PEARSON mathematics

S.B.

AUSTRALIAN CURRICULUM

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The poem on the inside front cover is called a Fib. The sequence of syllables in the lines follows a Fibonacci sequence: 1, 1, 2, 3, 5, 8.

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PEARSON mathematics

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The student book has been designed for maximum student engagement while providing teachers, particularly those lacking a strong mathematical background, with everything they need to teach for deeper understanding. It has been structured according to the latest research on how students learn mathematics and on how to avoid common misunderstandings, making it easy for teachers to cater for mixed abilities and ask good questions.

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Bringing the heart and soul of the Australian Mathematics Curriculum to life

Mathspace

Our Global village

Facts about our global village

There are over 6.9 billion people on earth today. Who is this large community composed of?

1 Convert the following facts to percentages to find out.

Race
- 0.61 Asian
- $\frac{12}{100}$ European
- 0.08 North American
- 0.05 South American & Caribbean
- 0.13 African
- $\frac{1}{100}$ Oceania (this includes Australia)

Conditions
- 0.43 live without basic sanitation
- $\frac{1}{8}$ don't have access to clean, safe drinking water
- 0.13 are hungry and/or malnourished

Wealth
- 0.06 of the population own 59% of the entire wealth
- $\frac{1}{5}$ of the population have 75% of the income
- 0.53 of the population live on $2.50 or less per day

Education and technology
- 0.14 can't read
- 0.07 have a secondary education
- 0.12 have a computer
- 0.03 have the Internet

How rich are you?

If you have a bed to sleep on, food to eat and a roof over your head, you're richer than what percentage of the entire world's population?

2 To find out, complete this decimal calculation and *convert your answer to a percentage.*

1.2 – 2.7 + 3.5 – 1.25 =

How many people?

Currently there are approximately 6 900 000 000 people living on earth.

3 Using the facts you now know about our world, *estimate* how many people:

(a) Don't have access to clean water

(b) Struggle to live on $2.50 or less per day

(c) Have a computer

(d) Don't have a secondary school education

I want to help

There are many charities out there that are trying to create more equality in the world—Oxfam is one of these. Charities often rely on the generosity of everyday people. Here is a pie chart of the age groups of Oxfam volunteers.

Volunteers by age group

18 – 25, 26 – 30, 31 – 40, 41 – 50, 51 – 65, 65+

43%, 28%, 12%, 8%, 7%, 2%

Oxfam Annual Report 07-08

4 Using the pie chart above, find out the following:

(a) What percentage of volunteers are between 18 and 30?

(b) Why do you think most volunteers are in this age range?

(c) One of the challenges charities face is the need to pay for fundraising and administration costs. If Oxfam spends 24.4% of their income on fundraising and 8.9% on administration costs, what percentage of their income is available to go directly to community aid projects?

In the future

5 Imagine it's the year 2055. How do you hope the 'Global Village' percentages have changed? Write down your predictions and ideas.

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The Australian Curriculum: Mathematics aims to ensure that students:

- *are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and work lives and as active citizens.*
- *develop an increasingly sophisticated understanding of mathematical concepts and fluency with processes, and are able to pose and solve problems and reason in* Number and Algebra, Measurement and Geometry, *and* Statistics and Probability.
- *recognise connections between the areas of mathematics and other disciplines and appreciate mathematics as an accessible and enjoyable discipline to study.*

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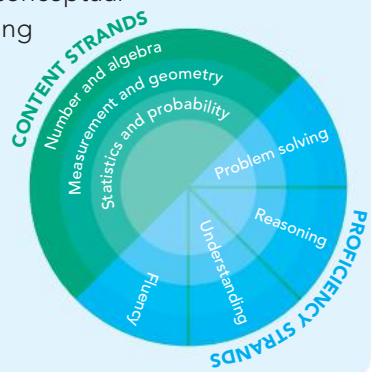
Pearson Mathematics shares these aims.

Pearson Mathematics is a compelling research-based series, written by experienced and practising Australian teachers with the support of Australia's leading mathematics education experts. It has been specifically designed to scaffold students' fluency development, conceptual understanding, reasoning and problem-solving skills, in addition to addressing the Australian Curriculum's General capabilities and Cross-curriculum priorities.



Making teachers' lives easier

Differentiation

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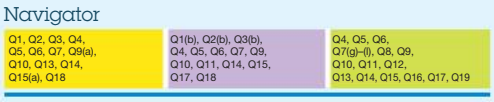


Navigator—Three optional graded pathways through *every* Exercise.

Recall—Each chapter begins with a review of assumed and necessary knowledge, with Recall worksheets available for each skill for those students needing extra revision.

Challenge—Every chapter has a full page challenge section for earlier finishers and those students who need to be extended.

Scaffolded Exercises—All Exercises are split into the Australian Curriculum Proficiency Strands: Fluency, Understanding and Reasoning. They have been carefully paced to help students build skills, develop deep conceptual understanding and apply learning. Every Exercise also has Open-ended questions to encourage students' creative thinking and ability to communicate mathematics effectively.

Regular revision and reinforcement

Pearson Mathematics has a broad range of cumulative and chapter-based revision.

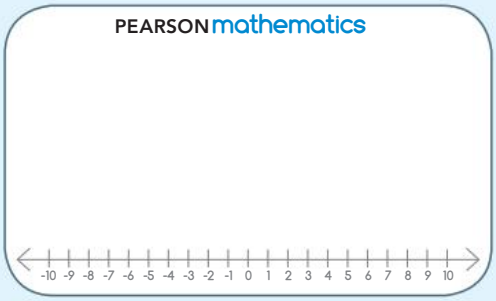
NAPLAN practice—An end-of-chapter section that helps students prepare for their NAPLAN tests

Half-time—A mid-chapter review

Chapter review—An end-of-chapter review

Mixed review—Cumulative revision for every second chapter

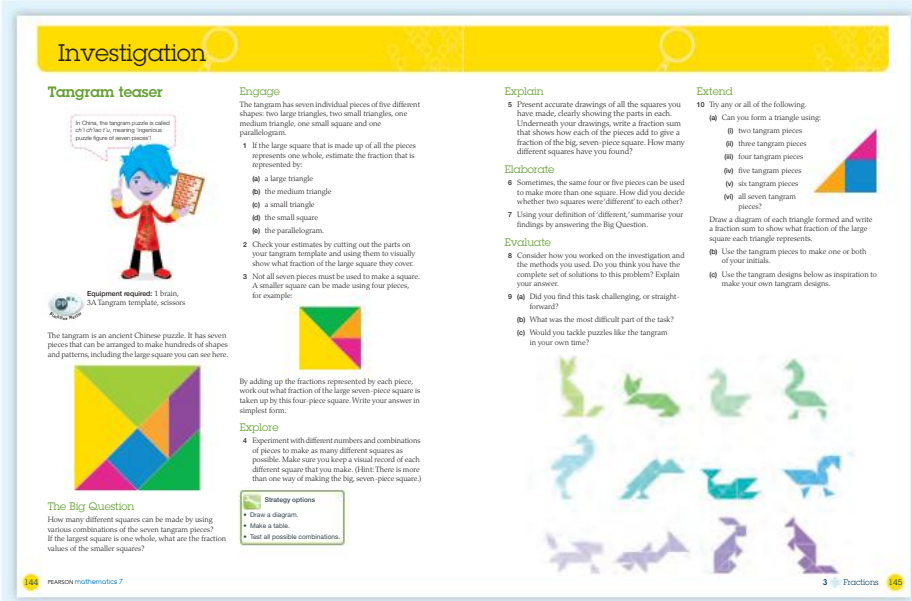
Active participation



Mini-whiteboard—A modern teaching and learning tool used internationally for full class involvement, allowing every student to participate in giving answers.

Every copy of ***Pearson Mathematics 7*** and ***Pearson Mathematics 8*** has a mini-whiteboard in the back cover pocket.

Encouraging inquiry and problem solving



Investigation

Tangram teaser

In China, the tangram puzzle is called *chi t ch tao* Tu, meaning 'ingenious puzzle figure of seven pieces'!

Equipment required: 1 brain, 3A Tangram template, scissors

The tangram is an ancient Chinese puzzle. It has seven pieces that can be arranged to make hundreds of shapes and patterns, including the large square you can see here.

The Big Question

How many different squares can be made by using various combinations of the seven tangram pieces? If the largest square is one whole, what are the fraction values of the smaller squares?

Engage

The tangram has seven individual pieces of five different shapes: two large triangles, two small triangles, one medium triangle, one small square and one parallelogram.

1 If the large square that is made up of all the pieces represents one whole, estimate the fraction that is represented by:
   (a) a large triangle
   (b) the medium triangle
   (c) a small triangle
   (d) the small square
   (e) the parallelogram.
2 Check your estimates by cutting out the parts on your tangram template and using them to visually show what fraction of the large square they cover.
3 Not all seven pieces must be used to make a square. A smaller square can be made using four pieces, for example:

By adding up the fractions represented by each piece, work out what fraction of the large seven-piece square is taken up by this four-piece square. Write your answer in simplest form.

Explore

4 Experiment with different numbers and combinations of pieces to make as many different squares as possible. Make sure you keep a visual record of each different square that you make. (Hint: There is more than one way of making the big, seven-piece square.)

Strategy options
- Draw a diagram.
- Make a table.
- Test all possible combinations.

Explain

5 Present accurate drawings of all the squares you have made, clearly showing the parts in each. Underneath your drawings, write a fraction sum that shows how each of the pieces add to give a fraction of the big, seven-piece square. How many different squares have you found?

Elaborate

6 Sometimes, the same four or five pieces can be used to make more than one square. How did you decide whether two squares were 'different' to each other?
7 Using your definition of 'different,' summarise your findings by answering the Big Question.

Evaluate

8 Consider how you worked on the investigation and the methods you used. Do you think you have the complete set of solutions to this problem? Explain your answer.
9 (a) Did you find this task challenging, or straight-forward?
   (b) What was the most difficult part of the task?
   (c) Would you tackle puzzles like the tangram in your own time?

Extend

10 Try any or all of the following.
   (a) Can you form a triangle using:
      (i) two tangram pieces
      (ii) three tangram pieces
      (iii) four tangram pieces
      (iv) five tangram pieces
      (v) six tangram pieces
      (vi) all seven tangram pieces?

   Draw a diagram of each triangle formed and write a fraction sum to show what fraction of the large square each triangle represents.
   (b) Use the tangram pieces to make one or both of your initials.
   (c) Use the tangram designs below as inspiration to make your own tangram designs.

144 PEARSON mathematics 7 — 3 Fractions 145

The Australian Mathematics Curriculum focuses on students developing problem-solving skills, in particular the ability to employ mathematical strategies to solve problems efficiently and to be able to respond to unfamiliar situations. **Pearson Mathematics** is designed to develop these skills.

Problem solving toolbox

12 key problem-solving strategies are included inside the front cover of the Student Book and these **strategy options** are referenced throughout investigations and problem solving.

Outside the Square: Problem solving

These are located throughout every chapter and give students ample opportunity to develop their problem-solving skills and logical reasoning.

Outside the Square
Problem solving

Investigations

Investigation

These are scaffolded to be accessible to all learners and structured following a 5e+ format.

Technology Explorations

Technology Exploration

Pearson Mathematics understands the importance the Australian Curriculum places on students developing ICT literacy. Technology Explorations help develop students' deeper understanding of the mathematical concepts covered.

Make a table
Look for a pattern
Act it out
Solve a simpler probl
Draw a picture
Test all possible combinations
Work backwards
Bre

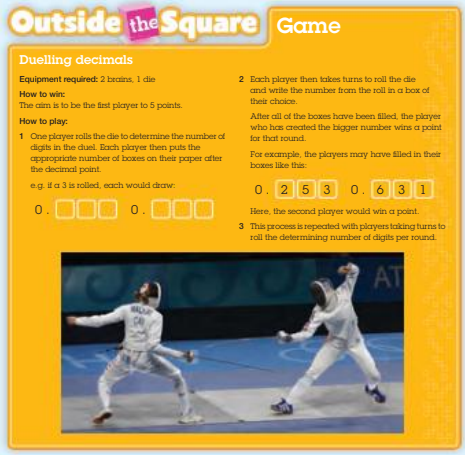
Engaging students

Pearson Mathematics has been designed to capture students' interest, with the incorporation of Mathspaces, Maths 4 Reals, Outside the Squares and Home Pages.

Maths 4 Reals

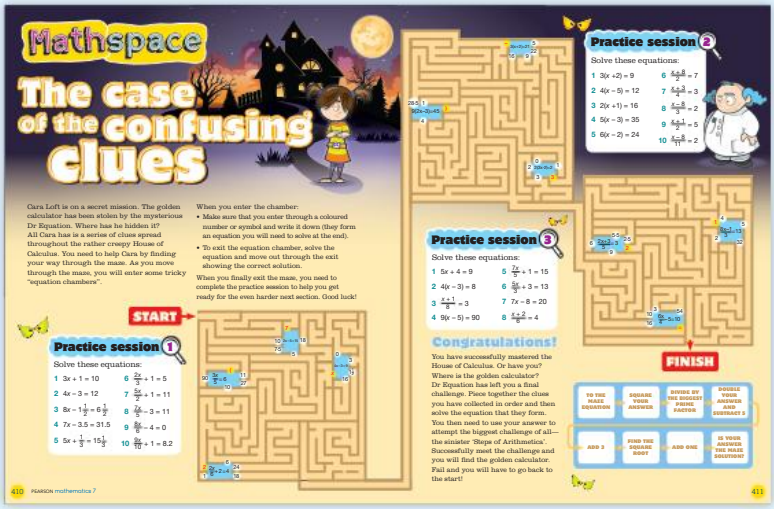
Scenarios that help students make connections with mathematics in the real world.

Outside the squares



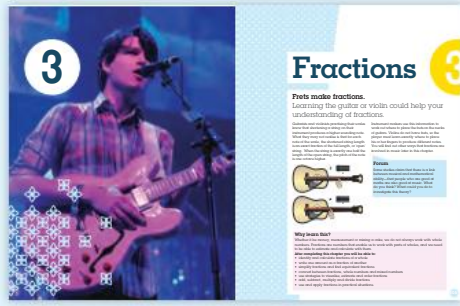
Small activities for early finishers involving problem solving, puzzles and topic-related games.

Mathspaces



Skill consolation and reinforcement wrapped up in fun and quirky scenarios, including multi-player maths board games, solve the riddle and find the clue style tasks.

Home Pages



Anticipate the 'Why learn this?' question and provide a motivating entry into a new topic, including discussion-provoking Forum questions.

Have I seen a similar problem?
Seek an exception
Guess and check
Draw a picture
Make a model
ak problem into manageable parts

1

Integers and indices

1

How low can you go? We have explored all over the surface of our planet, but we have seen very little of the vast depths of Earth's oceans.

The highest point on the Earth's surface is the summit of Mt Everest. A brave few have made it to this point, a height of 8848 m above sea level (+8848 m). The lowest point on Earth is the bottom of the Marianas Trench, in the Pacific Ocean, 11 022 m below sea level (-11 022 m). In 1960, naval lieutenant Don Walsh and engineer Jacques Piccard made the first expedition to the bottom of the trench, reaching a depth of -10 915 m. When they resurfaced, the windows of their submersible were cracked from the pressure of the water. Because of this safety issue, much of the Earth's oceans have only been explored to a depth of -300 m. There are only a few manned submersibles that can go beyond -3000 m (a naval submarine usually travels at around -500 m). At these depths, an amazing underwater landscape is waiting to be explored.

Forum

Have you ever experienced negative temperatures? If you have, where were you?

If you were able to pick up Mt Everest and place it in the Marianas trench, it would be completely covered by water. How much water would there be between the top of Mt Everest and sea level?

Why learn this?

Positive numbers are only half the story! You need to be able to work with numbers that are less than zero—the negative side of the number line. They are used to show actions that are the opposite to positives, such as a move backwards instead of forwards, or a fall in price instead of a rise. Indices (also known as powers or exponents) give us a convenient, 'shorthand' way of working with very large numbers. Having some rules for working with them helps make our calculations more efficient.

After completing this chapter you will be able to:

- assign directed numbers to everyday situations
- perform the four operations (+, –, ×, ÷) on directed numbers
- use a number line to assist with directed number calculations
- apply the order of operations rules to directed numbers
- understand and use index notation
- work efficiently with index numbers by developing and applying some rules.

Recall 1

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R1.1

1 To answer the following, it may help to draw a diagram.

(a) Cameron looked at the thermometer by his back door at 7a.m. on a winter morning. The temperature was 4°C. When he checked at midday, the temperature was 13°C. By midnight it had dropped to -2°C.

(i) By how many degrees did the temperature increase between 7a.m. and midday?

(ii) By how many degrees did the temperature decrease between midday and midnight?

(iii) What was the difference in the temperature at 7a.m. and the temperature at midnight?

(b) A city building has 5 storeys above, and 2 basement levels below, the ground floor.

(i) Jade parked her car in the 2nd basement level, got in a lift and moved up 5 floors. Which floor did she get out on?

(ii) Paula got in the lift on the 4th floor. She went down 5 floors, then up 2 floors. Which floor did she end up on?

Worksheet R1.2

2 Write the value of the following.

(a) 2^3 **(b)** 3^4 **(c)** 5^2 **(d)** 10^6

(e) $\sqrt{36}$ **(f)** $\sqrt{49}$ **(g)** $\sqrt[3]{64}$ **(h)** $\sqrt[3]{8}$

Worksheet R1.3

3 Imagine that you are standing at -4 on a large number line. Which number would you land on if you walked:

(a) 7 places to the right **(b)** 5 places to the left?

Worksheet R1.4

4 Calculate:

(a) 2×8 **(b)** 7×9 **(c)** 38×6 **(d)** 15×19

(e) $18 \div 3$ **(f)** $54 \div 9$ **(g)** $324 \div 3$ **(h)** $616 \div 4$

Worksheet R1.5

5 Calculate:

(a) $(8 \div 4) + (35 \div 5)$ **(b)** $6 \times 8 + 2 \times 3$ **(c)** $50 - 5 \times 2 + 3$

Worksheet R1.6

6 Use a factor tree to find the prime factors of the following, then write each number as a product of its prime factors in index form.

(a) 24 **(b)** 36 **(c)** 120

Worksheet R1.7

7 Evaluate the following.

(a) $4^2 + 3^2$ **(b)** $2^2 + 5^2$ **(c)** $8^2 - 6^2$ **(d)** $9^2 - 3^2$

(e) $5^2 \times 2^4$ **(f)** $3^3 \times 5 \times 7$ **(g)** $9^2 \div 3^2$ **(h)** $2^3 \times 7 \div 2^2$

Key Words

base	exponent	integers	simplify
cube	index	negative	square
evaluate	index form	positive	
expanded form	indices	power	

Integers review

1.1

A **positive** (+) or a **negative** (-) sign is used to show the direction of a number. Some words associated with negative numbers are: down, loss, below, decrease, lose, withdrawal. Some words associated with positive numbers are: up, profit, above, increase, gain, deposit.

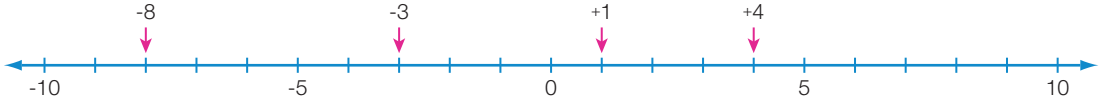
For example, a deposit of $50 into your bank account could be written as +50, whereas a withdrawal of $180 could be written as -180.

Date	Transaction	Balance
		$500
17 July	ATM Withdrawal -180	$320
20 July	Birthday money from Mum +50	$370

What is an integer?

The **integers** are all of the positive whole numbers 1, 2, 3 … , the negative whole numbers -1, -2, -3 … , and zero. (The use of '…' in mathematics shows that the sequence of numbers continues forever.)

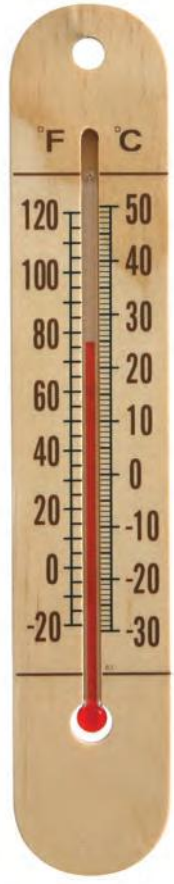
Integers can be represented on a number line:



The numbers on a number line get larger in value as we move from left to right.

If a number has no sign in front of it, we assume it is positive. 4 = +4

From the number line above, we can write the statement +4 > -8. We would say this as 'positive 4 is greater than negative 8'. We could also write the statement -3 < +1, which we would say as 'negative three is less than positive one'. Number lines can also be written vertically, with the positive numbers above the negative ones. A thermometer is an example of a vertical number line.



Adding and subtracting integers

Adding and subtracting positive integers is the straightforward addition you have been doing for years.

To *add a positive integer*, we can imagine moving that many places to the *right* on the number line (or upwards on a vertical number line), in the positive direction.

To *subtract a positive integer*, we can imagine moving that many places to the *left* on the number line (or downwards on a vertical number line), in the negative direction.

Adding a negative integer is the opposite of adding a positive one. We move in the negative direction, to the *left* on the number line (or downwards on a vertical number line). We can see that this is the same movement as subtracting a positive integer.

Subtracting a negative integer is the opposite of subtracting a positive one. We move in the positive direction, to the *right* on the number line (or upwards on a vertical number line).

Simplifying addition

When adding and subtracting integers, brackets are often placed around the second number and its sign, to separate it from the addition or subtraction sign; for example, +5 + (+9) or +7 – (+3). However, as we can write a positive integer without the + sign in front, we can drop the positive signs in front of the numbers, remove the brackets, and simply write 5 + 9 or 7 – 3.

We can see from the previous number lines that subtracting a negative integer is the same as adding a positive one. This means that we could write 8 – (-2) as 8 + 2.

We can also see that adding a negative integer is the same as subtracting a positive one. This means we could write 4 + (-10) as 4 – 10.

+ (+) and – (-) can be replaced with +	– (+) and + (-) can be replaced with –
We could also say:	
When the two signs are the *same*: add.	When the two signs are *different*: subtract.

Worked Example 1

WE1

Simplify the following by writing a single sign between the two numbers, then calculate the answer.

(a) -4 + (+11) **(b)** -9 + (+3) **(c)** +2 – (+8) **(d)** -5 – (+11)

Thinking		Working
(a) **1**	Simplify the addition by writing positive integers without their signs, and removing the brackets.	(a) -4 + (+11) = -4 + 11
2	Imagine (or draw) a number line. Start at the first number (-4), then move the number of places indicated by the second number (11) in the positive direction (to the right). Write your answer.	= 7
(b) **1**	Simplify the addition by writing positive integers without their signs, and removing the brackets.	(b) -9 + (+3) = -9 + 3
2	Imagine (or draw) a number line. Start at the first number (-9), then move the number of places indicated by the second number (3) in the positive direction (to the right). Write your answer.	= -6
(c) **1**	Simplify the subtraction by writing positive integers without their signs, and removing the brackets.	(c) +2 – (+8) = 2 – 8
2	Imagine (or draw) a number line. Start at the first number (2), then move the number of places indicated by the second number (8) in the negative direction (to the left). Write your answer.	= -6

(d)	**1**	Simplify the subtraction by writing positive integers without their signs, and removing the brackets.	(d) -5 – (+11) = -5 – 11
	2	Imagine (or draw) a number line. Start at the first number (-5), then move the number of places indicated by the second number (11) in the negative direction (to the left). Write your answer.	= -16

Worked Example 2

WE2

Simplify the following by writing a single sign between the two numbers, then calculate the answer.

(a) +9 + (-7) **(b)** -12 + (-1) **(c)** +4 – (-6) **(d)** -5 – (-7)

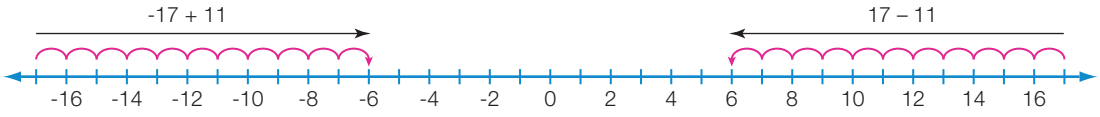
Thinking			Working
(a)	**1**	Remove the brackets and write a single subtraction sign between the numbers. Write positive integers without their signs.	(a) +9 + (-7) = 9 – 7
	2	Complete this straightforward subtraction. Write your answer.	= 2
(b)	**1**	Remove the brackets and write a single subtraction sign between the numbers. Write positive integers without their signs.	(b) -12 + (-1) = -12 – 1
	2	Imagine (or draw) a number line. Start at the first number (-12), then move the number of places indicated by the second number (1) in the negative direction (to the left). Write your answer.	= -13
(c)	**1**	Remove the brackets and write a single addition sign between the numbers. Write positive integers without their signs.	(c) +4 – (-6) = 4 + 6
	2	Complete this straightforward addition.	= 10
(d)	**1**	Remove the brackets and write a single addition sign between the numbers. Write positive integers without their signs.	(d) -5 – (-7) = -5 + 7

2 Imagine (or draw) a number line. Start at the first number (-5), then move the number of places indicated by the second number (7) in the positive direction (to the right). Write your answer. = 2

Using number line symmetry

The number line is symmetrical about zero. A positive number is the same distance from zero as its negative opposite; e.g. negative 6 is the same distance from zero as positive 6. This can be useful when adding integers, especially large ones.

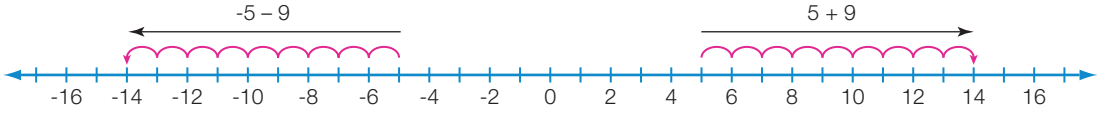
For example, the number line journey that shows -17 + 11 is the mirror image of the journey that shows 17 – 11.



So, to calculate -97 + 64, we could simply do 97 – 64, then change the sign of the answer.

97 – 64 = 33 -97 + 64 = -33

Similarly, to calculate -5 – 9, we could consider the mirror image journey of 5 + 9.



So, to calculate -53 – 49, we could simply do 53 + 49, and change the sign of the answer.

53 + 49 = 102 -53 – 49 = -102

1.1 Integers review

Navigator

Answers page 592

Q1 Columns 1–3, Q2 Columns 1–3, Q3, Q4 Column 1, Q6, Q7, Q11, Q13	Q1 Columns 2–4, Q2 Columns 2–4, Q3, Q4 Column 2, Q5, Q6, Q7, Q8, Q10(a), Q11, Q12, Q13	Q1 Columns 3 & 4, Q2 Columns 3 & 4, Q3, Q4 Column 2, Q5, Q6, Q7, Q8, Q9, Q10(b), Q11, Q12, Q14, Q15

Fluency

WE1

1 Simplify the following by writing a single sign between the two numbers, then calculate the answer.

(a) +4 + (+9) (b) -7 + (+5) (c) +5 + (+3) (d) +2 – (+6)

(e) +10 – (+13) (f) -3 + (+8) (g) +12 + (+6) (h) -1 – (+9)

(i) -15 + (+8) (j) +8 – (+9) (k) -8 – (+13) (l) -19 + (+11)

(m) +16 – (+9) (n) -14 + (+7) (o) -4 – (+22) (p) -17 + (+23)

2 Simplify the following by writing a single sign between the two numbers, then calculate the answer. WE2

(a) +5 + (-4) (b) +7 + (-9) (c) +5 – (-11) (d) -7 – (-6)

(e) -4 – (-9) (f) -3 – (+8) (g) -14 – (+3) (h) +12 – (-8)

(i) +19 – (+13) (j) 0 – (-3) (k) -8 – (-16) (l) +15 + (-22)

(m) -11 – (-7) (n) -13 + (-9) (o) +25 + (-31) (p) -27 – (-16)

3 Write a negative or a positive integer to describe the following situations.

(a) 350 m above sea level

(b) a loss of $4800

(c) rewinding a TV program 6 minutes

(d) depositing $73 into your bank account

(e) 2 levels below the ground floor of a building

(f) 19 metres under water

(g) a company profit of $10 750

(h) fast forwarding a TV program 34 minutes

(i) withdrawing $200 from your bank account

(j) a plane flying at an altitude of 8100 metres

4 Place the following in descending order (largest to smallest).

(a) +4, 0, -7, +11, -2 (b) -23, 1, 0, -9, +7

(c) -3, 4, 0, 11, -15, 1 (d) -5, 8, 19, -43, -2, 6

(e) 14, -72, 5, 26, -1, -38 (f) 32, -19, 0, 17, -56, 4

5 Calculate:

(a) 2 + 7 – 5 (b) -3 + 10 – 5 (c) -6 + 4 – 8 (d) -15 + 9 + 8

(e) 11 + 14 – 23 (f) -7 – 8 – (-9) (g) 4 + 5 – (-5) (h) -6 + (-9) – (+9)

Understanding

6 (a) Copy the diagram and label it according to the following information.

The bird is 24 metres above the water.

A scuba diver is 17 metres below the surface.

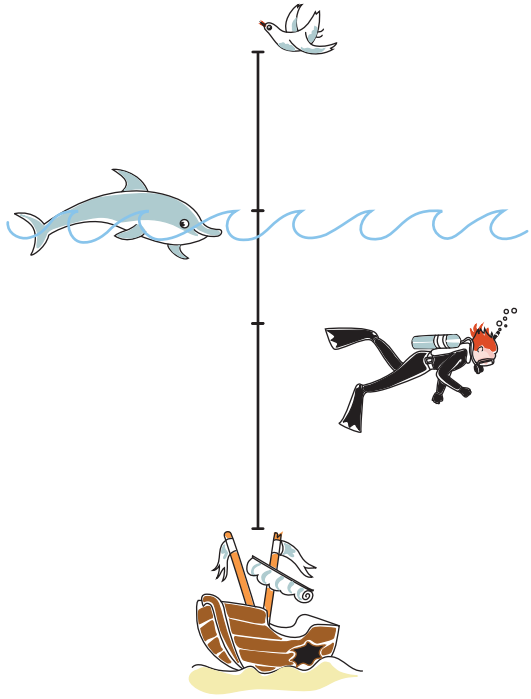
The wreck of a ship is 31 metres below the diver.

A dolphin has just broken the surface of the water.

(b) Use your diagram to determine:

(i) the distance between the bird and the diver

(ii) the distance between the wreck and the dolphin.



7 The maximum and minimum temperatures recorded during 1 week of June on Mount Kosciuszko were:

Day	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Maximum (°C)	11	13	8	6	9	12	13
Minimum (°C)	-3	-1	-7	-5	-4	-1	0

(a) On which day was the lowest minimum recorded?

(b) One which day was the highest minimum recorded?

(c) What was the difference between the maximum and the minimum temperatures on:

(i) Thursday **(ii)** Friday?

(d) On which day was the difference between the maximum and the minimum temperatures the greatest?

8 On Monday, Rachel withdrew $120 from her bank account at an ATM. On Tuesday, she used her bankcard to pay $87 for her shopping from the same account. On Thursday, her employer deposited her salary of $243 into the account, and her mum also deposited $50 for her birthday. On Friday, Rachel used the account to pay a $109 bill online.

(a) Write one long integer calculation that shows each of the events mentioned above as an addition or a subtraction. (Hint: Begin by writing the $120 withdrawal as -120.)

(b) Complete your calculation to determine the following.

(i) Did Rachel have more or less money in her account by the end of the week?

(ii) How much more or less?

9 Puerto Rico Trench is the deepest trench on the floor of the Atlantic Ocean. The bottom of the trench is 8605 metres below sea level. Mount McKinley is the highest mountain in North America at 6194 metres above sea level. If Mount McKinley could be picked up and dropped into the Puerto Rico Trench, what depth of water would be between the top of the mountain and sea level?

Reasoning

10 A magic square is one for which the numbers in every row, column and diagonal add up to the same 'magic' total.

Complete the following magic squares, by first working out the magic total.

(a)

-6		-2
	-3	
		0

(b)

6			-18
	2		8
	0	-8	
12		-2	-12

11 Calculate the following by considering their 'mirror image' journeys on the opposite side of the number line.

(a) -27 + 14 **(b)** -59 + 36 **(c)** -87 + 62

(d) -31 – 29 **(e)** -68 – 43 **(f)** -75 – 58

(g) -47 + (-62) **(h)** -71 – (-26) **(i)** -96 – (+31)

12 **(a)** Complete the following calculations.

(i) -2 + 3 **(ii)** -5 + 11 **(iii)** -8 + 17 **(iv)** -21 + 34

(b) Now, reverse the order of the two numbers in the above additions, and calculate the resulting subtractions (e.g. -4 + 6 becomes 6 + -4, or 6 – 4).

(c) What do you notice? Comment on your observation.

(d) What do we call the property of numbers that enables us to add them or multiply them in any order?

(e) Use your observation from part **(c)** to calculate the following.

(i) -34 + 45 **(ii)** -53 + 69 **(iii)** -72 + 99 **(iv)** -98 + 113

Open-ended

13 The thermometer at the weather station on Mount Wellington read -4°C at 5 a.m. one day in July. By 2 p.m., the temperature had reached the day's maximum of 9°C. Suggest what the thermometer reading might have been at:

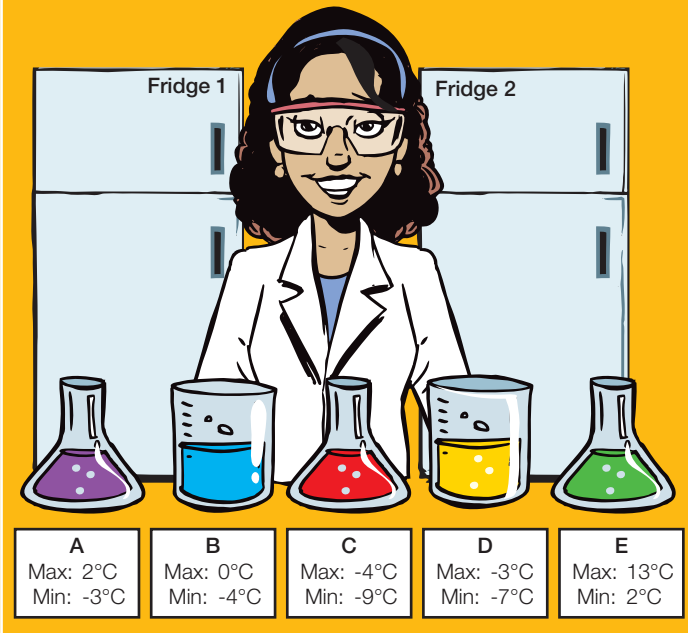
(a) 8.30 a.m. **(b)** midday.

14 List three integers that would give a negative answer when added to -17.

15 List three integers that would give a positive answer when subtracted from -17.

Outside the Square — Problem solving

Lab maths



Professor Berner has two fridges in her laboratory in which she needs to safely store five different chemicals, A, B, C, D and E.

Each chemical has a maximum and a minimum safe temperature at which it can be stored—anything outside this range and it will explode!

Each fridge can be set to any temperature Professor Berner chooses.

Use the information on the labels below to decide the temperature to which Professor Berner needs to set each fridge and which chemicals go in which fridge.

Strategy options

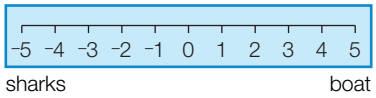
- Guess and check.
- Work backwards.

Investigation

Walking the plank

Equipment required: 1 or 2 brains, 1 large number line from -5 to +5 (optional), 1 die marked with +3, +2, +1, -1, -2, -3 and 1 die marked with +, +, +, – , –, –

You have been captured by a bloodthirsty gang of pirates who have decided you are to walk the plank. However, the pirate captain decides to give you a chance to save yourself. The plank is marked as shown below.



The pirate captain has two dice: an operations die with the sides marked +, +, +, –, –, –, and a number die marked -3, -2, -1, +1, +2, +3. He places you at point 0 and rolls the dice.

Depending on the roll of the dice, you'll have to either move forwards or backwards, according to the following instructions.

If the operations die shows a '+' you turn to face the boat (the positive end of the number line). If it shows a '–' you face the sharks (the negative end).

If the number die shows a positive number, you walk forwards that number of paces in the direction you are facing. If it shows a negative number, you walk backwards that number of paces.

For example, if the captain rolls – +1, you would face the sharks and walk forward one step to -1. If he rolled – -1, you would face the sharks and walk backwards one step to +1.

The captain will keep rolling and you will keep moving until you end up moving past +5 or -5. If you go beyond +5, you are safely back on the boat, but if you go past -5, the sharks will have you.

The Big Question

Is this game fair? How many moves will it take to save you?

Engage

1 Copy the diagram of the plank and, starting at 0, use it to find where you would be if the pirate captain rolled a '+' and a '-3', followed by a '–' and a '-2'.

2 Return to 0. Roll the two dice again, and move accordingly. Keep rolling and moving until you are either in with the sharks or back on the boat. Keep track of how many moves it takes.

Explore

3 (a) A fair game means that both you and the captain have an equal chance of winning. To decide this, make a list of all the possible outcomes of rolling the dice.

(b) To determine a reasonable number of moves in which you could expect to get a result (sharks or boat), you will need to play the game several times. (Take turns being the pirate captain and the 'victim'!)

Strategy options

- Make a table.
- Act it out.

Explain

4 (a) What is the minimum number of moves possible that would bring you to safety?

(b) What was the largest number of moves you needed to complete a game?

Elaborate

5 Complete the following to answer the Big Question.

(a) State whether you and the captain have an equal chance of winning, and explain how you decided.

(b) What is a 'reasonable' number of moves needed to complete the game? How did you decide this?

(c) How often would you expect to get a result in the 'reasonable' number of moves you have chosen?

6 Imagine you are teetering on the edge of the plank at -5. Show how it is possible to still be on -5 after four different rolls of the dice.

Evaluate

7 How confident are you in the answer you gave for a 'reasonable number of moves' in question **5(b)**? What could you do to become more confident?

8 Did playing this game help you become more familiar with the processes for adding and subtracting integers? Could it be useful for someone new to the topic? Explain your answer.

Extend

9 How would you expect your 'reasonable number of moves' to change if the plank extended from -8 to +8? Make a prediction and then test it by playing the game.

Integer multiplication

1.2

If you know your tables, you are familiar with multiplying positive numbers. For example:

$+4 \times +5 = +20$ can be written simply as $4 \times 5 = 20$

What happens when one of these numbers is negative? To help you understand this situation, imagine that you owe a friend \$5. We could write this as -5. If you owed 4 friends \$5 each, you owe 4 lots of -5, which we can write as $-5 + -5 + -5 + -5 = -20$. This means that you would have \$20 less than you did before.

PP Discovery Task

We can also write this as:

$4 \times -5 = -20$, or
$-5 \times 4 = -20$

Multiplying a positive number by a negative number gives a negative result.

What happens when both of the numbers being multiplied are negative? Consider the following number pattern.

$3 \times -5 = -15$
$2 \times -5 = -10$
$1 \times -5 = -5$
$0 \times -5 = 0$
$-1 \times -5 = ?$

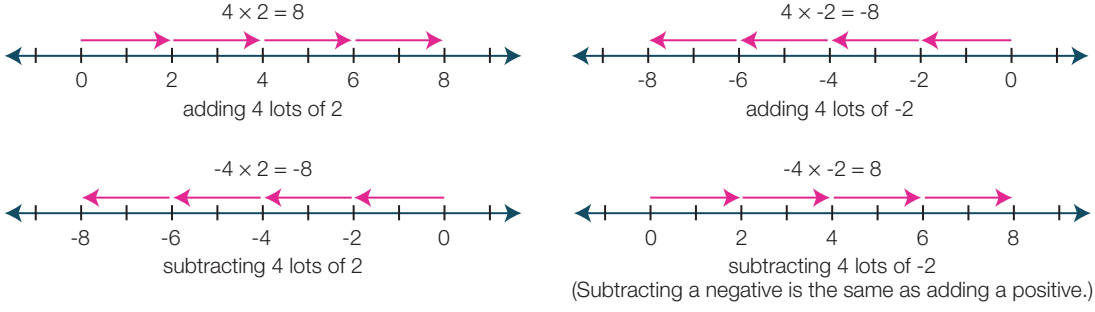
The numbers on the right-hand side of the equals sign are increasing by 5. We need to replace the ? with +5 to continue the pattern.

$-1 \times -5 = +5$
$-2 \times -5 = +10$
$-3 \times -5 = +15$ and so on.

Multiplying a negative number by another negative number gives a positive result.

Using a number line

We can use a number line to demonstrate integer multiplication, if we think of multiplication as repeated addition:



Multiplying two numbers with like signs gives a positive answer.
Multiplying two numbers with unlike signs gives a negative answer.
This can be summarised as:

$+ \times + = +$ $\quad$ $+ \times - = -$
$- \times - = +$ $\quad$ $- \times + = -$

Worked Example 3

WE3

Calculate:

(a) -5×-7 **(b)** $-6 \times +9$

Thinking	Working
(a) Perform the multiplication as though both numbers are positive ($5 \times 7 = 35$), then determine the sign of the answer ($- \times - = +$). If the final answer is positive you can leave out the + sign.	(a) $-5 \times -7 = 35$
(b) Perform the multiplication as though both numbers are positive ($6 \times 9 = 54$), then determine the sign of the answer ($- \times + = -$).	(b) $-6 \times +9 = -54$

When finding the product of more than two numbers we just extend the process already developed.

Worked Example 4

WE4

Calculate: $+4 \times -3 \times -1$

Thinking	Working
1 Multiply the first two numbers, placing the correct sign in front of the answer. (Here, $+ \times - = -$)	$+4 \times -3 \times -1$ $= -12 \times -1$
2 Multiply the product of the first two numbers by the third number, again ensuring the correct sign is in front of the answer. (Here, $- \times - = +$, so we can leave the sign off the final answer.)	$= 12$

Just as multiplying a number by 1 does not change the size of a number, multiplying by -1 also does not change the size of a number. However, it does change the sign of the number, as shown in the Worked Example above.

Squaring integers—the difference between $(-3)^2$ and -3^2

We 'square' a number by multiplying it by itself; for example, $5^2 = 5 \times 5 = 25$.

When we square a negative number, we need to place brackets around the number and the sign to avoid confusion.

$(-3)^2$ is read as 'the square of -3'.
$(-3)^2 = -3 \times -3 = +9$.

-3^2 is different to $(-3)^2$. Without the brackets, -3^2 is read as 'the negative of 3 squared'.
$-3^2 = -(3 \times 3) = -9$.

1.2 Integer multiplication

Navigator

Q1 Columns 1 & 2, Q2 Column 1, Q3, Q5 Columns 1 & 2, Q6, Q7 (a)–(d), Q8, Q9, Q11	Q1 Columns 2 & 3, Q2 Column 2, Q3, Q4, Q5 Columns 1–3, Q6, Q7 Column 1, Q8, Q9, Q10, Q11	Q1 Column 4, Q2 Columns 2 & 3, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12

Answers page 592

Fluency

1 Calculate: WE3

(a) $+6 \times +5$ **(b)** $+7 \times +3$ **(c)** $+11 \times +7$ **(d)** $+6 \times +9$

(e) $+8 \times -5$ **(f)** $+12 \times -2$ **(g)** $+9 \times -3$ **(h)** $+5 \times -7$

(i) $-2 \times +4$ **(j)** $-2 \times +8$ **(k)** $-5 \times +5$ **(l)** $-4 \times +4$

(m) -6×-5 **(n)** -2×-4 **(o)** -3×-2 **(p)** -5×-3

(q) -12×5 **(r)** 7×-20 **(s)** 9×-10 **(t)** -11×-12

2 Calculate: WE4

(a) $-2 \times -4 \times 3$ **(b)** $-1 \times 3 \times -4$ **(c)** $3 \times -5 \times -1$

(d) $4 \times 5 \times -2$ **(e)** $-9 \times 1 \times 2$ **(f)** $8 \times -1 \times 3$

(g) $-2 \times -5 \times -5$ **(h)** $-5 \times -2 \times -2$ **(i)** $-8 \times -1 \times -4$

(j) $-3 \times -3 \times -3$ **(k)** $-2 \times -2 \times -2$ **(l)** $-10 \times -10 \times -10$

3 Copy and complete by following the pattern.

(a)
$3 \times 2 = 6$
$3 \times 1 = 3$
$3 \times 0 = 0$
$3 \times -1 =$ _____
$3 \times -2 =$ _____
$3 \times -3 =$ _____

(b)
$-2 \times 3 = -6$
$-2 \times 2 = -4$
$-2 \times 1 = -2$
$-2 \times 0 =$ _____
$-2 \times -1 =$ _____
$-2 \times -2 =$ _____

(c)
$-2 \times -2 = 4$
$-2 \times -1 = 2$
$-2 \times 0 = 0$
$-2 \times 1 =$ _____
$-2 \times 2 =$ _____
$-2 \times 3 =$ _____

4 **(a)** $2 \times -2 \times 2 \times -2$ equals:

A -16 **B** 0 **C** 8 **D** 16

(b) $4 \times -3 \times -3 \times 2$ equals:

A -72 **B** -12 **C** 12 **D** 72

5 Calculate:

(a) $(-6)^2$ **(b)** -6^2 **(c)** $(-10)^2$ **(d)** -10^2

(e) $2 \times (-5)^2$ **(f)** $-5^2 \times -3$ **(g)** 4×-10^2 **(h)** $(-10)^2 \times -7$

(i) $(-2)^2 \times (-2)^2$ **(j)** $-2^2 \times (-5)^2$ **(k)** $-3^2 \times -3^2$ **(l)** $(-8)^2 \times -2^2$

Understanding

6 An electronic goods store is selling off old models of its DVD and CD players. To clear this stock, they are selling them below 'cost price' (the price the store bought them for).

DVD player: cost price \$56, sale price \$49
CD player: cost price \$45, sale price \$39

For the following questions, write your answers as negative integers.

(a) Calculate the loss the store will make on:

(i) a DVD player, and

(ii) a CD player, by finding the difference between the cost and sale prices.

(b) The store sold 23 DVD players. Calculate the loss made.

(c) The store sold 19 CD players. Calculate the loss made.

(d) Calculate the total loss made by the store on these goods.

7 In the card game 'Krummy' each player's final score is obtained by counting up the number of cards of each suit, and allocating points as shown in the table.

Card	Score
Hearts	-5 points each
Diamonds	-3 points each
Clubs	+2 points each
Spades	+4 points each

Find the score of a player who finishes with:

(a) 3 diamonds

(b) 7 hearts

(c) 5 hearts, 6 clubs

(d) 6 diamonds, 9 spades

(e) 4 hearts, 5 clubs, 3 spades

(f) 3 hearts, 3 diamonds, 8 clubs

(g) 6 hearts, 2 diamonds, 4 spades

(h) 4 hearts, 4 diamonds, 5 clubs, 3 spades

Reasoning

8 **(a)** Calculate the following.

(i) -1×-1

(ii) $-1 \times -1 \times -1$

(iii) $-1 \times -1 \times -1 \times -1$

(iv) $-1 \times -1 \times -1 \times -1 \times -1$

(v) $-1 \times -1 \times -1 \times -1 \times -1 \times -1$

(vi) $-1 \times -1 \times -1 \times -1 \times -1 \times -1 \times -1$

(b) Copy and complete.

In a multiplication chain involving directed numbers, the answer is positive if there is an _____ number of negative values and negative if there is an _____ number of negative values.

(c) Use the above result to evaluate the following.

(i) eighty-six -1s multiplied together

(ii) ninety-nine -1s multiplied together

(iii) $(-1)^{37}$

(iv) $(-1)^{50}$

9 Write the following in expanded form, then use your result from Question **8(b)** to evaluate them. (Hint: The expanded form of $(-4)^3$ is $-4 \times -4 \times -4$.)

(a) $(-4)^3$ **(b)** $(-3)^4$ **(c)** $(-2)^5$ **(d)** $-(-6)^2$

10 Sandra pays her gas and electricity bills by 'direct debit'. An amount is automatically taken out of her bank account each month by the energy suppliers. Sandra believes this method of payment is better than receiving a large bill every 3 months.

Every month, \$25 for gas and \$35 for electricity is 'debited' from Sandra's account.

(a) Calculate the amount that will have been debited after 3 months for **(i)** gas and **(ii)** electricity. Write your answers as negative integers.

(b) Sandra receives a statement from her gas and electricity companies after 3 months, telling her the cost of the energy she has actually used.

Her statements are as follows: Gas \$93, Electricity \$102.

(i) Calculate the difference between the amounts on Sandra's statements and your answers to part **(a)**.

(ii) Should Sandra adjust her monthly direct debit amounts for her gas or electricity bills? Give reasons for your answer.

Open-ended

11 Copy and complete. Give at least three different answers.

_____ × _____ × _____ = -24

12 Kemahl is playing the card game Krummy (see Question **7**). At the end of a round, Kemahl has -22 points and cards from three of the four suits in his hand. What could the cards be? Give two possible combinations, stating the number of cards of each suit.

Outside the Square Puzzle

The 1 dilemma

Draw a 3 × 3 grid into your exercise book. Then, arrange five 1s and four -1s in the grid so that the product of every row, column and diagonal is 1.

Mathspace

CRYOGENIC CRISIS

While cruising the outer reaches of the galaxy, you have been captured by a scary looking trio of aliens who bring you to their home planet, Zush.

They have placed you in a vat of a mysterious liquid, which is freezing cold at -20°C. If it gets to -40°C you will be frozen alive.

The aliens challenge you to an 'integers duel' to determine your fate. You have no choice but to accept.

EQUIPMENT REQUIRED:

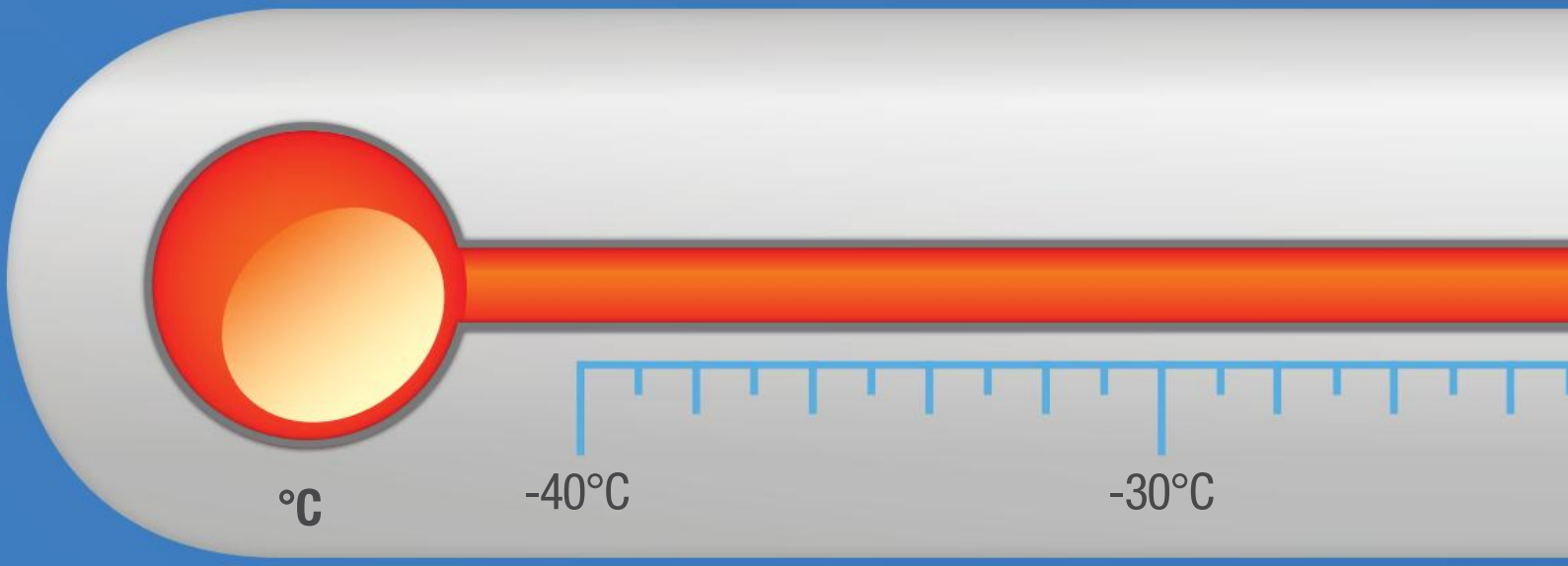
2 brains, 1 counter (or something to use as a marker), 1 die, 1 coin

HOW TO PLAY:

- Place the marker on the thermometer at -20°C.
- Decide who will be the victim (the person in the vat) and who will be the alien.
- Roll the die to decide who goes first (higher number first).

On your turn:

- Toss the coin to determine the operation.
  - If you toss a head you add (+).
  - If you toss a tail you subtract (—).
- To determine the number that you will be adding or subtracting, roll the die.
- Find the number you have rolled in the following table.



Die roll	Choose from these numbers	Die roll	Choose from these numbers
⚀	-1, -2, 5	⚅	0, 6, -2
⚁	2, 1, -6	⚄	1, -3, -6
⚂	3, 0, -4	⚃	5, -1, 4

- Choose any two numbers from the list next to your die roll and find their product (multiply them together).
- Take the product and, depending on your coin toss, either add it to, or subtract it from, the number the marker is sitting on.
- When you have calculated the answer, move the marker up or down the temperature scale to this number.

Sample move:

'Alien' player goes first and tosses a head, so they add. They roll a ⚂, and choose -4 and 3.

$-4 \times 3 = -12$

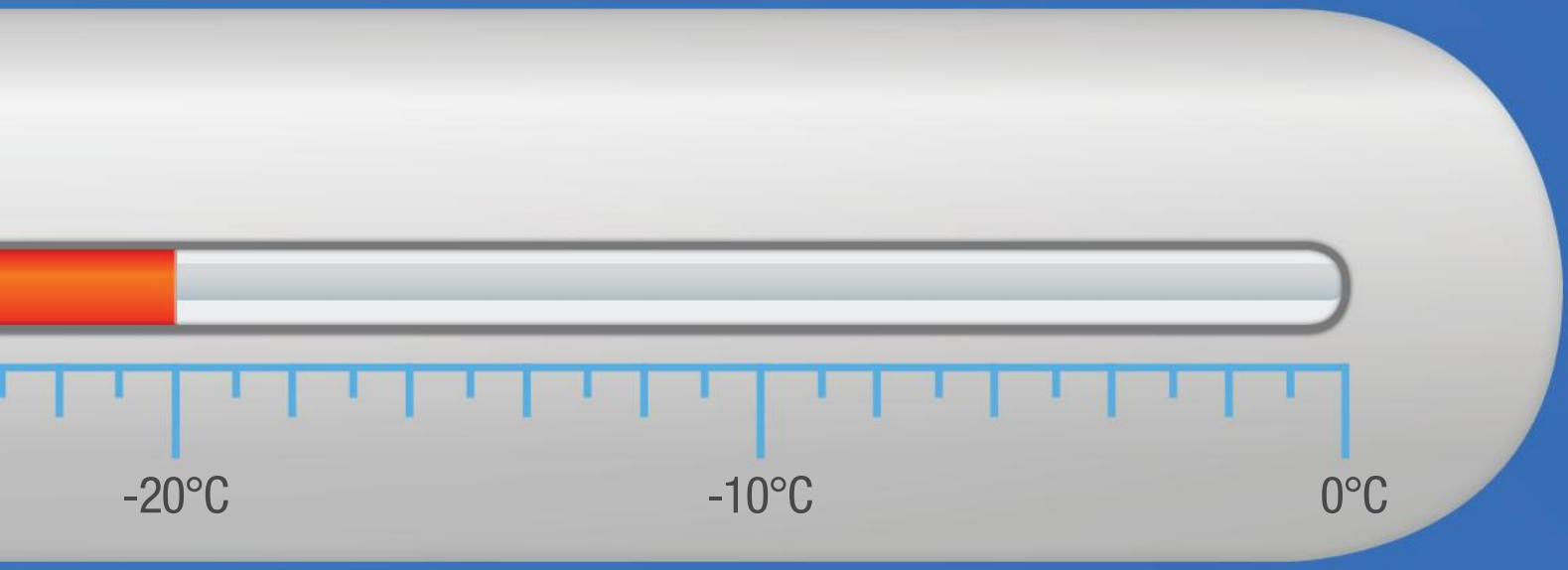
$-20 + (-12) = -32$

They move the marker down to -32°C.

HOW TO WIN:

The victim wins if they can move the marker up to or beyond 0°C.

The alien wins if they can move the marker down to or beyond -40°C.



1.3 Integer division

Division is the inverse operation to multiplication.

Because we know that $4 \times 3 = 12$, we also know that $12 \div 3 = 4$, and $12 \div 4 = 3$.

Similarly, if we know that $-2 \times 3 = -6$, we also know that $-6 \div -2 = 3$, and $-6 \div 3 = -2$.

The same rules that apply to multiplying negative numbers also apply to dividing negative numbers.

Dividing two numbers with 'like' (same) signs gives a positive answer.

$+ \div + = +$ $\qquad$ $- \div - = +$

Dividing two numbers with 'unlike' (different) signs gives a negative answer.

$+ \div - = -$ $\qquad$ $- \div + = -$

Worked Example 5

WE 5

Calculate:

(a) $-56 \div -7$ $\qquad$ **(b)** $20 \div -4$

Thinking	Working
(a) Perform the division as though both numbers are positive ($56 \div 7 = 8$), then determine the sign of the answer ($- \div - = +$). If an answer is positive, leave out the '+' sign.	(a) $-56 \div -7 = 8$
(b) Perform the division as though both numbers are positive ($20 \div 4 = 5$), then determine the sign of the answer ($+ \div - = -$).	(b) $20 \div -4 = -5$

Division calculations can also be written as fractions. For example, $-60 \div 4$ can be written as $\frac{-60}{4}$. In this case, as the result of the division will be negative, we can place the sign in front of the fraction; i.e. $-\frac{60}{4}$. This is also the case if the denominator is the negative number; i.e. $\frac{60}{-4} = -\frac{60}{4}$.

1.3 Integer division

Navigator

Q1 Columns 1 & 2, Q2 Columns 1 & 2, Q3, Q4, Q5, Q7, Q8, Q10	Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3, Q4, Q5, Q6, Q7, Q8, Q10	Q1 Column 4, Q2 Columns 3 & 4, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11

Answers page 593

Fluency

WE 5

1 Calculate:

(a) 9 ÷ -3 **(b)** 6 ÷ -2 **(c)** 20 ÷ -5 **(d)** 21 ÷ -7

(e) -8 ÷ 4 **(f)** -30 ÷ 5 **(g)** -54 ÷ 6 **(h)** -32 ÷ 4

(i) -12 ÷ -3 **(j)** -18 ÷ -2 **(k)** -49 ÷ -7 **(l)** -48 ÷ -8

(m) -63 ÷ -9 **(n)** 45 ÷ -5 **(o)** -16 ÷ 4 **(p)** 70 ÷ -7

(q) -240 ÷ 8 **(r)** -600 ÷ 6 **(s)** 240 ÷ -12 **(t)** -2300 ÷ -10

2 Calculate the following.

(a) $\frac{48}{-3}$ **(b)** $\frac{-77}{11}$ **(c)** $\frac{-65}{5}$ **(d)** $\frac{120}{-8}$

(e) $\frac{52}{-2}$ **(f)** $\frac{-93}{-3}$ **(g)** $\frac{-84}{-4}$ **(h)** $\frac{-96}{4}$

(i) $\frac{-90}{-6}$ **(j)** $\frac{100}{-5}$ **(k)** $\frac{-360}{40}$ **(l)** $\frac{-240}{-80}$

3 **(a)** $\frac{-28}{-4}$ equals:

A -32 **B** -24 **C** -7 **D** 7

(b) 162 ÷ -3 equals:

A -159 **B** -54 **C** 54 **D** 159

Understanding

4 A department store has 4 floors below ground level and 11 floors above ground. A shopper on the top level takes a lift to the bottom level, stopping every 3 floors to look around.

(a) Draw a diagram to represent the situation, clearly showing the lift and floor numbers.

(b) How many times will the shopper stop on his way to the lowest floor?

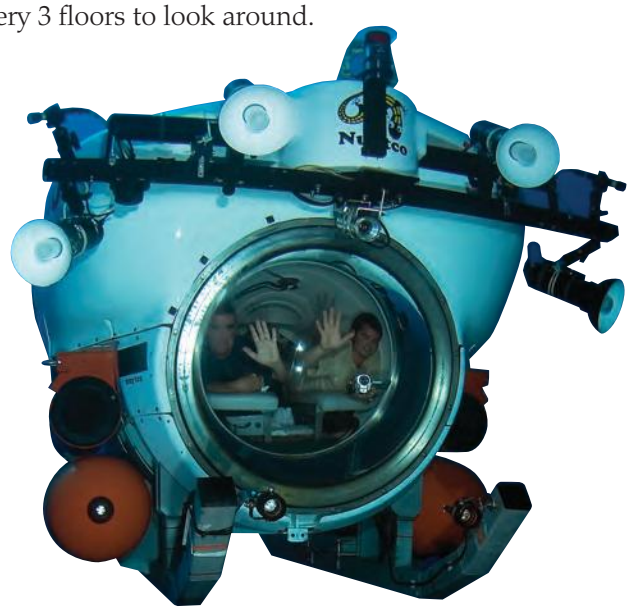
(c) List the floor numbers the shopper stops at.

5 A group of 6 people loses $240 trying to win a lottery. Write each person's share of the loss as a negative integer.

6 A submersible begins at the ocean surface and is lowered 6 metres every minute. If the ocean is 96 metres deep in this area, calculate:

(a) the time taken for the submersible to reach the ocean floor

(b) the time taken to rise to the surface if the submersible rises at a rate of 2 metres per minute.



7 The temperature falls from 24°C to -4°C between the hours of 7 p.m. and 2 a.m. one night in the Central Australian desert. Find the average hourly temperature change by dividing the overall change by the number of hours. Write your answer as a negative integer.

Reasoning

8 Beau has bought a home theatre system and some new furniture from a store offering a 'buy now, pay later' scheme. The total cost of the home theatre system and furniture was $8600. Beau plans to pay $200 every month.

(a) How many months will it take Beau to pay off his debt?

(b) How many years and months is this?

(c) After 1 year, the store sends Beau a statement showing how much he still owes. Write a negative integer to show this amount.

9 You are playing a game that involves rolling two dice.

The green die shows:	+24	-36	+12	-48	-60	+72
The blue die shows:	-1	+2	-3	+4	-6	+12

To find out the score obtained on your roll you need to divide the value showing on the green die by the value showing on the blue die.

Make a list, in order from smallest to largest, of all the possible scores you can obtain.

Open-ended

10 Write three numbers that when divided by 8 give a negative integer as the answer.

11 $\square \div \square > -1$

Place a different integer from the following list in each box to make the above statement true. Make at least three different true statements. (Remember, '>' means 'is greater than'.)

-2, 3, -4, 5, 6, -7, -8

Outside the Square Game

3 in a row

Equipment required: 2 brains, 1 pack of cards with the picture cards (Jacks, Queens, Kings) removed, about 10 coloured counters each (or 10 small pieces of paper with the player's initials on them)

How to win:
The first player to get three of their counters in a row (horizontally, vertically, or diagonally) is the winner.

How to play:
In this game, red card (Hearts and Diamonds) values are negative, whereas black card (Spades and Clubs) values are positive.

Shuffle the cards and place them face down on the table. Each player draws one card from the top of the pack. The player with the higher value card goes first. (Aces are worth 1 point.)

On their turn, a player turns over the top two cards from the deck and places them face up in front of themselves. They can either add, subtract, multiply or divide the two values on the cards. They say the calculation, and the answer, out loud, then (if they have stated it correctly), they may place a counter over the square containing the answer to the calculation.

If you have used all the cards and have not yet found the winner, reshuffle the pack and continue.

-2	-7	-16	-2	6	-1	12	3
8	10	1	-12	-4	18	5	-7
36	2	-24	6	0	-8	-3	14
-4	30	7	3	16	-9	2	-5
-6	-16	-10	5	-1	-24	4	8
24	4	0	-18	-36	7	-12	1
15	9	-3	10	-2	-30	11	16
12	-5	-8	13	24	-6	-10	-4

Half-time 1

Ex.1.1

1 Simplify the following by writing a single sign between the two numbers, then calculate the answer.

(a) 9 – (+6) **(b)** 14 – (+21) **(c)** -9 – (+8) **(d)** -31 – (+27)

(e) -8 + (-8) **(f)** -14 + (-20) **(g)** 7 – (-10) **(h)** -35 – (-21)

Ex.1.2

2 Calculate:

(a) -4×6 **(b)** -7×-8 **(c)** 10×-14 **(d)** -2×-12

(e) $(-3)^2$ **(f)** -5^2 **(g)** -2^3 **(h)** $(-4)^3$

Ex.1.1

3 The temperature at 6 a.m. on a cold winter's morning in Moscow was -11°C. By midday, the temperature had risen to 3°C. By 6 p.m., the temperature had dropped to -5°C.

(a) Write the increase in temperature from 6 a.m. to midday as a positive integer.

(b) Write the decrease in temperature from midday to 6 p.m. as a negative integer.

Ex.1.3

4 Calculate:

(a) $-40 \div -8$ **(b)** $-63 \div 7$ **(c)** $55 \div -11$ **(d)** $-7 \div -1$

(e) $\frac{-32}{4}$ **(f)** $\frac{-72}{9}$ **(g)** $\frac{-36}{-12}$ **(h)** $\frac{-60}{-15}$

Ex.1.1

5 Write the following sets of integers in ascending order (smallest to largest).

(a) -37, 7, -30, 0, -3, 3 **(b)** 54, -20, -1, -5, -40

Ex.1.1

6 Calculate the following.

(a) -3 + 4 **(b)** -9 + 7 **(c)** 4 – 5 **(d)** -8 – 6

(e) +4 – (+2) + (-5) **(f)** -7 + (-8) – (-10) **(g)** -9 + 5 – 7 **(h)** 18 – 25 + 11

Ex.1.2

7 Cheryl spends $5 every week buying a lottery ticket. After 16 weeks, she won a $50 prize.

(a) How much had Cheryl spent on tickets at the end of the 16-week period?

(b) Write the overall amount Cheryl had gained or lost at the end of the same period as a positive or a negative integer.

Ex.1.1

8 Write an integer to represent each of the following situations.

(a) a withdrawal of $120 from a bank account

(b) an increase of 7 cm in the height of a child

Ex.1.3

9 A group of 4 friends each invested $30 000 to start up a business. At the end of 1 year, the business made a profit of $80 000.

(a) What was the share of the profit for each friend?

(b) Considering their initial investment, write the overall profit or loss made by each friend as a positive or a negative number.

Ex.1.1

10 Calculate the following by considering the 'mirror image' of the calculation on the positive side of the number line.

(a) -67 + 45 **(b)** -83 + 57 **(c)** -39 – (+24) **(d)** -19 – (-31)

Combined operations with integers

1.4

The order of operations for working with negative integers is the same as the one we use when working with positive integers.

The order of operations rules:

- Perform any calculation inside brackets first.
- Then evaluate any numbers in index form (powers and roots).
- Then perform division and multiplication in the order they appear, moving from left to right.
- Then perform addition and subtraction in the order they appear, moving from left to right.

Worked Example 6

WE 6

Evaluate:

(a) $-7 \times 4 \div -2 \times -1$

(b) $-3 \times (-2 + -6) - 4$

Thinking	Working
(a) As there is only multiplication and division involved, work from left to right.	(a) $-7 \times 4 \div -2 \times -1$ $= -28 \div -2 \times -1$ $= 14 \times -1$ $= -14$
(b) **1** Evaluate brackets first. If the answer is negative, keep it inside the brackets to avoid confusion.	(b) $-3 \times (-2 + -6) - 4$ $= -3 \times (-8) - 4$
2 Perform multiplication and division as you move from left to right.	$= 24 - 4$
3 Perform subtraction and addition as you move from left to right.	$= 20$

Using the '-' sign

We have seen that the '-' sign can be used to indicate subtraction and also to indicate a negative number. To avoid any confusion, we indicate a negative number by placing brackets around the number and the sign wherever necessary. This is very important when applying the order of operations, as any powers must be calculated before subtracting or adding, and squaring a negative number changes its sign.

For example: $5 + (-4)^2 = 5 + 16$
$= 21$

$5 - 4^2 = 5 - 16$
$= -11$

Worked Example 7 WE7

Evaluate:

(a) $6 - 3^2 + 7 \times -2$ **(b)** $9 + (-4)^2 \times (7 - 2)$

Thinking	Working
(a) 1 As there are no calculations in brackets, calculate indices first.	(a) $6 - 3^2 + 7 \times -2$ $= 6 - 9 + 7 \times -2$
2 Next, multiply and divide from left to right.	$= 6 - 9 + -14$
3 Simplify any addition or subtraction of directed numbers and perform it from left to right.	$= 6 - 9 - 14$ $= -3 - 14$ $= -17$
(b) 1 Perform the calculation in brackets first.	(b) $9 + (-4)^2 \times (7 - 2)$ $= 9 + (-4)^2 \times 5$
2 Evaluate numbers in index form.	$= 9 + 16 \times 5$
3 Next, multiply and divide from left to right.	$= 9 + 80$
4 Perform addition and subtraction from left to right.	$= 89$

Calculators and integers

To show a negative number on the calculator, press the (–) key before entering the number. On some calculators, the key looks like this: +/–. Pressing this key repeatedly changes the sign of an entered number from positive to negative and back again.

You should also locate and know how to use the 'open bracket' and 'close bracket' keys, (and), the 'square root' key, √, and the 'square' key, x^2. If your calculator does not have a key like this, press ^ 2. All but the most basic calculators have the order of operations built into them, so we can enter the numbers and operations as they are written on the page, making sure we enter brackets in the correct positions.

1.4 Combined operations with integers

Navigator

Answers page 594

Q1 Columns 1 & 2, Q2 Column 1, Q3 Column 1, Q4, Q6, Q7, Q9, Q11	Q1 Columns 2 & 3, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q8, Q9, Q10, Q11	Q1 Column 3, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q12

Equipment required: Calculator for Questions **3** and **11**

Fluency

1 Evaluate: WE6

(a) $45 \div -9 \times -2$
(b) $-32 \div -8 \times 5$
(c) $56 \div {}^{-}7 \div -2$
(d) $4 + (6 \times -3) - 2$
(e) $-2 + (-64 \div -8)$
(f) $-5 \times (-7 - 4)$
(g) $7 + (14 \div -2) - 3$
(h) $44 \div (-12 + 1)$
(i) $(-6 \times 5) + (-24 \div -6)$
(j) $(-8 + 2) \times (4 - 10)$
(k) $(12 \times -3) \div (-2 - 2)$
(l) $(-5 \times -9) - (7 - 10)$

2 Evaluate: WE7

(a) $20 - 5^2 + 10 \div -2$
(b) $7 + (-4)^2 \times (3 - 5)$
(c) $30 \div -6 \times 5 + 5^2$
(d) $5 \times 8 \div -2 + (-8)^2$
(e) $(7 - 10) \times (20 \div \sqrt{4})$
(f) $(18 - \sqrt{16}) \times -2 + 4$
(g) $\sqrt{100} - 5^2 \times (1 - 3)$
(h) $(\sqrt{81} - \sqrt{36}) \times (-10)^2$
(i) $\dfrac{-4^2}{2} - 3 \times -6$
(j) $\dfrac{(17 - 5)}{-2} \times -3 + 4$

3 Evaluate the following. You may like to use a calculator to check your answers.

(a) $81 \div (9 - 18) + (16 - 25)$
(b) $-27 - (42 - 91) + 17$
(c) $40 \div (3 - 8) + 7^2$
(d) $-19 + 10 \times -3 + (-8)^2$
(e) $(-37 + 21)^2 - \sqrt{169}$
(f) $-14 \times (-6)^2 \div -2$
(g) $-(27 - 42)^2 \div -5$
(h) $(57 - 4^2) \times -3 - 21$
(i) $-18 \times -5 \div 3^2 - 54$
(j) $-18 \times -5 \div (3^2 - 54)$

4 (a) The expression $-4 - (2 \times -3) + 6$ equals:

A -2 B -12 C 8 D 24

(b) $5 + 6 \times (-3) + (-4)^2$ is equal to:

A 3 B -17 C -29 D -49

Understanding

5 Chen has \$250 in his bank account at the start of the month. Over the course of the month, he makes the following transactions.

Week 1: three \$50 withdrawals
Week 2: two \$80 withdrawals
Week 3: one \$100 deposit
Week 4: one \$100 deposit, one \$80 withdrawal

Chen — Bank Statement
STATEMENT PERIOD: **1/10/10–30/10/10**

Account Transactions

Transaction Details	Withdrawal	Deposit	Balance
Opening balance			**\$250.00**
Week 1	\$50.00		
	\$50.00		
	\$50.00		
Week 2	\$80.00		
	\$80.00		
Week 3		\$100.00	
Week 4	\$80.00	\$100.00	
Closing Balance			

(a) How much did Chen have in his account at the end of Week 1? Use numbers, brackets (if necessary), and the operation symbols (×, ÷, +, –) to write an expression that shows how to calculate this amount.

(b) How much did Chen have in his account at the end of Week 3? Use numbers, brackets (if necessary), and the operation symbols to write an expression that shows how to calculate this amount.

(c) During the month, Chen got a letter from his bank saying he had 'overdrawn' on his account—he now owed the bank! After which withdrawal would Chen have received this letter? How much did he owe the bank?

(d) What was Chen's final closing balance at the end of Week 4?

6 A company made a profit of $3 million per month for 7 months, then lost $9 million per month for 5 months. What was their result for the year?

7 Jade places some water at 24°C in the freezer. The freezer is able to lower the water's temperature by 4°C per hour. What will be the temperature of the water after:

(a) 3 hours

(b) 6 hours

(c) 7 hours

(d) 10 hours?

8 A round of golf consists of 18 holes of a course. Each hole has an expected number of shots needed to get the ball in the hole. This is called 'par' for the hole and has a score of zero. If you take one, two or three shots less than par, or 'under par', you score -1, -2 or -3, respectively. If you take one, two or three shots more than par, or 'over par', you score +1, +2 or +3, respectively.

Polly plays a round of golf at a course where 'par' for the whole course is 72 shots. Polly has the following results on her round.

- 4 holes at 1 under par
- 1 hole at 2 under par
- 5 holes at par
- 7 holes at 1 over par
- 1 'disastrous' hole of 10 over par.

(a) Write a single calculation that Polly can use to determine her finishing score.

(b) **(i)** What was Polly's finishing score?

(ii) How many shots over or under par for the course is this?

(c) **(i)** Polly has a handicap of 14, which means that 14 is subtracted from her finishing score to determine her official score. What was Polly's official score, taking into account her handicap?

(ii) How many shots over or under par for the course is this?

Reasoning

9 Evaluate the following.

(a) 19 – 7 – 15 + 14

(b) 19 – (7 – 15) + 14

(c) Explain why the presence of the brackets in **(b)** gives a very different answer to **(a)**.

10 Two of the following number sentences contain a pair of brackets that is unnecessary. Find them and rewrite the expression without them, checking that the same answer is still achieved.

(a) $8 + (16 - 23) - 9 = -8$

(b) $14 - (8 - 19) - 10 = 15$

(c) $(11 \times -8) + (24 \div -2) = -100$

(d) $(5 - 2) \times 4 - (3 - 7) = 16$

Open-ended

11 **(a)** Use your calculator to evaluate $(-15 + 11) \times -2 + (-4)^2$, and write down your answer.

(b) Re-enter the expression into your calculator, leaving out all of the brackets.

(c) Explain how your calculator gave you two different answers, given that it follows the rules for the order of operations.

12 Copy and complete. (Different answers are possible.)

$-3 - (_____ \div _____) = 5$

Outside the Square Puzzle

Animal speed challenge

Equipment required: 1 brain, 1 stopwatch

Time yourself to see how quickly you can complete each of the following chains of calculations.

Check your answers with some classmates.

Look at the scoring scale below to see which animal you are!

1 Start with 4 → Multiply by 7 → Double it → Divide by -8 → Subtract 10 → Add 3 → Multiply by 2

2 Start with 3 → Cube it → Divide by 9 → Add -6 → Multiply by 4 → Add 20 → Add 10

3 Start with 10 → Add 17 → Multiply by 2 → Subtract -10 → Divide by -8 → Square it → Add 6

How did you go?
Less than 30 seconds: Champion Cheetah
30 seconds to 1 minute: Galloping Giraffe
1 minute to 2 minutes: Zippy Zebra
More than 2 minutes: Leisurely Lion

Now, write one of your own for a friend to complete. Make sure it has at least seven steps. Also, make sure that you are able to complete it yourself, first!

Maths 4 Real

The ultimate cool

How cold can things get? The lowest temperature ever recorded on Earth was -89°C at the Russian research station Vostok in Antarctica. By comparison, the lowest temperature in the northern hemisphere, recorded in Oimekon, Siberia, was a mere -78°C.

The average Antarctic temperatures for different seasons are shown in the following table.

Season	Inland temperature (°C)	Coastal temperature (°C)
Autumn	-58	-9
Winter	-60	-12
Spring	-48	-8
Summer	-32	-2

At these sorts of inland temperatures, it becomes painful to breathe because the moisture on the tiny hairs inside your nose freezes instantly. However, other parts of the universe can get even colder. The coldest places in our Solar System are the bottom of the craters at the south pole of the Moon. Here, the temperature has been measured at -240°C. Parts of deep space, at -270°C, have the lowest temperature of anything in nature.

1 How much warmer is:

(a) the lowest temperature ever recorded in the northern hemisphere compared to the lowest temperature recorded in Antarctica?

(b) the average winter inland temperature in Antarctica compared to the temperature at the bottom of the craters at the south pole of the Moon?

2 What is the difference between the average coastal and inland temperatures in Antarctica in:

(a) autumn **(b)** winter **(c)** spring **(d)** summer?

What is the 'ultimate' cool? Scientists have long been interested in finding the lowest temperature possible. In early 2001, scientists at the Australian National University produced the coldest known substance for the first time in Australia. It was a super-cold cloud of atoms reaching very close to -273°C. To create the cloud, the temperature of one million atoms was lowered using laser beams.

Theoretically, the coldest temperature possible is -273°C, which is also called 'absolute zero' on the Kelvin temperature scale. This is where atoms reach their lowest possible level of movement and even air is frozen solid. The Australian scientists didn't quite get there. They were about 100 billionths of a degree above absolute zero.

3 A United Nations report predicted that the rising levels of greenhouse gases in the atmosphere will cause the average global surface temperature to rise between 1°C and 6°C by the end of the century. How would this affect Antarctic temperatures? For each season, show what would happen to the average inland temperatures if the temperature rose by:

(a) 1°C

(b) 6°C

4 Look at the following table of temperatures.

Temperature (°C)	Temperature fact
+5600	Temperature at the Sun's surface
+1064	Gold melts
+100	Boiling point of water (at sea level)
+58	Highest recorded temperature on Earth
+37	Human body temperature
0	Water freezes
-37	Car antifreeze freezes
-89	Lowest recorded temperature on Earth
-273	Absolute zero

Find the difference between:

(a) human body temperature and the temperature at which car antifreeze freezes

(b) the temperature of the Sun's surface and absolute zero

(c) the temperature at which gold melts and water freezes

(d) the highest and lowest temperatures recorded on Earth.

Research

Present a report on other temperature scales, such as the Fahrenheit and Kelvin scales. Describe how to convert these to degrees Celsius. Explain why -273°C is called absolute zero.

1.5 Multiplying and dividing numbers in index form

Index form

We **square** a number by multiplying it by itself. We **cube** a number by multiplying it by itself, then multiplying it by itself again.

For example: '4 squared' $= 4 \times 4 = 4^2$ '4 cubed' $= 4 \times 4 \times 4 = 4^3$

We have a shorthand way of writing the multiplication, and can extend it to any number of times that the number is multiplied by itself. A smaller number called the **index** (or **power**, or **exponent**) is written to the top right of the number. The index tells us how many times the number (called the **base**) appears in the multiplication; for example, $4^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4$.

Numbers written in this way are said to be written in **index form**. The plural of index is **indices**.

index or power or exponent

$$4^3 = 4 \times 4 \times 4 = 64$$

base

index form — **expanded form**

Two or more different base numbers must be written individually as numbers in index form, as shown in part **(b)** of the following Worked Example.

Worked Example 8 WE 8

Write the following in index form.

(a) $7 \times 7 \times 7 \times 7$ **(b)** $2 \times 2 \times 2 \times 5 \times 5$

Thinking	Working
(a) Count the number of times the base appears in the string of multiplications and use this number as the index. (Here, there are four 7s multiplied together, so 4 is the index.)	(a) $7 \times 7 \times 7 \times 7$ $= 7^4$
(b) Count the number of times each base appears in the string of multiplications and use these numbers as the index of each base. (Here, we have the product of three 2s and two 5s.)	(b) $2 \times 2 \times 2 \times 5 \times 5$ $= 2^3 \times 5^2$

Expanded form

Numbers in index form can be written in expanded form by writing a string of multiplications equal to the number of the index. Index numbers with different bases must be treated separately.

Worked Example 9 WE9

Write the following in expanded form.

(a) 8^5 **(b)** $3^2 \times 4^3$

Thinking	Working
(a) Use the index to determine the number of times the base will appear in the string of multiplications. (Here, an index of 5 means there are five 8s multiplied together.)	(a) 8^5 $= 8 \times 8 \times 8 \times 8 \times 8$
(b) Use the index of each base to determine the number of times each base will appear in the string of multiplications. (Here, the index of 3 is 2 and the index of 4 is 3, so we have two 3s and three 4s multiplied together.)	(b) $3^2 \times 4^3$ $= 3 \times 3 \times 4 \times 4 \times 4$

Indices on the calculator

We can use our calculator to find the value of numbers expressed in index form. Your calculator should have a button that looks like: [^] or [x^y]. They both work in the same way.

For example, to find 6^4 you would key in [6] [^] [4] [=], or [6] [x^y] [4] [=], to get 1296.

Multiplying index numbers

How could we calculate $2^3 \times 2^5$?

Method 1

We can **evaluate** each part separately and find the product. We evaluate a number or an expression by calculating its value.

$2^3 = 8$ and $2^5 = 32$

$2^3 \times 2^5 = 8 \times 32$

$= 256$

Method 2

We can **simplify** the calculation by writing each number in expanded form and counting the number of times the base appears. We rewrite the product in index form and use a calculator to find its value.

$2^3 \times 2^5 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= 2^8$

$= 256$

We will use this second method to investigate multiplying index numbers further.

Developing a rule for multiplying index numbers

Complete the following table by writing the product of the numbers in the first column, first in expanded form, then as a single number in index form.

Product	In expanded form	As an index number
$3^2 \times 3^4$		
$4^3 \times 4^4$		
$7^2 \times 7$		
$11^2 \times 11^2$		
$20^3 \times 20^5$		
$6^5 \times 6$		

For each row of your completed table, look closely at the index numbers in the first and third columns.

Can you see a pattern, or connection, between the numbers? Describe it. (For two rows, it might help to remember that $7 = 7^1$, and $6 = 6^1$.)

Did you notice that the index of the final product is equal to the sum of the indices of the numbers being multiplied?

$$\begin{aligned} & 3^2 \times 3^4 \\ = & 3^{2+4} \\ = & 3^6 \end{aligned}$$

$$\begin{aligned} & 4^3 \times 4^4 \\ = & 4^{3+4} \\ = & 4^7 \end{aligned}$$

We can state this pattern as a rule that we can use.

When multiplying numbers in index form with the same base, keep the base and add the indices.

If the numbers do not all have the same base, the indices cannot be added. For example, $2^3 \times 3^2$ cannot be written more simply in index form.

Worked Example 10

WE10

Simplify the following.

(a) $3^7 \times 3^5$

(b) $2^3 \times 3^5 \times 2^4 \times 3^6$

Thinking	Working
(a) Are the bases all the same? If yes, keep the base and add the indices.	(a) $3^7 \times 3^5$ $= 3^{7+5}$ $= 3^{12}$
(b) Are the bases all the same? If not, group the numbers that have the same base and add their indices. Write the answer as a product of the index numbers with different bases.	(b) $2^3 \times 3^5 \times 2^4 \times 3^6$ $= 2^3 \times 2^4 \times 3^5 \times 3^6$ $= 2^{3+4} \times 3^{5+6}$ $= 2^7 \times 3^{11}$

Odd and even powers

Recall that multiplying two negative numbers together gives a positive product, and multiplying a positive and a negative number together gives a negative product. Consider these powers of -2, written in expanded form and multiplied together:

$(-2)^2 = -2 \times -2 = 4$
$(-2)^3 = -2 \times -2 \times -2 = -8$
$(-2)^4 = -2 \times -2 \times -2 \times -2 = 16$
$(-2)^5 = -2 \times -2 \times -2 \times -2 \times -2 = -32$

Can you see a pattern here? Will $(-2)^6$ be a positive or a negative number? What about $(-2)^7$?

Negative numbers raised to an even power give a positive result.

Negative numbers raised to an odd power give a negative result.

Worked Example 11

WE 11

Simplify, if possible, then evaluate the following.

(a) $(-3)^2 \times (-3)^3$ **(b)** $-7^2 \times (-4^4)$ **(c)** $-2^2 \times -2^5$

Thinking		Working	
(a) **1**	Are the bases the same? (Yes.)	(a)	$(-3)^2 \times (-3)^3$
2	If yes, simplify by keeping the base (-3) and adding the indices.		$= (-3)^5$
3	Use a calculator to find the value of the product in index form, noting that the power is an odd number, so the answer will be negative.		$= -243$
(b) **1**	Are the bases the same? (No.)	(b)	$-7^2 \times (-4^4)$
2	If not, evaluate each index number separately. (Note here that the value of one will be negative (-7^2) and one will be positive (-4^4).)		$= -49 \times 256$
3	Use a calculator to find the value of the final product.		$= -12\,544$
(c) **1**	Are the bases the same? (Yes.)	(c)	$-2^2 \times -2^5$
2	If yes, simplify by keeping the base (2) and adding the indices. Note that multiplying the two negative signs in front of the bases will give a positive answer.		$= +2^7$
3	Use a calculator to find the value of the index number.		$= 128$

Dividing index numbers

How can we calculate $5^8 \div 5^5$? As we saw for multiplying index numbers, there are two methods we can use.

Method 1

We could evaluate each number separately and find the quotient using a calculator:

$$5^8 \div 5^5 = 390\,625 \div 3125$$
$$= 125$$

Method 2

We could write each number in expanded form and divide by cancelling common factors before simplifying the indices:

$$5^8 \div 5^5 = \frac{{}^{1}\cancel{5} \times {}^{1}\cancel{5} \times {}^{1}\cancel{5} \times {}^{1}\cancel{5} \times {}^{1}\cancel{5} \times 5 \times 5 \times 5}{{}_{1}\cancel{5} \times {}_{1}\cancel{5} \times {}_{1}\cancel{5} \times {}_{1}\cancel{5} \times {}_{1}\cancel{5}}$$
$$= \frac{5 \times 5 \times 5}{1}$$
$$= 5^3$$
$$= 125$$

We see that the division of numbers with the same base in index form also has a pattern we can use.

$$5^8 \div 5^5 = 5^{8-5}$$
$$= 5^3$$

When dividing numbers with the same base in index form, keep the base and subtract the indices.

Worked Example 12

WE12

Simplify the following.

(a) $3^9 \div 3^4$

(b) $(2^{11} \times 5^4) \div (2^9 \times 5^3)$

Thinking	Working
(a) Are the bases the same? (Yes.) If yes, write the base (3) and subtract the indices.	(a) $3^9 \div 3^4$ $= 3^{9-4}$ $= 3^5$
(b) 1 Are the bases the same? (No.) If not, rewrite the division as a fraction. Subtract the indices of numbers with the same base.	(b) $(2^{11} \times 5^4) \div (2^9 \times 5^3)$ $= \dfrac{2^{11} \times 5^4}{2^9 \times 5^3}$ $= 2^{11-9} \times 5^{4-3}$
2 Write the answer as a product of index numbers with different bases.	$= 2^2 \times 5$

The difference between 'simplify' and 'evaluate'

To simplify an expression involving index numbers, we apply the rules for multiplying and dividing them, writing it in a simpler form. Note that Worked Examples 10 and 12 asked for expressions to be simplified. The answer was given in index form—its actual value was not calculated.

To evaluate an expression involving index numbers, we find the value of the answer as an actual number. Worked Example 11 asked for expressions to be simplified first, then evaluated. The answer was given as a number.

1.5 Multiplying and dividing numbers in index form

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Equipment required: A calculator may be used for Questions **4**, **8(b)**, **12(a)**, **14(c)** and **16**

Fluency

1 Write the following in index form. WE 8

(a) $2 \times 2 \times 2 \times 2$
(b) $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$
(c) $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
(d) $11 \times 11 \times 11 \times 11 \times 11 \times 11$
(e) $13 \times 13 \times 13 \times 13 \times 13 \times 13$
(f) $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$
(g) $5 \times 5 \times 5 \times 5 \times 6 \times 6 \times 6$
(h) $2 \times 2 \times 2 \times 3 \times 3$
(i) $4 \times 4 \times 4 \times 4 \times 10 \times 10 \times 10 \times 10$
(j) $7 \times 7 \times 7 \times 7 \times 7 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
(k) $2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5$
(l) $11 \times 11 \times 11 \times 13 \times 13 \times 13 \times 13 \times 13 \times 13$
(m) $6 \times 6 \times 6 \times 6 \times 7 \times 7 \times 7 \times 9 \times 9$
(n) $12 \times 14 \times 14 \times 14$

2 Write the following in expanded form. WE 9

(a) 5^7
(b) 4^6
(c) 7^3
(d) 9^6
(e) 10^4
(f) 3^7
(g) $2^3 \times 5^4$
(h) $3^7 \times 11^5$
(i) $9^5 \times 7^5$
(j) $2 \times 4^6 \times 6^4$
(k) $3^4 \times 7^2 \times 12$
(l) $9^3 \times 11 \times 17^2$

3 Simplify the following. WE 10

(a) $5^2 \times 5^3$
(b) $3^7 \times 3$
(c) $6^4 \times 6^2$
(d) $4^5 \times 4^2$
(e) $9^2 \times 9^7$
(f) $17^5 \times 17^6$
(g) $63^4 \times 63^{12}$
(h) $4^3 \times 5^2 \times 4^6 \times 5^3$
(i) $2^7 \times 6^3 \times 2^3 \times 6^4$
(j) $7^3 \times 8^4 \times 7 \times 8^8$
(k) $11^2 \times 9^2 \times 11^5 \times 9^5$
(l) $10^{10} \times 3^8 \times 10 \times 3^7$

4 Simplify, if possible, then evaluate the following. WE 11

(a) $(-2)^2 \times (-2)^5$
(b) $(-6)^7 \times (-6)^2$
(c) $(-3)^4 \times (-3)^5$
(d) $(-7)^2 \times (-7)^2$
(e) $-5^2 \times 3^3$
(f) $(-2)^2 \times -4^2$
(g) $-6^2 \times (-8)^4$
(h) $-7^2 \times -9^2$

5 Simplify the following. WE 12

(a) $7^4 \div 7^2$
(b) $3^9 \div 3^4$
(c) $2^8 \div 2^5$
(d) $13^4 \div 13^2$
(e) $17^6 \div 17^3$
(f) $25^8 \div 25^5$
(g) $(-2)^5 \div (-2)^3$
(h) $(-4)^4 \times (-4)^3$
(i) $(-6)^3 \div (-6)^2$
(j) $(3^4 \times 2^6) \div (3^2 \times 2^3)$
(k) $(6^5 \times 4^3) \div (6^3 \times 4^2)$
(l) $(9^{11} \times 13^5) \div (9^9 \times 13^4)$
(m) $(5^{12} \times 7^6) \div 7^5$
(n) $(11^2 \times 8^3) \div (11 \times 8)$
(o) $(2^7 \times 3^5 \times 4^2) \div (2^5 \times 4)$

Understanding

6 (a) In index form $4 \times 4 \times 4 \times 4 \times 4 \times 4$ is:

A 24 B 6^4 C 4^6 D 1024

(b) In expanded form 3^5 is:

A 3×5 B $3 \times 3 \times 3 \times 3$ C $5 \times 5 \times 5$ D $3 \times 3 \times 3 \times 3 \times 3$

(c) The value of 7^5 is:

A 35 B 2401 C 16 807 D 78 125

(d) The value of -5^4 is:

A -625 B -20 C 20 D 625

7 Copy and complete the following table.

Words	Index form	Expanded form	Value
			49
		$6 \times 6 \times 6 \times 6 \times 6$	
	8^3		
Eleven to the power of six			

8 A scientist studying a population of insects has found that the population doubles every week.

If the scientist begins with 50 insects, at the end of 1 week she will have $50 \times 2 = 100$ insects.
At the end of 2 weeks, the population will be:

Week 1 population $\times 2$
$= 100 \times 2$
$= (50 \times 2) \times 2$
$= 50 \times 2^2$ insects.

(a) Use index notation to write down how many insects there will be after:

(i) 3 weeks (ii) 5 weeks (iii) 8 weeks.

(b) Evaluate each of your answers to **(a)** to find the actual number of insects after each period.

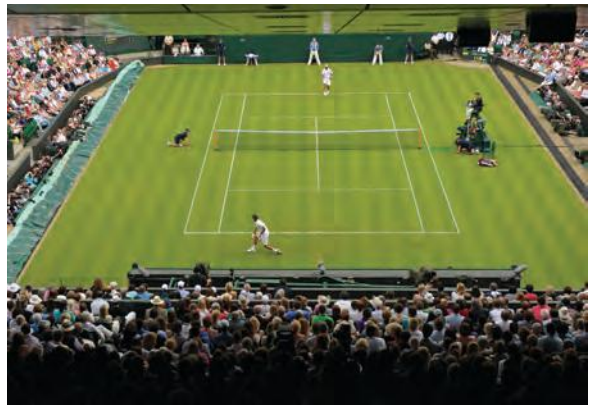
9 (a) Complete the table of powers of 5.

$(-5)^1$	$(-5)^2$	$(-5)^3$	$(-5)^4$	$(-5)^5$	$(-5)^6$	$(-5)^7$	$(-5)^8$	$(-5)^9$	$(-5)^{10}$	$(-5)^{11}$	$(-5)^{12}$
-5	25	-125									

(b) What is the pattern?

(c) How can you predict whether the answer is positive or negative?

10 The Wimbledon tennis championship consists of 7 rounds of matches: Rounds 1, 2, 3 and 4, the quarterfinals, the semifinals and the final. Half the players (the losers of their matches) are removed after each round, so that there are only 2 players remaining in the final.



(a) Starting at the final and working back through each round, write the number of players playing in each round as a number in index form.

(b) How many players are there at the start of the championship?

11 Simplify the multiplications first, then the divisions, then evaluate the following.

(a) $\frac{8^2 \times 8^3}{8^4}$ **(b)** $\frac{5^5 \times 5^4}{5^6}$ **(c)** $\frac{1^{13} \times 1^5}{1^9}$

(d) $\frac{(-4)^2 \times (-4)^5}{(-4)^4}$ **(e)** $\frac{(-3)^3 \times (-3)^2}{-3}$ **(f)** $\frac{(-2)^5 \times (-2)^3}{-2^2}$

12 **(a)** Evaluate each index number separately, then find the product or quotient.

(i) $4^5 \times 4^3$ **(ii)** $7^5 \div 7^2$

(iii) $(9^2 \times 9^3) \div 9^4$ **(iv)** $(5^3 \times 5^4) \div (5^4 \times 5^3)$

(b) Repeat part **(a)**, this time using the rules for multiplying and/or dividing index numbers to simplify, then evaluate the result.

(c) Which process is more efficient—the one used in part **(a)**, or in part **(b)**?

Reasoning

13 **(a)** Write the value of each of the following.

(i) 1^5 **(ii)** 1^7 **(iii)** 1^{15}

(iv) 0^4 **(v)** 0^8 **(vi)** 0^{11}

(b) What conclusions can you draw from this about:

(i) powers of 1 **(ii)** powers of 0?

14 **(a)** State whether the value of the following will be positive or negative.

(i) $(-4)^3$ **(ii)** -4^3 **(iii)** $(-6)^6$ **(iv)** -6^6

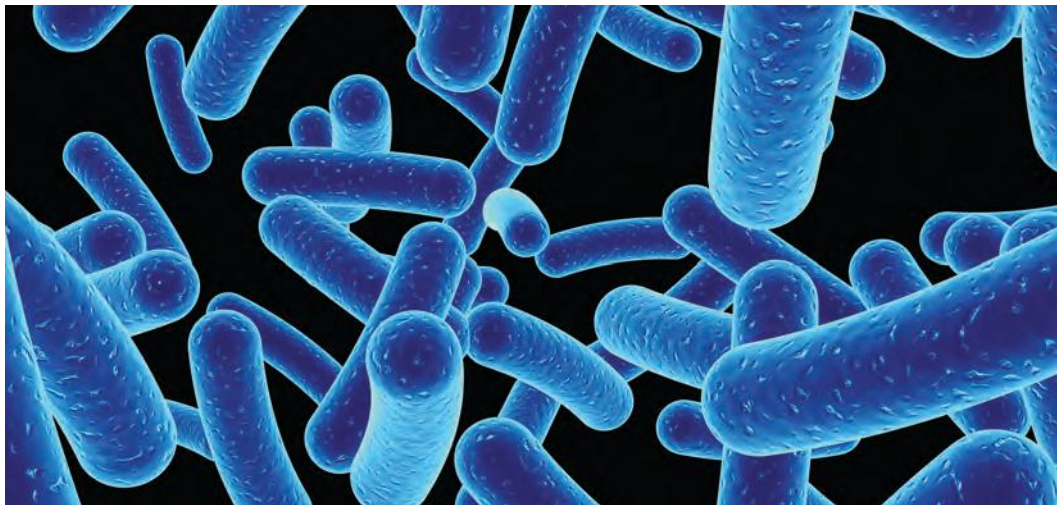
(b) Explain how brackets may affect the signs of the answers for **(a) (i)** and **(ii)** and for **(a) (iii)** and **(iv)**.

(c) Find the value of the following.

(i) $-3^3 \times 4^2$ **(ii)** $(-2)^4 \times (-3)^2$ **(iii)** $-2^5 \times -5^2$

(iv) $(-4)^3 \times (-5)^2$ **(v)** $-2^3 \times (2^4)$ **(vi)** $-2^3 \times (-2)^4$

15 Bacteria A splits into two after every 20 minutes and Bacteria B splits into two after every 40 minutes.



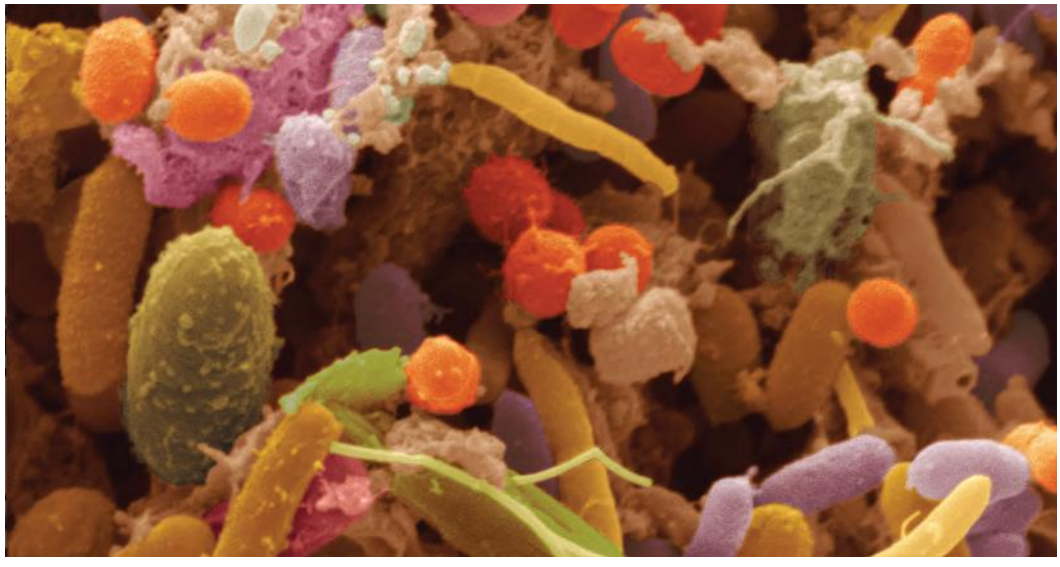
(a) Complete the following bacteria population table.

Time (min)	Number of Bacteria A	Number of Bacteria B
0	1	1
20		
40		
60		
80		
100		
120		
140		

(b) What will be the population of each after 3 hours?

(c) How many more of Bacteria A than Bacteria B will there be after 3 hours?

(d) If Bacteria A was competing for food and space with Bacteria B, which bacteria would be the winner? Give a reason for your answer.



Open-ended

16 Write down one example of each of the following that has a value between 20 and 100.

(a) a number that is a power of 2 **(b)** a number that is a power of 3

(c) a number that is a power of 4 and a power of 2

17 Here is some of Joel's homework on index numbers. Unfortunately, he only has one question correct.

'Write the value of the following index numbers'.

1 $4^6 = 24$ **4** $7^2 = 14$

2 $3^3 = 9$ **5** $5^4 = 20$

3 $2^5 = 10$ **6** $9^1 = 9$

(a) Explain how Joel achieved his answers.

(b) Which question did he 'accidentally' get correct? Why was this?

(c) What would you tell Joel to help him have a better understanding of index numbers?

18 Hayden and Tao were arguing about the solution to $3^3 \times 3^3 + 3^3 \times 3^3$. Hayden said it was 2×3^6 and Tao said it was 18^6. Who was correct, and why?

19 **(a)** Write a division between any two of the eight numbers below so that the answer is 2.
$10^2, 2^3, 4^2, 2^5, 4^3, 2 \times 5^2, 2 \times 3^2, 6^2$

(b) Write two more divisions that have an answer of 2, using two different pairs of the above numbers.

Outside the Square Game

Closest to 500

Equipment required: 2 or more brains, 2 dice, calculator

How to win:
The winner of the game is the player who gets closest to 500 after 10 rolls.

How to play:
Copy a blank version of the table shown for each person.

Roll both dice. Choose one number as the base, the other as the exponent (power).

Six is a 'wild card'. If a player rolls a six, they must choose any whole number between 1 and 5 to take its place.

Determine the resulting value (using the calculator if necessary) and then add or subtract your new number to your current total. It is possible to go into the negative numbers if you wish.

An example of a player's table after three rolls is shown below.

Roll	Base	Exponent	Value	Add/Subtract
1	2	3	$2^3 = 8$	8
2	6	2	$6^2 = 36$	8 + 36 = 44
3	4	5	$4^5 = 1024$	44 – 1024 = –980
4				
5				
6				
7				
8				
9				
10				

Do this for 10 rolls and then compare your final total with that of your playing partner(s).

Investigation

Cyclic powers

Equipment required: 1 brain, 1 scientific calculator

Do you recognise the sequence of numbers 2, 4, 6, 8, 10, 12, 14, 16, 18, 20…? They are the multiples of 2 (the even numbers). If we look at the last digit of each number, we can see a repeating, or cyclic, pattern: 2, 4, 6, 8, 0, 2, 4, 6, 8, 0…

The multiples of 5 form this sequence: 5, 10, 15, 20, 25, 30… Here, the last digits form a shorter repeating pattern: 5, 0, 5, 0, 5, 0…

In this investigation we will look for patterns in the sequences of powers.

The Big Question

Do sequences of powers contain patterns? Can we use these patterns to make predictions?

Engage

Here are the powers of 2.

$2^1 = 2 \quad (2)$
$2^2 = 4 \quad (2 \times 2)$
$2^3 = 8 \quad (2 \times 2 \times 2)$
$2^4 = 16 \quad (2 \times 2 \times 2 \times 2)$
$2^5 = 32 \quad (2 \times 2 \times 2 \times 2 \times 2)$

We can write the powers as a sequence: 2, 4, 8, 16, 32…

The next number in the sequence is obtained by multiplying the previous number by 2. Notice that the numbers in this sequence get very big very quickly.

1 (a) Write out the first 5 powers of 2 listed above, then calculate the next 5 numbers in the sequence. (Your calculator should have a button that calculates powers. It may look like this: '^'.)

(b) Write out the last digit in each of the 10 numbers. Can you see a pattern? If not, calculate more powers until the pattern becomes clear. Predict what the last digit of 2^{16} will be, then calculate it to check your prediction.

2 How many numbers are there in one 'cycle' of the pattern? (That is, how many numbers does the pattern go through before beginning again?)What power of 2 will begin the fifth cycle of the pattern?

Explore

3 Are there patterns in the sequences of other powers? Write out the sequences of the powers of 3, 4, 5, 6, 7, 8, 9. Write down any patterns formed by the last digits of the numbers in the sequence.

Strategy options

- Guess and check.
- Make a table.
- Look for a pattern.

Explain

4 (a) For each sequence of powers, describe any patterns you found in the last digits.

(b) The patterns vary in the number of digits that appear before the pattern begins again, or 'cycles'. How many digits in:

(i) the shortest cycle

(ii) the longest cycle?

5 Can you explain why some patterns have short cycles and some have long cycles? Think about the multiplications done to generate each number in the sequence of powers.

Elaborate

6 Use your pattern to predict the last digits of the 23rd and 39th powers of each of the numbers from 2 to 9. Explain how you arrived at your prediction. (You can try to check your predictions, but your calculator may struggle with some of the bigger numbers!)

7 What is unusual about the sequences of powers for 1 and 10?

Evaluate

8 Approximately how many numbers did you need to write before you 'spotted' a pattern in the sequence? Did you get better at spotting patterns as you went along?

Extend

9 Some numbers, such as 16, 81, 256 and 729, appear in more than one sequence of powers. What are the connections between these numbers and the sequences in which they appear?

Powers of powers, products and quotients

1.6

Raising a number in index form to a power

Consider $(7^2)^3$. We have 7^2 multiplied by itself three times.
We can write this out as follows.

$$\begin{aligned}(7^2)^3 &= 7^2 \times 7^2 \times 7^2 \\ &= 7^{2+2+2} \\ &= 7^6\end{aligned}$$

or:
$$\begin{aligned}&= (7^2)^3 \\ &= (7 \times 7) \times (7 \times 7) \times (7 \times 7) \\ &= 7^6\end{aligned}$$

Notice that 2×3 equals 6. So, we can simplify by multiplying the indices together:

$$\begin{aligned}(7^2)^3 &= 7^{2 \times 3} \\ &= 7^6\end{aligned}$$

When raising a number in index form to a power, keep the base and multiply the indices; e.g. $(3^4)^3 = 3^{12}$.

Worked Example 13

WE13

Simplify the following.

$(2^3)^5$

Thinking	Working
Keep the base and multiply the indices.	$(2^3)^5$ $= 2^{3 \times 5}$ $= 2^{15}$

Raising a product to a power

How could we calculate $(4 \times 7)^3$?

We could find the product in brackets first, then raise it to the power:

$$\begin{aligned}(4 \times 7)^3 &= (28)^3 \\ &= 21\,952\end{aligned}$$

or

We could raise each factor in the product to the power. We can show this by writing in expanded form first:

$$\begin{aligned}(4 \times 7)^3 &= (4 \times 7) \times (4 \times 7) \times (4 \times 7) \\ &= 4 \times 4 \times 4 \times 7 \times 7 \times 7 \\ &= 4^3 \times 7^3 \\ &= 64 \times 343 \\ &= 21\,952\end{aligned}$$

If a product of factors in brackets has been raised to a power, each factor in the brackets is raised to that power.

$$\text{e.g. } (3 \times 5)^4 = 3^4 \times 5^4$$

Raising a quotient to a power

What does $\left(\frac{2}{5}\right)^3$ simplify to? We have a base of $\frac{2}{5}$ and an index of 3.

$$\begin{aligned}\left(\frac{2}{5}\right)^3 &= \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \\ &= \frac{2 \times 2 \times 2}{5 \times 5 \times 5} \\ &= \frac{2^3}{5^3} \\ \left(\frac{2}{5}\right)^3 &= \frac{2^3}{5^3}\end{aligned}$$

Each number in the brackets has been raised to the power of 3.

If a quotient in brackets has been raised to a power, each number in the brackets is raised to that power.

$$\text{e.g. } \left(\frac{3}{4}\right)^5 = \frac{3^5}{4^5}$$

Worked Example 14

WE14

Expand the brackets in the following.

(a) $(2 \times 3)^4$

(b) $\left(\frac{5}{7}\right)^2$

Thinking	Working
(a) Raise every factor in the brackets to the power outside the brackets.	(a) $(2 \times 3)^4$ $= 2^4 \times 3^4$
(b) Raise every number in the brackets to the power outside the brackets.	(b) $\left(\frac{5}{7}\right)^2$ $= \frac{5^2}{7^2}$

The zero power

To simplify $5^3 \div 5^3$, we can use one of two methods.

Method 1

Use the rule for dividing index numbers, which is to keep the base and subtract the indices.

$$\begin{aligned}5^3 \div 5^3 &= 5^{3-3} \\ &= 5^0\end{aligned}$$

Method 2

Write in expanded form and cancel common factors.

$$5^3 \div 5^3 = \frac{5^3}{5^3} = \frac{\cancel{5} \times \cancel{5} \times \cancel{5}}{\cancel{5} \times \cancel{5} \times \cancel{5}} = 1$$

(We should also realise that any number divided by itself is 1.)

From Method 1, we see that $5^3 \div 5^3 = 5^0$

From Method 2, we see that $5^3 \div 5^3 = 1$

Therefore, $5^0 = 1$.

Try it with $\frac{8^4}{8^4}$.

Did you get $8^0 = 1$?

Any number raised to the power of zero equals one.
(The exception is zero itself: 0^0 cannot be defined.)

1.6 Powers of powers, products and quotients

Navigator

Q1, Q2, Q3, Q4, Q5 Columns 1 & 2, Q6, Q9, Q11, Q14, Q16, Q17	Q1, Q2 Columns 1–3, Q3, Q4, Q5 Columns 1–3, Q6, Q7, Q8, Q9, Q11, Q12, Q14, Q16, Q17	Q1, Q2 Columns 2–4, Q3, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13, Q15, Q16, Q17

Answers page 595

Equipment required: Calculator for Questions **9** and **15**

Fluency

1 Simplify the following. WE13

(a) $(9^6)^2$ **(b)** $(14^7)^3$ **(c)** $(6^8)^8$ **(d)** $(10^5)^5$

(e) $(4^2)^3$ **(f)** $(5^2)^0$ **(g)** $(2^4)^5$ **(h)** $(7^0)^2$

2 Expand the brackets in the following. WE14

(a) $(4 \times 3)^2$ **(b)** $(2 \times 5)^3$ **(c)** $(3 \times 2)^5$ **(d)** $(4 \times 7)^4$

(e) $(7 \times 10)^3$ **(f)** $(8 \times 9)^1$ **(g)** $(2 \times 3 \times 1)^6$ **(h)** $(4 \times 5 \times 6)^2$

(i) $\left(\frac{3}{5}\right)^3$ **(j)** $\left(\frac{2}{9}\right)^2$ **(k)** $\left(\frac{10}{13}\right)^3$ **(l)** $\left(\frac{12}{13}\right)^2$

(m) $\left(\frac{1}{2}\right)^5$ **(n)** $\left(\frac{1}{10}\right)^3$ **(o)** $\left(\frac{1}{11}\right)^6$ **(p)** $\left(\frac{1}{12}\right)^2$

3 Evaluate the following.

(a) 7^0 **(b)** 20^0 **(c)** $\left(\frac{5}{6}\right)^0$ **(d)** $\left(-3\frac{1}{2}\right)^0$ **(e)** $(-0.5234)^0$

(f) 3×5^0 **(g)** $2^2 \times 6^0$ **(h)** -3^0 **(i)** -6×5^0 **(j)** $3 \times (-6)^0$

4 **(a)** $(5^6)^2$ simplifies to:

A 5^8 **B** 5^4 **C** 5^3 **D** 5^{12}

(b) $(4 \times 3)^0$ simplifies to:

A 1 **B** 2 **C** 7 **D** 12

Understanding

5 Use a combination of two or more of the rules for working with index numbers to simplify the following. Leave your answers in index form (do not evaluate).

(a) $(8^2 \times 7^3)^2$ **(b)** $\left(\frac{6^2}{7^2}\right)^3$ **(c)** $\frac{(2 \times 4)^5}{2}$ **(d)** $\frac{(5^3)^2}{5^2}$

(e) $\frac{3^4 \times 4^7}{9^2 \times 3^4}$ **(f)** $\frac{3^7 \times 4^0}{3^5 \times 4^5}$ **(g)** $\frac{(7 \times 8)^4}{7^2}$ **(h)** $\frac{(8 \times 9 \times 10)^3}{(9 \times 8)^3}$

(i) $\frac{(4^3 \times 5)^4}{(5^2 \times 4)^2}$ **(j)** $(2 \times 3)^4 \times \left(\frac{1}{3}\right)^2$ **(k)** $\left(\frac{3}{2}\right)^4 \times \left(\frac{2}{3}\right)^3$ **(l)** $\left(\frac{1}{3^5}\right)^2 \times \left(\frac{2}{3}\right)^2$

6 Write the following as the product of two or three prime factors, raised to a single power; e.g. $15^2 = (5 \times 3)^2$.

(a) 10^6 **(b)** 21^3 **(c)** 35^2 **(d)** 45^4 **(e)** 70^5 **(f)** 77^7

7 Write the following as the product of two or more prime factors in index form. The index of each prime factor may be different; e.g. $24 = 2^3 \times 3$.

(a) 36 **(b)** 48 **(c)** 63 **(d)** 72 **(e)** 75 **(f)** 84

8 Evaluate the following without a calculator by writing the numerators and denominators as products of prime factors, then simplify.

(a) $\frac{9 \times 14}{15 \times 21}$ **(b)** $\frac{12 \times 18}{27 \times 24}$ **(c)** $\frac{45 \times 21}{35 \times 36}$ **(d)** $\frac{54 \times 60}{48 \times 33}$

9 **(a)** By trial and error, find the largest number that is a power of 2 that will fit on a 10-digit calculator display.

(b) Find the largest power of 9 that will fit on a 10-digit calculator display.

10 Pronumerals are the symbols we use to represent unknown numbers. We can apply the rules for working with index numbers to pronumerals. Use one of the index number rules you have learnt in this section to simplify the following.

(a) $(a^3)^2$ **(b)** $\left(\frac{c}{d}\right)^4$ **(c)** $(xy)^5$ **(d)** m^0

(e) $(2x)^2$ **(f)** $5p^0$ **(g)** $\left(\frac{b^2}{c}\right)^5$ **(h)** $\left(\frac{3a}{b}\right)^3$

Reasoning

11 Kelvin and Terry were given the following question as part of their indices homework.

Evaluate: $\left(\frac{3^3 \times 2^5}{3^2 \times 2^3}\right)^2$

Here is Kelvin's working:

$\left(\frac{3^3 \times 2^5}{3^2 \times 2^3}\right)^2$

$= \left(\frac{27 \times 32}{9 \times 8}\right)^2$

$= \frac{729 \times 1024}{81 \times 64}$

$= \frac{746\,496}{5184}$

$= 144$

Here is Terry's working:

$\left(\frac{3^3 \times 2^5}{3^2 \times 2^3}\right)^2$

$= (3 \times 2^2)^2$

$= (3 \times 4)^2$

$= 12^2$

$= 144$

'Wow', said Kelvin. 'We both got the same answer but your working doesn't look as complicated as mine'. 'No', said Terry. 'And I didn't have to use my calculator either!'

(a) How is Terry's method different to Kelvin's?

(b) Both boys answered the question correctly, but who did it more efficiently? Give a reason for your choice.

12 Use the appropriate index law to find the value of x in each of the following.

(a) $10^{12} \div 10^5 = 10^x$ **(b)** $(5^6)^3 = 5^x$ **(c)** $(6^x)^2 = 6^{20}$

(d) $\frac{7^4 \times 7^x}{7^{11}} = 1$ **(e)** $\frac{(3^4)^x}{3^{12}} = 1$ **(f)** $\frac{(5^6)^x \times (5^x)^2}{5^4} = 5^4$

13 Write the following numbers as index numbers raised to a second index; e.g. $64 = (2^2)^3$.

(a) 16 **(b)** 81 **(c)** 625

(d) 729 **(e)** 256 **(f)** 1 000 000

Open-ended

14 Find two or three ways of writing the following as a product of two or three factors, raised to a single power.

(a) 24^3 **(b)** 36^5 **(c)** 48^2 **(d)** 80^4

15 1000 can be written as 10^3, or as $5^3 \times 2^3$. Find at least two different ways of writing 625 000 using indices.

16 Write three different sets of values for m and n so that $(a^m)^n$ simplifies to a^{24}.

17 The index laws are sometimes used incorrectly. Below is a set of calculations that 'look' reasonable, but are actually incorrect.

In each case, explain the mistake that has been made and give the correct answer in its place. Explain why your answer is correct.

(a) $3^2 \times 3^4 = 3^8$ (b) $\frac{7^9}{7^3} = 7^3$ (c) $(5^2)^4 = 5^{2^4} = 5^{16}$ (d) $13 \times 4^0 = 1$

Challenge 1

1 The mean of -5, -3, 0, 4 and 9 is:

A -1 B 1 C $3\frac{4}{5}$ D $4\frac{3}{4}$

2 Reece bought a new table for \$100. He then changed his mind and sold it for \$110. He changed his mind again and bought it back for \$130, then sold it again for \$150. What overall profit or loss did Reece make?

A \$30 loss B \$10 loss C \$20 profit D \$30 profit

3 Three consecutive numbers are such that twice the greatest added to three times the least is -31. Find the numbers.

4 My father is 30 years older than I am, and my mother is 24 years older than I am. How old was I when my father's age was double my mother's? Explain your answer.

5 If $x^y = 64$, find all the possible pairs of values for x and y.

6 (a) By writing 2^{15} as $(2^3)^5$ and 3^{10} as $(3^2)^5$, determine whether $2^{15} > 3^{10}$ is a true or a false statement. Do not use a calculator.

(b) Use the same method to decide which is larger, 2^{27} or 3^{18}.

7 If the numbers $a = 2^{80}$, $b = 3^{60}$ and $c = 5^{40}$ are written in ascending order, then they are written in the order:

A a, b, c B a, c, b C b, c, a D c, a, b

8 What is the value of $(-1)^5 - (-1)^4$?

A 1 B 0 C -1 D -2

9 $2^3 \times 2^2 \times 3^3 \times 3^2$ is equal to:

A 6^5 B 6^6 C 36^5 D 36^{10}

10 If $10^x - 10 = 99990$, then x is equal to:

A 4 B 5 C 6 D 9

11 The value of $\frac{8^2}{2^8}$ is:

A $\frac{1}{4}$ B $\frac{1}{2}$ C 2 D 4

12 If $y^x = 256$, find all the possible whole number pairs of values for x and y.

13 If $x = -3$, which of the following expressions has the largest value?

A $x^2 - 3$ B $(x - 3)^2$ C $(x + 3)^3$ D $x^2 + 3$

14 What is the last digit in the number represented by 4^{3827}?

15 What is the last digit in the number represented by 3^{2004}?

Chapter review 1

D.I.Y. Summary

Key Words

base	exponent	integers	simplify
cube	index	negative	square
evaluate	index form	positive	
expanded form	indices	power	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 The _____ are all of the positive and negative whole numbers, and zero, which is neither positive nor negative.

2 Two negative numbers multiply to give a _____ result.

3 A _____ number divided by a positive number gives a negative result.

4 5^3 is written in _____ whereas $5 \times 5 \times 5$ is written in _____.

5 6^7 is read as 'six to the _____ of 7'.

6 The number that is raised to a power is called the _____.

7 Other names for 'power' are _____ and _____.

8 The addition of two negative numbers will always give a _____ answer.

9 To _____ means to find a value by performing a calculation.

10 Any number (except zero) raised to the _____ of zero is equal to 1.

11 When we _____ an expression, we do not need to calculate an actual value.

Fluency

1 Write a positive or a negative integer to represent each of the following. Ex 1.1

(a) The bottom of a lake is 23 metres below sea level.

(b) A business made a profit of $840 000.

(c) Depositing $350 into your bank account.

(d) The value of a share in a mining company rose by $4.

2 Place the following in ascending order. Ex 1.1

(a) -6, 9, 14, -23, 0 (b) 8, -15, 5, -7, -2 (c) 34, -11, 0, 6, 12

3 Calculate: Ex 1.1

(a) -3 + (+10) (b) 7 – (+9) (c) -5 – (+6) (d) -11 + (+4)

(e) 8 + (-5) (f) -13 + (-11) (g) 2 – (-18) (h) -12 – (-4)

4 Calculate: Ex 1.1

(a) 4 + 7 – 9 (b) -2 + 5 – 1 (c) -15 + 23 – 8

(d) 6 + (-3) – (+10) (e) -7 – (-8) + 3 (f) 5 – (-14) – 20

5 Find the following products. **Ex. 1.2**

(a) -14×2 (b) 4×-5 (c) -6×-6 (d) -7×-12

(e) 15×-9 (f) -22×8 (g) -60×30 (h) -28×-200

(i) $(-6)^2$ (j) -7^2 (k) $-2^2 \times 3^2$ (l) $(-4)^2 \times (-1)^2$

6 Find the following quotients. **Ex. 1.3**

(a) $-36 \div 3$ (b) $55 \div -11$ (c) $\frac{-28}{-4}$ (d) $-27 \div 3$

(e) $66 \div -11$ (f) $\frac{-91}{-7}$ (g) $72 \div -9$ (h) $-45 \div -5$

(i) $\frac{-80}{4}$ (j) $440 \div -10$ (k) $-320 \div 8$ (l) $\frac{-78}{6}$

7 Evaluate the following expressions. **Ex. 1.4**

(a) $45 \div 9 \times -2 - 4$ (b) $46 + (-6 \times 7) + 20 \div -5$

(c) $-9 \times -5 - 3 \times 4 + 2$ (d) $-8 + (-18) \div -3 - 4 \times -4$

(e) $-4 + (-6)^2 \div 9$ (f) $-7 \times -8 - 5^2 + (-10)$

8 Write the following in index form and find the value of each. **Ex. 1.5**

(a) $9 \times 9 \times 9 \times 9 \times 9$ (b) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$

(c) $3 \times 3 \times 3 \times 3 \times 4 \times 4 \times 4 \times 4 \times 4$ (d) $8 \times 8 \times 10 \times 10 \times 10$

9 Write the following in expanded form. **Ex. 1.5**

(a) $2^4 \times 5^3$ (b) $13^3 \times 8^6$ (c) $4^2 \times 7^2 \times 9^5$ (d) $6^6 \times 10^2 \times 17$

10 Simplify the following. **Ex. 1.5**

(a) $7^3 \times 7^2$ (b) $3^6 \times 3^2$ (c) $5^2 \times 5^2$ (d) $2^2 \times 2^6$

(e) $3^6 \times 2^6 \times 3^3 \times 2^4$ (f) $4^2 \times 7^2 \times 4^3 \times 7$ (g) $5^2 \times 2^3 \times 2^3 \times 5^4$ (h) $7^5 \times 11^5 \times 7^2 \times 11$

11 Simplify the following. **Ex. 1.5**

(a) $5^6 \div 5^2$ (b) $\frac{7^4}{7}$ (c) $11^5 \div 11^3$ (d) $\frac{2^6}{2^3}$

(e) $(3^6 \times 4^3) \div (3^5 \times 4^2)$ (f) $\frac{7^3 \times 10^5}{7 \times 10^3}$ (g) $(8^2 \times 5^7) \div (8 \times 5^4)$ (h) $\frac{2 \times 9^3 \times 13^3}{9^2 \times 13}$

12 Evaluate the following. **Ex. 1.5**

(a) $(-3)^4$ (b) -3^4 (c) $(-5)^3$

(d) -1^7 (e) $-4^2 \times (-3)^2$ (f) $(-2)^4 \times (-3)^4$

13 Simplify, then evaluate the following. **Ex. 1.6**

(a) $(3^5)^2$ (b) $(7^2)^3$ (c) $(3^5)^3$ (d) $(2^4)^4$

(e) 2^0 (f) 6×3^0 (g) $\frac{5^6}{5^6}$ (h) $\frac{2^{10}}{2^{10}}$

14 Expand the brackets: **Ex. 1.6**

(a) $(4 \times 11)^3$ (b) $(8 \times 9)^5$ (c) $(3 \times 5)^7$ (d) $(7 \times 10)^4$

(e) $\left(\frac{1}{2}\right)^3$ (f) $\left(\frac{3}{4}\right)^2$ (g) $\left(\frac{5}{6}\right)^4$ (h) $\left(\frac{8}{9}\right)^7$

Understanding

15 The minimum overnight temperatures for 1 week at Mt Hotham were -2°C, -3°C, 1°C, 2°C, -3°C, -2°C, 0°C. Find the mean minimum overnight temperature for the week. (The mean is found by adding all values, then dividing the result by the number of values.)

16 In the game called Count 'Em Up, red tokens are worth 5 points, black tokens are worth -3 points and white tokens are worth -1 point. Calculate the total point score at the end of a round for each of the following players.

Felicity: 2 red, 3 black and 1 white
Georgia: 3 red, 4 black and 2 white
Rosalie: 3 red, 5 black and 4 white
Doreen: 2 red, 5 black and 2 white

17 A scientist is observing the behaviour of bacterial cells. She has found that each cell divides itself into two every 24 hours.

(a) If the scientist isolates a single cell in a dish, complete the following table to show how many cells there will be after a certain number of days.

Time (number of days)	0	1	2	3	4
Number of cells	1				

(b) Extend the pattern in your table to determine how many cells there will be after 1 week.

(c) Write the 'Number of cells' row of the table as a series of index numbers.

(d) Use the pattern to determine how many cells there would be after 2 weeks.

18 Three friends invest a total of $270 in a lottery draw (they put in $90 each). If they collect prizes worth a total of:

(a) $60, find the loss for each friend

(b) $300, find the profit for each friend

(c) $28 500, find the profit for each friend.

19 An ice cube tray of water at a temperature of 21°C is placed in the freezer, where it takes 3 hours to freeze solid at 0°C. What is the average hourly drop in temperature?

20 Choose the best answer.

(a) $5^3 \times 2^3$ is the same as:

A 10^3 **B** $(5 \times 2)^6$ **C** 10^6 **D** both B and and C

(b) $7^6 \times 7^6$ is the same as:

A 49^6 **B** 7^{12} **C** both A and and B **D** 49^{12}

21 Simplify if possible, then evaluate:

(a) $\left(\frac{3}{7}\right)^0$ **(b)** $(-8)^0$ **(c)** $\frac{(-6)^5}{(-6)^5}$ **(d)** $\left(-\frac{2}{5}\right)^0$

(e) $\frac{(-3)^6}{(-3)^5}$ **(f)** $\frac{(-5)^8}{(-5)^6}$ **(g)** $\frac{(-2)^{10}}{(-2)^5}$ **(h)** $\frac{(-3)^{20}}{(-3)^{17}}$

(i) $\frac{2^3 \times 2^4}{2^5}$ **(j)** $\frac{10^5 \times 10}{-10^2}$ **(k)** $\frac{3^4 \times 4^2 \times 3^2 \times 4}{(3 \times 4)^3}$ **(l)** $\left(\frac{1}{2}\right)^3 \times \left(\frac{3}{4}\right)^2$

22 Evaluate the following without using a calculator by writing the numbers as products of prime factors and simplifying the calculation.

(a) $\frac{25 \times 12}{15 \times 18}$ **(b)** $\frac{10^2 \times 45}{21^2 \times 25}$ **(c)** $\frac{9 \times 7}{10^4 \times 21}$

Reasoning

23 Calculate the value of x in the following equations.

(a) $15^7 \div 15^4 = 15^x$ **(b)** $12^{12} \div 12^x = 12^4$ **(c)** $34^x \div 34^{10} = 1$

24 16 can be written as $(2^2)^2$. Rewrite the following as an index number raised to a second index, using the base given in brackets.

(a) 36^2 (6) **(b)** 128^4 (2) **(c)** $(1000^3)^2$ (10) **(d)** 4900^7 (70)

25 **(a)** Find the magic sum for the following 4×4 magic square.

(b) Complete the magic square in part **(a)**.

(c) Make a new 4×4 magic square by dividing each number in part **(b)** by -2. What is the magic sum of this magic square?

(d) Make another new 4×4 magic square by multiplying each number in the original magic square by -2. What is the magic sum of this magic square?

		2	-20
-18		-8	
	-2		8
10	-12	-4	

26 Van is organising a table tennis tournament for the local clubs in his area. The tournament will have 3 rounds, then the semifinals, then the final. The losers in each round leave the competition. How many players will Van need to invite to compete in round 1 in order to end up with 2 players in the final?

27 Insert brackets where necessary to make the following statements true.

(a) $-3 - 4 \times -2 = 14$ **(b)** $2 + -3 \times 4 + -2 = -6$

(c) $16 \div -4 + 3 \times -2 + 1 = -7$ **(d)** $4 + -3 \times -2 \div -6 \times -1 + 5 = 0$

(e) $2 - 3 + 6 \times 3 + 2 \times -1 - 6 = -37$ **(f)** $24 \div -2 \div -3 + 4 \times -2 - 5 = -24$

28 State whether the following are TRUE or FALSE.

(a) $5^3 \times 5^2 = 5^6$ **(b)** $8^3 \times 8^4 \times 8^6 > 8^9 \times 8^4$ **(c)** $17 + 17 + 17 + 17 = 17^4$

(d) $(3^6)^3 = (3^3)^6$ **(e)** $(4 + 6)^4 = (10^2)^2$ **(f)** $(4 + 3)^2 = 4^2 + 3^2$

NAPLAN practice 1

Numeracy: Non-calculator

1 Two numbers added together equal -5. The same two numbers multiply together to give -24. What are the two numbers?

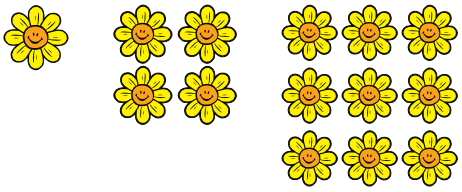
2 You have \$5 in your pocket. You owe a friend \$8, and another friend owes you \$12. If everyone pays up, how much money do you have now?

3 Which of the following is equal to $9^2 \times 3^3$?

A $9 \times 2 \times 3 \times 3$ **B** $3 \times 3 \times 3 \times 3 \times 3$

C $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ **D** 27^5

4 Dora is making this flower pattern.



(a) Complete the table to show the number of flowers she needs for each shape in her pattern.

Shape	1	2	3	4	5
Number of flowers					

(b) What sequence of numbers does the second row of the table form?

(c) How many flowers would she need for a Shape 10?

Numeracy: Calculator allowed

5 $(-2)^2 - 9$ is equal to:

A -13 **B** -5 **C** 5 **D** 13

6 A microwave oven can heat a frozen ready meal by 10°C every minute. How many minutes should a meal taken out of a freezer at -20°C be heated for, to ensure it is at least 50°C when served?

7 If $A^3 = B^2 = C^6 = 64$, what must A, B and C be?

8 Which of the following does not equal $\sqrt{4096}$?

A $\frac{2^8}{2^2}$ **B** $(2^3)^2$ **C** 2048 **D** 4^3

2

Fractions, decimals and percentages

2

Diamonds lose their glitter.

The 2008–09 global financial crisis severely affected Botswana's diamond industry.

The country of Botswana is the largest producer of diamonds in the world. Diamonds are the main source of income for the country, accounting for 70% of the total value of Botswana's exports. In the global financial crisis of 2008–09, a decrease in demand caused Botswana's diamond exports to drop by 90%. This, along with a drop in other mineral exports, resulted in a loss of 50% of the country's income and forced the government to ask for a $1.5 billion loan from the African Development Bank, so it could continue to import food, fuel and other basic needs. The second half of 2009 saw some closed mines reopen as demand slowly rose and production gradually returned to previous levels.

Forum

Why do you think the fall in exports and loss of income were reported using percentages instead of dollar amounts?

How can a drop in the export of just one particular product (diamonds) have such a large effect on the economy of the whole country?

What might be the effect of the drop in diamond exports on individual people and communities in Botswana?

Why learn this?

If you need to work out which discounted jacket is the best deal, or how much profit you made by selling your skateboard on eBay, then good skills in fractions, decimals and percentages are essential. They are not just important in the world of business and money, either. We use fractions, decimals and percentages to analyse and present all kinds of information, which can help us decide how useful or meaningful it is.

After completing this chapter you will be able to:

- convert between fractions, decimals and percentages
- identify different types of decimal numbers and use the correct notation for writing them
- calculate with negative fractions and decimals
- estimate percentages
- write one amount as a percentage of another
- find percentages of amounts
- use percentages to calculate discounts, mark ups, profit and loss
- solve problems involving fractions, decimals and percentages.

Recall 2

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R2.1

1 Evaluate the following, writing your answers in simplest form.

(a) $\frac{2}{3}+\frac{3}{5}$ **(b)** $\frac{5}{6}-\frac{1}{4}$ **(c)** $\frac{4}{9}\times\frac{3}{8}$ **(d)** $\frac{4}{15}\div\frac{4}{5}$

Worksheet R2.2

2 Evaluate the following.

(a) $2.6+3.04+5.3+2.101$ **(b)** $5.29-4.831$ **(c)** 12.7×5

(d) 4.2×3.1 **(e)** $3.72\div 8$ **(f)** $12.64\div 0.04$

Worksheet R2.3

3 Round the following decimals to the number of decimal places shown in brackets.

(a) 2.0335 (2) **(b)** 3.1884 (2)

(c) 2.9096 (3) **(d)** 4.086 19 (3)

Worksheet R2.4

4 Simplify the following fractions.

(a) $\frac{27}{45}$ **(b)** $\frac{16}{92}$ **(c)** $\frac{56}{21}$

Worksheet R2.5

5 Write the following mixed numbers as improper fractions.

(a) $2\frac{1}{2}$ **(b)** $4\frac{2}{9}$ **(c)** $3\frac{7}{20}$

Worksheet R2.6

6 Write the following improper fractions as mixed numbers in simplest form.

(a) $\frac{17}{7}$ **(b)** $\frac{40}{9}$ **(c)** $\frac{95}{10}$

Worksheet R2.7

7 Evaluate:

(a) 7.08×10 **(b)** 0.49×1000 **(c)** 0.8×100

(d) $5.21\div 100$ **(e)** $34.65\div 10$ **(f)** $4.037\div 1000$

Worksheet R2.8

8 Calculate the following.

(a) $\frac{5}{6}$ of \$90 **(b)** $\frac{7}{12}$ of 180 kg **(c)** $\frac{9}{8}$ of 200 m

Worksheet R2.9

9 Write the following fractions as percentages.

(a) $\frac{1}{2}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{2}{5}$ **(d)** $\frac{7}{10}$

Worksheet R2.10

10 Write the following decimals as percentages.

(a) 0.35 **(b)** 0.9 **(c)** 0.04 **(d)** 1.27

Key Words

cost price (CP)	irrational number	per cent	sale price
discount	loss	percentage	selling price (SP)
exact decimal form	mark up	profit	surd
Goods and Services Tax (GST)	non-terminating decimal	recurring decimal	terminating decimal

Working with fractions and decimals

2.1

Skills in working with fractions and decimals are important in all aspects of mathematics, so it is useful to revise and practise them.

Converting fractions to decimals

Fractions and decimals are used to represent parts of wholes. They are two different ways of writing a number. A fraction has a line or bar, which separates the top number (the numerator) from the bottom number (the denominator). This line can be thought of as a division sign, ÷.

So, $\frac{1}{2} = 1 \div 2$, or 1 shared between 2. If we perform this division, we obtain the decimal equivalent for $\frac{1}{2}$, 0.5.

$\frac{9}{5}$ means 9 wholes shared between 5, or $9 \div 5$, which is $1\frac{4}{5}$ or 1.8.

To convert a fraction to a decimal, perform the division; e.g. $\frac{3}{4} = 3 \div 4 = 0.75$.

Some fractions are used so often that it is useful to know the decimal equivalent. If you don't already know them, try to learn the following.

Common fractions and their decimal equivalents

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
0.5	$0.\dot{3}$	$0.\dot{6}$	0.25	0.75	0.2	0.125	0.1	0.01	0.001

Notice that the decimal equivalents for $\frac{1}{3}$ and $\frac{2}{3}$ have a dot above the decimal digit. This is to show that the digit is repeated forever; i.e. $\frac{1}{3} = 0.333\ldots$ These types of decimals are called **recurring decimals**. We will learn more about these in the next section.

In order to compare quantities expressed as fractions and decimals we need to change all values to the same format. It is often easier to convert all values to decimal form.

Fractions and decimals can most easily be compared by first converting the fractions to decimals.

Worked Example 1

WE1

Arrange the following list into ascending order (smallest to largest) by converting the fraction values to decimals.

$\frac{5}{8}, \frac{3}{5}, \frac{3}{4}$, 0.69, 0.686

Thinking	Working
1 Convert each of the fractions into decimals. (You could use your calculator for this step.)	$\frac{5}{8} = 5 \div 8 = 0.625$ $\frac{3}{5} = 3 \div 5 = 0.6$ $\frac{3}{4} = 3 \div 4 = 0.75$
2 List the numbers in order from smallest to largest, by comparing the decimal digits in each place value column.	0.6, 0.625, 0.686, 0.69, 0.75
3 Substitute the fraction values back into the list.	$\frac{3}{5}, \frac{5}{8}$, 0.686, 0.69, $\frac{3}{4}$

Converting decimals to fractions

Decimals use place value to represent the fractional parts of a number. Knowledge of place value allows you to convert decimals to fractions.

For example, we can write the number 13.2452 in a place value table, like this:

tens 10	ones 1	•	tenths $\frac{1}{10}$	hundredths $\frac{1}{100}$	thousandths $\frac{1}{1000}$	ten-thousandths $\frac{1}{10\,000}$
1	3	.	2	4	5	2

From this, 13.2452 can be written in expanded fractional form as

$13 + \frac{2}{10} + \frac{4}{100} + \frac{5}{1000} + \frac{2}{10\,000}$ or $13\frac{2452}{10\,000}$ (which simplifies to $13\frac{613}{2500}$ when we cancel a common factor of 4).

To convert a decimal to a fraction:

- use the place value column of the last digit to get the denominator of the fraction
- write all the digits of the decimal part in the numerator
- simplify the fraction if possible.

Worked Example 2

WE2

Write the following decimal as a fraction in simplest form: 0.384

Thinking	Working
1 The last digit in the decimal is in the thousandths column, so 1000 is the denominator. Write the other digits as the numerator.	$0.384 = \frac{384}{1000}$
2 Simplify the fraction. (Here, we have cancelled common factors of 4 and 2.)	$= \frac{96}{250}$ $= \frac{48}{125}$

Fractions and decimals on the calculator

Scientific calculators have keys that enable you to enter fractions and convert between fraction and decimal form. They will usually look something like this [▬] or this [$a^b/_c$].

To enter a fraction, either:

- press [▬], then key in the numerator and the denominator, using the cursor to move between them; or key in the numerator, press [$a^b/_c$], then key in the denominator.

For example, to enter $\frac{13}{25}$, you would press [▬] [1] [3] [⊕] [2] [5] [=] or press [1] [3] [$a^b/_c$] [2] [5] [=].

To enter a mixed number, such as $1\frac{3}{7}$, either:

- press [SHIFT] [▬] [1] [⊕] [3] [⊕] [7] [=] or
- press [1] [$a^b/_c$] [3] [$a^b/_c$] [7] [=].

Pressing [S⇔D] or [$a^b/_c$] will switch a number entered in fraction form to decimal form, and vice versa.

Pressing [SHIFT] [S⇔D] or [SHIFT] [$a^b/_c$] will convert an improper fraction to a mixed number.

Rounding decimals

Some decimal numbers have a lot of decimal places. Often we don't need all of the decimal places or it doesn't make sense to use them. For example, we might calculate that we need 1.697 214 metres of wood for a project. The third decimal digit represents millimetres (thousandths of a metre). The fourth, fifth and sixth decimal places represent such a tiny amount that they are impossible to measure accurately and are not necessary. In these cases, we 'round' the number to a certain number of decimal places.

It is often important to state how many decimal places a number has been rounded to. We do this by using the abbreviation 'd.p.' We can also say that a rounded number has been written 'correct to' a certain number of decimal places.

To round a number to a given number of decimal places, look at the digit to the right of the one being rounded. If it is:

- 0, 1, 2, 3 or 4, round down. Keep the digit in the place to be rounded the same, and drop the digits following it;

  e.g. 1.697 214 rounded to 3 d.p. is 1.697.

- 5, 6, 7, 8 or 9, round up. Increase the digit in the place to be rounded by 1, and drop the digits following it. If it is a 9, write the zero of the 10, and carry the one across to the next column;

  e.g. 1.789 24 rounded to 2 d.p. is 1.79
  24.5999 rounded to 3 d.p. is 24.600

Remember, if the digit to the right of the one being rounded is 5 or more, round up. If it is less than 5, round down.

Working with remainders

Division calculations do not always give us neat, whole number answers. On a calculator, remainders are usually given in decimal form. It is important to be able to interpret what the remainder means for that particular question.

Worked Example 3 WE3

Use a calculator, if necessary, to answer the following question. A length of wood is $2\frac{1}{2}$ m long. How many pieces, each 600 mm long, can be cut from this length, and how much will be left over? Ignore any wastage that might occur in cutting.

Thinking	Working
1 Write the amounts using the same unit. (In most cases, the smaller unit is more convenient.)	$2\frac{1}{2}$ m = 2500 mm
2 Determine the operation required. 'How many' tells us this is a division question. Perform the division.	25ØØ ÷ 6ØØ = 25 ÷ 6 = 4.166 666 667
3 Write down the whole number of pieces.	4
4 Multiply the whole number of pieces (4) by the size of each piece (600 mm) and use this to calculate the number of mm left over.	4 × 600 = 2400 2500 − 2400 = 100
5 Write the answer.	We will get 4 pieces 600 mm long and will have 100 mm of wood left over.

In the Worked Example above, the decimal remainder after the division represents the length of wood left over after all the pieces of the required length (four 600 mm pieces) had been cut off. 0.166 666 667 of 2500 m is 100 mm.

2.1 Working with fractions and decimals

Navigator

Q1 Column 1, Q2 Columns 1 & 2, Q3 (a) & (b), Q4 (a)–(d), Q5, Q6, Q7, Q9 Columns 1 & 2, Q11, Q12, Q15, Q18, Q21	Q1 Column 1, Q2 Columns 3 & 4, Q3, Q4, Q5, Q8, Q9 Columns 2 & 3, Q11, Q12, Q14, Q15, Q16, Q18, Q20, Q21	Q1 Column 2, Q2 Columns 3 & 4, Q3, Q4 (e)–(h), Q5 (e)–(h), Q6, Q8, Q9 Columns 2 & 3, Q10, Q12, Q13, Q15, Q16, Q17, Q19, Q20, Q21

Answers page 597

Equipment required: A calculator may be used for Questions **1**, **3**, **4**, **9**, **13**–**15** and **17**

Fluency

1 Arrange the following lists into ascending order (smallest to largest) by converting the fraction values to decimals. WE1

(a) $\frac{2}{5}$, 0.399, $\frac{4}{5}$, 0.382, $\frac{3}{4}$

(b) $\frac{9}{10}$, $\frac{9}{8}$, 0.88, 0.89, 0.899

(c) $\frac{1}{8}$, 0.112, 0.099, $\frac{1}{4}$, 0.07

(d) $\frac{2}{5}$, $\frac{1}{3}$, 0.3, $\frac{3}{8}$, 0.2

(e) $\frac{1}{2}$, 0.555, 0.58, $\frac{3}{5}$, 0.55

(f) 0.291, 0.302, $\frac{3}{10}$, $\frac{2}{9}$, $\frac{2}{3}$

(g) $2\frac{4}{5}$, $2\frac{3}{4}$, 2.278, 2.932, $2\frac{9}{10}$

(h) $1\frac{1}{5}$, $1\frac{2}{3}$, $1\frac{3}{8}$, 1.029, 1.243

(i) $4\frac{1}{5}$, $4\frac{1}{4}$, 4.295, 4.199, 4.201

(j) $3\frac{1}{5}$, 3.45, 3.439, $3\frac{2}{3}$, 3.482

2 Write the following decimals as fractions in simplest form. WE2

(a) 0.8 **(b)** 0.05 **(c)** 0.002 **(d)** 0.0009

(e) 0.14 **(f)** 0.62 **(g)** 0.31 **(h)** 0.85

(i) 0.711 **(j)** 0.684 **(k)** 0.625 **(l)** 0.128

(m) 0.203 **(n)** 0.094 **(o)** 0.1560 **(p)** 0.7009

3 Use a calculator, if necessary, to answer the following questions. WE3

(a) A length of wood is 2.6 m long. How many pieces, each 40 cm long, can be cut from this length, and how much wood will be left over?

(b) A bag holds 750 g of flour. How many cups, each containing 120 g of flour, can be filled from the bag, and how much will be left over?

(c) A container holds $3\frac{1}{4}$ litres of juice. How many 200 mL cups can be filled from the container, and how much juice will be left over? (1 litre = 1000 mL)

(d) A bus can carry 52 passengers. If every bus except the last one is filled to capacity, how many buses will be needed to transport 650 people, and how many people will be on the last bus?

Understanding

4 Write the following mixed numbers as decimals using a calculator where necessary.

(a) $51\frac{7}{10}$ **(b)** $5\frac{3}{20}$ **(c)** $7\frac{1}{8}$ **(d)** $67\frac{7}{25}$

(e) $39\frac{1}{16}$ **(f)** $28\frac{24}{200}$ **(g)** $14\frac{13}{80}$ **(h)** $24\frac{31}{400}$

5 Write the following decimals as mixed numbers in simplest form.

(a) 7.5 **(b)** 3.4 **(c)** 1.25 **(d)** 5.64

(e) 13.02 **(f)** 27.96 **(g)** 9.045 **(h)** 124.706

6 **(a)** $1\frac{5}{8}$ written as a decimal is:

A 0.16 **B** 1.58 **C** 1.625 **D** 1.63

(b) $2\frac{1}{3}$ written as a decimal, correct to two decimal places, is:

A 0.34 **B** 2.13 **C** 2.33 **D** 2.34

7 For each set of numbers, draw the number line below and indicate the position of each number with a labelled arrow. (Hint: First, determine the value of the smallest interval on the number line.)

(a) $0.5, 1.3, 1\frac{1}{2}, \frac{1}{5}$ **(b)** $\frac{2}{2}, 0.8, 1.6, 1\frac{4}{5}$

8 For each set of numbers, draw the number line below and indicate the position of each number with a labelled arrow. (You may need to estimate the position of some numbers.)

(a) $\frac{3}{4}, \frac{7}{10}, 0.34, 0.47, 1\frac{1}{3}$ **(b)** $1.05, \frac{7}{8}, 0.58, \frac{8}{5}, \frac{6}{4}$

9 Use a calculator to convert the fractions to decimal form, then place a > (greater than), < (less than) or = (equal to) symbol between the pairs of numbers to make the following statements correct.

(a) 0.84 ____ $\frac{18}{21}$ **(b)** 2.29 ____ $2\frac{12}{39}$ **(c)** 0.912 ____ $\frac{114}{125}$

(d) 0.64 ____ $\frac{16}{25}$ **(e)** 1.83 ____ $1\frac{18}{23}$ **(f)** 0.97 ____ $\frac{98}{99}$

(g) $\frac{1}{3}$ ____ 0.3 **(h)** $\frac{2}{9}$ ____ 0.23 **(i)** $\frac{2}{3}$ ____ 0.67

10 **(a)** Express each first quantity as a fraction of the second quantity. Write the fraction in simplest terms.

(i) first quantity 1.5 hours; second quantity 2.4 hours

(ii) first quantity 200 m; second quantity 322.5 m

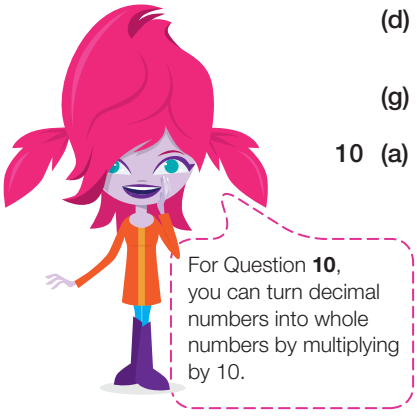
(iii) first quantity 80.6 mL; second quantity 200 mL

(iv) first quantity 20.6 kg; second quantity 12.5 kg

(v) first quantity 18.6 m^2; second quantity 12 m^2

(vi) first quantity 14.4 g; second quantity 1.5 g

(b) Express each fraction from part **(a)** as a decimal, rounding your answers to three decimal places where necessary.



11 Mr Scully is out buying materials to build the set for the school production. Calculate how much he will pay for the following materials. Round your answers to the nearest 5 cents.

(a) 6 lengths of timber, at $8.99 per length

(b) 9.4 metres of canvas, at $11.20 per metre

(c) 5.6 metres of ribbon, at $0.95 per metre

12 The exchange rate between the Australian and American dollars varies daily. Suppose that according to the current rate A$1 is worth US$0.72. How much are the following amounts worth in US dollars?

(a) A$10 **(b)** A$50 **(c)** A$100 **(d)** A$267 **(e)** A$1845

13 The most overdue library book ever was a copy of *Febrile Diseases* by Dr J. Currie. The book was borrowed from the University of Cincinnati Medical Library in 1823 by Mr M. Dodd, and was returned by his great-grandson in 1968. If the fines for late returns were $18.30 a year, how big was the fine his great-grandson had to pay?

14 The largest crab in the world is the giant spider crab, which is found off the south-eastern coast of Japan. It has a claw span of 2.74 m. If an average person has a width of 36 cm across the waist, how many people could fit in the claw span of the giant spider crab?

15 The highest wave ever recorded in Australia was 24.9 m, off Macquarie Island. If the average surfer is 180 cm tall, how many surfers standing on top of one another would it take to reach the top of the wave?

Reasoning

16 An insect was climbing a wall 2.7 m high. In the first 20 minutes the insect started at the bottom and climbed $\frac{1}{3}$ of the height of the wall. In the second 20 minutes it climbed $\frac{1}{4}$ of the remaining height and in the third 20 minutes it climbed $\frac{1}{5}$ of the remaining height.

(a) Calculate the distances climbed in each 20 minute period.

(b) Calculate how far the insect still had to climb to reach the top of the wall.

(c) Express the distance remaining as a fraction of the height of the wall.

17 Angela has to fill 50 gift bags with an equal amount of lollies in each. She has 4 packets of lollies that weigh 375 g each. Angela knows that the average mass of one lolly is 2.8 g.

(a) Explain how Angela can use this information to determine how many lollies she can put in each gift bag.

(b) Use your method from **(a)** to find the answer.

(c) About how many lollies will Angela have left over? Explain why this answer may not be exact.

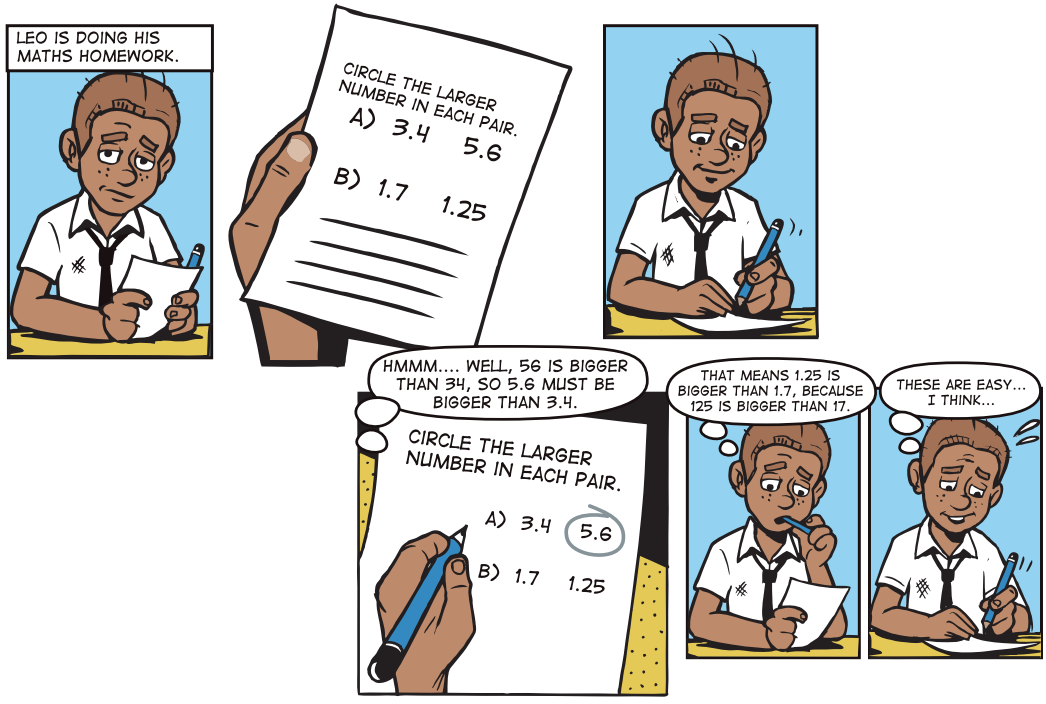
18 The force of gravity varies from planet to planet and so the weight of an object can vary, depending on which planet it is on. Objects on Jupiter would weigh 2.6 times their weight on Earth. Objects on Mars would weigh 0.38 times their 'Earth weight'. Calculate the weight of a 5 kg bag of potatoes on **(a)** Jupiter **(b)** Mars.

Open-ended

19 Write three fractions with three different denominators that have decimal values between 0.2 and 0.4.

20 Write three decimals, each with a different number of decimal places, that have fraction values between $\frac{9}{10}$ and $\frac{9}{8}$.

21



(a) Explain to Leo why his reasoning is incorrect.

(b) Describe to Leo how to compare two decimal numbers.

Outside the Square Puzzle

Sudoku

Equipment required: 1 brain, 1-cm grid paper

To solve a Sudoku you need to use the digits 1–9 to fill in the blank squares so that each row, column and small 3 × 3 box contains the digits 1–9 only once.

Copy the grid into your book and complete it.

5		8	2				6	
	7	2			6			5
6			1			4		
	8					2		4
			8		1			
	6	4	3			5	8	
			7		8	1		3
2						6		
		1	6		3		7	2

Types of decimals

2.2

Terminating decimals

Dividing two whole numbers can give another whole number; for example, $\frac{12}{3} = 4$.

(Recall that we can write a division using fraction notation.) Sometimes, the result of a division will be a decimal number with a clearly defined number of digits. These are called **terminating decimals**.

Examples of terminating decimals: $\frac{18}{5} = 18 \div 5 = 3.6$

$$\frac{9}{8} = 9 \div 8 = 1.125$$

$$\frac{88}{200} = 88 \div 200 = 0.44$$

'Terminating' is a similar word to 'terminal' which describes an end point. For example, aeroplanes depart from and arrive at 'terminals'. Can you see the similarity here?

Non-terminating decimals

Sometimes, the division of one number by another gives a decimal number with an infinite number of digits (the digits continue forever). These are called **non-terminating decimals**. If some, or all, of the decimal digits follow a repeating pattern, it is a recurring decimal.

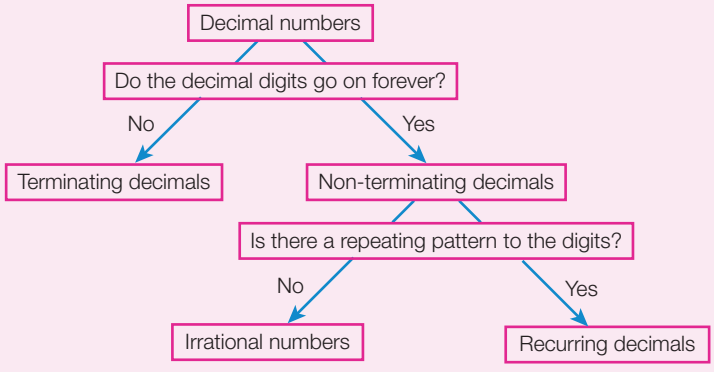
Examples of recurring decimals: $\frac{1}{3} = 1 \div 3 = 0.333\,333\,3\ldots$

$$\frac{4}{15} = 4 \div 15 = 0.266\,666\ldots$$

$$\frac{11}{7} = 11 \div 7 = 1.571\,428\,571\ldots$$

Some decimals are both non-terminating and non-recurring; that is, they have an infinite number of digits and no pattern. These decimals are called **irrational numbers**. These numbers cannot be written as fractions. Irrational numbers include special numbers, such as π (which you will learn about in Chapter 5) and the square roots of some numbers such as 2, 3 and 5 ($\sqrt{2}$, $\sqrt{3}$ and $\sqrt{5}$). Square roots that are irrational numbers are called **surds**. Because they are non-terminating decimals, we can only write an approximate value for an irrational number in decimal form, correct to a certain number of decimal places. The only way to write the exact value of a surd is to write it in surd form; that is ($\sqrt{2}$, $\sqrt{3}$ or $\sqrt{5}$).

For example: $\sqrt{2} \approx 1.4$ (1 d.p.)
≈ 1.41 (2 d.p.)
$\approx 1.414\,214$ (6 d.p.) ('≈' means 'is approximately equal to')



Worked Example 4

WE4

Use a calculator to express the following as decimals and classify them as either terminating, recurring, or irrational.

(a) $12 \div 7$ **(b)** $\frac{5}{16}$ **(c)** $\sqrt{50}$ **(d)** $\sqrt{156.25}$

Thinking	Working
(a) 1 Use the calculator to perform the division.	(a) $12 \div 7 = 1.714\,285\,714\ldots$
2 Do the decimal digits seem to go on forever? (Yes.) Is there a repeating pattern to the digits? (Yes, the repeating pattern is 714 285.) If both answers are yes, the number is a recurring decimal.	Recurring
(b) 1 Use the calculator to perform the division.	(b) $\frac{5}{16} = 0.3125$
2 Do the decimal digits seem to go on forever? (No.) If there is a definite number of decimal places, the number is a terminating decimal.	Terminating
(c) 1 Use the calculator to find the value of the square root.	(c) $\sqrt{50} = 7.071\,067\,812\ldots$
2 Do the decimal digits seem to go on forever? (Yes.) Is there a repeating pattern to the digits? (No.) If there is no repeating pattern, the number is irrational.	Irrational
(d) 1 Use the calculator to find the value of the square root.	(d) $\sqrt{156.25} = 12.5$
2 Do the decimal digits seem to go on forever? (No.) If there is a definite number of decimal places, the number is a terminating decimal.	Terminating

Writing recurring decimals

To write a recurring decimal:

- write any non-recurring digits, then the digits that form the repeating pattern
- place a dot on top of the first and last digits of the repeating pattern, or draw a dash across the whole of the repeating pattern. For example, 0.357 357 357 … can be written as $0.\dot{3}5\dot{7}$ or $0.\overline{357}$.

  Using a dot or dash notation is called writing the number in **exact decimal form**.

Examples of recurring decimals written in exact decimal form are:

$\frac{1}{3} = 1 \div 3 = 0.333\,333\ldots$ and is written as $0.\dot{3}$ or $0.\overline{3}$

$\frac{4}{15} = 4 \div 15 = 0.266\,666\,666\ldots$ and is written as $0.2\dot{6}$ or $0.2\overline{6}$

$\frac{1547}{9999} = 1547 \div 9999 = 0.154\,715\,471\,547\ldots$ and is written as $0.\dot{1}54\dot{7}$ or $0.\overline{1547}$.

Worked Example 5

WE 5

Use a calculator to write each of the following fractions in exact decimal form.

(a) $\frac{1}{7}$ (b) $\frac{1}{13}$ (c) $\frac{1}{12}$

Thinking	Working
(a) 1 Use a calculator to do the division.	(a) $\frac{1}{7} = 1 \div 7 = 0.142\,857\,142$
2 Look for the repeating pattern.	The repeating pattern is 142 857.
3 Write the answer in exact decimal form by placing a dot above the first and last digits in the repeating pattern.	$\frac{1}{7} = 0.\dot{1}42\,85\dot{7}$
(b) 1 Use a calculator to do the division.	(b) $\frac{1}{13} = 1 \div 13 = 0.076\,923\,076$
2 Look for the repeating pattern.	The repeating pattern is 076 923.
3 Write the answer in exact decimal form by placing a dot above the first and last digits in the repeating pattern.	$\frac{1}{13} = 0.\dot{0}76\,92\dot{3}$
(c) 1 Use a calculator to do the division.	(c) $\frac{1}{12} = 1 \div 12 = 0.083\,333\,333$
2 Look for the repeating pattern.	The repeating pattern is just 3.
3 Write the answer in exact decimal form by placing a dot above the repeating digit.	$\frac{1}{12} = 0.08\dot{3}$

Writing recurring decimals as fractions

Previously, we have converted decimals to fractions by using the place value of the last digit. Given that a recurring decimal does not have a last digit, we need a new method for converting recurring decimals to fractions.

We create two expressions with the same decimal part so that the decimal part is eliminated when the two expressions are subtracted. For example, 2.4444… – 0.4444… = 2.

Worked Example 6

WE 6

Write the following recurring decimals as fractions.

(a) $0.\dot{1}\dot{5}$ **(b)** $1.2\dot{4}$

Thinking	Working
(a) 1 Let x = the recurring decimal.	(a) $x = 0.151515\ldots$ (1)
2 Create a second expression by multiplying by the power of 10 that has the same number of zeroes as the number of digits in the repeating pattern. (Here, we multiply by 100.)	$100x = 15.151515\ldots$ (2)
3 Subtract the expressions created in the previous steps.	(2) − (1): $100x = 15.151515\ldots$ − $x = 0.151515\ldots$ $99x = 15$
4 Solve the equation that results and simplify your answer if necessary.	$x = \frac{15}{99}$ $= \frac{5}{33}$
(b) 1 Let x = the recurring decimal.	(b) $x = 1.244\,44\ldots$ (1)
2 Create a second expression by multiplying by the power of 10 that has the same number of zeroes as the number of digits in the repeating pattern.	$10x = 12.444\,4\ldots$ (2)
3 Subtract the expressions created in the previous steps.	(2) − (1): $10x = 12.4444\ldots$ − $x = 1.2444\ldots$ $9x = 11.2$
4 Solve the equation that results and simplify your answer if necessary. (Here, we multiply the numerator and denominator by 10 to eliminate the decimal place.)	$x = \frac{11.2}{9}$ $= \frac{112}{90}$ $= \frac{56}{45}$ or $1\frac{11}{45}$

2.2 Types of decimals

Navigator

Q1, Q2, Q4, Q5, Q6, Q7, Q9 (a) & (b), Q10, Q12, Q14	Q1, Q2 Columns 2–4, Q3 Columns 1 & 2, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12, Q14, Q15	Q1, Q2 Columns 3 & 4, Q3 Columns 2–4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q15

Answers page 598

Equipment required: Calculator for Questions **1**, **2**, **4(b)**, **5**, **6**, **7**, **9–13** and **15**

Fluency

1 Use a calculator to express the following as decimals and classify them as either terminating, recurring, or irrational. WE4

(a) $9 \div 8$ **(b)** $\frac{26}{4}$ **(c)** $\sqrt{449}$ **(d)** $\sqrt{84}$

(e) $\frac{5}{9}$ **(f)** $\sqrt{58.4}$ **(g)** $\frac{2}{11}$ **(h)** $22 \div 3$

(i) $\sqrt{216}$ **(j)** $15 \div 6$ **(k)** $\sqrt{533.61}$ **(l)** $\frac{11}{12}$

(m) $\sqrt{342.25}$ **(n)** $\sqrt{360}$ **(o)** $35 \div 9$ **(p)** $\sqrt{96.4}$

2 Use a calculator to write each of the following fractions in exact decimal form. WE5

(a) $\frac{1}{6}$ **(b)** $\frac{5}{6}$ **(c)** $\frac{1}{9}$ **(d)** $\frac{5}{9}$

(e) $\frac{1}{11}$ **(f)** $\frac{2}{11}$ **(g)** $\frac{3}{11}$ **(h)** $\frac{2}{13}$

(i) $\frac{1}{15}$ **(j)** $\frac{2}{15}$ **(k)** $\frac{4}{15}$ **(l)** $\frac{7}{15}$

(m) $\frac{1}{18}$ **(n)** $\frac{5}{18}$ **(o)** $\frac{7}{18}$ **(p)** $\frac{11}{18}$

3 Write the following recurring decimals as fractions. WE6

(a) $0.\dot{4}$ **(b)** $0.\dot{1}$ **(c)** $0.\dot{2}$ **(d)** $0.\dot{7}$

(e) $0.\dot{4}\dot{2}$ **(f)** $0.\dot{3}\dot{2}$ **(g)** $0.\dot{1}\dot{7}$ **(h)** $0.\dot{2}\dot{4}$

(i) $0.\dot{0}1\dot{2}$ **(j)** $0.\dot{2}1\dot{3}$ **(k)** $0.\dot{3}3\dot{2}$ **(l)** $0.\dot{2}2\dot{4}$

(m) $0.12\dot{5}$ **(n)** $0.1\dot{2}\dot{6}$ **(o)** $0.3\dot{2}1\dot{4}$ **(p)** $4.1\dot{0}2\dot{6}$

4 **(a)** The recurring decimal 4.563 213 213 21… written in exact decimal form is:

A $4.\overline{563\,21}$ **B** $4.563\,2\dot{1}$ **C** $4.563\,\overline{21}$ **D** $4.56\overline{3\,21}$

(b) The fraction $\frac{27}{55}$ written in exact decimal form is:

A $0.\overline{490}$ **B** $0.\dot{4}\dot{9}\dot{0}$ **C** $0.4\overline{90}$ **D** $0.\dot{4}9\dot{0}$

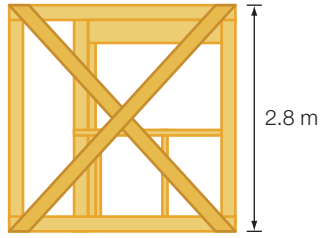
Understanding

5 If we evaluate $\sqrt{2}$ on a calculator, we get a non-terminating decimal value of 1.414 213…. Although we cannot write an exact decimal value, we can state that the value of $\sqrt{2}$ lies in between the values of 1.4 and 1.5, and is closer to 1.4. State the two decimal values that the following lie between, correct to one decimal place, also stating which of the two values is closer to the exact value.

(a) $\sqrt{3}$ **(b)** $\sqrt{5}$ **(c)** $\sqrt{12}$ **(d)** $\sqrt{20}$

(e) $\sqrt{8}$ **(f)** $\sqrt{7}$ **(g)** $\sqrt{47}$ **(h)** $\sqrt{68}$

6 The diagonal of a square is equal to its side length multiplied by $\sqrt{2}$. Jason is strengthening a large house frame by placing two diagonals across it, as shown. Using a calculator, determine the length of wood Jason needs for the diagonals, correct to three decimal places.



7 A shop is selling a packet of 11 lollipops for \$7.

(a) How much does each lollipop cost? Write your answer in exact decimal form, then as a price (rounded to two decimal places).

(b) If you can buy one lollipop individually, how much will you pay?

8 Find the fraction equivalent of $0.\dot{9}$.

9 Arrange the following in ascending order.

(a) $\frac{1}{3}$, $\sqrt{9}$, $\sqrt{3}$, 1.3, 0.9 **(b)** 0.5, $\sqrt{5}$, $\frac{5}{9}$, $\sqrt{59}$, $\frac{9}{5}$

(c) $\frac{4}{11}$, 0.4, $\sqrt{4}$, 0.36, $\sqrt{11}$ **(d)** 1.6, $\frac{1}{6}$, $\sqrt{6}$, $\frac{1}{16}$, $\sqrt{16}$

Reasoning

10 For each set of numbers, draw the number line below and indicate the position of each number with a labelled arrow. (You may need to estimate the position of some numbers.)

(a) $\frac{4}{3}$, $0.\dot{6}$, $\frac{11}{6}$, 0.3, $\frac{3}{5}$ **(b)** 1.04, 0.4, $\frac{4}{7}$, $0.\dot{4}$, $\frac{7}{4}$

11 For each set of numbers, draw the number line below, and indicate the approximate position of each number with a labelled arrow.

(a) $\sqrt{3}$, $\frac{13}{7}$, $\frac{7}{6}$, $\sqrt{6}$, $\sqrt{7}$ **(b)** $\sqrt{5}$, $\frac{15}{14}$, $\sqrt{11}$, $\frac{40}{11}$, $\sqrt{15}$

12 **(a)** The value of $\frac{11}{19}$ is closest to:

A 0.5 **B** 0.579 **C** 0.58 **D** 0.6

(b) The value of $\sqrt{75}$ is closest to:

A 8 **B** 8.66 **C** 8.7 **D** 9

13 **(a)** Use a calculator to write $\frac{1}{9}$ and $\frac{1}{11}$ as decimals with eight digits after the decimal point.

(b) Add together the two decimal values.

(c) Add the two fraction values together.

(d) Do you get the same answer?

(e) Repeat parts **(a)–(c)** with $\frac{2}{3}$ and $\frac{5}{9}$.

(f) Explain the difference between the fraction and decimal sums.

Open-ended

14 Which is bigger, 0.3 or $0.\dot{3}$? Explain your answer.

15 Write five proper fractions, with varying numbers of digits in both numerator and denominator, that have only 9s in the denominator. Can you predict the recurring digits in the decimal equivalent? Check your prediction with your calculator.

Outside the Square Puzzle

Hitori

Hitori is a Japanese one-person logic puzzle.

Copy the following grid into your book.

4	4	1	1	3
3	5	3	1	2
2	4	5	4	2
1	2	3	3	4
2	1	2	3	5

Rules:

Colour in selected squares, so that:

- Each number only appears once in each row and column. Where a number is repeated, all but one must be coloured in.
- No square that has been coloured in can have another coloured-in square beside it except diagonally.
- The squares that remain must all be joined. You should be able to trace an unbroken path, moving either horizontally, vertically or diagonally between all of them.
- It may be helpful to circle numbers that have to be included; e.g. numbers directly next to coloured-in cells.

Investigation

Terminating and recurring decimals

Equipment required: 1 brain (2 brains may be used for Question **2**), scientific calculator

This investigation focuses on unit fractions. Unit fractions are fractions with a numerator of 1.

The Big Question

Which decimal equivalents of the unit fractions are terminating and which are recurring?

Can we predict whether a fraction will produce a terminating or a recurring decimal?

Engage

1 **(a)** **(i)** Convert $\frac{1}{2}$ to its decimal equivalent by calculating $1 \div 2$. Is it a terminating or a recurring decimal?

(ii) Convert $\frac{1}{3}$ to its decimal equivalent by calculating $1 \div 3$. Is it a terminating or a recurring decimal?

(b) $\frac{1}{4}$ is $\frac{1}{2}$ of $\frac{1}{2}$. Find the decimal equivalent of $\frac{1}{4}$ by calculating $1 \div 4$. Now, find it another way by taking the decimal equivalent of $\frac{1}{2}$ and halving it (by dividing by 2). How do the decimal digits show that $\frac{1}{4}$ is $\frac{1}{2} \div 2$?

Explore

2 Investigate the unit fractions with denominators from 2 to 50. Which ones produce terminating decimals? Which ones produce recurring decimals?

Strategy options

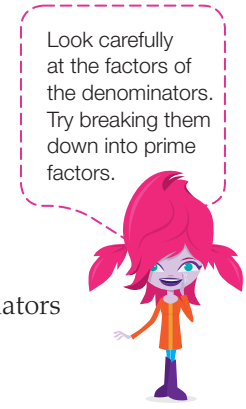
- Guess and check.
- Make a table.
- Look for a pattern.

Explain

3 **(a)** Collect together all the fractions that produced terminating decimals, together with their decimal equivalents. Describe any patterns you can see:

(i) in the fraction denominators

(ii) in the decimal digits.



(b) Collect together some of the fractions that produced recurring decimals, together with their decimal equivalents.

Describe any patterns you can see:

(i) in the fraction denominators

(ii) in the decimal digits.

Elaborate

4 **(a)** Use these patterns to predict a fraction with a denominator greater than 50 that will produce a terminating decimal. Write down a predicted value for this decimal. Test your prediction. Were you correct?

(b) Answer the Big Question by writing a general rule that enables you to tell whether a fraction will produce a terminating or a recurring decimal, just by looking at the denominator.

(c) Using examples, describe how:

(i) you can operate on (add, subtract, multiply or divide) a recurring decimal to turn it into another recurring decimal

(ii) you can operate on (add, subtract, multiply or divide) a terminating decimal to turn it into a recurring decimal.

(d) Is it possible to turn a recurring decimal into a terminating decimal?

Evaluate

5 **(a)** Have you found all of the terminating decimals for the fractions between $\frac{1}{2}$ and $\frac{1}{50}$? How confident are you of this?

(b) How far into the process of changing fractions into decimals were you when you began to see patterns? Were you confident enough to rely on these patterns to predict the answers or did you continue to test individual fractions?

Extend

6 Investigate the fraction family of 'sevenths' ($\frac{1}{7}, \frac{2}{7}, \frac{3}{7} \ldots \frac{6}{7}$), 'ninths' ($\frac{1}{9}, \frac{2}{9}, \frac{3}{9} \ldots \frac{8}{9}$), or 'elevenths' ($\frac{1}{11}, \frac{2}{11}, \frac{3}{11} \ldots \frac{10}{11}$) and their decimal equivalents.

(a) Describe any number patterns you find as you go from one fraction to the next.

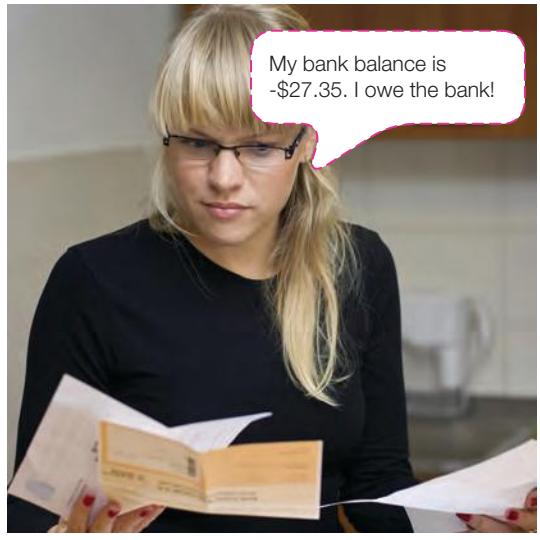
(b) Write the decimal equivalents of fraction pairs that add to 1; for example, $\frac{2}{7}$ and $\frac{5}{7}$. Describe any patterns or connections you can see between the two.

Negative fractions and decimals

2.3

Like integers, fractions and decimals can be positive or negative.

Here are some examples of negative fractions or decimals:



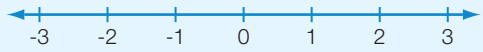
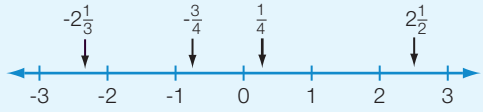
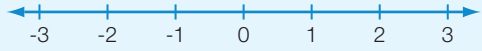
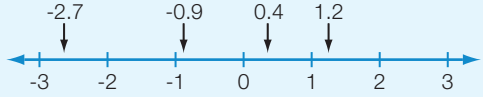
Negative fractions and decimals can be located on a number line in much the same way that we locate positive fractions and decimals.

Worked Example 7 WE7

Place the following sets of numbers on separate number lines.

(a) $2\frac{1}{2}, -\frac{3}{4}, \frac{1}{4}, -2\frac{1}{3}$

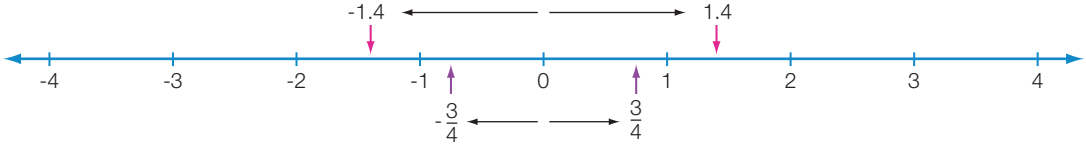
(b) -2.7, 0.4, 1.2, -0.9

Thinking	Working
(a) 1 Construct a number line and place zero in the middle. Mark off the positive integers at even intervals, starting at zero and moving to the right. Mark off the negative integers at even intervals, starting at zero and moving to the left.	*(a)* 
2 Locate each of the numbers by first determining whether to move left or right from zero. Use the marked integers and the intervals between them as a guide to where to locate the numbers.	
(b) 1 Construct a number line, place zero in the middle and mark off the positive and negative integers at even intervals, as before.	*(b)* 
2 Locate each of the numbers by first determining whether to move left or right from zero. Use the marked whole numbers and the intervals between them as a guide to where to locate the numbers.	

Number line symmetry

The number line is symmetrical about zero, which means that the negative equivalent of a number will be the same distance from zero as the positive number.

For example: -1.4 will be the same distance to the left of zero as 1.4 is to the right of zero.
$-\frac{3}{4}$ will be the same distance to the left of zero as $\frac{3}{4}$ is to the right of zero.



Adding and subtracting negative decimals

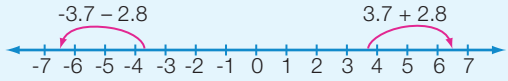
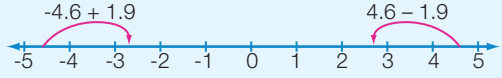
The symmetry of the number line can also help us add and subtract negative decimal numbers. Any calculation on one side of the number line has an equivalent 'mirror image' calculation on the opposite side.

Worked Example 8

WE 8

Calculate the following. It may be useful to visualise the equivalent 'mirror image' calculation on the number line.

(a) -3.7 + (-2.8) **(b)** -4.6 – (-1.9)

Thinking	Working
(a) 1 Simplify the addition by writing a single sign between the two numbers.	(a) -3.7 + (-2.8) = -3.7 – 2.8
2 Visualise the calculation on the number line. Visualise the 'mirror image' of this calculation on the opposite side of the number line. Notice that the original calculation moves to the left on the number line (subtraction), whereas the mirror image calculation moves to the right on the number line (addition).	
3 Complete the 'mirror image' calculation.	3.7 + 2.8 = 6.5
4 The answer to the original calculation has the opposite sign to the 'mirror image' calculation.	-3.7 – 2.8 = -6.5
(b) 1 Simplify the addition by writing a single sign between the two numbers.	(b) -4.6 – (-1.9) = -4.6 + 1.9
2 Visualise the calculation on the number line. Visualise the 'mirror image' of this calculation on the opposite side of the number line. Notice that the original calculation moves up the number line (addition), whereas the mirror image calculation moves down the number line (subtraction).	
3 Complete the 'mirror image' calculation.	4.6 – 1.9 = 2.7
4 The answer to the original calculation has the opposite sign to the 'mirror image' calculation.	-4.6 + 1.9 = -2.7

Adding and subtracting negative fractions

To add and subtract negative fractions, we need to apply the methods we have previously learnt for adding and subtracting fractions, along with the rules for adding and subtracting negative numbers.

Worked Example 9

WE 9

Calculate the following, writing your answers in simplest form.

(a) $-\frac{4}{5}+\frac{1}{2}$

(b) $-1\frac{3}{4}+\left(-\frac{5}{6}\right)$

Thinking	Working
(a) 1 Write the fractions as equivalent fractions with a common denominator.	(a) $-\frac{4}{5}+\frac{1}{2}$
2 Add the numerators and simplify the answer if possible.	$=-\frac{8}{10}+\frac{5}{10}$ $=-\frac{3}{10}$
(b) 1 Simplify the addition by writing a single sign between the two fractions.	(b) $-1\frac{3}{4}+\left(-\frac{5}{6}\right)$ $=-1\frac{3}{4}-\frac{5}{6}$
2 Write any mixed numbers as improper fractions.	$=-\frac{7}{4}-\frac{5}{6}$
3 Write the fractions as equivalent fractions with a common denominator.	$=-\frac{21}{12}-\frac{10}{12}$
4 Subtract the numerators and simplify the answer if possible.	$=-\frac{31}{12}$ or $-2\frac{7}{12}$

Multiplying and dividing negative fractions and decimals

To multiply or divide negative fractions and decimals:

1. Ignore any positive or negative signs, and apply the usual methods for multiplication and division.
2. Determine the sign of the answer by applying the rules for multiplying and dividing integers.

   Two 'like' signs give a positive answer: $+\times+$ or $-\times-=+$

   Two 'unlike' signs give a negative answer: $+\times-$ or $-\times+=-$

Negative numbers on the calculator

To enter a negative fraction or decimal on your calculator press the (–) or +/– key, then enter the fraction or decimal in the usual way.

2.3 Negative fractions and decimals

Navigator

Q1 (a)–(d), Q2 Columns 1–3, Q3 Columns 1 & 2, Q4 Columns 1 & 2, Q5 Columns 1 & 2, Q6, Q7, Q8, Q9, Q11, Q13 (a)–(f), Q16	Q1 (c)–(f), Q2 Columns 2–4, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5 Columns 2 & 3, Q6, Q8, Q9, Q10, Q11, Q13 Column 1, Q14, Q15, Q16, Q18	Q1 (c)–(f), Q2 Columns 3 & 4, Q3 Columns 3 & 4, Q4 Columns 3 & 4, Q5 Columns 3 & 4, Q6, Q8, Q9, Q10, Q12, Q13 Column 2, Q14, Q15, Q17, Q18

Answers page 599

Equipment required: Calculator for Question **13**

Fluency

1 Place the following sets of numbers on separate number lines. WE7

(a) $-3\frac{1}{2}, -2\frac{3}{4}, -\frac{1}{2}, 1\frac{1}{4}$ **(b)** -1.4, -1.9, 0.4, 2.2 **(c)** $-\frac{4}{5}, \frac{1}{5}, -2\frac{1}{4}, 1\frac{2}{5}$

(d) -3.6, -2.6, -6.1, 1.6 **(e)** $-\frac{6}{5}, -\frac{5}{6}, -2\frac{1}{6}, \frac{1}{5}$ **(f)** -0.5, 0.05, -5.2, 0.75

2 Calculate the following. It may be useful to visualise the equivalent 'mirror image' calculation on the number line. WE8

(a) -0.5 + 0.4 **(b)** -0.3 – 0.7 **(c)** -1.5 + (-1.6) **(d)** -2.4 – (-1.3)

(e) -7.2 + 6.8 **(f)** -5.9 – 8.3 **(g)** -4.6 + (-3.9) **(h)** 5.2 – (-4.3)

(i) 7.61 + (-5.92) **(j)** 3.47 – (-10.98) **(k)** 0.8 – 2.34 **(l)** -9.87 + 11.3

(m) 19.3 – 21.27 **(n)** -31.4 + 39.2 **(o)** -26.04 – (-15.97) **(p)** 13.92 – (-17.64)

3 Calculate the following, writing your answers in simplest form. WE9

(a) $-\frac{3}{4} - \frac{3}{4}$ **(b)** $-\frac{1}{5} + \left(-\frac{3}{5}\right)$ **(c)** $\frac{2}{7} - \frac{5}{7}$ **(d)** $-\frac{8}{9} - \left(-\frac{1}{9}\right)$

(e) $-\frac{3}{5} + \left(-\frac{1}{4}\right)$ **(f)** $\frac{7}{8} - \frac{9}{10}$ **(g)** $-\frac{1}{6} - \left(-\frac{1}{5}\right)$ **(h)** $-\frac{2}{3} + \frac{5}{6}$

(i) $-\frac{2}{5} + 1$ **(j)** $-\frac{1}{2} + 3$ **(k)** $-1\frac{1}{3} - (-2)$ **(l)** $1 + \left(-1\frac{1}{4}\right)$

(m) $-1\frac{1}{2} + 2\frac{1}{4}$ **(n)** $-3\frac{1}{3} - 2\frac{1}{2}$ **(o)** $4\frac{1}{5} - \left(-\frac{7}{10}\right)$ **(p)** $1\frac{3}{4} + \left(-\frac{5}{6}\right)$

4 Calculate the following, writing your answers in simplest form.

(a) $-\frac{3}{7} \times \frac{1}{2}$ **(b)** $-\frac{1}{2} \div \frac{1}{4}$ **(c)** $\frac{5}{2} \times -\frac{1}{3}$ **(d)** $\frac{3}{5} \div -\frac{1}{3}$

(e) $-4 \div \frac{1}{8}$ **(f)** $-\frac{1}{6} \times -\frac{3}{4}$ **(g)** $6 \div -\frac{2}{3}$ **(h)** $-\frac{5}{8} \times -\frac{4}{5}$

(i) $-2\frac{1}{2} \times -1\frac{3}{5}$ **(j)** $3\frac{2}{3} \div -\frac{1}{2}$ **(k)** $-4\frac{1}{6} \times 1\frac{3}{5}$ **(l)** $-6\frac{3}{4} \div \frac{9}{10}$

5 Calculate the following, rounding answers to two decimal places where necessary.

(a) -0.4×0.8 **(b)** $-0.6 \div 2$ **(c)** -3×1.5 **(d)** $2.7 \div -3$

(e) $-0.72 \div -0.3$ **(f)** -12.5×-7.2 **(g)** $-1.2 \div 0.5$ **(h)** 0.05×-7.4

(i) 14.25×6.3 **(j)** $24 \div -3.4$ **(k)** 29.5×-11.23 **(l)** $-56.2 \div 10.1$

6 Copy the following number line. Use arrows to show the locations of the following fractions. (You may need to estimate the position of some numbers.)

$-\frac{2}{3}, \frac{4}{3}, -1\frac{1}{2}, \frac{11}{6}, -\frac{7}{3}$

7 Copy the following number line. Use arrows to show the locations of the following decimals. (You may need to estimate the position of some numbers.)

-0.7, 1.3, -1.1, 0.65, -0.05

8 The number closest to the position indicated by the arrow on the number line is:

A -2.6 **B** -1.4 **C** -0.3 **D** 1.4

Understanding

9 The following table shows the maximum and minimum temperatures recorded at Thredbo during 5 days in winter.

Temp (°C)	Mon	Tues	Wed	Thur	Fri
Maximum	5.2	-0.8	6.1	3.5	-1.2
Minimum	-1.1	-4.3	-1.7	-2.9	-5.6

(a) On which day was the coolest maximum recorded?

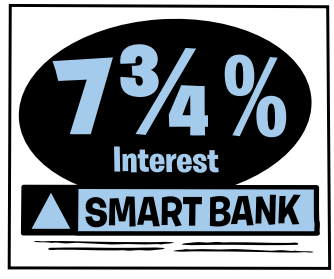
(b) On which day was the warmest minimum recorded?

(c) On which day was the difference between the maximum and minimum temperatures the greatest?

(d) Calculate **(i)** the mean maximum temperature and **(ii)** the mean minimum temperature, to one decimal place.

10 The executives of a bank meet every month to decide whether to raise or lower interest rates. Their decisions over an 8-month period were reported as follows.

Increase by 1.2%, no change, increase by $\frac{3}{4}$ %, decrease by 0.8%, decrease by $1\frac{1}{4}$ %, no change, decrease by $\frac{1}{2}$ %, decrease by 0.6%.



(a) Write each of the percentage changes, using a positive or a negative sign to show increases or decreases.

(b) Work out the overall increase or decrease in interest rates at the end of the 8-month period. Write your answer in **(i)** fraction form and **(ii)** decimal form, using a negative sign if necessary.

11 Write the following in order from smallest to largest: -3, 0, $\frac{3}{10}$, 1.7, -3.4, $-\frac{3}{4}$, -0.9, 1. Use a number line, if necessary, to help you.

12 Write the following in order from smallest to largest. Use a number line, if necessary, to help you. $-2\frac{1}{5}$, 0, -1.8, $-\frac{5}{4}$, 1.1, $\frac{4}{5}$, -2, $\frac{3}{4}$

13 Use your calculator to evaluate the following. Write answers correct to three decimal places.

(a) $(78.9 \times 3.4) - (54.8 \times 7.77)$

(b) $(54.9 \times 5.6) - (6.2 \times 103.7)$

(c) $(-14.7 + 5.8) \times (4.3 - 8.25)$

(d) $(56 - 72.8) \times (-12.3 + 17.1)$

(e) $\sqrt{7} - \sqrt{3} - \sqrt{6}$

(f) $\sqrt{3} - \sqrt{2} - (\sqrt{5})^2$

(g) $(2.3)^4 - (5.9)^5 + (1.5)^2$

(h) $(\sqrt{6})^2 - (10.5)^4 - (1.005)^2$

(i) $(3.8 - 5.7)^2 - (\sqrt{8})$

(j) $(2\sqrt{10})^2 - (53.66)^2$

Reasoning

14 Chelsea receives a statement from her bank every month. The statement is a list of all the transactions (deposits and withdrawals) made on the account, and the account balance (how much money is in the account after each transaction). Here is part of Chelsea's statement:

Date	Transaction	Balance
1 May		$257.89
7 May	ATM Withdrawal $180.00	$77.89
12 May	Cheque deposit $65.00	$142.89
16 May	Shopping paid for by EFTPOS $97.63	$45.26
23 May	Insurance, paid by direct debit $58.17	-$12.91
30 May	Salary deposit _____	$460.94

(a) Locate the negative number on Chelsea's statement. What is the '-' symbol used to indicate here?

(b) On 27 May, Chelsea went to an ATM and tried to withdraw $50, but the machine would not give her any money! Why not?

(c) What was the value of Chelsea's salary that was deposited on 30 May?

15 Use a number line to explain why $\frac{1}{2} > \frac{1}{4}$ but $-\frac{1}{2} < -\frac{1}{4}$.

Open-ended

16 Write down three decimal numbers that are greater than -2.1 but less than -1.2.

17 Write down three numbers in fraction form that are greater than $-\frac{7}{8}$ but less than 0.

18 Douglas is having trouble with negative decimals. He can add and subtract positive numbers successfully, but often gets questions with negatives wrong. Here are two questions that Douglas has done incorrectly:

$-8.3 + (-4.5) = -3.8$

$-6.7 - (-2.9) = -9.6$

(a) Explain the errors that Douglas has made.

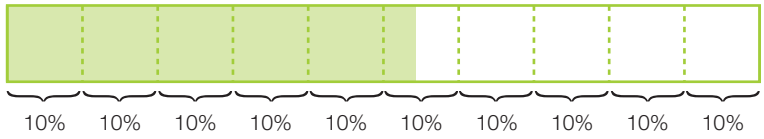
(b) Give him some tips as to how he can do these types of questions successfully.

2.4 Estimating percentages

The word **per cent** means 'for every hundred' or 'out of a hundred'. The symbol % is used for per cent—it even looks a little like a rearranged 100. A **percentage** is a value out of 100.

It's sometimes useful to imagine what a particular percentage looks like. For example, you might describe the painting of this wall as being about 60% complete.

A strategy for estimating a percentage of an amount is to mentally split the object up into 10 lots of 10%. (Recall that 100% represents the whole object.) If we divide up the rectangle below in such a way, we can see that the shaded area is somewhere between 50% and 60%.



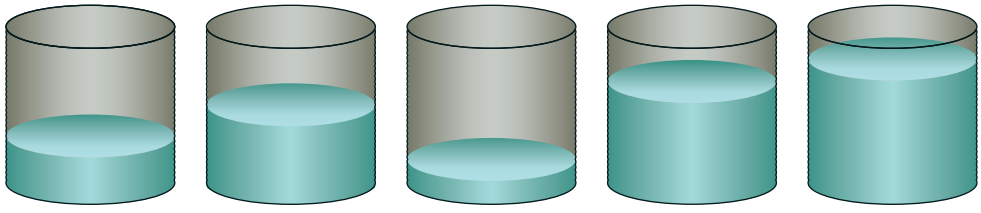
As the shading falls a little short of halfway between 50% and 60%, a better estimate of the shading in the rectangle above would be 53%.

Worked Example 10 WE10

Which of the following water tanks is approximately:

(a) 50% full **(b)** 15% full **(c)** 80% full?

(i) **(ii)** **(iii)** **(iv)** **(v)**



Thinking	Working
(a) 1 Is the percentage required bigger, smaller or equal to a half?	(a) 50% = half
2 50% is half full. Which tank looks half full?	Tank (ii) is 50% full.
(b) 1 Is the percentage required bigger, smaller or equal to a half?	(b) 15% < 50%

2	15% is less than one half, so there are two possibilities—tanks **(i)** and **(iii)**. Compare them by mentally dividing them into 10% pieces and counting how many are full.	Tank (iii) is 15% full.
(c) 1	Is the percentage required bigger, smaller or equal to a half?	(c) 80% > 50%
2	80% is greater than one-half, so there are two possibilities—tanks **(iv)** and **(v)**. Compare them by mentally dividing them into 10% pieces and counting how many are full.	Tank (v) is 80% full.

2.4 Estimating percentages

Navigator

Q1, Q2, Q3, Q4, Q5, Q7, Q9, Q11	Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11	Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11

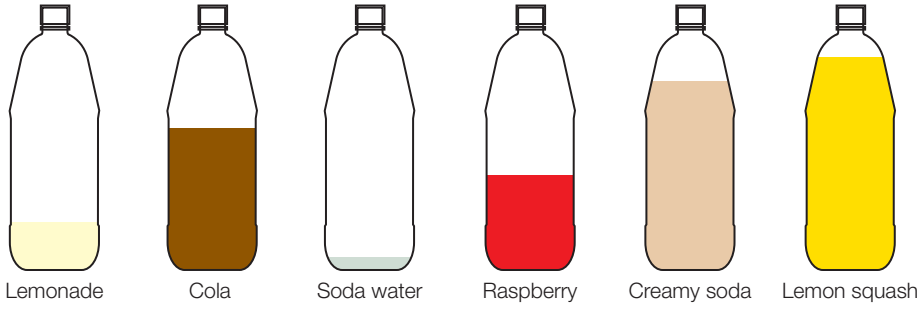
Answers page 600

Fluency

WE10

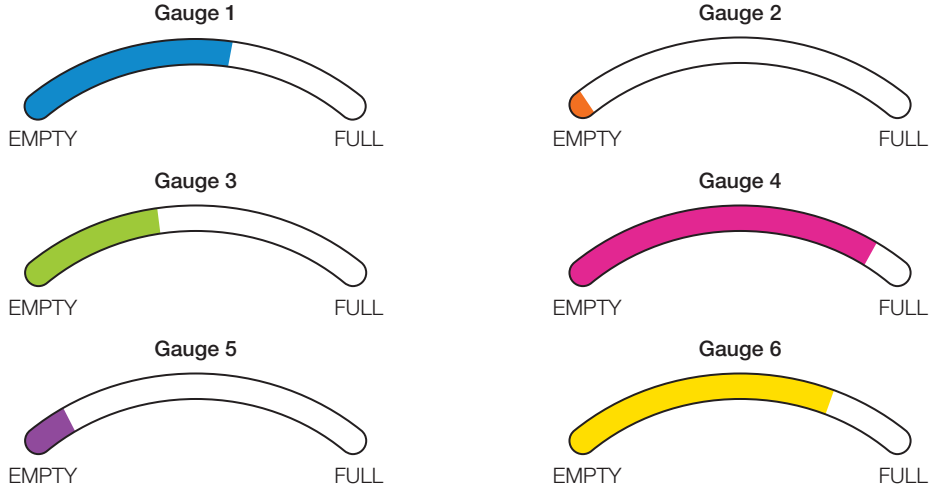
1 Which of the following bottles of soft drink is approximately:

(a) 5% full **(b)** 90% full **(c)** 80% full **(d)** 20% full?



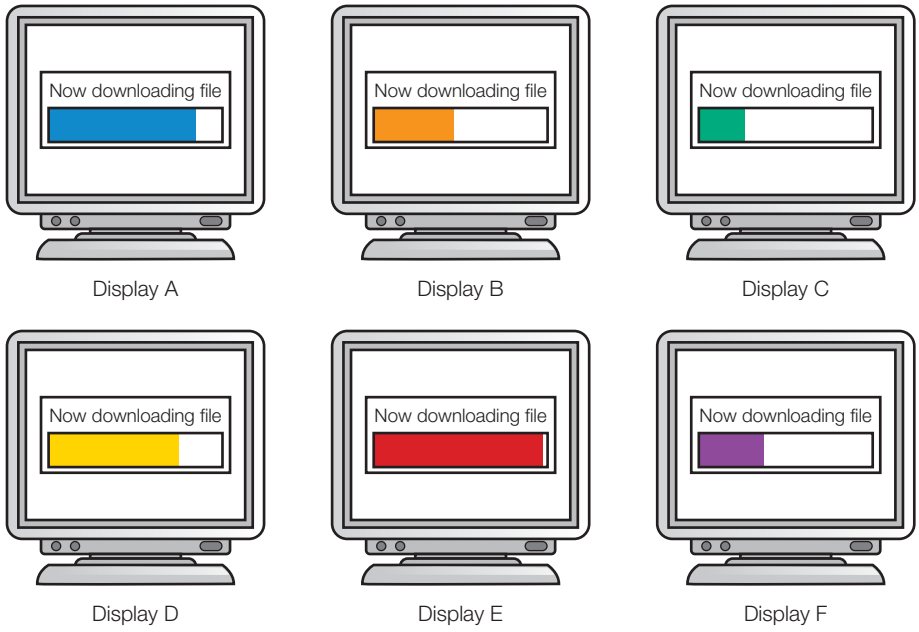
2 Which of the following fuel gauges shows a petrol tank that is approximately:

(a) 40% full **(b)** 15% full **(c)** 60% full **(d)** 75% full?



3 A computer displays the amount of a file it has downloaded by gradually filling in a bar until the file has been fully saved. Which display shows approximately:

(a) 33% saved **(b)** 70% saved **(c)** 82% saved **(d)** 22% saved?



4 Draw six displays similar to those in Question **3** that show approximately:

(a) 10% of a file downloaded **(b)** 70% of a file downloaded

(c) 25% of a file downloaded **(d)** 57% of a file downloaded

(e) 88% of a file downloaded **(f)** 31% of a file downloaded.

5 **(a)** Approximately what percentage of the rectangle is shaded?

A 10% **B** 30% **C** 60% **D** 80%

(b) Approximately what percentage of the square is shaded?

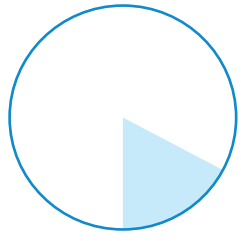
A 15% **B** 30%

C 45% **D** 75%

(c) Approximately what percentage of the circle is shaded?

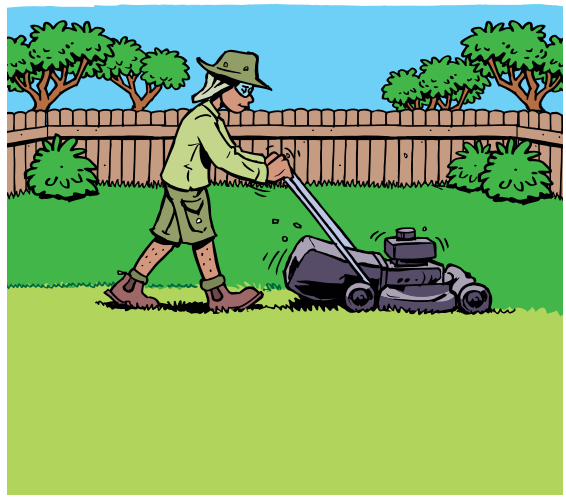
A 16% **B** 27%

C 55% **D** 78%



Understanding

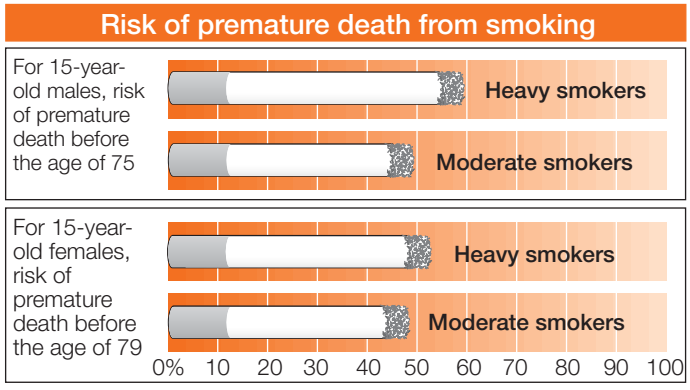
6 What percentage of lawn has John got left to mow?



7 The water dispenser in Larry's office appears as shown. Describe how full the dispenser is, using a percentage value.

Reasoning

8 The graph shows the risk of premature death for 15-year-old smokers.



Estimate the percentage risk of premature death for:

(a) 15-year-old male heavy smokers

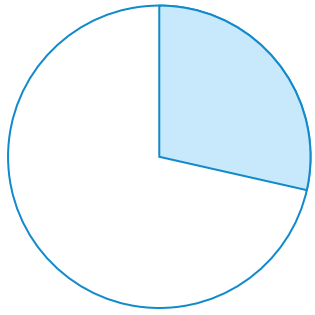
(b) 15-year-old male moderate smokers

(c) 15-year-old female heavy smokers

(d) 15-year-old female moderate smokers.

(e) Calculate the percentage difference between the risks for heavy and moderate smokers in males and females.

9 28% of this circle is shaded.



(a) Draw three similar sized circles, then shade in your estimate of:

(i) 40% of the circle

(ii) $\frac{5}{6}$ of the circle

(iii) 0.7 of the circle.

(b) In part **(a)**, did you find it easier to estimate the percentage, the decimal or the fraction amount? Why do you think this was the case?

Open-ended

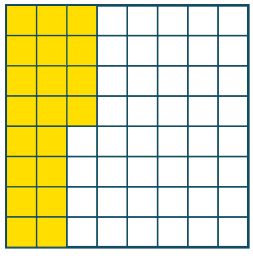
10 **(a)** Draw at least three different shapes and shade 25% of each of them.

(b) Redraw the shapes from **(a)** and shade a different 25% of them.

(c) Redraw the shapes from **(a)** and shade 65% of them.

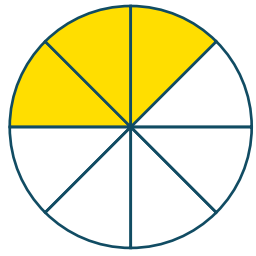
11 Helga has worked out the percentage of the areas shaded as follows.

(a)



24% shaded

(b)



30% shaded

Explain Helga's mistake in her calculations for each shape.

Outside the Square Game

Best estimate

Equipment required: 2 brains

How to win:
The aim is to get the most points in five rounds.

How to play:
On a piece of paper, draw a bar 10 cm long and 1 cm wide. This bar is going to represent 100%.

100%

Using a ruler, Player One draws another bar on a separate piece of paper that is shorter than the 100% bar. They must know the exact percentage it represents.

Player Two has to decide what percentage the second bar represents. (No rulers allowed, but you can move the paper about.)

Once Player Two has guessed, Player One states the actual answer and the error is written down.

For example: If Player Two guesses 80% when the actual answer is 83%, then the error is 3.

The roles then swap roles.

The player with the lowest error earns a point for that round.

Play best of five rounds.

Alternatives:
Change the length of the 100% bar to 20 cm.

Try guessing percentages that are greater than 100%.

Writing fractions and decimals as percentages

2.5

As percentages are easy to compare and are often easier than fractions for people to visualise, it is useful to be able to convert fractions and decimals into percentages. Some percentages are used so frequently that it is useful to be able to automatically recall their fraction or decimal equivalent. If you don't already know them, try to learn the following.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{1}{50}$	$\frac{1}{100}$
Decimal	0.5	0.25	0.75	$0.\dot{3}$	$0.\dot{6}$	0.2	0.1	0.05	0.02	0.01
Percentage	50%	25%	75%	$33.\dot{3}\%$	$66.\dot{6}\%$	20%	10%	5%	2%	1%

With the exception of $\frac{1}{3}$ and $\frac{2}{3}$, writing the fractions in the above table as percentages is straightforward. Because the denominators are factors of 100, we can simply convert them to fractions out of 100. For example:

$$\frac{3}{4} \overset{\times 25}{\underset{\times 25}{=}} \frac{75}{100} = 75\%$$

$$\frac{1}{5} \overset{\times 20}{\underset{\times 20}{=}} \frac{20}{100} = 20\%$$

However, for fractions that have denominators other than 2, 5, 10, 25 or 50 (or fractions that can be simplified to these denominators), we need to use another method.

To write a fraction or decimal as a percentage, we multiply the fraction or decimal by 100%.

There are two methods we can use to multiply a fraction by 100%. We can multiply two fractions together, as shown in Method 1 of the following Worked Example, or we can convert the fraction to a decimal, then multiply by 100%, as shown in Method 2 of the Worked Example. Method 1 is a more convenient 'pen and paper' method, whereas Method 2 is useful if you have a calculator.

Multiplying by 100% is equivalent to multiplying by 1. If we multiply a fraction or a decimal by 100%, the value of the fraction or decimal is not being changed, it is just being represented in a different way.

Worked Example 11

WE 11

Write the following fractions as percentages. Write any answers that are not whole numbers in both fraction and exact decimal form.

(a) $\frac{7}{8}$

(b) $\frac{11}{15}$

Method 1: Fraction multiplication

Thinking	Working
(a) 1 Multiply the fraction by 100% by writing 100% as an improper fraction, then performing the multiplication. Simplify the multiplication by cancelling common factors first. (Here, the common factor is 4.)	(a) $\frac{7}{8} \times 100\%$ $= \frac{7}{\cancel{8}_{2}} \times \frac{\cancel{100}^{25}}{1}\%$ $= \frac{175}{2}\%$
2 Simplify the answer.	$= 87\frac{1}{2}\%$ or 87.5%
(b) 1 Multiply the fraction by 100% by writing 100% as an improper fraction, then performing the multiplication. Simplify the multiplication by cancelling common factors first. (Here, the common factor is 5.)	(b) $\frac{11}{15} \times 100\%$ $= \frac{11}{\cancel{15}_{3}} \times \frac{\cancel{100}^{20}}{1}\%$ $= \frac{220}{3}$
2 Simplify the answer.	$= 73\frac{1}{3}\%$ or $73.\dot{3}\%$

Method 2: Convert to a decimal, then multiply

Thinking	Working
(a) Convert the fraction to decimal form first, then multiply the decimal by 100%.	(a) $\frac{7}{8} \times 100\%$ $= 7 \div 8 \times 100\%$ $= 0.875 \times 100\%$ $= 87.5\%$ or $87\frac{1}{2}\%$
(b) Convert the fraction to decimal form first, then multiply the decimal by 100%. (When working with recurring decimals, write out a couple of decimal places so you can place the decimal point in the correct position.)	(b) $\frac{11}{15} \times 100\%$ $= 11 \div 15 \times 100\%$ $= 0.7333... \times 100\%$ $= 73.\dot{3}\%$ or $73\frac{1}{3}\%$

Worked Example 12

WE 12

Write the following as a percentage: $3\frac{1}{2}$

Method 1: Fraction multiplication

Thinking	Working
1 Write the mixed number as an improper fraction. Write 100% as an improper fraction, then multiply the two fractions. Simplify the multiplication by cancelling common factors first. (Here, the common factor is 2.)	$3\frac{1}{2} \times 100\%$ $= \frac{7}{\cancel{2}_{1}} \times \frac{\cancel{100}^{50}}{1}\%$ $= \frac{350}{1}\%$
2 Simplify the answer.	$= 350\%$

Method 2: Convert to a decimal, then multiply

Thinking	Working
Convert the mixed number or decimal form first, then multiply the decimal by 100%.	$3\frac{1}{2} \times 100\%$ $= 3.5 \times 100\%$ $= 350\%$

Worked Example 13

WE 13

Find the percentage equivalent of each of the following.

(a) 0.8 **(b)** 1.65 **(c)** 0.032

Thinking	Working
(a) Multiply by 100%. Show this by moving the decimal point two places to the right. Fill in empty place value columns with zeroes.	(a) $0.8 = 0.8 \times 100\%$ $= 80\%$
(b) Multiply by 100%. Show this by moving the decimal point two places to the right.	(b) $1.65 = 1.65 \times 100\%$ $= 165\%$
(c) Multiply by 100%. Show this by moving the decimal point two places to the right.	(c) $0.032 = 0.032 \times 100\%$ $= 3.2\%$

2.5 Writing fractions and decimals as percentages

Navigator

Answers page 601

Q1 Columns 1–3, Q2 Columns 1 & 2, Q3 Columns 1–3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q13, Q17, Q18	Q1 Columns 2–4, Q2 Columns 2 & 3, Q3 Columns 3 & 4, Q5, Q6, Q8, Q9, Q10, Q11, Q12, Q13, Q15, Q17, Q18	Q1 Columns 3 & 4, Q2 Columns 3 & 4, Q3 Columns 3 & 4, Q5, Q6, Q8, Q9, Q11, Q12, Q14, Q15, Q16, Q18, Q19

Equipment required: A calculator may be used for Questions **9**, **11**, **15**, **16**, **17** and **19**

Fluency

WE11

1 Write the following fractions as percentages. Write any answers that are not whole numbers in both fraction and exact decimal form.

(a) $\frac{9}{100}$ **(b)** $\frac{1}{10}$ **(c)** $\frac{7}{50}$ **(d)** $\frac{19}{20}$

(e) $\frac{3}{5}$ **(f)** $\frac{1}{4}$ **(g)** $\frac{3}{2}$ **(h)** $\frac{16}{8}$

(i) $\frac{1}{3}$ **(j)** $\frac{2}{3}$ **(k)** $\frac{1}{6}$ **(l)** $\frac{2}{9}$

(m) $\frac{32}{40}$ **(n)** $\frac{9}{16}$ **(o)** $\frac{56}{80}$ **(p)** $\frac{14}{32}$

(q) $\frac{73}{80}$ **(r)** $\frac{13}{60}$ **(s)** $\frac{55}{66}$ **(t)** $\frac{23}{90}$

WE12

2 Write the following as percentages. Write any answers that are not whole numbers in exact decimal form.

(a) $1\frac{1}{4}$ **(b)** $1\frac{2}{5}$ **(c)** $3\frac{1}{5}$ **(d)** $5\frac{1}{2}$

(e) $2\frac{3}{8}$ **(f)** $2\frac{3}{4}$ **(g)** $5\frac{7}{10}$ **(h)** $4\frac{5}{8}$

(i) $4\frac{2}{3}$ **(j)** $5\frac{1}{3}$ **(k)** $1\frac{7}{9}$ **(l)** $7\frac{5}{6}$

WE13

3 Find the percentage equivalent of each of the following.

(a) 0.9 **(b)** 0.4 **(c)** 0.8 **(d)** 0.6

(e) 0.17 **(f)** 0.47 **(g)** 0.82 **(h)** 0.53

(i) 0.051 **(j)** 0.438 **(k)** 0.007 **(l)** 0.342

(m) 9.2 **(n)** 5.1 **(o)** 2.02 **(p)** 9.01

4 **(a)** The percentage equivalent of 0.7 is:

A 0.07% **B** 0.70% **C** 7% **D** 70%

(b) The percentage equivalent of 3.45 is:

A 0.0345% **B** 0.345% **C** 34.5% **D** 345%

5 (a) Which of the following is equivalent to $\frac{9}{50}$?

A 9% B 18% C 45% D 90%

(b) Which of the following is equivalent to $\frac{3}{20}$?

A 3% B 12% C 15% D 30%

6 Copy and complete the table of commonly used fractions, decimals and percentages.

Percentage	5%			20%	25%	$33\frac{1}{3}$%	40%		60%	$66\frac{2}{3}$%		80%	100%
Fraction			$\frac{1}{8}$								$\frac{3}{4}$		
Decimal		0.1						0.5					1

Understanding

7 Lou estimated that $\frac{7}{10}$ of the crowd at a rugby match were Broncos supporters. What percentage is this?

8 Due to a cyclone wiping out most of the crop, the price of bananas rose to $3\frac{3}{4}$ times the original price. What percentage of the old price was this?

9 In one term, Patrick achieved the following test results in maths.

Measurement $\frac{33}{40}$ Algebra $\frac{26}{30}$ Geometry $\frac{12}{15}$

(a) Convert Patrick's test results to percentages. Round your answers to the nearest whole percentage.

(b) In which topic did Patrick achieve his best result?

10 A teacher calculates her students' scores for a test by dividing the marks obtained by the total marks to get a decimal score. Part of her mark book appears as follows.

Student	Result
Agnes	0.675
Basil	0.95
Carlotta	0.9
Daphne	0.55
Elmer	0.425
Felix	0.825

Find the percentage mark obtained by:

(a) Agnes (b) Basil (c) Carlotta

(d) Daphne (e) Elmer (f) Felix

11 Rebecca needs to score at least 80% on the theory test for her Learner's Permit. She knows that the test contains 40 questions. What is the minimum number of questions Rebecca must get correct in order to receive her Learner's Permit?

12 A chemist testing a rock sample found that it contained 0.246 parts of calcium, 0.034 parts of magnesium and 0.0012 parts of potassium. Determine the percentage equivalent of each of these three elements.

13 For each of these sets of numbers, construct the percentage number line shown and indicate the position

of each number with an arrow. (You may need to estimate the position of some numbers.)

(a) 25%, $\frac{2}{5}$, 0.04, $\frac{13}{20}$

(b) 3%, 0.3, $\frac{3}{4}$, 34%

14 For each of these sets of numbers, construct the number line shown and indicate the position of each number with an arrow. (You may need to estimate the position of some numbers.)

(a) 120%, 0.8, 52.5%, 1.95, $\frac{11}{10}$

(b) 115%, 1.01, $\frac{5}{3}$, 270%, $\frac{14}{25}$

15 An advertisement for a particular brand of bread claims that it now has '20% more fibre!' The nutritional information table on the packaging shows that a 40 g serve of bread contains 3.4 g of fibre. Before the improvement was made, the same 40 g serving contained 2.8 g of fibre.

(a) Write the increase in the amount of fibre as a fraction of the original amount, in simplest form.

(b) Convert this fraction to a percentage, rounding your answer to two decimal places.

(c) Is the bread manufacturer's claim of '20% more fibre' justified? Explain your answer.

Reasoning

16 Imogen was comparing two toy stores to determine in which store the greater percentage of people entering the store actually bought goods. She wanted to know if the suburban store had a greater percentage of paying customers than the store in the city. She wrote the number of customers who bought goods as a fraction of the total number of customers who entered the store. The results for the suburban store over a 3-week period were $\frac{122}{200}$, $\frac{143}{220}$ and $\frac{275}{330}$. The results for the city store were $\frac{192}{400}$, $\frac{168}{320}$ and $\frac{336}{480}$.

(a) Calculate the percentage of paying customers for each week for the suburban store.

(b) Calculate the percentage of paying customers for each week for the city store.

(c) Which store had the greater percentage of paying customers, and what was the percentage difference for each week?

(d) For each store, calculate the fraction of the total number of people entering the store who bought goods over the 3-week period.

(e) Which store had the greater percentage over the 3-week period, and by what percentage?

17 A nutritionist analysing a sample of savoury biscuits obtained the following results.

Nutrient	Proportion
Protein	0.133 parts
Dietary fibre	0.03 parts
Sodium	0.0073 parts
Carbohydrate (of which 0.005 is sugar)	0.594 parts
Fat (of which 0.084 is saturated)	0.172 parts

(a) Calculate the percentage equivalent of protein, dietary fibre, sodium, carbohydrate and fat in the sample.

(b) Calculate the percentage equivalent of the fat content in the sample that is not made up of saturated fats.

(c) Calculate the percentage equivalent of the carbohydrate content in the sample that is not sugar.

(d) A batch is rejected if it is found to contain greater than 1% sodium. The next batch analysed was found to have an extra 0.0052 parts of sodium compared with the batch above. Calculate the percentage equivalent of sodium of this batch to determine if this batch will be accepted.

Open-ended

18 Fill in each box below with one of the digits 0–9 to make the equation correct. Try to find three different combinations.

□.3□ = □□□%

19 Write at least three different fractions that have equivalent percentage values between 68% and 75%. Only one of them can have a denominator of 100.

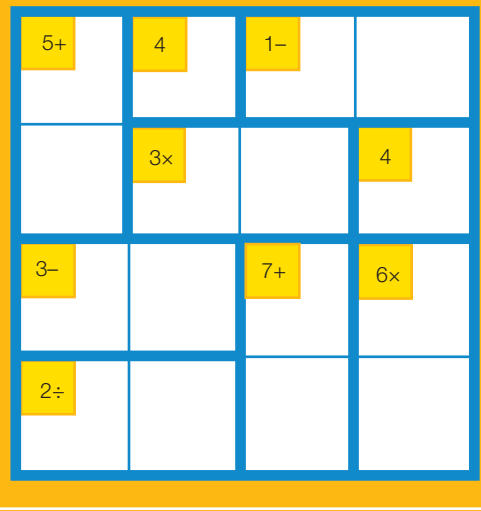
Outside the Square Puzzle

KenKen

Equipment required: 1 brain, 1-cm grid paper

To complete this puzzle, each row and column must contain the numbers 1–4 inclusive once and only once. The clue at the top left-hand corner of the outlined shapes means that the numbers in that shape need to do what is written. For example, 3+ means that the numbers add to 3 and so they have to be 2 and 1. However, you don't yet know the order in which they appear. An outlined shape that contains only one yellow square with a number in it must contain that number.

Copy this puzzle onto your grid paper and try to solve it.



Half-time 2

Ex. 2.4

1 Estimate the percentage of the jug that contains cordial.

Ex. 2.2

2 Express the following in decimal form, then state whether they are terminating decimals, recurring decimals, or irrational numbers.

(a) $15 \div 6$ (b) $\frac{7}{11}$

(c) $\sqrt{54}$ (d) $\sqrt{174.24}$

(e) $83 \div 12$ (f) $\sqrt{75}$

Ex. 2.1

3 Hasharan is cutting ribbon to wrap around gift boxes. She has a 4 m roll of ribbon, and needs 35 cm of ribbon per box.

(a) How many gift boxes will Hasharan be able to wrap?

(b) What length of ribbon will she have left over?

Ex. 2.5

4 Write each of the following fractions and decimals as percentages.

(a) 4.56 (b) $\frac{3}{4}$ (c) $\frac{71}{80}$ (d) 0.432 (e) $1\frac{2}{5}$

Ex. 2.2

5 Write the following recurring decimals in fraction form.

(a) $1.\dot{3}$ (b) $3.2\dot{2}\dot{6}$ (c) $5.1\overline{902}$

Ex. 2.1–2.3

6 Write the following in order from smallest to largest.

(a) $-\frac{5}{9}$, $\sqrt{5}$, $\frac{3}{8}$, -0.7, $\frac{1}{2}$, 1.099 (b) -1.23, 0.874, $-\frac{2}{3}$, $\frac{17}{8}$, $\sqrt{13}$

Ex. 2.2

7 Write each of the following in exact decimal form.

(a) $\frac{2}{3}$ (b) $\frac{5}{7}$ (c) $\frac{1}{9}$ (d) $\frac{5}{11}$

Ex. 2.1

8 Write the following decimals as fractions or as mixed numbers in simplest form.

(a) 0.438 (b) 1.072 (c) 2.0506 (d) 13.7005

Ex. 2.3

9 Calculate the following.

(a) $-\frac{1}{5}+\frac{1}{2}$ (b) $-\frac{1}{4}\times\left(-\frac{6}{7}\right)$ (c) $-\frac{3}{10}-\left(-\frac{7}{12}\right)$ (d) $-\frac{5}{4}\div\frac{2}{3}$

(e) $-7.8 + (-5.49)$ (f) $0.3 - (-11.46)$ (g) -2.6×3.05 (h) $18 \div -1.2$

Ex. 2.5

10 The fraction of the human body that is water varies between $\frac{11}{20}$ and $\frac{13}{20}$. Write this fraction range as a percentage range.

Writing percentages as fractions and decimals

2.6

Many shoppers know that a '50% off' sale is the same as a half-price sale. We can write 50% as $\frac{50}{100}$ or to 50 ÷ 100, which is equivalent to the decimal 0.5, or to the fraction $\frac{1}{2}$.

To write a percentage as a fraction or a decimal:

- Divide the value of the percentage by 100.
- For a fraction, write the division by 100 in fraction form, then simplify if possible.
- For a decimal, divide by 100. Show this by moving the decimal point two places to the left.

Worked Example 14

WE14

Write the following percentages as fractions in simplest form.

(a) 16% **(b)** 120%

Thinking	Working
(a) 1 Divide the value of the percentage by 100.	(a) 16% $= 16 \div 100$
2 Write the division as a fraction and simplify if possible.	$= \frac{16}{100}$ $= \frac{4}{25}$
(b) 1 Divide the value of the percentage by 100.	(b) 120% $= 120 \div 100$
2 Write the division as a fraction and simplify if possible.	$= \frac{120}{100}$ $= \frac{6}{5}$ or $1\frac{1}{5}$

Worked Example 15

WE15

Write the following 'fractional percentages' as fractions in simplest form.

(a) $\frac{1}{4}\%$ **(b)** $6\frac{2}{3}\%$

Thinking	Working
(a) 1 Divide the value of the percentage by 100.	(a) $\frac{1}{4}\% = \frac{1}{4} \div 100$
2 Perform the division (recall that to divide by a fraction, you must multiply by its inverse).	$= \frac{1}{4} \div \frac{100}{1}$ $= \frac{1}{4} \times \frac{1}{100}$
3 Write the answer, simplifying if possible.	$= \frac{1}{400}$
(b) 1 Divide the value of the percentage by 100.	(b) $6\frac{2}{3}\% = 6\frac{2}{3} \div 100$
2 Write both values as improper fractions so that the division can be performed.	$= \frac{20}{3} \div \frac{100}{1}$
3 Perform the division (recall that to divide by a fraction, you must multiply by its inverse).	$= \frac{20}{3} \times \frac{1}{100}$ $= \frac{20}{300}$
4 Write the answer, simplifying if possible.	$= \frac{2}{30}$ $= \frac{1}{15}$

Worked Example 16

WE16

Write the following percentages as fractions in simplest form.

(a) 48.2% **(b)** 3.75%

Thinking	Working
(a) 1 Divide the value of the percentage by 100.	(a) $48.2\% = 48.2 \div 100$
2 Write the division as a fraction, then multiply the numerator and denominator by a power of 10 to remove the decimal point. (Here, we multiply by 10.)	$= \frac{48.2}{100}$ $= \frac{482}{1000}$
3 Simplify the answer if possible. (Here, we divide by a common factor of 2.)	$= \frac{241}{500}$

(b) **1**	Divide the value of the percentage by 100.	(b) $3.75\% = 3.75 \div 100$
2	Write the division as a fraction, then multiply the numerator and denominator by a power of 10 to remove the decimal point. (Here, we multiply by 100.)	$= \frac{3.75}{100}$ $= \frac{375}{10000}$
3	Simplify the answer if possible. (Here, we divide by a common factor of 25, then 5.)	$= \frac{15}{400}$ $= \frac{3}{80}$

Worked Example 17

WE17

Write the following percentages as decimals.

(a) 23% **(b)** 171% **(c)** 3.49%

Thinking	Working
(a) Divide the value of the percentage by 100. Show this by moving the decimal point two places to the left.	(a) $23\% = 23 \div 100$ $= 0.23$
(b) Divide the value of the percentage by 100. Show this by moving the decimal point two places to the left.	(b) $171\% = 171 \div 100$ $= 1.71$
(c) Divide the value of the percentage by 100. Show this by moving the decimal point two places to the left. Fill in any empty place value columns with zeroes.	(c) $3.49\% = 3.49 \div 100$ $= 0.0349$

Worked Example 18

WE18

Write the following 'fractional percentages' as decimals.

(a) $\frac{1}{2}\%$ **(b)** $3\frac{1}{5}\%$

Thinking	Working
(a) **1** Write the fraction value as a decimal value.	(a) $\frac{1}{2}\% = 0.5\%$
2 Divide the decimal value by 100.	$0.5 \div 100$ $= 0.005$
(b) **1** Write the mixed number value as a decimal value.	(b) $3\frac{1}{5}\% = 3.2\%$
2 Divide the decimal value by 100.	$3.2 \div 100$ $= 0.032$

It is important to remember the difference between $\frac{1}{2}$ % and $\frac{1}{2}$. $\frac{1}{2}$% means 'one half of 1%'. This is one half of 0.01, or 0.005. $\frac{1}{2}$ means 'one half of the whole', which is 50%.

2.6 Writing percentages as fractions and decimals

Navigator

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Q1, Q2 Columns 1 & 2, Q3 Columns 1 & 2, Q4 Columns 1–3, Q5 Columns 1 & 2, Q8, Q9, Q10, Q11, Q12, Q14 (a) & (b), Q15, Q16, Q19	Q1 Columns 2–4, Q2 Columns 1–3, Q3 Columns 1–3, Q4 Columns 3 & 4, Q5 Columns 1–3, Q6, Q7, Q10, Q12, Q13, Q14, Q15, Q16, Q17, Q18, Q19	Q1 Columns 3 & 4, Q2 Columns 2–4, Q3 Columns 2–4, Q4 Columns 3 & 4, Q5 Columns 2–4, Q7, Q10, Q12, Q13, Q14 (c) & (d) Q15, Q16, Q17, Q18, Q19

Equipment required: A calculator may be used for Questions **5** and **18(a)**

Fluency

WE14

1 Write the following percentages as fractions in simplest form.

(a) 17% **(b)** 48% **(c)** 9% **(d)** 65%

(e) 58% **(f)** 76% **(g)** 117% **(h)** 129%

(i) 240% **(j)** 315% **(k)** 138% **(l)** 360%

WE15

2 Write the following 'fractional percentages' as fractions in simplest form.

(a) $\frac{1}{2}$% **(b)** $\frac{1}{5}$% **(c)** $\frac{3}{7}$% **(d)** $\frac{3}{8}$%

(e) $\frac{1}{10}$% **(f)** $\frac{2}{3}$% **(g)** $\frac{5}{6}$% **(h)** $\frac{2}{9}$%

(i) $2\frac{1}{4}$% **(j)** $6\frac{1}{2}$% **(k)** $5\frac{3}{5}$% **(l)** $8\frac{3}{4}$%

WE16

3 Write the following percentages as fractions in simplest form.

(a) 8.3% **(b)** 12.29% **(c)** 50.5% **(d)** 62.5%

(e) 2.1% **(f)** 3.4% **(g)** 0.55% **(h)** 0.4%

(i) 10.25% **(j)** 87.5% **(k)** 150.2% **(l)** 420.5%

WE17

4 Write the following percentages as decimals.

(a) 80% **(b)** 27% **(c)** 40% **(d)** 15%

(e) 66% **(f)** 92% **(g)** 3% **(h)** 5%

(i) 110% **(j)** 265% **(k)** 700% **(l)** 304%

(m) 0.5% **(n)** 7.2% **(o)** 40.5% **(p)** 6.25%

WE18

5 Write the following 'fractional percentages' as decimals.

(a) $\frac{1}{4}$% **(b)** $\frac{1}{8}$% **(c)** $\frac{7}{10}$% **(d)** $\frac{2}{5}$%

(e) $2\frac{1}{2}$% **(f)** $10\frac{3}{4}$% **(g)** $37\frac{1}{5}$% **(h)** $18\frac{4}{5}$%

(i) $25\frac{1}{3}$% **(j)** $5\frac{2}{3}$% **(k)** $2\frac{5}{9}$% **(l)** $23\frac{5}{6}$%

6 2.8% in its simplest fraction form is:

A $\frac{14}{500}$ B $\frac{28}{1000}$ C $\frac{7}{250}$ D $2\frac{4}{5}$

7 $\frac{3}{4}$% expressed as a fraction is:

A $\frac{3}{400}$ B $\frac{34}{100}$ C $\frac{3}{4}$ D $3\frac{4}{100}$

8 The decimal equivalent of 500% is:

A 50 000 B 5 C 0.5 D 0.05

Understanding

9 A TV advertisement stated that 86% of dentists recommend using 'Squeeky Clean' toothpaste. What fraction of dentists is this?

10 Gina has a $9\frac{1}{2}$% pay rise. What fraction of her salary does this represent?

11 Monica travelled 45% of the journey across Europe from Paris to Istanbul by train. What is the decimal equivalent of the percentage travelled by train?

12 Jacob received 92.5% on his Australian History test. What is the decimal equivalent of Jacob's result?

13 In rugby league, bruisings and strains are 30% of injuries, whereas more serious injuries such as fractures are 16% and concussions are 9%.

(a) Write each of these percentages as fractions.

(b) What fraction of the total number of injuries do bruisings, strains, fractures and concussions make up altogether?

(c) What fraction of the total number of injuries are due to other causes?

14 Place the following in ascending order.

(a) 20%, $\frac{1}{4}$, 0.02, $\frac{2}{5}$ (b) 0.45, $\frac{4}{5}$, 4.5%, $\frac{5}{4}$

(c) $\frac{3}{10}$, 0.03, $\frac{1}{3}$, 33% (d) 0.72, $7\frac{1}{2}$%, 7.2%, $\frac{7}{2}$

15 Employment figures show that 28% of those employed are tradespeople. What fraction of the employed workforce are tradespeople?

Reasoning

16 One-quarter of the 200 Year 8 students surveyed at St Gabriel's College said that they were driven to school and 30% said that they arrived by bus. What fraction of students are not driven to school and do not take the bus?

17 The Burandoc Water Authority is trying to estimate the volume of water available in the Burandoc Shire Reservoir. At present, it is at 85% capacity and it is estimated that if there is no significant rainfall over the next 6 months the volume would be reduced to 48% of capacity. However, if the Shire receives a normal amount of rainfall during this period the volume would only decrease by 17% of capacity.

(a) What fraction of the reservoir currently contains water?

(b) What fraction of the reservoir will contain water if there is no significant rainfall during the next 6 months?

(c) By what fraction would the water volume be reduced if there is no significant rainfall during the next 6 months?

(d) What fraction of the reservoir would remain unfilled if there is a normal amount of rainfall during this 6 months?

Open-ended

18 **(a)** Write at least three fractions with denominators less than 100 that have a percentage equivalent between 50% and 60%.

(b) Write at least three decimals that have a percentage equivalent between 25% and 26%. Each of the decimals given should have a different number of decimal places.

19 Valentino writes: $3\frac{1}{4} = 3.25\%$

(a) Explain to Valentino the difference between $3\frac{1}{4}$ and 3.25%.

(b) Correctly convert $3\frac{1}{4}$ into its percentage form.

Outside the Square Game

First to change

Equipment required: 2 brains, calculator, 1 die

How to win:
Each point of the game is won by a player who is the first to convert a percentage to a fraction in simplest form.

The first person to get 5 points wins.

How to play:
Roll the die two times and form a percentage from the numbers rolled, in the order in which they are rolled. For example, if a 3 is rolled and then a 6 the percentage is 36%.

The two players must then convert the percentage to a fraction in simplest form, either mentally or with pen and paper. The first with an answer taps the table and immediately gives their answer.

The answer is then checked on the calculator using the fraction button.

If the player is correct they get a point. If they are incorrect, their opponent gets a point.

For the above example, if the player gives $\frac{36}{100}$ or $\frac{18}{50}$ as their answer, their opponent gets a point, because the answer must be $\frac{9}{25}$.

Writing one amount as a percentage of another

2.7

In a recent netball match, Kate scored 17 goals from 23 shots at goal, whereas her team mate Mel scored 13 goals from 19 shots at goal. Who was the more accurate goal shooter? Kate got more shots in, but she had more attempts. Both girls missed 6 of their shots at goal. What do you think?

Percentages are very useful in these situations. Because a percentage value is always 'out of 100', we can easily compare them.

To write one amount as a percentage of another:

1. make sure both amounts are the same type, or measured in the same units; convert units if necessary
2. write a fraction with the 'part amount' as the numerator and the 'whole amount' as the denominator
3. convert this fraction into a percentage.

Not all calculations of percentage give whole number answers. Remainders can be written in fraction form, exact decimal form or as rounded decimals, depending on the situation. The S⇔D or $a^b/_c$ button on your calculator is useful for converting between answers in decimal form and fraction form.

Worked Example 19

WE19

Express the first amount as a percentage of the second. Give your answers in **(i)** fractional and **(ii)** exact decimal form.

(a) 23, 40 **(b)** 35, 30

Method 1: Multiply the fraction by 100%

Thinking	Working
(a) 1 Write a fraction with the first amount as the numerator and the second amount as the denominator.	(a) $\frac{23}{40}$

2	Multiply by $\frac{100}{1}$%, simplifying the multiplication by cancelling common factors. (Here, the common factor is 20.)	$\frac{23}{{}_{2}\cancel{40}} \times \frac{{}^{5}\cancel{100}}{1}\%$
3	Perform the simplified multiplication.	$= \frac{115}{2}\%$
4	Write your answer as a mixed number (fractional form), and as the equivalent decimal.	= (i) $57\frac{1}{2}$% or (ii) 57.5%
(b) 1	Write a fraction with the first amount as the numerator and the second amount as the denominator.	(b) $\frac{35}{30}$
2	Multiply by $\frac{100}{1}$%, simplifying the multiplication by cancelling common factors. (Here, common factors of 5, then 2, are cancelled.)	$\frac{{}^{7}\cancel{35}}{{}_{6}\cancel{30}} \times \frac{100}{1}\%$ $= \frac{7}{{}_{3}\cancel{6}} \times \frac{{}^{50}\cancel{100}}{1}\%$
3	Perform the simplified multiplication.	$= \frac{350}{3}\%$
4	Write your answer as a mixed number (fractional form), and as the equivalent decimal.	= (i) $116\frac{2}{3}$% or (ii) $116.\dot{6}$%

Method 2: Convert to a decimal, then multiply by 100%

Thinking		Working
(a) 1	Write a fraction with the first amount as the numerator and the second amount as the denominator.	(a) $\frac{23}{40}$
2	Convert the fraction to a decimal by performing the division. (A calculator could be used for this step.) Keep all of the decimal places (do not round off).	= 0.575
3	Multiply the decimal value by 100%.	$0.575 \times 100\%$
4	Write your answer in decimal form. Convert any decimal remainder to a fraction, and write in fractional form.	= (i) 57.5% or (ii) $57\frac{1}{2}$%

(b) 1	Write a fraction with the first amount as the numerator and the second amount as the denominator.	(b) $\frac{35}{30}$
2	Convert the fraction to a decimal by performing the division. (A calculator could be used for this step.) Keep all of the decimal places (do not round off).	$= 1.1666...$
3	Multiply the decimal value by 100%.	$1.1666... \times 100\%$
4	Write your answer in exact decimal form. Convert any decimal remainder to a fraction, and write in fractional form.	$=$ (i) $116.\dot{6}\%$ or (ii) $116\frac{2}{3}\%$

Worked Example 20

WE 20

Express the first amount as a percentage of the second: 40 cents, \$5.

Method 1: Fraction multiplication

Thinking	Working
1 Write both quantities in the smaller unit.	40 cents, 500 cents
2 Write a fraction with the first amount as the numerator, and the second amount as the denominator, multiplied by $\frac{100}{1}\%$.	$\frac{40}{5\cancel{00}} \times \frac{1\cancel{00}}{1}\%$
3 Simplify by cancelling common factors. (Here, we have divided by 100, then by 5.)	$= \frac{\overset{8}{\cancel{40}}}{\underset{1}{\cancel{5}}} \times \frac{1}{1}\%$ $= \frac{8}{1} \times \frac{1}{1}\%$ $= 8\%$
4 Write the answer.	40 cents is 8% of \$5.

Method 2: Convert to a decimal, then multiply

Thinking	Working
1 Write both quantities in the smaller unit.	40 cents, 500 cents
2 Write a fraction with the first amount as the numerator, and the second amount as the denominator.	$\frac{40}{500}$
3 Convert this fraction to a decimal by performing the division.	$= 0.08$
4 Multiply by 100%.	$= 0.08 \times 100\%$ $= 8\%$
5 Write the answer.	40 cents is 8% of \$5.

2.7 Writing one amount as a percentage of another

Navigator

Answers page 603

Q1 Columns 1–3, Q2 Column 1, Q3, Q4, Q8, Q9, Q11, Q12, Q13, Q16, Q18, Q19	Q1 Columns 2–4, Q2 Column 2, Q3, Q4, Q6, Q7, Q9, Q10, Q12, Q15, Q16, Q17, Q18, Q19	Q1 Columns 3 & 4, Q2 Column 2, Q3, Q5, Q7, Q8, Q9, Q14, Q15, Q16, Q17, Q18, Q19, Q20

Equipment required: Calculator

Fluency

WE19

1 Express the first amount as a percentage of the second. Give your answers in **(i)** fractional and **(ii)** exact decimal form.

(a) 13, 50 **(b)** 16, 20 **(c)** 27, 45 **(d)** 42, 48

(e) 35, 80 **(f)** 15, 18 **(g)** 54, 96 **(h)** 15, 27

(i) 86, 54 **(j)** 90, 70 **(k)** 72, 66 **(l)** 26, 22

WE20

2 Express the first amount as a percentage of the second. Round your answer to two decimal places where necessary.

(a) 25 cents, \$4 **(b)** 40 m, 2 km

(c) 7 mm, 2 cm **(d)** 30 seconds, 4 minutes

(e) 750 g, 2.5 kg **(f)** 5 days, 4 weeks

(g) 80 kg, 1 tonne **(h)** 85 mL, 2 L

(i) 600 mm, 7 m **(j)** 50 minutes, 40 hours

Remember that units have to be the same, so convert both values to the smaller unit first.

3 **(a)** Which of the following shows how to calculate 17 as a percentage of 30?

A $\frac{17}{30} \times \frac{100}{1}\%$ **B** $\frac{30}{17} \times \frac{100}{1}\%$ **C** $\frac{17}{100} \times \frac{30}{1}\%$ **D** $\frac{30}{100} \times \frac{17}{1}\%$

(b) Which of the following is 40 cm as a percentage of 2 m?

A 2% **B** 5% **C** 20% **D** 500%

Understanding

4 Craig received 38 marks out of 40 for a maths test. Find his result as a percentage.

5 In an election for house captain, Jessica received 324 votes. If 540 students voted, what percentage of them voted for Jessica?

6 Karen planted 30 small trees in her new garden, but only 19 survived after a year. What percentage of the new trees survived? (Answer in exact decimal form.)

7 Stuart improved his best time for a 200 m sprint by 2 seconds during a training session. His previous best time was 25 seconds. Write Stuart's improvement as a percentage of his previous best time.

8 Hamish improved his shot-put throw from 20 m to 23 m. By what percentage of his 20 m throw did he increase his distance?

9 Lara has a part-time job that pays \$160 per week. She receives a pay rise of \$24 per week. Express Lara's pay rise amount as a percentage of her original pay.

10 During one particular season, Cuong records 18 wins from 27 sets of tennis played. What percentage of sets played did Cuong win that season? (Answer in fractional percentage form.)

11 Harry scored 5 of his team's 16 goals during a football match. What percentage of his team's goals did he score? (Answer in fractional percentage form.)

12 Adila spends $90 each week on food. If her total income is $400 per week, what percentage of it is spent on food? (Answer in fractional percentage form.)

13 According to *Guinness World Records*, Antonio Gomesdos Santos stood motionless (apart from involuntary blinking) for approximately 15 hours on 30 July 1988. What percentage of a day was this? (Answer in decimal form.)

14 According to *Guinness World Records*, the tallest man in reliably recorded history was Robert Wadlow of Illinois, USA, whose height was measured to be 272 cm. If the average male height today is about 1.8 m, express Robert Wadlow's measured height as a percentage of the average male height. (Answer in exact decimal form.)

15 A 30 gram serving of a breakfast cereal contains 2.5 grams of dietary fibre. What percentage of the serving of cereal is this? (Answer in fractional percentage form.)

16 A sofa is advertised in a furniture sale catalogue at the discounted price of $699, a saving of $350. Find the saving as a percentage of the original price of the sofa. Round your answer to the nearest whole number.

Reasoning

17 In their basketball game on Friday night, Nathan scored 12 goals from 16 shots, and his team mate Dayo scored 14 goals from 18 shots.

(a) Estimate which boy is the more accurate scorer, then use percentages to check your estimate (the more accurate scorer will have turned a greater percentage of their shots into goals). Round your answers to one decimal place if necessary.

(b) In their game the following week, Nathan scored 13 goals from 18 shots, and Dayo scored 11 goals from 14 shots. For each boy, add the number of goals and the number of shots to those of the previous week. Use your total to calculate a new, two-game percentage for Nathan and for Dayo. (Again, round your answers to one decimal place if necessary.)

(c) Based on your answer to **(b)**, which boy improved his scoring percentage, from one game to the next?

18 Round your answers to the following to one decimal place.

(a) Jaime estimates that she spends 2 hours a day online. What percentage of a day is this?

(b) Jaime also estimates that she spends $\frac{1}{3}$ of a day sleeping. What percentage of her waking hours does she spend online?

(c) Jaime does not include the 7 hours that she spends at school as part of her 'online time'. Recalculate her 'online' time as a percentage of her day, excluding school time and sleeping time.

(d) Describe what happens to the value of the percentage as you move from **(a)** to **(b)** to **(c)**. Explain why this happens.

Open-ended

19 Write three different amounts of money that when expressed as a percentage of $4 are greater than 32% and less than 39%.

20 Bella has calculated that she is 89% of her dad's height (to the nearest whole percentage). Give three possible pairs of heights for Bella and her dad, if they are both between 1 m and 2 m tall.

Outside the Square — Problem solving

League tables

In many sports, percentage is used to separate teams that have the same number of wins and losses. It is calculated by writing the points the team has scored as a fraction of the points scored against them, then multiplying by 100.

$$\% = \frac{\text{points for}}{\text{points against}} \times \frac{100}{1}$$

Halfway through round 4 of the competition, the Coburg Comets are third on the ladder, whereas the Mitcham Mozzies are fourth.

Team	Matches played	Points for	Points against	%
Comets	4	192	147	130.6
Mozzies	3	104	81	128.4

The Comets have already played their match for the round, whereas the Mozzies are yet to play. What would the final scores of the Mozzies match need to be if they are to overtake the Comets' percentage?

Strategy options

- Guess and check.
- Solve a simpler problem.

Finding a percentage of an amount

2.8

Being able to find a percentage of an amount is a very useful skill. Because percentages can be written as fractions out of 100, we can use fraction multiplication to find a percentage of an amount (Method 1 of Worked Example 21 below). Alternatively, we can multiply the amount by the decimal equivalent of the percentage (Method 2 of Worked Example 21).

The key thing to remember is that the word 'of' can be interpreted mathematically as multiplication.

To find a percentage of an amount:

1. replace the 'of' in the expression with '×'
2. convert the percentage to a fraction or a decimal
3. perform the multiplication and simplify your answer.

Fractions are often easier to work with when performing calculations by hand. Decimals are much quicker if you have a calculator available.

Worked Example 21 WE 21

Find the following, rounding answers to two decimal places where necessary.

(a) 12% of \$350 **(b)** 5.2% of 30 kg

Method 1: Fraction multiplication

Thinking	Working
(a) 1 Substitute '×' for 'of' and omit any units.	(a) 12% of \$350 $= 12\% \times 350$
2 Write the percentage as a fraction, and the amount as an improper fraction with a denominator of 1. Simplify the multiplication by cancelling common factors. (Here, we have divided by 10 and 5, then by 2.)	$= \frac{12}{\cancel{100}_{2}} \times \frac{\cancel{350}^{7}}{1}$ $= \frac{\cancel{12}^{6}}{\cancel{2}_{1}} \times \frac{7}{1}$
3 Simplify, then perform the multiplication.	$= 6 \times 7$
4 State your answer, including any units.	= \$42

(b) 1	Substitute '×' for 'of'.	(b) 5.2% of 30 kg $= 5.2\% \times 30$
2	Write the percentage as a fraction, and the amount as an improper fraction with a denominator of 1. Simplify by cancelling common factors. (Here, we have divided by 10, then by 4.)	$= \frac{\overset{13}{\cancel{52}}}{\underset{25}{\cancel{1000}}} \times \frac{\cancel{30}}{1}$
3	Simplify, then perform the multiplication and write the answer.	$= \frac{13}{25} \times 3$ $= \frac{39}{25}$
4	Write your answer in the required form with the correct units.	$= 1.56$ kg

Method 2: Decimal multiplication

Thinking		Working
(a) 1	Substitute '×' for 'of'.	(a) 12% of \$350 $= 12\% \times 350$
2	Write the percentage as a decimal.	$= 0.12 \times 350$
3	Perform the multiplication and write the answer.	= \$42
(b) 1	Substitute '×' for 'of'.	(b) 5.2% of 30 kg $= 5.2\% \times 30$
2	Write the percentage as a decimal.	$= 0.052 \times 30$
3	Perform the multiplication and write the answer with the correct units.	$= 1.56$ kg

If the percentage you are finding is less than 100%, then your answer will always be smaller than the original amount. It is important to check the reasonableness of your answer.

2.8 Finding a percentage of an amount

Navigator

Answers page 603

Q1 Columns 1 & 2, Q3, Q5, Q6, Q7, Q9, Q11(a), Q12, Q15, Q19	Q1 Columns 2 & 3, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q14, Q16, Q18, Q19	Q1 Column 3, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q13, Q14, Q16, Q17, Q18, Q19

Equipment required: A calculator may be used for Questions **1** and **8–18**

Fluency

1 Find the following, rounding answers to two decimal places where necessary. WE21

(a) 6% of \$200 (b) 20% of \$150 (c) 64% of \$50

(d) 80% of \$25.50 (e) 75% of \$279.95 (f) 120% of \$35.74

(g) 150% of 30 kg (h) 160% of 25 kg (i) 90% of 4 kg

(j) 23.2% of 40 min (k) 42.5% of 70 min (l) 15.7% of 60 min

(m) 3.4% of 50 L (n) 9.3% of 500 L (o) 0.6% of 3000 L

2 (a) Which of the following shows how to calculate 33% of 70?

A $\frac{33}{1} \times \frac{100}{1} \times \frac{70}{1}$ B $\frac{33}{100} \times \frac{70}{1}$ C $\frac{33}{70} \times \frac{100}{1}$ D $\frac{70}{30} \times \frac{100}{1}$

(b) Which of the following shows how to find $7\frac{1}{2}$% of 600?

A $\frac{7}{200} \times \frac{600}{1}$ B $\frac{3}{40} \times \frac{600}{1}$ C $\frac{15}{2} \times \frac{600}{1}$ D 7.5×600

3 (a) Which of the following shows how to calculate 3.5% of 15?

A 3.5×15 B 35×15 C 0.035×15 D 0.035×0.15

(b) Which of the following shows the calculation for finding 19% of 6000?

A 19×6000 B 19×60 C 0.19×6000 D 0.19×6

Understanding

4 Nick got 85% of the questions right in a test containing 80 questions. How many questions did Nick answer correctly?

5 A 750 gram packet of breakfast cereal is only 20% full. How many grams of cereal are in the packet?

6 About 65% of the mass of an adult human is water. Ollie weighs 64 kg. How much of Ollie's mass is water, to the nearest kilogram?

7 A survey of 5500 households reveals that 4 in every 5 regularly sort their rubbish for recycling.

(a) Write this result as a percentage.

(b) Calculate the number of households that sort their rubbish.

8 The Cancer Council estimates that 2 in 7 cancer deaths are related to tobacco (smoking).

(a) Write this figure as a percentage, rounded to one decimal place.

(b) Of 412 000 cancer deaths, how many will be related to tobacco?

9 A geologist finds a rock of mass 600 g containing a substantial amount of gold. She has the rock analysed and finds that the rock is 19.4% gold.

(a) How many grams of gold are in the rock?

(b) If the current price of gold is \$45 per gram, how much is her find worth?

10 Forty-five percent of the residents in a particular suburb read their local newspaper. If the suburb contains 23 000 people:

(a) how many people read the local paper

(b) how many people don't read the local paper?

11 **(a)** A furniture store advertises a 40% reduction on the price of its beds.

(i) Calculate the dollar value of the percentage reduction on a bed priced at $1800.

(ii) Calculate the new, reduced price of the bed.

(b) The same store advertises a $33\frac{1}{3}$ % reduction on the price of sofas.

(i) Calculate the dollar value of the percentage reduction on a sofa priced at $450.

(ii) Calculate the new, reduced price of the sofa.

12 The population of a small town increases by 6.5% during the summer holidays. If the town's population is normally 1600, by how many does it increase during the summer holidays?

13 The Republic of Banano increases its banana crop by 4.4% one year. If the previous year's crop was 4200 tonnes, how many tonnes were produced in the new crop? Answer to the nearest tonne.

14 A meeting of 700 residents votes on a proposed road closure. If 2.1% of the residents voted for the closure:

(a) how many residents voted for the closure

(b) what percentage did not vote for the closure

(c) how many residents did not vote for the closure?

15 A 250 mL bottle of fruit juice drink contains 25% real fruit juice. The rest of the juice is water and sugar.

(a) What is the volume of real fruit juice in the bottle?

(b) What is the volume made up by water and sugar?

Reasoning

16 Aaron is studying dance. 60% of his overall marks for the subject are allocated to the practical exam, and the remaining 40% is allocated to the theory exam. Aaron scored 84% in his practical exam and 77% in the theory exam. Calculate Aaron's overall percentage for the subject, correct to one decimal place.

17 A chemical company is planning to increase its production of photographic solution by 20%. At present, it produces 11 200 litres of solution per week. The Environmental Protection Agency is concerned that this increase in production will increase the company's waste water output to greater than the $1\frac{1}{2}$% of the volume of the total production allowable by law. At present, waste water makes up only $\frac{3}{5}$% of the volume of the total production. The company claims that with new production procedures the total waste water produced will amount to only 168 litres per week.

(a) **(i)** Calculate the amount of photographic solution represented by the 20% increase in production.

(ii) Calculate the total amount of solution that will be produced after the increase.

(b) Calculate the amount of waste water that is produced at present.

(c) Calculate the maximum amount of waste water that would be allowed after production is increased.

(d) What percentage of the total production does the company claim the waste water will amount to?

(e) Is the chemical company's planned increase in its waste water production illegal?

Open-ended

18 There are 60 books on Naomi's bookshelf.

(a) Naomi says that 21% of her books are recipe books. Explain why this is not an accurate statement.

(b) Find at least three percentage values that are possible for the percentage of recipe books.

19 Two Year 8 classes each have 25 students. In 8A, 20% of the students play basketball. In 8B, 20 of the students play basketball.

(a) Which of the two classes has more basketball players?

(b) Explain how '20% of the students' is different to '20 students'.

Outside the Square

Problem solving

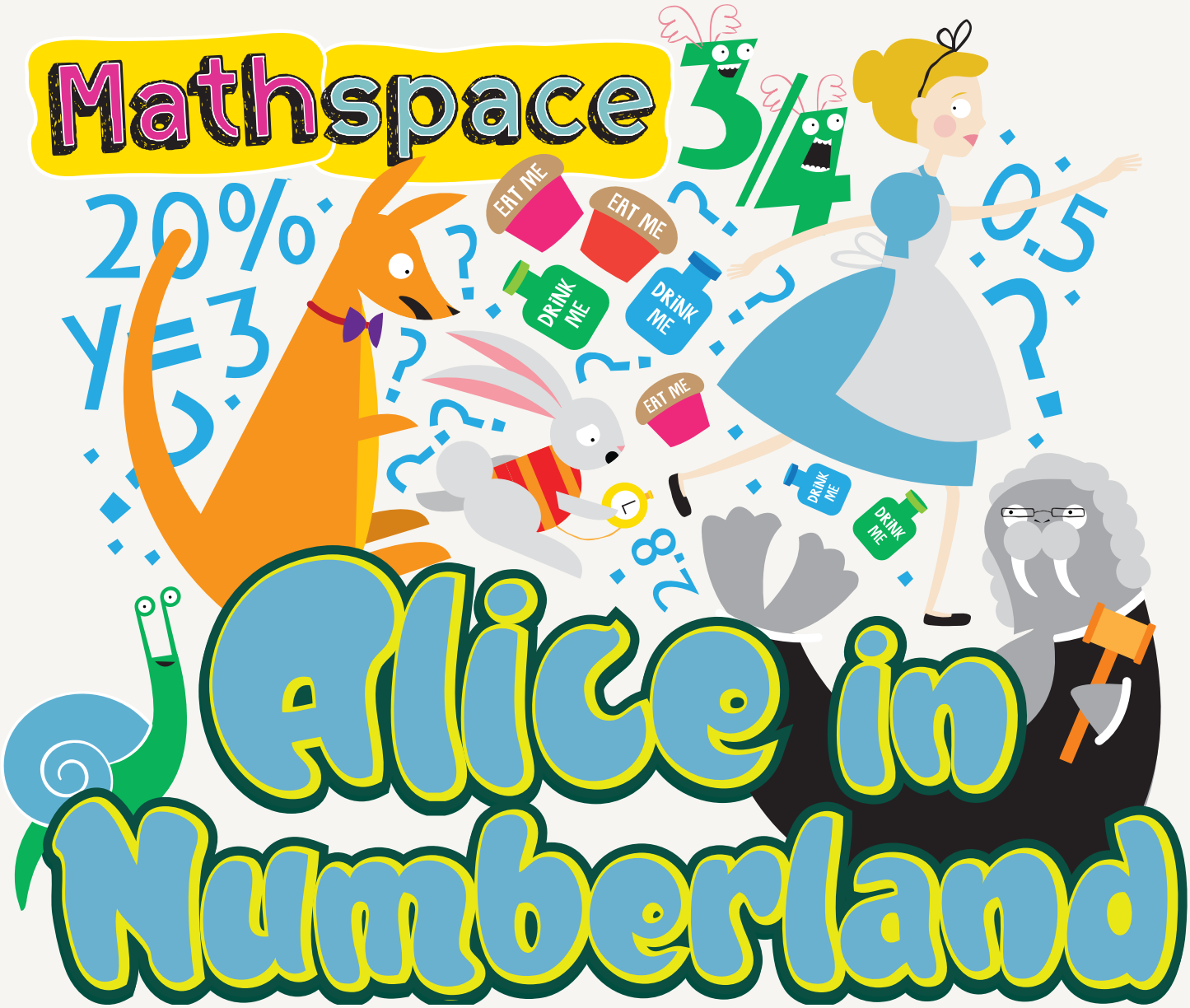
Cordial contents

Three jugs of cordial have volumes of 500 mL, 1250 mL and 1800 mL and they are each filled to capacity. The contents of the three jugs are combined together into another large jug. After this is completed the percentage of cordial in the large jug is 9.507%. Tilly knows that one of the original jugs contained 5% cordial, the second contained 7% cordial and the third contained 12.5% cordial. Work out which jug contained which percentage of cordial prior to mixing.

Strategy options

- Solve a simpler problem.
- Break problem into manageable parts.

Mathspace



Alice in Numberland

Alice was doing her homework while checking Facebook, messaging her friends and Googling for an English assignment. She fell asleep while waiting for one of her friends to reply …

She awoke in a strange land full of question marks, numbers and equations floating in the air.

I'm late!

A well-dressed rabbit went bounding by, muttering, 'I'm late, I'm late! How late you ask? I'm 20% of a minute late for the dentist!'

A kangaroo in a bow tie bounced past, crying, 'I'm later, I'm later, 15% of an hour late for my barbecue.'

'How strange', thought Alice.

Then she noticed a snail sliding along, whispering, 'I've just travelled 6 mm. That's 5% of my journey.'

(a) How late was the rabbit for the dentist (in seconds)?

(b) How late was the kangaroo for the bbq (in minutes)?

(c) How much further does the snail have to travel?

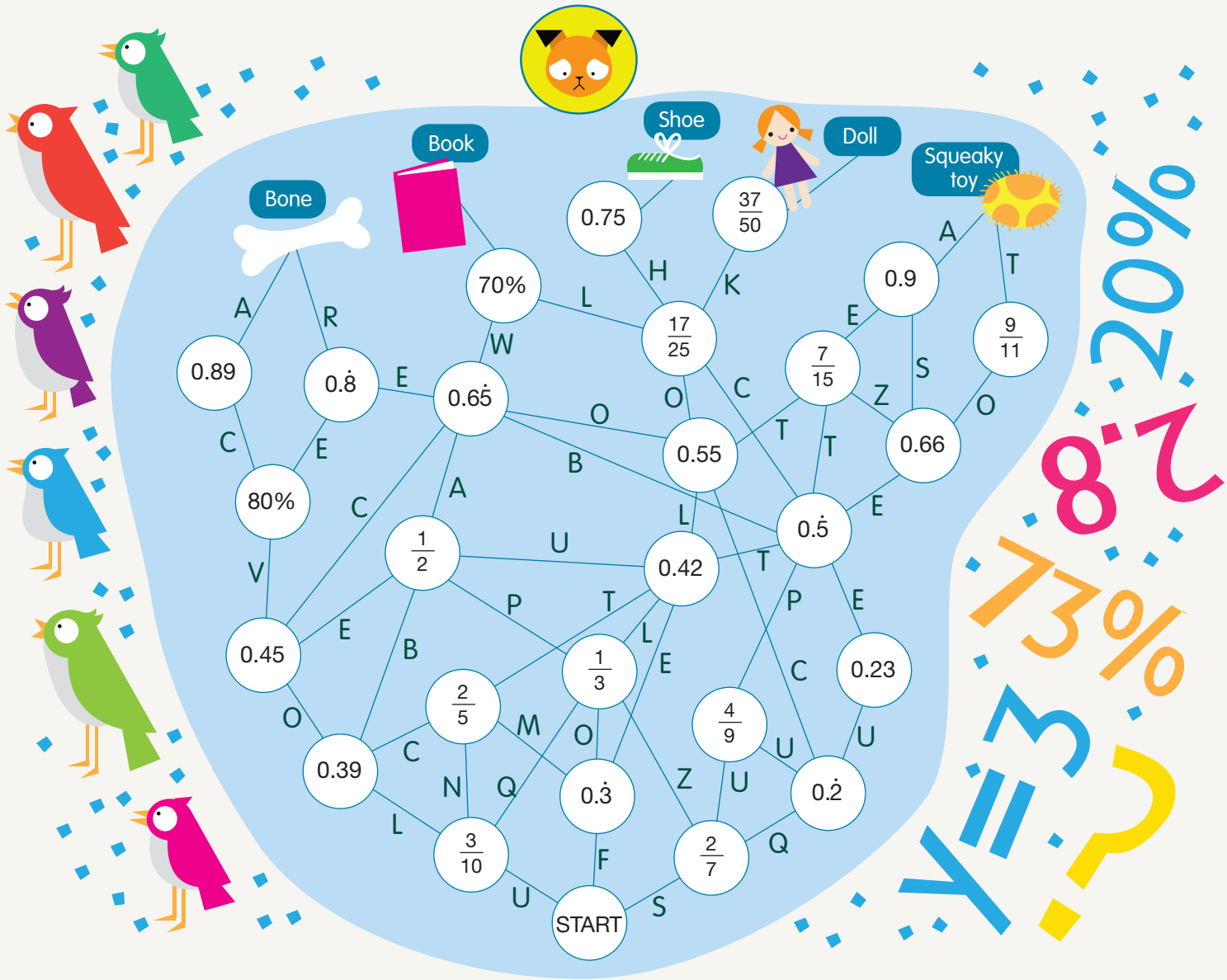
The pool of tears

Alice wandered until she saw an old dog sobbing in the far distance. He had forgotten where he had buried a favourite object and his tears had formed a large lake.

To find the lost object and the magic word to return it, Alice has to step on the stones in the correct order and collect the letters on each path she takes. At each stone, Alice must follow the path to the stone that has the largest number. Begin at START and find the path Alice must take.

(a) What object has the dog lost?

(b) What is the magic word that will return the object?



A fraction of her size

Then Alice stumbled upon some tiny muffins with 'eat me' written on them, and some little glass bottles with the words 'drink me' inscribed on them. 'Hmmm,' she thought, 'I am a little hungry,' and nibbled one of the muffins. She started shrinking …

(a) If Alice was originally 150 cm tall and shrank to 120 cm tall, by what percentage of her original size did the muffin make her shrink?

Alice didn't like her new, smaller height, so she sipped one of the drinks …

(b) She shot up from 120 cm to 300 cm! By what percentage did she increase in size?

(c) By what percentage of her current enlarged size does Alice need to decrease to return to her original size?

Off with her head!

Alice found herself in a mathematical court room, with a jury of birds and a walrus as the judge. Little fractions kept nipping Alice's head.

'CHOP OFF HER HEAD!' chorused the jury.

The judge declared, 'Unless you can convince the jury of the following mathematical statements, we will be left with no other option but to chop off your head'.

Alice must convince the jury that the following statements are true.

(a) 0.2% = 0.002

(b) If I have 50 tarts and I gain another 100, I have 200% extra.

How could Alice explain these statements to the jury?

Investigation

Supermarket specials

Equipment required: 1 brain, 1 calculator (optional), shopping catalogues (optional)

Many shops, particularly supermarkets, put certain items 'on special' by reducing the price of those items. These 'specials' are advertised in a number of different ways, such as percentage discounts, dollar savings and 'multi-buy' offers, such as 'buy two, get the third free'.

The Big Question

Is a 'multi-buy' deal better than a discounted price?

Engage

1 Imagine you are in the supermarket. Without doing any calculations, which of the following specials looks better value to you?

Deal A Tuna: 6 tins for the price of 5 or

Deal B Tuna: $1.08. Save 22c a tin

2 A supermarket is offering a 'buy two, get the third free', multi-buy deal on packets of biscuits. The usual price of the biscuits is $3 per packet.

(a) How much would you pay for the multi-buy deal?

(b) How much do you save in this deal?

(c) Calculate this saving as a percentage of the full price of the three packets.

(d) If you buy the multi-buy deal, what is the cost of each packet?

(e) How much do you save, per packet?

(f) Calculate the saving on one packet as a percentage of the original price of the packet.

(g) What do you notice?

Explore

3 To investigate the Big Question, choose a product and its price, and create several different 'multi-buy' deals, such as '2 for the price of 1' and '3 for the price of 2'. For each deal, calculate the amount saved as both a dollar saving and a percentage saving. You might like to use a shopping catalogue to get ideas, or to check the price of an item.

Strategy options

- Look for a pattern.
- Test all possible combinations.

Explain

4 For each of the 'multi-buy' deals you investigated:

(a) give a percentage reduction that the store could use to advertise an equivalent discount off the price of a single product

(b) calculate how much the individual price of the item would need to be reduced by to make it better value than the multi-buy.

Elaborate

5 Most multi-buy deals give you just 1 product for free, such as 3 for 2, 4 for 3, 5 for 4 etc. As the number of items in a multi-buy deal increases, what happens to:

(a) the dollar saving

(b) the overall percentage discount?

(c) Explain the difference between your answers to **(a)** and **(b)**.

6 Go back to the tuna deals you considered in Question **1**. Which deal is cheaper for buying 6 tins of tuna?

7 Answer the Big Question by summarising your results from Questions **3**–**6**.

8 Imagine that you are in the supermarket. You see one brand of corn chips with a discounted price of $1.30 per packet. Another brand of the same sized packet of corn chips has a regular price of $1.79, but has a '3 for 2' offer. Which special represents better value for money? Show how you arrived at your answer.

Evaluate

9 Give some reasons as to why stores might offer 'multi-buy' deals for certain products, instead of advertising a percentage or a dollar discount on individual items.

10 Imagine you are doing the grocery shopping for yourself or your family. List some pros and cons of purchasing 'multi-buy' items. For which types of item would it be good to take advantage of 'multi-buy' specials?

Extend

11 Another common promotion is the 'buy one and get the second one half-price'.

(a) When would store managers choose to use this type of promotion?

(b) When might this type of promotion be beneficial to you?

(c) Assuming both items are the same price, what percentage of the original price are you paying for each of them? Use an example to demonstrate how you arrived at your answer.

2.9 Increasing or decreasing by a given percentage

Calculating a percentage increase or decrease

A percentage can be used to represent a change in a quantity. We can calculate the result of a percentage increase or decrease by using the fact that the original amount represents 100%, which has a decimal equivalent of 1.0.

To increase or decrease an amount by a given percentage:

1. add the percentage to, or subtract it from, 100%, then write it as a decimal scale factor
2. multiply this decimal by the amount to be increased or decreased.

Worked Example 22 WE22

Increase or decrease the following amounts by the given percentages. Give answers in decimal form, rounding to two decimal places where necessary.

(a) Increase \$2700 by 22%.

(b) Decrease 163 kg by 45%.

Thinking	Working
(a) 1 Add the percentage increase to 100%. Write this new percentage as a decimal scale factor.	(a) 100% + 22% = 122% = 1.22
2 Multiply the scale factor by the original amount to find the new amount.	1.22 × 2700
3 Write the answer.	= \$3294
(b) 1 Subtract the percentage decrease from 100%. Write this new percentage as a decimal scale factor.	(b) 100% – 45% = 55% = 0.55
2 Multiply the scale factor by the original amount to find the new amount.	0.55 × 163
3 Write the answer.	= 89.65 kg

Expressing an increase or a decrease as a percentage

Often, an increase or a decrease is described as the percentage change of an original amount. To calculate this, we express the amount of increase or decrease as a fraction of the original amount, and convert it to a percentage.

Worked Example 23 WE23

Tori earns $950 per week as an electrician. She receives a pay increase of $32 per week. Write Tori's pay increase as a percentage of her original pay, correct to one decimal place.

Thinking	Working
1 Write the amount of the increase as a fraction of the original amount.	$\frac{32}{950} \times 100\%$
2 Convert the fraction to a percentage, rounding to the specified number of decimal places.	$= 0.033\,68\ldots \times 100\%$ $= 3.4\%$ (1 d.p.)

2.9 Increasing or decreasing by a given percentage

Navigator

Q1 (a) & (b), Q2, Q3, Q4, Q5, Q6, Q8, Q10, Q11, Q13, Q15, Q16	Q1 (b) & (c), Q2, Q3, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q13, Q15, Q16	Q1 (c) & (d), Q2, Q3, Q6, Q7, Q8, Q10, Q11, Q12, Q13, Q14, Q15, Q16

Answers page 604

Equipment required: Calculator

Fluency

1 Increase or decrease the following amounts by the given percentages. Give answers in decimal form, rounding to two decimal places where necessary. WE22

(a)	Increase 1500 by:	(i) 15%	(ii) 25%	(iii) 45%	(iv) 120%
(b)	Decrease 2200 by:	(i) 10%	(ii) 40%	(iii) 66%	(iv) 95%
(c)	Increase $83.75 by:	(i) 23%	(ii) 340%	(iii) 3.5%	(iv) $3\frac{2}{3}$ %
(d)	Decrease 5.2 km by:	(i) 6%	(ii) 74%	(iii) 2.8%	(iv) $1\frac{3}{4}$ %

2 (a) Georgia earns $1253 per week working as a designer. She receives a pay increase of $49 per week. Write Georgia's pay increase as a percentage of her original pay, correct to one decimal place. WE23

(b) Jayden earns $67 982 per year working as a lawyer. He receives a pay increase of $2850 per year. Write Jayden's pay increase as a percentage of his original pay, correct to one decimal place.

(c) This month, the Ling family have reduced their electricity bill by $15.76 compared to last month. If last month's bill was $109.43, calculate the reduction in this month's bill as a percentage of the previous bill, correct to one decimal place.

(d) This month, the Edwards family have used 13 kL (kilolitres) less water than last month. If they used 190 kL last month, calculate the reduction in this month's water usage as a percentage of last month's usage, correct to one decimal place.

3 Which of the following calculations decreases $129.50 by 40%?

A 129.5×0.4 B 1.295×40 C 0.6×129.5 D 1.4×129.5

Understanding

4 Summer's monthly rent is set to increase by 15%. If she currently pays $750 a month, what will she pay after the increase?

5 Last year, 87 453 people attended the Royal Show. This year, only 79 014 people attended. Write the drop in attendance as a percentage decrease.

6 Yang earns $834.72 a week working on a building site. His pay increases to $886.70 a week. Write the amount of Yang's pay rise as a percentage increase of his original pay, correct to one decimal place.

7 Sam buys a new sports car for $45 000. As soon as he drives it out of the showroom its value decreases by 12%.

(a) Find the value of the car after it leaves the showroom.

After just 3 years, the value of the car will have decreased by 35% of its original value.

(b) What will the car be worth in 3 years time?

(c) Write the 3-year decrease in value as an average yearly loss, in dollars per year.

8 Lizzie is a swimmer whose personal best time for her favourite race is 1 min 8.23 s.

(a) Write Lizzie's time in seconds.

(b) Lizzie is aiming to decrease her time by 5%. Use your answer to **(a)** to calculate this decrease as a number of seconds, correct to the nearest hundredth of a second.

(c) At the state titles, Lizzie lowered her time by 2.65 seconds. Write this improvement as a percentage of her previous best time, to the nearest whole number.

9 **(a)** Margaret purchased a three-bedroom house for $385 000. A year later it was reported that house prices had increased by 9%. What is Margaret's house worth now?

(b) The following year, it was reported that house prices had risen by 7.5% compared to the previous year. What is Margaret's house worth now?

(c) Margaret later sold her house for $469 000. What percentage increase on the original price is this, to the nearest whole number?

Reasoning

10 At the end of the first week of trading on the share market, shares in Deepak's company had increased by 12%. The shares were worth $2.50 each at the beginning of the week.

(a) What was the value of a share at the end of the first week?

(b) By the end of the second week, the share price had decreased by 7% compared to its price at the start of that week. Calculate the value of one share at the end of the second week.

(c) Calculate the overall increase or decrease at the end of the 2 weeks as a percentage of the original price of $2.50 per share.

11 The population of the small town of Greenhills decreased in one year by 4%. The population at the beginning of the year was 17 000.

(a) Calculate the population at the end of the year.

(b) If the population continues to drop by 4% for the next 2 years, calculate the population:

(i) at the end of the second year **(ii)** at the end of the third year.

(c) Do the three 4% decreases in population give the same result as one 12% decrease? Explain why or why not.

12 Pheap improves his maths test result by 20%.

(a) If his first result was 27 out of 60 marks, what was his improved result, if the second test was also out of 60?

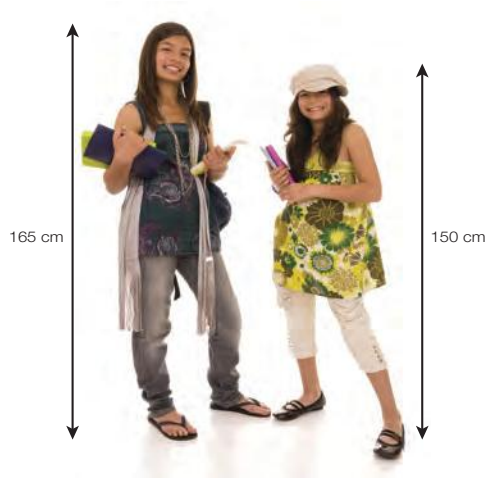
(b) If Pheap continues to improve by this percentage with each test and all tests are out of 60 marks, how many more tests must he take to achieve over 90%?

13 Melissa is 165 cm tall, and her younger sister Laura is 150 cm tall.

(a) Calculate Melissa's height as a percentage increase of Laura's height, to the nearest per cent.

(b) Calculate Laura's height as a percentage decrease of Melissa's height, to the nearest per cent.

(c) Explain how the same height difference of 15 cm can produce two different answers for **(a)** and **(b)**.



14 The amount of metal ore produced on a remote island mine is 6900 tonnes per year.

(a) Calculate the reduced amount of ore mined if production drops by 15%.

(b) If production continues to drop by 15% each year, how many more years will it be before the amount mined falls below 4000 tonnes?

(c) At this rate, will the mine ever stop producing ore? Explain.

Open-ended

15 **(a)** Calculate the number of girls in your class as a percentage of the whole class.

(b) Calculate the increase in this percentage if one extra girl joins the class.

(c) Repeat **(a)** for the boys in the class.

(d) Repeat **(b)** for the boys in the class.

(e) Were the percentage increases you calculated in **(b)** and **(d)** the same? Explain why or why not.

16 How many centimetres taller would you be if your height increased by 8%? (Answer to the nearest cm.)

Outside the Square Puzzle

Break the code

CABLE is the word needed to crack the safe. The letters C, A, B, L, E each represent a value from 1 to 5.

Using the following equations, find the values of *C*, *A*, *B*, *L* and *E*.

$L - 2 = B$
$E + B = L$
$C + E = B$

2.10 Financial applications of percentages

Businesses selling goods are called retailers. People buying these goods are called consumers. Retailers either manufacture their products or purchase them from suppliers. The cost of buying or manufacturing the goods (including things such as the cost of materials and transporting the goods) is called the **cost price (CP)**. In order to make a **profit**, the retailer needs to sell their product at a higher price than the cost price. This higher price is called the **selling price (SP)**. The amount added onto the cost price is called the **mark up**, and is often calculated as a percentage of the cost price. If the selling price is less than the cost of the item plus any associated costs (such as transport), then a **loss** is made.

As well as adding their mark up to the cost of a product, a business must also add a government tax called the **Goods and Services Tax (GST)**, which is an extra 10% of the selling price. A business will sometimes offer goods at a lower price, or **sale price**, to make them more attractive to consumers. This reduction in price is called a **discount**.

Cost price (CP): the total cost of making or buying the goods

Selling price (SP): cost price + mark up

Mark up: amount added to cost price in order to make a profit

Discount: decrease in the selling price to attract more buyers

Sale price = selling price – discount amount

GST: Goods and Services Tax of 10% added to selling price

Profit: difference between SP and CP, where SP > CP

Loss: difference between SP and CP where SP < CP

Calculating discounts and mark ups

Worked Example 24 WE24

Calculate the sale price of the following item, rounding your answer to the nearest cent.
A $350 stereo discounted by 25%.

Method 1: Find and subtract the discount amount

Thinking	Working
1 Calculate the dollar amount of the current price represented by the discount.	25% of $350 = 0.25 × 350 = $87.50
2 Subtract the discount amount from the original price to find the sale price.	Sale price = $350 – $87.50 = $262.50

Method 2: Calculate the remaining percentage

Thinking	Working
1 Subtract the percentage discount from 100%.	100% − 25% = 75%
2 Find the remaining percentage value of the price.	Sale price = 75% of \$350 = 0.75 × 350 = \$262.50

Worked Example 25

WE 25

An electrical goods store determines the selling price of appliances by applying a 68% mark up. Calculate the selling price of a toaster bought at a cost price of \$23.20.

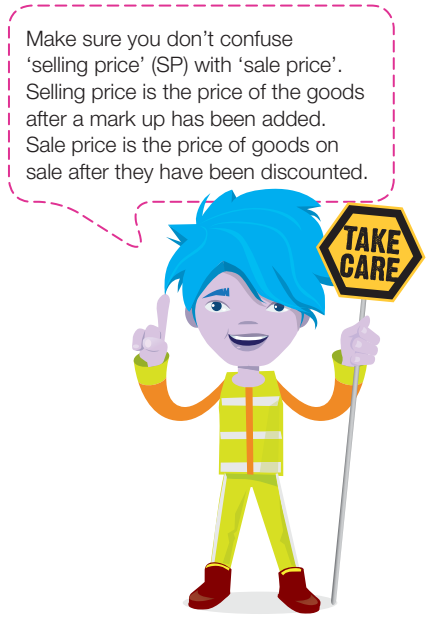
Method 1: Find and add on the mark up amount

Thinking	Working
1 Calculate the dollar amount of the cost price represented by the mark up (to the nearest cent).	68% of \$23.20 = 0.68 × 23.20 = \$15.78
2 Add the mark up amount onto the cost price to find the selling price.	SP = CP + mark up = \$23.20 + \$15.78 = \$38.98

Method 2: Calculate the percentage increase

Thinking	Working
1 Add the percentage mark up onto 100%.	100% + 68% = 168%
2 Calculate this new, increased percentage of the cost price, by multiplying by the percentage in decimal form.	SP = 168% × CP = 1.68 × 23.20 = \$38.98

Note from the previous two Worked Examples that Method 1 is useful if you need to know the actual dollar amount of the discount or mark up, whereas Method 2 is a more direct way of finding the reduced or increased price.



Calculating the original price

As consumers, we see only the final price of goods after the mark up has been added or discounts subtracted. If we know the value of the mark up or discount, we can calculate the price of the goods before the discount or mark up was applied. We simply reverse the process used to find the selling price.

In Method 2 of Worked Example 25, we multiplied the cost price of a toaster by the mark up to find the selling price. If we divide the selling price by the mark up, we arrive back at the cost price.

\$23.20 $\xrightarrow{\times 1.68}$ \$38.98

Cost price $\xleftarrow[\div 1.68]{}$ Selling price

In Method 2 of Worked Example 24, we calculated the sale price by multiplying the original selling price (often called the 'marked price') by the reduced percentage value. If we divide the sale price by this percentage value, we arrive back at the original price.

$350 $\xrightarrow{\times 0.75}$ $262.50

Marked price $\xleftarrow{\div 0.75}$ Sale price

For mark ups:

Cost price (CP) $\xrightarrow{\times (100\% + \text{mark up } \%)}$ Selling price (SP)

Cost price (CP) $\xleftarrow{\div (100\% + \text{mark up } \%)}$ Selling price (SP)

For discounts:

Marked price $\xrightarrow{\times (100\% - \text{discount } \%)}$ Sale price

Marked price $\xleftarrow{\div (100\% - \text{discount } \%)}$ Sale price

Convert percentages to their decimal forms before multiplying or dividing.

Worked Example 26

WE 26

(a) 'Great Groceries' supermarket has a policy of marking up all of its bakery products by 75%. If the selling price of a loaf of bread is $3.49, calculate the cost price.

(b) The 'Discount Computers' store is having a 20% sale on all items. Calculate the original marked price of a hard drive that has been discounted to $135.

Thinking		Working
(a) **1**	Summarise the question by writing an equation.	(a) 175% of cost price = $3.49
2	Rewrite the equation, replacing the percentage with its decimal value, and 'of' with a multiplication sign.	1.75 × cost price = $3.49
3	Solve the equation by dividing both sides by the decimal value of the percentage.	Cost price = 3.49 ÷ 1.75 = $1.99
(b) **1**	Summarise the question by writing an equation.	(b) 80% of marked price = $135
2	Rewrite the equation, replacing the percentage with its decimal value, and 'of' with a multiplication sign.	0.8 × marked price = $135
3	Solve the equation by dividing both sides by the decimal value of the percentage.	Marked price = 135 ÷ 0.8 = $168.75

Profit and loss

The amount of profit or loss made on a sale is the difference between the cost price and the selling price. Profit or loss is usually expressed as a percentage of the cost price or the selling price.

You make a profit when you make money on the price you paid. You make a loss when you lose money on the price you paid.

$$\% \text{ Profit} = \frac{\text{profit in \$}}{\text{CP}} \times 100\% \text{ or } \frac{\text{profit in \$}}{\text{SP}} \times 100\%$$

where CP = cost price, SP = selling price.

Replace 'profit' with 'loss' in these formulas to calculate the percentage loss.

Worked Example 27

WE 27

(a) Laura's wedding dress cost \$1450. After her wedding, she sold it for \$900.

(i) Calculate the loss Laura made on the dress.

(ii) Express the loss as a percentage of the cost price, rounded to the nearest per cent.

(b) Ajay bought a tent for \$125 and then resold it on eBay for \$139.

(i) How much profit did Ajay make on the tent?

(ii) Express Ajay's profit as a percentage of the selling price, rounded to the nearest per cent.

Thinking	Working
(a) (i) Find the loss by finding the difference between the cost price (CP) and the selling price (SP).	(a) Loss = CP − SP = \$1450 − \$900 = \$550
(ii) Write the loss as a fraction of the cost price, then convert it to a %. Round your answer to the nearest whole number.	$\frac{550}{1450} \times 100\%$ = 38%
(b) (i) Find the profit by finding the difference between the cost price (CP) and the selling price (SP).	(b) Profit = SP − CP = 139 − 125 = \$14
(ii) Write the profit as a fraction of the selling price, then convert it to a %. Round your answer to the nearest whole number.	$\frac{14}{139} \times 100\%$ = 10%

2.10 Financial applications of percentages

Navigator

Answers page 604

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q17, Q19, Q20, Q21, Q24, Q26, Q27, Q28, Q29	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q13, Q14, Q15, Q16, Q17, Q18, Q19, Q20, Q21, Q23, Q24, Q26, Q27, Q28, Q29	Q1 (c) & (d), Q2 (c) & (d), Q3 (c) & (d), Q4, Q6, Q7, Q8, Q9, Q10, Q11 (c) & (d), Q13, Q14, Q15, Q16, Q17, Q18, Q19, Q20, Q21, Q22, Q23, Q24, Q25, Q26, Q28, Q29

Equipment required: Calculator

Fluency

WE24

1 Calculate the sale price of the following items, rounding your answers to the nearest cent.

(a) A $520 surfboard discounted by 20%.

(b) A DVD originally priced at $18 discounted by 10%.

(c) A $34.50 book discounted by 12.5%.

(d) A digital camera with a marked price of $399.95 discounted by 35%.

WE25

2 (a) A clothing store determines the selling price of T-shirts by applying a 53% mark up. Calculate the selling price of a T-shirt bought at a cost price of $15.

(b) A mark up of 74% applies to all goods at Shelley's Shoe Shop. Find the selling price of a pair of shoes with a cost price of $25.

(c) Glen adds a mark up of 60% to the cost price of the wooden toys he makes before selling them at his market stall. Find the selling price of a toy that cost $12.50 to make.

(d) A computer games store adds a mark up of 78.5% to the cost price of all of its console games. Find the selling price of a game that cost the store $30.25.

WE26

3 (a) Deb's Designer Dresses has a policy of marking up the cost price of all of its dresses by 82%. If the selling price of a dress is $200, calculate the cost price.

(b) Amanda's DVD store offers a storewide discount of 15% on all DVDs. Calculate the original marked price of a boxed set that has been discounted to $49.30.

(c) Calculate the cost price of a pair of sunglasses that has been marked up by 76% to a selling price of $69.95.

(d) Calculate the original marked price of a cricket bat discounted by 30% and on sale for $169.

WE27

4 (a) Rosa buys a ticket to a concert for $65, but is unable to go and sells it to a friend for $39.

(i) Calculate the loss Rosa made on the ticket.

(ii) Express the loss as a percentage of the cost price, rounded to the nearest per cent.

(b) Travis bought an old car for $1500 and then resold it for $1950 without spending any money on it.

(i) How much profit did Travis make on the car?

(ii) Express Travis' profit as a percentage of the selling price, rounded to the nearest per cent.

5 For each of the following, express the profit or loss as a percentage of the cost price, rounded to the nearest whole number.

(a) A painting purchased for $750 and sold for $2000.

(b) A book purchased for $45 and sold for $20.

(c) Sporting goods purchased for $320 and sold for $520.

(d) Jewellery purchased for $980 and sold for $1750.

6 An electrical goods retailer buys a washing machine for $580 and applies a standard mark up of 70% to all of its goods. The selling price for one of its washing machines is:

A $174 **B** $406 **C** $754 **D** $986

7 A magazine is advertised for 20% off the marked price and you are charged $7.60. The original marked price of the magazine is closest to:

A $6.10 **B** $7.80 **C** $9.10 **D** $9.50

8 A bookstore applies a mark up of 92% to the cost price of all of its books. The cost price of a book with a selling price of $35 is closest to:

A $2.80 **B** $32.20 **C** $18.23 **D** $67.20

9 Calculate **(i)** the dollar value of the discount and **(ii)** the discount sale price of the following items. (Round your answers to the nearest cent.)

(a) A Wii console originally priced at $379 on sale at a 10% discount.

(b) A suitcase with an original price of $199 discounted by 40%.

(c) A $20 T-shirt discounted by 30%.

(d) A discount of 25% on a $99.95 pair of sunglasses.

10 Sanjay invests $30 000 in the sharemarket only to see his shares plummet in value to $22 000.

(a) Find the amount of Sanjay's loss.

(b) Find his percentage loss by expressing the loss as a percentage of the original amount invested.

Understanding

11 In Australia, a Goods and Services Tax (GST) of 10% is added to the price of all products and services purchased by consumers (with the exception of basic grocery items).

Calculate **(i)** the amount of GST and **(ii)** the new, GST-inclusive price of the following items.

(a) a $560 bike **(b)** a $64 pair of jeans

(c) a $19 calculator **(d)** a $2640 TV

12 The amount calculated by a builder for a roof restoration is $2450 before adding GST. Calculate:

(a) the 10% GST to be added

(b) the final amount (including GST).

13 Mitchell buys an antique desk for $150. He spends $45 to restore it to its original condition and sells it for $320.

(a) How much profit did Mitchell make, taking the cost of the restoration into account?

(b) Calculate the value of the profit as a percentage of the total cost. (Write your answer rounded to one decimal place.)

14 It costs a furniture company $180 to manufacture a sofa. The cost of transport to the company's stores is $7.50 per sofa. The company then sells the sofas for $350.

(a) How much profit does the company make on the sofas, taking transport and manufacturing costs into account?

(b) Express the profit as a percentage of the selling price to two decimal places.

15 An exclusive boutique applies a 200% mark up to all of the clothing it sells. What would the selling price be of a gown that had a cost price of $150?

16 A store purchases a bulk order of 72 photo frames for $324.

(a) Calculate the profit made if the photo frames are sold individually for $7 each.

(b) Calculate the profit as a percentage of the cost price to two decimal places.

17 Most of the time, the marked prices we see on goods in shops have the GST already added. Find the original, pre-GST price of:

(a) a book with a price tag of $32.95

(b) a microwave with a price tag of $279

(c) a boat priced at $29 999

(d) a T-shirt priced at $39.95.

18 A store takes delivery of a large container with 1000 packets of soap which cost the store $2750. The store policy is to add a 70% mark up and then add the GST. Find the selling price of each packet of soap.

Reasoning

19 When out shopping, you may have noticed that some percentages are used more commonly than others, such as 10%, 20%, 25% and 50%. Explain why these percentage values are used, rather than, say, 17%, 36% or 42%.

20 Julia makes and sells jars of jam to a local cafe, which then sells the jam to customers. Julia adds a 70% mark up to the cost of making the jam before selling it to the cafe. The cafe adds its own mark up of 85% to obtain the price at which the jam is sold to cafe customers. Express your answers to the following to the nearest 5 cents.

(a) If a jar of jam costs Julia $1.35 to make, at what price does she sell each jar to the cafe?

(b) At what price does the cafe sell the jam to its customers?

(c) Express the cafe's selling price of the jam as a percentage of Julia's initial cost price, correct to the nearest whole number.

21 **(a)** For the following situations, calculate the profit made as a percentage of the cost price, rounding your answers to two decimal places where necessary.

(i) CP = \$45, SP = \$54

(ii) CP = \$360, SP = \$504

(iii) CP = \$4200, SP = \$4350

(iv) CP = \$480, SP = \$600

(b) Repeat **(a)**, this time calculating the profit as a percentage of the selling price.

(c) For each of the situations **(i)**–**(iv)**, compare your answers to **(a)** and **(b)**. Which gives a higher percentage—calculating profit using the cost price or using the selling price?

(d) Explain your answer to **(c)**.

22 Billy Bob's Bargain Basement is discounting the selling price of its toys by 15%. If the store's mark up on toys is 50% of the cost price, find:

(a) the discounted sale price of a toy that has a cost price of \$45

(b) the percentage profit made by the store on the sale of the discounted toy

(c) the decrease in percentage profit made by discounting the sale price of the toy.

23 Tariq sees an item in a bargain store initially marked down by 50% with a sign that says 'take an extra 20% off'. If, after these two discounts, the price Tariq paid was \$11.20, what was the original price of the item?

24 Martha's favourite clothing shop is offering a 'buy two, get the third half price' offer on T-shirts. The shop next door is offering 20% off the same brand of T-shirts. If the regular retail price of the T-shirts in both stores is \$19.95, which store is the cheaper place for Martha to buy three T-shirts? How much would she save, compared to the other store?

25 A number of people own an earth-moving business. Each person owns a certain percentage of the business. Profits are divided according to the percentage of the money they invested. For example, someone investing 7% of the operating capital (the amount of money needed to run the business) would receive 7% of the profits. The operating capital is $675 000.

(a) Calculate the amount contributed by a person who has invested 7% of the operating capital.

(b) If the company makes $150 000 yearly profit, how much would the person who invested 7% receive?

(c) If the company continued to make this profit each year, how many years would it be before the person has received back their investment?

26 It is very useful to be able to calculate some common percentages mentally. Knowing their fraction equivalents is very helpful.

(a) Write down the fraction equivalents of:

(i) 25% **(ii)** 10% **(iii)** 20% **(iv)** $33\frac{1}{3}$%

(b) Chantal knows that 50% = $\frac{1}{2}$. When she sees a '50% off' sale sign, she halves the original price to find the new price. Use your answers to part **(a)** to help Chantal extend her method so she can calculate 25%, 10%, 20% and $33\frac{1}{3}$% off.

Open-ended

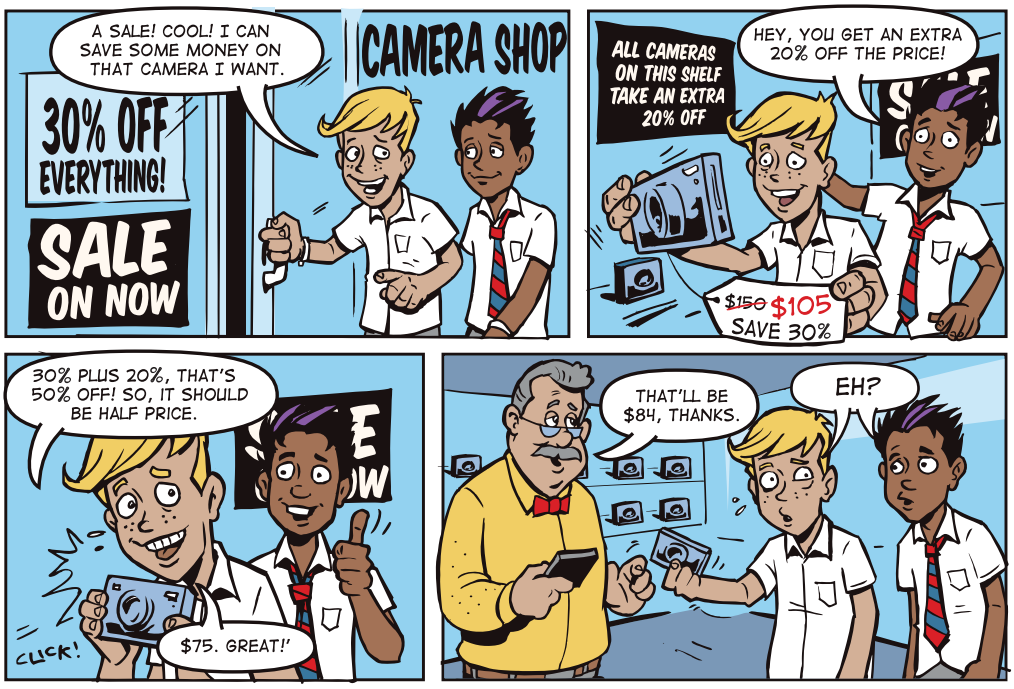
27 A sports store is having a sale on all sporting equipment. If a tennis racquet is normally priced at $299, write three different, common discount percentages and find the sale price for each.

28 Many stores, such as department stores, advertise sale items using percentage discounts, as seen in this catalogue:

At other times, stores advertise sale items using dollar disounts as seen in this catalogue:

Why do you think this is? Try to give a couple of reasons for your answer.

29



The boys think the camera should only cost $75. The shop assistant says the camera now costs $84. What mistake have the boys made? Explain how the shop assistant calculated the new price of $84.

Outside the Square Problem solving

The plummeting price

Frida puts a price tag of $45 on a Superman costume for sale in her fancy-dress store. Unfortunately, there has been little interest in the costume since the death of Superman in his comic, so she reduces the price to $27. As the costume still does not sell at the reduced price, Frida reduces the price again, by the same percentage as the first reduction.

1 What is the new reduced price of the costume?

2 How many reductions at the same rate would reduce the price to less than one-quarter of the original price?

3 If the initial price was different would it take the same number of reductions?

Strategy options

- Break problem into manageable parts.
- Test all possible combinations.

Technology Exploration Excel

Equipment required: 1 brain, 1 computer with Excel or a spreadsheet on a CAS calculator

Versions of this Exploration for other technologies are available in Pearson Reader.

Sam's Skate Shop

A spreadsheet can be a very useful tool to help businesses keep track of their sales and profits. In this task we will set up a spreadsheet for Sam, who has just opened a shop selling skateboards, protective gear, skate clothing and skating DVDs.

1 Open a new spreadsheet file and enter the following details.

D2

	A	B	C	D	E	F	G
1	Item	Cost Price ($)	% Mark Up	Selling Price	Profit	Volume Sold	Volume Profit
2	Skateboard	103	65				
3	Jeans	35	110				
4	Hoodie	20	95				
5	Skate Shoes	42	125				
6	DVD	8.5	50				
7	Helmet	17	76.4				
8							

2 To calculate the selling price, enter the formula =B2*(1+(C2/100)) into cell D2. Copy this formula down the column by placing the cursor over the bottom right-hand corner of the cell. When the thin black cross appears, click and drag the cursor to the bottom of the column. By referring particularly to the part in brackets, explain how this formula calculates the selling price.

3 As we are working with money, we will format the cells in column D to show two decimal places. Highlight the column by clicking on the letter D. Right click on the highlighted column, and select 'Format Cells' from the menu that appears. Select the 'Number' tab and in the 'Number' category, enter '2' in the 'Decimal places' box, then click 'OK'.

4 Click on cell E2 and enter a formula that will calculate the dollar profit on that item. Make sure your formula begins with the '=' sign. Copy this formula down the column, and format the cells as you did in step **3**. If you have entered the formula correctly, your table should look like the one at the top of the next column. Do not work out the profits and enter them manually, as we will be relying on the use of formulas later on.

F2

	A	B	C	D	E	F	G
1	Item	Cost Price ($)	% Mark Up	Selling Price	Profit	Volume Sold	Volume Profit
2	Skateboard	103	65	169.95	66.95		
3	Jeans	35	110	73.50	38.50		
4	Hoodie	20	95	39.00	19.00		
5	Skate Shoes	42	125	94.50	52.50		
6	DVD	8.5	50	12.75	4.25		
7	Helmet	17	76.4	29.99	12.99		

5 In one fairly typical week, Sam sold 9 skateboards, 27 pairs of jeans, 42 hoodies, 38 pairs of skate shoes, 86 DVDs and 15 helmets. Enter this information into your spreadsheet under the 'Volume sold' heading. To calculate the profit made on the volume of each item sold, enter a formula into cell G2 that will multiply the profit made on one item by the volume of that item sold. Format the cells as you did in step **3**. If you have entered the formula correctly, your table should look like this:

H1

	A	B	C	D	E	F	G	H
1	Item	Cost Price ($)	% Mark Up	Selling Price	Profit	Volume Sold	Volume Profit	
2	Skateboard	103	65	169.95	66.95	9	602.55	
3	Jeans	35	110	73.50	38.50	27	1039.5	
4	Hoodie	20	95	39.00	19.00	42	798	
5	Skate Shoes	42	125	94.50	52.50	38	1995	
6	DVD	8.5	50	12.75	4.25	86	365.5	
7	Helmet	17	76.4	29.99	12.99	15	194.85	

6 Find Sam's total profit on all the items sold for the week. Highlight (by clicking and dragging) all of the cells in column G of the table, plus the empty cell below them. Find the Σ symbol on the far right of the toolbar under the 'Home' tab and click on the downward arrowhead on the right. Click on 'Sum' from the list that appears. The total profit will appear at the bottom of the column.

Answer the following questions by changing the numbers on your spreadsheet and noting the effects.

7 Sam has found that he must make a profit of at least $4300 a week in order to pay the rent on his shop, his electricity, water and phone bills and to pay Will, his casual employee who works on weekends. By changing the number in the 'Volume Sold' column (but keeping them in roughly the same proportion as in Question **5**), find a minimum number of each item that Sam must sell to make the minimum profit.

8 Sam sells out of DVDs and will not receive a new order until next week. How many extra skateboards would he need to sell to make up for the loss of DVD profits, assuming he sells the same number of the other items as in Question **5**?

9 Sam believes that the mark up on his jeans and skate shoes is too high, and that he would sell more at 30% less mark up. Write down the selling price, the profit, the volume sold and the volume profit for the jeans and skate shoes from the original spreadsheet in Question **5** before using the spreadsheet to answer the following questions.

(a) Find the new selling prices of the jeans and skate shoes after lowering their mark up by 30%.

(b) How many extra pairs of jeans and shoes will Sam have to sell at this price to make the same volume profit as before?

Taking it further

10 Insert some extra columns and a formula into your spreadsheet so that you are able to enter the number of items in stock at the beginning of the week, and calculate the number left after the 'Volume Sold' data is entered.

11 Insert a 'Discount %' column and a 'Sale Price' column after the 'Selling Price' column. Enter a number of common discounts (such as 10% or 25%) into the 'Discount %' column. Enter and fill down a formula that will calculate the sale price for you.

(a) If Sam has a '20% off everything' sale, calculate a minimum combination of items that need to be sold to obtain his minimum profit of $4300.

(b) Explain why a 20% discount on the selling price of an item is not the same as 20% less mark up on the item.

Maths 4 Real

Relative Wealth

Are you wealthy?

Many people aspire to be wealthy. Many of their choices, such as which career path to follow, are based on whether they think it will help them to become wealthier. You might not think of yourself or your family as particularly wealthy. Here are some facts to consider.

In 2005, there were 6.46 billion people living in the world. Of these:

- 1 in 5 lived on less than US $1.25 a day.
- Nearly 1 in 2 lived on less than US $2.50 a day.
- Nearly 4 in 5 lived on less than US $10 a day.

Many of these people live in Africa, Southern Asia, South America and Eastern Europe.

1 (a) Write 1 in 5 as:
(i) a fraction (ii) a percentage.

(b) Repeat (a) for 1 in 2 and 4 in 5.

(c) Why do you think these statistics are reported using the phrase '1 in …' instead of using a fraction or a percentage?

2 In 2006, the median income of an Australian household was $1139 per week. How many dollars a day is this?

3 In 2010, the minimum wage (required by law) that an Australian adult should be paid was $570 a week. How many dollars a day is this?

4 In 2010, a single person on the Commonwealth government's unemployment allowance was entitled to $231.40 per week. How many dollars a day is this?

5 Look again at the facts stated in the red box, then at your answers to Questions 2, 3, and 4. If the countries of the world were ranked from richest to poorest, where do you think Australia might be positioned? Give some reasons for your answer.

6 If Australia is one of the world's wealthier countries, how is it that there are people in Australian society who we would say are poor, or living in poverty?

7 In 2007, the poorest 40% of the world's population received 5% of global income. The richest 20% received 75% of global income. (Global income can be thought of as the money received by all of the people working around the world.)

(a) Rewrite the above statement using fractions.

(b) Which statement do you think has more impact—the one with fractions or percentages?

The Millennium Development Goals

In 2000, the world's leaders came together for a meeting of the United Nations (UN).They agreed to work towards a set of goals designed to make the world a better and fairer place, known as the Millennium Development Goals. The first Goal was to eradicate extreme poverty and hunger, or, more specifically, to take the number of people living on less than US $1 per day in 1990, and halve it by 2015. Other Goals included stopping the spread of diseases such as malaria and AIDS, and ensuring that all children could complete a primary education.

In order to achieve this goal, the UN suggested that the wealthier, developed countries should give 0.7% of their national income in aid to developing countries. However, a 2006 report stated that the average amount given by a country was only 0.33%.

8 Write: (a) 0.7% and (b) 0.33% as fractions.

Despite this, the UN reported that in 2005 the number of people in extreme poverty was 1.4 billion, down from 1.8 billion in 1990.

9 (a) Write the reduction in the number of people in extreme poverty as a percentage of the 1990 figure.

(b) How many more millions of people need to be helped out of extreme poverty by 2015 for the Millennium Development Goal to be achieved?

It's not just about money

Poverty is not just about a lack of money and the inability to buy things such as food, fuel, clothes or tools. People in poverty often do not get access to healthcare (such as immunisations) or medical treatment, so their years of a healthy life are reduced. Children in poverty are often sent out to work instead of school, missing out on an education that would provide them with the basic literacy and numeracy skills that would enable them to get a better paying job.

10 Suggest some ways in which the aid money from the richer nations could be used to help move people out of extreme poverty.

How can I help?

There are many, many organisations devoted to eliminating the poverty and inequality that exists in the world, including: The Red Cross, World Vision, UNICEF and Oxfam. As well as directly helping those in need, these organisations work to raise other people's awareness of the issues faced, and suggest ways they can help, either by donating money, sponsoring a child or buying products made or grown by people in poverty. Other organisations, such as Doctors Without Borders, and Engineers Without Borders, are groups of professional people who work as volunteers in poorer countries, in their areas of expertise.

Research

Find out about the other Millennium Development Goals, and the progress that has been made towards achieving them.

Find out more about the organisations devoted to helping people in poverty, and make a list of practical ways in which you can support them. Share this list with your class.

Find out some other facts about poverty and inequality, either within Australia or within other countries. Create an online advertisement or campaign to raise awareness of the issues, and what people can do to help.

Challenge 2

1 The average (mean) of $\frac{2}{3}$, 0.7 and 55% is:

A $\frac{23}{36}$ **B** 175% **C** $1\frac{11}{12}$ **D** 5.75

2 Jake bought a large tub of yoghurt. On the first day he ate $\frac{1}{4}$ of the yoghurt. The next day he ate $\frac{1}{3}$ of what was left, and on the third day he ate $\frac{2}{3}$ of what was left. What fraction of the yoghurt remained after the third day?

3 What fraction is halfway between $\frac{1}{4}$ and $\frac{1}{2}$?

4 A piece of fabric cost Michael \$25. He purchases another piece of similar fabric that is twice as long and twice as wide as the first piece. How much should this second piece cost him?

5 Which of the following is the best value deal for a box of chocolates that normally sells for \$5.04?

A save \$1.70 **B** cost price of \$3.20, +5%

C discount of $33\frac{1}{3}$ % **D** buy 2, get 1 free

6 Write an expression using four 8s and any of the operation symbols (+, –, ×, ÷) that equals 65.

7 Find the sum of $\frac{1}{2} + \frac{0.1}{2} + \frac{1}{0.2}$, expressing your answer in decimal form.

8 Michael picks three *different* digits from the set {1, 2, 3, 4, 5} and forms a mixed number by placing the digits in the spaces $\square\frac{\square}{\square}$. The fractional part of the mixed number must be less than 1. What is the difference between the largest and the smallest mixed numbers that can be formed?

A $4\frac{4}{15}$ **B** $4\frac{3}{10}$ **C** $4\frac{7}{20}$ **D** $4\frac{3}{5}$

9 Suppose that x^* means $\frac{1}{x}$ which is called the inverse of x. For example, $5^* = \frac{1}{5}$. How many of the following statements are true?

(i) $2^* + 4^* = 6^*$ **(ii)** $3^* \times 5^* = 15^*$ **(iii)** $7^* - 3^* = 4^*$ **(iv)** $12^* \div 3^* = 4^*$

A 0 **B** 1 **C** 2 **D** 3

10 The sum of seven consecutive integers is always:

A odd **B** a multiple of 7 **C** a multiple of 3 **D** even.

11 Find the following sum, without using a calculator.

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128} + \frac{1}{256} + \frac{1}{512} + \frac{1}{1024} + \frac{1}{2048} + \frac{1}{4096} + \frac{1}{8192}$$

12 One liquid contains 22.5% water, another liquid contains 27% water. A glass is filled with 5 parts of the first liquid and 7 parts of the second liquid. What percentage of the liquid in the glass is water?

Chapter review 2

D.I.Y. Summary

Key Words

cost price (CP)	irrational number	per cent	sale price
discount	loss	percentage	selling price (SP)
exact decimal form	mark up	profit	surd
Goods and Services Tax (GST)	non-terminating decimal	recurring decimal	terminating decimal

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 The word _____ literally means per hundred or 'out of a hundred'.

2 A decimal such as $2.6\dot{3}$ is written in _____.

3 A decimal that has an infinite number of digits after the decimal point is called a _____.

4 To convert a decimal to a _____, you multiply by 100 and add the percentage symbol.

5 The amount added to the cost price of goods before selling them is known as the _____.

6 A number with a defined number of digits after the decimal point is called a _____.

7 An _____ is a non-terminating decimal that cannot be written as a fraction.

8 A _____ is applied to goods by decreasing the marked price by a given percentage.

9 An irrational number written in the form of a square root is called a _____.

10 To make a profit, the _____ must be higher than the _____.

Fluency

Equipment required: Calculator for Questions **2**, **5**, **6**, **15–32** and **34–37**

1 Write the following decimals as fractions or mixed numbers in simplest form. Ex. 2.1

(a) 0.58 **(b)** 0.0124 **(c)** 2.605

2 Put the following into ascending order. Ex. 2.1–2.3

(a) $\frac{2}{9}$, $-\frac{3}{10}$, -0.283, 0.26, $\frac{3}{4}$, $\sqrt{7}$ **(b)** $-\frac{1}{3}$, $\frac{7}{10}$, $\sqrt{2}$, 0.22, -0.26, 0.253

3 Write the following mixed numbers as decimals. Ex. 2.1

(a) $28\frac{5}{8}$ **(b)** $15\frac{3}{40}$ **(c)** $53\frac{261}{320}$

4 How many 60 cm lengths of wood can be cut from a 2.5 m long plank? What length of wood is left over? Ex. 2.1

5 Use a calculator to express the following numbers in decimal form, then classify them as terminating decimals, recurring decimals, or irrational numbers. Ex. 2.2

(a) $53 \div 4$ **(b)** $\sqrt{85.2}$ **(c)** $\frac{11}{9}$ **(d)** $\sqrt{125}$ **(e)** $\sqrt{30.25}$

6 The fraction $\frac{19}{11}$, written in exact decimal form, is: Ex 2.2

A $1.\overline{72}$ B $1.\overline{72}$ C $1.\dot{7}\dot{2}$ D $1.7\dot{2}$

7 Write the following recurring decimals in fraction form. Ex 2.2

(a) $0.\dot{2}$ (b) $0.3\dot{2}$ (c) $0.\dot{3}\dot{2}$

8 Calculate the following, giving your answers as mixed numbers in simplest form. Ex 2.3

(a) $-\frac{1}{2}+\frac{3}{4}$ (b) $\frac{8}{9}-1\frac{1}{6}$ (c) $-\frac{3}{10}\times-\frac{5}{6}$ (d) $-5\div\frac{1}{3}$

9 Calculate the following. Ex 2.3

(a) $-3.4+2.8$ (b) $8.6-10.2$ (c) 4.2×-6.1 (d) $20.4\div-5$

10 Approximately what percentage of the shape is shaded? Ex 2.4

A 20% B 34% C 67% D 75%

11 Convert the following to percentages. Express any remainders as fractions. Ex 2.5

(a) $\frac{23}{50}$ (b) $\frac{6}{5}$ (c) $2\frac{4}{5}$ (d) $\frac{25}{40}$

12 Convert the following to percentages. Ex 2.5

(a) 0.6 (b) 0.35 (c) 1.12 (d) 0.03

13 Convert the following to fractions in simplest form. Ex 2.6

(a) 65% (b) 154% (c) $5\frac{5}{6}$% (d) 4.7%

14 Convert the following to decimals. Ex 2.6

(a) 80% (b) 7% (c) 2.09% (d) $6\frac{1}{4}$ %

15 Express the first amount as a percentage of the second, giving answers rounded to two decimal places if necessary. Ex 2.7

(a) 27, 30 (b) 35, 16 (c) 4 cm, 2 m (d) 30 c, \$4.50

16 Find the following, giving answers rounded to two decimal places if necessary. Ex 2.8

(a) 35% of 84 m (b) $4\frac{2}{5}$% of 2000 kg (c) $7\frac{1}{3}$% of 4500 L

(d) 14% of \$48 (e) 0.59% of \$4000 (f) 9.5% of \$90

17 (a) Increase \$450 by (i) 35% and (ii) 67.2%. Ex 2.9

(b) Decrease \$19.95 by (i) 20% and (ii) 7.5%.

Round answers to two decimal places where necessary.

18 Calculate: Ex 2.10

(a) the sale price of a \$90 tennis racquet discounted by 15%

(b) the selling price of a \$950 ring that will be marked up by 80%

(c) the original marked price of a pair of sunglasses discounted by 30% to \$62.30

(d) the cost price of a dress that has been marked up by 85% to \$130.

19 Kayla bought a concert ticket for \$145, but was unable to go, so she sold it on eBay for \$120. Calculate Kayla's profit or loss as a percentage of what she paid for the ticket, correct to two decimal places.

Ex. 2.10

Understanding

20 The number of pre-booked tickets for a 1-day cricket match is 62% of the 75 242 tickets available. How many tickets were pre-booked?

A 4665 **B** 28 592 **C** 46 650 **D** 121 358

21 Michelle made deposits of \$210, \$25, \$45.50 and \$66.75 into her bank account during one month and withdrawals of \$35.75, \$56.90, \$214 and \$102.50 during the same period. Use a negative number to show how much her balance decreased during this period.

22 Franco has a 5 kg bag of nuts that he needs to use to fill smaller 375 g bags.

(a) How many bags can he fill?

(b) What mass of nuts will he have left over?

23 The value of a property increases by 17.5% one year. By what fraction did the property's value increase?

24 Approximately 85% of an iceberg lies under water. The average mass of an iceberg is 150 000 tonnes. What mass of the iceberg sits above water?

25 A geography book states that 0.7092 of the Earth's surface is covered by ocean. Write this value as a percentage.

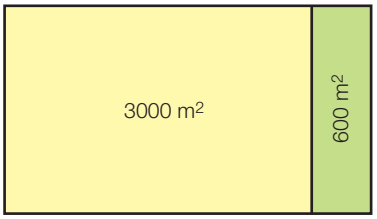
26 The Australian desert is 42.556% as large as the Sahara Desert in Africa, which has an area of 9 000 000 square kilometres. Find the area of the Australian desert to the nearest square kilometre.

27 Tom bought an old car for \$3500. He spent \$2700 to repair and restore it, then sold it for \$9400.

(a) Calculate the profit Tom made, taking all costs into account.

(b) Express the profit as a percentage of the selling price, rounding your answer to the nearest percent.

28 Tye buys a neighbouring 600 m^2 property to add to his 3000 m^2 farm. By what percentage has the area of his farm increased?



29 Gao currently earns \$600 a week. If he receives a pay rise of $2\frac{1}{3}$%, what will his new weekly pay be?

30 Arala runs a newsagency. She buys boxes of pens, then sells the pens individually. A box of 60 pens costs her \$17.

(a) To work out what she will sell them for individually, Arala needs to know what she paid per pen. Calculate the cost of each pen, to the nearest cent.

(b) Arala wants to sell each pen for 25% more than what she paid for them. Use your answer from **(a)** to calculate the price Arala should sell the pens for.

31 Nazim had 62 friends on his social networking web page. One month later, he had 97 friends. Write this increase as a percentage of his initial number of friends. Round your answer to one decimal place.

32 The minimum overnight temperatures for one week at Mt Hotham were -2°C, -3°C, -1°C, 2°C, -3°C, -2°C, 0°C. Find the mean minimum overnight temperature for the week, rounding your answer to one decimal place. (The mean is found by adding all values, then dividing the result by the number of values.)

33 Copy the number line below and place the following percentages on it: 14%, 140%, 4%, 104%, $\frac{1}{4}$%

Reasoning

34 A certain cheese contains $7\frac{1}{2}$% fat. How much fat is in a 600 gram piece?

35 **(a)** The decimal value of $\frac{4}{7}$ is closest to:

A 0.57 **B** 0.571 **C** 0.58 **D** 0.6

(b) The decimal approximation of $\sqrt{23}$ is closest to:

A 4.7 **B** 4.796 **C** 4.8 **D** 5

36 The combined capacity of all of the dams that supply water to Melbourne is 1 810 500 ML (1 ML = 1 000 000 L). In May 2010, the dams were 33.3% full of water, higher than the 27.2% recorded on the same day the previous year.

(a) How many ML of water was in the dams in May 2010?

(b) How many more ML was present in May 2010 than in May 2009?

(c) How many more ML would need to flow into the dams to achieve a capacity of 35% full?

37 A games store applies a mark up of 80% to the cost price of all of its products, then adds 10% GST.

(a) Find the final price of a game that had a cost price of $33.25, to the nearest 5 cents.

(b) The GST charged by the store must be given to the government. What is the amount of GST in the final price of the game?

(c) What profit does the store make on the game, allowing for GST?

(d) The store now discounts the price of the game by 25%. What is the new price of the game?

(e) What is the amount of GST in this new price?

(f) What profit does the store make on the game now, allowing for GST?

38 A packet of 12 chocolate frogs is on sale for $4.75.

(a) How much does each frog in the packet cost? Write your answer in exact decimal form.

(b) Write the cost of each frog rounded to the nearest cent.

(c) How much extra will you pay if you buy 12 frogs individually at this price?

(d) Where does this 'extra' amount come from?

NAPLAN practice 2

Numeracy: Non-calculator

1 Jye cuts a pizza into 12 equal slices. He eats 75% of the slices.

How many slices of pizza are left? _____

2 A ticket costs $75. A fee of 10% is added to the price. Which calculation will give the new price?

A $75 + 10$ **B** $75 + 0.1$ **C** 75×0.1 **D** 75×1.1

3 Vicky bought a house for $400 000. At the end of 1 year, its value had increased by 7%. At the end of the next year, the value had increased by another 5%. What was Vicky's house worth after 2 years?

4 A skateboard is on sale for 20% off the marked price. If the marked price is $140, how much is the sale price?

A $102 **B** $112 **C** $120 **D** $160

Numeracy: Calculator allowed

5 At midday during winter in Canberra the temperature was 11.7°C. At 6 p.m., it had dropped by 5.4°C. By midnight, the temperature was 15.2°C cooler than at midday. What was the temperature at midnight?

A 6.3°C **B** -3.5°C **C** -35°C **D** -9.8°C

6 There were only 13 students in Joseph's class on Wednesday. The other 7 were absent.

What percentage of Joseph's class was absent?

A 7% **B** 35% **C** 54% **D** 65%

7 Justin bought a multi-bag of chocolate bars. The bag contained 15 chocolate bars and it cost $3.70. What is the price of each chocolate bar, correct to the nearest cent?

8 Maryam scored 28 goals from 40 shots during netball training. What percentage was this?

Mixed review A

Fluency

1 Calculate the following. Ex. 1.1, 1.2, 1.3

(a) $7 - (-9)$ **(b)** $-3 + (-10)$ **(c)** $-14 - (+8)$ **(d)** $-21 + 9$

(e) -4×-9 **(f)** 7×-3 **(g)** $-60 \div 15$ **(h)** $-\frac{48}{8}$

2 Write the following decimals as fractions in simplest form. Ex. 2.1

(a) 0.36 **(b)** 0.05 **(c)** 0.75 **(d)** 0.002

3 A TV has been bought for a cost price of $560 by a retailer, who will add a mark up of 78% before selling it in the store. Calculate: Ex. 2.10

(a) the value of the mark up

(b) the selling price after the mark up has been applied.

4 Evaluate the following by simplifying first where possible. Ex. 1.5

(a) $2^3 \times 5^2$ **(b)** $3^4 \times 10^3$ **(c)** $-4^2 \times 5$

(d) $-3 \times (-2)^4$ **(e)** $(5^2 \times 3^3) \div (5 \times 3)$ **(f)** $(7^4 \times 8^4) \div (7^2 \times 8^2)$

(g) $\frac{4^2 \times 6}{2^3}$ **(h)** $\frac{8^2 \times 2^3}{4^2}$ **(i)** $\frac{3^3 \times 4^4}{2^6}$

5 Convert the following fractions to percentages. (Round answers to two decimal places if necessary.) Ex. 2.5

(a) $\frac{17}{20}$ **(b)** $\frac{3}{5}$ **(c)** $\frac{5}{8}$ **(d)** $\frac{20}{35}$

6 Write the following fractions as recurring decimals using the correct notation. Ex. 2.2

(a) $\frac{1}{3}$ **(b)** $\frac{2}{3}$ **(c)** $\frac{1}{6}$ **(d)** $\frac{5}{9}$

7 Write the following percentages **(i)** as decimals and **(ii)** as fractions in simplest form. Ex. 2.6

(a) 70% **(b)** 45% **(c)** 120% **(d)** 8%

8 Find the following percentages. (Round answers to two decimal places if necessary.) Ex. 2.8

(a) 20% of $240 **(b)** 65% of 700 kg **(c)** 42% of 8.5 m

(d) 3% of 16 seconds **(e)** 78% of $19.40 **(f)** 110% of 400 m

9 For each of the following, draw a number line from -3 to 3 and indicate the positions of the numbers (you may need to estimate the positions of some). Ex. 2.3

(a) $-\frac{1}{2}, -\frac{4}{3}, 1\frac{1}{2}, \frac{2}{3}$ **(b)** 0.7, -1.2, -0.5, 2.1, 0.02

10 Which of the following is equivalent to 10^4? Ex. 1.5

A 40 **B** 100 000 **C** 10 000 **D** 400

11 Evaluate: Ex. 1.4

(a) $-9 + 2 \times -5 + 7$ **(b)** $-4 + (-3)^2 + 10 \div 5$

12 A pair of sunglasses that has a marked price of \$74 will be discounted by 40%. Calculate:

Ex. 2.10

(a) the value of the discount

(b) the sale price after the discount has been applied.

13 Simplify the following, then evaluate.

Ex. 1.5, 1.6

(a) $6^2 \times 5^2 \times 5^4 \times 6^1 \times 5^3 \times 6^2$

(b) $(7^2 \times 10^1 \times 10^3 \times 7^6) \div (7^2 \times 5^2 \times 5 \times 2^2)$

(c) $(10^2 \times 10^1 \times 10^0) \div (2^0 \times 5^0 \times 5 \times 2)$

(d) $\dfrac{3^0 \times 4 \times 5^2}{2^0} \times \dfrac{(2^3 \times 5^2)^0}{(12 \times 10 \times 5)^0}$

Understanding

14 Dhanya has a group of 30 insects she is studying in her science lab. After 1 month, the number of insects had doubled, giving 30×2 (60) insects. After 2 months, the number had doubled again, giving $30 \times 2 \times 2$, or 30×2^2 insects. After 3 months, there were $30 \times 2 \times 2 \times 2$, or 30×2^3 insects. The group continues to double in number every month.

(a) Show, using a number in index form, how many insects there will be after **(i)** 6 and **(ii)** 12 months.

(b) Evaluate each of your answers to **(a)** to give an actual number of insects after 6 and 12 months.

15 An ice cube tray of water at room temperature (20°C) is placed in a freezer, where it takes 3 hours to freeze solid. What is the average hourly drop in temperature? Give your answer as a negative fraction in simplest form.

16 Show by evaluating that $(2 \times 3)^2 = 2^2 \times 3^2$.

17 Ella works in a clothing shop. Her manager has asked her to place the following items on sale at a 20% discount. Find the sale price for the following.

(a) Jeans, \$65 **(b)** Jacket, \$89 **(c)** T-shirts, \$24 **(d)** Socks, \$8.50

Reasoning

18 Casey is keeping track of how many goals she scores in her netball matches. She wants to steadily improve her accuracy.

Match 1: 10 goals out of 14 shots
Match 2: 15 goals out of 20 shots
Match 3: 13 goals out of 18 shots
Match 4: 12 goals out of 16 shots

Convert each of Casey's results to a percentage, then use them to rank the matches from her best performance to her worst. Round the answers to one decimal place where necessary. Would you say that Casey is steadily improving her accuracy?

19 **(a)** As a decimal, the value of $\frac{8}{17}$ is closest to:

A 0.4 **B** 0.47 **C** 0.48 **D** 0.5

(b) As a decimal, the value of $\sqrt{78}$ is closest to:

A 8 **B** 8.8 **C** 8.9 **D** 9

20 Ali has performed the following calculation in a maths assignment.

$2^4 + 3^3 + 5^2$
$= 8 + 9 + 10$
$= 27$

His teacher has marked this as incorrect. Explain to Ali what he has done wrong, and what he needs to remember when working with numbers in index form.

3

Algebra

3

The humble soap bubble may be much more important than you think.

Bubbles always try to make the smallest possible surface. As a sphere has the smallest surface area for its volume of any solid, bubbles will always be in the shape of a sphere. Mathematicians use algebra to explain how the surface tension and the pressure difference between the outside and the inside of the bubble determines the radius of the sphere. They use the following formula.

$$P_{in} - P_{out} = \frac{4T}{r}$$

where

P_{in} = the pressure inside the bubble

P_{out} = the pressure outside the bubble

T = the surface tension

r = the radius of the bubble.

The study of bubbles is important in research on the bends, a dangerous medical condition that occurs when nitrogen bubbles form in the tissues of scuba divers if they surface too quickly.

According to some current scientific ideas, our entire universe is merely one bubble in an ocean of bubbles.

Forum

What ingredients are needed to make a good bubble solution? Why are they necessary?

Our universe has been described as a bubble in an ocean of bubbles. What do you think this means? How could you find out more about this?

Why learn this?

Algebra gives us a symbolic way to describe and generalise patterns that occur in the real world. For example, we can develop formulas to describe where and when birds migrate, or how a bridge supports the weight of cars on it, or how oceans move so that we can predict tides. Modelling relationships using algebra allows us to identify patterns and make predictions from these patterns. This chapter will enable you to continue to develop your skills in using and manipulating algebraic expressions.

After completing this chapter you will be able to:

- recognise variables and form algebraic expressions
- simplify algebraic expressions using addition, subtraction, multiplication and division
- recognise like and unlike terms
- simplify algebraic expressions
- substitute values into algebraic expressions and formulas and then evaluate the expression
- expand algebraic expressions using the distributive law
- factorise algebraic expressions by identifying the highest numerical common factor (HCF)
- solve simple problems using algebra.

Recall 3

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R3.1

1 Write algebraic expressions to represent each of the following.

(a) ten more than x **(b)** m divided by three

(c) two lots of l **(d)** y less than twenty

Worksheet R3.2

2 Calculate:

(a) $6 + 3 \times 4$ **(b)** $2 \times -3 - 6 \times 5$

(c) $5 \times (14 - 10) \div 10$ **(d)** $5 - 11 + (16 - 15) \times 8 + 2$

Worksheet R3.3

3 Substitute each of the values of m into the formula $l = 2m + 1$ to find l.

(a) $m = 4$ **(b)** $m = -2$

Worksheet R3.4

4 Answer TRUE or FALSE for each of the following statements.

(a) $y = 3x - 4$ is an expression **(b)** $3m$ is a term

(c) $2x + 3 - 4a$ is an equation **(d)** $7y - 10$ is an expression

Worksheet R3.5

5 Express each number as a product of its prime factors.

(a) 18 **(b)** 28 **(c)** 42 **(d)** 660

Worksheet R3.6

6 Write each of the following in index form.

(a) $3 \times 3 \times 3 \times 3$ **(b)** six cubed

(c) fourteen squared **(d)** five to the power seven

Worksheet R3.7

7 Write these numbers in expanded form, then evaluate.

(a) 9^2 **(b)** $(-3)^4$ **(c)** $10^4 - 2^5$

Worksheet R3.8

8 Write down the highest common factor (HCF) of each of the following.

(a) 6 and 10 **(b)** 15 and 55 **(c)** 10, 30 and 42

Key Words

associative law	equation	factorising	substitution
coefficient	expanding	formulas	term
constant	expression	Highest Common Factor (HCF)	unlike terms
distributive law	factorised form	like terms	variable

Variables and expressions

3.1

The language of algebra

To be able to use algebra to solve problems, you need to understand the language. In Year 7 you were introduced to many new words and conventions.

A **variable** describes an unknown amount in an algebraic expression or equation. Variables are represented by a letter or a symbol. For example, the average amount of water used by a Year 8 student per day is a variable that can be represented by symbols such as a, x, θ or β. These symbols can also be referred to as pronumerals.

A **term** can be:

- a number or a variable by itself: 7, x
- a number multiplied by a variable: $12c$, $5k$
- different variables multiplied together: mn, abc
- a variable multiplied by itself a number of times: a^3, b^2
- the product of a number and several variables: $3xy$, $4a^2b$.

An **expression** is formed when terms are added or subtracted; for example, $3x + 4y$.

A **coefficient** of x is a number by which the variable is multiplied. The coefficient is written in front of any variable and includes any negative sign in front of the number. 8 is the coefficient of $8x$ and -6 is the coefficient of $-6y$. Note that the coefficient of x is 1.

A **constant** is a term that does not contain any variable factors. It is a number written by itself. 6 is the constant in $3x + 6$.

An **equation** joins two expressions with an equals sign: $3x + 6 = 2x - 4$.

Worked Example 1 WE1

For $6a + 3b - 4 = 2ab - c + 7$

(a) identify whether this is an equation or an expression

(b) write down the coefficient of b

(c) write down all the terms in the equation or expression

(d) list the variables

(e) write down any constants.

Thinking	Working
(a) Look for an equals sign. If there is one, it is an equation. If there is not, it is an expression.	(a) It is an equation because it has an equals sign.
(b) Look for the number in front of the b. This is the coefficient of b.	(b) 3 is the coefficient of b.

(c) Look for parts of the equation separated by addition or equals signs. (Subtraction is the addition of a negative term (-4 = + (-4)).	(c) 6a, 3b, -4, 2ab, -c and 7
(d) Look for the letters or symbols in the equation.	(d) a, b and c are variables.
(e) Look for numbers that are by themselves.	(e) -4 and 7 are constants.

Algebraic conventions

A convention is a rule mathematicians agree to work by.

- A multiplication sign is not required between variables and numbers or other variables ($5 \times b = 5b$, $a \times d \times f = adf$).
- Division can be shown as a fraction ($3a \div b = \frac{3a}{b}$).
- Indices show variables being multiplied by themselves ($c \times c = c^2$).
- Numbers are written before a variable ($a \times 4 = 4a$).
- Variables are written in alphabetical order ($y \times z \times x = xyz$).

Worked Example 2

WE2

Rewrite the following using algebraic conventions.

$t \times m \times 6 \times m \div 5$

Thinking	Working
1 Replace the division sign with a fraction bar.	$\frac{t \times m \times 6 \times m}{5}$
2 Rewrite with the numbers in the multiplication at the start.	$= \frac{6 \times t \times m \times m}{5}$
3 Remove multiplication signs.	$= \frac{6tmm}{5}$
4 Write in alphabetical order.	$= \frac{6mmt}{5}$
5 Use indices to show variables being multiplied by themselves.	$= \frac{6m^2t}{5}$
Note that the steps can be done in a different order as long as they are all completed.	

Worked Example 3 WE3

Pete has b blue pens and r red pens. Sue has three times as many blue pens as Pete. She has 2 fewer red ones than Pete.

(a) Write an expression for the number of pens Pete has altogether.

(b) If Pete has 16 pens to start with, write an equation to show this information.

(c) Write an expression for the number of blue pens Sue has.

(d) Write an expression for the number of red pens Sue has.

(e) If Sue has 28 pens altogether, write an equation to show this information.

(f) If Pete loses 4 of his pens, write an expression in terms of b and r to show how many pens Pete has now.

Thinking	Working
(a) Identify the variables (b and r) that need to be used and the operation needed to be performed on them (+).	(a) $b + r$
(b) Equate the expression in part **(a)** to the information given.	(b) $b + r = 16$
(c) Decide what needs to be done to the first variable ($3 \times b$).	(c) $3b$
(d) Decide what needs to be done to the second variable ($r - 2$).	(d) $r - 2$
(e) Use the information in parts **(c)** and **(d)** to form an expression ($3b + r - 2$) and equate to the information given.	(e) $3b + r - 2 = 28$
(f) Identify what operation needs to be performed on the expression in part **(a)** and write a new expression.	(f) $b + r - 4$

3.1 Variables and expressions

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q13, Q17	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q13, Q15, Q16	Q1, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16

Answers page 607

Fluency

1 For each of the following: WE1

(i) $3d + 4f - 3bd - 3 + 7b$ **(ii)** $5g - 3r + 5rf = 7 - 3b$ **(iii)** $b + f - 3bf + 8$

(a) identify whether this is an equation or an expression

(b) write down the coefficient of b

(c) write down all the terms in the equation or expression

(d) list the variables

(e) write down any constants.

WE2

2 Rewrite the following using algebraic conventions.

(a) $d \times 4 \times c$ (b) $2 \times t \div 7$ (c) $k \times g \times 4 \times k$ (d) $6 \div (f \times g)$

WE3

3 Nerida has w watches and r rings. Donna has two times as many watches and 2 fewer rings than Nerida.

(a) Write an expression for the number of watches and rings Nerida has altogether.

(b) If Nerida has 8 watches and rings to start with, write an equation to show this information.

(c) Write an expression for the number of watches Donna has.

(d) Write an expression for the number of rings Donna has.

(e) If Donna has 14 watches and rings altogether, write an equation to show this information.

(f) If Nerida loses 2 of her rings, write an expression in terms of w and r to show how many watches and rings Nerida has now.

4 (a) The coefficient of x in $6y + 7xy + 5x$ is:

A 5 B 7 C $5x$ D $6y$

(b) The coefficient of xy in $5 + 8xy - 2y + 4x$ is:

A 2 B 4 C 8 D $8xy$

(c) The coefficient of a in $5b + 2ab + 6$ is:

A 0 B 2 C 5 D 6

(d) The coefficient of x in $4xy - 6y + x + 8$ is:

A 0 B 1 C 4 D 6

5 (a) The constant in the expression $\frac{1}{2}x^2y - 3 + 5xy$ is:

A -3 B $\frac{1}{2}$ C 3 D 5

(b) The constant in $3ef + 7efg + 12 + 11ef + 4e$ is:

A 2 B 7 C 11 D 12

6 Write the following without division and multiplication signs or brackets. Do not simplify your expressions.

(a) $x \div 6$ (b) $h \div 9$

(c) $6 \times a \div 11$ (d) $15 \div (3 \times r)$

(e) $21 \div (12 \times v)$ (f) $4 \times s \div 19$

(g) $8 \div x - u \div 6$ (h) $h \div 5 + 4 \div i$

(i) $c \times u \div 5 + 9 \times y$ (j) $q \div (7 \times c) - g \times h \div 4$

(k) $v \times z \div 6 - 8 \div (f \times s)$ (l) $3 \div (t \times r) + 6 \times w \div (y \times z)$

(m) $4 \times h \times b \div (2 \times r)$ (n) $6 \times c \times a \div (5 \times e \times u)$

7 There are a apples and p pears in a fruit dish. There are 5 apples and 4 pears in a second dish. The total number of pieces of fruit is:

A $a + p$ B $5a + 4p$ C $a + 5a + p + 4p$ D $a + p + 9$

8 Andrew has y number of pencils in his pencil case.

(a) Dina has $y + 7$ pencils in her pencil case. What does this mean?

(b) Simon has $2y$ pencils in his pencil case. What does this mean?

(c) Suppose Cindy has $2y - 2$ pencils in her pencil case. Does she have more or fewer pencils than Simon?

(d) If Andrew took out 3 pencils from his pencil case, how many would he have left?

(e) If Andrew gave the 3 pencils to Dina and she put them in her pencil case, how many would she have?

(f) If Simon took 3 pencils out of his pencil case, how many would he have left?

(g) If Simon gave the 3 pencils to Cindy and she put them in her pencil case, how many would Cindy now have?

9 Suppose the number of cups you have is m and the number of glasses you have is n.

(a) How many cups and glasses do you have altogether?

(b) If you broke 2 glasses and threw them away, how many glasses would you now have?

(c) How many cups and glasses would you now have altogether?

(d) If you then bought another set of 8 cups, how many cups would you have?

(e) How many cups and glasses would you now have altogether?

10 Suppose x is the number of people in Sweats Sports Store at 3 p.m. on a certain Friday afternoon.

(a) If there are twice as many people at 3.30, write this in terms of x?

(b) If the number of people at 4.00 p.m. is $3x$, what does this mean?

(c) If five people leave the store between 4.00 p.m. and 4.15 p.m., and no one comes in, write an expression for the number of people in the store at 4.15 p.m.

(d) If nine people come into the store and two leave between 4.15 p.m. and 4.30 p.m., write an expression for the number of people in the store at 4.30 p.m.

Understanding

11 In a glass jar there are some different coloured jelly beans. There are g green jelly beans.

(a) There are twice as many red jelly beans as green ones. Write an expression for the number of red jelly beans in terms of g.

(b) The number of blue jelly beans is five less than the number of green ones. Write an expression for the number of blue jelly beans in terms of g.

(c) The number of black jelly beans is four more than the number of green ones. Write an expression for the number of black jelly beans in terms of g.

12 On a supermarket shelf there are different varieties of chocolate. There are m kinds of milk chocolate.

(a) The number of soft-centred varieties is one more than twice the number of milk chocolate varieties. Write an expression for the number of soft-centred varieties in terms of m.

(b) There are three more varieties of chocolate-nut than soft-centred varieties. Write an expression for the number of chocolate-nut varieties in terms of m.

(c) The number of varieties of white chocolate is two more than one-third of the amount of varieties of milk chocolate. Write an expression for the number of white chocolate varieties in terms of m.

13 Lauren takes h hours and m minutes to finish her mathematics project. Tim takes twice as long as Lauren to complete his project. The time Tim takes in minutes is:

A $2h + 2m$ **B** $2h + 120m$ **C** $60h + 2m$ **D** $120h + 2m$

Reasoning

14 Raymond is playing Crazy Maze on his games console. He starts with p points. Raymond plays the game for 45 minutes and every 5 minutes he notes how many points he has. The number of points he has is shown in the table below.

(a) At what time did Raymond have the most points?

(b) How many more points than he started with would Raymond have had if he had stopped playing at this time?

(c) At what time did Raymond have the fewest points?

(d) How many points would Raymond have lost if he had stopped playing at this time?

(e) At what times should Raymond have stopped playing if he was happy with breaking even?

Time	Number of points
10.00 p.m.	p
10.05 p.m.	$p + 2$
10.10 p.m.	$p + 7$
10.15 p.m.	$p + 1$
10.20 p.m.	$p + 4$
10.25 p.m.	p
10.30 p.m.	$p - 4$
10.35 p.m.	p
10.40 p.m.	$p - 10$
10.45 p.m.	$p - 2$

15 Suppose e stands for the number of people expected to be at a party. The table on the right tells us how many people were at the party during the evening.

Write a paragraph describing what happened during the evening.

Time	Number of people
6.00 p.m.	$e - 20$
6.10 p.m.	$e - 18$
6.15 p.m.	$e - 15$
6.20 p.m.	$e - 11$
6.25 p.m.	$e - 12$
6.30 p.m.	$e - 6$
7.00 p.m.	$e - 3$
8.00 p.m.	$e - 13$
9.30 p.m.	$e - 18$
11.00 p.m.	$e - 20$

Open-ended

16 Ruth has been answering some questions involving algebra, with k representing the variable in the expression.

(a) What could k represent?

(b) In this case, what would $2k$ represent?

(c) What would $k + 4$ represent?

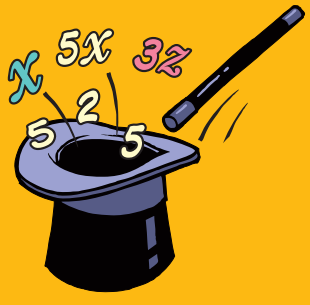
(d) What would $\frac{k}{2} - 1$ represent?

17 **(a)** Write a question of your own that is similar to Question **3**.

(b) Answer the question you have written.

Outside the Square — Problem solving

Tricky algebra



Instructions

1. Pick a number (keep it secret).
2. Multiply your number by 3.
3. Add 6.
4. Take away the number you started with.
5. Divide by 2.
6. Again, take away the number you started with.

You have finished with 3.

Use algebra to show how this works.

Strategy options

- Work backwards.
- Make a model.

Hint: Let your number be 'x'.

Create your own algebra trick and give this to a friend for them to work out.

Substitution for variables

3.2

Variables in an expression have unknown values. If we want to know the value for an expression when the variables have certain values, we can replace the variables in the expression with those values. This process is called **substitution**.

In the expression $3x + 2y$, x and y are variables. When $x = 5$ and $y = 1$, the expression can be evaluated by substituting 5 for x and 1 for y so:

$$\begin{aligned} 3x + 2y &= 3 \times (5) + 2 \times (1) \\ &= 17 \end{aligned}$$

Because the variables can vary, different values for the variable will give different values for the expression. When $x = -3$ and $y = 7$, the expression can be evaluated by substituting -3 for x and 7 for y so:

$$\begin{aligned} 3x + 2y &= 3 \times (-3) + 2 \times (7) \\ &= 5 \end{aligned}$$

Worked Example 4 WE4

Evaluate the expression $2b - a$ when $a = 3$ and $b = 10$.

Thinking	Working
1 Substitute the values for the variables, taking care to insert multiplication signs where necessary.	When $a = 3$ and $b = 10$ $2b - a = 2 \times 10 - 3$
2 Evaluate.	$= 20 - 3$ $= 17$

When substituting negative numbers, it is useful to place brackets around the number and its sign.

Worked Example 5 WE5

Evaluate the expression $2x^2 - 3y$ when $x = -3$ and $y = -1$.

Thinking	Working
1 Substitute the values for the variables, taking care to place brackets around the negative numbers and to insert multiplication signs where necessary.	When $x = -3$ and $y = -1$ $2x^2 - 3y = 2 \times (-3)^2 - 3 \times (-1)$
2 Evaluate.	$= 2 \times 9 + 3$ $= 18 + 3$ $= 21$

We can then use these skills to complete a table of values.

Worked Example 6 WE6

Use the rule $y = 3x$ to complete the following table.

x	-1	0	2
y			

Thinking	Working
1 Substitute each of the x-values into the rule to find the y-value.	When $x = -1$, $y = 3x$ $= 3 \times (-1)$ $= -3$ When $x = 0$, $y = 3x$ $= 3 \times (0)$ $= 0$ When $x = 2$, $y = 3x$ $= 3 \times (2)$ $= 6$
2 Complete the table using the y-values calculated.	

x	-1	0	2
y	-3	0	6

3.2 Substitution for variables

Navigator

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Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4 Column 1, Q5 Column 1, Q7, Q9, Q10, Q11, Q13	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4 Column 2, Q5 Column 2, Q6, Q7, Q9, Q10, Q12, Q13, Q14	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4 Column 3, Q5 Column 3, Q6, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15

Fluency

WE4

1 Evaluate the following expressions when $a = 3$ and $b = 6$.

(a) $a + 2b$ **(b)** $3a + 2b$ **(c)** $2a + 5b$

(d) ab **(e)** $3ab$ **(f)** $10ab - a$

(g) $8b - 16a$ **(h)** $10ab - b$ **(i)** $6b - ab$

(j) $12a^2$ **(k)** $2b^3$ **(l)** $4a^3$

(m) $a^2 - b^3$ **(n)** $3a^3 - 2b^2$ **(o)** $2a^5 - 2b^3$

(p) $\frac{2a^2}{b}$ **(q)** $\frac{a^3}{b^2}$ **(r)** $\frac{144a^2}{3b^3}$

(s) $a^2 - b - \frac{a}{3}$ **(t)** $a^3 + 2b^2 - \frac{b}{2}$ **(u)** $3b^2 + \frac{a^5}{2} - 5$

2 Evaluate the following expressions when $x = -2$ and $y = -5$.

WE5

(a) $x + y$ **(b)** $2x + 12y$ **(c)** $10x + 5y$

(d) $y - 4x$ **(e)** $2y - 3x$ **(f)** $2x - 3y$

(g) $-5y - 3 + x$ **(h)** $8 - 6y + 2x$ **(i)** $7y - 4 - 4x$

(j) $y^2 - x^2$ **(k)** $x^3 + y^3$ **(l)** $-5y^2$

3 Use the rules given to complete the following tables.

WE6

(a) $y = x - 2$

x	-2	0	-4	-8
y				

(b) $y = x - 4$

x	-5	-3	-2	-7
y				

(c) $y = x - 9$

x	-7	-1	-5	-9
y				

(d) $y = 4x - 1$

x	1	2	3	4
y				

(e) $y = 2x - 4$

x	-2	0	1	2
y				

(f) $y = 3x + 14$

x	-4	-7	-20	11
y				

(g) $y = -6x$

x	-3	-2	-1	-5
y				

(h) $y = -5x - 3$

x	-1	0	3	5
y				

(i) $y = 3 - 4x$

x	-3	-1	2	4
y				

(j) $y = \frac{x}{2}$

x	8	6	12	20
y				

(k) $y = \frac{x}{3} - 2$

x	15	9	12	30
y				

(l) $y = \frac{x}{4} - 1$

x	12	-4	8	-24
y				

(m) $y = x^2$

x	0	1	2	3
y				

(n) $y = 2x^3 - 1$

x	-2	0	2	4
y				

(o) $y = x^2 + 3x + 1$

x	0	2	4	8
y				

4 Evaluate these expressions when $d = -2$ and $e = 5$.

(a) $-3de - 6$ **(b)** $-5de + 100$ **(c)** $-8de + 15$

(d) $\frac{10}{e}$ **(e)** $\frac{40}{d}$ **(f)** $\frac{e}{d}$

(g) $\frac{2}{d} - \frac{5}{e}$ **(h)** $\frac{12}{d} - \frac{30}{e}$ **(i)** $\frac{15}{e} - \frac{18}{d}$

(j) $e^3 - 3d^2$ **(k)** $e^2 - 3d^2$ **(l)** $2e^3 - 3d^2$

5 Evaluate the following expressions for $m = 3$ and $n = -2$.

(a) $2(3m - n)$ (b) $4n(m - 3)$ (c) $m(6 + 3n)$

(d) $6m(8 - n)$ (e) $3m(n - 2)$ (f) $6n(3m - 4)$

(g) $\frac{m + n}{3}$ (h) $\frac{2m + n}{5}$ (i) $\frac{6m - n}{4}$

(j) $\frac{2m + n}{n}$ (k) $\frac{5(m + 2n)}{5m}$ (l) $\frac{3(m - 4n)}{4n}$

6 When $u = 6$ and $v = 8$ are substituted into the expression $\frac{v(3u + v - 2)}{u}$, the result is:

A 16 B 20 C 25 D 32

7 When the values $a = -1$, $b = 2$ and $c = -3$ are substituted into the expression $ab + bc$, the answer is:

A -8 B -4 C -2 D 4

8 If $d = 3$ and $e = -4$, then $de(d + e)$ is:

A -12 B 84 C 12 D -84

Understanding

9 Boon buys d DVDs at \$$y$ each.

(a) Write an expression to show how much Boon spent.

(b) Write an expression to show how much Boon received if he sold k DVDs for \$$m$ each.

(c) Write an expression to calculate the profit Boon made.

(d) If $d = 7$, $y = 12$, $k = 5$ and $m = 21$, find the profit Boon made.

(e) One DVD is returned to Boon as it is faulty. Does Boon still make a profit? Explain why or why not.

10 Asha has a dollars, Mayoarini has $2b$ dollars and Danni has $3c$ dollars.

(a) What is the difference between Danni's amount of money and Asha's amount of money if Danni has more money than Asha?

(b) Mayoarini gives her money to Asha. What is the difference now between Danni's amount of money and Asha's amount of money if Asha now has more money than Danni?

(c) How much money is there altogether if $a = 3$, $b = 5$ and $c = 4$?

11 Michael has scored r runs in a cricket game. His friend, Ravi, has scored $2r - 4$ runs.

(a) What is the smallest value that r could be?

(b) Show that Ravi and Michael score the same number of runs if $r = 4$. Explain.

(c) If $r > 4$, did Ravi or Michael score more runs? Explain.

Reasoning

12 It takes $\frac{3N}{P+1}$ minutes to package biscuits, where N is the number of biscuits and P is the number of packing machines.

(a) How long does it take to package 1500 biscuits if 99 machines are used?

(b) Forty-nine machines are to be used to package 2200 biscuits in 2 hours. Show why this is not possible.

(c) Five faulty machines have been repaired so that now there are 54 machines available. Show whether it is now possible to package the 2200 biscuits in 2 hours.

Open-ended

13 Find two different sets of values for the pronumerals M and N so that $\frac{3M+N}{2M-N}$ gives a whole number when M and N are substituted and the expression evaluated.

14 Lisa, Anna and Saji each choose an integer between -10 and +10.

Let a represent Lisa's number, b represent Anna's number and c represent Saji's number.

(a) Find one set of possible values for a, b and c if the sum of the three numbers is 0.

(b) Find three different sets of possible values for each girl's number if the product of the three numbers is -100 and a, b and c take different values.

15 Find three sets of whole number values for a and b so that $a^3 - 3b^2$ has a result that is less than 20.

Outside the Square Game

4 in a row

Equipment required: 2–4 brains, 1 die

Each player needs to copy the game board into their exercise book.

Game board

-4	-3	-2	-1	0
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

Expression list

$x^2 + 2$

$3x - 4$

$5x + 1$

$\frac{x^2}{2}$

x^3

$x + 3$

$2x - 1$

$x + 10$

$4x - 1$

$5x - 3$

$2 - x$

How to win:
The winner is the first player to get 4 numbers in a row.

How to play:
Player 1 rolls the die and then has 30 seconds to choose an expression from the Expression list. They substitute the number rolled into the expression, and circle the answer on their game board.

The other players check that Player 1 has correctly substituted their number. If the player has incorrectly circled a number, then the player to spot the mistake gets to circle the correct number.

It is then the next player's turn.

If your answer is a decimal, round to the nearest whole number.

3.3 Using formulas

Formulas (or formulae) are algebraic equations that are used in practical situations. Formulas are used to calculate one quantity from others. You have already used formulas to find the perimeter and area of simple shapes.

A rectangle with length l and width w has perimeter P given by the formula $P = 2(l + w)$ or $P = 2l + 2w$ and area A given by the formula $A = lw$, where l stands for the length and w for the width.

Worked Example 7 WE7

The formula $F = \frac{9C}{5} + 32$ is used to convert temperatures in degrees Celsius, C, to degrees in Fahrenheit, F. Use this formula to convert 29°C into degrees Fahrenheit.

Thinking	Working
1 Write the formula.	$F = \frac{9C}{5} + 32$
2 Substitute the value or values given ($C = 29$).	$= \frac{9(29)}{5} + 32$
3 Evaluate.	$= 84.2°F$

3.3 Using formulas

Navigator

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Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9 (a) & (b), Q10 (a) & (b), Q12	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13

Fluency

WE7

1 The formula $F = \frac{9C}{5} + 32$ is used to convert temperatures in degrees Celsius, C, to degrees in Fahrenheit, F. Use this formula to convert the following degrees Celsius into degrees Fahrenheit.

(a) 10°C **(b)** 5°C **(c)** 35°C

2 The formula $C = \frac{5F - 160}{9}$ converts degrees Fahrenheit, F, to degrees Celsius, C. Find the value of C when F is:

(a) 32°F **(b)** 41°F **(c)** 50°F

3 The distance travelled by a moving object travelling at a constant speed can be calculated using the formula $d = st$ where

d = distance travelled in metres
s = speed of object in metres/second
t = time in seconds that the object is in motion.

Calculate the value of d when:

(a) $s = 8, t = 4$ **(b)** $s = 4\frac{1}{2}, t = 1\frac{1}{2}$

4 The time, t seconds, needed for an elevator of power w kilowatts to lift m kilograms h metres is given by the formula $t = \frac{mh}{25w}$. If $w = 20$, the time in seconds it takes to lift a group of people of total mass 400 kilograms to a height of 50 metres is:

A 20 seconds **B** 25 seconds **C** 35 seconds **D** 40 seconds

5 The amount, C dollars, to be paid for a mobile phone bill is found from the formula:
$C = R + 0.5L + 0.2T$

where R is the rental of the equipment
L is the number of calls made
T is the number of text messages sent.

When the rental is \$32 and there are 110 calls and 25 text messages, the amount of the bill is:

A \$55.40 **B** \$92.00 **C** \$66.50 **D** \$42.50

Understanding

6 The salespeople who work at Crazy Joe's Motors are paid according to the formula $P = 250 + 100c$ where P is the total pay for one week in dollars and c is the number of cars the salesperson sells in the week.

(a) Warren didn't sell any cars one week. How much money would he have earnt that week?

(b) Caitlin sold five cars in the same week. How much would she have earnt?

(c) Claudio wants to earn over \$1000 a week. What is the minimum number of cars he would have to sell to earn this much?

7 The formula $d = 4.9t^2$ tells you how far a dropped object has travelled after a certain time, where d *is* the distance the object has fallen in metres and t is the time it has been falling in seconds.

(a) Suppose a football has been dropped from Sydney Tower. What distance would it fall in 2 seconds?

(b) Divers on the high cliffs in Acapulco, Mexico, usually take 4 seconds to hit the water. Approximately how high are the cliffs?

(c) Wayne the skydiver always counts out 10 seconds just after he jumps out of the plane before he pulls the rip cord to open his parachute. How far does he fall in this time?

8 Temperatures can be converted from degrees Celsius, C, to degrees Fahrenheit, F, using a less accurate but simpler formula than the formula used in Question **1**. The formula is $F = 2C + 30$.

(a) Use this formula to convert 10°C to degrees Fahrenheit.

(b) Compare your answer to part **(a)** to your answer to Question **1 (a)**.

(c) Use this formula to convert 5°C to degrees Fahrenheit.

(d) Compare your answer to part **(c)** to your answer to Question **1 (b)**.

(e) Use this formula to convert 35°C to degrees Fahrenheit.

(f) Compare your answer to part **(e)** to your answer to Question **1 (c)**.

(g) In most parts of Australia daily minimum temperatures rarely fall below 0°C and daily maximum temperatures rarely exceed 40°C. For the temperature range of 0°–40°, how close to the accurate temperatures in Fahrenheit, calculated using $F = \frac{9}{5}C + 32$, are the approximate temperatures calculated using $F = 2C + 30$?

9 The formula for finding how much water goes into a rectangular fish tank is $V = ldw$

where V = the volume of water (in cubic centimetres)

l = the length (in cm) of the tank

d = the depth (in cm) of the water in the tank

w = the width (in cm) of the tank.

(a) How much water does a tank hold if its length is 80 cm, its width is 30 cm and its depth is 20 cm?

(b) How much water would you put in the tank if you want the water level to be 5 cm from the top?

(c) Lucia is pumping the water from her smaller fish tank into her larger one. The smaller tank is 50 cm long, 20 cm wide and 20 cm deep. The larger tank is 100 cm long, 50 cm wide and 40 cm deep. After she's finished, how much *more* water will she need to add to the large tank to fill it completely?

Reasoning

10 Banks use the formula $I = PRT$ to calculate simple interest, which is the amount of money in interest that the money in your account earns.

I stands for the *simple interest*.

P stands for the *principal*, which is the amount of money you have in an account.

R stands for the *rate* of interest the bank is offering. This is given as a percentage per annum 'p.a.' (for a year). (Percentages need to be converted to a decimal value.)

T stands for the *time* the money has been in the bank account. This is usually measured in years and is the term of the investment.

(a) How much interest would you earn on $100 if you had it in an account for 1 year at an interest rate of 7% p.a.?

(b) How much interest would you earn on $1000 if you had it in an account for 5 years at an interest rate of 12% p.a.?

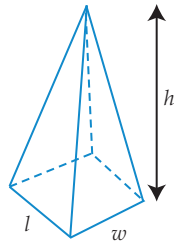
(c) Suppose you have $1000 that you want to invest for 5 years, and you have a choice between two banks: Aussie Bank and Matilda Bank which both offer a 5-year term investment deal. Aussie Bank offers a simple interest rate of 11% p.a., but charges a once-only set-up fee of $100. Matilda Bank's simple interest rate is 12% p.a., but it charges a once-only set-up fee of $160.

(i) Which bank should you invest your money in?

(ii) How much better off would you be with this bank?

(iii) Suppose you have $2000 that you want to invest for 5 years. Which bank should you invest your money in? How much better off would you be with this bank?

11 The volume V cubic metres of a pyramid of height h metres that has a rectangular base is given by the formula $V = \frac{lwh}{3}$, where l is the length of the base and w is the width.



(a) The Miza pyramid in Egypt is 30 metres long, 25 metres wide, and 60 metres high. What is its volume?

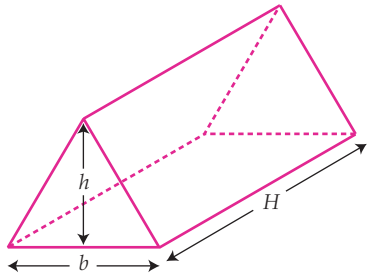
(b) The pyramid is made from stone blocks with a volume of five cubic metres each. If it is known that Miza is completely solid, how many blocks were used to build the pyramid?

Open-ended

12 The formula $A = l \times w$ is used to find the area of a rectangle. Given that the area is 20 cm^2 and the length, l, is longer than the width, w, find the possible dimensions of two different rectangles.

13 The volume of a triangular prism is given by $V = \frac{bhH}{2}$ where b is the base length, h is the perpendicular height of the triangle and H is the height of the prism.

Find two different sets of whole-number values for b, h and H if V is a whole number between 12 000 and 25 000, and each of b, h and H must be at least 20.



Outside the Square Puzzle

How old am I?

- Three years ago, my age was a prime number.
- This year, my age is a multiple of 4.
- I have a brother whose age twenty years ago was half my current age.
- The sum of the digits of my age is equal to one more than the product of the digits.

What is my age?

What is my brother's age?

Investigation

Take your medicine

Equipment required: 1 brain, calculator

It is critical that the amount of medicine required by a sick person is calculated correctly by a doctor or a nurse. The correct amount is called the dose, or dosage. Medicine doses for children are usually smaller than those for adults. There are some formulas that help doctors to work out the dose to give children. Here are two such formulas that use a child's age to decide the required dose:

Formula 1

$$X = \frac{A}{A + 12} \times N$$

where

X = the child's dosage in mL

A = the child's age

N = the normal adult's dosage in mL

Formula 2

$$Y = \frac{4 \times A + 20}{100} \times N$$

where

Y = the child's dosage in mL

A = the child's age

N = the normal adult's dosage in mL

A more accurate way of deciding a child's dosage uses body surface area (BSA). This formula uses the height and weight of a child to calculate the BSA of a child compared to a normal adult's BSA. This formula is the one most commonly used by health professionals.

Formula 3

$$Z = \frac{w^{0.425} \times h^{0.725}}{240} \times N$$

where

Z = the child's dosage in mL

w = the child's weight in kilograms

h = the child's height in centimetres

N = the normal adult's dosage in mL

The Big Question

According to these three dosage formulas, when should a young person be given an adult dose of medicine?

Engage

1 Use Formula 1 to find what fraction of the normal adult's dose would be needed by someone who is:

(a) 1 year old

(b) 7 years old

(c) 12 years old

(d) 18 years old.

Write your answers as decimal values (round to two decimal places).

2 Repeat Question **1** using Formula 2.

Explore

3 Use Formula 3 to find what fraction of the normal adult's dose would be needed by someone who has:

(a) weight 4 kg and height 50 cm

(b) weight 10 kg and height 75 cm

(c) weight 25 kg and height 115 cm.

Write your answers as decimal values (round to two decimal places).

4 Substitute the different ages, heights and weights given in the table below into the three formulas to find the dosage amounts for each. Use an adult dose of 50 mL each time.

	Age	Weight	Height
(a)	2	11 kg	88 cm
(b)	6	22 kg	106 cm
(c)	10	35 kg	132 cm
(d)	14	50 kg	156 cm
(e)	18	64 kg	170 cm

Strategy options

- Make a table.
- Look for a pattern.

Explain

5 Consider the dosages you calculated using Formula 1. Describe how the dosage changes as the child gets older. Is it a steady increase?

6 Repeat Question **5**, this time considering the dosages you calculated using Formula 2.

7 Repeat Question **5**, this time considering the dosages you calculated using Formula 3.

8 Use your answers to Questions **5–7** to answer the Big Question.

9 Compare the differences in the dosages calculated by the three different formulas. Comment on any pattern or trend you can see.

Elaborate

10 Children are usually considered to be adults at age 18. Which formula best reflects this?

11 Both Formulas 1 and 2 give answers that are not sensible when certain values are used and doctors and nurses should be aware of this.

We say the formulas have limitations, because we need to limit the values to certain ages.

(a) Describe the limitations of Formula 1. (If you are unsure, try calculating the dosage for 'children' aged 20, 30 and 40 years.)

(b) Describe the limitations of Formula 2 (once again, use dosages for a wide range of ages).

(c) Why do you think Formula 3 is the most commonly used formula?

Evaluate

12 Do you think you need to know how to substitute correctly into these formulas? Why or why not?

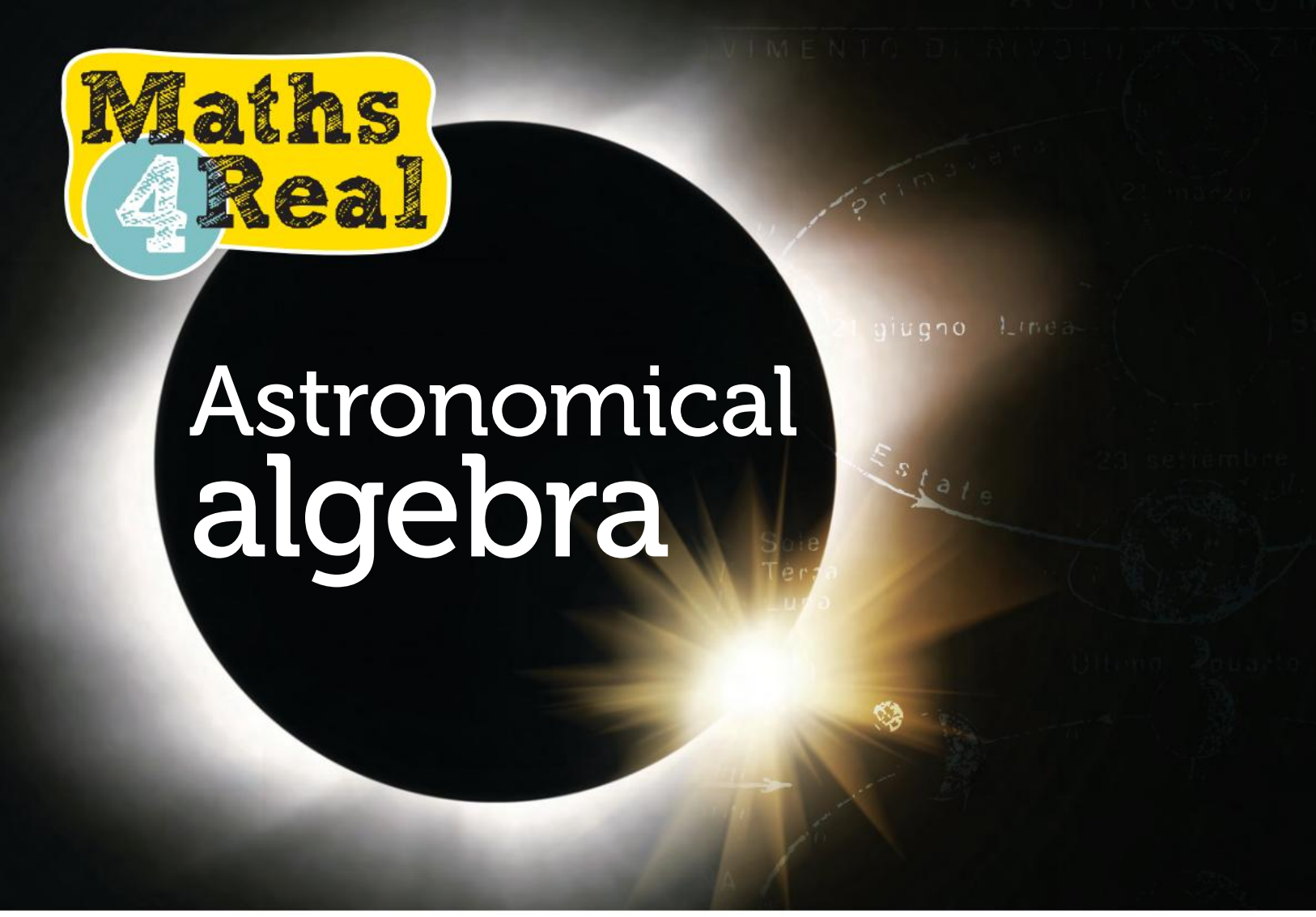
13 Do you think it is important for doctors and nurses to be able to use formulas correctly? Why or why not?

14 Which formula would be best for you if you needed to take some medicine? Explain why?

Extend

15 Research some other formulas that are used to find a child's dose based on an adult's dose.

16 Based on your results above, create your own formula to quickly find what fraction of medicine should be given to children.



Maths 4 Real

Astronomical algebra

Throughout history, mathematicians and astronomers have tried to predict the alignment of objects in the night sky. The most spectacular of these alignments is a solar eclipse, and algebra has proved to be crucial for making accurate predictions of when these occur.

A solar eclipse occurs when the Moon crosses the path between the Sun and the Earth, and the Moon's shadow crosses the surface of the Earth. There will be 224 solar eclipses this century. The longest, at 6 minutes 39 seconds, occurred on 22 July 2009.

Around 300 BC, the Greek astronomer Aristarchus of Samos used eclipses to calculate the sizes of the Sun and the Moon (relative to the size of the Earth), and their distances from the Earth. He is also believed to be the first to come up with the theory that the planets move around the Sun, not the Earth. In 62 AD, another Greek named Heron accurately described the physical situation that causes a lunar eclipse.

In 1619, the German astronomer Johannes Kepler published an important formula that connected the average distance of the planets from the Sun (known as the radius of the orbit, R), with the time it took the planet to make one complete orbit (known as the period, T).

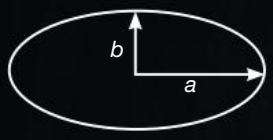
The formula stated that:

$$\frac{R^3}{T^2} = K, \text{ where: } R = \text{average radius, in astronomical units}$$
$$T = \text{period, in days}$$
$$K = \text{a constant number}$$

1. Earth's orbit can be said to have an average radius of 100 astronomical units and a period of 365.25 days. Find the value of K accurate to two decimal places.
2. The orbit of Mars has an average radius of 152.4 astronomical units and a period of 686.98 days. Find the value of K accurate to two decimal places.
3. What do you notice about your answers to **1** and **2**?
4. Using your value of K, calculate the period of Saturn using Kepler's modified formula $T^2 = \frac{R^3}{K}$ if the average radius of Saturn's orbit is 951 astronomical units.

One of the most important algebraic discoveries in relation to eclipses was that the planets move around the Sun in a path that is an ellipse (a 'flattened' circle) rather than a perfect circle, and that the Sun is at one of the two points called the foci of the ellipse, rather than at the centre (foci is plural of focus). Comets also travel in elliptical paths, but the ellipse is generally flatter than for a planet.

The *eccentricity* of an ellipse describes how flat it is and is a number between 0 and 1, inclusive. The flatter an ellipse, the greater its eccentricity.

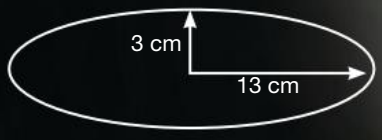


The formula for the eccentricity of an ellipse is $e = \frac{\sqrt{a^2 - b^2}}{a}$, where a and b are the lengths as shown in the diagram; a is referred to as the semi-major axis and b is the semi-minor axis.

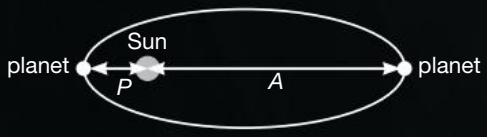
5 Find the eccentricity of the following ellipses, correct to one decimal place.

(a)

(b)



If we know the planet's closest distance to the Sun, P, and its furthest distance from the Sun, A, we can calculate the eccentricity of its orbit using the formula $e = \frac{A - P}{A + P}$.



6 Below are the closest and furthest distances from the Sun of three planets and Halley's Comet. Calculate the eccentricity of the orbit for each one, correct to one decimal place.

(a) Earth: $P \approx 147.1$ million km, $A \approx 152.1$ million km

(b) Mars: $P \approx 206.6$ million km, $A \approx 249.2$ million km

(c) Pluto: $P \approx 4435.0$ million km, $A \approx 7304.3$ million km

(d) Halley's Comet: $P \approx 89$ million km, $A \approx 5250$ million km

Research

Find out more about an astronomer, such as Aristarchus of Samos, Heron, Kepler, Galileo or Sir Isaac Newton, and the way they worked with algebra. Present your findings to the class using a poster or multimedia.

3.4 Simplifying expressions

Like terms

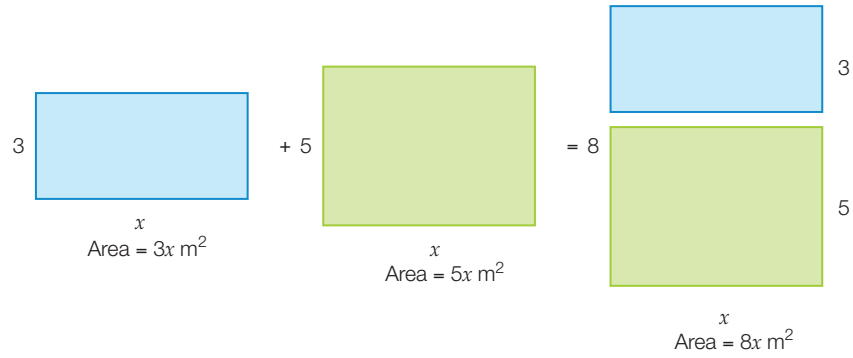
Algebraic terms that have exactly the same variables are called **like terms**.

We can simplify algebraic expressions by collecting like terms. We collect like terms by adding their coefficients.

$3x$ and $5x$ are like terms and can be simplified by adding the coefficients of x.

$3x + 5x = 8x$

This can be demonstrated by adding two rectangles of the same length, x m, together if one has a width of 3 m and the other as a width of 5 m.



If $x = 4$

$$\begin{aligned} \text{LHS} &= 3x + 5x \\ &= 3 \times 4 + 5 \times 4 \\ &= 12 + 20 \\ &= 32 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= 8 \times x \\ &= 8 \times 4 \\ &= 32 \end{aligned}$$

$3x + 4y$ are **unlike terms** and cannot be simplified.

> Like terms can be simplified by adding their coefficients.
>
> Unlike terms cannot be simplified.

Care must be taken when several variables are involved. To help us identify like terms, we write terms with more than one variable in alphabetical order.

- $5ab$ and $2ac$ are not like terms, as $ab \neq ac$. ($\neq$ means is not equal to.)
- $3xy$ and $5xyz$ are not like terms, as $xy \neq xyz$.
- $6x$ and $4x^2$ are not like terms, as $x \neq x^2$.

However:

- $3xy$ and $5yx$ are like terms as $5yx = 5xy$.
- $11x^2$ and $5x^2$ are like terms.
- $3p^3q^4$ and $-5p^3q^4$ are like terms.

Worked Example 8 WE8

In each of the following, write down whether the pair of terms is like or unlike.

(a) $5t$ and $3t$ **(b)** $2xy$ and $2x$ **(c)** $6m^2n$ and $4mn^2$ **(d)** $3xy^2$ and $7y^2x$

Thinking	Working
(a) Do the terms have exactly the same variables? (Both terms have t as their only variable.)	(a) $5t$ and $3t$ are like terms.
(b) Do the terms have exactly the same variables? (The first term has $x \times y$, but the second term only has x.)	(b) $2xy$ and $2x$ are unlike terms.
(c) Do the terms have exactly the same variables? (The first term has $m \times m \times n$ whereas the second term has $m \times n \times n$.)	(c) $6m^2n$ and $4mn^2$ are unlike terms.
(d) Do the terms have exactly the same variables? (The first term has $x \times y \times y$ and the second term has $y \times y \times x$. So, it has exactly the same variables, just in a different order.)	(d) $3xy^2$ and $7y^2x$ are like terms.

An expression can be simplified by collecting like terms.

Worked Example 9 WE9

Simplify the following expressions, if possible, then check your answers by substituting the value for the variables given in brackets.

(a) $5x + 3y - x + 7y$ $(x = 1, y = 2)$ **(b)** $7x^2 + 4xy + 8yx - 2x$ $(x = 3, y = 2)$

Thinking	Working
(a) **1** Rearrange the expression so that all the like terms are grouped together. Remember to keep the sign in front of each term with that term.	(a) $5x + 3y - x + 7y$ $= 5x - x + 3y + 7y$
2 Collect like terms.	$= 4x + 10y$

3	Substitute the given values into both expressions.	$5x + 3y - x + 7y$ $= 5 \times 1 + 3 \times 2 - 1 + 7 \times 2$ $= 5 + 6 - 1 + 14$ $= 24$ $4x + 10y$ $= 4 \times 1 + 10 \times 2$ $= 24$
4	Check that both expressions give the same value.	As both expressions equal 24, the simplification is correct.
(b) 1	Rearrange the expression so that all the like terms are grouped together. Reorder variables in the terms if necessary to make the like terms look the same. Remember to keep the sign in front of each term with that term. Note that $7x^2$ and $-2x$ are unlike terms.	(b) $7x^2 + 4xy + 8yx - 2x$ $= 7x^2 - 2x + 4xy + 8yx$ $= 7x^2 - 2x + 4xy + 8xy$
2	Collect like terms.	$7x^2 - 2x + 12xy$
3	Substitute the given values into both expressions.	$7x^2 + 4xy + 8yx - 2x$ $= 7 \times 3^2 + 4 \times 3 \times 2 + 8 \times 2 \times 3 - 2 \times 3$ $= 63 + 24 + 48 - 6$ $= 129$ $7x^2 - 2x + 12xy$ $= 7 \times 3^2 - 2 \times 3 + 12 \times 3 \times 2$ $= 63 - 6 + 72$ $= 129$
4	Check that both expressions give the same value.	As both expressions equal 129, the simplification is correct.

3.4 Simplifying expressions

Navigator

Answers page 608

Q1 Column 1, Q2 Column 1, Q3 (a–d), Q4, Q5, Q6, Q7, Q10, Q11, Q12, Q16	Q1 Column 2, Q2 Column 2, Q3 (c–f), Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q14	Q1 Column 2, Q2 Column 2, Q3 (e–h), Q4, Q5, Q6, Q9, Q10, Q11, Q12, Q13, Q14, Q15

Fluency

WE8

1 In each of the following, write down whether the pair of terms is like or unlike.

(a) $4w$ and $6w$

(b) $7u$ and $7w$

(c) $7t$ and $6i$

(d) $3e$ and $5e$

(e) $6d$ and $3de$

(f) yx and $33xy$

(g) $9xyz$ and $5xyz$

(h) $6x$ and $52xyz$

(i) $6xy$ and $8yx$

(j) $2xyz$ and $4zyx$

(k) $4x^2y$ and $7x^2y$

(l) $8xy^2$ and $12yx^2$

2 Simplify the following expressions, if possible, then check your answers by substituting the value for the variables given in brackets. WE9

(a) $6a + 11a$	$(a = 1)$	**(b)** $15f + 6f$	$(f = 6)$
(c) $7v - 2v$	$(v = 4)$	**(d)** $5v - 8v$	$(v = 2)$
(e) $-7w + 17w$	$(w = -2)$	**(f)** $-12d + 15d$	$(d = -3)$
(g) $8t + 7t + 2d + 5d$	$(d = 1, t = 3)$	**(h)** $40w + 50w + 100v + 34v$	$(v = 2, w = 1)$
(i) $6a + 5b + 4a - 3b$	$(a = 2, b = 1)$	**(j)** $18f - 5f + 6g - 3g$	$(f = 1, g = 4)$
(k) $-6b - 8b + 10b$	$(b = 5)$	**(l)** $-9b - 4b + 18b$	$(b = 4)$
(m) $7d + 8d + 9 - 3d$	$(d = 3)$	**(n)** $9f + 5 - 2f + 5f$	$(f = 3)$
(o) $-3r + 6u + 10r - 9u$	$(r = 2, u = 5)$	**(p)** $6m - 9n - 13m - 2n$	$(m = -1, n = 4)$
(q) $7f + 8fg - 4f - 6fg$	$(f = 4, g = 2)$	**(r)** $8ij + 14j - 2ij + 6j$	$(i = 2, j = 7)$

3 Simplify these expressions:

(a) $3hdw + 6hd + 7d - 4hd + 8 - 2d$

(b) $4x + 15 + 6x + 7xyz - 7 - 3xyz$

(c) $5x^2 - 6y - 13x^2 - 4y + 2$

(d) $7x^3 - y - 12x^3 - 5y + xy$

(e) $28ab + 40 + 30a - 10ab - 6a + 15b$

(f) $9s + 40st + 5 - 100st - 5s + 15$

(g) $-9b - 4b^3 + ab - b^3 + 8b - 6ab$

(h) $6a^3 - 8a^2 - 4a - 5a - 4a^3 - 61$

4 Which of these is a like term for $6y$?

A $6x$ **B** $14y$ **C** $6 + y$ **D** 6

5 Which of these is a like term for mn?

A $9n$ **B** mnp **C** amn **D** $8nm$

6 Which of these is a like term for $5xyz$?

A $22xy$ **B** $2xzj$ **C** $xy + z$ **D** $6zxy$

Understanding

7 Answer TRUE or FALSE for each of these statements.

(a) These two rules are really the same:

Rule 1: 'Take a number and multiply it by three. Take the same number and multiply it by seven. Then add the two answers together.'

Rule 2: 'Take a number and multiply it by ten.'

(b) These two rules are really the same:

Rule 1: 'Take a number and multiply it by ten. Take another number and multiply it by five. Then add the two answers together.'

Rule 2: 'Take a number and multiply it by fifteen.'

(c) These two rules are really the same:

Rule 1: 'Take a number and multiply it by fourteen. Take the same number and multiply it by ten. Then subtract the second answer from the first.'

Rule 2: 'Take a number and multiply it by four.'

8 A store is selling all its CDs for c dollars and all its DVDs for d dollars each. Tom bought eight CDs and five DVDs. His friend Ayman bought seven CDs and six DVDs.

(a) Write the expression for the amount Tom paid for his collection.

(b) Write the expression for the amount Ayman paid for his collection.

(c) Write the expression for the amount they paid altogether. Simplify this expression.

9 For each of her birthdays, Alysia is given as a present three times as much money as she was given for her previous birthday. On her first birthday she received d dollars.

(a) Write the expression for the amount of money Alysia received on her fourth birthday.

(b) Write an expression for the total amount of money Alysia received for her first four birthdays.

(c) How much money did Alysia receive altogether for her first four birthdays if $d = 5$?

10 (a) By drawing three rectangles each with a width of 2 cm, show that $2 \times 3 + 2 \times 4 = 2 \times 7$.

(b) Substitute 'y' for '2' and rewrite the statement above in algebra.

Reasoning

11 (a) Which of the following expressions cannot be simplified by collecting like terms?

(i) $5x^2 + 4x$ (ii) $3x^2 + 2x^2$ (iii) $2xy + 6yx$

(iv) $4xy^2 + 5x^2y$ (v) $5xy + xy$ (vi) $4xy - xy$

(b) Simplify the remaining expressions. Explain why these expressions can be simplified.

12 Jess has x ten cent coins and $10x$ fifty cent coins.

(a) Write an expression for the total number of coins Jess has.

(b) Ryan has $44x$ coins. Explain what this means, in terms of the coins that Jess has.

(c) Write an expression for the value of Jess' coins in dollars.

(d) Can you find the value of Ryan's coins? Why or why not?

13 A number of coins are placed on a square of a chessboard. Twice as many coins are then placed on the next square. On the third square, there are twice as many coins stacked as there are on the second square. This process is continued for six squares.

(a) If x is the number of coins on the first square, how many are stacked on the sixth square?

(b) How many coins in total are there on the board in these six squares?

(c) If each coin's value is \$$v$, what is the total value of the coins on the board?

(d) Find a number for x and a number for v that will make the total value of the coins between \$300 and \$400.

Open-ended

14 Consider the algebraic expressions $a + b$, $2a + b$, $3a + b$.

Find two sets of values for a and b so that the sum of the three expressions is a multiple of ten.

15 The sequence of terms: $x + y$, $3x + 2y$, $5x + 3y$, $7x + 4y$, $9x + 5y$, ... follows a pattern.

Find two sets of values for x and y so that the sum of the first five terms in this sequence is a multiple of ten.

16 Lindy was simplifying $7xy - xy$ and wrote: $7xy - xy = 7$.

(a) What mistake did Lindy make?

(b) Give Lindy a suggestion to help her avoid this mistake in the future.

Half-time 3

1 If Michael has n \$2 coins and m \$1 coins, write an expression for how much money he has altogether. **Ex.3.1**

2 Simplify: **Ex.3.4**

(a) $7x + 10x$ **(b)** $14y - 9y$ **(c)** $3x + 2y - x + 4y$

(d) $4x^2 + 3xy - 2x^2 + 4x^2$ **(e)** $10a + 13b - 3b - 26a$ **(f)** $4x + 2a - 3x + 2a^2$

3 If $x = 3$, $y = -4$ and $z = 5$, find: **Ex.3.2**

(a) $3x + 2y$ **(b)** $2z - 3y$ **(c)** $7x + 2y - z$

(d) $6x^2 - y$ **(e)** $10xy^2 + z$ **(f)** $2xyz + 3z^3$

4 Use the rules to complete the following tables. **Ex.3.2**

(a) $y = 3x - 4$

x	-2	0	1	2
y				

(b) $y = 3 + 2x$

x	-3	-1	2	4
y				

(c) $y = 2x^2 + 3$

x	-1	0	3	5
y				

5 The following formulas are often used in mathematics and science. **Ex.3.3**

(a) For $V = IR$, find V when $I = 1.8$ and $R = 20$.

(b) For $v = u + at$, find v when $u = 20$, $a = 1.2$ and $t = 15$.

(c) For $t = a + (n - 1)d$, find t when $a = 2.6$, $n = 10$ and $d = -0.4$.

(d) For $f = \dfrac{vu}{v + u}$, find f when $v = 25$ and $u = 20$.

(e) For $s = ut + \frac{1}{2}at^2$, find s when $u = 3$, $t = 10$ and $a = 9.8$.

6 Susan gets a ticket standing in a queue at the deli section of her local supermarket. The number on her ticket is t. Assume the tickets are given out in consecutive order (i.e. 1, 2, 3, ...). **Ex.3.1**

(a) John gets the ticket after Susan. Write an expression for the number on John's ticket.

(b) Jill is the person who is served just before Susan. Write an expression for the number on Jill's ticket.

7 Simplify the following expressions. **Ex.3.4**

(a) $4xy + 3x - 2y + 5xy$ **(b)** $2x^2 - 4x - 3xy + 5x^2$

(c) $8m^2 - 12m + m^2 + 8m + 5$ **(d)** $9pq - qp + 4p^2q - 3p^2q^2 + 4pq^2$

8 For each expression state: **Ex.3.1**

(i) the number of terms it contains

(ii) the coefficient of x

(iii) the constant.

(a) $2x + 3$ **(b)** $a - 5x - 4$ **(c)** $x + 7 + 4x^2$

(d) $-2x - 14xy$ **(e)** $6ax - 5 - \frac{1}{2}x$ **(f)** $3x + 4y + 6$

(g) $4x^2y - 2 + x$ **(h)** $-10 - 12x$ **(i)** $7x$

Mathspace

MAGIC ALGEBRA

MAGIC TRICK 1

Think of a number.
Double it.
Add the original number.
Divide by 3

Abracadabra, your answer is the number you started with.

Magician's secret: How does this trick work? Let the original number you thought of be 'x'. Now perform each of the steps on x. Write down each step as you go.

MAGIC TRICK 2

Think of a number.
Double it.
Add 8.
Multiply by 3.
Divide by 6.
Subtract your original number.

Alakazoom, your answer is 4.

Magician's secret: Discover how this works by letting the number you think of be x and performing each of the steps on it.

MAGIC TRICK 3: MIND READER

0 ♍	1 ◆	2 ●	3 ♋	4 △	5 ●	6 ◆	7 ♐
8 ♋	9 ✋	10 ♐	11 ♍	12 □	13 ⌂	14 ●	15 △
16 ♐	17 △	18 ✋	19 ◆	20 ♐	21 ♍	22 ♋	23 ♍
24 □	25 ♍	26 ⌂	27 ✋	28 ♋	29 ♐	30 ⌂	31 ◆
32 ●	33 ♐	34 ♋	35 ♐	36 ✋	37 □	38 □	39 ●
40 ♋	41 □	42 ◆	43 ●	44 ♍	45 ✋	46 ♐	47 ♋
48 ◆	49 ✋	50 ♍	51 △	52 ◆	53 △	54 ✋	55 □
56 ♍	57 ♋	58 ♐	59 ✋	60 ♐	61 ♋	62 △	63 ✋
64 △	65 △	66 ◆	67 ♋	68 ●	69 ◆	70 ♍	71 △
72 ✋	73 ♋	74 ●	75 □	76 △	77 ♋	78 ✋	79 ♐
80 ♋	81 ✋	82 ♍	83 ♍	84 ⌂	85 ♐	86 ◆	87 ♋
88 □	89 □	90 ✋	91 ⌂	92 □	93 ●	94 ⌂	95 ✋

Choose any two-digit number and add together both the digits. Subtract the total from your original number. Look up your answer on the chart to find out the magic symbol. Now stare at the symbol and think about it really hard. Can you find your symbol near the page number on this page? Try it again with a different starting number.

Magician's secret: Can you work out the secret?

Use algebra to show how the trick works.

(Hint: Any two-digit number can be broken down into tens and units: 42 = 40 + 2. So, if a is the tens digit and b is the units digit, then the two-digit number can be written as $10a + b$.)

MAGIC TRICK 4: WHEN IS MY BIRTHDAY?

Take the month number of your birthday (i.e. Jan. = 1, Feb. = 2 etc.).

Multiply your number by 4.

Add 14.

Multiply the result by 25.

Subtract 230.

Add the day number of your birthday, (e.g. if born on 19 May, add 19).

Subtract 120 from your answer.

Hey presto, the first digit is the month you were born in and the next two digits are the day you were born on.

Magician's secret: How does this work? Choose m as the month number and d as the day number to help you find out.

MAGIC TRICK 5: CREATE YOUR OWN

Create a "When is my birthday?" or "Guess the number" magic algebra trick.

3.5 Multiplying and dividing algebraic terms

We know that $5 \times 6 \times 2 = 5 \times 2 \times 6$
$= 60$

This is known as the **associative law.** When multiplying algebraic terms, we can rearrange the multiplication to group numbers and pronumerals separately. Variables are written in alphabetical order.

$5a \times 2 = 5 \times a \times 2$
$= 5 \times 2 \times a$
$= 10a$

$2d \times 3c = 2 \times d \times 3 \times c$
$= 2 \times 3 \times c \times d$
$= 6cd$

When multiplying terms:

- multiply the numbers
- then multiply the variables (algebraic factors)
- write variables in alphabetical order.

Worked Example 10 WE10

Simplify:

(a) $6a \times 3$ **(b)** $5a \times -2c \times b \times 3$

Thinking	Working
(a) Multiply the numbers ($6 \times 3 = 18$) and multiply this product by the variable (a).	(a) $6a \times 3 = 18a$
(b) Multiply the numbers ($5 \times -2 \times 3 = -30$) and multiply this product by the product of the variables ($a \times c \times b = abc$), writing them in alphabetical order.	(b) $5a \times -2c \times b \times 3 = -30abc$

When dividing algebraic terms:

- write as a fraction first
- then simplify the fraction by dividing both the numerator and the denominator by the **Highest Common Factor (HCF)** of the numerator and the denominator
- common factors can be numbers and/or variables.

Worked Example 11

WE11

Simplify: $\frac{-15ef}{10f}$

Thinking	Working
1 Identify the HCF of the numerator and the denominator (HCF = $5f$), then divide the HCF into both the numerator and the denominator.	$\frac{{}^{3}\cancel{-15}}{{}_{2}\cancel{10}} \times \frac{e\,{}^{1}\cancel{f}}{{}_{1}\cancel{f}}$
2 Write the simplified expression.	$= -\frac{3e}{2}$

Worked Example 12

WE12

Simplify: $14ab \div 7bc$

Thinking	Working
1 Write as a fraction.	$14ab \div 7bc = \frac{14ab}{7bc}$
2 Identify the HCF of the numerator and the denominator ($7b$), then divide it into both the numerator and the denominator.	$= \frac{{}^{2}\cancel{14}a\,{}^{1}\cancel{b}}{{}_{1}\cancel{7}\,{}_{1}\cancel{b}c}$
3 Write the simplified expression.	$= \frac{2a}{c}$

3.5 Multiplying and dividing algebraic terms

Navigator

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q7, Q9, Q10, Q11	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12

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Fluency

WE10

1 Simplify:

(a) $5 \times 3a$ **(b)** $6 \times 2a$ **(c)** $9g \times 2$

(d) $7 \times 8z$ **(e)** $3z \times 12$ **(f)** $9p \times 6$

(g) $11e \times 6f$ **(h)** $9m \times 8n$ **(i)** $12j \times 11k$

(j) $6gh \times k \times 3$ **(k)** $8gh \times 3 \times 2k$ **(l)** $6b \times 5eh \times 3$

(m) $4 \times -5y$ **(n)** $-3 \times -7y$ **(o)** $-2a \times -8$

(p) $11a \times -4b$ **(q)** $-3r \times -6uq$ **(r)** $-7u \times 8rq$

(s) $-4j \times -2 \times 10k$ **(t)** $10j \times -2k \times 3$ **(u)** $8 \times -u \times 3v \times 2$

WE11

2 Simplify:

(a) $\frac{14a}{7}$ (b) $\frac{12b}{3}$ (c) $\frac{63e}{9}$

(d) $\frac{15f}{5f}$ (e) $\frac{10x}{12x}$ (f) $\frac{x}{15x}$

(g) $\frac{15ab}{5}$ (h) $\frac{24ef}{8e}$ (i) $\frac{26ab}{13a}$

(j) $\frac{ef}{8e}$ (k) $\frac{7mn}{28m}$ (l) $\frac{11ef}{77f}$

(m) $\frac{-22ab}{11ab}$ (n) $\frac{-6ab}{3ba}$ (o) $\frac{-16ef}{-4ef}$

(p) $\frac{-9h}{-45gh}$ (q) $\frac{-7g}{-77gh}$ (r) $\frac{-9i}{-36ij}$

WE12

3 Simplify:

(a) $20c \div 5$ (b) $40d \div 4$ (c) $12g \div 6$

(d) $60h \div 10h$ (e) $18x \div 27x$ (f) $20x \div 22x$

(g) $18cd \div 9$ (h) $56cd \div 8$ (i) $30gh \div 6$

(j) $gh \div 3g$ (k) $45gh \div 18g$ (l) $50gh \div 16g$

(m) $8cd \div -4c$ (n) $-50ab \div 30a$ (o) $60cd \div -12d$

(p) $12ab \div -8ba$ (q) $-ij \div -6ij$ (r) $-11ij \div -ji$

4 $-2 \times 5g \times -a$ simplifies to:

A $-7ga$ B $-10ag$ C $7ga$ D $10ag$

5 $\frac{-12mad}{-9md}$ simplifies to:

A $\frac{4a}{3}$ B $-\frac{4m}{3}$ C $\frac{4m}{3}$ D $-\frac{4a}{3}$

6 $-a \times -2b \times -3c \div -6bad$ simplifies to:

A $-12a$ B $\frac{c}{d}$ C $6cd$ D $-\frac{6d}{c}$

Understanding

7 Write and simplify the product of the three terms $3abc$, $-5d$ and -4.

8 Below are two toolboxes. The volume of each is given by the product of the length, width and height.

(a) What is the volume of Toolbox 1?

(b) What is the volume of Toolbox 2?

(c) Find the volume of each toolbox if $w = 4$ and $j = 12$.

(d) How many small toolboxes will fit inside the large toolbox?

Toolbox 1

Toolbox 2

Reasoning

9 A necklace is to be made from 3 strands of gold wire. The shortest strand is x mm long. Each strand is 1.2 times longer than the previous one.

(a) Write expressions to show the lengths in mm of each strand.

(b) Write an expression in terms of x to show the total length in mm of gold wire required.

(c) If gold wire costs \$$d$ per mm, write an expression to show the total cost of gold used.

(d) There are $6y$ small diamonds to be added to the necklace and each strand will contain the same number of diamonds. Write an expression to show how many diamonds will be inserted in each strand.

(e) If each diamond costs \$$c$, write an expression to show the total cost of all the diamonds.

(f) Write an expression to show the total cost of materials required to make the necklace if a fastener costs \$$f$.

(g) If $x = 30$, $d = 5$, $y = 4$, $c = 40$ and $f = 34$, find the total cost of materials required to make the necklace.

Open-ended

10 Joshua simplified an expression. His work is shown below. Explain the mistake he has made and write the correct answer, so that he does not make a similar mistake in the future.

$3k \times -2k = k$

11 Choose two terms, each containing the variables p and r, so that the quotient of the two terms gives a result of $-\frac{3p}{7r}$. (Remember, the quotient is the result after one term has been divided by the other.)

12 Choose three terms, each containing the variables m and n, so that the product of two terms divided by the third gives a result of $6mn$.

Outside the Square — Problem solving

Make a profit

Strategy options

- Guess and check.
- Make a table.
- Look for a pattern.

A T-shirt manufacturer in Australia pays fixed expenses of \$400 per week to run his factory. In addition to this, it costs \$12 per T-shirt manufactured. The information in the table shows how many T-shirts he will sell at certain prices.

Profit = income from selling T-shirts – cost of manufacturing the T-shirts

1 What is the number of T-shirts that need to be manufactured in order to make a profit?

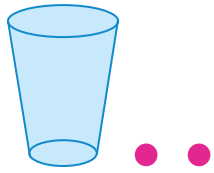
2 What is the number of T-shirts that need to be manufactured to make the largest profit?

Sale price per T-shirt	Number of T-shirts sold
\$10	45
\$15	40
\$20	35
\$25	30
\$30	25
\$35	20
\$40	15
\$45	10

3.6 Expanding brackets

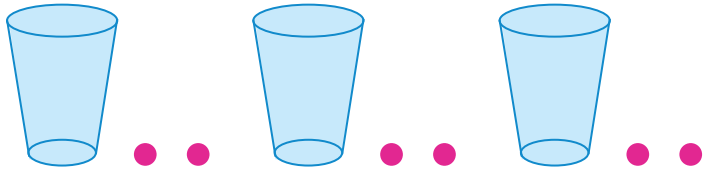
What do we mean by $3(x + 2)$?

We can show what this expression means by letting x be the number of counters in a cup, so that $x + 2$ is the number of counters in the cup plus two more.



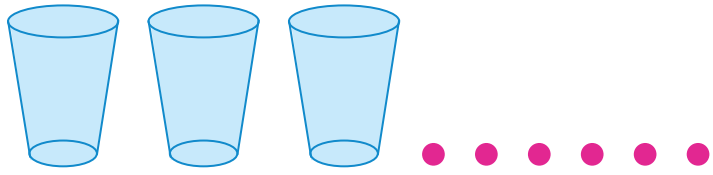
$x + 2$

If we have $3(x + 2)$, this represents 3 lots of $(x + 2)$ or $3 \times (x + 2)$ and can be shown by 3 cups with x counters in each and 2 more counters with each cup.



$(x + 2) \quad + \quad (x + 2) \quad + \quad (x + 2)$

This can be rearranged to give us 3 cups, each with x counters, and 6 more counters, which will give a total of $3x + 6$ counters.



$(x \quad + \quad x \quad + \quad x) + (2 \quad + \quad 2 \quad + \quad 2)$

Therefore, we can write $3(x + 2) = 3x + 6$.

Expanding is the process of removing brackets using the **distributive law**. We have seen this before with numbers.

For example: $3 \times 102 = 3 \times (100 + 2)$
$= 3 \times 100 + 3 \times 2$
$= 300 + 6$
$= 306$

Remember that variables in algebra are simply representing unknown numbers, so the distributive law must work in algebra if it works with numbers.

So, we can use the distributive law to expand algebraic expressions.

$3(x + 2) = 3 \times x + 3 \times 2$
$= 3x + 6$

Expanding using the distributive law:

$a(b + c) = ab + ac$ and $a(b - c) = ab - ac$

Worked Example 13 WE13

Use the distributive law to find 24×99.

Thinking	Working
1 Write one number in the product as the sum or difference of two numbers that are easy to multiply by ($99 = 100 - 1$).	24×99 $= 24 \times (100 - 1)$
2 Expand the brackets.	$= 24 \times 100 - 24 \times 1$
3 Simplify.	$= 2400 - 24$ $= 2376$

Worked Example 14 WE14

Expand each of the following expressions.

(a) $5(a - 2)$ **(b)** $2a(a + 3b)$ **(c)** $-3(a - b)$

Thinking	Working
(a) 1 Multiply each term in the bracket by the term in front of the bracket.	(a) $5(a - 2)$ $= 5 \times a - 5 \times 2$
2 Simplify.	$= 5a - 10$
(b) 1 Multiply each term in the bracket by the term in front of the bracket.	(b) $2a(a + 3b)$ $= 2a \times a + 2a \times 3b$
2 Simplify.	$= 2a^2 + 6ab$
(c) 1 Multiply each term in the bracket by the term in front of the bracket, making sure to change signs if necessary.	(c) $-3(a - b)$ $= -3 \times a - 3 \times -b$
2 Simplify.	$= -3a + 3b$

Expanding and collecting like terms

After an expression is expanded using the distributive law, it may contain like terms. These can be collected and the expression simplified.

Worked Example 15 WE15

Simplify the following expression by expanding the brackets and collecting like terms.

(a) $3(r - 4) - 8r$ **(b)** $5(a + 6b) + a(9 - 10a)$

Thinking	Working
(a) 1 Expand all brackets.	(a) $3(r - 4) - 8r$ $= 3r - 12 - 8r$
2 Simplify by collecting like terms.	$= -5r - 12$

(b) 1	Expand all brackets.	(b) $5(a+6b)+a(9-10a)$ $=5a+30b+9a-10a^2$
2	Simplify by collecting like terms.	$=14a+30b-10a^2$

Worked Example 16 WE16

Simplify the following expressions by expanding the brackets and collecting like terms.

(a) $8d-(2d+3)$ **(b)** $7(a-3b)-5(a+3b)$ **(c)** $3m(3n-4t)-4n(2m-t)$

Thinking		Working
(a) 1	Rewrite the expression, inserting a 1 and a × sign in front of the brackets.	(a) $8d-(2d+3)$ $=8d-1\times(2d+3)$
2	Expand the brackets, taking care to change signs where required.	$=8d-2d-3$
3	Simplify by collecting like terms.	$=6d-3$
(b) 1	Expand all brackets, taking care to change signs where required.	(b) $7(a-3b)-5(a+3b)$ $=7a-21b-5a-15b$
2	Simplify by collecting like terms.	$=2a-36b$
(c) 1	Expand all brackets, taking care to change signs where required.	(c) $3m(3n-4t)-4n(2m-t)$ $=9mn-12mt-8mn+4nt$
2	Simplify by collecting like terms.	$=mn-12mt+4nt$

3.6 Expanding brackets

Navigator

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Q1 Column 1, Q2 Columns 1 & 2, Q3 Column 1, Q4 Column 1, Q5, Q6, Q7, Q8, Q9, Q12	Q1 Column 2, Q2 Columns 2 & 3, Q3 Column 2, Q4 Column 2, Q5, Q6, Q7, Q8, Q9, Q10, Q12	Q1 Column 3, Q2 Columns 2 & 3, Q3 Column 2, Q4 Column 2, Q5, Q6, Q8, Q10, Q11, Q12, Q13

Fluency

WE13

1 Use the distributive law to find:

(a) 21×101 **(b)** 17×102 **(c)** 16×101

(d) 23×98 **(e)** 41×99 **(f)** 37×97

(g) 54×11 **(h)** 38×1010 **(i)** 64×1100

WE14

2 Expand each of the following expressions.

(a) $5(x+12)$ **(b)** $2(s-5)$ **(c)** $12(a+5)$

(d) $s(4+t)$ **(e)** $l(6-r)$ **(f)** $a(2+h)$

(g) $3(4x+5)$ **(h)** $9(6x-1)$ **(i)** $2(3h+4)$

(j) $2m(n+r)$ **(k)** $3p(r-z)$ **(l)** $6p(r+t)$

(m) $5t(t+2)$ **(n)** $2p(5p-7)$ **(o)** $4m(4+2m)$

(p) $3m(5m+7n)$ **(q)** $4k(p-5k)$ **(r)** $10x(2x-5y)$

3 Simplify the following expressions by expanding the brackets and collecting like terms. WE15

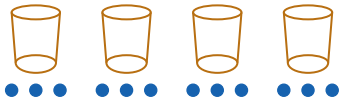
(a) $5(7 - d) + 11d$
(b) $8(3 - e) - 12e$
(c) $6n(m - 4) - 12nm$
(d) $4x(y - 2) - 7xy$
(e) $5t(7 - 4k) - 40tk$
(f) $9v(5 - 10w) - 50v$
(g) $6m(7n - 4) + 4(8mn - 4m)$
(h) $2a(5b - 8) + 2(6a - 12ab)$
(i) $9x(5 - y) + 3(10x - 4xy)$
(j) $3t(4 - 8z) + 6(7tz - 20t)$

4 Simplify the following expressions by expanding the brackets and collecting like terms. WE16

(a) $3x - 4(x + 2)$
(b) $5x - (2x + 2)$
(c) $8 - 3(x + 2)$
(d) $14 - (3x - 7)$
(e) $5x - 3(5x - 2)$
(f) $7x - 3(x - 2)$
(g) $5(x + 2) - 3(2x + 1)$
(h) $4(3x - 2) - 12(x + 5)$
(i) $3(3x + 4) - 5(7x - 3)$
(j) $5(x - 3) - (5x - 1)$
(k) $5(2x - 3y) - 3(3x + 4y)$
(l) $4(3x - 5y) - 2(5x + 2y)$
(m) $c(3d + 4e) - 2c(d + 5e)$
(n) $d(5e + 2f) - 4d(e + 4b)$
(o) $m(2n - 3t) - t(2m - n)$
(p) $a(2p - q) - q(2p - 3a)$

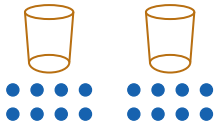
5 The expression that can be modelled by the cups and counters in the diagram is:

(a)



A $4x + 3$ B $4(x + 12)$ C $4(x + 3)$ D $2x + 12$

(b)



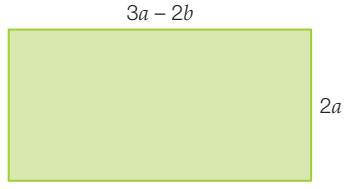
A $x + 16$ B $2x + 8$ C $2x + 16$ D $2(x + 16)$

6 Expanding and simplifying $3y(2y + a) - a(3y - a)$ gives:

A $6y^2 - ay$ B $6y^2 - 3ay + a^2$ C $3ay - 6y^2$ D $6y^2 + a^2$

Understanding

7 A rectangle has width $2a$ metres and length $3a - 2b$ metres. Write the expression for the area and expand the brackets.



8 (a) (i) Expand $33(x + 5)$. (ii) Substitute $x = 100$ and evaluate.

(b) (i) Expand $105(y + 3)$. (ii) Substitute $y = 30$ and evaluate.

(c) Explain why your answers to **(a) (ii)** and **(b) (ii)** are the same.

Reasoning

9 A rectangle has length $(5 + x)$ m and width 5 m.

(a) Draw a diagram to represent this information.

(b) Find the area of the rectangle.

(c) Split your diagram up into two parts, with one part being a square of side length 5 m.

(d) Find the area of each part of your diagram.

(e) Use your diagram and the answers to parts **(b)** and **(d)** to explain why $5 \times (5 + x) = 25 + 5x$.

10 Lucy amazed her friends with her quick computational skills. When asked to multiply 8 by 8.125, she immediately answered 65. She told her friends it was easy, and that they should investigate $20 \times 20.05 = 401$ and $4 \times 4.25 = 17$. She also said that she used the distributive law. How did she do the arithmetic so quickly?

Open-ended

11 Quentin was working on algebraic expressions, expanding brackets and then collecting like terms.

His answer was: $20x - xy$.

Write a question that Quentin might have been answering, assuming he answered correctly.

12 Michael was simplifying by expanding and collecting like terms. His working is shown below.

$$\begin{aligned} 3(2x-1)-(5x-2) &= 6x-3-5x-2 \\ &= x-5 \end{aligned}$$

(a) Identify the mistake that Michael made.

(b) Correct his mistake by completing the question.

(c) Suggest how Michael can avoid making this mistake in the future.

13 Write three expressions containing brackets that can be expanded to give the first term as $12ef$.

Outside the Square Puzzle

The rat race

It was the end of the National Rodent Racing season, but during a freak accident involving a llama and a cigar the main results table had been burned.

Luckily, a number of people could remember the following about the race results.

There were eight rats in the race.

- Little Wiggle finished after Burnside and Coddlington.
- Coddlington finished before Jurnpa, Little Wiggle, Friendly, Tigger and Burnside.
- Zipper finished before Darling and Coddlington.
- Tigger finished before Darling, Friendly and Little Wiggle.
- Friendly finished after Little Wiggle, Coddlington, Tigger, Burnside and Darling.
- Jurnpa finished after Burnside and Coddlington.
- Jurnpa finished before Little Wiggle and Friendly.
- Darling finished after Coddlington, Tigger and Little Wiggle.
- Tigger finished after Jurnpa and Coddlington.

Can you restore the rodents to the correct position in the results table and find out which rat won the race?

Technology Exploration Excel

Equipment required: 1 brain, 1 computer with an Excel spreadsheet program or a spreadsheet program on a CAS calculator

Versions of this Exploration for other technologies are available in Pearson Reader.

Farmer Jones' dilemma

Farmer Jones has a problem she would like you to solve. She wants to construct a rectangular pig pen for her prize pigs so that they will have the largest possible area to move around in.

Farmer Jones thinks she should fence a rectangular paddock for her prize bull first. She has 400 m of fencing wire and wants to also give her bull the largest possible area to move around in. Her son, Joe, offered to help his mum work out the dimensions. He first wrote down the formula for finding the perimeter of a rectangle:

$P = 2L + 2W$

Next, he rearranged the formula to find the width (W):

$$W = \frac{P - 2L}{2}$$

Then he used a spreadsheet to calculate W for different lengths, starting with $L = 10$ m and increasing the length by 10 m each time. He then found the area for each length and width:

$A = LW$

When he looked at his results, he was amazed at what he found.

The formulas Joe used are shown in the spreadsheet below.

(Hint: Remember that in Excel, the $ sign locks in a particular cell row or column. * means multiplication. Two dollar signs A2 are needed to lock in a particular cell.)

	A	B	C
1	Perimeter (m)		
2	400		
3			
4	L(m)	W (in m)	A(m2)
5	10	= (A2-2*A5)/2	=A5*B5
6	20		
7	30		
8	40		
9	50		
10	60		
11	70		
12	80		
13	90		
14	100		
15	110		
16	120		
17	130		
18	140		
19	150		
20	160		
21	170		
22	180		
23	190		

By following the same steps as Joe, find the dimensions of the rectangle that would enclose the largest area.

1 **(a)** Open a new spreadsheet file and enter the 'Perimeter' heading in A1 and the perimeter length (400) in A2. (Do not add units to the numbers.) Enter the column headings and start with 10 as the first length. Increase each length by 10 units up to 190. (Enter 20, highlight 10 and 20 and drag down to 190 with the little black cross in the bottom right-hand corner.)

Enter the formulas to find the width and the area and fill down.

(b) What was special about the rectangle that Joe found?

2 Joe wonders whether this shape would always give him the largest area. He wants to build a chicken coop with 50 m of chicken wire for his pet chickens.

(a) By using a spreadsheet, find the dimensions of the chicken coop that would enclose the largest area.

(Hint: You can use a similar format to the spreadsheet shown but you will need to change the perimeter and use smaller amounts for the length. Don't forget that length does not have to be a whole number.)

(b) What do you notice about this rectangle? Is it the same type of rectangle as Joe found in **1(b)**?

(c) What area does this rectangle now enclose?

3 Farmer Jones is impressed with her son's findings, but wants to check them for herself. She has found some more wire and decides to build the pig pen with it to maximise the area enclosed. She sets up a spreadsheet similar to the one her son used.

(a) Choose a suitable length of fencing wire (100 m to 200 m) for Farmer Jones and use this length and a spreadsheet to show that the type of rectangle with the biggest area is always the same.

(b) Using your chosen length of fencing wire, what are the dimensions of the pig pen she built and the area enclosed?

Taking it further

4 Farmer Jones is so happy with her new spreadsheet skills that she wants to try something new. She now wants to enclose a rectangular paddock with 800 m of fencing wire for her sheep, but one side of the paddock will be the river that runs through her property and this will not need to be fenced. She wants to ensure that she allows her sheep the largest area possible within the paddock.

(a) What is the new formula she needs to use:

(i) for the perimeter

(ii) for the width

(iii) for the area?

(b) Use a spreadsheet to find the dimensions of this new enclosure?

(c) What do you notice about this rectangle?

Good luck!! Farmer Jones is counting on you.

Factorising

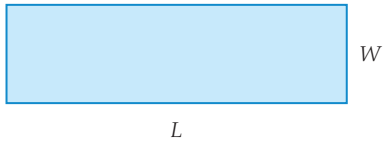
3.7

Factorising means writing an expression as the product of factors.

$10 = 2 \times 5$
$2a = 2 \times a$

$2(a + b)$ is a factorised expression, as it is the product of 2 and $(a + b)$.

How do we find the perimeter of a rectangle?



One method is to double the length and double the width and add these two amounts. Writing this as a formula, we get:

$P = 2L + 2W$ where L = length and W = width. This is the expanded form.

However, we could add the length and width together first and then double that amount. Writing this as a formula, we get:

$P = 2(L + W)$.

This is the **factorised form**.

This means that $2L + 2W = 2(L + W)$.

The expanded form = the factorised form.

We also know that $2(L + W) = 2L + 2W$ by expanding.

The factorised form = the expanded form.

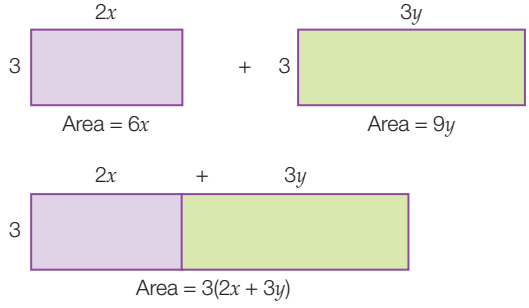
This shows us that factorising is the reverse of expanding.

What is the sum of the areas of the two rectangles on the right?

The total area can be written in two equally correct forms:

Expanded form: $6x + 9y$

Factorised form: $3(2x + 3y)$



Factorising an expression

- Write each term in the expression as a product of prime factors.
- Circle all common factors to find the highest common factor (HCF) of all the terms in the expression.
- Write the HCF in front of the brackets.
- Write the sum or difference of the other factors inside the brackets.
- Write the expression in expanded form.
- Check your answer by expanding the brackets.

$6x + 9y = 3 \times 2 \times x + 3 \times 3 \times y$, 3 is the HCF of $6x$ and $9y$
$= 3(2x + 3y)$

Check $3(2x + 3y) = 6x + 9y$.

Worked Example 17 WE17

Find the highest common factor (HCF) of the following pairs of algebraic terms.

(a) $10x$ and 15 **(b)** $12a$ and $18b$ **(c)** $6c^2d$ and $8cd^2$

Thinking		Working
(a) 1	Write each term as the product of prime factors, circling the common factors.	(a) $10x = 2 \times 5 \times x$ $15 = 3 \times 5$
2	Write the HCF.	HCF = 5
(b) 1	Write each term as the product of prime factors, circling the common factors.	(b) $12a = 2 \times 2 \times 3 \times a$ $18b = 2 \times 3 \times 3 \times b$
2	Multiply the common factors to find the HCF.	HCF $= 2 \times 3$ $= 6$
(c) 1	Write each term as the product of prime factors, circling the common factors.	(c) $6c^2d = 2 \times 3 \times c \times c \times d$ $8cd^2 = 2 \times 2 \times 2 \times c \times d \times d$
2	The HCF is the product of the common factors. (Convert to index form.)	HCF $= 2 \times c \times d$ $= 2cd$

It is sometimes convenient to factorise by using a negative integer as a common factor. Care needs to be taken to obtain the correct sign inside the brackets. We can do this using $+ = - -$ and $- = + -$. The negative sign can then be included in the HCF.

$$-2x^2 - 4xy = -2 \times x \times x + -2 \times 2 \times x \times y$$
$$= -2x(x + 2y)$$

Worked Example 18 WE18

Factorise the following expressions.

(a) $3y + 3$ **(b)** $10a - 12$ **(c)** $-15b + 10c$

Thinking		Working
(a) 1	Write each term as the product of its prime factors, circling the common factors. If all the factors in a term are circled, write 1 as another factor.	(a) $3y + 3$ $= 3 \times y + 3 \times 1$
2	Write the HCF outside the brackets and the remaining terms inside the brackets.	$= 3 \times (y + 1)$
3	Leave out the multiplication signs.	$= 3(y + 1)$

(b)	1	Write each term as the product of its prime factors, circling the common factors.	(b) $10a - 12$ $= 2 \times 5 \times a - 2 \times 2 \times 3$
	2	Write the HCF outside the brackets and the remaining terms inside the brackets.	$= 2 \times (5 \times a - 2 \times 3)$
	3	Leave out the multiplication signs.	$= 2(5a - 6)$
(c)	1	Write each term as the product of its prime factors. Change the + sign to –– so that a negative sign can be included in the common factor. Circle the common factors.	(c) $-15b + 10c$ $= -3 \times 5 \times b - -2 \times 5 \times c$
	2	Write the HCF, including the negative sign, outside the brackets and the remaining terms inside the brackets.	$= -5 \times (3 \times b - 2 \times c)$
	3	Leave out the multiplication signs.	$= -5(3b - 2c)$

3.7 Factorising

Navigator

Q1 Column 1, Q2 Column 1, Q4, Q6, Q10, Q12	Q1 Column 2, Q2 Column 2, Q4, Q5, Q6, Q9, Q10, Q12	Q1 Column 3, Q2 Column 3, Q3, Q4, Q5, Q7, Q8, Q9, Q11, Q12

Answers page 610

Fluency

WE17

1 Find the highest common factor (HCF) of the following pairs of algebraic terms.

(a) $6m$ and 9 (b) $14p$ and 21 (c) $22d$ and 55
(d) $8x$ and $36y$ (e) $30d$ and $45c$ (f) $24a$ and $54b$
(g) $2p^2$ and $8p^4$ (h) $9t^5$ and $12t$ (i) $15d^5$ and $10d^3z^3$
(j) fm^4 and f^4m^2 (k) a^2b and a^3b^2 (l) x^4y^2 and x^2y^3
(m) $20mn^4$ and $25m^3n^2$ (n) $18s^3t^6$ and $15st$ (o) $6g^2h$ and $15gh^3i$

WE18

2 Factorise the following expressions.

(a) $3t + 15$ (b) $2h + 14$ (c) $2a - 8$
(d) $9p + 3$ (e) $20p - 5$ (f) $15z + 3$
(g) $12 - 20b$ (h) $24 + 10y$ (i) $18 - 14q$
(j) $3px + 12$ (k) $2kt - 20$ (l) $4hs + 14$
(m) $-8ab - 16$ (n) $-5mn - 15$ (o) $-12kr - 132$

3 Factorise the following expressions.

(a) $3lx + 15x$ (b) $10pm - 20m$ (c) $14pq + 28q$
(d) $6vx - 3x$ (e) $30ab + 20a$ (f) $18xy - 40y$
(g) $16p + 14pq$ (h) $15l - 20lm$ (i) $22j - 55jl$
(j) $30mz - 20m$ (k) $32hk + 36k$ (l) $40h + 28hi$
(m) $-12ab + 16b$ (n) $-14xy + 21x$ (o) $-72m - 56mn$

You can check your answers by expanding the brackets. You should get the original expression back again.

4 The highest common factor of $20x$ and $14xy$ is:

A $140xy$ **B** $7y$ **C** $2y$ **D** $2x$

5 The HCF of $2mn$ and $2m^6$ is:

A m^6n **B** $2m$ **C** $2m^6n$ **D** $2mn$

Understanding

6 Fred has \$$5ab$ and Barney has \$$25c$. Write down the total amount of money they have between them, in factorised form.

7 The kinetic energy of a moving object that has a mass of m kilograms and a speed of v metres per second is given by $\frac{mv^2}{2}$. The object's gravitational potential energy h metres above the ground is given by mgh, where g is the acceleration of the object due to gravity at that point.

(a) Write down the total energy of the object (kinetic energy plus gravitational potential energy).

(b) Write down the total energy in factorised form.

8 Factorise:

(a) $3(x + 1) - y(x + 1)$ **(b)** $7(3x - 4) + y(3x - 4)$ **(c)** $5(x + 7) + 2y(x + 7)$

Reasoning

9 The expression $ax + ay + bx + by$ contains four terms.

(a) Factorise $ax + ay$.

(b) Factorise $bx + by$.

(c) Write down the expression for the sum of your factorised answers for **(a)** and **(b)**.

(d) Factorise the expression in **(c)** by taking the expression in brackets to be the highest common factor.

(e) Use the method above to factorise $3x + 3b + xy + by$.

Open-ended

10 Write down three pairs of terms whose highest common factor is 8.

11 A rectangle has a width of x cm and an area given by the formula $A = 10x - x^2$.

(a) Factorise the expression for the area to find an expression for the length of the rectangle.

(b) If the length is greater than the width, find a possible value for x.

12 Stephen has factorised $9x + 3$ as shown below.

$9x + 3 = 3(3x + 3)$

His friend Mohammed tells him that he is wrong.

Mohammed has written $9x + 3 = 3(3x + 1)$.

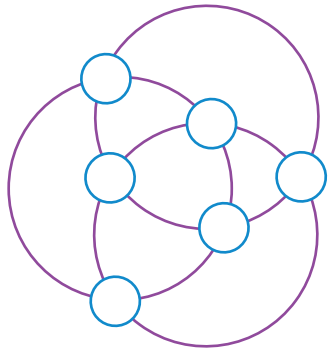
State who is correct. Explain where the other person has made a mistake and what can be done to avoid this mistake in the future.

Challenge 3

1 When four consecutive even numbers are added the total is 140. Use algebra to find the numbers.

2 In how many different ways can you add four positive odd numbers to total 16? You can use any positive odd number as many times as you like.

3 Three large circles overlap. Six smaller circles are located on the points where the circles cross.



(a) Copy this diagram and then write the digits 0, 1, 2, 3, 4 and 5 in the blue circles so that the totals of the numbers in the blue circles around each large circle are the same.

(b) Copy this diagram again and then write the expressions x, $x + 1$, $x + 2$, $x + 3$, $x + 4$ and $x + 5$ in the blue circles so that the totals of the numbers in the blue circles around each large circle are the same.

(c) Find six other integers or expressions for which this will work. (Integers are positive and negative whole numbers and zero; e.g. -7, 0, 4.)

4 Given the factorisations $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$ and $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$, answer the following questions, without using a calculator.

(a) Write 3^6 and 4^6 as perfect cubes.

(b) Show that 25 is a factor of $3^6 + 4^6$.

(c) Show that 4 is a factor of $3^6 - 5^3$.

5 Write an algebraic expression in each empty cell so that you create a magic square in which the sum of each row, column and diagonal is $(4x + 2)$.

$x + 8$	$x - 6$		$x + 5$
		$x + 2$	x
$x - 4$	$x + 6$	$x + 7$	

6 $\dfrac{a + \frac{1}{b}}{b + \frac{1}{a}}$ equals

A $\dfrac{a}{b}$ **B** $\dfrac{b}{a}$ **C** ab **D** 1

Chapter review 3

D.I.Y. Summary

Key Words

associative law	equation	factorising	substitution
coefficient	expanding	formulas	term
constant	expression	Highest Common Factor (HCF)	unlike terms
distributive law	factorised form	like terms	variable

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 $4b + 5$ is called an _____. The letter b in $4b + 5$ is called a _____ because it can take the place of any number. The number 5 is called a _____ and the number 4 is the _____.

2 When we replace x with a certain number, this is called _____.

3 To simplify $4e + 3f - 2e + 6f$ we collect _____. The expression then simplifies to $2e + 9f$.

4 When we obtain the expression $4c + 12$ from $4(c + 3)$ we are _____ the bracket using the _____.

5 When we write $6a + 18$ as $6(a + 3)$ we are _____ the expression.

6 Algebraic equations used in practical situations are called _____.

Fluency

1 You have x number of power points in your house. Ex 3.1

(a) The house next door has $x + 3$ power points. Comment on how this compares to the number in your house.

(b) The house on the other side of you has $x - 2$ power points. Comment on how this compares to the number in your house.

(c) The block of flats across the road has $6x$ power points. Comment on how this compares to the number in your house.

2 Evaluate the following expressions when $a = 4$ and $b = 5$. Ex 3.2

(a) $3a + 2b$ (b) $2ab - 12$ (c) $\frac{8b}{a} + 2b$ (d) $\frac{10}{b} + \frac{ab}{10}$

3 Evaluate the following expressions when $a = 2$ and $b = 3$. Ex 3.2

(a) $7(2a - b)$ (b) $3a(b + 17)$ (c) $2a(3 - 5b)$

4 Evaluate the following expressions when $a = -2$ and $b = -3$. Ex 3.2

(a) $-2a + b - 5$ (b) $ab - 4 - \frac{6b}{2}$

5 Evaluate the following expressions when $x = 2$ and $y = 12$. Ex 3.2

(a) $\frac{x + y}{2}$ (b) $\frac{2x + 3y}{5}$ (c) $\frac{12(x - y)}{y}$

6 (a) Using the formula $F = ma$, find F when $m = 100$ and $a = 6.3$.

(b) Using the formula $v^2 = u^2 + 2as$, find v when $u = 1$, $a = 10$ and $s = 4$.

Ex.3.3

7 Simplify the following, if possible, by collecting like terms.

Ex.3.4

(a) $-j - 9j$ (b) $5ty - 2ty$ (c) $7ghi - 4ghi$

(d) $24ab - 5ab + 7$ (e) $6a + 12b - 7a + 11b$ (f) $2a^2 - 4a$

8 Simplify:

Ex.3.5

(a) $15a \times 2b$ (b) $6ab \times c \times 11$ (c) $-2a \times -7 \times b$

9 Simplify:

Ex.3.5

(a) $24a \div 20a$ (b) $\frac{25ab}{5a}$ (c) $-4ab \div -44a$

10 Expand:

Ex.3.6

(a) $7(2 + a)$ (b) $a(2b + 6)$ (c) $4a(3a + 5)$

11 Simplify by expanding and collecting like terms:

Ex.3.6

(a) $2(4 + a) + 7a$

(b) $10a(6 + 2b) + 7(2a - ab)$

(c) $2a^2(3a^4 + 11a) + 3a(5a^5 - 2a^2)$

12 Factorise:

Ex.3.7

(a) $3a - 15$ (b) $20ab + 16$ (c) $-15p + 20q$

Understanding

13 Suppose there are m jelly beans in a packet. Melissa, Roderigo and Huong have bought some packets of jelly beans. The following tells us how many jelly beans each has at certain times.

Time	Melissa	Roderigo	Huong
12.30	m	0	0
12.31	$m - 2$	$2m$	0
12.32	$m - 4$	$2m - 10$	$3m$
12.33	$m - 6$	m	$3m - 5$
12.34	$m - 8$	$m - 6$	$2m$
12.35	$m - 14$	0	$2m - 8$

Write a paragraph telling a story about what happened during the 5 minutes.

14 In a school hall there are t teachers, g female students and b male students.

(a) How many people are there in total?

(b) Eight teachers, six female students and half the boys exit the hall. How many people remain inside?

(c) Half of the number of teachers and female students who had left re-enter the hall. How many people are now in the hall?

15 In centimetres, the heights of three Year 8 students, Troy, Barry and Tenielle, are, in order, $3a + 2$, $4b - 5$, and $6c + 1$.

If $a = 52$, $b = 41$, and $c = 26$, what is each person's height?

16 Who is the tallest, and what is his/her height if $d = 6$ and the heights of three Year 8 students, William, Stuart and Yumi, are, in order, $4d^2 - 8$, $d^3 - 45$, $2d^2 + 18d$?

17 If $a = 1$, $b = 2$, $c = 3$ and so on, with the number increasing by 1 for each letter, find the value of $ab - bc + cd - de + ef - fg$.

18 Fatima draws a design consisting of straight lines. She first draws a line of length u cm straight up, then another line of length r cm to the right and a line of length d cm straight down.

(a) Write an expression for the total length of the three lines.

(b) If Fatima continues this design ten times, what is the total length of the lines?

(c) If $r = d$, $r = \frac{1}{2}u$ and $u = 7$, what is your answer to part **(b)**?

19 The speed v metres per second at which a wheel of radius r metres turns is given by the formula $v = \dfrac{6.28r}{t}$, where t seconds is the time the wheel takes to make one turn. Find the speed of the wheel if $r = 0.35$ metres and $t = 1.57$ seconds.

20 Apples are sold at a market. A standard bag of apples contains x apples. A bonus bag of apples has twice the number of apples of a standard bag plus an extra 4 apples. A case of apples contains 2 fewer apples than 6 standard bags of apples. Three customers each bought a bonus bag of apples and five customers each bought a case of apples.

(a) Write an expression to show the number of apples in a bonus bag.

(b) Write an expression to show the number of apples in a case of apples.

(c) Write an expression to show the number of apples sold.

(d) Expand and simplify your answer.

(e) How many apples were sold if a standard bag contained a dozen apples ($x = 12$)?

21 A special tint is used to protect glass in a rectangular window of length L in metres and width W in metres. The cost in dollars of applying the tint is $7 \times$ area of glass.

(a) Write an expression for the cost of tinting the window in terms of L and W.

(b) Find the cost of tinting a window 2.5 m long and 0.8 m wide.

22 The area of a triangle is half the product of its base length and its height: $A = \dfrac{bh}{2}$.

(a) Write the expression for the area of a triangle whose base is $4x$ cm and whose height is $(3x + 2)$ cm.

(b) Expand and simplify the expression obtained in part **(a)**.

(c) What is the area of the triangle if the height is 14 cm?

Reasoning

23 A stencil is made by cutting a triangle out of a rectangle of length l cm and width w cm.

(a) If the base of the triangle is half the length of the rectangle and the height of the triangle is 6 cm less than the width of the rectangle, find the area of the triangle in terms of l and w in: **(i)** factorised form and **(ii)** expanded form.

(b) Show that the area of the rectangle that is left once the triangle is cut out is given by $\dfrac{3lw}{4} + \dfrac{3l}{2}$.

(c) Factorise this answer.

(d) Find the area of the rectangle that is left once the triangle is cut out if $l = 20$ cm and $w = 16$ cm by:

(i) subtracting the area of the triangle from the area of the rectangle

(ii) substituting the values for l and w into your factorised answer in part **(c)**.

24 a, b, c represent three numbers.

(a) Write the expression that represents b being increased by 1.

(b) Write the expression for the product of a and your answer to part **(a)**.

(c) Write the expression for the product of c and your answer to part **(a)**.

(d) Write the expression for the sum of the answers to parts **(b)** and **(c)**.

(e) Factorise the answer obtained in part **(d)**.

(f) **(i)** Write the expression for the answer to part **(e)** divided by $a + c$.

(ii) Simplify your answer to part **(i)**.

(iii) Subtract 1 from your answer to part **(ii)**.

(iv) What do you notice about your answer to part **(iii)**?

(g) Using the values $a = 3$, $b = 7$, $c = 8$, repeat steps **(a)** to **(f)**.

NAPLAN practice 3

Numeracy: Non-calculator

1 What is the value of $4a^2b$ when $a = -3$ and $b = 2$?

A -72 **B** -48 **C** 48 **D** 72

2 Which one of the following expressions is equivalent to $5(2x - 1)$?

A $10x - 1$ **B** $10x - 5$ **C** $10x + 1$ **D** $10x + 5$

3 Which one of the following expressions is equivalent to $3xy - 8x^2 + 2yx - 4x^2$?

A $3xy - 12x^2 + 2yx$ **B** $5xy - 12x^2$ **C** $-7x^2y$ **D** $5xy - 4x^2$

4 Which expression is equivalent to $4 - 8k$?

A $8k - 4$ **B** $8k + 4$ **C** $-8k + 4$ **D** $-8k - 4$

Numeracy: Calculator allowed

5 Which one of the following expressions is equivalent to $12x - 9y$?

A $9(3x - y)$ **B** $12(x - 9y)$ **C** $3(4x - 3)$ **D** $3(4x - 3y)$

6 If $a = 2$, what is the value of $\frac{3a - 4}{2a}$?

A 2 **B** $\frac{1}{2}$ **C** -2 **D** $-\frac{1}{2}$

7 A rule for y in terms of x is $y = 12 - 3x$. When $x = 2.3$ the value of y is:

A 5.1 **B** 16.6 **C** 20.7 **D** 34.5

8 Dylan is x years old. His brother, Liam, is 8 years old. John's age is three times Dylan's age, plus Liam's age. Find an expression in terms of x for John's age.

A $x + 11$ **B** $8x + 3$ **C** $3x + 8$ **D** $3x - 8$

4

Ratio and rate 4

What a dummy! Would you create an imaginary friend just so you could travel faster on the freeway?

In peak-hour traffic on freeways there can be up to 20 'driver-only' vehicles for every 1 vehicle carrying passengers. To encourage car pooling, transport authorities in many states and countries have set aside a transit lane, which is a lane only for vehicles with passengers. Because there are fewer cars, travel in this lane is much quicker and, by sharing transport, the commuters save time, petrol, money and the environment.

One enterprising driver placed a crash dummy in the passenger seat to make it look like there was a passenger so he could drive in the transit lane. He was caught and paid a large fine. An internet search of 'dummy passenger transit lane' revealed that numerous people around the world from places including New Zealand, Australia and the USA have all had the same idea – and have been caught!.

Forum

How do you think the comparison of driver-only vehicles to those carrying passengers might change outside of peak hour?

Can you think of a time or a place where the situation might be the opposite – 1 'driver-only' vehicle to 20 vehicles carrying passengers?

Why learn this?

An understanding of ratio is important for completing many tasks. A baker making a cake, a builder mixing concrete, a photographer enlarging an image or a cartographer drawing a map all need skills in working with ratios and scale factors. Rates enable us to compare quantities of different types, or how an amount, such as the population of a city or a country, is changing over time.

After completing this chapter you will be able to:

- write and simplify ratios
- understand the relationship between ratios, fractions and percentages
- use the scale factor of unit ratios to find unknown quantities
- use ratios to calculate one quantity given the other
- share a quantity in a given ratio
- use scale ratios to interpret diagrams and maps
- calculate with rates such as speed
- use the unitary method to compare different rates
- calculate population growth rates.

Recall 4

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R4.1

1 Copy and complete the following conversions.

(a) 2000 mm = _____ cm **(b)** 3.6 km = _____ m

(c) 4 t = _____ kg **(d)** 350 mL = _____ L

Worksheet R4.2

2 Copy and complete the following pairs of equivalent fractions.

(a) $\frac{2}{3} = \frac{8}{\square}$ **(b)** $\frac{\square}{2} = \frac{21}{6}$

Worksheet R4.3

3 Simplify each of the following by cancelling common factors and converting improper fractions to mixed numbers.

(a) $\frac{36}{48}$ **(b)** $\frac{25}{15}$ **(c)** $\frac{35}{49}$ **(d)** $\frac{100}{76}$

Worksheet R4.4

4 Convert to improper fractions. **(a)** $2\frac{1}{7}$ **(b)** $2\frac{3}{8}$

Worksheet R4.5

5 Write the following:

(a) as mixed numbers in simplest form

(b) as decimals.

(i) $\frac{12}{5}$ **(ii)** $\frac{9}{2}$ **(iii)** $\frac{25}{8}$ **(iv)** $\frac{52}{10}$

Worksheet R4.6

6 Round the following to two decimal places.

(a) 0.525 17 **(b)** 3.899

Worksheet R4.7

7 Find:

(a) 30% of 150 **(b)** 8% of 40 **(c)** 102% of 76

Worksheet R4.8

8 **(a)** 5 out of 20 students buy their lunch at the school canteen. What percentage is this?

(b) 9 out of 16 pet owners own dogs. What percentage is this?

Worksheet R4.9

9 Solve the following equations. **(a)** $\frac{x}{3} = 7$ **(b)** $\frac{d}{8} = \frac{3}{4}$

Key Words

equivalent ratios	rate	scale ratio	unit ratio
part:part ratio	ratio	simplest form	unitary method
part:whole ratio	scale factor	speed	

Writing ratios

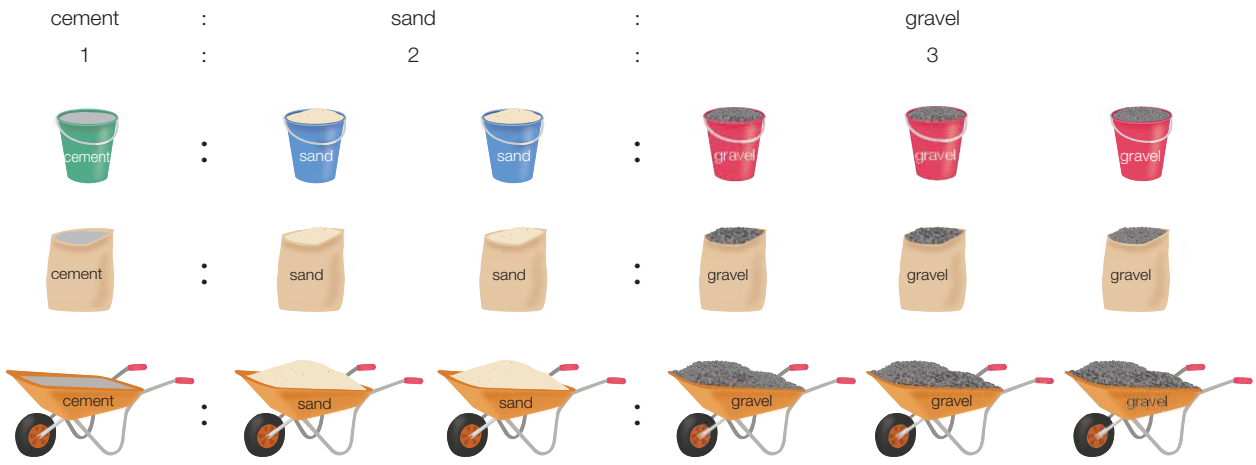
4.1

A **ratio** is a comparison of two or more amounts of the same type, written using a ':' symbol.

The order in which a ratio is stated must be made clear. If the ratio of teachers to students in a class is 1 : 20, then the ratio of students to teachers is 20 : 1.

A common ratio used for making concrete is to mix 1 part cement with 2 parts sand and 3 parts gravel. We can write this ratio as cement : sand : gravel = 1 : 2 : 3.

The actual measured amounts of these ingredients don't matter. It is the ratio, or proportion of each ingredient compared with the others, that is important. We could measure them out using buckets, bags or wheelbarrows and still get the correct mixture (provided the buckets, bags or wheelbarrows are all the same size).



Part : part and part : whole ratios

Suppose that an animal shelter has 13 dogs, 11 cats and six birds. We could write a ratio that compares the number of cats to the number of dogs as 11 : 13. This type of ratio compares two separate parts of a whole and is called a **part : part ratio**. We could write another part : part ratio that compared the number of cats, birds and dogs as 11 : 6 : 13. (Note that the order of the numbers in the ratio is the same as the written order of the animals.)

We can also write a ratio that compares the number of cats to the total number of animals as 11 : 30. This type of ratio compares one part to the whole and is called a **part : whole ratio**.

A part : part ratio compares separate parts of a whole.

A part : whole ratio compares a part (or parts) to a whole.

Ratios, fractions and percentages

It is often useful to express part : whole ratios as fractions or percentages of the whole. For example, the ratio cats : total animals of 11 : 30 means that $\frac{11}{30}$ of the animals are cats, which can be converted to $36.\dot{6}\%$. The number of dogs could be written as $\frac{13}{30}$, or $43.\dot{3}\%$. Part : part ratios can also be expressed as fractions or percentages. If the ratio of cats to dogs is 11 : 13, then we can say that the number of cats is $\frac{11}{13}$, or 84.6% of the number of dogs. The number of dogs is $\frac{13}{11}$, or 118.2% of the number of cats. The important thing to realise here is that we are writing one part as a fraction of the other part, not the overall whole.

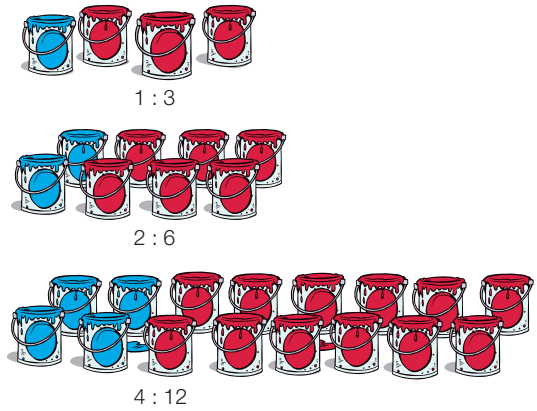
Simplifying ratios

When the paint in these three groups of tins is mixed, each group will give the same shade of purple. The ratio of blue paint tins to red paint tins is 1 : 3, 2 : 6 and 4 : 12, respectively. These ratios are called **equivalent ratios**.

When the paint in each group of tins is mixed, the fraction of red paint in each mixture will be $\frac{3}{4}, \frac{6}{8}, \frac{12}{16}$, respectively. These three fractions are equivalent and all simplify to $\frac{3}{4}$.

Ratios can be simplified in a similar way to fractions—by dividing each number in the ratio by a common factor. If we divide by the highest common factor (HCF), we obtain the ratio in **simplest form**. For example, the HCF of the numbers in the ratio 4 : 12 is 4.

$$\begin{array}{ccc} 4 & : & 12 \\ \div 4 \downarrow & & \downarrow \div 4 \\ = 1 & : & 3 \end{array}$$



To write a ratio in simplest form, divide each number in the ratio by the highest common factor (HCF).

Worked Example 1

WE1

A box of chocolates has seven plain, five nut and three soft-centred chocolates.

(a) Write a ratio that compares the number of nut chocolates to the number of plain chocolates, in simplest form.

(b) Write a ratio that compares the number of soft-centred chocolates to the total number in the box, in simplest form.

(c) Write the number of soft-centred chocolates as a percentage of the total number in the box.

(d) Write the number of soft-centred chocolates as a fraction of the number of plain chocolates.

Thinking	Working
(a) Write the ratio in words, in the correct order, with the numbers underneath. Simplify by dividing by a common factor, if possible. (Here, there are no common factors—the ratio is already in simplest form.)	(a) Nut : plain = 5 : 7
(b) **1** Find the number of parts in the whole (the total number of chocolates).	(b) $7 + 5 + 3 = 15$
2 Write the ratio in words, in the correct order, then write the numbers underneath. Simplify by dividing by the HCF. (Here, the HCF is 3.)	Soft-centred : total = 3 : 15 = 1 : 5 (÷ 3)

(c) Write the part : whole ratio as a fraction, then convert the fraction to a percentage.	(c) $\frac{1}{5}$ $= \frac{20}{100}$ $= 20\%$
(d) Write the part : part ratio, then use the numbers in the ratio to write the fraction. Simplify the fraction if possible.	(d) Soft-centred : plain = 3 : 7 = $\frac{3}{7}$

4.1 Writing ratios

Navigator

Q1, Q2 Columns 1 & 2, Q3, Q4, Q6, Q9, Q10, Q11, Q13	Q1, Q2 Column 2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q14	Q1, Q2 Column 3, Q3, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13, Q14

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Fluency

WE1

1 A greengrocer has nine boxes of apples, seven boxes of bananas and four boxes of pears.

(a) Write a ratio that compares the number of boxes of pears to the number of boxes of apples, in simplest form.

(b) Write a ratio that compares the number of boxes of pears to the total number of boxes of fruit, in simplest form.

(c) Write the number of boxes of pears as a percentage of the total number of boxes of fruit.

(d) Write the number of boxes of bananas as a fraction of the number of boxes of apples.

2 Write each of the following ratios in simplest form.

(a) 2 : 8	**(b)** 3 : 15	**(c)** 4 : 16
(d) 4 : 10	**(e)** 12 : 34	**(f)** 180 : 150
(g) 20 : 300	**(h)** 500 : 4000	**(i)** 225 : 325
(j) 25 : 10 : 5	**(k)** 24 : 4 : 8	**(l)** 33 : 15 : 27
(m) 3 : 12 : 6	**(n)** 36 : 4 : 16	**(o)** 670 : 500 : 215

3 There are 26 students in a Year 8 maths class. 15 of them are boys.

(a) The ratio of girls to boys is:

A 15 : 26 **B** 26 : 15 **C** 15 : 11 **D** 11 : 15

(b) The percentage of the class that are girls (to one decimal place) is:

A 42.3% **B** 57.7% **C** 73.3% **D** 136.4%

(c) The number of girls as a fraction of the number of boys is:

A $\frac{11}{26}$ **B** $\frac{15}{26}$ **C** $\frac{11}{15}$ **D** $\frac{15}{11}$

4 Lara mixes 8 litres of white paint with 3 litres of yellow and 2 litres of red to get the colour she wants to paint her living room. Find, in simplest form:

(a) the ratio of yellow paint to white paint to red paint

(b) the ratio of white paint to the total amount of paint in the mixture

(c) the ratio of yellow paint to the total amount of paint in the mixture

(d) the fraction of red paint in the mixture.

5 (a) Aimi estimates that for every 2 Sydney supporters at an AFL match there are 3 Brisbane supporters. Write her estimate as a ratio.

(b) Write the number of Brisbane supporters as a fraction of the number of Sydney supporters.

(c) Write the number of Sydney supporters as a fraction of the number of Brisbane supporters.

Understanding

6 A plank of wood 24 m long has an 8 m long piece cut off it. Express, in simplest form:

(a) the ratio of the length remaining to the length cut off

(b) the ratio of the length cut off to the length of the original piece

(c) the length of the shorter piece as a fraction of the longer piece

(d) the length of the longer piece as a percentage of the original whole.

7 4% of the fuel mixture used for a lawnmower is oil. The rest of the mixture is petrol.

(a) Write the ratio of petrol to oil in simplest form.

(b) Oliver has 200 mL of oil. How much petrol should he add to make the correct fuel mixture for his lawnmower?

8 Gek is a travel agent selling return airfares to London for $1950 and to Bangkok for $1150.

(a) Write a ratio comparing the London fare to the Bangkok fare, in simplest form.

(b) Write the London fare as a percentage of the Bangkok fare to the nearest whole number.

9 Mira mixes 24 buckets of sand with 6 buckets of cement to prepare a mortar mix.

(a) Write the ratio of sand to cement in simplest form.

(b) What fraction of her mortar mixture is cement?

10 Ryan uses a mixture of petrol and oil to power his go-kart. 2% of the mixture is oil, the rest is petrol.

(a) Write the ratio of oil to petrol in simplest form.

(b) What fraction of the mixture is oil?

Reasoning

11 In his Art lesson, Milan made pink paint by mixing 4 cups of red paint with 4 cups of white. His classmate Ali mixed 6 cups of red with 3 cups of white.

(a) Write the ratio, in simplest form, of red paint to white paint for:

(i) Milan **(ii)** Ali

(b) Whose mixture would have produced a paler shade of pink? Explain your answer.

12 Jo is making cordial. She follows the instructions on the side of the bottle and mixes 100 mL of cordial concentrate with 400 mL of water.

(a) What percentage of the total mixture is concentrate?

(b) Jo tastes it and finds the cordial is not strong enough. She adds another 50 mL of concentrate. What fraction of the resulting mixture is concentrate?

Open-ended

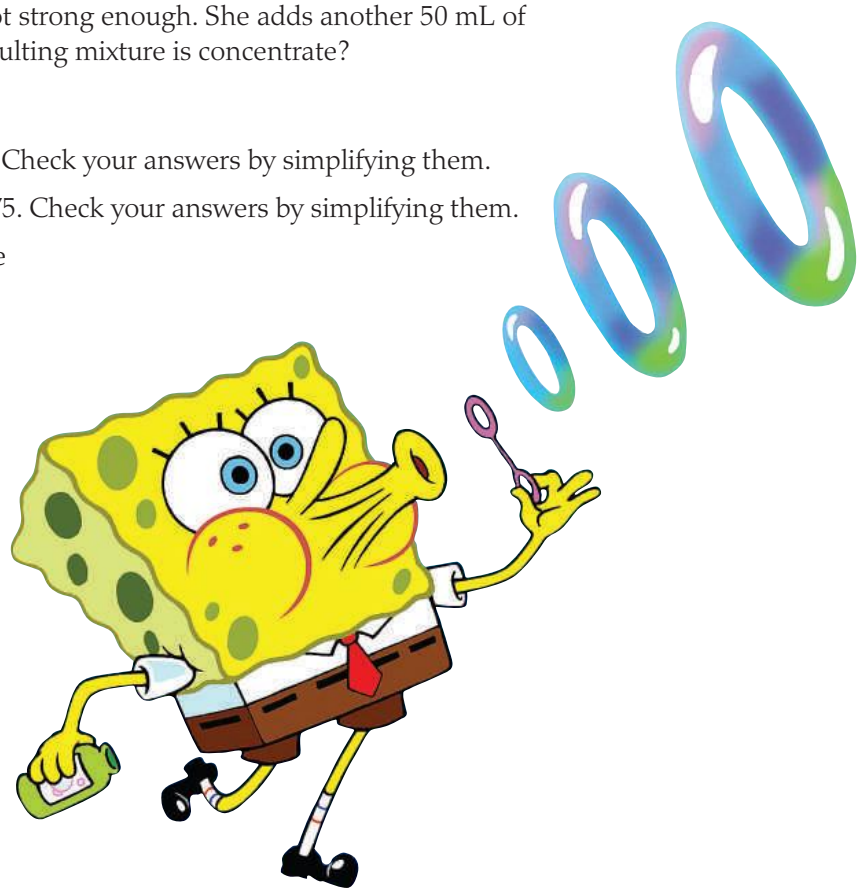
13 **(a)** Write three ratios equivalent to 5 : 2. Check your answers by simplifying them.

(b) Write three ratios equivalent to 50 : 75. Check your answers by simplifying them.

14 A group of primary school students were asked to name their favourite cartoon. The results were:

The Simpsons	3
Dora the Explorer	5
Scooby Doo	7
SpongeBob SquarePants	8

Use these results to write two fractions and two ratios that give some information about the group of students.



Outside the Square Game

The Tziansitzi stones

Equipment required: 2 brains, a heap of counters or stones

Tziansitzi is an ancient Chinese game that means 'the choosing of the stones'.

How to play:
Make two randomly sized piles of stones.

When it is your turn, you can either take the same number of stones from *both* heaps, or a specific number of stones from *one* heap.

How to win:
The player who takes the last stone is the winner.

4.2 Simplifying ratios

Ratios and units

We use ratios to compare quantities of the same type, such as lengths, masses or numbers of students or animals.

When writing ratios, any units of the quantities being compared are not included. It is assumed that the units are the same. To write a ratio comparing quantities with different units, we must first convert one or both amounts to the same unit (the smaller unit is usually more convenient). Once the units are the same they can be left out.

Ratios should be written in simplest form, using whole numbers only.

Each quantity in the ratio must be expressed in the same units, which should then be left out.

If a ratio contains fractions or decimals, multiply all parts in the ratio by the same number to convert them to an equivalent whole number ratio.

Worked Example 2 WE2

Write the ratio 25 m : 1500 cm in simplest form.

Thinking	Working
1 Are the units the same? If not, write both quantities in the smaller unit.	25 m : 1500 cm = 2500 cm : 1500 cm
2 Remove the units.	= 2500 : 1500
3 Divide both numbers by the HCF (500).	= 5 : 3

Worked Example 3 WE3

Write the ratio $2\frac{1}{2} : 1 : 3$ in simplest form.

Thinking	Working
1 Write any mixed numbers as improper fractions.	$2\frac{1}{2} : 1 : 3$ $= \frac{5}{2} : 1 : 3$
2 Multiply by the denominator to turn the fraction into a whole number. Multiply the other numbers in the ratio by the denominator as well to maintain the ratio.	$= \frac{5}{2} \times 2 : 1 \times 2 : 3 \times 2$ $= 5 : 2 : 6$

Worked Example 4 WE4

Write the ratio 3.5 : 4.25 in simplest form.

Thinking	Working
1 Multiply both sides by 10. Keep multiplying by 10 until all parts in the ratio are whole numbers.	3.5 : 4.25 = 35 : 42.5 = 350 : 425
2 Simplify if possible by cancelling common factors. (In this case, both numbers have been divided by 5, then by 5 again.)	= 70 : 85 = 14 : 17

4.2 Simplifying ratios

Navigator

Q1 Column 1, Q2 Columns 1–3, Q3 Columns 1–3, Q4, Q5, Q6, Q7, Q8, Q9, Q13, Q16	Q1 Column 2, Q2 Columns 2–4, Q3 Columns 2–4, Q4, Q5, Q7, Q8, Q10, Q12, Q13, Q15, Q16, Q17	Q1 Column 2, Q2 Columns 3 & 4, Q3 Columns 3 & 4, Q5 Column 2, Q7, Q8, Q9, Q10, Q11, Q12, Q14, Q15, Q16, Q17

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Equipment required: Ruler for Question **16**

Fluency

1 Write each of the following ratios in simplest form. WE2

(a) 5 cm : 3 mm
(b) 75 cm : 1.5 m
(c) 5 km : 600 m : 300 m
(d) 8000 m : 5 km : 4.5 km
(e) 60 cents : \$2
(f) \$3.50 : 45 cents
(g) 650 g : 3.5 kg
(h) 6 L : 500 mL
(i) 45 seconds : 2 minutes
(j) 480 kg : 3 tonnes
(k) 8 months : 1 year
(l) 2.5 L : 2000 mL
(m) 8 hours : 180 minutes
(n) 10 minutes : 20 seconds
(o) 5400 g : 6 kg : 1.2 kg
(p) 4 days : 4 weeks : 1 fortnight
(q) 80 cm : 2.5 m : 20 cm
(r) \$19 : \$20 : 60 cents

2 Write each of the following ratios in simplest form. WE3

(a) $3\frac{1}{2} : 1$
(b) $4 : 3\frac{2}{3}$
(c) $4 : 2\frac{1}{3}$
(d) $5\frac{1}{8} : 2$
(e) $3\frac{1}{3} : 2$
(f) $4 : 3\frac{1}{5}$
(g) $5 : 2\frac{1}{4}$
(h) $7\frac{3}{4} : 6$
(i) $5\frac{1}{7} : 3$
(j) $3\frac{5}{6} : 9$
(k) $6\frac{1}{8} : 9$
(l) $6 : 6\frac{2}{3}$

WE4

3 Write each of the following ratios in simplest form.

(a) 4.7 : 3.1 (b) 1.7 : 5.2 (c) 2.9 : 3.4 (d) 4.5 : 7.2

(e) 0.75 : 1.55 (f) 2.04 : 2.52 (g) 2.82 : 3.09 (h) 3.96 : 2.61

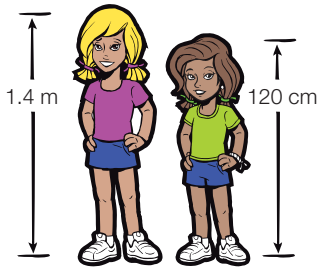
(i) 5.6 : 3.52 (j) 8.1 : 2.75 (k) 2.2 : 6.64 (l) 2.84 : 6.8

4 The simplest form of the ratio 6 : 33 is:

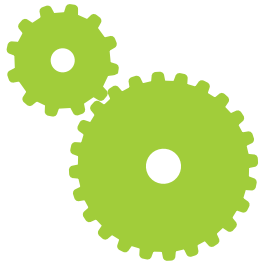
A 0.18 : 1 B 2 : 11 C 11 : 2 D $1 : 5\frac{1}{2}$

5 Express each of the stated ratios in simplest form.

(a) The ratio of the taller girl's height to the shorter girl's height.

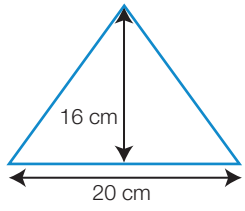


(b) The ratio of teeth on the small gear to the big gear.

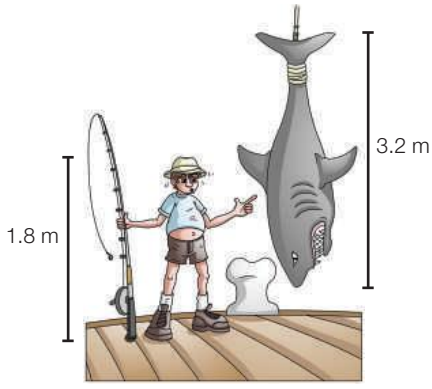


(c) The ratio of the larger can's mass to the smaller can's mass.

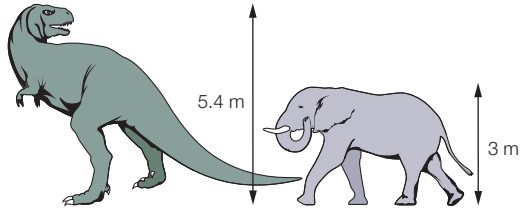
(d) The ratio of the triangle's height to its base length.



(e) The ratio of the shark's length to the person's height.



(f) The ratio of the dinosaur's height to the elephant's height.



Understanding

6 An unopened box of Megafibre breakfast cereal contains 1.3 kg of cereal. Another box that has been opened and used contains 450 grams.

(a) What is the ratio of the contents of the opened box to the contents of the unopened box? Write your answer in simplest form.

(b) Write the mass of the opened box as a fraction of the mass of the unopened box.

7 (a) A carpenter makes shelves in three different lengths: 45 cm, 75 cm and 1.05 m. Write a ratio comparing the lengths, from shortest to longest, in simplest form.

(b) Orange juice comes in three different-sized bottles of 250 mL, 600 mL and 1 L. Write the ratio comparing the bottle sizes, from smallest to largest, in simplest form.

8 Nancy is 57 years old. Her grandson Will is 11 months old. Nancy is 1.7 m tall, whereas Will is 89 cm tall.

(a) Write the ratio of Nancy's age to Will's age.

(b) Write Will's age as a fraction of his grandma's age.

(c) Write the ratio of Nancy's height to Will's height.

(d) Write Nancy's height as a percentage of Will's height.

9 Last year, Gloria grew 2 tonnes of peaches in her orchard, but her crop this year is down by 800 kg.

(a) Write the ratio of last year's crop to this year's crop, in simplest form.

(b) Write this year's crop as a fraction of last year's crop.

(c) Write the decrease in the mass of the crop this year as a percentage of last year's crop.

10 Tasso has three part-time jobs which he works at for 20 hours, 12.5 hours and 4.5 hours per week, respectively.

(a) Write a ratio to compare the amount of time he spends at his three jobs.

(b) What is the ratio of total hours worked to the hours at his first job?

(c) What percentage of his total hours does he spend at his second job? Write your answer to the nearest whole per cent.

11 Find the simplest ratio of two children's ages if they are:

(a) 14 and $10\frac{1}{2}$ years old

(b) 12 years 3 months and 7 years old.

12 Silvio's fruit salad contains $6\frac{3}{4}$ apples and $4\frac{1}{2}$ kiwi fruit. (He ate the missing fractions while he was making the fruit salad.) Silvio is delighted with the taste of the dessert, and wishes to determine the ratio of the fruits in terms of whole pieces of fruit. What is the ratio of apples to kiwi fruit in simplest form?

Reasoning

13 A fruit juice mixture contains 1.2 L of orange juice and 0.8 L of pineapple juice.

(a) What is the simplest ratio of orange to pineapple juice in the mixture?

(b) If 1 L of water is added to the mixture, what is the new ratio of orange juice to pineapple juice to water?

(c) What fraction of the final mixture is orange juice?

14 Pardeep is making a fruit punch for a party. She mixes 2.5 L juice with 1.25 L soda water and 750 mL ginger beer.

(a) Write the ratio for the three parts of the mixture, in simplest form.

(b) What fraction of the mixture is ginger beer? Write your answer in simplest form.

(c) Pardeep mixes the punch thoroughly, then divides it equally into 3 smaller containers. How much ginger beer is there in each container?

15 Troy completes a triathlon in 2 hours and 10 minutes. The triathlon consists of 3 'legs' (parts): a 1500 m swim, a 40 km ride, and a 10 km run. The swim and run take 20 minutes and 40 minutes, respectively.

(a) Write the ratio of swim distance : ride distance : run distance in simplest form.

(b) Write the ratio of swim time : ride time : run time in simplest form.

(c) Which 'leg' of the triathlon takes the longest time? Write the time taken for this leg as a percentage of the total time to the nearest whole number.

(d) Consider the two ratios you wrote in **(a)** and **(b)**. Explain why the ratios are not equivalent; that is, why the times to complete each leg are not in the same proportion as the distances of the legs.

Open-ended

16 Use a ruler to draw a rectangle that has a length 1.5 times as long as its width. Write the ratio length : width in simplest form. Compare your rectangle and ratio with that of a classmate. Is their rectangle the same size as yours? Is their ratio the same?

17 Troy wants to improve his time in the triathlon (see Question **15**). Suggest two ways he could allocate his time between the three sports when devising a monthly training program.

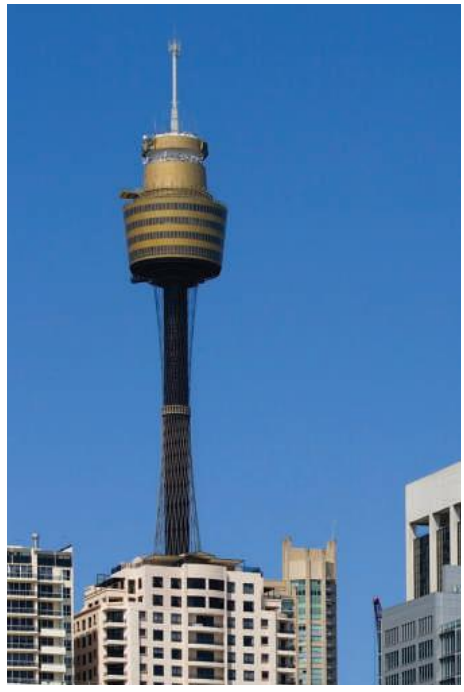
Unit ratios and scale factors

4.3

Discovery Task

Ratios that contain large numbers or numbers that cannot be simplified can be more challenging to understand and interpret. For example, consider the two buildings shown.

The Empire State Building is 443 m tall.



The Sydney Tower is 305 m tall.

We can write the ratio of their heights in simplest form as 443 : 305. This is correct, but it does not really give us a 'feel' for their relative sizes. The Empire State Building is taller, but how many times taller?

Writing the ratio of their heights as a **unit ratio** makes the comparison more meaningful.

To write a unit ratio, divide both numbers in the ratio by the second number.

Empire State Building : Sydney Tower
443 : 305
÷305 ÷305
1.45 : 1 (to two decimal places)

The unit ratio tells us that the Empire State Building is 1.45, or close to $1\frac{1}{2}$ times, the height of the Sydney Tower. Because 1.0 = 100%, we can also say that the Empire State Building's height is 145% of the height of the Sydney Tower or that its height is 45% taller than the height of the Sydney Tower.

1.45 is the **scale factor** we multiply the height of the Sydney Tower by to get the height of the Empire State Building:

305 m × 1.45 ≈ 442 m (allowing for a slight error due to rounding)

Dividing the height of the Empire State Building by 1.45 gives us the height of the Sydney Tower:

443 ÷ 1.45 ≈ 306 m (again, a slight rounding error)

- To write the ratio $a:b$ as a unit ratio, calculate $a \div b$ and express the ratio as $\frac{a}{b}:1$ (divide both of the numbers in the ratio by b).
- The quotient $\frac{a}{b}$ is known as the scale factor. It tells us the size of a in terms of how many times the size of b it is. It is the number we must multiply b by to get a.
- If a is greater than b ($a > b$), then $\frac{a}{b}$ is greater than 1, and more than 100%.
- If a is smaller than b ($a < b$), then $\frac{a}{b}$ is less than 1, and less than 100%.

Worked Example 5

WE 5

Write 34:19 as a unit ratio, rounded to two decimal places.

Thinking	Working
1 Divide both numbers in the ratio by the second number (19 in this case).	$34:19$ $= \frac{34}{19}:\frac{19}{19}$
2 Write your answer, rounding to the specified number of decimal places.	$= 1.79:1$ (to 2 d.p.)

Using the scale factor to find an unknown quantity

If we have a unit ratio or a scale factor that compares two quantities, we can use it to determine one of the quantities if we know the other one. The following Worked Examples show different types of scale factor questions and different methods of using the scale factor to find an unknown quantity.

Worked Example 6

WE 6

The ratio of the height of an elephant to the height of a lion is 3.07 : 1. If the elephant's height is 4 m, what is the height of the lion? Write your answer correct to one decimal place.

Method 1: Use the scale factor

Thinking	Working
1 Write the unit ratio, first in words, then in numbers underneath.	height of elephant : height of lion $= 3.07:1$
2 We know the quantity on the left side of the ratio (the height of the elephant). To find the quantity on the right side of the ratio, divide by the scale factor. Round your answer to one decimal place.	$4 \div 3.07$ $= 1.3$ m (to 1 d.p.)
3 Write your answer in words.	The lion is 1.3 m tall.

Method 2: Write and solve an equation

Thinking	Working
1 Write the ratio in words with the unit ratio underneath. Write a ratio that compares the known quantity (the height of the elephant) with the unknown. Use a pronumeral to represent the unknown. (Here, L represents the height of the lion.)	elephant's height : lion's height $= 3.07:1$ $= 4:L$
2 Write the ratio in fraction form with the unknown, the height of the lion (L), as the numerator.	$\frac{\text{lion's height}}{\text{elephant's height}} = \frac{L}{4}$
3 Form an equation by writing the unit ratio as an equivalent fraction on the other side of an equals sign.	$\frac{L}{4} = \frac{1}{3.07}$
4 Solve the equation. (Here, we multiply both sides by 4 to solve for L.) Round your answer to one decimal place.	$L = \frac{1}{3.07} \times 4$ $= 1.3$ (to 1 d.p.)
5 Write your answer in words.	The lion is 1.3 m tall.

Worked Example 7

WE7

In a recipe for pancakes there is four times as much flour as sugar and 1.6 times as much flour as milk.

(a) If we use 3 cups of sugar how much flour is required?

(b) If we use 2 cups of flour how much milk is required?

Method 1: Use a unit ratio

Thinking	Working
(a) 1 Write a unit ratio, first in words, then in numbers underneath.	(a) flour : sugar $= 4:1$
2 We know the quantity on the right side of the ratio (the cups of sugar). To find the quantity on the left side of the ratio, multiply by the scale factor.	3 cups of sugar × 4 = 12 cups of flour
(b) 1 Write a unit ratio, first in words, then in numbers underneath.	(b) flour : milk $= 1.6:1$
2 We know the quantity on the left side of the ratio (the cups of flour). To find the quantity on the right side of the ratio, divide by the scale factor.	2 cups of flour ÷ 1.6 = 1.25 cups of milk

Method 2: Write and solve an equation

Thinking	Working
(a) 1 Assign pronumerals to the two quantities.	(a) Let f = number of cups of flour Let s = number of cups of sugar
2 Use the given scale factor to write a formula that shows the relationship between these two quantities.	$f = 4s$
3 Substitute the known value (the amount of sugar) into the formula and evaluate.	$f = 4 \times 3$ $f = 12$
4 Write the answer in words.	12 cups of flour are needed.
(b) 1 Assign pronumerals to the two quantities.	(b) Let f = number of cups of flour Let m = number of cups of milk
2 Write a formula that shows the relationship between these two quantities.	$f = 1.6m$
3 Substitute the known value (the amount of flour) into the formula.	$2 = 1.6 \times m$
4 Solve the equation. (Here, we solve for m by dividing both sides by 1.6.)	$m = \frac{2}{1.6}$ $m = 1.25$
5 Write the answer in words.	1.25 cups of milk are needed.

Worked Example 8

WE 8

A stegosaurus was thought to weigh about 5 tonnes and yet its brain weighed only 50 g. How many times heavier is the stegosaurus than its brain?

Thinking	Working
1 Write the ratio and express both quantities in the same unit (grams).	5 tonnes : 50 g = 5 000 000 g : 50 g
2 Convert to a unit ratio by dividing both numbers in the ratio by the second number (50) and leaving out the units.	$= \frac{5\,000\,000}{50} : \frac{50}{50}$ = 100 000 : 1
3 Write your answer in words.	The stegosaurus is 100 000 times heavier than its brain.

4.3 Unit ratios and scale factors

Navigator

Q1 Columns 1 & 2, Q2, Q3, Q4, Q5, Q6, Q9, Q10, Q11, Q13, Q14, Q15, Q17	Q1 Columns 2 & 3, Q2, Q3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q16, Q17	Q1 Column 3, Q2 (b) & (d), Q3 (b) & (d), Q4 (b) & (d), Q6, Q7, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17

Answers page 612

Equipment required: Calculator

Fluency

1 Write each of the following as a unit ratio, rounded to two decimal places. WE5

(a) 17 : 16	**(b)** 733 : 525	**(c)** 766 : 329
(d) 29 : 37	**(e)** 1087 : 5230	**(f)** 3078 : 2100
(g) 65.5 : 13.4	**(h)** 4.25 : 0.8	**(i)** 3.56 : 5
(j) 6.25 : 8.33	**(k)** 20.375 : 18.116	**(l)** 651.9 : 300.7

2 **(a)** The ratio of Thanh-Duy's height to Hamish's height is 1.16 : 1. If Hamish has a height of 147 cm, what is Thanh-Duy's height? Write your answer correct to one decimal place. WE6

(b) The ratio of the value of the Australian dollar to the British pound is 2.38 : 1. How many Australian dollars will Patrick get for 250 pounds?

(c) The times taken for the planets Venus and Mercury to revolve around the Sun are in the ratio 2.55 : 1. If Venus takes 224.65 days to revolve around the Sun, how many days does Mercury take?

(d) At Callum's Pizza Shop the unit ratio of ham to pineapple on a Hawaiian pizza is 3.5 : 1. If 200 g of ham is used on a large pizza, how much pineapple is used?

3 **(a)** Megan's recipe for concrete requires 3.7 kg of cement for every litre of water. WE7

(i) If Megan uses 10 kg of cement how much water is required?

(ii) If Megan uses 6 litres of water how much cement is required?

(b) There are 11.5 sheep for every person in New Zealand. If the population of New Zealand is 4.2 million, how many sheep are there?

(c) Miriam saves three times as much money as Shannon does. If Miriam saves $150 how much does Shannon save?

(d) The circumference (perimeter) of a circle is approximately 3.14 times the length of the diameter (the distance across the circle through the centre). What is the circumference of a circle whose diameter is 14.5 cm?

WE8

4 Round your answers to the following questions to two decimal places.

(a) An adult female elephant weighs 3400 kg. A baby elephant weighs 160 kg. How many times heavier is the adult elephant than the baby?

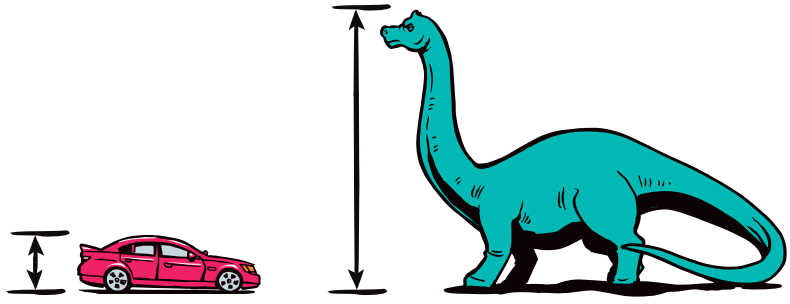
(b) The populations of Australia and Indonesia are shown below.

Australia: 22 million people Indonesia: 238 million people

How many times greater than the population of Australia is the population of Indonesia?

(c) A Formula One car has a top speed of 415 kilometres per hour (km/h). A family sedan has a top speed of 230 km/h. How many times faster is the top speed of the Formula One car than the family sedan?

(d) Make the measurements indicated on the picture below. How many times taller than the model car is the model dinosaur?



Understanding

5 The two longest rivers in the world are the Nile and the Amazon. The Nile is 1.034 times as long as the Amazon. If the Amazon is 6448 km long, how long is the Nile? Give your answer to the nearest km.

6 The diameters of Jupiter and Earth are in the ratio 20.93 : 1. If Earth has a diameter of 12 760 km:

(a) what is the diameter of Jupiter to the nearest km?

(b) what diameter should a model of Jupiter have so it is in the correct proportion to a tennis-ball sized (6.36 cm diameter) model of the Earth?

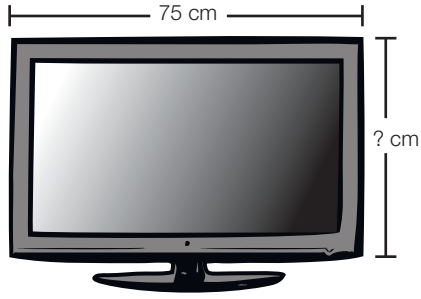
7 Lawnmower fuel is a mixture of petrol and oil. 25 litres of petrol are needed for every litre of oil in the mixture. Dion has 8 litres of petrol. How much oil does he need to add?

8 (a) Express the ratio of the heights of Mt Kosciuszko (2228 m) to Mt Everest (8848 m) as a unit ratio. Round your answer to two decimal places.

(b) How many times taller than Mt Kosciuszko is Mt Everest?

9 The ratio of the top speeds of the gazelle and the cheetah is 0.724 : 1. If a gazelle can run at a top speed of 76 kilometres per hour, how fast can a cheetah run? Give your answer to the nearest km/h.

10 A plasma screen TV has an 'aspect ratio' of 16 : 9. The aspect ratio is the ratio of the screen's length to its width.



(a) Write the aspect ratio as a unit ratio.

(b) How tall is a TV screen that is 75 cm long?

11 Copy and complete the following table.

Quantities	Ratio	Unit ratio	Scale factor	Explanation
$a : b$	4 : 5	0.8 : 1	0.8	a is 0.8 times the size of b
$x : y$	7 : 1	7 : 1		x is seven times as large as y
$m : p$	12 : 4			
	240 : 12			h is one-twentieth the size of g
	3.8 : 15.2			r is one-quarter the size of j
$d : e$	4000 : 20			

Reasoning

12 The system of measurement used before the metric system was called the imperial system. The scale factors for converting some of the units of length are as follows.

Conversion	Scale factor
feet to inches	$\times 12$
feet to yards	$\times \frac{1}{3}$
miles to yards	$\times 1760$

(a) How many inches are in 3 feet?

(b) How many feet are in 10 yards?

(c) How many inches are in 6 feet?

(d) How many yards are in 3 miles?

(e) How many miles are in 10 560 feet?

(f) What would I do to convert from yards to inches?

(g) What would I do to convert from miles to feet?

13 A travel agent adds 3.5% to the cash price of a plane ticket if it is paid for with a credit card.

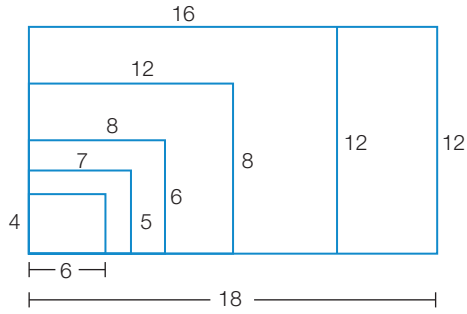
(a) Write the unit ratio for the ratio of the credit card price to the cash price.

(b) Meena paid \$361.20 for a ticket with her credit card. What would she have paid if she had paid in cash? (Answer to the nearest cent.)

14 The scale factor between the height of a 1.75 m tall father and the height of his 12-year-old son is 1.08. Find the two possible heights for the son and explain how you arrived at each. Which do you think is more likely to be the correct height?

15 The aspect ratio of an image produced by a camera is the ratio of the images's length to its width. Images taken with most digital cameras have an aspect ratio of 3:2. The paper used to print photos comes in the following range of sizes (the old unit of measurement, the inch, is still used to describe paper sizes). The relative sizes of the paper are shown in the diagram below.

Length (inches)	Width (inches)
6	4
7	5
8	6
12	8
16	12
18	12



(a) Copy the table and add two extra columns, one for the ratio $L:W$ and one for the unit ratio $\frac{L}{W}:1$. Complete both columns for each paper size.

(b) Use your table to decide which paper sizes provide the best 'fit' for an image that has an aspect ratio of 3:2? Explain your choice.

(c) Imagine that you work in a photographic printing business. A customer has brought in some images with an aspect ratio of 3:2 and asked for them to be printed on '7 by 5' paper. You sit down at your computer and start with an image in the smallest standard size of 6 by 4. To increase the width of the print from 4 to 5, you find that you must multiply by a scale factor of 1.25 ($4 \times 1.25 = 5$).

(i) If you increase the length of the standard sized image by the same scale factor, what is the new length of the image?

(ii) What is the problem with this new image length?

(d) Describe a potential solution to this problem.

Open-ended

16 The following table gives the approximate unit ratio of human life expectancy to that of some common pets (i.e. human life expectancy : animal's life expectancy).

Animal	Unit ratio
Dog	5.7:1
Cat	6.3:1
Rabbit	11.7:1
Budgie	7.6:1
Horse	3.8:1
Guinea-pig	16.9:1
Blue-tongue lizard	3.6:1
Mouse	25:1

These ratios can be used to calculate how old an animal is in 'human years'.

(a) Calculate the ages in human years of any pets you have (if you know how old they are).

(b) Use your knowledge of ratios and an average human life expectancy of 76 years to calculate the life expectancies to the nearest year of the animals listed in the previous table. Present your answers in a table.

(c) What does it mean if a particular animal has a high value for this unit ratio?

17 Brett has a space on his lounge room wall 1.7 m long and 1.2 m wide to hang a new plasma screen TV.

The 'aspect ratio' of a TV screen is the ratio of its length to its width. The two most common screen aspect ratios are 16 : 9 and 4 : 3.

(a) Find three sets of screen dimensions (length and width) in the ratio 4 : 3 that would fit in the wall space.

(b) Find three sets of screen dimensions (length and width) in the ratio 16 : 9 that would fit in the wall space.

(c) Which aspect ratio would be best for Brett's space? Explain your reasoning.

(d) Use your answer to **(c)** to find the dimensions of the screen that takes up the greatest amount of wall space.

Outside the Square — Problem solving

Cartoon capers

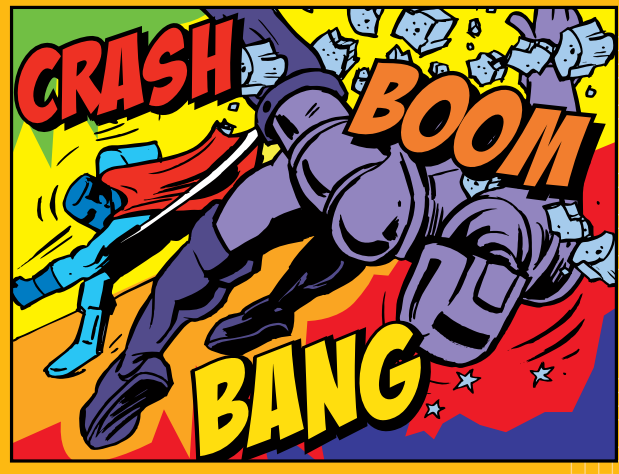
Mario has one scene from a cartoon strip that he wants to enlarge by a scale factor of 10. The cartoon frame is a rectangle 10 cm long and 7.5 cm high.

The poster paper is also rectangular, measuring 1.2 m long and 84 cm high.

1 What area of poster paper will be left over after Mario has made his enlargement?

2 **(a)** What scale factor should Mario use so that he uses the biggest area of poster paper possible (without losing any of the image)?

(b) What area of poster paper will be left over if Mario uses this new scale factor to enlarge the cartoon?



Strategy options

- Draw a diagram.
- Break problem into manageable parts.

Investigation

Bicycle gears

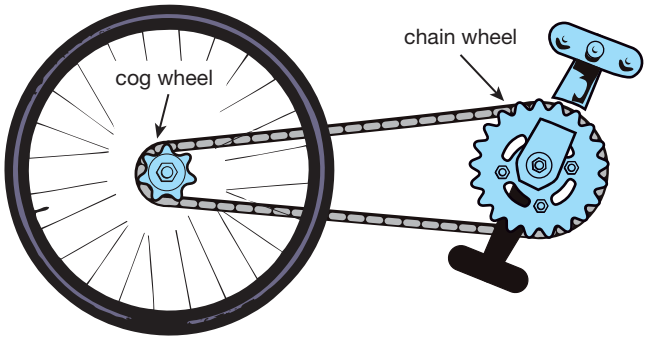
Equipment required: 1 brain, 1 bike with gears (optional)

The Big Question

How many bike gears are necessary to compete in the Tour de France?

Engage

Single-speed, or fixed wheel bicycles have a single chain wheel (sometimes called the front sprocket) attached to the pedals and a rear cogwheel (sometimes called the rear sprocket) attached to the rear bike wheel. The chain wheel is connected to the cogwheel by a chain.



Multi-speed bicycles often have two or more chain wheels that can combine with any of several differently sized rear cogwheels. These different combinations are called gears. A bicycle with three front chain wheels and six rear cogwheels would have 18 different gear combinations or speeds.

1 How many gears would a bicycle have if it has two front chain wheels and five rear cogwheels?

2 How many rear cogwheels would a 24-speed bicycle have if it has three chain wheels?

3 How many chain wheels would a 21-speed bicycle have if it has seven rear cogwheels?

The 'gear ratio' is found by counting the number of teeth on both the chain wheel and the cogwheel and calculating the unit ratio. For example, the chain wheel in the diagram on this page has 21 teeth and the rear cogwheel has 7 teeth.

So, chain wheel : cog wheel
= 21 : 7
Gear ratio = 3 : 1

When the pedals are turned, the smaller cogwheel will need to rotate faster than the chain wheel. For every rotation of the chain wheel, the rear wheel will rotate three times.

Although the back wheel turns three times faster, the effort required to make it turn is also three times greater. Lower gear ratios (often achieved by selecting a smaller chain wheel) require less effort to turn the pedals; however, the pedals must be turned more times to travel the same distance.

The gear specifications for a mountain bike are shown in the following table.

Front chain wheel	Rear cogwheel
Number of teeth	Number of teeth
48	28
38	24
28	21
	18
	16
	14

4 (a) How many gear combinations are possible for the mountain bike?

(b) Find the gear ratios for each combination and order them from largest to smallest. Are any very close to each other and could therefore be considered the same?

Explore

The Tour de France is a cycling endurance race held over a 3-week period around France and other parts of Europe. The bike that seven-time winner Lance Armstrong rode had the following gear arrangement: two front chain wheels with 39 and 53 teeth, respectively, and 10 rear cogwheels with 11, 12, 13, 14, 15, 16, 17, 18, 19 and 21 teeth.

5 **(a)** Calculate the gear ratio for every gear combination.

(b) How many different gears did Lance have to choose from?

(c) Does your answer to **(b)** depend on your definition of 'different'?

Strategy options

- Make a table.
- Test all possible combinations.
- Break problem into manageable parts.

Explain

6 Explain how you decided whether two gear ratios were different or the same.

7 Which gears would Lance use when going up a hill? Why? Which gear would Lance choose when travelling at top speed downhill? Why?

Elaborate

8 Why would Lance ride a bike with some gears that have almost the same ratio? (Hint: When changing gears, separate changes must be made for the front chain wheel and the rear cogwheel as they cannot be changed at the same time.)

9 Use your answers from Questions **5**, **6** and **8** to answer the Big Question.

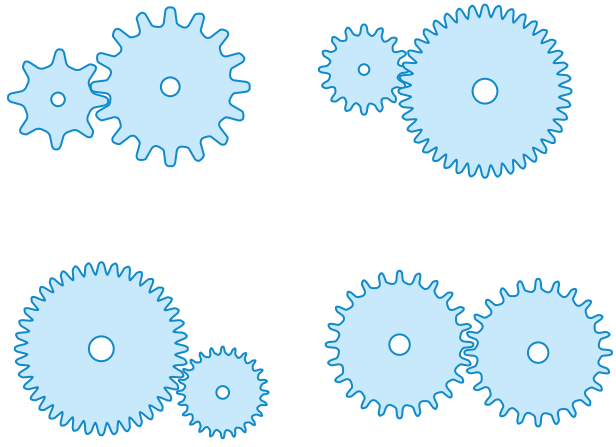
Evaluate

10 Consider how you went about answering the Big Question. Which strategies did you find useful?

11 'When solving problems, mathematicians not only need to define the problem, but also need to define the solution.' How does this statement apply to this investigation?

Extend

12 The letters rpm stand for 'revolutions per minute'. If the first gear is spinning at 12 rpm, use gear ratios to find the speed of the second cog for each of the following.



13 If you wanted to create a gear ratio of exactly 3.5 : 1, list three possibilities for the number of teeth on each cog. Which combination would you choose to put on a bicycle? Why?

14 Research the gears used by competition road cyclists and track riders. Can you explain why they use different gear ratios?

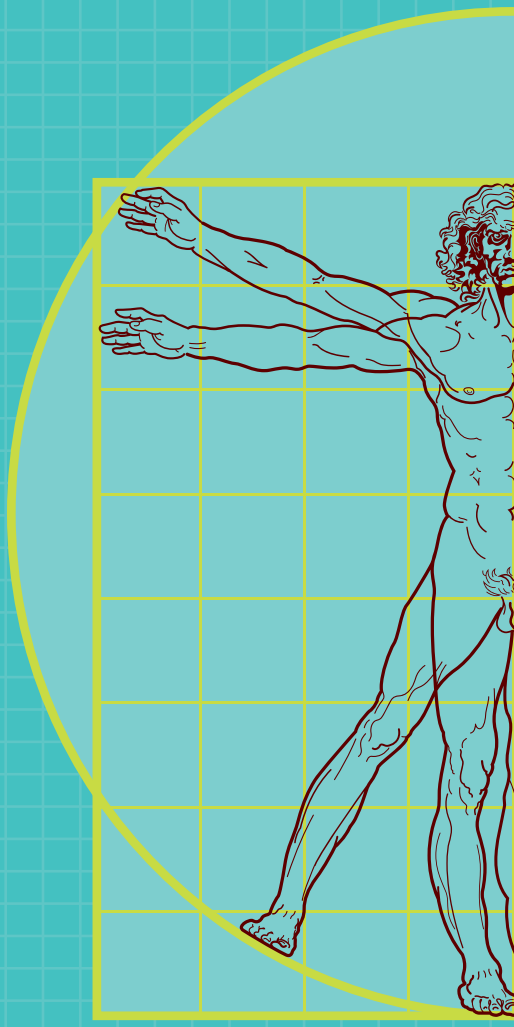
15 Research how the gears in a car work. Why do some cars have 3 forward gears, some 4 and others 5? How does reverse gear work? Why is there only one reverse gear?

Mathspace

Human ratios

Equipment required: 1–2 brains, a ruler and string or a tape measure

Throughout history, humans have had some interesting ideas about what makes a correctly proportioned, 'beautiful' face or body. Check out these so-called facts for yourself.



Gulliver's Travels

In Jonathan Swift's classic story *Gulliver's Travels*, Lemuel Gulliver is shipwrecked and swims to the island of Lilliput, where the people are an average height of 14 cm. As Gulliver's only clothes were those he was wearing, the Lilliputians had to make new clothing for him. They took only two measurements – the distance from his neck to knee and then the distance around his thumb. From this they calculated his shape using these ratios.

"*twice round the thumb is once around the wrist, twice around the wrist is once around the neck and twice around the neck is once around the waist.*"

1 Write three ratios from the sentence above.

2 Check yourself and some of your classmates to see if this works.

Vitruvian Man

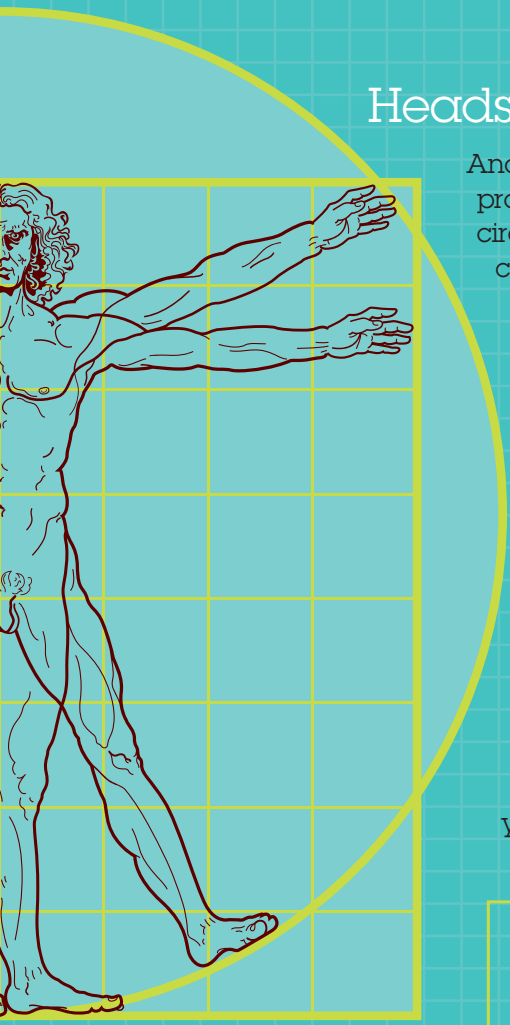
Many people believe Leonardo Da Vinci used a number of body ratios in his famous drawing *Vitruvian Man*, shown on this page.

- The maximum width of a person's shoulders is always a quarter of their height.
- The length of a person's outstretched arms is equal to their height.
- The length of a person's hand, from wrist to the end of their longest finger, is one-tenth of their height.

3 Write these sentences as ratios.

4 Which of these ratios do you think is likely to be true?

5 Test them on yourselves and your classmates.



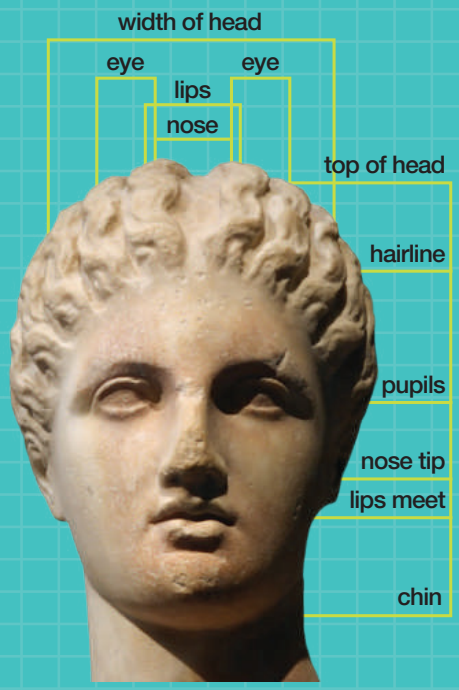
Heads up

Another interesting body proportion is that of the circumference of a person's head compared to their height. The ratio is supposed to be 1 : 3. Test this out using a tape measure or some string and a ruler.

Seven up

The ancient Greeks had high ideals about what the 'perfect' human was like.

They believed that the 'perfect' human body was 7 heads tall. Find out if you are 'perfect' by measuring from your chin to the top of your head, and comparing to your height.



The perfect Greek face

The ancient Greeks believed the perfect face was in proportion to the 'golden ratio' of 1.618. (See the Maths4Real section later to investigate this further.)

6 Using your ruler and the Greek face pictured, find each measurement below to the nearest millimetre. Remember, you are measuring the distance between the two locations. Write your measurements alongside their respective letters in the table below.

a = top-of-head to chin
b = top-of-head to pupil
c = pupil to nose tip
d = pupil to lip
e = width of nose
f = outside distance between eyes
g = width of head
h = hairline to pupil
i = nose tip to chin
j = lips to chin
k = length of lips
l = nose tip to lips

7 Now, use the measurements calculated in Question **1** to complete the following table. Remember: $\frac{a}{g}$ means calculate measurement a divided by measurement g (round to 3 d.p.).

ratio	value
$\frac{a}{g}$	
$\frac{b}{g}$	
$\frac{i}{j}$	
$\frac{i}{c}$	
$\frac{e}{l}$	
$\frac{f}{h}$	
$\frac{k}{e}$	

Are your ratios in the table the same as the golden ratio of 1.618?

What can you conclude about facial proportions? Is the ancient Greek idea of the 'perfect face' realistic?

4.4 Using ratios to find amounts

A painter is mixing two different coloured paints to get a desired shade. If the desired shade requires red paint to be mixed with yellow paint in the ratio 3 : 2 and the painter used 21 litres of red paint in the mixture, how would we calculate the volume of yellow paint required? If we know the ratio and one of the amounts, the other amount can be calculated.

One way is simply to list multiples of the given ratio (equivalent ratios) until 21 appears in the 'red paint' position in the ratio:

red : yellow
3 : 2
6 : 4
9 : 6
12 : 8
15 : 10
18 : 12
21 : 14

So, 14 litres of yellow paint are required.

A more efficient method is to use a pronumeral, such as 'n', to represent the unknown amount, and use the other numbers in the ratios to find the value of n. This is shown in the following Worked Example.

Worked Example 9 WE9

Find the value of the unknown, n, if $n : 24 = 9 : 4$.

Thinking	Working
1 Write the ratio with the unknown value underneath the ratio with the two known values.	$9:4$ $= n:24$
2 Consider the side of the ratios in which both values are known (the right-hand side). Use these values to find the multiplier that turns the first ratio into the second. (Here, we multiply 4 by 6 to get 24.)	$\times 6$ $9:4$ $\times 6$ $n:24$
3 Because the ratios are equivalent, multiplying the known value on the other side of the ratio (9) by the same multiplier (6) gives the value of the unknown (n).	$n = 9 \times 6$
4 Write the answer.	$n = 54$

Worked Example 10

WE10

A chemist needs to make a batch of medicine by mixing a chemical with water in the ratio 3 : 7. If the chemist begins with 600 mL of the chemical, how much water must be added?

Thinking	Working
1 Write the ratio in words, then the two numerical ratios underneath each other. Use a pronumeral to represent the unknown value.	chemical : water = 3 : 7 = 600 : w (where w = volume of water)
2 Consider the side of the ratios in which both values are known (the left-hand side). Use these values to find the multiplier that turns the first ratio into the second. (Here, we multiply 3 by 200 to get 600.)	$3:7$ $\times 200$ $\times 200$ $600:w$
3 Because the ratios are equivalent, multiplying the known value on the other side of the ratio (7) by the same multiplier (200) gives the value of the unknown (w).	$w = 7 \times 200$ $w = 1400$
4 Write the answer in words.	1400 mL of water must be added.

In the previous two Worked Examples, the number that we multiplied the first ratio by to get the second (the 'multiplier') was an easily identified whole number. In some ratios, this number is not so easily found. Methods for finding the unknown quantity in these situations are shown in the following Worked Examples.

Worked Example 11

WE11

Find the value of b if $3 : b = 5 : 7$.

Method 1: Use an intermediate unit ratio

Thinking	Working
1 Write the ratio containing the unknown value underneath the ratio with the two known values.	$5:7$ $= 3:b$
2 Divide both numbers in the top ratio by the number that is above the known value in the second ratio (in this case 5, which is above 3). This gives a unit ratio. Show the division using arrows.	$5:7$ $\div 5$ $\div 5$ $1:\frac{7}{5}$
3 Now, multiply the unit ratio to obtain the known amount in the second ratio. (Here, we multiply by 3.)	$\times 3$ $\times 3$ $3:\frac{7}{5} \times 3$ $3:b$
4 Use the sequence of operations used to turn the first ratio into the second (shown here in red) to find b.	$b = 7 \div 5 \times 3$ $b = 4.2$

Method 2: Use fractions to solve an equation

Thinking	Working
1 Write each ratio as a fraction, ensuring that the pronumeral is a numerator. (This means that the second number in the other ratio, 7, is also a numerator.) Because the ratios are equivalent, the fractions are also equivalent.	$5:7$ $=3:b$ $\frac{b}{3}=\frac{7}{5}$
2 Solve the equation for b. (Here, we multiply both sides by 3.)	$b=\frac{7}{5}\times 3$
3 Write the answer.	$b=4.2$

Worked Example 12

WE12

The ratio of Uppma's height to Brett's height is 9:10. If Brett is 152 cm tall, use a calculator to find Uppma's height. Write your answer to the nearest centimetre.

Method 1: Use an intermediate unit ratio

Thinking	Working
1 Write the ratio in words, then the numerical ratios underneath each other. Use a pronumeral to represent the unknown value. (Here, we use h to represent Uppma's height.)	Uppma : Brett $=$ 9 : 10 $=$ h : 152 (where h = Uppma's height)
2 Divide both numbers in the top ratio by the number that is sitting above the known number in the second ratio (in this case 10, which is above 152). This gives a unit ratio. Show the division using arrows.	$9:10$ $\div 10$ $\div 10$ $\frac{9}{10}:1$
3 Now, multiply the unit ratio to obtain the known amount in the second ratio. (Here, we multiply by 152.)	$\frac{9}{10}:1$ $\times 152$ $\times 152$ $\frac{9}{10}\times 152$
4 Use the sequence of operations used to turn the first ratio into the second (shown here in red) to find h.	$h=9\div 10\times 152$ $h=137$ cm
5 Write your answer in words.	Uppma is 137 cm tall.

Method 2: Use fractions to solve an equation

Thinking	Working
1 Write the ratio in words, then the numerical ratios underneath each other. Use a pronumeral to represent the unknown value. (Here, we use h to represent Uppma's height.)	Uppma : Brett = 9 : 10 = h : 152 (where h = Uppma's height)
2 Write each ratio as a fraction, ensuring that the pronumeral and the corresponding number in the other ratio (h and 9 in this case) are both numerators. Because the ratios are equivalent, the fractions are also equivalent.	$\frac{h}{152} = \frac{9}{10}$
3 Solve the equation for h. (Here, we multiply both sides of the equation by 152.)	$h = \frac{9}{10} \times 152$ $= 137$
4 Write your answer in words.	Uppma is 137 cm tall.

4.4 Using ratios to find amounts

Navigator

Q1 Columns 1 & 2, Q2, Q3 Columns 1 & 2, Q4 (b) & (c), Q5, Q7, Q8, Q10, Q13, Q15	Q1 Columns 2 & 3, Q2, Q3 Columns 2 & 3, Q4, Q6, Q7, Q8, Q9, Q10, Q13, Q14, Q15	Q1 Column 3, Q2 (c) & (d), Q3 Column 3, Q4, Q6, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q15, Q16

Answers page 613

Equipment required: Calculator for Questions **3**, **4** and **6–16**

Fluency

1 Find the value of each unknown:

WE 9

(a) $21 : g = 7 : 3$ **(b)** $8 : h = 1 : 7$ **(c)** $i : 72 = 5 : 8$

(d) $a : 15 = 2 : 5$ **(e)** $b : 6 = 5 : 3$ **(f)** $c : 18 = 5 : 3$

(g) $13 : 2 = p : 18$ **(h)** $5 : 3 = u : 300$ **(i)** $4 : 7 = v : 700$

(j) $2 : 9 = 24 : r$ **(k)** $18 : 5 = 180 : t$ **(l)** $11 : 6 = 330 : w$

2 **(a)** Tacky-bond glue comes in two tubes containing Part A and Part B pastes. These have to be mixed in the ratio 1 : 4 for maximum strength. If Elio measures out 5 mL of Part A, how much Part B should he mix with it for maximum strength?

WE 10

(b) A training college handbook describes a course as having 3 practical lessons for every 8 theory lessons. If the course involves 24 theory lessons, how many lessons will be spent on practical work?

(c) The legal requirement for a particular outdoor activity involving students is that the teacher–student ratio be 1:6. If 96 students are to participate, how many teachers should be present at the activity?

(d) A thick-shake is made of milk and ice-cream in the ratio 5 : 2, and contains 400 mL of milk. Find the amount of ice-cream in the shake.

WE11

3 Find the value of each unknown.

(a) $a : 44 = 3 : 8$ **(b)** $c : 32 = 2 : 5$ **(c)** $b : 20 = 2 : 9$

(d) $i : 10 = 11 : 4$ **(e)** $h : 12 = 9 : 8$ **(f)** $g : 30 = 7 : 4$

(g) $6 : x = 10 : 1$ **(h)** $2 : x = 5 : 11$ **(i)** $3 : x = 2 : 7$

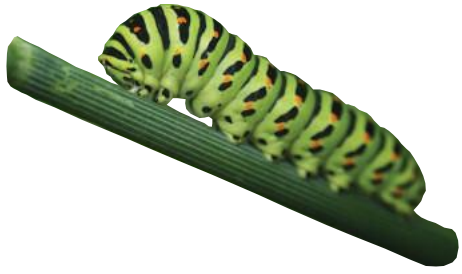
(j) $2 : x = 8 : 13$ **(k)** $4 : x = 8 : 15$ **(l)** $4 : x = 5 : 9$

WE12

4 (a) The ratio of an average African elephant's height to the prehistoric mammoth's height is 5 : 7. If an elephant's average height is 321 cm, find the height of the mammoth. Write your answer to the nearest centimetre.

(b) When correctly proportioned, the Australian flag has a width to length ratio of 10 : 19. For a project, Harley wishes to make an Australian flag that is 80 cm long. How wide should Harley's flag be, to the nearest centimetre?

(c) An entomologist found that the ratio of caterpillars to moths on a particular tree was 6 : 5. If there were 35 moths, how many caterpillars were there?



(d) Sunbeam Island Resort advertises that the daylight 'sun to cloud' ratio for the island is 20 : 3. If the average number of hours of sun is 12 during summer, what is the average number of hours of cloudy conditions each day? Give your answer as **(i)** a decimal and **(ii)** in hours and minutes.

5 (a) If $a : 16 = 5 : 4$, then a equals:

A 1 B 4 C 16 D 20

(b) If $40 : 9 = 800 : b$, then b equals:

A 900 B 450 C 180 D 40

6 (a) If $x : 4 = 9 : 7$, then x equals:

A $\frac{9}{7} \times 4$ B $\frac{9}{7} \times \frac{1}{4}$ C $\frac{7}{9} \times 4$ D $\frac{7}{9} \times \frac{1}{4}$

(b) If $19 : y = 2 : 5$, then y equals:

A $\frac{2}{5} \times 19$ B $\frac{2}{5} \times \frac{1}{19}$ C $\frac{5}{2} \times 19$ D $\frac{5}{2} \times \frac{1}{19}$

Understanding

7 The ratio of the dimensions of the rug shown is 9 : 5. Find the length of the rug.

8 Amber and Yusuf each put in some money to buy a bag of lollies. Amber puts in 90c, and Yusuf puts in 50c. They weigh their shares, which are in the same ratio as the amount of money each contributed.

(a) If Yusuf gets 75 grams, what mass of lollies does Amber receive?

(b) What was the mass of the lolly bag?

9 Linda and Jean own 35% and 65% of a company, respectively. When yearly profits are divided in proportion to each owner's investment in the company, Linda receives $450 000.

(a) How much does Jean receive (to the nearest dollar)?

(b) What was the company's total profit?

10 Bob's 'recipe' for concrete is to mix cement, sand and gravel in the ratio 1 : 2 : 4. He has one 10 kg bag of gravel left, and wants to use it all to make concrete. How much cement and sand should he add?

11 A jam manufacturer sells jam in 250 g and 450 g jars. The prices for each jar are to be in the same ratio as the jar sizes. If the price of the larger jar is $3.69, what should be the price of the smaller jar?

Reasoning

12 Voters in the electorate of Wallyville had the choice of voting for the Action Party, the Barry Party, the Coalition for Saving the Whale Party and the Democratic Party. Election results showed the ratio of votes received was 5 : 1 : 2 : 6, respectively.

(a) If the Barry Party received 850 votes, how many votes did the Democratic Party receive?

(b) The Coalition for Saving the Whale Party received 1700 votes. Find the number of votes for the Action Party.

(c) What fraction of voters voted for the Barry Party?

(d) How many voters were there in the Wallyville election?

13 Jesse's local petrol station sells three types of petrol: e10 petrol (a mixture of petrol and ethanol), 91% octane petrol and 95% octane petrol. The prices were \$1.25, \$1.32 and \$1.38 per litre, respectively.

(a) Write the ratio of the three prices in simplest form.

(b) Assuming the ratio between the prices remains constant and e10 petrol goes up in price to \$1.30 per litre, find the cost of the other two types of petrol to the nearest cent.

(c) Assuming the ratio between the prices remains constant and 91% octane petrol drops in price to \$1.25 per litre, find the cost of the other two types of petrol to the nearest cent.

Open-ended

14 Classes 8A, 8B and 8C are going on a skiing trip. The maximum allowed ratio of students to skiing instructors is 9 : 1. Classes with fewer than 7 students cannot run. If there are up to 10 instructors available, suggest different group sizes of students able to take a skiiing lesson with no students missing out.

15 The ratio of Mary's height to her daughter Alexi's height is 3 : 2. List three pairs of reasonable heights that fit this ratio.

16 On an exam, the ratio of the marks allocated for multiple choice to short answer to long answer questions is 5 : 6 : 4.

(a) Give two different whole number examples of how many marks could be allocated to each section of the test.

(b) To make it easier to record the results as percentages, the teachers want the total marks to add up to 100 exactly. Is this possible? Explain your reasoning.

Outside the Square Puzzle

Sudoku

Equipment required: 1 brain, grid paper

Copy the grid and numbers.

Fill in the spaces with the digits 1 to 9, so that each row, column, and bold square contains the numbers 1–9 only once.

	6		7		2		3	
	7	2				9		
	8		9	3		1		7
5		8	1	9				
1			6		5			3
				4	7	5		2
7		9		8	4		5	
		6				2	4	
	3		5		1		8	

Half-time 4

1 Use the following diagram to write the ratio of: Ex 4.1

(a) dogs to people (b) legs to heads (c) leads to dogs.

2 Roberto has planted potatoes and carrots in his market garden and the area they take up is in the ratio of 2 : 5. If the potatoes take up 50 m^2, what area is devoted to the carrots? Ex 4.4

3 Write the following ratios in simplest form. Ex 4.2

(a) 28 : 32 (b) 180 mm : 120 mm (c) 10 kg : 350 g

(d) $2\frac{1}{2} : \frac{1}{3}$ (e) 2.5 : 0.45 (f) $3\frac{1}{4}$ years : 9 months

4 In a small business, Lucas owns 5 shares, Jenny 2 and Mia 3. If they are the only shareholders find: Ex 4.1

(a) the ratio of shares owned by Lucas to those owned by Jenny to those owned by Mia

(b) the fraction of all the shares owned by Jenny, in simplest form

(c) the combined percentage of the business owned by the two women.

5 Find the value of each unknown. Ex 4.4

(a) $d : 28 = 6 : 7$ (b) $27 : f = 9 : 2$ (c) $1 : 5 = 12 : q$ (d) $4 : 13 = 88 : t$

6 The scale factor of the life expectancy of Australian females to males is currently 1.06. If the life expectancy of the average male is 79 years, what is the life expectancy of the average female? Give your answer to the nearest year. Ex 4.3

7 The tallest building in the world is currently the Burj Khalifa in the United Arab Emirates. It stands at 818 m high and is 1.81 times as tall as a previous record holder of the world's tallest building, the Petronas Towers in Malaysia. How tall is the Petronas Towers? (Answer to the nearest metre.) Ex 4.3

8 Due to short supply, Pedro has had to raise the price of bananas in his fruit shop from \$2.75 kg to \$3.50 kg. Ex 4.1, 4.2

(a) Write the ratio of the old price to the new price, in simplest form.

(b) Write the new price as a percentage of the old price, to the nearest percentage.

(c) What is the percentage increase of the old price?

Maths 4 Real

The golden ratio – fact or fiction?

Throughout history, people have been interested in mysteries and patterns in nature. The number 1.618, known as the 'golden ratio' or the 'divine proportion' has fascinated humans for centuries. It is represented by the symbol ϕ, the Greek letter 'phi'. (Actually, 1.618 is a rounded approximation of ϕ. The actual value is $\frac{1+\sqrt{5}}{2}$, which is an irrational number with an infinite number of decimal places.)

In Dan Brown's top-selling novel *The Da Vinci Code*[1] he describes the following situations in which he states that the golden ratio appears.

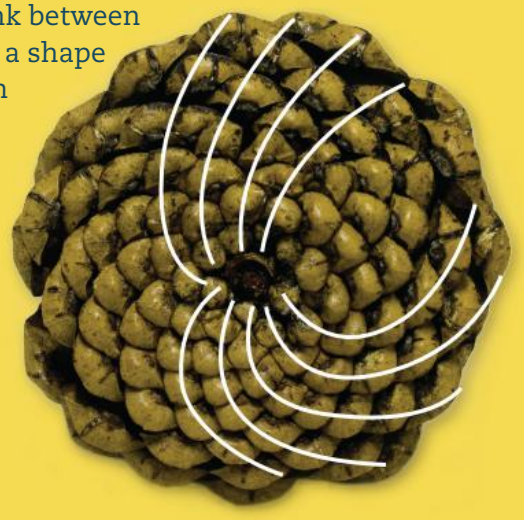
Statement	Fact or fiction?
Human proportions – 'Da Vinci actually exhumed corpses to measure the exact proportions of human bone structure. He was the first to show that the human body is literally made up of building blocks whose proportional ratios always equal phi.' For example, the ratio of a person's height to the distance from their navel to the ground is 1.618.	✓ Fact: Da Vinci did exhume bodies and measure bones as part of his studies of human anatomy. He was taught maths by Pacioli, a mathematician who was fascinated by the golden ratio. Da Vinci illustrated some of Pacioli's work. ✗ Fiction: There is no evidence that Leonardo believed that human proportions always equalled phi. The ratio of height to the distance between the navel and the ground varies from person to person.
The nautilus – A spiral-shelled sea creature 'Can you guess what the ratio is of each spiral's diameter to the next?' 'Phi, the divine proportion.'	✓ Fact: The growth of the nautilus shell is governed by the golden ratio. The shape of the spiral is the Fibonacci spiral, shown elsewhere on this page. ✗ Fiction: The diameter ratios are not in this ratio and vary from spiral to spiral.
Honey bees – 'Did you know that if you divide the number of female bees by the number of male bees in any beehive in the world, you always get the same number?' 'Phi.'	✗ Fiction: There is nothing factual about this statement. The ratio of male to female bees is estimated by bee keepers to be anything from 5 to 1 to 50 to 1. ✗ Fiction: The numbers vary from hive to hive, region to region and season to season.

[1] Dan Brown, *The Da Vinci Code*, Random House Australia, 2004.

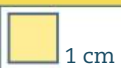
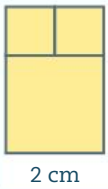
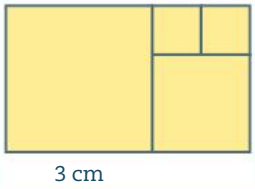
Here are some other commonly held beliefs about the golden ratio:

Belief	Fact or fiction?
Beauty – The golden proportion is what is most pleasing to the eye. 'Beautiful' objects, people, plants and animals are in the proportions of the golden ratio.	✓ Fact: Some cultures have used the golden ratio to define perfection. Ancient Greeks believed that the 'perfect face' was in the golden ratio. ✗ Fiction: Recent research has shown a wide range of ratios that are considered beautiful. The concept of beauty varies dramatically from culture to culture and throughout time.
Architecture – Ancient Greek architects used the golden ratio in their building designs. The Egyptian pyramids were built using the golden ratio.	✓ Fact: Euclid, an ancient Greek mathematician, discovered the golden ratio. ✗ Fiction: There is no evidence that the ratio was deliberately used in Ancient Greek architecture or by the Egyptians.
Art – Artists have used the golden ratio in their paintings.	✓ Fact: The artists Sérusier, Gris, Severini and Salvador Dali deliberately used the golden ratio in their art work. ✗ Fiction: There is no evidence that artists such as Michelangelo and Dürer used the golden ratio in their art.

Although there is little evidence to support many of the claims made about the golden ratio, there is a very clear link between the golden ratio, a shape called the golden rectangle and a very famous group of numbers called the Fibonacci sequence.



The golden rectangle

1	Draw a square of side length 1 cm. Calculate length ÷ width for this square.	
2	Attach another square to form a rectangle. Calculate length ÷ width for this rectangle.	
3	Now, attach a square to the larger side of the rectangle to form a new rectangle as shown. Calculate length ÷ width for this new rectangle.	
4	Continue attaching larger squares to the longer side of the rectangles formed at the previous stage until you run out of space. Calculate length ÷ width for the new rectangle formed at each stage.	

5 What do you notice about your answers to the length ÷ width calculation you did at each stage? If you can't draw a conclusion yet, continue creating larger rectangles and calculating length ÷ width.

The Fibonacci sequence

If you write down the side lengths of each of the squares drawn above, you should get the following sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34…

This is known as the Fibonacci sequence (named after the Italian mathematician who was the first to study it in detail). The terms (numbers) in the sequence are known as Fibonacci numbers.

6 Can you see how each term in the sequence is generated? (Hint: Look at the previous two terms.)

7 What are the 10th and 11th terms in the Fibonacci sequence?

8 Calculate the 11th term ÷ 10th term, and round your answer to three decimal places. What do you notice?

Fibonacci numbers in nature

Fibonacci numbers (and therefore the golden ratio) appear in nature, most notably, in the spiral patterns seen in plants such as sunflowers, pine cones and pineapples. The pine cone at the bottom of the previous page has two sets of spirals, one set curving anticlockwise, the other clockwise. Scientists have observed that the number of spirals in each set are two consecutive Fibonacci numbers. ('Consecutive' means one number follows the other in the sequence.)

9 Count the number of clockwise and anticlockwise spirals on the pine cone (some have been marked in for you). Do you come up with two consecutive Fibonacci numbers?

10 Draw a 'Fibonacci spiral' by drawing quarter circles inside each of the squares used to make the golden rectangle, as shown.

Research

11 Research the work of Fibonacci and other occurrences of Fibonacci numbers in nature.

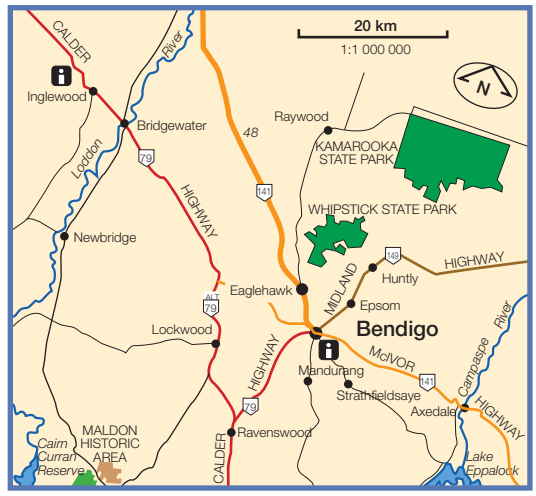
12 Two common beliefs about the golden ratio are that it has been used in art and architecture for centuries.

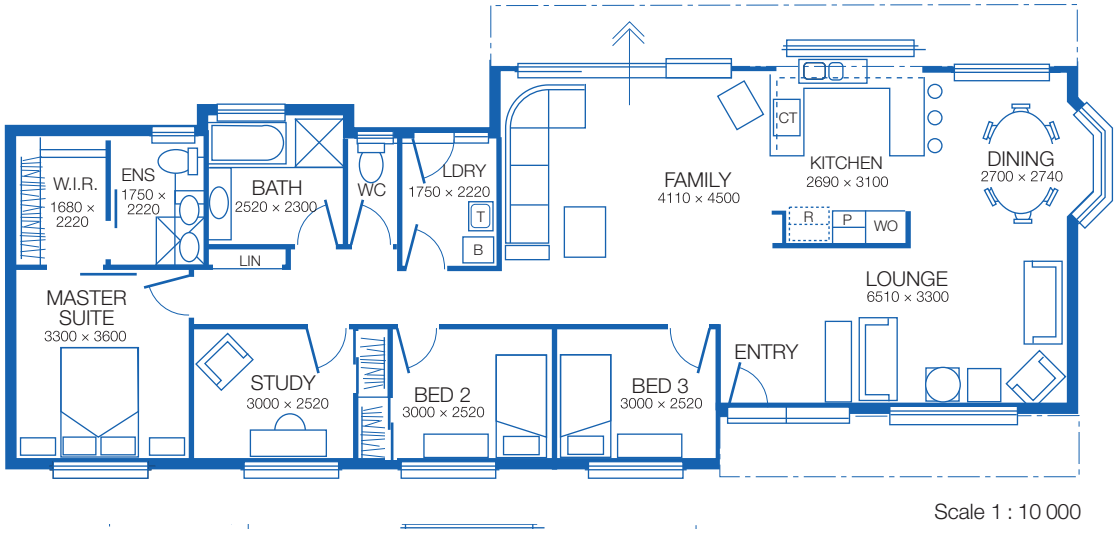
(a) Do some research to see if you can find any evidence that Ancient Greek buildings (such as the Parthenon) or Leonardo da Vinci's paintings (such as the Mona Lisa) contain golden rectangles.

(b) Can you find evidence that we use the golden ratio today? Take some measurements of doors, windows, TV screens, picture frames, paper sizes, vases etc.

4.5 Scale drawings

Scale drawings use ratios to present accurate information about distances and still have a diagram of a convenient size. Maps and building plans are two examples of scale drawings.





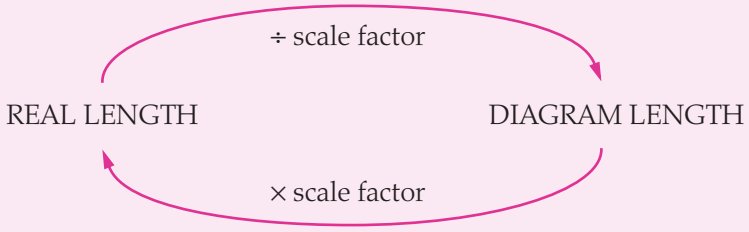
A scale drawing contains a ratio; for example, 1 : 1 000 000. The first number in the ratio is the distance on the diagram. The second is how many times bigger the distance is in real life.

As with other ratios, **scale ratios** should be written without units and in simplest form. For example, a plan of a house may use a scale of 5 millimetres to represent a metre or 5 mm : 1 m. To convert this to a scale ratio, we express both values in the same units, and simplify:

e.g. 5 mm : 1 m
= 5 mm : 1000 mm
= 5 : 1000
= 1 : 200

If the scale ratio is a unit ratio, the larger value in the scale ratio is called the scale factor. A scale factor of 200 means that in real life the object is 200 times as big as in the diagram. Another way to think of this is that the object or length was shrunk by a factor of 200 to make it fit on the page.

To write a ratio as a scale ratio, convert to appropriate smaller units and simplify by cancelling common factors.



When very small objects are enlarged to a scale, the operations in the diagram above are reversed.

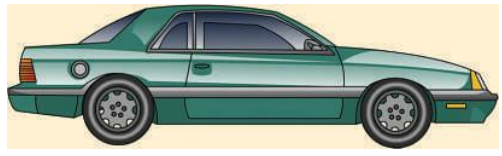
Worked Example 13 WE13

Convert 2 mm : 1 m to a scale ratio and underline the scale factor.

Thinking	Working
1 Write both quantities in the smaller units (mm).	2 mm : 1 m = 2 mm : 1000 mm
2 Simplify by dropping the units and any common factors. Underline the scale factor.	= 1 : <u>500</u>

Worked Example 14 WE14

The diagram at right uses the scale of 1 : 80. Find the real length of the car in metres.



Thinking	Working
1 Measure the length of the car (in mm) on the diagram (65 mm).	Diagram length = 65 mm
2 Multiply the diagram length by the scale factor to get the real length.	Real length = 65 mm × 80 = 5200 mm
3 Convert your answer to metres.	= 5.2 m

Worked Example 15 WE15

The height of a tower is 70 m. How high should it be drawn on a diagram with a scale of 1 : 200? Answer in millimetres.

Thinking	Working
1 Divide the real length by the scale factor to find the diagram height.	70 m ÷ 200 = 0.35 m
2 Convert the diagram height to mm.	= 350 mm

4.5 Scale drawings

Navigator

Answers page 609

Q1 Columns 1–3, Q2, Q3 Columns 1 & 2, Q4, Q5, Q6, Q7 Columns 1 & 2, Q8 Columns 1 & 2, Q9, Q11, Q13, Q15, Q19	Q1 Columns 2 & 3, Q2 Columns 2 & 3, Q3 Columns 2 & 3, Q4 Columns 2 & 3, Q5, Q7 Columns 2 & 3, Q8 Columns 2 & 3, Q9, Q11, Q13, Q14, Q15, Q16, Q17, Q19	Q1 Columns 3 & 4, Q2 Column 3, Q3 Column 3, Q4 Column 3, Q5, Q7 Column 3, Q8 Column 3, Q9, Q10, Q11, Q12, Q14, Q15, Q16, Q17, Q18, Q19

Equipment required: A ruler for Questions **9**, **10**, **11**, **14** and **15**; a calculator may be used for Questions **10**, **11** and **16–18**

Fluency

WE13

1 Convert the following to scale ratios and underline the scale factor.

(a) 1 cm : 6 m **(b)** 1 mm : 40 cm **(c)** 2 mm : 50 cm **(d)** 5 cm : 3 m

(e) 5 mm : 4 m **(f)** 2 mm : 1 m **(g)** 4 cm : 1 m **(h)** 4 mm : 50 cm

(i) 1 cm : 1 km **(j)** 1 cm : 500 m **(k)** 5 cm : 2 km **(l)** 4 cm : 1 km

WE14

2 A council plan is drawn to a scale of 1 : 100. Find the real length in metres for each of the following diagram lengths.

(a) 2 cm **(b)** 3 cm **(c)** 5 cm

(d) 8 mm **(e)** 23 mm **(f)** 19 mm

(g) 40.3 cm **(h)** 92.5 cm **(i)** 1.2 m

WE15

3 A two-storey house is to be drawn to scale.
Various lengths in the house are given below.
How long should each length be drawn on a diagram with a scale of 1 : 400? Answer in millimetres.

(a) 2 m **(b)** 4 m **(c)** 8 m

(d) 40 cm **(e)** 40 m **(f)** 80 m

(g) 320 cm **(h)** 1200 mm **(i)** 5200 mm

(j) 480 cm **(k)** 0.8 m **(l)** 0.24 m

4 A map is drawn to the scale of 1 : 2000. Find the real distances in metres for each of the following lengths measured on the map.

(a) 3 cm **(b)** 5 cm **(c)** 10 cm

(d) 2.6 cm **(e)** 38.4 cm **(f)** 43 mm

(g) 9 mm **(h)** 17.9 mm **(i)** 51 mm

5 Are the following scale ratios? Answer Yes or No.

(a) 4 : 7000 **(b)** 6 : 120 000 **(c)** 1 : 400

(d) 1 : 500 000 **(e)** 65 : 500 **(f)** 1 cm : 17 mm

6 In the ratio 1 : 3500, the scale factor is:

A $\frac{1}{3500}$ **B** 1 **C** 35 **D** 3500

7 Find the real length in metres for each of the following diagram lengths. The scale ratio is given in brackets in each case.

(a) 7 cm (1 : 100) **(b)** 5 mm (1 : 1000) **(c)** 4 cm (1 : 2000)

(d) 8 cm (1 : 500) **(e)** 6 mm (1 : 50) **(f)** 40 mm (1 : 3000)

(g) 21 mm (1 : 30 000) **(h)** 13 cm (1 : 40 000) **(i)** 8.8 cm (1 : 2000)

8 Find the diagram length in millimetres for each of the following real lengths. The scale ratio is given in brackets in each case.

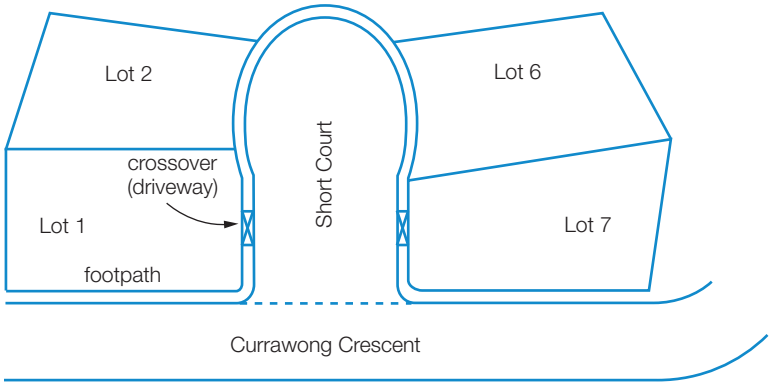
(a) 6 m (1 : 300) **(b)** 16 m (1 : 2000) **(c)** 700 m (1 : 200)

(d) 16 km (1 : 4000) **(e)** 12 km (1 : 50 000) **(f)** 180 m (1 : 60 000)

(g) 50 cm (1 : 20) **(h)** 400 cm (1 : 500) **(i)** 36 m (1 : 400)

Understanding

9 Part of a map of a new subdivision drawn to a scale of 1 : 1000 is shown. Find the real length corresponding to each of the following.



(a) the frontage (length of the block on Short Court) of Lot 1

(b) the length of the longest boundary of Lot 7

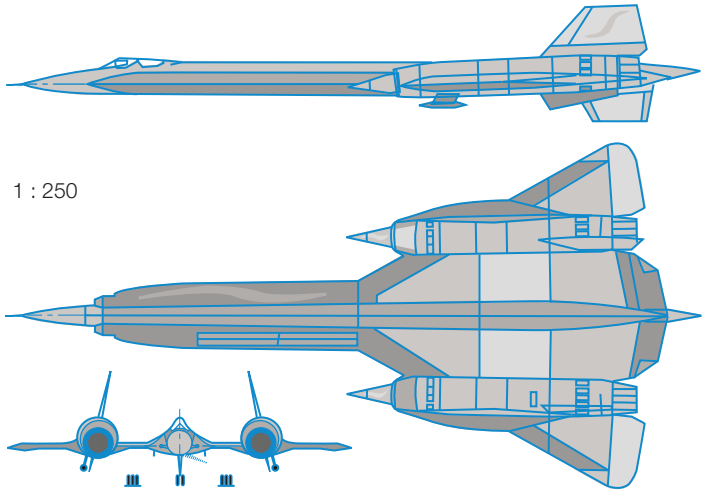
(c) the width of Currawong Crescent

(d) the width of the footpath

10 An engineer's drawing of an aircraft is shown below. Find, to the nearest metre:

(a) the length of the aircraft

(b) the wing-span.



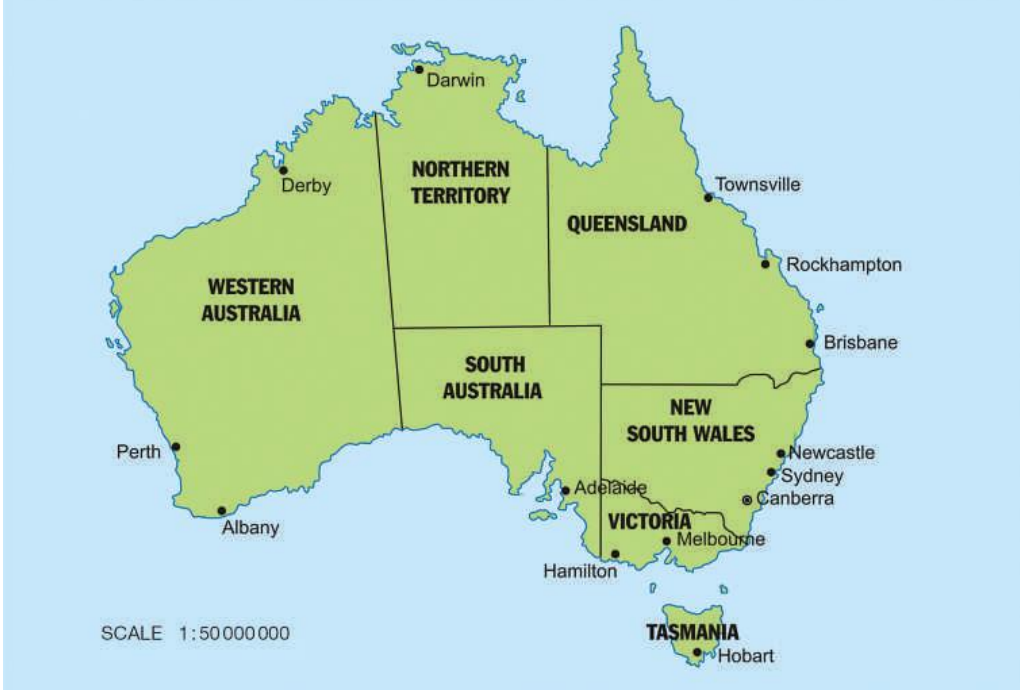
11 Use the scale of the map below and a ruler to estimate the distance (to the nearest km) between:

(a) Melbourne and Sydney

(b) Brisbane and Townsville

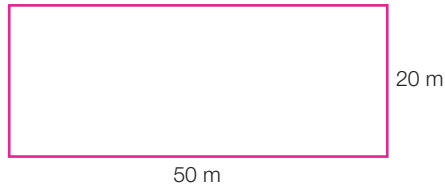
(c) Adelaide and Darwin

(d) Hobart and Perth.



Reasoning

12 Franjo is about to draw an accurate scale diagram of his block of land, which has the measurements shown (not to scale).



(a) Find the dimensions of Franjo's diagram if he uses a scale of:

(i) 1 cm : 5 m

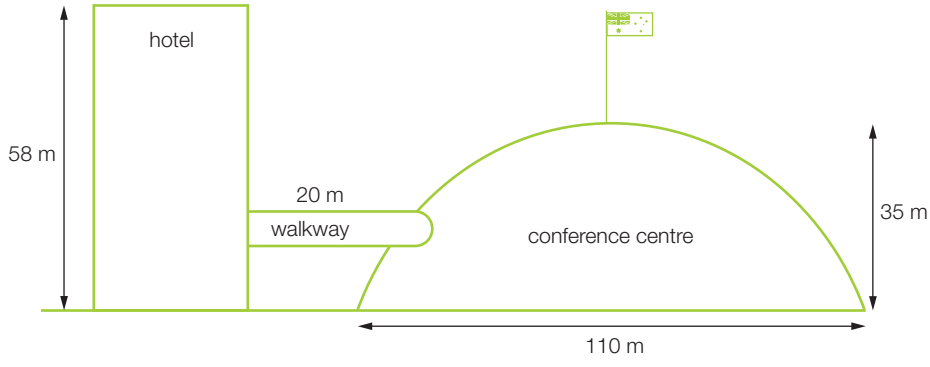
(ii) 4 mm : 1 m

(iii) 1 : 200

(iv) 1 : 400

(b) Which of the above scales would best cover an A4 sheet of paper?

13 A developer has a proposal for a city project. A section of her rough plans for the project appears below. If a scale model is to be made of the proposed development, what will be the lengths on the model of the following if a scale of 1 : 2000 is used? Answer in millimetres.



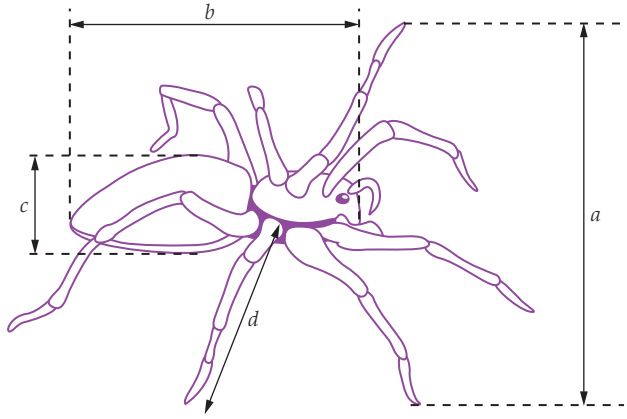
(a) the height of the hotel

(b) the length of the conference centre

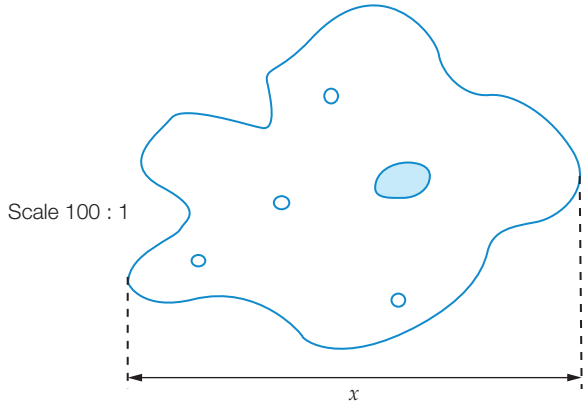
(c) the height of the conference centre

(d) the length of the connecting walkway

14 Scale drawings aren't always smaller than the real thing. The diagram opposite is of a spider, drawn to a scale of 2 : 1 (i.e. the real size of the spider is 0.5 or $\frac{1}{2}$ the diagram size). Find the real length of each distance marked with a variable on the diagram. (Answer in mm.)



15 Find the real length of the amoeba (a microscopic single-celled organism) using the scale shown.



16 The scale ratio for a housing estate plan is 1 : 900. A rectangular block measures 3 cm by 2.4 cm on the plan.

(a) Find the area of the block on the plan in cm^2.

(b) Find the actual dimensions of the block in cm.

(c) Find the actual area of the block in cm^2.

(d) Multiply the area of the block on the plan by the scale factor. Is the answer the same as the actual area? Explain why or why not.

(e) Find the actual dimensions of the block in m.

(f) Find the actual area of the block in m^2.

(g) What is the relationship between the actual area in cm^2 and the area in m^2?

Open-ended

17 An architect requires a scale model of a 140 m high building for a presentation she is making. The finished model must be between 20 cm and 50 cm high. List three whole-number scale factors that could be used.

18 A cartographer (map maker) wishes to fit an entire suburb on a single A4 piece of paper. Allowing for borders, the maximum map size is 18 cm by 25 cm. The dimensions of the suburb are roughly 5 km by 7 km. List two proper scale ratios the cartographer could use. Which would you recommend and why?

19 Sometimes, the scale drawing of an object requires a scale to increase the size of the drawn object rather than scale it down. List three examples of where this might be the case.

4.6 Sharing an amount in a given ratio

To share an amount in a given ratio:

1. Add the values in the ratio to find the total number of parts.
2. Divide the amount by the total number of parts to find the size of one part.
3. Multiply the values in the ratio by the size of one part.

Worked Example 16 WE16

Share 45 in the ratio stated in the brackets (2 : 7).

Method 1: Find the size of one part

Thinking	Working
1 Find the total number of parts in the given ratio.	Number of parts $= 2 + 7$ $= 9$
2 Divide the full amount by the number of parts to find the size of one part.	Size of one part $= \frac{45}{9}$ $= 5$
3 Multiply the ratio by the size of one part.	$2 : 7$ $= 5 \times 2 : 5 \times 7$ $= 10 : 35$
4 Check that the new amounts add up to the original amount.	$10 + 35 = 45$ ✓

Method 2: Find the fraction

Thinking	Working
1 Find the total number of parts in the given ratio.	Number of parts $= 2 + 7$ $= 9$
2 Express each part as a fraction by using the part as the numerator and the total as the denominator.	$\frac{2}{9}$ $\frac{7}{9}$
3 To find each part, multiply the full amount by the fraction.	$\frac{2}{9} \times 45 = 10$ $\frac{7}{9} \times 45 = 35$
4 Check that the new amounts add up to the original total.	$10 + 35 = 45$ ✓

Worked Example 17

WE17

Jake, Abdul and Pendo share a box containing 24 chocolates in the ratio 4 : 3 : 1. How many chocolates does each receive?

Method 1: Find the size of one part

Thinking	Working
1 Find the total number of parts in the given ratio.	Number of parts $= 4 + 3 + 1$ $= 8$
2 Divide the amount by the number of parts to find the size of one part.	Size of one part $= \frac{24}{8}$ $= 3$
3 Multiply the ratio by the size of one part.	$4 : 3 : 1$ $= 4 \times 3 : 3 \times 3 : 1 \times 3$ $= 12 : 9 : 3$
4 Check that the new amounts add up to the original total.	$12 + 9 + 3 = 24$ ✓
5 Explain the answer.	Jake receives 12 chocolates. Abdul receives 9 chocolates. Pendo receives 3 chocolates.

Method 2: Find the fraction

Thinking	Working
1 Find the total number of parts in the given ratio.	Number of parts $= 4 + 3 + 1$ $= 8$
2 Express each part as a fraction by using the part as the numerator and the total as the denominator.	$\frac{4}{8}$ $\frac{3}{8}$ $\frac{1}{8}$
3 To find each part, multiply the amount by the fraction.	$\frac{4}{8} \times 24 = 12$ $\frac{3}{8} \times 24 = 9$ $\frac{1}{8} \times 24 = 3$
4 Check that the new amounts add up to the original total.	$12 + 9 + 3 = 24$ ✓
5 Explain the answer.	Jake receives 12 chocolates. Abdul receives 9 chocolates. Pendo receives 3 chocolates.

4.6 Sharing an amount in a given ratio

Navigator

Answers page 614

Q1 Columns 1 & 2, Q2, Q3, Q5, Q7, Q8, Q10, Q11, Q12	Q1 Columns 2 & 3, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13	Q1 Column 3, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q12, Q13

Equipment required: Calculator for Questions **8** and **9**

Fluency

WE16

1 Share the following amounts in the ratios stated in the brackets.

(a) 30 (2 : 3) **(b)** 20 (3 : 1) **(c)** 42 (5 : 1)

(d) 39 (2 : 11) **(e)** 84 (16 : 5) **(f)** 38 (12 : 7)

(g) 56 (4 : 3 : 1) **(h)** 42 (2 : 4 : 1) **(i)** 54 (2 : 7 : 9)

(j) 33 (2 : 5 : 4) **(k)** 50 (2 : 3 : 5) **(l)** 96 (1 : 7 : 4)

WE17

2 **(a)** Kayla has a packet of 45 jelly beans that she has been instructed to share with her little brother in the ratio 3 : 2. How many jelly beans does each child receive?

(b) Mrs Everage makes a batch of 84 lamingtons which she wishes to divide in the ratio 2 : 7 : 3 and place in separate containers for afternoon tea engagements on three successive days. How many lamingtons should she place in each container?

3 How many parts are in the ratio 10 : 5 : 3?

A 3 **B** 5 **C** 12 **D** 18

Understanding

4 After 21 matches, the win:loss ratio of Adrian's cricket team is 5:2. How many matches has the team won, and how many have they lost?

5 The instructions on a container of orange juice concentrate say to mix with water in the ratio 1 : 4. If Boutros wishes to make 2 L of juice according to the instructions, how many mL of concentrate and water should he mix?

6 Two-stroke motor fuel is made by mixing 1 part oil with 24 parts petrol. Anastasia wishes to mix up 5 litres of two-stroke fuel. How much oil and petrol does she need?

7 The ratio of boys to girls in Year 8 at MacGregor High is 7 : 8. If there are 300 students in Year 8, how many of each gender are there?

8 A lotto win is to be divided among the following people in the same ratio as the amounts they contributed to tickets, which are as follows:

Karine	\$5
André	\$3
Vivian	\$2
Julian	\$2

If the group won \$57 000, determine how much each person should receive.

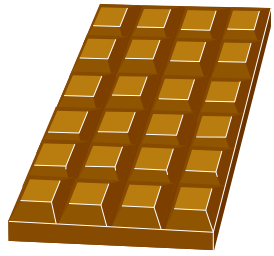
Reasoning

9 A nursery sells a fertiliser with an N:P:K (nitrogen : phosphorus : potassium) ratio of 3 : 2 : 4. (The capital letters are the chemical symbols for these elements.)

(a) How many kilograms of each element are in a 2.7 tonne load of the fertiliser?

(b) If 300 kg of phosphorus (P) was added to the 2.7 tonne load, what would be the new ratio of elements?

10 The block of chocolate shown is to be divided between two groups of students according to the number of students in each group.



(a) If there are 4 students in the first group and 2 in the second group, how many pieces of chocolate does each group get?

(b) How many pieces does each student get?

(c) One member of each group decides not to have any chocolate. If the chocolate is redistributed according to the new groups, how many more pieces of chocolate does each student get now?

11 A 6.4 m length of timber is to be cut so that the two pieces are in the ratio 2:3. How far along should the cut be made? (Answer in centimetres.)

Open-ended

12 Glenda, Joan and Petra have lunch together and when the bill comes it is not itemised, which means that individual prices for each course are not listed. The total bill is $115. All three had a main course and a drink but Joan also had a dessert. Suggest two ways the bill could be divided and explain your reasoning.

13 Ms Footsie and Ms Nikkei form a partnership and invest $5 million and $9 million, respectively, to buy a television station. They sell the station for $42 million some time later.

(a) How should the $42 million be divided between the two partners?

(b) Ms Footsie is claiming she should receive $19 million from the sale, only $4 million less than her partner. Can you suggest a reason why?

Outside the Square — Problem solving

Catch 22

1 Write down any three different single-digit numbers and add them up. Keep this number for later.

2 Use those numbers to write out six possible two-digit numbers, no repetition allowed.

3 Add the six two-digit numbers.

4 Divide the number you have in step **3** by the total from step **1** and record your answer.

5 Try this procedure again, starting with a different set of three numbers. What do you notice? Why do you think this works?

Strategy options

- Guess and check.
- Work backwards.
- Test all possible combinations.

4.7 Rates

A **rate** is a way of comparing two quantities that use different units. They usually involve the word *per* which means 'for each' and has the symbol / or the letter 'p'. A rate is similar to a unit ratio—it is the number of the first quantity for every 1 of the second quantity. Some examples of rates are as follows.

If the price of apples is \$4.96/kg, you will pay \$4.96 for 1 kg of apples.

Hair grows at an average rate of 1.3 cm per month.

Speed

Speed is a rate that compares distance and time. It is used to describe how fast something is travelling; for example, a car could be travelling at 60 kilometres per hour (kph or km/h), or a cheetah could be running at 25 metres per second (m/s or mps).

A car speed of 60 km/h means that in 1 hour, a car would travel a distance of 60 km provided it stayed at a constant speed. It is more likely that a car would speed up and slow down at various times during the hour. In this case, we would be calculating an average speed.

$$\text{average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

This can also be written as the formula $s = \frac{d}{t}$.

Worked Example 18 WE18

Calculate the average speed for the following. Express your answers correct to two decimal places where necessary.

(a) a car that travels 360 km in $4\frac{1}{2}$ hours

(b) a female athlete who sprinted 100 metres in 10.49 seconds

Thinking	Working
(a) 1 Write the formula for calculating average speed.	(a) $s = \frac{d}{t}$ $\quad d = 360$ km, $t = 4.5$ h
2 Substitute the values for distance and time.	$s = \frac{360}{4.5}$
3 Evaluate. Note that as the original units are km and hours, the speed unit is km/h.	average speed = 80 km/h
(b) 1 Write the formula for calculating average speed.	(b) $s = \frac{d}{t}$ $\quad d = 100$ m, $t = 10.49$ s
2 Substitute the values for distance and time.	$s = \frac{100}{10.49}$
3 Evaluate, rounding your answer to the stated number of decimal places. Note that as the original units are metres and seconds, the speed unit is m/s.	average speed = 9.53 m/s

The unitary method

A rate represents a certain number of a first quantity for every one of a second quantity. Calculating a number per 'unit' (where a 'unit' might be litres, kilograms, boxes or people) is a useful method for solving problems and is known as the **unitary method**. The first part of the method is similar to calculating a unit ratio, which was discussed in Section 4.3.

Worked Example 19 WE19

(a) A recipe for blueberry muffins requires 150 g of blueberries. The recipe makes 12 muffins. What mass of blueberries is required to make 40 muffins?

(b) 5 kg of potatoes costs $12.99. How much would 16 kg cost?

Thinking	Working
(a) 1 Calculate the amount required for one 'unit' (one muffin).	(a) 150 g for 12 muffins ÷ 12 ↓ ↓ ÷ 12 = 12.5 g for 1 muffin
2 Multiply the amount per unit by the number of units (40 muffins).	× 40 ↓ ↓ × 40 = 500 g for 40 muffins
(b) 1 Calculate the price per unit (per kilo).	(b) $12.99 for 5 kg ÷ 5 ↓ ↓ ÷ 5 = $2.598 for 1 kg
2 Multiply the price per unit by the number of units (16 kg).	× 16 ↓ ↓ × 16 = $41.57 for 16 kg

We can modify the unitary method slightly if we are working with numbers that have common factors. Instead of finding the cost or amount per one unit, we can find it for, say, 10 or 20 units or a number of units that is a common factor. For example, if 30 L of petrol costs \$45, then we can calculate the cost of 50 L by first finding the cost of 10 L.

\$45 for 30 L
= \$15 for 10 L (÷ 3)
= \$75 for 50 L (× 5)

Similarly, if one number is a multiple of another, our task is made simpler. For example, if 100 g of mixed nuts costs \$2.50, then we can simply multiply by 3 to find the cost of 300 g.

\$2.50 for 100 g
= \$7.50 for 300 g (× 3)

Percentage rates of change

A rate is often used to describe how quickly or slowly a quantity is changing over time. Speed is an example of distance changing over time. In order to track change over periods such as months, years or decades, the rate is often calculated as a percentage increase in the previous quantity. Percentage rates are commonly used to describe the yearly growth in a population (of a country, city or school), or financial growth (such as the value of shares in a business or the price of oil). A percentage rate is calculated by writing the change (the amount of increase or decrease) as a fraction of the previous amount, then converting to a percentage.

For example, if the population of a suburb increased in one year from 250 to 270 people, the percentage rate of change is: $\frac{20}{250} \times 100\%$, or 8% per year.

If the population continued to grow at this rate in the following year, the population at the end of that year would be 108% of 270: $1.08 \times 270 = 292$ (rounded to a whole number).

4.7 Rates

Navigator

Answers page 614

Q1, Q2, Q3, Q5, Q6, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q18 (a), Q19, Q21, Q22	Q1 (c)–(h), Q2, Q3, Q5, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q15, Q18, Q20, Q21, Q22	Q1 (e)–(h), Q2, Q3, Q4, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q17, Q18, Q19, Q20, Q21

Equipment required: Calculator

Fluency

WE18

1 Calculate the average speed for the following. Express your answers correct to two decimal places where necessary.

(a) 80 kilometres travelled in 2 hours

(b) 200 kilometres travelled in 4 hours

(c) 160 kilometres travelled in 2.4 hours

(d) 150 kilometres travelled in 3.9 hours

(e) 1120 kilometres travelled in 12 hours

(f) Cathy Dreemin running 400 m in 48.80 seconds

(g) Michael Quik swimming the 100 m freestyle in 49.2 seconds

(h) a parasail floating 3.5 km in 20 minutes

2 (a) A recipe for ANZAC biscuits requires 125 g of butter. The recipe makes 24 cookies. What mass of butter is required to make 100 cookies? (Answer to the nearest gram.) WE19

(b) 4 kg of tomatoes costs $19.75. How much would 10 kg cost?

(c) A box of 24 mangoes costs $65. How much would 20 mangoes cost?

(d) A recipe for vegetable soup requires three stalks of celery. The recipe feeds four people. How much celery is required if soup is to be made for 11 people?

3 The following table shows the change in population of Australia's three largest cities, as well as Australia's total population in the year from June 2008 to June 2009.

City	Population 30 June 2008	Population 30 June 2009	Change
Sydney	4 419 075	4 504 469	85 394
Melbourne	3 902 059	3 995 537	93 478
Brisbane	1 952 158	2 004 262	52 104
Australia – Total	21 498 540	21 955 256	456 716

For each of the cities, and also for the whole of Australia:

(a) calculate the increase in population as a number of people per week

(b) express the change in population as a percentage increase in the 2008 population. (Round answers to two decimal places.)

(c) If the population continued to grow by the rate calculated in **(b)**, what would the population have been by June 2010?

4 (a) Mal has just started his annual get-fit campaign. He wishes to jog 5 km a day in the first week. If the 5 km took him 45 minutes to complete, his average speed would be closest to:

A 0.15 km/h **B** 3.75 km/h **C** 5.45 km/h **D** 6.7 km/h

(b) Angela has just completed her daily 4 km walk. Today, it took her 40 minutes. Angela's average speed, in metres per second, was:

A 100 m/s **B** 1.67 m/s **C** 1.5 m/s **D** 0.0017 m/s

5 Find the cost of petrol, in cents/litre, for each of the following situations. Give your answers correct to the nearest tenth of a cent.

(a) It costs $37 for 30 litres.

(b) It costs $31.40 for 25 litres.

(c) It costs $22.84 for 15.7 litres.

(d) It costs $66.52 for 46.8 litres.

6 A typical healthy resting heart rate for an adult is in the range 60–80 beats per minute (b.p.m.). Find the number of beats per day if the rate is:

(a) 60 b.p.m.

(b) 80 b.p.m.

Understanding

7 (a) Emily walked 22 kilometres at a constant speed of 4 km/h. How long did it take her?

(b) Luan averaged 80 km/h in her car for 3 hours. How far did she travel?

8 Here are Martha's ingredients for scones. The quantities shown make 12 scones. Rewrite the quantities of ingredients to make 40 scones.

4 cups self-raising flour
2 tablespoons icing sugar mixture
$\frac{3}{4}$ cup milk
$\frac{3}{4}$ cup thickened cream
1 egg, lightly beaten

9 In one-day cricket, the 'run rate' is the number of runs that need to be scored each over by the batting team in order to win the match. Australia needed to score 215 runs to win a 50-over cricket match against England.

(a) Find the run rate required at the start of the innings.

(b) If Australia had scored 42 runs in the first 10 overs, find the run rate required for the next 40 overs for Australia to win the match.

(c) Shane Watson scored 20 runs from 16 balls. Express his run rate as the number of runs per ball.

(d) David Hussey scored 68 runs from 89 balls. Express his run rate as the number of runs per ball.

10 Calculate the following. Use mental strategies wherever possible.

(a) Ham costs \$13/kg. How much does it cost for:

(i) 500 g **(ii)** 250 g?

(b) Dried apricots cost \$9/kg. How much does it cost for:

(i) 100 g **(ii)** 300 g?

(c) 30 L of petrol costs \$42. How much does it cost for:

(i) 20 L **(ii)** 50 L?

(d) Chocolate-coated nuts cost \$3/100 g. How much does it cost for:

(i) 250 g **(ii)** 350 g?

11 Water costs \$1.7924 per kilolitre (kL). For each of the following charges for water supply, find the amount of water used, correct to four decimal places.

(a) \$52.40 **(b)** \$16.55 **(c)** \$22.40 **(d)** \$18.90

12 The following currency conversion table gives some conversion rates for the Australian dollar to other currencies on a particular day. The first column tells you how much of the other currency you will get if you exchange one Australian dollar (1 AUD). The second column tells you how many Australian dollars you will get if you exchange one unit (dollar, real, pound or yen) of the other currencies.

	1 AUD	In AUD
American dollar	0.989 139	1.010 98
Brazilian real	1.652 45	0.605 163
British pound	0.618 406	1.617 06
Japanese yen	80.770 5	0.012 229 4
New Zealand dollar	1.305 34	0.766 085

(a) Calculate how much $500 Australian dollars would be worth in:

(i) American dollars

(ii) Brazilian reals

(iii) British pounds

(b) You have 250 000 Japanese yen. How much would you get in Australian dollars for this amount?

(c) You have 750 New Zealand dollars. How much would you get in Australian dollars for this amount?

13 The following table gives the population and the percentage growth rate of that population in July 2010 for the world and a number of countries in the Asia–Pacific region.

Country	Population (million)	Growth rate (%)
World	6768	1.133
Australia	22.3	1.7
China	1330	0.494
India	1173	1.376
Indonesia	243	1.097

(a) Assuming the population growth rate remains constant for each country, find an estimate for the population of the world and each of the countries listed above for:

(i) 2011 **(ii)** 2012

Round your answers to one decimal place (0.1 million).

(b) Which country has the:

(i) fastest growth rate **(ii)** slowest growth rate?

(c) What would a growth rate of zero tell you about the population of a country?

(d) What would a negative growth rate tell you about the population of a country?

14 The nutritional information table on a packet of cream biscuits looks like this:

	Per serving: 17 g (one biscuit)	Per 100 g
Protein		5.2 g
Fat – total		26.7 g
Fat – saturated		13.1 g
Carbohydrate – total		66.1 g
Carbohydrate – sugars		27.6 g

Use the unitary method to complete the 'per serving' column.

15 Emily and Josh went bushwalking and travelled approximately 6.5 km in the first 3 hours. Correct to two decimal places:

(a) what was their average speed?

(b) If they maintained this speed for the next $5\frac{1}{2}$ hours, how far would they travel in total over the day?

Reasoning

16 Kieran and Jess walk to the bus stop each day. The stop is approximately 1.8 km away. If it takes them 20 minutes, calculate their average speed in:

(a) metres/minute **(b)** km/h.

17 In May 2010 the following world records stood. All events are athletics except for the 100 m swimming.

	100 m	100 m swimming	400 m	5000 m	Marathon (42 km)
Men	9.58 s	46.91 s	43.18 s	12 min 37.35 s	2 h 3 min 59 s
Women	10.49 s	52.07 s	47.60 s	14 min 11.15 s	2 h 15 min 25 s

(a) What is the average speed of the men's 100 m running record in m/s to one decimal place?

(b) If the men could maintain the speed calculated in part **(a)**, how long would it take to cover the 42 km of the full marathon? (Answer to the nearest minute.)

(c) What is the average speed in m/s for both the women's 100 m and 5000 m running events, to two decimal places?

(d) In which event is the ratio of the men's time to the women's time closest to 1:1?

(e) Complete the following sentence. (Write the answer correct to two decimal places.)

Men can run 100 m _____ times faster than they can swim 100 m.

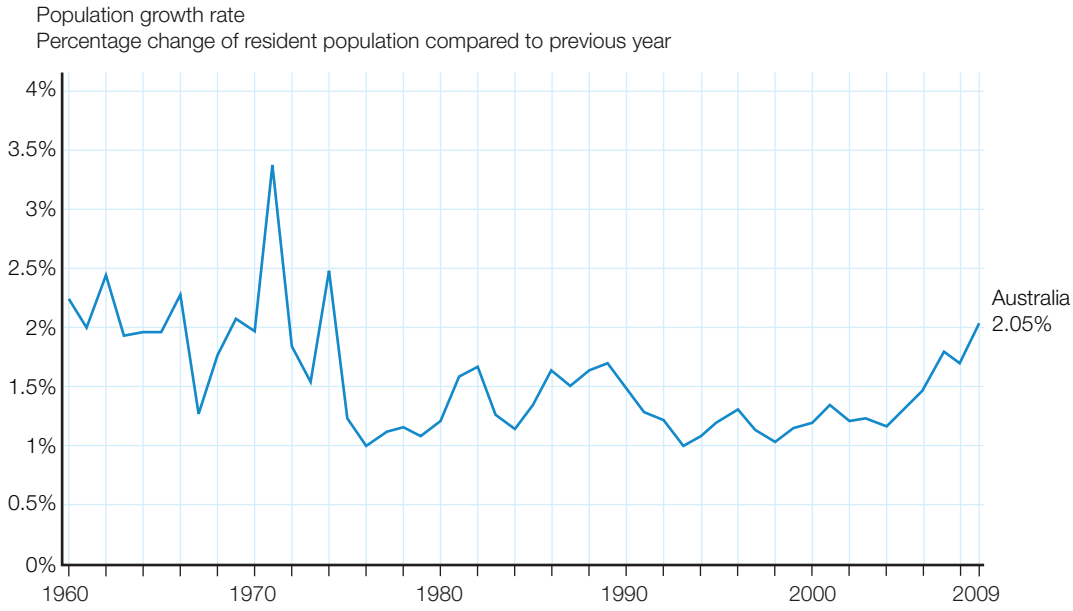
18 **(a)** As the use of electricity increases so does its cost. For the first 1000 kilowatt hours (kWh) the cost is 19.05 cents per kWh. Any extra energy consumed is charged at 19.51 cents per kWh. Find the cost of:

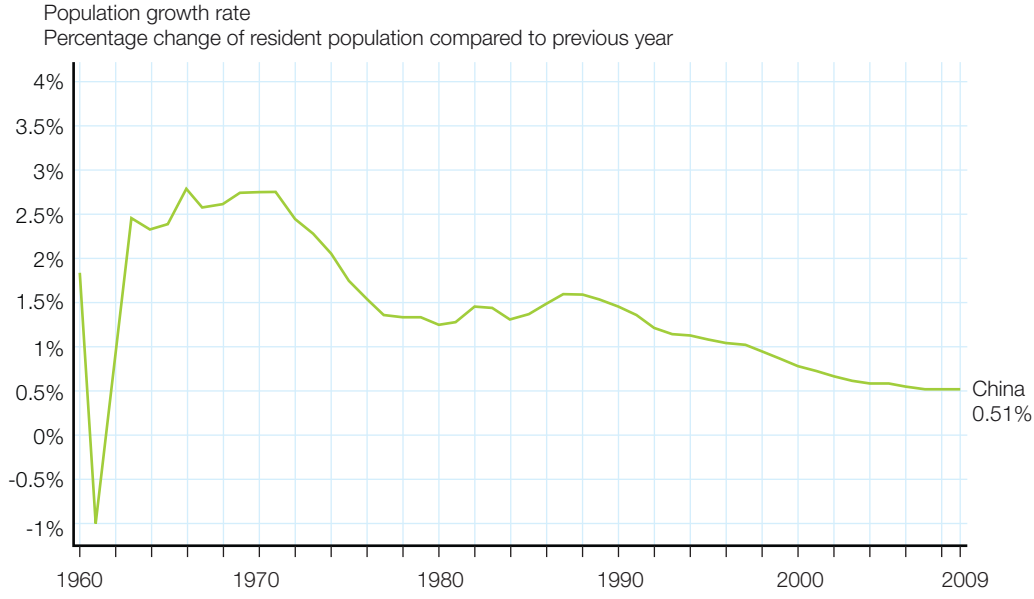
(i) 1203 kWh **(ii)** 1424 kWh **(iii)** 1046 kWh **(iv)** 1822 kWh

(b) Now, find, correct to two decimal places, the number of kWh used if the total cost is:

(i) \$172.50 **(ii)** \$199.85 **(iii)** \$203.45 **(iv)** \$220.90

19 The graphs below show the population growth rates for Australia (blue) and China (green) over the past 50 years.





(a) What does a growth rate of 0% indicate?

(b) What unusual thing happened to the Chinese population in the early 1960s?

(c) Summarise the population growth rate of Australia over the past 50 years.

(d) Summarise the population growth rate of China over the past 50 years.

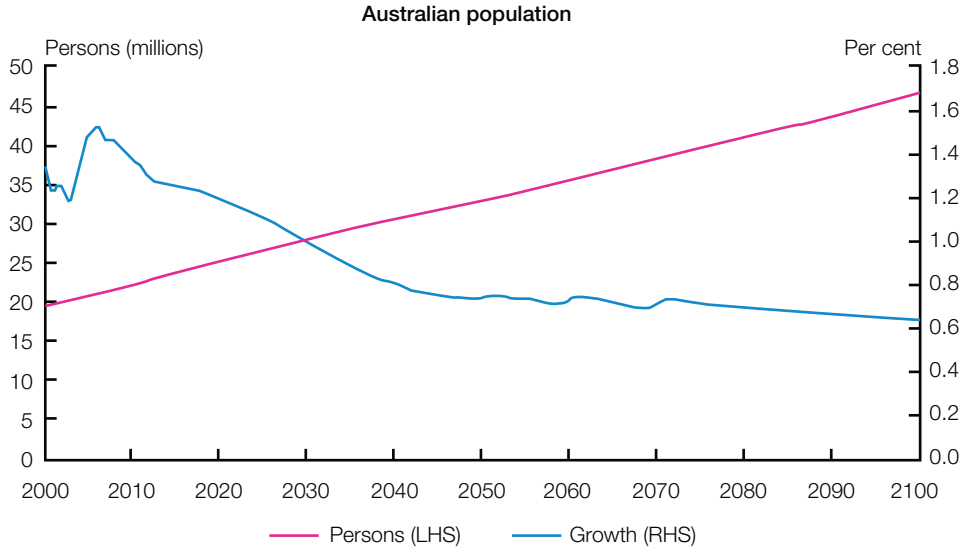
(e) List some possible reasons for the differences between the two countries.

(f) What further information do you require to be able to calculate how many people were added to each population in 2009?

20 The pink line on the graph below shows the population of Australia steadily increasing. It represents the actual number of people over the period 2000–2100. (Note that the part of the graph after 2010 is the predicted population.) The blue line on the graph shows the percentage growth rate of the population. After a sharp increase near the beginning, it decreases for most of the time period shown.

(a) What are the two main ways that the population of a country can grow?

(b) Explain how the population can keep increasing even though the growth rate is decreasing.



Open-ended

21 Make a list of factors that could influence the rate at which a population of a city or a country could grow.

22 By estimating the time between haircuts and the length of hair the hairdresser cuts each time, calculate the growth rate of your hair in mm/week.

Outside the Square — Problem solving

Happy numbers

1 Write down your favourite single-digit number.

2 Multiply that number by 9.

3 Multiply this new number by 12 345 679. (Unless your calculator has more than 8 digits in its display you will need to do this by hand to see the full effect.)

4 If you did choose your favourite number, then the answer should make you very happy. Does it work for other single-digit numbers? Can you explain why it works?

Strategy options

- Guess and check.
- Work backwards.
- Test all possible combinations.

Challenge 4

1 A train is travelling from Melbourne to Sydney at 80 km/h. Another train is travelling from Sydney to Melbourne at 90 km/h. How far apart are they half an hour before they pass each other?

2 Kevin and Sanna take 6 hours to clean their house. If Sanna does twice as much work as Kevin, how long would it take Kevin to clean the house on his own?

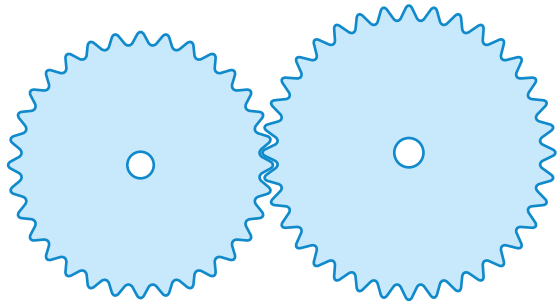
3 If six cats eat nine tins of cat food in three days, how many tins of cat food will two cats eat in six days?

4 One liquid contains 22.5% water, another liquid contains 27% water. A glass is filled with 5 parts of the first liquid and 7 parts of the second liquid. What percentage of the liquid in the glass is water?

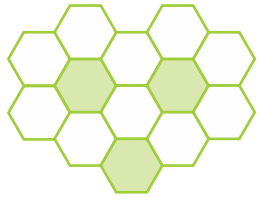
5 A train travels from A to B at a speed of 40 km/h, and from B to A at a speed of 60 km/h. What was its average speed for the whole journey?

6 A tank can be filled by one pipe in 3 hours, by a second pipe in 5 hours, and by a third pipe in 10 hours. If all three pipes are used together, how long will it take to fill the tank?

7 Two gears work together. There are 32 teeth on one gear and 36 on the other gear. The first gear makes 128 revolutions per second. How often, during a 24 hour period, will the same two teeth from each gear be together?



8 A square floor is tiled, as partially shown, with a large number of regular hexagonal tiles. Each green tile is surrounded by 6 white tiles and each white tile is surrounded by 3 white and 3 green tiles.



(a) Explain why the ratio of the number of green tiles to white tiles is 1 : 2.

(b) Justify your answer in **(a)** by completing the pattern in two ways, starting with a green tile in the top left-hand corner, then starting with a white tile in the top left-hand corner.

9 It takes 10 employees 25 days to drain a lake and remove the weeds. Assuming a constant relationship between the number of employees and the time taken:

(a) how long will it take five employees?

(b) how long will it take three employees?

(c) how long will it take 30 employees?

(d) how long will it take 100 employees?

(e) how long will it take 300 employees?

(f) What is the problem with our assumption that the relationship is constant?

Chapter review 4

D.I.Y. Summary

Key Words

equivalent ratios	rate	scale ratio	unit ratio
part:part ratio	ratio	simplest form	unitary method
part:whole ratio	scale factor	speed	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 A _____ is a comparison of the relative amounts of two quantities that are measured in the same units.

2 The _____ helps us solve problems by calculating a number per unit.

3 A _____ compares two parts of a whole.

4 _____ are created if you multiply or divide every quantity in a ratio by the same number.

5 A _____ is found by dividing the first quantity in a ratio, a, by the second quantity, b, and then expressing the ratio as $\frac{a}{b}:1$.

6 In a unit ratio, the number that is not 1 is called the _____.

7 The comparison of two quantities that use different units is called a _____.

8 An example of a rate is average _____ which is calculated using $\frac{\text{distance travelled}}{\text{time taken}}$.

9 A _____ is used on diagrams that are drawn smaller or larger than their actual size.

10 A _____ compares a part to the whole.

Fluency

1 A vet's waiting room contains 4 dogs, 5 cats and 3 budgies. Ex.4.1

(a) Find the ratio of:

(i) the number of dogs to the number of cats

(ii) the number of cats to the number of dogs to the number of budgies

(iii) the number of budgies to the number of dogs

(iv) the number of cats to the total number of animals.

(b) What percentage of the animals are cats? (Round your answer to the nearest whole number.)

(c) Write the number of budgies as a fraction of the number of dogs.

2 Simplify the following ratios. Ex.4.1, 4.2

(a) 14 : 24 (b) 80 : 16 : 32 (c) 5 cm : 20 mm

(d) 3 hours : 40 minutes (e) 4 days : 1 week (f) 3 L : 1500 mL

3 The ratio $2\frac{1}{4} : 3\frac{1}{2}$ simplifies to: Ex 4.2

A 2 : 1 **B** 3 : 2 **C** 9 : 7 **D** 9 : 14

4 Express the following as unit ratios to two decimal places. Ex 4.3

(a) 220 : 75 **(b)** 15 : 29 **(c)** 12 : 20

5 The Yangtze River in China is 6300 km long, whereas the Colorado River in the USA is 2333 km long. How many times longer is the Yangtze than the Colorado? (Answer correct to one decimal place.) Ex 4.3

6 Anna is using a recipe from a very old recipe book. The recipe calls for 2.5 pints of water, but Anna's measuring jug is in litres. How many litres of water should Anna use, if 1 pint = 0.568 L? Ex 4.3

7 Find the value of the pronumeral in each case. Ex 4.4

(a) $a : 42 = 11 : 6$ **(b)** $5 : 7 = 100 : b$ **(c)** $7 : c = 49 : 77$

(d) $12 : 10 = a : 15$ **(e)** $9 : 21 = 24 : b$ **(f)** $c : 3 = 9 : 2$

8 **(a)** Find a if $a : b = 1.6$ and $b = 5$. **(b)** Find q if $p : q = 5.2$ and $p = 26$. Ex 4.4

9 Instructions for making lawnmower fuel mixture state that 200 mL of oil should be added to 5 L of petrol. Ex 4.2, 4.4

(a) Write the ratio oil : petrol in simplest form.

(b) Robert has 30 L of petrol. What volume of oil should he add to get the correct fuel mixture?

10 Convert the following to proper scales, and underline the scale factor. Ex 4.5

(a) 1 cm : 2 m **(b)** 5 mm : 12 km **(c)** 3 m : 15 km

11 Find the real length in metres for each of the following diagram lengths. The scale ratio is given in brackets in each case. Ex 4.5

(a) 4 cm (1 : 100) **(b)** 14 mm (1 : 20 000) **(c)** 5.2 cm (1 : 5000)

12 Find the diagram length in millimetres for each of the following real lengths. The scale ratio is given in brackets in each case. Ex 4.5

(a) 5 m (17 : 7500) **(b)** 38 km (1 : 200 000) **(c)** 245 km (1 : 500 000)

13 Share each amount given below in the ratio stated in brackets. Ex 4.6

(a) 50 (2 : 3) **(b)** 72 (7 : 5) **(c)** 32 (3 : 5)

14 **(a)** Find Jake's average speed if he travels 6 km in 1.5 hours. Ex 4.7

(b) Find the average speed of a car that has travelled 450 km in 6 hours.

(c) Gazza earns \$600 000 a year for 25 games of football. How much does he earn per game?

(d) If it costs \$75.45 to put 45.3 litres of petrol in a tank, what was the price per litre?

(e) 3 kg of onions costs \$4.50. How much would it cost for:

(i) 6 kg **(ii)** 5 kg?

(f) A recipe for minestrone soup requires three potatoes and 200 g of pumpkin. The recipe feeds six people. What amounts of potato and pumpkin would be required to make soup for 21 people?

Understanding

15 Maxine runs a 12 km fun run at an average speed of 8 km per hour. If she begins at 9 a.m., at what time will she finish?

16 Isobel spends 12 minutes of her half-hour recess in the queue at the canteen.

(a) What is the ratio of time spent in the queue to the duration of recess, in simplest form?

(b) What percentage of her recess time did Isobel spend in the queue?

17 Francoise's fruit punch is made from chopped fruit and juice in the ratio 2 : 5. If she uses 24 cups of juice, how many cups of fruit should she chop?

18 Akio and Tsutomu are sumo wrestlers whose masses are in the ratio 8 : 5. If Akio has a mass of 200 kg, what is Tsutomu's mass?

19 Water flows through a water-efficient showerhead at a rate of 7.5 L/min. How much water would be used during a 6-minute shower?

20 An analysis of a fertiliser reveals a ratio of carbon to nitrogen of 2.9 : 1. If the fertiliser contained 335 g of nitrogen, how much carbon did it contain?

21 A hockey team scores a total of 137 goals one season, compared to 88 goals scored against them. For every goal scored against them, how many goals does the team score? (Answer to one decimal place.)

22 Three friends, Adrian, Gavin and Kevin, invest funds in the ratio 3 : 4 : 6, respectively, to finance a search for sunken treasure. They share what they find according to this ratio. If they are successful and discover a chest containing 520 identical gold coins, how should their find be shared?

23 The floor plan of a house has a scale of 1 : 125.

(a) What are the real dimensions in metres of a bedroom that measures 2.7 cm by 2.6 cm on the plan?

(b) What is the area of the bedroom on the plan, in square centimetres?

(c) What is the real area of the room, in square centimetres?

(d) How many times larger is the real area of the room than the area of the room on the plan?

Reasoning

24 At the beginning of the year, the small town of Splitsville had a population of 14 500. By the end of the year, 178 babies had been born, 69 people had died, 103 people had moved away and 152 people had moved into the town.

(a) Calculate the overall change in population as a percentage change in the population at the start of the year. Round your answer to two decimal places.

(b) If the population continued to grow at the rate you calculated in **(a)**, determine the population in 2 years time.

25 Concentrated disinfectant needs to be diluted so that the final solution contains 10% disinfectant. How much water should be added to 35 mL of disinfectant?

26 In a recent twenty20 cricket match, Australia made a score of 185 from its 20 overs.

(a) When they begin their innings, at what rate must the Pakistani team score per over if they are to win the match?

(b) After 8 overs, Pakistan has scored 63 runs. At what rate must they score (runs per over) if they are to win the match?

(c) After 18 overs, the Pakistani team has averaged 8.5 runs per over. How many runs do they need to win off the last two overs?

NAPLAN practice 4

Numeracy: Non-calculator

1 The ratio $3\frac{1}{3}:2$, expressed in whole numbers in simplest form, would be:

A 5:3 **B** 10:2 **C** 10:3 **D** 10:6

2 A soccer team consists of three defenders, five midfielders and three strikers. What fraction of the team are midfielders?

A $\frac{3}{11}$ **B** $\frac{5}{11}$ **C** $\frac{3}{11}$ **D** $\frac{5}{6}$

3 A map is drawn using the scale ratio 1:5000. If a distance on the map is 20 cm, how long is the actual distance?

A 20 m **B** 10 000 cm **C** 250 m **D** 1 km

4 The exchange rate between the Australian and New Zealand dollars is 1 : 1.15. If $200 Australian dollars is exchanged, how many New Zealand dollars would this be?

A $174 **B** $200 **C** $200.15 **D** $230

5 Lawn mower fuel consists of petrol and oil, mixed in the ratio 5:2. Luke has 3.5 L of oil. How much petrol does he need to make a fuel mixture?

Numeracy: Calculator allowed

6 Angelo achieved 60% for his first maths test and after studying hard for the next two tests increased his mark by 25% both times. His mark on the third test is closest to:

A 48% **B** 75% **C** 82% **D** 94%

7 A recipe for choc chip muffins requires 180 g of choc chips. The recipe makes 12 muffins. What mass of choc chips is required to make 20 muffins?

8 In a class of 22 students, the ratio of girls to boys is 6:5. Three extra girls and two extra boys join the class. What is the new ratio of girls to boys?

A 3 : 2 **B** 5 : 4 **C** 6 : 5 **D** 9 : 7

9 Greg, Jan and Laura own 40%, 35% and 25% of a company, respectively. The company profits are to be divided according to the share each has in ownership. If the company makes $240 000, the amount Laura is paid to the nearest dollar is:

A $60 000 **B** $84 000 **C** $156 000 **D** $171 429

10 Jacob intends to take three hours for his 6 km bushwalk. He walks only 1 km in the first hour. What speed will he need to average in the final two hours to make the trip in the planned three hours?

A 2.5 m/s **B** 2.5 km/h **C** 4 km/h **D** 5 km/h

Mixed review B

Fluency

1 Convert each of the following percentages to fractions in simplest form. Ex. 2.6

(a) 23% (b) $\frac{2}{5}$% (c) 12.5% (d) 0.7%

2 Convert each of the following decimals to percentages. Ex. 2.5

(a) 0.14 (b) 0.03 (c) 0.286 (d) 1.9

3 Calculate: Ex. 2.8

(a) 30% of 24 (b) 4.8% of 65 (c) 86% of 330

4 Calculate the sale price of each of the following items if they are discounted by 15%. Ex. 2.10

(a) a pair of jeans marked at \$80 (b) a stereo marked at \$650

5 Simplify, then evaluate: Ex. 1.5, 1.6

(a) $(2^3)^2$ (b) $\frac{5^8}{5^6}$ (c) $15^9 \div 15^9$

6 Simplify each of the following expressions. Ex. 3.4

(a) $7a^2 - 5a^2$ (b) $5t^2 + 7t - t + 2t^2$ (c) $12q + 7q^2 - 3p^2 + q^2$

7 Evaluate each of the following expressions when $g = 2$ and $h = 5$. Ex. 3.2

(a) $gh - \frac{16}{g}$ (b) $7h - 8g$ (c) $30gh + g(h + 4)$

8 Simplify: Ex. 3.5

(a) $5j \times 7k$ (b) $14pq \div 2q$ (c) $2a^2 \times 3ab$

9 Factorise the following expressions. Ex. 3.7

(a) $5h - 45$ (b) $y^2 + 7y$ (c) $12mn + 8m$

10 Convert the following to decimals. Ex. 2.6

(a) 17% (b) 6.5% (c) 400% (d) 0.006%

11 Calculate, giving answers in simplest form, where necessary. Ex. 1.1–1.3, 2.3

(a) $-7 + (-8)$ (b) $-3 - (-12)$ (c) -5×6 (d) $-\frac{28}{7}$

(e) $-\frac{3}{8} + \left(-\frac{7}{10}\right)$ (f) $-1\frac{1}{8} - \left(-\frac{5}{6}\right)$ (g) $-12.83 - (-7.5)$ (h) $0.46 + (-1.9)$

12 Find the value of the unknown in each of the following. Ex. 4.4

(a) $a : 16 = 3 : 4$ (b) $8 : h = 88 : 121$ (c) $120 : 42 = 20 : n$

(d) $c : 6 = 20 : 8$ (e) $p : 12 = 3 : 5$ (f) $k : 14 = 27 : 35$

13 Share each amount given according to the ratio stated in brackets. Ex. 4.6

(a) 48 (5 : 3) (b) 630 (2 : 7) (c) 88 (2 : 3 : 6)

14 An orange juice drink has 16% actual orange juice, with the rest made up of sugar and water. What percentage of the drink is not actual juice? Ex. 2.8

15 Convert the following fractions to percentages. (Express your answer as fractional percentages where appropriate.)

Ex. 2.5

(a) $\frac{9}{100}$ **(b)** $\frac{17}{20}$ **(c)** $\frac{3}{16}$ **(d)** $2\frac{1}{4}$

16 Increase the following amounts by the percentages given.

Ex. 2.9

(a) 600 by 9% **(b)** 54 by 8% **(c)** 150 by 80%

17 Simplify each of the following ratios.

Ex. 4.1, 4.2

(a) 15 : 33 **(b)** 14 weeks : 6 weeks **(c)** 50 seconds : 2 minutes

(d) $3\frac{1}{2} : 5$ **(e)** 6 : 4.4 **(f)** 0.2 : 6.32

Understanding

18 Mustafa drove 175 km in 2 hours, then covered the next 120 km in 1.5 hours. Calculate his overall average speed, to the nearest whole number.

19 Order the following values from smallest to largest.

(a) 76%, 0.7, $\frac{7}{9}$ **(b)** 15.2%, $\frac{3}{20}$, 0.105 **(c)** $5\frac{1}{5}$, 5.06, 500%

20 A car valued at \$6000 decreases in value by 6.5% in the first year, then by 5% in the second year.

(a) What is the value of the car at the end of the first year?

(b) What is the value of the car after 2 years?

21 Sophie's and Dean's heights are in the ratio 8 : 9. If Sophie is 166 cm tall, how tall is Dean to the nearest centimetre?

22 Jenny buys a new fridge for \$529. She then moves into a share house where there is already a fridge, so she sells her new fridge for \$425.

(a) What loss has Jenny made in the sale of her fridge?

(b) Find Jenny's percentage loss by expressing the loss as a percentage of the cost price.

23 Dzung slept for $7\frac{1}{2}$ hours last night. What percentage of a day is this?

24 Michelle's pay is increased by $7\frac{1}{2}$ %. If her fortnightly pay was orginally \$1250, what is her income now?

25 The formula for converting degrees Celsius into degrees Fahrenheit is $F = \frac{9C}{5} + 32$.

Use this formula to convert the following Celsius temperatures into degrees Fahrenheit.

(a) 0°C **(b)** 20°C **(c)** 35°C **(d)** 150°C

Reasoning

26 Gerard and Hao invest \$12 000 and \$18 000, respectively, to start their small business. After the first year, the profits come to \$12 500. How should they split the profits?

27 An amount of money increases by 10% and becomes \$121. It is then increased by a further 10%.

(a) What was the final amount of money?

(b) What was the original amount of money?

(c) What single percentage increase would take you from the original amount to the final amount in one step?

5

NORTHWEST MONSOONS
SEASON
BARRA'MIRRI MAYALTHA
STRONG WINDS HEAVY RAIN LIGHT WIND
HIGH TIDES GROWTH PERIOD
MAIN FEATURES
EAST SOUTH EAST
MIDAWARR
FLAT WATER & CLOUDS
BUSH FRUITS
ROOT CROPS
BARRAMUNDI
MAGPIE GEESE
CYCAS MEDIA
SHELLFISH
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
FOOD SOURCES
MUD CRABS
TURTLE EGGS
HONEY
SMALL SHARKS AND STINGRAYS
TURTLES
THREADFIN SALMON
CLOUDY
HUMID
THUNDER AND LIGHTING
NORTH & NORTHWEST
DHULUDUR'
WIND
RARRANHDHARR
GROUND HOT TO WALK ON
NO WATER MANGO FRUIT
SOUTHEAST & NORTHEAST
CALENDAR MONTHS
COOL NIGHTS HEAVY DEWS
BURNING OFF, FOGS
DHARRATHARRA
SOUTH SOUTHEAST

Measurement 5

Timelines and timecircles. If we can measure something, we know what it is, right? Maybe.

Mathematicians can measure time. That's the easy part. Atomic clocks use the regular vibrations of atoms to achieve a high degree of accuracy – losing less than 1 second every 1 million years. But what exactly *is* time? Great minds throughout history have tried to come up with the answer to that question. Western cultures see time as a straight line, whereas Indigenous Australians think of time as a circle. For Aboriginal people, the past isn't dead and gone – the Dreaming is both part of the past and part of the present. The events themselves and what they mean for people now are important to them, not the precise time the events occurred. While Westerners talk of time*lines*, traditional Aboriginal cultures see time as the cycles and rhythms of nature: the changing seasons that bring different types of food, the rise and fall of the tides, and the waxing and waning of the Moon. The calendar opposite shows how the Milingimbi people of north-east Arnhem Land see a year of time compared with non-Indigenous Australians.

Forum

Do you think time is like a straight line or like a circle? Or is it like something else? Why do you think this?

Why learn this?

All of us use measurement in some form on a daily basis. Some professions depend on having competent measurement skills. Builders, electricians, chefs, farmers, town planners, fashion designers and environmental scientists all need to be able to calculate length, area, volume and capacity efficiently. A landscape gardener, for example, may need to calculate the length of wood required for the border of a garden bed, the volume of soil required to fill it, the area she can cover with lawn or seed, and the capacity of the rainwater tank needed to water the garden. Surveyors and cartographers use many measurements of the landscape to create accurate maps that others will rely upon. Our ability to accurately measure time allows us to determine which athlete won the race and whether they broke a record. Accurate measurement is pretty important!

After completing this chapter you will be able to:

- calculate the perimeter and area of triangles, parallelograms, rectangles, trapeziums and kites
- investigate the relationships between the radius, diameter and circumference of a circle
- understand and apply the formulas for calculating circumference and area of circles
- calculate the perimeter and area of composite shapes
- calculate the volume of prisms and cylinders
- convert between units of length, area, volume and capacity
- write and calculate elapsed time using both 24-hour and 'a.m./p.m.' notation.

Recall 5

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R5.1

1 Round each value correct to two decimal places.

(a) 7.823 **(b)** 15.0047

(c) 0.986 **(d)** 106.4971

Worksheet R5.2

2 Calculate each of the following, correct to two decimal places if appropriate.

(a) 4.13×6 **(b)** 5.25^2

(c) 3.14×12 **(d)** 7.9×8.52

(e) $6.3 \times (5.1 + 4.4)$ **(f)** 13.9×7.8^2

Worksheet R5.3

3 Copy and complete each of the following conversions.

(a) 4.5 km = _____ m **(b)** 69 cm = _____ m

(c) 4.8 cm = _____ mm **(d)** 46 mm = _____ cm

(e) 6780 mm = _____ m **(f)** 90 000 mm = _____ km

Worksheet R5.4

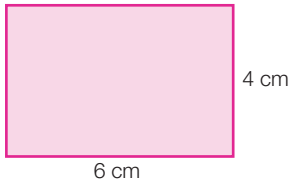
4 Calculate each of the following, expressing your answers in metres.

(a) 1.2 m + 40 cm + 5 cm + 3 m **(b)** 3.245 m + 73 cm + 108.9 cm

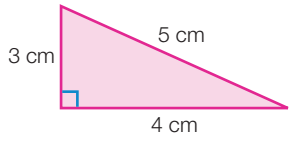
Worksheet R5.5

5 Find **(i)** the perimeter and **(ii)** the area of the following shapes.

(a)



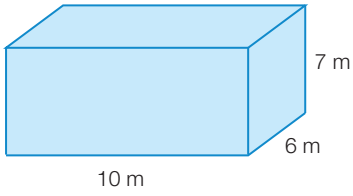
(b)



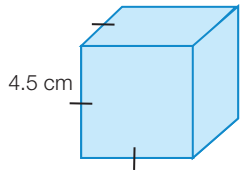
Worksheet R5.6

6 Find the volume of the following solids.

(a)



(b)



Key Words

24-hour time	diameter	perpendicular	trapezium
area	elapsed time	prism	triangle
base	hectare	radius	uniform cross-section
capacity	height	rectangle	volume
circumference	kite	rhombus	
composite shape	parallelogram	square	
cylinder	perimeter	time zone	

Perimeter

5.1

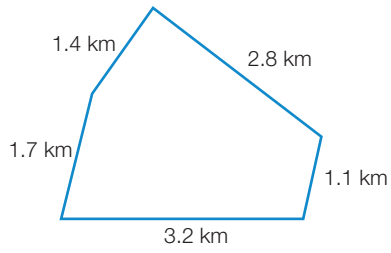
Perimeter is the total length of the boundary of a shape. It is measured using km, m, cm or mm. We often use markings such as | and || to indicate equal lengths when drawing diagrams or shapes. For example, two lines marked | are the same length.

Worked Example 1

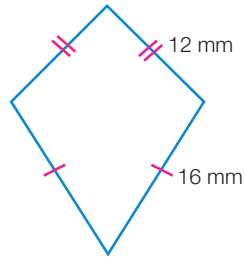
WE1

Find the perimeter of each of the following shapes.

(a)



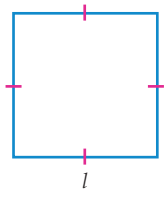
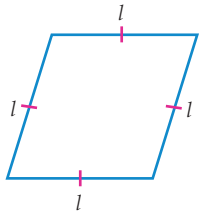
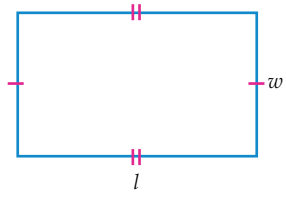
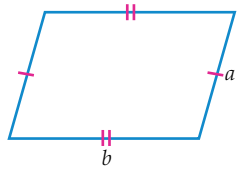
(b)



Thinking		Working
(a) 1	Add the individual lengths.	(a) $P = 1.7 + 1.4 + 2.8 + 1.1 + 3.2$
2	Write the total length, including the unit.	$= 10.2$ km
(b) 1	The sides marked with the same symbol have equal lengths. Multiply the lengths by the number of times each appears.	(b) $P = 2 \times 12 + 2 \times 16$
2	Add the products.	$= 24 + 32$
3	Write the total length, including the unit.	$= 56$ mm

Perimeter formulas

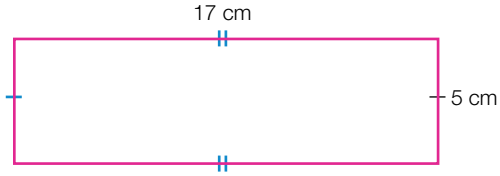
We can use formulas to find perimeters, using the same pronumeral for sides of equal length. Any pronumeral can be used; however, we usually use the letters l (length) and w (width) for rectangles, and b (base) for distances along the bottom of some shapes, such as triangles or parallelograms. If there is no convenient formula to use, we simply add all the side lengths, using multiplication to simplify the calculation when equal lengths are added.

Square	Rhombus	Rectangle	Parallelogram
 $P = 4l$	 $P = 4l$	 $P = 2l + 2w$ or $P = 2(l + w)$	 $P = 2a + 2b$ or $P = 2(a + b)$

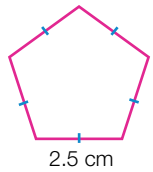
Worked Example 2 WE2

Use a formula to calculate the perimeter of each of the following shapes.

(a)



(b)



Thinking	Working
(a) 1 Write the formula for the perimeter of a rectangle.	(a) $P = 2l + 2w$
2 Substitute values for length and width.	$= 2 \times 17 + 2 \times 5$
3 Add the products.	$= 34 + 10$
4 Write the total length, including the unit.	$= 44$ cm
(b) 1 Let s be the length of the side marked I. Write a formula that multiplies s by the number of equal sides.	(b) $P = 5s$
2 Substitute the side length.	$= 5 \times 2.5$
3 Write the product with units of length.	$= 12.5$ cm

5.1 Perimeter

Navigator

Answers page 617

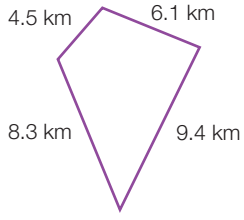
Q1 Column 1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q11	Q1 Column 2, Q2, Q3, Q4, Q5, Q7, Q8, Q9, Q11	Q1 Column 3, Q2, Q3, Q5, Q7(b), Q8, Q9, Q10, Q12

Fluency

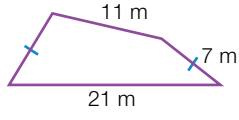
WE1

1 Find the perimeter of each of the following shapes.

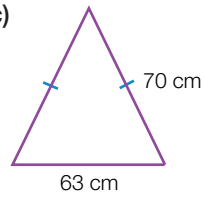
(a)



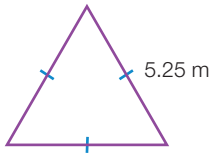
(b)



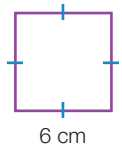
(c)



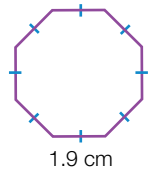
(d)



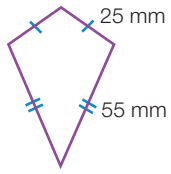
(e)



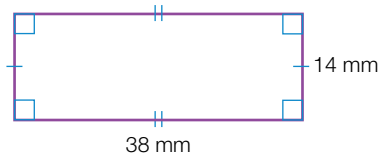
(f)



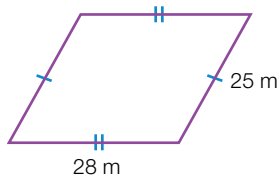
(g)



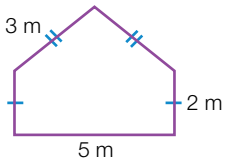
(h)



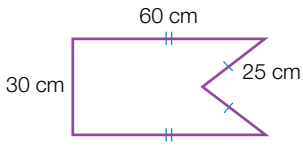
(i)



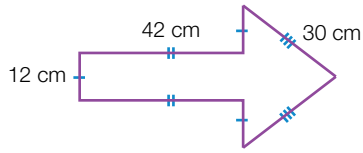
(j)



(k)



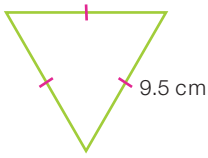
(l)



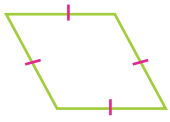
2 Use a formula to calculate the perimeter of each of the following shapes.

WE2

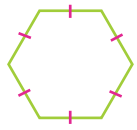
(a)



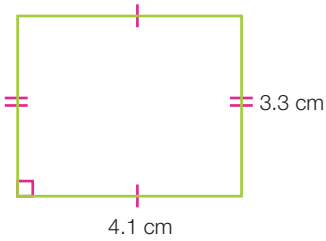
(b)



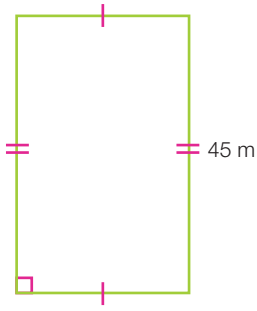
(c)



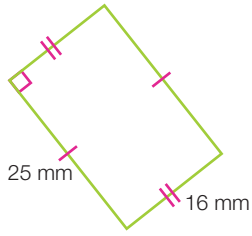
(d)



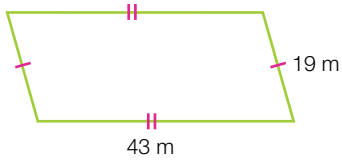
(e)



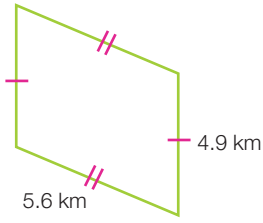
(f)



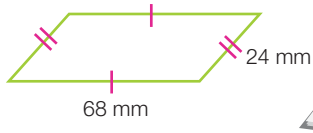
(g)



(h)



(i)



3 Find the perimeter of the following (draw a diagram if necessary).

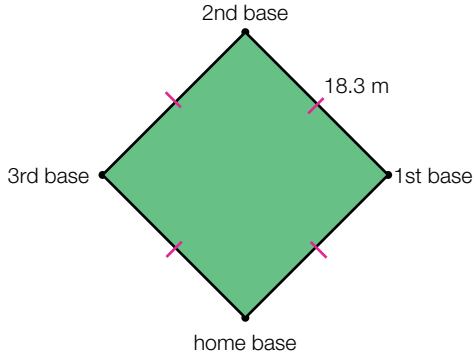
(a) a square of side length 57 mm

(b) a rectangle of width 5.8 m and length 10.7 m

(c) a parallelogram of base 66 cm and a sloping side length of 32 cm

(d) a rhombus of side length 4.25 mm.

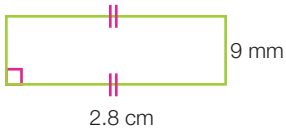
4 How far is it around the softball diamond shown?



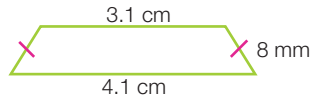
Understanding

5 Find the perimeter of each shape below.

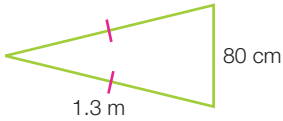
(a)



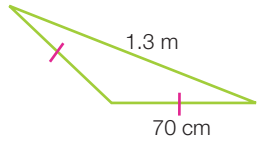
(b)



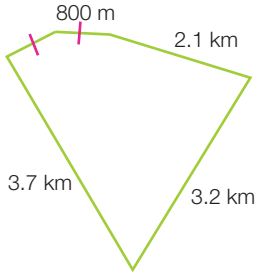
(c)



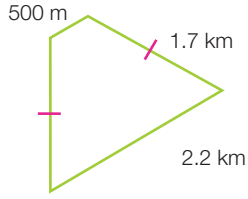
(d)



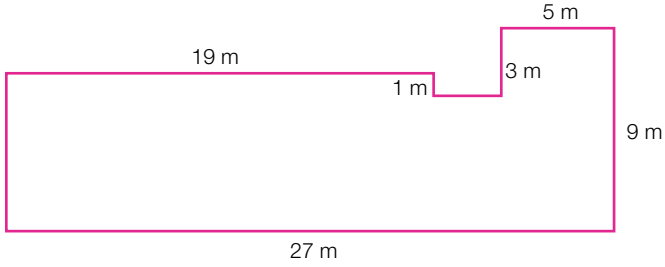
(e)



(f)



6 The roof plan for a new house appears here. Use it to calculate the length of guttering required if guttering is to be attached along each edge of the roof.



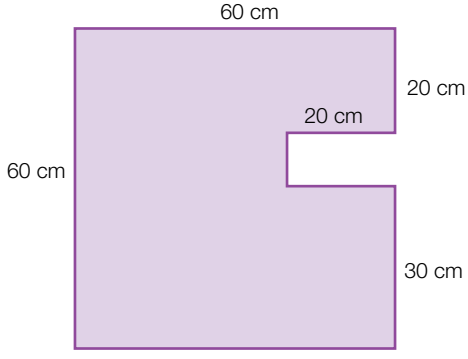
7 **(a)** The length of tape needed to go along the perimeter of the figure shown in the diagram is:

A 190 cm

B 200 cm

C 220 cm

D 280 cm



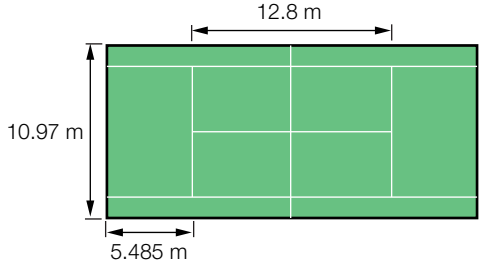
(b) The perimeter of the tennis court shown here is:

A 23.77 m

B 29.255 m

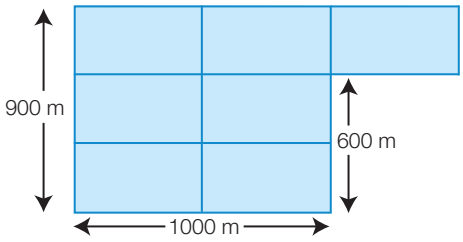
C 58.51 m

D 69.48 m



Reasoning

8 A farmer needs to fence a new property. The fencing needs to go along the boundary of the property, and also divide it up into rectangles, as shown.



(a) What is the total length of fence required?

(b) If the type of fence used by the farmer costs $5.75 per metre, what will it cost to fence the property?

(c) If the fencing contractor can build the fence at a rate of 20 metres per hour, how long will the job take?

9 **(a)** Find the side length of a square if it has the same perimeter as a rectangle of length 19 cm and width 11 cm.

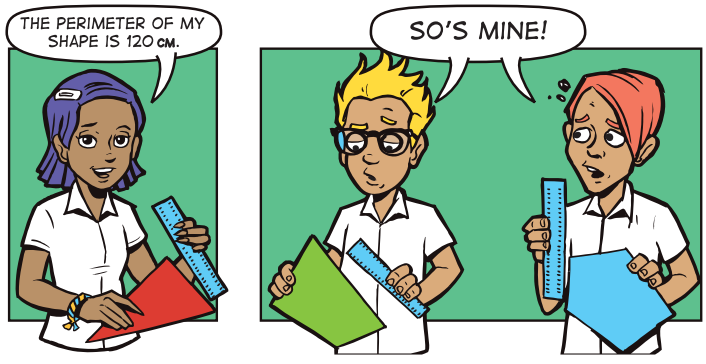
(b) Find the width of a rectangle if its length is 28 cm and it has the same perimeter as a square of side 20 cm.

10 **(a)** Find the perimeter of a square tiled area containing 25 square tiles if each tile is 40 cm wide.

(b) If the number of tiles is doubled, find the perimeter of all the different rectangles that can be formed, using every tile.

Open-ended

11 Write possible side lengths for the triangle, quadrilateral and pentagon that the students are holding below.



12 Brett is designing a new vegetable garden for his backyard.

(a) Brett's friend Emma is helping him and needs to know the dimensions of the garden. Brett simply tells Emma that the garden will be in the shape of an isosceles triangle, and will have a perimeter of 56 m. Find three possible sets of measurements for Brett's vegetable garden if the side lengths have to be whole numbers of metres.

(b) If Brett tells Emma that the perimeter can be any length from 56 m to 100 m, find two new sets of dimensions for Brett's vegetable garden if the two equal sides of the isosceles triangle are to be twice as long as the third side. All side lengths need to be whole metres.

Outside the Square Game

That formula is mine

How to win:
The winner is the first to 100 points.

List of formulas

$P = 2a + 2b + c$

$P = 4a + b$

$P = 4a + 2b$

$P = 2a + b$

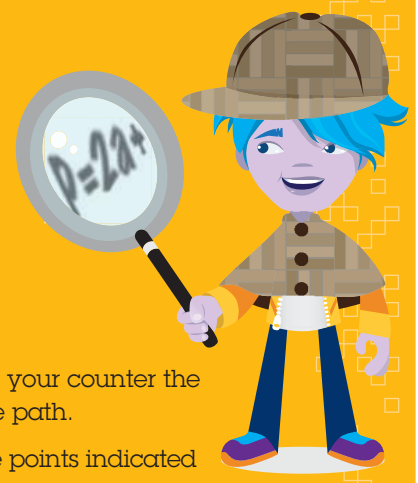
$P = 3a + 3b$

$P = 6a + 2b + c$

$P = 4a + b + c$

$P = 3a + 2b + c$

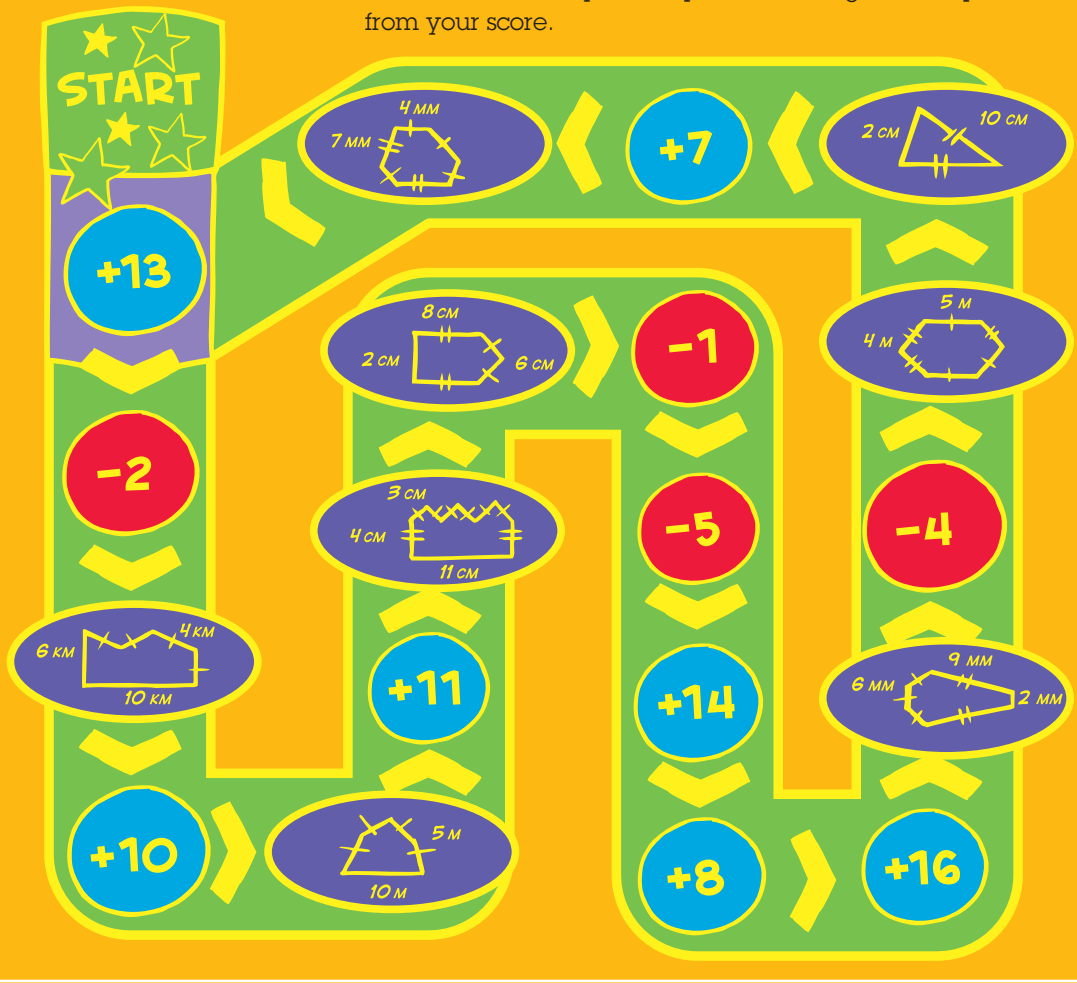
Equipment required: 2 brains, 1 die, 2 counters



How to play:

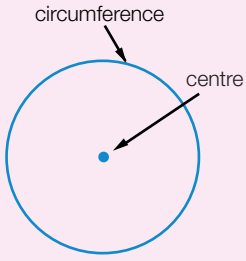
1. Take turns choosing a formula from the list until all formulas are taken.
2. Each person starts with 30 points.
3. On your turn, roll the die and move your counter the indicated number of steps along the path.

- If you land on a blue circle, add the points indicated to your score.
- If you land on a red circle, subtract the points indicated from your score.
- If you land on a shape for which you own the formula for its perimeter, score points equal to the length of the perimeter.
- If you land on a shape that goes with one of your opponent's formulas, subtract points equal to the length of the perimeter from your score.

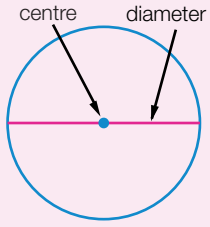


Circle relationships

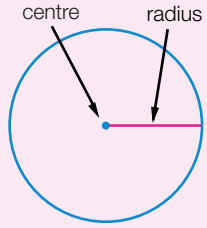
5.2



The perimeter of a circle is called the **circumference**.



The **diameter** of a circle is any straight line from the circumference through the centre to the circumference at the other side.



The **radius** of a circle is any straight line from the centre to the circumference.

PP Discovery Task

In order to calculate the perimeters and areas of circles and shapes that have circular parts, we need to understand how the circumference, diameter and radius of a circle are related to each other. In this exercise, we will investigate and establish these relationships.

5.2 Circle relationships

Navigator

Q1, Q2, Q3, Q4, Q6(a), Q7, Q9	Q1, Q2, Q3, Q4, Q5, Q6(a), Q7, Q9	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8 Q9

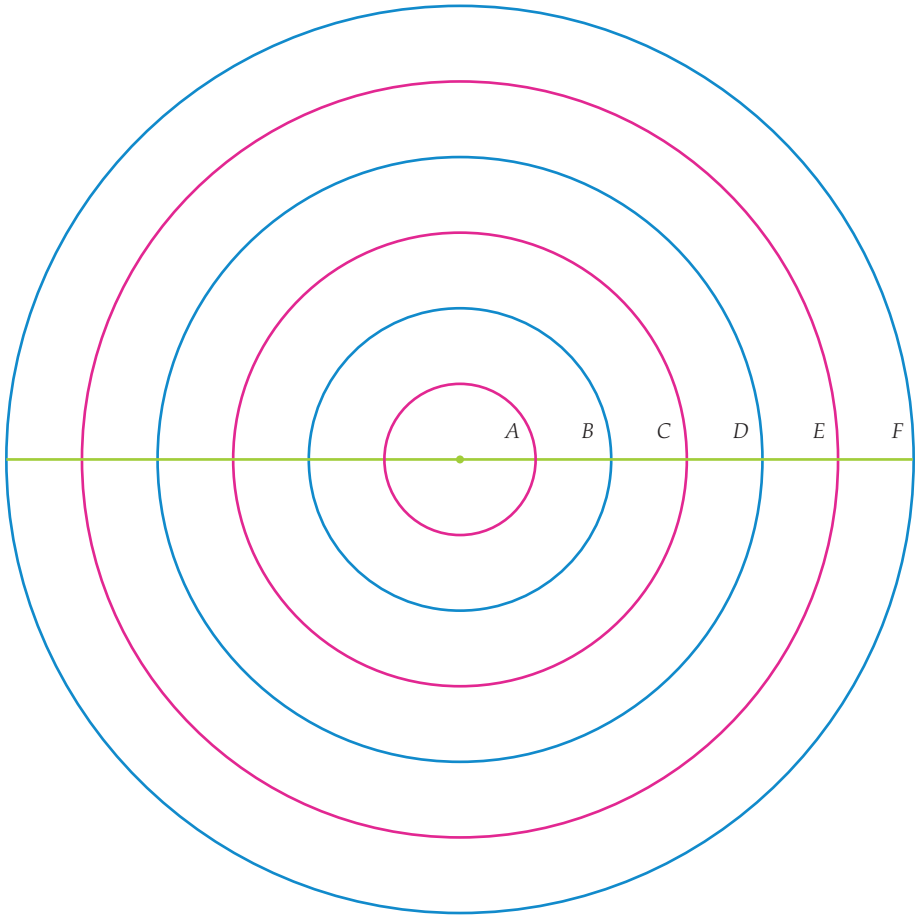
Answers page 617

Equipment required: Ruler, string, compass (optional), circular objects such as cups, plates, lids, drink bottles, wheels (optional) for Question **1**; a scientific calculator for Question **7(b)**

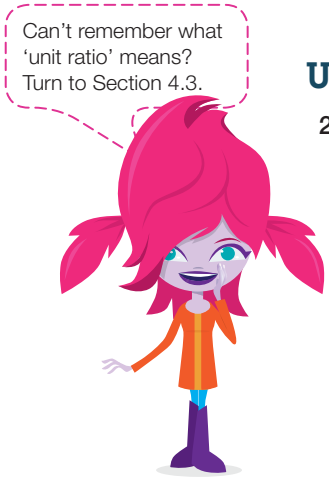
Fluency

1 Use your ruler to measure the diameter and radius of circles *A*–*F* given on the following page. Record your measurements in the table underneath. Measure and record the circumference of each circle by carefully laying the string along it. (A classmate can help you do this reasonably accurately.) Mark the beginning and end of the circumference on the string, then hold it straight against a ruler and read off the length of the circumference.

Alternatively, choose six circular objects, or use your compass to draw six circles of different sizes, then measure and record the diameter, radius and circumference using your ruler and string. (If you are using a compass, remember that the distance to which you open your compass is the same as the radius of the circle.)



Circle	Radius r (cm)	Diameter d (cm)	Circumference C (cm)
A			
B			
C			
D			
E			
F			



Understanding

2 **(a)** Use your completed table from Question **1** to describe any patterns or connections you can see between the numbers in the 'radius', 'diameter' and 'circumference' columns.

(b) To see the connections more clearly, add two columns to the end of your table, with the headings $\frac{C}{d}$ and $\frac{C}{r}$. Calculate these two unit ratios for each circle that you measured, round off to two decimal places, and record your results in the new columns. These unit ratios give us a scale factor for the length of the circumference compared to the diameter or radius.

3 Use your completed table from Question **1** and the unit ratios you calculated in Question **2** to copy and complete the following sentences.

(a) The circumference of a circle is approximately _____ times the size of its diameter.

(b) The circumference of a circle is approximately _____ times the size of its radius.

(c) The length of the diameter of a circle is _____ the length of its radius.

Reasoning

4 Based on your answers to Questions **2** and **3**, if a circle has a diameter of 4 m, predict the lengths of the:

(a) radius **(b)** circumference.

5 Based on your answers to Questions **2** and **3**, predict what will happen to:

(a) the diameter **(b)** the circumference

when the radius of the circle is doubled.

6 **(a)** Explain whether or not the measurements you obtained for the circumference by measuring with string and a ruler are accurate.

(b) How would your calculations of $\frac{C}{d}$ and $\frac{C}{r}$ be affected if you measured the circumference as being:

(i) smaller than actual size **(ii)** bigger than actual size?

7 **(a)** The exact value of the ratio $\frac{\text{circumference}}{\text{diameter}}$ is represented by the symbol π (the Greek letter pi).

π is an irrational number, which means that if written as a decimal, it is a non-terminating, non-recurring decimal. Describe what the digits in a 'non-terminating, non-recurring' decimal look like.

(b) Find and press the 'π' key on your calculator. Write the value of π as a decimal approximation, rounded to:

(i) two decimal places **(ii)** three decimal places **(iii)** four decimal places.

Open-ended

8 **(a)** Choose a length for the diameter of a circle that lies in between the lengths of two diameters listed in the table you drew in Question **1**. (For example, if your table contains diameters of 4 cm and 6 cm, you could choose 5 cm or 5.5 cm.) Use your circumference measurements for the two known diameters to estimate the circumference of a circle with your chosen diameter.

(b) Repeat part **(a)** for a different value that lies in between two different diameters.

9 Cricket grounds vary in diameter and circumference. (For example, the SCG is much smaller than the MCG.) Estimate the diameter of a cricket ground (perhaps your school oval) and then find the approximate distance a cricket team will run when they do a lap of honour after winning a championship on that ground. Assume the cricket ground is circular.

Equipment required: 1 brain, 1 computer with GeoGebra, 1 calculator

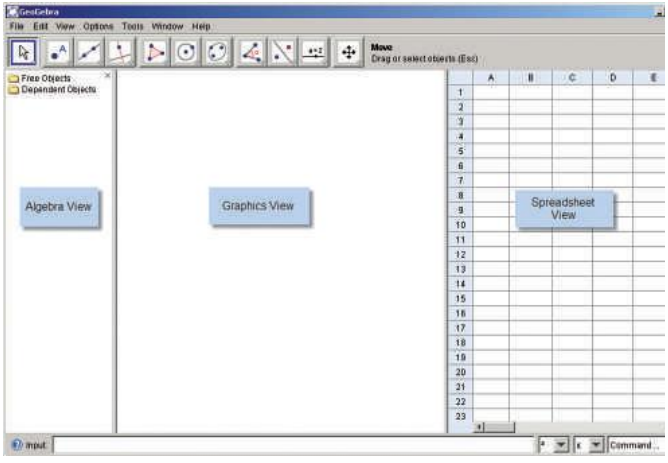
The relationship between the circumference and diameter of a circle

Open the GeoGebra program. You will see seven menu options (File, Edit etc.) at the top of the screen. Below these are 11 icons called tools. In this task, tools will be referred to by their position from the left-hand side. By clicking on the small arrow in the bottom right-hand corner of a tool icon, a drop-down list of more tools appears. The arrow turns red when you hover the cursor over it. If you hover over a tool, the tool's name and how to use it will appear in the top right-hand corner of the screen. (Some versions of GeoGebra differ in the name given to a tool; however, the icon remains the same. The names used in this activity are taken from GeoGebra Webstart.)

1 Click on the View menu. Deselect 'Axes' and 'Grid'. (Alternatively, you can right click in the 'Graphics View', the space where the drawings appear, to do the same.)

2 Click on the View menu. Select 'Spreadsheet View'. Resize the Spreadsheet View by placing the cursor over the line separating the spreadsheet from the Graphics View. When it becomes a double arrow, click and drag to the right. Leave five columns visible.

3 If a larger font is required, click on the options menu and select 'Font Size'. Choose an appropriate size from the list provided. The screen shot below shows how your screen should appear. The three parts of the screen have been labelled Algebra View, Graphics View and Spreadsheet View.



Investigating the relationship between the diameter and circumference of a circle

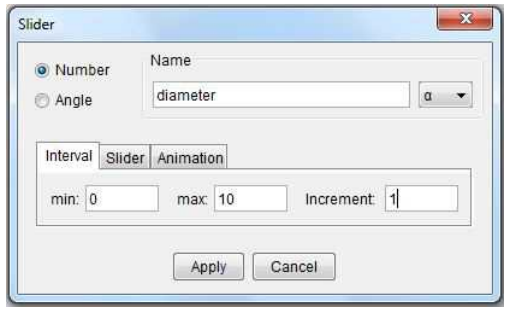
The perimeter of a circle is called the circumference.

The radius of a circle is the distance from the centre of the circle to the circumference.

The diameter of a circle is any straight line from the circumference through the centre to the circumference at the other side. It is double the size of the radius.

4 Create a slider. From icon 10, select the 'Slider' tool . Click in the Graphics View where you want the slider to appear (at the top of the page is best).

5 In the Slider dialog box, name the slider 'diameter'. Set the minimum value to 0, the maximum to 10 and the increment to 1. Now, select the 'Slider' tab and set the width to 200. (This makes the slider look longer on the screen.) Click 'Apply'.



6 From icon 6, select the 'Circle with centre and Radius' tool . Click in the centre of the Graphics View. For the value of the radius, type 'diameter/2' into the dialog box.

7 From icon 8, select the 'Distance or Length' tool . Measure the circumference by clicking on the circle. The measurement will appear next to the circle and also in the Algebra View, where it is called

'circumference*c*'. Note that the letter '*c*' appears on the end of the word circumference because it refers to the circumference of circle *c*. A symbol instead of a number may appear for your circumference. This will be investigated later.

8 Click on the 'Select' tool , then move the slider to change the length of the diameter. What does it mean when it says 'circumference = undefined' for a diameter of 0?

We now want to copy these measurements to the Spreadsheet View. To do this, one option is to create a point that uses the measurements for diameter and circumference and use the 'Trace Point' feature of GeoGebra.

9 Set the slider to 10. In the 'Input' bar at the bottom of the screen, type '(diameter, circumference*c*)' (don't forget the letter *c* on the end of the word circumference). Press 'Enter'.

If you look at the Algebra View on the left-hand side of the screen, a new point *B* will appear under 'Dependent Objects'.

10 Right click on point *B* in the Algebra View and select 'Trace to Spreadsheet'. You will notice that the coordinates of point *B* now appear in the spreadsheet on the right-hand side of the screen.

11 Move the slider across to the left and notice how the spreadsheet fills with data. The screen shot below shows a partially filled spreadsheet.

(a) What do the values in column A represent?

(b) What do the values in column B represent?

12 Use your calculator to divide the circumference by the diameter for each row (i.e. divide the value in cell B1 by the value in cell A1). Do this for all pairs of values. Type the answer to your division, rounded to two decimal places, into the cell next to the corresponding pair. What do you notice about the values you have calculated?

13 We will now get the spreadsheet to perform this calculation for us. Click in cell D1 and type '=B1/A1', then press Enter.

14 Click on the blue square on the right-hand side of cell D1 and drag the cursor down to the last row of your calculations.

(a) What do you notice about the values that were calculated by the spreadsheet?

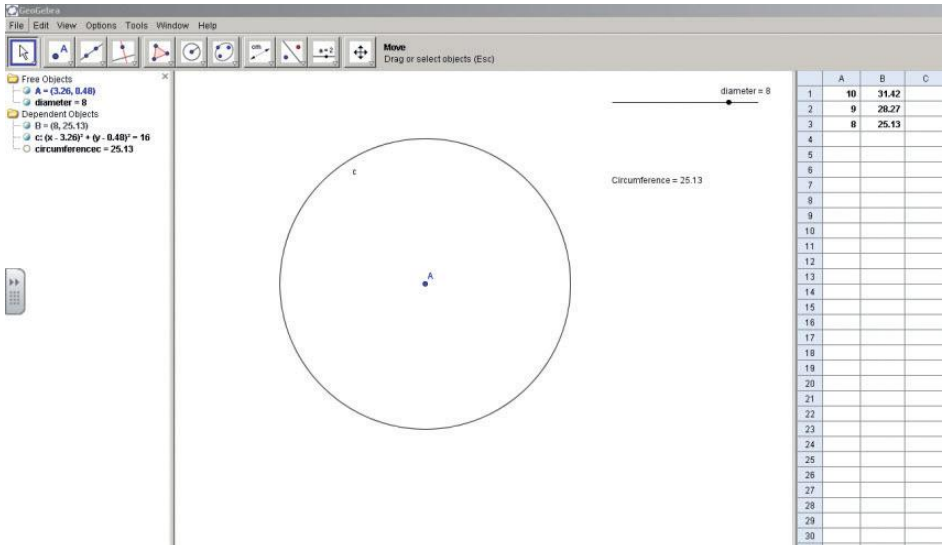
(b) How do your calculations compare to the calculations made by the GeoGebra spreadsheet?

15 (a) Write a sentence about the relationship between the circumference and the diameter of a circle.

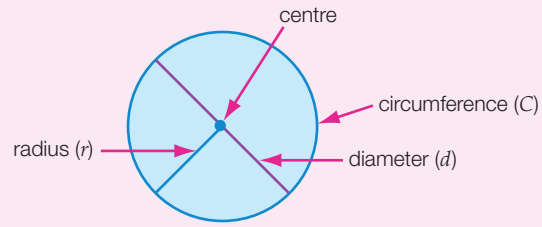
(b) Write a formula to link the circumference (*C*) and the diameter (*d*).

Taking it further

Research the special symbol that appeared in this spreadsheet investigation. Write a paragraph summarising your findings.



5.3 Circumference



For any circle:

- the length of the diameter, d, is twice the length of the radius: $d = 2r$
- the length of the circumference, C, is a bit more than three times the diameter: $C \approx 3d$. (Remember that the ≈ sign means 'approximately equal to'.)

The exact formula for circumference is given by: $C = \pi d$, where π (the Greek letter *pi*) is an irrational number (a non-terminating, non-recurring decimal).

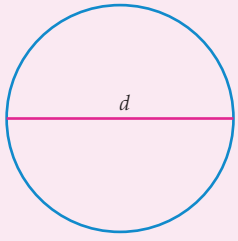
Calculators give an approximate value of π of 3.141 592 654. Before the widespread use of calculators it was common to use $\pi \approx 3\frac{1}{7}$ or $\frac{22}{7}$ or $\pi \approx 3.14$ (correct to two decimal places). You should use the π key on your calculator instead of these, unless you are estimating circle measurements or your calculator does not have a π button.

The value of π cannot be written as an exact decimal, but computers have been able to approximate it correct to more than a trillion digits!

Formulas for calculating the circumference of a circle

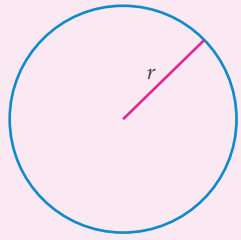
$C = \pi d$

where d is the length of the diameter



$C = 2\pi r$

where r is the length of the radius

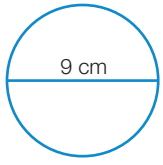


Worked Example 3

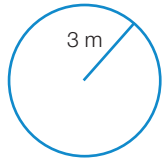
WE3

Calculate the circumference of the following circles correct to two decimal places.

(a)



(b)



Thinking	Working
(a) 1 Choose the formula that involves diameter.	(a) $C = \pi d$

2	Substitute the value for d into the formula.	$C = \pi \times 9$
3	Use the π button on your calculator to evaluate.	$\approx 28.274\,333\,88$
4	Round the answer to two decimal places and write it with the correct unit.	$C = 28.27$ cm (correct to 2 d.p.)
(b) 1	Choose the formula that involves radius.	(b) $C = 2\pi r$
2	Substitute the value for r into the formula.	$C = 2 \times \pi \times 3$
3	Use the π button on your calculator to evaluate.	$\approx 18.849\,555\,92$
4	Round the answer to two decimal places and write it with the correct unit.	$C = 18.85$ m (correct to 2 d.p.)

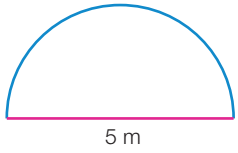
The perimeters of shapes may involve both curved sections and straight sections. If the curved sections are parts of circles, we can calculate their length as a fraction of the circumference and then find the total length of the perimeter.

Worked Example 4

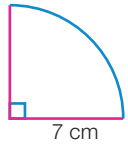
WE4

Find the perimeter of each of the following shapes correct to two decimal places.

(a)



(b)



Thinking		Working
(a) 1	Identify the parts that make up the perimeter (half of a circumference and a straight line equal to the diameter). Use the diameter formula for circumference to write a formula for perimeter.	(a) $P = \frac{C}{2} + d$ where $C = \pi d$ $P = \frac{\pi d}{2} + d$
2	Substitute for d.	$= \frac{\pi \times 5}{2} + 5$
3	Evaluate the curved length, checking that it is a reasonable value. Leave the answer as a long, unrounded decimal.	$\approx 7.853\ldots + 5$
4	Add the straight length to find the total and write the answer rounded to the correct number of decimal places with the correct units.	$= 12.85$ m (correct to 2 d.p.)

(b) 1	Identify the parts that make up the perimeter (a quarter of a circumference and two straight lines equal to the radius). Use the radius formula for circumference to write a formula for perimeter.	(b) $P = \frac{C}{4} + 2r$ where $C = 2\pi r$ $P = \frac{2\pi r}{4} + 2r$
2	Substitute for r.	$= \frac{2 \times \pi \times 7}{4} + 2 \times 7$
3	Evaluate the curved length and the total of the lines. Leave the answer as a long, unrounded decimal.	$\approx 10.995\ldots + 14$
4	Add the straight lengths to find the total and write the answer rounded to the correct number of decimal places with the correct units.	$= 25.00$ cm (correct to 2 d.p.)

It is more accurate to keep all of the decimal places on your calculator until you reach your final answer, then round off.

5.3 Circumference

Navigator

Answers page 618

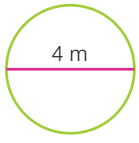
Q1 Columns 1 & 2, Q2 Column 1, Q3, Q4, Q5, Q6, Q8, Q11	Q1 Columns 1 & 2, Q2 Column 2, Q3, Q4, Q5, Q6, Q8, Q10, Q11	Q1 Column 3, Q2, Q3, Q5, Q6, Q7, Q8, Q9, Q11

Equipment required: Scientific calculator

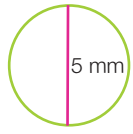
Fluency

WE3

1 Calculate the circumference of the following circles correct to two decimal places.

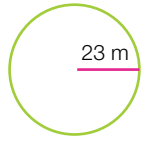
(a)


(b)

(c)


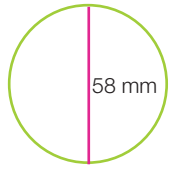
(d)

(e)

(f)


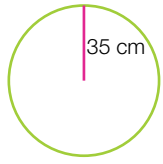
(g)

(h)

(i)


(j)

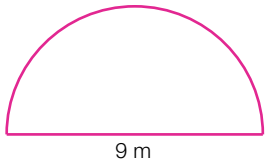
(k)

(l)


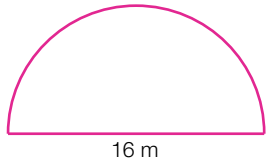
2 Find the perimeter of each of the following shapes correct to two decimal places.

WE4

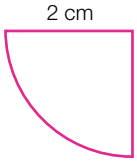
(a)



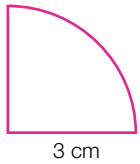
(b)



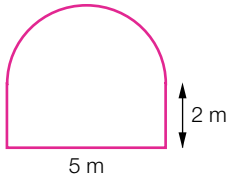
(c)



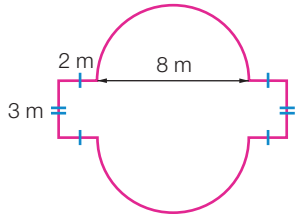
(d)



(e)



(f)



Understanding

3 (a) The distance around the outside of a round biscuit tin lid, if the lid has a radius of 21 cm, is closest to:

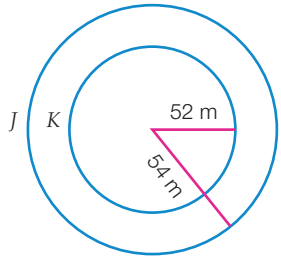
A 66 cm **B** 131 cm **C** 132 cm **D** 133 cm

(b) A bike tyre has a diameter of 70 cm. The circumference of the tyre is closest to:

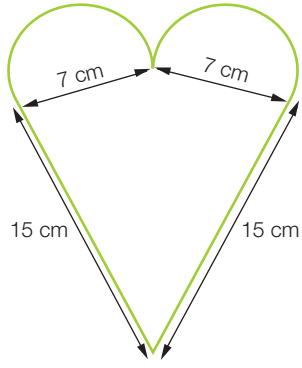
A 110 cm **B** 220 cm **C** 290 cm **D** 440 cm

4 A bicycle wheel has a diameter of 60 cm. How far would it move if it is rolled through four revolutions (complete turns)? Give your answer in metres, correct to two decimal places.

5 Jane and Kate run once around the circular track shown. Jane runs along path *J*, while Kate takes path *K*. How much further than Kate does Jane run? Give your answer correct to two decimal places.



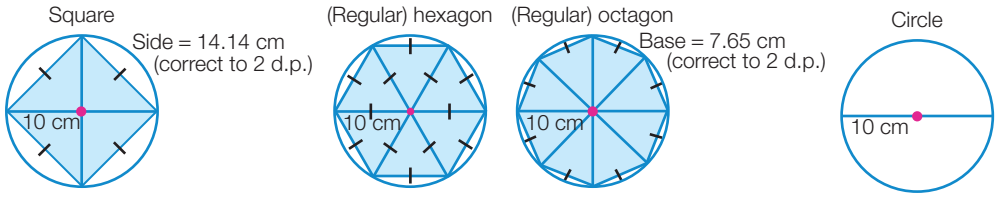
6 A box of chocolates is shaped like a heart as shown. Find the perimeter of the box correct to the nearest mm.



Reasoning

7 **(a)** For the shapes below, calculate the length of the perimeter. Use the values to copy and complete the table, rounding values to two decimal places where appropriate.

	Perimeter or circumference, *P* (cm)	Diagonal length or diameter, *D* (cm)	Ratio $\frac{P}{D}$
Square		20	
Hexagon		20	
Octagon		20	
Circle		20	



(b) For a circle, $\frac{P}{D}$, or $\frac{\text{circumference}}{\text{diameter}} = \pi$, which is slightly more than 3. For which shape was the ratio $\frac{P}{D}$ exactly 3?

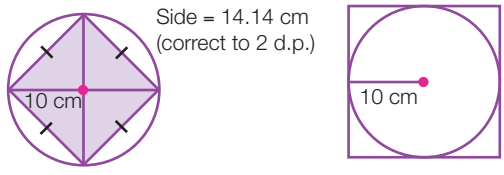
(c) Which of the three polygons above has a ratio of $\frac{P}{D}$ closest to π?

(d) Name another polygon for which the ratio $\frac{P}{D}$ would be even closer to π.

8 **(a)** Find the approximate perimeter of the internal square.

(b) Find the perimeter of the external square.

(c) Calculate C using $C = 2\pi r$. Comment on this value in comparison to the values you found for the perimeters of the two squares in parts **(a)** and **(b)**.

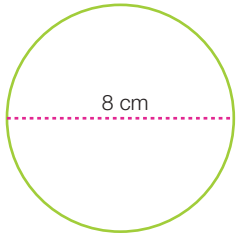


Open-ended

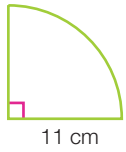
9 A semi-circle has a perimeter between 50 cm and 60 cm. Write down a possible value for the radius of the semi-circle.

10 Riya found the circumference of a circle, but lost her working and could only remember part of the answer. Her answer began with the digits 314 followed by some zeroes. Riya's friend said she could tell that the diameter must have been approximately a power of 10, such as 100 or 1000. Explain how her friend knew this.

11 Yogi was asked to find the perimeter of the following two shapes. Here is his working:



$C = 2 \times \pi \times 8$
$= 50.27$ cm (2 d.p.)



$P = \dfrac{2 \times \pi \times 11}{4}$
$= 17.28$ cm (2 d.p.)

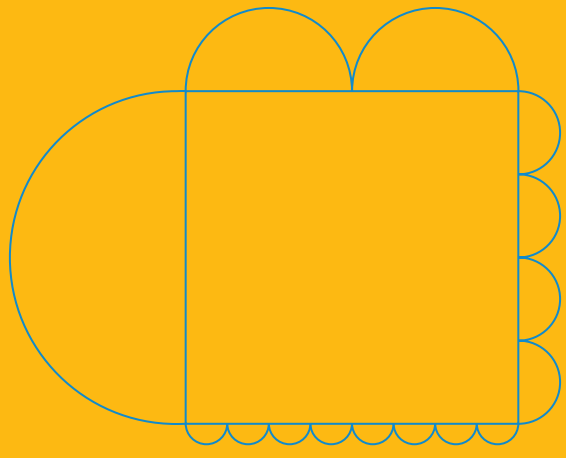
(a) Explain how you can tell, without using a calculator, that each answer is incorrect.

(b) For each one, explain the mistake that Yogi has made.

(c) Calculate the correct answers.

Outside the Square — Problem solving

The icing on the cake



A large square cake is being decorated with a series of curved pieces of icing, each in the shape of a semicircle. The semicircles along any edge are the same size as each other.

1 If the biggest curve of icing is 80 cm long, what are the lengths of each of the other curved pieces?

2 What is the total length of icing around the cake?

Strategy options

- Make a table.
- Break problem into manageable parts.

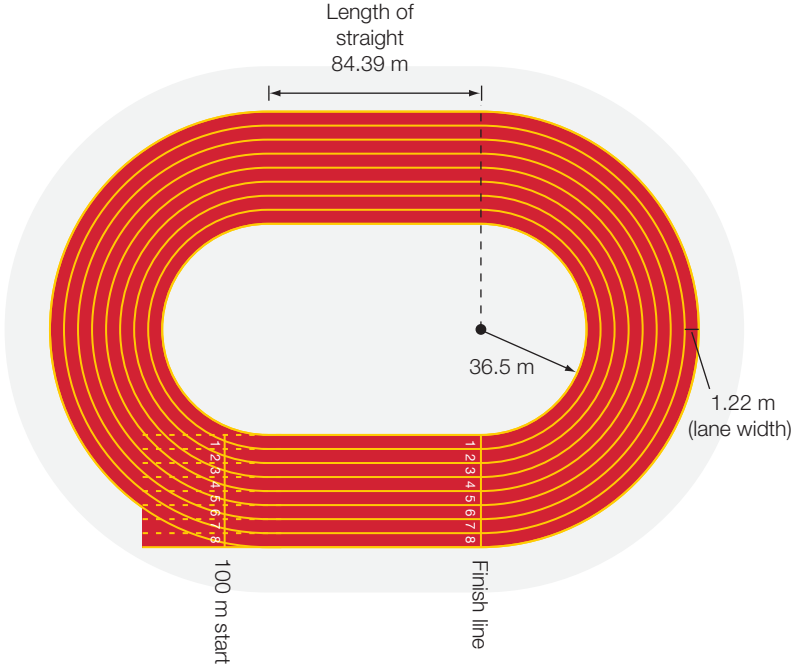
Investigation

Staggered starts

Equipment required: 1 brain, 1 scientific calculator

An Olympic-standard running track must be constructed according to very strict guidelines. The International Association of Athletics Federations (IAAF) specifies that it must be made up of two straight sections that are 84.39 m in length, and two curved sections that have an internal radius of 36.5 m. The running lanes must be 1.22 m wide.

Athletes in the 200 m and 400 m races line up in a 'staggered start' formation, in which the competitor in Lane 2 starts in front of Lane 1, Lane 3 starts in front of Lane 2, and so on, to Lane 8. Can you see why this is necessary?



The Big Question

How far in front of the competitor in Lane 1 should the competitor in Lane 8 start to ensure a 'fair' race?

Engage

1 Athletes must remain within their lane when running or they will be disqualified. Top athletes train to run as close as possible to the inside line of their lane (the one closer to the inside of the track). Why is this?

Explore

2 Use the measurements provided by the IAAF to calculate and compare the distances run by athletes in Lane 1 and Lane 8 if they run one complete lap of the track. (You may have to make some assumptions about how far the athlete runs from the lane lines. If you do, make sure you state any assumptions clearly.)

Strategy options

- Draw a diagram.
- Have I seen a similar problem?
- Break problem into manageable parts.

Explain

3 **(a)** If the perimeter of the track is calculated using the measurements for the internal radius and straight sections given on the diagram, it is less than 400 m. Why is this?

(b) By how much should you adjust the radius so that one lap of the track is exactly 400 m?

(c) How do your answers to **(a)** and **(b)** apply to an athlete running in that lane?

4 Describe how to determine the radius of the bend run by the athlete in Lane 8.

Elaborate

5 Use your answers to Questions **2**, **3** and **4** to answer the Big Question. Explain any assumptions you have made.

6 The distance that the athlete in Lane 8 stands in front of the athlete in Lane 1 at the start of the race is called the 'stagger distance'. Use the 'stagger distance' you have found for Lane 8 to determine the stagger distances for each of Lanes 2 to 7.

Evaluate

7 Describe how you approached this problem. List the main steps or tasks you undertook in order to solve it.

8 Did you need to make any assumptions in order to solve the problem? What were they? Do you think they were reasonable?

9 In races such as the 200 m and the 400 m, the middle two lanes (Lane 4 and Lane 5) are seen as being the best lanes to run in, and are usually given to the fastest qualifiers for the race. Suggest some possible reasons for this.

10 For an athletics track to be rated as 'Olympic standard', it must meet very strict and specific measurement standards. Why do you think this is?

11 If you were the person assigned the task of marking the starting positions for each lane on the track, how would you go about it?

Extend

12 Imagine that the workers constructing the track decided to simply round up the measurements for the radius and the straights to 37 m and 85 m, respectively.

(a) What effect would this have on the total length of the track?

(b) The formula $t = \frac{d}{s}$ can be used to calculate the time taken to complete a race, where d = distance and s = average speed. Olympic-standard 400 m runners can run at an average speed of 8.9 m/s. Use the formula to calculate the time they would take to complete 400 m, running at that speed.

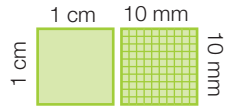
(c) Now, calculate how long it would take the same runner to complete a race on the track that is the length you calculated the length in **(a)**.

(d) Find the difference in your answers to **(b)** and **(c)**. Would you say it is a significant difference in terms of comparing the results of the race with other races?

5.4 Area

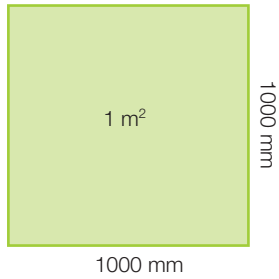
Converting units of area

Area is the amount of surface within a boundary. Area is measured in 'square' units; that is, square millimetres (mm^2), square centimetres (cm^2), square metres (m^2) or square kilometres (km^2). When we find the area of a shape, we are counting the number of whole squares and parts of whole squares that would cover the surface exactly.



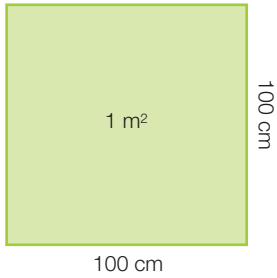
1 cm = 10 mm

$1\ cm^2 = 10\ mm \times 10\ mm$
$= 100\ mm^2$



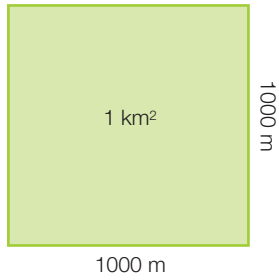
1 m = 1000 mm

$1\ m^2 = 1000\ mm \times 1000\ mm$
$= 1\,000\,000\ mm^2$



1 m = 100 cm

$1\ m^2 = 100\ cm \times 100\ cm$
$= 10\,000\ cm^2$

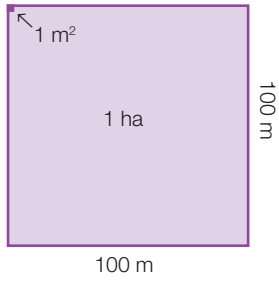


1 km = 1000 m

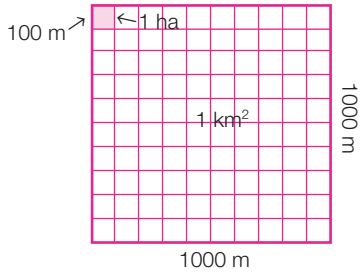
$1\ km^2 = 1000\ m \times 1000\ m$
$= 1\,000\,000\ m^2$

Area can also be measured in units called **hectares**. A hectare (ha) is an area of $10\,000\ m^2$, a square that is 100 m long and 100 m wide. Hectares are often used to describe the size of a property, such as a farm. A hectare is smaller than a square kilometre. It is approximately equal to the combined area of two hockey or soccer pitches placed side by side. A square kilometre is the approximate size of a small suburb.

$1\ ha = 10\,000\ m^2$



$1\ km^2 = 100\ ha$



It is important to remember that converting between units of area is *not* the same as converting between units of length. Although 1 m is equal to 100 cm, $1\ m^2$ is *not* equal to $100\ cm^2$. Because the units are square units, the conversion factor (the number we multiply or divide by) is the square of the conversion factor for length.

Units of length:

km $\xrightarrow{\times 1000}$ m $\xrightarrow{\times 100}$ cm $\xrightarrow{\times 10}$ mm

km ← m ← cm ← mm: $\div 1000$, $\div 100$, $\div 10$

Units of area:

$km^2 \xrightarrow{\times 1000^2} m^2 \xrightarrow{\times 100^2} cm^2 \xrightarrow{\times 10^2} mm^2$

$km^2 \leftarrow m^2$: $\div 1000^2$; $cm^2 \leftarrow mm^2$: $\div 10^2$

$km^2 \xrightarrow{\times 10^2} ha \xrightarrow{\times 100^2} m^2$

$km^2 \leftarrow ha$: $\div 10^2$; $ha \leftarrow m^2$: $\div 100^2$

Worked Example 5 WE5

Copy and complete the following conversions.

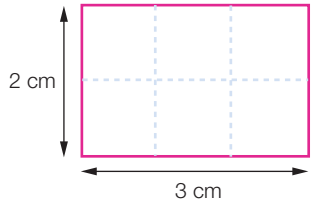
(a) $3.5\ m^2 =$ _____ mm^2 **(b)** $27\,000\ m^2 =$ _____ ha

Thinking	Working
(a) We are converting from a larger unit to a smaller unit, so we need to multiply. There are 1000^2 mm^2 in every m^2, so multiply by 1000^2 to convert m^2 to mm^2.	(a) 3.5×1000^2 $= 3.5 \times 1\,000\,000$ $= 3\,500\,000\ mm^2$
(b) We are converting from a smaller unit to a larger unit, so we need to divide. There are 100^2 m^2 in every ha, so divide by 100^2.	(b) $27\,000 \div 100^2$ $= 27\,000 \div 10\,000$ $= 2.7$ ha

If you are asked to calculate the area of a shape in units different to the ones given, it is often easier to convert the dimensions of the shape into the required units of length, then calculate the area.

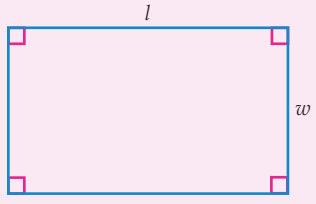
Area of rectangles and squares

The area of a **rectangle** is found simply by multiplying the length by the width. We can see two rows of three 1-centimetre squares inside this rectangle: $2 \times 3 = 6\ cm^2$.



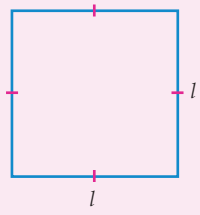
The area of a **square** is found by multiplying the side length by itself (because length = width).

Area of a rectangle



$A = lw$

Area of a square

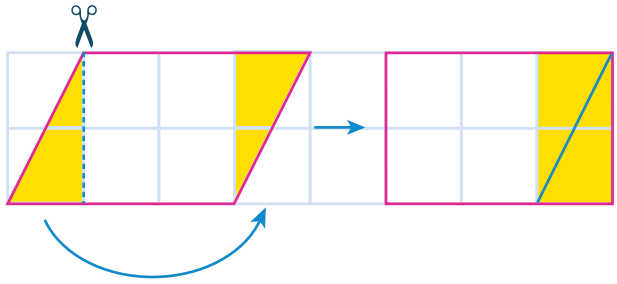


$A = l^2$

Parallelograms

A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.

We can cut and rearrange a parallelogram to form a rectangle as shown:

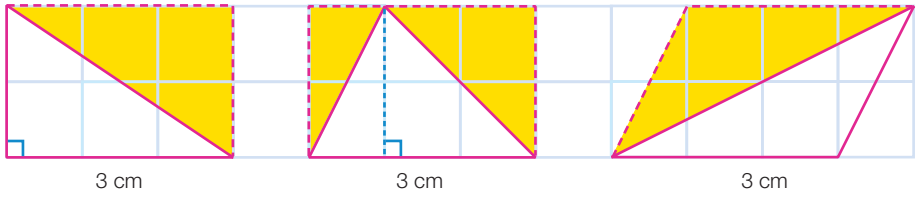


We can see from the diagram that a parallelogram with a **base** length of 3 cm and a **height** of 2 cm has the same area as a 3 cm × 2 cm rectangle. The height is **perpendicular** to the base. ('Perpendicular' means 'at right angles to'.)

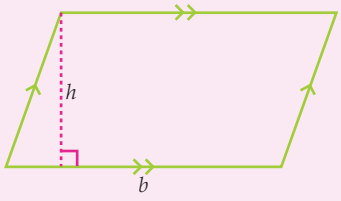
Triangles

Triangles have an area equivalent to either half of a rectangle or half of a parallelogram. The rectangles and parallelogram below each have an area of 6 cm^2 (3 cm × 2 cm).

Each of the triangles below has an area of 3 cm^2; i.e. half of 6 cm^2.

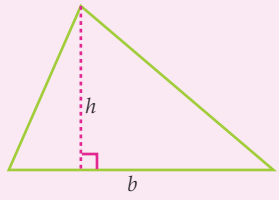


Area of a parallelogram



$A = bh$

Area of a triangle



$A = \frac{1}{2}bh$ or $\frac{bh}{2}$

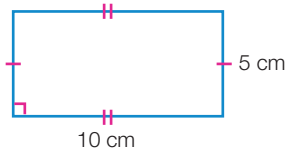
where b = base length, h = height perpendicular to base

Worked Example 6

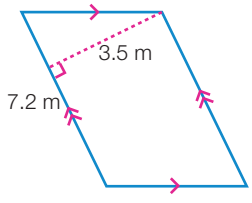
WE 6

Find the area of each of the following shapes.

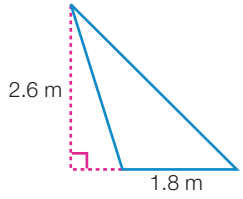
(a)



(b)



(c)

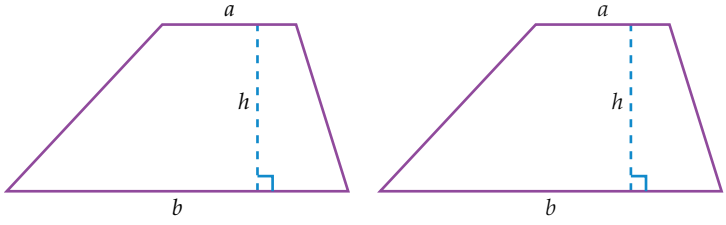


Thinking		Working
(a) 1	Write the formula for the area of a rectangle.	(a) $A = lw$
2	Substitute the values for l and w.	$= 10 \times 5$
3	Evaluate, including units of area.	$= 50\ cm^2$
(b) 1	Write the formula for the area of a parallelogram.	(b) $A = bh$
2	Substitute the values for b and h.	$= 7.2 \times 3.5$
3	Evaluate, including units of area.	$= 25.2\ m^2$
(c) 1	Write the formula for the area of a triangle.	(c) $A = \frac{1}{2}bh$
2	Substitute the values for b and h.	$= \frac{1}{2} \times 1.8 \times 2.6$
3	Evaluate, including units of area.	$= 2.34\ m^2$

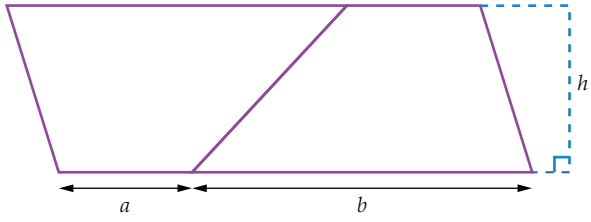
Trapeziums

A **trapezium** is a quadrilateral with only one pair of opposite sides parallel.

The two trapeziums below are identical.



If the first shape is rotated around 180° and joined to the other, they form a parallelogram, as shown below.



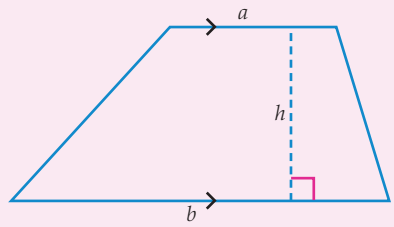
The area of this parallelogram is (base length) × height

$= (a + b) \times h$

$= (a + b)h$

However, this parallelogram contains two of the same trapezium, so the area of one trapezium is half of this amount, $\frac{(a + b) \times h}{2}$ or $\frac{1}{2}(a + b) \times h$

Area of a trapezium



$$A = \frac{1}{2}(a + b)h \qquad \text{or} \qquad \frac{(a + b)h}{2}$$

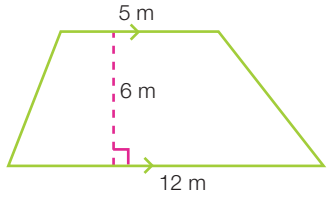
where a and b = the lengths of the parallel sides and h = height perpendicular to a and b.

The area of a trapezium equals half the sum of the lengths of the parallel sides multiplied by the perpendicular height.

Worked Example 7

WE7

Find the area of the following trapezium.

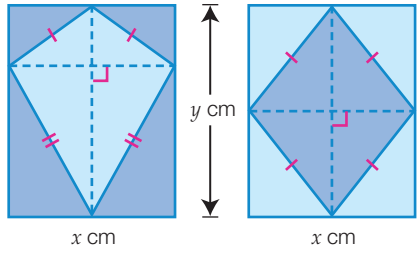


Thinking	Working
1 Write the formula for the area of a trapezium.	$A = \frac{1}{2}(a + b)h$
2 Substitute the values for a, b and h.	$= \frac{1}{2} \times (5 + 12) \times 6$
3 Evaluate, remembering the order of operations. Write your answer with the correct units of area.	$= 8.5 \times 6$ $= 51\ \text{m}^2$

Kites

A **kite** is a quadrilateral with two pairs of equal adjacent sides.

A **rhombus** is a special kind of kite that has four equal sides. (Because both pairs of its sides are parallel, a rhombus is also a special parallelogram—one with four equal sides.) The diagonals of both a kite and a rhombus intersect at right angles.



The area of a kite or rhombus is equal to half the area of the rectangle that just encloses it, such as the ones shown here. (Can you see why?) The length and width of such a rectangle are equivalent to the diagonals of the rhombus or kite (shown below as the lengths x and y).

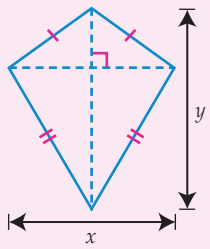
If $x = 2$ cm and $y = 4$ cm

$$A = \frac{1}{2}xy$$

$$= \frac{1}{2} \times 2 \times 4$$

$$= 4 \text{ cm}^2 \text{ for both shapes}$$

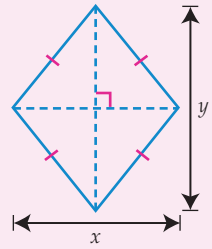
Area of a kite



Kite

$A = \frac{1}{2}xy$

Area of a rhombus



Rhombus

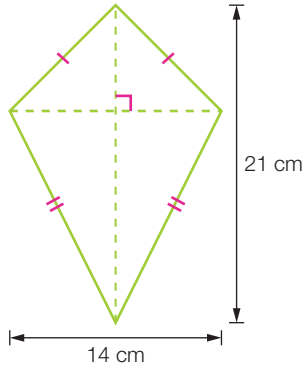
$A = \frac{1}{2}xy$

$A = \frac{1}{2}xy$, or $A = \frac{xy}{2}$, where x and y are the lengths of the diagonals.

Worked Example 8

WE 8

Find the area of the following kite.



Thinking	Working
1 Write the formula for the area of a kite.	$A = \frac{1}{2}xy$
2 Substitute the values for x and y.	$= \frac{1}{2} \times 14 \times 21$
3 Evaluate, writing your answer with the correct units of area.	$= 147 \text{ cm}^2$

5.4 Area

Navigator

Answers page 618

Q1 Column 1, Q2, Q3, Q4, Q6, Q7, Q9, Q11, Q13, Q17 (a), Q20	Q1 Column 1, Q2 Column 1, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q13, Q14, Q15, Q17, Q18, Q19, Q22	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4 Column 2, Q5, Q8, Q9, Q11, Q12, Q13, Q15, Q14, Q16, Q17, Q18, Q19, Q21, Q22

Equipment required: Calculator for Questions **2–22**

Fluency

WE5

1 Copy and complete the following conversions.

(a) $5\ m^2 =$ ____ cm^2

(b) $0.0065\ m^2 =$ ____ cm^2

(c) $94\ cm^2 =$ ____ mm^2

(d) $9.76\ cm^2 =$ ____ mm^2

(e) $12\ ha =$ ____ m^2

(f) $1.003\ ha =$ ____ m^2

(g) $50\,000\ m^2 =$ ____ ha

(h) $9800\ mm^2 =$ ____ cm^2

(i) $67\,000\ cm^2 =$ ____ m^2

(j) $950\ cm^2 =$ ____ m^2

(k) $23.6\ ha =$ ____ m^2

(l) $3400\ m^2 =$ ____ ha

(m) $460\ cm^2 =$ ____ m^2

(n) $345\ cm^2 =$ ____ mm^2

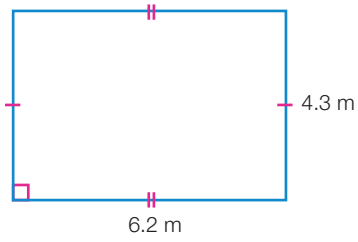
(o) $9\,000\,000\ m^2 =$ ____ km^2

(p) $5570\ ha =$ ____ km^2

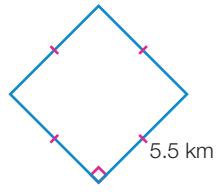
WE6

2 Find the area of each of the following shapes.

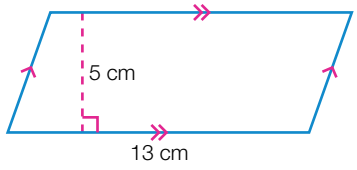
(a)



(b)



(c)



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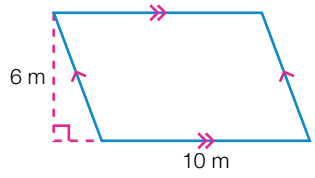
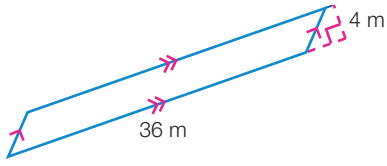
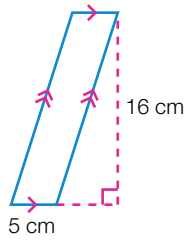


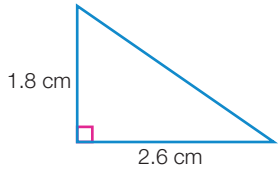
(e)



(f)



(g)



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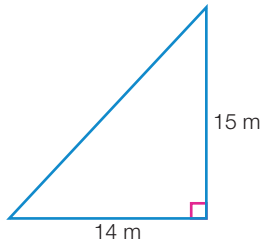
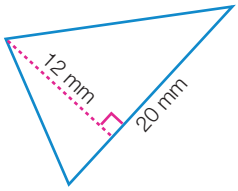
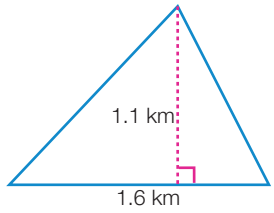


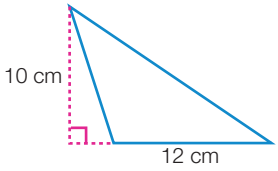
(i)



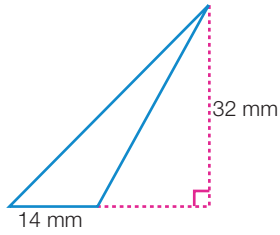
(j)



(k)



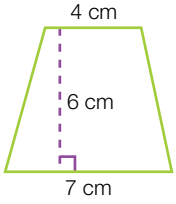
(l)



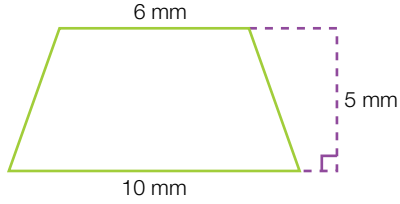
3 Find the area of each of the following trapeziums.

WE7

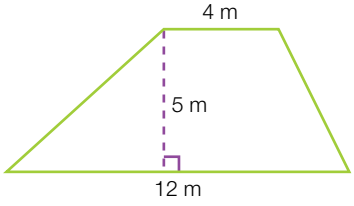
(a)



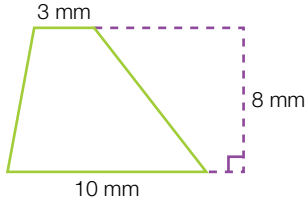
(b)



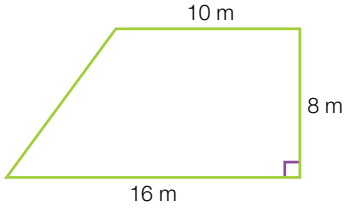
(c)



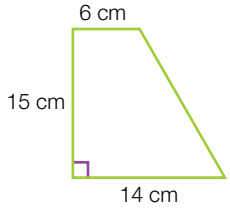
(d)



(e)



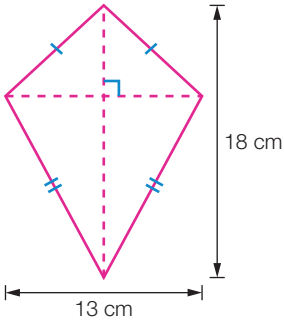
(f)



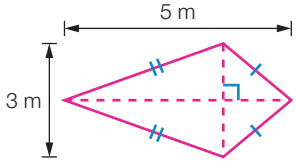
4 Find the area of each of the following kites.

WE8

(a)



(b)



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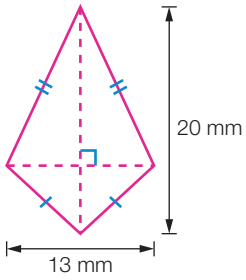
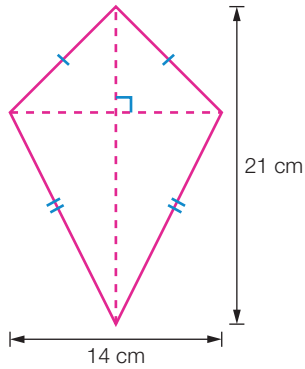
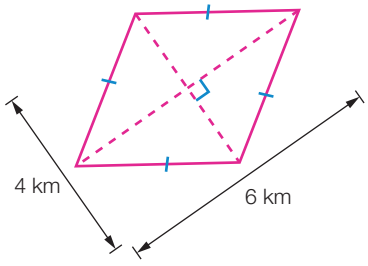


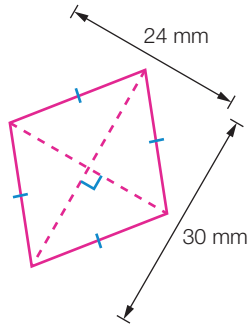
(d)



(e)

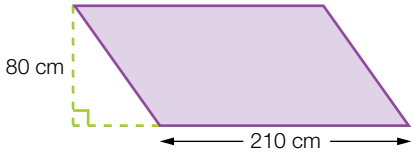


(f)

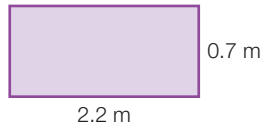


5 Find the area of the following figures in: **(i)** m^2 **(ii)** cm^2.

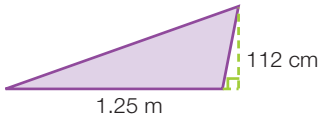
(a)



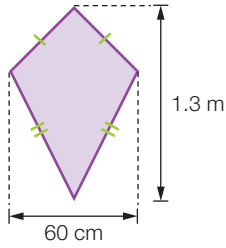
(b)



(c)



(d)



6 **(a)** A trapezium with perpendicular height 8 cm and parallel sides measuring 10 cm and 18 cm has an area of:

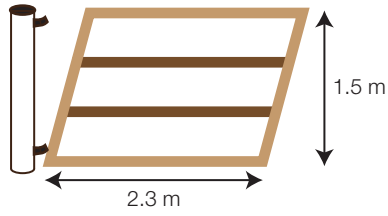
A 224 cm^2 **B** 180 cm^2 **C** 162 cm^2 **D** 112 cm^2

(b) A rhombus with side length 20 cm has diagonals 24 cm and 32 cm long. Its area is:

A 384 cm^2 **B** 400 cm^2 **C** 560 cm^2 **D** 768 cm^2

Understanding

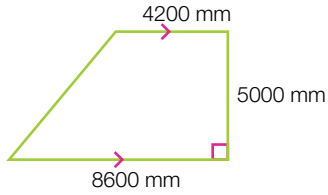
7 A very old gate has slumped over to appear as shown. What area does the gate's frame now enclose?



8 A parallelogram with base length 25 cm has an area of 375 cm^2. The height of the parallelogram is:

A 30 cm **B** 20 cm **C** 15 cm **D** 7.5 cm

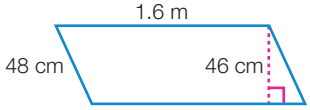
9 Sam wants to pave an area using slate costing $32 per square metre. The area he wishes to pave is shaped as shown.



(a) Find the area to be paved in m^2.

(b) What will be the total cost of the slate?

10 Which of the following calculations shows how to calculate the area of the parallelogram, in cm^2?



A $\frac{46 \times 48}{2}$ **B** 1.6×0.48

C 160×48 **D** 160×46

11 A beachside town has houses along 5 km of beachfront, occupying land that stretches up to 150 m inland from the beach. Find the area occupied by the town in:

(a) square metres **(b)** hectares **(c)** square kilometres.

12 This parallelogram-shaped building is found on the banks of the Elbe river in Hamburg, Germany. If the base length of the parallelogram shape enclosing the windows is 85 m, and each of the five floors is 3.5 m high, what is the area of glass on the front face of the building shown?

13 This mosaic was found on the floor of an ancient Roman building. The mosaic is made up of tiny square tiles, which are approximately 2 cm by 2 cm. At the centre of the floor is a pattern made up of eight white rhombuses with a side length of 40 cm and diagonal lengths of 36 cm and 72 cm. In the middle of each big rhombus is a smaller rhombus whose dimensions are one-third the size.

Calculate:

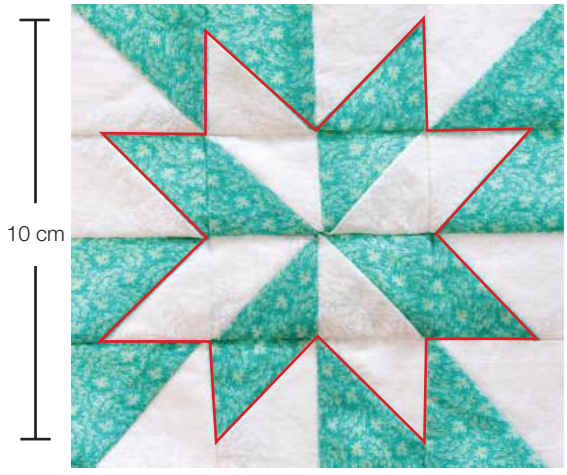
(a) the total area of the eight-rhombus pattern

(b) the perimeter of the eight-rhombus pattern

(c) the number of white tiles needed to make the pattern.

Reasoning

14 In this patchwork quilt design, a square 10 cm long is divided up into 32 identical green and white isosceles triangles. Two triangles make up a parallelogram. Find the area of the star figure made up by eight parallelograms.



15 Kwan has cut out a square of side length 8 cm from a sheet of metal.

(a) Hugh has cut out a parallelogram of the same area and with a base length of 10 cm. What will be the height of Hugh's parallelogram?

(b) Daniella wants to cut out a triangle with the same area and with a base length of 8 cm. What will be the height of her triangle?

16 **(a)** A rectangle of width 6 cm and length 8 cm has the same area as a trapezium of height 4 cm. What do the lengths of the parallel sides of the trapezium add to?

(b) The areas of a rectangle, a triangle and a trapezium are all 128 cm^2. If the heights of all three shapes are 8 cm, what are the lengths of each shape?

17 **(a)** **(i)** Find the side length of a square if it has the same area as a rectangle measuring 18 cm by 50 cm.

(ii) Find the difference in the perimeters of the two shapes.

(b) **(i)** Find the width of a rectangle that is 24 cm long if its area is the same as that of a square of side 18 cm.

(ii) Find the difference in the perimeters of the two shapes.

18 Omar has to design a kite that has an area of 2 m^2. He knows that for the kite to fly well the length of one diagonal has to be 2.5 times the length of the other diagonal. How long does he need to make each of the diagonals of his kite? Write answers in metres correct to one decimal place.

19 This pattern is found on the ceiling of a cathedral in Spain. It has eight identical kites arranged in a circle at its centre. The long diagonal of each kite is 1.5 times as long as the short diagonal, which is 3.4 m long.

(a) Calculate the total area of the kites, in m^2.

(b) If a circle is drawn so that the outermost vertex of each kite lies on the circumference of the circle, what would the circumference of the circle be, correct to two decimal places?

Open-ended

20 Amanda's restaurant has a floor plan in the shape of a parallelogram. She knows that the floor has an area of 48 m^2. Give three possible combinations for the base length and height (in whole numbers of metres) for the floor of Amanda's restaurant.

21 **(a)** Muhammed has built a new deck at the rear of his house. It is in the shape of a trapezium with an area of 80 m^2. He needs to buy some railing for the parallel sides of the deck but has forgotten the lengths of these two sides. He knows that these lengths are whole numbers of metres and that the distance between the parallel sides is 10 metres. Give three possible combinations for the lengths of the parallel sides.

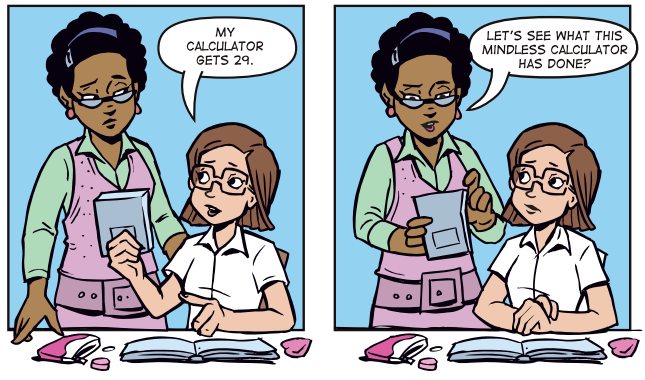
(b) Calculate the length of railing he will need to buy for one of your combinations of lengths.

22 Antoinette was trying to find the area in cm^2 for a trapezium in which $a = 10$ cm, $b = 4$ cm and $h = 6$ cm.

Antoinette's calculator working shows: $\frac{1}{2} \times 10 + 4 \times 6$.

(a) State the error that Antoinette has made.

(b) Find the actual area of the trapezium.



Outside the Square — Problem solving

Tiling a floor

A rectangular floor area is to be tiled. The floor has a perimeter of 280 cm. Its length is 2.5 times its width. 160 square tiles will be used.

What is the area of each square tile, to the nearest cm^2?

Strategy options

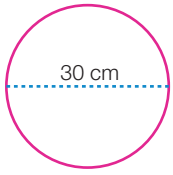
- Guess and check.
- Work backwards.
- Look for a pattern.

Half-time 5

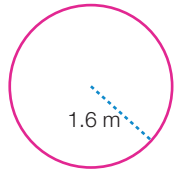
Ex.5.3

1 Find the circumference of the following circles.

(a)

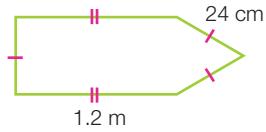


(b)



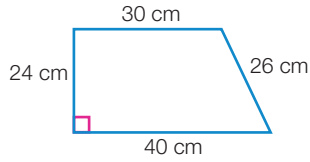
Ex.5.1

2 Find the perimeter of the shape in cm.



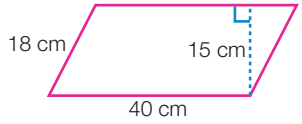
Ex.5.1, 5.4

3 Find the area and perimeter of the trapezium.



Ex.5.4

4 Draw a rectangle that has the same base length and area as the parallelogram.



Ex.5.1

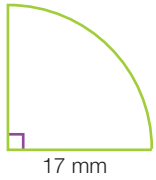
5 Find the perimeter of a regular octagon of side length 16 mm.

Ex.5.3

6 Find the perimeter of the following shapes, correct to two decimal places.

(a)

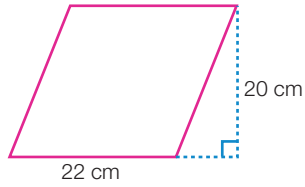
(b)



Ex.5.4

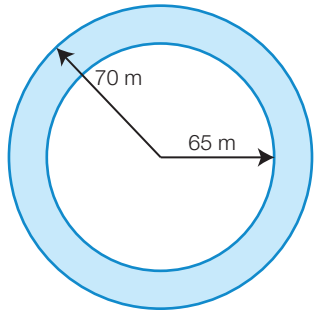
7 (a) Find the area of the parallelogram.

(b) Draw two triangles that have the same base length and same area as the parallelogram.



Ex.5.3

8 A running track has an inside radius of 65 m and an outside radius of 70 m. How much further would an athlete run if they ran on the outside of the track instead of on the inside of the track? (Give your answer correct to the nearest metre.)



Ex.5.3

9 A circle has a radius of 11 cm. Find the side length of a square with the same perimeter as the circumference of the circle. Round your answer correct to two decimal places.

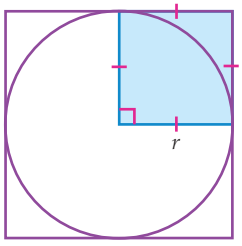
Ex.5.2

10 Estimate (do not calculate) the circumference of a circle with a diameter of 25 cm.

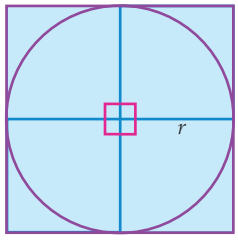
Area of a circle

5.5

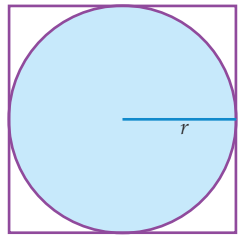
To find an approximation for the area of a circle, we can first consider the circle enclosed by a square whose side length is the same as the diameter of the circle. (The sides of the square just touch the circle at one point on each side.) This square can be divided into four identical smaller squares.



The area of the shaded square is r^2.

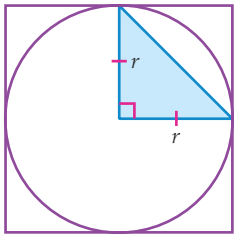


The area of the square that encloses the circle is $4r^2$ ($r^2 \times 4$).

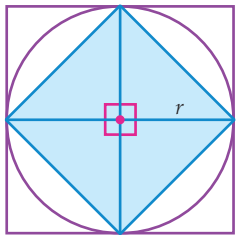


The area of the circle is clearly less than $4r^2$.

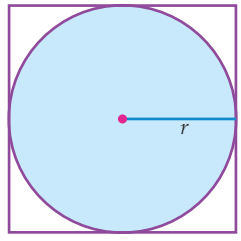
Now we consider a square drawn inside the circle, whose vertices touch the circumference. This square can be divided into four identical smaller triangles.



The area of the shaded triangle is $\frac{r^2}{2}$.



The area of the square inside the circle is $4 \times \frac{r^2}{2} = 2r^2$.

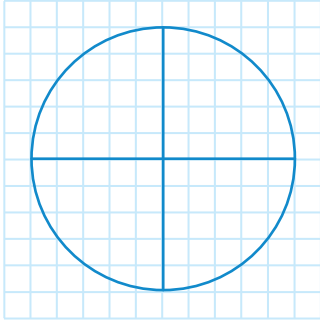


The area of the circle is clearly more than $2r^2$.

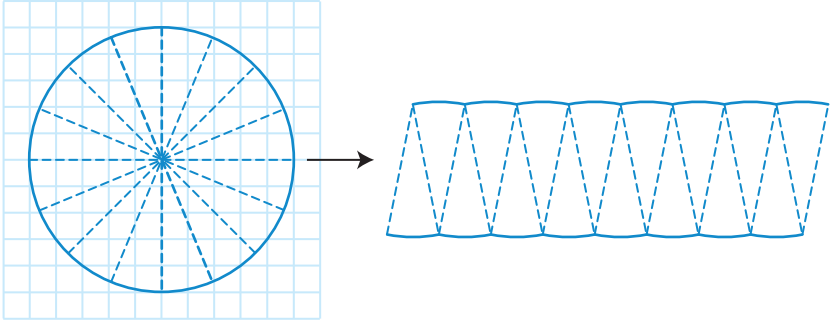
So, the area of the circle is greater than $2r^2$, but less than $4r^2$. We can write this mathematically as $2r^2 <$ Area of circle $< 4r^2$.

To develop a more accurate rule for finding the area of a circle, we can cut the circle into pieces and rearrange them to form a shape we know how to find the area of.

Step 1 Use a compass to draw three circles of the same size on centimetre grid paper. A radius of 4 or 5 cm is a convenient size. Divide the circle into four quarters. Estimate the area by counting the squares in a quarter and multiplying by 4. Write down your estimate for the area of your circle.



Step 2 Divide the circle into sixteenths by dividing the quarters in half, then in half again. Cut them out and glue them onto plain paper or into your workbook as shown below, making sure that you alternate the curved edge between the top and bottom.



Step 3 You should notice that the shape formed by the sectors of the circle resembles a rectangle. Write down the rule for finding the area of a rectangle.

Step 4 The circumference of the original circle has been chopped up into the small curved edges along the top and bottom sides of the rectangle, so the length of the rectangle, l, is equal to half the circumference, $2\pi r$. We can write:

$$l = \frac{\cancel{2}\pi r}{\cancel{2}} = \pi r$$

Step 5 The two shorter sides of the rectangle correspond to the length of the radius in the original circle. We can write w (width) $= r$.

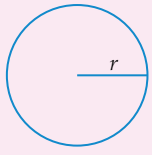
Step 6 Write down the rule for the area of the rectangle again. Now, replace the two dimensions in the formula with $l = \pi r$ and $w = r$.

$$\begin{aligned} A &= l \times w \\ &= \pi r \times r \\ &= \pi r^2 \end{aligned}$$

Step 7 Use the formula $A = \pi r^2$ to calculate the area of the circle. Is the calculated area greater or less than the area you found by counting squares?

We stated earlier that the area of a circle would be between $2r^2$ and $4r^2$ (the 'inside' and 'outside' squares shown at the beginning of this section). As π has an approximate value of 3.141 592 654, $A = \pi r^2$ will give such an area.

Area of a circle



$A = \pi r^2$

Worked Example 9

WE 9

Find the area of the following circles, correct to two decimal places.

(a)

(b)

Thinking		Working
(a) 1	Write the formula.	(a) $A = \pi r^2$
2	Substitute for r.	$= \pi \times 77^2$
3	Evaluate the formula.	$= 18\,626.502\ldots$
4	Round the answer to the specified number of decimal places and include units of area.	$\approx 18\,626.50\ \text{mm}^2$ (correct to 2 d.p.)
(b) 1	Write the formula.	(b) $A = \pi r^2$
2	Halve the diameter to get the radius.	$d = 16$ cm, so $r = 8$ cm
3	Substitute for r.	$\therefore A = \pi \times 8^2$
4	Evaluate the formula.	$= 201.061\ldots$
5	Round the answer to the specified number of decimal places and include units of area.	$\approx 201.06\ \text{cm}^2$ (correct to 2 d.p.)

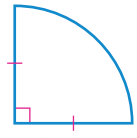
Worked Example 10

WE 10

Find the area of the following shapes, correct to two decimal places.

(a)

(b)



Thinking		Working
(a) 1	Modify the area formula to show the fraction of the whole circle being calculated (in this case, one half).	(a) $A = \dfrac{\pi r^2}{2}$
2	Find the radius by halving the diameter.	$d = 4.5$ m, so $r = 2.25$ m
3	Substitute into the formula.	$A = \dfrac{\pi \times 2.25^2}{2}$
4	Evaluate the formula.	$= 7.952\,156\ldots$
5	Round and write the answer, including the correct units.	$= 7.95\ \text{m}^2$
(b) 1	Modify the area formula to show the fraction of the whole circle being calculated (in this case, one quarter).	(b) $A = \dfrac{\pi r^2}{4}$
2	Substitute into the formula.	$A = \dfrac{\pi \times 27.3^2}{4}$
3	Evaluate the formula.	$= 585.349\,39\ldots$
4	Round and write the answer, including the correct units.	$= 585.35\ \text{cm}^2$

5.5 Area of a circle

Navigator

Answers page 619

Q1 Columns 1 & 2, Q2, Q3, Q5, Q6, Q7, Q8, Q11 (a) & (b), Q12, Q16	Q1 Columns 2 & 3, Q2, Q4, Q5, Q6, Q7, Q8, Q9, Q11 (a) & (b), Q12, Q13, Q15, Q16	Q1 Columns 2 & 3, Q2, Q4, Q6, Q7, Q8, Q10, Q11, Q12, Q13, Q14, Q15, Q16

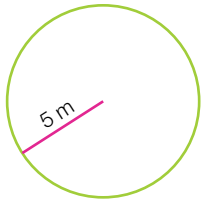
Equipment required: Scientific calculator

Fluency

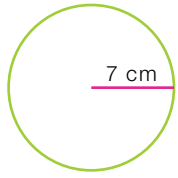
WE9

1 Find the area of the following circles, correct to two decimal places.

(a)

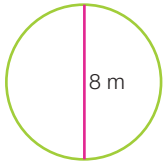


(b)



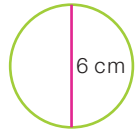
(c)

(d)

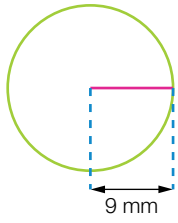


(e)

(f)

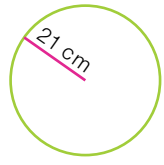


(g)



(h)

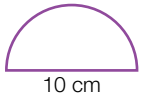
(i)



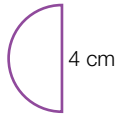
WE10

2 Find the area of the following shapes, correct to two decimal places.

(a)

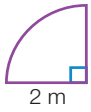


(b)

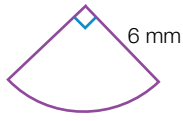


(c)

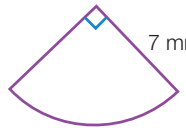
(d)



(e)



(f)



3 Calculate the area of each of the following circles, correct to the nearest cm^2.

(a) a circle of radius 35 cm

(b) a circle of diameter 68 cm

4 **(a)** The area of a circle with a diameter of 25 m is closest to:

A 157 m^2 **B** 490 m^2 **C** 491 m^2 **D** 1963 m^2

(b) If a circle has a radius of 75 mm, then its area, correct to two decimal places, is:

A 471.24 mm^2 **B** 4417.86 mm^2 **C** 17 671.46 mm^2 **D** 70 685.83 mm^2

Understanding

5 Find the area, correct to two decimal places, of the glass in a pair of spectacles if the radius of each circular lens is 4.2 cm.

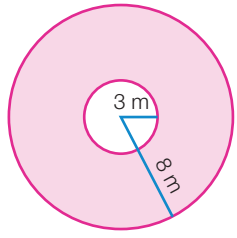
6 The playing field at Eastern Districts Cricket Club is a circle with a diameter of 106 m. Adam the groundskeeper wants to lay new turf on the entire playing surface.

(a) What area of turf should Adam order? (Answer to the nearest square metre.)

(b) If the turf costs $7.50 per square metre, how much will this cost the club?

7 Charlotte is a florist who has designed a new logo for her business using seven circles. If the central circle has a diameter of 4 cm and the smaller circles each have a diameter of 2 cm, calculate the total area of the circles used in the logo, correct to two decimal places.

8 An 'annulus' is the ring shape formed when a larger circle is drawn around a smaller one. It is the shaded area between the circumferences of the two circles, as shown in the diagram. For all questions below, give your answer correct to two decimal places.

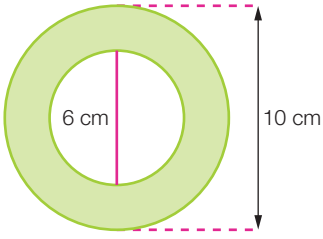


(a) Calculate the area of the larger circle.

(b) Calculate the area of the smaller circle.

(c) Subtract the smaller area from the larger one to find the area of the annulus.

9 The area of the annulus below, correct to the nearest whole number, is:



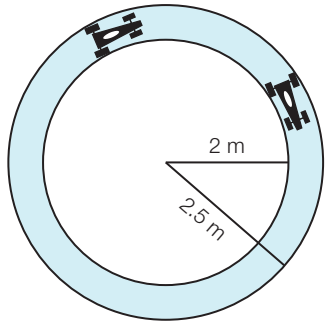
A 13 cm^2

B 50 cm^2

C 101 cm^2

D 201 cm^2

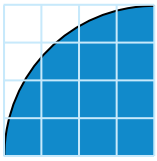
10 Find the area of a circular model racetrack of outer radius 2.5 m and inner radius 2 m. Give your answer correct to two decimal places.



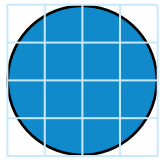
Reasoning

11 For each of the following shapes, estimate the area by considering grid squares that are covered or partially covered. Write your answer to the nearest whole square unit. Check your answers by calculation.

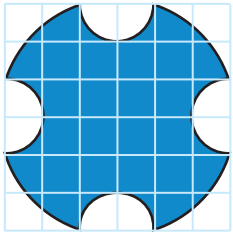
(a)



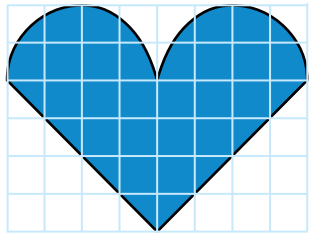
(b)



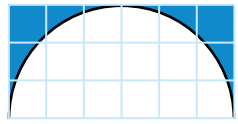
(c)



(d)



12 The fraction of the rectangle that is shaded is closest to:



A $\frac{1}{2}$ **B** $\frac{1}{3}$ **C** $\frac{1}{4}$ **D** $\frac{1}{5}$

13 A factory that produces soft drink cans cuts 150 circles out of a rectangular sheet of metal that is 1.68 m long and 1.13 m wide. The circles are 10 cm in diameter.

(a) Find the area of one circle, correct to two decimal places.

(b) Find the total area of all the circles in m^2, correct to two decimal places.

(c) Find the area of metal that remains after the circles have been removed in m^2, correct to two decimal places.

14 **(a)** The formula for the circumference of a circle is $C = 2\pi r$, where r is the radius of the circle. The formula for the area of a circle is $A = \pi r^2$.

Copy and complete the following table.

	Radius (r cm)	Circumference (C cm)	C^2 (cm^2)	Area (cm^2)	$\frac{C}{A}$	$\frac{C^2}{A}$
(i)	1					
(ii)	2					
(iii)	5					
(iv)	10					

What do you notice about the ratios $\frac{C}{A}$ and $\frac{C^2}{A}$?

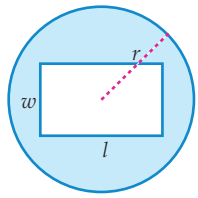
(b) Repeat part **(a)** for four more circles of different radius lengths. Which ratio seems to give a constant result?

(c) Divide the constant ratio by π. What number do you get?

(d) Write a formula connecting the circumference and area of a circle.

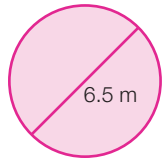
Open-ended

15 Write some dimensions for the rectangle and circle so that the shaded area is somewhere between 400 cm^2 and 500 cm^2.



16 Ajay and Simon were working together on their maths homework when they came across the following question.

Find the area of the following circle, correct to two decimal places.



Here is Simon's working:

$A = \pi r^2$
$= \pi \times 6.5^2$
$= 132.73\ m^2$

Here is Ajay's working:

$A = \pi r^2$
$= \pi \times 3.25 \times 2$
$= 40.42\ m^2$

(a) Is either student correct? What is the correct answer?

(b) Explain the mistakes that have been made, and give each boy a piece of advice to help him successfully answer these types of problems in the future.

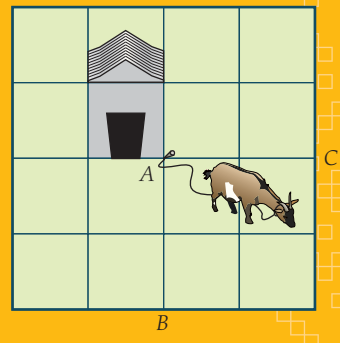
Outside the Square — Problem solving

Victa the goat

Victa, the grass-eating goat, lives in a square paddock that is 8 m long. He is tied to the corner of a square shelter at point *A*. The dimensions of the shelter are a quarter of the dimensions of the paddock. Victa is just able to reach the grass at the four edges of the paddock at points *B* and *C*. What percentage (to the nearest whole per cent) of the grass will still need to be mowed?

Strategy options

- Draw a diagram.
- Solve a simpler problem.
- Break problem into manageable parts.



Technology Exploration CAS

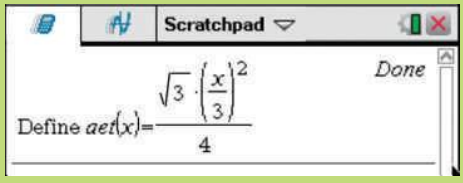
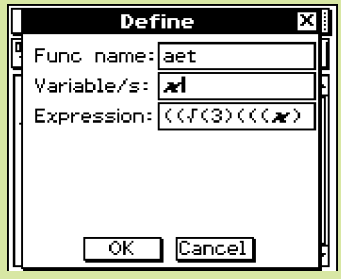
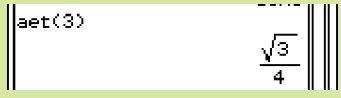
Equipment required: 1 brain, CAS calculator

Area of equilateral triangles

Sir Cumference, the famous mathematics professor, claims he has come up with a new rule that can be used to find the area of an equilateral triangle using just the perimeter of the triangle. He says the rule is:

Area of equilateral triangle = $\dfrac{\sqrt{3}\left(\dfrac{x}{3}\right)^2}{4}$, where x is the perimeter of the equilateral triangle.

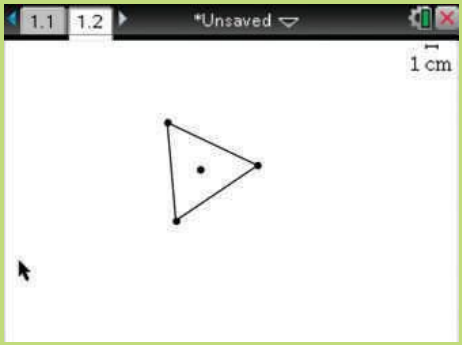
The first step in the process is to define the rule and store it on our CAS so we can use it whenever we like. We will use the name aet(x) to stand for area of equilateral triangle. It is a good idea to name functions so you can identify them again later.

Using the TI-Nspire CAS	Using the ClassPad
On the Home screen, select Calculate (under the scratchpad heading). Press menu > Actions > Define. Use the keypad, the ▣, √▢, and x^2 keys to enter aet(x) = $\dfrac{\sqrt{3}\left(\dfrac{x}{3}\right)^2}{4}$.  	Tap Main then press Keyboard and select the '2D' tab. Use the ▣, √▢ and x^2 keys to enter the expression $\dfrac{\sqrt{3}\left(\dfrac{x}{3}\right)^2}{4}$. Then highlight it and tap Interactive > Define and complete the dialog box as shown below. Note that the function name must be entered using the abc keypad but x is entered from the hard keypad. 
To check you have entered the rule correctly, enter aet(3). The answer should be $\dfrac{\sqrt{3}}{4}$. 	Tap OK to complete the definition of the variable. To check you have entered the rule correctly, enter aet(3). The answer should be $\dfrac{\sqrt{3}}{4}$. (If you got a decimal answer, change the setting at the bottom of the screen from 'Decimal' to 'Standard' by tapping it.) 

You will now use your CAS to explore whether or not the professor is right. You are going to draw an equilateral triangle, find its perimeter and area, and determine whether this gives the same answer as using the formula.

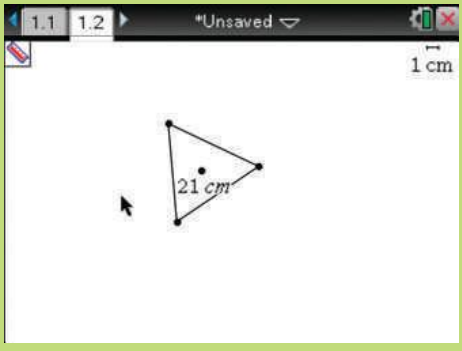
Using the TI-Nspire CAS

Now press (on) (or (home)) and use the cursor to select the icon (or select Geometry) and press (enter) (or (enter)). Press (menu) > Shapes > Regular Polygon > (enter) (or (enter)). Use the cursor to move the point to somewhere near the middle of the screen before pressing . Brush your finger over the touchpad (or use ▼) to make the polygon bigger. A diameter of about 1–2 cm is fine. Press again and then brush your finger over the touchpad to change the polygon to a triangle (or press ◀ to reduce the number of sides in the polygon until you get down to a triangle). Then press (enter) (or (enter)) to hold the triangle on the screen.



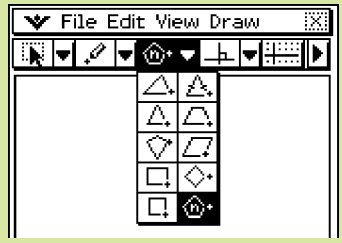
Press (esc) and move the cursor over the triangle. A hand will appear with the word triangle.

Press (menu) > Measurements > Length (enter) (or (enter)) and you will see a shadowy value on the screen. Press (enter) (or (enter)) twice to make it clearer and permanent. This is the perimeter of the triangle. (You will probably have a different value, as your triangle will not be exactly the same size as the one shown.)

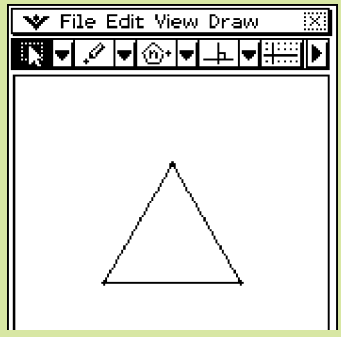


Using the ClassPad

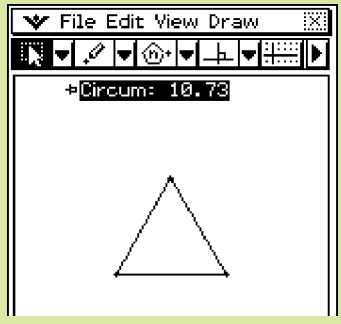
Tap Menu Geometry and then click the icon highlighted here in the screen shot.

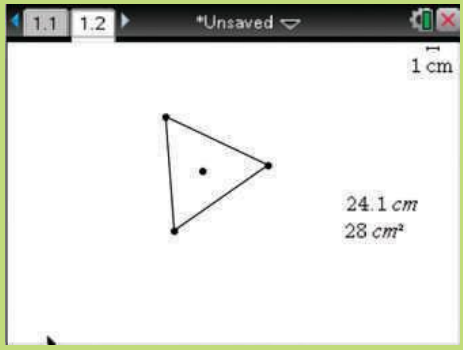
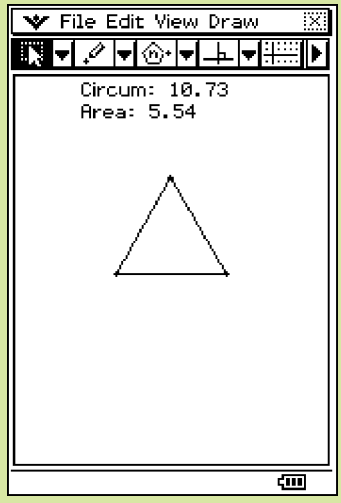
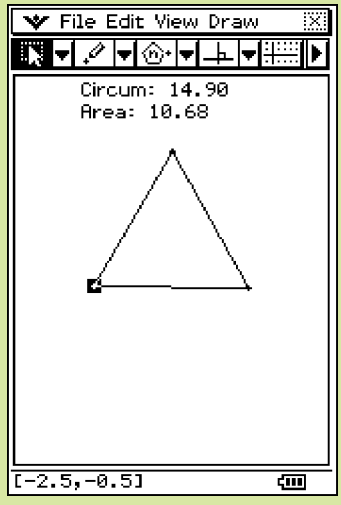
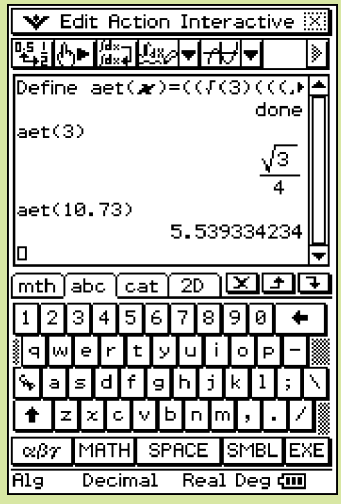


When you do this, a dialog box appears. You want to draw a triangle, so enter 3 and tap OK. This will then present you with a blank screen. Use the stylus to draw a box on the screen the size of the triangle you want. When you release the stylus the triangle will appear. We don't want the vertex names, so tap Edit > Properties > Hide Name.



We will now find the perimeter of this triangle. Just tap each of the sides with the stylus and then tap Draw > Measurement > Circumference and we see the perimeter of the triangle appear on the screen. You should remember that perimeter and circumference mean the same thing.



Using the TI-Nspire CAS	Using the ClassPad
Press (esc) and move the cursor over the value given, hold down until the hand closes (a message may appear).  Then use the cursor arrows to move the label to a place well away from the triangle and press (enter) (or (enter)). Now move the cursor back to the triangle but, this time, press (menu) > Measurement > Area and press (enter) (or (enter)) twice to make the number clearer and permanent. This is the area of the triangle. Move it in the same way to a more convenient location.	Then repeat the process but, this time, tap Area instead of Circumference. Record these values in your exercise book. 
Record these values for perimeter and area. If you now put the cursor on one of the vertices of the triangle (you will see the word 'point') and press you can then move the vertex to make a new triangle. Each time you do this you will get new values for the perimeter and area. Record at least six more pairs of values.	Now, tap one of the vertices and drag it to a new location. You will see new values for the perimeter and the area. 
Now use (calc) to return to the Calculator screen. It can be the one where you defined the rule earlier or you can get a new one from (on) (or (home)). From our initial screen shot, the perimeter is 6.58 cm and the area is 2.08 cm^2. If we enter aet(6.58), we get 2.0831. Check your other pairs of values in the same way and decide whether Sir Cumference has a correct formula.	Record at least six more pairs of values. Then go back to the calculator screen (Main) where we will check how accurate Sir Cumference's formula is. From our initial screen shot, the perimeter is 10.73 cm and the area is 5.54 cm^2. Enter aet(10.73) and we get 5.5393. Check your other pairs of values and decide whether Sir Cumference has a correct formula. 

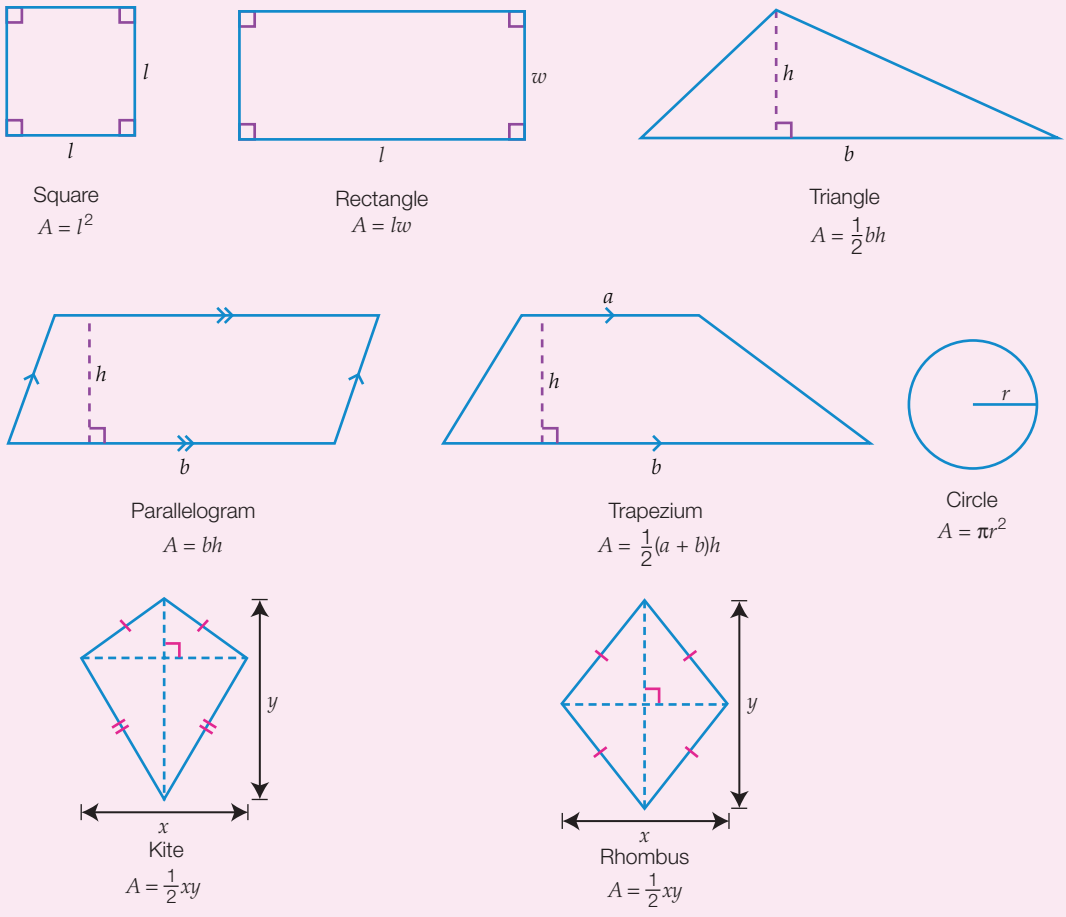
Finding the area of composite shapes

5.6

A **composite shape** is one made up of simpler shapes.

If we can find the areas of the simpler shapes, these can be added to calculate the total area.

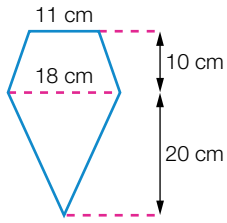
Area formula summary



Worked Example 11

WE11

Find the area of this composite shape.



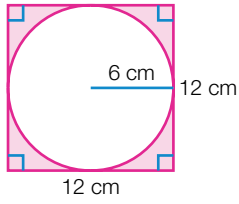
Thinking	Working
1 Identify the individual shapes that make up the total area.	$A_{total} = A_{trapezium} + A_{triangle}$
2 Write the formulas for each of the individual shapes. Use different pronumerals for the different heights (h for the trapezium and H for the triangle).	$= \frac{1}{2}(a+b)h + \frac{1}{2}bH$
3 Substitute the known lengths into the formulas. (Remember that multiplying by $\frac{1}{2}$ is the same as dividing by 2.)	$= \frac{1}{2}(11+18) \times 10 + \frac{1}{2} \times 18 \times 20$
4 Evaluate the total area, writing your answer with the correct units.	$= 325\ cm^2$

Areas of parts of shapes can be found by subtracting the areas that are not included from the overall area.

Worked Example 12

WE12

Find the shaded area in the following shape, correct to two decimal places.



Thinking	Working
1 Write the shaded area as the subtraction of the area of one shape from the area of the other.	$A_{shaded} = A_{square} - A_{circle}$
2 Write the formulas for each of the individual shapes.	$= l^2 - \pi r^2$
3 Substitute the known lengths into the formula.	$= 12^2 - \pi \times 6^2$ $= 144 - 36\pi$
4 Evaluate the difference, writing your answer with the correct units.	$= 30.90\ cm^2$

By not evaluating and writing down the areas of individual shapes (such as the circle in Worked Example 12), we avoid creating any rounding errors. Make sure you know how to enter these calculations into your calculator in one step, using the bracket keys as necessary to group calculations.

5.6 Finding the area of composite shapes

Navigator

Q1, Q2(a)–(d), Q3, Q4, Q6, Q7, Q8, Q10, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q13	Q1, Q2, Q4, Q5, Q8, Q9, Q10, Q11, Q12, Q13

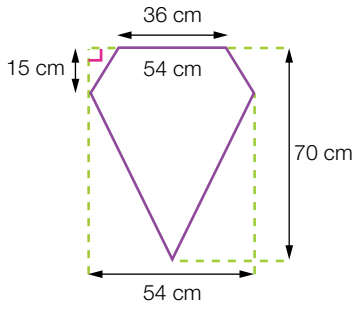
Answers page 619

Equipment required: Scientific calculator

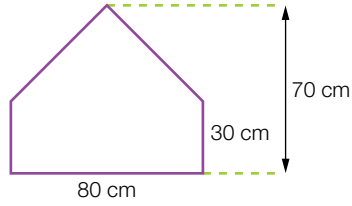
Fluency

1 Find the area of these composite shapes, correct to two decimal places where appropriate. WE11

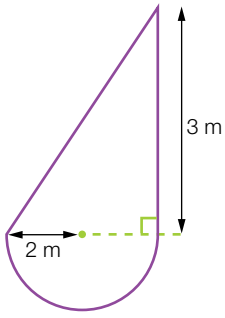
(a)



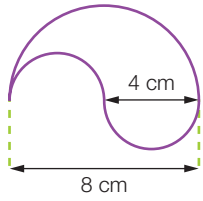
(b)



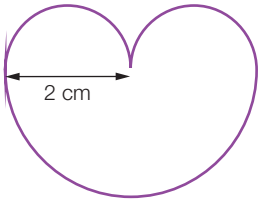
(c)



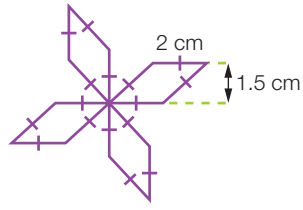
(d)



(e)

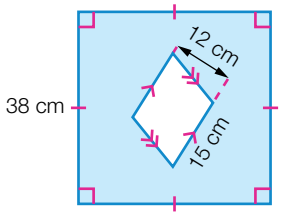


(f)

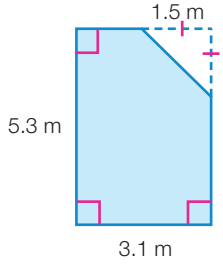


2 Find the shaded area in each of the following shapes, correct to two decimal places where appropriate. WE12

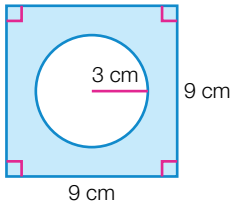
(a)



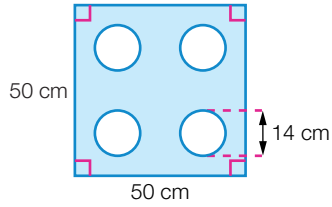
(b)



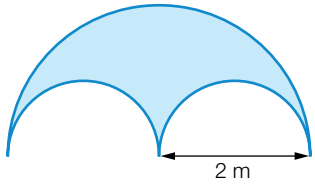
(c)



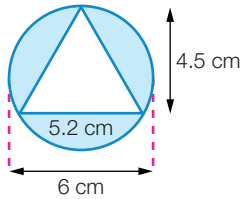
(d)



(e)



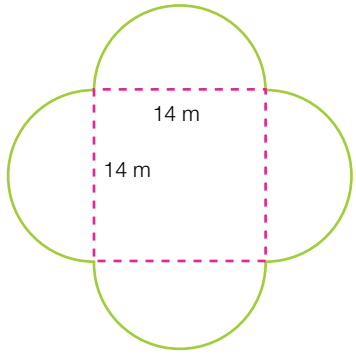
(f)



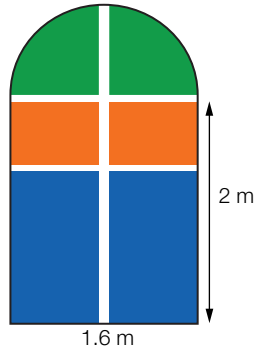
3 The area of the following shape, correct to the nearest m^2, is

A 196 m^2 **B** 308 m^2

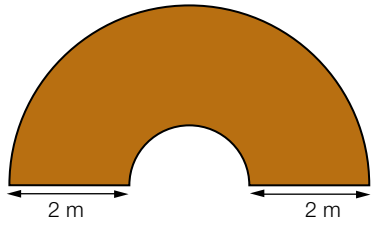
C 504 m^2 **D** 1232 m^2



4 A stained-glass church window is shaped as shown. What area of glass, correct to two decimal places, does it contain? (Ignore the parts of the frame between the glass areas.)

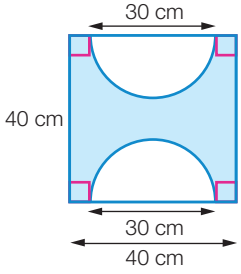


5 Michael needs to buy some pine-bark for a garden bed that is shaped as shown. What area does he need to cover? Give your answer correct to two decimal places.

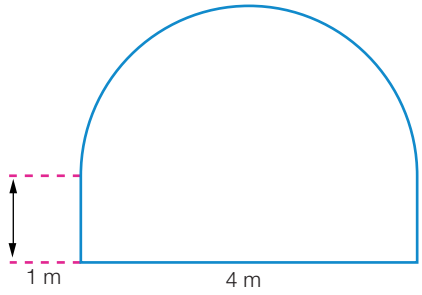


6 Find the perimeter and area of each shape correct to two decimal places.

(a)



(b)

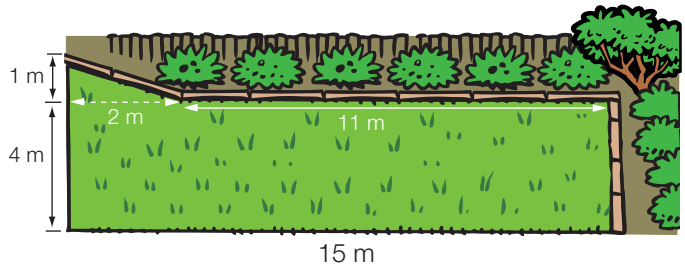


Understanding

7 A new home owner wishes to lay 'instant lawn' over her front yard, which has the shape below.

(a) What is the area of the yard?

(b) If instant lawn costs $6.50 per square metre, how much will she pay if she goes ahead with her plan?



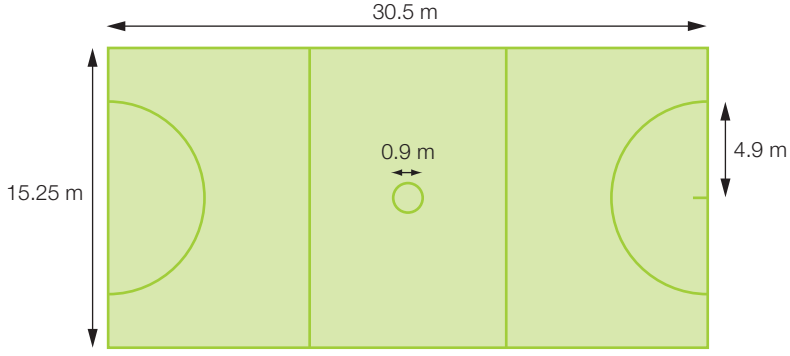
8 Below is a diagram of a netball court.

- The court is the shape of a rectangle 30.5 m in length and 15.25 m wide.
- The court is divided into thirds across its length.
- In the very centre of the middle third is a circle 0.9 m in diameter.
- At the ends of the court, in the middle of each of the shorter sides, are two semi-circles, with a radius of 4.9 m. These are called the 'goal circles'.

(a) Find the area of the court.

(b) The player in the position of 'goal attack' can only move across two of the thirds of the court. What area of the court is this? (Answer correct to two decimal places.)

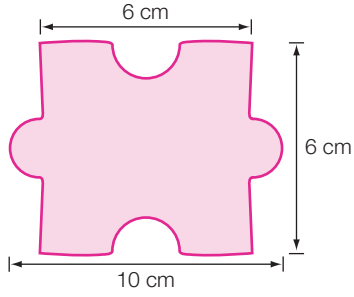
(c) The player in the position of 'centre' can move all over the court, except in the two goal circles. What area of the court is this? (Answer correct to two decimal places.)



9 A children's puzzle contains the piece shown. Its area is closest to:

A 28 cm^2 **B** 36 cm^2

C 48 cm^2 **D** 60 cm^2



Reasoning

10 Athletics running tracks are placed around the perimeter of a shape that is a rectangle with a semi-circle at each end. The rectangle is 85 m × 73 m. There are 8 lanes each 1.25 m wide around the perimeter of the shape. Answer the questions that follow correct to two decimal places.

(a) Find the distance you would run doing one lap of:

(i) the inside lane, running along the inside line

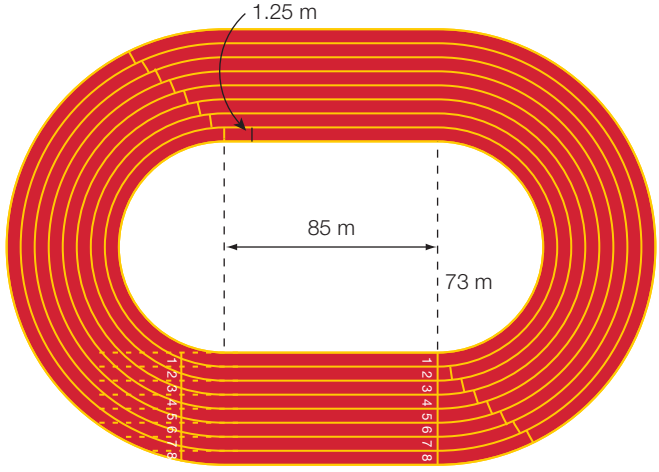
(ii) the outside lane, running along the outside line.

(b) Find the area of:

(i) the field inside the track

(ii) the area of the track and field

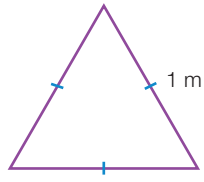
(iii) the area of the track.



11 **(a)** When the perpendicular height of a triangle is not known, the area can be calculated using the three side lengths by a formula known as Heron's formula.

$A = \sqrt{s(s-a)(s-b)(s-c)}$, where a, b and c are the side lengths and $s = \dfrac{a+b+c}{2}$, the semi-(or half) perimeter.

Use Heron's formula to find the area of the equilateral triangle shown, correct to two decimal places.



A regular hexagon is drawn inside a circle of radius 1 m. The hexagon is made up of six of the equilateral triangles from part **(a)**.

(b) Find the area of the hexagon, correct to two decimal places.

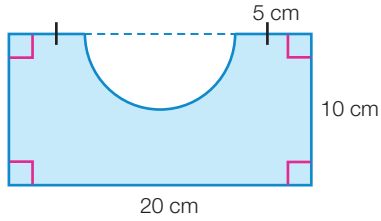
(c) Find the area of the circle, correct to two decimal places.

(d) Use your answers to **(b)** and **(c)** to find the area of the shaded region, correct to two decimal places.

Open-ended

12 Draw a composite shape, made up of a rectangle and a trapezium, that has an area of 60 cm^2. Show the appropriate dimensions.

13 Lucy was asked to find the perimeter and area of this shape, rounding her answer to the nearest whole number.



(a) Find all the errors Lucy has made in her working below.

(b) Calculate the correct answers.

Perimeter

Semicircle:

$\pi d + d$
$= \pi \times 10 + 10$
$\approx 31 + 10$
$= 41$ cm

$P = 5 + 10 \times 2 + 20 + 41$
$= 5 + 20 + 20 + 41$
$= 86$ cm

Area

Rectangle:

$A = 2l + 2w$
$= 2 \times 20 + 2 \times 10$
$= 40 + 20$
$= 60$ cm^2

Semicircle:

$A = \frac{\pi r^2}{2}$

$= \frac{\pi \times 10^2}{2}$

≈ 157 cm^2

Required area: $157 - 60 = 97$ cm^2

Outside the Square — Problem solving

Doubling a square

Farmer George owns one hectare of land in the shape of a square. One day, the king said to him "Farmer George, I will let you double the size of your land, but your farm must still be in the shape of a square".

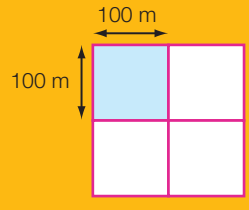
Farmer George jumped up and down in excitement, and then got busy with some maths. If he simply doubled the length and width of his farm, it would then be four times as big, not double.

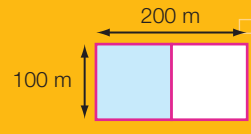
If he put two squares next to each other, the farm would be double in size, but would not be a square.

He finally worked it out, but how?

Strategy options

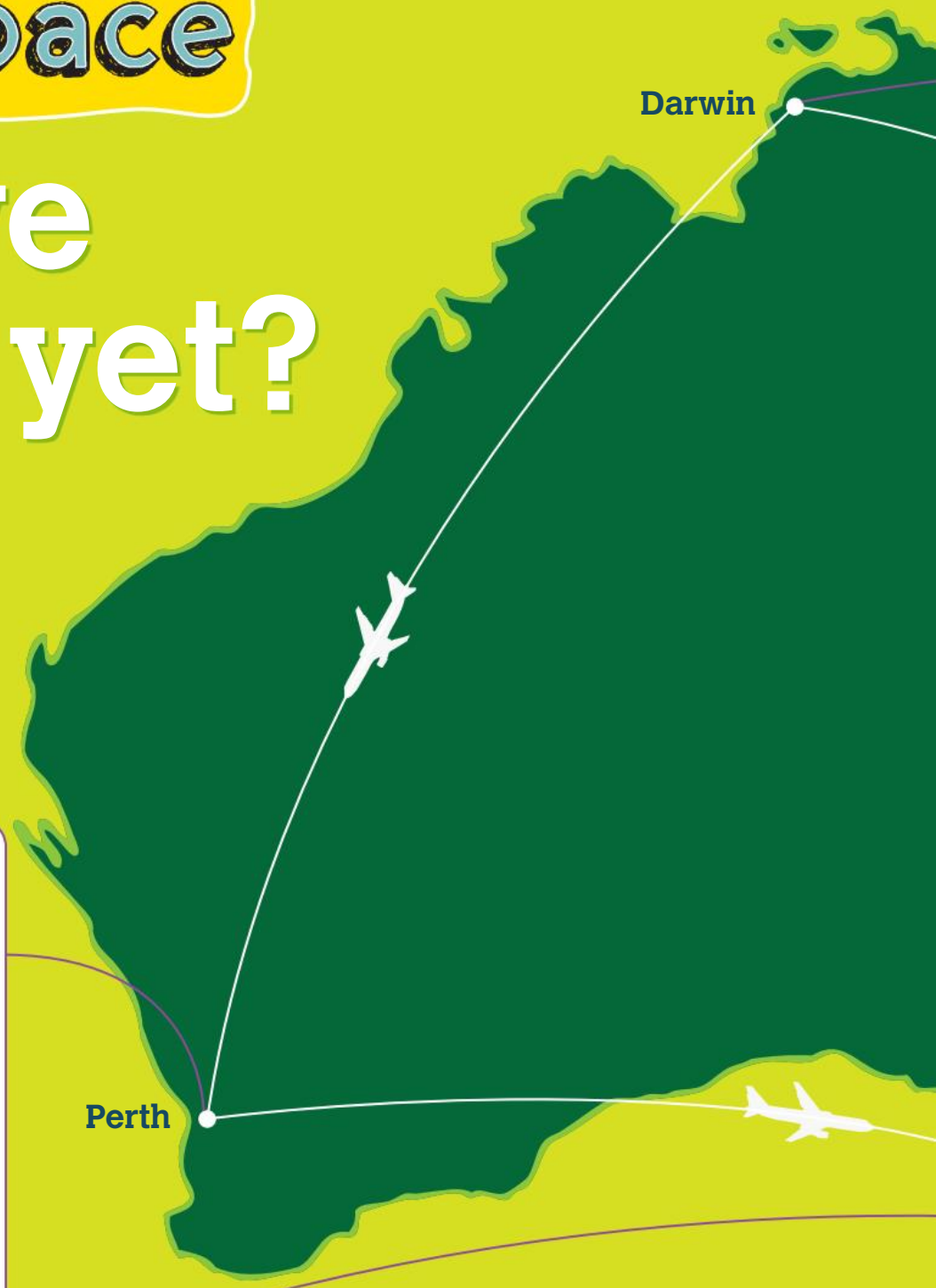
- Guess and check.
- Make a model.





Mathspace

Are we there yet?

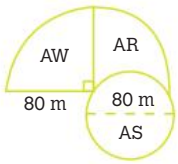


Your parents are taking you on a trip around Australia during term time. Your mathematics teacher happily provides a question for you to answer from each of the tourist attractions you plan to visit.

Start at the airport nearest your home and follow the flight paths in an anticlockwise direction, completing the tasks as you go.

Perth zoo – areas include Australian Walkabout (AW), Asian Rainforest (AR) and African Savannah (AS).

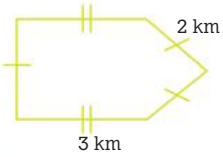
Which has more area, the Australian Walkabout or the African Savannah?



Adelaide Botanical Gardens

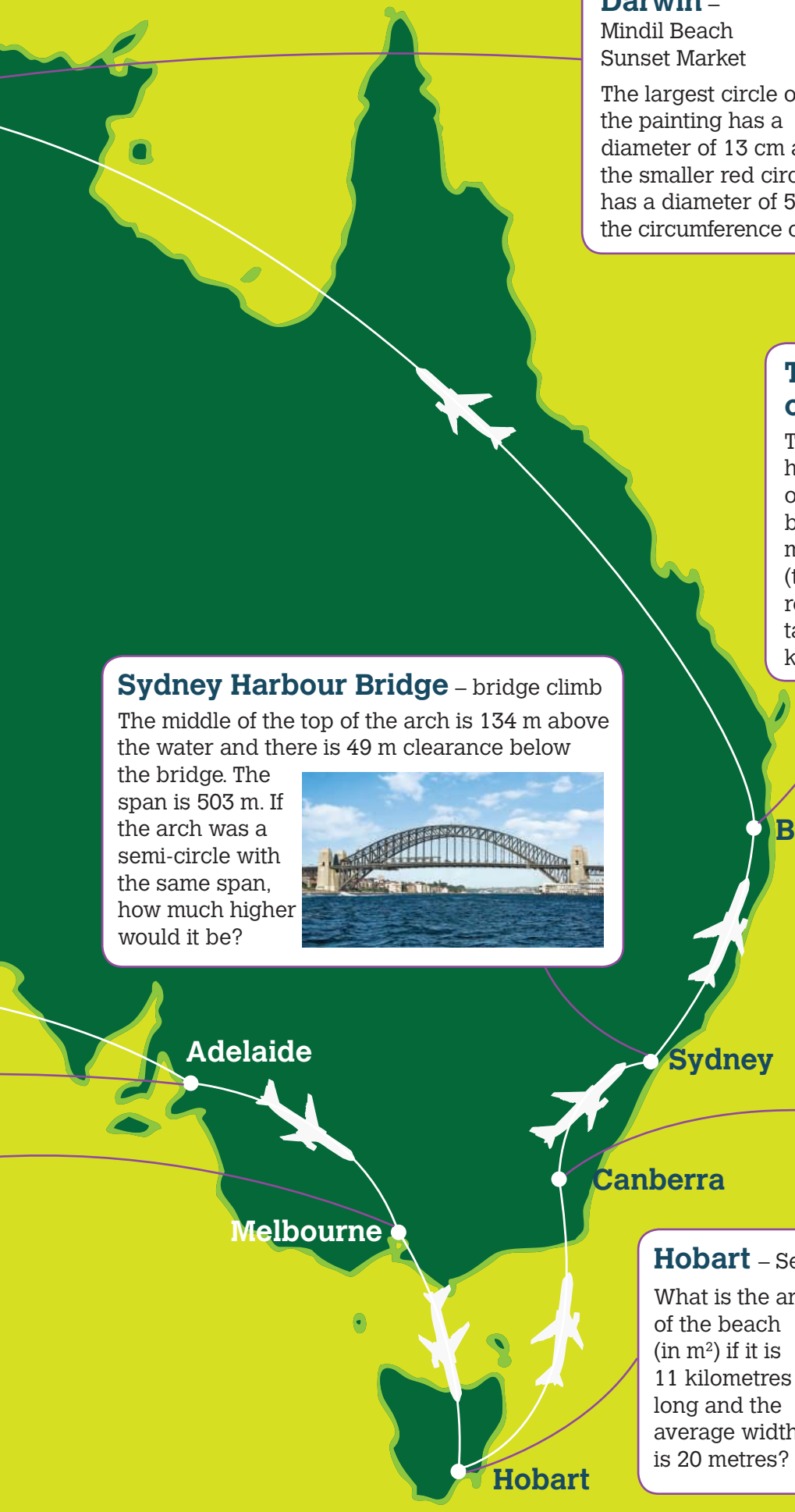
has bicycles for hire.

How far do you ride in completing one circuit of the trail shown?



Melbourne Observation Deck

Your parents take the elevator to the top of the 253 m tower while you walk up the 1254 stairs. How high is each stair in cm, correct to one decimal place?



Darwin – Mindil Beach Sunset Market

The largest circle on the painting has a diameter of 13 cm and the smaller red circle has a diameter of 5 cm. What is the difference between the circumference of the two circles (to the nearest cm)?

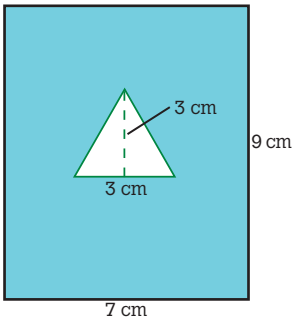
The Wheel of Brisbane

The wheel is 60 m high, so the diameter of the wheel would be about 58 m. How many revolutions (to the nearest half revolution) would it take to travel a whole kilometre?

Sydney Harbour Bridge – bridge climb

The middle of the top of the arch is 134 m above the water and there is 49 m clearance below the bridge. The span is 503 m. If the arch was a semi-circle with the same span, how much higher would it be?

Canberra – National Gallery of Australia



The centre of one of the pieces of art is as shown in the diagram. What is the area of the shaded section?

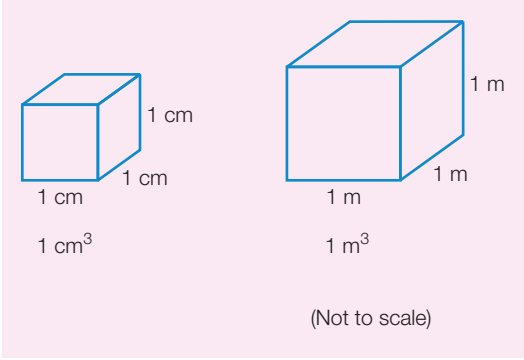
Hobart – Seven Mile Beach

What is the area of the beach (in m^2) if it is 11 kilometres long and the average width is 20 metres?

5.7 Volume and capacity

Units of volume

Volume is the amount of space taken up by three-dimensional objects, and is measured in units such as cubic centimetres (cm^3) and cubic metres (m^3).



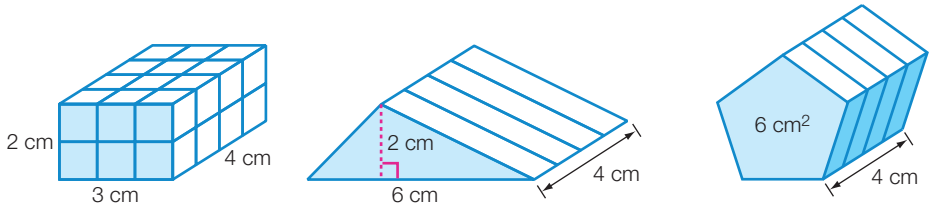
Volume is a measurement of the number of whole and parts of whole cubes that could be fitted into a solid (without any gaps).

Prisms

A **prism** is a solid that has a **uniform cross-section**. This means that the shape on one of the ends of the prism continues unchanged, throughout the height of the prism. This shape is called the base of the prism. The height of the prism is the perpendicular distance from the base to the top.

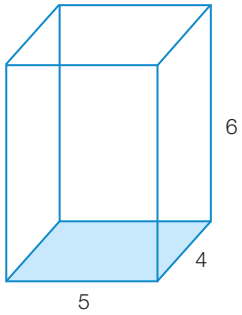
The base of a prism is a polygon.

Below are three prisms: a rectangular-based prism, a triangular-based prism and a pentagonal-based prism.

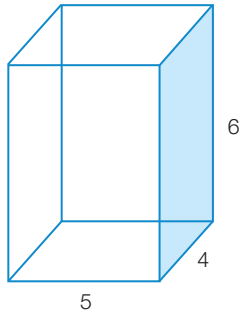


Finding the volume of a rectangular prism

Recall that to find the volume of a rectangular prism, we multiply together the length (l), width (w) and height (H). Using our definition of a prism, we can see that any one of the faces of the rectangular prism can act as the base.



$V = lwH$
$V = 5 \times 4 \times 6$
$= 120 \text{ cm}^3$



$V = lwH$
$V = 6 \times 4 \times 5$
$= 120 \text{ cm}^3$

If we look at the formula $V = lwH$, we can see that multiplying the values of l and w together gives us the area of the rectangular base: $A = lw$. So, we can write the formula

$V = lwH$ as

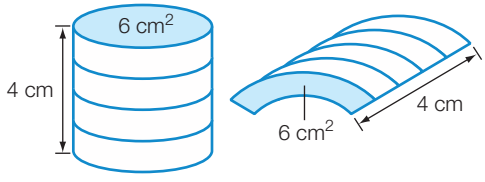
$V = AH$,

where A = the area of the base.

Because they all have a uniform cross-section, the volume of all prisms can be calculated in the same way: find the area of the base and multiply it by the height.

A general formula for the volume of solids with a uniform cross-section

There are other solids that are not prisms that have a uniform cross-section. A solid with a circular uniform cross-section is called a **cylinder**. The solid shown here with the odd-shaped curvy base has no special name, but does have a uniform cross-section.

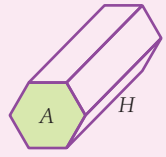


The rectangular, triangular and pentagonal prisms shown in blue on the previous page and the cylinder and the odd-shaped solid shown here all have the same volume, as they all have the same cross-sectional area (6 cm^2) and height (4 cm).

The formula for the volume of solids with a uniform cross-section:

$V = AH$

where A = area of the cross-section, H = height (H is perpendicular to A)



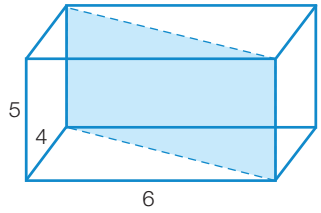
Triangular prisms

If we make a diagonal cut from corner to corner across the base and up through the height of the rectangular prism, we create a triangular prism that has half the volume of the rectangular prism.

We can write the volume of this triangular prism as:

$V = \frac{1}{2} \times 6 \times 4 \times 5$ or $V = \frac{6 \times 4 \times 5}{2}$

$= 60 \text{ cm}^3$ $\quad$ $= 60 \text{ cm}^3$



We could also use the formula $V = AH$ to find the volume, but, as the shape of the base of the prism is a triangle, its area is given by $A = \frac{1}{2}bh$, where b = length of base and h = perpendicular height to the base.

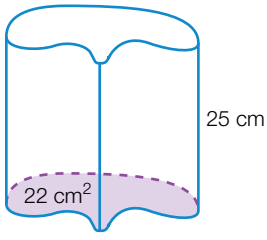
Substituting in our formula for A, we get: $V = \frac{1}{2}bhH$.

Note that we use h to represent the height of the triangular base (4 cm in the above diagram with a base of 6 cm) and H to represent the height of the prism (5 cm in the above diagram). This is demonstrated in part **(b)** of the Worked Example below.

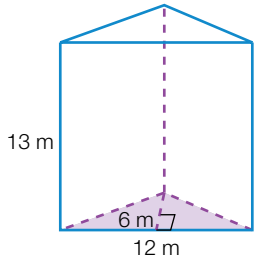
Worked Example 13 WE13

Find the volume of each solid. Give answers correct to two decimal places, where appropriate.

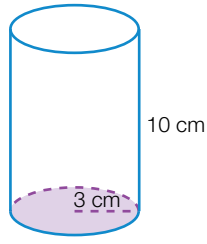
(a)



(b)



(c)

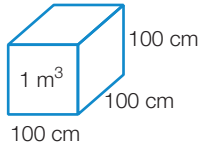


Thinking		Working
(a) **1**	Write the formula.	(a) $V = AH$
2	Substitute the values for A and H into the formula. Note that the base area A is already given.	$= 22 \times 25$
3	Evaluate, and write the answer with the appropriate units of volume.	$= 550 \text{ cm}^3$
(b) **1**	Write the general formula.	(b) $V = AH$
2	Substitute the area formula for the triangular base.	$= \frac{1}{2}bh \times H$
3	Substitute the values for b, h and H.	$A = \frac{1}{2} \times 12 \times 6 \times 13$
4	Evaluate the volume and write the answer with the appropriate units.	$= 468 \text{ m}^3$

(c) **1**	Write the general formula.	(c) $V = AH$
2	Substitute the area formula for the circular base.	$= \pi r^2 \times H$
3	Substitute the values for r and H.	$= \pi \times 3^2 \times 10$
4	Evaluate the volume. Round the answer correct to two decimal places and write it with the correct units.	$= 282.74\ \text{cm}^3$ (2 d.p.)

Converting units of volume

We know that one 'linear' metre (the unit of length) is equal to 100 cm. However, one cubic metre is equal to $100 \times 100 \times 100\ \text{cm}^3$.



$$\begin{aligned} 1\ \text{m}^3 &= (100)^3\ \text{cm}^3 \\ &= 100 \times 100 \times 100 \\ &= 1\,000\,000\ \text{cm}^3 \end{aligned}$$

Units of volume are cubic units. Converting between units of volume is not the same as converting units of length. The conversion factor (the number we multiply or divide by) is the cube of the conversion factor for length.

Units of length:	km	$\times 1000$ / $\div 1000$	m	$\times 100$ / $\div 100$	cm	$\times 10$ / $\div 100$	mm
Units of volume:	km^3	$\times 1000^3$ / $\div 1000^3$	m^3	$\times 100^3$ / $\div 100^3$	cm^3	$\times 10^3$ / $\div 10^3$	mm^3

Worked Example 14

WE14

Copy and complete the following conversions.

(a) $14.5\ \text{cm}^3 =$ _____ mm^3

(b) $37\,000\ \text{m}^3 =$ _____ km^3

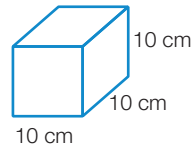
Thinking	Working
(a) Converting from a larger unit to a smaller unit, so multiply. There are $10^3\ \text{mm}^3$ in every cm^3, so multiply by 10^3.	(a) 14.5×10^3 $= 14.5 \times 1000$ $= 14\,500\ \text{mm}^3$
(b) Converting from a smaller unit to a larger unit, so divide. There are $1000^3\ \text{m}^3$ in every km^3, so divide by 1000^3.	(b) $37\,000 \div (1000)^3$ $= 37\,000 \div 1\,000\,000\,000$ $= 0.000\,037\ \text{km}^3$

Units of capacity

Capacity is a term used for the volume of space inside a container.

A litre (L) is the space inside a 10 cm × 10 cm × 10 cm cube.

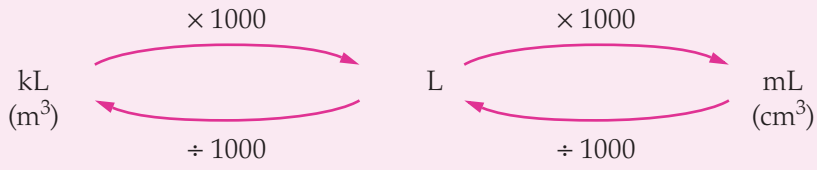
A millilitre (mL) is the space inside a cubic centimetre and a kilolitre (kL) is the space inside a cubic metre.



1 kL = 1000 L
= 1 m^3
= 1 000 000 cm^3

1 L = 1000 mL
= 1000 cm^3

1 cm^3 = 1 mL



Worked Example 15

WE15

Copy and complete the following conversions.

(a) 2.25 L to ____ mL **(b)** 600 cm^3 to ____ L **(c)** 40 000 cm^3 to ____ m^3

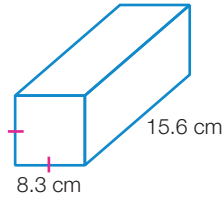
Thinking	Working
(a) Converting from a larger unit to a smaller unit, so multiply by 1000.	(a) 2.25 L = 2.25 × 1000 mL = 2250 mL
(b) **1** Change the volume to the equivalent capacity.	(b) 600 cm^3 = 600 mL
2 Converting from a larger unit to a smaller unit, so divide by 1000.	= 600 ÷ 1000 L = 0.6 L
(c) There are 1 000 000 cm^3 in 1 m^3, so divide by 1 000 000 (smaller unit to larger unit).	(c) 40 000 cm^3 ÷ 1 000 000 = 0.04 m^3

To find the capacity of a container or hollow solid, find the volume of the interior, then convert to units of capacity (mL, L, kL).

Worked Example 16

WE16

Find the capacity of the container, in litres. Round your answer correct to one decimal place.



Thinking	Working
1 Write the formula for volume.	$V = AH$
2 Substitute the formula for A, the area of the square base.	$= l^2 \times H$
3 Substitute the values.	$= 8.3^2 \times 15.6$
4 Evaluate the volume, including the units.	$= 1074.684\ cm^3$
5 Convert the answer to mL. ($1\ mL = 1\ cm^3$)	Capacity $= 1074.684$ mL
6 Convert mL to L by dividing by 1000.	$= 1.074\,684$ L
7 Round the answer to one decimal place.	$= 1.1$ L (correct to 1 d.p.)

5.7 Volume and capacity

Navigator

Q1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q13, Q14, Q15, Q16, Q20	Q1, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q16, Q18, Q20, Q21	Q1, Q2 Column 2, Q3 Column 2, Q4, Q5 (a) & (d), Q8, Q9, Q10, Q11, Q12, Q13, Q16, Q17, Q18, Q19, Q20, Q21

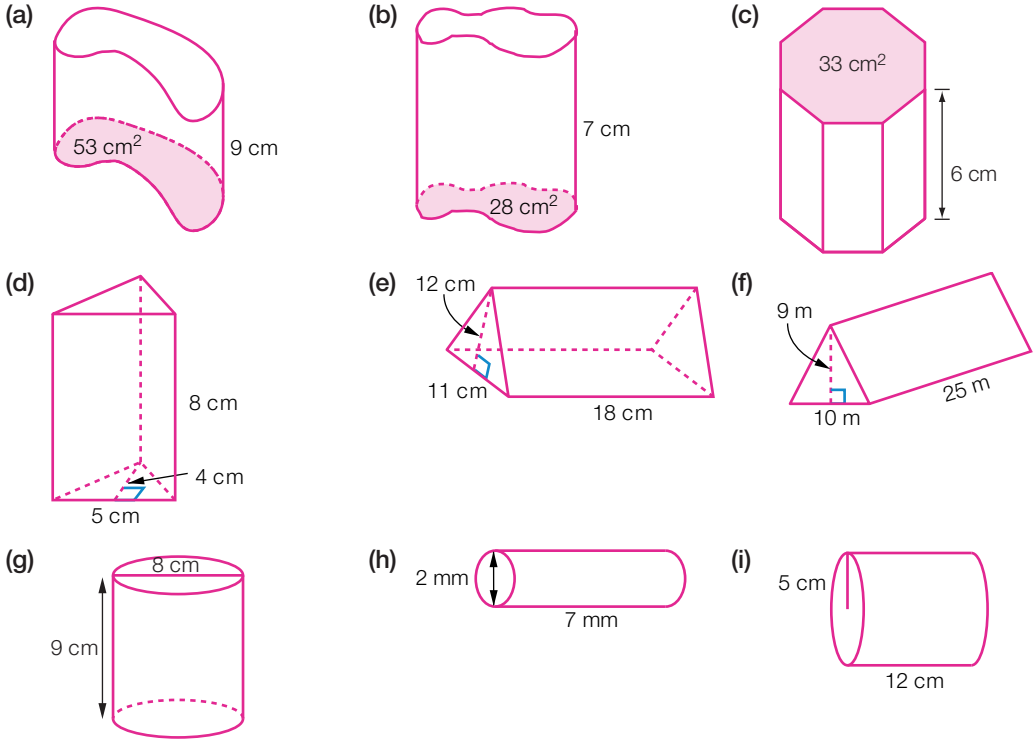
Answers page 619

Equipment required: Scientific calculator for Questions **1**, **4**, **5**, **7–17** and **19–21**

Fluency

1 Find the volume of each solid. Give answers correct to two decimal places, where appropriate.

WE13



WE14

2 Copy and complete the following conversions.

(a) 20 cm^3 = ____ mm^3
(b) 195 cm^3 = ____ mm^3
(c) 8.3 m^3 = ____ cm^3
(d) 25.46 m^3 = ____ cm^3
(e) 0.75 km^3 = ____ m^3
(f) 57 km^3 = ____ m^3
(g) 150 mm^3 = ____ cm^3
(h) 3830 mm^3 = ____ cm^3
(i) 47 900 cm^3 = ____ m^3
(j) 6070 cm^3 = ____ m^3
(k) 0.48 m^3 = ____ cm^3
(l) 97 500 m^3 = ____ km^3
(m) 360 000 m^3 = ____ km^3
(n) 0.0025 km^3 = ____ m^3
(o) 0.04 cm^3 = ____ mm^3
(p) 0.0356 m^3 = ____ cm^3

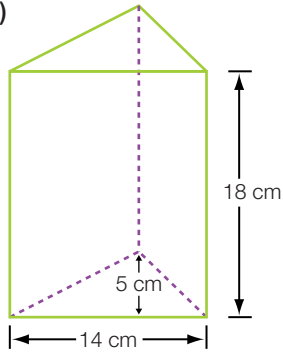
WE15

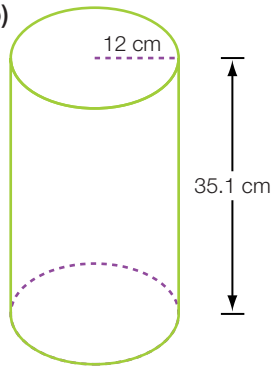
3 Copy and complete the following conversions.

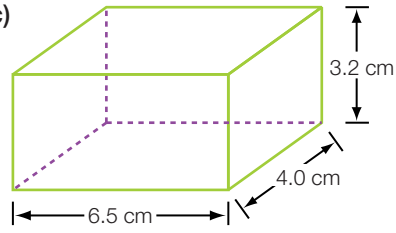
(a) 2000 mL = ____ L
(b) 55 000 mL = ____ L
(c) 800 mL = ____ L
(d) 40 mL = ____ L
(e) 5 L = ____ mL
(f) 95 L = ____ mL
(g) 0.03 L = ____ mL
(h) 0.5 L = ____ mL
(i) 25 cm^3 = ____ mL
(j) 48 cm^3 = ____ mL
(k) 33 mL = ____ cm^3
(l) 140 mL = ____ cm^3
(m) 5.8 m^3 = ____ kL
(n) 1.7 m^3 = ____ kL
(o) 0.13 kL = ____ m^3
(p) 0.05 kL = ____ m^3

WE16

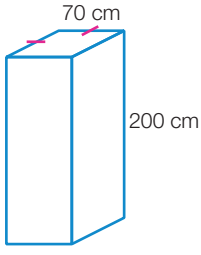
4 Find the capacity of the following containers, in millilitres. Round your answers correct to one decimal place where necessary.

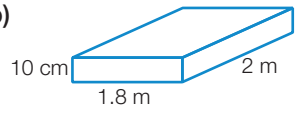
(a)


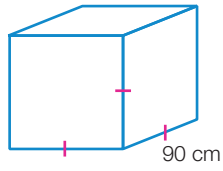
(b)


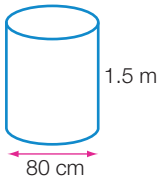
(c)


5 Find the volume of each of the following shapes **(i)** in cm^3 and **(ii)** in m^3. For the cylinders, round your answers correct to two decimal places.

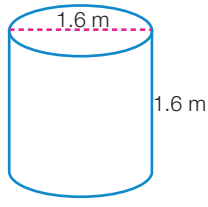
(a)


(b)


(c)


(d)


(e)

(f)


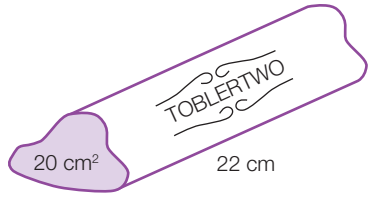
6 (a) 16 L is equal to:

A 0.016 kL B 160 mL C 0.16 kL D 1.6 kL

(b) 3450 cm^3 is equal to:

A 3.45 mm^3 B 34 500 mm^3 C 0.003 45 m^3 D 34.5 m^3

7 Find the volume of a chocolate bar shaped as shown.



Understanding

8 The shape of one of the fermenters shown can be modelled by a cylinder of height 1.5 m and a radius of 60 cm.

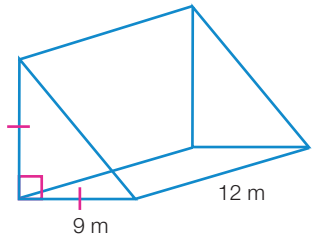
(a) Find the volume of a fermenter in m^3, correct to three decimal places.

(b) What is the capacity of the fermenter to the nearest litre?

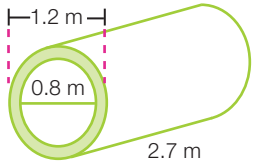
9 Calculate the volume of this triangular prism by:

(a) calculating the volume of a rectangular prism, then halving it

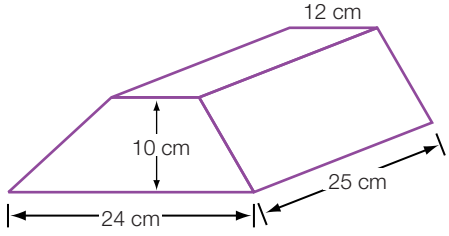
(b) finding the area of the triangular base and multiplying it by the height.



10 Find the volume of the hollowed-out cylinder, correct to two decimal places.



11 Find the capacity of this prism in litres.



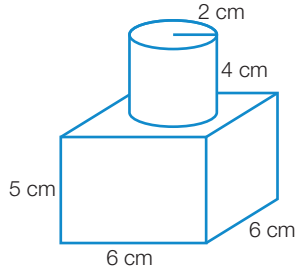
12 The capacity of the model aircraft fuel tank shown is closest to:

A 67.24 mL

B 205.12 mL

C 230.27 mL

D 1440 mL



13 During a science experiment, Allen pours 45 mL of acid into a flask already containing 0.75 L of acid. How much acid is now in the flask:

(a) in mL?

(b) in L?

14 What is the capacity of a soft drink can that has a diameter of 6 cm and a height of 13 cm? Answer to the nearest mL.

Reasoning

15 Larissa and Dave are installing a pool in their backyard. It is rectangular and will be 4 m wide, 12 m long, and have a depth of 1.5 m.

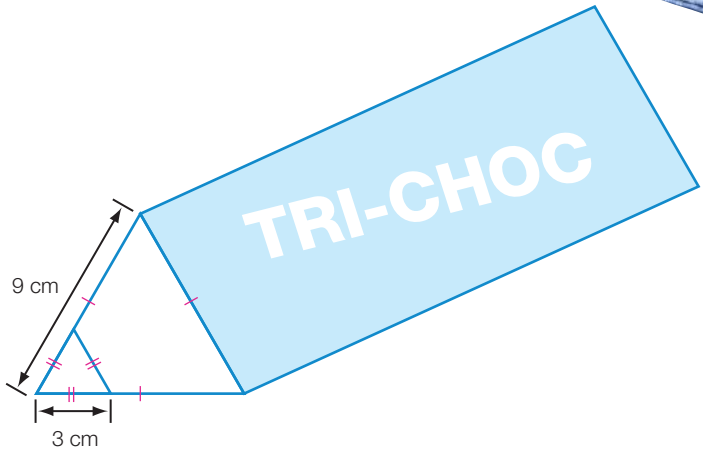
(a) Draw a diagram that shows this information.

(b) If it costs $1.68 for every 1000 litres of water, what will it cost to fill the pool?

16 A steel pin head has a volume of 1 mm^3. If pin heads are melted down, how many would it take to make a millilitre of steel?

17 A drink jug holds 2 litres of juice. Assuming they are cylindrical, how many glasses can be filled if their internal diameter is 8 cm and they are to be filled to a depth of 12 cm?

18 A confectioner makes triangular prism-shaped chocolate bars. The base of the prism is an equilateral triangle with a side length of 3 cm. The height of the prism is 12 cm long. The company also produces a giant 'party-pack' version, where a larger triangular prism-shaped box has the regular size bars packed tightly inside. The dimensions (base length and height) of this giant box are three times the size of the regular box. How many regular boxes will fit inside the giant box?



19 Rainfall is measured in millimetres. If 5 mm of rain is recorded in a particular place, any bucket or other container with straight sides will fill to a depth of 5 mm.

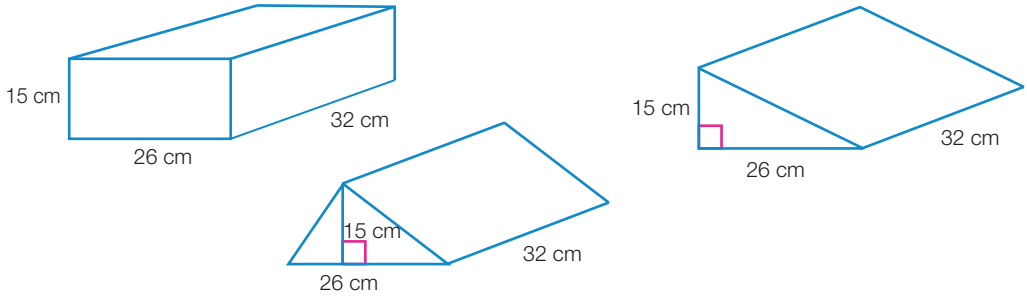
(a) When 5 mm of rain falls, how many litres of rain will be collected in a tank from a roof of equivalent horizontal area 200 m^2?

(b) What roof area is needed if 25 mm of rain is to fill a 10 000 L tank?

Open-ended

20 Sally is designing a cylindrical container for a new sports drink, PowerPlus, and the container is to have a capacity of 500 mL. Help Sally design three different containers for PowerPlus. How might she decide which is the best-shaped container? (You should consider both the height and the diameter of the container.)

21 The triangular prisms below would each fit snugly inside the rectangular prism.



(a) Explain how each of the triangular prisms could be produced from the rectangular prism, then compare the volume of the three solids.

(b) Draw another triangular prism of height (H) 26 cm that could be produced from the rectangular prism and has the same volume as the triangular prisms above.

Outside the Square — Problem solving

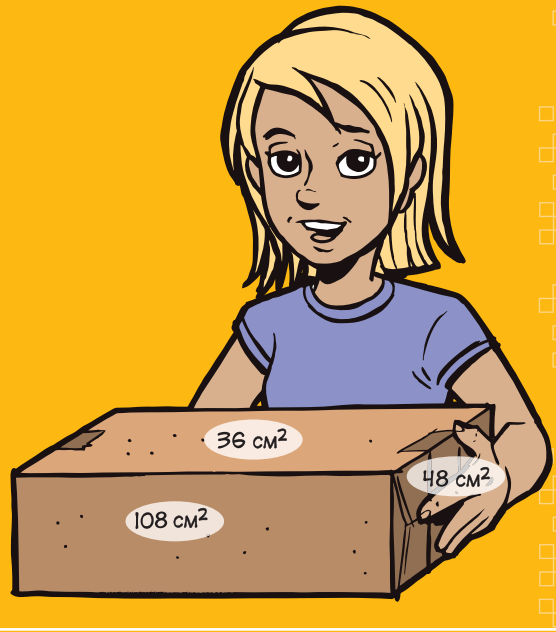
Baffling box

Chiara is making a wooden money box and has gone to a timber supplier to buy the wood. The trouble is, she can remember the area of each face, but not the dimensions.

1. Work out the dimensions for her from the information she remembers (see opposite).
2. Determine the volume of the proposed money box.
3. What is the total area of wood she will need for all six of the box's faces?

Strategy options

- Guess and check.
- Test all possible combinations.



5.8 Time

One day is the time taken for the Earth to complete one full rotation on its axis, and is broken into 24 hours. One year is the time taken for the Earth to complete one revolution around the Sun. Unlike other units of measurement, not all the units for time are based on the number 10. Some are, whereas others are based on the number 60.

1 minute = 60 seconds	2 weeks = a fortnight	1 year = 365 days*
1 hour = 60 minutes	1 month = 28–31 days	1 decade = 10 years
1 day = 24 hours	1 year = 12 months	1 century = 100 years
1 week = 7 days	1 year = 52 weeks	1 millennium = 1000 years

*366 in a 'leap year' (every 4 years)

24-hour time

Instead of dividing the day into two 12-hour blocks, **24-hour time** continues the numbering of the hours past 12 p.m. (midday), using a four-digit system:

12 a.m. (midnight)	0000	12 p.m. (midday)	1200
1 a.m.	0100	1 p.m.	1300
9.30 a.m.	0930	9.30 p.m.	2130
		11.59 p.m.	2359

Note that with 24-hour time, there is no need to use 'a.m.' or 'p.m.' after the time. The defence forces (army, navy and airforce) use only 24-hour time notation. Many European countries also use it in train and bus timetables, cinema times and shop opening hours.

Worked Example 17 WE17

Express the following in 24-hour time.

(a) 6.30 a.m. **(b)** 7.45 p.m.

Thinking	Working
(a) For times before 1 p.m., remove the point, and write it as a four-digit number (add a zero in front if you have only three digits).	(a) 0630
(b) For times from 1 p.m. onwards, remove the point, write it as a four-digit number, and add 1200.	(b) 0745 + 1200 = 1945

Worked Example 18

WE 18

Write the following 24-hour times as a.m. or p.m. times.

(a) 0852 **(b)** 1914

Thinking	Working
(a) 1 Do the first two digits make a number greater than 12? (No.)	(a) 0852
2 Rewrite the time and put a separating point between the hour and the minute number. If the first digit is a zero, it can be dropped.	8.52
3 As the hour number is less than 12, it is before midday, so write 'a.m.' after the time.	8.52 a.m.
(b) 1 Do the first two digits make a number greater than 12? (Yes.)	(b) 1914
2 Subtract 1200 from the number.	1914 – 1200 = 0714
3 Rewrite the time and put a separating point between the hour and the minute number.	7.14
4 As the original hour number is greater than 12, it is after midday, so write 'p.m.' after the time.	7.14 p.m.

Calculating elapsed time

A period of time that has passed is known as **elapsed time**.

Worked Example 19

WE 19

Work out how much time has passed between the times given.

10.56 a.m. to 3.33 p.m.

Thinking	Working
1 Look at the starting time and work out how many minutes to the next hour.	10.56 a.m. to 11 a.m. = 4 min
2 Work out how many whole hours occur before the finishing time.	11.00 a.m. to 3.00 p.m. = 4 h
3 Work out any remaining minutes to the finishing time.	3.00 p.m. to 3.33 p.m. = 33 min
4 Add up the hours and minutes separately.	4 min + 4 h + 33 min = 4h 37 min

Worked Example 20

WE 20

What is the time:

(a) $4\frac{1}{2}$ hours after 11.47 a.m.

(b) 9 hours and 25 minutes before 1315?

Thinking	Working
(a) 1 Ignore the fraction and add the whole number hours onto the time.	(a) 11.47 a.m. + 4 h = 3.47 p.m.
2 Work out how many minutes the fraction part is.	$\frac{1}{2}$ h = 30 min
3 Add the minutes onto the time. Make sure you go up to the next hour when you reach 60 minutes.	3.47 p.m. + 30 min = 3.47 p.m. + 13 min + 17 min = 4.00 p.m. + 17 min
4 Write down the answer.	= 4.17 p.m.
(b) 1 Subtract the number of hours from the time.	(b) 1315 – 9 h = 0415
2 Now, subtract the number of minutes, remembering to move into the previous hour if necessary.	0415 – 25 min = 0400 – 10 min = 0350

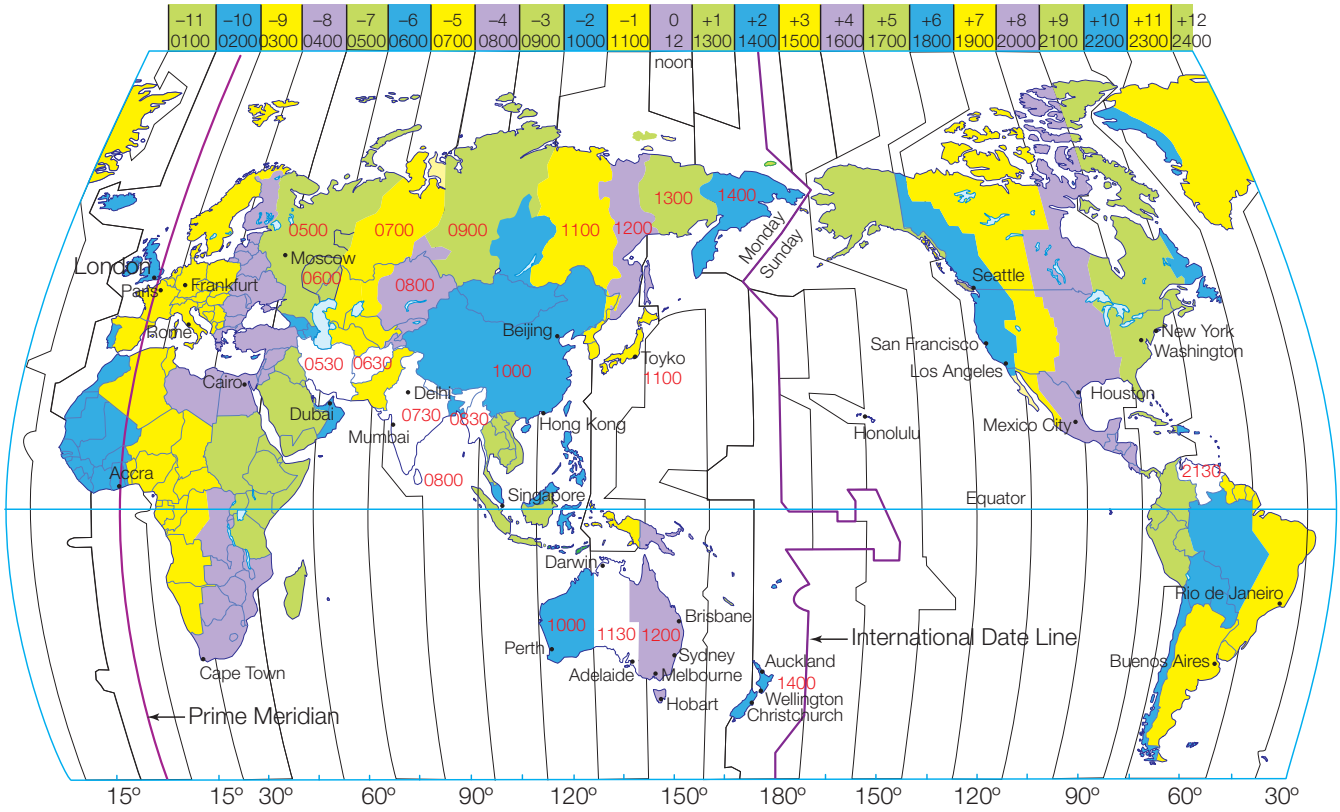
Time zones

The world is divided into 24 **time zones** (corresponding to the 24 hours in one day). These are the 24 columns at the top of this map of the world. Countries or states that are in the same time zone set their clocks to the same time. You can see from the map that some large countries, such as Australia and China, are spread across several time zones. Australia has three time zones, meaning that clocks in different cities are set to different times. China has chosen to have one time zone across the whole country, meaning that all clocks are set to the same time.

The first line of numbers across the top of the map on the following page shows how many hours ahead (+) or behind (–) each time zone is compared to Australian Eastern Standard Time. The second line of numbers shows what time it would be in each time zone if it was 1200 hours (noon) in Sydney, Melbourne or Brisbane.

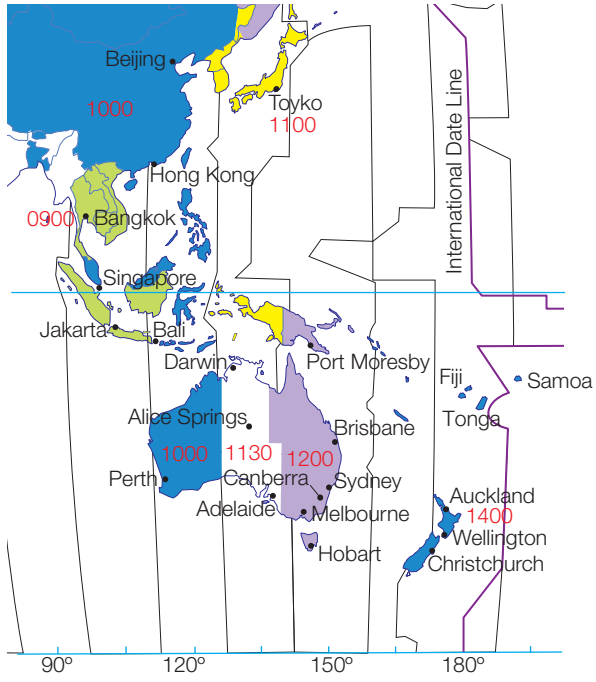
The reference point for setting the time is a place called Greenwich (pronounced Gren-itch) in London. The time in Greenwich is known as Greenwich Mean Time, or GMT. An imaginary vertical line running through Greenwich (called the Prime Meridian) divides the world into the Eastern and Western hemispheres. What hour the time is set to in another part of the world depends on how many time zones east or west of GMT it is. For example, Western Australia is 8 time zones east of Greenwich, so the time in WA is 8 hours in front of GMT. Brazil is 3 time zones to the west of Greenwich, so the time in Brazil is 3 hours behind GMT.

When working with different time zones around the world, it sometimes helps to think of whether a person in that time zone would be ahead of you or behind you, time wise. People in time zones to the east of you are ahead—they will be having breakfast while you are sleeping. People in time zones to the west of you are behind you—they may still be asleep by the time you have finished breakfast.



The International Date Line

The line shown on the map 12 hours later or 12 hours earlier than GMT (depending on whether you move east or west) is known as the International Date Line, or IDL. The local time in the places either side of the IDL is the same, but they are on different days. For example, when it is 7 a.m. on Monday in Tonga, just to the west of the IDL, it is 7 a.m. on Sunday in Samoa to the east of the IDL.



Australian and Asian time zones

Time zone	Time	States
Australian Western Standard Time (AWST)	GMT plus 8 hours	WA
Australian Central Standard Time (ACST)	GMT plus 9.5 hours	SA, NT
Australian Eastern Standard Time (AEST)	GMT plus 10 hours	Qld, NSW, Vic., Tas., ACT

Some countries to the north of Australia, such as Papua New Guinea, Japan and China, lie in the Australian time zones, so the time in these countries will be the same.

Daylight savings

Some states in Australia apply Daylight Savings Time (DST). This involves turning the clock forward by one hour near the start of spring (October) and turning the clock back one hour in autumn (April). A handy way to remember this is the phrase 'spring-forward, fall-back' ('fall' is another name for autumn). DST increases the hours of daylight in the evening.

5.8 Time

Navigator

Answers page 620

Q1, Q2, Q3 Column 1, Q4 Column 1, Q5, Q7, Q8, Q9, Q11, Q13, Q16 (a)–(c), Q17, Q20, Q22, Q23, Q24, Q25	Q1, Q2, Q3 Column 2, Q4 Column 2, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q19, Q20, Q21, Q22, Q23, Q24, Q25	Q1, Q2, Q3 Column 2, Q4 Column 2, Q6, Q8, Q10, Q11, Q12, Q14, Q15, Q16, Q17, Q18, Q19, Q20, Q21, Q22, Q23, Q24, Q25

Fluency

WE17

1 Express the following in 24-hour time.

(a) (i) 2.30 a.m. (ii) 2.30 p.m.
(b) (i) 6.15 a.m. (ii) 6.15 p.m.
(c) (i) 11.43 a.m. (ii) 11.43 p.m.
(d) (i) midday (ii) midnight

WE18

2 Write the following 24-hour times as a.m. or p.m. times.

(a) 1354 (b) 0833 (c) 0539 (d) 1634
(e) 1830 (f) 1902 (g) 0147 (h) 0320

WE19

3 Work out how much time has passed between the times given.

(a) 2.55 a.m. to 5.41 a.m.
(b) 1849 to 2304
(c) 1934 to 2220
(d) 1.22 a.m. to 6.13 a.m.
(e) 10.52 p.m. to 1.09 a.m.
(f) 2340 to 0243
(g) 0143 to 1152
(h) 2.03 a.m. to 12.44 a.m.
(i) 1027 to 0100
(j) 9.49 p.m. to 3.01 p.m.

WE20

4 What is the time:

(a) 3 hours after 4.15 a.m.

(b) 5 hours after 0123

(c) 6 hours and 10 min before 2012

(d) 4 hours and 40 min before 11.59 p.m.

(e) 5 hours and 23 min after 10.55 a.m.

(f) 7 hours and 13 min before 1443

(g) $3\frac{1}{2}$ hours before 1621

(h) $4\frac{1}{2}$ hours before 1.08 a.m.

(i) $7\frac{1}{4}$ hours after 9.49 a.m.

(j) $9\frac{1}{4}$ hours after 1158?

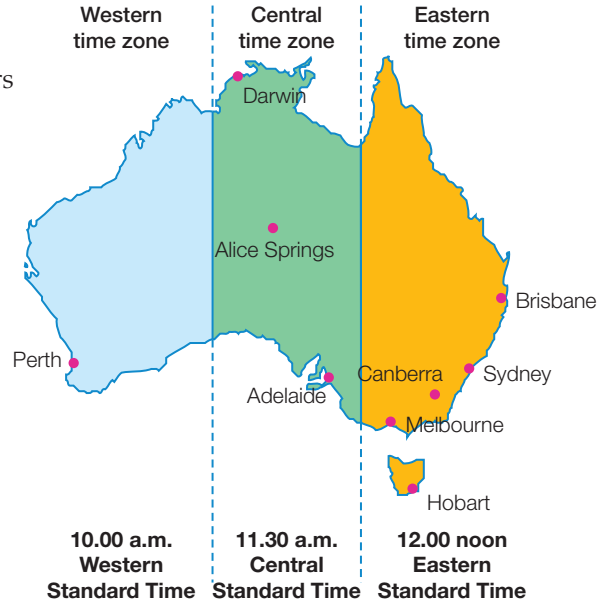
5 Look at this map showing the three Australian time zones. Central Standard Time (CST) is half an hour behind Eastern Standard Time (EST). Western Standard Time (WST) is 2 hours behind Eastern Standard Time.

(a) If it is 3.00 p.m. in Melbourne, what time is it in Perth?

(b) If it is 4.15 p.m. in Brisbane, what time is it in Adelaide?

(c) If it is 11.45 p.m. in Adelaide, what time is it in Canberra?

(d) If it is 7.10 a.m. in Perth, what time is it in Darwin?



6 Foxtel is televising live the AFL game between the West Coast Eagles and Collingwood. It goes to air at 1930 on a Friday night in June in Perth. Use this map of Australian time zones to determine:

(a) what time the broadcast starts in Melbourne

(b) what time the broadcast starts in Adelaide.

Understanding

7 About how long does it take you to:

(a) run 100 metres?

A 15 seconds B 15 minutes C 15 hours D 15 days

(b) boil an egg?

A 4 seconds B 4 minutes C 4 hours D 4 days

(c) drive from Melbourne to Sydney?

A 10 seconds B 10 minutes C 10 hours D 10 days

8 State TRUE or FALSE for the following.

(a) It takes about $365\frac{1}{4}$ days for the Earth to travel around the Sun.

(b) There are 2 weeks in a fortnight.

(c) A day is the time taken for the Earth to rotate once about its axis.

(d) There are exactly 52 weeks in a year.

(e) There are exactly 4 weeks in a month.

(f) There are exactly 10 years in a decade.

9 Jodie got to school at 8.40 a.m. If her journey to school took 63 minutes, at what time did she leave home?

10 Dirk left home at five past eight in the morning and caught the 8.13 train, which was one minute late. The train trip took twelve minutes, and it took him six minutes to walk from the station to the school.

(a) At what time did he get to school?

(b) How many minutes did it take him to get from his home to school?

11 Mark started a marathon at 10.49 a.m. and he finished at 2.36 p.m. The time that he took to run the course was:

A 3 h 47 min

B 4 h

C 4 h 13 min

D 13 h 25 min

12 Gordon's flight from Brisbane to Cairns arrived at 0915. If the flight took 2 hours and 20 mintues, at what 24-hour time did he leave Brisbane?

13 The late flight from Perth to Melbourne is called the 'Red-eye Special'. It leaves Perth at 12.30 a.m. and travels through the night. The flying time is 4 hours 20 minutes. When the plane lands, what will be the time in:

(a) Perth **(b)** Melbourne?

14 The following table shows when high and low tides were predicted at North Bondi Beach in Sydney over a period of 5 days. There are two high and two low tides each day.

June	Low tide		High tide	
8 M	0246	1400	0837	2040
9 T	0324	1438	0916	2116
10 W	0401	1517	0956	2153
11 T	0440	1557	1035	2230
12 F	0519	1639	1116	2309

Give your answers to these questions in a.m. or p.m. times.

(a) When was high tide at North Bondi Beach on 11 June?

(b) When was low tide at North Bondi Beach on 10 June?

(c) How much earlier was the a.m. high tide on 10 June than on 11 June?

(d) What was the elapsed time between the first low tide and the first high tide on 12 June?

(e) On which dates did low tide occur between 2 a.m. and 4 a.m.?

(f) How much earlier was the p.m. low tide on 8 June than on 12 June?

15 Grace works in Canberra. Her manager has asked her to attend a video link meeting between her office and the company's office in Beijing, China. If the phone meeting is scheduled for 10.00 a.m. Thursday in Canberra, for what time will the meeting be scheduled in Beijing? Use the time zone map on page 319 to help you determine the answer.

16 Western Australia, Queensland and the Northern Territory do not apply Daylight Savings Time. In all other states, clocks are moved forward one hour during spring (usually early October) until autumn (usually early April).

(a) Copy and complete the table to show the time during the Daylight Savings months. Use the time zone map on page 321 to help you.

State	Standard Time	Daylight Saving Time
WA		
NT		
SA		
Qld		
NSW	12 noon	
Vic		
Tas		

(b) At 3.30 p.m. in Sydney in January, what time is it in Darwin?

(c) At noon on Christmas day in Hobart, Dinara telephoned her Mum, who lives in Adelaide. What time was it in Adelaide?

(d) At the Perth Airport during summer, a row of clocks display the times in different cities in Australia. Copy and complete the table below.

Perth	Darwin	Adelaide	Sydney	Melbourne	Brisbane

Reasoning

17 Jamie is backpacking around Europe. He's on a train from Geneva (Switzerland) to Milan (Italy), a trip that will take 3 hours and 50 minutes. When he gets to Milan, he needs to change trains to go on to Rome. If the train left Geneva at 1415, will Jamie make the 1820 train to Rome? Explain your answer.

18 On 1 December 2010, what was the exact age in years, months and days of a woman who was born on 20 January 1985?

19 Corporal Green is writing a timetable for a training day for his army recruits. The day must start with a half-hour-long parade ground drill at 0800. This is to be followed by a one-hour lesson on navigation theory, a one-hour lesson on military history, then a one and a half hour fitness session. Recruits then get 35 minutes to shower and have lunch, before a three-quarters of an hour lesson on weapons handling and a two-and-a-quarter hour practice session on the firing range. Finally, recruits have a one-hour session on setting up a bush camp. Complete Corporal Green's timetable, writing all times as 24-hour times.

Recruit training day	
Activity	**Starting time**
Parade ground drill	0800

20 **(a)** If a plane leaves Brisbane at 0930 local time and arrives in Alice Springs at 1300 local time, how long has the trip taken?

(b) If a plane leaves Canberra at 1422 local time and arrives in Perth at 1715 local time, how long has the trip taken?

21 The following table shows the flying times from Sydney to various cities and the time differences between these cities and AEST (Australian Eastern Standard Time). If Yvonne leaves Sydney at 8.00 a.m., calculate her local arrival time at each city.

City	Time zone	Flight time
Hong Kong	AEST less 2 h	7 h 50 min
Tokyo	AEST less 1 h	9 h 30 min
Auckland	AEST plus 2 h	2 h 45 min
Bangkok	AEST less 3 h	9 h 30 min

Open-ended

22 Elizabeth recorded the amount of time she spent on homework in a week. Here is her record:

Day	Amount of time	
Monday	1 h 30 min	1.3 h
Tuesday	2 h	2.0 h
Wednesday	1 h	1.0 h
Thursday	1 h	1.0 h
Friday	45 min	0.45 h

Elizabeth added up her numbers in the last column and got a total time of 5.75 hours, which she converted to $5\frac{3}{4}$ hours. 'That's funny!' she thought. 'I'm sure I spent more time than that!'

What is wrong with Elizabeth's adding up? Give her some advice so that she can add time correctly in future.

23 A TV show has a total running time of 1 hour. It must include 22 minutes of advertisements. How many ad breaks would you have and how long would each ad break be?

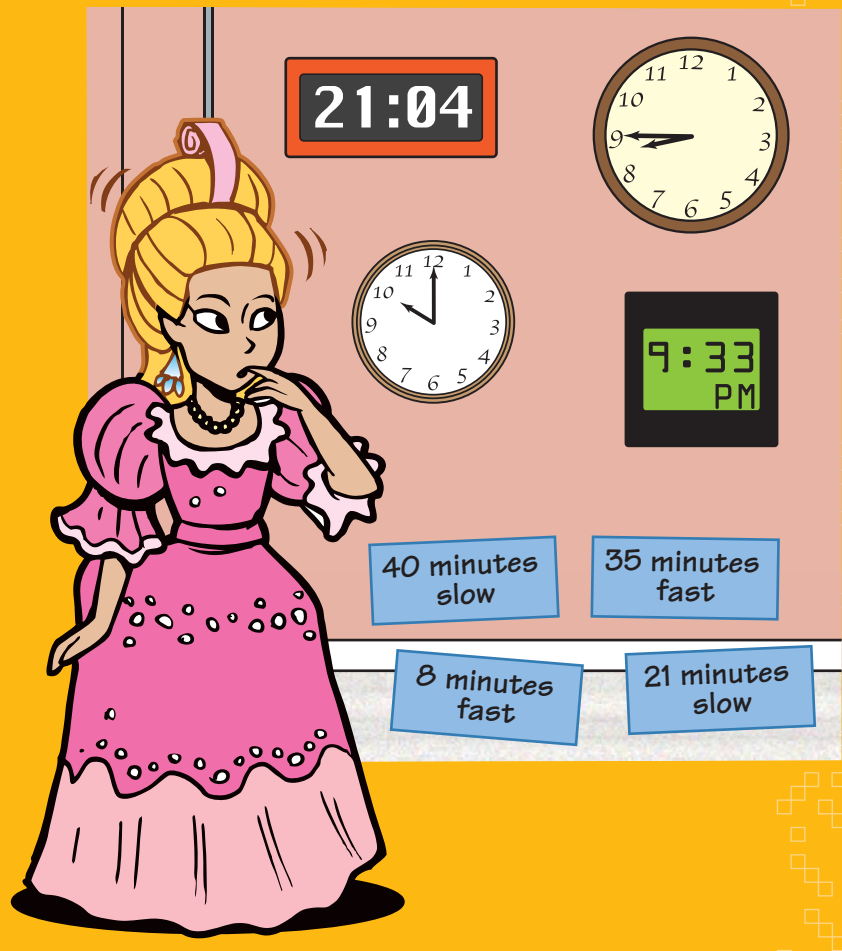
24 List four activities that would take about 2 hours each to complete.

25 In 2009, Western Australia voted against applying Daylight Savings Time. Make a list stating the advantages and disadvantages of turning the clocks forward by one hour over the hotter months.

Outside the Square Puzzle

Time gone cuckoo

As Cinderella arrived at the ball and stepped down from her carriage, she wondered what the time was. Cinderella ran over to the little clock shop that was next door to the Town Hall and looked in the window. In the window display of the shop there were four clocks and four signs that had been taken down and placed under the clocks. The signs read '40 minutes slow', '8 minutes fast', '35 minutes fast' and '21 minutes slow'. Cinderella has no idea which sign belongs to which clock. How many minutes does Cinderella have until midnight?



Maths 4 Real

Calendars are designed so they fit either the solar cycle (the time it takes the Earth to complete one orbit of the Sun) or the lunar cycle (the time it takes the Moon to complete one orbit of the Earth, which can be measured by observing the phases of the Moon).

The official calendar we use in Australia is known as the Gregorian calendar (named after Pope Gregory XIII, who introduced it). The Gregorian calendar is a solar calendar of 365 days, with an extra day added every four years to make a 'leap year'. This extra day is necessary because the length of the solar cycle is closer to 365.25 days.

A year is divided into 12 months. The length of one month corresponds approximately, but not exactly, to the lunar cycle.

The Islamic calendar

The Islamic calendar is a lunar calendar. It has 12 months, with the length of each month set to correspond to one lunar cycle. Because the timing of the Moon's orbit can vary, even numbered months have 29 days and odd numbered months have 30 days. Leap years in the Islamic calendar occur every two or three years, where an extra day is added onto the twelfth month to keep the months aligned with the lunar cycle.

1 **(a)** Is the Islamic calendar shorter or longer than the Gregorian calendar? (Assume regular, not leap, years.)

(b) What is the difference in the number of days?

Year 1 of the Islamic calendar is designated as the year that Muhammad fled from Mecca. This year corresponded to the year 622 AD on the Gregorian calendar, so the year numbers in the Islamic and Gregorian calendars are different. An Islamic year number can be converted to a Gregorian calendar year by applying the rule below:

Today's date is...

1 Finding the Gregorian year number: Multiply the Islamic year number by 970 224, divide this by one million, then add 621.5774. The whole number part of the resulting decimal is the Gregorian year number.

2 Finding the Gregorian day number: Multiply the decimal part of the number you obtained in **(1)** by 365 and round the answer down to the nearest whole number to find the day number of the Gregorian year on which the new Islamic year begins. (You will then need to determine which day of which month this is.)

2 **(a)** Apply the above rule to find out which Gregorian calendar year corresponds to the Islamic calendar year of 1434.

(b) On which day of the Gregorian year did the Islamic year of 1435 begin?

The Chinese calendar

The Chinese calendar is a 'lunisolar' calendar. This means that the length of the months corresponds to the lunar cycle, but the overall length of the year seeks to follow the solar cycle. To achieve this, an extra month is added every two or three years.

Unlike most other calendars, the Chinese calendar does not count the years in an infinite sequence. Instead, years go through a cycle of names. Each year has two parts to its name. The first part is called the 'Celestial Stem'. These ten words have no corresponding English word:

1	2	3	4	5	6	7	8	9	10
jia	*yi*	*bing*	*ding*	*wu*	*ji*	*geng*	*xin*	*ren*	*gui*

The second part is a 'Terrestrial Branch'. Each of these Chinese words is the name of an animal in the Chinese zodiac. (You may have heard certain years called, for example, 'The Year of the Dragon' or 'The Year of the Pig'.)

The first year of a cycle is named by using the first word in each of these tables: *jia-zi*. The second year is then *yi-chou*, the third year is *bing-yin* etc. When we reach the end of a table, we go back to the beginning. The tenth year is *gui-you*, the eleventh year is *jia-xu* (restarting the Celestial Stem), the twelfth year is *yi-hai*, and the thirteenth year is *bing-zi* (restarting the Terrestrial Branch).

3 The Gregorian calendar year 2009 had the Chinese name ***ji-chou***. Use this information to:

(a) determine the English name of the animal for this year

(b) determine the Chinese name of the year 2019. What do you notice about this name?

(c) determine the Chinese name of the year in which you were born. What is the animal name for this year?

(d) find a year before and after your birth year that has the same animal name. Write the names of the years using a Celestial Stem and Terrestrial Branch.

(e) work out how many years it will be before a year has the name ***ji-chou*** again. (Hint: Use your knowledge of lowest common multiples.)

Research

Find out about some of the calendars used by other civilisation and cultures, such as the Jewish, Mayan, Roman or Indian calendars. Are they lunar, solar or lunisolar? How many months in one year? What are the names of the months?

1	2	3	4	5	6	7	8	9	10	11	12
zi (rat)	*chou* (ox)	*yin* (tiger)	*mao* (rabbit)	*chen* (dragon)	*si* (snake)	*wu* (horse)	*wei* (sheep)	*shen* (monkey)	*you* (rooster)	*xu* (dog)	*hai* (pig)

Challenge 5

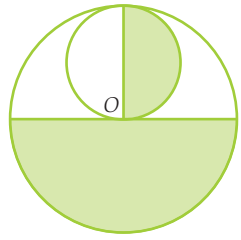
1 A certain substance doubles its volume every minute. At 9.20 a.m. a small amount is placed in a container and at 10.00 a.m. the container just fills. The time at which the container was one-quarter full was:

A 9.30 a.m. **B** 9.40 a.m. **C** 9.58 a.m. **D** 9.59 a.m.

2 A piece of string is used to form a square of area 196 cm^2. Rounded to the nearest whole number, the area of the largest circle that can be formed from this piece of string is:

A 16 cm^2 **B** 196 cm^2 **C** 250 cm^2 **D** 3057 cm^2

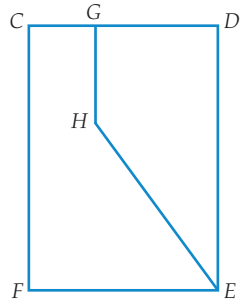
3 In the diagram, each circle is divided into two equal areas. O is the centre of the larger circle. The area of the larger circle is 144π cm^2. The total area of the shaded regions is:



A 54π cm^2 **B** 90π cm^2

C 108π cm^2 **D** 144π cm^2

4 In the diagram, the rectangle $CDEF$ is divided into two regions $CGHEF$ and $GDEH$ of equal area. If $GD = 60$ cm, $CF = 90$ cm and $GH = 30$ cm, what is the length of CG?



5 Stephen had a 10 a.m. appointment 60 km from his home. He drove at an average speed of 80 km/h for the trip and arrived 20 minutes late for the appointment. At what time did he leave his home?

A 9.35 a.m. **B** 9.20 a.m.

C 9.15 a.m. **D** 9 a.m.

6 If 1 August is a Thursday, what is the date of the second Saturday in September?

7 Petra's doctor has instructed her to take a course of 20 tablets. She has to take two tablets every 8 hours. How many days will it take Petra to complete the course?

8 A clock is set to the correct time at 9 a.m., but loses 3 minutes every hour. What is the correct time when the clock next shows 6.30?

9 If it takes 40 seconds to saw a log into five pieces, how long will it take to saw it into eight pieces?

10 Brett needs to cook rice for exactly 15 minutes, but he has only a 7-minute egg timer and an 11-minute egg timer. Explain how he can measure 15 minutes exactly.

11 How long will it take a 1 kilometre-long train going at 20 kilometres per hour to get completely through a 2 kilometre-long tunnel?

12 Kathryn's daughter Eleanor was born on Friday, 1 May 2009. What is the first year in which her birthday again falls on a Friday?

13 A cricket club always holds its board meetings on the second Tuesday of the month. What are the only possible dates for the meeting each month?

Chapter review 5

D.I.Y. Summary

Key Words

24-hour time	diameter	perpendicular	trapezium
area	elapsed time	prism	triangle
base	hectare	radius	uniform cross-section
capacity	height	rectangle	volume
circumference	kite	rhombus	
composite shape	parallelogram	square	
cylinder	perimeter	time zone	

Copy and complete the following using the words and phrases from the list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 The perimeter of a circle is called its _____.

2 The distance across a circle, through its centre, is called the _____. Half this distance is the _____.

3 The _____ of a shape is the amount of surface within the boundary.

4 A _____ is a quadrilateral with only two sides parallel. A _____ is a quadrilateral with two pairs of opposite sides parallel.

5 The amount of space occupied by a three-dimensional object is its _____. Its _____ is the amount of liquid or gas it can hold.

6 There are three _____s in Australia.

7 The volume of a _____ is the area of the base multiplied by the height.

8 A solid with a circular base and a _____ is called a cylinder.

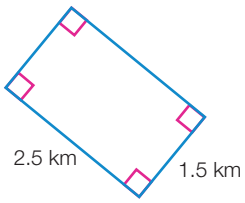
9 The diagonals of a _____ and a _____ intersect at right angles.

10 2.30 p.m. written in _____ is 1430.

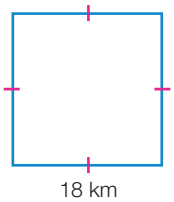
Fluency

1 Find the perimeter of the following shapes. Ex. 5.1

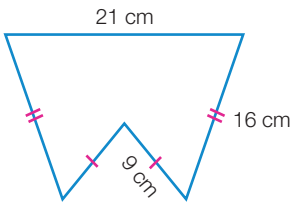
(a)



(b)



(c)



2 Estimate (do not calculate) to the nearest whole number the circumference of the following circles. Ex. 5.2

(a) diameter of 10 m

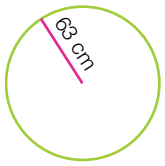
(b) radius of 4 cm

3 Calculate the circumference of each circle correct to two decimal places. **Ex. 5.3**

(a)

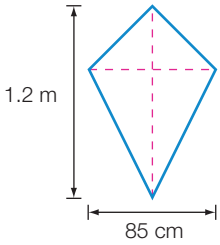
(b)

(c)



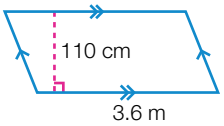
4 Find the area of each figure in m^2. State answers correct to two decimal places where appropriate. **Ex. 5.4, 5.5**

(a)

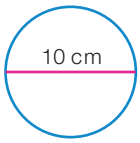


(b)

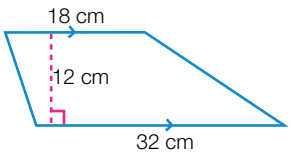
(c)



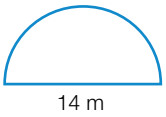
(d)



(e)

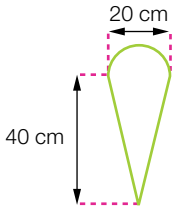


(f)

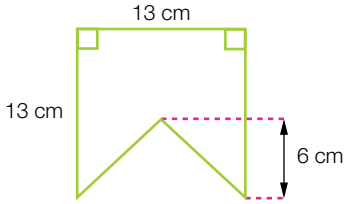


5 Calculate the area of these composite shapes to the nearest whole number. **Ex. 5.6**

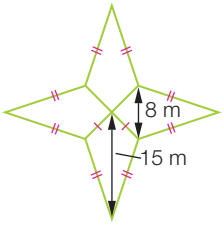
(a)



(b)

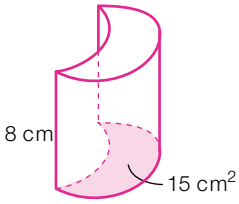


(c)



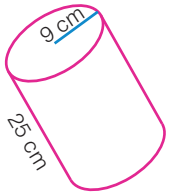
6 Calculate the volume of each solid correct to two decimal places. **Ex. 5.7**

(a)



(b)

(c)



7 Copy and complete the following conversions. **Ex. 5.4, 5.7**

(a) 0.045 L = _____ mL

(b) 27.5 cm^2 = _____ mm^2

(c) 16 m^3 = _____ cm^3

(d) 1575 mL = _____ L

(e) 3600 m^2 = _____ ha

(f) 275 cm^3 = _____ mm^3

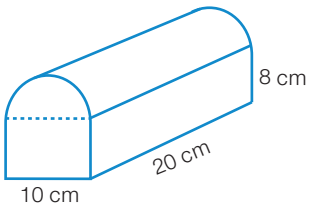
(g) 65 mL = _____ cm^3

(h) 0.053 km^3 = _____ m^3

(i) 250 L = _____ m^3

(j) 0.09 km^2 = _____ m^2

8 Find the volume of this solid, correct to two decimal places. **Ex. 5.7**



9 (a) How much time has passed from 4.58 a.m. to 3.41 p.m?

(b) What time is it $4\frac{1}{2}$ hours after 0914?

10 (a) Write 7.38 p.m. as a 24-hour time.

(b) Write 2321 as an a.m. or p.m. time.

Ex. 5.8

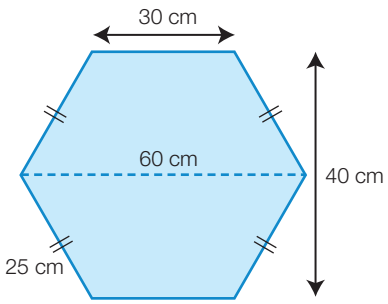
Ex. 5.8

Understanding

11 A basketball has a diameter of 23 cm. To the nearest whole number, how many times would it roll if it rolled from one end of the court to the other end 28 metres away?

12 Which of the following has a greater capacity, and by how many mL: a rectangular prism with length = 15 cm, width = 12 cm and height = 8 cm, or a cylindrical can that is 12 cm wide and 15 cm high?

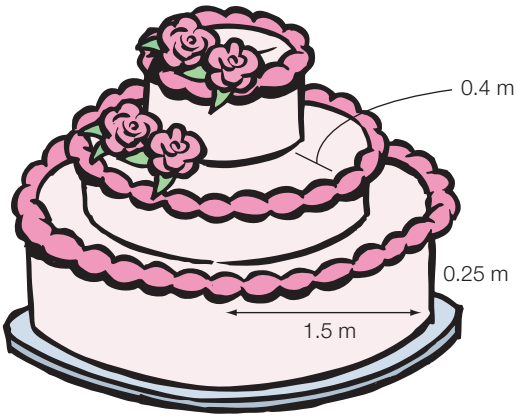
13 Find the perimeter and area of this hexagon.



14 Tom drove from Adelaide to Melbourne. He left Adelaide at 8.15 a.m. Tom spent $9\frac{1}{2}$ hours driving and also had three 20-minute breaks. What was the time in Melbourne when Tom arrived?

Reasoning

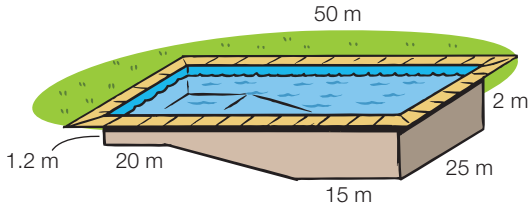
15 Wilma has to design a 'super' wedding cake for her niece's wedding. The cake is to be made of three tiers as shown. The base layer will have a radius of 1.5 m, then each additional layer's radius is reduced by 0.4 m. Each tier is 0.25 m high.



(a) Calculate, correct to two decimal places, the volume of cake needed.

(b) Each tier is to have a ribbon around its circumference. How many metres of ribbon will Wilma need to buy to place around the cake? (Answer to the nearest whole number.)

16 A new swimming pool is being built by the local council. Its design is shown in the diagram, with a shallow 1.2 m section of length 20 m, and a deep 2 m section of length 15 m, joined by a section of increasing depth.



(a) What is the volume, in cubic metres, of the new swimming pool?

Once the pool is built, the council has to fill the pool with water.

(b) How many kilolitres of water will be needed to fill the pool?

(c) The only way to fill the pool is to bring the water in by tanker. The tanker has a water tank in the shape of a cylinder, with diameter 2 metres and length 10 metres. How many tanker loads will be needed to fill the pool with water?

NAPLAN practice 5

Numeracy: Non-calculator

1 The area of a square with a perimeter of 80 cm is:

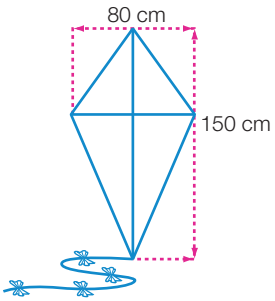
A 40 cm^2 **B** 80 cm^2 **C** 400 cm^2 **D** 1600 cm^2

2 Which metric unit would a landscape gardener use to measure the volume of soil in a pile like this?

A square metres **B** cubic metres

C cubic centimetres **D** square centimetres

3 The number of square centimetres of fabric needed to make the kite is:



A 600 **B** 3000

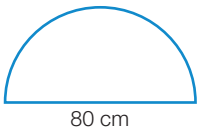
C 6000 **D** 12 000

4 Sam's favourite TV show starts at 5.35 p.m. If it is 12.47 p.m. now, how long until his favourite show starts?

A 4 h 38 min **B** 4 h 48 min **C** 5 h 12 min **D** 5 h 42 min

Numeracy: Calculator allowed

5 A semi-circular table has a diameter of 80 cm. The perimeter of the table is closest to:



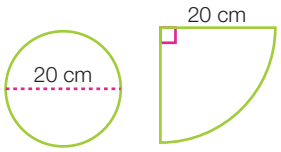
A 120 cm **B** 200 cm

C 250 cm **D** 320 cm

6 How many cylinders of diameter 10 cm and height 15 cm can be filled from a cubic tank of side measurement 50 cm?

A 53 **B** 106 **C** 159 **D** 212

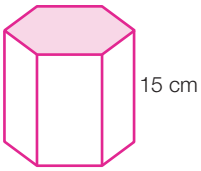
7 What is the difference between the perimeters of these two shapes, to the nearest centimetre?



A 3 cm **B** 9 cm

C 31 cm **D** 70 cm

8 The volume of the hexagonal prism is 500 cm^3. The area of cross-section is closest to:



A 2 cm^2 **B** 17 cm^2

C 20 cm^2 **D** 33 cm^2

6

Linear graphs

6

That's check, mate! One could argue that there are more possible chess positions than there are atoms in the universe, so how can players keep track of what moves have been made?

In blind chess, players verbally communicate their moves. One method is to imagine the chessboard has a grid labelled a to h along the top and bottom and numbered 1 to 8 along its sides. If N is the Knight, then Nb1-c3 means it will move from position b1 (near bottom left of the board) to position c3.

There are 400 possible positions after each player has made one move and about two hundred thousand positions after two moves. A grandmaster who can 'look ahead' five moves is faced with a trillion possibilities!

Forum

Why do you think the number of positions increases so quickly?

Are there other ways of referring to what pieces are moved?

Can this method of referencing be used in other games?

Why learn this?

Knowledge of the Cartesian plane is necessary in order to show relationships graphically when studying many areas of mathematics. In the real world, an atlas or a road map would be useless if there was no way of referring to the location of a country or a street. By using a systematic way of specifying position, important information can be displayed. A position marked on a grid may represent any quantity such as location, temperature, height or profit.

After completing this chapter you will be able to:

- interpret line graphs
- plot relationships using tables of values
- plot relationships using rules
- derive a rule for a set of points that lie on a straight line
- use linear relationships to solve problems
- compare graphs of different linear relationships
- identify different gradients
- recognise linear equations that generate graphs parallel to either axis.

Recall 6

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

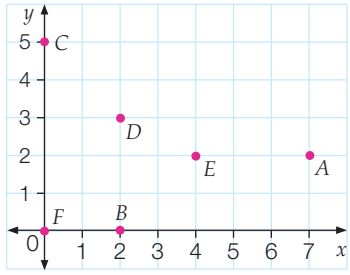
Worksheet R6.1

1 Draw a number plane to include x-values from -5 to 5 and y-values from -5 to 5, then plot and label each of the following points.

$P(3, -5)$, $Q(2, 0)$, $R(5, 1)$, $S(0, 4)$ and $T(-1, 5)$

Worksheet R6.2

2 From the diagram, list the coordinates of each of the points A to F.



Worksheet R6.3

3 Copy and complete the tables of values for the rules given.

(a) $y = 3x - 1$

x	1	2	3	10
y				

(b) $y = \frac{x}{2} + 4$

x	0	2	6	10
y				

Worksheet R6.4

4 **(a)** Substitute $T = 5$ into the equation $T = 3a - 7$, then solve for a.

(b) Substitute $x = 5$ and $y = 3$ into $y = mx + 8$ to find a value for m.

Worksheet R6.5

5 Solve each of the following equations for x.

(a) $5 = 2x - 3$ **(b)** $5 + 3x = 11$ **(c)** $\frac{x}{5} - 1 = 2$

Worksheet R6.6

6 If x represents a number, write an expression to represent:

(a) three times the number **(b)** eight more than double the number.

7 Simplify:

(a) $3x + 4y + 6(x - y)$ **(b)** $3(x - 2y) - 4(x + 2y)$

Key Words

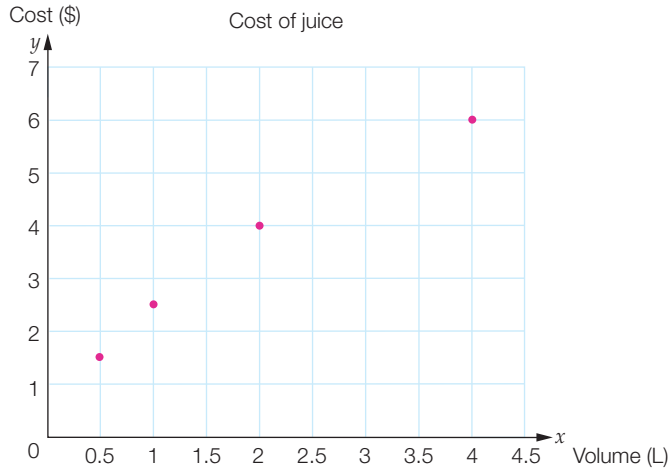
Cartesian plane	linear graph	line graph	origin	x-intercept
coordinate	linear relationship	ordered pair	slope	y-intercept
gradient				

Interpreting line graphs

6.1

In this section, we are going to investigate graphs in which points are connected with lines to make line graphs.

In Year 7, you investigated point graphs. These are graphs where the points cannot be joined in a way that makes sense. Look at the point graph opposite showing the relationship between the cost of juice and the volume of juice purchased.



It makes no sense to join these points, as the values in between have no meaning. Juice can only be purchased in certain volumes. This graph shows that this particular juice can only be purchased in 0.5 L, 1 L, 2 L and 4 L volumes.

However, there are many situations where connecting points on a graph with a straight line gives us valuable information about points in between and beyond the given points.

Line graphs show the relationship between two variables and can consist of several different straight line parts.

Worked Example 1

WE1

Selena leaves her home to go for a walk. The following line graph shows her distance from home.

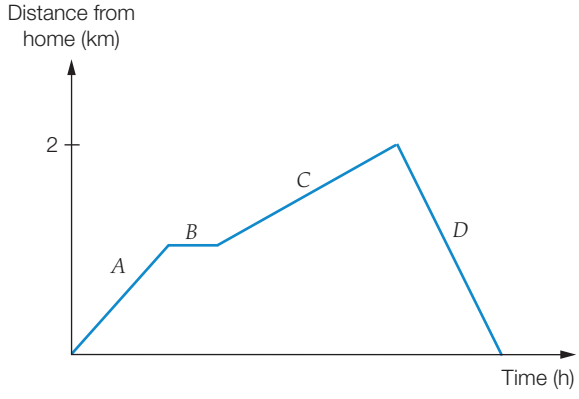
(a) Describe what information is being shown on the graph in:

- **(i)** section *A*
- **(ii)** section *B*
- **(iii)** section *D*.

(b) In which section was Selena walking the fastest?

(c) In which section was she walking the slowest?

(d) What was the total distance that she walked?



Thinking	Working
(a) **(i)** Decide where the travelling begins and then whether the distance from the starting point increases, decreases or stays the same.	(a) (i) Selena is starting from her home. As the distance from home is steadily increasing as time increases, Selena is moving away from her house at a steady rate in section A.
(ii) Decide whether the distance from the starting point increases, decreases or stays the same.	(ii) A horizontal line means that the distance from home is not changing as time increases. Selena may have stopped somewhere to rest.
(iii) Decide whether the distance from the starting point increases, decreases or stays the same.	(iii) As the distance from home is steadily decreasing as time increases, Selena is walking back to her house at a constant rate.
(b) As speed = $\frac{\text{distance}}{\text{time}}$, the steepness of the graph indicates travelling speed. Compare the steepness of each section. The steeper the graph, the faster the speed.	(b) Section D has the steepest line. The distance covered here is about twice the distance of section A or C, but it is covered in less time. Selena must be walking faster in section D than in section A. Perhaps, she was running home.
(c) As speed = $\frac{\text{distance}}{\text{time}}$, the steepness of the graph indicates travelling speed. Compare the steepness of each section. The steeper the graph, the faster the speed.	(c) Section A has a steeper line than section C. The total distance in section C is about the same as the total distance in section A but the time taken to cover the distance is greater, so Selena is walking slower in Section C than in section A.
(d) Decide how far Selena travelled away from home, then double this distance to find the total distance she travelled.	(d) Selena walked 2 km away from home, so she walked 4 km altogether.

6.1 Interpreting line graphs

Navigator

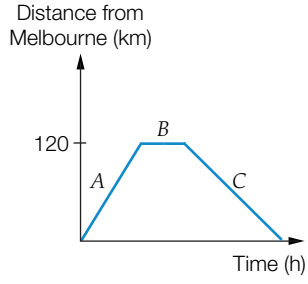
Answers page 622

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q12	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q12, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12

Fluency

WE1

1 A family is travelling in their car. The following line graph shows their distance from Melbourne.



(a) Describe what information is being shown on the graph in:

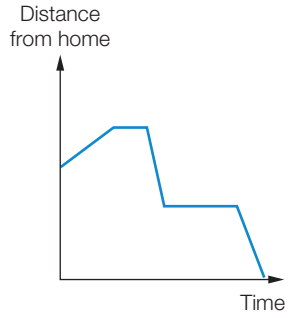
(i) section *A* **(ii)** section *B* **(iii)** section *C*.

(b) In which section was the car travelling the fastest?

(c) In which section was the car travelling the slowest?

(d) What was the total distance travelled by the family?

2 The graph opposite shows the distance Josie is from home while walking one afternoon.



Use the graph to answer the following questions.

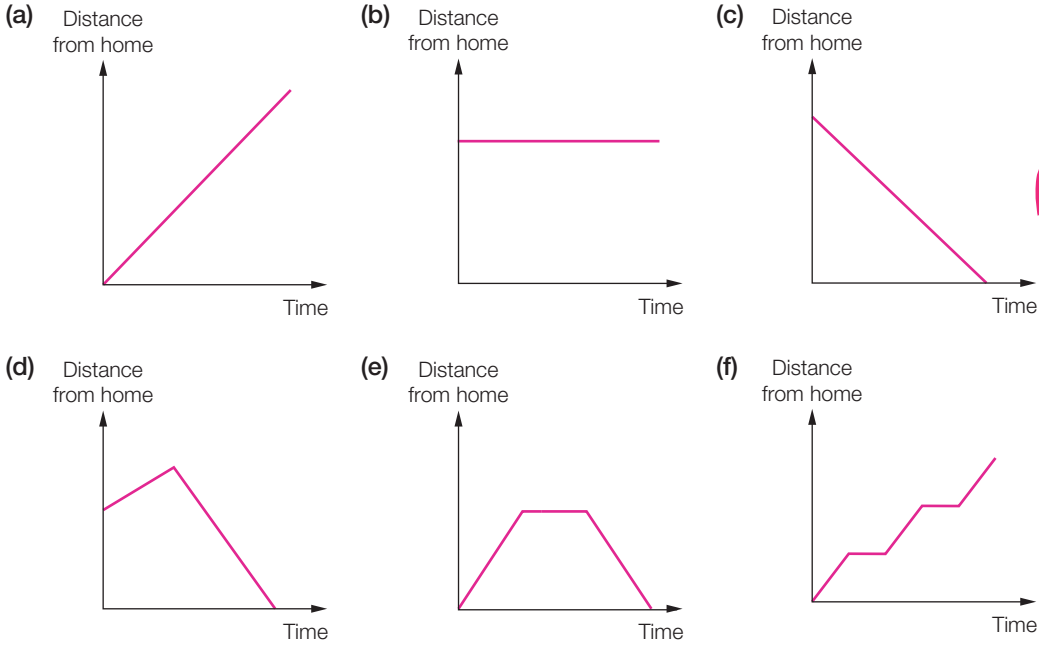
(a) Did Josie start her walk from home? Explain.

(b) Josie walked to the local shop and stopped to buy milk and bread. She then started to walk back home but stopped at a friend's place. What parts of the graph show when she stopped?

(c) Did she spend more time at the milk bar or at her friend's place?

(d) Did Josie reach her home at any time on her walk? Explain.

3 The following graphs show the distance Ahmid is from his home while out walking during different time intervals.



Match each graph **(a)**–**(f)** with the description of a journey **A**–**F** given below.

A Ahmid is walking and stopping at equal time intervals.

B Ahmid is walking directly home.

C Ahmid is walking away from his home at first, then he stops and returns home immediately without resting.

D Ahmid is stationary (not walking).

E Ahmid is walking away from his home, stops for a rest, then walks home.

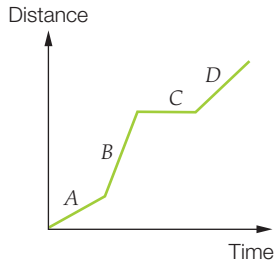
F Ahmid is walking away from his home.

4 Rhiannan is riding her bicycle.

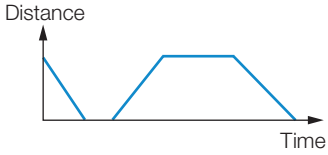
(a) In which section of the graph (*A*, *B*, *C* or *D*) is Rhiannan moving the fastest?

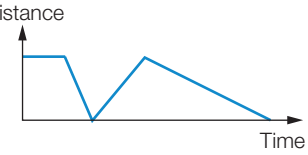
(b) In which section is she stationary?

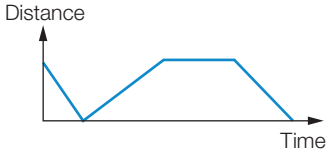
(c) In which section is she moving the slowest?

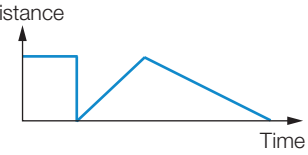


5 Josh is at work. He drives home at a constant speed, eats lunch and drives back to work at a constant speed. After working for several hours he drives home at a constant rate. The graph which describes this is:

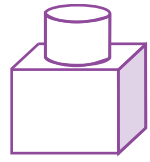
A


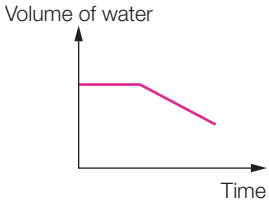
B


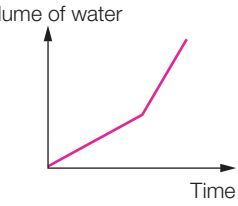
C


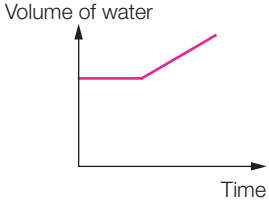
D


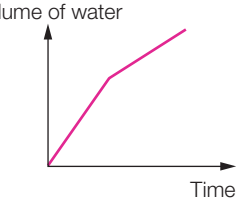
6 The container shown is initially empty. Water is then poured in at a constant rate. The graph that describes this is:



A


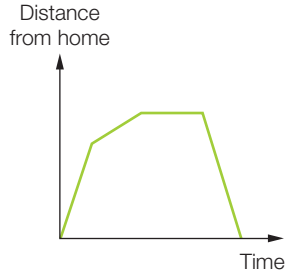
B


C


D


Understanding

7 State which journey illustrates Jason's trip as shown by the following graph.



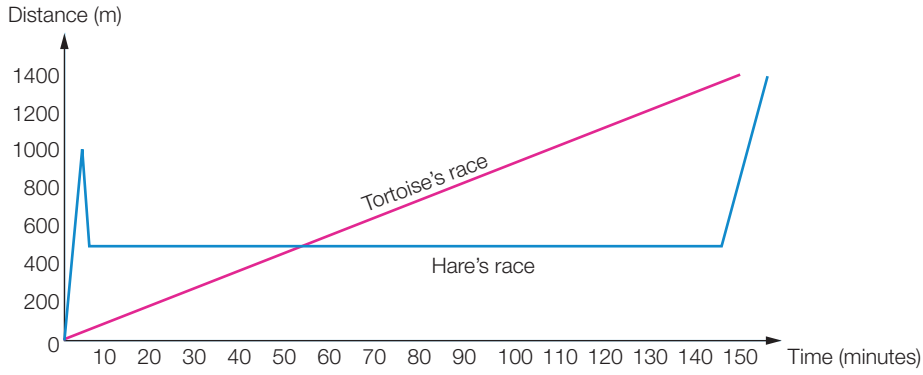
A Josh started from home, walked slowly through the park, then ran to the milk bar, bought an ice-cream and then walked back home.

B Jason started from home, ran through the park, then walked to the milk bar, bought an ice-cream and then ran back home.

C Jason started from the milk bar, ran through the park, then walked home, looked for a skateboard and then ran back to the milk bar.

D Jason started from home, ran through the park, then stopped at the milk bar, played with his skateboard and then walked back home.

8 The hare and the tortoise were having a race. Below is a graph of the race.



(a) What was the length of the race?

(b) Who won the race?

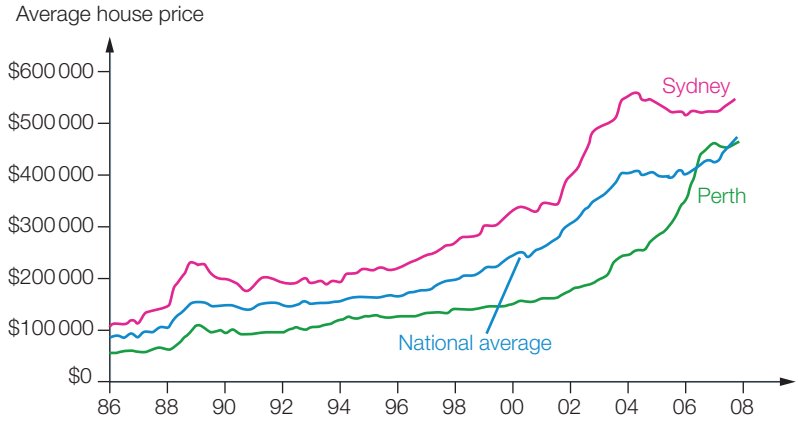
(c) What was the winning time?

(d) At what time and distance from the start were the hare and the tortoise at the same place?

(e) Describe the hare's race.

Reasoning

9 Look at the graph below.



(a) Explain what each of the three line graphs is measuring.

(b) Suggest a label for the horizontal axis.

(c) How many years does the graph cover?

(d) Would you sell your house for more money in Sydney or in Perth? Explain.

(e) In 2000, how much cheaper was it to buy a house in Perth than in Sydney?

(f) Sometimes, average house price graphs trend upwards and sometimes they trend downwards.

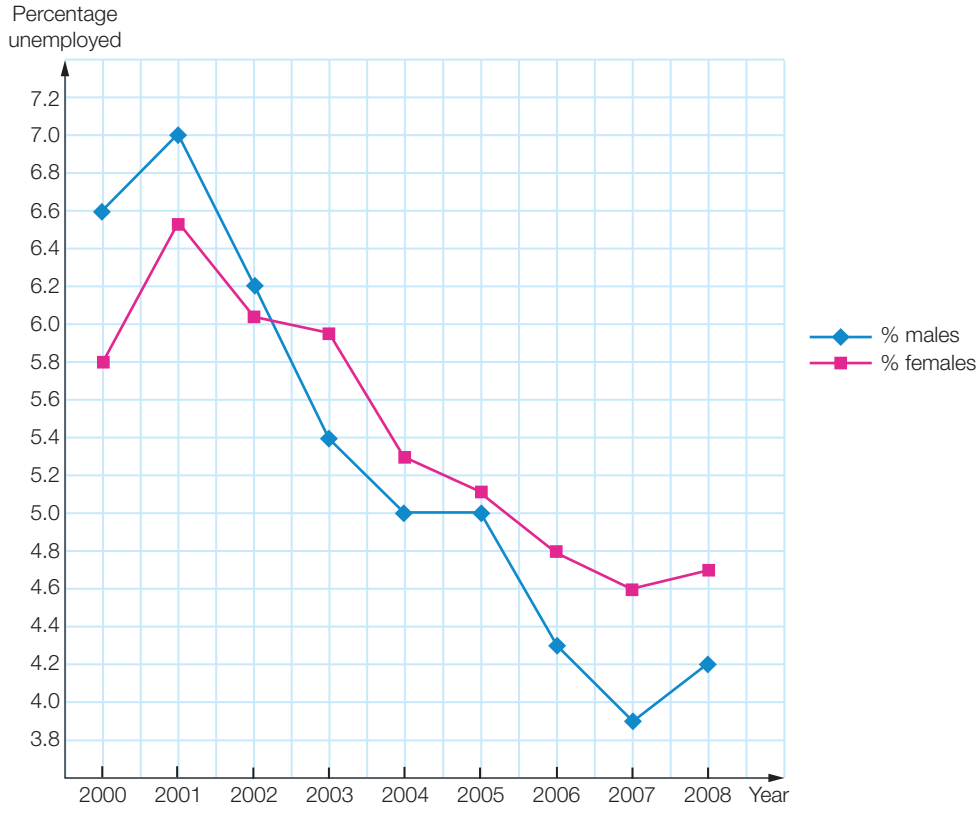
(i) Suggest a reason for an upward trend.

(ii) Suggest a reason for a downward trend.

(g) Is it possible to work out the average house price for Melbourne, using this graph? Explain.

(h) During which time period did the average house price in Perth increase the fastest?

10 The following graph shows information about unemployment in Australia.



(a) What information is provided by the values shown on the vertical axis?

(b) In what year was the percentage of male unemployment the greatest?

(c) In what year was there the least difference between the percentage of males and the percentage of females unemployed?

(d) During what period was there a higher percentage of male unemployment than of female unemployment?

(e) What was the greatest annual percentage change in female unemployment?

(f) For how many years was the male unemployment percentage below 5%?

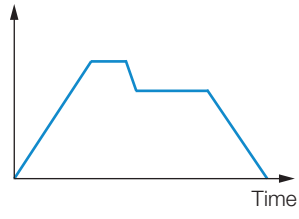
(g) State the number of years for which the female unemployment percentage was above 5.3%.

(h) The percentage of people unemployed is an indicator of a country's economy. A low unemployment percentage usually means strong economic growth. For one year in particular, the economy experienced slow growth. When do you think this occurred? Explain your answer.

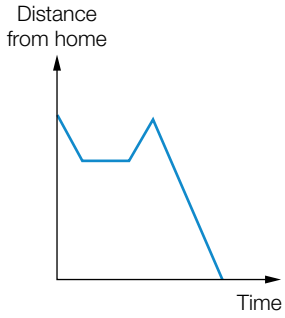
Open-ended

11 Jodie provided Andrew with some information that Andrew used to draw the line graph shown.

What might the information have been?



12 Write a story to describe the journey shown in this graph.



13 Draw graphs that describe each of the following situations.

(a) a train travelling from Rosebush station to Cedarwood station, stopping at Mapleleaf and Gumnut stations on the way

(b) a person travelling up and down in a lift that stops to let other people on and off at various floors

(c) Discuss your graphs with another student. Do they seem reasonable?

Outside the Square — Problem solving

Where is the missing area?

Equipment required: 1 brain, grid or graph paper

There are 64 small squares in Square A.

The four shapes are rearranged to make Shape B.

There are 65 squares in Shape B.

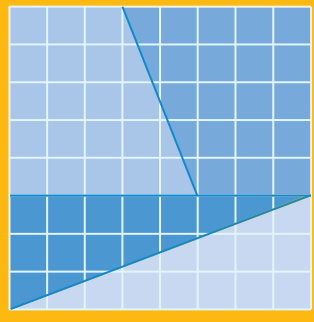
Square A and Square B are the same size, so they must have the same area.

Where did the extra small square come from?

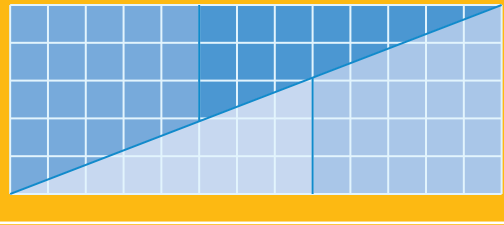
Strategy options

- Draw a diagram.
- Make a model.

Square A



Shape B



6.2 Linear relationships

Linear graphs

A **linear graph** is a straight line graph that shows the relationship between two variables. It can be drawn if we know the rule (e.g. $y = x + 2$) that connects the two variables. This rule shows a '**linear relationship**'. The rule or equation is used to complete a table of values that shows the x- and y-coordinates of some points on the line. These are plotted on the **Cartesian plane** and connected with a line.

Any point on the line can be represented by **coordinates**, an x-coordinate and a y-coordinate. These coordinates are written as an **ordered pair** (x, y).

The ordered pair (0, 0) is known as the **origin**.

Worked Example 2

WE2

(a) Copy and complete the following table of values for the rule $y = \frac{x}{2} + 4$ for values of x in the range -2 to 2.

x	-2	-1	0	1	2
y					
(x, y)					

(b) Use the table of values to draw a graph of the relationship.

Thinking

(a) 1 Substitute each value of x into the rule to find the corresponding value of y.

Working

(a) $y = \frac{x}{2} + 4$

$y = \frac{-2}{2} + 4$ when $x = -2$

$= 3$

$y = \frac{-1}{2} + 4$ when $x = -1$

$= 3.5$

$y = \frac{0}{2} + 4$ when $x = 0$

$= 4$

$y = \frac{1}{2} + 4$ when $x = 1$

$= 4.5$

$y = \frac{2}{2} + 4$ when $x = 2$

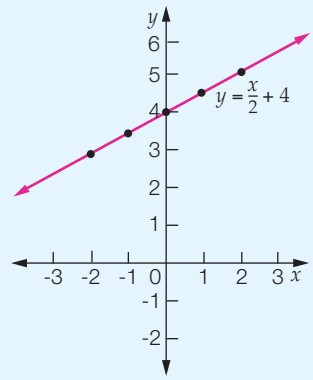
$= 5$

2 Write the y-values in the second row and enter the ordered pairs in the third row.

x	-2	-1	0	1	2
y	3	3.5	4	4.5	5
(x, y)	(-2, 3)	(-1, 3.5)	(0, 4)	(1, 4.5)	(2, 5)

(b) Draw and label the x and y Cartesian axes, plot the points, connect them with a straight line and label the graph with the rule.

(b)



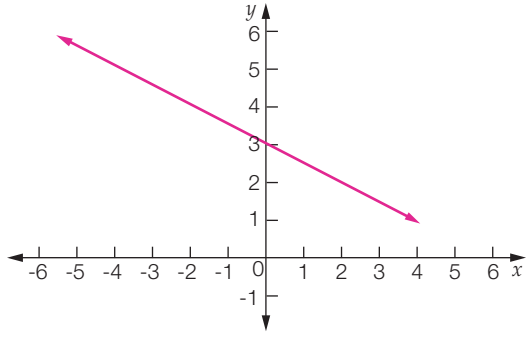
Worked Example 3

WE3

For the following graph, find:

(a) the value of y when $x = -4$

(b) the value of x when $y = 2$.

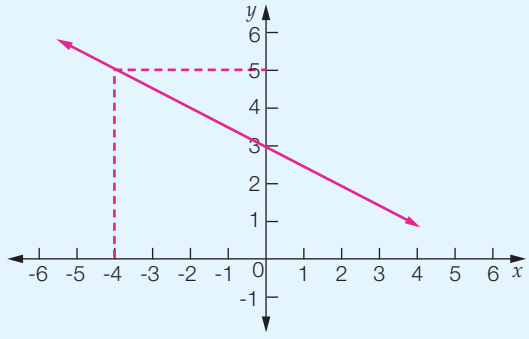


Thinking

Working

(a) Find the given point on the x-axis and draw a vertical line to meet the given line. From the point of intersection with this line, draw a horizontal line to meet the y-axis. The y-coordinate gives the required value.

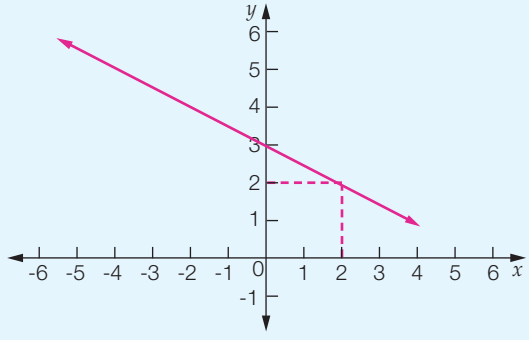
(a)



When $x = -4$, $y = 5$.

(b) Find the given point on the y-axis and draw a horizontal line to meet the given line. From the point of intersection with this line, draw a vertical line to meet the x-axis. The x-coordinate gives the required value.

(b)

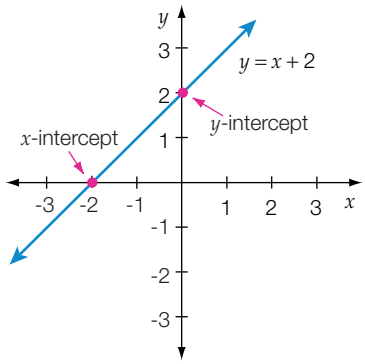


When $y = 2$, $x = 2$.

x- and y-intercepts

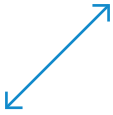
The ***x*-intercept** of a graph is the point where the line crosses the x-axis.

The ***y*-intercept** of a graph is the point where the line crosses the y-axis.



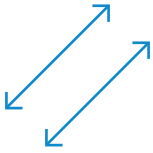
Gradient

The **gradient** tells us about the **slope** of a line.



This line has a positive gradient as it slopes upwards from left to right.

This line has a negative gradient as it slopes downwards from left to right.



Lines with the same gradient are parallel.

Worked Example 4

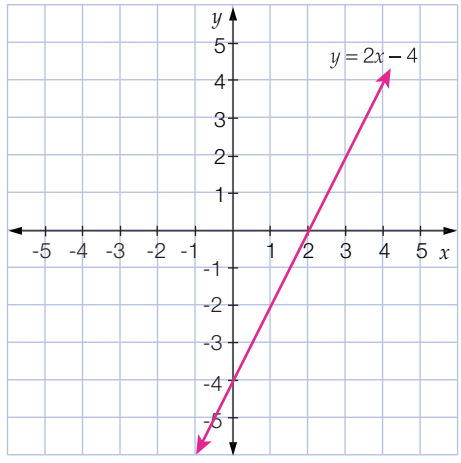
WE4

For the graph with the rule $y = 2x - 4$:

(a) state the x-intercept and write the coordinates of the point

(b) state the y-intercept and write the coordinates of the point

(c) state whether the gradient of the line is positive or negative.

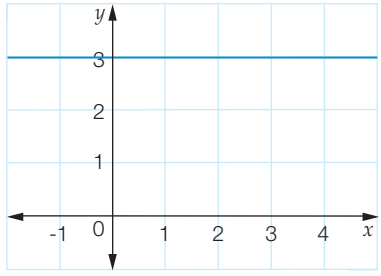


Thinking	Working
(a) The x-intercept is where the graph cuts the x-axis. Write the coordinate of the point.	(a) The x-intercept is 2; it has the coordinates (2, 0).
(b) The y-intercept is where the graph cuts the y-axis. Write the coordinate of the point.	(b) The y-intercept is -4; it has the coordinates (0, -4).
(c) The graph slopes upwards from left to right.	(c) The gradient is positive.

Special cases for the gradient

Below is the table of values for the equation $y = 0x + 3$ and its graph.

x	0	1	2	3	4
y	3	3	3	3	3



The equation produces a horizontal line passing through $y = 3$. The line is parallel to the x-axis. The gradient of the line is 0 (the coefficient of x) and the y-intercept is 3.

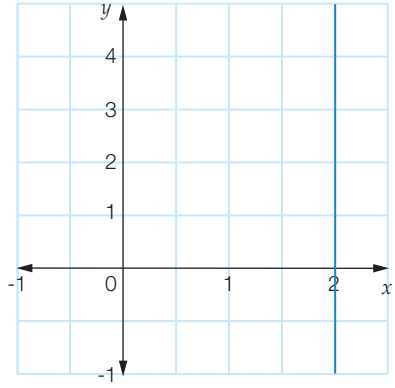
As $0x = 0$, we can write $y = 0x + 3$ as $y = 3$.

The graph of $y = c$ is a horizontal line parallel to the x-axis that passes through the point $(0, c)$. This point is the y-intercept.

Horizontal line graphs where $c \neq 0$ are the only linear graphs that do not have an x-intercept.

The graph of the values in the table below is a vertical line parallel to the y-axis and which passes through the point $x = 2$.

x	2	2	2	2	2
y	1	2	3	4	5



This gradient is the steepest possible for a straight line. We say the gradient is undefined and the x-intercept is 2.

The equation of the line is $x = 2$ and there is no y-intercept for equations of this type.

The graph of $x = a$ is a vertical line parallel to the y-axis that passes through the point $(a, 0)$. This point is the x-intercept.

Vertical line graphs where $a \neq 0$ are the only linear graphs that do not have a y-intercept.

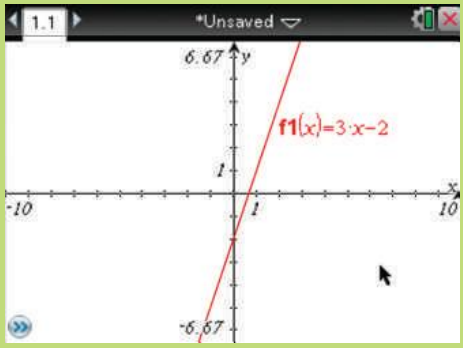
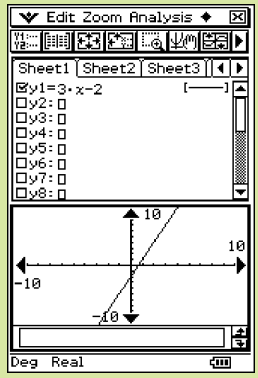
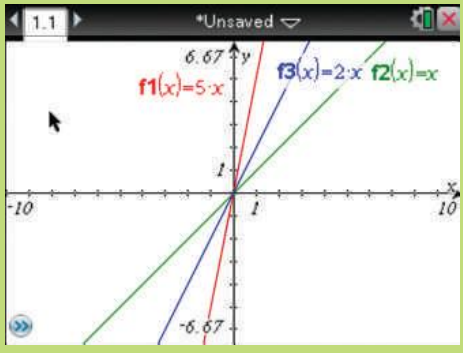
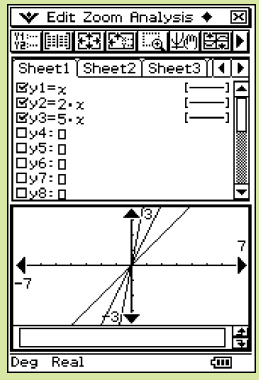
Gradient of special case graphs

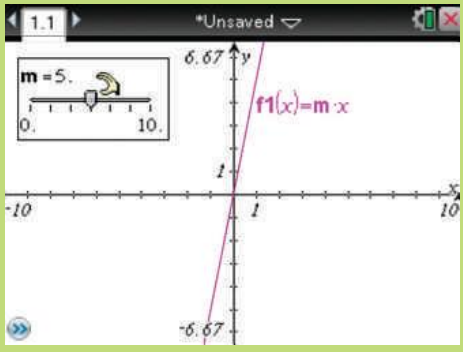
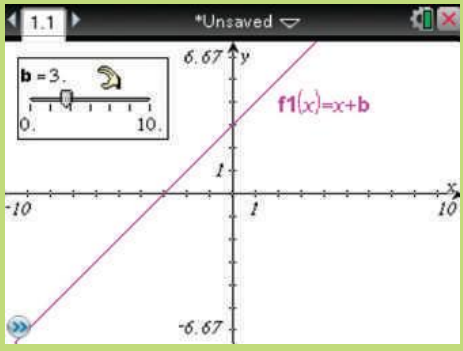
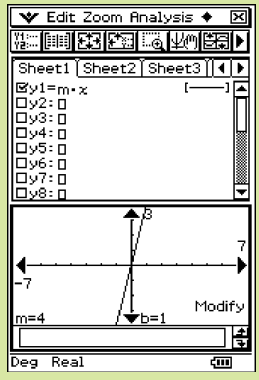
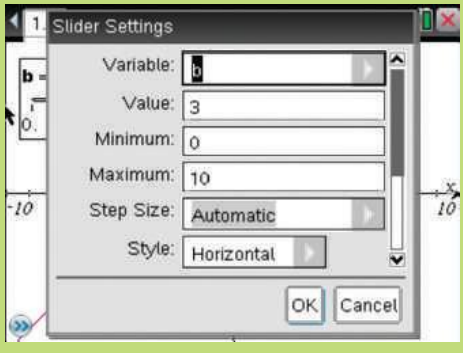
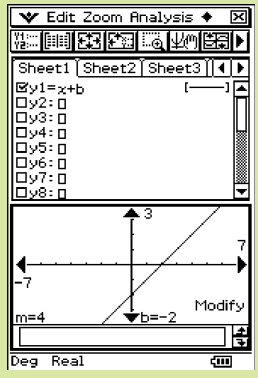
A horizontal line has a gradient of zero.

A vertical line has a gradient that cannot be evaluated. We say the gradient is undefined.

Using CAS to sketch linear graphs

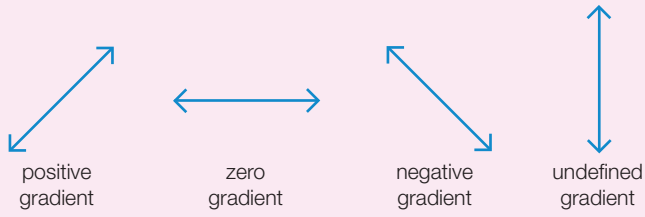
A CAS makes graphing straight lines very easy.

Using the TI-Nspire CAS	Using ClassPad
Add a Graph page to your document and enter the equation in the edit line at the bottom. We will use $y = 3x - 2$, so type $3x - 2$ and press (enter) (or (enter)). The graph appears on the screen with its equation beside it.  	At the screen enter the equation $3x - 2$ and tap (EXE). Then, tap to make the graph appear. 
Enter a few equations, such as $y = x$, $y = 2x$ and $y = 5x$. After the first graph is drawn you will need to press (ctrl) G (or (ctrl) G). This is shown below.  	Enter a few equations, such as $y = x$, $y = 2x$ and $y = 5x$. This is shown below. 

Using the TI-Nspire CAS	Using ClassPad
However, we can make the exploration more dynamic by using a slider. Start a new Graphs page and enter $m \times x$ as the rule. Then, press menu > Actions > Insert Slider and then press M enter (or M enter) to name the variable in the slider. Put the cursor (hand) over the slider and move it using the buttons on the touchpad. Watch what happens to the graph as the value of m changes. 	However, this can be easier to see if we draw a dynamic graph. Clear any graphs you already have and enter $m \times x$ as the rule beside y1. You will need to use m from the VAR keypad. Then, tap ◆ (next to GMem) and select Dynamic Graph. Change the variable to m and enter the desired values. You can just leave the other variable, as we are not using it.
In a similar way, by entering $y = x + b$, we can look at what happens as b changes. 	Tap OK and the graph appears. You then just tap ◀ and ▶ on the ends of the x-axis and the value of m changes and so does the graph. 
If you want to change the settings for the variable, press ctrl menu (or ctrl menu) highlight Settings and press enter (or enter). This will bring up a dialog box in which you can make whatever changes you like. 	We can then do a similar thing to check the effect of b by entering $y = x + b$.  This time, you control it by pressing ▲ and ▼ on the ends of the y-axis.

Linear graphs

- A linear relationship can be used to complete a table of values.
- A table of values gives coordinate points on a Cartesian plane that can be joined.
- The graph of a linear relationship is a straight line.
- The point where a line crosses the x-axis is called the x-intercept.
- The point where a line crosses the y-axis is called the y-intercept.
- The gradient of a line describes the slope or steepness of the line.



6.2 Linear relationships

Navigator

Answers page 623

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11 (a), Q12 Columns 1 & 2, Q14, Q15, Q19, Q20	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11 (b), Q12 Columns 2 & 3, Q14, Q15, Q16, Q17, Q19, Q20	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11 (b), Q12 Columns 2 & 3, Q13, Q14, Q15, Q16, Q17, Q18, Q20, Q21

Equipment required: Graph or grid paper

Fluency

WE2

1 **(a)** Copy and complete each of the following tables of values for the rules given for values of x in the range -2 to 2.

(b) Use the table of values to draw a graph of the relationship.

(i) $y = x + 1$

x	-2	-1	0	1	2
y	-1				
(x, y)	(-2, -1)				

(ii) $y = \frac{x}{2}$

x	-2	-1	0	1	2
y	-1				
(x, y)	(-2, -1)				

(iii) $y = 3x - 1$

x	-2	-1	0	1	2
$3x$	-6				
$3x - 1$	-7				
(x, y)	(-2,-7)				

(iv) $y = -x - 4$

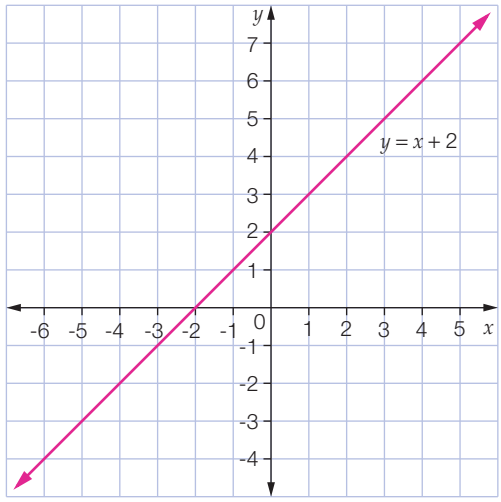
x	-2	-1	0	1	2
$-x$	2				
$-x - 4$	-2				
(x, y)	(-2, -2)				

2 For each of the following graphs, find:

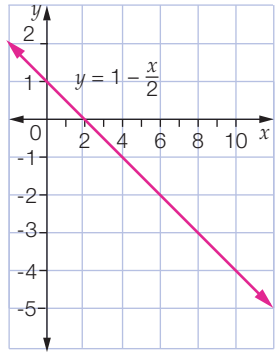
WE3

(a) the value of y when $x = 4$ **(b)** the value of x when $y = -3$.

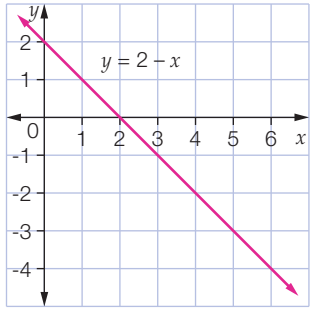
(i)



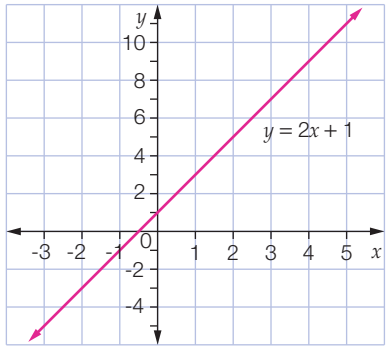
(ii)



(iii)



(iv)



3 For each of the graphs in Question **2**:

WE4

(a) state the x-intercept and write the coordinates of the point

(b) state the y-intercept and write the coordinates of the point

(c) state whether the gradient of the line is positive or negative.

4 The coordinates of a point that lies on the graph of $y = 5x - 4$ are:

A (1, 9) **B** (2, -6) **C** (0, 4) **D** (-1, -9)

5 Which table of values matches coordinates obtained using the equation $y = 2x - 3$?

A

x	-3	-2	0	2	3
y	-9	-7	-3	1	3

B

x	-3	-2	0	2	3
y	3	1	-3	-3	-9

C

x	-3	-2	0	2	3
y	-6	-5	-3	-1	0

D

x	-3	-2	0	2	3
y	3	-1	-3	7	9

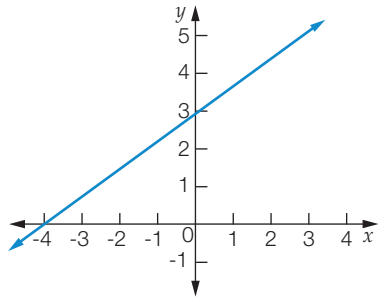
6 The coordinates of the x-intercept and the y-intercept are respectively:

A (3, -4) and (0, 0)

B (0, 3) and (-4, 0)

C (0, -4) and (3, 0)

D (-4, 0) and (0, 3)



7 Choose the correct answer.

A

B

C

D

(a) Which line is the steepest?

(b) Which line has a gradient of zero?

(c) Which line has the smallest positive gradient?

8 **(a)** Copy and complete the following sentences.

Lines with _____ gradient slope up to the right.

Lines with _____ gradient slope down to the right.

Horizontal lines have _____ gradient.

Parallel lines have the _____ gradient.

Vertical lines have _____ gradient.

(b) For the lines A–J shown below, list the letters of those:

(i) with positive gradient

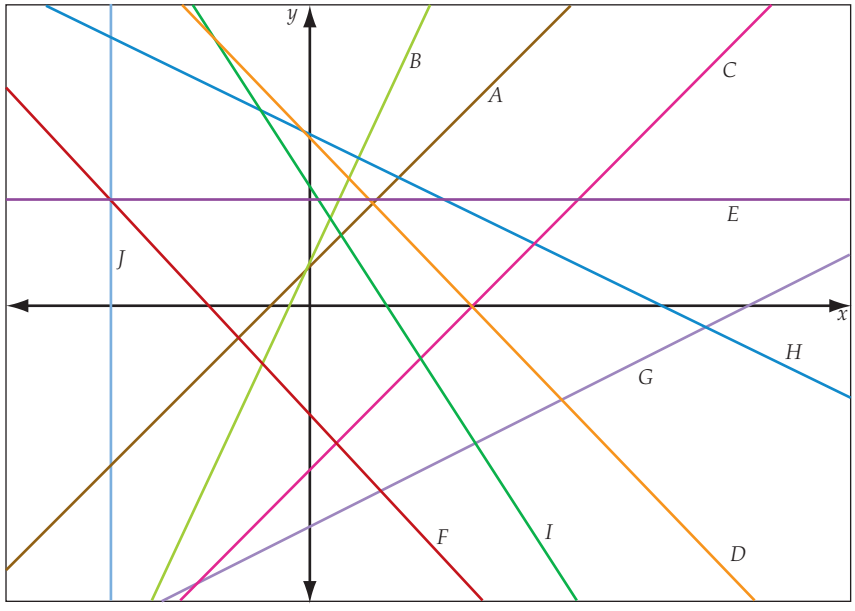
(ii) with negative gradient

(iii) with zero gradient

(iv) that are parallel

(v) with undefined gradient

(vi) with steepest positive gradient.



9 **(a)** Plot the graph of the following relationships by using the tables of values given.

(i) $y = 3$

x	0	1	2
y	3	3	3
(x, y)	(0, 3)	(1, 3)	(2, 3)

(ii) $y = -5$

x	0	1	2
y	-5	-5	-5
(x, y)	(0, -5)	(1, -5)	(2, -5)

(b) Are these relationships linear? Explain your answer.

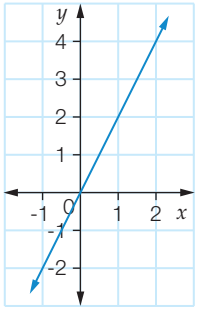
(c) What is the gradient of each line?

(d) Are the lines parallel to the x-axis or to the y-axis?

(e) State the x-intercept and the y-intercept for each graph.

Understanding

10 The point that lies on the line is:



A (4, 2)

B (-4, -2)

C (2, 4)

D (-2, 4)

11 Plot the graphs for each of the following linear relationships using only three points.

(a) $y = -2x$ **(b)** $y = -3x + 9$

12 Without plotting points, draw the graphs of each of the following relationships.

(a) $y = 4$ **(b)** $y = -2$ **(c)** $x = -6$

(d) $x = 3$ **(e)** $x = 2$ **(f)** $y = 7$

13 **(a)** Draw the graph of each linear equation on the same Cartesian plane for values of x from -2 to 2.

(i) $y = \frac{1}{2}x$ **(ii)** $y = 2x$ **(iii)** $y = 3x$

(b) What effect does the coefficient of x have on the steepness of the line?

14 A crane's boom length can be described by the equation $y = \frac{14}{5}x$, where x is the horizontal distance the boom can reach.

The origin (0, 0) is 1.5 metres above the ground.

(a) Complete a table of values for x-values from 0 to 8.

(b) Why don't we use negative values for x?

(c) Draw the graph using your results from **(a)**.

(d) The boom's full length is 20 metres. Use your graph to estimate the horizontal distance the boom can reach from the origin.

Reasoning

15 For the following linear relationships, state whether the slope of the graph would be positive, negative or zero.

(a) water running out of a water tank (x-axis = time, y-axis = volume in tank)

(b) a car accelerating at a constant rate (x-axis = time, y-axis = speed)

(c) the tray on a forklift being lowered (x-axis = time, y-axis = distance from ground)

(d) earning $20 per hour ($x$-axis = hours, y-axis = money earned)

(e) filling a cylindrical container with lollies (x-axis = time, y-axis = number of lollies)

(f) sitting on a park bench near your home watching a football match (x-axis = time, y-axis = distance from home)

16 Cameron and Moira share 10 lollies between them.

(a) Construct a table of values using at least three different combinations showing different ways in which they can share these lollies.

(b) Can any of the values be negative? What does this tell you about the graph?

(c) Plot the values from part **(a)** (Cameron's lollies on the x-axis and Moira's lollies on the y-axis) and join the points together. Extend the line to show all possible combinations.

(d) Is the gradient positive or negative?

(e) Explain the meaning of the slope in terms of the number of lollies that Cameron and Moira share.

(f) **(i)** What y-coordinate corresponds to x-coordinate 4.3?

(ii) What does this mean in terms of the number of lollies Cameron and Moira share?

(iii) Considering your answers to **(i)** and **(ii)**, is it sensible to join the points plotted to form a linear graph?

17 Zamira believes that the gradient of any line can be made opposite to what it is by using two of its ordered pairs and taking the negative of the x-coordinates. For example, the line joining (1, 4) to (3, 7) has a positive gradient, and changing 1 to -1 and 3 to -3 gives (-1, 4), (-3, 7), which has a negative gradient.

Is what Zamira saying always true? If not, explain when it is not true.

18 Liquid is being poured from a plastic container at a constant rate. The amount of liquid remaining is described by the rule $v = 6 - 2t$, where v is the volume of liquid in litres and t is the time in minutes.

At the same time, a glass container is being filled with liquid described by the rule $v = 2t$, where v is the volume of liquid in litres and t is the time in minutes.

(a) Use the rule $v = 6 - 2t$ to complete a table of values for values of t from 0 to 3.

(b) What has happened at $t = 3$?

(c) Use the rule $v = 2t$ to complete a table of values for values of t from 0 to 3.

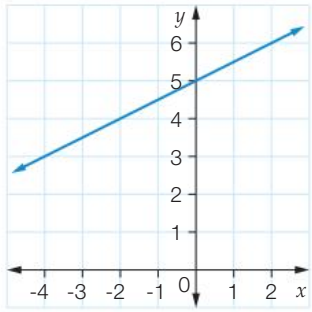
(d) Draw the two graphs on the same Cartesian plane.

(e) Why would it not be meaningful to use values of t less than 0 or values of t greater than 3?

(f) How long did it take for both containers to have the same amount of liquid? What volume was this?

Open-ended

19 Find any three points that lie on this line.



20 Rachelle completes a table for the rule $y = 2(x - 3)$. Her answers are the highlighted values.

x	3	$6\frac{1}{2}$	2
y	3	10	-2

(a) Which of Rachelle's answers are correct?

(b) Explain how she ignored the correct order of operations in each of the others.

(c) Write the correct values.

21 To do a question, Julian completed a table of values and drew a linear graph that included the point (2, 1).

What might the question have been? Write three different possibilities.

Outside the Square Puzzle

Who's who?

Coff, Salma, Flim, Troma, Plos, Zurf, Wada, Bleff, Kres and Alba are standing in a row, but not in that order.

Use the clues to place the ten people in order from shortest to tallest.

Clue 1 Counting from the left, Zurf is in an even-numbered position.

Clue 2 Wada and Alba are in the middle.

Clue 3 The greatest difference in height is between Bleff and Plos.

Clue 4 Flim is standing directly next to Plos and Salma is standing next to Bleff.

Clue 5 Coff is shorter than Zurf.

Clue 6 Troma is standing somewhere between Salma and Zurf.

Clue 7 Wada is shorter than Alba.

Clue 8 Kres is directly next to Troma.

Clue 9 Bleff is shorter than Plos.

Clue 10 There is one person standing between Troma and Wada.

Mathspace

ALGEBRA EV RACERS

In the foreseeable future major car manufacturers are planning to release cars in Australia that run on 100% electricity, called EVs ('Electric Vehicles'). If the electricity for these vehicles is sourced from renewable sources like sun, wind or water, these would be zero-emission vehicles.

Which car will win the race?

START
$s + b$

FINISH
$b + 4$

Mitsubishi EV

Toyota EV

Honda EV

$12 - 2b$

$s + 4$

$\frac{6}{s}$ (round to the nearest whole number)

$(b - s) \times (s - 1)$

$14 - 2s$

Speed out by moving forward a number of spaces equal to double the prime number you rolled.

$\frac{s + b}{2}$ (round to the nearest whole number)

$7 - b$

$s(b - s)$

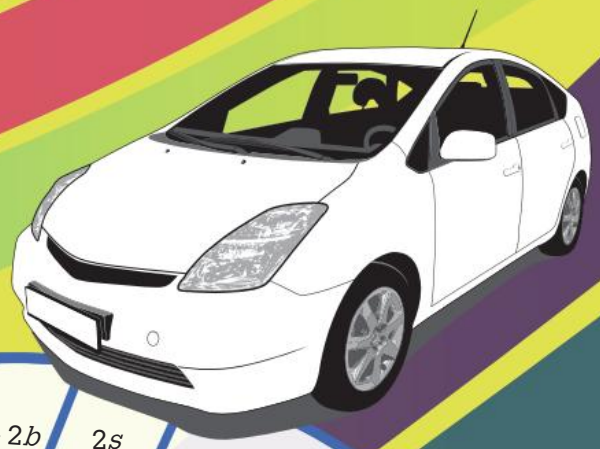
$6 - s$

$s - 1$

PITSTOP
CHARGE UP STATION

Roll one
You need to roll a prime number to move to the next space.

Equipment required: 2 dice, 2–3 brains, a counter for each player



$2b$ | $3s$ | $s + 1$ | $2b - s$ | $(s - 1)^2$ | $b - 1$ | $s + 3$ | $3b - 2b$ | $2s$ | $4b - 2s$ | $s + 2$ | $3b$ | $s + 4 + b - 3$ | $3(s - 1)$ | $\frac{4s}{2}$ | $b - s$ | $3b - s - 2b$ | $b + 2$ | $2s - 2$ | $3s + 5s - 6s$ | $\frac{2b}{3}$ (round to the nearest whole number) | $(b - s)^2$ | $\frac{b}{2} + 2$ (round to the nearest whole number) | $s^2 - 1$ | $1 + b - s$

Rules

- The aim is to be the first to complete 3 laps around the circuit.
- You must roll the dice and substitute the values shown into the expression that you are on to be able to move forward.
- Roll both dice at once and let b stand for the larger of the values you rolled and s stand for the smaller of the values you rolled.
- Move forward the number of spaces equal to the value of the expression after you have substituted in the values.
- Make sure you follow the order of operations.
- When you pass or land on the entry to the pit stop, if it's your second lap you must go in for a 'charge up'.

Half-time 6

Ex. 6.2

1 Raindrops falling into a container fill it at the rate of 4 millilitres per minute.

(a) Copy and complete the following table.

x (minutes)	3	4	5	6	7	8
y (volume of water in container, mL)	22					

(b) Draw a Cartesian plane showing values of x from 0 to 8 and values of y from 0 to 45.

(c) Plot and join the points with a straight line and extend the line until it intersects with the y-axis.

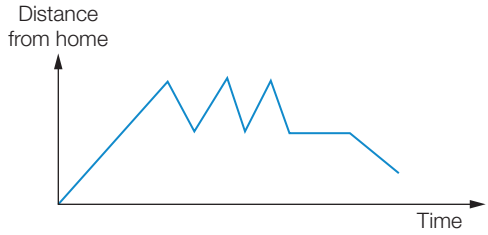
(d) Using the graph, state how much water was in the container initially.

Ex. 6.1

2 Melanie was 5 km from home. She waited at the bus stop for a time, then started to walk home at a constant rate. She stopped for a while to rest and then ran home at a constant rate. Draw a graph to show this information.

Ex. 6.1

3 The graph shows Dale's early morning run starting from his house.



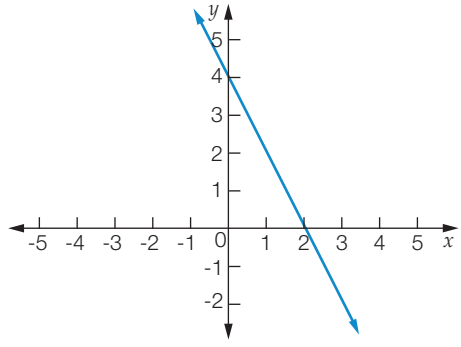
(a) The furthest point from home Dale reached was the oak tree in the park. How many times did he run to it?

(b) How many times did Dale rest?

(c) Did Dale return home? Explain your answer.

Ex. 6.2

4 Which point lies on the straight line shown?



A (0, 4)

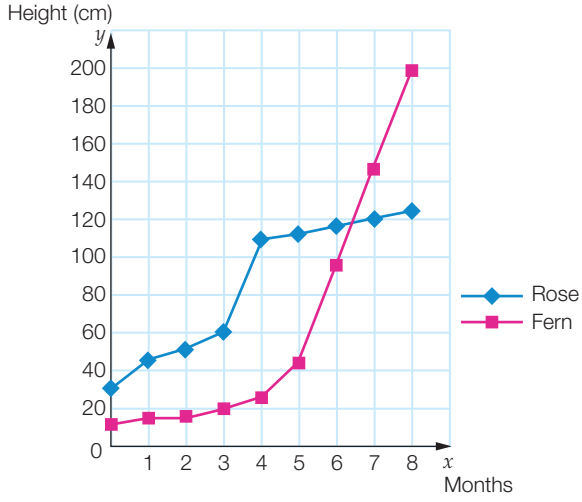
B (4, 0)

C (2, 2)

D (-2, 4)

Ex. 6.1

5 The graph shows the height of two seedlings over a period of months. Both were planted at the same time and were measured at the end of each month.



(a) Which seedling was taller when planted?

(b) When was the rose's rate of growth the fastest?

(c) During what time period was the rose taller than the fern?

(d) What was the difference between the two heights at 3 months?

(e) When did the fern reach a height of 40 cm?

(f) For how long was the rose less than 60 cm high?

Technology Exploration GeoGebra

Equipment required: 1 brain, 1 computer with GeoGebra, workbook

Versions of this Exploration for other technologies are available in Pearson Reader.

Exploring gradients and intercepts with technology

In the previous sections you have learnt to plot linear graphs from rules by making a table of values. These graphs were in the form of $y = mx + b$ where m is the coefficient of the variable x and b is a constant. In this exploration, we will show the effect that m and b have on linear graphs.

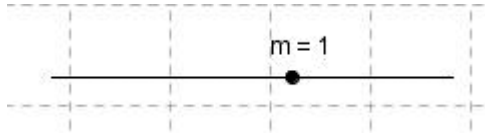
Open the GeoGebra program. You will see seven menu options (File, Edit etc.) at the top of the screen. Below these are 11 icons called tools. By clicking on the small arrow in the bottom right-hand corner of the tool icon, a drop-down list of more tools appears. The arrow turns red when you hover the cursor over it. If you hover over a tool, the tool's name and how to use it will appear in the top right-hand corner of the screen.

1. Click on the Options menu. Select 'Point Capturing' and 'On (Grid)'.
2. Click on View and tick Grid.
3. If a larger font is required, click on the Options menu and select 'Font Size'. Choose an appropriate size from the list provided.

Graphing a straight line

We will now create a line using $y = mx + b$ and look at how the graph changes when we change values of m and b.

4. Select the 'Slider' tool . Using the drop-down menu, select Slider from the list that appears. Click on the right-hand side of the drawing pad in the graphics view, the space where the axes and grid appear. A pop-up box will appear. Under 'Name' type m. Change 'Increment' to 1. Click on the 'Slider' tab and change 'Width' to 200. Click 'Apply'. A line like this will appear on the screen:



5. To construct a second slider, repeat step **4** naming it b.
6. In the 'Input' bar at the bottom of the screen type y=m*x+b (the * is needed to tell the program to multiply the values of m and x).

A straight line (a linear graph) should appear on the screen.

You will also notice that an equation will appear in the 'Algebra View' (on the left-hand side of the screen). See Figure 1 below.

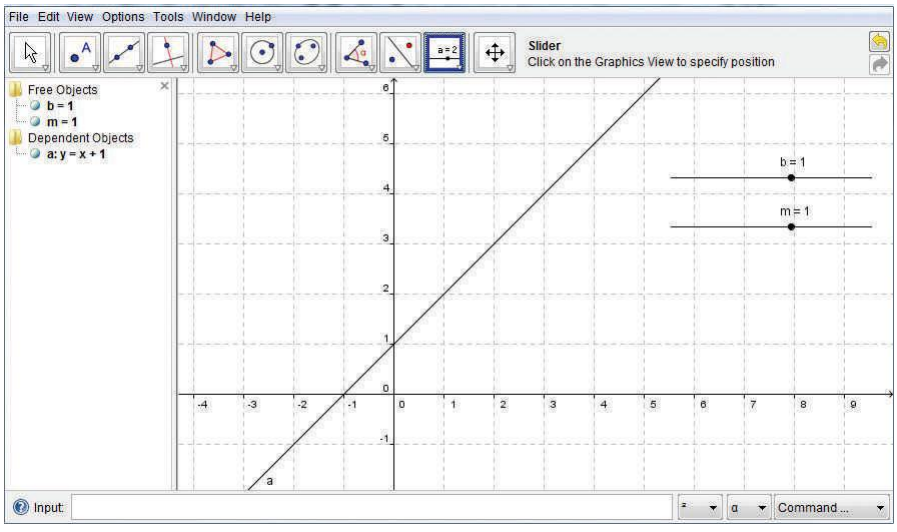


Figure 1

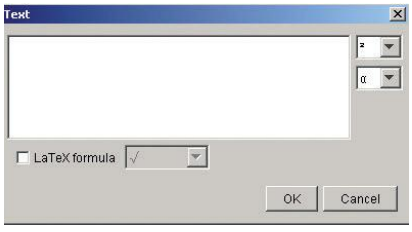
7 Click on the 'Move' tool . Now move the point on slider 'm' to the right (increasing the value of m). Take note of how the line changes on the screen as well as how the equation changes in the 'Algebra View'.

8 Can you predict what will happen to the line if you drag the slider to the left (decreasing the value of m)? Try it.

9 **(a)** Select File and New. You will be prompted to save. Save this file as 'Slider'. A new screen will appear. Use the input bar to graph the equations $y = x$, $y = 2x$ and $y = \frac{x}{2}$ on the same axes.

(Use * for multiplication and / for division.) All the graphs you draw should be labelled *a*, *b* and *c*.

(b) Right click on the graph and select 'show label' from the box that appears. This will remove the letter name from the screen. Select the Slider tool . Use the drop-down menu and select . In the text box that appears, write in the equation you have graphed.



10 **(a)** Select a new file, saving the graphs in Question **9**, then use the input bar to graph the equations $y = x$, $y = 2x$ and $y = \frac{x}{2}$ on the same axes. Label your graphs.

(b) All three graphs should be straight lines. Describe the gradients of these lines in terms of positive, negative, zero or undefined.

(c) Which graph has the steepest slope?

(d) Which graph has the smallest slope?

(e) What happened to the slope *m* as the coefficient of *x* increased?

(f) **(i)** What is the *y*-intercept of $y = x$?

(ii) What is the *y*-intercept of $y = 2x$?

(iii) What is the *y*-intercept of $y = \frac{x}{2}$?

(g) Use your answers to part **(f)** to find the point of intersection of $y = x$, $y = 2x$ and $y = \frac{x}{2}$.

11 **(a)** Select a new file, saving the graphs in Question **10**, then use the input bar to graph the equations $y = -x$, $y = -2x$ and $y = -\frac{x}{2}$ on the same axes. Label your graphs.

(b) All three graphs should be straight lines. Describe the gradients of these lines in terms of positive, negative, zero or undefined.

(c) Which graph has the steepest slope?

(d) Which graph has the smallest slope?

(e) What happened to the slope *m* as the coefficient of *x* increased?

(f) **(i)** What is the *y*-intercept of $y = -x$?

(ii) What is the *y*-intercept of $y = -2x$?

(iii) What is the *y*-intercept of $y = -\frac{x}{2}$?

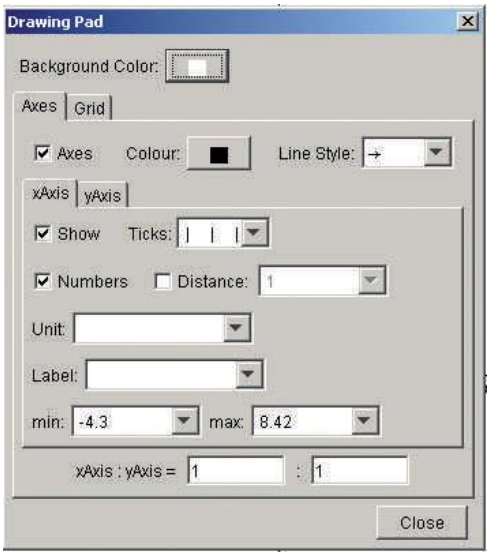
(g) Use your answers to parts **(d)**, **(e)** and **(f)** to find the point of intersection of $y = -x$, $y = -2x$ and $y = -\frac{x}{2}$.

12 Open the 'Slider' file, saving the graphs in Question **11**. In the input bar, input $y = mx + b$ again. Using $m = 1$, click on the 'Move' tool . Drag the point on slider 'b' to the right (increasing the value of b). Notice how the line changes and the equation changes.

13 Can you predict what will happen to the line if you drag the slider to the left (decreasing the value of b)? Try it.

14 **(a)** Select a new file, saving the graphs in Question **12** as 'Slider 2', then use the input bar to graph the equations $y = x + 3$, $y = x + 7$ and $y = x - 4$ on the same axes.

It is important to show the x- and y-intercepts of any graphs that you draw. If you need to change the x and/or y values on the axes, you need to click on Options and Drawing Pad. Use the min and max input boxes to change the values on your axes.



(b) All three graphs should be straight lines. Describe the gradients of these lines in terms of positive, negative, zero or undefined.

(c) Describe what you notice about these three graphs using words such as parallel, perpendicular, horizontal, vertical and intersect.

(d) What do you notice about the coefficient of x of these three graphs?

(e) **(i)** What is the y-intercept of $y = x + 3$?

(ii) What is the y-intercept of $y = x + 7$?

(iii) What is the y-intercept of $y = x - 4$?

(f) Use your answers to part **(e)** to explain the relationship of the y-intercept to its equation.

15 Repeat Question **14** but save Question **14** graphs with a new name and use the equations $y = 2x + 3$, $y = 2x + 7$ and $y = 2x - 4$.

16 What did you notice about your answers to Questions **14** and **15**? What was different about the two sets of graphs?

Compare **14 (i)** and **15 (i)** graphs. What was the same?

Compare **14 (ii)** and **15 (ii)** graphs. What was the same here?

Compare **14 (iii)** and **15 (iii)** graphs. What was the same here?

17 **(a)** Select a new file, saving the graphs in Question **15**. Use the input bar to graph the equations $y = 4$, $y = 9$ and $y = -3$ on the same axes.

(b) All three graphs should be straight lines. Describe the gradients of these lines in terms of positive, negative, zero or undefined.

(c) Describe what you notice about these three graphs using words such as parallel, perpendicular, horizontal, vertical and intersect.

(d) Is there an x-intercept for any of these graphs? What is the coefficient of x in the equations for these graphs?

(e) **(i)** What is the y-intercept of $y = 4$?

(ii) What is the y-intercept of $y = 9$?

(iii) What is the y-intercept of $y = -3$?

(f) Use your answers to part **(e)** to explain the relationship of the y-intercept to its equation.

18 **(a)** Select a new file, saving the graphs in Question **17**. Use the input bar to graph the equations $x = -5$, $x = 6$ and $x = -3$ on the same axes.

(b) All three graphs should be straight lines. Describe the gradients of these lines in terms of positive, negative, zero or undefined.

(c) Describe what you notice about the gradients of these three graphs using words such as parallel, perpendicular, horizontal, vertical and intersect.

(d) Is there a y-intercept for any of these graphs? What do you notice is different about the equations for these graphs?

(e) **(i)** What is the x-intercept of $x = 4$?

(ii) What is the x-intercept of $x = 6$?

(iii) What is the x-intercept of $x = -3$?

(f) Use your answers to part **(e)** to explain the relationship of the x-intercept to its equation.

19 **(a)** Select a new file, saving the graphs in Question **18**. Use the input bar to graph the equations $y = 2(x + 3)$ and $y = 2x + 6$.

(b) What is the y-intercept for each graph?

(c) What is the x-intercept for each graph?

(d) What is the relationship between the x-intercept and the m and b values in $y = 2x + 6$?

(e) Comment on the two graphs you have drawn.

20 (a) Select a new file, saving the graphs in Question **19**. Use the input bar to graph the equations $y = 3(2x + 1)$ and $y = 6x + 3$.

(b) What is the y-intercept for each graph?

(c) What is the x-intercept for each graph?

(d) What is the relationship between the x-intercept and m and b values in $y = 6x + 3$?

(e) Comment on the two graphs you have drawn.

Taking it further

Families of graphs all have something in common. For the following questions, we are going to create families of graphs using what you have learned. For these questions, do not use any equations used previously.

21 Write down three equations that will give parallel graphs. Graph these equations on the drawing pad. Were they parallel? If not, try again.

22 Write down three equations that will give graphs that all have the same y-intercept. Graph these equations on the drawing pad. Did they all have the same y-intercept? If not, try again.

23 Write down three equations that will give graphs that are all parallel to the x-axis. Graph these equations on the drawing pad. Were they all parallel to the x-axis? If not, try again.

24 Write down three equations that will give graphs that are all parallel to the y-axis. Graph these equations on the drawing pad. Were they all parallel to the y-axis? If not,try again.

Finding the rule

6.3

We can identify linear relationships and find their rule using any of the following methods.

Using ordered pairs

If we have a list of ordered pairs we can look for a pattern.

(1, 8), (2, 15), (3, 22), (4, 29), (5, 36), (6, 43)

We can see that:

- the value of y increases by 7 as the x-value increases by 1, so $7x$ is a term in our rule
- 1 needs to be added to complete our rule.

RULE: $y = 7x + 1$

Using a pattern and a table of values

In Year 7 you looked at patterns, tables of values and rules.

Number of octagons (n)	1	2	3	4	5	6
Number of lines (l)	8	15	22	29	36	43

We can see that:

- each new octagon is formed by adding 7 lines to the octagon before it
- the number of lines (l) in the table increases by 7, so $7n$ is a term in our rule
- 1 needs to be added because 1 line was needed to begin the pattern.

RULE: $l = 7n + 1$ where l is the number of lines and n is the number of octagons.

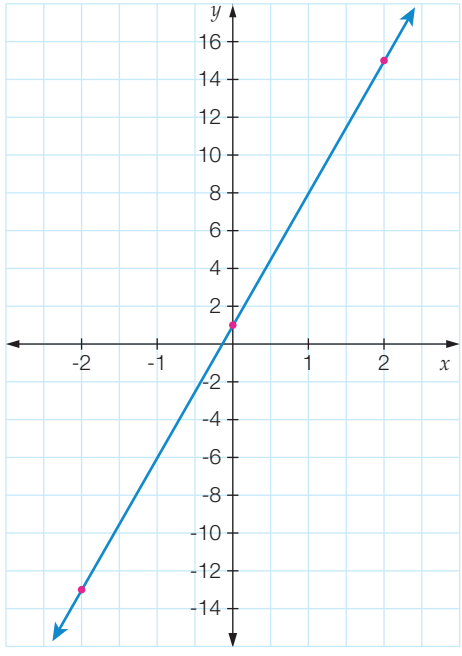
Using a graph

The ordered pairs (-2, -13), (0, 1) and (2, 15) plotted on a Cartesian plane can be connected with a straight line. They form a linear relationship.

We can see that:

- the y-value increases by 14 when the x-value increases by 2. That means that the y-value increases by 7 as the x-value increases by one. So, we need $7x$ in our rule.
- 1 needs to be added to find the y-value.

RULE: $y = 7x + 1$.



Satisfying the rule

Satisfying the rule means that the ordered pairs make true statements when they are substituted into the rule. By substituting (-2, -13), (0, 1) and (2, 15) into $y = 7x + 1$, we can see that all points satisfy this rule.

$-13 = 7 \times -2 + 1$ $\quad$ $1 = 7 \times 0 + 1$ $\quad$ $15 = 7 \times 2 + 1$

Sometimes, when given a list of ordered pairs (or a table of values) only one operation has been performed on x to obtain y. Finding the rule is very easy. Be careful to check that the rule applies to all points.

Worked Example 5 WE5

Find the rule for each of the following sets of points.

(a) (0, 3), (1, 4), (2, 5), (3, 6), (4, 7) **(b)** (-3, -6), (1, 2), (4, 8), (6, 12)

Thinking	Working
(a) 1 Look for a pattern. Can we perform one operation on x to obtain y? Yes (+ 3).	(a) 3 has been added to each x-value to obtain y.
2 Write the rule.	$y = x + 3$
3 Check the rule by substituting one x-value from the list into the equation to obtain the given y-value.	Check: Using the point (4, 7) $y = x + 3$ $= 4 + 3$ when $x = 4$ $= 7$ Our rule is correct.
(b) 1 Look for a pattern. Can we perform one operation on x to obtain y? Yes ($\times$ 2).	(b) Each x-value has been multiplied by 2 to obtain y.
2 Write the rule.	$y = 2x$
3 Check the rule by substituting one x-value from the list into the equation to obtain the given y-value.	Check: Using the point (4, 8) $y = 2x$ $= 2 \times 4$ when $x = 4$ $= 8$ Our rule is correct.

When more than one operation has been performed on x to obtain y, we cannot see the relationship so easily. We need to look for another pattern.

Worked Example 6

WE 6

Find the rule for the following table of values.

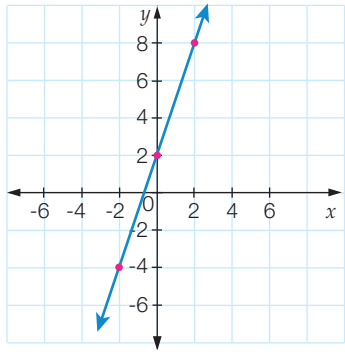
x	-3	1	5	9
y	-24	-4	16	36

Thinking	Working
1 Look for a pattern. Can we perform one operation on x to obtain y? No. (Subtracting 21 or multiplying by 8 works for the first point, but not for the others.)	We cannot perform one operation on x to obtain y.
2 Look for another pattern. By how much is the x-value increasing? By how much is the y-value increasing? Use this information to find the amount by which y is increasing as x increases by 1. This is the coefficient of x.	y increases by 20 as x increases by 4, so y increases by 5 as x increases by 1. We need 5x in our rule.
3 What number needs to be added or subtracted to find the y-value? This is the constant in the rule.	9 needs to be subtracted.
4 Write the rule.	$y = 5x - 9$
5 Check the rule by substituting one x-value from the table into the rule to obtain the corresponding y-value.	Check: use the values $x = 5$, $y = 16$ $y = 5x - 9$ $= 5 \times 5 - 9$ $= 25 - 9$ $= 16$ Our rule is correct.

Worked Example 7

WE 7

Find the rule that describes the following relationship.



Thinking	Working
1 List at least three points that lie on the line.	(-2, -4), (0, 2), (2, 8)

2 Look for a pattern. By how much is the x-value increasing? By how much is the y-value increasing? Use this information to find the amount by which y is increasing as x increases by 1. This number is the coefficient of x.	We cannot perform one operation on x to obtain y. y increases by 6 as x increases by 2, so y increases by 3 as x increases by 1. We need 3x in our rule.
3 After the x-value is multiplied by the coefficient found in step **2** (3), what number needs to be added or subtracted to find the corresponding y-value?	2 needs to be added.
4 Write the rule.	$y = 3x + 2$
5 Check the rule by substituting one x-value from the list into the equation to obtain the given y-value.	Check: Using the point (2, 8) $y = 3x + 2$ $= 3 \times 2 + 2$ when $x = 2$ $= 6 + 2$ $= 8$ Our rule is correct.

6.3 Finding the rule

Navigator

Answers page 625

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6 Column 1, Q7, Q8, Q9, Q12	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6 Column 2, Q7, Q8, Q9, Q10, Q13	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6 Column 2, Q7, Q9, Q10, Q11, Q12, Q13

Fluency

WE5

1 Find the rule for each of the following sets of points.

(a) (0, -1), (1, 0), (2, 1), (3, 2), (4, 3)

(b) (2, 3), (3, 4), (4, 5), (6, 7), (7, 8)

(c) (-3, 3), (-1, 5), (2, 8), (3, 9), (5, 11)

(d) (-2, -5), (0, -3), (2, -1), (5, 2), (6, 3)

(e) (1, 1), (3, 3), (4, 4), (7, 7), (10, 10)

(f) (-1, -4), (0, 0), (1, 4), (2, 8), (3, 12)

(g) (-2, -10), (0, 0), (2, 10), (4, 20), (6, 30)

(h) (-2, 2), (-1, 1), (0, 0), (1, -1), (2,-2)

WE6

2 Find the rule for the following tables of values.

(a)

x	-7	-5	1	4
y	-6	-4	2	5

(b)

x	-2	3	6	7
y	-5	0	3	4

(c)

x	-4	0	5	7
y	8	0	-10	-14

(d)

x	-2	-1	5	6
y	-14	-7	35	42

(e)

x	-6	-3	0	6
y	2	1	0	-2

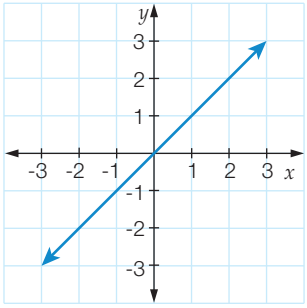
(f)

x	-5	0	10	20
y	-1	0	2	4

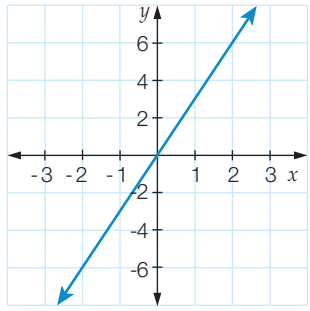
3 Find the rule that describes each of the following relationships.

WE7

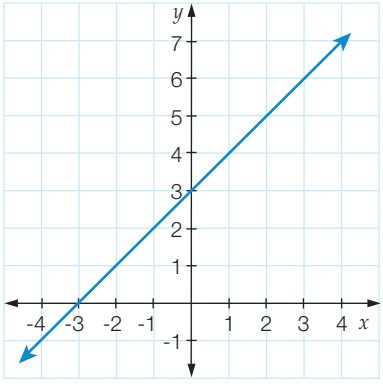
(a)



(b)



(c)



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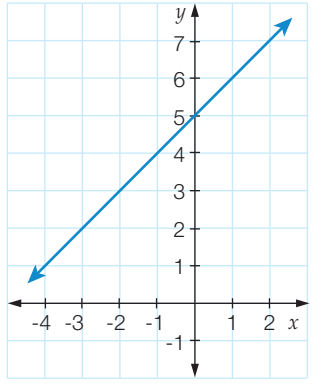

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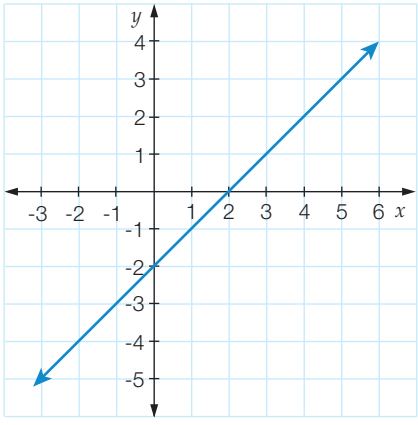
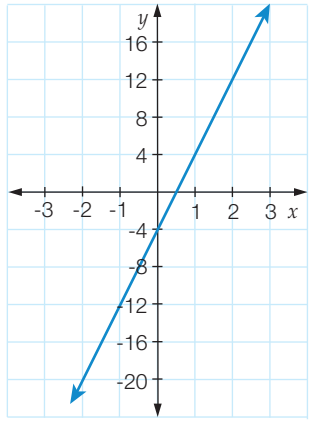
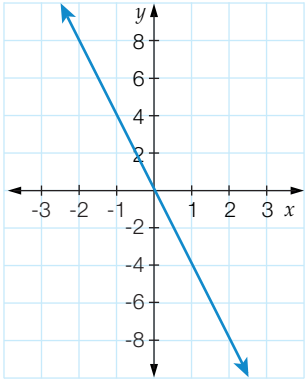


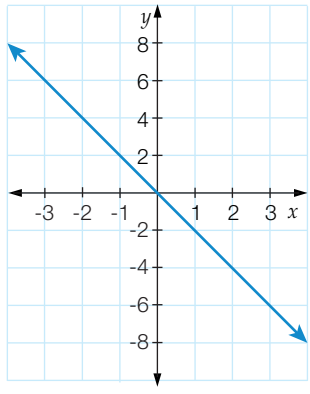
(f)



(g)



(h)



4 **(a)** (1, 5), (2, 6), (3, 7), (4, 8) are coordinate pairs that lie on the line with rule:

A $y = 5x$ **B** $y = x - 4$ **C** $y = 4x$ **D** $y = x + 4$

(b) (0, 0), (1, 6), (2, 12), (3, 18) are coordinate pairs that lie on the line with the equat

A $y = x + 5$ **B** $y = x + 6$ **C** $y = 6x$ **D** $y = 5x$

(c) (1, -4), (2, -3), (3, -2), (4, -1), (5, 0) are coordinate pairs that lie on the line with the rule:

A $y = x - 5$ **B** $y = 2x$ **C** $y = x + 6$ **D** $y = 3x$

5 The rule $y = x - 2$ is true for which ordered pairs?

A (5, 7) and (0, 2)

B (2, -2) and (3, 1)

C (-4, 2) and (1, 3)

D (-3, -5) and (4, 2)

6 Describe each of the following relationships with a rule. Hence, copy and complete each of the following tables.

(a)

x	-3	-2	-1	0	1	2	3
y	-2	-1	0	1			

(b)

x	-3	-2	-1	0	1	2	3
y	-1	0	1	2			

(c)

x	-3	-2	-1	0	1	2	3
y				-5	-4	-3	-2

(d)

x	-3	-2	-1	0	1	2	3
y				-3	-2	-1	0

(e)

x	-3	-2	-1	0	1	2	3
y	-6		-2		2		6

(f)

x	-3	-2	-1	0	1	2	3
y		-6		0	3		9

(g)

x	-3	-2	-1	0	1	2	3
y	24		8	0		-16	

(h)

x	-3	-2	-1	0	1	2	3
y		12		0	-6		-18

Understanding

7 Match each of the rules **(a)**, **(b)**, **(c)** and **(d)** with its worded description **E**, **F**, **G** and **H**.

(a) $y = x + 4$ **(b)** $y = 1 - x$ **(c)** $y = \frac{x}{4} + 1$ **(d)** $y = 1 - 4x$

E The sum of the x- and y-coordinates is 1.

F The y-coordinate is four more than the x-coordinate.

G The y-coordinate is the sum of 1 added to the x-coordinate divided by four.

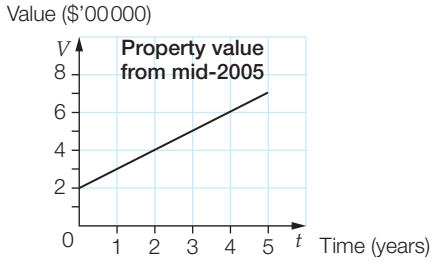
H The y-coordinate added to four times the x-coordinate less 1 is equal to zero.

Reasoning

8 The graph shows the value of a property over five years from the middle of 2005.

(a) Find the rule for the line.

(b) If the trend continues, what will the property be worth in the middle of 2017?



9 **(a)** Write a rule for the number of dots (D) in terms of the number of squares (S) in the pattern below.

(b) Explain how you found the coefficient of S in your rule.

(c) Explain how you found the amount to add on in your rule.

(d) Use your rule to find the number of dots needed to construct 8 squares with this pattern.

10 The contractors building a fence charge a set amount for materials and an hourly rate for labour. If it takes 6 hours to build the fence, the total cost will be \$880. If it takes 8 hours, the cost will be \$1040.

(a) What is the hourly rate for labour?

(b) Plot the two known points and use your graph to find the rule for the total cost in dollars (C) in terms of time in hours (t).

(c) What is the cost of materials?

11 Oil is leaking from a car engine at the rate of 250 millilitres per minute. Initially, the engine's oil volume was 2225 millilitres.

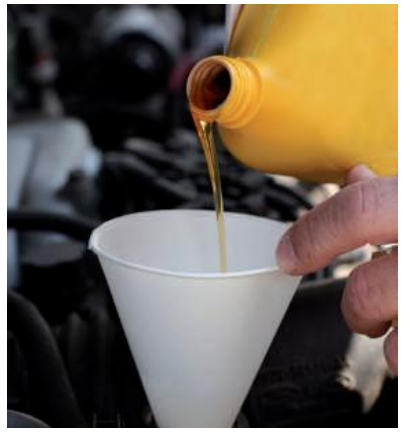
(a) Copy and complete the table.

x (minutes)	0	1	2	3	4	5
y (volume of oil in engine, mL)						

(b) Plot the points on the Cartesian plane and connect them with a straight line.

(c) Find the rule for y, the amount of oil remaining, after x minutes.

(d) How long does it take for the car's engine to lose all its oil?



Open-ended

12 Write the rules of two different straight line graphs that include the point (5, 2).

13 Write three rules whose graphs show decreasing values for y when x-values increase.

Outside the Square Puzzle

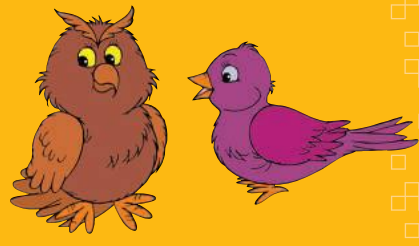
The Liar-bird and the Tru-tru bird

Jerry is lost on a mysterious island. The island contains two types of talking birds: The Liar-bird which always lies and the Tru-tru bird which always tells the truth. Jerry captures three birds, one Liar-bird and two Tru-tru birds, and wants to find out which is the Liar-bird.

He asks each of the birds what type they are:

Bird A answers but Jerry doesn't hear.

Bird B squawks frustratedly 'A said he's a Liar-bird'.



Bird C squawks impatiently 'Don't believe B, he's lying'.

Which bird is the Liar-bird?

Technology Exploration CAS

Equipment required: 1 brain, CAS calculator

Versions of this Exploration for other technologies are available in Pearson Reader.

Be still my beating heart

Medical professionals, elite athletes and the general public should all pay attention to their health. One measure of health is maximum heart rate (HR_{max}). This is a measure, in beats per minute, of heart activity after strenuous exercise. However, as with many health-related measures, there are no hard and fast rules as to what the maximum value should be. Even among elite athletes participating in the same sport HR_{max} can vary from 160 to 220 beats per minute.

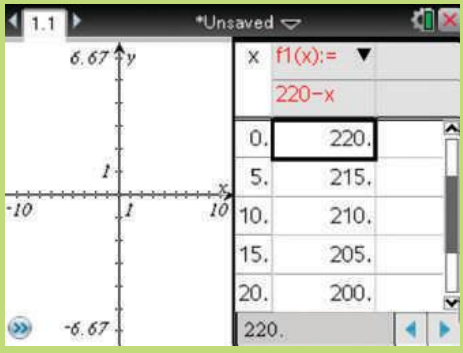
As it can require expensive equipment to measure HR_{max} accurately, use is frequently made of a formula to predict HR_{max}.

The most commonly used formula is: $HR_{max} = 220 - \text{age}$.

Let's look at how we can use our CAS to explore this and other formulas. We will make use of the table facility in this task.

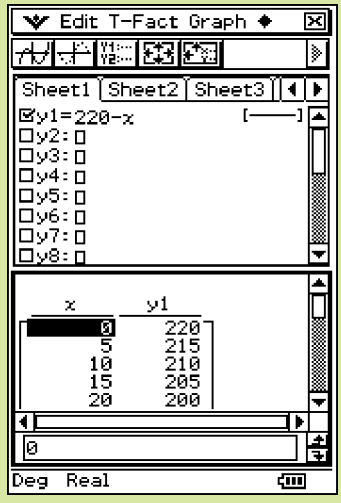
Using the TI-Nspire CAS

Open a new Graphs screen and enter the rule as $f1(x) = 220 - x$. Press enter (or enter) to confirm. Then press menu > View > Show Table. You may need to change the way the table is set up. We would prefer the x-values to change by 5, so press menu > Table > Edit Table Settings and make Table Start 0 and Table Step 5. Then, highlight OK and press ctrl enter (or enter clear).



Using the ClassPad

Open a new Graphs & Table (Graph&Tab...) screen and enter $y1$ as $220 - x$ and tap EXE when you are done. Then tap to make the table of values appear. You may need to change the way the table is set up. We would prefer the x-values to change by 5, so tap and make Start 0, End 100 and Step 5. Then, tap OK and the new table will appear.



This first formula is actually not thought of highly by many medical and fitness professionals who prefer: $HR_{max} = 205.8 - (0.685 \times \text{age})$.

There are several other formulas that can also be used.

They are:

$HR_{max} = 206.3 - (0.711 \times \text{age})$
$HR_{max} = 217 - (0.85 \times \text{age})$
and $HR_{max} = 208 - (0.7 \times \text{age})$.

1 Use your CAS to help you complete the following table. You will enter each new formula as a new equation and the table will grow accordingly.

Using the TI-Nspire CAS	Using the ClassPad
To return to the graph screen to enter the formulas, press ctrl tab (or ctrl tab). If the equation entry line is not showing, press ctrl G (or ctrl G) and enter the equations needed. You can then return to the table (ctrl tab or ctrl tab) and assign each formula $f1$, $f2$ etc. to a column, and scroll across the columns to get the values required.	Enter the four remaining formulas, by tapping in the appropriate equation line, $y2$, $y3$, $y4$, $y5$ etc. Then tap to make the table of values appear. You can then scroll across the columns to get the values required.

Formula/Age	$HR_{max} = 220 - age$	$HR_{max} = 205.8 - (0.685 \times age)$	$HR_{max} = 206.3 - (0.711 \times age)$	$HR_{max} = 217 - (0.85 \times age)$	$HR_{max} = 208 - (0.7 \times age)$
10					
15					
20					
25					
30					
35					
40					
50					
60					
70					
80					

2 Do you think there are any practical limitations with any of these formulas? Explain what they are.

When you are beginning a fitness program you should also be interested in your target heart rate (THR). This is the rate you want to get your heart to during your exercise session. One of the most effective ways of calculating this is known as the Karnoven method. To find this, you follow these steps:

(a) Find your resting heart rate (RHR): Count the number of heart beats in a minute each morning for three mornings as soon as you wake up. Find the mean of these three figures—this is your RHR.

(b) Find your HR_{max}. (You could use any of the methods above.)

(c) Calculate HR_{max} – RHR to get your heart rate reserve.

(d) Calculate the lower limit for your THR—this is 60% of your heart rate reserve.

(e) Calculate the upper limit for your THR—this is 80% of your heart rate reserve.

(f) Find the mean of these two values—this is your THR.

3 Can you think of a way of combining steps **(c)** and **(d)** into one calculation?

4 Imagine that you are 24 years of age and have a RHR of 80 beats per minute. Find your THR using each of the five methods for calculating the HR_{max}.

Taking it further

Use your CAS to draw the graphs of at least some of these functions. Then, write a few sentences describing what they tell you. In particular, pay attention to what heart rate they predict for very young and very old people.

6.4 Using linear relationships

Recognising linear relationships

There are many situations where two variables are connected that demonstrate a linear relationship such as those that involve a constant rate. For example, a constant flow rate of a liquid, the constant speed of a vehicle, or a daily rate of pay for work done.

Defining a variable

In mathematical word problems that contain variables we need to choose a suitable pronumeral to represent a variable. In measurement, we used 'h' to represent the variable 'height' and 'V' to represent the variable 'volume'. When 'time' is a variable we usually represent it using 't'.

Worked Example 8 WE 8

For each statement, define each of the two variables involved.

(a) The volume of water in a water tank increases during a day of heavy rain.

(b) The average speed of a car as it travels over a distance.

Thinking	Working
(a) **1** Identify the two variables.	(a) volume and time
2 Define each variable.	Let V be the volume of water in the tank. Let t be the time during which the tank is filling.
(b) **1** Identify the two variables.	(b) distance and speed
2 Define each variable.	Let d be the distance travelled. Let s be the speed the car travelled.

Worked Example 9 WE 9

Define all the variables you are using in the following situation and then write a linear equation to represent the total cost.

The cost of a building project at a school consists of a \$1000 fixed cost to cover the materials used and a variable cost of the labour, which is calculated by a fixed rate of \$30 per hour.

Thinking	Working
1 Identify and define the variables.	Let C be the cost of the project and h be the number of hours of labour.
2 Identify the fixed costs.	Cost of materials = \$1000. Hourly rate of labour = \$30.
3 Write the equation.	$C = 30h + 1000$

Using linear graphs

A straight line can be drawn through any two points, so if we know that a relationship between two variables is linear and we are given two pieces of information that connects the two variables, we can write that information as two ordered pairs. We can then use those ordered pairs as two points on a Cartesian plane, draw a linear graph on graph or grid paper and then use it to find the relationship between the variables for any other point on the line.

Worked Example 10 WE10

A tap is left dripping at a constant rate. The number of litres of water wasted is shown in the table below.

Number of hours the tap is left dripping, *d* (h)	0	2	6	11
Number of litres of water wasted, *w* (L)	0	10	30	55

(a) Plot the points shown in the table and join them to form a graph of this relationship. Label the axes with 'Number of hours tap was left dripping, *d*' on the *x*-axis and 'Number of litres wasted, *w*' on the *y*-axis.

(b) Is this relationship linear?

(c) From your graph, decide how much water would be wasted in:

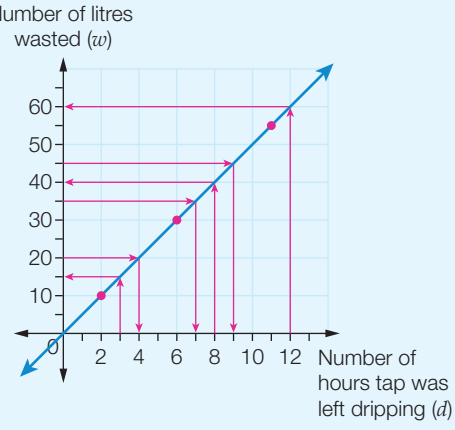
(i) 3 hours **(ii)** 8 hours **(iii)** 12 hours.

(d) From your graph, decide how long the tap had been dripping if the number of litres wasted was:

(i) 20 litres **(ii)** 35 litres **(iii)** 45 litres.

(e) Is it sensible to join the points in this situation?

(f) Write the rule that shows the relationship between the number of hours the tap is left dripping, *d*, and the number of litres of water wasted, *w*.

Thinking	Working
(a) Draw and label your *x*- and *y*-axes and plot the points. Join them in order with straight lines. Make sure your scale will allow you to read off the values required.	(a) 
(b) Decide whether all points lie on the same straight line. If so, the graph is linear.	(b) Yes, this is a linear graph.
(c) Read off the *y*-coordinate of the point on the line that has the given *x*-coordinate. This is your answer.	(c) (i) 15 litres (ii) 40 litres (iii) 60 litres

(d) Read off the x-coordinate of the point on the line that has the given y-coordinate. This is your answer.	(d) (i) 4 hours (ii) 7 hours (iii) 9 hours	
(e) Do we measure or count the variables? Variables that can be measured are continuous. In between values exist, so a linear graph is sensible.	(e) Because both time and volume are continuous variables, it is sensible to use a linear graph.	
(f) Look for a pattern between the number of hours the tap is left dripping (d) and the number of litres of water wasted (w).	(f) $w = 5d$	

Worked Example 11

WE11

A Year 8 class chose to support a cancer research charity by organising a school concert as a fundraiser. The following table shows the total money raised as the number of tickets sold increased. Their teachers made a donation to the fund.

Number of tickets sold (t)	50	70	160	180
Money raised ($\$M$)	2000	2200	3100	3300

(a) Plot the points shown in the table and join them to form a graph of this relationship. Label the axes with 'Number of tickets sold (t)' on the x-axis and 'Money raised ($\$M$)' on the y-axis.

(b) Is the relationship linear?

(c) From your graph, decide how much money would be raised if the following number of tickets was sold.

(i) 90 **(ii)** 170 **(iii)** 230

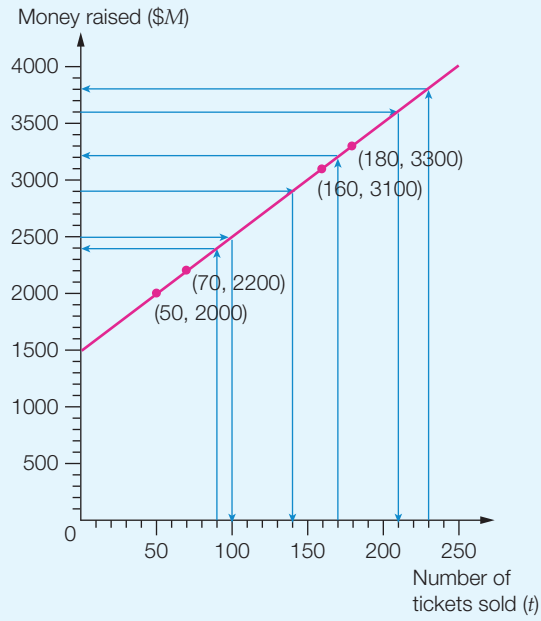
(d) From your graph, decide how many tickets would need to be sold to raise the following revenue.

(i) \$2500 **(ii)** \$2900 **(iii)** \$3600

(e) Is it sensible to join the points in this situation? If not, why do we use a linear graph?

(f) Write the rule that shows the relationship between the number of tickets sold (t) and the money raised ($\$M$).

(g) How much money did their teachers donate?

Thinking	Working
(a) Draw and label your x- and y-axes and plot the points. Join them in order with straight lines. Make sure your scale will allow you to read off the values required.	(a) 
(b) Decide whether all points lie on the same straight line. If so, the graph is linear.	(b) Yes
(c) Read off the point on the line that has an x-coordinate equal to the given value. The y-coordinate of the point is your answer.	(c) Amount of money raised (i) \$2400 (ii) \$3200 (iii) \$3800
(d) Read off the point on the line that has a y-coordinate equal to the given value. The x-coordinate of the point is your answer.	(d) Number of tickets sold (i) 100 (ii) 140 (iii) 210
(e) Do we measure or count the variables? Variables that are counted are not continuous and should be shown on a point graph, but we often model non-continuous variables with a continuous graph for convenience.	(e) These values are not continuous, but we use a linear graph for convenience as long as we realise that in-between values are meaningless.
(f) Look for a pattern between the number of tickets sold and the money raised. By how much is t increasing? By how much is M increasing? Use this information to find the amount by which M is increasing as t increases by 1. Check that this rule works for all points.	(f) As ticket numbers increase by 20, money raised increases by \$200, so if ticket numbers increase by 1, the money raised would increase by 10. We need $10t$ in our rule. \$1500 needs to be added on. $M = 10t + 1500$
(g) The constant in the rule in part **(f)** tells us the amount in the fund to start with. This is the teachers' donation.	(g) \$1500

6.4 Using linear relationships

Navigator

Answers page 626

Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q10, Q11	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q11, Q12	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q12

Fluency

Equipment required: Graph or grid paper

WE8

1 For each statement, define each of the two variables involved.

(a) The length of a section of tram track increases as the temperature increases.

(b) The volume of water in a swimming pool decreases when the pool develops a leak.

(c) Tyre pressure increases as the volume of air increases.

(d) The radius of a balloon increases as the volume increases.

(e) A student runs a distance during a race.

WE9

2 Define all the variables you are using in each of the following situations and then write a linear equation to represent the total cost of each.

(a) A solicitor charges a client a fixed fee of $50 for an initial consultation. She then charges $100 per hour for the documents she produces for her client.

(b) A taxi service charges a flagfall fee of $3.20, plus a fixed amount of $1.60 for each kilometre travelled.

(c) A call on a mobile phone incurs a flagfall charge of 37c. The call then incurs an additional charge of 40c per minute.

WE10

3 For a particular brand of cordial, the manufacturer recommends that the cordial syrup be diluted by adding a certain amount of water. The following table shows the corresponding amounts of cordial syrup and water needed.

Amount of cordial syrup, s (mL)	10	25	40	50
Amount of water needed, w (mL)	30	75	120	150

(a) Plot the points shown in the table and join them to form a graph of this relationship. Label the axes with 'Amount of cordial syrup' on the x-axis and 'Amount of water needed' on the y-axis.

(b) Is the relationship linear?

(c) From your graph, decide how much water should be added to the following quantities of cordial syrup.

(i) 15 mL **(ii)** 28 mL **(iii)** 55 mL

(d) From your graph, decide how much cordial syrup should be added to the following quantities of water.

(i) 24 mL **(ii)** 90 mL **(iii)** 135 mL

(e) Is it sensible to join the points in this situation?

(f) Write the rule that shows the relationship between the amount of cordial syrup (s) and the amount of water (w).

4 Yuni wanted to buy a number of golf clubs (g) all for the same price from an online golf store. The store adds a fixed postage and delivery cost to the cost of the clubs to find the total price (p) Yuni would be charged. The following table shows the amount Yuni could have paid for her clubs.

WE 11

Number of golf clubs (g)	2	5	7	9
Total price in dollars (p)	140	320	440	560

(a) Plot the points shown in the table and join them to form a graph of this relationship. Label the axes with 'Number of golf clubs (g)' on the x-axis and 'Total price in dollars (p)' on the y-axis.

(b) Is this relationship linear?

(c) From your graph, decide how much Yuni would pay for:

(i) four clubs (ii) six clubs (iii) ten clubs.

(d) From your graph, decide how many golf clubs Yuni could buy for:

(i) \$200 (ii) \$500 (iii) \$740.

(e) Is it sensible to join the points in this situation? If not, why do we use a linear graph?

(f) Write the rule that shows the relationship between the number of golf clubs Yuni could buy (g) and the total price she would pay (p).

(g) How much has Yumi paid for the postage and delivery of her clubs?

Understanding

5 Ribbon used to wrap Christmas gifts costs 55c a metre.

(a) Copy and complete the following table.

Length of ribbon (m)	0	1	2	3	4	5	6	7	8
Cost (\$)	0	0.55	1.10						

(b) Plot a graph of this relationship. Show the length of ribbon on the horizontal axis.

(c) How much would it cost to buy 4.2 m of ribbon?

(d) What length of ribbon could be bought for \$3.00?

(e) What length of ribbon could be bought for \$5.00?

6 Brent is driving his car at 70 km/h.

(a) Copy and complete the following table.

Time (h)	0	1	2	3	4	5	6
Distance travelled (km)	0	70					

(b) Plot a graph of distance travelled against time. Show time on the horizontal axis.

(c) From the graph, how far does Brent travel in:

(i) 3.5 hours (ii) 30 minutes (iii) 5 hours 15 minutes?

(d) From the graph, how long does it take Brent to travel:

(i) 175 km (ii) 298 km (iii) 400 km?

7 The table below shows the cost of buying different lengths of metal pipe.

Length of pipe (m)	2	5	8	10	15
Cost ($)	4.8	12	19.2	24	36

(a) Plot the points shown in the table and join them to form a graph of this relationship. Label the axes.

(b) Does the line pass through the point (0, 0)? Explain why you would expect it to do so.

(c) How much would it cost to buy a piece of pipe 12 m in length?

(d) How much would it cost to buy a 4 m length of pipe?

(e) Is it more economical to buy three 4 m lengths or one 12 m length?

(f) What length of pipe, to the nearest metre, can be bought for $50?

(g) What length of pipe, to the nearest metre, can be bought for $100?

8 To convert length measurements in inches (imperial system) to centimetres (metric system), we can draw a graph using two points.

(a) Rule a set of axes on graph paper. Label the horizontal axis 'Length in inches' and use a scale from 0 to 15. Label the vertical axis 'Length in centimetres' and use a scale from 0 to 40.

(b) What length in centimetres is equivalent to zero inches? Write this information as a coordinate point for your graph.

(c) 12 inches is equivalent to about 30.5 centimetres. Write this information as a coordinate point for your graph.

(d) Plot the two points you found in parts **(b)** and **(c)** onto your number plane and carefully rule a straight line through them.

Use your conversion graph to answer the following questions.

(e) The world's largest known spider is the goliath bird-eating spider. One specimen found in Venezuela in 1965 had a leg span of about 11 inches. What is this in centimetres?

(f) The smallest bird in the world is the bee hummingbird. If one of these birds has a length of 5.7 cm, how many inches long is it?

(g) The greatest reported diameter for a bubblegum bubble (under strict competition guidelines) is 22 inches. It was blown by Susan Williams of California, USA, in 1985. Convert this diameter to cm.

(h) The common flea (*Pulex irritans*) is a champion jumper. In 1910, one made a long jump of 33 cm and a high jump of 20 cm (130 times its own height). Convert these measurements to inches.

(i) The shortest venomous snake is the namaqua dwarf adder, which has a length of 8 inches. What is the length of this snake in cm?

(j) The world's largest cockroach measures about 3.8 inches in length and 1.7 inches across. Convert these measurements to cm.

Reasoning

9 Many countries in the world, such as Australia, use degrees Celsius (°C) as their unit of temperature, whereas others, such as the USA, use degrees Fahrenheit (°F). A conversion graph can be very useful in converting temperatures from one unit to the other.

(a) Rule a set of axes on graph paper. Label the horizontal axis 'Temperature in °C' and use a scale from -60 to 100. Label the vertical axis 'Temperature in °F' and use a scale from -80 to 220.

(b) The freezing point of water is 0°C or 32°F. Write this information as a coordinate point for your graph.

(c) The boiling point of water is 100°C or 212°F. Write this information as a coordinate point for your graph.

(d) Plot the two points you found in parts **(b)** and **(c)** onto your number plane and carefully rule a straight line through them.

Use your temperature conversion graph to answer the following questions.

(e) The highest shade temperature recorded in the world was 136°F at Al Aziziyan in Libya. What temperature is that in °C?

(f) The coldest place in the world is at Vostok, Antarctica, which recorded a temperature of -89°C. Convert this temperature into °F.

(g) Which is hotter: a temperature of 30°C or 80°F?

(h) Human body temperature is around 98°F. What is this in °C?

(i) An American tourist visiting Sydney reads that the maximum temperature forecast for the day is 35°C. What is this in °F?

(j) If you were visiting New York City, USA, and a weather forecaster predicted a maximum daily temperature of 50°F, describe what you might wear while sightseeing on this particular day.

10 Anika and Domenic leave the town of Graphville at the same time but in separate cars. They agree to meet at a camping site 60 km away. Anika travels to the camping site at 100 km/h whereas Domenic travels at 80 km/h along the same route.

(a) Rule a set of axes on graph paper. Label the horizontal axis 'Time (min)' and use a scale from 0 to 60. Label the vertical axis 'Distance (km)' and use a scale from 0 to 60.

(b) Draw a graph showing Anika's journey from Graphville to the camping site.

(c) On the same number plane, draw a graph showing Domenic's journey from Graphville to the camping site.

(d) How long does it take Anika to reach the camping site? Has Domenic arrived at the camping site yet?

(e) How long does Anika have to wait before Domenic arrives?

Open-ended

11 For the linear equation $y = ax + b$

(a) choose two different sets of values for a and b

(b) draw both graphs on the same Cartesian plane

(c) state the coordinates of the point of intersection of the two straight lines

(d) verify that this point lies on both lines by substituting the coordinates into both equations.

12 To convert centimetres to millimetres we use the equation $m = 10c$, where m is millimetres and c is centimetres.

(a) Write down two more linear equations that can be used to convert one unit of length to another unit of length.

(b) Place the larger of the two variables (the units of length) on the horizontal axis and draw each equation on Cartesian plane.

Outside the Square Puzzle

What number am I?

I am a three-digit number.

The sum of my digits is eighteen.

Five letters are used to spell my second digit.

The difference between my first two digits is one.

I have fewer even digits than odd digits.

My second digit is greater than my first digit.

I am a multiple of seventy-five.

If I add twenty-three, my third digit is eight.

Investigation

Captivating conversions

Equipment required: 1 brain, grid or graph paper, calculator

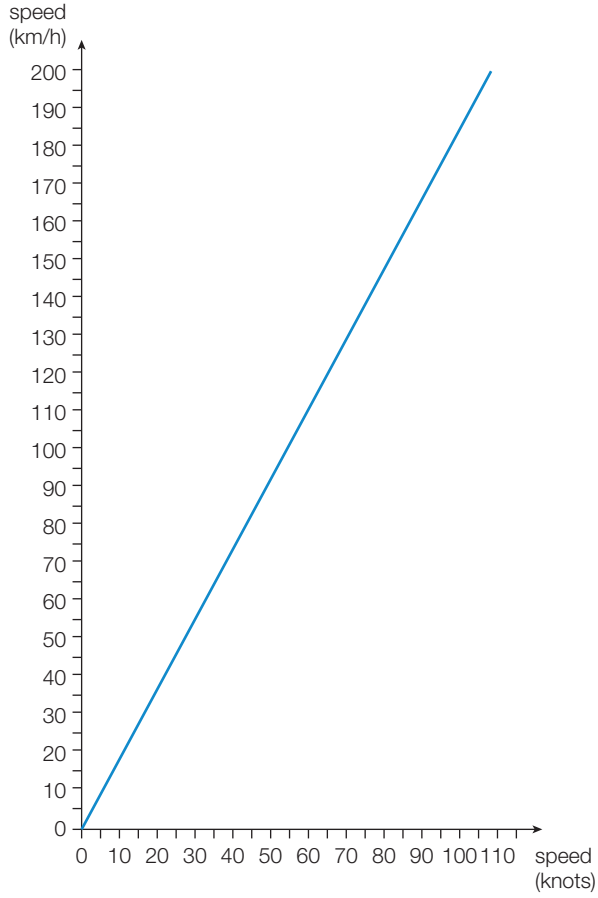
There are many units of measurement that are only used for specific purposes or situations. Converting them to more familiar metric measurement units can help us understand what they mean and get an idea of their size. Consider the following: 'The object was travelling at a rate of knots', 'the whale swam 1300 nautical miles', '5 inches of rain fell in 6 hours', '10 acres of land for sale', 'the plane flew at a height of 20 000 feet', 'crude oil costs $91 per barrel', 'a 3000-carat diamond', 'this school bag weighs a ton', 'these tyres need 24 psi of air pressure'.

The Big Question

Can conversion graphs help us understand the meaning of different units of measurement by finding their metric equivalents?

Engage

1 Sailor Sam loves sailing and has sailed around the world in his yacht. He is often asked how fast his yacht can travel. He has boasted that it has travelled at a speed of 40 knots. He recently took part in the Dynsey to Boarth yacht race which is 672 nautical miles long. He finished the race in 56 hours.



Conversion Graph 1 **Knots to km/h**

(a) Use Conversion Graph 1 to find how fast 40 knots is in km/h (to the nearest km/h).

(b) A speed of 1 knot is the same as a speed of 1 nautical mile per hour. Using speed $= \frac{\text{distance}}{\text{time}}$, calculate Sam's race speed in knots.

(c) Use Conversion Graph 1 to find Sam's race speed in km/h (to the nearest km/h).

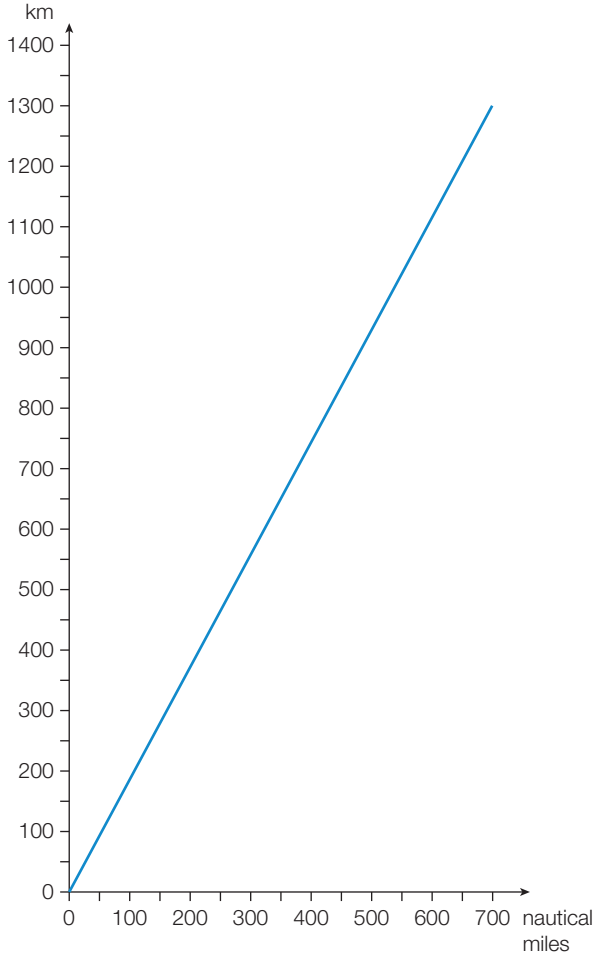
(d) Compare 40 knots with Sam's race speed, then comment on Sam's boast of travelling at 40 knots.

(e) Wind speeds of 4–27 knots are considered to be a breeze. Find this range of wind speeds in km/h (to the nearest km/h).

(f) If the wind speed was 50 knots, what is its speed in km/h?

(g) If the wind speed was 198 km/h, what is its speed in knots?

(h) Use Conversion Graph 2 to find the length of the race in kilometres (to the nearest km).



Conversion Graph 2 **Nautical miles to km**

Explore

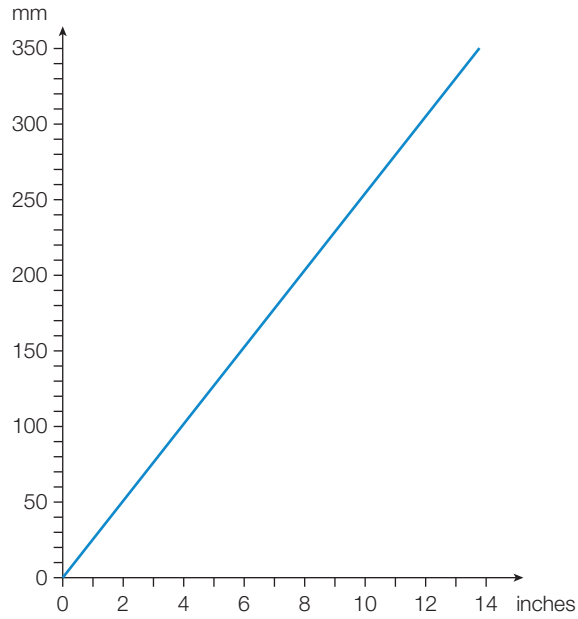
2 Farmer Joe has been growing crops for years on his farm of 100 acres. Recently it rained very heavily and Farmer Joe claimed he had measured 12 inches of rain in one 24-hour period.

(a) Use Conversion Graph 3 to find the number of mm of rain he had measured.

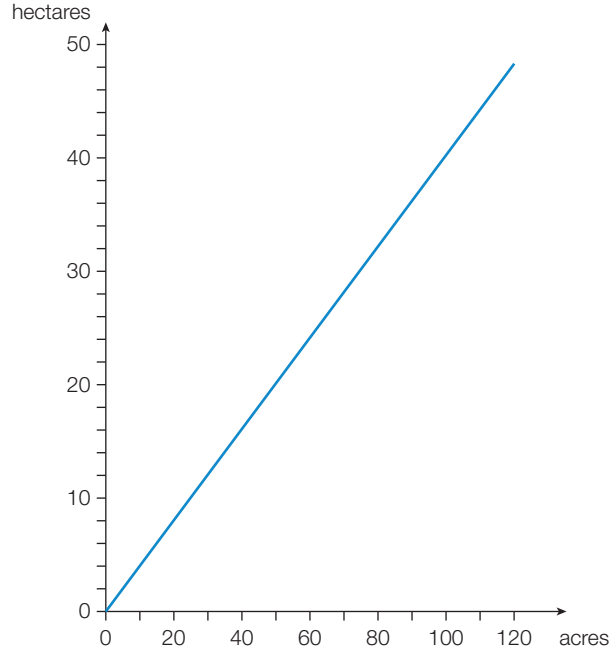
(b) If 50 mm of rain was recorded in a 24-hour period, how would Farmer Joe have recorded this in terms of inches (to the nearest inch)?

(c) Use Conversion Graph 4 to find how many hectares (correct to the nearest hectare) he farms.

(d) If 4 hectares was set aside for his home and garden, how many acres (correct to the nearest acre) would that be?

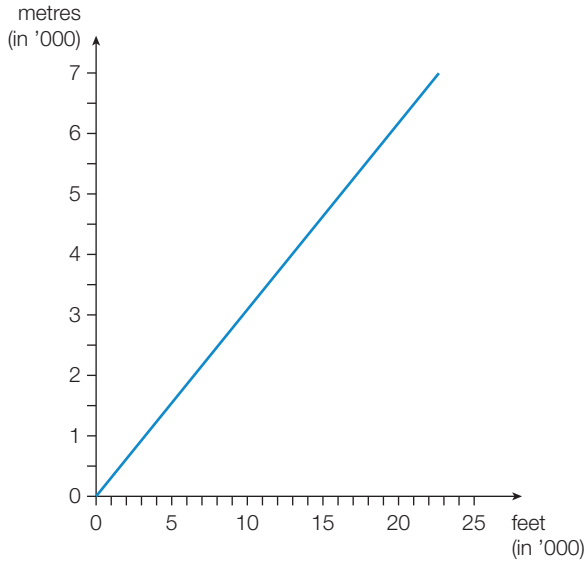


Conversion Graph 3 **Inches to mm**



Conversion Graph 4 **Acres to hectares**

3 The height or altitude above sea level is given to pilots in terms of feet. Captain Debbie is a pilot of a passenger aircraft with a cruising altitude between 18 000 and 24 000 feet.



Conversion Graph 5 **Feet to metres**

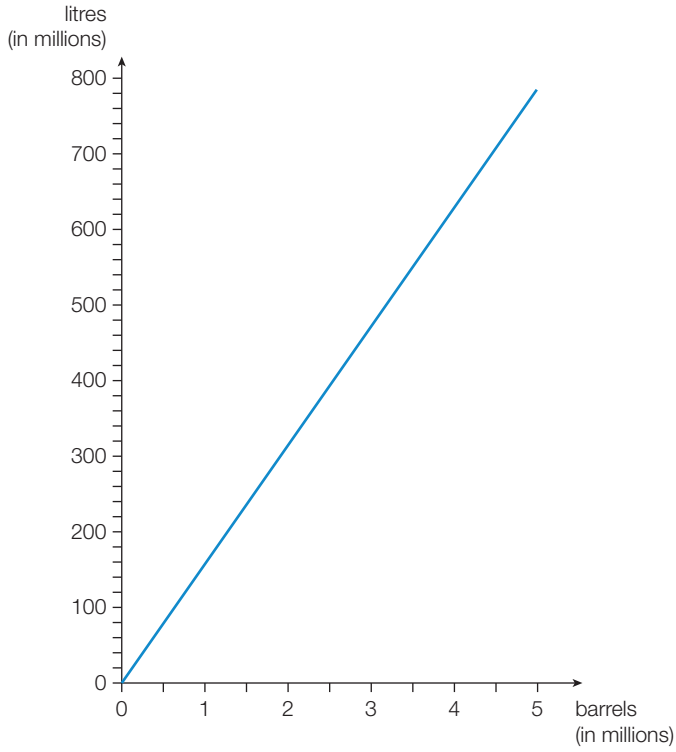
(a) Use Conversion Graph 5 to find the height above sea level, in metres, at which a plane would be flying for an altitude of:

(i) 18 000 feet **(ii)** 24 000 feet.

(b) Use Conversion Graph 5 to find the height above sea level, in feet, at which a plane would be flying for an altitude of:

(i) 6400 m **(ii)** 7500 m.

4 Oil companies measure oil by the barrel. A barrel of oil is equivalent to 159 litres. The largest accidental oil spill in history occurred on 20 April 2010. An explosion occurred aboard a drilling rig working on a well 1 mile (1.6 km) below the surface of the Gulf of Mexico. It took nearly 5 months to stop the spill, with an estimated five million barrels of oil being spilt into the surrounding ocean during that time.



Conversion Graph 6 **Barrels to litres**

(a) Use Conversion Graph 6 to find approximately how many litres of oil were spilt overall.

(b) If the spill lasted for 145 days, how many litres were spilt each day?

(c) An oil well typically produces 140 000 barrels per day. How many litres is this? Compare this amount with the amount lost per day during the spill.

5 The record for the largest single crystal emerald is 7025 carats. A carat is a measure of the mass of a gemstone.

(a) If 5 carats is equivalent to 1 g, draw a conversion graph using grid or graph paper with grams marked along the horizontal axis from 0 to 1800 and carats marked along the vertical axis from 0 to 9000. Choose an appropriate scale for each of your axes.

(b) Use your conversion graph to find the following, giving all units correct to the nearest multiple of 10 (e.g. 420 g, 1530 carats).

(i) the mass of the emerald described above

(ii) the mass of a 3106-carat uncut diamond called the 'Cullinan'

(iii) the number of carats in the 'Star of Africa', a diamond of mass 106 g cut from the 'Cullinan'

(iv) the mass of a 2302-carat sapphire found in Anakie, Queensland, in 1935

(v) the number of grams wasted after a 1318-carat head of Abraham Lincoln was carved from the sapphire in **(iv)**

(vi) the number of carats in a 304-g black opal called 'The Empress of Glengarry', cut from a stone found at Lightning Ridge, NSW, in 1972

(c) **(i)** Which has the greatest mass: a 6465-carat ruby called 'The Eminent Star' or the 'Liberty Bell', a ruby of mass 1700 g?

(ii) What is the difference in mass of the two rubies in part **(i)**?

6 Carats are used differently to describe the purity of gold by indicating the percentage of gold, not the mass.

(a) If 24 carats is 100% pure gold, draw a conversion graph to convert carats of gold into a percentage of purity.

(b) Pure gold (24 carats) is too soft to use in jewellery; however, a higher number of carats denotes a better quality of gold. 18-carat gold is considered the best quality gold. Use your conversion graph to find how pure this is.

(c) Which is the better quality gold, a 10-carat ring or a ring that is 45% gold?

Strategy options

- Make a table.
- Work backwards.
- Make a model.

Explain

7 **(a)** Explain why conversion graphs are a useful way of converting from one type of measurement to another.

(b) Explain the advantages and disadvantages of using a conversion graph rather than a conversion table.

(c) Explain why feet would still be used in the aviation industry.

Elaborate

8 **(a)** Answer the 'Big Question' by explaining how the conversion graphs helped you understand different types of measurement. Use your answers for Questions **1**–**7** to help you.

(b) How accurate do you think conversion graphs are?

(c) When is it reasonable to use estimated conversions? Give an example.

(d) When is it important for conversions to be accurate? Would you use a conversion graph for these purposes? If not, why not?

Evaluate

9 **(a)** Did you find Conversion Graphs 1–6 easy to use? What difficulties, if any, did you have?

(b) How accurate do you think your answers are? How could the accuracy be improved?

(c) Did you have difficulty drawing conversion graphs for Questions **5** and **6**? What difficulties did you encounter and what would you do differently if you had to draw them again?

Extend

10 **(a)** Investigate one other famous gemstone, such as, 'The Blue Hope' or 'The Excelsior', and find its mass in carats and grams.

(b) Investigate how gems are valued. Is their value determined just by their size? If not, what other factors are taken into account?

(c) Investigate another unit of measurement that is used today but is not metric, such as a ton, bushel (used for the weight of commodities, usually corn), psi of air pressure or troy ounces (used for the weight of gold and silver). Find the comparable metric unit and the conversion factor.

(d) Find an app or website that does conversions for you.

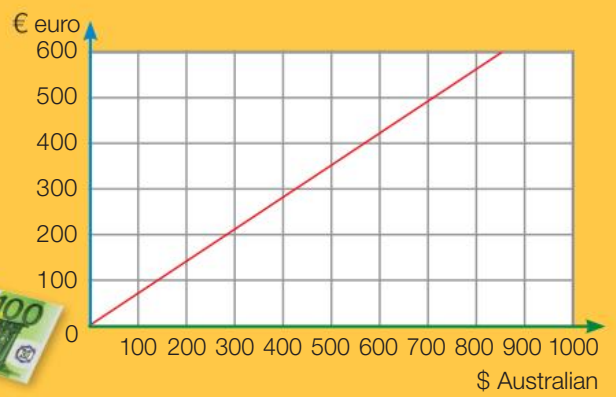
Maths 4 Real

Algebra – the

Travelling in Europe has been made easier with the introduction of a common currency, the euro. In most countries in Europe, you pay using the euro; however, in the United Kingdom you still pay using the British pound.

Before you travel you may wish to buy some of the currency that you need.

1 The graph shows the number of euros that you can buy for a given number of Australian dollars. The equation of this graph is $E = 0.70A$.



Use the graph to answer the following questions.

(a) How many euros would you get for:

(i) A$800 **(ii)** A$450 **(iii)** A$50?

(b) How many Australian dollars would you get for:

(i) €500 **(ii)** €300 **(iii)** €50?

When you travel you need to be able to quickly compare prices to decide if something is value for money. The exchange rate changes slightly every day; however, for quick mental calculations we can assume that:

€1 = A$1.50 €2 = A$3

So, our conversion rules are:

From A$ *to* €: $E = \frac{2}{3}A$ *From* € *to* A$: $A = \frac{3}{2}E$ (or $A = 1.5E$)

2 Use the quick conversion rules to find:

(a) the cost in A$ of a day tour advertised at €140

(b) the cost in A$ of a night's accommodation advertised as €110

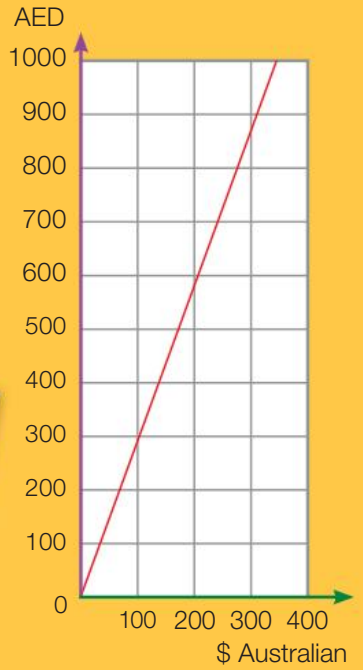
(c) the most you can spend on a camera priced in euros if you only have A$600 left

(d) the total cost in A$ of a rail pass costing €25, lunch costing €15, a theatre ticket costing €105 and dinner costing €35.

on move

When you fly from Sydney to Paris you decide to stop for 2 nights in Dubai. The currency used in Dubai is the United Arab Emirates dirham, or AED, often just called the dirham.

3 The graph shows the number of AED that you can buy for a given number of Australian dollars. The equation of this graph is $D = 2.95A$.



Use the graph to answer the following questions.

(a) How many AED would you get for:

(i) A$300 **(ii)** A$100 **(iii)** A$250?

(b) How many Australian dollars would you get for:

(i) AED800 **(ii)** AED350 **(iii)** AED100?

A quick and easy comparison to use would be that A$1 = AED3. The conversion rules are:

From A$ *to* AED: $D = 3A$ *From* AED *to* A$: $A = \frac{D}{3}$

4 Use the quick conversion method to find:

(a) the cost in A$ of a taxi fare of AED48

(b) the cost in A$ of a night's accommodation advertised as AED375

(c) the most you can spend on a camera if you only have A$200 left in your account.

After your holiday in Europe you have a 4 hour stop in Dubai on the way home to Australia. You find that you have a mixture of currencies left: some euros, some United Arab Emirates dirhams and your credit card. You wish to do some duty free shopping at the airport.

5 **(a)** You have €150 left and you wish to know how many dirhams this will give you. Use the first graph to change this to A$ and then use the second graph to change it to AED. How many dirhams do you get for €150?

(b) If you also had AED75 in your wallet, how many dirhams will you have to spend in total?

(c) The cost of your shopping comes to AED1000. How much, in Australian dollars, will you have to pay?

Research

- **Find the current exchange rates and see what difference it would make to your answers. Would you need to change your quick calculation conversion rates?**
- **Find a website or app that performs currency conversions for you. What other types of conversions can it do?**

Challenge 6

1 (a) Tom spent part of his morning visiting and playing with friends. The graph shows his travels. He went to his friends' houses and, if they were home, stopped to play. The number of houses at which he stopped to play is:

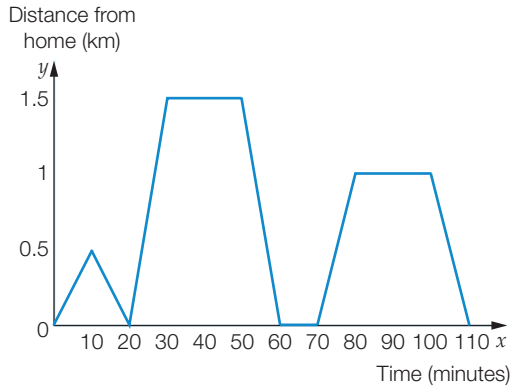
A 1 B 2

C 3 D 4

(b) How long did he spend at home between visits?

(c) How far did he travel altogether?

(d) Between what times was he travelling the fastest?

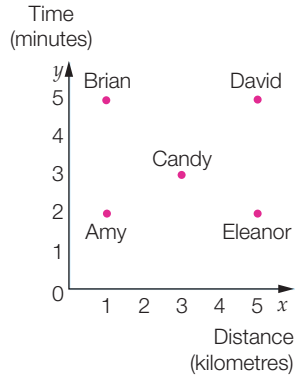


2 The graph shows the time taken by five people to travel various distances from the starting point (the origin).

(a) On average, which person travelled the fastest?

(b) On average, which person travelled the slowest?

(c) Farouk covers 4 kilometres in 4 minutes. Who else has the same average speed?



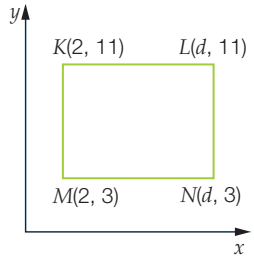
3 The coordinates of the vertices of the rectangle *KLNM* are given on the diagram. The area of the rectangle *KLNM* is 96 square units. The value of *d* is:

A 11

B 12

C 13

D 14



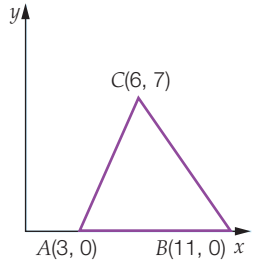
4 $A(3, 0)$, $B(11, 0)$ and $C(6, 7)$ are the vertices of a triangle as shown in the diagram. The area of ΔABC is:

A 56 unit^2

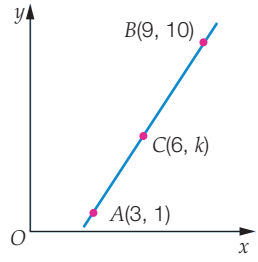
B 48 unit^2

C 28 unit^2

D 24 unit^2



5 The point *C* lies on the line *AB* as shown. If *A* and *B* are the points (3, 1) and (9, 10), respectively, and *C* is the point $(6, k)$, find the value of *k*.



Chapter review 6

D.I.Y. Summary

Key Words

Cartesian plane	linear graph	line graph	origin	*x*-intercept
coordinate	linear relationship	ordered pair	slope	*y*-intercept
gradient				

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 The point (0, 0) is also known as the _____ .

2 The _____ or _____ of a line is positive when it goes up from left to right.

3 A number plane that is divided into four quadrants is also known as a _____.

4 The location of a point on a Cartesian plane is given by an _____.

5 A _____ is a relationship that makes a straight line.

6 A _____ is a straight-line graph.

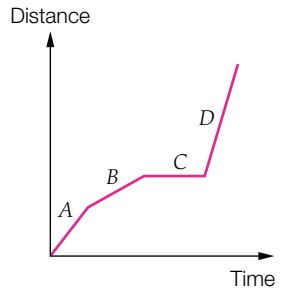
7 The point where a line cuts the x-axis is called the _____.

8 The point where a line cuts the y-axis is called the _____.

Equipment required: Grid or graph paper

Fluency

1 Liam is rollerblading along a path.



Ex. 6.1

(a) In which section of the graph (**A**, **B**, **C** or **D**) is Liam moving the fastest?

(b) In which section is he stationary?

(c) In which section is he moving the slowest, but is not stationary?

2 For each of the following:

Ex. 6.2

(i) $y = 2x - 2$ **(ii)** $y = -x + 4$ **(iii)** $y = -3x - 1$

(a) Copy and complete the table of values.

x	-2	-1	0	1	2
y					
(x, y)					

(b) Plot the points on a Cartesian plane. Join each set of points with a straight line.

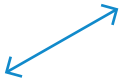
(c) Write down the coordinates of the x- and y-intercepts of each graph.

3 The line with the largest positive gradient is:

Ex.6.2

A

B

C 

D

4 Match the following equations with the descriptions given below.

Ex.6.2

An equation that:

(a) goes through the origin

(b) has a y-intercept of 4

(c) has a negative gradient

(d) has a zero gradient.

A $y = 8 - 5x$ **B** $y = -x$ **C** $y = 2(x + 2)$ **D** $y = -4$

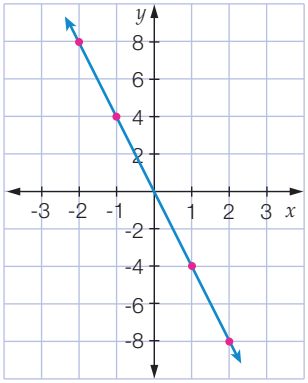
5 **(a)** Find the rule for these points:

(0, 3), (1, 4), (2, 5), (3, 6), (4, 7).

(b) Find the equation that describes the relationship shown by the graph.

(c) Find the rule for the linear relationship shown in the following table. Hence, copy and complete the table of values for x and y.

Ex.6.3



x	-2	-1	0	1	2
y	-4	-3	-2		

6 Write a linear equation to represent the following situation, defining all the variables you are using.

Ex.6.4

The drains at Bryce's house are blocked. A plumber charges a fixed fee for a call-out. He then charges an hourly rate to unblock the drains.

Understanding

7 **(a)** Sketch graphs of each of the following linear relationships.

(i) $y = x + 1$ **(ii)** $y = 5x$ **(iii)** $y = -3x + 6$

(b) State the x-intercept for each line.

(c) State the y-intercept for each line.

(d) For each, state whether the gradient is positive or negative.

8 **(a)** Without plotting, sketch the graphs of each of the following linear relationships.

(i) $y = 5$ **(ii)** $x = 2$ **(iii)** $y = -4$ **(iv)** $x = -1$

(b) State the x- and y-intercept for each graph.

9 The table below shows the cost of buying different lengths of timber.

Length of timber (m)	2	3	5	8	10
Cost ($)	10	13	19	28	34

The timber can be cut to whatever length the customer requires.

(a) Plot the points shown in the table and join them to form a graph of this relationship. Show length of timber on the horizontal axis.

(b) Is the relationship linear? Explain your answer.

(c) How much would it cost to buy a piece of timber of length:

(i) 4 m **(ii)** 9 m **(iii)** 11 m?

(d) What length of timber could be bought for:

(i) \$15 **(ii)** \$20 **(iii)** \$32?

10 A pool is being filled with water. The volume of water, v, in litres at any time, t, in hours is given by the equation $v = 600t$.

(a) Use the rule to complete a table of values of t from 0 to 40 (use increments of 10).

(b) Use the table to draw a graph for this rule.

(c) If the pool is full after 40 hours, use your graph to find how many litres it can hold.

(d) Use the graph to find:

(i) the volume after 18 hours

(ii) the time it would take for the volume to reach 18 000 L.

(e) The pool begins to leak after it is filled. The volume of water in the pool is given by the equation $v = 24\,000 - 30t$. Complete a table of volumes from 0 to 800 (use increments of 100).

(f) Use this table to draw a graph for this rule.

(g) Use the graph to find:

(i) the volume of water left in the pool after $12\frac{1}{2}$ days

(ii) the time it would take for the pool to lose $\frac{5}{8}$ of its volume. Give your answer in terms of days.

(h) What volume of water is lost every hour?

11 Here is a table of values:

x	-4	-2	0	2	4
y	29	19	9	-1	-11

(a) Find the rule connecting x and y. Explain how you found it.

(b) Plot the graph.

(c) Use the graph to find the value of y when $x = 3$.

(d) Use the graph to find the value of x when $y = 14$.

Reasoning

12 One of the metric units used for land area is the hectare. However, the unit 'acre' is still commonly used. A conversion graph can be useful in converting land area from one unit to the other.

(a) Rule a set of axes on graph paper. Label the horizontal axis 'Area in hectares' and use a scale from 0 to 12. (Use 1 cm on the graph to represent 1 hectare.) Label the vertical axis 'Area in acres' and use a scale from 0 to 30. (Use 1 cm on the graph to represent 2 acres.)

(b) What area in acres is equivalent to zero hectares? Write this information as a coordinate point for your graph.

(c) 10 hectares is equivalent to about 24.7 acres. Write this information as a coordinate point for your graph.

(d) Plot the two points you found in parts **(b)** and **(c)** onto your number plane and carefully rule a straight line through them.

Use your conversion graph to answer the following questions.

(e) The playing field of the MCG has an area of about 2 hectares. How many acres is this?

(f) A property is advertised as having an area of 20 acres. Express this area in hectares.

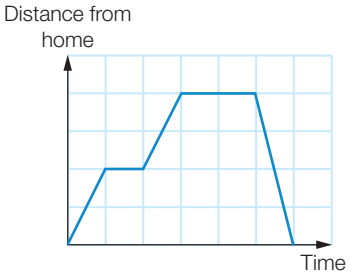
(g) Which is larger in area, 15 acres or 7 hectares?

NAPLAN practice 6

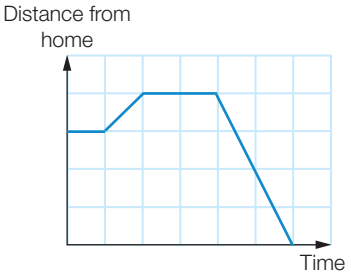
Numeracy: Non-calculator

1 Ivanka leaves her house to walk to the park. She stays there for a time and then goes to the shopping centre before returning home. The diagram that represents this information is:

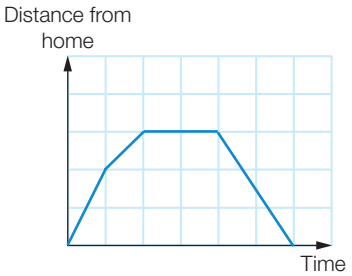
A



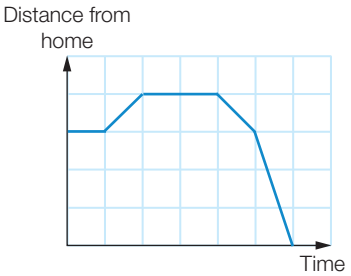
B



C

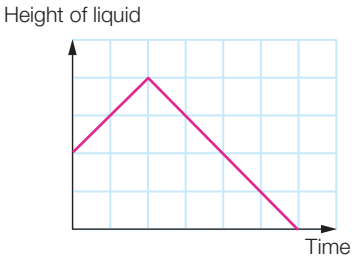


D

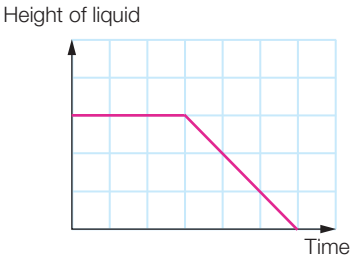


2 Jeremy noticed that water was leaking from his water bottle and that its volume was decreasing at a constant rate. He placed his finger to slow the leak, but the water continued to leak at a constant rate until the bottle was empty. The graph that describes this information is:

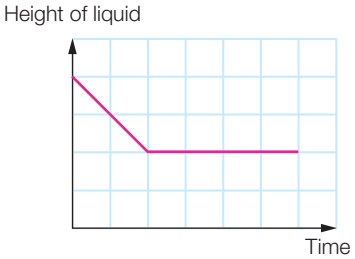
A



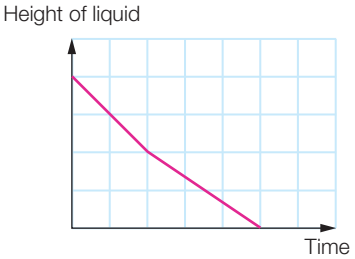
B



C



D



3 How many of the ordered pairs (7, 0), (2, -1), (-3, 5), (4, 2), (-1, 2), (0, 4), (-3, -3) lie in quadrant 2?

A 0 **B** 1 **C** 2 **D** 3

4 Which set of coordinates lies on a horizontal straight line?

A (7, -2), (-2, 7), (2,-7)
B (-1, 4), (5, 4), (-6, 4)
C (3, 1), (3, 4), (3, -6)
D (1, 2), (2, 1), (1, 1)

Numeracy: Calculator allowed

5 A rule for y in terms of x is $y = 3 - 2x$.

When $x = 4\frac{1}{2}$ the value of y is:

A 12 **B** 6 **C** -6 **D** $-21\frac{1}{2}$

6 Taxi fares are calculated using a flagfall and a cost/kilometre. The following table shows the cost of the taxi fare for several different distances.

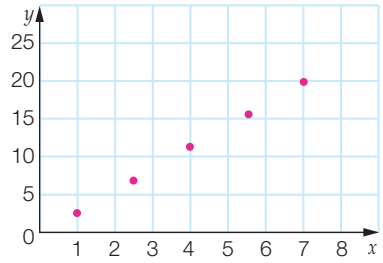
Distance	2	4	5	6
Taxi fare	\$6.40	\$9.60	\$11.20	\$12.80

Which of the following shows the flagfall and the cost/kilometre?

A \$1.60 flagfall + \$3.20/km
B \$1.60 flagfall + \$1.60/km
C \$3.20 flagfall + \$3.20/km
D \$3.20 flagfall + \$1.60/km

7 The points shown in the graph lie on a straight line. The y-coordinate when the x-coordinate is 6.2 is closest to:

A 5
B 7
C 18
D 20



Mixed review C

Fluency

1 Simplify:

(a) $2^4 \times 5^3 \times 2^5 \times 5^2$ **(b)** $\dfrac{6^3 \times 7^4}{6^2 \times 7}$ Ex. 1.5

2 Find the value of the pronumeral in each of the following. Ex. 4.4

(a) $d:10 = 3:8$ **(b)** $15:m = 12:5$ **(c)** $p:42 = 13:15$

3 Factorise: Ex. 3.7

(a) $6ab + 9bc$ **(b)** $24x^2y^2 - 3xy$

4 Write the following recurring decimals as fractions. Ex. 2.2

(a) $0.\dot{4}$ **(b)** $0.\dot{1}\dot{9}$

5 Expand and simplify: Ex. 3.6

(a) $x(3y - 2x) + y(4 - 2x)$ **(b)** $3m(2m - n) - n(n - 3m)$

6 Simplify: Ex. 1.6

(a) $(2^3 \times 3^4 \times 4)^2$ **(b)** $\left(\dfrac{6^2 \times 7^2}{6}\right)^3$

7 Evaluate the following expressions when $x = 3$ and $y = -4$. Ex. 3.2

(a) $5xy - 3x^2$ **(b)** $\dfrac{3y^3}{8x}$

8 Share each amount given according to the ratio stated in brackets. Ex. 4.6

(a) 60 (8 : 7) **(b)** 85 (7 : 10) **(c)** 21 000 (1 : 2 : 4)

9 Find the area of the following circles correct to one decimal place. Ex. 5.5

(a) radius of 1.7 cm **(b)** diameter of 2.6 cm

10 Write the following fractions as **(i)** decimals and **(ii)** percentages. Ex. 2.5

(a) $\dfrac{3}{8}$ **(b)** $\dfrac{13}{40}$ **(c)** $\dfrac{102}{125}$

11 Find the area of the following trapeziums correct to one decimal place. Ex. 5.4

(a) parallel sides 2 cm and 4 cm and perpendicular height 1.6 cm

(b) parallel sides 1.6 cm and 3.1 cm and perpendicular height 1.2 cm

12 Express the first amount as a percentage of the second amount. Ex. 2.7

(a) 15, 60 **(b)** 41, 200 **(c)** 1, 5 **(d)** 18, 75

13 **(a)** Construct a table of values for $y = 4x + 2$ from -2 to 2. Ex. 6.2

(b) Use the table of values to plot the graph of $y = 4x + 2$.

(c) Use the graph to find the x- and y-intercepts of $y = 4x + 2$.

14 Calculate the following, writing your answers in simplest form. Ex. 2.3

(a) $-\dfrac{2}{5} + \dfrac{3}{4}$ **(b)** $-\dfrac{3}{7} \times -\dfrac{17}{18}$ **(c)** $-2.3 + (-4.9)$ **(d)** $-5.2 \div 1.6$

Understanding

15 Find the equation for each of the following linear relationships.

(a) (-4, -2), (-2, -1), (0, 0), (6, 3)

(b) (1, 4), (2, 7), (3, 10), (4, 13)

16 Find the gradient, x-intercept and y-intercept for the following equations.

(a) $y = 3(2x - 5)$

(b) $y = \dfrac{3x + 2}{4}$

17 A store purchases a bulk order of 200 DVDs for $1575.

(a) Calculate the profit made if all of the DVDs are sold for $11.95 each.

(b) Calculate the profit as a percentage of the cost price.

18 The Empire State Building in New York is 443 m tall. The Eiffel Tower in Paris is 300 m tall.

(a) Express the ratio of the height of the Empire State Building to the height of the Eiffel Tower as a unit ratio. (Round decimals to two decimal places.)

(b) Rafal wants to make a scale model of the two buildings. If he has made the model of the Empire State Building 50 cm tall, how tall should he make the Eiffel Tower? (Answer to the nearest cm.)

Reasoning

19 A train is travelling at a speed of 120 km/h.

(a) Write an equation showing the relationship between distance travelled, d (km), and time, t (h).

(b) Draw a graph showing up to 2 hours of travel.

(c) Use your graph to estimate the distance travelled in:

(i) 50 minutes

(ii) 1.5 hours.

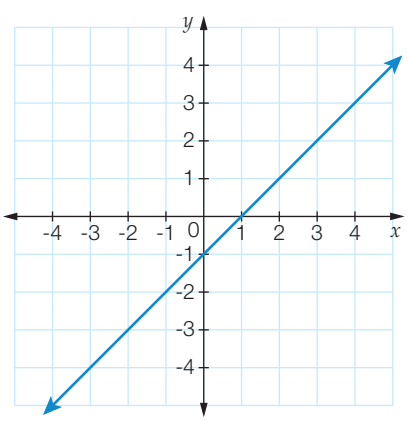
(d) Use your graph to estimate the time it takes the train to travel:

(i) 90 km

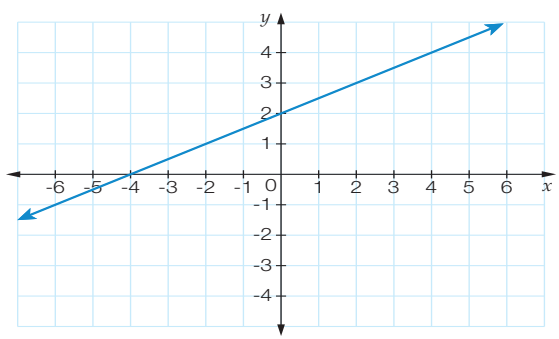
(ii) 160 km.

20 Find the equation for each of the following graphs.

(a)



(b)



21 Show that the points (0, 4), (1, 2) and (2, 0) lie on the line with equation $y = -2x + 4$.

7

Linear equations

7

The ups and downs of equations. Equations can find the speed of a roller coaster.

Imagine you are a structural engineer whose job it is to design a roller coaster that will be the most exciting ride in the show. You use algebra to determine the speed of the roller coaster during its fall. The roller coaster you have designed climbs to a great height, stops briefly, then drops rapidly. As the roller coaster falls, it will accelerate under gravity gaining speed. If we ignore the effects of friction, the speed of the roller coaster depends on the time it takes to reach the ground. Its speed can be found using the equation:

$v = gt$

where v = speed in metres per second,
g = acceleration due to gravity and
t = the time taken for the roller coaster to fall.

In 3 seconds, a roller coaster would have a speed of 29.7 m/s, which is over 100 km/h, and it would have fallen over 44 m. What a thrill!

Forum

What other calculations might the roller-coaster engineers need to do?

What could the potential consequences be if roller-coaster engineers made errors in their calculations?

If you halved the height of the roller-coaster, will the speed at the bottom also be halved?

Why learn this?

Equations are a shorthand way of describing something mathematically. The relationship between variables such as velocity and height from the ground can be expressed in an equation that describes the situation. We can then predict the effects of changing the different variables. A builder might use equations to find how much weight can be hung from a ceiling. A meteorologist might use equations to predict the likelihood of rain tomorrow.

After completing this chapter you will be able to:

- write an equation in algebra
- understand equivalence
- solve equations using a variety of techniques
- understand the relationship between graphs and equations
- solve equations with the variable on both sides of the equation
- use substitution to check solutions
- use equations to solve problems.

Recall 7

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Worksheet R7.1

1 Which of the following are equations?

(a) $3 = 7 - 4$ **(b)** $a + 5$ **(c)** $3m + 2 = m - 4$

Worksheet R7.2

2 Solve each of the following equations. (Find the value of the pronumeral that makes the statement true.)

(a) $a + 7 = 12$ **(b)** $\frac{b}{2} = 10$ **(c)** $6c = -18$

(d) $d - 4 = 13$ **(e)** $e - 3 = -8$ **(f)** $3f = 24$

Worksheet R7.3

3 Copy and complete these flowcharts:

(a) $\boxed{5} \xrightarrow{\times 3} \square \xrightarrow{-2} \square$

(b) $\square \xleftarrow{-2} \boxed{10} \xleftarrow{\times 5} \square$

(c) $\square \xrightarrow{+1} \square \xrightarrow{\times 4} \boxed{32}$

(d) $\square \xleftarrow{\div 7} \square \xleftarrow{+5} \boxed{9}$

Worksheet R7.4

4 Find the value of each of the following expressions by substituting the value given in the brackets.

(a) $\frac{x}{2} + 9 \quad (x = 10)$ **(b)** $4(x + 5) \quad (x = 3)$ **(c)** $\frac{5x + 1}{7} \quad (x = 4)$

Worksheet R7.5

5 What is the inverse (or opposite) operation to each of these?

(a) multiply by 2 **(b)** add -4 **(c)** divide by 5

Worksheet R7.6

6 Expand each of the following to remove the brackets.

(a) $2(x + 5)$ **(b)** $3(2x - 7)$

Key Words

backtracking	flowchart	inverse operation	solving
balanced	guess, check and improve	point of intersection	
equivalent	inspection	solution	

The language of equations

7.1

An equation is a mathematical sentence that contains two expressions connected by an equals sign. The equals sign tells us that the two expressions have the same value. Equations are used when some information is unknown. Unknown amounts are called variables and these are represented by letters or symbols.

For example, $x + 2 = 7$ is an equation where the pronumeral x represents a variable such as length.

Finding the value that makes the equation a true number sentence is called **solving** the equation. We can solve the equation $x + 2 = 7$ to obtain the **solution** $x = 5$.

When mathematical operations are written in words, we need to write the equations using mathematical symbols before we can begin the solution process.

Worked Example 1 WE1

Write an equation for each of the following. Use the letter in brackets to represent the variable.

(a) Seven is added to a number to give a result of ten. (a)

(b) A number is multiplied by five, then four is subtracted to give a result of sixteen. (b)

Thinking	Working
(a) Write an expression containing the variable showing the operations that have been performed on it and equate this expression to the unknown amount.	(a) $a + 7 = 10$
(b) **1** Write an expression containing the variable showing the operations that have been performed on it and equate this expression to the unknown amount.	(b) $b \times 5 - 4 = 16$
2 Simplify.	$5b - 4 = 16$

To check that the value obtained for a variable is the solution, we substitute the value into each side of the equation. If the left-hand side (LHS) is equal to the right-hand side (RHS), we know that we have correctly solved the equation. This is called checking by substitution. We know that $x = 5$ is the solution for $x + 2 = 7$ because $5 + 2 = 7$ is a true number sentence.

Worked Example 2 WE2

Check by substitution whether the value given in the brackets is the solution for the equation $\frac{x+4}{6} = 2$ $(x = 14)$. (Does it make the equation true?) Answer yes or no.

Thinking	Working
1 Substitute the x-value into the left-hand side of the equation. $(x = 14)$	$\text{LHS} = \frac{x+4}{6}$ $= \frac{14+4}{6}$ when $x = 14$
2 Simplify.	$= \frac{18}{6}$ $= 3$ $\neq \text{RHS}$
3 Check whether the left-hand side of the equation equals the right-hand side.	No The value given is not the solution.

7.1 The language of equations

Navigator

Answers page 630

Q1 (a)–(d), Q2 Column 1, Q3 (a)–(d), Q4 Column 1, Q5, Q7, Q8	Q1 (c)–(f), Q2 Column 1, Q3 (b)–(e), Q4 Column 2, Q5, Q6, Q7, Q8	Q1 (e)–(h), Q2 Column 2, Q3 (c)–(f), Q4 Column 3, Q5, Q6, Q7, Q8

Fluency

WE1

1 Write an equation for each of the following. Use the letter in brackets to represent the variable.

(a) Eight is added to a number to give a result of twelve. (a)

(b) Four is subtracted from a number to give a result of sixteen. (b)

(c) Nine times a number gives a result of sixty-three. (c)

(d) The sum of eleven and a number is zero. (f)

(e) Seven is added to three times a number to give a result of ten. (u)

(f) A number is multiplied by two, then seven is added to give a result of thirteen. (v)

(g) Nine is added to a number, then the result is divided by seven to give six. (x)

(h) The sum of six and a number is multiplied by eight to give a result of zero. (z)

WE2

2 Check by substitution whether the value given in the brackets is the solution for each of the following equations. (Does it make the equation true?) Answer yes or no.

(a) $m + 2 = 9$ $(m = 7)$

(b) $l - 2 = -9$ $(l = -11)$

(c) $n - 6 = 3$ $(n = 9)$

(d) $10 - p = 4$ $(p = 14)$

(e) $5q = 55$ $(q = 11)$ **(f)** $\frac{r}{8} = 9$ $(r = 72)$

(g) $6s - 7 = 23$ $(s = 4)$ **(h)** $3a + 5 = -10$ $(a = -5)$

(i) $3(u - 8) = 6$ $(u = 6)$ **(j)** $4(5 - v) = 12$ $(v = 8)$

(k) $\frac{w + 3}{4} = 8$ $(w = 29)$ **(l)** $\frac{6 - x}{7} = 4$ $(x = 34)$

3 **(a)** Which equation describes this sentence?
Five is added to a number to give a result of twelve.

A $5 + 12 = n$ **B** $n + 12 = 5$ **C** $n + 5 = 12$ **D** $5n = 12$

(b) Which equation describes this sentence?
A number is subtracted from nineteen to give a result of eight.

A $b - 19 = 8$ **B** $-b - 19 = 8$ **C** $b - 8 = 19$ **D** $19 - b = 8$

(c) Which equation describes this sentence?
Six is subtracted from a number to give a result of seven.

A $b - 6 = 7$ **B** $6 - b = 7$ **C** $b - 7 = 6$ **D** $7 - b = 6$

(d) Which equation describes this sentence?
Five is added to three times a number to give a result of twenty.

A $3p + 20 = 5$ **B** $5 + 20 = 3p$ **C** $3(p + 5) = 20$ **D** $3p + 5 = 20$

(e) Which equation describes this sentence?
A number is subtracted from six; this result is multiplied by four to give eight.

A $4(w - 6) = 8$ **B** $4(6 - w) = 8$ **C** $4w - 6 = 8$ **D** $8 - 4w = 6$

(f) Which equation describes this sentence?
Three is subtracted from five times a number to give a result of seven.

A $(5 - 3)k = 7$ **B** $5(k - 3) = 7$ **C** $3 - 5k = 7$ **D** $5k - 3 = 7$

4 Write each of these equations in words.

(a) $m + 2 = 9$ **(b)** $l - 2 = 9$ **(c)** $n - 6 = 3$

(d) $10 - p = 4$ **(e)** $5q = 55$ **(f)** $\frac{r}{8} = 9$

(g) $6s - 7 = 23$ **(h)** $3a - 5 = -10$ **(i)** $3(u - 8) = 6$

(j) $4(5 - v) = 12$ **(k)** $\frac{w + 3}{4} = 8$ **(l)** $\frac{6 - x}{7} = 4$

Understanding

5 Write an equation for each of these rules, using the given variables for each of the quantities described.

(a) The area (A) of a rectangle is equal to its length (l) multiplied by its width (w).

(b) The area (A) of a triangle is equal to half its base (b) multiplied by its perpendicular height (h).

(c) The average speed (s) of a car is equal to the distance it travels (d) divided by the time taken (t).

(d) The area of a trapezium (A) is equal to half the sum of the lengths of the parallel sides (a and b) multiplied by the distance between them (h).

(e) The cost in dollars (C) of a cruise is 200 times the number of nights (n) plus 300.

(f) The tax-free price (F) of an item is $\frac{10}{11}$ of its retail price (R).

Reasoning

6 A school wants to take a class of Year 8 students on an excursion to the zoo. The bus will cost \$150 and the zoo entrance is \$$x$ per person.

(a) Write an equation to show how much it will cost the school to take a class of 25 students and 2 staff members.

(b) If the zoo entrance fee is \$5 per person, how much will the excursion cost?

(c) Write an equation to show the total amount the students will pay for the excursion.

(d) If the students are charged \$10 each for the excursion and the staff do not pay, show that this will not cover the cost of the excursion.

(e) As the bus can carry up to 57 passengers, it was decided that another class of 25 students and another 2 staff should go on the same excursion. If the students are now to be charged \$8.50 each, use equations to show that this will cover the cost of the excursion.

(f) How much money will be left over?

7 Sarah wants to save enough money to buy a ticket to a concert. Tickets go on sale in 5 weeks and will cost \$95.00. She has saved \$20 already.

(a) Write an equation to show how much money she will have in 5 weeks if she gets \$$x$ pocket money each week for doing all her chores for the next 5 weeks.

(b) If she gets \$15 per week pocket money, use your equation to show that she will have enough money to buy the ticket.

(c) Sarah's friend, Rhiannon, wants to go to the same concert. She gets \$$y$ pocket money each week but owes her brother \$15. Write an equation to show how much money she will have in 5 weeks time if she repays her brother.

(d) If Rhiannon gets \$20 per week pocket money, use your equation to find whether she will have enough money to buy a ticket.

Open-ended

8 Write at least three equations that have $w = 3$ as a solution, using different operations +, –, × and ÷ in each equation.

Outside the Square Puzzle

Lots of letters

If:

$A + C = A$	$FD = F$
$B - G = G$	$A + H = E$
$B \div H = G$	$E - G = F$

and A–H represent the numbers from 0 to 7, find the values of A, B, C, D, E, F, G and H.

Solving linear equations

7.2

Solving an equation means finding a value for a variable that makes the equation true. There are many methods that can be used to solve a linear equation but they can be grouped into three categories: *numerical*, *algebraic* and *graphical*.

Solving equations numerically

We can solve equations numerically using methods such as inspection or guess, check and improve. **Inspection** is the method you used in primary school to solve simple equations: ■ + 4 = 7; ■ = 3. We simply look at the equation to find the solution. This method only works for very simple equations. We can use **guess, check and improve** for more complicated equations such as $\frac{5x-9}{2} = 8$. We guess a value for x and substitute this value into the equation.

If our guess is correct, the RHS will equal the LHS. If our guess is not correct, we continue to guess until we find the solution. This can be quite a lengthy process and it may be difficult to find the exact solution, which may involve fractions and negatives. Calculators that solve equations numerically, such as graphics calculators, use this method.

Worked Example 3 WE3

Solve the following equations numerically to find x.

(a) $5x = 35$

(b) $\frac{5x-9}{2} = 8$

Thinking	Working
(a) 1 Write the equation and identify whether this equation can be solved by inspection.	(a) $5x = 35$
2 If it can, write the answer.	$x = 7$
(b) 1 Write the equation and identify whether this equation can be solved by inspection.	(b) $\frac{5x-9}{2} = 8$
2 If it can't, use guess, check and improve. Make a guess.	Try $x = 1$ $\text{LHS} = \frac{5 \times 1 - 9}{2}$ $= \frac{5-9}{2}$ $= \frac{-4}{2}$ $= -2$ $\neq \text{RHS}$

3	If this is not the solution, guess again.	Try $x = 2$ $\text{LHS} = \frac{5 \times 2 - 9}{2}$ $= \frac{10 - 9}{2}$ $= \frac{1}{2}$ $\neq \text{RHS}$
4	Continue guessing until a solution is found. ($x = 2$ made the LHS closer to the RHS than $x = 1$, so we increase the value for x)	Try $x = 5$ $\text{LHS} = \frac{5 \times 5 - 9}{2}$ $= \frac{25 - 9}{2}$ $= \frac{16}{2}$ $= 8$ $= \text{RHS}$
5	Write the solution.	Because LHS = RHS, $x = 5$ is the solution.

Solving equations algebraically

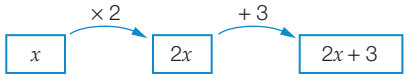
To solve equations algebraically we use **equivalent** equations.

Equivalent equations occur when a new equation can be written that leaves the value of the variable unchanged. The left-hand side of the equation remains the same as the right-hand side. To understand the process of equivalence you have previously learnt two strategies as follows.

Backtracking

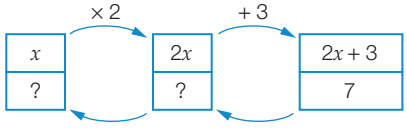
In Year 7, you learnt to solve equations by **backtracking** using a **flowchart**. Flowcharts can be used to show the sequence of steps required to build an expression and then to undo it by working backwards or backtracking. For example, to solve $2x + 3 = 7$, we build the expression $2x + 3$ using a flowchart and the necessary operations.

Remember that we undo addition and subtraction before multiplication and division.

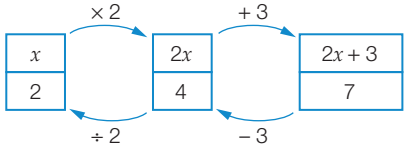


If you first added 3, then multiplied by 2 you would get a different equation, $2(x + 3) = 7$, with a different solution.

As $2x + 3$ equals 7, we can move backwards along the flowchart to find the x-value that makes the equation true. This process is called backtracking.



Remember that to 'undo' an operation we must use the **inverse** (or opposite) **operation**.



$x = 2$ is the solution.

When backtracking, we can 'undo' an operation by using the inverse operation.

+ and – are inverse operations. × and ÷ are inverse operations.

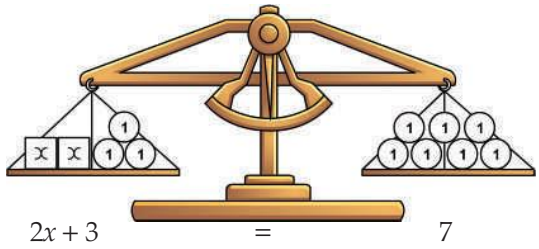
We undo + and – before × and ÷.

Balancing scales

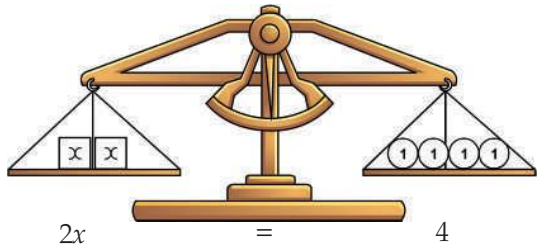
Another strategy you used to solve equations was to imagine an equation as a set of old-fashioned scales. Like the scales, an equation is **balanced** if its left- and right-hand sides are equal to each other or equivalent.
If you perform an operation (add, subtract, multiply or divide by a value) on only one side of an equation, it becomes unbalanced, as the two sides are no longer equivalent.

To balance the scales and the equations, you must always remember:
Whatever is done to one side of the equation must be done to the other side.

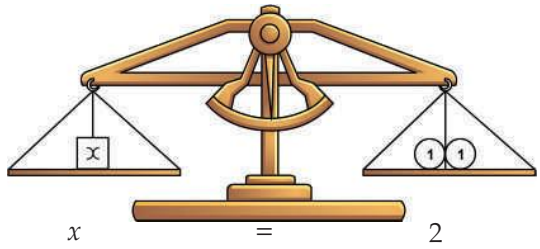
These scales show the equation $2x + 3 = 7$.



These scales show the result of removing 3 from both sides to give $2x = 4$.



These scales show the result of removing half the mass from both sides of the scales to give $x = 2$.



Solving equations using equivalence

Backtracking and balancing the scales are really the same process. We will compare the processes to solve the equation $2x + 3 = 7$. Both use inverse operations to work backwards to find the value of the unknown. Both methods use the steps subtract 3 and then divide by 2.

Drawing flowcharts and scales are time-consuming methods for solving equations and do not work for more complex equations. We can solve these equations more quickly by using the idea of equivalence and the use of inverse operations on both sides of the equals sign in reverse order to find the value of the variable. The following is the algebraic process used to solve the above equation.

$$2x + 3 = 7$$

(subtract 3 from both sides) $$2x + 3 - 3 = 7 - 3$$

$$2x = 4$$

(divide both sides by 2) $$\frac{2x}{2} = \frac{4}{2}$$

$$x = 2$$

Always check the signs in front of the constants and the coefficients. If they are negative, we need to undo the negative as well as the numerical value.

As $5 - 2x = -2x + 5$, we undo by subtracting 5 and dividing by -2.

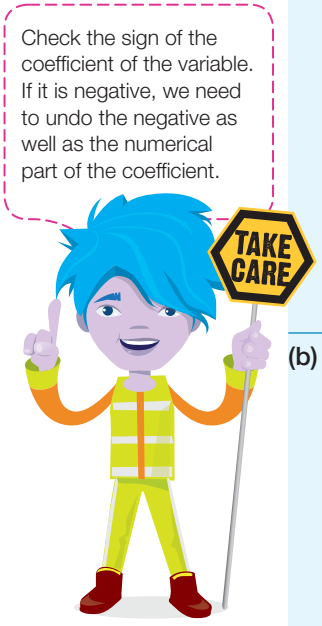
Worked Example 4

WE4

Solve each of the following equations using algebra.

(a) $4x - 5 = 7$ **(b)** $5 - 2x = 7$

Thinking	Working
(a) 1 Write the equation and identify the last operation to be performed on the given variable (-5). This is the first operation to be undone.	(a) $4x - 5 = 7$
2 Use the inverse operation for this operation on both sides of the equation and simplify the equation ($+5$).	$4x - 5 + 5 = 7 + 5$ $4x = 12$
3 Identify the next operation to be undone ($\times 4$) and apply the inverse operation ($\div 4$). If one side of the equation is now the variable by itself you have found the solution. State the solution.	$\frac{4x}{4} = \frac{12}{4}$ $x = 3$
4 Check by substitution that your answer is the solution.	Check: LHS $= 4x - 5$ $= 4 \times 3 - 5$ $= 12 - 5$ $= 7$ $=$ RHS
(b) 1 Write the equation and identify the last operation to be performed on the given variable ($+5$). This is the first operation to be undone.	(b) $5 - 2x = 7$
2 Use the inverse operation for this operation on both sides of the equation and simplify the equation (-5).	$5 - 5 - 2x = 7 - 5$ $-2x = 2$



3 Identify the next operation to be undone (× -2) and apply the inverse operation (÷ -2). If one side of the equation is now the variable by itself you have found the solution. State the solution.

$$\frac{-2x}{-2} = \frac{2}{-2}$$

$$x = -1$$

4 Check by substitution that your answer is the solution.

$$\begin{aligned} \text{LHS} &= 5 - 2x \\ &= 5 - 2 \times -1 \\ &= 5 + 2 \\ &= 7 \\ &= \text{RHS} \end{aligned}$$

Equations can be solved by performing inverse operations on both sides of the equals sign.

The order in which you perform the inverse operations is important and is the opposite of the order used in the construction process.

Solving equations graphically

One of the important uses of graphs is to be able to find solutions to equations.

If we draw a graph of a linear equation, all points on the graph will make the equation true. We can use the graph to find the y-coordinate for any given x-value and the x-coordinate for any given y-value.

- A graph can be used to evaluate an expression. If a value for x is used as the x-coordinate of the graph, the y-coordinate of the point gives the value of the equation for that x-value.
- A graph can be used to solve an equation. If a value for y is used as the y-coordinate of the graph, the x-coordinate of the point gives the solution of the equation.

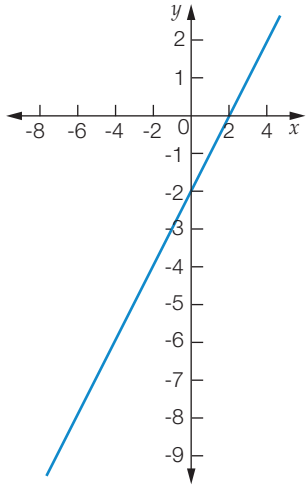
Worked Example 5

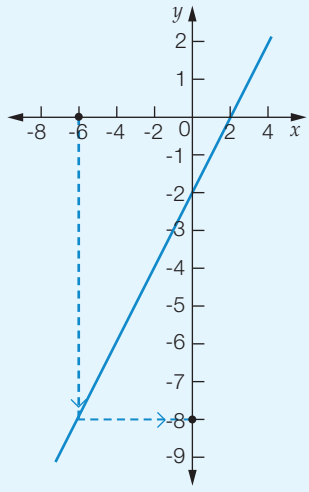
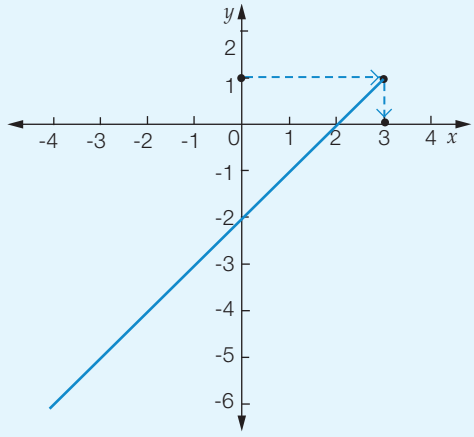
WE5

Use the following graph to find the value of:

(a) y when $x = -6$

(b) x when $y = 1$.



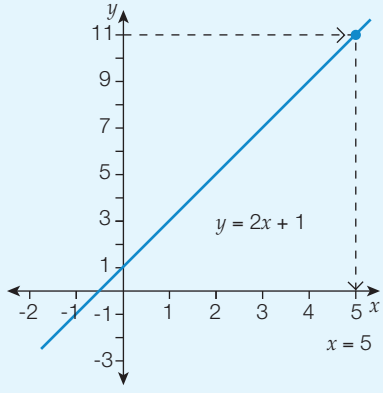
Thinking	Working
(a) 1 Locate the x-value you need to use along the x-axis ($x = -6$). Draw a vertical line from the x-value until you reach the graph, then draw a horizontal line to the y-axis.	(a) 
2 Read off the value for y.	When $x = -6$, $y = -8$.
(b) 1 Locate the y-value you need to use along the x-axis ($y = 1$). Draw a horizontal line from the y-value until you reach the graph and then draw a vertical line to the x-axis.	(b) 
2 Read off the value for x.	When $y = 1$, $x = 3$.

Worked Example 6

WE 6

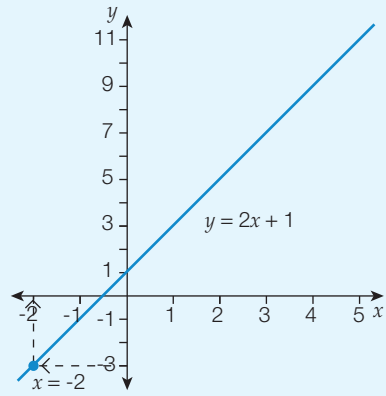
Use the graph of $y = 2x + 1$ to solve:

(a) $2x + 1 = 11$ **(b)** $2x + 1 = -3$

Thinking	Working
(a) 1 Use the number on the RHS of the equation as the y-value. This is the y-coordinate of a point on your graph.	(a) 

2 Find the x-coordinate of that point. This is your solution. $x = 5$

(b) 1 Use the number on the RHS of the equation as the y-value. This is the y-coordinate of a point on your graph. (b)



2 Find the x-coordinate of that point. This is your solution. $x = -2$

7.2 Solving linear equations

Navigator

Q1 Column 1, Q2, Q3, Q4, Q5, Q6, Q7, Q8 Column 1, Q9 Column 1, Q10 (a) & (b), Q11, Q12, Q13, Q14, Q19, Q20, Q22	Q1 Column 2, Q2, Q3, Q4, Q5, Q6, Q7, Q8 Column 2, Q9 Column 2, Q10 (a) & (b), Q11, Q12, Q13, Q14, Q15, Q17, Q18, Q19, Q21	Q1 Column 3, Q2, Q3, Q4, Q5, Q6, Q7, Q8 Column 3, Q9 Column 3, Q10 (c) & (d), Q11, Q12, Q13, Q14, Q15, Q16, Q17, Q18, Q19, Q21

Answers page 630

Fluency

1 Solve each of the following equations numerically to find x. WE3

(a) $x - 5 = 11$ (b) $x - 7 = 4$ (c) $x - 6 = -2$

(d) $2x = -16$ (e) $\frac{x}{5} = 3$ (f) $\frac{x}{7} = -11$

(g) $2x - 3 = -11$ (h) $6x + 5 = 47$ (i) $3x - 7 = -16$

(j) $\frac{3x + 7}{5} = 5$ (k) $\frac{4x - 3}{3} = 11$ (l) $\frac{7x - 4}{6} = -3$

2 Solve each of the following equations using algebra. WE4

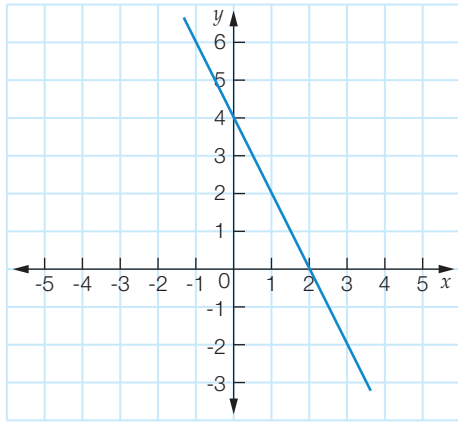
(a) $5a + 2 = 7$ (b) $3b - 8 = 4$ (c) $3c + 10 = 1$

(d) $8 - 3x = 11$ (e) $9 - 5x = 24$ (f) $13 - 7x = 69$

(g) $7 - 3x = 4$ (h) $11 - 2x = 5$ (i) $7 - 5x = -23$

WE5

3 Use the following graph to find the value of:



(a) y when $x = -1$ **(b)** y when $x = 0$ **(c)** y when $x = 2$

(d) x when $y = 4$ **(e)** x when $y = 2$ **(f)** x when $y = -2$

(g) y when $x = 1$ **(h)** y when $x = -0.5$ **(i)** y when $x = 2.5$

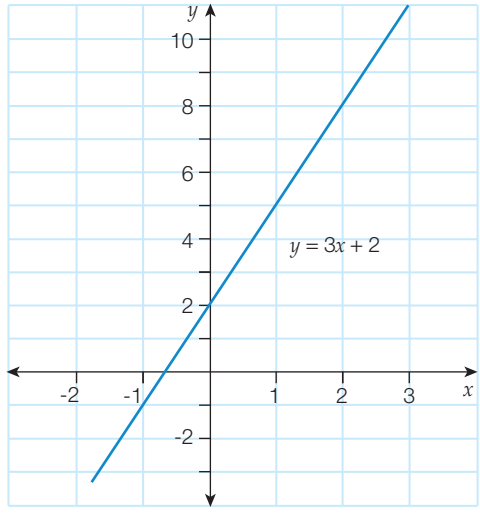
WE6

4 Use the graph of $y = 3x + 2$ to solve:

(a) $3x + 2 = 8$

(b) $3x + 2 = -1$

(c) $3x + 2 = 2$



5

(a) Write an equation that represents the set of scales shown.

(b) Write an equation to show the mass of the two boxes. (Let b = mass of a box.)

(c) Write an equation to show the mass of each box.

(d) What is the solution to the equation you found in part **(a)**?

6 **(a)** Which of the following equations is equivalent to $5z - 10 = 20$?

A $5z = 10$ **B** $5z = 30$ **C** $5z = -10$ **D** $5z = -30$

(b) Which of the following equations is equivalent to $\frac{5p - 3}{2} = 4$?

A $5p - 3 = 20$ **B** $5p - 3 = 6$ **C** $5p - 3 = 7$ **D** $5p - 3 = 8$

7 (a) To obtain x from $4x + 1$ you would need to:

A add 1, then multiply by 4
B divide by 4, then subtract 1
C subtract 1, then multiply by 4
D subtract 1, then divide by 4

(b) To obtain k from $-3k - 5$ you would need to:

A add 5, then divide by 3
B subtract 5, then multiply by 3
C add 5, then divide by -3
D subtract 5, then divide by -3

(c) To obtain m from $4 - 2m$ you would need to:

A subtract 2, then divide by 4
B subtract 4, then divide by -2
C add 2, then subtract 4
D add 4, then divide by -2

8 Solve each of the following using algebra.

(a) $3d + 5 = -10$
(b) $5e + 6 = -4$
(c) $2f + 7 = -13$
(d) $9 + 4k = 1$
(e) $28 + 5h = 18$
(f) $22 + 3g = 7$
(g) $8 + 9n = -1$
(h) $12 + 7c = -16$
(i) $5 + 9k = -13$
(j) $2r + 7 = 10$
(k) $3t + 8 = 15$
(l) $7l + 11 = 23$
(m) $3d + 2.4 = 8.9$
(n) $4p + 3.8 = 7.2$
(o) $12a + 5.7 = 11.3$
(p) $2g + 1\frac{3}{4} = 2\frac{1}{2}$
(q) $3r + \frac{5}{6} = 4\frac{1}{3}$
(r) $8p + 3\frac{1}{3} = 4\frac{2}{9}$

9 Solve each of the following equations using algebra.

(a) $-2a + 7 = 3$
(b) $-5b + 11 = 6$
(c) $-3c - 4 = 8$
(d) $-d - 1 = 7$
(e) $-e + 5 = 2$
(f) $-f + 9 = 12$
(g) $3 - 4g = -17$
(h) $8 - 3h = -10$
(i) $-9 - 2m = 7$
(j) $7 - 3r = 4$
(k) $13 - 7p = -8$
(l) $29 - 5t = -11$
(m) $2 - 3p = 4$
(n) $8 - 5l = 12$
(o) $9 - 7k = 19$
(p) $5.3 - 2a = -7.8$
(q) $6.5 - 5b = -2.1$
(r) $7.9 - 4c = -6.7$

Understanding

10 Write an equation for each of these word problems and use algebra to find the unknown number.

(a) A number is multiplied by 4, then 5 is subtracted to give a result of 23. (Let the number be n.)

(b) The sum of a number and seven is multiplied by three to give a result of 45. (Let the number be m.)

(c) The sum of five and a number is divided by nine to give a result of three. (Let the number be p.)

(d) A number is divided by four, then two is added to give a result of negative one. (Let the number be q.)

11 Match each equation in the left column with an equivalent equation in the right column.

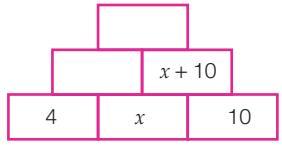
Equation	Equivalent equation
(a) $2x + 1 = 3$	**A** $x = 2$
(b) $3x - 4 = 5$	**B** $x = 9$
(c) $4x = 8$	**C** $x = -3$
(d) $\frac{x}{5} = 1$	**D** $2x = 8$
(e) $2x - 3 = 5$	**E** $2x = 2$
(f) $3x = -6$	**F** $3x = -3$
(g) $\frac{x}{3} = 3$	**G** $x = 5$
(h) $3x + 7 = 4$	**H** $x = -2$
(i) $2x - 9 = 3$	**I** $3x = 9$
(j) $-4x = 12$	**J** $2x = 12$

12 Anita's father is twice her weight plus 16 kilograms. Her father weighs 102 kg.

(a) Write an equation using a to represent Anita's weight.

(b) Solve the equation to find Anita's weight.

13 This is an addition pyramid. Each brick is the sum of the two bricks below it.



(a) Complete the rest of the pyramid.

(b) If the top brick has a value of 20, what is the value of x?

Reasoning

14 Jeremy has a water tank in his garden. One day, his tank had only 1000 litres left in it. During the day it rained heavily and the tank filled at a rate of 400 litres per hour. Write an equation and solve it to find how many hours it took for Jeremy's tank to contain 3600 L.

15 Jamal wants to buy a puppy. He has been saving all his pocket money each week for the last 5 weeks, and his grandmother has given him \$25 for his birthday. He started with no money and now has \$62.50. Using p to represent the amount of pocket money he gets each week, write an equation and solve it to find how much pocket money Jamal receives each week.

16 A light aircraft needs to be careful of its weight limits. The pilot has been told that as long as the passengers on board are of average weight the limit will not be exceeded if the passengers consist of 2 adults and 3 children, or 1 adult and 5 children.

(a) How many children balance one adult?

(b) If the pilot was to take only children on the plane, use an equation to find how many she could take.

17 Fatima drives to and from work each day for the five-day working week, and 75 kilometres on the weekend. She manages to travel 325 kilometres every week in her car. Use an equation to find how far it is from her home to her workplace.

18 Paul has to take 3 litres of water with him on a boat trip. He has one 1.25 L container and two other containers of equal capacity. Together, the three containers will hold 3 litres. Use an equation to find the capacity of each of the other containers.

Open-ended

19 Write an equation using:

(a) multiplication and addition

(b) multiplication and subtraction

and then solve it.

20 The scales have 2 apples with a bunch of 20 grapes on the left and 3 apples with a bunch of 10 grapes on the right.

Claire was trying to find how many grapes had a mass equivalent to one apple. She suggests the following as possible first steps.

- Remove 20 grapes from each side.
- Take off half of everything from both sides.

(a) Explain to Claire the faults in her suggested first steps.

(b) Solve the problem in two steps.

21 Write down at least three equations that are equivalent to $2x + 3 = 11$.

22 Jill adds a number to both sides of an equation, then divides both sides by a different number.

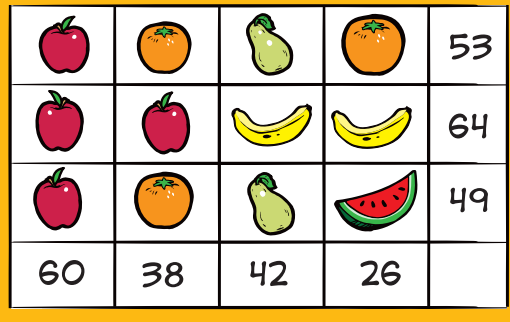
She gets the answer: $x = 2$.

What might the original equation be?

Outside the Square Puzzle

Algebra fruit bowl

Every object in the puzzle at right is equal to a number. Find the value of each of the given objects presented in the puzzle. The numbers given are the sum of the objects in each row or column. Sometimes, only one object will appear in a row or column. That makes the puzzle easier to solve. At other times, you will have to look for relationships among the objects.



7.3 Solving more complex equations

The equations in this section may look more complex than others you have solved so far. However, the process of performing inverse operations on both sides is the same. To find a solution to these equations, identify the order in which the equation was constructed around the variable, using the correct inverse order of operations.

The equations need to be deconstructed (undone) step by step with inverse operations in the reverse order to the way the equation was built. The number of operations that have been performed on the variable tells you how many operations need to be performed to solve the equation.

Worked Example 7

WE7

Solve each of the following using algebra.

(a) $\frac{y}{6} + 9 = 7$

(b) $\frac{t+3}{4} = -4$

Thinking	Working
(a) **1** Write the equation and count the number of operations that have been performed on the variable. This tells us how many inverse operations need to be performed to find the solution. (Here, there are two operations, ÷ 6 and + 9.)	(a) $\frac{y}{6} + 9 = 7$ Two operations need to be undone.
2 Perform the first inverse operation on both sides of the equation (– 9).	$\frac{y}{6} + 9 - 9 = 7 - 9$
3 Simplify the equation.	$\frac{y}{6} = -2$
4 Perform the second inverse operation on both sides of the equation (× 6).	$\frac{y}{6} \times 6 = -2 \times 6$
5 Write the solution to the equation.	$y = -12$
6 Check by substitution that you have found the solution.	$\text{LHS} = \frac{y}{6} + 9$ $= \frac{-12}{6} + 9$ $= -2 + 9$ $= 7$ $= \text{RHS}$

(b) **1**	Write the equation and count the number of operations that have been performed on the variable. This tells us how many inverse operations need to be performed to find the solution. (Here, there are two operations, + 3 and ÷ 4.)	(b) $\frac{t+3}{4} = -4$ Two operations need to be undone.
2	Perform the first inverse operation on both sides of the equation (× 4). Place brackets around numerator.	$\frac{(t+3)}{4} \times 4 = -4 \times 4$
3	Simplify the equation.	$t + 3 = -16$
4	Perform the second inverse operation on both sides of the equation (– 3).	$t + 3 - 3 = -16 - 3$
5	Write the solution to the equation.	$t = -19$
6	Check by substitution that you have found the solution.	$\text{LHS} = \frac{t+3}{4}$ $= \frac{-19+3}{4}$ $= \frac{-16}{4}$ $= -4$ $= \text{RHS}$

Worked Example 8

WE 8

Solve each of the following equations using algebra.

(a) $5(2a - 9) = 13$ **(b)** $6(3 - x) = 12$

Thinking		Working
(a) **1**	Write the equation.	(a) $5(2a - 9) = 13$
2	Remove the brackets first by expanding using the distributive law, then count the number of operations that have been performed on the variable. This tells us how many inverse operations need to be performed to find the solution. (Here, there are two operations, × 10 and – 45.)	$10a - 45 = 13$
3	Perform the first inverse operation on both sides of the equation (+ 45).	$10a - 45 + 45 = 13 + 45$
4	Simplify the equation.	$10a = 58$
5	Perform the second inverse operation on both sides of the equation (÷ 10).	$\frac{10a}{10} = \frac{58}{10}$
6	Write the solution to the equation.	$a = 5.8$ or $5\frac{4}{5}$

7	Check by substitution that you have found the solution.	LHS = 5(2a – 9) = 5(2 × 5.8 – 9) = 5(11.6 – 9) = 5 × 2.6 = 13 = RHS
(b) 1	Write the equation.	(b) $6(3 - x) = 12$
2	Remove the brackets first by expanding using the distributive law, then count the number of operations that have been performed on the variable. This tells us how many inverse operations need to be performed to find the solution. (Here, there are two operations, × – 6 and + 18.)	$18 - 6x = 12$
3	Perform inverse operations until the variable is by itself on one side of the equals sign (– 18 first, ÷ -6 next).	$18 - 18 - 6x = 12 - 18$ $-6x = -6$ $\frac{-6x}{-6} = \frac{-6}{-6}$
4	Write the solution to the equation.	$x = 1$
5	Check by substitution that you have found the solution.	LHS = 6(3 – x) = 6(3 – 1) = 6 × 2 = 12 = RHS

Worked Example 9

WE 9

Solve each of the following equations using algebra.

(a) $\frac{2a - 3}{7} = -3$

(b) $\frac{6 - 4k}{5} = -2$

Thinking		Working
(a) 1	Write the equation and count the number of operations that have been performed on the variable. This tells us how many inverse operations need to be performed to find the solution. (Here, there are three operations, × 2, – 3, and ÷ 7.)	(a) $\frac{2a - 3}{7} = -3$ Three operations need to be undone.
2	Multiply both sides of the equation by the number in the denominator to eliminate the fraction (× 7).	$\frac{2a - 3}{7} \times 7 = -3 \times 7$ $2a - 3 = -21$

3 Continue to use inverse operations until the variable is by itself on one side of the equation (+ 3, ÷ 2).

$$2a - 3 + 3 = -21 + 3$$
$$2a = -18$$
$$\frac{2a}{2} = \frac{-18}{2}$$
$$a = -9$$

4 Check by substitution that you have found the solution.

$$\text{LHS} = \frac{2a - 3}{7}$$
$$= \frac{2 \times -9 - 3}{7}$$
$$= \frac{-18 - 3}{7}$$
$$= \frac{-21}{7}$$
$$= -3$$
$$= \text{RHS}$$

(b) 1 Write the equation and count the number of operations that have been performed on the variable. This tells us how many inverse operations need to be performed to find the solution. (Here, there are three operations, × 4, + 6, and ÷ 5.)

(b) $$\frac{6 - 4k}{5} = -2$$

2 Multiply both sides of the equation by the number in the denominator to eliminate the fraction (× 5) and simplify the equation.

$$\frac{6 - 4k}{5} \times 5 = -2 \times 5$$
$$6 - 4k = -10$$

3 Continue to use inverse operations until the variable is by itself on one side of the equation (– 6, ÷ -4).

$$6 - 6 - 4k = -10 - 6$$
$$-4k = -16$$
$$\frac{-4k}{-4} = \frac{-16}{-4}$$
$$k = 4$$

4 Check by substitution that you have found the solution.

$$\text{LHS} = \frac{6 - 4k}{5}$$
$$= \frac{6 - 4 \times 4}{5}$$
$$= \frac{6 - 16}{5}$$
$$= \frac{-10}{5}$$
$$= -2$$
$$= \text{RHS}$$

- $-x$ can be thought of as $-1 \times x$.
- The fraction bar means division. It also acts as though there are brackets around everything above it. For example, $\frac{2x + 5}{3} = (2x + 5) \div 3$.

7.3 Solving more complex equations

Navigator

Answers page 631

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4, Q5 Column 1, Q6, Q7, Q8, Q9, Q13, Q14, Q15, Q17	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4, Q5 Column 2, Q6, Q7, Q8, Q9, Q11, Q13, Q14, Q16, Q17	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q5 Column 3, Q6, Q7, Q8, Q10, Q11, Q12, Q13, Q14, Q15, Q16

Fluency

WE7

1 Solve each of the following using algebra.

(a) $\frac{c}{6} + 8 = 10$ **(b)** $\frac{f}{7} + 5 = 8$ **(c)** $\frac{m}{9} + 3 = 12$

(d) $\frac{x}{2} - 3 = -5$ **(e)** $\frac{b}{3} - 4 = -7$ **(f)** $\frac{t}{4} - 5 = -9$

(g) $\frac{x+5}{3} = -3$ **(h)** $\frac{p+2}{5} = -7$ **(i)** $\frac{r+7}{9} = -8$

(j) $\frac{x-3}{4} = 3$ **(k)** $\frac{y-9}{5} = 7$ **(l)** $\frac{t-11}{12} = -5$

WE8

2 Solve each of the following equations using algebra.

(a) $4(x-2) = 20$ **(b)** $5(x+3) = -50$ **(c)** $8(x+2) = -24$

(d) $2(2x+1) = -9$ **(e)** $3(6x+4) = -16$ **(f)** $7(2x-5) = 41$

(g) $3(2-x) = 36$ **(h)** $5(9-x) = 30$ **(i)** $7(5-x) = 56$

(j) $4(2-3x) = -16$ **(k)** $5(3-2x) = -25$ **(l)** $9(5-3x) = -27$

WE9

3 Solve each of the following equations using algebra.

(a) $\frac{2a+5}{3} = -9$ **(b)** $\frac{3m+5}{4} = -7$ **(c)** $\frac{6r+1}{5} = -7$

(d) $\frac{3r-2}{5} = 8$ **(e)** $\frac{5n-3}{4} = 8$ **(f)** $\frac{7p-9}{12} = 8$

(g) $\frac{7-3r}{2} = 2$ **(h)** $\frac{28-5f}{4} = 2$ **(i)** $\frac{3-7m}{3} = 8$

(j) $\frac{4-5r}{3} = -2$ **(k)** $\frac{5-2p}{4} = -3$ **(l)** $\frac{7-4k}{5} = -9$

4 **(a)** To obtain x from $\frac{x}{2} + 7$ you would need to:

A multiply by 2, then subtract 7 **B** add 7, then multiply by 2

C subtract 7, then multiply by 2 **D** divide by 2, then add 7

(b) To obtain k from $\frac{k-8}{3}$ you would need to:

A add 8, then multiply by 3 **B** multiply by 3, then add 8

C subtract 8, then divide by 3 **D** multiply by 3, then subtract 8

5 Solve each of the following equations.

(a) $\frac{3a}{7} + 5 = 11$ **(b)** $\frac{5p}{3} + 8 = -7$ **(c)** $\frac{7m}{4} + 3 = -18$

(d) $5(3 - 2a) = -7$ **(e)** $4(5 - 3a) = 11$ **(f)** $3(7 - 4a) = 37$

(g) $\frac{2x + 3}{5} = 4$ **(h)** $\frac{3x + 5}{2} = 9$ **(i)** $\frac{4x - 7}{3} = 8$

(j) $\frac{1 - 2x}{3} = 3$ **(k)** $\frac{6 - 3x}{5} = 3$ **(l)** $\frac{8 - 5x}{7} = 4$

6 Match each equation (left column) with the appropriate list of inverse operations needed to solve the equation (right column).

Equation	Inverse operations needed to solve the equation
(a) $\frac{2x + 4}{3} = 5$	**A** $\times 4$, then $+ 2$, then $\div 3$
(b) $\frac{3x}{4} + 2 = 5$	**B** $- 4$, then $\times 3$, then $\div -2$
(c) $\frac{2x}{3} + 4 = 5$	**C** $\times 3$, then $- 4$, then $\div -2$
(d) $\frac{3x + 2}{4} = 5$	**D** $\times 3$, then $- 4$, then $\div 2$
(e) $\frac{2x - 4}{3} = 5$	**E** $- 2$, then $\times 4$, then $\div -3$
(f) $\frac{4 - 2x}{3} = 5$	**F** $- 2$, then $\times 4$, then $\div 3$
(g) $\frac{3x - 2}{4} = 5$	**G** $- 4$, then $\times 3$, then $\div 2$
(h) $\frac{2 - 3x}{4} = 5$	**H** $\times 3$, then $+ 4$, then $\div 2$
(i) $2 - \frac{3x}{4} = 5$	**I** $\times 4$, then $- 2$, then $\div 3$
(j) $4 - \frac{2x}{3} = 5$	**J** $\times 4$, then $- 2$, then $\div -3$

Understanding

7 Write the following statements as equations and then find the value of the unknown.

(a) A number is multiplied by three, seven is added, and the result is divided by four to give an answer of four. What is the number?

(b) A number is divided by four, one is subtracted , and the result is doubled to give an answer of six. What is the number?

(c) Two is subtracted from five times a number, then the result is divided by six to give an answer of negative twelve. What is the number?

(d) Seven is added to one and a half times a number to give a result of eight. What is the number?

8 Margaret has put aside a certain amount of money to buy Christmas presents for five relatives. She spends \$10 of the money on wrapping paper and cards, and intends to divide the remaining money equally between each relative. However, she then decides to add \$20 to her budget to buy a special present for one relative. This present costs \$48.

(a) Write an equation and solve it to find how much money Margaret initially put aside.

(b) How much money did Margaret spend altogether?

9 Amir has a job as a telemarketer to sell a particular product. He will earn two-elevenths of the total value of the product he sells each day plus an extra \$50.

(a) Write an equation and solve it to find the total value of the products sold in one day if he earns \$150.

(b) If Amir makes 22 sales of this product, what is the selling price of the product?

10 Rimesh planted some daffodil bulbs in his garden. The following year he found that the number of bulbs had doubled. He also planted another six bulbs. The year after that he found the number of bulbs had tripled to give him 108 daffodil bulbs.

Write an equation and solve it to find how many bulbs he planted initially.

Reasoning

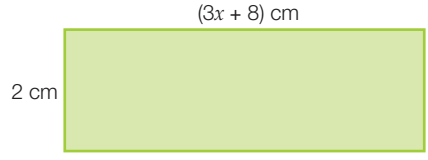
For each of the following, write an appropriate equation and solve it to answer the question.

11 The sum of the digits of a two-digit number is 12. If the first digit is three times as large as the second digit, what is the number?

12 The sum of the digits of a three-digit number is 15. If the first digit is twice as large as the third digit and the second digit is one less than the third digit, what is the number?

13 Lyndall goes to the markets each Sunday to sell flowers. At the end of each market day she spends some of her earnings, which she keeps in a special 'market' wallet. At the beginning of July, she has a certain amount of money in her wallet. On the first weekend she doubled this amount, then spent \$20. On the second weekend, she doubled the money in the wallet, then spent \$25. On the third weekend, she trebled the money in the wallet and spent \$15. At this point, Lyndall has \$150 in her market wallet. How much did she have at the beginning of July?

14 Given that the area of the following rectangle is 106 cm^2, determine the value of x.



Open-ended

15 Write down at least three equations that can be solved by first adding 3, then dividing by $\frac{2}{3}$.

16 (a) Starting with the number 3, perform a sequence of three different operations on it to obtain an answer of 5. Then, write your sequence as an equation with x instead of 3.

(b) Make up two more three-step equations. Give the equations to your friends and see whether they can solve them.

17 Caitlyn solves the equation $2 - 5x = 8$ as follows.

$2 - 5x = 8$	step 1
$5x = 8 - 2$	step 2
$5x = 6$	step 3
$x = \frac{6}{5}$	step 4
$x = 1\frac{1}{5}$	step 5

(a) At what step did Caitlyn make a mistake? Describe how you would correct this mistake.

(b) Solve, remembering to check your answer.

Outside the Square Puzzle

Magic algebra squares

Magic algebra squares work just like ordinary magic squares in that the sum of each row, column and diagonal must be equal.

Use these magic squares to answer the questions below.

4	9	a
$a + 1$	5	
$a + 6$	$a - 1$	6

$x + 1$	$x - 10$	$x + 6$
	$x - 1$	
		$x - 3$

$c + a$	$c - a - b$	
		$c + a - b$

1 **(a)** Find the value for a.

(b) Find the missing number.

(c) Substitute this number and the value you found for a into your magic square and copy and complete your magic square with numbers.

2 **(a)** Copy and complete the magic square in terms of x.

(b) Find the value for x if the magic sum is 9.

(c) Substitute the value you found for x into your magic square and redo your magic square with numbers.

3 **(a)** Complete the magic square in terms of c if the magic sum is $3c$.

(b) Find the value of a, b and c if the magic sum is 30, $c + a = 12$ and $c - b = 15$.

(c) Substitute the values you found for a, b and c into your magic square and redo your magic square with numbers.

Half-time 7

Ex. 7.3

1 Solve the following equations using algebra.

(a) $3(5 - x) = 24$

(b) $\frac{5x}{2} + 4 = 14$

(c) $\frac{3x - 6}{8} = 9$

(d) $3(x + 3) = -6$

(e) $\frac{7 - 4x}{5} = -7$

(f) $5 - \frac{x}{3} = 25$

Ex. 7.1

2 Check whether the solution given in the brackets is correct for the following equations.

(a) $x + 5 = 11$ $(x = 7)$

(b) $2x - 3 = 7$ $(x = 5)$

(c) $3(x - 9) = 15$ $(x = 14)$

(d) $\frac{x - 8}{3} = 8$ $(x = 23)$

Ex. 7.3

3 Phillip, James and Sam all contribute to a present worth $130 for Chris. If James contributes half the amount that Phillip does and Sam contributes $10 less than twice the amount that Phillip contributes, find the amount that each person gives towards the present.

Ex. 7.2

4 Solve the following equations using algebra.

(a) $x - 9 = 15$

(b) $2x + 12 = 16$

(c) $\frac{x}{4} + 7 = 12$

(d) $8 - \frac{x}{5} = 17$

Ex. 7.1

5 Write the following sentences as equations and find the value of the unknown number.

(a) Four is subtracted from a number to give a result of twelve.

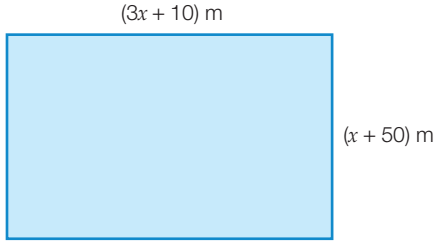
(b) A number multiplied by seven is equal to thirty-five.

(c) A number multiplied by six is equal to three times twelve.

(d) A number is multiplied by seven, then three is added to give a result of twenty-four.

Ex. 7.3

6 The total distance around this rectangular field is 480 metres. Find the value of x and use this result to find the length and width of the field.



Investigation

Magic calendar squares

Equipment required: 1 brain, 1 calendar (you can use the one in your school diary)

Magic squares have fascinated people for over 4000 years. A magic square is a grid of different numbers in which every row, column and diagonal add up to the same total. The total is called the 'magic sum'. People used to believe that these squares had mystical powers. The following 'Lo-Shu' magic square from ancient China is the simplest magic square and uses the numbers 1 to 9.

Lo-Shu magic square

4	9	2
3	5	7
8	1	6

A 'calendar' square is any 3×3 square drawn around a set of numbers on a calendar. Calendar squares are 'semi-magic' squares. Your exploration will show you why.

For example:

APRIL

M	T	W	Th	F	Sa	Su
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

The Big Question

Can all calendar squares be rearranged into magic squares?

Engage

1 Find the magic sum of the Lo-Shu magic square by adding the numbers in each row, column and diagonal.

2 Here is another magic square:

5	10	3
4	6	8
9	2	7

(a) What is the new magic sum?

(b) What operation was performed on the Lo-Shu magic square to get this magic square?

(c) If $x = 1$, then $x + 1 = 2$. Use this pattern to find an algebraic relationship to represent all the integers from 3 to 9.

(d) Now, replace all the numbers in the boxes in the Lo-Shu magic square with algebraic expressions in terms of x using the relationships you found in part **(c)**.

	x	

(e) (i) What is the magic sum in terms of x?

(ii) Write this in factorised form.

(f) By solving equations, find x for the following magic sums.

(i) 33 (ii) 54 (iii) -3

(g) Draw the grids for these three magic squares, calculate the numbers that will go into each square and then enter these numbers into the squares.

3 Here is another magic square:

		4
	10	
16		12

(a) Find the missing numbers.

(b) How is this square related to the Lo-Shu magic square?

(c) By substituting x for 1, write all the numbers in this new square in terms of x using the relationship you found in Question **3** part **(b)**.

Explore

4 (a) Select a calendar square and find the row, column and diagonal sum of it. Use these sums to explain why the squares are called 'semi-magic'.

(b) Substitute x for the earliest date on your chosen calendar square. As you did for the Lo-Shu magic square, write all the numbers in terms of x.

(c) Find the diagonal sum in terms of x.

(d) Find the sum of all the numbers in your calendar square in terms of x.

(e) What is the mean of this sum?

(f) Find the start date for these calendar square totals by solving an appropriate equation:

(i) 180 (ii) 153

(g) Rearrange the numbers in your calendar square to turn your 'semi-magic' square into a 'magic' square.

Strategy options

- Guess and check.
- Look for a pattern.
- Test all possible combinations.

Explain

5 (a) If x is an integer, explain, in terms of x, why the magic sum of an altered Lo-Shu magic square could never be 22, 38 or -2.

(b) Use the algebraic magic sum in Question **3** to explain why the magic sum of the squares in Question **3** will always be a multiple of 6.

(c) Explain why the total on a calendar square can never exceed 207 and will always be a multiple of 9.

(d) What is the smallest total? Explain how you found it.

Elaborate

6 Write your rearranged calendar square in terms of x. Substitute two other start dates into this square to generate two new calendar squares.

Evaluate

7 (a) Did you go about this task systematically?

(b) Did you record your work carefully?

(c) Did you find any patterns? What were they?

(d) Did algebra and equation solving help you to find the rearranged magic calendar square? If so, how did it help?

(e) Did you use any other strategies to find the rearranged magic calendar square? If so, what were they?

Extend

8 Investigate some famous magic squares, such as the artist Albrecht Dürer's 4×4 magic square in his woodcut *Melancholia* or Benjamin Franklin's 8×8 magic square. See what other magic sums and patterns you can find.

Solving equations where the unknown appears on both sides

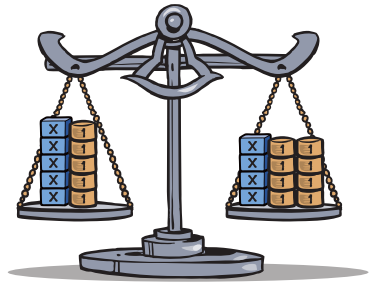
7.4

So far, we have solved equations with the unknown (variable) on only one side of the equals sign. However, many equations have the unknown on both sides of the equation; for example, $5x + 5 = 4x + 8$.

Backtracking cannot be used to solve these equations, but we can solve them using algebra. Graphs can also be used to solve these equations.

Using algebra

Consider the scales on the right. The left-hand side of the scales shows five blocks of mass x g and five blocks of mass 1 g. This can be written in algebraic form as $5x + 5$.



$5x + 5 \quad = \quad 4x + 8$

The right-hand side of the scales shows four blocks of mass x g and eight blocks of mass 1 g. This can be written in algebraic form as $4x + 8$.

How can we find the value of x? Because the scales are balanced, we can represent the situation with the equation $5x + 5 = 4x + 8$.

If all four blocks of mass x are removed from the right side of the balance, we will also need to remove four blocks of mass x from the left side to keep the scales balanced. This will leave one block of mass x on the left side of the equation and no blocks of mass x on the right.

$x + 5 \quad = \quad 8$

Writing in algebra:

$$5x - 4x + 5 = 4x - 4x + 8$$
$$x + 5 = 8$$

We have used the inverse operation ($-4x$) on both sides of the equation to remove the unknown from the right side of the equation.

We can now remove the five blocks of mass 1 g from both sides of the scales. They will stay balanced. One block of mass x will remain on the left side of the scales and three blocks of mass 1 g will remain on the right side.

$x \quad = \quad 3$

Writing in algebra:

$$x + 5 - 5 = 8 - 5$$
$$x = 3$$

We have used the inverse operation (-5) on both sides of the equation to remove the number from the left side of the equation.

We have found that the unknown mass x is equal to 3 g by solving the equation $5x + 5 = 4x + 8$ and finding that $x = 3$ is the solution to the equation. We can check by substitution that the value you have found, $x = 3$, is the solution.

$$\begin{aligned} \text{LHS} &= 5x + 5 \\ &= 5 \times 3 + 5 \\ &= 15 + 5 \\ &= 20 \end{aligned}$$

$$\begin{aligned} \text{RHS} &= 4x + 8 \\ &= 4 \times 3 + 8 \\ &= 12 + 8 \\ &= 20 \end{aligned}$$

LHS = RHS, so $x = 3$ is the solution.

To solve equations where the unknown appears on both sides of the equation:

1. Perform the same inverse operation on both sides of the equation to remove the variable from the RHS of the equation. This keeps the unknown on the LHS (left-hand side.)
2. Perform the same operation on both sides of the equation to remove any number on the LHS of the equation.
3. Continue to use inverse operations on both sides of the equation until a solution is found.
4. Always check by substitution that you have found the solution.

With some equations, this process will give a negative coefficient of x after the first step.

Consider $3x + 4 = 5x + 2$

Subtracting $5x$ from both sides:

$$\begin{aligned} 3x - 5x + 4 &= 5x - 5x + 2 \\ -2x + 4 &= 2 \end{aligned}$$

We prefer to keep the coefficient of the unknown positive. We can do this by interchanging (swapping) the sides of the equation before we start.

$$\begin{aligned} 3x + 4 &= 5x + 2 \\ 5x + 2 &= 3x + 4 \end{aligned}$$

Now, we subtract $3x$ from both sides of the equation:

$$\begin{aligned} 5x - 3x + 2 &= 3x - 3x + 4 \\ 2x + 2 &= 4 \end{aligned}$$

This equation is much easier to solve than $-2x + 4 = 2$.

If we think about the balanced scales, this is the same as turning the scales around so the left-hand side and the right-hand side of the scales are reversed.

If the coefficient of x is greater on the RHS of the equation than on the LHS, interchange the sides before beginning the solution process.

Worked Example 10 WE10

Solve the following equations using algebra.

(a) $5x - 2 = 3x + 8$

(b) $2x + 7 = 5x - 8$

Thinking		Working
(a) **1**	Write the equation.	(a) $5x - 2 = 3x + 8$
2	Use inverse operations to remove the variable from the RHS of the equals sign. ($-3x$)	$5x - 3x - 2 = 3x - 3x + 8$ $2x - 2 = 8$
3	Use inverse operations to solve for x. ($+2$ first, $\div 2$ next)	$2x - 2 + 2 = 8 + 2$ $2x = 10$ $\frac{2x}{2} = \frac{10}{2}$ $x = 5$
4	Check by substitution that you have found the solution.	LHS $= 5 \times 5 - 2$ $= 23$ RHS $= 3 \times 5 + 8$ $= 23$ $x = 5$ is the solution
(b) **1**	Write the equation.	(b) $2x + 7 = 5x - 8$
2	As the coefficient of the variable on the LHS is less than the coefficient of the variable on the RHS, interchange sides.	$5x - 8 = 2x + 7$
3	Use inverse operations to remove the variable from the RHS of the equals sign. ($-2x$)	$5x - 2x - 8 = 2x - 2x + 7$ $3x - 8 = 7$
4	Use inverse operations to solve for x.	$3x - 8 + 8 = 7 + 8$ $3x = 15$ $\frac{3x}{3} = \frac{15}{3}$ $x = 5$
5	Check by substitution that you have found the solution.	LHS $= 2 \times 5 + 7$ $= 17$ RHS $= 5 \times 5 - 8$ $= 17$ $x = 5$ is the solution

Worked Example 11

WE11

Solve the following equations using algebra.

(a) $3(x + 4) = 2(1 - x)$

(b) $\frac{3x + 4}{2} = \frac{2x - 1}{5}$

Thinking		Working
(a) **1**	Write the equation.	(a) $3(x + 4) = 2(1 - x)$
2	Expand the brackets first, then simplify.	$3x + 12 = 2 - 2x$
3	Use inverse operations to collect the variable onto the left-hand side of the equals sign. ($+2x$ to both sides)	$3x + 12 + 2x = 2 - 2x + 2x$ $5x + 12 = 2$

4	Use inverse operations to solve for x. (-12 first, then $\div 5$)	$5x + 12 - 12 = 2 - 12$ $5x = -10$ $\frac{5x}{5} = \frac{-10}{5}$ $x = -2$
5	Check by substitution that you have found the solution.	LHS $= 3(-2 + 4)$ $= 6$ RHS $= 2(1 - -2)$ $= 6$ $x = -2$ is the solution
(b) 1	Write the equation.	(b) $\left(\frac{3x+4}{2}\right) = \left(\frac{2x-1}{5}\right)$
2	Multiply both sides by the LCD (LCD = 10), expand and simplify.	${}^{5}\cancel{10} \times \left(\frac{3x+4}{{}_{1}\cancel{2}}\right) = {}^{2}\cancel{10} \times \left(\frac{2x-1}{{}_{1}\cancel{5}}\right)$ $5(3x + 4) = 2(2x - 1)$ $15x + 20 = 4x - 2$
3	Use inverse operations to collect the variable onto the left-hand side of the equals sign. ($-4x$ from both sides)	$15x - 4x + 20 = 4x - 4x - 2$ $11x + 20 = -2$
4	Use inverse operations to solve for x. (-20 first, then $\div 11$)	$11x + 20 - 20 = -2 - 20$ $11x = -22$ $\frac{11x}{11} = \frac{-22}{11}$ $x = -2$
5	Check by substitution that you have found the solution.	LHS $= \frac{3 \times -2 + 4}{2}$ $= -1$ RHS $= \frac{2 \times -2 - 1}{5}$ $= -1$ $x = -2$ is the solution

- Try to organise your equation so the variable is on the left-hand side and its coefficient is positive.
- If brackets are present in the equation, always expand first.
- If a fraction is present in the equation, always multiply both sides by the LCD (lowest common denominator) and expand the result.

Solving graphically

We can use graphs to solve equations when the variable appears on both sides of the equals sign by:

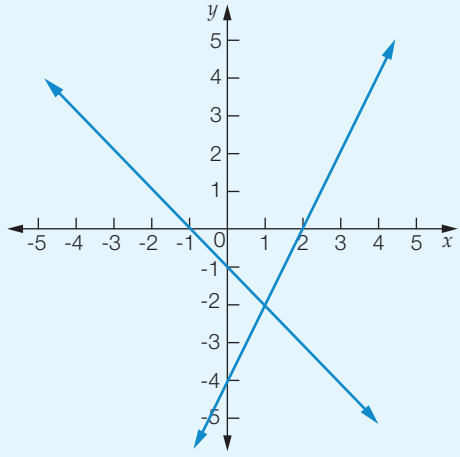
- equating the expression on each side of the equals sign to y
- sketching the graph of each linear equation
- locating the point of intersection.

The x-coordinate of the **point of intersection** of the two graphs gives us the solution of the original equation. It is the only x-value that makes the original equation true.

Worked Example 12

WE12

Solve $-x - 1 = 2x - 4$ graphically by finding the point of intersection of relevant graphs.

Thinking	Working
1 Because the expressions on each side of the equation equal each other, we can equate both of them to y. Write each expression in the form of $y = mx + b$.	$y = -x - 1$ and $y = 2x - 4$
2 Graph each linear equation on the same Cartesian plane (you can use by-hand techniques or technology).	
3 Find the point of intersection and read off the values for x and y.	Point of intersection is at (1, -2). Therefore, $x = 1$ and $y = -2$.
4 The x-value is the solution of your equation, so write this as the solution.	$x = 1$ is the solution
5 Check by substitution that you have found the solution. Substituting the x-value found will give the value of each expression and will give the y-coordinate of the point of intersection.	LHS $= -1 - 1$ $= -2$ RHS $= 2 \times 1 - 4$ $= -2$ $x = 1$ is the solution

- Finding the point of intersection of two linear graphs allows us to solve equations where the variable appears on both sides of the equation, as it is the only point that satisfies both equations.
- The y-coordinate of your graph gives the value of the expression on both sides of the equals sign.
- Some equations have no solutions. For instance, $2x + 4 = 2x - 7$ has no solution. Graphing $y = 2x + 4$ and $y = 2x - 7$ will show that they are parallel lines.

Lines that are parallel to each other have the same gradient, so they will never intersect. Hence, there is no solution for equations when the coefficients of the variable on both sides of the equation are the same.

7.4 Solving equations where the unknown appears on both sides

Navigator

Answers page 632

Q1 Column 1, Q2 Column 1, Q3 Column 1, Q4, Q5, Q6, Q8, Q9, Q14	Q1 Column 2, Q2 Column 2, Q3 Column 2, Q4, Q5, Q6, Q8, Q9, Q11, Q13, Q14	Q1 Column 3, Q2 Column 3, Q3 Column 3, Q4, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14

Equipment required: A graphing program can be used for Question **3**

Fluency

WE10

1 Solve the following equations using algebra.

(a) $7c + 2 = 6c - 5$ **(b)** $4b + 1 = 2b - 15$ **(c)** $3a - 5 = a + 7$

(d) $5d - 4 = 2d + 17$ **(e)** $5e - 3 = 2e + 9$ **(f)** $9f + 5 = 5f - 3$

(g) $3g + 12 = 2g + 5$ **(h)** $2h - 7 = h - 13$ **(i)** $4x - 3 = 7 - x$

(j) $k + 6 = 9 - k$ **(k)** $10j + 1 = 25 - 2j$ **(l)** $3y - 22 = -4 - 3y$

(m) $2m + 5 = 4m - 3$ **(n)** $5 + 4n = 7n - 1$ **(o)** $7x - 4 = 9x - 6$

(p) $2 - y = 3 - 2y$ **(q)** $p + 16 = 19 - 2p$ **(r)** $6 + 3x = 21 - 2x$

(s) $6 - 5p = 30 - p$ **(t)** $14 - 5x = 5 - 2x$ **(u)** $17 - 15a = 39 - 4a$

WE11

2 Solve the following equations using algebra.

(a) $3(x - 2) = 2x + 1$ **(b)** $4(x - 1) = x + 8$ **(c)** $5(x + 3) = 2x + 9$

(d) $5(2x + 3) = 2(3x + 1)$ **(e)** $7(x + 4) = 3(x - 4)$ **(f)** $4(3x - 2) = 3(2x + 9)$

(g) $2(3x + 7) = 3(5 - x)$ **(h)** $3(2x + 1) = 4(3 - x)$ **(i)** $4(2x + 3) = 3(4 - 5x)$

(j) $\frac{2x + 3}{3} = \frac{4x - 1}{5}$ **(k)** $\frac{4x + 2}{3} = \frac{3x - 4}{5}$ **(l)** $\frac{5x + 1}{3} = \frac{5x - 2}{4}$

(m) $\frac{3x + 1}{5} = \frac{2x + 7}{3}$ **(n)** $\frac{3x + 2}{4} = \frac{4x + 1}{5}$ **(o)** $\frac{2x + 5}{4} = \frac{4x + 1}{3}$

WE12

3 Solve the following equations graphically by finding the point of intersection of relevant graphs.

(a) $x + 6 = -x$ **(b)** $5x - 2 = 2x + 7$ **(c)** $2x - 6 = -2x + 10$

(d) $-2x + 7 = x - 5$ **(e)** $5x - 1 = 3x + 7$ **(f)** $x - 2 = 2x + 7$

(g) $7x - 2 = 2x + 3$ **(h)** $5x - 1 = -x + 11$ **(i)** $-2x + 1 = 4x - 5$

4 **(a)** $9x + 4 = 2x - 3$ has the solution:

A $x = 1$ **B** $x = -1$ **C** $x = 2$ **D** $x = 4$

(b) $2y + 7 = -8 - 3y$ has the solution:

A $y = 2$ **B** $y = 1$ **C** $y = -2$ **D** $y = -3$

(c) $4(3k - 2) = 5(k + 4)$ has the solution:

A $k = 3$ **B** $k = -2$ **C** $k = 1$ **D** $k = 4$

Understanding

5 **(a)** Write an expression to show 11 subtracted from four times a certain number.

(b) Write an expression to show two times the number in part **(a)** subtracted from 7.

(c) Equate the expressions in parts **(a)** and **(b)**.

(d) Solve the equation in part **(c)** to find the unknown number.

6 **(a)** Write an expression to show 36 added to a certain number.

(b) Write an expression to show the number in part **(a)** being added to 6 and the result doubled.

(c) Equate the expressions in parts **(a)** and **(b)**.

(d) Solve the equation in part **(c)** to find the unknown number.

7 **(a)** Write an expression to show five less than three-quarters of a certain number.

(b) Write an expression to show twice the result of subtracting ten from the number in part **(a)**.

(c) Equate the expressions in parts **(a)** and **(b)**.

(d) Solve the equation in part **(c)** to find the unknown number.

8 **(a)** Show that (-3, 2) is the point of intersection of $y = 2x + 8$ and $y = -x - 1$.

(b) Write an equation without y that is equivalent to the two equations in part **(a)**.

(c) Without using algebra or doing any calculations, write down the solution to your equation in part **(b)**.

(d) What does the y-coordinate of the point (-3, 2) tell you about your equation in part **(b)**.

Reasoning

9 Find the value of x in the following magic square. The rule for any magic square is that the sum of each row, column and diagonal are equivalent (or add to the same value).

$x + 3$	$x + 4$	$x - 1$
$x - 2$	6	$x + 6$
9	x	$x + 1$

10 Sarah has \$20 more than Frank, but \$40 less than Bruce. If Bruce has \$10 more than twice the amount that Frank has, write an equation to find how much money each person has.

11 Solve $2(x - 5) = \dfrac{x + 5}{3}$ by using graphs.

12 Three times Theo's age a year ago is the same as double his age in five years time. Write this information as an equation and then find Theo's age using graphs.

Open-ended

13 Make up three equations similar to those in Question **1**, where the solution is a positive whole number.

14 Barry solves the equation $6(x + 6) = 2(x - 4)$ as follows.

$6x + 36 = 2x - 4$	step 1
$4x + 36 = -4$	step 2
$4x = 32$	step 3
$x = 8$	step 4

(a) At what step/s did Barry make a mistake? Describe how you would correct this mistake.

(b) Solve the equation, remembering to check your answer.

Outside the Square — Problem solving

Alphametics

Alphametics are a type of mathematical puzzle in which digits are replaced by letters that form words and phrases. A famous example is:

```
    S E N D          9 5 6 7
+   M O R E  solution  +  1 0 8 5
-----------          -----------
  M O N E Y          1 0 6 5 2
```

The rules of alphametics are:

1 Each letter represents a unique digit.

2 Numbers must not start with a zero.

3 The solution is unique (unless otherwise stated).

Strategy options

- Guess and check.
- Work backwards.
- Look for a pattern.

Use the example and rules above to solve the following alphametics.

1
```
    E A T
+   T H A T
-----------
  A P P L E
```

2
```
    H E R E
+     S H E
-----------
  C O M E S
```

3
```
    M E M O
+   F R O M
-----------
  H O M E R
```

4
```
    C R O S S
+   R O A D S
-----------
  D A N G E R
```

Technology Exploration Excel

Equipment required: 1 brain (but you do need someone to try the trick on), 1 computer with an Excel spreadsheet program

Versions of this Exploration for other technologies are available in Pearson Reader.

How tricky was that!

You have probably seen people on television claiming to be able to read minds. Here, we will show you how you can read someone else's mind. Go home and amaze your parents tonight!

Step 1 Get your partner to think of a number from 1 to 10.

Step 2 Tell them to double it and then add ten.

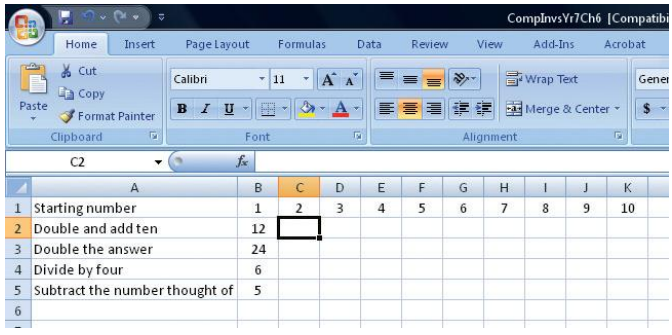
Step 3 Double this answer.

Step 4 Divide the new answer by four.

Step 5 Subtract the number you started with.

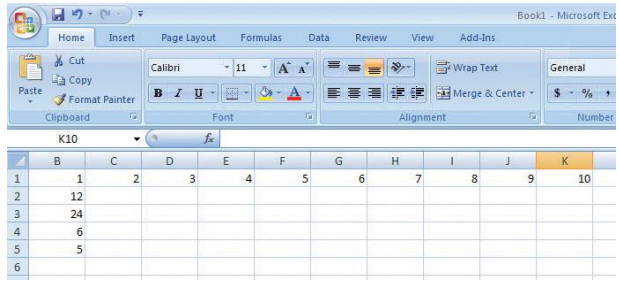
You should then wave your hands about a bit before announcing that the answer is five. Watch the amazed look on your partner's face.

We will set up a spreadsheet to check that this does, indeed, always give an answer of five.



	A	B	C	D	E	F	G	H	I	J	K
1	Starting number	1	2	3	4	5	6	7	8	9	10
2	Double and add ten	12									
3	Double the answer	24									
4	Divide by four	6									
5	Subtract the number thought of	5									
6											

1. In row 1, enter the numbers 1 to 10 in columns **B** to **K** as shown.
2. We need to enter a formula into each of the cells B2, B3, B4 and B5 that will perform the operation required in cells A2, A3, A4 and A5. Because the operation required in A2 is to double and add 10, the formula for B2 is '=B1*2+10'. Enter an appropriate formula for each of the other cells into column B. To check that you have them correct, you can fill in the rest of the cells using the fill tool. (To do this, highlight cells B2, B3, B4 and B5, then using the little black cross on the bottom right-hand corner of cell B5, drag the mouse across until column **K** is reached.) If you are correct you should get 5 as the last entry in every column.



	B	C	D	E	F	G	H	I	J	K
1	1	2	3	4	5	6	7	8	9	10
2	12									
3	24									
4	6									
5	5									
6										

How did this work? A little algebra will help us here. We will let x stand for the number our partner thought of initially.

The starting number	$= x$
Double the number and add ten	$= 2x + 10$
Double the result	$= 2(2x + 10)$
Factorise by taking out the common factor	$= 2 \times 2(x + 5)$
	$= 4(x + 5)$
Divide this result by four	$= \dfrac{4(x+5)}{4}$
	$= x + 5$
Subtract the number first thought of from your answer	$= x + 5 - x$
	$= 5$

Aha!

3. Here is another one of these tricks. Think of a number from 1 to 10 and triple it. Then, add nine and double that answer. Then, divide by six and take away three. You should be left with the number you started with.
   - **(a)** Construct a spreadsheet to demonstrate that this works.
   - **(b)** Use algebra to explain and prove how this works.
4. You should be getting good at these by now. Again, think of a number from 1 to 10. Add two to the number and then double the result. Add five and then add the number you started with. Divide by three and subtract three more than the number you started with. The answer should be zero.
   - **(a)** Construct a spreadsheet to demonstrate that this works.
   - **(b)** Use algebra to explain and prove how this works.

5 (a) Make up your own mind-reading number rule.

(b) Construct a spreadsheet to demonstrate that this works.

(c) Use algebra to explain and prove how your trick works.

Taking it further

Not much is known about Diophantus, the Greek mathematician who became known as the 'Father of Algebra'. Some scholars believe he lived sometime between 100 AD and 400 AD. One of his admirers wrote the following riddle about Diophantus.

Diophantus' youth lasted one-sixth of his life. He grew a beard after half more. After another one-seventh of his life he married. Five years later he had a son. His son lived exactly half as long as his father and Diophantus died only four years after his son. All of this adds up to the years Diophantus lived.

(Note that the sentence in the riddle '*He grew a beard after half more*' refers to half of his youth, not half of his life.)

6 Can you solve the riddle to find how long Diophantus lived?

Step 1 We are going to use Excel to help solve the riddle. On a new spreadsheet, in cell A1, enter a guess for the number of years Diophantus lived; for example, try 75.

Step 2 Reading the riddle you should be able to see that Diophantus' youth was one-sixth of his life.

Step 3 In cell B1, enter =A1/6. Because he grew a beard after half more, enter =A1/12 in cell C1.

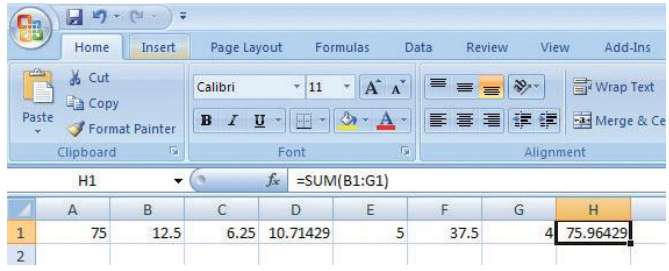
Step 4 Because he married after another one-seventh of his life, enter =A1/7 in cell D1.

Step 5 Because he had a son 5 years later, enter 5 in cell E1.

Step 6 Because his son lived exactly half as long as his father, enter =A1/2 into F1.

Step 7 Because Diophantus died 4 years later, enter 4 into cell G1.

Step 8 Add all the values from B1 to G1 using =SUM(B1:G1) in cell H1.



Step 9 If the answer in H1 is the same as the guess in A1, you have guessed correctly. As you can see, 75 is not the correct answer. Try another guess by changing the value in A1. Keep doing this until you guess correctly.

Step 10 Write down how old Diophantus was when he died.

7 Write the information as an equation and solve it to find how old Diophantus was when he died.

Solving problems using equations

7.5

In everyday life, we are faced with problems that can be solved with equations. We need to be able to write equations to represent the problem and then solve them with equation-solving techniques.

A simple process for solving problems using algebraic equations is:

Step 1: Identify the variable that needs to be found and represent it with a pronumeral.

Step 2: Use the information to write an equation, focusing on key phrases, such as 'is more than', 'is less than' and 'triple the amount'. Turn these phrases into corresponding operations, remembering the correct order of operations and ensuring that you convert all given amounts to the same units.

Step 3: Use inverse operations to solve the equation.

Step 4: Check that your answer is the solution by substituting it into the original equation and evaluating.

Worked Example 13

WE 13

For the following situation, write an equation to show the situation, define the pronumeral clearly, and then solve the equation.

Jared has \$6.40 to spend on lollipops that cost 40 cents each. How many lollipops can he buy?

Thinking	Working
1 Identify the variable and represent it with a pronumeral.	Let n represent the number of lollipops.
2 Use the information to write an equation ensuring that you convert all given amounts to the same units. (\$6.40 = 640c)	$40n = 640$
3 Use inverse operations to solve the equation. ($\div 40$)	$\frac{40n}{40} = \frac{640}{40}$ $n = 16$ Jared can buy 16 lollipops.
4 Check that your answer is the solution.	LHS $= 40n$ $= 40 \times 16$ $= 640$ $=$ RHS Therefore, $n = 16$ is the solution.

Worked Example 14 WE14

A repair company charges a fixed travel fee of \$50 and an hourly rate of \$30 for labour on each job completed in a client's home. If h is the number of hours a job takes:

(a) write an expression for the total cost of the job.

(b) Use an equation to find the number of hours the job took if the total cost of the job was \$200.

Thinking	Working
(a) 1 Identify the unknown and represent it with a pronumeral.	(a) Let h represent the number of hours.
2 Form an expression around the pronumeral.	$30h + 50$
(b) 1 Form an equation using the information given.	(b) $30h + 50 = 200$
2 Use inverse operations to solve the equation. (-50 first, then $\div 30$)	$30h + 50 - 50 = 200 - 50$ $30h = 150$ $\frac{30h}{30} = \frac{150}{30}$ $h = 5$ The job takes 5 hours.
3 Check by substitution that you have found the solution.	Substitute $h = 5$ into the original equation. LHS $= 30h + 50$ $= 30 \times 5 + 50$ $= 150 + 50$ $= 200$ $=$ RHS Therefore, $h = 5$ is the solution.

7.5 Solving problems using equations

Navigator

Answers page 633

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q13, Q15, Q16, Q17, Q22, Q24	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q14, Q15, Q18, Q19, Q20, Q23	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q11, Q12, Q15, Q16, Q17, Q19, Q21, Q23

Fluency

WE13

1 For each of the following situations, write an equation to show the situation, define the pronumeral clearly, and then solve the equation.

(a) Elena buys 3 apples for \$1.17. What is the cost of each apple?

(b) The mass of a shark pup is 935 kg less than that of its mother. If the pup has a mass of 9 kg, what is its mother's mass?

(c) A bus has the capacity to seat 49 passengers. If 21 passengers have boarded the bus, how many seats are still available?

(d) After a cool change, the outside temperature fell 9°C to a temperature of 27°C. What was the outside temperature before the cool change arrived?

2 Darren walked to the park while Ann rode her bicycle along the bike path to the park. If Darren walked x km while Ann rode 7 km: WE14

(a) write an expression for the total distance travelled by both.

(b) Use an equation to find the distance walked by Darren if together they travelled a total distance of 11 km.

3 Alan bought a salad roll and a container of orange juice for his lunch.

(a) If the salad roll cost \$$x$ and the orange juice cost \$2, write an expression for the total cost of Alan's lunch.

(b) Use an equation to find the cost of the salad roll if Alan spent \$4.50 on his lunch.

4 Olivia spent $\frac{3}{4}$ of her weekly pay from a part-time job on a new shirt.

(a) If Olivia's weekly pay is \$$w$, write an expression for the cost of the shirt.

(b) If the shirt cost \$24, solve an equation to find how much Olivia is paid each week.

5 The Kittens basketball team won 14 games more than it lost during the last season.

(a) If the team lost x games during the season, write an expression for the total number of games the team played.

(b) Use an equation to find the number of games won and lost by the Kittens if they played a total of 20 games in the season.

6 **(a)** Daniel is travelling at a speed of 28 km/h on his bicycle. How far does he travel in 2.5 hours if he maintains this speed?

If d represents the distance (in km) Daniel travels in 2.5 hours, the equation to solve is:

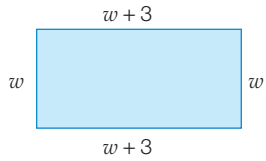
A $d + 2.5 = 28$ **B** $\frac{d}{2.5} = 28$ **C** $d = \frac{2.5}{28}$ **D** $2.5d = 28$

(b) Melita buys 5 lamingtons from the local bakery and receives \$5.50 change from \$10. Find the cost of each lamington.

If c represents the cost of a lamington (in cents), the equation to solve is:

A $5c + 10 = 5.5$ **B** $5c + 550 = 1000$ **C** $5c + 1000 = 550$ **D** $5c + 550 = 10$

(c) The perimeter of a rectangular swimming pool is 14 m. The pool is 3 m longer than it is wide, as shown in the diagram opposite.



If w represents the width of the pool (in m), the equation to find the width of the pool is:

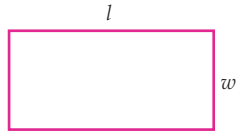
A $w + 3 = 14$ **B** $4w + 3 = 14$ **C** $4w + 14 = 6$ **D** $4w + 6 = 14$

7 Six friends share a bag of jelly beans equally. After dividing them up, there is one jelly bean left over.

(a) If each person received x jelly beans, write an expression for the total number of jelly beans in the bag.

(b) Use an equation to work out how many jelly beans each person received if there were 25 jelly beans in the bag.

8 **(a)** For the rectangle shown, where l is the length of the rectangle and w is the width, write an expression for the perimeter.



(b) Use the expression for perimeter to form an equation to solve for each of the following problems.

(i) Cameron has a 10 m length of wire fencing he wishes to use to protect a rectangular vegetable garden. If the width must be 2 m, how long can the vegetable garden be?

(ii) Cameron finds an extra 5 m of wire fencing, so, instead of having a width of 2 m, the vegetable garden is to have a width of 3 m. Find the new length of the vegetable garden.

For each of the following questions, form an equation and solve it to find the answer.

9 A birthday card and wrapping paper cost a total price of \$5.65. If the card cost \$2.90, how much did the wrapping paper cost? (Write 5.65 on the RHS of your equation.)

10 Samir has \$70. At a sale, he buys three CDs (marked at the same price) and receives \$11.50 change. How much did each CD cost? (Write 70 on the RHS of your equation.)

11 Megan buys three ice-creams and four drinks for her family at a cost of \$12. If each drink costs \$1.20, find the cost of each ice-cream. (Write 12 on the RHS of your equation.)

Understanding

12 It took Sasha two hours and twelve minutes to get between two towns while travelling to Queensland. The computer in the car told her that her average speed was 97 km/h. Sasha forgot to reset the odometer and she wants to know how far the two towns are apart.

(a) Write an equation to show this situation, using d to represent the distance in km.

(b) Find the distance that Sasha travelled between the two towns.

For each of the following, write an equation and solve it to find the answer.

13 A farmer needs to fence off a rectangular paddock, but must use 4 layers of wire at different heights to keep her piglets in. She uses a 110 m roll of wire and has 2 m of wire left over. Find the perimeter of the piglet pen.

14 Jack spent 7 hours at the beach. If he relaxed in the shade reading for three more hours than he surfed, for how long did he surf? Hence, find the time he spent reading.

15 The perimeter of a rectangular block of land is 74 m. The width is 7 m less than the length. Find the dimensions of the block of land. (First, draw a diagram.)

16 Natalie bought 5 cartons of milk and received \$2 in change. If she gave \$10 to the shopkeeper, find the cost of each carton of milk.

Reasoning

17 Yvonne buys a car that costs \$23 000. She pays a deposit of \$14 000 and then arranges to pay an amount each month for the next three years. Use an equation to find how much each equal monthly repayment should be if she wishes to pay off the car in three years.

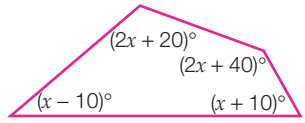
18 At the end of the term each student in the class was given their average test score in mathematics. During the term the class sat three tests. For two of her tests Leonie obtained results of 93% and 85%, but she has forgotten her result in the third. Use an equation to find this unknown result for Leonie if she scored an average of 91%.

19 A rectangle has an area of 27 cm^2. Its width is 4 cm, and its length is longer than its width. Write an equation to show this situation, and find its length.

20 A boy is twice as tall as his little sister and 30 cm shorter than his father. The combined height of the three family members is 3.8 m. Find (in cm) the height of each person.

21 The sum of three consecutive even numbers is 102. Find the numbers by using an equation.

22 The angles inside a quadrilateral add up to 360°. For the following quadrilateral, write an equation to determine the value of x and then find the value of the four angles.



Open-ended

23 Two plumbers quote for a job to renovate a bathroom. Plumber Alan quotes \$1050 for material and \$50 per hour for labour. Plumber Barry quotes \$1200 for material and \$45 per hour for labour. Choose different numbers of hours and decide which quote would be cheaper for those hours. For what number of hours will the quotes be the same?

24 The formula for the area of a triangle is:

$$A = \frac{1}{2} \times \text{base} \times \text{height}$$

Identify at least three different sets of base length (b) and height (h) to give an area of 64 m^2.

Outside the Square — Problem solving

The perfect holiday

The cost of a cruise ship holiday from Totally Oz Tours consists of a booking fee of \$150 and a charge of \$25 per day. The cost of food is an additional \$15 per day and there is a one-off beverage charge of \$100 for the duration of the cruise.

A rival company, Fun in the Sun Cruises, charges \$200 as a booking fee and \$45 per day (including food and beverage charge).

(a) After how many days does each company charge the same amount?

(b) Explain under which circumstances either company would be a cheaper option.

Strategy options

- Make a model.
- Guess and check.
- Test all possible combinations.

Mathspace

INTO THE MATRIX

"Wow, Alpha, check this out!" Raydon exclaims while pointing at the history text that he is reading. "Back in 2010, students used to play 3D computer games. How ancient is that?" "I know," says Alpha, "and did you know that they were outside of the game and had to watch it on a screen?" The boys are interrupted by Alpha's mother: "Boys, stop reading and go and play *4D Mario Sisters*; you've got a test tomorrow."

Level 1: Equation labyrinth

Alpha's character is stuck in the Equation Labyrinth. He needs to determine the correct order of operations to move to the next level.

Example move

$2 = \frac{\square}{3} - 6$

For the above equation, imagine Alpha is the variable. In order for Alpha to move you need to find the operations that would 'solve' him in the right order.

In this case, the operations are + 6, × 3

List the operations required (in the correct order) to solve each of the following:

(a) $5 = 3 \times \square$

(b) $3 = \frac{\square}{4}$

(c) $1 = \square + 3$

(d) $7 = 4 \times \square - 7$

(e) $6 = \frac{6 \times \square}{-2} - 3$

Alpha needs to escape the maze with the correct code in order to conquer the Level 1 enemy.

Use your answers for questions **(a)** to **(e)**, in the correct order, to find a path through the maze. You must start in a corner and you cannot move diagonally. The letters in each of the squares you pass through will spell the level 1 code.

M + 7	U − 3	Q ÷ -2	E ÷ 6	A × 3
I ÷ 4	T − 7	B + 3	L × -2	T × 6
R − 3	I + 7	A ÷ 4	S + 6	U × 2
A × 4	W + 3	N − 3	U + 7	N × 4
V ÷ 3	P ÷ 4	S + 3	Q ÷ 6	U − 5

What is the level 1 code?

Level 2: Troopers in disguise

The Equation Troopers and Nipping Solvers are hiding in disguise ready to capture players.

Raydon must hop on the right bricks along the path to avoid being captured.

If you substitute positive whole numbers for x and y into the following statements, then they will either *always* be true, *sometimes* be true or will *never* be true.

Equation Troopers are hiding behind the *sometimes* true statements and Nipping Solvers are hiding behind *never* true statements.

Help Raydon to find them so that he can identify the safe bricks and complete the level. Join the safe bricks to form a path from start to finish.

START

$x < 2x$	$x^2 < x^3$	$x^2 + y^2 < xy$	$2(x + 4) = 2x + 8$	$x + 3 > -4$
$3x > x^2$	$5x + 2 > 2(2x + 1)$	$y - 2 > y - 3$	$5 - y < y - 5$	$x^y < 0$

FINISH

Level 3: Who won?

To find who has the highest ★ score, solve the following equations (assume ★ scores must be positive).

Game scores:

Alpha's score: $5★ + 2 = 158 + 2★$

Raydon's score: $\frac{6★}{7} - 12 + 3★ = 30 + 3★$

Bonus challenge

$\frac{1}{3}\frac{(★^2 - 7)}{2} + 1 = 8$

Maths 4 Real

Virgin Galactic

'A brief moment of quiet before a wave of unimaginable, but controlled, power surges through the craft. You are instantly pinned back into your seat, overwhelmed but enthralled by the howl of the rocket motor and the eye watering acceleration.'

Virgin Galactic is the first spaceline (an airline that offers trips into outer space for paying passengers).

Escaping gravity

A rocket needs to reach a certain speed in order to escape a planet's gravity. Aeronautical engineers and astronauts need to take this into account when designing and piloting spacecraft.

The formula for working out the escape speed (in km/s) is:

$$v = \sqrt{\frac{2b}{r}}$$

where:

- v = the escape velocity (the speed, in km/s, that the rocket must reach in order to leave a planet's atmosphere)
- r = the radius of the planet in kilometres
- b = a constant, the value of which is dependent on the mass and gravitational force of the planet.

1 Calculate the escape velocity in kilometres per second (km/s) needed to leave these planets.

Planet	b	r (km)
Earth	398 600	6378
Mercury	23 916	2440
Mars	42 828	3397

2 Which planet has the slowest escape velocity? Why do you think this is so?

3 If the escape velocity of Venus is 10.37 km/s, and its b value is 325 156, what is the approximate radius of Venus, to the nearest km?

Cost of travel

If the cheapest tickets on Virgin Galactic cost \$200 000, and the space flight takes $2\frac{1}{2}$ hours:

4 how many dollars per minute does the space flight cost each passenger?

5 Write a formula for the income that Virgin Galactic receives in dollars (I), in relation to the number of paying passengers n it takes.

6 The rockets are designed to carry six passengers and Virgin Galactic only runs the flight if all the seats are booked.

(a) Using your formula from Question **5**, what income will Virgin Galactic make from one flight?

(b) Virgin Galactic initially plans to have one flight per week. What is the income Virgin will make in the first year?

(c) If, by the fifth year, they are running two flights per day, what is the income that Virgin Galactic will make? (Assume this is a non-leap year.)

(d) Imagine that one year Virgin Galactic's income was \$38 220 000. How many flights were completed in that year?

7 Building, maintaining and flying rockets costs millions of dollars. Imagine it costs Virgin Galactic approximately \$960 000 in maintenance, fuel and wages per flight. Assuming that each flight always carries six passengers, what would be the costs per paying passenger per flight?

8 On top of the costs mentioned in Question **7**, Virgin Galactic also plans to have annual development costs of \$6 million. Write a formula for the cost per annum (C) that Virgin Galactic would incur in relation to the number of paying passengers it takes, n.

9 Profit = income – costs. Using your formulas from Question **5** and Question **8**, create a formula for Virgin Galactic's annual profit (P).

10 What is the minimum number of passengers, and hence the minimum number of flights, that Virgin Galactic needs to run per year in order to break even (i.e. have a profit of \$0)?

Challenge 7

1 Mike is twice as old as his brother Jamie. When Mike turns 15, the sum of his and Jamie's ages will be 25. How old are Mike and Jamie now?

2 R, S, T and U are all whole, positive numbers. If $R \times S = 15$, $S \times T = 3$ and $T \times U = 7$, find the values of $R \times U$ and $S \times U$.

3 If $\frac{4}{h} = \frac{h}{36}$, find the value(s) of h.

4 Amber purchases 5 tyres for her car. If each tyre can be used for 16 000 km, how many kilometres can Amber travel using these five tyres?

5 The four numbers a, b, c and d add to give 30. b is double a, c is a third of b and d is 4 more than c. What are the numbers?

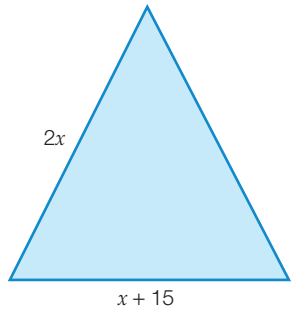
6 A square and an equilateral triangle have equal perimeters. If the side length of the triangle is 1 cm longer than the side of the square, what is the length of the side of the square?

7 If $d + e = 15$ and $e + f = 12$ and $d + e + f = 20$, find d, e and f.

8 The equilateral triangle has sides $2x$ cm and $(x + 15)$ cm as shown. The perimeter of the triangle is:

A 15 cm **B** 30 cm

C 60 cm **D** 90 cm



9 The height of a tree is measured annually. It was x metres high when planted, and has grown by exactly y metres between each annual measurement. At its latest annual measurement it was 23 metres high. Which of the following are possible?

A $x = 6, y = 2$ **B** $x = 7, y = 5$ **C** $x = 4, y = 7$ **D** $x = 5, y = 3$

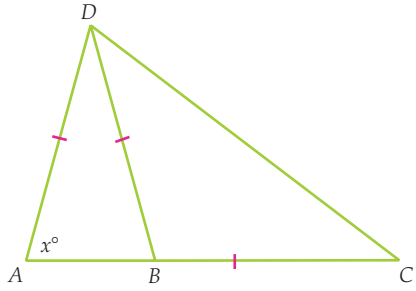
10 In a test, Sue answered 15 out of the first 20 questions correctly. Of the remaining questions, she answered exactly one-third of them correctly. If all the questions were worth the same number of marks and Sue scored 50% on the test, how many questions were in the test?

11 ACD is a triangle in which B lies on the side AC such that $AD = DB = BC$. $\angle DAC = x°$. All angles in the diagram are a positive whole number of degrees.

(a) Find the value of x for which $\angle ADC$ has its largest value.

(b) What is the largest value of $\angle ADC$?

(c) What is the smallest value of $\angle ADC$?



Chapter review 7

D.I.Y. Summary

Key Words

backtracking	flowchart	inverse operation	solving
balanced	guess, check and improve	point of intersection	
equivalent	inspection	solution	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 The _____ of an equation is the value of the variable that makes the equation true.

2 A _____ is a way of solving an equation by using a diagram and then _____ to find the solution.

3 When we solve an equation, we perform the same operations on each side of the equals sign so that the equation is _____.

4 Two equations are said to be _____ if they have the same solution.

5 Addition and subtraction are examples of _____s.

Fluency

1 Write an equation for each of these sentences. Ex.7.1

(a) A number is multiplied by three, then four is subtracted to give a result of two.

(b) A number is added to two, then the answer is divided by 5 to give a result of six.

2 Draw a flowchart and use backtracking to solve the following equations. Ex.7.2

(a) $4x + 3 = 11$ (b) $3(x - 2) = 9$ (c) $\frac{x+5}{4} = 2$ (d) $\frac{x}{7} - 6 = -3$

3 Solve each of the following by performing the same operation on both sides of the equation. Ex.7.2

(a) $2x + 5 = 3$ (b) $11 - 3b = 5$ (c) $19 = -4c - 1$

4 (a) The solution to the equation $3x - 2 = 7$ is: Ex.7.2

A $x = 1$ B $x = 2$ C $x = 3$ D $x = 4$

(b) The solution to the equation $\frac{x}{2} + 1 = -2$ is:

A $x = -2$ B $x = -3$ C $x = -4$ D $x = -6$

(c) The solution to the equation $\frac{x-4}{4} = 2$ is:

A $x = 12$ B $x = 24$ C $x = -4$ D $x = 8$

5 Solve each of the following by performing the same operation on both sides of the equation. Ex.7.2

(a) $\frac{a}{3} + 7 = 10$ (b) $\frac{b-6}{5} = -1$ (c) $\frac{3c}{2} = 12$

6 Solve each of the following equations using algebra. Ex. 7.3

(a) $\frac{4x-1}{3}=5$ (b) $6-\frac{x}{5}=1$ (c) $\frac{5x}{4}+3=13$ (d) $\frac{1-2x}{7}=3$

7 Solve each of the following equations using algebra. Ex. 7.3

(a) $\frac{3m+4}{2}=5$ (b) $3-\frac{2n}{5}=1$ (c) $4(x+5)=32$ (d) $\frac{3(m+11)}{8}=-6$

8 Solve each of the following equations using algebra. Ex. 7.4

(a) $7x+2=6x-5$ (b) $3x+7=15-5x$

(c) $7(x+1)=2(x-4)$ (d) $3(5x+4)=4(2-3x)$

9 Tony has \$25 and saves \$5 a week. Use an equation to find how many weeks it will take him to save \$80. Ex. 7.5

10 Use equations to solve each of the following problems. Ex. 7.5

(a) Janis passes a \$10 note to the shopkeeper to pay for 5 cans of cola. He receives \$2.25 in change. Find the cost of each can of cola.

(b) Almira has set up a hot dog stand at the school fete. The ingredients for the hotdogs have cost \$60. If Almira sells each hot dog for \$3, how many does she need to sell to make a profit of \$24?

(c) A stick 1 m long is broken into two parts. If one piece is 9 cm longer than the other, how long are the pieces?

Understanding

11 Sam buys a new suitcase that is 70 cm wide. He discovers that two of his folders (which are identical) will fit next to each other in the suitcase with only 5 cm to spare.

(a) Write an equation to describe this situation, using f to represent the width of the folder.

(b) Find the width of the folder.

12 Three consecutive integers add to 24. Use an equation to find the three numbers.

13 Robert has 50 coins, all 5 cent and 10 cent pieces, amounting to \$3.50. By using an equation, find how many of each type of coin he has.

14 Use an equation to find x in the magic square shown. As with all magic squares the sum of each row, column and diagonal is the same.

$4x$	$6x+2$	28
$6x-2$	30	$2(2x+1)$
$4(x+2)$	$4x-2$	$6x$

15 If Peter is 7 years older than Sharon and John is twice as old as Peter, find how old Peter is if the mean of their ages is 19.

Reasoning

Use an equation to solve each of the following.

16 (a) In 5 years time, Maja's age will be twice what it was 6 years ago. How old is Maja now?

(b) Jake rides a motorbike. He travels to and from work three times in a week. He also travels to and from his girlfriend's place, which is one-third of the distance to work, four times a week. In total, he travels 156 km each week. How far is it from his home to work?

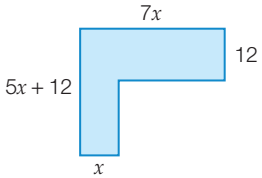
(c) Jung spent a quarter of his money on a new shirt. He then gave a donation to charity of \$3 and deposited half of the remaining money in the bank. If he is left with \$15, how much did he start with? Write an equation first using s to represent the starting money.

17 The sum of two numbers is 37. If the larger number is divided by the smaller number, the result is 3, remainder 5. Find the numbers.

18 An isosceles triangle has a perimeter of 45 cm. Its equal sides each measure 6 cm longer than the non-equal base. How long is the base of this particular triangle?

19 Three students take different routes home from school. Mark needs to walk twice as far as Tom, and Fred walks four times as far as Tom. If the three cover a total distance of 3.5 km (or 3500 m), how far does each person walk?

20 The shape at right has a perimeter of 120 cm. Use the information provided in the question to generate an algebraic equation, and then use it to find the value of x cm.



21 **(a)** Using the magic square, find the magic sum in terms of a, b and c. Hence, determine the expressions that belong in Box A and Box B. Remember, as with all magic squares, the sum of each row, column and diagonal is the same.

(b) If $a = 3$, $b = 2$ and the magic sum is 27, use an equation to find c. Hence, replace the pronumerals with numbers in each of the boxes.

$a + b$	$a + 2c$	$a + c + 2b$
$a + 2b + 2c$	$a + c + b$	Box A
$a + c$	Box B	$a + 2c + b$

22 A triangular garden bed has its three sides measured at x metres, $(x + 4)$ metres and $(x + 5)$ metres. The perimeter of the garden bed is measured to be 51 metres. Find the lengths of all three sides of the garden bed.

NAPLAN practice 7

Numeracy: Non-calculator

1 What is the value of p in this equation? $13p - 8 = 8p + 7$?

2 The equation that best represents the following 'I am thinking of a number—when I add 4 the number is 13' is:

A $x + 13 = 4$ **B** $x - 4 = 13$ **C** $x + 4 = 13$ **D** $x - 13 = 4$

3 What is the value of x in the equation $4x + 7 = 3$?

A $x = -1$ **B** $x = 1$ **C** $x = 2$ **D** $x = 11$

4 If $y = 2x - 1$ and $y = 3x + 2$, which value of x satisfies both of these equations?

A $x = -3$ **B** $x = -1$ **C** $x = 1$ **D** $x = 3$

5 The digits 4, 5 and 6 are placed in the equation $2x + y - z = 8$. If y is 4, what is the value of z?

Numeracy: Calculator allowed

6 Robert and Fergus shared a bag of Smarties. If Fergus ate 18 more Smarties than Robert did, and there were 66 Smarties in the bag, how many Smarties did Fergus eat?

7 The solution to the equation $6x - 1 = 4x + 5$ is:

A $x = -2$ **B** $x = 2$ **C** $x = 3$ **D** $x = 4$

8 Find the value of x that makes the equation $\frac{x + 3}{4} = 5$ true.

9 The cost in dollars to make k cakes is $8 + 3k$.

How many cakes are made for a cost of \$203?

10 Phil is 3 years older than his sister Bridget. The product of their ages is the same as their mother's age. If Phil's mother is 40 years old, how old is Bridget?

8

Geometry 8

What do we use to build a better stadium? **Triangles, of course.** Melbourne's newest stadium features the humble triangle in its award-winning design.

AAMI Park is Melbourne's most innovative sports stadium, hosting soccer and rugby matches as well as other events. The stadium seats 30 000 people and incorporates extensive training facilities, including gyms, a pool and a boxing ring. The architects used a design made up of geodesic domes—spherical shapes made from small equilateral triangles. These triangles give the structure strength and stability, and are extremely light. This design also required 50 per cent less steel than a typical stadium roof of the same size. The geodesic domes act as a roof, walls and supports, without the need for extra pillars or walls. This means that spectators can enjoy the events inside the stadium with a clear view from wherever they are sitting. The design also allows for future modification of the triangular panels to solar panels, making it possible for the stadium to generate its own electricity in a sustainable way.

Forum

Why do architects use triangles in their designs?

Can you think of any other buildings that have triangles in their design?

What makes a triangle such a strong shape?

If you were designing a stadium, what factors would you need to take into account?

Why learn this?

Don't underestimate the power of a simple triangle! A knowledge of geometry helps us to accurately describe the world we live in and the natural and man-made objects around us. Understanding geometric principles allows people to design and build things that are strong, efficient and safe. This chapter will help you develop your skills in identifying and using the properties of triangles and quadrilaterals.

After completing this chapter you will be able to:

- revise and develop skills in identifying and applying angle and side properties of triangles and quadrilaterals
- use geometric symbols and notation
- use transformations to produce congruent shapes
- identify corresponding sides and angles in congruent figures of different orientations
- use the four congruency tests to show that two triangles are congruent
- use congruent triangles and angle properties to prove the properties of special quadrilaterals.

Recall 8

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Equipment required: Protractor, ruler, grid paper

Worksheet R8.1

1 Draw angles of the following sizes using a protractor and a ruler.

(a) 48° **(b)** 127° **(c)** 235° **(d)** 315°

Worksheet R8.2

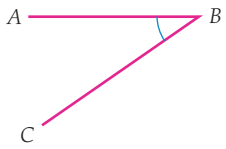
2 Name the type of each angle drawn in Question **1** from this list: acute; right; obtuse; straight; reflex.

Worksheet R8.3

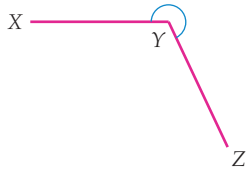
3 **(a)** Use a protractor to measure the following angles.

(b) Name the type of each angle.

(i)



(ii)



Worksheet R8.4

4 Solve for x.

(a) $x + 7 = 180$ **(b)** $2x + 40 + 3x = 360$ **(c)** $x + 20 + 5x - 80 = 360$

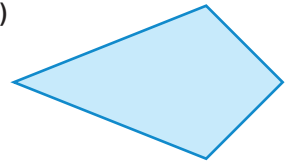
Worksheet R8.5

5 Copy the quadrilaterals below into your workbook. Give them their specific name and mark equal side lengths, parallel sides and equal angles.

(a)

(b)

(c)



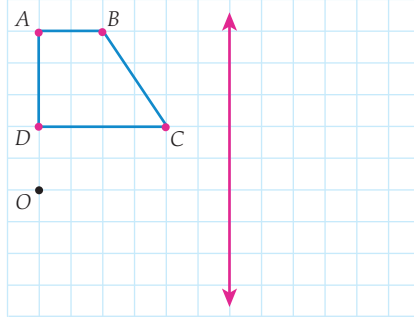
Worksheet R8.6

6 Use grid paper to perform the following transformations on the shape $ABCD$. Draw the transformed shape and label it $A'B'C'D'$.

(a) A translation of 5 units right and 3 units down.

(b) A reflection in the vertical line shown.

(c) A rotation of 90° clockwise around the point O.



Key Words

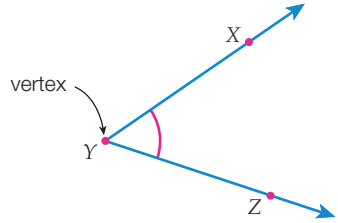
acute-angled triangle	congruence statement	hypotenuse	right-angled triangle
adjacent angles	congruent	included angle	scalene triangle
alternate angles	congruent triangle	interior angle	supplementary angles
bisect	corresponding angles	isosceles triangle	transversal
co-interior angles	equilateral triangle	obtuse-angled triangle	vertex
complementary angles	exterior angle	quadrilateral	vertically opposite angles

Angles review

8.1

An angle is formed when two lines (or rays, or line segments) touch or intersect at a point, known as the **vertex**. An angle is named using the symbol ∠ and the letters that mark the three points that form the angle. The letter at the vertex is written in the centre.

This angle is called $\angle XYZ$ or $\angle ZYX$.

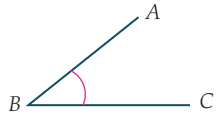


Angles at a point

Acute angle: Less than 90°

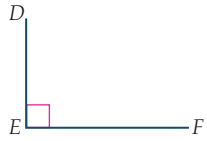
$0° < \angle ABC < 90°$

(We read the above symbols as: Angle *ABC* is greater than 0°, but less than 90°.)



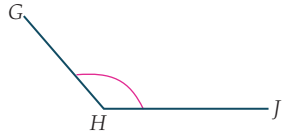
Right angle: Exactly 90°

$\angle DEF = 90°$



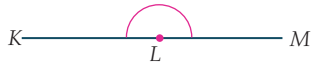
Obtuse angle: Greater than 90° but less than 180°

$90° < \angle GHJ < 180°$



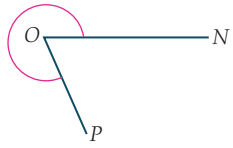
Straight angle: Exactly 180°

$\angle KLM = 180°$



Reflex angle: Greater than 180° but less than 360°

$180° < \angle NOP < 360°$

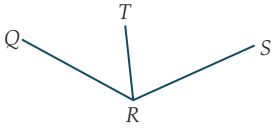


Revolution or angle of revolution: Exactly 360°

Pairs of angles

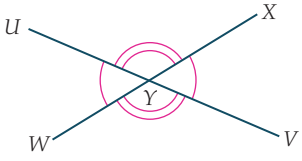
Adjacent angles have a common vertex and a common arm.

Vertically opposite angles are equal.



$\angle QRT$ and $\angle TRS$ are a pair of adjacent angles.

TR is an arm common to both angles.



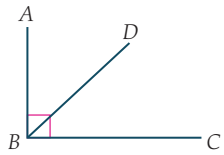
The lines *UV* and *WX* intersect at *Y*.

$\angle UYW = \angle XYV$ as they are a pair of vertically opposite angles.

$\angle UYX = \angle WYV$ as they are a pair of vertically opposite angles.

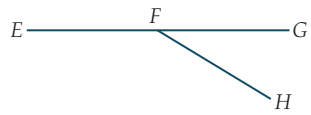
Complementary angles: a pair of angles that add to 90°.

Supplementary angles: a pair of angles that add to 180°.



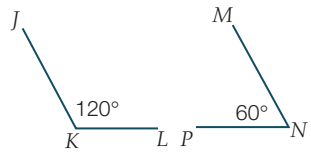
$\angle ABD + \angle DBC = 90°$

$\angle ABD$ and $\angle DBC$ are complementary angles.



$\angle EFH + \angle GFH = 180°$

$\angle EFH$ and $\angle GFH$ are supplementary angles.



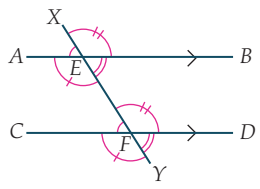
$\angle JKL$ and $\angle MNP$ are supplementary angles.

Angles on parallel lines

A **transversal** is a line that intersects two or more other lines. When the lines that the transversal intersects are parallel:

- pairs of **corresponding angles** are equal
- pairs of **alternate angles** are equal
- pairs of **co-interior angles** are supplementary.

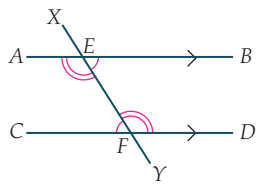
The lines AB and CD are parallel. We can write this as $AB \parallel CD$. The transversal XY intersects AD at point E and CD at point F.



Each of the four angles formed at E has an equal corresponding angle at F, lying on the same side of the transversal and the same position above or below the line. Each pair of corresponding angles are marked in the same way.

$\angle AEX = \angle CFE$ $\quad$ $\angle FEB = \angle YFD$

$\angle AEF = \angle CFY$ $\quad$ $\angle DFE = \angle BEX$



Two pairs of alternate angles are formed, lying on opposite sides of the transversal.

$\angle AEF = \angle EFD$

$\angle BEF = \angle EFC$

Two pairs of co-interior angles are formed, one on each side of the transversal, inside the parallel lines. Co-interior angles are sometimes called 'allied' angles.

$\angle AEF + \angle EFC = 180°$

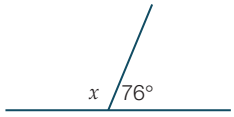
$\angle BEF + \angle EFD = 180°$

Worked Example 1 WE1

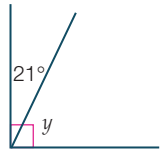
(a) What is the complement of 14°?

(b) What is the supplement of 53°?

(c) Find the value of x.



(d) Find the value of y.

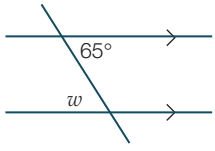


Thinking	Working
(a) Complementary angles add to 90°. Subtract the known angle from 90° to find the unknown.	(a) $90° - 14° = 76°$ The complement of 14° is 76°.
(b) Supplementary angles add to 180°. Subtract the known angle from 180° to find the unknown.	(b) $180° - 53° = 127°$ The supplement of 53° is 127°.
(c) x and 76° are supplementary angles. Write an equation that shows this, and solve it.	(c) $x + 76° = 180°$ (supplementary angles) $x = 104°$
(d) y and 21° are complementary angles. Write an equation that shows this, and solve it.	(d) $y + 21° = 90°$ (complementary angles) $y = 69°$

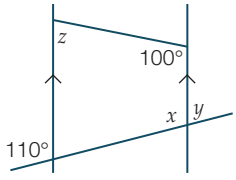
Worked Example 2 WE2

Find the value of the pronumerals in each diagram. Give reasons for your answer.

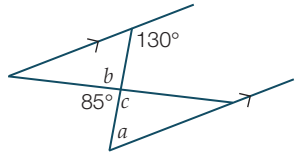
(a)



(b)



(c)



Thinking	Working
(a) Identify the relationship between the known and the unknown angle. Write this in brackets next to your answer as a reason for your answer.	(a) $w = 65°$ (alternate angles on parallel lines)
(b) 1 Identify relationships between known and unknown angles. Write these in brackets next to your answers as reasons for your answers.	(b) $x = 110°$ (corresponding angles on parallel lines) $z + 100° = 180°$ (co-interior angles on parallel lines) $z = 80°$
2 Once you have found the value of some unknowns, use them to determine others. (Here, we use the value of x to find y.)	$x + y = 180°$ (supplementary angles) $110° + y = 180°$ $y = 70°$

(c) 1	Identify relationships between known and unknown angles. Write these in brackets next to your answers as reasons for your answers.	(c) $a + 130° = 180°$	(co-interior angles on parallel lines)
		$a = 50°$	
2	Once you have found the value of some unknowns, use them to determine others. (Here, we find the value of c and use it to find b.)	$c + 85° = 180°$	(supplementary angles)
		$c = 95°$	
		$b = 95°$	(vertically opposite angles)

8.1 Angles review

Navigator

Answers page 635

Q1, Q2 (a)–(d), Q3, Q4, Q5, Q6, Q9, Q10, Q13	Q1, Q2, Q3 (a)–(d), Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q13, Q14	Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q11, Q12, Q13, Q14

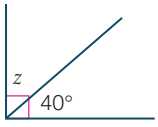
Fluency

WE1

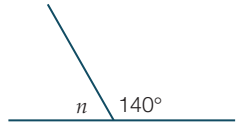
1 (a) What is the complement of 37°?

(b) What is the supplement of 105°?

(c) Find the value of z.



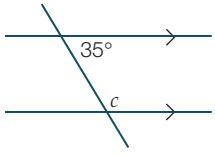
(d) Find the value of n.



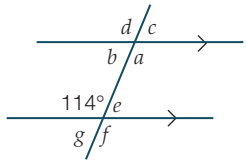
WE2

2 Find the value of the pronumerals in each diagram. Give reasons for your answer.

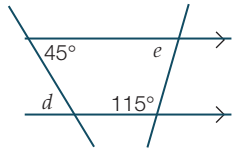
(a)



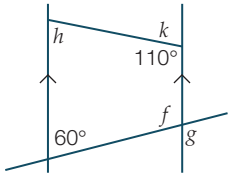
(b)



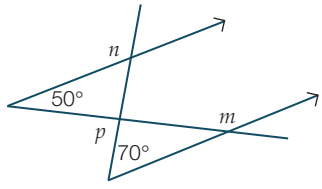
(c)



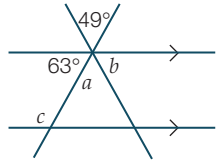
(d)



(e)

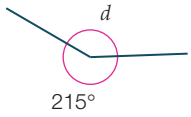


(f)

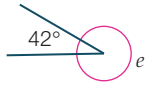


3 Find the value of the pronumerals in each diagram. Give reasons for your answer.

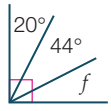
(a)



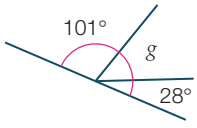
(b)



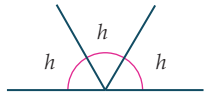
(c)



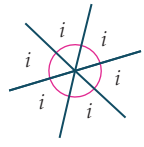
(d)



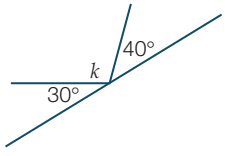
(e)



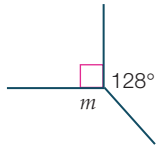
(f)



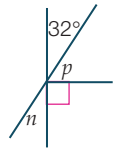
(g)



(h)

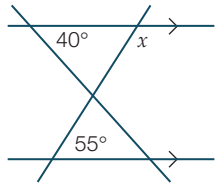


(i)



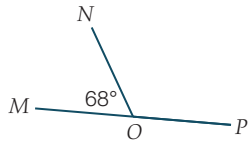
4 In the diagram, the value of x is:

A 85° B 95° C 125° D 140°



5 The value of $\angle NOP$ is:

A 102° B 112° C 122° D 132°



Understanding

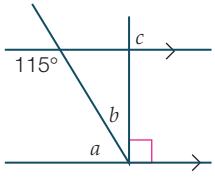
6 State whether the following sets of angles are complementary angles (C), supplementary angles (S) or neither (X).

(a) 30°, 60° (b) 50°, 80°, 50° (c) 45°, 55°

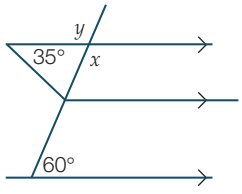
(d) 65°, 45°, 60° (e) 33°, 57° (f) 122°, 46°, 12°

7 Find the value of the pronumerals in each diagram. Give reasons for your answer.

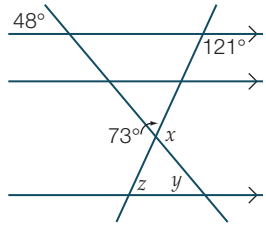
(a)



(b)



(c)



8 Write TRUE or FALSE for each of the following statements.

(a) Vertically opposite angles are always equal.

(b) Adjacent angles are always complementary.

(c) If a pair of adjacent angles is supplementary, then one of the angles must be an acute angle.

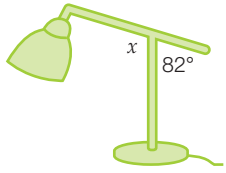
(d) If a pair of adjacent angles forms an angle of revolution, then one of the angles must be a reflex angle.

(e) Corresponding angles are always equal.

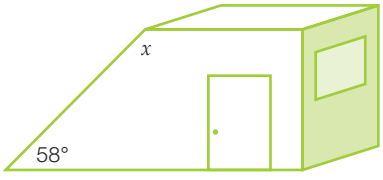
9 Find the missing angle in each of the following.

(a)

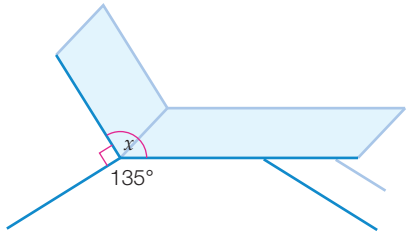
(b)



(c)



(d)

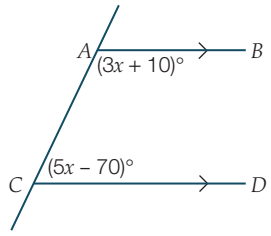


Reasoning

10 c and d are complementary angles. If c is 20° larger than d, what is the size of d?

11 The angles g and h are supplementary angles. g is twice as large as h. Find g and h.

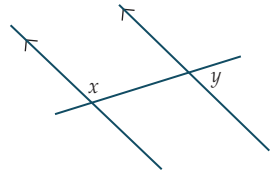
12 Find the value of x, giving reasons for your answer.



Open-ended

13 Write down three possible pairs of values for x and y in the diagram shown.

Compare your answers with those of others in your class. What do you notice?

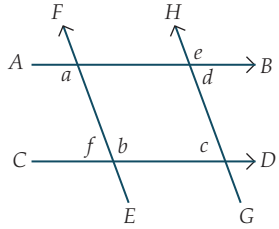


14 In the diagram, $AB \parallel CD$ (AB is parallel to CD) and $EF \parallel GH$ (EF is parallel to GH).

Some of a, b, c, d, e and f are equal.

(a) Write down all the pairs of equal angles you can find. Check your answer by substituting a value for a and calculating the values of the other pronumerals.

(b) Write all the pairs of supplementary angles you can find.

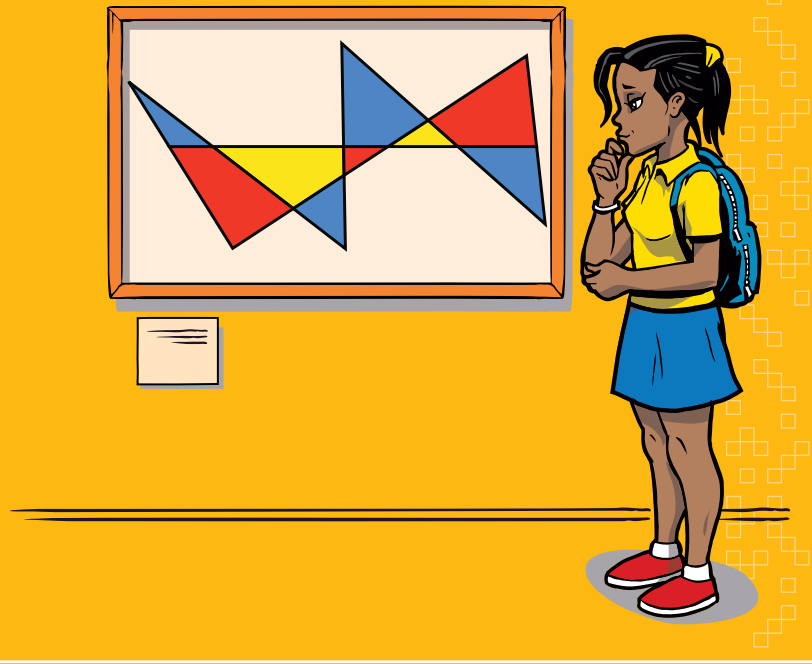


Outside the Square Puzzle

Angle art

Milla studied a painting at an art gallery. She noticed that the painting was created by using just seven straight lines, making nine closed regions. Each region was then coloured in.

Can you draw a design that has more than nine closed regions using just seven lines?



Properties of quadrilaterals

Versions of this Exploration for other technologies are available in Pearson Reader.

Equipment required: 1 brain, 1 computer with GeoGebra

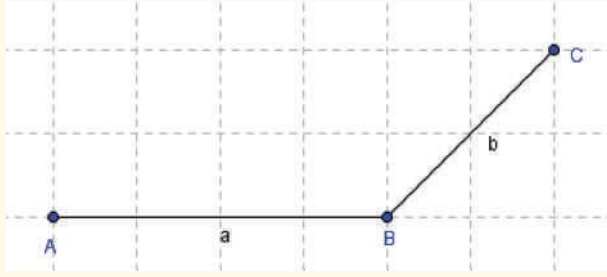
Open the GeoGebra program. You will see seven menu options (File, Edit etc.) at the top of the screen. Below these are 11 icons called tools. In this task, tools will be referred to by a number from the left. By clicking on the small arrow in the bottom right-hand corner of the tool icon, a drop-down list of more tools appears. The arrow turns red when you hover the cursor over it. If you hover over a tool, the tool's name and how to use it will appear in the top right-hand corner of the screen. (Note that some versions of GeoGebra differ in the name given to a tool; however, their icons remain the same.)

1. Click on the Options menu. Select 'Rounding', and '1 Decimal Place'.
2. Click on the View menu. Deselect 'Axes'. (If a larger font is required, click on the Options menu and select 'Font Size'. Choose an appropriate size from the list provided.)
3. Choose one shape to investigate (Parallelogram, Rectangle, Rhombus, Square, Kite and Trapezium).
4. Follow the steps for the construction of your shape. Then proceed through the 'Investigation' section.

Constructing a parallelogram

A parallelogram is a quadrilateral with both pairs of opposite sides parallel. We will construct a parallelogram and investigate its properties using the tools in GeoGebra.

(i) From the third tool, select the 'Interval Between two Points' tool . Create a horizontal interval *AB* from left to right. Create a second interval *BC* by first clicking on *B* and then on a new point *C*. Your diagram should look similar to this:



(ii) From the fourth tool, select the 'Parallel Line' tool . Click on interval *a* and then on point *C*. Repeat with interval *b* and Point *A*.

(iii) From the second tool, select the 'Intersect Two Objects' tool . Click directly on the intersection of the two new lines (lines *c* and *d*). A new point *D* will appear. Right click on line *c* and deselect 'Show Object'. Repeat this for line *d*.

(iv) Select the 'Interval' tool . Complete the parallelogram by clicking on points *A* and *D*, and *C* and *D* to create the intervals *AD* and *CD*.

Constructing a rectangle

A rectangle is a parallelogram with a right angle.

(i) From the third tool, select the 'Interval Between two Points' tool . Create a horizontal interval *AB* from left to right by clicking to create points *A* and *B*.

(ii) From the fourth tool, select the 'Perpendicular Line' tool . Click on point *B* and line *a*.

(iii) Select the 'Interval' tool . Click on point *B*, then on line *b*. Click on the grey circle next to the letter *b* in the Algebra View to hide line *b*. The circle should go clear. (The Algebra View is the panel on the left of the screen.) Because the opposite sides of a rectangle are parallel, we need to create two parallel lines. From the fourth tool, select the 'Parallel Line' tool . Click on interval *a* and then on *C*. Repeat with interval *c* and Point *A*.

(iv) From the second tool, select the 'Intersect Two Objects' tool . Click directly on the intersection of the two new lines (lines *d* and *e*). A new point *D* will appear. Right click on line *d* and deselect 'Show Object'. Repeat this for line *e*.

(v) Select the 'Interval' tool . Complete the rectangle by creating intervals joining points *A* and *D*, and *C* and *D*.

Constructing a rhombus

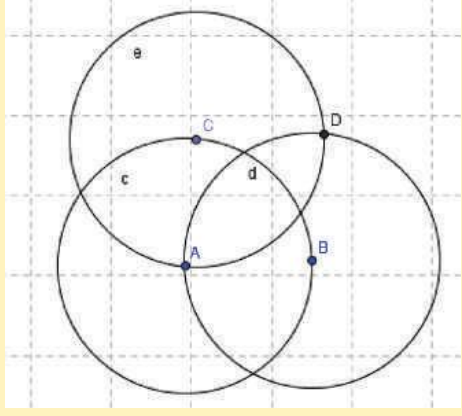
A rhombus is a parallelogram with a pair of adjacent sides equal.

(i) Select the 'Circle with Centre Through Point' tool . Draw a circle by clicking any two points. (The centre will be *A* and *B* and will lie in the circumference.)

(ii) From the second tool, select the 'New Point' tool and place a new point *C* on the circumference.

(iii) Select the 'Circle with Centre Through Point' tool . Draw a circle with its centre at point *B* and out to point *A*. Draw another circle within centre *C* and out to point *A*.

(iv) Click on the second tool from the left and select 'Intersect Two Objects' . Mark the point of intersection of the two new circles from step **(iii)**. Label this point *D*. Your diagram will look similar to this:



(v) Hide the circles by right clicking on them in turn and deselecting 'Show Object'.

(vi) For the purposes of the investigation later, we will need to rename points *C* and *D*. Right click on point *D* and select 'Rename'. Rename it *C*. This will change the original point *C* to *C'*. Right click on *C'* and rename it *D*.

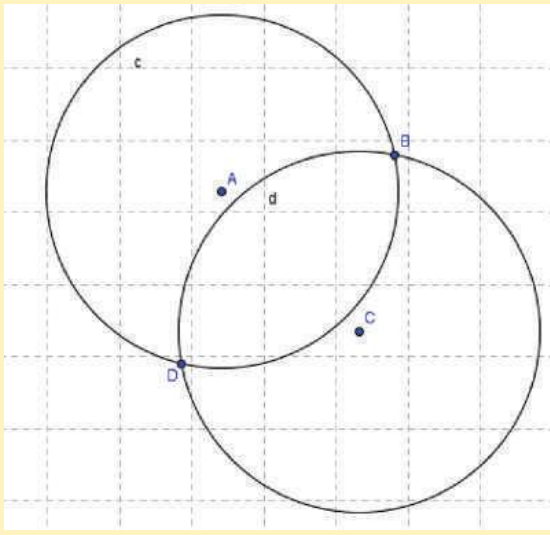
(vii) Select the 'Interval Between two Points' tool . Join *A* to *B*, *B* to *C*, *C* to *D* and *D* to *A* by clicking on these points in turn

Constructing a kite

A kite is a quadrilateral with two pairs of equal adjacent sides.

(i) Select the 'Circle with Centre Through Point' tool . Draw a circle by clicking any two points. (The centre will be point *A*, and point *B* will lie on the circumference.)

(ii) Repeat step **(i)** to create another circle with centre point *C* and out to point *B* as shown in the diagram.



(iii) From the second tool, select the 'Intersect Two Objects' tool . Click on the point of intersection of the two circles to create point *D*.

(iv) Hide the circles by right clicking on them in turn and deselecting 'Show Object'.

(v) Select the 'Interval Between two Points' tool . Join *A* to *B*, *B* to *C*, *C* to *D* and *D* to *A* by clicking on these points in turn.

Constructing a square

A square is a rectangle with a pair of adjacent sides equal.

It is also a regular quadrilateral (a regular polygon with four sides).

(i) From the fifth tool, select the 'Regular Polygon' tool . Click on two points in the same horizontal line from left to right. A pop-up box will appear. Make sure four points are selected and click 'OK'.

Constructing a trapezium

(i) From the third tool, select the 'Interval Between two Points' tool . Create a horizontal interval *AB* from left to right by clicking to create points *A* and *B*.

(ii) From the fourth tool, select the 'Parallel Line' tool . Click on interval *a* and anywhere above the line.

(iii) From the second tool, select the 'New Point' tool and place a new point *D* to the left of point *C*.

(iv) Right click on line *b* and deselect 'Show Object'.

(v) Select the 'Interval Between two Points' tool . Join *B* to *C*, *C* to *D* and *D* to *A*.

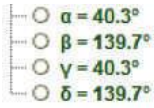
Once you have constructed a quadrilateral, complete the following.

5 From the eighth tool, select the 'Distance or Length' tool . Click on each of the intervals to display their lengths. What can you say about the lengths of opposite pairs of sides for your particular quadrilateral?

6 From the eighth tool, select the 'Angle' tool . Measure the size of each angle by clicking on pairs of adjacent lines. (If the reflex angle appears, use Ctrl + z to undo and then click on the lines in the opposite order.)

7 Resize the figure by clicking on the 'Move' tool and dragging any of the blue points. (Labels can also be moved in this way.) What can you say about the sizes of the opposite angles for your particular quadrilateral?

8 Hide the angle markers by clicking on the small grey circle next to each angle in the Algebra View. The circles become clear when the object is hidden from view.



9 Using the 'Interval' tool , create diagonals *AC* and *BD*.

10 Find the point of intersection *E* of the diagonals using the 'Intersect Two Objects' tool .

11 Measure the length of each diagonal using the 'Distance or Length' tool and clicking on the diagonal lines. (You may need to use the 'Move ' tool to move the labels away from the point of intersection.) Are the diagonals of your quadrilateral equal in length?

12 Using the 'Distance or Length' tool , measure the distances *AE*, *BE*, *CE* and *DE* by clicking on each pair of points. (You may need to use the 'Move' tool to move the labels to see them better.)

(a) The word bisect means to cut in half. Can we say that the diagonals bisect each other?

(b) Do you think that the diagonals meet at a right angle? Check by measuring $\angle AEB$.

Taking it further

From each group of properties, choose the best description for each of the quadrilaterals you considered (parallelogram, rectangle, rhombus, square, kite and trapezium).

Group A:

- Two pairs of opposite sides parallel
- One pair of opposite sides parallel
- No parallel sides

Group B:

- All sides equal
- Two pairs of opposite sides equal
- Two pairs of adjacent sides equal
- No sides equal

Group C:

- All angles are right angles
- Opposite angles are equal
- One pair of opposite angles equal
- No angles equal

Group D:

- Diagonals are not equal
- Diagonals bisect each other
- One diagonal is bisected
- Diagonals bisect each other at right angles
- Diagonals are equal and bisect each other
- Diagonals are equal and bisect each other at right angles

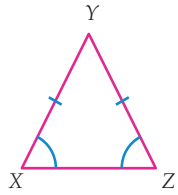
Shapes review

8.2

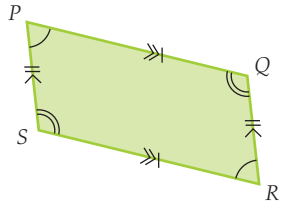
Using geometric symbols

We name a shape by assigning a letter to each of its vertices. We then list the vertices in order as we move around the shape, usually in a clockwise direction.

We write the name of this triangle as ΔXYZ:



We write the name of this quadrilateral as $PQRS$:



Sides that are equal in length are marked with the same number of dashes. ΔXYZ has two equal sides: $XY = YZ$. $PQRS$ has two pairs of equal sides: $PQ = SR$ and $PS = QR$. Angles that are equal in size are marked with the same number of arcs between the arms of the angle.

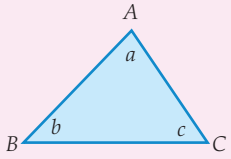
ΔXYZ has two equal angles: $\angle YXZ = \angle YZX$.

$PQRS$ has two pairs of equal angles: $\angle QPS = \angle QRS$, and $\angle PSR = \angle PQR$.

Sides that are parallel are marked with an equal number of arrows. On $PQRS$, sides PS and QR are parallel to each other, and sides PQ and SR are parallel to each other. We can write this as $PS \parallel QR$ and $PQ \parallel SR$.

The symbol '$\parallel$' means 'is parallel to'. The symbol ⊥ means 'is perpendicular to'.

Angle sum of a triangle



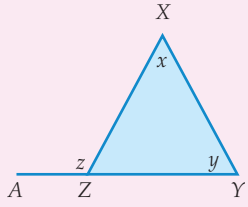
A triangle is a three-sided plane shape.

The sum of the angles of a triangle is 180°. We can write this angle sum as either:

$a + b + c = 180°$ or

$\angle BAC + \angle ABC + \angle BCA = 180°$

Exterior angle of a triangle



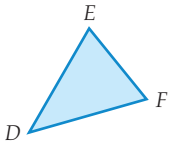
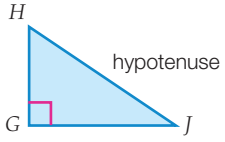
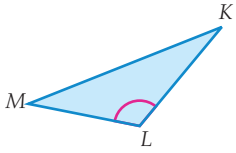
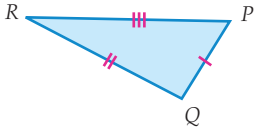
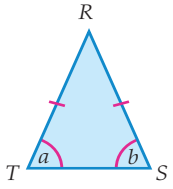
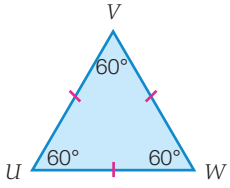
An exterior angle is produced when any side of a shape is extended. The exterior angle of a triangle is equal to the sum of the two opposite interior angles.

YZ is extended to A to form the **exterior angle** $\angle AZX$.

$\angle AZX = \angle ZXY + \angle XYZ$ or

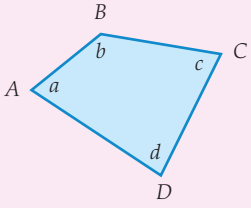
$z = x + y$

Triangle definitions and properties

Shape	Definition and properties
Acute-angled triangle 	All angles are acute. $\angle DFE < 90°$, $\angle FED < 90°$, $\angle EDF < 90°$
Right-angled triangle 	One angle equals 90°. The side opposite the right angle is called the **hypotenuse**. $\angle HGJ = 90°$ HJ is the hypotenuse.
Obtuse-angled triangle 	One angle is obtuse (the other two angles are acute). $\angle KLM > 90°$ $\angle LKM < 90°$, $\angle KML < 90°$
Scalene triangle 	All sides and angles are unequal. PQ, QR and RP are different lengths.
Isosceles triangle 	One pair of sides are equal in length. • The angles opposite the equal sides are also equal in size. $RS = RT$ $\angle RST = \angle RTS$ or $a = b$
Equilateral triangle 	All sides are equal in length, as are all angles. $UV = VW = WU$ • All angles are equal to 60°. $\angle UVW = \angle VWU = \angle WUV = 60°$

Quadrilaterals

Angle sum of a quadrilateral



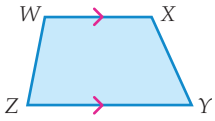
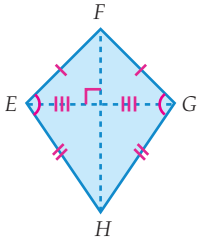
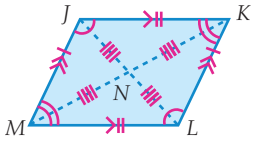
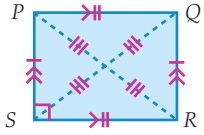
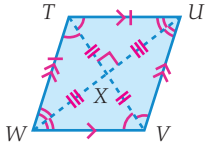
A **quadrilateral** is a four-sided plane shape.

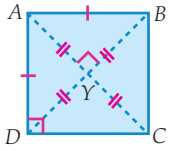
The sum of the angles of a quadrilateral is 360°. We can write this as:

$a + b + c + d = 360°$ or

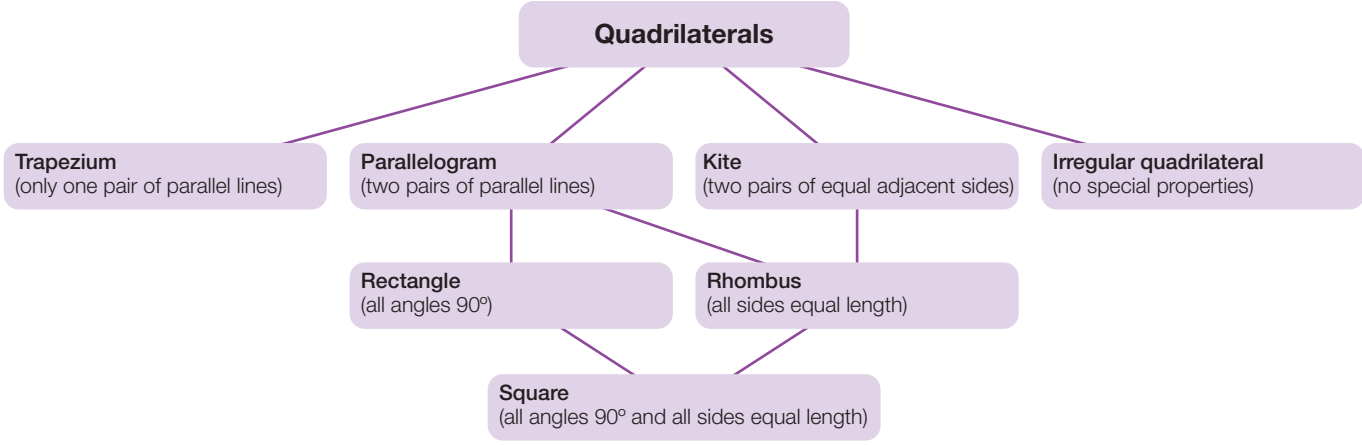
$\angle DAB + \angle ABC + \angle BCD + \angle CDA = 360°$

Properties of special quadrilaterals

Shape	Definition	Properties
Trapezium  	A trapezium is a quadrilateral with only one pair of opposite sides parallel. *WXYZ* is a trapezium in which $WX \parallel ZY$.	
Kite  	A kite is a quadrilateral with two pairs of equal adjacent sides. *EFGH* is a kite in which $EF = FG$, $EH = HG$.	• A kite has one pair of opposite angles that are equal: $\angle FEH = \angle FGH$ • The diagonals are perpendicular: $FH \perp EG$ • The diagonal that passes through the equal angles is **bisected** (cut in half) by the other diagonal.
Parallelogram  	A parallelogram is a quadrilateral with both pairs of opposite sides parallel. *JKLM* is a parallelogram in which $JK \parallel ML$, $JM \parallel KL$.	A parallelogram has: • both pairs of opposite sides equal in length: $JK = ML$, $JM = KL$ • both pairs of opposite angles equal: $\angle MJK = \angle MLK$, $\angle JML = \angle JKL$ • the diagonals bisect each other: $JN = NL$, $MN = NK$
Rectangle  	A rectangle is a parallelogram with a right angle. *PQRS* is a rectangle in which $\angle PSR = 90°$.	A rectangle has all the properties of a parallelogram, plus: • each angle is 90°: $\angle PSR = \angle SRQ = \angle RQP = \angle QPS = 90°$ • the diagonals are equal in length: $PR = QS$
Rhombus  	A rhombus is a parallelogram with two adjacent sides that are equal. *TUVW* is a rhombus in which $TU = TW$.	A rhombus has all the properties of a parallelogram, plus: • all the sides are equal in length: $TU = TW = WV = UV$ • the diagonals bisect the angles of the rhombus: $\angle WTV = \angle VTU$, $\angle WVT = \angle TVU$, $\angle TWU = \angle UWV$, $\angle TUW = \angle WUV$ • the diagonals bisect each other at right angles: $TX = XV$, $WX = XU$, $\angle WXT = \angle TXU = \angle UXV = \angle VXW = 90°$

Shape	Definition	Properties
Square  	A square is a rectangle with one pair of adjacent sides equal. *ABCD* is a square in which $AB = AD$.	A square has all the properties of a rectangle, plus: • all the sides are equal: $AB = AD = DC = BC$ • the diagonals are equal in length: $AC = BD$. • the diagonals bisect the angles of the square: $\angle DAC = \angle CAB$, $\angle DCA = \angle ACB$, $\angle ABD = \angle DBC$, $\angle ADB = \angle BDC$ • the diagonals bisect each other at right angles: $\angle AYD = \angle DYC = \angle CYB = \angle BYA = 90°$
Alternative definition of a square	A square is a rhombus with a right angle. *ABCD* is a square in which $\angle ADC = 90°$.	A square has all the properties of a rhombus, plus: • all the angles are right angles: $\angle ADC = \angle DCB = \angle CBA = \angle BAD = 90°$ • the diagonals are equal: $AC = BD$

Pictorial summary of the quadrilateral family



From the above diagram, we can see that:

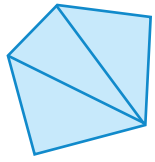
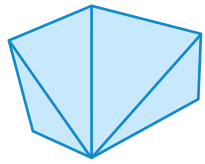
- rectangles, rhombuses and squares are special types of parallelograms
- a rhombus can also be classified as a special type of kite
- a square can also be classified as a special type of rectangle.

Polygons

A polygon is a plane shape with many sides. (When we use the term 'sides', we are referring to straight lines. A circle is not a polygon.) A triangle is a polygon with three sides, a quadrilateral is a polygon with four sides, a pentagon is a polygon with five sides, and so on. A regular polygon has all its sides and angles equal. Thus, an equilateral triangle is a regular three-sided polygon and a square is a regular four-sided polygon. There are no special names for other regular polygons—a regular five-sided figure is simply called a regular pentagon.

Angle sum of a polygon

We can find the angle sum of these polygons by dividing each shape into triangles as shown.

Triangle	Quadrilateral	Pentagon	Hexagon
			
Angle sum $= 180°$	$= 2 \times 180°$ $= 360°$	$= 3 \times 180°$ $= 540°$	$= 4 \times 180°$ $= 720°$

This could be written, in each case, as:

Angle sum $= (3 - 2) \times 180°$	$= (4 - 2) \times 180°$	$= (5 - 2) \times 180°$	$= (6 - 2) \times 180°$

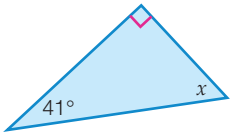
The sum of the angles of a polygon with n sides $= (n - 2) \times 180°$

The **interior angles** (angles inside the shape) of a regular polygon are equal.

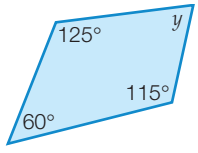
Worked Example 3 WE3

Find the value of the pronumerals in each shape.

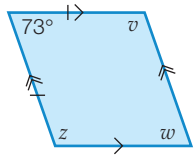
(a)



(b)



(c)



Thinking	Working
(a) 1 Use the angle properties of the shape to form an equation. Give a reason in brackets.	(a) $x + 41 + 90 = 180$ (angle sum of a triangle is 180°)
2 Solve the equation.	$x + 131 = 180$ $x = 180 - 131$ $x = 49°$
(b) 1 Use the angle properties of the shape to form an equation. Give a reason in brackets.	(b) $y + 115 + 60 + 125 = 360$ (angle sum of a quadrilateral)
2 Solve the equation.	$y + 300 = 360$ $y = 360 - 300$ $y = 60°$
(c) 1 Identify the shape.	(c) A parallelogram with one pair of adjacent sides equal is a rhombus.
2 Form an equation, with reason.	$v + 73 = 180$ (co-interior angles on parallel lines)
3 Solve the equation.	$v = 107°$

4	Repeat for the other angles.	$w = 73°$ (opposite angles of a rhombus are equal) $z = v$ (opposite angles of a rhombus are equal) $= 107°$
5	List all answers.	$v = 107°$, $w = 73°$, $z = 107°$

Worked Example 4 WE4

What is the most appropriate name for the quadrilateral *ABCD*?

Thinking	Working
1 Use the symbols given to list the information known about the shape. Work through the list of quadrilaterals to see which shape fits the information.	$AB \parallel DC$, $AD \parallel BC$, so $ABCD$ is a parallelogram; $\angle ADC = 90°$, so all other angles must be 90° (co-interior angles on parallel lines are supplementary)
2 State the answer.	$ABCD$ is a rectangle.

8.2 Shapes review

Navigator

Answers page 636

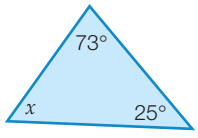
Q1, Q2, Q3, Q4, Q6 (a)–(d), Q8, Q9, Q11, Q13, Q16, Q17, Q19	Q1, Q2, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q17, Q20, Q21	Q1, Q2, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q14, Q15, Q17, Q18, Q20, Q21

Fluency

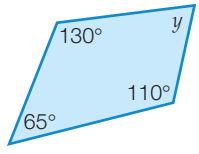
WE3

1 Find the value of the pronumerals in each shape.

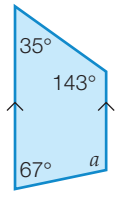
(a)



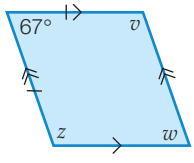
(b)



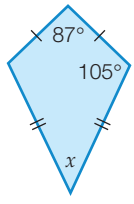
(c)



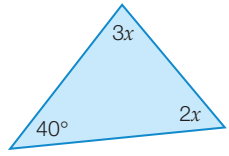
(d)



(e)

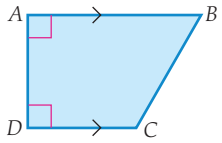


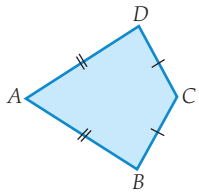
(f)

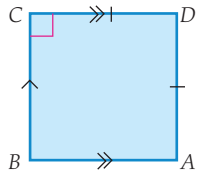


WE4

2 What is the most appropriate name for the quadrilateral *ABCD* in each diagram?

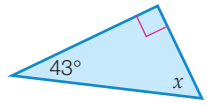
(a) 

(b) 

(c) 

3 The value of x is:

A 43° **B** 47° **C** 137° **D** 227°



4 Sketch a hexagon (a six-sided polygon) and use it to answer the following.

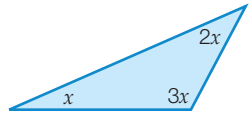
(a) How many triangles can it be divided into?

(b) What is the angle sum of a triangle?

(c) Use your answers to **(a)** and **(b)** to determine the sum of the angles in a hexagon.

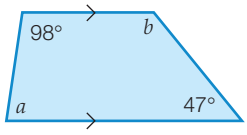
5 The three angles in a triangle are x, $2x$ and $3x$.

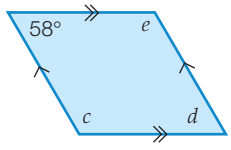
What is the size of each angle?

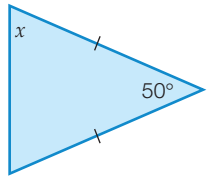


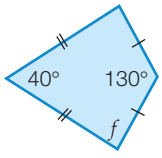
Understanding

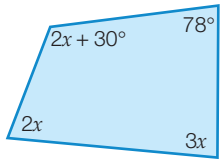
6 Find the value of the pronumerals in each shape.

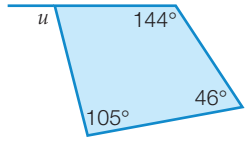
(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

7 Write TRUE or FALSE for each of the following statements.

(a) A triangle can have two obtuse angles.

(b) A square has all the properties of a rhombus.

(c) A kite has at least one pair of parallel sides.

(d) The diagonals of a rectangle bisect each other and also bisect the angles of the rectangle.

8 In which regular polygon is each angle 90°?

9 This geometric design was found in the brickwork of a wall of a house in the ancient Italian city of Pompeii. A regular hexagon has six squares joined to its sides. On the end of each square is an equilateral triangle. In between the squares are six rhombuses. One rhombus is made up of two equilateral triangles.

Sketch a copy of the design, then use the information given above to:

(a) mark every equal side.

(b) Use the properties of a rhombus to determine the angles in the rhombuses used in the design.

(c) Use the angle properties of the squares, triangles and rhombuses in the design to show that the interior angles of the hexagon are each 120°.

10 Which of the following statements is false?

A A kite has one pair of opposite angles equal.

B A trapezium cannot be a rectangle.

C A rhombus is a rectangle.

D A rhombus is a parallelogram.

11 Which statement is true?

A All quadrilaterals have at least one pair of parallel sides.

B All trapeziums have a pair of sides of equal length.

C All kites have a pair of equal angles.

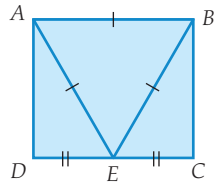
D All quadrilaterals have a right angle.

12 **(a)** What is the sum of the angles of a regular pentagon?

(b) What is the size of each interior angle?

Reasoning

13 The rectangle *ABCD* is shown. Point *E* is halfway between *C* and *D*. *ABE* is an equilateral triangle.



Find:

(a) $\angle DAE$

(b) $\angle BEC$.

14 The design of this cathedral ceiling has eight identical rhombuses with one common vertex (the point at the centre of the design). Sharing sides with the rhombuses are eight other identical quadrilaterals and, between each pair of these quadrilaterals, is a right-angled isosceles triangle.

(a) Determine the size of the internal angles of one rhombus.

(b) Use the side properties of rhombuses and isosceles triangles to demonstrate that the eight other quadrilaterals are kites.

(c) **(i)** What regular polygon is formed by joining the outermost vertices of the kites?

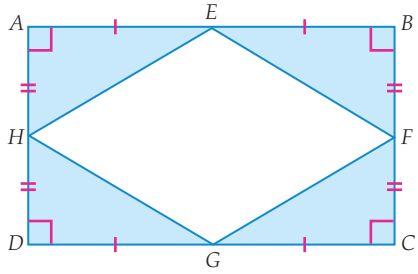
(ii) What is the sum of the angles in this polygon?

(iii) Use your answer to **(ii)** to determine the size of one of the interior angles of this polygon.

(d) Use your answers to **(a)** and **(c)** to determine the interior angles of one of the kites.

15 What is the greatest number of acute angles that you can have in a quadrilateral? Explain your answer using diagrams.

16 The midpoints of each side of a rectangle *ABCD* are joined in order to form the quadrilateral *EFGH*.



(a) Measure *EF*, *FG*, *GH* and *HE* and use this information to name *EFGH*.

(b) If *ABCD* had been a square, what sort of a quadrilateral would *EFGH* be?

17 The diagonals of a quadrilateral bisect each other. Draw all the possible different quadrilaterals, naming each type.

18 Both a rhombus and a rectangle are parallelograms, but a rectangle is not necessarily a rhombus. Explain.

Open-ended

19 Draw three different right-angled triangles, stating the size of each angle in them.

20 Using as few lines as possible, draw a shape that contains:

(a) a rectangle and two squares

(b) a kite and eight triangles

(c) two rectangles, a square, a trapezium and two triangles.

21 **(a)** An isosceles triangle has one 45° angle. What are some possible sizes for the other two angles?

(b) A triangle has one 60° angle. Must it be equilateral? Give examples of possible triangles.

(c) Draw some examples of an obtuse-angled scalene triangle.

Outside the Square Puzzle

Matchstick quadrilaterals

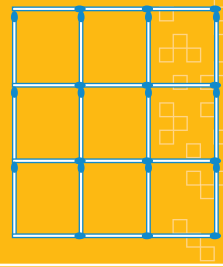
Equipment needed: matches (optional)

1 With five matchsticks, three quadrilaterals can be made, as shown.

(a) With six matchsticks, make six quadrilaterals.

(b) With six matches, make nine quadrilaterals

2 Which six matches should you remove, without changing the position of the others, so that only three squares are left.



8.3 Congruence and transformation

In Year 7, you learnt about the transformations of translation, reflection and rotation. These transformations all produce **congruent** figures—figures with an identical shape and size. We can use the Cartesian plane to plot points, join points to form shapes, then transform the shapes.

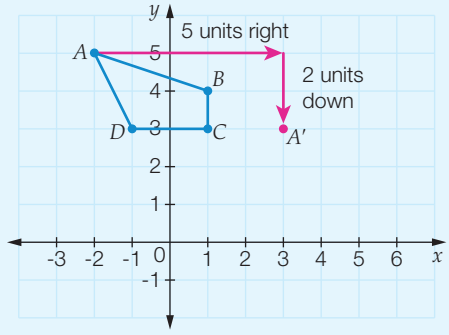
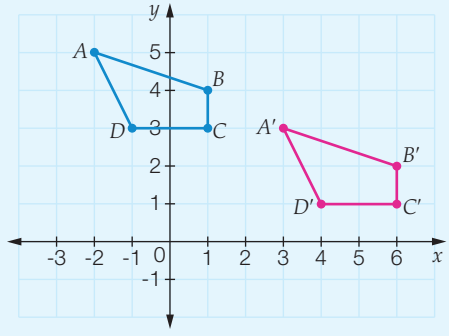
Worked Example 5 WE5

Plot the following set of coordinates on a Cartesian plane: (-2, 5), (1, 4), (1, 3), (-1, 3). Join and label them *A*–*D* in the order given to form the quadrilateral *ABCD*. Perform the following transformations on *ABCD*. Label the transformed figure *A′B′C′D′* and list the coordinates of figure *A′B′C′D′* for each transformation.

(a) The translation [5, -2]

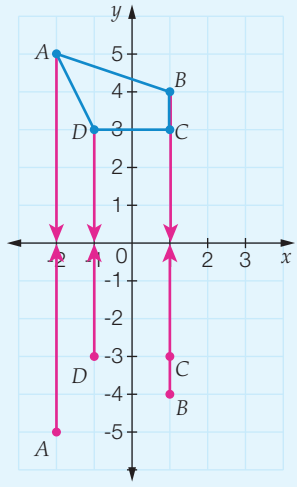
(b) A reflection in the *x*-axis

(c) A clockwise rotation of 90° about the point (-1, 1)

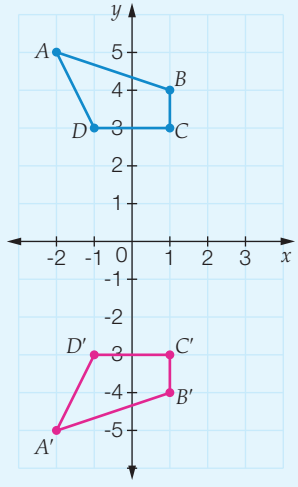
Thinking	Working
(a) **1** Plot and label the points, then join them to form the shape. Select one of the vertices and move it according to the translation given. Label it using image notation (*A′*).	(a) 
2 Repeat for the remaining vertices, label them *B′*, *C′* and *D′*, then join them to form the new image.	
3 Read off the coordinates of the new points *A′*, *B′*, *C′* and *D′* and write them down.	*A′*(3, 3) *B′*(6, 2) *C′*(6, 1) *D′*(4, 1)

(b) **1** Count or measure the perpendicular distance between each vertex and the line of reflection. Use this distance to plot the image of each vertex on the opposite side of the line.

(b)



2 Label and join each of the reflected vertices to form the image of the figure.

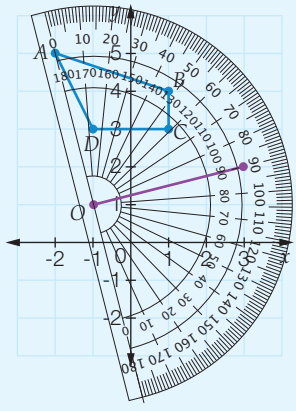


3 Read off the coordinates of the new points A', B', C' and D' and write them down.

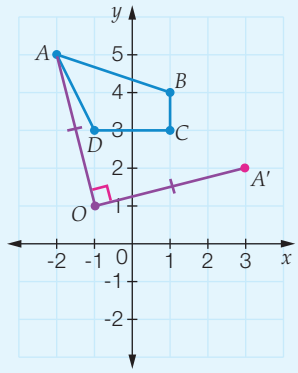
$A'(-2, -5)$ $B'(1, -4)$
$C'(1, -3)$ $D'(-1, -3)$

(c) **1** Plot the centre of rotation. Label it O. Place the centre of a protractor against the centre of rotation, with the chosen vertex (point A is shown here) on the base line. Measure the angle of rotation around from zero in the given direction and mark it off.

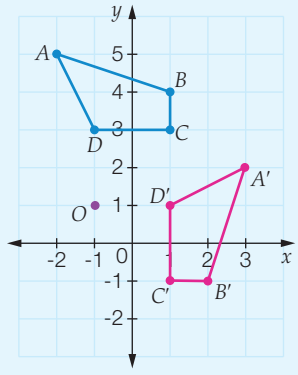
(c)



2 Plot and label the image point, using a ruler if necessary to make sure that it is the same distance from O as the original point.



3 Repeat Steps **1** and **2** for the other vertices.



4 Read off the coordinates of the new points $A'B'C'D'$ and write them down.

$A'(3, 2)$ $B'(2, -1)$
$C'(1, -1)$ $D'(1, 1)$

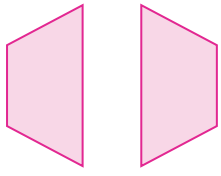
Congruent figures

If one figure can be placed on top of another, so that every side, angle and vertex matches up with the other figure, the two figures are said to be congruent.

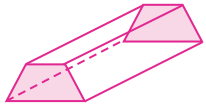
The symbols ≡ or ≅ mean 'is congruent to'.

Congruent figures are *exactly* the same shape and the same size, but they can be in different positions and have a different orientation.

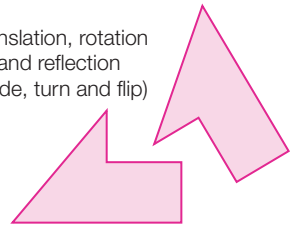
One congruent figure can be placed on top of another by one or more transformations, such as a translation (slide), reflection (flip) or rotation (turn). The pairs of figures below are congruent.



Reflection (flip)



Translation (slide)

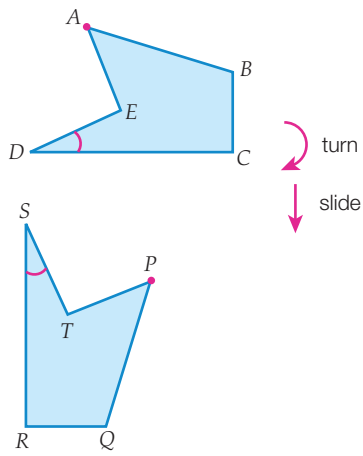


Translation, rotation and reflection (slide, turn and flip)

The sides, angles and vertices of one figure can be paired with the matching parts of a congruent figure. We use the ↔ symbol to show this match.

The two figures shown below are congruent. Figure *ABCDE* can be rotated 90° in a clockwise direction around point *A*, then translated downwards to sit on top of figure *PQRST*, so that:

- vertex *A* matches with vertex *P* — $A \leftrightarrow P$
- side *BC* matches with side *QR* — $BC \leftrightarrow QR$
- angle *CDE* matches with angle *RST* — $\angle CDE \leftrightarrow \angle RST$



As the two figures above are congruent to each other, we can write the following **congruence statement**: $ABCDE \equiv PQRST$.

When naming congruent figures, the vertices of both figures must be written in the same order, to allow matching vertices, angles and sides to be identified from the congruence statement.

Worked Example 6

WE 6

For the congruent figures shown:

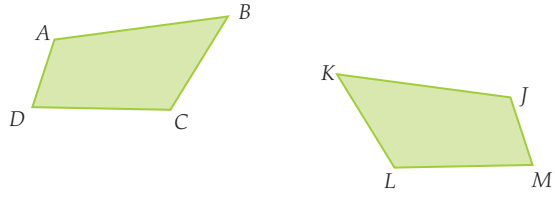
(a) identify the transformation(s) that places the first figure onto the second

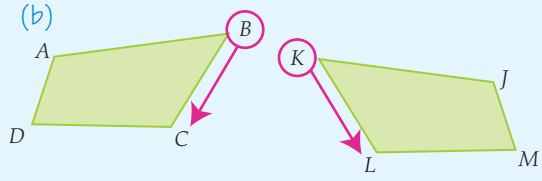
(b) pair the matching vertices

(c) pair the matching side lengths

(d) pair the matching angles

(e) write a congruence statement.

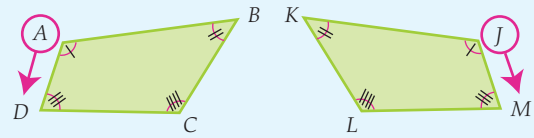


Thinking	Working
(a) Which transformation—reflection, rotation or translation—will place the first shape on top of the other?	(a) Reflection (in a vertical line), then translation
(b) Locate one part of the figure that can easily be identified in both congruent figures. (Here, we locate the smallest angle in each shape.) Pair the vertices at this point, then pair the rest by moving around the figure.	(b)  $B \leftrightarrow K$ $C \leftrightarrow L$ $D \leftrightarrow M$ $A \leftrightarrow J$
(c) Pair the side lengths starting at the same easily identified point as in **(b)**.	(c) $BC \leftrightarrow KL$ $CD \leftrightarrow LM$ $DA \leftrightarrow MJ$ $AB \leftrightarrow JK$

(d) Mark pairs of matching angles in the same way, by marking in the angle arc from one arm to the other, then placing one, two, three or four lines across it.

Three points are written to describe an angle. The vertex of the angle is at the point in the middle.

(d)

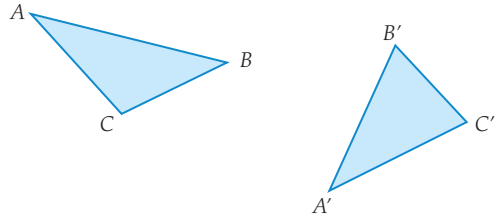


$\angle ABC \leftrightarrow \angle JKL$ $\angle BCD \leftrightarrow \angle KLM$
$\angle CDA \leftrightarrow \angle LMJ$ $\angle DAB \leftrightarrow \angle MJK$

(e) Move around the first shape, writing the vertices down in order. Start at the matching vertex in the second shape and move around it in the same order.

(e) $ABCD \equiv JKLM$

Matching vertices on congruent figures produced by transformations are shown by labelling the vertices of the second figure with a dash (also called a prime mark). We write the vertex that matches with A as A' and say *A dash* or *A prime*. We refer to the second triangle shown here as $A'B'C'$.



8.3 Congruence and transformation

Navigator

Answers page 633

Q1, Q2 (a) & (b), Q3, Q4, Q5, Q6, Q7 (a), Q8 (a)–(d), Q9, Q11, Q13	Q1, Q2 (b) & (c), Q3, Q4, Q5, Q6, Q7 (b), Q8, Q9, Q10, Q11, Q12, Q13	Q1, Q2 (c) & (d), Q3, Q4, Q5, Q6, Q7 (c), Q8, Q9, Q10, Q11, Q12, Q13

Equipment required: A ruler for Question **11**, grid paper for Question **4**, a protractor may be used for Question **1**

Fluency

WE5

1 **(a)** Plot the following set of coordinates on a Cartesian plane: (1, 3), (0, 4), (-2, 3), (-1, 1). Join and label them *A*–*D* in the order given to form the quadrilateral *ABCD*. Perform the following transformations on *ABCD*. Label the transformed figure *A′B′C′D′* and list the coordinates of *A′B′C′D′* for each transformation.

(i) The translation [2, -3]

(ii) A reflection in the x-axis

(iii) A clockwise rotation of 180° about the point (0, 1)

(b) Plot the following set of coordinates on a Cartesian plane: (-3, 1), (-4, -3), (-1, -2). Join and label them *D*–*F* in the order given to form the triangle *DEF*. Perform the following transformations on *DEF*. Label the transformed figure *D′E′F′* and list the coordinates of *D′E′F′* for each transformation.

(i) The translation [-1, 4]

(ii) A reflection in the y-axis

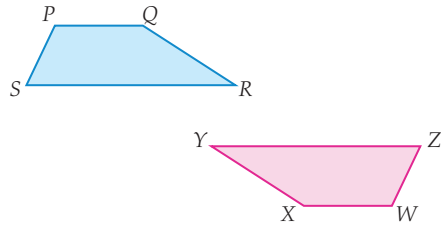
(iii) An anticlockwise rotation of 90° about the point (-2, 2)

2 For the congruent figures shown below:

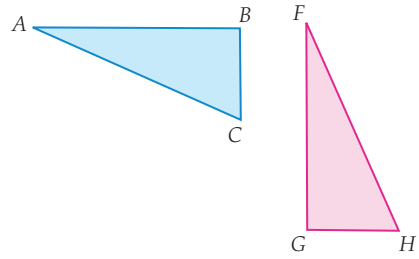
WE6

- **(i)** identify the transformation(s) that places the first figure onto the second
- **(ii)** pair the matching vertices
- **(iii)** pair the matching side lengths
- **(iv)** pair the matching angles
- **(v)** write a congruence statement.

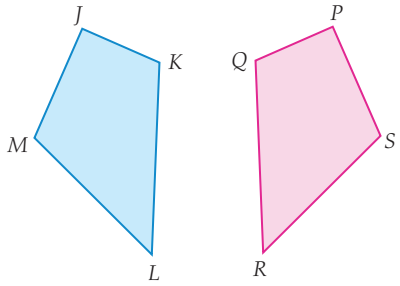
(a)



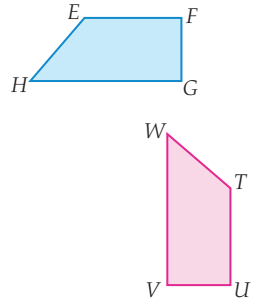
(b)



(c)

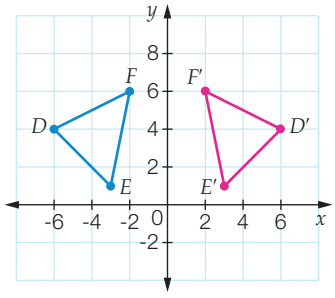


(d)

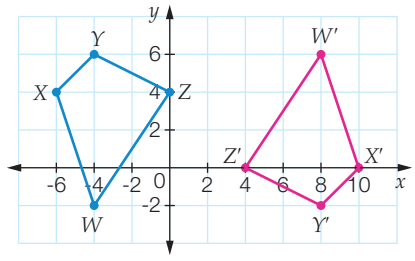


3 Describe the transformation that has been applied to produce the congruent figures shown.

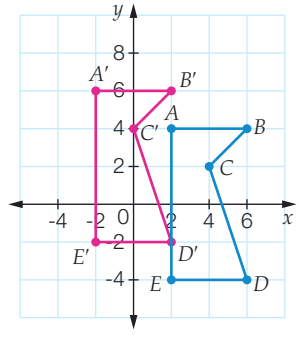
(a)



(b)



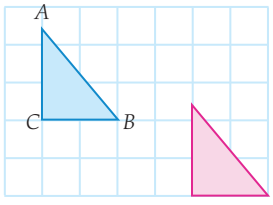
(c)



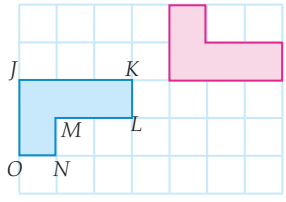
4 Copy the following congruent figures onto grid paper.

- **(i)** Describe the transformation(s) required to place the blue figure on top of the pink figure.
- **(ii)** Label the matching vertices on the transformed figure using 'dash' notation ($A \leftrightarrow A'$, $B \leftrightarrow B'$ etc.)

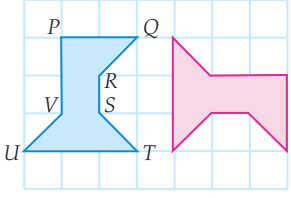
(a)



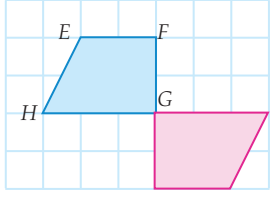
(b)

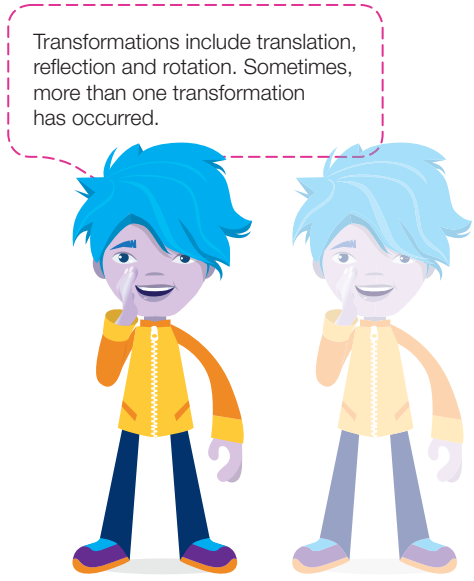


(c)



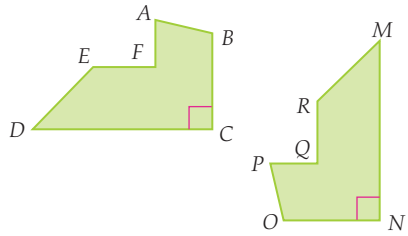
(d)





5 Refer to the congruent figures on the right.

(a) For the congruent figures shown, which pairing is NOT correct?



A $BC \leftrightarrow ON$

B $\angle NOP \leftrightarrow \angle CBA$

C $\angle DEF \leftrightarrow \angle RMN$

D $P \leftrightarrow A$

(b) Which congruence statement describes the figures shown?

A $ABCDEF \equiv MNOPQR$

B $ABCDEF \equiv PQRMNO$

C $CDEFAB \equiv NOPQRM$

D $CDEFAB \equiv NMRQPO$

Understanding

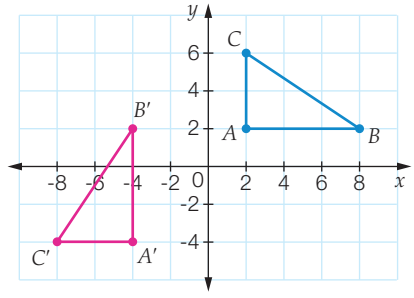
6 Triangle ABC has had two transformations applied to it. Which sequence of transformations will place triangle ABC onto the image $A'B'C'$?

A Reflection in the y-axis, then the translation [-2, -1].

B Rotation 90° anticlockwise about the point (2, 2), then the translation [-3, -3].

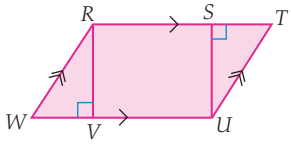
C Rotation 180° clockwise about the point (0, 0), then a reflection in the x-axis.

D Reflection in the x-axis, then the translation [-3, 2].

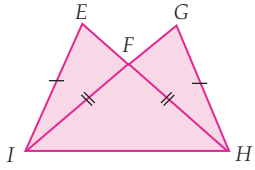


7 Write congruence statements for pairs of congruent triangles within the following figures. Some figures have more than one pair.

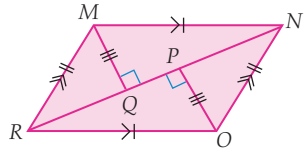
(a)



(b)



(c)



8 For the congruence statement $PQRS \equiv ABCD$, complete the following pairings.

(a) $R \leftrightarrow$

(b) $D \leftrightarrow$

(c) $PQ \leftrightarrow$

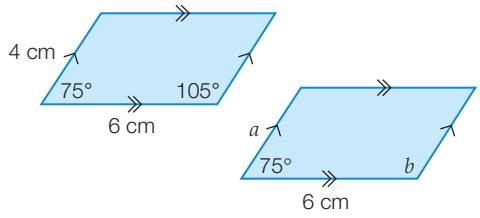
(d) $DA \leftrightarrow$

(e) $\angle PQR \leftrightarrow$

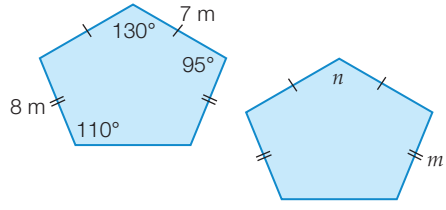
(f) $\angle CDA \leftrightarrow$

9 The pairs of figures below are congruent. Use your knowledge of matching angles and side lengths to find the value of the pronumerals.

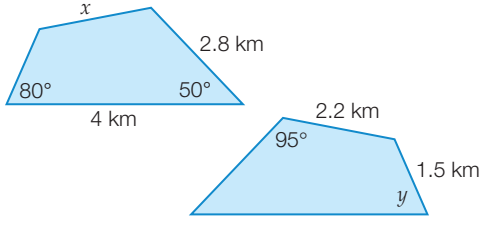
(a)



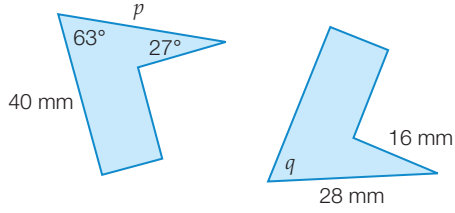
(b)



(c)

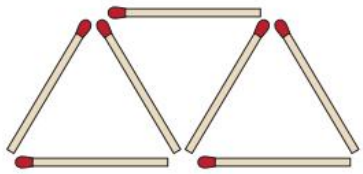


(d)



Reasoning

10 Seven matchsticks have been used to create the shape shown.



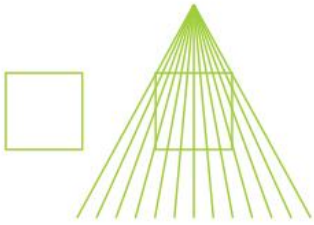
(a) How many congruent triangles are in the shape?

(b) If the shape is extended to 11 matchsticks, how many congruent triangles are there now?

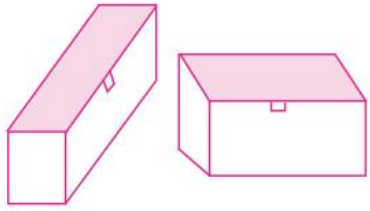
(c) In the diagram above, there are two congruent rhombuses. Draw each of these separately.

11 Look carefully at the figures below.

(a) Are the squares congruent?



(b) Are the lids of the boxes congruent?



Use a ruler to check your answers. Did the results surprise you? Explain the results.

Open-ended

12 Keira was answering some homework questions relating to two congruent figures with different orientations. Her answers are shown.

(a) $GH = 4$ cm (b) $\angle XYZ = 90°$ (c) $EFGH \equiv WXYZ$

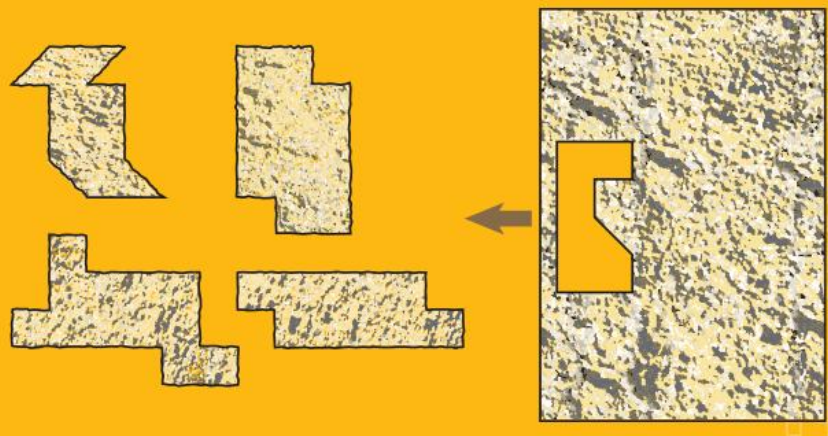
What might the two figures look like?

13 Two figures are congruent if they are exactly the same shape and size. State two real-life examples of congruent figures.

Outside the Square Puzzle

The ancient tomb

A clever archaeologist uncovered the door to the tomb of the ruler of an ancient civilisation. He uncovered a keyhole and an arrow pointing to four figures on the surrounding stone wall. This was puzzling. He decided he must push a stone to open the door, but if he pushed the wrong stone, he would surely be met by an ancient trap and possible death.



The archaeologist realised that each of the figures could be divided into two congruent halves and that one of the figures, when divided in this way formed the shape of the keyhole.

The archaeologist escaped death and unearthed treasures from the tomb. Which stone did the archaeologist push to open the door? Show how it could be decided.

Mathspace

The art of MC Escher

The use of tessellation in Art

Two or more shapes are said to tessellate if they fit together and cover a flat space without any gaps between them. A pattern made up of tessellating shapes is called a tessellation.

Tessellating geometric shapes are used in many areas of art and design. Examples can be found on the wall of Spain's Alhambra Palace, or in Indian Rangoli patterns. Maurits Cornelis Escher was a famous Dutch artist who often used transforming and tessellating shapes to dramatic effect.

1 Describe the transformation (flip, slide or turn) Escher has used to create the congruent copies of the fish in this tessellation.

2 The 'repeating unit' of a tessellation is the smallest figure or group of figures that can be used to create and continue the entire tessellation by using transformations (flips, slides or turns).

For each of the Escher designs shown below:

(a) identify the repeating unit

(b) identify the transformation that has been applied to the repeating unit to create the tessellation.

(i)

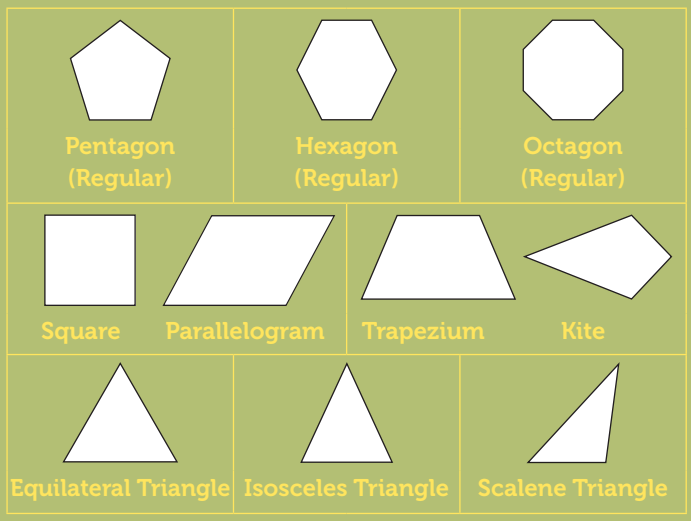
(ii)

Investigating tessellations

Equipment required: paper, scissors

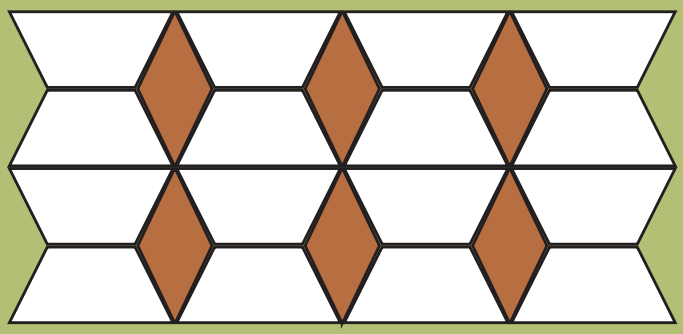
A perfect, or regular, tessellation uses shapes of only one type, which must all be congruent. To test and see whether a shape will tessellate, fold a piece of paper in half, then in half again, and again, so that there are eight layers of paper. Draw the shape in the middle of the top layer, and cut it out. If the eight shapes fit together with no gaps, then the shape forms a regular tessellation.

3 Which of the following shapes tessellate? Test them using the method described above.



4 What property do the tessellating shapes have in common?

A tessellation can also be made with two different shapes. This is called a *partial* or *semi-regular tessellation*.



5 Investigate shapes that will tessellate when paired with another shape.

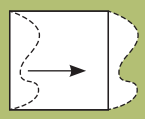
(a) Draw a semi-regular tessellation.

(b) Explain why the two shapes selected in (a) form a semi-regular tessellation.

6 Make an Escher-style drawing of your own.

(a) Start with a square.

(b) Move a shape from the left to the right.

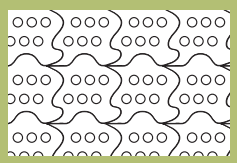


(c) Move a piece from the bottom to the top.

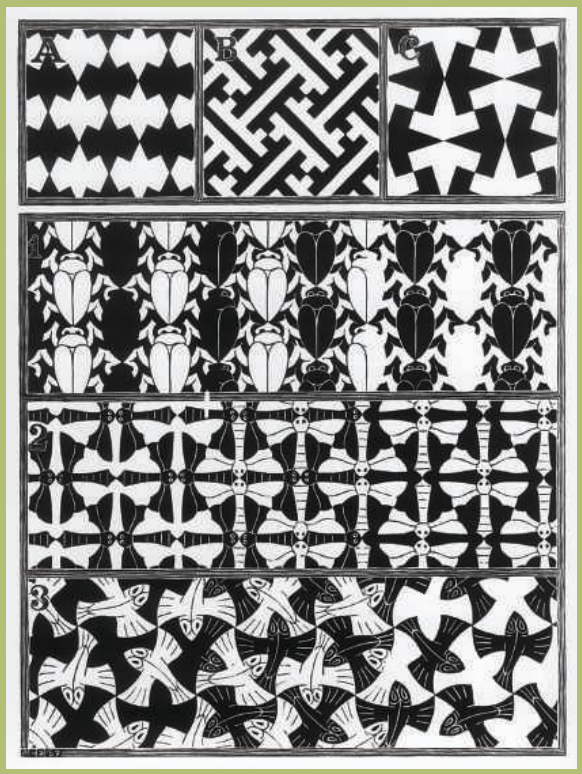
(d) You now have an unusual shape that will tessellate.

(e) Add detail to the inside of your shape.

(f) Reproduce your shape and place the shapes together to fill an area (the area could be any shape: square, rectangle, triangle, circle etc.).



7 Look at the print *Regular Division of the Plane II*. Discuss how Escher has used the basic shapes and patterns in the top three squares in the designs below.



Regular Division of the Plane by MC Escher, 1939–1940

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8 *Gravity* by MC Escher (1944) is a lithograph.

(a) What is a lithograph?

(b) What geometrical solid is shown here?

9 Find out more about MC Escher and his art.

10 Find out about artists who use tessellations or other mathematical properties in their art.

11 Find examples of tessellation being used in other areas, such as architecture or quilting designs.

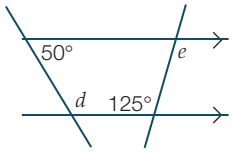
Gravity by MC Escher (1944)

Half-time 8

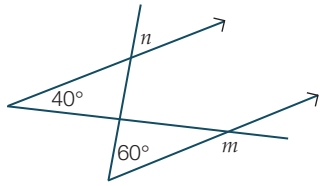
Ex. 8.1

1 Find the value of the pronumerals in each figure.

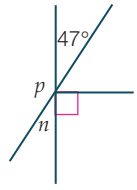
(a)



(b)



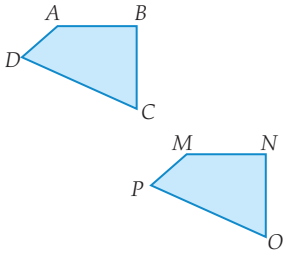
(c)



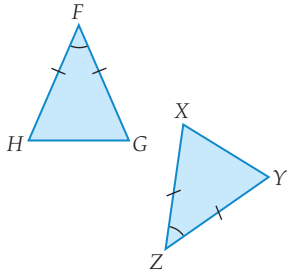
Ex. 8.3

2 Write a congruence statement to describe the congruent figures shown.

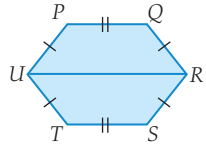
(a)



(b)



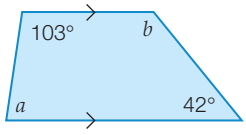
(c)



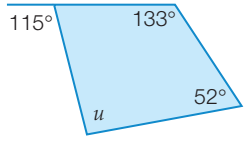
Ex. 8.2

3 Find the value of the pronumerals in each shape.

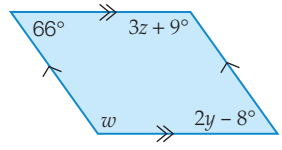
(a)



(b)



(c)



Ex. 8.1, 8.2

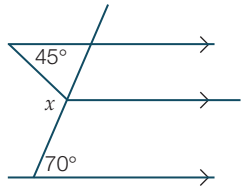
4 The value of x in the diagram is:

A 45°

B 65°

C 70°

D 115°



Ex. 8.3

5 Plot the points (1, 1), (3, 4), (5, 4) and (5, 2) on a Cartesian plane. Join them and label them A–D in the order given to form the quadrilateral $ABCD$. Perform the following transformations on $ABCD$. Label the transformed figure $A'B'C'D'$ and list the coordinates of $A'B'C'D'$ for each transformation.

(a) A reflection in the y-axis

(b) A rotation 90° clockwise about the point (0, 0)

(c) The translation [-5, -2]

Ex. 8.2

6 Which statements are true?

A All squares are rectangles.

B All parallelograms are quadrilaterals.

C All kites have equal diagonals.

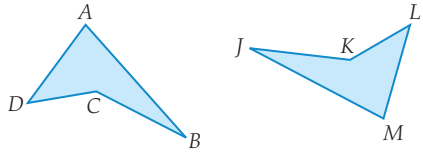
D The diagonals of a parallelogram bisect each other at right angles.

Ex. 8.3

7 Complete the following pairings for the congruent figures shown.

(a) $B \leftrightarrow$ **(b)** $AD \leftrightarrow$

(c) $JM \leftrightarrow$ **(d)** $\angle JKL \leftrightarrow$

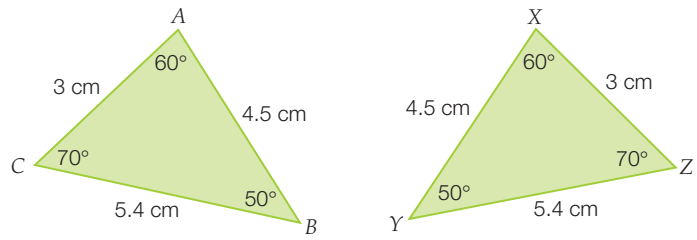


Congruent triangles

8.4

Triangles that have matching sides of the same length *and* matching angles of the same size are known as **congruent triangles**.

The triangles pictured below are congruent. The congruence statement to describe the two triangles is written as $\Delta ABC \equiv \Delta XYZ$. This is said as 'Triangle ABC is congruent to triangle XYZ'.



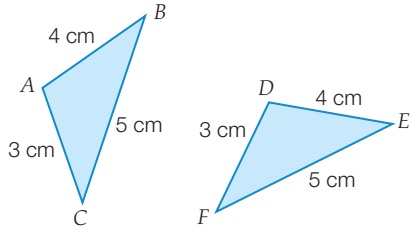
ΔXYZ is a reflection of ΔABC. All matching sides are the same length and all matching angles are the same size. It is not necessary, however, to know all side lengths and all angles to determine whether two triangles are congruent.

Tests for congruency

There are four tests we can apply to determine whether two triangles are congruent. Each of the tests requires three pieces of information.

- Side, Side, Side (SSS)

  Triangles are congruent if all pairs of corresponding sides are the same length.

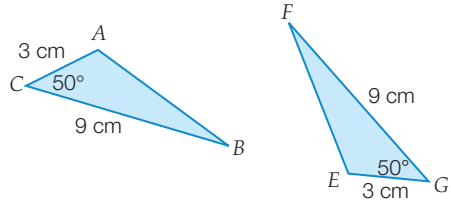


$AB = DE$
$BC = EF$
$AC = DF$

Therefore, $\Delta ABC \equiv \Delta DEF$ (SSS)

- Side, Angle, Side (SAS)

  Triangles are congruent if they have two sides of equal length and the angle formed by the two sides is equal (we call this the **included angle**).

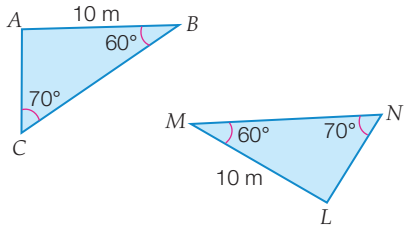


$AC = EG$
$\angle ACB = \angle EGF$
$BC = FG$

Therefore, $\Delta ABC \equiv \Delta EFG$ (SAS)

- Angle, Angle, Side (AAS)

  Triangles are congruent if two matching angles are the same size and one pair of matching sides are the same length.

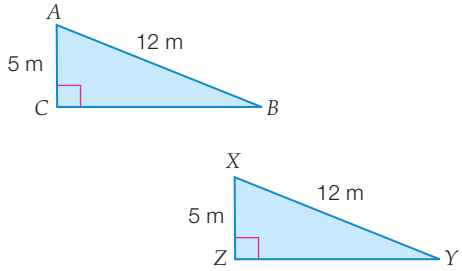


$\angle ABC = \angle LMN$
$\angle ACB = \angle LNM$
$AB = LM$

Therefore, $\Delta ABC \equiv \Delta LMN$ (AAS)

- Right angle, Hypotenuse, Side (RHS)

  Right-angled triangles are congruent if the matching hypotenuses (the sides opposite the right angles) and a pair of matching second sides are equal in length.



$AC = XZ$
$AB = XY$

Therefore, $\Delta ABC \equiv \Delta XYZ$ (RHS)

The four congruency tests are: Side, Side, Side (SSS), Side, Angle, Side (SAS), Angle, Angle, Side (AAS) and Right angle, Hypotenuse, Side (RHS).

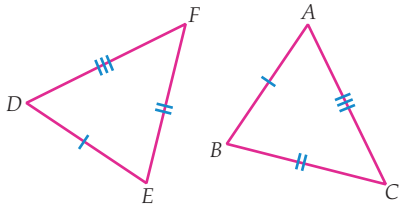
The initials of the test used to demonstrate congruency should be written in brackets after a congruence statement.

Worked Example 7

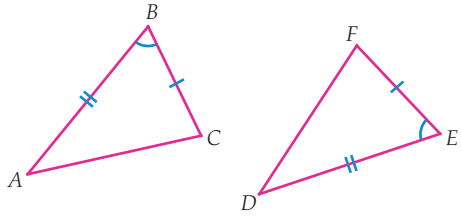
WE7

Show that each of the following pairs of triangles are congruent, stating the congruence test used.

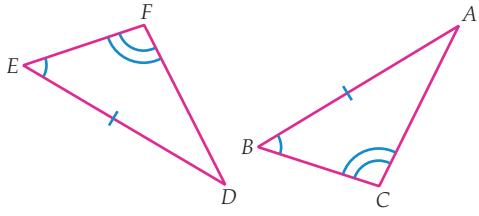
(a)



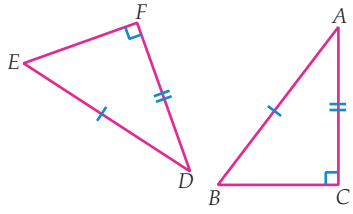
(b)



(c)



(d)



Thinking		Working	
(a) 1	State the matching sides that are equal.	(a) $AB = DE$	(Side)
		$BC = EF$	(Side)
		$CA = FD$	(Side)

	2	State that the triangles are congruent, taking care to state matching sides in the same order. In brackets, write the initials of the congruence test used.		Therefore $\Delta ABC \equiv \Delta DEF$ (SSS)
(b)	1	State the matching sides and angles that are equal.	(b)	$\angle ABC = \angle DEF$ (Angle) $AB = DE$ (Side) $BC = EF$ (Side)
	2	State that the triangles are congruent, taking care to state matching vertices in the same order. In brackets, write the initials of the congruence test used.		Therefore $\Delta ABC \equiv \Delta DEF$ (SAS)
(c)	1	State the matching sides and angles that are equal.	(c)	$\angle ABC = \angle DEF$ (Angle) $\angle ACB = \angle DFE$ (Angle) $AB = DE$ (Side)
	2	State that the triangles are congruent, taking care to state matching sides and angles in the same order. In brackets, write the initials of the congruence test used.		Therefore $\Delta ABC \equiv \Delta DEF$ (AAS)
(d)	1	State the matching sides and angles that are equal.	(d)	$\angle ACB = \angle DFE$ (Right angle) $AB = DE$ (Hypotenuse) $CA = FD$ (Side)
	2	State that the triangles are congruent, taking care to state matching sides and angles in the same order. In brackets, write the initials of the congruence test used.		Therefore $\Delta ABC \equiv \Delta DEF$ (RHS)

Constructing congruent triangles

SSS

We can construct congruent triangles with three given side lengths (SSS) by carrying out the following steps. The example below is for a triangle with sides of 10 cm, 8 cm and 4 cm.

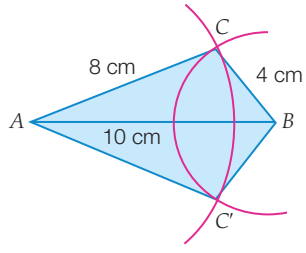
Step 1 Rule a side of length 10 cm and label it *AB*.

Step 2 Set your compass to a radius of 8 cm. Place the point on *A*. Draw an arc, extending it above and below the line *AB*.

Step 3 Set your compass to a radius of 4 cm. Place the point on *B*. Draw another arc, extending it again above and below *AB*. The two arcs should intersect at two points. Label these points *C* and *C′*.

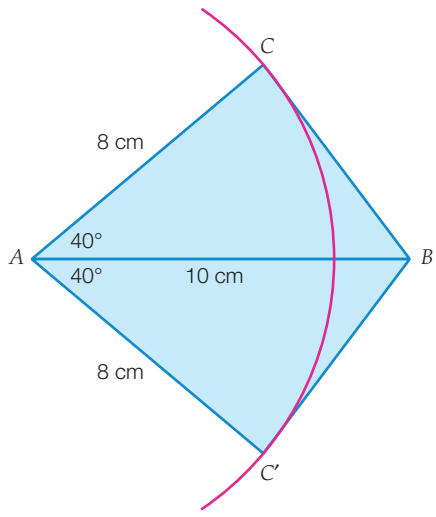
Step 4 Use your ruler to join the sides *AC*, *BC*, *AC′* and *BC′*.

Notice that you have drawn two congruent triangles: $\Delta ABC'$ is a reflection of ΔABC in the line *AB*.



SAS

We can construct congruent triangles with two given side lengths and a given included angle (SAS) in a similar way. The following steps produce triangles with sides of 10 cm, 8 cm and an included angle of 40°.



Step 1 Rule a side of length 10 cm and label it *AB*.

Step 2 Use a protractor to construct a 40° angle above and below the line at point *A*.

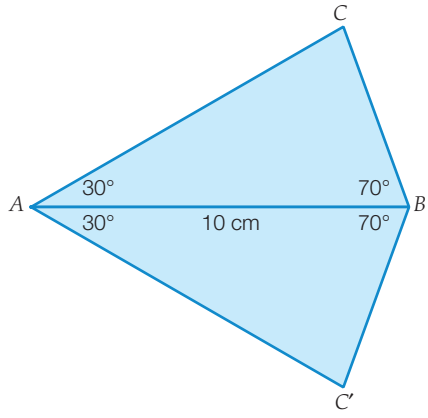
Step 3 Set your compass to a radius of 8 cm. Place the point on B and draw an arc above and below *AB* so that it intersects the angle arms drawn in Step 2. Label the points of intersection *C* and *C′* respectively.

Step 4 Use your ruler to join the sides *BC* and *BC′*.

You have now drawn two congruent triangles with Δ*ABC′* being a reflection of Δ*ABC* in the line *AB*.

AAS

We can construct congruent triangles with a given side length and two given angles (AAS). The following steps produce triangles with sides of 10 cm, and angles of 30° and 70°.



Step 1 Rule a side of length 10 cm and label it *AB*.

Step 2 Use a protractor to construct a 30° angle above and below the line at point *A*.

Step 3 Use a protractor to construct a 70° angle above and below the line at point *B*.

Step 4 Extend the arms of the angles until they intersect. Label the points of intersection *C* and *C′*.

You have now drawn two congruent triangles with Δ*ABC′* being a reflection of Δ*ABC* in the line *AB*.

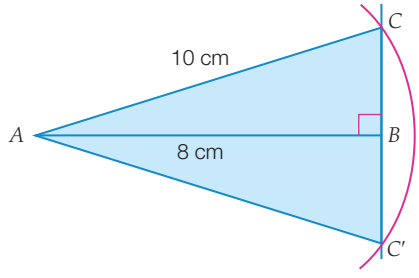
RHS

We can construct congruent right-angled triangles with a given hypotenuse and side (RHS). The following steps produce a triangle with a hypotenuse of 10 cm and a shorter side of 8 cm.

Step 1 Rule a side of length 8 cm and label it *AB*.

Step 2 Using a compass or a protractor, construct a line perpendicular to *AB* to pass through point *B*, extending above and below point *B*.

Step 3 Set your compass to a radius of 10 cm. Place the point on *A*. Draw an arc, extending it above and below the line *AB*. The arc should intersect the perpendicular line at two points. Label these points *C* and *C′*.



Step 4 Use your ruler to join the sides *AC* and *AC′*.

Again, you have drawn two congruent triangles, with Δ*ABC′* being a reflection of Δ*ABC* in the line *AB*.

8.4 Congruent triangles

Navigator

Q1, Q2, Q3, Q4, Q5 (a)–(d), Q6 (a) & (b), Q7, Q9, Q10, Q12	Q1, Q2, Q3, Q5 (b)–(e), Q6, Q7, Q8, Q9, Q10, Q12, Q13	Q1, Q2, Q5 (c)–(f), Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13

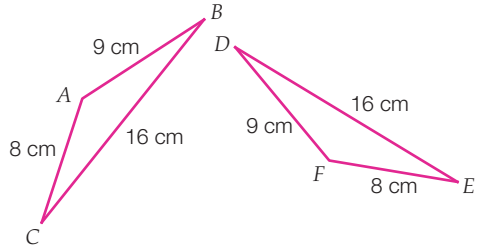
Answers page 639

Equipment required: Ruler, protractor and compass for Questions **6**, **8**, **9**, **10**, **11**, **12** and **13**

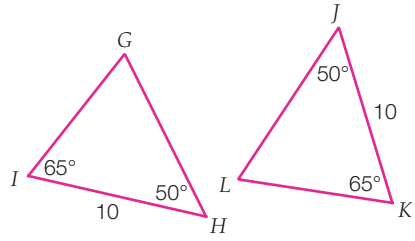
Fluency

1 Show that each of the following pairs of triangles are congruent, stating the congruence test used. WE7

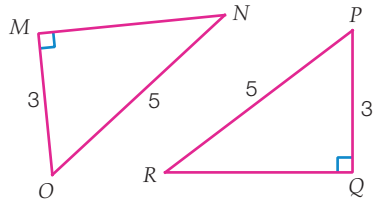
(a)



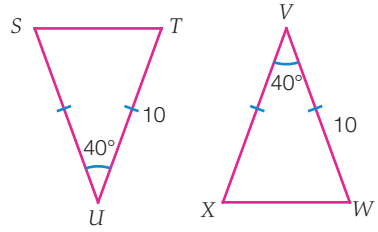
(b)



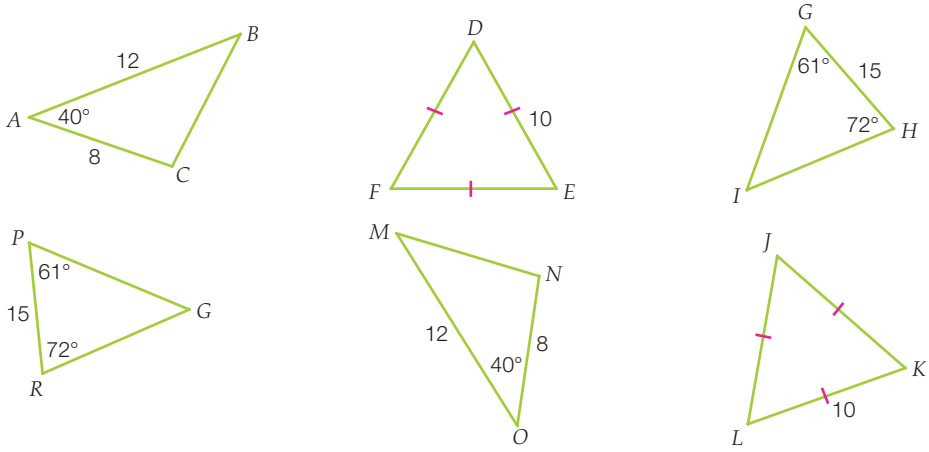
(c)



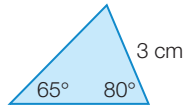
(d)



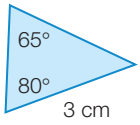
2 Find three pairs of congruent triangles among the following diagrams. Write a congruence statement for each pair and state the congruence test used.



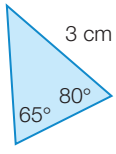
3 Which of the following triangles is not congruent to the triangle shown?



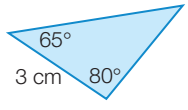
A



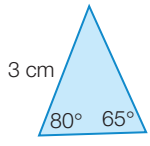
B



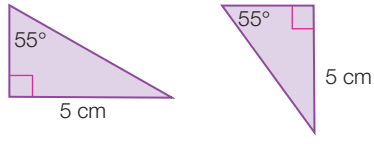
C



D



4 Which test could be used to show that these two triangles are congruent?



A SSS **B** SAS

C AAS **D** RHS

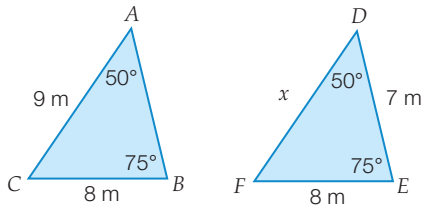
Understanding

5 For each pair of triangles below:

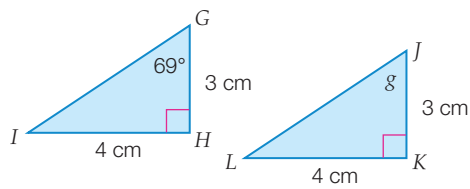
(i) show that they are congruent, stating the relevant congruency test.

(ii) Find the value of the pronumeral.

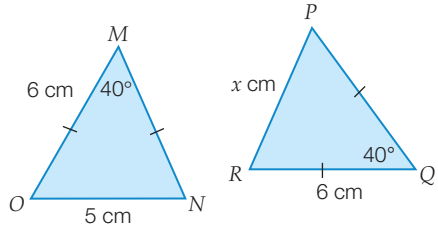
(a)



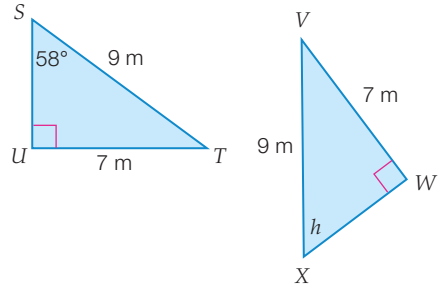
(b)



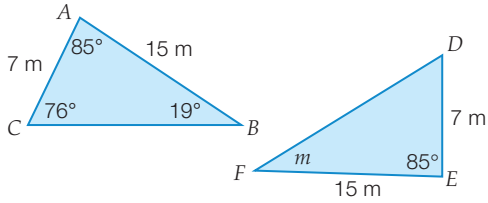
(c)



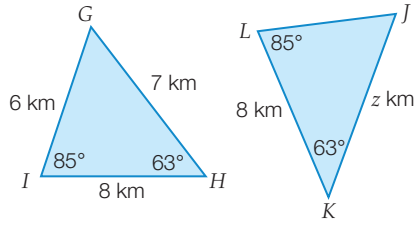
(d)



(e)



(f)



6 Use a protractor, compass and ruler to construct the following unique triangles.

(a) sides 7 cm and 4 cm, with included angle 30°

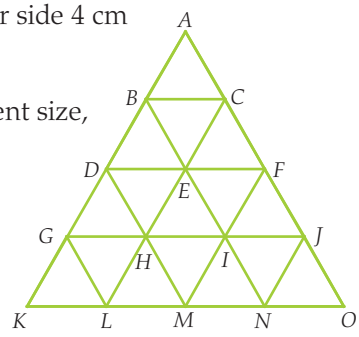
(b) angles of 20° and 100° with the side between the angles 8 cm in length

(c) right-angled triangle with hypotenuse 6 cm and another side 4 cm

(d) sides 3 cm, 5 cm and 8 cm

7 Find three pairs of congruent triangles, each pair of a different size, in this diagram. (The small triangles are all equilateral.)

State the congruence test that you used.



Reasoning

8 Triangle *ABC* has the following measurements:
$AB = 6$ cm, $\angle ABC = 50°$ and $AC = 8$ cm.

(a) Use a protractor and ruler to draw two different triangles with these measurements.

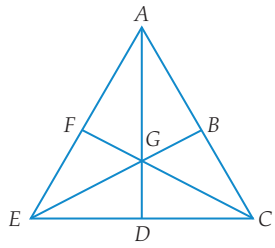
(b) Explain why two triangles with only these measurements stated do not have sufficient information to show that they are congruent.

(c) Write three further measurements for triangle *ABC* to make the triangle unique.

9 Explain why having two pairs of matching equal sides does not necessarily mean that a pair of triangles is congruent. Use a protractor, compass and ruler to draw a diagram to support your answer.

10 Explain why two triangles with three equal angles may not necessarily be congruent. Use a protractor, compass and ruler to draw a diagram to support your answer.

11 The word 'bisect' means to 'cut in half'. Each angle of the equilateral triangle shown has been bisected. Three different sizes of congruent triangle are formed. The side length of the triangle is 6 cm.



(a) Using a ruler and a protractor, draw the three different-sized triangles formed inside the equilateral triangle on the right.

(b) For each set of congruent triangles formed in the equilateral triangle above:

(i) write a congruency statement

(ii) state a congruency test to prove that they are congruent.

(c) Draw an example of the following types of triangles and bisect all angles using a protractor.

(i) isosceles triangle

(ii) scalene triangle

(d) How do the congruent triangles formed in **(c)** differ from the congruent triangles in an equilateral triangle? Why is this?

Open-ended

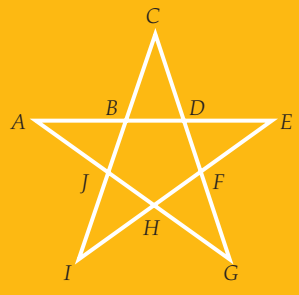
12 Construct a pair of congruent triangles of different orientation, using only a pencil, ruler and protractor. State the minimum information necessary for each triangle to show that they are congruent.

13 Construct a pair of congruent triangles of different orientation, using only a pencil, ruler and compass. State the minimum information necessary for each triangle to show that they are congruent.

Outside the Square — Problem solving

The magical pentagram

When all of the vertices of a regular pentagon are joined with a straight line to all the other vertices, a pentagram is formed. Within a pentagram there are different congruent figures.



(a) Find **(i)** two congruent kites

(ii) two large congruent isosceles triangles

(iii) two small congruent isoceles triangles.

If the vertices of the pentagram are now joined to form a regular pentagon more congruent shapes are formed.

(b) Find **(i)** two congruent rhombuses

(ii) two congruent scalene triangles

(iii) another two congruent isosceles triangles

(iv) two congruent trapeziums

(v) two more different-sized congruent trapeziums.

(c) How many different-sized isosceles triangles are there?

(d) How many congruent triangles are there of each size of isosceles triangle?

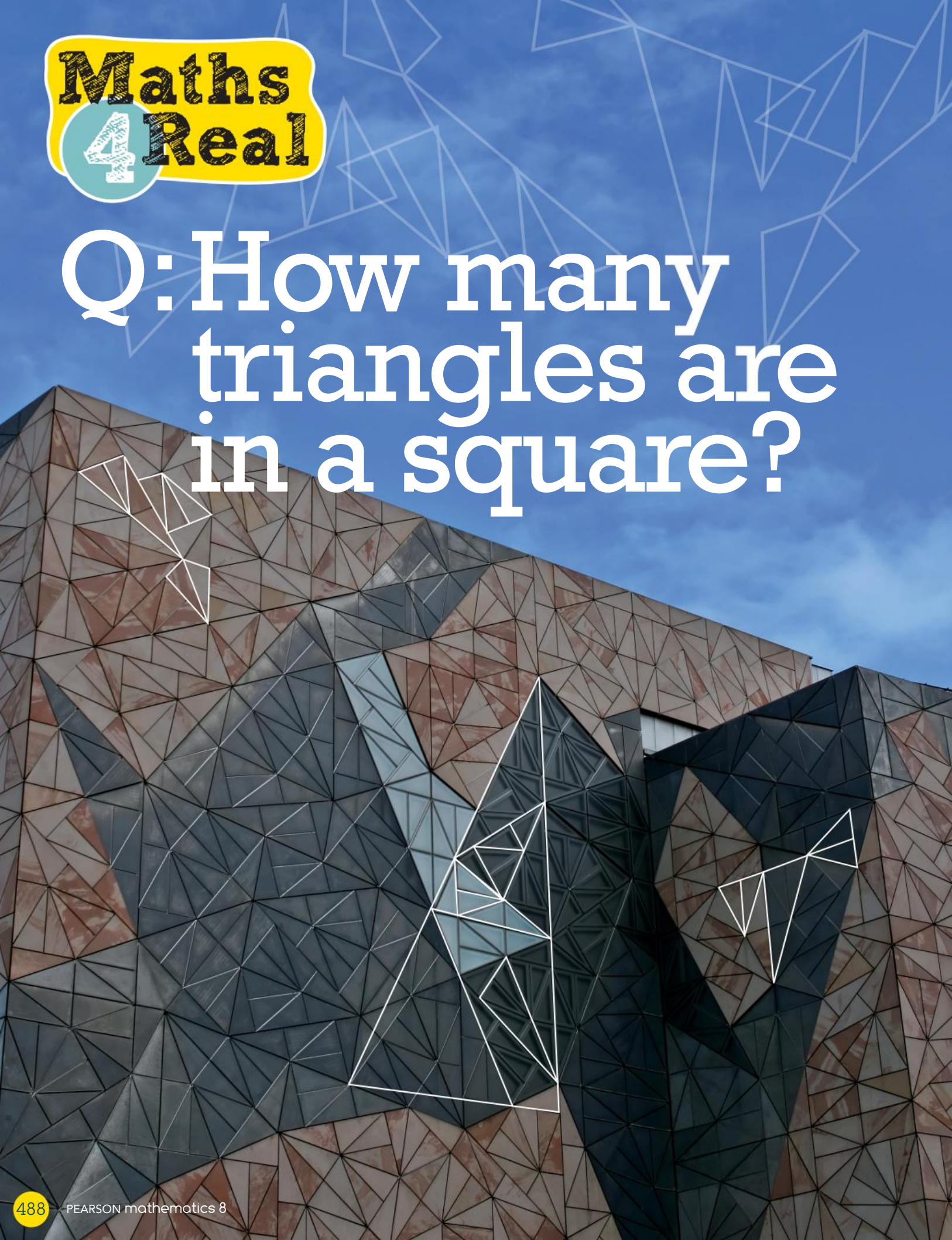
(e) How many triangles are there altogether in the diagram?

Strategy options

- Draw a diagram.
- Look for a pattern.

Maths 4 Real

Q: How many triangles are in a square?



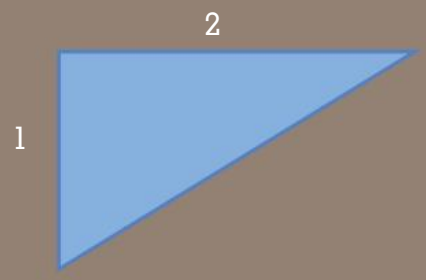
A: Far too many to count if the square we are talking about is Federation Square in Melbourne, the capital of Victoria. The buildings are covered with small right-angled triangles as seen in the photo. Many people love the design for its novel way of using a simple triangle to create patterns that have such a visual impact. Others hate it, believing that such stark and modern buildings have no place among the old and familiar architecture of yesteryear.

If you look closely you can see that the triangles join together to make many different shapes.

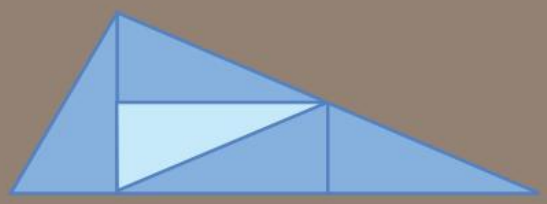
1 Find an isosceles triangle, an irregular quadrilateral, a parallelogram, a rectangle, a square, a kite, a trapezium and a pentagon in the design.

Draw diagrams to show what you have found.

How have all the triangular tiles been fitted together? We start with a right-angled triangle in which the middle-length side is twice the length of the shortest side. This is sometimes called a Golden triangle.

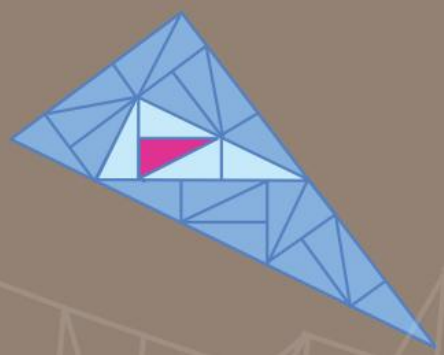


2 Draw four congruent triangles around this triangle to make a larger triangle as shown below.



3 How many triangles can you find in this shape? Make sure you find more than the five original triangles.

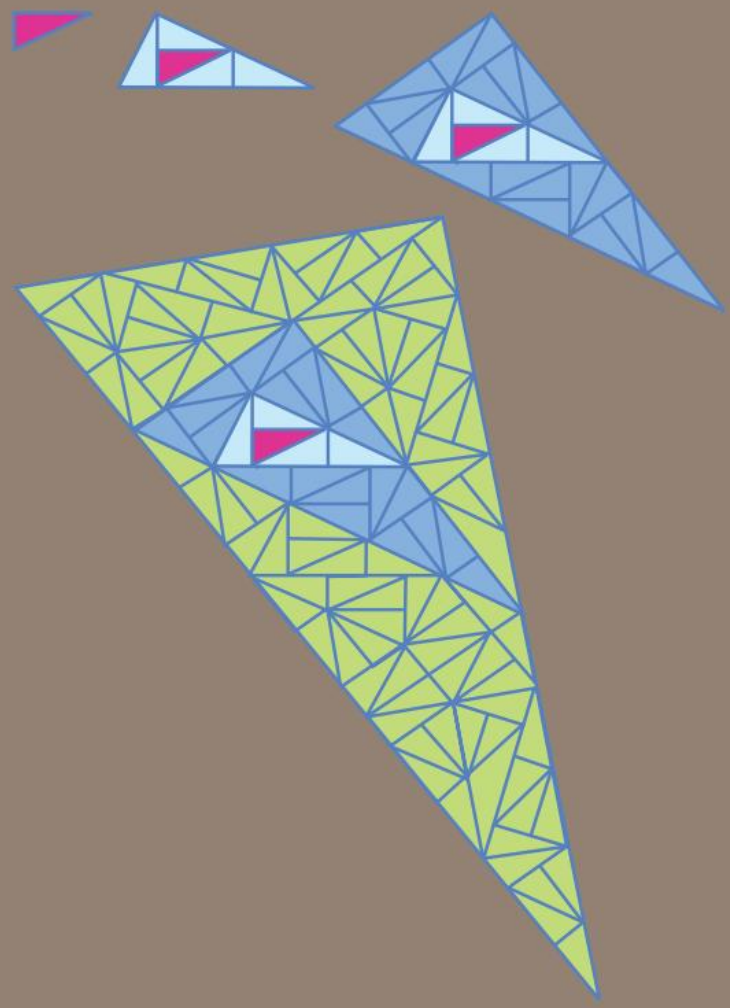
4 Using five of these new triangles, construct an even larger triangle as shown below.



5 How many triangles can you find now?

6 Find the number of triangles in the next size as shown below.

Use your answer to Question **5** to find a pattern that will help you.



7 Look closer at the sequence of diagrams above. What is happening to the orientation of the triangles as the largest triangle increases in size?

(Hint: This tiling is called pinwheel tiling after the pinwheel toy that rotates in the wind. This effect was discovered by the mathematician John Conway.)

8 Does pinwheel tiling give you a repeating (tessellating) pattern? Refer to the picture of Federation Square on the opposite page.

Research

Find another example of pinwheel tiling in your local area, your capital city or anywhere else in the world.

8.5 Congruence and quadrilaterals

Definitions and properties of geometric figures

It is important to know the difference between a definition and a property of a geometric figure.

A definition is the *minimum* amount of information necessary to describe a figure *exactly*.

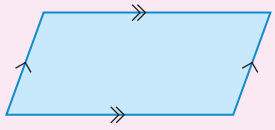
A property is any characteristic that a figure possesses after it has been defined.

Definitions of special quadrilaterals

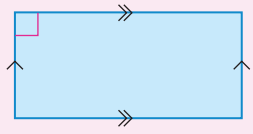
A quadrilateral is a polygon with four straight sides.

The definitions of six special quadrilaterals, namely a parallelogram, a rectangle, a kite, a rhombus, a square and a trapezium, are given below. We will use these definitions to prove angle and side properties for each of these quadrilaterals.

A parallelogram is a quadrilateral with both pairs of opposite sides parallel.



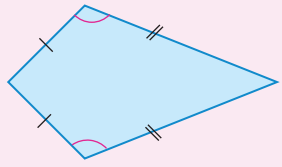
A rectangle is a quadrilateral with both pairs of opposite sides parallel and a right angle.



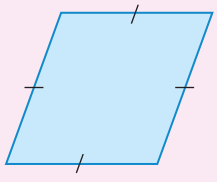
It can also be described as:

- a parallelogram with a right angle.

A kite is a quadrilateral with two pairs of adjacent sides equal in length.



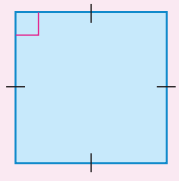
A rhombus is a quadrilateral with all sides equal in length.



It can also be described as:

- a parallelogram with adjacent sides equal
- a kite with all sides equal in length.

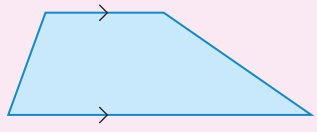
A square is a quadrilateral with all sides equal and a right angle.



It can also be described as:

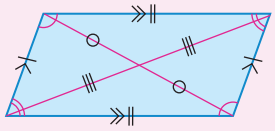
- a parallelogram with all sides equal and a right angle
- a rectangle with all sides equal
- a rhombus with right angles.

A trapezium is a quadrilateral with only one pair of parallel sides.



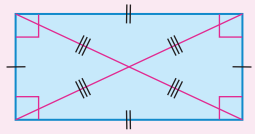
Properties of special quadrilaterals

Parallelogram



- Both pairs of opposite sides are equal in length.
- Both pairs of opposite angles are equal.
- The diagonals bisect each other.

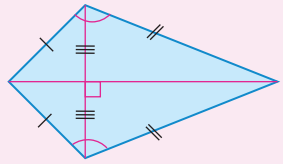
Rectangle



A special type of parallelogram

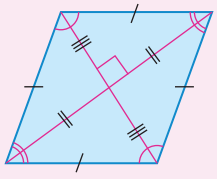
- Both pairs of opposite sides are equal in length.
- All angles are right angles.
- The diagonals are equal in length and bisect each other.

Kite



- One pair of opposite angles is equal.
- The diagonals intersect at right angles.
- The diagonal that passes through the two equal angles is bisected by the other diagonal.

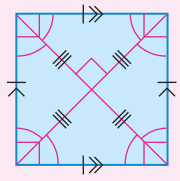
Rhombus



A special type of parallelogram and kite

- Both pairs of opposite sides are parallel.
- Both pairs of opposite angles are equal.
- The diagonals bisect each other at right angles.
- Each diagonal bisects the two interior angles through which it passes.

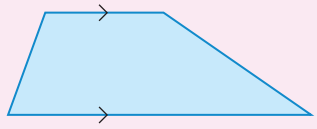
Square



A special type of parallelogram, rhombus, rectangle and kite

- All angles are right angles.
- The diagonals are equal in length and bisect each other at right angles.
- Each diagonal bisects the two interior angles through which it passes and each angle formed equals 45°.

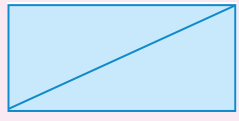
Trapezium



- A trapezium has no special angle, side or diagonal properties.

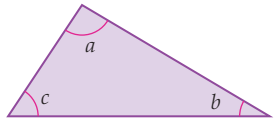
Proving special properties

Quadrilaterals can be divided into two triangles with a diagonal line. If we can prove that the two triangles are congruent using the definition of the special quadrilaterals, we can prove the properties of the special quadrilateral. To 'prove' something is to demonstrate that it is true for all cases.



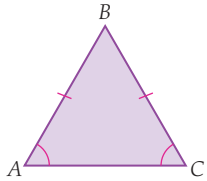
We will use our knowledge of angle sums and angles on parallel lines to help us investigate the properties of special quadrilaterals.

- Angles in a triangle add to 180°.



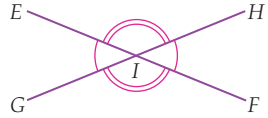
$a + b + c = 180°$

- Base angles of isosceles triangles are equal.



$\angle BAC = \angle BCA$

- Vertically opposite angles are equal.

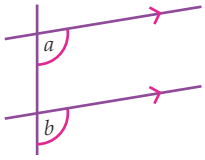


$\angle EIG = \angle FIH$

$\angle EIH = \angle GIF$

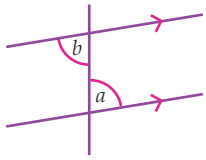
When two parallel line are crossed by a transversal (a third line):

- Corresponding angles are equal



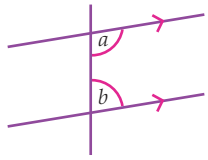
$\angle a = \angle b$

- Alternate angles are equal



$\angle a = \angle b$

- Co-interior (allied) angles add to 180°



$\angle a + \angle b = 180°$

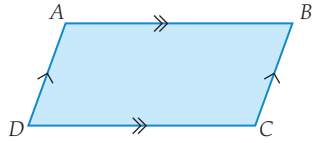
Worked Example 8

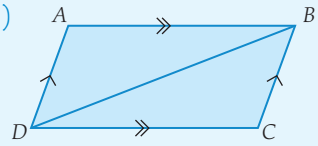
WE 8

Use the definition of a parallelogram and congruent triangles to prove that:

(a) both pairs of opposite sides are equal in length

(b) both pairs of opposite angles are equal.



Thinking	Working
(a) 1 Divide the parallelogram into two triangles by joining opposite vertices. (Join *B* to *D*.)	(a) 
2 Determine whether the two triangles are congruent by writing down known information about sides and angles. One side is common and two pairs of angles are equal because they are alternate angles on parallel lines ($\angle ABD$ and $\angle CDB$, $\angle ADB$ and $\angle CBD$). Therefore, the two triangles are congruent (AAS).	$\angle ABD = \angle CDB$ (alternate angles) $\angle ADB = \angle CBD$ (alternate angles) $BD = DB$ (common side)
3 Write a congruence statement with the relevant congruence test in brackets.	Therefore, $\Delta ABD \equiv \Delta CDB$ (AAS)
4 Matching sides of congruent triangles are equal.	Therefore, $AB = CD$ and $AD = CB$
5 State the property you have just proved.	Therefore, both pairs of opposite sides are equal in length.

(b) **1**	State the congruent triangles found in part **(a)**.	(b) $\Delta ABD \equiv \Delta CDB$
2	Matching angles of congruent triangles are equal.	Therefore, $\angle DAB = \angle BCD$
3	Use co-interior angles to show that the other pair of angles are equal.	$\angle ADC = 180° - \angle DAB$ (co-interior) and $\angle CBA = 180° - \angle DAB$ (co-interior) Therefore, $\angle ADC = \angle CBA$
4	State the property you have just proved.	Therefore, both pairs of opposite angles are equal.

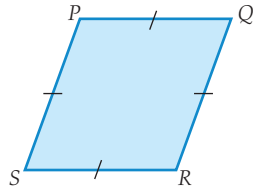
Worked Example 9

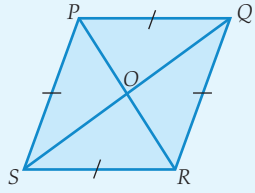
WE 9

Use the definition of a rhombus to prove that:

(a) the diagonals bisect each other at right angles

(b) each diagonal bisects the two interior angles it passes through.



Thinking		Working
(a) **1**	Divide the rhombus into four triangles by drawing both diagonals. (Join P to R and Q to S.) Label the point of intersection O.	(a) 
2	Prove that two large triangles are congruent by using the definition of a rhombus (four equal sides).	$PS = QR$ $SR = PQ$ PR is common
3	Write a congruence statement.	Therefore, $\Delta PSR \equiv \Delta RQP$ (SSS)
4	Prove two small triangles opposite each other are congruent, using properties of the larger congruent triangle if necessary.	$\angle POS = \angle QOR$ (vertically opposite) $PS = QR$ (given) $\angle SPR = \angle QRP$ ($\Delta PSR \equiv \Delta RQP$)
5	Write a congruence statement.	$\Delta POS \equiv \Delta ROQ$ (AAS)
6	Matching sides of the congruent triangles are equal.	Therefore, $PO = RO$ and $SO = QO$
7	Prove two small triangles adjacent to each other are congruent.	$PS = PQ$ (given) PO is common $SO = QO$ (proven)
8	Write a congruence statement.	Therefore, $\Delta POS \equiv \Delta POQ$ (SSS)
9	Because we have two congruent triangles with a common side, we can make a statement about the adjacent angles.	Therefore, $\angle POS = \angle POQ = 90°$ (angles on a straight line) Similarly, $\angle ROS = \angle ROQ = 90°$

10	State the property you have just proved.	The diagonals bisect each other at right angles.
(b) 1	Write down any information needed that was proved in part **(a)**.	(b) $\angle SPR = \angle QRP$
2	Use the base angles of congruent isosceles triangles to find more equal angles. (*PSR* and *PQR* are isosceles.)	$\angle SPR = \angle SRP$ (base angles of an isosceles triangle) and $\angle QRP = \angle QPR$ (base angles of isosceles triangle) Therefore, $\angle SPR = \angle QPR = \angle SRP = \angle QRP$
3	Show all angles that are equal to each other.	So, $\angle SPR = \angle QPR$ and $\angle SRP = \angle QRP$ Similarly, $\angle PSQ = \angle RSQ = \angle RQS = \angle PQS$
4	State the property you have just proved.	Each diagonal bisects the two interior angles it passes through.

8.5 Congruence and quadrilaterals

Navigator

Answers page 640

Q1, Q2, Q3, Q5, Q6 (a)–(d), Q7, Q8, Q10	Q1, Q2, Q3, Q4, Q5, Q6 (c)–(f), Q7, Q8, Q10, Q11	Q1, Q2, Q3, Q4, Q5, Q6 (e)–(h), Q7, Q9, Q10, Q11

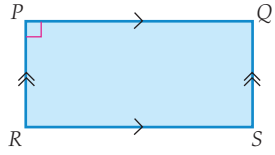
Fluency

WE8

1 Use the definition of a rectangle and congruent triangles to prove that:

(a) both pairs of opposite sides are equal in length

(b) both pairs of opposite angles are equal.

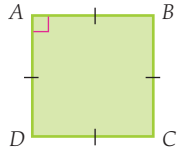


WE9

2 Use the definition of a square to prove that:

(a) the diagonals bisect each other at right angles

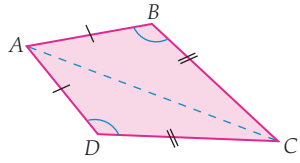
(b) each diagonal bisects the two interior angles it passes through.



3 A kite *ABCD* has been divided into two congruent triangles. Which of the following statements is true?

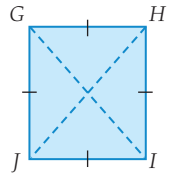
A $AD \| BC$ **B** $\angle ABC = \angle CDA$

C $\Delta BCD \equiv \Delta DAB$ **D** $CD = AB$

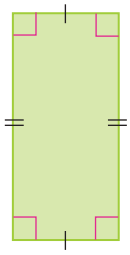


4 Prove that the diagonals of a kite intersect at right angles by first dividing it into two congruent triangles.

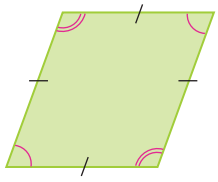
5 Square *GHIJ* is shown. Diagonals *GI* and *HJ* divide the square into four congruent triangles. Which of the following shapes will also divide into four congruent triangles when both diagonals are drawn?



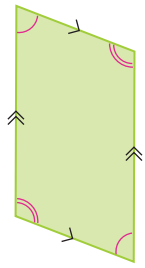
A



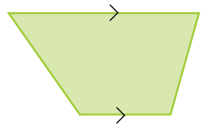
B



C



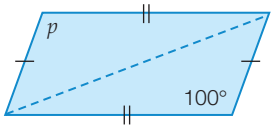
D



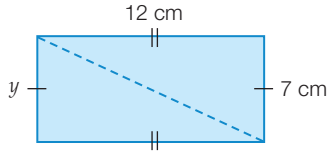
Understanding

6 Use congruent triangles and known angle facts to find the value of the pronumerals in the following quadrilaterals. State the reason for each of your answers.

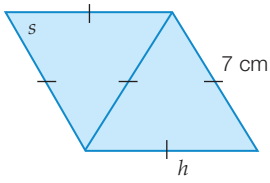
(a)



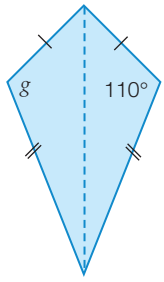
(b)



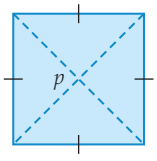
(c)



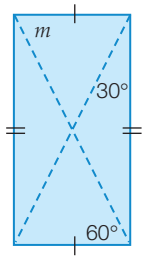
(d)



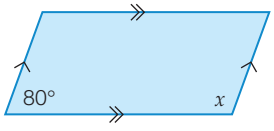
(e)



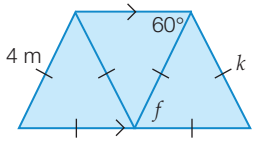
(f)



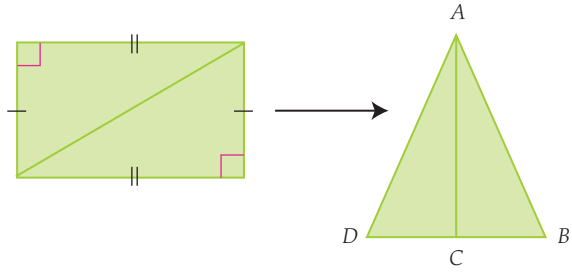
(g)



(h)



7 A rectangle has been cut along one diagonal to divide it into two triangles. The triangles are then rearranged to form another triangle.



(a) Mark equal side lengths and known angles on the triangle.

(b) Prove $\angle ADC = \angle ABC$.

(c) What type of triangle is this?

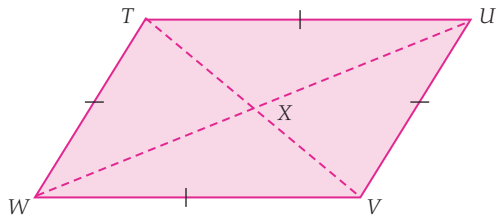
Reasoning

8 Draw a rectangle with length 6 cm and width 4 cm. Rule a diagonal line, dividing the rectangle into two triangles.

(a) Are the two triangles congruent? State a congruency test that proves the triangles will be congruent in all rectangles.

(b) The two congruent triangles in the rectangle you drew can be rearranged to form two other special types of quadrilateral. State the names of the two special quadrilaterals. Write a statement to show the congruent triangles formed within the quadrilaterals.

9 Rhombus *TUVW* is shown. Diagonals *UW* and *VT* divide it into four triangles, intersecting at point *X*.



(a) Prove $\angle TWV = \angle TUV$.

(b) What type of triangle is *TUW*?

(c) Prove $TX = XV$.

(d) Do any other quadrilaterals share these properties?

Open-ended

10 Draw a diagram to support each of the following statements.

(a) A parallelogram can be divided into two congruent parallelograms.

(b) A parallelogram can be divided into two parallelograms which are not congruent.

(c) A parallelogram can be divided into a rectangle and two congruent triangles.

11 Draw a quadrilateral in which one diagonal will divide the shape into congruent halves and the other diagonal will not.

Outside the Square — Problem solving

The tablet of Geo Met Tree

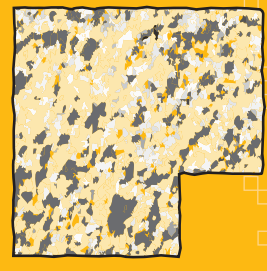
An archaeological dig has unearthed an ancient stone tablet. The tablet was wrapped inside a scroll that explained the mystical powers the tablet was believed to possess. The scroll read:

'This tablet may seem incomplete, yet a life of prosperity will be given to the one who, with two straight cuts, can arrange all pieces into a square.'

Can you cut the tablet and then arrange the pieces in the shape of a square?

Strategy options

- Guess and check.
- Make a model.



Investigation

The Möbius strip

Equipment required: 1 brain, paper, scissors, glue

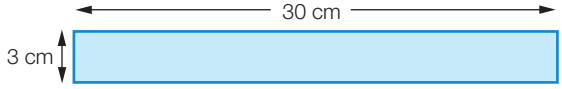
The Möbius strip is a loop with amazing properties discovered by the German mathematician and astronomer August Ferdinand Möbius (1790–1868).

The Big Question

Can a piece of paper have just one surface?

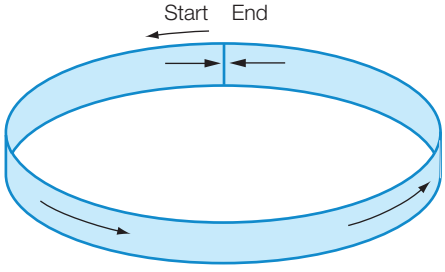
Engage

Cut several strips of paper about 3 cm wide and 30 cm long.



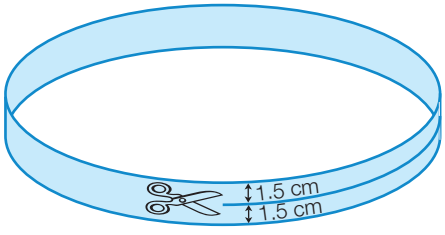
Take one of the strips and glue the ends together to form a loop as shown.

Draw a line all around the outside of the loop, starting at the join in the loop and not lifting your pencil until you get back to the join.



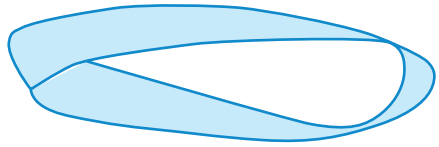
1 How many distinct (separate) surfaces does this loop have?

2 Cut the loop down the middle, right around. What do you end up with after cutting?



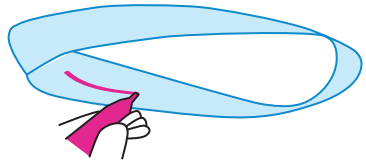
Explore

To investigate the Big Question, you need to make a Möbius strip. Take the second strip of paper, but before glueing the ends together, give one end a half-twist.



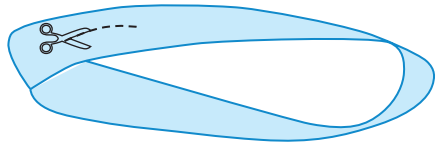
3 How many surfaces do you think the Möbius strip has?

To check your prediction, start at a point and begin drawing a line along the loop. Do not take the pencil off the loop until you reach the starting point again.



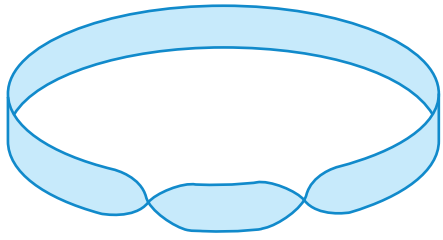
4 How many surfaces does a Möbius strip have?

Now, cut the Möbius strip down the middle lengthways, right around the loop.



5 What figure results from cutting the strip?

Take the third strip of paper. Give one end two half-twists (one complete 360° twist) and then glue the ends together.



Again, draw a line along the loop without lifting the pencil.

6 How many surfaces does this loop have?

Strategy options

- Make a model.
- Look for a pattern.

7 Predict what figure will result from cutting this loop along the middle, lengthways. Now, cut the loop.

Explain

8 Copy and complete the table below.

Number of half-twists	Number of surfaces	Figure resulting from cutting along middle
0		
1		
2		

9 Does a Möbius strip have an inside and an outside surface? Explain.

10 Describe the effect of cutting a Möbius strip along the middle. Why do you think this happens?

Elaborate

11 (a) Predict how many surfaces a loop with three half-twists will have.

(b) Check your prediction.

12 (a) If the strip of paper is given three half-twists to form a loop, what figure do you predict will result from cutting it along the middle all the way around? How confident are you that your prediction is correct?

(b) Check your prediction.

13 (a) Can you draw a conclusion that connects the number of twists with the number of surfaces? Do you have enough evidence for this conclusion?

(b) Answer the Big Question.

Evaluate

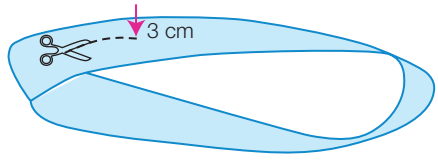
14 Were you surprised by some of your observations in this investigation? Explain why or why not.

15 How is the mathematical thinking that is needed for tasks like these different to the thinking required for other maths tasks, such as measurement calculations or algebra?

Extend

Investigate the properties of the Möbius strip further.

Start by making another Möbius strip, but instead of cutting it lengthways in the middle, cut the strip starting one-third from the edge.

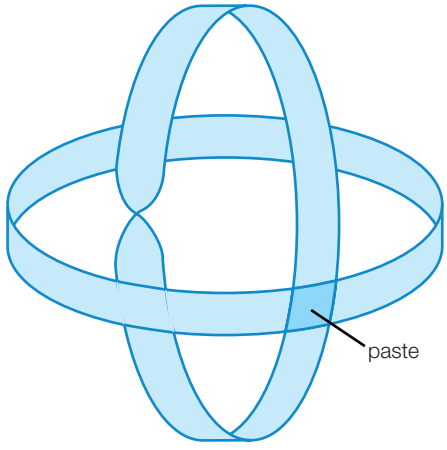


16 What figure was created when cutting a Möbius strip one-third from the edge?

17 (a) Predict the figure that will result from cutting a loop in this way for loops with different numbers of half-twists.

(b) Check your prediction.

Paste an ordinary loop and a Möbius strip together as shown.



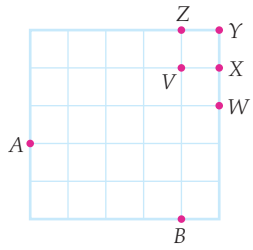
18 What happens if both are cut lengthways?

Challenge 8

1 The points A, B, V, W, X, Y, Z are marked on a square grid as shown.

When joined to the points A and B, some of the points W, X, Y and Z will form an isosceles triangle. These points are:

A V and X **B** X and Y **C** Y and Z **D** V and Z



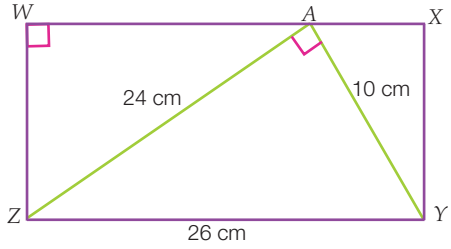
2 What is the area of the rectangle $WXYZ$?

A 120 cm^2

B 240 cm^2

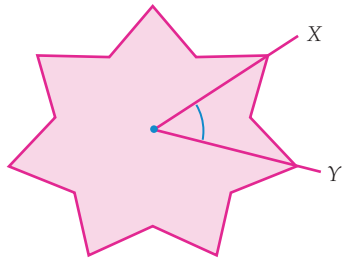
C 260 cm^2

D 624 cm^2



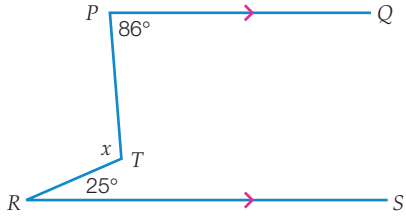
3 The large star on the Australian flag has 7 points, one for each state and one for the territories. The angle between X and Y as seen from the centre of the star is:

A 50° **B** $50\frac{3}{7}$° **C** $51\frac{3}{7}$° **D** $60\frac{3}{7}$°



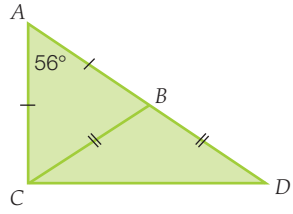
4 In the diagram, PQ is parallel to RS. The value of x is:

A 121° **B** 111° **C** 101° **D** 61°



5 In the diagram, $AB = AC$ and $BC = BD$. If $\angle BAC = 56°$, then $\angle CDB$ equals:

A 28° **B** 31° **C** 59° **D** 62°



6 Twenty-seven small cubes have been stacked to form this shape.

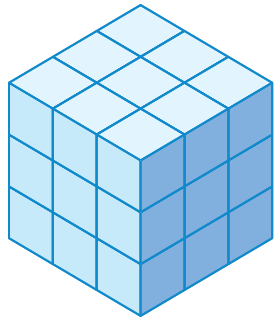
How many cubes have:

(a) no faces showing

(b) one face showing

(c) two faces showing

(d) three faces showing?



Chapter review 8

D.I.Y. Summary

Key Words

acute-angled triangle	congruence statement	hypotenuse	right-angled triangle
adjacent angles	congruent	included angle	scalene triangle
alternate angles	congruent triangle	interior angle	supplementary angles
bisect	corresponding angles	isosceles triangle	transversal
co-interior angles	equilateral triangle	obtuse-angled triangle	vertex
complementary angles	exterior angle	quadrilateral	vertically opposite angles

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 The point at which two arms of an angle meet is called the _____.

2 Shapes that can be placed on top of each other so that every side and vertex match up are said to be _____.

3 $EFGH \equiv PQRS$ is an example of a _____.

4 Triangles with one equal side length and two equal angles marked can be proven to be _____.

5 _____ add to 180°.

6 A _____ is a line that intersects two or more parallel lines.

7 A _____ has three sides, none of which are equal in length.

8 _____ are equal and lie on either side of a transversal.

9 _____ have a common arm and a common vertex.

10 An _____ has three equal sides and three equal angles.

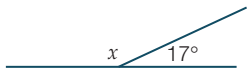
Fluency

Equipment required: A ruler, protractor and compass for Questions **13** and **18**

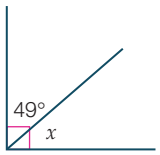
Ex. 8.1

1 Find the value of x in the following diagrams.

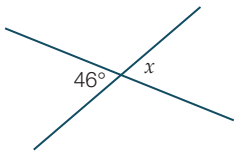
(a)



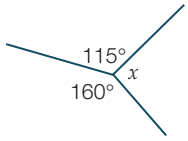
(b)



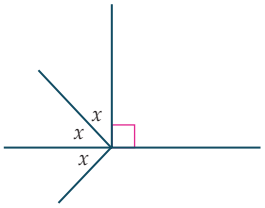
(c)



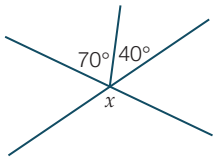
(d)



(e)

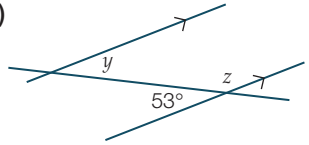


(f)

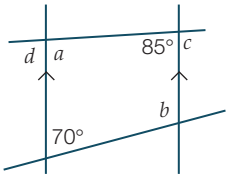


2 Find the value of the pronumerals in the following diagrams. **Ex. 8.1**

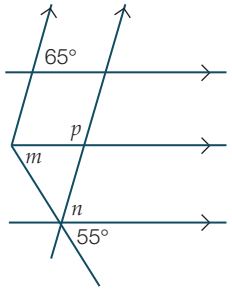
(a)



(b)

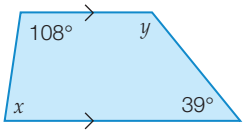


(c)

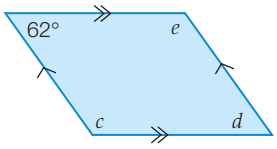


3 Find the value of the pronumerals in the following diagrams. **Ex. 8.2**

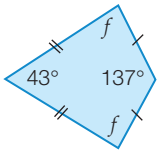
(a)



(b)



(c)



4 Match the name of the shape with its correct description. **Ex. 8.2**

(a)	Rectangle	A	Adjacent sides of equal length, no parallel sides
(b)	Kite	B	One pair of opposite sides parallel
(c)	Isosceles triangle	C	Opposite sides equal in length, all angles 90°
(d)	Trapezium	D	Two sides equal in length

5 Plot the following points on the Cartesian plane: (-2, 1), (-3, 4), (1, 5), (1, 2). Join and label them *W*–*Z* in the order given to form the quadrilateral *WXYZ*. Perform the following transformations on *WXYZ*. Label the transformed shape *W'X'Y'Z'*. **Ex. 8.3**

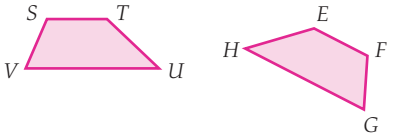
(a) A reflection in the *x*-axis

(b) The translation [3, -4]

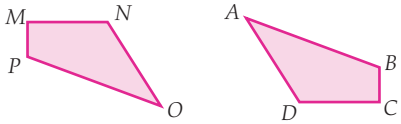
(c) A rotation 180° anticlockwise about the point (-3, 0)

6 For the two pairs of congruent figures shown below, complete the following pairings. **Ex. 8.3**

(i)



(ii)

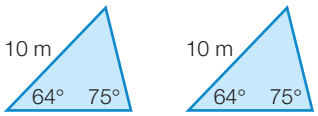


(a) $H\leftrightarrow$ (b) $N\leftrightarrow$ (c) $TU\leftrightarrow$

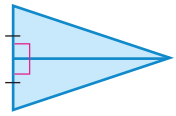
(d) $AD\leftrightarrow$ (e) $\angle CDA\leftrightarrow$ (f) $\angle EFG\leftrightarrow$

7 State the congruency test used to show that each of the following pairs of triangles are congruent. **Ex. 8.4**

(a)

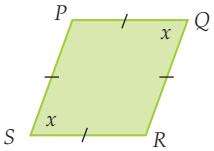


(b)

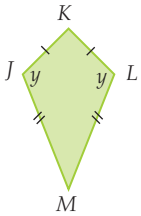


8 Use congruent triangles to prove that the opposite angles shown in the shapes below are equal: **Ex. 8.5**

(a)



(b)



Understanding

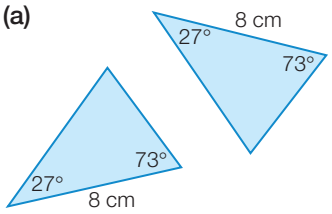
9 **(a)** How many triangles can a heptagon be divided into?

(b) Use your answer to **(a)** to determine the sum of the angles in a heptagon.

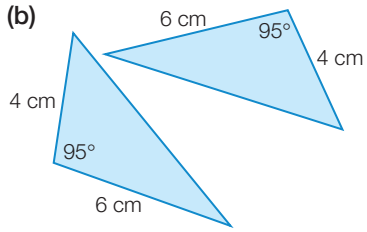
(c) Use your answer to **(b)** to determine the size of the interior angle in a regular heptagon.

10 Explain why each of the following pairs of triangles are congruent.

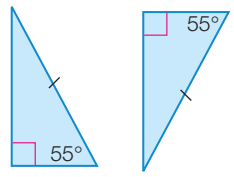
(a)



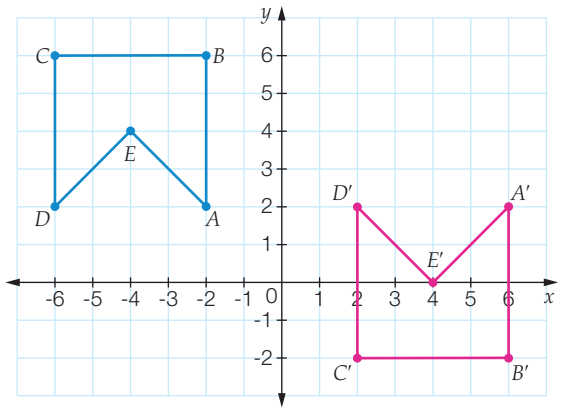
(b)



(c)



11 The shape *ABCDE* has had two transformations applied to it. Which sequence of transformations will place *ABCDE* onto the image *A′B′C′D′E′*?



A Reflection in the y-axis, then the translation [0,-2].

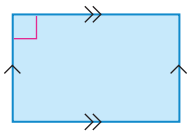
B Rotation 90° clockwise about the point (-2, 2), then the translation [2, 0].

C Rotation 180° anticlockwise about the point (0, 0), then the translation [0, 2].

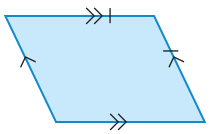
D Reflection in the x-axis, then the translation [8, 4].

12 Identify each of the following quadrilaterals using the name that corresponds to all the properties shown.

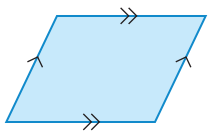
(a)



(b)



(c)



13 **(a)** Use a pencil, ruler and protractor to construct the following.

(i) a triangle with angles of 35° and 65°, and a side in between them of 7 cm

(ii) a triangle with sides of 9 and 6.5 cm, and an angle in between them of 48°

(b) Use a pencil, ruler and compass to construct the following.

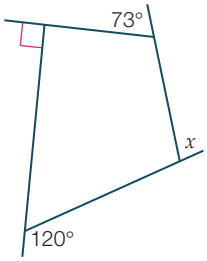
(i) a triangle with sides of 7, 9 and 11 cm

(ii) a right-angled triangle with an hypotenuse of 13 cm and a shorter side of 12 cm

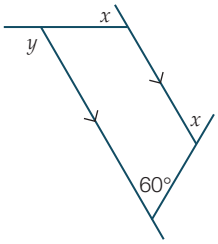
Reasoning

14 Find the value of the pronumerals in each of the following quadrilaterals.

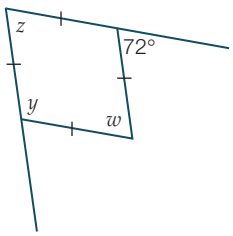
(a)



(b)



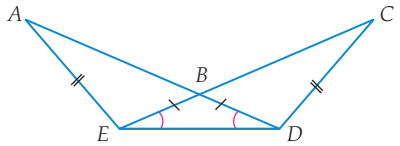
(c)



15 Which of the following special quadrilaterals will always divide into two congruent triangles along a diagonal?

kite, trapezium, rhombus, rectangle

16 Two triangles are placed so that an isosceles triangle forms where the two triangles overlap. Is there enough information to prove the triangles are congruent? Why or why not?



17 Triangle *ABC* has the following measurements: $AB = 8$ cm, $\angle ABC = 55°$, $AC = 10$ cm. Triangle *DEF* also has the same measurements.

(a) Use a protractor and ruler to draw two different triangles with these measurements.

(b) Explain why there is not enough information to show that ΔABC and ΔDEF are congruent.

(c) Give one extra piece of information that would ensure that the two triangles are congruent.

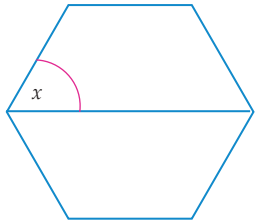
18 Explain how a square can be both a special type of rhombus and a special type of rectangle.

NAPLAN practice 8

Numeracy: Non-calculator

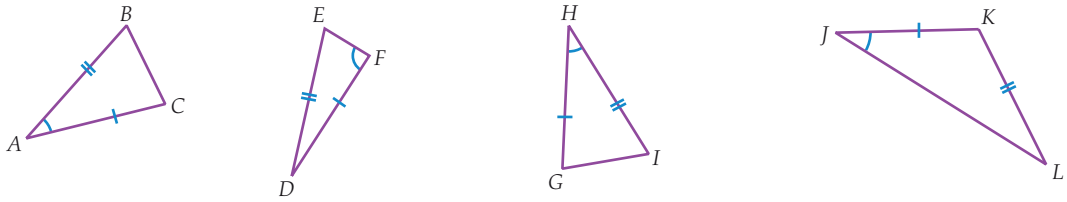
1 Two identical trapeziums fit together to make the regular hexagon shown.

What is the value of x?



2 Which pair of these triangles is congruent?

A ΔABC and ΔDEF **B** ΔGIH and ΔKLJ **C** ΔGHI and ΔCAB **D** ΔFDE and ΔJKL



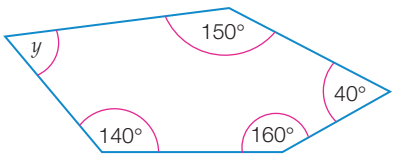
3 Which of these are not always parallel?

A the opposite sides of a square **B** the opposite sides of a parallelogram

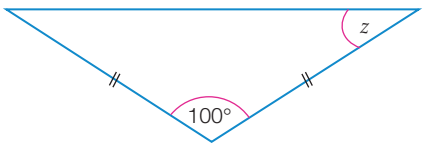
C the opposite sides of a rhombus **D** the opposite sides of a kite

Numeracy: Calculator allowed

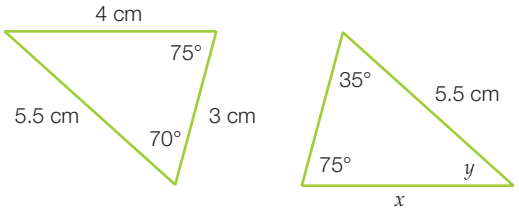
4 What is the value of y?



5 What is the value of z?



6 The following two triangles are congruent. Determine the value of the pronumerals.



9

Statistics and probability

9

Counting the catch. The harvesting of seafood, such as abalone, is a profitable business venture in which statistics plays a vital role.

Concern is often expressed about the effect of overfishing on the number of fish and other forms of seafood remaining in the ocean. Some effort must be made to keep the catch size to a sustainable limit. Statistical techniques are used to estimate the number of each species. Recently, a digital video survey program has been implemented to keep track of abalone numbers. It has been found that this method produces comparable results to previously used methods for 40% of the sampling cost and effort. Abalone can fetch more than $40 per kilogram, so it is easy to see why keeping this industry viable is so important.

Forum

How do you think catch limits are enforced when a species is in danger of over-harvesting? Does this apply only to fish? Why was it mentioned that the video survey method was 40% of the cost of other older methods?

Why learn this?

How we make sense of data is an important part of life. From checking tables to see which supermarket is the cheapest in our area to deciding whether or not we should insure the family car, we make decisions every day based on statistics and probability, sometimes without even knowing that we're doing so.

After completing this chapter you will be able to:

- use sample statistics to predict population statistics
- understand why random sampling is necessary to avoid bias
- construct frequency tables and graphs with and without technology
- calculate measures of centre and spread from grouped and ungrouped data
- describe the shape of data in terms of symmetry and skew
- understand that probabilities range between 0 and 1
- calculate complementary probabilities
- use Venn diagrams and two-way tables to calculate probabilities involving criteria of 'and', 'or' 'given' and 'not'.

Recall 9

Prepare for this chapter by attempting the following questions. If you have difficulty with a question, go to Pearson Places and download the Recall Worksheet from Pearson Reader.

Equipment required: Calculator for Questions **2** and **4**

Worksheet R9.1

1 Students in a Year 8 class were surveyed as to how many pets they had, and the results are shown below.

1, 4, 0, 1, 3, 6, 2, 1, 1, 1, 2, 3, 2, 1, 1, 2, 3, 5, 1, 1, 2, 2, 2, 1, 2

(a) Tally these results and construct a frequency table.

(b) Construct a frequency column graph showing this information.

Worksheet R9.2

2 Find the mean, median and mode for the following sets of data. Use a calculator when necessary and round off your answers to one decimal place.

(a) 2, 4, 2, 7, 3, 5, 4, 2, 3, 6, 1

(b) 40, 20, 60, 30, 50, 10

Worksheet R9.3

3 State which of the two probabilities represents a 'more likely' chance of an event.

(a) $\frac{1}{4}$ or $\frac{3}{4}$

(b) 0.6 or 0.8

Worksheet R9.4

4 Find the mean, median and mode for the following data set that represents the number of cars in each of the households for the students in a class. Use a calculator when necessary and round your answers to one decimal place.

Number of cars	Frequency
0	1
1	5
2	9
3	6
4	3
5	1

Worksheet R9.5

5 Find the median for the data set shown in the stem plot.

STEM	LEAF
2_L	2 2 4
2_H	6 7 7 9
3_L	0 0 0 1 1 3
3_H	5 5 6
4_L	0 0 1 1 2 3

Key 2 | 2 = 22

Key Words

bias	intersection	outcome	sample space
census	mean	outlier	sample statistics
class centre	measures of centre	poll	two-way table
class intervals	median	population	union
complement	median class interval	population statistics	universal set
frequency column graphs	midpoint	random sampling	Venn diagram
grouped data	modal class interval	range	
histogram	mode	sample	

Population sampling

9.1

Statistics can be calculated using data collected from a **population** or a **sample**.

- A population is the entire category and often refers to people but, in statistics, it is used to describe any defined category, such as birds, insects, light globes, vases, cars.
- A sample is a small proportion of the population.

When dealing with statistics, it is important to know whether we are using **population statistics** or **sample statistics** and that we can use data from samples instead of an entire population to gain information about the population.

Population statistics

- A **census** is the process of collecting data from every member of the population.
- Population statistics are statistics calculated from the data collected by a census.

Sample statistics

A census is expensive and time-consuming, and statisticians have found that reliable information can be gathered by using data from a sample instead of the entire population. Therefore, samples are frequently used by market research companies to find out whether a new product, a chocolate bar, for example, is likely to be successful if launched onto the market. The views of the selected people are taken to represent the views of the population as a whole.

- Sample statistics only use data from a sample.

Polls

- A **poll** is a survey of an issue given to a sample of the population. A poll is used to collect data that can be used to calculate sample statistics. These statistics are then used to predict the population statistics.

Throughout the year, we see the results of polls in the newspapers related to the performance of political parties. The frequency of these polls increases the closer we get to elections. These polls, which usually consist of about 1500 people, are then used to predict the result of the elections when more than 10 million votes may be cast. Some predictions are extremely accurate, whereas some get it wrong altogether. These election polls are excellent examples of sample statistics being used to predict the election result. The differences in predicted results are usually due to differences in the way the samples have been taken. Sample statistics from well-designed surveys should match the population statistics. This would mean that the poll accurately predicts the result of an event.

Uses of sampling

If we know the proportion of all the different categories in a sample and we know the population size we can estimate the number of each category in the population. We can expect that the range of values will vary by approximately 5% either side of our estimate.

Worked Example 1

WE1

A large box is filled with 500 building blocks all of the same size. The blocks are yellow, red and blue. A sample of 20 blocks is taken and it is found that 6 are yellow, 9 are red and the rest are blue. Make an estimate for the number of each colour of block in the box. Give a range of values that you think the number should lie between.

Thinking	Working
1 Find the proportion of each colour in the sample.	Proportion of yellow $= \frac{6}{20}$ $= \frac{3}{10}$ Proportion of red $= \frac{9}{20}$ Proportion of blue $= \frac{5}{20}$
2 Multiply the total number by the separate proportions to estimate the total population, rounding to the nearest whole number if necessary.	Estimated number of yellow $= \frac{3}{10} \times 500$ $= 150$ Estimated number of red $= \frac{9}{20} \times 500$ $= 225$ Estimated number of blue $= \frac{5}{20} \times 500$ $= 125$
3 Decide what would be a reasonable number to add and subtract from each number to give a range of values. 5% either side of each value would give a reasonable range. Answers must be rounded to the nearest whole number.	Amount added to or subtracted from yellow value = 5% of 150 = 7.5 = 8 (to the nearest whole number) Amount added to or subtracted from red value = 5% of 225 = 11.25 = 11 (to the nearest whole number) Amount added to or subtracted from blue value = 5% of 125 = 6.25 = 6 (to the nearest whole number)
4 Add these amounts to, and subtract these amounts from, your estimates.	Range for yellow = 142 to 158 Range for red = 214 to 236 Range for blue = 119 to 131

Another use of population sampling is in estimating wildlife populations. To do this, a sample of wildlife is captured, tagged and released. Later on, after the tagged wildlife has had a chance to reintegrate with the rest of the wildlife population, a second sample is taken. To estimate the population, the following proportion is used.

$$\frac{\text{tagged wildlife in 2nd sample}}{\text{total wildlife in 2nd sample}} = \frac{\text{tagged wildlife in population}}{\text{total wildlife in population}}$$

We can rearrange this to give:

$$\text{total wildlife in population} = \frac{\text{total wildlife in 2nd sample}}{\text{tagged wildlife in 2nd sample}} \times \text{tagged wildlife in population}$$

Worked Example 2

WE2

A sample of 200 fish is taken from a lake, tagged and released. Some time later, a second sample of 150 fish is taken and it is found that 15 of these are tagged. What is the estimated fish population for the lake?

Thinking	Working
1 Write the three known values.	Tagged fish in population: 200 Tagged fish in 2nd sample: 15 Total fish in 2nd sample: 150
2 Use the rule to estimate the population of fish in the lake, rounding to the nearest whole number if necessary.	Total fish in lake $= \frac{\text{total fish in 2nd sample}}{\text{tagged fish in 2nd sample}} \times \text{tagged fish in population}$ $= \frac{150}{15} \times 200$ $= 2000$
3 State the answer.	There are about 2000 fish in the lake.

Taking several samples and averaging the results give a more accurate estimate of the population. More samples help to eliminate any unusual or unrealistic results; for example, a sample having an unusually low or unusually high number of tagged fish that is not representative of the population.

Sampling bias

The word '**bias**' indicates a preference or belief that is not supported by evidence. In statistics, a sampling bias is used to describe a mathematical selection process in which unfairness or favouritism is included in some way in the collection of data so that the sample does not truly reflect the population.

Before we use a sample to make predictions about the population, we need to be sure that the sample we take is not biased. Using criteria such as age, gender, ethnicity, physical appearance, wealth and personal preferences can cause samples to be biased and should not be used in the selection process. Asking people coming out of a cinema where they have just seen an adventure movie a question such as 'Do you like going to the movies?' or 'What type of movie do you most enjoy watching?' would be considered to be biased sampling. If your sample is biased, you cannot draw any conclusions from the data.

It is possible to use unbiased data to support a biased belief.

Random sampling is the name given to sampling where there is no pattern involved in the sampling process. Random sampling eliminates the possibility of bias. Choosing names out of a hat is an example of random sampling.

Worked Example 3

WE3

For each of the following survey methods, state whether or not you think it will result in a biased sample. For those that you think are biased, give a reason why you think this is the case.

(a) You want to know the favourite sport of a city, so you ask people at an AFL football match.

(b) You are trying to find out how much time each day a person spends using a mobile phone, so you question people who are entering a mobile phone shop.

(c) You want to do a survey on the use of parks in your city, so a survey form is sent to all home and business owners in the city.

(d) You want to know about changes to speed zones in your local area, so you randomly select people from the local area phone book and conduct a phone poll.

(e) A very large business wants to know if its employees would use gym facilities if they were provided, so every third name on a list of employees is surveyed.

Thinking	Working
(a) Are people who could influence the results of your survey being excluded? Are the people being included more likely to respond in a particular way?	(a) Biased—People who are not at the match may prefer other sports. People who go to an AFL match are more likely to name AFL football as their favourite sport.
(b) Are people who could influence the results of your survey being excluded? Are the people being included more likely to respond in a particular way?	(b) Biased—There are a lot of people who own a mobile phone who do not need to go near a mobile phone store. Those who do go to the store may be higher mobile phone users than those who don't.
(c) Are people who could influence the results of your survey being excluded? Are the people being included more likely to respond in a particular way?	(c) Biased—There are people who live in the area that do not own a home (people who rent) or a business, so they are excluded. It also relies on people returning the survey they have been sent.
(d) Are people who could influence the results of your survey being excluded? Are the people being included more likely to respond in a particular way?	(d) Biased—Although people have been randomly selected, people with silent numbers or who do not have a landline phone are excluded.
(e) Are people who could influence the results of your survey being excluded? Are the people being included more likely to respond in a particular way?	(e) Fair—By selecting every third name, a random sample has been generated and, as the list contained all employees' names, everyone has an equal chance of being surveyed.

Bias in experiments

We often use coins or dice in experiments to collect data. We then use this data to calculate sample statistics. If we are going to use this data to predict the population statistics, we must be sure that the coins or dice we used are unbiased.

To be able to assess bias, we often need to run a number of trials. The size of the sample is very important in determining bias, as a small number of trials may seem to indicate bias, whereas a larger number can indicate that the coins or dice are fair.

Worked Example 4

WE4

A die is rolled 60 times and the result for each outcome is recorded. This experiment is repeated so that five sets of results are collected. These results are shown in the table opposite.

(a) Use each set of results separately to decide whether you think the die is fair.

(b) Using the separate decisions you made in part **(a)**, make an overall decision about the fairness of the die.

(c) Combine the results and comment again about the fairness of the die.

Outcome	Set 1	Set 2	Set 3	Set 4	Set 5
1	10	10	11	11	7
2	14	7	8	11	9
3	13	12	7	10	11
4	6	11	17	9	11
5	12	10	7	11	9
6	5	10	10	8	13

Thinking

Working

(a) 1 If the die is fair, we expect each outcome to occur the same number of times, so divide the total number of rolls by the number of possible outcomes (60 ÷ 6).

(a) Number of expected results $= \frac{60}{6}$
$= 10$

2 Compare this number to the results in each column in the table.

Set 1—Biased. There are too many 2s and 3s and too few 4s and 6s.

Set 2—Biased. There are too many 3s and too few 2s.

Set 3—Biased. There are too many 4s and too few 3s and 5s.

Set 4—Fair. All results are about the same value.

Set 5—Biased. There are too many 6s and too few 1s.

(b) Make a statement about the fairness of the die by looking at the decision made for each set.

(b) Based on the results of each set, it seems the die is not fair. However, the results are not either consistently low or consistently high for any particular outcome.

(c) 1 Total the numbers for all five sets for each outcome.

(c)

Number	Total
1	49
2	49
3	53
4	54
5	49
6	46

2 Find the result we expect by dividing the total number of rolls by the number of possible outcomes (300 ÷ 6).

Number of expected results $= \frac{300}{6}$
$= 50$

3 Compare the number to the total results and make a statement about the fairness of the die.

The die is clearly fair. All results are about the same value and close to the 50 we expected.

9.1 Population sampling

Navigator

Answers page 642

Q1, Q2, Q3, Q4, Q5, Q7, Q9, Q10, Q15, Q16, Q17, Q18	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q10, Q11, Q13, Q14, Q15, Q18	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q13, Q14, Q16, Q18

Equipment required: Coin for Question **15**, die for Question **17**

Fluency

WE1

1 **(a)** A large jar is filled with 2500 jelly beans. The jelly beans are red, green and black. A sample of 50 jelly beans is taken and it is found that 20 are green, 18 are red and the rest are black. Make an estimate for the number of each colour of jelly bean in the jar. Give a range of values that you think the number should lie between.

(b) Another jar is filled with 1800 jelly beans. The jelly beans are red, green and black. A sample of 60 jelly beans is taken and it is found that 24 are green, 15 are red and the rest are black. Make an estimate for the number of each colour of jelly bean in the jar. Give a range of values that you think the number should lie between.

WE2

2 **(a)** A sample of 300 fish is taken from a lake, tagged and released. Some time later, a second sample of 150 fish is taken and it is found that 20 of these are tagged. What is the estimated fish population for the lake?

(b) A sample of 100 koalas is taken from a national park, tagged and released. Months later, a second sample of 50 koalas is taken and it is found that 10 of these are tagged. What is the estimated koala population for the national park?

WE3

3 For each of the following survey methods, state whether or not you think it will result in a biased sample. For those that you think are biased, give a reason why you think this is the case.

(a) You want to find the average height of adults in your local area, so you measure the height of 100 adults chosen at random who are shopping at the local shopping centre.

(b) You are trying to find the average hand span of teenagers, so you measure the hand span of all the Year 8 students in your school.

(c) You want to find the average weekly income of families in your local area, so you question people who come into the local petrol station.

(d) You want to determine the health issues in your local community, so you question people who are attending the local medical clinic.

(e) You want to find the average age of cars on the road, so you get the local mechanic to record the age of each car she works on over a 1-month period.

(f) You are trying to find the preferred music style for your local community, so you question people chosen at random who are shopping at a local shopping centre.

4 A die is rolled 60 times and the result for each outcome is recorded. This experiment is repeated so that five sets of results are collected. These results are shown in the table below.

WE4

Outcome	Set 1	Set 2	Set 3	Set 4	Set 5
1	13	9	10	7	6
2	8	12	9	11	12
3	10	4	9	12	6
4	12	15	12	8	14
5	9	11	8	15	8
6	8	9	12	7	14

(a) Use each set of results separately to decide whether you think the die is fair.

(b) Using the separate decisions you made in part **(a)**, make an overall decision about the fairness of the die.

(c) Combine the results and comment again about the fairness of the die.

Understanding

5 Scientists capture and tag 300 penguins and then release them back into the general population. After several months, the scientists begin a monthly sampling program, with the following results.

	Month 1	Month 2	Month 3	Month 4
Sample size	150	175	200	125
Number of tagged penguins	55	65	72	51

(a) Estimate the size of the penguin population for each of the months.

(b) Combine the four samples into one big sample and use it to estimate the penguin population.

(c) Which answer for the total population do you think is more reliable? Give some reasons for your answer.

6 The following table shows the results from groups of 60 rolls of a die labelled 1–6 that you have been told is biased.

	#1	#2	#3	#4	#5
1	10	9	8	15	10
2	15	12	6	10	8
3	7	11	10	9	15
4	10	11	9	7	3
5	10	8	10	3	10
6	8	9	17	16	14

In what way do you think this die is biased? Does each sample show the same apparent bias?

7 Manufacturers should always be interested in quality control. However, they can usually test only a sample of their product.

(a) (i) Explain why a baker cannot taste test every loaf of bread.

(ii) What percentage of loaves do you think would be reasonable to test?

Matches are sold in boxes that state 'Average contents 50 safety matches'.

(b) (i) If you counted the contents of 50 such boxes of matches what range of values do you think you might find?

(ii) What do you think the word 'average' means?

(iii) How do you think the match manufacturer ensures that the statement 'Average contents 50 safety matches' is correct? There are serious penalties for false advertising.

Reasoning

8 The following article appeared on the website of *The Courier*.

Support to cut speed limits on country roads

NEW road safety research shows strong support for lower speed limits on country roads. The Monash University study of 4100 drivers showed the majority thought a speed limit of 100km/h for country roads was too high.

Seventy-five per cent believed there should be a reduction in speed limits on country roads from 100km/h to 90km/h and 92 per cent considered a limit of 80km/h appropriate for gravel roads.

Only 14 per cent of drivers said the 50km/h speed limit for local residential roads was too high.

However, there was some discrepancy between the views of city and country drivers to lower speed limits.

More than 80 per cent of city drivers backed a 90km/h limit for country roads compared to 61.8 per cent of country drivers. While 93.2 per cent of city drivers and 88.5 per cent of country drivers supported the 80km/h limit for gravel roads.

Senior research fellow Dr Bruce Corben of Monash University Accident Resource Centre said the difference in opinion could reflect a desire by country drivers to save travel time and greater familiarity with the roads.

He said lowering the speed limit was a way of reducing road trauma at a low cost.

"I think governments have been reluctant to move in that direction because there is a perception that the community is opposed to it."

"With these results is an opportunity to re-think what's the appropriate limit for rural roads."

Cathy Morris, *The Courier*, 01/04/2010

(a) The article does not state the numbers of country and city drivers who were surveyed. Does this make a difference to any interpretation of the percentages stated?

(b) The corresponding article in Victoria's *Herald Sun* indicated that 1217 of the drivers surveyed were Victorians. Do you think this is an unbiased or a biased survey? Give reasons for your answer.

(c) If this survey was to be conducted Australia-wide, how could you ensure that each state and territory had a fair representation?

(d) Further research at the website of the Monash University Accident Research Centre, the authors of the report, gave the following additional information: Victoria contributed 1217 (29.7%) of the respondents, South Australia 1175 (28.6%), West Australia 1135 (27.7%) and Tasmania 573 (14%) and the survey was web-based with individuals on existing data bases sent an invitation to participate in the survey.

Does this additional information change your opinion about the survey in any way? Why or why not? Give a detailed answer to this question.

9 The following article appeared in the *Herald Sun*.

Millions of workers have poor language, literacy and numeracy skills

An astonishing four million Australian workers have poor language, literacy and numeracy skills and cannot understand the meaning of some everyday words and their inability to follow basic instructions and warnings is causing a safety and productivity nightmare. Most are in labour-intensive and low-level service jobs. Among the terms that are too difficult for some workers are "hearing protection" and "personal protective equipment is required", according to a report by Skills Australia for the Rudd Government. The words that many do not understand include: immediately, authorised, procedure, deliberate, isolation, mandatory, recommended, experience, required and optional. Australian Industry Group chief executive Heather Ridout told the *Herald Sun* 46 per cent of workers had substandard literacy skills and 53 per cent had numeracy below the expected benchmark.

Phillip Hudson, *Herald Sun*, 06/04/2010

(a) Are you surprised by the percentages quoted in this article? Why or why not?

(b) Do you think the results would be based on a census of the entire working population or on a sample?

(c) Assuming a sample was used, does the article give any indication of what sampling method has been used to produce the statistics quoted? Explain your answer.

(d) Does the article give any indication as to what the expected levels of language, literacy and numeracy might be? Explain your answer.

(e) How would you go about choosing an unbiased sample to collect data to address the issues raised by this article?

10 Whaling, the killing of whales, is a controversial issue, with most countries today opposed to it for almost every purpose. In 1986, a ban was introduced on all whaling that is controlled by the IWC (International Whaling Commission), which is an organisation formed by 88 countries. Despite the ban, whales continue to be killed legally under three distinct categories: aboriginal subsistence, special permit and whaling under objection. Small communities in Greenland, USA, Russia and Canada are allowed to whale for 'aboriginal subsistence' purposes. This means that small numbers of whales can be killed by certain communities when it is necessary as a food source and where it is an important part of their cultural traditions. Japan and Norway are allowed to whale under a 'special permit' for scientific purposes. These two countries maintain that whale meat has been a significant part of their traditional diet for centuries. They argue that the data they collect will help maintain and increase whale numbers so that commerical whaling can resume in the future and that this data cannot be obtained any other way. There is a limit to the

number of whales they are allowed to catch each season. Iceland and Norway have objected to the ban and are allowed to continue 'whaling under objection'.

The table below shows the number of whales killed in 2008–2009 and in the summer of 2009.

Whale species	Aboriginal subsistence	Special permit	Whaling under objection	Illegal catch	Total
Blue	0	0	0	0	0
Bowhead	41	0	0	0	41
Bryde's	0	50	0	0	50
Fin	10	1	125	0	136
Gray	116	0	0	0	116
Humpback	1	0	0	0	1
Minke	168	845	565	16	1594
Right	0	0	0	0	0
Sei	0	101	0	0	101
Sperm	0	1	0	0	1

Use the table above to answer Questions **(a)**–**(f)**

(a) How many whales were killed in total?

(b) How many whales were killed for aboriginal subsistence?

(c) How many whales were killed with a special permit?

(d) How many whales were killed while whaling under objection?

(e) Use your answers to parts **(b)**–**(d)** to comment on the effect each reason for whaling has on the total number of whales killed.

(f) **(i)** What sort of data would be collected for scientific purposes?

(ii) Explain how this data could help maintain or increase whale numbers?

11 The table below shows the effect whaling had on whale numbers before the ban was introduced. The original level in the table gives the figures before whaling began at the beginning of the 20th century and the latest figures in the table are from 1997.

Species	Whale population estimates		
	Original level	Latest level	Year protected
Blue	228 000	11 700	1967
Bowhead	30 000	7800	1935
Bryde's	90 000	43 000	1986
Fin	648 000	110 000	1986
Gray	20 000	18 000	1935
Humpback	115 000	10 000	1966
Minke	490 000	880 000	1986
Right	100 000	3200	1935
Sei	256 000	54 000	1986
Sperm	2 400 000	1 950 000	1985

Use this table to answer Questions **(a)**–**(f)**

(a) Which whale species do you think was not hunted when whaling was allowed? Explain your answer.

(b) Which species of whale would be in danger today if commercial whaling was allowed to resume?

(c) Why do you think a ban was introduced on whaling for the right whale, gray whale and bowhead whale as early as 1935? Comment on the gray whale numbers.

(d) Which whale species still had a population of less than $\frac{1}{4}$ of the original number in 1997?

(e) Why do you think the minke whale is now the most hunted whale species?

(f) Populations of whales in different parts of the ocean are very hard to estimate. In Australia, the CSIRO conducts extensive surveys into whale numbers, but there is still a large range in the estimated whale populations. Another source gives the fin whale population as 300 000–600 000 originally and 123 000 in 1997. It also gave the right whale population as 2000 in 1997.

(i) How accurate do you think the figures in the table above would be?

(ii) Can the sampling methods you have explored for other wildlife populations, such as tag and release, be applied to whale populations? Explain why or why not.

(iii) How are whale population numbers estimated?

12 This table shows the number of whales killed since the ban started until 2010.

Country	No. of whales killed from 1986 to 2010
Japan	17 000
Norway	10 000
Iceland	821
Other countries	2179
Total	30 000

(a) Do you think the ban is effective if 30 000 whales have been killed from 1986 to 2010? On average, how many were killed each year over this 25-year period?

(b) Which country was responsible for nearly 60% of all whales killed from 1986 to 2010?

(c) Two main beliefs held by antiwhaling countries are that the killing of whales is cruel and unnecessary to conduct scientific research and that whaling threatens to cause the extinction of whales species.

(i) Do you think these beliefs could cause sampling bias and incorrect estimates of whale populations? Explain your answer.

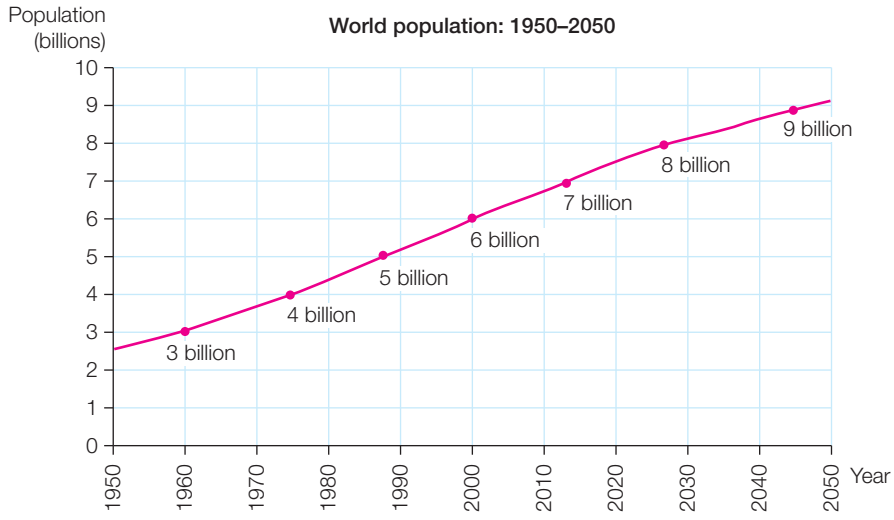
(ii) Do you think these beliefs could be used in a biased way to interpret unbiased data? Explain your answer.

(d) Two main beliefs held by whaling countries are that there is no threat to the extinction of any species of whale and that the whales must be killed to do the scientific research.

(i) Do you think these beliefs could cause sampling bias and incorrect estimates of whale populations? Explain your answer.

(ii) Do you think these beliefs could be used in a biased way to interpret unbiased data? Explain your answer.

13 Below is a line graph of the world population from 1950 to 2010. The line graph has been projected to predict the world population from 2010 to 2050.



(a) What was the world population in 1959?

(b) What happened to the world population in the 40 years from 1959 to 1999?

(c) To draw the graph into the future, assumptions have been made by statisticians. Name one assumption that has been made.

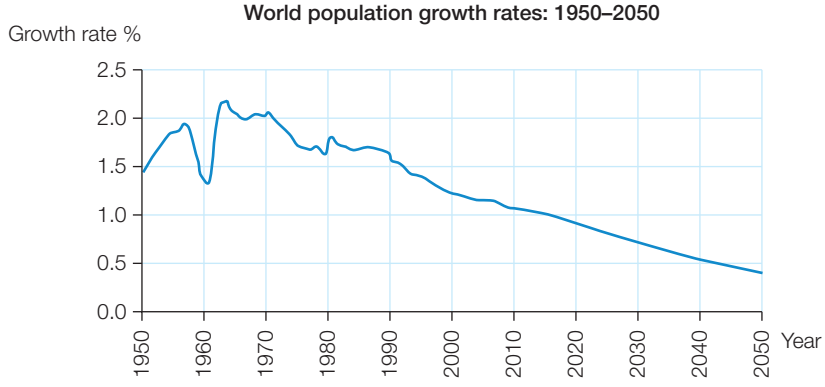
(d) Is an assumption the same as a bias? Explain.

(e) How many years after 1999 will it take for the population to increase by the same amount that it did from 1959 to 1999?

(f) What are some of the issues the world will face if the world population increases as this graph predicts?

(g) How would this data have been collected?

14 The graph below shows the rate at which the world population grew from 1950 to 2010 and is predicted to grow from 2010 to 2050.



(a) During what 10-year period was the world growth rate the greatest?

(b) Babies born in the years following World War II, which ended in 1945, were called the 'Baby Boomers'. What connection does this have to your answer in part **(a)**?

(c) What was the world growth rate in 2010?

(d) What has happened to the world growth rate from 1980 to 2010?

(e) If the prediction is correct, will the world population be increasing or decreasing in 2050? Explain your answer.

(f) A combination of natural disasters and a marked decrease in agricultural production in China affected the world population rate for a couple of years. What effect would this have had on the shape of the graph and when do you think this occurred?

15 Flip a coin 20 times, recording the number of heads and tails. Flip the coin another 20 times and compare the results. Do you think your coin is biased?

Open-ended

16 Public transport systems attract a lot of criticism related to late and cancelled services. Explain how you could choose an unbiased sample to survey the general public's opinions of the punctuality and reliability of public transport. Make sure you clearly state what population you are considering.

17 (a) If you rolled a normal die 30 times, how many times would you expect each number to appear?

(b) If you rolled a die 30 times and eight 3s turned up, would you think that the die was biased?

(c) How many of a particular number appearing would make you feel confident that the die was biased?

(d) What would you do to check your die if you thought it was biased?

(e) Roll a die 30 times and note the frequency for each value. Do you think your die is biased?

18 Lara and Michael were asked whether a poll on an issue conducted by a local TV station was biased or unbiased. Michael thought it was unbiased, but Lara decided it was biased. Who is correct? Explain why to the other person so they better understand the idea of sampling bias.

Outside the Square — Problem solving

How many trains?

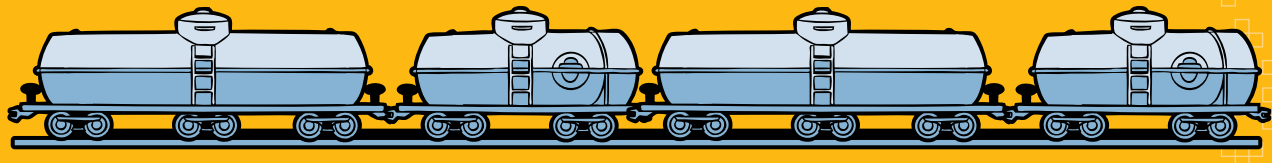
A new train company, AusTrain, is formed to carry freight around the country. They have freight cars of two different types—Roomy and Extra Roomy. An Extra Roomy carriage is twice as long as a Roomy carriage.

How many different combinations of carriages are there if the total length of the train is to be equivalent to the length of ten Roomy carriages?

(If the order of the carriages is different, then it is a different train, even if it contains the same number of each type as another train.)

Strategy options

- Make a table.
- Solve a simpler problem.
- Test all possible combinations.



9.2 Using sample measures of centre and spread

When we sample a population, we use the **mean**, **median** and **mode** to represent the **measures of centre** for the population, provided the sample is unbiased. Sometimes, it is difficult to select a sample that accurately represents a population. Statisticians not only need to minimise bias when sampling, but also need to take into account any underlying bias when analysing data. Consider the following.

Every five years in Australia, the Australian Bureau of Statistics, on behalf of the Federal Government, conducts a census of the Australian population. According to the census in 2009, the median age of the Australian population on 30 June, 2009 was 36.9 years, which is higher than the median age of 31.8 years on 30 June,1989. Could a random sample of Australians obtain the same median? The census median of 36.9 years represents Australians from 1 day old to over 100 years of age. Where would you go to select a sample that best represents the ages of the Australian population? Sports events, festivals or shopping centres where large crowds gather are unlikely to have the very old or very young and are biased in other ways. Hospitals may have the very old and very young, but do not include the healthy population.

Measures of centre

We use three measures of centre: the mean, median and mode.

Mean

The mean is the arithmetic average of a data set:

$$\text{Mean} = \frac{\text{sum of all the data values}}{\text{number of data values}}$$

Median

This is the middle value of a data set. If there are two middle values, the median is the average of these two values.

Mode

This is the value that occurs most often in a data set.

There may be no mode.

If there are two modes, the data set is bimodal.

If there are more than two modes, the data set is multimodal, but the mode is not of any use in these cases. A data set of this type is often considered to have no mode.

Worked Example 5 WE5

Sarah collected rewards points from her local supermarket. The points she has earned in the last 12 months are listed below.

72, 39, 57, 82, 36, 42, 64, 54, 62 48, 74, 42

Find the **(a)** mean, **(b)** median and **(c)** mode of this data set.

Thinking	Working
(a) To find the mean, add all the data values and divide by the number of data values.	(a) Mean $= \frac{72+39+57+82+36+42+64+54+62+48+74+42}{12}$ $= 56$
(b) To find the median, arrange in ascending or descending order and identify the middle value. If there are two middle values, average them.	(b) Ascending order: 36, 39, 42, 42 48, 54, 57, 62, 64, 72, 74, 82 Middle values: 54 and 57 Median $= \frac{54+57}{2}$ $= 55.5$
(c) To find the mode, identify the value that occurs the most frequently.	(c) Mode = 42

Measures of spread

To make sense of the data we collect, in addition to measures of centre, we measure the spread of the data using the **range**.

Range

You have used the range as a measure of spread in Year 7.

Range = highest value in the data set – lowest value in the data set.

The effect of outliers

In Year 7, you saw that, sometimes, some data is quite different from the rest of the data set. This could be because of some unusual circumstances or errors in collecting or recording the data. These are **outliers**. We need to know what effect an outlier or outliers have on the statistics we can calculate. The most reliable statistics will be those in which outliers do not have a great effect on the statistics.

Worked Example 6 WE6

(a) For each of the following data sets calculate the: **(i)** mean **(ii)** median and **(iii)** range.

Set 1: 1 1 2 3 4 4 4 5 6 7 8 9 10 12 14 16 30

Set 2: 1 1 2 3 4 4 4 5 6 7 8 9 10 12 14 16 16

(b) What effect did the change in data values from Set 1 to Set 2 have on each of the sample statistics you have calculated?

Thinking	Working
(a) (i) To find the mean, add the values and divide by the number of values.	(a) (i) Set 1: Mean $= \frac{136}{17} = 8$ Set 2: Mean $= \frac{122}{17} = 7.18$
(ii) The median is the middle value.	(ii) Set 1: 1 1 2 3 4 4 4 5 6 7 8 9 10 12 14 16 30 Median = 6 Set 2: 1 1 2 3 4 4 4 5 6 7 8 9 10 12 14 16 16 Median = 6
(iii) The range is the highest value – the lowest value.	(iii) Set 1: Range = 30 – 1 = 29 Set 2: Range = 16 – 1 = 15
(b) Compare the means, medians and ranges of Set 1 and Set 2 and comment on similarities and differences.	(b) The ranges are quite different. The means are a little different. There is no difference between the medians.

In the above Worked Example, the 30 in the data set in part **(a)** could be an outlier. It had no effect on the median. It had a slight effect on the mean and a significant effect on the range. This is why the median is often used to describe data sets in preference to the mean and the range.

9.2 Using sample measures of centre and spread

Navigator

Answers page 644

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q11	Q1, Q2, Q3, Q4, Q5, Q7, Q8, Q9, Q11	Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q10, Q11, Q12

Fluency

WE5

1 **(a)** Rohan recorded the points he won in a board game he played with several of his friends. The points he earned in the last 15 games are listed below.

23, 37, 42, 36, 45, 52, 25, 65, 47, 37, 54, 36, 50, 48, 33

Find the **(i)** mean, **(ii)** median and **(iii)** mode of this data set.

(b) Stevie recorded the number of points her netball team scored this season. The points her team earned in 18 games are listed below.

43, 45, 56, 26, 36, 44, 64, 54, 67, 34, 42, 63, 51, 29, 41, 30, 55, 39

Find the **(i)** mean, **(ii)** median and **(iii)** mode of this data set.

2 **(a)** For each of the following data sets calculate the: **(i)** mean **(ii)** median and **(iii)** range.

WE6

Set 1: 8 9 9 10 10 10 10 10 14 15 16 18 19 20 22

Set 2: 1 9 9 10 10 10 10 10 14 15 16 18 19 20 22

(b) What effect did the change in data values from Set 1 to Set 2 have on each of the sample statistics you have calculated?

3 In the 2009 NRL season, the Manly-Warringah team, in all games including the finals, scored the following points.

12	24	10	12	23
24	12	8	22	34
17	38	20	20	19
18	32	44	18	22
44	20	18	38	12

(a) Calculate the mean and median number of points scored for the season.

(b) Which of these statistics better represents their scores for the season?

4 **(a)** For the data set: 1 2 2 2 3 4 5 5 6 6 6 8 8 8 8 9 the median is:

A 5 **B** 5.5 **C** 6 **D** 8

(b) For the data set: 2 3 3 4 5 6 6 6 7 7 7 9 9 9 9 the mode is:

A 6 **B** 7 **C** 8 **D** 9

(c) For the data set: 1 1 1 2 2 3 3 3 3 3 5 6 6 6 6 6 6 7 7 7 7 8 8 9 9 the mean is:

A 3 **B** 5 **C** 6 **D** 7

Understanding

5 The Australian cricket team toured New Zealand in March 2010 and played two test matches. The following is a list of the scores made by each individual Australian batsman during the two test matches.

Batsman	1	2	3	4	5	6	7	8	9	10	11
1st test, 1st innings	20	79	41	1	108	112	11				
1st test, 2nd innings	86	18									
2nd test, 1st innings	12	88	22	22	28	9	12	0	12	10	4
2nd test, 2nd innings	65	106	6	67	63	90	48	0	41	18	

(a) Calculate the overall mean score of the batsmen's scores.

(b) Calculate the overall median score of the batsmen's scores.

(c) Which batsman's score was the closest to the overall mean score in each innings?

(d) Over time, would you expect the mean score for each batsman to be similar to the mean score for the team? Explain your answer.

6 An orange grove farmer expects to harvest all oranges at once. To help the farmer predict the total mass of oranges, he picks the oranges from one tree and weighs each one. The masses, in g to the nearest 5 g, recorded are:

175	180	190	195	175	175
180	165	185	175	175	175
175	175	175	180	175	175
175	185	165	180	175	175
195	190	185	190	175	165
175	170	165	195	190	185
180	190	175	170	165	155
185	180	185	175	175	190
195	175	175	185	170	185

(a) Calculate the mean, median and mode for these oranges.

(b) Which of the statistics do you think should be used to estimate the total mass of oranges in the grove?

(c) If there are 250 trees in the grove, make an estimate for the total mass of oranges harvested.

(d) Give a range within which you think the total mass will lie. How did you calculate this range?

(e) Four random samples of 5 oranges were taken and the weights recorded:

Set 1:	175	155	175	185	180
Set 2:	180	175	165	190	170
Set 3:	170	175	190	195	175
Set 4:	170	195	165	175	195

(i) Find the mean, median and mode for each set.

(ii) Which of these statistics stayed closest to the value of the statistics found in part **(a)** for the population of oranges on this tree?

(iii) Which of these statistics varied the most?

7 The following data set appears to have two outliers, one at the start of the data and one at the end.

3 10 12 12 14 14 15 16 17 18 18 19 28

We have already seen that one outlier affects the mean and the range, but what happens here where there are two outliers?

(a) Calculate the mean, median and range for the data set.

(b) Recalculate these values after changing the 3 to 10 and the 28 to 19.

(c) What conclusions can you draw from your calculations?

Reasoning

8 The following table shows the minimum and maximum temperatures recorded for Melbourne on each of the days in March 2010.

Date	1	2	3	4	5	6	7	8	9	10	11
Min °C	14.1	14.1	15.8	16.5	20.5	19.8	17.9	16.4	16.2	13.7	13.7
Max °C	21.4	22.4	27.7	31.3	26.7	26.8	27.3	22.5	21.2	20.4	22.2
Date	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
Min °C	11.1	13.7	13.8	15.3	18.5	21.5	20.9	23.0	18.8	17.5	13.1
Max °C	26.4	28.3	28.8	30.4	32.2	33.0	31.9	28.4	26.7	24.7	22.5
Date	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**		
Min °C	13.3	15.8	12.8	14.8	17.2	17.8	17.9	15.1	13.4		
Max °C	23.7	23.4	31.5	30.7	22.7	27.2	23.3	22.6	23.9		

(a) On the same set of axes:

(i) plot the minimum temperatures for each day in March 2010. Join the points to create a line graph.

(ii) plot the maximum temperatures for each day in March 2010. Join the points to create a line graph.

(b) Estimate the mean minimum and maximum temperatures for March 2010.

(c) Calculate the mean minimum and maximum temperatures for March 2010 in Melbourne correct to one decimal place.

Since record keeping began in 1855, the mean minimum temperature for Melbourne in March has been 13.2°C and the mean maximum temperature has been 23.9°C.

(d) How did the mean minimum and maximum temperatures in March 2010 in Melbourne compare to these long-term statistics?

If the mean March temperatures are calculated using just the figures from 1981 to the present day, then the mean minimum temperature is 14.5°C and the mean maximum temperature is 24.4°C.

(e) How did the mean minimum and maximum temperatures in March 2010 in Melbourne compare to these restricted statistics?

(f) Do you think it is better to use the historical means (from 1855) or the restricted means (from 1981)? Give as detailed an answer to this as you can.

9 A growth chart for boys aged 13 years indicates that 95% of them will be less than 169 cm, 90% less than 166.5 cm, 75% less than 163 cm, 50% less than 157 cm, 25% less than 151 cm, 10% less than 147 cm and 5% less than 143 cm.

(a) What height(s) would you consider to be outliers for 13-year-old boys?

(b) You are given the following set of heights and told that they are from boys of the same approximate age.

149	147	159	167	171	177	155	180
160	170	173	166	162	150	155	169
155	179	160	161	172	155	149	154
150	168	171	151	152	141	171	156
168	155	156	158	161	160	159	168

Do you think these boys are 13 year olds? Give a reason for your answer.

(c) A random sample of eight values was chosen from this data.

160, 155, 171, 168, 177, 159, 160, 161

(i) Find the mean, median and mode for this sample.

(ii) Which statistic do you think is the best measure of centre? Give a reason for your choice.

10 The following tables give information about the amount of caffeine in various coffee drinks and the amount of coffee consumed in various countries.

Coffee (225 mL cup)	Caffeine range (mg)
Brewed/plunger	80–350
Instant	60–100
Decaf, brewed	2–4
Decaf, instant	1–4
Cappuccino	30–50
Mocchaccino	35–55

Table 1

Country	Coffee consumption per capita (kg) per year
Slovakia	10.8
Iceland	9.2
France	5.5
Italy	5.3
Canada	4.7
USA	4.6
Australia	2.8
UK	2.5

Table 2

(a) What percentage is the average annual Australian consumption of caffeine of that in Slovakia? Answer correct to one decimal place, where necessary.

(b) Espresso served in a 250 mL cup has a caffeine range of 30 to 50 mg. State the range for the number of mg of caffeine each cup would contain after 25 mL has been consumed.

(c) If data from all eight countries listed in Table 2 was combined, do you think the coffee consumption per capita per year would be closest to 2.8, 4.7 or 9.2 kg? Justify your answer.

(d) Do you think the figures in Table 2 were obtained from sampling or from a census? Explain your answer.

(e) Do the range of values in Table 1 give you information about the average (mean) amount of caffeine in each type of coffee? Could you estimate the median or the mode? Explain your answer.

Open-ended

11 Make up 10 pieces of data so that the mean is 12.3, the median is 13 and the mode is 11.

12 Make up another 10 pieces of data so that the mean is 12.3, the median is 13 and the mode is 11 and there is a possible outlier of 32. (32 is one of your 10 pieces of data.)

Frequency tables and graphs

9.3

In this section, we will look at statistical graphs called **frequency column graphs**. If the data is discrete (counted), such as the number of cars in a car park, or categorical (in categories), such as the colours of different cars in a car park, the columns will have a gap between them. If the data is continuous (measured), such as the lengths of different cars in a car park, the graphs will look like column graphs except that the columns will have no gaps between them. These frequency column graphs are called **histograms**.

In Year 7, you used frequency column graphs to represent small amounts of data using individual numbers on the horizontal scale.

Worked Example 7

WE7

For the following question, find a suitable scale and draw a frequency column graph to represent the results.

A park ranger was asked to record the number of reported sightings for an endangered species of bird in four different sections of a national park over a certain time period. The results are shown at right.

Section	Frequency
1	12
2	9
3	16
4	13

Thinking

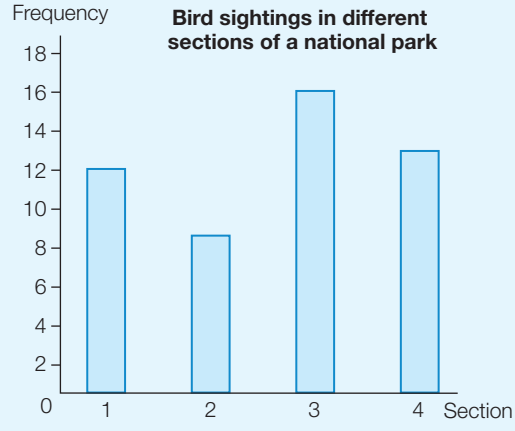
Working

1 Find a suitable vertical scale. Use the largest frequency (16) and divide it by 10 because we want a maximum of 10 divisions on the vertical axis. Round your answer to the nearest whole number.

$\frac{16}{10} = 1.6$

≈ 2

2 Draw a frequency column graph using the frequency table given, labelling the axes and naming the graph.



Grouping data

We need to use class intervals to group data if there is a large amount of data and the range is large (greater than 10).

The measures of centre, mean, mode and median, cannot be read off the graph for grouped data, but the graph does give us a good idea of the spread of the data, and the shape of the graph also gives us important information.

Finding class intervals

A **class interval** is found by dividing the range into a number of equally sized intervals, so we have between five and 10 columns in our frequency column graph. For example, if a discrete data set has values ranging from 10 to 75, the range is 65. If we use a class interval of 10, we would get seven columns with intervals 10–19, 20–29, ... 70–79.

All grouped data can be presented by a histogram, even if it is discrete data. The class intervals for discrete data are presented as discrete groups, as shown in the following Worked Example.

Worked Example 8

WE 8

The following data shows the number of customers that have dined at a restaurant each day for 50 days.

11, 45, 29, 43, 37, 30, 58, 80, 70, 82, 23, 31, 27, 74, 41, 48, 65, 11, 65, 77, 70, 36, 50, 50, 65, 50, 45, 60, 37, 67, 2, 41, 62, 43, 27, 81, 56, 35, 31, 50, 2, 48, 98, 51, 72, 43, 67, 43, 43, 26

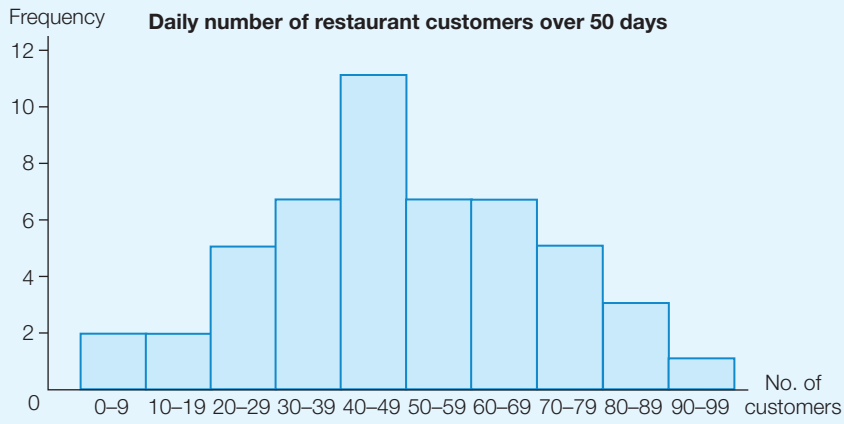
Select a suitable class interval, construct a frequency table with the grouped data and draw a histogram of the data.

Thinking	Working
1 Find the range and divide it by 5 and 10 to find a suitable class interval.	Range $= 98 - 2$ $= 96$ $\frac{96}{5} = 19.2$ and $\frac{96}{10} = 9.6$ Use a class interval of 10.
2 Decide where the class interval should start.	Use 0–9, 10–19, …

3 Construct a frequency table for the grouped data. Use a tally column to help you count the frequency in each class interval.

No. of customers	Tally	Frequency
0–9	II	2
10–19	II	2
20–29	~~IIII~~	5
30–39	~~IIII~~ II	7
40–49	~~IIII~~ ~~IIII~~ I	11
50–59	~~IIII~~ II	7
60–69	~~IIII~~ II	7
70–79	~~IIII~~	5
80–89	III	3
90–99	I	1

4 Construct a histogram for the grouped data. Label and number each axis.



Histograms are also used for continuous grouped data. The class intervals are presented as continuous groups.

Worked Example 9

WE 9

The following data shows the winning times recorded for the last fifty years at a school swimming carnival for the 50 m freestyle event. Times are in seconds.

32.12, 31.23, 31.56, 33.28, 33.73, 32.17, 34.49, 32.55, 34.28, 33.74, 31.58, 35.47, 30.44, 34.65, 33.89, 31.14, 35.36, 29.99, 30.69, 32.16, 33.75, 31.17, 30.45, 34.46, 30.78, 35.95, 33.35, 34.74, 30.12, 33.88, 32.19, 33.42, 31.57, 35.67, 32.96, 31.45, 32.64, 32.64, 33.68, 34.54, 32.14, 33.76, 30.45, 34.23, 34.28, 32.54, 31.99, 32.43, 30.76, 32.24

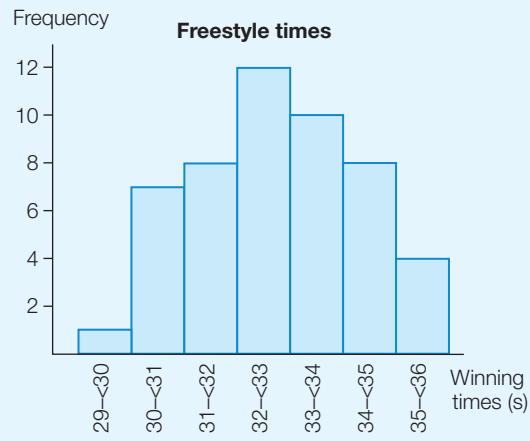
Select a suitable class interval, construct a frequency table with the grouped data and draw a histogram of the data.

Thinking	Working
1 Find the range and divide it by 5 and 10 to find a suitable class interval.	Range = 35.95 − 29.99 = 5.96 $\frac{5.96}{5} = 1.19$ and $\frac{5.96}{10} = 0.596$ Use a class interval of 1.
2 Decide where the class interval should start.	Use 29–<30, 30–<31, …

3 Construct a frequency table for the grouped data. Use a tally column to help you count the frequency in each class interval.

Winning times (s)	Tally	Frequency
29–<30	I	1
30–<31	卌 II	7
31–<32	卌 III	8
32–<33	卌 卌 II	12
33–<34	卌 卌	10
34–<35	卌 III	8
35–<36	IIII	4

4 Construct a histogram for the grouped data. Label and number each axis.



When drawing any statistical graph, as a general rule, you should ensure your graph has:

- labelled axes
- a consistent scale on both axes
- a title.

9.3 Frequency tables and graphs

Navigator

Answers page 645

Q1 (a) & (b), Q2, Q3, Q4 (a), Q5 (a), Q6, Q7 Column 1, Q9, Q10, Q12, Q13	Q1 (b) & (c), Q2, Q3, Q4 (b), Q5 (b), Q6, Q7 Column 1, Q8, Q9, Q10, Q12	Q1 (c) & (d), Q2, Q3, Q4 (c), Q5 (b), Q6, Q7 Column 2, Q8, Q9, Q10, Q11, Q12

Fluency

WE7

1 For the following questions, find a suitable scale and draw a frequency column graph to represent the results.

(a)

Age of cars in a car park (years)	Frequency
1	2
2	4
3	3
4	5
5	8
6	16

(b)

Number of goals scored	Frequency
10	4
11	3
12	6
13	3
14	2
15	0
16	4

(c)

Number of red lollies in different packets (same variety, size and manufacturer)	Frequency
10	5
11	6
12	14
13	8
14	7
15	6
16	3

(d)

Number of TVs sold each day	Frequency
0	14
1	3
2	3
3	5
4	10
5	11
6	4
7	3

2 (a) The following data shows the number of plants a nursery sells over a 60-day period.

52, 38, 65, 29, 109, 123, 63, 72, 118, 35, 76, 82, 44, 54, 85, 132, 153, 98, 42, 58, 77, 94, 105, 113, 104, 88, 74, 53, 67, 93, 73, 64, 23, 52, 118, 109, 73, 43, 95, 27, 83, 122, 143, 79, 81, 84, 35, 41, 107, 97, 115, 123, 145, 78, 42, 97, 133, 125, 102, 94

Select a suitable class interval, construct a frequency table with the grouped data and draw a histogram of the data.

WE8

(b) The following data shows the number of plant sales for the same nursery for the same 60-day period the previous year.

40, 43, 57, 64, 36, 74, 54, 21, 86, 64, 53, 86, 46, 75, 88, 75, 97, 47, 53, 22, 89, 96, 103, 110, 112, 52, 77, 94, 69, 99, 113, 87, 64, 78, 85, 102, 95, 89, 93, 108, 113, 114, 127, 88, 76, 55, 45, 37, 67, 89, 95, 75, 36, 59, 63, 79, 91, 72, 84, 92

Select a suitable class interval, construct a frequency table with the grouped data and draw a histogram of the data.

WE9

3 **(a)** The following data shows the heights of 50 Year 8 students. Heights are in cm.

127.5, 135.5, 130, 124.5, 129.5, 140, 132, 144, 141, 142.5, 130.5, 132, 135, 133, 144.5, 128.5, 141.5, 133, 132.5, 147, 138.5, 130, 140.5, 138, 133, 157, 143, 132, 129, 142, 152, 142.5, 144, 135, 155, 149.5, 146, 126.5, 132, 127, 153, 142.5, 139, 127.5, 145, 155, 149.5, 167, 156, 152.5

Select a suitable class interval, construct a frequency table with the grouped data and draw a histogram of the data.

(b) The following data shows the heights of the same 50 students measured a year later. Heights are in cm.

131.5, 140.5, 136, 129.5, 133, 145, 135.5, 149, 144, 147.5, 135, 136, 139.5, 138, 146.5, 138.5, 151.5, 141, 139.5, 155, 146.5, 137, 146.5, 146, 140, 161, 153, 140, 136, 149, 156, 144.5, 149, 145, 159, 153.5, 157, 138, 139.5, 137, 164, 150.5, 147, 135.5, 153, 161, 155.5, 172, 167, 161.5

Select a suitable class interval, construct a frequency table with the grouped data and draw a histogram of the data.

For Questions **4** and **5**, select a suitable size for the class interval, record the results on a frequency table, and display the data using a histogram.

4 **(a)** A class of 26 Year 8 students was asked how many hours they spent online on a 'typical' weekend.

4, 6, 10, 5, 4, 8, 12, 14, 18, 6, 4, 6, 13, 2, 16, 8, 6, 4, 10, 14, 6, 4, 7, 8, 9, 12

(b) A group of 22 students was asked how far away they lived from the school (km).

26, 8, 12, 34, 40, 14, 22, 36, 30, 10, 8, 4, 3, 8, 6, 18, 12, 10, 16, 1, 7, 11

(c) A class of 24 Year 8 students was asked how many hours of play station or computer games they played in a typical week.

14, 21, 12, 0, 6, 6, 10, 16, 2, 5, 9, 17, 13, 0, 4, 6, 16, 26, 14, 21, 19, 8, 5, 4

5 **(a)** A group of students measured the mass (grams) of various objects in the science laboratory: 0 < mass < 250 g.

26, 238, 16, 4, 202, 80, 96, 102, 84, 36, 44, 16, 12, 20, 88, 236, 3, 26, 75, 47

(b) A lolly shop attendant weighed out a number of bags of sweets for customers in the range 250 g < mass < 500 g.

383, 433, 280, 314, 251, 386, 450, 257, 296, 305, 442, 355, 450, 363, 411, 375, 262, 262, 287, 312, 330, 495, 405, 499

6 **(a)** If we had 70 data values spread between 15 and 105, a suitable first interval would be:

A 0–15 **B** 7–15 **C** 15–19 **D** 15–24

(b) If we had 200 data values spread between 0 and 50, a suitable first interval would be:

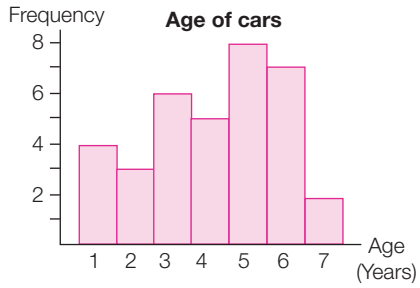
A 0–2 **B** 0–5 **C** 0–15 **D** 0–20

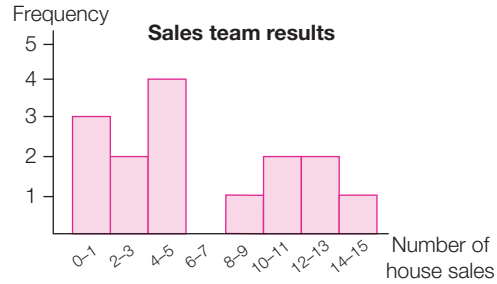
(c) If we had 500 data values spread between 70 and 270, a suitable first interval would be:

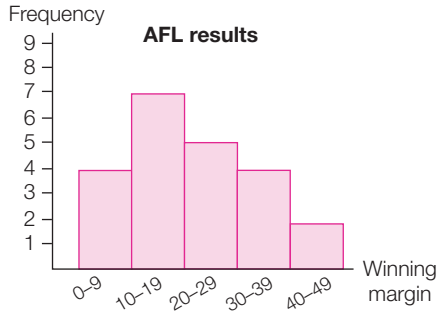
A 70–75 **B** 70–80 **C** 70–89 **D** 70–99

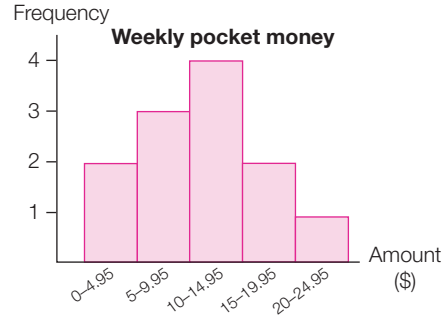
Understanding

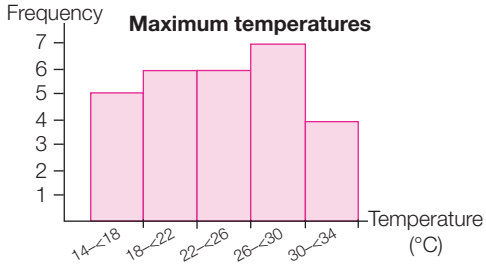
7 Construct a frequency table for each of the following graphs.

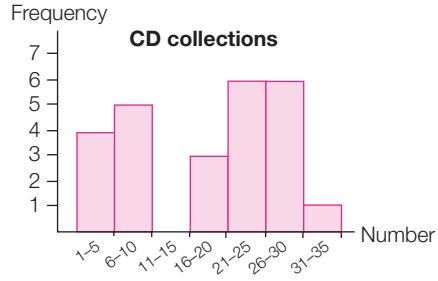
(a)


(b)


(c)


(d)


(e)


(f)


Reasoning

8 Refer to the two histograms constructed in Question **2**. Compare your histograms and comment on any similarities and differences.

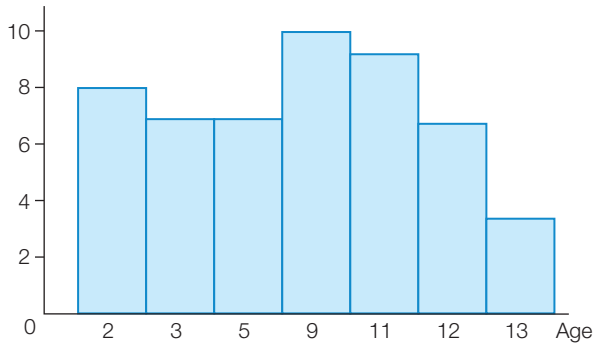
9 Refer to the two histograms constructed in Question **3**. Compare your histograms and comment on any similarities and differences.

Open-ended

10 Make up 30 pieces of discrete data that could be grouped into 6 classes with a class interval of 10.

11 Make up 40 pieces of continuous data that could be grouped into 8 classes with a class interval of 5.

12 What is wrong with the histogram shown?



13 Why might it be difficult to complete the following frequency table?

No. of customers	Frequency
0–10	
10–20	
20–30	
30–40	
40–50	

Outside the Square — Problem solving

Missing frequencies

1 Complete the frequency table so that the mean is $3\frac{11}{19}$, the median is 4 and there are 19 scores in total.

Score	Frequency
1	4
2	A
3	2
4	2
5	B
6	4

2 Complete the frequency table so that the mean is $6\frac{12}{29}$, the median is 6 and there are 29 scores in total.

Score	Frequency
4	3
5	C
6	4
7	5
8	2
9	D

Strategy options

- Work backwards.
- Test all possible combinations.

Technology Exploration Excel

Equipment required: 1 brain, 1 computer with an Excel spreadsheet program

Versions of this Exploration for other technologies are available in Pearson Reader.

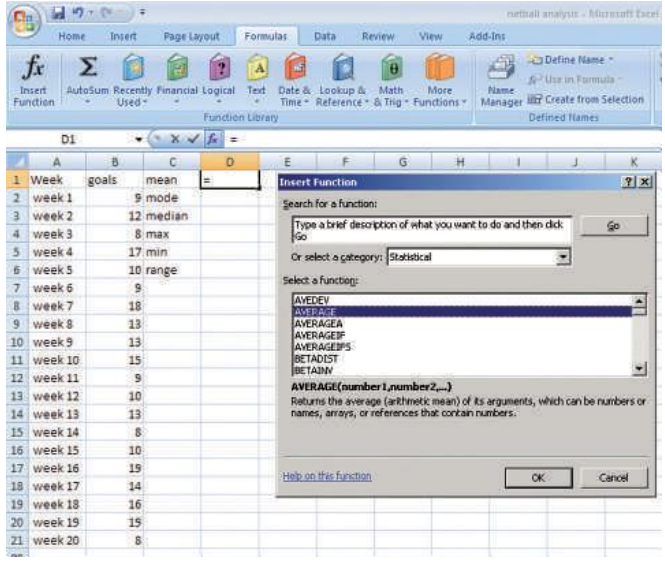
Can technology improve sporting achievement?

Lacey, an enthusiastic netballer, and her coach are very keen to improve her goal-shooting skills in netball. Her coach suggests that she should keep a record of her weekly performance. The following data represents the number of goals Lacey scored each week during the first 20 weeks of the netball season.

9, 12, 8, 17, 10, 9, 18, 13, 13, 15,
9, 10, 13, 8, 10, 19, 14, 16, 19, 8

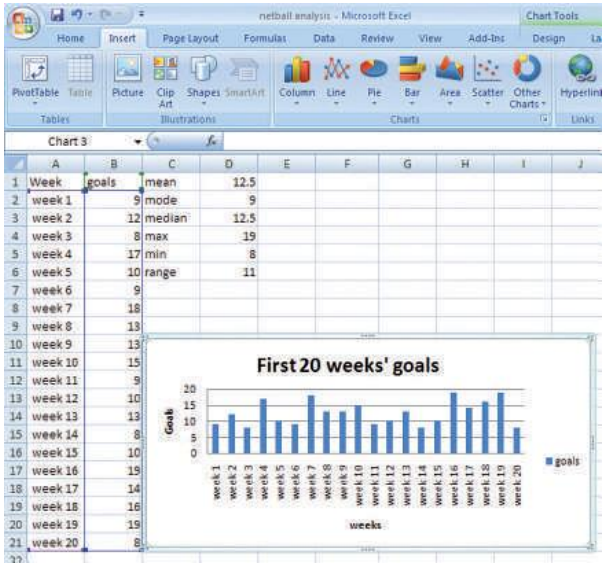
Lacey's coach reminds Lacey that some of the statistics she has learnt at school could help her. Lacey thinks that the mean, mode, median and range could be useful. Her coach suggests that she use a spreadsheet to analyse the data. She opens an Excel spreadsheet and inputs her data.

1 Use a spreadsheet to enter Lacey's results. Use the Formulas tab on the menu bar and select Insert Function. Choose the Statistical category and use the functions listed to find the mean, mode, max and min. List the labels in column C and enter the values in column D. The screen shot below shows you what your screen should look like. (Note that the mean is called the average and the range is not a function in the list and needs to be calculated with a separate formula.)



When the dialog box comes up, enter b2:b21 in the box beside number 1 and press OK.

2 Lacey would like to know how her season looks on a graph, so she uses Excel to plot goals against weeks. Highlight A1:B20 and choose Insert Column and then select the top left graph, Clustered Column in the 2-D Column. Before your graph is finished you need to do a few things. Using the Layout tab, choose Chart Title to name your graph and Axis Titles to label your axes. Your graph should look like this.



3 Over the last 6 weeks, Lacey has been using some new training techniques. In the last four games of the season, she scored 17, 15, 11 and 18. Add these to your spreadsheet and recalculate the statistics over the whole 24-week season, using your graph and the summary statistics.

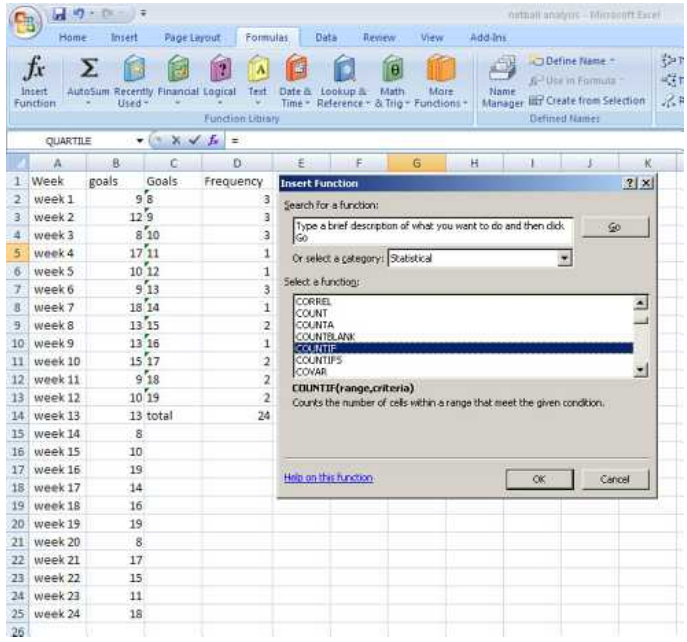
4 Draw a new graph with these statistics. Using your graph and the summary statistics, comment on whether you think the new training methods are effective.

5 Select the last 10 weeks of the season and redo the summary statistics. Comment on whether you think the new training methods are effective.

Taking it further

6 Lacey decided that she would like to produce a frequency graph using the season statistics.

To do this, she needed to set up a new table. She headed one column 'Number of goals' and listed her scores from the lowest to the highest, making them labels by using a ' in front of the number. Remember, you can drag down the numbers after you have entered the first two numbers. A green triangle will appear in the top left corner of the cell. Label the next column 'Frequency' and, using the countif function, fill in the Frequency column.



Use =countif(b2:b25,c2) as on the spreadsheet shown. The first input is all the data that you wish to use and the $ signs are used to fix the cells to be used for the data. The second input is the score that you want to tally. You can check that you have all 24 readings by using the sum function at the bottom of the frequency input. Type the word 'Total' at the bottom of the 'Number of goals' column.

You can now produce a frequency graph. Remember to give your graph a title and label your axes.

7 What does this graph tell you about Lacey's performance throughout the season?

8 Which graph do you think gave you the most information?

9 Which statistics gave you the most information?

10 Do you think the spreadsheet will help Lacey to improve her game? Give your reasons using graphs and/or statistics.

Statistics from grouped data

9.4

In Year 7, you found the mean, median and mode from a frequency table using ungrouped data. We will now find these statistics using **grouped data**.

Finding measures of centre from grouped data

To find the mean of grouped data, we carry out the following steps.

Step 1 We find the **class centre** or **midpoint** by adding the end points of the class interval and dividing by 2. We enter these values in a column labelled x.

Examples for discrete data

If the class interval is 0–9, the midpoint is 4.5. If the class interval is 0–4, the midpoint is 2.

Examples for continuous data

If the class interval is 0–<10, the midpoint is 5. If the class interval is 0–<5, the midpoint is 2.5.

Step 2 We then multiply the midpoint value, x, by the frequency, f, and enter this product in a new column labelled fx.

Step 3 We total all the values in the fx column and write the sum as Σfx (sigma fx).

Step 4 We then divide Σfx by the sum of all the frequencies Σf. This will give us the mean.

$$\text{mean} = \frac{\text{sum of } fx}{\text{sum of } f} = \frac{\Sigma fx}{\Sigma f}$$

We will not find the median and mode for grouped data yet, but we can find the **modal class interval** and the **median class interval**.

The modal class interval is the class interval that has the highest frequency. The median class interval is the class interval that contains the median value.

Worked Example 10

WE 10

A manufacturer has orders from 50 different retailers for a particular product each week. One week the number of orders were as follows.

207, 312, 333, 215, 145, 332, 177, 149, 263, 147, 238, 319, 336, 479, 284, 167, 352, 123, 198, 352, 346, 163, 133, 197, 227, 482, 265, 187, 316, 214, 287, 324, 469, 375, 471, 328, 183, 318, 236, 255, 353, 227, 165, 276, 175, 389, 299, 273, 365, 328

Use grouped data to:

(a) calculate the mean number of orders, giving your answer correct to two decimal places

(b) calculate the median class interval for the number of orders

(c) calculate the modal class interval for the number of orders

(d) draw a histogram of the orders

(e) identify any possible outliers.

Thinking	Working

(a) 1 To calculate the mean you need to find the class centre and multiply this value by the frequency.

(a)

Number of orders	x	f	fx
100–149	124.5	5	622.5
150–199	174.5	9	1570.5
200–249	224.5	7	1571.5
250–299	274.5	8	2196.0
300–349	324.5	11	3569.5
350–399	374.5	6	2247.0
400–449	424.5	0	0.0
450–499	474.5	4	1898.0
		$\Sigma f = 50$	$\Sigma fx = 13\,675$

2 The mean is then found by calculating $\frac{\Sigma fx}{\Sigma f}$. It does not need to be a whole number.

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{13\,675}{50} = 273.50$$

(b) Find the median in the usual way by counting down the frequency column until the middle is found. (Here, there are 50 values, so the median is between the 25th and 26th values.)

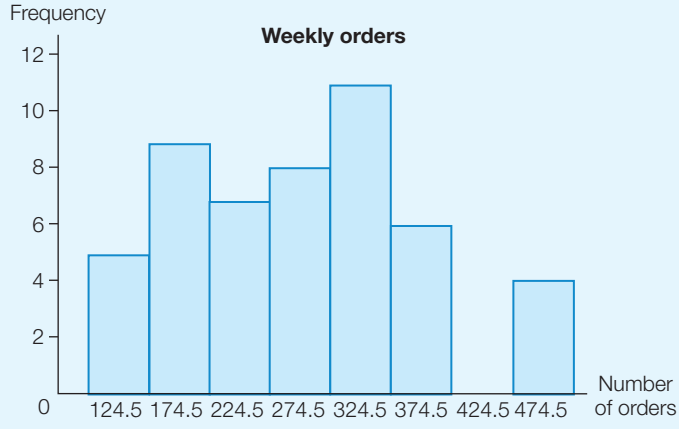
(b) Both the 25th and 26th values are in the 250–299 class, so this is the median class interval.

(c) The modal class interval is the class interval with the highest frequency.

(c) Modal class interval is 300–349.

(d) Draw the histogram in the usual way, remembering to leave half a column width before starting the first column. The midpoint can be used on the horizontal axis.

(d)



(e) Identify whether any data values appear to be quite different from the other data.

(e) There don't appear to be any outliers.

Worked Example 11

WE11

A wool producer wants to keep some of his lambs for breeding. To do this, the lambs must reach a certain breeding weight. He decides to use statistics to help him plan a breeding program. The data in the frequency table gives the weight, in kg, of the lambs in his flock.

Weight (kg)	Frequency
25–<30	5
30–<35	12
35–<40	14
40–<45	16
45–<50	13
50–<55	10
55–<60	2

(a) Calculate the mean weight of the lambs. Give your answer correct to two decimal places.

(b) Calculate the median class interval for the weights of the lambs.

(c) Calculate the modal class interval for the weights of the lambs.

(d) Draw a histogram of the data.

(e) Identify any possible outliers.

Thinking

Working

(a) **1** To calculate the mean you need to find the class centre and multiply this value by the frequency.

(a)

Weight (kg)	x	Frequency	fx
25–<30	27.5	5	137.5
30–<35	32.5	12	390
35–<40	37.5	14	525
40–<45	42.5	16	680
45–<50	47.5	13	617.5
50–<55	52.5	10	525
55–<60	57.5	2	115
		$\Sigma f = 72$	$\Sigma fx = 2990$

2 The mean is then found by calculating $\frac{\Sigma fx}{\Sigma f}$.

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{2990}{72} = 41.53 \text{ kg (correct to 2 d.p.)}$$

(b) Find the median in the usual way by counting down the frequency column until the middle is found. (Here, there are 72 values, so the median is between the 36th and 37th values.)

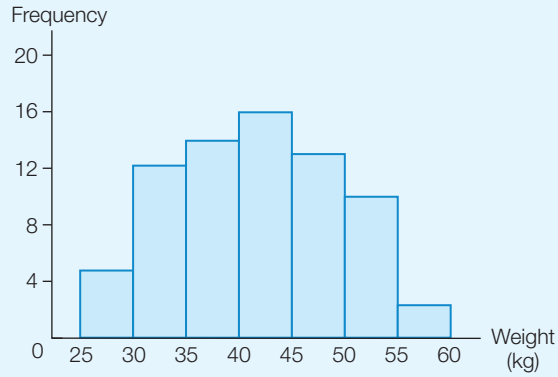
(b) Both the 36th and 37th values are in the 40–<45 class, so this is the median class interval.

(c) The modal class interval is the class interval with the highest frequency.

(c) Modal class interval is 40–<45 kg.

(d) Draw the histogram in the usual way, remembering to leave half a column width before starting the first column.

(d)



(e) Identify whether any data values appear to be quite different from the other data values.

(e) There don't appear to be any outliers.

Using technology

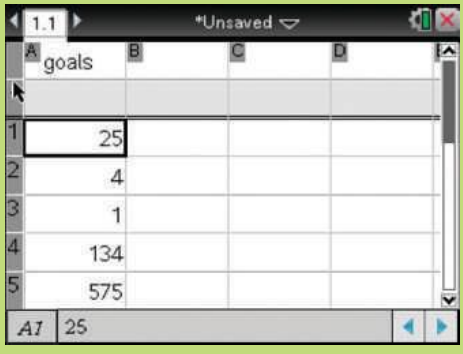
If we have a large list of raw data spread out over a large range it can take some time to sort it into class intervals and then draw a frequency graph. However, technology, such as a CAS calculator, makes this much easier. We will look at a relatively small data set to illustrate the process, but the size of the data set does not change the way we would go about the task.

We will use the number of goals scored prior to the beginning of the 2010 AFL season by each player on the list of the Brisbane Lions.

25	4	1	134	575	198	27	2	106	14	73	0	0
84	49	423	66	0	55	153	17	1	3	19	0	0
11	1	6	15	17	57	0	40	19	0	0	26	0
2	0	0	0	0	0	0						

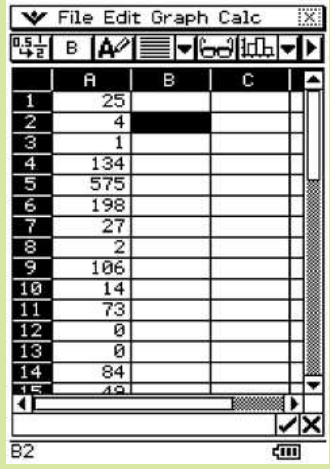
Using the TI-Nspire CAS

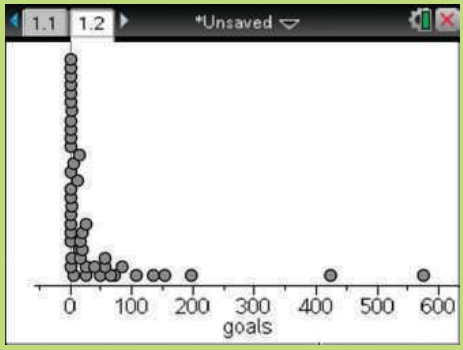
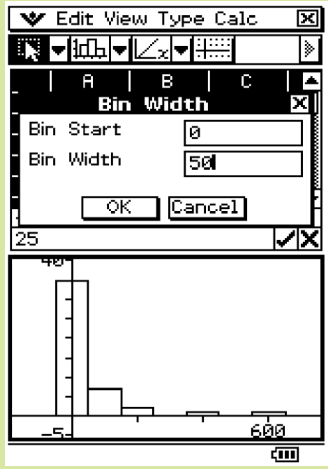
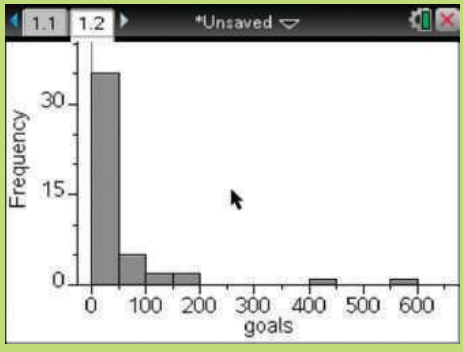
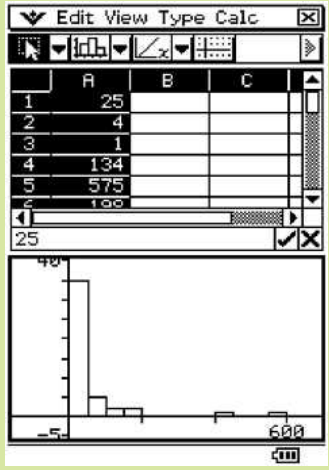
1 Press on and choose a new Lists & Spreadsheets page. Scroll up to beside the A and enter 'goals' as the name of the column. You need this to be able to draw a graph. Now, scroll down and enter the data, starting in the row labelled '1'.

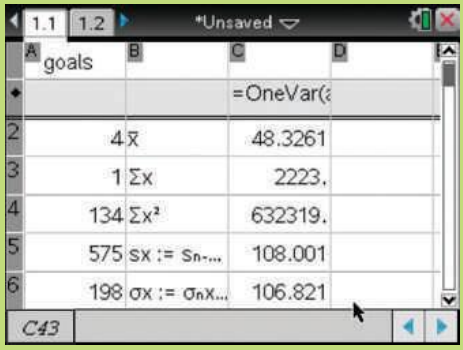
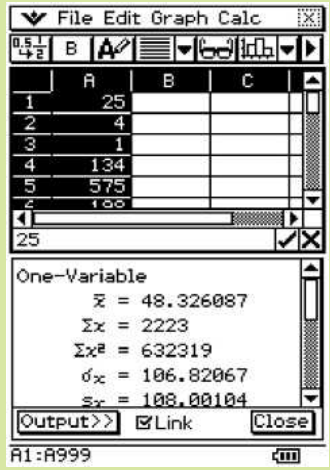
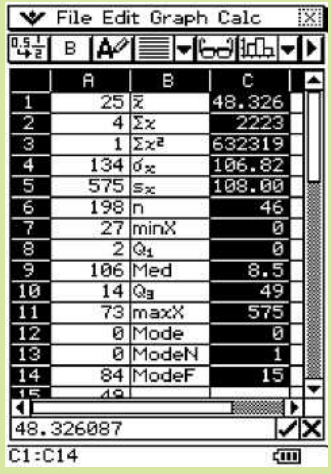


Using the ClassPad

1 Tap Menu and choose Spreadsheet. Enter the data in column A.



Using the TI-Nspire CAS	Using the ClassPad
2 Now, press ctrl I to insert a new page and choose Add Data & Statistics. This will show as a scattering of points. Move the cursor to below the horizontal axis and press and choose 'goals'. This will then create the equivalent of a dot plot.  	2 Highlight column A and select Graph Histogram. A histogram is drawn immediately. Select Calc >Bin Width. Make Bin Start 0 and bin Width 50.  Tap OK.
3 Now, press menu > Plot Type > Histogram to create the histogram.  	3 The histogram is now in the form you want. 
4 This has chosen the class interval for you, in this case, 50. If you want to change it, you can, by pressing menu > Plot Properties > Histogram Properties > Bin Settings and choose the width you want.	4 If we had wanted a different interval size we could have made that choice in the dialog box.
5 The histogram shows that there are probably two outliers, 423 and 575. Before we started we probably suspected these would be outliers!	5 The histogram shows that there are probably two outliers, 423 and 575. Before we started we probably suspected these would be outliers!

Using the TI-Nspire CAS	Using the ClassPad
6 Press (ctrl) ◀ to go back to the data screen and press (menu) > Statistics > Stat Calculations > One-Variable Statistics and say OK to Num of Lists being 1 and OK to the X1 List being in a [ ]. You will have to scroll up to see the results.  	**6** Now, with column A highlighted, tap Calc > One-Variable. All the summary statistics are listed. Tap on Output. You can choose where to paste your statistics. If you are happy with the displayed columns for Labels and Results, tap on Paste. 
7 Because the screen is not big enough to see the statistics all at the same time, we will summarise: Mean = 48.3261 Median = 8.5 Range = 575 (max – min)	**7**  In summary: Mean = 48.3261 Median = 8.5 Mode = 0 Range = 575 (max – min)

If we are dealing with grouped data, then we need to use the midpoint of the class interval to represent the score and enter this in the first column. We then enter the frequencies in the second column and proceed in much the same way.

9.4 Statistics from grouped data

Navigator

Q1 (a), Q2 (a), Q3, Q4, Q5, Q6, Q7, Q12, Q14 (a)	Q1 (a), Q2 (a), Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q14 (b)	Q1 (b), Q2 (b), Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q13, Q14

Answers page 648

Equipment required: CAS calculator or Excel spreadsheet may be used for Question **4**

Fluency

1 **(a)** A bakery records the number of sales of a new type of specialty bread over a 60-day period. The number of sales of the bread are recorded below.

WE10

78, 62, 68, 93, 123, 87, 92, 77, 113, 105, 85, 121, 113, 127, 105, 99, 108, 110, 116, 128, 123, 111, 109, 93, 132, 103, 94, 117, 124, 132, 117, 118, 122, 107, 110, 125, 106, 88, 138, 109, 128, 103, 138, 104, 128, 134, 122, 118, 99, 111, 119, 128, 121, 116, 105, 108, 135, 137, 127, 124

Use grouped data to:

(i) calculate the mean number of orders, giving your answer correct to two decimal places

(ii) calculate the median class interval for the number of orders

(iii) calculate the modal class interval for the number of orders

(iv) draw a histogram of the orders

(v) identify any possible outliers.

(b) The number of cars in a council car park was recorded every hour over a 48 hour period starting from midnight. The data is recorded below.

58, 17, 9, 23, 42, 67, 95, 177, 183, 205, 285, 221, 213, 227, 245, 299, 278, 270, 215, 228, 168, 141, 109, 93, 42, 18, 10, 17, 34, 58, 97, 118, 172, 207, 210, 225, 216, 248, 288, 269, 283, 273, 238, 174, 128, 134, 122, 118, 99, 111, 119, 128, 121, 116, 105, 108, 135, 125, 107, 74

Use grouped data to:

(i) calculate the mean number of cars, giving your answer correct to two decimal places

(ii) calculate the median class interval for the number of cars

(iii) calculate the modal class interval for the number of cars

(iv) draw a histogram of the cars

(v) identify any possible outliers.

WE11

2 (a) A horse trainer has a large stable of horses. He needs to keep records of each horse's weight. The weights are entered into a frequency table so that the trainer can gather some statistics on the horses in his stable. The data in the frequency table gives the weight, in kg, of the horses at their last weigh-in.

Weight (kg)	Frequency
250–<300	3
300–<350	13
350–<400	17
400–<450	20
450–<500	16
500–<550	14
550–<600	4

(i) Calculate the mean weight of the horses. Give your answer correct to two decimal places.

(ii) Calculate the median class interval for the weights of the horses.

(iii) Calculate the modal class interval for the weights of the horses.

(iv) Draw a histogram of the data.

(v) Identify any possible outliers.

(b) Another horse trainer has a stable of horses. He also keeps records of each horse's weight. The weights are entered into a frequency table so the trainer can gather some statistics on the horses in his stable. The data in the frequency table gives the weight, in kg, of the horses at their last weigh-in.

Weight (kg)	Frequency
250–<300	5
300–<350	11
350–<400	21
400–<450	27
450–<500	18
500–<550	11
550–<600	7

(i) Calculate the mean weight of the horses. Give your answer correct to two decimal places.

(ii) Calculate the median class interval for the weights of the horses.

(iii) Calculate the modal class interval for the weights of the horses.

(iv) Draw a histogram of the data.

(v) Identify any possible outliers.

3 For each of the following stem-and-leaf plots, calculate the **(i)** mean and **(ii)** median. Give your answers correct to two decimal places.

(a)

STEM	LEAF
4	8
5	2 3 4 5 6 7 7
6	4 5 5 6 7
7	3 4 5
8	2
9	1
10	9

(b)

STEM	LEAF
11	5 7
12	4 5
13	2 3 4 9
14	0 0 2 3 4
15	6 7 8 9 9 9
16	1 3 3 4 5 6
17	0 5 6 7 8

(c)

STEM	LEAF
21	0 2
22	1 8 8
23	0 1 2 3
24	1 2 5 6 7 9
25	1 2 3
26	1 1 3
27	1 2

4 If you have access to a CAS calculator or an Excel spreadsheet, use it to help you with this question, otherwise complete it using the normal methods.

The numbers that follow are the career points scored by the players on the list of the Brisbane Broncos at the start of the 2010 NRL season.

0	0	0	32	4	0	238	8	28	0
16	302	4	0	130	8	1163	114	16	8
88	8	0	0	582	0	16	44	0	0
60	62	82	0	0	56	36			

(a) Using grouped data, draw a histogram.

(b) Do you think any of the values might be possible outliers? If so, which ones?

(c) Using grouped data, calculate the mean, median and range of the data.

5 **(a)** The class centre for a class interval of 0–19 is:

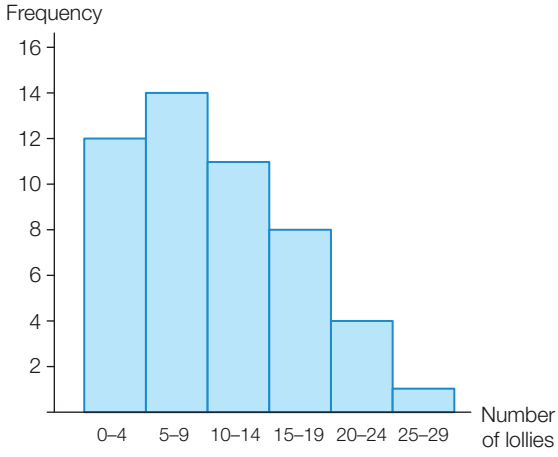
A 9 **B** 9.5 **C** 10 **D** 10.5

(b) The class centre for a class interval of 10–20 is:

A 14.5 **B** 15 **C** 15.25 **D** 15.5

Understanding

6 The graph opposite shows the number of lollies found by a group of children attending a birthday party in a treasure hunt.



(a) Estimate the mean number of lollies found.

(b) Calculate the mean from the grouped data. How close was your estimate?

(c) How many children attended the party?

(d) How could you check to see whether this was a realistic spread of data?

(e) Can you calculate the exact number of lollies found? Explain your answer.

7 Find the class intervals for each of the following lists of class centres for discrete data.

(a) 5.5, 15.5, 25.5, 35.5, 45.5

(b) 2, 7, 12, 17, 22

(c) 24.5, 74.5, 124.5, 174.5, 224.5

(d) 25.5, 75.5, 125.5, 175.5, 225.5

(e) 11, 14, 17, 20, 23

(f) 27, 32, 37, 42, 47

8 The following frequency table shows the time taken, in minutes, by the students at a particular school to complete a cross-country race.

Time (min)	Frequency
20–<22	2
22–<24	12
24–<26	27
26–<28	15
28–<30	8
30–<32	5
32–<34	1

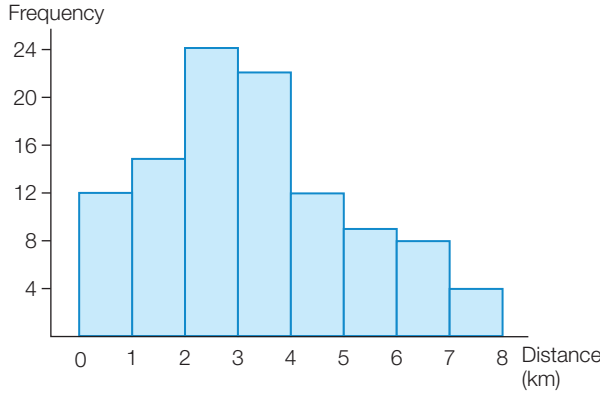
(a) Calculate the mean time taken by the students to complete the cross-country race.

(b) Draw a histogram.

(c) Five students did not complete the race within the time limit. How would their results affect the mean time taken?

(d) How might you record their times in order to include them in the histogram?

9 The following histogram shows the distance travelled to school, measured in km, by each of the students in Year 8 at Westbow High School.



(a) State the modal class.

(b) State the median class.

(c) Find the mean distance travelled to school (correct to one decimal place).

Reasoning

10 The following table shows the number of AFL games played by each of the 45 players on the list of the Sydney Swans at the beginning of the 2010 season.

Number of games	Frequency
0–39	22
40–79	7
80–119	7
120–159	3
160–199	2
200–239	3
240–279	1

(a) Calculate the mean number of games played.

(b) State the game interval that represents the modal class.

(c) State the game interval that represents the median class.

(d) Which of these measures of centre do you think does the best job of describing the number of games played?

(e) In fact, 13 of the players have yet to play a game. Recalculate the three measures of centre, leaving out these players.

(f) Does this give a better representation of the measures of centre for the number of games? Explain your reasoning.

11 **(a)** For discrete data, what can you tell about a class interval if the class centre is a whole number?

(b) For discrete data, what can you tell about a class interval if the class centre is a decimal number with a decimal part of .5?

(c) For discrete data, can a class centre ever be other than a whole number or a .5 number? Explain your reasoning.

Open-ended

12 **(a)** Write two different lists of class intervals that have class centres 5 apart.

(b) Write two different lists of class intervals that have class centres 10 apart.

13 Alphaville is a small town at an elevation of 221 m; Betaborough is a country town at an elevation of 107 m; and Deltatown is a coastal town at an elevation of 3 m. The following table gives some statistics related to the annual mean temperatures of these towns.

	Max (°C)	Max range (°C)	Min (°C)	Min range (°C)
Betaborough	28.2	18.7	13.4	18.1
Alphaville	20.6	17.6	6.6	9.2
Deltatown	25.1	7.5	17.2	9.8

Choose one of the towns and construct a table of the monthly maximum and minimum temperatures that fit the given statistics.

14 **(a)** Construct a stem-and-leaf plot with at least 20 values such that the median is bigger than the mean.

(b) Construct a stem-and-leaf plot with at least 20 values such that the median is smaller than the mean.

Outside the Square Puzzle

Complete the patterns

In each case, you need to find the next number in the pattern.

1

4	8	10	11	11.5	

2

3	4	9	24	81	144	

3

4	2	3	1	2	0	

4

4	5	9	14	23	

5 Now, see if you can make up another four of your own and get one of your classmates to try to solve them.

Maths 4 Real

Which statistic is best in sport?

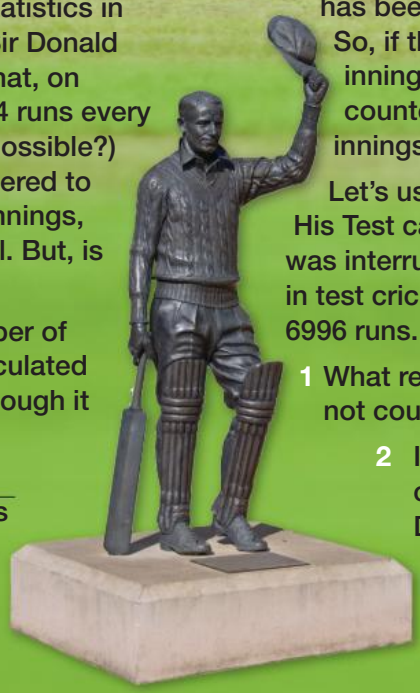
Cricket

Cricket uses lots of statistics, such as batting averages, bowling averages, run rates, and the greatest number of innings without scoring a 'duck' (zero).

One of the best known cricket statistics in Australia is the run average for Sir Donald Bradman of 99.94. This means that, on average, Sir Donald scored 99.94 runs every time he batted. (Is this actually possible?) As a century (100 runs) is considered to be an outstanding effort in any innings, this average really is very special. But, is it accurate?

The average (mean) for the number of runs scored by a batsman is calculated in a slightly strange fashion. Although it follows the basic formula of

$$\text{Batsman's average} = \frac{\text{total runs scored}}{\text{number of innings}}$$



only completed innings (that is where the batsman has been 'out') are counted in the denominator. So, if the batsman is not out when the team's innings is completed, then their runs are counted in the average calculation, but the innings is not added to the denominator value.

Let's use Sir Donald Bradman as an example. His Test career spanned the years 1928–1948, but was interrupted by World War II. He batted 80 times in test cricket, with 10 not outs, and accumulated 6996 runs. So, his average (mean) was $\frac{6996}{70} = 99.94$.

1 What reasons do you think there would be for not counting the 'not outs'?

2 If we did count the not outs as completed innings, what would be Sir Donald Bradman's batting average?

We also know how to calculate the median and the mode. The following is a list of Sir Donald's scores, innings by innings. A * means that score was a 'not out'.

18	1	79	112	40	58	123	37*	8	131
254	1	334	14	232	4	25	223	152	43
0	226	112	2	167	299*	0	103*	8	66
76	24	48	71	29	25	36	13	30	304
244	77	38	0	0	82	13	270	26	212
169	51	144*	18	102*	103	16	187	234	79
49	0	56*	12	63	185	13	132	127*	201
57*	138	0	38	89	7	30*	33	173*	0

3 Calculate the median and mode for Sir Donald's results.

4 Of the four numbers calculated (the average, the average including 'not outs', median and mode), which do you think is the fairest, that is, the number that best describes Sir Donald's career?

Now let's look at two cricketers at different stages in their careers.

The following are the results for Marcus North in his first 10 innings:

117	5	38	0	125*	0	6	12	96	110

The following are the results for Michael Hussey in his first 70 innings:

1	29	137	31*	133*	30*	23	58	122	31
45	6	14*	75	73	89	23	37	182	86
91	61*	74*	103	6	37	133	132	34*	2
36	41	145*	0	46	22	56	1	10	40
12	18	146	31	54	1	53	90	19	35
0	70	0	8	0	2	30	45*	4	0
50	19	20	39	3	51	27	0	64	10

Note that Hussey's 70 innings were completed in the time period of November 2005 to August 2009 – a much shorter period than Sir Donald.

5 Calculate the four different results for each of these players.

6 Now, make a few comments about which result you now think is the fairest one to use to analyse a batsman's performance.

AFL

The Coleman Medal is awarded to the player in the Australian Football League who kicks the greatest number of goals in the 22-round home and away season. In 2010, Jack Riewoldt of the Richmond football club kicked 78 goals to win the medal. A year earlier, his cousin, Nick Riewoldt of the St Kilda football club, kicked 68 goals to finish third in the medal tally.

Jack Riewoldt played 22 games in the 2010 home and away season and kicked the following number of goals:

2	0	3	3	2	3	2	4	6	4	6	10	3	5	5	3	2	2	1	2	7	3

Nick Riewoldt played 21 games in the 2009 home and away season and kicked the following number of goals:

0	2	1	5	4	4	5	3	4	4	3	5	2	3	1	4	5	2	2	3	6	

7 For each player, calculate the mean, median and modal number of goals.

8 If you were to comment on the number of goals Jack or Nick kicked in an 'average' game, what answer would you give?

9 Nick Riewoldt also played 3 games in the 2009 finals, scoring 5, 4 and 1 goals. How does this affect his mean, median and mode?

Another way of comparing the players is to look at their accuracy when kicking for goal. (In Australian Rules a goal is worth 6 points and a behind is worth 1 point.)

Over the course of four seasons these results were obtained by the two players:

	Jack Riewoldt		Nick Riewoldt	
	Goals	Behinds	Goals	Behinds
2007	7	3	42	26
2008	18	8	65	39
2009	32	27	78	47
2010	78	39	34	27

10 (a) Calculate the number of goals scored as a percentage of their scoring shots.

(b) Which player would you say is a 'better' kick for goal?

Half-time 9

Ex. 9.3

1 For the following data list find a suitable class interval, record the results in a frequency table, and then display the data using a frequency column graph.

25, 45, 67, 99, 18, 24, 67, 87, 56, 65, 43, 21, 19, 91, 37, 67, 54, 23, 19, 22, 54, 88, 93, 86, 54
67, 43, 22, 27, 28, 38, 46, 98, 25, 33, 62, 90, 30, 21, 28, 78, 18, 77, 71, 50, 43, 40, 20, 77, 91

Ex. 9.2

2 The following data values represent the number of games played by players on the list of the South Sydney Rabbitohs as of May 2010.

74	33	0	23	61	10	9	4	0	8
20	110	34	68	63	4	126	37	0	46
30	178	122	69	9	0	9	4	12	

Using the raw data, calculate the:

(a) mean number of games played correct to two decimal places

(b) median number of games played

(c) mode of the number of games played.

(d) Do you think any of the results represents an outlier? If so, which one(s). If not, how many games would need to be played for you to consider that result an outlier?

(e) Four players have yet to play a game. Remove these results from your data and recalculate the mean, median and mode. Did this make much of a difference to your statistics?

(f) Which of the above statistics do you think does the best job of measuring the centre of this data?

Half-time 9

3 Each student in a class was asked how much pocket money they received each week. The following table summarises the results.

(a) Find the mean amount of pocket money received by the students.

(b) Find the class interval that represents the median amount of pocket money received.

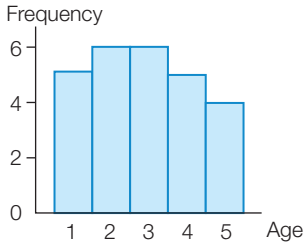
Amount of pocket money	Frequency
0–<5	6
5–<10	13
10–<15	3
15–<20	2
20–<25	1

Ex. 9.4

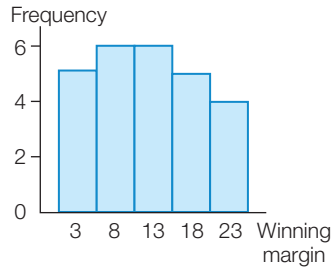
4 Construct a frequency table from each of the following histograms.

Ex. 9.3

(a)



(b)



5 Each student in a class was asked how many times they had travelled to a different Australian State or Territory. The following table summarises the results.

(a) Find the mean, correct to two decimal places.

(b) Find the median number of interstate visits for these students.

Number of interstate visits	Frequency
0	8
1	7
2	3
3	4
4	1
5	1
6	0
7	0
8	2

Ex. 9.2

6 Draw a histogram for the data displayed in the following table.

Ex. 9.3

Number of points scored	Frequency
8	12
9	8
10	13
11	11
12	9
13	16
14	10

7 The National Park Service is trying to determine the number of freshwater crocodiles in a particular lagoon. To do this, they have captured and tagged 30 crocodiles and then released them back into the wild. Some time later, they capture another 20 crocodiles and find that two of them are tagged. What should be their estimate for the total crocodile population in the lagoon?

Ex. 9.1

Investigation

Who's the best? Here's the test

Every year in November, the AFL holds a draft camp where young players eligible for the draft are brought together and taken through a series of physical tests.

The Big Question

Were the group of young footballers better in 2007 or in 2008?

Engage

One of the tests that the young players need to perform is the 'beep' test. They must run a 20 m distance in a time interval set by two electronically timed beeps. The time interval starts at 9 seconds for level 1, but the time gets shorter and shorter and the distance run gets longer and longer at each level as players progress through the levels of the test. Each level takes about 1 minute. The score is the highest level reached keeping in time with the beeps. The higher the level, the fitter the player.

The best ten results for beep test levels in 2008 were:
14.8, 14.5, 14.5, 14.4, 14.3, 14.3, 14.2, 14.2, 14.2, 14.1

In 2007, the best ten results were:
15.2, 15.1, 14.9, 14.9, 14.8, 14.8, 14.7, 14.5, 14.4, 14.3

1. Find the mean, mode and median for each group.
2. Find the range for each group.
3. Draw a frequency column graph for each set of data.

Explore

Using only one set of results does not really give us a balanced comparison of the two groups. The best ten results for the 3 km time trial (given in minutes and seconds) for the two years are given below.

2008: 10:17, 10:32, 10:41, 10:42, 10:42, 10:43, 10:44, 10:53, 10:57, 10:57

2007: 10:23, 10:25, 10:32, 10:42, 10:48, 10:49, 10:49, 10:49, 10:51, 10:54

4 Find the mean, mode and median for each group.

5 Find the range for each group.

6 Draw a frequency column graph for each set of data.

Another set of results for the agility test (seconds) is given below.

2008: 7.77, 8.03, 8.18, 8.25, 8.27, 8.31, 8.32, 8.32, 8.34, 8.35

2007: 7.91, 8.05, 8.07, 8.22, 8.26, 8.29, 8.32, 8.36, 8.38, 8.39

7 Find the mean, mode and median for each group.

8 Find the range for each group.

9 Draw a frequency column graph for each set of data.

Strategy options

- Have I seen a similar problem?
- Look for a pattern.

Explain

10 Using the 'beep' test statistics, which group do you think is better at this test?

11 Using the statistics from the 3 km time trial, which group do you think is better at this test? Give reasons for your decision.

12 Using the statistics from the agility trial, which group do you think is better at this test? Give reasons for your decision.

Elaborate

13 Using all three results, which group do you think is better overall? Give reasons for your decision.

14 What do you think 'better' means in this context?

Evaluate

15 Each year, 75 young players attend the camp. Do you think that using the best ten results is the fairest way to make a comparison between the groups? Can you suggest a better method for comparison?

16 Do you think you can make a fair comparison on the basis of three tests?

17 Which statistics were the most useful in helping you to make a decision about the best group?

18 Were the frequency column graphs helpful in making a decision about the better group?

19 What other graphs could you have used?

Extend

Here are some more results from the two camps.

Repeated 30 m sprint (seconds)
2008: 24.82, 24.97, 24.97, 25.10, 25.10, 25.14, 25.44, 25.44, 25.48, 25.49
2007: 23.59, 23.96, 24.01, 24.08, 24.17, 24.22, 24.22, 24.25, 24.37, 24.48

Vertical jump (cm)
2008: 78, 74, 72, 72, 70, 70, 69, 69, 69, 67
2007: 81, 74, 73, 72, 71, 70, 70, 70, 68, 68

20 m sprint (seconds)
2008: 2.80, 2.82, 2.83, 2.86, 2.89, 2.90, 2.91, 2.92, 2.92, 2.92
2007: 2.83, 2.88, 2.88, 2.89, 2.91, 2.91, 2.92, 2.92, 2.93, 2.94

20 Use this additional data to make a final decision about whether the 2007 or the 2008 group was better.

21 Do you think that these tests would be good predictors of future performance in AFL football? What other factors need to be considered when selecting a top AFL player?

Mathspace

Sales at Auction.
Sales by Auction
Land on North-Street.
Notice to the Proprietors of the Newburyport Turnpike Corporation.
M. CAREY,
MISCELLANY.
Real Estate.
Real Estate.
Valuable Estate on Broad-Street.
To be sold at Public Auction
To be sold at Public Auction,
NOTICE.
TO THE PUBLIC.
Hold the front page

The deadline is looming for tomorrow's edition of *The Morning Standard* but Katrina, the editor, is not happy about some of the stories her reporters have submitted. You will have to help her meet her deadline by writing some new stories!

Retention rates

This story is supposed to be based on the following information.

For a school, the retention rate compares the number of students still enrolled at that school at the end of a time period with the number at the beginning and expresses this as a percentage. As an example, if there were 100 students in Year 7 in 2006 and 94 students in Year 9 in 2008 then the retention rate is 94%.

The table shows the retention rate for Australian Government schools in 2008.

	NSW	Vic	Qld	SA	WA	Tas	NT	ACT
Year 7/8 to Year 9 retention rate	99.7	101.1	101.1	100.6	102.0	99.9	98.2	98.3
Year 10 to Year 12 retention rate	67.6	74.2	70.7	66.0	67.6	61.8	74.6	96.6
Year 7/8 to Year 12 retention rate	64.6	71.9	70.3	64.7	68.2	61.3	67.6	95.9

ABS, 4221.0 Schools, Australia 2008

The story, of about 300 words, should include the following, as a minimum:

- a graph showing a comparison on a state by state basis
- an explanation of how the rate could be more than 100%
- a discussion of why the rates are so different for the different time periods
- an attempt at deciding which state or territory does best at retaining its students in Government schools
- an explanation of where the students who are not retained go.

How good is Australia at sport?

The following table shows some information about medal results at the 2010 Commonwealth Games in New Delhi, India, and the 2006 Commonwealth Games in Melbourne.

	Gold		Silver		Bronze		**Total**	
	2010	2006	2010	2006	2010	2006	2010	2006
Australia	74	84	55	69	48	69	177	222
Canada	26	26	17	29	32	31	75	86
England	37	36	59	40	46	34	142	110
India	38	22	27	17	36	11	101	50
Jamaica	2	10	4	4	1	8	7	22
Kenya	12	6	11	5	9	7	32	18
Malaysia	12	7	10	12	13	10	35	29
New Zealand	6	6	22	12	8	13	36	31
Scotland	9	11	10	7	7	11	26	29
South Africa	12	12	11	13	10	13	33	38

You need to present this information in a more informative manner. At the moment, it is in alphabetical order only, but this makes it difficult to see which countries did best. You might like to draw some sort of graph as well. When you have done that you need to write a brief report, using as many statistical terms and values as you can. Come up with a headline for your report.

9.5 Understanding probability

An event in probability is simply something that occurs. Examples of events in probability are tossing a head with a coin, rolling a 4 on a die or having a day when it rains.

The probability of an event occurring is the *chance* of that event occurring. Probabilities range from 0, where the chance of an event is impossible, to 1, where the chance of the event occurring is *certain*. The probability of obtaining a 7 on a normal die would be 0, whereas the probability of getting a number less than 7 would be 1.

Any possible result of an event is called an **outcome**. When tossing a coin, there are two possible outcomes: obtaining a head or obtaining a tail. A successful outcome is the result that is required, such as getting a head on a coin toss. The probability of an event, Pr(event), can be written as a decimal, fraction or percentage.

Probabilities can be determined through experiments and by collecting long-term data, but we often determine probabilities from theory.

Sample space

To calculate a theoretical probability, we need to know the **sample space**. A sample space is a list of all possible outcomes and the outcomes must all be equally likely.

Successful outcomes must be some or all of the outcomes listed in the sample space. The sample space from tossing a coin is a head (H) and a tail (T). If a successful outcome is tossing a head, there is only one successful outcome possible out of two outcomes in the sample space, so:

$$\Pr(H) = \frac{1}{2}$$

The sample space from rolling a normal die is 1, 2, 3, 4, 5, 6. If a successful outcome is rolling a 2, then there is only one successful outcome possible out of six outcomes in the sample space, so:

$$\Pr(2) = \frac{1}{6}$$

When more than one outcome is considered a success, we can calculate the probability by counting the number of successes overall or calculating each probability separately and adding them.

For example, finding the probability of rolling a 1 or a 2 on a die can be calculated in two ways.

There are two successful outcomes possible out of six outcomes in the sample space, so:

$$\Pr(1 \text{ or } 2) = \frac{2}{6} = \frac{1}{3}$$

However, we could have calculated this by considering the probability of each event separately and adding them:

$$\Pr(1) = \frac{1}{6} \text{ and } \Pr(2) = \frac{1}{6} \text{ so } \Pr(1 \text{ or } 2) = \frac{1}{6} + \frac{1}{6}$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$

Calculating probability

$$\Pr(\text{event}) = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

All outcomes must be equally likely.

Worked Example 12

WE 12

In Kayla's class there are 24 students. Five were born in Cambodia, three in Vietnam, six in Australia, two in Croatia, five in Serbia, one in England, one in New Zealand and one in South Africa. A member of the class is selected at random. What is the probability that a student chosen at random was born in:

(a) Vietnam **(b)** Australia **(c)** Croatia or Serbia?

Thinking	Working
(a) 1 Write the number of successful outcomes.	(a) Number born in Vietnam = 3
2 Write the total number of outcomes.	Total = 24
3 Use the formula to express the probability as a fraction and then cancel down to simplest terms (if possible).	$\Pr(\text{born in Vietnam}) = \frac{3}{24} = \frac{1}{8}$
(b) 1 Write the number of successful outcomes.	(b) Number born in Australia = 6
2 Write the total number of outcomes.	Total = 24
3 Use the formula to express the probability as a fraction and then cancel down to simplest terms (if possible).	$\Pr(\text{born in Australia}) = \frac{6}{24} = \frac{1}{4}$
(c) 1 Calculate the number of successful outcomes.	(c) Number born in Croatia or Serbia = 2 + 5 = 7
2 Write the total number of outcomes.	Total = 24
3 Use the formula to express the probability as a fraction and then cancel down to simplest terms (if possible).	$\Pr(\text{born in Croatia or Serbia}) = \frac{7}{24}$

Note: if we added together each of the individual probabilities for country of birth they would have to equal 1. It is certain that each student was born in one of the listed countries.

Complements

All situations can be written so that there are only two possible outcomes. An event may happen or it may not happen. These two probabilities add up to 1. The outcome of a game of football could be win, lose or draw, but we could consider winning and not winning as the two outcomes. One outcome is the **complement** of the other. The probabilities of 'winning' or 'not winning' are called complementary probabilities.

Pr(event occurring) + Pr(event not occurring) = 1

Sometimes, the probability you want to calculate is easier to find if you calculate the probability of the complement and subtract this from 1.

Pr(event not occurring) = 1 – Pr(event occurring)

Consider the probability of getting a 2, 3, 4, 5 or 6 when rolling a die. This probability is the same as the probability of 'not getting a 1', so if we calculate the probability of 'getting a 1' and subtract this from 1 we will get the desired probability.

Worked Example 13

WE13

If I roll an unbiased six-sided die, what is the probability of:

(a) not rolling a 6 **(b)** not rolling a 5 or a 6?

Thinking	Working
(a) 1 Write the known probability. (In this case, the probability of rolling a 6.)	(a) $Pr(\text{rolling a } 6) = \frac{1}{6}$
2 Use the formula Pr(event not occurring) = 1 – Pr(event occurring) and calculate the answer.	$Pr(\text{not rolling a } 6) = 1 - Pr(\text{rolling a } 6)$ $= 1 - \frac{1}{6}$ $= \frac{5}{6}$
(b) 1 Write the known probabilities.	(b) $Pr(\text{rolling a } 5) = \frac{1}{6}$ and $Pr(\text{rolling a } 6) = \frac{1}{6}$
2 Find the total known probability.	$Pr(\text{rolling a 5 or a } 6) = \frac{1}{6} + \frac{1}{6}$ $= \frac{2}{6}$ $= \frac{1}{3}$
3 Use the formula Pr(event not occurring) = 1 – Pr(event occurring) and calculate the answer.	$Pr(\text{not rolling a 5 or a } 6) = 1 - \frac{1}{3}$ $= \frac{2}{3}$

9.5 Understanding probability

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q15, Q16, Q17	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q10, Q11, Q12, Q13, Q14, Q15, Q17	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q11, Q12, Q13, Q14, Q15, Q16, Q17

Answers page 650

Fluency

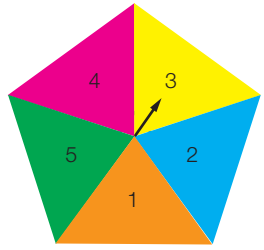
1 A class consists of 26 students. Ten study Chinese and the rest study Italian. What is the probability that a student chosen at random studies: WE12

(a) Chinese **(b)** Italian?

2 If I spin a five-sided spinner with numbers 1, 2, 3, 4, 5, what is the probability of: WE13

(a) not getting a 2

(b) not getting a 1 or a 2?



3 Illona is running in a race at the local athletics track. If her probability of winning is $\frac{1}{8}$ what is the probability that she will not win?

4 There are 13 teachers in the staff room: eight females and five males, one of whom is Mr Antonac. A student knocks at the door wanting to see Mr Antonac. What is the probability that the teacher who answers the door is:

(a) male **(b)** female **(c)** Mr Antonac?

5 The hockey training squad consists of eight forwards, five half-backs, four full-backs and Jarrod, who plays in goal. One player is chosen at random from the training squad to represent the team at a fundraising function.

What is the probability that the person is:

(a) a forward **(b)** a half-back **(c)** Jarrod, the goalkeeper?

6 There are 10 000 tickets sold in the school raffle. If Alf bought 10 tickets, find:

(a) Pr(Alf wins the raffle) **(b)** Pr(Alf does not win the raffle).

7 My fruit bowl contains five bananas, four apples, one orange and five peaches. If I choose one piece of fruit from the bowl at random, find:

(a) Pr(a banana)

(b) Pr(a peach or an apple)

(c) Pr(not an apple)

(d) Pr(not an orange or a banana).

8 A bag of potatoes contains five green ones and fourteen good ones. If a potato is chosen at random, the probability of it being green is:

A $\frac{5}{19}$ B $\frac{5}{14}$ C $\frac{1}{2}$ D $\frac{14}{19}$

Understanding

9 What is the probability that the first digit on a car number plate will be a 2, 3 or 4? (Ignore the possibility of personalised number plates without digits.)

10 For seven of the last twelve years, Dennis has received a birthday present of socks and handkerchiefs from his grandparents. Assuming his grandparents don't change their giving behaviour, what is the probability that he doesn't receive socks and handkerchiefs from his grandparents for his next birthday?

11 Dominic rolls a standard six-sided die. Find:

(a) Pr(a 2 or a 4)
(b) Pr(a number larger than 2)
(c) Pr(a number less than 2)
(d) Pr(a multiple of 3)
(e) Pr(not a 5)
(f) Pr(not a number larger than 3).

12 A bag contains 26 small tiles, each marked with a different letter of the alphabet. Celeste selects one tile from the bag. Choose the answer with the correct probability that the tile Celeste draws is:

(a) a vowel

A $\frac{2}{13}$ B $\frac{5}{26}$ C $\frac{1}{5}$ D $\frac{1}{4}$

(b) b, d, g or z

A $\frac{2}{13}$ B $\frac{4}{25}$ C $\frac{3}{13}$ D $\frac{1}{4}$

(c) one of the letters in the word 'chance'.

A $\frac{2}{13}$ B $\frac{5}{26}$ C $\frac{3}{13}$ D $\frac{6}{25}$

13 The numbers from one to twenty are written on separate counters and placed in a hat. If one counter is drawn at random, find:

(a) Pr(a number between 5 and 11)
(b) Pr(a number with a 1 in it)
(c) Pr(a number less than 30)
(d) Pr(a multiple of 5)
(e) Pr(not a multiple of 5)
(f) Pr(not odd).

Reasoning

14 The following table shows the number of vehicles sold across Australia in March 2010 divided into a number of vehicle types.

Vehicle type	Number sold in March 2010
Light car	12 153
Small car	22 463
Medium car	7256
Large car	8816
Upper large car	272
People mover	1234
Sports car	1523
SUV Compact	9651
SUV Medium	8690
SUV Large	1407
SUV Luxury	1895

Source: VFacts (official results issued by the Federal Chamber of Automotive Industries)

Express your answers to the following questions as percentages, correct to one decimal place. Find the probability that a new vehicle buyer in March 2010, chosen at random from purchasers of vehicles in the table, had purchased a:

(a) light car

(b) small car

(c) medium car

(d) SUV of any type

(e) not a people mover

(f) not a sports car nor a people mover.

(g) Do you think that the probabilities would change very much from year to year? Why or why not?

(h) Is it more probable that a new vehicle buyer in March 2010 bought a small car or an SUV of any type? Justify your answer.

15 **(a)** Draw a spinner for which Pr(green) = Pr(blue) = Pr(orange) = $\frac{1}{3}$.

(b) Draw a spinner for which Pr(red) = 3 × Pr(blue) and Pr(green) = $\frac{1}{5}$.

(c) Draw a spinner for which Pr(brown) = 5 × Pr(green), Pr(red) = 3 × Pr(green) and Pr(yellow) = Pr(green).

Open-ended

16 **(a)** Look at the following list of words and expressions very carefully and then put them in order from impossible on the left to certain on the right. Then, assign a percentage probability to each of them. Your teacher may ask you to discuss your order with some other members of the class.

50-50	certain	even chance
impossible	little chance	more often than not
most likely	most often	most unlikely
probably	sure thing	unlikely
unusual	usually	very unlikely

(b) Can you add some other words or expressions to the list?

17 Assign a probability to the following situations in your life.

(a) getting to school on time every day

(b) having an apple for lunch

(c) going to the movies at least once a month

(d) getting a phone call from a friend today

(e) running a marathon in your lifetime

(f) taking a space ride to the Moon

(g) playing sport on the weekend

(h) sleeping after 9 a.m. on a Saturday

Outside the Square Puzzle

Bracket placement

Insert brackets to make each of the following equations true.

1 $3 \times 4 - 5 + 6 \times 2 + 5 = 39$

2 $5 - 2 \times 3 - 4 \div 6 - 3 = 1$

3 $18 + 36 \div 3 - 3 \times 5 \times 2 - 3 \times 1 = 9$

9.6 Theoretical probability for single-step experiments

There are many situations where the probabilities of experiments can be calculated theoretically. This means we can calculate the probabilities without actually doing an experiment. Finding probabilities when tossing a coin or a die, using a spinner, picking a card from a pack of cards or a coloured marble out of a bag are all such examples.

We can list the sample space that gives the total number of outcomes, then count the number of successful outcomes. This allows us to calculate the required probability using:

$$\text{Pr(event)} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}.$$

Worked Example 14 WE14

Alvin has a normal pack of 52 playing cards and he selects one card from the pack.

(a) List the sample space.

(b) Find the probability of that card being:

(i) a diamond **(ii)** a king or a queen.

Thinking	Working
(a) 1 List the sample space.	(a) ♥: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K ♦: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K ♣: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K ♠: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K (♥ = Hearts, ♦ = Diamonds, ♣ = Clubs, ♠ = Spades, A = Ace, J = Jack, Q = Queen, K = King)
(b) (i) 1 Count the number of successful outcomes in the sample space. Count the total number of outcomes. **2** Calculate the probability by using $\text{Pr(event)} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$	(b) (i) Number of successful outcomes = 13 Total number of outcomes = 52 $\text{Pr(diamond)} = \frac{13}{52}$ $= \frac{1}{4}$

(ii) 1 Count the number of successful outcomes.

2 Calculate the probability by using

$$\Pr(\text{event}) = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

(ii) Number of ways of selecting a King or a Queen = 8

$$\Pr(\text{King or Queen}) = \frac{8}{52} = \frac{2}{13}$$

9.6 Theoretical probability for single-step experiments

Navigator

Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q13	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q9, Q10, Q11, Q12, Q13, Q14

Answers page 651

Fluency

WE14

1 Phoebe has a normal pack of 52 playing cards and she selects one card from the pack.

(a) List the sample space.

(b) Find the probability of that card being:

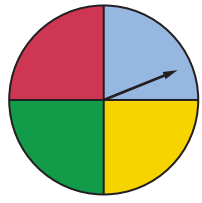
(i) a 7 **(ii)** an ace or a 10.

2 For each of the following questions **(i)** list the sample space and **(ii)** find the probability of the event occurring.

(a) tossing a coin and getting a head

(b) rolling a normal die and getting a 4

(c) spinning the spinner at right and the needle pointing to blue



(d) choosing an ace of hearts out of a normal pack of 52 playing cards

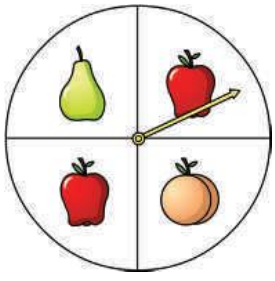
(e) choosing a red marble from a bag containing one red, one blue, one green and one orange marble

3 Find the probability of the following events occurring.

(a) rolling a normal die and getting a 3 or a 6

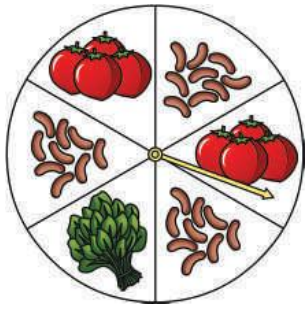
(b) rolling a normal die and getting an odd number

(c) spinning the following spinner and the needle pointing to apple



(d) spinning the following spinner and the needle pointing to a 4 or a 5

(e) spinning the following spinner and the needle pointing to tomatoes



(f) choosing a black card out of a normal pack of 52 playing cards

(g) choosing a spade out of a normal pack of 52 playing cards

(h) choosing a heart or a diamond out of a normal pack of 52 playing cards

(i) choosing a red marble out of a bag containing 2 red, 2 blue and 2 black marbles

(j) choosing a red marble out of a bag containing 2 red, 2 blue, 2 black and 2 green marbles

4 Find the probability of the following events occurring.

(a) rolling a normal die and not getting a 3 or a 6

(b) spinning the spinner in Question **3(d)** and the needle not pointing to 4 or 5

(c) choosing a card that is not a spade out of a normal pack of 52 playing cards

(d) choosing a marble that is not red out of a bag containing 2 red, 2 blue, 2 black and 2 green marbles

5 For the following spinner, work out the probability (Pr) of each event:

(a) Pr(4) **(b)** Pr(5)

(c) Pr(not 5) **(d)** Pr(6 or 7)

(e) Pr(1 or 2) **(f)** Pr(number less than 3)

(g) Pr(number ≥ 4) **(h)** Pr(number < 6).

6 For a packet of coloured jelly beans, it is known that:

Pr(green) = 0.2
Pr(red) = 0.3
Pr(purple) = 0.2
Pr(yellow) = 0.1
Pr(white) = 0.2

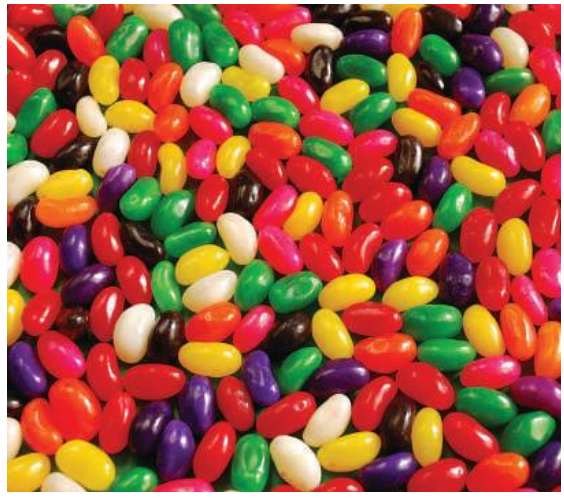
Find:

(a) Pr(red or purple)

(b) Pr(yellow or white)

(c) Pr(not yellow or white)

(d) Pr(not green or red).



7 You have a twelve-sided die, with faces numbered from 1 to 12.

(a) The probability of getting a multiple of 3 is:

A $\frac{1}{4}$ B $\frac{1}{3}$ C $\frac{1}{2}$ D $\frac{2}{3}$

(b) The probability of getting a prime number is:

A $\frac{5}{12}$ B $\frac{7}{12}$ C $\frac{2}{3}$ D $\frac{3}{4}$

(c) The probability of getting a multiple of 4 is:

A $\frac{1}{4}$ B $\frac{1}{3}$ C $\frac{1}{2}$ D $\frac{2}{3}$

(d) The probability of getting a number from 2 to 5 inclusive is:

A $\frac{1}{4}$ B $\frac{1}{3}$ C $\frac{1}{2}$ D $\frac{2}{3}$

(e) The probability of getting a perfect square number is:

A $\frac{1}{6}$ B $\frac{1}{5}$ C $\frac{1}{4}$ D $\frac{1}{3}$

Understanding

8 Draw and label a spinner that would produce the probabilities:

$\Pr(\text{blue}) = \frac{1}{6}$, $\Pr(\text{red}) = \frac{1}{2}$, $\Pr(\text{green}) = \frac{1}{3}$

9 For a six-sided die, show how you would colour the faces so that:

(a) $\Pr(\text{white}) = \frac{1}{2}$, $\Pr(\text{blue}) = \frac{1}{3}$, $\Pr(\text{green}) = \frac{1}{6}$

(b) $\Pr(\text{grey}) = \Pr(\text{orange}) = \frac{1}{3}$, $\Pr(\text{purple}) = \Pr(\text{silver}) = \frac{1}{6}$

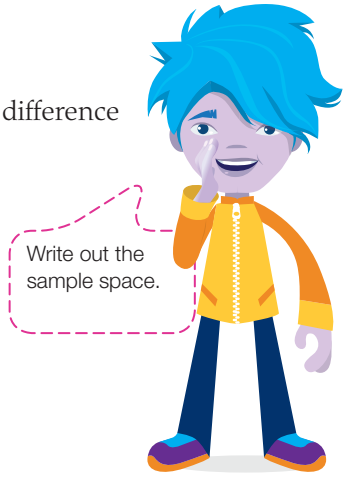
10 You are to set up a bag of at least twenty coloured counters. Show how you could do this so that:

(a) $\Pr(\text{yellow}) = 0.2$, $\Pr(\text{blue}) = 0.4$, $\Pr(\text{red}) = 0.3$, $\Pr(\text{black}) = 0.1$

(b) $\Pr(\text{yellow}) = \Pr(\text{pink}) = 0.25$, $\Pr(\text{purple}) = 0.15$, $\Pr(\text{green}) = 0.05$, $\Pr(\text{black}) = \Pr(\text{blue}) = \Pr(\text{white}) = 0.1$

Reasoning

11 Two normal dice are rolled. Find the probability that the difference between the two numbers is 2.



12 A Scrabble set consists of 100 pieces, 98 of which have a letter of the alphabet written on them, and two that are blank.

The distribution of letters is as follows.

A 9	B 2	C 2	D 4	E 12	F 2
G 3	H 2	I 9	J 1	K 1	L 4
M 2	N 6	O 8	P 2	Q 1	R 6
S 4	T 6	U 4	V 2	W 2	X 1
Y 2	Z 1	Blank 2			

(a) Why do you think the numbers of letters differ so much?

(b) Find the probability that the first piece selected is:

(i) a blank tile **(ii)** the letter O **(iii)** the letter R **(iv)** a vowel.

(c) You are selecting the first seven pieces, and you select in order E A E I E U O.

Find the probability of choosing:

(i) an E first **(ii)** then an A second

(iii) then another E third **(iv)** then an I fourth.

Open-ended

13 Draw and label a spinner that has at least four colours where the probability of one colour is at least twice the probability of one of the other colours.

14 You have a bag of 40 counters with 4 colours and at least 7 of each colour. Work out the possible contents of the bag if the probability of any particular colour is no greater than $\frac{1}{3}$.

Outside the Square — Problem solving

Random walks

Emily is playing a game in which she stands in the middle of a strip of rectangles (as on a footpath) and moves one rectangle according to the toss of a coin. This is called a random walk.

In each of the problems that follow, your task is to predict the number of tosses necessary to end the game.

Emily decides that a head means walk right one rectangle and a tail means walk left one rectangle.

1 Emily is standing in the middle rectangle of a strip of three rectangles. Emily keeps tossing the coin and walking according to the instructions until she leaves the strip of rectangles.

Play the game ten times. What was the mean number of tosses required before the walk ended?

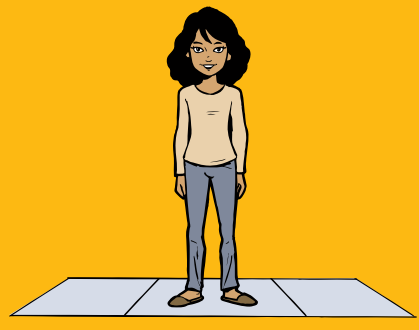
2 Emily is now standing in the middle rectangle of a strip of five rectangles. The rules of her walk are the same as before.

What is the potential problem this time? How many rectangles do you think there would need to be before it was too difficult to predict the number of tosses required?

Strategy options

- Make a table.
- Act it out.

You could simulate the tossing of the coin by using your calculator to produce random numbers—an even number could be heads and an odd number could be tails, for instance.



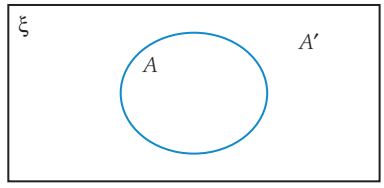
Venn diagrams and two-way tables

9.7

Sometimes, we want to know the probabilities involved when two different events, such as tossing a coin and rolling a die, occur within the same experiment. We might want to know the probability of both events occurring, of one or the other event occurring, of one event occurring given that the other event has occurred or the probability of neither event occurring. **Venn diagrams** or **two-way tables** help us to calculate these probabilities.

Venn diagrams

- A Venn diagram is usually represented by a rectangle that contains all the possible outcomes, the **universal set**, ξ (ξ is the Greek letter, epsilon).
- The outcomes of an event are contained in a circle with a letter name, such as A.
- A', the complement of A, lies outside the circle. This is the equivalent to saying '*not A*'.

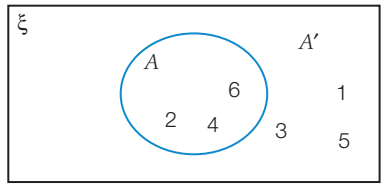


Consider rolling a die.

Then, the universal set $\xi = \{1, 2, 3, 4, 5, 6\}$ and all the outcomes are written inside the rectangle.

If A represents the outcomes of rolling an even number, then $A = \{2, 4, 6\}$ and these outcomes are contained in a circle labelled A.

A', the *complement* of A, would be rolling an odd number. (This is the same as saying not rolling an even number.) $A' = \{1, 3, 5\}$ and these outcomes would be outside circle A.



Finding probabilities with Venn diagrams

In probability, we are interested in the number of outcomes for an event. We show this in a Venn diagram and write it as $n(event)$; for example, $n(A)$.

$$\Pr(\text{event}) = \frac{\text{number of successful outcome}}{\text{total number of outcomes}}$$

$$\Pr(A) = \frac{\text{number of outcomes in set } A}{\text{number of outcomes in the universal set}}$$

$$= \frac{n(A)}{n(\xi)}$$

If we are counting the even numbers obtained when rolling a die, $n(A) = 3$.

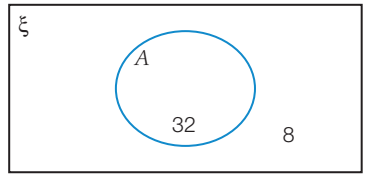
$$\Pr(A) = \frac{n(A)}{n(\xi)}$$

$$= \frac{3}{6}$$

$$= \frac{1}{2}$$

Worked Example 15 WE15

For this Venn diagram that shows the number of people in a group, write down:



(a) $n(A)$ **(b)** $n(\text{not } A)$ **(c)** $n(\xi)$

If a person is chosen at random, find:

(d) $\Pr(A)$ **(e)** $\Pr(\text{not } A)$

Thinking	Working
(a) Write the number inside the circle as the number in the required set (A).	(a) $n(A) = 32$
(b) Write the number outside the circle as the number in the complement of the required set (not A).	(b) $n(\text{not } A) = 8$
(c) Write the sum of these numbers as the number in the universal set (ξ).	(c) $n(\xi) = n(A) + n(\text{not } A)$ $= 32 + 8$ $= 40$
(d) Write the probability using: $\Pr(\text{event}) = \dfrac{n(\text{event})}{n(\xi)}$	(d) $\Pr(A) = \dfrac{n(A)}{n(\xi)}$ $= \dfrac{32}{40}$ $= \dfrac{4}{5}$
(e) Write the probability using: Pr(complement of the event) $= \dfrac{n(\text{complement})}{n(\xi)}$ or you can use $\Pr(\text{not } A) = 1 - \Pr(A)$	(e) $\Pr(\text{not } A) = \dfrac{n(\text{not } A)}{n(\xi)}$ or $\Pr(\text{not } A) = 1 - \Pr(A)$ $= \dfrac{8}{40}$ $\quad = 1 - \dfrac{4}{5}$ $= \dfrac{1}{5}$ $\quad = \dfrac{1}{5}$

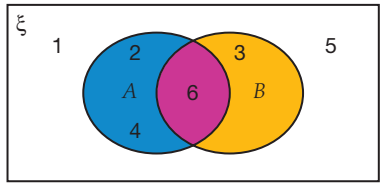
Unions and intersections

We may often want to represent two different events in a Venn diagram. The outcomes of the second event can be contained in another circle labelled B, for example.

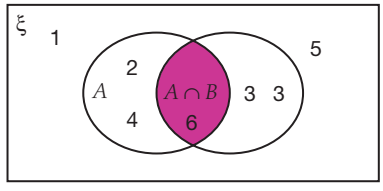
If we again consider rolling a die, where $\xi = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 4, 6\}$, and $B = \{\text{multiples of } 3\}$.

A and B will overlap as they both contain the number 6.

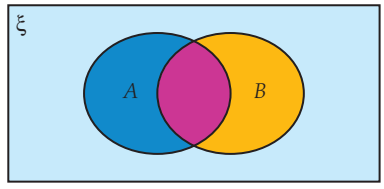
The **union** of A and B includes all the outcomes in A *or* B. A or $B = \{2, 3, 4, 6\}$



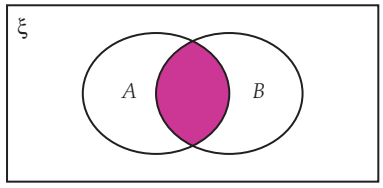
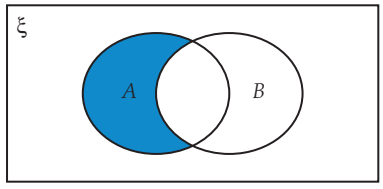
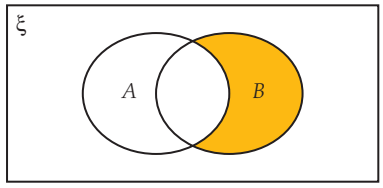
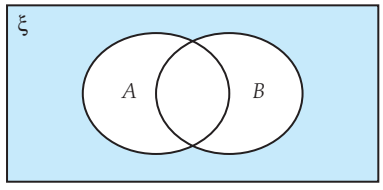
The **intersection** of A and B includes the outcomes that are in both A *and* B. A and $B = \{6\}$



Consider the following diagram.



There are four distinct areas in this diagram.

1 The area enclosed by the overlap of A and B: • is known as the intersection of A *and* B • represents the outcomes in which A *and* B both occur.	
2 The area enclosed by the remainder of A: • contains the outcomes in A not in the overlap with B • represents the outcomes in which A occurs but B does not.	
3 The area enclosed by the remainder of B: • contains the outcomes in B not in the overlap with A • represents the outcomes in which B occurs but A does not.	
4 The area outside both circles: • contains the outcomes in the universal set not in either A or B • represents the outcomes in which neither A nor B occurs.	

Summary of probability notation

ξ: universal set, all possible outcomes.

Intersection of sets A and B. A and B contains outcomes that are common to A and B.

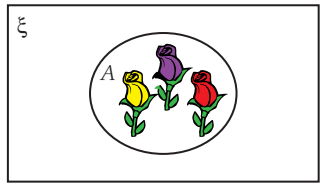
Union of sets A and B. A or B contains outcomes that are in A or B (some outcomes belong to both A and B.

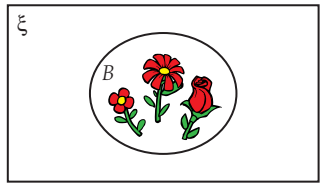
A': complement of A; also written as 'not A'

$n(\xi)$: number of elements in the universal set

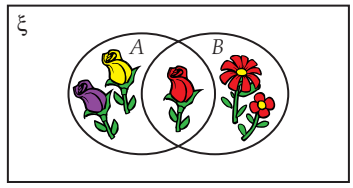
$$\Pr(A) = \frac{n(A)}{n(\xi)}$$

If ξ = {flowers}, A = {roses}, and B = {red flowers}, then

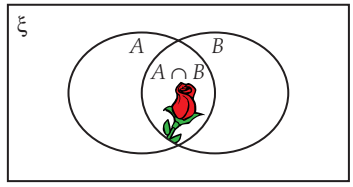




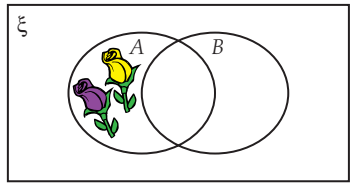
A or B = {roses of any colour *or* any red flowers}



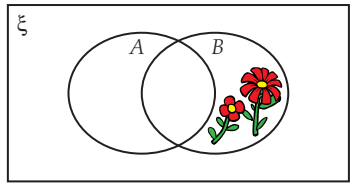
A and B = {flowers that are roses *and* that are red}
= {red roses}



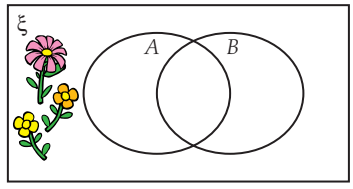
A and not B = {flowers that are roses *and* are not red}
= {roses of any colour except red}



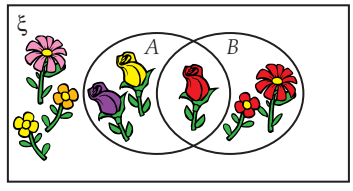
not A and B = {flowers that are not roses *and* that are red}
= {red flowers that are not roses}



not A and not B = {flowers that are not roses *and* are not red}
= {any flower that is not red or a rose}



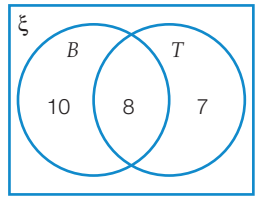
ξ = the universal set
= A and B + A and not B + not A and B + not A and not B



Worked Example 16

WE16

In a Year 8 class of 25 students, all of whom enjoy watching either basketball or tennis or both basketball and tennis, 18 enjoy watching basketball and 15 enjoy watching tennis. This information is shown in the Venn diagram at right. If a student is chosen at random, find the probability that they:



(a) enjoy watching both basketball and tennis

(b) only enjoy watching tennis.

Thinking	Working
1 Define the sets you will be using.	Let $B = \{\text{students who enjoy watching basketball}\}$ $T = \{\text{students who enjoy watching tennis}\}$ $\xi = \{\text{Year 8 class}\}$
2 Write the number in each of the sets defined, including the universal set and the intersection.	$n(B) = 18$, $n(T) = 15$, $n(\xi) = 25$, $n(B \text{ and } T) = 8$
(a) Find the probability of the intersection by dividing the number in the intersection by the total number.	(a) Pr(student enjoys basketball and tennis) $= \Pr(B \text{ and } T)$ $= \dfrac{n(B \text{ and } T)}{n(\xi)}$ $= \dfrac{8}{25}$
(b) Find the probability of the set required by dividing the number in the set by the number in the universal set.	(b) Pr(student enjoys tennis only) $= \Pr(T \text{ only})$ $= \dfrac{n(T \text{ only})}{n(\xi)}$ $= \dfrac{7}{25}$

Two-way tables

We can use a two-way table to help us work out the probability of any outcome occurring from separate events. For example, all the outcomes resulting from rolling two dice can be listed on a two-way table as shown below.

		Die 1					
		1	2	3	4	5	6
Die 2	6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)
	5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
	4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
	3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
	2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
	1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)

There are a total of 36 different possible outcomes, so $n(\xi) = 36$.

We can find probabilities by looking at the table and counting the successful outcomes.

To find Pr(rolling at least one 6), we count the number of outcomes that contain at least one 6 and divide by $n(\xi)$. Therefore, Pr(rolling at least one 6) $= \dfrac{11}{36}$.

Worked Example 17 WE17

A coin is tossed and a die is rolled. Use a two-way table to list all the possible outcomes.

From the table, determine the probability of obtaining:

(a) a head

(b) a head and a multiple of 2

(c) a tail and a number greater than 4.

Thinking	Working
(a) 1 Construct a two-way table.	(a)

COIN

DIE	H	T
6	(H, 6)	(T, 6)
5	(H, 5)	(T, 5)
4	(H, 4)	(T, 4)
3	(H, 3)	(T, 3)
2	(H, 2)	(T, 2)
1	(H, 1)	(T, 1)

Thinking	Working
2 Count the total number of outcomes.	$n(\xi) = 12$
Count the number of successful outcomes.	$n(\text{H}) = 6$
Write the probability.	$\Pr(\text{H}) = \frac{6}{12} = \frac{1}{2}$
(b) Count the number of successful outcomes using the table in **(a)**. Write the probability.	(b) $n(\text{H and a multiple of 2}) = 3$ $\Pr(\text{H and a multiple of 2}) = \frac{3}{12} = \frac{1}{4}$
(c) Count the number of successful outcomes using the table in **(a)**. Write the probability.	(c) $n(\text{T and a number greater than 4}) = 2$ $\Pr(\text{T and a number greater than 4}) = \frac{2}{12} = \frac{1}{6}$

Restricting the universal set

Sometimes, when we are doing a probability calculation, we are given some additional information that reduces the size of the universal set. If we see the word '*given*', this tells us that one event has already happened. Our universal set is now much smaller, as we only need to count successes for the second event using the first event as the universal set.

A flower is selected from a bunch of 20 flowers that are either red or roses or both.

If there are 10 red flowers and 12 roses:

ξ = {bunch of flowers} and $n(\xi) = 20$
A = {red flowers} and $n(A) = 10$
B = {roses} and $n(B) = 12$

A and B = {red roses} and
$n(A \text{ and } B) = n(A) + n(B) - n(\xi)$
$= 10 + 12 - 20$
$= 2$

$$\Pr(\text{a selected flower is a red rose}) = \frac{2}{20} = \frac{1}{10}$$

However, if it is given that the selected flower is red, we are now not selecting a rose out of all 20 flowers but a rose only from the 10 red flowers.

$$\Pr(\text{selected flower is a rose given that it is red}) = \frac{2}{10} = \frac{1}{5}$$

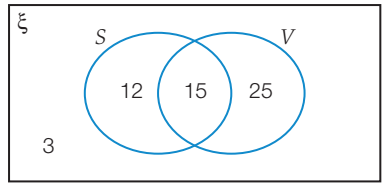
If it is given that the selected flower is a rose, we are selecting a red flower only out of the 12 roses.

$$\Pr(\text{selected flower is red given that it is a rose}) = \frac{2}{12} = \frac{1}{6}$$

Worked Example 18

WE18

Some students were asked their preferences regarding stamp collecting (S) and playing video games (V). The results are shown in the Venn diagram.



If one person is chosen at random from those surveyed, find:

(a) Pr(they like stamp collecting)

(b) Pr(they like stamp collecting but not video games)

(c) Pr(they don't like either of these two hobbies)

(d) Pr(they like stamp collecting given that they like video games)

(e) Pr(they like video games given that they like stamp collecting).

Thinking	Working
(a) Count the total number in the universal set ξ (all the students surveyed).	(a) $n(\xi) = 12 + 15 + 25 + 3$ $= 55$
Count the number in S (all the students who collect stamps).	$n(S) = 12 + 15$ $= 27$
Find the probability using: $\Pr(\text{event}) = \dfrac{\text{number of successful outcomes}}{\text{total number of outcomes}}$	$\Pr(S) = \dfrac{27}{55}$

(b)	State the total number in ξ. State the number in S but not V (all students who collect stamps but who do not like playing video games). Find the probability.	(b) $n(\xi) = 55$ $n(S \text{ and not } V) = 12$ $Pr(S \text{ and not } V) = \frac{12}{55}$
(c)	State the total number in ξ. State the number outside both S and V (students who don't like stamp collecting and who also don't like playing video games). Find the probability.	(c) $n(\xi) = 55$ $n(\text{not } S \text{ and not } V) = 3$ $Pr(\text{not } S \text{ and not } V) = \frac{3}{55}$
(d)	Count the total number in the restricted set V. State the number in the intersection of S and V (students who like stamp collecting who like playing video games). Find the probability using the restricted universal set as the denominator.	(d) $n(V) = 15 + 25$ $= 40$ $n(S \text{ and } V) = 15$ $Pr(S \text{ and } V \text{ given } V) = \frac{15}{40}$ $= \frac{3}{8}$
(e)	Count the total number in the restricted set (S). State the number in the intersection of S and V. Find the probability using the restricted universal set as the denominator.	(e) $n(S) = 12 + 15$ $= 27$ $n(S \text{ and } V) = 15$ $Pr(S \text{ and } V \text{ given } S) = \frac{15}{27}$ $= \frac{5}{9}$

Numerical information can be presented in two-way tables and these tables can assist us in calculating probabilities. Once again, we can restrict the universal set if it is 'given' that an event has already occurred.

Worked Example 19 WE19

A school has 280 girls and 320 boys. Of the 400 students who play school sport, 240 are boys.

(a) Construct a two-way table showing this information.

(b) Use the table to answer the following questions.

What is the probability that:

(i) a student selected at random is a girl?

(ii) a student selected at random plays school sport?

(iii) a girl who plays school sport is selected?

(iv) a girl is selected given that they play school sport?

(v) a student who does not play school sport is selected given that the student is a boy?

Thinking

Working

(a) 1 Set up a two-way table with one variable and its complement as the row headings (Plays school sport and Does not play school sport), and the other variable and its complement as the column headings (Boys and Girls).

2 Use the information given to fill in the table. (240 in Plays school sport row and Boys column). Subtract this number from 400 to find the number of girls who play school sport. Subtract 240 from 320 to find the number of boys who don't play school sport. Add the number of boys and girls to find the total number of students. Subtract 400 from this total to find the number of students who don't play school sport and finally fill in the last cell.)

(a)

	Boys	Girls	Total
Plays school sport	240	160	400
Does not play school sport	80	120	200
Total	320	280	600

(b) (i) Use the number of successful outcomes and the total number in ξ to find the probability (the number of girls and the total number of students).

(b) (i) $\Pr(\text{girl}) = \frac{280}{600}$

$= \frac{7}{15}$

(ii) Use the number of successful outcomes and the total number in ξ to find the probability (the number of students who play school sport and the total number of students).

(ii) $\Pr(\text{student who plays school sport}) = \frac{400}{600}$

$= \frac{2}{3}$

(iii) Use the number of successful outcomes and the total number in ξ to find the probability (the number of girls who play school sport and the total number of students).

(iii) $\Pr(\text{girl who plays school sport}) = \frac{160}{600}$

$= \frac{4}{15}$

(iv) Use the number of successful outcomes and the total number in the restricted universe to find the probability (the number of girls who play school sport and the total number of students who play school sport).	(iv) Pr(girl given they play shool sport) $= \frac{160}{400}$ $= \frac{2}{5}$
(v) Use the number of successful outcomes and the total number in the restricted universe to find the probability (the number of boys who don't play school sport and the total number of boys).	(v) Pr(does not play school sport given that the student is a boy) $= \frac{80}{320}$ $= \frac{1}{4}$

9.7 Venn diagrams and two-way tables

Navigator

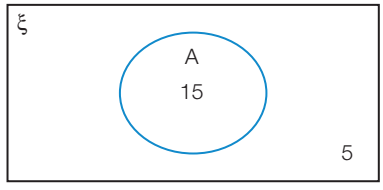
Answers page 651

Q1, Q2, Q3, Q6, Q7, Q8, Q9, Q11, Q13, Q17, Q18	Q1, Q2, Q3, Q6, Q7, Q8, Q9, Q11, Q13, Q14, Q15, Q17, Q18	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10, Q12, Q13, Q14, Q16, Q17, Q18

Fluency

WE15

1 For this Venn diagram that shows the number of people in a group, write down:



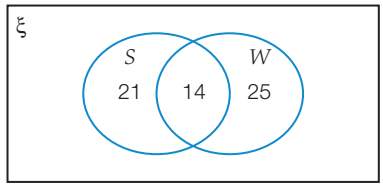
(a) $n(A)$ **(b)** $n(\text{not } A)$ **(c)** $n(\xi)$

If a person is chosen at random, find:

(d) $\Pr(A)$ **(e)** $\Pr(\text{not } A)$

WE16

2 In a survey of 60 music students at a school, all of whom play either a string instrument, a wind instrument or both, 35 play a string instrument and 39 play a wind instrument. This information is shown in the Venn diagram at right. If a student is chosen at random, find the probability that they:



(a) play both a string instrument and a wind instrument

(b) only play a wind instrument.

WE17

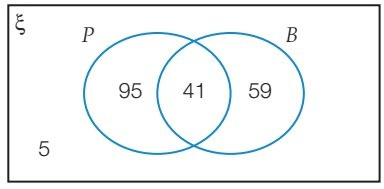
3 A coin is tossed and a die is rolled. Use a two-way table to list all the possible outcomes. From the table, determine the probability of obtaining:

(a) a tail

(b) a tail and a multiple of 3

(c) a head and a number less than 4.

4 A sample of 200 people was surveyed as to their access to pay TV (P) and broadband internet (B). The results are shown in the Venn diagram.



WE18

If one person is chosen at random from those surveyed, find:

(a) Pr(they have access to pay TV)

(b) Pr(they have access to broadband internet but not pay TV)

(c) Pr(they have access to neither of these technologies)

(d) Pr(they have access to pay TV given that they have access to broadband internet)

(e) Pr(they have access to broadband internet given that they have access to pay TV).

5 A mouse breeder has 18 white and 12 brown mice. Of the 14 males, 10 are white.

WE19

(a) Construct a two-way table showing this information.

(b) Use the table to answer the following questions.

What is the probability that:

(i) a mouse selected at random is white?

(ii) a mouse selected at random is male?

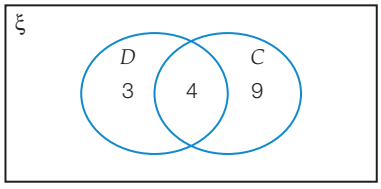
(iii) a mouse selected at random is a brown female?

(iv) a white mouse is selected given that it is a male?

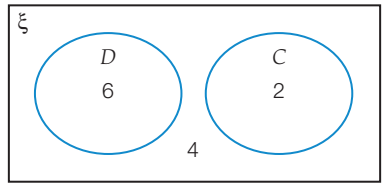
(v) a female mouse is selected given that it is brown?

6 Sally decided to survey some Year 8 students about whether they owned a dog, a cat, both or neither. She did this over 4 lunchtimes and recorded her results on the following Venn diagrams where D = {Year 8 students who own a dog} and C = {Year 8 students who own a cat}.

(a)



(b)



For each diagram:

(i) set up a two-way table showing numbers.

Hence, for each diagram, find:

(ii) Pr(D) **(iii)** Pr(D and C) **(iv)** Pr(D or C)

(v) Pr(C only) **(vi)** Pr(not D) **(vii)** Pr(not C)

7 A die is rolled. $\xi = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 4, 6\}$ and $B = \{1, 2\}$

(a) Draw a Venn diagram to represent this information.

(b) Hence, find:

(i) (A and B) **(ii)** (A or B) **(iii)** not A

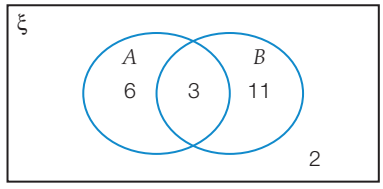
(iv) $n(A)$ **(v)** $n(B)$ **(vi)** $n(\xi)$

(vii) $n(A \text{ and } B)$ **(viii)** $n(A \text{ or } B)$ **(ix)** $n(\text{not } B)$

(c) If a number is chosen at random from ξ, find the probability that it came from:

(i) Pr(A) **(ii)** Pr(A and B) **(iii)** Pr(A or B)

8 This Venn diagram shows the number of elements in each of two sets A and B.



(a) $n(A)$ is:

A 3 **B** 6

C 9 **D** 22

(b) $n(B \text{ only})$ is:

A 3 **B** 9 **C** 11 **D** 14

(c) $n(A \text{ or } B)$ is:

A 3 **B** 17 **C** 20 **D** 22

(d) $n(A \text{ and } B)$ is:

A 2 **B** 3 **C** 9 **D** 20

(e) $n(\xi)$ is:

A 22 **B** 20 **C** 17 **D** 9

(f) $n(\text{not } B)$ is:

A 2 **B** 6 **C** 8 **D** 9

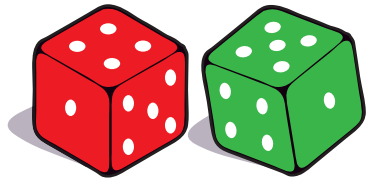
9 Two six-sided dice, one green and one red, are rolled. Construct a two-way table and use it to write down the outcomes that match the following descriptions.

(a) a 6 on either one of the dice

(b) a 6 on both dice

(c) a 6 on the green die

(d) a 6 on the green die but not the red die



10 Two dice, one green and one red, are rolled. Use your two-way table in Question **9** and the answers to Question **9** to find the probability of getting:

(a) a 6 on either one of the dice

(b) a 6 on both dice

(c) a 6 on the green die

(d) a 6 on the green die but not on the red die

(e) a 6 on the green die given that a 6 has been rolled on the red die.

Understanding

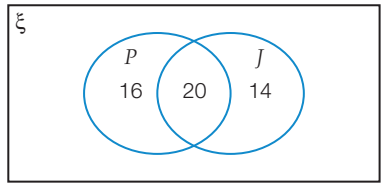
11 Two six-sided dice, one red and one green, are rolled. Find the probability of getting:

(a) a sum of 7

(b) a 3 on the green die

(c) even numbers on both dice

12 In a survey of 50 people, it was found that 36 people liked peanut butter and 34 people liked jam. Of all those surveyed, 20 said they liked both peanut butter and jam. This information is shown on a Venn diagram where P = {people who liked peanut butter} and J = {people who liked jam}. If one person is chosen at random, find:



(a) Pr(P) **(b)** Pr(P and J) **(c)** Pr(J)

(d) Pr(J only) **(e)** Pr(J given P) **(f)** Pr(P given J)

Reasoning

13 A shopkeeper found that, out of his first 20 customers, 13 bought milk and 15 bought a newspaper. Four customers bought neither milk nor a newspaper.

(a) Represent this information **(i)** in a two-way table and **(ii)** on a Venn diagram.

(b) How many customers bought both milk and a newspaper?

(c) Find the probability that a customer, chosen at random from the first 20:

(i) bought milk only **(ii)** bought milk and a newspaper

(iii) bought milk or a newspaper **(iv)** did not buy a newspaper.

14 A group of 40 Year 8 students were asked if they preferred reading books or magazines. Ten students said they liked reading both, seven said they liked reading books only, whereas two said they did not like reading either.

Find the probability that a student chosen at random from this group:

(a) liked reading magazines only

(b) liked reading books or magazines

(c) did not like reading magazines

(d) did not like reading books or magazines.

15 At a school camp, 50 Year 8 students were surveyed about whether they preferred pizza or fish and chips.

Ten students said they liked both and 4 students said they liked neither. The number of students who liked pizza was the same as the number who liked fish and chips. Find the probability that a student, selected at random:

(a) likes both pizza and fish and chips

(b) likes pizza only

(c) does not like fish and chips

16 In a Year 8 class of 25 students, 15 said they liked athletics, 17 said they liked swimming and 4 said they didn't like either. Find the probability that a student chosen at random:

(a) likes swimming

(b) likes athletics only

(c) likes swimming or athletics

(d) likes swimming given that they like athletics.

Open-ended

17 Survey a number of students about whether they travel to school by bus, train, both or neither. Construct a Venn diagram with this information and use it to find the probability that a student chosen at random travels by bus only.

18 Robbie was working out the probability of getting a 3 on at least one die when he tossed two dice. He decided that, because there were 6 outcomes in which a 3 could appear on one die and 6 more outcomes in which a 3 could appear on the second die, there were 12 successful outcomes out of a possible 36. He wrote:

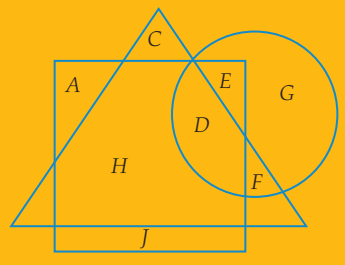
$$\Pr(3 \text{ on one die}) = \frac{12}{36}$$
$$= \frac{1}{3}$$

Show Robbie why he is wrong and find the correct answer.

Outside the Square — Problem solving

Reasoning diagram

All questions relate to the following diagram.



1 Which letter or letters appear in the intersection of all three shapes?

2 Which letter or letters appear in the triangle and the square but not in the circle?

3 Which letter or letters appear in the triangle only?

4 Which letter or letters appear in either the square or the triangle?

5 Which letter or letters appear in neither the circle nor the triangle?

6 How would you describe the location of *F*?

7 Make up another three similar questions based on the diagram.

Strategy options

- Act it out.
- Break problem into manageable parts.

Challenge 9

1 Georgina has green socks, red socks and blue socks that she keeps loose at the top of her wardrobe. She can't see the socks when she grabs them.

(a) How many socks does she need to select before she has a pair?

(b) How many more socks must she select to be sure she has a second pair?

2 Two children and one adult have to cross a river in a boat that can only hold one adult or two children at a time. How can they do this?

3 Three playing cards are placed in a row. The club is to the right of the heart and the diamond. The 5 is to the left of the heart. The eight is to the right of the 4. From left to right, the cards are:

A 5 of diamonds, 4 of hearts, 8 of clubs

B 5 of hearts, 4 of diamonds, 8 of clubs

C 8 of diamonds, 5 of clubs, 4 of hearts

D 4 of hearts, 8 of clubs, 5 of diamonds

4 **(a)** At a local fundraiser, a ring toss game is played. Three rings are tossed over any of three pegs. A ring over peg X is worth *one* point, over peg Y *three* points and over peg Z *five* points. If all three rings land on pegs, how many different point totals are possible?

A 6 **B** 7 **C** 8 **D** 9

(b) Which total(s) were the easiest to get?

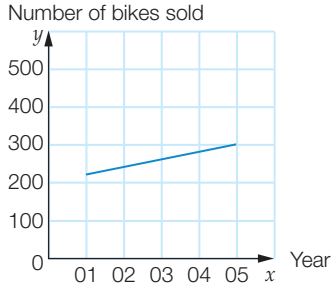
(c) How is it possible to get an even total?

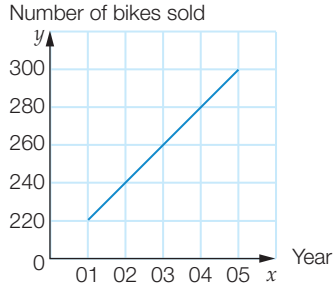
5 For the set of numbers 3, 4, 5, 8, x, the mean, median and the mode are all the same number.

(a) Considering the mode is known, what are the only possible values of x?

(b) What is the value of x?

6 Margaret claims the two graphs represent the same information. Do you agree? Give reasons for your answer.





Chapter review 9

D.I.Y. Summary

Key Words

bias	intersection	outcome	sample space
census	mean	outlier	sample statistics
class centre	measures of centre	poll	two-way table
class intervals	median	population	union
complement	median class interval	population statistics	universal set
frequency column graphs	midpoint	random sampling	Venn diagram
grouped data	modal class interval	range	
histogram	mode	sample	

Copy and complete the following using the words and phrases from this list, where appropriate, to write a summary for this chapter. A word or phrase may be used more than once.

1 When we are dealing with a large data set we usually group the data into _____.

2 To find the _____ of a class interval you add together the two end points and divide by two.

3 The difference between the largest value and the smallest value is called the _____.

4 The _____ is the list of outcomes possible in a mathematical experiment.

5 When we find the elements in common between two sets we are finding the _____.

6 Statistical information for two events that could be presented in a Venn diagram can also be presented in a _____.

7 The set that contains all the elements in the universal set not included in A is called the _____ of A.

8 A data value that is quite different from other values is called an _____.

9 When data is collected from all members of a population it is called a _____.

Fluency

1 A sample of 50 parrots are captured, tagged and released. Some time later, a second sample of 75 parrots are captured, of which 10 are found to be tagged. Use these values to estimate the parrot population in the area. Ex. 9.1

2 For each of the following data sets, calculate the: Ex. 9.2

(i) mean (correct to two decimal places)

(ii) median

(iii) range

(a) 12 23 23 25 26 27 28 29 29 31 32 33 35 35 36 38

(b) 1 23 23 25 26 27 28 29 29 31 32 33 35 35 36 38

(c) Comment on any similarities and differences you found.

3 Six Year 8 students have a worm farm. The following data shows the number of worms collected in each of the six separate worm farms over the same time period. Ex. 9.3

Worm farm	Worms collected
1	1044
2	986
3	569
4	1012
5	758
6	999

Select a suitable scale and draw a frequency column graph of the results.

4 For the following data, select a suitable size for the class interval, record the results in a frequency table, and draw a frequency column graph. Ex. 9.3

4, 6, 10, 5, 4, 8, 12, 14, 18, 6, 4, 6, 13, 2, 16, 8, 6, 4, 10, 14, 6, 4, 7, 8, 9, 12

5 The data in the following frequency table gives the weight, in grams, of fish of a particular species caught one day by a commerical fisherman. Ex. 9.4

Weight (g)	Frequency
325–<350	5
350–<375	7
375–<400	11
400–<425	24
425–<450	31
450–<475	29

(a) Calculate the mean weight of the fish correct to two decimal places.

(b) Calculate the median class interval for the weights of the fish.

(c) Calculate the modal class interval for the weights of the fish.

(d) Draw a histogram of the data.

6 A 10-sided die with the numbers 1 to 10 is rolled once. Find the probability of rolling: Ex. 9.5

(a) a five

(b) a six or a seven or an eight

(c) an eleven

(d) a prime number

(e) a number greater than 3

(f) a number that is not a multiple of 3.

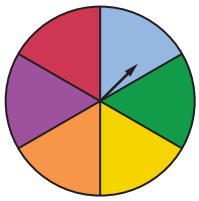
7 Find, after one spin of the spinner shown: Ex. 9.6

(a) Pr(blue)

(b) Pr(red)

(c) Pr(blue or red)

(d) Pr(not red).



8 When rolling a normal die,

Ex 9.6

(a) Pr(rolling a 5) is:

A $\frac{1}{6}$ **B** $\frac{1}{5}$ **C** $\frac{5}{6}$ **D** 1

(b) Pr(rolling a 4 or 6) is:

A $\frac{1}{3}$ **B** $\frac{1}{2}$ **C** $\frac{4}{5}$ **D** 1

(c) Pr(rolling a number that is not 4 or 6) is:

A $\frac{1}{4}$ **B** $\frac{1}{3}$ **C** $\frac{1}{2}$ **D** $\frac{2}{3}$

9 Draw and label a spinner that would produce the following probabilities.

Ex 9.6

$\Pr(\text{red}) = \frac{1}{3}$ $\Pr(\text{blue}) = \frac{1}{2}$ $\Pr(\text{yellow}) = \frac{1}{6}$

10 $\xi = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 3, 5, 7\}$ and $B = \{1, 2\}$

Ex 9.7

(a) Draw a Venn diagram to represent this information.

(b) Hence, find:

(i) A or B **(ii)** A and B **(iii)** not A

(c) Find:

(i) $\Pr(A \text{ or } B)$ **(ii)** $\Pr(A \text{ and } B)$ **(iii)** $\Pr(\text{not } A)$

Understanding

11 The Australian cricket team toured New Zealand in March 2010 and played two test matches. The following is a list of the scores made by each individual New Zealand batsman during the first innings of the first test match.

4 46 11 138 4 15 5 7 22 7 0

(a) Calculate the mean score correct to two decimal places.

(b) Calculate the median score.

(c) Calculate the range.

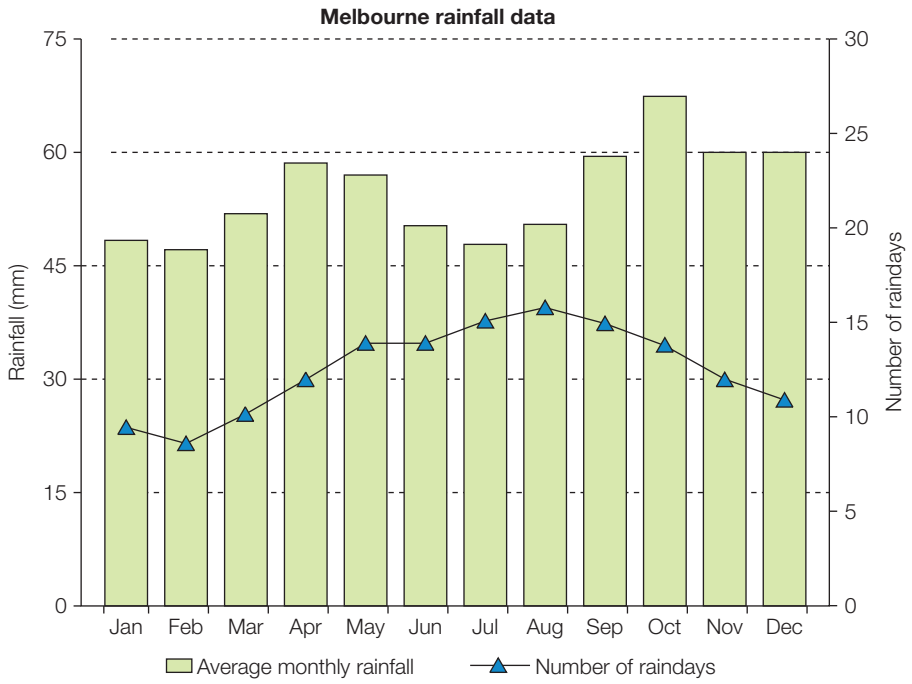
(d) Is there a possible outlier?

In the second innings of the same test match the New Zealand batsmen scored:

24 19 29 22 58 22 51 3 45 0 5

(e) Describe these scores using the mean, median and range as you did for the first innings.

12 The following graph shows rainfall data for the Melbourne weather station.



Bureau of Meteorology

(a) Why does the graph have two different vertical scales marked on it?

(b) Which month records the highest mean rainfall?

(c) Which month has the most rainy days?

(d) **(i)** Which month would you say produces, on average, the most rain on the days when it actually rains?

(ii) Which month would you say produces, on average, the least rain on the days when it actually rains?

13 A motor company has launched a new car, the Merlin. The car is available as either a sedan, a station wagon, a coupe or a ute. It is available in white, black, red, yellow or silver.

(a) How many different variations of the Merlin are available?

(b) If Anissa buys a Merlin at random, find:

(i) Pr(it is a yellow coupe)

(ii) Pr(it is not black)

(iii) Pr(it is red or a ute or both).

14 In a survey of 60 people it was found that 40 people liked lemonade and 30 people liked cola. Two people said they disliked both.

(a) Present this information as a Venn diagram.

(b) If one person is chosen at random, find the probability that this person:

(i) likes lemonade

(ii) likes lemonade only

(iii) likes cola

(iv) like both lemonade and cola

(v) does not like lemonade

(vi) likes lemonade or cola.

15 For a ten-sided die, describe how you would colour the faces so that:

(a) Pr(blue) = Pr(green) = 2 × Pr(red) where red, green and blue are the only available colours

(b) Pr(blue) = 5 × Pr(black) where blue, black and pink are the only available colours.

16 A string orchestra of 20 musicians has 12 members who play the violin. There are 16 members who are right-handed and 2 left-handers play the violin. If a member of the orchestra is chosen at random, find the probability that he or she:

(a) is right-handed and plays the violin

(b) is left-handed and doesn't play the violin

Reasoning

17 The following data values are the height (in cm) of the 46 players on the list of the Fremantle Dockers at the beginning of the 2010 AFL season.

174	191	176	182	188	185	189	180	190
182	186	181	186	189	190	192	194	193
186	193	194	188	191	191	199	184	193
192	203	211	184	188	181	180	194	177
187	182	184	191	191	192	189	194	184
186								

Using the raw data calculate the:

(a) mean height, correct to two decimal places

(b) median height

(c) modal height.

(d) Which of these do you think is the best measure of centre for height?

(e) Do you think any of the values represents a possible outlier? If so, which one(s). If not, what would the height need to be for you to consider it a possible outlier?

(f) Group the data in intervals beginning 171–175, 176–180 etc. Use the table to find an estimate for the mean height to the nearest cm.

(g) It should have been easy to do the calculation from the table, but was there an unacceptable loss of accuracy? Explain in some detail your thoughts on this matter.

NAPLAN practice 9

Numeracy: Non-calculator

Consider the following data set: 2, 3, 4, 2, 4, 6, 1, 5, 2, 9, 7

1 The median is:

A 2 **B** 4 **C** 4.5 **D** 6

2 The range is:

A 2 **B** 5 **C** 8 **D** 11

3 The mode is:

A 2 **B** 4 **C** 5 **D** 8

4 At an athletics meet, Julie recorded the heights jumped by 30 students in the high jump event. They are recorded below.

15_L	1 2 2 2 4 4
15_U	5 8 9 9
16_L	0 0 1 2 2 4 4
16_U	5 5 5 6 7 7 8 8 9 9
17_L	0 1 2

Key 17/2 means 172

What is the median height jumped?

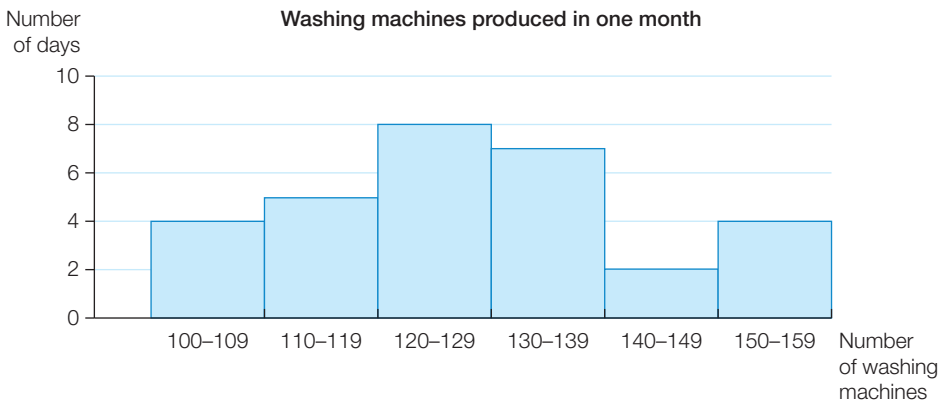
A 163 cm **B** 164 cm **C** 165 cm **D** 166 cm

Numeracy: Calculator allowed

5 Find an estimate for the mean of the following data set. Give your answer correct to two decimal places.

Score	Frequency
1–<2	4
2–<3	7
3–<4	11
4–<5	6
5–<6	9
6–<7	2

6 This graph shows data on how many washing machines a factory produces in one month.

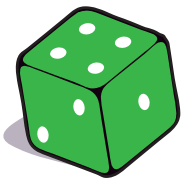


On how many days did the factory produce more than 129 washing machines?

A 8 **B** 9 **C** 13 **D** 21

7 A die has 6 faces that are numbered from 1 to 6.
Nicky throws the die 1000 times to determine whether it is fair.

Number on top face	1	2	3	4	5	6
Frequency	150	166	194	148	160	182



What is the probability of rolling a 5 based on Nicky's results?

A $\frac{1}{160}$ **B** $\frac{4}{25}$ **C** $\frac{1}{6}$ **D** $\frac{16}{25}$

Mixed review D

Fluency

1 Change the following percentages to fractions in simplest form. Ex. 2.6

(a) $16\frac{3}{8}\%$ (b) 31.55%

2 A card is drawn from a standard deck. What is the probability of: Ex. 9.5

(a) not getting the 2 of hearts (b) not getting a black card

(c) not getting a queen (d) not getting a diamond?

3 Evaluate each of the following expressions when $a = -2$ and $b = 5$. Ex. 3.2

(a) $ab + a$ (b) $6b - \frac{80}{ab}$ (c) $5(a - b) + 7b$

4 Change the following fractions to percentages. Ex. 2.5

(a) $\frac{31}{60}$ (b) $1\frac{5}{6}$

5 Increase 850 by the following percentages and give your answers to two decimal places. Ex. 2.9

(a) 13% (b) 8% (c) 56.5%

6 Solve each of the following equations. Ex. 7.2

(a) $5g - 6 = 4$ (b) $3(m + 7) = 27$ (c) $\frac{s}{9} + 2 = 7$

7 On a particular day one Australian dollar was worth \$0.55 US. Ex. 6.4

(a) Construct a set of axes showing \$A on the horizontal axis and \$US on the vertical axis. Extend the horizontal axis to \$100 and the vertical axis to \$60.

(b) \$0 Australian must always be worth \$0 US. What is this as a coordinate pair?

(c) How much would \$A10 be worth? Write this as a coordinate pair.

(d) Plot the two coordinate pairs on the graph and draw a straight line to connect them, extending the line to fit the axes.

(e) Use the graph to find how much \$A45 is worth in \$US.

(f) Use the graph to find how much \$US50 is worth in \$A.

8 Find the probability of each of the following events occurring. Ex. 9.5

(a) rolling a normal die and getting an odd number

(b) choosing a spade out of a normal pack of 52 playing cards

(c) choosing a red or white marble out of a bag containing 3 red, 5 white, 2 green and 6 orange marbles

9 Joanna and Stephen went bushwalking and travelled approximately 7.5 km in the first 3 hours. Ex. 4.7

(a) What was their average speed in the first 3 hours?

(b) If they continued at this speed for 4.5 more hours, what distance would they have travelled in the day?

10 Simplify the following. Ex 1.5, 1.6

(a) $\dfrac{2^4 \times 3^5}{(2 \times 3)^3}$ **(b)** $\left(\dfrac{2^2}{3}\right)^5$

11 Simplify the following. Ex 1.5, 1.6

(a) $(4^2)^3 \times 5^0$ **(b)** $\dfrac{(6 \times 8)^3}{6^2 \times 8}$

12 Simplify the following expressions by expanding the brackets and collecting like terms. Ex 3.6

(a) $5(2d + e) - 7d$ **(b)** $4j(3k + 2) + 5(jk - 7)$

13 Draw a frequency column graph or a histogram for each of the following data sets. Ex 9.3

(a)

Score	Frequency
1	5
2	7
3	8
4	6
5	2

(b)

Score	Frequency
15–<20	3
20–<25	5
25–<30	8
30–<35	2
35–<40	1

14 Solve for x. Ex 7.3

(a) $\dfrac{3x + 2}{5} = -11$ **(b)** $8(3x - 2) = 32$

15 A normal die is rolled. Ex 9.6

(a) List all possible outcomes.

(b) Find:

(i) Pr(4) **(ii)** Pr(12) **(iii)** Pr(2 or 5) **(iv)** Pr(0)

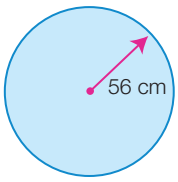
16 If $A = \frac{1}{2}(a + b)h$, find: Ex 3.2

(a) A when $a = 2$, $b = 6$ and $h = 10$ **(b)** h when $A = 40$, $a = 2$ and $b = 8$.

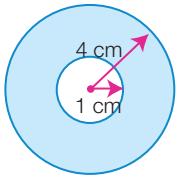
17 Show that these shapes are congruent and state the congruence test used. Ex 8.4

18 Find the shaded area of the following shapes correct to two decimal places. Ex 5.5, 5.6

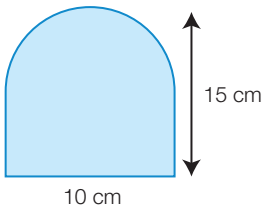
(a)



(b)



(c)



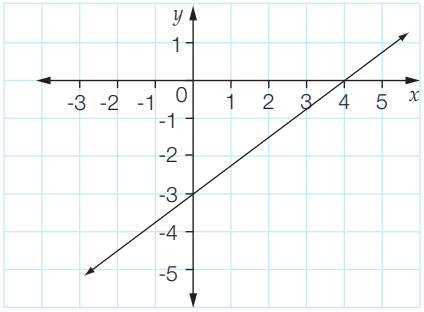
19 Solve for x. Ex 7.4

(a) $8x + 5 = 2x - 13$ **(b)** $\dfrac{x}{4} + 1 = \dfrac{x}{2} - 1$

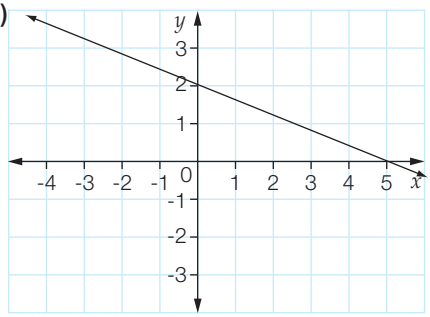
20 Find the rule that describes each of the following relationships.

Ex 6.3

(a)



(b)



Understanding

21 You have a bag of twenty coloured counters. Given that Pr(yellow) = 0.2, Pr(green) = 0.1, Pr(red) = 0.4 and Pr(brown) = 0.3, how many of each coloured counter do you have?

22 A car is travelling at an average speed of 90 km/h.

(a) Write an equation showing the relationship between distance travelled, d km, and time, t h.

(b) Draw a graph showing three hours of travel for the car.

(c) Use your graph to estimate the distance travelled in:

(i) 45 minutes **(ii)** 2 hours 20 minutes.

(d) Use your graph to estimate the time it takes to travel:

(i) 70 km **(ii)** 250 km.

23 Write equations to help you solve the following.

(a) Six less than a third of a number is 27. What is the number?

(b) A number is doubled, then divided by six and finally has ten added to it. The result is the number first thought of. Find the number.

24 Find the area of an annulus that has an inner diameter of 6 cm and an outer diameter of 13 cm. Give your answer correct to one decimal place.

25 In a sample of the staff at a particular firm, 78% said they were happy with the treatment they receive from their manager.

(a) If the company has 500 staff, approximately how many would you expect to be happy with the treatment they receive?

(b) What would you think if a census of the population revealed that 405 staff were happy with the treatment they received?

26 Michelle's pay is increased by 40%. If her fortnightly pay was $1480 what is her income now?

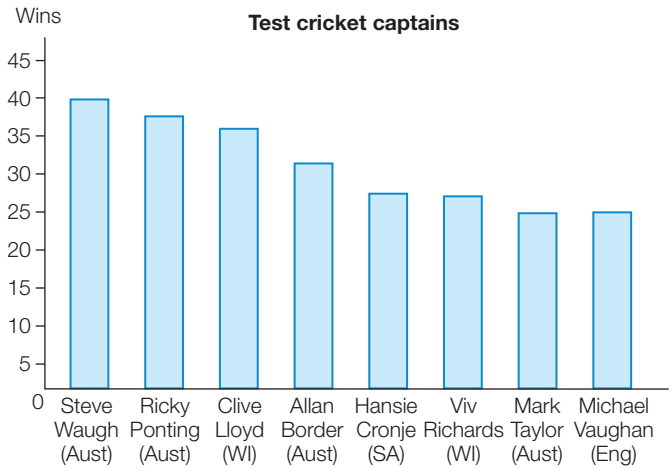
27 Billy slept for 8.4 hours, watched TV for 2.6 hours, was at school for 8.75 hours, played sport for 2.3 hours and spent the rest with his family. Write each of these as a percentage of a day, to the nearest whole number.

28 Twenty counters labelled 11 to 30 are placed in a container. If a counter is drawn at random, find:

(a) Pr(not 15) **(b)** Pr(not divisible by 5) **(c)** Pr(not less than 17).

Reasoning

29 The graph shows the record of the Test cricket captains who have won the most tests. The information was correct for the beginning of April 2009.



(a) Who appears to be the most successful captain?

(b) What other numerical information would be required to calculate the percentage of games won as captain?

Steve Waugh captained his team in 57 tests and the other values from left to right are 56, 74, 93, 53, 50, 50, 51.

(c) Use these values to calculate the percentage, to the nearest whole number, of games won as captain.

(d) Redraw the graph showing the winning percentage instead of the number of games won.

(e) Who is really the most successful captain?

30 The cost of hiring a bicycle is \$10 plus \$5 per half-hour.

(a) Copy and complete the table of costs for up to 8 hours of hire.

Time (*h*)	0	2	4	6	8
Cost (\$)					

(b) Draw a graph of this relationship.

(c) What does the y-intercept represent?

(d) A bike equivalent to the type hired costs \$350. For how many hours would you need to hire a bike to reach this purchase cost?

Answers

Worked solutions and answers to ***all*** activities appear in the Teacher Companion.

Chapter 1

Recall 1 (p. 4)

1 **(a)** **(i)** 9°C **(ii)** 15°C **(iii)** 6°C
(b) **(i)** 3rd **(ii)** 1st

2 **(a)** 8 **(b)** 81 **(c)** 25 **(d)** 1 000 000
(e) 6 **(f)** 7 **(g)** 4 **(h)** 2

3 **(a)** +3 **(b)** -9

4 **(a)** 16 **(b)** 63 **(c)** 228 **(d)** 285
(e) 6 **(f)** 6 **(g)** 108 **(h)** 154

5 **(a)** 9 **(b)** 54 **(c)** 43

6 **(a)** $2^3 \times 3$ **(b)** $2^2 \times 3^2$ **(c)** $2^3 \times 3 \times 5$

7 **(a)** 25 **(b)** 29 **(c)** 28 **(d)** 72
(e) 400 **(f)** 945 **(g)** 9 **(h)** 14

Exercise 1.1 (p. 8)

1 **(a)** +13 **(b)** -2 **(c)** +8 **(d)** -4
(e) -3 **(f)** +5 **(g)** +18 **(h)** -10
(i) -7 **(j)** -1 **(k)** -21 **(l)** -8
(m) +7 **(n)** -7 **(o)** -26 **(p)** +6

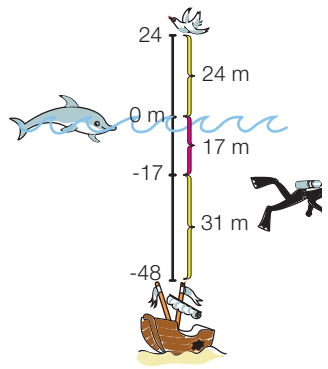
2 **(a)** +1 **(b)** -2 **(c)** +16 **(d)** -1
(e) +5 **(f)** -11 **(g)** -17 **(h)** +20
(i) +6 **(j)** +3 **(k)** +8 **(l)** -7
(m) -4 **(n)** -22 **(o)** -6 **(p)** -11

3 **(a)** +350 **(b)** -4800 **(c)** -6 **(d)** +73 **(e)** -2
(f) -19 **(g)** +10 750 **(h)** +34 **(i)** -200 **(j)** +8100

4 **(a)** +11, +4, 0, -2, -7 **(b)** +7, 1, 0, -9, -23
(c) 11, 4, 1, 0, -3, -15 **(d)** 19, 8, 6, -2, -5, -43
(e) 26, 14, 5, -1, -38, -72 **(f)** 32, 17, 4, 0, -19, -56

5 **(a)** 4 **(b)** 2 **(c)** -10 **(d)** 2
(e) 2 **(f)** -6 **(g)** 14 **(h)** -24

6 **(a)**



(b) **(i)** 41 m **(ii)** 48 m

7 **(a)** Wednesday **(b)** Sunday
(c) **(i)** 11°C **(ii)** 13°C
(d) Wednesday

8 **(a)** -120 – 87 + 243 + 50 – 109
(b) **(i)** less **(ii)** $23 less

9 2411 m

10 **(a)** magic total = -9

-6	-1	-2
1	-3	-7
-4	-5	0

(b) magic total = -12

6	-4	4	-18
-16	2	-6	8
-14	0	-8	10
12	-10	-2	-12

11 **(a)** -13 **(b)** -23 **(c)** -25
(d) -60 **(e)** -111 **(f)** -133
(g) -109 **(h)** -45 **(i)** -127

12 **(a)** **(i)** 1 **(ii)** 6 **(iii)** 9 **(iv)** 13
(b) **(i)** +3 – 2 = 1 **(ii)** +11 – 5 = 6
(iii) +17 – 8 = 9 **(iv)** +34 – 21 = 13
(c) The answers to both **(a)** and **(b)** are the same.
(d) commutative law
(e) **(i)** 11 **(ii)** 16 **(iii)** 27 **(iv)** 15

Open-ended – Sample answers

13 **(a)** 1°C **(b)** 6°C

14 +1, +5, -10 (or +10, -1, -5)

15 -18, -19, -20

Exercise 1.2 (p. 15)

1 **(a)** 30 **(b)** 21 **(c)** 77 **(d)** 54
(e) -40 **(f)** -24 **(g)** -27 **(h)** -35
(i) -8 **(j)** -16 **(k)** -25 **(l)** -16
(m) 30 **(n)** 8 **(o)** 6 **(p)** 15
(q) -60 **(r)** -140 **(s)** -90 **(t)** 132

2 (a) 24 (b) 12 (c) 15 (d) -40
(e) -18 (f) -24 (g) -50 (h) -20
(i) -32 (j) -27 (k) -8 (l) -1000

3 (a) -3, -6, -9 (b) 0, 2, 4 (c) -2, -4, -6

4 (a) D (b) D

5 (a) 36 (b) -36 (c) 100 (d) -100
(e) 50 (f) 75 (g) -400 (h) -700
(i) 16 (j) -100 (k) 81 (l) -256

6 (a) (i) -$7 (ii) -$6
(b) -$161 (c) -$114 (d) -$275

7 (a) -9 (b) -35 (c) -13 (d) 18
(e) 2 (f) -8 (g) -20 (h) -10

8 (a) (i) 1 (ii) -1 (iii) 1 (iv) -1 (v) 1 (vi) -1
(b) even, odd
(c) (i) 1 (ii) -1 (iii) -1 (iv) 1

9 (a) -64 (b) 81 (c) -32 (d) -36

10 (a) (i) 3 × -$25 = -$75 for gas
(ii) 3 × -$35 = -$105 for electricity
(b) (i) Gas: $93 – $75 = $18
Electricity $102 – $105 = -$3
(ii) Sandra's direct debit of $25 per month for gas has resulted in her account being underpaid by $18. She should consider increasing the direct debit amount to $30 per month, which would give a 3-monthly total very close to her current bill.
Sandra's direct debit of $35 for electricity has resulted in her account being slightly overpaid by $3. She doesn't need to adjust it now, but should monitor it closely.

Open-ended – Sample answers

11 3 × 4 × -2 = -24; 8 × 3 × -1 = -24; -6 × -2 × -2 = -24

12 3 hearts, 3 diamonds, 1 club (3 × -5 + 3 × -3 + 2 = -22)
4 hearts, 2 diamonds, 1 spade (4 × -5 + 2 × -3 + 4 = -22)

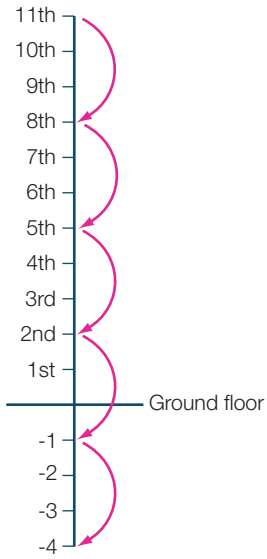
Exercise 1.3 (p. 21)

1 (a) -3 (b) -3 (c) -4 (d) -3
(e) -2 (f) -6 (g) -9 (h) -8
(i) 4 (j) 9 (k) 7 (l) 6
(m) 7 (n) -9 (o) -4 (p) -10
(q) -30 (r) -100 (s) -20 (t) 230

2 (a) -16 (b) -7 (c) -13 (d) -15
(e) -26 (f) 31 (g) 21 (h) -24
(i) 15 (j) -20 (k) -9 (l) 3

3 (a) D (b) B

4 (a)



(b) 4 stops between top and bottom levels
(c) floors: 8, 5, 2, -1, -4

5 -$40

6 (a) 16 minutes (b) 48 minutes

7 -4°C

8 (a) 8600 ÷ 200 = 43 months (b) 3 years and 7 months
(c) 8600 – (200 × 12) = 6200, -$6200 would appear on the account

9 -72, -30, -24, -18, -15, -12, -9, -8, -5, -4, -3, -2, 1, 2, 3, 6, 8, 10, 12, 16, 18, 20, 36, 48, 60

Open-ended – Sample answers

10 Must be a negative multiple of 8; e.g. -8, -24, -80.

11 6 ÷ 5 > -1
5 ÷ 6 > -1
5 ÷ 3 > -1
3 ÷ 5 > -1
-8 ÷ -7 > -1
-7 ÷ -8 > -1
-4 ÷ 5 > -1
3 ÷ -4 > -1

Half-time 1 (p. 24)

1 (a) +9 – 6 = +3 (b) +14 – 21 = -7
(c) -9 – 8 = -17 (d) -31 – 27 = -58
(e) -8 – 8 = -16 (f) -14 – 20 = -34
(g) +7 + 10 = +17 (h) -35 + 21 = -14

2 (a) -24 (b) 56 (c) -140 (d) 24
(e) 9 (f) -25 (g) -8 (h) -64

3 (a) +14°C (b) -8°C

4 (a) 5 (b) -9 (c) -5 (d) 7
(e) -8 (f) -8 (g) 3 (h) 4

5 (a) -37, -30, -3, 0, 3, 7 (b) -40, -20, -5, -1, 54

6 (a) 1 (b) -2 (c) -1 (d) -14
(e) -3 (f) -5 (g) -11 (h) 4

7 (a) \$80 (b) -30

8 (a) -120 (b) +7

9 (a) \$20 000 (b) -10 000

10 (a) -22 (b) -26 (c) -63 (d) 12

Exercise 1.4 (p. 26)

1 (a) 10 (b) 20 (c) 4 (d) -16
(e) 6 (f) 55 (g) -3 (h) -4
(i) -26 (j) 36 (k) 9 (l) 48

2 (a) -10 (b) -25 (c) 0 (d) 44
(e) -30 (f) -24 (g) 60 (h) 300
(i) 10 (j) 22

3 (a) -18 (b) 39 (c) 41 (d) 15
(e) 243 (f) 252 (g) 45 (h) -144
(i) -44 (j) -2

4 (a) C (b) A

5 (a) $250 - (3 \times 50) = \$100$
(b) $250 - (3 \times 50) - (2 \times 80) + 100 = \$40$
(c) After the 2nd of his \$80 withdrawals in Week 2, as at this point his account balance would have been -\$60.
(d) \$60

6 \$24 million loss

7 (a) 12°C (b) 0°C (c) -4°C (d) -16°C

8 (a) $72 + (1 \times -2) + (4 \times -1) + (5 \times 0) + (7 \times +1) + (1 \times +10)$
(b) (i) 83 (ii) 11 over par
(c) (i) 69 (ii) 3 under par

9 (a) 11 (b) 41
(c) Without brackets, the calculation, starting with 19 and moving from left to right is: subtract 7 (to give 12), then subtract 15 (to give -3), then add 14 (to give 11).

Calculating 7 – 15 in brackets first gives 19 – (-8), which gives 27. Adding 14 to this gives 41.

10 (a) $8 + 16 - 23 - 9 = -8$
(c) $11 \times -8 + 24 \div -2 = -100$

Open-ended – Sample answers

11 (a) 24 (b) -53
(c) Without the brackets, the calculation becomes:
$-15 + 11 \times -2 + -4^2$.

Evaluating the index number and the multiplication first gives:
$-15 + -22 + -16$
$= -37 + -16$
$= -53$

12 $-3 - (48 \div (-6)) = 5$
The brackets must give -8.

Exercise 1.5 (p. 37)

1 (a) 2^4 (b) 4^7 (c) 8^8
(d) 11^6 (e) 13^6 (f) 9^9
(g) $5^4 \times 6^3$ (h) $2^3 \times 3^2$ (i) $4^4 \times 10^4$
(j) $7^5 \times 8^6$ (k) $2^3 \times 3^4 \times 5^2$ (l) $11^3 \times 13^6$
(m) $6^4 \times 7^3 \times 9^2$ (n) 12×14^3

2 (a) $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$ (b) $4 \times 4 \times 4 \times 4 \times 4 \times 4$
(c) $7 \times 7 \times 7$ (d) $9 \times 9 \times 9 \times 9 \times 9 \times 9$
(e) $10 \times 10 \times 10 \times 10$
(f) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
(g) $2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5$
(h) $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 11 \times 11 \times 11 \times 11 \times 11$
(i) $9 \times 9 \times 9 \times 9 \times 9 \times 7 \times 7 \times 7 \times 7 \times 7$
(j) $2 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 6 \times 6 \times 6 \times 6 \times 6$
(k) $3 \times 3 \times 3 \times 3 \times 7 \times 7 \times 12$
(l) $9 \times 9 \times 9 \times 11 \times 17 \times 17$

3 (a) 5^5 (b) 3^8 (c) 6^6 (d) 4^7
(e) 9^9 (f) 17^{11} (g) 63^{16} (h) $4^9 \times 5^5$
(i) $2^{10} \times 6^7$ (j) $7^4 \times 8^{12}$ (k) $9^7 \times 11^7$ (l) $3^{15} \times 10^{11}$

4 (a) $(-2)^7 = -128$ (b) $(-6)^9 = -10\,077\,696$
(c) $(-3)^9 = -19\,683$ (d) $(-7)^4 = 2401$
(e) -675 (f) -64
(g) -147 456 (h) 3969

5 (a) 7^2 (b) 3^5 (c) 2^3 (d) 13^2
(e) 17^3 (f) 25^3 (g) $(-2)^2$ (h) -4
(i) -6 (j) $3^2 \times 2^3$ (k) $6^2 \times 4$ (l) $9^2 \times 13$
(m) $5^{12} \times 7$ (n) 11×8^2 (o) $2^2 \times 3^5 \times 4$

6 (a) C (b) D (c) C (d) A

7

Words	Index form	Expanded form	Value
Seven squared	7^2	7×7	49
Six to the power of five	6^5	$6 \times 6 \times 6 \times 6 \times 6$	7776
Eight to the power of three	8^3	$8 \times 8 \times 8$	512
Eleven to the power of six	11^6	$11 \times 11 \times 11 \times 11 \times 11 \times 11$	1 771 561

8 (a) (i) 50×2^3 (ii) 50×2^5 (iii) 50×2^8
(b) (i) 400 (ii) 1600 (iii) 12 800

9 (a)

$(-5)^1$	-5
$(-5)^2$	25
$(-5)^3$	-125
$(-5)^4$	625
$(-5)^5$	-3 125
$(-5)^6$	15 625
$(-5)^7$	-78 125
$(-5)^8$	390 625
$(-5)^9$	-1 953 125
$(-5)^{10}$	9 765 625
$(-5)^{11}$	-48 828 125
$(-5)^{12}$	244 140 625

(b) The answers alternate between negative and positive.

(c) Odd powers give a negative answer, even powers give a positive answer.

10 (a) $2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7$ (b) 128

11 (a) 8 (b) 125 (c) 1

(d) -64 (e) 81 (f) -64

12 (a) (i) $4^5 \times 4^3$
$= 1024 \times 64$
$= 65\,536$

(ii) $7^5 \div 7^2$
$= 16\,807 \div 49$
$= 343$

(iii) $(9^2 \times 9^3) \div 9^4$
$= (81 \times 729) \div 6561$
$= 59\,049 \div 656$
$= 9$

(iv) $(5^3 \times 5^4) \div (5^3 \times 5^4)$
$= (125 \times 625) \div (125 \times 625)$
$= 78\,125 \div 78\,125$
$= 1$

(b) (i) $4^5 \times 4^3$
$= 4^8$
$= 65\,536$

(ii) $7^5 \div 7^2$
$= 7^3$
$= 343$

(iii) $(9^2 \times 9^3) \div 9^4$
$= 9^5 \div 9^4$
$= 9$

(iv) $(5^3 \times 5^4) \div (5^4 \times 5^3)$
$= 5^7 \div 5^7$
$= 1$

(c) The process used in part (b) is more efficient.

13 (a) (i) 1 (ii) 1 (iii) 1

(iv) 0 (v) 0 (vi) 0

(b) (i) 1 raised to any positive whole number power is equal to 1.

(ii) 0 raised to any positive whole number power is 0.

14 (a) (i) negative (ii) negative

(iii) positive (iv) negative

(b) A negative number raised to an odd power will give a negative answer regardless of brackets, as seen in (i) and (ii). A negative number raised to an even power will give a positive answer when enclosed in brackets, as seen in (iii), but if not enclosed in brackets will remain negative, as seen in (iv).

(c) (i) -432 (ii) 144 (iii) 800

(iv) -1600 (v) -128 (vi) -128

15 (a)

Time (min)	No. of A	No. of B
0	1	1
20	2	1
40	4	2
60	8	2
80	16	4
100	32	4
120	64	8
140	128	8

(b) Bacteria A: 512, Bacteria B: 16

(c) 496

(d) Bacteria A, because after a period of time there would be many more of Bacteria A than Bacteria B.

Open-ended – Sample answers

16 (a) 32 (2^5), 64 (2^6) (b) 27 (3^3), 81 (3^4)

(c) 16 (2^4 and 4^2) or 64 (2^6 and 4^3)

17 (a) Joel has multiplied the base and the index together.

(b) He got 9^1 correct, as he would have calculated 9×1 to achieve an answer of 9.

(c) An index number is a shorthand way of writing repeated multiplication of the same number. Writing each number in expanded form might help him see this.

18 Hayden was correct because he added like terms. Tao was incorrect because he multiplied the bases and then added like terms.

19 $4^2 \div 2^3$; $10^2 \div (2 \times 5^2)$; $4^3 \div 2^5$; $6^2 \div (2 \times 3^2)$

Exercise 1.6 (p. 45)

1 (a) 9^{12} (b) 14^{21} (c) 6^{64} (d) 10^{25}

(e) 4^6 (f) 1 (5^0) (g) 2^{20} (h) 1 (7^0)

2 (a) $4^2 \times 3^2$ (b) $2^3 \times 5^3$ (c) $3^5 \times 2^5$ (d) $4^4 \times 7^4$

(e) $7^3 \times 10^3$ (f) 8×9 (g) $2^6 \times 3^6$ (h) $4^2 \times 5^2 \times 6^2$

(i) $3^3 \div 5^3$ (j) $2^2 \div 9^2$ (k) $10^3 \div 13^3$ (l) $12^2 \div 13^2$

(m) $1^5 \div 2^5$ (n) $1^3 \div 10^3$ (o) $1^6 \div 11^6$ (p) $1^2 \div 12^2$

3 (a) 1 (b) 1 (c) 1 (d) 1 (e) 1

(f) 3 (g) 4 (h) -1 (i) -6 (j) 3

4 (a) D (b) A

5 (a) $8^4 \times 7^6$ (b) $\frac{6^6}{7^6}$ (c) $2^4 \times 4^5$ (d) 5^4

(e) $\frac{4^7}{9^2}$ (f) $\frac{3^2}{4^5}$ (g) $7^2 \times 8^4$ (h) 10^3

(i) 4^{10} (j) $2^4 \times 3^2$ (k) $\frac{3}{2}$ (l) $\frac{2^2}{3^{12}}$

6 (a) $(5 \times 2)^6$ (b) $(3 \times 7)^3$

(c) $(7 \times 5)^2$ (d) $(3 \times 3 \times 5)^4$

(e) $(2 \times 5 \times 7)^5$ (f) $(7 \times 11)^7$

7 (a) $2^2 \times 3^2$ (b) $2^4 \times 3$ (c) $3^2 \times 7$
(d) $2^3 \times 3^2$ (e) 3×5^2 (f) $2^2 \times 3 \times 7$

8 (a) $\frac{2}{5}$ (b) $\frac{1}{3}$ (c) $\frac{3}{4}$ (d) $\frac{45}{22}$

9 (a) 2^{33} (8 589 934 592) is the largest power of 2 for which all digits can fit on a 10-digit calculator display.

(b) 9^{10} (3 486 784 401) is the largest power of 9 for which all digits can fit on a 10-digit calculator display.

10 (a) a^6 (b) $\frac{c^4}{d^4}$

(c) x^5y^5 (d) 1

(e) $2^2x^2 = 4x^2$ (f) 5

(g) $\frac{b^{10}}{c^5}$ (h) $\frac{3^3a^3}{b^3} = \frac{27a^3}{b^3}$

11 (a) Terry used the index number rules to simplify the fraction inside the brackets before raising it to the power of 2. Kelvin raised the numerator and the denominator to the power of 2, worked out their values and then simplified the answer.

(b) Terry answered the question more efficiently. By simplifying the fraction inside the brackets first, he didn't have to deal with any large numbers on a calculator.

12 (a) 7 (b) 18 (c) 10 (d) 7 (e) 3 (f) 1

13 (a) $(2^2)^2$ (b) $(3^2)^2$ (c) $(5^2)^2$
(d) $(3^2)^3$ or $(3^3)^2$ (e) $(4^2)^2$ or $(2^4)^2$ (f) $(10^3)^2$ or $(10^2)^3$

Open-ended – Sample answers

14 (a) $(4 \times 6)^3$, $(8 \times 3)^3$ or $(2 \times 12)^3$

(b) $(4 \times 9)^5$, $(3 \times 12)^5$ or $(2 \times 18)^5$

(c) $(6 \times 8)^2$, $(4 \times 12)^2$ or $(24 \times 2)^2$

(d) $(4 \times 20)^4$, $(16 \times 5)^4$ or $(40 \times 2)^4$

15 $625\,000 = 5^4 \times 10^3$ or $25^2 \times 10^3$

16 Values for m and n must multiply to give 24.

17 (a) powers were multiplied instead of added; 3^6

(b) powers were divided instead of subtracted; 7^6

(c) powers were raised to a power instead of multiplied; 5^8

(d) Any number raised to the power of 0 = 1. Therefore, $13 \times 1 = 13$.

Challenge 1 (p. 48)

1 B

2 D

Total Reece spent = \$100 + \$130 = \$230
Total Reece got back = \$110 + \$150 = \$260
Profit = \$260 – \$230 = \$30

3 Let the consecutive numbers be $x - 1$, x, $x + 1$.
$2(x + 1) + 3(x - 1) = -31$
$5x - 1 = -31$, $x = -6$
The numbers are -7, -6, -5.

4 I am x years old. Father is $x + 30$, mother is $x + 24$.
$x + 30 = 2(x + 24)$, $x = -18$. My father's age was double my mother's age 18 years before I was born.

5 $64 = 2^6 = 4^3 = 8^2$; hence, $x = 2$, $y = 6$; $x = 4$, $y = 3$; $x = 8$, $y = 2$; $x = 64$, $y = 1$.

6 (a) $2^{15} = (2^3)^5 = 8^5$. $3^{10} = (3^2)^5 = 9^5$. So, $2^{15} < 3^{10}$ and statement is false.

(b) $2^{27} = (2^3)^9 = 8^9$. $3^{18} = (3^2)^9 = 9^9$. So, $3^{18} > 2^{27}$.

7 B $a = (2^4)^{20} = 16^{20}$; $b = (3^3)^{20} = 27^{20}$; $c = (5^2)^{20} = 25^{20}$

8 D 9 A 10 B 11 A

12 $256 = 256^1 = 16^2 = 4^4 = 2^8$; hence $y = 256$, $x = 1$; $y = 16$, $x = 2$; $y = 4$, $x = 4$; $y = 2$, $x = 8$.

13 B

14 $4^1 = 4$, $4^2 = 16$, $4^3 = 64$, $4^4 = 256$, $4^5 = 1024$. If the power is odd, then the last digit in the expansion is 4, so 4^{3827} ends in 4.

15 $3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$, $3^5 = 243$, $3^6 = 729$, $3^7 = 2187$, $3^8 = 6561$.

There is a pattern in the last digit that is repeated every fourth number: 3, 9, 7, 1, 3, 9, 7, 1, …

$2004 \div 4 = 501$. As there is no remainder, 2004 is the 4th digit in the pattern, so 3^{2004} ends in 1.

Chapter review 1 (p. 49)

1 (a) -23 (b) +840 000 (c) +350 (d) +4

2 (a) -23, -6, 0, 9, 14 (b) -15, -7, -2, 5, 8 (c) -11, 0, 6, 12, 34

3 (a) +7 (b) -2 (c) -11 (d) -7
(e) +3 (f) -24 (g) +20 (h) -8

4 (a) 2 (b) 2 (c) 0 (d) -7
(e) 4 (f) -1

5 (a) -28 (b) -20 (c) 36 (d) 84
(e) -135 (f) -176 (g) -1800 (h) 5600
(i) 36 (j) −49 (k) -36 (l) 16

6 (a) -12 (b) -5 (c) 7 (d) -9
(e) -6 (f) 13 (g) -8 (h) 9
(i) -20 (j) -44 (k) -40 (l) -13

7 (a) -14 (b) 0 (c) 35 (d) 14
(e) 0 (f) 21

8 (a) 9^5, 59 049 (b) 6^7, 279 936
(c) $3^4 \times 4^5$, 82 944 (d) $8^2 \times 10^3$, 64 000

9 (a) $2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$
(b) $13 \times 13 \times 13 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
(c) $4 \times 4 \times 7 \times 7 \times 9 \times 9 \times 9 \times 9 \times 9$
(d) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 10 \times 10 \times 17$

10 (a) 7^5 (b) 3^8 (c) 5^4 (d) 2^8
(e) $3^9 \times 2^{10}$ (f) $4^5 \times 7^3$ (g) $5^6 \times 2^6$ (h) $7^7 \times 11^6$

11 (a) 5^4 (b) 7^3 (c) 11^2 (d) 2^3
(e) 3×4 (f) $7^2 \times 10^2$ (g) 8×5^3 (h) $2 \times 9 \times 13^2$

12 (a) 81 (b) -81 (c) -125

(d) -1 (e) -144 (f) 1296

13 (a) $3^{10} = 59\,049$ (b) $7^6 = 117\,649$

(c) $3^{15} = 14\,348\,907$ (d) $2^{16} = 65\,536$

(e) 1 (f) 6 (g) 1 (h) 1

14 (a) $4^3 \times 11^3$ (b) $8^5 \times 9^5$ (c) $3^7 \times 5^7$ (d) $7^4 \times 10^4$

(e) $\frac{1}{2^3}$ (f) $\frac{3^2}{4^2}$ (g) $\frac{5^4}{6^4}$ (h) $\frac{8^7}{9^7}$

15 -1°C

16 Felicity 0; Georgia 1; Rosalie -4; Doreen -7

17 (a)

Time (number of days)	0	1	2	3	4
Number of cells	1	2	4	8	16

(b)

Time (number of days)	0	1	2	3	4	5	6	7
Number of cells	1	2	4	8	16	32	64	128

(c) $2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7$

(d) $N = 2^{14} = 16\,384$

18 (a) \$70 (b) \$10 (c) \$9410

19 -7°C per hour

20 (a) A (b) C

21 (a) 1 (b) 1 (c) 1 (d) 1

(e) -3 (f) 25 (g) -32 (h) -27

(i) 4 (j) -10 000 (k) 27 (l) $\frac{9}{128}$

22 (a) $\frac{10}{9}$ (b) $\frac{20}{49}$ (c) $\frac{3}{10\,000}$

23 (a) 3 (b) 8 (c) 10

24 (a) $(6^2)^2$ (b) $(2^{14})^2$ (c) $(10^9)^2$ (d) $(70^2)^7$

25 (a) -20

(b)

4	-6	2	-20
-18	0	-8	6
-16	-2	-10	8
10	-12	-4	-14

(c)

-2	3	-1	10
9	0	4	-3
8	1	5	-4
-5	6	2	7

Magic sum is 10.

(d)

-8	12	-4	40
36	0	16	-12
32	4	20	-16
−20	24	8	28

Magic sum is 40.

26 32 players

27 (a) $(-3 - 4) \times -2 = 14$ (b) $(2 + -3) \times 4 + -2 = -6$

(c) $16 \div -4 + 3 \times (-2 + 1) = -7$

(d) $4 + -3 \times -2 \div -6 \times (-1 + 5) = 0$

(e) $2 - 3 + 6 \times (3 + 2) \times -1 - 6 = -37$

(f) $24 \div -2 \div -3 + 4 \times (-2 - 5) = -24$

28 (a) False (b) False (c) False

(d) True (e) True (f) False

NAPLAN practice 1

1 -8, 3 2 \$9 3 C

4 (a)

Shape	1	2	3	4	5
Number of flowers	1	4	9	16	25

(b) the perfect squares

(c) 100

5 B

6 7 minutes

7 $A = 4, B = 8, C = 2$

8 C

Chapter 2

Recall 2 (p. 56)

1 (a) $1\frac{4}{15}$ (b) $\frac{7}{12}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

2 (a) 13.041 (b) 0.459 (c) 63.5

(d) 13.02 (e) 0.465 (f) 316

3 (a) 2.03 (b) 3.19 (c) 2.910 (d) 4.086

4 (a) $\frac{3}{5}$ (b) $\frac{4}{23}$ (c) $\frac{8}{3}$

5 (a) $\frac{5}{2}$ (b) $\frac{38}{9}$ (c) $\frac{67}{20}$

6 (a) $2\frac{3}{7}$ (b) $4\frac{4}{9}$ (c) $9\frac{1}{2}$

7 (a) 70.8 (b) 490 (c) 80 (d) 0.0521

(e) 3.465 (f) 0.004 037

8 (a) \$75 (b) 105 kg (c) 225 m

9 (a) 50% (b) 75% (c) 40% (d) 70%

10 (a) 35% (b) 90% (c) 4% (d) 127%

Exercise 2.1 (p. 61)

1 (a) 0.382, 0.399, $\frac{2}{5}, \frac{3}{4}, \frac{4}{5}$

(b) 0.88, 0.89, 0.899, $\frac{9}{10}, \frac{9}{8}$

(c) 0.07, 0.099, 0.112, $\frac{1}{8}, \frac{1}{4}$

(d) 0.2, 0.3, $\frac{1}{3}, \frac{3}{8}, \frac{2}{5}$

(e) $\frac{1}{2}$, 0.55, 0.555, 0.58, $\frac{3}{5}$

(f) $\frac{2}{9}$, 0.291, $\frac{3}{10}$, 0.302, $\frac{2}{3}$

(g) 2.278, $2\frac{3}{4}, 2\frac{4}{5}, 2\frac{9}{10}$, 2.932

(h) $1.029, 1\frac{1}{5}, 1.243, 1\frac{3}{8}, 1\frac{2}{3}$

(i) $4.199, 4\frac{1}{5}, 4.201, 4\frac{1}{4}, 4.295$

(j) $3\frac{1}{5}, 3.439, 3.45, 3.482, 3\frac{2}{3}$

2 (a) $\frac{4}{5}$ (b) $\frac{1}{20}$ (c) $\frac{1}{500}$ (d) $\frac{9}{10\,000}$

(e) $\frac{7}{50}$ (f) $\frac{31}{50}$ (g) $\frac{31}{100}$ (h) $\frac{17}{20}$

(i) $\frac{711}{1000}$ (j) $\frac{171}{250}$ (k) $\frac{5}{8}$ (l) $\frac{16}{125}$

(m) $\frac{203}{1000}$ (n) $\frac{47}{500}$ (o) $\frac{39}{250}$ (p) $\frac{7009}{10\,000}$

3 (a) 6 pieces; 20 cm remaining

(b) 6 cups; 30 g remaining

(c) 16 cups; 50 mL remaining

(d) 13 buses; 26 people on the thirteenth bus

4 (a) 51.7 (b) 5.15 (c) 7.125 (d) 67.28

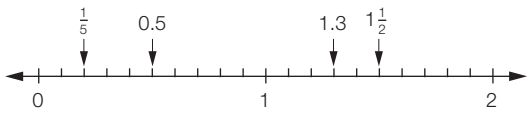
(e) 39.0625 (f) 28.12 (g) 14.1625 (h) 24.0775

5 (a) $7\frac{1}{2}$ (b) $3\frac{2}{5}$ (c) $1\frac{1}{4}$ (d) $5\frac{16}{25}$

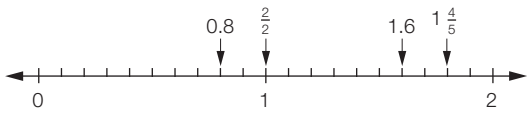
(e) $13\frac{1}{50}$ (f) $27\frac{24}{25}$ (g) $9\frac{9}{200}$ (h) $124\frac{353}{500}$

6 (a) C (b) C

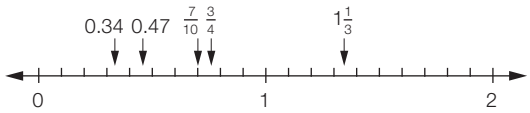
7 (a)



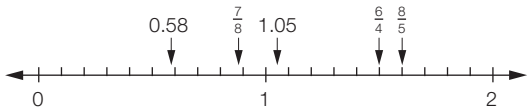
(b)



8 (a)



(b)



9 (a) < (b) < (c) = (d) = (e) >

(f) < (g) > (h) < (i) <

10 (a) (i) $\frac{5}{8}$ (ii) $\frac{80}{129}$ (iii) $\frac{403}{1000}$ (iv) $1\frac{81}{125}$

(v) $1\frac{11}{20}$ (vi) $9\frac{3}{5}$

(b) (i) 0.625 (ii) 0.620 (iii) 0.403 (iv) 1.648

(v) 1.55 (vi) 9.6

11 (a) \$53.95 (b) \$105.30 (c) \$5.30

12 (a) US\$7.20 (b) US\$36 (c) US\$72

(d) US\$192.24 (e) US\$1328.40

13 \$2653.50

14 7

15 14

16 (a) 0.9 m, 0.45 m, 0.27 m (b) 1.08 m (c) $\frac{2}{5}$

17 (a) Sample answer: Angela can calculate how many lollies are in each packet to find out how many she has in total, and then divide that amount among the 50 gift bags.

(b) 10

(c) Sample answer: 36; this answer may not be exact as we can't be sure how many lollies are in each packet.

18 (a) 13 kg (b) 1.9 kg

Open-ended – Sample answers

19 $\frac{1}{4}, \frac{3}{10}, \frac{7}{20}$

20 0.95, 0.99999, 1.102

21 (a) Leo is treating the decimals as whole numbers and not comparing place value. He needs to compare the place value of each digit in the number. 1.7 has 7 in the tenths place value, whereas 1.25 only has 2 in that same position. Leo's answer to the first question is only correct because the numbers have a digit in the same place value position (units and tenths).

(b) Compare the whole number parts of the two decimal numbers first. If one is larger than the other, then that decimal number is larger. If the whole number parts are the same, compare the digits in the tenths place value column. If one is larger than the other, then that decimal number is larger. If they are the same, compare the digits in the hundredths place value column. Keep moving down the place value columns comparing digits until you find one digit larger than the other corresponding digit. This digit belongs to the larger number.

Exercise 2.2 (p. 69)

1 (a) 1.125 Terminating (b) 6.5 Terminating

(c) 21.189 620 1… Irrational (d) 9.165 151 39… Irrational

(e) 0.555 5… Recurring (f) 7.641 989 27… Irrational

(g) 0.181 818 … Recurring (h) 7.333 3… Recurring

(i) 14.696 938 46… Irrational (j) 2.5 Terminating

(k) 23.1 Terminating (l) 0.916 66… Recurring

(m) 18.5 Terminating (n) 18.973 665 96… Irrational

(o) 3.888 8… Recurring (p) 9.818 350 16… Irrational

2 (a) $0.1\dot{6}$ (b) $0.8\dot{3}$ (c) $0.\dot{1}$ (d) $0.\dot{5}$

(e) $0.\dot{0}\dot{9}$ (f) $0.\dot{1}\dot{8}$ (g) $0.\dot{2}\dot{7}$ (h) $0.\dot{1}5384\dot{6}$

(i) $0.0\dot{6}$ (j) $0.1\dot{3}$ (k) $0.2\dot{6}$ (l) $0.4\dot{6}$

(m) $0.0\dot{5}$ (n) $0.2\dot{7}$ (o) $0.3\dot{8}$ (p) $0.6\dot{1}$

3 (a) $\frac{4}{9}$ (b) $\frac{1}{9}$ (c) $\frac{2}{9}$ (d) $\frac{7}{9}$

(e) $\frac{14}{33}$ (f) $\frac{32}{99}$ (g) $\frac{17}{99}$ (h) $\frac{8}{33}$

(i) $\frac{4}{333}$ (j) $\frac{71}{333}$ (k) $\frac{332}{999}$ (l) $\frac{224}{999}$

(m) $\frac{113}{900}$ (n) $\frac{25}{198}$ (o) $\frac{3211}{9990}$ (p) $\frac{8197}{1998}$

4 (a) D (b) C

5 (a) 1.7 and 1.8; closer to 1.7 (b) 2.2 and 2.3; closer to 2.2

(c) 3.4 and 3.5; closer to 3.5 (d) 4.4 and 4.5; closer to 4.5

(e) 2.8 and 2.9; closer to 2.8 (f) 2.6 and 2.7; closer to 2.6

(g) 6.8 and 6.9; closer to 6.9 (h) 8.2 and 8.3; closer to 8.2

6 7.920 m

7 (a) $0.\overline{63}$; 64 cents

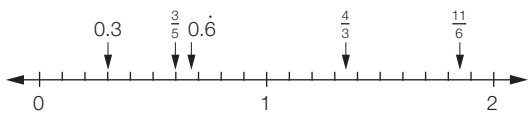
(b) 65 cents if you buy only one

8 1

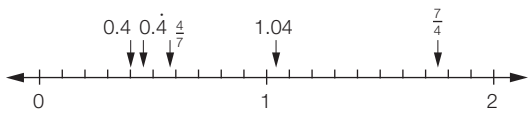
9 (a) $\frac{1}{3}$, 0.9, 1.3, $\sqrt{3}$, $\sqrt{9}$ (b) 0.5, $\frac{5}{9}$, $\frac{9}{5}$, $\sqrt{5}$, $\sqrt{59}$

(c) 0.36, $\frac{4}{11}$, 0.4, $\sqrt{4}$, $\sqrt{11}$ (d) $\frac{1}{16}$, $\frac{1}{6}$, 1.6, $\sqrt{6}$, $\sqrt{16}$

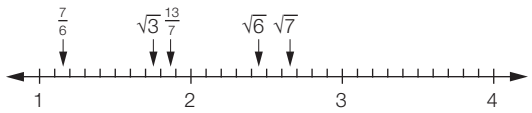
10 (a)



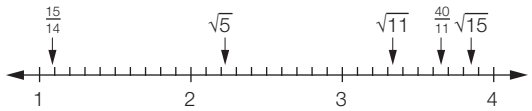
(b)



11 (a)



(b)



12 (a) B (b) B

13 (a) 0.111 111 11 and 0.090 909 09

(b) 0.202 020 2 (c) $\frac{20}{99} = 0.\dot{2}\dot{0}$ (d) Yes

(e) 0.666 666 66 and 0.555 555 55 which add to give 1.222 222 21; the fractional sum is $1\frac{2}{9} = 1.\dot{2}$.

(f) The final digit in the decimal sum is incorrect as there was no number to carry into this column. When we only expressed the fractions as decimals with 8 digits after the decimal point, we introduced this error.

Open-ended – Sample answers

14 $0.\dot{3}$ is bigger, it is more than $\frac{33}{100}$ and 0.3 is $\frac{30}{100}$

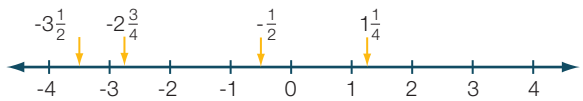
15 $\frac{9}{99} = 0.\dot{0}\dot{9}$, $\frac{7}{9} = 0.\dot{7}$, $\frac{13}{999} = 0.\dot{0}1\dot{3}$, $\frac{58}{9999} = 0.\dot{0}05\dot{8}$,

$\frac{78\,465}{99\,999} = 0.\dot{7}84\,6\dot{5}$

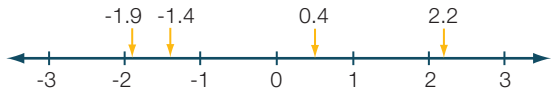
The recurring cycle is just the numerator of the fraction.

Exercise 2.3 (p. 77)

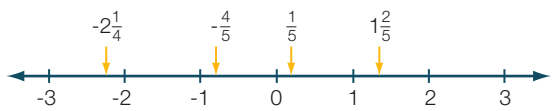
1 (a)



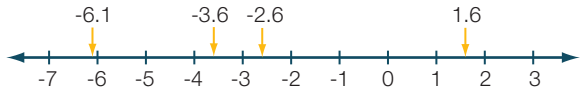
(b)



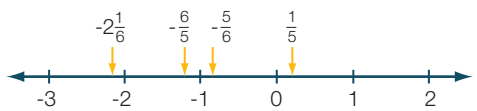
(c)



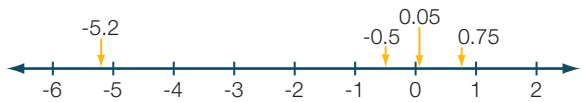
(d)



(e)



(f)



2 (a) -0.1 (b) -1 (c) -3.1 (d) -1.1

(e) -0.4 (f) -14.2 (g) -8.5 (h) 9.5

(i) 1.69 (j) 14.45 (k) -1.54 (l) 1.43

(m) -1.97 (n) 7.8 (o) -10.07 (p) 31.56

3 (a) $-\frac{3}{2}$ or $-1\frac{1}{2}$ (b) $-\frac{4}{5}$ (c) $-\frac{3}{7}$

(d) $-\frac{7}{9}$ (e) $-\frac{17}{20}$ (f) $-\frac{1}{40}$

(g) $\frac{1}{30}$ (h) $\frac{1}{6}$ (i) $\frac{3}{5}$

(j) $\frac{5}{2}$ or $2\frac{1}{2}$ (k) $\frac{2}{3}$ (l) $-\frac{1}{4}$

(m) $\frac{3}{4}$ (n) $-\frac{35}{6}$ or $-5\frac{5}{6}$ (o) $\frac{49}{10}$ or $4\frac{9}{10}$

(p) $\frac{11}{12}$

4 (a) $-\frac{3}{14}$ (b) -2 (c) $-\frac{5}{6}$

(d) $-\frac{9}{5}$ or $-1\frac{4}{5}$ (e) -32 (f) $\frac{1}{8}$

(g) -9 (h) $\frac{1}{2}$ (i) 4

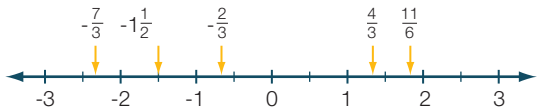
(j) $-7\frac{1}{3}$ (k) $-6\frac{2}{3}$ (l) $-7\frac{1}{2}$

5 (a) -0.32 (b) -0.3 (c) -4.5 (d) -0.9

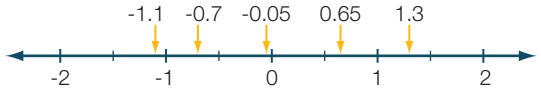
(e) 2.4 (f) 90 (g) -2.4 (h) -0.37

(i) 89.78 (j) -7.06 (k) -331.29 (l) -5.56

6



7



8 B

9 (a) Friday

(b) Monday

(c) Wednesday (difference of 7.8°C)

(d) (i) Average maximum: 2.6°C

(ii) Average minimum: -3.1°C

10 (a) +1.2, 0, $+\frac{3}{4}$, -0.8, $-1\frac{1}{4}$, 0, $-\frac{1}{2}$, -0.6

(b) (i) $-1\frac{1}{5}$ (ii) -1.2

11 -3.4, -3, -0.9, $-\frac{3}{4}$, 0, $\frac{3}{10}$, 1, 1.7

12 $-2\frac{1}{5}$, -2, -1.8, $-\frac{5}{4}$, 0, $\frac{3}{4}$, $\frac{4}{5}$, 1.1

13 (a) -157.536 (b) -335.5 (c) 35.155

(d) -80.64 (e) -1.536 (f) -4.682

(g) -7119.009 (h) -12 150.073 (i) 0.782

(j) -2839.396

14 (a) Chelsea's account is overdrawn. She owes the bank $12.91.

(b) Her account was still overdrawn. There was no money available.

(c) $473.85

15 The further a number lies to the right on the number line, the larger it is.

$\frac{1}{2}$ lies to the right of $\frac{1}{4}$, so $\frac{1}{2} > \frac{1}{4}$.

$-\frac{1}{4}$ lies to the right of $-\frac{1}{2}$, so $-\frac{1}{2} < -\frac{1}{4}$.

Open-ended – Sample answers

16 -1.9, -1.75, -1.24

17 $-\frac{5}{6}$, $-\frac{1}{2}$, $-\frac{1}{10}$

18 (a) Douglas has ignored the signs of the numbers he is adding and subtracting, and not applied the rules for adding and subtracting directed numbers. In the first problem, he has added 4.5, not -4.5. In the second problem, he has subtracted 2.9, not -2.9.

(b) When the two symbols are opposites (+ – or – +), replace them with one – symbol and subtract.

When the two symbols are the same (+ + or – –), replace them with one + symbol and add.

Exercise 2.4 (p. 81)

1 (a) Soda water (b) Lemon squash

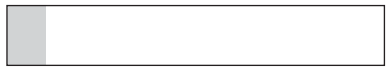
(c) Creamy soda (d) Lemonade

2 (a) Gauge 3 (b) Gauge 5 (c) Gauge 1 (d) Gauge 6

3 (a) F (b) D (c) A (d) C

4 The following diagrams are reduced versions of the answers required.

(a)



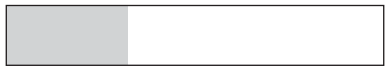
(b)

(c)

(d)

(e)

(f)



5 (a) D (b) B (c) A

6 35% (approx.)

7 55% (approx.)

8 (a) 59% (b) 49% (c) 53% (d) 48%

(e) males 10%, females 5%

9 (a) (i)

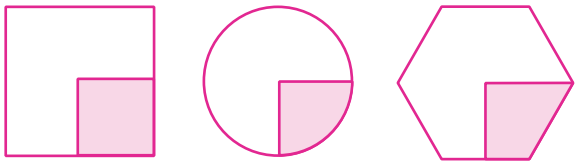
(ii)

(iii)

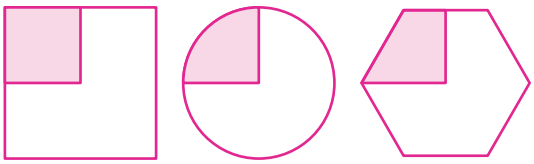
(b) Students' own answers. However, students will probably find it easier to estimate the percentage amount in (a).

Open-ended – Sample answers

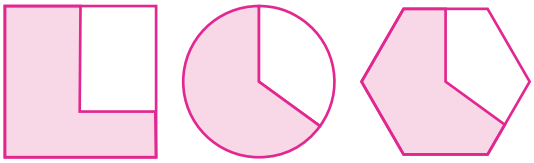
10 (a)



(b)



(c)



11 (a) Helga mistakenly believes that the large square contains 100 individual squares and so has shaded in 2 columns and 4 squares in the belief that each column is 10 units long and that she has shaded in 24 individual squares out of 100, or 24%. However, the large square contains only 64 individual squares, so by shading in 2 columns and 4 squares, Helga has actually shaded in 20 squares out of 64, or 31.25%.

(b) Helga mistakenly believes that the total circle contains 10 segments and so has shaded in 3 of the segments in the belief that she has shaded in three-tenths or 30%. However, the total circle contains only 8 segments, not 10, so shading in 3 out of 8 segments equals 37.5%, not 30%.

Exercise 2.5 (p. 88)

1 (a) 9% (b) 10% (c) 14% (d) 95%
(e) 60% (f) 25% (g) 150% (h) 200%
(i) $33.\dot{3}\%$ or $33\frac{1}{3}\%$ (j) $66.\dot{6}\%$ or $66\frac{2}{3}\%$
(k) $16.\dot{6}\%$ or $16\frac{2}{3}\%$ (l) $22.\dot{2}\%$ or $22\frac{2}{9}\%$
(m) 80% (n) 56.25% or $56\frac{1}{4}\%$
(o) 70% (p) 43.75% or $43\frac{3}{4}\%$
(q) 91.25% or $91\frac{1}{4}\%$ (r) $21.\dot{6}\%$ or $21\frac{2}{3}\%$
(s) $83.\dot{3}\%$ or $83\frac{1}{3}\%$ (t) $25.\dot{5}\%$ or $25\frac{5}{9}\%$

2 (a) 125% (b) 140% (c) 320% (d) 550%
(e) 237.5% (f) 275% (g) 570% (h) 462.5%
(i) $466.\dot{6}\%$ (j) $533.\dot{3}\%$ (k) $177.\dot{7}\%$ (l) $783.\dot{3}\%$

3 (a) 90% (b) 40% (c) 80% (d) 60%
(e) 17% (f) 47% (g) 82% (h) 53%
(i) 5.1% (j) 43.8% (k) 0.7% (l) 34.2%
(m) 920% (n) 510% (o) 202% (p) 901%

4 (a) D (b) D

5 (a) B (b) C

6

Percentage	Fraction	Decimal
5%	$\frac{1}{20}$	0.05
10%	$\frac{1}{10}$	0.1
$12\frac{1}{2}\%$	$\frac{1}{8}$	0.125
20%	$\frac{1}{5}$	0.2
25%	$\frac{1}{4}$	0.25
$33\frac{1}{3}\%$	$\frac{1}{3}$	$0.\dot{3}$
40%	$\frac{2}{5}$	0.4
50%	$\frac{1}{2}$	0.5
60%	$\frac{3}{5}$	0.6
$66\frac{2}{3}\%$	$\frac{2}{3}$	$0.\dot{6}$
75%	$\frac{3}{4}$	0.75
80%	$\frac{4}{5}$	0.8
100%	1	1

7 70%

8 375%

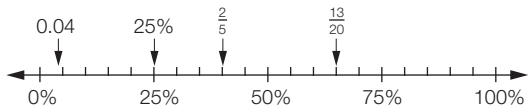
9 (a) Measurement 83%, Algebra 87%, Geometry 80%
(b) Algebra

10 (a) 67.5% (b) 95% (c) 90% (d) 55%
(e) 42.5% (f) 82.5%

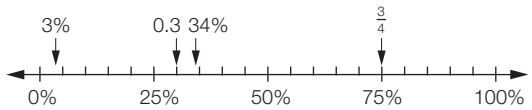
11 32

12 24.6% calcium, 3.4% magnesium, 0.12% potassium

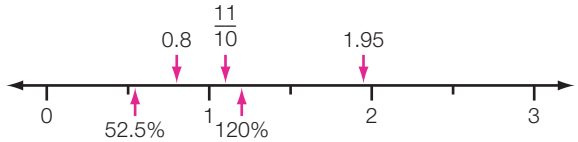
13 (a)



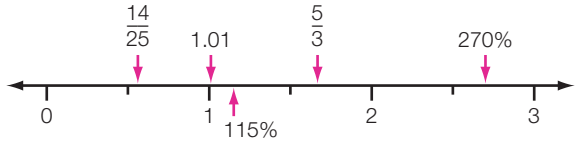
(b)



14 (a)



(b)



15 (a) $\frac{3}{14}$ (b) 21.43%

(c) Yes, the amount of extra fibre is more than 20% of the original amount.

16 (a) 61%, 65%, $83\frac{1}{3}$% (b) 48%, $52\frac{1}{2}$%, 70%

(c) Suburban store by 13%, $12\frac{1}{2}$%, $13\frac{1}{3}$%

(d) Suburban store: $\frac{18}{25}$, City store: $\frac{29}{50}$

(e) Suburban store by 14%

17 (a) 13.3%, 3%, 0.73%, 59.4%, 17.2%

(b) 8.8% (c) 58.9%

(d) Not accepted: 1.25% sodium

Open-ended – Sample answers

18 1.32 = 132%, 2.39 = 239%, 5.30 = 530%

19 $\frac{73}{100}, \frac{37}{50}, \frac{18}{25}$

Half-time 2 (p. 92)

1 65–70% full

2 (a) 2.5, terminating (b) $0.\overline{63}$, recurring

(c) 7.348 469 228…, irrational

(d) 13.2, terminating (e) $6.91\overline{6}$, recurring

(f) 8.660 254 038…, irrational

3 (a) 11 boxes (b) 15 cm left over

4 (a) 456% (b) 75% (c) $88\frac{3}{4}$% or 88.75%

(d) 43.2% (e) 140%

5 (a) $1\frac{1}{3}$ (b) $3\frac{112}{495}$ or $\frac{1597}{495}$ (c) $5\frac{1901}{9990}$ or $\frac{51\,851}{9990}$

6 (a) -0.7, $-\frac{5}{9}, \frac{3}{8}, \frac{1}{2}$, 1.099, $\sqrt{5}$

(b) -1.23, $-\frac{2}{3}$, 0.874, $\frac{17}{8}$, $\sqrt{13}$

7 (a) $0.\dot{6}$ (b) $0.\dot{7}1428\dot{5}$ (c) $0.\dot{1}$ (d) $0.\dot{4}\dot{5}$

8 (a) $\frac{219}{500}$ (b) $1\frac{9}{125}$ (c) $2\frac{253}{5000}$ (d) $13\frac{1401}{2000}$

9 (a) $\frac{3}{10}$ (b) $\frac{3}{14}$ (c) $\frac{17}{60}$ (d) $-1\frac{7}{8}$

(e) -13.29 (f) 11.76 (g) -7.93 (h) -15

10 55% to 65%

Exercise 2.6 (p. 96)

1 (a) $\frac{17}{100}$ (b) $\frac{12}{25}$ (c) $\frac{9}{100}$ (d) $\frac{13}{20}$

(e) $\frac{29}{50}$ (f) $\frac{19}{25}$ (g) $1\frac{17}{100}$ (h) $1\frac{29}{100}$

(i) $2\frac{2}{5}$ (j) $3\frac{3}{20}$ (k) $1\frac{19}{50}$ (l) $3\frac{3}{5}$

2 (a) $\frac{1}{200}$ (b) $\frac{1}{500}$ (c) $\frac{3}{700}$ (d) $\frac{3}{800}$

(e) $\frac{1}{1000}$ (f) $\frac{1}{150}$ (g) $\frac{1}{120}$ (h) $\frac{1}{450}$

(i) $\frac{9}{400}$ (j) $\frac{13}{200}$ (k) $\frac{7}{125}$ (l) $\frac{7}{80}$

3 (a) $\frac{83}{1000}$ (b) $\frac{1229}{10\,000}$ (c) $\frac{101}{200}$ (d) $\frac{5}{8}$

(e) $\frac{21}{1000}$ (f) $\frac{17}{500}$ (g) $\frac{11}{2000}$ (h) $\frac{1}{250}$

(i) $\frac{41}{400}$ (j) $\frac{7}{8}$ (k) $1\frac{251}{500}$ (l) $4\frac{41}{200}$

4 (a) 0.8 (b) 0.27 (c) 0.4 (d) 0.15

(e) 0.66 (f) 0.92 (g) 0.03 (h) 0.05

(i) 1.1 (j) 2.65 (k) 7 (l) 3.04

(m) 0.005 (n) 0.072 (o) 0.405 (p) 0.0625

5 (a) 0.0025 (b) 0.00125 (c) 0.007 (d) 0.004

(e) 0.025 (f) 0.1075 (g) 0.372 (h) 0.188

(i) $0.25\dot{3}$ (j) $0.05\dot{6}$ (k) $0.02\dot{5}$ (l) $0.238\dot{3}$

6 C **7** A **8** B

9 $\frac{43}{50}$ **10** $\frac{19}{200}$ **11** 0.45 **12** 0.925

13 (a) 30% = $\frac{3}{10}$; 16% = $\frac{4}{25}$; 9% = $\frac{9}{100}$

(b) 55% = $\frac{11}{20}$ (c) 45% = $\frac{9}{20}$

14 (a) 0.02, 20%, $\frac{1}{4}, \frac{2}{5}$ (b) 4.5%, 0.45, $\frac{4}{5}, \frac{5}{4}$

(c) 0.03, $\frac{3}{10}$, 33%, $\frac{1}{3}$ (d) 7.2%, $7\frac{1}{2}$%, 0.72, $\frac{7}{2}$

15 $\frac{7}{25}$

16 $\frac{9}{20}$ (45%)

17 (a) $\frac{17}{20}$ (b) $\frac{12}{25}$ (c) $\frac{37}{100}$ (d) $\frac{8}{25}$

Open-ended – Sample answers

18 (a) $\frac{13}{25}$ (52%), $\frac{27}{50}$ (54%), $\frac{11}{20}$ (55%)

(b) 0.255 (25.5%), 0.2504 (25.04%), 0.250 03 (25.003%)

19 (a, b) $3\frac{1}{4}$ is greater than 1 whole, so it is greater than 100%.

3 = 300%, so $3\frac{1}{4}$ = 300 + 25% = 325%.

3.25% is one-hundredth the size of 325% (325 ÷ 100 = 3.25). Its value lies between 3% and 4%.

Exercise 2.7 (p. 102)

1 (a) (i) 26% (ii) 26%

(b) (i) 80% (ii) 80%

(c) (i) 60% (ii) 60%

(d) (i) $87\frac{1}{2}\%$ (ii) 87.5%

(e) (i) $43\frac{3}{4}\%$ (ii) 43.75%

(f) (i) $83\frac{1}{3}\%$ (ii) $83.\dot{3}\ \%$

(g) (i) $56\frac{1}{4}\%$ (ii) 56.25%

(h) (i) $55\frac{5}{9}\%$ (ii) $55.\dot{5}\ \%$

(i) (i) $159\frac{7}{27}\%$ (ii) $159.\dot{2}5\dot{9}\ \%$

(j) (i) $128\frac{4}{7}\%$ (ii) $128.\dot{5}71\ 42\dot{8}\ \%$

(k) (i) $109\frac{1}{11}\%$ (ii) $109.\dot{0}\dot{9}\ \%$

(l) (i) $118\frac{2}{11}\%$ (ii) $118.\dot{1}\dot{8}\ \%$

2 (a) 6.25% (b) 2% (c) 35% (d) 12.5% (e) 30%

(f) 17.86% (g) 8% (h) 4.25% (i) 8.57% (j) 2.08%

3 (a) A (b) C

4 95% 5 60% 6 $63\frac{1}{3}\%$ 7 8%

8 15% 9 15% 10 $66\frac{2}{3}\%$ 11 $31\frac{1}{4}\%$

12 $22\frac{1}{2}\%$ 13 62.5% 14 $151.\dot{1}\ \%$ 15 $8\frac{1}{3}\ \%$

16 33%

17 (a) Nathan = 75%, Dayo = 77.8%.
Dayo is the more accurate scorer.

(b) Nathan = 73.5%, Dayo = 78.1%

(c) Dayo has improved his percentage from the first game to the second.

18 (a) 8.3% (b) 12.5% (c) 22.2%

(d) The value of the percentage increases because the number of hours representing the 'whole' is decreasing.

Open-ended – Sample answers

19 \$1.30, \$1.35, \$1.45

20 Bella 160 cm and dad 180 cm; Bella 156 cm and dad 175 cm; Bella 171 cm and dad 192 cm.

Exercise 2.8 (p. 106)

1 (a) \$12 (b) \$30 (c) \$32 (d) \$20.40

(e) \$209.96 (f) \$42.89 (g) 45 kg (h) 40 kg

(i) 3.6 kg (j) 9.28 min (k) 29.75 min (l) 9.42 min

(m) 1.7 L (n) 46.5 L (o) 18 L

2 (a) B (b) B

3 (a) C (b) C

4 68 questions

5 150 grams

6 42 kg

7 (a) 80% (b) 4400 households

8 (a) 28.6% (b) 117 714 people

9 (a) 116.4 grams

(b) \$5238

10 (a) 10 350 people (b) 12 650 people

11 (a) (i) \$720 (ii) \$1080

(b) (i) \$150 (ii) \$300

12 104 people

13 4385 tonnes

14 (a) 15 people (b) 97.9% (c) 685

15 (a) 62.5 mL (b) 187.5 mL

16 81.2%

17 (a) (i) 2240 litres (ii) 13 440 litres

(b) 67.2 litres (c) 201.6 litres (d) 1.25%

(e) No, the water production is less than 1.5% of total volume.

Open-ended – Sample answers

18 (a) That would be 12.6 books and the answer needs to be a whole number.

(b) 20%, 25%, 30%

19 (a) 8B has 15 more basketball players.

(b) 20% is the percentage amount and so means the same as one-fifth of the 25 students in the class, or 5 students. 20 students is the number of students out of the class, or 80% of the class. It is better to compare things when they are in the same form.

Mathspace (p. 110)

I'm late!

(a) 12 s (b) 9 min (c) 114 mm

The pool of tears

$0.\dot{3} \rightarrow 0.42 \rightarrow 0.\dot{5} \rightarrow \frac{17}{25} \rightarrow 0.75$

(a) shoe (b) FETCH

A fraction of her size

(a) 20% (b) 150% (c) 50%

Off with her head!

(a) 2% is $\frac{2}{100}$, or 0.02. 0.2% is one tenth of 2%, $0.02 \div 10 = 0.002$

(b) The whole is 100%, which is 50 tarts. Gaining another 50 tarts would be a 100% increase, so gaining 100 tarts, which is 2 lots of 50, is a 200% increase.

Exercise 2.9 (p. 115)

1 (a) (i) 1725 (ii) 1875 (iii) 2175 (iv) 3300
(b) (i) 1980 (ii) 1320 (iii) 748 (iv) 110
(c) (i) $103.01 (ii) $368.50 (iii) $86.68 (iv) $86.82
(d) (i) 4.89 km (ii) 1.35 km (iii) 5.05 km (iv) 5.11 km

2 (a) 3.9% (b) 4.2% (c) 14.4% (d) 6.8%

3 C 4 $862.50 5 9.65% 6 6.2%

7 (a) $39 600 (b) $29 250 (c) $5250

8 (a) 68.23 s
(b) 3.41 s
(c) 4%

9 (a) $419 650
(b) $451 123.75
(c) 22%

10 (a) $2.80
(b) $2.60
(c) 4% increase

11 (a) 16 320
(b) (i) 15 667 (ii) 15 040
(c) The three 4% decreases don't give the same result as one 12% decrease, because you are decreasing by 4% of a smaller number each time, rather than by 4% of the original number.

12 (a) 32 (b) three more tests

13 (a) 10% increase (b) 9% decrease
(c) In (a), 150 cm represented 100%.
In (b), 165 cm represented 100%.

14 (a) 5865 tonnes (b) 3 years
(c) Theoretically, no, because the decrease gets smaller every time without ever reaching zero. However, it would eventually become unprofitable to continue.

Open-ended – Sample answers

15 Sample answer for a class of 11 boys and 13 girls:
(a) 54.17% (b) 1.83% (c) 45.83% (d) 2.17%
(e) No, the percentage increase for each gender is not the same. The gender that represents the lower fraction of the class has a greater gain by adding one extra student.

16 Sample answer for a 160 cm student: 12.8 cm

Exercise 2.10 (p. 122)

1 (a) $416 (b) $16.20 (c) $30.19 (d) $259.97

2 (a) $22.95 (b) $43.50 (c) $20 (d) $54

3 (a) $109.89 (b) $58 (c) $39.74 (d) $241.43

4 (a) (i) $26 (ii) 40%
(b) (i) $450 (ii) 23%

5 (a) 167% profit (b) 56% loss
(c) 63% profit (d) 79% profit

6 D 7 D 8 C

9 (a) (i) $37.90 (ii) $341.10
(b) (i) $79.60 (ii) $119.40
(c) (i) $6 (ii) $14
(d) (i) $24.99 (ii) $74.96

10 (a) $8000 (b) 26.7%

11 (a) (i) $56 (ii) $616
(b) (i) $6.40 (ii) $70.40
(c) (i) $1.90 (ii) $20.90
(d) (i) $264 (ii) $2904

12 (a) $245 (b) $2695

13 (a) $125 (b) 64.1%

14 (a) $162.50 (b) 46.43%

15 $450

16 (a) $180 (b) $55.56%

17 (a) $29.95 (b) $253.64 (c) $27 271.82 (d) $36.32

18 $5.14

19 They are easy for people to estimate or calculate mentally, as they correspond to common fractions.

20 (a) $2.30 (b) $4.25 (c) 315%

21 (a) (i) 20% (ii) 40% (iii) 3.57% (iv) 25%
(b) (i) 16.67% (ii) 28.57% (iii) 3.45% (iv) 20%
(c) Calculating the profit using the cost price gives a higher percentage.
(d) This is because when you have made a profit the cost price will be lower than the selling price, so calculating the same amount as a percentage of the cost price will always give a higher figure.

22 (a) $57.38 (b) 27.5% (c) 22.5% less profit

23 $28

24 The second store that is offering 20% off. Here, the T-shirts will cost $47.88, $2 less than the other store's 'buy two, get the third half price' offer.

25 (a) $47 250 (b) $10 500
(c) 4.5 years, but if profit is paid only once a year, then 5 years.

26 (a) (i) $25\% = \frac{1}{4}$ (ii) $10\% = \frac{1}{10}$
(iii) $20\% = \frac{2}{10}$ or $\frac{1}{5}$ (iv) $33\frac{1}{3}\% = \frac{1}{3}$
(b) To find 25% off, Chantal can find a quarter of the price by dividing by 4 (or halving twice), then subtracting this amount. To find 10% off, Chantal should divide by 10, which is as simple as moving the decimal point 1 place to the left, then subtracting this amount. To find 20% off, Chantal could find 10%, then multiply this amount by 2, or divide by 5 and then subtract this amount. To find $33\frac{1}{3}$%, she needs to find a third of the price by dividing by 3, and subtracting this amount.

Open-ended – Sample answers

27 50% off, sale price = $0.5 \times \$299 = \$149.50$
25% off, sale price = $0.75 \times \$299 = \$224.25$
10% off, sale price = $0.9 \times \$299 = \$269.10$

28 Sometimes, it will look more impressive for a store to give a saving as a percentage, particularly when the price is high and a percentage is easily identifiable. In other cases, a dollar discount may look more attractive, particularly when the equivalent percentage discount is relatively small.

29 The second discount of 20% applies to the reduced price, not to the original price. The shop assistant discounted \$150 by 30% to give a sale price of \$105, which is then discounted by a further 20%; i.e. $0.8 \times \$105 = \$84$.

Challenge 2 (p. 132)

1 A

$$\frac{2}{3} + 0.7 + 55\% = \frac{2}{3} + \frac{7}{10} + \frac{55}{100} = \frac{40}{60} + \frac{42}{60} + \frac{33}{60} = \frac{115}{60}$$

$$\frac{115}{60} \div 3 = \frac{115}{180} = \frac{23}{36}$$

2 Ate $\frac{1}{4}$, then ate $\frac{1}{3} \times \frac{3}{4} = \frac{1}{4}$, then ate $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$.

Amount left $= 1 - \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{3}\right) = \frac{1}{6}$.

$\frac{1}{6}$ of the tub of yoghurt remained.

3 $\left(\frac{1}{4} + \frac{1}{2}\right) \div 2 = \frac{3}{4} \div 2 = \frac{3}{8}$

4 Area of new piece is 4 times original piece.
Cost $= \$25 \times 4 = \$100$.

5 A

Save \$1.70 gives a price of $5.04 - 1.70 = \$3.34$

Cost price of $\$3.20 + 5\% = 3.2 \times 1.05 = \$3.36$

Discount of $33\frac{1}{3}\% = 5.04 \times 66\frac{2}{3}\% = 5.04 \times \frac{2}{3} = \$3.36$

Buy 2, get 1 free $= \frac{5.04 \times 2}{3} = \$3.36$

6 $8 \times 8 + 8 \div 8 = 65$

7 $\frac{1}{2} + \frac{0.1}{2} + \frac{1}{0.2} = 0.5 + 0.05 + 5 = 5.55$

8 C Largest $= 5\frac{3}{4}$, smallest $= 1\frac{2}{5}$.

Difference $= 5\frac{3}{4} - 1\frac{2}{5} = 4\frac{7}{20}$.

9 C **(ii)** and **(iv)** are true

10 B $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$,
$2 + 3 + 4 + 5 + 6 + 7 + 8 = 28 + (-1 + 8) = 35$.

Each time I need to add 7 to the first of the seven consecutive numbers to get the last number in the next set.

11 $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

$\frac{3}{4} + \frac{1}{8} = \frac{6}{8} + \frac{1}{8} = \frac{7}{8}$

$\frac{7}{8} + \frac{1}{16} = \frac{14}{16} + \frac{1}{16} = \frac{15}{16}$

This pattern continues until we reach

$\frac{4095}{4096} + \frac{1}{8192} = \frac{8190}{8192} + \frac{1}{8192} = \frac{8191}{8192}$

12 Glass contains $\frac{5}{12}$ of the first liquid and $\frac{7}{12}$ of the second liquid.

Percentage water $= \frac{5}{12} \times 22.5\% + \frac{7}{12} \times 27\% = 25.125\%$

Chapter review 2 (p. 133)

1 **(a)** $\frac{29}{50}$ **(b)** $\frac{31}{2500}$ **(c)** $2\frac{121}{200}$

2 **(a)** $-\frac{3}{10}$, -0.283, $\frac{2}{9}$, 0.26, $\frac{3}{4}$, $\sqrt{7}$

(b) $-\frac{1}{3}$, -0.26, 0.22, 0.253, $\frac{7}{10}$, $\sqrt{2}$

3 **(a)** 28.625 **(b)** 15.075 **(c)** 53.815 625

4 Four 60 cm lengths, with 10 cm left over.

5 **(a)** 13.25 Terminating **(b)** 9.230 384 6… Irrational
(c) 1.222 222… Recurring **(d)** 11.180 339… Irrational
(e) 5.5 Terminating

6 B

7 **(a)** $\frac{2}{9}$ **(b)** $\frac{29}{90}$ **(c)** $\frac{32}{99}$

8 **(a)** $\frac{1}{4}$ **(b)** $-\frac{5}{18}$ **(c)** $\frac{1}{4}$ **(d)** -15

9 **(a)** -0.6 **(b)** -1.6 **(c)** -25.62 **(d)** -4.08

10 B

11 **(a)** 46% **(b)** 120% **(c)** 280% **(d)** $62\frac{1}{2}\%$

12 **(a)** 60% **(b)** 35% **(c)** 112% **(d)** 3%

13 **(a)** $\frac{13}{20}$ **(b)** $\frac{77}{50}$ or $1\frac{27}{50}$ **(c)** $\frac{7}{120}$ **(d)** $\frac{47}{1000}$

14 **(a)** 0.8 **(b)** 0.07 **(c)** 0.0209 **(d)** 0.0625

15 **(a)** 90% **(b)** 218.75% **(c)** 2% **(d)** 6.67%

16 **(a)** 29.4 m **(b)** 88 kg **(c)** 330 L **(d)** \$6.72
(e) \$23.60 **(f)** \$8.55

17 **(a)** **(i)** \$607.50 **(ii)** \$752.40
(b) **(i)** \$15.96 **(ii)** \$18.45

18 **(a)** \$76.50 **(b)** \$1710 **(c)** \$89 **(d)** \$70.27

19 17.24% loss **20** C **21** -\$61.90

22 (a) 13 bags (b) 125 g left

23 $\frac{7}{40}$ **24** 22 500 tonnes

25 70.92% **26** 3 830 040 km^2

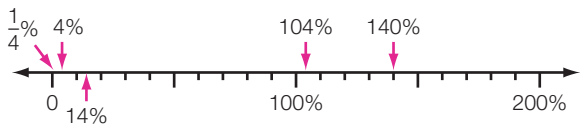
27 (a) \$3200 (b) 34%

28 20% **29** \$614

30 (a) 28 cents per pen (b) 35 cents per pen

31 56.5% **32** -1.3°C

33



34 45 g **35** (a) B (b) B

36 (a) 602 896.5 ML (b) 110 440.5 ML (c) 30 778.5 ML

37 (a) \$65.85 (b) \$5.99 (c) \$26.61
(d) \$49.40 (e) \$4.49 (f) \$11.66

38 (a) $39.58\dot{3}$ cents (b) 40 cents (c) 5 cents
(d) This extra amount comes from the difference between
$40 \times 12 - 39.58\dot{3} \times 12 = 5$

NAPLAN practice 2

1 3 **2** D **3** \$449 400 **4** B

5 B **6** B **7** 25 cents **8** 70%

Mixed review A (p.138)

1 (a) 16 (b) -13 (c) -22 (d) -12
(e) 36 (f) -21 (g) -4 (h) -6

2 (a) $\frac{9}{25}$ (b) $\frac{1}{20}$ (c) $\frac{3}{4}$ (d) $\frac{1}{500}$

3 (a) \$436.80 (b) \$996.80

4 (a) 200 (b) 81 000 (c) -80 (d) -48 (e) 45
(f) 3136 (g) 12 (h) 32 (i) 108

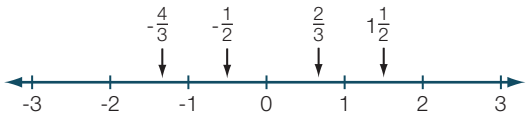
5 (a) 85% (b) 60% (c) 62.5% (d) 57.14%

6 (a) $0.\bar{3}$ (b) $0.\bar{6}$ (c) $0.1\bar{6}$ (d) $0.\bar{5}$

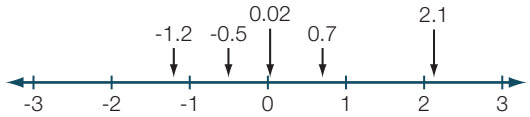
7 (a) (i) 0.7 (ii) $\frac{7}{10}$
(b) (i) 0.45 (ii) $\frac{9}{20}$
(c) (i) 1.2 (ii) $\frac{6}{5}$
(d) (i) 0.08 (ii) $\frac{2}{25}$

8 (a) \$48 (b) 455 kg (c) 3.57 m (d) 0.48 s
(e) \$15.13 (f) 440 m

9 (a)



(b)



10 C

11 (a) -12 (b) 7

12 (a) \$29.60 (b) \$44.40

13 (a) $6^5 \times 5^9 = 15\,187\,500\,000$
(b) $7^6 \times 10^4 \div (5^3 \times 2^2) = 2\,352\,980$
(c) $10^2 = 100$ (d) $4 \times 5^2 = 100$

14 (a) (i) 30×2^6 (ii) 30×2^{12}
(b) (i) 1920 (ii) 122 880

15 $-6\frac{2}{3}$°C

16 $(2 \times 3)^2 = 6^2$
$= 36$

$2^2 \times 3^2 = 4 \times 9$
$= 36$

17 (a) \$52 (b) \$71.20
(c) \$19.20 (d) \$6.80

18 Match 1: 71.4% Match 2: 75%
Match 3: 72.2% Match 4: 75%
Matches 2 & 4; Match 3; Match 1
Casey is not steadily improving—her accuracy fluctuates. She is fairly consistent, however.

19 (a) B (b) B

20 Ali has interpreted 2^4 as 2×4, 3^3 as 3×3 and 5^2 as 5×2. He needs to remember that the index number tells you how many of the base number are multiplied together; e.g. $2^4 = 2 \times 2 \times 2 \times 2$.

Chapter 3

Recall 3 (p. 142)

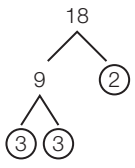
1 (a) $x + 10$ (b) $\frac{m}{3}$ (c) $2l$ (d) $20 - y$

2 (a) 18 (b) -36 (c) 2 (d) 4

3 (a) $l = 9$ (b) $l = -3$

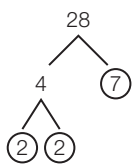
4 (a) False (b) True (c) False (d) True

5 (a)



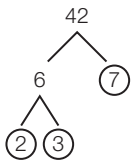
$18 = 2 \times 3^2$

(b)



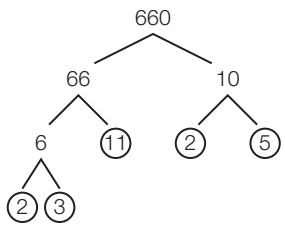
$28 = 2^2 \times 7$

(c)



$42 = 2 \times 3 \times 7$

(d)



$660 = 2^2 \times 3 \times 5 \times 11$

6 (a) 3^4 (b) 6^3 (c) 14^2 (d) 5^7

7 (a) $9 \times 9 = 81$

(b) $(-3) \times (-3) \times (-3) \times (-3) = 81$

(c) $10 \times 10 \times 10 \times 10 - 2 \times 2 \times 2 \times 2 \times 2 = 9968$

8 (a) 2 (b) 5 (c) 2

Exercise 3.1 (p. 145)

1 (a) (i) expression (ii) equation (iii) expression

(b) (i) 7 (ii) -3 (iii) 1

(c) (i) $3d$, $4f$, $-3bd$, -3, $7b$, (ii) $5g$, $-3r$, $5rf$, 7, $-3b$

(iii) b, f, $-3bf$, 8

(d) (i) b, d and f (ii) b, f, g and r (iii) b and f

(e) (i) -3 (ii) 7 (iii) 8

2 (a) $4cd$ (b) $\frac{2t}{7}$ (c) $4gk^2$ (d) $\frac{6}{fg}$

3 (a) $w + r$ (b) $w + r = 8$ (c) $2w$

(d) $r - 2$ (e) $2w + r - 2 = 14$ (f) $w + r - 2$

4 (a) A (b) C (c) A (d) B

5 (a) A (b) D

6 (a) $\frac{x}{6}$ (b) $\frac{h}{9}$ (c) $\frac{6a}{11}$ (d) $\frac{15}{3r}$

(e) $\frac{21}{12v}$ (f) $\frac{4s}{19}$ (g) $\frac{8}{x} - \frac{u}{6}$ (h) $\frac{h}{5} + \frac{4}{i}$

(i) $\frac{cu}{5} + 9y$ (j) $\frac{q}{7c} - \frac{gh}{4}$ (k) $\frac{vz}{6} - \frac{8}{fs}$ (l) $\frac{3}{rt} + \frac{6w}{yz}$

(m) $\frac{4bh}{2r}$ (n) $\frac{6ac}{5eu}$

7 D

8 (a) Dina has seven more pencils than Andrew.

(b) Simon has twice as many pencils as Andrew.

(c) Cindy has two fewer pencils than Simon.

(d) $y - 3$ (e) $y + 10$ (f) $2y - 3$ (g) $2y + 1$

9 (a) $m + n$ (b) $n - 2$ (c) $m + n - 2$

(d) $m + 8$ (e) $m + n + 6$

10 (a) $2x$

(b) There are 3 times as many people as at 3 p.m.

(c) $3x - 5$ (d) $3x + 2$

11 (a) $2g$ (b) $g - 5$ (c) $g + 4$

12 (a) $2m + 1$ (b) $2m + 4$ (c) $\frac{m}{3} + 2$

13 D

14 (a) 10.10 p.m. (b) 7 points (c) 10.40 p.m.

(d) 10 points (e) 10.25 p.m., 10.35 p.m.

15 At 6.00 p.m. there were 20 fewer people than expected. Two people arrived at 10 past, another three at quarter past, and four more at 20 past. Five minutes later one left, but another six people arrived five minutes after that. At 7.00 p.m. three more people arrived; this was the greatest number for the night. Ten left at 8.00 p.m., another five at 9.30 p.m., and the last two at 11.00 p.m.

Open-ended – Sample answers

16 (a) The number of kangaroos originally in the park.

(b) Twice the number of kangaroos originally in the park.

(c) Four more than the number of kangaroos originally in the park.

(d) One less than half the number of kangaroos originally in the park.

17 Refer to Exercise 3.1 Question **3** for sample answers.

Exercise 3.2 (p. 150)

1 (a) 15 (b) 21 (c) 36 (d) 18 (e) 54

(f) 177 (g) 0 (h) 174 (i) 18 (j) 108

(k) 432 (l) 108 (m) -207 (n) 9 (o) 54

(p) 3 (q) $\frac{3}{4}$ (r) 2 (s) 2 (t) 96

(u) $224\frac{1}{2}$

2 (a) -7 (b) -64 (c) -45 (d) 3 (e) -4

(f) 11 (g) 20 (h) 34 (i) -31 (j) 21

(k) -133 (l) -125

3 (a)

x	-2	0	-4	-8
y	-4	-2	-6	-10

(b)

x	-5	-3	-2	-7
y	-9	-7	-6	-11

(c)

x	-7	-1	-5	-9
y	-16	-10	-14	-18

(d)

x	1	2	3	4
y	3	7	11	15

(e)

x	-2	0	1	2
y	-8	-4	-2	0

(f)

x	-4	-7	-20	11
y	2	-7	-46	47

(g)

x	-3	-2	-1	-5
y	18	12	6	30

(h)

x	-1	0	3	5
y	2	-3	-18	-28

(i)

x	-3	-1	2	4
y	15	7	-5	-13

(j)

x	8	6	12	20
y	4	3	6	10

(k)

x	15	9	12	30
y	3	1	2	8

(l)

x	12	-4	8	-24
y	2	-2	1	-7

(m)

x	0	1	2	3
y	0	1	4	9

(n)

x	-2	0	2	4
y	-17	-1	15	127

(o)

x	0	2	4	8
y	1	11	29	89

4 **(a)** 24 **(b)** 150 **(c)** 95 **(d)** 2

(e) -20 **(f)** $-\frac{5}{2}$ **(g)** −2 **(h)** -12

(i) 12 **(j)** 113 **(k)** 13 **(l)** 238

5 **(a)** 22 **(b)** 0 **(c)** 0

(d) 180 **(e)** -36 **(f)** -60

(g) $\frac{1}{3}$ **(h)** $\frac{4}{5}$ **(i)** 5

(j) -2 **(k)** $-\frac{1}{3}$ **(l)** $-\frac{33}{8}$ or $-4\frac{1}{8}$

6 D **7** A **8** C

9 **(a)** dy **(b)** km **(c)** $(km - dy)$

(d) $(5 \times 21) - (12 \times 7) = 105 - 84 = \$21$ profit

(e) No, because 21 – 21 = -$0. Boon makes no profit.

10 **(a)** $3c - a$ **(b)** $a + 2b - 3c$ **(c)** $25

11 **(a)** 2, otherwise Ravi would have scored a negative number of runs.

(b) $4 = 2 \times 4 - 4$

(c) Ravi scored more runs, as when $r > 4$, $2r - 4 > r$.

12 **(a)** 45 minutes

(b) $\frac{6600}{50} = 132$ minutes, which is more than 2 hours.

(c) $\frac{6600}{55} = 120$ minutes, so it is now possible to package the 2200 biscuits in 2 hours.

Open-ended – Sample answers

13 $M = 1, N = 1$; $M = 3, N = 1$

14 **(a)** $a = -6, b = -2, c = 8$

(b) $a = -10, b = 10, c = 1$; $a = 2, b = 10, c = -5$; $a = 4, b = 5, c = -5$

15 $a = 2, b = 1$; $a = 3, b = 2$; $a = 4, b = 4$

Exercise 3.3 (p. 154)

1 **(a)** 50°F **(b)** 41°F **(c)** 95°F

2 **(a)** 0°C **(b)** 5°C **(c)** 10°C

3 **(a)** 32 **(b)** 6.75

4 D **5** B

6 **(a)** $250 **(b)** $750 **(c)** 8

7 **(a)** 19.6 m **(b)** 78.4 m **(c)** 490 m

8 **(a)** 50°F

(b) The answers are the same.

(c) 40°F

(d) The answer to part **(c)** is 1°F less than the answer to Question **1** **(b)**.

(e) 100°F

(f) The answer to part **(e)** is 5°F greater than the answer to Question **1** **(c)**.

(g) Sample answer: Around 10°C there is little difference. Lower than 10°C it gives slightly lower temperatures, but at higher temperatures it gives a much greater difference in temperature. At 0°C it is 2°F less, at 40°C it is 6°F more.

9 **(a)** 48 000 cm^3 **(b)** 36 000 cm^3 **(c)** 180 000 cm^3

10 **(a)** $7 **(b)** $600

(c) **(i)** Aussie Bank **(ii)** $10 **(iii)** Matilda Bank; $40

11 **(a)** 15 000 m^3 **(b)** 3000

Open-ended – Sample answers

12 10 cm × 2 cm; 5 cm × 4 cm

13 100, 20 and 21; 100, 20 and 24

Exercise 3.4 (p. 164)

1 **(a)** Like **(b)** Unlike **(c)** Unlike **(d)** Like **(e)** Unlike

(f) Like **(g)** Like **(h)** Unlike **(i)** Like **(j)** Like

(k) Like **(l)** Unlike

2 **(a)** $17a$, 17 **(b)** $21f$, 126 **(c)** $5v$, 20

(d) $-3v$, -6 **(e)** $10w$, -20 **(f)** $3d$, -9

(g) $15t + 7d$, 52 **(h)** $90w + 134v$, 358 **(i)** $10a + 2b$, 22

(j) $13f + 3g$, 25 **(k)** $-4b$, -20 **(l)** $5b$, 20

(m) $12d + 9$, 45 **(n)** $12f + 5$, 41 **(o)** $7r - 3u$, -1

(p) $-7m - 11n$, -37 **(q)** $3f + 2fg$, 28 **(r)** $6ij + 20j$, 224

3 **(a)** $3dhw + 2dh + 5d + 8$ **(b)** $10x + 4xyz + 8$

(c) $-8x^2 - 10y + 2$ **(d)** $-5x^3 - 6y + xy$

(e) $18ab + 24a + 15b + 40$ **(f)** $4s - 60st + 20$

(g) $-b - 5b^3 - 5ab$ **(h)** $2a^3 - 8a^2 - 9a - 61$

4 B **5** D **6** D

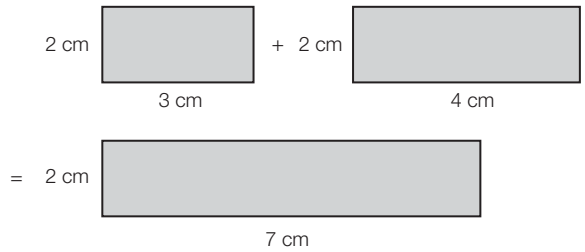
7 **(a)** True **(b)** False **(c)** True

8 **(a)** $8c + 5d$ **(b)** $7c + 6d$

(c) $8c + 5d + 7c + 6d = 15c + 11d$

9 **(a)** $27d$ **(b)** $40d$ **(c)** $200

10 **(a)**



(b) $3y + 4y = 7y$

11 (a) (i) x and x^2 are unlike; (iv) x^2y and xy^2 are unlike

(b) (ii) $5x^2$; x^2 and x^2 are like

(iii) $8xy$; xy and yx are like

(v) $6xy$; xy and xy are like

(vi) $3xy$; xy and xy are like

12 (a) $11x$

(b) Ryan has 4 times as many coins as Jess.

(c) $510x$ cents or $\$5.1x$

(d) No. We don't know how many coins he has of each value.

13 (a) $32x$ (b) $63x$ (c) $\$63vx$

(d) Sample answer: $x = 3$, $v = 2$

Open-ended – Sample answers

14 $a = 10$, $b = 10$; $a = 3$, $b = 4$

15 $x = 1$, $y = 1$; $x = 2$, $y = 2$

16 (a) Lindy forgot that xy means $x \times y$ and that $xy = 1xy$. Only coefficients can be added and subtracted.

(b) If Lindy wrote a 1 in front of the xy, it might remind her to add and subtract only the coefficients of like terms.

Half-time 3 (p. 167)

1 $\$(2n + m)$

2 (a) $17x$ (b) $5y$ (c) $2x + 6y$

(d) $6x^2 + 3xy$ (e) $10b - 16a$ (f) $x + 2a + 2a^2$

3 (a) 1 (b) 22 (c) 8 (d) 58 (e) 485 (f) 255

4 (a)

x	-2	0	1	2
y	-10	-4	-1	2

(b)

x	-3	-1	2	4
y	-3	1	7	11

(c)

x	-1	0	3	5
y	5	3	21	53

5 (a) 36 (b) 38 (c) -1 (d) $11\frac{1}{9}$ (e) 520

6 (a) $t + 1$ (b) $t - 1$

7 (a) $9xy + 3x - 2y$ (b) $7x^2 - 4x - 3xy$

(c) $9m^2 - 4m + 5$ (d) $8pq + 4p^2q - 3p^2q^2 + 4pq^2$

8 (a) (i) 2 (ii) 2 (iii) 3

(b) (i) 3 (ii) -5 (iii) -4

(c) (i) 3 (ii) 1 (iii) 7

(d) (i) 2 (ii) -2 (iii) 0

(e) (i) 3 (ii) $-\frac{1}{2}$ (iii) -5

(f) (i) 3 (ii) 3 (iii) 6

(g) (i) 3 (ii) 1 (iii) -2

(h) (i) 2 (ii) -12 (iii) -10

(i) (i) 1 (ii) 7 (iii) 0

Exercise 3.5 (p. 171)

1 (a) $15a$ (b) $12a$ (c) $18g$ (d) $56z$

(e) $36z$ (f) $54p$ (g) $66ef$ (h) $72mn$

(i) $132jk$ (j) $18ghk$ (k) $48ghk$ (l) $90beh$

(m) $-20y$ (n) $21y$ (o) $16a$ (p) $-44ab$

(q) $18qru$ (r) $-56qru$ (s) $80jk$ (t) $-60jk$

(u) $-48uv$

2 (a) $2a$ (b) $4b$ (c) $7e$ (d) 3

(e) $\frac{5}{6}$ (f) $\frac{1}{15}$ (g) $3ab$ (h) $3f$

(i) $2b$ (j) $\frac{f}{8}$ (k) $\frac{n}{4}$ (l) $\frac{e}{7}$

(m) -2 (n) -2 (o) 4 (p) $\frac{1}{5g}$

(q) $\frac{1}{11h}$ (r) $\frac{1}{4j}$

3 (a) $4c$ (b) $10d$ (c) $2g$ (d) 6

(e) $\frac{2}{3}$ (f) $\frac{10}{11}$ (g) $2cd$ (h) $7cd$

(i) $5gh$ (j) $\frac{h}{3}$ (k) $\frac{5h}{2}$ (l) $\frac{25h}{8}$

(m) $-2d$ (n) $-\frac{5b}{3}$ (o) $-5c$ (p) $-\frac{3}{2}$

(q) $\frac{1}{6}$ (r) 11

4 D 5 A 6 B 7 $60abcd$

8 (a) $18hlw$ (b) $3hjl$

(c) $72hl$ and $36hl$ (d) 2

9 (a) x, $1.2x$, $1.44x$ (b) $3.64x$ mm

(c) $\$3.64dx$ (d) $2y$

(e) $\$6cy$ (f) $\$(3.64dx + 6cy + f)$

(g) \$1540

Open-ended – Sample answers

10 Joshua missed the × sign. The correct answer is $-6k^2$.

11 $-3p^2r$ and $7pr^2$

12 $3mn$, $2mn^2$ and mn^2

Exercise 3.6 (p. 176)

1 (a) 2121 (b) 1734 (c) 1616 (d) 2254 (e) 4059

(f) 3589 (g) 594 (h) 38 380 (i) 70 400

2 (a) $5x + 60$ (b) $2s - 10$ (c) $12a + 60$

(d) $4s + st$ (e) $6l - lr$ (f) $2a + ah$

(g) $12x + 15$ (h) $54x - 9$ (i) $6h + 8$

(j) $2mn + 2mr$ (k) $3pr - 3pz$ (l) $6pr + 6pt$

(m) $5t^2 + 10t$ (n) $10p^2 - 14p$ (o) $16m + 8m^2$

(p) $15m^2 + 21mn$ (q) $4kp - 20k^2$ (r) $20x^2 - 50xy$

3 (a) $35 + 6d$ (b) $24 - 20e$ (c) $-6mn - 24n$
(d) $-3xy - 8x$ (e) $35t - 60kt$ (f) $-5v - 90vw$
(g) $74mn - 40m$ (h) $-14ab - 4a$ (i) $75x - 21xy$
(j) $18tz - 108t$

4 (a) $-x - 8$ (b) $3x - 2$ (c) $2 - 3x$
(d) $21 - 3x$ (e) $6 - 10x$ (f) $4x + 6$
(g) $7 - x$ (h) -68 (i) $-26x + 27$
(j) -14 (k) $x - 27y$ (l) $2x - 24y$
(m) $cd - 6ce$ (n) $de + 2df - 16bd$ (o) $2mn - 5mt + nt$
(p) $2ap + 2aq - 2pq$

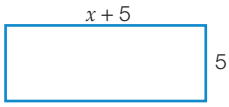
5 (a) C (b) C

6 D

7 $2a(3a - 2b) = 6a^2 - 4ab$

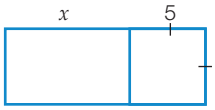
8 (a) (i) $33x + 165$ (ii) 3465
(b) (i) $105y + 315$ (ii) 3465
(c) Both are ways of evaluating 105×33 using the distributive law.

9 (a)



(b) $5(x + 5)$

(c)



(d) Area of left part = $5x$
Area of right part = 25

(e) The area of the first rectangle is $5 \times (5 + x)$ and the second rectangle is $25 + 5x$. Both rectangles are the same size, therefore $5 \times (5 + x) = 25 + 5x$.

10 She multiplied $x\left(x + \frac{1}{x}\right) = x^2 + 1$.

Open-ended – Sample answers

11 Simplify $x(18 - y) + 2x$.

12 (a) The −2 at the end of the first line should be +2.
(b) $3(2x - 1) - (5x - 2) = x - 1$
(c) If there is a minus in front of a bracket, we should write a 1 between the minus and the bracket; i.e. $3(2x - 1) - 1(5x - 2)$.

13 $6e(2f + 2)$, $2ef(6 + 2g)$, $f(12e + 6f + 4)$

Exercise 3.7 (p. 183)

1 (a) 3 (b) 7 (c) 11 (d) 4 (e) 15
(f) 6 (g) $2p^2$ (h) $3t$ (i) $5d^3$ (j) fm^2
(k) a^2b (l) x^2y^2 (m) $5mn^2$ (n) $3st$ (o) $3gh$

2 (a) $3(t + 5)$ (b) $2(h + 7)$ (c) $2(a - 4)$
(d) $3(3p + 1)$ (e) $5(4p - 1)$ (f) $3(5z + 1)$
(g) $4(3 - 5b)$ (h) $2(12 + 5y)$ (i) $2(9 - 7q)$
(j) $3(px + 4)$ (k) $2(kt - 10)$ (l) $2(2hs + 7)$
(m) $-8(ab + 2)$ (n) $-5(mn + 3)$ (o) $-12(kr + 11)$

3 (a) $3x(l + 5)$ (b) $10m(p - 2)$ (c) $14q(p + 2)$
(d) $3x(2v - 1)$ (e) $10a(3b + 2)$ (f) $2y(9x - 20)$
(g) $2p(8 + 7q)$ (h) $5l(3 - 4m)$ (i) $11j(2 - 5l)$
(j) $10m(3z - 2)$ (k) $4k(8h + 9)$ (l) $4h(10 + 7i)$
(m) $-4b(3a - 4)$ (n) $-7x(2y - 3)$ (o) $-8m(9 + 7n)$

4 D 5 B 6 $\$5(ab + 5c)$

7 (a) $\frac{mv^2}{2} + mgh$ (b) $m\left(\frac{v^2}{2} + gh\right)$

8 (a) $(x + 1)(3 - y)$ (b) $(3x - 4)(7 + y)$ (c) $(x + 7)(5 + 2y)$

9 (a) $a(x + y)$ (b) $b(x + y)$ (c) $a(x+y)+b(x+y)$
(d) $(x + y)(a + b)$ (e) $(3 + y)(x + b)$

Open-ended – Sample answers

10 $8a^2b$ and $16c$; $16b^3$ and $24c^2$; $48pr$ and $56s$

11 (a) Area = $x(10 - x) \Rightarrow L = 10 - x$.
(b) $x = 1, 2, 3, 4$

12 Mohammed is correct. Stephen has made the mistake of not taking the factor out of the constant term, 3. He can avoid this mistake by expanding the brackets to check that his factorised expression equals his orginal expression.

Challenge 3 (p. 185)

1 $x + (x + 2) + (x + 4) + (x + 6) = 140$, $4x + 12 = 140$, $4x = 128$, $x = 32$. Numbers are 32, 34, 36, 38.

2 $1 + 3 + 5 + 7$, $1 + 5 + 5 + 5$, $1 + 3 + 3 + 9$, $1 + 1 + 3 + 11$, $1 + 1 + 5 + 9$, $1 + 1 + 7 + 7$, $1 + 1 + 1 + 13$, $3 + 3 + 5 + 5$, $3 + 3 + 3 + 7$. Nine ways.

3 (a) The circles contain the numbers: 0, 2, 3, 5; 0, 1, 4, 5; 1, 2, 3, 4.
(b) Match the position of the expressions with the positions of 0, 1, 2, 3, 4, 5 in that order from (a).
(c) 3, 4, 5, 6, 7, 8 or $x + 1$, $x + 2$, $x + 3$, $x + 4$, $x + 5$, $x + 6$

4 (a) $3^6 = (3^2)^3 = 9$; $4^6 = (4^2)^3 = 16^3$
(b) $(3^2)^3 + (4^2)^3 = 9^3 + 16^3 = (9 + 16)(81 - 144 + 256) = 25 \times 193$. Hence, 25 is a factor.
(c) $(3^2)^3 - 5^3 = 9^3 - 5^3 = (9 - 5)(81 + 45 + 25) = 4 \times 151$. Hence, 4 is a factor.

5

$x + 8$	$x - 6$	$x - 5$	$x + 5$
$x - 3$	$x + 3$	$x + 2$	x
$x + 1$	$x - 1$	$x - 2$	$x + 4$
$x - 4$	$x + 6$	$x + 7$	$x - 7$

6 A

$$\frac{a + \frac{1}{b}}{b + \frac{1}{a}} = \frac{\frac{ab}{b} + \frac{1}{b}}{\frac{ab}{a} + \frac{1}{a}} = \frac{\frac{ab + 1}{b}}{\frac{ab + 1}{a}} = \frac{ab + 1}{b} \times \frac{a}{ab + 1} = \frac{a}{b}$$

Chapter review 3 (p. 186)

1 (a) They have three more power points in their house than you do.
(b) They have two fewer power points in their house than you do.
(c) They have six times as many power points as you do.

2 (a) 22 (b) 28 (c) 20 (d) 4

3 (a) 7 (b) 120 (c) -48

4 (a) -4 (b) 11

5 (a) 7 (b) 8 (c) -10

6 (a) 630 (b) ± 9

7 (a) $-10j$ (b) $3ty$ (c) $3ghi$ (d) $19ab + 7$
(e) $-a + 23b$ (f) Cannot be simplified.

8 (a) $30ab$ (b) $66abc$ (c) $14ab$

9 (a) $\frac{6}{5}$ (b) $5b$ (c) $\frac{b}{11}$

10 (a) $14 + 7a$ (b) $2ab + 6a$ (c) $12a^2 + 20a$

11 (a) $8 + 9a$ (b) $74a + 13ab$ (c) $21a^6 + 16a^3$

12 (a) $3(a - 5)$ (b) $4(5ab + 4)$ (c) $-5(3p - 4q)$

13 Melissa has one packet, and for the next four minutes she eats 2 jelly beans per minute, and 6 during the fifth minute. Roderigo buys two packets in the first minute. He then eats 10 in the second minute, finishes the first packet in the third minute, eats 6 more in the fourth minute, and finishes the second packet in the fifth minute. Huong buys three packets in the second minute. He then eats 5 in the third minute, finishes the first pack in the fourth minute, and eats 8 in the fifth minute.

14 (a) $t + g + b$ (b) $t + g + \frac{b}{2} - 14$ (c) $t + g + \frac{b}{2} - 7$

15 Troy 158 cm; Barry 159 cm, Tenielle 157 cm

16 Yumi 180 cm 17 -24

18 (a) $u + r + d$ (b) $10(u + r + d)$ (c) 140 cm

19 1.40 metres per second

20 (a) $2x + 4$ (b) $6x - 2$
(c) $3(2x + 4) + 5(6x - 2)$ (d) $36x + 2$
(e) 434 apples

21 (a) $7LW$ (b) \$14.00

22 (a) $2x(3x + 2)$ (b) $6x^2 + 4x$ (c) 112 cm^2

23 (a) (i) $\frac{l}{4}(w - 6)$ (ii) $\frac{lw}{4} - \frac{3l}{2}$

(b) $A_{\text{left}} = lw - \left(\frac{lw}{4} - \frac{3l}{2}\right)$

$= lw - \frac{lw}{4} + \frac{3l}{2}$

$= \frac{3lw}{4} + \frac{3l}{2}$

(c) $\frac{3l}{2}\left(\frac{w}{2} + 1\right)$

(d) (i) $A = 320 - 50 = 270$ cm^2

(ii) $A = \frac{3 \times 20}{2}\left(\frac{16}{2} + 1\right) = 270$ cm^2

24 (a) $b + 1$ (b) $a(b + 1)$ (c) $c(b + 1)$
(d) $a(b + 1) + c(b + 1)$ (e) $(a + c)(b + 1)$
(f) (i) $\frac{(a + c)(b + 1)}{a + c}$ (ii) $b + 1$ (iii) b
(iv) It is the same as the second number, b.
(g) Result should be 7, which is equal to b.

NAPLAN practice 3

1 D 2 B 3 B 4 C
5 D 6 B 7 A 8 C

Chapter 4

Recall 4 (p. 192)

1 (a) 200 cm (b) 3600 m (c) 4000 kg (d) 0.35 L

2 (a) $\frac{8}{12}$ (b) $\frac{7}{2}$

3 (a) $\frac{3}{4}$ (b) $1\frac{2}{3}$ (c) $\frac{5}{7}$ (d) $1\frac{6}{19}$

4 (a) $\frac{15}{7}$ (b) $\frac{19}{8}$

5 (a) (i) $2\frac{2}{5}$ (ii) $4\frac{1}{2}$ (iii) $3\frac{1}{8}$ (iv) $5\frac{1}{5}$
(b) (i) 2.4 (ii) 4.5 (iii) 3.125 (iv) 5.2

6 (a) 0.53 (b) 3.90

7 (a) 45 (b) 3.2 (c) 77.52

8 (a) 25% (b) 56.25%

9 (a) $x = 21$ (b) $d = 6$

Exercise. 4.1 (p.195)

1 (a) 4 : 9 (b) 1 : 5 (c) 20% (d) $\frac{7}{9}$

2 (a) 1 : 4 (b) 1 : 5 (c) 1 : 4 (d) 2 : 5
(e) 6 : 17 (f) 6 : 5 (g) 1 : 15 (h) 1 : 8
(i) 9 : 13 (j) 5 : 2 : 1 (k) 6 : 1 : 2 (l) 11 : 5 : 9
(m) 1 : 4 : 2 (n) 9 : 1 : 4 (o) 134 : 100 : 43

3 (a) D (b) A (c) C

4 (a) 3 : 8 : 2 (b) 8 : 13 (c) 3 : 13 (d) $\frac{2}{13}$

5 (a) 2 : 3 (b) $\frac{3}{2}$ (c) $\frac{2}{3}$

6 (a) 2 : 1 (b) 1 : 3 (c) $\frac{1}{2}$ (d) $66.\dot{6}\%$

7 (a) 24 : 1 (b) 4800 mL (4.8 L)

8 (a) 39 : 23 (b) 170%

9 (a) 4 : 1 (b) $\frac{1}{5}$

10 (a) 1 : 49 (b) $\frac{1}{50}$

11 (a) (i) 1 : 1 (ii) 2 : 1

(b) Milan's mixture will produce a paler shade of pink because he has equal parts of red and white, whereas Ali has twice as much red as white.

12 (a) 20% (b) $\frac{3}{11}$

Open-ended – Sample answers

13 (a) 10 : 4; 15 : 6; 20 : 8 (b) 10 : 15; 2 : 3; 100 : 150

14 The ratio of *Simpsons* fans to *Scooby Doo* fans is 3 : 7. The ratio of *SpongeBob* fans to *Dora the Explorer* fans is 8 : 5. The fraction of the total group who liked *Scooby Doo* best is $\frac{7}{23}$.

The number of *Dora the Explorer* fans is $\frac{5}{7}$ of *Scooby Doo* fans.

Exercise 4.2 (p.199)

1 (a) 50 : 3 (b) 1 : 2 (c) 50 : 6 : 3 (d) 16 : 10 : 9
(e) 3 : 10 (f) 70 : 9 (g) 13 : 70 (h) 12 : 1
(i) 3 : 8 (j) 4 : 25 (k) 2 : 3 (l) 5 : 4
(m) 8 : 3 (n) 30 : 1 (o) 9 : 10 : 2 (p) 2 : 14 : 7
(q) 8 : 25 : 2 (r) 95 : 100 : 3

2 (a) 7 : 2 (b) 12 : 11 (c) 12 : 7 (d) 41 : 16
(e) 5 : 3 (f) 5 : 4 (g) 20 : 9 (h) 31 : 24
(i) 12 : 7 (j) 23 : 54 (k) 49 : 72 (l) 9 : 10

3 (a) 47 : 31 (b) 17 : 52 (c) 29 : 34 (d) 5 : 8
(e) 15 : 31 (f) 17 : 21 (g) 94 : 103 (h) 44 : 29
(i) 35 : 22 (j) 162 : 55 (k) 55 : 166 (l) 71 : 170

4 B

5 (a) 7 : 6 (b) 1 : 2 (c) 11 : 5 (d) 4 : 5
(e) 16 : 9 (f) 9 : 5

6 (a) 9 : 26 (b) $\frac{9}{26}$

7 (a) 3 : 5 : 7 (b) 250 : 600 : 1000 (or 5 : 12 : 20)

8 (a) 684 : 11 (b) $\frac{11}{684}$ (c) 170 : 89 (d) 191%

9 (a) 5 : 3 (b) $\frac{3}{5}$ (c) 40%

10 (a) 40 : 25 : 9 (b) 37 : 20 (c) 34%

11 (a) 4 : 3 (b) 7 : 4

12 3 : 2

13 (a) 3 : 2 (b) 6 : 4 : 5 (c) $\frac{2}{5}$

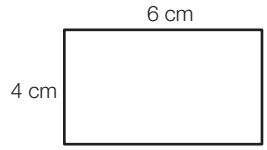
14 (a) 10 : 5 : 3 (b) $\frac{1}{6}$ (c) 250 mL

15 (a) 3 : 80 : 20 (b) 2 : 7 : 4 (c) ride, 54%

(d) You can cycle faster than you can run, and you can run faster than you can swim.

Open-ended – Sample answers

16 Your classmate's rectangle may be a different size, but the ratio of side lengths will be the same.



17 Troy could spend equal time training for each discipline, or he could use one of the ratios found in Question **15** to structure his training. Alternatively, if he feels that he is weaker in one particular discipline, then he may wish to focus his training on that.

Exercise 4.3 (p. 207)

1 (a) 1.06 : 1 (b) 1.40 : 1 (c) 2.33 : 1 (d) 0.78 : 1
(e) 0.21 : 1 (f) 1.47 : 1 (g) 4.89 : 1 (h) 5.31 : 1
(i) 0.71 : 1 (j) 0.75 : 1 (k) 1.12 : 1 (l) 2.17 : 1

2 (a) 170.5 cm (b) $595
(c) 88.1 days (d) 57.14 g

3 (a) (i) 2.7 L (ii) 22.2 kg
(b) 48 300 000 sheep
(c) $50 (d) 45.53 cm

4 (a) 21.25 (b) 10.82 (c) 1.80 (d) 5.29

5 6667 km

6 (a) 267 067 km (b) 133.11 cm or 1.33 m

7 0.32 L or 320 mL

8 (a) 0.25 : 1 (b) 3.97

9 105 kilometres per hour

10 (a) 1.78 : 1 (b) 42.19 cm

11

Quantities	Ratio	Unit ratio	Scale factor	Explanation
$a : b$	4 : 5	0.8 : 1	0.8	a is 0.8 of b
$x : y$	7 : 1	7 : 1	7	x is seven times as large as y
$m : p$	12 : 4	3 : 1	3	m is three times as large as p
$g : h$	240 : 12	20 : 1	20	h is one-twentieth the size of g
$r : j$	3.8 : 15.2	0.25 : 1	0.25	r is one-quarter the size of j
$d : e$	4000 : 20	200 : 1	200	d is 200 times as large as e

12 (a) 36 (b) 30 (c) 72 (d) 5280 (e) 2
(f) multiply by 36 (g) multiply by 5280

13 (a) 1.035 : 1 (b) $348.99

14 1.89 m tall son or 1.62 m tall son. The 1.62 m is the more likely as the 12-year-old son is more likely to be shorter and not taller than his dad.

15 (a)

Length (inches)	Width (inches)	*L* : *W*	Unit ratio
6	4	6 : 4	1.5 : 1
7	5	7 : 5	1.4 : 1
8	6	8 : 6	1.33 : 1
12	8	12 : 8	1.5 : 1
16	12	16 : 12	1.33 : 1
18	12	18 : 12	1.5 : 1

(b) 6 by 4, 12 by 8 or 18 by 12. These paper sizes have the same unit aspect ratio as the image, 1.5 : 1.

(c) (i) 7.5 inches

(ii) This image length is too long for the paper.

(d) The extra 0.5 inches of length could be 'cropped' from the image. Alternatively, recommend that the image be printed on different-sized paper.

Open-ended – Sample answers

16 (a) 2-year-old dog = 11.4; 10-year-old cat = 63; 6-month-old mouse = 12.5

(b)

Animal	Life expectancy (years)
Dog	13
Cat	12
Rabbit	6
Budgie	10
Horse	20
Guinea-pig	4
Blue-tongue lizard	21
Mouse	3

(c) A high value means a short-lived animal.

17 (a) Possible dimensions in the ratio 4 : 3 are: 100 cm long and 75 cm wide, 80 cm long and 60 cm wide, 120 cm long and 90 cm wide.

(b) Possible dimensions in the ratio 16 : 9 are: 112 cm long and 63 cm wide, 160 cm long and 90 cm wide, 168 cm long and 94.5 cm wide.

(c) The aspect ratio of the wall space dimensions is 1.4 : 1—closer to the aspect ratio of 4 : 3 ($1.\dot{3} : 1$).

(d) Multiplying the width of 1.2 m by the aspect ratio 4 : 3 gives a length of 1.6 m, only 10 cm short of the maximum amount of wall space.

Exercise 4.4 (p. 219)

1 (a) 9 (b) 56 (c) 45 (d) 6
(e) 10 (f) 30 (g) 117 (h) 500
(i) 400 (j) 108 (k) 50 (l) 180

2 (a) 20 mL (b) 9 lessons
(c) 16 teachers (d) 160 mL

3 (a) 16.5 (b) 12.8 (c) 4.44 (d) 27.5
(e) 13.5 (f) 52.5 (g) 0.6 (h) 4.4
(i) 10.5 (j) 3.25 (k) 7.5 (l) 7.2

4 (a) 449 cm (b) 42 cm (c) 42 hairy caterpillars
(d) (i) 1.8 hours (ii) 1 hour 48 minutes

5 (a) D (b) C

6 (a) A (b) C

7 5.4 m

8 (a) 135 g (b) 210 g

9 (a) $835 714 (b) $1 285 714

10 2.5 kg of cement, 5 kg of sand

11 $2.05

12 (a) 5100 (b) 4250 (c) $\frac{1}{14}$ (d) 11 900

13 (a) 125 : 132 : 138 (b) $1.37, $1.44 (c) $1.18, $1.31

Open-ended – Sample answers

14 Multiples of nine would be ideal but the following examples show how classes could be arranged if we assume that seven or eight in a class is economically viable. Fewer than 6 isn't economically viable. 7–9: 1 class, 10–13: only one class (some miss out), 14–18: 2 classes, 19 and 20: take only 2 classes (some miss out), all other numbers up to 90 can be arranged so that the class sizes are a minimum of 7 without anyone missing out.

15 171 cm, 144 cm; 162 cm, 108 cm; 168 cm, 112 cm

16 (a) 10, 12, 8; 25, 30, 20

(b) As 15 does not divide evenly into 100 the test cannot be made to add to exactly 100; 90 and 105 are the closest.

Half-time 4 (p. 223)

1 (a) 7 : 3 (b) 17 : 5 (c) 1 : 1

2 125 m^2

3 (a) 7 : 8 (b) 3 : 2 (c) 200 : 7 (d) 15 : 2 (e) 50 : 9 (f) 13 : 3

4 (a) 5 : 2 : 3 (b) $\frac{1}{5}$ (c) $\frac{1}{2}$

5 (a) 24 (b) 6 (c) 60 (d) 286

6 84 years

7 452 m

8 (a) 11 : 14 (b) 127% (c) 27%

Exercise 4.5 (p. 228)

1 (a) 1 : 600 (b) 1 : 400 (c) 1 : 250
(d) 1 : 60 (e) 1 : 800 (f) 1 : 500
(g) 1 : 25 (h) 1 : 125 (i) 1 : 100 000
(j) 1 : 50 000 (k) 1 : 40 000 (l) 1 : 25 000

2 (a) 2 m (b) 3 m (c) 5 m
(d) 0.8 m (e) 2.3 m (f) 1.9 m
(g) 40.3 m (h) 92.5 m (i) 120 m

3 (a) 5 mm (b) 10 mm (c) 20 mm (d) 1 mm
(e) 100 mm (f) 200 mm (g) 8 mm (h) 3 mm
(i) 13 mm (j) 12 mm (k) 2 mm (l) 0.6 mm

4 (a) 60 m (b) 100 m (c) 200 m
(d) 52 m (e) 768 m (f) 86 m
(g) 18 m (h) 35.8 m (i) 102 m

5 (a) No (b) No (c) Yes (d) Yes (e) No (f) No

6 D

7 (a) 7 m (b) 5 m (c) 80 m (d) 40 m
(e) 0.3 m (f) 120 m (g) 630 m (h) 5200 m
(i) 176 m

8 (a) 20 mm (b) 8 mm (c) 3500 mm
(d) 4000 mm (e) 240 mm (f) 3 mm
(g) 25 mm (h) 8 mm (i) 90 mm

9 (a) 19 m (b) 35 m (c) 10 m (d) 2 m

10 (a) 23 m (b) 11.5 m

11 (a) 850 km (b) 1200 km (c) 3000 km (d) 3700 km

12 (a) (i) 40 mm × 100 mm (or 4 cm × 10 cm)
(ii) 80 mm × 200 mm (or 8 cm × 20 cm)
(iii) 100 mm × 250 mm (or 10 cm × 25 cm)
(iv) 50 mm × 125 mm (or 5 cm × 12.5 cm)
(b) 1 : 200

13 (a) 29 mm (b) 55 mm (c) 17.5 mm (d) 10 mm

14 (a) 25 mm (b) 19 mm (c) 6.5 mm (d) 13.5 mm

15 0.6 mm

16 (a) 7.2 cm^2 (b) 2700 cm by 2160 cm
(c) 5 832 000 cm^2
(d) 6480 cm^2. No. To find the actual area, both the length and the width have been multiplied by 900, so the actual area has been multiplied by 900^2.
(e) 27 m by 21.6 m
(f) 583.2 m^2
(g) Answer in m^2 = 10 000 × area in cm^2

Open-ended – Sample answers

17 (minimum) 1 : 700; 1 : 500; (maximum) 1 : 280

18 1 : 300 000, 1 : 380 000
Even though the second scale will fill the page more completely, the first scale is easier to work with.

19 drawing plans of small pieces of machinery, drawing insects, drawing microscopic objects

Exercise 4.6 (p. 234)

1 (a) 12 : 18 (b) 15 : 5 (c) 35 : 7 (d) 6 : 33
(e) 64 : 20 (f) 24 : 14 (g) 28 : 21 : 7 (h) 12 : 24 : 6
(i) 6 : 21 : 27 (j) 6 : 15 : 12 (k) 10 : 15 : 25 (l) 8 : 56 : 32

2 (a) Kayla 27 jelly beans; little brother 18 jelly beans
(b) 14 lamingtons in one container, 49 in another and 21 in the final

3 D

4 won 15, lost 6

5 400 mL of concentrate and 1600 mL of water

6 200 mL oil, 4800 mL petrol (or 0.2 L oil, 4.8 L petrol)

7 140 boys and 160 girls

8 Karine $23 750; André $14 250; Vivian $9500; Julian $9500

9 (a) N 900 kg, P 600 kg, K 1200 kg
(b) 3 : 3 : 4

10 (a) first group 16 squares; second group 8 squares
(b) 4 (c) 2

11 256 cm or 384 cm from one end

Open-ended – Sample answers

12 The bill could be divided equally and each pays approximately $38.33. 3 drinks, 3 main courses and 1 dessert means there is a total of 7 parts. Glenda and Petra could pay $\frac{2}{7}$ or $32.86 each, whereas Joan could pay $\frac{3}{7}$ or $49.29. A fairer method might be to let a main course be 2 parts, and drinks and dessert 1 part each, making a total of 10 parts. Joan pays $\frac{4}{10}$ or $46, while the others pay $\frac{3}{10}$ or $34.50 each.

13 (a) Ms Footsie $15 million, Ms Nikkei $27 million
(b) That way, they each get back their investment and then share the profit equally.

Exercise 4.7 (p. 238)

1 (a) 40 km/h (b) 50 km/h (c) 66.67 km/h
(d) 38.46 km/h (e) 93.33 km/h (f) 8.20 m/s
(g) 2.03 m/s (h) 10.5 km/h

2 (a) 521 g (b) $49.38 (c) $54.17 (d) 8.25 stalks

3 (a) Sydney 1642 people/week, Melbourne 1798 people/week, Brisbane 1002 people/week, Australia 8783 people/week
(b) Sydney 1.93%, Melbourne 2.40%, Brisbane 2.67%, Australia 2.12%
(c) Sydney 4 591 405, Melbourne 4 091 430, Brisbane 2 057 776, Australia 22 420 707

4 (a) D (b) B

5 (a) $123.3 c/L (b) $125.6 c/L
(c) $145.5 c/L (d) $142.1 c/L

6 (a) 86 400 beats per day (b) 115 200 beats per day

7 (a) 5.5 hours (b) 240 km

8 $13\frac{1}{3}$ cups self-raising flour, $6\frac{2}{3}$ tablespoons icing sugar, $2\frac{1}{2}$ cups milk, $2\frac{1}{2}$ cups thickened cream, 3 eggs (impossible to have $3\frac{1}{3}$ eggs)

9 (a) 4.3 runs per over (b) 4.325 runs per over
(c) 1.25 runs per ball (d) 0.764 runs per ball

10 (a) (i) \$6.50 (ii) \$3.25

(b) (i) \$0.90 (ii) \$2.70

(c) (i) \$28 (ii) \$70

(d) (i) \$7.50 (ii) \$10.50

11 (a) 29.2345 kL (b) 9.2334 kL

(c) 12.4972 kL (d) 10.5445 kL

12 (a) (i) US\$494.57 (ii) 826.23 Brazilian reals

(iii) 309.20 British pounds

(b) \$3057.35 (c) \$574.56

13 (a)

World	(i) 6844.7 million	(ii) 6922.2 million
Australia	(i) 22.7 million	(ii) 23.1 million
China	(i) 1336.6 million	(ii) 1343.2 million
India	(i) 1189.1 million	(ii) 1205.5 million
Indonesia	(i) 245.7 million	(ii) 248.4 million

(b) (i) Australia (ii) China

(c) The population is not changing.

(d) The population is decreasing.

14

	Per serving: 17 g (one biscuit)	Per 100 g
Protein	0.884 g	5.2 g
Fat – total	4.539 g	26.7 g
Fat – saturated	2.227 g	13.1 g
Carbohydrate – total	11.237 g	66.1 g
Carbohydrate – sugars	4.692 g	27.6 g

15 (a) 2.17 km/h (b) 18.42 km

16 (a) 90 m/min (b) 5.4 km/h

17 (a) 10.4 m/s (b) 1 hour 7 minutes

(c) 9.53 m/s, 5.87 m/s (d) Marathon, 0.92

(e) 4.90

18 (a) (i) \$230.11 (ii) \$273.22

(iii) \$199.47 (iv) \$350.87

(b) (i) 905.51 kWh (ii) 1047.92 kWh

(iii) 1066.38 kWh (iv) 1155.82 kWh

19 (a) No change in population.

(b) The population actually decreased (when the rate dropped below zero).

(c) A decade around 2%, briefly reached up over 3%, then dropped to be around 1%–1.5% for several decades, until recently climbing back up over 2%.

(d) A sharp drop down to nearly -1%, then a sharp climb back up to nearly 3% for a decade. Slowly decreased to around 1–1.5%, before decreasing further in recent years to around 0.5%.

(e) Severe famine and natural disasters drove the population of China in the 1960s. Later on, the 'one-child' policy brought the rate down slowly. Australia has never had such a policy. Australia also had a lot of migration in the 1960s and 1970s. Australia also has an aging population, meaning that people are living longer.

(f) The actual populations of the two countries in 2009.

20 (a) Babies being born, people moving to Australia (immigration)

(b) Even though the growth rate is getting smaller, it is still positive. The population is still growing, it is just growing more slowly.

Open-ended – Sample answers

21 A high birth rate, people moving to or from the city or country, people living longer (and therefore a slower death rate), natural disaster.

22 If your hair grows 2 cm and you go to the hairdresser every 8 weeks, your hair grows at a rate of 2.5 mm/week.

Challenge 4 (p. 245)

1 To be half an hour apart they must each have to travel for 30 minutes. At 80 km/h, 30 min is 40 km; at 90 km/h, 30 min is 45 km. They are 85 km apart.

2 Kevin takes 6 hours to do one-third of the work, Sanna takes 6 hours to do two-thirds of the work. Kevin takes 18 hours on his own.

3 Two cats eat three tins of cat food in three days. Two cats eat six tins of cat food in six days.

4 Glass contains $\frac{5}{12}$ of the first liquid and $\frac{7}{12}$ of the second liquid.

Percentage water $= \frac{5}{12} \times 22.5\% + \frac{7}{12} \times 27\% = 25.125$

5 Let the distance from A to B be x km.

Time from A to B $= \frac{x}{40}$ hours. Time from B to A $= \frac{x}{60}$ hours.

Total time to cover $2x$ km $= \frac{x}{40} + \frac{x}{60} = \frac{3x + 2x}{120} = \frac{5x}{120}$

$= \frac{x}{24}$ hours.

Average speed $= 2x \div \left(\frac{x}{24}\right) = \frac{48x}{x} = 48$ km/h.

6 In one hour, the first pipe would fill $\frac{1}{3}$ of the tank, the second pipe would fill $\frac{1}{5}$ of the tank and the third pipe would fill $\frac{1}{10}$ of the tank.

Hence, in 1 hour you fill $\frac{1}{3} + \frac{1}{5} + \frac{1}{10} = \frac{10 + 6 + 3}{30} = \frac{19}{30}$ of the tank.

Hence, it takes $\frac{30}{19} = 1\frac{11}{19}$ hours or 1 hour 34 minutes 44 seconds to fill the tank using all three pipes.

7 Each revolution of the first wheel results in $\frac{32}{36} = \frac{8}{9}$ of a revolution of the second wheel.

Thus, the first wheel rotates 9 times for the second wheel to rotate 8 times, and the cogs would again come together if they had started together.

First wheel: 128 rev/s = 128 × 60 rpm = 128 × 60 × 60 rev/hour = 128 × 60 × 60 × 24 revolutions in a 24-h period.

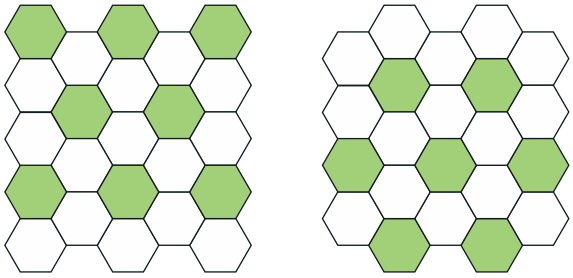
Number of times together if they start apart

$= \frac{128 \times 60 \times 60 \times 24}{9} = 1\,228\,800$ times.

If they started together, they would come together 1 228 801 times.

8 (a) Each green tile is surrounded by 6 white tiles. Each white tile is surrounded by 3 white and 3 green tiles. Hence, each white tile is used with 3 different green tiles, so the ratio of green tiles to white tiles is 1 : 2.

(b) Each diagram shows all the tiles needed to cover an approximately square area.



First diagram has 8 green tiles and 15 white tiles, ratio green : white = 8 : 15 ≈ 1 : 2.

The second diagram has 7 green tiles and 15 white tiles, ratio green : white = 7 : 15 ≈ 1 : 2.

9 (a) 50 days (b) 83.3 days (c) 8.3 days
(d) 2.5 days (e) 0.83 days
(f) The more people you have the more likely they will get in each other's way and, eventually, adding more people will not help.

Chapter review 4 (p. 246)

1 (a) (i) 4 : 5 (ii) 5 : 4 : 3 (iii) 3 : 4 (iv) 5 : 12
(b) 42% (c) $\frac{3}{4}$

2 (a) 7 : 12 (b) 5 : 1 : 2 (c) 5 : 2
(d) 9 : 2 (e) 4 : 7 (f) 2 : 1

3 D

4 (a) 2.93 : 1 (b) 0.52 : 1 (c) 0.60 : 1

5 2.7 6 1.42 L

7 (a) 77 (b) 140 (c) 11
(d) 18 (e) 56 (f) 13.5

8 (a) 8 (b) 5

9 (a) 1 : 25 (b) 1.2 L (1200 mL)

10 (a) 1 : 200 (b) 1 : 2 400 000 (c) 1 : 5000

11 (a) 4 m (b) 280 m (c) 260 m

12 (a) 11.3 mm (b) 190 mm (c) 490 mm

13 (a) 20 : 30 (b) 42 : 30 (c) 12 : 20

14 (a) 4 km/h (b) 75 km/h
(c) $24 000 per game (d) $1.67 per litre
(e) (i) $9 (ii) $7.50
(f) $10\frac{1}{2}$ potatoes, 700 g pumpkin

15 10:30 a.m.

16 (a) 2 : 5 (b) 40%

17 9.6 cups of fruit

18 125 kg

19 45 L

20 971.5 g

21 1.6

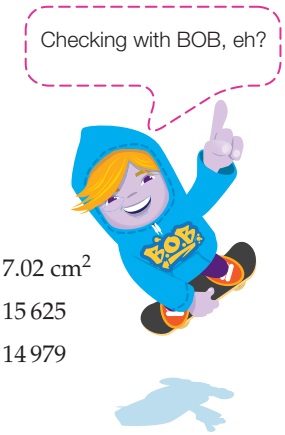
22 120 coins, 160 coins and 240 coins

23 (a) 3.375 m × 3.25 m (b) 7.02 cm^2
(c) 109 687.5 cm^2 (d) 15 625

24 (a) 1.09% (b) 14 979

25 315 mL

26 (a) at least 9.3 runs per over
(b) 10.25 runs per over (c) 33 runs



NAPLAN practice 4

1 A 2 B 3 D 4 D
5 8.75 L 6 D 7 300 g
8 B 9 A 10 B

Mixed review B (p. 250)

1 (a) $\frac{23}{100}$ (b) $\frac{1}{250}$ (c) $\frac{1}{8}$ (d) $\frac{7}{1000}$

2 (a) 14% (b) 3% (c) 28.6% (d) 190%

3 (a) 7.2 (b) 3.12 (c) 283.8

4 (a) $68 (b) $552.50

5 (a) $2^6 = 64$ (b) $5^2 = 25$ (c) $15^0 = 1$

6 (a) $2a^2$ (b) $7t^2 + 6t$ (c) $12q + 8q^2 - 3p^2$

7 (a) 2 (b) 19 (c) 318

8 (a) $35jk$ (b) $7p$ (c) $6a^3b$

9 (a) $5(h - 9)$ (b) $y(y + 7)$ (c) $4m(3n + 2)$

10 (a) 0.17 (b) 0.065 (c) 4 (d) 0.000 06

11 (a) -15 (b) 9 (c) -30 (d) -4
(e) $-\frac{43}{40}$ or $-1\frac{43}{40}$ (f) $\frac{-7}{24}$ (g) -5.33 (h) -1.44

12 (a) 12 (b) 11 (c) 7 (d) 15 (e) 7.2 (f) 10.8

13 (a) 30 : 18 (b) 140 : 490 (c) 16 : 24 : 48

14 84%

15 (a) 9% (b) 85% (c) $18\frac{3}{4}$% (d) 225%

16 (a) 654 (b) 58.32 (c) 270

17 (a) 5 : 11 (b) 7 : 3 (c) 5 : 12 (d) 7 : 10
(e) 15 : 11 (f) 5 : 158

18 84 km/h

19 (a) 0.7, 76%, $\frac{7}{9}$ (b) 0.105, $\frac{3}{20}$, 15.2% (c) 500%, 5.06, $5\frac{1}{5}$

20 (a) \$5610 (b) \$5329.50

21 187 cm

22 (a) \$104 (b) 19.66%

23 $31\frac{1}{4}$% 24 \$1343.75

25 (a) 32°F (b) 68°F (c) 95°F (d) 302°F

26 Gerard \$5000, Hao \$7500

27 (a) \$133.10 (b) \$110 (c) 21%

Chapter 5

Recall 5 (p. 254)

1 (a) 7.82 (b) 15.00 (c) 0.99 (d) 106.50

2 (a) 24.78 (b) 27.56 (c) 37.68 (d) 67.31
(e) 59.85 (f) 845.68

3 (a) 4500 m (b) 0.69 m (c) 48 mm (d) 4.6 cm
(e) 6.78 m (f) 0.09 km

4 (a) 4.65 m (b) 5.064 m

5 (a) (i) $P = 20$ cm (ii) $A = 24$ cm^2
(b) (i) $P = 12$ cm (ii) $A = 6$ cm^2

6 (a) 420 m^3 (b) 91.125 cm^3

Exercise 5.1 (p. 256)

1 (a) 28.3 km (b) 46 m (c) 203 cm
(d) 15.75 m (e) 24 cm (f) 15.2 cm
(g) 160 mm (h) 104 mm (i) 106 m
(j) 15 m (k) 200 cm (l) 180 cm

2 (a) 28.5 cm (b) 96 mm (c) 7.2 cm
(d) 14.8 cm (e) 146 m (f) 82 mm
(g) 124 m (h) 21 km (i) 184 mm

3 (a) 228 mm (b) 33 m (c) 196 cm (d) 17 mm

4 73.2 m

5 (a) 74 mm or 7.4 cm (b) 88 mm or 8.8 cm
(c) 340 cm or 3.4 m (d) 270 cm or 2.7 m
(e) 10 600 m or 10.6 km (f) 6100 m or 6.1 km

6 74 m

7 (a) D (b) D

8 (a) 8000 m (b) \$46 000 (c) 400 h

9 (a) 15 cm (b) 12 cm

10 (a) 800 cm (8 m)
(b) 1200 cm (12 m), 2160 cm (21.6 m), 4080 cm (40.8 m)

Open-ended – Sample answers

11 Triangle: 3 different lengths that add to 120 cm where the total of the 2 smallest must be more than the longest; e.g. 35 cm, 40 cm, 45 cm. Quadrilateral: 4 different lengths that add to 120 cm where the total of the 3 smallest must be more than the longest; e.g. 20 cm, 25 cm, 30 cm, 45 cm. Pentagon: 5 different lengths that add to 120 cm where the total of the 4 smallest must be more than the longest; e.g. 10 cm, 20 cm, 25 cm, 30 cm, 35 cm.

12 (a) 20 m, 20 m, 16 m; 16 m, 16 m, 24 m; 22 m, 22 m, 12 m
(b) 15, 30, 30 or 19, 38, 38

Exercise 5.2 (p. 261)

1 Approximate answers:

Circle	Radius r (cm)	Diameter d (cm)	Circumference C (cm)
A	1	2	6.3
B	2	4	12.6
C	3	6	18.8
D	4	8	25.1
E	5	10	31.4
F	6	12	37.7

2 (a) The diameter is twice the radius. The circumference is approximately three times the diameter, or six times the radius.
(b) Students' own answers.

3 (a) 3 (b) 6 (c) double

4 (a) 2 m (b) 12 m

5 (a) doubled (b) doubled

6 (a) hard to follow curve of circle with string, measuring instruments not very accurate; human error
(b) (i) They would both be smaller.
(ii) They would both be bigger.

7 (a) In a non-terminating, non-recurring number, the digits do not form a repeating pattern and there is never a final digit.
(b) (i) 3.14 (ii) 3.142 (iii) 3.1416

Open-ended – Sample answers

8 (a) $d = 10$ cm, $C = 31.4$ cm
$d = 16$ cm, $C = 50.2$ cm
$d = 13$ cm, $C = 40.8$ cm
(b) $d = 4$ cm, $C = 12.6$ cm
$d = 6$ cm, $C = 18.8$ cm
$d = 5$ cm, $C = 15.7$ cm

9 diameter = 75 m, lap of honour = 235 m

Exercise 5.3 (p. 268)

1 (a) 12.57 m (b) 6.28 cm (c) 15.71 mm
(d) 94.25 cm (e) 113.10 m (f) 144.51 m
(g) 141.37 m (h) 94.25 mm (i) 182.21 mm
(j) 62.83 cm (k) 502.65 km (l) 219.91 cm

2 (a) 23.14 m (b) 41.13 m (c) 7.14 cm
(d) 10.71 cm (e) 16.85 m (f) 39.13 m

3 (a) C (b) B

4 7.54 m

5 12.57 m

6 520 mm

7 (a) Square: $P = 56.57$ cm, $\frac{P}{D} = 2.83$

Hexagon: $P = 60$ cm, $\frac{P}{D} = 3$

Octagon: $P = 61.2$ cm, $\frac{P}{D} = 3.06$

Circle: $P \approx 62.83$ cm, $\frac{P}{D} \approx 3.14$

(b) hexagon (c) octagon

(d) Regular nonagon or decagon, or any other regular shape with more than 8 sides.

8 (a) 56.56 cm (b) 80 cm (c) 62.83 cm

The value seems suitable, as the circumference should be somewhere between the perimeter of the inside square and the perimeter of the outside square.

Open-ended – Sample answers

9 Any value between 9.72 cm and 11.67 cm is suitable.

10 $C = \pi d$ where $\pi \approx 3.14$, so $\times 100 \approx 314$, $\pi \times 1000 \approx 3140$ etc., so $d = 1000, 10\,000, \ldots$

11 (a) $8 \times 4 = 32$ cm gives perimeter of a square that is bigger than the circle, so 50.27 cm is too big. $11 \times 3 = 33$ cm gives perimeter of an equilateral triangle smaller than the quarter circle, so 17.28 cm is too small.

(b) First: $2\pi r$ has been used where d was given, not r. Second: Straight parts were not included.

(c) 25.13 cm, 39.28 cm

Exercise 5.4 (p. 280)

1 (a) 50 000 cm^2 (b) 65 cm^2 (c) 9400 mm^2
(d) 976 mm^2 (e) 120 000 m^2 (f) 10 030 m^2
(g) 5 ha (h) 98 cm^2 (i) 6.7 m^2
(j) 0.095 m^2 (k) 236 000 m^2 (l) 0.34 ha
(m) 0.046 m^2 (n) 34 500 mm^2 (o) 9 km^2
(p) 55.7 km^2

2 (a) 26.66 m^2 (b) 30.25 km^2 (c) 65 cm^2
(d) 60 m^2 (e) 144 m^2 (f) 80 cm^2
(g) 2.34 cm^2 (h) 105 m^2 (i) 120 mm^2
(j) 0.88 km^2 (k) 60 cm^2 (l) 224 mm^2

3 (a) 33 cm^2 (b) 40 mm^2 (c) 40 m^2
(d) 52 mm^2 (e) 104 m^2 (f) 150 cm^2

4 (a) 117 cm^2 (b) 7.5 m^2 (c) 130 mm^2
(d) 147 cm^2 (e) 12 km^2 (f) 360 mm^2

5 (a) (i) 1.68 m^2 (ii) 16 800 cm^2
(b) (i) 1.54 m^2 (ii) 15 400 cm^2
(c) (i) 0.7 m^2 (ii) 7000 cm^2
(d) (i) 0.39 m^2 (ii) 3900 cm^2

6 (a) D (b) A

7 3.45 m^2

8 C

9 (a) 32 m^2 (b) $1024

10 D

11 (a) 750 000 m^2 (b) 75 ha (c) 0.75 km^2

12 1487.5 m^2

13 (a) 10 368 cm^2 (b) 640 cm (c) 2592

14 50 cm^2

15 (a) 6.4 cm (b) 16 cm

16 (a) 24 cm
(b) 16 cm, 32 cm, parallel sides add to 32 cm

17 (a) (i) 30 cm (ii) 16 cm
(b) (i) 13.5 cm (ii) 3 cm

18 1.3 m and 3.2 m

19 (a) 69.36 m^2 (b) 32.04 m

Open-ended – Sample answers

20 6 m × 8 m, 12 m × 4 m, 8 m × 6 m

21 (a) 9 m + 7 m, 10 m + 6 m, 11 m + 5 m, 12 m + 4 m, 13 m + 3 m, 14 m + 2 m, 8 m + 8 m
(b) 16 m

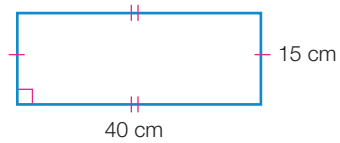
22 (a) Missed the brackets around 10 + 4.
(b) 42 cm^2

Half-time 5 (p. 286)

1 (a) 94.25 cm (b) 10.05 m

2 312 cm

3 840 cm^2, 120 cm

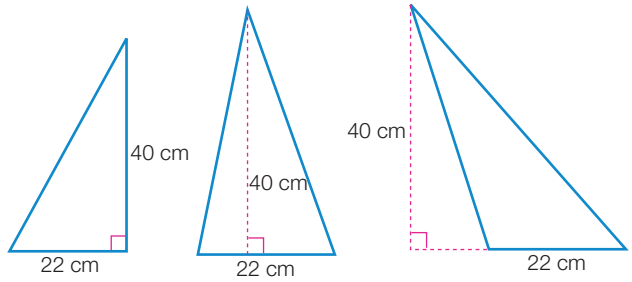
4


5 128 mm

6 (a) 205.66 cm (b) 60.70 mm

7 (a) 440 cm^2

(b) Possible triangles:



8 31 m

9 17.28 cm

10 an estimate between 75 cm and 85 cm

Exercise 5.5 (p. 290)

1 (a) 78.54 m^2 (b) 153.94 cm^2 (c) 12.57 cm^2
(d) 50.27 m^2 (e) 113.10 km^2 (f) 28.27 cm^2
(g) 254.47 mm^2 (h) 615.75 m^2 (i) 1385.44 cm^2

2 (a) 39.27 cm^2 (b) 6.28 cm^2 (c) 14.14 m^2
(d) 3.14 m^2 (e) 28.27 mm^2 (f) 38.48 mm^2

3 (a) 3848 cm^2 (b) 3632 cm^2

4 (a) C (b) C 5 110.84 cm^2

6 (a) 8825 m^2 (b) \$66 187.50

7 31.42 cm^2

8 (a) 201.06 m^2 (b) 28.27 m^2 (c) 172.79 m^2

9 B 10 7.07 m^2

11 (a) 13 $units^2$ (b) 13 $units^2$ (c) 25 $units^2$ (d) 29 $units^2$

12 D

13 (a) 78.54 cm^2 (b) 1.18 m^2 (c) 0.72 m^2

14 (a)

	r	C	C^2	A	$\frac{C}{A}$	$\frac{C^2}{A}$
(i)	1	6.28	39.48	3.14	2	12.57
(ii)	2	12.57	157.91	12.57	1	12.57
(iii)	5	31.42	986.96	78.54	0.4	12.57
(iv)	10	62.83	3947.84	314.16	0.2	12.57

$\frac{C}{A}$ is not constant; $\frac{C^2}{A} = 12.57$

(b) $\frac{C^2}{A} = 12.57$ (c) $\frac{12.57}{\pi} = 4$ (d) $\frac{C^2}{A} = 4\pi$ or $C^2 = 4\pi A$

Open-ended – Sample answers

15 $r = 16$ cm, $l = 21$ cm, $w = 17$ cm gives $A \approx 447$ cm^2

16 (a) No, the correct answer is 33.18 m^2

(b) Simon has used the diameter instead of the radius in his working. He must be careful to check what value he is substituting into his formula. Ajay has doubled the radius instead of squaring it. He must be sure he knows the difference between multiplying a number by 2 and multiplying a number by itself.

Exercise 5.6 (p. 299)

1 (a) 2160 cm^2 (b) 4000 cm^2 (c) 12.28 m^2
(d) 25.13 cm^2 (e) 9.42 cm^2 (f) 12 cm^2

2 (a) 1264 cm^2 (b) 15.31 m^2 (c) 52.73 cm^2
(d) 1884.25 cm^2 (e) 3.14 m^2 (f) 16.57 cm^2

3 C 4 4.21 m^2 5 12.57 m^2

6 (a) 194.25 cm, 893.14 cm^2 (b) 12.28 m, 10.28 m^2

7 (a) 53 m^2 (b) \$344.50

8 (a) 465.13 m^2 (b) 310.08 m^2 (c) 389.70 m^2

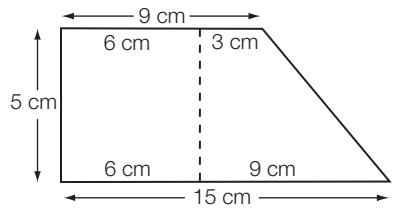
9 B

10 (a) (i) 399.34 m (ii) 462.17 m
(b) (i) 10 390.39 m^2 (ii) 14 697.91 m^2 (iii) 4307.52 m^2

11 (a) 0.43 m^2 (b) 2.60 m^2 (c) 3.14 m^2 (d) 0.54 m^2

Open-ended – Sample answers

12



13 (a) Perimeter: for the semi-circle, diameter is not part of the boundary; the curve is only half of a circumference; for the total, 5 should be added twice. Area: for the rectangle, the perimeter has been used instead of that for area; for the semi-circle, the formula for diameter has been used instead of radius; the final subtraction has been performed in the wrong order.

(b) $P = 66$ cm, $A = 161$ cm^2

Mathspace (p. 304)

Darwin: 25 cm; Perth: the same, i.e. 5027 m^2; Adelaide: 12 km; Melbourne: 20.2 cm; Hobart: 220 000 m^2; Canberra: 58.5 cm^2; Sydney: 166.5 m; Brisbane: 5.5.

Exercise 5.7 (p. 311)

1 (a) 477 cm^3 (b) 196 cm^3 (c) 198 cm^3
(d) 80 cm^3 (e) 1188 cm^3 (f) 1125 m^3
(g) 452.39 cm^3 (h) 21.99 mm^3 (i) 942.48 cm^3

2 (a) 20 000 mm^3 (b) 195 000 mm^3 (c) 8 300 000 cm^3
(d) 25 460 000 cm^3 (e) 750 000 000 m^3
(f) 57 000 000 000 m^3 (g) 0.15 cm^3
(h) 3.83 cm^3 (i) 0.047 9 m^3 (j) 0.006 07 m^3
(k) 480 000 cm^3 (l) 0.000 097 5 km^3 (m) 0.000 36 km^3
(n) 2 500 000 m^3 (o) 40 mm^3 (p) 35 600 cm^3

3 (a) 2 L (b) 55 L (c) 0.8 L
(d) 0.04 L (e) 5000 mL (f) 95 000 mL
(g) 30 mL (h) 500 mL (i) 25 mL
(j) 48 mL (k) 33 cm^3 (l) 140 cm^3

(m) 5.8 kL (n) 1.7 kL (o) 0.13 m^3

(p) 0.05 m^3

4 (a) 630.0 mL (b) 15 878.9 mL (c) 83.2 mL

5 (a) (i) 980 000 cm^3 (ii) 0.98 m^3

(b) (i) 360 000 cm^3 (ii) 0.36 m^3

(c) (i) 729 000 cm^3 (ii) 0.729 m^3

(d) (i) 753 982.24 cm^3 (ii) 0.75 m^3

(e) (i) 3 716 504.11 cm^3 (ii) 3.72 m^3

(f) (i) 3 216 990.88 cm^3 (ii) 3.22 m^3

6 (a) A (b) C

7 440 cm^3

8 (a) 1.696 m^3 (b) 1696 L

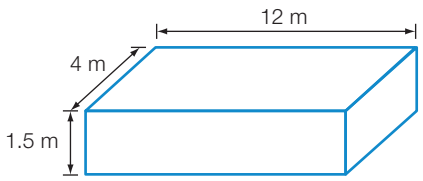
9 (a) 486 m^2 (b) 486 m^2

10 1.70 m^3 11 4.5 L 12 C

13 (a) 795 mL (b) 0.795 L

14 368 mL

15 (a)



(b) $120.96

16 1000 17 3 with about 190 mL left over

18 27

19 (a) 1000 L (b) 400 m^2

Open-ended – Sample answers

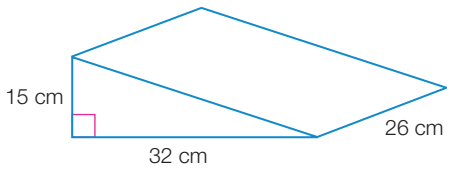
20

d (cm)	3	4	5	6	7
H (cm)	70.7	39.8	25.5	17.7	13.0

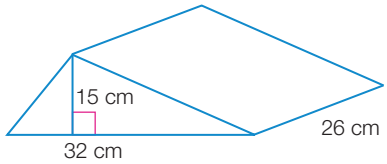
A 'handy' size would be best.

21 (a) The bases of the triangular prisms have the same dimensions as the base of the rectangular prism (15 cm and 26 cm). The base of the rectangular prism can be cut to produce the base of both triangular prisms. Extending this cut along the height of the rectangular prism (32 cm) gives the triangular prisms. The volume of each triangular prism is half of the volume of the rectangular prism.

(b)



or



Exercise 5.8 (p. 320)

1 (a) (i) 0230 (ii) 1430

(b) (i) 0615 (ii) 1815

(c) (i) 1143 (ii) 2343

(d) (i) 1200 (ii) 0000

2 (a) 1.54 p.m. (b) 8.33 a.m. (c) 5.39 a.m.

(d) 4.34 p.m. (e) 6.30 p.m. (f) 7.02 p.m.

(g) 1.47 a.m. (h) 3.20 a.m.

3 (a) 2 h 46 min (b) 4 h 15 min (c) 2 h 46 min

(d) 4 h 51 min (e) 2 h 17 min (f) 3 h 3 min

(g) 10 h 9 min (h) 22 h 41 min (i) 14 h 33 min

(j) 17 h 12 min

4 (a) 7.15 a.m. (b) 0623 (c) 1402 (d) 7.19 p.m.

(e) 4.18 p.m. (f) 0730 (g) 1251 (h) 8.38 p.m.

(i) 5.04 p.m. (j) 2113

5 (a) 1.00 p.m. (b) 3.45 p.m. (c) 12.15 a.m. (d) 8.40 a.m.

6 (a) 2130 (b) 2100

7 (a) A (b) B (c) C

8 (a) True (b) True (c) True

(d) False (e) False (f) True

9 7.37 a.m.

10 (a) 8.32 a.m. (b) 27 min

11 A 12 0655

13 (a) 4.50 a.m. (b) 6.50 a.m.

14 (a) 10:35 a.m. and 10:30 p.m. (b) 4:01 a.m. and 3:17 p.m.

(c) 39 min (d) 5 hours and 57 min

(e) 8 June and 9 June (f) 2 hours and 39 min

15 8 a.m. Thursday

16 (a)

State	Standard Time	Daylight Saving Time
WA	10 a.m.	10 a.m.
NT	11.30 a.m.	11.30 a.m.
SA	11.30 a.m.	12.30 p.m.
Qld	12 noon	12 noon
NSW	12 noon	1 p.m.
Vic	12 noon	1 p.m.
Tas	12 noon	1 p.m.

(b) 2.00 p.m. (c) 11.30 a.m.

(d)

Perth	09:45 a.m.
Darwin	11:15 a.m.
Adelaide	12:15 p.m.
Sydney	12:45 p.m.
Melbourne	12:45 p.m.
Brisbane	11:45 a.m.

17 Yes he will, as he arrives in Geneva at 1805.

18 25 years, 10 months, 11 days (or 12 days depending on how the months and days are counted)

19

Recruit training day	
Activity	**Starting time**
Parade ground drill	0800
Navigation lesson	0830
Military history lesson	0930
Fitness session	1030
Lunch/shower	1200
Weapons handling lesson	1235
Weapons handling practice	1320
Bush camp	1535
End	1635

20 (a) 4 h (b) 4 h 53 min

21 arrive Hong Kong at 1.50 p.m., arrive Tokyo at 4.30 p.m., arrive Auckland at 12.45 p.m., arrive Bangkok at 2.30 p.m.

Open-ended – Sample answers

22 Elizabeth has written two times incorrectly as decimal numbers. 1 h 30 min is $1\frac{1}{2}$, or 1.5 h, not 1.3 h. 45 min is $\frac{3}{4}$ of an hour, or 0.75, not 0.45 h. There are 60 minutes in 1 hour, not 100, so times cannot be written as decimals—the 'point' between hours and minutes is a separator, not a decimal point.

A correct way to add up the times would be to add the hours: $1 + 2 + 1 + 1 = 5$, and then add the minutes: $30 + 45 = 75$ min $= 1\frac{1}{4}$ hours, so the total time is $6\frac{1}{4}$ hours.

23 Five breaks of 4 min 24 s each

24 Trip to see a movie; swim and walk at the beach; a leisurely lunch; a football game

25 Advantage: Get up soon after daybreak, giving more daylight time in the evening.
Disadvantage: Difficult to fall asleep when the Sun is up late and it's still hot.

Challenge 5 (p. 328)

1 C Half full 1 minute before 10 a.m., a quarter full 1 minute before that.

2 C $2\pi r = 14 \times 4$. $r = \frac{28}{\pi}$. $A = \pi \times \left(\frac{28}{\pi}\right)^2 = 249.55$

3 B

4 Area $GDEH = \frac{1}{2}(30 + 90) \times 60 = 3600$. Let $CG = x$.
$90x + 1800 = 3600$. $CG = 20$ cm.

5 A Journey takes 45 minutes.

6 29 August is a Thursday, so 31 August is a Saturday. 7 September is the first Saturday, so 14 September is the second Saturday.

7 $20 \div 2 = 10$, so she will take the tablets on 10 occasions, the last one being $9 \times 8 = 72$ hours after she takes the first ones, or 3 days.

8 10 hours after 9 a.m. the time is 7 p.m., the clock will have lost $10 \times 3 = 30$ minutes, so it will show 6.30 p.m. Correct time is 7 p.m.

9 To get 5 pieces you need 4 cuts, so each cut takes $40 \div 4 = 10$ seconds. To get 8 pieces you need 7 cuts, so it takes $7 \times 10 = 70$ seconds, or 1 min 10 seconds.

10 Turn them both before cooking the rice. When the 7-minute timer runs out, start cooking, as there are 4 minutes left in the 11-minute timer. After the 4 minutes is over (the 11-minute timer has run out) turn the 11-minute timer upside-down to cook for another 11 minutes.

11 From when the train enters the tunnel to when the end just leaves the tunnel, the front of the train will have travelled $2 + 1 = 3$ km. At 20 km/h it takes $3 \div 20$ hours; that is, 9 minutes.

12 A normal year has 52 weeks and 1 day, so a given date falls a day later each year. A leap year has 52 weeks and 2 days, so a given date falls 2 days later from March of a leap year.

2009, Friday; 2010 Saturday; 2011 Sunday; 2012 leap year, Tuesday; 2013 Wednesday; 2014 Thursday; 2015 Friday. The year is 2015.

13 8th to 14th of the month

Chapter review 5 (p. 329)

1 (a) 8 km (b) 72 km (c) 71 cm

2 (a) 31 m (b) 24, 25 or 26 cm (all acceptable)

3 (a) 182.21 cm (b) 232.48 mm (c) 395.84 cm

4 (a) 0.51 m^2 (b) 5 309 291.59 m^2 (c) 3.96 m^2
(d) 0.01 m^2 (e) 0.03 m^2 (f) 76.97 m^2

5 (a) 557 cm^2 (b) 130 cm^2 (c) 240 m^2

6 (a) 120 cm^3 (b) 99 m^3 (c) 6361.73 cm^3

7 (a) 45 mL (b) 2750 mm^2 (c) 16 000 000 cm^3
(d) 1.575 L (e) 0.36 ha (f) 275 000 mm^3
(g) 65 cm^3 (h) 53 000 000 m^3 (i) 0.25 m^3
(j) 90 000 m^2

8 2385.40 cm^3

9 (a) 10 h 43 min (b) 1344

10 (a) 1938 hours (b) 11.21 p.m.

11 39 times

12 The can has the greater capacity by 256 mL.

13 160 cm, 1800 cm^2 14 7.15 p.m.

15 (a) 3.10 m^3 (b) 21 m

16 (a) 1950 m^3 (b) 1950 kL (c) 63

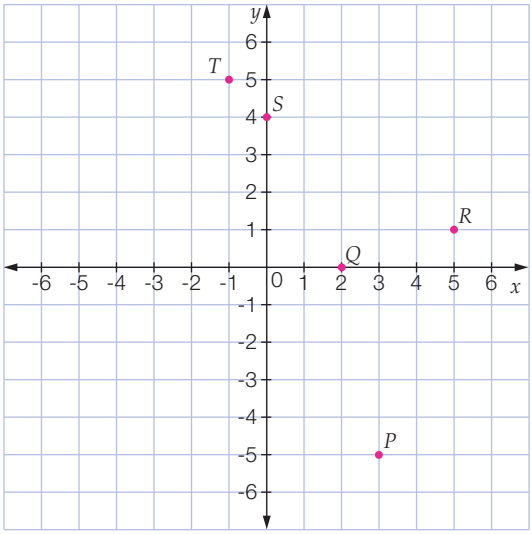
NAPLAN practice 5

1 C 2 B 3 C 4 B
5 B 6 B 7 B 8 D

Chapter 6

Recall 6 (p. 336)

1



2 $A(7, 2)$, $B(2, 0)$, $C(0, 5)$, $D(2, 3)$, $E(4, 2)$, $F(0, 0)$

3 (a)

x	1	2	3	10
y	2	5	8	29

(b)

x	0	2	6	10
y	4	5	7	9

4 (a) $a = 4$ (b) $m = -1$

5 (a) $x = 4$ (b) $x = 2$

(c) $x = 15$

6 (a) $3x$ (b) $2x + 8$

7 (a) $9x - 2y$ (b) $-x - 14y$

Exercise 6.1 (p. 338)

1 (a) (i) The family are starting their journey in Melbourne. They are travelling away from Melbourne (constant speed).

(ii) The car is stationary.

(iii) The family are travelling back to Melbourne (constant speed).

(b) section A (c) section C (d) 240 km

2 (a) No, as the line does not start at zero on the 'Distance from home' axis.

(b) The horizontal parts of the graph show when Josie stopped.

(c) Friend's place.

(d) Yes, as the line finishes at zero distance from home.

3 (a) F (b) D (c) B (d) C (e) E (f) A

4 (a) B (b) C (c) A

5 A 6 B 7 B

8 (a) 1400 m (b) tortoise (c) 150 min

(d) 55 min into the race, 500 m from the start

(e) The hare ran very quickly for 1000 m, then went back 500 m to see where the tortoise was. He decided to wait for the tortoise and probably fell asleep for about 140 min. He then tried to catch the tortoise, but was too late. The tortoise won the race.

9 (a) Red line: average house prices in Sydney from 1986 to 2008.

Green line: average house prices in Perth from 1986 to 2008.

Blue line: average house prices in Australia from 1986 to 2008.

(b) Year

(c) 22 years

(d) Sydney. The sale price will be higher than in Perth.

(e) Sydney: \$340 000, Perth: \$160 000; save \$180 000.

(f) (i) increased demand, lower interest rates

(ii) decreased demand, higher interest rates

(g) No, because Melbourne prices are part of the national average.

(h) 2006–2007

10 (a) unemployment rate, expressed as a percentage

(b) 2001

(c) 2005

(d) 2000 to 2002

(e) 0.7%

(f) 3 years

(g) 4 years

(h) 2001, the year of greatest unemployment.

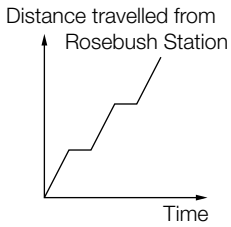
Open-ended – Sample answers

11 Students' own answers. For example:

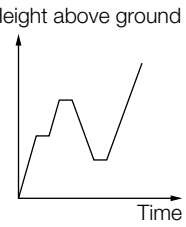
Let the vertical axis represent Jodie's distance from school. Jodie walks at a constant rate away from school. She reaches the shop where she stops for a time. She then walks at a constant rate back towards school. She stops to rest at the park and then walks at a constant rate all the way back to school.

12 At a particular distance from home, a person travels towards home, then stops at a position for a period of time, then returns to the place they started from. From there, the person travels directly home.

13 (a)



(b)



(c) Students' own answers.

Exercise 6.2 (p. 350)

1 (a) (i)

x	-2	-1	0	1	2
y	-1	0	1	2	3
x, y	(-2, -1)	(-1, 0)	(0, 1)	(1, 2)	(2, 3)

(ii)

x	-2	-1	0	1	2
y	-1	-0.5	0	0.5	1
x, y	(-2, -1)	(-1, -0.5)	(0, 0)	(1, 0.5)	(2, 1)

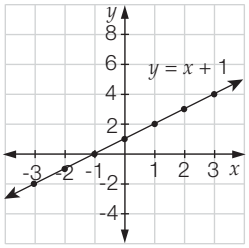
(iii)

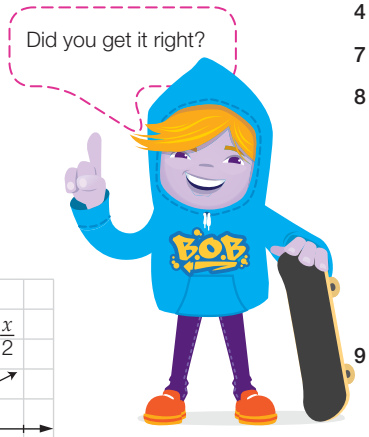
x	-2	-1	0	1	2
$3x$	-6	-3	0	3	6
$3x - 1$	-7	-4	-1	2	5
(x, y)	(-2, -7)	(-1, -4)	(0, -1)	(1, 2)	(2, 5)

(iv)

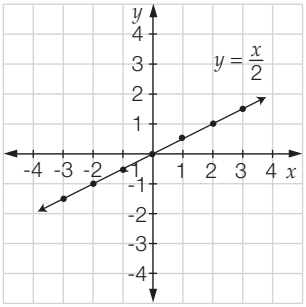
x	-2	-1	0	1	2
$-x$	2	1	0	-1	-2
$-x - 4$	-2	-3	-4	-5	-6
(x, y)	(-2, -2)	(-1, -3)	(0, -4)	(1, -5)	(2, -6)

(b) (i)

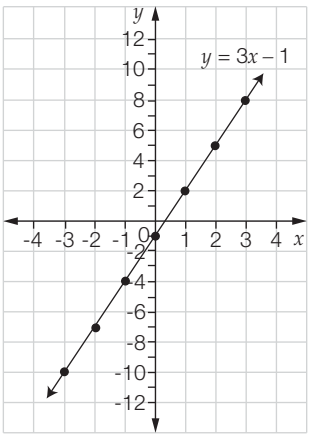




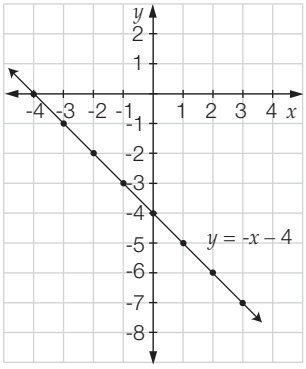
(ii)



(iii)



(iv)



2 (a) (i) 6 (ii) -1 (iii) -2 (iv) 9

(b) (i) -5 (ii) 8 (iii) 5 (iv) -2

3 (a) (i) -2, (-2, 0) (ii) 2, (2, 0) (iii) 2, (2, 0) (iv) -0.5, (-0.5, 0)

(b) (i) 2, (0, 2) (ii) 1, (0, 1) (iii) 2, (0, 2) (iv) 1, (0, 1)

(c) (i) positive (ii) negative (iii) negative (iv) positive

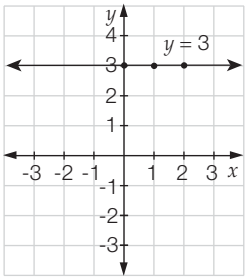
4 D **5** A **6** D

7 (a) A **(b)** C **(c)** D

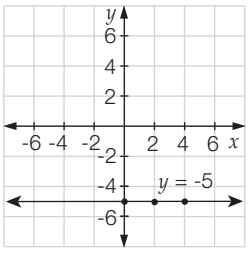
8 (a) Lines with positive gradient slope up to the right.
Lines with negative gradient slope down to the right.
Horizontal lines have zero gradient.
Parallel lines have the same gradient.
Vertical lines have undefined gradient.

(b) (i) A, B, C, G (ii) D, F, H, I (iii) E
(iv) A and C; F and D (v) J (vi) B

9 (a) (i)



(ii)



(b) Each relationship is linear, as the graph is a straight line.

(c) zero

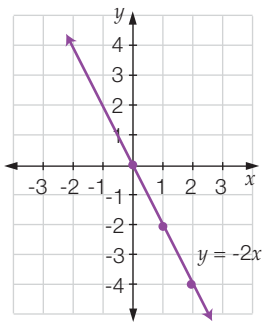
(d) parallel to the x-axis

(e) (i) no x-intercept; y-intercept is 3

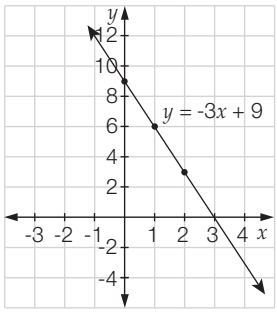
(ii) no x-intercept; y-intercept is -5

10 C

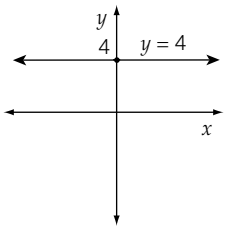
11 (a)



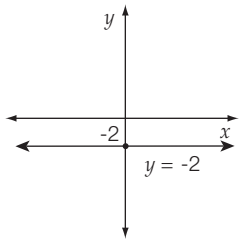
(b)



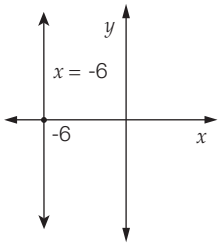
12 (a)



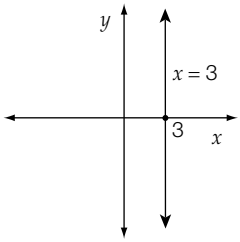
(b)



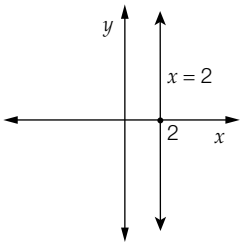
(c)



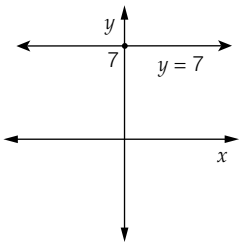
(d)



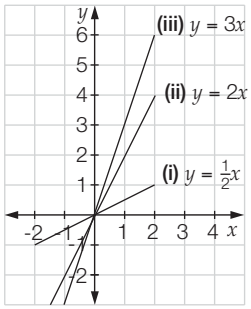
(e)



(f)



13 (a)



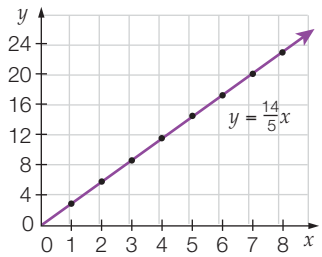
(b) The larger the coefficient, the steeper the line for positive coefficients, but for negative coefficients, the smaller the coefficient the steeper the line.

14 (a)

x	y	(x, y)
0	0	(0, 0)
1	2.8	(1, 2.8)
2	5.6	(2, 5.6)
3	8.4	(3, 8.4)
4	11.2	(4, 11.2)
5	14	(5, 14)
6	16.8	(6, 16.8)
7	19.6	(7, 19.6)
8	22.4	(8, 22.4)

(b) The horizontal distance cannot be less than zero.

(c)



(d) 7.1 metres

15 (a) negative **(b)** positive **(c)** negative **(d)** positive

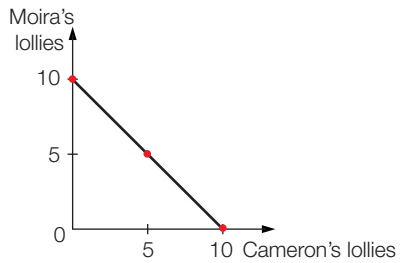
(e) positive **(f)** zero

16 (a) Sample answer:

Cameron	10	5	0
Moira	0	5	10

(b) Neither value can be negative, so the graph can't cross the x- or y-axis.

(c)



(d) negative

(e) As Moira gets fewer lollies, Cameron gets more.

(f) **(i)** 5.7

(ii) Cameron would have 4.3 lollies, Moira would have 5.7 lollies.

(iii) No, because it is not possible to have a decimal number of lollies.

17 It is true when the line is not vertical or horizontal.

18 (a)

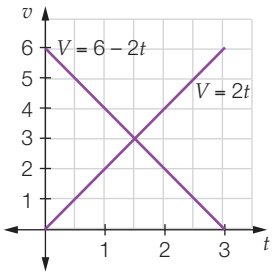
t	0	1	2	3
v	6	4	2	0
(t, v)	(0, 6)	(1, 4)	(2, 2)	(3, 0)

(b) The container is empty.

(c)

t	0	1	2	3
v	0	2	4	6
(t, v)	(0, 0)	(1, 2)	(2, 4)	(3, 6)

(d)



(e) Time and volume must be positive quantities. Therefore, it does not make sense to use values of t less than 0 or greater than 3.

(f) 1.5 minutes, 3 litres

Open-ended – Sample answers

19 (-2, 4), (1, $5\frac{1}{2}$), (-4, 3), (2, 6)

20 (a) $x = 2, y = -2$

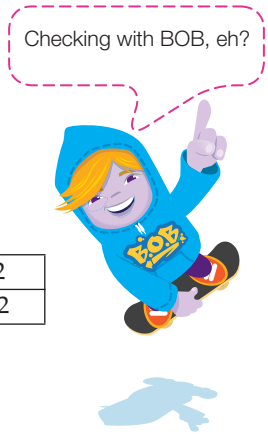
(b) $x = 3, y = 2 \times 3 - 3 = 3$

$x = 6\frac{1}{2}, y = 2 \times 6\frac{1}{2} - 3 = 10$

Rachelle ignored the brackets.

(c)

x	3	8	2
y	0	10	-2



21 Students' own answers.

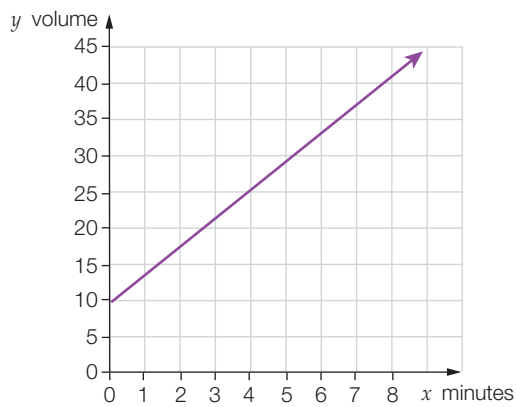
For example: Complete a table of values and draw the graph using the rule $y = x - 1$.

Half-time 6 (p. 358)

1 (a)

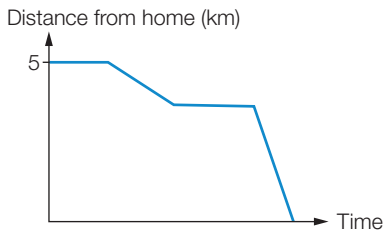
x (minutes)	3	4	5	6	7	8
y (volume of water in container, mL)	22	26	30	34	38	42

(b), (c)



(d) 10 millilitres

2



3 (a) 3 (b) Dale stopped to rest once.

(c) No. The end of the graph does not finish on the horizontal axis.

4 A

5 (a) Rose (b) During month 4.

(c) Between months 1 and 7 (the fern overtook the rose during month 7)

(d) 40 cm

(e) month 5

(f) 3 months

Exercise 6.3 (p. 366)

1 (a) $y = x - 1$ (b) $y = x + 1$ (c) $y = x + 6$

(d) $y = x - 3$ (e) $y = x$ (f) $y = 4x$

(g) $y = 5x$ (h) $y = -x$

2 (a) $y = x + 1$ (b) $y = x - 3$ (c) $y = -2x$

(d) $y = 7x$ (e) $y = -\frac{x}{3}$ (f) $y = \frac{x}{5}$

3 (a) $y = x$ (b) $y = 3x$ (c) $y = x + 3$

(d) $y = x + 5$ (e) $y = x - 2$ (f) $y = 8x - 4$

(g) $y = -4x$ (h) $y = -2x$

4 (a) D (b) C (c) A

5 D

6 (a) $y = x + 1$

x	-3	-2	-1	0	1	2	3
y	-2	-1	0	1	2	3	4

(b) $y = x + 2$

x	-3	-2	-1	0	1	2	3
y	-1	0	1	2	3	4	5

(c) $y = x - 5$

x	-3	-2	-1	0	1	2	3
y	-8	-7	-6	-5	-4	-3	-2

(d) $y = x - 3$

x	-3	-2	-1	0	1	2	3
y	-6	-5	-4	-3	-2	-1	0

(e) $y = 2x$

x	-3	-2	-1	0	1	2	3
y	-6	-4	-2	0	2	4	6

(f) $y = 3x$

x	-3	-2	-1	0	1	2	3
y	-9	-6	-3	0	3	6	9

(g) $y = -8x$

x	-3	-2	-1	0	1	2	3
y	24	16	8	0	-8	-16	-24

(h) $y = -6x$

x	-3	-2	-1	0	1	2	3
y	18	12	6	0	-6	-12	-18

7 (a) F (b) E (c) G (d) H

8 (a) $V = t + 2$ (b) \$1 400 000

9 (a) $D = 2S + 4$

(b) 2 dots are added to make each new square.

(c) 4 dots needed: 3 at one end and 1 at the other end.

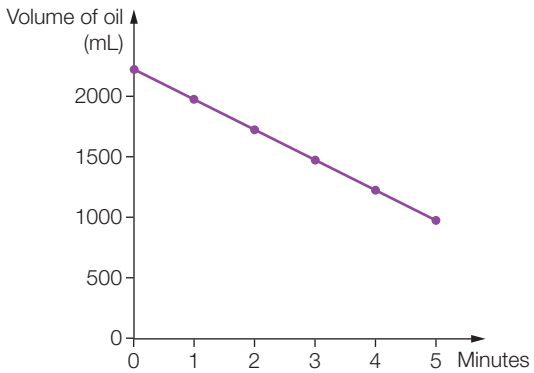
(d) 20 dots

10 (a) \$80 per hour (b) $C = 80t + 400$ (c) \$400

11 (a)

x (minutes)	0	1	2	3	4	5
y (volume of oil in engine, mL)	2225	1975	1725	1475	1225	975

(b)



(c) $y = 2225 - 250x$

(d) 8.9 minutes (8 minutes, 54 seconds)

Open-ended – Sample answers

12 $y = x - 3$; $y = \frac{x}{5} + 1$; $y = \frac{x+1}{3}$

13 Students' own answers.
For example: $y = -x$; $y = 1 - x$; $y = 2 - x$

Exercise 6.4 (p. 376)

1 (a) l = length of track, T = temperature

(b) V = volume of water, t = time

(c) p = tyre pressure, V = volume of air

(d) r = radius of balloon, V = volume

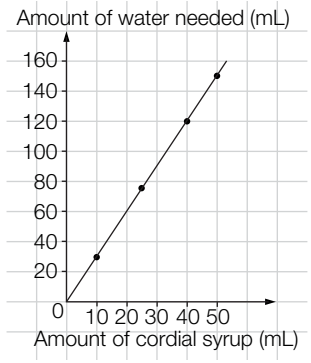
(e) d = distance ran, t = time taken

2 (a) s = solicitor's fee in \$
h = hours spent producing documents
$s = 50 + 100h$

(b) f = taxi fee in \$
k = number of km travelled
$f = 3.2 + 1.6k$

(c) c = cost of a phone call in \$
m = number of minutes spent on phone
$c = 0.37 + 0.4m$

3 (a)



(b) Yes

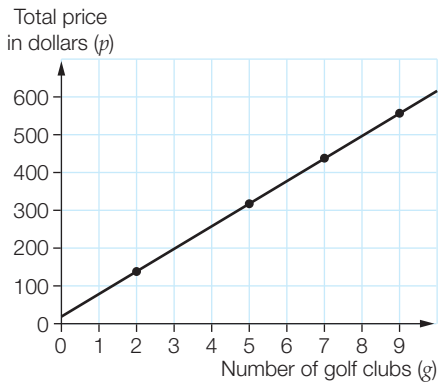
(c) (i) 45 mL (ii) 84 mL (iii) 165 mL

(d) (i) 8 mL (ii) 30 mL (iii) 45 mL

(e) Yes, because both variables are continuous.

(f) $w = 3s$

4 (a)



(b) Yes

(c) (i) \$260 (ii) \$380 (iii) \$620

(d) (i) 3 (ii) 8 (iii) 12

(e) No, because the number of golf clubs isn't a continuous variable. We use a linear graph for convenience.

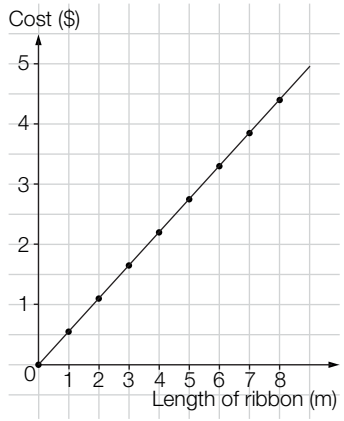
(f) $p = 20 + 60g$

(g) \$20

5 (a)

Length of ribbon (m)	0	1	2	3	4	5	6	7	8
Cost (\$)	0	0.55	1.10	1.65	2.20	2.75	3.30	3.85	4.40

(b)

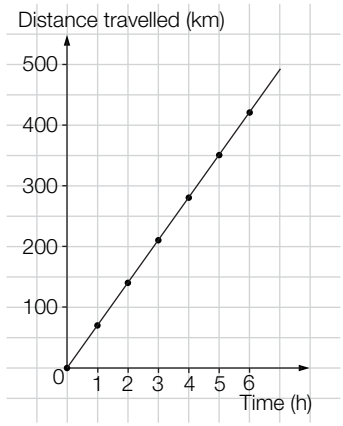


(c) $2.30 (d) 5.5 m (e) 9 m (9.1 m)

6 (a)

Time (h)	0	1	2	3	4	5	6
Distance travelled (km)	0	70	140	210	280	350	420

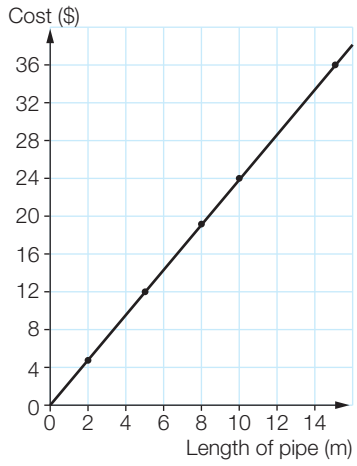
(b)



(c) (i) 245 km (ii) 35 km (iii) 370 km (367.5 km)

(d) (i) 2.5 h (ii) 4.3 h (iii) 5.7 h

7 (a)



(b) Yes, it doesn't cost anything to buy 0 m of pipe.

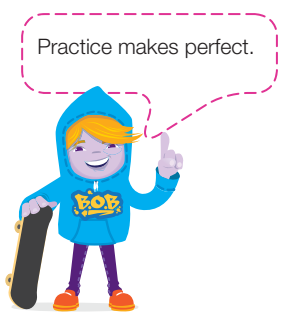
(c) $28.80

(d) $9.60

(e) It costs the same amount.

(f) 21 m

(g) 42 m



8 (b) (0, 0) (c) (12, 30.5) (e) 28 cm

(f) 2.2 in (g) 56 cm (h) 13 in, 8 in

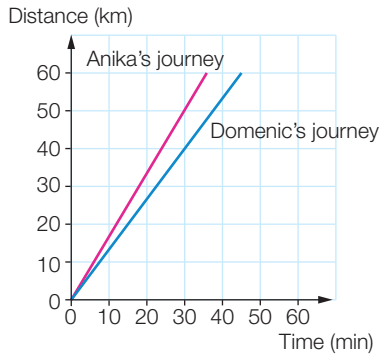
(i) 20 cm (j) 9.7 cm, 4.3 cm

9 (b) (0, 32) (c) (100, 212) (e) 58°C (f) -128°F

(g) 30°C (86°F) is hotter than 80°F (h) 37°C

(i) 95°F (j) 10°C, wear something warm

10 (a–c)



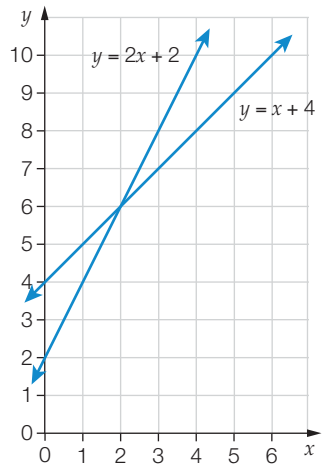
(d) 36 min, No (e) 45 – 36 = 9 min

Open-ended – Sample answers

11 Students' own answers. For example:

(a) $a = 2, b = 2$ and $a = 1, b = 4$

(b)



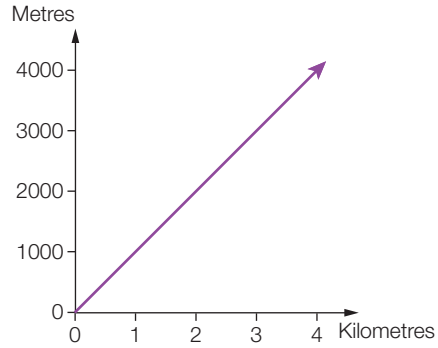
(c) (2, 6)

(d) When $x = 2$,
$y = 2x + 2 = 4 + 2 = 6$
$y = x + 4 = 2 + 4 = 6$

12 Students' own answers. For example:

(a) $m = 1000k$, where m is metres and k is kilometres

(b)



Challenge 6 (p. 386)

1 (a) B (b) 10 minutes (c) 6 km

(d) Between 20 and 30 minutes and 50 and 60 minutes.

2 (a) Eleanor (b) Brian (c) Candy and David

3 D 4 C

5 $k = 5.5$. Using the link between gradients, $\frac{k-1}{6-3} = \frac{10-1}{9-3}$.

Or, find the equation of AB and substitute the coordinates of C.

Chapter review 6 (p. 389)

1 (a) D (b) C (c) B

2 (a) (i)

x	-2	-1	0	1	2
y	-6	-4	-2	0	2
(x, y)	(-2, -6)	(-1, -4)	(0, -2)	(1, 0)	(2, 2)

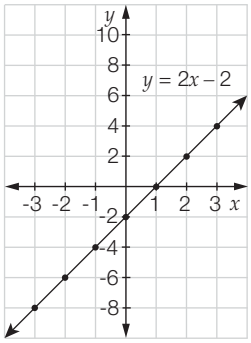
(ii)

x	-2	-1	0	1	2
y	6	5	4	3	2
(x, y)	(-2, 6)	(-1, 5)	(0, 4)	(1, 3)	(2, 2)

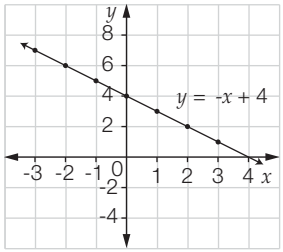
(iii)

x	-2	-1	0	1	2
y	5	2	-1	-4	-7
(x, y)	(-2, 5)	(-1, 2)	(0, -1)	(1, -4)	(2, -7)

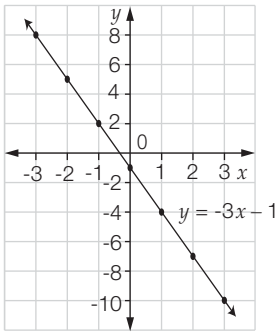
(b) (i)



(ii)



(iii)



(c) (i) x-intercept (1, 0); y-intercept (0, -2)

(ii) x-intercept (4, 0); y-intercept (0, 4)

(iii) x-intercept $(-\frac{1}{3}, 0)$; y-intercept (0, -1)

3 C

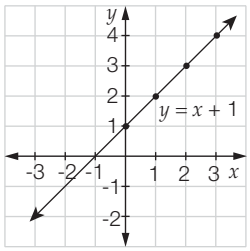
4 (a) B (b) C (c) A (d) D

5 (a) $y = x + 3$ (b) $y = -4x$ (c) $y = x - 2$

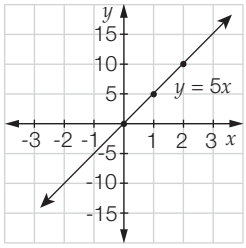
x	-2	-1	0	1	2
y	-4	-3	-2	-1	0

6 Sample answer: $C = a + bt$, where C is the total cost in dollars, a = fixed fee, b = hourly charge, t = time in hours.

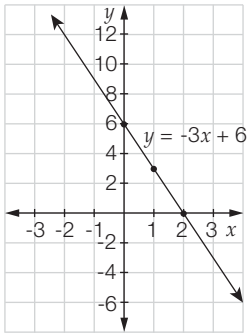
7 (a) (i)



(ii)



(iii)

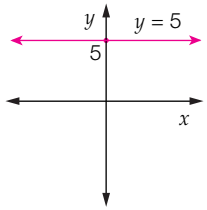


(b) (i) -1 (ii) 0 (iii) 2

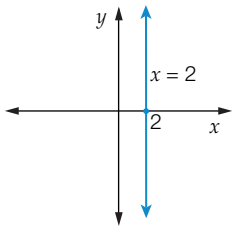
(c) (i) 1 (ii) 0 (iii) 6

(d) (i) positive (ii) positive (iii) negative

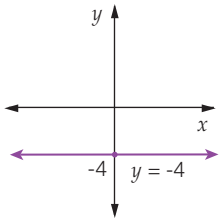
8 (a) (i)



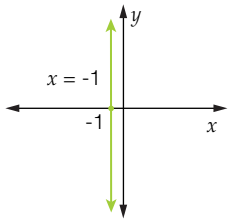
(ii)



(iii)



(iv)



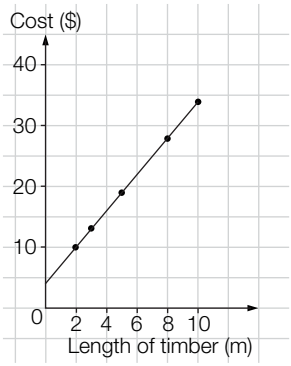
(b) (i) No x-intercept, y-intercept is 5

(ii) x-intercept is 2, no y-intercept

(iii) No x-intercept, y-intercept is -4

(iv) x-intercept is -1, no y-intercept

9 (a)



(b) The relationship is linear because the graph is a straight line.

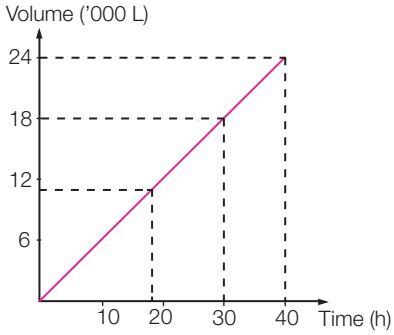
(c) (i) $16 (ii) $31 (iii) $37

(d) (i) 3.7 m (ii) 5.3 m (iii) 9.3 m

10 (a)

t	0	10	20	30	40
V	0	6000	12 000	18 000	24 000

(b)



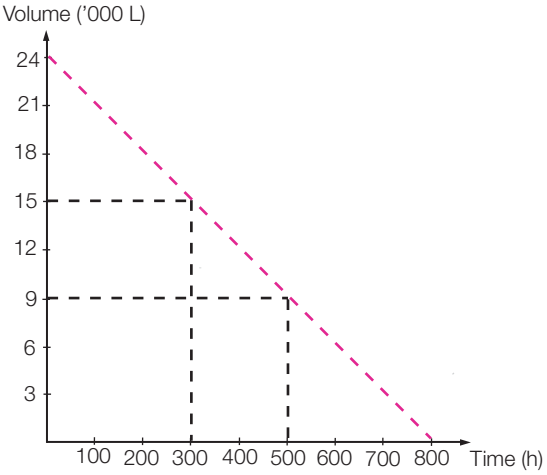
(c) 24 000 L

(d) (i) 10 800 L (ii) 30 hours

(e)

t	0	100	200	300	400	500	600	700	800
v	24 000	21 000	18 000	15 000	12 000	9000	6000	3000	0

(f)

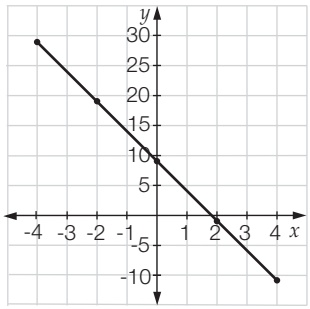


(g) (i) 15 000 L (ii) 500 hours $= 20\frac{5}{6}$ days

(h) 30 L

11 (a) $y = 9 - 5x$
y-intercept is 9, so the constant is 9.
When x goes up 2, y goes down 10, so the coefficient of $x = -5$.

(b)



(c) -6 (d) -1

12 (b) (0, 0) (c) (10, 24.7)

(e) approximately 5 acres (f) approximately 8 hectares

(g) 7 hectares is larger than 15 acres (≈ 6 hectares).

NAPLAN practice 6

1 A 2 D 3 C 4 B

5 C 6 D 7 C

Mixed review C (p. 394)

1 (a) $2^9 \times 5^5$ (b) 6×7^3

2 (a) $d = 3.75$ (b) $m = 6.25$ (c) $p = 36.4$

3 (a) $3b(2a + 3c)$ (b) $3xy(8xy - 1)$

4 (a) $\frac{4}{9}$ (b) $\frac{19}{99}$

5 (a) $xy - 2x^2 + 4y$ (b) $6m^2 - n^2$

6 (a) $2^6 \times 3^8 \times 4^2$ (b) $6^3 \times 7^6$

7 (a) -87 (b) -8

8 (a) 32 : 28 (b) 35 : 50 (c) 3000 : 6000 : 12 000

9 (a) 9.1 cm^2 (b) 5.3 cm^2

10 (a) (i) 0.375 (ii) 37.5%

(b) (i) 0.325 (ii) 32.5%

(c) (i) 0.816 (ii) 81.6%

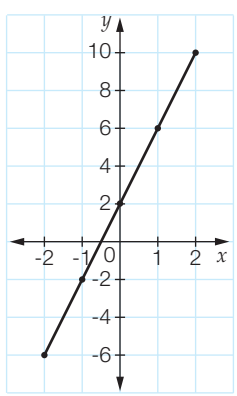
11 (a) 4.8 cm^2 (b) 2.8 cm^2

12 (a) 25% (b) 20.5% (c) 20% (d) 24%

13 (a)

x	-2	-1	0	1	2
y	-6	-2	2	6	10

(b)



(c) x-intercept = -0.5, y-intercept = 2

14 (a) $\frac{7}{20}$ (b) $\frac{17}{42}$ (c) -7.2 (d) -3.25

15 (a) $y = \frac{1}{2}x$ (b) $y = 3x + 1$

16 (a) $m = 6$, x-intercept $= \frac{5}{2}$, y-intercept $= -15$

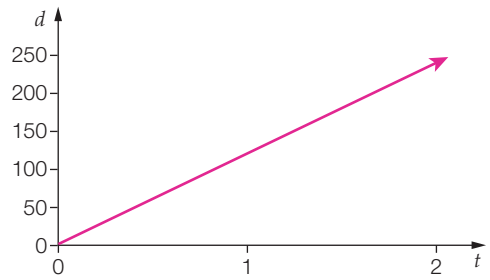
(b) $m = \frac{3}{4}$, x-intercept $= -\frac{2}{3}$, y-intercept $= \frac{1}{2}$

17 (a) \$815 (b) 52%

18 (a) 1.48 : 1 (b) 34 cm

19 (a) $d = 120t$

(b)



(c) (i) 100 km (ii) 180 km

(d) (i) 45 minutes (ii) 1 hour, 20 minutes

20 (a) $y = x - 1$ (b) $y = \frac{x}{2} + 2$

21 (0, 4), (1, 2) and (2, 0)
$x = 0, y = -2 \times 0 + 4 = 4$
$x = 1, y = -2 \times 1 + 4 = 2$
$x = 2, y = -2 \times 2 + 4 = 0$

Chapter 7

Recall 7 (p. 398)

1 (a) and (c)

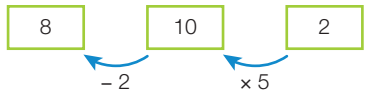
2 (a) $a = 5$ (b) $b = 20$

(c) $c = -3$ (d) $d = 17$

(e) $e = -5$ (f) $f = 8$

3 (a)

(b)



(c)

(d)

4 (a) 14 (b) 32 (c) 3

5 (a) divide by 2 (b) subtract −4 (c) multiply by 5

6 (a) $2x + 10$ (b) $6x - 21$

Exercise 7.1 (p. 400)

1 (a) $8 + a = 12$ (b) $b - 4 = 16$ (c) $9c = 63$

(d) $11 + f = 0$ (e) $7 + 3u = 10$ (f) $2v + 7 = 13$

(g) $\frac{9 + x}{7} = 6$ (h) $8(6 + z) = 0$

2 (a) Yes (b) No (c) Yes (d) No

(e) Yes (f) Yes (g) No (h) Yes

(i) No (j) No (k) Yes (l) No

3 (a) C (b) D (c) A (d) D (e) B (f) D

4 (a) A number is added to two to give the result nine.

(b) Two less than a number is equal to nine.

(c) Six is taken away from a number to give the result three.

(d) A number is taken away from ten and the result is four.

(e) A number is multiplied by five and the result is fifty-five.

(f) A number is divided by eight and the result is nine.

(g) Seven is taken away from six times a number and the result is twenty-three.

(h) Five less than three times a number equals negative ten.

(i) Eight is taken away from a number, then the result is multiplied by three to give six.

(j) A number is taken away from five, then the result is multiplied by four to give twelve.

(k) Three is added to a number, then this answer is divided by four and the result is eight.

(l) A number is taken away from six, then this answer is divided by seven to give the result of four.

5 (a) $A = l \times w$ (b) $A = \frac{1}{2}bh$ (c) $s = \frac{d}{t}$

(d) $A = \frac{1}{2}h(a + b)$ (e) $C = 200n + 300$ (f) $F = \frac{10}{11}R$

6 (a) $C = 150 + 27x$ (b) \$285 (c) $A = 25y$

(d) $25 \times \$10 = \$250$, which will not cover the cost.

(e) Cost = \$420, Charge = \$425 (f) \$5

7 (a) $S = 20 + 5x$ (b) $20 + 5 \times 15 = 95$

(c) $R = 5y - 15$ (d) No, $5 \times 20 - 15 = 85$

Open-ended – Sample answers

8 $w + 5 = 8$; $5w = 15$; $\frac{w}{3} = 1$

Exercise 7.2 (p. 409)

1 (a) $x = 16$ (b) $x = 11$ (c) $x = 4$

(d) $x = -8$ (e) $x = 15$ (f) $x = -77$

(g) $x = -4$ (h) $x = 7$ (i) $x = -3$

(j) $x = 6$ (k) $x = 9$ (l) $x = -2$

2 (a) $a = 1$ (b) $b = 4$ (c) $c = -3$
(d) $x = -1$ (e) $x = -3$ (f) $x = -8$
(g) $x = 1$ (h) $x = 3$ (i) $x = 6$

3 (a) $y = 6$ (b) $y = 4$ (c) $y = 0$
(d) $x = 0$ (e) $x = 1$ (f) $x = 3$
(g) $y = 2$ (h) $y = 5$ (i) $y = -1$

4 (a) $x = 2$ (b) $x = -1$ (c) $x = 0$

5 (a) $2b + 3 = 7$ (b) $2b = 4$ (c) $b = \frac{4}{2}$ (d) $b = 2$

6 (a) B (b) D

7 (a) D (b) C (c) B

8 (a) $d = -5$ (b) $e = -2$ (c) $f = -10$
(d) $k = -2$ (e) $h = -2$ (f) $g = -5$
(g) $n = -1$ (h) $c = -4$ (i) $k = -2$
(j) $r = \frac{3}{2}$ (k) $t = \frac{7}{3}$ (l) $l = \frac{12}{7}$
(m) $d = \frac{13}{6}$ (n) $p = 0.85$ (o) $a = \frac{7}{15}$
(p) $g = \frac{3}{8}$ (q) $r = \frac{7}{6}$ (r) $p = \frac{1}{9}$

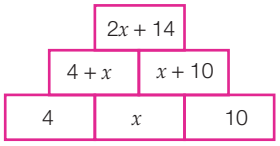
9 (a) $a = 2$ (b) $b = 1$ (c) $c = -4$
(d) $d = -8$ (e) $e = 3$ (f) $f = -3$
(g) $g = 5$ (h) $h = 6$ (i) $m = -8$
(j) $r = 1$ (k) $p = 3$ (l) $t = 8$
(m) $p = \frac{-2}{3}$ (n) $l = \frac{-4}{5}$ (o) $k = \frac{-10}{7}$
(p) $a = 6.55$ (q) $b = 1.72$ (r) $c = 3.65$

10 (a) $4n - 5 = 23$; $n = 7$ (b) $3(m + 7) = 45$; $m = 8$
(c) $\frac{p+5}{9} = 3$; $p = 22$ (d) $\frac{q}{4} + 2 = -1$; $q = -12$

11 (a) E (b) I (c) A (d) G (e) D
(f) H (g) B (h) F (i) J (j) C

12 (a) $2a + 16 = 102$ (b) $a = 43$ kg

13 (a)



(b) $x = 3$

14 $6\frac{1}{2}$ hours

15 $5p + 25 = 62.5$; $p = 7.5$. He receives \$7.50 each week.

16 (a) 2 (b) 7 children

17 25 km

18 $1.25 + 2m = 3$; $m = 0.875$ L = 875 mL each

Open-ended – Sample answers

19 (a) $2x + 5 = 13$, $x = 4$
(b) $3x - 7 = 5$, $x = 4$

20 (a) No, the steps will result in negative 10 grapes and one-and-a-half apples.
(b) Remove 10 grapes on each side, then remove 2 apples on each side. The answer is 1 apple = 10 grapes.

21 $4x + 6 = 22$; $2x = 8$; $2x + 12 = 20$

22 $3x - 4 = 2$

Exercise 7.3 (p. 418)

1 (a) $c = 12$ (b) $f = 21$ (c) $m = 81$ (d) $x = -4$
(e) $b = -9$ (f) $t = -16$ (g) $x = -14$ (h) $p = -37$
(i) $r = -79$ (j) $x = 15$ (k) $y = 44$ (l) $t = -49$

2 (a) $x = 7$ (b) $x = -13$ (c) $x = -5$ (d) $x = -2\frac{3}{4}$
(e) $x = -1\frac{5}{9}$ (f) $x = 5\frac{3}{7}$ (g) $x = -10$ (h) $x = 3$
(i) $x = -3$ (j) $x = 2$ (k) $x = 4$ (l) $x = 2\frac{2}{3}$

3 (a) $a = -16$ (b) $m = -11$ (c) $r = -6$ (d) $r = 14$
(e) $n = 7$ (f) $p = 15$ (g) $r = 1$ (h) $f = 4$
(i) $m = -3$ (j) $r = 2$ (k) $p = 8.5$ (l) $k = 13$

4 (a) C (b) B

5 (a) $a = 14$ (b) $p = -9$ (c) $m = -12$ (d) $a = 2\frac{1}{5}$
(e) $a = \frac{3}{4}$ (f) $a = -1\frac{1}{3}$ (g) $x = 8.5$ (h) $x = 4\frac{1}{3}$
(i) $x = 7\frac{3}{4}$ (j) $x = -4$ (k) $x = -3$ (l) $x = -4$

6 (a) D (b) F (c) G (d) I (e) H
(f) C (g) A (h) J (i) E (j) B

7 (a) $\frac{3x+7}{4} = 4$, $x = 3$ (b) $2\left(\frac{x}{4} - 1\right) = 6$, $x = 16$
(c) $\frac{5x-2}{6} = -12$, $x = -14$ (d) $\frac{3x}{2} + 7 = 8$, $x = \frac{2}{3}$

8 (a) $\frac{x-10}{5} + 20 = 48$, $x = \$150$ (b) \$170

9 (a) $\frac{2x}{11} + 50 = 150$, $x = \$550$ (b) \$25

10 (a) $3(2x + 6) = 108$, $x = 15$

11 93

12 834

13 Money in wallet = \$30

14 $x = 15$

Open-ended – Sample answers

15 $\frac{2x}{3} - 3 = 0$; $\frac{2y}{3} - 3 = 21$; $\frac{2m}{3} - 3 = -1$

16 **(a)** $\frac{7x+4}{5} = 5$ **(b)** Students' own answers.

17 **(a)** Step 2—The $5x$ must remain negative and then you divide both sides by -5.

(b) $x = -1\frac{1}{5}$

Half-time 7 (p. 422)

1 **(a)** $x = -3$ **(b)** $x = 4$ **(c)** $x = 26$
(d) $x = -5$ **(e)** $x = 10.5$ **(f)** $x = -60$

2 **(a)** Incorrect **(b)** Correct **(c)** Correct **(d)** Incorrect

3 Phillip = \$40, James = \$20, Sam = \$70

4 **(a)** $x = 24$ **(b)** $x = 2$ **(c)** $x = 20$ **(d)** $x = -45$

5 **(a)** $x - 4 = 12$, $x = 16$ **(b)** $7x = 35$, $x = 5$
(c) $6x = 3 \times 12$, $x = 6$ **(d)** $7x + 3 = 24$, $x = 3$

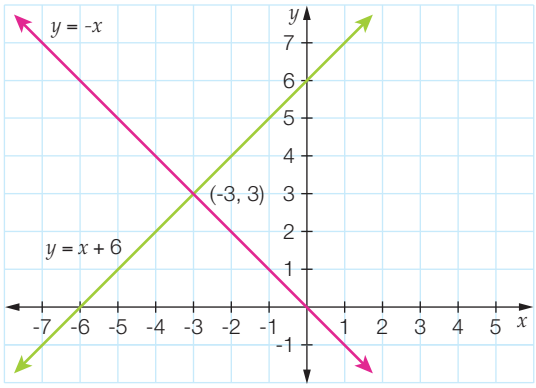
6 $x = 45$, length = 145 m, width = 95 m

Exercise 7.4 (p. 430)

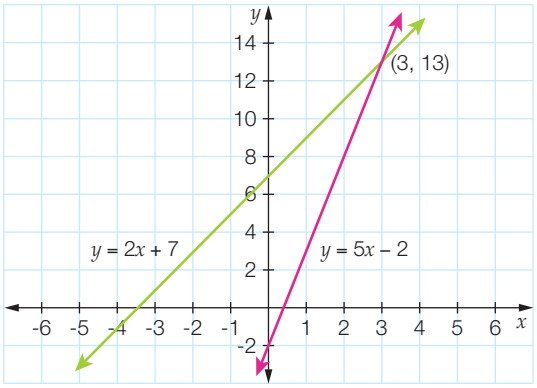
1 **(a)** $c = -7$ **(b)** $b = -8$ **(c)** $a = 6$ **(d)** $d = 7$
(e) $e = 4$ **(f)** $f = -2$ **(g)** $g = -7$ **(h)** $h = -6$
(i) $x = 2$ **(j)** $k = 1\frac{1}{2}$ **(k)** $j = 2$ **(l)** $y = 3$
(m) $m = 4$ **(n)** $n = 2$ **(o)** $x = 1$ **(p)** $y = 1$
(q) $p = 1$ **(r)** $x = 3$ **(s)** $p = -6$ **(t)** $x = 3$
(u) $a = -2$

2 **(a)** $x = 7$ **(b)** $x = 4$ **(c)** $x = -2$ **(d)** $x = \frac{-13}{4}$
(e) $x = -10$ **(f)** $x = \frac{35}{6}$ **(g)** $x = \frac{1}{9}$ **(h)** $x = \frac{9}{10}$
(i) $x = 0$ **(j)** $x = 9$ **(k)** $x = -2$ **(l)** $x = -2$
(m) $x = -32$ **(n)** $x = 6$ **(o)** $x = \frac{11}{10}$

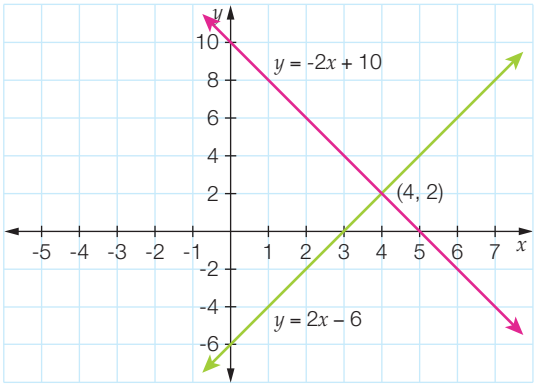
3 **(a)**



(b)



(c)



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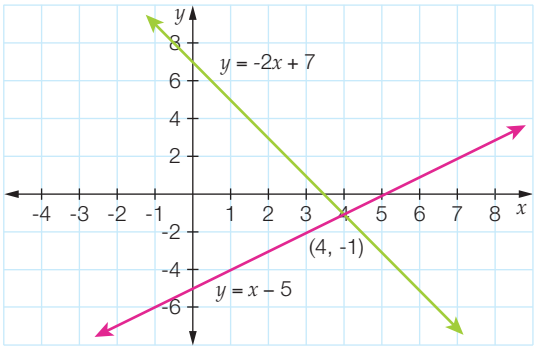

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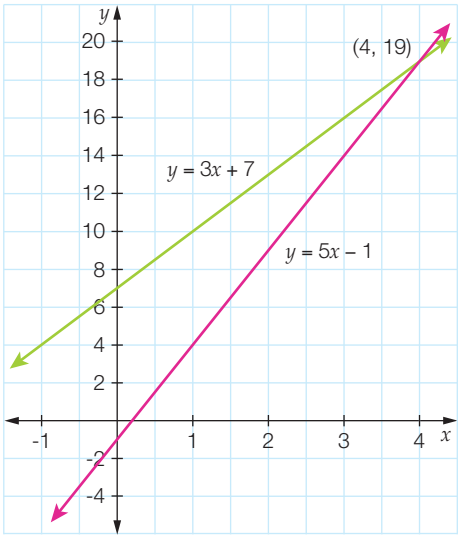
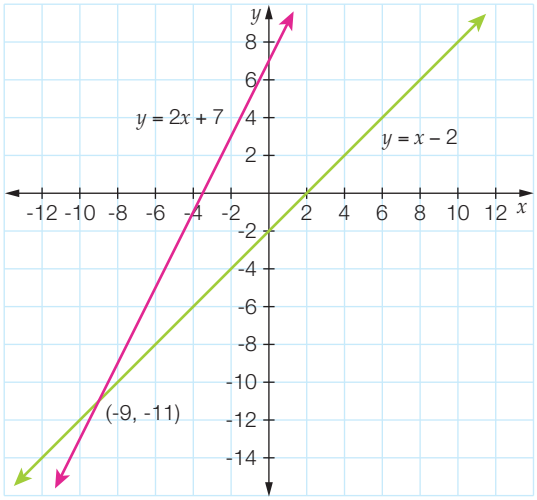
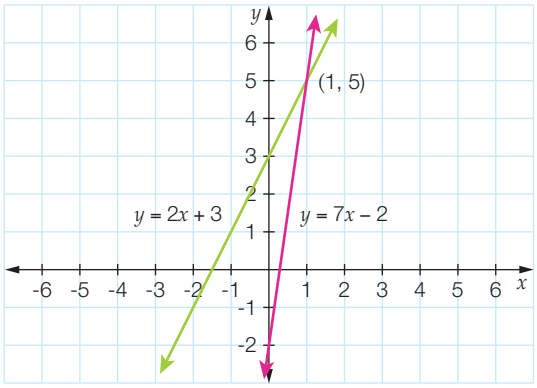


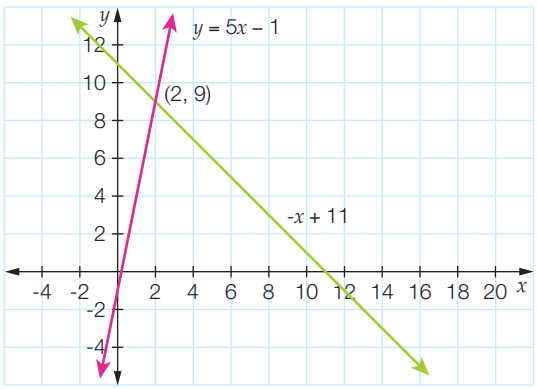
(f)



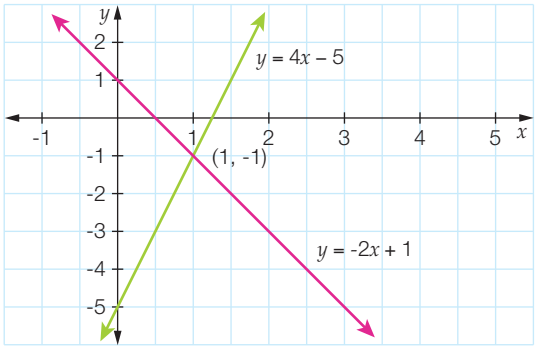
(g)



(h)



(i)



4 (a) B (b) D (c) D

5 (a) $4x - 11$ (b) $7 - 2x$

(c) $4x - 11 = 7 - 2x$ (d) $x = 3$

6 (a) $x + 36$ (b) $2(x + 6)$

(c) $x + 36 = 2(x + 6)$ (d) $x = 24$

7 (a) $\frac{3x}{4} - 5$ (b) $2(x - 10)$

(c) $\frac{3x}{4} - 5 = 2(x - 10)$ (d) $x = 12$

8 (a) $y = 2x + 8$
$= 2 \times -3 + 8$, when $x = -3$
$= 2$

(-3, 2) lies on $y = 2x + 4$

$y = -x - 1$
$= - -3 - 1$ when $x = -3$
$= 2$

(-3, 2) lies on $y = -x - 1$

Therefore, (-3, 2) must be the point of intersection.

(b) $2x + 8 = -x - 1$

(c) $x = -3$

(d) 2 is the value of $2x + 8$ and $-x - 1$ when $x = -3$

9 $x = 4$

10 Sarah = \$70, Frank = \$50, Bruce = \$110

11 $x = 7$

12 $x = 13$

Open-ended – Sample answers

13 $2x + 3 = 8 - 3x$ ($x = 1$); $4x - 5 = 2x + 3$ ($x = 4$); $3x - 2 = 2x + 7$ ($x = 9$)

14 (a) Step 1—error in expanding the right bracket. Correction—expand both terms by 2 to get the result $2x - 8$.

Step 3—error in inverse operation of +36. Correction—subtract 36 from both sides.

(b) $x = -11$

Exercise 7.5 (p. 436)

1 (a) $3c = 117$; $c = 39$ cents (b) $m - 935 = 9$; $m = 944$ kg

(c) $n + 21 = 49$; $n = 28$ seats (d) $t - 9 = 27$; $t = 36°C$

2 (a) $x + 7$ (b) $x + 7 = 11$; Darren walked 4 km.

3 (a) $x + 2$ (b) $x + 2 = 4.5$; cost of salad roll is \$2.50.

4 (a) $\frac{3w}{4}$ (b) $\frac{3w}{4} = 24$; Olivia is paid \$32.

5 (a) $x + x + 14$

(b) $2x + 14 = 20$; The Kittens lost 3 games and won 17 games.

6 (a) B (b) B (c) D

7 (a) $6x + 1$

(b) $6x + 1 = 25$; Each person receives 4 jelly beans.

8 (a) $2l + 2w$

(b) (i) $2l + 4 = 10$; The vegetable garden can be 3 m long.

(ii) $2l + 6 = 15$; Now, the vegetable garden can be 4.5 m long.

9 The wrapping costs \$2.75.

10 Each CD costs \$19.50.

11 Each ice-cream costs \$2.40.

12 (a) $s = \frac{d}{t}$; $97 = \frac{d}{2.2}$ (b) $d = 213.4$ km

13 $4p + 2 = 110$; the perimeter of the pen is 27 m.

14 $2s + 3 = 7$; Jack surfed for 2 hours and read for 5 hours.

15 $2(2l - 7) = 74$; the block of land has length 22 m and width 15 m.

16 $5m + 2 = 10$; the carton of milk costs \$1.60.

17 \$250 each month for 3 years.

18 95%

19 $4l = 27$

$l = \frac{27}{4}$

$l = 6.75$ cm

20 Sister = 70 cm, brother = 140 cm and dad = 170 cm.

21 33, 34 and 35

22 $x = 50°$; angles = 40°, 60°, 120°, 140°

Open-ended – Sample answers

23 $h = 20$, plumber Alan is cheaper
$h = 50$, plumber Barry is cheaper
$h = 30$, both quotes will be the same.

24 $b = 16$ cm and $h = 8$ cm; $b = 32$ cm and $h = 4$ cm; $b = 64$ m and $h = 2$ cm; $b = 128$ cm and $h = 1$ cm; $b = 8$ cm and $h = 16$ cm

MathSpace (p. 440)

Level 1

(a) $\div 3$ (b) $\times 4$ (c) -3

(d) $+7, \div 4$ (e) $+3, \times -2, \div 6$

Level 2

Safe bricks are: $x < 2x$, $5x + 2 > 2(2x + 1)$, $y - 2 > y - 3$, $2(x + 4) = 2x + 8$, $x + 3 > -4$

Level 3

Alpha's score: 52, Raydon's score: 49

Bonus Challenge: 7

Challenge 7 (p. 444)

1 10 and 5

2 $R = 5$, $S = 3$, $T = 1$, $U = 7$, so $R \times U = 35$ and $S \times U = 21$

3 ±12 4 20 000 km 5 $a = 6, b = 12, c = 4, d = 8$

6 3 cm 7 $d = 8, e = 7, f = 5$

8 D 9 D 10 50

11 (a) $\angle DBA = \angle DAB = x°$, $\angle ADB = (180 - 2x)°$,

$\angle DBC = 180 - x°$, $\angle BDC = \angle BCD = \frac{x°}{2}$,

$\angle ADC = 180° - 2x° + \frac{x°}{2} = \left(\frac{360 - 3x}{2}\right)°$. For $\angle ADC$ to be a whole number of degrees, x must be even. The smallest even value for x is 2. This gives the largest value for $\angle ADC$.

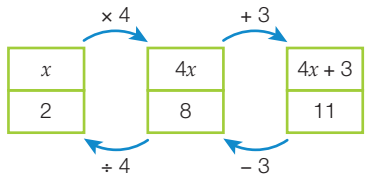
(b) Largest value for $\angle ADC = 177°$.

(c) The largest even value for x is 88. The smallest value of $\angle ADC = 48°$.

Chapter review 7 (p. 445)

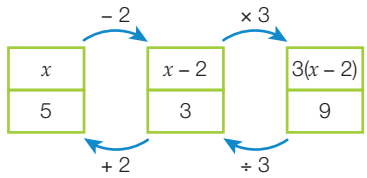
1 (a) $3x - 4 = 2$ (b) $\frac{2 + x}{5} = 6$

2 (a)



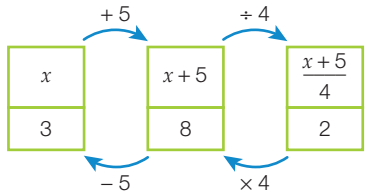
Solution is $x = 2$

(b)



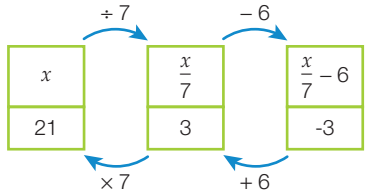
Solution is $x = 5$

(c)



Solution is $x = 3$

(d)



Solution is $x = 21$

3 (a) $x = -1$ (b) $b = 2$ (c) $c = -5$

4 (a) C (b) D (c) A

5 (a) $a = 9$ (b) $b = 1$ (c) $c = 8$

6 (a) $x = 4$ (b) $x = 25$ (c) $x = 8$ (d) $x = -10$

7 (a) $m = 2$ (b) $n = 5$ (c) $x = 3$ (d) $m = -27$

8 (a) $x = -7$ (b) $x = 1$ (c) $x = -3$ (d) $x = \frac{-4}{27}$

9 in 11 weeks

10 (a) Each can of cola costs \$1.55 ($5c + 2.25 = 10$).

(b) Almira needs to sell 28 hot dogs to make a profit of \$24. ($3h - 60 = 24$)

(c) One piece is 45.5 cm long and the other is 54.5 cm long. ($x + x + 9 = 100$)

11 (a) $2f + 5 = 70$ (b) $f = 32.5$ cm

12 7, 8 and 9 13 30 and 20 14 $x = 6$

15 16 years old

16 **(a)** 17 years old

(b) 18 km

(c) $s = 44$

17 8, 29

18 base = 11 cm

19 Tom = 500 m, Mark = 1000 m and Fred = 2000 m

20 $7x + (5x + 12) + 12 + x + (7x - x) + (5x + 12 - 12) = 120$

$24x + 24 = 120$

$24x = 96$

$x = 4$ cm

21 **(a)** Box A: a; Box B: $a + 2b$

(b)

5	11	11
15	9	3
7	7	13

22 $x = 14$; Side 1 = 14 cm, Side 2 = 18 cm and Side 3 = 19 cm

NAPLAN practice 7

1 $p = 3$ **2** C **3** A **4** A

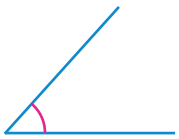
5 $z = 6$ **6** 42 **7** C **8** $x = 17$

9 65 **10** 5

Chapter 8

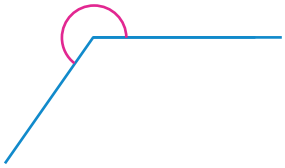
Recall 8 (p. 450)

1 **(a)**

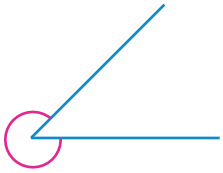


(b)

(c)



(d)



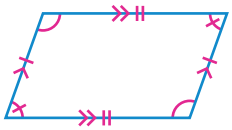
2 **(a)** acute **(b)** obtuse **(c)** reflex **(d)** reflex

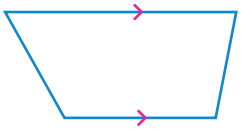
3 **(a)** **(i)** 35° **(ii)** 245°

(b) **(i)** acute **(ii)** reflex

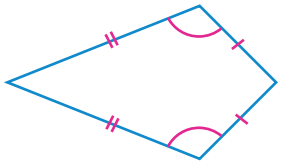
4 **(a)** $x = 173$ **(b)** $5x = 320$, $x = 64$ **(c)** $6x = 420$, $x = 70$

5 **(a)** parallelogram **(b)** trapezium

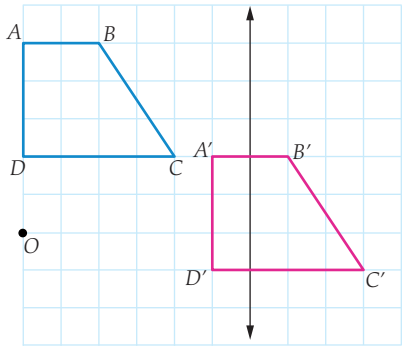




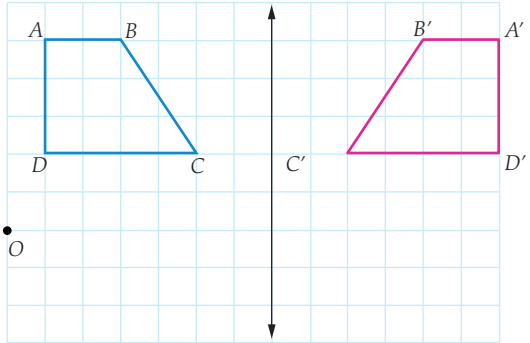
(c) kite



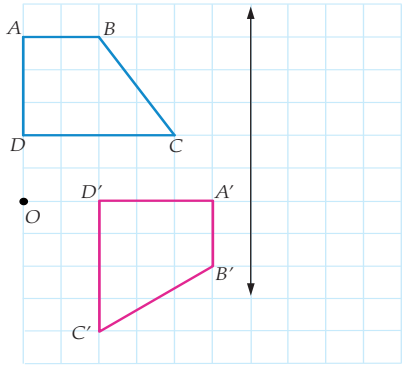
6 **(a)**



(b)



(c)



Exercise 8.1 (p. 454)

1 **(a)** 53° **(b)** 75° **(c)** 50° **(d)** 40°

2 **(a)** $c = 145°$, co-interior angles supplementary on parallel lines

(b) $a = 114°$, alternate angles on parallel lines; $b = 66°$, co-interior angles supplementary on parallel lines; $c = 66°$, vertically opposite angles equal; $d = 114°$, straight angle; $e = 66°$, straight angle; $f = 114°$, vertically opposite angles equal; $g = 66°$, straight angle.

(c) $d = 45°$, alternate angles on parallel lines; $e = 65°$, co-interior angles supplementary on parallel lines.

(d) $f = 120°$, co-interior angles supplementary on parallel lines; $g = 120°$, vertically opposite angles equal; $h = 70°$, co-interior angles supplementary on parallel lines; $k = 70°$, straight angle.

(e) $m = 130°$, co-interior angles supplementary on parallel lines; $n = 110°$, alternate angles equal and supplementary adjacent angles; $p = 50° + 70° = 120°$, exterior angle of triangle.

(f) $a = 49°$, vertically opposite angles equal; $b = 68°$, straight angle; $c = 117°$, co-interior angles supplementary on parallel lines.

3 (a) $d = 145°$, angle of revolution

(b) $e = 318°$, angle of revolution

(c) $f = 26°$, right angle

(d) $g = 51°$, straight angle

(e) $h = 51°$, straight angle

(f) $i = 60°$, angle of revolution

(g) $k = 110°$, straight angle

(h) $m = 142°$, angle of revolution

(i) $p = 58°$, complementary angles; $n = 32°$, vertically opposite angles

4 C 5 B

6 (a) C (b) S (c) X (d) X (e) C (f) S

7 (a) $a = 65°$, co-interior angles supplementary on parallel lines; $b = 25°$, complementary angles; $c = 90°$, corresponding angles on parallel lines.

(b) $x = 120°$, co-interior angles supplementary on parallel lines; $y = 120°$, vertically opposite angles equal.

(c) $x = 107°$, straight angle; $y = 48°$, corresponding angles on parallel lines; $z = 59°$, co-interior angles supplementary on parallel lines.

8 (a) True

(b) False

(c) False (they could both be right angles)

(d) False (they could both be straight angles)

(e) True

9 (a) 65° (b) 98° (c) 122° (d) 135°

10 $d = 35°$

11 $g = 120°$, $h = 60°$

12 $(3x + 10)° + (5x - 70)° = 180°$, allied angles supplementary on parallel lines; $8x - 60° = 180°$, $8x = 240°$, $x = 30°$

Open-ended – Sample answers

13 $x = 100°$, $y = 80°$; $x = 105°$, $y = 75°$; $x = 110°$, $y = 70°$
The pairs of values are always supplementary.

14 (a) $a = b = e$, $c = d = f$

(b) a and c, a and d, a and f, b and c, b and d, b and f, e and c, e and d, e and f

Exercise 8.2 (p. 466)

1 (a) $x = 82°$ (b) $y = 55°$

(c) $a = 115°$ (d) $w = 67°$, $v = z = 113°$

(e) $x = 63°$ (f) $x = 28°$

2 (a) Trapezium (b) Kite (c) Square

3 B

4 (a) 4 (b) 180° (c) $4 \times 180° = 720°$

5 30°, 60°, 90°

6 (a) $a = 82°$, $b = 133°$ (b) $d = 58°$, $c = e = 122°$

(c) $x = 65°$ (d) $f = 95°$

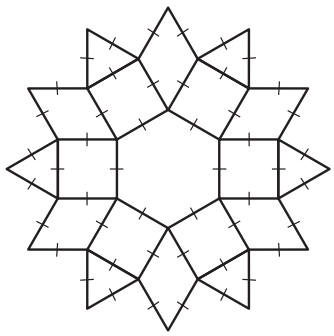
(e) $2x + 3x + 78° + (2x + 30°) = 360°$, $7x + 108° = 360°$, $7x = 252°$, $x = 36°$

(f) $u = 180° - 65° = 115°$

7 (a) False (b) True (c) False (d) False

8 Square

9 (a)



(b) Two angles 60°, two angles 120°

(c) $2 \times 90°$ (squares) $+ 60°$ (rhombus) $= 240°$
$360° - 240° = 120°$

10 C

11 C

12 (a) 540° (b) 108°

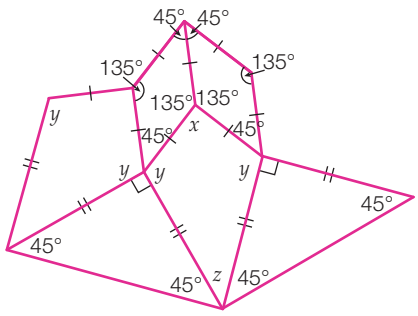
13 (a) 30° (b) 60°

14 (a) 45°, 45°, 135°, 135°

(b) Two adjacent sides of the kite are the sides of a rhombus, which must be equal. The other pair of adjacent sides are the sides of the isosceles triangle, which also must be equal. Two pairs of equal adjacent sides means that the quadrilateral is a kite.

(c) (i) Octagon (ii) 1080° (iii) 135°

(d)



$x = 360° - (2 \times 135°)$
$= 90°$

$2y = 360° - (45° + 90°)$
$= 225°$

$y = 112.5°$
$z = 360° - 315°$
$= 45°$

15 A rectangle has four 90° angles; three of them can be made acute, making the fourth obtuse. An example would be a quadrilateral with angles of 75°, 80°, 85°, 120°.

16 **(a)** *EFGH* is a rhombus

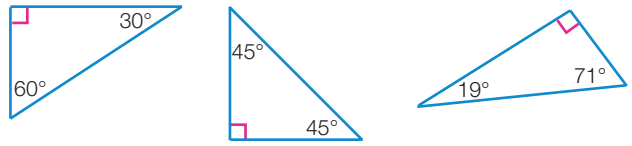
(b) Square (regular quadrilateral)

17 Types are: parallelogram, rectangle, rhombus, square (and, for the thinkers, an isosceles trapezium)

18 A rectangle has angles of 90°, whereas a rhombus does not have 90° angles.

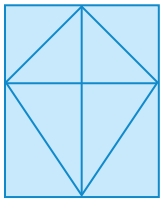
Open-ended – Sample answers

19

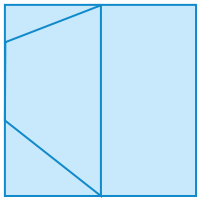


20 **(a)**

(b)



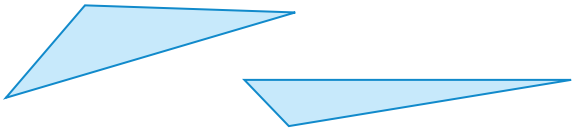
(c)



21 **(a)** 90°, 45°; 67.5°, 67.5°

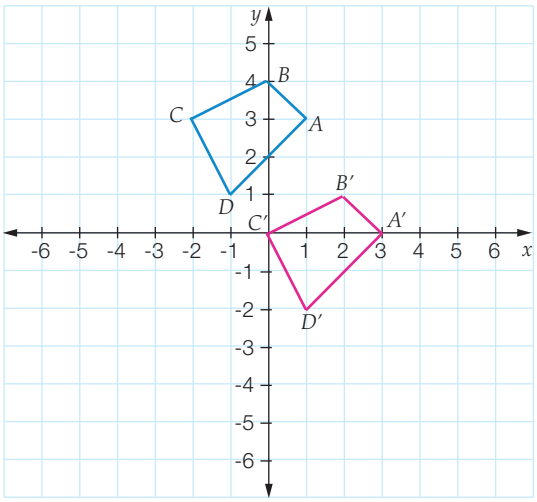
(b) No. Angles could be 60°, 30°, 90°; or 60°, 40°, 80°

(c)



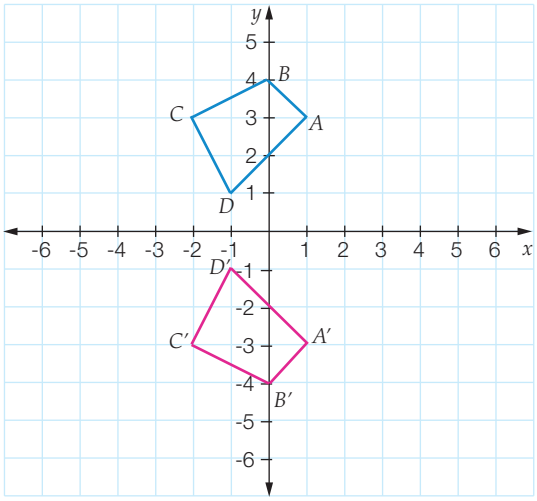
Exercise 8.3 (p. 474)

1 **(a)** **(i)**



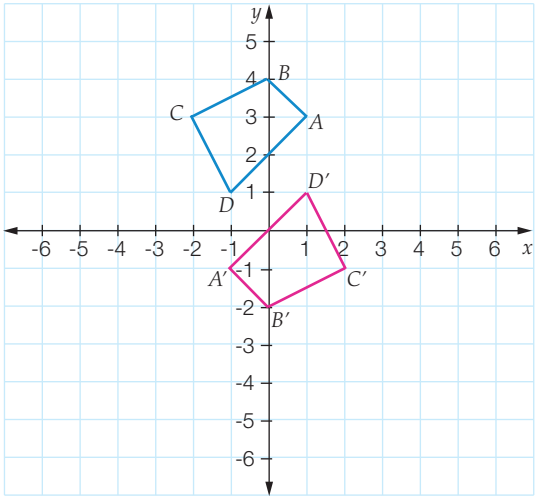
A′(3, 0), *B*′(2, 1), *C*′(0, 0), *D*′(1, -2)

(ii)



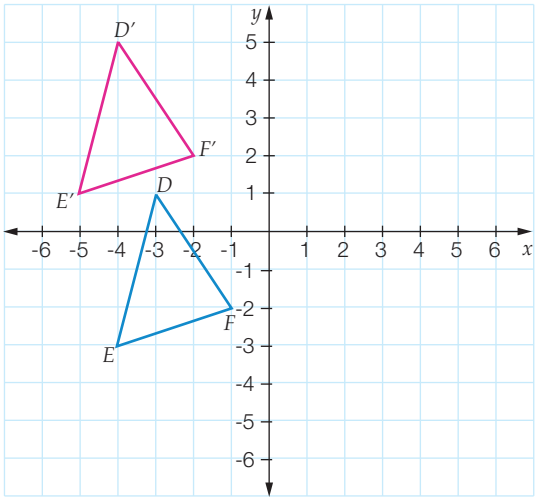
A′(1, -3), *B*′(0, -4), *C*′(-2, -3), *D*′(-1, -1)

(iii)



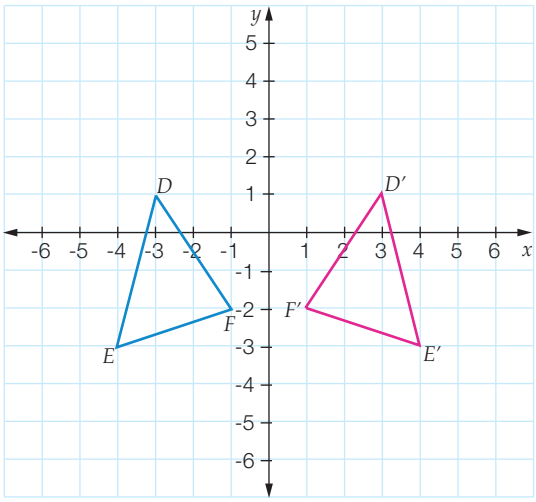
A′(-1, -1), *B*′(0, -2), *C*′(2, -1), *D*′(1, 1)

(b) **(i)**



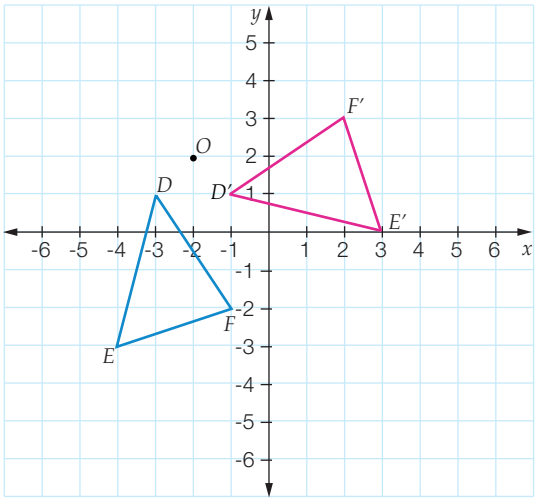
D′(-4, 5), *E*′(-5, 1), *F*′(-2, 2)

(ii)



$D'(3, 1)$, $E'(4, -3)$, $F'(1, -2)$

(iii)



$D'(-1, 1)$, $E'(3, 0)$, $F'(2, 3)$

2 (a) (i) Rotation (180°) and translation

(ii) $P \leftrightarrow W$, $Q \leftrightarrow X$, $R \leftrightarrow Y$, $S \leftrightarrow Z$

(iii) $PQ \leftrightarrow WX$, $QR \leftrightarrow XY$, $RS \leftrightarrow YZ$, $SP \leftrightarrow ZW$

(iv) $\angle PQR \leftrightarrow \angle WXY$, $\angle QRS \leftrightarrow \angle XYZ$, $\angle RSP \leftrightarrow \angle YZW$, $\angle SPQ \leftrightarrow \angle ZWX$

(v) $PQRS \equiv WXYZ$

(b) (i) Rotation (90° clockwise), reflection (in a vertical line) and translation

(ii) $A \leftrightarrow F$, $B \leftrightarrow G$, $C \leftrightarrow H$

(iii) $AB \leftrightarrow FG$, $BC \leftrightarrow GH$, $AC \leftrightarrow FH$

(iv) $\angle ABC \leftrightarrow \angle FGH$, $\angle BCA \leftrightarrow \angle GHF$, $\angle BAC \leftrightarrow \angle GFH$

(v) $\Delta ABC \equiv \Delta FGH$

(c) (i) Reflection (in a vertical line)

(ii) $J \leftrightarrow P$, $K \leftrightarrow Q$, $L \leftrightarrow R$, $M \leftrightarrow S$

(iii) $JK \leftrightarrow PQ$, $KL \leftrightarrow QR$, $LM \leftrightarrow RS$, $MJ \leftrightarrow SP$

(iv) $\angle MJK \leftrightarrow \angle SPQ$, $\angle JKL \leftrightarrow \angle PQR$, $\angle KLM \leftrightarrow \angle QRS$, $\angle LMJ \leftrightarrow \angle RSP$

(v) $JKLM \equiv PQRS$

(d) (i) Rotation (90° clockwise) and translation

(ii) $E \leftrightarrow T$, $F \leftrightarrow U$, $G \leftrightarrow V$, $H \leftrightarrow W$

(iii) $EF \leftrightarrow TU$, $FG \leftrightarrow UV$, $GH \leftrightarrow VW$, $HE \leftrightarrow WT$

(iv) $\angle HEF \leftrightarrow \angle WTU$, $\angle EFG \leftrightarrow \angle TUV$, $\angle FGH \leftrightarrow \angle UVW$, $\angle GHE \leftrightarrow \angle VWT$

(v) $EFGH \equiv TUVW$

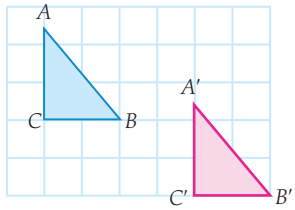
3 (a) Reflection in the y-axis.

(b) Rotation 180° clockwise about the point (2, 2).

(c) Translation of [-2, 1].

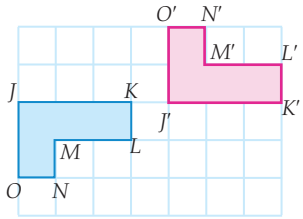
4 (a) (i) Translation 4 units right, 2 units down.

(ii)



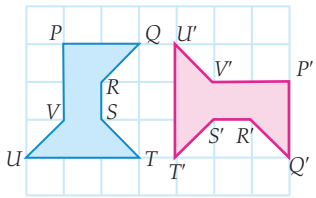
(b) (i) Reflection along the line JK, then translation 4 units right.

(ii)



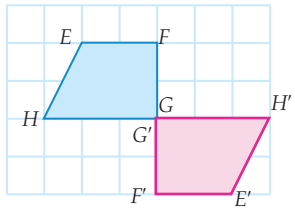
(c) (i) Rotation 90° clockwise around T, then translation 1 unit right.

(ii)



(d) (i) Rotation 180° about G.

(ii)



5 (a) C (b) D 6 B

7 (a) $RVW \equiv UST$

(b) $IFE \equiv HFG$, $IEH \equiv HGI$

(c) $RMQ \equiv NOP$, $ROP \equiv NMQ$, $RNO \equiv NRM$

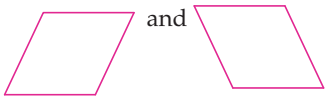
8 (a) C (b) S (c) AB

(d) SP (e) $\angle ABC$ (f) $\angle RSP$

9 (a) $a = 4$ cm, $b = 105°$ (b) $m = 8$ m, $n = 130°$

(c) $x = 2.2$ km, $y = 80°$ (d) $p = 28$ mm, $q = 63°$

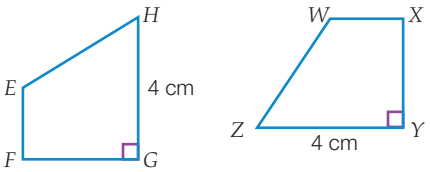
10 (a) 3 (b) 5

(c) 

11 (a) Yes (b) Yes

In (a) the lines make the second square appear as a trapezium and in (b) the perspective makes the first lid appear longer and thinner than the second.

Open-ended – Sample answers

12 

13 squares on a chessboard, opposite faces of boxes and cans

Mathspace (p. 478)

1 A rotation of 120° clockwise has been applied to both the white and the black fish.

2 (a) (i) The repeating unit is one bird that has been translated and alternately coloured black and white.

(ii) The repeating unit is one horse and one fish attached to it from above or below. This repeating unit has been translated and reflected.

3 The hexagon, square, parallelogram and equilateral triangle all tessellate. The isosceles triangle will tessellate if it is right-angled. The other shapes do not tesselate.

4 Shapes that tessellate have interior angles that are factors of 360°. This means that when the vertices of these shapes are placed together, the angle sum is 360°— an entire revolution around a point.

5 Students' own answers.

6 Students' own answers.

7 Students' own answers.

8 (a) A lithograph is a print that has been created by engraving a surface such as wax or plastic. Ink is then applied to the surface and the paper laid on top.

(b) A stellated dodecahedron (a dodecahedron with a pyramid attached to each of its faces)

9 Students' own answers.

10 Students' own answers.

11 Students' own answers.

Half-time 8 (p. 480)

1 (a) $d = 130°$, $e = 125°$ (b) $m = 140°$, $n = 60°$

(c) $n = 47°$, $p = 133°$

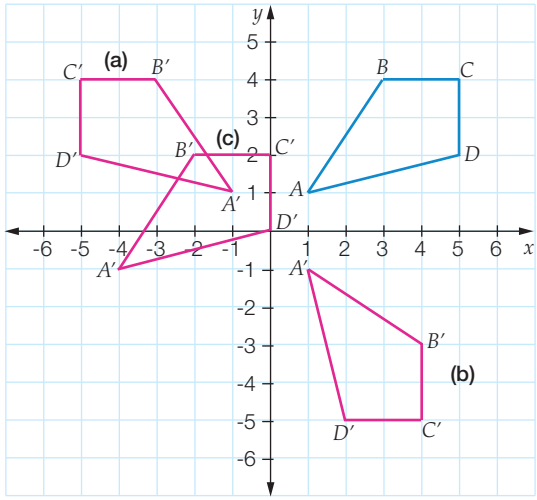
2 (a) $ABCD \equiv MNOP$ (b) $FGH \equiv ZXY$

(c) $UPQR \equiv RSTU$

3 (a) $a = 77°$, $b = 138°$ (b) $u = 110°$

(c) $w = 114°$, $y = 37°$, $z = 35°$

4 D

5 

(a) $A'(-1, 1)$, $B'(-3, 4)$, $C'(-5, 4)$, $D'(-5, 2)$

(b) $A'(1, -1)$, $B'(4, -3)$, $C'(4, -5)$, $D'(2, -5)$

(c) $A'(-4, -1)$, $B'(-2, 2)$, $C'(0, 2)$, $D'(0, 0)$

6 A, B

7 (a) $B \leftrightarrow J$ (b) $AD \leftrightarrow ML$

(c) $JM \leftrightarrow BA$ (d) $\angle JKL \leftrightarrow \angle BCD$

Exercise 8.4 (p. 485)

1 (a) SSS (b) AAS (c) RHS (d) SAS

2 $BAC \equiv MON$ (SAS)

$DEF \equiv KLJ$ (SSS)

$HGI \equiv RPG$ (AAS)

3 C

4 C

5 (a) (i) AAS (ii) $x = 9$ m

(b) (i) SAS (ii) $g° = 69°$

(c) (i) SAS (ii) $x = 5$ cm

(d) (i) RHS (ii) $h° = 58°$

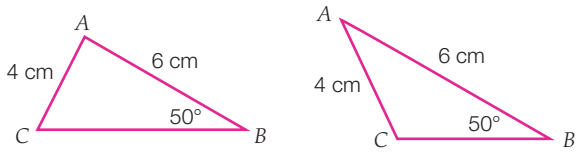
(e) (i) SAS (ii) $m° = 19°$

(f) (i) AAS (ii) $z = 7$ km

6 Students' own answers; many possible solutions.

7 Students' own answers.

8 (a) Possible answer:

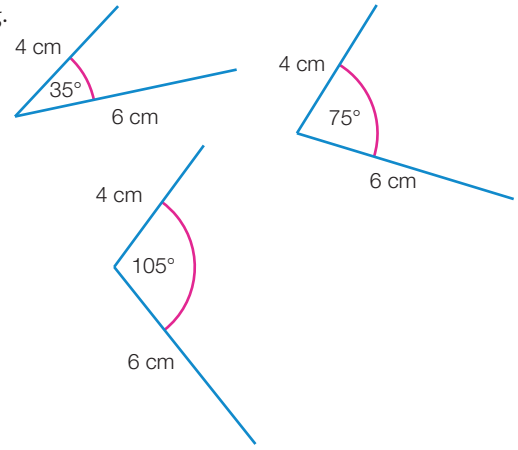


(b) If the lengths of two sides are given, the included angle must be stated to create a unique triangle.

(c) Another angle or side measurement will make the triangle unique; e.g. $\angle A = 95°$, $\angle C = 45°$ or $BC = 10.5$ cm.

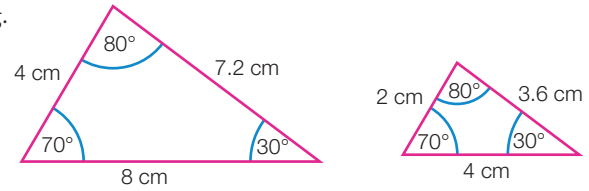
9 The corresponding sides can meet at a variety of angles—like opening or closing a pair of scissors. Specifying the angle in between these two sides 'fixes', or 'anchors' them into position (the SAS test).

e.g.

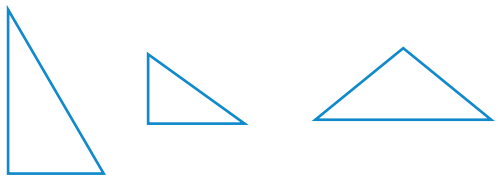


10 Three equal angles means that the triangles will be the same shape, but not necessarily the same size.

e.g.



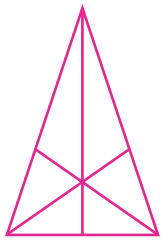
11 **(a)**



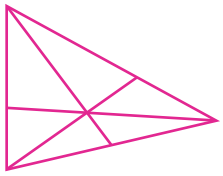
(b) **(i)** Answers may vary: $\Delta ADC \equiv \Delta ADE$; $\Delta AGF \equiv \Delta AGB$; $\Delta EGC \equiv \Delta CGA$.

(ii) All congruency tests can be applied to prove that the different-sized triangles are congruent.

(c) **(i)** An isosceles triangle has six pairs of congruent triangles of different sizes.



(ii) A scalene triangle has three pairs of congruent triangles of different sizes.



(d) Bisecting the sides of an equilateral triangle produces six congruent triangles because the three sides of the original triangle are all the same length. Bisecting the sides of an isosceles triangle produces four congruent triangles of one size, and two of another size, as two of the original triangle's sides are equal. Bisecting the sides of a scalene triangle produces three pairs of congruent triangles, as none of the original triangle's sides are equal.

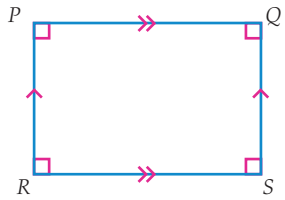
Open-ended – Sample answers

12 Students' own answers. Triangles constructed in this way are SAS or AAS.

13 Students' own answers. Triangles constructed in this way are SSS or RHS.

Exercise 8.5 (p. 494)

1



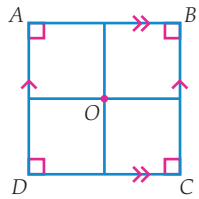
(a) $\angle PQR = \angle SRQ$ (alternate angles)
$\angle PRQ = \angle SQR$ (alternate angles)
$RQ = QR$ (common side)

Therefore, $\Delta PQR \equiv \Delta SRQ$ (AAS)
Therefore, $PR = QS$ and $PQ = RS$
Therefore, both pairs of opposite sides are equal in length.

(b) $\Delta PQR = \Delta SRQ$
Therefore, $\angle RPQ = \angle QSR$
$\angle PRS = 180° - \angle RPQ$ (corresponding angles)
but $\angle SQP = 180° - \angle RPQ$ (corresponding angles)

Therefore, $\angle PRS = \angle SQP$
Therefore, both pairs of opposite angles are equal.

2

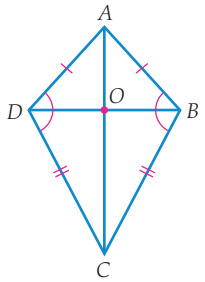


(a) $AD = BC$, $AB = DC$
Therefore, $\Delta ABC \equiv \Delta ADC$ (SSS)
$\angle DAC = \angle BCA$
$\angle AOD = \angle BOC$ (vertically opposite)
$AD = BC$ (given)
Therefore, $\Delta AOD = \Delta BOC$ (AAS)
Therefore, $AO = CO$ and $DO = BO$
$AD = AB$ (given)
Therefore, $\Delta AOD = \Delta AOB$ (SSS)
Therefore, $\angle AOD = \angle AOB = 90°$(angles on a straight line).
Similarly, $\angle COD = \angle COB = 90°$
Therefore, the diagonals bisect each other at right angles.

(b) $\angle DAC = \angle BCA$
$\angle DAC = \angle DCA$ (base angles of isosceles triangles)
$\angle BCA = \angle BAC$
Therefore, $\angle DAC = \angle BCA = \angle BAC = \angle BCA$
So, $\angle DAC = \angle BAC$ and $\angle DCA = \angle BCA$
Similarly, $\angle ADB = \angle CDB = \angle CBD = \angle ABD$
Therefore, each diagonal bisects the two interior angles it passes through.

3 B

4



$AB = AD$
$BC = CD$
Therefore, $\Delta ACD \equiv \Delta ACB$ (SAS)
Therefore, $\angle DAC = \angle BAC$
Therefore, $\Delta AOB \equiv \Delta AOD$ (SAS)
Therefore, $\angle DOA = \angle BOA$
$\angle DOA + \angle BOA = 180°$ (angles on a straight line)
Therefore, the diagonals intersect at right angles.

5 B

6 (a) $p = 100°$, opposite angles are equal

(b) $y = 7$ cm, opposite side lengths are equal

(c) $s = 60°$, $h = 7$ m, all angles and side lengths are equal in an equilateral triangle

(d) $g = 110°$ (SSS)

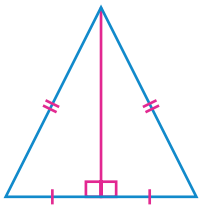
(e) $p = 90°$ (SSS), diagonals cross at right angles in a square

(f) $m = 60°$, alternate angles are equal

(g) $x = 100°$, co-interior angles add to 180°

(h) $f = 60°$, $k = 4$ m, equilateral triangles

7 (a)



(b) $\Delta ABC \equiv \Delta ADC$ (SSS), $\angle ADC \leftrightarrow \angle ABC$, therefore, angles are equal.

(c) isosceles

8 (a) Yes (SSS)

(b) parallelogram (opposite sides and angles are equal) and kite (one pair of equal angles)

9 (a) $\Delta TWV \equiv \Delta TUV$ (AAS), $\angle TWV \leftrightarrow \angle TUV$, therefore, opposite angles are equal.

(b) isosceles

(c) $\angle TUX = \angle VUX$ (diagnonals bisect), therefore, $\Delta TUX \equiv \Delta VUX$ (SAS), $TX \leftrightarrow VX$ (corresponding sides are equal).

(d) The diagonals of a rhombus intersect at right angles. A square has this property.

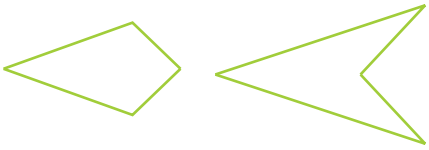
Open-ended – Sample answers

10 (a)

(b)

(c)

11



Challenge 8 (p. 499)

1 $AB = \sqrt{20}$, $BW = \sqrt{10}$, $BX = \sqrt{17}$, $BY = \sqrt{26}$, $BV = 4$, $BZ = 5$, $AW = \sqrt{26}$, $AX = \sqrt{29}$, $AY = \sqrt{34}$, $AV = \sqrt{20}$, $AZ = 5$.

$AB = AV = \sqrt{20}$, hence ΔABV is isosceles. $AZ = BZ = 5$, hence ΔABZ is isosceles.

Answer D

2 B

3 C 4 B 5 B

6 (a) 1 (b) 6 (c) 12 (d) 8

Chapter review 8 (p. 500)

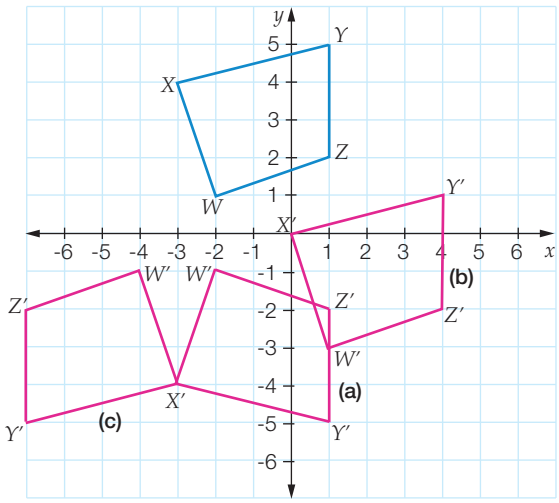
1 (a) $x = 163°$ (b) $x = 41°$ (c) $x = 46°$
(d) $x = 85°$ (e) $x = 45°$ (f) $x = 110°$

2 (a) $y = 53°$, $z = 127°$ (b) $a = 95°$, $b = 110°$, $c = 95°$, $d = 85°$
(c) $m = 55°$, $n = 65°$, $p = 115°$

3 (a) $x = 72°$, $y = 141°$ (b) $c = 118°$, $d = 62°$, $e = 118°$
(c) $f = 90°$

4 (a) C (b) A (c) D (d) B

5



6 (a) $H \leftrightarrow U$ (b) $N \leftrightarrow D$ (c) $TU \leftrightarrow EH$

(d) $AD \leftrightarrow ON$ (e) $\angle CDA \leftrightarrow \angle MNO$

(f) $\angle EFG \leftrightarrow \angle TSV$

7 (a) AAS (b) SAS

8 (a) Join PR, then SSS. (b) Join KM, then SSS.

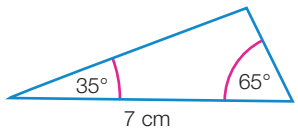
9 (a) 5 (b) $5 \times 180° = 900°$ (c) $900 \div 7 = 128.6°$

10 (a) AAS (b) SAS (c) AAS

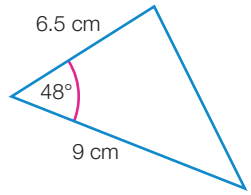
11 D

12 (a) rectangle (b) rhombus (c) parallelogram

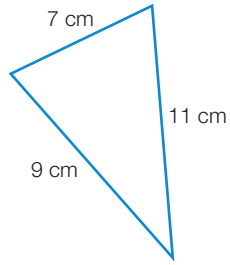
13 (a) (i)



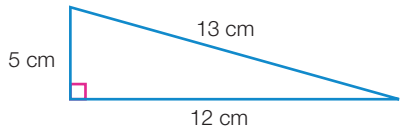
(ii)



(b) (i)



(ii)



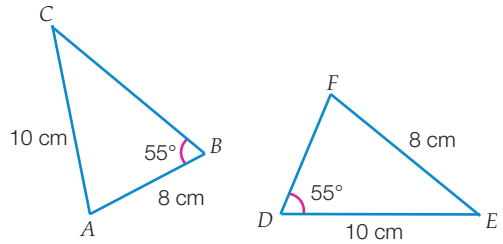
14 (a) $x = 77°$ (b) $x = 60°, y = 120°$

(c) $w = 108°, y = 108°, z = 72°$

15 rhombus, rectangle

16 Triangles are congruent—two sides are equal and one angle, but as it is not the included angle, more information is needed. Triangles overlapping to make an isosceles triangle show that EC and AD are equal in length and that $\angle A = \angle C$.

17 (a)



(b) For the triangles to be congruent, the positions of the sides and angle must be specified—the angle needs to be the angle between the sides (the included angle), then we have SAS congruence.

(c) BC is 7 cm.

18 A square is a rhombus with 90° angles. A square is a rectangle with all four sides equal in length.

NAPLAN practice 8

1 60° 2 B 3 D 4 $y = 50°$

5 $z = 40°$ 6 $x = 4$ cm, $y = 70°$

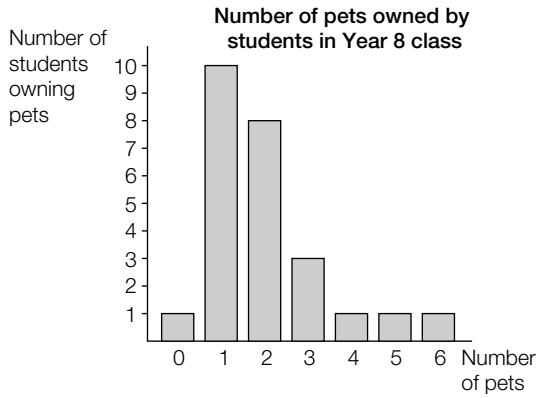
Chapter 9

Recall 9 (p. 506)

1 (a) Number of pets owned by students in Year 8 class

Number of pets	Tally	Frequency
0	|	1
1	卌 卌	10
2	卌 |||	8
3	|||	3
4	|	1
5	|	1
6	|	1

(b)



2 (a) mean = 3.5, median = 3, mode = 2

(b) mean = 35, median = 35, there is no mode

3 (a) $\frac{3}{4}$ (b) 0.8

4 Mean: 2.4 Median: 2 Mode: 2

5 The median is 31.

Exercise 9.1 (p. 512)

1 (a) Estimated number of green = 1000
Estimated number of red = 900
Estimated number of black = 600
Range for green = 950 to 1050
Range for red = 855 to 945
Range for black = 570 to 630

(b) Estimated number of green = 720
Estimated number of red = 450
Estimated number of black = 630
Range for green = 684 to 756
Range for red = 427 to 473
Range for black = 598 to 662

2 (a) 2250 fish (b) 500 koalas

3 (a) Not biased

(b) Biased – Not all teenagers are in Year 8 and, as you get older, your hand span increases.

(c) Biased – Poorer families may not have a car.

(d) Biased – Healthy people are less likely to be at the medical centre, so the rate of illness may be overstated.

(e) Biased – New cars under warranty are less likely to be serviced by the local mechanic.

(f) Not biased

4 (a) Set 1 - Biased – too many 1s
Set 2 - Biased – too many 4s
too few 3s
Set 3 - Fair
Set 4 - Biased – too many 5s
too few 1s + 6s
Set 5 - Biased – too many 4s + 6s
too few 1s + 3s

(b) Based on the individual results it seems that the die is not fair. The number 4 seems to appear too often.

(c) Number of expected results $= \frac{300}{6}$
$= 50$

Number	Total
1	45
2	52
3	41
4	61
5	51
6	50

The number 3 appears 9 times fewer than it should and the number 4 appears 11 times more than it should. This makes us think that the die is biased, and a higher number of rolls would confirm or reject this conclusion.

5 (a) Month 1: 818, Month 2: 808, Month 3: 833, Month 4: 735

(b) 802

(c) The estimate of all the samples put together is likely to be the more reliable, as you are using a larger sample size to calculate the population.

6 It would seem that more 6s occur than we would expect from a fair die. However, each sample certainly does not show this explicitly. If we total all 300 rolls, we find there are 64 6s, whereas the theoretical expected value is 50. The next highest total is 52. We also have two very low totals—4 with 40 and 5 with 41. In summary, the die does appear to be biased.

7 (a) (i) If every loaf was tested there would be none left to sell.

(ii) Students' own answers.

(b) (i) between 48 and 52

(ii) The manufacturer probably means the mean. In this context, it wouldn't really make much difference, as there would be a small range of values with the mean, median and mode being very close to each other.

(iii) The machinery is probably set up to ensure that there are at least 50 matches in each box, and the boxes could be weighed to check this.

8 (a) If it is expected that views will be different, then the difference in the numbers surveyed should be proportional to the population.

(b) This shows that the people surveyed were from more than just one state, so the survey could be unbiased.

(c) To ensure that each state had a fair representation, you could take a proportional number of people to be surveyed in each state according to the population of that state.

(d) The additional information probably makes you more wary about accepting the conclusions. As a starting point, it does not include any people from the most populous state (NSW), and people voluntarily opted into the survey, which is not usually a sound method.

9 (a) The percentages seem very high.

(b) The results would be based on a sample.

(c) No indication is given in the article.

(d) No indication is given in the article.

(e) Students should provide their own answers, but this should include comments about sample size and how people would be selected.

10 (a) 2040 (b) 336 (c) 998 (d) 690

(e) Scientific research accounts for nearly $\frac{1}{2}$ of all the whales killed. Whaling under objection accounts for approximately $\frac{1}{3}$, whereas aboriginal subsistence is only approximately $\frac{1}{6}$ of the total.

(f) (i) Length and weight of adult males and females, birth weight, life span, food source, habitat and reproductive maturity.

(ii) Ensuring their food source is not depleted, that their habitat is not polluted or threatened, that pregnant females and immature whales are not killed and that no more than necessary are killed to supply whale meat demands.

11 (a) Minke whale. This is the only species that had an increase in population.

(b) Right whale. This species had the greatest decrease in population.

(c) They were all in danger of extinction. The gray whale numbers have recovered faster than the other two species.

(d) Blue, fin, humpback, right, sei.

(e) Its numbers have increased significantly from the original level, so it is not in any danger of extinction.

(f) (i) Because populations of whales are very hard to estimate, these figures are not very accurate.

(ii) Tag and release is impossible because of the size of the whales.

(iii) Whales must surface to breathe, so planes and boats are constantly counting them when they surface and comparing their data.

12 (a) Although 30 000 is a large number, the ban has prevented a great deal more and has saved some species from extinction. An average of 1200 whales were killed each year over this 25-year period.

(b) Japan

(c) (i) Yes, numbers could be underestimated.

(ii) Yes, blaming whaling for reduced numbers when other factors such as disease, depleted food supply or global warming may be responsible.

(d) (i) Yes, numbers of whales could be overestimated.

(ii) Yes, blaming other factors such as disease, global warming or a depleted food supply.

13 (a) 3 billion

(b) The world population doubled.

(c) There will be no unusual events that will reduce the world population dramatically, for instance Earth being hit by a meteorite.

(d) No, an assumption is a belief that is based on evidence of what has happened in the past; a bias is a belief that has no evidence to support it.

(e) 45 years

(f) Food and housing shortages, environment damage – loss of species, global warming, running out of resources.

(g) Many countries submit census data to be collated. This would be very accurate. The population of other countries is estimated through sampling and other statistical techniques.

14 (a) 1962–1971

(b) The large number of babies born after the war would themselves be having children.

(c) 1.1%

(d) It has declined steadily.

(e) The population would still be increasing, but more slowly than at anytime in this period.

(f) The growth rate would have decreased. This would have happened from 1957 to 1960.

15 If values anywhere near 10 come up in each sample we should not claim bias.

Open-ended – Sample answers

16 Students should provide their own answers, but this should include comments about sample size and how people would be selected.

17 (a) 5 times

(b) 30 rolls is a fairly small sample, so we would not claim bias.

(c) 15

(d) Create more samples to see if any number was consistently overrepresented.

(e) Unless half or more of a particular number comes up we should not claim bias.

18 Biased. Not everyone watches TV or that particular TV station. Not everyone who watches would bother to respond (only those with strong opions or those who like responding to TV polls would respond).

Exercise 9.2 (p. 522)

1 (a) (i) 42 (ii) 42 (iii) bimodal (36 and 37)

(b) (i) 45.5 (ii) 43.5 (iii) no mode

2 (a) (i) Set 1: Mean = 13.33 Set 2: Mean = 12.87

(ii) Set 1: Median = 10 Set 2: Median = 10

(iii) Set 1: Range = 14 Set 2: Range = 21

(b) Changing the first piece of data has an effect on both the mean and range of the data, but has no effect on the median.

3 (a) The mean score is 22.44 and the median is 20 points.

(b) The median is probably the better predictor of scores as there are four scores of 40 or above, which will push the mean score out a bit too far.

4 (a) A (b) D (c) B

5 (a) 39.97 (b) 22

(c) 1st test, 1st innings: Batsman 3
1st test, 2nd innings: Batsman 2
2nd test, 1st innings: Batsman 5
2nd test, 2nd innings: Batsman 9

(d) No—some batsmen are specialist batsmen and would expect to average a higher score than the other batsmen who may be specialist bowlers.

6 (a) Mean: 178.6 g Median: 175 g Mode: 175 g

(b) The mean

(c) $250 \times 54 \times 178.6 = 2\,411\,1100$ g = 2411.1 kg

(d) 2290–2532 kg, within 5% of the estimated mean

(e) (i)

	Mean	Median	Mode
Set 1	174	175	175
Set 2	176	175	175
Set 3	181	175	175
Set 4	180	175	190

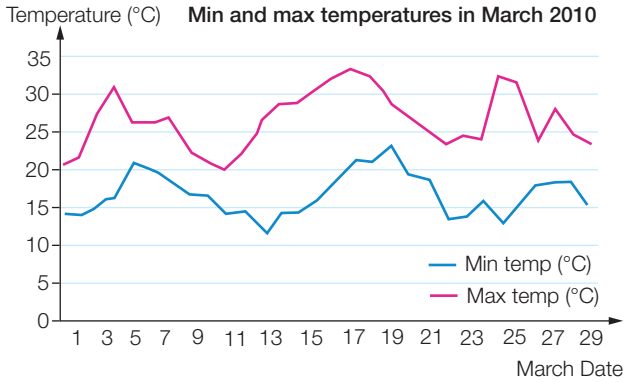
(ii) The median (iii) The mean

7 (a) Mean: 15.08 Median: 15 Range: 25

(b) Mean: 14.92 Median: 15 Range: 9

(c) Having an outlier at each end of the data set can balance out the mean, but there is a major impact on the range. The median is not affected.

8 **(a)**



(b) Estimated max temp (°C) = 26–27
Estimated min temp (°C) = 16–17

(c) The mean minimum temperature was 16.3°C and the mean maximum was 26.2°C.

(d) Both of these figures are well above the historical mean values, so you would have to say that March in 2010 was warmer than usual in Melbourne.

(e) Even compared to the restricted figures, which are both higher than the historical means, Melbourne was warmer than usual in March 2010.

(f) A good argument could be mounted for either point of view. The historical mean has the advantage of taking into account every year for which records have been kept, but the restricted means might better reflect the current trend in temperatures. If we are passing through a warm spell in general, as far as temperatures are concerned, we might be able to better compare a particular year to its era.

9 **(a)** Students' own answers, but it would probably be somewhere in the region of 140 cm or less and 172 cm or greater.

(b) The mean and median values are larger than expected for 13-year-old boys, so these boys are probably older than 13 years. The median is 160 cm, the mean is 161 cm and the mode is 155 cm.

(c) **(i)** mean 163.9, median 160.5, mode 160

(ii) The median. It is close to the median for the data set given. The mean and mode are quite different. The inclusion of a possible outlier of 177 cm made no difference to the median.

10 **(a)** 25.9% **(b)** 27–45 mg

(c) The combined coffee consumption would be closest to 4.7 kg per capita per year, because this is closest to the quantity in the countries with the largest populations (USA, France, Italy and UK), and the countries with a high consumption have small populations.

(d) The figures were obtained from a sample, as a census would have been prohibitively expensive and time-consuming.

(e) As we don't know about the spread of the data in the provided range, we can't accurately estimate any of the averages. However, when the range is smaller, we can provide more accurate estimates than with a larger range.

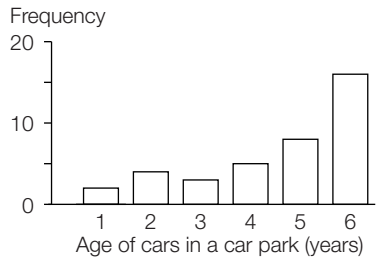
Open-ended – Sample answers

11 2, 4, 10, 11, 11, 15, 16, 17, 18, 19

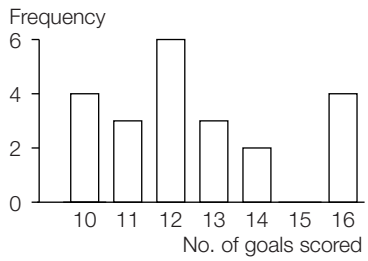
12 0, 1, 2, 11, 11, 15, 16, 17, 18, 32

Exercise 9.3 (p. 530)

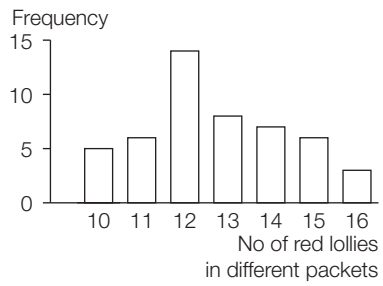
1 **(a)**



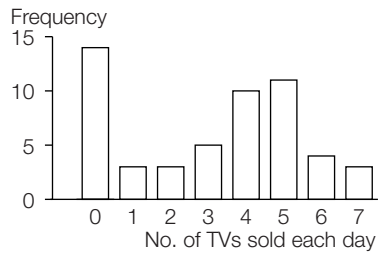
(b)



(c)

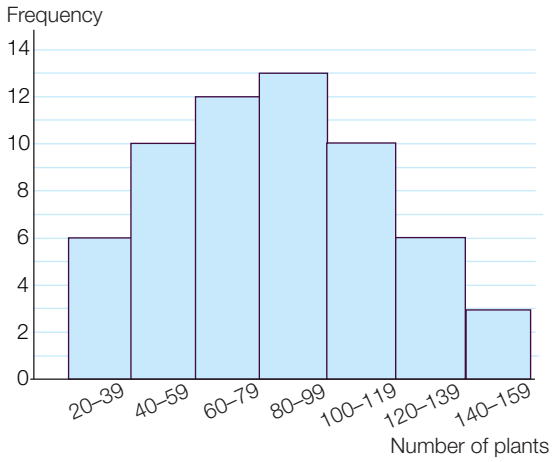


(d)



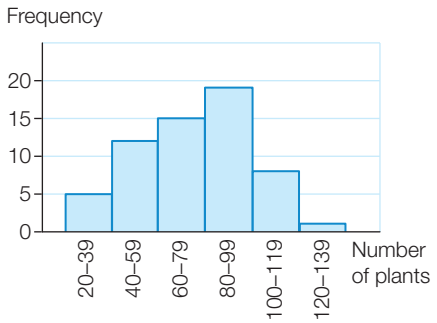
2 **(a)**

Number of plants	Frequency
20–39	6
40–59	10
60–79	12
80–99	13
100–119	10
120–139	6
140–159	3



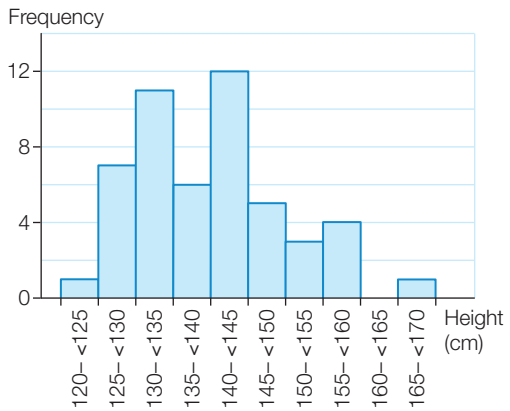
(b)

Number of plants	Frequency
20–39	5
40–59	12
60–79	15
80–99	19
100–119	8
120–139	1



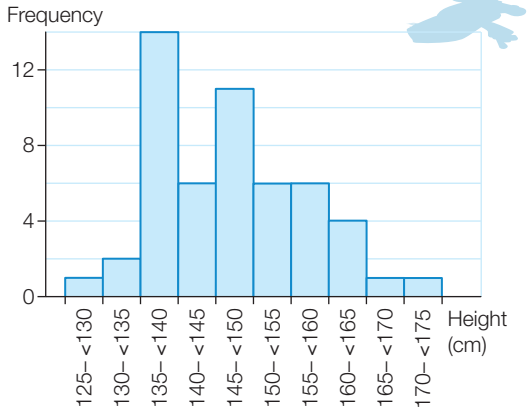
3 (a)

Height in cm	Frequency
120–<125	1
125–<130	7
130–<135	11
135–<140	6
140–<145	12
145–<150	5
150–<155	3
155–<160	4
160–<165	0
165–<170	1



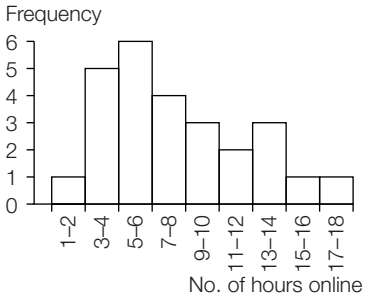
(b)

Height in cm	Frequency
125–<130	1
130–<135	2
135–<140	14
140–<145	6
145–<150	11
150–<155	5
155–<160	5
160–<165	4
165–<170	1
170–<175	1



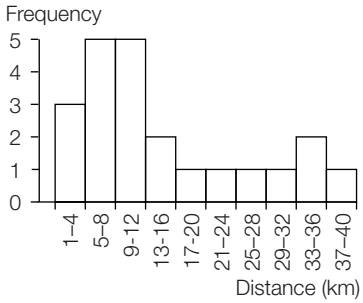
4 (a)

Hours online	Frequency
1–2	1
3–4	5
5–6	6
7–8	4
9–10	3
11–12	2
13–14	3
15–16	1
17–18	1



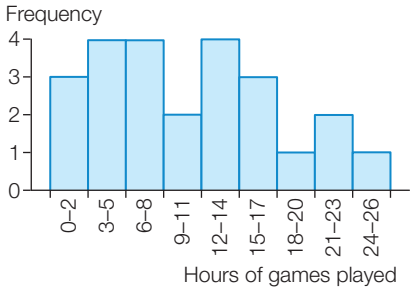
(b)

Distance (km)	Frequency
1–4	3
5–8	5
9–12	5
13–16	2
17–20	1
21–24	1
25–28	1
29–32	1
33–36	2
37–40	1



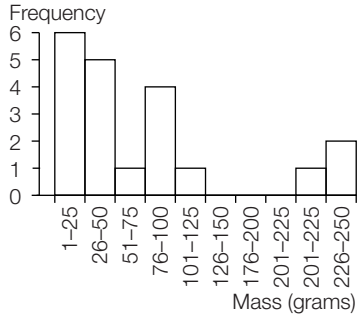
(c)

Hours of games played	Frequency
0–2	3
3–5	4
6–8	4
9–11	2
12–14	4
15–17	3
18–20	1
21–23	2
24–26	1



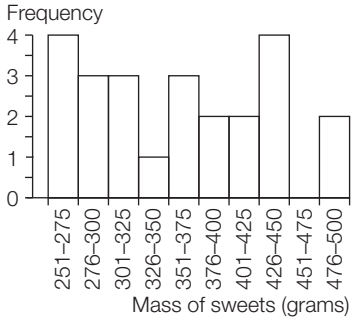
5 (a)

Mass (grams)	Frequency
1–25	6
26–50	5
51–75	1
76–100	4
101–125	1
126–150	0
151–175	0
176–200	0
201–225	1
226–250	2



(b)

Mass of sweets (g)	Frequency
251–275	4
276–300	3
301–325	3
326–350	1
351–375	3
376–400	2
401–425	2
426–450	4
451–475	0
476–500	2



6 (a) D (b) B (c) C

7 (a)

Age (years)	Frequency
1	4
2	3
3	6
4	5
5	8
6	7
7	2

(b)

No. of house sales	Frequency
0–1	3
2–3	2
4–5	4
6–7	0
8–9	1
10–11	2
12–13	2
14–15	1

(c)

Winning margin	Frequency
0–9	4
10–19	7
20–29	5
30–39	4
40–49	2

(d)

Amount ($)	Frequency
0–4.95	2
5–9.95	3
10–14.95	4
15–19.95	2
20–24.95	1

(e)

Temperature (°C)	Frequency
14–<18	5
18–<22	6
22–<26	6
26–<30	7
30–<34	4

(f)

Number	Frequency
1–5	4
6–10	5
11–15	0
16–20	3
21–25	6
26–30	6
31–35	1

8 The histogram in **(a)** is symmetrical, whereas the one in **(b)** has a negative skew. There are no outliers in either graph.

9 Both histograms have a positive skew, whereas the histogram in **(a)** appears to have an outlier.

Open-ended – Sample answers

10 25, 18, 33, 27, 12, 30, 45, 41, 21, 57, 5, 33, 47, 21, 15, 51, 36, 48, 9, 20, 19, 31, 35, 22, 46, 30, 15, 27, 10, 41

11 41.5, 47.3, 36.8, 53.3, 31.2, 40.5, 51.2, 39.9, 41.4, 44.9, 27.1, 58.3, 38.7, 48.0, 60.0, 44.4, 39.6, 54.7, 52.3, 47.1, 44.5, 34.6, 62.2, 49.1, 37.6, 42.2, 46.5, 57.8, 33.7, 31.4, 61.3, 40.0, 25.9, 50.6, 47.7, 30.5, 45.3, 52.1, 62.4, 30.8

12 The numbers along the horizontal axis are not consistently scaled.

13 There is overlap of the class intervals. Should 10 be entered on the first line, the second line, or both?

Exercise 9.4 (p. 543)

1 (a)

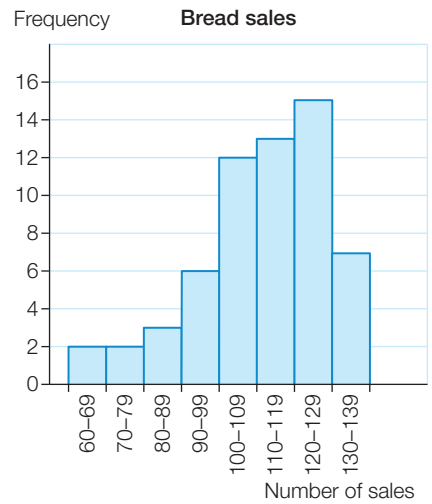
Class interval	Midpoint	Frequency	*fx*
60–69	64.5	2	129
70–79	74.5	2	149
80–89	84.5	3	253.5
90–99	94.5	6	567
100–109	104.5	12	1254
110–119	114.5	13	1488.5
120–129	124.5	15	1867.5
130–139	134.5	7	941.5
		60	6650

(i) mean = 110.83

(ii) median class interval = 110–119

(iii) modal class interval = 120–129

(iv)



(v) No outliers.

(b)

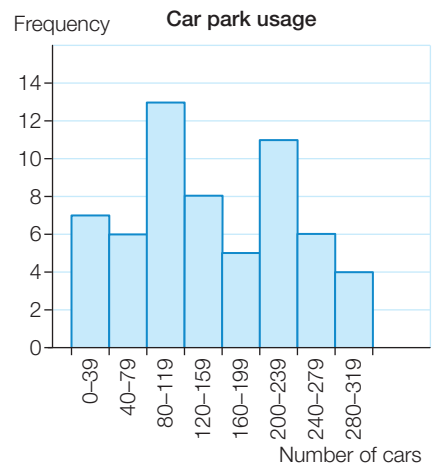
Class interval	Midpoint	Frequency	*fx*
0–39	19.5	7	136.5
40–79	59.5	6	357
80–119	99.5	13	1293.5
120–159	139.5	8	1116
160–199	179.5	5	897.5
200–239	219.5	11	2414.5
240–279	259.5	6	1557
280–319	299.5	4	1198
		60	8970

(i) mean = 149.5

(ii) median class interval = 120–159

(iii) modal class interval = 80–119

(iv)



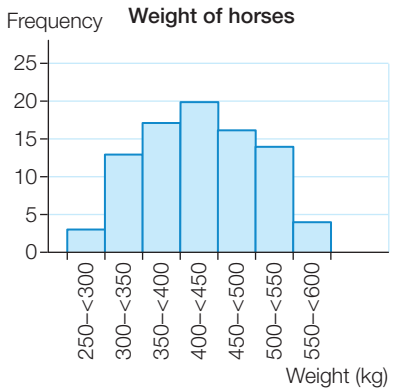
(v) No outliers

2 (a) (i) mean = 427.30 kg

(ii) median class interval = 400–<450 kg

(iii) modal class interval = 400–<450 kg

(iv)



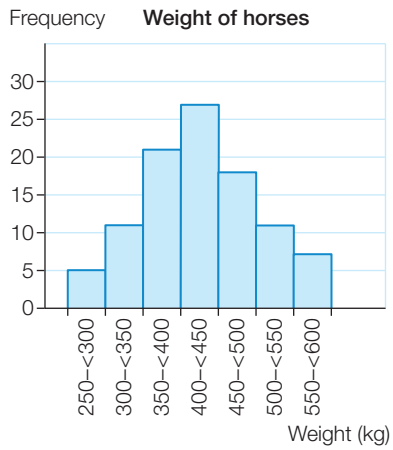
(v) No outliers.

(b) (i) mean = 426.50 kg

(ii) median class interval = 400–<450 kg

(iii) modal class interval = 400–<450 kg

(iv)



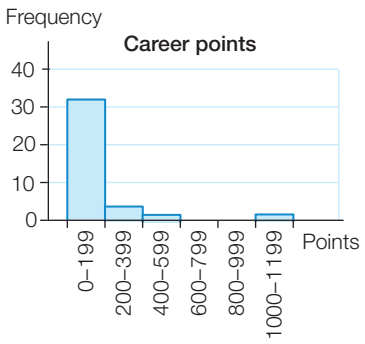
(v) The data distribution is symmetrical with no outliers.

3 (a) (i) 66.47 (ii) 65

(b) (i) 151.13 (ii) 157.5

(c) (i) 242.57 (ii) 245

4 (a)



(b) Certainly, 1163 is an outlier, and an argument could be made for 582 and 302 as well.

(c) mean = 148.15, median class = 0–199, range = 0–1199

5 (a) B (b) B

6 (a) 10 (b) 10.1, very close (c) 50 children

(d) Organise a treasure hunt for 50 children with about 500 sweets and record the results; repeat several times.

(e) No, from the data, we don't know the exact number each child found.

7 (a) 1–10, 11–20, 21–30, 31–40, 41–50

(b) 0–4, 5–9, 10–14, 15–19, 20–24

(c) 0–49, 50–99, 100–149, 150–199, 200–249

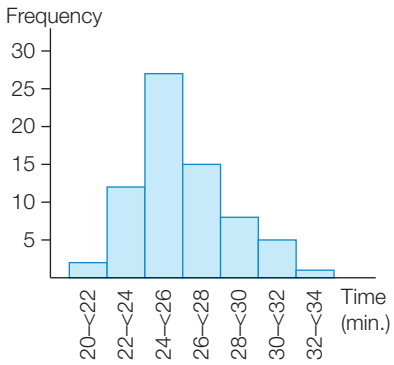
(d) 1–50, 51–100, 101–150, 151–200, 201–250

(e) 10–12, 13–15, 16–18, 19–21, 22–24

(f) 25–29, 30–34, 35–39, 40–44, 45–49

8 (a) 25.97 minutes

(b)



(c) Their times would increase the mean.

(d) You might just assign them a time, say 35 minutes.

9 (a) (2–<3) km (b) (3–<4) km (c) 3.3 km

10 (a) 71.94 (b) 0–39 games (c) 40–79 games

(d) The mean

(e) mean: 93.25 mode: 0–39 games
median: 79.5 (between 40–79 and 80–119)

(f) Yes, 13 out of 45 is nearly one-third of the team that hasn't played a game yet. Saying the players have played 93 games each 'on average' makes more sense than saying 72 games including non-starters.

11 (a) The class interval has an odd number of values in it.

(b) The class interval has an even number of values in it.

(c) No other outcome is possible because there are only odd numbers and even numbers and both of these have been covered.

Open-ended – Sample answers

12 (a) 20–24, 25–29, 30–34; 11–15, 16–20, 21–25

(b) 0–9, 10–19, 20–29; 51–60, 61–70, 71–80

13 Students will produce their own set of temperatures for one of these places. The Bureau of Meteorology website can be used to find the actual data for each place.

14 (a)

STEM	LEAF
0	3 6 8
1	0 1 2
2	0 5 8 9
3	3 3 5 6 7 9
4	1 3 3 4 6
5	0 1

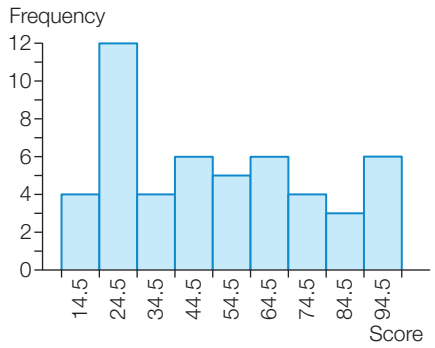
(b)

STEM	LEAF
2	1 3 4 7 9
3	0 5 5 5 8 9
4	1 4 7
5	3 6 8 8
6	6 8 9

Half-time 9 (p. 550)

1

Score	Frequency
10–19	4
20–29	12
30–39	4
40–49	6
50–59	5
60–69	6
70–79	4
80–89	3
90–99	6



2 (a) 40.10 (b) 23 (c) 0

(d) 178, as this value is quite a bit higher than all the other values. It is unusual for any player to play so many games.

(e) mean: 46.52, median: 33, mode: 4 and 9. Yes, all of the statistics changed significantly.

(f) The median didn't change much when the outlier was removed, so is probably the best measure of centre.

3 (a) \$8.30 (b) \$(5–<10)

4 (a)

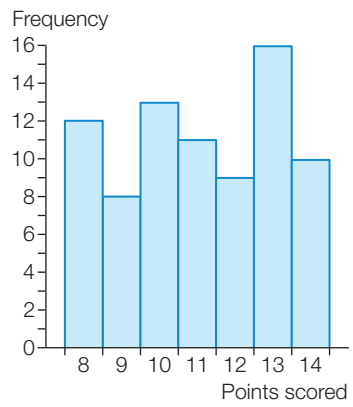
Age	Frequency
1	5
2	6
3	6
4	5
5	4

(b)

Margin	Frequency
1–5	5
6–10	6
11–15	6
16–20	5
21–25	4

5 (a) mean: 1.92 (b) median: 1

6



7 300 crocodiles

Exercise 9.5 (p. 557)

1 (a) $\frac{5}{13}$ (b) $\frac{8}{13}$

2 (a) $\frac{4}{5}$ (b) $\frac{3}{5}$

3 $\frac{7}{8}$

4 (a) $\frac{5}{13}$ (b) $\frac{8}{13}$ (c) $\frac{1}{13}$

5 (a) $\frac{4}{9}$ (b) $\frac{5}{18}$ (c) $\frac{1}{18}$

6 (a) $\frac{1}{1000}$ (b) $\frac{999}{1000}$

7 (a) $\frac{1}{3}$ (b) $\frac{3}{5}$ (c) $\frac{11}{15}$ (d) $\frac{3}{5}$

8 A 9 $\frac{3}{10}$ 10 $\frac{5}{12}$

11 (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{1}{6}$ (d) $\frac{1}{3}$ (e) $\frac{5}{6}$ (f) $\frac{1}{2}$

12 (a) B (b) A (c) B

13 (a) $\frac{1}{4}$ (b) $\frac{11}{20}$ (c) 1 (d) $\frac{1}{5}$ (e) $\frac{4}{5}$ (f) $\frac{1}{2}$

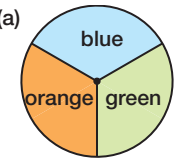
14 (a) 16.1% (b) 29.8% (c) 9.6%

(d) 28.7% (e) 98.4% (f) 96.3%

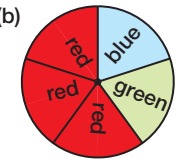
(g) Only very slightly — the types of cars people buy won't change much year on year.

(h) It is slightly more probable that a small car was bought as more were sold.

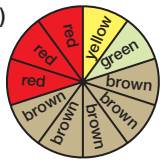
15 (a)



(b)



(c)



Open-ended – Sample answers

16 (a) impossible – 0%, unusual – 30%, usually – 70%, very unlikely – 5%, even chance – 50%, most likely – 75%, most unlikely – 10%, 50-50 – 50%, most often – 80%, little chance – 20%, more often than not – 60%, sure thing – 99%, unlikely – 25%, probably – 65%, certain – 100%

(b) rare, occasionally, frequently, possibly, good chance

17 (a) 95% (b) 70% (c) 90% (d) 65%
(e) 50% (f) 1% (g) 60% (h) 70%

Exercise 9.6 (p. 563)

1 (a) ♥ : Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King
♦ : Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King
♣ : Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King
♠ : Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King

(b) (i) $\frac{1}{13}$ (ii) $\frac{2}{13}$

2 (a) (i) T, H (ii) $\frac{1}{2}$

(b) (i) 1, 2, 3, 4, 5, 6 (ii) $\frac{1}{6}$

(c) (i) red, blue, green, yellow (ii) $\frac{1}{4}$

(d) (i) A♥, A♦, A♠, A♣;
2♥, 2♦, 2♠, 2♣;
3♥, 3♦, 3♠, 3♣;
4♥, 4♦, 4♠, 4♣;
5♥, 5♦, 5♠, 5♣;
6♥, 6♦, 6♠, 6♣;
7♥, 7♦, 7♠, 7♣;
8♥, 8♦, 8♠, 8♣;
9♥, 9♦, 9♠, 9♣;
10♥, 10♦, 10♠, 10♣;
J♥, J♦, J♠, J♣;
Q♥, Q♦, Q♠, Q♣;
K♥, K♦, K♠, K♣

(ii) $\frac{1}{52}$

(e) (i) red, blue, green, orange (ii) $\frac{1}{4}$

3 (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{3}$ (e) $\frac{1}{3}$
(f) $\frac{1}{2}$ (g) $\frac{1}{4}$ (h) $\frac{1}{2}$ (i) $\frac{1}{3}$ (j) $\frac{1}{4}$

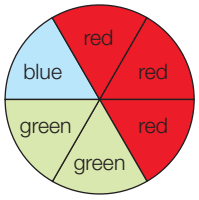
4 (a) $\frac{2}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{4}$ (d) $\frac{3}{4}$

5 (a) $\frac{1}{8}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{1}{4}$ (e) $\frac{1}{4}$
(f) $\frac{1}{4}$ (g) $\frac{5}{8}$ (h) $\frac{5}{8}$

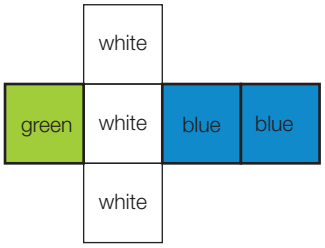
6 (a) 0.5 (b) 0.3 (c) 0.7 (d) 0.5

7 (a) B (b) A (c) A (d) B (e) C

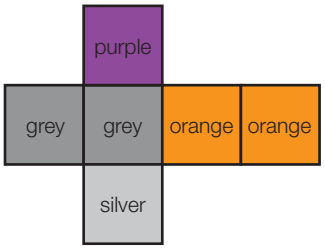
8



9 (a)



(b)



10 (a) For 20 counters: 4 yellow, 8 blue, 6 red, 2 black

(b) For 20 counters: 5 yellow, 5 pink, 3 purple, 1 green, 2 black, 2 blue, 2 white

11 $\frac{2}{9}$

12 (a) The letters that are used most frequently in words have more pieces so that words can be formed right through the game.

(b) (i) $\frac{1}{50}$ (ii) $\frac{2}{25}$ (iii) $\frac{3}{50}$ (iv) $\frac{21}{50}$

(c) (i) $\frac{3}{25}$ (ii) $\frac{1}{11}$ (iii) $\frac{11}{98}$ (iv) $\frac{9}{97}$

Open-ended – Sample answers

13

14 One-third of 40 is $13\frac{1}{3}$, so the most popular colour can have no more than 13. A possible solution is 13 red, 13 yellow, 7 black and 7 blue. In essence, it can be any combination where all values lie between 7 and 13 with a total of 40.

Exercise 9.7 (p. 576)

1 (a) 15 people (b) 5 people (c) 20 people
(d) $\frac{3}{4}$ (e) $\frac{1}{4}$

2 (a) $\frac{7}{30}$ (b) $\frac{5}{12}$

3

H	H,1	H,2	H,3	H,4	H,5	H,6
T	T,1	T,2	T,3	T,4	T,5	T,6
	1	2	3	4	5	6

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$

4 (a) $\frac{136}{200} = \frac{17}{25}$ (b) $\frac{59}{200}$ (c) $\frac{5}{200} = \frac{1}{40}$

(d) $\frac{41}{100}$ (e) $\frac{41}{136}$

5 (a)

	White	Brown
Male	10	4
Female	8	8

(b) (i) $\frac{3}{5}$ (ii) $\frac{7}{15}$ (iii) $\frac{4}{15}$ (iv) $\frac{5}{7}$ (v) $\frac{2}{3}$

6 (a) (i)

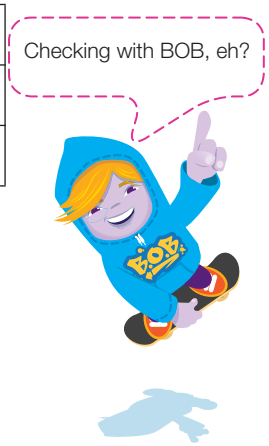
	D	not D
C	$\frac{1}{4}$	$\frac{9}{16}$
not C	$\frac{3}{16}$	0

(ii) $\frac{7}{16}$ (iii) $\frac{1}{4}$ (iv) 1

(v) $\frac{9}{16}$ (vi) $\frac{9}{16}$ (vii) $\frac{3}{16}$

(b) (i)

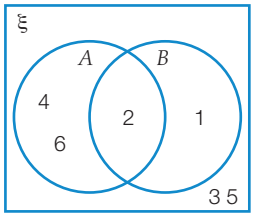
	D	not D
C	0	$\frac{1}{6}$
not C	$\frac{1}{2}$	$\frac{1}{3}$



(ii) $\frac{1}{2}$ (iii) 0 (iv) $\frac{2}{3}$

(v) $\frac{1}{6}$ (vi) $\frac{1}{2}$ (vii) $\frac{5}{6}$

7 (a)



(b) (i) {2} (ii) {1, 2, 4, 6} (iii) {1, 3, 5}

(iv) 3 (v) 2 (vi) 6

(vii) 1 (viii) 4 (ix) 4

(c) (i) $\frac{1}{2}$ (ii) $\frac{1}{6}$ (iii) $\frac{2}{3}$

8 (a) C (b) C (c) C (d) B (e) A (f) C

9 (a) (6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6) (1, 6) (2, 6) (3, 6) (4, 6) (5, 6)

(b) (6, 6)

(c) (6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)

(d) (6, 1) (6, 2) (6, 3) (6, 4) (6, 5)

10 (a) $\frac{11}{36}$ (b) $\frac{1}{36}$ (c) $\frac{1}{6}$ (d) $\frac{5}{36}$ (e) $\frac{1}{6}$

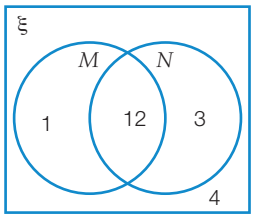
11 (a) $\frac{1}{6}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$

12 (a) $\frac{18}{25}$ (b) $\frac{2}{5}$ (c) $\frac{17}{25}$ (d) $\frac{7}{25}$ (e) $\frac{5}{9}$ (f) $\frac{10}{17}$

13 (a) (i)

	Bought milk	Didn't buy milk
Bought a newspaper	12	3
Didn't buy a newspaper	1	4

(ii)



(b) 12

(c) (i) $\frac{1}{20}$ (ii) $\frac{3}{5}$ (iii) $\frac{4}{5}$ (iv) $\frac{1}{4}$

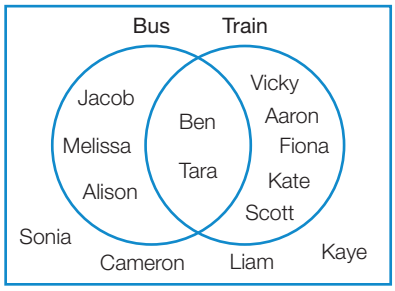
14 (a) $\frac{21}{40}$ (b) $\frac{19}{20}$ (c) $\frac{9}{40}$ (d) $\frac{1}{20}$

15 (a) $\frac{1}{5}$ (b) $\frac{9}{25}$ (c) $\frac{11}{25}$

16 (a) $\frac{17}{25}$ (b) $\frac{4}{25}$ (c) $\frac{21}{25}$ (d) $\frac{11}{15}$

Open-ended – Sample answers

17



$\frac{3}{14}$

18 Robbie double-counted the (3, 3) outcome.

Pr(3 on one die) = $\frac{11}{36}$

Challenge 9 (p. 581)

1 (a) 4

(b) 2

If she only selected one sock, it might be the same as the pair she already had, so she would not have a second pair.

2 The two children cross first, one comes back. The adult crosses, the child comes back. The two children now cross together.

3 A

4 (a) B 3, 5, 7, 9, 11, 13, 15

(b) 7, 9 or 11. Each occurred twice.

(c) The only way to get an even total is if one of the rings misses a peg.

5 (a) 3, 4, 5, 8 (b) $x = 5$

6 The only difference between the graphs is the vertical scale. The second graph appears to be growing faster. They represent the same information.

Chapter review 9 (p. 582)

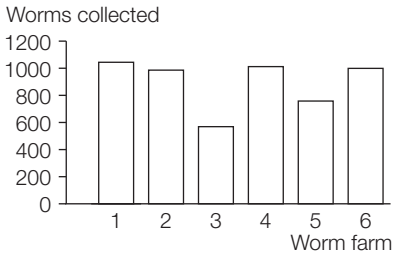
1 375 parrots

2 (a) (i) 28.88 (ii) 29 (iii) 26

(b) (i) 28.19 (ii) 29 (iii) 37

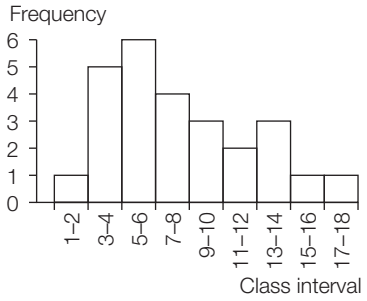
(c) The mean has been reduced slightly and the range has increased significantly. The median is unaffected. The outlier (1) has caused the changes, again showing that the median is unaffected by outliers.

3



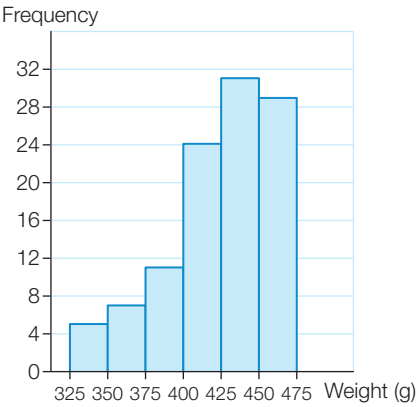
4

Class interval	Frequency
1–2	1
3–4	5
5–6	6
7–8	4
9–10	3
11–12	2
13–14	3
15–16	1
17–18	1



5 (a) 423.95 g (b) 425–<450 g (c) 425–<450 g

(d)

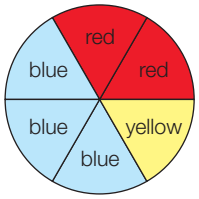


6 (a) $\frac{1}{10}$ (b) $\frac{3}{10}$ (c) 0 (d) $\frac{2}{5}$ (e) $\frac{7}{10}$ (f) $\frac{7}{10}$

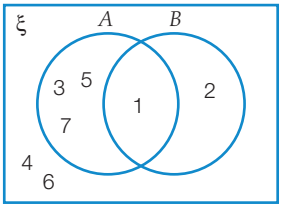
7 (a) $\frac{1}{6}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{5}{6}$

8 (a) A (b) A (c) D

9



10 (a)



(b) (i) {1, 2, 3, 5, 7} (ii) {1} (iii) {2, 4, 6}

(c) (i) $\frac{5}{7}$ (ii) $\frac{1}{7}$ (iii) $\frac{3}{7}$

11 (a) 23.55 (b) 7 (c) 138

(d) 138 is clearly an outlier.

(e) Mean is 25.27, median is 22 and range is 58. There doesn't appear to be an outlier.

12 (a) Two pieces of information on the graph—each has its own scale.

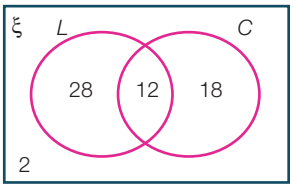
(b) October (c) August

(d) (i) December (ii) August

13 (a) 20 cars

(b) (i) $\frac{1}{20}$ (ii) $\frac{4}{5}$ (iii) $\frac{2}{5}$

14 (a)



(b) (i) $\frac{2}{3}$ (ii) $\frac{7}{15}$ (iii) $\frac{1}{2}$

(iv) $\frac{1}{5}$ (v) $\frac{1}{3}$ (vi) $\frac{29}{30}$

15 (a) four blue, four green and two red

(b) five blue, one black and four pink

16 (a) $\frac{1}{2}$ (b) $\frac{1}{10}$

17 (a) 188.20 cm (b) 188.5 cm (c) 191 cm

(d) Either the mean or the median. The two values are very similar.

(e) 211 could be an outlier.

(f)

Interval class (cm)	Frequency
171–175	1
176–180	4
181–185	10
186–190	13
191–195	15
196–200	1
201–205	1
206–210	0
211–215	1

The estimate for the mean is 188.43 cm.

(g) In this case, there was almost no loss of accuracy. In part, this was because of the tight bunching of the data.

NAPLAN practice 9

1 B **2** C **3** A **4** A

5 3.88 **6** C **7** B

Mixed review D (p. 588)

1 (a) $\frac{131}{800}$ (b) $\frac{631}{2000}$

2 (a) $\frac{51}{52}$ (b) $\frac{1}{2}$ (c) $\frac{12}{13}$ (d) $\frac{3}{4}$

3 (a) -12 (b) 38 (c) 0

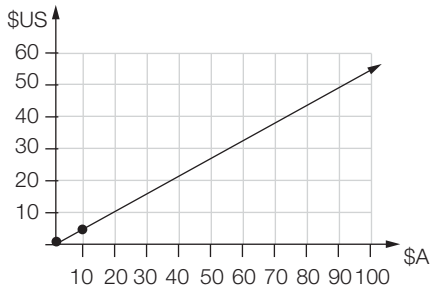
4 (a) $51\frac{2}{3}\%$ (b) $183\frac{1}{3}\%$

5 (a) 960.5 (b) 918 (c) 1330.25

6 (a) $g = 2$ (b) $m = 2$ (c) $s = 45$

7 (b) (0, 0) (c) US \$5.50, (10, 5.5)

(a) and (d)



(e) \$A45 is worth approximately \$US25

(f) \$US50 is worth approximately \$A90

8 (a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$

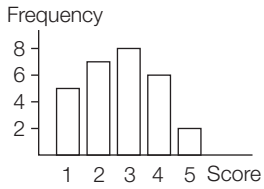
9 (a) 2.5 km/h (b) 18.75 km

10 (a) 2×3^2 (b) $\frac{2^{10}}{3^5}$

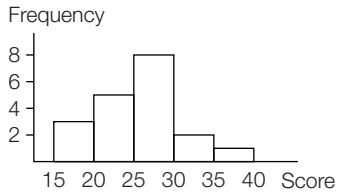
11 (a) 4^6 (b) 6×8^2

12 (a) $3d + 5e$ (b) $17jk + 8j - 35$

13 (a)



(b)



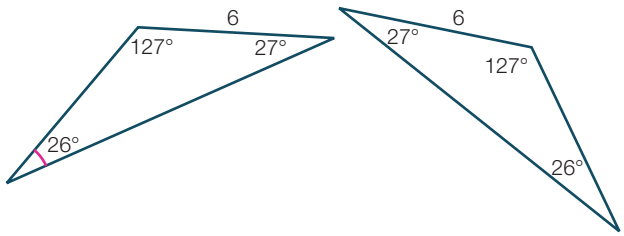
14 (a) $x = -19$ (b) $x = 2$

15 (a) 1, 2, 3, 4, 5, 6

(b) (i) $\frac{1}{6}$ (ii) 0 (iii) $\frac{1}{3}$ (iv) 0

16 (a) 40 (b) 8

17



Find the missing angle on the second figure to show ASA congruency.

18 (a) 86.59 cm^2 (b) 47.12 cm^2 (c) 178.54 cm^2

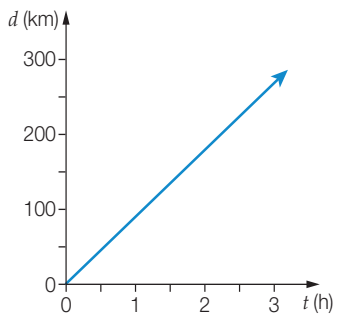
19 (a) $x = -3$ (b) $x = 8$

20 (a) $y = \frac{3x}{4} - 3$ (b) $y = -\frac{2x}{5} + 2$

21 4 yellow, 2 green, 8 red, 6 brown

22 (a) $d = 90t$

(b)



(c) (i) 67.5 km (ii) 210 km

(d) (i) 48 min (ii) 2 h 48 min

23 (a) $\frac{x}{3} - 6 = 27$, the number is 99

(b) $\frac{2x}{6} + 10 = x$, the number is 15

24 104.5 cm^2

25 (a) 390

(b) The survey had done a reasonable job of representing the population.

26 \$2072 per fortnight

27 Sleep: 35%; TV: 11%; School: 36%; Sport: 10%; Family: 8%

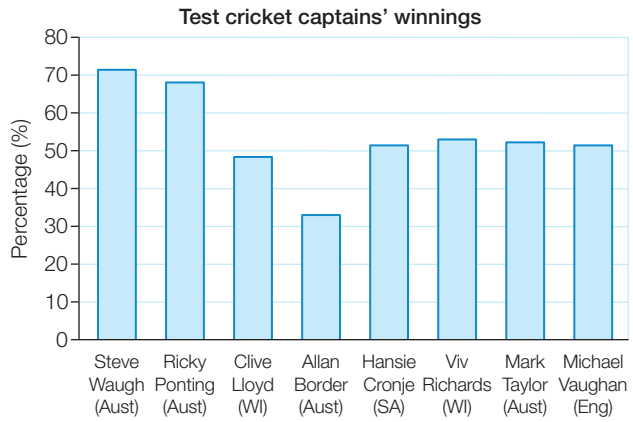
28 (a) $\frac{19}{20}$ (b) $\frac{4}{5}$ (c) $\frac{7}{10}$

29 (a) Steve Waugh appears to be the most successful captain.

(b) You would need to know for how many games they captained the team.

(c) Waugh 72%, Ponting 68%, Lloyd 49%, Border 34%, Cronje 51%, Richards 54%, Taylor 52%, Vaughan 51%

(d)

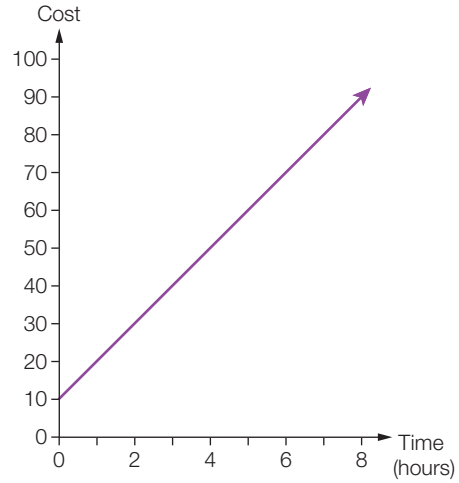


(e) Steve Waugh is still the most successful captain, although the other captains have not necessarily retained their positions on the list.

30 (a)

Time (h)	0	2	4	6	8
Cost (\$)	10	30	50	70	90

(b)



(c) Cost for 0 hours hire

(d) 34 hours

Glossary and index

24-hour time a system of writing time that numbers the hours from 0 to 23. It is not necessary to write a.m. or p.m. (p. 318)

acute-angled triangle a triangle in which all angles are acute; i.e. less than 90° (p. 462)

adjacent angles two angles that share a common arm, and a common vertex (p. 451)

alternate angles a pair of angles within parallel lines and on opposite sides of the transversal (p. 452)

area the amount of space contained within a plane shape (p. 274)

associative law the order in which numbers are added or multiplied is not important; e.g. $33 \times 5 \times 2$ could be calculated as 165×2, or 33×10.
$29 + 13 + 17$ could be calculated as $42 + 17$, or $29 + 30$. This can be written in algebra as $a + (b + c) = (a + b) + c$ (p. 170)

backtracking working backwards along a flowchart (p. 404)

balanced even, the same on both sides of scales or an equation (p. 405)

base **1.** the number that is raised to an index or power (p. 32)
2. the base of a triangle, parallelogram or trapezium is the side length that is perpendicular to the height. The face of the prism that has a uniform cross-section and is perpendicular to the height of the prism (p. 276)

bias preference or belief not supported by evidence (p. 313)

bisect to cut in half (p. 463)

capacity volume of produce (often liquid) that a container can hold (unit is litre) (p. 310)

Cartesian plane divides the plane into four quadrants; used to locate points (p. 344)

census the process of collecting data from the entire population (p. 507)

circumference the perimeter of a circle (p. 261)

class centre the middle of a class interval (p. 537)

class intervals the value ranges used to construct graphs such as histograms for grouped data (p. 528)

coefficient the number factor in a term (p. 143)

co-interior angles a pair of angles within parallel lines and together on the same side of the transversal; they are supplementary (p. 452)

complement (of a set A) the set of elements inside the universal set that are not in A; written as A' (p. 555)

complimentary angles a pair of angles that add to 90° (p. 450)

composite shape a shape made up of more than one basic shape (p. 297)

congruence statement if two shapes $ABCD$ and $EFGH$ are congruent we write the congruence statement $ABCD \equiv EFGH$. The symbol '≡' means 'is congruent to'. The vertices of the shapes must be written in the same order so that matching vertices, sides and angles can be identified from the statement. (p. 473)

congruent alike in all respects except position; same shape and size (p. 468)

congruent triangles triangles that are exactly the same shape and size. There are four tests that can be used to determine if two triangles are congruent: AAA (Angle, Angle, Angle), SAS (Side, Angle, Side), AAS (Angle, Angle, Side), and RHS (Right angle, Hypotenuse, Side) (p. 481)

constant a term with no pronumeral part (p. 143)

coordinate an x-value or a y-value used to locate a point (p. 344)

corresponding angles term used to describe matching angles and sides in congruent or similar figures (p. 452)

cost price (CP) the amount a retailer buys an item for (p. 118)

cube to multiply a number by itself, then multiply by itself again (p. 32)

cylinder a solid with circular ends and cross-section (p. 307)

diameter the distance from one side of a circle to the other side through the centre (p. 261)

discount a reduction in the cost of goods to attract customers (p. 118)

distributive law the way to 'multiply out the brackets', $a(b + c) = ab + ac$ (p. 174)

elapsed time a period of time that has passed (p. 317)

equation two expressions either side of an equals sign (p. 143)

equilateral triangle a triangle in which all sides are equal in length, as are all angles (p. 462)

equivalent the same as (but not necessarily written the same way) (p. 404)

equivalent ratios a pair of ratios in which the numbers in one ratio are multiples of the numbers in the other (p. 194)

evaluate to calculate the value of a formula or expression (p. 33)

exact decimal form using a dash or dot(s) to indicate the repeating digits of a recurring decimal (p. 66)

expanded form a power expression written as a string of multiplications (p. 32)

expanding removing brackets using the distributive law (p. 174)

exponent *see power* (p. 32)

expression is formed when terms are added or subtracted; e.g. $3x + 4y$ (p. 143)

exterior angle an angle on the outside of a shape (p. 461)

factorised form an expression written as a product of factors (p. 181)

factorising writing an expression as the product of factors (p. 181)

flowchart a diagram that shows the order in which tasks are to be attempted (p. 404)

formulas plural of formula (also formulae) (p. 154)

frequency column graphs column graphs used to show the frequency of a variable in a data set (p. 527)

Goods and Services Tax (GST) an extra 10% of the selling price that businesses must add onto the cost of a product (p. 118)

gradient the steepness of a line (p. 346)

grouped data data that has been combined into class intervals to simplify the data and the calculation of statistics from the data (p. 537)

guess, check and improve solving equations by choosing a number, substituting it into the equation and repeating this process until the LHS equals the RHS (p. 403)

hectare a unit of area equal to 10 000 square metres (p. 274)

height the height of a triangle, parallelogram or trapezium is the distance to the top of the shape, perpendicular to the base. The height of a prism is also perpendicular to the base. (p. 276)

Highest Common Factor (HCF) the largest factor that occurs in each of the sets of factors of a group of terms or expressions (p. 170)

histogram a graph in which side-by-side columns represent the frequency of numerical data (p. 527)

hypotenuse the longest side on a right-angled triangle; it is always opposite the right angle (p. 462)

included angle the angle between the two sides used in the SAS test for congruent triangles (p. 481)

index *see* power (p. 32)

index form short way of writing a number multiplied by itself several times using a base raised to an index (p. 32)

indices plural of index (p. 32)

inspection solving an equation by looking at the equation and seeing the solution (p. 403)

integers all of the positive whole numbers 1,2, 3, …, the negative whole numbers -1, -2, -3 …, and zero (p. 5)

interior angle an angle on the inside of a shape (p. 465)

intersection the elements that are common to two or more sets (p. 568)

inverse operation each operation has an inverse that produces the opposite operation; e.g. the inverse of + 2 is – 2 (p. 404)

irrational number decimal number that is both non-terminating and non-recurring; i.e. it has an infinite number of digits (p. 65)

isosceles triangle a triangle in which one pair of sides are equal in length; the angles opposite the equal sides are also equal in size (p. 462)

kite a quadrilateral with two pairs of adjacent sides equal in length (p. 278)

like terms terms that have exactly the same pronumeral part (order not important) even though their coefficients may differ (p. 162)

linear graph a straight-line graph (p. 344)

linear relationship a relationship between two quantities that produces a straight line when plotted on the number plane (p. 344)

line graph a graph that shows the relationship between two variables and can consist of several straight line sections (p. 337)

loss difference between the selling price (SP) of a product (or investment) and the purchase price (cost

price, CP) of the product (or investment) after taking account of all costs associated with the purchase and sale, when SP < CP (p. 118)

mark up the amount added onto the cost price of an item sold (p. 118)

mean the arithmetic average of a data set. All data values are added and the total is divided by the number of data values (p. 520)

measures of centre statistics that indicate the middle of a data set. There are three measures of centre we can use, the mean, the median and the mode. (p. 520)

median the middle value of an ordered data set (p. 520)

median class interval the class interval that contains the median (p. 537)

midpoint the middle of a class interval (p. 537)

modal class interval the class interval that contains the mode (p. 537)

mode the most frequently occurring data value (p. 520)

negative usually associated with losses or decreases (p. 5)

non-terminating decimal a decimal that goes on forever (p. 65)

obtuse-angled triangle a triangle in which one angle is obtuse (the other two angles are acute) (p. 462)

ordered pair an x-value followed by a y-value that uniquely determines the location of a point (p. 344)

origin the point where the x-axis and the y-axis intersect; it has the coordinate pair (0, 0) (p. 344)

outcome the result of a probability experiment (p. 554)

outlier an unusual data value, quite different from other values in the data set (p. 521)

parallelogram quadrilateral with both pairs of opposite sides parallel (p. 276)

part : part ratio a ratio that compares two or more parts of a whole (p. 193)

part : whole ratio a ratio that compares a part (or parts) to a whole (p. 193)

percent literally 'for each hundred' (p. 80)

percentage out of one hundred (p. 80)

perimeter the distance around the outside of a shape (p. 255)

perpendicular at right angles to (p. 276)

point of intersection the point where two or more lines, rays or line segments meet (p. 428)

poll a survey given to a sample of a population. It is used to collect sample data so that sample statistics can be calculated (p. 507)

population the entire category or group (p. 507)

population statistics statistics calculated from the data collected by a census (p. 507)

positive usually associated with gains or increases (p. 5)

power a number, usually written smaller and above another number (the base), that tells how many times the base is multiplied by itself (p. 32)

prism solid with parallel polygonal ends and sides that are parallelograms (p. 306)

profit difference between the selling price (SP) of a product (or investment) and the purchase price (cost price, CP) of the product (or investment) after taking account of all costs associated with the purchase and sale, when SP > CP (p. 118)

quadrilateral a four-sided plane shape (p. 482)

radius the distance from the centre of a circle to its edge (p. 261)

random sampling sampling in which there is no pattern involved in the sampling process (p. 509)

range the difference between the largest and smallest value in a data set (p. 521)

rate a way of comparing two quantities that use different units (p. 236)

ratio a pair of numbers used to show the relative sizes of two amounts (p. 193)

rectangle a parallelogram with right angles (p. 275)

recurring decimal a decimal with a pattern of digits that repeats (p. 57)

rhombus a parallelogram with adjacent sides equal in length (p. 278)

right-angled triangle a triangle with one right (90°) angle (p. 462)

sale price selling price – discount amount of an item sold to the public (p. 118)

sample a small proportion of a population (p. 507)

sample space a list of all the possible outcomes of an event (p. 554)

sample statistics statistics calculated from sample data (p. 507)

scale factor the number by which the side lengths of a shape is multiplied to obtain a larger or smaller similar shape (p. 203)

scale ratio a ratio placed on a map or diagram used to indicate the actual lengths of the lengths shown on the diagram. For example, a scale ratio of 1 : 100 indicates that 1 cm on the map corresponds to a distance of 100 cm in real life. (p. 226)

scalene triangle a triangle in which all sides and angles are unequal (p. 462)

selling price (SP) the amount for which an item is sold (p. 118)

simplest form a fraction or a ratio in which the numbers have no common factors is said to be in simplest form (p. 194)

simplify to write in simplest form by either cancelling common factors (fractions and ratios), or by collecting like terms (algebra) (p. 33)

slope the steepness of a line (p. 346)

solution the value of a pronumeral that makes the equation true; the answer (p. 399)

solving finding a value that makes an equation a true number sentence (p. 399)

speed the ratio of distance travelled to time taken (p. 236)

square **1.** to multiply a number by itself (p. 32)
2. a square is a rectangle with one pair of adjacent sides equal (p. 275)

substitution replacing a pronumeral with a given number (p. 149)

supplementary angles a pair of angles that add to 180° (p. 452)

surd a square root that is an irrational number (p. 65)

term a number or a pronumeral by itself: 7, x; a number multiplied by a pronumeral: $12c$, $5k$; different pronumerals multiplied together: mn, abc; a pronumeral multiplied by itself a number of times: a^3, b^2; or the product of a number and several pronumerals: $3xy$, $4a^2$, b (p. 143)

terminating decimal a decimal number with a clearly defined number of digits (p. 65)

time zone one of 24 imaginary vertical columns that the world has been divided into. Within a time zone, all clocks are set to the same time. The difference between two adjacent time zones is 1 hour. (p. 320)

transversal a line that crosses (transverses) other lines (p. 452)

trapezium a quadrilateral with only one pair of opposite sides parallel (p. 277)

triangle a three-sided plane shape (p. 276)

two-way table information presented in rows and columns (p. 567)

uniform cross-section a prism in which the shape on the end continues unchanged throughout the height of the prism has a uniform cross-section (p. 306)

union the combining of two or more sets; the contents of all the sets are included in the union (p. 568)

unit ratio a ratio expressed in the form decimal : 1 (p. 203)

unitary method a method for solving rate problems that involves a number per unit (such as litres, boxes or people) being calculated first (p. 237)

universal set the set of all elements; it is symbolised by ξ (p. 567)

unlike terms terms that have different pronumeral parts (p. 162)

variable a quantity or pronumeral that can take a set of values (p. 143)

Venn diagram a diagram used to show the relationships between sets (p. 567)

vertex (pl. vertices) the point where two or more lines (or edges) meet (p. 451)

vertically opposite angles when two lines intersect, four angles are formed at the point of intersection. Pairs of angles that are opposite each other are equal (p. 451)

volume the amount of space a solid occupies (p. 306)

***x*-intercept** the point where a graph crosses the x-axis (p. 346)

***y*-intercept** the point where a graph crosses the y-axis (p. 346)