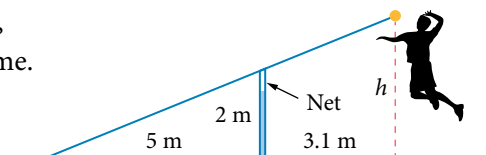
- 19 Determine the value of the pronumerals in each of the following, and calculate the scale factor k , where the larger triangle is the image in each case.



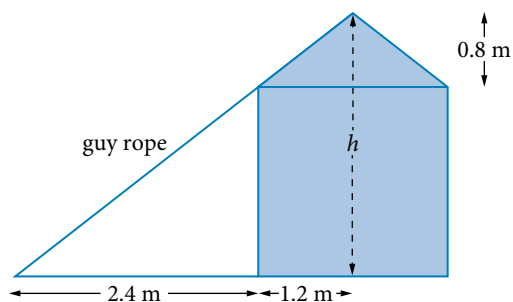
- 20 Hassan climbs a 5.5 m long ladder, which is leaning against his house wall, to clean the gutters. Hassan has fallen from ladders before, so his doctor says that Hassan's feet should never be more than 3.5 m from the ground. Hassan is 1.7 m tall and his head is level with the top of the ladder. Is Hassan obeying his doctor's ladder rule? Explain any assumptions you had to make.



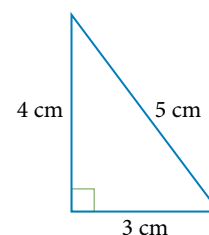
- 21 Vonda is standing 3.1 m horizontally from the volleyball net, when she jumps high into the air to spike a ball during a game. How high is the ball when Vonda strikes it, if it just skims the 2 m high net then hits the ground 5 m from the net? Assume the path of the ball is a straight line.



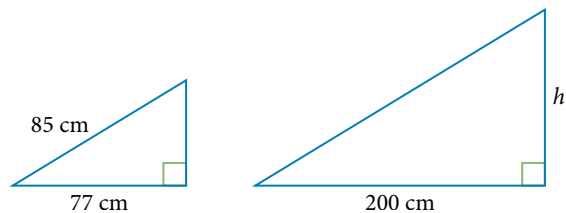
- 22 Warren pitches a tent and attaches a guy rope parallel with the slope of the roof, between the top of the tent and the ground, as shown. Determine the length of the guy rope used.



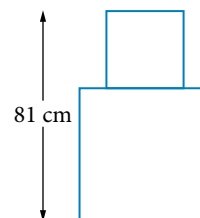
- 23 A triangle with dimensions 3 cm, 4 cm and 5 cm is shown. If a triangle is drawn similar to this triangle, with a perimeter of 15 cm, by what scale factor is the original enlarged?



- 24 The triangles shown are similar. Determine the height h of the larger triangle.



- 25 Two stacked cubes have a combined height of 81 cm. If the larger cube's side length is 1.7 times that of the smaller cube, what is the side length of each cube?



3.2

Scale drawing

Scale ratios and scale factors

Scale drawings may involve reductions of actual distances, as in maps.

For maps and plans of buildings, the convention is to avoid using decimals. For a map that reduces the size 500 times, the scale ratio of 1 : 500 is used, not the decimal 0.002. The scale factor is 500, the number you need to multiply the map distance by to find the actual distance.

Usually, but not always, one of the values in the ratio is 1.

There are no units in a scale ratio, because the values represent the number of parts.

As an example, 1 : 1000 can mean

‘1 mm on the diagram represents

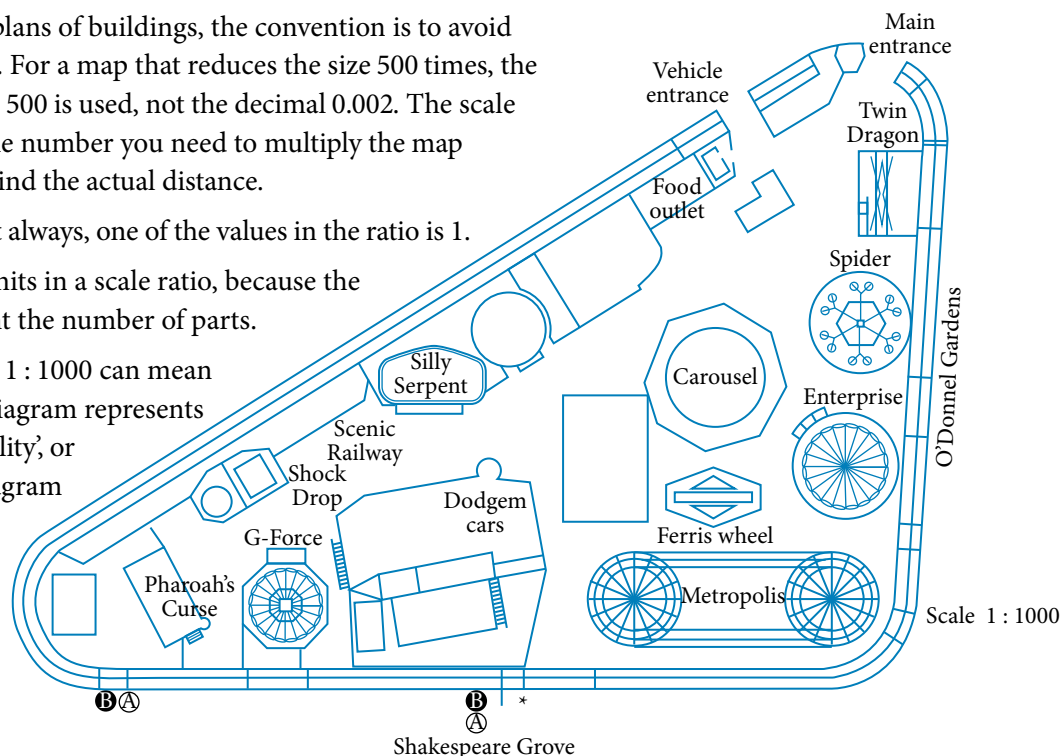
1000 mm in reality’, or

‘1 cm on the diagram

represents

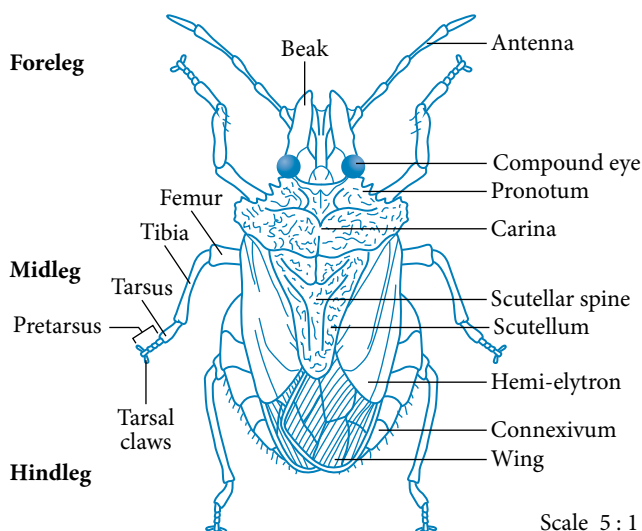
1000 cm in

reality’.



For enlargements, such as scientific diagrams of insects or microscopic organisms, a scale ratio such as 10 : 1 may be referred to as a magnification, seen on the microscope lens as 10 $\times$, or a scale factor of 10 as defined in the first section of this chapter.

The scale ratio is written in the form:
distance on drawing : actual distance



The ratio indicated on the key or legend of the diagram represents the scale. The scale indicates the relationship between the dimensions of the diagram and the dimensions of the real object—that is, whether the dimensions on the diagram are enlarged or reduced to give the actual dimensions.

When the scale drawing is a reduction of the actual subject, the scale ratio is 1 : k .

When the scale drawing is an enlargement of the actual subject, the scale ratio is k : 1.

The scale factor is k , where $k \geq 1$.

7 Determining a scale ratio

Determine the scale ratio and hence the scale factor in each case. Determine whether the situation represents a reduction or an enlargement.

- (a) a map in which 4 cm represents 1 km

THINKING

- 1 Identify the ratio of one amount to the other amount, including the units.
- 2 Convert one of the sides of the ratio so that the units are the same.
- 3 Simplify the ratio.
Cancel the units from both parts.
- 4 Interpret the scale ratio and check the reasonableness as a reduction $1 : k$ or an enlargement $k : 1$.

WORKING

Map distance : actual distance
4 cm : 1 km

4 cm : $1 \times 100 \times 1000$ cm
4 cm : 100 000 cm

$$\frac{4}{4} : \frac{100\,000}{4} = 1 : 25\,000$$

The scale ratio is $1 : 25\,000$.

The scale factor is 25 000.

The map is a reduction of the actual distance, so the scale ratio $1 : 25\,000$ seems reasonable.

- (b) a diagram of a bug in which 2.5 cm represents 2 mm

- 1 Identify the ratio of one amount to the other amount, including the units.
- 2 Convert one of the sides of the ratio so that the units are the same.
- 3 Simplify the ratio.
Cancel the units from both parts.
- 4 Interpret the scale ratio and check the reasonableness as a reduction $1 : k$ or an enlargement $k : 1$.

Drawing distance : actual distance
2.5 cm : 2 mm

2.5×10 mm : 2 mm
250 mm : 2 mm

$$\frac{250}{2} : \frac{2}{2} = 125 : 1$$

The scale ratio is $125 : 1$.

The scale factor is 125.

The diagram gives an enlargement of the actual bug, so the scale ratio $125 : 1$ seems reasonable.

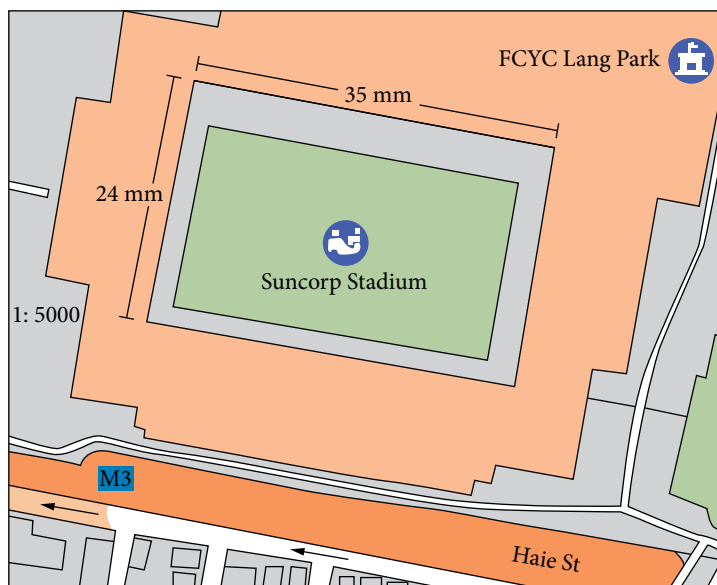
Actual dimensions

You can determine actual distances from a map or a plan by multiplying known lengths by the scale factor.

$$\text{Actual distance} = k \times \text{drawing distance}$$

8 Using a scale to calculate a real distance

A scale drawing of Suncorp Stadium (formerly Lang Park) is shown. The scale of the drawing is 1 : 5000.



Use the scale drawing to determine the length and width of the stadium.

METHOD 1

THINKING

- 1 Identify the dimensions and scale factor from the drawing.
- 2 Recall the scale formula.
- 3 Calculate the real dimensions, using the scale formula.
- 4 Convert to appropriate units and interpret the result.

WORKING

Drawing:

$$\text{Length} = 35 \text{ mm}$$

$$\text{Width} = 24 \text{ mm}$$

$$k = 5000$$

$$\text{Actual distance} = k \times \text{drawing distance}$$

Reality:

$$\text{Length} = 5000 \times 35 \text{ mm}$$

$$= 175\,000 \text{ mm}$$

$$\text{Width} = 5000 \times 24 \text{ mm}$$

$$= 120\,000 \text{ mm}$$

$$\text{Length} = 175\,000 \div 1000 \text{ m}$$

$$= 175 \text{ m}$$

$$\text{Width} = 120\,000 \div 1000 \text{ m}$$

$$= 120 \text{ m}$$

The actual dimensions of the stadium are 175 m by 120 m.

Alternatively, work directly from the scale.

METHOD 2

- 1 Multiply both parts of the scale ratio by the dimensions on the diagram or map.

Simplify to appropriate units.

Length on the diagram: 35 mm

1 : 5000

$$= 1 \times 35 : 5000 \times 35$$

$$= 35 \text{ mm} : 175\,000 \text{ mm}$$

$$= 35 \text{ mm} : 175 \text{ m}$$

Width on the diagram: 24 mm

1 : 5000

$$= 1 \times 24 : 5000 \times 24$$

$$= 24 \text{ mm} : 120\,000 \text{ mm}$$

$$= 24 \text{ mm} : 120 \text{ m}$$

- 2 Interpret the result.

The actual dimensions of the stadium are

175 m by 120 m.

A third method is to use the scale to calculate how much real distance is represented by each millimetre on the diagram.

METHOD 3

- 1 Convert the scale so that each part represents 1 unit and then simplify to the most appropriate units.

1 : 5000

$$= 1 \text{ mm} : 5000 \text{ mm}$$

$$= 1 \text{ mm} : 5 \text{ m}$$

- 2 Multiply both parts by the measured distances to find the actual distances.

Length

$$1 \text{ mm} \times 35 : 5 \text{ m} \times 35$$

$$35 \text{ mm} : 175 \text{ m}$$

35 mm on the map represents an actual length of 175 m.

Width

$$1 \text{ mm} \times 24 : 5 \text{ m} \times 24$$

$$24 \text{ mm} : 120 \text{ m}$$

24 mm on the map represents an actual width of 120 m.

- 3 Interpret the result.

The actual dimensions of the stadium are

175 m by 120 m.

1 mm : 5 m can be written $1 \text{ mm} \leftrightarrow 5 \text{ m}$, which means 1 mm on the map represents an actual distance of 5 m.

i Additional information**Calculating distances with ratios**

Review converting units of length and area, and using map scales to calculate distances.

Dimensions on maps and diagrams

You can find map, plan or diagram distances by multiplying or dividing the real distances by the scale factor.

9 Using a scale to calculate a distance on a drawing

Determine the diagram distance in each case.

- (a) An architect is preparing a house plan using a scale of 1 : 50.

The architect's client has asked for a spa of length 2.6 m and width 3.1 m to be shown on the plan. What length and width should the spa have on the plans (in centimetres)?

THINKING

1 Identify the value of the scale factor from the scale ratio.

2 Recall the formula.

3 Calculate the smaller distances using the scale formula.

4 Convert to appropriate units. The question asks for the answers in centimetres.

5 Interpret the result.

WORKING

Scale 1 : 50, so $k = 50$

Drawing distance = actual distance $\div k$

Length:

$$\begin{aligned}\text{Smaller distance (on plan)} &= 2.6 \text{ m} \div 50 \\ &= 0.052 \text{ m}\end{aligned}$$

Width:

$$\begin{aligned}\text{Smaller distance (on plan)} &= 3.1 \text{ m} \div 50 \\ &= 0.062 \text{ m}\end{aligned}$$

Length on plan:

$$\begin{aligned}0.052 \text{ m} &= 0.052 \times 100 \text{ cm} \\ &= 5.2 \text{ cm}\end{aligned}$$

Width on plan:

$$\begin{aligned}0.062 \text{ m} &= 0.062 \times 100 \text{ cm} \\ &= 6.2 \text{ cm}\end{aligned}$$

On the plan the spa should have a length of 5.2 cm and a width of 6.2 cm.

- (b) An optometry student is creating a schematic diagram of a human eye with a scale of 6 : 1. She measures the diameter of the pupil to be 3.4 mm. Determine the diameter of the pupil on the diagram (in centimetres).

1 Identify the value of the scale factor.

Scale 6 : 1, so $k = 6$

2 Recall the formula.

Drawing distance = actual distance $\times k$

3 Calculate the larger diagram distance using the scale formula.

$$\begin{aligned}\text{Larger diameter (on diagram)} &= 3.4 \text{ mm} \times 6 \\ &= 20.4 \text{ mm}\end{aligned}$$

- 4 Convert to appropriate units. The question asks for the answer in centimetres.

$$\begin{aligned}\text{Diameter on diagram:} \\ 20.4 \text{ mm} &= 20.4 \div 10 \text{ cm} \\ &= 2.04 \text{ cm}\end{aligned}$$

- 5 Interpret the result.

On the diagram, the diameter of the pupil should be 2.04 cm.

Enlargement techniques

Technology has made enlargement of shapes, especially plane shapes, easy and convenient. To carry out the process yourself you need a technique that preserves all the angles and enlarges each length by the required scale factor.

The following worked example shows steps that do not require special techniques.

10 Enlarging a polygon

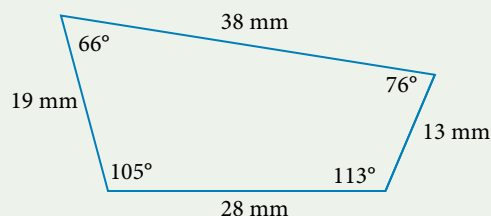
Enlarge the quadrilateral using a scale factor of 1.2.



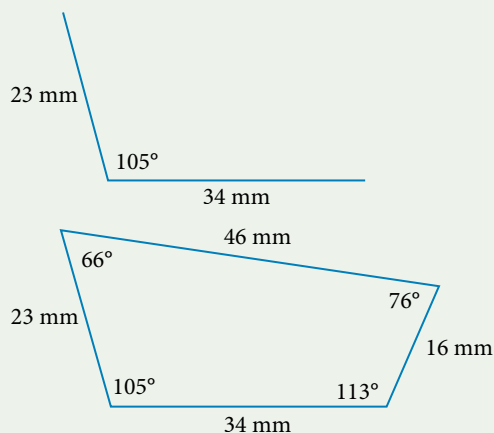
THINKING

- 1 Measure every length and angle.
- 2 Calculate the value of the new lengths to the nearest millimetre by multiplying the original lengths by the scale factor.
- 3 Construct the image by drawing two image lengths forming the same angle as the original shape. Use a protractor to construct the angles.
- 4 Complete the image by drawing each side of the image to form the same angles as the matching original sides.
- 5 Compare the two shapes to check size and shape.

WORKING



$$\begin{aligned}28 \text{ mm} \times 1.2 &\approx 34 \text{ mm} \\ 19 \text{ mm} \times 1.2 &\approx 23 \text{ mm} \\ 38 \text{ mm} \times 1.2 &\approx 46 \text{ mm} \\ 13 \text{ mm} \times 1.2 &\approx 16 \text{ mm}\end{aligned}$$



EXERCISE

3.2 Scale drawing

In the following questions, round answers to 2 decimal places where necessary.

Worked
Example

7

- 1 Determine the map scale as a ratio for each of the following, and determine whether the scale represents a reduction or an enlargement from the real object to the drawing.

- (a) 1 cm : 1 m (b) 2 mm : 5 m (c) 10 mm : 1 mm (d) 1 cm : 1 km
 (e) 1 mm : 10 m (f) 2 cm : 1 mm (g) 1 cm : 15 m (h) 5 mm : 1 m
 (i) 4 m : 1 cm (j) 0.8 cm : 1 mm (k) 4 cm : 2 m (l) 5 m : 1 mm
 (m) 15 mm : 300 m (n) 25 mm : 1000 000 mm

8

- 2 The scale on a particular plan is 1 : 500. Calculate the actual distance, in metres, that each of the following map distances represents.

- (a) 2 cm (b) 5 cm (c) 12 cm (d) 25 mm

9

- 3 A map is to be constructed showing key features of the Relaxville tourist resort. The scale for the map is to be 1 : 5000. Calculate the distance on the map, in millimetres, for each of the following real distances.

- (a) 100 m (b) 250 m (c) 0.7 km (d) 550 cm

- 4 A map scale is written as 2 cm $\leftrightarrow$ 5 m. Select the option that best shows the scale factor k .

A $\frac{1}{1000}$

B $\frac{1}{250}$

C 250

D 1000

- 5 Plans for a micro-circuit are drawn so that 1 cm represents 0.001 mm.

- (a) Select the option that best shows the equivalent plan scale.

A 10 000 : 1

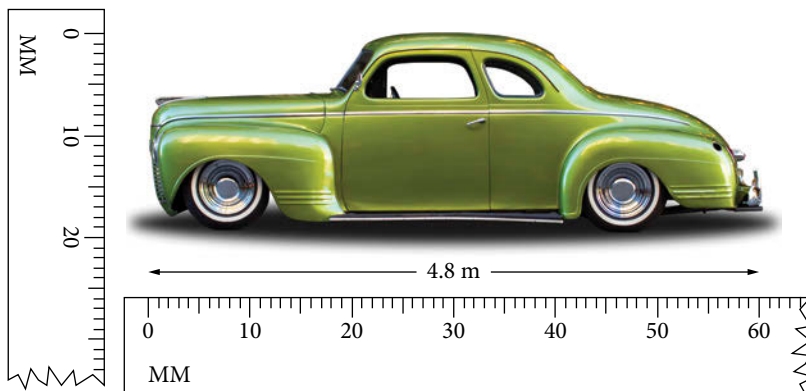
B 1000 : 1

C 100 : 1

D 1 : 100

- (b) Explain two common errors that would lead to a student choosing the last of the incorrect options.

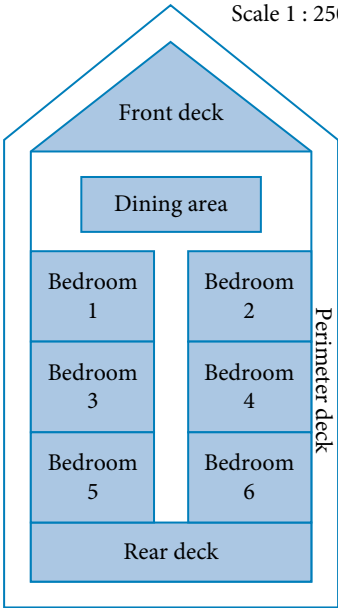
- 6 The profile of a new car and the actual length are shown in the diagram. Determine the scale of the drawing.



7 The plan for a houseboat is shown.

Calculate the following measurements of the actual houseboat, given the measurements provided from the plan.

- (a) the length of the houseboat (length on plan: 8 cm)
- (b) the width of the houseboat (width on plan: 3.9 cm)
- (c) the width of the perimeter deck (width on plan: 0.3 cm)
- (d) the dimensions of the dining area, putting the larger value first (dimensions on plan: 2.2 cm × 0.7 cm)



Scale 1 : 250

Houseboat plan view

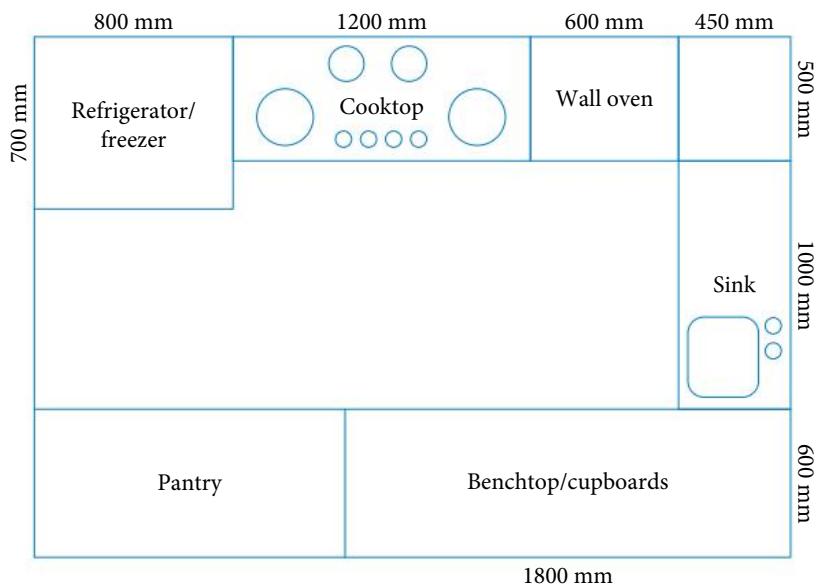
8 The map below shows several major Australian cities.

- (a) If the scale marker in the bottom left corner of the map is 2 cm long, determine the scale of the map.
- (b) Assuming the measurements between cities on the map are as listed in the table, determine the actual distances between the cities, to the nearest 10 km.



Between cities	Distance on map (cm)	Actual distance (km)
Perth–Melbourne	5.80	
Perth–Brisbane	7.28	
Melbourne–Brisbane	2.84	
Sydney–Brisbane	1.50	
Brisbane–Adelaide	2.96	

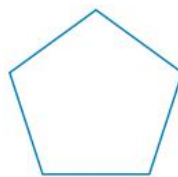
- 9 This plan of a new kitchen is *not* drawn to scale. Redraw the plan using a scale of 1 : 30. The distances on the plan are actual distances.



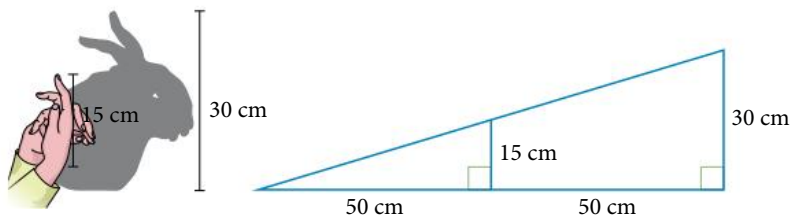
Worked
Example

10

- 10 Enlarge this regular pentagon using a scale factor of 1.6.

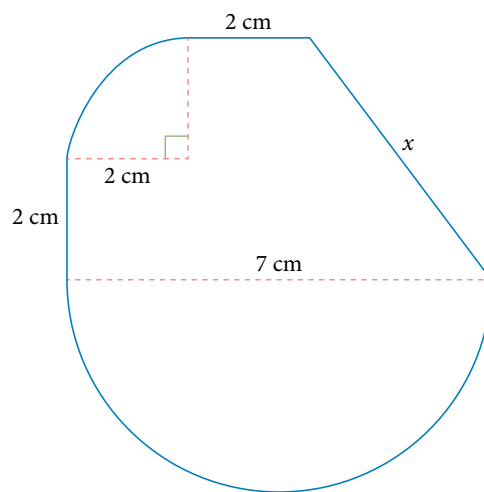


- 11 A light source is a metre from a vertical screen. Ben makes 'bunny ears' with his hand positioned midway between the light source and the screen. From Ben's wrist to his fingertip is 15 cm and the shadow of the bunny's neck to ear tip is 30 cm.



By how many centimetres will the shadow grow when Ben moves his hand 5 cm closer to the light source? Give your answer to the nearest millimetre.

- 12 Enlarge the diagram with scale factor 1.5, then measure the unknown side length in the drawing. Verify your measurement with a calculation.



- 13 Decide on an appropriate scale and draw a scale map of a square $40\text{ m} \times 40\text{ m}$ section of a fun park. Your map should include the following:

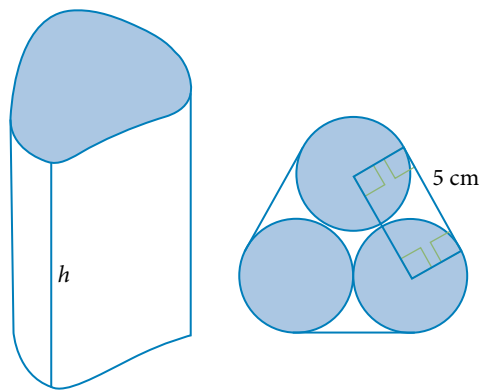
- the scale you have used for the drawing
- a circular carousel of diameter 8 m
- a rectangular dodgem car area of length 16 m and width 12 m
- a $10\text{ m} \times 5\text{ m}$ rectangular café area
- a 50 m long path joining an entrance and exit to this section of the park.

Choose a scale that will allow your diagram to fit easily onto an A5 page, which is $148\text{ mm} \times 210\text{ mm}$.

- 14 Melanie has designed a carry bag with a base the shape of three packed circles with straight connecting lines, as shown.

Melanie has made the carry bag aesthetically pleasing by making height : width for the front rectangle in the 'golden ratio' $1.62 : 1$.

The carry bag is resized to make a snug fit for three cylindrical cans of soft drink of height 13 cm and diameter 6.5 cm. How much will the cans protrude above the top of the carrier?



- 15 Queensland is approximately 1800 km from west to east, and 2000 km from north to south. A map of Queensland is downloaded from the Internet.

Calculate the scale ratio for the map, if it is to fit on each of the following screens. State any assumptions you make in your calculations. Write your answers to the nearest whole number.

- laptop of screen size 34 cm by 19 cm
- smartphone of screen size 10 cm by 6 cm

3.3

Similar shapes and area

Scale factor

The scale factor k describes the ratio of corresponding lengths in two similar objects, but does *not* describe the ratio of areas, as this example involving two squares demonstrates.

The scale factor for the lengths is $k = \frac{L}{l} = \frac{6}{2}$. Therefore $k = 3$.

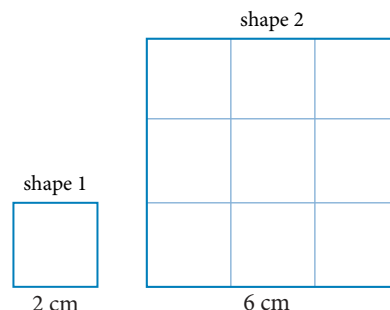
Let A denote the area of the larger square, and a denote the area of the smaller square.

Now consider the ratio of the *areas* of shapes 1 and 2.

$$\frac{A}{a} = 9 = 3^2.$$

This value is the ratio of areas, and is obtained by squaring the scale factor k .

$$\text{Ratio of areas} = k^2$$

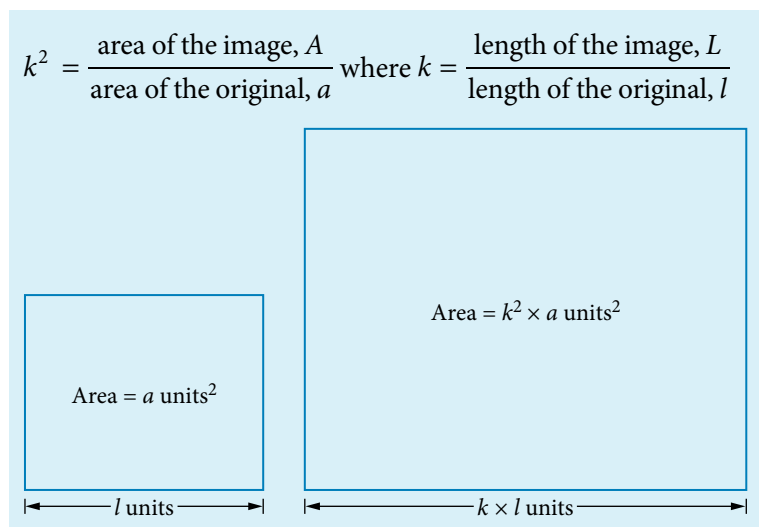


Explore further

Creating similar shapes

Explore the relationship between scale factor and area.

Scale factor and area



Additional information

Scale factors and area

Practise using the scale factor to calculate the area of similar figures.

For any similar figures, if A and a are the areas of an image and an original shape respectively, then these areas and k are related in the following ways, where $k > 0$.

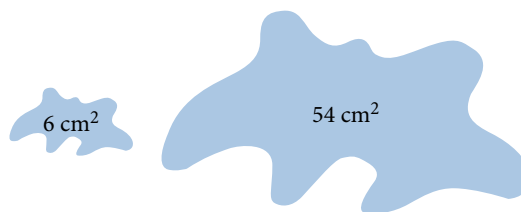
$$k^2 = \frac{A}{a}, \text{ where } a \text{ and } A \text{ have the same units.}$$

$A = k^2 a$ means that the area of the image may be found by multiplying the area of the original shape by k^2 .

$a = \frac{A}{k^2}$ means that the area of the original shape may be found by dividing the area of the image by k^2 .

11 Calculating the scale factor, using the areas of original shape and image

Calculate the scale factor k of similar shapes, given that the area of the original is 6 cm^2 and the area of the image is 54 cm^2 .



THINKING

1 Recall the formula relating area and scale factor.

2 Substitute the given values and solve for k .

For the scope of this course, k is always positive, that is, determine k , where $k > 0$.

3 Interpret the result.

WORKING

$$k^2 = \frac{\text{area of the image}}{\text{area of the original}}$$

$$\begin{aligned} k^2 &= \frac{54}{6} \\ &= 9 \end{aligned}$$

Given that $k > 0$:

$$\begin{aligned} k &= \sqrt{9} \\ &= 3 \end{aligned}$$

The lengths in the original shape have been increased by a factor of 3.

12 Calculating the area of an image, using the area of an original shape and the scale factor

Calculate the image area, given that the original area 25 mm^2 of a similar shape has been increased by a scale factor $k = 2$.



THINKING

1 Recall the formula required to calculate the image area A .

2 Identify the given information.

3 Substitute the given values for k and a into the formula. Determine the value of A .

4 Interpret the result.

WORKING

$$A = k^2 a$$

$$\text{Original area: } a = 25 \text{ mm}^2$$

$$\text{For } a = 25 \text{ mm}^2 \text{ and } k = 2$$

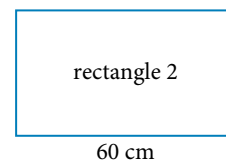
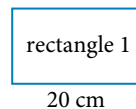
$$\begin{aligned} A &= 2^2 \times 25 \\ &= 100 \text{ mm}^2 \end{aligned}$$

The image area is 100 mm^2 .

13 Using diagram lengths to calculate k and then area

The two rectangles shown here are similar. The area of rectangle 1 is 200 cm^2 .

Calculate the scale factor k , then use k to calculate the area of rectangle 2.



THINKING

- 1 Recall the formula for scale factor.
- 2 Calculate the scale factor by substituting appropriate values, and interpret the result.
- 3 Recall the formula for calculating the area of an image.
- 4 Substitute the given value for the area and the value for k to calculate the image area.
- 5 Interpret the result.

WORKING

$$k = \frac{L}{l}$$

Let $L = 60$ and $l = 20$

$$k = \frac{60}{20} = 3$$

Rectangle 2 is 3 times longer than Rectangle 1.

$$A = k^2 a$$

Let $a = 200 \text{ cm}^2$ and $k = 3$

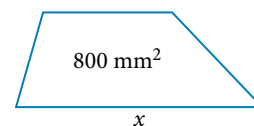
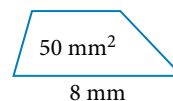
$$A = 3^2 \times 200 \text{ cm}^2 = 1800 \text{ cm}^2$$

The area of rectangle 2 is 1800 cm^2 .

14 Using diagram areas to calculate k and a side length

The two trapeziums shown here are similar. The area of the original trapezium is 50 mm^2 and the area of the image trapezium is 800 mm^2 .

Calculate the scale factor k , then use k to calculate the value of x .



THINKING

- 1 Identify an appropriate formula relating area and scale factor.
- 2 Substitute the given values into the formula.
- 3 Calculate k by taking the positive square root, and interpret the result.

WORKING

$$k^2 = \frac{\text{area of the image } (A)}{\text{area of the original } (a)}$$

$$k^2 = \frac{800}{50} = 16$$

$$\therefore k = \sqrt{16} = 4$$

The length of the image is 4 times the length of the original.

4	Recall the formula for calculating the length of the image.	$L = kl$
5	Substitute the values into the formula to determine the length of the image.	$\text{Let } k = 4 \text{ and } l = 8$ $x = 4 \times 8 \text{ mm}$ $= 32 \text{ mm}$
6	Interpret the result.	The length of the base of the image trapezium is 32 mm.

i Additional information

Enlarged areas

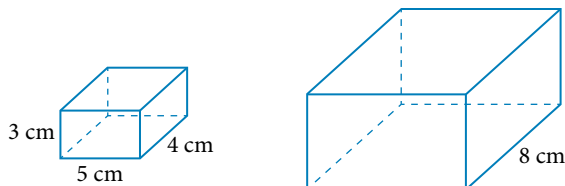
Review areas of figures under enlargement by doing the activity.

Surface area and scale factor

The rules for surface area and scale factor are the same as the rules for scale factor and area.

15 Determine the surface area of a similar shape

A solid rectangular prism is enlarged to scale.



- (a) Calculate the scale factor k .

THINKING

- 1 Recall the scale factor formula.
- 2 Substitute a pair of corresponding lengths to determine the scale factor.
- 3 Interpret the result.

WORKING

$$k = \frac{L}{l}$$

$$\text{Let } L = 8 \text{ and } l = 4$$

$$k = \frac{8}{4}$$

$$= 2$$

The dimensions of the rectangular prism image are twice the length of the dimensions of the original.

- (b) Calculate the surface area (SA) of the original, smaller prism.

- 1 Recall an appropriate formula that gives the total area of all faces.

$$SA = 2LW + 2LH + 2WH$$

- 2 Substitute the dimensions of the original prism to calculate the surface area.

Let $L = 3$, $W = 5$ and $H = 4$

$$SA_1 = 2(3 \times 5) + 2(3 \times 4) + 2(5 \times 4) \\ = 94 \text{ cm}^2$$

- 3 Interpret the result.

The original rectangular prism has a total surface area of 94 cm^2 .

- (c) Calculate the surface area of the larger prism.

- 1 Recall the formula for calculating the area of an image when the area of the original is known.

$$A = k^2 a$$

where $A = SA_2$ and $a = SA_1$

- 2 Substitute the known scale factor and original area into the formula, calculating the answer.

Let $k = 2$ and $a = SA_1 = 94$

$$A = k^2 \times 94 \\ = 2^2 \times 94 \\ = 376 \text{ cm}^2 \\ \therefore SA_2 = 376 \text{ cm}^2$$

- 3 Interpret the result.

The enlarged rectangular prism has a total surface area of 376 cm^2 .

EXERCISE

3.3

Similar shapes and area

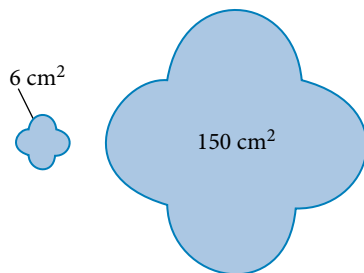
Worked
Example

In the following questions, round your answers to 2 decimal places where necessary, unless otherwise specified.

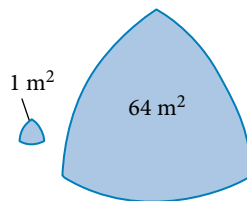
11

- 1 Calculate the scale factor k for the following similar shapes, given the original area a and the area of the image A .

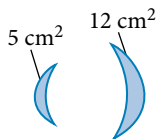
(a) $a = 6 \text{ cm}^2$, $A = 150 \text{ cm}^2$



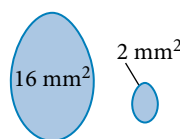
(b) $a = 1 \text{ m}^2$, $A = 64 \text{ m}^2$



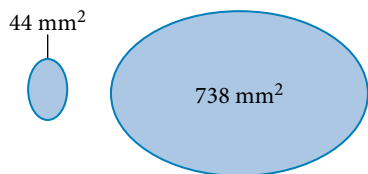
(c) $a = 5 \text{ cm}^2$, $A = 12 \text{ cm}^2$



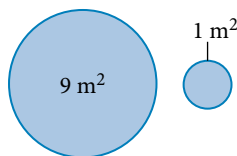
(d) $a = 16 \text{ mm}^2$, $A = 2 \text{ mm}^2$



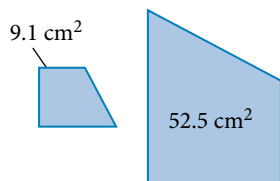
(e) $a = 44 \text{ mm}^2$, $A = 738 \text{ mm}^2$



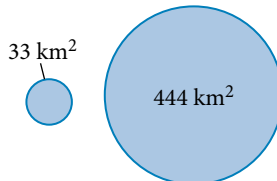
(f) $a = 9 \text{ m}^2$, $A = 1 \text{ m}^2$



(g) $a = 9.1 \text{ m}^2$, $A = 52.5 \text{ cm}^2$



(h) $a = 33 \text{ km}^2$, $A = 444 \text{ km}^2$

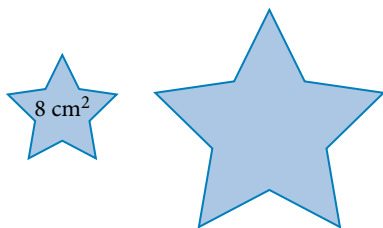


Worked
Example

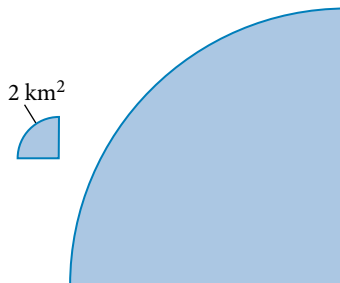
12

- 2 Calculate the area of the image A of a shape, given the original area a and the scale factor k in each case.

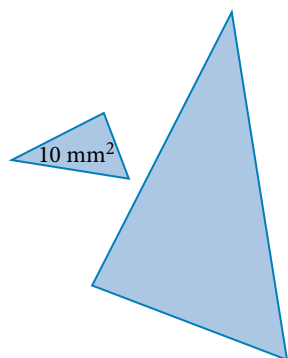
(a) $a = 8 \text{ cm}^2$, $k = 2$



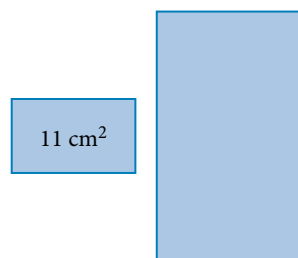
(b) $a = 11 \text{ cm}^2$, $k = 7$



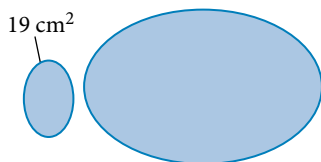
(c) $a = 10 \text{ mm}^2$, $k = 3$



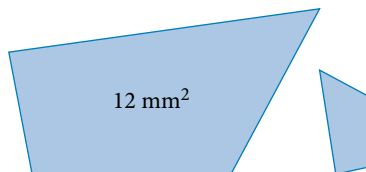
(d) $a = 11 \text{ cm}^2$, $k = 25.4$



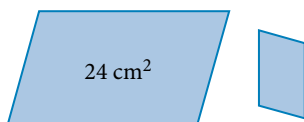
(e) $a = 19 \text{ cm}^2$, $k = 60$



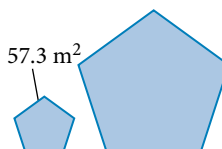
(f) $a = 12 \text{ mm}^2$, $k = \frac{1}{10}$



(g) $a = 24 \text{ cm}^2$, $k = 0.4$

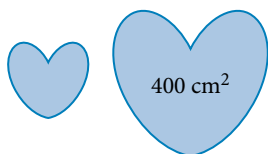


(h) $a = 57.3 \text{ m}^2$, $k = 8.33$

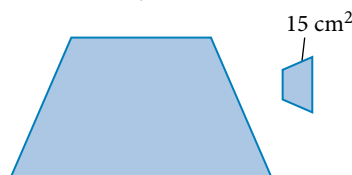


3 Calculate the original area a of a shape, given the image area A and the scale factor k in each case.

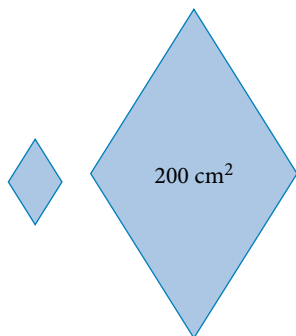
(a) $A = 400 \text{ cm}^2$, $k = 2$



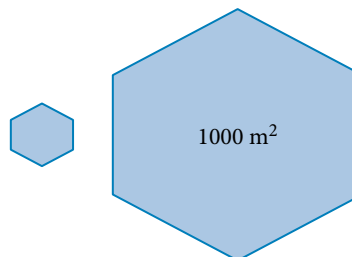
(b) $A = 15 \text{ cm}^2$, $k = 0.2$



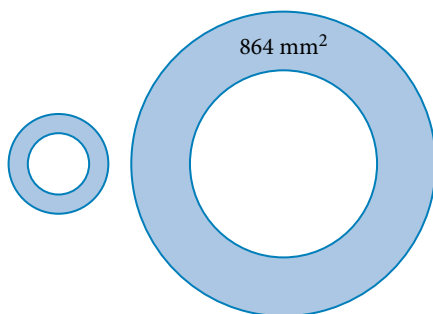
(c) $A = 200 \text{ cm}^2$, $k = 5$



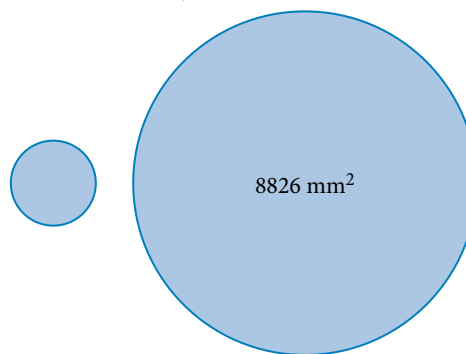
(d) $A = 1000 \text{ m}^2$, $k = 10$



(e) $A = 864 \text{ mm}^2$, $k = 12$



(f) $A = 8826 \text{ mm}^2$, $k = 7.4$



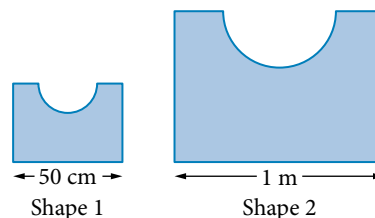
4 Calculate the ratio $\frac{\text{Area of shape 1}}{\text{Area of shape 2}}$ for the shapes shown.

A 0.25

B 0.5

C 2

D 4



5 An ellipse is scaled up by a scale factor of 2.5 to form a larger, similar ellipse of area 500 mm^2 .



(a) Select the option that gives the area of the smaller ellipse.

A 31.25 cm^2

B 80 mm^2

C 125 mm^2

D 200 mm^2

(b) Explain the common error made by a student who selected the last of the incorrect options.

6 A teacher projects an image of her notes onto a screen at the front of a classroom.

If the page of notes has an area of 600 cm^2 , and the projected image has an area of $15\,000 \text{ cm}^2$, what is the scale factor by which the original notes have been enlarged?

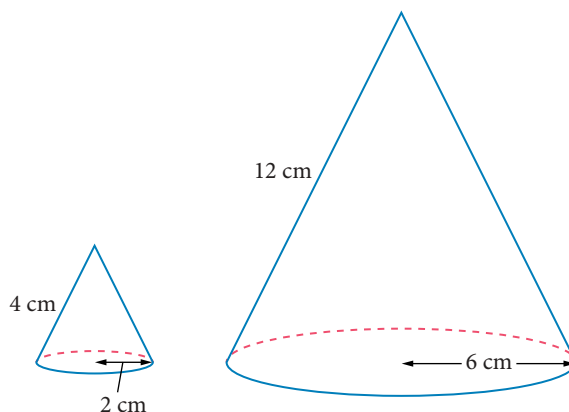
7 A stamp collector's brochure contains an image of a stamp that takes up 30 cm^2 of space on the page.

The actual stamp has an area of 6 cm^2 . By what scale factor has the image of the stamp been scaled up?

15

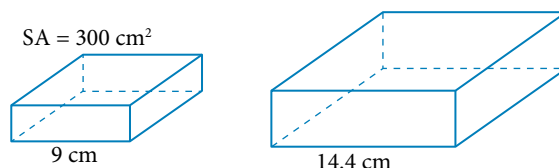
8 Two similar solid cones are shown.

- Calculate the scale factor k .
- Calculate the surface area (SA) of the small cone.
- Calculate the surface area (SA) of the large cone.
- Calculate $\frac{\text{SA (large cone)}}{\text{SA (small cone)}}$.
- Comparing your answers for (a) and (c), write a statement about scale factor and surface area for similar shapes.

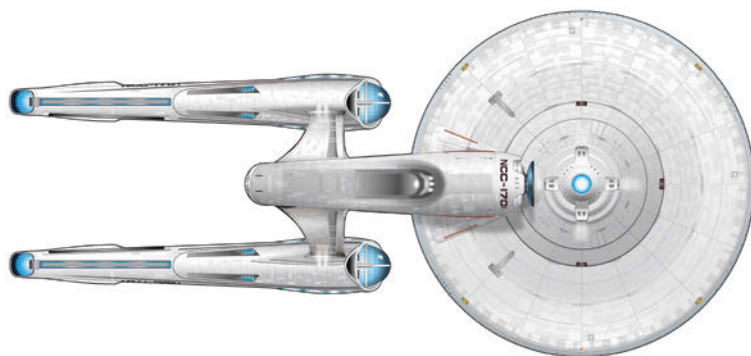


9 A pair of similar rectangular prisms are shown.

- Calculate the scale factor k .
- Use k to determine the surface area (SA) of the larger solid.



10 A model of the Starship *Enterprise* is 18 cm long.



The surface area of the model disc-shaped saucer section is 543 cm^2 . If the real *Enterprise* were 300 m long, calculate the surface area of its saucer section, to the nearest square metre.

..

.. .. .

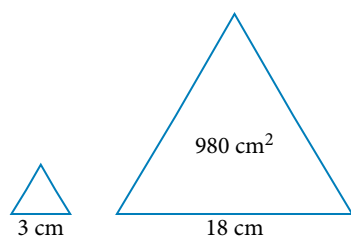
11 For each pair of similar shapes, calculate the scale factor k and the area of the image on the right.

13

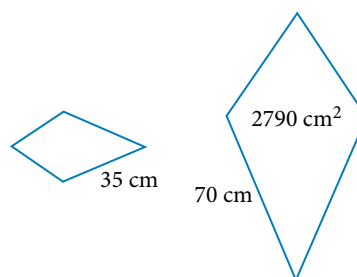
-
-
-
-
-
-

- 12 For each pair of similar shapes, calculate the scale factor k and the area of the smaller shape. Assume the image is on the right.

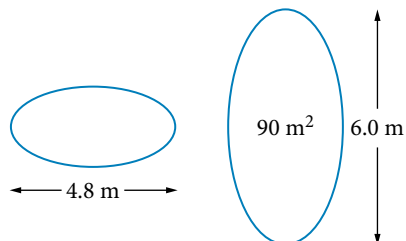
(a)



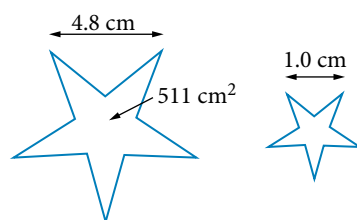
(b)



(c)



(d)

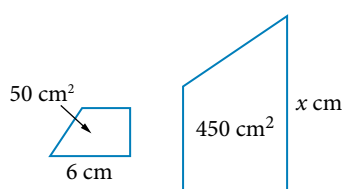


Worked Example

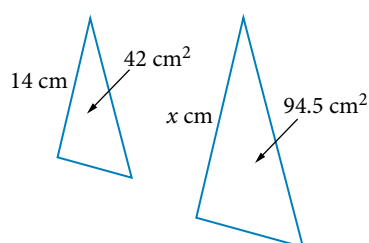
14

- 13 For each pair of similar shapes, calculate the scale factor k and the length x in the larger shape. Assume the image is on the right.

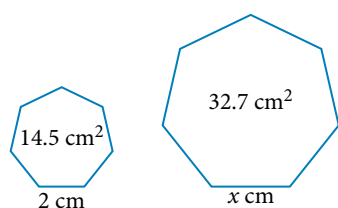
(a)



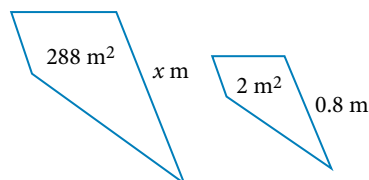
(b)



(c)

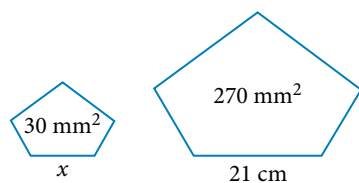


(d)

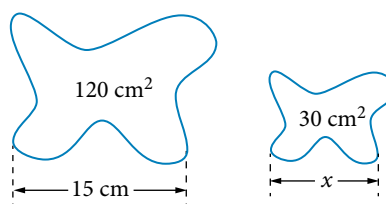


- 14 For each pair of similar shapes, calculate the scale factor k and the length x in the smaller shape. Assume the image is on the right.

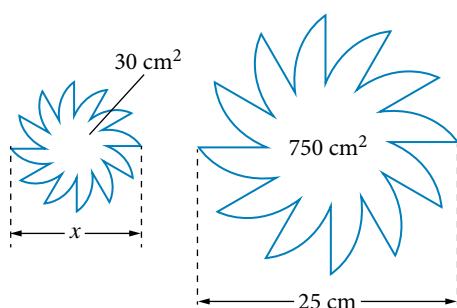
(a)



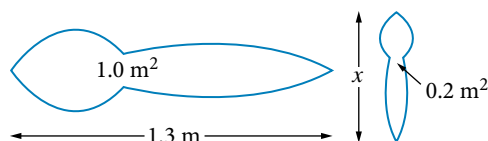
(b)



(c)



(d)



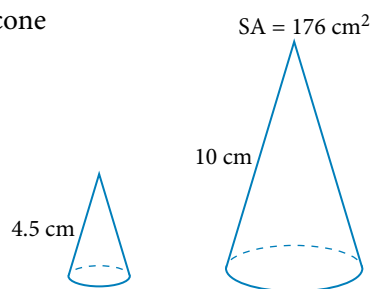
- 15 An entomologist believes that adult butterflies of a particular species are all similar in shape. She makes the measurements shown for two specimens.

The entomologist estimates the area of the smaller specimen's wings to be 670 mm^2 .

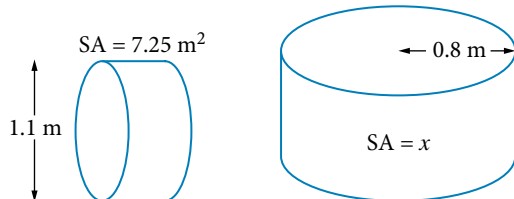
- (a) Determine the scale factor relating the specimens, assuming the image is on the right. Give your answer to 3 decimal places.
- (b) Estimate, to the nearest square millimetre, the area of the larger specimen's wings.



- 16 Calculate the surface area (SA) of the smaller cone to the nearest square centimetre.

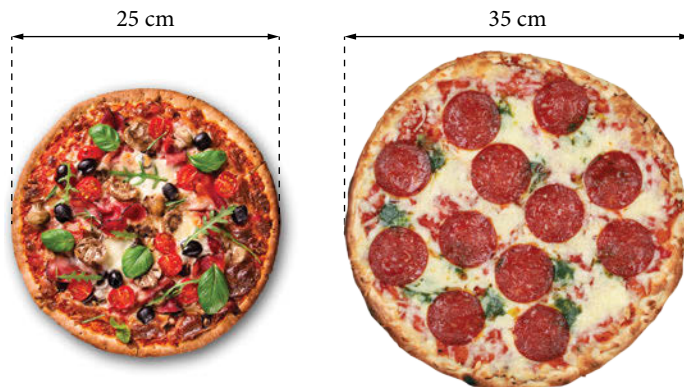


- 17 Calculate the value of x .



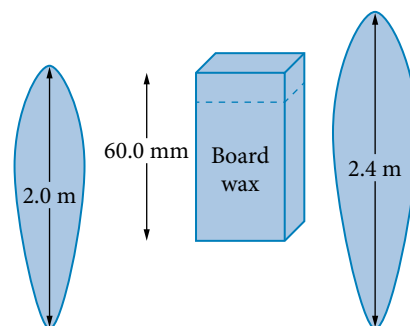
- 18 A disaster movie involves a city being destroyed by aliens. The *real* city is represented by a *model* city constructed over a base area of 3.5 m^2 . If the real city is 500 times longer than the model, determine the area covered by the real city. Answer in hectares.
- 19 Angelina makes statuettes of superheroes, her latest being a 15 cm tall statue of Batman. If the real Batman is 190 cm tall, and the surface area of his cape is 3.8 m^2 , determine the area of material there will be in the scaled-down version of the cape that Angelina attaches to her model Batman. Answer in square centimetres to the nearest whole number.
- 20 A plan of a new regional housing estate shows a block of land of area 16 cm^2 on the map. If the scale of the plan is 1 : 1250, determine the actual area of the block of land in square metres.

- 21 Dirk is a pizza lover and compares the value of two different-sized pizzas: a medium and a family size. The medium pizza costs \$15.00 and the family size pizza costs \$24.00. Their diameters are shown in the diagram.



- (a) Calculate the scale factor for the two pizzas, assuming the image is on the right. Explain any assumptions you make.
- (b) Calculate the scale factor for the price.
- (c) Use your answers to (a) and (b) to determine which pizza is the better value for money. Explain any assumptions you made.
- 22 A family has a photographic portrait taken and receives a large version of area 650 cm^2 and a smaller copy measuring 12 cm by 8 cm. Calculate the dimensions of the larger portrait, to 1 decimal place.

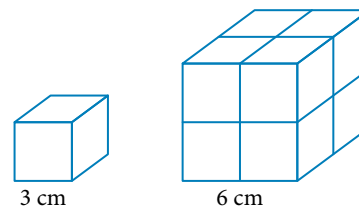
- 23 Sarah hopes that she has bought enough wax to wax both her surfboards now, before summer, and again in a few months at the end of the surf season.
- She has one board of length 2.0 m and a larger one 2.4 m long. When she waxes the smaller board she uses 12 mm of the 60 mm wax block. Determine whether or not she will have enough wax to wax the boards twice.



Similar solids and volume

Scale factor

The scale factor k describes the ratio of corresponding lengths in two similar objects, and the ratio of areas is the square of the scale factor k^2 . It follows that the ratio of volumes of similar solids is given by the cube of the scale factor k^3 . For example, consider two cubes:



The scale factor is $k = \frac{L}{l} = \frac{6}{3} = 2$

Now consider the ratio of the *volumes* of the two solids.

Let V be the volume of the larger solid, and v be the volume of the smaller solid.

Ratio of volumes: $\frac{V}{v} = \frac{6^3}{3^3} = 8$ or 2^3

Scale factor and volume

For any similar solids, if V and v are the volumes of the image and the original respectively, then these volumes and k are related in the following ways:

$k^3 = \frac{V}{v}$, where v and V have the same units.

$V = k^3 v$ means that the volume of the image may be found by multiplying the volume of the original solid by k^3 .

$v = \frac{V}{k^3}$ means that the volume of the original solid may be found by dividing the volume of the image by k^3 .

$$k^3 = \frac{\text{volume of the image } (V)}{\text{volume of the original } (v)}$$

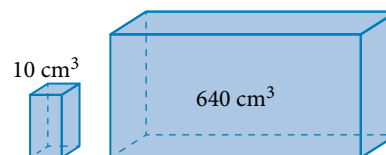
i Additional information

Enlarging solids

Explore how the volume of a solid changes when it is enlarged.

16 Determining the scale factor using volumes of similar solids

Determine the scale factor k , given the original volume $v = 10 \text{ cm}^3$ and volume of the image $V = 640 \text{ cm}^3$ for similar solids.



THINKING

- 1 Recall the formula relating volume and scale factor.
- 2 Substitute the given values and solve for k .
- 3 Interpret the result.

WORKING

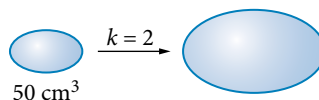
$$k^3 = \frac{\text{volume of the image}}{\text{volume of the original}}$$

$$\begin{aligned} k^3 &= \frac{640}{10} \\ &= 64 \\ k &= \sqrt[3]{64} \\ &= 4 \end{aligned}$$

This means that lengths in the original solid have been increased by a factor of 4.

17 Calculating the volume of a solid using the volume of an original solid and the scale factor

Two similar solids have the scale factor $k = 2$.
Calculate the volume of the image V , given the original volume $v = 50 \text{ cm}^3$.



THINKING

- 1 Recall the formula relating volume and scale factor k .
- 2 Substitute the given values for k and v into the formula and calculate V .
- 3 Interpret the answer.

WORKING

$$V = k^3 v$$

$$\begin{aligned} V &= 2^3 \times 50 \text{ cm}^3 \\ &= 400 \text{ cm}^3 \end{aligned}$$

The volume of the image is 400 cm^3 .

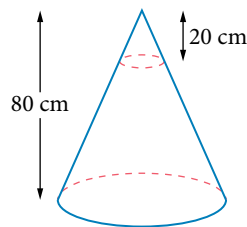
Embedded objects

Embedded objects are often similar because of shared angles and parallel faces.

18 Volume ratio of embedded objects

The volume of the cone shown here is 5000 cm^3 .
The top 20 cm of the cone is cut off to give a smaller, similar cone.

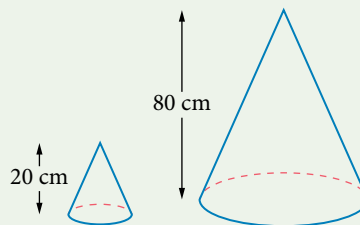
Determine the volume of the cut-off top.



THINKING

- 1 Construct a diagram to show the original solid and its cut-off top as two different solids.

WORKING



- 2 Recall the scale factor formula.

Substitute the two given lengths to calculate the scale factor.

With the image to the right:

$$\begin{aligned} k &= \frac{L}{l} \\ &= \frac{80}{20} \\ &= 4 \end{aligned}$$

- 3 Determine the volume of the original solid v by dividing the volume of the image V by k^3 .

$$\begin{aligned} v &= \frac{V}{k^3} \\ &= \frac{5000}{4^3} \\ &= 78.125 \text{ cm}^3 \end{aligned}$$

- 4 Interpret the answer.

The volume of the cut-off top is 78.125 cm^3 .

i Additional information

Similar spheres, cylinders and cones

Practise determining similarity of spheres, cylinders and cones.

EXERCISE

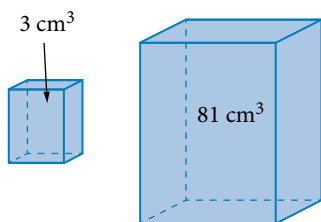
3.4

Similar solids and volume

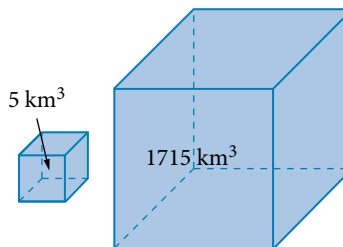
In the following questions round answers to 2 decimal places where necessary, unless otherwise specified.

- 1 Determine the scale factor k , given the original volume v and the image volume V of similar solids.

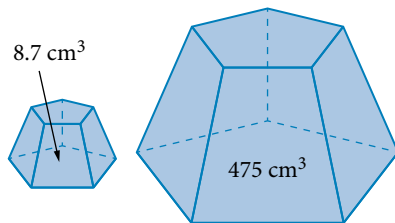
(a) $v = 3 \text{ cm}^3$, $V = 81 \text{ cm}^3$



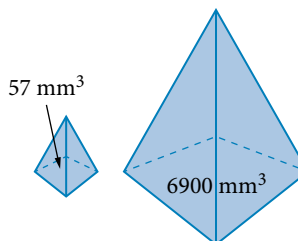
(b) $v = 5 \text{ km}^3$, $V = 1715 \text{ km}^3$



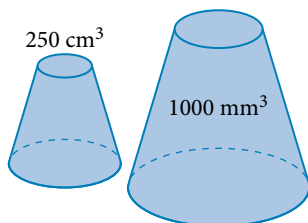
(c) $v = 8.7 \text{ cm}^3$, $V = 475 \text{ cm}^3$



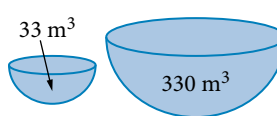
(d) $v = 57 \text{ mm}^3$, $V = 6900 \text{ mm}^3$



(e) $v = 250 \text{ cm}^3$, $V = 1000 \text{ cm}^3$



(f) $v = 33 \text{ m}^3$, $V = 330 \text{ m}^3$



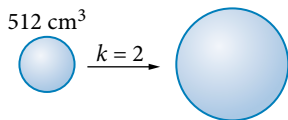
Worked
Example

16

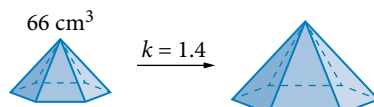
17

- 2 Calculate the volume V of a solid, given the volume v of a smaller similar solid, and the scale factor k in each case.

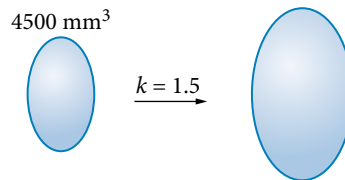
(a) $v = 512 \text{ cm}^3, k = 2$



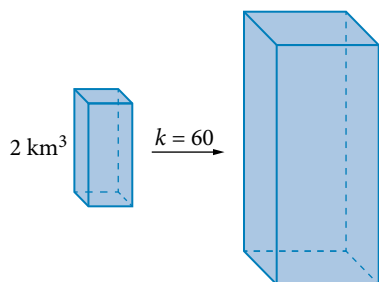
(b) $v = 66 \text{ cm}^3, k = 1.4$



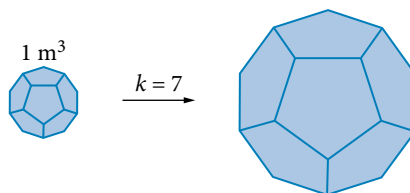
(c) $v = 4500 \text{ mm}^3, k = 1.5$



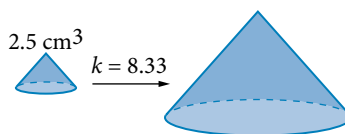
(d) $v = 2 \text{ km}^3, k = 60$



(e) $v = 1 \text{ m}^3, k = 7$

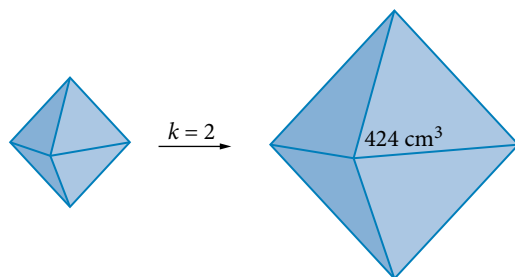


(f) $v = 2.5 \text{ m}^3, k = 8.33$

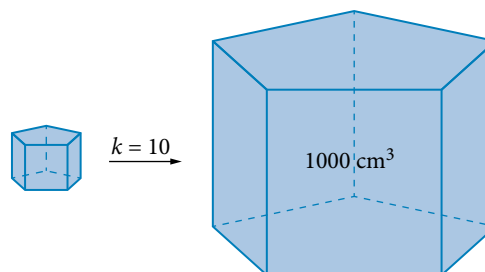


- 3 Calculate the volume v of a solid, given the volume V of an image solid and the scale factor k in each case.

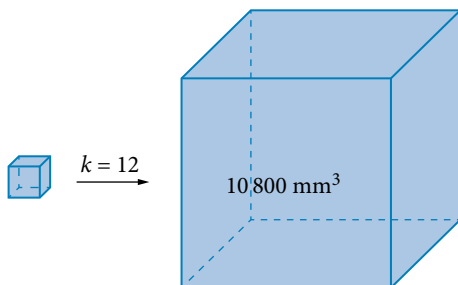
(a) $V = 424 \text{ cm}^3, k = 2$



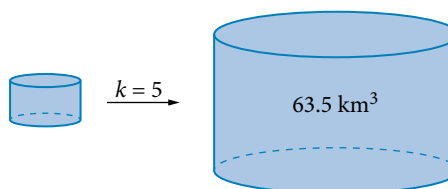
(b) $V = 1000 \text{ cm}^3, k = 10$



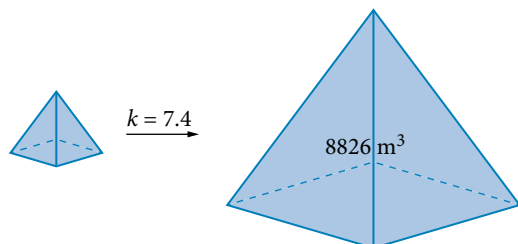
(c) $V = 10\,800 \text{ mm}^3, k = 12$



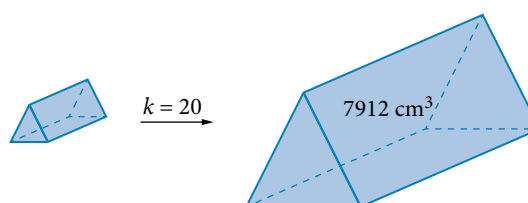
(d) $V = 63.5 \text{ km}^3, k = 5$



(e) $V = 8826 \text{ m}^3, k = 7.4$

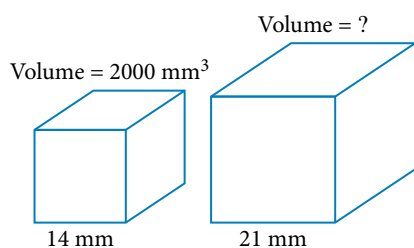


(f) $V = 7912 \text{ cm}^3, k = 20$

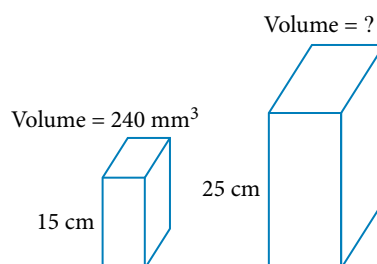


- 4 The solids shown are similar. In each case determine the scale factor k and the unknown volume. Let the solid on the right be the image. Round each volume to the nearest whole number if necessary.

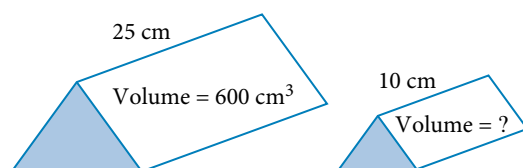
(a)



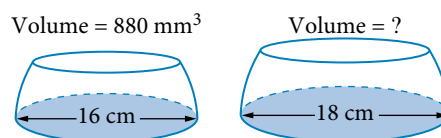
(b)



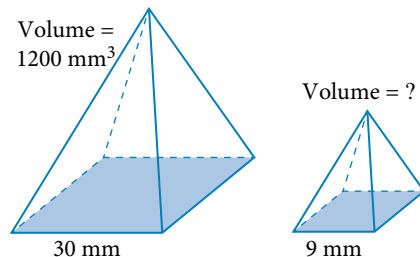
(c)



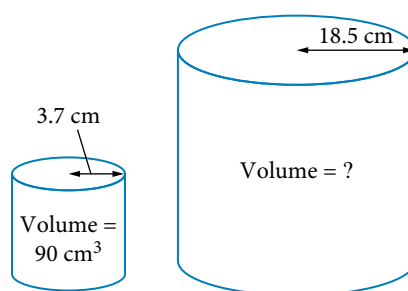
(d)



(e)

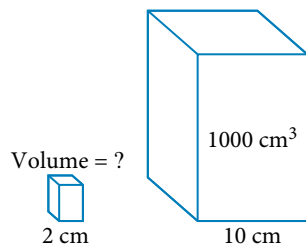


(f)

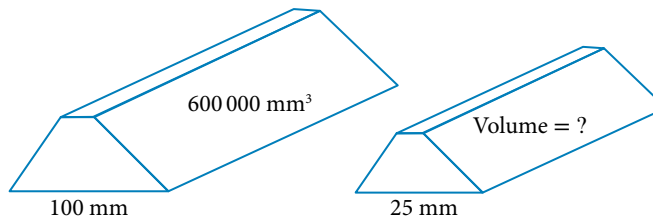


- 5 Determine the scale factor k and the volume of the smaller solid v for the following pairs of similar solids. Let the solid on the right be the image.

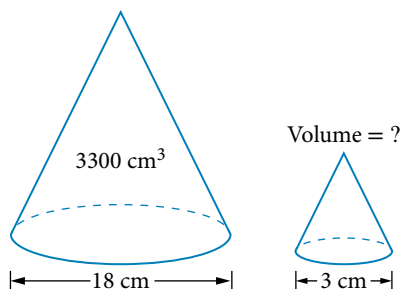
(a)



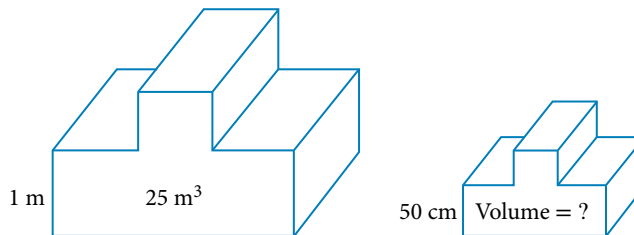
(b)



(c)

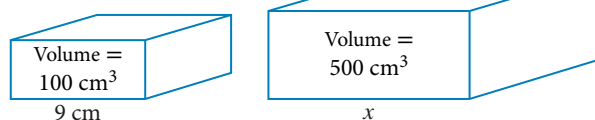


(d)

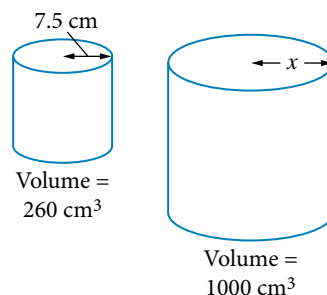
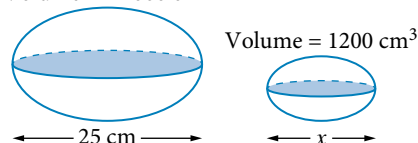


- 6 In each of the following pairs, the objects are similar. Determine the value of x in each case.

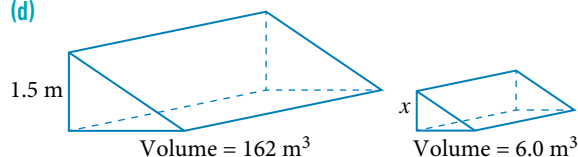
(a)



(b)

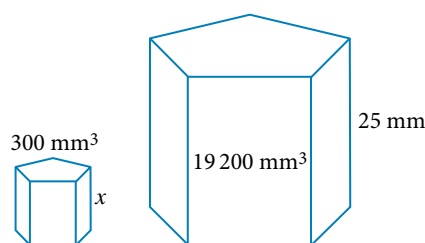
(c) Volume = 12 000 cm³

(d)

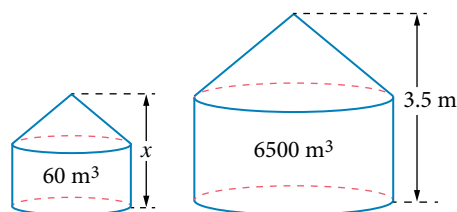


- 7 In each of the following pairs, the solids are similar. In each case, determine the scale factor k and the length x of the smaller solid. Let the image be the solid on the right.

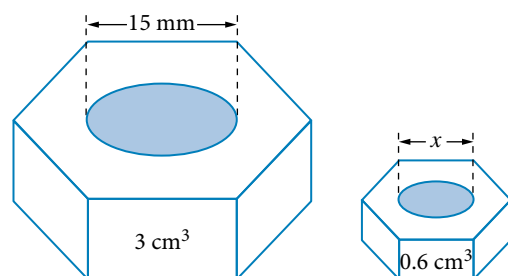
(a)



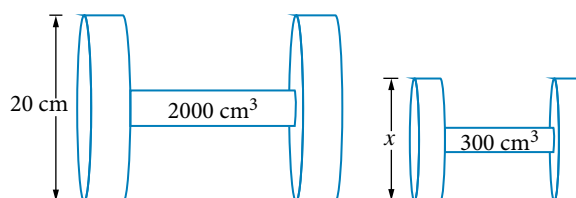
(b)



(c)



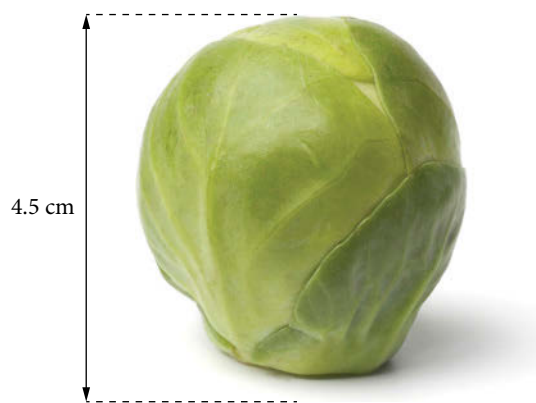
(d)



- 8 Automotive engineers are designing a new car. The length of the car will be 4.2 m. The engineers use a 90 cm long scale model to simulate the air-conditioning system of the proposed new car. They determine (by submerging the model under water with and without its windows closed) that the interior of the model car holds approximately 0.01 m^3 of air.

- Calculate the scale factor, assuming $k > 1$.
 - Determine the ratio of the volumes.
 - How much air will the real car's air-conditioner need to keep cool?
- 9 A chocolate maker decides to make a larger, similar-shaped version of its famous triangular prism-shaped product. If the original bar contains 320 cm^3 of chocolate, and the larger product contains 10 times the amount of chocolate, how many times longer is the larger product?

- 10 Kris is an entrepreneur and healthy eater who has an idea to create a tourist attraction involving a giant Brussels sprout. She measures the height of a typical Brussels sprout at 4.5 cm, and calculates its volume to be 52 cm^3 . Her planned giant version is to be 3.6 m tall.



- (a) What will the volume of the giant sprout be in cubic metres?
 (b) How many times greater will the surface area of the giant sprout be than a normal-sized sprout?

- 11 The ratio between the surface areas of two solids is 3.2. Select the value that is closest to the ratio between the volumes.

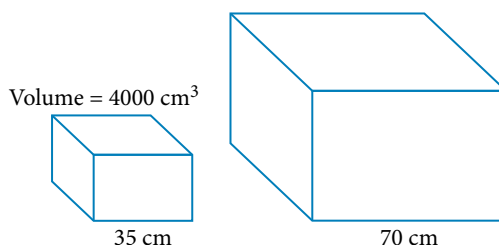
A 1.34 B 1.79 C 4.096 D 5.72

- 12 Two similar rectangular prisms are shown.

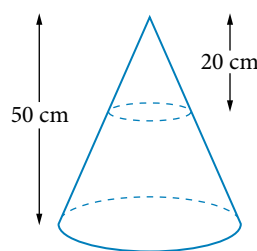
- (a) Calculate the difference between the volumes of the prisms.

A 4000 cm^3 B 8000 cm^3
 C 12000 cm^3 D 28000 cm^3

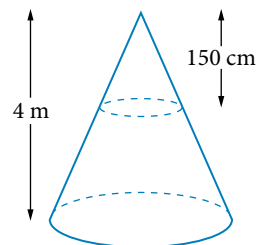
- (b) Explain the common error made by a student who chose the first of the incorrect options.



- 13 The top of a cone of height 50 cm and volume 1000 cm^3 is to be cut off 20 cm from the top, parallel to its base. Determine the volume of the cut-off top.



- 14 A cone of height 4 m has its top 150 cm cut off. If the volume of the cut-off top is 0.5 m^3 , calculate the volume of the original cone.

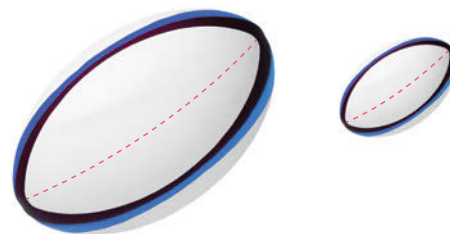


- 15 The bottom 2 cm of a solid cone-shaped ice-cream (at the pointy end) contains 1 cm^3 of chocolate. The rest of the cone contains ice-cream. If the cone is 12 cm long, what volume of ice-cream does it contain? Explain any assumptions you make.
- 16 Palaeontologists theorise that a newly discovered dinosaur, when living, was similar in shape to a modern-day reptile of length 34 cm and volume 750 cm^3 . Fossil remains indicate that the newly discovered specimen was approximately 2.6 m long. If each cubic metre of dinosaur weighed on average 1000 kg, estimate the mass of the dinosaur to the nearest kilogram.

Worked
Example

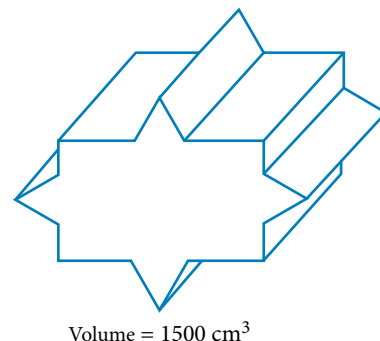
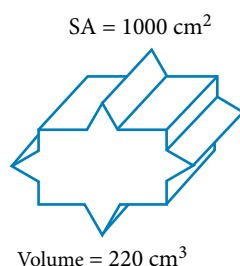
18

- 17 A football manufacturer's standard ball is made of leather of area $11\,300\text{ cm}^2$ and occupies a volume of $36\,800\text{ cm}^3$. A smaller version of volume $15\,525\text{ cm}^3$ is planned. What area of leather will the smaller version have? Answer to the nearest square centimetre.



- 18 The volumes of two similar solids and the surface area of the smaller solid are given.

Determine the surface area of the larger solid to the nearest hundred square centimetres.



- 19 A cylinder's height and diameter are the same. A scale model of the cylinder is made using a scale factor of 2.3. Given that the original volume was 500 cm^3 , determine the volume and height of the scale model.

..

... ..

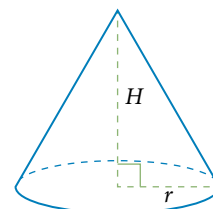
- 20 Ruby builds a LEGO® model using 350 identical LEGO® bricks. She is later invited to build a larger, similar-shaped model, using the same-sized bricks, for an exhibition. If the length of the original model must be scaled up by a factor of 20, how many LEGO® bricks are required to build the larger version? State any assumptions you made.

- 21 Hugh has had 2 m^3 of garden mulch delivered to his driveway. The height of the pile is 1.4 m. He decides that he has drastically underestimated how much he needs, and orders another 3 m^3 .



- (a) If the second load of mulch forms a pile of similar shape to the first one, how much higher will it be? Answer to the nearest centimetre.
- (b) Hugh is overwhelmed by the area of his driveway that has been taken up by the mulch. If the second load was dumped on top of the first, by what percentage would this reduce the area of his driveway that is covered? Answer to the nearest whole per cent. *Hint:* Assume that the original pile covers an area of 1 m^2 .
- (c) Hugh's house occupies an area of 243 m^2 of his 750 m^2 block. He is going to apply the mulch to a depth of 6 cm. Determine the percentage, to the nearest per cent, of his yard that will be covered in mulch.
- 22 Jarli compares two different-sized cylindrical tubs of popcorn that are similar. One has a base diameter 10 cm and costs \$5, the other has a base diameter 13 cm and costs \$9. He determines that the larger tub contains 20 handfuls of popcorn.
- Which tub is better value? Show calculations to justify your choice.

- 23 What percentage of the volume of a cone is contained in the bottom 50%? In other words, what percentage is left if a cone of height $\frac{H}{2}$ is cut off the top?



Summary

Similar shapes and length

Similar shapes have exactly the same shape but are not necessarily the same size.

Two triangles are similar if they satisfy any of the following conditions.

- The lengths of their corresponding sides are in the same ratio (SSS).
- At least two of their angles are congruent (AA).
- The lengths of two corresponding sides are in the same ratio and the angles between these two sides are congruent (SAS).

If L = length in an image, l = corresponding length in an original of that shape, and k = scale factor,

$$\text{then: } k = \frac{L}{l}$$

Scale drawing

Scale is represented by the ratio of the distance on a plan to the real distance, or $1 : k$ where k is the scale factor, or $k : 1$ for an enlargement.

For maps and plans:

$$\text{Actual distance} = k \times \text{plan distance}$$

$$\text{Plan distance} = \text{actual distance} \div k$$

Similar shapes and area

If A = area of image shape, a = area of the original, and k = scale factor, then: $k^2 = \frac{A}{a}$

Similar solids and surface area use the same formulas as those for similar shapes and area.

Similar solids and volume

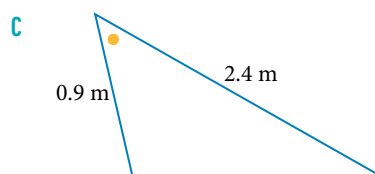
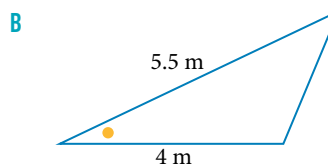
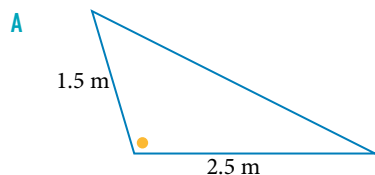
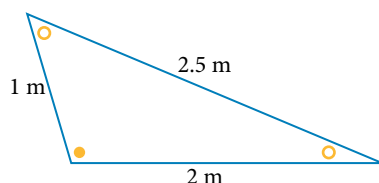
If V = volume of image solid, v = volume of the original, and k = scale factor, then: $k^3 = \frac{V}{v}$

Chapter review

3

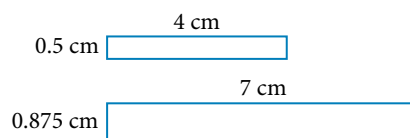
Exercise 3.1

- 1 Select the triangle that is similar to this one.



Exercise 3.1

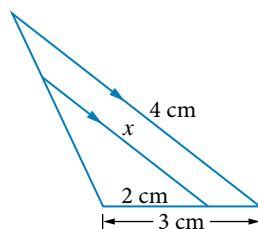
- 2 The following rectangles are similar. Select the option that gives the scale factor where the lower rectangle is the image.



- A** 0.572 **B** 1.75 **C** 2 **D** 3.063

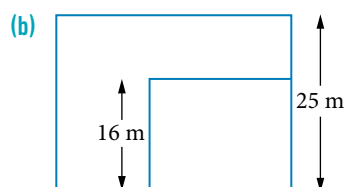
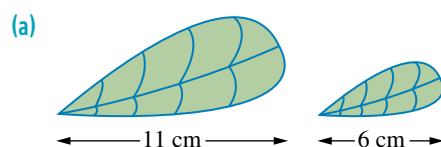
Exercise 3.1

- 3 Calculate the value of x , to 2 decimal places.



Exercise 3.1

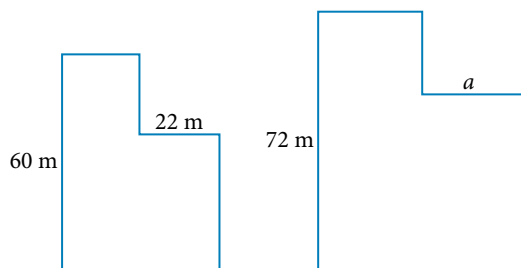
- 4 For each pair of similar figures, calculate the scale factor k required to increase the smaller shape to the size of the larger shape. Make your answer accurate to 3 decimal places.



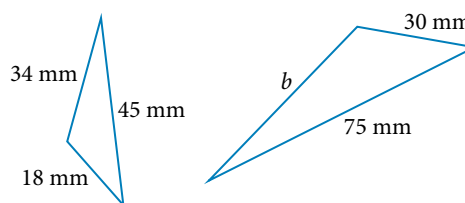
- 5 For each pair of similar shapes, calculate the value of the pronumeral to 1 decimal place.

Exercise 3.1

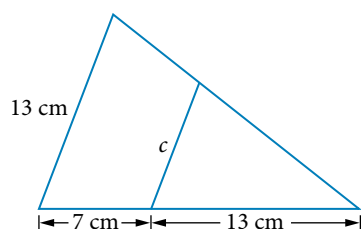
(a)



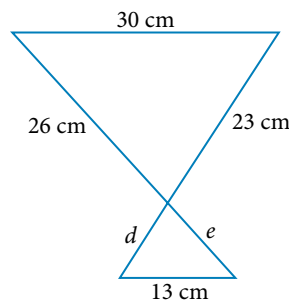
(b)



(c)



(d)



- 6 A groundskeeper is making a plan of the school athletics track, and uses a scale of 1 : 400. If the length of the track is 150 m, select the answer that shows the length she should draw the track.

Exercise 3.2

A 3.75 cm B 5.5 cm C 6 cm D 37.5 cm

- 7 A map has a scale in which 2 cm represents 1 km. Determine the scale of the map. Write your answer in the form 1 : k .

Exercise 3.2

- 8 Select the option that shows the actual distance represented by 5 cm on a map with a scale of 1 : 8000.

Exercise 3.2

A 4 m B 40 m C 400 m D 4000 m

- 9 A tourist brochure shows two points of interest separated by 16 cm on a map. If the map uses a scale of 1 : 650, how far apart are the points of interest in reality? Give your answer in metres.

Exercise 3.2

- 10 A rectangle is drawn to represent a block of land measuring 42 m by 35 m, using a scale of 1 : 100. State the dimensions of the rectangle in the drawing.

Exercise 3.2

- 11 The shapes shown are similar, with a scale factor of 1.4. Select the option that is closest to the area of the larger shape.

Exercise 3.3

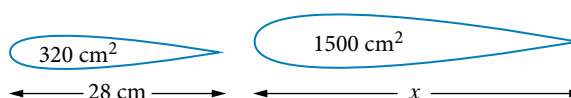


A 77 mm² B 107 mm² C 210 mm² D 294 mm²

- 12 An aircraft designer draws two similar wing cross-sections as shown.

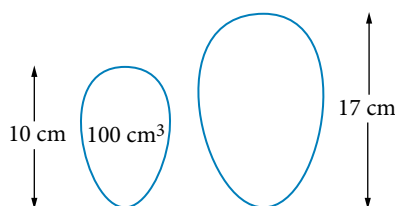
Exercise 3.3

Calculate the value of x , to the nearest centimetre.



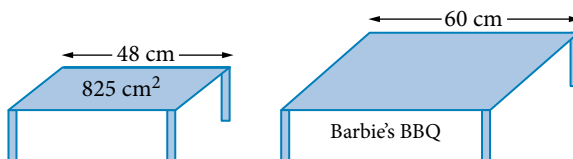
Exercise 3.3

- 13 Eggbert collects data relating to a hen's egg and an emu's egg of similar shape. Determine the volume of the emu's egg (the larger egg), to the nearest cubic centimetre.



Exercise 3.3

- 14 Barbie wants to copy a friend's barbecue design, but to increase the width of the hotplate from 48 cm to 60 cm, while keeping the same proportions.



Calculate the area of the larger hotplate. Express your answer to the nearest square centimetre.

Exercise 3.3

- 15 A tropical cyclone has a circular diameter of 600 km and is expected to double in area over the next 24 h. Determine the estimated diameter of the cyclone after 24 h. Round your answer to the nearest 10 km.

Exercise 3.3

- 16 A polygon is enlarged so its area is increased by a factor of 6. By what scale factor has the polygon been enlarged? Express your answer to 2 decimal places.

Exercise 3.3

- 17 Zane has sewn himself an adult-sized surf life-saving cap using 340 cm^2 of lycra material, and wants to make a child-sized one for his daughter. He determines that the circumference of his daughter's head is $\frac{3}{4}$ that of his own.

Adult size (340 cm^2)

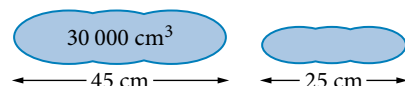
Child size



- (a) If Zane's head circumference is 60 cm, what is his daughter's head circumference?
- (b) What area of lycra does Zane need, to make his daughter's cap? Give your answer to the nearest square centimetre.

Exercise 3.4

- 18 Brutus has made a cushion of width 45 cm, and wants to make another one in the same proportions of width 25 cm. If his original cushion required $30\,000 \text{ cm}^3$ of stuffing, how much stuffing will he need to make the new cushion? Round your answer to the nearest 10 cm^3 .

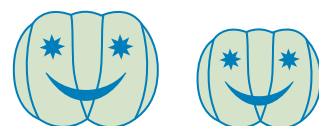


Exercise 3.4

- 19 Two similar water tanks hold 40 000 L and 100 000 L. If the height of the smaller tank is 1.8 m, how tall is the larger tank (to 2 decimal places)?

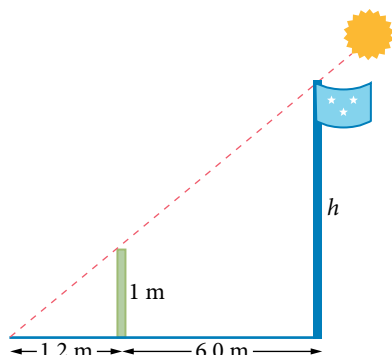
Exercise 3.4

- 20 Two friends have hollowed out similar pumpkins for Halloween. If the pulp removed from the larger pumpkin was 3.7 times the amount removed from the smaller one, calculate the ratio (to 2 decimal places): $\frac{\text{diameter of large pumpkin}}{\text{diameter of small pumpkin}}$.



- 21 A flagpole casts a shadow of length 7.2 m. A 1 m ruler is placed 6 m from the flagpole and casts a shadow of length 1.2 m as shown. Determine the height h of the flagpole.

Exercise 3.1



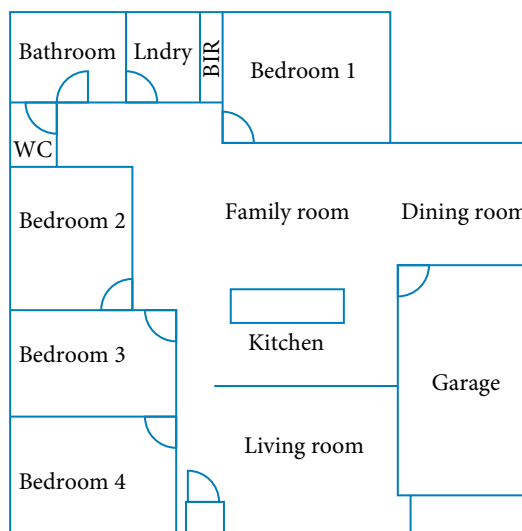
- 22 An area burnt by a small bushfire measures approximately 0.6 cm^2 on a map. If the scale of the map is 1 : 200 000, determine how many hectares of bushland were affected.
- 23 Plans for a new hotel are drawn to a scale of 1 : 50. If the base of the hotel measures 60 m in length and 40 m in width, determine the area of paper taken up by the base of the hotel on the plan. Answer in square metres.
- 24 The house plan shown in the diagram uses a scale of 1 : 250.

Exercise 3.2

Exercise 3.2, 3.3

Exercise 3.2, 3.3

- (a) Determine the dimensions of bedroom 1 ($15 \text{ mm} \times 20 \text{ mm}$ on the map).
- (b) Determine the area of the garage ($27 \text{ mm} \times 15 \text{ mm}$ on the map) to the nearest square metre.
- (c) The area of the entire house on the map is 3654 mm^2 . Determine the number of 'house squares' the house occupies, given that one 'house square' is equivalent to 9.29 m^2 (or 100 square feet in imperial units). Give your answer to the nearest whole number.

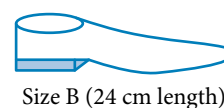
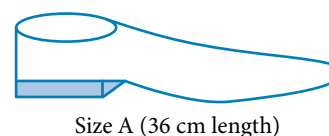


- 25 Tania owns a shoe store and also designs shoes. She has made a prototype of a shoe, in what she calls size A, which is 36 cm long and is made from 950 cm^2 of fake crocodile leather, with a 500 m^3 rubber heel.

Exercise 3.3, 3.4

Tania has available 10 m^2 of fake crocodile leather and 10000 cm^3 of heel rubber.

- (a) How many size A shoes could Tania produce?
- (b) How many size B shoes could she produce?



Exercise 3.4

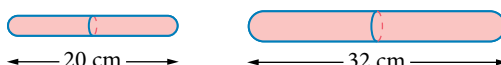
- 26 Chester produces small blocks of cheese of length 20 cm, width 15 cm and height 10 cm. His customers have requested a larger block that contains triple the volume of cheese. If the larger block is to have a similar shape, calculate its dimensions to the nearest centimetre.

Exercise 3.3

- 27 The length, width and height of a rectangular prism are in the ratio 1.2 : 1 : 1.4. A scale model of the prism has a volume of 2 m^3 , which is 50% more than the original volume. Determine the volume and dimensions of the original prism, to 2 decimal places.

Exercise 3.3, 3.4

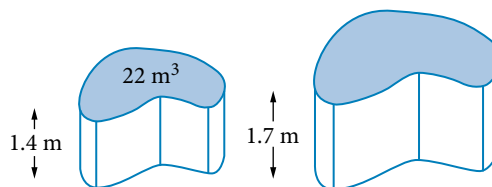
- 28 Sophia makes her own sausages. It costs her 0.12 cents per square centimetre of skin, and 0.2 cents per cubic centimetre for the meat filling, which she gets from the same supplier. She makes two sizes of sausage: standard (20 cm) and jumbo (32 cm). The sausages are similar in shape.



- (a) The standard sausage requires 120 cm^2 of skin and 115 cm^3 of filling, and sells for \$1.00. What price should jumbo sausages be sold at, to make the same percentage profit?
- (b) Sophia typically sells 1000 standard sausages and 750 jumbo sausages per week. What is her weekly profit, to the nearest \$100? Assume the standard sausage and the jumbo sausage are sold at the prices given in (a). Use profit amounts rounded to the nearest cent per sausage.

Exercise 3.3, 3.4

- 29 Chen must decide between two sizes of backyard wading pools. The two pools are similar in shape. The shop assistant says that the smaller, 1.4 m deep model holds 22 m^3 of water. The larger model is 1.7 m deep, but the assistant is unsure how much water it holds.

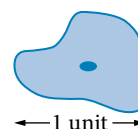


- (a) Chen is able to fill a pool at the rate of 40 L/min. How long would it take to fill the large pool? Give your answer in hours and minutes, to the nearest minute.
- (b) An insulating cover may be custom made for any sized pool, and costs \$4.80 per square metre of coverage. Calculate the cost of a cover for the larger pool. Explain any assumptions you made.

Exercise 3.3, 3.4

- 30 Living things obtain energy by taking in nutrients and releasing heat and waste. For a cell, the amount of inwards and outwards flow depends on the cell's surface area and volume. Consider a cell of length l units, surface area $SA = 3l^2$ and volume $V = 0.5l^3$.

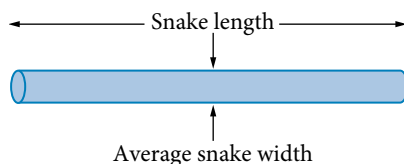
The value of $\frac{\text{cell volume}}{\text{cell surface area}}$ for a cell of length 1 unit would be:

$$\frac{0.5 \text{ cubic units}}{3 \text{ square units}} \approx 0.17 \text{ cubic units/square unit}$$


- (a) Calculate the value of $\frac{\text{cell volume}}{\text{cell surface area}}$ for each of the following scenarios.
- The cell doubles in length.
 - The cell triples in length (compared with its original size).
 - The cell's length increases to 10 times its original length.
- (b) Suppose a cell can only be supplied with enough energy if the value of $\frac{\text{cell volume}}{\text{cell surface area}}$ is no more than 5 cubic units / square unit. What would be the maximum length of such a cell?

- 31 A toy company is contracted to produce long rubber snakes that are similar in shape to a real taipan on display in a reptile park. The volume of each toy snake is 60 cm^3 . The park's herpetologist determines the length of the real snake to be 180 cm and its average width to be 7.4 cm.

Exercise 3.3, 3.4

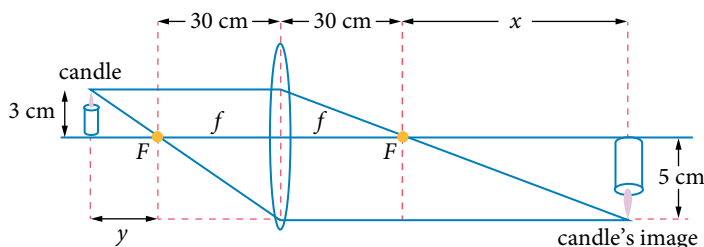


- A snake may be approximated geometrically to a cylinder of constant width, with a diameter equal to the average width of the snake. Use this approximation to estimate the volume of the real snake, to the nearest cubic centimetre.
- Determine the length of the toy snake, to the nearest centimetre.
- Estimate the surface area of the toy snake, to the nearest square centimetre.
- What length must the toy snake be in order to produce 150 snakes from a 20 cm cube of rubber, assuming no wastage? Round your answer down to the next whole centimetre.

- 32 Miranda is conducting a physics experiment. She shines rays of light through a lens and records the information shown in the diagram. The focal length f is the distance from each focal point F to the lens.

Exercise 3.1

If x is the distance of the candle's image from the focal point F on the right of the lens and y is the distance of the candle from F on the left of the lens, construct an equation connecting the product of x and y with the value of f . Assume all pronumerals represent lengths in the same units.



- 33 Marid is planning to build a gabion wall, which is a wire-framed rectangular prism filled with rocks that are all about the same size. His original plans call for a wire prism of particular dimensions, but he decides the wall should be 50% longer, 25% wider and 10% higher. Determine the factor he should use to calculate the number of rocks needed, from the number in the original plan. Interpret your answer.

Exercise 3.4





4

Linear equations and their graphs

.....

Recall	210
4.1 Linear equations	211
4.2 Linear graphs	220
4.3 Sketching linear graphs	231
4.4 Simultaneous linear equations	246
4.5 Piece-wise linear graphs and step graphs	259
Summary	268
Chapter review	270

Recall

Substitute values into a formula

1 If $x = 4$ and $y = -2$, calculate the value of the following expressions.

(a) $5x - 7$

(b) $4(x - y)$

(c) $x^2 + 3xy$

Use a formula to complete a table of values

2 Complete a table of values of x and y for each equation.

(a) $y = 5x - 8$ where $x = -2, x = 0, x = 1, x = 4, x = 10$.

(b) $y = \frac{1}{2}x + 3$ where $x = -2, x = 0, x = 2, x = 4, x = 6$.

(c) $y = 1 - 3x$ where $x = -10, x = -5, x = 0, x = 5, x = 10$.

Identify like terms

3 Determine whether the following pairs are like or not like terms.

(a) $4c$ and $-3c$

(b) $7xy^2$ and x^2y

Add and subtract like terms

4 Simplify the following expressions by collecting the like terms.

(a) $3x + 7y + 4x - 9$

(b) $xf^2 + 3xf^2 + 7x - 3f - 10$

Substituting for y

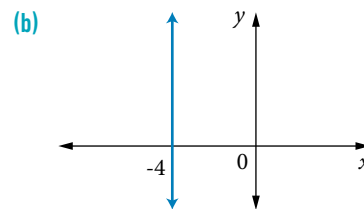
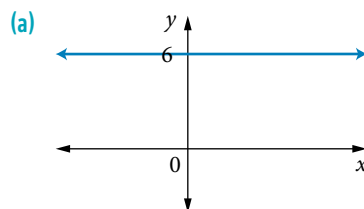
5 For the equation $y = 2x - 1$, determine the value of x where:

(a) $y = 0$

(b) $y = 1$

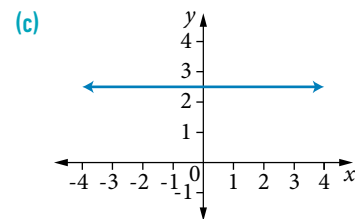
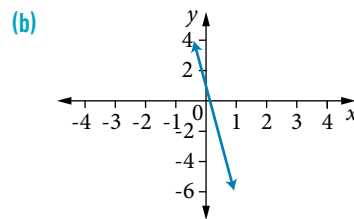
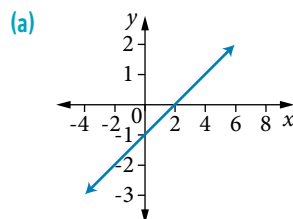
Horizontal and vertical graphs

6 Determine the equation for each graph.



Distinguish between a negative, zero and positive gradient

7 Determine whether the gradient of the linear function is negative, zero or positive.



Linear equations

Identifying linear equations

Linear equations contain an equals symbol (=) and one or more variables raised to the power of 1. For example, $2x + 5 = 0$, $3(x - 2) = 4$ and $a + b - 7 = -1$ are linear equations.

Non-linear equations include terms such as x^2 , y^3 , $\frac{1}{x}$, $\sin(\theta)$ and $\sqrt{x}$.

Linear expressions have no equals symbol. For example, $2x + 5$, $3(x - 2)$ and $a + b - 7$ are linear expressions.

1 Identifying linear equations

Classify each of the following as a linear equation, a non-linear equation or a linear expression.

$x^2 + 5 = 30$

$45 + 8k$

$\sqrt{y} + 5 = 30$

$3x - 7 = 14$

$5(a - 4) = 45$

$6m + 14 = 21 + 5m$

THINKING

- 1 Identify linear equations by the presence of an equals symbol and variables raised to the power of 1.
- 2 Identify non-linear equations by the presence of an equals symbol but the variables do not have powers of 1.
- 3 An expression has no equals symbol.

WORKING

Linear equations: $3x - 7 = 14$, $5(a - 4) = 45$, $6m + 14 = 21 + 5m$

Non-linear equations: $x^2 + 5 = 30$, $\sqrt{y} + 5 = 30$

Linear expression: $45 + 8k$

Solving linear equations

A linear equation is only true for one value of the variable.

For example, the linear equation $5(a - 4) = 45$ is only true when $a = 13$.

You can demonstrate that $a = 13$ is a solution by substituting into the left-hand expression:

$$\begin{aligned} 5(a - 4) &= 5 \times (13 - 4) \\ &= 5 \times 9 \\ &= 45 \end{aligned}$$

Finding the value that makes an equation true is called *solving the equation*.

One way of solving an equation is to identify the operations that have been done to create the equation, and then work backwards, reversing the operations, to determine the solution.

The reverse of an operation is known as an *inverse operation*.

i Additional information

Equations

Practise identifying whether an equation is linear or non-linear.

Operation	Inverse operation
+	−
−	+
×	÷
÷	×

To create the left-hand side of the equation $5(a - 4) = 45$:

$$\begin{array}{ccc} \boxed{a} & = & \boxed{?} \\ \downarrow - 4 & & \downarrow - 4 \\ \boxed{a - 4} & = & \boxed{?} \\ \downarrow \times 5 & & \downarrow \times 5 \\ \boxed{5(a - 4)} & = & \boxed{45} \end{array}$$

To solve the equation $5(a - 4) = 45$ perform the inverse operations in the opposite order:

$$\begin{array}{ccc} \boxed{5(a - 4)} & = & \boxed{45} \\ \downarrow \div 5 & & \downarrow \div 5 \\ \boxed{a - 4} & = & \boxed{9} \\ \downarrow + 4 & & \downarrow + 4 \\ \boxed{a} & = & \boxed{13} \end{array}$$

This should be checked by substituting the result $a = 13$ into the original equation: $5(13 - 4) = 45$

Alternatively, you can multiply out the brackets and solve: $5a - 20 = 45$

2 Solving a simple linear equation

Solve the following linear equations.

(a) $3a - 5 = 25$

THINKING

- 1 Solve the equation by using the inverse operations on both sides of the equals symbol.
- 2 As you gain confidence you can solve equations in a more streamlined manner, without showing the inverse operations at each step.
- 3 Check your answer by substituting the solution into the original equation.

WORKING

$$\begin{aligned} 3a - 5 &= 25 \\ 3a - 5 + 5 &= 25 + 5 \\ 3a &= 30 \\ \frac{3a}{3} &= \frac{30}{3} \\ a &= 10 \end{aligned}$$

$$\begin{aligned} 3a - 5 &= 25 \\ 3a &= 30 \\ a &= 10 \end{aligned}$$

Substitute $a = 10$ into $3a - 5$:

$$3 \times 10 - 5 = 25$$

(b) $6(b + 4) = 30$

- 1 Solve the equation by using the inverse operations on both sides of the equals symbol.
- 2 As you gain confidence you can solve equations in a more streamlined manner, without showing the inverse operations at each step.
- 3 Check your answer by substituting the solution into the original equation.

$$\begin{aligned} 6(b + 4) &= 30 \\ \frac{6(b + 4)}{6} &= \frac{30}{6} \\ b + 4 &= 5 \\ b + 4 - 4 &= 5 - 4 \\ b &= 1 \end{aligned}$$

$$\begin{aligned} 6(b + 4) &= 30 \\ b + 4 &= 5 \\ b &= 1 \end{aligned}$$

Substitute $b = 1$ into $6(b + 4)$:

$$6 \times (1 + 4) = 30$$

(c) $\frac{2c-8}{3} = -10$

- 1 Solve the equation by using the inverse operations on both sides of the equals symbol.

$$\begin{aligned}\frac{2c-8}{3} &= -10 \\ \frac{2c-8}{3} \times 3 &= -10 \times 3 \\ 2c-8 &= -30 \\ 2c-8+8 &= -30+8 \\ 2c &= -22 \\ \frac{2c}{2} &= \frac{-22}{2} \\ c &= -11\end{aligned}$$

- 2 As you gain confidence you can solve equations in a more streamlined manner, without showing the inverse operations at each step.

$$\begin{aligned}\frac{2c-8}{3} &= -10 \\ 2c-8 &= -30 \\ 2c &= -22 \\ c &= -11\end{aligned}$$

- 3 Check your answer by substituting the solution into the original equation.

$$\begin{aligned}\text{Substitute } c = -11 \text{ into } \frac{2c-8}{3}: \\ \frac{2 \times -11 - 8}{3} &= -10\end{aligned}$$

Collecting like terms before solving

Sometimes a variable appears in more than one place in an equation. The first step in solving these equations is to add or subtract like terms so that the variable only appears once.

3 Collecting like terms before solving

Solve each of the following linear equations.

(a) $5d - 8 = 3d + 14$

THINKING

- 1 Eliminate the variable from one side of the equation by adding or subtracting like terms.
- 2 Solve the equation by using the inverse operations.
- 3 Check your answer by substituting the solution into each side of the original equation.

WORKING

$$\begin{aligned}5d - 8 &= 3d + 14 \\ 5d - 8 - 3d &= 3d + 14 - 3d \\ 2d - 8 &= 14 \\ 2d - 8 + 8 &= 14 + 8 \\ 2d &= 22 \\ \frac{2d}{2} &= \frac{22}{2} \\ d &= 11\end{aligned}$$

$$\begin{array}{ll}\text{LHS} = 5 \times 11 - 8 & \text{RHS} = 3 \times 11 + 14 \\ = 47 & = 47\end{array}$$

Given that LHS = RHS, the solution is correct.

(b) $3(e - 5) = 8e$

- 1 Expand the brackets, then add or subtract like terms to eliminate the variable from one side of the equation.

$$\begin{aligned} 3(e - 5) &= 8e \\ 3e - 15 &= 8e \\ 3e - 15 - 3e &= 8e - 3e \\ -15 &= 5e \end{aligned}$$

- 2 Solve the equation by using the inverse operations.

$$\begin{aligned} \frac{-15}{5} &= \frac{5e}{5} \\ -3 &= e \\ e &= -3 \end{aligned}$$

- 3 Check your answer by substituting the solution into the original equation.

$$\begin{aligned} \text{LHS} &= 3(-3 - 5) & \text{RHS} &= 8 \times -3 \\ &= -24 & &= -24 \end{aligned}$$

Given that LHS = RHS, the solution is correct.

Explore further

Solving equations

Solve equations using a spreadsheet.

Additional information

Solving equations

Practise solving equations with the variable on both sides of the equals symbol. This activity also highlights some useful shortcuts.

Writing a linear equation to represent a problem

Many problems can be solved by forming and then solving an equation.

4 Writing a linear equation to represent a problem

Solve each of the following by first defining a suitable variable and then writing an equation to express the problem.

- (a) The total cost of a 2 minute mobile call was \$2.90. If the connection fee was 40 cents, calculate the cost per minute.

THINKING

- 1 Define the variables, including units, if appropriate.

- 2 Ensure all units are the same.

WORKING

Let C be the total cost in dollars.
Let x be cost per minute in dollars.
Let n be the number of minutes.

$$40\text{c} = \$0.40$$

3 Write an equation using your defined variable.	$C = 0.40 + nx$ The total cost is found by adding the connection fee to 2 lots of the 1 minute call periods.
4 Substitute the given values and calculate the unknown value.	$C = \$2.90$ and $n = 2$: $2.90 = 0.40 + 2x$ $2.50 = 2x$ $x = \$1.25/\text{minute}$
5 Check your results by substituting the solution into the original equation, including the units.	$\$0.40 + 2 \times \$1.25 = \$2.90$
6 Interpret your answer.	The cost per minute is \$1.25. 40c connection fee + \$1.25 for the first minute + \$1.25 for the second minute = \$2.90 for the phone call.

- (b) Dominique is 5 years older than her sister Sonya, and 25 years younger than her mother. If the sum of all their ages is 50 years, determine each of their ages.

1 Define the variable. Define any other unknown quantities in terms of the variable.	Let d be Dominique's age, then $d - 5$ is Sonya's age and $d + 25$ is Mother's age.
2 Write an equation using your defined variable.	$d + (d - 5) + (d + 25) = 50$
3 Collect like terms and calculate the unknown value.	$3d + 20 = 50$ $3d + 20 - 20 = 50 - 20$ $3d = 30$ $d = 10$
4 Interpret the result.	Dominique is 10 years old.
5 Determine the values for the other defined expressions, interpreting each value.	Sonya's age: $d - 5 = 5$ years Mother's age: $d + 25 = 35$ years
6 Check the results by substituting the solution into the original equation.	$10 + 5 + 35 = 50$
7 Interpret the answer.	Dominique is 10 years old, Sonya is 5 years old, Their mother is 35 years old.

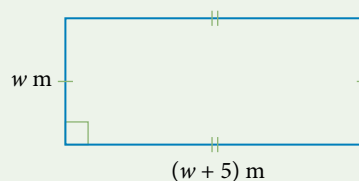
- (c) The length of a rectangular paddock is 5 m longer than the width. If the perimeter is 380 m, determine the dimensions of the paddock.

- 1 Define the variable.

Define any other unknown quantities in terms of the same variable.

Construct a diagram, labelling the chosen variables.

Let w be the width of the paddock in metres, then $w + 5$ is the length in metres.



- 2 Write an equation using your defined variable.

The formula for the perimeter of a rectangle is $P = 2L + 2W$.

$$380 = 2(w + 5) + 2w$$

- 3 Expand brackets and collect like terms.

$$\begin{aligned} 380 &= 2w + 10 + 2w \\ &= 4w + 10 \end{aligned}$$

- 4 Solve the equation.

$$380 - 10 = 4w + 10 - 10$$

$$\begin{aligned} \frac{370}{4} &= \frac{4w}{4} \\ w &= 92.5 \end{aligned}$$

$$w + 5 = 97.5$$

- 5 Calculate values for the other defined expressions and interpret your results.

Width is 92.5 m Length: $w + 5 = 97.5$ m

- 6 Check your answer by substituting the solution into the original equation.

$$\begin{aligned} P &= 2L + 2W \\ &= 2 \times 97.5 + 2 \times 92.5 \\ &= 380\text{m} \end{aligned}$$

- 7 Interpret the answer.

The length of the paddock is 97.5 m and the width is 92.5 m.

- (d) The sum of three consecutive odd numbers is 63. Determine the value of the three numbers.

- 1 Define the variable.

Define any other unknown quantities in terms of the variable.

Let n be the smallest number.

The second odd number is $n + 2$.

The third odd number is $n + 4$.

- 2 Write an equation using your defined variable.

$$n + n + 2 + n + 4 = 63$$

- 3 Collect like terms.

$$3n + 6 = 63$$

4 Solve the equation.

$$3n + 6 - 6 = 63 - 6$$

$$\frac{3n}{3} = \frac{57}{3}$$

$$n = 19$$

5 Interpret the results.

The first odd number is 19.

6 Calculate values for the other defined expressions, interpreting each value.

The second odd number: $n + 2 = 21$

The third odd number: $n + 4 = 23$

7 Check your results by substituting the solution into the original equation.

The sum of three consecutive odd numbers:
 $19 + 21 + 23 = 63$

8 Interpret the answer.

The three consecutive odd numbers that add to 63 are 19, 21 and 23.

EXERCISE

4.1

Linear equations

Worked
Example

1 Classify each of the following as a linear equation, a non-linear equation or a linear expression.

(a) $5w^2 - 8 = 36$

(b) $3c - 12$

(c) $15n + 12n = 21 + 6n$

(d) $\sqrt{s} + 5 = 100$

(e) $2(s + 3) = 25$

(f) $7 + 3k = 28$

2 Solve each of the following linear equations.

(a) $4g + 8 = 32$

(b) $7(k - 4) = 28$

(c) $\frac{9b + 6}{4} = 15$

(d) $5n - 7 = 33$

(e) $-4(m + 5) = -24$

(f) $\frac{6e - 18}{3} = 2$

(g) $3m - 2 = -17$

(h) $\frac{1 - 6b}{2} = -1$

(i) $-7t + 6 = -18.15$

3 Sally is 42 years older than her daughter Kate and 2 years younger than her husband Bill. Their ages add to 116 years. If s represents Sally's current age, which of the following can be used to determine everyone's age?

A $s - 2 + k + b = 116$

B $s + s + 42 + s - 2 = 116$

C $3s - 40 = 116$

D $s + k + b - 2 = 116$

4 A rectangular pool is 38 m longer than it is wide. The perimeter is 124 m. Determine an expression (in expanded form) representative of the perimeter in terms of the length l , in metres, and hence the equation you could use to find l .

5 Solve each of the following equations.

(a) $\frac{3(f - 6)}{2} = 6$

(b) $5(x + 2) = 12x$

(c) $3 - 2a = 7a - 51$

(d) $10y + 7 - 3(y + 1) = 39$

- 6 For the equation $-3(4 + g) = 18$:
- Select the inverse operations needed to isolate g .
 - Multiply by -3 then subtract 4 .
 - Divide by -3 then subtract 4 .
 - Subtract 4 then divide by -3 .
 - Add 3 then subtract 4 .
 - Explain the common error made by a student who chooses the second incorrect option.
- 7 If you square Andrea's age, the answer is 380 more than Ken's age, and Ken is 40 years older than Andrea.
- Write an equation to represent this information.
 - Explain why the equation you created in part (a) is not linear.

Worked
Example

..

..

3

- 8 Solve each of the following linear equations.

(a) $7h - 6 = 3h + 14$	(b) $10e - 20 = 6e$	(c) $b + 3(b + 4) = 4$
(d) $3x - 5 = 2(x + 1)$	(e) $-3(2 - 3f) = 11(f + 3)$	(f) $2 + 3(h - 7) = 4 + h$

4

- 9 Solve each of the following by first defining a suitable variable and then writing an equation to express the problem.
- The total cost for a 4 min 20 s mobile phone call was \$4.50. The connection fee is 50 cents per call. Determine the cost rate for each period of 1 minute in \$ / min.
 - Jana is 3 years older than her sister Meredith, but 7 years younger than her sister Lin. The sum of their ages is 169. How old is each sister?
 - The length of a rectangular piece of cardboard is 10 cm longer than the width. The perimeter is 100 cm. Determine the length and width of the rectangle.
 - The sum of three consecutive even numbers is 144. Determine the values of the numbers.
 - Asok is 3 times as old as Billie and 2 years younger than Cassie. The total of their ages is 44 years. How old is each person?
 - A father's age is three times that of his son. Their ages add to 56. How old are father and son?
- 10 Solve each of the following by first defining a suitable variable and then writing an equation to express the problem.
- Three consecutive odd numbers add to 129. What are the numbers?
 - Four consecutive even numbers add to 508. What are the numbers?
- 11 Solve each of the following by first expressing the problem as an equation.
- Andreas, Sofie and their three children inherit \$140 000. The money is divided so that each parent receives twice the amount of each child. How much does each parent receive?
 - Ashton and Carol spend a total of \$1080 for five nights in a motel and two nights in a luxury hotel. The luxury hotel costs twice as much as the motel. What is the cost of one night in the luxury hotel?
 - To construct a new fence Jai has 1000 new bricks, and 2000 old recycled bricks that need to be cleaned. The cost of a new brick is 50 cents more than the cost to clean an old brick. The total cost of the bricks for the job is \$2600. Determine the cost of cleaning an old brick.
- 12 Three consecutive negative numbers add to make -153 . Create and solve an equation to determine the values of the numbers.
- 13 Seven less than five times a certain number is the same as twice the number plus 26. Create and solve an equation to calculate the number.

- 14 Consider this number trick.

Think of a whole number between 0 and 10.

Double your number.

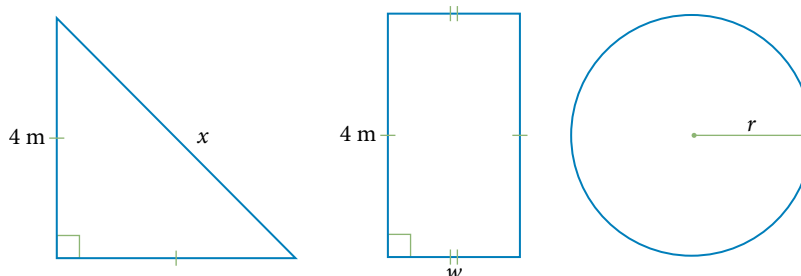
Add 12.

Divide by 2.

Subtract your original number.

Repeat using a different number.

- Let the number be x and create an equation by following the instructions to demonstrate why the answer is always 6.
 - The first instruction was to choose a whole number between 0 and 10. Explain what would have happened if a fraction such as $\frac{5}{9}$ was used, or a negative number such as -73 . Justify your answer with calculations.
 - Design your own trick and demonstrate why it works.
- 15 The cost of taking a taxi is based on a flagfall of \$4.30 plus \$2.17 per kilometre for the journey. Use algebraic equations to solve the following. Include any assumptions you need to make.
- What is the maximum distance, to the nearest tenth of a kilometre, a passenger can travel for \$50?
 - By how much is this distance reduced if a booking fee of \$1.50 is included?
 - Would the fare in part (a) be reduced or increased if the driver waived the flagfall and instead increased the rate per kilometre by 20%?
- 16 Britta designs and cuts the fencing to build the triangular enclosure shown. She then designs a rectangular enclosure and a circular enclosure to use the fencing that has already been cut.



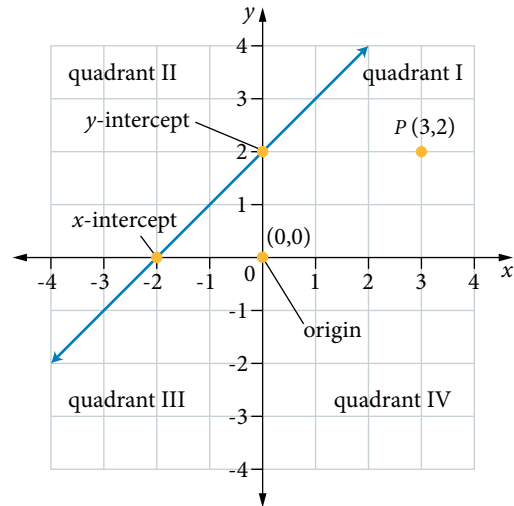
- Calculate the length of fencing Britta has to construct the enclosure, and use this to determine the unknown dimensions of each of the three designs, to the nearest millimetre.
 - Britta wants to choose the design with the largest area, so her animals can move around easily. Compare the options, justifying your choice as the correct option for Britta.
- 17 Kelly and Gino are cyclists training for a triathlon. Kelly rides the 68 km from Cairns to Port Douglas at an average speed of 25 km/h. Gino begins his ride at the same time in Port Douglas and rides at an average speed of 32 km/h to Cairns.
- Create an equation demonstrating the position of each cyclist relative to Cairns, where t is the time elapsed in hours.
 - Determine how long the two cyclists will ride (to the nearest second) until they meet and how far out of Cairns they will meet, to the nearest metre.

4.2

Linear graphs

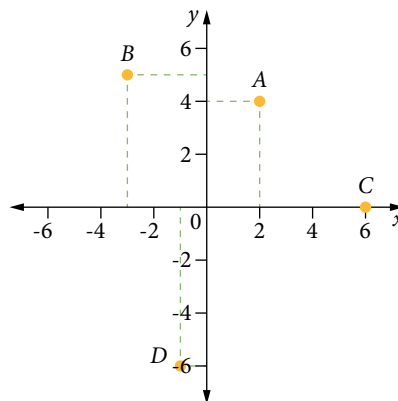
Locating points on a grid

The horizontal x -axis and the vertical y -axis divide the Cartesian plane into four quadrants. The intersection of the two axes is called the *origin*. The point P is located by its x - and y -coordinates $(3, 2)$.



5 Locating points

Determine the coordinates of the points on the following graph.



THINKING

The coordinates of a point show the horizontal and vertical distance, measured in 'units' from the origin in the order (x, y) .

WORKING

$A(2, 4)$

$B(-3, 5)$

$C(6, 0)$

$D(-1, -6)$

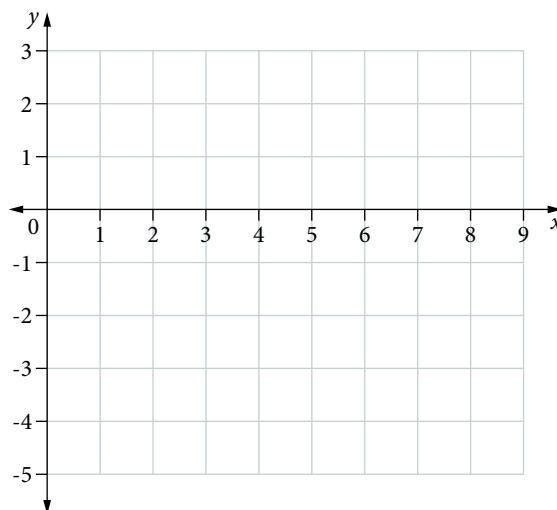
The values included along the vertical axis of a plane are called the *range*, and the values included along the horizontal axis of a plane are called the *domain*.

i Additional information

Reading coordinates

Practise locating coordinates on a Cartesian plane.

The Cartesian plane shown here has a range from -5 to 3, which can be written as $-5 \leq y \leq 3$, and a domain from 0 to 9, or $0 \leq x \leq 9$.



6 Plotting points

Plot the points $E(2,4)$, $F(-2,-3)$ and $G(0,3)$ on a Cartesian plane.

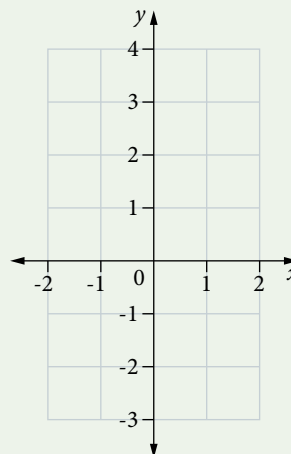
THINKING

- 1 Construct a set of axes and label each axis.

Consider the x values in each of the coordinates to determine the domain for the horizontal axis,
 $-2 \leq x \leq 2$ here.

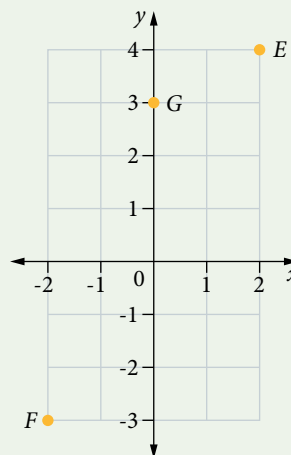
Consider the y values in each of the coordinates to determine the range for the vertical axis,
 $-3 \leq y \leq 4$ here.

WORKING



- 2 Locate and label each point using the given coordinates.

From the origin, move horizontally for the x -coordinate and then vertically for the y -coordinate.



A linear relationship can be represented on a graph by choosing appropriate values for x , and then substituting these into the equation to calculate the corresponding values of y . You can choose a minimum of two points, but it is better to find a third point as a check.

Choosing a scale for your graph

Unless a scale for the axes has been given, choose a convenient scale that gives between 5 and 10 scale markings on both axes. The values on the horizontal axis may need to be restricted in practical situations to keep the graph realistic. For example, if the horizontal axis represents *number of people*, then only positive numbers are needed because it is not possible to have a negative number of people. Many practical situations therefore require a graph that shows only the first quadrant of the Cartesian plane.

The horizontal values that can be substituted into the equation are known as the *domain*.

The possible values for the vertical scale are known as the *range*.

7 Sketching a graph

The cost for a delivery from Cedric's Cement is \$110 plus \$220 for each cubic metre of cement. A single truck can carry up to 10 m^3 of cement.

(a) Create an equation for this linear relationship.

THINKING

- 1 Define the variables.
- 2 Create an equation linking the defined variables.

WORKING

Let d be the cost, in \$, of delivery and n be the number of m^3 of cement.

$$d = 110 + 220n$$

(b) Calculate the costs for 0, 5 and 10 m^3 .

- 1 Substitute the given values into the equation to calculate the cost.

For $n = 0$:

$$\begin{aligned} d &= 110 + 220 \times 0 \\ &= 110 \end{aligned}$$

For $n = 5$:

$$\begin{aligned} d &= 110 + 220 \times 5 \\ &= 1210 \end{aligned}$$

For $n = 10$:

$$\begin{aligned} d &= 110 + 220 \times 10 \\ &= 2310 \end{aligned}$$

- 2 Record the values in a table.

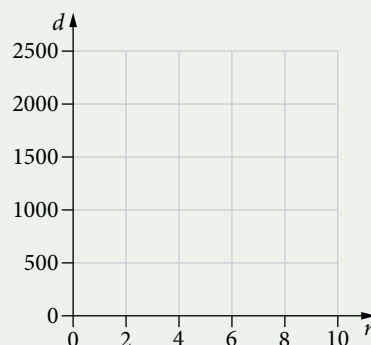
n	0	5	10
d	110	1210	2310

(c) Sketch the graph on a suitable scale using the three calculated points.

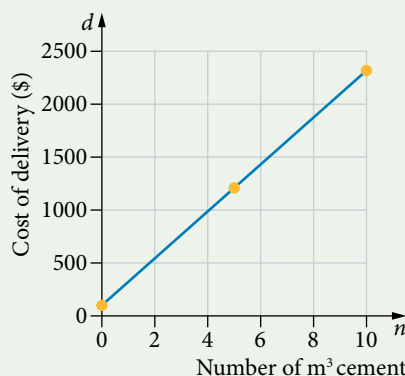
- 1 Choose a suitable set of axes based on the table of values.

The horizontal axis domain of n must extend from 0 to 10 as 10 is the maximum load a single truck can carry.

The vertical axis range of d must include the values from 0 to the maximum price of \$2310.



- 2 Plot the ordered pairs and draw a straight line through the points.
Add context labels to each axis.



Confirm the accuracy of your graphs by drawing them with technology.

i Additional information

Linear graphs

Watch the animation to see how to sketch the graphs of linear equations.

e Explore further

Creating linear functions with technology

Draw linear graphs from two points and from an equation.

Gradient-intercept form of a linear equation $y = a + bx$

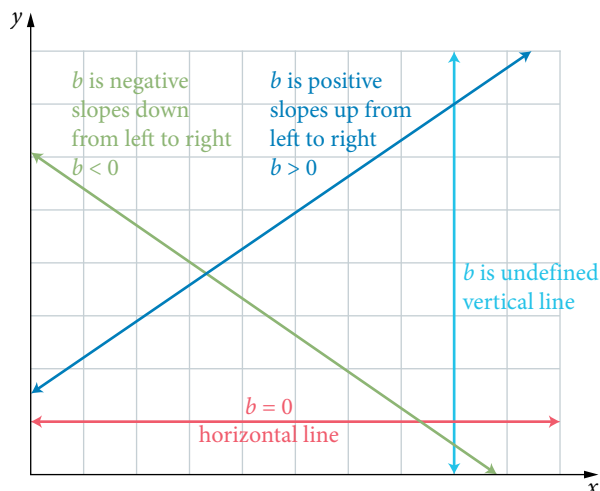
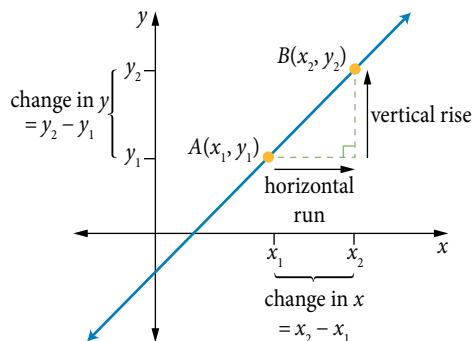
Linear equations are often written as either $y = mx + c$ or $y = a + bx$. In later units of the General Mathematics course, you will spend a great deal of time focusing on statistical analysis involving linear regression. Linear regression models relationships between two variables by fitting a line with an equation in the form $y = a + bx$, where a represents the y -coordinate of the y -intercept and b represents the slope or gradient of a straight line. To avoid confusion between variables, familiarise yourself with the gradient-intercept form $y = a + bx$.

Gradient, b

A linear graph has a constant slope or *gradient*. The gradient is a measure of the steepness and direction of a graph.

To determine the gradient of a line, locate two points that lie on the line. The gradient is the ratio of the change in the vertical distance to the change in the horizontal distance. For example, a gradient $b = 5$ represents the ratio $b = \frac{5}{1}$, a vertical increase of 5 units per unit of horizontal increase.

Whereas a gradient $b = -3$ represents the ratio $b = -\frac{3}{1}$, a vertical decrease of 3 units per unit of horizontal increase.



The gradient b of a line that passes through two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$\begin{aligned} b &= \frac{\text{vertical rise}}{\text{horizontal run}} \\ &= \frac{\text{change in } y}{\text{change in } x} \\ &= \frac{y_2 - y_1}{x_2 - x_1} \end{aligned}$$

8 Determining the gradient from coordinate pairs

Determine the gradient of the line passing through points $A(2, 8)$ and $B(4, 22)$.

THINKING

- 1 Define the points (x_1, y_1) and (x_2, y_2) .
- 2 Calculate the gradient by substituting the points into the formula $b = \frac{y_2 - y_1}{x_2 - x_1}$ and simplifying to determine the value of b .
- 3 Interpret the result.

WORKING

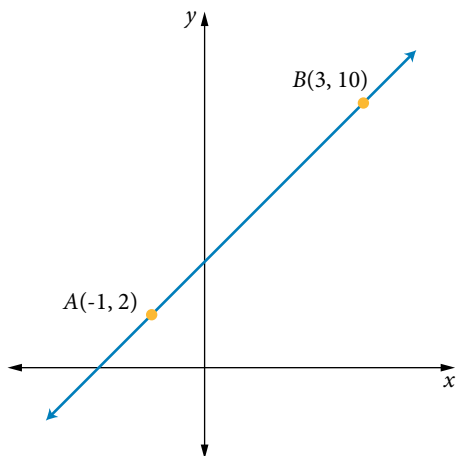
$$\begin{aligned} (x_1, y_1) &\rightarrow (2, 8) \\ (x_2, y_2) &\rightarrow (4, 22) \\ b &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{22 - 8}{4 - 2} \\ &= \frac{14}{2} \\ &= 7 \end{aligned}$$

For $b = 7$, this represents the ratio $b = \frac{7}{1}$. The graph has a vertical increase of 7 units per unit of horizontal increase.

9 Determining the gradient from a graph

Determine the gradient of the line shown in each graph.

(a)



THINKING

- 1 Define the points (x_1, y_1) and (x_2, y_2) .
- 2 Calculate the gradient by substituting the points into the formula $b = \frac{y_2 - y_1}{x_2 - x_1}$ and simplify to determine the value of b .
- 3 Interpret the result.
- 4 Check that your algebraic calculations match the 'visual' formula:

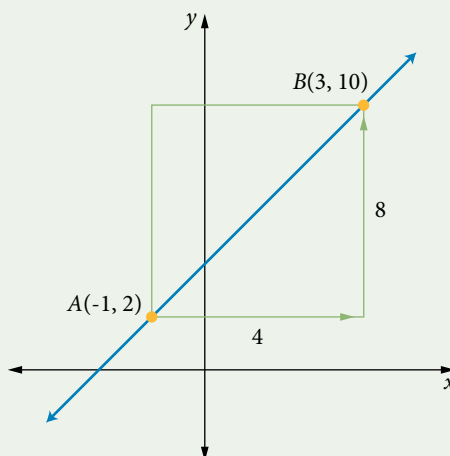
$$b = \frac{\text{vertical rise}}{\text{horizontal run}}$$

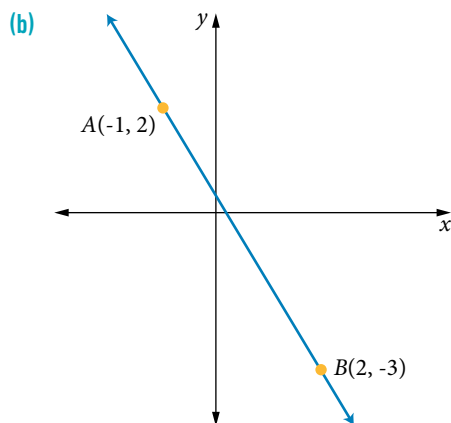
WORKING

$$\begin{aligned}(x_1, y_1) &\rightarrow (-1, 2) \\ (x_2, y_2) &\rightarrow (3, 10)\end{aligned}$$

$$\begin{aligned}b &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{10 - 2}{3 - (-1)} \\ &= \frac{8}{4} \\ &= 2\end{aligned}$$

For $b = 2$, this represents the ratio $b = \frac{2}{1}$.
The graph has a vertical increase of 2 units per unit of horizontal increase.





1 Define the points (x_1, y_1) and (x_2, y_2) .

$$(x_1, y_1) \rightarrow (-1, 2)$$

$$(x_2, y_2) \rightarrow (2, -3)$$

2 Calculate the gradient by substituting the points into the formula $b = \frac{y_2 - y_1}{x_2 - x_1}$.

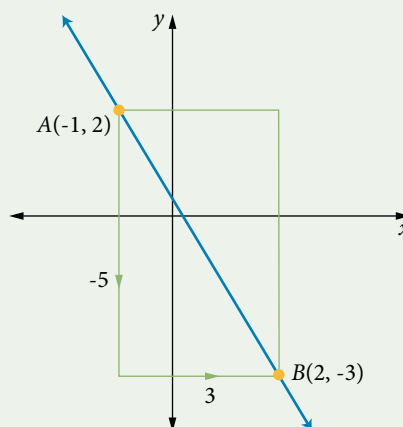
$$\begin{aligned} b &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-3 - 2}{2 - (-1)} \\ &= \frac{-5}{3} \\ &= -\frac{5}{3} \end{aligned}$$

3 Interpret the result.

For $b = -\frac{5}{3}$, the graph has a vertical decrease of 5 units for every 3 units of horizontal increase.

4 Check that your algebraic calculations match the 'visual' formula:

$$b = \frac{\text{vertical rise}}{\text{horizontal run}}$$



Notice that when reading a gradient value in the form $b = -\frac{p}{q}$ the gradient is read as a vertical decrease of p units per q units of horizontal increase.

When interpreting the gradient of a graph, you are investigating the properties of the slope from left to right, where a positive gradient represents a vertical increase per unit of horizontal increase and a negative gradient represents a vertical decrease per unit of horizontal increase.

EXERCISE

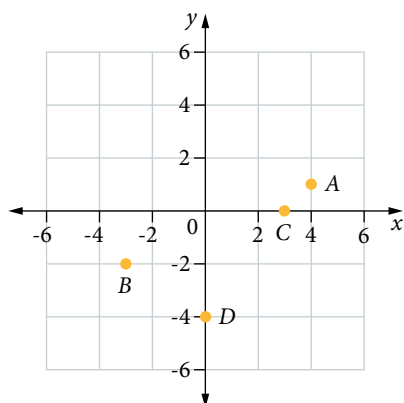
4.2

Linear graphs

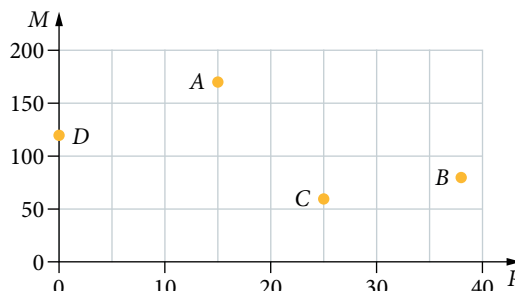
Worked
Example

- 1 Determine the coordinates of points A to D in the following diagrams.

(a)



(b)



- 2 Plot the points $E(2, 4)$, $F(-2, 3)$ and $G(0, 3)$ on a suitable set of axes.

- 3 Plot the points from the table on a suitable set of axes.

x	-2	0	2	4
y	5	2	-1	-4

- 4 The cost C (\$) of renting a jumping castle for a child's party is \$200 for the delivery and collection, plus \$30 per hour. The party will go for a maximum of 8 hours. Let n = the number of hours the party lasts.

(a) Which equation defines this linear relationship?

A $C = 30 + 200n$ B $C = 200n - 30$ C $C = 200 + 30n$ D $C = 30n - 200$

(b) Calculate the cost of renting the jumping castle for 0, 4 and 8 hours.

(c) Plot the graph of the cost for up to 8 hours.

- 5 Calculate and interpret the gradient of the line passing through each pair of points.

(a) $A(4, 1)$ and $B(10, 19)$ (b) $A(2, 5)$ and $B(8, 29)$

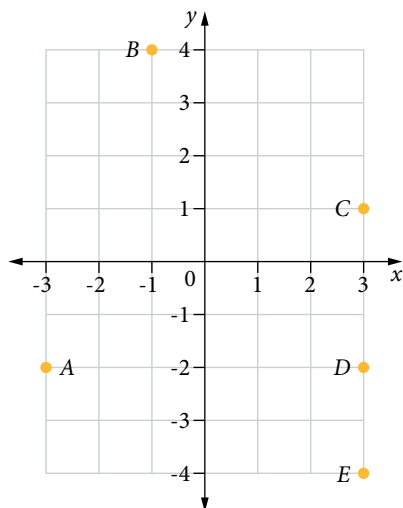
(c) $A(-2, 1)$ and $B(3, -9)$ (d) $A(-3, -4)$ and $B(3, 0)$

- 6 Determine the common error made in the following calculation and calculate the correct answer.

$A(-2, 8)$ and $B(5, -6)$

$$\begin{aligned}
 b &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{-6 - 8}{5 - 2} \\
 &= \frac{-14}{3}
 \end{aligned}$$

- 7 Determine the gradient of a line passing through the following points.



(a) AB

(b) AC

(c) AD

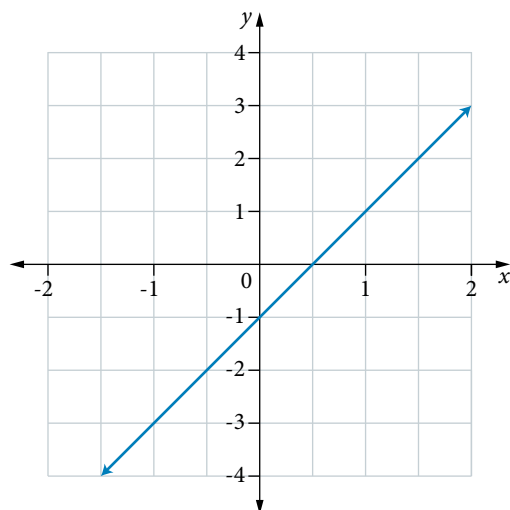
(d) AE

Worked
Example

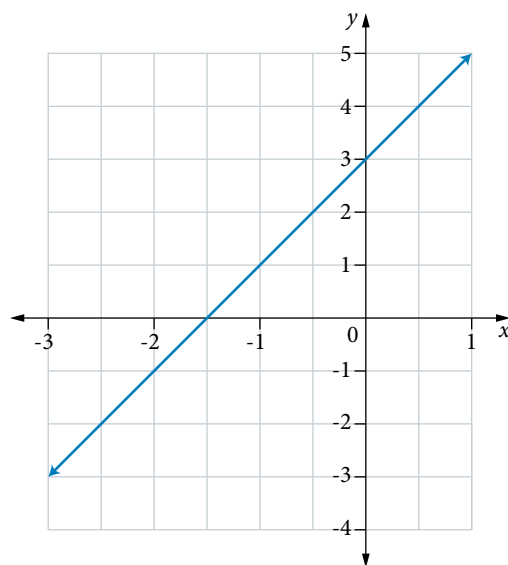
9

- 8 Determine the gradient of the line shown in each graph.

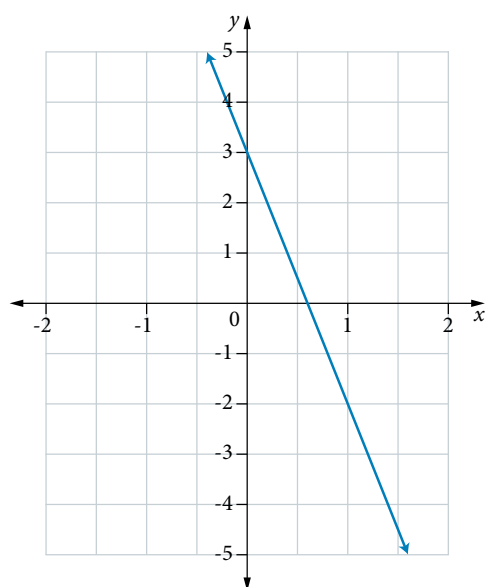
(a)



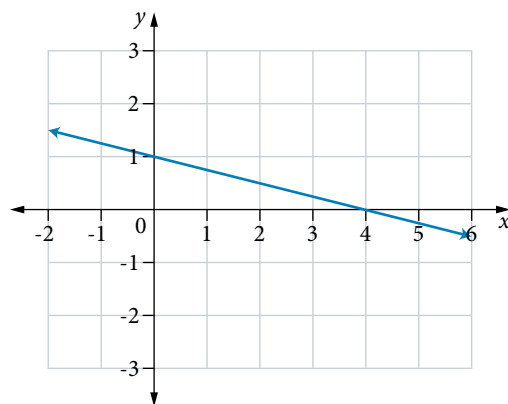
(b)



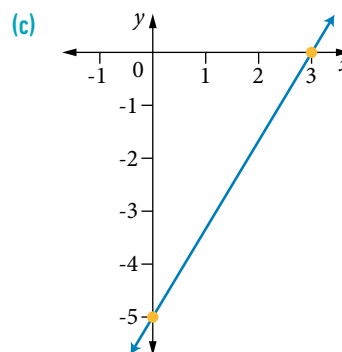
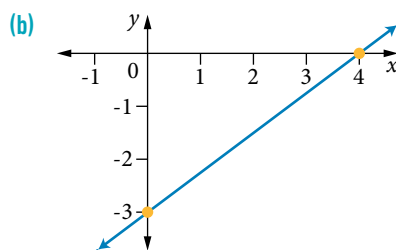
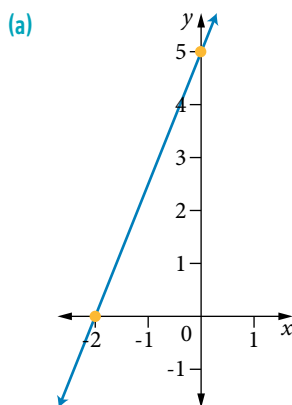
(c)



(d)



- 9 Determine the gradient of the line shown in each graph.



- 10 An electrician charges \$50 for a callout, and then \$20 every 15 minutes. Given that c is the cost (\$) of the visit and n is the number of 15 minute periods, create an algebraic equation for cost in terms of the number of 15 minute periods.

- 11 Albert investigates the relationship between the temperature and the distance from a radiator heater. He constructs the following table of data.

Distance (cm)	0.5	1	2	10	20
Temperature ($^{\circ}\text{C}$)	39	34	30	26	25

- (a) Plot the points on a suitable set of axes.
- (b) Does this graph support the idea that this is a linear relationship? Explain your answer.
- (c) The temperature of the room during the experiment is 24°C . How does this help explain the levelling off of the graph?
- 12 Three customers bought E10 petrol from the same service station. Each recorded the amount they spent and the amount of petrol showing on the bowser, although this was rounded to the nearest tenth of a litre: \$10 bought 6.7 L; 40 L cost \$59.96; a full tank of 62.4 L cost \$93.58.
- (a) Plot points on a set of axes showing cost, C (\$) against amount, A (L). Draw an appropriate line through the points.
- (b) Write an approximate equation for the line passing through the points, allowing for discrepancies due to rounding.
- (c) Determine the cost of the petrol in c/L .
- 13 Arlia's weekly wage (\$) at the tool hire store is calculated by the formula $W = 400 + 8n$, where n is the number of tools she hires out per week. Each week she hires out up to 80 tools.
- (a) Copy and complete the table of values for her wage if she hires 0, 20, 40 and 60 tools per week.

Number of tools	0	20	40	60
Weekly wage (\$)				

- (b) Sketch a graph on a suitable set of axes to show the relationship between the number of tools she hires out and her wage.
- (c) Extend your graph to estimate Arlia's wage if she hired 100 tools for the week.
- (d) Is this a reasonable estimate?

- 14 A private fishing trip costs \$100 to hire the boat, and then \$20 per person to pay for bait and the hire of a rod. The boat can take a maximum of 8 people.
- Calculate the cost of the trip for just one person.
 - Calculate the cost if 8 people are on the boat.
 - Define the variables as c the cost of the trip (\$) and n the number of people fishing. Create an equation for the cost of the trip in terms of the number of people fishing.
 - Plot a graph of the relationship.
 - Determine the cost of the trip for 5 people.

..

..

- 15 Two lines are drawn to cut the x -axis at -3 and 3 respectively. The lines meet in the first quadrant at a point with whole number coordinates. Given that the gradient of one of the lines is $\frac{6}{5}$, determine:
- the gradient of the second line, given that its gradient is negative
 - the two possible points of intersection for the lines, given that the other line has a positive, whole number gradient greater than 1.
- 16 A plumber charged \$107 for a job that took just 3 minutes and \$331 for a job that took just under 4 hours. The cost of each job involves a call-out fee and a set rate for each half hour, so the 3 minute job incurred a charge for 1 half hour.

Use algebra and graphs to determine and demonstrate the following:

- the cost of unblocking a toilet: 1 h 10 min
- the cost of installing a new bathroom: 6 h 20 min.

Sketching linear graphs

Linear graphs using the intercept method

The position where the graph crosses the x -axis is called the x -intercept.

The position where the graph crosses the y -axis is called the y -intercept.

To determine the coordinates of the x -intercept, substitute $y = 0$ into the equation and solve for x .

To determine the coordinates of the y -intercept, substitute $x = 0$ into the equation and solve for y .

10 Sketching a graph using the x - and y -intercepts

Sketch the following graph by first finding the coordinates of the x - and y -intercepts.

(a) $3x - 8y = 24$

THINKING

- 1 Determine the coordinates of the x -intercept, by substituting $y = 0$ into the equation and solving for x .

Write the intercept as coordinates.

- 2 Determine the coordinates of the y -intercept by substituting $x = 0$ into the equation and solving for y .

Write the intercept as coordinates.

- 3 Construct a suitable set of axes, plot and label the two intercepts and then draw a straight line through the points.

WORKING

$$3x - 8y = 24$$

$$3x - 8 \times 0 = 24$$

$$3x = 24$$

$$x = 8$$

The coordinates of the x -intercept are $(8, 0)$.

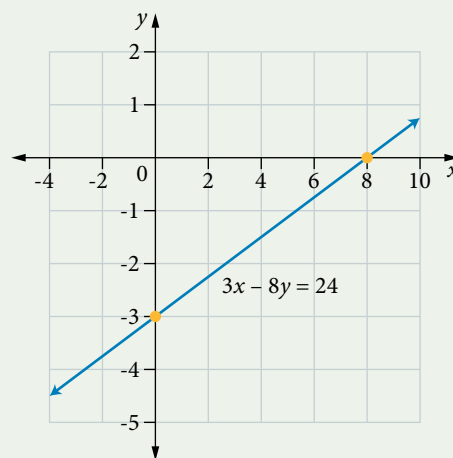
$$3x - 8y = 24$$

$$3 \times 0 - 8y = 24$$

$$-8y = 24$$

$$y = -3$$

The coordinates of the y -intercept are $(0, -3)$.



(b) $y = 10 + 4x$

- 1 Determine the coordinates of the x -intercept by substituting $y = 0$ into the equation and solving for x .

$$\begin{aligned} y &= 10 + 4x \\ 0 &= 10 + 4x \\ 4x &= -10 \\ x &= -\frac{10}{4} \\ &= -2.5 \end{aligned}$$

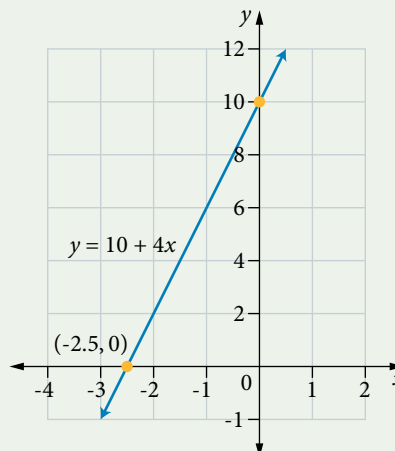
The coordinates of the x -intercept are $(-2.5, 0)$.

- 2 Determine the coordinates of the y -intercept by substituting $x = 0$ into the equation and solving for y .

$$\begin{aligned} y &= 10 + 4x \\ &= 10 + 4 \times 0 \\ &= 10 \end{aligned}$$

The coordinates of the y -intercept are $(0, 10)$.

- 3 Construct a suitable set of axes, plot the two intercepts and draw a straight line through the points.



i Additional information

Axial intercepts

Practise finding the x - and y -intercepts of a given linear function.

General form of a linear equation $y = a + bx$

Linear graphs can be written in the form $y = a + bx$, where b is the gradient and a is the y -coordinate of the y -intercept. This form provides a method of sketching a linear graph.

STEP 1: Mark the y -intercept, a , on the y -axis.

STEP 2: Express the gradient, b , as a simple fraction. Starting at the y -intercept, locate a second point by moving horizontally to the right by the number of units represented by the denominator, and then vertically up (+) or down (−) by the number of units in the numerator.

STEP 3: Draw a straight line passing through the two points.

e Explore further

A function in the form $y = a + bx$

Explore the effect of changing each variable on a linear function.

Linear graphs can be written in the form $y = a + bx$, where b is the gradient and a is the y -coordinate of the y -intercept.

11 Sketching a graph using the y -intercept and gradient

Determine the y -intercept and gradient, and use them to sketch the graph of the following equations.

(a) $y = 2 + 3x$

THINKING

- 1 Identify the values of a and b .
- 2 Convert the gradient, b , to a simple fraction and interpret.
- 3 From the a value, state the coordinates of the y -intercept.
- 4 Locate a second point by starting at the y -intercept and moving to the right by the number of units in the denominator, and then up (+) or down (–) by the number of units in the numerator of the gradient.
- 5 Plot the two points and draw a straight line through the points.

WORKING

$$y = a + bx$$

$$y = 2 + 3x$$

$$a = 2 \text{ and } b = 3$$

$$b = 3$$

$$= \frac{3}{1}$$

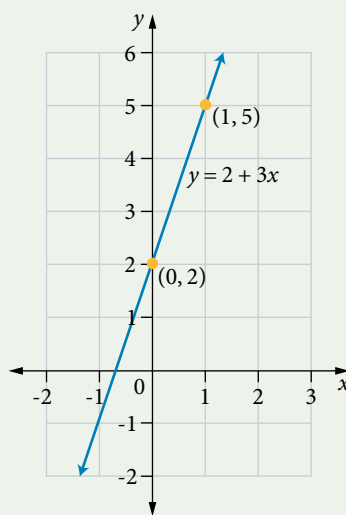
$b = \frac{3}{1}$ represents a slope with a vertical increase of 3 units per unit of horizontal increase.

The coordinates of the y -intercept are $(0, 2)$.

$$b = \frac{3}{1}$$

From the y -intercept, move 1 unit to the right and 3 units up.

$$(0 + 1, 2 + 3) \rightarrow (1, 5)$$



(b) $3x + 4y = 8$

- 1 Convert the equation into the form $y = a + bx$.

$$\begin{aligned} 3x + 4y &= 8 \\ 4y &= 8 - 3x \\ \frac{4y}{4} &= \frac{8 - 3x}{4} \\ y &= 2 - \frac{3}{4}x \end{aligned}$$

- 2 Identify the values of a and b .

$$\begin{aligned} y &= a + bx \\ a &= 2 \text{ and } b = -\frac{3}{4} \end{aligned}$$

- 3 Interpret the gradient.

$b = -\frac{3}{4}$ represents a slope with a vertical decrease of 3 units for every 4 units of horizontal increase.

- 4 From the a value, state the coordinates of the y -intercept.

The coordinates of the y -intercept are $(0, 2)$.

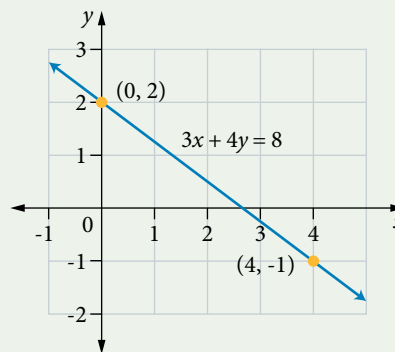
- 5 Locate a second point by starting at the y -intercept and moving to the right by the number of units in the denominator, and then up (+) or down (-) by the number of units in the numerator.

$$b = \frac{\text{rise}}{\text{run}} = \frac{-3}{4}$$

From the y -intercept, move 4 units to the right and 3 units down.

$$(0 + 4, 2 - 3) \rightarrow (4, -1)$$

- 6 Plot the two points and draw a straight line through the points.



Additional information

Linear functions

Consolidate your understanding of linear functions using the slider and completing the activities.

Additional information

Linear relationships

Try the activity to practise using linear relationships between two variables to make predictions.

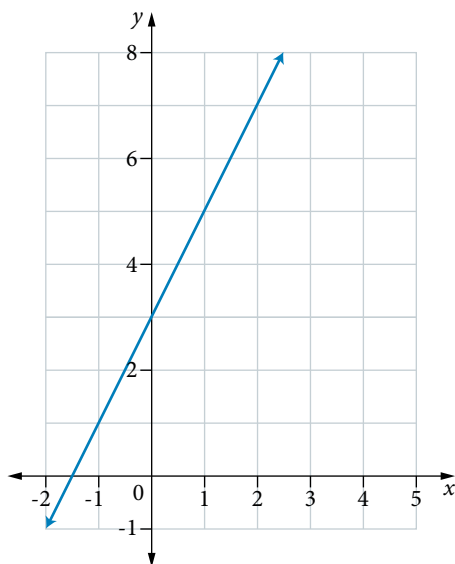
Using models to make predictions

A linear mathematical model represents a relationship between variables, usually in the form $y = a + bx$, and can be used to make predictions.

12 Determining the rule of a function from a graph

Determine the rule of each line from the graph.

(a)

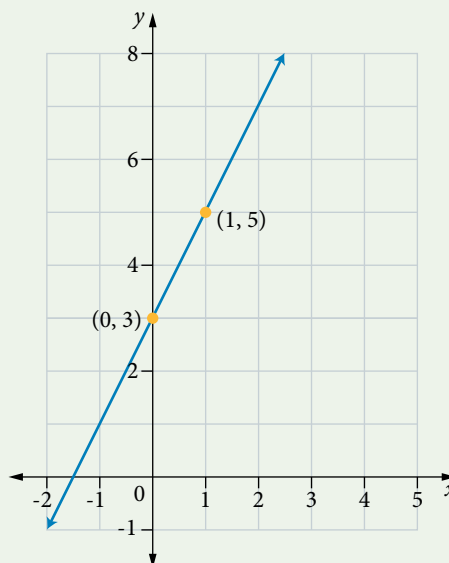


THINKING

- 1 Identify and label the coordinates of the y -intercept and one other point.

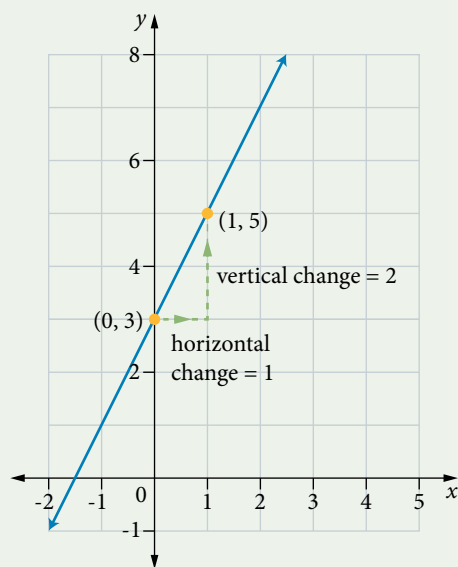
- 2 Substitute the coordinates of the y -intercept into $y = a + bx$.

WORKING



For $(0, 3)$:
 $y = a + bx$
 $3 = a + b(0)$
 $a = 3$
 $\therefore y = 3 + bx$

- 3 Calculate the gradient of the graph from two points on the graph.

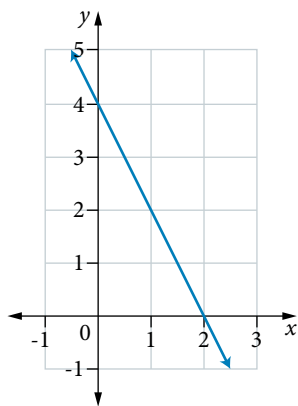


$$\begin{aligned}
 b &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{5 - 3}{1 - 0} \\
 &= 2
 \end{aligned}$$

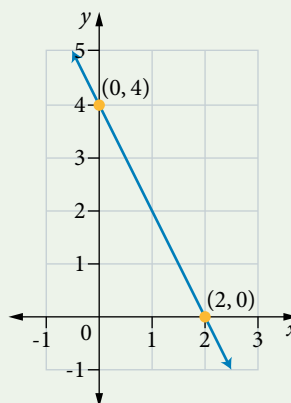
- 4 Interpret the rule of the function in the form $y = a + bx$.

The rule of the linear function is $y = 3 + 2x$.

(b)



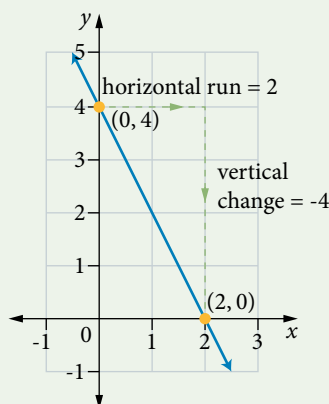
- 1 Identify and label the coordinates of the y -intercept and one other point.



- 2 Substitute the coordinates of the y -intercept into $y = a + bx$.

$$\begin{aligned}\text{For } (0, 4): \\ y &= a + bx \\ 4 &= a + b(0) \\ a &= 4 \\ \therefore y &= 4 + bx\end{aligned}$$

- 3 Calculate the gradient of the graph from two points on the graph.

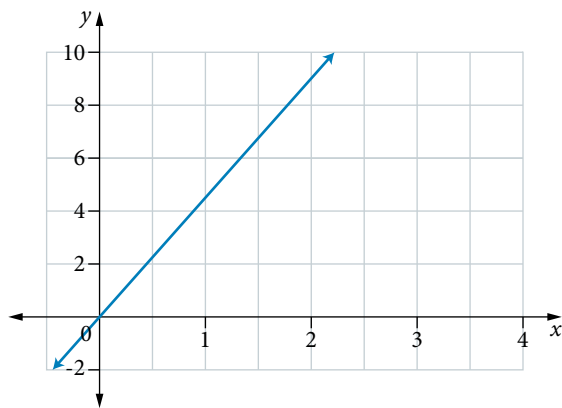


$$\begin{aligned}b &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0 - 4}{2 - 0} \\ &= -2\end{aligned}$$

- 4 Interpret the rule of the function in the form $y = a + bx$.

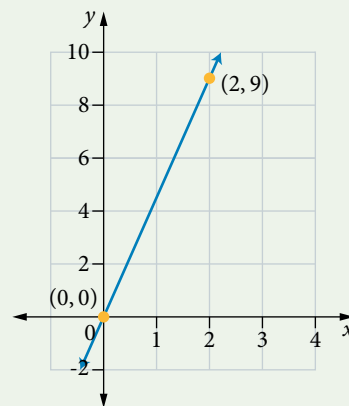
The rule of the linear function is $y = 4 - 2x$.

(c)

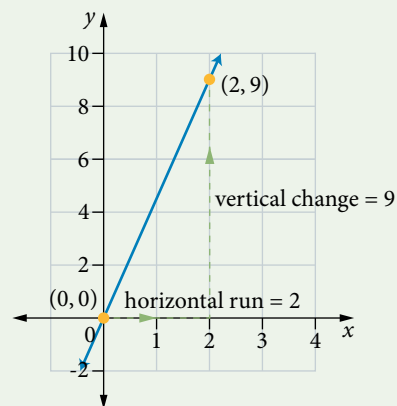


- 1 For graphs passing through the origin, the rule of the function will be in the form $y = bx$ because the coordinates of the y -intercept are $(0,0)$.

Identify and label the origin and one other point.



- 2 Calculate the gradient of the line.



$$\begin{aligned} b &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{9 - 0}{2 - 0} \\ &= 4.5 \end{aligned}$$

- 3 Interpret the rule of the linear function in the form $y = bx$.

The rule of the function is $y = 4.5x$.

13 A simple model of a real-life situation

The makers of a new eco-friendly car with a 60 L tank claim that it has a fuel consumption of 5 L per 100 km. Create a linear graph to model this situation by following the steps below.

- (a) If the fuel consumed (L) is x and the distance travelled (km) is y , express two known amounts using coordinate pairs.

THINKING

Convert the information into coordinates (x, y) .

WORKING

Initially the car has used 0 L of fuel and has travelled 0 km: $(0, 0)$

The car would use 5 L of fuel after 100 km: $(5, 100)$

- (b) Determine the relationship between the fuel consumed and the distance travelled.

- 1 In the form $y = a + bx$, identify the value of a .

When fuel consumption x is 0 L, distance travelled y is 0 km, so $a = 0$.

- 2 Interpret the value of b using the rate given.

For every 100 km travelled the car uses 5 L of fuel, so $b = \frac{100}{5} = 20$ km/L.

- 3 Alternatively, calculate the gradient by substituting values from determined coordinates into $b = \frac{y_2 - y_1}{x_2 - x_1}$.

Using $(0, 0)$ and $(5, 100)$:

$$\begin{aligned} b &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{100 - 0}{5 - 0} \\ &= 20 \text{ km/L} \end{aligned}$$

- 4 Create an equation in the form $y = a + bx$.

$$y = 0 + 20x$$

$$\therefore y = 20x$$

- 5 Interpret the result.

Distance travelled (km) = $20 \times$ fuel consumed (L)

For every litre of fuel consumed, the car travels 20 km.

- (c) Determine the maximum distance the car would travel on a tank of fuel.

- 1 Substitute the maximum value for x to calculate the maximum value for y .

Maximum amount of fuel is 60 L: $x = 60$

$$\begin{aligned} y &= 20x \\ &= 20 \times 60 \\ &= 1200 \end{aligned}$$

- 2 Interpret the answer.

The maximum distance the car can travel on a full 60 L tank of fuel is 1200 km.

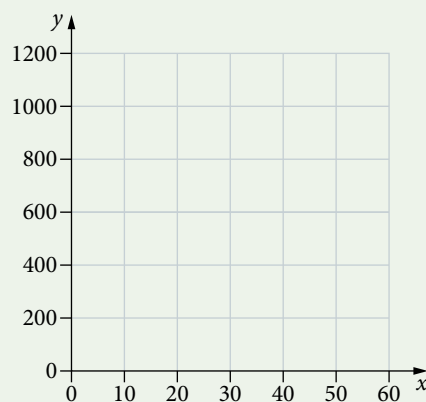
- (d) Choose a suitable scale and draw the graph.

- 1 Construct a suitable set of axes.

Starting at zero on each axis is usual for many practical situations. Extend the axes to include maximum values.

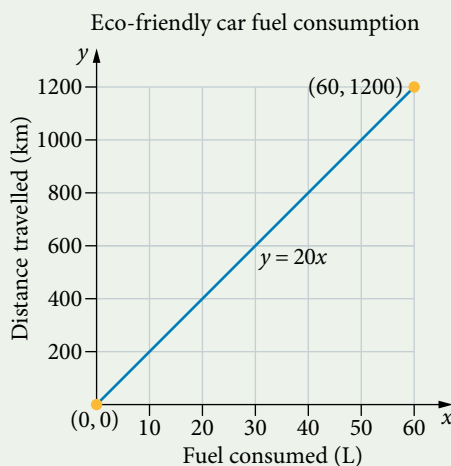
Distance travelled is from 0 to 1200 km.

The petrol used is from 0 to 60 L.



- 2 Plot two points and draw a straight line through them.

Label the axes to give the context.



14 A complex model of linear relationships

A private tour of the Lakes District costs \$200 to hire a boat, and then \$25 per person for lunch. The boat can take a maximum of 12 people.

- (a) Determine the relationship between the number of people x and the cost (\$) y .

THINKING

- 1 Identify the y -intercept in the form $y = a + bx$.
- 2 Interpret the value of b using the rate given.
- 3 Determine the cost of hiring a boat for 1 person.

WORKING

The y -intercept is the cost for 0 passengers, i.e. the hire fee of \$200: $a = 200$

\$25 per person for lunch, so $b = 25$

For 1 person

\$200 hire + \$25 for lunch = \$225

- 4 Verify the gradient by substituting values from determined coordinates into

$$b = \frac{y_2 - y_1}{x_2 - x_1}$$

- 5 Create the equation in the form $y = a + bx$.

- 6 Interpret the result.

Using (0, 200) and (1, 225):

$$\begin{aligned} b &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{225 - 200}{1 - 0} \\ &= 25 \end{aligned}$$

$$y = 200 + 25x$$

$$\text{Cost}(\$) = 200 + 25 \times \text{number of passengers}$$

The cost of the tour is \$200 plus \$25 per passenger.

- (b) Determine the maximum cost of the trip.

- 1 Substitute the maximum value for x to find the maximum value for y .

The maximum number of people is 12.

$$x = 12:$$

$$\begin{aligned} y &= 200 + 12 \times 25 \\ &= 500 \end{aligned}$$

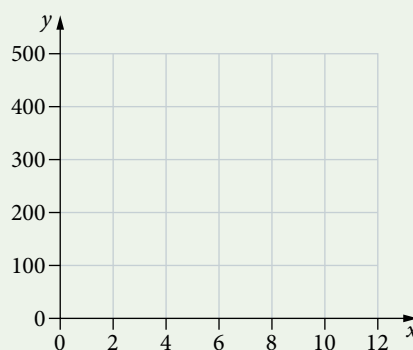
- 2 Interpret the result.

The maximum cost of the tour is \$500.

- (c) Draw a graph of the relationship.

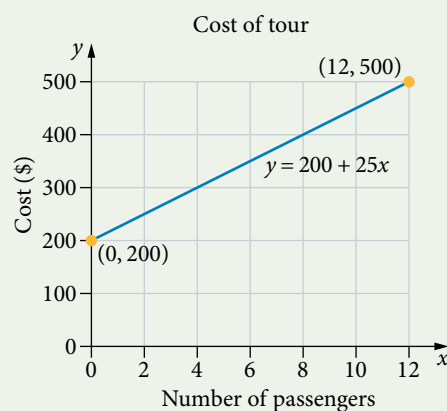
- 1 Construct a suitable set of axes.

Starting at zero on each axis is usual for many practical situations. Extend the axes to include maximum values.



- 2 Plot two points and draw a straight line through them.

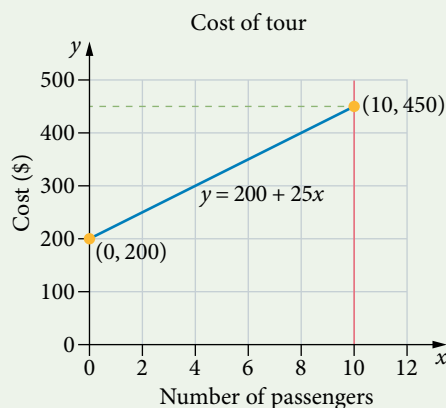
Label the axes to give the context.



- (d) From the graph, determine the cost of a tour for 10 people.

- 1 Extend the known x -coordinate to the line and read off the y -coordinate.

Interpret the coordinate.



The cost for 10 people is \$450.

- 2 Check your answer by substituting the value into the equation formed.

$$\begin{aligned}\text{For } x = 10: \\ y &= 200 + 25x \\ &= 200 + 25 \times 10 \\ &= 450\end{aligned}$$

EXERCISE

4.3

Sketching linear graphs

Worked
Example

10

- 1 Sketch the following graphs by determining and plotting the x - and y -intercepts.

(a) $5x - 2y = 20$

(b) $y = 16 + 8x$

(c) $3x + 5y = -30$

(d) $y = 24 + 3x$

(e) $x + 4y = 12$

(f) $y = 7 + 2x$

- 2 For the line $y = 5x - 3$:

- (a) Determine the x - and y -intercepts for its graph.

A $(5, 0)$ and $(0, -3)$ B $\left(-\frac{5}{3}, 0\right)$ and $(0, -3)$ C $\left(\frac{3}{5}, 0\right)$ and $(0, -3)$ D $\left(\frac{5}{3}, 0\right)$ and $(0, -3)$

- (b) Explain the common error made by a student who chose the first of the incorrect options.

- 3 The graph with intercepts $(0, -7)$ and $(7, 0)$ has equation:

A $y = x - 7$

B $y = x + 7$

C $y = -x - 7$

D $y = -x + 7$

- 4 Create the equation in the form $y = a + bx$ of the graph identified by the following values.

(a) $b = 4$ and $a = 3$

(b) $b = \frac{2}{3}$ and $a = 5$

(c) $b = -4$ and $a = 3$

(d) $b = 1$ and $a = 0$

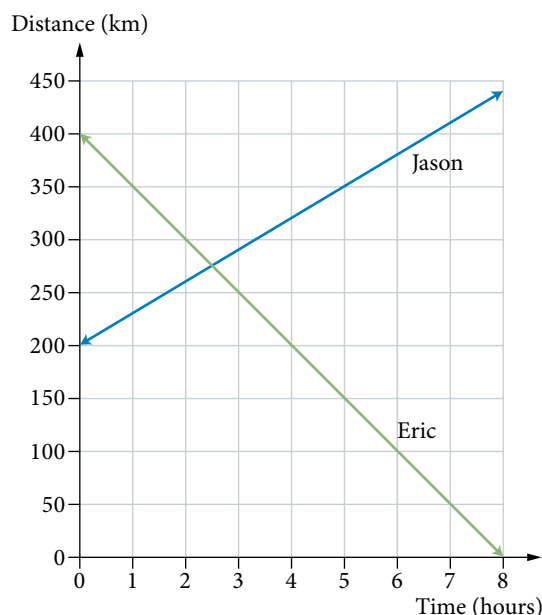
(e) $b = -\frac{5}{7}$ and $a = -2$

(f) $b = 0$ and $a = 3$

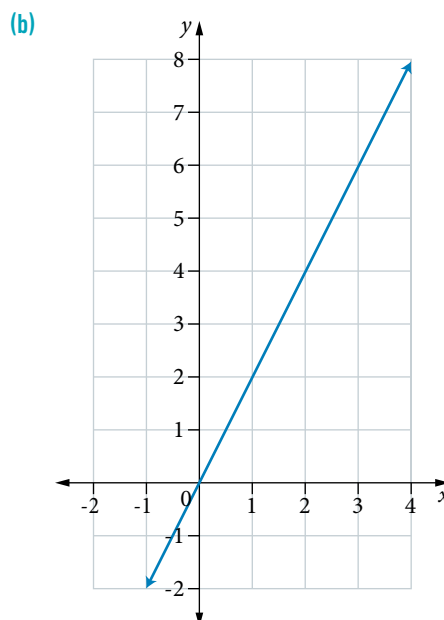
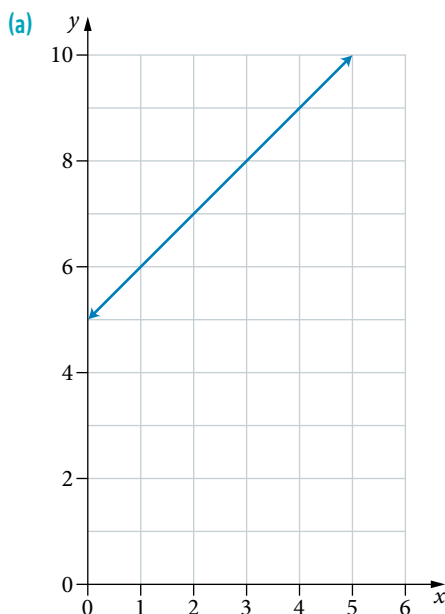
13

- 5 A 2L bottle of cordial concentrate makes 12 L of diluted cordial. It is expected that no more than 48 litres of diluted cordial will be required for a child's birthday party.
- If the amount of cordial concentrate used (L) is x and the amount of diluted cordial made (L) is y , express two known amounts using coordinate pairs.
 - Determine the relationship between the concentrate used and the volume of cordial made. Express your answer algebraically and in words.
 - Calculate the maximum amount of cordial concentrate needed for the party.
 - Plot a graph of the relationship.

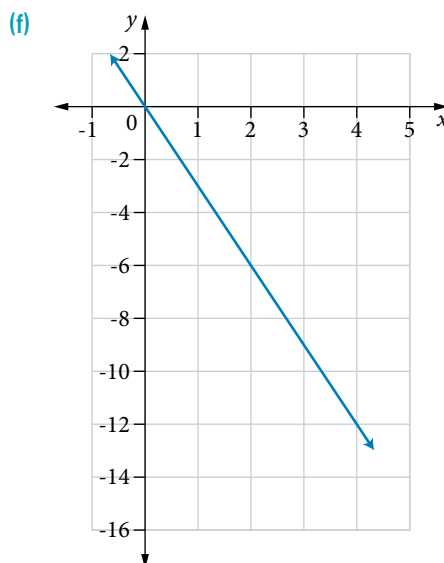
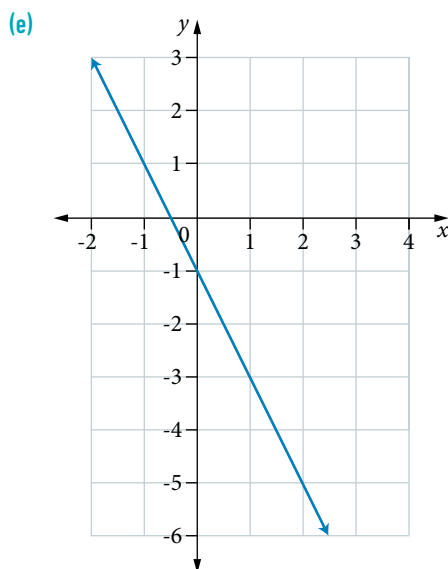
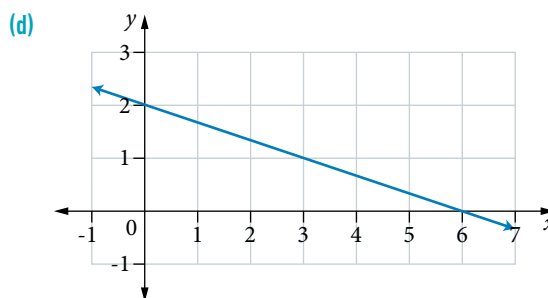
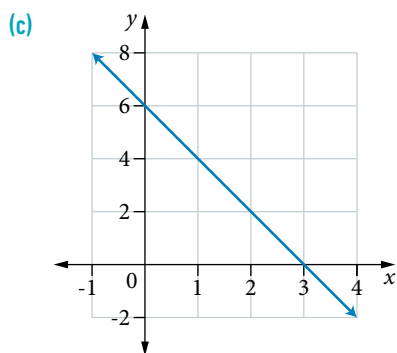
- 6 The distance of Eric and Jason from their homes is shown in the graph.
- Determine the distance each person is from home at the beginning of the journey.
 - Who is travelling towards home?
 - Determine the time when Eric and Jason are the same distance from home.
 - Calculate Eric's speed.
 - Create an equation for Eric's distance from home d (km) in terms of time t (hours).



- 7 Determine the rule of each graph.



12



Worked
Example

..

..

11

8 Determine the y -intercept and gradient, and then sketch the graph of each equation.

(a) $y = 2 + 3x$

(b) $3x + 4y = 8$

(c) $y = 3 + \frac{3}{4}x$

(d) $x + 5y = 10$

9 Consider the line that passes through the coordinates $(2, 5)$ and $(6, 4)$.

(a) Determine the rule of the line joining the points.

(b) Determine the coordinates of the x -intercept.

(c) Plot the points and join them with a line, and from this discuss the reasonableness of your answers.

14

10 The tap on a 2000 L water tank is left slightly open and 400 L of water leaks over 10 minutes.

(a) Assuming that the water leaks out at a constant rate, calculate the gradient of the graph. What does this represent in terms of the time elapsed and the volume of water?

(b) Write a linear relationship between time t (min) and the volume of water remaining in the tank V (L).

(c) Sketch a graph to show the volume remaining over the first 10 minutes.

(d) Reading from the graph, determine the volume of water:

(i) after 2 minutes (ii) after 5 minutes.

(e) In reality, it is incorrect to assume that the water leaks out at a constant rate. The flow of water actually slows as the volume of water remaining reduces. What effect would this have on the shape of the graph?

- 11 Reggie's Redgum Supply business delivers firewood for home heating. The charge is \$200 per cubic metre plus \$75 delivery. The truck can carry a maximum of 4 m^3 of wood.
- Define appropriate variables and create an equation to describe the cost of firewood.
 - Construct a graph of cost versus volume of wood delivered over an appropriate domain.
 - Use your graph to estimate the cost of delivering 2.5 m^3 of firewood. Check your answer by substituting into the equation you created in part (a).
 - Use your graph to estimate the volume of wood that would be delivered for \$500, to 1 decimal place. Check your answer by substituting into the equation you created in part (a).
- 12 The cost of having photocopier paper delivered to a business is shown in the table of values below. There is a minimum of 100 reams per delivery.

Number of reams n	100	200	300	400
Cost of delivery C (\$)	425	815	1205	1595



- Plot a graph of C as a function of n .
 - Create a linear equation for the relationship for C in terms of n .
 - What does the gradient represent?
 - What does the C -intercept represent?
 - Determine the cost of delivering:
 - 150 reams
 - 280 reams.
- 13 Joshua buys his first car for \$14 000. He takes a loan for the whole amount from his parents and agrees to pay \$1600 interest plus the initial money borrowed with 48 equal monthly repayments.
- Calculate the initial amount that Joshua owes, including interest.
 - Calculate the amount of each monthly repayment.
 - Create a linear equation linking the amount Joshua owes his parents A (\$) and the number of months since the loan began n .
 - Sketch the graph of this relationship over an appropriate domain.
 - How much does Joshua owe after 18 months?
- 14 The Speedy delivery company will pick up and deliver packages overnight for a collection fee of \$20 plus \$5 for every kilogram. The maximum weight of a package that can be delivered is 12 kg.
- Define appropriate variables and create an equation to describe the cost of delivering a package.
 - Construct a graph of cost versus weight of parcel delivered over an appropriate domain.
 - Determine the cost of delivering three 4 kg packages.
 - A charge of \$80 is made for the three packages. Explain why this might have been the case.
- 15 Use algebra and graphs to demonstrate your selected response to the following.
- Create equations for two different oblique (not horizontal or vertical) lines through $(2,1)$. Graph the lines on the same set of axes.
 - Choose two different pairs of points whose connecting line has a gradient of 1.5. Make sure the two lines are not the same. Graph the lines and determine the equation for each.

4.4

Simultaneous linear equations

Simultaneous equations

Simultaneous equations are a set of two or more equations that are both true at the same time. Solve the simultaneous linear equations to find the pair of values for the variables that make this happen. The coordinate pair of the variables represents the point of intersection on a graph of the two equations.

15 Solving simultaneous equations algebraically

Solve the following pairs of simultaneous equations algebraically.

(a) $y = x + 5$, $4x - 2y = -2$

THINKING

- 1 Identify the equations.
- 2 Select the equation with the isolated variable and substitute into the other equation.
- 3 Simplify the equation and then solve for the unknown variable.
- 4 Substitute the value into the simpler equation and calculate the unknown variable.
- 5 Interpret the answer.
- 6 Check the reasonableness of the answer by substituting both values into each equation.

WORKING

$$\begin{aligned}
 y &= x + 5, 4x - 2y = -2 \\
 \text{Substitute } y &= x + 5 \text{ into } 4x - 2y = -2. \\
 4x - 2y &= -2 \\
 4x - 2(x + 5) &= -2 \\
 4x - 2(x + 5) &= -2 \\
 4x - 2x - 10 &= -2 \\
 2x - 10 &= -2 \\
 2x &= 8 \\
 x &= 4 \\
 y &= x + 5 \\
 &= 4 + 5 \\
 &= 9 \\
 \text{The solution to the pair of equations is } x &= 4 \text{ and } y = 9. \\
 9 &= 4 + 5 \\
 4 \times 4 - 2 \times 9 &= -2
 \end{aligned}$$

(b) $x + y = 7$, $y - 2x = 4$

- 1 Identify the equations, rearranging one equation so that a variable is isolated.
- 2 Select the equation with the isolated variable and substitute into the other equation.

$$\begin{aligned}
 x + y &= 7 \text{ is the same as } x = 7 - y \\
 y - 2x &= 4 \\
 \text{Substitute } x &= 7 - y \text{ into } y - 2x = 4. \\
 y - 2x &= 4 \\
 y - 2(7 - y) &= 4
 \end{aligned}$$

- 3 Simplify the equation and then solve for the unknown variable.

$$\begin{aligned}y - 2(7 - y) &= 4 \\y - 14 + 2y &= 4 \\3y - 14 &= 4 \\3y &= 18 \\y &= 6\end{aligned}$$

- 4 Substitute the value into the simpler equation.

$$\begin{aligned}x &= 7 - y \\&= 7 - 6 \\&= 1\end{aligned}$$

- 5 Interpret the answer.

The solution to the pair of simultaneous equations is $x = 1$ and $y = 6$.

- 6 Check the reasonableness of the answer by substituting both values into each equation.

$$\begin{aligned}1 &= 7 - 6 \\6 - 2 \times 1 &= 4\end{aligned}$$

In part (b) you could have isolated y instead.

$x + y = 7$ could have been rearranged as $y = 7 - x$ and substituted into $y - 2x = 4$.

$$\begin{aligned}7 - x - 2x &= 4 \\7 - 3x &= 4 \\-3x &= -3 \\x &= 1\end{aligned}$$

Or $y - 2x = 4$ could have been rearranged as $y = 4 + 2x$ and substituted into $x + y = 7$.

$$\begin{aligned}x + (4 + 2x) &= 7 \\3x + 4 &= 7 \\3x &= 3 \\x &= 1\end{aligned}$$

Either way the solution remains the same: $x = 1$ and $y = 6$.

The solution to simultaneous equations can be displayed graphically as the point of intersection.

16 Solving simultaneous equations graphically

Solve the following pair of simultaneous equations graphically: $y = 2x$, $y = 6 - x$

THINKING

- 1 Determine two points for each graph.

WORKING

Consider the function $y = 2x$.

$$\begin{aligned}\text{For } x &= 0, \\y &= 2 \times (0) \\&= 0\end{aligned}$$

The coordinates of the point are $(0, 0)$.

$$\begin{aligned}\text{For } x &= 1, \\y &= 2 \times (1) \\&= 2\end{aligned}$$

The coordinates of the point are $(1, 2)$.

- 2 Graph the equations of the straight lines on the same set of axes, using an appropriate method.

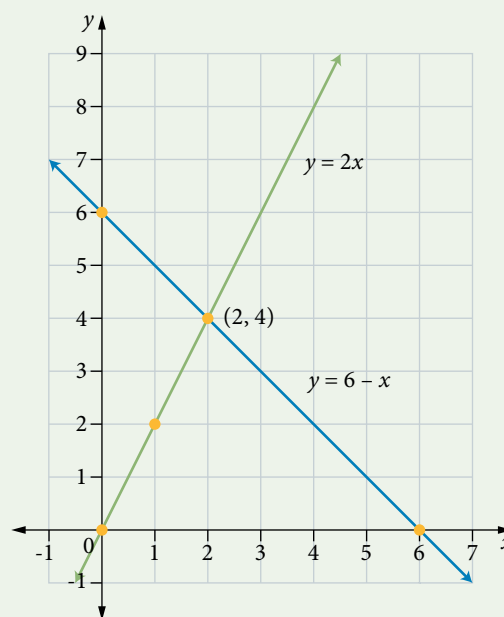
Consider the function $y = 6 - x$.

$$\begin{aligned}\text{For } x = 0, \\ y &= 6 - 0 \\ &= 6\end{aligned}$$

The coordinates of the point are $(0, 6)$.

$$\begin{aligned}\text{For } x = 6, \\ y &= 6 - 6 \\ &= 0\end{aligned}$$

The coordinates of the point are $(6, 0)$.



- 3 Identify the point of intersection.
- 4 Interpret the answer.
- 5 Check the reasonableness of the answer by substituting both values into each equation.

The coordinates of the point of intersection are $(2, 4)$.

Therefore the solution is: $x = 2$ and $y = 4$.

Check:

Let $x = 2$ and $y = 4$ to check that LHS = RHS.

For $y = 2x$

$$4 = 2 \times 2$$

For $y = 6 - x$

$$4 = 6 - 2$$

Word problems

Solving simultaneous equations is a technique that can be used to tackle problems involving two variable quantities. Extract the relevant information from the worded description, define variables and then write the equations.

17 Writing simultaneous equations

Write a pair of simultaneous equations to represent the following situation.

- (a) A drink and a pie cost \$6, while 2 drinks and 3 pies cost \$14.

THINKING

- 1 Define the variables.
- 2 Create each equation using the information supplied.

WORKING

Let d be the cost of a drink (\$),
 p be the cost of a pie (\$).

A drink and a pie cost \$6:

$$d + p = 6$$

2 drinks and 3 pies cost \$14:

$$2d + 3p = 14$$

- (b) A paddock contains emus and sheep. There is a total of 60 legs and 30 heads.

- 1 Define the variables.
- 2 Create each equation using the information supplied.

Let e be the number of emus,
 s be the number of sheep.

There is a total of 60 legs, assuming each emu has 2 legs and each sheep has 4 legs:

$$2e + 4s = 60$$

There is a total of 30 heads:

$$e + s = 30$$

18 Solving simultaneous equation word problems using algebra

Two tradespeople and an apprentice earn a total of \$950 for one 10 hour job. The same rate of pay earns one tradesperson and one apprentice a total of \$275 for a 5 hour job.

Write and solve a pair of simultaneous equations to find the hourly rate of pay for the tradesperson and for the apprentice.

THINKING

- 1 Define the variables and include the units of each rate.
- 2 Create each equation using the information supplied.

WORKING

Let t be the pay rate (\$/h) for a tradesperson,
 a be the pay rate (\$/h) for an apprentice.

Two tradespeople and an apprentice earn \$950 for a 10 hour job:

$$10(2t + a) = 950$$

One tradesperson and one apprentice earn \$275 for a 5 hour job:

$$5(t + a) = 275$$

- 3 Isolate a variable in the simpler equation.

$$\begin{aligned} 5(t + a) &= 275 \\ t + a &= \frac{275}{5} \\ &= 55 \\ a &= 55 - t \end{aligned}$$

- 4 Substitute the expression into the other equation and solve for the unknown variable.

$$\begin{aligned} 10(2t + a) &= 950 \\ 10(2t + 55 - t) &= 950 \\ t + 55 &= 95 \\ t &= 40 \end{aligned}$$

- 5 Substitute the value into the simpler equation.

$$\begin{aligned} a &= 55 - t \\ &= 55 - 40 \\ &= 15 \end{aligned}$$

- 6 Interpret the answer.

The tradesperson is paid \$40 an hour.

The apprentice is paid \$15 an hour.

- 7 Check that the values are reasonable and fit with the original scenario.

$$\begin{aligned} &10 \text{ hour job:} \\ &(\$40 \times 2 + \$15) \times 10 = \$950 \\ &5 \text{ hour job:} \\ &(\$40 + \$15) \times 5 = \$275 \end{aligned}$$

WARNING

When you have an algebraic solution, always check that the matching worded solution is reasonable.



Additional information

Simultaneous equations

Watch the video to see how to set up a system of linear equations to solve real-life applications. Then try the activity.

The break-even point

The break-even point is often thought of as the point at which a business goes from making a loss to making a profit.

More generally, if you have to choose between two options, and the best answer you can give is 'it depends', there may be a point at which you could be persuaded to change. For example, if asked to wear moccasins to their school formal most people would say 'no', but if they were offered \$1 million to wear them then the answer would probably be 'yes'. The answer depends on other factors, and there is a point where the decision will change.

The *break-even point* is where the decision changes.

A more realistic example would be when comparing two mobile phone plans. Which plan is the best for you? The answer depends on how you use your phone.

19 Break-even point

Jess is offered two mobile plans:

- Sensational Syd, which has a flat rate of 15 cents for every call
- Tellus, which charges \$10 per month plus 10 cents for every call.

- (a) If Jess makes an average of 50 calls per month, which plan should she choose?

THINKING

- 1 Create a worded equation to express the cost of calls per month on each plan.
- 2 Define appropriate variables and convert the worded equation to an algebraic equation in the form $y = a + bx$.
- 3 Calculate the cost of making 50 calls on each plan.
- 4 Compare the costs and interpret the answer.

WORKING

Cost for Sensational Syd is 15 cents for each call.
Cost for Tellus is \$10 plus 10 cents for each call.

Let n be the number of calls.
Let C be the monthly cost (\$) where C_S will represent the cost of Sensational Syd's plan and C_T will represent the cost of Tellus' plan.

$$C_S = 0.15n$$

$$C_T = 10 + 0.1n$$

$$\begin{aligned} C_S &= 0.15 \times 50 \\ &= \$7.50 \end{aligned}$$

$$\begin{aligned} C_T &= 10 + 0.1 \times 50 \\ &= \$15 \end{aligned}$$

Jess should choose Sensational Syd's plan if 50 calls are made per month. Sensational Syd's plan will cost $(\$15 - \$7.50)$, \$7.50 less than Tellus.

- (b) If Jess makes an average of 300 calls per month, which plan is cheaper and by how much?

- 1 Calculate the cost of making 300 calls, by substituting $n = 300$ into the equations created in part (a).

$$\begin{aligned} C_S &= 0.15 \times 300 \\ &= \$45 \end{aligned}$$

$$\begin{aligned} C_T &= 10 + 0.1 \times 300 \\ &= \$40 \end{aligned}$$

- 2 Compare the costs and interpret the answer.

The Tellus plan is $(\$45 - \$40)$, \$5 cheaper than Sensation Syd if 300 calls are made per month.

- (c) Determine the break-even point for the two plans, and explain Jess's options in terms of this value.

- 1 Equate the expressions and solve for n to calculate the number of calls made where the cost will be the same.

Break even when $C_S = C_T$

$$0.15n = 10 + 0.1n$$

$$0.05n = 10$$

$$\begin{aligned} n &= \frac{10}{0.05} \\ &= 200 \end{aligned}$$

- 2 Explain which option is cheaper in terms of the break-even point.

For 200 calls the cost is the same for both companies. Therefore, if Jess makes fewer than 200 calls per month she should go with Sensational Syd, but if she makes more than 200 calls then Tellus is cheaper.

Graphical solution for the break-even point

Another way to find the break-even point is to draw the two graphs on the same axes. The break-even point is found at the intersection of the two lines.

20 Calculating the break-even point graphically

Two Year 12 students are running an end-of-year social for the Year 8 students. The costs are \$800 for the hire and decoration of the hall, and a further \$5 per person for drinks and food. Up to 250 students may attend, and the cost of a ticket is \$15.

- (a) Create the equation for the cost of the social in the form $y = a + bx$.

THINKING

- 1 Create a worded equation for the cost.
- 2 Define the variables to create an algebraic equation.

WORKING

The cost is \$800 plus an additional \$5 for every student who attends.

Let x be the number of students attending and y be the cost of the event (\$).

$$y = 800 + 5x$$

- (b) Create the equation for the income of the social.

- 1 Create a worded equation for income.
- 2 Define the variables and form an algebraic equation.

The income is \$15 for every student who attends.

Let x be the number of people attending and y be the income (\$).

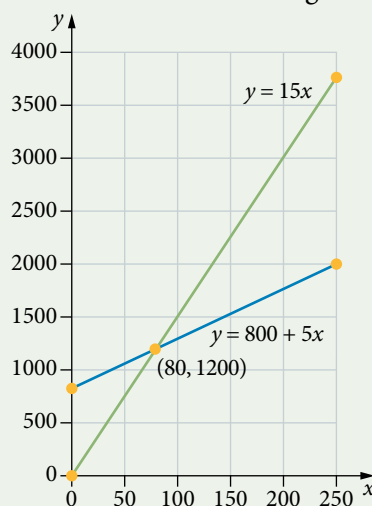
$$y = 15x$$

- (c) Graph the equations on the same set of axes and estimate the point of intersection.

- 1 Construct a suitable set of axes of an appropriate size, and plot both graphs over an appropriate domain.

Domain: $0 \leq x \leq 250$

$15 \times 250 = \$3750$ so range could be $0 \leq y \leq 3750$



- 2 Identify the point of intersection of the two graphs and read off the coordinates. Explain the significance of the break-even point for this situation.

The coordinates of the point of intersection are (80, 1200). The break-even point signifies that if fewer than 80 people attend, the event will make a loss. If more than 80 attend, the event will make a profit.

- (d) Use algebra to check the attendance at the break-even point found graphically.

At the break-even point the cost and income are equal. Equate the expressions for cost and income and solve for the unknown.

$$\begin{aligned}800 + 5x &= 15x \\800 &= 15x - 5x \\800 &= 10x \\x &= \frac{800}{10} \\x &= 80\end{aligned}$$

The break-even point for the event is 80 students.

Explore further

A graphical solution to simultaneous equations

Explore the solution to the system of simultaneous linear equations in the form $y = a + bx$, identifying the point of intersection.

Additional information

Solving simultaneous equations graphically

Determine the solution to a system of linear equations.

Spreadsheet solution for the break-even point

A spreadsheet can be used to find the point of intersection of two graphs if the break-even point is required.

21 Using a spreadsheet to determine the break-even point

A football club wants to hire caterers for its presentation night. A catering company charges \$12 per head plus a \$400 flat fee. Between 100 and 200 people are expected to attend. Use a spreadsheet to determine the break-even point, and from this calculate an appropriate entry fee for the presentation night.

THINKING

- 1 Create the worded equation for the cost and income for the club.

- 2 Open a spreadsheet and enter the following headings.

In column A, put the heading Number, then enter the sequence from the minimum to the maximum number going up in 1 s or 10 s or other suitable increments.

Head the other columns Income, Cost and Profit.

In the cells below the entered values, input a trial value for the entry fee, here: \$15.

WORKING

The cost is \$400 plus an additional \$12 for every person who attends.

Income is the entry fee for each person who attends.

	A	B	C	D
1	Number	Income	Cost	Profit
2	100			
3	110			
...	...			
10	180			
11	190			
12	200			
13	Entry fee	15		

- 3 Enter the formula for Income in the B column by entering $= B\$13 \times A2$ into B2 and filling down to the maximum value of the number column.

Note: The \$ sign in the formula keeps the value of the entry fee constant for each calculation.

Input the equation for Cost in the C column by entering $= 400 + 12 \times A2$ into C2 and filling down.

Input the equation for Profit in the D column by entering $= B2 - A2$ into D2 and filling down.

	A	B	C	D
1	Number	Income	Cost	Profit
		$B\$13 \times A2$	$400 + 12 \times A2$	$B2 - A2$
2	100	1500	1600	-100
3	110	1650	1720	-70
4	120	1800	1840	-40
5	130	1950	1960	-10
6	140	2100	2080	20
7	150	2250	2200	50
8	160	2400	2320	80
9	170	2550	2440	110
10	180	2700	2560	140
1	190	2850	2680	170
12	200	3000	2800	200
13	Entry fee	15		

- 4 Interpret the results.

If the entry fee is \$15 then the break-even point is between 130 and 140 people attending.

- 5 Change the value for the entry fee and reinterpret the results.

Number	Income	Cost	Profit
100	1600	1600	0
110	1760	1720	40
120	1920	1840	80
130	2080	1960	120
140	2240	2080	160
150	2400	2200	200
160	2560	2320	240
170	2720	2440	280
180	2880	2560	320
190	3040	2680	360
200	3200	2800	400
Entry fee	16		

A profit will be made if the entry fee is \$16 and more than 100 people attend.

- 6 Try other values for the entry fee and summarise your findings to provide advice to the club.

Number	Income	Cost	Profit
100	1400	1600	-200
110	1540	1720	-180
120	1680	1840	-160
130	1820	1960	-140
140	1960	2080	-120
150	2100	2200	-100
160	2240	2320	-80
170	2380	2440	-60
180	2520	2560	-40
190	2660	2680	-20
200	2800	2800	0
Entry fee	14		

If only 100 people are expected, an entry fee of \$16 will cover all costs. If you are confident that 200 people will attend, then the entry fee could be as low as \$14 to break even.

e Explore further

Solving simultaneous equations algebraically

Solve simultaneous equations algebraically using a spreadsheet.

EXERCISE

4.4

Simultaneous linear equations

Worked
Example

- 1 Solve the following pairs of simultaneous equations algebraically.

(a) $y = 3 + x$, $3x - 2y = -2$

(b) $3a + 4b = 22$, $a = 2b + 4$

(c) $m = 2 - n$, $2m - n = 16$

(d) $a - b = 2$, $3a - b = 6$

(e) $5x - y = 26$, $x - 3y = -6$

(f) $2c + d = 18$, $3c - d = 2$

(g) $2t + p = 5$, $p = 4t - 16$

(h) $2a - 4b = -7$, $a - b = -2$

(i) $z - \frac{t}{3} = -4$, $2t - 3z = 3$

- 2 Solve the following pairs of simultaneous equations graphically.

(a) $y = 4 + x$, $y = 5x$

(b) $y = 3 + x$, $y = 7 - x$

(c) $y = 7 - 2x$, $y = x - 2$

(d) $x + y = 10$, $y = 4x$

(e) $y = 4x - 1$ and $y = 7$

(f) $2x - y = 6$ and $x = -3$

- 3 A coffee and a cake cost \$8, and 3 cakes and a coffee cost \$14. Let x be the cost of a cake (\$) and y be the cost of a coffee (\$).

(a) Create a pair of simultaneous equations to represent the situation.

(b) Solve the pair of equations, to determine the cost of cake and coffee.

19

- 4 Ben has the choice of two mobile plans. Opt-in offers a rate of 20 cents for every call. Opt-out offers \$15 per month plus 10 cents for every call.
- If Ben makes an average of 60 calls per month, which plan should he choose?
 - If Ben makes an average of 200 calls per month, which plan should he choose?
 - Determine the break-even point for the two plans.
- 5 A company makes a pair of sunglasses for \$10 and sells them for \$40 a pair. It also has fixed costs of \$60 000 each month for sales and distribution.
- Select the option that shows how many pairs of glasses must be sold each month to break even.
 A 1200 B 1500 C 2000 D 6000
 - Which option would a student choose if they had incorrectly combined the cost price and selling price?
- 6 The cost of materials and production of an MP3 player is \$75 per unit and the fixed costs are \$125 000. The selling price for each player is \$110.
- Select the option that shows the cost C (\$) of producing x units.
 A $C = 75x + 125000$ B $C = 75x + 110$
 C $C = 75x - 110$ D $C = 75x - 125000$
 - Select the option that shows the income (\$) from selling x units.
 A income = $110x - 125000$ B income = $110x$
 C income = $75x + 125000$ D income = $35x - 125000$
 - Determine the number of sales, rounding up to the next whole number, for the company to break even.
 - Calculate the income at the break-even point.
- 7 The income, I (\$), from selling x T-shirts is given by $I = 30x$. The cost C (\$) of production is given by $C = 2500 + 10x$.
- Determine the selling price of a single T-shirt.
 - Identify the fixed cost of production.
 - Determine the individual cost of production for a T-shirt.
 - How many T-shirts must be sold to break even?
 - Calculate the profit if 200 T-shirts are sold.
 - How many T-shirts must be sold to make a profit of \$10 000?
- 8 The organising committee for the Year 11 formal has two quotes. The Lakehouse will charge a base fee of \$250 for set-up, plus \$40 per head for food and drink. The Shed is offering a \$300 base fee, plus \$35 per head for food and drink.
- Express the information as a pair of simultaneous equations, with c representing the total cost in dollars and n representing the number of people attending.
 - Solve the simultaneous equations to find the number of people who must attend for the costs to be the same.
 - If 100 people attend, will The Shed be cheaper or more expensive than The Lakehouse?

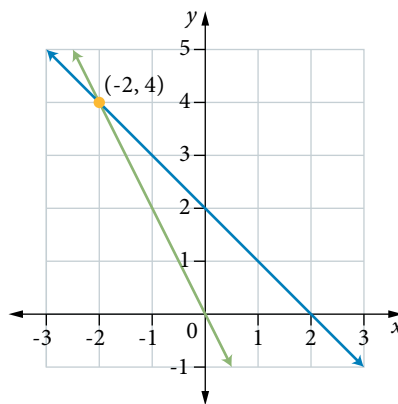
- 9 A bag contains 22 pieces of fruit. If you removed a lemon there would be exactly twice the number of mandarins as lemons remaining. Let M be the number of mandarins and L be the number of lemons in the bag.

- Create a pair of simultaneous equations to represent the situation.
- Solve the pair of equations.

- 10 Viv and Pym are planning a costume ball as a fundraiser for the local football team. They intend to hire the clubrooms for \$400, and a DJ for \$800, and spend \$500 on decorations, ticketing, marketing and printing. In addition, food and drinks will cost \$38 a head. The clubrooms can cater for up to 100 guests. They decide to sell tickets for \$70 each.

- Create an equation for the costume ball expenses.
- Create an equation for the income from the costume ball.
- Graph both equations on the same set of axes to find the break-even point, to the nearest whole number.
- Determine the break-even point algebraically.

- 11 Determine the rule for each of the graphs. To justify your answers, demonstrate that the point of intersection is valid for each line.



- 12 A textbook costs \$20 per book to print and has an initial set-up cost of \$120 000 for writing and editing. It has potential sales of up to 10 000 books. Use a spreadsheet or other technology to calculate the sale price of a single book in order to break even, if:

- 4000 copies are printed and sold
- 6000 copies are printed and sold
- 200 copies are printed and sold.

- 13 Porky Pizzas sells family-sized gourmet pizzas for \$12 each. The cost of ingredients for each pizza is \$4.80 and the running costs of the shop are \$840 per day.

- Create an equation for the income per day, in the form $y = a + bx$.
- Create the cost equation in terms of pizzas sold per day.
- Plot the graphs for daily cost and income on the same set of axes. Determine the approximate coordinates of the point of intersection.
- From the graph, determine the number of pizzas Porky Pizzas must sell in a day to break even.
- Using algebra, calculate the number of pizza sales needed to break even, to the nearest whole number.
- Plot the graph of daily profit against sales for the same domain as the previous graphs. Determine the coordinates of the point on the graph that gives the break-even point for pizza sales.
- Given that the annual sales are 100 000 and they are open for 360 days per year, what is the annual profit to the nearest dollar? Write any assumptions you made.

- 14 Rebecca is arranging a party for her daughter and is considering two options for catering. Mobile Pancakes charges a callout fee of \$80 and then \$10 a head to serve two pancakes for each person. Finger Foodz will supply finger food for a flat \$12 a head. There will be between 30 and 80 people attending.
- How many people must attend for the costs to be the same for the two companies?
 - Assuming that cost is the only issue, provide Rebecca with advice on which company she should employ, depending on the number of guests who attend.
- 15 A community centre is considering offering a beginner's course in using a computer. The costs will be \$5000 to hire a tutor, rental costs of \$1800 and an advertising cost of \$250. Between 10 and 30 people are expected to enrol. Courses must run at a profit if they are to go ahead. Determine the break-even point for the fee in terms of enrolment numbers and provide advice to the board on how much should be charged.
- ...
- 16 The straight line joining the points $(4, 3)$ and $(6, 7)$ is in the form $y = a + bx$. Create two simultaneous equations and solve to determine values of a and b to identify the equation of the line.
- 17 Solve the following pairs of simultaneous equations algebraically.
- $4b + 3j = 30$, $3b - 4j = 10$
 - $5p - 4q = 14$, $3p + 2q = 4$
 - $4a - 3b = 14.3$, $2a + 4b = 0$
 - $3m - 5n = -1$, $2n - 6m = -2.8$
 - $2x + 3y = 14.5$, $5x + 7y = 38$
- 18 Marissa wishes to hire a car for a day and has two rental car options. Star Cars charges \$70 per day plus 10 cents per km travelled. Roger's Rental charges \$50 a day plus 40 cents per km.
- Let C represent the cost in dollars, and d represent the distance travelled in kilometres. Demonstrate on a graph the distance that needs to be travelled for the cost to be the same. State the distance to the nearest 10 km.
 - Marissa is expecting to travel 180 km a day. Use the graph to demonstrate which company is offering the cheaper deal. Support your answer with algebraic calculations and a summary in words.
- 19 Adam is considering expanding his small farm by buying some more calves. The fixed costs are \$6000 per year. He decides to buy beef calves, which he will fatten and sell after 12 months. The cost of each calf is \$250 and he spends an extra \$45 per animal on feed and transport per year.
- If Adam buys 40 calves and all goes as planned:
 - Calculate the break-even price for the sale of the calves.
 - Determine the sale price that will yield a profit of \$2000.
 - Unfortunately, two of the calves die from a disease, and the visit from the veterinarian costs \$200. Recalculate the sale price for both the break-even point and the \$2000 profit outcome. Round up to the nearest dollar each time.
- 20 A company manufactures MP3 players for \$50 per unit plus \$400 000 fixed costs per year. The annual sales and marketing cost is \$50 000. Last year, the company sold 20 000 players at \$95 each.
- Calculate the profit made last year.
 - The company is considering a marketing proposal that will cost an additional \$1 million and is predicted to double sales. Decide whether it should accept this proposal.

Piece-wise linear graphs and step graphs

Varying rates

Consider the following situation.

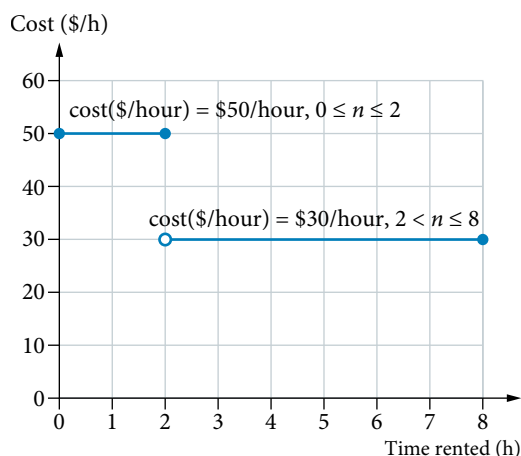
The cost of hiring a small boat is \$50 per hour for the first 2 hours, and \$30 per hour for each hour after that. The maximum rental time is 8 hours.

There is a specific period of time for which each rate is valid. This is the *domain* for each of the linear functions.

This information can be displayed graphically in two ways: using a step graph to show the rate for each period of time, or a piece-wise linear graph to show the actual amounts.

Step graph

A step graph consists of one or more non-overlapping horizontal lines that follow a step-like pattern. The step graph below shows the hourly rate depending on how long the boat is rented.



A closed circle indicates an included point. This will be referenced with a $\leq$ or $\geq$ symbol in an inequality.



An open circle indicates an end-point that is not included. This will be referenced with a $<$ or $>$ symbol in an inequality.

WARNING

Take care when reading step graphs, as they may provide information as per unit or total amount on the vertical axis.

Explore further

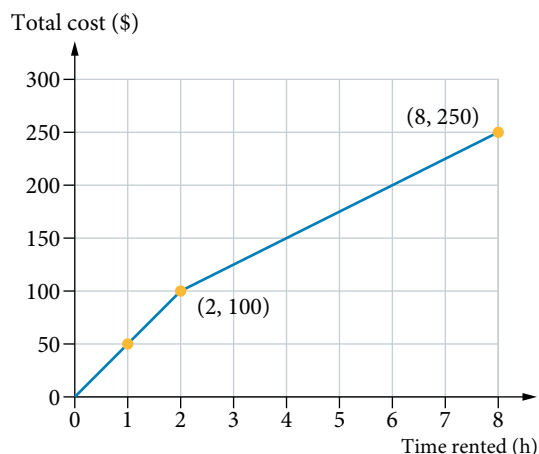
Piece-wise functions and step graphs

Create and analyse step graphs and piece-wise functions using graphing software.

Piece-wise linear graph (line-segment graph)

A piece-wise linear graph is a graph consisting of one or more non-overlapping line segments. It is sometimes called a line segment graph. Each segment follows a different rule depending on the domain. The piece-wise linear graph at right shows the total cost of the rental according to the time the boat is rented.

Consider the following example concerning a water bill.



22 A step graph

To encourage water conservation, the local water supply company charges different rates depending on how much water a household uses. The charges increase for just the portion of water that exceeds the lower rate range.

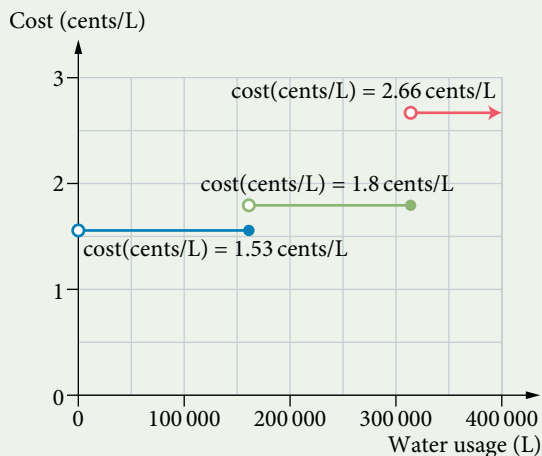
Water usage (L)	Cost (c/L)
0–160 000	1.53
160 000–320 000	1.8
More than 320 000	2.66

(a) Construct a step graph to display this information.

THINKING

Construct a suitably scaled set of axes. For each separate domain, plot the cost per litre.

WORKING



(b) Determine the cost of 120 000 L.

1 Identify the rate from the table and multiply this by the number of litres used.

$$120\,000 \times 1.53 = 183\,600 \text{ c}$$

2 Convert the answer from cents to dollars and interpret the result.

Cost of 120 000 L: \$1836

The total cost of 120 000 L is \$1836.

(c) Determine the cost of 200 000 L.

- 1 Calculate the cost of the first 160 000 L of water at the cheapest rate.

$$160\,000 \times 1.53 = 244\,800 \text{ c}$$

Cost of the first 160 000 L: \$2448

- 2 Calculate the charge for the volume of water above 160 000 L at the medium rate.

Volume above 160 000 L:

$$200\,000 - 160\,000 = 40\,000 \text{ L}$$

40 000 L of water is charged at the higher rate of 1.8 c/L.

$$40\,000 \times 1.8 = 72\,000 \text{ c}$$

The cost for the final 40 000 L is \$720.

- 3 Determine the total cost by adding the two charges together.

The total cost of 200 000 L is

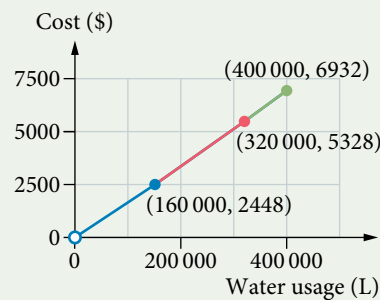
$$\$2448 + \$720 = \$3168.$$

(d) Construct a piece-wise linear graph to represent the cost of consumption from 0 to 400 000 L.

- 1 Calculate the cost of usage at the endpoints of the graph and the critical points where the costs change.

Water usage	Cost
0 L	0
160 000 L	$160\,000 \times 1.53 = 244\,800 \text{ c}$ = \$2448
320 000 L	$244\,800 + 160\,000 \times 1.8 = 532\,800 \text{ c}$ = \$5328
400 000 L	$532\,800 + 40\,000 \times 2.66 = 639\,200 \text{ c}$ = \$6932

- 2 Construct a suitably scaled set of axes and plot the critical points. These segments are linear, so connect the points with straight lines.



In the previous example, the equation for the piece-wise graph changes at each critical point, becoming steeper as you move from left to right. Graphs that have a different rule for different segments of the domain are called *piece-wise* (or *hybrid*) *graphs*. The information for the previous worked example is shown in the table.

Water usage (L)	Cost (c/L)
0–160 000	1.53
160 000–320 000	1.8
More than 320 000	2.66

If the volume of water in litres is w , then the relationship could be written as:

$$\text{Cost} = \begin{cases} 1.53w, & 0 < w \leq 160\,000 \\ 2448 + 1.8w, & 160\,000 < w \leq 320\,000 \\ 6932 + 2.66w, & w > 320\,000 \end{cases}$$

To sketch a piece-wise graph, calculate the key points where the rule changes at the end of each segment, and then plot the segments.

The first rule, $\text{Cost} = 1.53w$, is used for volumes greater than 0 L up to and including 160 000 L, so $0 < w \leq 160\,000$ is used to include the end value but not include the value at the start.

23 A continuous piece-wise graph

Plot the following piece-wise graph.

$$y = \begin{cases} 2 + x, & x \leq 0 \\ 2x + 2, & 0 < x \leq 10 \\ 32 - x, & x > 10 \end{cases}$$

THINKING

- 1 Calculate the endpoint for each segment by substituting in the extreme values of the domain. Where the extreme is not specified, choose a representative value.
- 2 Construct a suitable set of axes, scaled to cover the required domain, then plot the calculated points and join them. Include arrowheads where required.

WORKING

For $x \leq 0$: $y = 2 + x$

$$x = 0 \rightarrow y = 2 + 0 = 2: (0, 2)$$

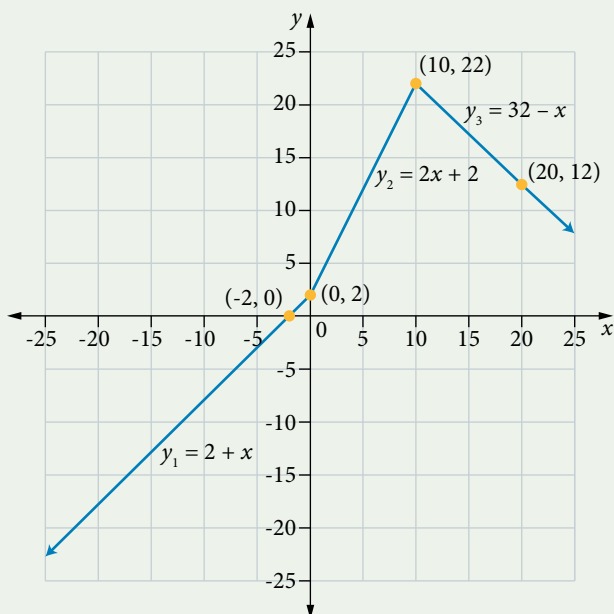
$$x = -2 \rightarrow y = 2 - 2 = 0: (-2, 0)$$

For $0 < x \leq 10$: $y = 2x + 2$

$$x = 10 \rightarrow y = 2 \times 10 + 2 = 22: (10, 22)$$

For $x > 10$: $y = 32 - x$

$$x = 20 \rightarrow y = 32 - 20 = 12: (20, 12)$$



In the previous example, the graph is considered to be *continuous* because the coordinates at the critical points are the same for both segments of the graph. In other words, the line for a continuous graph can be drawn without lifting your pen.

This is not always the case. Some graphs are *discontinuous* at a critical x value, where substituting the same value of x gives a different y value for the rules either side of that x value. Calculate the values in the same way but use each rule separately for the same x value.

24 A discontinuous piece-wise graph

Plot the following piece-wise graph.

$$y = \begin{cases} x, & x < 0 \\ 5 + x, & 0 \leq x \leq 4 \\ 6 - x, & 4 < x < 9 \end{cases}$$

THINKING

- 1 Calculate the endpoints for each segment by substituting in the extreme values of the domain.

- 2 Construct a suitable set of axes, scaled to cover the required domain, then plot the calculated points and join them. *Note:* An open circle is used to indicate a point that is not included and a closed circle indicates an included point. Include arrowheads where required. In this case:

Line 1 goes through the points $(-3, -3)$ and $(0, 0)$, continuing through to the left past $x = -3$ with an arrow at the left end and a hollow dot at the right.

Line 2 joins the points $(0, 5)$ and $(4, 9)$ with solid dots at both ends.

Line 3 joins the points $(4, 2)$ and $(9, -3)$ with an open dot at both ends.

WORKING

For $x < 0$: $y = x$

$$x = -3 \rightarrow y = -3: (-3, -3)$$

$$x = 0 \rightarrow y = 0: (0, 0)$$

For $0 \leq x \leq 4$: $y = 5 + x$

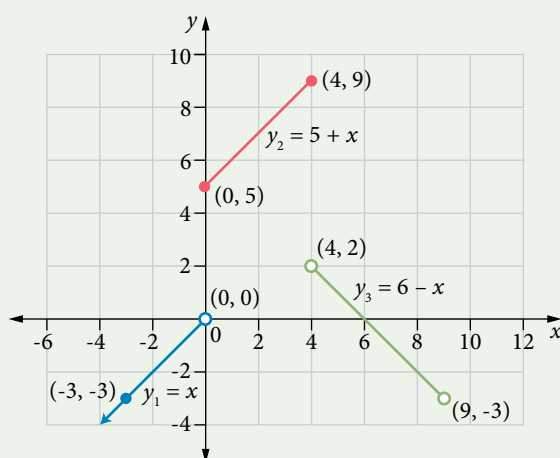
$$x = 0 \rightarrow y = 5 + 0 = 5: (0, 5)$$

$$x = 4 \rightarrow y = 5 + 4 = 9: (4, 9)$$

For $4 < x < 9$: $y = -x + 6$

$$x = 4 \rightarrow y = 6 - 4 = 2: (4, 2)$$

$$x = 9 \rightarrow y = 6 - 9 = -3: (9, -3)$$



EXERCISE

4.5

Piece-wise linear graphs and step graphs

Worked
Example

22

- 1 The table gives information on variable speed over time.

- (a) Construct a step graph of speed versus time over the given domain.
(b) Construct a piece-wise linear graph of total distance travelled against time.

Time (seconds)	Speed (m/s)
$0 \leq t < 5$	6
$5 \leq t < 15$	4
$15 \leq t \leq 30$	2

- 2 The table gives information on variable rate of pay over a 16 h day.

- (a) Draw the step graph of rate of pay versus time over the given domain.
(b) Draw the piece-wise linear graph of total pay against time.

Time (hours)	Rate of pay (\$/h)
$0 \leq t \leq 8$	16
$8 < t \leq 12$	24
$12 < t \leq 16$	32

- 3 Natural gas bills are charged at the following rates according to usage over a 2 month period.

Usage (MJ)	Cost
0 to 4000	0.8457 c/MJ for each MJ
More than 4000 to 8000	3382.8 c plus 0.9709 c for each MJ over 4000
More than 8000	7266.4 plus 0.8182 c for each MJ over 8000

- (a) Copy and complete the table and then plot a piece-wise graph for cost (cents) for up to 10 000 MJ of electricity.

Usage (MJ)	0	4000	8000	10 000
Cost				

- (b) Let x be the volume of gas used in MJ and C be the cost of gas in cents.

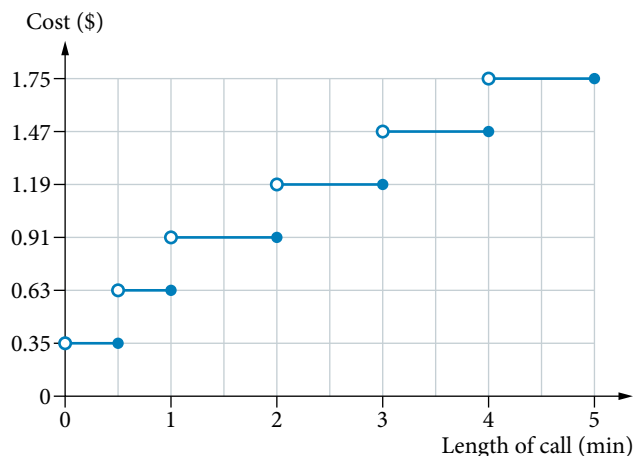
The cost equation for $0 < x \leq 4000$ is $C = 0.8457x$ and the cost equation for $4000 < x \leq 8000$ is $C = 3382.8 + 0.9709(x - 4000)$.

Create a cost equation for $x > 8000$.

- (c) Using the graph, determine the approximate cost of using 5000 MJ of gas. Write your answer to the nearest dollar.
(d) Use the formula to calculate the cost of 5000 MJ of gas, to the nearest cent.

- 4 In the plan offered by a mobile phone company, the cost of a call depends on the length of the call, as shown in the graph.

- (a) Determine the cost of a 50 s call.
(b) Determine the cost of a 1 minute call.
(c) Calculate the longest call that can be made for under \$1.50.



- (d) On a particular day, calls are made lasting 30 s, 65 s, 90 s and 4 m 20 s. Determine the total cost of the calls.
- (e) For expressing the step graph algebraically, let C be the cost of a call (\$) and let x be the length of a call (minutes).
- (i) Select the correct equation and relevant domain for calls between 0 and 5 minutes.
- A $C = 0.63, 0 < x \leq 1$ B $C = 0.91, 1 \leq x < 2$
 C $C = 1.47, 2 < x \leq 3$ D $C = 1.75, 4 < x \leq 5$
- (ii) Correct the domains for the incorrect options and write the complete set of equations with relevant domains.

- 5 The income tax schedule for Italy in a certain year is shown in the following table.

Income bracket (euro)	Tax rate
0–15 000	€0.23 for every euro
15 001–28 000	€3450 + €0.27 for every euro over 15 000
28 001–55 000	€6960 + €0.38 for every euro over 28 000
55 001–75 000	€17 220 + €0.41 for every euro over 55 000
75 001 and over	€25 420 + €0.43 for every euro over 75 000

- (a) Choose the option that shows the correct step equation and relevant domain for the taxation rate on income from 0 to €55 000, where y is the tax rate in euro/euro earned and x is taxable income in euro.

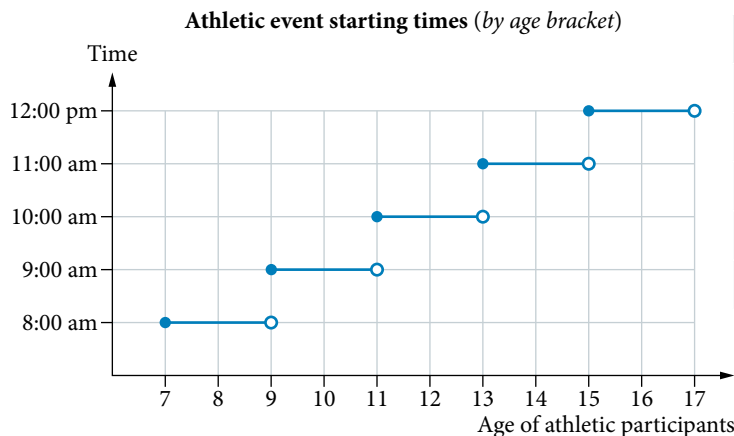
A $y = \begin{cases} 0.23, & 0 < x \leq 15000 \\ 3450, & 15000 < x \leq 28000 \\ 6960, & 28000 < x \leq 55000 \end{cases}$

B $y = \begin{cases} 0.23, & 0 < x \leq 15000 \\ 0.27, & 15000 < x \leq 28000 \\ 0.38, & 28000 < x \leq 55000 \end{cases}$

C $y = \begin{cases} 0.23, & 0 < x < 15000 \\ 0.27, & 15000 \leq x < 28000 \\ 0.38, & 28000 \leq x < 55000 \end{cases}$

D $y = \begin{cases} 0, & 0 < x \leq 15000 \\ 0.23, & 15000 < x \leq 28000 \\ 0.27, & 28000 < x \leq 55000 \end{cases}$

- (b) Draw a step graph showing the rate of tax for each income level up to 55 000 euro.
- (c) Is this graph continuous?
- 6 Using the data in the graph, copy and complete the table showing the starting time for each age group.



Age	Starting time	Age	Starting time
7		12	
8		13	
9		14	
10		15	
11		16	

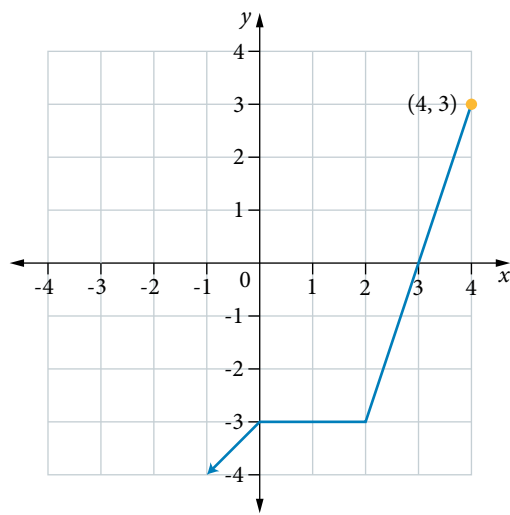
- 7 Electricity prices increase according to usage every 3 months, as outlined in the table.

First 1000 kWh	21.8680 c/kWh
Remaining kWh	23.2760 c/kWh

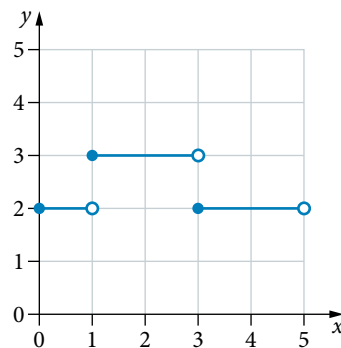
- (a) Calculate the cost of using 800 kWh.
 (b) Calculate the cost of using 1600 kWh.
 (c) Draw a piece-wise graph to show the cost of electricity for usage between 0 and 1600 kWh.
 (d) Why do you think the price increases after 1000 kWh?

- 8 Determine the piece-wise equation and domain for each graph.

(a)



(b)



- 9 The local water supply company charges different rates depending on the volume of water a household uses. The charges are shown in the table.

Consumption	Charge
0–150 000 L	1.2 c/L
More than 150 000–350 000 L	1.9 c/L
More than 350 000 L	3.2 c/L

- (a) Draw a step graph to show the rates charged at the different consumption levels up to 500 000 L. Only the charge rate is required, not the total cost.
 (b) Construct a piece-wise linear graph to represent the total cost (\$) for consumption from 0 to 500 000 L.

Worked
Example

23

- 10 Plot the following continuous piece-wise graphs.

(a)

$$y = \begin{cases} 1 + x, & x \leq 0 \\ 5x + 1, & 0 < x \leq 10 \\ 61 - x, & x > 10 \end{cases}$$

(b)

$$y = \begin{cases} 4x, & x \leq 0 \\ -x, & 0 < x \leq 5 \\ 3x - 20, & 5 < x \leq 8 \end{cases}$$

24

- 11 Plot the following discontinuous piece-wise graphs.

(a)

$$y = \begin{cases} 2, & x < 0 \\ 2x - 3, & 0 \leq x \leq 5 \\ \frac{9}{5}x - 16, & 5 < x < 10 \end{cases}$$

(b)

$$y = \begin{cases} -x, & x \leq 0 \\ 4, & 0 < x \leq 5 \\ 8 - \frac{2}{5}x, & 5 < x \leq 10 \end{cases}$$

- 12 Waterworks supplies water to households. They charge a fixed quarterly maintenance fee of \$40, and then \$2.50 a kilolitre for up to 50 kL of water supplied per quarter. After 50 kL the price rises to \$2.95 for every additional kilolitre.
- Create a system of linear equations that links the cost c (\$) of water per quarter and the number of kilolitres supplied n .
 - Sketch a graph to show the situation for water for up to 100 kL supplied per quarter.
 - Ashan receives a bill for close to \$200. Use your graph to determine his water usage, to the nearest kilolitre, for the quarter. Check your answer algebraically.

- 13 The income tax rates in Australia are said to be progressive because the rate increases as your income goes above set thresholds. The table shows the rates of tax that applied to the lowest three thresholds, almost a decade ago.

Income bracket	Tax
0 – \$6000	\$0
\$6001 – \$37 000	15 c for each \$1 over \$6000
\$37 001 – \$80 000	\$4650 plus 30c for each \$1 over \$37 000

- Plot a step graph to display the rate of taxation for each tax bracket from \$0 to \$80 000.
 - Plot a piece-wise graph to show the tax paid for incomes from 0 to \$50 000.
- 14 Gemma installs a cylindrical water tank to collect rainwater for her garden. It has a 10 000 L capacity and as part of the installation has 2000 L of water pumped in. Over the next 100 days, rain falls on 6 days, as shown in the table.

Day	12	20	35	75	85	90
Water collected	1000 L	1400 L	800 L	300 L	2400 L	800 L

- Plot a graph to show the volume of water over the first 100 days if no water is taken out of the tank.
 - There is a dry spell between days 35 and 75, so Emma waters her garden on days 55 and 65. Redraw the graph, assuming each watering uses 1000 L.
- 15 In 2009 a new flat taxation rate of 20 cents per dollar was proposed to replace the progressive tax rate for 2009–2010 shown in the table below.

Income bracket	Tax
\$0–6000	\$0
\$6001–35 000	15 c for each \$1 over \$6000
\$35 001–80 000	\$4350 plus 30c for each \$1 over \$35 000
\$80 001–180 000	\$17 850 plus 38c for each \$1 over \$80 000
\$180 001 and over	\$55 850 plus 45c for each \$1 over \$180 000

- Plot a graph of income versus tax paid, for the progressive and the flat tax rate. Draw both graphs on the same axes for incomes from \$0 to \$100 000.
- Calculate the break-even point or points and discuss the income levels that would be advantaged and disadvantaged by this proposal.

Summary

Linear equations

A linear equation contains an equals symbol ($=$), and one or more variables that are all raised to the power of 1.

To solve an equation, visualise the steps necessary to create the equation and then reverse the operations to find the solution. The steps used to create an equation must be reversed in the opposite order in which they were applied. These reverse operations are more formally known as inverse operations.

Operation	Inverse operation
+	−
−	+
$\times$	$\div$
$\div$	$\times$

Linear graphs

The horizontal x -axis and the vertical y -axis divide the grid into four quadrants. The intersection of the two axes is called the origin.

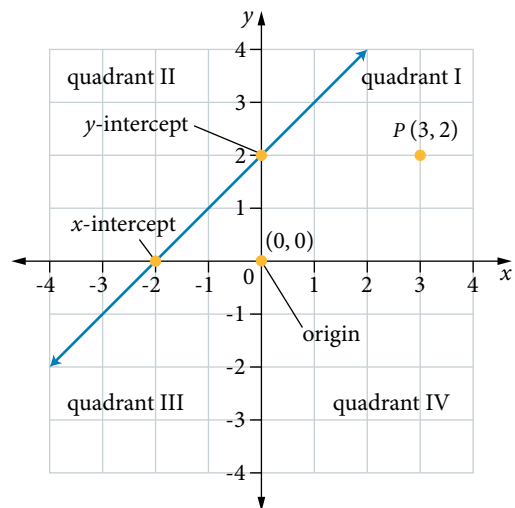
A point is located by a coordinate pair (x, y) .

The point where the graph crosses the x -axis is called the x -intercept. To find the x -intercept, substitute $y = 0$ into the equation and solve for x .

The point where the graph crosses the y -axis is called the y -intercept. To find the y -intercept, substitute $x = 0$ into the equation and solve for y .

A linear graph is represented by the general form of $y = a + bx$ where a is the y -coordinate of the y -intercept and b is the gradient.

A straight-line graph has a constant slope or gradient. The gradient b is a measure of the steepness of a graph and is found using $b = \frac{\text{change in } y}{\text{change in } x}$. From two points (x_1, y_1) and (x_2, y_2) the formula becomes $b = \frac{y_2 - y_1}{x_2 - x_1}$.



Simultaneous equations

The simultaneous solution to two equations with two variables can be found algebraically by the method of substitution. This involves isolating a variable in one equation and substituting into the other.

The intersection of the graphs of the two equations displays the simultaneous solution – that is, the value for each variable that satisfies both equations.

Break-even analysis

For graphs comparing the costs of two plans, the point where the graphs intersect is called the break-even point. This is the number (of phone calls, for example) where the two plans cost the same.

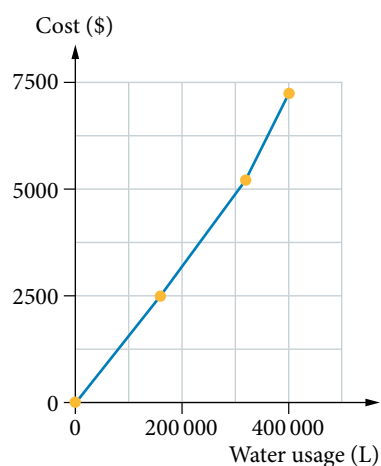
For graphs comparing income with costs, the break-even point shows the number of sales at which income = costs.

Piece-wise linear graphs and step graphs

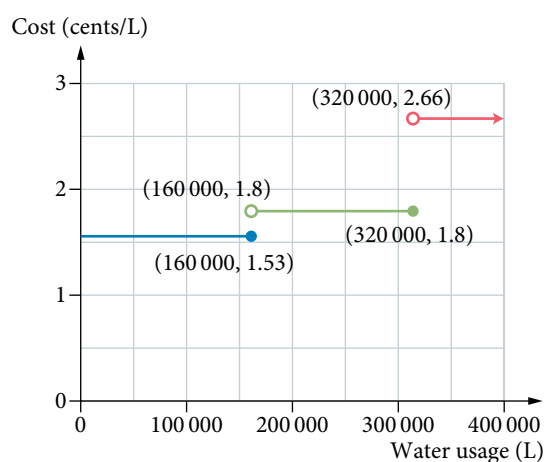
Many graphs are useful for a specific set of x values. The specific set of x values is called the domain. A graph with different rules for different segments of the domain is called a piece-wise graph.

If the lines of a piece-wise graph are all horizontal, this is a step graph.

Piece-wise graph



Step graph



$$\text{Cost} = \begin{cases} 1.53w, & 0 < w \leq 160\,000 \\ 2448 + 1.8w, & 160\,000 < w \leq 320\,000 \\ 6932 + 2.66w, & w > 320\,000 \end{cases}$$



A closed circle indicates an included point. This will be referenced with a $\leq$ or $\geq$ symbol in an inequality.



An open circle indicates an end-point, which is not included. This will be referenced with a $<$ or $>$ symbol in an inequality.

Chapter review

4

- Exercise 4.1 1 For the graph of $y = 5x - 6$ determine the coordinates of the point where $x = 1$.
- Exercise 4.1 2 For the graph of $4x - 5y = 20$ determine the coordinates of the point where $x = 5$.
- Exercise 4.3 3 Determine the equation of the linear function with:
- (a) a gradient of 5 and a y -intercept of 3
 - (b) a gradient of $-\frac{2}{3}$ and coordinates of the y -intercept $(0, 4)$
 - (c) a y -intercept at $y = -1$ and rate of decrease of 0.6 per horizontal unit of increase.
- Exercise 4.3 4 Determine the equation of a graph with an x -intercept of -4 and a y -intercept of 6.
- Exercise 4.3 5 Determine the coordinates of the y -intercept of the graph $3x + 6y = 24$.
- Exercise 4.1 6 Solve the equation $6x - 8 = 16$ for x .
- Exercise 4.1 7 Solve the equation $2(j + 3) = 18$ for j .
- Exercise 4.1 8 Solve the equation $\frac{2(f - 6)}{5} = -2$ for f .
- Exercise 4.4 9 Determine the solution to the pair of simultaneous equations $a + b = 7$ and $a - 2b = 4$.
- Exercise 4.2 10 Calculate the gradient of the line joining the points $(1, 4)$ and $(5, 8)$.
- Exercise 4.1 11 Select the option that is not a linear equation.
- A $6x - 8$ B $5x - 10 = 30$ C $\frac{2(m - 3)}{5} = -2$ D $7x + 3y = 21$
- Exercise 4.5 12 Select the statement that best describes the difference between a continuous and a discontinuous piece-wise graph.
- A A continuous graph is a straight line.
 - B A continuous graph has a value of y for every value of x .
 - C A discontinuous graph has limits at one or both ends.
 - D A discontinuous graph cannot be drawn without lifting your pencil from the page.
- Exercise 4.1 13 Dan is currently d years of age. Create an expression to represent:
- (a) Dan's age in 6 years' time
 - (b) half Dan's age
 - (c) four times Dan's age plus 3 years.
- Exercise 4.1 14 Sonja receives \$1.85 in change after buying five chocolate bars with a \$10 note. What is the average price of the chocolate bars? Create an algebraic equation to support your solution.
- Exercise 4.3 15 Sketch the following graphs.
- (a) $y = 1 + \frac{4}{5}x$ (b) $y = -3 - \frac{x}{2}$
 - (c) $2y - 5x = 20$ (d) $3y + 6x = 18$

- 16 Sketch the piece-wise graph showing:

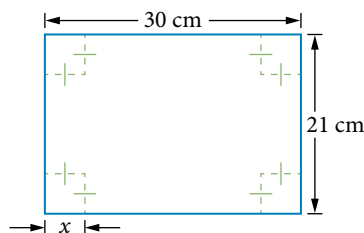
$$y = \begin{cases} x, & x < 5 \\ 5, & 5 \leq x \leq 10 \\ 15 - x, & x > 10 \end{cases}$$

Exercise 4.5

- 17 An open box is to be made from a sheet of A4 paper. (Assume the dimensions of the A4 sheet are 21 cm by 30 cm.)

Exercise 4.1

A square, measuring x by x , is cut from each corner and the sides folded up to make the box.



- Express all the dimensions of the box in terms of x .
- Show that volume = $x(21 - 2x)(30 - 2x)$.
- Determine the volume for $x = 2$, $x = 5$ and $x = 10$.
- Calculate the maximum and minimum values of x for the box to exist.
- Is $V = x(21 - 2x)(30 - 2x)$ a linear equation? Explain your answer.

- 18 Solve the equation $4(e - 5) + 6 = 2e$ for e .

Exercise 4.1

- 19 Solve the pair of simultaneous equations $2x + 3y = 17$ and $x - 2y = -2$. Display the solution graphically.

Exercise 4.4

- 20 The perimeter of a rectangular paddock is 248 m. The length l (m) is 8 m longer than the width.

Exercise 4.4

- Create an equation in terms of l to represent the situation.
- Calculate the dimensions of the paddock.

- 21 In 6 years' time Mang Mang will be twice as old as her daughter Christy. Currently their ages add to 54 years.

Exercise 4.4

- Express the information as a pair of simultaneous equations with two variables.
- Display the equations graphically, and solve algebraically to find the ages of the mother and daughter.

- 22 For each of the following, write a pair of simultaneous equations with two variables and then solve.

Exercise 4.4

- The sum of two numbers is 29 and their difference is 11. Determine the numbers.
- Jacob scores 78 for 18 holes of golf. He scores 6 less on the back nine holes than on the front nine. Calculate his score on each of the back nine holes and the front nine holes.
- A garage contains some bicycles and cars. The 28 vehicles have 88 wheels. How many bicycles are there, and how many cars?

Exercise 4.5

- 23 A household electricity bill is shown.

Account period:		18 Apr 2020 to 16 Jul 2020 – Bill days 89				
Next anticipated reading between:		15 Oct 2020 & 18 Oct 2020				
Electricity Tariff – Home						
	Meter number	Previous reading	Current reading	Usage kWh	@Rate c/kWh	Amount \$
From 18 Apr 2020 to 16 Jul 2020 – Bill days 89						
Peak use						
Energy	6 129 894	90 896	92 545	1020	13.4600	137.29
				629	12.5500	78.94
Total kWh				1649		
Off peak use						
Off peak	6 672 518	108 688	110 019	1331	7.2800	96.90
Other charges						
Service to property						37.38
Subtotal						\$350.51
Electricity Tariff – Green						
			Your green contribution			Amount \$
From 18 Apr 2020 to 16 Jul 2020 – Bill days 89						
20% Green power				\$1.818/week		23.07
Subtotal						\$23.07
Total electricity charges						\$373.58
GST						\$37.36
Vic Gov network tariff rebate		Peak use		1000	0.20c	2.00
		Off-peak use		1000	0.35c	3.50
Total current account charges						\$405.44

- Identify the charge per kWh for the first 1020 kWh of peak energy used.
- Determine the charge per kWh for peak energy used above 1020 kWh.
- Determine the charge per kWh for off peak energy.
- Draw a step-wise graph to show the cost per kWh for the different levels of peak usage.
- Explain how the charges are designed to reduce the use of peak energy.

Exercise 4.4, 4.5

- 24 An internet company is offering two internet plans.

Plan 1 has a base charge of \$20 for 500 MB download and then an additional 50 cents for every MB in excess of 500 MB.

Plan 2 has a base charge of \$35 for 800 MB download and then an additional 40 cents for every MB in excess of 800 MB.

- Express the cost of each plan as an equation, where n is the number of MB used.
- Determine the break-even point by solving simultaneous equations.

Exercise 4.4, 4.5

- 25 The income tax rates for single taxpayers earning less than \$70 000 in the United States and in New Zealand for a certain year are shown in the table (continued next page).

US tax bracket	Tax
\$0–35 020	15 cents per dollar
\$35 021–84 872	\$5253 plus 28 cents for every \$1 over \$35 020

Exercise 1.2

- 1 Calculate the hourly rate of pay for Amy, who earns \$287.50 for 25 h of work.

Exercise 1.4

- 2 Logan invested \$24 000 at a simple interest rate of 5.25% p.a. for 3 years.

- (a) Determine the total interest earned.
- (b) Calculate the balance of his investment.

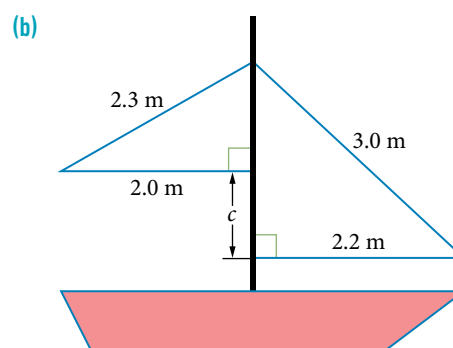
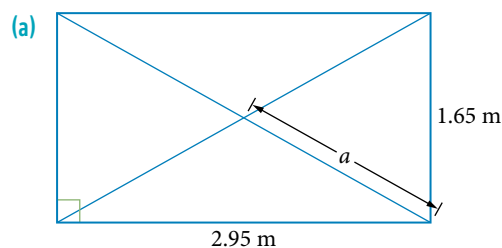
Exercise 1.6

- 3 Sara needs to make some quick sales, so she decides to reduce prices on a variety of goods. For each of the following, calculate the price reduction to be applied.

- (a) 7% off \$450
- (b) 37.5% off \$880

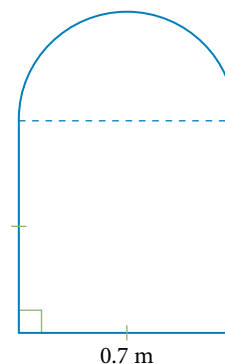
Exercise 2.1

- 4 Determine the value of each pronumeral, to 2 decimal places.



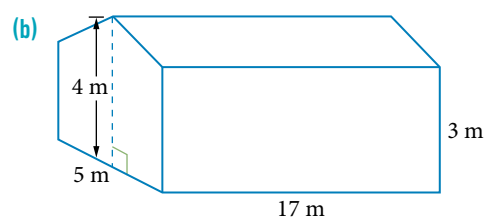
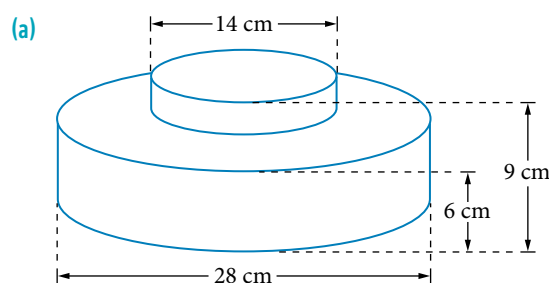
Exercise 2.2

- 5 Determine the perimeter and area of the shape. Give your answers to 2 decimal places.



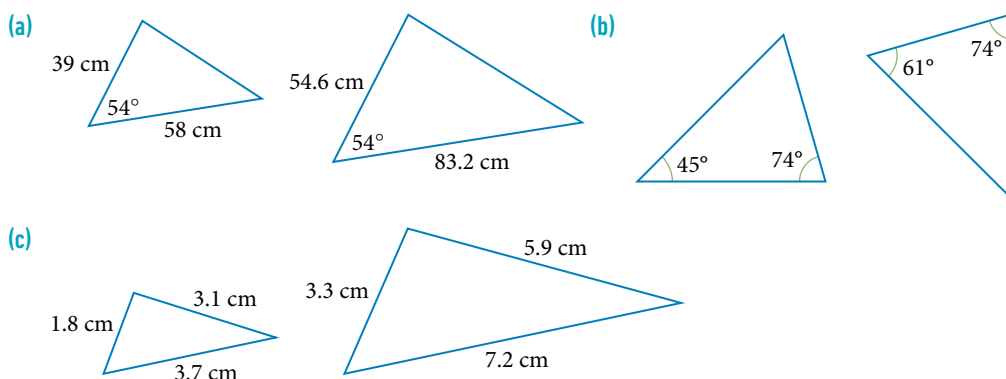
Exercise 2.3

- 6 Calculate the volume of each of the following solids. Give your answers to 1 decimal place.



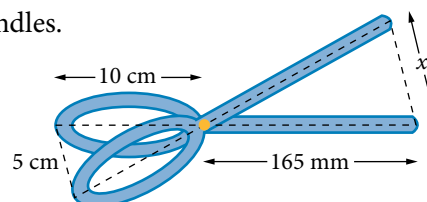
- 7 Identify whether or not the pairs of shapes in each diagram are similar shapes. Explain your answer using mathematical reasoning.

Exercise 3.1



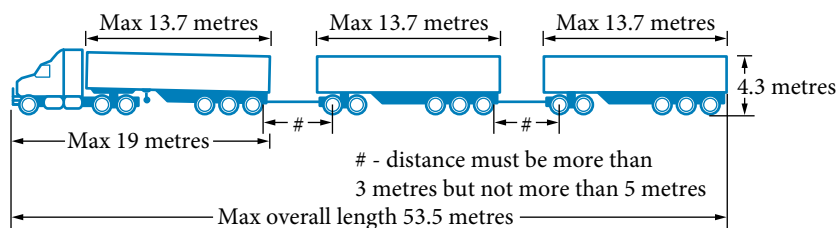
- 8 A pair of scissors is opened by 5 cm at the top of the handles. Determine the opening distance of the blades (x).

Exercise 3.1



- 9 The diagram shows the maximum dimensions of a road train allowed on NSW roads. A scale model of this is to be built using the 1:87 scale that is common for transport models.

Exercise 3.2



Calculate the maximum overall length and the maximum height (in millimetres) for the scale model. Give your answers to the nearest millimetre.

- 10 In each case below, calculate the area in square centimetres on the plan, given the plan scale and actual area.

Exercise 3.3

- (a) Plan scale 1:100; Actual area 60 m^2 (b) Plan scale 1:500; Actual area 100 m^2
 (c) Plan scale 1:20000; Actual area 4 km^2 (d) Plan scale 1:8500; Actual area $50\,000 \text{ m}^2$

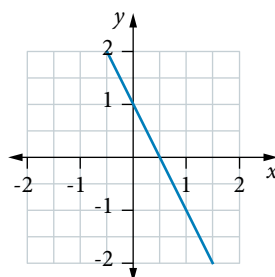
- 11 Giaan wins a maths competition, and is presented with a giant cheque by her maths teacher as a publicity stunt designed to increase numbers of senior maths students.

Exercise 3.3

The area of the giant cheque is 1.76 m^2 . If the area of an actual cheque is 126 cm^2 , determine the scale factor by which the length of the cheque has been increased. Give your answer to 2 decimal places.

Exercise 4.2

- 12 Determine the gradient of the line shown in the graph.



Exercise 4.3

- 13 Calculate the x - and y -intercepts and sketch the graph of $y = 4x - 28$.

Exercise 4.4

- 14 Solve the following pairs of simultaneous equations algebraically.

(a) $a + b = 7, a - b = 1$

(b) $x + 2y = 15, 5x + 2y = 19$

(c) $3c + 5d = 34, c - 5d = -2$

(d) $x = y + 3, 3x - 5y = 1$

Exercise 4.5

- 15 To encourage water conservation, the rural water supply company charges different rates depending on the volume of water used by a household. The charges are shown in the table.

Water usage (L)	Cost (c/L)
0–150 000	1.4
150 000–350 000	2.0
More than 350 000	2.5

(a) Calculate the cost of 150 000 L.

(b) Calculate the cost of 200 000 L.

(c) Draw a step graph to show this information.

(d) Draw a piece-wise linear graph to represent the cost of consumption from 0 to 400 000 L.

Exercise 1.1

- 16 Calculate the original price before a discount of 45% was given on a washing machine sold for \$627.

Exercise 1.3

- 17 Use the *Tax brackets and tax rates table* (page 23) for this question.

Naomi is an electrical engineer who earned \$89 540 in the financial year. She has \$4200 of deductions. Her group certificate shows that she has already paid \$20 876 in tax. Calculate the amount of her refund or bill.

Exercise 2.5

- 18 The surface area of a cylindrical tin can is 340 cm^2 and its diameter is 7 cm. Calculate the height of the can to the nearest centimetre.

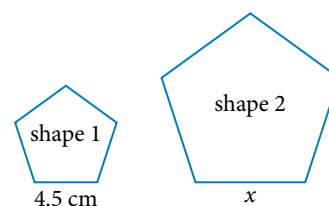
Exercise 2.6

- 19 A cone has both diameter and height of 7 cm. Calculate the diameter of a sphere with the same volume as the cone. Give your answer to the nearest millimetre.

Exercise 3.3

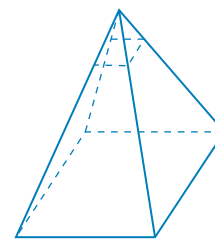
- 20 The two shapes shown here are similar. The area of shape 1 is 60 mm^2 and the area of shape 2 is 240 mm^2 .

Determine the value of x .



- 21 The volume of the square pyramid is 2000 cm^3 and its height is 60 cm.

The top 10 cm of the pyramid is cut off to give a smaller, similar pyramid. Determine the volume that remains after the top has been cut off, to the nearest cubic centimetre.



Exercise 3.4

- 22 Lucas is 3 years older than his brother Nathan, and 7 years younger than his sister-in-law Hayley. The sum of their ages is 79 years. How old are Lucas, Nathan and Hayley?
- 23 Two tradespeople and an apprentice earn a total of \$880 for an 8 hour job. The same rate of pay earns one tradesperson and an apprentice \$260 for a 4 hour job.
- (a) Write a pair of simultaneous equations to represent the situation, where T is the pay rate of the tradesperson and A is the pay rate of the apprentice.
- (b) Solve the pair of equations to determine the hourly rate of pay for the tradesperson and for the apprentice.

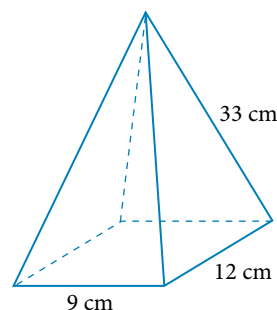
Exercise 4.1

Exercise 4.4

- 24 Hien has saved \$15000 to buy a car. The purchase price is \$21500, with finance available at a flat rate of 9% p.a. over 6 months. Hien decides to take a cash advance on his credit card instead and pay back an amount equivalent to the monthly car repayments (rounded up to the next whole dollar) every 30 days. If the credit card rate is 18.4% p.a. on daily balances, calculate how many months it will take Hien to clear his debt.

Exercise 1.8

- 25 Determine the height of the pyramid shown. State any assumptions you needed to make to carry out your calculations. Give your answer to 2 decimal places.



Exercise 2.1

- 26 The fixed costs for a company making souvenir surfboards are \$4000 each month. Each surfboard costs \$2.50 in raw materials and sells for \$4.10.

Exercise 4.4

A year later the fixed costs go up by \$1000 and the cost of raw materials increases by 50%, all at the same time. By what percentage (to the nearest whole number) will the selling price need to increase, to keep the break-even point the same?





5

Applications of trigonometry

• • • • •

Recall	280
5.1 Trigonometric ratios	281
5.2 Angles of elevation and depression	289
5.3 Directions and bearings	296
5.4 The sine rule	305
5.5 The cosine rule	313
5.6 Alternative formulas for triangle area	322
Summary	330
Chapter review	332

Recall

Convert metric units of length

1 Convert the following measurements to metres.

(a) 87.6 km

(b) 560 mm

(c) 6700 cm

Round calculations to a given number of decimal places

2 Calculate the decimal value, rounding your answers to the specified number of decimal places.

(a) $\frac{11}{8}$ (1 d.p.)

(b) $\frac{30 \times 0.473}{220}$ (2 d.p.)

(c) $\frac{15.25}{0.628}$ (3 d.p.)

Solve equations involving ratios

3 Solve for x , to 3 decimal places.

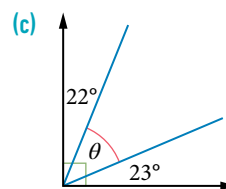
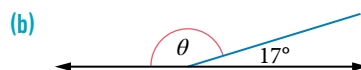
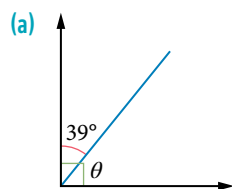
(a) $\frac{x}{7} = \frac{5}{13}$

(b) $\frac{75}{x} = 9.6$

(c) $2.45x = 82.19$

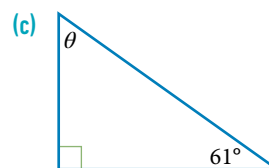
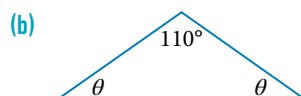
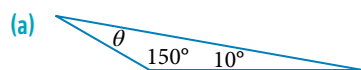
Use supplementary and complementary angles to calculate unknown angles

4 Determine the value of angle θ in each case.



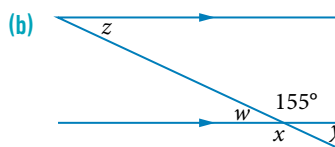
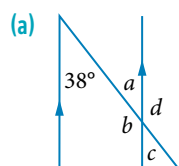
Calculating unknown angles in triangles

5 Calculate the value of θ in the following triangles.



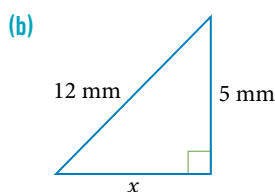
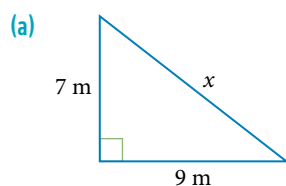
Determine unknown angles on parallel lines

6 Determine the value of each pronumeral.



Calculating the side lengths of right-angled triangles using Pythagoras' theorem

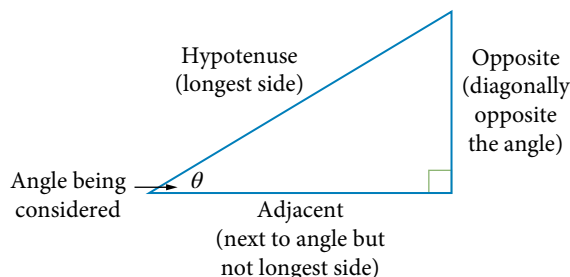
7 Using Pythagoras' theorem, determine the missing length to 1 decimal place.



Trigonometric ratios

Sine, cosine and tangent

You may recall that sine (sin), cosine (cos) and tangent (tan) can be used to calculate quantities in right-angled triangles. The sides of right-angled triangles are named according to their position relative to the angle being considered. The symbol θ (theta) is often used to denote the angle being considered in a right-angled triangle.



Here are the rules for these three trigonometric ratios.

$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$	$\sin(\theta) = \frac{O}{H}$
$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\cos(\theta) = \frac{A}{H}$
$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$	$\tan(\theta) = \frac{O}{A}$

i Additional information

Right-angled triangles

These videos and activities revise the angles, sides and ratios of right-angled triangles.

Use SOH CAH TOA to help remind you of these three rules.

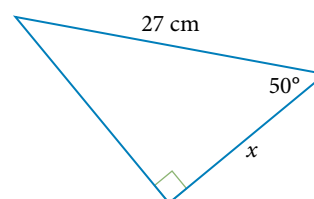
Solving sides

Solve any trigonometry problem involving a right-angled triangle in just four steps:

- 1 Label the sides of the triangle (Opposite, Adjacent, Hypotenuse).
- 2 Recall an appropriate rule to use.
- 3 Substitute the given values, using a pronumeral (e.g. x) for the unknown.
- 4 Solve for the unknown quantity.

1 Determining side lengths: unknown in the numerator

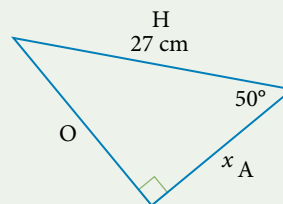
Calculate the length of the side labelled x in the triangle to 2 decimal places.



THINKING

- 1 Draw the triangle and label the sides O (Opposite), A (Adjacent) and H (Hypotenuse).
- 2 Recall the appropriate rule.
Here you know H and are asked to find A.
- 3 Substitute the given values, using a pronumeral for the unknown.
- 4 Solve for the unknown, rounding to the required number of decimal places.
- 5 Check that the answer is reasonable.

WORKING



$$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

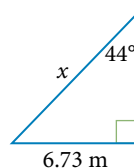
$$\cos(50^\circ) = \frac{x}{27}$$

$$\begin{aligned}\frac{x}{27} &= \cos(50^\circ) \\ x &= 27 \cos(50^\circ) \\ &= 17.36 \text{ cm (2 d.p.)}\end{aligned}$$

The answer represents one of the two shorter sides and is smaller than the hypotenuse, as expected.

2 Determining side lengths: unknown in the denominator

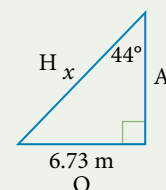
Calculate the length of the side labelled x for the triangle to 2 decimal places.



THINKING

- 1 Draw the triangle and label the sides O (Opposite), A (Adjacent) and H (Hypotenuse).
- 2 Recall the appropriate rule.
Here you know O and are asked to find H.
- 3 Substitute the given values, using a pronumeral for the unknown.
- 4 Solve for the unknown, rounding to the required number of decimal places.
- 5 Check that the answer is reasonable.

WORKING



$$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\sin(44^\circ) = \frac{6.73}{x}$$

$$\begin{aligned}x \sin(44^\circ) &= 6.73 \\ x &= \frac{6.73}{\sin(44^\circ)} \\ &= 9.69 \text{ m (2 d.p.)}\end{aligned}$$

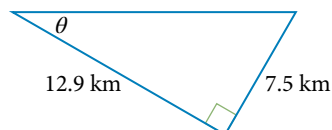
The answer is larger than the given side, because it is the hypotenuse.

Solving angles

For $\sin(\theta) = \frac{O}{H}$ the angle can be determined using the inverse of sine: $\theta = \sin^{-1}\left(\frac{O}{H}\right)$.
 For $\cos(\theta) = \frac{A}{H}$ the angle can be determined using the inverse of cosine: $\theta = \cos^{-1}\left(\frac{A}{H}\right)$.
 For $\tan(\theta) = \frac{O}{A}$ the angle can be determined using the inverse of tangent: $\theta = \tan^{-1}\left(\frac{O}{A}\right)$.

3 Determining the size of an angle

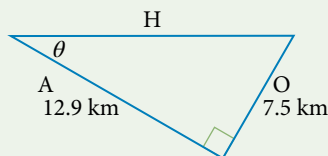
Calculate the value of the angle θ , to the nearest degree.



THINKING

- 1 Draw the triangle and label the sides O (Opposite), A (Adjacent) and H (Hypotenuse).
- 2 Recall the appropriate rule.
Here you know O and A.
- 3 Substitute the given values.
- 4 Solve for θ , rounding to the required number of decimal places.
- 5 Check that the answer is reasonable.

WORKING



$$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\tan(\theta) = \frac{7.5}{12.9}$$

$$\theta = \tan^{-1}\left(\frac{7.5}{12.9}\right)$$

$$= 30^\circ \text{ (nearest degree)}$$

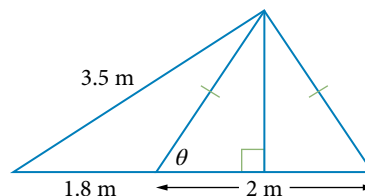
The angle is less than 45° because it is opposite the smaller of the two sides.

Compound shapes

To solve quantities in a right-angled triangle you need to know two measurements. Sometimes it is necessary to solve sides or angles of another triangle first.

4 Solving triangles with shared sides

A 2 m wide tent has a 3.5 m guy rope, which meets the ground 1.8 m from the base. Calculate the angle, to the nearest degree, that the side of the tent makes with the ground.



THINKING

- 1 Draw a right-angled triangle for which two measurements are known or can be found.

Mark the required unknown measurement with a pronumeral.

- 2 When two sides are given in a right-angled triangle, use Pythagoras' theorem to determine the length of the third.

- 3 Check that the value is reasonable.

- 4 Draw the right-angled triangle with the required unknown, marking the known measurements.

- 5 Solve using trigonometric ratios.

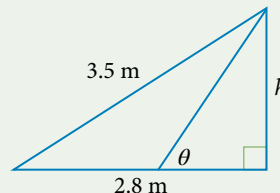
- 6 Check that the answer is reasonable.

WORKING

$$2\text{ m} \div 2 = 1\text{ m}$$

$$1.8\text{ m} + 1\text{ m} = 2.8\text{ m}$$

The horizontal distance between the guy rope peg and the centre of the tent is 2.8 m.



In the larger triangle:

$$h^2 + 2.8^2 = 3.5^2$$

$$h^2 = 3.5^2 - 2.8^2$$

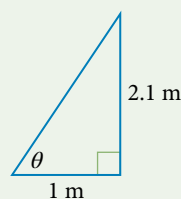
$$= 4.41$$

$$\therefore h = \sqrt{4.41}$$

$$= 2.1\text{ m}$$

The height of the tent is 2.1 m.

The height is less than the length of the guy rope (hypotenuse), as expected.



$$\begin{aligned}\tan(\theta) &= \frac{\text{O}}{\text{A}} \\ &= \frac{2.1}{1}\end{aligned}$$

$$\tan(\theta) = 2.1$$

$$\theta = \tan^{-1}(2.1)$$

$$= 64.536\dots^\circ$$

$$= 65^\circ \text{ (to the nearest degree)}$$

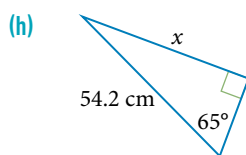
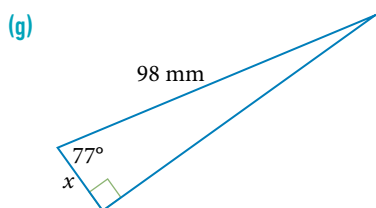
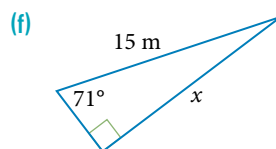
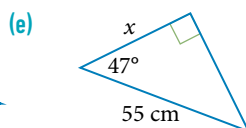
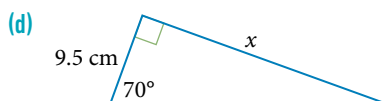
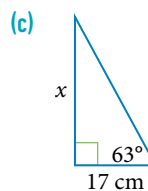
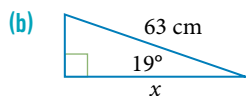
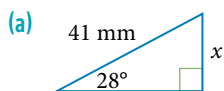
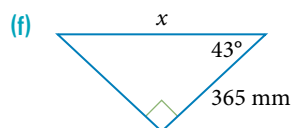
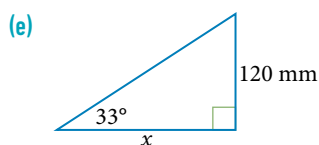
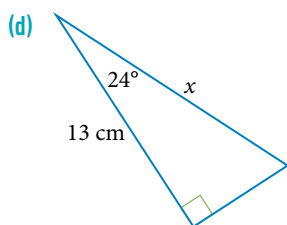
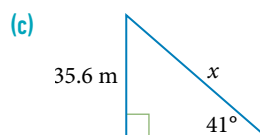
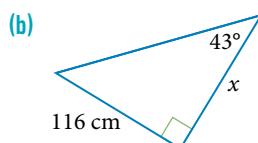
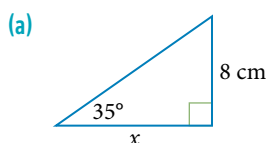
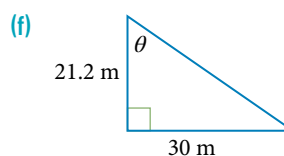
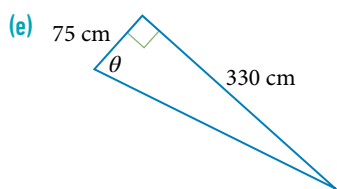
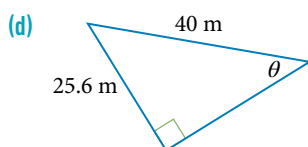
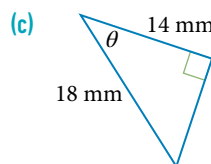
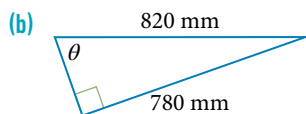
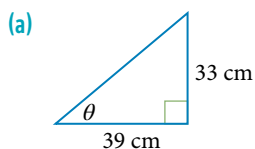
Assuming the shape of the tent is perfectly triangular, the tent makes an angle of 65° with the ground.

The angle is greater than 45° because it is opposite the larger of the two sides.

EXERCISE

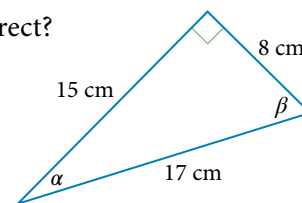
5.1

Trigonometric ratios

Worked
Example1 Calculate the length of the side labelled x , to 2 decimal places.2 Determine the length of the side labelled x , to 2 decimal places.3 Calculate the size of the angle θ for each triangle, to the nearest degree.

- 4 Which of the following statements concerning the triangle shown is *not* correct?

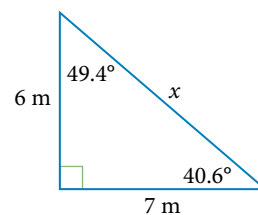
A $\sin(\alpha) = \frac{8}{17}$ B $\cos(\alpha) = \frac{15}{17}$
 C $\sin(\beta) = \frac{15}{17}$ D $\cos(\beta) = \frac{8}{15}$



- 5 Consider the triangle shown at right.

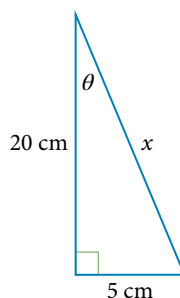
- (a) Which one of the following statements about the triangle is true?

A $x^2 + 6^2 = 7^2$ B $x = \frac{6}{\cos(49.4^\circ)}$
 C $x = \frac{6}{\sin(49.4^\circ)}$ D $x = \frac{7}{\tan(49.4^\circ)}$



- (b) Which two incorrect alternatives will result in the hypotenuse being shorter than 7 m? Your reasoning should not require any calculations.

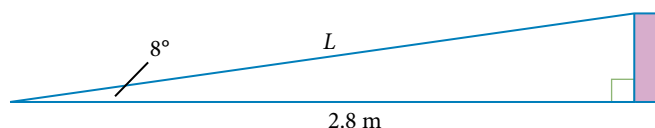
- 6 Solve the triangle for both pronumerals. Give the length to 2 decimal places and the angle to the nearest degree.



- 7 Calculate the size of the equal angles in an isosceles triangle with a base length 10 cm and two equal sides 13 cm in length.
- 8 Rod places a 3 m ladder against a wall of his house, with the base of the ladder 0.9 m from the wall. Calculate the size of the angle θ between the ladder and the wall, to the nearest degree.

- 9 A wheelchair ramp is to be built at an angle of 8° to the horizontal and must span a horizontal distance of 2.8 m.

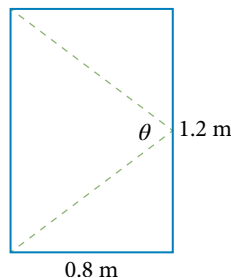
Determine the length L of the proposed ramp, to 2 decimal places.



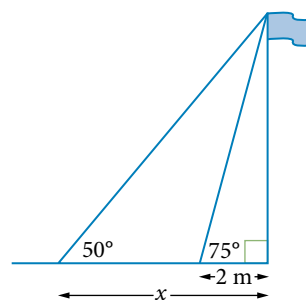
..

..

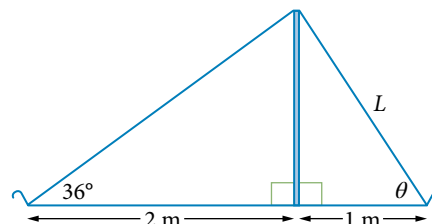
- 10 Gina strengthens a 1.2 m tall rectangular metal gate by adding two diagonal struts (shown as dashed lines). She welds the struts together half-way along the vertical side of the gate. Calculate the angle θ to the nearest degree.



- 11 Patoot stands 2 m from the base of a flagpole. Using a clinometer, he measures the angle the support cable makes with the ground to be 75° . How far from the pole should Patoot stand so that the angle of elevation is 50° ?

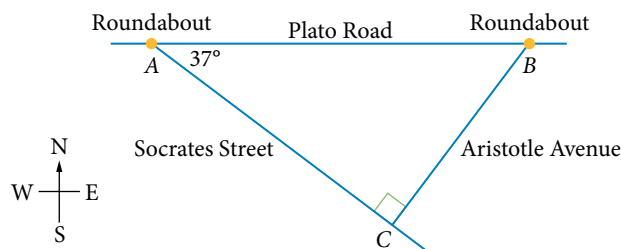


- 12 Kami has secured a tent pole using two ropes pegged to the ground. The pegs are 2 m and 1 m either side of the base of the pole. The longer rope makes an angle of 36° with the ground.
- (a) Determine the angle θ that the shorter rope makes with the ground. Give your answer to the nearest degree.
- (b) Calculate the length L of the shorter rope, to 2 decimal places.

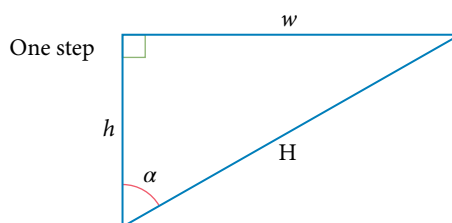
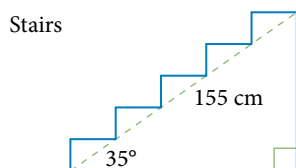


- 13 The roads bordering part of a planned new housing estate are shown here. The distance along Plato Road between the two roundabouts is 13.5 km.

How much shorter is it to travel from roundabout A to roundabout B along Plato Road than to go along Socrates Street and Aristotle Avenue? Give your answer to 2 decimal places.

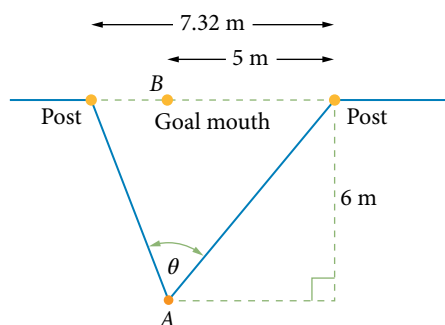


- 14 Korra wishes to build a set of five stairs according to the plan below.

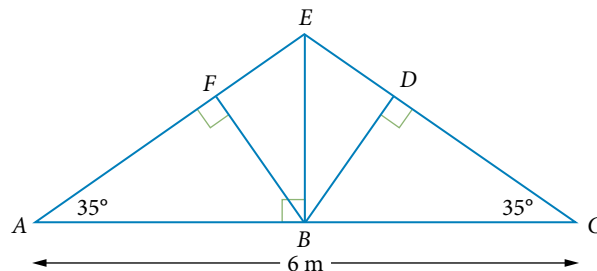


Determine the value of each pronumeral on the diagram of an individual step, shown above. Round to 2 decimal places where necessary.

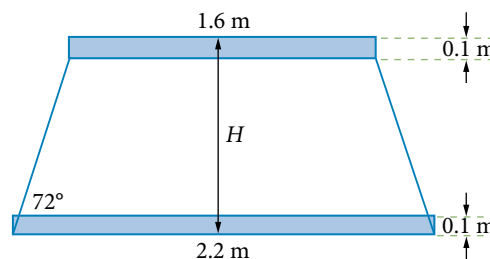
- 15 Archibella strikes a soccer ball 6 m out from goal at point A. She is directly in line with point B on the goal line, which is 5 m from the right-hand goal post.
- Any straight kick within the angle θ will result in a goal. Determine the size of angle θ , to the nearest degree, that Archibella should strike within in order to score a goal from point A.



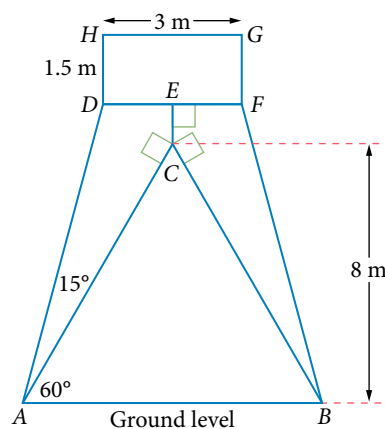
- 16 Tyler is building a garage roof using pre-made trusses, as shown here.
- Determine the total length of timber in the truss, to 2 decimal places.



- 17 Mr Geronimo has built some hurdles for a school sports day. His technical drawing of his hurdle design is shown here.
- How high are Mr Geronimo's hurdles? Express your answer to 2 decimal places.

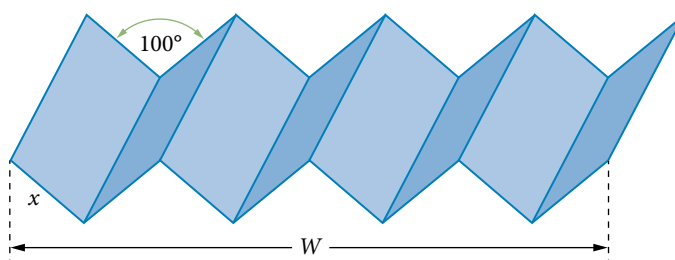


- 18 A birdwatchers' group builds an observation tower according to the specifications shown here.
- Calculate the length CE to 2 decimal places.



- 19 A 40 cm section of flat sheet metal is bent to form the regular corrugated shape shown here.

- (a) Determine the length x .
- (b) How wide (W) is the entire sheet of corrugated metal, to 2 decimal places?

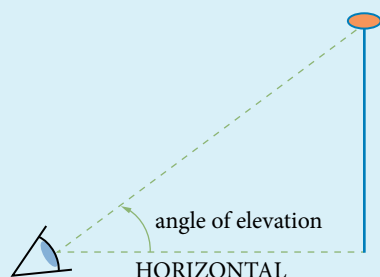


- 20 From the top of a tower are suspended two cables with clips at the end, forming flying foxes. The first cable ends by clipping to a fixture on the ground 20 m from the base of the tower. The second cable begins at the same height as the first cable, and ends by clipping into a fixture an additional 16 m from the base of the tower. The longer cable is 1.56 times the length of the shorter cable.
- Determine the difference in angle between each of the journeys down the flying foxes.

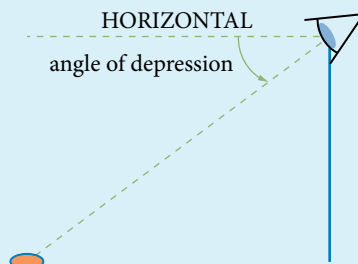
Angles of elevation and depression

Defining angles of elevation and depression

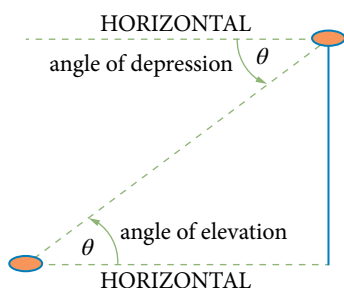
An *angle of elevation* is measured upwards from the horizontal.



An *angle of depression* is measured downwards from the horizontal.



Angles of elevation and depression involve two points, such as a point on the ground and the top of a streetlight.



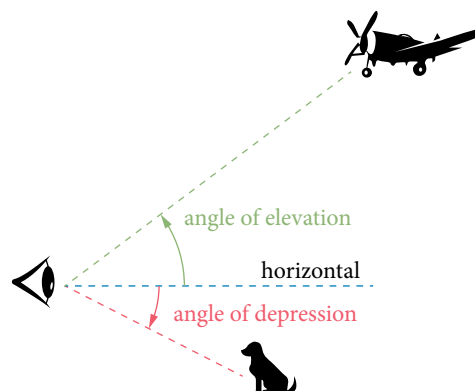
The angle of elevation is the angle that is formed from the horizontal ground up to the streetlight.

The angle of depression is the angle that is formed horizontally from the streetlight down to the ground.

These angles are identical because they are alternate (or Z) angles.

If the observation point is above a given object, the *angle of depression* is the angle at which the observer must look down to see the object.

If the observation point is below the given object, the *angle of elevation* is the angle at which the observer must look up to see the object.



Additional information

Angles of elevation and depression

The activities review the concept of angles of elevation and depression.

Calculations with angles of elevation and depression

5 Angles of elevation and depression

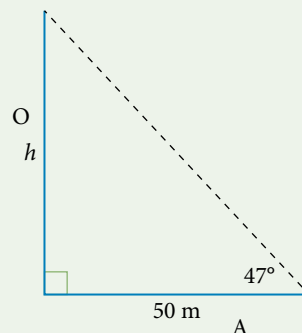
A surveyor stands 50 m from the foot of a skyscraper. Using a clinometer, she measures the angle of elevation from ground level to the top of the building to be 47° .

- (a) Determine the height of the skyscraper, to the nearest tenth of a metre.

THINKING

- 1 Draw the triangle, using horizontal and vertical lines to form right angles.
Mark the known measurements.
Use a suitable pronumeral to indicate the measurement to be found.

WORKING



- 2 Recall the appropriate rule.
- 3 Use the tangent ratio to calculate the length of the missing side.
- 4 Interpret the answer, confirming that it is reasonable.

$$\tan(\theta) = \frac{O}{A}$$

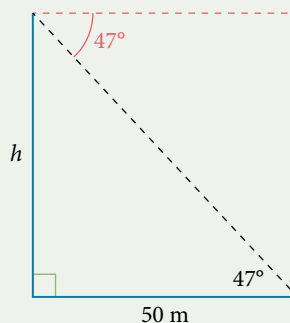
$$\tan(47^\circ) = \frac{h}{50}$$

$$h = 50 \times \tan(47^\circ) \\ = 53.6 \text{ m (to one-tenth of a metre)}$$

The height of the skyscraper is 53.6 m.
It is slightly more than 50 m, as expected, because the angle of elevation is slightly greater than 45° .

- (b) Determine the angle of depression from the top of the building to the surveyor's position at ground level.

The angle of depression is equal to the angle of elevation (alternate angles).



Angle of elevation is 47° .

$\therefore$ Angle of depression is 47° .

6 Forming the right-angled triangle first

A tree is 10 m tall. Evelyn stands 20 m from the tree and uses a hand-held clinometer at a height of 160 cm above the ground to measure the angle of elevation to the top of the tree.

- (a) Draw a labelled diagram of the situation, showing the measurements of the right-angled triangle formed. Mark the angle of elevation. List any assumptions you made.

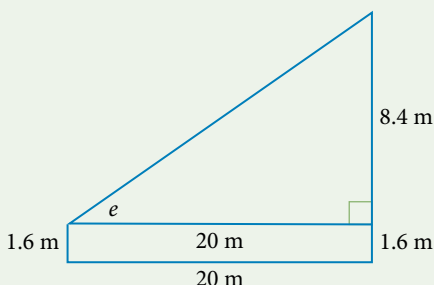
THINKING

- 1 State assumptions.
- 2 Convert values to the same units.
Use subtraction to find the side lengths of the triangle.
- 3 Draw the diagram, using horizontal and vertical lines to form right angles.

WORKING

Assume the ground is level and the distance from the tree is given to a point directly below the highest point.

$$\begin{aligned} 170 \text{ cm} - 10 \text{ cm} &= 160 \text{ cm} \\ &= 1.6 \text{ m} \\ 10 \text{ m} - 1.6 \text{ m} &= 8.4 \text{ m} \end{aligned}$$



- (b) Calculate the angle of elevation to the nearest degree.

- 1 Recall the appropriate formula.

$$\tan(\theta) = \frac{O}{A}$$

- 2 Substitute the known values into the formula.

$$\begin{aligned} \tan(e) &= \frac{8.4}{20} \\ e &= \tan^{-1}\left(\frac{8.4}{20}\right) \\ &= 23^\circ \text{ (to the nearest degree)} \end{aligned}$$

- 3 Interpret the answer, confirming that it is reasonable.

The angle of elevation to the top of the tree is 23° .

It is less than 45° , as expected.

i Additional information

Angles of elevation and depression in real life

This activity reviews angles of depression in a real-life context.

EXERCISE

5.2

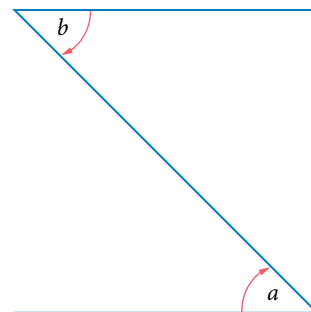
Angles of elevation and depression

Worked
Example

5

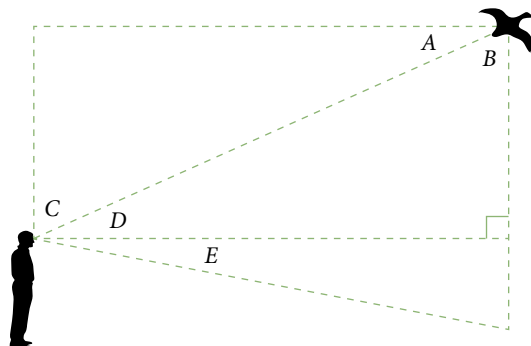
- 1 An ant is a horizontal distance of 85 cm from the base of a vertical garden stake. The angle of elevation from the ant to the top of the stake is 72° . How high from the ground is the top of the stake, to the nearest centimetre?

- 2 An angle of elevation a and an angle of depression b are shown here. Select the statement that is correct for angles a and b .
- A They always add to 90° .
 B They are alternate angles and therefore equal.
 C They are corresponding angles and therefore equal.
 D They are co-interior angles and therefore add to 180° .



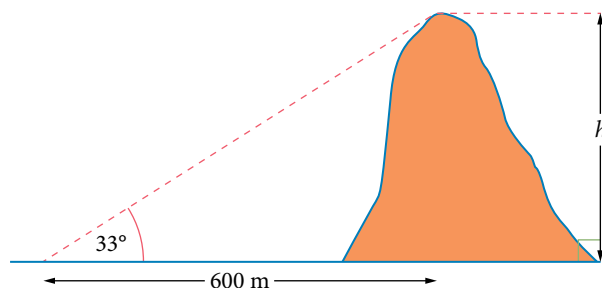
- 3 Sami looks up at a bird flying overhead. The angle of elevation and the angle of depression, in that order, are:

- A A and B
 B C and B
 C D and A
 D D and E

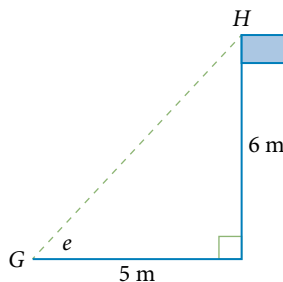


- 4 Kosy is 600 m horizontally from the peak of Mount Mathematica. The angle of elevation to the top of the mountain has been measured and is 33° .

How high (h) is the summit of Mount Mathematica, to the nearest metre?



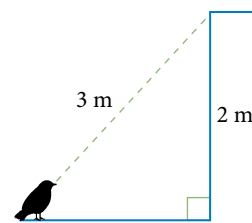
- 5 Determine the angle of elevation e from point G to point H at the top of the 6 m tall flagpole, to the nearest degree.



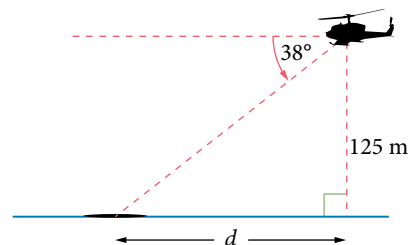
- 6 A flagpole is 4.6 m high. The flagpole casts a shadow 6 m across the ground. Determine the angle of elevation from the tip of the shadow to the top of the flagpole, to the nearest degree.

- 7 Ava's bird feeder consists of a dish mounted on top of a 2 m high pole. A bird flies 3.0 m along the dashed path shown, to pick up some spilled seed on the ground before contemplating flying back to the dish.

Determine the angle of elevation from the bird on the ground to the top of the pole, to the nearest degree.

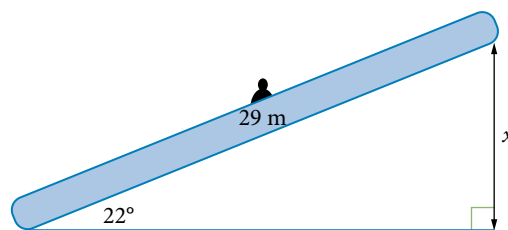


- 8 A rescue helicopter is descending at an angle of depression of 38° towards a landing pad from an altitude of 125 m. How far, to the nearest metre, will the helicopter travel horizontally before landing?



- 9 An aircraft is flying over a large horizontal area at an altitude of 700 m. Determine the angle of depression, to 2 decimal places, from the aircraft to a point on the ground 1 km away horizontally.

- 10 Nigel is riding an escalator at the airport. If the angle of elevation of the escalator is 22° , and he travels a distance of 29 m along the escalator, determine the vertical distance Nigel climbs, to the nearest centimetre.



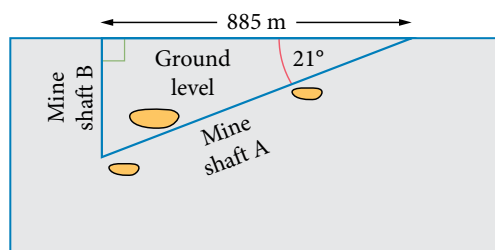
- 11 A wheelchair ramp is constructed so that it rises 20 cm for every 1 m horizontal increase. Determine the angle of elevation of the ramp, to 2 decimal places.

- 12 A gold mining company constructs two mine shafts to access deposits of gold. One shaft is vertical and the other descends at an angle of depression of 21° .

(a) Determine the length of the sloping shaft, to the nearest metre.

(b) How deep does the vertical shaft reach?

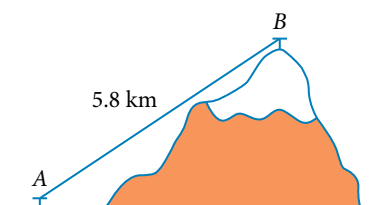
Answer to the nearest metre.



- 13 A 5.8 km skyrail climbs from station A to station B at an angle of elevation of 28° . Express your answer in kilometres to 2 decimal places.

(a) How high vertically above station A is station B?

(b) Determine the horizontal distance between the two stations.

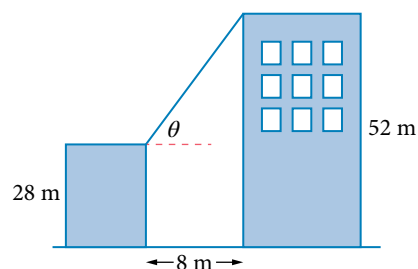


- 14 Katerina is skiing down a slope that drops at an angle of depression of 17° .
- If she skis a direct distance of 325 m, how far vertically below her starting point does she move, to the nearest metre?
 - Explain the common error made by a student who calculated the height to be $325 \times \cos(17^\circ)$.
- 15 Mike and Manpreet are driving their four wheel drive vehicle up a very long straight road that rises at an angle of elevation of 6° . According to their map, they must rise vertically 68 m from their current position to reach their destination, the site of a rare geological formation.
- Draw a diagram with all known information, depicting Mike and Manpreet's journey.
 - How much further along the road must Mike and Manpreet travel to reach their destination? (Round to the nearest metre.)

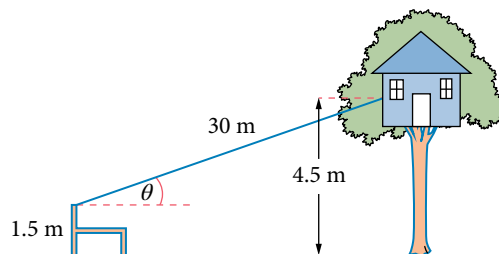
Worked
Example

6

- 16 Jeremiah works in a small building. While on the rooftop terrace one day, he notices Janine enjoying a coffee on the top of her taller building across the street.
- Calculate the angle of elevation θ from Jeremiah to Janine, to the nearest degree.
 - Jeremiah waves to Janine. Janine is short-sighted and can only see up to 28 m away. Can Janine see Jeremiah wave?

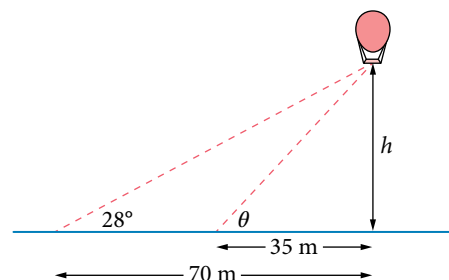


- 17 An adventure park zip line has a cable that connects a 1.5 m high platform to a point on a cubby house 4.5 m above the ground. The length of the cable is 30 m. Calculate the angle of elevation θ from the top of the platform to the connection point on the cubby, to 2 decimal places.



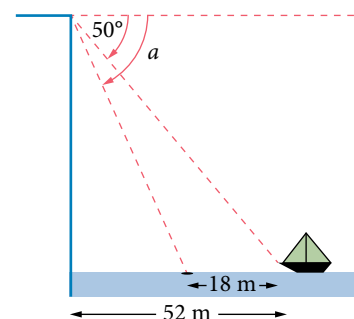
- 18 A backup team follows a hot air balloon in a car. At a horizontal distance of 70 m the team measures an angle of elevation to the balloon of 28° .

If the ground vehicle reduces the horizontal distance from the balloon by half, determine the new angle of elevation θ to the nearest degree.

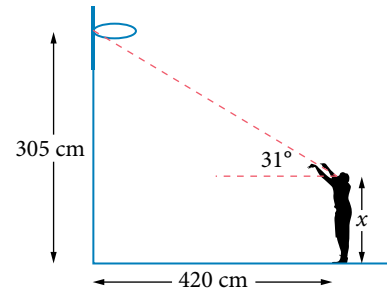


- 19 An observer at the top of a cliff spots a yacht at an angle of depression of 50° when the yacht is 52 m from the base of the cliff. A shark is spotted 18 m horizontally from the yacht, between the yacht and the cliff.

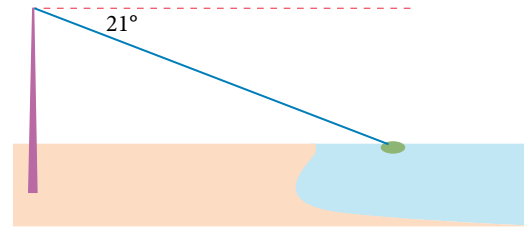
- Determine the height of the cliff, to 2 decimal places.
- Determine the angle of depression to the shark from the observer's position, to the nearest degree.



- 20 Zoe estimates that the angle of elevation from her eye to the back of a basketball hoop is 31° when she is positioned 420 cm horizontally from the pole supporting the hoop. Calculate the height of Zoe's eye-level above the ground (x). Answer in centimetres, to the nearest centimetre.

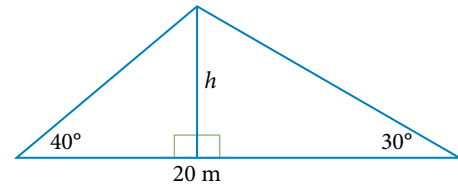


- 21 Venkat secures his 2.5 m fishing rod in a vertical position by burying the lower 30 cm in the sand so he can sleep on the beach nearby. The fishing line is pulled tight when a toadfish is hooked near the surface, level with the base of the rod where it meets the sand.



Determine the length of fishing line between the fish and the rod, if the angle of depression from the top of the rod to the fish is 21° ? Round your answer to 1 decimal place.

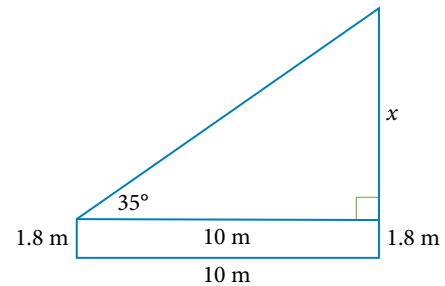
- 22 When Barack looks up to the top of a pole, the angle of elevation is 40° . When he walks towards the pole for a total of 20 m he finds, looking back, that the angle of elevation is now 30° . Determine the height of the pole to the nearest centimetre.



Hint: Create and solve simultaneous equations.

- 23 An entire class of students set out to estimate the height of a tree using hand-held clinometers. The teacher finds great variation in the estimates, not surprising when he realises that the angles of elevation were being recorded with accuracy to the nearest 5° . The diagram from one group is shown.

Assuming other measurements were more accurate, calculate a range of values for the actual height of the tree, and determine how far out the students' answers could be. Use appropriate rounding.



5.3

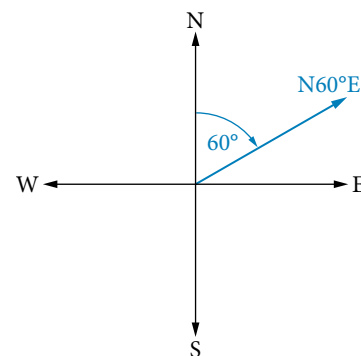
Directions and bearings

Standard compass directions

A compass direction describes a direction relative to the four key compass points: north (N), south (S), east (E) and west (W).

Standard compass directions are measured from north or south, and begin with N or S followed by an acute angle, then either E or W.

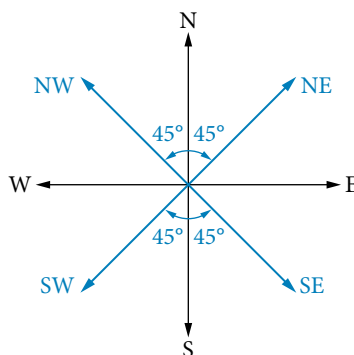
For example, N60°E describes the direction you face if you begin facing north, and then rotate your view by 60° towards the east.



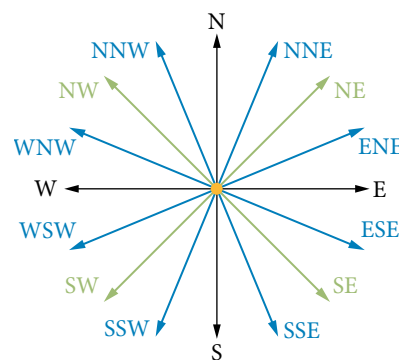
Compass points

Directions that follow N, S, E or W directly are described as due north, due south, due east or due west.

Directions 45° from north or south are described by the compass points they divide (always starting with N or S: NE, NW, SE or SW).



Directions half-way between these eight compass points use an extra direction letter to describe the change in direction.



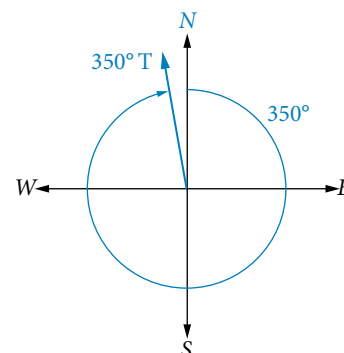
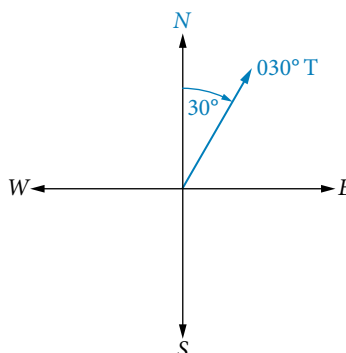
True bearings

The word 'true' is used to distinguish true north from magnetic north and grid north. True north uses the North Pole, where the Earth's axis of rotation meets the surface of the Earth. Magnetic north is near the North Pole, in Canada, and gradually changes position all the time. Grid or map north is always slightly different from true north as flat maps are used to approximate the curved surface of the Earth.

Bearings are always measured clockwise, starting from north, and are written as three-digit angles (also called three-figure bearings) followed by the letter T (for True).

For example, 030°T describes a bearing 30° clockwise from north, and both 350° and 350°T describe a bearing 350° clockwise from north.

Even though the direction is only 10° west from north, the true bearing must be measured in the clockwise direction from north.



C Making connections

True and compass bearings

Explore the relationship between compass bearings and true bearings.

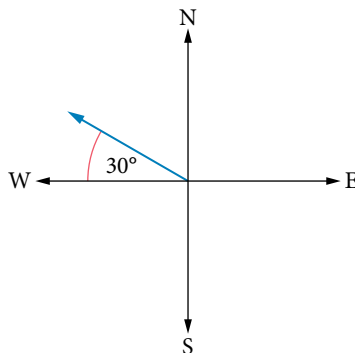
i Additional information

Bearings

Try the activity to see examples of true bearings.

7 Directions and bearings from a diagram

Write the following as a compass direction and as a true bearing.

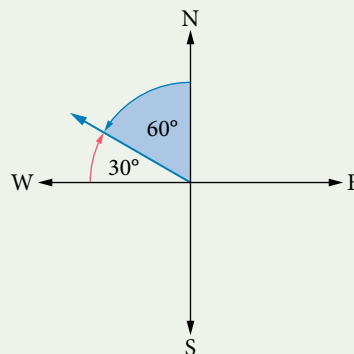


THINKING

- 1 To convert the angle to a compass direction, calculate the angle from the nearest north or south direction.

In this case, measure from north towards west.

WORKING

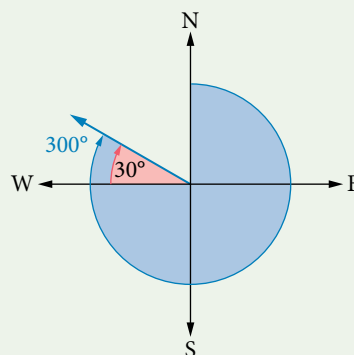


- 2 State the standard compass direction.

The angle represents a compass direction of $N60^\circ W$.

- 3 To convert the angle to a true bearing, measure clockwise from north and state the true bearing.

$$360^\circ - 60^\circ = 300^\circ$$



The angle represents a true bearing of $300^\circ T$.

8 Converting to a true bearing

Convert the following compass directions to true bearings.

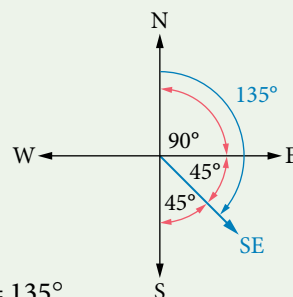
(a) SE

THINKING

- 1 Draw a diagram showing the angle required. Mark relevant angles and determine the size of the angle measured in a clockwise direction from north 0°T .

- 2 State the true bearing.

WORKING



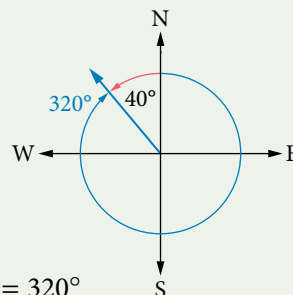
$$90^\circ + 45^\circ = 135^\circ$$

The compass direction SE or $S45^\circ\text{E}$ is equivalent to a true bearing of 135°T .

(b) $N40^\circ\text{W}$

- 1 Draw a diagram showing the angle required. Mark relevant angles and determine the size of the angle measured in a clockwise direction from north 0°T .

- 2 State the true bearing.



$$360^\circ - 40^\circ = 320^\circ$$

The compass direction $N40^\circ\text{W}$ is equivalent to a true bearing of 320°T .

9 Converting to a compass direction

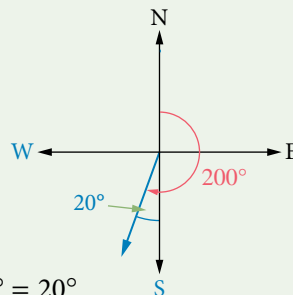
Convert 200°T to a compass direction.

THINKING

- 1 Draw a diagram showing the bearing and determine the size of the angle measured from either north or south.

- 2 State the standard compass direction.

WORKING



$$200^\circ - 180^\circ = 20^\circ$$

The true bearing 200°T is equivalent to a compass direction of $S20^\circ\text{W}$.

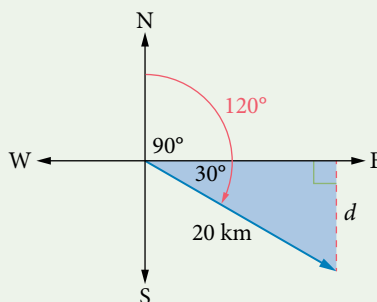
10 Calculating the distance in a particular direction

A car travels 20 km on a bearing 120°T . How far south does the car travel?

THINKING

- 1 Draw a diagram showing the information given and include a right-angled triangle.

WORKING



$$120^\circ - 90^\circ = 30^\circ$$

$$\sin(\theta) = \frac{O}{H}$$

$$\sin(30^\circ) = \frac{d}{20}$$

$$\begin{aligned} d &= 20 \sin(30^\circ) \\ &= 10 \text{ km} \end{aligned}$$

- 3 Interpret the result.

The car travels 10 km south.

11 Direction and distance

Mateo walks from his house 85 m in a southerly direction, turns right at the intersection and walks a further 154 m to his friend Finn's house. The friends then fly their drone from Finn's house directly back to Mateo's house.

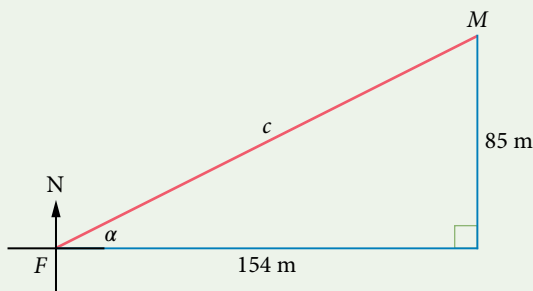
How far and in what direction do they fly the drone?

THINKING

- 1 Draw a diagram displaying the information given and include a right-angled triangle.

Place pronumerals where useful measurements can be calculated.

WORKING



- 2 Use Pythagoras' theorem $c^2 = a^2 + b^2$ to calculate the direct distance between Mateo and Finn's homes.

$$c^2 = a^2 + b^2$$

$$c^2 = 154^2 + 85^2$$

$$\begin{aligned} \therefore c &= \sqrt{154^2 + 85^2} \\ &= 176 \text{ m (nearest m)} \end{aligned}$$

- 3 Use the tangent ratio $\tan(\theta) = \frac{O}{A}$ to calculate the size of the unknown angle in the diagram.

$$\begin{aligned}\tan(\alpha) &= \frac{85}{154} \\ \alpha &= \tan^{-1}\left(\frac{85}{154}\right) \\ &= 29^\circ \text{ to the nearest degree}\end{aligned}$$

- 4 Calculate the direction from the acute angle.

$$90 - 29 = 61^\circ, \text{ so } N61^\circ E \text{ or } 61^\circ T$$

- 5 Interpret the answer, commenting on any assumptions you had to make.

The drone flies 176 m in the direction $061^\circ T$ or $N61^\circ E$, assuming there are no obstructions in its path.

12 Return journey

Patrick flies a light plane from Cloncurry to Longreach, a distance of 495 km on a bearing of $129^\circ T$, then continues due east to Rockhampton, a further 639 km. Patrick then returns directly to Cloncurry.

Determine the distance and bearing of Patrick's return journey.

THINKING

- 1 Draw a diagram showing the information given. Include a right-angled triangle and place pronumerals where useful measurements can be calculated.

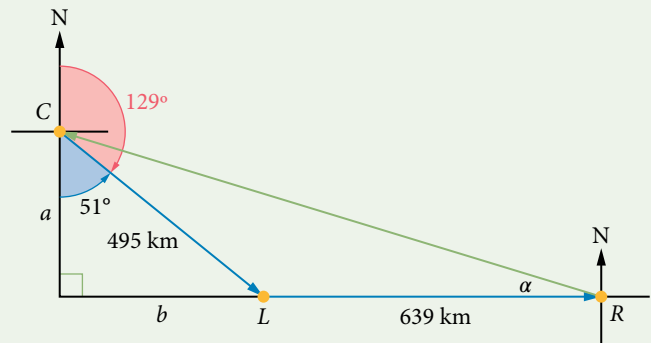
- 2 Use the right-angled triangle to determine the distance travelled in a southerly direction (a) and an easterly direction (b).

- 3 Interpret the values in terms of the return journey.

- 4 Use Pythagoras' theorem to find the diagonal length.

- 5 Use the tangent ratio to calculate the size of the angle in the right-angled triangle of the return journey.

WORKING



$$\begin{aligned}\cos(51^\circ) &= \frac{a}{495} \\ a &= 495 \times \cos(51^\circ) \\ &= 311.51... \text{ km}\end{aligned}$$

$$\begin{aligned}\sin(51^\circ) &= \frac{b}{495} \\ b &= 495 \times \sin(51^\circ) \\ &= 384.68... \text{ km}\end{aligned}$$

From Cloncurry to Rockhampton, Patrick has flown 311.51 km south and $384.68 + 639 = 1023.68$ km west.

$$\begin{aligned}d^2 &= 311.51...^2 + (1023.68...)^2 \\ d &= \sqrt{311.51^2 + 1023.68^2} \\ &= 1070 \text{ km (nearest km)}\end{aligned}$$

$$\begin{aligned}\tan(\alpha) &= \frac{311.51}{1023.68...} \\ \alpha &= \tan^{-1}\left(\frac{311.51}{1023.68...}\right) \\ &= 17^\circ \text{ (to the nearest degree)}\end{aligned}$$

- 6 Calculate the bearing from the acute angle. Patrick flies 17° from west in a northerly direction.
 $17 + 270 = 287^\circ$, so 287°T
- 7 Interpret the answer, commenting on any assumptions you made. Patrick must fly 1070 km on a true bearing of 287°T , assuming that the curvature of the Earth does not have a significant effect on the calculations.

EXERCISE

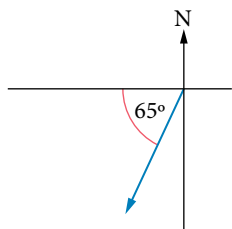
5.3

Directions and bearings

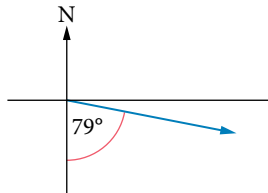
Worked
Example

- 1 Determine the compass direction and true bearing of the following.

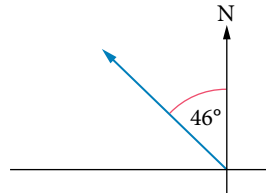
(a)



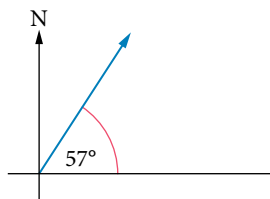
(b)



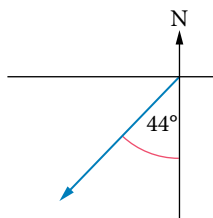
(c)



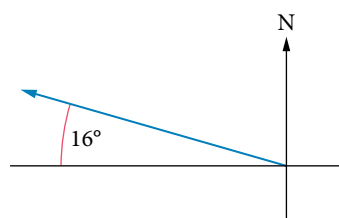
(d)



(e)



(f)



- 2 Convert the following directions to true bearings.

(a) NW

(b) $S30^\circ\text{E}$ (c) $N72^\circ\text{W}$

(d) SE

(e) $S69^\circ\text{W}$ (f) $N32^\circ\text{E}$

- 3 Convert the following bearings to compass directions. Use compass points where appropriate.

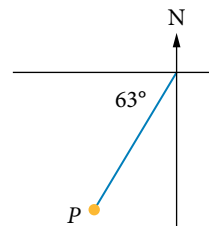
(a) 090°T (b) 135°T (c) 290°T (d) 256°T (e) 057°T (f) 351°T

- 4 Consider point P in the diagram.

(a) Determine the compass direction of point P.

(b) Explain the common error/s made by a student who gave a compass direction including an angle of 63° .

(c) Determine the true bearing of point P.

A 027°T B 063°T C 153°T D 207°T 

Worked
Example

5 For the direction S44°E:

(a) Choose the equivalent bearing.

A 044°T

B 046°T

C 136°T

D 224°T

(b) Which option would be chosen by a student who confuses east and west?

10

6 Answer each of the following to 1 decimal place.

(a) Denzil walks 15 m on a bearing 072°T, as shown.

How far east has he walked?

(b) A bird flies 25 km on a bearing 250°T. How far south did it fly?

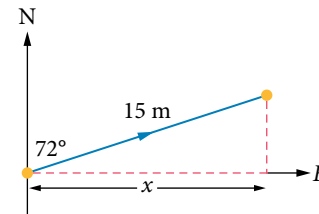
(c) An escaped hot air balloon travels 250 km on a bearing 350°T away from its starting point before it finally comes to rest on the ground.

How far north did the balloon travel?

(d) Amal drives 67 km on a straight road on a bearing N63°W. How far west does he travel?

(e) A cyclist on the Tour de Maths rides 87 km on a bearing 105°T. How far east does she ride?

(f) A groundskeeper marks a straight chalk line on a school athletics track that is 80 m long and follows a bearing of S20°W. How far south of its starting point does the line extend?



11

7 Carter is swimming 70 m due north of the shore when he is caught in a rip. He conserves energy and drifts 100 m east parallel to the shore before being rescued. What is Carter's direction from his starting point when he is rescued? Round your answer to the nearest degree.

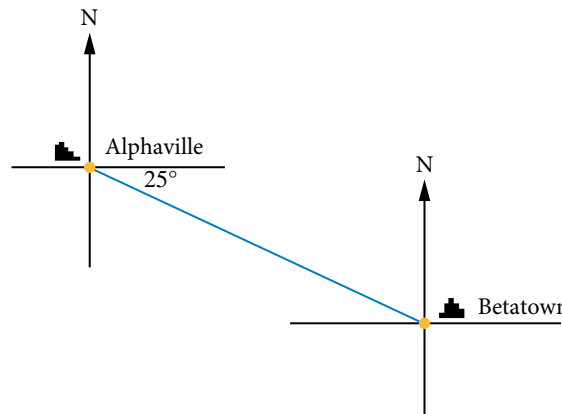
8 Duncan is flying his kite when the line snaps and the kite blows 60 m due west. The wind then changes direction, and the kite is blown due south before it eventually crashes to ground. Duncan has not moved during this time, and notes that the direction of the crashed kite is S50°W from his position. How far south did the wind blow his kite? Give your answer to the nearest metre.

9 A pilot flying over the town of Alphaville plots a course to take her over the nearby city of Betatown.

(a) Determine the true bearing of Betatown from Alphaville.

(b) Determine the standard compass direction of Alphaville from Betatown.

(c) Given that Alphaville is 22 km west of Betatown, how far apart are the towns, to 1 decimal place?

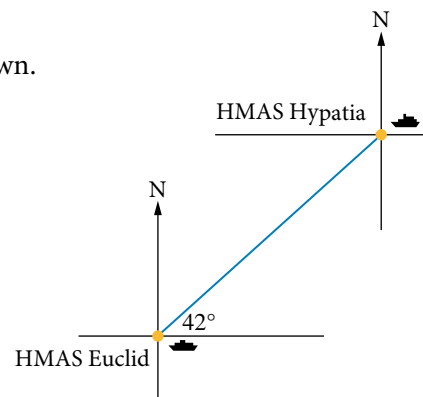


10 Two navy vessels are on manoeuvres, and their positions are shown.

(a) Determine the true bearing of *HMAS Euclid* from *HMAS Hypatia*.

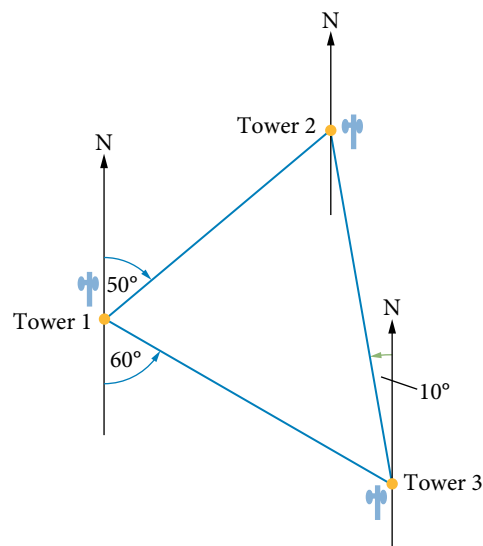
(b) Determine the direction of *HMAS Hypatia* from *HMAS Euclid*.

(c) Given that *HMAS Hypatia* is 850 m north of *HMAS Euclid*, how far apart are the vessels? Give your answer in metres, to the nearest metre.



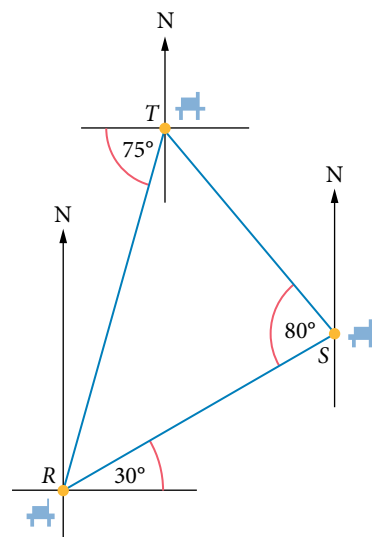
- 11 Various directions between three mobile phone towers are shown in the diagram. Determine the standard compass direction and true bearing of the following.

- (a) Tower 2 from Tower 1
- (b) Tower 3 from Tower 1
- (c) Tower 3 from Tower 2
- (d) Tower 1 from Tower 2
- (e) Tower 1 from Tower 3
- (f) Tower 2 from Tower 3



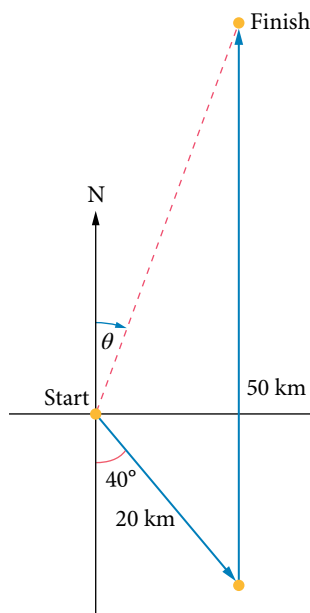
- 12 The positions of offshore oilrigs R , S and T are shown. Determine the true bearing of the following.

- (a) S from R
- (b) T from R
- (c) T from S
- (d) R from S
- (e) R from T
- (f) S from T



- 13 During a car rally, a competitor drives 20 km from the start in the direction $S40^\circ E$, then due north for 50 km.

What is the competitor's distance (to 2 decimal places) and true bearing (nearest degree) from the start position?

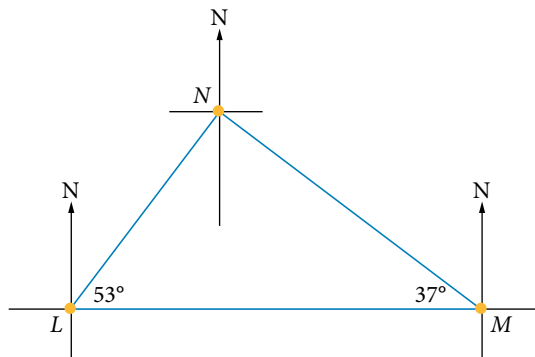


Worked
Example

12

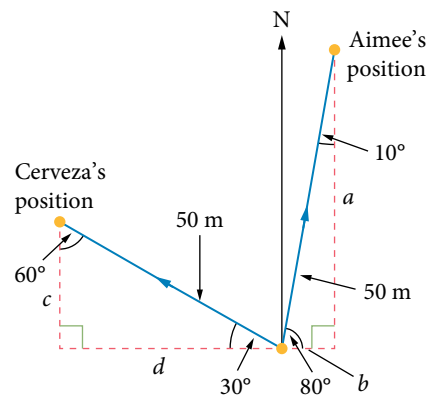
- 14 A skier skis 40 m on a bearing $N70^\circ W$, then executes a sharp turn and skis 100 m due east. What is the distance (to 2 decimal places) and compass direction (to the nearest degree) of the skier from his starting point after completing these movements?

- 15 Directions from Mount Lagrange (L) and Mount Monge (M) to a third peak, Mount Neumann (N), are shown in the diagram.



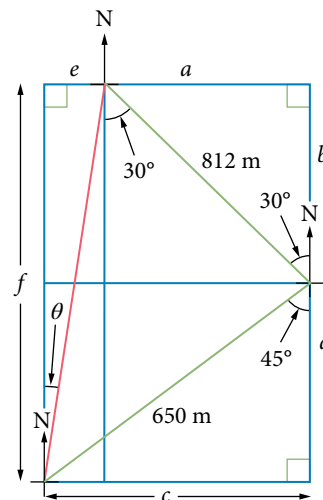
- (a) Determine the following directions and bearings.
- The standard compass direction of Mount Lagrange from Mount Neumann
 - The true bearing of Mount Monge from Mount Neumann.
- (b) Given that Mount Monge is 64 km east of Mount Lagrange, how far is Mount Neumann from each of the other peaks? Give answers to 1 decimal place.

- 16 During an orienteering activity, Aimee walks 50 m on a bearing $010^\circ T$ from a base point B . At the same time, her classmate Cerveza leaves the same base point and walks 50 m on bearing $300^\circ T$.



- (a) What is Aimee's direction from Cerveza?
- (b) How far apart are the girls, to 1 decimal place?

- 17 A honey bee makes a beeline for some eucalyptus blossom 812 m away in the direction $S30^\circ E$. On arrival she is directed by another worker to go SW to a better spot 650 m away.



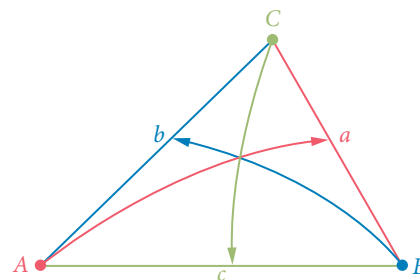
- (a) After collecting some nectar, what bearing must she take to make a beeline for the hive? Answer to the nearest degree.
- (b) Bees fly at about 24 km/h. How long will it take the bee to return, to the nearest minute?

The sine rule

Naming conventions

In the next sections you will learn rules that enable you to solve sides and angles of triangles without having to draw a right-angled triangle.

To make the rules clear, label the triangle using the same pronumeral for the side and the angle that are opposite each other, but use lowercase for the side and UPPER CASE for the angle. So side a is always opposite angle A and so on.



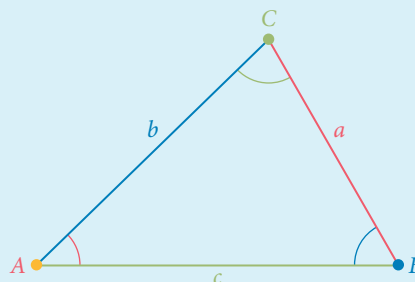
The sine rule

The sine rule is used when you know the values of either two angles and one side, or two sides and an angle opposite one of the sides.

The sine rule states:

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \quad \text{or} \quad \frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

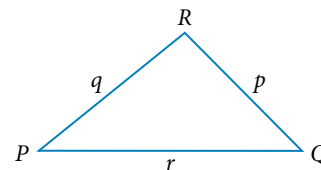
where A , B and C are angles and a , b and c are the corresponding opposite sides.



The pronumerals used for sides and angles can be any letters or symbols. For example, you could write the sine rule as

$$\frac{p}{\sin(P)} = \frac{q}{\sin(Q)} = \frac{r}{\sin(R)} \quad \text{or} \quad \frac{\sin(P)}{p} = \frac{\sin(Q)}{q} = \frac{\sin(R)}{r}$$

as long as side p is opposite angle P , and so on.



The sine rule is useful for calculating side lengths and angles in triangles that do not include a right-angle.

In checking whether your answers are reasonable, be aware that the ratios in the rule tell you that if $a > b > c$ then $\sin(A) > \sin(B) > \sin(C)$ and $A > B > C$.

In any triangle:

- the larger the side, the larger the angle opposite
- the larger the angle, the larger the side opposite
- the larger the sine of the angle, the larger the side opposite.

WARNING

When using a calculator, ensure it is in degrees mode.

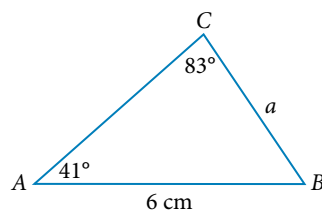
i Additional information

Sine rule

This activity gives a demonstration and practice in calculating missing side lengths using the sine rule.

13 Calculating side lengths using the sine rule

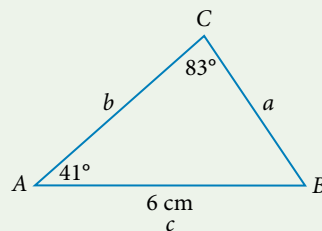
Calculate the side length a , stating your answer to 2 decimal places.



THINKING

- Two angles and one side are given, so you can use the sine rule. Redraw and label the triangle if required.
- Identify the part of the sine rule that applies in this case.
- Substitute the known values.
- Solve the equation to find the length of the side labelled a . Round answer to 2 decimal places.

WORKING



$$\frac{a}{\sin(A)} = \frac{c}{\sin(C)}$$

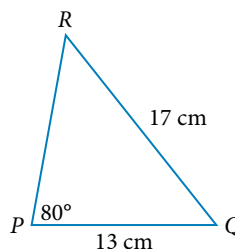
$$\frac{a}{\sin(41^\circ)} = \frac{6}{\sin(83^\circ)}$$

$$a = \frac{6}{\sin(83^\circ)} \times \sin(41^\circ)$$

$$= 3.97\text{ cm (2 d.p.)}$$

14 Calculating angles with the sine rule

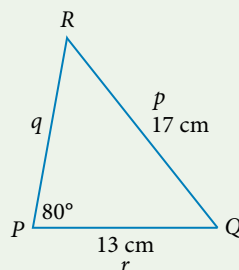
Calculate the size of the angles marked R and Q , rounding to the nearest degree.



THINKING

- Two sides and a non-included angle are given, so you can use the sine rule. Redraw and label the triangle if required.
- Identify the part of the sine rule that applies in this case.
Note: It is easier to use the version that has the unknown ($\sin(R)$) as a numerator.

WORKING



$$\frac{\sin(R)}{r} = \frac{\sin(P)}{p}$$

3 Substitute the known values.

$$\frac{\sin(R)}{13} = \frac{\sin(80^\circ)}{17}$$

4 Solve the equation to find the size of the unknown angle. At the final step, round to the nearest degree as requested.

$$\sin(R) = \frac{\sin(80^\circ)}{17} \times 13$$

$$= 0.753\dots$$

$$R = \sin^{-1}(0.753\dots)$$

$$= 49^\circ \text{ (nearest degree)}$$

5 To calculate the size of the remaining angle, subtract the sum of the other two angles from 180° .

Angle sum of a triangle is 180° .

$$Q = 180 - (80 + 49)$$

$$= 51^\circ$$

i Additional information

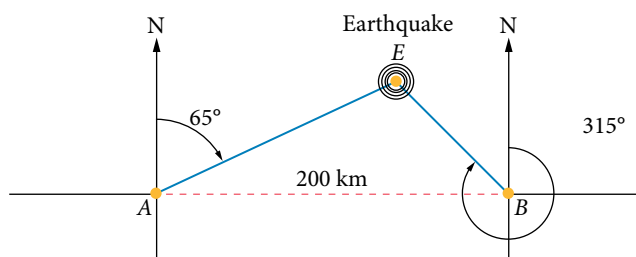
Review the sine rule

Try the activity to review solving problems using the sine rule.

15 Application of the sine rule

Two earthquake detection stations, A and B , are situated 200 km apart, with station B due east of station A . Station A detects an earthquake on a bearing of 065°T . Station B detects the same earthquake on a bearing of 315°T .

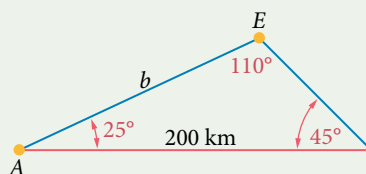
Determine the distance of the earthquake from station A , to 1 decimal place.



THINKING

1 Calculate the size of the angles within the triangle.

WORKING



$$A = 90 - 65$$

$$= 25^\circ$$

$$B = 315 - 270$$

$$= 45^\circ$$

$$E = 180 - (25 + 45)$$

$$= 110^\circ$$

2 Identify the sine rule and substitute the given values.

$$\frac{b}{\sin(B)} = \frac{e}{\sin(E)}$$

$$\frac{b}{\sin(45^\circ)} = \frac{200}{\sin(110^\circ)}$$

- 3 Solve to determine the length of side b .

Round the answer to 1 decimal place.

$$b = \frac{200}{\sin(110^\circ)} \times \sin(45^\circ)$$

$$= 150.5 \text{ km (1 d.p.)}$$

- 4 Interpret the answer.

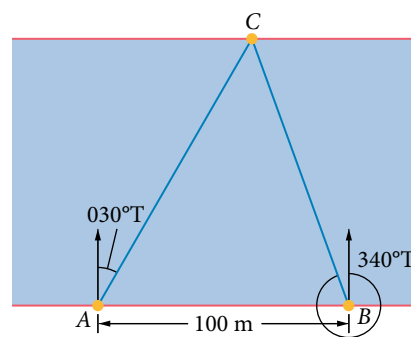
The earthquake is 150.5 km from station A.

Triangulation

Surveyors are able to measure an inaccessible distance indirectly by taking bearings from two points a known distance apart, as the following example demonstrates.

16 Triangulation

Trent is standing on one side of a river and has measured the distance between two power poles A and B on the river's edge to be 100 m. He measures bearings at A and B to pole C to be 030°T and 340°T as shown.

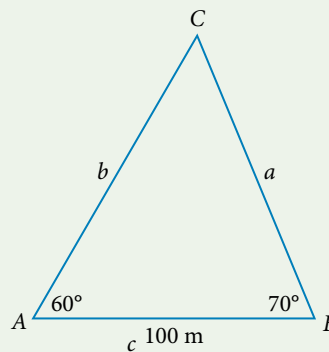


- (a) Calculate the distance from pole A to pole C , on the river's edge on the other side of the river, to 2 decimal places.

THINKING

- 1 Draw and label the triangle formed and recall the appropriate formula.

WORKING



Angle C is $180 - (60 + 70) = 50^\circ$.

$$\frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

$$\frac{AC}{\sin(70^\circ)} = \frac{100}{\sin(50^\circ)}$$

$$AC = \frac{100}{\sin(50^\circ)} \times \sin(70^\circ)$$

$$= 122.67 \text{ m (2 d.p.)}$$

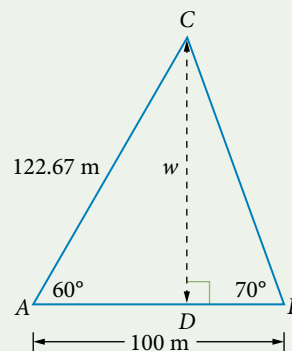
- 2 Substitute known values into the sine rule and solve for the unknown variable, accurate to 2 decimal places.

- 3 Interpret the answer.

The distance from pole A to pole C is 122.67 m.

(b) Calculate the width of the river to the nearest metre.

- 1 Show the width of the river in a right-angled triangle.



- 2 Use the sine ratio to find the width, to the nearest metre.

In triangle ACD:

$$\sin(60^\circ) = \frac{w}{122.67}$$

$$w = 122.67 \times \sin(60^\circ)$$

$$= 106 \text{ m (nearest m)}$$

- 3 Interpret the answer.

The width of the river is 106 m.

EXERCISE

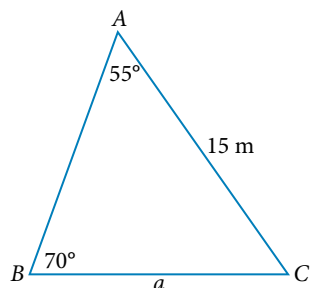
5.4

The sine rule

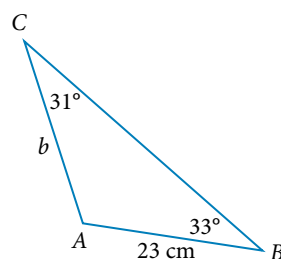
Unless otherwise stated, give all answers to 2 decimal places.

- 1 Determine the length indicated by the pronumeral.

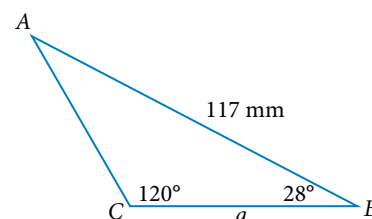
(a)



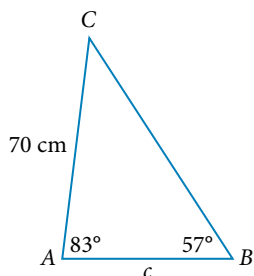
(b)



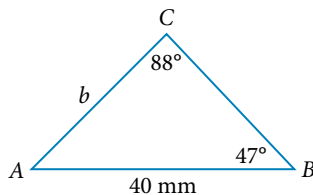
(c)



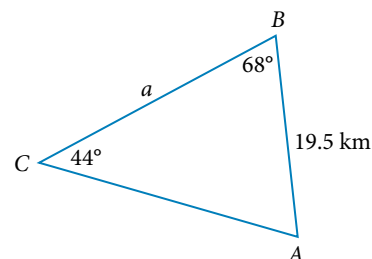
(d)



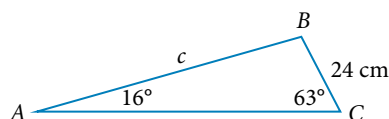
(e)



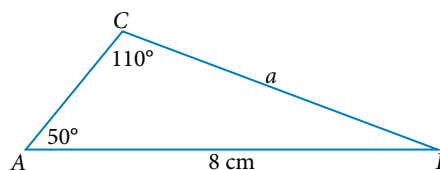
(f)



(g)



(h)

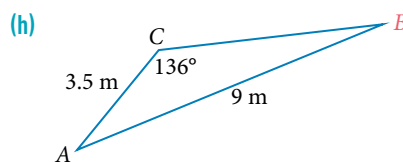
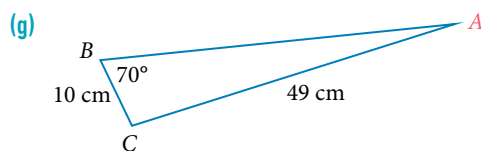
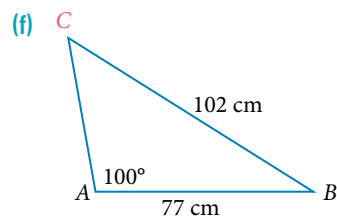
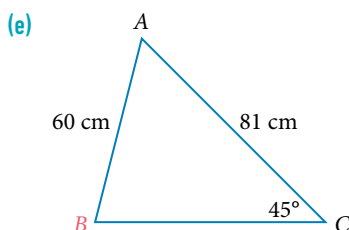
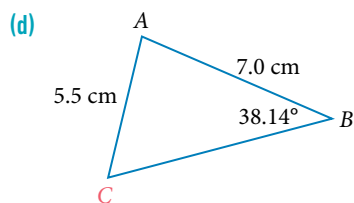
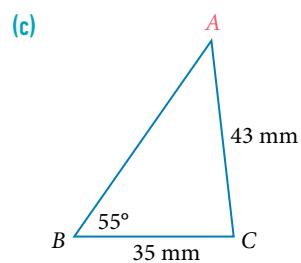
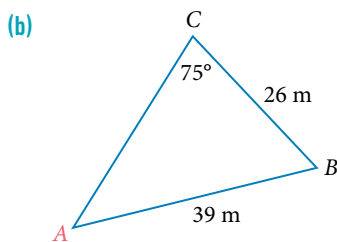
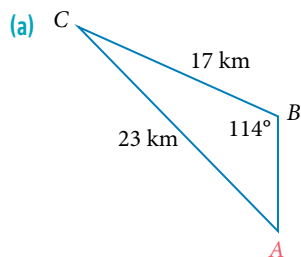


Worked
Example

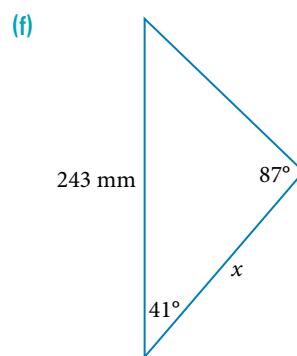
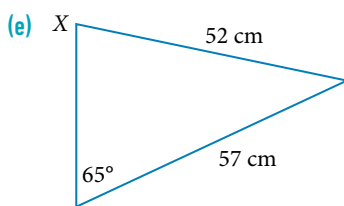
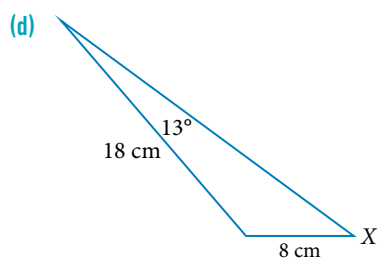
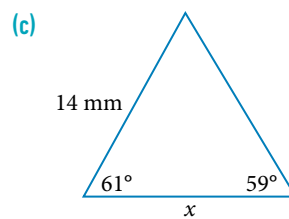
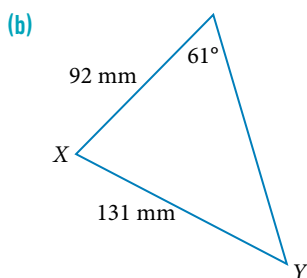
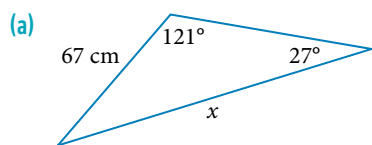
13

14

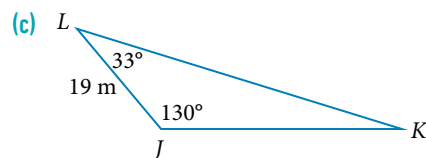
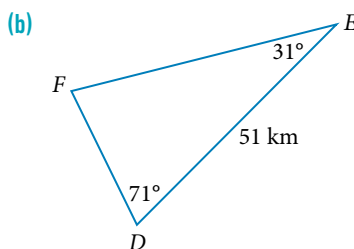
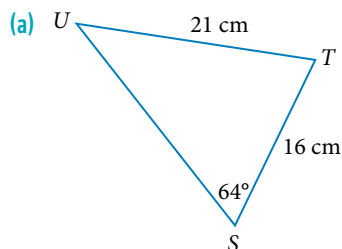
2 Calculate the size of the angles shown in red, to the nearest degree.



3 Determine the value of the pronumeral(s) in each triangle.



4 Solve the triangle to determine every unknown angle and side length in each triangle.



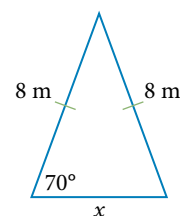
- 5 In relation to the triangle shown, which of the following statements is correct?

A $\frac{x}{\sin(40^\circ)} = \frac{8}{\sin(70^\circ)}$

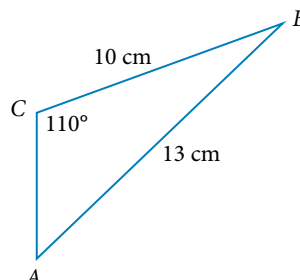
B $\frac{x}{\sin(70^\circ)} = \frac{8}{\sin(40^\circ)}$

C $\frac{x}{\sin(70^\circ)} = \frac{\sin(40^\circ)}{8}$

D $\frac{x}{\sin(40^\circ)} = \frac{16}{\sin(140^\circ)}$



- 6 Determine angles A and B to the nearest degree.



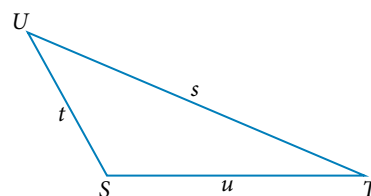
- 7 (a) Which of the following statements is correct?

A $\frac{\sin(S)}{u} = \frac{\sin(T)}{s}$

B $\frac{\sin(S)}{u} = \frac{\sin(U)}{s}$

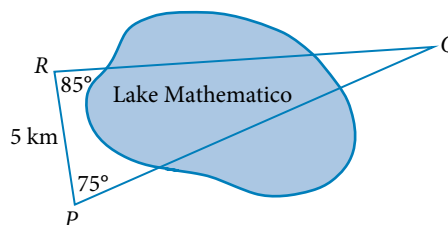
C $\frac{\sin(T)}{\sin(U)} = \frac{t}{u}$

D $\frac{\sin(T)}{\sin(S)} = \frac{s}{t}$



- (b) If the triangle has been drawn to scale, explain how the last incorrect response can be rejected on the basis of size.

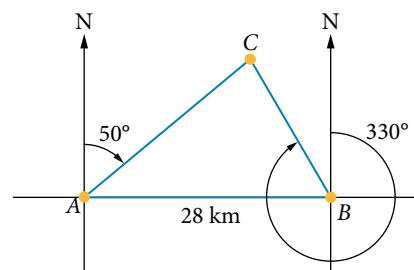
- 8 Lin is mapping an artificial lake using survey markers P, Q and R, and measures the distance PR to be 5 km. She is unable to measure PQ, as PQ intersects Lake Mathematico, but she does make the angle measurements shown here.



- (a) Calculate the distance PQ.
(b) Calculate the distance RQ.

- 9 Three friends, Aisha, Borak and Cassie, use a satellite navigation system to locate each other. Aisha is 28 km due west of Borak. Cassie is on a bearing of 050°T from Aisha, and on a bearing of 330°T from Borak.

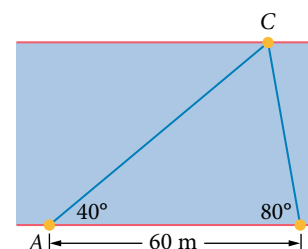
- (a) Calculate the angles A, B and C in the triangle depicting the locations.
(b) How far apart are Cassie and Aisha?
(c) How far apart are Cassie and Borak?



- 10 Ray surveys three points A, B and C, and the distance AB on a river's edge, as shown.

Use Ray's results to determine:

- (a) the distance AC
(b) the width of the river, to the nearest metre.

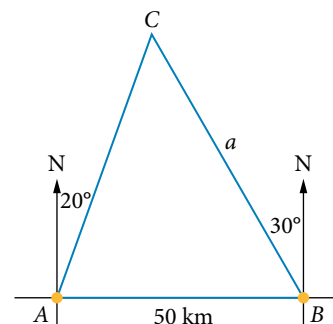


Worked
Example

15

16

- 11 Petra (P) measures the angle of elevation to the top R of a transmission tower to be 35° . Her friend Qiana (Q) is 560 m closer to the tower and measures the angle of elevation to the top to be 75° . Calculate the distance between Petra and the top of the tower.
- 12 The sewerage system for a particular property has three inspection points W , X and Y . Point Y is 40 m due north of W , and point X is $S76^\circ E$ from Y and $N46^\circ E$ from W . How far apart are points W and X ?
- 13 The bearing of point C from two positions A and B is shown here. Determine the distance a .

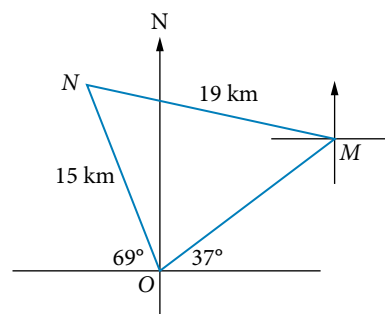


- 14 A triangular section ABC of a park has been fenced off to assist revegetation. Corner C is 20 m due east of corner A and 35 m from corner B . Side AB runs south-west. Determine the size of the angle at corner C . Express your answer to the nearest degree.

..

..

- 15 Oscar sets up base camp at O while two friends, Neil and Marcus, hike to landmarks N and M along the directions shown. GPS data indicates that Neil hikes 15 km, and is 19 km from Marcus' position. How far is Marcus from base camp?



- 16 A lookout at a fire station detects two fires. Fire A is 16 km from the station on a bearing of $053^\circ T$. Fire B is on a bearing of $074^\circ T$ from the station and is 19 km from fire A .
- (a) How far is fire B from the station?
- (b) What is the direction of fire A from fire B , to the nearest degree?
- 17 A council inspector maps the positions of three trees in a park. Tree B is 120 m due east of tree A . Tree C is north-east of tree A , and is on a bearing of $N345^\circ T$ from tree B . How far apart are trees B and C ?

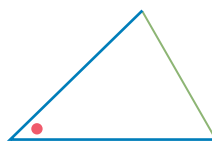
The cosine rule

5.5

The cosine rule

The cosine rule is used when you know the values of either all three sides, or two sides and the angle between them (i.e. an included angle).

Two sides and an included angle

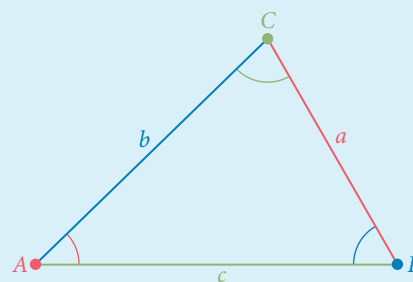


All three sides



There are three versions of the cosine rule that apply to any triangle, each one involving a different angle. A , B and C are the angles and a , b and c are the corresponding opposite sides.

Side version	Angle version
$a^2 = b^2 + c^2 - 2bc \cos(A)$	$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$
$b^2 = a^2 + c^2 - 2ac \cos(B)$	$\cos(B) = \frac{a^2 + c^2 - b^2}{2ac}$
$c^2 = a^2 + b^2 - 2ab \cos(C)$	$\cos(C) = \frac{a^2 + b^2 - c^2}{2ab}$

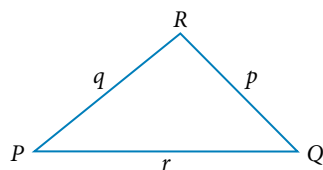


The pronumerals used for sides and angles can be any letters or symbols.

For the following triangle, the cosine rule could be written as:

$$p^2 = q^2 + r^2 - 2qr \cos(P)$$

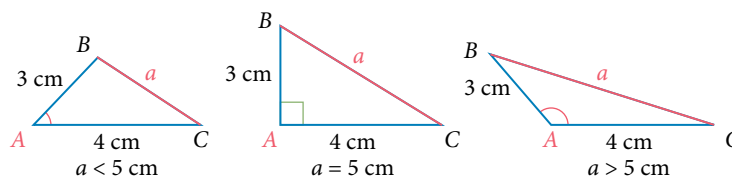
$$\text{or } \cos(P) = \frac{q^2 + r^2 - p^2}{2qr}$$



The cosine rule is useful for calculating side lengths and angles in triangles that do not include a right angle.

$\cos(90^\circ) = 0$, which leads to the connection between the cosine rule and Pythagoras' theorem.

In checking whether your answers are reasonable, remember that values of cosine are negative when $90^\circ < \theta < 180^\circ$, and that the side opposite an obtuse angle is larger than the hypotenuse would be if the triangle was right-angled.



In obtuse-angled triangles with obtuse angle θ :

- $\cos(\theta)$ is negative
- the longest side is opposite the obtuse angle.

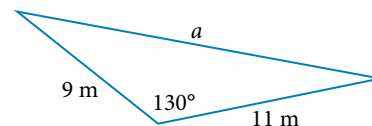
Additional information

The cosine rule

Complete the activities for a demonstration and practice in calculating an unknown side length using the cosine rule.

17 Calculating a side length using the cosine rule

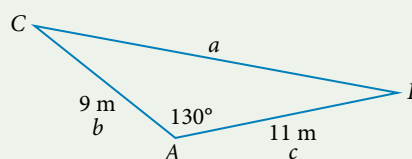
Calculate the side length a , accurate to 2 decimal places.



THINKING

- Two sides and the included angle are given, so you can use the cosine rule. Redraw and label the triangle if required.
- Identify the known angle and recall the cosine rule.
- Substitute the known values.
- Solve the equation to find the length of side a . Given that a represents the side length of a triangle, reject the negative a value.
- Check the reasonableness of your answer.
When calculating the side length opposite an angle greater than or equal to 90° , check that you have calculated a value representative of the longest side length.

WORKING



Known angle: $A = 130^\circ$

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

$$= 9^2 + 11^2 - 2 \times 9 \times 11 \times \cos(130^\circ)$$

$$a^2 = 329.271\dots$$

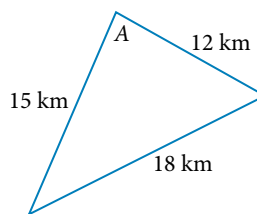
$$\therefore a = \sqrt{329.271\dots}$$

$$= 18.15\text{ m (2 d.p.)}$$

Given that $A = 130^\circ$, which is greater than 90° , $a > 11\text{ m}$ so $a = 18.15\text{ m}$ is a reasonable calculation.

18 Calculating the size of an angle using the cosine rule

Calculate the size of angle A , to 1 decimal place.



THINKING

- 1 Three sides are given, so use the cosine rule.
Redraw and label the triangle if required.
Note: Angles may be marked on the inside of the triangle.

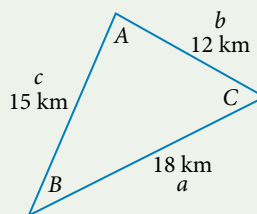
- 2 Recall the cosine rule.

- 3 Substitute the known values.

- 4 Simplify and use the inverse cosine to find angle A .
Round answer to 1 decimal place.

- 5 Check that your answer is reasonable.
When given three side lengths, the angle opposite the longest side length must be greater than 60° (an angle opposite the shortest length must be smaller than 60°).

WORKING



$$\cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos(A) = \frac{12^2 + 15^2 - 18^2}{2 \times 12 \times 15}$$

$$\cos(A) = 0.125$$

$$A = \cos^{-1}(0.125) \\ = 82.8^\circ \text{ (1 d.p.)}$$

Given that angle A is opposite the longest side length, $A = 82.8^\circ$ is reasonable because A must be greater than 60° .

i Additional information

Cosine rule – angles

Try the activity, to review the process of calculating an unknown angle using the cosine rule.

19 Applications of the cosine rule

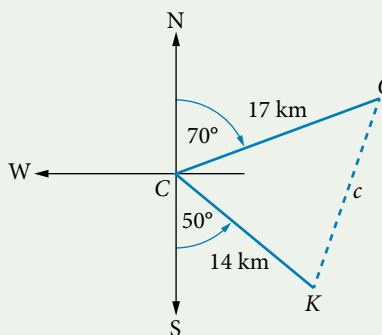
Two hikers leave the same base camp C for different destinations. Olga travels 17 km on a bearing 070°T before setting up a new camp for the night. Katrina hikes 14 km on a bearing $S50^\circ\text{E}$ before pitching her tent.

- (a) How far apart are the hikers' new campsites? Round your answer to 1 decimal place.

THINKING

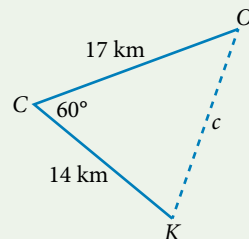
- 1 Draw a diagram using appropriate pronumerals for locations and include the given information. Redraw the triangle with all known information.
Note that the cosine rule uses two side lengths and the angle between. Calculate that angle.

WORKING



Calculate angle C in the triangle OCK .

$$\begin{aligned} C &= 180 - (70 + 50) \\ &= 60^\circ \end{aligned}$$



- 2 Recall the cosine rule and solve to find the length of the unknown side.

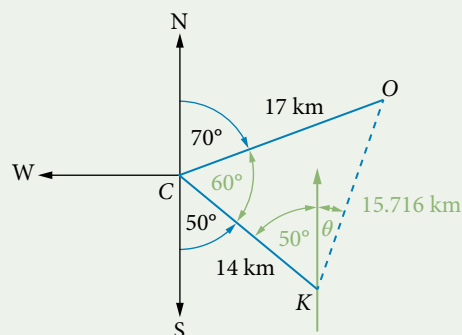
$$\begin{aligned} c^2 &= o^2 + k^2 - 2ok \cos(C) \\ &= 14^2 + 17^2 - 2 \times 14 \times 17 \cos(60^\circ) \\ &= 247 \\ \therefore c &= \sqrt{247} \\ &= 15.7 \text{ (1 d.p.)} \end{aligned}$$

- 3 Interpret the answer.

The hikers' new campsites are 15.7 km apart.

- (b) What is the true bearing of Olga from Katrina's position? Answer to the nearest degree.

- 1 Draw a north-south line segment, and mark the required bearing (θ) and the additional information.



- 2 Use the sine rule to determine the size of the unknown angle.

Note the use of alternate angles.

$$\begin{aligned} \frac{\sin(K)}{17} &= \frac{\sin(60^\circ)}{15.716} \\ \sin(K) &= \frac{\sin(60^\circ)}{15.716} \times 17 \\ &= 0.936... \\ K &= \sin^{-1}(0.936...) \\ &= 70^\circ \text{ (nearest degree)} \end{aligned}$$

- 3 Calculate the angle needed for the bearing.

$$\begin{aligned} \theta &= 70 - 50 \\ &= 20^\circ \end{aligned}$$

- 4 Interpret the answer.

Olga is at a true bearing of 020°T from Katrina.

i Additional information

Sine and cosine applications

Try the activity, to practise using the sine and cosine rules in a real-life application.

EXERCISE

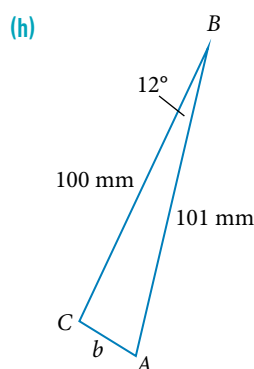
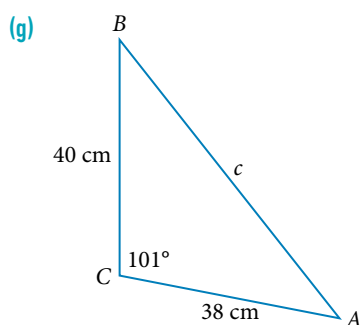
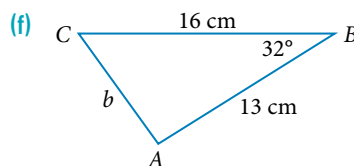
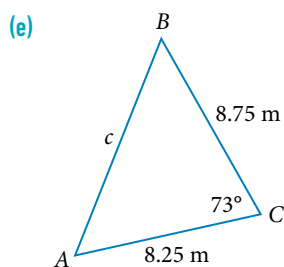
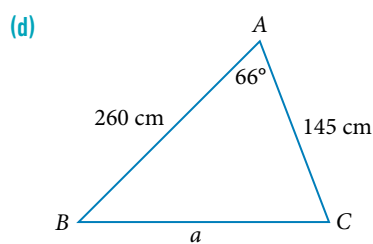
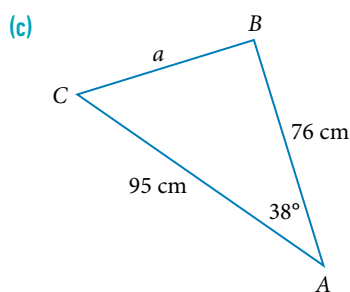
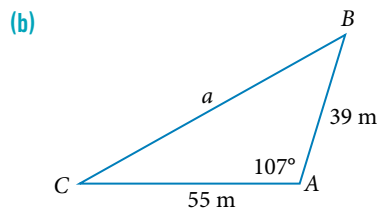
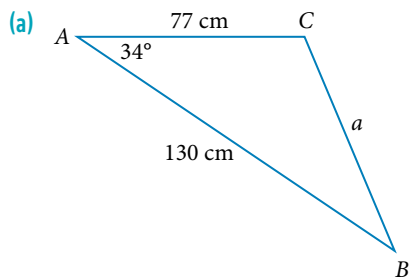
5.5

The cosine rule

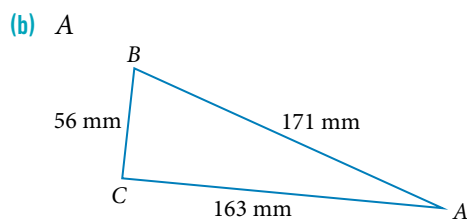
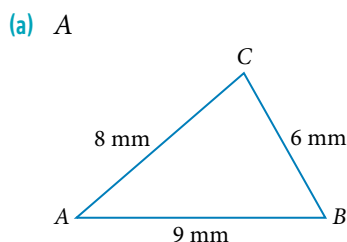
Worked
Example

17

- 1 Determine the length indicated by the pronumeral in each case.

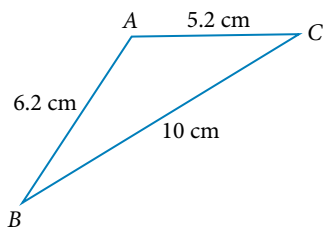


- 2 Determine the size of the nominated angle, to 2 decimal places.

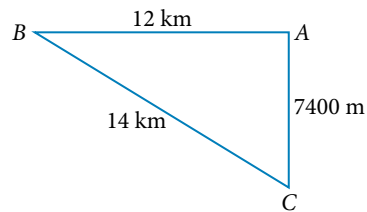


18

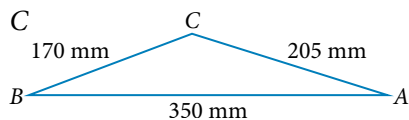
(c) A



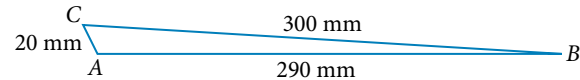
(d) A



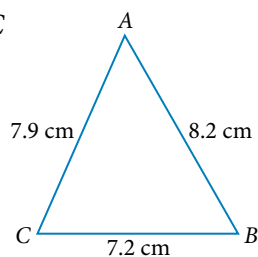
(e) C



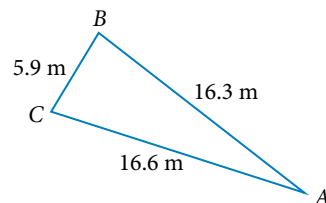
(f) B



(g) C

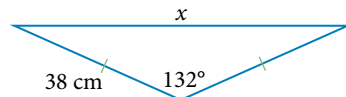


(h) B

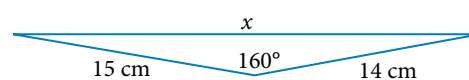


3 Determine the value of x to 2 decimal places.

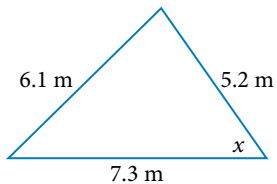
(a)



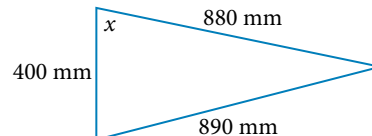
(b)



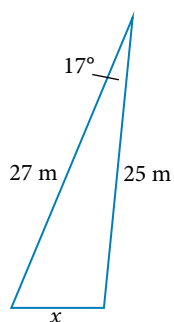
(c)



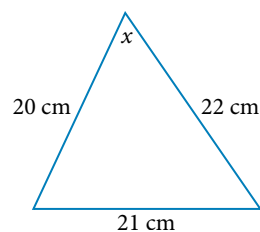
(d)



(e)



(f)

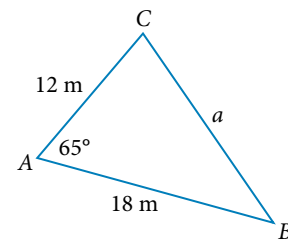


4 Consider the triangle shown.

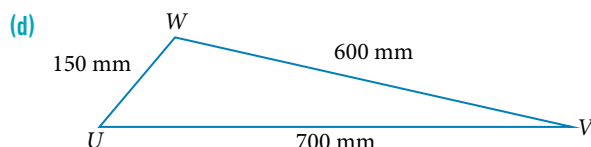
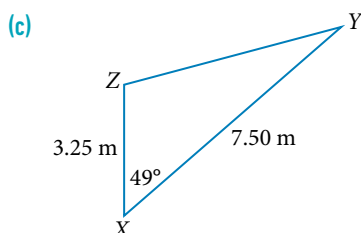
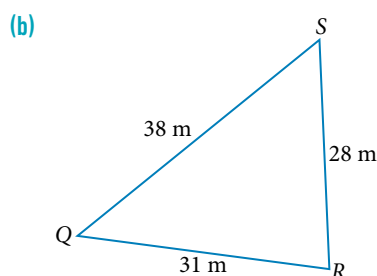
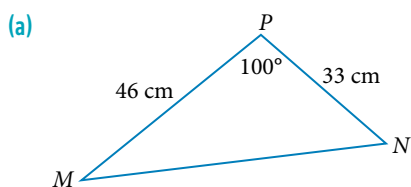
(a) Determine the length of a .

(b) Explain the common error made by a student who determined the length of b to be 28543 m.

(c) Give a strategy for testing the reasonableness of the answer, to avoid the error in (b).



- 5 Determine every unknown angle and side length in each triangle below. (This is referred to as solving the triangle.) Give your answers to 1 decimal place.



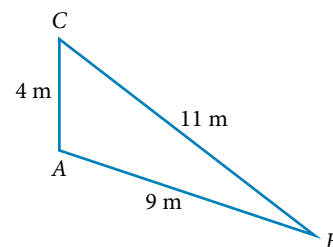
- 6 (a) Which of the following is correct for the triangle shown?

A $\cos(A) = \frac{4^2 + 11^2 - 9^2}{2 \times 4 \times 11}$

B $\cos(A) = \frac{4^2 + 9^2 - 11^2}{2 \times 11 \times 9}$

C $\cos(B) = \frac{9 + 11 - 4}{2 \times 9^2 \times 11^2}$

D $\cos(B) = \frac{9^2 + 11^2 - 4^2}{2 \times 9 \times 11}$



- (b) Which of the incorrect options gives a calculation that would be correct for the cosine of a different angle?

- 7 Consider the triangle shown.

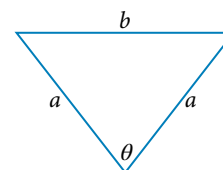
- (a) Which of the following is correct?

A $a^2 = b^2 - 2ab \cos(\theta)$

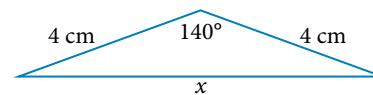
B $b^2 = 2a^2 - 2a^2 \cos(\theta)$

C $\cos(\theta) = \frac{b^2 - 2a^2}{2a^2}$

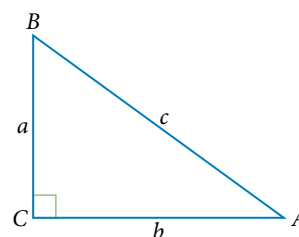
D $\cos(\theta) = \frac{2a^2 - b^2}{2b^2}$



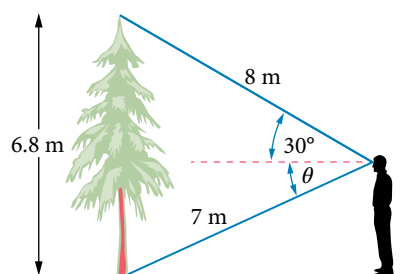
- (b) Using the formula from (a), or otherwise, solve for x , to 2 decimal places.



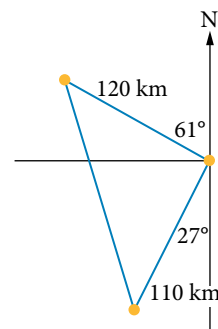
- 8 Simplify the expression $c^2 = a^2 + b^2 - 2ab \cos(C)$ for the right-angled triangle shown.



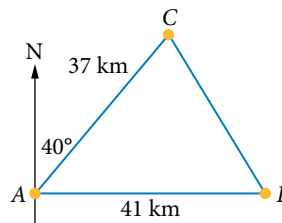
- 9 Monty the lumberjack is going to cut down a 6.8 m tall tree, and measures the distances shown. He also measures the angle of elevation from his eye to the tree top to be 30° . What is the angle of depression θ from Monty's eye to the base of the tree, to the nearest degree?



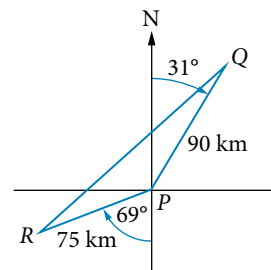
- 10 Two cyclists are riding in different directions as they compete in a time trial around a triangular course. One cyclist rides the 120 km stage in the direction $N61^\circ W$. The other rides the 110 km stage in the direction $S27^\circ W$. How long is the third side of the triangular course, to the nearest kilometre?



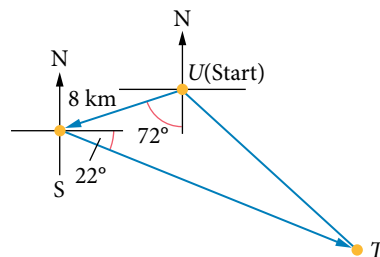
- 11 The new suburb of Triangula has three borders, as shown in the diagram. How long is boundary CB , to the nearest kilometre?



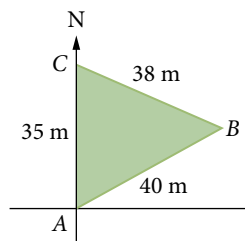
- 12 Two sailboats leave from a pier P and reach points Q and R simultaneously, as shown. Calculate the distance between the boats, to the nearest kilometre, when they are at points Q and R .



- 13 A bushwalker walks from point U , 8 km in the direction $S72^\circ W$ to point S , then a further 20 km in the direction $S68^\circ E$ to point T . How far must the bushwalker walk to return to point U , to the nearest kilometre?



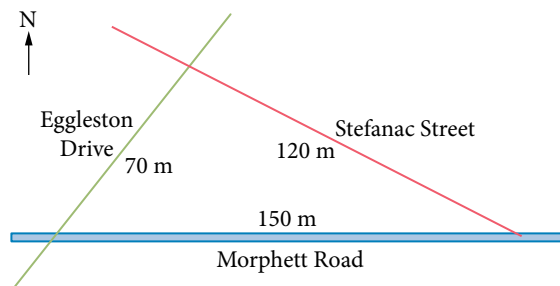
- 14 Three fences surround a triangular paddock, as shown. What is the true bearing of corner A from corner B , to the nearest degree?



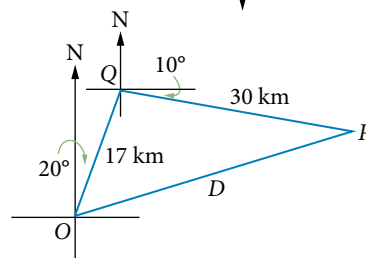
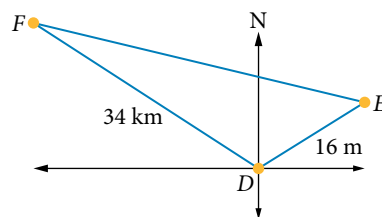
- 15 The distances between the intersections of three roads are shown in the diagram.

Determine the following angles, to the nearest degree:

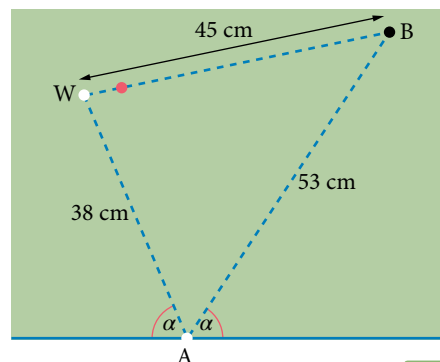
- the acute angle that Morphet Road makes with Eggleston Drive
- the acute angle that Stefanac Street makes with Morphet Road
- the obtuse angle between Eggleston Drive and Stefanac Street.



- 16 Two hikers leave the same base camp A for different destinations. Breeana (B) travels 13 km on a bearing 050°T before setting up a new camp for the night. Campbell (C) hikes 9 km on a bearing $S20^\circ\text{E}$ before pitching his tent.
- What is the distance between the new campsites? Round your answer to 1 decimal place.
 - What is the bearing of Breeana from Campbell's position? Answer to the nearest degree.
- 17 During the testing of a new game plan, a football coach instructs two players to run in different directions as shown. Frederico (F) runs 34 m on a bearing of 304°T . Evan (E) runs 16 m on a bearing of 044°T before both players stop.
- At what angle D do the players separate? Express your answer to the nearest degree.
 - How far apart are Frederico and Evan, to the nearest metre?
 - What is the standard compass direction of Frederico from Evan's position, to the nearest degree?
- 18 Consider the triangle shown here.
- Calculate the distance D to the nearest kilometre.
 - Determine the true bearing of point P from point O to the nearest degree.

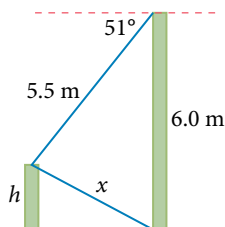


- 19 Wa-Lin is about to attempt a shot in a game of billiards. A red ball blocks her direct line towards the black ball. Her plan is to bounce the white ball W off the edge of the table at point A at an angle α so that it hits the black ball B . Calculate the angle α that the white ball should make with the edge of the table. Give your answer to the nearest degree.



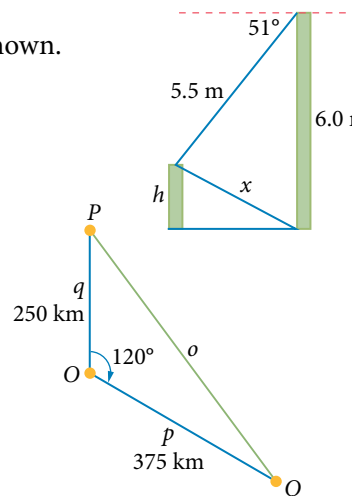
- 20 Jai hikes 6 km on a bearing of 240°T , then changes direction and hikes 19 km on a bearing 100°T . How far is Jai now from his starting point, to 2 decimal places?

- 21 Builders are bracing a 6.0 m wall by connecting it to a shorter wall as shown. Calculate the values x and h for the builders. Give your answers to 3 decimal places.



- 22 Aircraft P and aircraft Q depart the same airport at the same time. When aircraft P is 250 km due north of the airport, aircraft Q has travelled 375 km on a bearing 120°T .

- How far apart are the two aircraft at this time, to the nearest kilometre?
- What direction is aircraft P from aircraft Q , to the nearest degree?



5.6

Alternative formulas for triangle area

Area of a triangle, given two sides and the angle between them

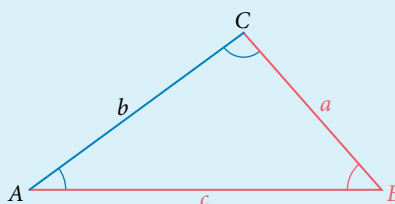
Previously you used the formula $A = \frac{1}{2}bh$ to determine the area of a triangle when given the base length and height.

Triangles can be defined by other measurements, and other formulas can be used to determine the area in such cases.

WARNING

In this section you may have to use the word Area rather than A for area of a triangle, because you will be using the naming convention for triangles that uses A for an angle.

Use the formulas to find the area of a triangle when two sides and the angle between them are known.



$$\text{Area} = \frac{1}{2}ab \sin(C)$$

$$\text{Area} = \frac{1}{2}bc \sin(A)$$

$$\text{Area} = \frac{1}{2}ac \sin(B)$$

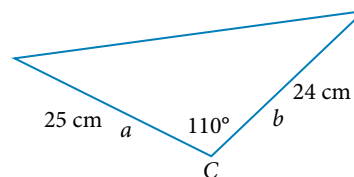
i Additional information

Area of a triangle

Review the formula for finding the area of a triangle when two sides and the angle between them are given.

20 Area of a triangle, given two sides and the angle between them

Determine the area of the triangle, to 2 decimal places.



THINKING

- Two sides and the angle between them are given. Use $\text{Area} = \frac{1}{2}ab \sin(C)$.
- Identify a , b and C from the triangle.
- Substitute for a , b and C .
- Simplify, express the answer to the nominated number of decimal places (2) and include units.
- Interpret the result.

WORKING

$$\text{Area} = \frac{1}{2}ab \sin(C)$$

$$a = 25, b = 24, C = 110^\circ$$

$$\text{Area} = \frac{1}{2} \times 25 \times 24 \times \sin(110^\circ)$$

$$\text{Area} = 281.91 \text{ cm}^2 \text{ (2 d.p.)}$$

The area of the triangle is 281.91 cm^2 .

Area of a triangle, given three sides

It would be possible to use $\text{Area} = \frac{1}{2}ab \sin(C)$ if you first solved an included angle using the cosine rule. However, Heron's rule uses the three sides directly.

i Additional information

Heron's rule

Practise using Heron's rule.

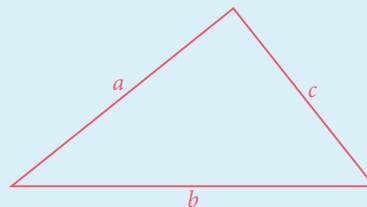
Heron's rule for area of a triangle, given three sides

If the lengths of all three sides (a , b and c) in a triangle are known:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

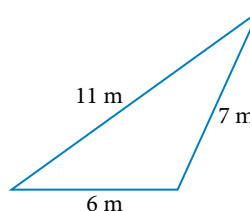
where s is the semi-perimeter, equal to half the perimeter:

$$s = \frac{a+b+c}{2}$$



21 Area of a triangle, given three sides

Determine the area of the triangle, to 2 decimal places.



THINKING

- 1 Three sides are given, so Heron's rule applies. First calculate the semi-perimeter s .
- 2 Use Heron's rule, substitute values for s and the three sides a , b and c , and simplify.
- 3 Calculate the answer, expressing it to the nominated number of decimal places (2) and including units.
- 4 Interpret the result.

WORKING

$$\begin{aligned} s &= \frac{a+b+c}{2} \\ &= \frac{6+7+11}{2} \\ &= \frac{24}{2} \\ &= 12 \end{aligned}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\begin{aligned} \text{Area} &= \sqrt{12(12-6)(12-7)(12-11)} \\ &= \sqrt{12 \times 6 \times 5 \times 1} \\ &= 18.97 \text{ cm}^2 \quad (2 \text{ d.p.}) \end{aligned}$$

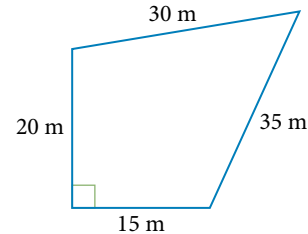
The area of the triangle is 18.97 cm^2 .

Area of polynomials

Every polynomial can be divided into triangles. This means that, given the necessary measurements, the area of any polynomial can be found.

22 Area of a quadrilateral

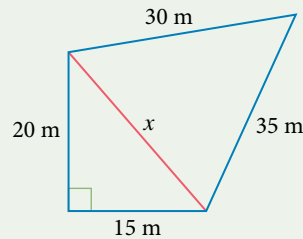
Determine the area of the quadrilateral, to 2 decimal places.



THINKING

- 1 Draw a diagonal so the shape is broken into triangles. Choose the diagonal carefully so that individual areas can be found.
- 2 Use Pythagoras' theorem to determine the length of the diagonal. This length will be needed to use Heron's rule for the triangle on the right.
- 3 Calculate the area of the right-angled triangle.
- 4 Use Heron's rule to find the area of the second triangle.
- 5 Add individual areas to determine the total area.

WORKING



Pythagoras' theorem:

$$\begin{aligned}x^2 &= 15^2 + 20^2 \\ \therefore x &= \sqrt{15^2 + 20^2} \\ &= 25 \text{ m}\end{aligned}$$

Triangle on the left:

$$\begin{aligned}A &= \frac{1}{2}bh \\ &= \frac{1}{2} \times 15 \times 20 \\ &= 150 \text{ m}^2\end{aligned}$$

Triangle on the right:

$$\begin{aligned}s &= \frac{a + b + c}{2} \\ &= \frac{25 + 30 + 35}{2} \\ &= 45\end{aligned}$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\begin{aligned}\text{Area} &= \sqrt{45(45-25)(45-30)(45-35)} \\ &= \sqrt{45 \times 20 \times 15 \times 10} \\ &= 367.42 \text{ m}^2 \quad (2 \text{ d.p.})\end{aligned}$$

$$\begin{aligned}A_{\text{total}} &= 150 + 367.42 \\ &= 517.42 \text{ m}^2\end{aligned}$$

The area of the quadrilateral is 367.42 m^2 .

EXERCISE

5.6

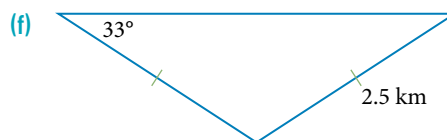
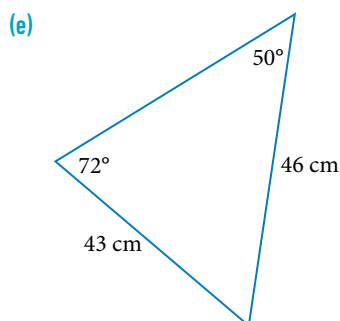
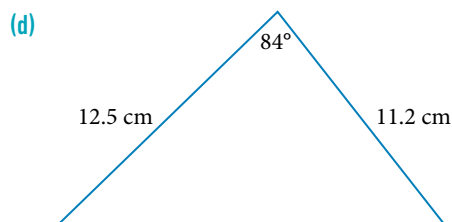
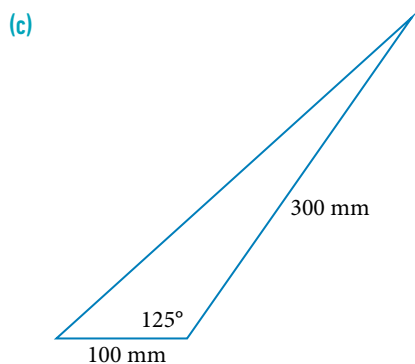
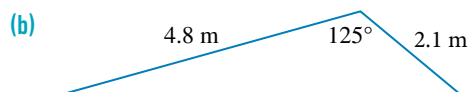
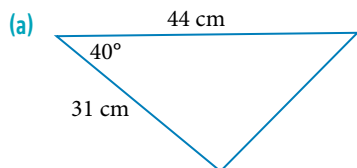
Alternative formulas for triangle area

In this exercise, express answers to 2 decimal places unless otherwise stated.

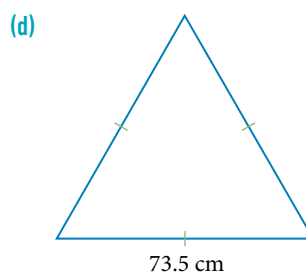
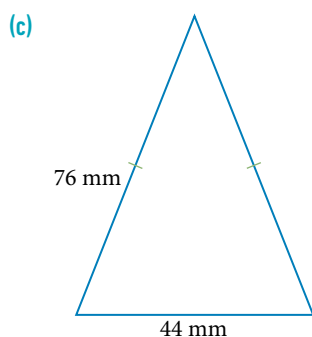
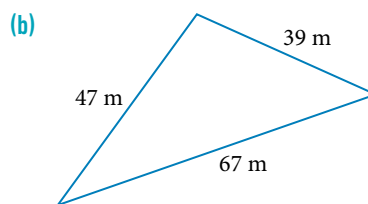
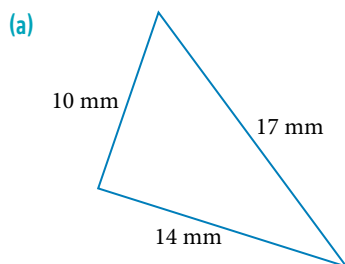
Worked
Example

20

- 1 Calculate the area of each triangle.

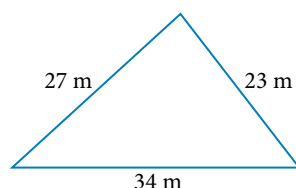


- 2 Use Heron's rule to calculate the area of each triangle.

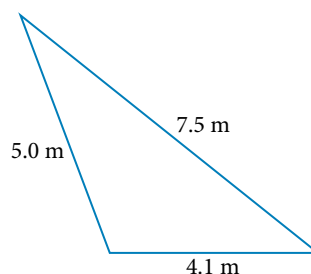


21

(e)



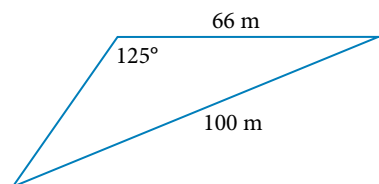
(f)



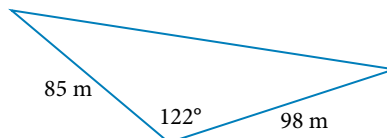
3 Consider the triangles below.

(a) The area of which triangle can be calculated directly using $\text{Area} = \frac{1}{2}ab \sin(C)$?

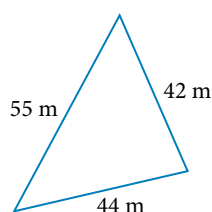
A



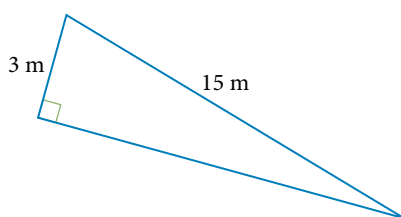
B



C



D



(b) Explain why the formula could not be used for the incorrect options where two sides and an angle were known.

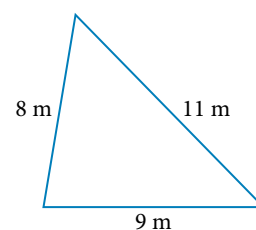
4 Which expression below could be used to calculate the area of this triangle?

A $\frac{1}{2} \times 9 \times 8$

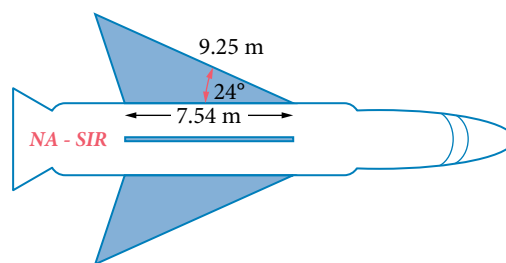
B $\frac{1}{2} \times 8 \times 9 \times \sin(11)$

C $14 \times 6 \times 5 \times 3$

D $\sqrt{14(14-8)(14-9)(14-11)}$



5 A new design for a rocket is shown. It includes four flat fins made from a special carbon-fibre compound. Calculate the total area of carbon fibre required for the four fins, to 2 decimal places.

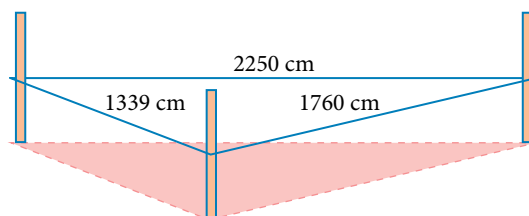


6 A triangular garden has edges of length 5.0 m, 7.5 m and 6.9 m. Calculate the area of the garden.

7 Police place a triangular barrier around a crime scene as shown. (Assume the ground inside the barrier is level.)

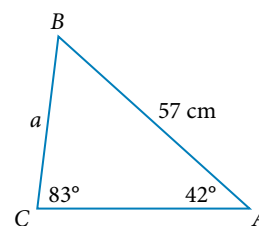
(a) Determine the perimeter of the crime scene in metres.

(b) What area of ground is within the perimeter?
Answer to the nearest square metre.



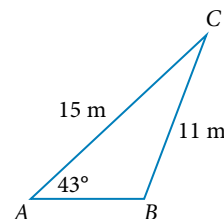
8 Consider the triangle ABC .

- Determine the angle B .
- Use the sine rule to calculate a .
- Use a triangle area formula to calculate the area of the triangle.



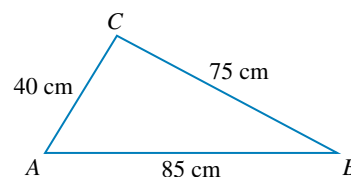
9 Consider the triangle ABC .

- Use the sine rule to calculate the size of angle B .
- Calculate the size of angle C .
- Use a triangle area formula to calculate the area of the triangle.



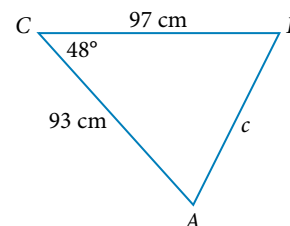
10 The following questions refer to the triangle ABC .

- Use Heron's rule to calculate the area of the triangle.
- Use the cosine rule to calculate the size of angle A .
- Use $\text{Area} = \frac{1}{2}bc \sin(A)$ to check the area of the triangle.
- Determine the angle C , using $\text{Area} = \frac{1}{2}ab \sin(C)$
- Use $A = \frac{1}{2}bh$ to verify the area of the triangle for a third time.

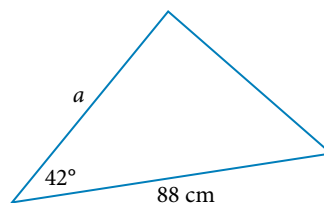


11 The following questions refer to the triangle ABC .

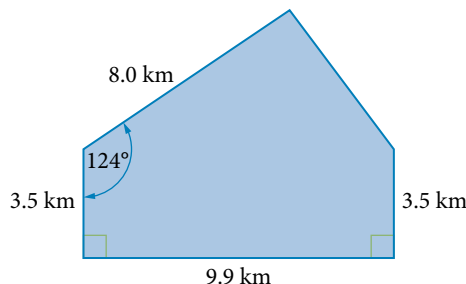
- Calculate the area of the triangle using $\text{Area} = \frac{1}{2}ab \sin(C)$ to the nearest cm^2 .
- Use the cosine rule to determine the length of c .
- Use Heron's rule to calculate the area of the triangle (to the nearest cm^2) using all side lengths. Compare your answer to the area calculated in part (a).



12 Determine the value of a if the area of the triangle is 2500 cm^2 .



13 A satellite photograph of a large wheat farm provides the data shown. Calculate the area of the farm.

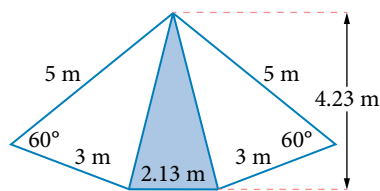


Worked
Example

22

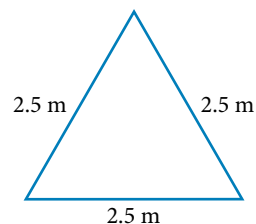
- 14 Builders of the set for the sci-fi series *StarGrate* must join three triangular panels made of sheet metal to form part of a spacecraft.

Determine the total area of sheet metal required for the finished section. Give your answer to 2 decimal places.



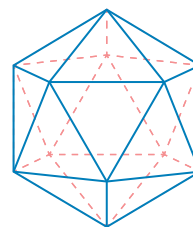
- 15 A tent manufacturer assembles tents from panels of canvas shaped as shown.

If the cost of raw material for each panel is \$250, what is the price per square metre for the canvas, assuming no wastage?



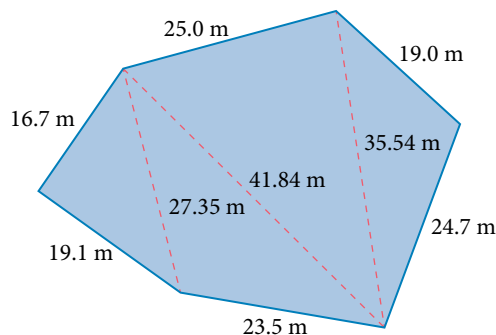
- 16 Moukthik has constructed a three-dimensional cardboard geometric model called an icosahedron. The model consists of 20 joined equilateral triangles, each of side length 15 cm.

Determine the surface area of Moukthik's completed model, to the nearest cm^2 .

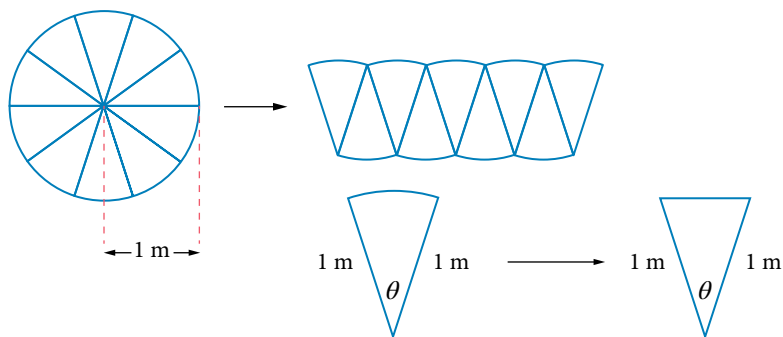


- 17 A property developer has acquired a non-conventional shaped block of land.

The developer is wondering how to find the area of this hexagonal shape. She knows Heron's rule, and so she divides the area into four triangles and measures the distances shown on the dashed line segments. Calculate the area of this block of land to the nearest square metre.



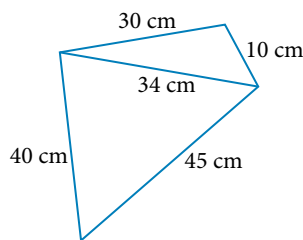
- 18 The formula $\text{Area} = \pi r^2$ can be used to calculate the area of a circle. The area is $\pi \text{ m}^2$ for $r = 1 \text{ m}$. If you didn't know the formula, you could think of the circle divided into, say, 10 sectors.



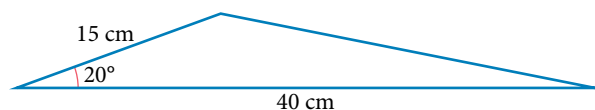
The small sectors can each be approximated by a triangle.

- Determine the approximate area of the circle of radius 1 m by calculating the total area of the 10 isosceles triangles.
- Use a similar procedure to estimate the area of a circle of radius 1 m using 100 isosceles triangles.

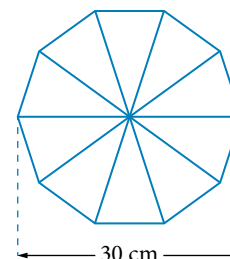
- 19 A quadrilateral frame is to be made with side lengths of 10 cm, 30 cm, 40 cm and 45 cm. The frame is to be braced as shown. What area of the frame is lost if the brace is shortened by 10 cm?



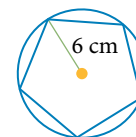
- 20 A triangle has side lengths 15 cm and 40 cm as shown. By what percentage does the area of the triangle increase when the angle between the given sides doubles in size? Write your answer to the nearest whole percentage.



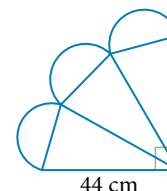
- 21 Determine the area of the regular decagon with diameter length 30 cm.



- 22 A regular pentagon is drawn within a circle of radius 6 cm as shown. Calculate the percentage of the circle occupied by the pentagon. Write your answer to the nearest whole percentage.

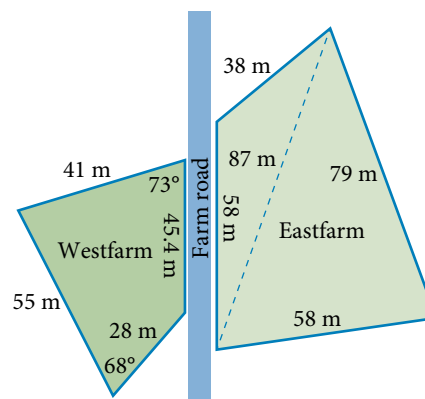


- 23 Three identical isosceles triangles of equal sides 44 cm and attached semi-circles form the shape shown here. Determine the area of the shape.



- 24 A farmer decides to retire from active farming and transfers ownership of her two small farms to her daughter and son, with her daughter receiving Eastfarm and her son receiving Westfarm. Because the farms are of different size (and hence value), she also gives the two children cash totalling \$1 000 000, divided so that the total value of property and cash is the same for the two farms. She values the farms at \$200 per square metre, rounding the area of each farm to the nearest square metre.

- (a) Calculate the total value (of cash and property) to be divided between the son and daughter, and use this to determine the value that each will receive.
- (b) How much cash will the son receive?

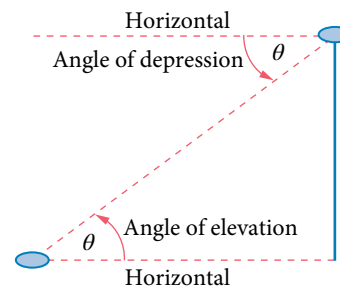
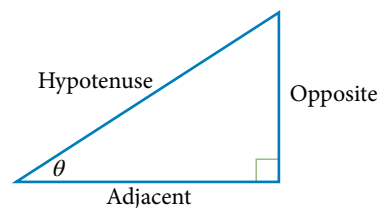


Summary

Sine, cosine and tangent

The rules for the three trigonometric ratios for right-angled triangles:

$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}$	$\sin(\theta) = \frac{O}{H}$
$\cos(\theta) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\cos(\theta) = \frac{A}{H}$
$\tan(\theta) = \frac{\text{Opposite}}{\text{Adjacent}}$	$\tan(\theta) = \frac{O}{A}$



Angles of elevation and depression

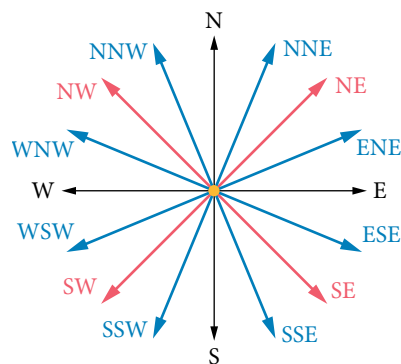
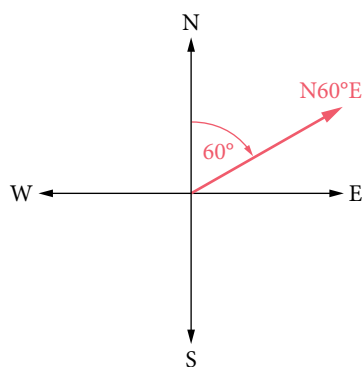
Angle of elevation = angle of depression

Bearings

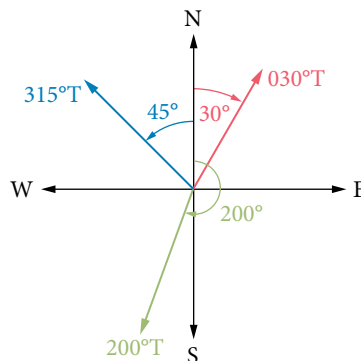
Standard compass directions are measured from N or S.

For example: N60°E, S50°W

NE is equivalent to N45°E.

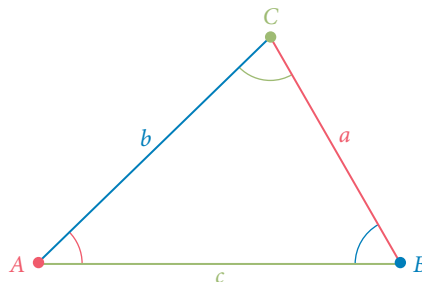


True bearings are measured clockwise from north. Use three digits, even for angles less than 100°. For example: 030°T.



Non-right-angled triangles

The labelling convention for non-right-angled triangles is that angles have pronumerals to match the side opposite.



Non-right-angled triangles and the sine rule

Use the sine rule when you know:

- two angles and one side length
- two sides and a non-included angle (i.e. an angle that is not between the given sides).

$$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \quad \text{or} \quad \frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$$

Non-right-angled triangles and the cosine rule

Use the cosine rule when you know:

- two sides and the angle between them (i.e. an included angle)
- all three sides.

$$a^2 = b^2 + c^2 - 2bc \cos(A) \quad \text{or} \quad \cos(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

Applications of the sine and cosine rules

Draw a neat and labelled diagram.

Recall that the angles in a triangle add to 180° .

Make use of alternate, equal angles.

Mark the four key compass directions on your diagram.

Alternative formulas to calculate the area of a triangle

If given two sides and the angle between them, then you can use:

$$\text{Area} = \frac{1}{2} ab \sin(C)$$

If given all three side lengths, then you can use:

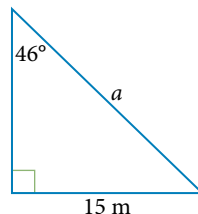
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where} \quad s = \frac{a+b+c}{2}$$

Chapter review

5

Exercise 5.1

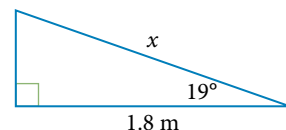
- 1 Calculate the value of a to 2 decimal places.



Exercise 5.1

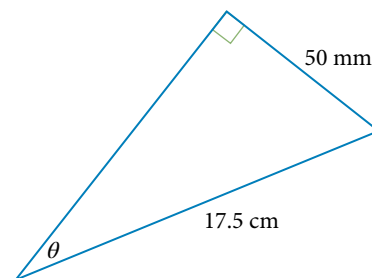
- 2 To determine the length of x , which calculation would you use?

- A $1.8 \times \cos(19^\circ)$ B $1.8 \times \tan(19^\circ)$
 C $\frac{1.8}{\sin(19^\circ)}$ D $\frac{1.8}{\cos(19^\circ)}$



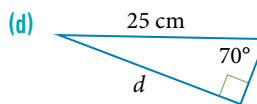
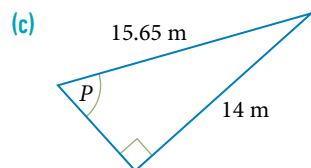
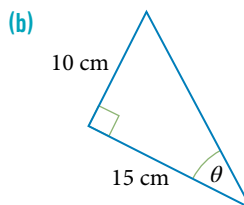
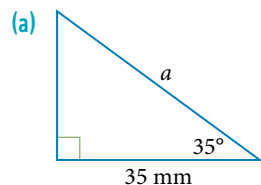
Exercise 5.1

- 3 Calculate the value of θ , rounded to the nearest degree.



Exercise 5.1

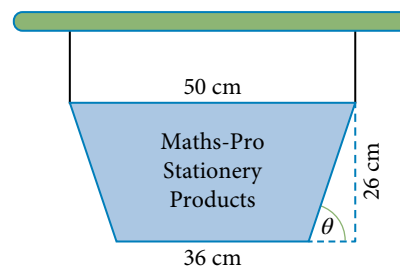
- 4 Determine the value of the pronumeral in each diagram, to 2 decimal places.



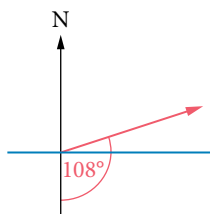
Exercise 5.1

- 5 A trapezium-shaped sign is shown. The left and right sides of the sign are of equal length, and the top and bottom edges are parallel.

Calculate the angle θ to 2 decimal places.



- 6 Give the bearing shown in the diagram.



Exercise 5.3

- 7 Narayan walks 220 m west, then 140 m south. Determine his direction from his starting position, written as a compass bearing.

Exercise 5.3

- 8 Write NW as a true bearing.

Exercise 5.3

- 9 Convert the following directions and bearings, as specified.

Exercise 5.3

- (a) S25°E to a true bearing (b) 280°T to a conventional compass direction
(c) SW to a true bearing

- 10 A remote-controlled model helicopter flies 80 m on a bearing N22°W. Calculate the following distances to 2 decimal places.

Exercise 5.3

- (a) How far north does it fly? (b) How far west does it fly?

- 11 Which of the following can be used to solve triangle GHI ?

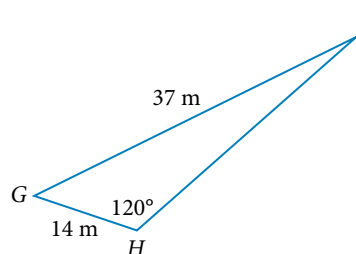
Exercise 5.4

A $\frac{14}{\sin(120^\circ)} = \frac{37}{\sin(I)}$

B $\frac{37}{\sin(120^\circ)} = \frac{14}{\sin(G)}$

C $\frac{37}{\sin(120^\circ)} = \frac{14}{\sin(I)}$

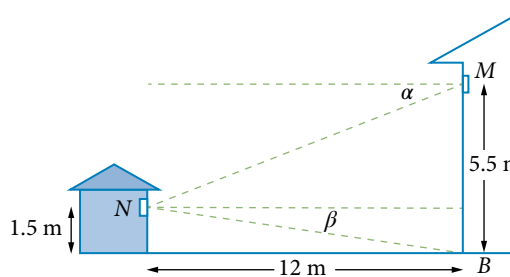
D $\frac{14}{\sin(120^\circ)} = \frac{37}{\sin(G)}$



- 12 Two buoys, B_1 and B_2 , are anchored to the ocean floor, with B_2 located 44 m due north of B_1 . An angler at A anchors his boat at a bearing of N52°W from B_1 . The bearing of A from B_2 is S68°W. How far is the angler from B_1 ? Give your answer to 2 decimal places.

Exercise 5.4

- 13 Neville is annoyed that his neighbour has built a triple-storey building next door. He is even more annoyed that his neighbour is able to look down into his kitchen window. In the diagram below, N is the centre of Neville's kitchen window, and M is the centre of his neighbour's window. B is a point at the base of Neville's neighbour's house. Give your answers to the following questions to 2 decimal places.

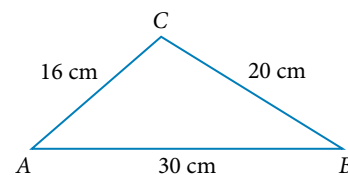
Exercise 5.1,
5.2, 5.5

- (a) Calculate the angle of depression α from the middle of the neighbour's window M to the middle of Neville's kitchen window N .
(b) Calculate the angle of depression, β , from the middle of Neville's window to the base of his neighbour's house.
(c) Determine the size of the angle MNB .
(d) Determine the distance from Neville's window to his neighbour's window NM .
(e) Calculate the direct distance from Neville's window to the base of his neighbour's house NB .

Exercise 5.4, 5.5

- 14 Solve the following triangle by finding the unknown angles. Give your answers to 2 decimal places.

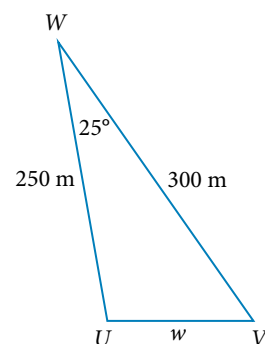
- (a) angle A
 (b) angle B
 (c) angle C



Exercise 5.4, 5.5

- 15 Solve the following triangle by finding all the unknown angles and sides. Give your answers to 2 decimal places.

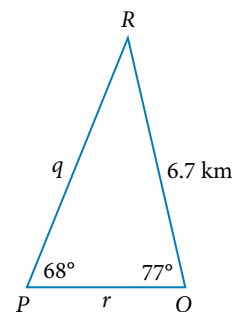
- (a) side w
 (b) angle U
 (c) angle V



Exercise 5.4, 5.5

- 16 Solve the following triangle by finding all the unknown angles and sides. Give your answers to 2 decimal places.

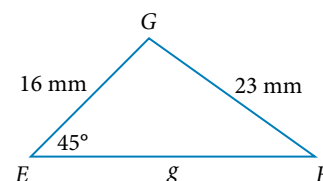
- (a) angle R
 (b) side r
 (c) side q



Exercise 5.4, 5.5

- 17 Solve the following triangle by determining all the unknown angles and sides. Give your answers to 2 decimal places.

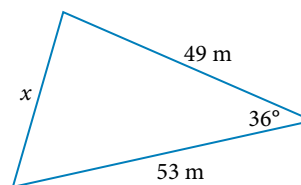
- (a) angle F
 (b) angle G
 (c) side g



Exercise 5.5

- 18 Which of the equations below can be used to solve for x ?

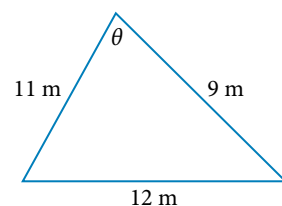
- A $x^2 = 49^2 + 53^2$
 B $x = 49 + 53 - 2 \times 49 \times 53 \cos(36^\circ)$
 C $x^2 = 49^2 + 53^2 - 2 \times 49 \times 53 \sin(36^\circ)$
 D $x^2 = 49^2 + 53^2 - 2 \times 49 \times 53 \cos(36^\circ)$



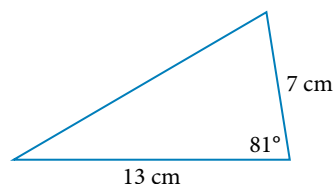
Exercise 5.5

- 19 Which expression can be used to calculate the angle θ ?

- A $\sin(\theta) = \frac{11^2 + 12^2 - 9^2}{2 \times 11 \times 12}$
 B $\cos(\theta) = \frac{11^2 + 12^2 - 9^2}{2 \times 9 \times 12}$
 C $\cos(\theta) = \frac{9^2 + 12^2 - 11^2}{2 \times 11 \times 9}$
 D $\cos(\theta) = \frac{9^2 + 11^2 - 12^2}{2 \times 9 \times 11}$

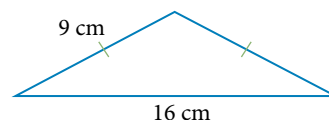


- 20 Calculate the area of the triangle to 2 decimal places.



Exercise 5.6

- 21 Calculate the value of the semi-perimeter of the triangle.



Exercise 5.6

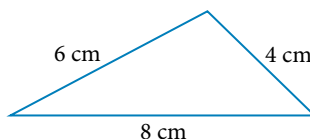
- 22 Which expression can be used to calculate the area of the triangle?

A $\sqrt{4 \times 8 \times 6 \times 9}$

B $\sqrt{9^2 + 4^2 + 8^2 + 6^2}$

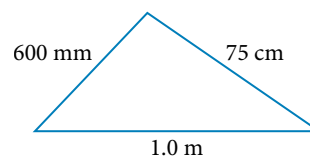
C $\sqrt{9(9-4)(9-8)(9-6)}$

D $\sqrt{18(18-4)(16-8)(18-6)}$



Exercise 5.6

- 23 Calculate the area of the triangle, in square centimetres. Give your answer to 2 decimal places.



Exercise 5.6

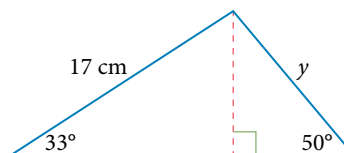
- 24 Which calculation gives the value of y ?

A $\frac{17 \sin(33^\circ)}{\sin(50^\circ)}$

B $\frac{17 \cos(33^\circ)}{\sin(50^\circ)}$

C $\frac{17 \cos(33^\circ)}{\cos(50^\circ)}$

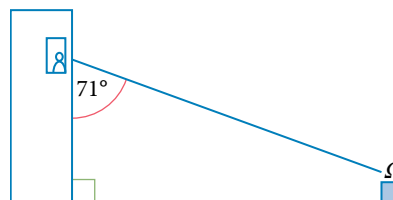
D $\frac{17 \tan(33^\circ)}{\tan(50^\circ)}$



Exercise 5.1

- 25 Eyela looks down from the centre of her office window at a monument to Mathematics in a nearby park.

- (a) What is the angle of depression?
- (b) If Eyela's eye level is 36 m above the top of the monument, how far is the monument from the building, to the nearest metre?
- (c) To comply with new park regulations, the monument is moved to a position 117.75 m from Eyela's building. She can still see the top of the monument. What is the change in the angle of depression to the top of the monument?



Exercise 5.2

- 26 Gordon hits a golf ball 180 m due east. Unfortunately, this is off course and the ball goes into the rough. He then chips the ball 32 m due north to get back onto the fairway. Finally, he gets the ball into the hole with an 8 m putt due east.

Exercise 5.3

Calculate the true bearing of the hole from Gordon's tee-off (start) position, to the nearest degree.

Exercise 5.5

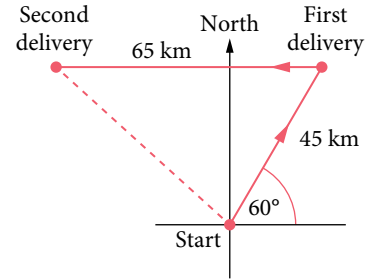
- 27 A delivery van drives 45 km from a warehouse on a bearing $N30^\circ E$ for one delivery, then travels 65 km due west to make another delivery.

(a) How far does the van travel on its journey back to the warehouse after the second delivery?

- A 46.9 km B 57.7 km
C 79.1 km D 167.7 km

(b) As the van is making the second delivery, what is the true bearing of its position from the starting point?

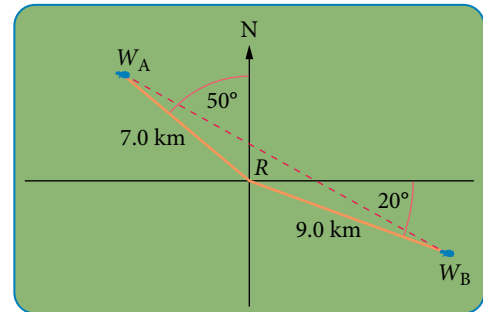
- A $043^\circ T$ B $048^\circ T$ C $143^\circ T$ D $313^\circ T$



Exercise 5.5

- 28 Two wombats, W_A and W_B , are tagged with GPS locators and released into a nature reserve by a park ranger. A week later the ranger (R) pinpoints the positions of the wombats relative to herself and displays this information on her tablet using special software. The tablet's display is shown.

It is hoped that the wombats will mate, but the ranger is concerned that they are drifting apart. How far apart are the wombats according to the display? Give your answer to 2 decimal places.



Exercise 5.6

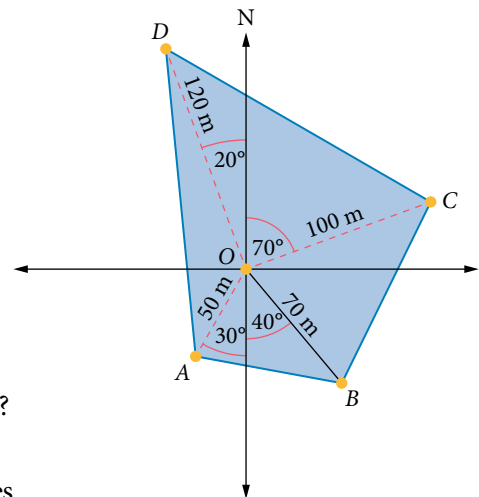
- 29 A pastry chef has a triangular pastry cutter with two side lengths of 16 cm and 13 cm. The angle between the two sides is 80° .

- (a) What is the area of the piece of pastry in cm^2 ? Give your answer to 2 decimal places.
(b) The chef discovers he has only 100 cm^2 of pastry, and decides to adjust the 80° angle. What should the angle be changed to? Give your answer to the nearest degree.

Exercise 5.5, 5.6

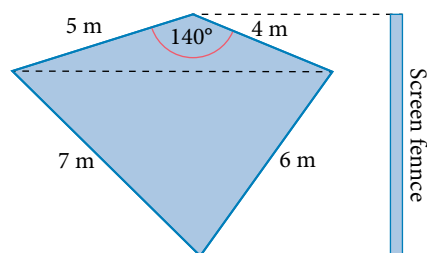
- 30 Organisers are setting up an orienteering course in the school grounds. They place checkpoints at A, B, C and D to define the area of the course. The bearing and distance of each checkpoint from a central command point O are shown.

- (a) Determine the perimeter of the course to the nearest metre.
(b) If checkpoint A moved along the same bearing so that the perimeter of the course was 500 m, would the distance AO be less than 60 m or more than 60 m? Assume all other checkpoints remain fixed.
(c) Calculate the original area within the course boundaries (that is, the area of shape ABCD) to the nearest square metre.
(d) Checkpoint A is to be moved along the same bearing so that the area of the course will become 15000 m^2 . Assuming all other checkpoints remain fixed, what is the new length OA (to 1 decimal place)?



- 31 A landscape gardener is given plans for a section of garden, as shown.

- (a) Calculate the area of the section of garden. Express your answer to 1 decimal place.
 (b) Calculate the length of screen fencing required. Round up to the next whole metre.

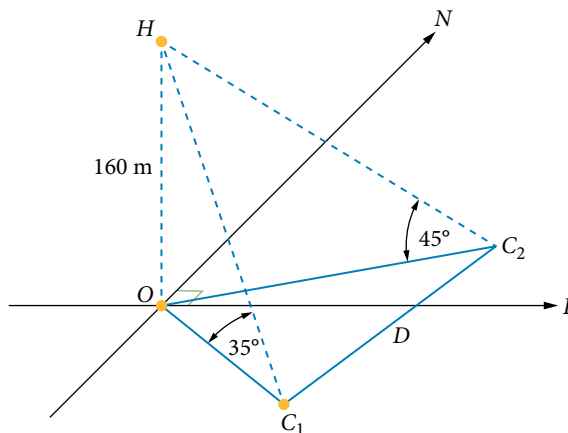


Exercise 5.6

- 32 A police helicopter (H) is hovering 160 m above point O on the ground, and sights two escaped prisoners C_1 and C_2 at angles of depression 35° and 45° respectively.

The three-dimensional situation is shown here, including the angles of elevation.

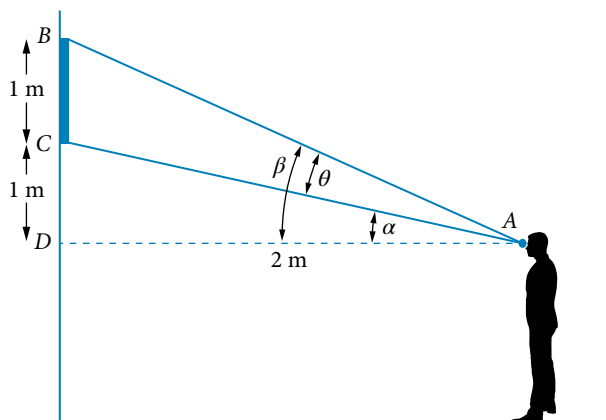
If the ground bearings from O to the escaped prisoners are 070°T , and 130°T respectively, determine the distance D between the prisoners (to the nearest metre).



Exercise 5.3, 5.5

- 33 Aziz stands 2 m from the wall to view a painting.

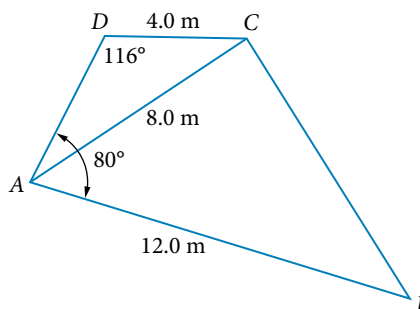
- (a) Calculate the viewing angle θ , to 1 decimal place, that Aziz has by standing this distance from the painting.
 (b) If Aziz moves 60 cm closer to the painting, by how many degrees does his viewing angle increase, to 1 decimal place?



Exercise 5.2

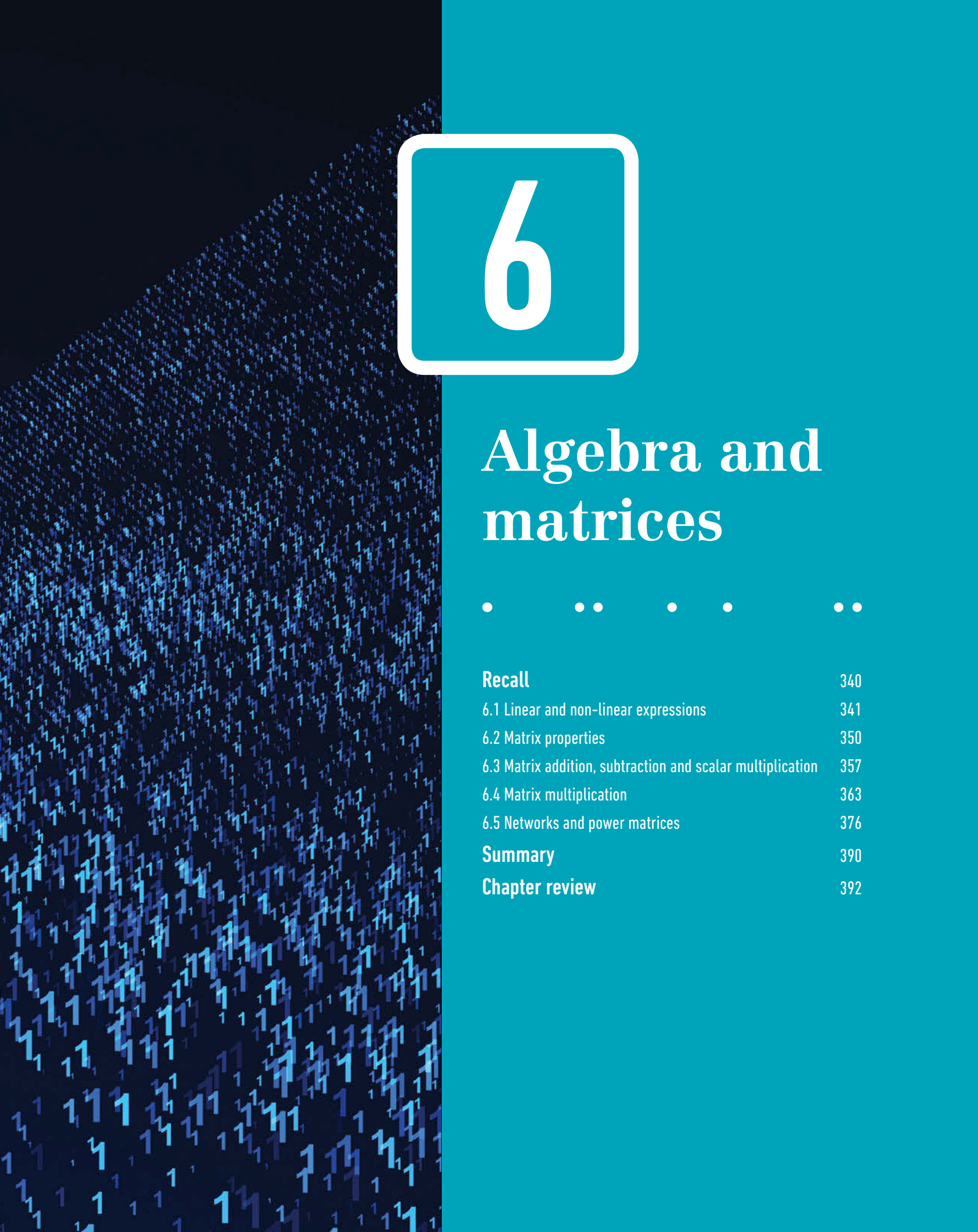
- 34 A shade-sail is being installed in a school's grounds. It consists of two triangular sections stitched together.

The principal decides that a larger shade-sail is required. By what factor should the sails be scaled up so they total 100 m^2 in area, to 2 decimal places?



Exercise 5.4, 5.6





6

Algebra and matrices

• • • • •

Recall	340
6.1 Linear and non-linear expressions	341
6.2 Matrix properties	350
6.3 Matrix addition, subtraction and scalar multiplication	357
6.4 Matrix multiplication	363
6.5 Networks and power matrices	376
Summary	390
Chapter review	392

Recall

Solve linear equations

1 Solve the following linear equations for x .

(a) $2x - 1 = 7$

(b) $-2(x + 3) = 10$

(c) $\frac{6x - 1}{5} = 7$

Apply the order of operations rules to calculations with integers

2 Use the order of operations rules to calculate the following.

(a) $30 \div (9 - 4) \times 3$

(b) $7 \times 3 - 4^2 + 1$

(c) $\frac{-3(-15 + 7)}{2} + 9$

Understand the meaning of key words used in algebra

3 Place the correct word or pair of words from the list below into each of the following sentences.

like terms

unlike terms

terms

constant

coefficient

pronumeral

expressions

equations

The unknown value in an expression or equation is represented by a _____, such as x .

Individual parts of an expression, such as $2x$, $-7y$, 5 , ab and -3 , are called _____.

The number in front of a pronumeral is multiplied by the pronumeral. It is called the _____.

Terms are added or subtracted to form _____. Terms that have exactly the same pronumeral part are called _____.

Expand brackets

4 Expand the brackets and simplify the expressions.

(a) $4(x - 5)$

(b) $2a(b + 7)$

(c) $-3c(4t - c)$

(d) $7a + 3(a + b) - 5b$

(e) $x^2 + x(y - x) + y(x - 1)$

Evaluate expressions that contain indices

5 Determine the value of the following expressions.

(a) a^5 when $a = 3$

(b) a^2b^3 when $a = -2$ and $b = 4$

Linear and non-linear expressions

Linear formulas

Formulas are used to solve practical problems. A formula can be useful to quickly convert temperatures measured in degrees Fahrenheit to degrees Celsius and vice versa. In order to perform such a conversion, two equations involving the appropriate pronumerals can be established: $C = \frac{5}{9}(F - 32)$ and $F = \frac{9}{5}C + 32$ where C represents the temperature in Celsius and F the temperature in Fahrenheit. These equations are called *linear formulas* because the pronumerals F and C are raised to the power of one.

A *formula* is a mathematical rule describing the relationship between variable quantities.

Substituting numerical values into formulas

Substitution into a formula involves replacing pronumerals with known numerical values and simplifying the result.

1 Substituting numerical values into a linear formula

The formulas for converting temperature between Fahrenheit and Celsius are given by $C = \frac{5}{9}(F - 32)$ and $F = \frac{9}{5}C + 32$. Convert the following temperatures.

(a) Convert 50°F to degrees Celsius.

THINKING

1 Recall the formula for converting temperature from Fahrenheit to Celsius.

2 Replace the pronumerals F with the value given and evaluate the expression.

3 Interpret the answer.

WORKING

$$C = \frac{5}{9}(F - 32)$$

$$\begin{aligned} C &= \frac{5}{9}(50 - 32) \\ &= \frac{5 \times 18}{9} \\ &= 10 \end{aligned}$$

50°F is equivalent to 10°C .

(b) Convert -10°C to degrees Fahrenheit.

1 Recall the formula for converting temperature from Celsius to Fahrenheit.

2 Substitute $C = -10$ into the formula and calculate the value of the expression.

3 Interpret the answer.

$$F = \frac{9}{5}C + 32$$

$$\begin{aligned} F &= \frac{9}{5}(-10) + 32 \\ &= -18 + 32 \\ &= 14 \end{aligned}$$

-10°C is equivalent to 14°F .

2 Substituting numerical values into a non-linear formula

Use the formulas to find each of the following, to 1 decimal place.

- (a) The area A of a circle of diameter d can be written as $A = \frac{\pi d^2}{4}$. Calculate the area in square metres (m^2) of a circle of diameter 6 m.

THINKING

- 1 Recall the formula for the area of a circle.
- 2 Substitute the value for d into the formula and evaluate the expression.
- 3 Interpret the answer.

WORKING

$$A = \frac{\pi d^2}{4}$$

$$A = \frac{\pi \times 6^2}{4}$$

$$= 28.3$$

The area of the circle is 28.3 m^2 .

- (b) The volume V of a cone of base radius r and height h is $V = \frac{1}{3}\pi r^2 h$. Calculate the volume of a cone with base radius 3 m and height 1.5 m.

- 1 Recall the formula for the volume of a cone.
- 2 Substitute the values for r and h into the formula and evaluate the expression.
- 3 Interpret the answer.

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi (3)^2 \times 1.5$$

$$= 14.1$$

The volume of the cone is 14.1 m^3 .

Transposing a formula

A formula, usually in the form of an equation, has one variable isolated on one side of the equation. An isolated variable is said to be the *subject* of the equation (formula).

For example, if $A = \frac{h}{2}(a + b)$ then A is the subject of the equation (formula).

A formula can also be used to determine the unknown value of a variable other than the subject. In this situation, the formula can be rearranged or *transposed* so that the variable required becomes the subject of the formula. The process of transposing a formula is similar to solving a linear equation. Inverse operations can be applied in reverse order on both sides of the equation.

Non-linear formulas can also be rearranged, or transposed, so that the variable required becomes the subject of the formula.

To rearrange, or transpose, the terms in a formula, the same operations are performed on both sides of the formula. Whatever is done to one side of the equation must also be done to the other.

i Additional information

Transposing formulas

Watch the video and try the activity for more practice with changing the subject in a formula.

i Additional information

Solving equations

Watch the video and try the activity for more practice with changing the subject in a more complex formula.

3 Calculating the value of the subject of a formula

Transpose the formula to make the unknown value the subject and solve.

- (a) The perimeter of a particular pentagon is given by $P = 2(a + b) + c$. Determine the value of side length b when $P = 40$ m, $a = 8$ m and $c = 11$ m.

THINKING

- 1 Identify the formula required.
- 2 Solve the equation for the unknown variable.
In this case, make b the subject.
- 3 Simplify the formula, if possible.
- 4 Substitute the pronumerals P , a and c with the values given and calculate the value of b , rounding if necessary and attaching appropriate units.
- 5 Check that your answer is correct by substituting back into the original equation to ensure that the left-hand side of the equation is equal to the right-hand side of the equation.
- 6 Interpret the answer.

WORKING

$$P = 2(a + b) + c$$

$$2(a + b) = P - c$$

$$a + b = \frac{P - c}{2}$$

$$b = \frac{P - c}{2} - a$$

$$b = \frac{P - c - 2a}{2}$$

$$\begin{aligned} b &= \frac{P - c - 2a}{2} \\ &= \frac{40 - 11 - 2 \times 8}{2} \\ &= \frac{13}{2} = 6.5 \text{ cm} \end{aligned}$$

$$P = 2(a + b) + c$$

Left-hand side (LHS):

$$P = 40$$

Right-hand side (RHS):

$$\begin{aligned} 2(a + b) + c &= 2 \times (8 + 6.5) + 11 \\ &= 2 \times (14.5) + 11 \\ &= 40 \end{aligned}$$

Given that LHS = RHS, the answer is correct.

The unknown side length b is 6.5 cm.

METHOD 2

- 1 Identify the formula required.
- 2 Substitute the known values into the formula.
- 3 Solve the equation for the unknown. In this case, make b the subject.
- 4 Interpret the answer.

$$P = 2(a + b) + c$$

$$40 = 2(8 + b) + 11$$

$$40 = 2(8 + b) + 11$$

$$29 = 2(8 + b)$$

$$\frac{29}{2} = 8 + b$$

$$b + 8 = 14.5$$

$$b = 6.5$$

The unknown side length b is 6.5 cm.

- (b) The area A of a circular path of inner radius r and outer radius R is given by $A = \pi R^2 - \pi r^2$. Determine, to 2 decimal places, the circular path's outer radius when the inner radius is 30 m and the area is 1800 m^2 .

1 Identify the formula required.

$$A = \pi R^2 - \pi r^2$$

2 Solve the equation for the unknown variable.

$$\pi R^2 - \pi r^2 = A$$

In this case, make R the subject.

$$\pi R^2 = A + \pi r^2$$

$$R^2 = \frac{A + \pi r^2}{\pi}$$

$$R = \sqrt{\frac{A + \pi r^2}{\pi}}$$

R represents a length $R > 0$, and therefore take the positive square root of both sides.

3 Substitute the pronumerals r and A with the values given and calculate the value of R , rounding if necessary with appropriate units.

$$\begin{aligned} R &= \sqrt{\frac{1800 + \pi \times 30^2}{\pi}} \\ &= 38.38 \text{ m (2 d.p.)} \end{aligned}$$

4 Interpret the answer.

The circular path's outer radius is 38.38 m.

Using a spreadsheet to construct a table of values

A table of values consists of a set of input values (independent variables) and a set of resulting output values (dependent variables).

4 Constructing a table from a formula

Construct a table to display temperatures measured in degrees Fahrenheit and their equivalent temperatures in degrees Celsius.

- (a) Use $C = \frac{5}{9}(F - 32)$ for values of F from -20 to 40 in increments of 10. Display values of C to 1 decimal place.

THINKING

1 Create a heading row with the independent variable first and a formula for the dependent variable, as a visual aid.

2 Enter the first two values of the independent value, then click and drag to fill the column to the required maximum value.

3 Enter a formula for the first value of the dependent variable.

WORKING

	A	B
1	F	$C=5/9*(F-32)$

	A	B
1	F	$C=5/9*(F-32)$
2	-20	
3	-10	
	$\vdots$	
8	40	

$$= 5/9*(A2-32)$$

Note: click on the cell for the first value in case you have not started at top left.

- 4 Click and drag to fill the column to match the first column. Adjust the decimal places as directed.

	A	B
1	F	$C = 5/9(F - 32)$
2	-20	-28.9
3	-10	-23.3
4	0	-17.8
5	10	-12.2
6	20	-6.7
7	30	-1.1
8	40	4.4

- (b) Use your table to determine the value of the freezing point for water in °F.

- 1 Locate the required values on the table.

30	-1.1
40	4.4

- 2 Verify the outcome using algebra.

$$\begin{aligned}
 C &= \frac{5}{9}(F - 32) \\
 &= \frac{5}{9}(32 - 32) \\
 &= 0
 \end{aligned}$$

- 3 Interpret the answer.

Freezing point (0°C) occurs between 30 and 40°F, at 32°F.

A spreadsheet or an equivalent technology can also be used to construct two-by-two tables for formulas with two variable quantities.

5 Construct a two-by-two table of values for a formula using technology

The volume of a cone $V \text{ m}^3$ of base radius $r \text{ m}$ and height $h \text{ m}$ is given by the formula $V = \frac{1}{3}\pi r^2 h$.

- (a) Use a spreadsheet to construct a table of values that displays the volumes of cones, to 2 decimal places, of radii 1 m, 2 m, 4 m and 8 m, and heights 1 m, 2 m, 4 m and 8 m.

THINKING

- 1 Create a heading row and column with the independent variables and a formula for the dependent variable, as a visual aid.
Enter the independent values.

WORKING

	A	B	C	D	E	F
1	$V = 1/3 \pi r^2 h$		Radius			
2			1	2	4	8
3	Height	1				
4		2				
5		4				
6		8				

- 2 Input a formula for the first value of the dependent variable.

Note: Click on the cell for the first value of radius and height in case you have not started at top left.

- 3 Alter the formula so that the row reference remains constant for the radii and the column reference remains constant for the heights.

- 4 Click and drag to fill the column to match the height values.

Click and drag the column of volume values to fill the rows to match the radius values. Adjust the decimal places as directed.

In the cell where radius is 1 and the row where height is 1 (see C3):

$$=1/3*PI()*C2^2*B3$$

Note: π is spelled out and followed by a set of brackets.

$$=1/3*PI()*C\$2^2*\$B3$$

The \$ symbol used with the cell reference denotes the constant row/column.

Volumes in m^3 :

	A	B	C	D	E	F
1	$V=1/3*\pi*r^2*h$		Radius			
2			1	2	4	8
3	Height	1	1.05	4.19	16.76	67.02
4		2	2.09	8.38	33.51	134.04
5		4	4.19	16.76	67.02	268.08
6		8	8.38	33.51	134.04	536.17

- (b) If the base radius and height of a cone are doubled, determine the effect this has on the volume of a cone.

- 1 Locate cells in the table where the height and radius are double that of an existing cell.

Note: the face value of the cells has been rounded but the spreadsheet retains the unrounded values.

- 2 Go to a vacant cell to the right of the table and key in a formula to divide the larger volume by the smaller volume. Let V_1 be initial volume and V_2 be the volume of a cone with double the height and double the radius.

Click and drag the cell with the formula twice to make a 3×3 block of cells.

- 3 Check the answer.

- 4 Interpret the result.

Height and radius have both doubled in the cell to the right and down from an original cell.

1.05	4.19
2.09	8.38

Formula for first pair of V_2/V_1 : $=D4/C3$

Final formula of V_2/V_1 : $=F6/E5$

D4/C3	E4/D3	F4/E3
D5/C4	E5/D4	F5/E4
D6/C5	E6/D5	F6/E5

Table of values for V_2/V_1 :

8.0	8.0	8.0
8.0	8.0	8.0
8.0	8.0	8.0

If the base radius and height of a cone are doubled, the volume of the cone increases by a factor of 8.

EXERCISE

6.1

Linear and non-linear expressions

Worked
Example

- 1 In each of the following, substitute the given values and calculate the formula unknown, to 1 decimal place if necessary.

(a) $C = 2\pi r$, $r = 3$ (b) $C = \frac{5}{9}(F - 32)$, $F = 68$ (c) $x = \frac{y - c}{m}$, $y = 30$, $c = 10$, $m = 5$

- 2 The formula used to calculate simple interest is given by $I = \frac{Pni}{100}$ where P is the principal, $i\%$ is the annual interest rate and n is the number of years.

Determine the interest earned on a loan of \$10 000 at 2.5% p.a. simple interest for 2 years.

- 3 In each of the following, substitute the given values and calculate the value of the subject of the formula. Express each answer to 1 decimal place.

(a) $A = \pi r^2$, $r = 1.7$ (b) $c = \sqrt{a^2 + b^2}$, $a = 13$, $b = 3$ (c) $E = \frac{1}{2}mv^2$, $m = 3$, $v = 49$

- 4 Substitute the stated values into each non-linear formula and calculate the value of the subject. Where appropriate, give each answer to 1 decimal place.

(a) $V = \frac{1}{3}\pi r^2 h$, $r = 2.7$, $h = 2.8$

(b) $S = \frac{n}{2}(2a + (n - 1)d)$, $a = 12$, $n = 19$, $d = -4$

- 5 For each of the following, transpose the equation to make the variable in brackets the subject.

(a) $P = 2(l + w)$, (l)

(b) $v = u + at$, (t)

(c) $c^2 = a^2 + b^2$, (b) where $b > 0$

(d) $F = \frac{mv^2}{r}$, (r)

(e) $6 - 4y = 3x$, (y)

- 6 The number of diagonals D of an n -sided polygon is given by $D = \frac{n}{2}(n - 3)$. Use a spreadsheet to complete the following.

(a) Construct a table of values that displays the number of diagonals for polygons of 4, 5, 6, 7 and 8 sides.

(b) If a polygon has 170 diagonals, determine the number of sides it has.

- 7 A plumber charges \$45 per hour plus a \$50 transport fee.

Create a formula for calculating the cost \$ C charged by this plumber for completing h hours of work.

- 8 A spreadsheet is set up to calculate the perimeter of a rectangle given the length and width. The formula to be used is $P = 2(L + W)$.

	A	B	C	D
1	Length			
2	Width	30	40	50
3	20			
4	30			
5	40			

- (a) Identify an appropriate formula to go into cell B3 to enable click-and-drag filling of the rows and columns.
- (b) When you check cell D4, what formula will you expect to find?

- 9 Create a two-by-two spreadsheet to produce a multiplication table in the style shown here.

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

- (a) Complete the spreadsheet for whole numbers from 21 to 30.
- (b) Complete the spreadsheet for 0.5 to 5 in increments of 0.5.

..

..

- 10 The area $A \text{ cm}^2$ of a trapezium of height $h \text{ cm}$ and parallel sides $a \text{ cm}$ and $b \text{ cm}$ can be expressed as

$$A = \frac{h}{2}(a + b).$$

Worked Example

3

Transpose this formula to calculate the height of the trapezium, to 1 decimal place, when the area is 60 cm^2 and the parallel sides have lengths of 3.2 cm and 5.1 cm.

- 11 The formula used to calculate simple interest is given by $I = \frac{Pni}{100}$ where P is the principal, $i\%$ is the annual interest rate and n is the number of years.

- (a) Transpose the formula to calculate the interest rate if a principal of \$5000 yields \$600 interest in 3 years.

- (b) Choose and explain the option that shows a common keying-in error for the calculation $\frac{200 \times 400}{5 \times 8000}$.

A $(200 \times 400) \div (5 \times 8000)$

B $200 \times 400 \div 5 \div 8000$

C $200 \times 400 \div (5 \times 8000)$

D $200 \times 400 \div 5 \times 8000$

- 12 If $d = 8\sqrt{\frac{h}{5}}$, select the option that shows the equation transposed to make h the subject.

A $h = \frac{5d^2}{64}$

B $h = \frac{5d^2}{8}$

C $h = \frac{64}{5d^2}$

D $h = 320d^2$

5

- 13 The surface area of a cylinder $S \text{ cm}^2$ is given by $S = 2\pi r^2 + 2\pi rh$ where r is the base radius and h is the height. The base radius and height are measured in centimetres.
- Use a spreadsheet or similar technology to construct a table of values that gives the surface area of cylinders of base radii 1 m, 2 m, 4 m and 8 m and heights 1 m, 2 m, 4 m and 8 m with 2 decimal places.
 - If the base radius and height of a cylinder are doubled, determine the effect this has on the surface area of a cylinder.
 - Transpose $S = 2\pi r^2 + 2\pi rh$ to find, to 1 decimal place, the height of a cylinder with surface area of 200 cm^2 and base radius of 5 cm.

- 14 A rectangular box with a lid has length l m, width w m and height h m.

The surface area $S \text{ m}^2$ of this rectangular box is given by $S = 2(lw + hl + hw)$.

- A rectangular box is required to have a surface area between 120 m^2 and 130 m^2 and for the length, width and height measurements to be integers.

Determine the surface area and the integer length, width and height measurements so that $l < w < h$.

- A second rectangular box with a lid has length $\frac{l}{2}$ m, width $\frac{w}{2}$ m and height $\frac{h}{2}$ m.

Determine the formula for the surface area, $T \text{ m}^2$, of this rectangular box in terms of l , w and h .

- Calculate the ratio of the surface area of the original box to that of the box with each dimension half of the original lengths.

- 15 Create a spreadsheet that converts temperatures in $^{\circ}\text{C}$ to temperatures in $^{\circ}\text{F}$ using the formula

$F = \frac{9}{5}C + 32$. Extend or alter the values in your table until it displays the numerical value that is the same for both temperature scales. Check your answer algebraically.

- 16 The formula for body mass index (BMI) requires the input of two numbers, mass (kg) and height (m).

- Set up a two-by-two spreadsheet for mass in the range 40 kg to 150 kg with increments of 5 kg, and heights from 1 m to 2 m with increments of 5 cm.

Enter a suitable formula given $\text{BMI} = \frac{\text{mass}}{\text{height}^2}$.

- Suggested BMI values with descriptions are given in the table.

Use the table to determine the following:

- The healthy weight range for a person whose height is 170 cm
- The maximum weight to avoid obesity for a person whose height is 150 cm
- The minimum weight to avoid being underweight for a person whose height is 165 cm.

Description	BMI
Severely underweight	< 16.5
Underweight	< 18.5
Overweight	> 25
Obese	> 30

6.2

Matrix properties

Concept of a matrix

Matrices can be used to represent and process tables of numerical data. In addition, matrices can be used to:

- solve equations
- find solutions to problems in social or road networks
- encode and decode information.

A *matrix* is a rectangular array of numbers enclosed by square brackets and denoted by a capital letter. A matrix is displayed in the form of rows (horizontal) and columns (vertical).

$$\mathbf{A} = \begin{matrix} & \begin{matrix} \text{C1} & \text{C2} & \text{C3} \end{matrix} \\ \begin{matrix} \text{Row 1} \\ \text{Row 2} \end{matrix} & \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \end{matrix}$$

i Additional information

What is a matrix?

Watch the video as an introduction to how information can be represented in matrices.

Representing tables of information in matrix form

A furniture company sells sofas, lounge chairs and coffee tables at two different stores. In May, Store 1 sold 5 sofas, 8 lounge chairs and 10 coffee tables. In the same month, Store 2 sold 7 sofas, 12 lounge chairs and 20 coffee tables.

This sales information can be summarised in a table.

	Sofas (<i>S</i>)	Lounge chairs (<i>L</i>)	Coffee tables (<i>C</i>)
Store 1	5	8	10
Store 2	7	12	20

The table above can be expressed as the matrix **A**.

$$\mathbf{A} = \begin{bmatrix} \begin{matrix} S & L & C \end{matrix} \\ \begin{matrix} 5 & 8 & 10 \\ 7 & 12 & 20 \end{matrix} \end{bmatrix} \begin{matrix} \text{Store 1} \\ \text{Store 2} \end{matrix}$$

Without the labels, matrix **A** can be represented as:

$$\mathbf{A} = \begin{bmatrix} 5 & 8 & 10 \\ 7 & 12 & 20 \end{bmatrix}$$

Order of a matrix

The *order* (or *size* or *dimensions*) of a matrix is determined by the number of rows and columns in the matrix, in that order.

For example, the matrix $\mathbf{A} = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix}$ has 2 rows and 3 columns.

Such a matrix is described as a ‘two-by-three’ matrix or a 2×3 matrix.

A matrix that has m rows and n columns is said to be an $m \times n$ matrix.

Two matrices are said to have the same order (or size) if they have the same number of rows and columns.

For example, $\mathbf{A} = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & 3 & 5 \\ 7 & 9 & 11 \end{bmatrix}$

These two matrices have the same order. They are both 2×3 matrices.

Elements of a matrix

The numbers within a matrix are known as its *elements*.

For example, $\mathbf{A} = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix}$

$\mathbf{A}$ is a matrix with six elements.

Each matrix element can be described according to its position in the matrix, in terms of the row and column it is situated in.

The symbol a_{ij} represents the element occurring in the row i and column j of matrix $\mathbf{A}$.

For example, a general 2×3 matrix is $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}$.

So for the matrix $\mathbf{A} = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix}$, a_{23} is the element in the *second* row and *third* column (read as ‘two, three’), which is 12.

So $a_{23} = 12$.

Additional information

The elements of a matrix

Watch the video as an introduction to the elements of a matrix.

6 Representing data in matrix form

Hannah keeps a record of her LOTE test results. Her first four test results in French are 85, 76, 88 and 71. Her first four test results in German are 73, 68, 76 and 73.

Express Hannah's test results in the form of a 2×4 matrix $\mathbf{A}$.

THINKING

- 1 Create and label a matrix with appropriate dimensions, where the first number of the order corresponds to the number of rows and the second number corresponds to the number of columns.
- 2 Enter the data into the appropriate positions according to the labels.

WORKING

A 2×4 matrix means 2 rows and 4 columns.

$$\mathbf{A} = \begin{matrix} & T1 & T2 & T3 & T4 \\ \begin{matrix} F \\ G \end{matrix} & \begin{bmatrix} & & & \end{bmatrix} \end{matrix}$$

$$\mathbf{A} = \begin{bmatrix} 85 & 76 & 88 & 71 \\ 73 & 68 & 76 & 73 \end{bmatrix}$$

7 Order and elements of a matrix

Answer the following questions for the matrix $\mathbf{A} = \begin{bmatrix} 16 & 6 & 12 \\ 6 & 7 & 5 \end{bmatrix}$.

- (a) Identify the order of $\mathbf{A}$.

THINKING

Count the number of rows and columns and express the order in the form rows $\times$ columns.

WORKING

$\mathbf{A}$ has 2 rows and 3 columns, so $\mathbf{A}$ is a 2×3 matrix.

- (b) Determine the value of a_{13} .

For element a_{ij} , the value of i corresponds to the row position and the value of j corresponds to the column position.

a_{13} is the element in the first row and the third column.

$$a_{13} = 12$$

- (c) Which element of matrix $\mathbf{A}$ is the number 7?

Identify the row that the element is in and then identify the column that the element is in.

The number 7 is in the second row and the second column.

$$7 \text{ is } a_{22}.$$

Some special types of matrices

Some special types of matrices are defined below.

Special matrix	Definition	Example
Row matrix	A matrix that has only one row is known as a <i>row matrix</i> . In general, a row matrix is a $1 \times n$ matrix.	$\mathbf{A} = \begin{bmatrix} 2 & -4 & 3 & 5 \end{bmatrix}$
Column matrix	A matrix that has only one column is known as a <i>column matrix</i> . In general, a column matrix is an $m \times 1$ matrix.	$\mathbf{B} = \begin{bmatrix} 0 \\ 8 \end{bmatrix}$
Square matrix	A <i>square matrix</i> is a matrix with the same number of rows and columns. In general, a square matrix is an $n \times n$ matrix.	$\mathbf{C} = \begin{bmatrix} 4 & 7 & -2 \\ 3 & 4 & 8 \\ 5 & 1 & -6 \end{bmatrix}$
Equal matrices	Two matrices are <i>equal</i> if they have the same order and their corresponding elements are equal. If $\mathbf{A} = \mathbf{B}$ and $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, then $a_{11} = b_{11}$, $a_{12} = b_{12}$, $a_{21} = b_{21}$ and $a_{22} = b_{22}$.	$\begin{bmatrix} 1 & 4 \\ 9 & 16 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 9 & 16 \end{bmatrix}$ The two matrices above are equal. The two matrices below have the same order as each other. However, the corresponding elements are different. They are not equal. $\begin{bmatrix} 16 & 9 \\ 4 & 1 \end{bmatrix} \neq \begin{bmatrix} 1 & 4 \\ 9 & 16 \end{bmatrix}$
Zero matrix	A <i>zero matrix</i> $\mathbf{O}$ is a matrix where each element is zero.	$[0]$, $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$
Identity matrix	An <i>identity matrix</i> $\mathbf{I}$ is a square matrix where every element in the lead diagonal is 1 and all other elements are 0.	$[1]$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Equal matrices

Additional information

Equal matrices

Watch the video as an introduction to the concept of equal matrices.

8

Equal matrices

Consider the following two matrices, $A = \begin{bmatrix} p & 3q \\ r-2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 9 \\ 8 & s \end{bmatrix}$.

If $A = B$, determine the values of p , q , r and s .

THINKING

WORKING

1

Equate the matrices.

$A = B$

$$\begin{bmatrix} p & 3q \\ r-2 & 5 \end{bmatrix} = \begin{bmatrix} -3 & 9 \\ 8 & s \end{bmatrix}$$

2

Identify pairs of corresponding elements from both matrices to form and then solve equations.

$p = -3$

$3q = 9$ and so $q = 3$.

$r - 2 = 8$ and so $r = 10$.

$5 = s$ and so $s = 5$.

Hence $p = -3$, $q = 3$, $r = 10$ and $s = 5$.

EXERCISE

6.2

Matrix properties

Worked Example

6

1

2

3

4

5

6

7

8

9

10

11

12

13

14

15

16

17

18

19

20

21

22

23

24

25

26

27

28

29

30

31

32

33

34

35

36

37

38

39

40

41

42

43

44

45

46

47

48

49

50

51

52

53

54

55

56

57

58

59

60

61

62

63

64

65

66

67

68

69

70

71

72

73

74

75

76

77

78

79

80

81

82

83

84

85

86

87

88

89

90

91

92

93

94

95

96

97

98

99

100

1 Anne keeps a record of her LOTE test results. Her first three test results in Indonesian (I) were 68%, 71% and 79%. Her first three test results in Japanese (J) were 59%, 63% and 70%. Express Anne's test results in the form of a 2×3 matrix A .

2 Consider the matrix $M = \begin{bmatrix} 1 & -2 \\ 4 & 0 \\ 6 & 2 \\ 1 & -5 \end{bmatrix}$.

- (a) Select the option that represents the order of matrix M .
- A 2×4 B 4×2 C $4 + 2$ D $2 + 4$
- (b) Suggest a way to remember the order in which number of rows and number of columns should be written.
- 3 Complete the table for the following matrices.

Matrix	$A = \begin{bmatrix} 4 & -3 & 7 \end{bmatrix}$	$B = \begin{bmatrix} -2 & 1 \\ 6 & \frac{1}{2} \end{bmatrix}$	$C = \begin{bmatrix} 5 \\ a \end{bmatrix}$	$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$E = \begin{bmatrix} 0 & 0 \end{bmatrix}$
Type					
Order					
Number of elements					

4 Consider $A = \begin{bmatrix} 9 & 3 \\ 4 & 1 \\ 8 & 2 \end{bmatrix}$.

- (a) Determine the order of A .
- (b) Identify the common misconception by a student who stated that the order of A is 6.
- (c) Determine the value of a_{22} .
- (d) Identify which element of matrix A is the number 2.
- (e) Calculate $a_{12} + a_{31}$.

5 Create the following matrices.

- (a) Matrix A is a row matrix with 3 columns, where $a_{12} = 5$ and all other elements are 1.
- (b) Matrix B is a 2×2 identity matrix.
- (c) Matrix C is a 3×2 zero matrix.

6 Supermarket 1 sells cartons of apple juice for \$4.90 each and cartons of orange juice for \$4.65 each. Supermarket 2 sells the same cartons of apple juice for \$5.15 each and the same cartons of orange juice for \$4.95 each.

Construct a matrix to represent the cost of fruit juice at these two supermarkets, where the rows represent the supermarket the juice was purchased from.

7 Consider $A = \begin{bmatrix} 3 & -6 & 5 \\ 0 & -4 & 0 \\ -2 & 0 & 2 \end{bmatrix}$.

- (a) State the order of A .
- (b) Determine the value of a_{31} .
- (c) Identify the common error made by a student who gave $a_{31} = 5$ as the answer to part (b).
- (d) Which element of matrix A is the number -6?

8 The table shows average maximum daily temperatures in degrees Celsius for Alpha, Garfield, Kalpowar and Pony Hills for the months of June, July and August.

	June	July	August
Alpha	17.0	16.8	17.3
Garfield	21.5	20.6	21.1
Kalpowar	26.2	25.6	29.9
Pony Hills	19.6	19.4	20.1

- (a) Express the above information in a matrix T of order 4×3 . Give headings for the rows and columns.
- (b) Which city had the highest average maximum daily temperature in July?
- (c) Which city had the lowest average maximum daily temperature in August?
- (d) (i) Identify the value of t_{32} . (ii) What does t_{32} represent?
- (e) (i) Identify the value of t_{21} . (ii) What does t_{21} represent?
- (f) According to matrix T , on average, how much warmer is Pony Hills in August compared with June?

- 9 Four students were surveyed to determine the number of males (M) and females (F) living in the family home. The results are shown in matrix G .

Which one of the following statements is *incorrect*?

- A There is one male in Maria's family.
 B There are equal numbers of males and females in Marc's family.
 C There are two females in Ray's family.
 D There are more females than males in Eva's family.

$$G = \begin{matrix} & \begin{matrix} M & F \end{matrix} \\ \begin{bmatrix} 1 & 3 \\ 2 & 1 \\ 3 & 3 \\ 1 & 5 \end{bmatrix} & \begin{matrix} \text{Eva} \\ \text{Ray} \\ \text{Marc} \\ \text{Maria} \end{matrix} \end{matrix}$$

Worked
Example

8

- 10 For the following matrices, determine the value of the unknowns if the matrices are equal.

(a) $A = \begin{bmatrix} 2t & -2u \\ v+3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & -6 \\ 2 & w \end{bmatrix}$

(b) $A = \begin{bmatrix} 2a-1 & a+3 \\ -2 & -6 \end{bmatrix}$, $B = \begin{bmatrix} 0 & \frac{7}{2} \\ b+2 & b-2 \end{bmatrix}$

(c) $P = \begin{bmatrix} a & 2a \\ 5 & c \end{bmatrix}$ and $Q = \begin{bmatrix} 2b & d \\ b & 3b \end{bmatrix}$

- 11 To show measurement formulas for a square using matrices, you could write the perimeter P and area A in terms of the side length L of the square as:

$$\begin{bmatrix} P \\ A \end{bmatrix} = \begin{bmatrix} 4L \\ L^2 \end{bmatrix}$$

- (a) Write a similar equation to show similar formulas for a parallelogram using b for the length of the base, a for the length of the adjacent side length and h for the height of the parallelogram.

- (b) Give the matrix of actual values for $\begin{bmatrix} P \\ A \end{bmatrix}$, for a parallelogram with base 15 cm, height 10 cm and adjacent side 12 cm. Do not show units in the matrix.

- (c) For a rectangle of length 9 cm the equation is $\begin{bmatrix} P \\ A \end{bmatrix} = \begin{bmatrix} x \\ 36 \end{bmatrix}$. Determine the value of x , with units.

- (d) For a circle the equation is $\begin{bmatrix} C \\ A \end{bmatrix} = \begin{bmatrix} 314 \\ y \end{bmatrix}$. Determine the value of y , to the nearest whole number, given that the circumference was given in metres.

- 12 Determine the values of x , y and z if $\begin{bmatrix} y-x \\ z \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 2y \\ x+y \end{bmatrix}$.

Matrix addition, subtraction and scalar multiplication

6.3

Matrix addition and subtraction

Matrices can be added and subtracted. To add and subtract matrices, the order of the matrices must be the same.

For addition, add the corresponding elements. For subtraction, subtract the corresponding elements.

Additional information

Matrix addition

The video demonstrates the addition of matrices.

$$\text{If } \mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}, \text{ then } \mathbf{A} + \mathbf{B} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix} \text{ and } \mathbf{A} - \mathbf{B} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}.$$

9 Adding matrices

The sales of sofas (S), lounge chairs (L) and coffee tables (C) at a company's three furniture stores for the month of May is represented by the matrix $\mathbf{M}$ and the corresponding sales at the three stores for June is represented by the matrix $\mathbf{J}$ where:

$$\mathbf{M} = \begin{array}{ccc|l} & S & L & C \\ \hline & 5 & 8 & 10 & \text{Store 1} \\ & 7 & 12 & 20 & \text{Store 2} \\ & 4 & 11 & 15 & \text{Store 3} \end{array}$$

$$\mathbf{J} = \begin{array}{ccc|l} & S & L & C \\ \hline & 7 & 6 & 9 & \text{Store 1} \\ & 8 & 14 & 17 & \text{Store 2} \\ & 3 & 11 & 13 & \text{Store 3} \end{array}$$

- (a) Determine the sales of sofas, lounge chairs and coffee tables for the three furniture stores for the 2 months combined. Give your answer in matrix form.

THINKING

- 1 Add the corresponding elements of each matrix.

WORKING

$$\begin{aligned} \mathbf{M} + \mathbf{J} &= \begin{bmatrix} 5 & 8 & 10 \\ 7 & 12 & 20 \\ 4 & 11 & 15 \end{bmatrix} + \begin{bmatrix} 7 & 6 & 9 \\ 8 & 14 & 17 \\ 3 & 11 & 13 \end{bmatrix} \\ &= \begin{bmatrix} 5+7 & 8+6 & 10+9 \\ 7+8 & 12+14 & 20+17 \\ 4+3 & 11+11 & 15+13 \end{bmatrix} \\ &= \begin{bmatrix} 12 & 14 & 19 \\ 15 & 26 & 37 \\ 7 & 22 & 28 \end{bmatrix} \end{aligned}$$

- 2 Add the labels S , L and C and the stores numbers to the matrix.

$$\mathbf{M} + \mathbf{J} = \begin{array}{ccc|l} & S & L & C \\ \hline & 12 & 14 & 19 & \text{Store 1} \\ & 15 & 26 & 37 & \text{Store 2} \\ & 7 & 22 & 28 & \text{Store 3} \end{array}$$

3 Interpret the answer.

Over the month of May:

Store 1 sold 12 sofas, 14 lounges and 19 coffee tables.

Store 2 sold 15 sofas, 26 lounges and 37 coffee tables.

Store 3 sold 7 sofas, 22 lounges and 28 coffee tables.

(b) What does the matrix element $(\mathbf{M} + \mathbf{J})_{11}$ represent?

Note the row value and the column value.

$(\mathbf{M} + \mathbf{J})_{11}$ indicates that store 1 sold 12 sofas over the 2 months of May and June.

As a general rule, if $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$, then $\mathbf{A} \pm \mathbf{B} = \begin{bmatrix} a \pm e & b \pm f \\ c \pm g & d \pm h \end{bmatrix}$.

Matrix addition is said to be *commutative* (the order in which the matrices are added is not important). In other words, $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.

For example, if $\mathbf{A} = \begin{bmatrix} 7 & 4 \\ 8 & 2 \\ 3 & 5 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 9 & 6 \\ -2 & 4 \\ 1 & 8 \end{bmatrix}$, then:

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 7+9 & 4+6 \\ 8+(-2) & 2+4 \\ 3+1 & 5+8 \end{bmatrix} = \begin{bmatrix} 16 & 10 \\ 6 & 6 \\ 4 & 13 \end{bmatrix} \quad \mathbf{B} + \mathbf{A} = \begin{bmatrix} 9+7 & 6+4 \\ -2+8 & 4+2 \\ 1+3 & 8+5 \end{bmatrix} = \begin{bmatrix} 16 & 10 \\ 6 & 6 \\ 4 & 13 \end{bmatrix}$$

10 Subtracting matrices

Consider matrices $\mathbf{A} = \begin{bmatrix} -3 & 2 & 5 \\ 6 & 1 & 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & 4 & 7 \\ -2 & -6 & 0 \end{bmatrix}$.

(a) Calculate $\mathbf{A} - \mathbf{B}$.

THINKING

Subtract the corresponding elements.

WORKING

$$\begin{aligned} \mathbf{A} - \mathbf{B} &= \begin{bmatrix} -3 & 2 & 5 \\ 6 & 1 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 4 & 7 \\ -2 & -6 & 0 \end{bmatrix} \\ &= \begin{bmatrix} -3-1 & 2-4 & 5-7 \\ 6-(-2) & 1-(-6) & 3-0 \end{bmatrix} \\ &= \begin{bmatrix} -4 & -2 & -2 \\ 8 & 7 & 3 \end{bmatrix} \end{aligned}$$

(b) Calculate $\mathbf{B} - \mathbf{A}$.

Subtract the corresponding elements.

$$\begin{aligned}\mathbf{B} - \mathbf{A} &= \begin{bmatrix} 1 & 4 & 7 \\ -2 & -6 & 0 \end{bmatrix} - \begin{bmatrix} -3 & 2 & 5 \\ 6 & 1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 1 - (-3) & 4 - 2 & 7 - 5 \\ -2 - 6 & -6 - 1 & 0 - 3 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 2 & 2 \\ -8 & -7 & -3 \end{bmatrix}\end{aligned}$$

In general, if $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$, then $\mathbf{A} - \mathbf{B} = \begin{bmatrix} a - e & b - f \\ c - g & d - h \end{bmatrix}$.

However, $\mathbf{A} - \mathbf{B} \neq \mathbf{B} - \mathbf{A}$.

Scalar multiplication of matrices

When a matrix is multiplied by a real number, known as a *scalar*, each element in that matrix is multiplied by that scalar. This is known as *scalar multiplication*.

If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $k\mathbf{A} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix}$ where k is a scalar.

A scalar works like a scale factor to increase or decrease every element in that matrix by the same factor.

In a previous worked example, the sales figures of sofas (S), lounge chairs (L) and coffee tables (C) at a company's three furniture stores for the month of May were represented by the matrix $\mathbf{M}$ where:

$$\mathbf{M} = \begin{matrix} & \begin{matrix} S & L & C \end{matrix} \\ \begin{bmatrix} 5 & 8 & 10 \\ 7 & 12 & 20 \\ 4 & 11 & 15 \end{bmatrix} & \begin{matrix} \text{Store 1} \\ \text{Store 2} \\ \text{Store 3} \end{matrix} \end{matrix}$$

If the sales figures for December were exactly double those for May in each category, these could be represented by the matrix $\mathbf{D}$ where $\mathbf{D} = 2\mathbf{M}$.

$$\mathbf{D} = 2 \begin{bmatrix} 5 & 8 & 10 \\ 7 & 12 & 20 \\ 4 & 11 & 15 \end{bmatrix} = \begin{bmatrix} 2 \times 5 & 2 \times 8 & 2 \times 10 \\ 2 \times 7 & 2 \times 12 & 2 \times 20 \\ 2 \times 4 & 2 \times 11 & 2 \times 15 \end{bmatrix} = \begin{bmatrix} 10 & 16 & 20 \\ 14 & 24 & 40 \\ 8 & 22 & 30 \end{bmatrix}$$

$$\text{So } \mathbf{D} = \begin{matrix} & \begin{matrix} S & L & C \end{matrix} \\ \begin{bmatrix} 10 & 16 & 20 \\ 14 & 24 & 40 \\ 8 & 22 & 30 \end{bmatrix} & \begin{matrix} \text{Store 1} \\ \text{Store 2} \\ \text{Store 3} \end{matrix} \end{matrix}$$

i Additional information
Scalar multiplication of matrices
The video explores scalar multiplication and scale factor.

11 Scalar multiplication

If $\mathbf{A} = \begin{bmatrix} 4 & -8 \\ 2 & 6 \end{bmatrix}$, calculate $3\mathbf{A}$.

THINKING

- 1 Write in expanded matrix form.
- 2 Multiply the elements in $\mathbf{A}$ by 3.

WORKING

$$3\mathbf{A} = 3 \times \begin{bmatrix} 4 & -8 \\ 2 & 6 \end{bmatrix}$$

$$3\mathbf{A} = \begin{bmatrix} 3 \times 4 & 3 \times -8 \\ 3 \times 2 & 3 \times 6 \end{bmatrix} = \begin{bmatrix} 12 & -24 \\ 6 & 18 \end{bmatrix}$$

12 Scalar multiplication and addition

If $\mathbf{A} = \begin{bmatrix} 4 & -8 \\ 2 & 6 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & 0 \\ 3 & -8 \end{bmatrix}$, calculate $\frac{1}{2}\mathbf{A} + 3\mathbf{B}$

THINKING

- 1 Write the multiplication in expanded matrix form.
- 2 Multiply the elements in $\mathbf{A}$ by $\frac{1}{2}$ and the elements in $\mathbf{B}$ by 3.
- 3 Add the elements in corresponding positions.

WORKING

$$\frac{1}{2}\mathbf{A} + 3\mathbf{B} = \frac{1}{2} \begin{bmatrix} 4 & -8 \\ 2 & 6 \end{bmatrix} + 3 \begin{bmatrix} 1 & 0 \\ 3 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} \times 4 & \frac{1}{2} \times -8 \\ \frac{1}{2} \times 2 & \frac{1}{2} \times 6 \end{bmatrix} + \begin{bmatrix} 3 \times 1 & 3 \times 0 \\ 3 \times 3 & 3 \times -8 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -4 \\ 1 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 9 & -24 \end{bmatrix}$$

$$= \begin{bmatrix} 2+3 & -4+0 \\ 1+9 & 3+(-24) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -4 \\ 10 & -21 \end{bmatrix}$$

EXERCISE

6.3

Matrix addition, subtraction and scalar multiplication

Worked
Example

10

1 Let $\mathbf{A} = \begin{bmatrix} -2 & 4 \\ 2 & 5 \\ -1 & 8 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -5 & 0 \\ 3 & 6 \\ -4 & 1 \end{bmatrix}$.

(a) Calculate $\mathbf{A} - \mathbf{B}$.

(b) Calculate $\mathbf{B} - \mathbf{A}$.

9

- $$\mathbf{J} = \begin{bmatrix} P & S & C \\ 100 & 160 & 80 \\ 140 & 240 & 65 \\ 175 & 220 & 90 \end{bmatrix} \begin{matrix} \text{Store 1} \\ \text{Store 2} \\ \text{Store 3} \end{matrix}$$

$$\mathbf{N} = \begin{bmatrix} P & S & C \\ 75 & 125 & 80 \\ 125 & 160 & 55 \\ 135 & 150 & 95 \end{bmatrix} \begin{matrix} \text{Store 1} \\ \text{Store 2} \\ \text{Store 3} \end{matrix}$$

- $$\mathbf{D}_1 = \begin{bmatrix} & F & S & W \\ 12 & 3 & 5 & \\ 9 & 1 & 2 & \end{bmatrix} \begin{matrix} \text{Shop 1} \\ \text{Shop 2} \end{matrix}$$

$$\mathbf{D}_2 = \begin{bmatrix} F & S & W \\ 16 & 2 & 6 \\ 11 & 3 & 2 \end{bmatrix} \begin{matrix} \text{Shop 1} \\ \text{Shop 2} \end{matrix}$$

- 4 Let $\mathbf{A} = \begin{bmatrix} 1 & 5 \\ 2 & 4 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 5 & -4 \\ 2 & 10 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 4 & 1 \\ -2 & -6 \end{bmatrix}$.

- 5 Let $\mathbf{A} = \begin{bmatrix} -1 & 4 \\ 3 & 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 & -4 \\ 0 & -5 \end{bmatrix}$. Calculate the following.

- 6** Let $\mathbf{A} = \begin{bmatrix} 2 & 5 \\ 3 & 6 \\ 4 & 7 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 & 1 & 2 \\ -2 & -1 & 0 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 3 \\ 2 \\ 6 \end{bmatrix}$, $\mathbf{D} = \begin{bmatrix} 5 & -9 & 6 \\ 7 & 2 & 4 \end{bmatrix}$ and $\mathbf{E} = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix}$.

- D** **A + D**

- •

- 7** Given $\mathbf{A} = \begin{bmatrix} -1 & 4 \\ 3 & 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 2 & -4 \\ 0 & -5 \end{bmatrix}$, determine the resultant matrix.

- (a) $3\mathbf{A} - 2\mathbf{B}$ (b) $\frac{1}{2}\mathbf{A} + \frac{1}{2}\mathbf{B}$ (c) $\frac{1}{2}(\mathbf{A} + \mathbf{B})$ (d) $3\mathbf{B} - 6\mathbf{A}$

1

1

8 Let $\mathbf{A} = \begin{bmatrix} -2 & -1 & 0 \\ 2 & 3 & 4 \\ 6 & 7 & 8 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & -3 & 4 \\ 3 & 0 & 1 \\ 0 & 1 & -3 \end{bmatrix}$. Calculate $-(3\mathbf{A} - 2\mathbf{B})$.

9 Select the correct answer to complete the equation.

$$3 \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix} + 2 \begin{bmatrix} -2 & 1 \\ 0 & -4 \end{bmatrix} =$$

A $\begin{bmatrix} -1 & 11 \\ 3 & -2 \end{bmatrix}$

B $6 \begin{bmatrix} -1 & 4 \\ 1 & -2 \end{bmatrix}$

C $\begin{bmatrix} -1 & 11 \\ 3 & 2 \end{bmatrix}$

D $5 \begin{bmatrix} -1 & 4 \\ 1 & -2 \end{bmatrix}$

10 Tonia and Jamilah study first-year biology and chemistry at university. Their results are shown in the table.

	Semester 1:		Semester 2:	
	Tonia	Jamilah	Tonia	Jamilah
Biology	79	66	72	68
Chemistry	71	81	75	79

(a) Construct matrix $\mathbf{H}$ to represent their Semester 1 results.

(b) Construct matrix $\mathbf{F}$ to represent their Semester 2 results.

(c) (i) Calculate $\frac{1}{2}(\mathbf{H} + \mathbf{F})$.

(ii) What does this matrix represent?

(d) (i) Identify the element in row 2 and column 1 of $\frac{1}{2}(\mathbf{H} + \mathbf{F})$.

(ii) What does this element represent?

11 If $\mathbf{A} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 4 \\ 1 \\ 7 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} -3 \\ 0 \\ 5 \end{bmatrix}$, determine the values of x , y and z when:

(a) $\mathbf{A} = 2\mathbf{B} - \mathbf{C}$

(b) $\mathbf{C} + \mathbf{A} = 2\mathbf{B}$

12 If $\mathbf{A} = \begin{bmatrix} 3x \\ y+2 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 1-2x \\ 4y \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$, determine the values of x and y when $\mathbf{A} = 2\mathbf{B} + \mathbf{C}$.

..

..

13 The table shows monthly sales figures for jeans and shorts at two department stores for March and August.

	March: Jeans	March: Shorts	August: Jeans	August: Shorts
Store 1	80	180	240	120
Store 2	160	200	300	80

(a) Construct matrix $\mathbf{M}$ and matrix $\mathbf{A}$ where $\mathbf{M}$ represents March sales and $\mathbf{A}$ represents August sales.

(b) Determine $\mathbf{M} + \mathbf{A}$ and state what it represents.

(c) Which store had the better sales figures for each of the items for the 2 months combined?

(d) If sales in December for both stores increased by 20% from August's sales figures, construct the resultant matrix showing the total sales for March, August and December.

14 If $\mathbf{A} = \begin{bmatrix} 2x \\ y \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 4y \\ 3x \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$, determine the values of x and y when $2\mathbf{A} + \mathbf{B} = \mathbf{C}$.

Matrix multiplication

Matrix multiplication

In the previous section, you saw that scalar multiplication involves the multiplication of a matrix by a scalar (a real number). The effect of scalar multiplication is that each element in the matrix is multiplied by that scalar.

Matrix multiplication is different from scalar multiplication. Matrix multiplication involves the multiplication of a matrix by another matrix.

Consider the following situation. A fruit and vegetable company produces two different fruit boxes consisting of apples, bananas and oranges. Each apple is sold for \$0.90, each banana is sold for \$0.50 and each orange is sold for \$0.65.

- Box 1 contains 4 apples, 5 bananas and 4 oranges.
- Box 2 contains 6 apples, 3 bananas and 6 oranges.

Given this information, you can calculate the total selling price of each fruit box.

- The selling price for box 1: $(4 \times 0.90) + (5 \times 0.50) + (4 \times 0.65) = \$8.70$
- The selling price for box 2: $(6 \times 0.90) + (3 \times 0.50) + (6 \times 0.65) = \$10.80$

This can easily be calculated with matrices, where the aim is to calculate the cost of each of the fruit boxes.

The resultant, or product, matrix will be in the form:

$$\begin{array}{l} \$ \\ \text{Box 1} \left[\begin{array}{c} \\ \\ \end{array} \right] \\ \text{Box 2} \left[\begin{array}{c} \\ \\ \end{array} \right] \end{array}$$

The two fruit boxes can be represented by a matrix **N**:

$$\mathbf{N} = \begin{array}{ccc} & A & B & O \\ \left[\begin{array}{ccc} 4 & 5 & 4 \\ 6 & 3 & 6 \end{array} \right] & \begin{array}{l} \text{Box 1} \\ \text{Box 2} \end{array} \end{array}$$

The selling price for each type of fruit can be represented by the column matrix **S**:

$$\begin{array}{c} \$ \\ \mathbf{S} = \left[\begin{array}{c} 0.90 \\ 0.50 \\ 0.65 \end{array} \right] \begin{array}{l} A \\ B \\ O \end{array} \end{array}$$

Note: The number of columns in the initial matrix **N** is equal to the number of rows in the second matrix **S**.

Multiplying the elements in row 1 of **N** by the elements in **S** gives the selling price for box 1.

$$\begin{aligned}
 \mathbf{N} \times \mathbf{S} &= \begin{matrix} \text{Box 1} \\ \text{Box 2} \end{matrix} \begin{matrix} A & B & O \\ \left[\begin{array}{ccc} 4 & 5 & 4 \end{array} \right] \end{matrix} = \begin{matrix} \$ \\ \left[\begin{array}{c} 0.90 \\ 0.50 \\ 0.65 \end{array} \right] \end{matrix} \begin{matrix} A \\ B \\ O \end{matrix} \\
 &= \begin{matrix} \text{Box 1} \\ \text{Box 2} \end{matrix} \begin{matrix} \$ \\ \left[\begin{array}{c} (4 \times 0.90) + (5 \times 0.50) + (4 \times 0.65) \end{array} \right] \end{matrix} \\
 &= \begin{matrix} \text{Box 1} \\ \text{Box 2} \end{matrix} \begin{matrix} \$ \\ \left[\begin{array}{c} 8.70 \end{array} \right] \end{matrix}
 \end{aligned}$$

As you move across row 1 of matrix **N**, move down the column of matrix **S**, forming each product as you go. Then add each product to obtain the selling price of box 1 (\$8.70).

To form the element in the first row and first column (\$8.70) of the final matrix (**N** × **S**), add the results of multiplying each element of row 1 with the corresponding element of column 1.

Multiplying the elements in row 2 of **N** by the elements in **S** gives the selling price for box 2.

$$\begin{aligned}
 \mathbf{N} \times \mathbf{S} &= \begin{matrix} \text{Box 1} \\ \text{Box 2} \end{matrix} \begin{matrix} \left[\begin{array}{ccc} 6 & 3 & 6 \end{array} \right] \end{matrix} = \begin{matrix} \$ \\ \left[\begin{array}{c} 0.90 \\ 0.50 \\ 0.65 \end{array} \right] \end{matrix} \begin{matrix} A \\ B \\ O \end{matrix} \\
 &= \begin{matrix} \text{Box 1} \\ \text{Box 2} \end{matrix} \begin{matrix} \$ \\ \left[\begin{array}{c} (6 \times 0.90) + (3 \times 0.50) + (6 \times 0.65) \end{array} \right] \end{matrix} \\
 &= \begin{matrix} \text{Box 1} \\ \text{Box 2} \end{matrix} \begin{matrix} \$ \\ \left[\begin{array}{c} 10.80 \end{array} \right] \end{matrix}
 \end{aligned}$$

Move across row 2 of matrix **N** and down the column of matrix **S**, forming each product as you go. Then add each product to obtain the selling price of box 2 (\$10.80).

To form the element in the second row and first column (\$10.80) of the final matrix, add the results of multiplying each element of row 2 with the corresponding element of column 1.

In matrix form, the calculations can be summarised in the following way:

$$\begin{aligned}
 \mathbf{N} \times \mathbf{S} &= \begin{matrix} \text{Box 1} \\ \text{Box 2} \end{matrix} \begin{bmatrix} 4 & 5 & 4 \\ 6 & 3 & 6 \end{bmatrix} = \begin{matrix} \$ \\ \left[\begin{array}{c} 0.90 \\ 0.50 \\ 0.65 \end{array} \right] \end{matrix} \begin{matrix} A \\ B \\ O \end{matrix} \\
 &= \begin{matrix} \text{Box 1} \\ \text{Box 2} \end{matrix} \begin{matrix} \$ \\ \left[\begin{array}{c} (4 \times 0.90) + (5 \times 0.50) + (4 \times 0.65) \\ (6 \times 0.90) + (3 \times 0.50) + (6 \times 0.65) \end{array} \right] \end{matrix} \\
 &= \begin{matrix} \text{Box 1} \\ \text{Box 2} \end{matrix} \begin{matrix} \$ \\ \left[\begin{array}{c} 8.70 \\ 10.80 \end{array} \right] \end{matrix}
 \end{aligned}$$

The cost of fruit in Box 1 is \$8.70.

The cost of fruit in Box 2 is \$10.80.

Rules for matrix multiplication

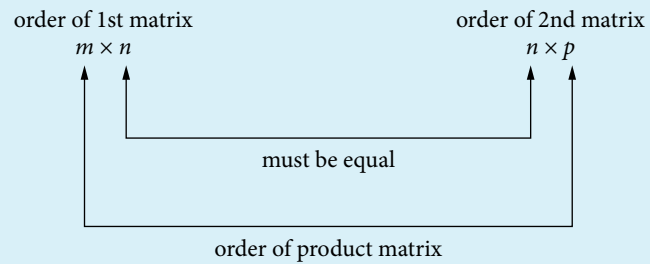
Matrix multiplication is only possible under certain conditions.

For matrix multiplication to be defined, the number of columns in the first matrix must equal the number of rows in the second matrix.

In general, if an $m \times n$ matrix is multiplied by an $n \times p$ matrix, the product matrix will be of order $m \times p$.

$$[m \times n] \times [n \times p] = m \times p$$

If the number of columns in the first matrix does not equal the number of rows in the second matrix, the matrix product cannot be found and the matrix product is described as *undefined*.



You can use the order of matrices to find the order of the resultant matrix. Using the matrices from the previous example, **N** and **S**:

$$\mathbf{N} \times \mathbf{S} = \begin{bmatrix} 4 & 5 & 4 \\ 6 & 3 & 6 \end{bmatrix} \begin{bmatrix} 0.90 \\ 0.50 \\ 0.65 \end{bmatrix} = \begin{bmatrix} 8.70 \\ 10.80 \end{bmatrix}$$

$\underline{2} \times \underline{3} \quad \quad \underline{3} \times \underline{1} \quad \quad \underline{2} \times \underline{1}$

In this instance, the number of columns (**3**) in matrix **N** is equal to the number of rows (**3**) in matrix **S**.

The order of the resultant matrix is given by the number of rows in matrix **N** and by the number of columns in matrix **S**. That is, the resultant matrix has order 2×1 .

i Additional information

Does the product matrix exist?

The video explains how to determine whether two matrices can be multiplied together.

For 2×2 matrices, if $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$, then $\mathbf{A} \times \mathbf{B} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$.

To calculate the element in the first row and first column of the product matrix **AB**, multiply the elements in the first row of **A** by the elements in the first column of **B**.

e Explore further

Multiplying matrices

Discover matrix multiplication with Excel.

13 Order of a matrix product

Let $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} -2 \\ 4 \\ -6 \end{bmatrix}$.

Determine whether or not the following matrix products can be found.

Where the matrix product can be found, identify the order of the matrix product.

(a) **BA**

THINKING

- 1 State the order of each matrix.
- 2 State the number of columns in the first matrix and the number of rows in the second matrix.

WORKING

B is a 2×2 matrix and **A** is a 2×2 matrix.
B has 2 columns and **A** has 2 rows.
 So the matrix product **BA** can be found.
BA is a 2×2 matrix.

(b) **AC**

- 1 State the order of each matrix.
- 2 State the number of columns in the first matrix and the number of rows in the second matrix.

A is a 2×2 matrix and **C** is a 2×1 matrix.
A has 2 columns and **C** has 2 rows.
 So the matrix product **AC** can be found.
AC is a 2×1 matrix.

(c) **BD**

- 1 State the order of each matrix.
- 2 State the number of columns in the first matrix and the number of rows in the second.

B is a 2×2 matrix and **D** is a 3×1 matrix.
B has 2 columns and **D** has 3 rows.
 So the matrix product **BD** *cannot* be found.
BD is undefined.

(d) **CD**

- 1 State the order of each matrix.
- 2 State the number of columns in the first matrix and the number of rows in the second.

C is a 2×1 matrix and **D** is a 3×1 matrix.
C has 1 column and **D** has 3 rows.
 So the matrix product **CD** *cannot* be found.
CD is undefined.

14 Matrix multiplication

Let $\mathbf{A} = \begin{bmatrix} -1 & 3 \\ 2 & -4 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 8 & 5 \\ -2 & 3 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 2 & 1 \end{bmatrix}$.

Determine the following matrix products (if they exist).

(a) $\mathbf{AB}$

THINKING

- 1 Identify the order of each matrix.
- 2 State the number of columns in the first matrix and the number of rows in the second matrix.
- 3 Move across each row of the first matrix and down each column of the second matrix. Form each product and add.

WORKING

$\mathbf{A}$ is a 2×2 matrix and $\mathbf{B}$ is a 2×2 matrix.

$\mathbf{A}$ has 2 columns and $\mathbf{B}$ has 2 rows, therefore the matrix exists.

The matrix product $\mathbf{AB}$ is a 2×2 matrix.

$$\begin{aligned} \mathbf{AB} &= \begin{bmatrix} -1 & 3 \\ 2 & -4 \end{bmatrix} \times \begin{bmatrix} 8 & 5 \\ -2 & 3 \end{bmatrix} \\ &= \begin{bmatrix} (-1 \times 8) + (3 \times -2) & (-1 \times 5) + (3 \times 3) \\ (2 \times 8) + (-4 \times -2) & (2 \times 5) + (-4 \times 3) \end{bmatrix} \\ &= \begin{bmatrix} -8 - 6 & -5 + 9 \\ 16 + 8 & 10 - 12 \end{bmatrix} \\ &= \begin{bmatrix} -14 & 4 \\ 24 & -2 \end{bmatrix} \end{aligned}$$

(b) $\mathbf{BA}$

- 1 Identify the order of each matrix.
- 2 State the number of columns in the first matrix and the number of rows in the second matrix.
- 3 Move across each row of the first matrix and down each column of the second matrix. Form each product and add.

$\mathbf{B}$ is a 2×2 matrix and $\mathbf{A}$ is a 2×2 matrix.

$\mathbf{B}$ has 2 columns and $\mathbf{A}$ has 2 rows, therefore the matrix exists.

The matrix product $\mathbf{BA}$ is a 2×2 matrix.

$$\begin{aligned} \mathbf{BA} &= \begin{bmatrix} 8 & 5 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ 2 & -4 \end{bmatrix} \\ &= \begin{bmatrix} (8 \times -1) + (5 \times 2) & (8 \times 3) + (5 \times -4) \\ (-2 \times -1) + (3 \times 2) & (-2 \times 3) + (3 \times -4) \end{bmatrix} \\ &= \begin{bmatrix} -8 + 10 & 24 - 20 \\ 2 + 6 & -6 - 12 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 4 \\ 8 & -18 \end{bmatrix} \end{aligned}$$

(c) DC

- 1 Identify the order of each matrix.
- 2 State the number of columns in the first matrix and the number of rows in the second matrix.
- 3 Move across each row of the first matrix and down each column of the second matrix. Form each product and add.

D is a 3×2 matrix and **C** is a 2×1 matrix.

D has 2 columns and **C** has 2 rows, therefore the matrix exists.

The matrix product **DC** is a 3×1 matrix.

$$\begin{aligned} \mathbf{DC} &= \begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -6 \end{bmatrix} \\ &= \begin{bmatrix} (0 \times 2) + (1 \times -6) \\ (2 \times 2) + (3 \times -6) \\ (2 \times 2) + (1 \times -6) \end{bmatrix} \\ &= \begin{bmatrix} -6 \\ -14 \\ -2 \end{bmatrix} \end{aligned}$$

In arithmetic, multiplication is commutative (order is not important); i.e. $a \times b = b \times a$.

In the previous worked example you saw that $\mathbf{AB} = \begin{bmatrix} -14 & 4 \\ 24 & -2 \end{bmatrix}$ and $\mathbf{BA} = \begin{bmatrix} 2 & 4 \\ 8 & -18 \end{bmatrix}$.

That is, $\mathbf{AB} \neq \mathbf{BA}$.

Matrix multiplication must be performed in the required order. In general, matrix multiplication is *not* commutative.

$$\mathbf{AB} \neq \mathbf{BA}$$

The identity matrix

In arithmetic, the identity element for multiplication is 1, which simply means that multiplying by 1 allows a number to keep its identity, or stay the same: $1 \times 6 = 6 \times 1 = 6$.

In matrices, if you multiply by a square matrix whose leading diagonal elements are 1s and all other elements are 0, the matrix is unchanged.

If **A** is a square matrix then **I** is a square matrix with the same dimensions.

If **A** is not a square matrix then the identity matrix required to multiply by **A** from the left is square, but has different dimensions from the identity matrix required to multiply by **A** from the right.

Identity matrix **I**.

If **A** is a square matrix: $\mathbf{IA} = \mathbf{A}$ and $\mathbf{AI} = \mathbf{A}$ where $\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ etc.

If **A** is an $m \times n$ matrix then $\mathbf{I}_m \mathbf{A} = \mathbf{A}$ and $\mathbf{A} \mathbf{I}_n = \mathbf{A}$ where $\mathbf{I}_n$ is the $n \times n$ identity matrix.

15 Multiplication by the identity matrix

Choose the correct dimensions for the identity matrix to demonstrate that $\mathbf{IA} = \mathbf{A}$ and $\mathbf{AI} = \mathbf{A}$ for each matrix $\mathbf{A}$.

(a) $\mathbf{A} = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 4 & 4 \\ -2 & 1 & 6 \end{bmatrix}$

THINKING

- 1 Because $\mathbf{A}$ is a square matrix, the identity matrix has the same dimensions. Show $\mathbf{IA} = \mathbf{A}$.

WORKING

$$\begin{aligned} \mathbf{IA} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ -1 & 4 & 4 \\ -2 & 1 & 6 \end{bmatrix} \\ &= \begin{bmatrix} 1 \times 2 & 1 \times 1 & 1 \times 3 \\ 1 \times -1 & 1 \times 4 & 1 \times 4 \\ 1 \times -2 & 1 \times 1 & 1 \times 6 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 1 & 3 \\ -1 & 4 & 4 \\ -2 & 1 & 6 \end{bmatrix} \end{aligned}$$

- 2 Show $\mathbf{AI} = \mathbf{A}$.

$$\begin{aligned} \mathbf{AI} &= \begin{bmatrix} 2 & 1 & 3 \\ -1 & 4 & 4 \\ -2 & 1 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 \times 1 & 1 \times 1 & 3 \times 1 \\ -1 \times 1 & 4 \times 1 & 4 \times 1 \\ -2 \times 1 & 1 \times 1 & 6 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 1 & 3 \\ -1 & 4 & 4 \\ -2 & 1 & 6 \end{bmatrix} \\ &= \mathbf{A} \end{aligned}$$

(b) $\mathbf{A} = \begin{bmatrix} 4 & 6 & -2 \\ 1 & 3 & 5 \end{bmatrix}$

- 1 Because $\mathbf{A}$ is a 2×3 matrix, to show that $\mathbf{IA} = \mathbf{A}$, $\mathbf{I}$ must be a 2×2 matrix.

$$\begin{aligned} \mathbf{I}_2 \mathbf{A} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 6 & -2 \\ 1 & 3 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 1 \times 4 & 1 \times 6 & 1 \times -2 \\ 1 \times 1 & 1 \times 3 & 1 \times 5 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 6 & -2 \\ 1 & 3 & 5 \end{bmatrix} \\ &= \mathbf{A} \end{aligned}$$

2 Show $\mathbf{AI}_3 = \mathbf{A}$.

$$\begin{aligned}\mathbf{AI}_3 &= \begin{bmatrix} 4 & 6 & -2 \\ 1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 4 \times 1 & 6 \times 1 & -2 \times 1 \\ 1 \times 1 & 3 \times 1 & 5 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 6 & -2 \\ 1 & 3 & 5 \end{bmatrix} \\ &= \mathbf{A}\end{aligned}$$

16 Matrix multiplication to calculate a semester result

Pia scores the following marks (out of 100) for her assessment components in chemistry: topic tests 80; practical work 86; semester examination 70.

The final semester result for chemistry is weighted as follows: topic tests 30%; practical work 30%; semester examination 40%.

Use a matrix multiplication method to calculate Pia's final semester result (out of 100) for chemistry.

THINKING

- 1 Represent the marks for each assessment component as a 1×3 matrix.
- 2 Represent the weighting of each assessment component as a 3×1 matrix.
- 3 Calculate $\mathbf{AW}$.

- 4 State the final answer and interpret the result (answers must not be left in a matrix).

WORKING

Let $\mathbf{A} = \begin{bmatrix} 80 & 86 & 70 \end{bmatrix}$.

Let $\mathbf{W} = \begin{bmatrix} 0.3 \\ 0.3 \\ 0.4 \end{bmatrix}$.

$\mathbf{A}$ is a 1×3 matrix and $\mathbf{W}$ is a 3×1 matrix.

So the matrix product $\mathbf{AW}$ is a 1×1 matrix.

$$\begin{aligned}\mathbf{AW} &= \begin{bmatrix} 80 & 86 & 70 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.3 \\ 0.4 \end{bmatrix} \\ &= [77.8]\end{aligned}$$

Pia's final semester result (out of 100) for chemistry was 77.8.

Note: $[77.8]$ is not the same as 77.8.

$[77.8]$ is a *matrix* with one element in one row and one column. 77.8 is the *element* of the matrix in this row and column.

17 Matrix multiplication to calculate construction costs

A manufacturer makes three products A , B and C , each requiring different stages of construction P , Q , R and S . Matrix $\mathbf{X}$ shows the construction time, in hours, required to make one of each product.

Each stage of construction, P , Q , R and S , costs the manufacturer \$100, \$50, \$80 and \$250 per hour respectively.

$$\mathbf{X} = \begin{matrix} & \begin{matrix} P & Q & R & S \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 1 & 2 & 0 & 1.5 \\ 0.5 & 1 & 2 & 1 \\ 0 & 1.5 & 3 & 0 \end{bmatrix} \end{matrix}$$

- (a) Express each stage of hourly construction costs as a column matrix $\mathbf{Y}$.

THINKING

Represent each stage of hourly construction costs as a 4×1 matrix.

WORKING

$$\mathbf{Y} = \begin{matrix} \$ \\ \begin{bmatrix} 100 \\ 50 \\ 80 \\ 250 \end{bmatrix} \end{matrix}$$

- (b) Determine $\mathbf{XY}$ and state what information it displays.

- 1 Calculate $\mathbf{XY}$.

$\mathbf{X}$ is a 3×4 matrix and $\mathbf{Y}$ is a 4×1 matrix.

So the matrix product $\mathbf{XY}$ is a 3×1 matrix.

$$\begin{aligned} \mathbf{XY} &= \begin{bmatrix} 1 & 2 & 0 & 1.5 \\ 0.5 & 1 & 2 & 1 \\ 0 & 1.5 & 3 & 0 \end{bmatrix} \begin{bmatrix} 100 \\ 50 \\ 80 \\ 250 \end{bmatrix} \\ &= \begin{bmatrix} 575 \\ 510 \\ 315 \end{bmatrix} \end{aligned}$$

- 2 State what information $\mathbf{XY}$ displays.

Product A costs \$575, product B costs \$510 and product C costs \$315 to manufacture.

EXERCISE

6.4

Matrix multiplication

Worked
Example

13

1 Let $\mathbf{A} = \begin{bmatrix} -3 & 4 \\ 2 & 2 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 2 \\ 1 \\ -3 \\ 5 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ and $\mathbf{D} = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$.

Determine whether or not the following matrix products exist.

Where the matrix product can be found, determine the order of the matrix product.

(a) $\mathbf{AD}$ (b) $\mathbf{BD}$ (c) $\mathbf{AC}$

14

2 Let $\mathbf{P} = \begin{bmatrix} 1 & 0 \\ 4 & 7 \end{bmatrix}$, $\mathbf{Q} = \begin{bmatrix} 3 & -1 \\ -5 & -1 \end{bmatrix}$, $\mathbf{R} = \begin{bmatrix} 6 \\ -1 \end{bmatrix}$ and $\mathbf{S} = \begin{bmatrix} -2 & -1 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}$.

Determine the following matrix products (if they exist).

(a) $\mathbf{QP}$ (b) $\mathbf{PQ}$ (c) $\mathbf{SR}$

3 Calculate the following matrix products.

(a) $\begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$

(b) $\begin{bmatrix} -3 & 4 \end{bmatrix} \begin{bmatrix} 6 \\ 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 5 & 1 \\ -3 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} -2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 8 & 9 \\ 0 & 3 & 0 & 3 \end{bmatrix}$

(e) $\begin{bmatrix} 4 & 3 \\ -2 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

(f) $\begin{bmatrix} 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 9 & 1 \\ -1 & -2 \\ 4 & 3 \end{bmatrix}$

(g) $\begin{bmatrix} -1 & 0 & 4 \\ 3 & -1 & 0 \\ 6 & 2 & 0 \\ 0 & 4 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

(h) $\begin{bmatrix} 2 \\ -5 \end{bmatrix} \begin{bmatrix} 3 & 1 \end{bmatrix}$

4 Explain why each matrix product exists in the previous question.

15

5 Consider the following matrices.

$\mathbf{A} = \begin{bmatrix} 2 \\ -1 \\ 7 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 3 & 0 \\ -2 & 4 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 8 & -6 \end{bmatrix}$

Complete the following multiplications where $\mathbf{I}$ represents an appropriate identity matrix. State the dimensions of $\mathbf{I}$ in your answer, so $\mathbf{I}_1$, $\mathbf{I}_2$ or $\mathbf{I}_3$.

(a) $\mathbf{AI}$ (b) $\mathbf{BI}$ (c) $\mathbf{CI}$ (d) $\mathbf{IA}$ (e) $\mathbf{IB}$ (f) $\mathbf{IC}$

6 Determine the following matrix products.

$$(a) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (b) \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (c) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(d) \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (e) \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (f) \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$(g) \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (h) \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (i) \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

7 For $\mathbf{A} = \begin{bmatrix} 7 & -2 \\ 1 & 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -1 & 4 \\ 2 & 8 \end{bmatrix}$

(a) Calculate $\mathbf{AB}$.

(b) Identify the common error made by students who perform the calculation.

$$\mathbf{AB} = \begin{bmatrix} 7 \times -1 & -2 \times 4 \\ 1 \times 2 & 3 \times 8 \end{bmatrix}$$

8 Apples cost \$5.50 per kilogram, bananas cost \$5.30 per kilogram and pears cost \$6.10 per kilogram. Andrew buys 5 kg of apples, 3.5 kg of bananas and 2 kg of pears.

(a) Which of the following can be used to determine the total cost of this fruit?

$$\mathbf{A} \begin{bmatrix} 5.50 \\ 5.30 \\ 6.10 \end{bmatrix} \begin{bmatrix} 5 \\ 3.5 \\ 2 \end{bmatrix}$$

$$\mathbf{B} \begin{bmatrix} 5.50 & 5.30 & 6.10 \end{bmatrix} \begin{bmatrix} 5 & 3.5 & 2 \end{bmatrix}$$

$$\mathbf{C} \begin{bmatrix} 5 \\ 3.5 \\ 2 \end{bmatrix} \begin{bmatrix} 5.50 & 5.30 & 6.10 \end{bmatrix}$$

$$\mathbf{D} \begin{bmatrix} 5.50 & 5.30 & 6.10 \end{bmatrix} \begin{bmatrix} 5 \\ 3.5 \\ 2 \end{bmatrix}$$

(b) Which of the other options has a defined matrix product and what does this product mean?

9 Let $\mathbf{A} = \begin{bmatrix} 2 & 1 & 0 \\ -3 & 1 & 5 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} -2 & 4 \\ 5 & 1 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 3 & 1 \\ -2 & 0 \\ -1 & 1 \end{bmatrix}$, $\mathbf{D} = \begin{bmatrix} 3 & 5 \end{bmatrix}$ and $\mathbf{E} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$.

(a) State the order of each matrix.

(b) Calculate the following matrix products (if they exist). If a matrix product does not exist, explain why.

(i) $\mathbf{AB}$

(ii) $\mathbf{BA}$

(iii) $\mathbf{AD}$

(iv) $\mathbf{CE}$

Worked
Example

10 Corey scored the following marks (out of 100) for his assessment components in biology: topic tests 76; practical work 90; semester examination 72.

The final semester result for biology is weighted as follows: topic tests 20%; practical work 30%; semester examination 50%.

Use a matrix multiplication method to calculate Corey's final semester result (out of 100) for biology. Give your answer to the nearest whole number.

16

- 11 Gillian earned the following amounts of money for the last three financial years: 2015–2016: \$54 522; 2016–2017: \$45 124; 2017–2018: \$64 119.

The percentage Gillian paid in tax each financial year was as follows: 2015–2016: 17.0%; 2016–2017: 13.8%; 2017–2018: 19.3%.

Use a matrix multiplication method to calculate the total amount of tax paid by Gillian over the three financial years. Round each annual amount to the nearest cent.

Worked
Example

17

- 12 A manufacturer makes three products X , Y and Z , each requiring different amounts of construction time for stages A , B , C , D and E . Matrix $\mathbf{M}$ shows the different amounts of construction time, in hours, required to make one of each product.

$$\mathbf{M} = \begin{bmatrix} 3 & 2 & 0 & 1 & 1 \\ 2 & 1 & 2 & 1 & 1 \\ 3 & 0 & 3 & 1 & 0 \end{bmatrix} \begin{matrix} X \\ Y \\ Z \end{matrix}$$

Each stage of construction, A , B , C , D and E , costs the manufacturer \$300, \$400, \$50, \$100 and \$200 per hour respectively.

- (a) Express the hourly construction cost for each stage as a column matrix $\mathbf{R}$.
- (b) Calculate $\mathbf{MR}$ to determine how much each product costs to manufacture.
- 13 A school tuckshop makes three types of sandwiches: cheese and salad, ham and salad, and cheese and tomato. Each sandwich requires different unit amounts of the ingredients bread, B , lettuce, L , ham, H , cheese, C and tomato, T . Matrix $\mathbf{M}$ shows the different number of units of each ingredient needed to make each kind of sandwich.

$$\mathbf{M} = \begin{bmatrix} 2 & 2 & 0 & 1 & 2 \\ 2 & 2 & 1 & 0 & 1 \\ 2 & 0 & 0 & 2 & 2 \end{bmatrix} \begin{matrix} \text{cheese and salad} \\ \text{ham and salad} \\ \text{cheese and tomato} \end{matrix}$$

Each unit of ingredient costs the tuckshop the following amounts: bread 10 cents, lettuce 10 cents, ham 80 cents, cheese 40 cents, tomato 20 cents.

- (a) Express each unit of ingredient cost (cents) as a column matrix $\mathbf{R}$.
- (b) Determine $\mathbf{MR}$ to calculate how much each sandwich costs to produce.

- 14 Hafiz, Peter and Winston repair electrical equipment.

The matrix $\mathbf{T}$ shows the time it takes (in minutes) for each of Hafiz (H), Peter (P) and Winston (W) to repair a microwave (M), a DVD player (D) and a CD player (C).

$$\mathbf{T} = \begin{bmatrix} 20 & 25 & 15 \\ 40 & 45 & 30 \\ 60 & 70 & 70 \end{bmatrix} \begin{matrix} M \\ D \\ C \end{matrix}$$

The matrix $\mathbf{E}$ shows the number of microwaves, DVD players and CD players repaired in three suburbs, Belmont B , Fortitude Valley F and Norman Park N , over a week.

$$\mathbf{E} = \begin{bmatrix} 1 & 3 & 7 \\ 2 & 6 & 5 \\ 1 & 4 & 3 \end{bmatrix} \begin{matrix} B \\ F \\ N \end{matrix}$$

Construct a matrix that displays the total time (in minutes) it would take for each of Hafiz, Peter and Winston to repair the electrical equipment in Belmont (B), Fortitude Valley (F) and Norman Park (N).

- 15 A gift shop sells bathroom gift packs for Father's Day.

Gift pack 1 (*GP1*) contains two bottles of talcum powder, one roll-on deodorant and two bottles of shower gel.

Gift pack 2 (*GP2*) contains one bottle of talcum powder, two roll-on deodorants and one bottle of shower gel.

Gift pack 3 (*GP3*) contains two roll-on deodorants and two bottles of shower gel.

- Express this information in a matrix **G**, where each row represents a different gift pack.
- Individually, each bottle of talcum powder (*T*) sells for \$12, each roll-on deodorant (*D*) sells for \$7 and each bottle of shower gel (*S*) sells for \$15.

Represent these individual sales prices in a 3×1 matrix **S**.

- Ignoring packaging costs and potential discounts, calculate the selling price for each bathroom gift pack. Use a matrix product in your working.
- As a goodwill gesture, a local business orders a total of nine gift packs, one for each of its male employees. The breakdown of the order is: three of gift pack 1, two of gift pack 2 and four of gift pack 3.

Use a matrix method to determine the total cost of the order.

- 16 An ice-cream shop sells ice-cream in three sizes. A single-scoop (*S*) sells for \$3, a double-scoop (*D*) sells for \$5 and a mega-scoop (*M*) sells for \$6.

- Express these prices in a 3×1 matrix **S**.
- In a single day, the shop sells 80 single-scoops, 50 double-scoops and 30 mega-scoops. Express these sales in a 1×3 matrix **N**.
- The product of the two earlier matrices is **NS**. Calculate **NS** and interpret the results.
- The table below displays the number of single-scoop, double-scoop and mega-scoop ice-creams sold at four ice-cream shops in a single day. Express this information in a 4×3 matrix **T**.

	Shop A	Shop B	Shop C	Shop D
Single-scoop (<i>S</i>)	50	30	100	80
Double-scoop (<i>D</i>)	40	90	60	20
Mega-scoop (<i>M</i>)	0	10	20	15

- The product of the two earlier matrices is **TS**. Calculate **TS** and interpret the result.
- Identify the ice-cream shop that made the most money from sales.
- In one week's time, the selling price of each type of ice-cream is due to rise by 50c per ice-cream. Determine the new selling price matrix **S**.
- Calculate how much each ice-cream shop would make on sales of ice-cream in a single day if they sold the same amount in one week's time.
- Calculate the increase in the amount of money each ice-cream shop would make on sales in a single day if they sold the same amount of ice-cream in one week's time.

6.5

Networks and power matrices

Networks and matrices

Matrices have many applications in science, engineering and commerce. One application that involves physical situations is networks, and may involve roads, rail, flights or lines of communication.

18 Creating a matrix from a diagram

Consider the following map showing the number of roads connecting three towns, A, B and C.



Construct a matrix **R** displaying this information.

THINKING

- 1 A matrix depicting the connecting roads between towns is a square matrix with the names of each town on both the horizontal and vertical headers.

WORKING

$$\mathbf{R} = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{matrix}$$

- 2 Count the roads directly connecting one town to another (without travelling through another town) and fill in the matrix with the correct numerical values.

$$\mathbf{R} = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 0 & 3 & 1 \\ 3 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

Notice that the matrix is symmetrical about the leading diagonal.

Using matrices and powers of matrices to analyse a transport network

The matrix **C** displays whether or not a pair of towns are linked directly by a road. The towns are labelled T, U, V and W.

$$\mathbf{C} = \begin{matrix} & \begin{matrix} T & U & V & W \end{matrix} \\ \begin{matrix} T \\ U \\ V \\ W \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{matrix} \begin{matrix} T \\ U \\ V \\ W \end{matrix}$$

T

U

V

W

0

1

0

1

T

1

0

0

0

U

0

0

0

1

V

1

0

1

0

W

The elements on the main diagonal are 0. This shows that no town is connected to itself.

T

U

V

W

0

1

0

1

T

1

0

0

0

U

0

0

0

1

V

1

0

1

0

W

The element 1 indicates that a direct road exists between the two towns. For example, $c_{21} = c_{12} = 1$ indicates that a road directly links town T and town U .

T

U

V

W

0

1

0

1

T

1

0

0

0

U

0

0

0

1

V

1

0

1

0

W

The element 0 indicates that there is no direct road link between the two towns. For example, $c_{31} = c_{13} = 0$ indicates that no road directly links town T and town V .

Based on the matrix, the towns can be sorted into categories.

Direct road link	No direct road link
Town T and town U	Town T and town V
Town W and town T	Town U and town W
Town W and town V	Town V and town U

You can use matrix $\mathbf{C}$ to construct a diagram.

STEP 1: How many towns are there?

There are four towns.

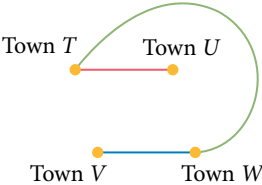
STEP 2: How many roads are connecting the towns?

Adding the elements in the matrix and then dividing by 2 gives the number of roads in the network. You must divide by 2 because each road has been counted twice, e.g. town T to town U , and town U to town T .

So from matrix $\mathbf{C}$ you can see that: number of roads = $\frac{2 + 1 + 1 + 2}{2} = 3$.

There are three roads in the network.

STEP 3: Draw a network diagram using this information.



Powers of matrices

When considering networks, for towns that do not have a road directly connecting them, matrix $\mathbf{C}^2$ show whether you can travel from one town to another town via a third town.

First, calculate the matrix $\mathbf{C}$ raised to the power of 2, $\mathbf{C}^2$.

You can use an online matrix calculator or other technology with matrix arithmetic capabilities to save time.

Using technology to calculate $\mathbf{C}^2$ gives:

$$\mathbf{C}^2 = \begin{matrix} & \begin{matrix} T & U & V & W \end{matrix} \\ \begin{matrix} T \\ U \\ V \\ W \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{matrix} = \begin{matrix} & \begin{matrix} T & U & V & W \end{matrix} \\ \begin{matrix} T \\ U \\ V \\ W \end{matrix} & \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix} \end{matrix}$$

What do the elements in the first row of matrix $\mathbf{C}^2$ tell you?

The element $c_{11} = 2$ of matrix $\mathbf{C}^2$ informs you that there are two ways to start and finish at town T , passing through one other town. The first way is to start at town T , travel to town U and then travel back to town T , i.e. follow the path TUT .

The second way is to start at town T , travel to town W and then travel back to town T , i.e. follow the path TWT .

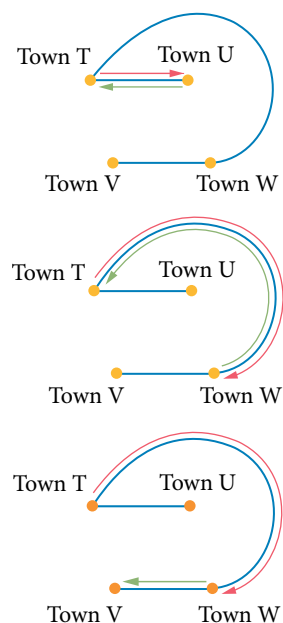
The element $c_{12} = 0$ of matrix $\mathbf{C}^2$ informs you that it is not possible to travel from town T to town U via one other town.

The element $c_{13} = 1$ of matrix $\mathbf{C}^2$ informs you that there is one way to travel from town T to town V via another town, namely via town W , i.e. follow the path TWV .

The element $c_{14} = 0$ of matrix $\mathbf{C}^2$ informs you that it is not possible to travel from town T to town W via another town.

Extending the analysis to the remaining rows of matrix $\mathbf{C}^2$, you can conclude that finding the second power of matrix $\mathbf{C}$, i.e. $\mathbf{C}^2$, represents the number of direct road links between two towns via another (or third) town.

The matrix $\mathbf{C}^3$ would tell you the number of ways you can travel from one town to another via two other towns.



Matrix $\mathbf{C}$ is an example of a matrix known as an *incidence matrix*. The element in row i and column j is 1 if i and j are related (incident) and 0 if they are not.

Matrix $\mathbf{C}$ represents the number of direct links between two objects, e.g. towns.

$\mathbf{C}^2$ represents the number of links between two objects via one other object.

WARNING

$$\text{If } \mathbf{C} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ then } \mathbf{C}^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} aa + bc & ab + bd \\ ca + dc & cb + dd \end{bmatrix}$$

Note: This is not equal to simply squaring each element.

19 Using powers of matrices to analyse a transport network

The matrix $\mathbf{C}$ displays direct air routes between Mt Isa, Rockhampton and Cairns on Mondays.

$$\mathbf{C} = \begin{array}{ccc} & \begin{matrix} M & R & C \end{matrix} \\ \begin{matrix} M \\ R \\ C \end{matrix} & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

(a) Calculate matrix $\mathbf{C}^2$.

THINKING

Multiply matrix $\mathbf{C}$ by itself.

WORKING

$$\mathbf{C}^2 = \begin{array}{ccc} & \begin{matrix} M & R & C \end{matrix} \\ \begin{matrix} M \\ R \\ C \end{matrix} & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array} \begin{array}{ccc} & \begin{matrix} M & R & C \end{matrix} \\ \begin{matrix} M \\ R \\ C \end{matrix} & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{array} \begin{matrix} M \\ R \\ C \end{matrix}$$

$$= \begin{array}{ccc} & \begin{matrix} M & R & C \end{matrix} \\ \begin{matrix} M \\ R \\ C \end{matrix} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array} \begin{matrix} M \\ R \\ C \end{matrix}$$

(b) What is represented by element c_{13} of matrix $\mathbf{C}^2$?

1 Interpret matrix $\mathbf{C}^2$.

The matrix $\mathbf{C}^2$ represents the number of air routes between two towns with one stopover.

2 Look at element c_{13} of matrix $\mathbf{C}^2$.

$$c_{13} = 1$$

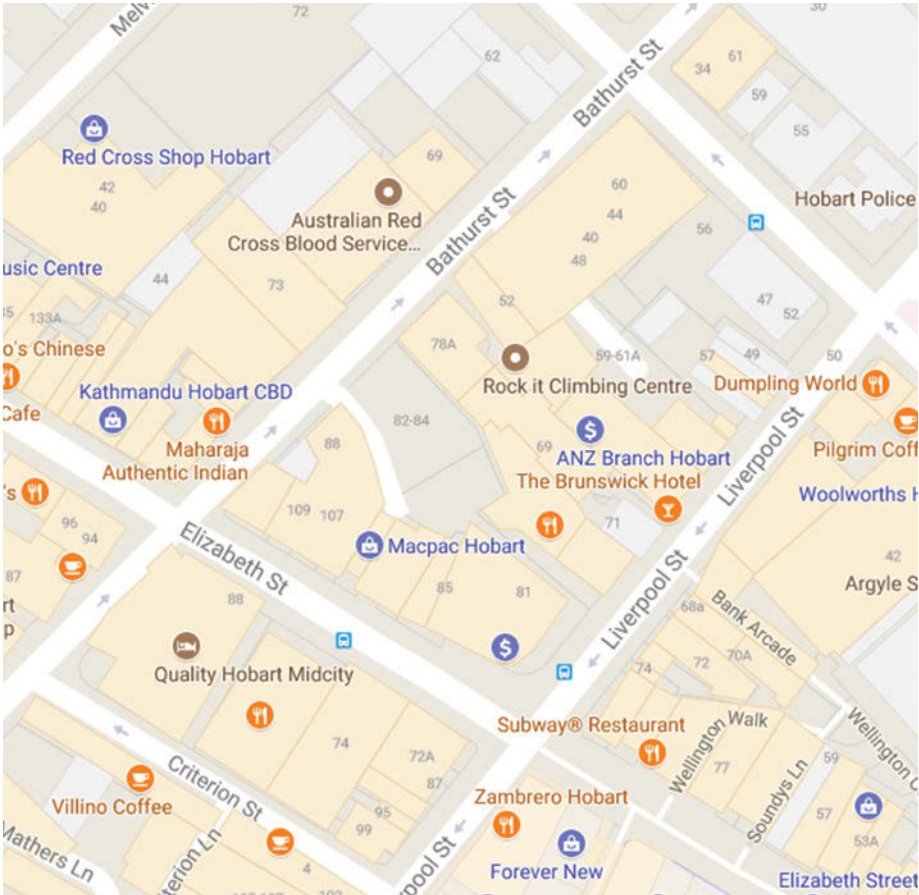
There is one air route with a single stopover between Mt Isa and Cairns.

Fly from Mt Isa to Rockhampton and then fly from Rockhampton to Cairns.

Reachability matrices

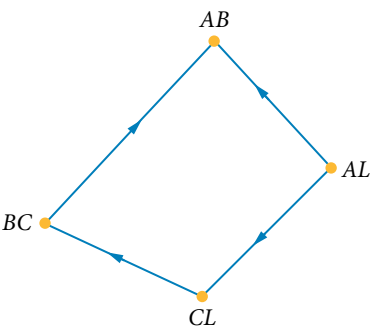
A directed graph, or *digraph*, is a directed network, meaning that the edges of a diagram have arrows indicating the direction of flow, movement or communication.

The map of Hobart shows an array of one-way streets that can cause confusion for visiting drivers.



The direction of the flow of traffic is represented by the edges of the digraph; the vertices in this case indicate the intersections of the respective streets.

If you want to travel from the intersection of Argyle St and Liverpool St (*AL*) to the intersection of Argyle St and Bathurst St (*AB*), there are two possible options. The first is the direct route from *AL* to *AB*. The second is to drive three streets from *AL* to *CL* to *BC* and *AB*.



The direct paths between vertices/intersections can be expressed as an adjacency matrix A^1 .

The rows indicate the flow of traffic, i.e. Row 1 shows that from *AL* you are able to move to *AB* and *CL*. Row 2 shows that if you start at intersection *AB*, you are unable to reach any of the other three intersections.

To

AL AB BC CL

$$A^1 = \text{From} \begin{bmatrix} AL & 0 & 1 & 0 & 1 \\ AB & 0 & 0 & 0 & 0 \\ BC & 0 & 1 & 0 & 0 \\ CL & 0 & 0 & 1 & 0 \end{bmatrix}$$

The columns indicate the reachability, i.e. Column 2 shows that you are able to reach AB if you start at AL or BC .

By calculating A^3 , you can see that there is a three-step route from the intersection of AL to that of AB .

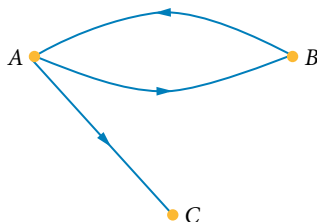
Using the digraph, you can determine the route to be:
 $AL \rightarrow CL \rightarrow BC \rightarrow AB$.

$$A^3 = \begin{matrix} & \begin{matrix} AL & AB & BC & CL \end{matrix} \\ \begin{matrix} AL \\ AB \\ BC \\ CL \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

The arrows of a reachability digraph and the rows of the adjacency matrix describe the path joining two subjects and can be read as 'will reach'. The sum of a column in a reachability matrix describes the number of ways each subject is reachable from another.

20 Reachability matrices and their application

Consider the digraph shown.



(a) Create the reachability matrix R^1 .

THINKING

1 Label a square matrix based on the number of subjects in the reachability diagram (vertices).

WORKING

$$R^1 = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{matrix}$$

2 Complete each row, remembering that the arrows represent 'will reach'.

A reaches B and C .

$$R^1 = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 0 & 1 & 1 \\ & & \\ & & \end{bmatrix} \end{matrix}$$

B reaches A .

$$R^1 = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ & & \end{bmatrix} \end{matrix}$$

C reaches no one.

$$R^1 = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

- (b) Use the matrix to determine the number of subjects A will reach.

- 1 Adding the elements of each row together gives the number of ways each subject will reach another.

$$\mathbf{R}^1 = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{matrix} \begin{matrix} = 2 \\ = 1 \\ = 0 \end{matrix}$$

- 2 Interpret the results.

A will reach 2 other subjects.

- (c) Use the matrix to determine the number of subjects B is reachable from.

- 1 Adding the elements of each column together gives the number of ways each subject is reachable from another.

$$\mathbf{R}^1 = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$\begin{matrix} \swarrow \\ \searrow \\ \searrow \end{matrix} \begin{matrix} = 1 \\ = 1 \\ = 1 \end{matrix}$

- 2 Interpret the results.

B is reachable from 1 other subject.

- (d) Calculate $\mathbf{R}^2$ and interpret the element r_{23} .

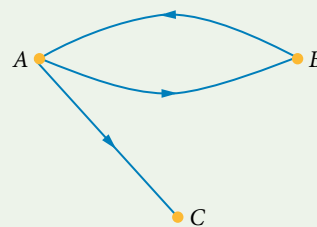
- 1 Calculate $\mathbf{R}^2$ by multiplying matrix $\mathbf{R}^1$ by itself.

$$\mathbf{R}^2 = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \boxed{1} \\ 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

- 2 Identify the element.

$$r_{23} = 1$$

- 3 Use the diagram to interpret the result.



$r_{23} = 1$ represents the two-step path where B can reach C through subject A . So you would say C is reachable from B , through A .

(e) Calculate the total one- and two-step reachability matrix $\mathbf{R}$.

1 $\mathbf{R} = \mathbf{R}^1 + \mathbf{R}^2$

$$\mathbf{R} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

A B C

$$= \begin{matrix} A \\ B \\ C \end{matrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

2 Interpret the results by adding the columns.

Vertex	Reachability
A	2
B	2
C	2

Communication matrices

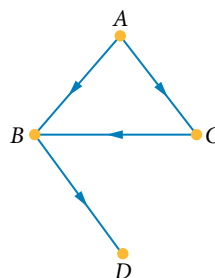
Communication matrices also stem from a digraph. The matrix will be binary, where the number 1 represents a line of communication and 0 represents no communication.

Communication can be one- or two-way. There will never be loops, meaning that the lead diagonal will always equal 0 in a communication matrix $\mathbf{C}^1$ as there is no need to communicate to oneself.

The arrows of a communication diagram and the rows of the adjacency matrix describe the path of communication between objects and can be read as 'passes communication to'. The columns of a communication matrix describe where the communication has come from.

21 Communication matrices and their application

Consider the communication digraph shown.



- (a) Create an adjacency matrix for the digraph.

THINKING

- 1 Label a square matrix based on the number of subjects in the communication diagram (vertices).
- 2 Complete each row, remembering that the arrows represent 'communicates to'.

WORKING

$$C^1 = \begin{matrix} & A & B & C & D \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{matrix}$$

A passes communication to B and C.

$$C^1 = \begin{matrix} & A & B & C & D \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{matrix}$$

B passes communication to D.

$$C^1 = \begin{matrix} & A & B & C & D \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ & & & \\ & & & \end{bmatrix} \end{matrix}$$

C passes communication to B.

$$C^1 = \begin{matrix} & A & B & C & D \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ & & & \end{bmatrix} \end{matrix}$$

D passes communication to no one.

$$C^1 = \begin{matrix} & A & B & C & D \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

(b) Determine the number of people each passes communication to.

1 Add the elements of each row together.

$$C^1 = \begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{array}{c} A \ B \ C \ D \\ \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \end{array} \begin{array}{l} = 2 \\ = 1 \\ = 1 \\ = 0 \end{array}$$

2 Interpret the results.

A passes communication to 2 other subjects.

(c) Determine the number of subjects passing communication to B.

1 Add the elements of each column together.

$$C^1 = \begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{array}{c} A \ B \ C \ D \\ \left[\begin{array}{cccc} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \end{array} \begin{array}{l} \swarrow \\ \swarrow \\ \swarrow \\ \swarrow \end{array} \begin{array}{l} = 1 \\ = 1 \\ = 2 \\ = 0 \end{array}$$

2 Interpret the results.

B receives communication from 2 other subjects.

(d) Calculate C^2 and interpret the element c_{34} .

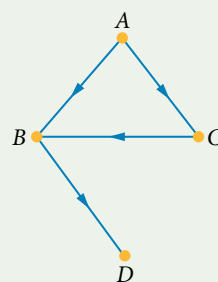
1 Calculate C^2 .

$$C^2 = \begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{array}{c} A \ B \ C \ D \\ \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \boxed{1} \\ 0 & 0 & 0 & 0 \end{array} \right] \end{array}$$

2 Identify the element.

$$c_{34} = 1$$

3 Use the diagram to interpret the result.



$c_{34} = 1$ represents the path of communication from C to D through subject B.

- (e) Calculate the total one- and two-step communication matrix C .

THINKING

1 $C = C^1 + C^2$

WORKING

$$C = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$A \quad B \quad C \quad D$

$$= \begin{matrix} A \\ B \\ C \\ D \end{matrix} \begin{bmatrix} 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- 2 Interpret the results by adding the rows.

Vertex	Communicates with
A	$0 + 2 + 1 + 1 = 4$
B	$0 + 0 + 0 + 1 = 1$
C	$0 + 1 + 0 + 1 = 2$
D	$0 + 0 + 0 + 0 = 0$

EXERCISE

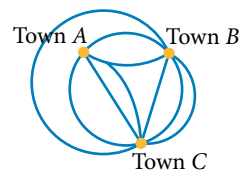
6.5

Networks and power matrices

Worked
Example

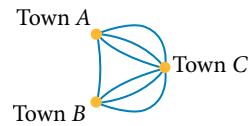
18

- 1 The diagram shows the number of roads that connect three towns, A , B and C . Construct a square matrix to show the number of roads connecting each pair of towns.



- 2 The matrix shows the number of roads connecting the three towns: Copy and complete the diagram so that it matches the matrix.

$$\begin{bmatrix} 0 & 2 & 3 \\ 2 & 0 & 3 \\ 3 & 3 & 0 \end{bmatrix}$$

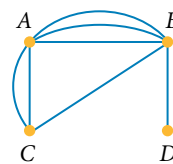


- 3 The matrix below gives the number of roads connecting A , B , C and D .

$$\begin{bmatrix} 0 & v & v & 0 \\ 1 & 0 & x & y \\ w & w & 0 & 3 \\ 0 & 2 & z & 0 \end{bmatrix}$$

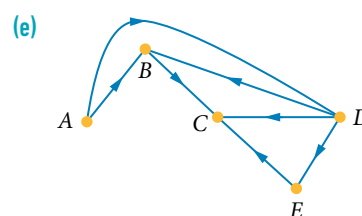
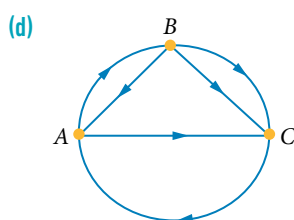
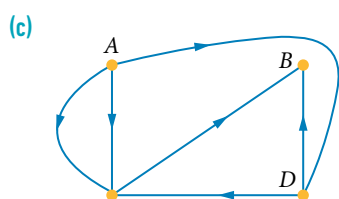
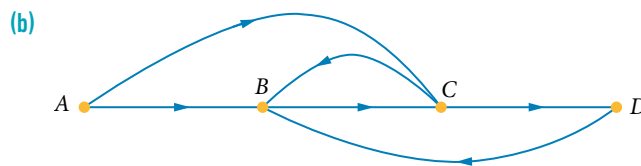
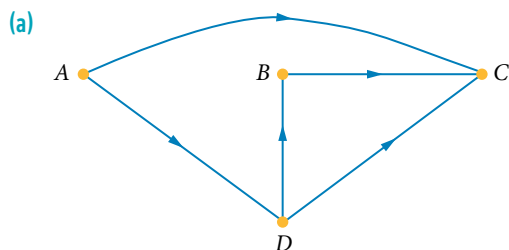
Determine the values of the pronumerals, and use this information to draw a map showing the connections between the towns.

- 4 The map shows the number of roads connecting towns A, B, C and D.



- (a) Construct matrix $\mathbf{R}$ to display this information.
 (b) State the value of r_{22} and explain why r_{22} has this value.
 (c) State the value of r_{13} and explain why $r_{13} = r_{31}$.

- 5 Create an adjacency matrix $\mathbf{Z}$ for each of the following digraphs.



- 6 Calculate the matrix $\mathbf{C}^2$.

(a) $\mathbf{C} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

(b) $\mathbf{C} = \begin{bmatrix} 0 & 6 \\ 1 & 0 \end{bmatrix}$

(c) $\mathbf{C} = \begin{bmatrix} -4 & 1 \\ 1 & -2 \end{bmatrix}$

(d) $\mathbf{C} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

(e) $\mathbf{C} = \begin{bmatrix} 0 & 5 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

(f) $\mathbf{C} = \begin{bmatrix} 0 & -2 & 2 \\ 1 & -1 & 0 \\ -1 & 1 & 0 \end{bmatrix}$

- 7 The matrix $\mathbf{C}$ displays direct air routes between Albany, Esperance and Kalgoorlie.

- (a) Calculate matrix $\mathbf{C}^2$.
 (b) What does matrix $\mathbf{C}^2$ represent?
 (c) Describe the one-stopover route from Albany to Kalgoorlie.

$$\mathbf{C} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{matrix} A \\ E \\ K \end{matrix}$$

- 8 The matrix $\mathbf{C}$ displays the direct air routes proposed by a small airline company between Rockhampton, Gladstone, Maroochydore and Bundaberg.

- (a) Which pair of towns will not be connected by a direct flight?
 (b) Calculate $\mathbf{C}^2$.
 (c) For $\mathbf{C}^2$, what is represented by the value of c_{14} ?

$$\mathbf{C} = \begin{bmatrix} R & G & M & B \\ R & 0 & 1 & 1 \\ G & 1 & 0 & 0 \\ M & 1 & 0 & 0 \\ B & 1 & 1 & 0 \end{bmatrix}$$

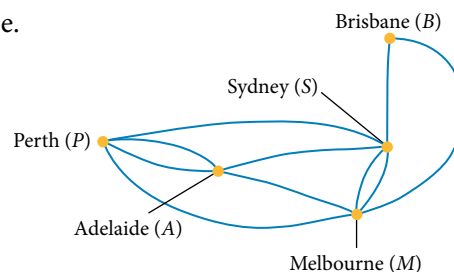
Worked
Example

19

- 9 The diagram at right shows the flight paths offered by an airline.

- (a) Construct a matrix $\mathbf{R}$ showing all available direct routes.
Order the cities alphabetically in your rows and columns.
- (b) The matrix $\mathbf{R}^2$ is given by:

$$\mathbf{R}^2 = \begin{bmatrix} 6 & 2 & 4 & 2 & 4 \\ 2 & 2 & 2 & 2 & 2 \\ 4 & 2 & 7 & 4 & 3 \\ 2 & 2 & 4 & 6 & 4 \\ 4 & 2 & 3 & 4 & 7 \end{bmatrix} \begin{matrix} A \\ B \\ M \\ P \\ S \end{matrix}$$



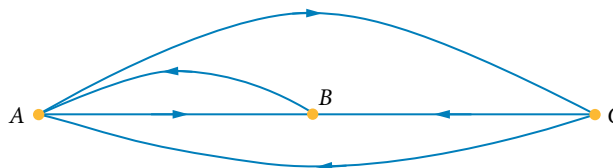
- (i) How many routes are there from Melbourne to Sydney with one stopover?
- (ii) The element $r_{12} = 2$ in the matrix $\mathbf{R}^2$ describes which two routes?
A AMB and ASB **B** AMSB and APSB **C** BPA and BMA **D** AMA and APA
- (iii) Given that the element $r_{35} = 3$ in the matrix $\mathbf{R}^2$, there are three possible stopovers available for this journey with this airline. What are these possible stopovers?

Worked
Example

20

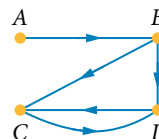
- 10 Complete the following questions regarding the reachability digraph shown.

- (a) Create a matrix $\mathbf{R}^1$.
- (b) Interpret the reachability of vertex B .
- (c) Calculate $\mathbf{R}^2$.
- (d) Calculate the total one- and two-step reachability matrix $\mathbf{R}$.
- (e) Determine the reachability of each vertex.



- 11 The digraph shows the reachability between points A , B , C and D .

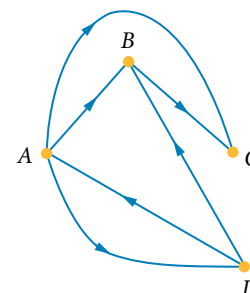
- (a) Create the reachability matrix $\mathbf{R}^1$.
- (b) Use the matrix to calculate how many places B can reach.
- (c) Use the matrix to calculate how many places D is reachable from.
- (d) Calculate $\mathbf{R}^2$ and interpret the value of r_{14} from this matrix.
- (e) Calculate the total reachability matrix $\mathbf{R} = \mathbf{R}^1 + \mathbf{R}^2$ and use this matrix to calculate the reachability of A , B , C and D by 1 or 2 steps.



21

- 12 Complete the following questions regarding the communication digraph shown between subjects A , B , C and D .

- (a) Create a matrix $\mathbf{C}^1$ to show the direct path of communication.
- (b) Interpret the communication matrix, stating the number of ways each person passes on and receives communication.
- (c) Calculate $\mathbf{C}^2$.
- (d) Calculate $\mathbf{C}^3$.
- (e) Calculate the single communication matrix $\mathbf{C}$, stating the one-, two- and three-step communication paths.
- (f) Interpret the paths represented by the element c_{12} .



- 13 Communication matrix C^1 shows who passes information to another person in the workplace containing persons A, B, C and D.

$$C^1 = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

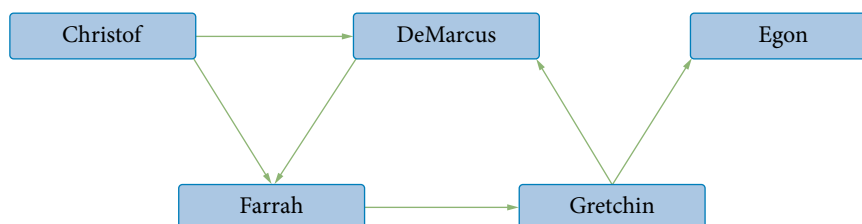
- (a) Determine the number of people to whom person A passes information.
 (b) Determine the number of people who pass information to B.
 (c) Calculate C^2 .
 (d) State the value of each of the following elements of C^2 and interpret the meaning of this value.

(i) c_{13} (ii) c_{23} (iii) c_{43}

- 14 The combined one- and two-step reachability for four vertices is: A 4; B 0; C 3; D 5.

Construct a digraph consistent with these values, and use matrices to confirm your result.

- 15 A take on 'Chinese Whispers' is played, where students select names out of a hat to determine the participants they may share the message with.

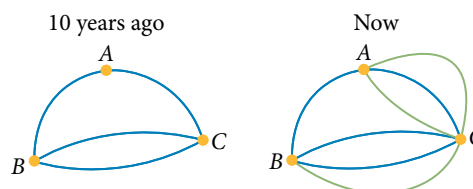


- (a) If you wanted to ensure that every person in the group was given the opportunity to receive the message, who needs to start it?
 (b) Identify the longest chain of communication if each person may only receive the message once.
 (c) Complete the adjacency matrix C^1 for the game of Chinese Whispers.

- (d) Given that $C^4 = \begin{matrix} & \begin{matrix} C & D & E & F & G \end{matrix} \\ \begin{matrix} C \\ D \\ E \\ F \\ G \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$ determine the element that denotes the path in part (a).

- 16 The two diagrams show the roads connecting three towns, 10 years ago and now.

Determine the three matrices for the matrix addition $B = A + X$ where A shows the original situation and B shows the present-day connections.



Summary

Linear and non-linear formulas

A *formula* is a mathematical rule describing the relationship between variable quantities.

Substitution into a formula involves replacing pronumerals with numerical values and simplifying the result.

A formula usually has one variable isolated on one side of the equation. An isolated variable is said to be the *subject* of the equation (or formula).

To rearrange or transpose the terms in a formula, the same operations are performed on both sides of the formula, i.e. whatever is done to one side of the formula must also be done to the other side.

Tables of values

A table of values or spreadsheet consists of a set of input values (independent variable) and a set of resulting output values (dependent variable).

Matrix properties

A *matrix* is a rectangular array of numbers enclosed by square brackets and denoted by a capital, bold letter. A matrix is displayed in the form of rows and columns.

A matrix that has m rows and n columns is described as an $m \times n$ matrix. This is known as the order, or size, or dimension of the matrix.

An element in the i th row and the j th column of a matrix is represented as a_{ij} .

A row matrix has order or size $1 \times n$.

A column matrix has order or size $m \times 1$.

A square matrix is a matrix with the same number of rows and columns.

Two matrices are equal if they have the same order and their corresponding elements are equal.

Matrix addition, subtraction and scalar multiplication

Two matrices can be added or subtracted if they have the same order. Addition and subtraction are done by adding and subtracting the corresponding elements.

$$\text{If } \mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}, \text{ then } \mathbf{A} \pm \mathbf{B} = \begin{bmatrix} a \pm e & b \pm f \\ c \pm g & d \pm h \end{bmatrix}.$$

For matrix addition, order is not important, i.e. $A + B = B + A$.

A *zero matrix* $\mathbf{O}$ is a matrix where each element is zero.

When a matrix is multiplied by a *scalar*, each element in the matrix is multiplied by that scalar. This is known as *scalar multiplication*.

$$\text{If } \mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{ then } k\mathbf{A} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix} \text{ where } k \text{ is a scalar.}$$

Matrix multiplication

Two matrices can be multiplied together if the number of columns in the first matrix equals the number of rows in the second matrix.

If an $m \times n$ matrix is multiplied by an $n \times p$ matrix, the resulting matrix will be of the order $m \times p$, i.e. $[m \times n] \times [n \times p] \rightarrow m \times p$.

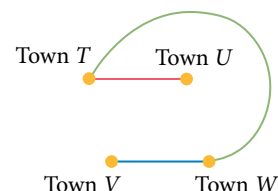
$$\text{If } \mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} e & f \\ g & h \end{bmatrix}, \text{ then } \mathbf{AB} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}.$$

In general, matrix multiplication is not commutative; in other words, it must be done in the order given. So, in general, $\mathbf{AB} \neq \mathbf{BA}$.

$$\text{Identity matrix } \mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ etc. where } \mathbf{IA} = \mathbf{A} \text{ and } \mathbf{AI} = \mathbf{A}.$$

An *incidence matrix* has the element 1 in row i and column j if i and j are related (incident), and the element 0 if they are not.

$$\mathbf{C} = \begin{matrix} & \begin{matrix} T & U & V & W \end{matrix} \\ \begin{matrix} T \\ U \\ V \\ W \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

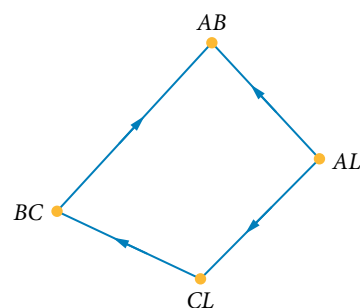


If matrix $\mathbf{C}$ represents the number of direct links between two objects, e.g. the roads linking two towns, then, in general, the matrix $\mathbf{C}^2$ represents the number of links between two objects via one other object.

$$\mathbf{C}^2 = \begin{matrix} & \begin{matrix} T & U & V & W \end{matrix} \\ \begin{matrix} T \\ U \\ V \\ W \end{matrix} & \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix} \end{matrix}$$

The arrows of a reachability digraph and rows of the adjacency matrix describe the path joining two subjects and can be read as 'will reach'. The sum of a column in a reachability matrix describes the number of ways each subject is reachable from another.

If matrix $\mathbf{R}^1$ represents the communication links from one subject to another, then the matrix $\mathbf{R}^2$ represents the communication links from one subject to another via one other subject.



$$\mathbf{R}^1 = \begin{matrix} & \begin{matrix} \text{To} \\ AL & AB & BC & CL \end{matrix} \\ \begin{matrix} \text{From} \\ AL \\ AB \\ BC \\ CL \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

$$\mathbf{R}^2 = \begin{matrix} & \begin{matrix} \text{To} \\ AL & AB & BC & CL \end{matrix} \\ \begin{matrix} \text{From} \\ AL \\ AB \\ BC \\ CL \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

Chapter review

6

Exercise 6.1

- 1 Select the option that shows the equation $y = mx + c$ correctly transposed to make m the subject.

A $m = \frac{y}{c} - x$ B $m = \frac{c - y}{x}$ C $m = \frac{y - c}{x}$ D $m = \frac{x}{y - c}$

Exercise 6.1

- 2 For each of the following, transpose the non-linear formula to make it correct for the new subject.

(a) $E = mc^2$, subject c where $c > 0$ (b) $T = \sqrt{a + b}$, subject b

Exercise 6.1

- 3 For the quadratic equation $y = 10 + 3x - x^2$, calculate the value of y when:

(a) $x = 0$ (b) $x = 5$ (c) $x = -2$

Exercise 6.2

- 4 If $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix}$ what is the size of matrix $\mathbf{A}$?

A 2×2 B 2×4 C 4×2 D 8

Exercise 6.2

- 5 Four pet owners were surveyed to determine the number of dogs (D) and cats (C) owned at the family home. The results are shown in the matrix.

Select the statement that is false.

- A Jenny owns one cat.
B Paul owns an equal number of dogs and cats.
C Allan owns four dogs.
D Kellie owns more cats than dogs.

$$\mathbf{P} = \begin{matrix} & \begin{matrix} D & C \end{matrix} \\ \begin{bmatrix} 3 & 1 \\ 1 & 2 \\ 3 & 3 \\ 1 & 4 \end{bmatrix} & \begin{matrix} \text{Jenny} \\ \text{Kellie} \\ \text{Paul} \\ \text{Allan} \end{matrix} \end{matrix}$$

Exercise 6.2

- 6 The matrix shows the number of roads that connect three towns P , Q and R .

How many roads connect town Q to town R ?

$$\begin{matrix} & \begin{matrix} P & Q & R \end{matrix} \\ \begin{matrix} P \\ Q \\ R \end{matrix} & \begin{bmatrix} 0 & 5 & 2 \\ 5 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix} \end{matrix}$$

Exercise 6.2

- 7 Let $\mathbf{A} = \begin{bmatrix} 2 & -3 & 0 \\ 1 & -4 & 5 \\ -6 & 0 & 1 \end{bmatrix}$.

- (a) State the value of element a_{32} .
(b) Which element is the number 5? State your answer in the form a_{ij} .
(c) State the order of $\mathbf{A}$.

- 8 The table shows monthly rainfall in millimetres for Birdsville, Cairns, Mount Isa and Townsville for the months of January, February and March in a given year.

	January	February	March
Birdsville	0.0	15.0	23.2
Cairns	326.8	116.8	183.6
Mount Isa	7.6	14.6	17.8
Townsville	286.0	83.6	104.6

Exercise 6.2

- (a) Express the information in the table in a matrix $\mathbf{R}$ of order 4×3 . Give headings for the rows and columns.
- (b) Let r_{ij} represent the element in the i th row and the j th column of matrix $\mathbf{R}$.
- (i) Interpret the value of r_{41} . (ii) Interpret the value of r_{12} .
- (c) According to matrix $\mathbf{R}$, how much more rain fell in Cairns over the 3 month period than in Townsville?

- 9 Determine the order of each matrix.

Exercise 6.2

(a) $\begin{bmatrix} 4 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -1 & 3 & 4 & 0 \end{bmatrix}$

10 Let $\mathbf{A} = \begin{bmatrix} 0 & 2 \\ -3 & 1 \\ 2 & 5 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 3 & 4 & 5 \\ 0 & 1 & 2 \end{bmatrix}$, $\mathbf{C} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{D} = \begin{bmatrix} 1 & -3 & 5 \\ 6 & 2 & 3 \end{bmatrix}$ and $\mathbf{E} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

Exercise 6.3

Select the matrix operation that is not possible.

A $\mathbf{B} - \mathbf{D}$

B $\mathbf{E} + \mathbf{C}$

C $2\mathbf{D} - 4\mathbf{B}$

D $\mathbf{D} + \mathbf{A}$

11 If $\mathbf{A} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, calculate $-3\mathbf{A}$.

Exercise 6.3

- 12 Perform the following matrix operations.

Exercise 6.3

(a) $\begin{bmatrix} 1 & 3 & 0 \\ 5 & -2 & 4 \end{bmatrix} + \begin{bmatrix} 9 & 2 & 3 \\ 4 & 1 & -4 \end{bmatrix}$

(b) $3 \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$

- 13 Calculate $\mathbf{A} - \mathbf{B}$ given:

Exercise 6.3

(a) $\mathbf{A} = \begin{bmatrix} 3 \\ 6 \\ 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 5 \\ 9 \\ 0 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} 2 & 4 & 6 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 6 & 4 & 2 \end{bmatrix}$

14 Let $\mathbf{P} = \begin{bmatrix} -2 & 3 \\ 4 & 1 \end{bmatrix}$, $\mathbf{Q} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\mathbf{R} = \begin{bmatrix} -7 & 5 \\ 2 & 0 \\ 1 & -3 \end{bmatrix}$, $\mathbf{S} = \begin{bmatrix} 3 & 2 \end{bmatrix}$ and $\mathbf{T} = \begin{bmatrix} 0 & -2 & 4 \end{bmatrix}$.

Exercise 6.3, 6.4

Determine whether the following can be found and state the solution if it exists.

(a) $\mathbf{PQ}$

(b) $\mathbf{P} + \mathbf{Q}$

(c) $\mathbf{QS}$

(d) $\mathbf{SQ}$

(e) $\mathbf{RQ} - \mathbf{T}$

Exercise 6.4

15 Let $\mathbf{P} = \begin{bmatrix} 2 & 3 \\ -5 & 1 \end{bmatrix}$, $\mathbf{Q} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$ and $\mathbf{R} = \begin{bmatrix} -5 & 0 & 2 \\ 1 & -3 & -4 \end{bmatrix}$.

Determine whether the following can be found, and state the solution if it exists.

(a) $\mathbf{PQ}$ (b) $\mathbf{RQ}$ (c) $\mathbf{PR}$ (d) $\mathbf{RP}$

Exercise 6.4

16 Consider the matrices $\mathbf{A} = \begin{bmatrix} 2 & 5 & -3 \\ 4 & 7 & 1 \\ 2 & -1 & 6 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 3 & 1 \\ -4 & 6 \\ 8 & 2 \end{bmatrix}$.

Complete the following multiplications where $\mathbf{I}$ represents an appropriate identity matrix.

Determine the dimensions of $\mathbf{I}$ and include this in your answer ($\mathbf{I}_2$ or $\mathbf{I}_3$).

(a) $\mathbf{AI}$ (b) $\mathbf{BI}$ (c) $\mathbf{IB}$

Exercise 6.4

17 Apples cost \$5.60 per kilogram, bananas cost \$5.20 per kilogram and pears cost \$6.00 per kilogram.

Julia buys 4.5 kg of apples, 3.5 kg of bananas and 2.5 kg of pears.

Select the matrix product that represents the total cost of this fruit.

A $\begin{bmatrix} 5.60 \\ 5.20 \\ 6.00 \end{bmatrix} \begin{bmatrix} 4.5 \\ 3.5 \\ 2.5 \end{bmatrix}$

B $\begin{bmatrix} 5.60 & 5.20 & 6.00 \end{bmatrix} \begin{bmatrix} 4.5 & 3.5 & 2.5 \end{bmatrix}$

C $\begin{bmatrix} 4.5 \\ 3.5 \\ 2.5 \end{bmatrix} \begin{bmatrix} 5.60 & 5.20 & 6.00 \end{bmatrix}$

D $\begin{bmatrix} 5.60 & 5.20 & 6.00 \end{bmatrix} \begin{bmatrix} 4.5 \\ 3.5 \\ 2.5 \end{bmatrix}$

Exercise 6.4

18 Calculate the following matrix products.

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix}$

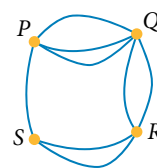
(b) $\begin{bmatrix} 6 & -1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Exercise 6.5

19 The diagram shows the number of roads that connect four towns

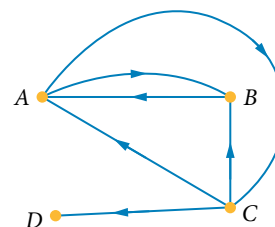
P , Q , R and S .

Construct a matrix to show the number of roads connecting each pair of towns.



Exercise 6.5

20 Create an adjacency matrix $\mathbf{Z}$ for the digraph.



Exercise 6.5

21 Determine the square of each matrix.

(a) $\begin{bmatrix} 0 & 5 \\ 2 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 3 & 2 \\ 1 & 0 & -1 \end{bmatrix}$

- 22 The equation $V = \frac{4}{3}\pi r^3$ is transposed to make r the subject. Select the correct equation.

Exercise 6.1

A $r = \sqrt[3]{\frac{V}{4\pi}}$ B $r = \frac{1}{3}\sqrt[3]{\frac{3V}{4\pi}}$ C $r = \sqrt[3]{\frac{3V}{4\pi}}$ D $r = \sqrt[3]{\frac{3V}{4\pi}}$

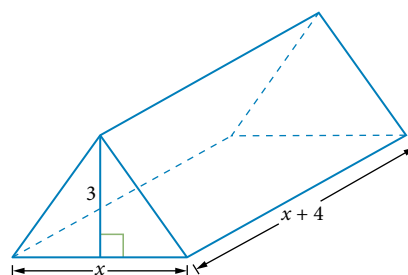
- 23 The perimeter P m of a rectangle of length l m and width w m is given by $P = 2l + 2w$.
Transpose the formula to find the width when the perimeter is 16 m and the length is 2 m.

Exercise 6.1

- 24 A tent in the shape of a triangular prism has volume V m³ and all measurements in metres, as shown in the diagram.

Exercise 6.1

- (a) Create a formula for V in terms of x .
(b) Calculate the volume enclosed by the tent when $x = 8$ m.



- 25 Select the correct answer to $2 \begin{bmatrix} -1 & 3 \\ 2 & 1 \end{bmatrix} + 3 \begin{bmatrix} 0 & -5 \\ 1 & 2 \end{bmatrix}$.

Exercise 6.3

A $\begin{bmatrix} -2 & -9 \\ 7 & 8 \end{bmatrix}$ B $\begin{bmatrix} -3 & -1 \\ 8 & 7 \end{bmatrix}$ C $5 \begin{bmatrix} -1 & -2 \\ 3 & 3 \end{bmatrix}$ D $\begin{bmatrix} -2 & 21 \\ 1 & -4 \end{bmatrix}$

- 26 If $\mathbf{B} = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$ and $\mathbf{C} = \begin{bmatrix} 1 & 3 \\ 5 & 3 \end{bmatrix}$, calculate $3\mathbf{B} - 2\mathbf{C}$.

Exercise 6.3

- 27 The monthly sales figures for men's ties and polo shirts at two department stores for December and January are shown in the table.

Exercise 6.3

	December: Ties	December: Shirts	January: Ties	January: Shirts
Store 1	60	80	40	36
Store 2	48	60	50	24

Let matrix $\mathbf{D}$ represent the December sales and let matrix $\mathbf{J}$ represent the January sales.

- (a) Determine $\mathbf{D} + \mathbf{J}$ and describe what this matrix represents.
(b) Which store had the better sales figures for each of the items for the 2 months combined?
(c) If sales in February for both stores decreased by 50% from January's sales figures, find the following:
(i) a matrix $\mathbf{F}$ that shows February's sales figures for both stores
(ii) a matrix that shows the total sales for December, January and February.
- 28 Maddi scored the following marks for her assessment components (out of 100) in Chemistry: topic tests 80; practical work 86; semester examination 70.
The final semester result for Chemistry is weighted as follows: topic tests 30%; practical work 30%; semester examination 40%.
Use a matrix method to calculate Maddi's final semester result (out of 100) for Chemistry. Give your answer to 1 decimal place.

Exercise 6.4

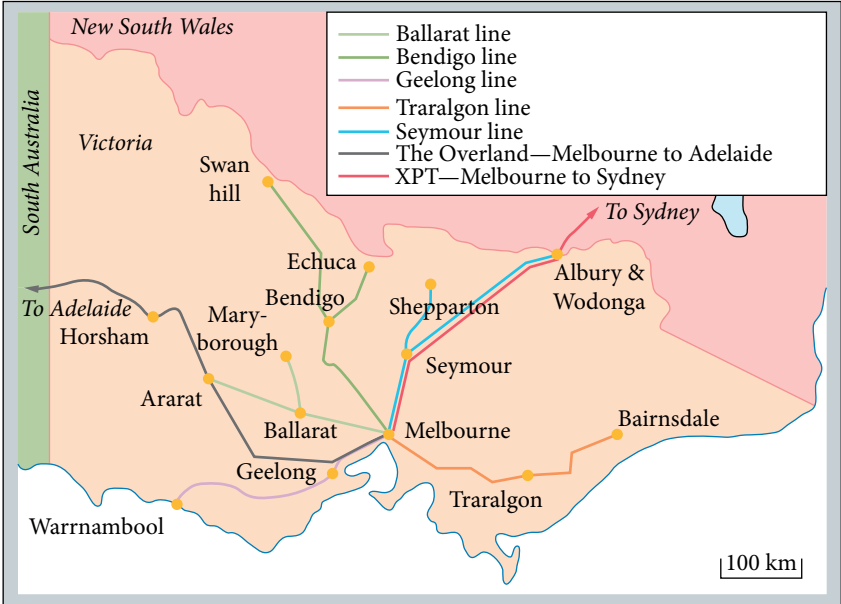
Exercise 6.4

- 29 The matrix C displays direct air routes between A, B, C and D .
- (a) Calculate C^2 .
 - (b) What does matrix C^2 represent?
 - (c) Describe both one-stopover routes from B to C .

$$C = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

Exercise 6.5

- 30 Jonas and Lily are visiting the state of Victoria. They wish to see a few of the western cities in the first week of their stay. They choose to follow the Geelong and Ballarat lines for this week. The map of Victorian train lines is shown.



- (a) Complete the Victorian train line adjacency matrix V for the Ballarat and Geelong lines. (Note: M = Melbourne, m = Maryborough)
- (b) Use technology to calculate V^3 .

- (c) Lily and Jonas wish to end up in Maryborough, the birthplace of NBA player Matthew Dellavedova, in exactly three train rides. Identify the town/s at which they should begin their journey.

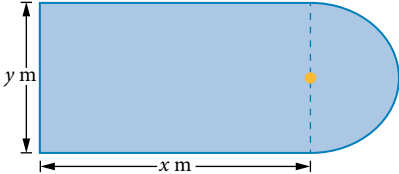
- A** Ballarat **B** Geelong
- C** Melbourne **D** Warrnambool

$$V = \begin{matrix} & \begin{matrix} A & B & G & m & M & W \end{matrix} \\ \begin{matrix} A \\ B \\ G \\ m \\ M \\ W \end{matrix} & \begin{bmatrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{bmatrix} \end{matrix}$$

- (d) From part (c), identify the route that will allow them to see the most of Victoria.
- (e) Calculate V , the one, two and three train stop reachability matrix for the western Victoria lines.
- (f) Determine the reachability of each city along these lines, in fewer than four stops.

Exercise 6.1

- 31 A swimming pool in the shape of a rectangle and a semicircle combined is shown. The rectangular part of the swimming pool has length x m and width y m.



- (a) Determine an appropriate formula for A , the area of the swimming pool, in m^2 .
- (b) The swimming pool has volume V m^3 and a uniform depth of d m. Determine, in terms of d, x and y , a formula for the volume V of water in the swimming pool.
- (c) A pump fills the swimming pool with water at a rate of r L per min. Determine, in terms of r, d, x and y , a formula for the time t min taken to fill the swimming pool.

- (d) The swimming pool has length 50 m, width 15 m and uniform depth 2 m, and water is pumped into it at a rate of 3000 L per min. Calculate, to the nearest minute, the time it takes to fill the swimming pool.

- 32 A triangular number is any number found from the first n terms of the sequence $1 + 2 + 3 + \dots$. The n th triangular number T_n is given by the formula $T_n = \frac{n}{2}(n + 1)$

Exercise 6.1

- (a) Use a spreadsheet or similar technology to determine the first five triangular numbers.
 (b) Use a spreadsheet or similar technology to calculate the 10th, 25th, 50th and 100th triangular numbers.
 (c) If the n th triangular number is 300, find the value of n .
 (d) Calculate the values of $T_1 + T_2$, $T_2 + T_3$ and $T_3 + T_4$. What do you notice about the results obtained? Determine a formula for $T_n + T_{n+1}$.

- 33 A convenience store sells water in three different sized plastic bottles. A 300 mL bottle sells for \$1.50, a 600 mL bottle sells for \$2.50 and a 125 L bottle sells for \$3.50.

Exercise 6.4

- (a) Express these prices in a column matrix **S**.
 (b) In a single day, the convenience store sells thirty 300 mL bottles, twenty 600 mL bottles and ten 125 L bottles.

Express these sales in a row matrix **N**.

- (c) Calculate the product matrix **NS** and interpret the result.

- (d) The table displays the number of 300 mL, 600 mL and 125 L bottles of water sold at four convenience stores in a single day.

	Store A	Store B	Store C	Store D
300 mL bottle	35	40	60	75
600 mL bottle	30	60	75	50
1.25 L bottle	10	15	20	30

Express this information in a 4×3 matrix **T**.

- (e) Calculate the matrix product **TS** and interpret the result.
 (f) Identify the convenience store that made the most money from sales.
 (g) In a week's time, the selling price of each size of bottled water will rise by 25c per bottle. Determine the new selling price matrix **S**.
 (h) Determine how much each convenience store would make on sales of bottled water in a single day if they sold the same amount in a week's time. Give your answer as a matrix.
 (i) Calculate the increase in the amount of money each convenience store would make on sales in a single day if they sold the same amount of bottled water in a week's time.

- 34 The matrix **C** displays whether or not a pair of towns are connected directly by a road. The towns are labelled P , Q , R , S and T .

Exercise 6.5

- (a) Calculate and interpret matrix $\mathbf{C}^2$.
 (b) Determine the number of road links between town R and town T via another town, and also determine which town this is.
 (c) Identify the ways a driver could travel from town R and back to town R via another town.
 (d) Calculate and interpret matrix $\mathbf{C}^3$.
 (e) State how a driver could travel from town R to town P via two other towns.

$$\mathbf{C} = \begin{matrix} & \begin{matrix} P & Q & R & S & T \end{matrix} \\ \begin{matrix} P \\ Q \\ R \\ S \\ T \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$$





7

Univariate data analysis

• • • • •

Recall	400
7.1 Categorising and organising data	401
7.2 Categorical data	409
7.3 Numerical data	417
7.4 Median, range and interquartile range	429
7.5 Cumulative frequency	443
7.6 Mean and standard deviation	454
7.7 Comparing data	466
Summary	476
Chapter review	479

Recall

Calculate a percentage of the whole amount

- 1 For each of the following lists, calculate the first quantity as a percentage of the sum of the data. Express your answer to the nearest per cent.

(a) 12, 20, 25 (b) 290, 430, 120, 650, 200, 150 (c) 43.5, 27.6, 33.3, 27.0

Calculate the mean of a simple data set

- 2 For each of the following data sets, determine the mean, to 1 decimal place where necessary.

(a) 1, 2, 3, 3, 4, 5, 6 (b) 2, 2, 2, 3, 3, 4, 4, 4, 5, 5 (c) 10, 11, 22, 19, 12, 18, 15, 11

Calculate the median of a simple data set

- 3 For each of the following data sets, determine the median.

(a) 8, 9, 11, 11, 15, 18, 18 (b) 15, 15, 15, 21, 21, 22, 23, 23, 23, 25
(c) 5, 15, 22, 35, 19, 27, 25, 15

Identify the mode of a simple data set

- 4 For each of the following data sets, determine the mode.

(a) 51, 53, 56, 56, 58, 60, 66 (b) 9, 9, 9, 10, 10, 11, 11, 11, 12, 13 (c) 17, 18, 29, 26, 19, 22, 18

Calculate the range of a simple data set

- 5 Calculate the range of each of the following data sets.

(a) 3, 6, 8, 9, 11, 15, 19, 22, 24 (b) 5, 7, 7, 8, 9, 9, 10, 11, 13, 15
(c) 4, 8, 12, 13, 16, 23, 15, 19, 17, 16

Draw a pie graph from a spreadsheet

- 6 The data in the table shows the number of students in a class who travelled to school by different methods of transport. Use a spreadsheet to display the data as a pie graph.

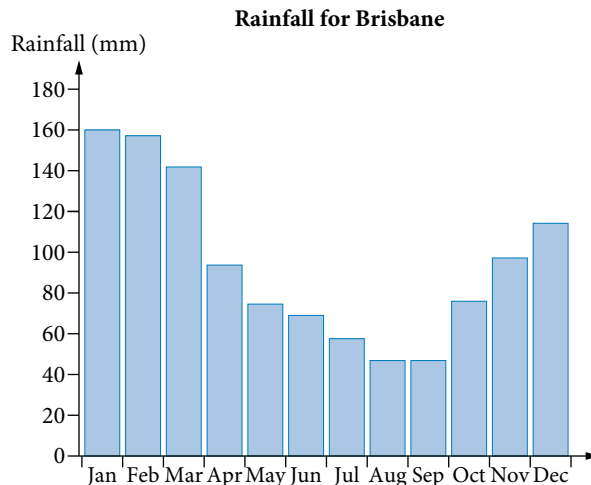
Walk	Bicycle	Car	Bus	Train
3	3	15	4	5

Interpret a bar or column chart

- 7 The graph shows the average monthly rainfall for Brisbane.

- (a) On average, which is the wettest month?
(b) On average, which is the driest month?
(c) Which season would be called the wet season?
(d) The rainfall for the first four months of 2017 is shown in the following table.

Month	Rainfall (mm)
Jan	218.4
Feb	30.6
Mar	297.6
Apr	5.8



How does the rainfall in these months compare to the first four months of average rainfall?

Categorising and organising data

Univariate data

Data is information, most often raw facts, that can be collected and/or measured.

Univariate data is the data collected from a single variable. The purpose of univariate analysis is to use the frequency of each group of data to describe, find patterns and summarise. The data comes in many different data types. For example, the univariate data that you might collect about class members could be their eye colour, arm span, whether they are left- or right-handed or how many siblings they have.

Types of data

There are two main types of univariate data: categorical and numerical.

Categorical data

Categorical data is data that can be grouped into categories. Examples are hair colour and grading on a report. There are two types of categorical data: nominal and ordinal.

Nominal data

Nominal data has no special order associated with the possible responses. Examples are hair colour, type of pet owned and gender.

☐ Mr ☐ Mrs ☐ Ms ☐ Master ☐ Miss

Ordinal data

Ordinal data does have some sort of order associated with it. Examples are ratings of service, behavioural grades on a report card, and rating your happiness or the likelihood of an event.

☐ Impossible ☐ Unlikely ☐ Even chance ☐ Likely ☐ Certain

Numerical data

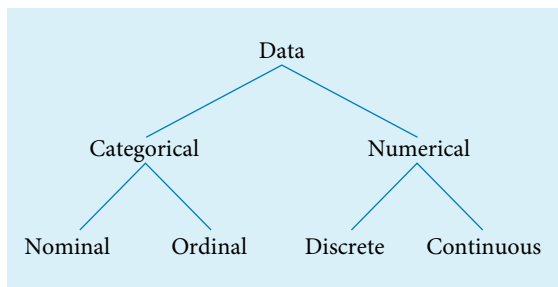
Numerical data has values that can be counted or measured. Examples are number of siblings and height. There are two types of numerical data: discrete and continuous.

Discrete data

Discrete data has a countable or finite number of numerical values. The values cannot be subdivided and don't have to be whole numbers. Examples are the number of brothers and sisters, or the age of your teacher.

Continuous data

Continuous data has an infinite number of possibilities within a particular range. This type of data is measured using some sort of instrument. The accuracy with which you record the data may vary but this does not alter the fact that there are an infinite number of possible values. Examples are the heights of a group of people or the times taken to complete a task.



i Additional information

Classifying data

Watch the videos and try the activities to revise your knowledge about different variable types.

1 Data types

Decide whether these data samples are ordinal or nominal (categorical), or discrete or continuous (numerical).

- (a) The number of students present in your mathematics class each lesson over a 3 week period.

THINKING

- 1 Identify whether the data is categorical or numerical.
- 2 Will the data be collected by counting (discrete) or measuring (continuous)?

WORKING

The data is numerical because the data recorded will be a numerical value.

The data can only be whole numbers. To collect the data, the number of students will be counted. The data is discrete.

- (b) The favourite music style for each student in your mathematics class.

- 1 Identify whether the data is categorical or numerical.
- 2 Is the order of the data important? If yes, the data is ordinal, otherwise it is nominal.

As numbers are not involved, the data is categorical. Answers may include the categories: classical, rock, rap, jazz.

The data is nominal.

- (c) The team level for basketball players (level A, B, C or D).

- 1 Identify whether the data is categorical or numerical.
- 2 Is the order of the data important? If yes, the data is ordinal, otherwise it is nominal.

As numbers are not involved, the data is categorical.

The data is ordinal, as the levels will be based on ability or age and players will progress through them accordingly.

- (d) The weights of newborn kittens.

- 1 Identify whether the data is categorical or numerical.
- 2 Will the data be collected by counting (discrete) or measuring (continuous)?

The data is numerical because the data recorded will be a numerical value.

The data can have an infinite number of values, even though you have to approximate according to the accuracy of the scales used. To collect the data, the weights will be measured. The data is continuous.

Organising data

Raw data of most categories can be organised into a frequency table. Frequencies enable comparison between the amounts of each data or data category. Relative frequencies can be expressed as fractions, decimals or percentages of the total frequency, and enable more definitive comparisons. Entering the data into a spreadsheet will enable you to calculate relative frequencies more efficiently, or you can simply choose to display the percentages on a pie chart.

2 Frequency tables

The following data gives the number of various types of birds observed drinking at a bird bath in a suburban park.

crow, crow, crow, magpie, magpie, crow, magpie, magpie, koel, koel, dove, dove, crow, crow, miner, miner, miner, magpie, magpie

(a) Organise the data into a frequency table.

THINKING

Create a table with each category listed in the first column (in any order) and its frequency in the second column. Tally or count the number of each kind of bird.

WORKING

Kind of bird	Frequency
crow	6
magpie	6
koel	2
dove	2
miner	3

(b) Determine the relative frequencies, as decimals correct to 2 decimal places, for each kind of bird.

1 Calculate the total frequency and then express each frequency as a fraction of the total frequency and convert to a decimal.

Kind of bird	Frequency	Relative frequency
crow	6	$\frac{6}{19} = 0.32$ (2 d.p.)
magpie	6	$\frac{6}{19} = 0.32$ (2 d.p.)
koel	2	$\frac{2}{19} = 0.11$ (2 d.p.)
dove	2	$\frac{2}{19} = 0.11$ (2 d.p.)
miner	3	$\frac{3}{19} = 0.16$ (2 d.p.)
Total	19	$1.02 \approx 1$

2 Check that the total relative frequency is 1, allowing for rounding error.

Here, all values are rounded up, so it makes sense that the rounded relative frequency is slightly higher than 1.

(c) Determine the percentage frequency of each bird, to the nearest percentage.

1 Determine the percentage frequencies by multiplying the relative frequency by 100%. Add an additional column to the frequency table. Check that the total percentage frequency (the sum) is 100%, allowing for rounding error.

Kind of bird	Frequency	Relative frequency	Percentage frequency
crow	6	0.32	$0.32 \times 100 = 32\%$
magpie	6	0.32	$0.32 \times 100 = 32\%$
koel	2	0.11	$0.11 \times 100 = 11\%$
dove	2	0.11	$0.11 \times 100 = 11\%$
miner	3	0.16	$0.16 \times 100 = 16\%$
Total	19	$1.02 \approx 1$	$32 + 32 + 11 + 11 + 16 = 102\%$

The error is from the rounding that occurred to determine the relative frequency. The error could be reduced by rounding to a higher degree of accuracy or stating exact answers (fractions).

(d) Use technology to construct a pie graph of the types of birds, showing percentages.

Enter the original table values into an Excel spreadsheet. Insert a pie chart. Select a style that displays percentages.

Kind of bird at a water fountain

Kind of bird	Percentage frequency
crow	32%
magpie	32%
miner	16%
koel	10%
dove	10%

Explore further

Frequency tables

Create frequency tables and graphs with an Excel spreadsheet.

EXERCISE

7.1

Categorising and organising data

Worked Example

1

- 1 Decide whether these data examples are categorical or numerical.
- (a) The amount of money carried by each student in your mathematics class on a particular day
 - (b) The favourite chocolate truffle flavours, selected from a sample provided, for 100 people chosen at random in a shopping centre
 - (c) The hair colour of the teachers at your school
 - (d) The ages, in years, of the parents of the students in your class
 - (e) The individual diameter of each orange in a fruit bowl
 - (f) The size of running shoes worn by each competitor in the 100 m race at the last Olympic Games

- 2 Decide whether these categorical data examples are nominal or ordinal.
 - (a) The favourite radio station of each member of your extended family
☐ B105 ☐ 4MMM ☐ NOVA ☐ JJJ ☐ ABC612
 - (b) The degree of support for the new jumper design for your local sporting team as recorded by the members of the club
☐ Love it ☐ It's OK ☐ Unsure ☐ Don't really like it ☐ No way
 - (c) The starting letter of the number plate for each vehicle in the staff car park at your school
 - (d) The makes of cars in a used-car lot
 - (e) The brand of oil used by a sample of masseurs
 - (f) The rating on a report card: A, B, C, etc
- 3 Decide whether these numerical data examples are continuous or discrete.
 - (a) The volume of milk taken from each cow in a herd of dairy cattle
 - (b) The number of brands of clothing available in a shopping centre
 - (c) The number of spectators at each of the Brisbane Lions home games
 - (d) The distance thrown by each of the competitors in the Olympic shot put for women
 - (e) The weight of fruit taken from each individual tree in an orchard
 - (f) The distance between consecutive stations on the exercise circuit in the local park
- 4 Decide whether these examples are nominal, ordinal, discrete or continuous.
 - (a) The strength of agreement with a number of statements is recorded using a scale of 1 to 5, where 1 represents strongly disagree and 5 represents strongly agree.
 - (b) The height of the high tide at Sydney Heads is recorded for 30 consecutive days.
 - (c) The number of brothers and sisters of each member of the teaching staff at your school is recorded.
- 5 Organise each of the following sets of categorical data into a frequency table.
 - (a) Colour chosen for plastic lunchboxes: blue, red, blue, yellow, yellow, yellow, yellow, blue, white, yellow, yellow, white, red, blue, red, white, yellow, red, white, white, red
 - (b) Favourite weekend viewing: sport, sport, movies, sport, drama, comedy, drama, comedy, movies, movies, sport, drama, drama, comedy, drama, sport, movies, movies, movies, sport, sport, movies, movies
 - (c) Main family car: Hyundai, Mazda, Mazda, VW, Toyota, Ford, Holden, Ford, Holden, Toyota, Holden, Hyundai, Toyota, Ford, Toyota, Holden, Mazda, VW, VW, Holden, Ford, Holden, Ford, Holden
- 6 Organise each of the following sets of discrete numerical data into a frequency table. For each set of data collected, determine the number of people in the survey.
 - (a) Number of questions completed in a lesson: 3, 3, 3, 4, 5, 3, 5, 4, 5, 3, 3, 5, 3, 5, 4, 6, 6, 4, 5, 3, 4, 3
 - (b) Number of brothers: 0, 5, 0, 1, 1, 0, 0, 2, 3, 0, 0, 0, 2, 1, 1, 2, 0, 2, 0, 0, 0, 1, 1, 0

- 7 Determine the percentage frequencies, to the nearest whole percentage, for each of the frequency tables.

- (a) Responses to the need for more covered parking at the shopping mall

Opinion	Frequency
Strongly agree	53
Agree	142
Neutral	68
Disagree	35
Strongly disagree	7

- (b) Students using the library during lunch break on a particular day

Year level	Frequency
7	10
8	15
9	5
10	20
11	18
12	6

- (c) The five most popular lunch choices at the student cafe on a particular day

Lunch choice	Frequency
Sandwich	35
Pie	20
Pizza	15
Wrap	12
Salad	10

- 8 Determine the relative frequencies, as decimals to 2 decimal places, for each of the frequency tables.

- (a) Year 11 girls' shoe sizes for a school

Shoe size	Frequency
6	20
7	37
8	40
9	15
10	9
11	6

- (b) Initial opinion poll for a new flag

Alternative flag	Frequency
Choice 1	5 268
Choice 2	372 635
Choice 3	23 591
Choice 4	93
Choice 5	824

- (c) Times for the open boys 100 m swim at a school carnival

Time (seconds)	Frequency
60–69.99	2
70–79.99	5
80–89.99	20
90–99.99	14
100–109.99	1



9 Consider the frequency table shown here.

(a) Choose the false statement.

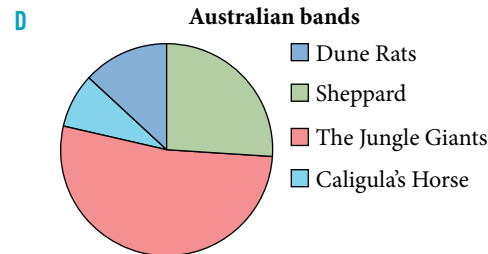
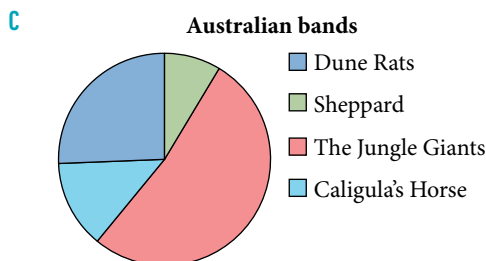
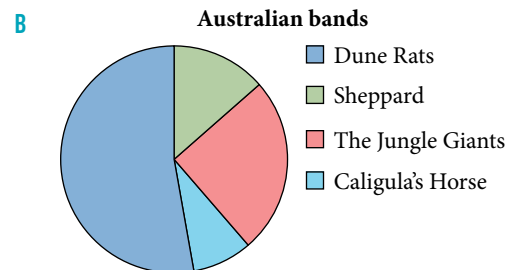
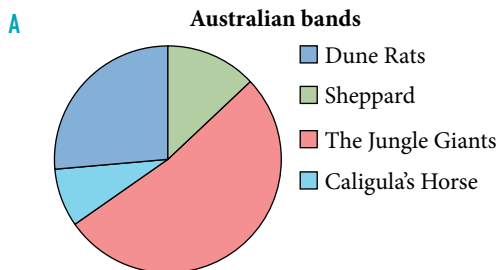
- A The frequency of 'kangaroo' is 25.
- B The relative frequency of 'cockatoo' is about 30%.
- C The relative frequency of 'goanna' is about 31%.
- D The frequency of 'cockatoo' is 30.

Australian wildlife	Frequency
Kangaroo	25
Goanna	25
Cockatoo	30

(b) Explain the common error made by a student who chose the last incorrect option as the false statement.

10 Determine which of the pie graphs matches the frequency table on a vote for the most popular Australian band.

Australian band	Frequency
Sheppard	20
The Jungle Giants	80
Caligula's Horse	13
Dune Rats	40



11 During the club Best and Fairest awards ceremony, finals votes were given by different tables at the celebratory dinner. These are shown in the table.

Create a frequency table to show the number of votes for each player, and use this information to determine which player was voted Best and Fairest in the finals.

Name	Table 1	Table 2	Table 3	Table 4
Arlo	3		3	1
Denji		2		2
Praveen	2	3		
Jamie	1	1	1	3
Kendall			2	

12 Design a 'tick the box' survey to collect the following types of data about an issue relevant to the school, such as a new school lunch menu or uniform.

- (a) nominal data
- (b) ordinal data

2

- 13 The following data gives the number of particular colours of cars observed in a shopping centre car park.
- black, black, white, yellow, blue, grey, white, grey, red, green, white, blue, red, red, white, grey, black, red, black, black, blue, black, white, blue, red, red, black, black, white, grey, white, white, black, red, white, blue, black, grey, white, black, white, white, green, grey, red, grey, white, red, blue, white, black, red, grey.
- (a) Organise the data into a frequency table.
 - (b) Determine the relative frequencies, as decimals to 2 decimal places, for each colour.
 - (c) Use technology to draw a pie graph of the car colours, showing percentages.

- 14 Entries to a local sports stadium are tallied using a tally counter, to get an estimate of the number of people likely to attend a planned twilight match.
- A member of the administration team stands at each of four gates and uses a tally counter to determine attendance numbers. The results are shown in the table.

Gate	Number of people
A	741
B	238
C	591
D	650



- (a) Calculate the percentage of spectators entering the venue through each gate. Give your answers to the nearest per cent.
- (b) The venue has a maximum capacity of 8500 spectators, which generally occurs for finals matches. If the proportion of people entering each gate remains the same, determine the anticipated number of spectators entering through each gate. Use the rounded values from (a) and round these numbers to the nearest hundred.
- (c) Use your estimates from (b) to determine the anticipated revenue for a sell-out game, where Gates A, C and D are general admission tickets selling for an average price of \$28 and Gate B is the VIP entrance to the front tier, with tickets selling for an average price of \$72.

- 15 The spectators at a school soccer game were surveyed and the data tallied to produce the frequency table shown.
- The sports captains have proposed a scheme whereby students score points for their 'house' by attending the soccer games. Their target is for 80% of the spectators to be students from the school.

Type of spectator	Frequency
Related to a player	16
Injured player	2
Friend of a player	8
Other student from school	1

- Determine the number of extra students the scheme must attract to the next game in order to meet the target. Explain any assumptions you had to make to answer the question.
- 16 Students at a school put their names on a list for an excursion. The choice was between Dreamworld and Wet and Wild. The numbers so far are given in the two-way frequency table.
- Another 30 girls said they would go, but didn't mind which venue they went to.
- Allocate the 30 girls to the venues so that the relative frequency of girls is the same for each venue.

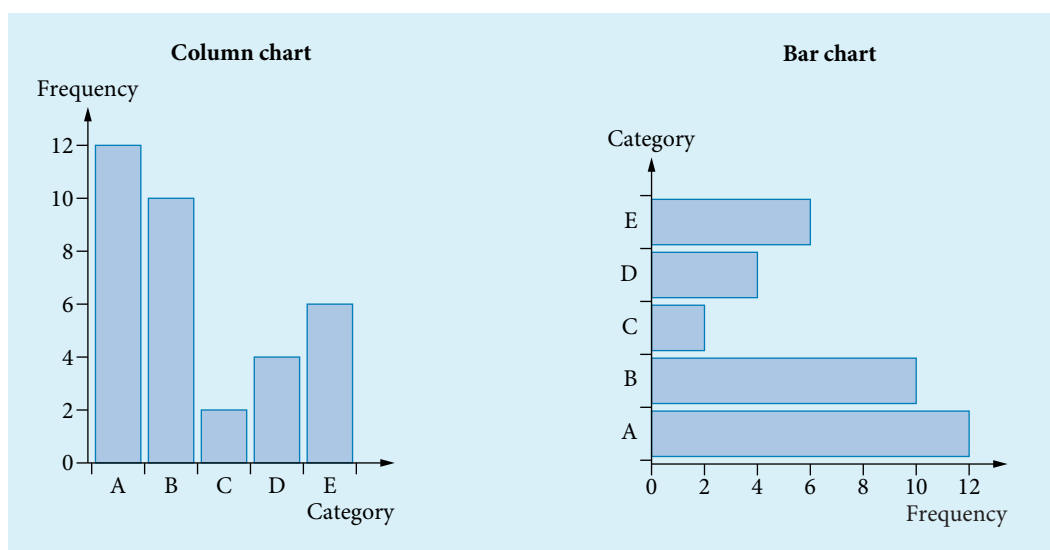
	Dreamworld	Wet and Wild
Boys	32	45
Girls	15	20

Displaying categorical data

All types of data can be organised into frequency tables. Categorical data, either ordinal or nominal, can be displayed as a column or bar chart; for a small number of data, a dot plot can be used. Pie charts are especially good for displaying nominal data.

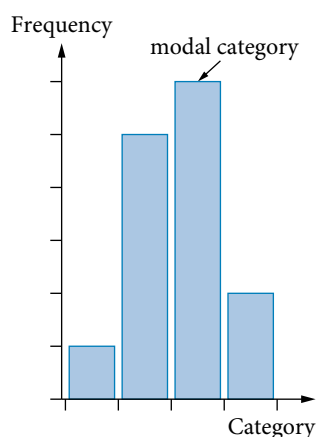
Column and bar charts

A *bar chart* is used to display categorical information. Values are read from the graph, using the frequency scale. Bar charts can have vertical or horizontal bars, which are spaced evenly with a gap between each of the categories. When vertical bars are used, the graph is sometimes called a column chart.

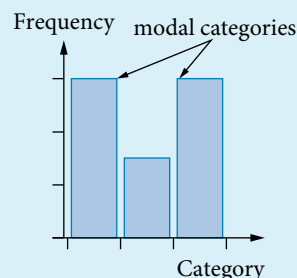


The *mode* (the category with the highest frequency) is easily identified from the graph.

For nominal data, the categories can be arranged in any order, so there is no significance to any symmetry the graph may have. Conversely, ordinal data is arranged in ascending or descending order.



If there are two categories with the highest frequency, the data is said to be bimodal and both categories are listed as modal categories.



3 Constructing a bar chart

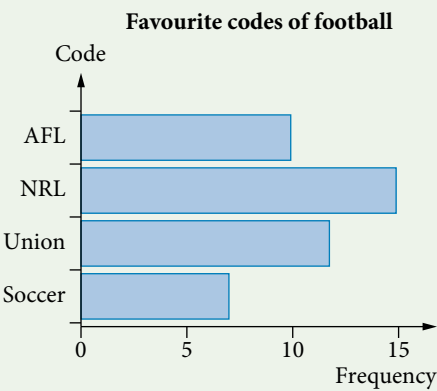
Construct a bar chart to represent the following data, which was collected in a survey asking for respondents' favourite code of football. Determine the modal category.

Football code	Frequency
AFL	10
NRL	15
Union	12
Soccer	7

THINKING

- 1 Draw and label the bar chart, remembering to leave a gap between the categories.
- Note:* The categories can be displayed in any order because the data is nominal.

WORKING



- 2 Identify the modal category and interpret the meaning.

For this group of people, NRL is the modal category. More people chose NRL as their favourite than any other code of football.

Explore further

Bar and column charts

Create bar and column charts to display data.

4 Constructing a column chart

Construct a column chart to represent the following data, which was collected using the survey statement: 'I prepared well for my assessment.'

Strongly agree	Agree	Unsure	Disagree	Strongly disagree
57	32	19	12	1

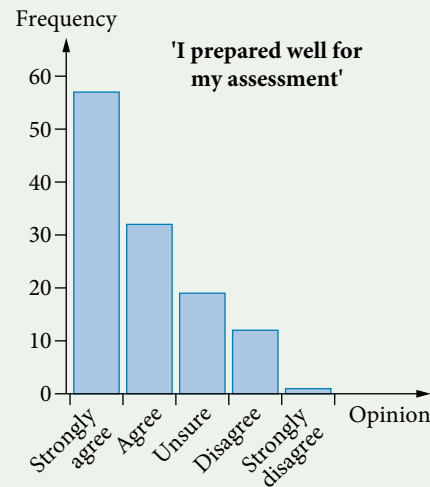
THINKING

Construct a column chart with the categories listed across the horizontal axis and the frequency on the vertical axis.

Remember to leave a space between each category. If the data is ordinal, list the categories in order.

WORKING

The frequency will run from 0 to 60.



Dot plots

A *dot plot* can be used to display categorical data. The categories are written along the horizontal axis, and the dots are arranged vertically to represent the frequency of each category.

5 Drawing a dot plot

Draw a dot plot to represent the country of birth for a group of friends. Use your graph to identify the mode.

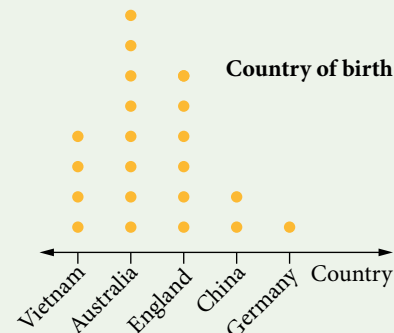
Score	Frequency
Vietnam	4
Australia	8
England	6
China	2
Germany	1
Total	21

THINKING

- 1 Draw a horizontal line marked with the values from the 'Score' column.

Plot, in the appropriate places, the number of dots represented by the 'Frequency' value.

WORKING



2 Identify the modal category (the most frequently occurring score) and interpret the result.	The highest number of dots are plotted above the Australia category, with a frequency of 8. From the group of friends, Australia is the most common birthplace.

A dot plot should not be used if there are too many values. It would be best to limit this type of display to no more than 10 categories, with individual frequencies of no more than 10.

If you are working with raw data, there is no need to draw a frequency table for a dot plot. Simply place a dot above the data value for each occurrence of the score or datum.

Pie charts

A *pie chart* is a circular graph where each category is represented by a sector demonstrating the proportion of a whole. Pie charts are not suitable for displaying ordinal data, as there is no way of presenting the order on the chart.

To construct a pie chart by hand, calculate the angle for each sector as a proportion of the total central angle of 360° .

6 Drawing a pie chart

Draw pie charts, by hand, to represent each of the following.

- (a) The proportion of each type of school in Queensland, where 66% are primary schools, 16% are combined primary/secondary schools, 14% are secondary schools and 4% are special schools.

THINKING

1

Create a relative frequency table by dividing each frequency by the total.

2

Calculate the angle of each sector by multiplying the relative frequency (portion of the whole) by 360° .

WORKING

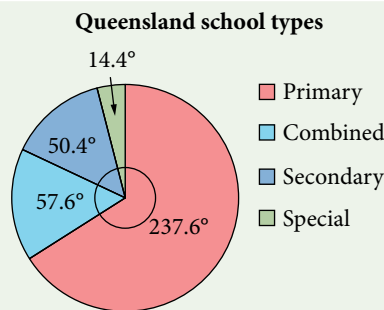
Type of school	Percentage frequency	Relative frequency
Primary	66%	0.66
Combined	16%	0.16
Secondary	14%	0.14
Special	4%	0.04

Type of school	Percentage frequency	Relative frequency	Angle
Primary	66%	0.66	$0.66 \times 360 = 237.6^\circ$
Combined	16%	0.16	$0.16 \times 360 = 57.6^\circ$
Secondary	14%	0.14	$0.14 \times 360 = 50.4^\circ$
Special	4%	0.04	$0.04 \times 360 = 14.4^\circ$

- 3 Draw a circle to represent the pie chart and mark the centre.

Draw a vertical line from the centre, then use a protractor to measure descending angles in a clockwise direction.

Add a legend or labels.



- (b) The proportions of ingredients in a cereal, if a 40 g cereal serving has 20.8 g of grains, 7.2 g of almonds, 6.4 g of shredded coconut, 4 g of cranberries, 1.6 g of other (honey, oil and sugar).

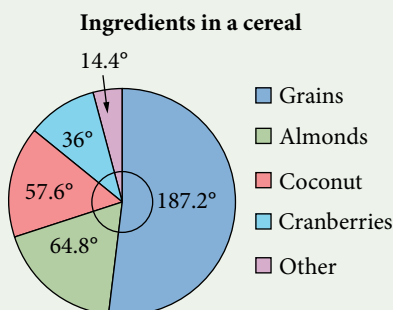
- 1 Create a relative frequency table by dividing each mass by the total and then calculate the angle of each sector by multiplying the proportion by 360° .

Ingredient	Mass (g)	Proportion	Angle
Grains	20.8	$\frac{20.8}{40} = 0.52$	$0.52 \times 360 = 187.2^\circ$
Almonds	7.2	$\frac{7.2}{40} = 0.18$	$0.18 \times 360 = 64.8^\circ$
Coconut	6.4	$\frac{6.4}{40} = 0.16$	$0.16 \times 360 = 57.6^\circ$
Cranberries	4	$\frac{4}{40} = 0.10$	$0.10 \times 360 = 36^\circ$
Other	1.6	$\frac{1.6}{40} = 0.04$	$0.04 \times 360 = 14.4^\circ$
TOTAL	40	1	360°

- 2 Draw a circle to represent the pie chart and mark the centre.

Draw a vertical line from the centre to the top of the circle.

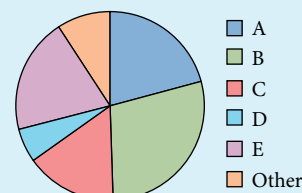
Use a protractor to measure the angles in descending order in a clockwise direction.



Pie charts are a way of summarising a set of categorical data where all possible categories are included. Categories are represented by sectors of the chart. To calculate the size of a sector:

$$\text{Central angle} = \text{proportion} \times 360^\circ$$

If there are many sectors with small frequencies, combine them into a single category called 'Other'.



Explore further

Pie charts

Use an Excel spreadsheet to make a pie chart to demonstrate the bone composition in the human body.

EXERCISE

7.2

Categorical data

Worked
Example

3

- 1 For the following data sets, construct a bar chart and identify the modal category.

(a)

Colour	Frequency
Red	8
Blue	8
Green	7
Yellow	7

(b)

Colour of hair	Frequency
Blond	9
Brown	10
Red	6

- 2 Consider the following data, which shows the types of cars parked in a school carpark.

Holden, BMW, Mazda, BMW, Holden, Ford, Ford, Ford, Holden, BMW, Mazda, Mazda, Holden, Ford, Holden, Ford, Ford, Ford, Mazda, BMW, Holden, Ford, Ford, Ford

(a) Construct a frequency table and a column chart to represent the set of data.

(b) Identify the modal category.

4

- 3 The satisfaction of customers with their newspaper delivery service was rated as shown. Construct a column graph to display the data.

Satisfaction level	Frequency
Very satisfied	152
Satisfied	90
Neutral	10
Unsatisfied	57
Very unsatisfied	10

- 4 Patients brought into the emergency room at a local hospital are analysed in triage and their degree of illness is assessed. The results for a particular night are summarised in the frequency table.

Severity of illness	Minor	Moderate	Severe	Acute
Frequency	15	10	4	1

(a) Construct a bar chart to display the data.

(b) Determine the percentage of patients who came into the emergency room with illnesses more severe than 'minor'.

5

- 5 In a Year 11 class, 7 students spent their holidays in Qld, 4 in NSW, 3 in Vic, 5 in WA, 3 in Tas, 1 overseas, 1 in NT and 2 in SA. Summarise the data as a dot plot and state the modal destination.

- 6 Construct a dot plot for a class that owns 4 guinea pigs, 8 cats, 7 dogs, 1 horse and 3 budgies, and comment on the modal category.

- 7 Create a dot plot to show the popularity of four movie snacks. Determine the modal snack.

9 nachos 12 popcorn 5 choc-tops 2 pretzels

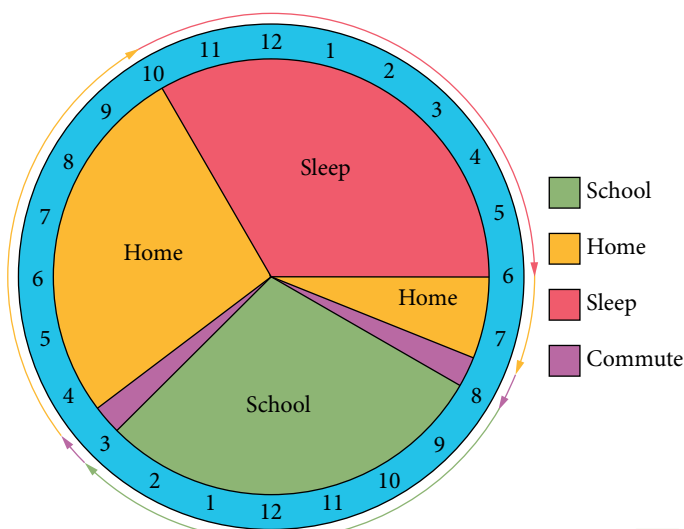
- 8 The diagram represents the way in which a teenager might spend a typical school day.

(a) The sector angle for 'school' would be:

- A 80° B 85°
C 95° D 105°

(b) The two angles for 'home' would be:

- A 22.5° and 97.5°
B 30.5° and 90°
C 15° and 65°
D 22.5° and 65°

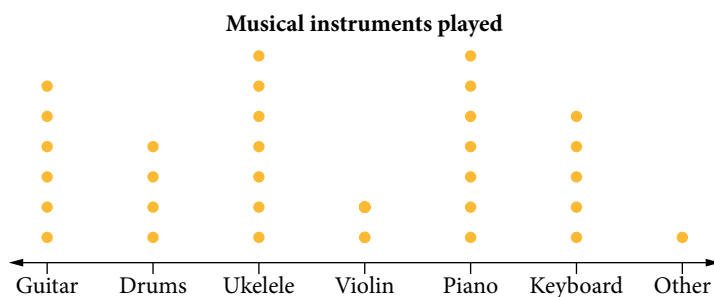


Worked
Example

- 9 The musical instruments played by 20 musicians are displayed in the dot plot.

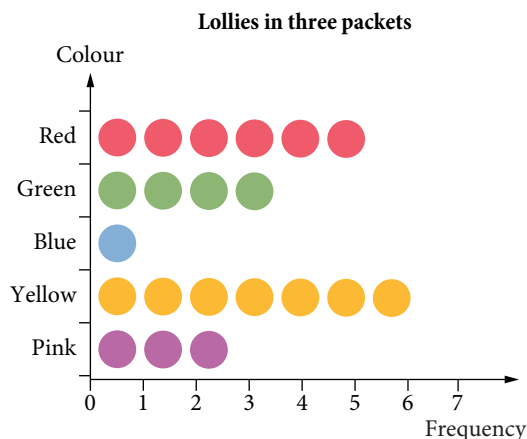
(a) Summarise the information given and explain why the data is ideal for presentation in a pie graph.

(b) Construct a pie graph by hand, showing the angles rounded to the nearest degree.



- 10 Three students empty their packets of lollies to create the bar graph shown.

- (a) Estimate the number of lollies present in each packet.
(b) Each of 22 students in the class has a similar packet of lollies. How many of each colour would you expect there to be?
(c) If a student picks a lolly at random, what is the probability of choosing a green one?



- 11 The pie graph shows the proportion of students in three sub-schools of a P-12 school.

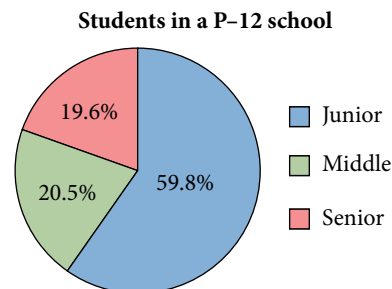
(a) The sector angle for the Senior School will be closest to:

- A 20° B 70°
C 80° D 120°

(b) Explain the error made by a student who chose the first incorrect option.

(c) If the total enrolment of the school is 1345, determine the number of students in the Junior School.

(d) If the Middle School enrolment is 416, determine the number of students in the Senior School.



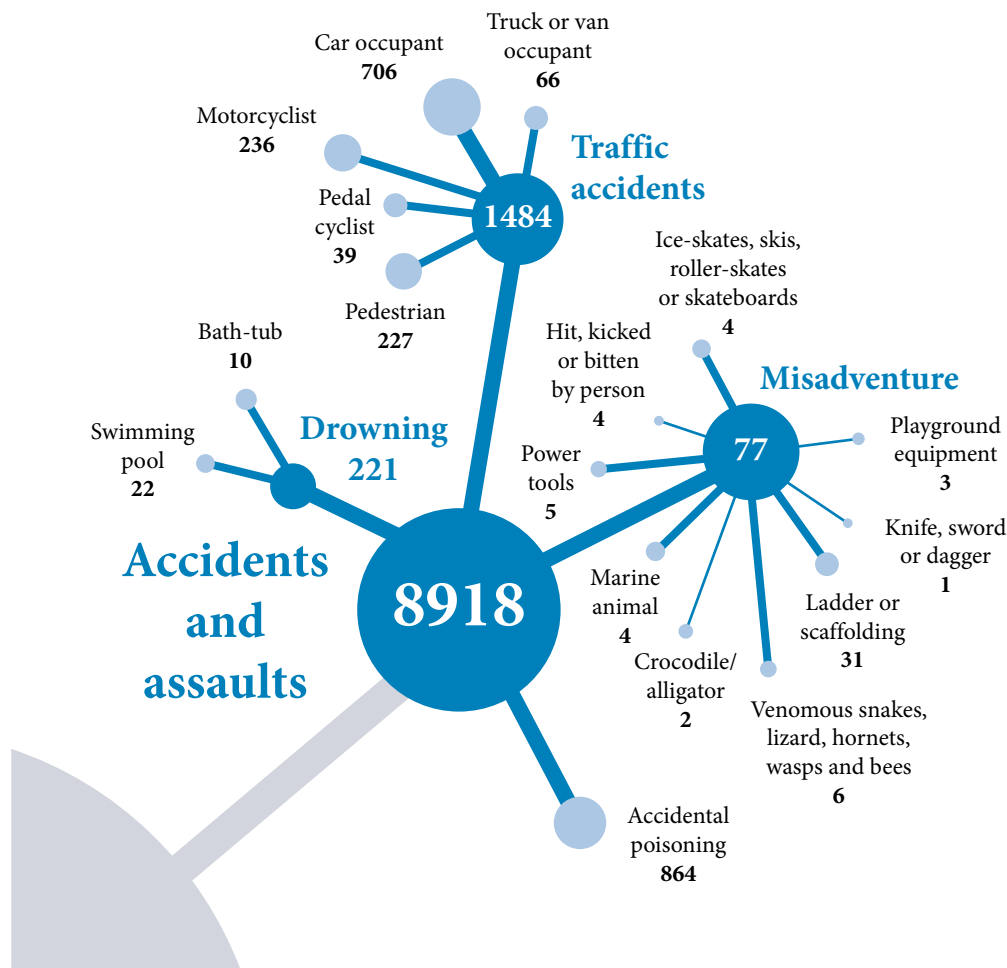
- 12 The results of a survey where people responded to the statement: 'Great customer service was received' are shown below.

28 Strongly agree 19 Agree 4 No opinion 3 Disagree 1 Strongly disagree

Create a dot plot, a column chart and a pie chart of the results, and discuss the appropriateness of each type of display.

- 13 A pie graph is drawn to show the distribution of meat, bone and fat in a beef carcass. Meat is known to be 70%, bone is shown as an angle of 68.4° and a whole carcass is assumed to be around 223 kg. Construct a pie chart that shows the mass of each part.

- 14 Consider the following extract from a cause-of-death graphic.



- Present the 'Traffic accidents' portion of the graphic in two other ways.
- Create a frequency table, broken into as many categories as possible. Use the table to determine the three categories with the highest frequencies. List the categories in descending order.
- Explain how the graphic provided above gives a clearer picture of the data than a column graph, and how, at first glance, the graphic can mislead.

Numerical data

The shape of numerical data distributions

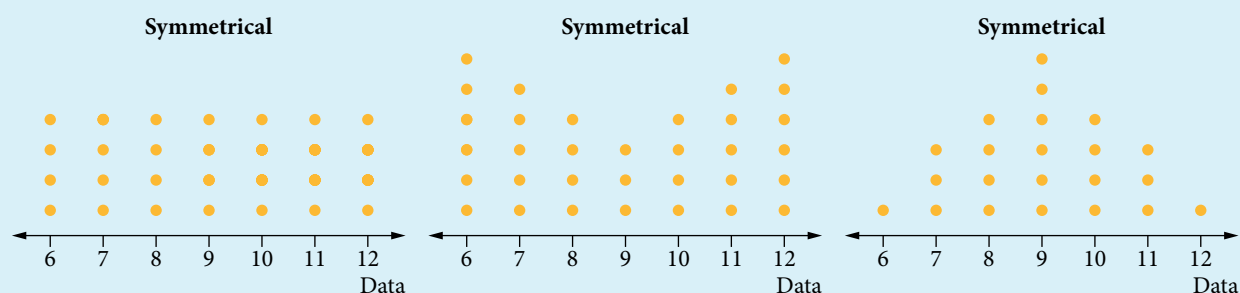
Frequency distributions of numerical data are conventionally drawn with the data on the horizontal axis, sequentially if possible, from smallest to largest.

Dot plots can be used for discrete data as long as the frequency for each datum is not too large.

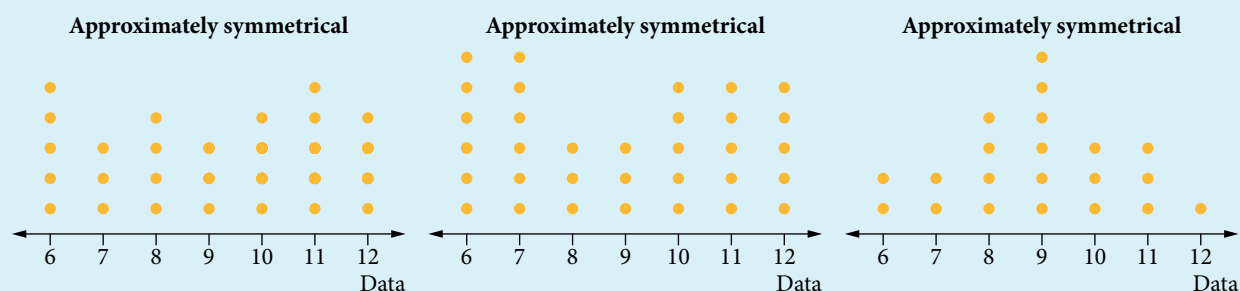
Here dot plots are being used to explain the shape of numerical data distributions.

The shape of numerical distributions is said to be symmetrical, approximately symmetrical or skewed.

A symmetrical distribution is symmetrical about the centre.

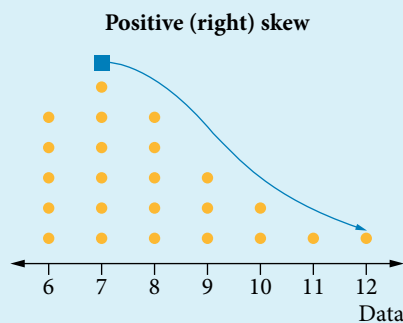


A distribution can be approximately symmetrical about the centre.

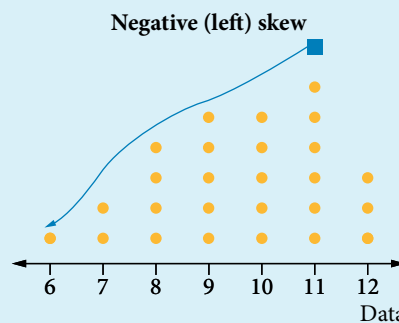


If the data is not distributed symmetrically, it is said to be skewed.

Positively skewed data has frequencies that trail off in the positive direction, to the right.



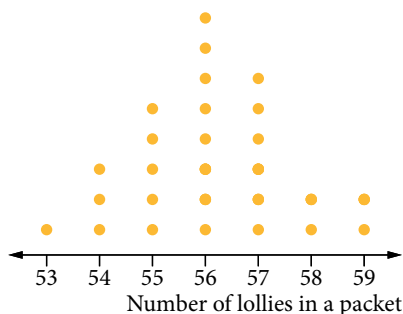
Negatively skewed data has frequencies that trail off in the negative direction, to the left.



7 The shape of a data distribution

For each of the following dot plots state the mode, then identify the shape of the distribution.

(a)



THINKING

1 Identify the mode, the data value with the highest frequency.

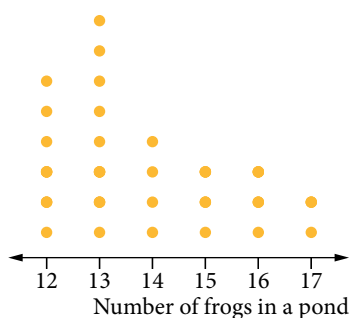
2 Determine the shape of the distribution.

WORKING

Mode = 56

The data is symmetrical.

(b)



1 Identify the mode, the data value with the highest frequency.

2 Determine the shape of the distribution.

Mode = 13

The data is positively skewed.

Stem-and-leaf plots

A *stem-and-leaf plot* is a useful way of grouping numerical data, while still showing individual values. The data may be discrete, or it may be continuous data with rounded values.

A stem-and-leaf plot consists of two parts: a stem and a leaf.

Typically, the leaf contains the last digit of the number and the stem contains all the other digits. For example, the leaf may represent the 'ones' – single-digit numbers 1 to 9 – and the stem will represent the rest of the number (tens, hundreds, thousands etc). Use spaces, not commas, to separate each entry.

The values in the leaf section are ordered before final presentation. You must include a key that explains the type of values contained in the plot. The key includes a vertical line to show how the numbers have been divided into stem and leaf.

Stem-and-leaf plots display grouped discrete or rounded continuous data while still showing the ordered data values.

The key explains the values and their units contained in the stem and leaf parts.

Stem	Leaf
0	1 4 8
1	1 1 2 3 4 5 5 6 7 8 9
2	0 2 3 5 7 9
3	4 7 8 8
4	0 1 5 7

Key: 2 | 3 = 2.3

Stem	Leaf
0	1 4 8
1	1 1 2 3 4 5 5 6 7 8 9
2	0 2 3 5 7 9
3	4 7 8 8
4	0 1 5 7

Key: 2 | 3 = 23

Stem	Leaf
0	1 4 8
1	1 1 2 3 4 5 5 6 7 8 9
2	0 2 3 5 7 9
3	4 7 8 8
4	0 1 5 7

Key: 2 | 3 = 230

8 Constructing a stem-and-leaf plot

Construct a stem-and-leaf plot to display the following data.

- (a) A class obtained the following marks, out of 100, on a mathematics test.

42, 55, 46, 78, 83, 85, 99, 78, 83, 66, 50, 49, 72, 84, 92, 81, 60, 56, 57, 82, 78, 57, 93, 58

THINKING

- 1 Determine the maximum and minimum data values.
- 2 Decide which values should appear in the stem column. List them down the page in ascending order.

Write the key near the plot.

Organise the data values into appropriate rows by placing the units (leaf) values beside their tens digit.

- 3 Organise the plot with the leaf values in order.
If a context is known, give an appropriate title to the plot.

WORKING

Maximum = 99

Minimum = 42

Stem	Leaf
4	2 6 9
5	5 0 6 7 7 8
6	6 0
7	8 8 2 8
8	3 5 3 4 1 2
9	9 2 3

Key: 4 | 2 = 42 marks

Mathematics test scores

Stem	Leaf
4	2 6 9
5	0 5 6 7 7 8
6	0 6
7	2 8 8 8
8	1 2 3 3 4 5
9	2 3 9

Key: 4 | 2 = 42 marks

- (b) The masses of newborn babies, in kilograms, born in a hospital birthing suite in a particular week were:

2.8, 5.0, 3.5, 5.2, 4.6, 2.7, 2.3, 2.7, 2.6, 3.1, 2.8, 2.9, 3.7, 2.5, 3.0, 3.4, 2.4

- 1 Determine the maximum and minimum data values.

Maximum = 5.2 kg

Minimum = 2.3 kg

- 2 Decide which values should appear in the stem column. List them down the page in ascending order. Write the key near the plot.

Organise the data values into appropriate rows by placing the units (leaf) values beside their units digit.

Stem	Leaf
2	8 7 3 7 6 8 9 5 4
3	5 1 7 0 4
4	6
5	0 2

Key: 2 | 7 = 2.7 kg

3 Organise the plot with the leaf values in order.

If a context is known, give an appropriate title to the plot.

Mass of newborn babies

Stem	Leaf
2	3 4 5 6 7 7 8 8 9
3	0 1 4 5 7
4	6
5	0 2

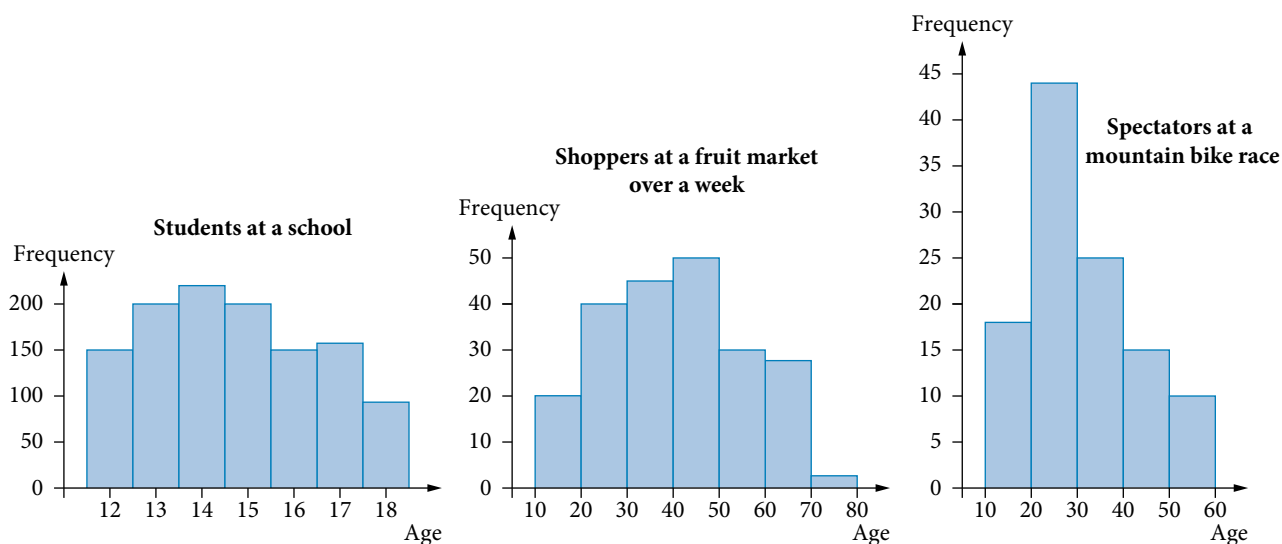
Key: 2 | 7 = 2.7 kg

Histograms

A histogram is a column chart for numerical data, with no gaps between the columns.

Both discrete and continuous data can be displayed using a histogram, as long as the data is sequential or can be grouped into sequential classes of equal width.

If the data does not conform to this type of treatment then a column graph, with ordered classes on the horizontal axis, may be appropriate.



In a histogram the data values do not generally start at 0. The first column starts one-half column width from the vertical axis.

As you saw with dot plots of numerical data, histograms give a visual impression of the distribution of the data, such as the mode, or modal class, and tell you whether the data is symmetrical or skewed.

The data values on the horizontal axis can be written in classes, as shown above, or as an evenly spaced scale.

9 Constructing a histogram for continuous data

Draw a histogram of the following data, which represents the haemoglobin levels of a sample of 25 students. Use your graph to identify the modal class and to determine whether the data is symmetrical or skewed.

Haemoglobin level (g/dL)	Frequency
9.0–<10.0	1
10.0–<11.0	2
11.0–<12.0	5
12.0–<13.0	2
13.0–<14.0	7
14.0–<15.0	5
15.0–<16.0	2
16.0–<17.0	1

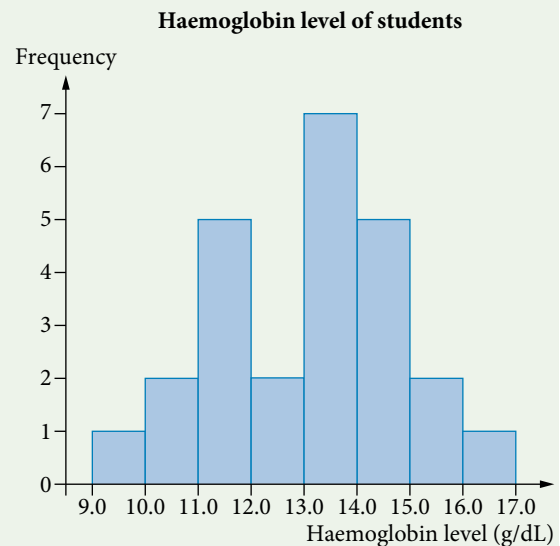
THINKING

- 1 Determine the minimum and maximum values, to determine the range to be marked on the axes.
- 2 Construct a set of axes, marking haemoglobin level on the horizontal axis and frequency on the vertical axis. Leave a half column-width gap at the start of the data axis. Complete the graph.
- 3 Identify the modal class, and interpret what this means.
- 4 Comment on the shape of the graph.

WORKING

The data ranges from 9 to 17.

The maximum frequency is 7.



The modal class is 13.0–< 14.0. The most common level of haemoglobin for this group is between 13.0 and 14.0 (g/dL).

The graph has no distinct skew. It is approximately symmetrical.

Histograms can be used for discrete data by treating the data as if it were continuous. For example, the discrete value of 5 is treated as a continuous group of 4.5 to 5.5.

10

Constructing a histogram for discrete data

Draw a histogram for each of the following sets of data.

- (a) The data represents the number of TVs per household for a class of 24 students.

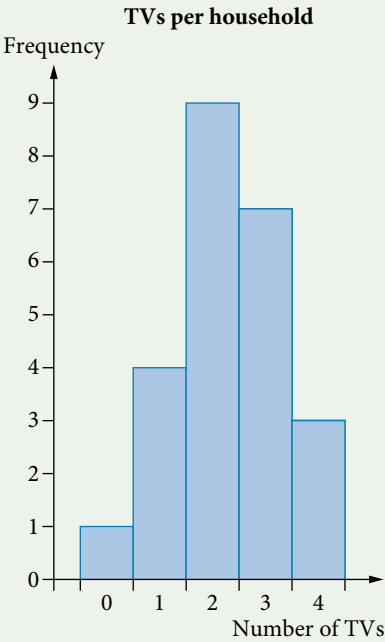
Number of TVs	Frequency
0	1
1	4
2	9
3	7
4	3

THINKING

- 1
- Identify the minimum and maximum values, to determine the range to be marked on the axes, remembering to extend the lowest and highest data values by a half at each end.
- 2
- Draw a set of axes, marking Number of TVs on the horizontal axis and Frequency on the vertical axis. Leave a half column-width gap at the start of the data axis and use the whole-number values of the data. Construct the graph, include a title and labelled axes.

WORKING

The data will be graphed from -0.5 to 4.5.
The maximum frequency is 9.



(b) The data represents the number of albums owned by a sample of 17 students.

Number of albums	Frequency
0–9	3
10–19	5
20–29	6
30–39	2
40–49	1

- 1 Identify the minimum and maximum values to determine the range to be marked on the axes, remembering to extend the lowest and highest data values by a half at each end.

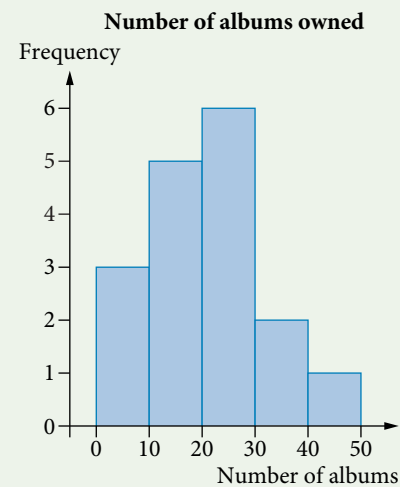
The data will be graphed from -0.5 to 49.5.

The maximum frequency is 6.

- 2 Draw a set of axes on grid paper, marking Number of albums on the horizontal axis and Frequency on the vertical axis. Leave a half column-width gap at the start of the data axis.

Choose a suitable scale for the data values. Here starting at 0 and going up in 10s is the most appropriate.

Draw the rectangles to go a half each side for each class, so the true boundaries are -0.5, 9.5, 19.5 and so on.



Histograms display single or grouped discrete or continuous data from a frequency distribution.

The data, or groups, must be sequential, with classes of equal width.

The data values on the horizontal axis can be written as single values or classes in the middle of the base of each column, or as an evenly spaced scale with edges of columns drawn to meet the true boundaries.

EXERCISE

7.3

Numerical data

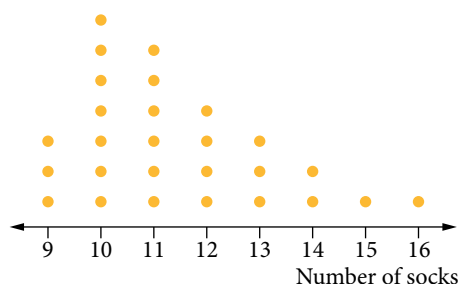
Worked
Example

7

- 1 For each of the following dot plots, identify the mode and the shape of the distribution.

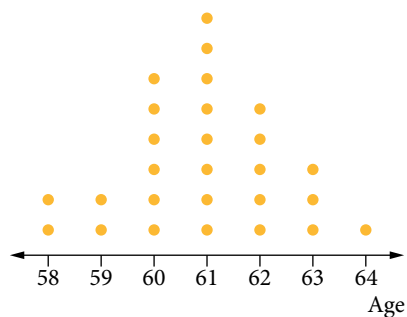
(a)

Socks in the wash



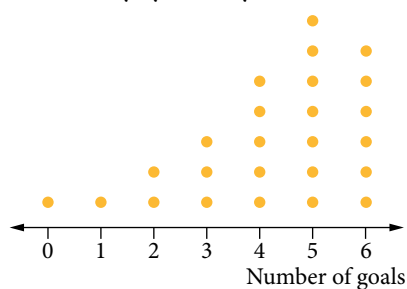
(b)

Superannuation seminar audience



(c)

Hayley's hockey scores



- 2 For the following stem-and-leaf plots, identify the modal class and the shape of the distribution.

(a)

Stem	Leaf
14	2 5 7 9
15	0 1 3 3 4 5 6 7 7 8 9
16	0 1 2 3 3 5 8 9
17	2 3 5 6
18	0 8 9
19	1 3

Key: 19 | 1 = 191

(b)

Stem	Leaf
1	4 5
2	1 1 4 6 9
3	0 1 2 2 3 7 9
4	3 6 8 8 9
5	2 3
6	8

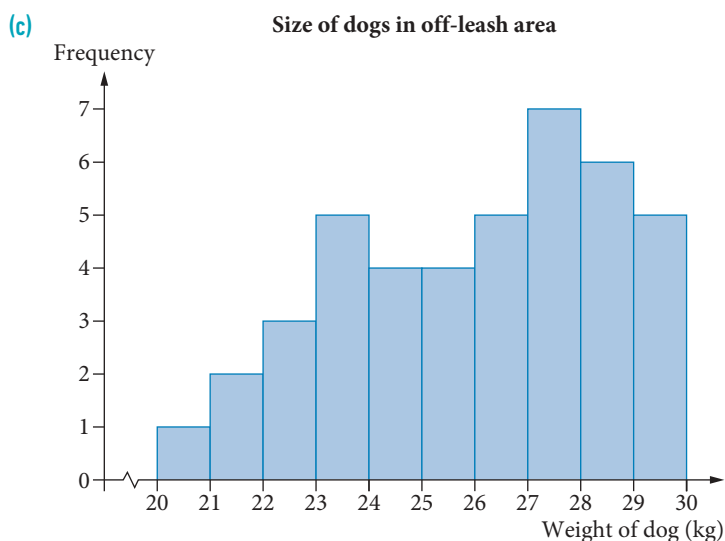
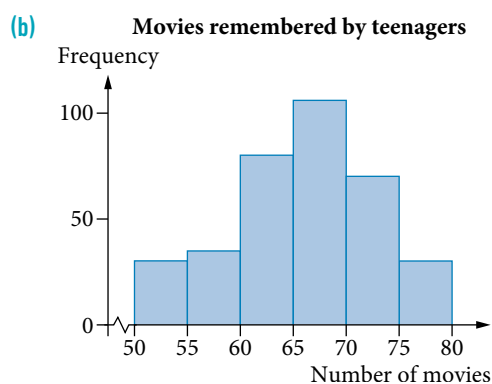
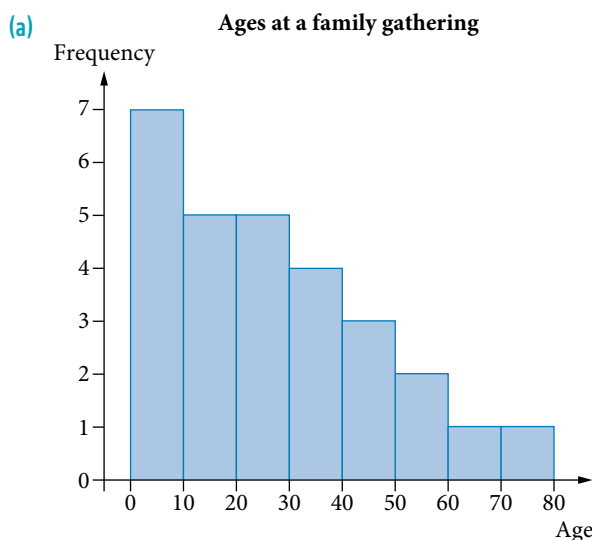
Key: 1 | 5 = 150

(c)

Stem	Leaf
1	0 1 2
2	5 5 6 7
3	0 2 3 3 4
4	5 5 6 7 7 8 8 9
5	1 1 2 2 2 3 3 4 4
6	6 7 7 7 8 9

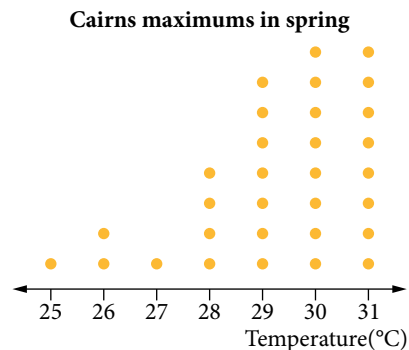
Key: 1 | 2 = 1.2

- 3 For each of the following histograms, state the mode or modal class, then identify the shape of the distribution.



- 4 Identify the following features of the dot plot showing the maximum temperatures reached in Cairns throughout a month in spring.

- Identify the hottest and coldest maximum temperatures recorded for the month, and use this to state the range of maximum temperatures for this particular month.
- Assuming the temperature was recorded on every day of the month, which month is displayed?
- Identify the shape of the graph and interpret it in terms of the weather in Cairns in spring.



- 5 Construct dot plots to represent the following data sets. Use your graph to determine the mode, and comment on the shape of the distribution.

(a) Soccer scores for the C team:

2, 0, 1, 5, 3, 4, 2, 1, 0, 2, 3, 0, 1, 3, 1, 2, 2, 4, 3, 1, 1, 2, 4, 2

(b) Sanjay, the best bowler in the local cricket team, recorded the number of wickets he took in each innings of the season. The following values were recorded:

4, 3, 5, 2, 1, 0, 3, 4, 2, 5, 6, 3, 2, 4, 5, 3, 0, 3, 5, 4

- 6 The stem-and-leaf plot shows the maximum number of tries ever scored by an NRL player who has played in a Queensland team.

Stem	Leaf
7	9
8	1 1 4 5 5 6 6
9	0 0 0 9 9
10	0 0 2 7
11	1 5
12	0 2 7
13	0
14	2
15	9

Key: 9 | 0 = 90

(a) Calculate and interpret the range of scores achieved by the top 25 players.

(b) Identify the shape of the graph and interpret it in terms of the number of tries scored by the NRL players.

Worked
Example

8

- 7 Construct a stem-and-leaf plot for the following.

(a) A class obtained the following marks out of 100 on a mathematics test.

36, 47, 44, 93, 82, 80, 70, 75, 66, 51, 56, 84, 92, 97, 35, 48, 91, 90, 82, 67, 54, 77, 82, 84

(b) A class obtained the following marks out of 100 on a science test.

42, 67, 81, 37, 78, 98, 35, 42, 66, 78, 50, 72, 71, 92, 67, 56, 66, 87, 92, 93, 91, 65, 39, 32

In your stem-and-leaf plot, arrange the data with a class interval of 10.

- 8 The scores in the first round of a club golf championship were as follows:

68, 72, 76, 84, 82, 80, 69, 71, 70, 73, 77, 76, 81, 80, 87, 88, 79, 76, 69, 71, 74, 75, 77, 77, 78, 82, 84, 83, 82, 77, 76, 70, 88, 73, 72, 71, 74, 66, 77, 79, 64

(a) Construct a stem-and-leaf plot to display the data.

(b) Break the stem-and-leaf plot into the smaller classes of 60–64, 65–69, 70–74, 75–79, 80–84 and 85–89 by using stems of 6_L for the lower values and 6_H for the higher values.

- 9 Construct a stem-and-leaf plot to represent each of the following data sets.

(a) Diameters of pebbles in centimetres, measured to the nearest millimetre:

2.2, 2.4, 2.6, 4.5, 4.6, 1.5, 2.4, 2.6, 3.1, 3.7, 4.2, 3.0, 4.0, 4.4, 2.5, 4.1, 2.7, 2.2, 3.0, 3.1, 3.5, 4.5, 1.8, 2.1

(b) Weight of birds in grams to the nearest 10g:

320, 410, 300, 200, 170, 220, 310, 430, 290, 330, 350, 420, 410, 450, 470, 220, 160, 190, 310, 300, 290, 270, 360, 400

Waiting time for a bus (minutes)	Frequency
0 – < 10	9
10 – < 20	18
20 – < 30	13
30 – < 40	15
40 – < 50	14
50 – < 60	16

Monthly salary (\$'000)	Number of employees
2-<4	16
4-<6	8
6-<8	5
8-<10	2
10-<12	1

- (e) Describe the data distribution in terms of symmetry or skew.

Number of siblings	Frequency
0	3
1	6
2	2
3	3
4	1
5	2
6	1

- 13** Construct a histogram to represent the heights of Year 8 students given in the table. Scale the data axis using multiples of 5, so 140, 145 and so on. (Use grid paper if necessary.) Use your graph to identify the modal class and identify whether the data is symmetrical or skewed.

Height (cm)	141-<146	146-<151	151-<156	156-<161	161-<166	166-<171
Frequency	22	31	21	26	22	24

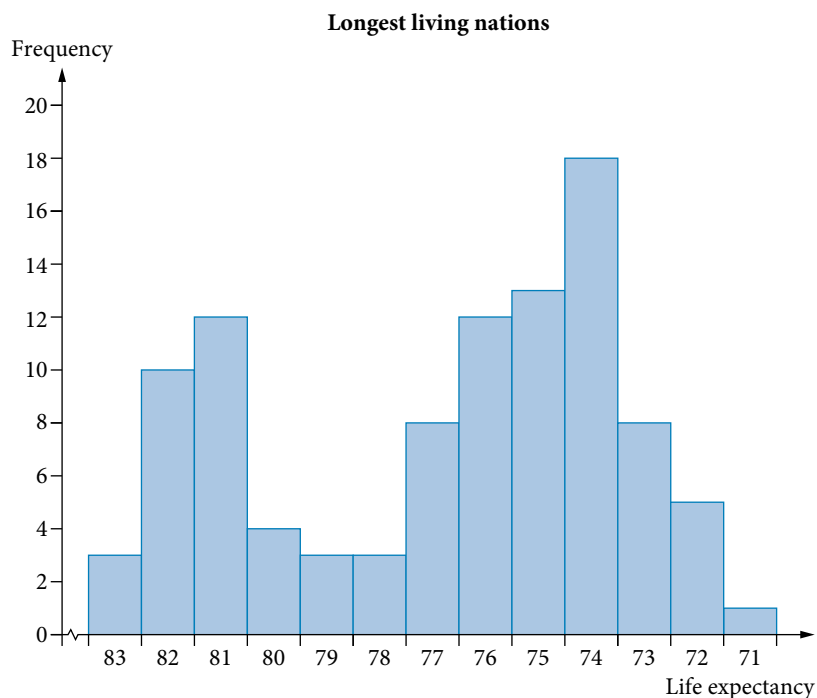
- 14 The following data represents the distance travelled (km) each day by a solar vehicle. The distances have been rounded to the nearest whole number of kilometres.

112, 95, 122, 78, 99, 145, 133, 160, 145, 144, 78, 66, 68, 79, 93, 125, 133, 89, 67, 78, 114, 134, 80, 65, 105, 115, 127, 134, 117, 84

- Do you consider the data discrete or continuous? Give a brief justification for your answer.
- Construct a frequency table for the data, using a class interval of 20 and a starting value of 61.
- Looking only at the frequency table, what can you say about the maximum distance travelled on any one day?
- Construct a stem-and-leaf plot to display the data.
- What can this stem-and-leaf plot tell you that the frequency table cannot?

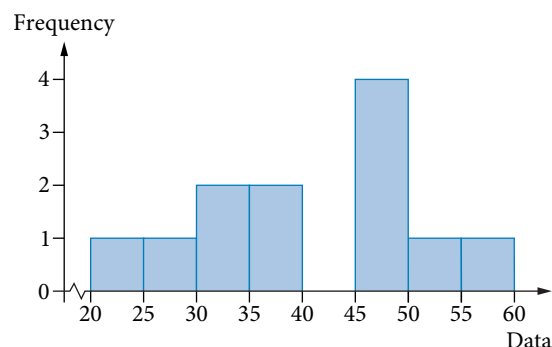
- 15 The column chart shows life expectancies in nations of the world where people live the longest.

- Determine the number of countries that are represented.
- Discuss the findings of the graph. Use the shape of the graph in your discussion, including anything unusual about the way it is drawn.
- Predict what the graph would look like if all countries of the world were included. Use the shape of the predicted graph in your discussion.



- 16 Consider the histogram shown here.

- Describe the data represented in the histogram, based on the shape of the histogram.
- Redraw the histogram using fewer classes, to produce a different view of the way the data is distributed.
- Describe the data again, this time based on the shape of the new histogram.



Median, range and interquartile range

Measures of central tendency

When dealing with data, the three measures of central tendency are mean, median and mode. A measure of central tendency is a value that can be used to represent the data set. This section features the median, which is the middle datum of a set of ordered data.

Median

The median is the middle score of an ordered data list. The method for finding the median depends on whether there is an odd or even number of data values.

For odd n , the median is the $\left(\frac{n+1}{2}\right)$ th value.

2 5 8 **10** 11 12 12 ; median = 10

For even n , the median is the mean of the $\left(\frac{n}{2}\right)$ th and the next value.

3 4 4 **5** | **6** 6 8 9 ; median = 5.5

When data values are grouped, you should state the class interval in which the median falls.

11 Calculating the median

Determine the median of the following data sets.

- (a) Number of cars that passed the school gate in a number of 5 minute periods

10, 5, 12, 13, 14, 7, 12, 15, 7, 3, 2, 8

THINKING

- 1 Organise the data in ascending order (from smallest to largest).
- 2 There is an even number of data values, so identify the $\left(\frac{n}{2}\right)$ th and the next value.
- 3 Calculate the median by finding the mean of these two values.
- 4 Interpret the answer.

WORKING

2, 3, 5, 7, 7, 8, 10, 12, 12, 13, 14, 15

2 3 5 7 7 8 | 10 12 12 13 14 15

$\left(\frac{n}{2}\right)$ th = $\left(\frac{12}{2}\right)$ th = 6th value

Count from the left until you get to the 6th value: 8

7th value: 10

$\frac{8 + 10}{2} = 9$

The median is 9 cars.

- (b) Number of brothers and sisters for the students in two Year 11 classes

No. brothers and sisters	Frequency
0	7
1	11
2	15
3	3
4	1

- 1 The frequency table has effectively put the data in order already. Calculate Σf , the total frequency, to decide whether there is an odd or even number of data values.

No. brothers and sisters (x)	Frequency (f)
0	7
1	11
2	15
3	3
4	1
	$\Sigma f = 7 + 11 + 15 + 3 + 1$ $= 37$

- 2 When there is an odd number of data values, identify the $\left(\frac{n+1}{2}\right)$ th value.

$$\left(\frac{n+1}{2}\right)\text{th value: } \left(\frac{37+1}{2}\right)\text{th} = 19\text{th}$$

- 3 Count down the frequency column to find the required value, and state the answer.

$$7 + 11 = 18: \text{not far enough}$$

$$18 + 15 = 33: \text{past the 19th}$$

$$2 \text{ is the 19th to 33rd value}$$

$$19\text{th value is } 2.$$

The median number of brothers and sisters is 2.

- 4 Check that your answer is reasonable.

A median of 2 is reasonable, because there are 18 that are smaller, 4 that are larger and 15 that are the same.

If data values are grouped into classes, you can find the median class using the frequency values in the table.

12 Calculating the median class

Determine the median time interval for competitors in the under-17 100m dash at an athletics carnival.

Time (s)	Frequency (f)
12–<12.5	3
12.5–<13	5
13–<13.5	12
13.5–<14	28
14–<14.5	10

THINKING

- 1 The frequency table has ordered data. Calculate Σf and see whether there is an odd or even number of data values.

WORKING

Time (s)	Frequency (f)
12–<12.5	3
12.5–<13	5
13–<13.5	12
13.5–<14	28
14–<14.5	10
	$\Sigma f = 3 + 5 + 12 + 28 + 10$ $= 58$

- 2 There is an even number of data values, so identify the $\left(\frac{n}{2}\right)$ th and the next value.

$$\left(\frac{n}{2}\right)\text{th} = \left(\frac{58}{2}\right)\text{th} = 29\text{th value}$$

Count down the frequency column until you get to the 29th value: in the interval 13.5–<14

The next value is also in the interval 13.5–<14

The median interval is 13.5–<14 s.

- 3 Interpret the answer.

Quartiles

The median divides the data set into two equal ‘halves’. The data set could be divided into many equal groups, called quantiles. In this subject you will only consider dividing the data into four groups, called quartiles.

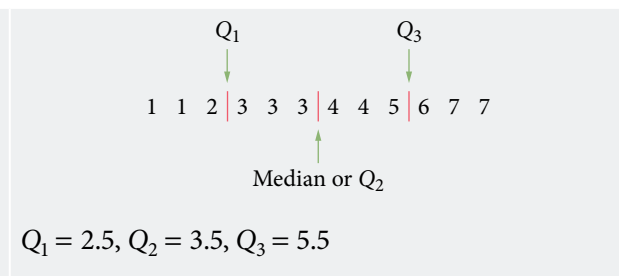
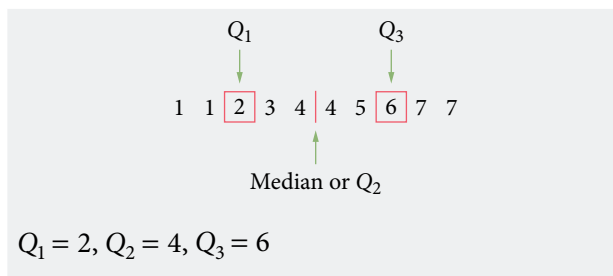
The median is referred to as the second quartile, Q_2 .

The median of the lower half of the data is called the first or lower quartile, Q_1 .

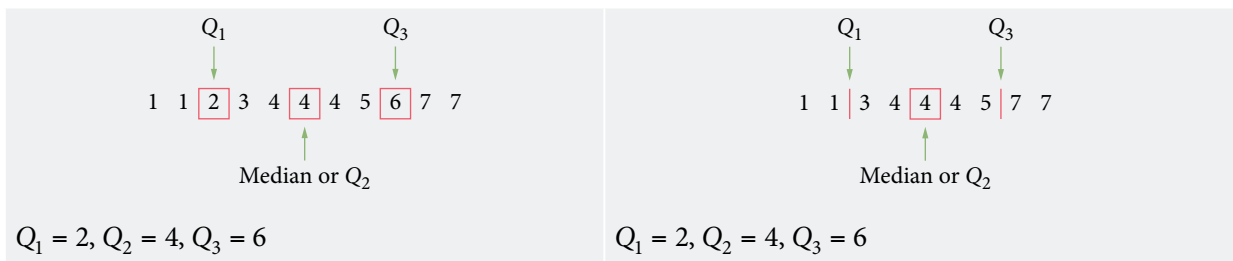
The median of the upper half of the data is called the third or upper quartile, Q_3 .

Remember to order the data from smallest to largest before you find these values.

For an even number of data values, the median divides the data into equal halves. The first and third quartiles are then effectively the medians of each half of the data. This may be an actual data value or the average of the middle two.



When the number of data values is odd, the median is an actual datum (data value). The data values above and below the median are used to locate the first and third quartiles, and again these may be actual data values or the average of the middle two values.



Measures of spread

The median is more powerful in describing a set of data when it is accompanied by two measures of variation, or spread: the range, and the interquartile range.

Range

The range is the difference between the highest and lowest values of the data.

$$\text{Range} = \text{maximum value} - \text{minimum value}$$

The range is influenced by extreme values. For example, the table shows the number of pets owned by each of the students in a class.

$$\text{Range} = 6 - 0 = 6 \text{ pets}$$

A new student joins the class. She has an aviary that contains 25 birds.

$$\text{Range} = 25 - 0 = 25 \text{ pets}$$

This additional value has distorted the range. This could be regarded as not being a fair representation of the class as a whole. You will further consider this issue of outlier (or extreme) values later in this chapter.

Number of pets (x)	Frequency (f)
0	5
1	9
2	4
3	2
4	2
5	1
6	1

Interquartile range (IQR)

The interquartile range is a measure of spread based on the quartile values. It is often referred to by the abbreviation IQR.

$$\text{Interquartile range} = \text{third quartile} - \text{first quartile}$$

$$\text{IQR} = Q_3 - Q_1$$

The IQR represents the middle 50% of the ordered data set. It is a more reliable measure of spread because it does not include extreme values at the start or end of the ordered data list.

Additional information

IQR

Listen to the description of how to determine the interquartile range (IQR) from data. Then try the activity.

13 Calculate the interquartile range

Determine the interquartile range for the data set: 1, 14, 20, 1, 105, 30, 27, 16, 24, 24, 29.

THINKING

- 1 Order the data from smallest to largest.
- 2 Identify Q_2 (median), then Q_1 and Q_3 for the data set.
- 3 Calculate the interquartile range, the difference between the upper and lower quartiles.
- 4 Interpret the result.

WORKING

1, 1, 14, 16, 20, 24, 24, 27, 29, 30, 105

1 1 14 16 20 24 24 27 29 30 105

$Q_1 = 14$, median = 24, $Q_3 = 29$

$$\begin{aligned} \text{IQR} &= Q_3 - Q_1 \\ &= 29 - 14 \\ &= 15 \end{aligned}$$

The range of the middle 50% is 15; specifically, 50% of the data values lie between 14 and 29.

Five-figure summary

The five-figure summary for a data set consists of:

- the minimum value
- the lower quartile
- the median
- the upper quartile
- the maximum value.

For the data in the previous worked example, the five-figure summary is:

1 1 14 16 20 24 24 27 29 30 105

Minimum = 1, $Q_1 = 14$, Median $Q_2 = 24$, $Q_3 = 29$, Maximum = 105

i Additional information

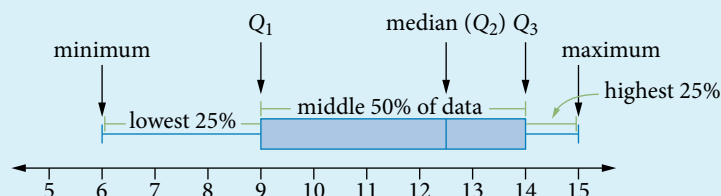
Five-figure summary

Practise finding the five-figure summary.

Box plots

The five-figure summary can be used to draw a statistical graph called a *box plot*. This is short for 'box-and-whisker plot'.

A box plot needs to have a scale line associated with it. The box is the central box structure and the whiskers are the lines going out from the box to the extreme values.



Additional information

Box plots

Watch the animation demonstrating how to read the five-figure summary, IQR and range from a box plot. Then try the activity.

Explore further

Five-figure summary and box plots

Use Excel to determine the five-figure summary and create a box plot.

14 Determine the five-figure summary and create a box plot

Determine the five-figure summary and then construct a box plot for this data set. Comment on the shape of the distribution. 2, 3, 5, 4, 5, 5, 12, 9, 2, 6, 7, 5

THINKING

- 1 Write the data in ascending order.
- 2 Identify the location of Q_1 , Q_2 and Q_3 .
- 3 State the five-figure summary.
- 4 Construct the box plot. Include a scale.
- 5 Comment on the spread of the data.

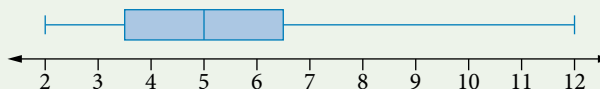
WORKING

2, 2, 3, 4, 5, 5, 5, 5, 6, 7, 9, 12

There are 12 data values.

2 2 3 | 4 5 5 | 5 6 | 7 9 12

Min = 2, $Q_1 = 3.5$, Median = 5, $Q_3 = 6.5$, Max = 12



The data has a tail to the right, so it is positively skewed. The data ranges from 2 to 12, with the middle 50% of the data between 3.5 and 6.5.

You can also find the five-figure summary if the data is presented in a frequency table or a stem plot.

15 Using a frequency table to find the five-figure summary

Determine the interquartile range and draw a box plot for each of the following. Comment on the shape of the distribution.

(a)

Score	3	4	5	6	7	8
Frequency	2	8	9	6	4	1

THINKING

- 1 Calculate the sum of the frequencies, n .
- 2 Identify the position of the median, and its value.

WORKING

$$\begin{aligned}
 n &= \sum f \\
 &= 2 + 9 + 9 + 6 + 4 + 1 \\
 &= 30
 \end{aligned}$$

The median is between the 15th and 16th values.

The 15th and 16th values are both 5, so:

Median = 5

- 3 Identify the position of Q_1 and Q_3 .

There are 15 values in each of the upper and lower set of data. Therefore the quartiles are the 8th value from either end.

$$Q_1 = 4 \text{ and } Q_3 = 6$$

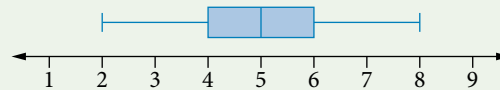
- 4 Calculate the interquartile range.

$$\begin{aligned} \text{IQR} &= Q_3 - Q_1 \\ &= 6 - 4 \\ &= 2 \end{aligned}$$

- 5 List the five-figure summary as a basis for drawing the box plot.

$$\text{Min} = 2, Q_1 = 4, \text{Median} = 5, Q_3 = 6, \text{Max} = 8$$

- 6 Construct the box plot. (Include a scale line.)



- 7 Comment on the spread of the data.

The data is evenly spread to the left and right of the median, so it is symmetrical. The data ranges from 2 to 8, with the middle 50% of the data between 4 and 6.

(b)

Stem	Leaf
3	0 3 6
4	1 3 5 7 9
5	4 5 7 8
6	2 4 7 9 9
7	0 0 0 1 4 7 8
8	3 5 7 8
9	0 0 0 0 0 1

Key: 3 | 0 = 3.0

- 1 Count the number of values, n , in the data set.

$$n = 34$$

- 2 Identify the position of the median and its value.

Stem	Leaf
3	0 3 6
4	1 3 5 7 9
5	4 5 7 8
6	2 4 7 9 9
7	0 0 0 1 4 7 8
8	3 5 7 8
9	0 0 0 0 0 1

Key: 3 | 0 = 3.0

- 3 Identify the positions of Q_1 and Q_3 and find their values.

The median is between the 17th and 18th values.

$$\begin{aligned}\text{Median} &= \frac{6.9 + 7.0}{2} \\ &= 6.95\end{aligned}$$

Stem	Leaf
3	0 3 6
4	1 3 5 7 9
5	4 5 7 8
6	2 4 7 9 9
7	0 0 0 1 4 7 8
8	3 5 7 8
9	0 0 0 0 0 1

Key: 3 | 0 = 3.0

There are 17 values in the lower half, so the quartiles are the 9th score from each end.

$$Q_1 = 5.4, Q_3 = 8.5$$

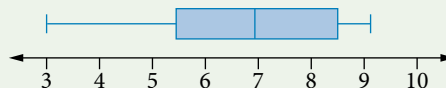
- 4 Calculate the interquartile range.

$$\begin{aligned}\text{IQR} &= Q_3 - Q_1 \\ &= 8.5 - 5.4 \\ &= 3.1\end{aligned}$$

- 5 List the five-figure summary.

Min = 3.0, $Q_1 = 5.4$, Median = 6.95, $Q_3 = 8.5$, Max = 9.1

- 6 Construct the box plot.



- 7 Comment on the spread of the data.

The data has a tail to the left, so it is negatively skewed. The data ranges from 3.0 to 9.1, with the middle 50% of the data lying between 5.4 and 8.5.

Outliers

An outlier is a value that is distant from the rest of the data set. There is a mathematical measure that is used to allow you to be consistent in identifying outliers.

An outlier is an extreme value that lies outside the fences set up, as follows:

$$\text{Lower fence} = Q_1 - 1.5 \times \text{IQR}$$

$$\text{Upper fence} = Q_3 + 1.5 \times \text{IQR}$$

16 Identifying outliers

Determine whether any of the values in the following data set would be regarded as outliers.

1, 5, 7, 8, 8, 8, 9, 11, 13, 18, 23

THINKING

1 Determine the median, Q_1 and Q_3 .

2 Calculate the interquartile range.

3 Calculate the fence values.

4 Interpret the results.

WORKING

1 5 7 8 8 8 9 11 13 18 23

$Q_1 = 7$, Median = 8 and $Q_3 = 13$

$$\begin{aligned} \text{IQR} &= Q_3 - Q_1 \\ &= 13 - 7 \\ &= 6 \end{aligned}$$

Lower fence = $Q_1 - 1.5 \times \text{IQR}$

$$\begin{aligned} &= 7 - 1.5 \times 6 \\ &= -2 \end{aligned}$$

Upper fence = $Q_3 + 1.5 \times \text{IQR}$

$$\begin{aligned} &= 13 + 1.5 \times 6 \\ &= 22 \end{aligned}$$

The value 23 is an outlier because it is above the upper fence.

It is possible for outliers to be present at both ends of the data list and for there to be more than one outlier at either or both ends.

In a box plot the whiskers do not extend to outliers. Any outlier is marked with a $\star$ or $\bullet$ and the whisker is drawn only as far as the last data value before the fence.

17 Construct and interpret a box plot with outliers

Consider the frequency table of marks for a test.

Mark	20	19	18	17	16	15	14	13	12	6
Frequency	1	1	4	2	6	8	10	6	4	1

(a) Construct the box plot and comment on its shape.

THINKING

1 Calculate the sum of the frequencies, n .

2 Identify the position of the median, and its value.

WORKING

$$\begin{aligned} n &= \sum f \\ &= 1 + 1 + 4 + 2 + 6 + 8 + 10 + 6 + 4 + 1 \\ &= 43 \end{aligned}$$

The median is the 22nd value.

Median = 15

3 Identify the position of Q_1 and Q_3 . There are 21 values in the upper and lower half of the data. The quartiles are the 11th position from the end.
 $Q_1 = 13$ and $Q_3 = 16$.

4 Calculate the interquartile range.

$$\begin{aligned} \text{IQR} &= Q_3 - Q_1 \\ &= 16 - 13 \\ &= 3 \end{aligned}$$

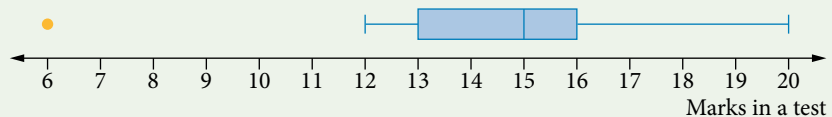
5 Calculate the fence values.

$$\begin{aligned} \text{Lower fence} &= Q_1 - 1.5 \times \text{IQR} \\ &= 13 - 1.5 \times 3 \\ &= 8.5 \\ \text{Upper fence} &= Q_3 + 1.5 \times \text{IQR} \\ &= 16 + 1.5 \times 3 \\ &= 20.5 \end{aligned}$$

6 Interpret the results. The value 6 is an outlier because it is below the lower fence.

7 List the five-figure summary.
 $\text{Min} = 12, Q_1 = 13, \text{Median} = 15, Q_3 = 16, \text{Max} = 20$

8 Construct the box plot.



9 Interpret the data. The data has a tail to the right of the median, so it is positively skewed. The data ranges from 6 to 20, but has an outlier, 6. The middle 50% of the data is between 13 and 16.

(b) Determine the cut-off mark for a B if at least 75% of students are given a B or higher.

1 Identify the top 75% of data. This is equal to or above the first quartile.

$$Q_1 = 13$$

If the cut-off mark for a B is 13, at least 75% would be rated B or better.

2 Check with the table to see if the answer is reasonable.

From the table, only $\frac{5}{43} \times 100\% = 12\%$ scored less than 13.

If you include students who scored 13:

$$\frac{11}{43} \times 100\% = 26\%, \text{ so less than 75\% would be included.}$$

EXERCISE

7.4

Median, range and interquartile range

Worked
Example

- 1 Determine the median of the following data sets.

(a) 1, 4, 3, 2, 6, 4, 8, 9, 4, 3, 1

(b) 2, 3, 6, 2, 9, 6, 9, 6, 3, 2, 5, 2

(c)

Score	Frequency (f)
0	3
1	6
2	7
3	4
4	3
5	1

(d)

x	f
1	5
2	8
3	3
4	2
5	4
6	5

- 2 Determine the median class interval for the following data sets.

(a)

Time (s)	Frequency (f)
12 – < 12.5	3
12.5 – < 13	5
13 – < 13.5	4
13.5 – < 14	11
14 – < 14.5	6

(b)

Time (s)	Frequency (f)
12 – < 12.5	6
12.5 – < 13	8
13 – < 13.5	4
13.5 – < 14	5
14 – < 14.5	7

- 3 Determine the median for each of the following discrete data sets.

(a)

Score	Frequency (f)
4	15
5	23
6	14
7	23
8	17
9	19
10	20

(b)

Score	Frequency (f)
201	49
202	83
203	99
204	121
205	143
206	117

- 4 Calculate the interquartile range for the following data sets.

(a) 2, 3, 5, 8, 9, 11, 15, 16, 18, 25, 36

(b) 4, 5, 2, 6, 7, 9, 1, 9, 11, 12, 16, 18, 21, 17, 16, 17

(c) 1, 7, 2, 7, 3, 1, 9, 11, 14, 12, 16, 17, 22, 11

- 5 Determine the five-figure summary and use it to construct a box plot for each of the following data sets. Comment on the shape of the distribution.

(a) 2, 3, 5, 5, 5, 6, 8, 9, 10, 11, 12, 15

(b) 2, 5, 9, 11, 12, 9, 8, 7, 12, 14, 13, 22, 15, 18

- 6 Determine the five-figure summary and use it to create a box plot for each of the following data sets. Use technology if possible. Comment on the shape of the distribution.

(a) 1, 4, 6, 3, 2, 4, 5, 7, 8, 9, 10, 2, 5, 6, 7, 4, 5, 3, 5, 1, 7, 9, 6, 9, 2, 4, 8, 10

(b) 8, 12, 15, 18, 23, 34, 32, 12, 16, 13, 18, 19, 13, 15, 21, 32, 23, 34, 23, 12, 18, 19, 17, 15

(c)

Score	Frequency
8	10
9	12
10	1
11	11
12	5
13	7
14	9

(d)

Score	Frequency
23	8
24	11
25	14
26	12
27	13
28	7
29	3

- 7 Determine the five-figure summary and use it to create a box plot for each of the following data sets. Use technology if possible.

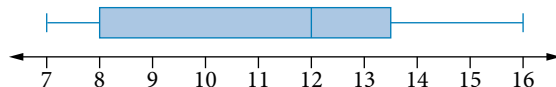
(a) 2, 3, 4, 6, 6, 6, 8

(b) 2, 2, 4, 5, 6, 7, 9

(c) 2, 3, 4, 5, 8, 9, 9

(d) 2, 2, 2, 2, 5, 7, 8

- 8 Consider the following box plot.



The IQR is closest to:

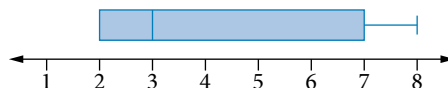
A 9

B 1

C 5.5

D 1.5

- 9 Consider the following box plot.



(a) The range is closest to:

A 7

B 8

C 5

D 6

(b) Determine what each of the incorrect options is referring to.

Worked
Example

16

- 10 Determine whether any of the values in the following data sets would be regarded as outliers.

(a) 1, 2, 17, 18, 19, 22, 24, 26, 27, 28

(b) 5, 9, 11, 13, 17, 18, 19, 19, 29

(c) 7, 22, 23, 26, 29, 31, 34, 38, 58

(d) 3, 23, 24, 27, 28, 29, 32, 34, 48

- 11 For each of the following data sets, calculate:

- the value below which outliers would occur
- the value above which outliers would occur.

(a) 13, 15, 22, 27, 36, 41, 28, 17, 25, 36, 15

(b) 33, 35, 28, 27, 46, 51, 29, 36, 49, 26

(c)

x	f
5	3
6	4
7	2
8	1

(d)

x	f
11	4
12	5
13	3
14	2

Worked
Example

- 12 Determine the five-figure summary, IQR and range for each of the following data sets. Then create a box plot.

(a)

Score	Frequency
0	3
1	6
2	7
3	4
4	3
5	1

(b)

Score	Frequency
4	15
5	23
6	14
7	23
8	17
9	19
10	20

(c)

Stem	Leaf
1	1 2 3 3 4 4 5 8
2	2 3 4 5
3	0 1 6 6 2 1
4	2 2 3 9
5	0 9
6	1 8

Key: 5 | 9 = 59

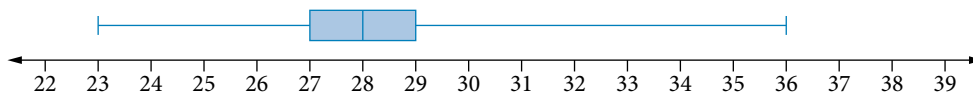
(d)

Stem	Leaf
15	2 5 8
16	0 1 3 4 5
17	1 8 9 9
18	9 9
19	8 9
20	0 2

Key: 18 | 9 = 189

- 13 Describe the common error made by a student who analysed the following data set to create the box plot below it. Then construct the correct box plot.

23, 25, 27, 27, 28, 28, 28, 29, 33, 36



- 14 The data gives the number of times each of a group of students from Coolangatta went to NSW during the summer holidays.

22, 24, 25, 19, 45, 73, 21, 25, 18, 16, 22, 25, 28, 19, 17, 16, 23

- (a) Draw a box plot for the data, identifying any outliers.
(b) Determine between what numbers of times the middle 50% of students went to NSW.

-
- (a) A number line from 4 to 19. A box plot has a minimum at 6, Q1 at 7, Median at 8, Q3 at 10, and Maximum at 14. An outlier is at 18.
- (b) A number line from 20 to 55. A box plot has a minimum at 23, Q1 at 30, Median at 35, Q3 at 38, and Maximum at 43. An outlier is at 52.
- (c) A number line from 1 to 9. A box plot has a minimum at 3, Q1 at 5, Median at 7, Q3 at 7.5, and Maximum at 8. An outlier is at 1.

- 18** The data gives the number of texts sent in a week by a group of teenagers.
113, 134, 123, 143, 123, 129, 127, 137, 136, 139, 129, 130, 140, 76, 72, 112, 142, 132, 141, 132, 137
- (a) Construct a box plot for the data, identifying any outliers.
- (b) Determine the number of texts exceeded by 75% of teenagers.

-
- A box plot showing the distribution of the number of green leaves per plant. The number line ranges from 4 to 14. The box plot has a minimum at 4, Q1 at 8, Median at 11, Q3 at 12, and Maximum at 14. There are 15 data points plotted as green dots above the number line.

- 60, 72, 75, 76, 76, 80, 81, 85

- 442 Pearson General Mathematics 11 Queensland

Cumulative frequency

7.5

Cumulative frequency for continuous data

Cumulative frequency values give a running total of how much data has accumulated up to any given data amount.

It is usual to insert the cumulative frequency column into a frequency table, to the right of the frequency column.

Cumulative frequency for continuous data allows you to determine how many are *less than* a particular data value.

18 Cumulative frequency from a table

The table at right shows the birth weights of 50 children.

Insert a cumulative frequency column into the table, and interpret the result to determine the number of children born with a birth weight less than 4.5 kg.

Birth weight (kg)	Number of children
2 – < 2.5	1
2.5 – < 3	3
3 – < 3.5	18
3.5 – < 4	15
4 – < 4.5	7
4.5 – < 5	4
5 – < 5.5	2

THINKING

- 1 The cumulative frequency is the cumulative sum of the frequency column.

Write the first frequency value as the first cumulative frequency value. For the second cumulative frequency value, add the second frequency to the first cumulative frequency value.

Continue adding the next frequency until the column is filled. The final value should be the total frequency.

- 2 Identify the cumulative frequency up to but not including the given value.

WORKING

Birth weight (kg)	Number of children	Cumulative frequency
2 – < 2.5	1	1
2.5 – < 3	3	$3 + 1 = 4$
3 – < 3.5	18	$18 + 4 = 22$
3.5 – < 4	15	$22 + 15 = 37$
4 – < 4.5	7	$37 + 7 = 44$
4.5 – < 5	4	$44 + 4 = 48$
5 – < 5.5	2	$48 + 2 = 50$

Consider the row:

Birth weight (kg)	Cumulative frequency
4 – < 4.5	44

3 Interpret the result.

44 of the 50 children were born weighing less than 4.5 kg.

This can be verified by finding the sum of the number of children born weighing 2–4.5 kg:
 $(1 + 3 + 18 + 15 + 7 = 44)$

Cumulative frequency for discrete data

If the data has not been grouped, the five-figure summary can be read from the relevant cumulative frequency positions.

Cumulative frequency for discrete data allows you to determine how many are *less than or equal to* a particular data value.

19 Cumulative frequency for ungrouped discrete data

The frequency distribution below gives the number of customers at a popular cafe during lunchtime over several months.

Number of customers	Frequency
18	14
19	40
20	35
21	20
22	2

(a) Determine the number of days on which the cafe had no more than 20 customers at lunchtime.

THINKING

1 Insert a cumulative frequency column in the table.

2 Interpret the question in terms of cumulative frequency.

Read the value from the table.

WORKING

Number of customers	Frequency	Cumulative frequency
18	14	14
19	40	$14 + 40 = 54$
20	35	$54 + 35 = 89$
21	20	$89 + 20 = 109$
22	2	$109 + 2 = 111$

No more than 20 customers is the same as 20 customers or fewer.

Number of customers	Cumulative frequency
≤ 20	89

So on 89 days there were no more than 20 customers at lunchtime.

- (b) Determine the five-figure summary for the data. Because the total frequency is large, use $\frac{n}{2}$ to locate the median and $\frac{n}{4}, \frac{3n}{4}$ to locate the respective quartiles.

- 1 Identify n from the table, then calculate $\frac{n}{4}, \frac{n}{2}, \frac{3n}{4}$. Round to whole numbers if necessary.

From the table, $n = 111$.

To the nearest whole number:

$$Q_1: \frac{n}{4} = \frac{111}{4} = 28\text{th}$$

$$\text{Median: } \frac{n}{2} = \frac{111}{2} = 56\text{th}$$

$$Q_3: \frac{3n}{4} = \frac{3}{4} \times 111 = 83\text{rd}$$

- 2 To locate the quartiles, start from the top. Here the first 14 values are 18, then values in positions 15–54 are the number 19. Q_1 is the value 28th in line and therefore is a 19. The number 20 occupies all values from position 55 to 89, the median is the 56th data value or the second 20, Q_3 is the 83rd data value, which is also a 20.

Number of customers	Cumulative frequency
18	14
19: Q_1	54: (28th)
20: Median, Q_3	89: (56th), (83rd)
21	109
22	111

- 3 List the five-figure summary by including the minimum and maximum values of the data.

Five-figure summary:

Minimum = 18, $Q_1 = 19$, Median = 20, $Q_3 = 20$, Maximum = 22

Cumulative frequency polygons (graphs)

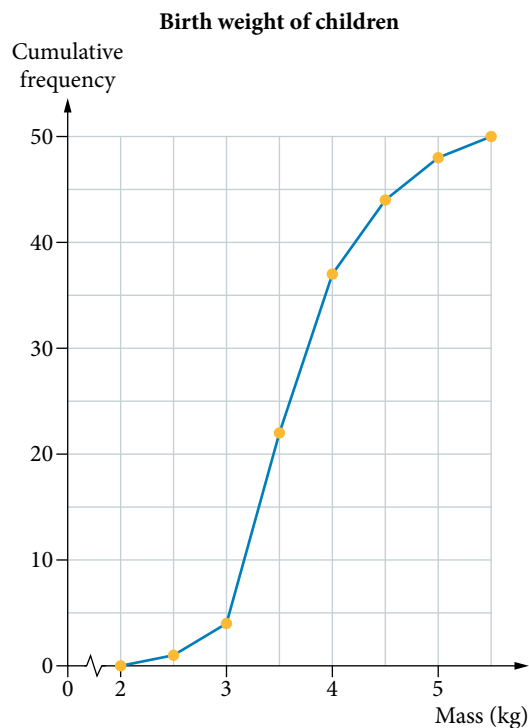
For continuous data or grouped discrete data, you can only read how many are less than a particular amount that appears in the table. Reading values from a graph allows you to estimate how many are less than any value in the range of data given.

Conversely, the median and quartiles can be read from the graph.

A cumulative frequency polygon is drawn from the points $(x, \text{cumulative frequency})$ where x is the data values at the end points of each class, joined with straight lines.

20 Reading values from a cumulative frequency graph

The cumulative frequency polygon for the birth weights of 50 children is given.



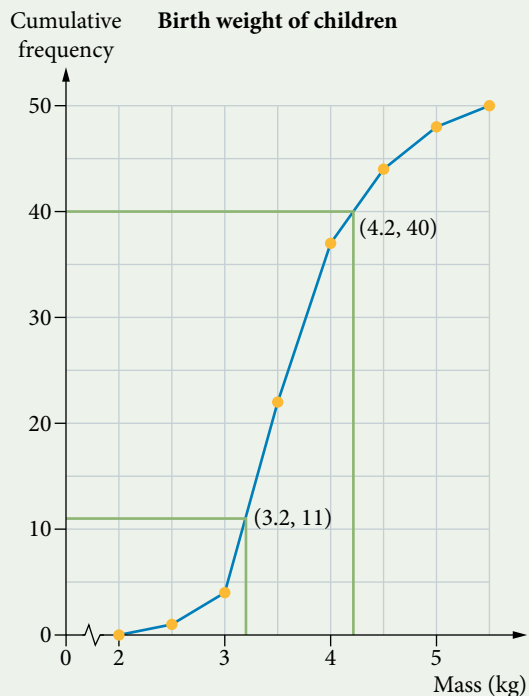
Estimate the number of children with a birth weight:

- (a) less than 3.2 kg (b) more than 4.2 kg

THINKING

- 1 Draw a line up from 3.2 kg and across to the cumulative frequency axis.

WORKING



- 2 Interpret the result.

The point $(3.2, 11)$ means that 11 children weighed less than 3.2 kg at birth. The point $(4.2, 40)$ means that 40 children weighed less than 4.2 kg at birth.

Therefore there were $50 - 40 = 10$ children who weighed more than 4.2 kg at birth.

Estimating the five-figure summary from a graph

You have just seen that the median and quartiles can be read from a cumulative frequency graph. The five-figure summary requires the minimum and maximum values as well. For both extreme values, assume that the data goes to the lower end of the first class and the higher end of the last class.

WARNING

For large amounts of grouped data you can read off the quartiles at $0.25n$, $0.5n$ (the median) and $0.75n$.

Without the raw data your summary will always contain estimated values only.

21 Five-figure summary from a cumulative frequency graph

Draw a cumulative frequency polygon and construct a box plot for the heights of a sample of 24 girls in Year 11.

Height (cm)	Frequency
155 – <160	2
160 – <165	8
165 – <170	10
170 – <175	4

THINKING

- 1 Insert a cumulative frequency column.

WORKING

Classes	f	x	Cumulative frequency
		155	0
155 – <160	2	160	2
160 – <165	8	165	10
165 – <170	10	170	20
170 – <175	4	175	24

- 2 Identify the positions of the median and quartiles.

Positions of the quartiles:

$$Q_1: 0.25n = 0.25 \times 24 = 6$$

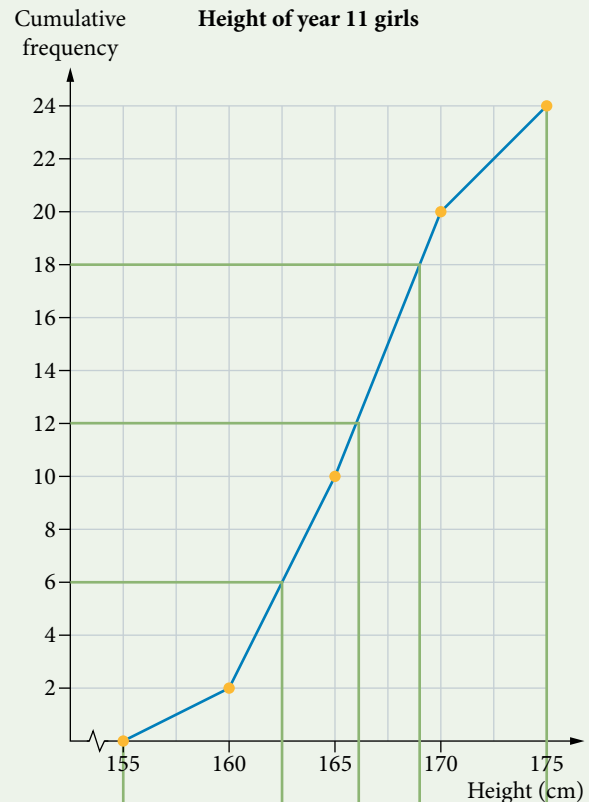
$$\text{Median: } 0.5n = 0.5 \times 24 = 12$$

$$Q_3: 0.75n = 0.75 \times 24 = 18$$

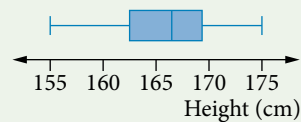
- 3 Use grid paper to draw the graph of cumulative frequency against data values, x .

Read across from the positions of the quartiles (dividing the cumulative frequency into four equal amounts) and down to the data axis.

Construct a box plot from five points on the data axis.



- 4 Construct the box plot.
(The scale line is the same as the data axis on the graph.)



e Explore further

Cumulative frequency

Use Excel to create cumulative frequency tables and curves.

EXERCISE

7.5

Cumulative frequency

Worked
Example

18

- 1 The table gives the birth weights of 43 baby elephants.
- Insert a cumulative frequency column into the table.
 - Determine the number of baby elephants born with a birth weight less than 110 kg.
 - Determine the number of baby elephants born with a birth weight more than 100 kg.



Birth weight (kg)	Number of baby elephants
90–<95	2
95–<100	3
100–<105	14
105–<110	10
110–<115	10
115–<120	4

- 2 The table gives the height of 10 six-year-old boys.
- Determine the number of boys who are less than 118 cm tall.
 - 4
 - 6
 - 9
 - 24

Height (cm)	Cumulative frequency
105	0
110	1
115	4
120	9
125	10

- Determine the number of boys who are between 110 cm and 115 cm tall.
 - 1
 - 2
 - 3
 - 5

- What error has been made by a student who chose the last of the incorrect options in part (b)?

- 3 The table gives the weight of 18 mountain bikes.
- Insert a cumulative frequency column into the table.
 - Determine the number of mountain bikes weighing less than 12 kg.
 - Determine the number of mountain bikes weighing more than 13 kg.

Weight (kg)	Number of mountain bikes
10.5–<11	2
11–<11.5	2
11.5–<12	2
12–<12.5	7
12.5–<13	2
13–<13.5	1
13.5–<14	2

- 4 The table gives the number of siblings for a group of students.
- (a) The median and quartiles, in ascending order, are:
- A 0, 2, 3 B 0, 1, 2
C 1, 3, 4 D 1, 1, 3
- (b) Determine the number of siblings in the given data that was not true for any of the students in the class. Explain how you know this.
- (c) Determine the number of students who have 2 siblings.

Number of siblings	Cumulative frequency
0	7
1	15
2	20
3	24
4	24
5	26

- 5 The table gives the number of cars in the household for a group of teachers.
- (a) Determine the median and quartiles.
- (b) Determine the modal number of cars per teacher household.
- (c) Determine the number of teacher households with more than 2 cars.

Number of cars	Cumulative frequency
0	2
1	20
2	40
3	48
4	50
5	51

- 6 The table gives the number of crocodiles thought to inhabit a number of waterholes along the Daintree River.
- (a) Determine the median and quartiles.
- (b) Determine the modal number of crocodiles per waterhole.
- (c) Determine the number of waterholes with more than 5 crocodiles.

Number of crocodiles	Cumulative frequency
0	1
1	11
2	15
3	28
4	38
5	45
6	60
7	68
8	70

Worked
Example

19

- 7 The frequency distribution gives the number of students per class in a high school.
- (a) Insert a cumulative frequency column into the table.
- (b) Determine the number of classes at the school with no more than 23 students.
- (c) Determine the five-figure summary for the data.
- (d) Comment on the shape of the data.

Number of students	Frequency
22	10
23	8
24	8
25	9
26	10

- 8 The frequency distribution gives the number of days of rain in Townsville each month in a particular year.

- Insert a cumulative frequency column into the table.
- Determine the number of months that have at least 10 days of rain.
- Determine the five-figure summary for the data.

Number of days of rain	Frequency
2	2
3	1
4	1
5	1
6	2
7	1
9	1
13	1
14	1
16	1

- 9 The frequency distribution gives the number of dogs registered in households for a suburb in Ipswich.

- Insert a cumulative frequency column into the table.
- Determine the number of households in the suburb that have no more than two dogs registered.
- Determine the five-figure summary for the data.
Use $\frac{n}{2}$ to locate the median and $\frac{n}{4}$, $\frac{3n}{4}$ to locate the respective quartiles.

Number of dogs	Number of households
0	1142
1	419
2	357
3	31
4	24

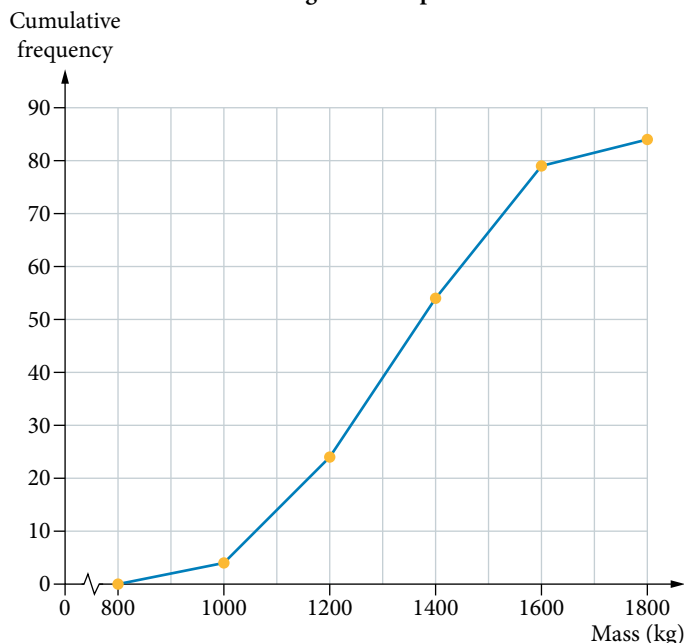
- Comment on the shape of the distribution, suggesting reasons for this shape.

Worked
Example

- 10 The cumulative frequency polygon for the birth weights of 84 chimpanzees is given.

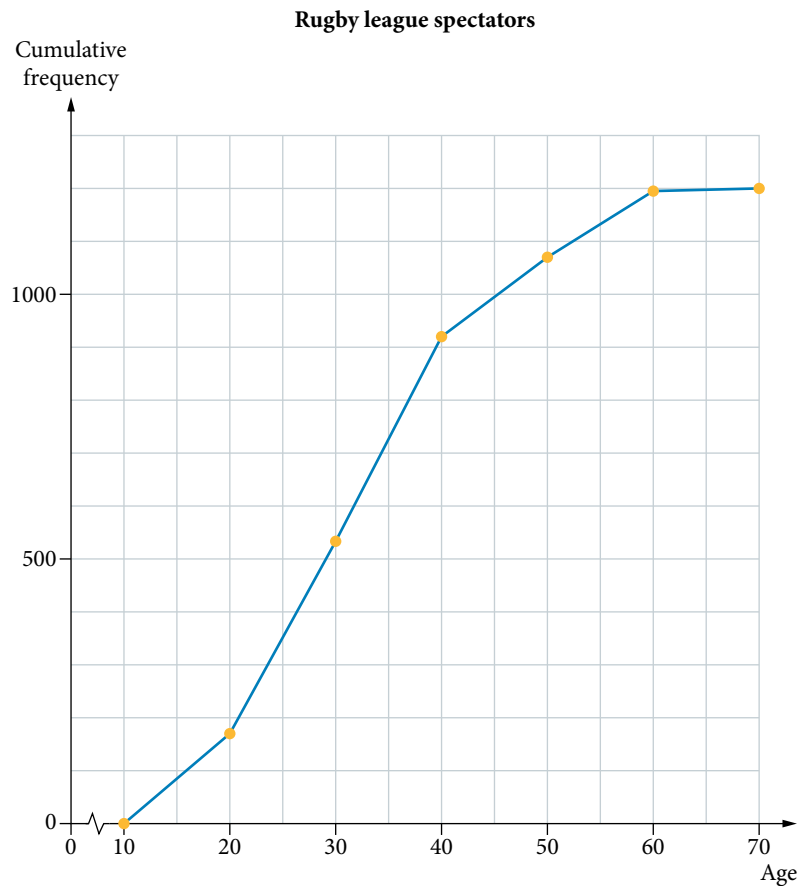
- Estimate the number of chimpanzees born weighing less than 1500 g.
- Estimate the number of chimpanzees born weighing more than 1100 g. What percentage is this, to the nearest per cent?
- Estimate the upper quartile for the birth weights and interpret what this means.
Use the rough estimate for quartile position, $\frac{3}{4}n$.

Birth weight of chimpanzees



20

- 11 The cumulative frequency polygon for the ages of 1200 spectators at a rugby league match is given. Estimate the five-point summary from the graph.



..



- 12 The table gives the number of towns (some now cities) established in Australia over the first 100 years after the arrival of the First Fleet in 1788.
- (a) Draw the cumulative frequency polygon for the data.
 - (b) Estimate the time it took for 100 towns to be established.
 - (c) Brisbane was established in 1825. Estimate the number of towns that preceded Brisbane.
 - (d) Determine the percentage of towns, to the nearest per cent, that were established in the first 50 years.

Years since 1788	Number of towns
0 – < 10	4
10 – < 20	6
20 – < 30	4
30 – < 40	10
40 – < 50	26
50 – < 60	26
60 – < 70	34
70 – < 80	28
80 – < 90	26
90 – < 100	28

- 13 Draw a cumulative frequency graph and construct a box plot for each of the monthly averages for Brisbane weather. Use the simple estimates from frequencies of $0.25n$, $0.5n$, $0.75n$ for reading the quartiles (and median) from the graph.

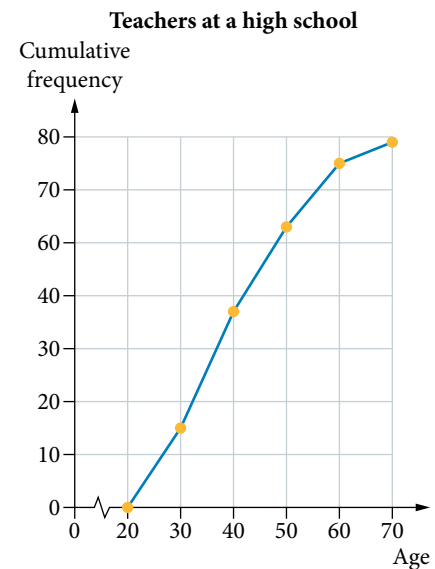
(a)

Rainfall (mm)	Frequency
40 – < 60	3
60 – < 80	3
80 – < 100	2
100 – < 120	0
120 – < 140	1
140 – < 160	3

(b)

Temperature (°C)	Frequency
20 – < 22	3
22 – < 24	1
24 – < 26	1
26 – < 28	3
28 – < 30	4

- 14 The cumulative frequency polygon gives the ages of teachers at a high school.
Construct a histogram for the data and read the modal age group from your graph.



- 15 The data table gives the monthly average hours of sunshine for Brisbane.

- (a) Draw a cumulative frequency graph and construct a box plot for the data. Use the simple estimates from frequencies of $0.25n$, $0.5n$, $0.75n$ for reading the quartiles (and median) from the graph.
- (b) If you had the data of hours of sunshine each day for an entire year, you could draw a box plot of this data. Alter both your cumulative frequency graph and your box plot to an estimated view of what this might look like. Justify your choice of values, including the use of outliers.

Hours of sunshine	Frequency
6.5 – < 7	4
7 – < 7.5	3
7.5 – < 8	1
8 – < 8.5	4

7.6

Mean and standard deviation

Statistics that use all the data

In the previous section you described data using the five-figure summary. The median and interquartile range are unaffected by values in the highest 25% of data values and the lowest 25%. In this section you will concentrate on mean and standard deviation, measures of centre and spread that involve, and are affected by, all data values.

Mean

To calculate the mean ($\bar{x}$) of a data set, use the formula:

$$\begin{aligned}\bar{x} &= \frac{\text{sum of the data values}}{\text{number of data values}} \\ &= \frac{\sum x}{n}\end{aligned}$$

To calculate the mean of data in a frequency table, use the formula:

$$\bar{x} = \frac{\sum xf}{\sum f}, \text{ where } \Sigma \text{ (capital Greek letter sigma) stands for 'the sum of' and } f \text{ stands for 'frequency'}$$

i Additional information

Mean of discrete data

Revise your knowledge of calculating the mean of discrete data.

22 Calculating the mean

Calculate the mean of the following data sets. Give your answer to 1 decimal place, if necessary.

- (a) Number of cars that passed the school gate in a number of 5 minute periods:

10, 5, 12, 13, 14, 7, 12, 15, 7, 3, 2, 8

THINKING

- 1 Calculate the mean by adding up the data values and then dividing by the number of data values.

- 2 Interpret the answer, writing your response in a sentence.

WORKING

$$\begin{aligned}\bar{x} &= \frac{\sum x}{n} \\ &= \frac{10 + 5 + 12 + 13 + 14 + 7 + 12 + 15 + 7 + 3 + 2 + 8}{12} \\ &= \frac{108}{12} \\ &= 9\end{aligned}$$

The mean number of cars was 9.

- (b) Number of brothers and sisters of the students in two Year 11 classes.

No. brothers and sisters (x)	Frequency (f)
0	7
1	11
2	15
3	3
4	1

- 1 Add a new column to the frequency table, label it xf (x stands for the data value, f for frequency), and fill it in by multiplying the data values (in this case, the number of brothers and sisters) by the frequency for that value.

No. brothers and sisters (x)	Frequency (f)	xf
0	7	$0 \times 7 = 0$
1	11	$1 \times 11 = 11$
2	15	$2 \times 15 = 30$
3	3	$3 \times 3 = 9$
4	1	$4 \times 1 = 4$

- 2 Add up the frequency column. This gives the number of data values.

Add up the xf column. This shows the sum of the data values.

$\Sigma f = 7 + 11 + 15 + 3 + 1$ $= 37$	$\Sigma xf = 0 + 11 + 30 + 9 + 4$ $= 54$

- 3 Substitute the values into the rule for determining the mean of data in a frequency table.

$$\begin{aligned}\bar{x} &= \frac{\Sigma xf}{\Sigma f} \\ &= \frac{54}{37} \\ &= 1.46 \text{ (2 d.p.)}\end{aligned}$$

- 4 Interpret the result, writing the answer in a sentence.

The mean number of brothers and sisters was 1.46.

- 5 Check that the answer is reasonable.

The mean is within the data values from 0 to 4, so it is reasonable.

Grouped data

The mean for grouped data can be estimated by treating the midpoint of each class as the representative of that class.

To calculate the mean of grouped data, use the formula:

$\bar{x} = \frac{\Sigma x_m f}{\Sigma f}$ where the mid-point (x_m) of the class interval represents the data value.

i Additional information

Mean of continuous data

Revise your knowledge of calculating the mean of continuous data.

23 Mean of grouped data

Determine the mean of the data, which represents the time in seconds taken by competitors in the under-17 100 m dash at the school athletics carnival. Give your answer correct to 2 decimal places, if necessary.

Time (s)	Frequency (f)
12 – < 12.5	3
12.5 – < 13	5
13 – < 13.5	12
13.5 – < 14	28
14 – < 14.5	10

THINKING

- 1 Add two new columns to the frequency table, one labelled x_m and the other labelled $x_m f$.

The values for x_m are calculated by adding together the end points of the class interval and dividing by 2 (calculating the mean/median of the interval).

- 2 Add up the frequency column. This gives the number of data values.

Add the $x_m f$ column. This gives the sum of the data values.

- 3 Substitute the values into the rule to calculate the mean of the grouped data.

- 4 Interpret the result, writing the answer in a sentence.

- 5 Check that the answer is reasonable.

WORKING

Time (s)	Frequency (f)	x_m	$x_m f$
12 – < 12.5	3	12.25	$3 \times 12.25 = 36.75$
12.5 – < 13	5	12.75	$5 \times 12.75 = 63.75$
13 – < 13.5	12	13.25	$12 \times 13.25 = 159$
13.5 – < 14	28	13.75	$28 \times 13.75 = 385$
14 – < 14.5	10	14.25	$10 \times 14.25 = 142.5$

$$\Sigma f = 3 + 5 + 12 + 28 + 10 = 58$$

$$\Sigma x_m f = 787$$

$$\begin{aligned}\bar{x} &= \frac{\Sigma x_m f}{\Sigma f} \\ &= \frac{787}{58} \\ &= 13.57 \text{ s}\end{aligned}$$

The mean time for the race was 13.57 s.

The mean is within the data values from 12 to 14.5, so it is reasonable.

Scientific calculators and summary statistics

You can calculate the values of $\bar{x}$, Σx or n . Note that the calculator will give n rather than Σf , and Σx rather than Σxf .

To calculate summary statistics:

STEP 1: Clear previous data first.

STEP 2: In statistics mode, choose whether you have single data or a frequency distribution. If there is no frequency column you can turn the frequency off, or allow the calculator to insert frequency of 1 for each data value.

STEP 3: Exit the data entry screen to use function keys, to read the summary statistics.

STEP 4: Go online if you do not know how to access the statistics for your calculator.

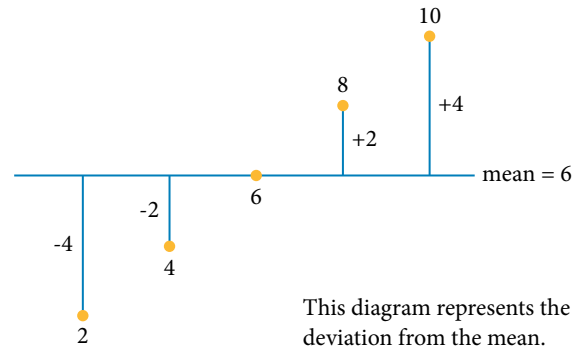
Standard deviation

The *standard deviation* is a measure of the spread of data relative to the mean. Because the standard deviation considers all data values, it is not influenced by outliers as much as the range.

Consider this very simple data set: 2, 4, 6, 8, 10.

Here, the mean is:

$$\begin{aligned}\bar{x} &= \frac{\sum x}{n} \\ &= \frac{2 + 4 + 6 + 8 + 10}{5} \\ &= 6\end{aligned}$$



The (signed) differences between each data value and the mean, in order from left to right, are: -4, -2, 0, 2, 4. The sum of these differences is always 0.

x	$(x - \bar{x})$
2	$2 - 6 = -4$
4	$4 - 6 = -2$
6	$6 - 6 = 0$
8	$8 - 6 = 2$
10	$10 - 6 = 4$
	$\sum(x - \bar{x}) = -4 + -2 + 0 + 2 + 4 = 0$

To avoid the sum of the deviations equalling 0, square the deviations to make them all positive: 16, 4, 0, 4, 16.

x	$(x - \bar{x})$	$(x - \bar{x})^2$
2	$2 - 6 = -4$	$(-4)^2 = 16$
4	$4 - 6 = -2$	$(-2)^2 = 4$
6	$6 - 6 = 0$	$(0)^2 = 0$
8	$8 - 6 = 2$	$(2)^2 = 4$
10	$10 - 6 = 4$	$(4)^2 = 16$
	$\sum(x - \bar{x}) = 0$	$\sum(x - \bar{x})^2 = 16 + 4 + 0 + 4 + 16 = 40$

Now you must find the mean of the squared deviations. Normally you would divide by n to find the mean, but because you will be dealing with sample data in this course, divide by $n - 1$ to make the value slightly larger, giving the best possible estimate for the actual value if all the data was available to you.

Hence, the 'adjusted' mean of these squared deviations is:

$$\begin{aligned}\frac{\sum(x - \bar{x})^2}{n - 1} &= \frac{16 + 4 + 0 + 4 + 16}{4} \\ &= \frac{40}{4} \\ &= 10\end{aligned}$$

Now calculate the square root of this value (to undo the squaring you did earlier):

$$\begin{aligned}s &= \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}} \\ &= \sqrt{10} \\ &= 3.16 \text{ (2 d.p.)}\end{aligned}$$

So the standard deviation has a value of 3.16, correct to 2 decimal places.

Examine the effect of increasing the spread of the data from the mean by changing the datum 10 to 100.

The mean is:

$$\begin{aligned}\bar{x} &= \frac{\sum x}{n} \\ &= \frac{2 + 4 + 6 + 8 + 100}{5} \\ &= 24\end{aligned}$$

$$\begin{aligned}s &= \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}} \\ &= \sqrt{\frac{7240}{4}} \\ &= 42.54 \text{ (2 d.p.)}\end{aligned}$$

x	$(x - \bar{x})$	$(x - \bar{x})^2$
2	$2 - 24 = -22$	$(-22)^2 = 484$
4	$4 - 24 = -20$	$(-20)^2 = 400$
6	$6 - 24 = -18$	$(-18)^2 = 324$
8	$8 - 24 = -16$	$(-16)^2 = 256$
100	$100 - 24 = 76$	$(76)^2 = 5776$
	$\sum(x - \bar{x}) = 0$	$\sum(x - \bar{x})^2 = 484 + 400 + 324 + 256 + 5776 = 7240$

The standard deviation of the data set is 42.54. This value tells you that the data is more spread out in comparison to the previous data set, which has a standard deviation of 3.16.

And lastly, examine the effect of decreasing the spread of the data from the mean by changing the datum 10 to 5.

The mean is:

$$\begin{aligned}\bar{x} &= \frac{\sum x}{n} \\ &= \frac{2 + 4 + 6 + 8 + 5}{5} \\ &= 5\end{aligned}$$

$$\begin{aligned}s &= \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}} \\ &= \sqrt{\frac{20}{4}} \\ &= 2.24 \text{ (2 d.p.)}\end{aligned}$$

x	$(x - \bar{x})$	$(x - \bar{x})^2$
2	$2 - 5 = -3$	$(-3)^2 = 9$
4	$4 - 5 = -1$	$(-1)^2 = 1$
6	$6 - 5 = 1$	$(1)^2 = 1$
8	$8 - 5 = 3$	$(3)^2 = 9$
5	$5 - 5 = 0$	$(0)^2 = 0$
	$\sum (x - \bar{x}) = 0$	$\sum (x - \bar{x})^2 = 9 + 1 + 1 + 9 + 0 = 20$

By decreasing the spread of the data (by decreasing the value of the maximum data value), the standard deviation also decreased from the original standard deviation of 3.16.

The value of the standard deviation by itself does not tell you very much. When you compare similar sets of data, the set with the higher standard deviation will have data that is more spread out.

You will be using a calculator for all standard deviation calculations, so the procedure above will not need to be memorised.

The *population standard deviation*, represented by the Greek letter sigma σ , uses all data values in a population.

The *sample standard deviation*, represented by s , represents the standard deviation of a sample taken from the population. The sample standard deviation is the greater value on your calculator as it predicts the spread of the population to be greater.

For the purpose of this course, you will be calculating the sample standard deviation.

When using technology, the population standard deviation is shown as σx and the sample standard deviation is shown as sx .

24 Using technology to calculate the standard deviation

The results of a survey show the number of siblings for each member of the class.

1, 3, 4, 2, 1, 0, 3, 1, 2, 4, 0, 1, 0, 2, 1, 0, 1, 2, 2, 3

Use your calculator to determine the mean and standard deviation, correct to 2 decimal places.

THINKING

1

Clear previous data.
Turn the frequency off.
Enter the single data as x values.
Use function keys to find all relevant summary values: n , $\bar{x}$, σx , sx .

WORKING

$n = 20$
 $\bar{x} = 1.65$
 $\sigma x = 1.23592\dots$
 $sx = 1.26802\dots$

The mean number of siblings for the class is 1.65 with a standard deviation of 1.27.

The mean is within the data values from 0 to 4, so it is reasonable. The standard deviation is less than half of the range ($\text{range} = 4 - 0 = 4$) as expected.

Explore further

Standard deviations

Calculate and compare the mean and standard deviation of heights of groups of students.

25

Calculating the mean and standard deviation of grouped data using technology

Members of a running club keep track of the distance, to the nearest whole kilometre, covered in training over the course of a week. The frequency table summarises the data collected. Calculate the estimated mean and the standard deviation for the club.

Distance covered (nearest km)	No. runners
21–30	3
31–40	5
41–50	4
51–60	2
61–70	1
71–80	1

THINKING

1

Calculate the midpoint for each class interval.
Label this column x_m , to stand for the data value.

WORKING

Distance covered (nearest km)	No. runners (f)	x_m
21–30	3	25.5
31–40	5	35.5
41–50	4	45.5
51–60	2	55.5
61–70	1	65.5
71–80	1	75.5

2 Clear previous data.	$n = 16$
Turn the frequency on.	$\bar{x} = 43$
Enter the single data as x values.	$\sigma x = 13.9194\dots$
Use function keys to find all relevant summary values: n , $\bar{x}$, σx , sx .	$sx = 14.3759\dots$
3 Interpret the result, choosing the sample standard deviation sx .	The estimated mean distance covered is 43 km with a standard deviation of 14.38 km.

Effect of changes to data on standard deviation

Calculations for the standard deviation and the mean both use every piece of data, so each are affected by any change to the data.

26 Investigating the standard deviation

The number of goals Eloise scored in 15 games of a netball competition was recorded.

22, 27, 32, 48, 29, 36, 29, 41, 44, 29, 38, 26, 39, 42, 40

- (a) Calculate the mean and the standard deviation for the data. Give the values rounded to 2 decimal places, if necessary.

THINKING

- 1 Enter the data into your technology and obtain the values in the normal way.

WORKING

$$\begin{aligned} n &= 15 \\ \bar{x} &= 34.8 \\ \sigma x &= 7.377\dots \\ sx &= 7.636\dots \end{aligned}$$

- 2 Write the answer in a sentence, assuming the sample value for standard deviation.

In the first 15 games Eloise has a mean score of 34.8 goals with a standard deviation of 7.64.

- (b) In the following week, Eloise is injured early in the game and scores only 4 goals. Determine the new mean and standard deviation and comment on the changes.

- 1 Add the new value to the end of your data list and read off the new values.

$$\begin{aligned} n &= 16 \\ \bar{x} &= 32.875 \\ \sigma x &= 10.325\dots \\ sx &= 10.663\dots \end{aligned}$$

- 2 Write the answer in a sentence, assuming the sample value for standard deviation.

After 16 games Eloise has a mean score of 32.88 goals with a standard deviation of 10.66.

- 3 Comment on whether the value of each statistic increased or decreased, and why.

The mean decreased because Eloise scored fewer goals than her previous average. The standard deviation increased because the latest score produced a greater spread of data values.

EXERCISE

7.6

Mean and standard deviation

Worked
Example

22

- 1 Calculate the mean of the following data sets.

Do not use the STAT function on your calculator, except to check your answers. Where necessary, state your answers to 1 decimal place.

- (a) The following data represents the number of cars that passed the school gate in a number of 5 minute periods.

7, 14, 12, 11, 13, 16, 17, 9, 7, 12, 13, 15

- (b) The data in the table represents the number of brothers and sisters of the students in two Year 11 classes.

No. brothers and sisters (x)	Frequency (f)
0	6
1	11
2	18
3	9
4	3

- 2 Calculate the mean of each of the following discrete data sets. Where necessary, state your answers to 1 decimal place.

(a)

Score	Frequency
4	15
5	23
6	14
7	23
8	17
9	19
10	20

(b)

Score	Frequency
201	49
202	83
203	99
204	121
205	143
206	117

(c)

x	f
1	5
2	8
3	3
4	2
5	4
6	5

- 3 Use your calculator to determine the mean, median and mode for each of the following data sets.

(a) 2, 3, 4, 5, 6, 6, 7, 8, 9, 10

(b) 2, 2, 2, 2, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13

(c) 2, 3, 4, 5, 6, 7, 8, 9, 21

(d) 2, 3, 4, 5, 6, 7, 11, 11

24

- 4 You conduct a survey of your class to find the number of pets each student has.

Number of pets: 0, 4, 1, 1, 6, 3, 8, 4, 2, 0, 0, 0, 1, 1, 4, 6, 2, 3, 4, 1

Use your calculator to determine the mean and standard deviation for the data.

- 5 Calculate the standard deviation for the following data sets to 1 decimal place.

(a) 3, 3, 4, 6, 2, 1, 3, 4, 6, 7, 5, 3, 2, 1, 7, 9

(b) 25, 21, 22, 25, 26, 29, 28, 22, 24, 25, 27, 26, 30

(c) 7, 5, 2, 4, 7, 8, 11, 13, 16, 15, 11, 16, 17, 18, 13, 12

(d) 15, 19, 22, 28, 21, 23, 26, 27, 21, 20, 17, 13, 18, 17

- (g)

(h)

Worked Example

- 26

- •

- 23

Chapter 7 Univariate data analysis 463

- 10 The table gives the maximum temperature ($^{\circ}\text{C}$) reached on a particular day in a number of cities around the world. Answer the following questions about this data set.

City	Temp ($^{\circ}\text{C}$)	City	Temp ($^{\circ}\text{C}$)
Amsterdam	3	Los Angeles	14
Athens	18	Madrid	6
Barcelona	9	Moscow	1
Belgrade	3	New York	13
Budapest	1	Oslo	-3
Cairo	34	Prague	0
Chicago	13	Singapore	33
Copenhagen	0	Taipei	27
Helsinki	-4	Toronto	10
Istanbul	18	Warsaw	-1
London	5		

- (a) Calculate the mean of the temperatures. Give your answer to 1 decimal place.
- (b) Draw a grouped data frequency table using $-5 - < 0$ as the first class interval. Use the frequency table to find an estimate for the mean temperature and comment on the result. Give your answer to 1 decimal place.

Worked
Example

25

- 11 The data set represents the length of a sample of fish, measured to the nearest centimetre, caught by a commercial fishing boat.

Use technology to find the mean and standard deviation, to 2 decimal places.

Length of fish (nearest cm)	Number
20–29	8
30–39	12
40–49	28
50–59	38
60–69	54
70–79	49
80–89	21
90–99	7

- 12 A teacher recorded the results for a test, marked out of 50.

Test results: 26, 29, 31, 32, 34, 36, 36, 39, 40, 40, 41, 42, 43, 44, 46, 46, 47

Before the tests were returned, a student who had been absent sat the test, and scored 27.

- (a) Which of the following statements correctly describes what would happen to the class statistics?
- A The standard deviation and mean would both decrease.
- B The standard deviation and mean would both increase.
- C The standard deviation would increase and the mean would decrease.
- D The standard deviation would decrease and the mean would increase.
- (b) Explain the common error made by a student who chose the first incorrect option.

- (a)

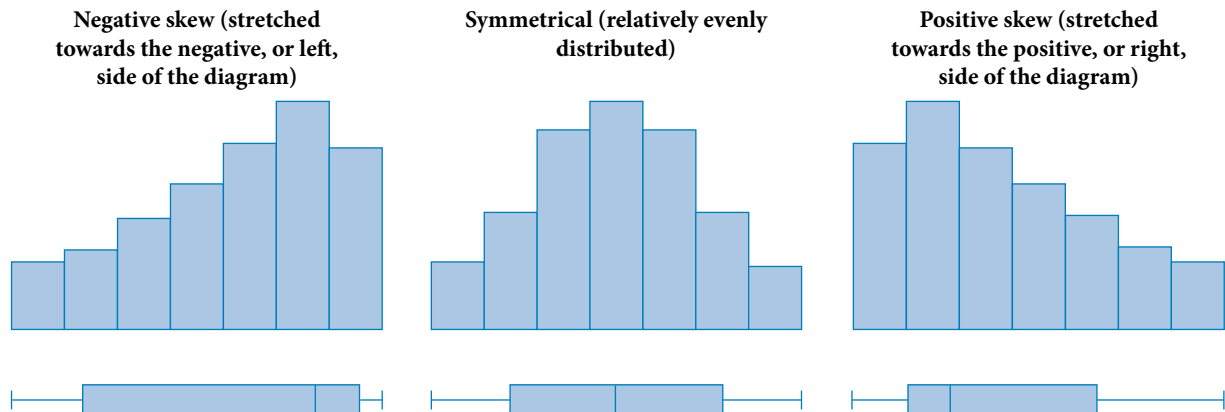
(b)

• • • • • • • • • • •

- 465

Statistical displays

Statistical displays are used to analyse data and to determine the shape of the data distribution. A data set can be symmetrical or skewed in the negative (left) or positive (right) direction. The histograms and box plots below illustrate these terms.



If data is skewed, the mean is located towards the tail (or skew) of the distribution.

Negatively skewed data has mean $<$ median.
Positively skewed data has mean $>$ median.

i Additional information

Skewed graphs

Consolidate your visual understanding of skewed graphs.

Analysing a data set

When you analyse a set of data, try to use any summaries you have been able to make, and describe the data according to the summaries.

To analyse a data set, include the following:

- 1 Centre
 - Mean
 - Median
- 2 Spread
 - Range (max – min)
 - IQR ($Q_3 - Q_1$) in which the middle 50% of data lies
 - Standard deviation
- 3 Shape
 - Positively skewed, negatively skewed or approximately symmetrical
- 4 Presence of outliers

27 Describing a data set

The following data set represents the marks obtained on a test out of 50, by a class of 20 students.

14, 26, 49, 45, 46, 23, 24, 25, 25, 48, 49, 28, 33, 35, 37, 38, 38, 39, 41, 43

Draw a box plot and describe the data set, including a comment about its shape.

THINKING

- 1 Determine the five-figure summary, the mean and the standard deviation.
- 2 Determine whether there are any outliers in the data set.
- 3 Construct a box plot.
- 4 Describe the data set.

WORKING

Minimum = 14, $Q_1 = 25.5$, Median = 37.5, $Q_3 = 44$,
Maximum = 49.

Mean = 35.3, Standard deviation = 10.15

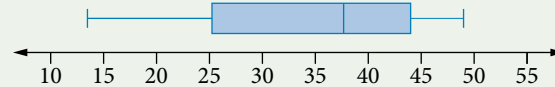
$IQR = 44 - 25.5$
 $= 18.5$

$Q_1 - 1.5 \times IQR = 25.5 - 1.5 \times 18.5$
 $= -2.250$

$Q_3 + 1.5 \times IQR = 44 + 1.5 \times 18.5$
 $= 71.75$

All marks are in the interval from -2.25 to 71.75.

Therefore, there are no outliers.



The data is slightly negatively skewed, with a mean of 35.3 and 50% of the students achieving 37.5 or more. The middle half of students scored between 25.5 and 44.

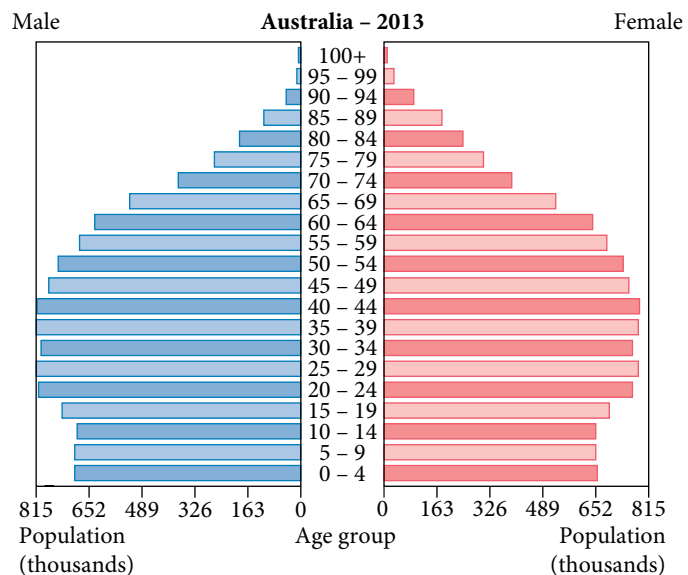
Composite bar charts

A *composite bar chart* allows two data sets to be visually compared. The example at right shows a comparison of a population based on gender and age. This type of graph is sometimes referred to as a population pyramid.

i Additional information

Double bar graph

View the demonstration of how to compare two sets of data as a double bar graph.



Back-to-back stem plots

Another way of comparing two data sets is by using a *back-to-back stem plot*. These have a central stem with the leaves moving away from the stem in either direction.

28 Back-to-back stem plots

The following data gives the heights, measured to the nearest centimetre, of members of two different hockey teams.

Team A: 155, 158, 159, 161, 164, 164, 165, 166, 175, 176, 178, 179

Team B: 151, 154, 156, 157, 160, 168, 172, 174, 174, 176, 183, 186, 190

Draw a back-to-back stem plot of the data. Comment on the comparative heights of the two teams.

THINKING

- 1 Enter the stems in order.

Place the leaves so that both sides have the lowest leaf values next to the stem column.
So the left-hand data set is written right to left, but the right-hand data set is written left to right.

- 2 Comment, describing centre, spread, maximum and minimum values.

WORKING

Team A		Team B
9 8 5	15	1 4 6 7
6 5 4 4 1	16	0 8
9 8 8 6 5	17	2 4 4 6
	18	3 6
	19	0

$$17 \mid 9 = 179 \text{ cm}$$

Team B heights are more diverse. They have both the tallest and the shortest players of the two teams.
Overall Team B would appear to be taller, with three players over 180 cm, whereas Team A has none.

Parallel box plots

A *parallel box plot* draws two or more box plots on the same scale.

i Additional information

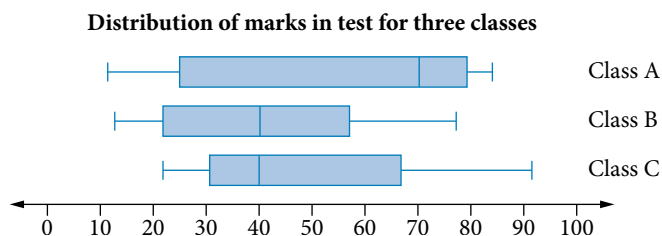
Parallel box plots

Watch the video for a demonstration of how to create and compare box plots. Then try the activity.

29 Comparing box plots

The following parallel box plots show the distribution of marks in a test for three classes.

Compare the performance of the three classes based on the statistics given in the box plots.



THINKING

Write a comparative description of the data sets, mentioning as many relevant points as you can.

Recall that 25% of the data is in each section of a box plot.

WORKING

The box plots indicate that Class A is negatively skewed. Class B is close to symmetrical and Class C is positively skewed.

The highest mark was in Class C and the lowest was in Class A.

At least 50% of Class A obtained 70 or more, whereas less than 25% of Class B or Class C obtained a score greater than 70.

Classes B and C have the same median mark of 40, which is much less than the median mark for Class A (70).

30 Drawing parallel box plots

Describe the performance of each class using statistics shown in the box plots.

Class A: 22, 25, 34, 12, 49, 50, 44, 48, 27, 32, 42, 44, 28, 50

Class B: 33, 37, 30, 40, 48, 49, 50, 42, 46, 21, 17, 18, 22, 25

THINKING

- 1 For each set, enter the data into your technology and list the five-figure summary.

- 2 Determine the presence of any outliers.

- 3 Draw the box plot for each, using the same axis. Label the box plots.

- 4 Describe the data set, using any of the five key values to compare.
- Recall that 25% of the data is in each section of a box plot.

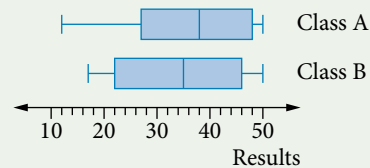
WORKING

Class A: Min = 12, $Q_1 = 27$, Median = 38, $Q_3 = 48$, Max = 50.

Class B: Min = 17, $Q_1 = 22$, Median = 35, $Q_3 = 46$, Max = 50.

$$\begin{aligned} \text{Class A: } A_{LF} &= Q_1 - 1.5 \times IQR & A_{UF} &= Q_3 + 1.5 \times IQR \\ &= 27 - 1.5 \times (48 - 27) & &= 48 + 1.5 \times (48 - 27) \\ &= -4.5 & &= 79.5 \\ \text{Class B: } B_{LF} &= Q_1 - 1.5 \times IQR & B_{UF} &= Q_3 + 1.5 \times IQR \\ &= 22 - 1.5 \times (46 - 22) & &= 46 + 1.5 \times (46 - 22) \\ &= -14 & &= 82 \end{aligned}$$

Neither data set contains an outlier.



Class A and Class B both have the highest mark of 50, and Class A had the lowest mark of 12. Class A's top 25% of students all had similar results, whereas the lowest 25% of results are quite spread out. Of Class A, 75% scored 27 or more, compared to a score of 22 or more for Class B.

Mean and standard deviation

When mean and standard deviation are known for comparative sets of data, your analysis can compare the centre and spread of the data.

31 Mean and standard deviation

The heights of players in two teams of Australian netball players is summarised as:

Thunderbirds: mean = 183.3 cm, SD = 6.1 cm

Lightning: mean = 180.9 cm, SD = 9.3 cm

Compare the overall height distribution of the two teams.

THINKING

1 Compare the data sets, using the means.

2 Compare the data sets, using the standard deviations.

WORKING

While both teams have players generally taller (on average more than 180 cm) than Australian women generally, the Thunderbirds are, on average $183.3 - 180.9 = 2.4$ cm taller than Lightning players.

There is much greater variation in heights of Lightning players.

It is possible to use mean and standard deviation to estimate an equivalent result in one set of data, given the result in another related data set.

The expression $\frac{(x - \bar{x})}{s}$ gives the number of standard deviations above (or below) the mean for an individual score, x .

32 Comparing individual scores in data sets

The results for a Term 1 problem-solving test are:

19, 6, 14, 20, 14, 19, 10, 10, 17, 18, 15, 15, 16, 17, 16, 5, 20, 13, 12, 9, 4, 15, 5

Chloe was absent for the Term 2 problem-solving test, with the results for the others being:

15, 5, 14, 13, 14, 11, 12, 16, 16, 15, 7, 12, 13, 12, 7, 16, 12, 13, 9, 5, 11, 9

If Chloe's result is the third value in the first list, use mean and standard deviation to estimate a score for Chloe for the second test.

THINKING

1 Enter the data into your technology and write down the mean and sample standard deviation for each test.

Remove Chloe's mark from the first list so the results match the same group of people each time.

WORKING

Term 1 (without Chloe's result): 19, 6, 20, 14, 19, 10, 10, 17, 18, 15, 15, 16, 17, 16, 5, 20, 13, 12, 9, 4, 15, 5

mean = 13.4, SD = 5.1

Term 2: 15, 5, 14, 13, 14, 11, 12, 16, 16, 15, 7, 12, 13, 12, 7, 16, 12, 13, 9, 5, 11, 9

mean = 11.7, SD = 3.4

2 Determine how many SDs Chloe's mark is above the mean.	SDs above the mean: $\frac{(x_1 - \bar{x}_1)}{s} = \frac{14 - 13.4}{5} = 0.12$
3 Estimate a result for Chloe in the second test. Ensure the result is the same number of SDs above the new mean.	Chloe's estimated score: $11.7 + 0.12 \times 3.4 = 12.1$
4 Interpret the answer.	The best estimate for Chloe's result in the Term 2 test is 12.

e Explore further

Comparing data

Create and compare summary statistics in a spreadsheet.

EXERCISE

7.7

Comparing data

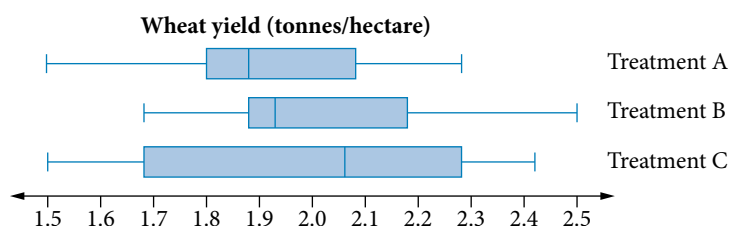
Worked
Example

- 1 The following data set represents the marks obtained on a test, out of 66, by a class of 21 students.
60, 51, 47, 42, 53, 34, 47, 39, 56, 63, 35, 34, 50, 35, 41, 19, 48, 42, 37, 45, 29
 - (a) Determine the following statistics. Round to 1 decimal place, where necessary:
mean, standard deviation, minimum, Q_1 , median, Q_3 , maximum.
 - (b) Construct a box plot and describe the data set, including a comment on its shape.
- 2 The following data set represents the high jump results (m) for the open age group boys at a school carnival.
1.25, 1.35, 1.40, 1.40, 1.51, 1.26, 1.28, 1.29, 1.53, 1.41, 1.44, 1.34, 1.26, 1.25, 1.25, 1.29, 1.28
 - (a) Use technology to determine summary statistics. Round to 3 decimal places if necessary.
 - (b) Construct a box plot and describe the data set, including a comment on its shape.
- 3 The following data set represents the 100 m sprint results (s) for the open age group girls at a school carnival.
12.3, 13.8, 14.6, 14.5, 12.9, 13.9, 14.7, 14.2, 13.7, 14.5, 14.8, 14.0, 14.8
 - (a) Use technology to determine summary statistics, to 2 decimal places.
 - (b) Draw a box plot and describe the data set, including a comment on its shape.

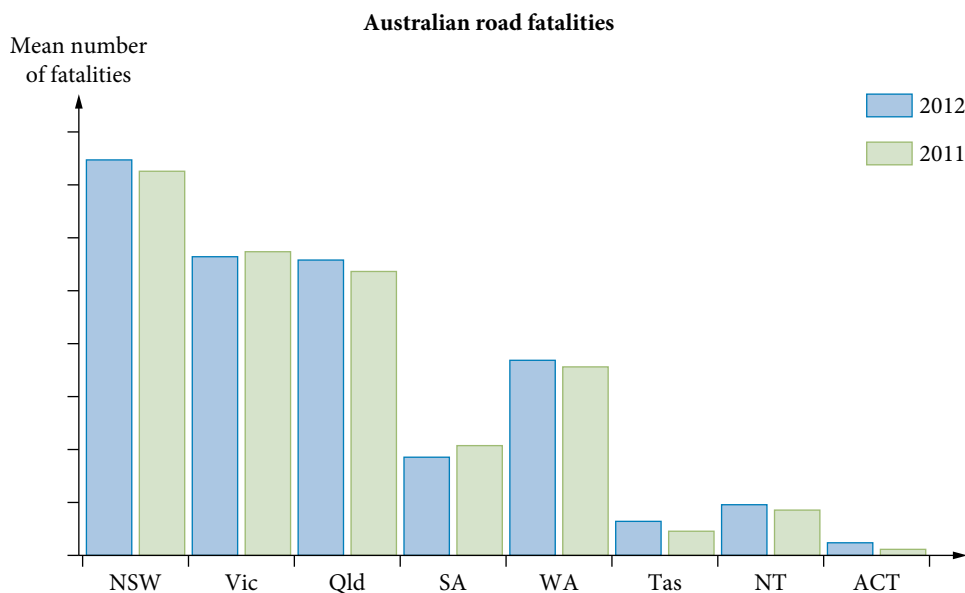
27

29

- 4 The parallel box plots compare the yield per hectare of wheat for paddocks treated in various ways. Which of the following statements is *incorrect*?



- A The interquartile range for treatment A is the smallest.
- B The highest-yielding paddock has been given treatment B.
- C About 50% of the paddocks receiving treatment B had a yield of less than 2.2 tonnes/hectare.
- D About 25% of the paddocks receiving treatment B had a yield between 1.875 and 1.925 tonnes/hectare.
- 5 The composite column chart below shows the number of people killed in traffic accidents in Australia in the years 2011 and 2012 by state and territory. Compare the two years.



- 6 The following table shows the average batting performances (number of runs scored) by the Waugh twins, Steve and Mark, in cricket Test matches against other Test-playing nations. The figures cover the entire careers of both players. Draw a composite column chart and use it to assist you in deciding who was the better batsman.

Although Steve played one Test against Bangladesh, making 256 without being dismissed, Mark never played against Bangladesh, so those figures are not included.

	England	India	New Zealand	Pakistan	South Africa	Sri Lanka	West Indies	Zimbabwe	Career
Steve	58.18	41.92	38.52	34.59	49.87	87.63	49.82	145	51.06
Mark	50.09	33.24	42.56	42.41	42.04	24.64	41.29	90	41.82

28

- 2003:**

2013:

31

- •

-
Month	Recent 30 year av	1941-1970
Jan	78	142
Feb	110	162
Mar	74	110
Apr	56	38
May	52	44
Jun	40	37
Jul	22	31
Aug	31	20
Sep	21	20
Oct	50	43
Nov	70	58
Dec	108	97

30

- 11 The table below gives the life expectancies of males and females in 20 of the most densely populated countries in the world. Construct parallel box plots for males and females, and compare the data.

Country	Male	Female	Country	Male	Female
Bangladesh	54	53	Japan	76	82
Brazil	62	68	Mexico	68	76
China	68	72	Pakistan	56	57
Egypt	60	61	Philippines	62	67
Ethiopia	50	53	Russia	64	74
France	74	82	Thailand	66	71
Germany	73	79	Turkey	68	72
India	57	58	UK	73	79
Indonesia	59	63	United States	72	79
Iran	64	65	Vietnam	63	67

- 12 The arm spans of a sample of men and women are given in the table.

Construct cumulative frequency graphs and use the simple values of $0.25n$, $0.5n$ and $0.75n$ to read off the estimated values of the median and quartiles. Draw parallel box plots of the data and comment on the similarities and differences between the two sets of data.

Arm span (cm)	Male	Female
150 – < 160	11	18
160 – < 170	14	23
170 – < 180	16	15
180 – < 190	12	4
190 – < 200	7	0

- 13 Two tests give the following statistics:

Test 1: $\bar{x} = 45$, $s = 15$

Test 2: $\bar{x} = 50$, $s = 10$

- (a) Determine which of the following people achieved comparable marks in the two tests.

A Ben: 60 for both tests

B John: 40 for both tests

C May: Test 1: 40, Test 2: 45

D Both Ben and John

- (b) Explain the common error in choosing the second incorrect response.

32

- 14 The results of two tests are given in each of the following questions. Use the mean and standard deviation to estimate a score for a student who was absent for the second test.

- (a) Test 1: 32, 12, 30, 34, 30, 37, 22, 34, 36, 35, 32, 27, 34, 34, 32, 14, 35, 32, 29, 24. Dylan's result is the first value.

Test 2: 20, 61, 65, 51, 66, 35, 46, 52, 66, 65, 45, 65, 59, 67, 23, 69, 58, 64, 18

- (b) Test 1: 31, 36, 32, 35, 19, 33, 31, 32, 32, 30, 27, 32, 35, 35, 19, 20, 31, 28, 13. Aiden's result is the last value.

Test 2: 17, 19, 17, 17, 5, 11, 13, 13, 8, 8, 12, 14, 20, 13, 9, 13, 19, 8

- (c) Test 1: 93, 109, 122, 120, 112, 128, 127, 120, 119, 108, 121, 122, 117. Hannah's result is the fourth value.

Test 2: 8, 10, 14, 3, 11, 14, 14, 14, 13, 11, 14, 13

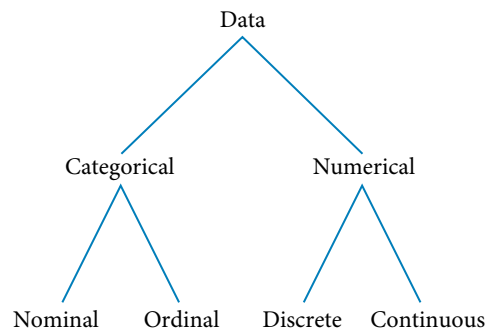
- 15 Michael's boss organises a health check for employees.
- (a) Michael's diastolic blood pressure is 85. The diastolic blood pressure readings of a sample of 50 of his co-workers are shown below. Use the formula $z = \frac{(x - \bar{x})}{s}$ to determine Michael's z -score, the number of standard deviations his blood pressure is above or below the mean, to 4 decimal places.
- 80, 70, 80, 80, 110, 88, 94, 74, 80, 80, 80, 90, 75, 80, 70, 90, 80, 82, 110, 70, 70, 90, 80, 70, 78, 80, 80, 70, 74, 80, 82, 82, 85, 100, 70, 80, 80, 90, 80, 80, 90, 70, 70, 100, 55, 60, 76, 90, 88, 88
- (b) Michael's cholesterol reading is 296. The sample of 50 co-workers gave the readings shown below. Calculate Michael's z -score for cholesterol, to 4 decimal places.
- 254, 240, 279, 289, 315, 250, 298, 384, 310, 337, 367, 273, 273, 314, 243, 341, 245, 302, 302, 386, 312, 302, 394, 358, 336, 251, 322, 281, 336, 314, 383, 360, 369, 333, 253, 268, 257, 474, 391, 248, 520, 285, 352, 428, 334, 278, 264, 243, 348, 290
- (c) Compare Michael's two readings with those of the rest of the group.
- 16 The table gives data for the population of Australia in 1998 and the projected population for 2025.

Age	Population in 1998 ('000s)		Population in 2025 ('000s)	
	Male	Female	Male	Female
0–4	661	628	724	726
5–9	682	650	914	868
10–14	680	648	893	849
15–19	645	613	867	826
20–24	686	654	880	848
25–29	728	703	950	930
30–34	729	713	1007	1000
35–39	733	724	1025	1020
40–44	688	689	997	991
45–49	665	658	848	854
50–54	572	554	852	870
55–59	445	433	777	798
60–64	360	365	763	798
65–69	328	347	680	722
70–74	290	338	579	623
75–79	198	269	488	532
80+	188	348	534	702

Construct four parallel box plots for the data. Make any assumptions you need to in order to complete the task. Discuss your findings.

Summary

Data types

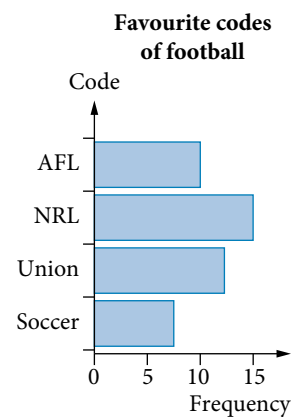
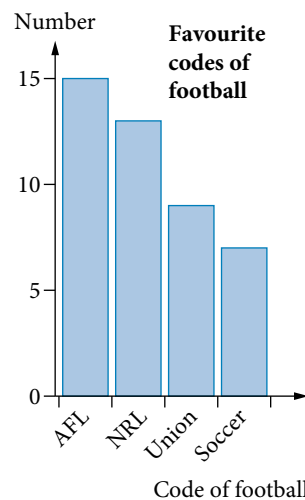


Simple data displays

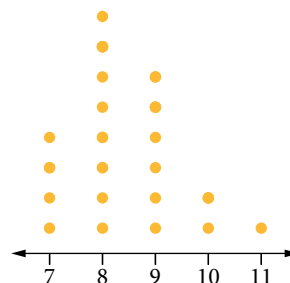
A frequency table is used to collect numerical data. It can be interpreted on its own or used as an aid in drawing a statistical graph.

x	f
1	3
2	5
3	2
4	4
5	2

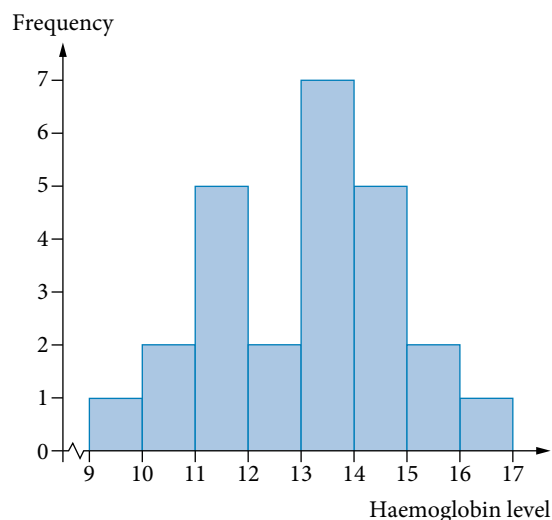
A column chart or bar chart is used to represent categorical data.



A dot plot can be used with either numerical or categorical data.



A histogram is used to display numerical data.



A stem plot is used to display numerical data. A stem plot should include a key to enable the values to be interpreted. A stem plot does not lose any of the original information.

Stem	Leaf
4	2 6 9
5	0 5 6 7 7 8
6	0 6
7	2 8 8 8
8	1 2 3 3 4 5
9	2 3 9

Key: 5 | 0 = 50

Measures of central tendency and spread

There are three measures of central tendency: mean, median and mode.

For ungrouped data the mean, $\bar{x}$, is found using:

$$\bar{x} = \frac{\text{sum of the data values}}{\text{number of data values}} = \frac{\sum x}{n}$$

For ungrouped data presented in a frequency table, use:

$$\bar{x} = \frac{\sum xf}{\sum f}$$

For grouped data, determine the midpoint of the class interval x_m and multiply this by f to find the approximate total for each interval.

$$\bar{x} = \frac{\sum x_m f}{\sum f}$$

The median is the physical middle of the data set, when the data values are in ascending order. For odd values of n , the median is the $\left(\frac{n+1}{2}\right)$ th value; for even values of n , the median is the mean of the $\left(\frac{n}{2}\right)$ th and the next value.

The mode is the most frequently occurring data value.

A bimodal data set has two values with the same highest frequency. If there are more than two values with the highest frequency, then the set has no mode or is multimodal.

There are three measures of spread: range, interquartile range (IQR) and standard deviation.

Range = maximum value – minimum value

To determine the IQR, first divide the data set into four pieces, called quartiles. The dividing lines are called Q_1 , median (Q_2) and Q_3 .

$$\text{IQR} = Q_3 - Q_1$$

The IQR, unlike the range, is not affected by extreme values.

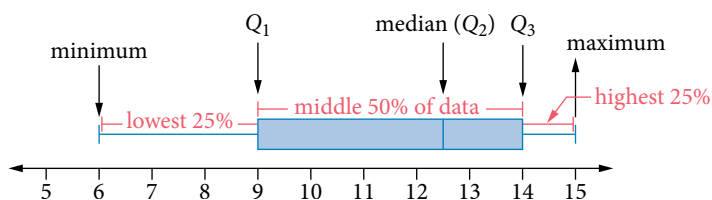
An outlier lies below $Q_1 - 1.5 \times \text{IQR}$ or above $Q_3 + 1.5 \times \text{IQR}$.

Finding the median and quartiles of grouped data is more difficult. You can record the class interval that contains the value, or draw a cumulative frequency graph to estimate the values by reading across from $0.25n$, $0.5n$, $0.75n$ and down to the data axis.

Standard deviation is a measure of spread that uses all the data values. It can be affected by extreme values. Use sample standard deviation, s_x on your calculator, for all standard deviation calculations.

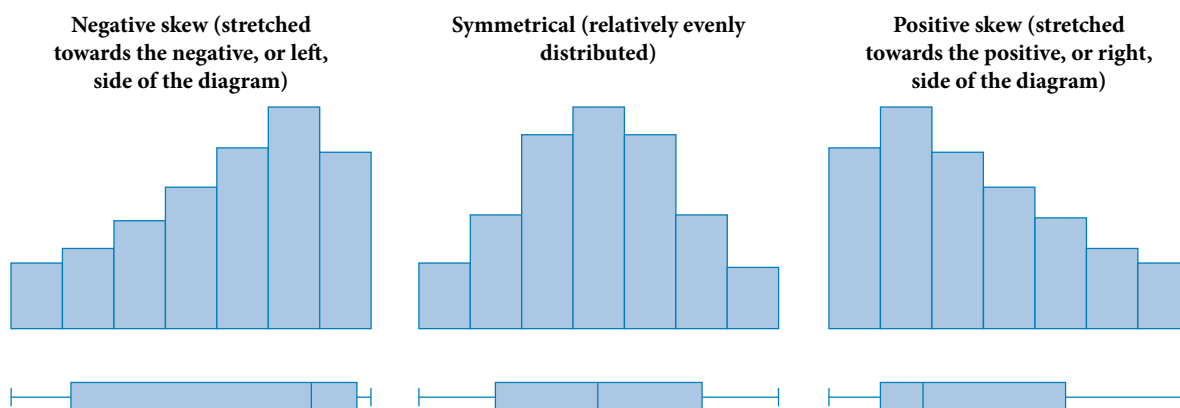
Data analysis methods and tools

The five-figure summary arises from dividing the data set into quartiles. It consists of the lowest value, Q_1 , median, Q_3 and the maximum value.



The statistical graph that uses the five-figure summary is the box plot.

The shape of a data set can be described using terms such as negative skew, symmetrical and positive skew.



For negatively skewed data, mean $<$ median. For positively skewed data, mean $>$ median.

Composite bar charts, back-to-back stem plots and parallel box plots are all used to assist in the analysis of data sets.

Mean and standard deviation can be used together to compare scores, by showing how many standard deviations a value is above the mean: $\frac{(x - \bar{x})}{s}$.

Chapter review

7

1 Classify each of the following data types.

Exercise 7.1

- (a) The size of shirts worn by the pupils in a class
- (b) The height jumped by each of the competitors in the school pole vault competition

2 State the data type for each of the following examples.

Exercise 7.1

- (a) The favourite colour of each of the students in your class
- (b) The highest level of education achieved by each of the respondents to a survey

3 Consider the data set shown in the table.

Exercise 7.1

x	f
82	7
83	16
84	12
85	1
86	2

- (a) Determine the mode.
- (b) Calculate the relative frequency of each data value as a percentage, to the nearest per cent.

4 Construct a stem-and-leaf plot for each of the following data sets.

Exercise 7.3

- (a) 25, 38, 65, 32, 77, 21, 79, 81, 66, 50, 47, 53, 25, 30, 42, 60, 70, 29, 51, 63, 68, 82, 40, 33, 22, 45, 37, 65, 74, 70, 35, 61, 81, 77, 65, 66
- (b) 5.8, 2.1, 7.6, 3.0, 4.1, 2.1, 6.5, 6.4, 3.4, 2.8, 6.3, 3.2, 6.0, 5.9, 3.1, 5.7, 3.8, 4.6, 7.2, 7.1, 4.4, 4.5, 3.4, 7.5, 2.9, 4.6, 3.8

5 Construct a dot plot for the data set shown.

Exercise 7.3

Number of pets: 2, 0, 1, 3, 3, 2, 5, 2, 0, 3, 6, 0, 5, 1, 3, 3, 1, 2, 2, 4, 3

6 Determine the median of the data set.

Exercise 7.4

x	f
16	3
17	5
18	2
19	4
20	2

Exercise 7.4

7 For the following data sets, calculate the five-figure summary, the range and the IQR.

(a) 2, 4, 5, 2, 4, 3, 1, 4, 6, 5, 4, 7, 8, 6, 8, 4, 5, 6, 2, 4, 3, 1, 3, 5, 7, 9, 1, 3, 2, 4, 6, 5, 3, 2, 1, 5, 2, 5, 6

(b)

Stem	Leaf
3	0 1 4 6 7
4	3 3 3 3 4 5
5	8 9
6	1 3 4 6 6 7 8
7	0 6 7 7 9
8	1 2 2 2 5 6
9	2 2 2 5 7

Key: 5 | 8 = 58

(c)

x	f
8	4
13	8
15	6
19	3
21	1
25	2

Exercise 7.4

8 Determine the mean of the data set to 1 decimal place.

x	f
1	3
2	5
3	2
4	4
5	2

Exercise 7.6

9 A teacher recorded the results for a test, marked out of 50, as follows:

12, 17, 22, 23, 26, 26, 27, 29, 30, 31, 31, 32, 32, 35, 35, 37, 38, 40, 41, 42, 42, 49

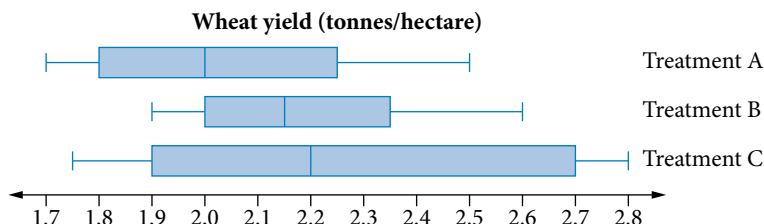
After the tests were returned, the student who had scored 38 pointed out that several pages had not been marked. This increased their mark to 48. Which of the following statements correctly describes what would happen to the class statistics?

- A The standard deviation and mean would both decrease.
- B The standard deviation and mean would both increase.
- C The standard deviation would increase and the mean would decrease.
- D The standard deviation would decrease and the mean would increase.

- 10 Calculate the standard deviation of the following data sets, to 2 decimal places.
- (a) 20, 26, 24, 26, 19, 18, 10, 33, 28, 42, 5, 6, 18, 12, 16, 32, 41, 38, 25, 26, 15, 10, 29, 30
- (b) 1, 1, 2, 2, 2, 2, 3, 4, 4, 5, 6, 7, 8, 8, 8, 9, 9, 9, 9, 9
- 11 The parallel box plots below compare the yield per hectare of wheat for paddocks treated in various ways.

Exercise 7.6

Exercise 7.7



Which of the following statements is *incorrect*?

- A The interquartile range for treatment B is the smallest.
- B Treatment C produced the most consistent results.
- C About 50% of the paddocks given treatment C had a yield greater than 2.2 tonnes/hectare.
- D About 25% of the paddocks given treatment A had a yield between 1.7 and 1.8 tonnes/hectare.
- 12 The ages of players in two Australian netball teams are summarised in terms of mean and standard deviation.

Exercise 7.7

Thunderbirds: mean = 26.5, SD = 4.5. Swifts: mean = 22.9, SD = 2.4

Compare the overall age distribution of the two teams.

- 13 Determine which result is better for each of the following. Justify your answer with calculations.
- (a) A score of 15 when the mean is 12.3 and standard deviation is 5.23, or a score of 79 when the mean is 65.5 and standard deviation is 12.44.
- (b) A score of 10 when the mean is 12.3 and standard deviation is 5.23, or a score of 60 when the mean is 65.5 and standard deviation is 12.44.

Exercise 7.7

- 14 The data below represents the weight (kg) of the senior list players for Western Bulldogs, in the AFL, for the years 2003 and 2013.

Exercise 7.7

2003:

77, 98, 91, 77, 89, 81, 90, 76, 94, 80, 98, 84, 79, 72, 77, 75, 75, 98, 94, 82, 94, 96, 84, 89, 85, 72, 94, 74, 81, 88, 83, 85, 82, 83, 78, 101, 83, 85

2013:

84, 83, 84, 87, 88, 76, 89, 85, 92, 84, 80, 100, 81, 83, 77, 87, 87, 88, 97, 77, 81, 84, 100, 77, 83, 80, 107, 76, 82, 82, 92, 90, 87, 93, 90, 77, 80, 104, 99

Construct a back-to-back stem plot with class interval of 5 and use it to help make comparisons between the two sets of data.

Exercise 7.3

- 15 A sample of 24 students provided data about the number of pets they had.

- (a) Group the data as 0–1, 2–3, 4–5, 6–7. Draw a histogram of the discrete data, treating the values as continuous by going a half each side. Place whole-number data values on the horizontal axis. Use grid paper for accuracy.
- (b) Describe the data distribution in terms of symmetry or skew.

Number of pets	Frequency
0	2
1	9
2	5
3	6
4	1
5	0
6	1

Exercise 7.4

- 16 Construct a box plot for each of the following data sets, identifying any outliers.

- (a) 47, 49, 50, 44, 70, 98, 46, 50, 43, 41, 47, 50, 53, 44, 42, 41, 48
- (b) 27, 27, 26, 28, 30, 31, 31, 34, 32, 33, 30, 31, 35, 7, 31, 34, 36, 61, 21, 32, 30, 29, 31, 34

Exercise 7.5

- 17 Construct a cumulative frequency polygon and a box plot for the ages of women in Australian netball teams. Use the simple estimates from frequencies of $0.25n$, $0.5n$, $0.75n$ for reading the quartiles (and median) from the graph.

Age	Frequency
18 – < 21	17
21 – < 25	22
25 – < 30	30
30 – < 35	11

Exercise 7.4, 7.6

- 18 Determine the five-figure summary, the mean and the standard deviation of the tabulated data. Round values to 1 decimal place, where necessary. Draw a box plot and comment on the shape of the distribution.

x	f
43	20
44	30
45	15
46	8
47	0
48	2

Exercise 7.6

- 19 Estimate the mean of the data set, then select the closest answer.

- A 25
- B 25.5
- C 29
- D 24.5

x	f
0 – 9	6
10 – 19	2
20 – 29	7
30 – 39	8
40 – 49	3

- 20 The height of every student in the school was recorded. The results are shown in the table.
- What is the best estimate for the standard deviation of the heights?
- A 11.27
B 9.66
C 50
D 5.00

Height (cm)	Frequency
140 – <145	5
145 – <150	11
150 – <155	18
155 – <160	26
160 – <165	44
165 – <170	92
170 – <175	64
175 – <180	32
180 – <185	17
185 – <190	9
190 – <195	4

Exercise 7.6

..

.....

..



- 21 To qualify for javelin in the school athletics squad, a student's best throw must be at least 49.5 m. The state record is 62.8 m and no-one at the school is likely to challenge that distance. Ten boys qualify for the squad with a median throw of 55.35 m. Emilio's throw was so good, it was an outlier from the other best throws.
- Create a set of possible distances, to 1 decimal place, for the best throw for the 10 members of the squad.
- 22 The following set of data has 13 scores. Change three scores so that the mean stays the same and the median increases by 1.

Exercise 7.6

x	4	7	8	9	10
f	1	2	5	4	1

Exercise 7.4, 7.6

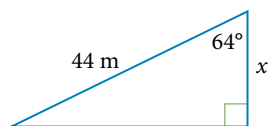
- 23 Two brands of rechargeable batteries were tested to compare their lifetimes, in hours, before they required recharging. Samples of 10 batteries of each brand were tested in the same equipment, producing the following times before requiring recharging.
- Big Zap: 7, 22, 9, 24, 16, 22, 25, 26, 23, 26
- Sparky: 19, 20, 21, 15, 17, 15, 39, 23, 14, 15
- (a) Calculate the mean, median, standard deviation and interquartile range of lifetime before recharging, for each brand of battery.
- (b) Represent the data as a back-to-back stem plot.
- (c) Construct parallel box plots for the lifetimes of the two brands of batteries.
- (d) Which brand of battery would last at least 14 hours?
- (e) Which brand of battery is most likely to last over 20 hours?
- (f) Which brand of battery tends to last longer? Justify your answer with reference to the summary statistics and statistical displays.

Exercise 7.7

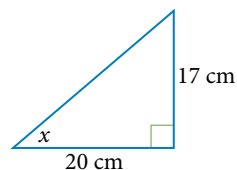
Exercise 5.1

- 1 Determine the value of x in the triangle, to 2 decimal places.

(a)

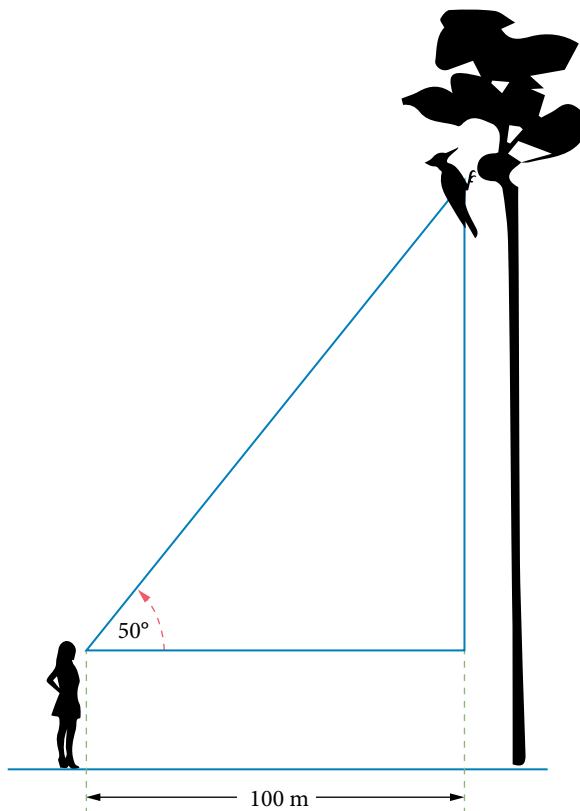


(b)



Exercise 5.2

- 2 Ava is observing a rare breed of speckled woodpecker from a horizontal distance of 100 m . The angle of elevation from Ava's eye to the woodpecker is 50° . Determine the height of the bird above Ava's eye, to the nearest metre.

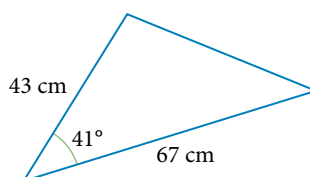


Exercise 5.3

- 3 Convert $S71^\circ W$ to a true bearing.

Exercise 5.6

- 4 Calculate the area of the triangle, to 1 decimal place.



- 5 Copy and complete the table of values by calculating the surface areas. Round the values of surface area to 2 decimal places.

Exercise 6.1

$SA(\text{cm}^2) = 2\pi r^2 + 2\pi rh$		Radius (cm)	
		6	30
Height (cm)	10		
	15		

- 6 Consider the matrix $\mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 0 & -1 & -2 \\ -2 & -1 & 0 \end{bmatrix}$.

Exercise 6.1

What is the order of matrix $\mathbf{B}$?

- 7 Consider the two matrices $\mathbf{A} = \begin{bmatrix} 4u & v \\ 7 & w+4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -4 & 0 \\ -x & 1 \end{bmatrix}$.

Exercise 6.2

If $\mathbf{A} = \mathbf{B}$, determine the values of u , v , w and x .

- 8 The matrix $\mathbf{C}$ displays direct air routes between Ballarat, Anglesea and Casterton.

Exercise 6.4

$$\mathbf{C} = \begin{bmatrix} B & A & C \\ 0 & 2 & 1 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{matrix} B \\ A \\ C \end{matrix}$$

- (a) Determine matrix $\mathbf{C}^2$.
- (b) Describe what matrix $\mathbf{C}^2$ represents.
- (c) What is represented by element c_{12} of matrix $\mathbf{C}^2$?
- 9 The height of each member of the class is recorded to the nearest centimetre. Identify the type of data collected.
- 10 The following data set represents the number of brothers and sisters of each student in a class.
0, 1, 4, 3, 1, 1, 2, 0, 2, 2, 4, 2, 3, 0, 1, 2, 1, 0, 2, 1, 3, 1, 0, 0
Construct a dot plot to represent the data.
- 11 For the following data set, identify any outliers.
1, 4, 6, 3, 5, 7, 11, 1, 3, 2, 4, 6, 5, 3, 2, 1, 5, 2, 5, 2, 4, 3, 10, 4, 3, 4, 5, 6, 2

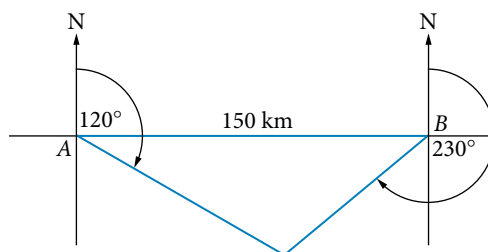
Exercise 7.1

Exercise 7.2

Exercise 7.4

Exercise 5.4

- 12 Two earthquake detection stations A and B are situated 150 km apart, with station B due east of station A . Station A detects an earthquake on a bearing of 120° T. Station B detects the same earthquake on a bearing of 230° T.



How far is the earthquake from station A ? Express your answer to 1 decimal place.

Exercise 6.1

- 13 The area $A \text{ m}^2$ of a circular path of inner radius $r \text{ m}$ and outer radius $R \text{ m}$ is given by $A = \pi R^2 - \pi r^2$.

A circular path has an inner radius of 40 m and area of 2000 m^2 . Transpose the formula to determine the circular path's outer radius, to 2 decimal places.

Exercise 6.3

- 14 If $\mathbf{A} = \begin{bmatrix} 6 & -2 \\ 4 & 2 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} -1 & 4 \\ 2 & -7 \end{bmatrix}$, calculate $5\mathbf{A} - 3\mathbf{B}$.

Exercise 6.4

- 15 Poh scores the following marks (out of 100) for her assessment components in chemistry: topic tests, 74; practical work, 81; semester examination, 85.

The final semester result for chemistry is weighted as follows:

topic tests, 25%; practical work, 25%; semester examination, 50%.

Use a matrix multiplication method to calculate Poh's final semester result (out of 100) for chemistry, to the nearest whole number.

Exercise 7.7

- 16 Calculate the mean and standard deviation, to 1 decimal place, of each set of sample data and use the statistics to compare the distributions of the two sets.

Class	0–9	10–19	20–29	30–39	40–49
Set A Frequency	7	8	7	3	2
Set B Frequency	1	15	5	5	1

Exercise 5.3

- 17 Eamonn hikes on a bearing 056° T for 410 m, then turns 90° right and hikes 520 m before setting up his tent.

(a) How far is Eamonn from his starting point, to 2 decimal places?

(b) Along what bearing, to the nearest degree, should he hike to return directly to his starting point?

Exercise 7.4

- 18 Add two data values to the set of values below so that the median and interquartile range are both 6.

5, 8, 1, 7, 9

- 1 Kitty is a professional soccer player who is paid a base salary of \$45 000 a season and also earns a bonus of \$3000 for every goal she scores. She scores a total of 18 goals over the 20 week season. What is her average weekly wage during the season?

Exercise 1.2

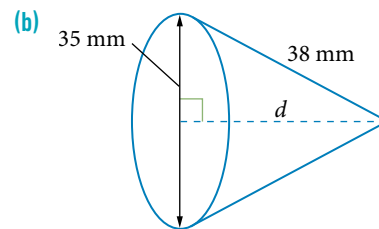
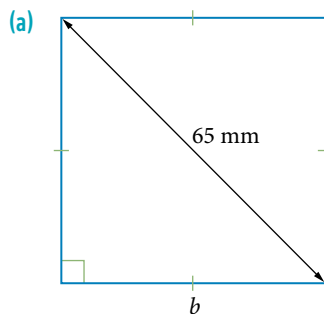
- 2 Calculate the following amounts involving GST.

Exercise 1.3

- (a) the GST to be added to a car service costing \$280
(b) the pre-GST cost of a bicycle advertised for \$379.50 including GST

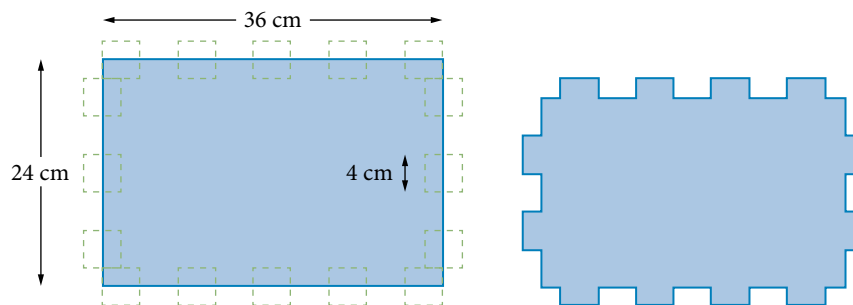
- 3 Determine the value of each pronumeral, to 2 decimal places.

Exercise 2.1



- 4 Eleanor is making a personalised birthday card for a friend. She starts with a rectangular piece of card measuring $36\text{ cm} \times 24\text{ cm}$, then uses a square cutter measuring $4\text{ cm} \times 4\text{ cm}$ to remove half-squares from the edges, as shown.

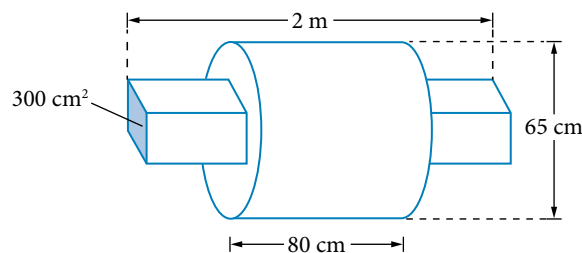
Exercise 2.2



Determine the perimeter and the area of the final card shape.

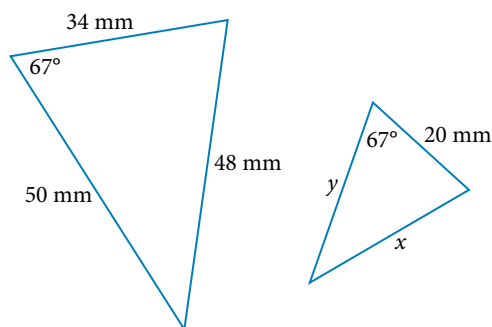
- 5 Calculate the volume of the symmetrical solid, to the nearest cubic centimetre.

Exercise 2.3



Exercise 3.1

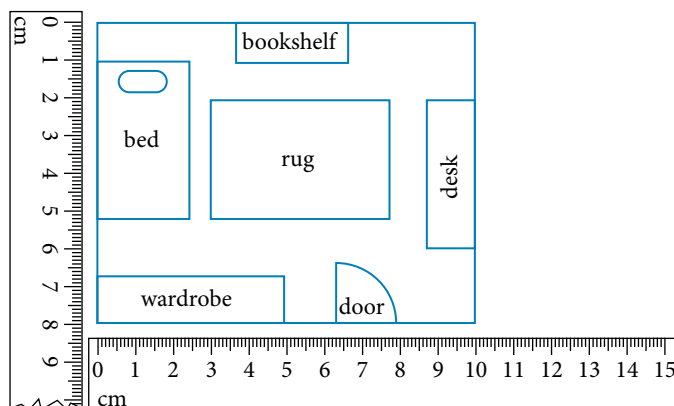
- 6 The triangles shown below are similar.



Determine the value of each unknown, to the nearest millimetre.

Exercises 3.1, 3.3

- 7 Kuppar has drawn a plan of how he would like to arrange his room. He uses a scale in which 10 cm represents 5 m. Determine the following, rounding your answers to 1 decimal place where necessary.



- Calculate the scale of Kuppar's drawing.
- Determine the dimensions of each of the five items in Kuppar's room, in metres.
- What is the actual area of the room?
- Determine the area of floor not covered by the rug or furniture.

Exercise 3.4

- 8 Two similar solids have the scale factor $k = 3.2$. Calculate the original volume v to 2 decimal places, given the larger volume $V = 1750 \text{ m}^3$.

Exercise 4.3

- 9 Calculate the x - and y -intercepts and sketch the graph of $2y = 3x + 24$.

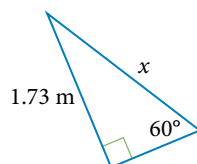
Exercise 4.3

- 10 Solve the following pairs of simultaneous equations algebraically.

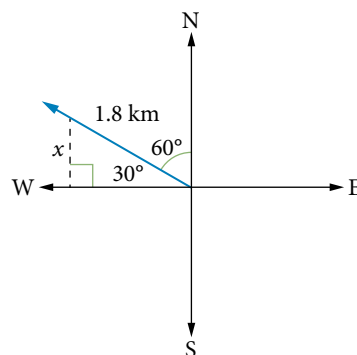
- $3x + 2y = 22$, $x - y = -1$
- $4j - 3k = -19$, $2j + k = 13$

Exercise 5.1

- 11 Determine the value of x in the triangle, to 2 decimal places.

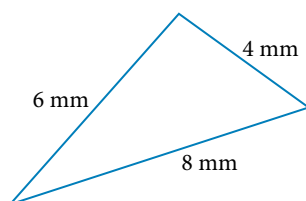


- 12 A hiker walks 1.8 km on a bearing N 60° W.
How far north does the hiker travel?



Exercise 5.3

- 13 Calculate the area of the triangle, to 2 decimal places.

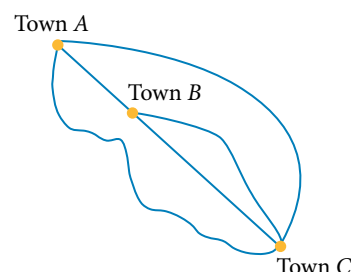


Exercise 5.6

- 14 The formula for the area A of a circle of diameter d is $A = \frac{\pi d^2}{4}$.
Calculate, to 1 decimal place, the area in square metres (m^2) of a circle of diameter 8.6 m.

Exercise 6.1

- 15 The map shows the number of roads connecting three towns A , B and C .
Construct a matrix $\mathbf{R}$ displaying this information.



Exercise 6.5

- 16 The sales of sofas (S), lounge chairs (L) and coffee tables (C) at a company's three furniture stores for the month of August are represented by the matrix $\mathbf{A}$ where:

Exercise 6.3

$$\mathbf{A} = \begin{matrix} & \begin{matrix} S & L & C \end{matrix} \\ \begin{bmatrix} 7 & 4 & 9 \\ 3 & 8 & 15 \\ 6 & 11 & 12 \end{bmatrix} & \begin{matrix} \text{Store 1} \\ \text{Store 2} \\ \text{Store 3} \end{matrix} \end{matrix}$$

The corresponding sales at the three stores for September are represented by the matrix $\mathbf{S}$ where:

$$\mathbf{S} = \begin{matrix} & \begin{matrix} S & L & C \end{matrix} \\ \begin{bmatrix} 5 & 6 & 8 \\ 9 & 12 & 14 \\ 2 & 7 & 13 \end{bmatrix} & \begin{matrix} \text{Store 1} \\ \text{Store 2} \\ \text{Store 3} \end{matrix} \end{matrix}$$

- (a) Calculate the sales of sofas, lounge chairs and coffee tables for the three furniture stores for the 2 months combined. Give your answer in matrix form.
(b) What does the matrix element $(\mathbf{A} + \mathbf{S})_{31}$ represent?

Exercise 7.1

- 17 Decide whether these categorical data examples are nominal or ordinal.
- (a) The favourite actor, chosen from a list of 20, for each of the students in your year level
 - (b) The predicted likelihood of people, chosen at random, buying a new car within the next two years, recorded on a six-point scale from *certain* to *definitely not*

Exercise 7.1

- 18 Draw a bar chart to represent the following data set, which shows the favourite holiday destinations of a randomly selected group of people.

Gold Coast, NZ, Europe, NZ, Gold Coast, Bali

Thailand, NZ, NZ, Bali, Bali, Gold Coast

Bali, Bali, NZ, Europe, Thailand, Bali

Exercise 7.4

- 19 Calculate the five-figure summary for the data set below.

Stem	Leaf
13	1 1 2 3 5 6 7
14	2 3 3 3 4 5 7 9 9
15	0 0 0 1 2 3 3 3 3
16	2 3 4 4 5 8 9
17	0 0 1 2 3 4
18	2 2 2 5 7 8 9
19	0 0 1

Key: 13 | 2 = 132

Exercise 1.3

- 20 In Queensland, the state government requires a one-off tax, called stamp duty, to be paid on the transfer of ownership of motor vehicles, at the rates shown in the table below.

Ekbal is going to purchase a four-cylinder hybrid car valued at \$17 850. Marissa is purchasing a six-cylinder sports car for \$21 960.

Type of motor vehicle	Stamp duty rate (% of purchase price)	Registration fee	Fuel economy	Fuel source
Hybrid and electric vehicles	2%	\$300	4.2L/100 km	LPG: \$0.78/L
5 to 6 cylinders, 3 rotors	3.5%	\$475	16.5 L/100 km	Unleaded: \$1.20/L

- (a) Calculate the stamp duty payable on each purchase.
- (b) To register a motor vehicle, drivers must also pay an annual Traffic Improvement Fee of \$52.30 and a compulsory third-party insurance premium of \$352.60, as well as a registration fee as outlined in the table.
Calculate Ekbal and Marissa's registration costs over the span of 5 years.
- (c) Ekbal and Marissa both drive approximately 175 km weekly in approximately the same travel conditions.
Determine the anticipated fuel costs (to the nearest dollar) for each person over the 5 year period.
- (d) Compare the 5 year total outlay for Ekbal and Marissa.

- (e) Ekbal is a university student and has a casual job as a retail sales assistant earning \$19.50 per hour, working on average 15 hours per week. After tax, Marissa's job as a carpenter pays \$30.80 per hour and she works a standard 38 hour week.

Determine, to the nearest number of weeks, the amount of whole wage each will spend on paying for their car ownership and running costs (excluding maintenance) over the 5 year period.

- 21 A linen importer buys a carton of towels for \$400 and sells them after 3 years for \$650. Annual inflation averages 2.8% over that time period. Calculate the importer's effective percentage profit, to 1 decimal place.

Exercise 1.6

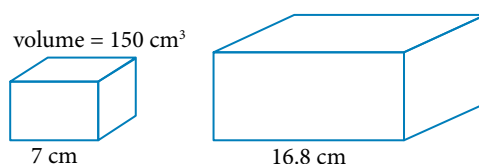
- 22 A 34 m tall lighthouse is to be repainted, and requires enough white paint to cover 350 m^2 of its surface. Don has completed painting a model of the same lighthouse, and painted 350 cm^2 of his model white.

Exercise 3.3

How tall, in centimetres, is Don's model lighthouse?

- 23 The rectangular prisms shown below are similar. The volume of the smaller prism is 150 cm^3 .

Exercise 3.4



Determine the volume of the larger rectangular prism.

- 24 Three consecutive numbers add to 336. What are the numbers?

Exercise 4.5

- 25 Plot the following piece-wise graph.

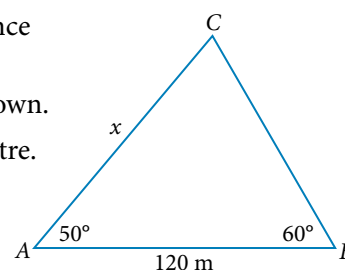
Exercise 4.4

$$y = \begin{cases} -x, & x < 0 \\ x + 2, & 0 \leq x \leq 3 \\ 3x - 6, & 3 < x < 5 \end{cases}$$

- 26 Trent is standing on one side of a river and has measured the distance between two power poles A and B on the river's edge to be 120 m. He measures bearings at A and B to pole C to be 50° and 60° , as shown. Calculate the following distances, to the nearest hundredth of a metre.

Exercises 5.1, 5.4

- (a) The distance from pole A to a third pole, C , on the river's edge on the other side of the river
(b) The width of the river



- 27 A manufacturer makes three products A , B and C , each requiring different amounts of construction time for stages P , Q , R and S . Matrix $\mathbf{X}$ shows the construction time, in hours, required to make one of each product.

Exercise 6.4

Each stage of construction P , Q , R and S costs the manufacturer \$120, \$65, \$40 and \$100 per hour, respectively.

$$\mathbf{X} = \begin{matrix} & \begin{matrix} P & Q & R & S \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 2 & 3 & 1 & 2.5 \\ 0 & 1.5 & 1 & 3 \\ 1 & 2.5 & 4 & 2 \end{bmatrix} \end{matrix}$$

- (a) Express the hourly costs of each stage of construction as a column matrix $\mathbf{Y}$.
(b) Determine $\mathbf{XY}$ and identify what information it displays.



Exercise 1.4

- 28** Rajid invested \$15 000 at an interest rate of 6.4% p.a. compounding monthly for 18 months. During that time he was unable to pay the full amount due on his credit card, and so he had to pay simple interest on an average of \$520 for 12 days of each month at a rate of 19% p.a. The interest on his investment was taxed at 32.5 cents in the dollar.

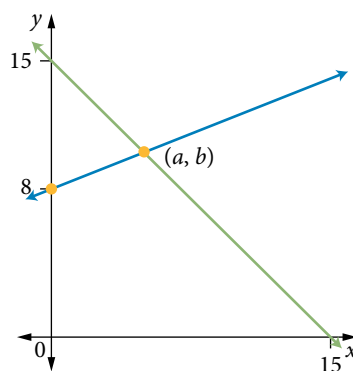
Would Rajid have been better off leaving his money in an accessible, low-interest account and paying the credit card in full each month? Justify your answer, and explain any assumptions you made.

Exercises 2.4, 2.6

- 29** A rectangular-based pyramid has a $15\text{ cm} \times 20\text{ cm}$ base and a volume of 700 cm^3 . If all four slanted edges are the same length, determine the surface area of the visible part of the pyramid, to the nearest square centimetre.

Exercises 4.2, 4.3

- 30** For the graph below, given that $b = 2a$, determine the equation for the line passing through $(0, 8)$.



Exercise 5.3

- 31** In a race, a yacht must sail $\text{N}40^\circ\text{E}$ for 40 km to the first checkpoint, then alter course and sail $\text{S}60^\circ\text{E}$ for a further 30 km, to the finishing point.
- How far apart are the starting point and the finishing point? Round your answer to 1 decimal place.
 - What is the true bearing (to the nearest degree) of the finishing point from the starting point?

Answers

Chapter 1 Consumer arithmetic

RECALL

- 1 (a) $\frac{12}{5}$ (b) $\frac{11}{8}$
 2 (a) 0.68 (b) 0.846 (c) 9.90
 3 (a) $y = 26$ (b) $a = 17$ (c) $i = 5$
 4 (a) 7 (b) 35 (c) 3.45
 5 (a) 0.15 (b) 0.035 (c) 1.52 (d) 0.0075
 6 (a) 73% (b) 2.6% (c) 109%

EXERCISE 1.1

- 1 (a) 40% (b) 44.44% (c) 37.5%
 (d) 35% (e) 90% (f) 75.56%
 (g) 61.82% (h) 0.6% (i) 28%
 (j) 87.6% (k) 66.67% (l) 0.12%
- 2 (a) 240% (b) 122% (c) 3545%
 (d) 343% (e) 2457% (f) 780%
 (g) 483% (h) 224% (i) 188%
- 3 (a) 75.6 g (b) \$30.60 (c) 72 cm
 (d) \$40.25 (e) 422.2 m (f) 90.2 kg
 (g) \$17 (h) 2 ms (i) 55 c
- 4 (a) 336 (b) 20 (c) 48.88 (d) 70.14
 (e) 0.12 (f) 0.04 (g) 24.66 (h) 54.59
- 5 (a) 540 km (b) 39 L
- 6 (a) \$11.50/h (b) \$18.50/h (c) \$37.65/h
 (d) \$36.44/h, assuming 52 weeks have been worked
- 7 (a) 41.7 L/h (b) 2000 days
- 8 (a) \$2.34 (b) \$97.50 (c) \$8
- 9 \$701.25
- 10 \$382.50
- 11 (a) D
 (b) A is the first incorrect calculation. While it is true that the original amount would be more than the sale price, the original is unknown. Increasing the sale price by 15% is not the same as decreasing the original price by 15%, because 15% of the sale price is not the same as 15% of the original price.
 (c) B and C

- 12 (a) \$50 (b) \$65 (c) \$26.34
 (d) \$80 (e) \$45 (f) \$72
 (g) \$3.50 (h) \$10.80 (i) \$152.76

- 13 (a) C
 (b) The student incorrectly increased the sale price by 15%.
- 14 (a) 0.07 L/km (b) 7.0 L per 100 km (c) \$1.80/L

15 (a)

	Percentages	
	Mosi	Liam
Test 1	75%	85%
Test 2	86%	64%
Test 3	80%	93.3%

- (b) 80.3% (c) 80.8% (d) Liam
- (e) Mosi's mark = 82, Liam's mark = 77. Mosi achieved a final mark 5 marks higher than Liam and therefore has the better result.
- (f) The tests do not contribute equally to the result. Taking the average of the three percentage results assumes each test is worth one third of the marks.
- 16 Lump sum, \$1550
- 17 (a) 2% (b) 77% (c) 23%
- 18 (a) Alina should trial Gym and swim, which offers the greatest saving over 5 sessions (\$22.50).
 (b) \$4
 (c) Spin and boxing: save a total of \$126.
 (d) \$7

19 (a) (i)

Region	Percentage of area of Australia
ACT	0%
NSW	10%
NT	18%
Qld	23%
SA	13%
Tas	1%
Vic	3%
WA	33%

- (ii) Largest: Western Australia, smallest: Victoria

(b) (i)	Region	Population density (persons/km ²)
	ACT	174.3
	NSW	9.3
	NT	0.2
	Qld	2.7
	SA	1.7
	Tas	7.5
	Vic	26.1
	WA	1.0
	Australia	3.0

(iii) Queensland

- 20 (a) 188% (b) 3% (c) 3%
(d) 619% (e) 971% (f) 0.03%

(g) Sample answer: No, the distances between planets are too great for the sizes of the planets and the distances between them to be drawn on the same scale. Jupiter's diameter is about 10 times the diameter of Earth, but in the drawing it is only about 3 times the size.

EXERCISE 1.2

- 1 (a) \$490 (b) \$658 (c) \$427 (d) Less, \$63
2 (a) \$564 (b) \$625 (c) \$615
(d) Collecting trolleys
3 \$518
4 \$4974
5 (a) 9 days (b) \$1226 (c) \$1058 (d) \$254
6 base + commission, \$330
7 C
8 (a) Djaali: \$540, Bridie: \$958.50, Namish: \$180
(b) \$81 (c) \$135
9 \$3540
10 \$4167
11 (a) B (b) D, Commission would be \$263.25
12 \$29000
13 12%
14 (a) \$2955.60
(b) (i) \$985.20 (ii) \$448.60
(iii) \$338.60 (iv) \$688.60
15 (a) \$18512 (b) \$47252.40
16 (a) 14 (b) 17 h 30 min (c) \$68.60/h
17 \$1284.40
18 More than \$4000

EXERCISE 1.3

- 1 (a) \$8.50 (b) \$620.40 (c) \$5254.55

- 2 (a) \$495 (b) \$132 (c) \$30.80
3 \$46800
4 (a) \$27222 (b) \$6497 (c) \$0
(d) \$15207 (e) \$0 (f) \$24669
(g) \$16897 (h) \$0 (i) \$13060.70
5 \$45200
6 (a) \$5538 (b) \$0 (c) \$2208
(d) \$7620 (e) \$0 (f) \$2268.52
7 (a) \$2272.80 (b) \$43.71

8 B

9 A

10 (a) D

(b) The student has incorrectly calculated the monthly gross income by multiplying the gross weekly income by 4.

11 \$13049.65

12 (a) The HELP repayment should be on the entire taxable income.

$$\$70000 \times 0.05 = \$3500$$

(b) The tax on taxable income should be an initial amount, then just the amount above the threshold at the rate given.

$$\begin{aligned} & \$3572 + \$0.325 \times (70000 - 37000) \\ &= \$3572 + \$10725 \\ &= \$14297 \end{aligned}$$

13 (a) \$32130 (b) \$2646.70 (c) Refund of \$698.30

14 \$209

15 (a) \$50.60 (b) \$13.20 (c) \$33

16 (a) \$92943 (b) \$22021.91 (c) Refund: \$1579.09

17 Refund: \$1586.32

18 (a) \$700 (b) \$36400 (c) \$3458 (d) \$66.50

(e) \$633.50 (f) \$14700 (g) \$0 (h) \$1396.50

19 (a) \$62141 (b) \$60698 (c) \$11273.85

(d) Refund: \$581.15

(e) HELP payment: \$2427.92; debt: \$1846.77

20 \$1164.37

21 (a) \$3572 (b) \$13975 (c) \$37000

(d) The answer to part (a) is the same as the tax paid on amounts up to \$37000.

(e) The total of answers to parts (a) and (b) is the same as the tax paid on amounts up to \$87000.

(f) The total of answers to parts (a), (b) and (c) is the same as the tax paid on amounts up to \$180000.

(g) The base tax payable in each income bracket is the maximum amount payable in the previous bracket.

EXERCISE 1.4

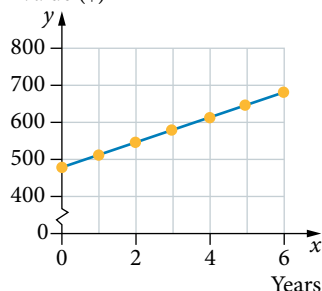
1 (a) \$270 (b) \$10800 (c) \$420 (d) \$6666.67

2 (a) \$134.27 (b) \$209.59 (c) \$29538.46 (d) \$689.33

(e) \$22350.19

3 (a) A

(b) Value (\$)



The graph is linear because the value increases by the same amount each year.

4 (a) \$6592 (b) \$1003.89

(c) \$83622.40 (d) \$211400.82

5 (a) \$600 (b) \$618

(c) The principal changes after 1 year and interest is earned on the first year's interest.

6 (a) (i) \$10137.98 (ii) \$10191.97

(iii) \$10204.45 (iv) \$10210.55

(b) The shorter the compounding period, the higher the final balance.

If period and rate are the same, the sooner the principal begins to grow, the higher the amount at the end.

7 (a) \$10210.55 (b) \$2210.55

8 (a) \$2577.20 (b) \$2291.20 (c) \$19393.10

(d) \$5144.00 (e) \$18599.39 (f) \$230479

Time (years)	Balance
0	\$40000.00
0.5	\$40900.00
1	\$41820.25
1.5	\$42761.21
2	\$43723.23
2.5	\$44707.11
3	\$45713.02
3.5	\$46741.02
4	\$47793.25
4.5	\$48868.59
5	\$49968.14

10 (a) 8.5% p.a. (b) 4.2% p.a. (c) 2.9% p.a.

11 (a) \$2380.95 (b) \$45000 (c) \$3900

12 (a) 3 years 6 months

(b) 1 year 3 months

(c) 6 years 9 months

13 (a) \$3326.57

(b) \$3413.62

14 (a) B

(b) A: The time units for i and n do not match.(c) C and D: n must be a whole number.

15 5.5% p.a. simple interest

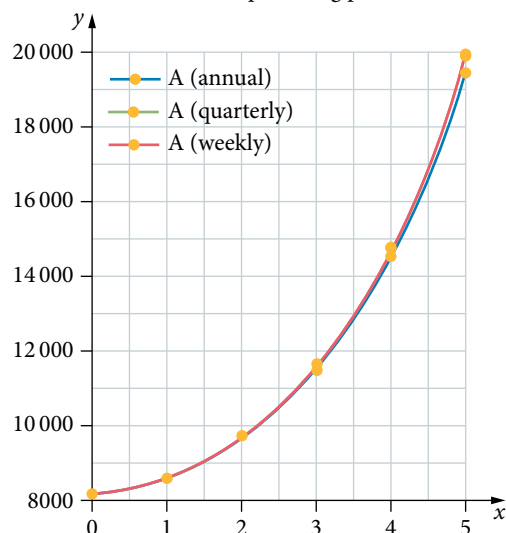
Month	Account balance (\$)	Interest earned in the month (\$)
1	12 060.00	60.00
2	12 120.30	60.30
3	12 180.90	60.60
4	12 241.81	60.90
5	12 303.02	61.21
6	12 364.53	61.52
7	12 426.35	61.82
8	12 488.48	62.13
9	12 550.93	62.44
10	12 613.68	62.75
11	12 676.75	63.07
12	12 740.13	63.38

(b) Sample answer: The interest increases each month because interest is being earned on the previously added interest.

17

	A	B	C	D
1	Time (years)	A (annual)	A (quarterly)	A (weekly)
2	0	8200	8200	8200
3	1	8601.80	8609.24	8611.61
4	2	9664.98	9703.99	9709.47
5	3	11511.15	11612.58	11624.17
6	4	14532.56	14753.66	14776.92
7	5	19447.84	19900.47	19946.27

Varied compounding periods



For all three compounding periods the growth was slow at first then much more rapid. From the table you can see that the balance for the annual compounding situation is always less than for quarterly compounding, and that the balance for quarterly compounding is less than for weekly compounding. The difference increases as time goes on.

- 18 (a) 10 days: \$25 045.93; 20 days: \$25 091.94;
30 days: \$25 138.04; 40 days: \$25 184.22
- (b) 10 days: \$25 053.48; 20 days: \$25 107.07;
30 days: \$25 160.77; 40 days: \$25 214.59

19 Time (years)	Balance (\$)
0	490
1	500.19
2	514.20
3	536.31

EXERCISE 1.5

- 1 (a) \$0.25 (b) \$1250
- 2 (a) \$0.17 (b) \$1.20 (c) \$11.82
(d) \$0.08 (e) D
- 3 (a) \$2760 (b) \$755 (c) \$5910
(d) \$9975 (e) \$7420 (f) \$22 470
- 4 \$1500
- 5 (a) 5.7% (b) 4.0% (c) 1.5%
(d) 0.7% (e) 6.2% (f) 1.4%
- 6 B
- 7 (a) \$43.90 (b) \$103.06 (c) \$12.88
(d) \$40.86 (e) \$50.09 (f) \$32.32
- 8 10%
- 9 (a) \$4145 (b) \$25 loss (c) \$4430
(d) \$7160 (e) \$82 loss (f) \$3630
- 10 (a) 103.0 (b) 7.4 (c) 38.8
- 11 Yes
- 12 (a) 8000 (b) \$480 (c) \$10 400
(d) \$10830 (e) Profit, \$50
- 13 Yes, Eden is \$15 877.11 better off
- 14 \$9.03, assuming no dividends paid and that Georgia had enough money each year to pay interest.
- 15 (a) \$2590
(b) Brushwell Pty Ltd, as it is predicted to take the shortest time of 4.7 years for the dividend to pay for the price of the shares.
(c) \$4640

EXERCISE 1.6

- 1 (a) T-shirt: \$4.90, jacket: \$5.20, pen: \$2.50, hamburger: \$2.30
(b) T-shirt: 46.7%, jacket: 14.9%, pen: 108.7%, hamburger: 191.7%
- 2 (a) TV: \$20, chocolate bar: \$0.48, boat: \$22500, book: \$27.50
(b) TV: 2.5%, chocolate bar: 40%, boat: 34.6%, book: 84.6%
- 3 Jeans: \$79.90, shoes: \$136, jacket: \$210.80, scarf: \$9.69
- 4 \$519145.85
- 5 D
- 6 (a) 20% (b) 96% (c) 39.1% (d) 150%
- 7 (a) \$19.55 (b) \$3.22 (c) \$11.73 (d) \$26.45
- 8 (a) \$263 (b) \$704 (c) \$1577
- 9 (a) 2.9% (b) 3.4%
(c) The Reserve Bank is likely to have left interest rates unchanged in the first year and raised interest rates in the second year.
(d) \$465.30
- 10 (a) 9.5% (b) \$10.11 (c) 1.1%
- 11 $\frac{300 - 250 \times 1.022^3}{250 \times 1.022^3} \times 100\% = 12.42\%$
- 12 (a) \$850 (b) 34.7% (c) \$406.84 (d) 14.1%
- 13 (a) \$8.89 (b) \$4.16
(c) (i) \$1.59 (ii) \$1.64 (iii) \$1.68
(d) \$3494.40. To simplify the calculation, the estimate had the cost stay the same throughout the year. If the change was a small amount each week, the calculation would have been very complicated and the result would have been higher.
- 14 Profit, 2.2%
- 15 Effective profit: \$496.35
- 16 (a) \$2750 (b) \$687.50 (c) 56.7%
(d) The effect of inflation is very small in only 4 weeks.
- 17 (a) \$5123.25 (b) 77.1%
- 18 33% p.a.

EXERCISE 1.7

- 1 1.25 kg
- 2 B
- 3 (a) 800 g (b) Rice is Nice
(c) Crispex, 800 g (d) Rice is Nice
- 4 (a) Brand B, 800 g (b) Brand B, 410 g (c) 1 can of 810 g
- 5 (a) \$75
(b) (i) \$390 (ii) \$140 (iii) \$169
(c) \$27

- 6 (a) \$5380 (b) \$2880 (c) \$170 (d) \$179
 7 \$12316
 8 \$170
 9 (a) \$1.90 (b) \$5.40 (c) \$225 (d) No
 10 (a) \$17.99 (b) \$17.99 (c) \$436.89
 11 (a) \$385 (b) \$192
 12 250 g of tuna, 250 g of rice and 5 individual apples; \$6.55
 13 Sample answer: Other expenses: water bills, education costs (e.g. textbooks if studying).
 Costs for moving in to a new apartment, sharing with two others: Set-up costs (bond, furniture, appliances):
 $\$3300 \div 3 = \$1100$. Weekly costs: rent: \$100; food: \$100; bills \$25; transport \$50; clothes: \$20; education: \$10. Total cost \$1100 to set up, plus \$305 per week ongoing.
 14 4 months
 15 Sample answer: Saving: \$100; sport, entertainment and holidays: \$32; clothes: \$20; music: \$10; food: \$30
 16 Must be less than \$15.22. Sample answer: \$13.99.

EXERCISE 1.8

- 1 (a) \$450 (b) \$9.32
 2 (a) NZ \$654 (b) ¥49962 (c) US\$450
 3 (a) \$116.28 (b) \$422.48 (c) \$72.46
 4 (a) \$30233 (b) \$586500 (c) \$997.50
 5 (a) \$630.57
 If A\$1 will be given for €0.7 and £0.6, then Virginia will end up with more AUD than the combined number of pounds and euro.
 (b) NZ \$378
 The exchange rate of A\$1 = NZ\$1.26 means that she will receive a higher number of NZD than AUD.
 6 C
 7 \$3.82
 8 (a) \$1799.60 (b) \$8400 (c) \$14199.60
 (d) \$5201.60 (e) 14.45%
 9 (a) (i) \$400 (ii) \$480 (iii) \$86.67 (iv) \$2480
 (b) (i) \$200 (ii) \$200 (iii) \$540 (iv) \$89.17
 (v) \$2540
 (c) Kathryn had to pay less as her initial deposit, but more for her monthly repayments. Therefore, the total cost of the TV was more with the balloon payment.
 10 Loss, \$97.24
 11 (a) Sample answer: Money deposited during the month earns no interest until the next month. If money is withdrawn during the month, there is no interest on that money while it was there earlier in the month. The

company would have had the benefit of these amounts of money without paying the client anything.

- (b) Sample answer: The client would only benefit if they did not have to use the account very much, otherwise Lenny has the use of their money without having to pay any interest.
 12 \$651.70
 13 (a) \$99.20 (b) \$3891.20
 14 (a) \$440 251.57 (b) \$1509.43
 (c) The Australian property represents a slightly better buy. The price and rent are similar but the rent yields a slightly higher return (4.4% p.a. compared with 4.1% p.a.) for the Australian property.
 15 (a) \$1200 (b) \$192
 (c) Sample answer: It could be argued either way. Yes, \$192 interest when a \$50 payment is late sounds excessive. No, the conditions of the deal were known at the start and the payment was late. The company may have to pay a lawyer to help retrieve the money.

CHAPTER REVIEW 1

- 1 C
 2 (a) \$1092 (b) \$688.80 (c) \$800
 3 (a) (i) 195 km (ii) 10.8 km (iii) 90.3 m
 (b) (i) 4.6 h or 4 h 37 min (ii) 9.2 min or 9 min 14 s
 (iii) 0.1 s
 4 \$542.50
 5 B
 6 \$8864
 7 Chen: \$612.50, Sanjeev: \$315.00, Sasha: \$722.40
 8 (a) \$73974 (b) \$15547 (c) \$15255.43
 (d) Refund: \$291.57
 9 \$12526.73
 10 \$2069.50
 11 \$19980
 12 C
 13 \$99.18
 14 (a) \$55230.05 (b) \$55333.48 (c) \$55357.21
 (d) \$55368.78
 15 D
 16 (a) 66.7% (b) \$703.95
 17 \$4.57
 18 B
 19 500 mL
 20 \$315.51

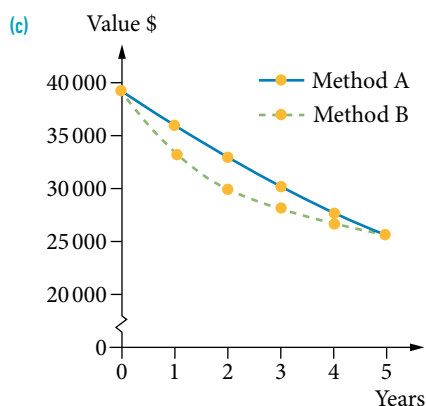
- 21 (a) \$4.68 (b) \$5.15
(c) \$4.95. The GST is 10% of \$4.68 which is more than 10% of \$2.75.
- 22 \$540
- 23 (a) \$109 670 (b) \$7880 (c) 7.2%
(d) Mike's Corporation: 2.88, Sunstart Communications: 6.25, CIO Mining: 21.64, YHSC Bank: 16.50. Mike's Corporation represents the best investment.
(e) Loss: \$5200
- 24 \$372.80 profit
- 25 13% p.a. assuming simple interest, 16% p.a. assuming compound interest
- 26 (a) \$1032 (b) \$4500
(c) \$1043, assuming the vet bills were not higher due to ill health.
- 27 7.7% p.a.

28 (a)

Age (years)	0	1	2	3	4	5
Value	\$38 950	\$35 834	\$32 967	\$30 330	\$27 904	\$25 672

(b)

Age (years)	0	1	2	3	4	5
Value	\$38 950	\$33 108	\$29 797	\$28 307	\$26 892	\$25 547



- 29 (a) \$90119.41 (b) \$26300 (c) Owes: \$63819.41
- 30 (a) \$81236 (b) \$17948.70 (c) \$4874.16
(d) Refund, \$1027.14
- 31 (a) \$511764 (b) \$76843.56 (c) \$420000
(d) \$343156.44 (e) Make: \$243236
(f) Sample answer: Anna and Juan already make a profit in choosing buying over renting. If the rent rose over the 25 years then the buying option would be even more attractive. Tax paid on the investment would make the renting option even less attractive. Costs of rates and body corporate fees were not estimated. These would remove much of the earnings from the buying option.

Chapter 2 Measurement

RECALL

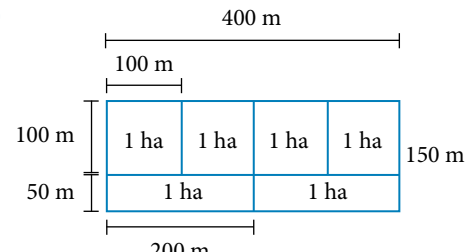
- 1 (a) AC (b) EF
- 2 (a) 8.56 km (b) 34 m (c) 140 mm
- 3 (a) 30.55 (b) 6.000 (c) 3.14
- 4 (a) $A = 70$ (b) $V = 226.2$ (c) $A = 98.1$
- 5 (a) $a = 9$ (b) $x = 6.86$ (c) $c = 13.60$
- 6 (a) $l = 9$ (b) $w = 3$
- 7 35%
- 8 (a) Increase of 18% (b) Decrease of 8%

EXERCISE 2.1

- 1 (a) Right angled (b) Not right angled
(c) Right angled (d) Not right angled
(e) Right angled (f) Not right angled
(g) Right angled (h) Right angled
- 2 D
- 3 (a) $x = 10.44$ cm (b) $x = 41.34$ cm (c) $x = 100$ mm
(d) $x = 9.44$ km (e) $x = 1.34$ m
(f) $x = 1.69$ m or $x = 169.18$ cm
- 4 (a) $x = 8$ cm (b) $x = 36.33$ cm (c) $x = 21.86$ m
(d) $x = 31.81$ km (e) $x = 13.10$ mm (f) $x = 1.90$ cm
- 5 19.3 m
- 6 25.3 m
- 7 7.5 m
- 8 (a) 4.49 m
(b) The calculated height, 5.46 m, would be greater than the length of the rope.
- 9 20.59 km
- 10 18.0 cm
- 11 39.6 cm or shorter to compensate for the width
- 12 Yes, the slab will be declared 'square'.
- 13 (a) $s = 25$ mm (b) $y = 16.3$ cm
- 14 $e = 155$ cm
- 15 $L = 3.1$ m
- 16 $H = 1.44$ m
- 17 (a) $AC = 3.606$ m (b) $DG = 8.544$ m
(c) $AH = 8.246$ m (d) $AG = 8.775$ m
- 18 $PQ = 79$ cm
- 19 1.66 m or smaller to allow for width, and the pair
- 20 $s = 7.0$ m
- 21 $x = 194$ cm

- 22 4.76 m
 23 11.35 cm
 24 7.07 cm
 25 Clearance: 9 cm

EXERCISE 2.2

- 1 (a) 110 cm (b) 24.6 km (c) 168 cm
 (d) 92.55 mm (e) 94 cm (f) 387 mm
- 2 C
- 3 (a) 1209.6 cm^2 (b) 573.4 cm^2 (c) 28.84 m^2
 (d) $196\,349.54 \text{ cm}^2$ or 19.63 m^2
 (e) 19.82 m^2
- 4 (a) 6 ha
 (b)
- 
- 5 (a) 200 m^2 (b) 1518.75 cm^2 (c) 97.5 m^2
 (d) 2.65 m^2 (e) 106.91 m^2
- 6 4070 cm^2
- 7 2527.38 cm^2
- 8 (a) $d = 10.2 \text{ m}$ (b) 50.5 m^2 (c) 69.7 m^2
 (d) 19.2 m^2
- 9 (a) 7700 mm^2 (b) 7300 mm^2 (c) 490 mm
- 10 (a) $P = 326.43 \text{ cm}$, $A = 5059.16 \text{ cm}^2$
 (b) $P = 58.01 \text{ m}$, $A = 210.09 \text{ m}^2$
 (c) $P = 17.27 \text{ cm}$, $A = 17.16 \text{ cm}^2$
- 11 (a) $l = 7 \text{ m}$ (b) $l = 9 \text{ cm}$ (c) $b = 14 \text{ mm}$
 (d) $r = 1.95 \text{ cm}$ (e) $c = 10 \text{ m}$ (f) $l = 1.81 \text{ km}$
- 12 5
- 13 (a) 1250 cm^2 (b) 209 m^2 (c) 753.98 cm^2
 (d) 1.45 m^2 (e) 970.01 cm^2
- 14 2.0175 ha
- 15 D
- 16 (a) $A_{\max} = \frac{3xy}{2}$ (b) $\frac{xy}{4}$
 (c) (i) $xy = 6$ (ii) $x = 2$, $y = 3$ and $P_{\max} = 12.61$ (2 d.p.)
- 17 (a) 297.4 m (b) 6.3 m (c) 3706.9 m^2 (d) 601.1 m^2
- 18 6 m^2
- 19 1.27 m
- 20 2.688 m^2

- 21 3.3 cm
 22 Pink: 21.46 cm^2 ; Yellow: 57.08 cm^2
 23 17 800

EXERCISE 2.3

- 1 (a) 2744 cm^3 (b) 840 cm^3 (c) 6 m^3
 (d) 771.9 cm^3 (e) $V = 461.81 \text{ mm}^3$ (f) $V = 72 \text{ cm}^3$
- 2 (a) 18 L (b) 7944 L (c) 312 000 L (d) 10 L
- 3 B
- 4 (a) $x = 2 \text{ m}$ (b) $x = 2.1 \text{ cm}$ (c) $x = 5.20 \text{ m}$
- 5 (a) A
 (b) The student would have incorrectly equated cubic centimetres with litres.
- 6 (a) 168 mL (b) 7252 L or 7 kL
- 7 1696 cm^3
- 8 \$810
- 9 The semicircular tent is larger by 0.04 m^3 .
- 10 4.08 m^3
- 11 3949 cm^3
- 12 $x = 5 \text{ mm}$
- 13 7920 cm^3
- 14 4320 cm^3
- 15 (a) 1440 000 L (b) 33 days
- 16 (a) 15 m^3 (b) 65 cm
- 17 0.58 m^3
- 18 14.66 cm
- 19 1.116 m^3
- 20 (a) $800\,000 \text{ m}^2$
 (b) 67 km. The harvester will complete a circular turn at each end, therefore stopping 6 m from the end to turn the draper front without hitting the fence. Marshall would choose to harvest with the longest strip to minimise turns. The assumption is that the paddock is perfectly rectangular with no trees or other obstructions to drive around.
- (c) 13 h 24 min (d) 240 tonnes
 (e) 298.51 kg/min, 40.2 min (f) 30 tonnes
 (g) 28.215 tonnes (h) 27 tonnes
 (i) 51 tonnes (j) 7.46 days
- 21 19 cm
- 22 45%
- 23 (a) 3168 m^3
 (b) 420 cylinders assuming a grid pattern and all upright. More would fit if cylinders could be on their sides, or in a hexagonal pattern.

24 1.33 m, 9.39 m

25 (a) 41.2 mL

(b) 32 mm, 5 mm

26 (a) $V_2 = 8V_1$

(b) $V_3 = 27V_1$

(c) $V_5 = 125V_1$

(d) $V_k = k^3 \times V_1$

EXERCISE 2.4

1 (a) 960 cm^3

(b) 210 cm^3

(c) 46.67 cm^3

(d) 51.84 cm^3 or 51836.28 mm^3

(e) 0.18 m^3

(f) 115.17 cm^3 or 115171.13 mm^3

2 (a) 31059.36 mm^3 or 31.06 cm^3

(b) 7.07 m^3

3 (a) $A = 84\text{ m}^2$

(b) $H = 15\text{ mm}$

(c) $r = 8.60\text{ cm}$

4 (a) 111.26 m^3

(b) 1044315.66 cm^3

(c) 333494.72 mm^3

(d) 26666.67 mm^3

5 (a) 4974 cm^3

(b) 2094 mL

6 30630.53 cm^3

7 (a) 953 m^3

(b) 256 m^3

(c) 27%

8 (a) 328.52 cm^3

(b) 102.18 m^3

(c) 34.30 cm^3

9 33888 cm^3

10 (a) C

(b) A: Radius and diameter have not been distinguished.

11 2.88 cm

12 (a) 11

(b) 5.8 cm

13 37.5 mL

14 15.8 cm

15 49

16 (a) 94.1 m^3

(b) 420 m^3

(c) 90 cm

(d) 5

17 8.5 m

18 19 mm

19 (a) 2592276 m^3

(b) 3054.38 kg (2 d.p.)

(c) 99.86%

20 33%

21 \$8.63

22 (a) 6056 cm^3

(b) 1125 mL

23 32 cm

24 (a) 7.66 cm

(b) 5.64 cm

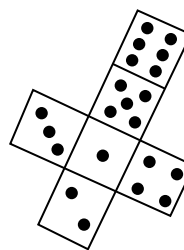
EXERCISE 2.5

1 (a) 250 cm^2

(b) 2120 cm^2

2 2.43 m^2

3 Possible solution



4 (a) 39.7 m^2

(b) 1046.2 mm^2

(c) 59.9 m^2

5 (a) 874 cm^2

(b) 193 cm^2

6 8780 cm^2

7 (a) 269 m^2

(b) 801 cm^2

8 23885 mm^2

9 68 m^2 , assuming no overlaps or gaps

10 Both designs use the same amount of plastic. The first one is big enough to walk through. With the second option you would have to plant, cover with the plastic and then leave. An irrigation system would be required.

11 (a) 11.43 m^2

(b) The tent is student ready.

The vent area is 12.5% of the base area.

12 (a) C

(b) B

13 23 cm, assuming that SA referred to the outer surface of the vase, not including the rim at the top. If the inner surface was included, the calculation would have indicated a much shorter vase.

14 174 cm^2 , 181.25%

15 444 cm^2

16 58%

17 (a) 19%

(b) Yes, 27%

EXERCISE 2.6

1 (a) 208 cm^2

(b) 210 cm^2

2 (a) 11150 mm^2

(b) 7.9 m^2

3 (a) 1018 m^2

(b) 8495 mm^2

(c) 38 m^2

(d) 1816 cm^2

4 (a) 534 cm^2

(b) 547 cm^2

(c) 919 cm^2

5 330 m^2

6 (a) 186 cm^2

(b) 8495 mm^2

7 (a) D

(b) A

8 6.1 m^2

9 (a) 63300000 km^2

(b) 44400000 km^2

(c) 5%

10 5.50 m^2

11 124 cm^2

12 10.4 cm; Diameter would be about 21 cm, which is a bit more than the edge length of the cube.

- 13 The new surface area is 157 mm^2 more than the original, i.e. more than two and a half times the original area.
- 14 The new shape has an extra half as much SA as the original.

CHAPTER REVIEW 2

- 1 50.9 cm
- 2 78.3 cm or 91.2 cm
- 3 B
- 4 31 cm
- 5 25.7 m
- 6 C
- 7 10 m^2
- 8 (a) 49 mm (b) 19.73 m
- 9 (a) 4.62 cm^2 (b) 703 cm^2
- 10 16285 cm^2
- 11 900 mL
- 12 A
- 13 201 L
- 14 221 mL
- 15 D
- 16 15.09 m^2
- 17 B
- 18 (a) 146 cm^3 (b) 144 cm^2
- 19 146 m
- 20 38.3 m
- 21 3.56 m^2
- 22 (a) 1198 cm^2 (b) 5088 cm^3 (c) 7%
- 23 (a) 151 cm^3 (b) 161 cm^2
- 24 The volume remains the same. The SA increases by a factor of 3.
- 25 (a) 30 m (b) 33 m^2 (c) 9000 L
- 26 (a) 7.14 m
(b) Inner circle: 314 cm^2 ; middle ring: 2513 cm^2 ; outer ring: 3534 cm^2
(c) 12723 cm^3
(d) 6927 cm^2
- 27 (a) 69.1 cm (b) 322 cm^2 (c) 290 cm^3 (d) 8710 cm^3
- 28 7.76 m
- 29 21 kg
- 30 9.3 cm
- 31 (a) 6427 cm^2 (b) 79% (c) 22450 cm^2 (d) 1.5 cm

Chapter 3 Similar figures

RECALL

- 1 (a) 2070 cm (b) 0.35 m (c) 5200 m
- 2 (a) 9.09 (b) 5.657
- 3 (a) 2.5 (b) 0.214
- 4 (a) $x = \pm 17$ (b) $x = 5.59$
- 5 (a) $x = 50$ (b) $x = 2.53$
- 6 (a) $a = 56^\circ$ (b) $a = 51^\circ$
- 7 (a) $a = 53^\circ, b = 80^\circ, c = 47^\circ, d = 47^\circ$
(b) $e = 52^\circ, f = 78^\circ, g = 52^\circ$

EXERCISE 3.1

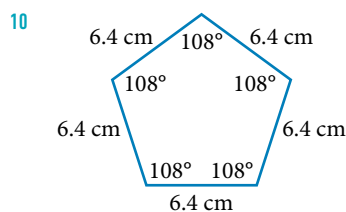
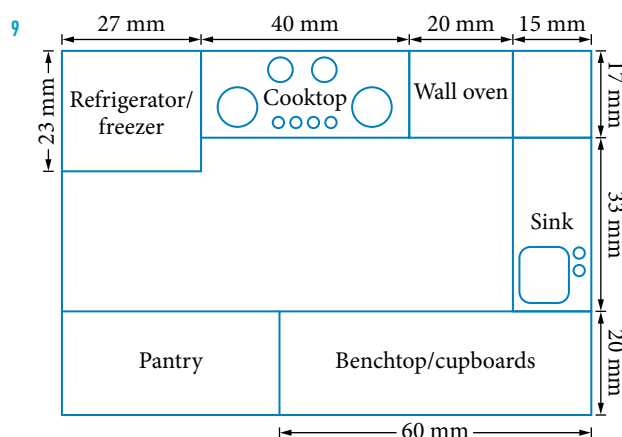
- 1 (a) $k = 4$ (b) $k = 0.05$ (c) $k = 80$
(d) $k = 2.91$ (e) $k = 9.23$ (f) $k = 0.08$
- 2 (a) 60 m (b) 240 cm (c) 25 cm
(d) 293.04 mm (e) 3 m (f) 165 km
- 3 (a) 6.5 km (b) 80 cm (c) 0.51 m
(d) 10.99 cm (e) 7.06 mm (f) 2.23 m
- 4 (a) $k = 4$ (b) $k = 0.25$ (c) $k = 0.17$
(d) $k = 3$ (e) $k = 1.65$ (f) $k = 6.5$
- 5 (a) Similar (b) Not similar
(c) Not necessarily similar (d) Not similar
(e) Not necessarily similar (f) Not similar
(g) Similar (h) Similar (i) Similar
(j) Not similar (k) Similar (l) Similar
- 6 (a) 2.89 (b) 0.59 (c) 4
(d) 1.90 (e) 0.2 (f) 1.23
- 7 C
- 8 B
- 9 (a) 13.33 (b) 9.6 cm
- 10 6.24 m
- 11 (a) $x = 13.33 \text{ m}$ (b) $x = 3.75 \text{ cm}$
(c) $x = 12.14, y = 12.14$ (d) $x = 7.5 \text{ mm}$
(e) $x = 14.4 \text{ cm}$ (f) $x = 6.43 \text{ cm}, y = 7.71 \text{ cm}$
(g) $x = 4.27 \text{ cm}, y = 7.47 \text{ cm}$ (h) $x = 4.62 \text{ cm}$
(i) $x = 24 \text{ m}$ (j) $x = 10.18 \text{ cm}, y = 11.64 \text{ cm}$
(k) $x = 171.88 \text{ cm}$ (l) $x = 735 \text{ mm}$
- 12 4.08 m
- 13 All pairs of angles are equal, meeting the similarity test for triangles, $W = 25 \text{ m}$
- 14 8.25 m, similarity test for triangles where all pairs of angles are equal
- 15 120 cm
- 16 3.26 m

- 17 $d = 30\text{ cm}$
 18 37.5 cm
 19 (a) $x = 11\text{ m}$, $k = 3.75$
 (b) $x = 3\text{ km}$, $k = 2$
 (c) $x = 4\text{ cm}$, $y = 5\text{ cm}$, $k = 1.5$
 20 Yes, Hassan is obeying the doctor's ladder rule.
 21 3.24 m
 22 4.3 m
 23 1.25
 24 $h = 93.51\text{ cm}$
 25 30 cm , 51 cm

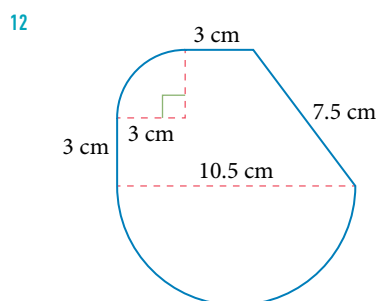
EXERCISE 3.2

- 1 (a) $1 : 100$, reduction (b) $1 : 2500$, reduction
 (c) $10 : 1$, enlargement (d) $1 : 100\,000$, reduction
 (e) $1 : 10\,000$, reduction (f) $20 : 1$, enlargement
 (g) $1 : 1500$, reduction (h) $1 : 200$, reduction
 (i) $400 : 1$, enlargement (j) $8 : 1$, enlargement
 (k) $1 : 50$, reduction (l) $5000 : 1$, enlargement
 (m) $1 : 20\,000$, reduction (n) $1 : 40\,000$, reduction
 2 (a) 10 m (b) 25 m (c) 60 m (d) 12.5 m
 3 (a) 20 mm (b) 50 mm (c) 140 mm (d) 1.1 mm
 4 C
 5 (a) A
 (b) D: real and drawing parts in the wrong order, cm to mm conversion multiply or divide incorrectly applied
 6 $1 : 80$
 7 (a) 20 m (b) 9.75 m
 (c) 0.75 m (d) $5.5\text{ m} \times 1.75\text{ m}$
 8 (a) $1 : 50\,000\,000$

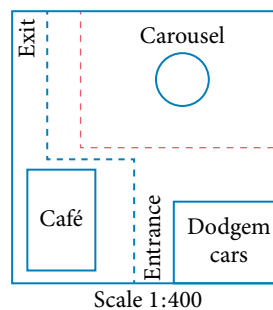
Between cities	Distance on map (cm)	Actual distance (km)
Perth–Melbourne	5.80	2900
Perth–Brisbane	7.28	3640
Melbourne–Brisbane	2.84	1420
Sydney–Brisbane	1.50	750
Brisbane–Adelaide	2.96	1480



- 11 3.3 cm or 33 mm



- 13 Answers may vary. Sample answer:



- 14 2.47 cm
 15 (a) $1 : 10\,526\,316$ (b) $1 : 30\,000\,000$

EXERCISE 3.3

- 1 (a) $k = 5$ (b) $k = 8$ (c) $k = 1.55$ (d) $k = 0.35$
 (e) $k = 4.10$ (f) $k = 0.33$ (g) $k = 2.40$ (h) $k = 3.67$
 2 (a) 32 cm^2 (b) 98 km^2 (c) 90 mm^2
 (d) 7096.76 cm^2 (e) $68\,400\text{ cm}^2$ (f) 0.12 mm^2
 (g) 3.84 cm^2 (h) 3975.98 m^2

- 3 (a) 100cm^2 (b) 375cm^2 (c) 8cm^2
 (d) 10m^2 (e) 6mm^2 (f) 161.18mm^2
- 4 A
- 5 (a) B (b) D, division by k instead of k^2
- 6 5
- 7 2.24
- 8 (a) $k = 3$ (b) 37.70cm^2 (c) 339.29cm^2 (d) 9
 (e) The ratio between surface areas for similar shapes is the square of the scale factor.
- 9 (a) 1.6 (b) 768cm^2
- 10 150833m^2
- 11 (a) $k = 2$, $A = 72\text{m}^2$ (b) $k = 1.67$, $A = 36.11\text{mm}^2$
 (c) $k = 2.22$, $A = 296.30\text{cm}^2$ (d) $k = 4$, $A = 5.63\text{m}^2$
 (e) $k = 0.71$, 49.83cm^2
 (f) $k = 20$, 20000cm^2 or 2m^2
- 12 (a) $k = 6$, $a = 27.22\text{cm}^2$ (b) $k = 2$, $a = 697.5\text{cm}^2$
 (c) $k = 1.25$, $a = 57.6\text{m}^2$ (d) $k = 0.21$, $A = 22.18\text{cm}^2$
- 13 (a) $k = 3$, $x = 18\text{cm}$ (b) $k = 1.5$, $x = 21\text{cm}$
 (c) $k = 1.50$, $x = 3.00\text{cm}$ (d) $k = 0.08$, $x = 9.6\text{m}$
- 14 (a) $k = 3$, $x = 7\text{mm}$ (b) $k = \frac{1}{2}$, $x = 7.5\text{cm}$
 (c) $k = 5$, $x = 5\text{cm}$ (d) $k = 0.45$, $x = 0.58\text{m}$
- 15 (a) 1.645 (b) 1813mm^2
- 16 36cm^2
- 17 $x = 15.34\text{m}^2$
- 18 87.5 ha
- 19 237cm^2
- 20 2500m^2
- 21 (a) 1.4 is the scale factor for the circular top, not the whole pizza.
 (b) 1.6
 (c) The family size pizza is better value, as you are getting 1.96 times the amount for only 1.6 times the price.
- 22 31.2cm by 20.8cm
- 23 Yes, she will have enough wax: 29.28 mm used, 30.72 mm remaining

EXERCISE 3.4

- 1 (a) 3 (b) 7 (c) 3.79
 (d) 4.95 (e) 1.59 (f) 2.15
- 2 (a) 4096cm^3 (b) 181.10cm^3 (c) 15187.5mm^3
 (d) 432000km^3 (e) 343m^3 (f) 1445.02m^3
- 3 (a) 53cm^3 (b) 1cm^3 (c) 6.25mm^3
 (d) 0.51km^3 (e) 21.78m^3 (f) 0.99cm^3

- 4 (a) $k = 1.5$, 6750mm^3 (b) $k = 1.67$, $V = 1111\text{mm}^3$
 (c) $k = 0.4$, 38cm^3 (d) $k = 1.13$, 1253cm^3
 (e) $k = 0.3$, 32mm^3 (f) $k = 5$, 11250cm^3
- 5 (a) $k = 5$, 8cm^3 (b) $k = 0.25$, 9375mm^3
 (c) $k = 0.17$, 15.28cm^3 (d) $k = 0.5$, 3.13m^3
- 6 (a) $x = 15.39\text{cm}$ (b) $x = 11.75\text{cm}$
 (c) $x = 11.60\text{cm}$ (d) $x = 0.5\text{m}$
- 7 (a) $k = 4$, $x = 6.25\text{mm}$ (b) $k = 4.77$, $x = 0.73\text{m}$
 (c) $k = 0.58$, $x = 8.77\text{cm}$ (d) $k = 0.53$, $x = 10.63\text{cm}$
- 8 (a) 4.67 (b) 101.63 (c) 1.02m^3
- 9 2.15
- 10 (a) 26.62m^3 (b) 6400
- 11 D
- 12 (a) D, 28000cm^3
 (b) D, false assumption that the volume has doubled because the lengths have doubled.
- 13 64cm^3
- 14 9.48m^3
- 15 215cm^3 , assuming internal dimensions given
- 16 335 kg
- 17 6356cm^2
- 18 3600cm^2
- 19 Height: 19.79cm, volume: 6083.5cm^3
- 20 2800000, assuming the bricks are solidly packed and not merely a shell; overlap to hold the shape together is not a problem.
- 21 (a) 20cm (b) 20% (c) 16%
- 22 The large tub costs about 45 cents per handful, while the small tub costs about 55 cents per handful, so the large tub represents better value.
- 23 87.5%

CHAPTER REVIEW 3

- 1 D
- 2 B
- 3 $x = 2.67\text{cm}$
- 4 (a) $k = 1.833$ (b) $k = 1.563$
- 5 (a) $a = 26.4\text{m}$ (b) $b = 56.7\text{mm}$
 (c) $c = 8.5\text{cm}$ (d) $d = 10.0\text{cm}$, $e = 11.3\text{cm}$
- 6 D
- 7 1 : 50000
- 8 C
- 9 104m

- 10 42 cm by 35 cm
 11 D
 12 $x = 61$ cm
 13 491 cm^3
 14 1289 cm^2
 15 850 km
 16 2.45
 17 (a) 45 cm (b) 191 cm^2
 18 5140 cm^3
 19 2.44 m
 20 1.55
 21 $h = 6$ m
 22 240 ha
 23 0.96 m^2
 24 (a) $3.75 \text{ m} \times 5 \text{ m}$ (b) 25 m^2 (c) 25 squares
 25 (a) 20 (b) 67
 26 $29 \text{ cm} \times 22 \text{ cm} \times 14 \text{ cm}$
 27 Volume: 1.33 m^3 , dimensions: 1.11 m, 0.93 m, 1.30 m
 28 (a) \$3.50 (b) \$2300
 29 (a) 16 h 25 min
 (b) \$115.20, assuming area must be rounded up to a whole number.
 30 (a) (i) 0.33 cubic units/square unit
 (ii) 0.5 cubic units/square unit
 (iii) 1.67 cubic units/square unit
 (b) 30 units
 31 (a) 7742 cm^3 (b) 36 cm (c) 167 cm^2 (d) 34 cm
 32 $xy = f^2$
 33 2.0625; Marid needs double the number of rocks.

Chapter 4 Linear equations and their graphs

RECALL

- 1 (a) 13 (b) 24 (c) -8
 2 (a)

x	-2	0	1	4	10
y	-18	-8	-3	12	42

 (b)

x	-2	0	2	4	6
y	2	3	4	5	6

 (c)

x	-10	-5	0	5	10
y	31	16	1	-14	29

 3 (a) like terms (b) not like terms

- 4 (a) $7x + 7y - 9$ (b) $4xf^2 + 7x - 3f - 10$
 5 (a) $x = \frac{1}{2}$ (b) $x = 1$
 6 (a) $y = 6$ (b) $x = -4$
 7 (a) Positive (b) Negative (c) Zero

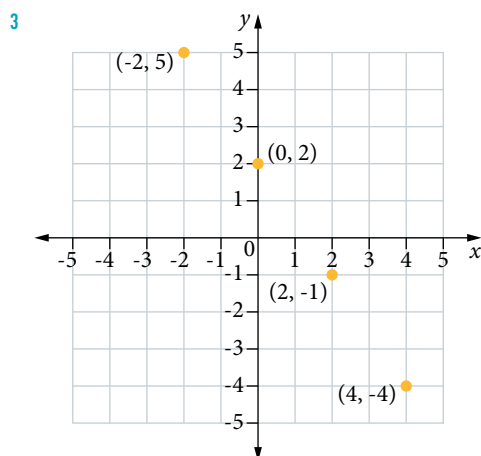
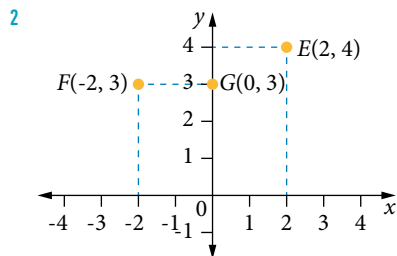
EXERCISE 4.1

- 1 (a) Non-linear equation (b) Linear expression
 (c) Linear equation (d) Non-linear equation
 (e) Linear equation (f) Linear equation
 2 (a) $g = 6$ (b) $k = 8$ (c) $b = 6$
 (d) $n = 8$ (e) $m = 1$ (f) $e = 4$
 (g) $m = -5$ (h) $b = \frac{1}{2}$ (i) $t = 3.45$
 3 C
 4 $4l - 76 = 124$
 5 (a) $f = 10$ (b) $x = \frac{10}{7}$ (c) $a = 6$ (d) $y = 5$
 6 (a) B
 (b) C: When solving, reverse order of operations applies.
 7 (a) $a^2 = a + 420$, where a is Andrea's age
 (b) One of the terms contains a variable to the power of 2.
 8 (a) $h = 5$ (b) $e = 5$ (c) $b = -2$
 (d) $x = 7$ (e) $f = -19\frac{1}{2}$ (f) $h = 11\frac{1}{2}$
 9 (a) \$0.80/min
 (b) Jana is 55, Meredith is 52, Lin is 62.
 (c) Length is 30 cm, width is 20 cm.
 (d) 46, 48, 50
 (e) Billie is 6, Asok is 18, Cassie is 20.
 (f) The son is 14, the father is 42.
 10 (a) 41, 43, 45 (b) 124, 126, 128, 130
 11 (a) \$40 000 (b) \$240 (c) 70 cents
 12 -52, -51, -50
 13 11
 14 (a) The x and $-x$ cancel each other out, and so the answer is always 6.
 (b) The solution did not involve x so it will not matter what the starting value is.
 (c) Various answers.
 15 (a) 21 km, assuming tip not included and no other hidden costs.
 (b) 0.7 km
 (c) Increased by \$4.81.
 16 (a) Length of fencing: 13.657 m, $x = 5.657$ m, $w = 2.828$ m, $r = 2.173$ m
 (b) Circular enclosure, area of 14.834 m^2 .

- 17 (a) Kelly: $25t$, Gino: $68 - 32t$
 (b) 1 h 11 min 35 s, 29.825 km from Cairns

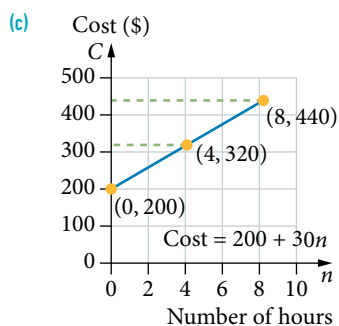
EXERCISE 4.2

- 1 (a) $A(4, 1)$, $B(-3, -2)$, $C(3, 0)$, $D(0, -4)$
 (b) $A(15, 170)$, $B(38, 80)$, $C(25, 60)$, $D(0, 120)$



- 4 (a) C

(b) Hours	0	4	8
Cost	\$200	\$320	\$440



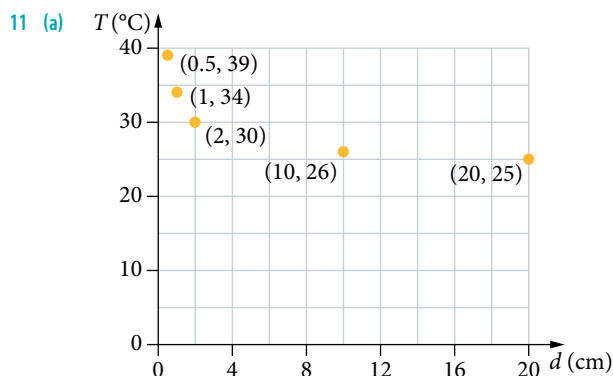
- 5 (a) 3, a vertical increase of 3 units per unit of horizontal increase.
 (b) 4, a vertical increase of 4 units per unit of horizontal increase.
 (c) -2, a vertical decrease of 2 units per unit of horizontal increase.
 (d) $\frac{2}{3}$, a vertical increase of 2 units per 3 units of horizontal increase.

- 6 The student has not included the negative, in the value of x_1 .

$$b = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-6 - 8}{5 - (-2)} = \frac{-14}{7} = -2$$

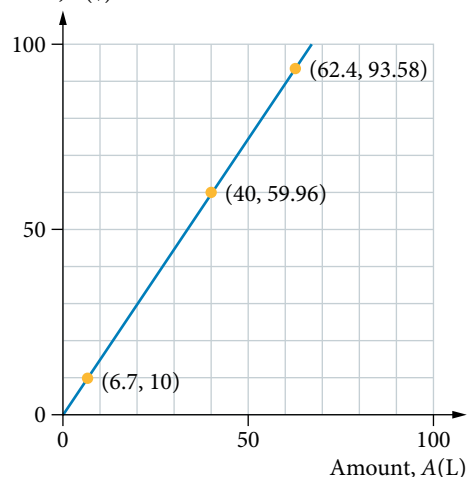
- 7 (a) 3 (b) $\frac{1}{2}$ (c) 0 (d) $-\frac{1}{3}$
 8 (a) 2 (b) 2 (c) -5 (d) $-\frac{1}{4}$
 9 (a) $\frac{5}{2}$ (b) $\frac{3}{4}$ (c) $\frac{5}{3}$

10 $c = 50 + 20n$



- (b) No, the temperature does not drop at a constant rate as you move away from the heater. The plotted points are not in a straight line.
 (c) The heater has little effect beyond 5 cm and the room temperature becomes the major factor.

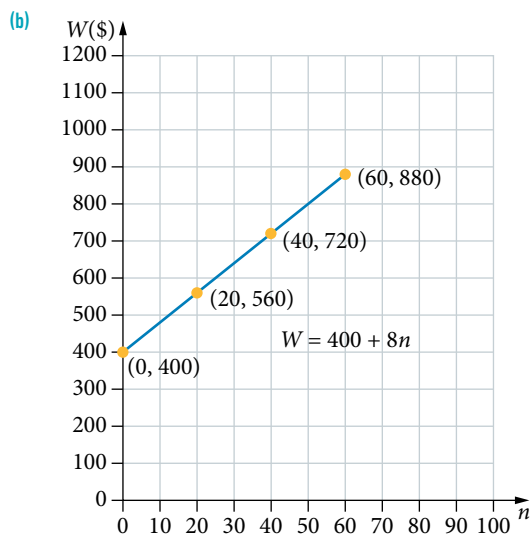
- 12 (a) Cost, $C(\text{\$})$



- (b) $C = 1.5A$
 (c) 150 c/L

13 (a)

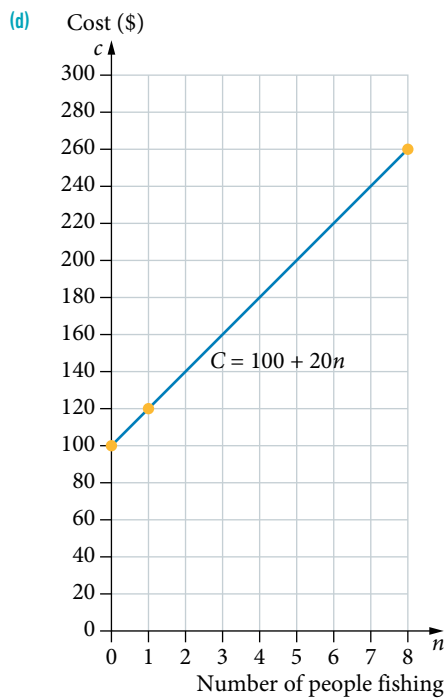
Number of tools	0	20	40	60
Weekly wage (\\$)	400	560	720	880



(c) \$1200

(d) 100 hired tools is outside the normal conditions so the rule may not apply.

14 (a) \$120 (b) \$260 (c) $c = 100 + 20n$



(e) \$200

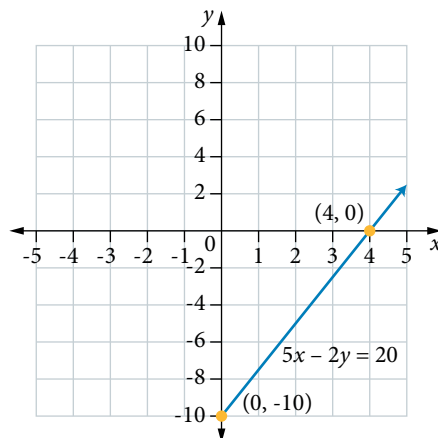
15 (a) -6

(b) (7,12) gradient 3; (12,18) gradient 2

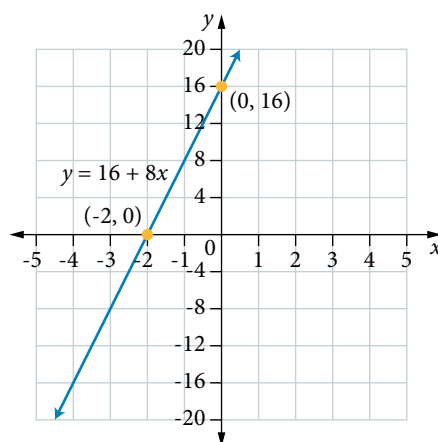
16 (a) \$171 (b) \$491

EXERCISE 4.3

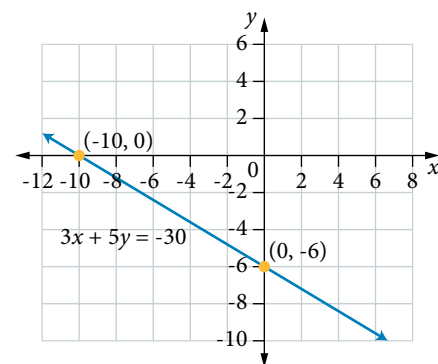
1 (a)



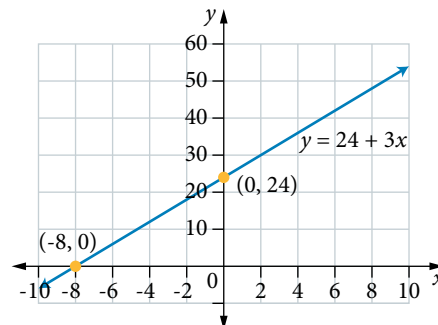
(b)

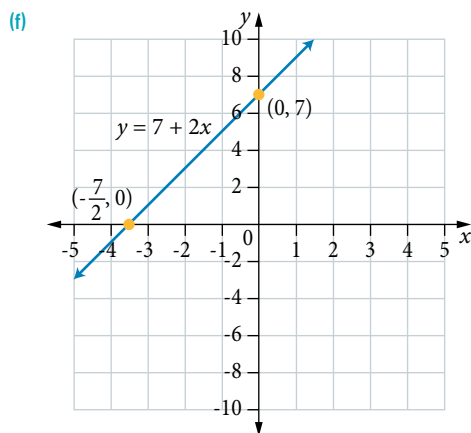
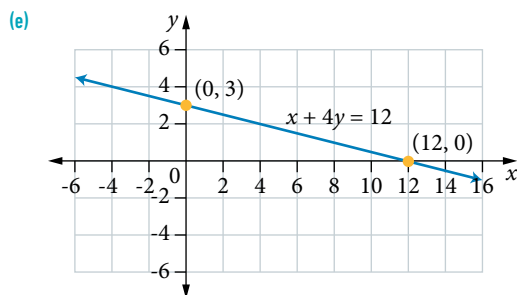


(c)



(d)





2 (a) C

(b) A: Incorrect assumption that the value of b in $y = a + bx$ gives an intercept, as a does.

3 A

4 (a) $y = 3 + 4x$ (b) $y = 5 + \frac{2}{3}x$ (c) $y = 3 - 4x$

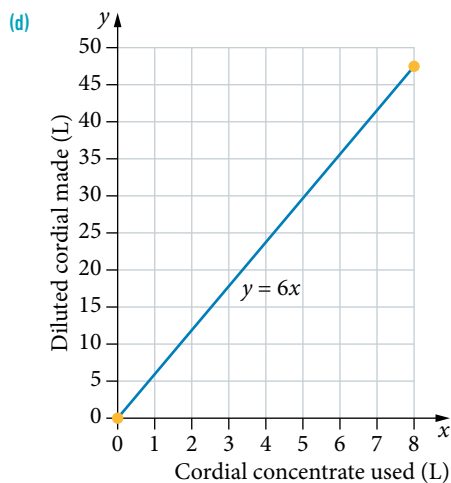
(d) $y = x$ (e) $y = -2 - \frac{5}{7}x$ (f) $y = 3$

5 (a) $(0, 0)$ and $(2, 12)$

(b) $y = 6x$,

Diluted cordial made (L) = $6 \times$ cordial concentrate used (L)

(c) 8L



6 (a) Eric: 400 km, Jason: 200 km

(b) Eric

(c) 2.5 hours

(d) 50 km/h

(e) $d = 400 - 50t$

7 (a) $y = 5 + x$

(b) $y = 2x$

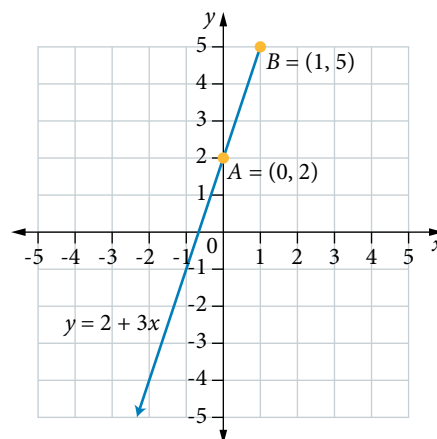
(c) $y = 6 - 2x$

(d) $y = 2 - \frac{1}{3}x$

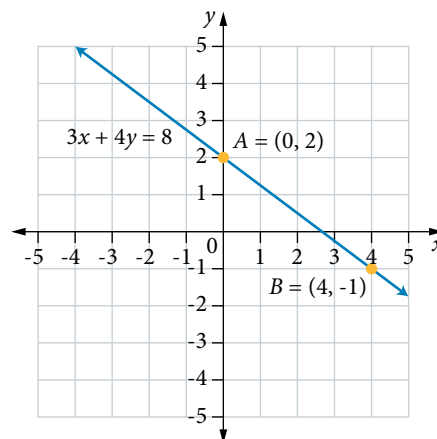
(e) $y = -1 - 2x$

(f) $y = -3x$

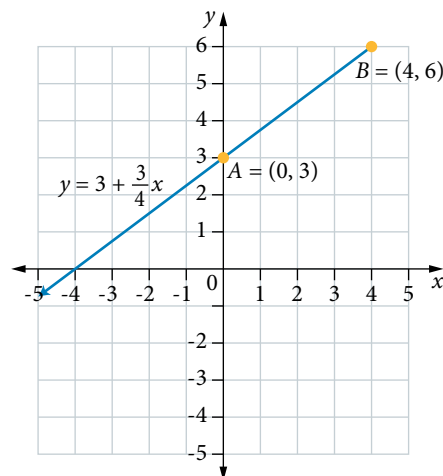
8 (a) $a = 2$ and $b = 3$



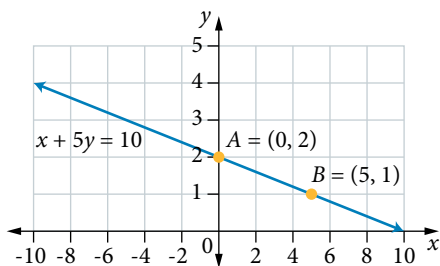
(b) $a = 2$ and $b = -\frac{3}{4}$



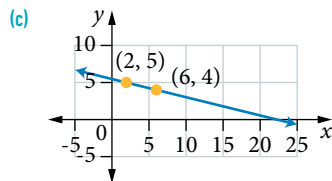
(c) $a = 3$ and $b = \frac{3}{4}$



(d) $a = 2$ and $b = -\frac{1}{5}$



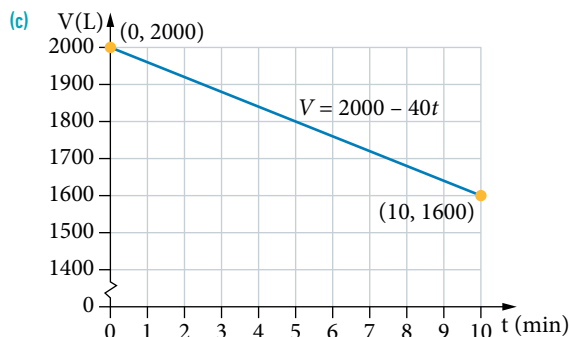
9 (a) $y = 5.5 - 0.25x$ (b) $(22, 0)$



The line joining the two points crosses the y -axis between 5 and 6 and crosses the x -axis at 22. The line is sloping downwards (from left to right) and therefore $b < 0$.

10 (a) -40; Every minute 40 L leaks from the tank.

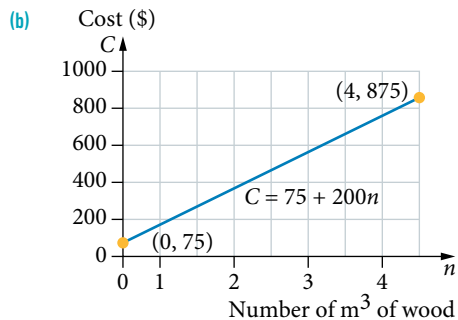
(b) $V = 2000 - 40t$



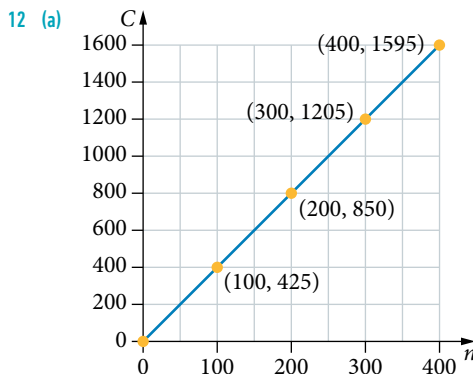
(d) (i) 1920 L (ii) 1800 L

(e) The graph would not be linear. It would be steeper at the start.

11 (a) If n is the volume, measured as the number of cubic metres of wood, and c (\$) is the total cost: $c = 200n + 75$ or $c = 75 + 200n$.



(c) \$575 (d) 2.1m^3



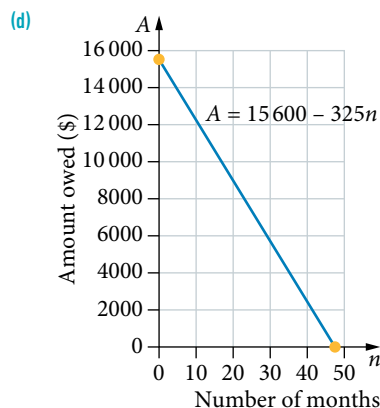
(b) $C = 35 + 3.9n$ (c) Cost (\$) per ream of paper

(d) The fixed delivery cost

(e) (i) \$620 (ii) \$1127

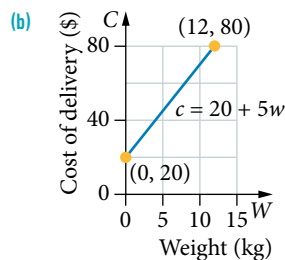
13 (a) \$15600 (b) \$325

(c) $A = 15600 - 325n$



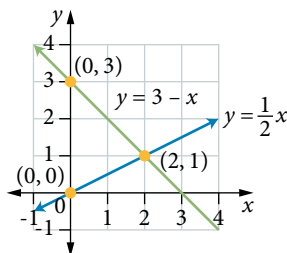
(e) \$9750

14 (a) If w is weight (kg) and c is the collection fee (\$), then $c = 20 + 5w$.

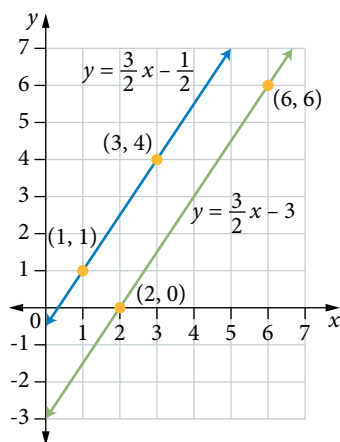


(c) \$120 (d) The collection fee is only charged once.

15 (a) Sample answer: $y = \frac{1}{2}x$ and $y = 3 - x$



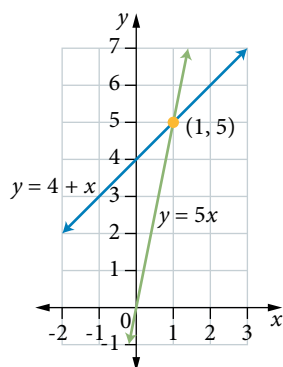
(b) Sample answers:



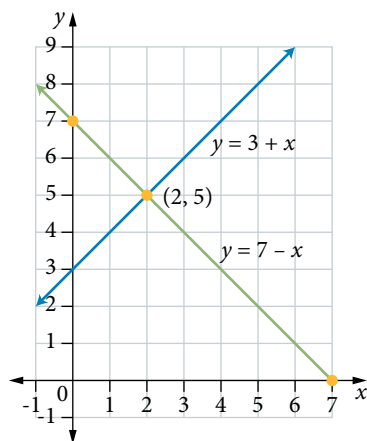
EXERCISE 4.4

1 (a) $x = 4, y = 7$ (b) $a = 6, b = 1$ (c) $m = 6, n = -4$ (d) $a = 2, b = 0$ (e) $x = 6, y = 4$ (f) $c = 4, d = 10$ (g) $p = -2, t = 3.5$ (h) $a = -0.5, b = 1.5$ (i) $t = -9, z = -7$

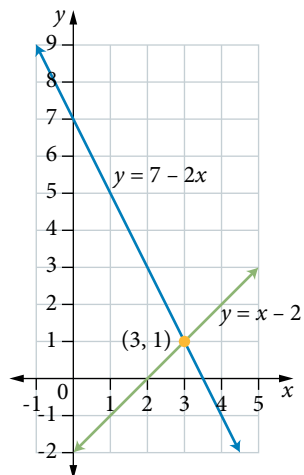
2 (a)

 $x = 1, y = 5$

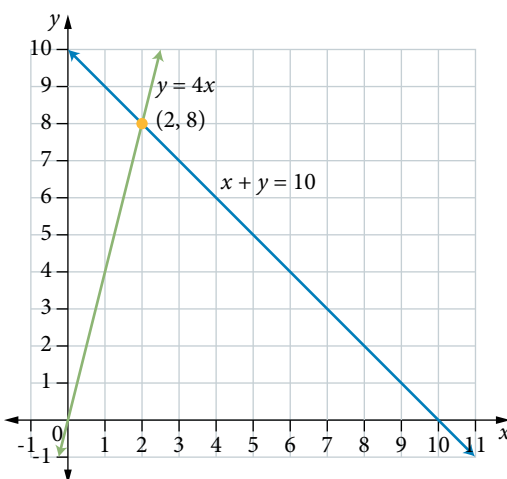
(b)

 $x = 2, y = 5$

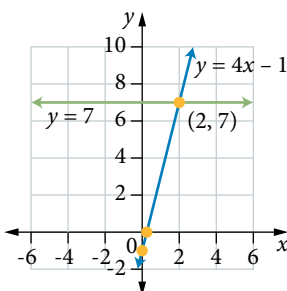
(c)

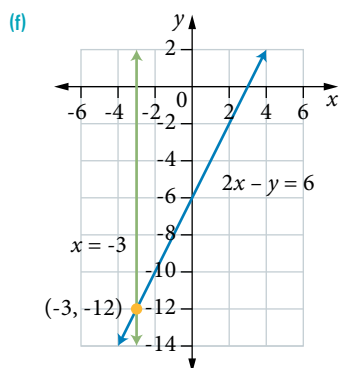
 $x = 3, y = 1$

(d)

 $x = 2, y = 8$

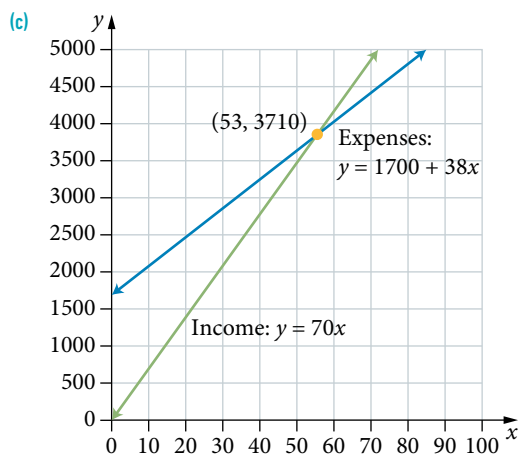
(e)

 $x = 2$ and $y = 7$



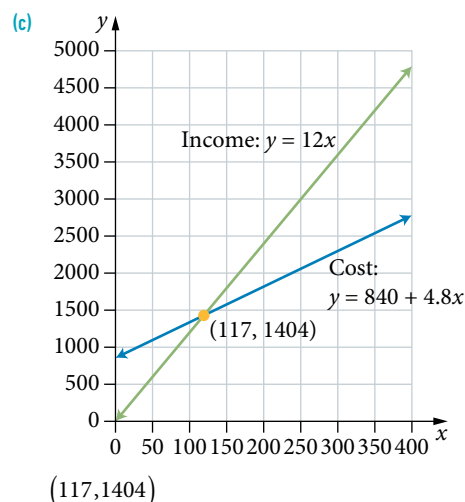
$x = -3$ and $y = -12$

- 3 (a) $x + y = 8$, $3x + y = 14$ (b) Cake: \$3, coffee: \$5
 4 (a) Op-tin (b) Opt-out (c) 150 calls per month
 5 (a) C (b) A
 6 (a) A (b) B (c) 3572 (d) \$392 920
 7 (a) \$30 (b) \$2500 (c) \$10
 (d) 125 (e) \$1500 (f) 625
 8 (a) The Lakehouse: $c_1 = 250 + 40n$, The Shed: $c_2 = 300 + 35n$
 (b) 10 (c) cheaper
 9 (a) $M + L = 22$, $M = 2(L - 1)$ (b) $M = 14$, $L = 8$
 10 (a) Expenses = $1700 + 38x$ (b) Income = $70x$

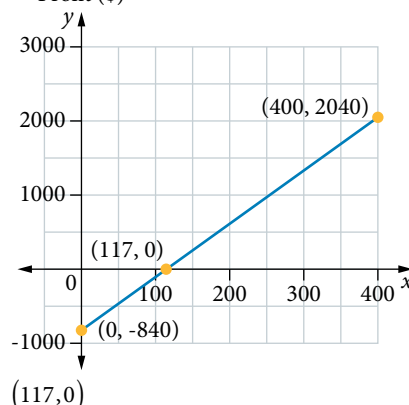


53 people

- (d) 53 people attending makes a loss, 54 people attending makes a profit.
 11 The solutions to the system of linear equations $y = -2x$ and $y = 2 - x$ are $x = -2$ and $y = 4$.
 For $y = -2x$, $4 = -2(-2)$
 For $y = 2 - x$, $4 = 2 - (-2)$
 12 (a) \$50 (b) \$40 (c) \$620
 13 (a) $y = 12x$ (b) $y = 840 + 4.8x$

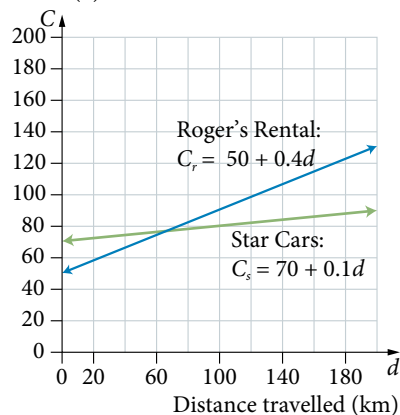


- (d) 117
 (e) 117
 (f) Profit (\$)

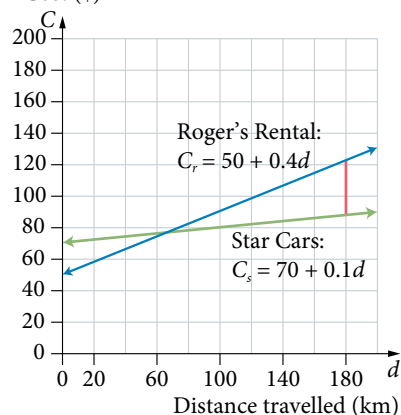


- (g) \$417 600
 14 (a) 40
 (b) For fewer than 40 people use Finger Foodz, for more than 40 people use Mobile Pancakes.
 15 Must charge more than \$705 per person to be sure of making a profit, but the price can be lowered to little more than \$235 if maximum numbers are achieved.
 16 $y = 2x - 5$
 17 (a) $b = 6$, $j = 2$
 (b) $p = 2$, $q = -1$
 (c) $a = 2.6$, $b = -1.3$
 (d) $m = \frac{2}{3}$, $n = 0.6$
 (e) $x = 12.5$, $y = -3.5$

18 (a) Cost (\$)



(b) Cost (\$)



Star Cars is cheaper by \$34/day when daily travel averages 180 km.

19 (a) (i) \$445 (ii) \$495

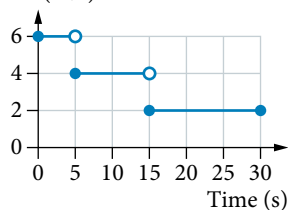
(b) \$474 to break even, \$527 to make a \$2000 profit

20 (a) \$450000

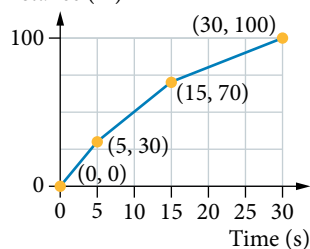
(b) Profit will decrease from \$450000 to \$350000, so the marketing proposal is not a good idea.

EXERCISE 4.5

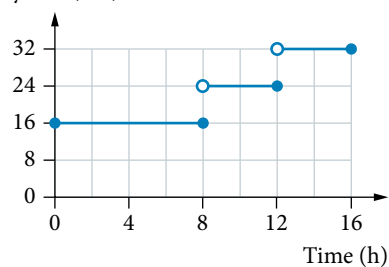
1 (a) Speed (m/s)



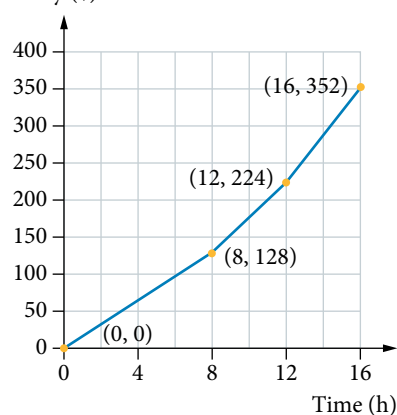
(b) Distance (m)



2 (a) Pay rate (\$/h)



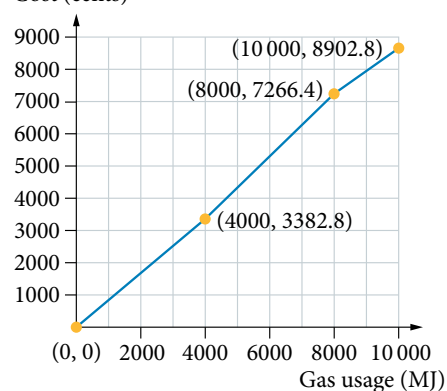
(b) Pay (\$)



3 (a)

Usage (MJ)	0	4000	8000	10000
Cost	0	3382.8	7266.4	8902.8

Cost (cents)

(b) $C = 7266.4 + 0.8182(x - 8000)$

(c) \$45

(d) \$43.54

4 (a) 63 cents

(b) 63 cents

(c) 4 minutes

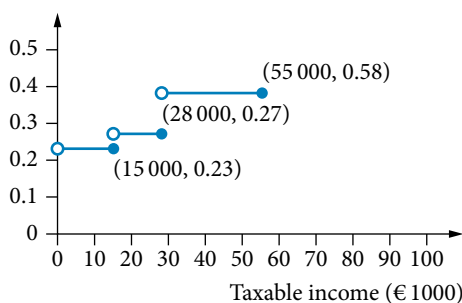
(d) \$3.92

(e) (i) D

$$(ii) \quad C = \begin{cases} 0.35, & 0 < x \leq 0.5 \\ 0.63, & 0.5 < x \leq 1 \\ 0.91, & 1 < x \leq 2 \\ 1.19, & 2 < x \leq 3 \\ 1.47, & 3 < x \leq 4 \\ 1.75, & 4 < x \leq 5 \end{cases}$$

5 (a) B

(b) Taxation rate



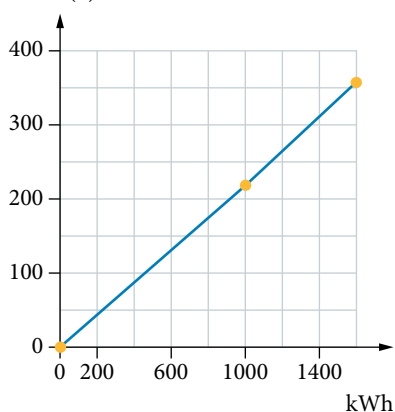
(c) No

6	Age	Starting time	Age	Starting time
	7	8:00 am	12	10:00 am
	8	8:00 am	13	11:00 am
	9	9:00 am	14	11:00 am
	10	9:00 am	15	12:00 pm
	11	10:00 am	16	12:00 pm

7 (a) \$174.94

(b) \$358.34

(c) Cost(\$)

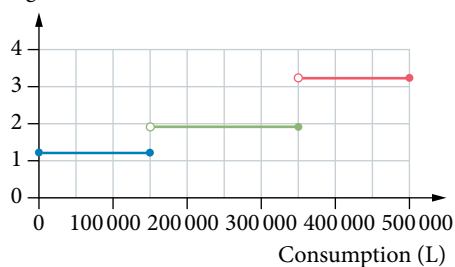


(d) The rate increases as consumption increases, to encourage people to use less electricity.

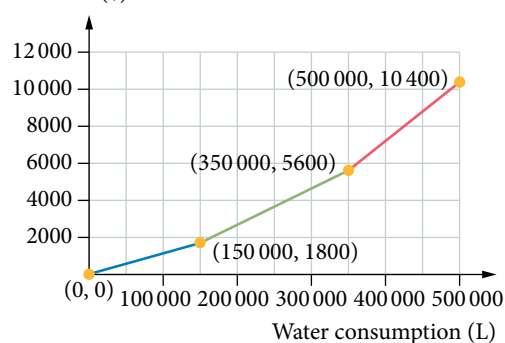
$$8 \text{ (a) } y = \begin{cases} x-3, & x \leq 0 \\ -3, & 0 < x \leq 2 \\ 3x-9, & 2 < x \leq 4 \end{cases}$$

$$(b) y = \begin{cases} 2, & 0 \leq x < 1 \\ 3, & 1 \leq x < 3 \\ 2, & 3 \leq x < 5 \end{cases}$$

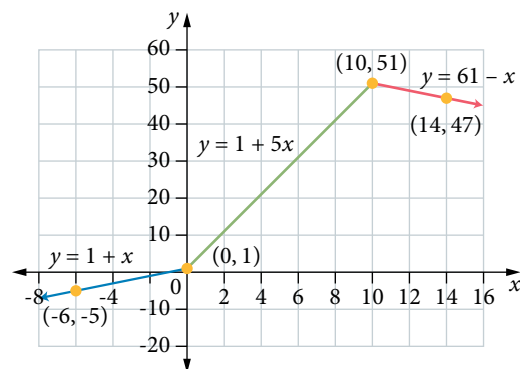
9 (a) Charge (c/L)



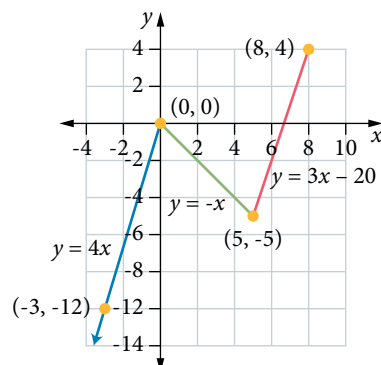
(b) Cost (\$)



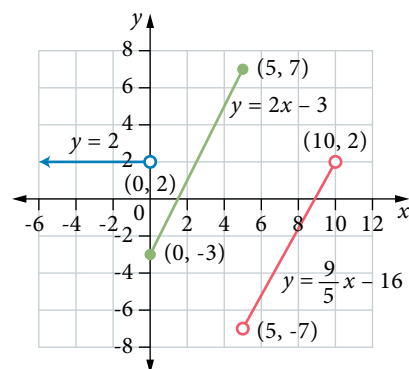
10 (a)

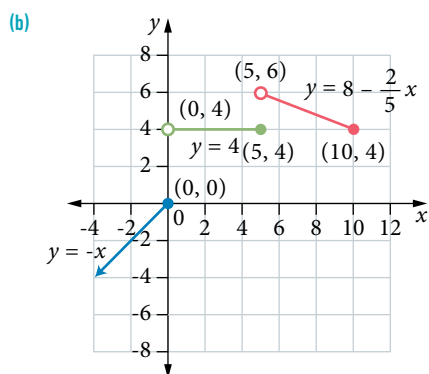


(b)

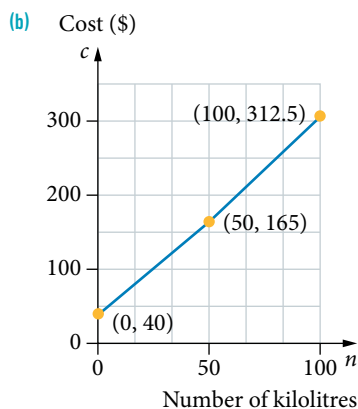


11 (a)

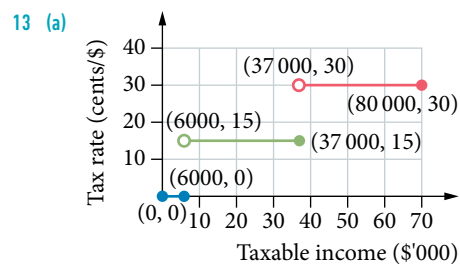




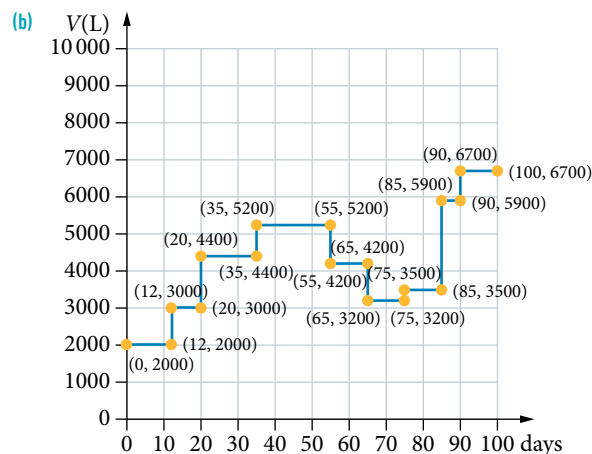
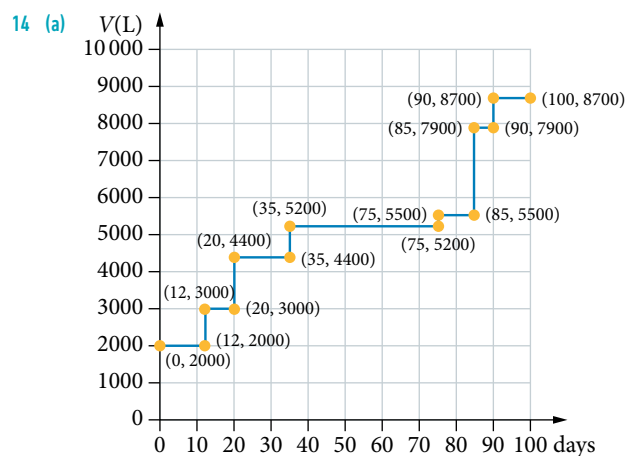
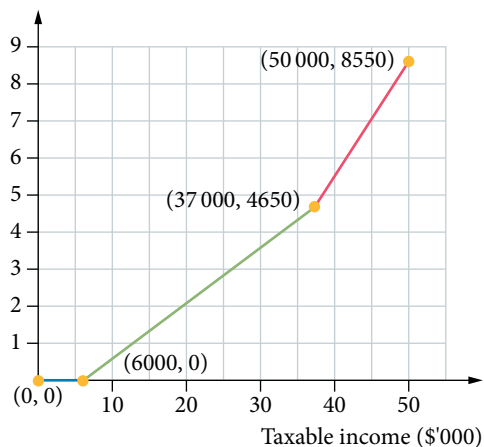
12 (a)
$$c = \begin{cases} 40 + 2.5n, & 0 < n \leq 50 \\ 165 + 2.95(n - 50), & n > 50 \end{cases}$$



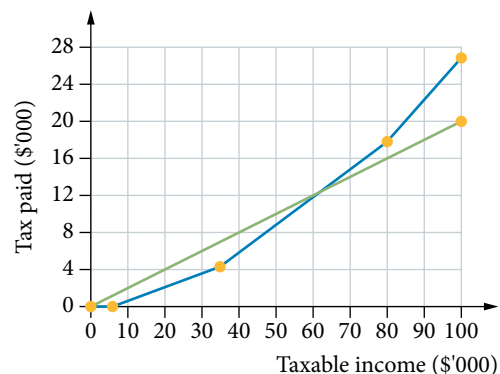
(c) 62 kL



(b) Tax paid (\$'000)



15 (a)



(b) Taxpayers with an income less than \$61 500 will pay more tax on the proposed system.

Taxpayers with an income of \$61 500 will pay the same amount of tax: \$12 300.

Taxpayers with an income greater than \$61 500 will pay less tax.

CHAPTER REVIEW 4

1 (1, -1)

2 (5, 0)

3 (a) $y = 3 + 5x$ (b) $y = 4 - \frac{2}{3}x$ (c) $y = -1 - 0.6x$

4 $y = 6 + \frac{3}{2}x$

5 $(0, 4)$

6 $x = 4$

7 $j = 6$

8 $f = 1$

9 $a = 6, b = 1$

10 1

11 A

12 D

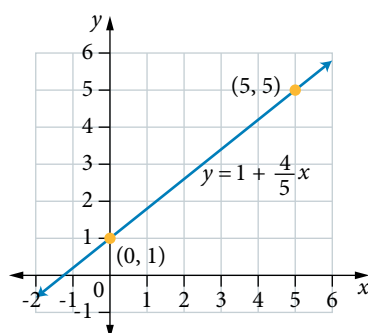
13 (a) $d + 6$

(b) $d \div 2$

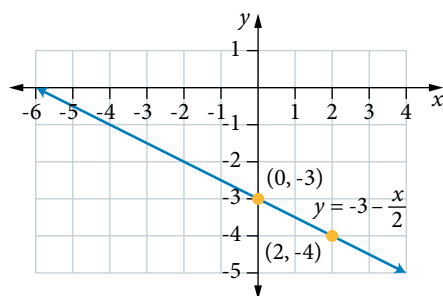
(c) $4d + 3$

14 \$1.63

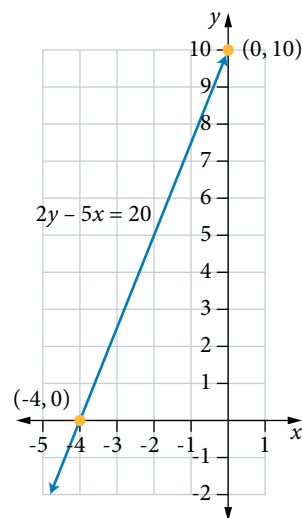
15 (a)



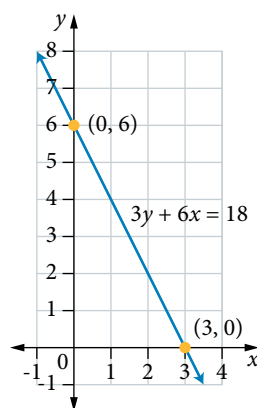
(b)



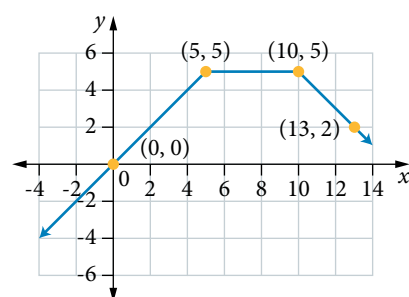
(c)



(d)



16



17 (a) Height = x , length = $30 - 2x$, width = $21 - 2x$

(b) Volume = $(30 - 2x)(21 - 2x)(x)$

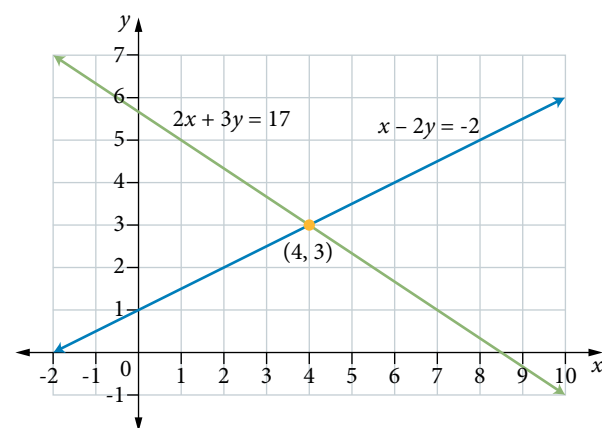
(c) $x = 2$: 884 cm^3 , $x = 5$: 1100 cm^3 , $x = 10$: 100 cm^3

(d) $0 < x < 10.5$

(e) Sample answer: This is not a linear equation because x has a power of 3 when the brackets are expanded. A linear equation has all variables to the power of 1.

18 $e = 7$

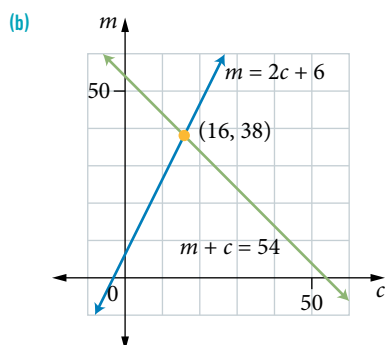
19 $x = 4, y = 3$



20 (a) $2l + 2(l - 8) = 248$ or $4l - 16 = 248$

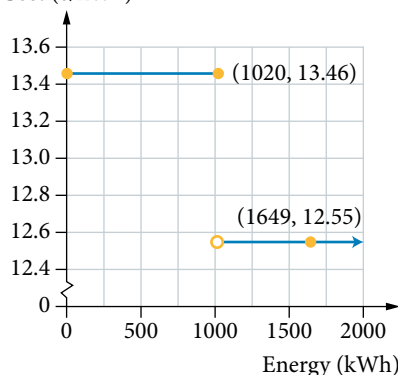
(b) Length = 66 m, width = 58 m

21 (a) $m + c = 54, m = 2c + 6$



Christy is 16, Mang Mang is 38

- 22 (a) 9, 20 (b) Back: 36, front: 42
 (c) 12 bicycles, 16 cars
 23 (a) 13.4600 c/kWh (b) 12.5500 c/kWh (c) 7.28 c/kWh
 (d) Cost (c/kWh)



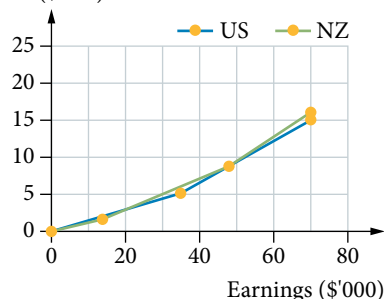
- (e) Off peak is cheaper, to encourage people to use the electricity when the power is not in high demand.

- 24 (a) Plan 1: $\text{Cost}_1 = \begin{cases} 20, & 0 \leq n \leq 500 \\ 20 + 0.5(n - 500), & n > 500 \end{cases}$
 Plan 2: $\text{Cost}_2 = \begin{cases} 35, & 0 \leq n \leq 800 \\ 35 + 0.4(n - 800), & n > 800 \end{cases}$

- (b) 530 MB

- 25 (a) (i) \$0 (ii) \$5253 (iii) \$15 047.40
 (b) (i) \$0 (ii) \$1750 (iii) \$8890 (iv) \$16 150

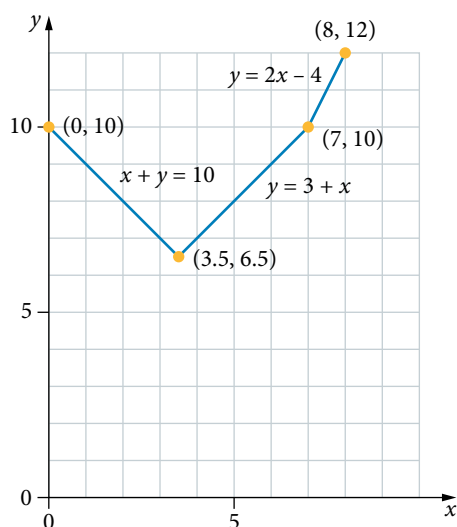
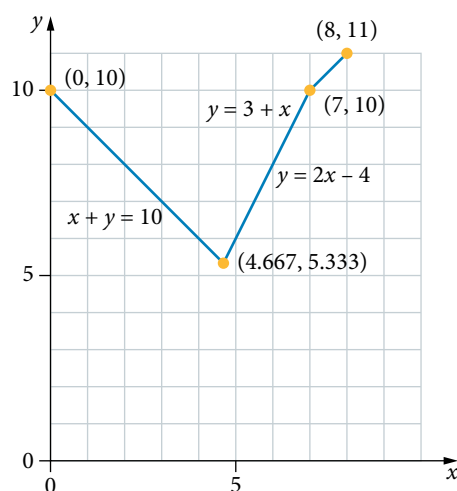
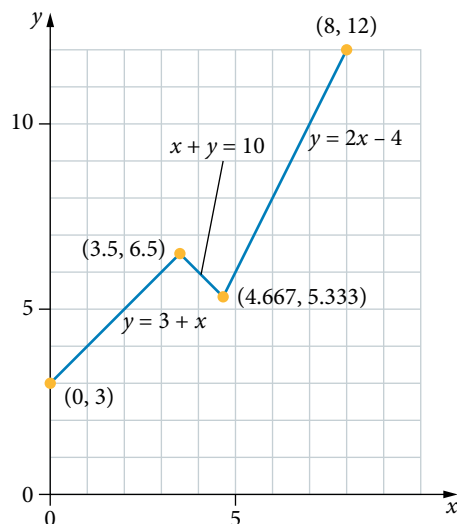
- (c) Tax (\$'000)



- (d) These graphs are continuous as each graph can be drawn without lifting the pen from the page. There are no jumps from one tax bracket to the next.
 (e) 0.125: the tax rate for the earnings below \$14 000 is 12.5 cents per dollar.

- (f) NZ tax is lower for earnings less than \$20 000.
 US tax is lower for earnings greater than \$20 000.

- 26 Any of

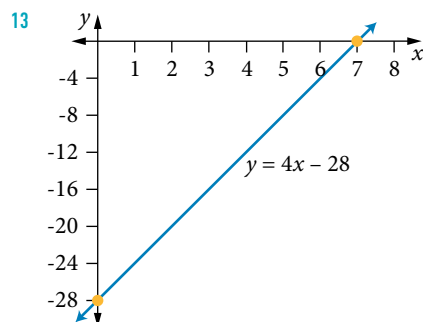


- 27 Working algebraically, the final expression is a constant, 3.

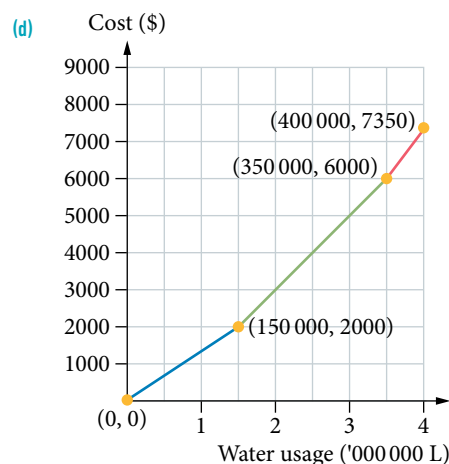
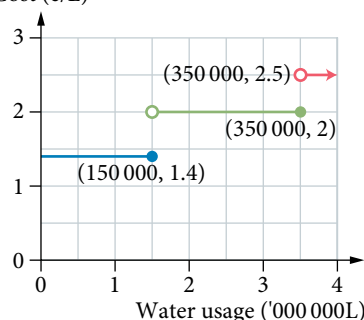
$$\frac{5x + 15}{5} - x = 3$$
- 28 (a) 3449 (b) \$950 000 (c) 4667
 (d) No; the profit would decrease from \$950 000 to \$800 000.

Exam review: Unit 1

- 1 \$11.50/h
- 2 (a) \$3780 (b) \$27 780
- 3 (a) \$31.50 (b) \$330
- 4 (a) $a = 1.69$ m (b) $c = 0.90$ m
- 5 $P = 3.20$ m, $A = 0.68$ m²
- 6 (a) 4156.3 cm³ (b) 297.5 m³
- 7 (a) The proportions between pairs of sides are not constant, so the triangles are not similar.
 (b) At least two pairs of angles are equal, so the triangles are similar.
 (c) The ratio for the pairs of corresponding sides is not constant, so the triangles are not similar.
- 8 82.5 mm
- 9 Maximum overall length: 615 mm; Maximum height: 49 mm
- 10 (a) 60 cm² (b) 4 cm² (c) 100 cm² (d) 6.92 cm²
- 11 11.82
- 12 -2



- 14 (a) $a = 4$, $b = 3$ (b) $x = 1$, $y = 7$
 (c) $c = 8$, $d = 2$ (d) $x = 7$, $y = 4$
- 15 (a) \$2100 (b) \$3100
 (c) Cost (c/L)



- 16 \$1140
- 17 Refund: \$1593.82
- 18 12 cm
- 19 5.6 cm
- 20 $x = 9$ cm
- 21 1991 cm³
- 22 Lucas: 25, Nathan: 22, Hayley: 32
- 23 (a) $8(2T + A) = 880$, $4(T + A) = 260$
 (b) Tradesperson: \$45 / h, apprentice: \$20 / h
- 24 6 months
- 25 32.14 cm
- 26 40%

Chapter 5 Applications of trigonometry

RECALL

- 1 (a) 87 600 m (b) 0.56 m (c) 67 m
- 2 (a) 1.4 (b) 0.06 (c) 24.283
- 3 (a) $x = 2.692$ (b) $x = 7.813$ (c) $x = 33.547$
- 4 (a) $\theta = 51^\circ$ (b) $\theta = 163^\circ$ (c) $\theta = 45^\circ$
- 5 (a) $\theta = 20^\circ$ (b) $\theta = 35^\circ$ (c) $\theta = 29^\circ$
- 6 (a) $a = 38^\circ$, $b = 142^\circ$, $c = 38^\circ$, $d = 142^\circ$
 (b) $w = 25^\circ$, $x = 155^\circ$, $y = 25^\circ$, $z = 25^\circ$
- 7 (a) $x = 11.4$ m (b) $x = 10.9$ mm

EXERCISE 5.1

- 1 (a) $x = 19.25$ mm (b) $x = 59.57$ cm (c) $x = 33.36$ cm
 (d) $x = 26.10$ cm (e) $x = 37.51$ cm (f) $x = 14.18$ m
 (g) $x = 22.05$ mm (h) $x = 49.12$ cm
- 2 (a) $x = 11.43$ cm (b) $x = 124.39$ cm (c) $x = 54.26$ m
 (d) $x = 14.23$ cm (e) $x = 184.78$ mm (f) $x = 499.07$ mm

- 3 (a) $\theta = 40^\circ$ (b) $\theta = 72^\circ$ (c) $\theta = 39^\circ$
 (d) $\theta = 40^\circ$ (e) $\theta = 77^\circ$ (f) $\theta = 55^\circ$

4 D

5 (a) B (b) A and D

6 $x = 20.62$ cm, $\theta = 14^\circ$

7 67°

8 $\theta = 17^\circ$

9 $L = 2.83$ m

10 $\theta = 74^\circ$

11 6.26 m

12 (a) $\theta = 55^\circ$ (b) $L = 1.76$ m

13 5.41 km

14 $H = 31$ cm, $\alpha = 55^\circ$, $h = 17.78$ cm, $w = 25.39$ cm

15 $\theta = 61^\circ$

16 18.87 m

17 1.02 m

18 $CE = 1.97$ m

19 (a) $x = 5$ cm (b) $W = 30.64$ cm

20 14.25°

EXERCISE 5.2

1 262 cm

2 B

3 C

4 390 m

5 $e = 50^\circ$

6 37°

7 42°

8 $d = 160$ m

9 34.99°

10 $x = 10.86$ m

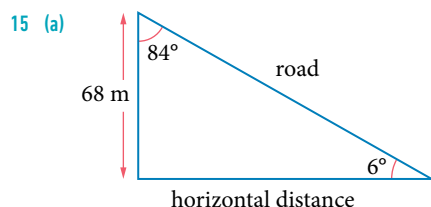
11 11.31°

12 (a) 948 m (b) 340 m

13 (a) 2.72 km (b) 5.12 km

14 (a) 95 m

(b) The student has incorrectly identified the angle of depression as the angle between the slope and the vertical.



(b) 651 m

16 (a) 72° (b) Yes, 25.30 m $<$ 28 m

17 $\theta = 5.74^\circ$

18 $\theta = 47^\circ$

19 (a) 61.97 m (b) 61°

20 $x = 153$ cm

21 6.1 m

22 6.84 m

23 Tree: 8.2 m to 9.5 m high, inaccuracy up to 70 cm

EXERCISE 5.3

1 (a) S 25° W, 205° T (b) S 79° E, 101° T (c) N 46° W, 314° T

(d) N 33° E, 033° T (e) S 44° W, 224° T (f) N 74° W, 286° T

2 (a) 315° T (b) 150° T (c) 288° T

(d) 135° T (e) 249° T (f) 032° T

3 (a) E (b) SE (c) N 70° W

(d) S 76° W (e) N 57° E (f) N 9° W

4 (a) S 27° W

(b) The student has either measured from west to south or has failed to recognise that point P is located 27° from south.

(c) D

5 (a) C (b) D

6 (a) 14.3 m (b) 8.6 km (c) 246.2 km

(d) 59.7 km (e) 84.0 km (f) 75.2 m

7 N 55° E

8 50 m

9 (a) 115° T (b) N 65° W (c) 24.3 km

10 (a) 228° T (b) N 48° E (c) 1270 m

11 (a) N 50° E, 50° T (b) S 60° E, 120° T (c) S 10° E, 170° T

(d) S 50° W, 230° T (e) N 60° W, 300° T (f) N 10° W, 350° T

12 (a) 060° T (b) 015° T (c) 320° T

(d) 240° T (e) 195° T (f) 140° T

13 36.99 km, 020° T

14 63.89 m, N 78° E

15 (a) (i) S 37° W (ii) 127° T

(b) 38.5 km from Mount Lagrange, 51.1 km from Mount Monge

16 (a) N 65° E (b) 57.4 m

17 (a) 003° T (b) 3 minutes

EXERCISE 5.4

1 (a) $a = 13.08$ m (b) $b = 24.32$ cm

(c) $a = 71.59$ mm (d) $c = 53.65$ cm

(e) $b = 29.27$ mm (f) $a = 26.03$ km

(g) $c = 77.58$ cm (h) $a = 6.52$ cm

- 2 (a) $A = 42^\circ$ (b) $A = 40^\circ$ (c) $A = 42^\circ$ (d) $C = 52^\circ$
 (e) $B = 73^\circ$ (f) $C = 48^\circ$ (g) $A = 11^\circ$ (h) $B = 16^\circ$
- 3 (a) $x = 126.50$ cm (b) $X = 81.10^\circ$, $Y = 37.90^\circ$
 (c) $x = 14.14$ mm (d) $X = 30.41^\circ$
 (e) $X = 83.44^\circ$ (f) $x = 191.75$ mm
- 4 (a) $U = 43.22^\circ$, $T = 72.78^\circ$, $t = 22.32$ cm
 (b) $F = 78^\circ$, $d = 49.30$ km, $e = 26.85$ km
 (c) $K = 17^\circ$, $j = 49.78$ m, $l = 35.39$ m
- 5 A
- 6 $A = 46^\circ$, $B = 24^\circ$
- 7 (a) C (b) D, $\frac{\sin(T)}{\sin(S)} < 1$ and $\frac{s}{t} > 1$
- 8 (a) 14.56 km (b) 14.12 km
- 9 (a) $A = 40^\circ$, $B = 60^\circ$, $C = 80^\circ$ (b) 24.62 km
 (c) 18.28 km
- 10 (a) 68.23 m (b) 44 m
- 11 841.52 m
- 12 45.77 m
- 13 61.33 km
- 14 21°
- 15 16.51 km
- 16 (a) 33.05 km (b) N 88° W
- 17 97.98 m

EXERCISE 5.5

- 1 (a) 78.94 cm (b) 76.16 m (c) 58.50 cm
 (d) 240.74 cm (e) 10.12 cm (f) 8.50 cm
 (g) 60.20 cm (h) 21.03 mm
- 2 (a) $A = 40.80^\circ$ (b) $A = 19.11^\circ$ (c) $A = 122.37^\circ$
 (d) $A = 89.11^\circ$ (e) $C = 137.73^\circ$ (f) $B = 3.37^\circ$
 (g) $C = 65.59^\circ$ (h) $B = 82.55^\circ$
- 3 (a) $x = 69.43$ cm (b) $x = 28.56$ cm (c) $x = 55.39^\circ$
 (d) $x = 78.34^\circ$ (e) $x = 7.94$ m (f) $x = 59.77^\circ$
- 4 (a) 16.89 m
 (b) The student has forgotten to take the square root of x^2 .
 (c) The maximum length of any side in a triangle is less than the sum of the other two side lengths.
- 5 (a) $M = 32.1^\circ$, $N = 47.9^\circ$, $p = 61.1$ cm
 (b) $Q = 46.5^\circ$, $R = 80.0^\circ$, $S = 53.5^\circ$
 (c) $Y = 24.6^\circ$, $Z = 106.4^\circ$, $x = 5.9$ m
 (d) $U = 43.4^\circ$, $V = 9.9^\circ$, $W = 126.7^\circ$
- 6 (a) D (b) A, correct for $\cos(C)$
- 7 (a) B (b) $x = 7.52$ cm

- 8 $c^2 = a^2 + b^2$
- 9 $\theta = 23.41^\circ$
- 10 166 km
- 11 $CB = 33.16$ km
- 12 156 km
- 13 15 km
- 14 240° T
- 15 (a) 52° (b) 27° (c) 101°
- 16 (a) 18.2 km (b) 022° T
- 17 (a) $D = 100^\circ$ (b) 40 m (c) N 79° W
- 18 (a) $D = 37$ km (b) 073° T
- 19 62°
- 20 14.91 km
- 21 $x = 3.868$ m, $h = 1.726$ m
- 22 (a) 545 km (b) N 37° W

EXERCISE 5.6

- 1 (a) 438.38 cm² (b) 4.13 m² (c) $12\,287.28$ mm²
 (d) 69.62 cm² (e) 838.72 cm² (f) 2.85 m²
- 2 (a) 69.98 mm² (b) 896.64 m² (c) 1600.41 mm²
 (d) 2339.24 cm² (e) 309.45 m² (f) 9.59 m²
- 3 (a) B
 (b) A and D, because the known angle is not between the known sides.
- 4 D
- 5 56.74 m²
- 6 16.76 m²
- 7 (a) 53.49 m (b) 118 m²
- 8 (a) $B = 55^\circ$ (b) $a = 38.43$ cm (c) 897.11 cm²
- 9 (a) $B = 68.43^\circ$ (b) $C = 68.57^\circ$ (c) 76.72 m²
- 10 (a) 1500 cm² (b) $A = 61.93^\circ$ (c) 1500 cm²
 (d) $C = 90^\circ$ (e) 1500 cm²
- 11 (a) 3352 cm² (b) $c = 77.37$ cm
 (c) Area = 3352 cm² both times
- 12 $a = 84.91$ cm
- 13 56.79 km²
- 14 17.50 m²
- 15 \$92.38
- 16 1949 cm²
- 17 1124 m²
- 18 (a) 2.94 m² (b) 3.14 m²
- 19 215.18 m²
- 20 88%

- 21 661.26 cm^2
 22 76%
 23 2063.14 cm^2
 24 (a) \$969 700 (b) \$648 900

CHAPTER REVIEW 5

- 1 $a = 20.85 \text{ m}$
 2 D
 3 $\theta = 17^\circ$
 4 (a) $a = 42.73 \text{ mm}$ (b) $\theta = 33.69^\circ$
 (c) $P = 63.45^\circ$ (d) $d = 23.49 \text{ cm}$
 5 $\theta = 74.93^\circ$
 6 072° T
 7 $S58^\circ \text{ W}$
 8 315° T
 9 (a) 155° T (b) $N80^\circ \text{ W}$ (c) 225° T
 10 (a) 74.17 m (b) 29.97 m
 11 C
 12 47.11 m
 13 (a) $\alpha = 18.43^\circ$ (b) $\beta = 7.13^\circ$ (c) $\angle MNB = 25.56^\circ$
 (d) $NM = 12.65 \text{ m}$ (e) $NB = 12.09 \text{ m}$
 14 (a) $A = 38.05^\circ$ (b) $B = 29.54^\circ$ (c) $C = 112.41^\circ$
 15 (a) $w = 128.66 \text{ m}$ (b) $U = 80.20^\circ$ (c) $V = 74.80^\circ$
 16 (a) $R = 35^\circ$ (b) $r = 4.14 \text{ km}$ (c) $q = 7.04 \text{ km}$
 17 (a) $F = 29.47^\circ$ (b) $G = 105.53^\circ$ (c) $g = 31.34 \text{ mm}$
 18 D
 19 D
 20 44.94 cm^2
 21 17 cm
 22 C
 23 2241.64 cm^2
 24 A
 25 (a) 19° (b) 105 m (c) Decrease of 2°
 26 080° T
 27 (a) B (b) D
 28 15.76 km
 29 (a) 102.42 cm^2 (b) 74°
 30 (a) 484 m (b) More (c) $13\,232 \text{ m}^2$ (d) 72.4 m
 31 (a) 27.2 m^2 (b) 7 m
 32 $D = 203 \text{ m}$
 33 (a) $\theta = 18.4^\circ$ (b) 1.1°
 34 1.44

Chapter 6 Algebra and matrices

RECALL

- 1 (a) $x = 4$ (b) $x = -8$ (c) $x = 6$
 2 (a) 18 (b) 6 (c) 21
 3 pronumeral, terms, coefficient, expressions, like terms
 4 (a) $4x - 20$ (b) $2ab + 14a$ (c) $-12ct + 3c^2$
 (d) $10a - 2b$ (e) $2xy - y$
 5 (a) 243 (b) 256

EXERCISE 6.1

- 1 (a) $C = 18.8$ (b) $C = 20$ (c) $x = 4$
 2 \$500
 3 (a) $A = 9.1$ (b) $c = 13.3$ (c) $E = 36.0$
 4 (a) $V = 21.4$ (b) $S = -456$
 5 (a) $l = \frac{P}{2} - w$ or $l = \frac{P - 2w}{2}$ (b) $t = \frac{v - u}{a}$
 (c) $b = \sqrt{c^2 - a^2}$ (d) $r = \frac{mv^2}{F}$
 (e) $y = \frac{6 - 3x}{4}$ or $y = \frac{3(2 - x)}{4}$

6 (a)

n	$D = n/2 * (n - 3)$
4	2
5	5
6	9
7	14
8	20

(b) 20

7 $C = 50 + 45h$

8 (a) $= 2 * (\text{B\$2} + \text{\$A3})$ (b) $= 2 * (\text{D\$2} + \text{\$A5})$

9 (a)

$\times$	21	22	23	24	25	26	27	28	29	30
21	441	462	483	504	525	546	567	588	609	630
22	462	484	506	528	550	572	594	616	638	660
23	483	506	529	552	575	598	621	644	667	690
24	504	528	552	576	600	624	648	672	696	720
25	525	550	575	600	625	650	675	700	725	750
26	546	572	598	624	650	676	702	728	754	780
27	567	594	621	648	675	702	729	756	783	810
28	588	616	644	672	700	728	756	784	812	840
29	609	638	667	696	725	754	783	812	841	870
30	630	660	690	720	750	780	810	840	870	900

(b)

$\times$	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
0.5	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5
1	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5
1.5	0.75	1.5	2.25	3	3.75	4.5	5.25	6	6.75	7.5
2	1	2	3	4	5	6	7	8	9	10
2.5	1.25	2.5	3.75	5	6.25	7.5	8.75	10	11.25	12.5
3	1.5	3	4.5	6	7.5	9	10.5	12	13.5	15
3.5	1.75	3.5	5.25	7	8.75	10.5	12.25	14	15.75	17.5
4	2	4	6	8	10	12	14	16	18	20
4.5	2.25	4.5	6.75	9	11.25	13.5	15.75	18	20.25	22.5
5	2.5	5	7.5	10	12.5	15	17.5	20	22.5	25

10 14.5 cm

11 (a) 4% p.a.

(b) D

12 A

13 (a)

	Radius	1	2	4	8
Height	$S=2\pi r^2+2\pi r h$				
1		12.57	37.70	125.66	452.39
2		18.85	50.27	150.80	502.65
4		31.42	75.40	201.06	603.19
8		56.55	125.66	301.59	804.25

(b) increased by a factor of 4

$$(c) h = \frac{S - 2\pi r^2}{2\pi r} \text{ or } h = \frac{S}{2\pi r} - r, h = 1.4 \text{ cm}$$

14 (a) $S = 126 \text{ m}^2$ with $l = 3 \text{ m}$, $w = 5 \text{ m}$, $h = 6 \text{ m}$

$$(b) T = \frac{1}{2}(lw + hl + hw)$$

$$(c) \frac{S}{T} = 4$$

15 $-40^\circ \text{C} = -40^\circ \text{F}$

16 (a)

	Mass (kg)	40	45
Height (m)	$\text{BMI} = m/h^2$		
1		40	45
1.05		36	41

(b) (i) 55–73 kg

(ii) 56 kg

(iii) 50 kg

EXERCISE 6.2

$$1 \quad A = \begin{bmatrix} 68 & 71 & 79 \\ 59 & 63 & 70 \end{bmatrix}$$

2 (a) B

(b) Horizontal before vertical, just like coordinate geometry.

3

	A	B	C	D	E
Matrix	$\begin{bmatrix} 4 & -3 & 7 \end{bmatrix}$	$\begin{bmatrix} -2 & 1 \\ 6 & \frac{1}{2} \end{bmatrix}$	$\begin{bmatrix} 5 \\ a \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 \end{bmatrix}$
Type	Row	Square	Column	Identity	Zero/Row
Order	1×3	2×2	2×1	3×3	1×2
Number of elements	3	4	2	9	2

4 (a) 3×2

(b) The student either calculated the order as a multiplication or misinterpreted the term 'order' to mean the number of elements within the matrix.

(c) $a_{22} = 1$

(d) a_{32}

(e) 11

5 (a) $A = \begin{bmatrix} 1 & 5 & 1 \end{bmatrix}$

(b) $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

apple orange

6 $\begin{bmatrix} 4.90 & 4.65 \\ 5.15 & 4.95 \end{bmatrix}$ Supermarket 1
Supermarket 2

7 (a) 3×3

(b) $a_{31} = -2$

(c) The student has mistakenly identified element a_{13} instead of a_{31} , by misinterpreting the row and column values.

(d) a_{12}

$$8 \quad (a) \quad \begin{matrix} & \text{Jun} & \text{Jul} & \text{Aug} \\ \begin{matrix} A \\ G \\ K \\ P \end{matrix} & \begin{bmatrix} 17.0 & 16.8 & 17.3 \\ 21.5 & 20.6 & 21.1 \\ 26.2 & 25.6 & 29.9 \\ 19.6 & 19.4 & 20.1 \end{bmatrix} \end{matrix}$$

(b) Kalpowar

(c) Alpha

(d) (i) $t_{32} = 25.6$

(ii) The average maximum daily temperature in Kalpowar in July.

(e) (i) $t_{21} = 21.5$

(ii) The average maximum daily temperature in Garfield in June.

(f) 0.5°C

9 C

10 (a) $t = 4, u = 3, v = -1, w = 7$

(b) $a = \frac{1}{2}, b = -4$

(c) $a = 10, b = 5, c = 15, d = 20$

11 (a) $\begin{bmatrix} P \\ A \end{bmatrix} = \begin{bmatrix} 2a + 2b \\ bh \end{bmatrix}$

(b) $\begin{bmatrix} P \\ A \end{bmatrix} = \begin{bmatrix} 54 \\ 150 \end{bmatrix}$

(c) $x = 26\text{ cm}$

(d) $y = 7846\text{ m}^2$

12 $x = -2, y = 4, z = 8$

EXERCISE 6.3

1 (a) $\begin{bmatrix} 3 & 4 \\ -1 & -1 \\ 3 & 7 \end{bmatrix}$

(b) $\begin{bmatrix} -3 & -4 \\ 1 & 1 \\ -3 & -7 \end{bmatrix}$

2 (a) $\begin{bmatrix} 175 & 285 & 160 \\ 265 & 400 & 120 \\ 310 & 370 & 185 \end{bmatrix}$

(b) 400

(c) 280

3 (a) $\begin{bmatrix} 28 & 5 & 11 \\ 20 & 4 & 4 \end{bmatrix}$

(b) 4

4 (a) $\begin{bmatrix} 6 & 1 \\ 4 & 14 \end{bmatrix}$

(b) $\begin{bmatrix} 6 & 1 \\ 4 & 14 \end{bmatrix}$

(c) $\begin{bmatrix} 9 & -3 \\ 0 & 4 \end{bmatrix}$

(d) $\begin{bmatrix} 10 & 2 \\ 2 & 8 \end{bmatrix}$

5 (a) $\begin{bmatrix} 2 & -8 \\ -6 & -6 \end{bmatrix}$

(b) $\begin{bmatrix} 8 & -16 \\ 0 & -20 \end{bmatrix}$

6 (a) D

(b) Matrix A

7 (a) $\begin{bmatrix} -7 & 20 \\ 9 & 19 \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{2} & 0 \\ \frac{3}{2} & -1 \end{bmatrix}$

(c) $\begin{bmatrix} \frac{1}{2} & 0 \\ \frac{3}{2} & -1 \end{bmatrix}$

(d) $\begin{bmatrix} 12 & -36 \\ -18 & -33 \end{bmatrix}$

8 $\begin{bmatrix} 8 & -3 & 8 \\ 0 & -9 & -10 \\ -18 & -23 & -30 \end{bmatrix}$

9 A

10 (a) $\begin{bmatrix} 79 & 66 \\ 71 & 81 \end{bmatrix}$

(b) $\begin{bmatrix} 72 & 68 \\ 75 & 79 \end{bmatrix}$

(c) (i) $\begin{bmatrix} 75.5 & 67 \\ 73 & 80 \end{bmatrix}$

(ii) This matrix represents the average result for each person in each subject across the two semesters.

(d) (i) 73

(ii) The element 73 represents Tonia's average result for Chemistry.

11 (a) $x = 11, y = 2, z = 9$

(b) $x = 11, y = 2, z = 9$

12 $x = 1, y = -1$

13 (a) $\mathbf{M} = \begin{bmatrix} 80 & 180 \\ 160 & 200 \end{bmatrix} \begin{matrix} J \\ S \end{matrix}, \mathbf{A} = \begin{bmatrix} 240 & 120 \\ 300 & 80 \end{bmatrix} \begin{matrix} J \\ S \end{matrix} \begin{matrix} 1 \\ 2 \end{matrix}$

(b) $\mathbf{M} + \mathbf{A} = \begin{bmatrix} 320 & 300 \\ 460 & 280 \end{bmatrix}$

This matrix represents the total sales figures for jeans and shorts for each store over 2 months.

(c) Store 2 had better sales for jeans and store 1 had better sales for shorts.

(d) $\begin{bmatrix} 608 & 444 \\ 820 & 376 \end{bmatrix}$

14 $x = -2, y = 4$

EXERCISE 6.4

1 (a) 2×2

(b) cannot be found

(c) 2×1

2 (a) $\begin{bmatrix} -1 & -7 \\ -9 & -7 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & -1 \\ -23 & -11 \end{bmatrix}$

(c) $\begin{bmatrix} -11 \\ -1 \\ 1 \end{bmatrix}$

3 (a) $\begin{bmatrix} 22 \\ 4 \end{bmatrix}$

(b) $[-14]$

(c) $\begin{bmatrix} -17 & 2 \end{bmatrix}$

(d) $\begin{bmatrix} -2 & 9 & -16 & -9 \end{bmatrix}$ (e) $\begin{bmatrix} 14 \\ -2 \\ 8 \end{bmatrix}$

(f) $\begin{bmatrix} 23 & 9 \end{bmatrix}$

(g) $\begin{bmatrix} -5 \\ 1 \\ 10 \\ 9 \end{bmatrix}$

(h) $\begin{bmatrix} 6 & 2 \\ -15 & -5 \end{bmatrix}$

4 The number of columns in the first matrix equals the number of rows in the second matrix, e.g. for (a): $2 \times \boxed{2}$ times $\boxed{2} \times 1$.

$$5 \text{ (a)} \begin{bmatrix} 2 \\ -1 \\ 7 \end{bmatrix}, \mathbf{I}_1 \quad \text{(b)} \begin{bmatrix} 3 & 0 \\ -2 & 4 \end{bmatrix}, \mathbf{I}_2 \quad \text{(c)} \begin{bmatrix} 8 & -6 \end{bmatrix}, \mathbf{I}_2$$

$$\text{(d)} \begin{bmatrix} 2 \\ -1 \\ 7 \end{bmatrix}, \mathbf{I}_3 \quad \text{(e)} \begin{bmatrix} 3 & 0 \\ -2 & 4 \end{bmatrix}, \mathbf{I}_2 \quad \text{(f)} \begin{bmatrix} 8 & -6 \end{bmatrix}$$

$$6 \text{ (a)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{(b)} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{(c)} \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{(d)} [1] \quad \text{(e)} [1] \quad \text{(f)} [1] \quad \text{(g)} [0] \quad \text{(h)} [0] \quad \text{(i)} [0]$$

$$7 \text{ (a)} \begin{bmatrix} -11 & 12 \\ 5 & 28 \end{bmatrix}$$

(b) The student has mistakenly tried to multiply corresponding elements.

8 (a) D

(b) C: This product gives a matrix of total prices for individual types of fruit.

9 (a) $\mathbf{A}: 2 \times 3$, $\mathbf{B}: 2 \times 2$, $\mathbf{C}: 3 \times 2$, $\mathbf{D}: 1 \times 2$, $\mathbf{E}: 2 \times 1$

(b) (i) $\mathbf{A}$ has 3 columns and $\mathbf{B}$ has 2 rows. The matrix product $\mathbf{AB}$ does not exist because the number of columns in the first matrix does not equal the number of rows in the second matrix.

$$\text{(ii)} \begin{bmatrix} -16 & 2 & 20 \\ 7 & 6 & 5 \end{bmatrix}$$

(iii) $\mathbf{A}$ has 3 columns and $\mathbf{D}$ has 1 row. The matrix product $\mathbf{AD}$ does not exist because the number of columns in the first matrix does not equal the number of rows in the second matrix.

$$\text{(iv)} \begin{bmatrix} 4 \\ -4 \\ -4 \end{bmatrix}$$

10 78

11 \$27 870.82

$$12 \text{ (a)} \begin{bmatrix} 300 \\ 400 \\ 50 \\ 100 \\ 200 \end{bmatrix}$$

(b) X: \$2000, Y: \$1400, Z: \$1150

$$13 \text{ (a)} \begin{bmatrix} 10 \\ 10 \\ 80 \\ 40 \\ 20 \end{bmatrix}$$

(b) Cheese and salad sandwich: \$1.20; ham and salad: \$1.40; cheese and tomato: \$1.40

$$14 \begin{bmatrix} 560 & 650 & 595 \\ 580 & 670 & 560 \\ 360 & 415 & 345 \end{bmatrix}$$

$$15 \text{ (a)} \mathbf{G} = \begin{matrix} & T & D & S \\ \begin{bmatrix} 2 & 1 & 2 \\ 1 & 2 & 1 \\ 0 & 2 & 2 \end{bmatrix} & GP1 \\ & GP2 \\ & GP3 \end{matrix}$$

$$\text{(b)} \mathbf{S} = \begin{matrix} \$ \\ \begin{bmatrix} 12 \\ 7 \\ 15 \end{bmatrix} & T \\ & D \\ & S \end{matrix}$$

(c) GP1: \$61, GP2: \$41, GP3: \$44

(d) \$441

$$16 \text{ (a)} \mathbf{S} = \begin{matrix} \$ \\ \begin{bmatrix} 3 \\ 5 \\ 6 \end{bmatrix} & S \\ & D \\ & M \end{matrix}$$

$$\text{(b)} \mathbf{N} = \begin{matrix} S & D & M \\ \begin{bmatrix} 80 & 50 & 30 \end{bmatrix} \end{matrix}$$

(c) $\mathbf{NS} = [670]$: The total sales are \$670.

$$\text{(d)} \mathbf{T} = \begin{matrix} & S & D & M \\ \begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} & \begin{bmatrix} 50 & 40 & 0 \\ 30 & 90 & 10 \\ 100 & 60 & 20 \\ 80 & 20 & 15 \end{bmatrix} \end{matrix}$$

$$\text{(e)} \mathbf{TS} = \begin{matrix} \$ \\ \begin{bmatrix} 350 \\ 600 \\ 720 \\ 430 \end{bmatrix} & A \\ & B \\ & C \\ & D \end{matrix}$$

The matrix product $\mathbf{TS}$ gives the total sales at each store.

(f) Shop C

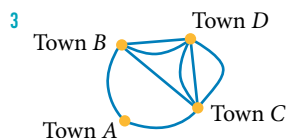
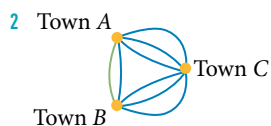
$$\text{(g)} \mathbf{S} = \begin{matrix} \$ \\ \begin{bmatrix} S \\ D \\ M \end{bmatrix} & \begin{bmatrix} 3.50 \\ 5.50 \\ 6.50 \end{bmatrix} \end{matrix}$$

(h) A: \$395, B: \$665, C: \$810, D: \$487.50

(i) A: \$45, B: \$65, C: \$90, D: \$57.50

EXERCISE 6.5

$$1 \mathbf{R} = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 0 & 4 \\ 3 & 4 & 0 \end{bmatrix}$$



4 (a) $R = \begin{bmatrix} 0 & 3 & 2 & 0 \\ 3 & 0 & 1 & 1 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

(b) $r_{22} = 0$ because town B is not connected to itself.

(c) $r_{13} = 2$; both elements give the number of connections between towns A and C.

5 (a) $Z = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$

(b) $Z = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$

(c) $Z = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$

(d) $Z = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 0 & 0 \end{bmatrix} \end{matrix}$

(e) $Z = \begin{matrix} & \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$

6 (a) $\begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

(b) $\begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 17 & -6 \\ -6 & 5 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} 5 & 5 & 5 \\ 1 & 7 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(f) $\begin{bmatrix} -4 & 4 & 0 \\ -1 & -1 & 2 \\ 1 & 1 & -2 \end{bmatrix}$

7 (a) $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

(b) The number of air routes with one stopover.

(c) AEK

8 (a) Gladstone and Maroochydore

(b) $C^2 = \begin{bmatrix} 3 & 1 & 1 & 2 \\ 1 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 3 \end{bmatrix}$

(c) $c_{14} = 2$; there will be two ways to go from Rockhampton to Bundaberg via another town.

9 (a) $\begin{bmatrix} 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 2 \\ 2 & 0 & 1 & 0 & 1 \\ 1 & 1 & 2 & 1 & 0 \end{bmatrix}$

(b) (i) 3

(ii) A

(iii) Adelaide, Brisbane and Perth

10 (a) $R^1 = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \end{matrix}$

(b) B is reachable from A and C.

(c) $R^2 = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$

(e)

Vertex	Reachability
A	5
B	3
C	5

$$11 \text{ (a) } R^1 = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

(b) 2

(c) 2

$$(d) R^2 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(e) R = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Reachability: A: 0; B: 1; C: 5; D: 5

$$12 \text{ (a) } C^1 = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

(b)	Person	Number of ways to pass on communication	Number of ways to receive communication
	A	3	1
	B	1	2
	C	0	2
	D	2	1

$$(c) C^2 = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix} \end{matrix}$$

$$(d) C^3 = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{matrix}$$

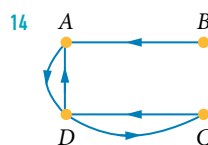
$$(e) \begin{bmatrix} 1 & 3 & 4 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 3 & 3 & 1 \end{bmatrix}$$

(f) $c_{12} = 3$ represents communication AB, ADB and ADAB.

13 (a) 3

(b) 1

$$(c) C^2 = \begin{bmatrix} 0 & 0 & 2 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(d) (i) $c_{13} = 2$; 2 communications from A to C (via B and D).(ii) $c_{23} = 1$; 1 communications from B to C (via D).(iii) $c_{43} = 0$; no communications from D to C via an intermediary.

14 (a) Christof

(b) 4, Christof $\rightarrow$ DeMarcus $\rightarrow$ Farrah $\rightarrow$ Gretchin $\rightarrow$ Egon

$$(c) C^1 = \begin{matrix} & \begin{matrix} C & D & E & F & G \end{matrix} \\ \begin{matrix} C \\ D \\ E \\ F \\ G \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

(d) $c_{13} = 1$

$$16 \text{ (a) } A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 0 & 3 \\ 3 & 3 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

CHAPTER REVIEW 6

1 C

$$2 \text{ (a) } c = \sqrt{\frac{E}{m}} \quad (b) b = T^2 - a$$

$$3 \text{ (a) } y = 10 \quad (b) y = 0 \quad (c) y = 0$$

4 C

5 C

6 3

$$7 \text{ (a) } 0 \quad (b) a_{23} \quad (c) 3 \times 3$$

$$8 \text{ (a) } \begin{matrix} & \begin{matrix} \text{Jan} & \text{Feb} & \text{Mar} \end{matrix} \\ \begin{matrix} B \\ C \\ M \\ T \end{matrix} & \begin{bmatrix} 0.0 & 15.0 & 23.2 \\ 326.8 & 116.8 & 183.6 \\ 7.6 & 14.6 & 17.8 \\ 286.0 & 83.6 & 104.6 \end{bmatrix} \end{matrix}$$

(b) (i) $r_{41} = 286.0$, 286.0 mm of rainfall occurred during January in Townsville.(ii) $r_{12} = 15.0$, 15.0 mm of rainfall occurred during February in Birdsville.

(c) 153.0 mm

$$9 \text{ (a) } 2 \times 3 \quad (b) 3 \times 1 \quad (c) 1 \times 5$$

10 D

$$11 \begin{bmatrix} -6 \\ -3 \end{bmatrix}$$

$$12 \text{ (a) } \begin{bmatrix} 10 & 5 & 3 \\ 9 & -1 & 0 \end{bmatrix} \quad (b) \begin{bmatrix} 3 \\ 12 \\ -6 \end{bmatrix}$$

$$13 \quad (a) \begin{bmatrix} -2 \\ -3 \\ 4 \end{bmatrix} \quad (b) \begin{bmatrix} -4 & 0 & 4 \end{bmatrix}$$

$$14 \quad (a) \begin{bmatrix} 1 \\ 5 \end{bmatrix} \quad (b) \mathbf{P} + \mathbf{Q} \text{ cannot be found.}$$

$$(c) \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix} \quad (d) [5]$$

$$(e) \mathbf{RQ} - \mathbf{T} \text{ cannot be found.}$$

$$15 \quad (a) \begin{bmatrix} 5 \\ -21 \end{bmatrix} \quad (b) \mathbf{RQ} \text{ cannot be found.}$$

$$(c) \begin{bmatrix} -7 & -9 & -8 \\ 26 & -3 & -14 \end{bmatrix} \quad (d) \mathbf{RP} \text{ cannot be found.}$$

$$16 \quad (a) \begin{bmatrix} 2 & 5 & -3 \\ 4 & 7 & 1 \\ 2 & -1 & 6 \end{bmatrix}, \mathbf{I}_3 \quad (b) \begin{bmatrix} 3 & 1 \\ -4 & 6 \\ 8 & 2 \end{bmatrix}, \mathbf{I}_2$$

$$(c) \begin{bmatrix} 3 & 1 \\ -4 & 6 \\ 1 & 2 \end{bmatrix}, \mathbf{I}_3$$

$$17 \quad \mathbf{D}$$

$$18 \quad (a) \begin{bmatrix} 4 & 3 \\ -1 & 2 \end{bmatrix} \quad (b) \begin{bmatrix} 6 & -1 \\ -2 & 0 \end{bmatrix}$$

$$19 \quad \begin{bmatrix} 0 & 3 & 0 & 1 \\ 3 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 \\ 1 & 0 & 2 & 0 \end{bmatrix}$$

$$20 \quad \mathbf{Z} = \begin{matrix} & A & B & C & D \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$21 \quad (a) \begin{bmatrix} 10 & 5 \\ 2 & 11 \end{bmatrix} \quad (b) \begin{bmatrix} 0 & 8 & 4 \\ 2 & 9 & 4 \\ 0 & 2 & 0 \end{bmatrix}$$

$$22 \quad \mathbf{D}$$

$$23 \quad w = \frac{P-2l}{2} \text{ or } w = \frac{P}{2} - l, w = 6 \text{ m}$$

$$24 \quad (a) V = \frac{3x}{2}(x+4) \quad (b) V = 144 \text{ m}^3$$

$$25 \quad \mathbf{A}$$

$$26 \quad \begin{bmatrix} 4 & -15 \\ -7 & 6 \end{bmatrix}$$

$$27 \quad (a) \mathbf{D} + \mathbf{J} = \begin{matrix} & T & S \\ \begin{matrix} \text{Store 1} \\ \text{Store 2} \end{matrix} & \begin{bmatrix} 100 & 116 \\ 98 & 84 \end{bmatrix} \end{matrix}$$

Total sales figures for men's ties and polo shirts for each store in December and January.

$$(b) \text{ Store 1 for both items}$$

$$T \quad S$$

$$(c) \quad (i) \quad \mathbf{F} = \begin{bmatrix} 20 & 18 \\ 25 & 12 \end{bmatrix} \begin{matrix} \text{Store 1} \\ \text{Store 2} \end{matrix}$$

$$(ii) \quad \mathbf{D} + \mathbf{J} + \mathbf{F} = \begin{matrix} & \text{Ties} & \text{Shirts} \\ \begin{matrix} \text{Store 1} \\ \text{Store 2} \end{matrix} & \begin{bmatrix} 120 & 134 \\ 123 & 96 \end{bmatrix} \end{matrix}$$

$$28 \quad 77.8$$

$$29 \quad (a) \begin{bmatrix} 2 & 1 & 1 & 2 \\ 1 & 3 & 2 & 1 \\ 1 & 2 & 3 & 1 \\ 2 & 1 & 1 & 2 \end{bmatrix}$$

$$(b) \text{ The number of air routes with one stopover.}$$

$$(c) \mathbf{BAC}, \mathbf{BDC}$$

$$30 \quad (a) \quad \mathbf{V} = \begin{matrix} & A & B & G & m & M & W \\ \begin{matrix} A \\ B \\ G \\ m \\ M \\ W \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$(b) \quad \mathbf{V}^3 = \begin{matrix} & A & B & G & m & M & W \\ \begin{matrix} A \\ B \\ G \\ m \\ M \\ W \end{matrix} & \begin{bmatrix} 0 & 3 & 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 3 & 4 & 1 \\ 1 & 0 & 0 & 1 & 3 & 2 \\ 0 & 3 & 1 & 0 & 0 & 0 \\ 0 & 4 & 3 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$(c) \mathbf{A} \text{ and } \mathbf{B}$$

$$(d) \text{ Geelong - Melbourne - Ballarat - Maryborough}$$

$$(e) \quad \mathbf{V} = \begin{bmatrix} 1 & 4 & 1 & 1 & 1 & 0 \\ 4 & 3 & 1 & 4 & 5 & 1 \\ 1 & 1 & 2 & 1 & 4 & 3 \\ 1 & 4 & 1 & 1 & 1 & 0 \\ 1 & 5 & 4 & 1 & 2 & 1 \\ 0 & 1 & 3 & 0 & 1 & 1 \end{bmatrix}$$

(f) City	Reachability
Ararat	8
Ballarat	18
Geelong	12
Maryborough	8
Melbourne	14
Warrnambool	6

- 31 (a) $A = xy + \frac{\pi y^2}{8}$
 (b) $V = dy \left(x + \frac{\pi y}{8} \right)$ or $V = dxy + \frac{\pi dy^2}{8}$
 (c) $t = \frac{1000 dy}{r} \left(x + \frac{\pi y}{8} \right)$ or $t = \frac{1000 dxy}{r} + \frac{1000 \pi dy^2}{8r}$
 (d) 9 h 19 min
- 32 (a) 1, 3, 6, 10, 15 (b) 55, 325, 1275, 5050
 (c) $n = 24$ (d) $T_n + T_{n+1} = (n+1)^2$

33 (a) $S = \begin{bmatrix} 1.50 \\ 2.50 \\ 3.50 \end{bmatrix} \begin{matrix} 300 \text{ mL} \\ 600 \text{ mL} \\ 1.25 \text{ L} \end{matrix}$

(b) $N = \begin{bmatrix} 30 & 20 & 10 \end{bmatrix}$

(c) $NS = [130]$, total sales are \$130.

(d) $T = \begin{bmatrix} 35 & 30 & 10 \\ 40 & 60 & 15 \\ 60 & 75 & 20 \\ 75 & 50 & 30 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \end{matrix}$

(e) $TS = \begin{bmatrix} 162.5 \\ 262.5 \\ 347.5 \\ 342.5 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \end{matrix}$

Revenue from sales at convenience stores A, B, C and D.

(f) Store C

(g) $S = \begin{bmatrix} 1.75 \\ 2.75 \\ 3.75 \end{bmatrix} \begin{matrix} 300 \text{ mL} \\ 600 \text{ mL} \\ 1.25 \text{ L} \end{matrix}$

(h) $TS = \begin{bmatrix} 181.25 \\ 291.25 \\ 386.25 \\ 381.25 \end{bmatrix} \begin{matrix} A \\ B \\ C \\ D \end{matrix}$

(i) A: \$18.75, B: \$28.75, C: \$38.75, D: \$38.75

34 (a) $C^2 = \begin{bmatrix} 2 & 1 & 2 & 0 & 0 \\ 1 & 2 & 1 & 1 & 1 \\ 2 & 1 & 3 & 0 & 1 \\ 0 & 1 & 0 & 2 & 2 \\ 0 & 1 & 1 & 2 & 3 \end{bmatrix}$

The number of road links between any two towns via another town.

- (b) 1, town Q
 (c) 3, RQR, RSR and RTR.

(d) $C^3 = \begin{matrix} & P & Q & R & S & T \\ \begin{matrix} P \\ Q \\ R \\ S \\ T \end{matrix} & \begin{bmatrix} 0 & 2 & 1 & 4 & 5 \\ 2 & 2 & 4 & 2 & 4 \\ 1 & 4 & 2 & 5 & 6 \\ 4 & 2 & 5 & 0 & 1 \\ 5 & 4 & 6 & 1 & 2 \end{bmatrix} \end{matrix}$

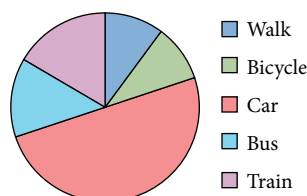
The number of road links between any two towns via two other towns.

(e) RQTP

Chapter 7 Univariate data analysis

RECALL

- 1 (a) 21% (b) 16% (c) 33%
 2 (a) 3.4 (b) 3.4 (c) 14.8
 3 (a) 11 (b) 21.5 (c) 20.5
 4 (a) 56 (b) 9, 11 (c) 18
 5 (a) 21 (b) 10 (c) 19
 6 Transport to school



- 7 (a) January, closely followed by February
 (b) August and September
 (c) Summer
 (d) January and March were much wetter than average.
 February and April were much drier than average.
 Overall the rainfall for the first four months of 2017 was average.

EXERCISE 7.1

- 1 (a) Numerical (b) Categorical (c) Categorical
 (d) Numerical (e) Numerical (f) Categorical
 2 (a) Nominal (b) Ordinal (c) Nominal
 (d) Nominal (e) Nominal (f) Ordinal
 3 (a) Continuous (b) Discrete (c) Discrete
 (d) Continuous (e) Continuous (f) Continuous
 4 (a) Ordinal (b) Continuous (c) Discrete

5 (a)

Colour	Frequency
blue	4
red	5
yellow	7
white	5

(b)

Favourite weekend viewing	Frequency
sport	7
movies	8
drama	5
comedy	3

(c)

Type of car	Frequency
Hyundai	2
Mazda	3
VW	3
Toyota	4
Ford	5
Holden	7

6 (a)

Number of questions	Frequency
3	9
4	5
5	6
6	2
	22

(b)

Number of brothers	Frequency
0	12
1	6
2	4
3	1
5	1
Total	24
	24

7 (a)

Opinion	Percentage frequency
Strongly agree	17%
Agree	47%
Neutral	22%
Disagree	11%
Strongly disagree	2%

(b)

Year level	Percentage frequency
7	14%
8	20%
9	7%
10	27%
11	24%
12	8%

(c)

Lunch choice	Relative frequency
sandwich	38%
pie	22%
pizza	16%
wrap	13%
salad	11%

8 (a)

Shoe size	Relative frequency
6	0.16
7	0.29
8	0.31
9	0.12
10	0.07
11	0.05

(b)

Alternative flag	Relative frequency
Choice 1	0.01
Choice 2	0.93
Choice 3	0.06
Choice 4	0.00
Choice 5	0.00

(c)

Time (seconds)	Relative frequency
60–69.99	0.05
70–79.99	0.12
80–89.99	0.48
90–99.99	0.33
100–109.99	0.02

9 (a) B

(b) Frequency and the relative frequency have been mixed up.

10 A

11

Name	Votes
Arlo	7
Denji	4
Praveen	5
Jamie	6
Kendall	2

Arlo

12 (a) Sample solution: nominal data

Would you like to see fried rice on the tuckshop menu once a week?

☐ Yes ☐ No

- (b) Sample solution: ordinal data

Respond to the statement: 'Our school needs a more colourful sports uniform.'

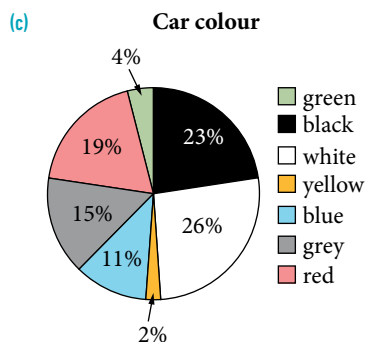
- ☐ Strongly agree
- ☐ Agree
- ☐ Neutral
- ☐ Disagree
- ☐ Strongly disagree

13 (a)

Car colour	Frequency
black	12
white	14
yellow	1
blue	6
grey	8
red	10
green	2

(b)

Car colour	Relative frequency
black	0.23
white	0.26
yellow	0.02
blue	0.11
grey	0.15
red	0.19
green	0.04



14 (a)

Gate	Percentage
A	33%
B	11%
C	27%
D	29%

(b)

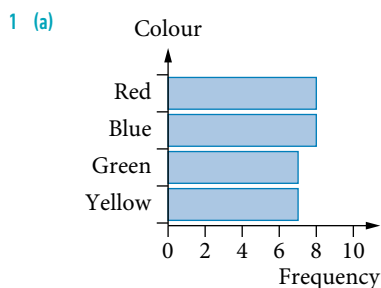
Gate	Anticipated
A	2800
B	900
C	2300
D	2500

(c) \$277 600

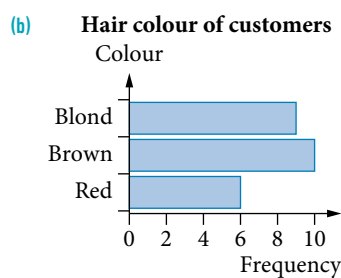
- 15 53, assuming none of the relatives were students, and that the numbers this week safely predict the numbers for next week, other than the proposed incentive.

- 16 12 to Dreamworld, 18 to Wet and Wild

EXERCISE 7.2



Modal categories: red and blue

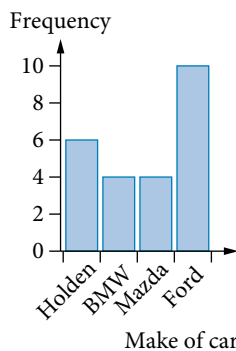


Modal category: brown

2 (a)

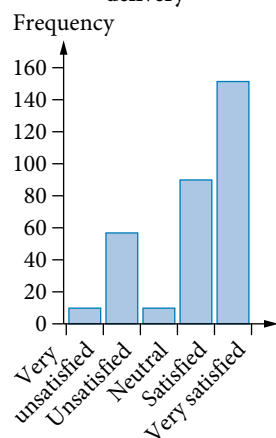
Make of car	Frequency
Holden	6
BMW	4
Mazda	4
Ford	10

Cars parked in carpark at school

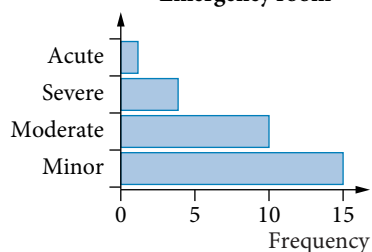


- (b) Ford

3 Customer opinion of newspaper delivery

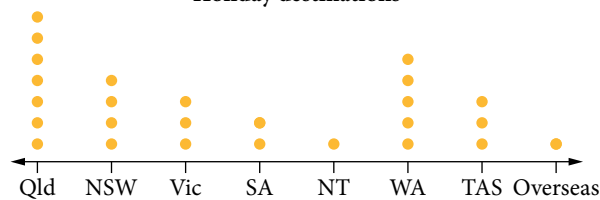


4 (a) Emergency room



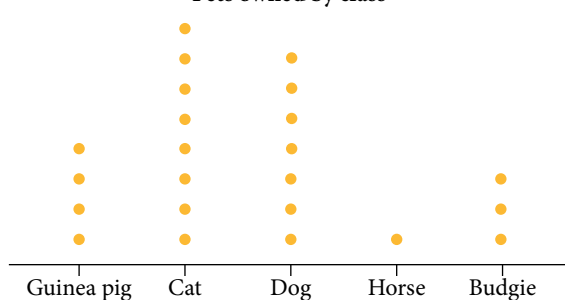
(b) 50%

5 Holiday destinations



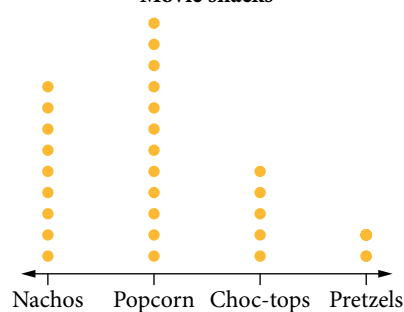
Mode: Qld

6 Pets owned by class



The modal pet is the cat. However, the number of dog owners is almost the same.

7 Movie snacks



Modal snack: popcorn

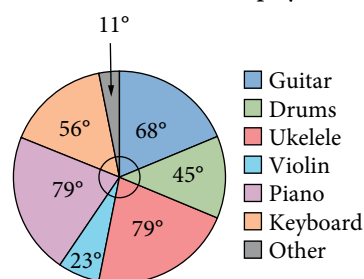
8 (a) D (b) A

9 (a) Some students must play more than one instrument.

Ukulele and piano are the most common instruments played by students.

The data is ideal for a pie graph as the components make up the whole of instruments played.

(b) Musical instrument played



10 (a) 7, assuming the same number of lollies in each packet

(b) Red: 44, green: 29, blue: 7, yellow: 51, pink: 22

(c) $\frac{4}{21}$ or about 19%

11 (a) B

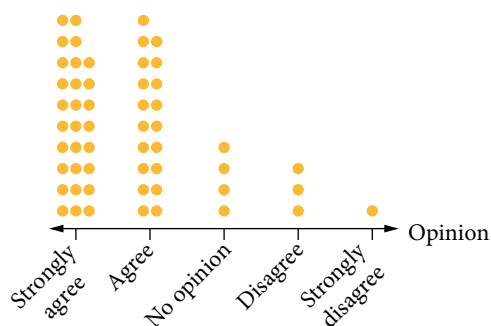
(b) Degrees have been confused with percentage.

(c) 804

(d) 398

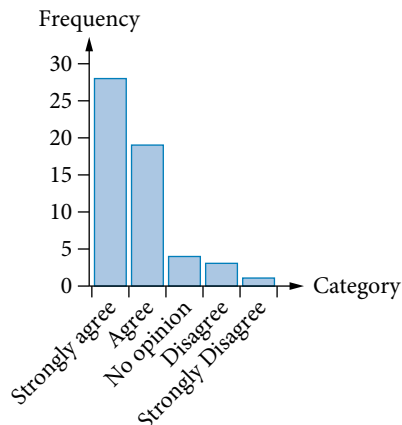
12 Dot plot: too many dots for the first two categories, so not really suitable. Mode is clearly shown as Strongly agree.

Great customer service was received

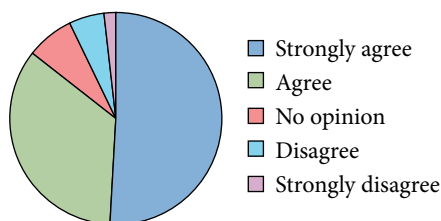


Column graph: much more effective than the dot plot, as it clearly shows the modal category and can be graphed quickly and easily.

Great customer service received

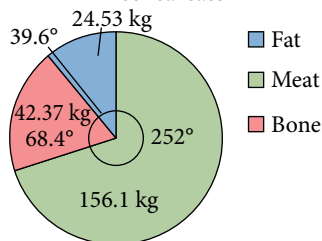


Great customer service received

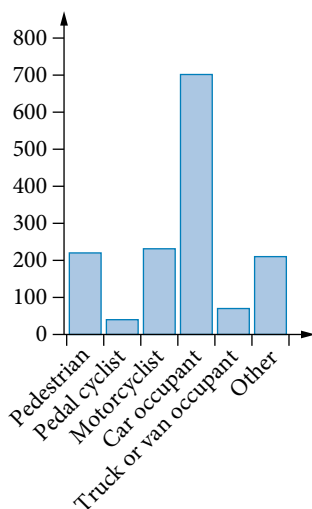


Although the pie chart shows the modal category clearly, it is not suitable because the ordered nature of the data cannot be seen.

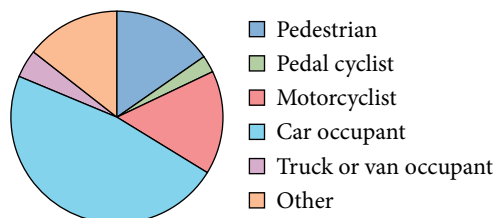
13 Beef carcass



14 (a) Cause of death: traffic accidents



Cause of death: traffic accidents



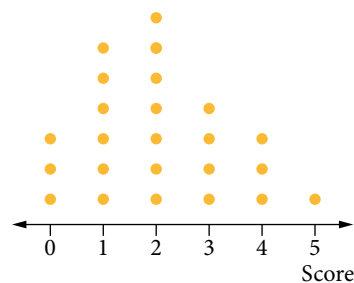
- (b) Other accidents and assaults: 6272, Accidental poisoning: 864, Traffic accidents, car occupant: 706
- (c) On a column graph many of the smaller categories would not be seen. The 'other' category would dominate, where it is usually one of the smaller categories.

The graphic provided features 'Misadventure' with as large a circle as 'Traffic accidents', yet the numbers are much smaller. By featuring Swimming pools and Bath-tub, these categories would appear to dominate, and yet the numbers are comparatively small. Some calculations need to be done to realise that the individual numbers do not add up to the total numbers.

EXERCISE 7.3

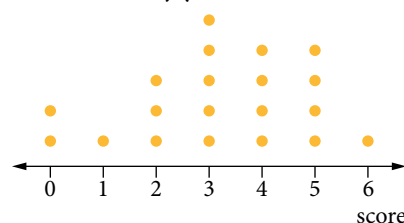
- (a) 10, positively skewed (b) 61, symmetrical
(c) 5, negatively skewed
- (a) 150–159, positively skewed
(b) 300–390, symmetrical
(c) 5.0–5.9, negatively skewed
- (a) 0–<10 years, positively skewed
(b) 65–69 movies, negatively skewed
(c) 27–<28 kg, negatively skewed
- (a) 6°C (b) October
(c) Negatively skewed, Cairns in October was predominantly 28–31°C with a few cooler days.

5 (a) Soccer scores for the C team



Mode = 2, symmetrical

(b) Sanjay's wickets



Mode = 3, symmetrical

- 6 (a) 80. The highest rank scored 80 more tries than the 25th rank.
- (b) Positively skewed. Most of the top 25 players scored less than 130 tries and the top three scored 130, 142 and 159 career tries.

- 7 (a) Marks on a mathematics test

Stem	Leaf
3	56
4	478
5	146
6	67
7	057
8	022244
9	01237

Key: 9|0 = 90

- (b) Marks on a science test

Stem	Leaf
3	2579
4	22
5	06
6	56677
7	1288
8	17
9	12238

Key: 9|1 = 91

- 8 (a) Golf club championship scores

Stem	Leaf
6	46899
7	001115666
8	0012788

Key: 8|0 = 80

- (b) Golf club championship scores

Stem	Leaf
6 _L	4
6 _H	6899
7 _L	00111
7 _H	5666
8 _L	0012
8 _H	788

Key: 8|0 = 80

- 9 (a) Diameters of pebbles

Stem	Leaf
1	58
2	122445667
3	001157
4	0124556

Key: 3|1 = 3.1 cm

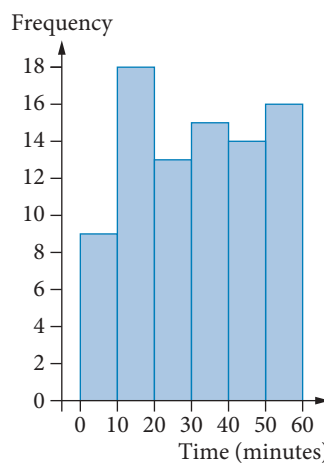
- (b) Weight of birds

Stem	Leaf
1	679
2	022799
3	00112356
4	0112357

Key: 3|1 = 310 g

10

Wait times for a bus

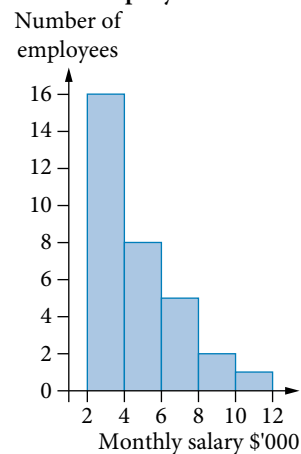


Modal class: 10–<20 minutes, symmetrical

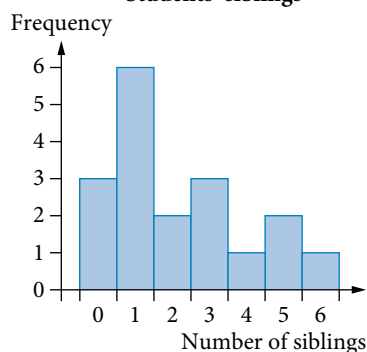
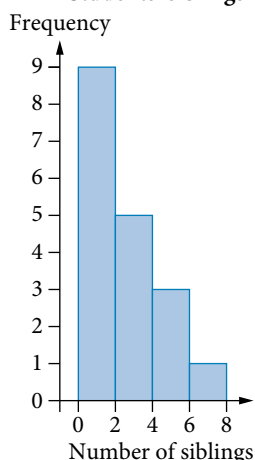
- 11 (a) C

- (b) B

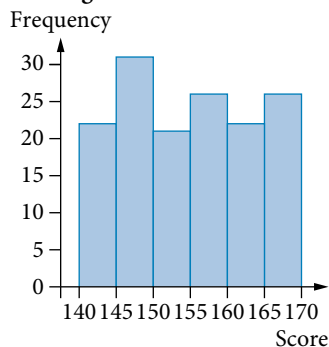
- (c) Company salaries



- (d) 2–<4, which means that more people in the company earn between \$2000 and \$4000 per month than any other similarly sized salary range.
- (e) The data is positively skewed.

12 (a) **Students' siblings**(b) **Students' siblings**

(c) The data is positively skewed.

13 **Heights of Year 8 students**

Modal class: 146–<151cm, symmetrical

14 (a) The data is continuous, because it has been measured.

Kilometres travelled in a day	Frequency
61–80	9
81–100	5
101–120	5
121–140	7
141–160	4

(c) 140.5–160.5 km

(d) Daily travels of a solar car

Stem	Leaf
6	5 6 7 8
7	8 8 8 9
8	0 4 9
9	3 5 9
10	5
11	2 4 5 7
12	2 5 7
13	3 3 4 4
14	4 5 5
15	0
16	

Key: 10|5 = 105

(e) The stem-and-leaf plot gives the actual data values.

15 (a) 100

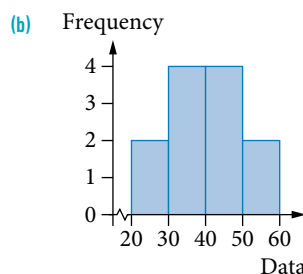
(b) The data is positively skewed (to the left are getting higher).

The modal life expectancy is 74 with a range of 12 years spread more towards the higher ages. The maximum life expectancy for any country is 83, shared by three countries.

(c) More columns would appear to the right. The minimum life expectancy for any country would determine the data value furthest to the right. (In 2017 this was 31.)

The data will have a tail to the right, in this case a negative skew because the age is decreasing to the right.

16 (a) The histogram is asymmetrical.



(c) The new histogram suggests that the data is symmetrical.

EXERCISE 7.4

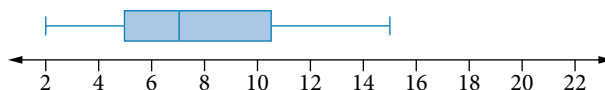
1 (a) 4 (b) 4 (c) 2 (d) 3

2 (a) 13.5–<14 s (b) 13–<13.5 s

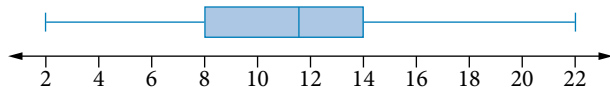
3 (a) 7 (b) 204

4 (a) 13 (b) 11 (c) 11

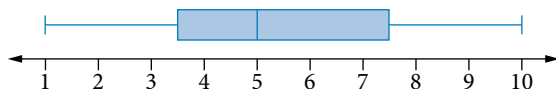
5 (a) Minimum = 2, $Q_1 = 5$, Median = 7, $Q_3 = 10.5$,
Maximum = 15, Positive skew



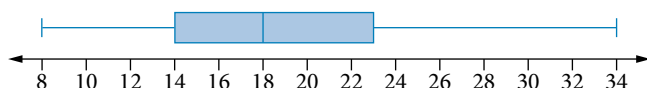
- (b) Minimum = 2, $Q_1 = 8$, Median = 11.5, $Q_3 = 14$,
Maximum = 22, Symmetrical



- 6 (a) Minimum = 1, $Q_1 = 3.5$, Median = 5, $Q_3 = 7.5$,
Maximum = 10, Positive skew



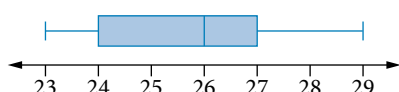
- (b) Minimum = 8, $Q_1 = 14$, Median = 18, $Q_3 = 23$,
Maximum = 34, Positive skew



- (c) Minimum = 8, $Q_1 = 9$, Median = 11, $Q_3 = 13$,
Maximum = 14, Symmetrical

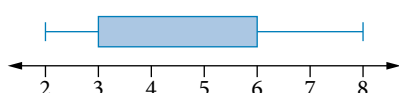


- (d) Minimum = 23, $Q_1 = 24$, Median = 26, $Q_3 = 27$,
Maximum = 29, Symmetrical



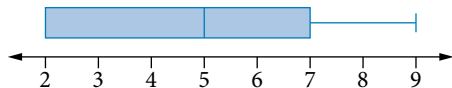
- 7 (a) Minimum = 2, $Q_1 = 3$, Median = 6, $Q_3 = 6$,
Maximum = 8

The median is equal to the upper quartile.



- (b) Minimum = 2, $Q_1 = 2$, Median = 5, $Q_3 = 7$,
Maximum = 9

The minimum is equal to the lower quartile.



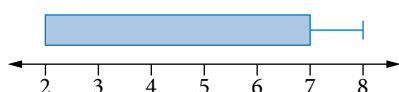
- (c) Minimum = 2, $Q_1 = 3$, Median = 5, $Q_3 = 9$,
Maximum = 9

The upper quartile is equal to the maximum.



- (d) Minimum = 2, $Q_1 = 2$, Median = 2, $Q_3 = 7$,
Maximum = 8

The minimum is equal to the lower quartile and the median.



8 C

9 (a) D

(b) A: the upper quartile, B: the maximum value, C: the interquartile range

- 10 (a) 1 and 2 (b) No outliers

(c) 58 (d) 3 and 48

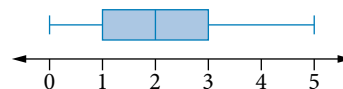
- 11 (a) -16.5, 67.5

(b) 1, 73

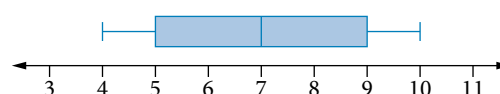
(c) 2, 10

(d) 8, 16

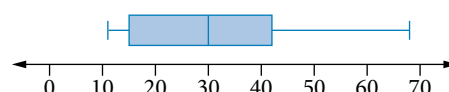
- 12 (a) Minimum = 0, $Q_1 = 1$, Median = 2, $Q_3 = 3$,
Maximum = 5, IQR = 2, Range = 5



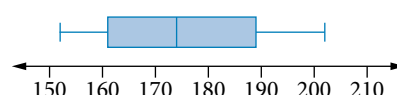
- (b) Minimum = 4, $Q_1 = 5$, Median = 7, $Q_3 = 9$,
Maximum = 10, IQR = 4, Range = 6



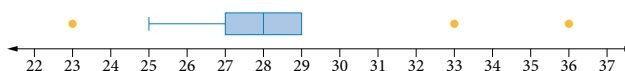
- (c) Minimum = 11, $Q_1 = 15$, Median = 30.5, $Q_3 = 42$,
Maximum = 68, IQR = 27, Range = 57



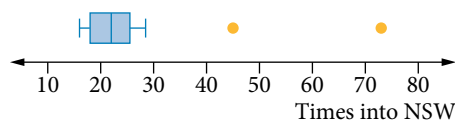
- (d) Minimum = 152, $Q_1 = 161$, Median = 174.5, $Q_3 = 189$,
Maximum = 202, IQR = 28, Range = 50



- 13 The student failed to calculate the interval for outliers, which lie outside $24 \leq x \leq 32$.

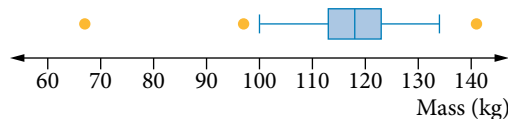


- 14 Outliers are 45 and 73.



(b) 19 to 25

- 15 (a) Outliers are 67, 97 and 141.



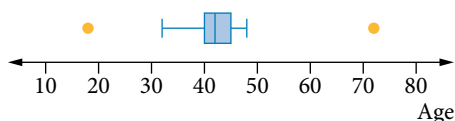
(b) 113 kg

- 16 (a) The minimum is 6 and maximum is 18, range of 12.
Outlier at 18. Median is 8, middle 50% lies between 7 and 14. Outliers exist outside the interval $2.5 \leq x \leq 14.5$. Distribution is positively skewed.

(b) The minimum is 23 and maximum is 52, range of 29. Outlier at 52. Median is 33, middle 50% lies between 29 and 38. Outliers exist outside the interval $15.5 \leq x \leq 51.5$. Distribution is approximately symmetrical.

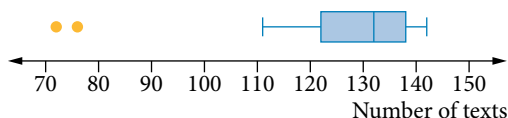
(c) The minimum is 1 and maximum is 7.5, range of 6.5. Outlier at 1. Median is 7, middle 50% lies between 5 and 7.5. Outliers exist outside the interval $1.25 \leq x \leq 11.25$. Distribution is negatively skewed.

- 17 (a) Outliers are 18 and 72.



(b) 40 to 44

- 18 (a) Outliers are 72 and 76.



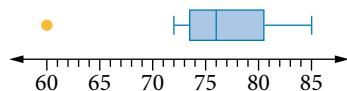
(b) 123

- 19 (a) Possible solutions: 4, 5, 6, 8, 8, 8, 9, 11, 11, 11, 11, 12, 13, 14, 14 or 4, 4, 7, 8, 9, 10, 11, 11, 12, 12, 12, 12, 12, 14

(b) Possible solutions: 4, 6, 8, 10, 11, 11, 12, 12, 13, 14 or 4, 4, 8, 10, 10, 12, 12, 12, 14, 14

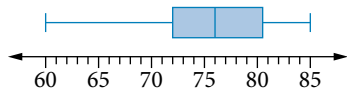
(c) Possible solutions: 4, 6, 8, 8, 10, 11, 11, 11, 12, 12, 13, 14 or 4, 6, 7, 9, 9, 10, 12, 12, 12, 12, 14

- 20 Original data:



(a) Smallest change: $75 \rightarrow 72$ or $72 \rightarrow 69$

(b) Altered data (same box plot for each):



EXERCISE 7.5

1 (a)	Birth weight (kg)	Number of baby elephants	Cumulative frequency
	90–<95	2	2
	95–<100	3	5
	100–<105	14	19
	105–<110	10	29
	110–<115	10	39
	115–<120	4	43

(b) 29

(c) 38

- 2 (a) D

(b) C

(c) The student may have added cumulative frequencies that won't ever make sense.

3 (a)	Weight (kg)	Number of mountain bikes	Cumulative frequency
	10.5–<11	2	2
	11–<11.5	2	4
	11.5–<12	2	6
	12–<12.5	7	13
	12.5–<13	2	15
	13–<13.5	1	16
	13.5–<14	2	18

(b) 6

(c) 3

- 4 (a) B

(b) 4

(c) 5

- 5 (a) $Q_1 = 1$, Median = 2, $Q_3 = 2$

(b) 2

(c) 11

- 6 (a) $Q_1 = 3$, Median = 4, $Q_3 = 6$

(b) 6

(c) 25

7 (a)	Number of students	Frequency	Cumulative frequency
	22	10	10
	23	8	18
	24	8	26
	25	9	35
	26	10	45

(b) 18

(c) minimum = 22, $Q_1 = 23$, median = 24, $Q_3 = 25$, maximum = 26

(d) The data is symmetrical and fairly uniform.

8 (a)	Number of days of rain	Frequency	Cumulative frequency
	2	2	2
	3	1	3
	4	1	4
	5	1	5
	6	2	7
	7	1	8
	9	1	9
	13	1	10
	14	1	11
	16	1	12

(b) 3

(c) minimum = 2, $Q_1 = 3.5$, median = 6, $Q_3 = 11$, maximum = 16

9 (a)	Number of dogs	Number of households	Cumulative frequency
	0	1142	1142
	1	419	1561
	2	357	1918
	3	31	1949
	4	24	1973

(b) 1918

(c) Minimum = 0, $Q_1 = 0$, median = 0, $Q_3 = 1$, maximum = 4

(d) Severely positively skewed. Many households have no dogs. The small number with more than 2 dogs registered is due to a special permit (and more land area) being required.

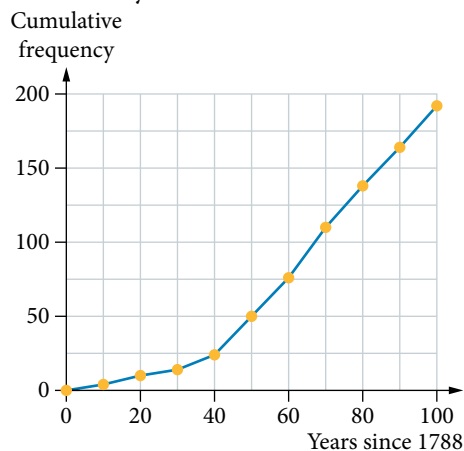
10 (a) 66

(b) 70, 83%

(c) 1472 g, three-quarters of the chimpanzees weighed less than 1472 g at birth.

11 Ages: minimum = 10, $Q_1 = 24$, median = 32, $Q_3 = 39$, maximum = 70

12 (a) **First 100 years of Australian towns and cities**

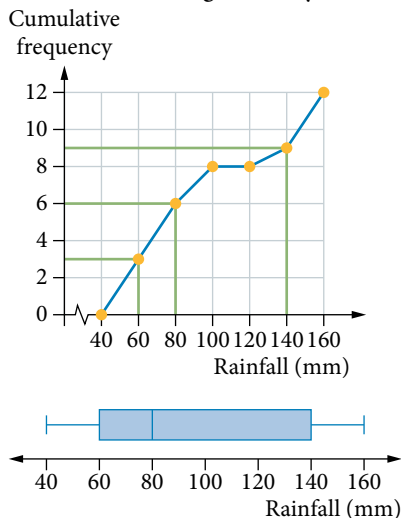


(b) 67 years

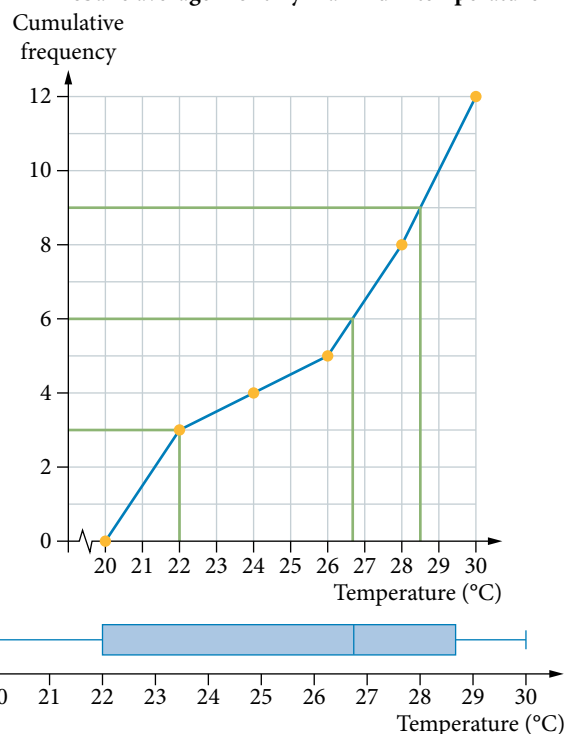
(c) 20

(d) 26%

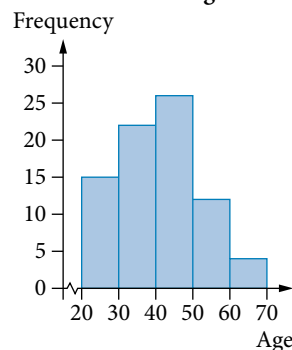
13 (a) **Brisbane average monthly rainfall**



(b) **Brisbane average monthly maximum temperature**

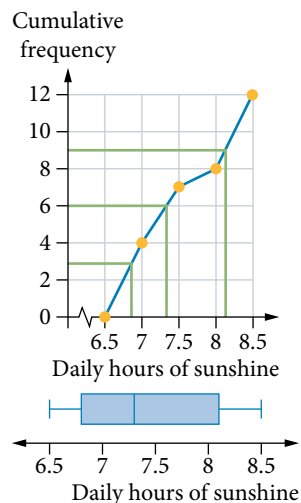


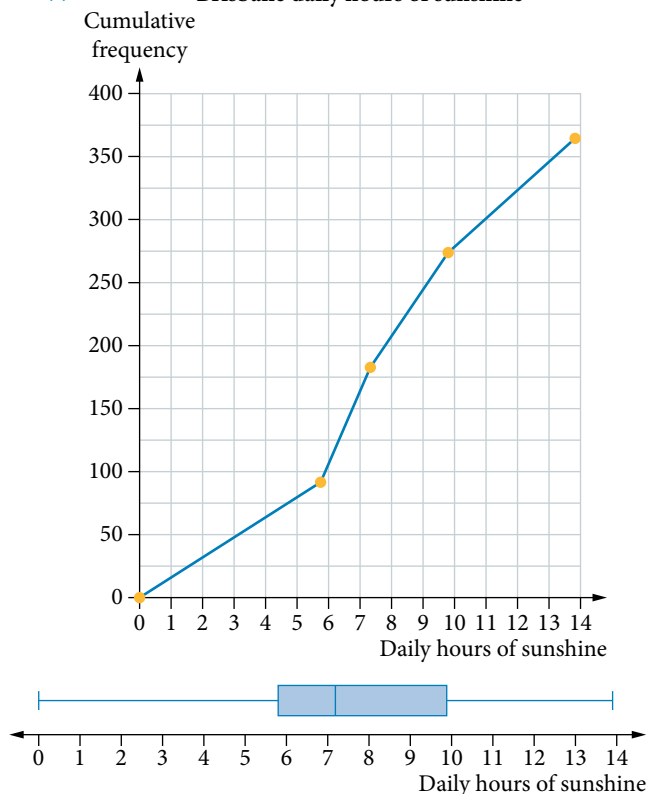
14 **Teachers at a high school**



Modal age group: 40 – <50

15 (a) **Brisbane average hours of sunshine**



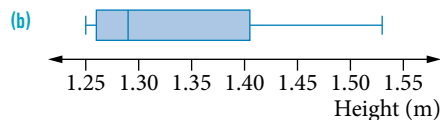
(b) Brisbane daily hours of sunshine**EXERCISE 7.6**

- 1 (a) 12.2 (b) 1.8
- 2 (a) 7.1 (b) 203.9 (c) 3.3
- 3 (a) Mean: 6, median: 6, mode: 6
(b) Mean: 6.125, median: 5.5, mode: 2
(c) Mean: 7.2, median: 6, no mode
(d) Mean: 6.125, median: 5.5, mode: 11
- 4 Mean: 2.55, SD: 2.31
- 5 (a) 2.3 (b) 2.7 (c) 4.9 (d) 4.5
(e) 5.6 (f) 5.2 (g) 1.5 (h) 1.6
- 6 (a) Mean: 37.2, standard deviation: 6.43
(b) Mean: 35.06 (a decrease), standard deviation: 10.57 (an increase)
- 7 (a) 4 people (b) 2.06 people
- 8 B
- 9 13.54 s
- 10 (a) 9.5°C
(b) 9.9°C. The estimated answer varied from the actual mean by about 0.4°C.
- 11 Mean: 62.20 cm, SD: 16.25 cm
- 12 (a) C
(b) Mean is a measure of centre, whereas standard deviation is a measure of spread.
- 13 (a) 6.8 (b) 16.3

- 14 (a) Neither machine would be shut down.
(b) Machine A would be shut down.
- 15 Interquartile range is unaffected by changes to data at either end of an ordered data set. Standard deviation increases if either a smaller value at the lower end or a larger value at the higher end replaces an existing value.
- 16 (a) Sample answer: 10, 11, 12, 13, 15, 15, 17, 18, 19, 20; $s = 3.46$
(b) Sample answer: 45, 50, 59, 60, 60, 62, 63, 70, 75, 80; $s = 10.57$

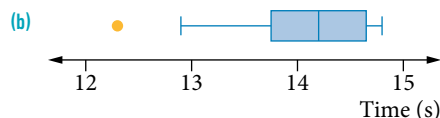
EXERCISE 7.7

- 1 (a) Mean = 43.2, Standard deviation = 10.6, Minimum = 19, $Q_1 = 35$, Median = 42, $Q_3 = 50.5$, Maximum = 63
(b)
- 2 (a) Mean = 1.341 m, SD = 0.092 m, min = 1.25 m, $Q_1 = 1.26$ m, median = 1.29 m, $Q_3 = 1.405$ m, max = 1.53 m



The data is positively skewed. 50% of the students jumped 1.29 m or less. Only 25% jumped more than 1.40 m. The highest jump was 1.53 m.

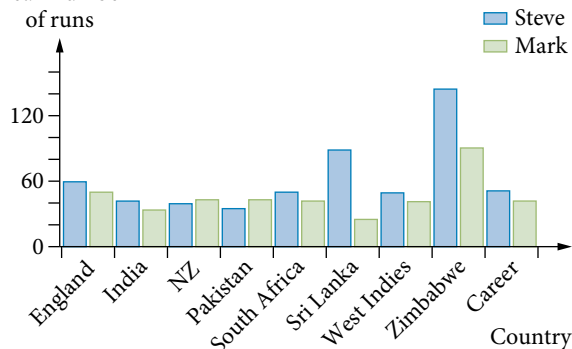
- 3 (a) Mean = 14.05 s, SD = 0.757 s, min = 12.3 s, $Q_1 = 13.75$ s, median = 14.2 s, $Q_3 = 14.65$ s, max = 14.8 s



The data is negatively skewed, with the fastest time an outlier, at 12.3 s. 50% of the students had times of 14.2 s or more. Only 25% had times less than 13.8 s.

- 4 C
- 5 Sample answer: The states with larger populations have a higher number of fatalities. Road fatalities changed by small amounts in each case, with an increase in 6 of the 8 states and territories.

- 6 Mean number of runs



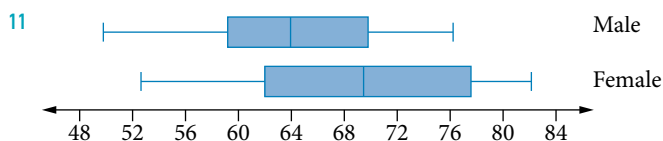
Sample answer: From the graph it can be seen that the batting averages for the brothers were similar for several countries. However, Steve had much better averages against Sri Lanka and Zimbabwe (87.63 and 145 respectively) compared with Mark's (24.64 and 90). This helped Steve maintain a higher overall career average.

2003		2013
9 6	17 _H	5 7 7 9
4 3 3 3 2 1 1 1	18 _L	0 2 2 2 3 3 3 4
9 9 9 9 7 6 6 5 5	18 _H	5 6 6 6 6 7 7 7 8 8 9 9
4 3 3 2 2 2 2 2 0 0 0 0	19 _L	0 0 0 1 2 2 3 3 4
8 7 7 6 6 5	19 _H	6 7 7 8 9
2	20 _L	3
8	20 _H	6

Key: 17 | 9 = 179

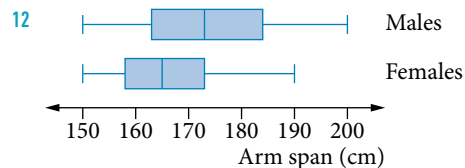
Sample answer: Both sets of data are approximately symmetrical, with very similar minimums and maximums. The playing group in 2013 had two more players shorter than 180cm than the group in 2003.

- 8 (a) The Vixens are on average 1.5cm taller than Swifts players. There is greater variation in the heights of Swifts players.
- (b) The Firebirds are on average 1.2cm taller than Lightning players. The variation in heights is virtually the same in both teams.
- (c) The Giants are on average 0.8cm taller than Fever players. There is greater variation in the heights of Fever players.
- 9 (a) Test 1: $\bar{x} = 13.1$, $s = 4.2$. Test 2: $\bar{x} = 12.1$, $s = 3.2$.
- The Test 2 results are lower than Test 1 by an average of 1 mark, and there is less variation in the marks. It was possibly easier to score well in Test 1, or perhaps the students were better prepared.
- (b) Test 1: $\bar{x} = 11.7$, $s = 4.6$. Test 2: $\bar{x} = 11.9$, $s = 4.5$.
- The two tests have similar means and variation.
- (c) Test 1: $\bar{x} = 13.0$, $s = 3.1$. Test 2: $\bar{x} = 16.6$, $s = 2.7$.
- The Test 2 results are higher than Test 1 by an average of 3.6 marks, and there is slightly less variation in the marks. It was possibly easier to score well in Test 2, or perhaps the students were better prepared.
- 10 The wet season seems to be starting earlier in recent years, with December now expected to have the highest totals. Rainfall later in the season is lower than in the past, and overall total rainfall for the year is, on average, lower.



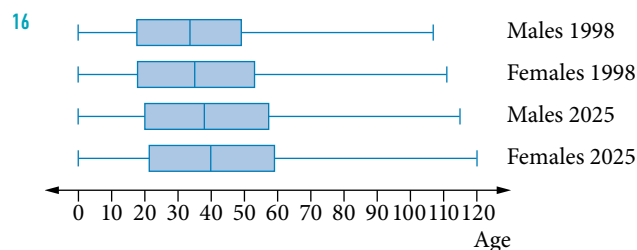
For the set of countries represented, the sets of data are approximately symmetrical and the data is similarly spread.

The median life expectancy for females is 5.5 years greater than for males. All countries have life expectancy for males at 76 years or less, whereas more than 25% of the countries have a life expectancy for females of longer than this. In summary, females are expected to live longer than males.



The arm span for both males and females is symmetrical data (remembering that maximum and minimum values are rough estimates with grouped data). For this set of data, 50% of men and 75% of women have arm spans of 173cm or less. The median arm span for men is 8cm greater than for women.

- 13 (a) A
- (b) Scoring the same amount below the mean is comparable only when the standard deviation is the same.
- 14 (a) 58 (b) 0 (c) 13
- 15 (a) 0.3720 (b) -0.3576
- (c) Michael's blood pressure reading is above the mean. His cholesterol reading is below the mean by the same relative amount.



All sets of data are skewed to the right, indicating that there is a large spread of ages for the upper 25% of people.

For each pair of years, a larger portion of women reach any particular age, especially past 50 years old. For the projection into the future, a larger portion of men and women are expected to reach any particular age, especially past 50 years old.

CHAPTER REVIEW 7

- 1 (a) Categorical, ordinal (b) Numerical, continuous
- 2 (a) Categorical, nominal (b) Categorical, ordinal
- 3 (a) 83

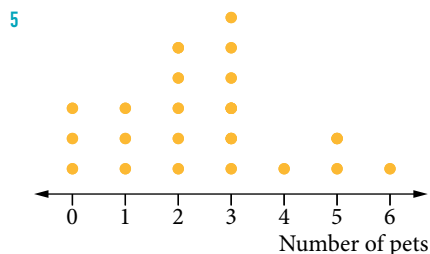
(b) x	Relative frequency
82	18%
83	42%
84	32%
85	3%
86	5%

4 (a)	<table><tr><th>Stem</th><th>Leaf</th></tr><tr><td>2</td><td>1 2 5 5 9</td></tr><tr><td>3</td><td>0 2 3 5 7 8</td></tr><tr><td>4</td><td>0 2 5 7</td></tr><tr><td>5</td><td>0 1 3</td></tr><tr><td>6</td><td>0 1 3 5 5 5 6 6 8</td></tr><tr><td>7</td><td>0 0 4 7 7 9</td></tr><tr><td>8</td><td>1 1 2</td></tr></table>	Stem	Leaf	2	1 2 5 5 9	3	0 2 3 5 7 8	4	0 2 5 7	5	0 1 3	6	0 1 3 5 5 5 6 6 8	7	0 0 4 7 7 9	8	1 1 2	(b)	<table><tr><th>Stem</th><th>Leaf</th></tr><tr><td>2</td><td>1 1 8 9</td></tr><tr><td>3</td><td>0 1 2 4 4 8 8</td></tr><tr><td>4</td><td>1 4 5 6 6</td></tr><tr><td>5</td><td>7 8 9</td></tr><tr><td>6</td><td>0 3 4 5</td></tr><tr><td>7</td><td>1 2 5 6</td></tr></table>	Stem	Leaf	2	1 1 8 9	3	0 1 2 4 4 8 8	4	1 4 5 6 6	5	7 8 9	6	0 3 4 5	7	1 2 5 6
Stem	Leaf																																
2	1 2 5 5 9																																
3	0 2 3 5 7 8																																
4	0 2 5 7																																
5	0 1 3																																
6	0 1 3 5 5 5 6 6 8																																
7	0 0 4 7 7 9																																
8	1 1 2																																
Stem	Leaf																																
2	1 1 8 9																																
3	0 1 2 4 4 8 8																																
4	1 4 5 6 6																																
5	7 8 9																																
6	0 3 4 5																																
7	1 2 5 6																																

Key: 6 | 5 = 6.5

Key: 7 | 0 = 70

Key: 6 | 5 = 6.5



6 17.5

7 (a) Five-figure summary: 1, 2, 4, 6, 9; range = 8; IQR = 4

(b) Five-figure summary: 30, 43.5, 66.5, 82, 97; range = 67, IQR = 38.5

(c) Five-figure summary: 8, 13, 14, 17, 25; range = 17, IQR = 4

8 2.8

9 B

10 (a) 10.41

(b) 3.10

11 B

12 The Swifts are on average 3.6 years younger than Thunderbirds players. There is much less variation in the ages of Swifts players.

13 (a) The second result, 79, is better because 1.1SD above the mean is better than 0.5SD above the mean.

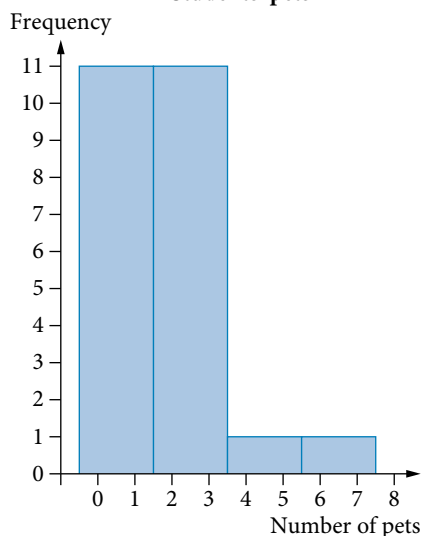
(b) The results are virtually the same, 0.44SD below the mean for both.

14	2003		2013
	4 2 2	7 _L	
	9 8 7 7 6 5 5	7 _H	6 6 7 7 7 7
	4 4 3 3 3 2 2 1 1 0	8 _L	0 0 0 1 1 2 2 3 3 3 4 4 4 4
	9 9 8 5 5 5	8 _H	5 7 7 7 8 8 9
	4 4 4 4 1 0	9 _L	0 0 2 2 3
	8 8 8 6	9 _H	7 9
	1	10 _L	0 0 4
		10 _H	7

Key: 7 | 2 = 72

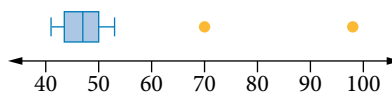
Both sets of data are positively skewed. There are more players over 100 kg and fewer under 80 kg in 2013 (greater variation) compared to 2003 and yet the median weight has remained at 84 kg.

15 (a) Students' pets

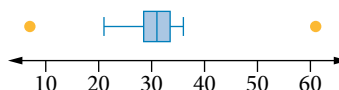


(b) The data is positively skewed.

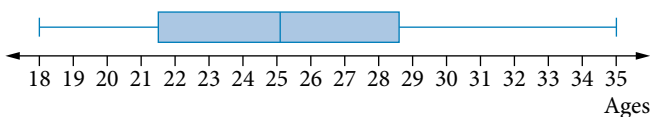
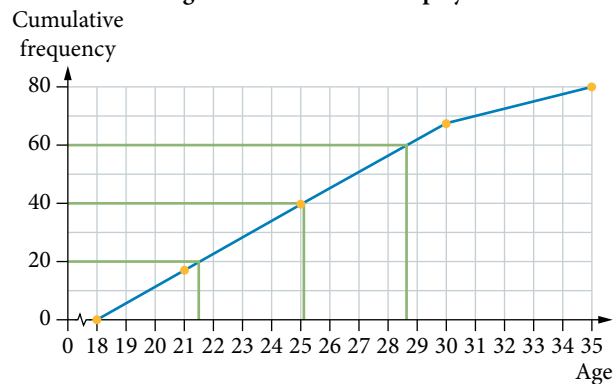
16 (a) Outliers are 45 and 73.



(b) Outliers are 18 and 72.

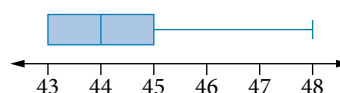


17 Age of Australian netball players



18 Minimum = 43, $Q_1 = 43$, Median = 44, $Q_3 = 45$, Maximum = 48

$\bar{x} = 44.3$, $s = 1.1$



19 D

20 B

21 Sample answer: 49.5 m, 49.8 m, 54.4 m, 54.6 m, 55.3 m, 55.4 m, 55.6 m, 55.8 m, 56.0 m, 59.8 m

22 Possible solution: replace 7, 8, 8 with 5, 9, 9.

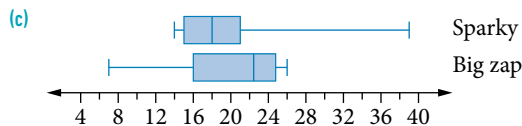
23 (a) Big Zap: Mean: 20, Median: 22.5, Standard deviation: 6.96, IQR: 9

Sparky: Mean: 19.8, Median: 18, Standard deviation: 7.39, IQR: 6

(b)

Big Zap		Sparky
9 7	0 _H	
	1 _L	4
6	1 _H	5 5 5 7 9
4 3 2 2	2 _L	0 1 3
6 6 5	2 _H	
	3 _L	
	3 _H	9

Key: 1 | 9 = 19



(d) Sparky batteries, because 100% have lives over 14 hours.

(e) Big Zap batteries, because more than 50% have lives over 20 hours.

(f) As Big Zap batteries have a more consistent lifetime and a higher median, they generally tend to last longer.

Exam review: Unit 2

1 (a) $x = 19.29$ m

(b) $x = 40.36^\circ$

2 119 m

3 251° T

4 945.1 cm^2

5

$SA(\text{cm}^2) = 2\pi r^2 + 2\pi rh$		Radius (cm)	
		6	30
Height (cm)	10	603.19	7539.82
	15	791.68	8482.30

6 4×3

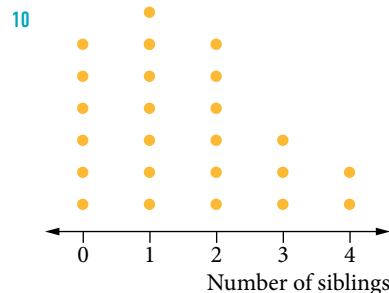
7 $u = -1, v = 0, w = -3, x = -7$

8 (a) $\begin{bmatrix} 5 & 1 & 2 \\ 1 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}$

(b) The matrix C^2 represents how many air routes there are with 1 stopover.

(c) $c_{12} = 1$, so there is 1 route with a single stopover between Ballarat and Anglesea. Fly from Ballarat to Casterton to Anglesea.

9 Numerical, continuous



11 10 and 11

12 102.6 km

13 47.29 m

14 $\begin{bmatrix} 33 & -22 \\ 14 & 31 \end{bmatrix}$

15 81

16 Set A: $\bar{x} = 18.9, s_x = 12.2$

Set B: $\bar{x} = 20.8, s_x = 9.7$

Both data sets are skewed to the right and have the same (estimated) minimum and maximum values. Set B has a mean value 1.9 greater than Set A, and there is much less variation in the data.

17 (a) 654.37 m

(b) 288° T

18 2 and 6

Exam review: Units 1 & 2

1 \$4950

2 (a) \$28

(b) \$345

3 (a) $b = 45.96$ mm

(b) $d = 33.73$ mm

4 $P = 152 \text{ cm}, A = 736 \text{ cm}^2$

5 301465 cm^3

6 $x = 28 \text{ mm}, y = 29 \text{ mm}$

7 (a) 1 : 50

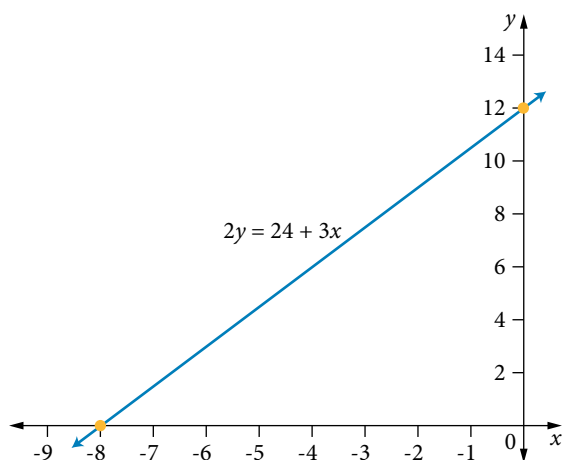
(b) Bookshelf: 1.4 m, 0.5 m; Bed: 2.1 m, 1.2 m; Rug: 2.4 m, 1.6 m; Desk: 2.0 m, 0.6 m; Wardrobe: 2.5 m, 0.6 m

(c) 20 m^2

(d) 10.9 m^2

8 $v = 53.41 \text{ m}^3$

9



10 (a) $x = 4, y = 5$

(b) $j = 2, k = 9$

11 $x = 2.00 \text{ m}$

12 0.9 km

13 11.62 mm^2

14 58.1 m^2

15
$$\mathbf{R} = \begin{matrix} & \begin{matrix} A & B & C \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix} \end{matrix}$$

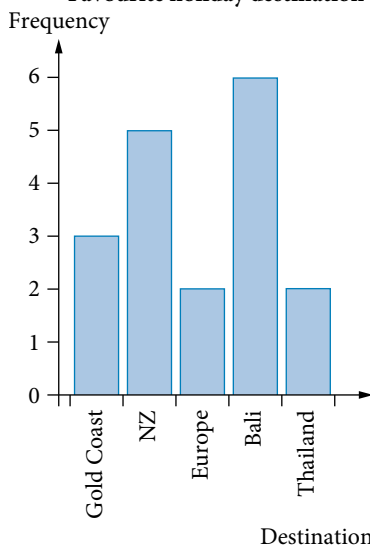
16 (a)
$$\mathbf{A} + \mathbf{S} = \begin{bmatrix} 12 & 10 & 17 \\ 12 & 20 & 29 \\ 8 & 18 & 25 \end{bmatrix}$$

(b) Store 3 sold 8 sofas over the 2 months of August and September.

17 (a) Nominal

(b) Ordinal

18 **Favourite holiday destination**



19 Five-figure summary: 131, 145, 153, 173, 191

20 (a) Ekbal: \$357, Marissa: \$768.60

(b) Ekbal: \$3524.50, Marissa \$4399.50

(c) Ekbal: \$1490.58, Marissa: \$9009

(d) Ekbal's hybrid will cost: \$23141

Marissa's six-cylinder will cost: \$36137

Marissa will pay \$12996 more than Ekbal over 5 years.

(e) Ekbal and Marissa will contribute approximately 79 and 31 weeks of income towards their car ownership and running costs (excluding maintenance) for their respective vehicles over the 5 year period.

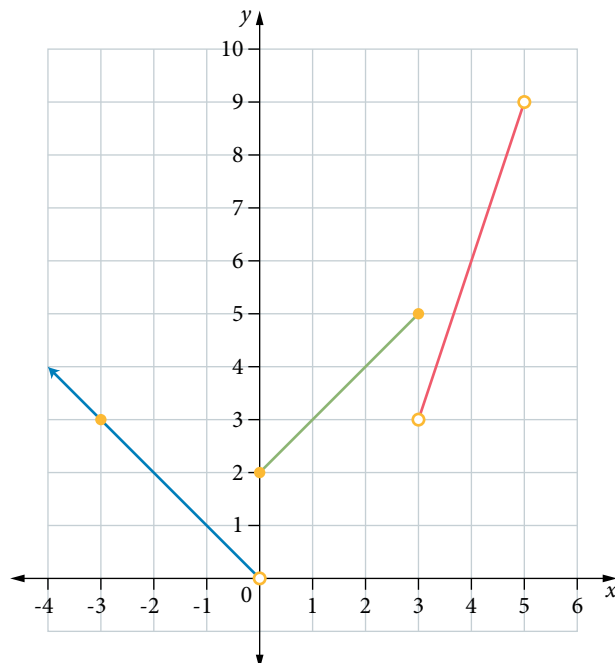
21 49.6%

22 34 cm

23 2073.6 cm^3

24 111, 112, 113

25



26 (a) 110.59 m

(b) 84.72 m

27 (a)
$$\mathbf{Y} = \begin{bmatrix} 120 \\ 65 \\ 40 \\ 100 \end{bmatrix}$$

(b)
$$\mathbf{XY} = \begin{bmatrix} 725 \\ 437.5 \\ 642.5 \end{bmatrix}$$

Product A costs \$725, product B costs \$437.50 and product C costs \$642.50 to manufacture.

28 Rajid should invest, as this puts him ahead by \$958.92.

However, he would be better off leaving a small amount in an accessible account, such as \$520, to pay his credit card on time.

29 388 cm^2

30 $y = 8 + \frac{2}{5}x$

31 (a) 54.0 km

(b) 073° T

Pearson Australia

(a division of Pearson Australia Group Pty Ltd)
707 Collins Street, Melbourne, Victoria 3008
PO Box 23360, Melbourne, Victoria 8012
www.pearson.com.au

Copyright © Pearson Australia 2018

(a division of Pearson Australia Group Pty Ltd)
First published 2018 by Pearson Australia

eBook credits

Resource authors: Rodney Anderson, David Coffey, Emily Hobbs, Robyn Johnson

Publisher: Daniel Hernandez

Project Manager: Sarah Daglish

Production Editor: Natalie Lincoln

Development Editor: Lindy Bayles

Editor: Kay Waters

Rights & Permissions: Amirah Fatin

Illustrator: diacriTech

eBook Production Coordinator: Louise Grundy

Digital Administrator: Jenny Smirnov

ISBN: 978 1 4886 2148 2

Pearson Australia Group Pty Ltd ABN 40 004 245 943

eBook attributions

General Mathematics 2017 v1.1 General Senior Syllabus © Queensland Curriculum and Assessment Authority (QCAA). This syllabus forms part of a new senior assessment and tertiary entrance system in Queensland. Along with other senior Syllabuses, it is still being refined in preparation for implementation in schools from 2019. For the most current syllabus versions and curriculum information please refer to the QCAA website <https://www.qcaa.qld.edu.au/>. The QCAA's permission does not imply permission to reproduce non-QCAA material. For permission to reproduce the material for which the QCAA owns copyright, please contact the QCAA, PO Box 307, Spring Hill Qld 4004; publishing@qcaa.qld.edu.au.

Learnetic www.learnetic.com Copyright © 2014 Learnetic S.A.

Virtual Nerd www.virtualnerd.com

Disclaimer

Some of the images used in *Pearson General Maths 11 (Units 1 & 2) Queensland Reader+* might have associations with deceased Indigenous Australians. Please be aware that these images might cause sadness or distress in Aboriginal or Torres Strait Islander communities.